Lecture 2: Symmetric Key Cryptography

-COMP 6712 Advanced Security and Privacy

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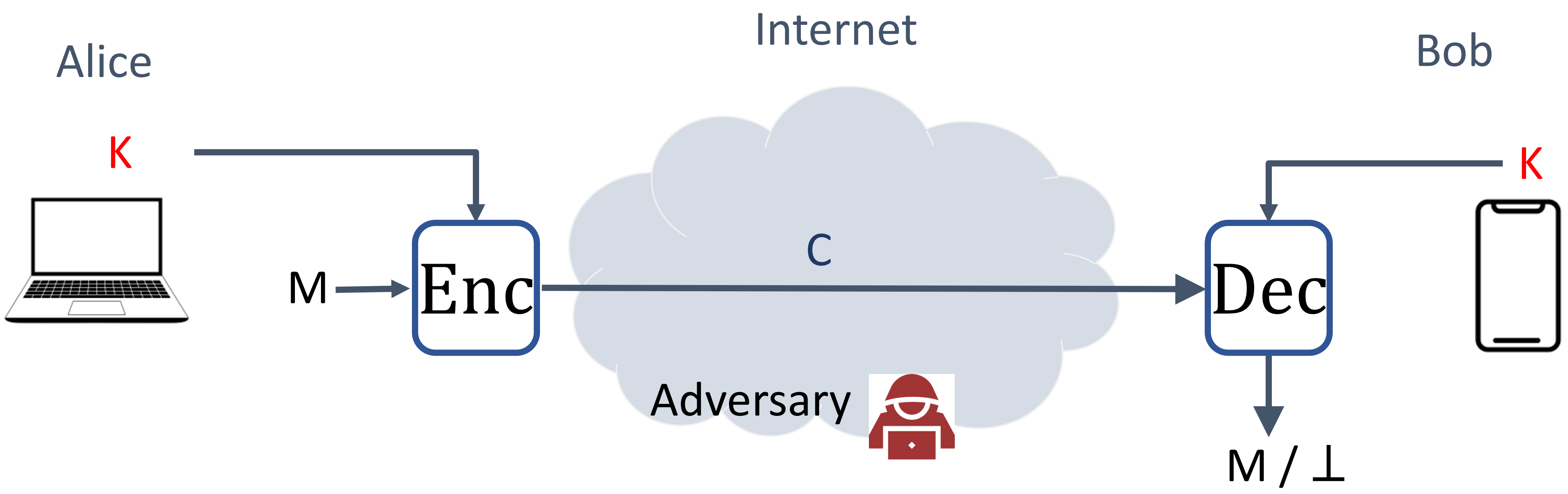
Symmetric-key cryptography



Enc : encryption algorithm (public)

Dec : decryption algorithm (public)

K : shared key between Alice and Bob

Outline of this lecture

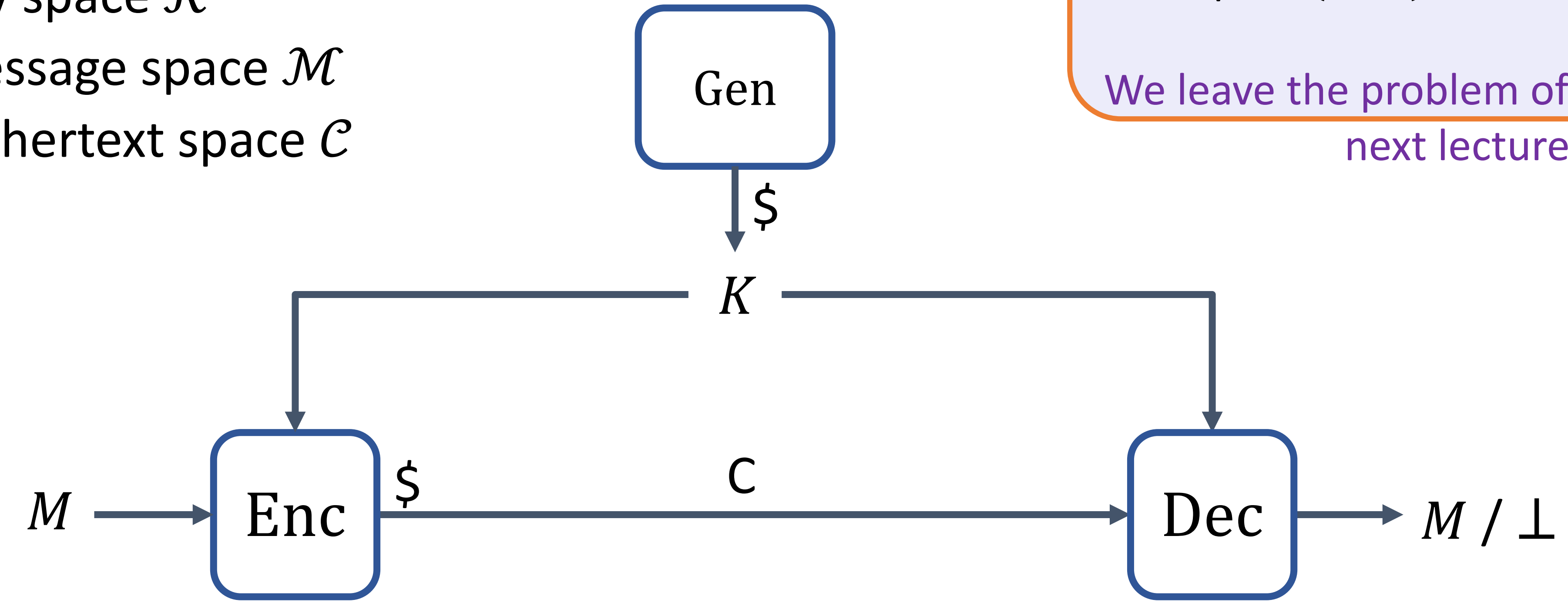
- Syntax and security of symmetric-key cryptography
- Perfect security and one-time pad
- Stream cipher, block cipher and MAC
- Hash function
- Constructions

Syntax of symmetric encryption scheme

- A **symmetric encryption** $\Pi = (\mathrm{Gen}, \mathrm{Enc}, \mathrm{Dec})$ consists of three public algorithms:
- with
  - Key space $\mathcal{K}$
  - Message space $\mathcal{M}$
  - Ciphertext space $\mathcal{C}$

Key Generation: on input security parameter and randomness,
Outputs (K, K) as the secret keys

We leave the problem of sending K to next lecture



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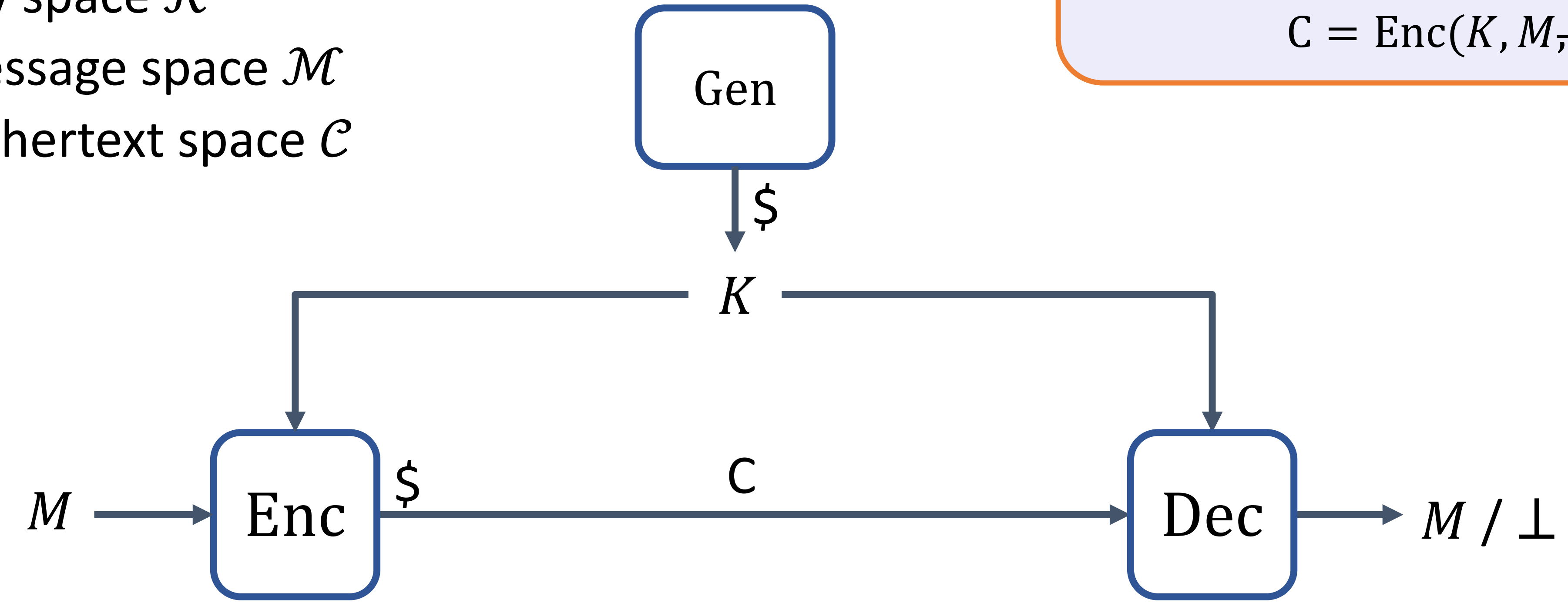
- with
  - Key space $\mathcal{K}$
  - Message space $\mathcal{M}$
  - Ciphertext space $\mathcal{C}$

Encryption: on input M from $\mathcal{M}$ and K, (and randomness r)

$$\mathrm{C} = \mathrm{Enc}(K, M\cancel{, r})$$



Syntax of symmetric encryption scheme

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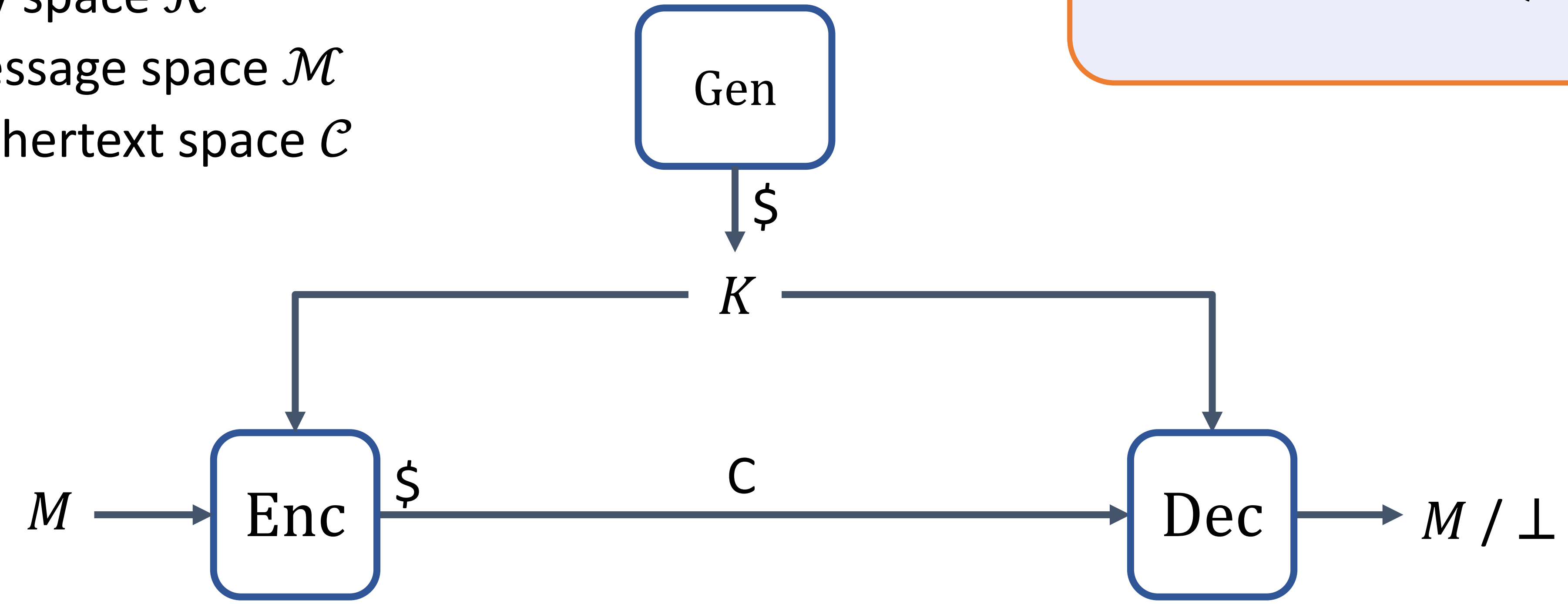
- with
  - Key space $\mathcal{K}$
  - Message space $\mathcal{M}$
  - Ciphertext space $\mathcal{C}$

Decryption: on input C from $\mathcal{C}$ and K,

$$\mathrm{M}/\perp = \mathrm{Dec}(K, C)$$



Syntax of symmetric encryption scheme

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- with
  - Key space $\mathcal{K}$
  - Message space $\mathcal{M}$
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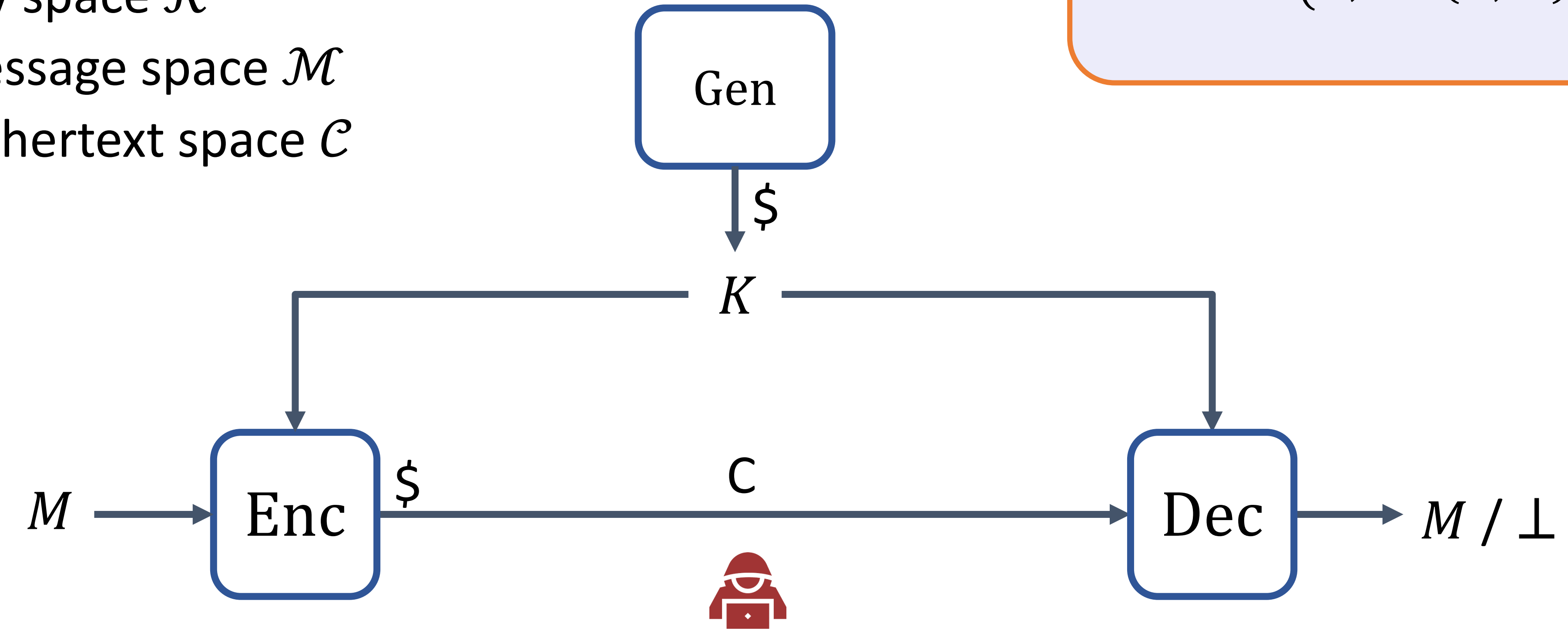
Correctness: For all $K \leftarrow \mathrm{Gen}$:

$$\mathrm{Dec}\big(K, \mathrm{Enc}(K, M)\big) = M$$



Is it possible to be secure against an adversary with unbounded computational power???

Perfect security and one-time pad

- If an enc is secure against an adversary with unbounded computational power, it satisfies Perfect security

Definition: $\Pi = (\mathrm{Gen}, \mathrm{Enc}, \mathrm{Dec})$ is said to be **perfectly secret** if for every distribution over $\mathcal{M}$, any $m \in \mathcal{M}$, any $c \in \mathcal{C}$

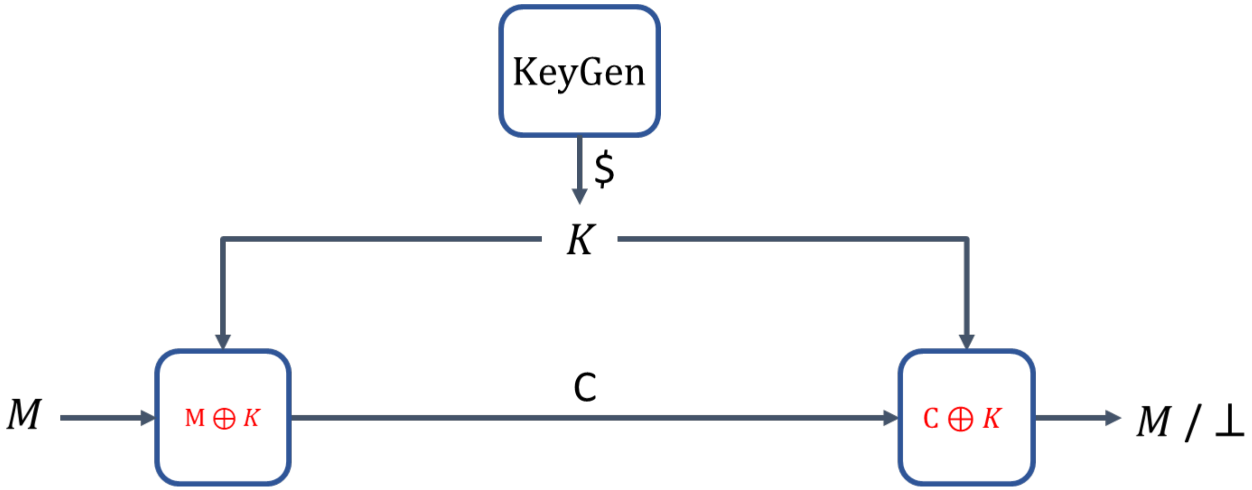
$$\Pr[M = m \mid C = c] = \Pr[M = \mathrm{m}]$$

with probability taken over the random choice $K \leftarrow \mathcal{K}$ and the random coins used by Enc (if any))

- The ciphertext gives nothing about the message (even for unbounded adversary)

Is perfect security possible? One-time Pad

- $\mathcal{K} = \{0,1\}^n$
- $\mathcal{M} = \{0,1\}^n$
- $\mathcal{C} = \{0,1\}^n$



$Gen:$ $\qquad$ $Enc: \mathcal{K} \times \mathcal{M} \rightarrow \mathcal{C}$ $\qquad$ $Dec : \mathcal{K} \times \mathcal{C} \rightarrow \mathcal{M}$

$K \leftarrow \{0,1\}^n$ $\qquad$ $Enc(K, M) = M \oplus K$ $\qquad$ $Dec(K, C) = C \oplus K$

Is perfect security possible? One-time Pad

- $\mathcal{K} = \{0,1\}^n$
- $\mathcal{M} = \{0,1\}^n$
- $\mathcal{C} = \{0,1\}^n$

Gen:

$K \leftarrow \{0,1\}^n$

1110001101

$Enc: \mathcal{K} \times \mathcal{M} \rightarrow \mathcal{C}$

$Enc(K, M) = M \oplus K$

$$\begin{array}{rll} & 0101100100 & M \\ \oplus & 1110001101 & K \\ \hline = & 1011101001 & C \end{array}$$

$Dec: \mathcal{K} \times \mathcal{C} \rightarrow \mathcal{M}$

$Dec(K, C) = C \oplus K$

$$\begin{array}{rll} & 1011101001 & C \\ \oplus & 1110001101 & K \\ \hline = & 0101100100 & M \end{array}$$

One-time Pad

Theorem: The One-time Pad encryption scheme has perfect security

- **Have to show:** $\Pr[M = m \mid C = c] = \Pr[M = m]$

$$\Pr[C = c \mid M = m] = \Pr[m \oplus K = c] = \Pr[K = m \oplus c] = \frac{1}{2^n}$$

$$\Pr[C = c] = \sum_{m \in \mathcal{M}} \Pr[C = c \mid M = m] \Pr[M = m] = \frac{1}{2^n} \sum_{m \in \mathcal{M}} \Pr[M = m] = \frac{1}{2^n}$$

$$\Pr[M = m \mid C = c] = \frac{\Pr[C = c | M = m] \Pr[M = m]}{\Pr[C = c]} = \frac{\frac{1}{2^n} \Pr[M = m]}{\frac{1}{2^n}}$$

Limitation

- But $|\mathcal{K} = \{0,1\}^n| = |\mathcal{M} = \{0,1\}^n|$?
- If we find a way to deliver K, why not deliver M directly?

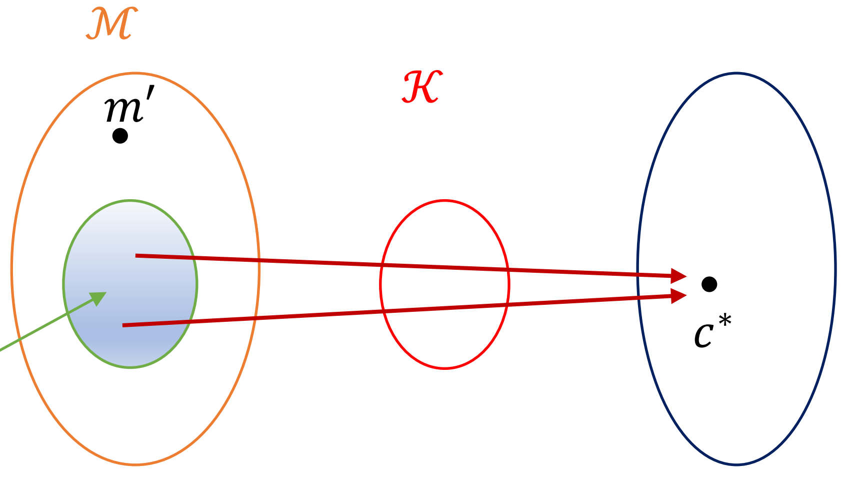
Theorem: If Π is a perfectly secret enc with key space $\mathcal{K}$ and message space $\mathcal{M}$

$$|\mathcal{K}| \geq |\mathcal{M}|$$

We show: if $|\mathcal{K}| < |\mathcal{M}|$, Π can not be perfectly secret

We have $|M(c^*)| \leq |\mathcal{K}| < |\mathcal{M}|$,

$\Pr[M = m'] \neq 0$, while $\Pr[M = m' \mid C = c^*] = 0$



$M(c^*) = \{m | m = Dec(K, c^*), K \in \mathcal{K}\}$

A short summary

- perfect security against the unbounded adversary
- could be achieved via the one-time pad
- Inherent limitation, key space $\geq$ message space
- How to break the limitation?

Break the limitation

- Aim low
- ~~Unbounded adversary~~

- Guarantee against efficient adversaries that run for some feasible amount of time. (ex. probabilistic polynomial time (PPT))

- Adversaries can potentially succeed with a small probability

small probability- negligible function

Definition: A positive function f is said to be **negligible** if for every positive polynomial p, and sufficiently large n

$$f(n) \leq \frac{1}{p(n)}.$$

- Ex

$$2^{-n}$$

$$2^{-\sqrt{n}}$$

$$\frac{1}{n^{1000}}??$$

Theorem: for every positive polynomial q, if f is **negligible**, so does $q(n) \cdot f(n)$.

Necessary of PPT and negligible

- probability polynomial time
  - If $|\mathcal{K}| < |\mathcal{M}|$, ciphertext must leak some information to UNBUOUNDed adversary

- Negligible success probability
  - Adversary runs in constant time can win with probability $\frac{1}{|\mathcal{K}|}$

Computational security

Definition: A scheme is (t, ε)-secure if any adversary running for a time at most t succeeds in breaking the scheme with probability at most ε.

Definition: A scheme Π is said to be **computationally secure** if any PPT adversary succeeds in breaking the scheme with negligible probability.

IND-eavesdropper

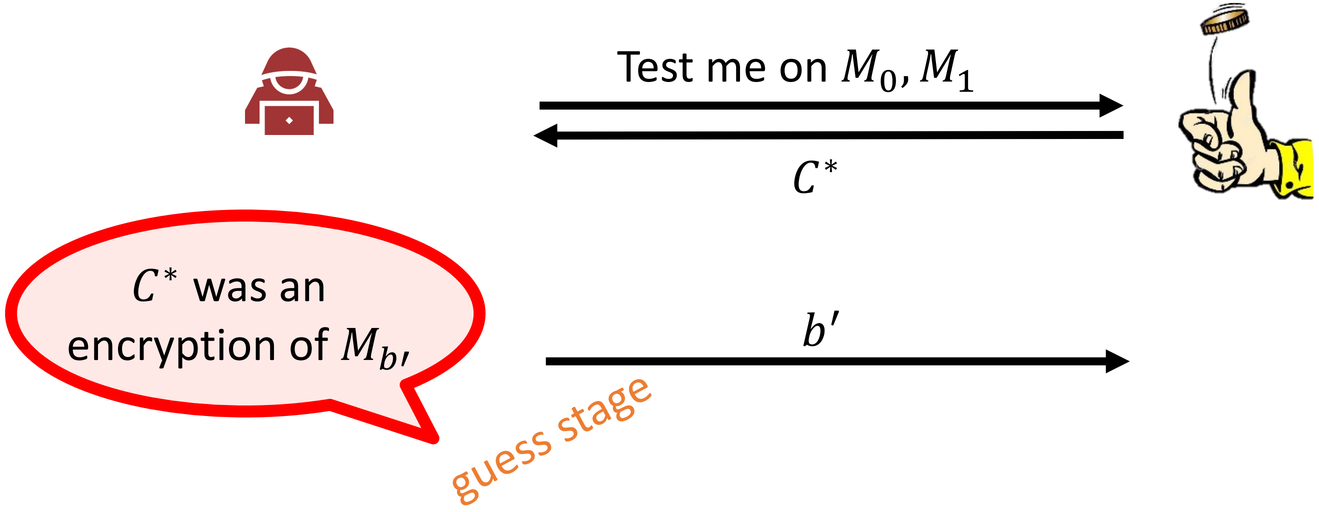
$\mathbf{Exp}_{\Pi}^{\text{ind-eav}}(A)$

$b \xleftarrow{\$} \{0,1\}$
$K \xleftarrow{\$} \Pi.\mathrm{Gen}$
$M_0, M_1 \leftarrow A()$ // find stage
  if $|M_0| \neq |M_1|$ **then**
  return $\perp$

$C^* \leftarrow \Pi.\mathrm{Enc}(K, M_b)$

$b' \leftarrow A\,(C^*)$ // guess stage

return $b' \overset{?}{=} b$



Definition: The **IND-eav-advantage** of an adversary A is

$$\mathbf{Adv}_{\Pi}^{\text{ind-eav}}(A) = \left|\Pr\left[\mathbf{Exp}_{\Pi}^{\text{ind-eav}}(A) \Rightarrow 1\right] - 1/2\right|$$

Construction of IND-eavesdropper secure enc

- We could construct a secure enc from PRG

- PRG is generally a function to extends k random bits to $k + l$ pseudo-random bits

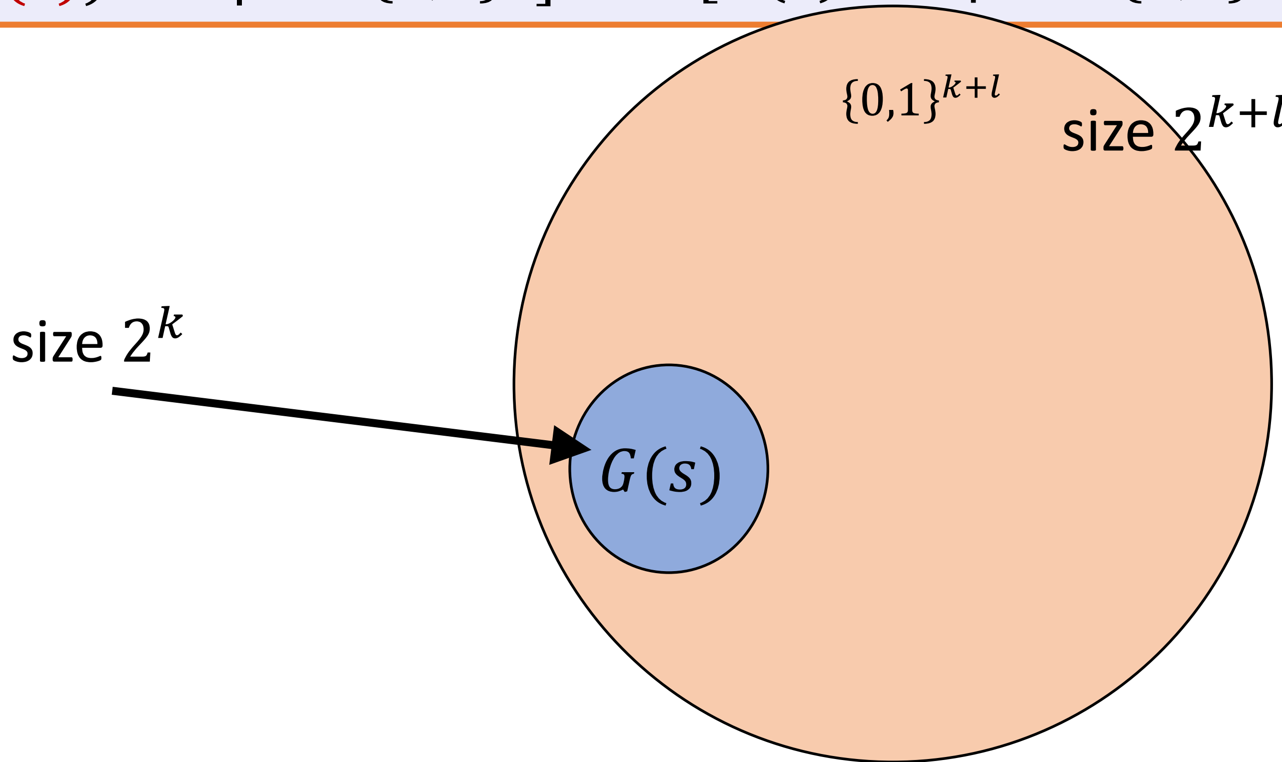
pseudo-random generator (PRG)

Definition: A **pseudorandom random generator (PRG)** is a function

$$G : \{0,1\}^k \to \{0,1\}^{k+l}$$

Such that

- $0 < l < poly\ (k)$
- For any PPT A, $\Pr[A(G(s)) = 1 | s \leftarrow \{0,1\}^k] - \Pr[A(r) = 1 | \, r \leftarrow \{0,1\}^{k+l}] < negl$



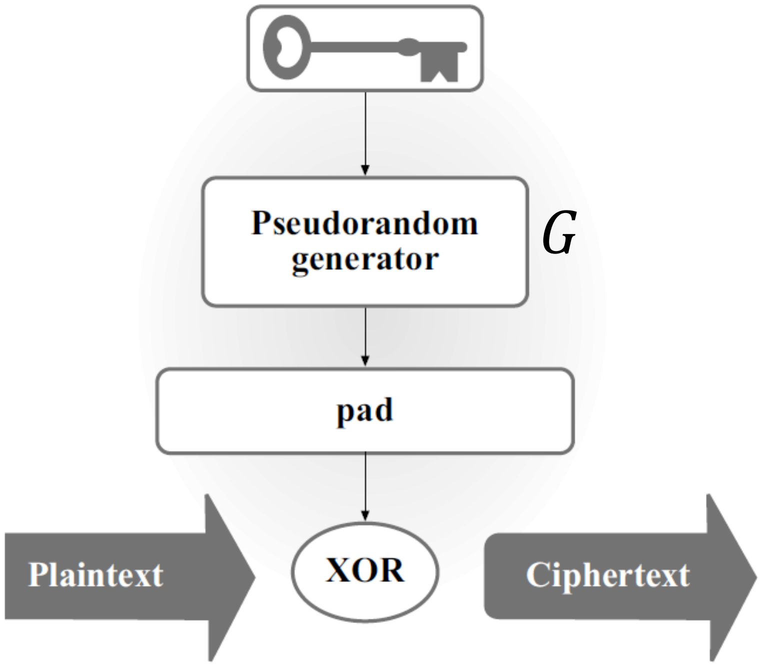
IND-eavesdropper Enc (with fix length) from PRG

- Let $G : \{0,1\}^k \rightarrow \{0,1\}^{k+l}$ be a PRG

- $\Pi 1.\mathrm{Gen}$: $K \leftarrow \{0,1\}^k$

- $\Pi 1.\mathrm{Enc}(\mathrm{K},\mathrm{M})$: $C = G(K) \oplus M$

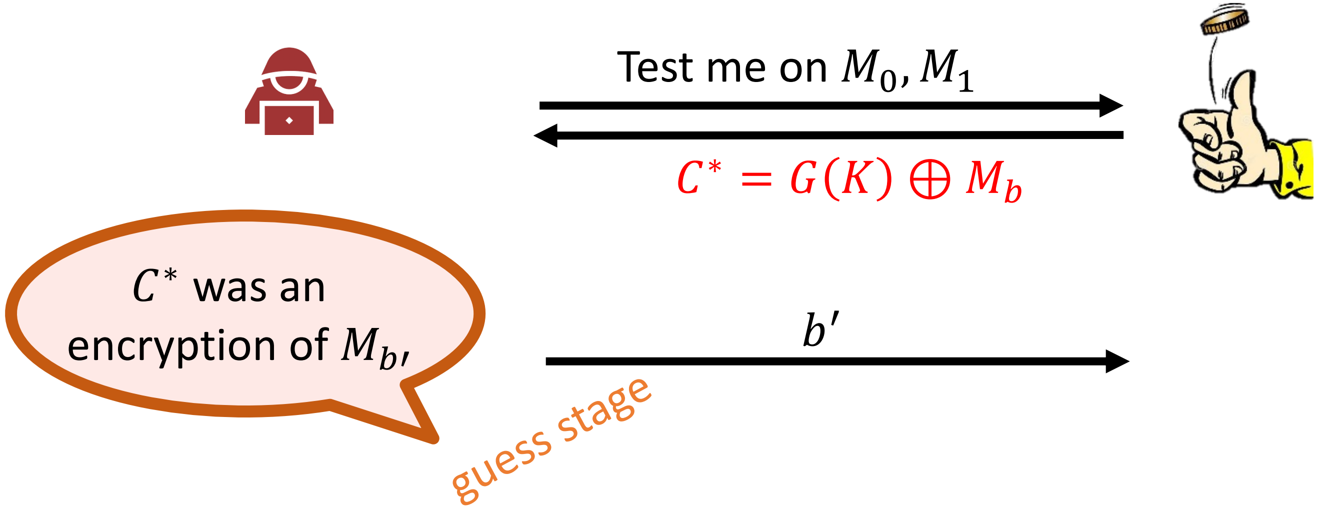
- $\Pi 1.\mathrm{Dec}(\mathrm{K},\mathrm{C})$: $M = G(K) \oplus C$



PROOF idea: IND-eavesdropper

$\mathbf{Exp}_{\Pi 1}^{\text{ind-eav}}(A)$

1. $b \xleftarrow{\$} \{0,1\}$
2. $K \xleftarrow{\$} \Pi 1.\mathrm{Gen}$
3. $M_0, M_1 \leftarrow A()$ // find stage
4. **if** $|M_0| \neq |M_1|$ **then**
5. **return** $\perp$
6. $C^* \leftarrow G(K) \oplus M_b$
7. $b' \leftarrow A\,(C^*)$ // guess stage
8. **return** $b' \stackrel{?}{=} b$



PROOF idea: IND-eavesdropper

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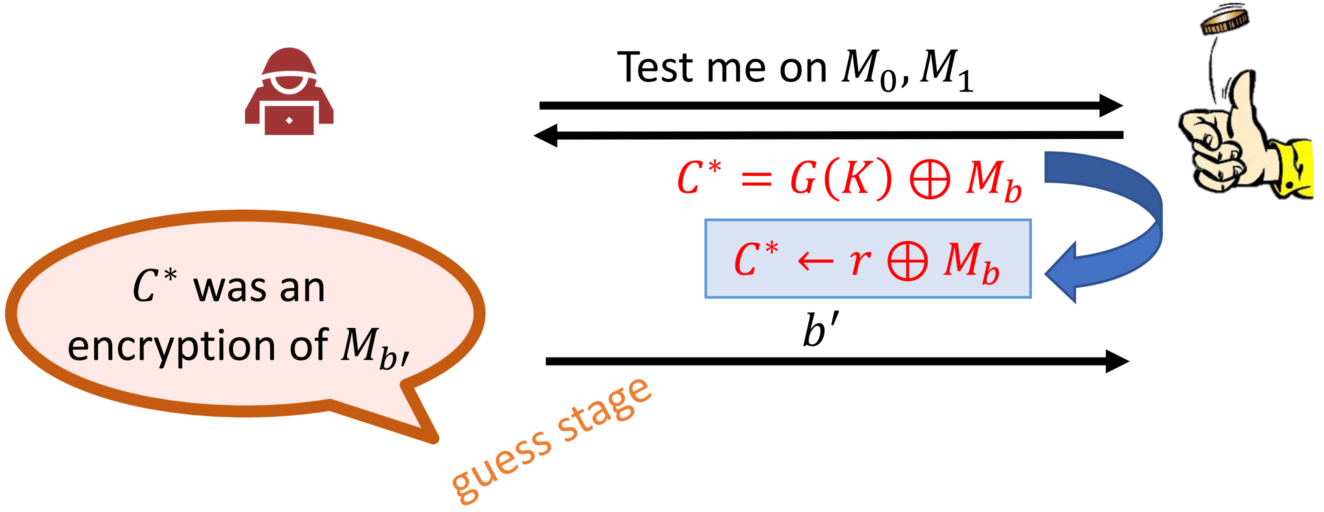
$M_0, M_1 \leftarrow A()$ // find stage
  if $|M_0| \neq |M_1|$ **then**
  return $\perp$
$C^* \leftarrow G(K) \oplus M_b$ $\boxed{C^* \leftarrow r \oplus M_b}$
$b' \leftarrow A\,(C^*)$ // guess stage
return $b' \stackrel{?}{=} b$



Now, this is an **one-time pad** and the **IND-eav-advantage** of an adversary A is

$$\mathbf{Adv}_{\Pi 1}^{\text{ind-eav}}(A) = 0$$

PROOF idea: IND-eavesdropper

$\mathbf{Exp}_{\Pi1}^{\text{ind-eav}}(A)$

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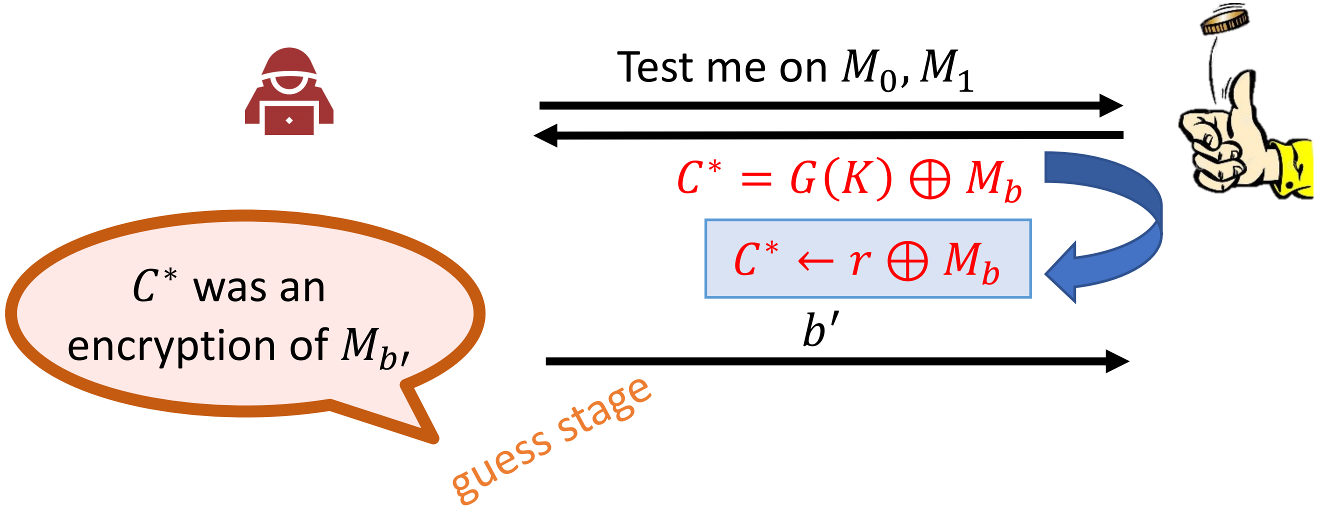
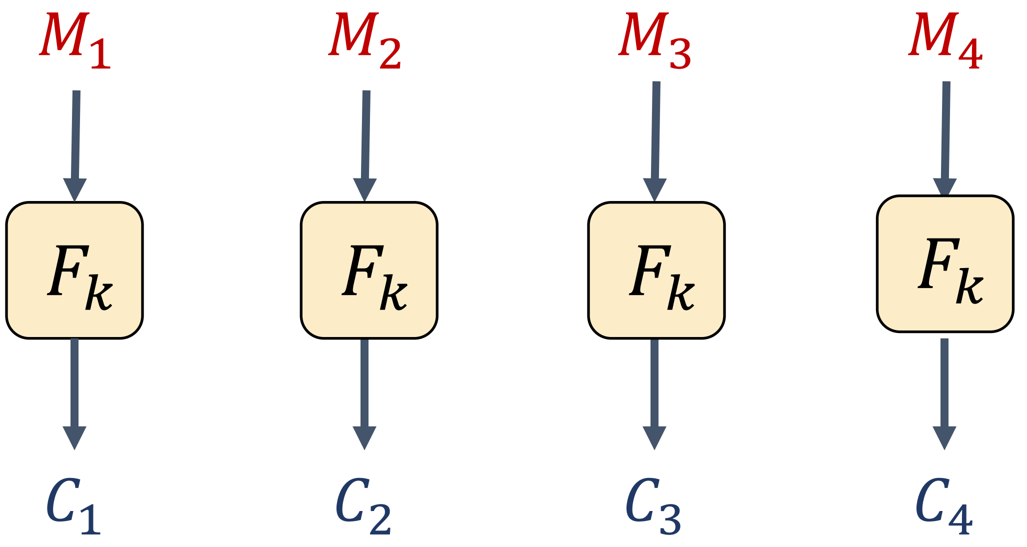


Any PPT adversary can not find the switch, since G is a PRG

Electronic Code Book (ECB) mode (for longer message)

- Given a block cipher $F_k\colon \{0,1\}^n \to \{0,1\}^n$ which is the encryption of $\Pi 1$

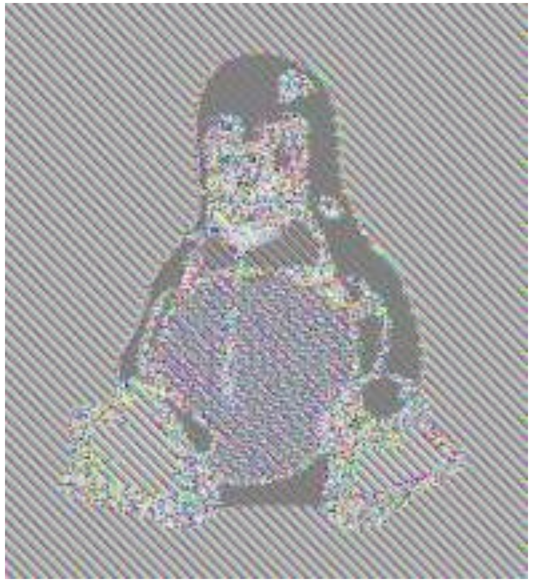
- $\mathrm{ECB}[F_k] = (\mathrm{Gen}, \mathrm{Enc}, \mathrm{Dec})$



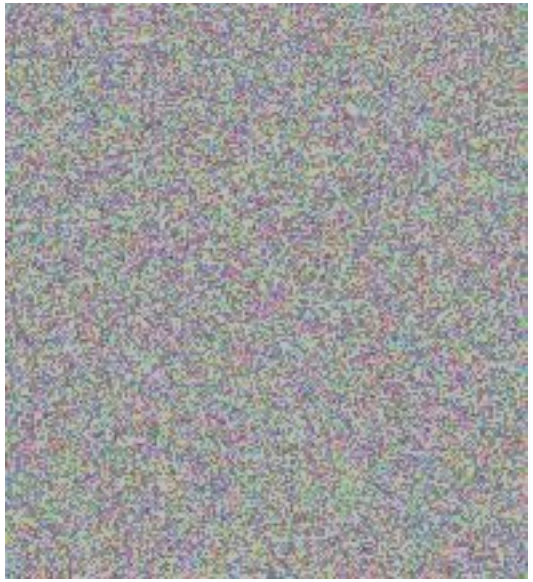
Weakness of ECB

Plaintext

ECB encrypted

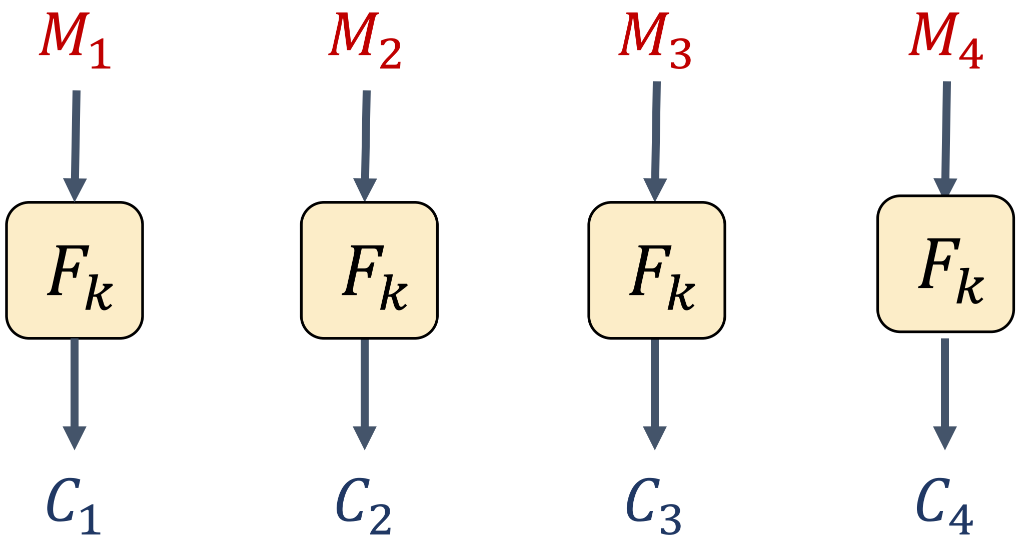


Properly encrypted

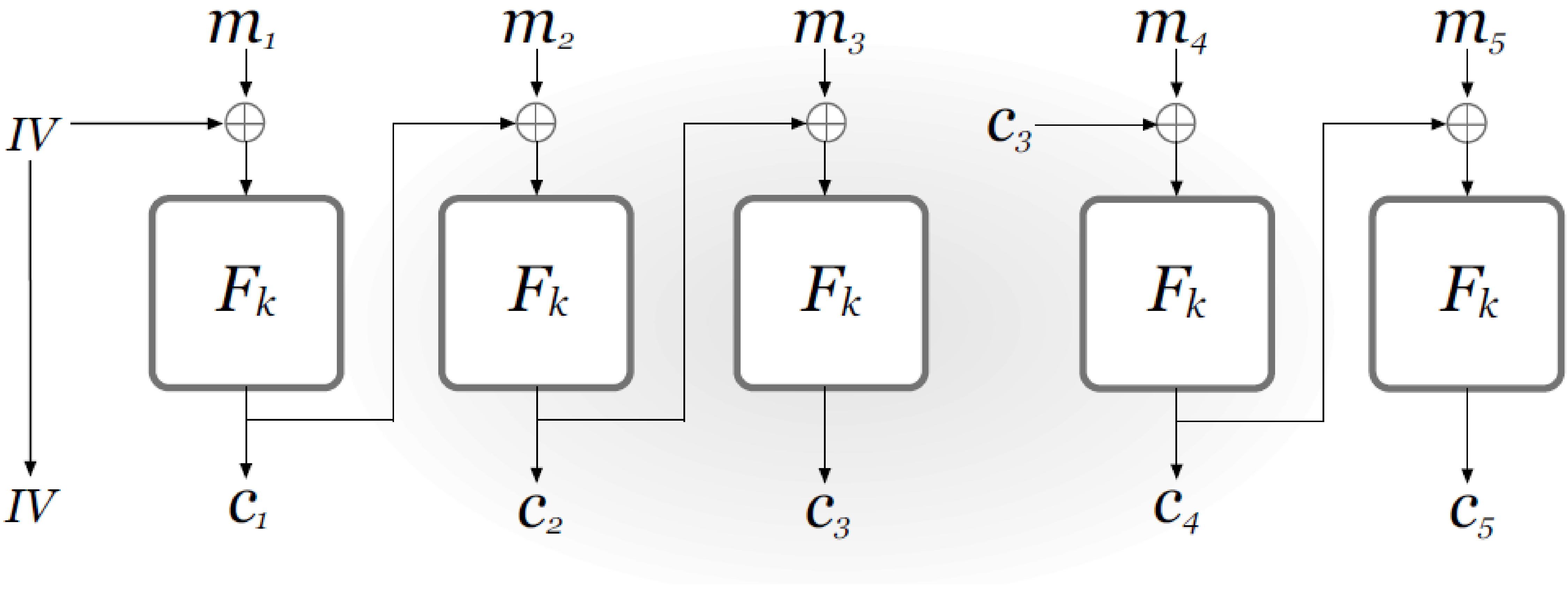


Electronic Code Book (ECB) mode (for longer message)

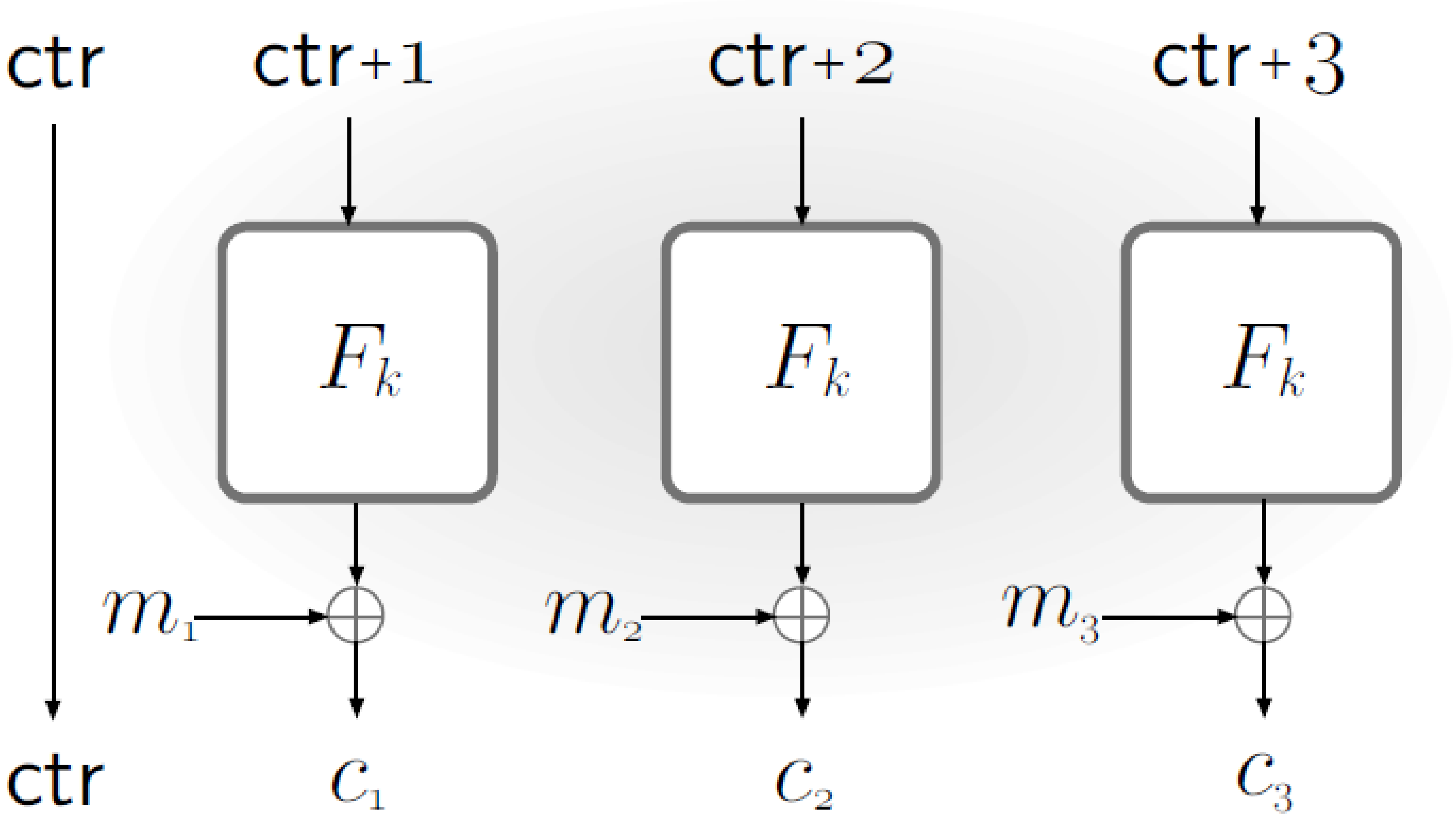
- This is because if $M_1 = M_2$, then $C_1 = C_2$



Cipher Block Chaining (CBC) mode

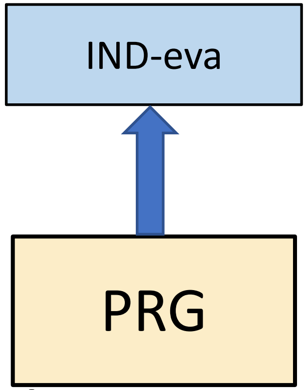


Counter (CTR) mode



A short summary

- With aim of computational security, we can encrypt a long message with a short key

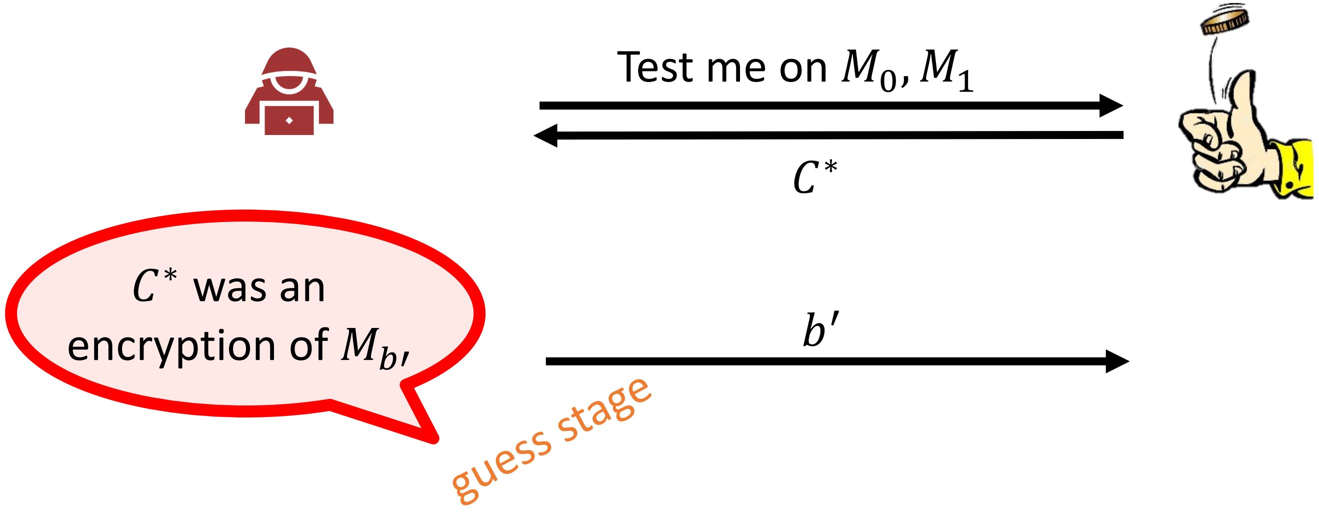


- With PRG, we could build IND-eavesdropper Enc

- We can further encrypt a longer message by splitting the message in blocks. It may operate in several models, EBC, CBC, CTR etc.

- IND-eavesdropper is a very weak security aim.

IND-eavesdropper is weak



Definition: The **IND-eav-advantage** of an adversary A is

$$\mathbf{Adv}_{\Pi}^{\text{ind-eav}}(A) = \left|\Pr\left[\mathbf{Exp}_{\Pi}^{\text{ind-eav}}(A) \Rightarrow 1\right] - 1/2\right|$$

Strong Security: IND-CPA

- In World War II
- British placed naval mines at certain locations, knowing that the Germans—when finding those mines—would encrypt the locations and send them back to Germany

- C = Enc (location of mines)

https://en.wikipedia.org/wiki/Naval_mine

An adversary may have the capability to choose a message and get the ciphertext

IND-CPA (choose plaintext attack)

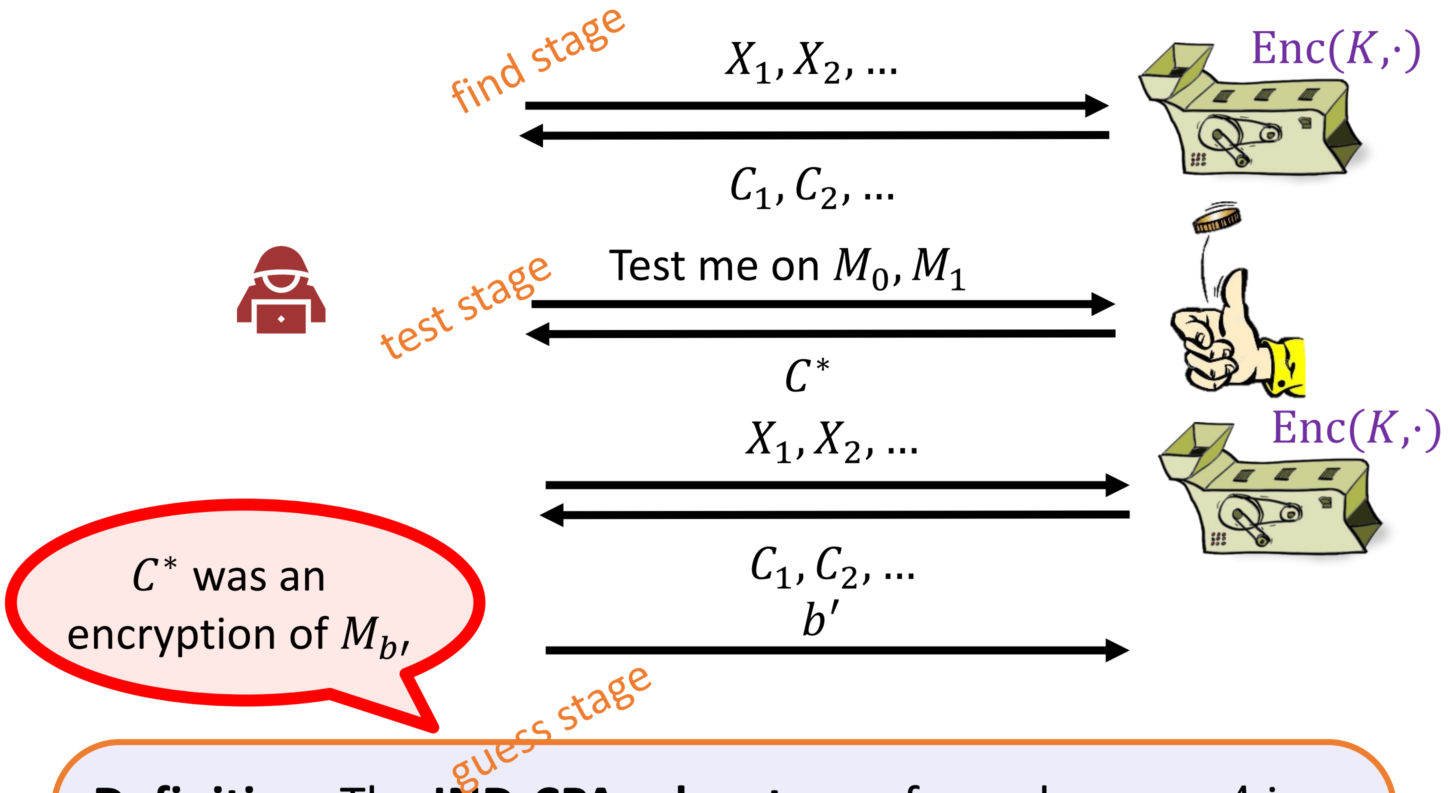
$\mathbf{Exp}_{\Pi}^{\text{ind-cpa}}(A)$

$b \xleftarrow{\$} \{0,1\}$
$K \xleftarrow{\$} \Pi.\mathrm{Gen}$
$M_0, M_1 \leftarrow A^{Enc(K,\cdot)}$ // find stage
  if $|M_0| \neq |M_1|$ **then**
  return $\perp$

$C^* \leftarrow \Pi.\mathrm{Enc}(K, M_b)$ // test stage

$b' \leftarrow A^{Enc(K,\cdot)}(C^*)$ // guess stage
return $b' \stackrel{?}{=} b$

$Enc(K, M)$
----
return $\Pi.\mathrm{Enc}(K, M)$



Definition: The **IND-CPA-advantage** of an adversary A is

$$\mathbf{Adv}_{\Pi}^{\text{ind-cpa}}(A) = \left| \Pr\left[\mathbf{Exp}_{\Pi}^{\text{ind-cpa}}(A) \Rightarrow 1 \right] - 1/2 \right|$$

IND-CPA Insecurity of $\Pi 1$

Adversary A

1. Query $C \leftarrow \Pi 1.\mathrm{Enc}(K, 0^{128})$ in the find stage
2. Submit $M_0 = 0^{128}$ and $M_1 = 1^{128}$
3. Receive challenge C^*
4. if $C^* = C$ output 0
5. else, output 1

Actually, this attack works for any DETERMINISTIC Enc

Construction of IND-CPA secure enc

- We could construct an IND-CPA secure enc from PRF

- PRF generalizes the notion of PRG

- instead of considering “random-looking” strings we consider “random-looking” functions

pseudorandom function (PRF)

Definition: A **pseudorandom function (PRF)** is a function

$$F : \{0,1\}^k \times \{0,1\}^{in} \to \{0,1\}^{out}$$

satisfying security in next page

- k, in, out are called **key-length**, **input-length**, and **output-length** of F

- Think of a PRF as a *family* of functions:
  - For each $K \in \{0,1\}^k$ we get a function $F_K : \{0,1\}^{in} \to \{0,1\}^{out}$ defined by $F_K(X) = F(K, X)$

Secure PRFs

- Let $F : \{0,1\}^k \times \{0,1\}^{in} \to \{0,1\}^{out}$

- $S_F = \{\ F_K \ \big|\, K \in \{0,1\}^k\} \subseteq \text{Func}[in, out]$
- $\text{Func}[in, out]$: the set of *all* functions from $\{0,1\}^{in}$ to $\{0,1\}^{out}$

- F_K is **secure** if

$$\Pr\left[A^{F_K(\cdot)}(\quad) = 1 \,\middle|\, F_K \leftarrow S_F\right] - \Pr\left[A^{\tilde{F}(\cdot)}(\quad) = 1 \,\middle|\, \tilde{F} \leftarrow \text{Func}[in, out]\right] < negl$$

- Size of $\mathrm{Func}[in, out]$

size $2^{out \cdot 2^{in}}$

$\mathrm{Func}[in, out]$

If $out = in$ =128, size is $(2^{128})^{2^{128}}$

2^{in}

X	$\tilde{F}(X)$
$000 \ldots 000$	$101 \ldots 111$
$000 \ldots 001$	$001 \ldots 001$
$000 \ldots 010$	$111 \ldots 100$
$000 \ldots 011$	$101 \ldots 000$
$\vdots$	$\vdots$
$111 \ldots 111$	$100 \ldots 010$

out

S_F

size 2^k

AES-128: 2^{128}

Concrete PRF

- AES-128/256/512
- S_F=$2^{128}, 2^{256}, 2^{512}$

IND-CPA secure $\Pi 2$

Let F_k be a PRF

Alg $\Pi 2.\mathrm{Enc}(K, M)$

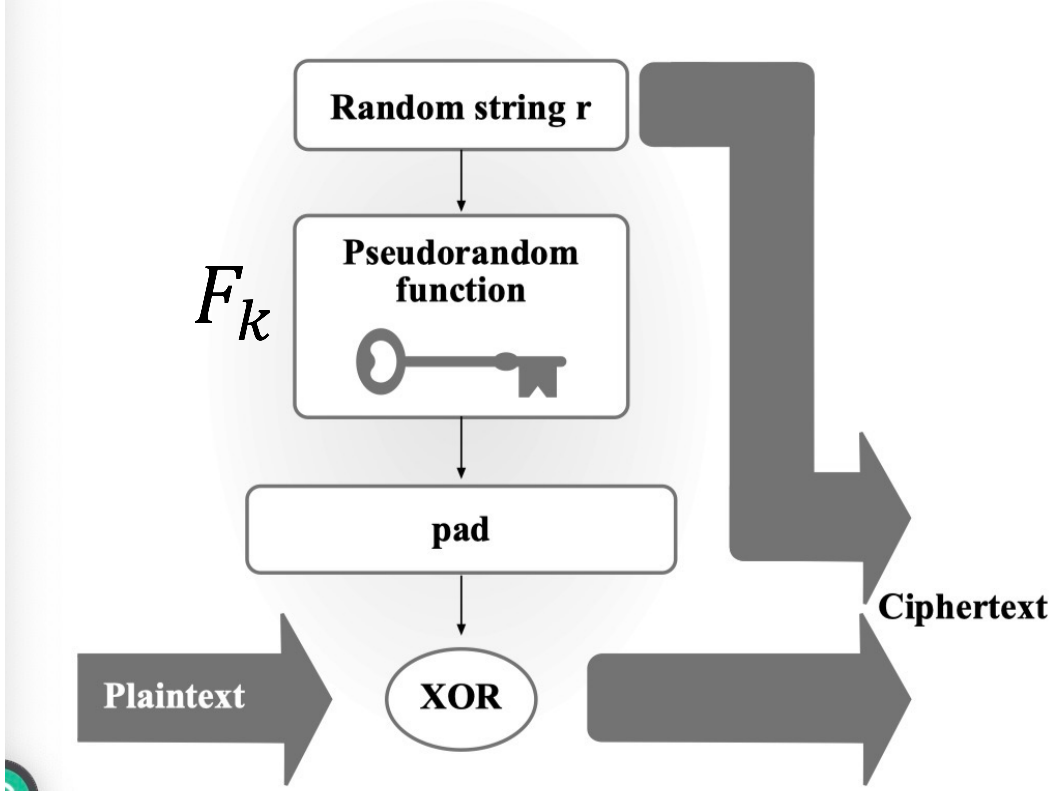
----

1. $r \leftarrow \{0,1\}^\wedge n$
2. $c_2 = F_k(r) \oplus M$
3. **return** $< r, c_2 >$

Alg $\Pi 2.\mathrm{Dec}(K, C)$

----

1. **return** $c_2 \oplus F_k(r)$



Proof idea: IND-CPA (choose plaintext attack)

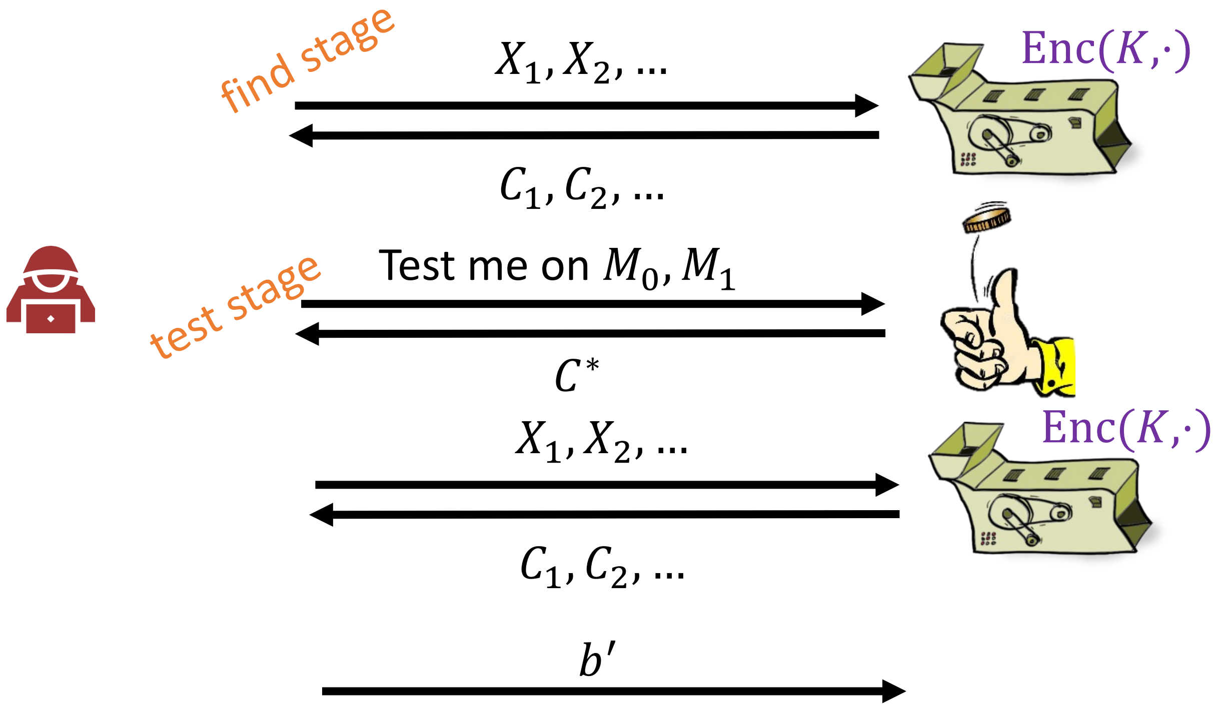
$\mathbf{Exp}_{\Pi 2}^{\text{ind-cpa}}(A)$

1. $b \stackrel{\$}{\leftarrow} \{0,1\}$
2. $K \stackrel{\$}{\leftarrow} \Pi 2.\text{Gen}$
3. $M_0, M_1 \leftarrow A^{Enc(K,\cdot)}$ // find stage
4.
5.
6. $C^* \leftarrow < r^*, F_K(r^*) \oplus M_b >$ // test stage
7. $b' \leftarrow A^{Enc(K,\cdot)}(C^*)$ // guess stage
8. **return** $b' \stackrel{?}{=} b$

$Enc(K, M)$

----

1. **return** $< r, F_K(r) \oplus M >$



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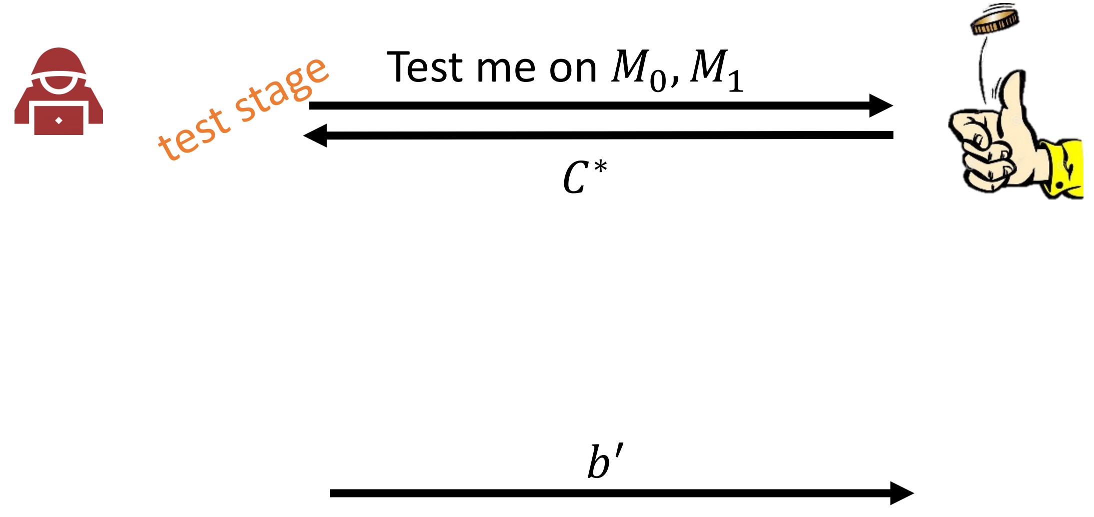
8. **return** $b' \stackrel{?}{=} b$

$Enc(K, M)$

----

1. **return** $< r, F_K(r) \oplus M >$ $\boxed{< r, \tilde{F}(r) \oplus M >}$



Step 1: Due to PRF

Proof idea: IND-CPA (choose plaintext attack)

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$M_0, M_1 \leftarrow A^{Enc(K,\cdot)}$ // find stage

$C^* \leftarrow < r^*, F_K(r^*) \oplus M_b >$ /

$< r^*, \tilde{F}(r^*) \oplus M_b >$

$b' \leftarrow A^{Enc(K,\cdot)}(C^*)$ // guess stage

return $b' \stackrel{?}{=} b$

$Enc(K, M)$

----

return $< r, F_K(r) \oplus M >$ $< r, \tilde{F}(r) \oplus M >$

test stage

Test me on M_0, M_1

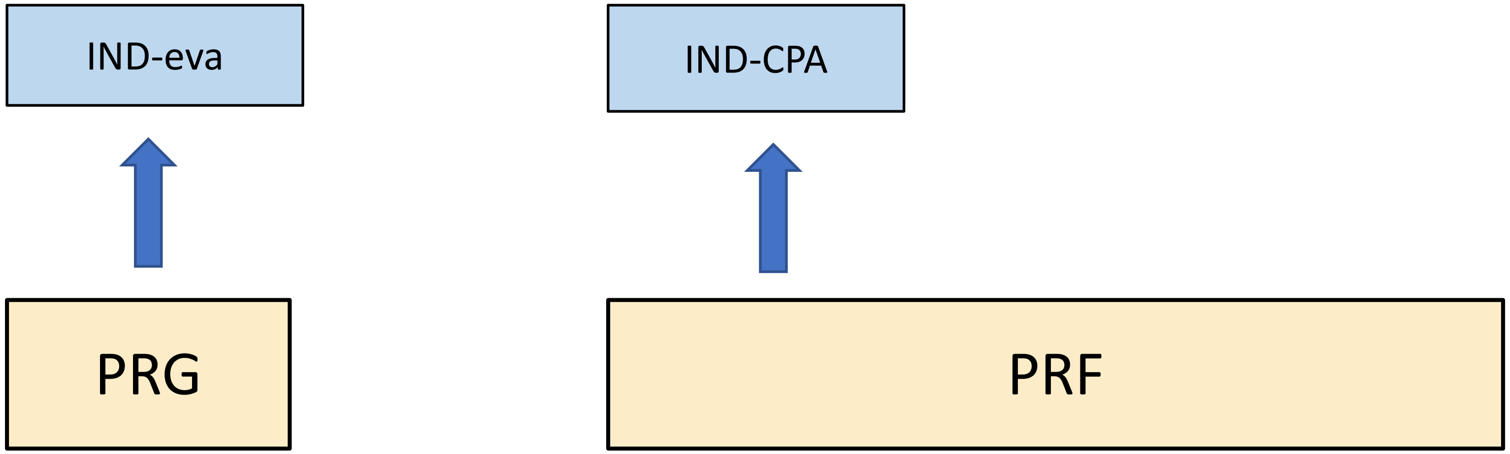
C^*

b'

Step 1: Due to PRF

Step 2: Due to PRF

A short summary

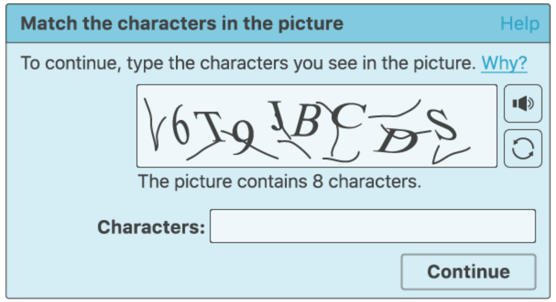


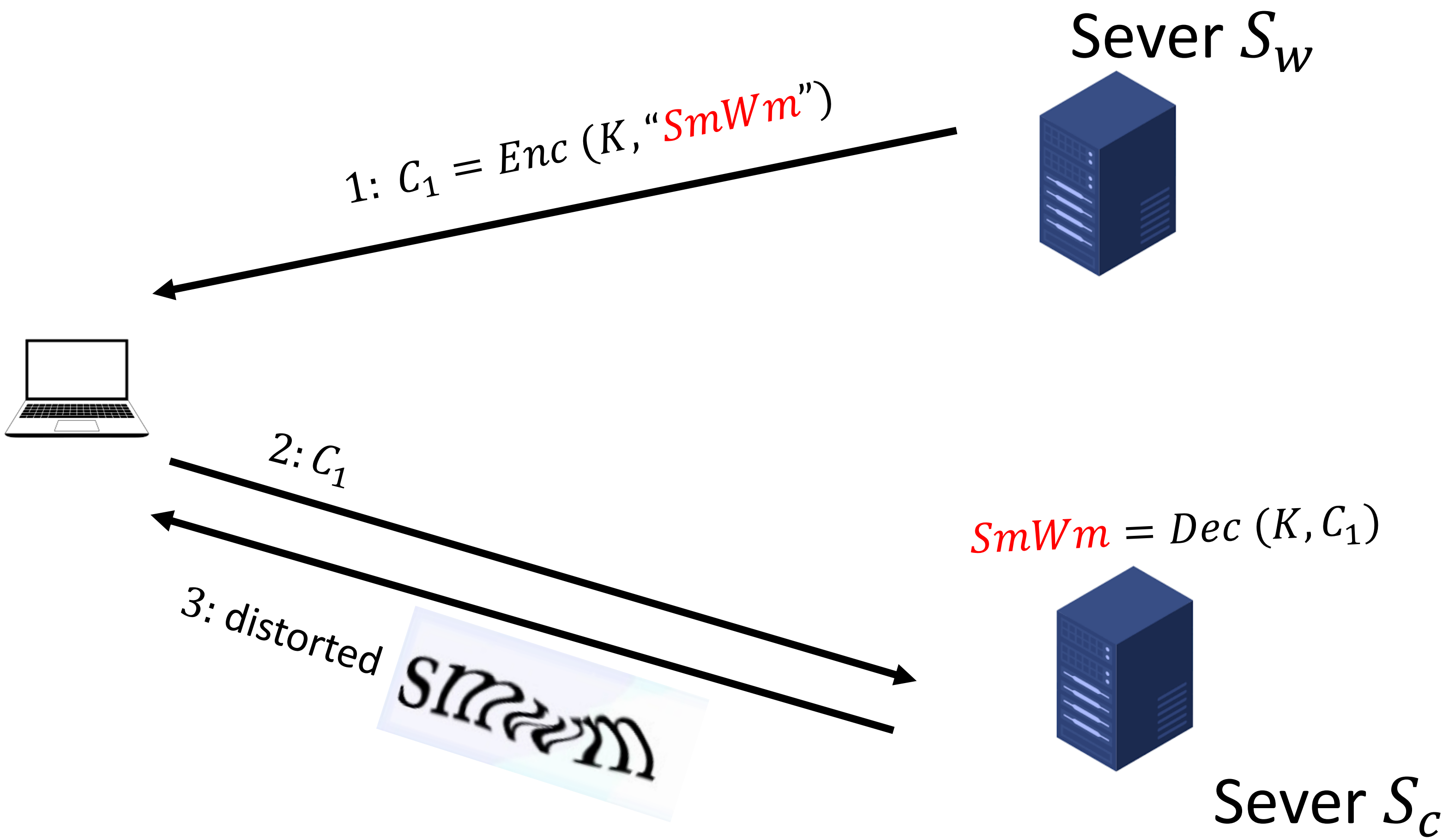
A short summary

- Define IND-CPA is necessary
- $\Pi 1$ is not IND-CPA secure
- With PRF in hand, we can construct generic IND-CPA secure Enc
- Stronger security????

Stronger Security: IND-CCA

- Example CAPTCHA





An adversary may have the capability to choose a ciphetext and get the message

IND-CCA (choose ciphertext attack)

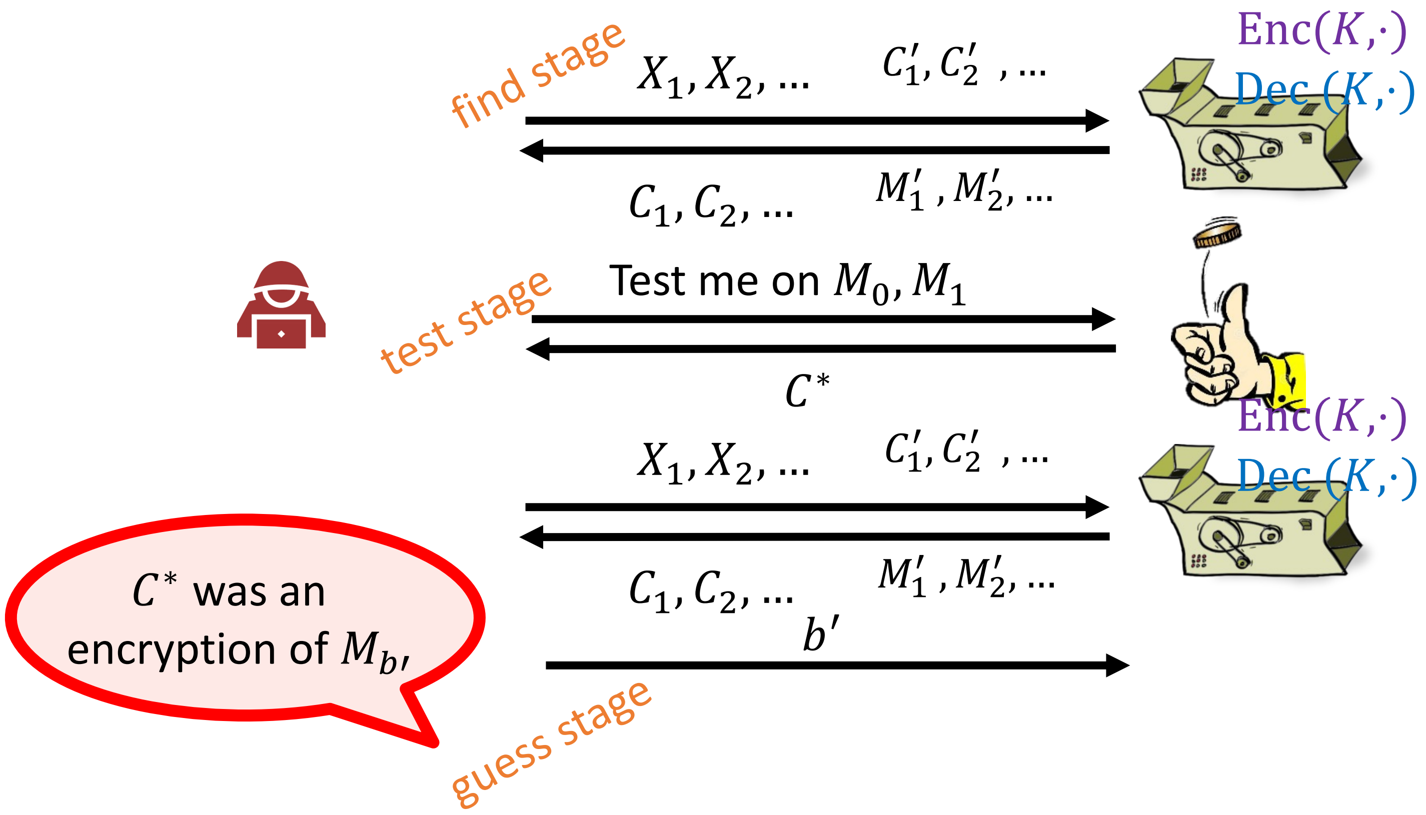
$\mathbf{Exp}_{\Pi}^{\text{ind-cpa}}(A)$

$b \xleftarrow{\$} \{0,1\}$
$K \xleftarrow{\$} \Pi.\mathrm{Gen}$
$M_0, M_1 \leftarrow A^{Enc(K,\cdot)}$ // find
  if $|M_0| \neq |M_1|$ **then**
  return $\perp$
$C^* \leftarrow \Pi.\mathrm{Enc}(K, M_b)$ // test
$b' \leftarrow A^{Enc(K,\cdot)}(C^*)$ // guess
return $b' \stackrel{?}{=} b$

$Enc(K, M)$

----

return $\Pi.\mathrm{Enc}(K, M)$



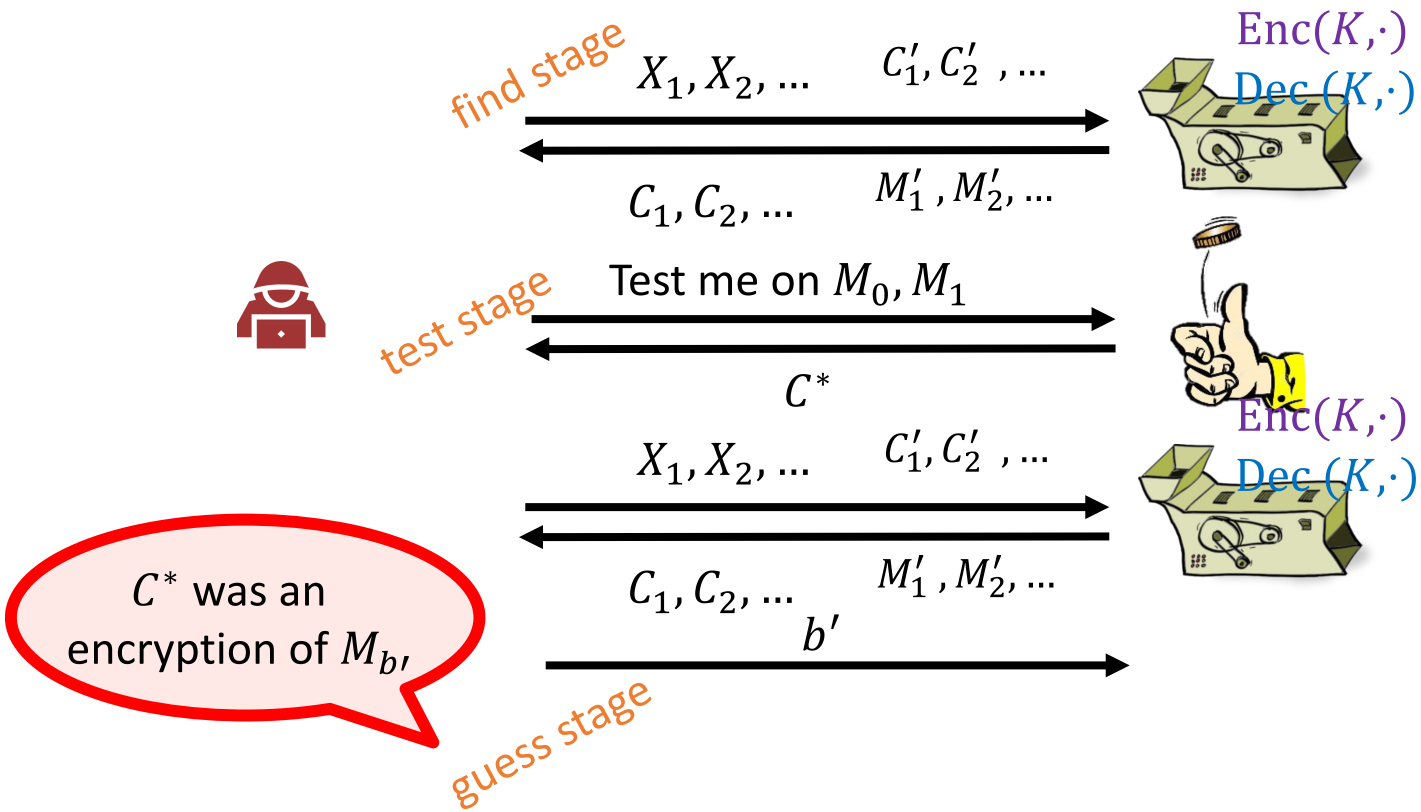
IND-CCA (choose ciphertext attack)

$\mathbf{Exp}_{\Pi}^{\text{ind-cca}}(A)$

$b \xleftarrow{\$} \{0,1\}$
$K \xleftarrow{\$} \Pi.\mathrm{Gen}$
$M_0, M_1 \leftarrow A^{Enc(K,\cdot)Dec(K,\cdot)}$ // find
  if $|M_0| \neq |M_1|$ **then**
  return $\perp$
$C^* \leftarrow \Pi.\mathrm{Enc}(K, M_b)$ // test
$b' \leftarrow A^{Enc(K,\cdot)Dec(K,\cdot)}(C^*)$ // guess
return $b' \stackrel{?}{=} b$

$Enc(K, M)$
----
return $\Pi.\mathrm{Enc}(K, M)$

$Dec(K, C), C \neq C^*$
----
return $\Pi.\mathrm{Dec}(K, C)$



Definition: The **IND-CCA-advantage** of an adversary A is

$$\mathbf{Adv}_{\Pi}^{\text{ind-cca}}(A) = \left|\Pr\left[\mathbf{Exp}_{\Pi}^{\text{ind-cca}}(A) \Rightarrow 1\right] - 1/2\right|$$

IND-CCA Insecurity of $\Pi2$

Adversary A

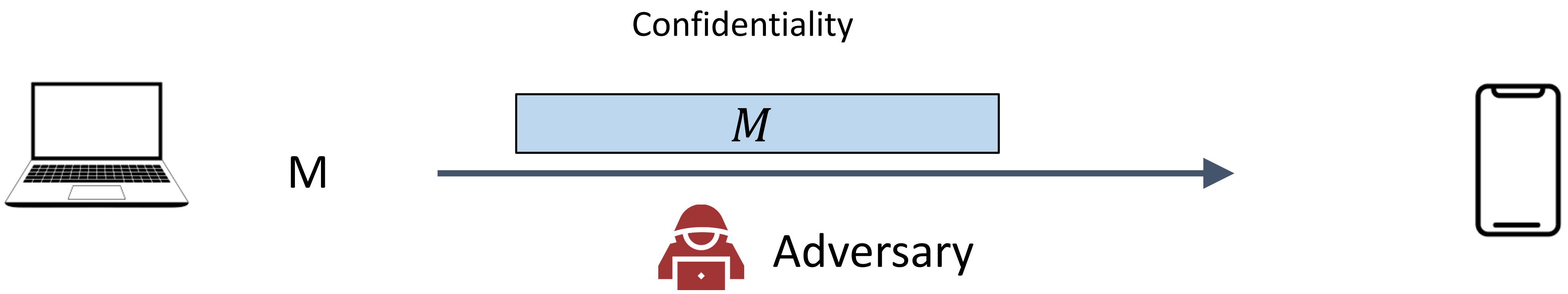
1. On receiving $C^* =< r^*, F_K(r^*) \oplus M_b >$
2. Query $C =< r^*, F_K(r^*) \oplus M_b \oplus M_0 >$ to Dec
3. On receiving $M_0 \oplus M_0$, set b=0
4. otherwise, b=1

Constructions

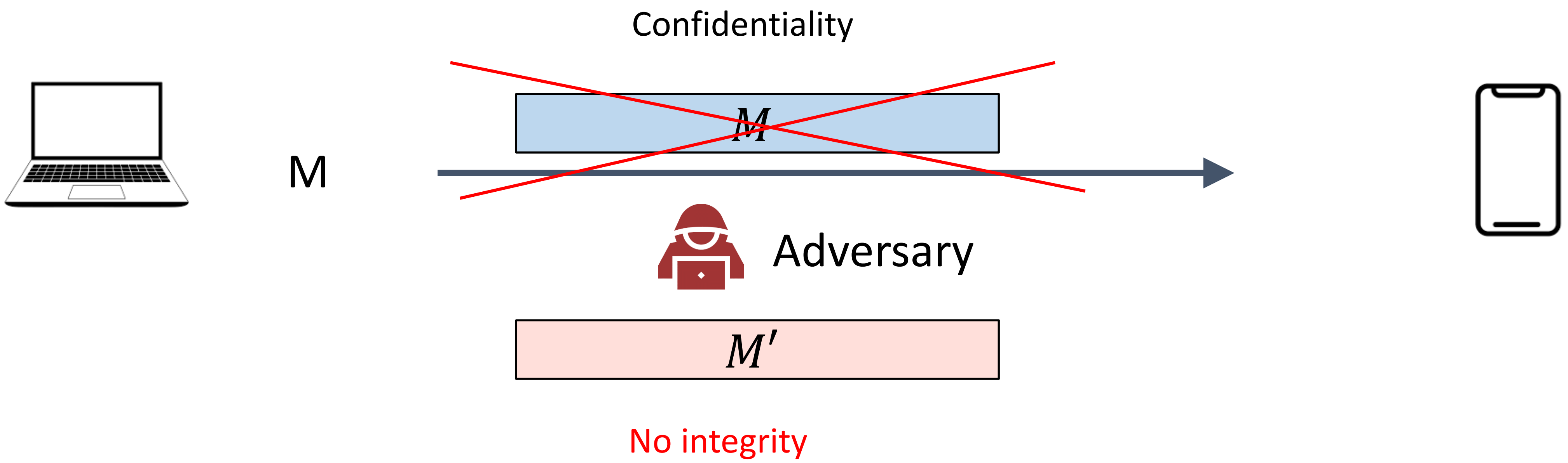
- We leave the construction of CCA secure Enc in the following part

- after introducing MAC

Massage Authenticated Code



Massage Authenticated Code

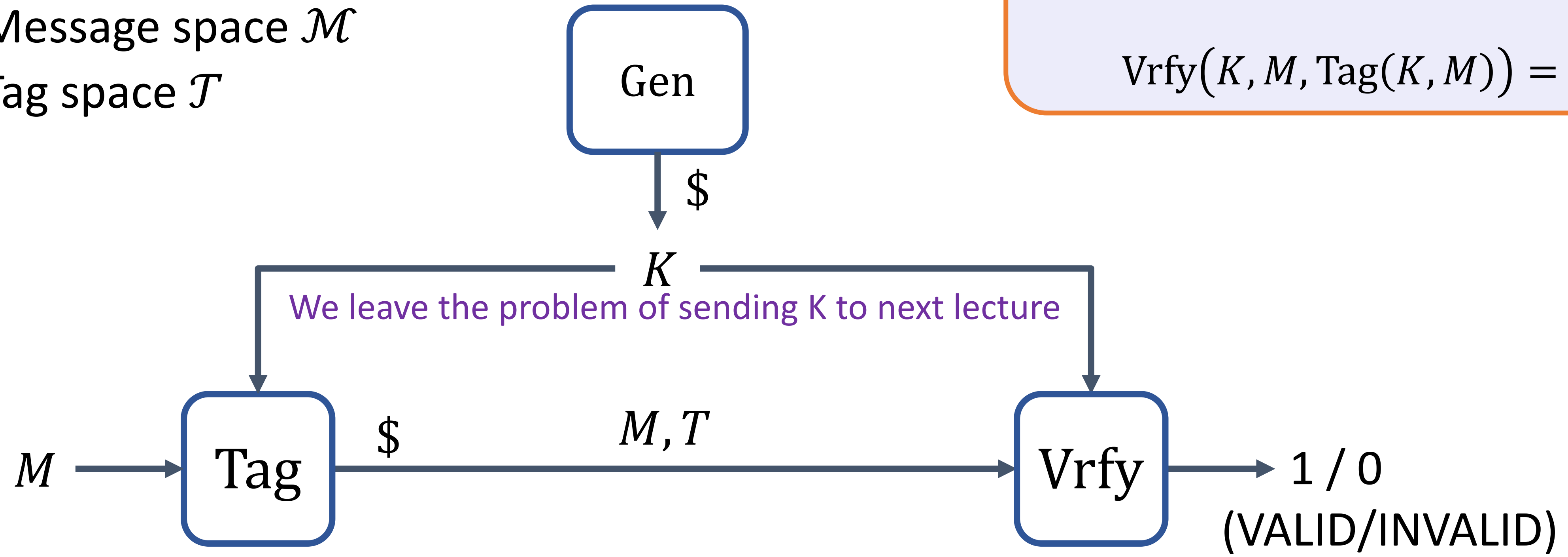


Message authentication code (MAC)– syntax

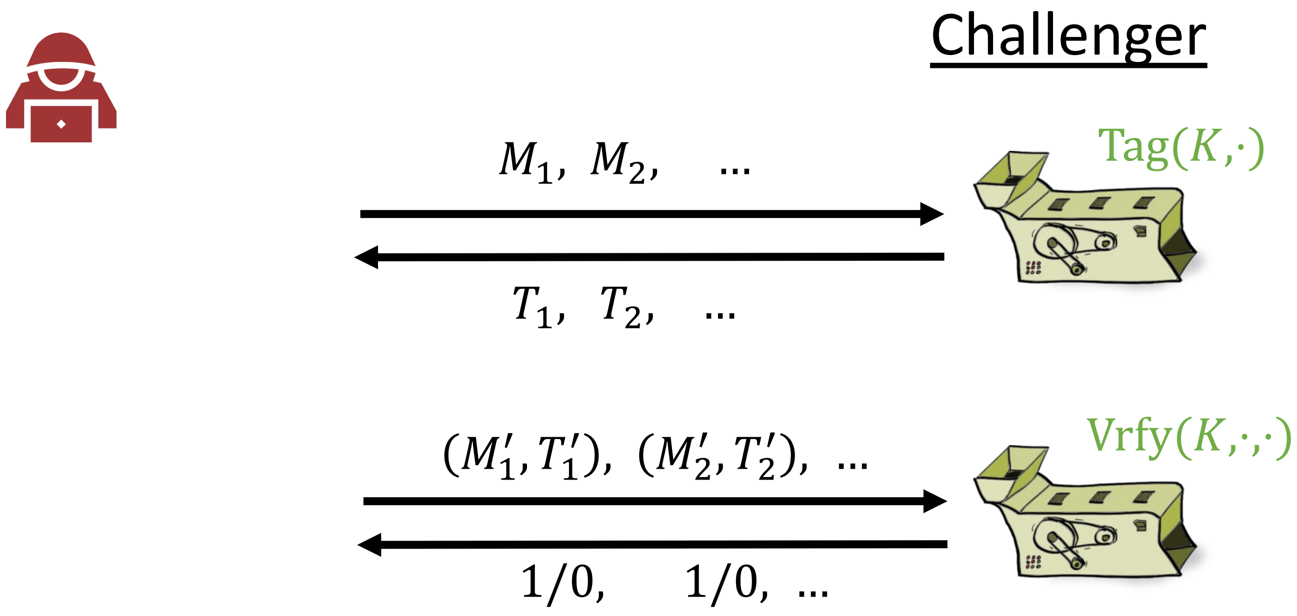
- A **message authentication scheme** $\Pi = (\mathrm{Gen}, \mathrm{Tag}, \mathrm{Vrfy})$ consists of three public algorithms:
- Associated to Π:
  - Key space $\mathcal{K}$
  - Message space $\mathcal{M}$
  - Tag space $\mathcal{T}$

Correctness requirement: For all $K \leftarrow \mathrm{Gen}$ and all $M \in \mathcal{M}$

$$\mathrm{Vrfy}\big(K, M, \mathrm{Tag}(K, M)\big) = 1$$



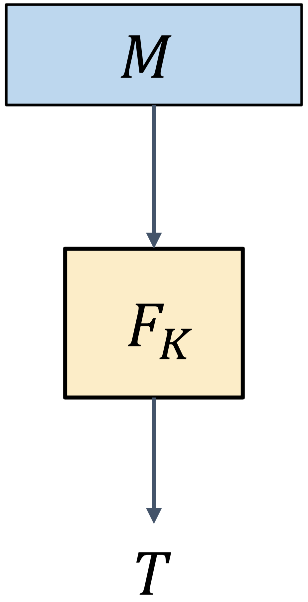
UF-CMA secure MAC



Adversary *wins* if a pair (M_i', T_i') is valid,
and was not among the pairs $(M_1, T_1), (M_2, T_2), \ldots$

PRFs are good MACs

$$\underbrace{F : \{0,1\}^k \times \{0,1\}^{in} \to \{0,1\}^{out}}_{\text{PRF}}$$



Alg $\Sigma_{\mathrm{PRF}}.\mathrm{Tag}(K, M)$

---

if $M \notin \{0,1\}^{in}$ **then**
  return $\perp$
return $F_K(M)$

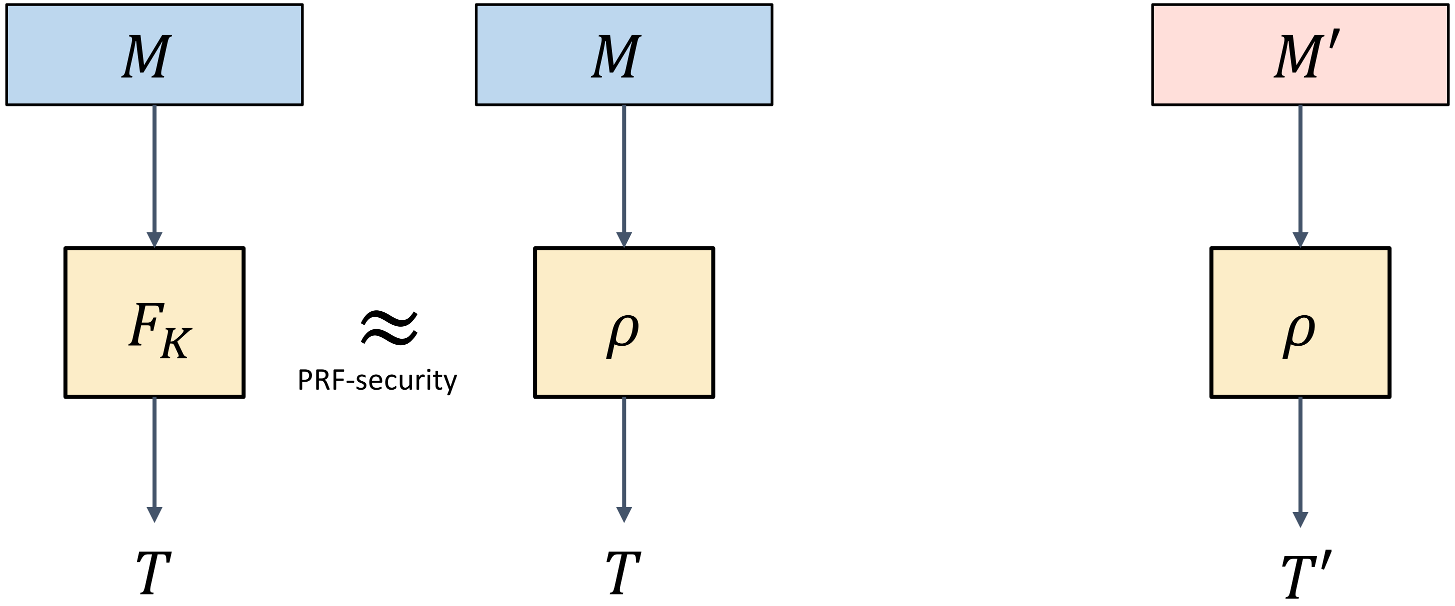
Alg $\Sigma_{\mathrm{PRF}}.\mathrm{Vrfy}(K, M, T)$

$T' \leftarrow F_K(M)$
return $T' \stackrel{?}{=} T$

Theorem: If F is a secure PRF then Σ_{PRF} is UF-CMA secure for *fixed-length* messages $M \in \{0,1\}^{in}$

PRFs are good MACs – proof sketch

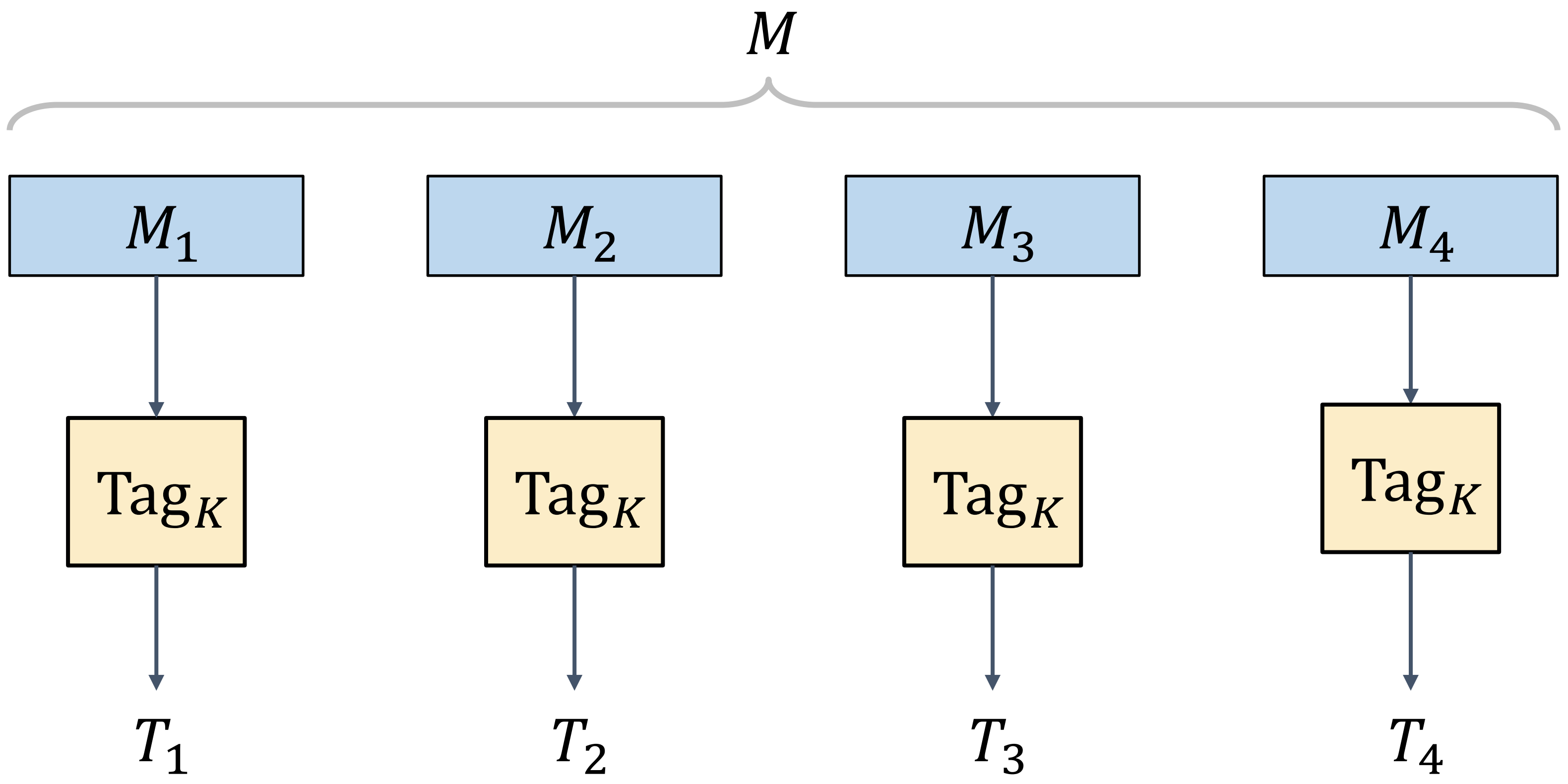
Theorem: If F is a secure PRF then Σ_{PRF} is UF-CMA secure for *fixed-length* messages $M \in \{0,1\}^{in}$



$$\Pr[\rho(M') = T'] = \frac{1}{2^{out}}$$

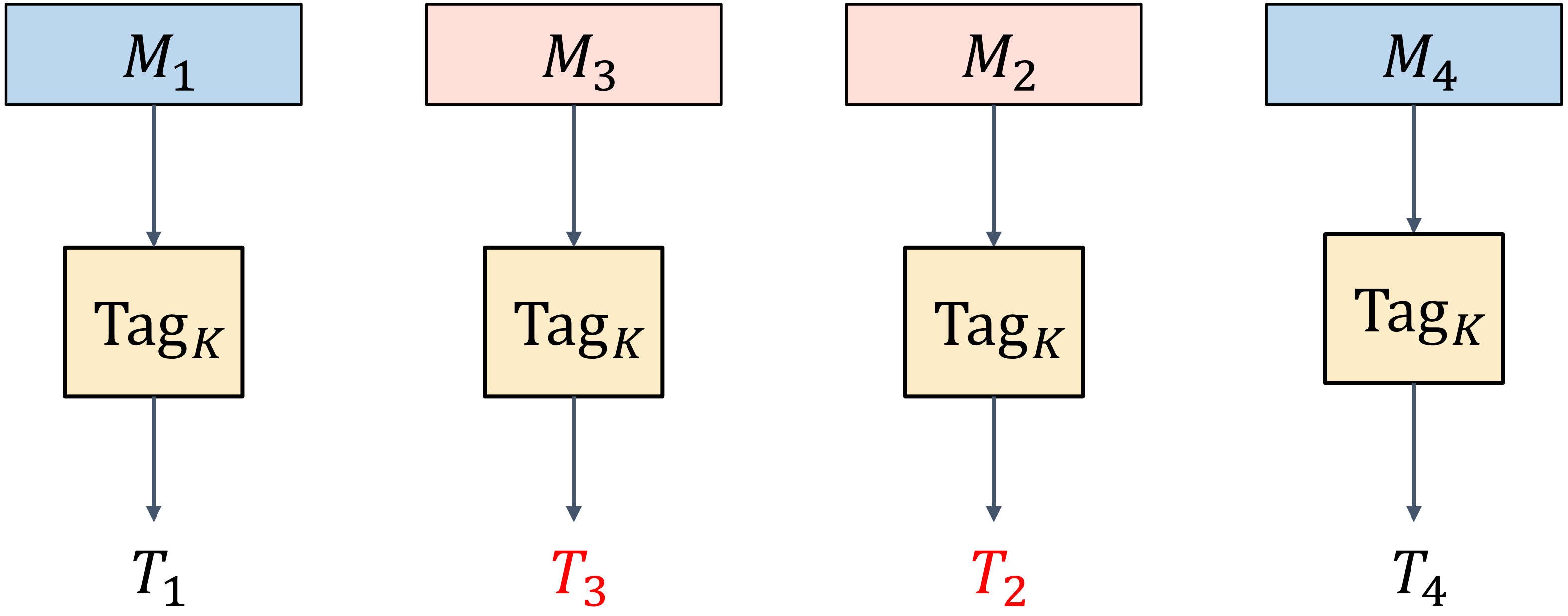
$$\rho \xleftarrow{\$} \text{Func}[in, out]$$

MAC for longer message Attempt 1:EBC



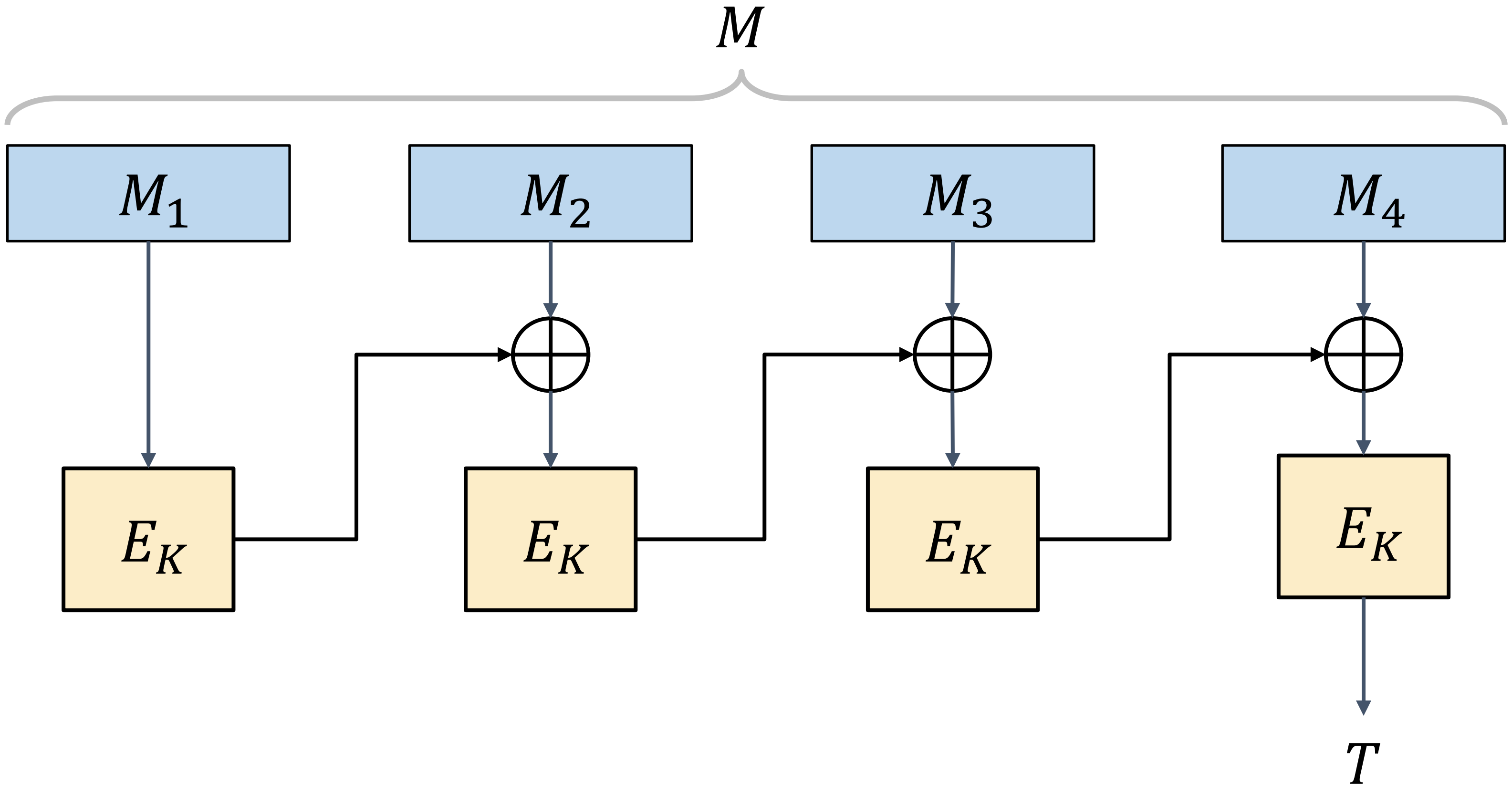
$$T = T_1||T_2||T_3||T_4$$

Attempt 1 – an attack



$$T = T_1 || T_3 || T_2 || T_4$$

CBC-MAC

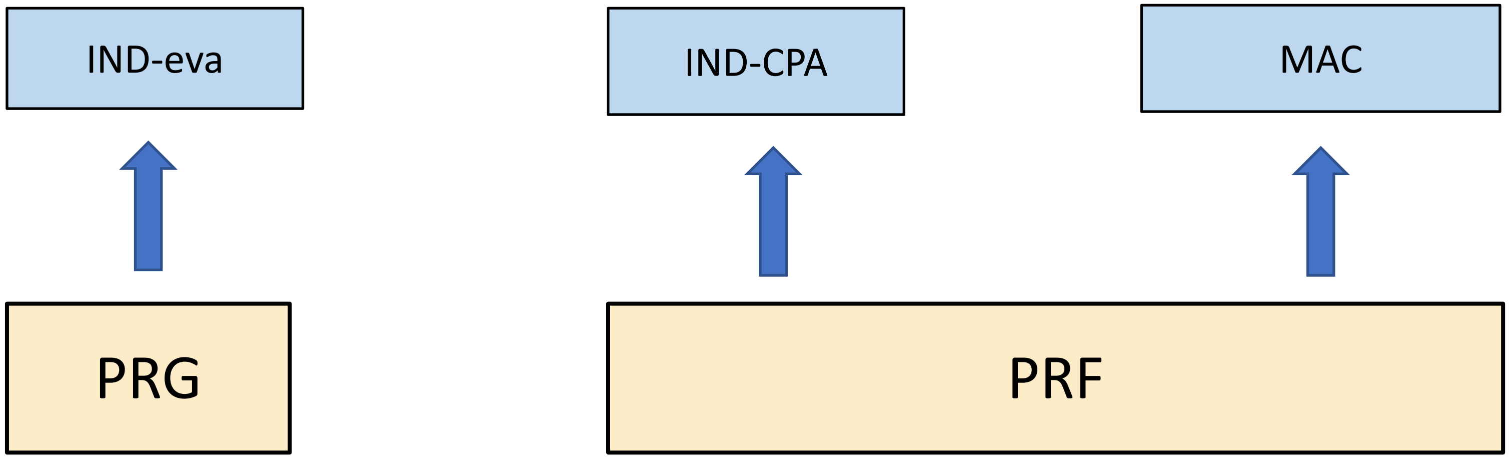


✓ Secure

A short summary

- IND-CCA security is necessary
- Existing studied schemes are not IND-CCA secure
- MAC could be used to provide integrity.
- With IND-CPA enc and MAC, we are ready to construct IND-CCA

A short summary



Recall IND-CCA

$\mathbf{Exp}_{\Pi}^{\text{ind-cpa}}(A)$

$b \xleftarrow{\$} \{0,1\}$

$K \xleftarrow{\$} \Pi.\mathrm{Gen}$

$M_0, M_1 \leftarrow A^{Enc(K,\cdot)Dec(K,\cdot)}$ // find

$C^* \leftarrow \Pi.\mathrm{Enc}(K, M_b)$ // test

$b' \leftarrow A^{Enc(K,\cdot)Dec(K,\cdot)}(C^*)$ // guess

return $b' \stackrel{?}{=} b$

$Enc(K, M)$

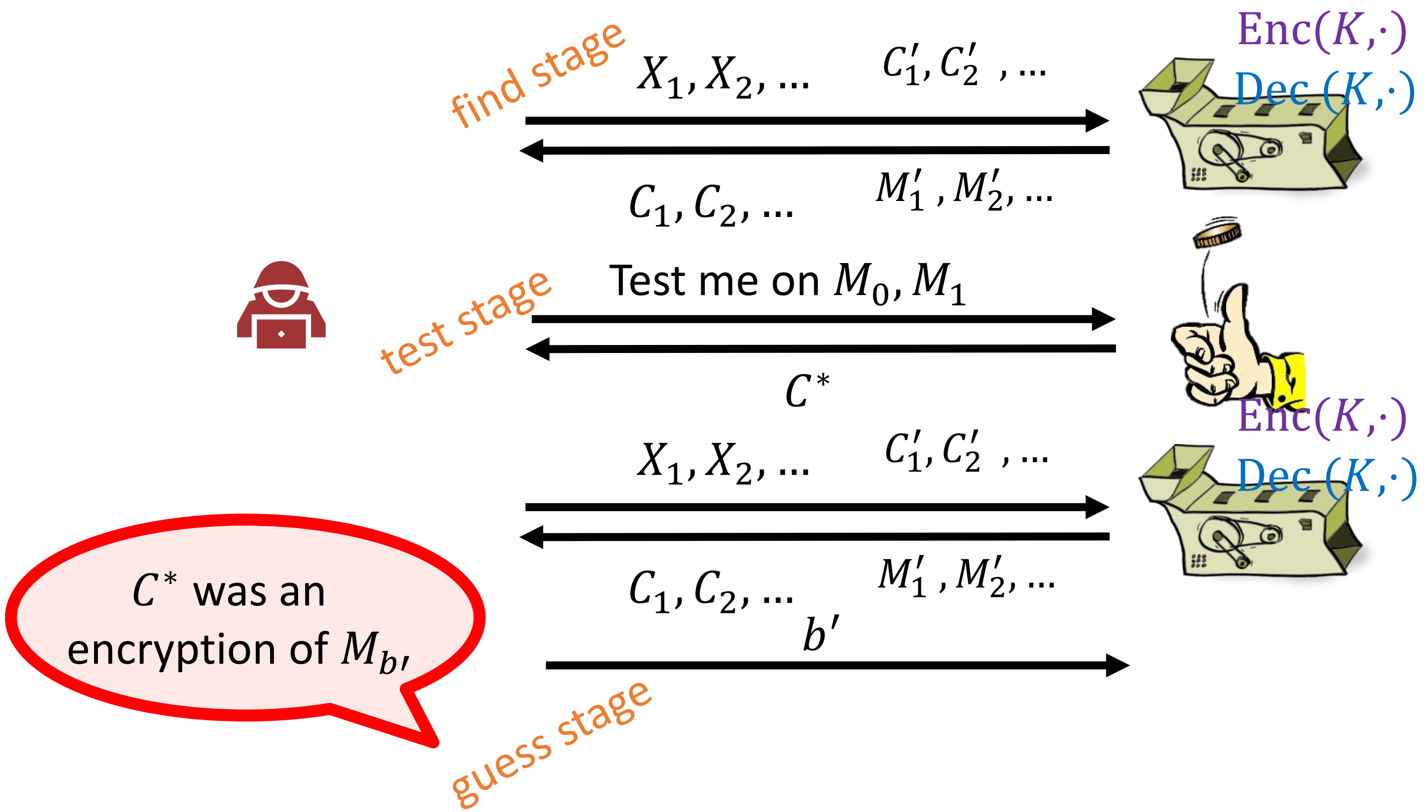
---

return $\Pi.\mathrm{Enc}(K, M)$

$Dec(K, C), C \neq C^*$

---

return $\Pi.\mathrm{Dec}(K, C)$

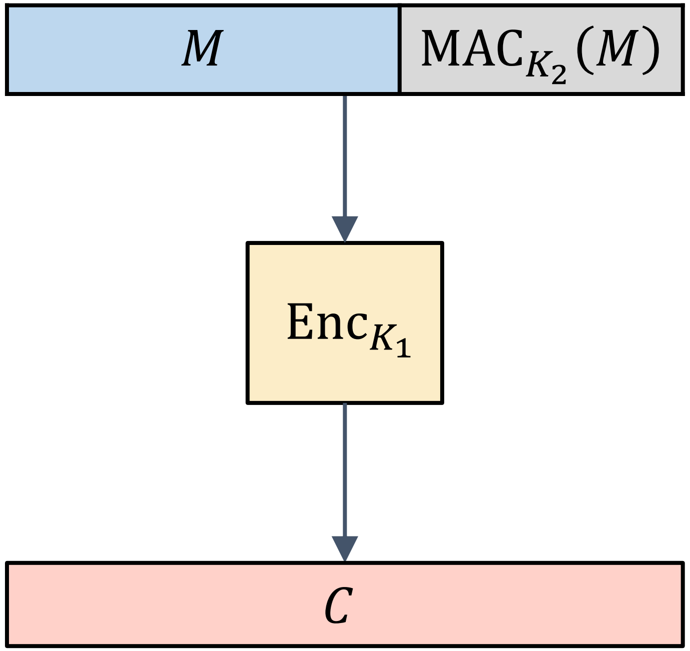


Definition: The **IND-CCA-advantage** of an adversary A is

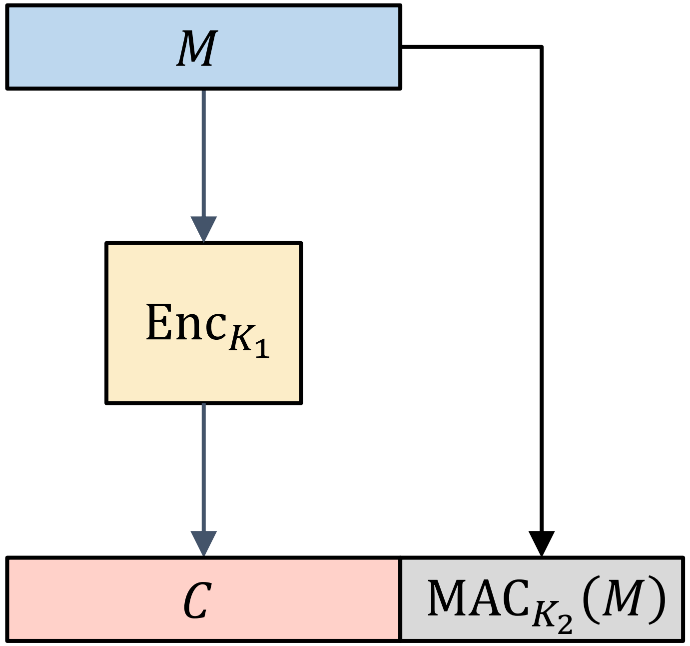
$$\mathbf{Adv}_{\Pi}^{\text{ind-cca}}(A) = \left|\Pr\left[\mathbf{Exp}_{\Pi}^{\text{ind-cca}}(A) \Rightarrow 1\right] - 1/2\right|$$

Generic composition: IND-CPA + MAC?→ IND-CCA

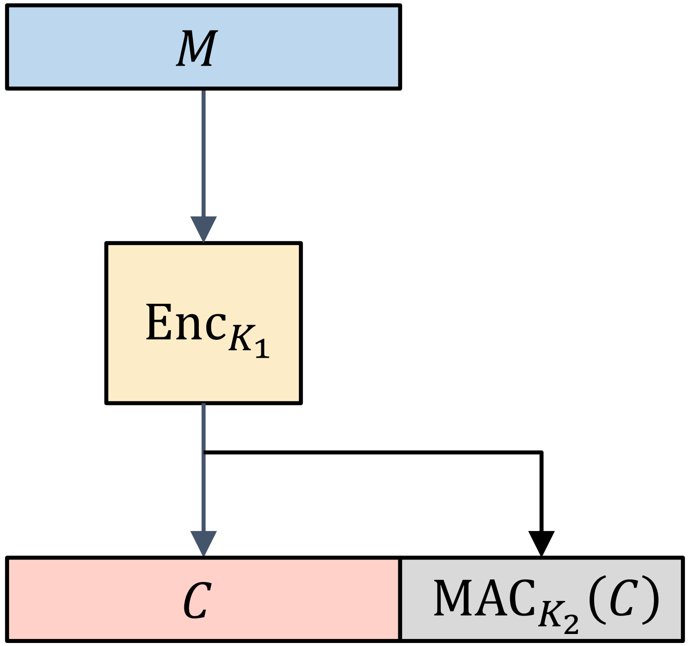
MAC-then-Encrypt (MtE)



Encrypt-and-MAC (E&M)



Encrypt-then-MAC (EtM)



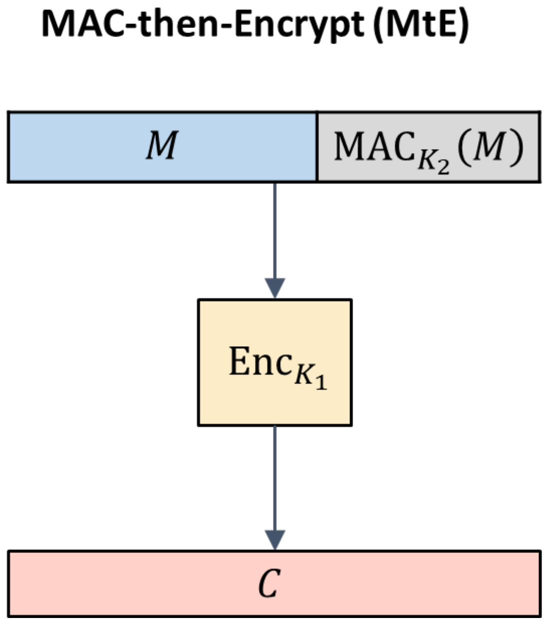
First Attempt: MAC-then-Encrypt (MtE)

- If $Enc(K, M)$ is IND-CPA secure,
- r $||Enc(K, M)$is also IND-CPA secure, where r is a random bit

- If $\mathrm{Enc}_{\mathrm{K}}(\cdot) = r||Enc(K,\cdot)$

CCA Adversary A

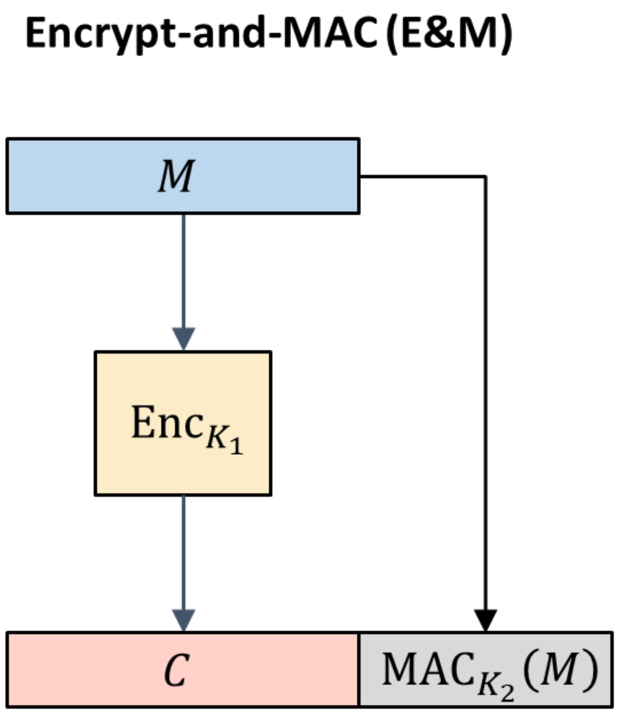
1. Query $\bar{r}||Enc(K, M, MAC_{k_2}(M))$ to Dec



Second Attempt: Encrypt-and-MAC (E&M)

- If $MAC_k(M)$ is a UF secure MAC,
- M || $MAC_k(M)$ is also a UF secure MAC

MAC does not provide confidentiality to the input



Encrypt-then-MAC (EtM)

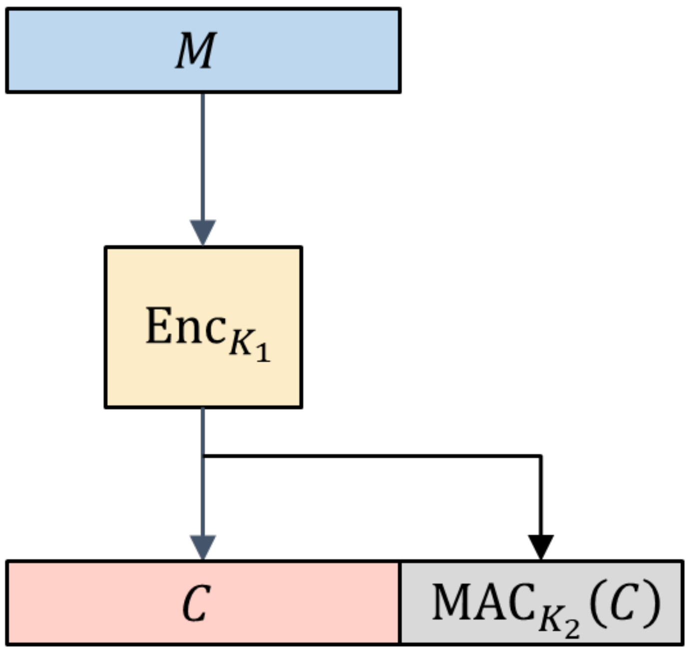
Let $\Pi 2 = (\text{Enc}, \text{Dec})$ be an IND-CPA enc
Let $\Pi_\text{m} = (\text{Tag}, \text{Vrfy})$ be a secure MAC

Alg $\Pi 3.\text{Gen}$
----
1. **return** random $\text{K} = (K_1, K_2)$

Alg $\Pi 3.\text{Enc}(K, M)$
----
1. $C = \Pi 2.\text{Enc}(K_1, M)$
2. **return** $< C, \text{Tag}(\text{K}_2, \text{C}) >$

Alg $\Pi 3.\text{Dec}(K, c_1 || c_2)$
----
1. **return** $\Pi 2.\text{Dec}(\text{K}_2, \text{c}_1)$ if $\text{Vrfy}(\text{K}_2, \text{c}_1, \text{c}_2) = 1$

Encrypt-then-MAC (EtM)



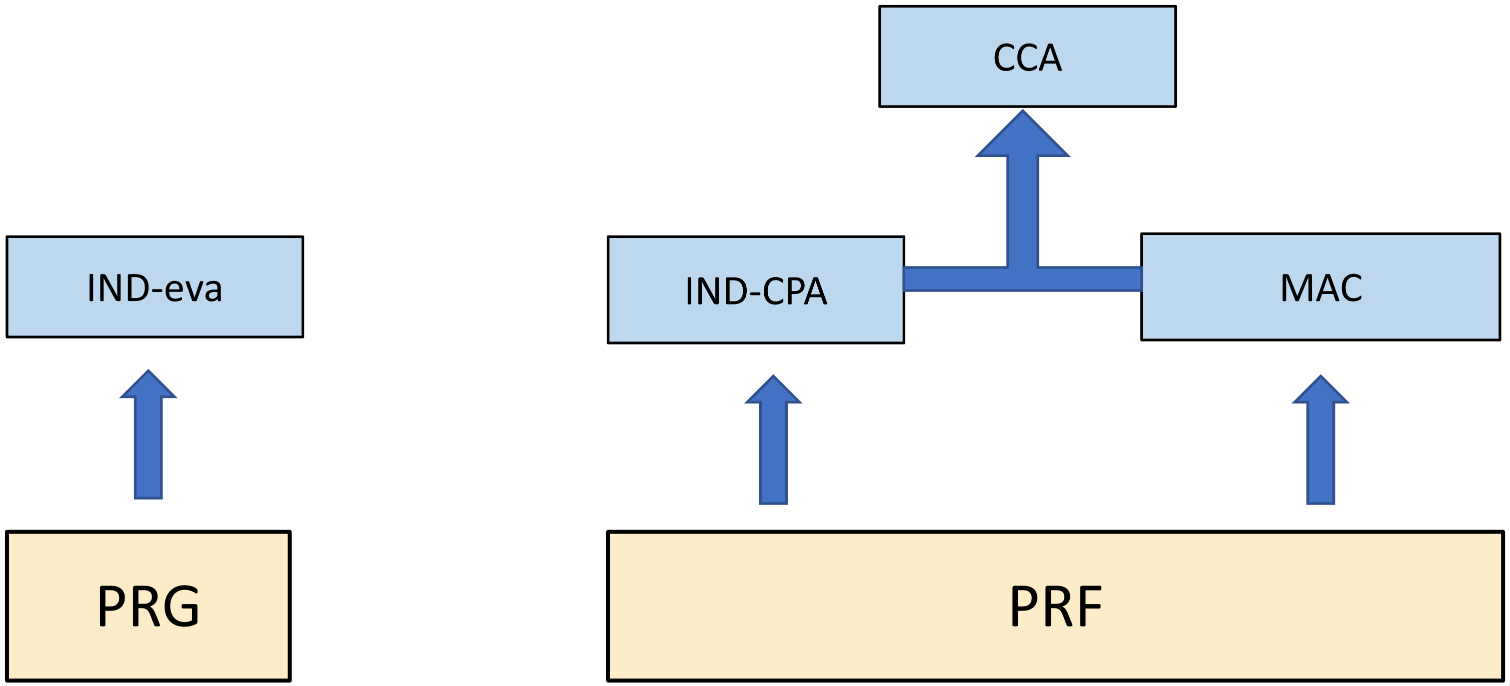
Proof idea: IND-CCA

Please refer to [KL20, Theorem 4.19] for the proof

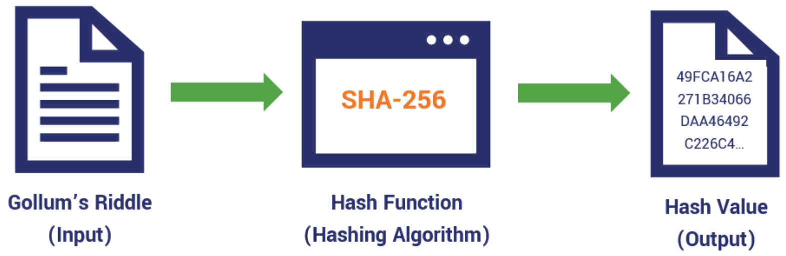
A short summary

- IND-CCA security is necessary
- We could construct an IND-CCA secure scheme from IND-CAP + MAC using Encrypt-then-MAC (EtM)

A short summary



Hash function



https://www.thesslstore.com/blog/what-is-a-hash-function-in-cryptography-a-beginners-guide/

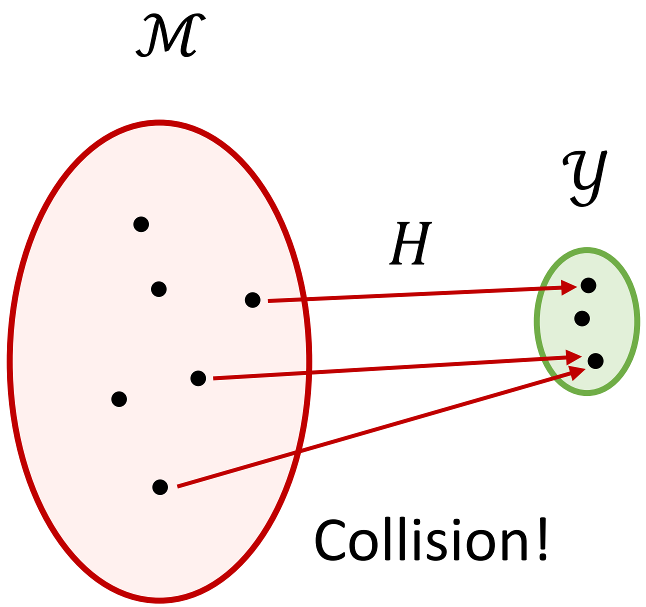
Hash functions

$H : \mathcal{M} \to \mathcal{Y}$

Keyless function

$|\mathcal{M}| \gg |\mathcal{Y}|$

Compressing



- SHA1 *: $\{0,1\}^{<2^{64}} \to \{0,1\}^{160}$
- $\text{SHA2}-256 : \{0,1\}^{<2^{64}} \to \{0,1\}^{256}$
- $\text{SHA3}-512 : \{0,1\}^{<2^{128}} \to \{0,1\}^{512}$

Collision Resistant

One way

* Insecure now. https://en.wikipedia.org/wiki/SHA-1#Attacks

Collision resistance

$\mathbf{Exp}_H^{\mathrm{cr}}(A)$
1. $(X_1, X_2) \leftarrow A_H$
2. **if** $X_1 \neq X_2$ **and** $H(X_1) = H(X_2)$ **then**
3. **return** 1
4. **else**
5. **return** 0

$\boldsymbol{A}$
1. Output (X_1, X_2) where X_1, X_2 is a collision for H

X_1, X_2 must *exist* since $|\mathcal{M}| \gg |\mathcal{Y}|$

hence $\mathbf{Adv}_H^{\mathrm{cr}}(A) = 1$ for unbounded A

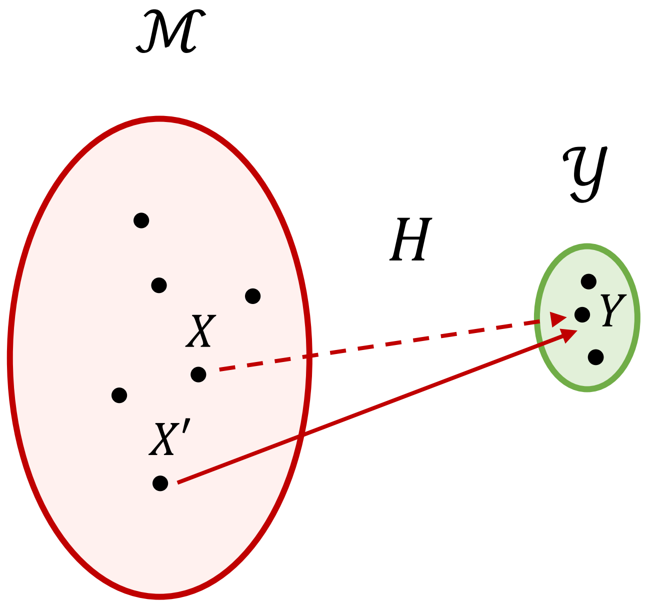
…but how do we actually find X_1, X_2?!

Definition: The **CR-advantage** of an adversary A against H is

$$\mathbf{Adv}_H^{\mathrm{cr}}(A) = \Pr[\mathbf{Exp}_H^{\mathrm{cr}}(A) \Rightarrow 1]$$

One-way security

$\mathbf{Exp}_H^{\mathrm{ow}}(A)$	
1.	$X \xleftarrow{\$} \mathcal{M}$
2.	$Y \leftarrow H(X)$
3.	$X' \leftarrow A_H(Y)$
4.	**return** $H(X') \stackrel{?}{=} Y$



Definition: The **OW-advantage** of an adversary A against H is

$$\mathbf{Adv}_H^{\mathrm{ow}}(A) = \Pr[\mathbf{Exp}_H^{\mathrm{cr}}(A) \Rightarrow 1]$$

Relation between notions

$\mathbf{Exp}_H^{\mathrm{cr}}(A)$
1. $(X_1, X_2) \leftarrow A_H$
2. **if** $X_1 \neq X_2$ **and** $H(X_1) = H(X_2)$ **then**
3. **return** 1
4. **else**
5. **return** 0

$\mathbf{Exp}_H^{\mathrm{ow}}(A)$
1. $X \xleftarrow{\$} \mathcal{M}$
2. $Y \leftarrow H(X)$
3. $X' \leftarrow A_H(Y)$
4. **return** $H(X') \stackrel{?}{=} Y$

Collision-resistance $\Longrightarrow$ One-wayness

Proof idea: suppose A_{ow} is an algorithm that breaks one-wayness

1. Pick $X \xleftarrow{\$} \mathcal{M}$ and give $Y \leftarrow H(X)$ to A_{ow}
2. A_{ow} outputs X'
3. output (X, X') as a collision ($H(X') = Y = H(X)$)

Problem: what if $X' = X$? Very unlikely assuming $|\mathcal{M}| \gg |\mathcal{Y}|$

Relation between notions

$\mathbf{Exp}_H^{\mathrm{cr}}(A)$
1. $(X_1, X_2) \leftarrow A_H$
2. **if** $X_1 \neq X_2$ **and** $H(X_1) = H(X_2)$ **then**
3. **return** 1
4. **else**
5. **return** 0

$\mathbf{Exp}_H^{\mathrm{ow}}(A)$
1. $X \xleftarrow{\$} \mathcal{M}$
2. $Y \leftarrow H(X)$
3. $X' \leftarrow A_H(Y)$
4. **return** $H(X') \stackrel{?}{=} Y$

Collision-resistance $\Longrightarrow$ One-wayness

Collision-resistance $\not\Longleftarrow$ One-wayness

Suppose $H : \mathcal{M} \rightarrow \{0,1\}^{256}$ is one-way. Define

$$H'(X) = \begin{cases} 0^{256} & \text{if } X = 0 \text{ or } X = 1 \\ H(X) & \text{otherwise} \end{cases}$$

H' is one-way

H' is **not** collision-resistant

Application– MAC domain extension (HMAC)

$\mathrm{MAC} : \mathcal{K} \times \{0,1\}^n \to \mathcal{T}$ $\qquad H : \{0,1\}^* \to \{0,1\}^n$

$\mathrm{MAC}' : \mathcal{K} \times \{0,1\}^* \to \mathcal{T}$

$\mathrm{MAC}'(K, M) = \mathrm{MAC}\big(K, H(M)\big)$ ⟵ Hash-then-MAC paradigm

Theorem: If H is collision-resistant and MAC is UF-CMA secure, then MAC' is UF-CMA secure

A short summary

- Hash functions are compressing functions
- Collision resistance and one-wayness are two properties of hash function
- Hash could be used to build HMAC

Summary

- Syntax and security of symmetric-key cryptography
- Perfect security and one-time pad
- Stream cipher, block cipher and MAC
- Hash function
- Constructions

Recap

Primitives	**Security**	**Examples**
Pseudorandom function (PRF)	Indistinguishability from random function	AES-128/256/512 HMAC
Encryption	IND-eva IND-CPA IND-CCA	PRG $+PRF Enc-t-Mac
MAC	Integrity	PRF CBC-MAC HMAC
Authenticated Encryption	IND-CCA (+ unforgeable encryption)	IND-CPA+MAC AES-256-GCM
Hash function	Collision-resistance + one-wayness	SHA2-256 SHA2-512 SHA3

- Symmetric key encryption assumes two paries have a shared key K

We will talk in the next lecture the problem of sending K

Thank you

Questions