Paper Summary: Loan Prime Rate Options

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1 Introduction

The paper *Loan Prime Rate Options* provides an idea of pricing vanilla interest-rate options with underlying asset RMB loan prime rates (LPRs). The LPR is the lending rate provided by commercial banks to their highest-quality customers and prime clients. It serves as the benchmark for lending rates provided for other loans by adding or subtracting basis points based on it. The panel banks submit quotations to the National Interbank Funding Center (NIFC) with 5 basis points (bps) as tick size, before 9am (GMT+8) on the 20th day of each month (holiday postpone). Some stylised features of the randomness in LPR time series could be summarised as:

- non-normal distribution for underlying returns
- pure-jump process without diffusion
- deterministic jump timing on the 20th day of each month (holidays postpone)
- discrete-state jump sizes (**changes can only be the multiples of 5 bps**)

The structure of paper is to firstly introduce benchmark model for the LPR time series and summarise its key properties. Secondly, the LPR swap market would be explained and algorithms for constructing a simple forward LPR curve from swap rate data would be obtained as it is the infrastructure for option market development. Thirdly, a new market model called integer-valued Skellam distribution for pricing vanilla interest-rate options would be introduced.

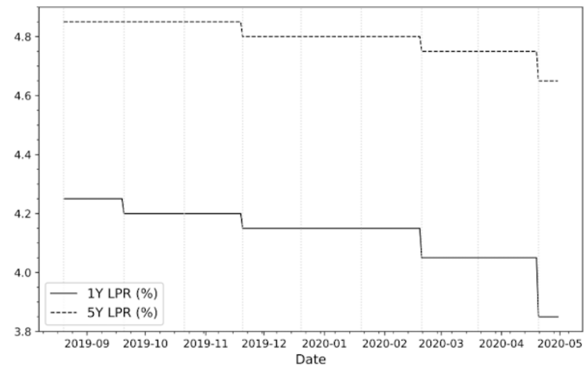

Figure 1: Daily time series of spot loan prime rates (1Y LPR and 5Y LPR since 2019/8/20)

The Figure 1 is a plot of LPR.

2 Benchmark Model for LPR

2.1 Skellam Model Introduction

To simulate the features of LPR appears in Figure 1, we firstly introduce a continuous-time model for the dynamics of discrete-valued spot LPR. Let $L(t, t+\tau)$ be the time-t spot LPR that lend to borrowers for the period $[t, t+\tau]$ with a fixed period τ prevailing at time t. There are only two choices of τ: $\tau = 1, 5$ in the unit of year. For simplicity, we denote both by

$$L(t) := L(t, t+\tau), \qquad t \geq 0$$

and it is characterised by the stochastic differential equation (SDE):

$$dL(t) = bd\mathbb{D}(t) \tag{2.1}$$

where

- $\mathbb{D}(t)$ is a pure-jump process on a filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$,

$$\mathbb{D}(t) := \sum_{k=0}^{n(t)} D(k), \qquad D(.) \in \mathbb{Z}$$

- $b > 0$ is a constant representing the minumum amount that the LPR can change, i.e., tick size $b = 0.05$;
- $n(t)$ is a deterministic right-continuous point process with $n(0) = 0$, counting the total number of LPR-announcement times within the period $[0, t]$, i.e.

$$n(t) := \sum_{k=1}^{\infty} \mathbb{I}\{a_k \leq t\}$$

- $\{a_k\}_{k=1,...}$ are the ordered sequence of predetermined LPR-announcement times and the Figure 2 reflects the $n(t)$ process;
- tick change $D(k)$ is an integer on the set of integers $\mathbb{Z} := \{0, \pm 1, \pm 2, ...\}$ representing the number of ticks that the spot LPR changes at LPR-announcement arrival time a_k and $D(0) := 0$.

To solve SDE (2.1) conditional on initial level $L(0)$ gives the dynamics of LPR.

$$L(t) := L(0) + b\sum_{k=0}^{n(t)} D(k) = L(0) + b\mathbb{D}(t), \tag{2.2}$$

And the discrete-valued random jump size at each LPR-announcement arrival time can be iteratively expressed by

$$\Delta L(a_{k+1}) := L(a_{k+1}) - L(a_{k+1}^-) = bD(k+1), \qquad k = 1, ...$$

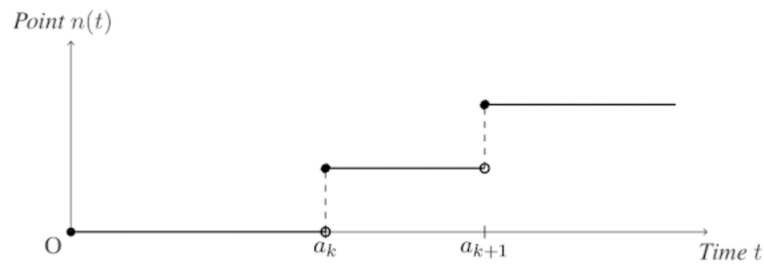

Figure 2: Deterministic right-continuous point process n(t)

such that $L(a_{k+1}^-) = L(a_k)$.

Since the LPR announcements occur monthly and we ignore any delays due to weekends or holidays, then $a_{k+1} - a_k = \frac{1}{12}$ for any $k = 1, ...$ and $a_1 \in (0, \frac{1}{12}]$, and thus $n(t)$ is a deterministic function of t and can be written explicitly as:

$$n(t) = \lfloor 12(t - a_1) \rfloor + 1$$

if $a_1 = 1/12$, then it is simplified as

$$n(t) = \lfloor 12t \rfloor, \qquad t \geq 0$$

According to take the floor, we could conclude that $n(t) - n(s) \neq n(t - s)$ for all $s \in [0, t)$ and thus we could not use Levy processes models to describe $L(t)$. And we can see that $\{D(k)\}_{k=1,...}$ are the only random components in LPR dynamics (2.2). Instead, we introduce Skellam distribution (Poisson-difference) distribution to model the tick change $D(.)$. The Skellam distribution construct the difference between two independent Poisson-distributed random variables N^+, N^- with constant rate parameters $\lambda^+, \lambda^- > 0$ respectively. Here λ^+ can be considered as the number of upward jumps while λ^- is the number of downward jumps.

$$D := N^+ - N^-$$

we denote $D \sim SK(\lambda^+, \lambda^-)$ on $\mathbb{Z}$ with pmf:

$$Pr(D = d; \lambda^+, \lambda^-) = e^{-(\lambda^+ + \lambda^-)} (\frac{\lambda^+}{\lambda^-})^{\frac{d}{2}} I_{|d|}(2\sqrt{\lambda^+ \lambda^-}), \qquad d \in \mathbb{Z} \tag{2.3}$$

and cdf:

$$\Psi(u) := Pr(D \leq u) = \sum_{d=-\infty}^{\lfloor u \rfloor} e^{-(\lambda^+ + \lambda^-)} I_{|d|}(2\sqrt{\lambda^+ \lambda^-}), \qquad u \in \mathbb{R}$$

with mean and variance of D:

$$\mathbb{E}(D) = \lambda^+ - \lambda^-, \qquad Var(D) = \lambda^+ + \lambda^-$$

In case of symmetric Skellam distribution, i.e., $\lambda^+ = \lambda^- = \lambda > 0$, the pmf can be rewritten as:

$$Pr(D = d; \lambda) = e^{-2\lambda} I_{|d|}(2\lambda), \qquad d \in \mathbb{Z} \tag{2.4}$$

2.2 Normal Approximation

Here we explore the relationship between the Skellam-based distribution of $L(t)$ and normal distribution. The result is helpful for explaining the links between Skellam-based option pricing models and Bachelier model in implied volatility.

Proposition 2.2 A normal approximation for $L(t)$ is given by

$$L(t)|L(0) \overset{D}{\approx} N(L(0) + n(t)b(\lambda^+ - \lambda^-), n(t)b^2(\lambda^+ + \lambda^-))$$

and an asymptotics when $\lambda^+ = \lambda^- = \lambda$ is given by

$$\frac{L(t)}{b\sqrt{2\lambda n(t)}} \overset{D}{\to} N(0,1), \qquad t \to \infty$$

Since the tick changes $\{D(k)\}_{k=1,\dots}$ are observable, the sample mean and variance could be approximated by method of moment estimation. Since

$$\hat{\mu}_D = \hat{\lambda^+} - \hat{\lambda^-}, \qquad s_D^2 = \hat{\lambda^+} + \hat{\lambda^-}$$

we have

$$\hat{\lambda^+} = \frac{1}{2}(s_D^2 + \hat{\mu}_D), \qquad \hat{\lambda^-} = \frac{1}{2}(s_D^2 - \hat{\mu}_D)$$

3 LPR Swap and Forward Rate

The LPR Swap market provides the key infrastructure for further development of LPR option market. Currently traded swaps are 1Y-LPR and 5Y-LPR with 9 tenors include 6, 9 months and 1, 2, 3, 4, 5, 7, 10 years, and interests are paid quarterly. The market data could be used for discount rate curve (yield curve) and forward rate curve. The yield curve is used to discount cash flows to the present while the forward rate is used for option pricing. Using forward rate for option pricing is common as the instantaneous spot rates or forward rates are unobservable.

Based on the relationship between market-quoted swap rates and simple forward rate, **we want to find out an algorithm to construct simple forward LPR curve**. The forward rates are used as inputs in pricing interest-rate options.

3.1 Notations

For the i^{th} swap, $i = 1, ..., 9$, there are a set of increasing dates (i.e. a discrete tenor structure) $t \leq T_0 < T_1 < ... < T_{n_i}$, where $T_0, ..., T_{n_i-1}$ are the floating-leg reset dates, $T_1, ..., T_{n_i}$ are settlement (payment) dates, T_{n_i} is swap maturity date.

Denote the time grids $\mathbb{T}_i := \{T_0, ..., T_{n_i}\}$ for the i^{th} swap covering the period $[T_0, T_{n_i}]$, $\quad i = 1, ..., 9$. T_0 is the common start date for all swaps, and the lengths of 9 swaps follows an increasing order as $n_1 < n_2 < ... < n_9$, and define $n_0 := 1$. Here, we have $n_1 = 2, n_2 = 3, n_3 = 4, n_4 = 8, n_5 = 12, n_6 = 16 n_7 = 20, n_8 = 28, n_9 = 40$ for the 9 LPR swaps. The time grids are appeared in Figure 3.

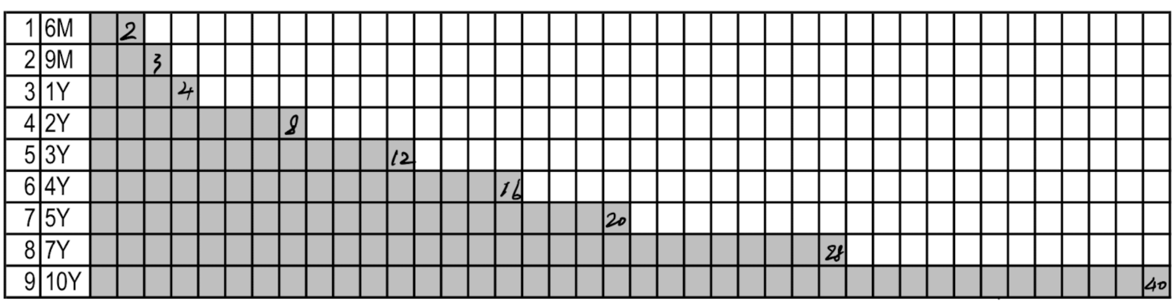

Figure 3: Visualization for the time grids of 9 swaps (one grid = one quarter)

Denote **forward tenors**, the time intervals between two consecutive time grids (i.e. accrual periods) that is normally three months.

$$\eta_j := T_j - T_{j-1}, \qquad j = 1, ..., n_9 \tag{3.1}$$

Denote **swap tenors**, the time intervals between two consecutive swap maturity dates:

$$\zeta_i := T_{n_i} - T_{n_i-1}, \qquad i = 1, ..., 9 \tag{3.2}$$

and define $n_0 := 1$.

3.2 Forward swap rates and simple forward rates

By definition, the i^{th} simple forward swap rate $S_{0,n_i}(t) := S_i(t)$ at spot time t is set such that the spot time-t value of the i^{th} swap covering the period $[T_0, T_{n_i}]$ is zero, i.e.,

$$P(t,T_0) - P(t,T_{n_i}) = S_i(t) \sum_{j=1}^{n_i} \eta_j P(t,T_j), \qquad t \in [0,T_0], \qquad i = 1,..,9 \tag{3.3}$$

where $P(t,T)$ is the time-t discount bond price. And the forward swap rate can be expressed in terms of spanning simple forward rates, i.e., the i^{th} simple forward swap rate $S_i(t)$ in (3.3) **can be rewritten as a nonlinear function of simple forward rates** $\{F_j(t)\}_{j=1,...,n_i}$:

$$\begin{aligned} S_i(t) &= \frac{P(t,T_0) - P(t,T_{n_i})}{\sum_{j=1}^{n_i} \eta_j P(t,T_j)} \\ &= \frac{1 - \frac{P(t,T_{n_i})}{P(t,T_0)}}{\sum_{j=1}^{n_i} \eta_j \frac{P(t,T_j)}{P(t,T_0)}} \\ &= \frac{1 - \prod_{j=1}^{n_i} \frac{1}{1+\eta_j F_j(t)}}{\sum_{j=1}^{n_i} \eta_j \prod_{m=1}^{j} \frac{1}{1+\eta_m F_m(t)}}, \qquad t \in [0,T_0], \qquad i = 1,...,9 \end{aligned} \tag{3.4}$$

where $F_j(t)$ is the simple forward rate for the future period $[T_{j-1}, T_j]$ at time t:

$$F_j(t) := F(t; T_{j-1}, T_j) = \frac{P(t,T_{j-1}) - P(t,T_j)}{\eta_j P(t,T_j)}, \qquad t \in [0, T_{j-1}]$$

Proposition 3.1 . A recursive relationship between the i^{th} swap rate $S_i(T_0)$ and spanning forward rates $\{F_j(T_0)\}_{j=1,...,n_i}$ is given by

$$\begin{aligned} &(1 - S_i(T_0) \sum_{j=1}^{n_i - 1} \eta_j \prod_{m=1}^{j} \frac{1}{1+\eta_m F_m(T_0)}) \prod_{m=1}^{n_i-1} (1 + \eta_m F_m(T_0)) \\ = &\prod_{j=n_{i-1}+1}^{n_i} \frac{1}{1+\eta_j F_j(T_0)} + S_i(T_0) \sum_{j=n_{i-1}+1}^{j} \eta_j \prod_{m=n_{i-1}+1}^{j} \frac{1}{1+\eta_m F_m(T_0)} \end{aligned} \tag{3.5}$$

Given the observations of spot swap rates at the common start date T_0, we could back out all forward rates within one year iteratively in closed forms as Lemma3.2. But the forward rates beyond one year cannot be uniquely backed out without assumption. We make an assumption that the forward rates are piecewise-constant between two consecutive swap-maturity dates beyond one year. i.e.,

$$F_j(T_0) = f_i, \qquad j = n_{i-1}+1, ..., n_i, \qquad i = 4, ..., 9 \tag{3.6}$$

all forward rates beyond one year can be uniquely solved numerically in Proposition 3.2 or approximated by Lemma 3.2. We denote the following such that the notation is consistence with (3.6):

$$F_{i+1}(T_0) := f_i, \qquad i = 0, 1, 2, 3 \tag{3.7}$$

i.e.,

$$F_j(T_0) := f_{j-1}, \qquad j = 1, 2, 3, 4$$

and $f_0 := F_1(T_0) = L(T_0)$ is known at T_0. Given f_0, all forward rates $\{f_i\}_{i=1,...,9}$ can be solved:

Lemma 3.2 . For $i = 1, 2, 3$, given the i^{th} swap rate $S_i(T_0)$ and forward rates $\{F_j(T_0)\}_{i=1,...,n_{i-1}}$, the forward rate f_i as defined by (3.7) can be solved analytically:

$$f_i = \frac{\frac{1}{\eta_{n_i}} + S_i(T_0)}{(1 - S_i(T_0) \sum_{j=1}^{n_i-1} \eta_j \prod_{m=1}^{j} \frac{1}{1+\eta_m F_m(T_0)}) \prod_{m=1}^{n_i-1}(1 + \eta_m F_m(T_0))} - \frac{1}{\eta_{n_i}}, \qquad i = 1, 2, 3 \tag{3.8}$$

Proposition 3.3 . For $i = 4, .., 9$, given the i^{th} swap rate $S_i(T_0)$ and all forward rates $\{F_j(T_0)\}_{j=1,...,n_{i-1}}$, we have

$$\begin{aligned}
&(1 - S_i(T_0) \sum_{j=1}^{n_i-1} \eta_j \prod_{m=1}^{j} \frac{1}{1 + \eta_m F_m(T_0)}) \prod_{m=1}^{n_i-1} (1 + \eta_m F_m(T_0)) \\
&= \prod_{j=n_{i-1}+1}^{n_i} \frac{1}{1 + \eta_j F_j(T_0)} + S_i(T_0) \sum_{j=n_{i-1}+1}^{j} \eta_j \prod_{m=n_{i-1}+1}^{j} \frac{1}{1 + \eta_m F_m(T_0)}
\end{aligned} \tag{3.9}$$

where the forward rate f_i as assumed by (3.6) can be solved numerically.

Lemma 3.4 . Alternatively, the forward rate f_i as assumed by (3.6) can be solved approximately in a closed form as

$$f_i \approx \frac{\frac{1}{T_{n_i} - T_{n_{i-1}}} + S_i(T_0)}{(1 - S_i(T_0) \sum_{j=1}^{n_i-1} \eta_j \prod_{m=1}^{j} \frac{1}{1+\eta_m F_m(T_0)}) \prod_{m=1}^{n_i-1}(1 + \eta_m F_m(T_0))} - \frac{1}{T_{n_i} - T_{n_{i-1}}}, \qquad i = 4, ..., 9 \tag{3.10}$$

If we combine (3.8) and (3.10) into expression (3.11):

Theorem 3.5 . Forward rate f_i can be solved (exactly for $i = 1, 2, 3$, and approximately for $i = 4, ..., 9$) in closed form as:

$$f_i = \frac{\frac{1}{\zeta_i} + S_i(T_0)}{(1 - S_i(T_0) \sum_{j=1}^{n_i-1} \eta_j \prod_{m=1}^{j} \frac{1}{1+\eta_m F_m(T_0)}) \prod_{m=1}^{n_i-1}(1 + \eta_m F_m(T_0))} - \frac{1}{\zeta_i}, \qquad i = 1, ..., 9 \tag{3.11}$$

where $\zeta_i := T_{n_i} - T_{n_i-1}$ is the $i - th$ swap tenor.

4 Pricing LPR Options

The key idea for pricing LPR options is to extend the classical LIBOR market model by replacing the log-normal distribution by the shifted and scaled Skellam distribution. Therefore, we denote the new model as **Skellam market model**.

4.1 Pricing LPR Caps and Floors

The forward measure is used to price interest-rate options as the we could generate martingale under the forward measure such that it is useful for option pricing. More precisely, if we take the tradable asset T_i-bond as the numeraire associated with T_i-*forward measure* $\mathbb{Q}_i \sim \mathbb{P}$ and then the simple forward rate process $F_i(t)$ is a $\mathbb{Q}_i$-martingale. We actually need the marginal distribution at the time point T_{i-1} under the associated measure $\mathbb{Q}_i$. In the classical LIBOR market, it is log-normal distributed. In our case, we assume that under measure $\mathbb{Q}_i$:

$$L(T_{i-1}) = F_i(T_{i-1}) = F_i(0) + b\mathbf{D}_i(T_{i-1}) \tag{4.1}$$

where

$$\mathbf{D}_i(T_{i-1}) := \sum_{k=1}^{n(T_{i-1})} D_i(k)$$

and $\{D_i(k)\}_{k=1,...}$ are i.i.d Skellam random variables under measure $\mathbb{Q}_i$, which is based on the change of measure for the sum of Skellam random variables.

Caplets are paid in arrears. The $i-th$ caplet settles at time T_{i-1} and is paid three months later at time T_i. More precisely, the time-T_i payoff of the $i-th$ caplet with strike K and unit notional amount, starting at T_{i-1} and maturing at T_i is:

$$Cpl_i(T_i) = \eta_i(F_i(T_{i-1}) - K)^+$$

and we want to calculate its current price:

Theorem 4.1 . Based on our Skellam market model (4.1), the closed-form time-0 price for the ith caplet is given by:

$$Cpl_i(0) = b\eta_i P(0, T_i) \sum_{d=\lceil \kappa_i \rceil}^{\infty} (d - \kappa_i) e^{-2n(T_{i-1})\lambda_i} I_{|d|}(2n(T_{i-1})\lambda_i) \tag{4.2}$$

$$\kappa_i := \frac{K - F_i(0)}{b}$$

λ_i is the intensity parameter under T_i-forward measure $\mathbb{Q}_i$ and $n(T_{i-1})$ is the total number of LPR-announcement times within the period $[0, T_{i-1}]$.

The infinity sum can be easily calculated with high accuracy by truncation. And thus we only have one parameter λ_i i.e., the Skellam-implied rate from our pricing formula, similar as the implied volatility from the classical Black formula.

Given the caplet price in (4.2), the associated floorlet then can be priced immediately by *caplet-floorlet parity*:

$$(F_i(T_{i-1}) - K)^+ - (K - F_i(T_{i-1}))^+ = F_i(T_{i-1}) - K$$

the right-hand-side is the payoff of a payer FRA (Forward Rate Agreement) (swaplet). A cap (floor) is just a portfolio of caplets (floorlets) whose maturities are three months apart.

4.2 Implied Volatility

4.2.1 Black-Implied Volatility

As the world-wide industrial convention for quoting the prices of vanilla interest-rate options, such as capsk/floors and swaptions, is using the Black-implied volatility (or lognormal-implied volatility) based on the standard model of lognormal market model. The Black model for interest-rate options assumes that, under the forward measure $\mathbb{Q}_i$,

$$lnF_i(T_{i-1})|F_i(0) \sim N(lnF_i(0) - \frac{1}{2}\sigma_{LN}^2 T_{i-1}, \sigma_{LN}^2 T_{i-1})$$

where σ_{LN} is the lognormal volatility of $F_i(t)$. The Black formula for pricing the ith caplet is given by

$$Cpl_{LN}(0) = \eta_i P(0, T_i)(F_i(0)\Phi(d_+) - K\Phi(d_-)) \tag{4.3}$$

where

$$d_{LN}^{\pm} := \frac{ln\frac{F_i(0)}{K} \pm \frac{1}{2}\sigma_{LN}^2 T_{i-1}}{\sigma\sqrt{T_{i-1}}}, \qquad d_{LN}^{+} = d_{LN}^{-} + \sigma_{LN}\sqrt{T_{i-1}}$$

4.2.2 Bachelier-Implied Volatility

Normal market model (Bachelier market model) is recently used due to the possible presence of negative interest rates.

Assumption: under the forward measure $\mathbb{Q}_i$:

$$F_i(T_{i-1})|F_i(0) \sim N(F_i(0), \sigma_N^2 T_{i-1})$$

where σ_N is the normal-implied volatility (or Bachelier-implied volatility) of $F_i(t)$, then, the formula for pricing the *ith* caplet is given by:

$$Cpl_N(0) = \eta_i P(0, T_i)\sigma_N\sqrt{T_{i-1}}(d_N\Phi(d_N) + \phi(d_N)) \tag{4.4}$$

where

$$d_N := \frac{F_i(0) - K}{\sigma_N \sqrt{T_{i-1}}}$$

The Bachelier-implied volatility from our Skellam market model presents a symmetric volatility smile. The theoretical relation between the distribution of the LPR based on Skellam model and a normal distribution is provided in Proposition 2.2 and it explains the implied symmetric smile.

Caps and floor prices ae quoted by normal-implied or Bachelier-implied volatility. We can plot the volatility surface for the Skellam-implied intensity and Bachelier-implied volatility and if we fix the maturity time to 9-month, both of them present smile (frown).

4.3 Pricing LPR Swaption

The $i-th$ simple forward swap rate $S_{0,n_i}(t) := S_i(t)$ as implied by (3.3) can be expressed as:

$$S_i(t) = \frac{P(t,T_0) - P(t,T_{n_i})}{A_i(t)}, \qquad t \in [0, T_0], \qquad i = 1, ..., 9$$

where $A_i(t)$ is the swap annuity or present value of basis point, i.e.,

$$A_i(t) := \sum_{j=1}^{n_i} \eta_j P(t, T_j)$$

and can be considered as a tradable asset. Then we can take it as the numeraire associated with forward-swap measure $\mathbb{S}_i$, and $S_i(t)$ is $\mathbb{S}_i$-martingale.

Besides, the simple forward swap rate (3.4) can be interpreted as a weighted linear combination of spanning forward rates and can be approximated by weight freezing at time 0. i.e.,

$$S_i(t) = \sum_{j=1}^{n_i} w_j(t) F_j(t) \approx \sum_{j=1}^{n_i} w_j(0) F_j(t)$$

where

$$w_j(t) := \frac{\eta_j P(t,T_j)}{\sum_{m=1}^{n_i} \eta_m P(t,T_m)}$$

the variability of weights is small in comparison with the variability of forwards, so the weight terms can be approximated by their initial values. So the weight terms can be approximated by their initial values.

To be consistent with previous assumption in (4.1):

$$F_j(T_0) = F_j(0) + b\mathbf{D}_j(T_0), \qquad j = 1, ..., n_i$$

then

$$
\begin{aligned}
S_i(T_0) &\approx \sum_{j=1}^{n_i} w_j(0) F_j(T_0) \\
&= \sum_{j=1}^{n_i} w_j(0)(F_j(0) + b\mathbf{D}_j(T_0)) \\
&= S_{n_i}(0) + b \sum_{j=1}^{n_i} w_j(0) \mathbf{D}_j(T_0)
\end{aligned} \tag{4.5}
$$

where

$$
\mathbf{D}_j(T_0) := \sum_{k=0}^{n(T_0)} D_j(k)
$$

similarly, we assume that

$$
S_{n_i}(T_0) = S_{n_i}(0) + b\mathbf{D}^{\mathbb{S}_i}(T_0) \tag{4.6}
$$

Swaption is an European call or put option on interest rate swap. The swaption maturity is the first reset date T_0 of the underlying interest rate swap, and the holder of payer swaption has the right (but not the obligation) to enter a payer swap at T_0. The payoff of the i^{th} payer swaption (i.e. T_0-into-($T_{n_i} - T_0$) swaption, or $T_0 \times (T_{n_i} - T_0) swaption$) with strike K and unit notional amount at the swaption maturity T_0:

$$
PSwpt_i(T_0) = (S_{n_i}(T_0) - K)^+ \sum_{j=1}^{n_i} \eta_j P(T_0, T_j)
$$

and its current price in analytical form is:

$$
PSwpt_i(0) = bA_i(0) \sum_{d=\lceil \kappa_i \rceil}^{\infty} (d - \kappa_i) e^{-2n(T_0)\lambda^{\mathbb{S}_i}} I_{|d|}(2n(T_0)\lambda^{\mathbb{S}_i}) \tag{4.7}
$$

$\lambda^{\mathbb{S}_i}$ is the rate parameter under forward-swap measure $\mathbb{S}_i$, and $n(T_0)$ is the total number of LPR-announcement times within the period $[0, T_0]$.

Swaption prices are quoted by normal-implied or Bachelier-implied swap volatility. And the Skellam-implied intensity and Bachelier-implied volatility present smile(skew).

The intensity skew and intensity frown shows that our Skellam-based market model cannot fully capture the real data just by using a single parameter.

5 Reference

Chen, Z. Y., Zhang, K., and Zhao, H. B. (2020). Loan Prime Rate Options, https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3605156.