INTRODUCTION TO COMPUTER VISION

Lecture 5 – Epipolar Geometry

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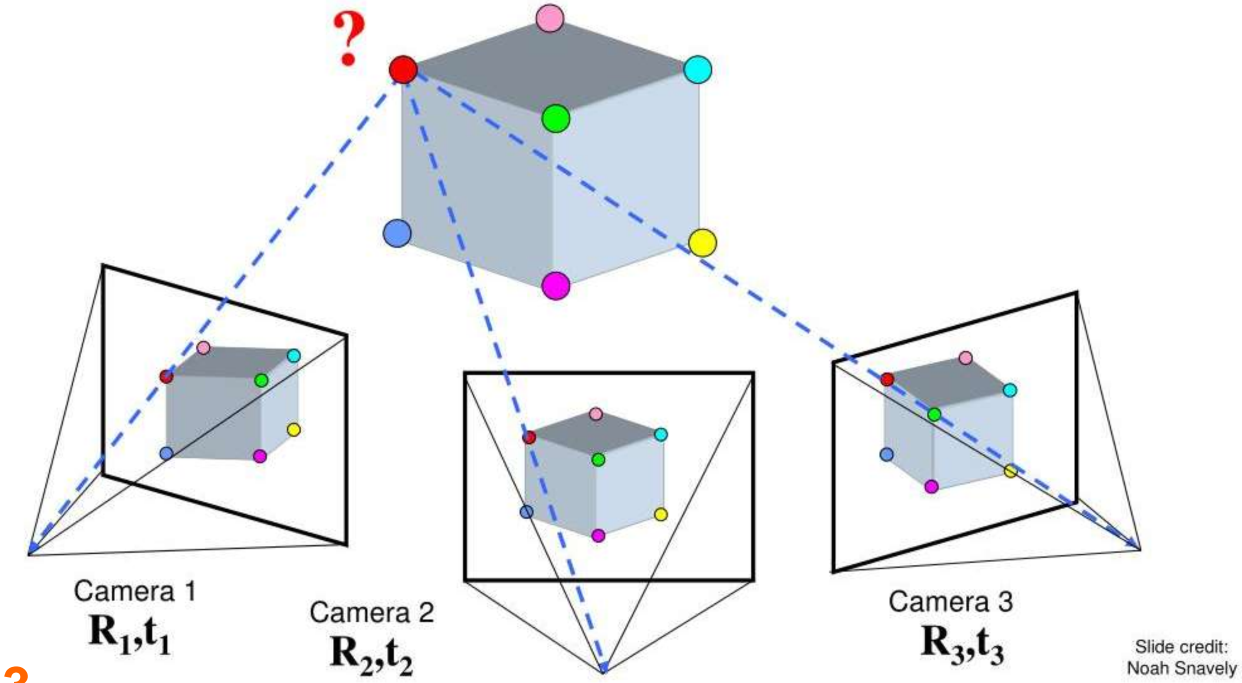
Multi-View Geometry (MVG) Theory

According to MVG (we'll learn this later), if we know

- Camera intrinsic/extrinsic parameters (we learnt what are they in prev. classes)
- *Matched points across multiple viewpoints (same-colored dots in images)*

, then, we can lift the multi-view observations to the 3D space



Slide from Lecture 3

Epipolar Geometry

- We learned both 1) camera calibration to get camera parameters and 2) image matching to get coherent points from multiple images
- Finally, we can lift observations from multiple images to 3D space! triangulation.

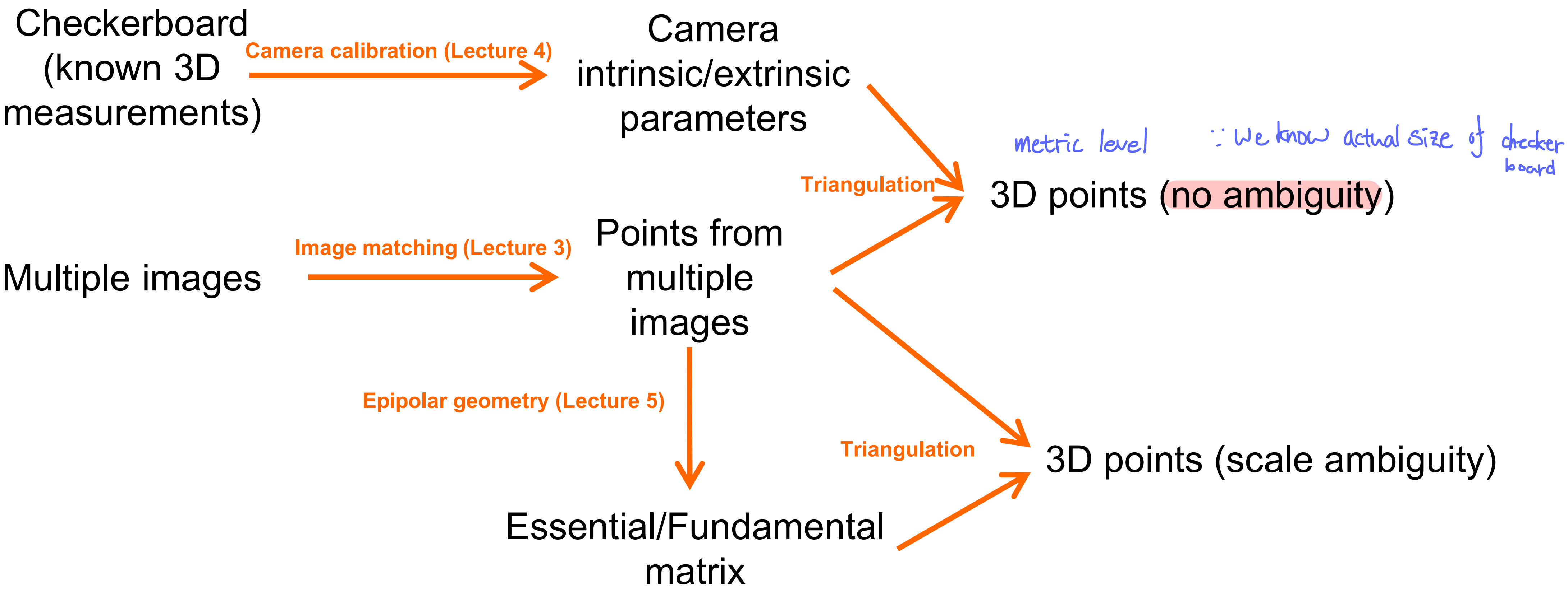
- But what if we don't want to (or can't) calibrate cameras? eg. images from internet.
    - E.g., taking a video with your mobile phone without camera calibration
- How? -> epipolar geometry
- Epipolar geometry: only two-view images
- Multi-view geometry: multi-view images (# of images >= 2)

Camera Calibration vs. Epipolar Geometry

- If we know extrinsics with camera calibrations and checkerboard
  - E.g., you calibrate cameras with checkerboards
  - Skip all things we'll learn today
  - Just triangulate (we'll learn this later) 2D points to the 3D space

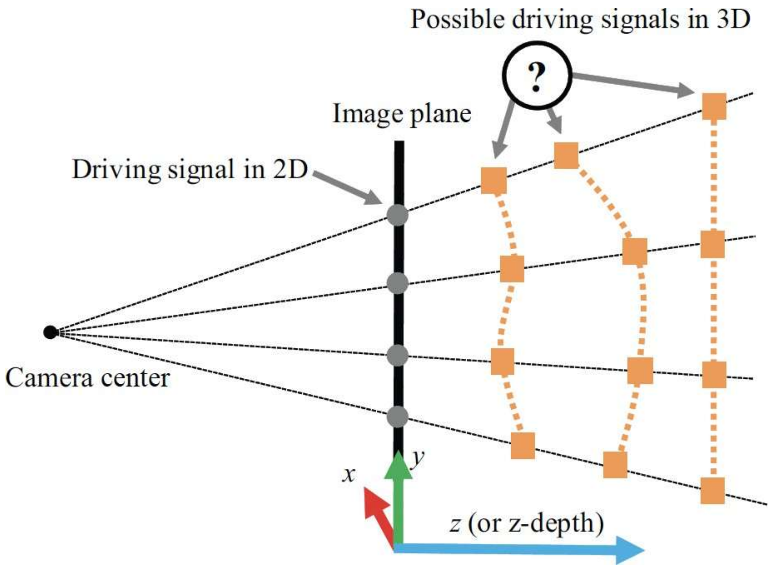
- If we do not know extrinsics (today's focus)
  - E.g., taking a video with your mobile phone without any calibrations/checkerboards
  - Epipolar geometry!
  - We get essential/fundamental matrices
  - We do not have checkerboard, which provides actual 3D measurements -> we get extrinsics up to scale

- All the entire pipeline: multi-view geometry theory



Recovering 3D Geometry

- Camera calibration from a single view
- Recover 3D geometry from a single view?
  - No: due to ambiguity of 3D -> 2D mapping



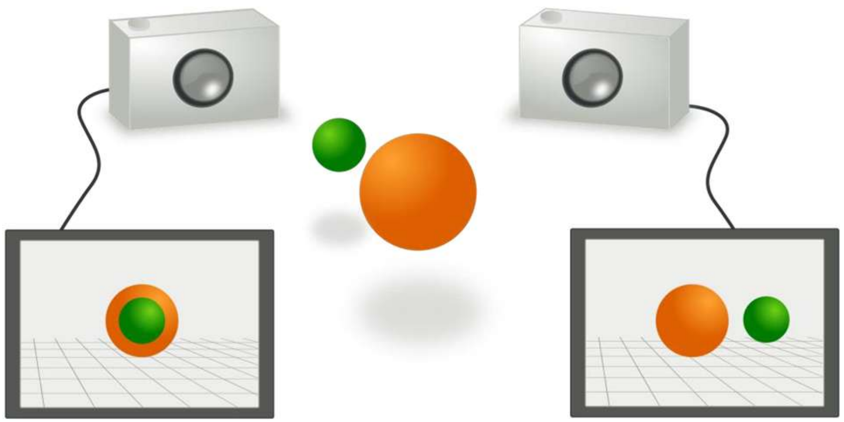
depth ambiguity

Epipolar Geometry

monocular — 1 view

2 views

- The geometry of stereo vision
  - Given 2D images of two views
  - What is the relationship between pixels of the images?
  - Can we recover the 3D structure of the world from the 2D images?

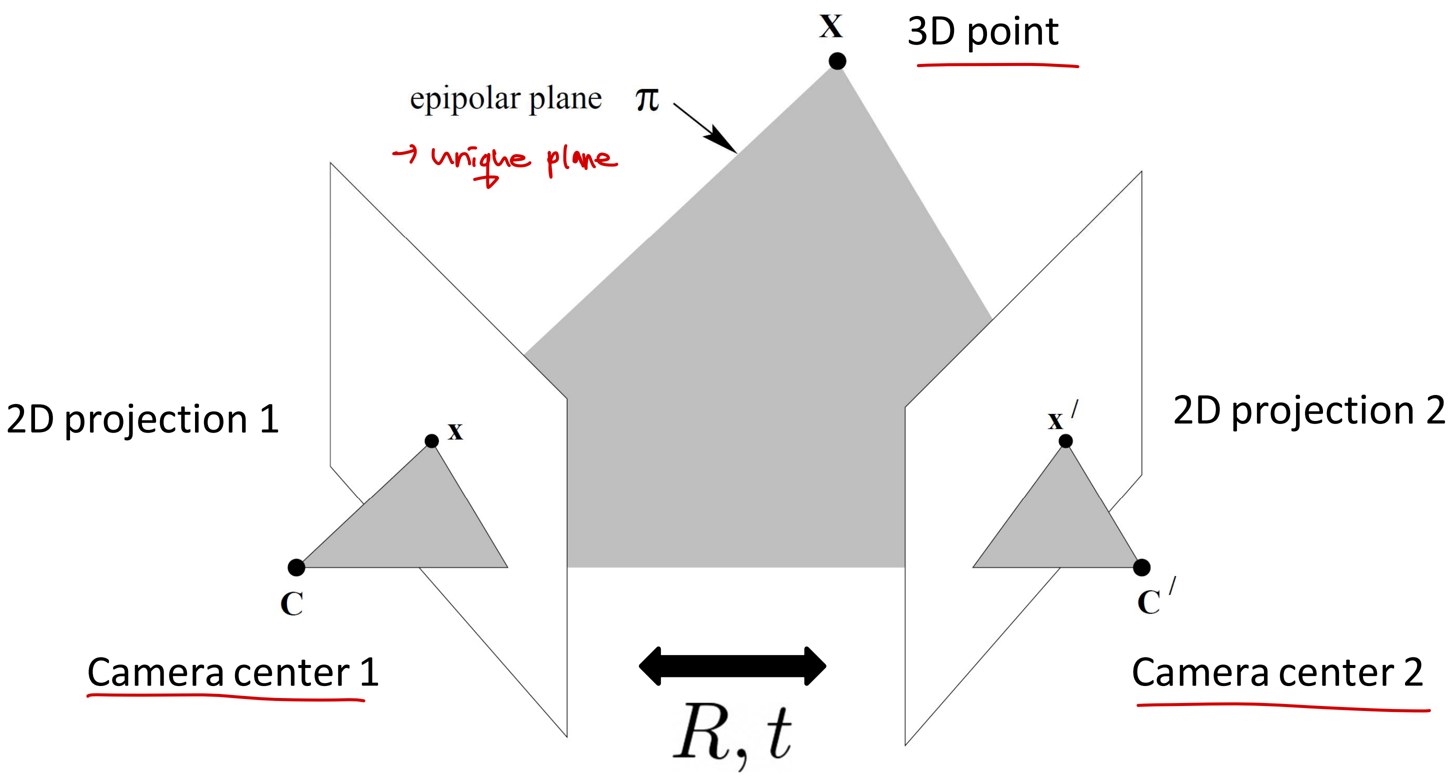


Wikipedia

Epipolar Geometry

uniquely

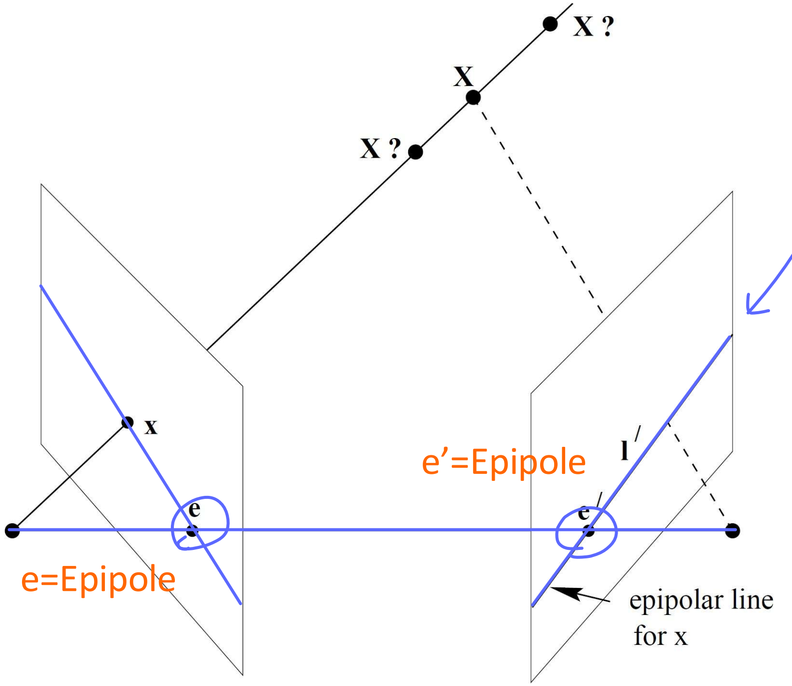
- The **epipolar plane** is defined by two camera centers and a 3D point.



Epipolar Geometry

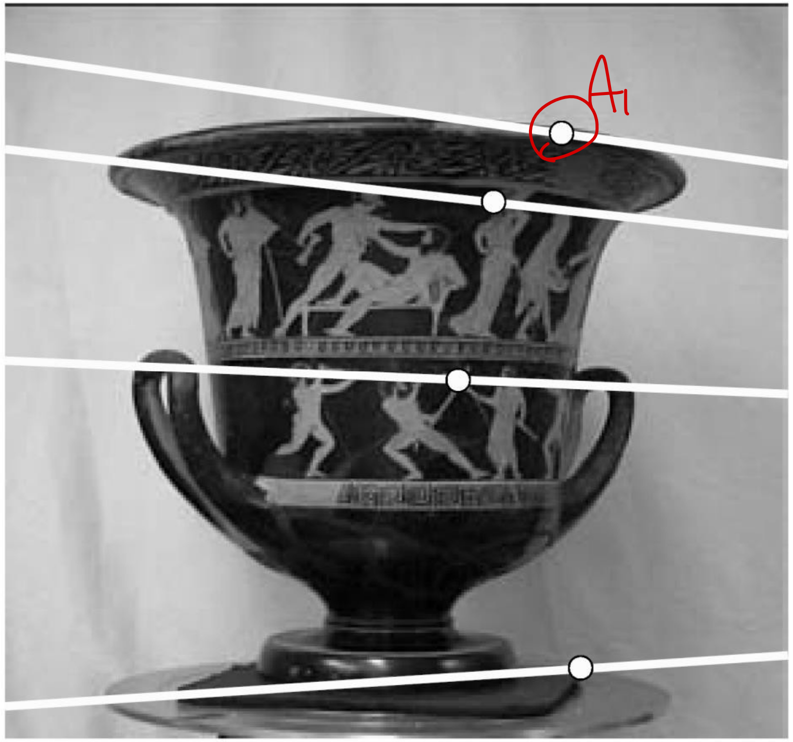
- The **epipolar plane** is defined by two camera centers and a 3D point.
- Its intersection with an image plane gives the corresponding **epipolar line**.

- All epipolar lines pass through the **epipole**.
- The epipole is the projection of one camera center onto the other camera's image plane.
- It is the intersection point of all epipolar lines in an image.



Epipolar Geometry

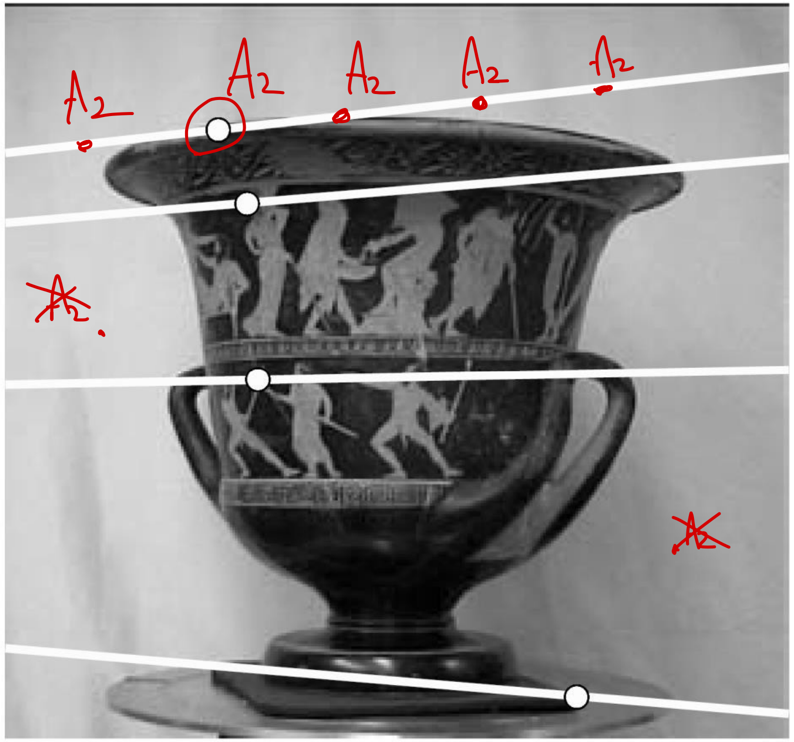
- If we know 1) the intrinsic and extrinsic parameters of two cameras, and 2) the 2D image position of a point in one camera, then the corresponding epipolar line in the other camera defines the location locus of all potential 2D positions where the point could appear. 저 line 위에서만 찾으면 됨



I_1

Epipolar lines

A1과 corresponding 하는 A2는 Epipolar line 위에만 있을 수 있음.

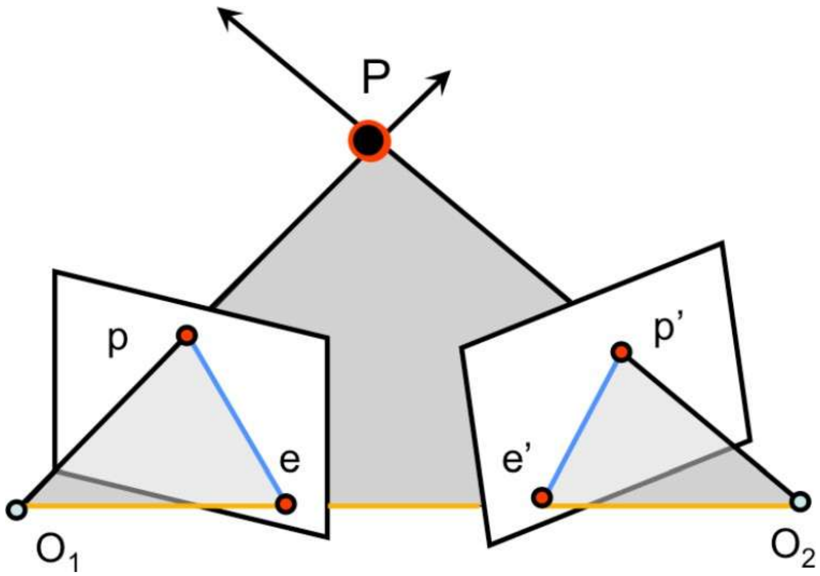


I_2

Rotation and Translation between two views

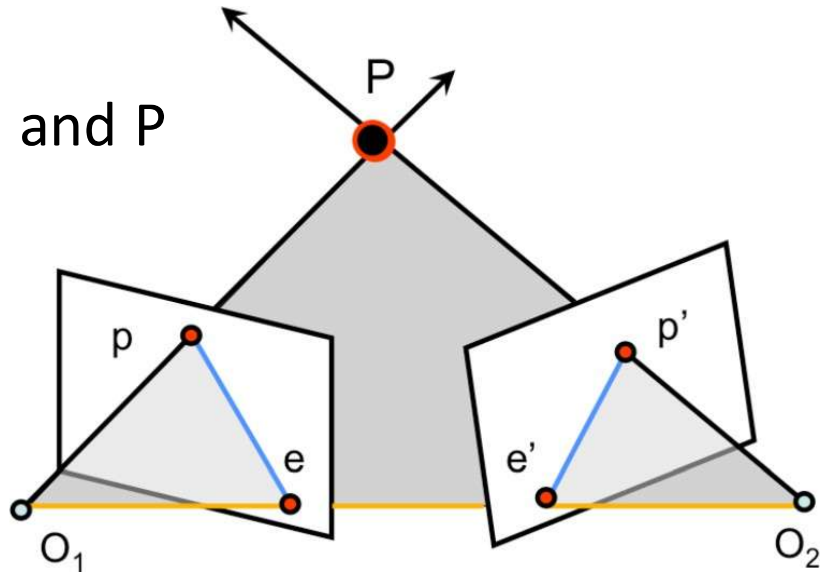
Epipolar Geometry

- Baseline (Yellow line)
  - The line between the two camera centers O_1 and O_2



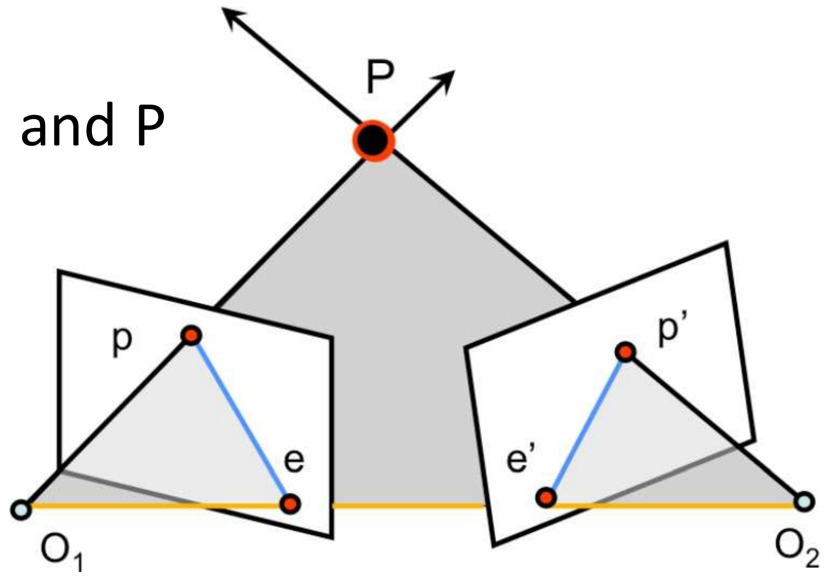
Epipolar Geometry

- Baseline (Yellow line)
  - The line between the two camera centers O_1 and O_2
- Epipolar plane (gray plane)
  - Defined by P, O_1, and O_2; contains baseline and P



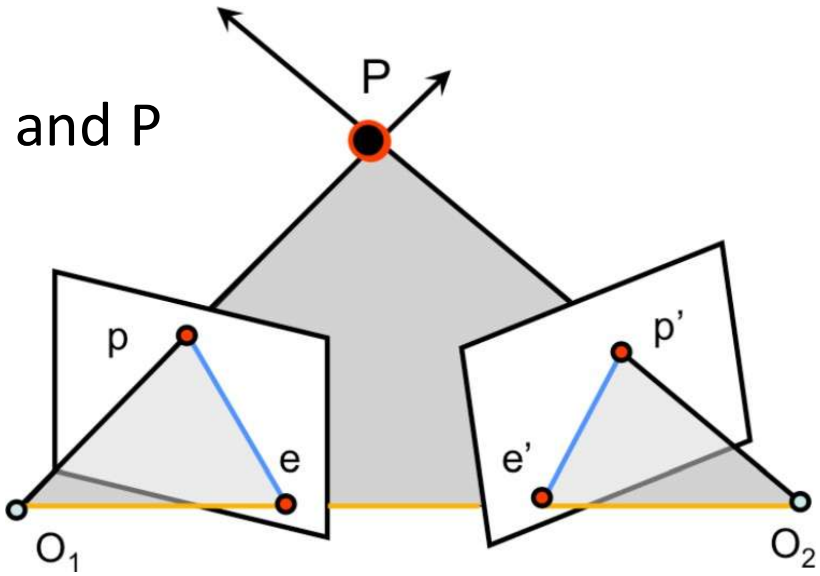
Epipolar Geometry

- Baseline (Yellow line)
  - The line between the two camera centers O_1 and O_2
- Epipolar plane (gray plane)
  - Defined by P, O_1, and O_2; contains baseline and P
- Epipoles (e and e')
  - ∩ of baseline and image plane: e and e'
  - Projection of the other camera center



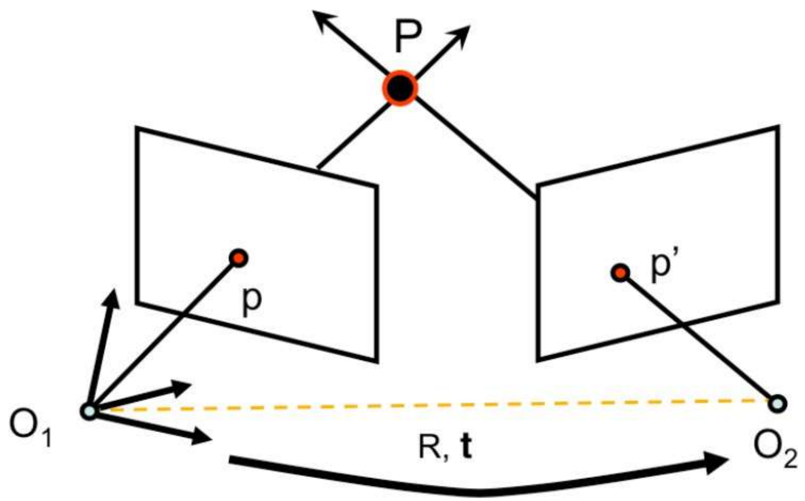
Epipolar Geometry

- Baseline (Yellow line)
  - The line between the two camera centers O_1 and O_2
- Epipolar plane (gray plane)
  - Defined by P, O_1, and O_2; contains baseline and P
- Epipoles (e and e’)
  - ∩ of baseline and image plane
  - Projection of the other camera center
- Epipolar lines (Blue lines)
  - ∩ of epipolar plane with the image plane



Epipolar Constraint

- The relationship between corresponding image points
  - The world reference system aligned with the left camera
  - The right camera has orientation R and offset $\mathbf{t}$



Camera projection matrices

Left camera

$$M = K[I \quad 0]$$

$$\mathbf{p} = M\mathbf{P} = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}$$

Right camera

extrinsic

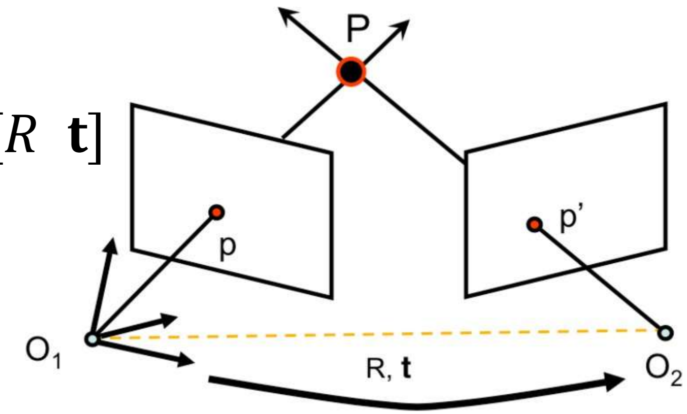
$$M' = K'[R \quad \mathbf{t}]$$

$$\mathbf{p}' = M'\mathbf{P} = \begin{bmatrix} u' \\ v' \\ 1 \end{bmatrix}$$

Epipolar Constraint

- The relationship between corresponding image points
  - Canonical cameras ($K = K' = I$)

    $M = K[I \ \ 0] \rightarrow M = [I \ \ 0] \qquad M' = K'[R \ \ \mathbf{t}] \rightarrow M' = [R \ \ \mathbf{t}]$



p' in world coordinate system

Epipolar Constraint

$p' = RX + t$ $\quad$ X: p' in world

$R^{-1}(p' - t) = X$

R이 orthogonal 이라

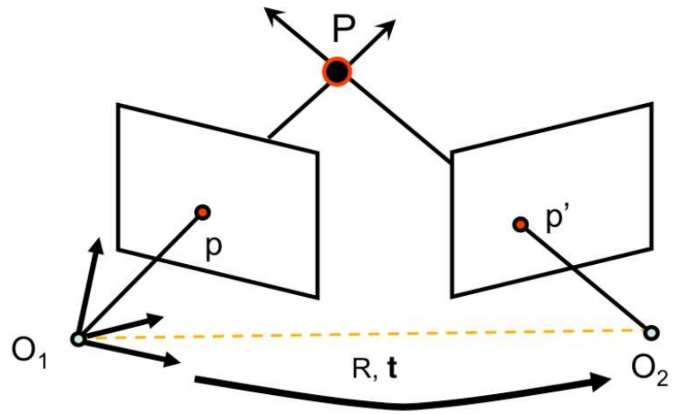
$R^T R = RR^T = I$ $\quad$ $R^T = R^{-1}$

$\|Rx\| = \|x\|$

$(Rx)\cdot(Ry) = x \cdot y$

- The relationship between corresponding image points
  - Canonical cameras ($K = K' = I$)

    $M = [I \ \ 0] \quad M' = [R \ \ \mathbf{t}]$



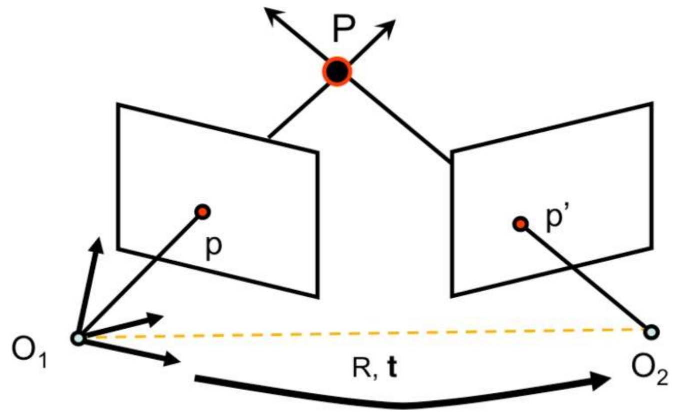
p' in world coordinate system $\quad$ $R^T(\mathbf{p}' - \mathbf{t})$

O_2 in world coordinate system

Epipolar Constraint

- The relationship between corresponding image points
  - Canonical cameras ($K = K' = I$)

    $M = [I \;\; 0] \quad M' = [R \;\; \mathbf{t}]$



p' in world coordinate system $\quad R^T(\mathbf{p}' - \mathbf{t})$

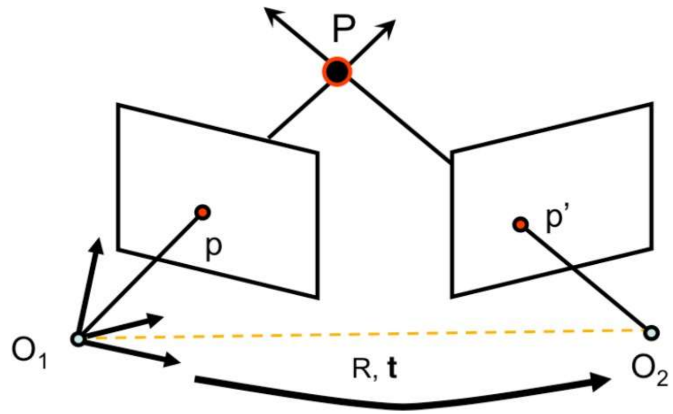
$\mathbf{O_2}$ in world coordinate system $\quad R^T(\mathbf{O_2} - \mathbf{t}) = -R^T\mathbf{t}$

Normal of the epipolar plane

Epipolar Constraint

- The relationship between corresponding image points
  - Canonical cameras $(K = K' = I)$

    $M = [I \;\; 0] \quad M' = [R \;\; \mathbf{t}]$



p' in world coordinate system $\qquad R^T(\mathbf{p}' - \mathbf{t})$

O_2 in world coordinate system $\qquad R^T(\mathbf{O_2} - \mathbf{t}) = -R^T\mathbf{t}$

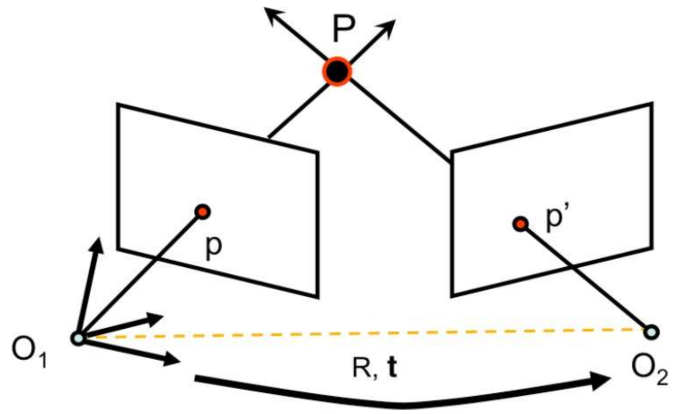
Normal of the epipolar plane $\qquad R^T\mathbf{t} \underset{\text{외적}}{\times} [R^T(\mathbf{p}' - \mathbf{t})] = R^T(\mathbf{t} \times \mathbf{p}')$

O_1p lies in the epipolar plane: so the dot product of the normal of the Epipolar plane and **O_1p** is 0

Epipolar Constraint

- The relationship between corresponding image points
  - Canonical cameras ($K = K' = I$)

$$M = [I \ \ 0] \qquad M' = [R \ \ \mathbf{t}]$$



p' in world coordinate system $R^T(\mathbf{p}' - \mathbf{t})$

O_2 in world coordinate system $R^T(\mathbf{O_2} - \mathbf{t}) = -R^T\mathbf{t}$

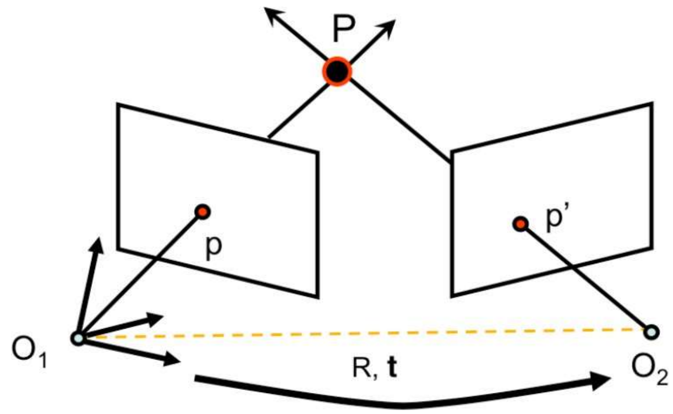
Normal of the epipolar plane $R^T\mathbf{t} \times [R^T(\mathbf{p}' - \mathbf{t})] = R^T(\mathbf{t} \times \mathbf{p}')$

O_1p lies in the epipolar plane $[R^T(\mathbf{t} \times \mathbf{p}')]^T \mathrm{p} = 0$

Epipolar Constraint

- The relationship between corresponding image points
  - Canonical cameras ($K = K' = I$)

$$M = [I \;\; 0] \qquad M' = [R \;\; \mathbf{t}]$$



p' in world coordinate system $\quad R^T(\mathbf{p}' - \mathbf{t})$

$\mathbf{O_2}$ in world coordinate system $\quad R^T(\mathbf{O_2} - \mathbf{t}) = -R^T\mathbf{t}$

Normal of the epipolar plane $\quad R^T\mathbf{t} \times [R^T(\mathbf{p}' - \mathbf{t})] = R^T(\mathbf{t} \times \mathbf{p}')$

$\mathbf{O_1 p}$ lies in the epipolar plane $\quad [R^T(\mathbf{t} \times \mathbf{p}')]^T \mathrm{p} = 0 \Rightarrow (\mathbf{t} \times \mathbf{p}')^T R \mathrm{p} = 0$

Epipolar Constraint

- The relationship between corresponding image points
  - Canonical cameras ($K = K' = I$)

$$M = [I \;\; 0] \qquad M' = [R \;\; \mathbf{t}]$$

Cross product as matrix-vector multiplication

$$\boxed{\mathbf{a} \times \mathbf{b} = \begin{bmatrix} 0 & -\mathbf{a}_z & \mathbf{a}_y \\ \mathbf{a}_z & 0 & -\mathbf{a}_x \\ -\mathbf{a}_y & \mathbf{a}_x & 0 \end{bmatrix} \begin{bmatrix} \mathbf{b}_x \\ \mathbf{b}_y \\ \mathbf{b}_z \end{bmatrix} = [\mathbf{a}_\times]\mathbf{b}}$$

$[a_\times]^T = -[a_\times]$

Skew-symmetric matrix (rank=2)

$a^T = -a$

p' in world coordinate system $\quad R^T(\mathbf{p}' - \mathbf{t})$

O_2 in world coordinate system $\quad R^T(\mathbf{O_2} - \mathbf{t}) = -R^T\mathbf{t}$

Normal of the epipolar plane $\quad R^T\mathbf{t} \times [R^T(\mathbf{p}' - \mathbf{t})] = R^T(\mathbf{t} \times \mathbf{p}')$

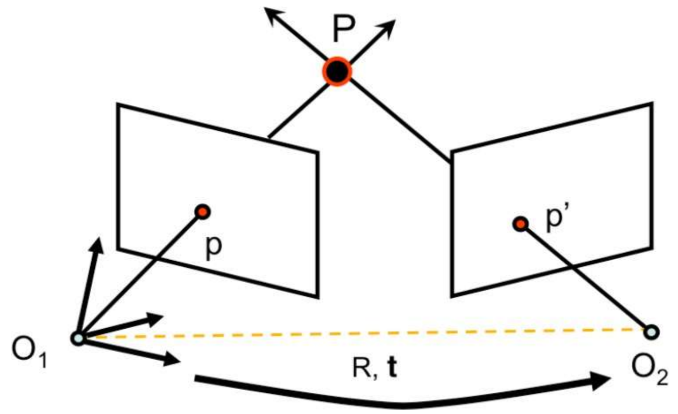
O_1p lies in the epipolar plane $\quad [R^T(\mathbf{t} \times \mathbf{p}')]^T \mathrm{p} = 0 \Rightarrow (\mathbf{t} \times \mathbf{p}')^T R \mathrm{p} = 0 \Rightarrow ([\mathbf{t}_\times]\,\mathbf{p}')^T R \mathrm{p} = 0$

항상 rank 2

Epipolar Constraint

- The relationship between corresponding image points
  - Canonical cameras ($K = K' = I$)

$$M = [I \;\; 0] \qquad M' = [R \;\; \mathbf{t}]$$



p' in world coordinate system $R^T(\mathbf{p}' - \mathbf{t})$

O_2 in world coordinate system $R^T(\mathbf{O_2} - \mathbf{t}) = -R^T\mathbf{t}$

Normal of the epipolar plane $R^T\mathbf{t} \times [R^T(\mathbf{p}' - \mathbf{t})] = R^T(\mathbf{t} \times \mathbf{p}')$

O_1p lies in the epipolar plane $[R^T(\mathbf{t} \times \mathbf{p}')]^T \mathrm{p} = 0 \Rightarrow (\mathbf{t} \times \mathbf{p}')^T R\mathrm{p} = 0 \Rightarrow ([\mathbf{t}_\times]\,\mathbf{p}')^T R\mathrm{p} = 0$

$\Rightarrow \mathbf{p}'^T[\mathbf{t}_\times]R\mathrm{p} = 0$

Epipolar Constraint

$\lambda . p'^T E p = 0$

$p'^T (\lambda E) p = 0 \quad \lambda = 1, 2, \cdots ?$

- Essential matrix
  - Establish constraints between matching image points
  - Determine relative position and orientation of two cameras
  - 5 degrees of freedom (R: 3, $\mathbf{t}$: 3, but scale is not known)

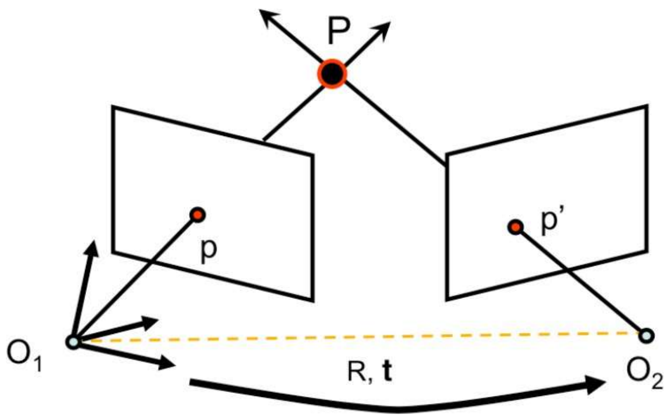
$6 - 1 = 5$

$$\mathbf{p}'^T \boxed{[\mathbf{t}_\times] R} \mathbf{p} = 0$$

$$E = [\mathbf{t}_\times] R$$

$$\mathbf{p}'^T \boxed{E} \mathbf{p} = 0$$

Essential matrix



Epipolar Constraint

- How to generalize Essential matrix?
  - Canonical cameras → general cameras

$$M = K[I \;\; 0]$$
$$M' = K'[R \;\; \mathbf{t}]$$

Essential: $K = K' = I$

$$M = [I \;\; 0] \qquad M' = [R \;\; \mathbf{t}]$$

$$\mathrm{p} = M\mathbf{P} = [I \;\; 0]\mathbf{P} \qquad \mathbf{p}' = M'\mathbf{P} = [R \;\; \mathbf{t}]\mathbf{P}$$

$$\mathbf{p}'^{T}E\mathrm{p} = 0 \qquad E = [\mathbf{t}_{\times}]R$$

Fundamental: $K \neq I, K' \neq I$

Epipolar Constrai nt

- How to generalize Essential matrix?
  - Canonical cameras → general cameras

Canonical cameras: the image points in homogeneous coordinates are actually the 3D point expressed in the camera coordinate system

$$M = K[I \;\; 0]$$
$$M' = K'[R \;\; \mathbf{t}]$$

$K = K' = I$

$$M = [I \;\; 0]$$
$$M' = [R \;\; \mathbf{t}]$$

$$\mathrm{p} = M\mathbf{P} = [I \;\; 0]\mathbf{P}$$
$$\mathbf{p}' = M'\mathbf{P} = [R \;\; \mathbf{t}]\mathbf{P}$$

$$\mathbf{p}'^T E \mathrm{p} = 0$$
$$E = [\mathbf{t}_{\times}]R$$

$K \neq I, K' \neq I$

$$\mathrm{p} = M\mathbf{P} = K[I \;\; 0]\mathbf{P}$$
$$\mathbf{p}' = M'\mathbf{P} = K'[R \;\; \mathbf{t}]\mathbf{P}$$

$\mathrm{p} \rightarrow K^{-1}\mathbf{p}$

$\mathbf{p}' \rightarrow K'^{-1}\mathbf{p}'$

$$\mathbf{p}'^T K'^{-T} E K^{-1} \mathrm{p} = 0$$
$$\Downarrow$$
$$\mathbf{p}'^T F \mathrm{p} = 0$$
$$F = K'^{-T}[\mathbf{t}_{\times}]RK^{-1}$$

no detail

Epipolar Constraint

- Essential matrix vs. Fundamental matrix
  - Similarity
    - Both relate the matching image points
      - – Encode epipolar geometry of two views & camera parameters
  - Differences
    - E (essential matrix) encodes only the camera extrinsic parameter
    - F (fundamental matrix) also encodes the intrinsic parameters

$$\mathbf{p}'^{T} E \mathrm{p} = 0$$

$$E = [\mathbf{t}_{\times}]R$$

Essential matrix

$$\mathbf{p}'^{T} F \mathrm{p} = 0$$

$$F = K'^{-T}[\mathbf{t}_{\times}]RK^{-1}$$

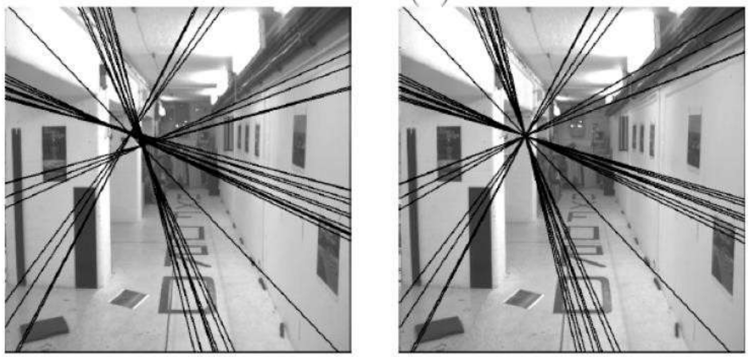
Fundamental matrix

Epipolar Constraint

- Properties of the Fundamental matrix
  - 3 by 3
  - homogeneous (has scale ambiguity)
  - rank(F) = 2
    - The potential matching point is located on a line
  - F has 7 degrees of freedom (3x3 – 1 (rank2) – 1 (scale ambiguity) = 7)

$$\mathbf{p}'^{T} F \mathrm{p} = 0 \qquad F = K'^{-T} [\mathbf{t}_{\times}] R K^{-1}$$

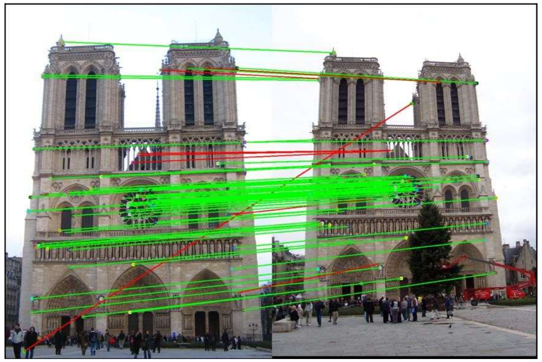
Fundamental matrix has rank 2 : $\det(\mathtt{F}) = 0$.



Left : Uncorrected F – epipolar lines are not coincident.

Right : Epipolar lines from corrected F.

Epipolar Constraint



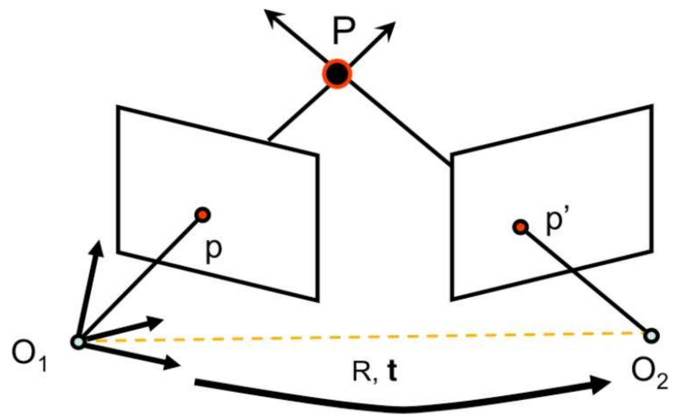
- How can we use the fundamental matrix?
  - Given a 2D point in one image and fundamental matrix, we can get epipolar line from the other image (candidate positions of matched points)
  - Without knowing camera intrinsic and extrinsic parameters
  - If we know p , epipolar line is $l = F\mathrm{p}$ F를 알면 epipolar line 알수 있음.
  - If we know $\mathbf{p}'$, epipolar line is $l' = F^T\mathbf{p}'$
  - With the epipolar line, we can reduce the matching space
    - Not entire image, only search points on the epipolar line
    - Find matched points and lift them to the 3D space with triangulation

$$\mathbf{p}'^T F \mathrm{p} = 0 \qquad F = K'^{-T}[\mathbf{t}_{\times}]RK^{-1}$$

Recovering Fundamental Matrix

- How to recover F?
  - From image correspondences

$$\mathbf{p}'^T F \mathrm{p} = 0 \qquad F = K'^{-T}[\mathbf{t}_{\times}]RK^{-1}$$

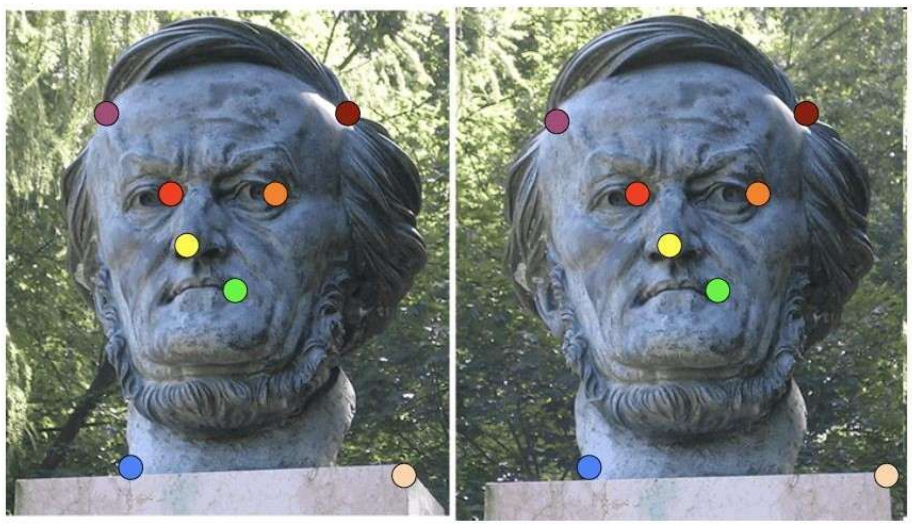


Recovering Fundamental Matrix

- How to recover F?
  - From image correspondences
  - How many point pairs needed?

$\mathbf{p}'^{T}F\mathrm{p} = 0$ $\quad\quad$ $F = K'^{-T}[\mathbf{t}_{\times}]RK^{-1}$

3



Recovering Fundamental Matrix

- How to recover F?
  - From image correspondences
  - 8-point pairs required
    - Each point pair gives one equation
    - F is known up to scale
      - The linear system is homogeneous

$$\begin{cases} \mathbf{p}_i = (u_i, v_i, 1) \\ \mathbf{p}'_i = (u'_i, v'_i, 1) \end{cases} \qquad \mathbf{p}'^T F \, \mathbf{p} = 0$$

$$\begin{bmatrix} u_i u'_i & v_i u'_i & u'_i & u_i v'_i & v_i v'_i & v'_i & u_i & v_i & 1 \end{bmatrix} \begin{bmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{bmatrix} = 0$$

Recovering Fundamental Matrix

- How to recover F?
  - From image correspondences
  - 8-point pairs required
    - Each point pair gives one equation
    - F is known up to scale
      - The linear system is homogeneous

$$\begin{cases} \mathbf{p}_i = (u_i, v_i, 1) \\ \mathbf{p}_i' = (u_i', v_i', 1) \end{cases} \qquad \mathbf{p}'^T F \ \mathbf{p} = 0$$

$$\begin{bmatrix} u_i u_i' & v_i u_i' & u_i' & u_i v_i' & v_i v_i' & v_i' & u_i & v_i & 1 \end{bmatrix} \begin{bmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{bmatrix} = 0$$

Recovering Fundamental Matrix

F has 7 degrees of freedom Are 7-point pairs sufficient?

- Yes in theory
- We need to use det(F)=0 as an additional constraint to use only 7 points
- det(F)=0 gives us cubic equation, so there could be more than one solution

  non-linear
- Unstable, so we simply use 8 points

Recovering Fundamental Matrix

- 8-point algorithm

$$\begin{bmatrix} u_i u_i' & v_i u_i' & u_i' & u_i v_i' & v_i v_i' & v_i' & u_i & v_i & 1 \end{bmatrix} \begin{bmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{bmatrix} = 0$$

$$\boxed{\begin{bmatrix} u_1 u_1' & v_1 u_1' & u_1' & u_1 v_1' & v_1 v_1' & v_1' & u_1 & v_1 & 1 \\ u_2 u_2' & v_2 u_2' & u_2' & u_2 v_2' & v_2 v_2' & v_2' & u_2 & v_2 & 1 \\ u_3 u_3' & v_3 u_3' & u_3' & u_3 v_3' & v_3 v_3' & v_3' & u_3 & v_3 & 1 \\ u_4 u_4' & v_4 u_4' & u_4' & u_4 v_4' & v_4 v_4' & v_4' & u_4 & v_4 & 1 \\ u_5 u_5' & v_5 u_5' & u_5' & u_5 v_5' & v_5 v_5' & v_5' & u_5 & v_5 & 1 \\ u_6 u_6' & v_6 u_6' & u_6' & u_6 v_6' & v_6 v_6' & v_6' & u_6 & v_6 & 1 \\ u_7 u_7' & v_7 u_7' & u_7' & u_7 v_7' & v_7 v_7' & v_7' & u_7 & v_7 & 1 \\ u_8 u_8' & v_8 u_8' & u_8' & u_8 v_8' & v_8 v_8' & v_8' & u_8 & v_8 & 1 \end{bmatrix} \begin{bmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{bmatrix} = 0}$$

Recovering Fundamental Matrix

- 8-point algorithm
  - Construct linear system using corresponding image points

$$W\mathbf{f} = 0$$

How to solve it?

$$\begin{bmatrix} u_1u_1' & v_1u_1' & u_1' & u_1v_1' & v_1v_1' & v_1' & u_1 & v_1 & 1 \\ u_2u_2' & v_2u_2' & u_2' & u_2v_2' & v_2v_2' & v_2' & u_2 & v_2 & 1 \\ u_3u_3' & v_3u_3' & u_3' & u_3v_3' & v_3v_3' & v_3' & u_3 & v_3 & 1 \\ u_4u_4' & v_4u_4' & u_4' & u_4v_4' & v_4v_4' & v_4' & u_4 & v_4 & 1 \\ u_5u_5' & v_5u_5' & u_5' & u_5v_5' & v_5v_5' & v_5' & u_5 & v_5 & 1 \\ u_6u_6' & v_6u_6' & u_6' & u_6v_6' & v_6v_6' & v_6' & u_6 & v_6 & 1 \\ u_7u_7' & v_7u_7' & u_7' & u_7v_7' & v_7v_7' & v_7' & u_7 & v_7 & 1 \\ u_8u_8' & v_8u_8' & u_8' & u_8v_8' & v_8v_8' & v_8' & u_8 & v_8 & 1 \end{bmatrix} \begin{bmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{bmatrix} = 0$$

Recovering Fundamental Matrix

- 8-point algorithm
  - Construct linear system using corresponding image points
  - Solve for $\mathbf{f}$ using SVD

$$W\mathbf{f} = 0$$

$$W = USV^T$$

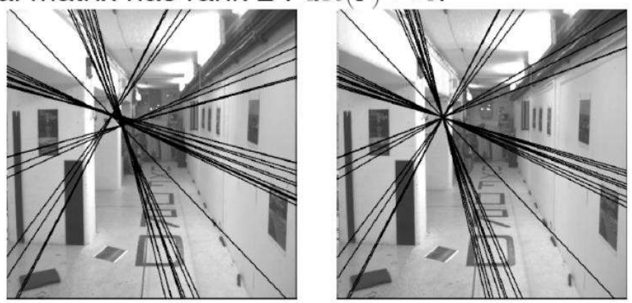
Last column of V gives $\mathbf{f}$

$$\begin{bmatrix} u_1u_1' & v_1u_1' & u_1' & u_1v_1' & v_1v_1' & v_1' & u_1 & v_1 & 1 \\ u_2u_2' & v_2u_2' & u_2' & u_2v_2' & v_2v_2' & v_2' & u_2 & v_2 & 1 \\ u_3u_3' & v_3u_3' & u_3' & u_3v_3' & v_3v_3' & v_3' & u_3 & v_3 & 1 \\ u_4u_4' & v_4u_4' & u_4' & u_4v_4' & v_4v_4' & v_4' & u_4 & v_4 & 1 \\ u_5u_5' & v_5u_5' & u_5' & u_5v_5' & v_5v_5' & v_5' & u_5 & v_5 & 1 \\ u_6u_6' & v_6u_6' & u_6' & u_6v_6' & v_6v_6' & v_6' & u_6 & v_6 & 1 \\ u_7u_7' & v_7u_7' & u_7' & u_7v_7' & v_7v_7' & v_7' & u_7 & v_7 & 1 \\ u_8u_8' & v_8u_8' & u_8' & u_8v_8' & v_8v_8' & v_8' & u_8 & v_8 & 1 \end{bmatrix} \begin{bmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{bmatrix} = 0$$

Recovering Fundamental Matrix

- 8-point algorithm
  - Construct linear system using corresponding image points
  - Solve for $\mathbf{f}$ using SVD
  - Constraint enforcement (essential step)
    - rank(F) = 2

Fundamental matrix has rank 2 : $\det(\mathtt{F}) = 0$.



Left : Uncorrected $\mathtt{F}$ – epipolar lines are not coincident.

Right : Epipolar lines from corrected $\mathtt{F}$.

Left: without constraint rank=2
Right: with constraint rank=2

Recovering Fundamental Matrix

- 8-point algorithm
  - Construct linear system using corresponding image points
  - Solve for $\mathbf{f}$ using SVD
  - Constraint enforcement (essential step)
    - rank(F) = 2

$$\hat{F} = UDV^T \qquad D = \begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{bmatrix} \Rightarrow F = U \begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & 0 \end{bmatrix} V^T$$