QF600 Asset Pricing Homework 4

Stochastic Discount Factor

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Contents

Suppose that consumption growth has log normal distribution with the possibility of rare disasters:

$$\ln \tilde{g} = 0.02 + 0.02\tilde{\epsilon} + \tilde{\nu}$$

Here ϵ is a standard normal random variable, while ν is an independent random variable that has value of either zero (with probability of 98.3%) or ln(0.65) (with probability of 1.7%).

Simulate ϵ with (at least) 10ˆ4 random draws from a standard normal distribution, and simulate ν with (at least) 10ˆ4 random draws from a standard uniform distribution.

Use the simulated distribution of consumption growth to find the simulated distribution of the pricing kernel for power utility:

$$\tilde{M} = 0.99\tilde{g}^{-\gamma}$$

Repeat this process for all values of γ in the range from 1 to 4, in increments of 0.1 (or less). (Note that you can reuse the same simulated distribution of consumption growth for all values of γ).

Question 1: Calculate μ_M and σ_M for each value of γ, and plot σ_M/μ_M (on the vertical line) vs γ (on the horizontal line)

Table 1: Mean and Standard Deviation of Pricing Kernel for First Ten Relative Risk Aversion

Mean of Pricing Kernel	Standard Deviation of Pricing Kernel
0.9804314	0.0735210
0.9804243	0.0736090
0.9804172	0.0736970
0.9804101	0.0737851
0.9804030	0.0738732
0.9803959	0.0739613
0.9803888	0.0740494
0.9803818	0.0741376

Mean of Pricing Kernel	Standard Deviation of Pricing Kernel
0.9803747	0.0742259
0.9803676	0.0743141

Table 1 shows the mean and standard deviation of pricing kernel for the first ten value of relative risk aversion.

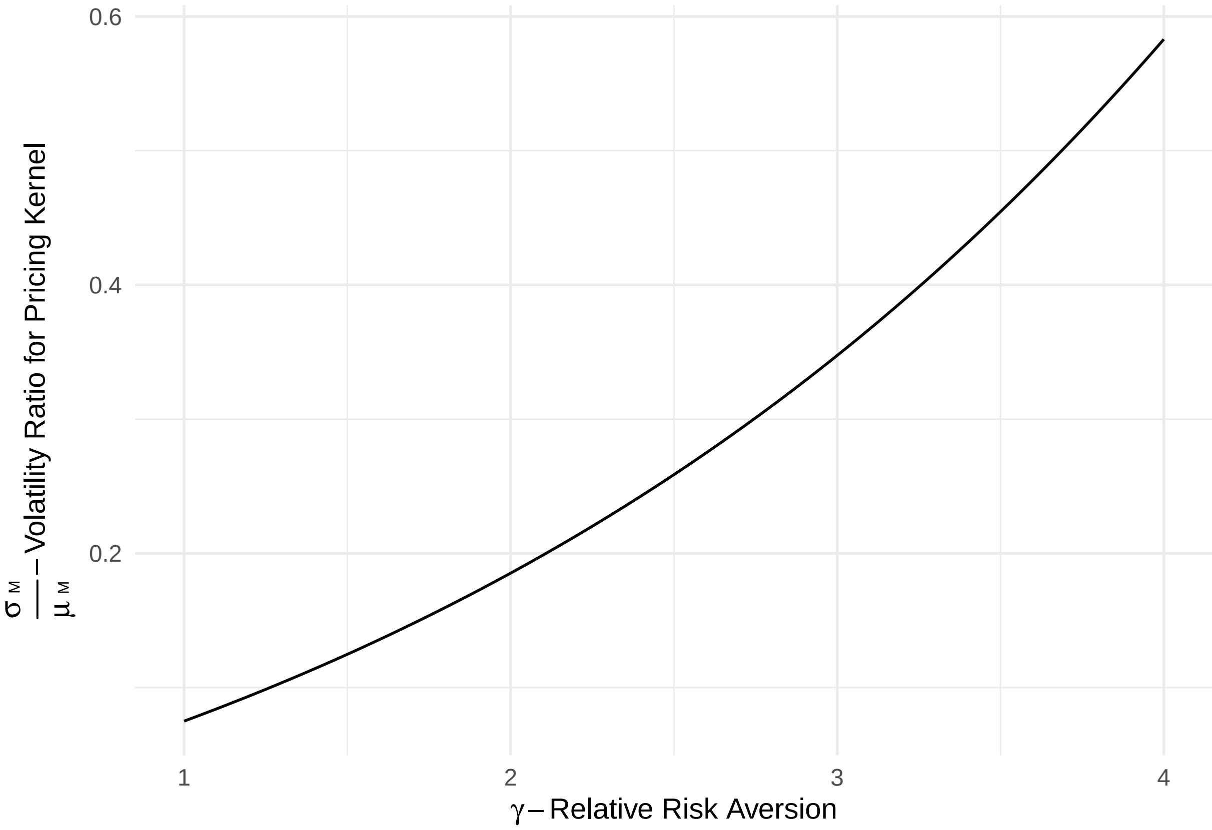

Figure 1: The Plot for Volatility Ratio for Pricing Kernel and Relative Risk Aversion

According to **Figure 1**, which shows the volatility ratio for pricing kernel with the different value of γ relative risk aversion.

Question 2: Find the smallest value of γ (in your data) for which $\frac{\sigma_M}{\mu_M}$ > 0.4, so that the Hansen-Jagannathan bound is satisfied. Explain the economic significant of the results.

After we perform the computation for volatility ratio for pricing kernel again, and then **we could conclude that the smallest value of γ for which $\frac{\sigma_M}{\mu_M} > 0.4$ is 3.257.** The result of minimum value of γ for which, $\frac{\sigma_M}{\mu_M}$ is greater than 0.4, is similar with the plot above.

The γ in pricing kernel with power utility represents the relative risk aversion, therefore the goal for minimize the γ is to figure out the smallest value and the acceptable degrees of relative risk aversion as well. Furthermore, the $\frac{\sigma_M}{\mu_M}$ represents the volatility ratio for pricing kernel and by the Hansen-Jagannathan bound, then the volatility ratio cannot less than the Sharpe ratio. As the Sharpe ratio is around 0.4 for U.S. stock market, then the volatility ratio must be greater than 0.4 for the U.S. stock market.

Then the result for question 2, the minimum value for relative risk aversion is 3.257 for the volatility ratio of pricing kernel which greater than the 0.4 of Sharpe ratio for U.S. stock market, which represents the acceptable degree of relative risk aversion is greater or equal to 3.257. As well, that means there is no equity premium puzzle based on the Sharpe ratio of U.S. stock market because there is no unreasonablely high degree of relative risk aversion.

Appendix

Set up Code

```
# import the library
library(tidyverse)
library(ggplot2)
library(knitr)
```

Question 1 Code

```
# set seed in order to get the same result for simulation
set.seed(600)
# number of simulation for epsilon and nu
n_simulation = 10^4
```

```
# create a data frame to save the consumption growth after exponentiation
g_df = data.frame()

# simulate the consumption growth after exponentiation
for (i in 1:n_simulation){
  # simulate epsilon from standard normal distribution
  epsilon = rnorm(1, mean = 0, sd = 1)
  # simulate probability of rare disaster occurs from standard uniform distribution
  random_nu = runif(1, 0, 1)
  # with probability of 1.7% for rare disaster occurs
  if (random_nu <= 0.017){
    # the effect of rare disaster for consumption growth is log(0.65)
    nu = log(0.65)
  }
  # with probability of 98.3% for rare disaster does not occurs
  else{
    # the effect of rare disaster for consumption growth is 0
    nu = 0
  }
  # compute consumption growth after exponentiation
  random_g = exp(0.02 + 0.02 * epsilon + nu)
  # save them into the data frame
```

```r
  g_df[i, 1] = random_g
}
```

```r
# set up the given value for relative risk aversion
gamma = seq(1, 4, 0.001)
# create a data frame for mean of pricing kernel
mean_m = data.frame()
# create a data frame for standard deviation for pricing kernel
sd_m = data.frame()
# compute pricing kernel with each value of relative risk aversion
for (i in gamma){
  M = 0.99 * g_df ^ (-i)
  mean_m[nrow(mean_m) + 1, 1] = colMeans(M)
  sd_m[nrow(sd_m) + 1, 1] = apply(M, 2, sd)
}
# compute the volatility ratio for pricing kernel
ratio_m = sd_m / mean_m
```

```r
# the mean of pricing kernel for the first ten relative risk aversion
mean_df = head(mean_m, 10)
# the standard deviation of pricing kernel for the first ten relative risk aversion
sd_df = head(sd_m, 10)
# put the mean and standard deviation of pricing kernel into one data frame
table_df = data.frame(mean_df, sd_df)
# show the table of mean and standard deviation of pricing kernel for the first ten
↪  relative risk aversion
kable(table_df,
      col.names = c("Mean of Pricing Kernel", "Standard Deviation of Pricing Kernel"),
      ↪
      caption = "Mean and Standard Deviation of Pricing Kernel for First Ten Relative
      ↪  Risk Aversion")
```

```r
# save the volatility ratio for pricing kernel and relative risk aversion into one
↪  data frame
plot_df = data.frame(ratio_m, gamma)
# rename the column of data frame
colnames(plot_df) = c("ratio_M", "gamma")
```

```r
# plot the volatility ratio for pricing kernel verus relative risk aversion
plot_df %>%
  # relative risk aversion on x-axis and volatility ratio for pricing kernel on y-axis
  ggplot(aes(x = gamma, y = ratio_M)) +
  # connect the observation in the order in data frame
  geom_path() +
  # use the minimalistic theme for the plot
  theme_minimal() +
  # add x label
  xlab(expression(gamma - "Relative Risk Aversion")) +
  # add y label
  ylab(expression(frac(sigma~scriptscriptstyle(M), mu~scriptscriptstyle(M)) -
  ↪  "Volatility Ratio for Pricing Kernel"))
```

Question 2 Code

```r
# set up the given value for relative risk aversion
gamma = seq(1, 4, 0.001)
# create a data frame for the relative risk aversion with volatility ratio is greater
↪  than 0.4
HJ_df = data.frame()

# compute pricing kernel with each value of relative risk aversion
for (i in gamma){
  M = 0.99 * g_df ^ (-i)
  # if the volatility ratio for pricing kernel is greater than 0.4
  if ((apply(M, 2, sd) / colMeans(M)) > 0.4){
    # then put the gamma into the data frame
    HJ_df[nrow(HJ_df) + 1, 1] = i
  }
}
# the smallest value for relative risk aversion with volatility ratio for pricing
↪  kernel is greater than 0.4
min(HJ_df)
```