

Possible Project: Probability
UConn-PCS: Data Science, 2022

You may choose to use simulation to find answers to one (or more) of the following problems.

1. A King offers a death row prisoner a chance to live by playing a simple game. He gives 50 black marbles, 50 white marbles, and 2 empty bowls to the prisoner. The King asks the prisoner to divide these 100 marbles into these 2 bowls in any way as long as all the marbles are used. He will then blindfold the prisoner and mix the bowls around. After that, the prisoner can choose one bowl and pick one marble randomly. If the marble is white the prisoner will live, but if the marble is black the prisoner will die. Use simulation to find the probability of living for the following ways of dividing.
 - (a) Put the 50 white balls in one bowl and the 50 black balls in another bowl.
 - (b) Put one white ball in one bowl and the other 99 bowl balls in another bowl.
2. Tom and Jerry, each put 30 dollar in a jackpot to start a game. Suppose that they have equal chance to win and the one who wins three times takes all the 60 dollars. Now, Tom has won twice and Jerry has won once, but then something happens and the game must be stopped. How should they split the 60 dollars according their probabilities of winning if the game was finished.
3. Three players enter a room and a red or blue hat is placed on each person's head. The color of each hat is determined by an independent coin toss. No communication of any sort is allowed, except for an initial strategy session before the game begins. Once they have had a chance to look at the other hats but not their own, the players must simultaneously guess the color of their own hats or pass. The puzzle is to find a group strategy that maximizes the probability that at least one person guesses correctly and no-one guesses incorrectly.
 - (a) One strategy is for the group to agree that one person should guess and the others pass. What is the probability of success?
 - (b) Here is another strategy. For each person, guess the color he/she does not see if he/she see same colors, and pass is he/she sees different colors. What is the probability of success?
4. Henry has been caught stealing cattle, and is brought into town for justice. The judge is his ex-wife Gretchen, who wants to show him some sympathy, but the law clearly calls for two shots to be taken at Henry from close range. To make things a little better for Henry, Gretchen tells him she will place two bullets into a six-chambered revolver in successive order. She will spin the chamber, close it, and take one shot. If Henry is still alive, she will then either take another shot, or spin the chamber again before shooting. Henry is a bit incredulous that his own ex-wife would carry out the punishment, and a bit sad that she was always such a rule follower. He steels himself as Gretchen loads the chambers, spins the revolver, and pulls the trigger. Whew! It was blank. Then Gretchen asks, "Do

you want me to pull the trigger again, or should I spin the chamber a second time before pulling the trigger?” What should Henry choose?

5. In a prison, there are 100 death row prisoners who are numbered from 1 to 100, and there is a room with 100 drawers labeled from 1 to 100. The director randomly puts one prisoner's number in each closed drawer and offers a last chance. The prisoners enter the room, one after another. Each prisoner may open and look into 50 drawers in any order. The drawers are closed again afterwards. If, during this search, every prisoner finds his number in one of the drawers, all prisoners are pardoned. If some prisoner does not find his number, all prisoners die. Before the first prisoner enters the room, the prisoners may discuss strategy, but they cannot communicate once the first prisoner enters the room. Which of the following strategies is better for the prisoners?

(a) Strategy 1 (simple): every prisoner selects 50 drawers at random.

(b) Strategy 2 (complicated):

- i. Each prisoner first opens the drawer with his own number.
- ii. If this drawer contains his number he is done and was successful.
- iii. Otherwise, the drawer contains the number of another prisoner and he next opens the drawer with this number.
- iv. The prisoner repeats steps 2 and 3 until he finds his own number or has opened 50 drawers.