IM316 Management Science - homework 04

1.

Reboot, Inc., is a manufacturer of hiking boots. Demand for boots is highly seasonal. In particular, the demand in the next year is expected to be 3,000, 4,000, 8,000, and 7,000 pairs of boots in quarters 1, 2, 3, and 4, respectively. With its current production facility, the company can produce at most 6,000 pairs of boots in any quarter. Reboot would like to meet all the expected demand, so it will need to carry inventory to meet demand in the later quarters. Each pair of boots sold generates a profit of $20 per pair. Each pair of boots in inventory at the end of a quarter incurs $8 in storage and capital recovery costs. Reboot has 1,000 pairs of boots in inventory at the start of quarter 1. Reboot’s top management has given you the assignment of doing some spreadsheet modeling to analyze what the production schedule should be for the next four quarters and make a recommendation.

Question

1. Formulate and solve a linear programming model for this problem on a spreadsheet.

Assume that Q1 is the amount of boots in quarter 1 producted, Q2 is the amount of boots in quarter 2 producted, Q3 is the amount of boots in quarter 3 producted, and Q4 is the amount of boots in quarter 4 producted

Max Z=20\*(Q1+Q2+Q3+Q4)-8(1000+I1+I2+I3)

St.

Q1-I1=2000

Q2+I1-I2=4000

Q3+I2-I3=8000

Q4+I3-I4=7000

Q1, Q2, Q3, Q4 <= 6000

Q1, Q2, Q3, Q4 >= 0, int

Solver:

一張含有 文字, 螢幕擷取畫面, 數字, 軟體 的圖片

自動產生的描述

The final result:

一張含有 文字, 螢幕擷取畫面, 數字, 行 的圖片

自動產生的描述

=> Q1:3000 pairs、Q2:6000 pairs、Q3:6000 pairs、Q4:6000 pairs

2.

The Northern Airplane Company builds commercial airplanes for various airline companies around the world. The last stage in the production process is to produce the jet engines and then to install them (a very fast operation) in the completed airplane frame. The company has been working under some contracts to deliver a considerable number of airplanes in the near future, and the production of the jet engines for these planes must now be scheduled for the next four months. To meet the contracted dates for delivery, the company must supply engines for installation in the quantities indicated in the second column of table below. Thus, the cumulative number of engines produced by the end of months 1, 2, 3, and 4 must be at least 10, 25, 50, and 70, respectively.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  |  | Maximum Production | | Unit Cost of Production ($million) | | Unit Cost of Storage |
| Month | **Scheduled Installations** | **Regular Time** | **Overtime** | **Regular Time** | **Overtime** | **($thousand)** |
| 1 | 10 | 20 | 10 | 1.08 | 1.10 | 15 |
| 2 | 15 | 30 | 15 | 1.11 | 1.12 | 15 |
| 3 | 25 | 25 | 10 | 1.10 | 1.11 | 15 |
| 4 | 20 | 5 | 10 | 1.13 | 1.15 |  |

The facilities that will be available for producing the engines vary according to other production, maintenance, and renovation work scheduled during this period. The resulting monthly differences in the maximum number of engines that can be produced during regular time hours (no overtime) are shown in the third column of table above, and the additional numbers that can be produced during overtime hours are shown in the fourth column. The cost of producing each one on either regular time or overtime is given in the fifth and sixth columns. Because of the variations in production costs, it may well be worthwhile to produce some of the engines a month or more before they are scheduled for installation, and this possibility is being considered. The drawback is that such engines must be stored until the scheduled installation (the airplane frames will not be ready early) at a storage cost of $15,000 per month (including interest on expended capital) for each engine1 , as shown in the rightmost column of table above. The production manager wants a schedule developed for the number of engines to be produced in each of the four months so that the total of the production and storage costs will be minimized.

Question

1. Formulate and solve a linear programming model for this problem on a spreadsheet.

Min Z = 1080\*x1+1110\*x2+1100\*x3+1130\*x4+1100\*y1+1120\*y2+1110\*y3+1150\*y4+15\*z1+

15\*z2+15\*z3

Subject to.

x1+y1-z1>=10

x2+y2+z1-z2>=15

x3+y3+z2-z3>=25

x4+y4+z3=20

x1 <= 20、x2 <= 30、x3 <= 25、x4 <= 5

y1 <= 10、y2 <= 15、y3 <= 10、y4 <= 10

xi, yi, zi >= 0, i=1~4

Solver:

一張含有 文字, 螢幕擷取畫面, 數字, 軟體 的圖片

自動產生的描述

The final result(單位為 thousands):

一張含有 文字, 螢幕擷取畫面, 數字, 行 的圖片

自動產生的描述

3.

A water theme park plays seven games depending on the demand during weekdays and weekends. The park earns revenues (in thousands of euros) from playing these games and the revenue varies with the days. Certain games generate more revenue during weekends and certain games do not yield any revenue. The park analyzed its revenue data and prepared a revenue matrix as follows (values in euros):

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Games | Monday | Tuesday | Wednesday | Thursday | Friday | Saturday | Sunday |
| A | 2 | 2 | 0 | 3 | 8 | 7 | 4 |
| B | 2 | 0 | 0 | 3 | 5 | 5 | 5 |
| C | 2 | 0 | 0 | 3 | 7 | 4 | 5 |
| D | 0 | 0 | 0 | 2 | 8 | 5 | 6 |
| E | 0 | 0 | 1 | 0 | 5 | 8 | 8 |
| F | 0 | 0 | 1 | 0 | 6 | 4 | 8 |
| G | 0 | 2 | 1 | 3 | 5 | 4 | 8 |

Besides, the machines used for these games require maintenance and have other costs associat with them. The park realizes the games F and G can be played at most 2 times a week; while other games can be played up to three times a week. On any given day, a game cannot be play more than once. The park also plans to play a maximum of 2 games daily from Monday to Thursday to amuse the few visitors during weekdays. During the weekend (Friday, Saturday, Sunday), to amuse the large number of visitors, the park plays a maximum of 5 games. The park wants to know which game should be played on which days to maximize revenue.

Question

1. Formulate and solve a linear programming model for this problem on a spreadsheet.

Assume that

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Games | Monday | Tuesday | Wednesday | Thursday | Friday | Saturday | Sunday |
| A |  |  |  |  |  |  |  |
| B |  |  |  |  |  |  |  |
| C |  |  |  |  |  |  |  |
| D |  |  |  |  |  |  |  |
| E |  |  |  |  |  |  |  |
| F |  |  |  |  |  |  |  |
| G |  |  |  |  |  |  |  |

Max Z=2\*X\_A1+2\*X\_A2+0\*X\_A3+3\* X\_A4+8\* X\_A5+7\* X\_A6+4\* X\_A7+2\*

X\_B1+0\* X\_B2+0\* X\_B3+3\* X\_B4+5\* X\_B5+5\* X\_B6+5\* X\_B7+2\*

X\_C1+0\* X\_C2+0\* X\_C3+3\* X\_C4+7\* X\_C5+4\* X\_C6+5\* X\_C7+0\*

X\_D1+0\* X\_D2+0\* X\_D3+2\* X\_D4+8\* X\_D5+5\* X\_D6+6\* X\_D7+0\*

X\_E1+0\* X\_E2+1\* X\_E3+0\* X\_E4+5\* X\_E5+8\* X\_E6+8\* X\_E7+0\*

X\_F1+0\* X\_F2+1\* X\_F3+0\* X\_F4+6\* X\_F5+4\* X\_F6+8\* X\_F7+0\*

X\_G1+2\* X\_G2+1\* X\_G3+3\* X\_G4+5\* X\_G5+4\* X\_G6+8\* X\_G7

St.

X\_F1+X\_F2+X\_F3+X\_F4+X\_F5+X\_F6+X\_F7 <=2

X\_G1+X\_G2+X\_G3+X\_G4+X\_G5+X\_G6+X\_G7 <=2

X\_A1+X\_A2+X\_A3+X\_A4+X\_A5+X\_A6+X\_A7 <=3

X\_B1+X\_B2+X\_B3+X\_B4+X\_B5+X\_B6+X\_B7 <=3

X\_C1+X\_C2+X\_C3+X\_C4+X\_C5+X\_C6 +X\_C7 <=3

X\_D1+X\_D2+X\_D3+X\_D4+X\_D5+X\_D6 +X\_D7 <=3

X\_E1+X\_E2+X\_E3+X\_E4+X\_E5+X\_E6+X\_E7 <=3

X\_A1+X\_B1+X\_C1+X\_D1+X\_E1+X\_F1+X\_G1<=2

X\_A2+X\_B2+X\_C2+X\_D2+X\_E2+X\_F2+X\_G2<=2

X\_A3+X\_B3+X\_C3+X\_D3+X\_E3+X\_F3+X\_G3<=2

X\_A4+X\_B4+X\_C4+X\_D4+X\_E4+X\_F4+X\_G4<=2

X\_A5+X\_B5+X\_C5+X\_D5+X\_E5+X\_F5+X\_G5<=5

X\_A6+X\_B6+X\_C3+X\_D6+X\_E6+X\_F6+X\_G6<=5

X\_A7+X\_B7+X\_C4+X\_D7+X\_E7+X\_F7+X\_G7<=5

Xij=0 or 1, i=A~G, j=1~7

Solver:

一張含有 文字, 螢幕擷取畫面, 軟體, 陳列 的圖片

自動產生的描述

The final result:

一張含有 文字, 螢幕擷取畫面, 數字, Rectangle 的圖片

自動產生的描述

4.

The city of Salem has four police stations, with the following inputs and outputs:

Input 1 = number of police officers

Input 2 = number of patrol vehicles

Input 3 = space (100s ft.)

Output 1 = calls responded to (100s)

Output 2 = traffic citations (100s)

Output 3 = convictions

The monthly output and input data for each station are

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | Outputs | | | Inputs | | |
| Police Station | **1** | **2** | **3** | **1** | **2** | **3** |
| A | 12.7 | 3.6 | 35 | 34 | 18 | 54 |
| B | 14.2 | 4.9 | 42 | 29 | 22 | 62 |
| C | 13.8 | 5.2 | 56 | 38 | 16 | 50 |
| D | 15.1 | 4.2 | 39 | 35 | 24 | 57 |

Question

1. Formulate and use Solver to help the city council determine which of the police stations are relatively inefficient.

Police A:input:34\*y1+18\*y2+54\*y3、output:12.7\*x1+3.6\*x2+35\*x3

* 12.7\*x1+3.6\*x2+35\*x3 <= 34\*y1+18\*y2+54\*y3

Police B: input:29\*y1+22\*y2+62\*y3、output:14.2\*x1+4.9\*x2+42\*x3

* 14.2\*x1+4.9\*x2+42\*x3 <= 29\*y1+22\*y2+62\*y3

Police C: input:38\*y1+16\*y2+50\*y3、output:13.8\*x1+5.2\*x2+56\*x3

* 13.8\*x1+5.2\*x2+56\*x3 <= 38\*y1+16\*y2+50\*y3

Police D: input:35\*y1+24\*y2+57\*y3、output:15.1\*x1+4.2\*x2+39\*x3

* 15.1\*x1+4.2\*x2+39\*x3 <= 35\*y1+24\*y2+57\*y3

The efficiency of police A:

Max Z=12.7\*x1+3.6\*x2+35\*x3

Subject to.

34\*y1+18\*y2+54\*y3=1

(12.7\*x1+3.6\*x2+35\*x3) - (34\*y1+18\*y2+54\*y3) <= 0

(14.2\*x1+4.9\*x2+42\*x3) - (29\*y1+22\*y2+62\*y3) <= 0

(13.8\*x1+5.2\*x2+56\*x3) – (38\*y1+16\*y2+50\*y3) <= 0

(15.1\*x1+4.2\*x2+39\*x3) – (35\*y1+24\*y2+57\*y3) <= 0

Xi, yi >= 0, i=1~3

Solver:

一張含有 文字, 螢幕擷取畫面, 數字, 軟體 的圖片

自動產生的描述

The final result:

一張含有 文字, 螢幕擷取畫面, 數字, 字型 的圖片

自動產生的描述

The efficiency of police B:

Max Z=14.2\*x1+4.9\*x2+42\*x3

Subject to.

29\*y1+22\*y2+62\*y3=1

(12.7\*x1+3.6\*x2+35\*x3) - (34\*y1+18\*y2+54\*y3) <= 0

(14.2\*x1+4.9\*x2+42\*x3) - (29\*y1+22\*y2+62\*y3) <= 0

(13.8\*x1+5.2\*x2+56\*x3) – (38\*y1+16\*y2+50\*y3) <= 0

(15.1\*x1+4.2\*x2+39\*x3) – (35\*y1+24\*y2+57\*y3) <= 0

Xi, yi >= 0, i=1~3

Solver:

一張含有 文字, 螢幕擷取畫面, 軟體, 數字 的圖片

自動產生的描述

The final result:

一張含有 文字, 螢幕擷取畫面, 數字, 字型 的圖片

自動產生的描述

The efficiency of police C:

Max Z=13.8\*x1+5.2\*x2+56\*x3

Subject to.

38\*y1+16\*y2+50\*y3=1

(12.7\*x1+3.6\*x2+35\*x3) - (34\*y1+18\*y2+54\*y3) <= 0

(14.2\*x1+4.9\*x2+42\*x3) - (29\*y1+22\*y2+62\*y3) <= 0

(13.8\*x1+5.2\*x2+56\*x3) – (38\*y1+16\*y2+50\*y3) <= 0

(15.1\*x1+4.2\*x2+39\*x3) – (35\*y1+24\*y2+57\*y3) <= 0

Xi, yi >= 0, i=1~3

Solver:

一張含有 文字, 螢幕擷取畫面, 軟體, 數字 的圖片

自動產生的描述

The final result:

一張含有 文字, 螢幕擷取畫面, 數字, 字型 的圖片

自動產生的描述

The efficiency of police D:

Max Z=15.1\*x1+4.2\*x2+39\*x3

Subject to.

35\*y1+24\*y2+57\*y3=1

(12.7\*x1+3.6\*x2+35\*x3) - (34\*y1+18\*y2+54\*y3) <= 0

(14.2\*x1+4.9\*x2+42\*x3) - (29\*y1+22\*y2+62\*y3) <= 0

(13.8\*x1+5.2\*x2+56\*x3) – (38\*y1+16\*y2+50\*y3) <= 0

(15.1\*x1+4.2\*x2+39\*x3) – (35\*y1+24\*y2+57\*y3) <= 0

Xi, yi >= 0, i=1~3

Solver:

一張含有 文字, 螢幕擷取畫面, 軟體, 數字 的圖片

自動產生的描述

The final result:

一張含有 文字, 螢幕擷取畫面, 數字, 字型 的圖片

自動產生的描述

Because efficiency of police A is the lowest, so the police A is relatively inefficient.