week3 quiz

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6/5/2021

1 Consider a random variable XX that has a t distribution with 5 degrees of freedom. Calculate P[|X| > 2.1].

```
pt(2.1, 5, lower.tail = FALSE) * 2
```

```
## [1] 0.08975325
```

2 Calculate the critical value used for a 90% confidence interval about the slope parameter of a simple linear regression model that is fit to 10 observations. (Your answer should be a positive value.)

```
qt(0.95, (10 - 2))
```

```
## [1] 1.859548
```

3 Consider the true simple linear regression model, $Y_i = 5 + 4x_i + \epsilon_i, \epsilon_i \sim N(0, \sigma^2 = 4), i = 1, 2, ...20$ Given $S_{xx} = 1.5$ calculate the probability of observing data according to this model, fitting the SLR model, and obtaining an estimate of the slope parameter greater than 4.2. In other words, calculate $P[\hat{\beta}_1 > 4.2]$

```
pnorm(0.2, 0, 2/sqrt(1.5), lower.tail = FALSE)
```

```
## [1] 0.4512616
```

4 Suppose we would like to predict the duration of an eruption of the Old Faithful geyser in Yellowstone National Park based on the waiting time before an eruption. Fit a simple linear model in R that accomplishes this task.

What is the value of $SE[\hat{\beta}_1]$?

```
faithul_model = lm(eruptions~waiting, data = faithful)
summary(faithul_model)
```

```
##
## Call:
## lm(formula = eruptions ~ waiting, data = faithful)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.29917 -0.37689  0.03508  0.34909  1.19329
##
```

```
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -1.874016   0.160143  -11.70   <2e-16 ***
## waiting      0.075628   0.002219   34.09   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.4965 on 270 degrees of freedom
## Multiple R-squared:  0.8115, Adjusted R-squared:  0.8108
## F-statistic:  1162 on 1 and 270 DF,  p-value: < 2.2e-16
```

```r
summary(faithul_model)$coefficients[1,3]
```

```
## [1] -11.70212
```

```r
summary(faithul_model)$coefficients[2,3]
```

```
## [1] 34.08904
```

```r
summary(faithul_model)$coefficients[2,2]
```

```
## [1] 0.002218541
```

8 Calculate a 90% confidence interval for β_0. Report the upper bound of this interval.

```r
qt(0.95, length(faithful$waiting) - 2) * 0.160143 + (-1.874016)
```

```
## [1] -1.609697
```

9 Calculate a 95% confidence interval for β_1. Report the length of the margin of this interval.

```r
qt(0.975, length(faithful$waiting) - 2) * summary(faithul_model)$coefficients[2,2]
```

```
## [1] 0.00436784
```

```r
(output = confint(faithul_model, level = 0.95))
```

```
##                   2.5 %      97.5 %
## (Intercept) -2.18930436 -1.55872761
## waiting      0.07126011  0.07999579
```

```r
(output[2,2] - output[2,1]) / 2
```

```
## [1] 0.00436784
```

10 Create a 90% confidence interval for the mean eruption duration for a waiting time of 81 minutes. Report the lower bound of this interval.

```r
predict(faithul_model, newdata = data.frame(waiting = c(81)), interval = c("confidence"), level = 0.9)
```

```
##        fit      lwr      upr
## 1 4.251848 4.189899 4.313797
```

11 Create a 99% prediction interval for a new observation's eruption duration for a waiting time of 72 minutes. Report the upper bound of this interval.

```r
predict(faithul_model, newdata = data.frame(waiting = c(72)), interval = c("prediction"), level = 0.99)
```

```
##        fit      lwr      upr
## 1 3.571196 2.280781 4.861612
```

12 Consider a 90% confidence interval for the mean response and a 90% prediction interval, both at the same xx value. Which interval is narrower?

```r
set.seed(100)
x = runif(1, min(faithful$waiting), max(faithful$waiting))
predict(faithul_model, newdata = data.frame(waiting = c(x)), interval = c("confidence"), level = 0.9)
```

```
##        fit      lwr      upr
## 1 2.611599 2.546263 2.676935
```

```r
predict(faithul_model, newdata = data.frame(waiting = c(x)), interval = c("prediction"), level = 0.9)
```

```
##        fit      lwr      upr
## 1 2.611599 1.789496 3.433702
```

13 Fail to reject the null hypothesis