M2177.003100
Deep Learning

[6: Convolutional Neural Nets (Part 1)]

Electrical and Computer Engineering
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(last compiled at 20:52:00 on 2019/10/06)

Outline

References

- *Deep Learning* by Goodfellow, Bengio and Courville ▸ Link
  - Chapter 9
- online resources:
  - *Deep Learning Specialization (coursera)* ▸ Link
  - *Stanford CS231n: CNN for Visual Recognition* ▸ Link
- note:
  - you should open this file in Adobe Acrobat to see animated images (other types of pdf readers will not work)

Outline

Convolutional (neural) networks (CNNs)

- simply neural nets that use **convolution** in their layers
  - in place of general matrix multiplication
- employ a mathematical operation called *convolution*
  - convolution[1]: a special kind of linear operation
- tremendously successful in practical applications
  - especially for data with a known, grid-like topology

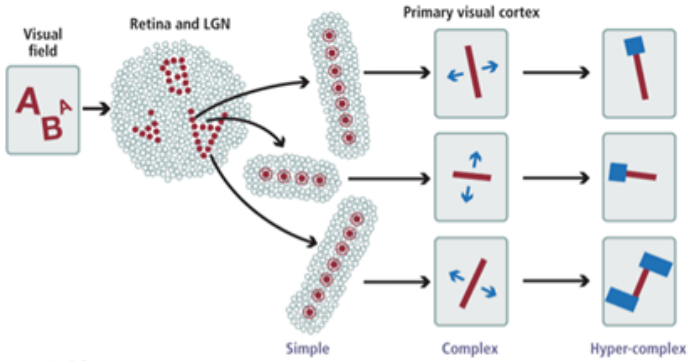
  e.g. time series (1D grid), image (2D grid of pixels), video (3D grid)
- explicit assumption: inputs have grid topology
  - allow us to encode certain properties into architecture
  - make forward function more efficient to implement
  - significantly _____ the amount of parameters

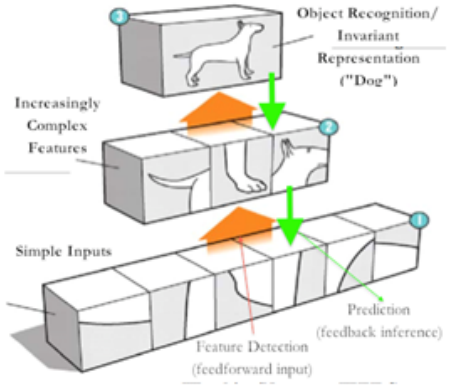
[1] usually does not correspond precisely to the definition of convolution as used in other fields

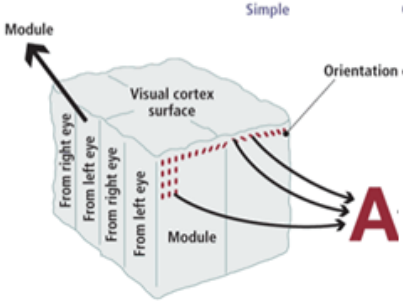
- an example of neuroscientific principles influencing deep learning

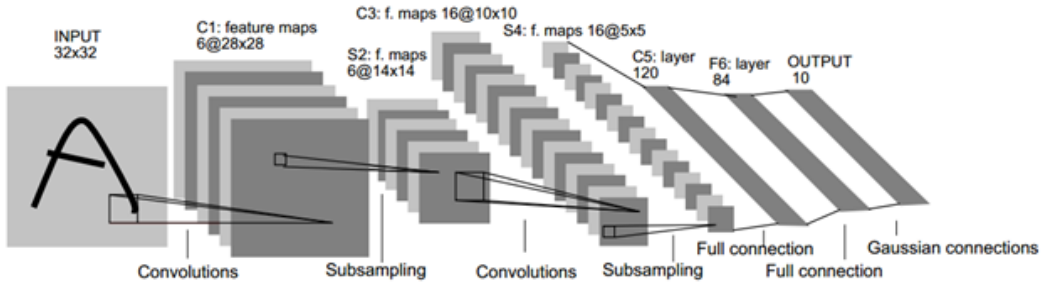
  e.g. human vision system:

  - simple cells $\rightarrow$ complex cells $\rightarrow$ hyper-complex cells





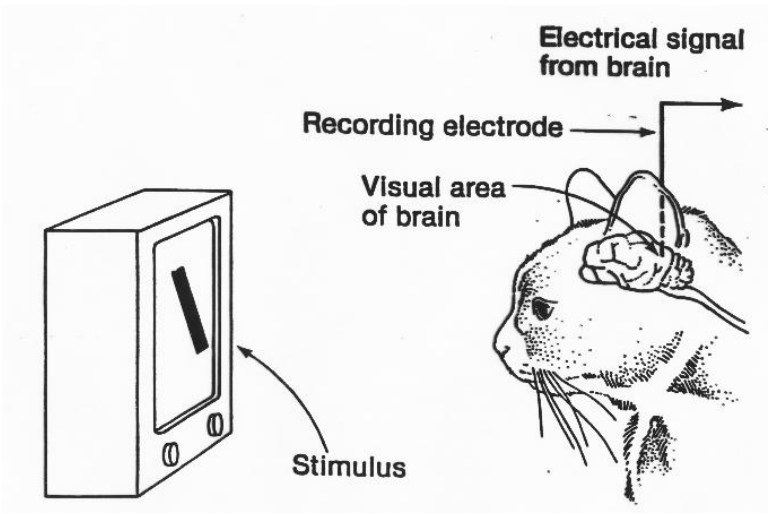




(source: LeCun)

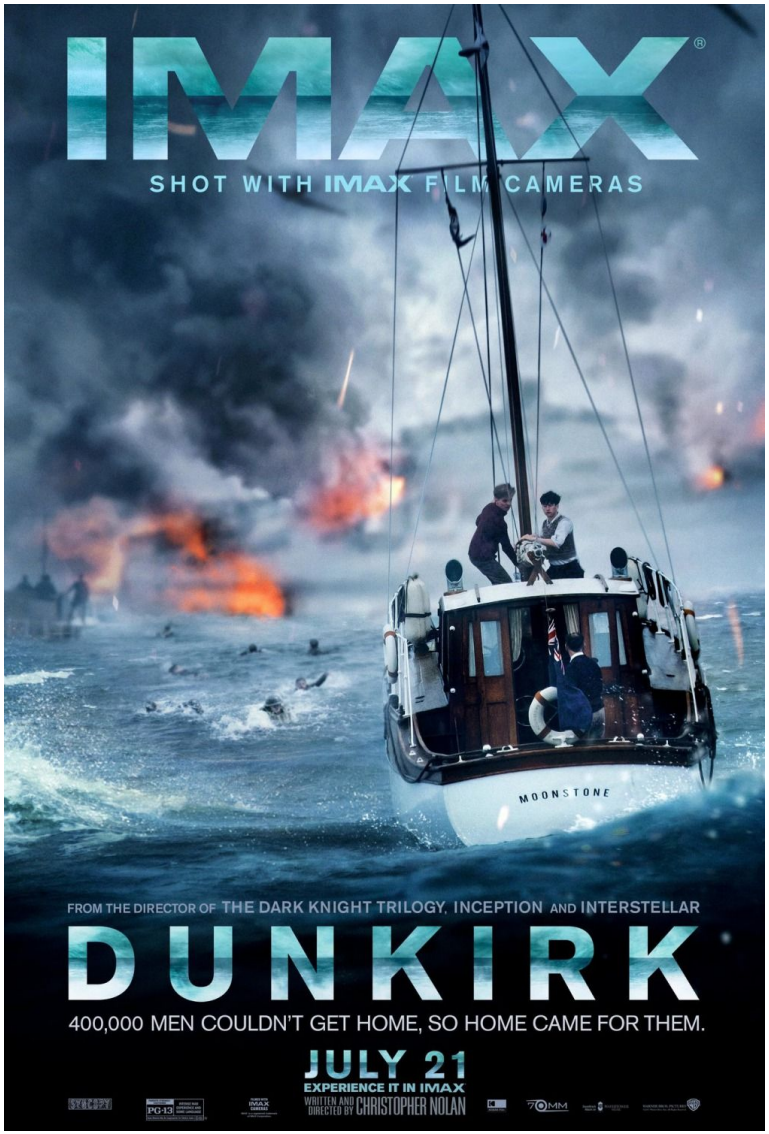
Hubel and Wiesel

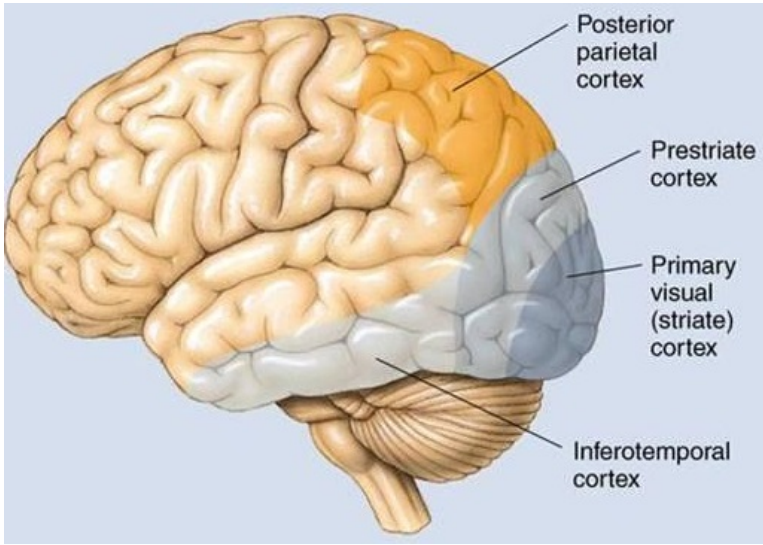
- neurophysiologists David Hubel and Torsten Wiesel
  - determined many facts about the mammalian vision system
- their findings with greatest influence on deep learning models:
  - based on recording the activity of individual neurons in cats

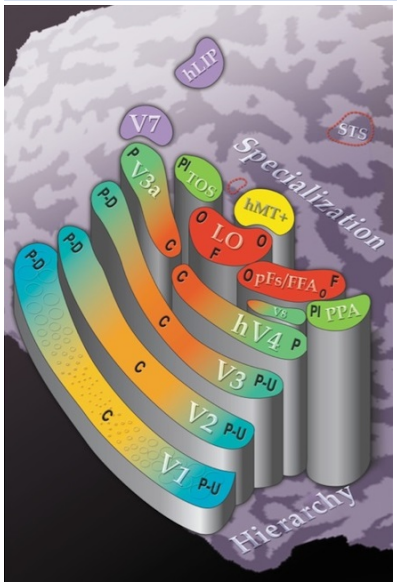


(source: Harvard U)

Human vision

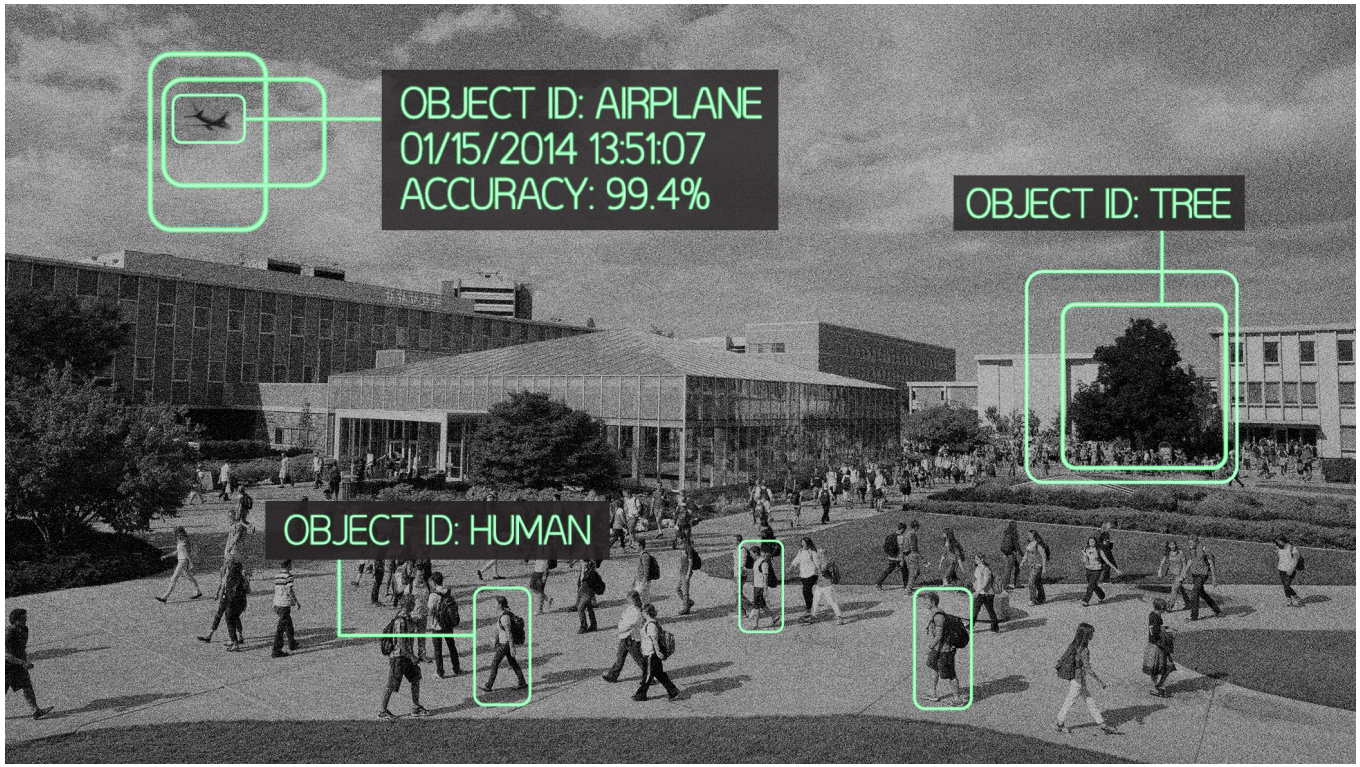






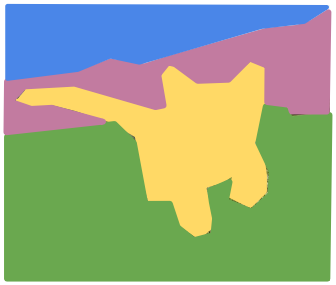
Computer vision

- methods to acquire/process/analyze/understand
  - images and high-dimensional data from the real world
  - in order to produce numerical or symbolic information



(source: BYU Photo)

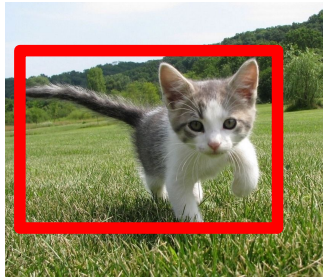
semantic segmentation



GRASS, CAT, TREE, SKY

no objects, just pixels

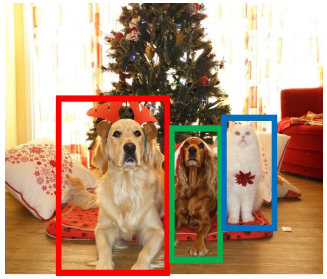
classification + localization



CAT

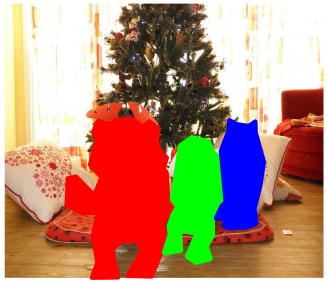
single object

object detection



DOG, DOG, CAT

instance segmentation



DOG, DOG, CAT

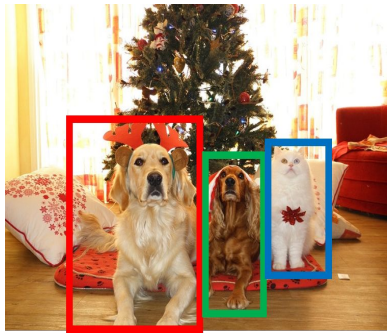
multiple objects

semantic segmentation

GRASS, CAT, TREE, SKY

no objects, just pixels

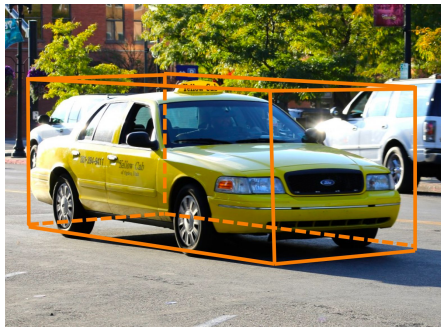
2D object detection



DOG, DOG, CAT

object categories + 2D bounding boxes

3D object detection



Car

object categories + 3D bounding boxes

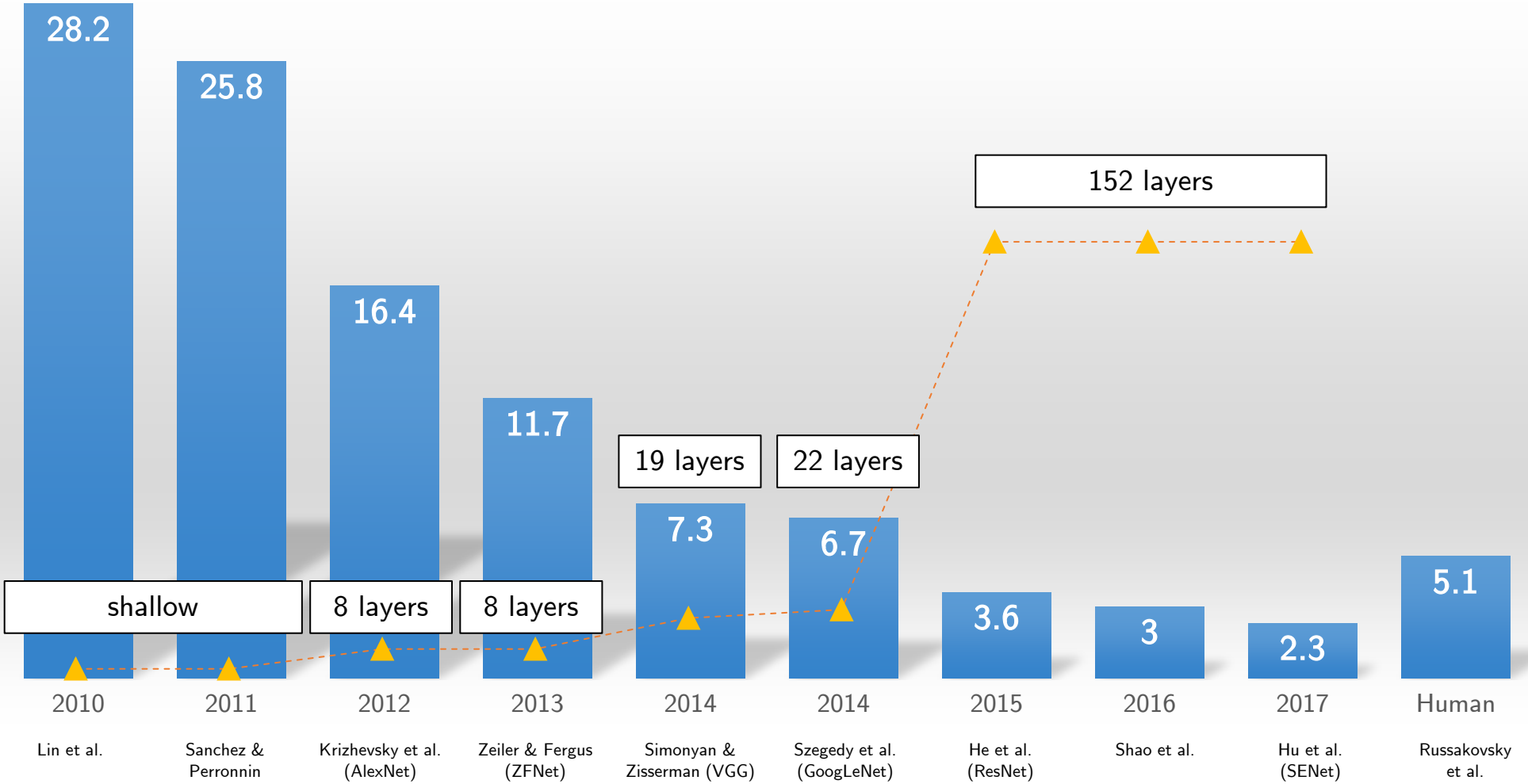
(source: cs231n)

Neural networks specifically adapted for CV

- ideal properties
  - must deal with very high-dimensional inputs
    (*e.g.* $150 \times 150 = 22,500$-dimensional inputs per channel)
  - can exploit 2D/3D topology of pixels
  - invariance to certain variations (translation, illumination)

- to this end, CNNs exploit
  1. ____ connectivity
  2. parameter sharing
  3. pooling/subsampling hidden units

ImageNet challenge winners



Outline

Motivating example

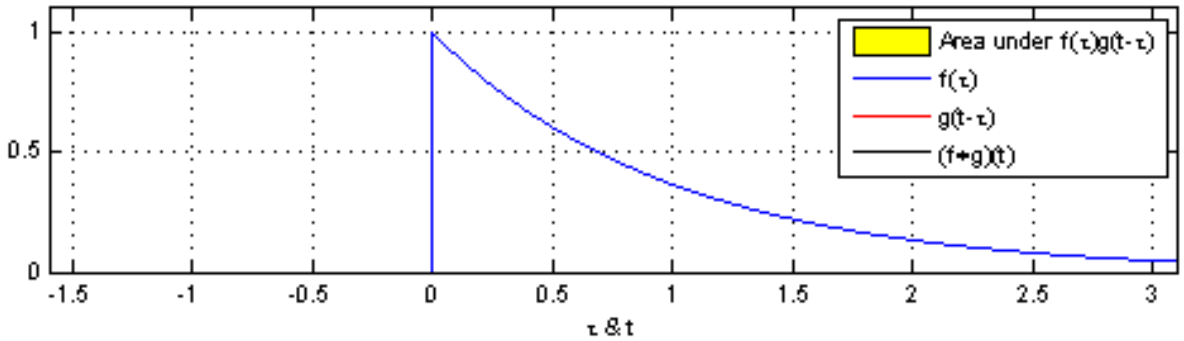
- suppose we are tracking the location of a plane with a laser sensor
  - the sensor provides a single output $x(t)$: position of the plane at time t
  - both x and t : real-valued
- now suppose: the sensor is noisy
- to get less noisy estimate of x
  - we average together several measurements
  - more _____ measurements are more relevant
- we thus introduce a weighting function $w(a)$
  - to give more weight to recent measurements
  - a : the age of a measurement

Convolution

- if we apply such a weighted average operation at every moment
  - we get a new function s providing a smoothed estimate of x

$$s(t) = \sum_{-\infty}^{\infty} x(a)w(t-a)$$
$$\triangleq (x * w)(t)$$

- in CNN terminology
  - first argument (function x): *input*
  - second argument (function w): *kernel* or *filter*
  - output (function s): *feature map* or *activation map*



(source: Wikipedia)

Multi-dimensional convolution

- convolutions over more than one axis at a time

e.g. if we use 2D image I as input

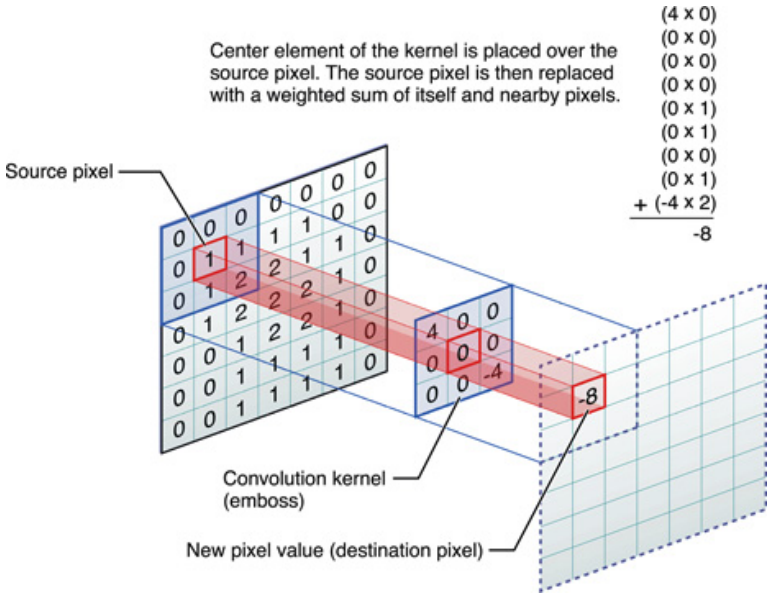
  - use 2D kernel $K \Rightarrow$ 2D convolution:

$$S(i,j) = (I * K)(i,j) = \sum_m \sum_n I(m,n) K(i-m, j-n)$$

- note

  - convolution: commutative

$$(I * K)(i,j) = (K * I)(i,j)$$



(source: Apple)

Cross-correlation

- the same as convolution but without flipping the kernel:

$$S(i,j) = (I * K)(i,j)$$
$$= \sum_m \sum_n I(m,n)K(i+m, j+n)$$

- neural net libraries implement cross-correlation but call it convolution
  - learned kernel will be in essence the same

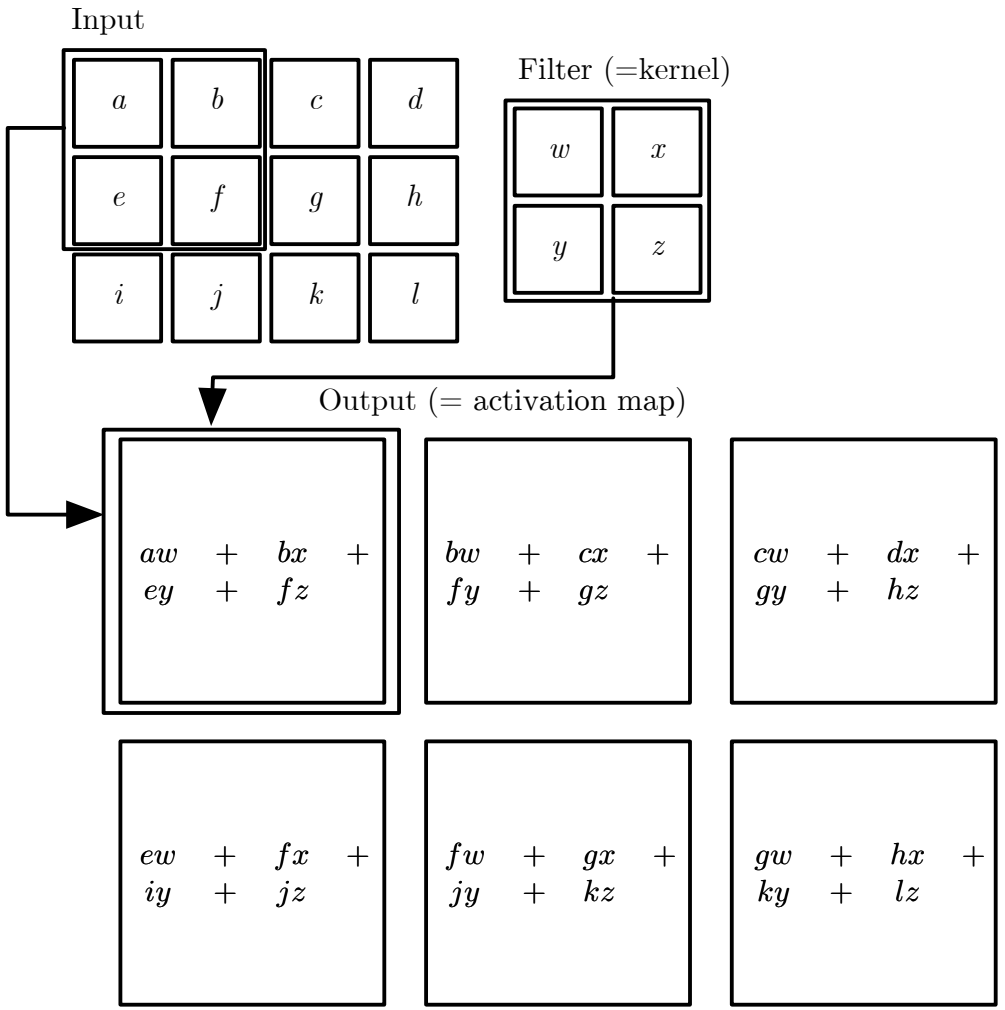
- we call both operations convolution
  - specify whether ___ (180° rotate) the kernel or not if needed

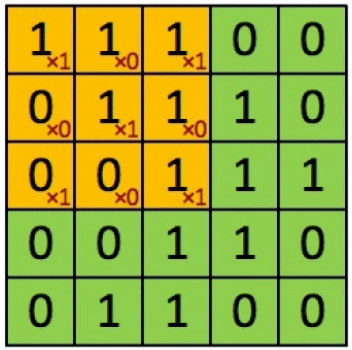
1	2	3
4	5	6
7	8	9

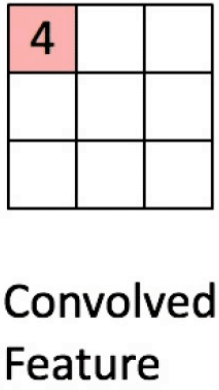
$\Longleftrightarrow$
flip

9	8	7
6	5	4
3	2	1

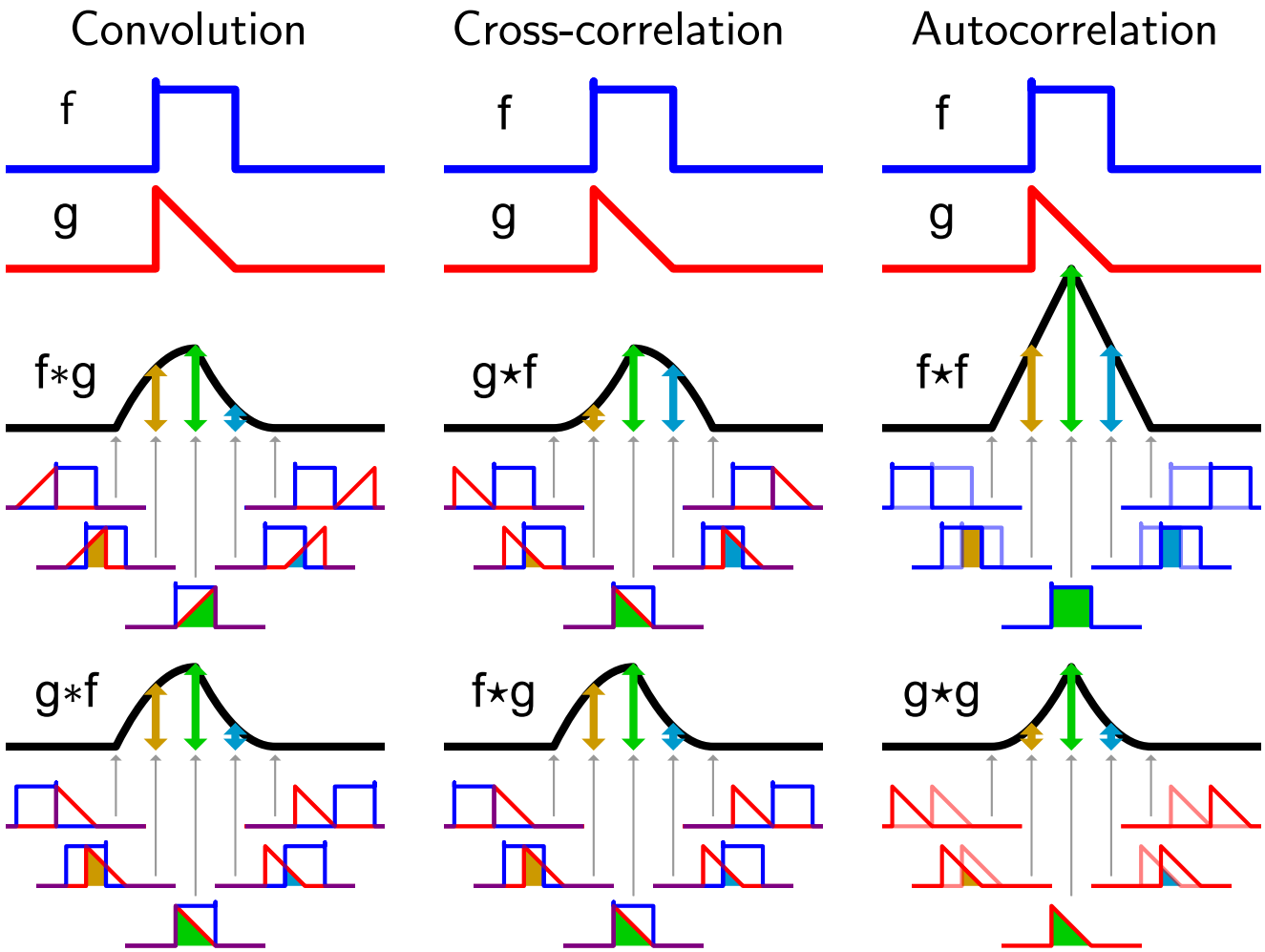
- example: convolution applied to a 2D tensor (without kernel flipping)







Comparison



(source: Wikipedia)

Filters

- feature detectors

  e.g. vertical edge detection

image

10	10	10	0	0	0
10	10	10	0	0	0
10	10	10	0	0	0
10	10	10	0	0	0
10	10	10	0	0	0
10	10	10	0	0	0

$*$

filter

1	0	-1
1	0	-1
1	0	-1

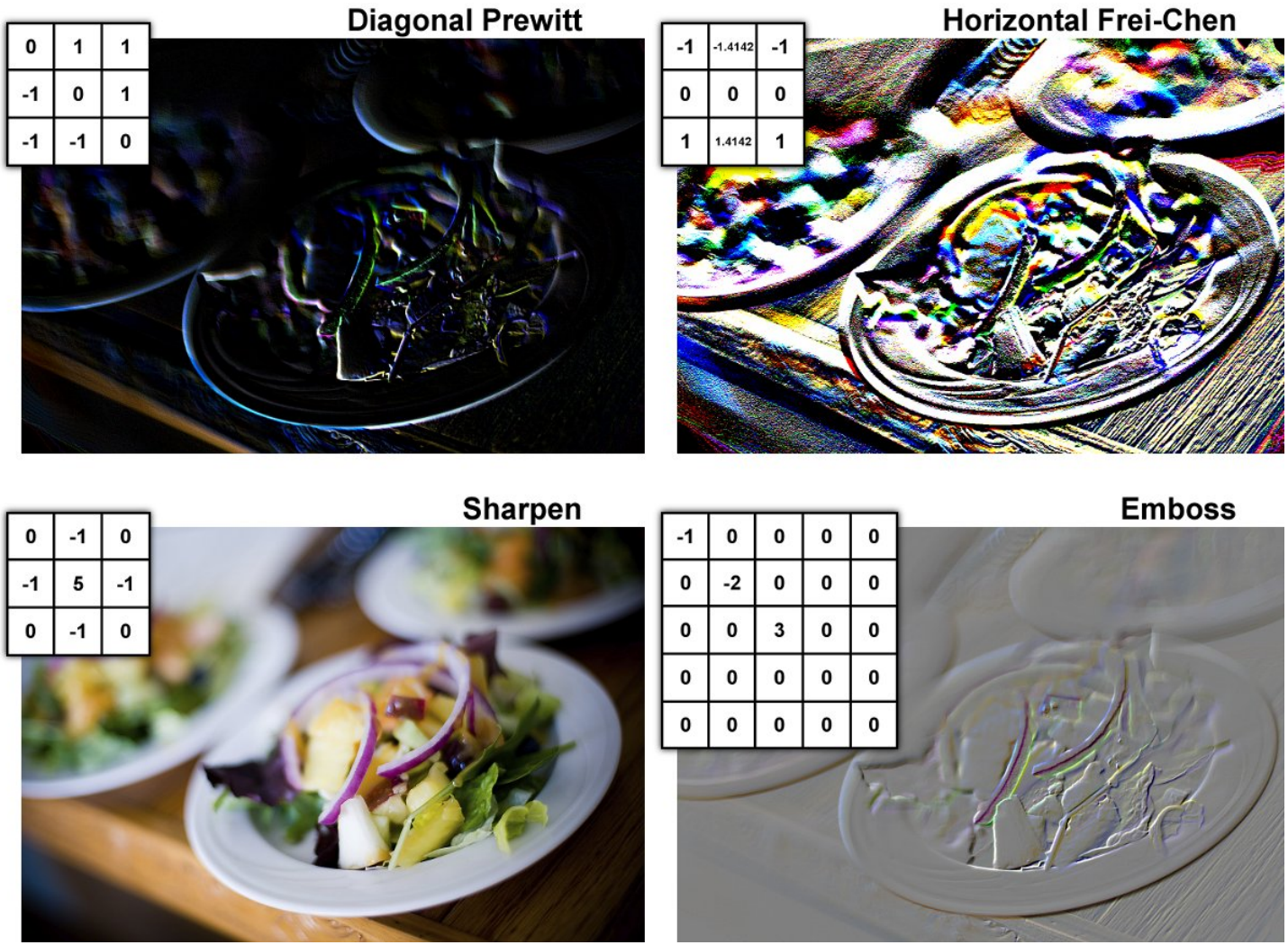
$=$

activation map

0	30	30	0
0	30	30	0
0	30	30	0
0	30	30	0

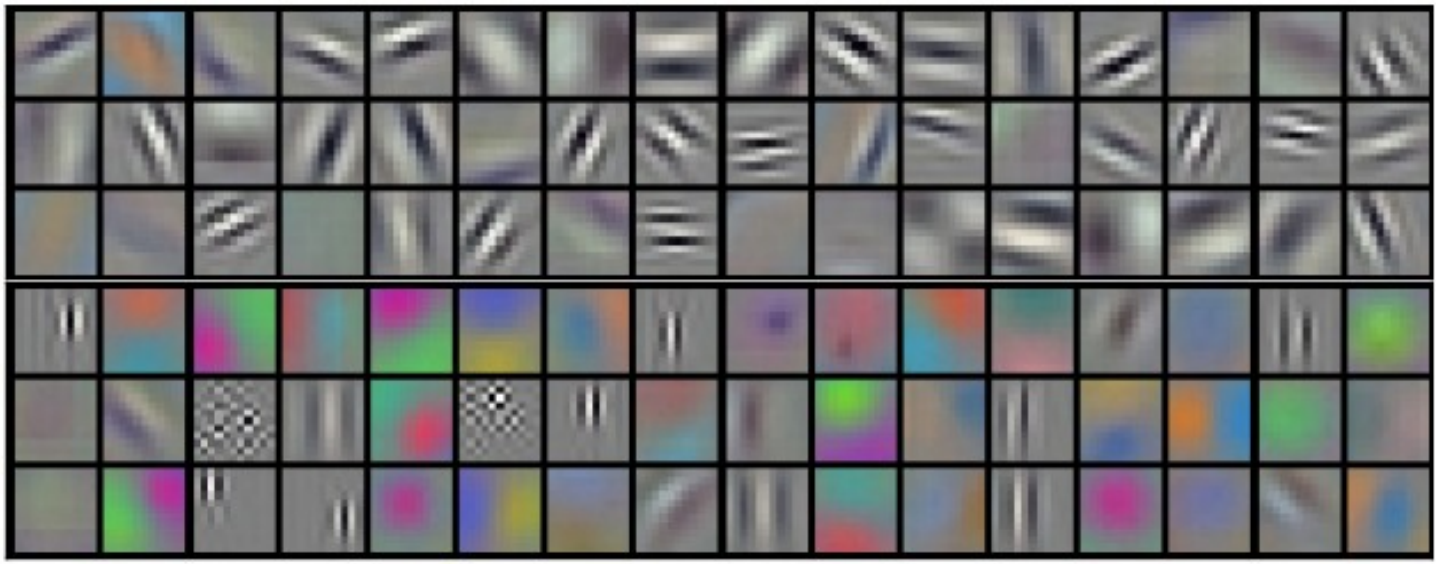
- hand-designed (conventional ML) vs _____ (CNN)

- filter examples (hand-designed):



(source: www.gimpbible.com)

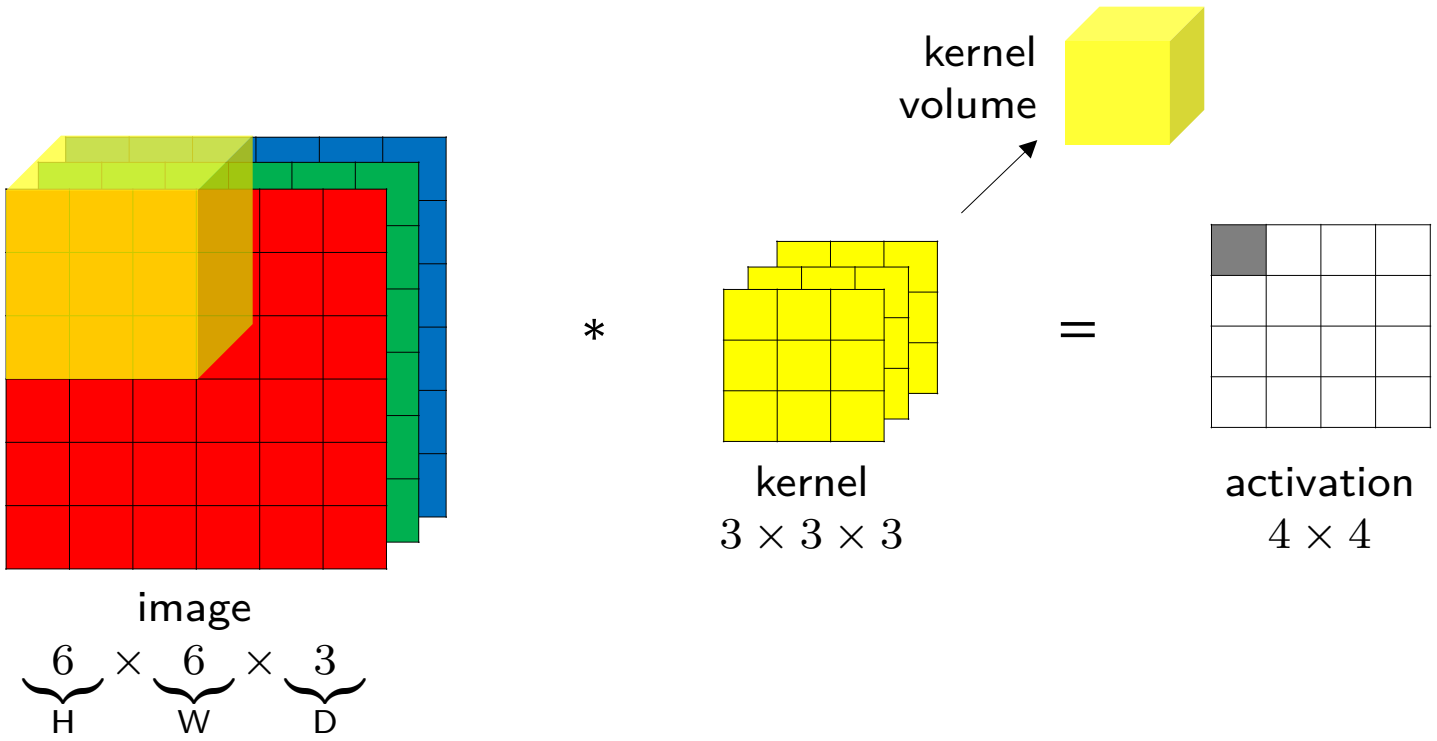
- filter examples (learned):
  - 96 filters (size: 11×11) learned by AlexNet



(source: Krizhevsky et al.)

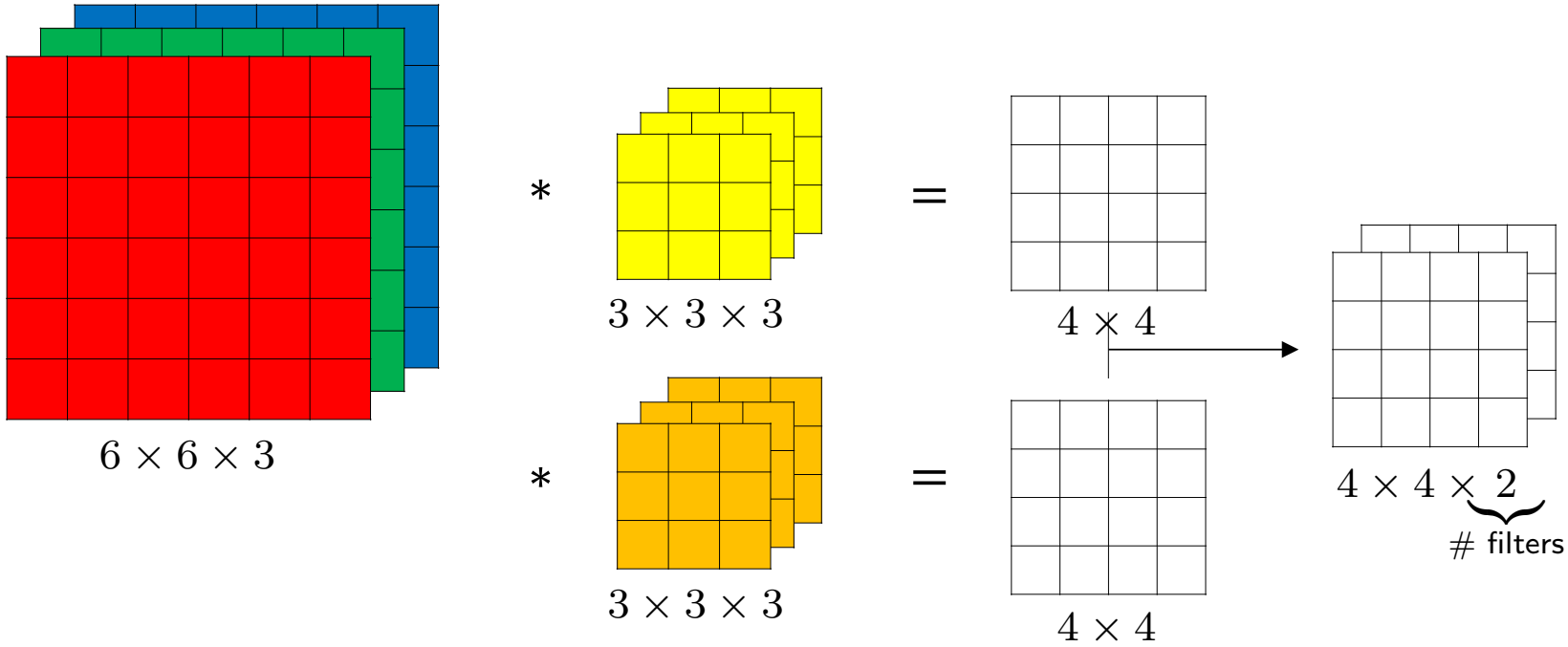
Outline

Convolutions over RGB image



(source: Coursera)

Multiple filters



(source: Coursera)

Tensor convolution (textbook notation)

- representations
  - kernel: 4D tensor **K**

$$\mathsf{K}_{\underbrace{i}_{\substack{\text{output}\\\text{channel}}},\underbrace{j}_{\substack{\text{input}\\\text{channel}}},\underbrace{k}_{\substack{\text{row}\\\text{index}}},\underbrace{l}_{\substack{\text{column}\\\text{index}}}}$$

  - input (observed data): 3D tensor **V**

$$\mathsf{V}_{\underbrace{i}_{\text{channel}},\underbrace{j}_{\text{row}},\underbrace{k}_{\text{column}}}$$

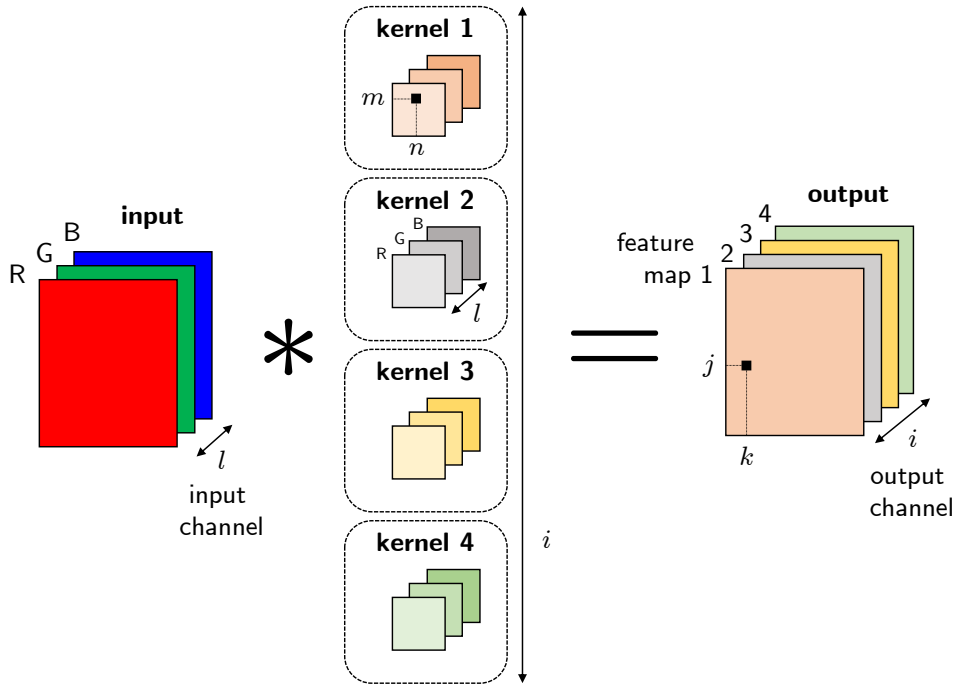
  - output: 3D tensor **Z**

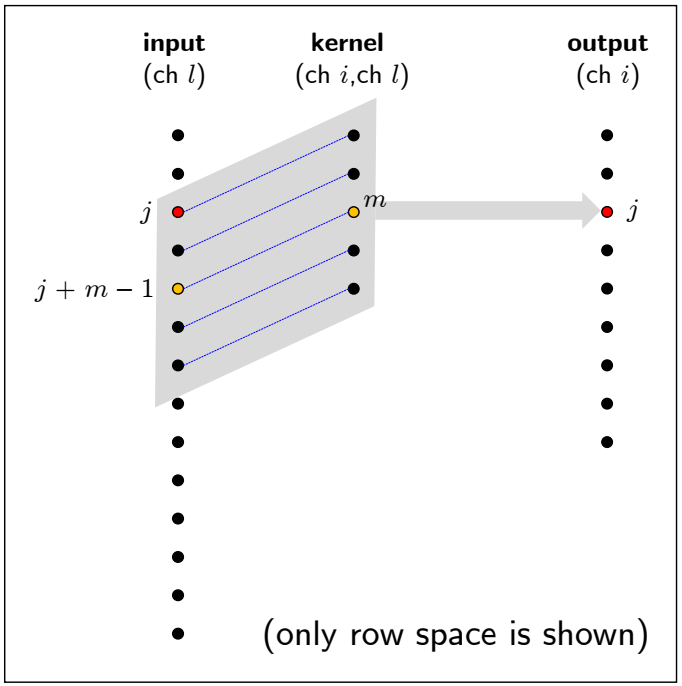
$$\mathsf{Z}_{\underbrace{i}_{\text{channel}},\underbrace{j}_{\text{row}},\underbrace{k}_{\text{column}}}$$

- if **Z** is produced by convolving **K** across **V** without flipping **K** :

$$\mathsf{Z}_{\underbrace{i}_{\substack{\text{out}\\\text{ch}}},\underbrace{j}_{\text{row}},\underbrace{k}_{\text{col}}} = \sum_{l,m,n} \mathsf{V}_{\underbrace{l}_{\substack{\text{in}\\\text{ch}}},\underbrace{j+m-1}_{\text{row}},\underbrace{k+n-1}_{\text{col}}} \mathsf{K}_{\underbrace{i}_{\substack{\text{out}\\\text{ch}}},\underbrace{l}_{\substack{\text{in}\\\text{ch}}},\underbrace{m}_{\text{row}},\underbrace{n}_{\text{col}}}$$

  - the summation[2] over l, m and n : over valid indices

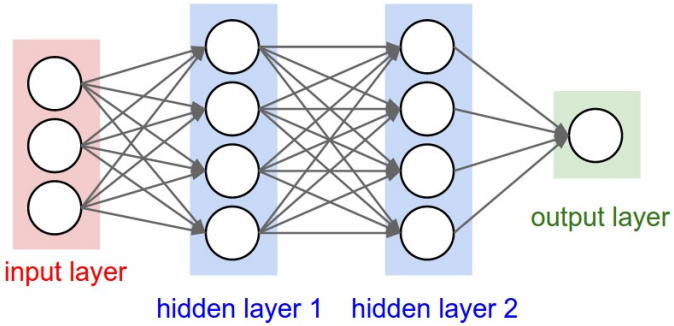




[2] in linear algebra notation, we index into arrays using a 1 for the first entry, which necessitates the -1 above; programming languages (such as C and Python) index starting from 0, rendering the above expression even simpler

Outline

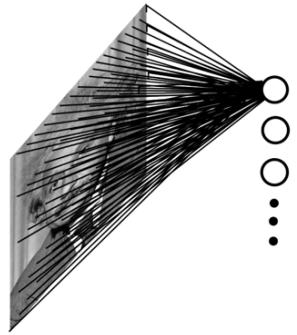
Review



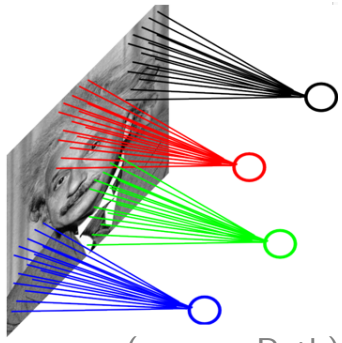
(source: cs231n)

- feedforward neural nets
  - receive an input (a single vector), and
  - transform it through a series of hidden layers
- each hidden layer: made up of a set of neurons
  - each neuron: ____ connected to all neurons in the previous layer
  - neurons in a single layer
    - function completely independently
    - do not share any connections
- the last fully connected layer: called the output layer
  - in classification: represents the class scores

Motivation



- regular neural nets do not scale well to images
  - reason: full connectivity



- consider a fully connected neuron in the first hidden layer
  - to handle a color image of size 1000×1000
  - $\Rightarrow$ the neuron needs $1000 \times 1000 \times 3 = 3 \times 10^6$ weights

(source: Roth)

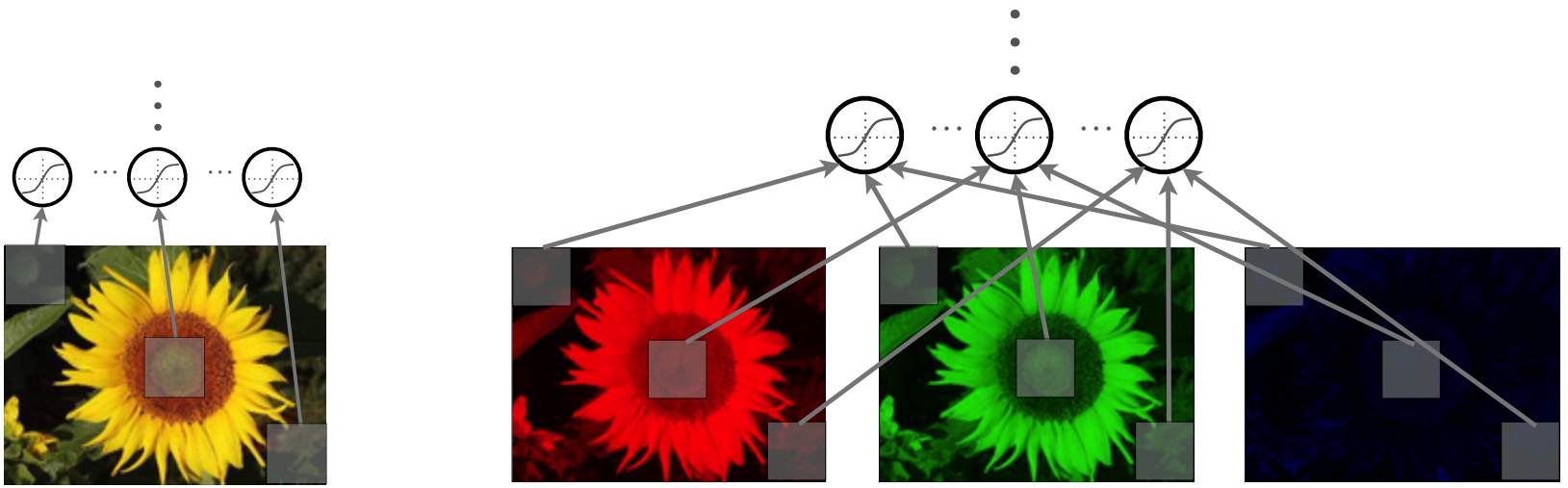
- this full connectivity: wasteful
  - the huge number of parameters $\Rightarrow$ quickly lead to _____
- CNNs
  - take advantage of the fact that input consists of images
  - constrain the architecture in a more sensible way

Local connectivity

- connectivity of each neuron in CNN
  - spatial (height and width) axes: only a local region of the input volume
    - ▷ filter size = size of this local region (called _____ field)

      ↑

      hyperparameter
  - depth axis: the same as input depth (# ch)



r = receptive field

(source: Larochelle)

example:

- input volume: $32 \times 32 \times 3$ (*e.g.* an RGB CIFAR-10 image)
  - if receptive field (or filter size) is 5×5, then
  - each neuron will have weights to a $5 \times 5 \times 3$ region in input volume
  - $\#$ weights $= 5 \times 5 \times 3 = 75$ (and $+1$ ___ parameter)

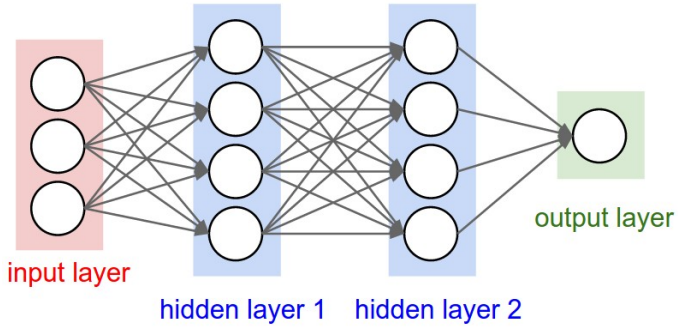
- the extent of the connectivity along the depth axis
  - must be 3 (this is the depth of the input volume)

3D volumes of neurons

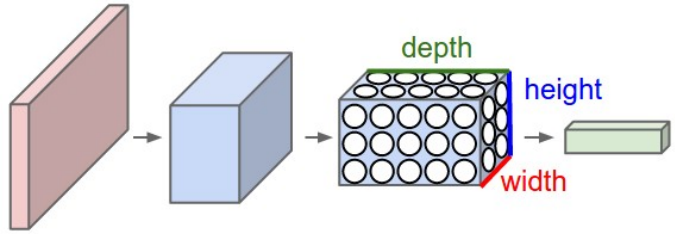
- neurons in a CNN
  - arranged in 3D: height, width, depth[3]

  e.g. a CIFAR-10 image $\Rightarrow$ an input volume of $32 \times 32 \times 3$

- every layer of a CNN
  - transforms 3D input volume to 3D output volume of neuron activations



(a) regular 3-layer neural net



(b) CNN

(source: cs231n)

[3] the word *depth* here refers to the third dimension of an activation volume, not to the depth of a full neural net (= total number of layers in a network)

- the neurons in a layer
  - will only be connected to a small region of the layer before it

    (instead of all of the neurons in a fully connected manner)

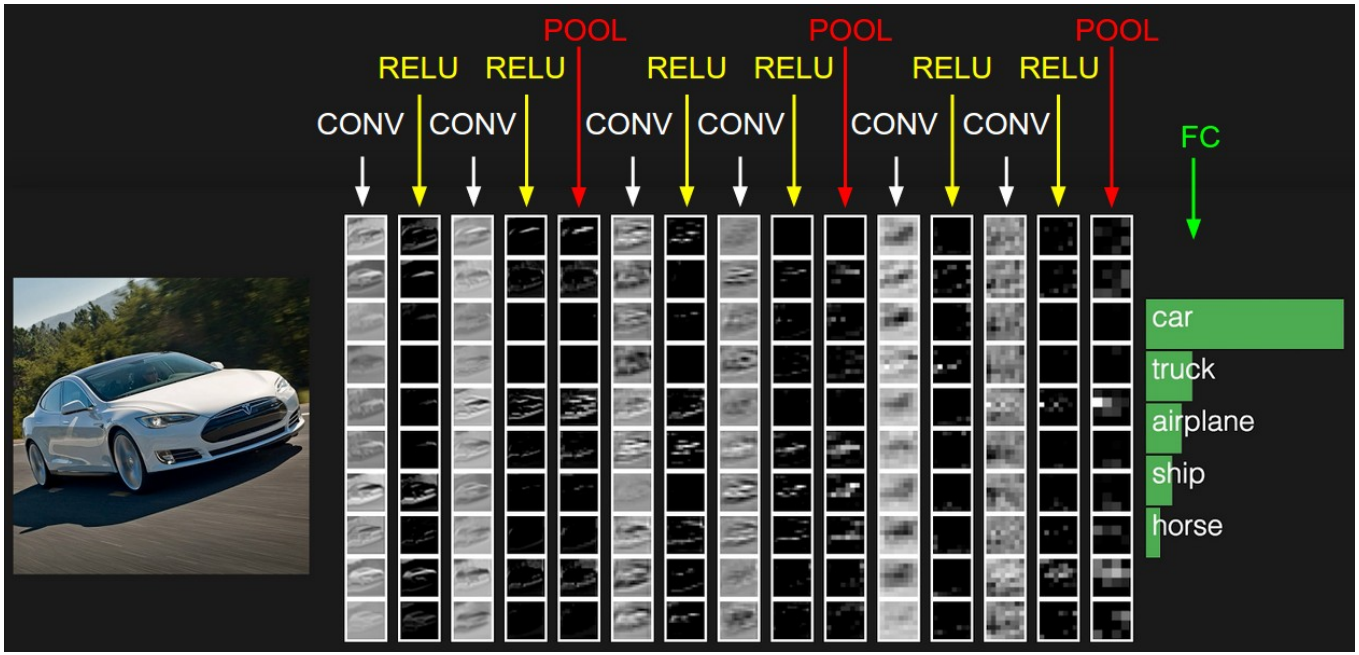
- the final output layer: a single vector of class scores (arranged along the depth)

  e.g. for CIFAR-10: $1 \times 1 \times 10$
  - CNN reduces the full image into a vector of class scores

- bottom line
  - a CNN is made up of layers
  - each layer $=$ a volume transformer
    - $\boxed{\text{3D volume} \xrightarrow{\text{transform}} \text{3D volume}}$
    - uses a _____ function that may or may not have parameters

Building a CNN

- four main types of layers $\Rightarrow$ _____ them gives a full CNN architecture
  1. convolutional layer
  2. RELU layer
  3. pooling layer
  4. fully connected layer



(source: cs231n)

- each layer may or may not have (hyper)parameters

layer	parameters	hyperparameters
CONV		○
RELU		×
POOL		○
FC		○

- parameters
  - trained with gradient descent[4] (in a supervised fashion)

[4] the backward pass for a convolution operation (for both the data and the weights): also a convolution (but with spatially flipped filters)

Hierarchical feature learning

- CNN _____ filters that activate when seeing some type of visual feature
  - implements hierarchical representation learning

e.g. an edge of some orientation or a blotch of some color (first layer)

more complicated patterns (higher layers)

(source: Coursera)

Outline

Convolutional layer

- the core building block of a CNN
  - does most of the computational heavy lifting

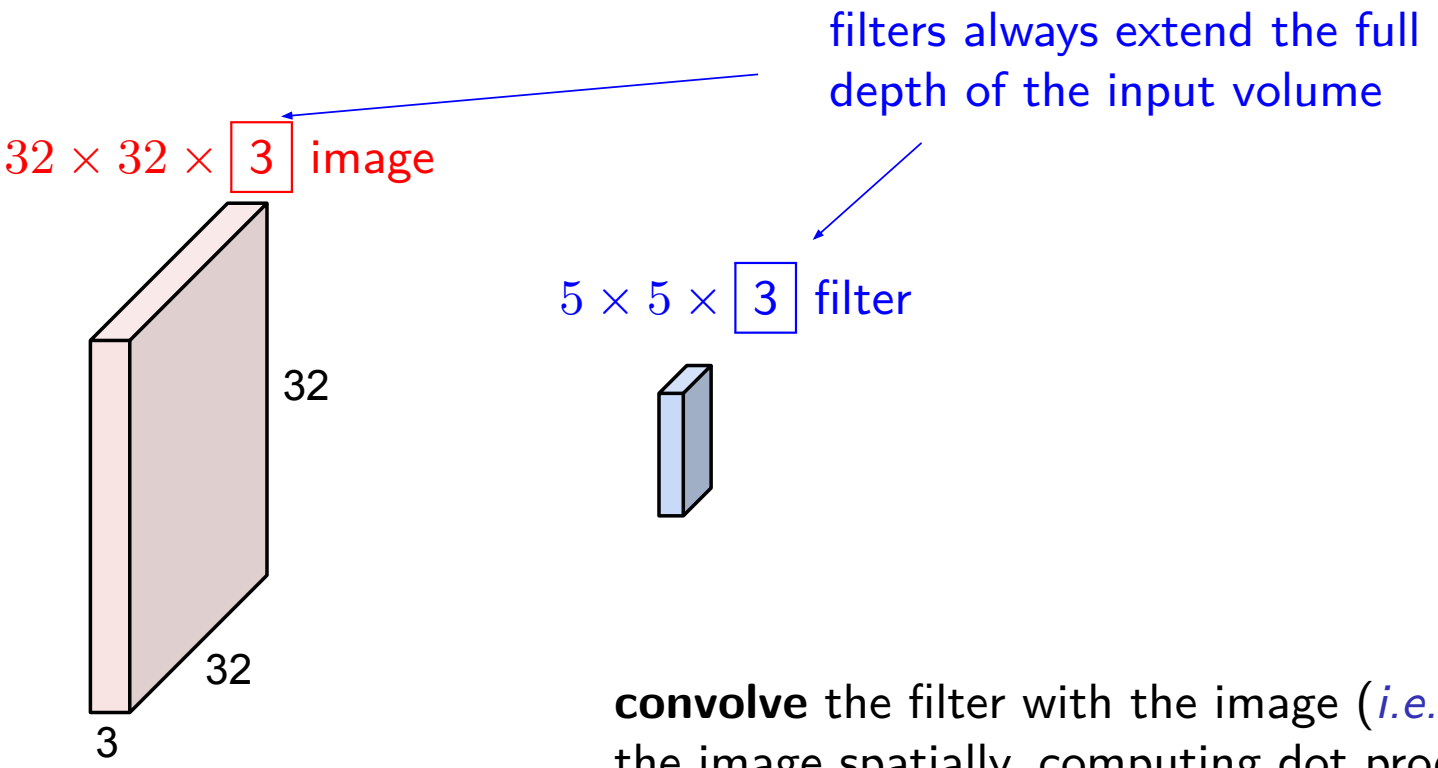
    input volume (3D) $*$ a filter $=$ output map (2D)

    input volume (3D) $*$ filters $=$ output volume (3D)

- each filter (3D):
  - is small spatially along height and width
  - but extends through the _____ of the input volume

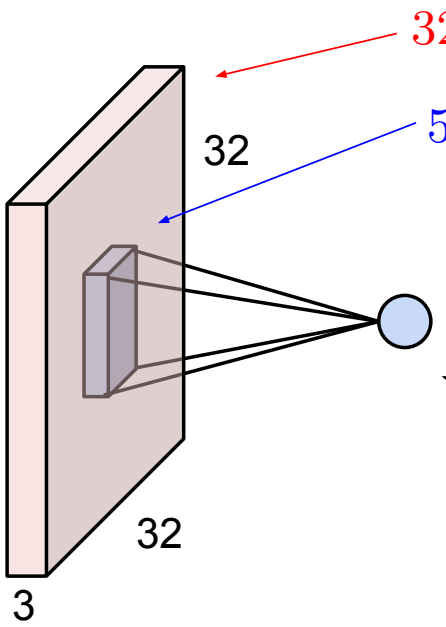
- *e.g.* a typical filter on a first layer of a CNN
  - might have size $5 \times 5 \times 3$ (height $\times$ width $\times$ $\underbrace{\text{depth}}_{\uparrow}$)

    RGB color channels



convolve the filter with the image (*i.e.* slide over the image spatially, computing dot products)

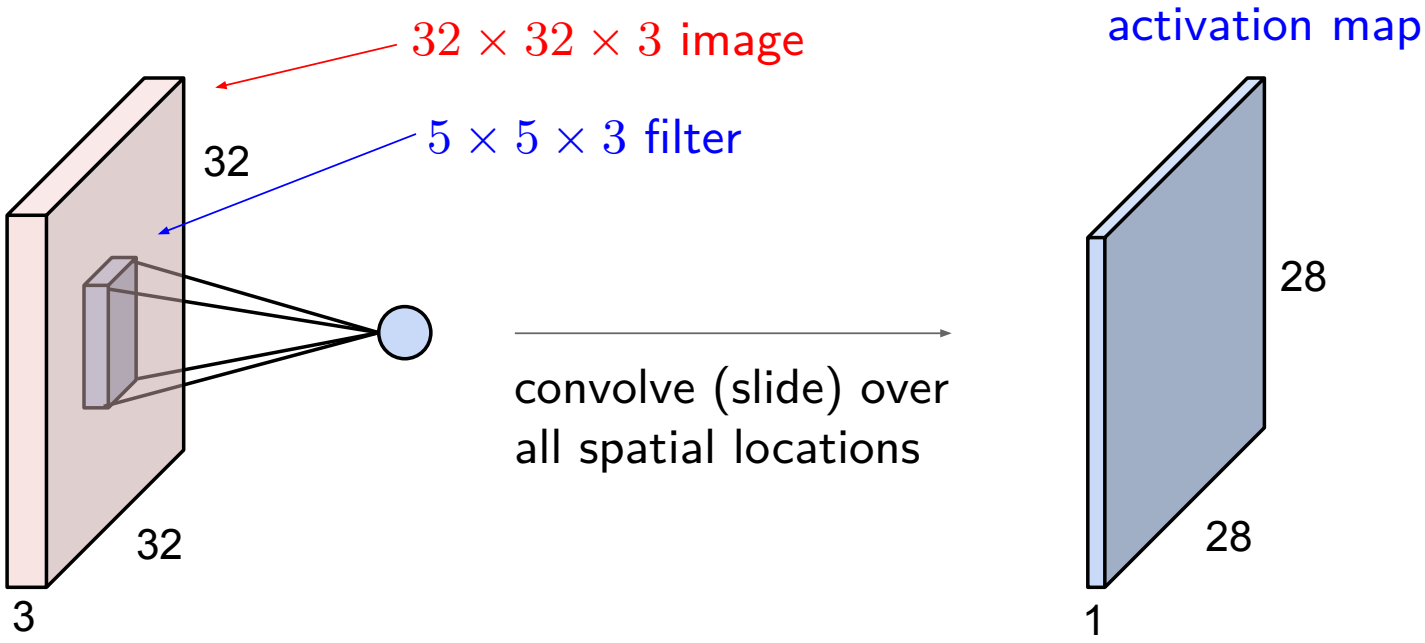
(source: cs231n)



1 number:

the result of taking a dot product between the filter and a small $5 \times 5 \times 3$ chunk of the image

(source: cs231n)



(source: cs231n)

Why convolution?

1. parameter sharing[5]
   - a feature detector (*i.e.* filter) useful in one part of an image

   $\Rightarrow$ probably useful in another part of the image

2. sparse interactions (aka sparse connectivity/weights)
   - in each layer, each output value depends only on a small # inputs

3. flexibility
   - convolution allows us to work with inputs of _____ size

- taken together
  - significant reduction in number of parameters
  - significant gain in performance (generalization & efficiency)

  e.g. 16,000,000,000 ops $\rightarrow$ 267,960 ops (fig 9.6 in textbook)

[5] causes the conv layer to have a property called equivariance to translation

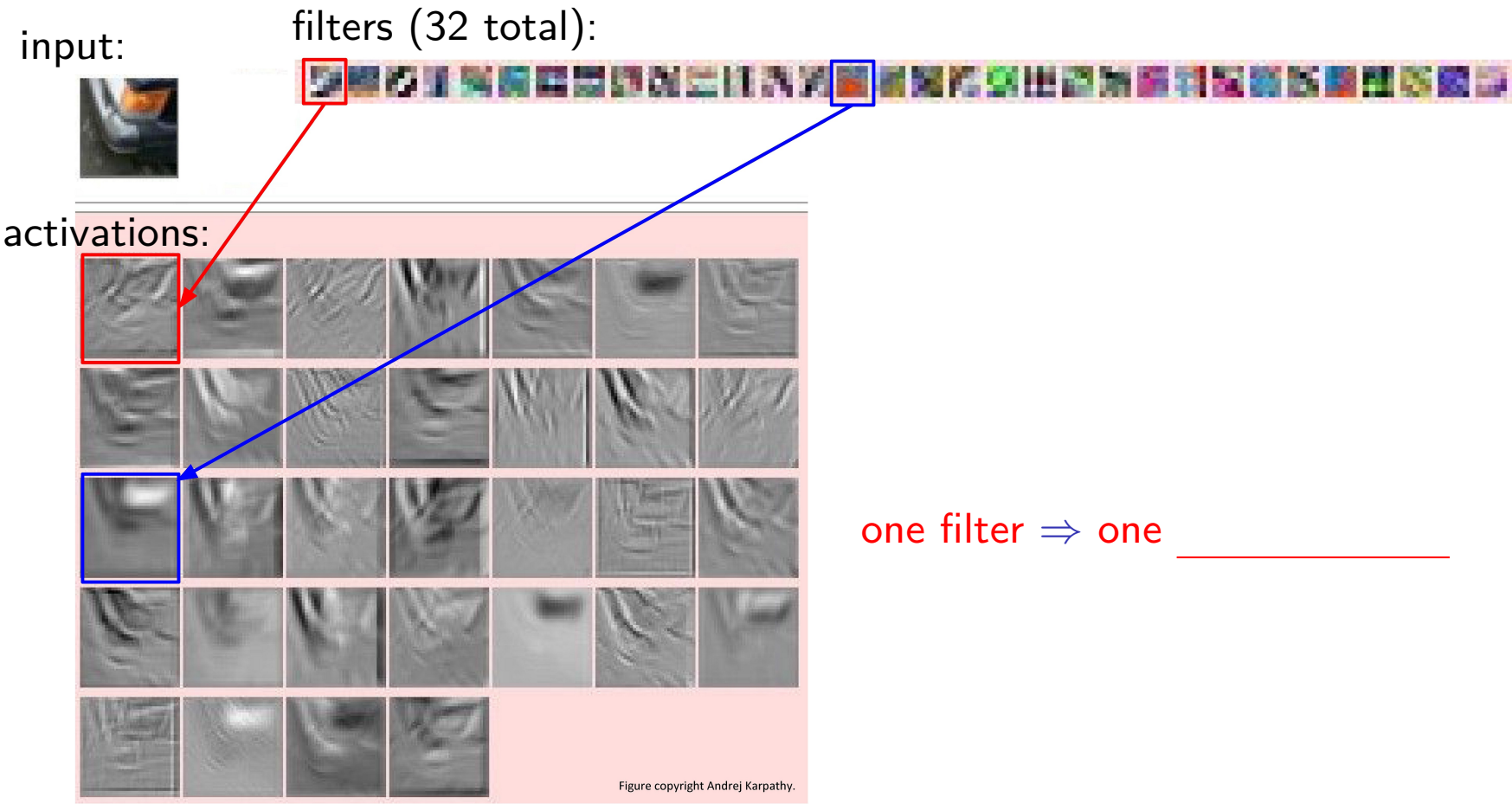
Producing output volume

- each CONV layer
  - has a set of $\underbrace{\text{filters}}_{\uparrow}$ (*e.g.* 12 filters)

    each of them will produce a separate 2D activation map
  - stacks these activation maps along the depth dimension

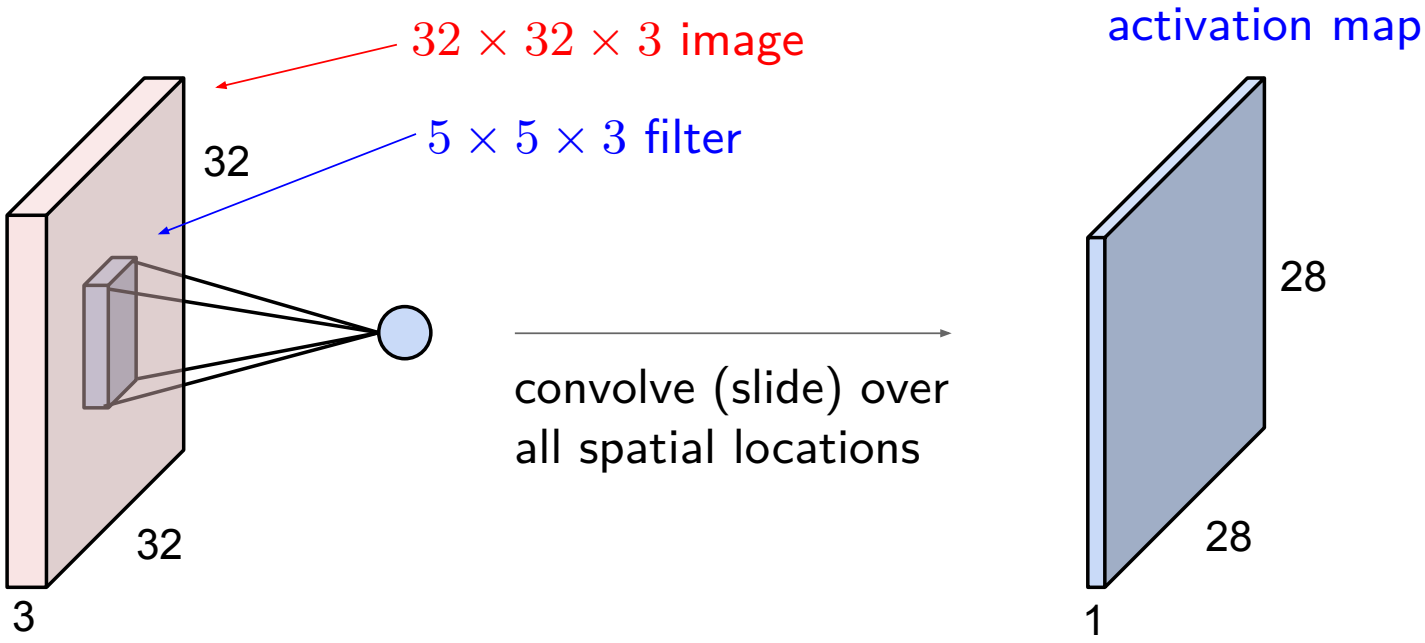
  $\Rightarrow$ produces output volume

- example: applying 32 filters $\Rightarrow$ output depth $= 32$

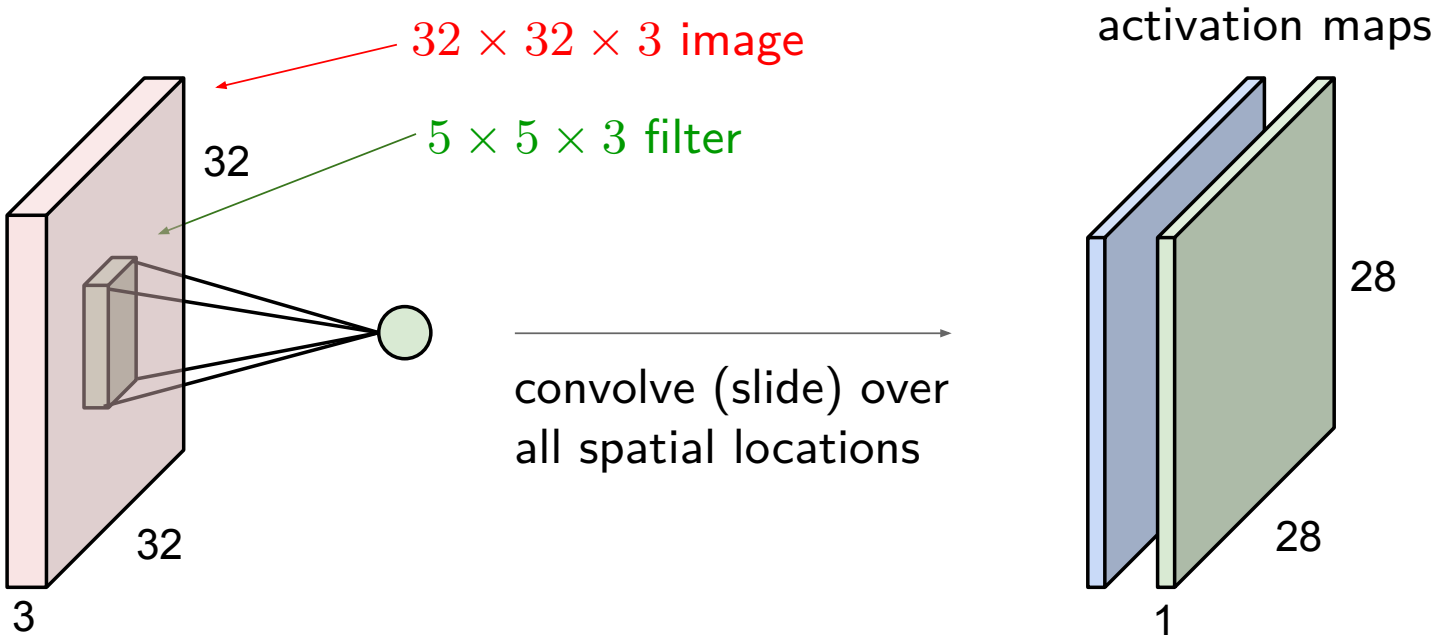


one filter $\Rightarrow$ one _____

(source: cs231n)

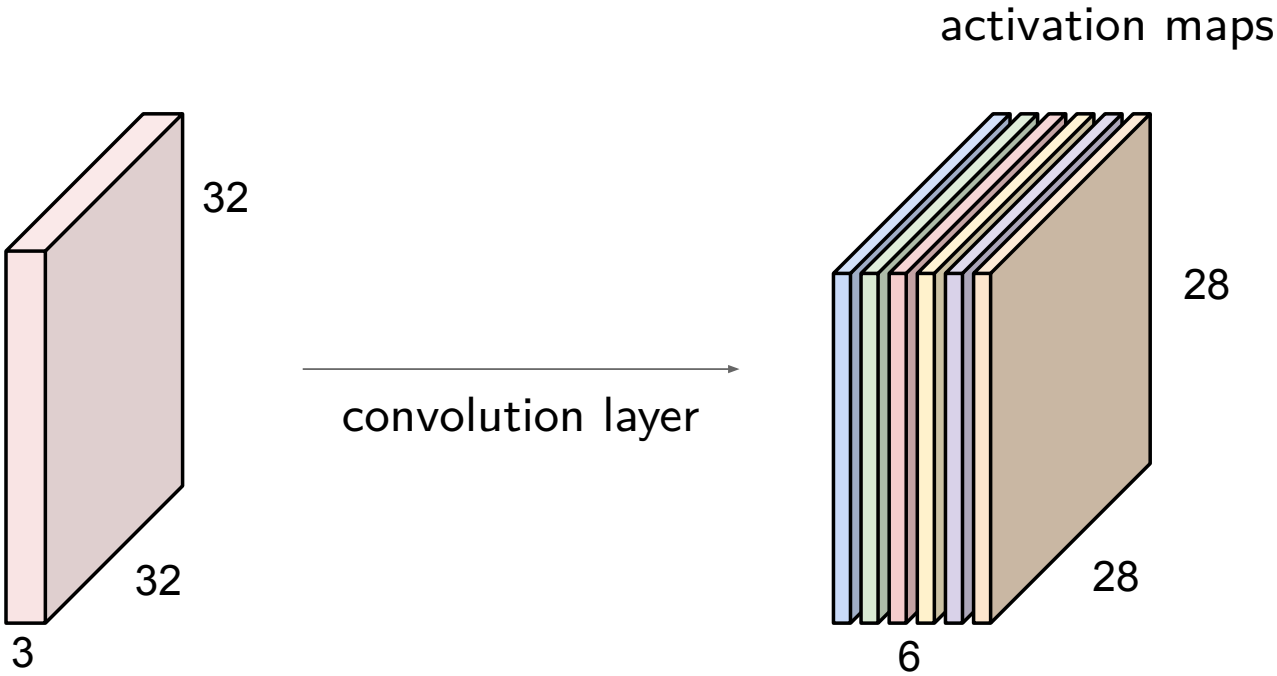


(source: cs231n)



(source: cs231n)

- *e.g.* if we had 6 5×5 filters $\Rightarrow$ get 6 separate activation maps:



(source: cs231n)

- we stack these up to get a "new _____" of size $28 \times 28 \times 6$

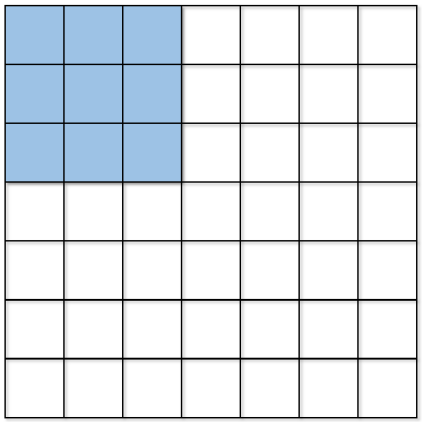
Spatial arrangement

- connectivity of each neuron in CONV layer
  - to the input volume: explained (*i.e.* spatially local but full depth)
  - to the output volume: controlled by three hyperparameters

1. depth
   - means depth of the output volume = # filters we would like to use
2. _____
   - specifies how many pixels to jump when sliding each filter
3. _____
   - how to pad the input volume with zeros around the border
   - controls the spatial size (*i.e.* width and height) of the output volume

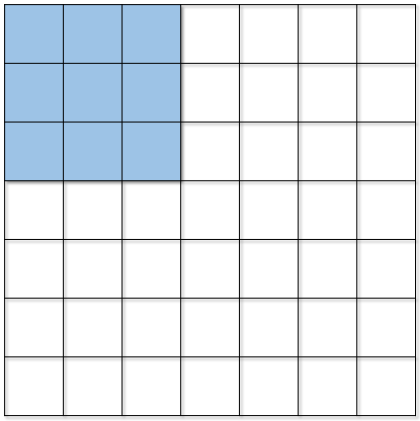
Example

- 7×7 input (spatially)



- filter size: 3×3
- stride: 1

$\Rightarrow$ 5×5 output

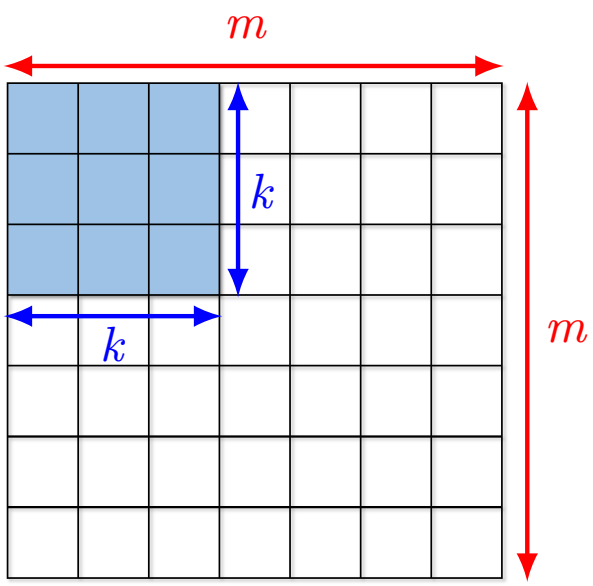


- filter size: 3×3
- stride: 2

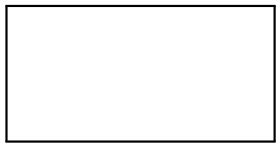
$\Rightarrow$ 3×3 output

Output size

- hyperparameters (spatially)
  - input size: $m \times m$
  - filter size: $k \times k$
  - stride: s



- output size:



- *e.g.* $m = 7,\ k = 3$

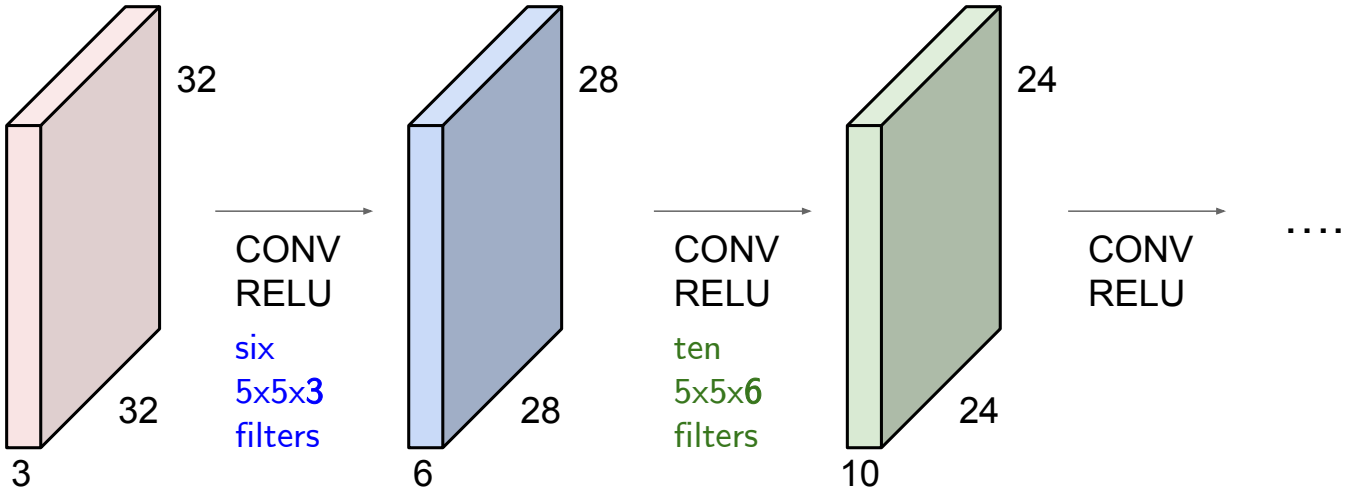
$s = 1 \Rightarrow (7-3)/1+1 = 5$

$s = 2 \Rightarrow (7-3)/2+1 = 3$

$s = 3 \Rightarrow (7-3)/3+1 = 2.3$ (not fit)

Motivations for zero-padding

- consider a 32×32 input convolved repeatedly with 5×5 filters
  - $\Rightarrow$ volume shrinks spatially ($32 \rightarrow 28 \rightarrow 24 \cdots$)
    - shrinking too fast: not good (does not work well)



(source: cs231n)

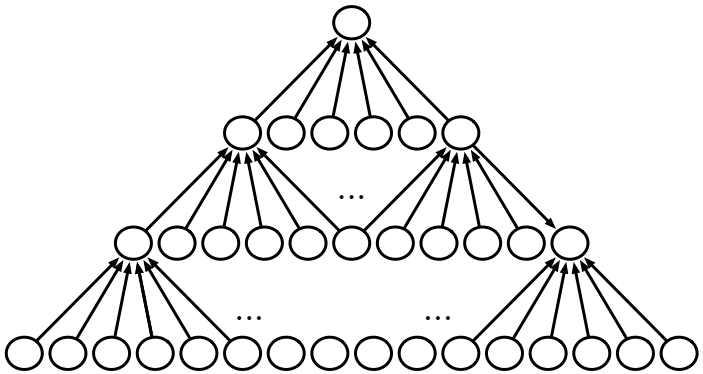
- pixels along the edges: less used for convolutions $\Rightarrow$ potential information loss

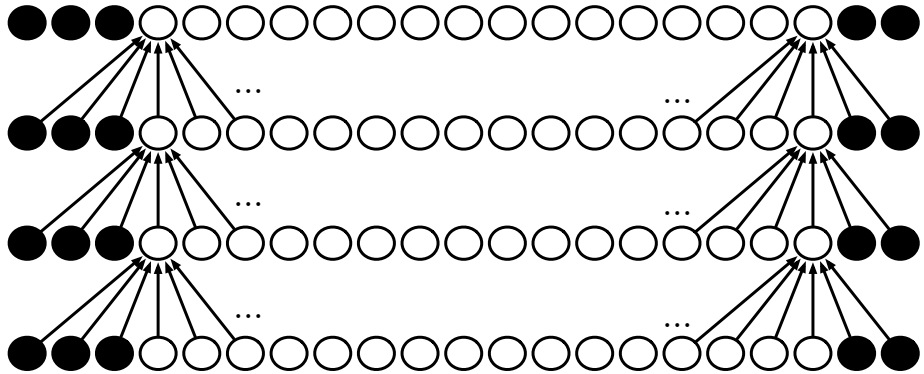
Zero-padding

- one essential feature of any CNN implementation
  - implicitly zero-pad input to make it _____
  - allows us to control kernel width and output size independently

- without this feature, we must
  - shrink the spatial extent of the net rapidly or use small filters

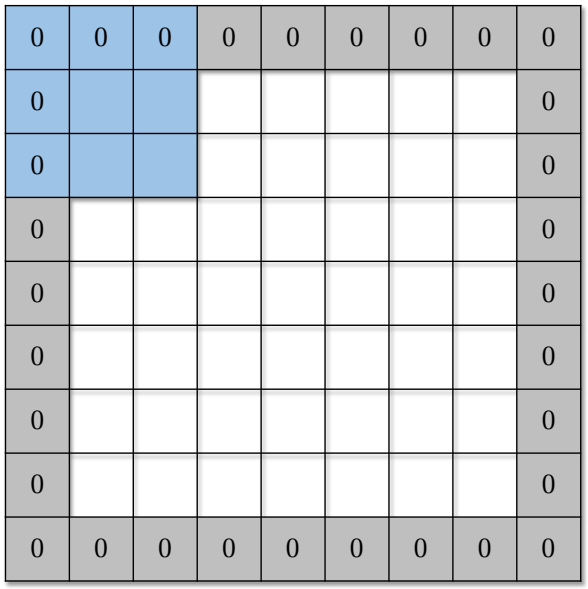
  $\Rightarrow$ both significantly limit expressive power of the net





- also helpful for preserving information along the edges

Example



- 7×7 input, 3×3 filter, stride 1
  - zero-pad with 1 pixel border
  - otuput size: $(7 + 2 - 3)/1 + 1 = 7$

  $\Rightarrow$ input size preserved after convolution

- in general, to preserve spatial size
  - zero-pad with $\boxed{}$

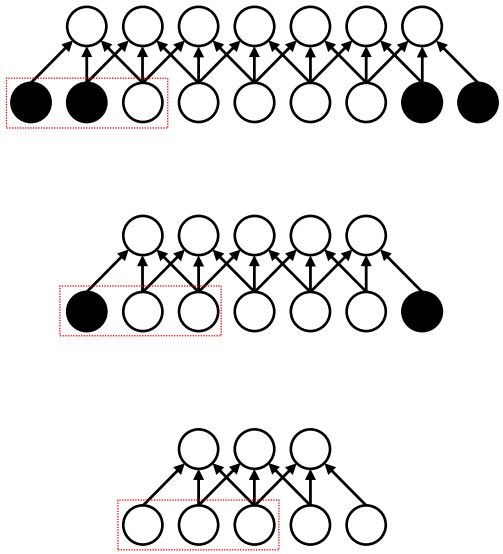
  e.g. $k = 3 \Rightarrow$ zero pad with 1

  $k = 5 \Rightarrow$ zero pad with 2

Three types of zero-padding schemes

type	output	# zeros padded		
		left	right	total
full	$m + (k-1)$	$k-1$	$k-1$	$2(k-1)$
same	m	$\lfloor \frac{k-1}{2} \rfloor$	$\lfloor \frac{k-1}{2} \rfloor + 1$	$k-1$
		$\lfloor \frac{k-1}{2} \rfloor + 1$	$\lfloor \frac{k-1}{2} \rfloor$	
		k even		
		k odd	$\frac{k-1}{2}$	$\frac{k-1}{2}$
valid	$m - (k-1)$	0	0	0

(m: input width; k: kernel width; stride $s = 1$)



optimal zero padding (in terms of test accuracy)

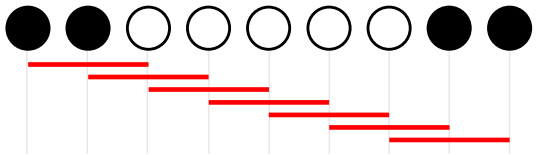
- usually lies somewhere between "valid" and "same" convolution

✳ value of k: typically odd

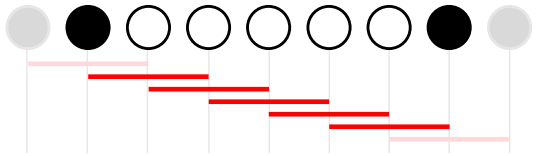
examples:

- $m = 5, k = 3, s = 1$

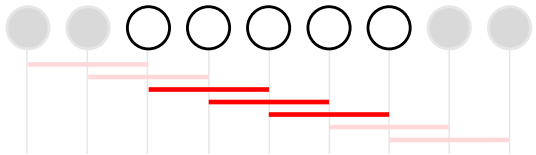
full



same

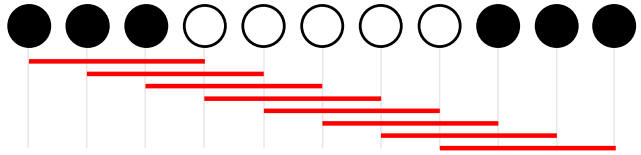


valid

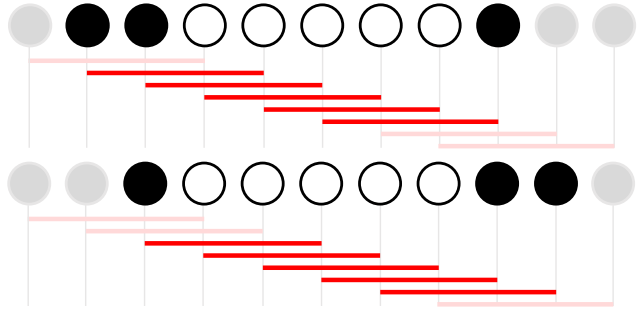


- $m = 5, k = 4, s = 1$

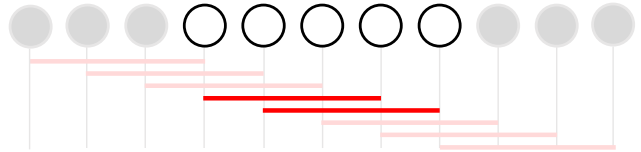
full



same



valid



Summary: activation map size

- input size: $m \times m$
- hyperparameters
  - filter size: $k \times k$ (k : normally odd)
  - padding: p per side
  - stride: s
- output size:

$$\underbrace{\left\lfloor \frac{m+2p-k}{s} + 1 \right\rfloor}_{\text{height}} \times \underbrace{\left\lfloor \frac{m+2p-k}{s} + 1 \right\rfloor}_{\text{width}}$$

Summary: convolution layer l

- notation
  - $f^{[l]}$: filter size
  - $p^{[l]}$: zero padding size
  - $s^{[l]}$: stride size
  - $n_c^{[l]}$: total # filters in l
- total # parameters in l
  - # weights $= f^{[l]} \cdot f^{[l]} \cdot n_c^{[l-1]} \cdot n_c^{[l]}$
  - # biases $= n_c^{[l]}$ (1 per filter)

dimensions

- input: $n_H^{[l-1]} \times n_W^{[l-1]} \times n_c^{[l-1]}$
- each filter: $f^{[l]} \times f^{[l]} \times n_c^{[l-1]}$
- output: $n_H^{[l]} \times n_W^{[l]} \times n_c^{[l]}$
  - height & width:

$$n_H^{[l]} = \left\lfloor \frac{n_H^{[l-1]} + 2p^{[l]} - f^{[l]}}{s^{[l]}} + 1 \right\rfloor$$

$$n_W^{[l]} = \left\lfloor \frac{n_W^{[l-1]} + 2p^{[l]} - f^{[l]}}{s^{[l]}} + 1 \right\rfloor$$

  - for size-m minibatch:

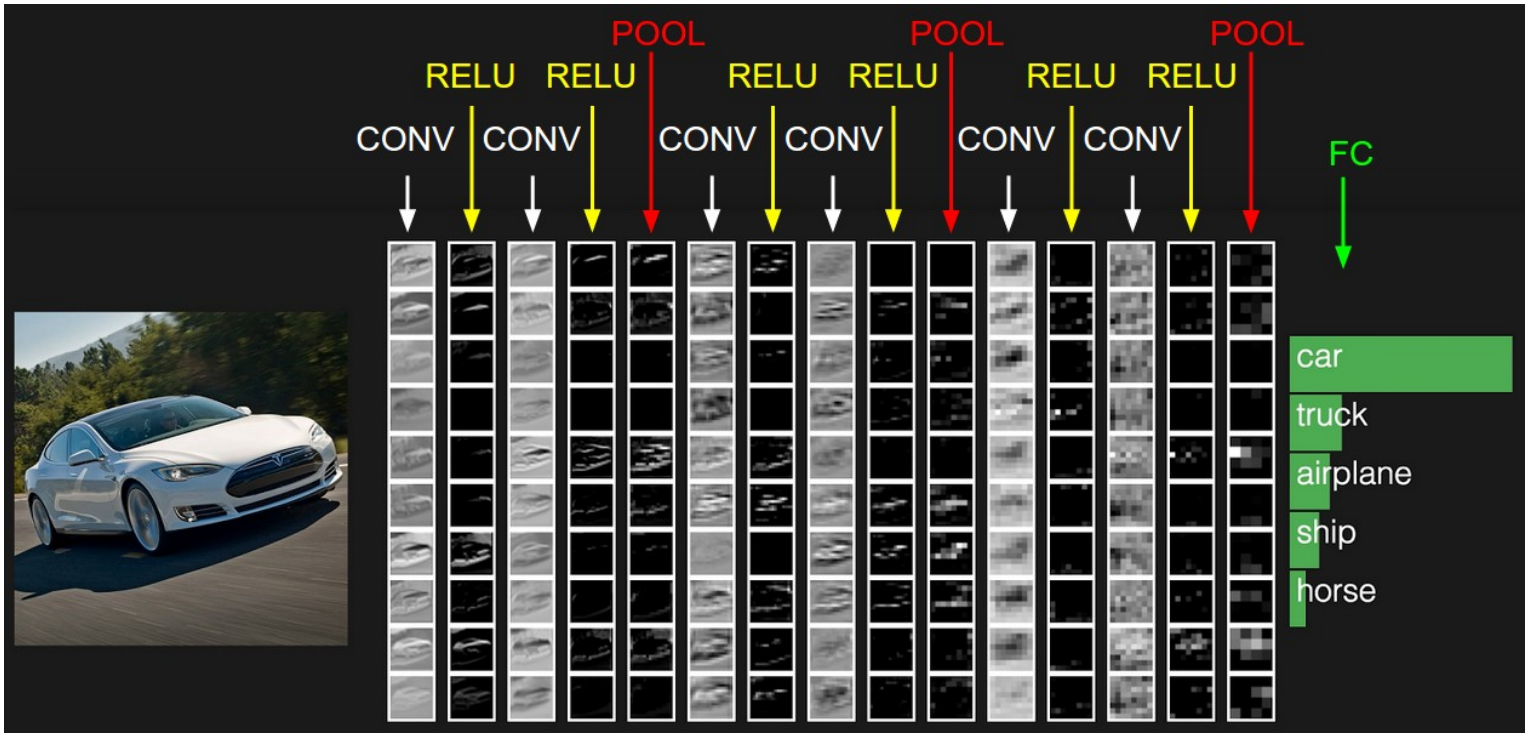
    $\mathbf{A}^{[l]} : m \times n_H^{[l]} \times n_W^{[l]} \times n_c^{[l]}$

    (default order in TF)

Outline

Other layers

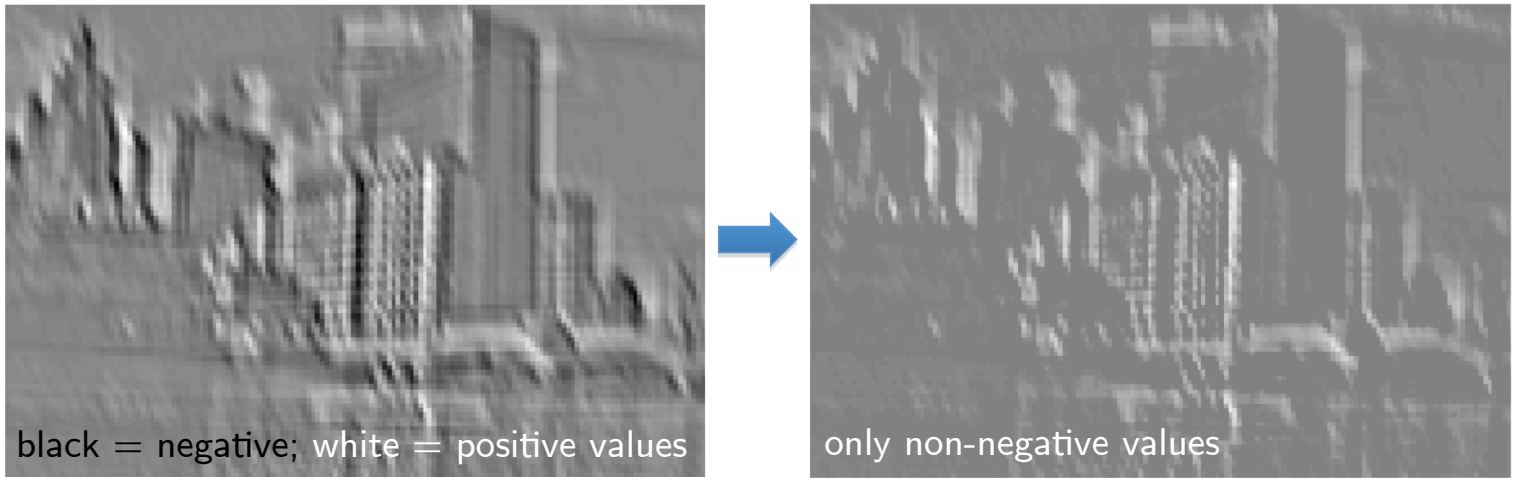
- three more layers to go:
  - RELU layer
  - pooling layer
  - fully connected layer



(source: cs231n)

RELU layer

- purpose
  - increase the _____ properties (of decision function and of overall net)
- facts
  - often called detector (or nonlinear) stage
  - introduces no (hyper)parameters



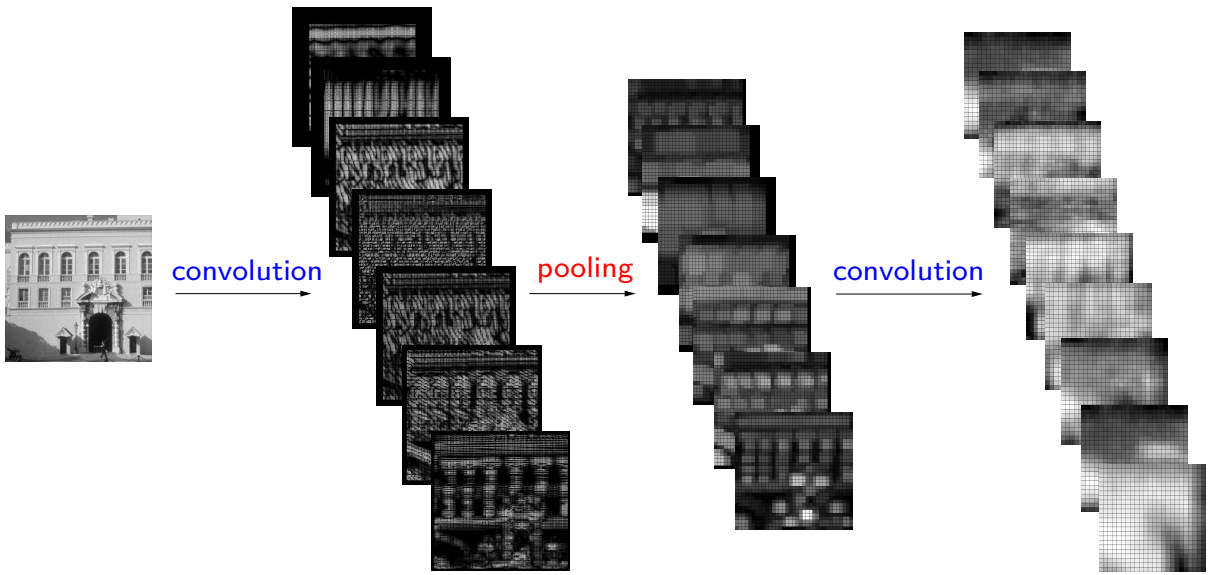
(source: Fergus)

Pooling layer

- common practice:
  - periodically insert a pooling layer in-between successive conv layers

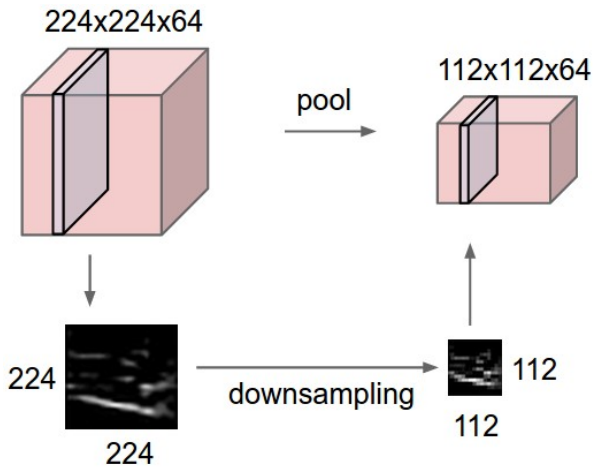
- the function of pooling layers:
  - progressively reduce the _____ size of the representation

  $\Rightarrow$ reduce the amount of parameters and computation + control overfitting



(source: Thériault, 2012)

- each pooling layer
  - operates independently on every depth slice of the input
  - resizes it spatially (using *e.g.* max operation)

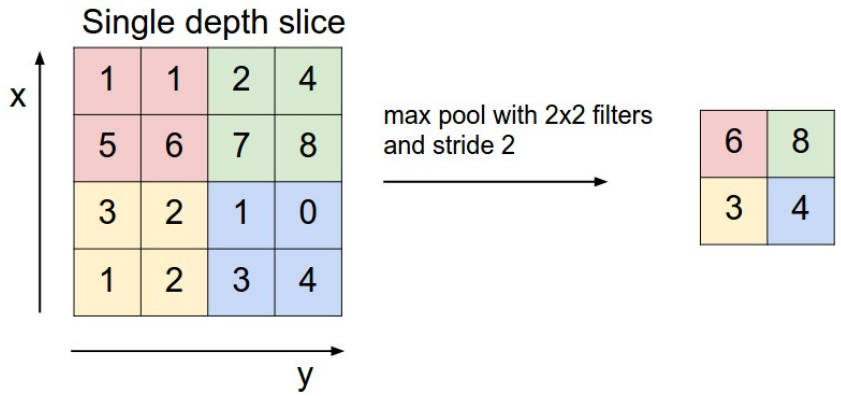


(source: cs231n)

- facts
  - the depth dimension remains _____
  - pooling introduces no parameters
  - pooling gives invariance to (local) translation

Max pooling

- the most common form: 2×2 max pooling with stride 2
  - discards 75% of activations



(source: cs231n)

- other types of pooling: pooling, L^2-norm pooling, ...

↑ often used historically but phased out by max pooling

Eliminating pooling

- some researchers proposes to discard pooling layer
  - in favor of architecture that only consists of repeated CONV layers

  i.e. use _____ in CONV layer once in a while

- also been found important in training good generative models

  e.g. generative adversarial nets (GANs), variational autoencoders (VAEs)

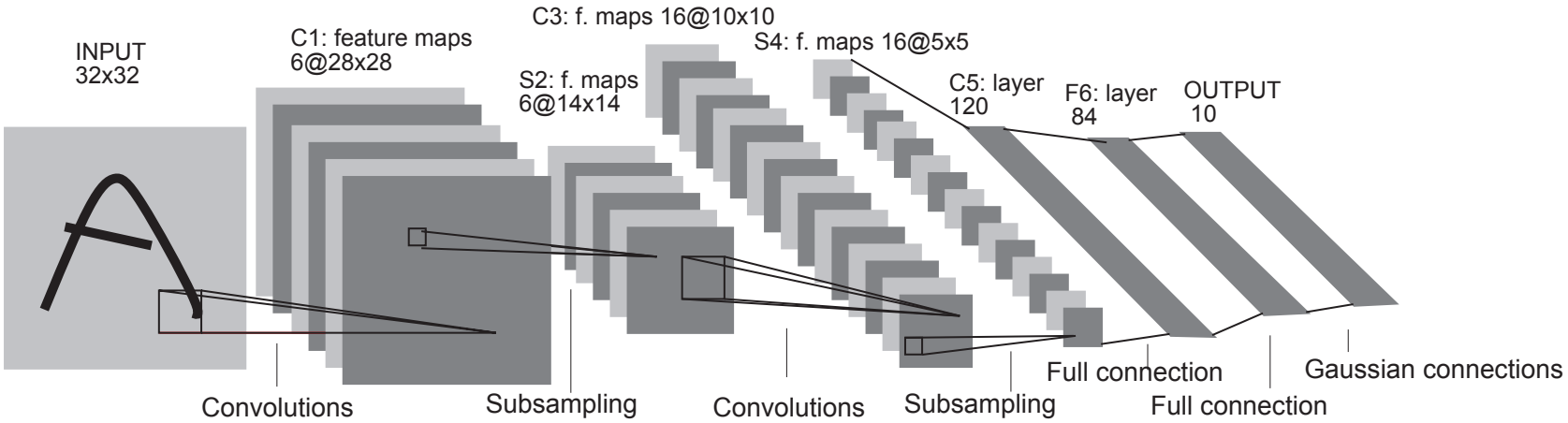
- future architectures
  - likely to feature very few to no pooling layers

Fully connected (FC) layer

- connections:
  - the same as in regular multilayer perceptrons
  - often dropout is used to reduce overfitting in FC layer
- role of FC layer
  - high-level reasoning such as classification and regression
  - often with the softmax function embedded
- remarks: FC layers
  - not spatially located any more $\Rightarrow$ no conv layers possible after a FC layer
  - may account for the majority of _____ in a CNN architecture
    - recent models try to minimize use of FC layers

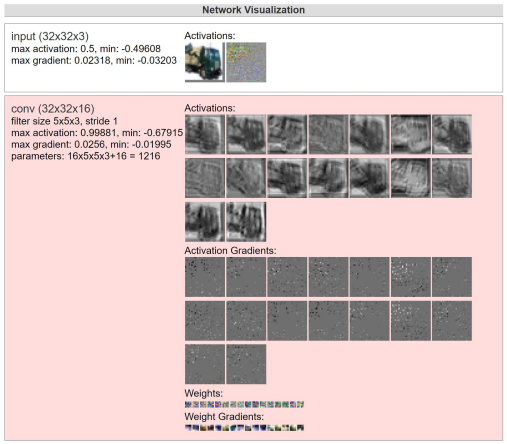
Putting it all together

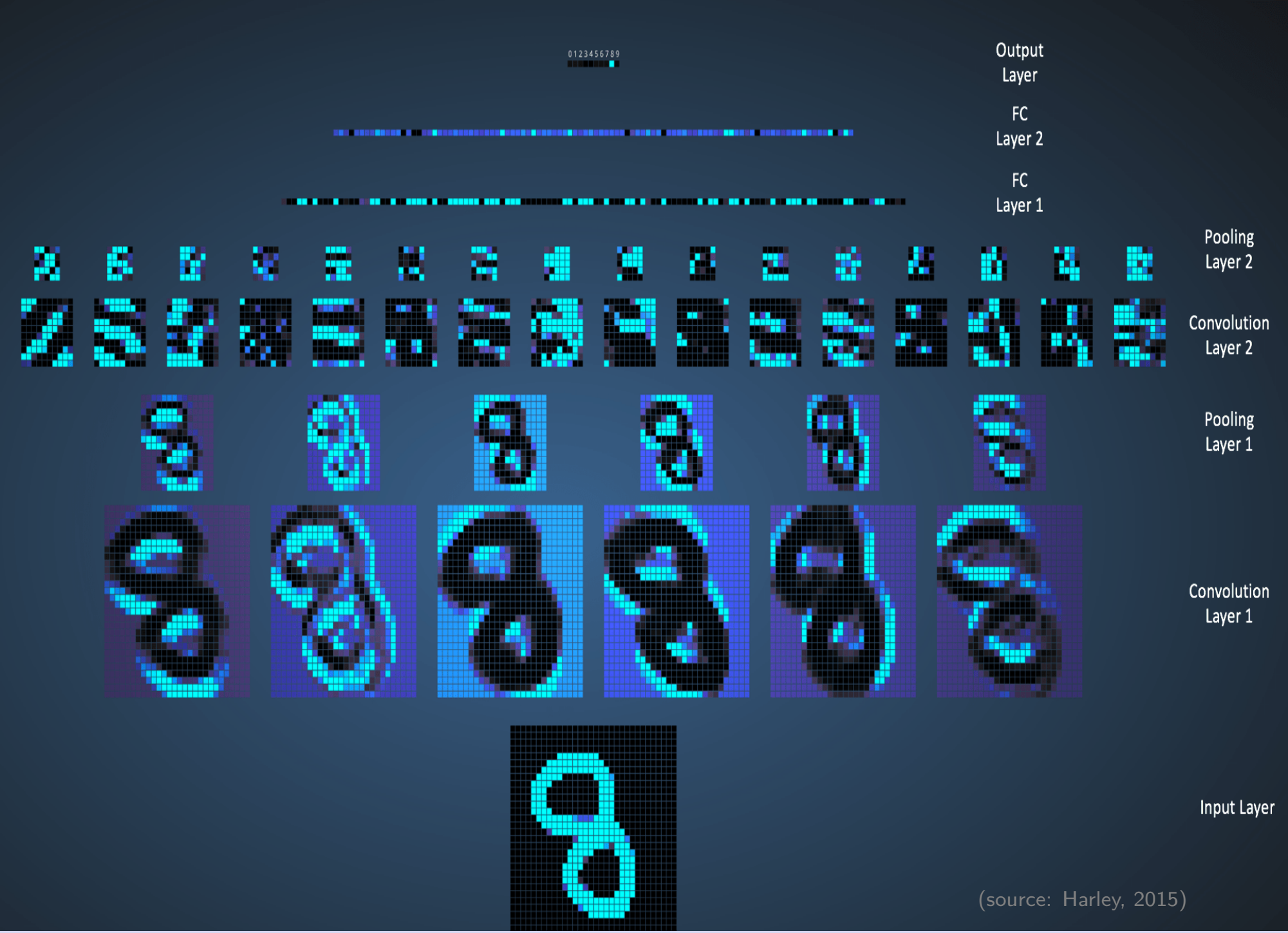
- Lenet-5 for digits recognition:
  - layer $1 \rightarrow$ layer l : input size $\downarrow$ but # filters $\uparrow$ (common trend in CNNs)



(source: LeCun, 1998)

- demo
  - ConvNetJS: training on CIFAR-10 ▸ Link




0 1 2 3 4 5 6 7 8 9
Output Layer
FC Layer 2
FC Layer 1
Pooling Layer 2
Convolution Layer 2
Pooling Layer 1
Convolution Layer 1
Input Layer
(source: Harley, 2015)

Outline

Summary

- convolution (neural) networks (CNNs)
  - neural networks with convolution operations
  - specialized for grid topology (*e.g.* images and time series)
  - tremendous commercial success (intense interest by industry)
- fundamental principles and properties
  - sparse interactions, parameter sharing $\Rightarrow$ efficiency
  - equivariance (convolution), invariance (pooling)
- CNN layers: convolution, RELU, pooling, and fully connected
  - transform 3D input $\rightarrow$ 3D output by differentiable function
  - may or may not have (hyper)parameters
  - trained by backpropagation