QF600 Asset Pricing Homework 2

Capital Asset Pricing Model (CAPM)

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Contents

Part 1: Market Model

Estimate the intercept coefficient (α) and slope coefficient (β) for each of the ten industry portfolio using the market model: regress the monthly excess returns for each industry portfolio on the monthly excess returns for the market portfolio.

`Question 1`: Create a table showing the intercept and slope coefficients for the ten industry portfolios.

Table 1: Intercept and Slope Coefficients for Ten Industry Portfolios

	alpha - Intercept	beta - Slope
NoDur	0.3694429	0.6526474
Durbl	-0.4155991	1.6485356
Manuf	0.1597708	1.1698463
Enrgy	0.5017186	0.9698500
HiTec	-0.0640195	1.1329694
Telcm	0.1946909	0.9007293
Shops	0.2754922	0.8264924
Hlth	0.2378412	0.6730357
Utils	0.4445846	0.5380861
Other	-0.3871345	1.2073095

Table 1 shows the intercept and slope coefficient for the models of ten industry portfolio and market portfolio respectively.

`Question 2`: Briefly explain the economic significant of intercept and slope coefficients.

The formula for capital asset pricing model (CAPM) is:

$$R_i - R_f = \beta_i(R_m - R_f) + \epsilon_i$$

The formula for market model is

$$R_i - R_f = \alpha_i + \beta_i(R_m - R_f) + \epsilon_i$$

where α is the intercept of the market model and that represents the pricing error compare to capital asset pricing model for passive portfolio. The intercept coefficient also considered as **Jensen's alpha**. That represents the α measured active portfolio that **difference of average return on portfolio above or below that predicted by capital asset pricing model with same systematic risk**. Thus, the coefficient of intercept α for the capital asset pricing model is 0. If the α closer to 0, that means the the average excess return on the industry portfolio is same as the excess return that predicted by capital asset pricing model. If the coefficient of α is **positive**, that demonstrates that the excess return on the industry portfolio above that predicted by CAPM. For the other case, the coefficient of α is **negative**, that demonstrates that the excess return on the industry portfolio below that predicted by CAPM. Moreover, if the coefficient of α is not 0, that demonstrates there is pricing error for the market model of specific industry portfolio relative to capital asset pricing model.

For the case of industry portfolio, there are three industry portfolios have the negative coefficient of α, which are Durbl, HiTec, Other industry portfolios. For these industry portfolios, **they have underperform the market.** For the other industry portfolios with positive coefficient of α, **they have outperform the market.** Furthermore, there is no zero coefficient of α for industry portfolios, that means the market model for all industry portfolios have pricing error relative to captial asset pricing model.

The β is the slope of the market model and that represents the amount of exposure to the **systematic risk** of the industry portfolio. The market portfolio has the β and systematic risk of 1. Therefore, the coefficient of β is greater than 1 and that means the industry portfolio has the greater systematic risk compare with the market. For the other cases, if the coefficient of β is less than 1 and greater than 0, that represents the industry portfolio has the smaller systematic risk compare with the market.

For the case of industry portfolio, there are four industry portfolios have coefficient of β greater than 1, that means these industry portfolios have **greater systematic risk relatively to the**

market. For the other industry portfolios with coefficient of β less than 1 and greater than 0, that demonstrate the industry portfolio has the **smaller systematic risk compare with the market.**

Part 2: Security Market Line (SML)

Calculate the mean monthly return for each of the ten industry portfolios, as well as the market portfolio.

Regress the mean monthly returns of the ten industry portfolios and the market portfolio on the corresponding β's. This will give you the intercept and slope coefficients for the SML.

`Question 1`: Use the estimated intercept and slope coefficients for the SML to plot the SML in the range of β from zero to two on the horizontal axis. Also plot the positions of the ten industry portfolios and the market portfolio. (You are NOT required to label the individual portfolios.)

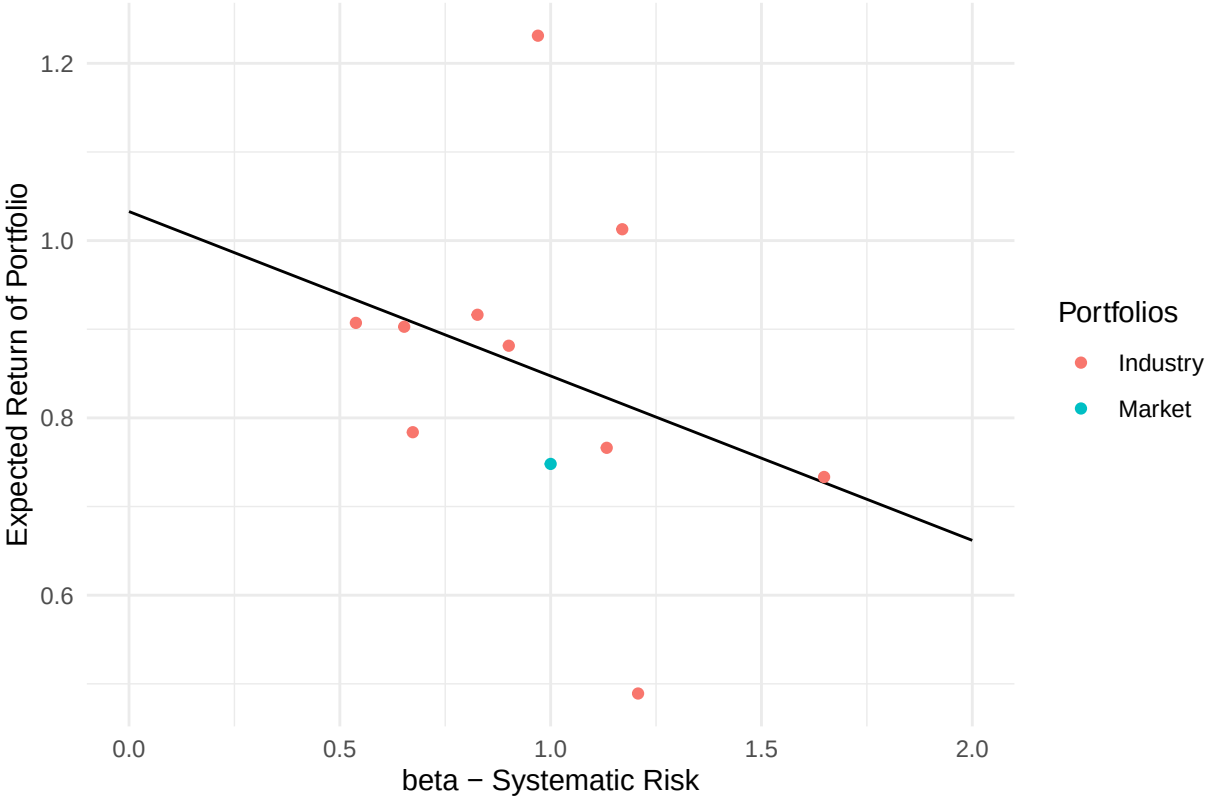

Figure 1. Security Market Line with the Position of 10 Industry Portfolios and Market Portfolio

The figure 1 shows the security market line with the position of 10 industry portfolios and the market portfolio. The security market line has the negative slope, and the red dot is the position of industry portfolios and the green dot is the position of market portfolio.

`Question 2`: Briefly explain the economic significance of the SML.

According to the defination of security market line, that is the grpahical representation of captial asset pricing model formula.

The capital asset pricing model formula is,

$$R_i - R_f = \beta(R_m - R_f)$$

where $R_i - R_f$ represents the excess return for specific industry portfolio, $R_m - R_f$ represents the excess return for market portfolio and β represents the exposure amount of systematic risk.

Capital asset pricing model shows the relationship between systematic risk of the portfolio with the expected return. Thus, the security market line also show the graphical relationship between the systematic risk of portfolio on x-axis and the expected return of portfolio on y-axis.

Theoretically, the intercept of security market line is risk-free rate and the slope is the ratio of risk premium to beta and that considered as Treynor ratio of the market portfolio. Thus, the risky assets on the security market line have the same Treynor ratio.

Furthermore, the security market line could determine if the investment portfolio has the favourable expetation of return with the given level of sysmetic risk. If the portfolio lies above security market line is **underpriced**, then the investor could **buy the portfolio and that would cause price to rise** back to security market line. On the other hand, if the portfolio lies below security market line is **overpriced**, then the investor could **sell the portfolio and that would cause the price fall back on security market line.**

Appendix

Part 1 - Question 1 Code

```
# import the industry portfolio data
industry_df = read_excel("Industry_Portfolios.xlsx")
# remove the column of date from the industry portfolio data frame
industry_df = industry_df %>%
  select(!Date)
# import the market portfolio data
market_df = read_excel("Market_Portfolio.xlsx")
# remove the column of date from the market portfolio data frame
market_df = market_df %>%
  select(!Date)
```

```
# set up the risk-free rate
return_f = 0.13
# compute the excess return for assets
asset_excessr = industry_df - return_f
# compute the excess return for market
market_excessr = market_df - return_f
# extract excess return for assets and market into one data frame
return_df = data.frame(asset_excessr, market_excessr)
```

```
# regress the industry NoDur excess return on market portfolio excess return
model1 = lm(NoDur ~ Market, data = return_df)
# extract the intercept (alpha) for the model of industry NoDur
alpha1 = summary(model1)$coefficients[1]
# extract the slope (beta) for the model of industry NoDur
beta1 = summary(model1)$coefficients[2]
```

```
# regress the industry Durbl excess return on market portfolio excess return
model2 = lm(Durbl ~ Market, data = return_df)
# extract the intercept (alpha) for the model of industry Durbl
alpha2 = summary(model2)$coefficients[1]
# extract the slope (beta) for the model of industry Durbl
beta2 = summary(model2)$coefficients[2]
```

```r
# regress the industry Manuf excess return on market portfolio excess return
model3 = lm(Manuf ~ Market, data = return_df)
# extract the intercept (alpha) for the model of industry Manuf
alpha3 = summary(model3)$coefficients[1]
# extract the slope (beta) for the model of industry Manuf
beta3 = summary(model3)$coefficients[2]
```

```r
# regress the industry Enrgy excess return on market portfolio excess return
model4 = lm(Enrgy ~ Market, data = return_df)
# extract the intercept (alpha) of the model of industry Enrgy
alpha4 = summary(model4)$coefficients[1]
# extract the slope (beta) of the model of industry Enrgy
beta4 = summary(model4)$coefficients[2]
```

```r
# regress the industry HiTec excess return on market portfolio excess return
model5 = lm(HiTec ~ Market, data = return_df)
# extract the intercept (alpha) of the model of industry HiTec
alpha5 = summary(model5)$coefficients[1]
# extract the slope (beta) of the model of industry HiTec
beta5 = summary(model5)$coefficients[2]
```

```r
# regress the industry Telcm excess return on market portfolio excess return
model6 = lm(Telcm ~ Market, data = return_df)
# extract the intercept (alpha) of the model of industry Telcm
alpha6 = summary(model6)$coefficients[1]
# extract the slope (beta) of the model of industry Telcm
beta6 = summary(model6)$coefficients[2]
```

```r
# regress the industry Shops excess return on market portfolio excess return
model7 = lm(Shops ~ Market, data = return_df)
# extract the intercept (alpha) of the model of industry Shops
alpha7 = summary(model7)$coefficients[1]
# extract the slope (beta) of the model of industry Shops
beta7 = summary(model7)$coefficients[2]
```

```r
# regress the industry Hlth excess return on market portfolio excess return
model8 = lm(Hlth ~ Market, data = return_df)
# extract the intercept (alpha) of the model of industry Hlth
alpha8 = summary(model8)$coefficients[1]
```

```
# extract the slope (beta) of the model of industry Hlth
beta8 = summary(model8)$coefficients[2]
```

```
# regress the industry Utils excess return on market portfolio excess return
model9 = lm(Utils ~ Market, data = return_df)
# extract the intercept (alpha) of the model of industry Utils
alpha9 = summary(model9)$coefficients[1]
# extract the slope (beta) of the model of industry Utils
beta9 = summary(model9)$coefficients[2]
```

```
# regress the other industry excess return on market portfolio excess return
model10 = lm(Other ~ Market, data = return_df)
# extract the intercept (alpha) of the model of other industry
alpha10 = summary(model10)$coefficients[1]
# extract the slope (beta) of the model of other industry
beta10 = summary(model10)$coefficients[2]
```

```
# combine the alpha and beta for each portfolio into data frame respectively
alpha_df = rbind(alpha1, alpha2, alpha3, alpha4, alpha5, alpha6, alpha7, alpha8,
↪  alpha9,alpha10)
beta_df = rbind(beta1, beta2, beta3, beta4, beta5, beta6, beta7, beta8, beta9, beta10)
```

```
# combine the alpha and beta data frame
alpha_beta_df = data.frame(alpha_df, beta_df,
                           row.names = c("NoDur", "Durbl", "Manuf", "Enrgy", "HiTec",
                           ↪  "Telcm", "Shops", "Hlth", "Utils", "Other"))
# perform the table of intercept and slope coefficients
kable(alpha_beta_df,
      # add columns name
      col.names = c("alpha - Intercept", "beta - Slope"),
      # add caption for the table
      caption = "Intercept and Slope Coefficients for Ten Industry Portfolios",
      # adjust the column alignment to center
      align = "c")
```

Part 2 - Question 1 Code

```
# compute the mean return for 10 industry portfolios
portfolio_mean = colMeans(industry_df)
# compute the mean return for market portfolio
market_mean = colMeans(market_df)
# combine the mean return for industry portfolio and market portfolio into one data
↪  frame
capm_return_df = c(portfolio_mean, market_mean)
# extract the value of beta from part 1
# the beta for market portfolio is 1
beta_df = c(alpha_beta_df$beta_df, 1)
# combine the mean return and beta of portfolios into one data frame
capm_df = data.frame(capm_return_df, beta_df)
# change the column name of data frame
colnames(capm_df) = c("return", "beta")
```

```
# regress the return and beta of portfolio
sml_model = lm(return ~ beta, data = capm_df)
# compute the beta value for sml
risk_sml = seq(0, 2, by = 0.01)
# extract the intercept (alpha) for the sml model
alpha_sml = rep(summary(sml_model)$coefficients[1], times = length(risk_sml))
# extract the slope (beta) for sml model
beta_sml = summary(sml_model)$coefficients[2]
# compute the return for sml
return_sml = t(alpha_sml + beta_sml %*% risk_sml)
# combine the beta and return for sml into one data frame
# prepare for the plot
plt_df = data.frame(risk_sml, return_sml)
```

```
# add label to industry portfolio and market portfolio respectively
point_df = capm_df %>%
  mutate(label = c("Industry", "Industry", "Industry", "Industry", "Industry",
  ↪  "Industry", "Industry", "Industry", "Industry", "Industry", "Market"))
```

```
# plot the security market line and plot the position of each portfolio
plt_df %>%
  # beta as x-axis and return as y-axis
  ggplot(aes(x = risk_sml, y = return_sml)) +
```

```
# plot as line figure
geom_line() +
# plot the position of each portfolio as point figure
geom_point(data = point_df, aes(x = beta, y = return, color = factor(label))) +
# use the minimalistic theme with no background annotations
theme_minimal() +
# add title to figure
labs(caption = str_c("Figure 1. Security Market Line with the Position of 10
↪  Industry Portfolios and Market Portfolio"),
     # add x label of beta
     x = "beta - Systematic Risk",
     # add y label of return
     y = "Expected Return of Portfolio",
     # add legend title
     color = "Portfolios")
```