Strategic Voting in TV Game Show "*The Weakest Link*"

By

Haishan Yuan

hyua0289@usyd.edu.au

Supervisors: Kunal Sengupta and Hajime Katayama

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Abstract

This paper utilizes the data generated from Television game shows *The Weakest Link* to study real life strategic voting with large stake payoffs. Theoretical models are built to analyze the three-player voting possibly with second move of tie breaker. With reasonable specifications, equilibria can be constructed to support all observed outcomes. When the game structure is relatively simple with single possible tie breaker, the empirical results are in line with theoretical implications. When the tie breaker is uncertain, the predictability of basic game theory analysis is largely compromised. Regression results suggest that, with presence of increased complexity and uncertainty in the voting game, naive coordination strategies are instead adapted. Strategic untruthful performance is found as a source of inaccurate prediction of theory.

Keywords: The Weakest Link, Strategic voting, Strategic uncertainty, experimental game theory.

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1 Introduction

Though Reinhard Selten, one of laureates who shared the Nobel prize in Economics 1994, stated that "Game theory is for proving theorems, not for playing games.", regarding game theory as normative theory about what rational players would do, rather than a positive theory about facts. Works on testing and advancing game theory as a predicting tool has been numerous. Game theory as a decision theory in social science is expected to have close reality relevance with human behavior and is demanded to be applicable in a considerable level of precision. However, due to the complexity of real life situation, the study of applicability of game theory as a predicting tool in the real life is largely constrained. The burgeoning experimental methodology in game theory exploits the advantages of exact control and discretionary design of games as in laboratory environment. However, due to the limiting financial resources available for the experimental studies, stakes and payoff levels in this class of study are normally small. As in Kahneman and Tversky (1979), under uncertainty, decision making is rather reference point dependent. Payoffs are related to agents' initial positions when decisions are to be made. Also, by calibrating risk-aversion magnitudes when stakes are small and moderately large in an expected utility framework, Rabin (2000) showed, the conventional practices of the expected utility theory could lead to absurd results. On the ground that essentially all decision makings are associated with uncertainty, the stake sizes in a game could have significant influence on how people actually play games.

The Weakest Link is a series of Television game shows providing nice sources for study in people's strategic behavior in real life situation with large stake rewards. In such quiz-like game shows with strategic voting, the rules are well familiar to the contestants and the incentive of strategic playing is strong, particularly in the final voting round that three contestants are left, given that only one contestant can be rewarded to the final prize after all. Beside large stakes involvement, TV game shows as natural experimental study for game theory also has the advantage that the participants are generated from a wider, hence more representative, population, unlike many experimental studies conducted around educational and research institutes and subjects pools are limited to some narrower groups such as college students.

This paper focuses on the last voting round in which three players vote to decide which two are going to stay and compete for the final prize. At this stage, information for decision making is available to certain amount and incentive for strategic voting is at its summit. Theoretical models with various specifications on information symmetry, possible revenge motive and tie breaker identity, are built to analyze the voting behavior. Although all three possible voting outcomes in such voting are observed with significant proportions in the data, only the outcome of eliminating the strongest player universally exists in all theoretical specifications. Outcomes such as eliminating the weakest player can be only supported by equilibria in particular specifications with restrictions. In particular, in the simplest specification when information is symmetric and players are free from revenge motive in voting, eliminating the strongest player is the only outcome survives Iterative Elimination of Weakly Dominated Strategy. It is found that the predictions of theoretical models are relatively well when the game is relatively simple and involves less uncertainty. When a larger level of uncertainty complicates the game, players tend to utilize native coordinating strategy so that sophisticated equilibrium analyze loses its power of prediction. A moderate level of strategic pretence is also found, which is attributed as a source of uncertainty that compromises the effectiveness of expected utility maximizing play.

This paper proceeds as following. Next subsection provides a brief survey on related literature. Section two introduces the TV game shows The Weakest Link and provides basic observations and summary statistics on the data sets. Section three constructs theoretical models to analysis the interested three-person voting game. Section four provides empirical evidence on how individuals shape their voting decisions, what factors affect the voting outcome, and a discussion on the discrepancy between theoretical and empirical results. Section five concludes.

1.1 Related literature

There have been several papers studying TV game shows to pursue economic understanding. By studying TV shows "The Price is Right", Bennett and Hickman (1993) and Berk, Hughson and Vandezande (1996), documented the existence of sub-optimal behavior and improvement of strategic effectiveness through learning respectively. Blavatskyy and Pogrebna (2006) tested loss-aversion using the data from the TV shows “Affari Tuoi”. Bertner (1993), Metrick (1995), and Beetsma and Schotman (2001) focused on the revealed risk preference as in TV shows “Card Sharks”, “Jeopardy” and “Lingo”. With U.S. version of The Weakest Link, discrimination is studied by Levitt (2004) and Antonovics, Arcidiancono and Walsh (2005) independently; With data from the British version, Haan et. al (2003) studied the sub-optimal strategy on the so-called banking decisions in the process on cooperative prize money accumulation; Pogrebna (2006) studies the pure strategy Nash plays in all but last round and the mixed strategy in the last voting round. Overconfidence bias and prevalence of proper mixed strategy Nash equilibrium was inferred by posited equilibrium play and successful coordination over multiple equilibria in the last voting round. Févriera and Linnemerb (2006) also studied the three-person voting with data generated from French version of The Weakest Link. Risk dominance and Payoff dominance criteria for equilibrium selection in a coordination game was assessed by modeling the voting as a two-signal one-shot game. However, they failed to generalize with the three-signal situation, which accounts for a major proportion in our data set collected by Levitt (2004). More importantly, their basic assumption regarding current round performance as signals is at odd with our larger data set.

There is a large set of experimental literature on whether and how well individual plays equilibrium, subject to refinement criteria if applicable. The conclusions critically depend on the structure of the games. Laboratory results in Costa-Gomes, Crawford and Broseta (2001) illustrated that in the simple one-shot 3×2 normal form games, which have complete information and were dominant solvable, around 90% of time players managed to play dominant strategy immediately available, 60% of time players succeeded to play the dominant strategy after one round's elimination of dominated strategy by other's, and figure decreased to 20% if dominant strategy stands out after two rounds of iterative deletions of dominated strategy. In Stahl and Wilson (1994), 60% of time responses in the 3×3 symmetric normal form games with unique pure strategy equilibrium complied with Nash equilibrium prediction. In the games with similar structures but unique mixed strategy equilibrium, the observed distribution of actions differs from the mixed equilibrium implied distribution at 5% level of significance. The Nash prediction however is quite poor when it comes to the class of games known as "beauty contest", special those gave zero equilibrium value according to backward deduction. A "beauty contest" game is for example a game of two players simultaneously choosing a number between 0 and 100. The one who chose the number closest to 2/3 of the average wins a fixed prize. Backward deduction would predict a unique corner solution that zero is chosen by both in equilibrium. However, laboratory results summarized by Camerer, Ho and Chong (2004) indicated that observed actions substantially deviated from the predicted value. In the same paper, it was shown that games with higher stake in general had closer distance between observed and predicted outcome, indicating a cost-benefit analysis for cognitive effort and confirming the significance of large stake involvement in experimental study. In general, the more complicated the game structure was, the less likely the players played as predicted by equilibrium.

In modeling the strategic voting of The Weakest Link in the last voting game in which three players are left, the payoffs are uncertain. Although the payoff for the final winner is approximately known, the outcome directly associated with the strategy voting is not. The voting is to determine who is going to be eliminated from the game and which pair of players is left to compete for the final prize. Since one can only estimate the probability of winning when one competes against the other, payoffs are largely ranked and not precise. Given that the subjective estimate of payoff is based on signals received by players, changes in the strength of signals may have large impact on the strategic behavior. This is suggested by the experimental results from Goeree and Holt (2001), in which it was shown that the payoff structure could dramatically alter the observed outcome even if the unique equilibrium or the unique equilibrium survived appropriate refinement concepts is unchanged. Also, Costa-Gomes, Crawford and Broseta (2001) provided evidence that subject's capability to play simple equilibrium in the laboratory environment could be reduced moderately if the payoffs were to be looked up by simple mouse clicking individually without time and frequency constraint, instead of common practice of presenting all payoffs in a matrix within a screen for straight forward comparison. Hence, it is interesting to see how well players play equilibrium in such game with ranked payoffs.

Decisions in The Weakest Link were also subject to framing effect, as discussed in

Févriera and Linnemer (2006). The name of the TV game shows, The Weakest Link, implicitly pointed to the direction of voting against the weakest player. This suggestion, however, may not be self-enforceable. Voting off a weak candidate means one is to be matched with a strong contestant. It lowers one's chance to gain the winner-take-all prize. Though, it is plausible that matching with a stronger candidate could increase the money pool more in the post-voting round, such gain may not large enough to compensate the drawback in low winner chance. Brandts and MacLeod (1995) experimentally tested the stability of equilibrium with various concepts, when it is subject to recommended strategy. With present of dominant strategy, players in their experiment did not player weakly dominated strategy, regardless the recommendations. However, when there was strategic uncertainty, players did not always follow the recommended strategy, even if it pointed to a Pareto-superior equilibrium. Nevertheless, recommendation did help coordination sometime. Given that effect of recommendation is indecisive and inconclusive in game with strategic uncertainty, I hope that this paper would provide some insight on strategic behavior when implicit suggestion presents.

2 The Weakest Link TV shows

2.1 Format and Rules

The data set used in this thesis was generated from the syndicated version of The Weakest Link broadcasted by NBC in the U.S.A. It featured with six contestants at the beginning of a game show. The game consisted of six rounds. Each of first four rounds was divided into two phrases, prize accumulation phrase and voting phrase. In the prize accumulation phrase, the contestants cooperated to build up the money pot for the final winner. They alternatively gave answers to general quiz questions. A correct answer constructed a "link" in the "money chain", in which amount of money grown exponentially. Once a so-called Banking decision was made, the amount of money in the money chain would be added to the money pot reserved for the final winner and a new money chain would be constructed. Each contestant had the chance to make Banking decision before s/he attempted the quiz question, simply by saying "Bank". However, an incorrect attempt destroyed the whole money chain and a new chain needed to be constructed from next attempt. Thus, the cooperation of building up a high amount of prize for the final winner required correct attempts to the questions and good timing of bank decisions.

After the prize accumulation phrase within a time frame, a voting phrase followed. Each player remained in that round cast a vote to vote off a contestant. The one received maximum votes would be eliminated from the game and got no monetary reward. In case that more than one had maximum number of votes, a tie breaker rule applied. The one who had maximum number of correct attempts in that round should be the tie breaker. In case that this rule did not give unique identification, the one with minimum attempts, among those who had maximum correct attempts, gained this tie-breaking privilege. If the tie-breaker identity was still undetermined, it was resorted to the banking decisions, in favor of the one who banked highest amount of money in total in that round. The last resort was randomization. In the final voting round with three players left, the first two criterions is equivalent to choosing the one with maximum correct rate, given that in the data set, the maximum and minimum number of attempts were four and two respectively, and they did not coexist in the same game. Such tie breaker was called the *Strongest Link*. Correspondingly, the opposite player was called the Weakest Link. The *Strongest Link* and *Weakest Link* identities were known to audience through an announcer's narration. However, contestant were not told of such identities explicitly, nor any statistics.

After four rounds of voting and elimination, two players were left in the game and there was an additional round, round 5, for prize accumulation without voting. Money accumulated in this round would be triple before being added into the money pool. After this, the two players entered into the final head-to-head round to compete for the prize for final winner. In this round, each player would be asked three questions in alternative fashion. The *Strongest Link* from the last round, or second strongest link if the first strongest link was eliminated, could choose between first move and second move. The one with more correct attempts won. If winner did not come out within the three questions each, the fourth, fifth, sixth question and so on would be asked until the one gave correct answer while the other did not. The winner received all the money accumulated in the game and else received nothing.

The primary data set used in this thesis is the same data set used in Levitt (2004). It contains 136 broadcasted episodes. Variables include the demographic characteristics of each contestant, such as age, gender, occupation, race and geographic origin, numbers of correct answers and total attempts in each round, the votes cast by players in each round with voting, and who was the final winner. A supplementary data set is also used. This is the data set used in Antonovics, Arcidiacono and Walsh (2005) (AAW(2005) hereafter), containing 75 episodes of syndicated version. The episodes in AAW(2005)'s data set are mostly covered in the former one. But it contains additional variable recording what amount of money was banked by each contestant in each round.

In AAW(2005)'s dataset, though the prize for the winner was capped at $75,000, this ceiling was never reached. The average amount of money for the final winner is $8400, with minimum, maximum and median at $2,750, $23,750 and $7,000. The average money pot accumulated up to the voting phrase in round 4 (last voting round) is $7,323. The average increment of money pot after all voting is $1,077.

2.2 Myopic or not?

To analyze the strategic behavior in the last voting round, the first question might be, to what extent were players able to infer the strength, i.e. capability to give correct answers, of her/his opponents from past performance, so that one can estimate the payoffs associated with different outcomes. In Févriera and Linnemer (2006), in which the French version of The Weakest Link was studied, players were presumed to be myopic, i.e. have imperfect recall on other contestants' past performance. The conclusion that players can only recall the performance of contestants' current round performance was drawn by the multinomial logit regressions. These regressions modeled the probability of a candidate being voted off in a voting as a function of the candidate's correct rates of attempts in each of current and previous rounds, as well as the candidate's age and gender. Févriera and Linnemer (2006) found that only current round performance significantly affect the voting behavior, except in Round 3 where the correct rates in the previous rounds were also determining factor. However, this is not the case with our regression results as we will discuss.

To examine whether players are myopic, it is better to model the individual voting decisions than the voting outcomes (i.e., which candidate was voted off). Assume that the utility of voting against a particular candidate is a linear function of the covariates and the error term follows a standard Type I extreme value distribution. This leads to the conditional logit model; the probability of voter i voting against candidate j can be expressed as:

$$p_{ij} = \frac{exp(x_j\alpha + z_j\beta)}{\sum_{k=1}^{n} exp(x_k\alpha + z_k\beta)} \tag{1}$$

where x_j is a vector of candidate j's correct rates in the current and past round(s), z_j is a vector of variables controlling for candidate j's demographic characteristics such as education level, occupation, age, gender, and race. For each round, equation (1) is estimated by the maximum likelihood method.

Estimation results are reported in Table 1. From Round 1 to Round 3 all current and past correct rates of candidates are significant at the one percent levels. This contrasts with evidence provided by Févriera and Linnemer (2006). In Round 4, unlike Round 1 to Round 3, only current round correct rates are significant at the one percent level. To further examine whether correct rates in previous rounds did not affect the individual voting decisions in Round 4, the likelihood ratio test is conducted. The null hypothesis that all coefficients of correct rates in previous rounds are zero is not rejected at 10% level of significance. However, it is quite implausible that players suddenly cannot recollect the past performance when s/he comes to Round 4. The insignificance of correct rates in previous rounds in Round 4 could be due to the strategic behavior. Incentive to vote off the stronger candidate was strongest in this final voting round, while the consideration of keeping strong contestants in game to help accumulate prize pool could dominate in previous rounds. Also, since a single vote is much less influential in previous rounds given a large number of voters, it is difficult for players to coordinate to vote off a strong candidate even if they have strong incentive to do so. Thus, the framing effect as in the TV show's name "The Weakest Link" would have heavily shaped the voting decisions.

The divergent conclusions between Févriera and Linnemerb (2006) and this thesis can be attributed to two reasons. Firstly, in the French version of The Weakest Link studied by Févriera and Linnemerb (2006), there are nine contestants at the first round and hence seven rounds of elimination in total. There are three more rounds than in the U.S. version. So given in the same order of round, say Round 3, three more correct rates in each previous round were to be recalled. Such increased requirement on working memory could lead to imperfect recall for contestants in the French version. Secondly, the findings in Févriera and Linnemerb (2006) may be due to a small sample. Their dataset recorded 36 episodes aired in France. Since they model the voting outcomes, only 36 observations were used for each round-specific regression. The degree of freedom becomes even 26 as a consequence of including 9 explanatory variables when they analyzed Round 7 voting. With this small degree of freedom, it is hard to accurately estimate the effects of correct rates in previous rounds. On the other hand, our conditional logit model utilizes 544 voting decisions from 136 episodes, giving 1632 voter-candidate cases. This makes it possible to have more accurate estimates than those in Févriera and Linnemerb (2006).

2.3 Performance index

With the assumption that players were capable of recalling others' past performance, the following performance index formula in Pogrebna (2006) is applied to represent the individual strength to give correct answers.

$$S_i^r = \frac{\sum_{k=1}^{r} w_k x_{ik}}{\sum_{k=1}^{r} w_k y_{ik}} \tag{2}$$

The above performance index is essentially a weighted average of correct rate of individual i as in Round r. x_{ik} is the correct attempts of individual i in Round k. y_{ik} is the total attempts of individual i in Round k. Weight for Round k performance is given by the incorrect rate of all contestants in that round, denoted by w_k. Therefore, this performance index is able to capture players' impression of strength of other contestants.

The calculated performance index up to the last voting in Round 4 then gives unique identities for the three players who survive to this stage, except in two episodes[1] For convenience of subsequent analysis, let "A", "B", and "C" denote the player with highest, median and lowest performance index respectively. Table 2 gives summary statistics on this

[1] In each of these two episodes, there are two players with identical correct rates in each previous rounds. In the subsequent regressions, the (ordinal) identities among these two players are randomly assigned.

performance index as in the end of Round 4.

2.4 Voting results and tie breaker identity

In the last voting round of interest, three players cast their votes simultaneously. If they vote randomly with equal probability, one would expect tie happens once in four voting games.[2] The identity of tie breaker is critical to the structure of the voting game. In 82 episodes out of 136 in Levitt's dataset, tie breaker is determined by the primary criteria in the tie breaker identification. That is, in 60% of time, a single player has the maximum correct rate in Round 4. In the data set of Antonovics etc., it is rare that tie breaker identity remains unsolved after consider the money banked by player in that round. However, it seems to unrealistic to assume that contestants are able to remember the money banked by each person. Banking decision needed to be made quickly before the host started a new question, and the value of that answer and the amount of money in the chain was not explicitly stated to the contestants. Unlike the correct rates of other contestants, banking decisions provided little information about one's probability of winning if a particular contestant would be confronted in the final head-to-head[3]. Given the intense environment of the show scene, it is assumed that when two or three contestants had equal maximum correct rates, the assignment of tie breaker identity is perceived to be random to those who got maximum correct rates. The Table 3 gives the distribution of tie breaker identity profile under this assumption in Levitt's data set. As shown in Table 4, the most likely cases are (1) where tie breaker is player A for sure and (2) where tie breaker is player A or player B, presumed to be with equal chance. The former accounts for 40% of time while the latter for 21%. The distribution of voting outcomes in the whole data set (of Levitt) and in various tie breaker identity profiles is presented in the Table 4. Each possible outcome is found to share a significant portion. However, intuitively, one would expect that the strongest player, player A, would have a high chance to be eliminated, which was not the case. In fact, player A has the highest chance to stay, 74%, among all contestants. Since the above two tie breaker situations account for the major share in the dataset, the following section analyzes these

[2] Two voting profiles in $2^3 = 8$ possible voting profiles raise ties.

[3] A simple linear regression does not find any significant correlation of performance index with amount of money banked, nor with number of banking decisions

two situations theoretically.

3 Theoretical analysis

As shown in Table 4, when the tie beaker is known to be player A for sure, voting outcomes of (A, B) match account for the largest proportion, 43%, while (B, C) matches account for the smallest proportion, 23%. When tie breaker can be either player A or player B with equal chance, (A, B) match remains the most likely outcome, accounting for 48%, though (A, C) match now takes place of (B, C) match as the least likely outcome. The following section first provides a baseline model to illustrate the intuitive results that only (B, C) match should be formed. Then, more sophisticated theoretical models are constructed to support the observation that all three outcomes accounts for significant proportions. In particular, equilibria are characterized to supported to most dominant outcomes of (A, B) match.

Suppose there are two types of players, High type (H) and Low type (L). High type players are more capable to give correct answer for a quiz question than Low type players do. Thus, when a High type player competes with a Low type player in the final head-to-head, High type has a better chance to win. Let the probability of a High type beating a Low type in the head-to-head to be $\alpha > 1/2$. Assume that two players with the same type have equal chance to win.

At the final voting round, let the three players left denoted by i, $i = A, B, C$. The strength of player i is denoted by p_i, the probability of player i being High type. As before, player A is perceived to be the strongest player while player C is the weakest, so that

$$p_A \geq p_B \geq p_C \tag{3}$$

Let X be the prize money for the final winner, $U(\cdot)$ be the utility from the prize money for the winner. Given that the prize pool increment in the post-voting round, Round 5, is small, and the proportional increment should not be too sensitive to strengths of survived contestants, we treat X as fixed at the time of voting. $U(0)$ is normalized to zero. Therefore, the expected utility of a High type player to compete with a player with strength p_j is

$$\pi_H(p_j) = [\frac{1}{2}p_j + \alpha(1 - p_j)]U(X) \tag{4}$$

Similarly, the expected utility for Low type is

$$\pi_L(p_j) = [(1 - \alpha)p_j + \frac{1}{2}(1 - p_j)]U(X) \tag{5}$$

In this voting game, each player chooses to vote against one candidate out of two. In case of tie that each player receives one vote, the tie breaker finalizes. Changing from initial vote is allowed. Two cases are considered here. One is for those episodes there is only one player scores maximum correct rate in the round and this unique tie breaker is player A, who is perceived to be the strongest player according to the overall performance in the game. The other is that two players, player A and player B, both score the maximum correct rate in that round. And they are presumed to have equal chance to be revealed as tie breaker when tie happens. The tie breaker situation is common knowledge to all players. Through out the theoretical section, these two tie breaker identity situations are focused for their major share in the data set. The three possible voting outcomes, given by the formation of (A, C) match, (A, B) match and (B, C) match in the head-to-head round, are associated with player B, C, A being eliminated respectively.

3.1 Symmetric information[4]

Assume that the strength distribution is common knowledge. No player has private information regarding his/her own type. Then, player i's expected utility when s/he competes with player j is given by

$$\pi_i(p_j) = p_i\pi_H(p_j) + (1 - p_i)\pi_L(p_j) \tag{6}$$

[4]Proofs of propositions in this section are included in Appendix A.

3.1.1 Baseline models

In the baseline model, players are merely motivated by expected utility maximization. Given that expected utility $\pi_i(p_j)$ is a strictly decreasing function of opponent's strength, p_j, the tie breaker will always choose to eliminate the stronger candidate.

Proposition 1: If tie breaker is player A for sure and $p_A > p_B > p_C$, the only voting outcome survives Iterative Elimination of Weakly Dominated Strategy is that player A will be voted off.

Proposition 2: If it is common knowledge that tie breaker is player A or player B with equal chance and $p_A > p_B > p_C$, the unique voting outcome survives Iterative Elimination of Weakly Dominated Strategy is (B, C) match.

3.1.2 Vengeful tie breaker

Now suppose that tie breakers are all vengeful, i.e. in case of tie, the tie breaker will eliminate the one who just voted against him/her, regardless the associated payoffs. A tie breaker or potential tie breaker awards that s/he will revenge in a tie before s/he votes.

Proposition 3: If it is common knowledge that player A is tie breaker for sure, tie breaker is vengeful, and $p_A > p_B > p_C$, (B, C) match is the unique voting outcome survives Iterative Elimination of Weakly Dominated Strategy.

Proposition 4: If it is common knowledge that player A or player B is tie breaker with equal chance, tie breaker is vengeful, and $\pi_C(p_B) \geq 2\pi_C(p_A)$, (B, C) match is the unique voting outcome survives Iterative Elimination of Weakly Dominated Strategy.

Proposition 5: If it is common knowledge that player A or player B is tie breaker with equal chance, and tie breaker is vengeful, and the following conditions are satisfied:

$$\pi_C(p_B) < 2\pi_C(p_A) \tag{7}$$

$$p_A \geq p_B > p_C \tag{8}$$

(A, C) match and (B, C) match can be supported by pure strategy Nash equilibrium.

Proposition 6: If it is common knowledge that player A or player B is tie breaker with equal chance, tie breaker is vengeful, and $\pi_C(p_B) < 2\pi_C(p_A)$, (A, B) match can be supported by mixed strategy Nash equilibrium.

Thus, (A, C) match and (A, B) match can be only supported by equilibrium when (1) tie breaker is uncertain; (2) all tie breakers are presumed to be vengeful; and (3) the strength profile of the three players satisfies certain condition. In other cases, eliminating player A is the unique outcome. In Proposition 5 & 6, $\pi_C(p_B) < 2\pi_C(p_A)$, is solved to be $(\alpha - \frac{1}{2})(2p_A - p_B - p_C) < \frac{1}{2}$. Thus, higher strengths of player B and player C, and lower strength of player A are in favor of the existence of (A, C) match and (A, B) match in the above specifications. Given that (A, B) match is still the most likely outcome when tie breaker is player A for sure, the following subsection turns to analysis on asymmetric information for less restrictive results.

3.2 Asymmetric information[5]

In this subsection, asymmetric information is introduced and revenge motive is placed in a more general setting. In the previous subsection, players are assumed to be vengeful with probability 0 or 1. Here, suppose there are two types of tie breaker, Vengeful and Non-vengeful. As before, Vengeful players in a tie will switch his/her initial vote to eliminate the player who just voted against him/her, while Non-vengeful players are only interested in maximizing expected utility from prize money. However, vengeful players recognize such emotional constraint imposed by themselves, and maximize expected utility subject to constraints. Suppose that the prior belief of play i being Vengeful type is $\gamma_i \in (0, 1)$, which is a non-increasing function of the strength ratio p_B/p_C for player $i = A$ and strength ratio p_A/p_C for player $i = B$. This specification captures the fact that the higher is the strength ratio of the two candidates, the higher is the expected utility difference from matching with one versus the other. And higher cost of this non-strategic behavior reduces one's incline to revenge. Of cause this includes the special case that γ is just a constant. Let γ_i to be common knowledge. While strength distribution p_i, $i = A, B, C.$, remains common knowledge, players now know their own type, both strength type and Vengeful-or-not type. Suppose one's strength type is distributed independent on his/her tie breaker tie. Firstly, Proposition 7-10 characterize results when revenge motive is absent, i.e. γ_i is restricted to be zero for

[5]Proofs of propositions in this section are included in Appendix B.

$i = A, B$.

Proposition 7: If tie breaker is player A for sure and revenge motive is absent, there are pooling sequential equilibria supporting (A, C) match and (B, C) match.

Proposition 8: If tie breaker is player A for sure and revenge motive is absent, there is no pure strategy sequential equilibrium (A, B) match.

Proposition 9: If tie breaker is player A or player B with equal chance, revenge motive is absent, there are pure strategy sequential equilibria supporting (A, C) match and (B, C) match[6].

Proposition 10: If tie breaker is player A or player B with equal chance, revenge motive is absent, there is mixed strategy sequential equilibria supporting (A, B) match.

Then, more general results with $\gamma_i \in (0, 1)$ are presented in Proposition 11-14.

Proposition 11: If tie breaker is player A for sure, there are pooling Sequential equilibria supporting (A, C) match and (B, C) match.

Proposition 12: If tie breaker is player A for sure, and $\gamma_A \pi_L(p_A) \geq (1 - p_C)\pi_L(0)$, the following strategy profile is a sequential equilibrium supporting (A, C) match and (A, B) match[7]:

1. Non-vengeful player A votes against player B; Vengeful player A votes against player C;

2. Both types of player B votes against player C;

3. High type player C votes against player B; Low type player C votes against player A;

4. when tie $A \to B \to C \to A$ [8] happens, player A chooses to go with player C;

[6] It is possible to construct a mixed strategic equilibrium such that (1) player A, regardless of type, is indifferent between voting against either candidate, and hence randomizes; (2) Low type of player B and player C play proper mixed strategy; (3) High type of player B and player C vote against each other. Such mixed strategy equilibrium supports all three possible outcomes. However, it is believed that players are not able to carry out such delicated calculation of mixed strategy, exspecially time allowed for casting vote is tight in the show scene.

[7] Similar as before, given restriction on parameters, it is possible to construct a mixed strategic equilibrium such that (1) player A of non-vengeful tie, regardless of strength types, votes against player B; (2) player A of vengeful type, regardless of strength types, is indifferent between voting against either candidate, and hence randomizes; (3) Low type of player B and player C play proper mixed strategy; (4) High type of player B and player C vote against each other. Such mixed strategy equilibrium supports all three possible outcomes. However, it is believed that players are not able to carry out such delicated calculation of mixed strategy, exspecially time allowed for casting vote is tight in the show scene.

[8] $i \to j$ indicates that player i votes against player j

5. The associated equilibrium belief is that player C is of Low type, and player B with probability p_B is High type.

6. The off equilibrium belief of player A in tie $A \to C \to B \to A$ is that player C is of High type, and player B's type is unrestricted.

The condition $\pi_L(p_A) \geq (1 - p_C)\pi_L(0)$ seems to be quite restrictive. However, if mixed strategy is allowed for player C, this condition can be relaxed a bit. As long as High type and Low type player C plays mixed strategy in such a way that, conditional on voting against player B, player C's probability of being High type is higher than that of player B ex ante (p_B), other two players will not deviate. Player C remains indifferent since s/he will be still matched to non-vengeful player A. For example, let High type C votes against player B, Low type C vote against player A with probability $q_C/(1-p_C)$, against player B with probability $1 - \frac{q_C}{1-p_C}$. So that conditional on voting against player B, the probability of player C being High type is $p_C/(1 - q_C)$. Let $\frac{p_C}{1-q_C} = p_B + \epsilon$, for some $\epsilon > 0$. Thus, $q_C = \frac{p_B - p_C + \epsilon}{p_B + \epsilon}$ and $\gamma_A \pi_L(p_A) \geq q_C \pi_L(0)$ is required for this equilibrium to exist. In fact, as ϵ converges to zero, it gives the necessary condition for this equilibrium:

$$\gamma_A \pi_L(p_A) > \frac{p_B - p_C}{p_B} \pi_L(0) \tag{9}$$

By looking at the both sides of the above inequality, weakness of player A and player B, and strength of player C would be in favour of the existence of such equilibrium supporting both (A, C) match and (A, B) match.

Proposition 13: Given tie breaker is player A or player B with equal chance, (B, C) match can be supported by pooling sequential equilibrium; (A, C) match can be supported by pooling sequential equilibrium if $\pi_L(p_A) \geq \gamma_B \pi_L(p_B)$.

Proposition 14: Given tie breaker is player A and player B with equal chance, and $(\alpha - \frac{1}{2})(2p_A + 2p_B - 3p_C) \leq \frac{1}{2}$, (A, B) match can be supported by sequential equilibrium with strategy profile that both player A and player B vote against player C, and both types of player C randomize in the same way.

In this equilibrium, one would expect that the higher strength of player A or player B is, the less likely the above condition is satisfied and hence the less likely (A,B) match is observed. In contrast, the higher strength of player C implies higher likelihood for (A, B) match, given all else constant.

In sum, in both symmetric and asymmetric information setting, the theory predicts that (B, C) should always exist in both tie breaker situations. Moreover, in symmetric information setting, if breaker is known to be player A, (B, C) match is the only possible outcome. If tie breaker is player A or player B with equal chance, the theoretical setting with symmetric information but no revenge motive still predicts (B, C) match to be the unique outcome. While tie breaker is either player A or player B with equal chance, and revenge motive is presumed and dominates in the tie breaking decisions, (A, C) match and (B, C) match can be supported by pure strategy equilibrium if certain conditions are satisfied. Weaker strongest player, player A, and stronger weak players, player B and C, are in favor of the existence of these two voting outcomes. (A, B) match can be supported by mixed strategy equilibrium, in which player C randomizes. In asymmetric information setting, while tie breaker is player A for sure, (A, C) match, in addition to (B, C) match, is supported by equilibrium without much restriction. Existence of (A, B) match requires that player A is not too strong and player C is not too weak. The strength of player B may affect the existence condition through its affect on the probability of player A being vengeful. While tie breaker is either player A or player B with equal chance, for (A, C) match to be supported by equilibrium, it needs strength difference of player A and player B to be small enough[9] . Player C's strength may affect the existence condition through its impact on the probability of player B being vengeful. (A, B) match's existence also requires that player C's strength is not too far behind from player A and player B.

In spite of the universal existence of (B, C) match, (B, C) match is not the outcome observed most frequently in the data. Indeed, in tie breaker situation that player A is the tie breaker for sure, (B, C) match accounts for the smallest share. On the other hand, (A, B) match, though its existence is always subject to constraints, is observed most frequently in both tie breaker situations. The following section provides an empirical analysis to see what factors influence individual voting decisions and voting outcomes, and to explore the reason of the relative prevalence of (A, B) match.

[9]According to the condition in Proposition 13, $\pi_L(p_A) \geq \gamma_B \pi_L(p_B)$

4 Empirical analysis

4.1 How do individuals vote?

In section 2, a conditional logit model is provided to argue that players are able to update information regarding opponents' strengths according to their past performance. However, the voting decisions in this regression are modeled without identifying the relative position of a player. Strategic interaction in this voting game implies that voter in different strength position may vote differently. Here, the voting decisions of player A, B and C are estimated by three binary probit regressions. For each regression, the voter's probability of voting against the stronger candidate in episodes k, e.g. player A voting against player B, is given by the following equation:

$$Pr_k(A \rightarrow B) = \Phi(x_k \beta') \tag{10}$$

where x_k is a vector of explanatory variables and $\Phi(\cdot)$ in the estimated equation is given by cumulative probability function of Gauss distribution. x_k includes six binary variables indicating the tie breaker identity situations, six binary variables indicating the revenge motive originated from voting history in previous rounds, six variables on performance indices of three players interacted with the two major tie breaker identity situations, and a set of variables controlling for possible discrimination effect. It also includes a binary variable indicating whether the stronger candidate is also the one who scores the single minimum correct rate in the voting round, controlling for possible framing effect. A revenge motive of player i against player j is also controlled for by including a dummy variable which takes one if, in one of three previous rounds, player j has voted against player i at least once. In addition, the revenge motives of candidate against voter and revenge motives between candidates are considered. In the two major tie breaker situations discussed in the theoretical section, the effects of performance index on voting are estimated. This is done by multiplying performance indices of the three players with the dummy variable indication tie breaker situations. Six variables are hence yielded. Due to small sub-sample size, for example only about ten observations are available for other tie breaker situations, the effects of performance indices in other tie breaker situations are omitted. Summary description on explanatory variables is provided in Table 5. And estimated results for player A, player B and player C are presented in Table 6.

For player A, B and C, the revenge motives to vote against the stronger candidate are all positive and significant at 5% level of significance. If Player A has a revenge motive against player B, the probability of player A voting against player B increases by 43.5% (evaluated at the sample means of explanatory variables). The corresponding estimates for player B to vote against player A, and for player C to vote against player A are 37% and 36.8% respectively. The results justify the assumption that a considerable proportion of players are vengeful in tie breaking. Moreover, coefficients indicating revenge motive against stronger candidate have high significant level than those against weak candidate. It seems that revenge motive is affected by the strength of target, justifying the assumption that the proportion of vengeful tie breaker depends on the strength ratio of two candidates due to its association with revenge cost.

A remarkable feature suggested by the probit models is that voting behavior is somehow inertial. A player i who has voted against player j before is likely to vote against player j again. For example, in the regression for player A, the revenge motive of player B against player A indicates that player A has voted against player B before. This variable being positive and significant means that the probability of player A voting against player B increases if player A has done so previously. Similar observation can be found in the regressions for player B and player C. If player B has voted against player A, s/he is likely to vote against player A again. If player C has voted against player B before, s/he is likely to vote against player B again. Such inertia in voting choice may reflect personal preference on unobserved characteristics such as appearance and manner, or it reflects some private signals received by individual regarding to strength of a particular opponent. These private signals, for example being confident, may cast one's belief of a candidate being strong in spite of statistical performance.

Being the unique possible tie breaker significantly increase player B and player C's probability to vote against player A. This suggests that the tie breaker identity is crucial in the voting. Players are capable to consider the voting as a two stage game, in contrast with the presumption in Févriera and Linnemerb (2006) that tie breaker randomizes in a tie, which essentially reduces it into a one-shot game.

When tie breaker is player A for sure, only the performance index of player A has a statistically significant impact on player C's voting decision. As expected, the higher is the performance index of player A, the more likely player C would vote against player A. To voting off player A, it requires both player B and C to vote against player A. It is risky in the sense that failure of coordination may result elimination by vengeful tie breaker. High performance index of player A is associated with a larger payoff difference between matching with player A and with player B. Thus, ceteris paribus, higher performance index of player A attracts player C's incline for (B, C) match outcome.

When tie breaker is player A or player B with equal chance, higher performance index of player C significant increase the chance of player A voting against player B. However, the asymmetric information model suggests the opposite. For (A, B) match to exist, $(\alpha - \frac{1}{2})(2p_A + 2p_B - 3p_C) \leq \frac{1}{2}$ needs to be satisfied. Higher strength of player C is in favor of this condition and hence reduces the chance for player A to vote against player B. For (A, C) match to exist, one needs $\pi_L(p_A) \geq \gamma_B \pi_L(p_B)$. If the assumption that γ_B is a non-increasing function of p_A/p_C holds, γ_B increase as p_C. (A, C) match is harder to hold and player A is less likely to vote against player B. In either case, the probability of player A voting against player B decrease as strength of player C increases. Of course, analysis above is static. Existence conditions may not be binding. And changes in strengths of players may also affect the coordination and choice of equilibrium. The following section explores strengths and other factors' influence on the voting outcomes.

4.2 What affects voting outcome?

To examine what factors affect the voting outcomes, this study uses a multinomial logit model. The probability that the voting outcome in episodes k is j is given by

$$Prob(Y_k = j) = \frac{exp(x_k \beta_j)}{\sum_{l=1}^{3} exp(x_k \beta_l)} \tag{11}$$

where j = 1, 2, 3 indicate (A, B), (A, C) and (B,C) match respectively. x_k is a vector of variables describing a voting game. In this model, the three players in one game are treated as an agent making collective decision over the possible voting outcomes. The interested subset of explanatory variables is the same as those used in the binary probit models, but the set of control variables are changed. The control variables now include distribution of gender, occupation and race to control for the possible discrimination. Details are presented in Table 7. Since underlying justification of multinomial logit model is the random utility, the associated indeterminacy is to be solved by artificially setting a base outcome, so that the predicted probabilities of other outcomes are compared to the base outcome. Given that (B, C) match is the voting outcome universally exist, it is chosen as base outcome.

Player B being tie breaker for sure significantly reduces the probability of observing (A, C) match, and player C being tie breaker significantly reduces the probability of observing (A, B) match, both compared to (B, C) match. Also, when tie breaker is player A for sure, it increases the probability of (A, C) match compared to (B, C) match. That is, being a tie breaker for sure significantly lowers one's probability of being eliminated. The marginal effect for player B and player C around mean values of explanatory variables are more than 30%, which is too high to be explained solely by the advantage of survival for the tie breaker after a tie happens. This confirms the fact that tie breaker identity is crucial in the strategic voting.

Historical revenge motive remains significant. A revenge motive between player A and player B in either direction has a negative marginal effect greater than 40% on observing (A, B) match. The probability of (B, C) match is lower if a revenge motive of player B against player C presents. A revenge motive of player C against player B also has a positive impact on (A, C) match and a negative impact on (A, B) match, when marginal effects on observing these matching outcomes are evaluated around sample means.

When player A is tie breaker for sure, the coefficient of performance index of play A is negative and significant. An increment of 0.15 in performance index of player A, which is close to the mean strength difference between player A and player B is roughly associated with a 0.25 decrease in the probability of (A, C) match in related to (B, C) match. This is in line with the implication in the asymmetric information model. Outstanding performance of player A makes the existent condition of the equilibrium supporting both (A, C) match and (A, B) match more restrictive. Also, it is intuitive that, given strength of player B and player C, higher strength of player A provides large incentive for player B and C to coordinate to eliminate player A.

When player A or player B can both be tie breaker, it is striking that, the marginal effects of three strength variables on observing (A, C) match are all significant with a large magnitude. In contrast with situation in which player A is tie breaker for sure, the probability of observing (A, C) match now increases as strength of player A. Moreover, performance index of player B has a negative marginal effect, and performance index of player C has a positive marginal effect, both on the probability of observing (A, C) match. Only the marginal effect of strength of player C (on observing (A, C) match) is in the same direction as in the symmetric information model with revenge motive. For strategy profile supporting (A, C) match to survive dominant eliminations, $(\alpha - \frac{1}{2})(2p_A - p_B - p_C) < \frac{1}{2}$ needs to be satisfied. If this condition is binding, weakness of player A, and strength of player B and player C increase the probability of (A, C) match. In the asymmetric information model, the pooling equilibrium supporting (A, C) match requires $\pi_L(p_A) \geq \gamma_B \pi_L(p_B)$. Higher strength of player A lowers the LHS of above inequality. Though it potentially lowers γ_B as well, one might expect that the linear decrease in the LHS utility dominates the indirect decrease through γ_B on the RHS. So it is more likely that probability of observing (A, C) match decreases as strength of player A increase. High strength of player B lowers the RHS and hence is in favor of (A, C) match's existence. Through its possible affect on γ_B, higher strength of player C may increase the RHS of above inequality and constrain the existence for this equilibrium. In sum, the two models agree that strength of player A should have a negative marginal effect on observing (A, C) match, which is rejected by the regression results. Neither model could give implication correctly on more than one strength variable.

For (A, B) match, the symmetric information model with revenge motive suggests that marginal effects of strength of player B and player C on observing (A, B) match should be positive, and marginal effect of player A should be negative. In the asymmetric information model, the marginal effects are negative for the strength of player A and player B, and are positive for player C. The two models agree on the negative effect of player A's strength and positive effect on player C's strength. However, the coefficient of p_A is significantly positive, though the significance is gone when marginal effect around sample mean is calculated.

Also, the marginal effect of player C's strength is negative and significant. Both estimates are not in line with theoretical implications.

The poor predictability of theoretical models when tie breaker is uncertain suggests that distribution of strength as an instrument for coordination plays a significant role in voting. The increased strategic uncertainty induces players to be more keen to coordinate to eliminate a particular opponent before tie happens, and less concerned about the associated payoff difference in different matches. Successful coordination secures one's proceeding in the game and hence a minimum expected payoff. Given the implicit suggestion in the TV show's name, the successful coordination on eliminating weakest contestants in previous round, and the show's pitiless highlight on eliminated contestants in each round, the strength of players is a natural focus point for such coordination. Indeed, the regression result suggests that, when tie breaker could be player A or player B, higher past performance lowers one's chance to be eliminated, which does not exhibit when tie breaker is player A for sure. The coordination effect dominates the existent condition effects in equilibrium analysis.

4.3 Strategic pretence

In the simplest situation where player A is tie breaker for sure, and information regarding strengths is symmetric, player A will always be eliminated. Through out the theoretical analysis, equilibrium eliminating player C to have (A, B) match requires more restrictive condition and is less straight forward to be constructed. Even without any background in game theory, it is not difficult for contestants to have a prior that strong contestant would be considered as a threat and more tend to be voted off. It thus creates incentive for a player to intentionally give incorrect attempt even if one knew the answer. Then, a question would be whether and to what extent such strategic pretence exists and, if exists, how it affects voting. Table 9 shows the correct rates in each round of players who survive all voting. As shown, the average correct rates of player A and player B across the sample experience a mild decrease from Round 3 to Round 4. While contemporary decrease for player C is much more dramatic. However, when it comes to Round 5, the average correct rates of player A and player B continue to decline. But there is a significant raise for the correct rate of player C. Since there is no elimination in Round 5, contestants should have clear incentive to give correct answers for prize accumulation. It suggests that player C is likely to pretend weak strategically. Given that the correct rates are averaged over those who survive all voting, it should not be due to selection bias from the voting result of Round 4 that strong player A and/or player B have been eliminated[10] It suggest. Moreover, the drop in correct rate is significant for player A but mild for player B. If it reflects a significant increase in question difficulty, it implies that player A has little strategic pretending but there could be some for player B, though the proportion should be much small than player C.

Figure 1 illustrates a non-parametric regression of wining probability of the stronger candidate over the weaker one, on the difference of performance index they have as in Round 4. One would expect that the wining probability is an increasing function of such strength gap and the probability being winner for the stronger contestant in a survived match is on average greater than 50%. However, the monotonic increase does not happen until the strength gap exceeds a certain value. Below such value, the stronger contestant could have winning chance less than a fair toss. Table 10 summarizes the performance index difference of the survived contestants as in Round 4, and the winning chance of the stronger contestant estimated by the sample mean. The performance index of player A in (A, B) match and (A, C) match are very close, while performance index gap is .1725785 on average between player A and player B, and .244332 on average for player A and player C. However, the probability of player A wining over player C is just slightly higher than that of player A winning over player B. The hypothesis that the winning chances of player A in these two matches are equal can not be rejected at any conventional level of significance. On the other hand, when player B head-to-heads against player C, who potentially can be very strong, s/he still has about 64% chance on average to win. Combining the non-parametric regression and the head-to-head outcome, it can be concluded that extent the strategic pretence of player C is moderate, i.e. strategic pretence only spans a narrow interval of real strength. It is rare that player C, who is identified as the weakest player according to past performance, actually turns out to be strongest player.

[10] A Heckman Selection model was also estimated. Correct rates of player A and player B in Round 5 significantly decreased, compared to their correct rates in all previous rounds. But there is insufficient evidence of such decrease for player C. The Heckman Selection model confirmed that this observation was not due to sample selection in the last voting.

When strategic pretence of player C and probably of player B exists, the past performance as a signal of strength for comparing real strength of player B and player C is invalid. It justifies the insensitivity of player A in respond to the strengths of player B and player C when s/he is tie breaker for sure. In this situation, though higher strengths of player B and player C increase their chances being voted against by player A in the probit model, the coefficient is nevertheless insignificant. It also provides an explanation why as large as 58% of time player A, being the tie breaking for sure, votes against player C, which is a dominated strategic if player A acts as if strength information is symmetric. However, on the ground that weak contestants are not able to pretend strong, the performance index of player A provides a signal of one's lower bound of strength. Therefore, though the possibility of strategic pretence is open, outstanding performance record of player A still increases his/her odds of being voted off.

5 Conclusion

The paper assessed the three-person strategic voting in TV game shows The Weakest Link. It is found that, when the game form is relatively simple, i.e. tie breaker identity is clear, basic game theory analysis fits the empirical observations quite well. When the tie breaker identity is unsure, actual voting behavior deviates from theoretical prediction. It is argued that coordinating behavior has a dominant effect. It suggests that strategic uncertainty, payoff uncertainty and game form complexity could significantly reduce predictability of game theory on actual strategic behavior. Strategic pretence is also found as a source partially invalidating public signals, which is also attributed for inaccurate predictability of theoretical analysis. Revenge motive is found to be significant in voting, suggesting players do not always maximize monetary payoff.

Appendix A

Proof of Proposition 1: If $p_B > p_C$, in case of tie, player A will always choose to go with player C. When both player B and player C vote against player A, player A is directly eliminated, player A's vote is immaterial. In other cases, voting against player B will ensure player A to go with player C in the head-to-head. However, voting against player C in some case leads player A to be matched with player B in head-to-head. In particular, this happens when player B votes against player C. Therefore, it is weakly dominant for player A to vote against player B. Given that player A will vote against player B and will eliminate player B in a tie, player B can only stay in the game and get positive expected utility when player C votes against A. This requires player B to vote against player A as well. Finally, when player A and player B vote against each other, it is optimal for player C to vote off player A.

Q.E.D.

Proof of Proposition 2: Since player C will not be eliminated in any tie, and matching with player A is strictly worse off for player C than matching with player B, voting against player A weakly dominates voting against player B. Then, it is optimal for player B to vote against A. As a result, (B, C) match forms and player A is indifferent between voting either.

Q.E.D.

Proof of Proposition 3: Given that player A is vengeful, it is still weakly dominated for player A to vote against player B. To see this, when player B and player C vote against each other, voting against player A results (A, C) match, giving player A strictly higher expected payoff than in (A, B) match if s/he votes against player C. In other strategy profile of player B and player C, player A's vote does not affect the voting outcome. Player A will be always matched to player B if player B votes against C, and player C votes against A, since player A's vote either eliminates player C or induces a tie, in which eventually player C is eliminated. The analogue argument applies to the strategy profile in which player B votes against A, and player C votes against player B. When player B and player C both votes against player A, player A will be voted off regardless. Given player A will vote against B, it is weakly dominant for player B to vote against player A. This is because, when player C votes against player A, voting against A matches player B to player C, while voting against player C induces a tie and match player B to player A. When player C votes against player B, player B will be voted off directly and player B's vote will be insignificant. Given player A and player B votes against each other, it is optimal for player C to votes against player A. Therefore, player A is eliminated.

Q.E.D.

Proof of Proposition 4: If $\pi_C(p_B) \geq 2\pi_C(p_A)$ holds, it is weakly dominant for player C to vote against player A. Then, it is strictly dominant for player B to vote against player A. Thus, player A will be eliminated.

Q.E.D.

Proof of Proposition 5: If $\pi_C(p_B) < 2\pi_C(p_A)$, no player has weakly dominated strategy. (B, C) match is supported by the strategy profile that both player B and player C vote against player A, and player A votes against either. It is optimal for player B and player C to match with each other, and player A' vote does not change the voting outcome. (B, C) match is Nash equilibrium.

(A, C) match is supported by the strategy profile that both player A and player C vote against player B, player B votes against player C. Now it is optimal for player A to match with player C in the head-to-head, and player B's unilateral deviation does not affect the voting outcomes. Player C's equilibrium expected utility is $\pi_C(p_A)$. If Player C deviates to vote against player A, there will be a tie. With probability 1/2, player A will be tie breaker and will eliminate player C vengefully. With probability 1/2, player B will be tie breaker and choose to go with player C. Thus, the expected utility of deviation for player C is $\pi_C(p_B)/2$. Therefore, given inequality (7), it is not profitable to deviate.

Q.E.D.

Proof of Proposition 6: (A, B) match is supported by the strategy profile that both player A and player B vote against player C, player C votes against player A with probability q_C, and with $(1 - q_C)$ against player B. If player A deviates, with probability $(1 - q_C)$ s/he is matched with player C; With probability q, deviation of player A induces a tie and player A will be eliminated when the tie breaker turns out to be player B. When the tie breaker turns out to be player A, player A chooses player B to go with due to the revenge motive against player C. Thus, for player A not to deviate, it requires

$$\pi_A(p_B) \geq (1 - q_C)\pi_A(p_C) + \frac{1}{2}q_C\pi_A(p_B) \tag{12}$$

Similarly, for player B to not deviate, it requires

$$\pi_B(p_A) \geq q_C\pi_B(p_C) + \frac{1}{2}(1 - q_C)\pi_B(p_A) \tag{13}$$

These two equations are reduced to:

$$\frac{\pi_A(p_C) - \pi_A(p_B)}{\pi_A(p_C) - \frac{1}{2}\pi_A(p_B)} \leq q_C \leq \frac{\pi_B(p_C) - \pi_B(p_A)}{\pi_B(p_C) - \frac{1}{2}\pi_B(p_A)} \tag{14}$$

Since $\frac{\pi_A(p_C)-\pi_A(p_B)}{\pi_A(p_C)-\frac{1}{2}\pi_A(p_B)} > 0$ and $\frac{\pi_B(p_C)-\pi_B(p_A)}{\pi_B(p_C)-\frac{1}{2}\pi_B(p_A)} < 1$, there exists $q \in (0, 1)$ if and only if

$$\frac{\pi_A(p_C) - \pi_A(p_B)}{\pi_A(p_C) - \frac{1}{2}\pi_A(p_B)} \leq \frac{\pi_B(p_C) - \pi_B(p_A)}{\pi_B(p_C) - \frac{1}{2}\pi_B(p_A)} \tag{15}$$

$$\Leftrightarrow \pi_A(p_C)\pi_B(p_A) \leq \pi_A(p_B)\pi_B(p_C) \tag{16}$$

By the definition of π_i, the above inequality always holds. Thus, (A, B) match can be supported by mixed strategy equilibrium.

Q.E.D.

Appendix B

Proof of Proposition 7: Firstly, no player has dominant strategy when the two types of players, H and L, can vote differently. The pooling sequential equilibrium giving (A, C) match has the strategy profiles that, regardless types, both player A and player C vote against player B, and player B votes against player C. The off-equilibrium belief of player A is unconstrained. In this equilibrium, player A gets expected utility $\pi_H(p_C)$ or $\pi_L(p_C)$, depending on his/her strength type. Player A's deviation will match player A with B, giving player A expected utility $\pi_H(p_B)$ or $\pi_L(p_B)$, which is lower for both strength types of player A. Unilateral deviation for player B does not change the voting outcome, player B remains to be voted off and get zero utility. Player C's deviation would induce a tie. Since tie breaker is player A for sure, player C will be matched to player A, with probability p_A being High type, or be eliminated. There is no chance player C can be matched to player B given this equilibrium strategy profile. Thus, player C does not have profitable deviation.

The pooling equilibrium supporting (B, C) match is that, both types of player B and player C vote against player A, and player A votes against player B or player C. Player A will be eliminated regardless how s/he votes, so there is no profitable deviation for player A. Suppose player A votes against player B, player C's deviation will immediately eliminate player B. This leaves player C matched with player A in the head-to-head, instead of with player B, which is worse off for player C. For player B, the deviation induces a tie. Matching with the tie breaker, player A, gives player B at most $\pi_H(p_A)$ or $\pi_L(p_A)$ (achieved if tie breaker always eliminates player C). Therefore, no one has profitable deviation. The argument is analogous when player A votes against player C.

Q.E.D.

Proof of Proposition 8: Firstly, (A, B) match can not be a result after tie in equilibrium. Whoever is eliminated in the tie can just switch her/his vote to ensure his/her survival in the game and yields positive expected payoff. At least one type in each of player A and player B must vote against player C. Moreover, it can not be the case that both types of player B vote against the same candidate. If both types of player B vote against player A, it is optimal for player C to do so as well. If both types of player B to vote against player C, voting against player B ensures player A at least $\pi_T(p_C)$, instead of $\pi_T(p_B)$ if s/he votes against player C. Thus, to have a pure strategy equilibrium supporting (A, B) match, different types of player B must vote differently. Also, it can not be the case that both type of player C vote against player A. Otherwise, it is optimal for player B to vote against player A and there is no (A, B) match.

Therefore, there are six strategy profiles of player B and player C left. In the strategy profile that High type player B and player C vote against player A, while Low types vote against each other, player A is indifferent between voting against either. However, this can not be an equilibrium since no matter how player A vote. High type of player B and player C can only survive when they are matched with each other. No matter how player A votes, at least one High type of player B or player C can gain by deviating to mimic his/her Low type. The two strategy profiles with which it is optimal for player A to vote against player C are not Nash equilibrium as well. In both strategy profiles, High type player B vote against player A while Low type votes against player C. In the strategy profile that both types of player C votes against player B, High type player B is always eliminated but Low type is not. So s/he will deviate. In the strategy profile that High type player C votes against player B while Low type votes against player A, it is strictly better off for High type player C to deviate[11]. In sum, there is no pure strategy equilibrium supporting (A, B) match.

Q.E.D.

Proof of Proposition 9: Again dominant strategy does not exist in asymmetric information situation. The equilibrium supporting (A, C) match is that both types of player A and player C vote against player B, and both types of player B vote against player C. If player C deviate to induce a tie, the off-equilibrium belief of the tie breaker is that the deviating player C is of High type while the other candidate's probability of being High type is as prior belief (strength signal). So player C will be eliminated in a tie and would not deviate. This off-equilibrium belief can be supported by the following complete mixed strategy. Let the probability High type player C assigns to vote against player A be given by sequence $\{\sigma_{CHi}\}$, the probability Low type player C assigns to vote against player A be given by

[11] Assume that the tie breaker randomizes when both candidates in a tie have the same type according to Bayesian update

sequence $\{\sigma_{CLi}\}$. They are positive and converge to zero. Let $\sigma_{CLi} = \sigma^2_{CHi}$, when both σ_{CLi} and σ_{CHi} converge to zero, the Bayesian update on the probability of deviating C being High type, when player C's strategy converge to equilibrium strategy is

$$\lim_{\sigma_{CHi}\to 0} \frac{p_C \sigma_{CHi}}{p_C \sigma_{CHi} + (1-p_C)\sigma^2_{CHi}} = 1 \tag{17}$$

It is easy to see that player A and player B do not have profitable deviation as well.

Q.E.D.

Proof of Proposition 10: Similar to proof of Proposition 6. The two types of each player vote in the same way as in the equilibrium in Proposition 6 The off-equilibrium belief on candidate strength is the same as the prior belief.

Q.E.D.

Proof of Proposition 11: The pooling sequential equilibrium giving (A, C) match has the strategy profiles that, regardless types, both player A and player C vote against player B, and player B votes against player C. The off-equilibrium belief of (Non-vengeful) player A is unconstrained. The proof is essentially the same as in Proposition 7, except that only non-vengeful player A has off-equilibrium belief. Non-vengeful player A will eliminate player C if player C deviates and induces a tie. As in the proof of Proposition 7, deviation makes player A worse off, for both tie-breaker types. Player B's deviation changes nothing. Given the existence of Vengeful tie breaker, deviating player C, who induces a tie, will be eliminated with some positive probability. Without player B's vote against player A, there is no chance for player C to match with player B, who is weaker than player A. Thus, deviation leads player C strictly worse off.

The pooling equilibrium supporting (B, C) match is similar to the one in Proposition 7. All type of player A, including strength type and vengeful-non-vengeful types, vote in the same way, either against player B or against player C. Both player B and player C of both types vote against player A. The proof is almost identical except that the deviating player who creates a tie will be eliminated by vengeful player A, and becomes worse off.

Q.E.D.

Proof of Proposition 12: In this equilibrium, non-vengeful player A will always go with player C. Deviating to vote against player C will match him/her with player B and becomes worse off. Vengeful player A is always match with player B, yielding expected utility $\pi_T(p_B)$, $T = H, L$ depending on player A's strength type. If Vengeful player A deviates to vote against B, with probability p_C, s/he will go with High type player C; with probability $(1-p_C)$, player C votes against player A and there is a tie $A \to B \to C \to A$. The vengeful nature will dictate player A to eliminate player C, even if player C is of Low type in this tie. The deviating expected utility will be $p_C\pi_T(1) + (1-p_C)\pi_T(p_B)$. As $\pi_T(1) < \pi_T(p_B)$, deviating makes Vengeful player strictly worse off, for both High and Low type. For player C, s/he stays if and only if player A is non-vengeful, who will vote against player B, regardless whether player C votes against player A or B. Hence, it is indifferent for both type of player C to vote against either. For player B, the equilibrium utility is $\gamma_A\pi_L(p_A)$. If s/he deviates, s/he can be stay only when player C votes against A. In other case, i.e. player C votes against B, s/he is either directly voted off when player A is non-vengeful or voted off after a tie when player A is Vengeful. Given Low type player C votes against player A, $\gamma_A\pi_L(p_A) \geq (1-p_C)\pi_L(0)$ prevents Low type player C to deviate. As $\pi_L(p_A) < \pi_L(0)$, γ_A must be greater than $(1-p_C)$. Given $\pi_H(x) = \pi_L(x) + (\alpha - \frac{1}{2})$, $\gamma_A\pi_L(p_A) \geq (1-p_C)\pi_L(0)$ implies

$$\gamma_A(\pi_L(p_A) + \alpha - \frac{1}{2}) \geq (1-p_C)(\pi_L(0) + \alpha - \frac{1}{2})$$

i.e. $\gamma_A\pi_H(p_A) \geq (1-p_C)\pi_H(0)$, High type player B will not deviate as well.

Q.E.D.

Proof of Proposition 13: (B, C) match is supported by the following strategy profile: both types of player B and player C vote against player A, all types of player A, including tie breaker types and strength types, votes against the same candidate, either player B or player C. Given that player B and player C both vote against A, deviating player B or player C will either match with player A or induce a tie. In either case, the deviating player (B or C) gets expected utility strictly lower than $\pi_T(p_A)$, $T = H, L$, as with positive probability they will be eliminated or positive probability of matching with player A. This does not depend on the off equilibrium belief.

(A, C) match can be supported by pooling equilibrium that all player A and C vote against player B, both types of player B vote against player C. The associated off equilibrium belief is that the deviating player C, who will induce tie $A \to B \to C \to A$, is of High type. The off-equilibrium belief can be supported by the complete mixed strategy sequence as in proof of Proposition 9. Given this belief, the deviating player C will not be eliminated only when the tie breaker is player B and s/he is of Vengeful type. So the non-deviating condition for player C is:

$$\pi_L(p_A) \geq \gamma_B \pi_L(p_B) \tag{18}$$

Since $\pi_H(p_A) = \pi_L(p_A) + \alpha - \frac{1}{2}$, and $\gamma_B < 1$, the non-deviating condition for High type C is automatically satisfied once the above non-deviating condition for Low type holds. Finally, player A and player B's deviation is not profitable. Thus, one has equilibrium gives (A, C) match.

Q.E.D.

Proof of Proposition 14: The off equilibrium associated with this sequential equilibrium is that the deviating player A or player B is believed to have strength as the prior by the other potential tie breaker, which can be supported by the complete mixed strategy sequence that both types of a player vote exactly the same. So that deviating player A will be eliminated when the tie breaker turns out to be player B, and vise versus, given that in either case player C is believed to be of strength p_C.

Let both types of player C vote against player A with probability q_C. The non-deviating conditions for player A and player B with non-vengeful Low type are

$$\pi_L(p_B) \geq (1 - q_C)\pi_L(p_C) + \frac{1}{2} q_C \pi_L(p_C) \tag{19}$$
$$\pi_L(p_A) \geq q_C \pi_L(p_C) + \frac{1}{2}(1 - q_C)\pi_L(p_C)$$

If the non-deviating conditions for non-vengeful players are satisfied, non-deviating conditions for Vengeful players below are automatically satisfied

$$\pi_L(p_B) \geq (1 - q_C)\pi_L(p_C) + \frac{1}{2} q_C \pi_L(p_B) \tag{21}$$

$$\pi_L(p_A) \geq q_C \pi_L(p_C) + \frac{1}{2}(1 - q_C)\pi_L(p_A) \tag{22}$$

For the same argument as in proof of Proposition 8, the non-deviating conditions of Low type implies the non-deviating conditions of High type. The non-deviating conditions for Low type player A and player B can be reduced to

$$\frac{2[\pi_L(p_C) - \pi_L(p_B)]}{\pi_L(p_C)} \leq q_C \leq \frac{2\pi_L(p_A) - \pi_L(p_C)}{\pi_L(p_C)} \tag{23}$$

$q_C \in (0, 1)$ exists if and only if:

$$2[\pi_L(p_C) - \pi_L(p_B)] \leq 2\pi_L(p_A) - \pi_L(p_C) \tag{24}$$

$$\Leftrightarrow (\alpha - \frac{1}{2})(2p_A + 2p_B - 3p_C) \leq \frac{1}{2} \tag{25}$$

Q.E.D.

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Table 1: Probability of a candidate being voted against [a] [b]

	Round 1	Round 2	Round 3	Round 4
	$\beta, (S.E.)$	$\beta, (S.E.)$	$\beta, (S.E.)$	$\beta, (S.E.)$
percentc	-2.840***	-2.105***	-1.960***	-0.778***
	(0.144)	(0.159)	(0.204)	(0.247)
percentclag1	-	-0.769***	-0.494***	-0.324
	-	(0.176)	(0.191)	(0.268)
percentclag2	-	-	-0.496**	-0.488*
	-	-	(0.218)	(0.287)
percentclag3	-	-	-	-0.231
	-	-	-	(0.291)
age	0.012**	0.009*	0.011	0.024**
	(0.005)	(0.006)	(0.007)	(0.010)
black	0.153	-0.110	-0.504**	-0.530*
	(0.147)	(0.193)	(0.232)	(0.280)
otherrace	-0.026	0.334	0.260	-0.826*
	(0.254)	(0.276)	(0.322)	(0.494)
fat	-0.001	0.163	0.005	0.206
	(0.148)	(0.162)	(0.206)	(0.266)
college	-0.014	-0.478***	0.068	0.133
	(0.111)	(0.123)	(0.150)	(0.205)
doctorate	-0.564	-1.386***	-1.199***	0.196
	(0.392)	(0.455)	(0.462)	(0.517)
uncertainedu	-0.135	-0.212	0.116	0.380
	(0.286)	(0.292)	(0.319)	(0.422)
profedu	0.166	-0.090	0.054	-0.132
	(0.196)	(0.210)	(0.276)	(0.383)
stillsch	0.369*	-0.210	0.574*	0.154
	(0.207)	(0.267)	(0.307)	(0.439)
samefemale	-0.399***	-0.363***	-0.158	-0.493**
	(0.119)	(0.128)	(0.142)	(0.203)
samemale	-0.152	-0.218*	-0.520***	0.023
	(0.116)	(0.129)	(0.167)	(0.207)
samerace	0.019	-0.189	-0.429*	-0.607**
	(0.146)	(0.194)	(0.230)	(0.276)
sameBcollar	-0.205	0.086	0.572	0.124
	(0.300)	(0.301)	(0.402)	(0.478)
sameprof	-0.022	0.218	-0.332	-0.667**
	(0.163)	(0.185)	(0.208)	(0.320)
sameedu	-0.013	-0.002	-0.135	0.271
	(0.114)	(0.124)	(0.155)	(0.208)
Pseudo R^2	0.200	0.144	0.131	0.068
Log-likelihood	-1049.014	-807.127	-517.620	-262.852
χ^2	525.355	271.105	155.656	38.519
Number of cases	4075	2720	1626	814

[a] β's give the point estimates of coefficients. Standard errors of coefficient estimates are included in the parentheses

[b] See Table 11 for explanation of variables

Table 2: Summary statistics of performance index in Round 4 [a]

Variable	Obs	Mean	Std. Dev.	Min	Max
$pfindex_A$	136	.7819255	.1348992	.4360146	1
$pfindex_B$	136	.6335508	.1454915	.2766506	.9461312
$pfindex_C$	136	.49683	.140341	.1533583	.8662421
$pfindex_A - pfindex_B$	136	.1483748	.1333252	0	.6179487
$pfindex_A - pfindex_C$	136	.2850955	.1554585	.017234	.6777432
$pfindex_B - pfindex_C$	136	.1367207	.1107028	0	.4627487
$pfindex_i$	408	.6374354	.1821476	.1533583	1

[a] $pfindex_i$ indiateds the performance index for Player i, $i = A, B, C$

Table 3: Distribution of tie breaker identity profile

	A	B	C	A or B	A or C	B or C	A or B or C
Observations	53	18	11	29	11	4	10
Percent	39%	13%	8%	21%	8%	3%	7%

Table 4: Distribution of voting outcomes

Survived pair	A & B		A & C		B & C	
Tie breaker identity	Obs.	Percent	Obs.	Percent	Obs.	Percent
A	23	43%	18	34%	12	23%
B	5	28%	6	33%	7	39%
C	4	36%	1	9%	6	55%
A or B	14	48%	6	21%	9	31%
A or C	4	36%	5	45%	2	18%
B or C	0	0%	4	100%	0	0%
A or B or C	3	30%	7	70%	0	0%
Overall	53	39%	47	35%	36	26%

Table 5: Summary statistics of explanatory variables in the Binary Probit Models

	Mean	S.D	Description
tbAsure	0.390	0.489	Dummy variable, 1 if tie breaker is player A for sure, 0 otherwise
tbBsure	0.132	0.340	Dummy variable, 1 if tie breaker is player B for sure, 0 otherwise
tbCsure	0.081	0.274	Dummy variable, 1 if tie breaker is player C for sure, 0 otherwise
tbAB	0.213	0.411	Dummy variable, 1 if tie breaker is either player A or B with equal chance
tbAC	0.081	0.274	Dummy variable, 1 if tie breaker is either player A or C with equal chance
tbBC	0.029	0.170	Dummy variable, 1 if tie breaker is either player B or C with equal chance
rmstrongA	0.140	0.348	Dummy variable, 1 if player A have revenge motive to vote against player B
rmstrongB	0.184	0.389	Dummy variable, 1 if player B have revenge motive to vote against player A
rmstrongC	0.140	0.348	Dummy variable, 1 if player C have revenge motive to vote against player B
rmweakA	0.125	0.332	Dummy variable, 1 if player A have revenge motive to vote against player C
rmweakB	0.132	0.340	Dummy variable, 1 if player B have revenge motive to vote against player C
rmweakC	0.184	0.389	Dummy variable, 1 if player C have revenge motive to vote against player B
XpfindexA	0.305	0.391	tbAsure × pfindexA, performance index of player A if tbAsure=1, 0 otherwise
XpfindexB	0.224	0.297	tbAsure × pfindexB, performance index of player B if tbAsure=1, 0 otherwise
XpfindexC	0.173	0.232	tbAsure × pfindexC, performance index of player C if tbAsure=1, 0 otherwise
tbABpfindexA	0.172	0.336	tbAB × pfindexA, performance index of player A if tbAB=1, 0 otherwise
tbABpfindexB	0.142	0.280	tbAB × pfindexB, performance index of player B if tbAB=1, 0 otherwise
tbABpfindexC	0.100	0.199	tbAB × pfindexC, performance index of player C if tbAB=1, 0 otherwise
samemale	0.206	0.406	Dummy variable, 1 if voter and the stronger candidate are both male, 0 otherwise
samefemale	0.191	0.395	Dummy variable, 1 if voter and the stronger candidate are both female, 0 otherwise
candiblack	0.213	0.411	Dummy variable, 1 if the stronger candidate is black, 0 otherwise
candicollege	0.397	0.491	Dummy variable, 1 if the stronger candidate's education level is college
candidoctorate	0.044	0.206	Dummy variable, 1 if the stronger candidate's education level is doctorate
candihighsch	0.404	0.493	Dummy variable, 1 if the stronger candidate's education level is high school
candiprofedu	0.066	0.250	Dummy variable, 1 if the stronger candidate's education level is postgraduate professional qualification
candiwlinksingle	0.162	0.370	Dummy variabe, 1 if the stronger candidate has unique minimum correct rate in Round 4
candiage	33.375	7.789	Age of the stronger candidate
candiBcollar	0.213	0.411	Dummy variable, 1 if the stronger candidate's occupation is blue collar
candiprof	0.338	0.475	Dummy variable, 1 if the stronger candidate's occupation is professional

Table 6: Probability of player i voting against the stronger candidate [a] [b] [c]

	player A		player B		player C	
	Coeff. $\beta/(S.E.)$	Marginal effect $\beta/(S.E.)$	Coeff. $\beta/(S.E.)$	Marginal effect $\beta/(S.E.)$	Coeff. $\beta/(S.E.)$	Marginal effect $\beta/(S.E.)$
tbAsure (d)	-0.196	-0.077	-2.349	-0.660**	-3.961*	-0.881***
	(1.410)	(0.548)	(1.667)	(0.300)	(2.069)	(0.170)
tbBsure (d)	-0.689	-0.248	1.079*	0.410*	0.843	0.326
	(0.601)	(0.188)	(0.620)	(0.215)	(0.802)	(0.296)
tbCsure (d)	-1.241*	-0.379***	0.545	0.211	3.744***	0.742***
	(0.705)	(0.137)	(0.795)	(0.313)	(1.441)	(0.071)
XpfindexA	-2.575	-1.014	2.587	0.948	6.785***	2.546***
	(1.739)	(0.685)	(1.797)	(0.655)	(2.315)	(0.870)
XpfindexB	2.448	0.963	1.302	0.477	-2.787	-1.046
	(2.363)	(0.930)	(2.125)	(0.780)	(2.702)	(1.014)
XpfindexC	-0.310	-0.122	-0.253	-0.093	2.042	0.766
	(2.525)	(0.994)	(2.389)	(0.876)	(3.071)	(1.150)
tbAB (d)	-0.393	-0.150	1.578	0.569	8.488***	0.985***
	(2.626)	(0.959)	(2.532)	(0.726)	(2.999)	(0.027)
tbAC (d)	-1.374	-0.402***	0.257	0.098	0.610	0.239
	(0.869)	(0.147)	(0.684)	(0.267)	(0.814)	(0.316)
tbBC (d)	-0.182	-0.070	-0.263	-0.091	1.313	0.473
	(0.900)	(0.339)	(1.218)	(0.390)	(1.521)	(0.403)
rmstrongA (d)	1.192***	0.435***	0.965**	0.370***	0.553	0.215
	(0.427)	(0.124)	(0.393)	(0.142)	(0.423)	(0.165)
rmstrongB (d)	-0.019	-0.007	0.908***	0.348***	-0.156	-0.057
	(0.362)	(0.142)	(0.334)	(0.124)	(0.409)	(0.147)
rmstrongC (d)	-0.743*	-0.265**	0.116	0.043	0.959**	0.368**
	(0.443)	(0.135)	(0.398)	(0.150)	(0.476)	(0.170)
rmweakA (d)	-0.239	-0.092	0.137	0.051	0.043	0.016
	(0.448)	(0.168)	(0.429)	(0.163)	(0.520)	(0.197)
rmweakB (d)	0.293	0.116	-0.395	-0.134	-3.250***	-0.531***
	(0.397)	(0.158)	(0.452)	(0.139)	(0.978)	(0.063)
rmweakC (d)	0.634*	0.249*	0.057	0.021	-0.367	-0.131
	(0.356)	(0.134)	(0.351)	(0.130)	(0.400)	(0.135)
tbABpfindexA	-0.457	-0.180	-0.523	-0.192	-10.313**	-3.870**
	(3.509)	(1.381)	(3.414)	(1.251)	(4.592)	(1.692)
tbABpfindexB	-4.258	-1.676	-2.914	-1.068	3.234	1.213
	(3.153)	(1.243)	(2.920)	(1.070)	(3.537)	(1.320)
tbABpfindexC	5.795*	2.281*	2.345	0.860	-2.660	-0.998
	(3.130)	(1.232)	(3.020)	(1.107)	(3.052)	(1.144)
Pseudo R^2	0.262	0.262	0.216	0.216	0.411	0.411
Log-likelihood	-69.017	-69.017	-70.154	-70.154	-54.958	-54.958
χ^2	49.059	49.059	38.584	38.584	76.734	76.734
N	136.000	136.000	136.000	136.000	136.000	136.000

[a] β's give the point estimates of coefficients or marginal effects. Standard errors of these estimates are included in the parentheses. Marginal effects on the voting decisions are evaulated at sample means of explantory variables.

[b] * indicates 10% L.O.S., ** indicates 5% L.O.S., *** indicates 1% L.O.S.

[c] (d) indicates that marginal effect is calculated for discrete change of dummy variable from 0 to 1

Table 7: Summary statistics of control variables in multinomial logit regressions

	Mean	S.D	Description
ABsameoccup	.1911765	.3946814	Dummy variable, 1 if player A and B have same class of occupation different from player C
ACsameoccup	.0735294	.2619684	Dummy variable, 1 if player A and C have same class of occupation different from player B
BCsameoccup	.0514706	.2217724	Dummy variable, 1 if player B and C have same class of occupation different from player A
ABwhite	.1397059	.3479633	Dummy variable, 1 if player A and B are both white while player C is not
ACwhite	.1764706	.3826294	Dummy variable, 1 if player A and C are both white while player B is not
BCwhite	.1617647	.3695961	Dummy variable, 1 if player B and C are both white while player A is not
ABmale	.1470588	.3554738	Dummy variable, 1 if player A and B are both male while player C is female
ABfemale	.0882353	.2846854	Dummy variable, 1 if player A and B are both female while player C is male
ACmale	.1323529	.3401269	Dummy variable, 1 if player A and C are both male while player B is female
ACfemale	.1617647	.3695961	Dummy variable, 1 if player A and C are both female while player B is male
BCmale	.125	.3319415	Dummy variable, 1 if player B and C are both male while player A is female
BCfemale	.1838235	.3887722	Dummy variable, 1 if player B and C are both female while player A is male

Table 8: Multinomial Logit Model - Probability of matching outcomes [a] [b] [c]

	(A, B) match		(A, C) match	
	Coefficient	Marginal effect	Coefficient	Marginal effect
tbAsure (d)	5.978	0.311	6.881*	0.464
	(3.788)	(0.539)	(4.066)	(0.511)
tbBsure (d)	-2.386*	-0.271	-3.707***	-0.327***
	(1.241)	(0.183)	(1.261)	(0.083)
tbCsure (d)	-2.749**	-0.328**	-4.379***	-0.332***
	(1.361)	(0.162)	(1.653)	(0.077)
tbAB (d)	-10.749**	-0.167	-20.088**	-0.823***
	(5.218)	(0.222)	(8.456)	(0.231)
XpfindexA	-6.584	0.083	-12.188**	-1.666**
	(4.427)	(0.791)	(5.036)	(0.798)
XpfindexB	-3.290	-0.411	-2.881	-0.149
	(4.189)	(0.956)	(5.428)	(1.019)
XpfindexC	0.752	-0.688	6.208	1.215
	(4.545)	(1.010)	(5.443)	(1.006)
rmstrongA (d)	-2.477**	-0.424***	-0.491	0.139
	(1.006)	(0.105)	(0.928)	(0.187)
rmstrongB (d)	-2.462***	-0.452***	-0.257	0.205
	(0.930)	(0.104)	(0.837)	(0.172)
rmweakA (d)	0.255	0.057	0.044	-0.027
	(0.892)	(0.198)	(1.173)	(0.206)
rmweakB (d)	2.421**	-0.091	3.567***	0.378**
	(1.230)	(0.183)	(1.295)	(0.187)
rmweakC (d)	-0.775	-0.288**	0.811	0.297*
	(0.800)	(0.126)	(0.824)	(0.156)
tbABpfindexA	15.418*	0.430	24.118**	2.960*
	(8.092)	(1.532)	(9.856)	(1.561)
tbABpfindexB	0.208	1.979	-13.680	-2.940*
	(6.821)	(1.530)	(9.419)	(1.714)
tbABpfindexC	-7.226	-3.567**	12.585	3.696**
	(6.484)	(1.655)	(10.244)	(1.884)
Pseudo R^2	0.336	0.336	0.336	0.336
Log-likelihood	-98.062	-98.062	-98.062	-98.062
χ^2	99.341	99.341	99.341	99.341
Observations	136	136	136	136

[a] β's give the point estimates of coefficients or marginal effects. Standard errors of these estimates are included in the parentheses. Marginal effects on the voting outcomes are evaulated at sample means of explantory variables.

[b] * indicates 10% L.O.S., ** indicates 5% L.O.S., *** indicates 1% L.O.S.

[c] (d) indicates that marginal effect is calculated for discrete change of dummy variable from 0 to 1

Table 9: Average correct rates of those survive all voting

	A	B	C	A, B & C
Round 1	0.88667	0.76326	0.66463	0.780443
Round 2	0.75167	0.69318	0.64228	0.698265
Round 3	0.81833	0.63636	0.59959	0.689937
Round 4	0.79833	0.61364	0.46646	0.637237
Round 5	0.67167	0.60393	0.64257	0.638069
Sample size	100	89	83	272

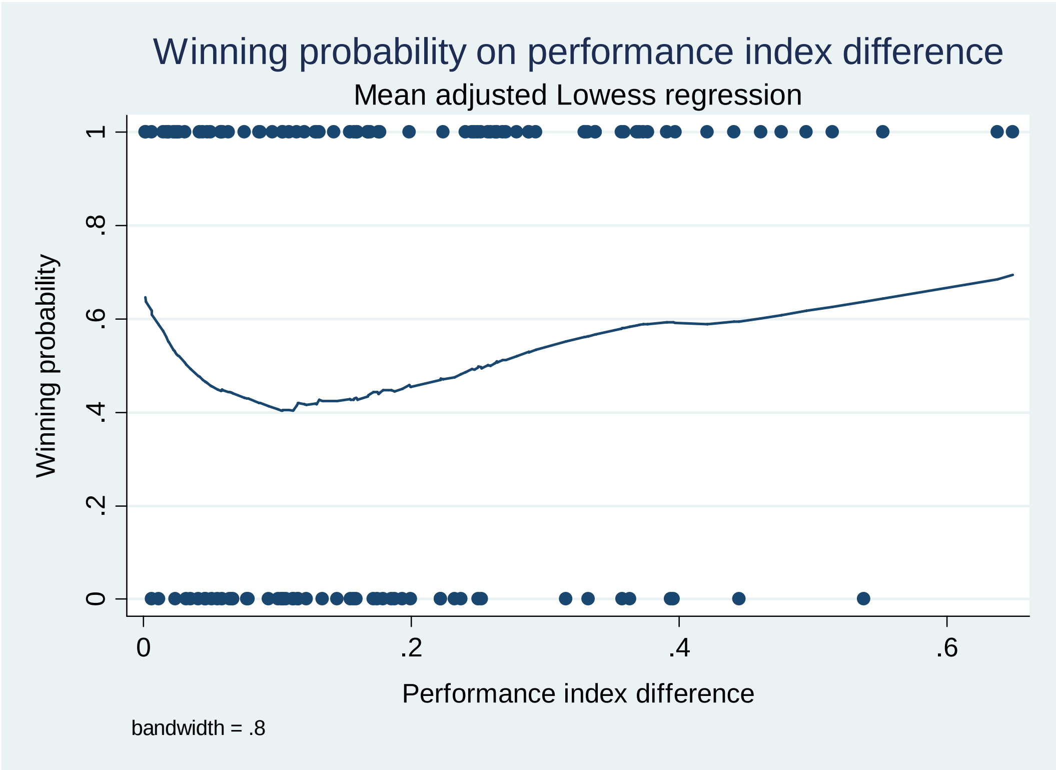
Table 10: Average performance index of those survive all voting

	(A, C)	(A, B)	(B, C)
$pfindexA$	.7837669	.7890374	.7690513
$pfindexB$	.6532661	.6164589	.6329742
$pfindexC$	.5394349	.4695422	.4813807
$pfindexA - pfindexB$	.1305008	.1725785	.1360771
$pfindexA - pfindexC$	.244332	.3194952	.2876706
$pfindexB - pfindexC$	.1138312	.1469167	.1515935
winning	.6170213	.5849057	.6388889
Observations	47	53	36

Table 11: Description of variables in Table 1

Variable	Description
percentc	The correct rate of answering questions in the curent voting round.
percentclag1	The correct rate of answering questions in one round before the voting round.
percentclag2	The correct rate of answering questions in two round before the voting round.
percentclag3	The correct rate of answering questions in three round before the voting round.
age	Age of the candidate.
black	Dummy variable, 1 if the candiate is black, 0 otherwise.
otherrace	Dummy variable, 1 if the candiate is not black or white, 0 otherwise.
fat	Dummy variable, 1 if the candiate is overweighted, 0 otherwise.
college	Dummy variable, 1 if the candidate has college education, 0 otherwise.
doctorate	Dummy variable, 1 if the candidate has a doctorate degree., 0 otherwise.
uncertainedu	Dummy variable, 1 if the candidate's education level is uncertain, 0 otherwise.
profedu	Dummy variable, 1 if the candidate has a professional degree, 0 otherwise.
stillsch	Dummy variable, 1 if the candidate is still in school, 0 otherwise.
samefemale	Dummy variable, 1 if the voter and candidate are both female, 0 otherwise.
samemale	Dummy variable, 1 if the voter and candidate are both male, 0 otherwise.
samerace	Dummy variable, 1 if the voter and candidate have the same race, 0 otherwise.
sameBcollar	Dummy variable, 1 if the voter and candidate are both blue collar, 0 otherwise.
sameprof	Dummy variable, 1 if both voter and candidate both have professional occupation, 0 otherwise.
sameedu	Dummy variable, 1 if both voter and candidate have the same education level, 0 otherwise.

Figure 1


Winning probability on performance index difference
Mean adjusted Lowess regression
Winning probability
0
.2
.4
.6
.8
1
0
.2
.4
.6
Performance index difference
bandwidth = .8