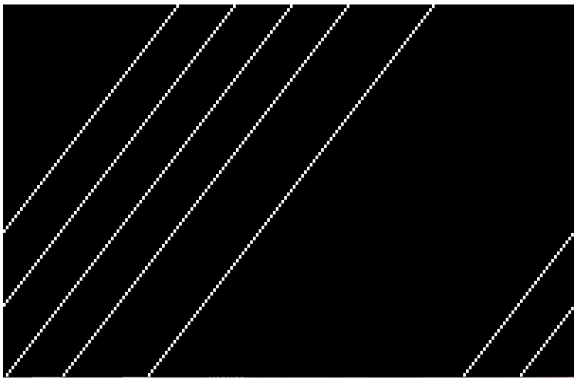
Task A - Using 1D Cellular Automata to investigate complexity

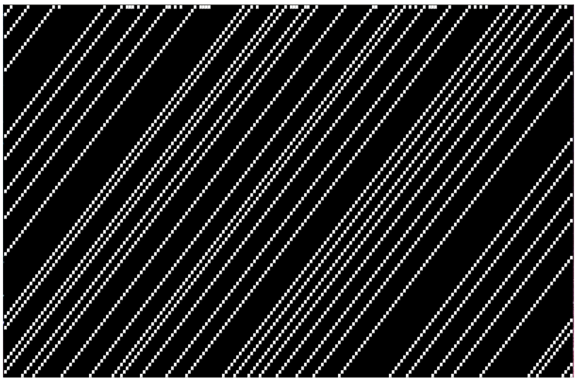
1) Classify:
For this task, We need to create the simulations for twelve rules, and then use these simulations to classify each rule into the four classes suggested by Wolfram [1].
Class 1: evolves to a *homogeneous state*.`
Class 2: evolves to simple *separated periodic structures*.
Class 3: evolves to *chaotic aperiodic patterns*.
Class 4: evolves to *complex patterns of localized structures*.

Rule 2

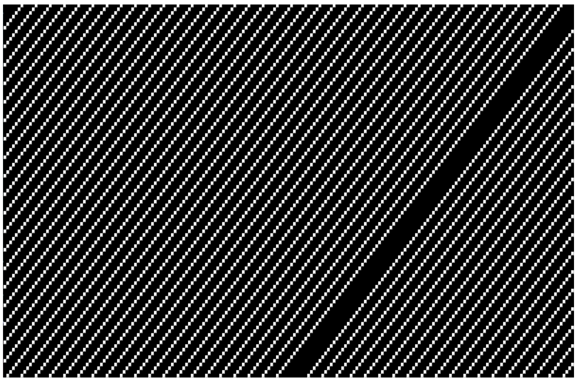
(a) a single seed



(b) five seeds



(c) random initial seeds

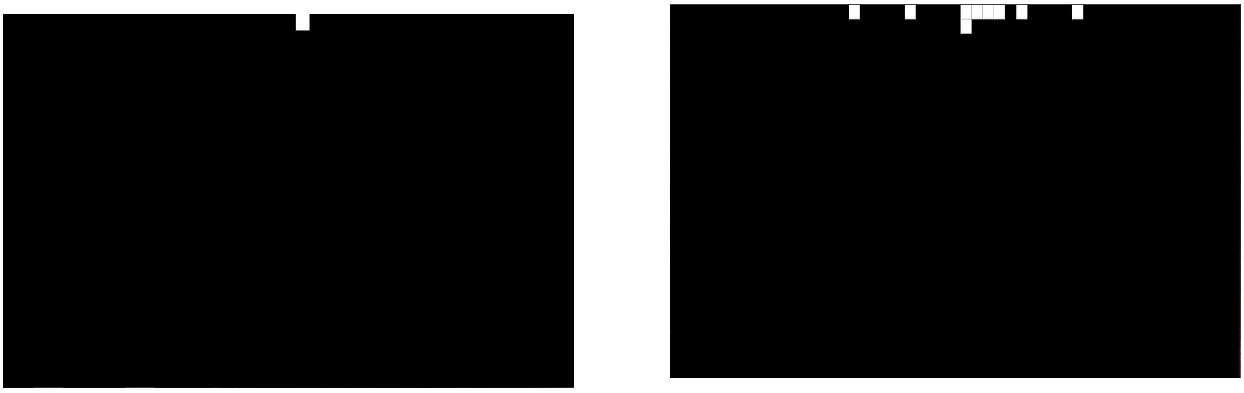


(d) evenly-separated seeds

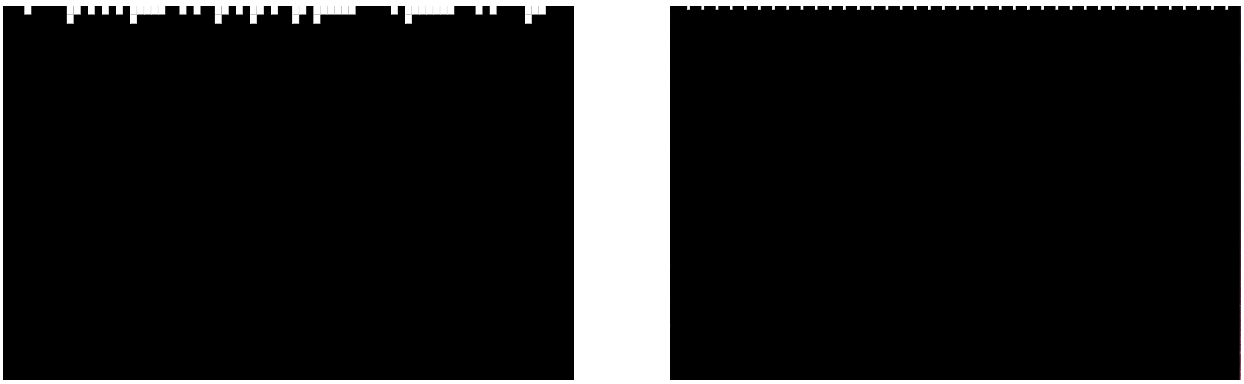
Figure 1: Simulations for rule 2

All configurations show similar behaviour with successive generations. Patterns are periodic. Rule 2 is in class 2.

Rule 8



(a) single seed **(b) 4-seed cluster**

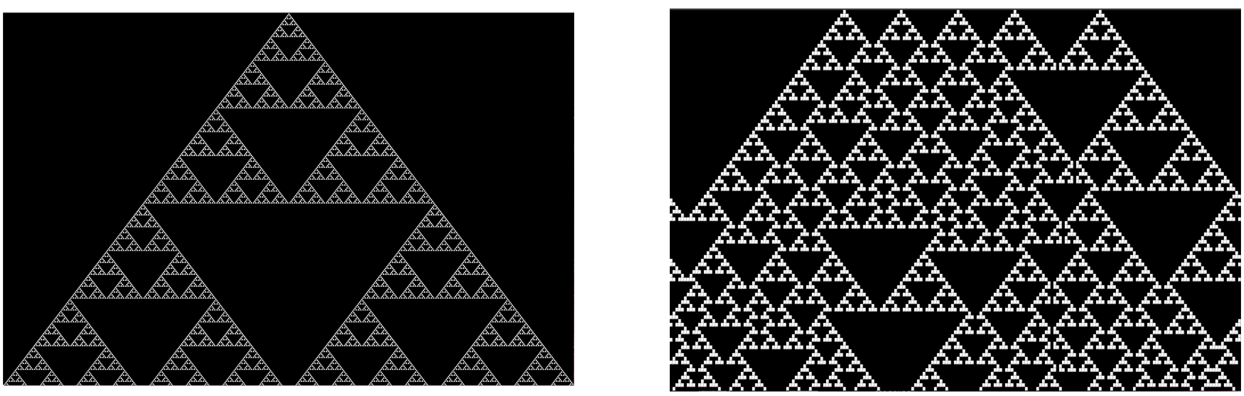


(c) initial random seeds **(d) evenly-separated seeds**

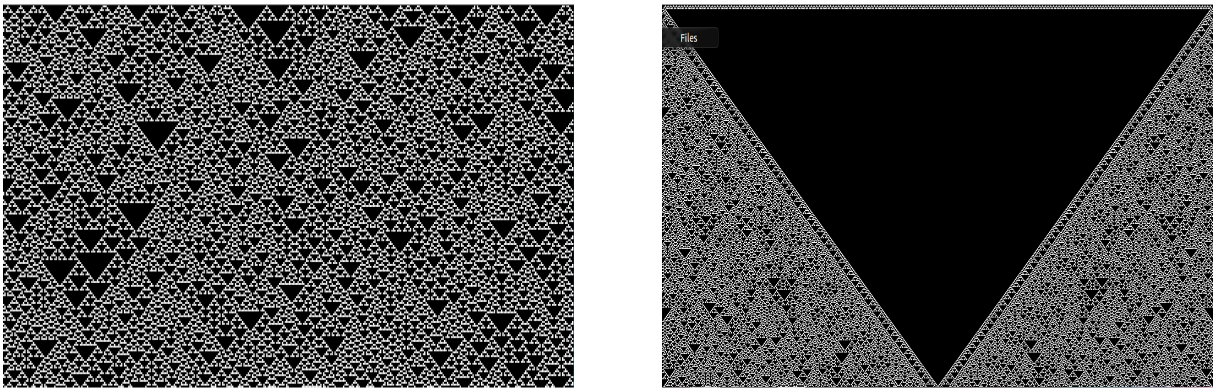
Figure 2: Simulations for rule 8.

Class 1, homogeneous final state, all configurations result in class 1 behaviour.

Rule 22:



(a) single seed **(b) five seeds**



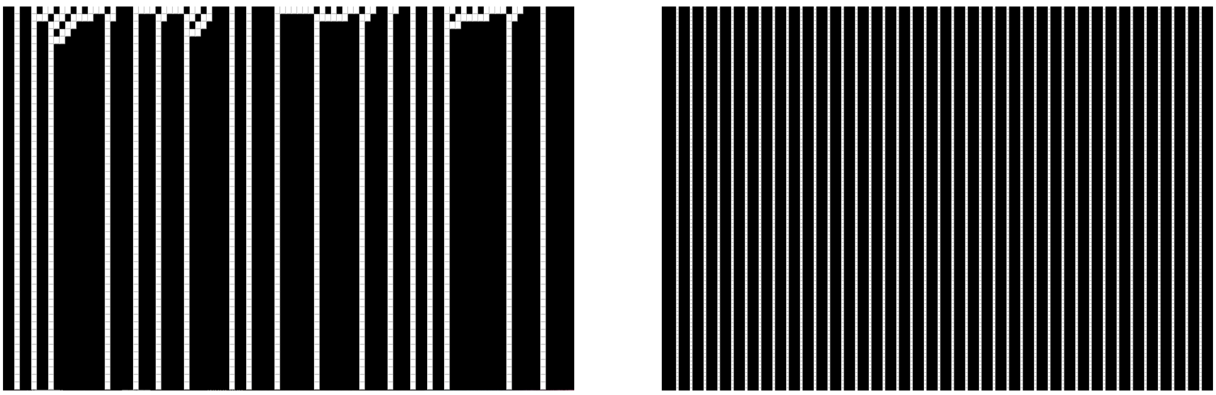
(c) random initial configuration **(d) evenly-separated seeds**

Figure 3: Simulations for Rule 22

Fig. 3a shows a fractal patterns (i.e. nested patterns that emerge), while the others behave randomly, especially in random initial configuration, no predictable pattern is found. Hence, Rule 22 is in class 3.

Rule 44

(a) single seed **(b) five seeds**

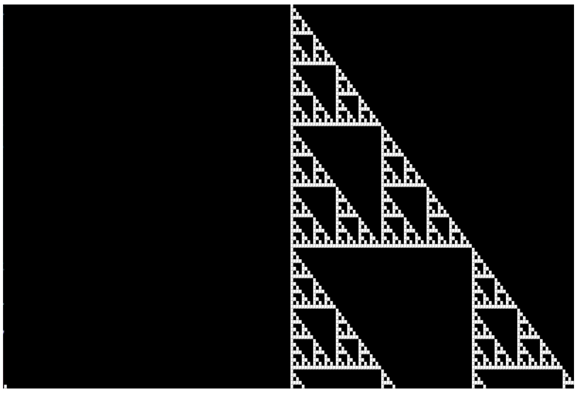


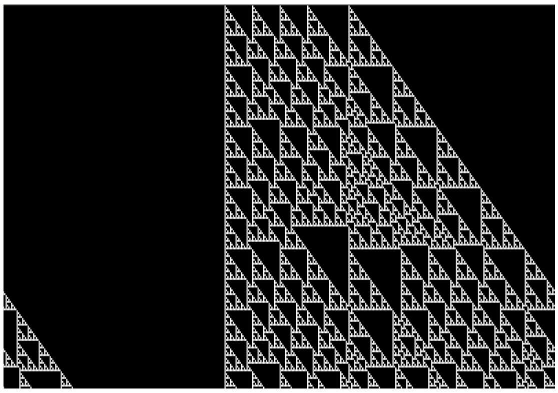
(c) random initial configuration **(d) evenly-separated seeds**

Figure 4: Simulations for Rule 44.

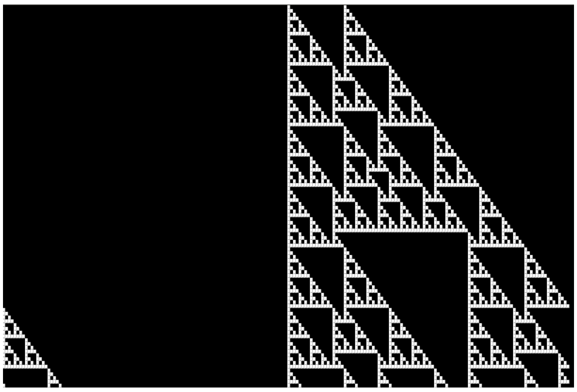
Class-2 rule, behaviour of states is periodic even when given random seeds the rule has class 2 behaviour with periodic structures.

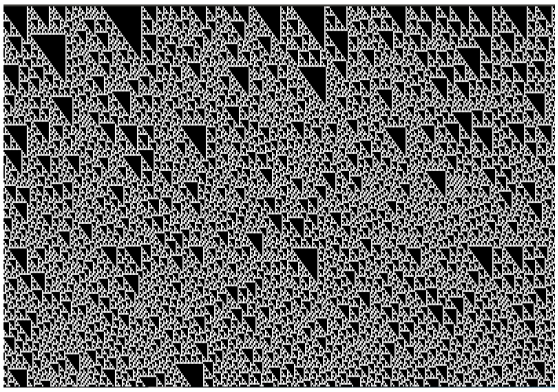
Rule 60:





(a) single seed **(b) five seeds**



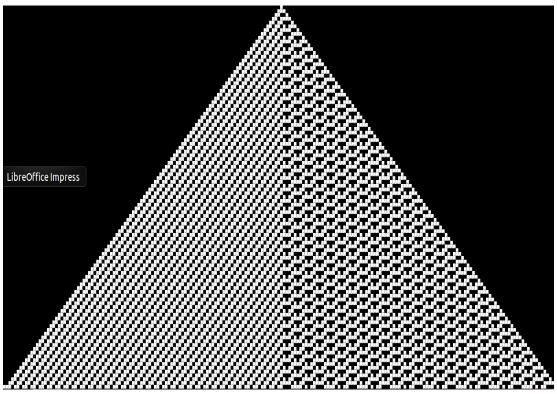


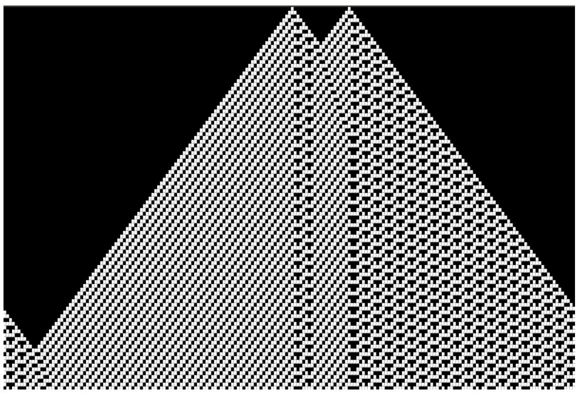
(c) two seeds **(d) random initial configuration**

Figure 5: Simulations for Rule 60

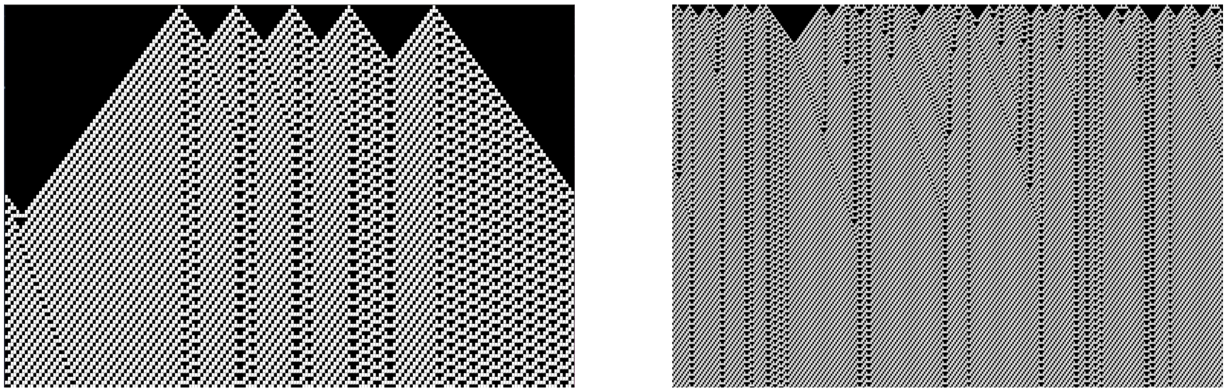
Fig. 5a shows fractal patterns, but random initial configuration shows clearly random behaviour. Rule 60 is a class-3 rule.

Rule 62





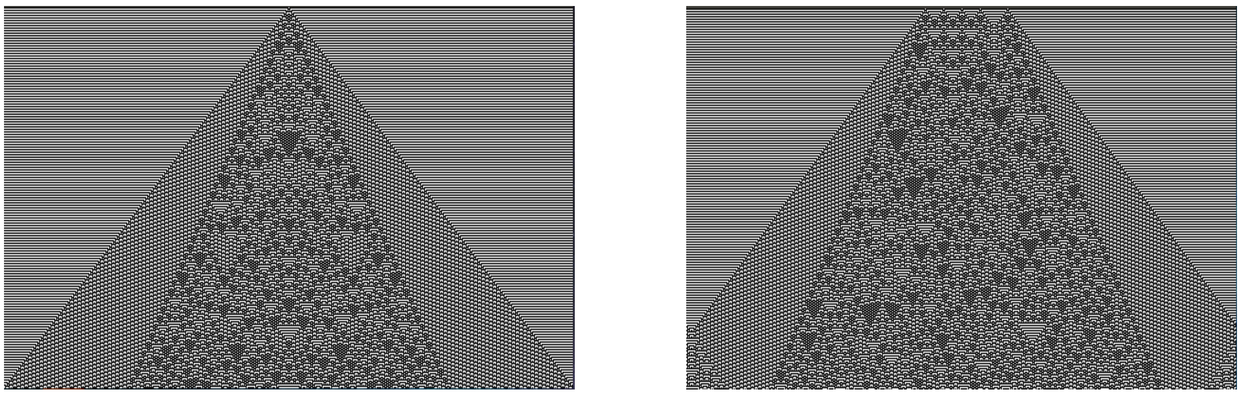
(a) single seed **(b) two seeds**



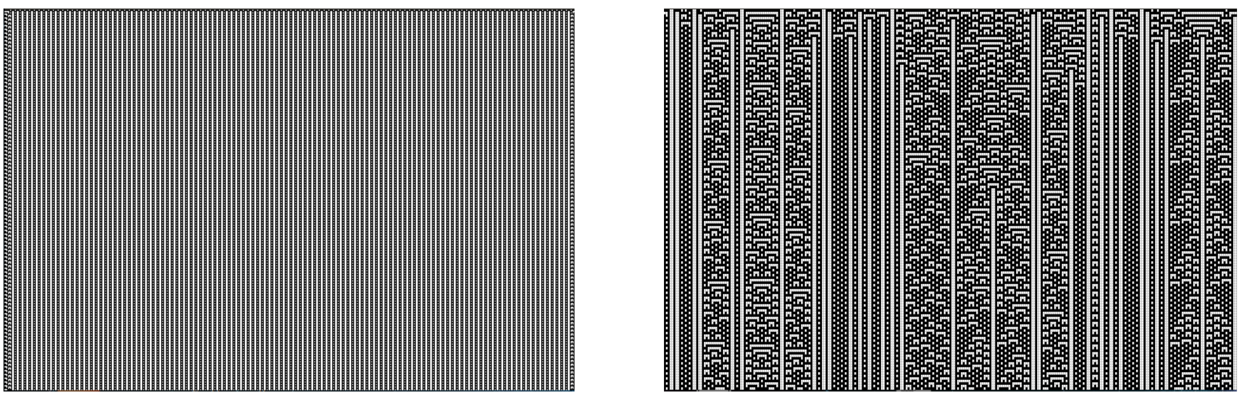
(c) five seeds **(d) random initial configuration**

Figure 6: Simulations for Rule 62.
Rule 2 shows class 2 behaviour which can be observed with the random initial configuration of seeds, pattern repeats with separated periodic structures.

Rule 73:



(a) single seed **(b) five seeds**

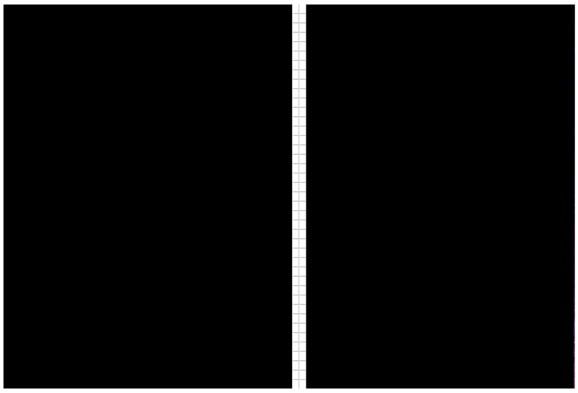


(c) evenly-separated seeds **(d) random initial configuration**

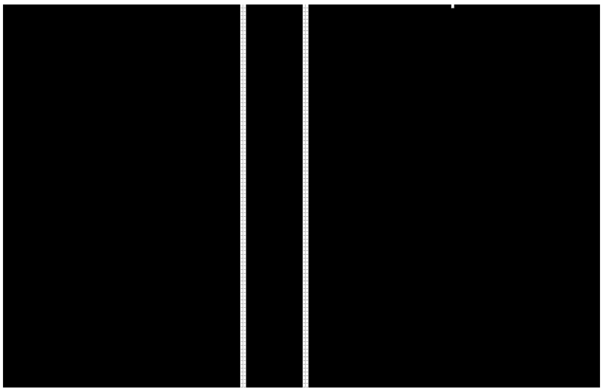
Figure 7: Simulations for Rule 73.

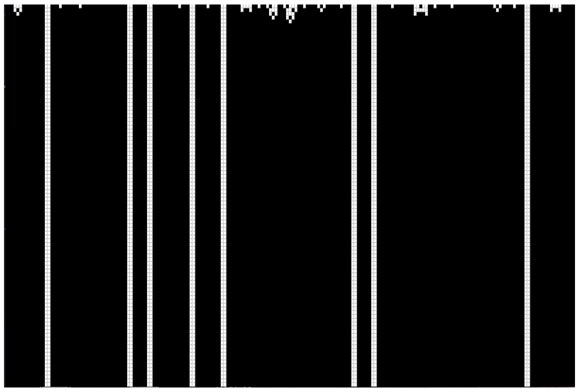
Class 2 rule, can be seen with random initial seeds, the pattern observed is separated periodic and repeats.

Rule 104:



(a) single seed **(b) double seed**



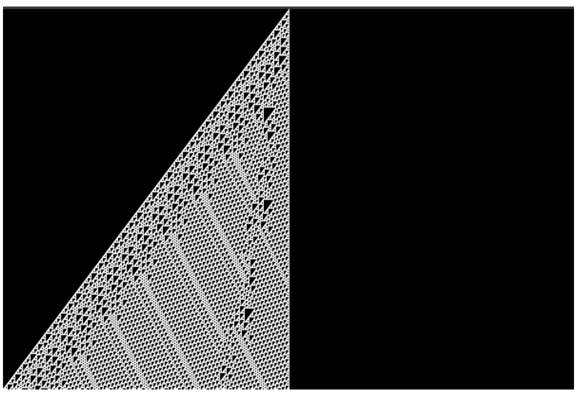


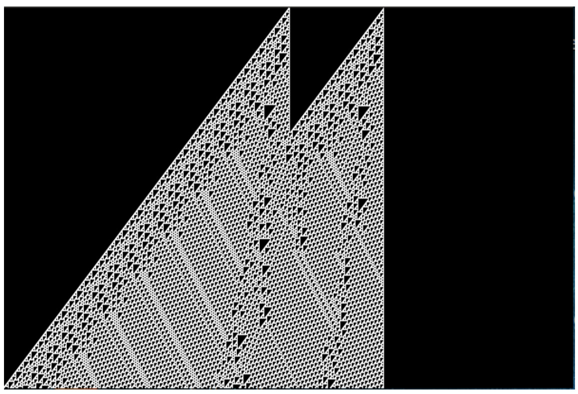
(c) two double-seeds **(d) random initial configuration**

Figure 8: Simulations for Rule 104.

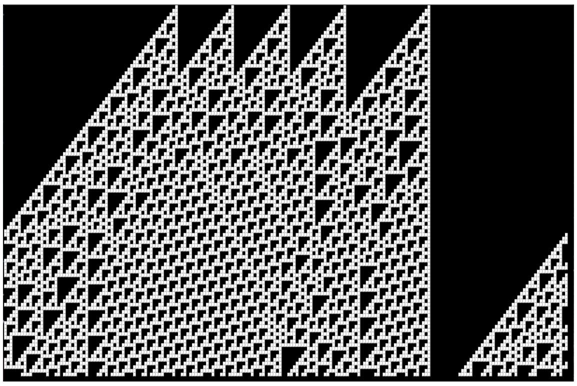
Initially, Fig. 8a gives an idea of homogeneous structure (i.e. class-1 rule), but when experimenting with double seeds, which contain two single seed sitting next to each other, it can be seen that the patterns is separately periodic, especially in random configuration. Clearly, Rule 104 is a class-2 rule.

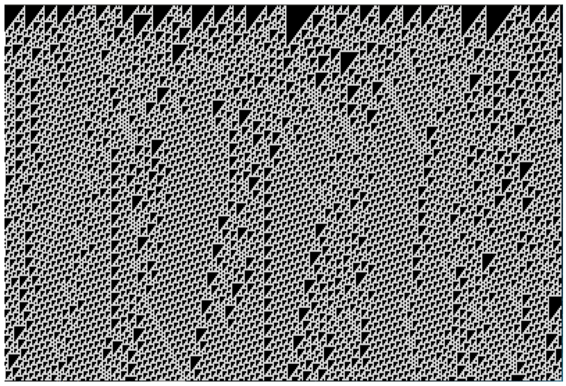
Rule 110





(a) single seed **(b) two seeds**





(c) five seeds

(d) random initial configuration

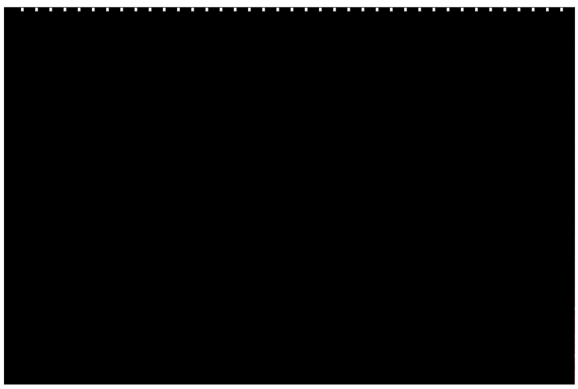
Figure 9: Simulations for Rule 110.

It can be seen from all simulations that the patterns are localized structures that interact in a complex way. Clearly, Rule 110 is in class 4.

Rule 136:

(a) single seed

(b) three seeds



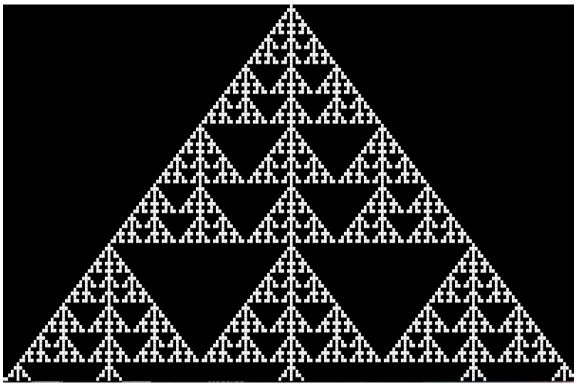
(c) random initial configuration

(d) evenly-separated seeds

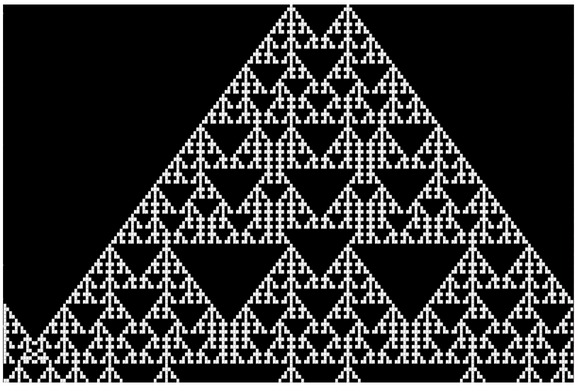
Figure 10: Simulations for Rule 136.

All configurations end up with homogeneous patterns (i.e. all cells become black). It becomes uniform state after different time steps, 1 time step for single seed, 4 for random initial configuration. Hence, Rule 136 is a class-1 rule.

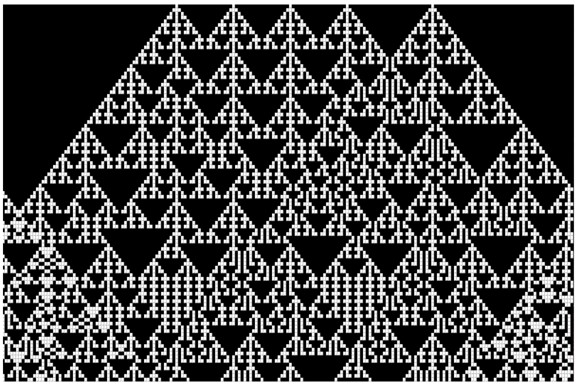
Rule 150:



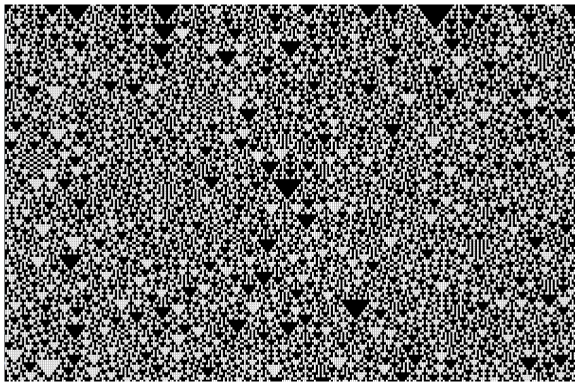
(a) single seed



(b) two seeds



(c) five seeds



(d) random initial configuration

Figure 11: Simulations for Rule 150.

Fig. 11a shows clearly fractal structure, but random configuration confirms patterns behave in a random respect. Clearly, It is a Class-3 rule

Rule 160:

(a) single seed

(b) three seeds

(c) cluster **(d) random initial configuration**
Figure 12: Simulations for Rule 160.

All simulations end up with uniform final state (i.e. all cells become black) after different step of times, 1 time step for single seed and 4 for random configuration. Rule 160 is a class-1 rule.

2) Calculate Langton's parameter
Langton [2] provided the formula to calculate Langton's parameter. Let's n be the number of transitions to special quiescent state in a rule, K is the number of state each cell can have, N is the number of neighbours.

$\lambda = \frac{K^N - n}{K}$

In this experiment
- Each cell has 2 states, K=2.
- Each cell has 3 neighbours (including itself), N=3.
- Quiescent state S_q = 0.

a) Class 1:
    8 = 0000 1000
    λ = (2^3 – 7)/(2^3) = (8-7)/8 = 1/8

b) Class 2:
    44 = 0010 1100
    λ = (2^3 – 5)/(2^3) = (8-5)/8 = 3/8

c) Class 3:
    150 = 1001 0110
    λ = (2^3 – 4)/(2^3) = (8-4)/8 = 1/2

d) Class 4:
    110 = 0110 1110
    λ = (2^3 – 3)/(2^3) = (8-3)/8 = 5/8

Comment:
The value of Langton's parameter falls into the range of from 0 to 1 as "the average behavior of the systems goes from freezing to periodic patterns to chaos" [3]. Langton [2] introduced the idea of "Edge of chaos", in which almost rules with $\lambda = 0.5$ turns to lie on the Edge. The edge of chaos represents the phase transition in Cellular Automata, at which the system turns from chaos into complex. In additions, rules which behave in homogeneous and separated periodic manner are expected to have value lower than this. Therefore, the values of langton's parameter we have just measured for each class above seem to be quite reasonable, as the values are in ascending order, starting from ⅛, ⅜, ½, to ⅝ as the behaviour of classes changed from homogeneous to periodic to chaotic and complex. The values for classes 3 and 4 are around 0.5, which is around the edge of chaos.

References:
[1] Wolfram, S., 1984, "Universality and complexity in cellular automata", Physica D 10 1-35.

[2] Langton, C., 1990, "Computation at the Edge of Chaos: Phase Transitions and Emergent Computation", *Physica D*, 42:12–37.

[3] Berto, Francesco and Tagliabue, Jacopo, "Cellular Automata", *The Stanford Encyclopedia of Philosophy* (Summer 2012 Edition), Edward N. Zalta (ed.), URL = <http://plato.stanford.edu/archives/sum2012/entries/cellular-automata/>.