1. **Question**:

For the following mean-field Hamiltonian, if we want to code and numerically solve the self-consistent equation, what is the dimension of the mean-field Hamiltonian matrix at a given momentum $k$?

Consider the mean-field Hamiltonian: $H = H_{\text{Kinetic}} + H_{\text{Hartree}} + H_{\text{Fock}}$, with each term defined below:
$H_{\text{Kinetic}} = \sum_{s,k} E_s(k) c_s^\dagger(k) c_s(k)$, where $E_s(k) = \sum_n t_s(n) e^{-ik\cdot n}$
$H_{\text{Hartree}} = \frac{1}{N} \sum_{s,s'} \sum_{k_1,k_2} U(0) \langle c_s^\dagger(k_1) c_s(k_1) \rangle c_{s'}^\dagger(k_2) c_{s'}(k_2)$
$H_{\text{Fock}} = -\frac{1}{N} \sum_{s,s'} \sum_{k_1,k_2} U(k_1 - k_2) \langle c_s^\dagger(k_1) c_{s'}(k_1) \rangle c_{s'}^\dagger(k_2) c_s(k_2)$
$U(k) = \sum_n U_n e^{-ik\cdot n}$, where $U_0$ is the on-site interaction, $U_1$ is the nearest neighbor interaction, and $U_n$ is the n-th shell neighbor interaction. Here, we consider only the on-site interaction and the nearest neighbor interaction.
Hopping parameters are $t_1 = 6$ meV for nearest-neighbor hopping and $t_2 = 1$ meV for next-nearest-neighbor hopping, which correspond to $t_s(n)$ in the kinetic energy term. $s = \{\uparrow, \downarrow\}$ is the spin index.
The lattice is a two-dimensional triangular lattice.

**Answer**:

$2 \times 2$ matrix.

2. **Question**:

For the following mean-field Hamiltonian, what do you think is the possible order parameter we can write down for the system without breaking the translational symmetry?

Consider the mean-field Hamiltonian: $H = H_{\text{Kinetic}} + H_{\text{Hartree}} + H_{\text{Fock}}$, with each term defined below:
$H_{\text{Kinetic}} = \sum_{s,k} E_s(k) c_s^\dagger(k) c_s(k)$, where $E_s(k) = \sum_n t_s(n) e^{-ik\cdot n}$
$H_{\text{Hartree}} = \frac{1}{N} \sum_{s,s'} \sum_{k_1,k_2} U(0) \langle c_s^\dagger(k_1) c_s(k_1) \rangle c_{s'}^\dagger(k_2) c_{s'}(k_2)$
$H_{\text{Fock}} = -\frac{1}{N} \sum_{s,s'} \sum_{k_1,k_2} U(k_1 - k_2) \langle c_s^\dagger(k_1) c_{s'}(k_1) \rangle c_{s'}^\dagger(k_2) c_s(k_2)$
$U(k) = \sum_n U_n e^{-ik\cdot n}$, where $U_0$ is the on-site interaction, $U_1$ is the nearest neighbor interaction, and $U_n$ is the n-th shell neighbor interaction. Here, we consider only the on-site interaction and the nearest neighbor interaction.
Hopping parameters are $t_1 = 6$ meV for nearest-neighbor hopping and $t_2 = 1$ meV for next-nearest-neighbor hopping, which correspond to $t_s(n)$ in the kinetic energy term. $s = \{\uparrow, \downarrow\}$ is the spin index.
The lattice is a two-dimensional triangular lattice.

**Answer**:

$\langle c_s^\dagger c_{s'} \rangle$ with $s, s' = \uparrow, \downarrow$.

3. **Question**:

For the following mean-field Hamiltonian, if we want to code and solve the problem in the momentum space, what are the coordinates of the 6 Brillouin zone corners, $K$ point? Round to 2 decimal places.

Consider the mean-field Hamiltonian: $H = H_{\text{Kinetic}} + H_{\text{Hartree}} + H_{\text{Fock}}$, with each term defined below:
$H_{\text{Kinetic}} = \sum_{s,k} E_s(k) c_s^\dagger(k) c_s(k)$, where $E_s(k) = \sum_n t_s(n) e^{-ik\cdot n}$
$H_{\text{Hartree}} = \frac{1}{N} \sum_{s,s'} \sum_{k_1,k_2} U(0) \langle c_s^\dagger(k_1) c_s(k_1) \rangle c_{s'}^\dagger(k_2) c_{s'}(k_2)$
$H_{\text{Fock}} = -\frac{1}{N} \sum_{s,s'} \sum_{k_1,k_2} U(k_1 - k_2) \langle c_s^\dagger(k_1) c_{s'}(k_1) \rangle c_{s'}^\dagger(k_2) c_s(k_2)$
$U(k) = \sum_n U_n e^{-ik\cdot n}$, where $U_0$ is the on-site interaction, $U_1$ is the nearest neighbor interaction, and $U_n$ is the n-th shell neighbor interaction. Here, we consider only the on-site interaction and the nearest neighbor interaction.
Hopping parameters are $t_1 = 6$ meV for nearest-neighbor hopping and $t_2 = 1$ meV for next-nearest-neighbor hopping, which correspond to $t_s(n)$ in the kinetic energy term. $s = \{\uparrow, \downarrow\}$ is the spin index.
The lattice is a two-dimensional triangular lattice.

**Answer**:

(0, 4.19)
(0, -4.19)
(3.62, 2.09)
(-3.62, 2.09)
(3.62, -2.09)
(-3.62, -2.09)

4. **Question**:

For the following mean-field Hamiltonian, if we want to code and solve the problem in the momentum space, what are the coordinates of the center of Brillouin zone, Γ point? Round to 2 decimal places.

Consider the mean-field Hamiltonian: $H = H_{\text{Kinetic}} + H_{\text{Hartree}} + H_{\text{Fock}}$, with each term defined below:
$H_{\text{Kinetic}} = \sum_{s,k} E_s(k) c_s^\dagger(k) c_s(k)$, where $E_s(k) = \sum_n t_s(n) e^{-ik\cdot n}$
$H_{\text{Hartree}} = \frac{1}{N} \sum_{s,s'} \sum_{k_1,k_2} U(0) \langle c_s^\dagger(k_1) c_s(k_1) \rangle c_{s'}^\dagger(k_2) c_{s'}(k_2)$
$H_{\text{Fock}} = -\frac{1}{N} \sum_{s,s'} \sum_{k_1,k_2} U(k_1 - k_2) \langle c_s^\dagger(k_1) c_{s'}(k_1) \rangle c_{s'}^\dagger(k_2) c_s(k_2)$
$U(k) = \sum_n U_n e^{-ik\cdot n}$, where $U_0$ is the on-site interaction, $U_1$ is the nearest neighbor interaction, and $U_n$ is the n-th shell neighbor interaction. Here, we consider only the on-site interaction and the nearest neighbor interaction.
Hopping parameters are $t_1 = 6$ meV for nearest-neighbor hopping and $t_2 = 1$ meV for next-nearest-neighbor hopping, which correspond to $t_s(n)$ in the kinetic energy term. $s = \{\uparrow, \downarrow\}$ is the spin index.
The lattice is a two-dimensional triangular lattice.

**Answer**:

(0,0)

5. **Question**:

For the following mean-field Hamiltonian, if we want to code and solve the noninteracting band structure, what are the energies at the center of Brillouin zone, Γ point, for all bands?

Consider the mean-field Hamiltonian: $H = H_{\text{Kinetic}} + H_{\text{Hartree}} + H_{\text{Fock}}$, with each term defined below:
$H_{\text{Kinetic}} = \sum_{s,k} E_s(k) c_s^\dagger(k) c_s(k)$, where $E_s(k) = \sum_n t_s(n) e^{-ik\cdot n}$
$H_{\text{Hartree}} = \frac{1}{N} \sum_{s,s'} \sum_{k_1,k_2} U(0) \langle c_s^\dagger(k_1) c_s(k_1) \rangle c_{s'}^\dagger(k_2) c_{s'}(k_2)$
$H_{\text{Fock}} = -\frac{1}{N} \sum_{s,s'} \sum_{k_1,k_2} U(k_1 - k_2) \langle c_s^\dagger(k_1) c_{s'}(k_1) \rangle c_{s'}^\dagger(k_2) c_s(k_2)$
$U(k) = \sum_n U_n e^{-ik\cdot n}$, where $U_0$ is the on-site interaction, $U_1$ is the nearest neighbor interaction, and $U_n$ is the n-th shell neighbor interaction. Here, we consider only the on-site interaction and the nearest neighbor interaction.
Hopping parameters are $t_1 = 6$ meV for nearest-neighbor hopping and $t_2 = 1$ meV for next-nearest-neighbor hopping, which correspond to $t_s(n)$ in the kinetic energy term. $s = \{\uparrow, \downarrow\}$ is the spin index.
The lattice is a two-dimensional triangular lattice.

**Answer**:

(48 meV, 48 meV)

6. **Question**:

For the following mean-field Hamiltonian, if we want to code and solve the noninteracting band structure, what are the energies at the first Brillouin zone corners, $K$ point, for all bands?

Consider the mean-field Hamiltonian: $H = H_{\text{Kinetic}} + H_{\text{Hartree}} + H_{\text{Fock}}$, with each term defined below:
$H_{\text{Kinetic}} = \sum_{s,k} E_s(k) c_s^\dagger(k) c_s(k)$, where $E_s(k) = \sum_n t_s(n) e^{-ik\cdot n}$
$H_{\text{Hartree}} = \frac{1}{N} \sum_{s,s'} \sum_{k_1,k_2} U(0) \langle c_s^\dagger(k_1) c_s(k_1) \rangle c_{s'}^\dagger(k_2) c_{s'}(k_2)$
$H_{\text{Fock}} = -\frac{1}{N} \sum_{s,s'} \sum_{k_1,k_2} U(k_1 - k_2) \langle c_s^\dagger(k_1) c_{s'}(k_1) \rangle c_{s'}^\dagger(k_2) c_s(k_2)$
$U(k) = \sum_n U_n e^{-ik\cdot n}$, where $U_0$ is the on-site interaction, $U_1$ is the nearest neighbor interaction, and $U_n$ is the n-th shell neighbor interaction. Here, we consider only the on-site interaction and the nearest neighbor interaction.
Hopping parameters are $t_1 = 6$ meV for nearest-neighbor hopping and $t_2 = 1$ meV for next-nearest-neighbor hopping, which correspond to $t_s(n)$ in the kinetic energy term. $s = \{\uparrow, \downarrow\}$ is the spin index.
The lattice is a two-dimensional triangular lattice.

**Answer**:

(-15 meV, -15 meV)

7. **Question**:

For the following problem, derive the kinetic term of the Hamiltonian in second-quantized form ($H_{TB}$) for this triangular lattice system. Express your final answer as the complete tight-binding Hamiltonian, clearly showing the summations and indices.

Consider a triangular lattice system where the degrees of freedom are spin states associated with $+K$ and $-K$ valleys. The kinetic term of the Hamiltonian can be described by a tight-binding model with the following properties:
Electrons hop between sites with amplitude $t_s(R_i - R_j)$.
$s = \uparrow, \downarrow$ represents spin states associated with $+K$ and $-K$ valleys respectively.
$R_i$ represents a site position in the triangular lattice.
$c_{R_i,s}$ is the electron annihilation operator at site with position $R_i$ with spin $s$.
$c^\dagger_{R_i,s}$ is the electron creation operator at site with position $R_i$ with spin $s$.
The hopping process occurs between sites at positions $R_i$ and $R_j$.
The summation should be taken over all spin states and all real space positions.

**Answer**:

$H_{TB} = -\sum_{R_i,R_j,s} t_s(R_i - R_j) c^\dagger_{R_i,s} c_{R_j,s}$

8. **Question**:

For the following problem, derive the interaction part of the Hamiltonian $H^{\text{int}}$ in the second-quantized form for this triangular lattice system. Express your final answer in terms of the number operators $n_s(R_i)$.

Consider a triangular lattice system with spin degrees of freedom associated with $+K$ and $-K$ valleys. We want to examine the interaction term of the Hamiltonian in the second-quantized form, which complements the kinetic term.

The system has a density-density interaction between sites with the following properties:

The interaction occurs between sites at positions $R_i$ and $R_j$ with interaction strength $U(R_i - R_j)$.

$R_i$ and $R_j$ represent site positions in the triangular lattice.

$c_{R_i,s}$ and $c^\dagger_{R_i,s}$ are the electron annihilation and creation operators.

The summation should be taken over all spin states $s, s'$ and all real space positions.

The number operator for electrons with spin $s$ at site position $R_i$ is defined as $n_s(R_i) = c^\dagger_{R_i,s} c_{R_i,s}$.

**Answer**:

$H^{\text{int}} = \frac{1}{2} \sum_{s,s'=\uparrow,\downarrow} \sum_{R_i,R_j} U(R_i - R_j) n_s(R_i) n_{s'}(R_j)$

9. **Question**:

For the following problem, express the noninteracting Hamiltonian $\hat{H}_{\text{Kinetic}}$ in terms of the second quantized operators $c_s^\dagger(k)$ and $c_s(k)$ in the momentum space. Simplify any summation indices if possible.

Consider a triangular lattice system with spin degrees of freedom associated with $+K$ and $-K$ valleys. We have a noninteracting Hamiltonian in the second-quantized form expressed in the real space basis as:
$\hat{H}_{\text{Kinetic}} = \sum_{R_i,R_j} \sum_{s=\uparrow,\downarrow} t_s(R_i - R_j) c_s^\dagger(R_i) c_s(R_j)$
where:
$c_s^\dagger(R_i)$ and $c_s(R_j)$ are creation and annihilation operators in real space
$t_s(R_i - R_j)$ is the hopping parameter
The summation is over all sites and spin states

The Fourier transformation from real space to momentum space is defined as:
$c_s^\dagger(k) = \frac{1}{\sqrt{N}} \sum_i c_s^\dagger(R_i) e^{ik\cdot R_i}$
where $N$ is the number of unit cells in the real space.

**Answer**:

$\hat{H}_{\text{Kinetic}} = \sum_{s=\uparrow,\downarrow} \sum_k E_s(k) c_s^\dagger(k) c_s(k)$
where $E_s(k) = \sum_n t_s(n) e^{-ik\cdot n}$

10. **Question**:

For the following problem, express the interaction Hamiltonian $\hat{H}^{\text{int}}$ in terms of second quantized operators $c_s^\dagger(k)$ and $c_s(k)$ in the momentum space. Simplify any summation indices if possible.

Consider a triangular lattice system with spin degrees of freedom associated with $+K$ and $-K$ valleys. We have an interaction Hamiltonian in the second-quantized form expressed in the real space basis as:
$\hat{H}^{\text{int}} = \frac{1}{2}\sum_{R_i,R_j}\sum_{s,s'=\uparrow,\downarrow} U(R_i - R_j)n_s(R_i)n_{s'}(R_j)$

where:
$n_s(R_i) = c_s^\dagger(R_i)c_s(R_i)$ is the number operator at site $R_i$ with spin $s$
$U(R_i - R_j)$ is the interaction strength between sites $i$ and $j$
The summation is over all sites and spin states

The Fourier transformation from real space to momentum space is defined as:
$c_s^\dagger(k) = \frac{1}{\sqrt{N}}\sum_i c_s^\dagger(R_i)e^{ik\cdot R_i}$

where $k$ is defined within the first Brillouin zone and $N$ is the number of unit cells in the real space.

**Answer**:

$\hat{H}^{\text{int}} = \frac{1}{2N}\sum_{s,s'=\uparrow,\downarrow}\sum_{k,k',k'',k'''} U(k-k')c_s^\dagger(k)c_s(k')c_{s'}^\dagger(k'')c_{s'}(k''')\sum_G \delta(k-k'+k''-k''',G)$

where $U(k) = \sum_n U(n)e^{-ik\cdot n}$ and $G$ is a reciprocal lattice vector.