Report on Value at Risk (VaR) Estimation and Backtesting

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1. Introduction

In the ever-evolving landscape of finance, the ability to accurately quantify and manage risk is paramount for institutions to thrive in dynamic markets. Among the myriad risk management tools available, Value at Risk (VaR) stands as a cornerstone, offering a quantitative measure of potential losses within a specified confidence level and time horizon. VaR estimation, however, is not without its challenges, as the accuracy and reliability of VaR models are subject to scrutiny and validation through rigorous backtesting procedures.

This comprehensive report delves into the intricate world of VaR estimation and backtesting, aiming to provide a thorough understanding of these essential components in risk management practices. Through a systematic exploration of various methodologies, techniques, and best practices, this paper endeavors to equip stakeholders with the knowledge and insights necessary to enhance their risk management frameworks.

Ultimately, this report serves as a comprehensive guide for professionals involved in risk management, providing a holistic understanding of VaR estimation and backtesting principles. By embracing the insights and recommendations outlined herein, institutions can fortify their risk management frameworks, fostering resilience and adaptability in an increasingly complex financial landscape.

2. Literature Review

2.1. Value at Risk Estimation

As Value at Risk (VaR) has remained a central focus of risk measurement and management, a vast body of literature has emerged on the subject. We have referenced several significant studies that align with the scope of our current investigation. Among earlier research, Crnkovic and Drachman (1995) introduced a metric and conducted a comparative analysis of the standard variance-covariance method and the historical simulation approach. Schinassi (1999) explored the dependency of VaR models on historical relationships between price movements in various markets and their susceptibility to breakdowns during periods of stress and turbulence marked by structural breaks in market relationships. While numerous studies exist, we have presented only a broad overview of the related literature here.

- **Parametric Method (Normal distribution):**

$$VaR(\alpha) = -\mu + \sigma \times z_\alpha$$

Where z_α is the standard normal variate corresponding to σ.

- **Historical Simulation Method:**

$$VaR_{0.99} = Return\ at\ the\ 1st\ percentile$$

- **Bootstrap historical simulation method**

  This method assumes that the distribution of returns will remain the same in the past and in the future and hence historical returns will be used in the forecast of VaR. Bootstrap historical simulation method (BHS) generates pseudo returns by sampling with replacement from the set of original returns. Then every bootstrap sample allows acquiringan estimate of VaR using (1). In the end, the average of all the VaR estimates gives us the bootstrap estimate of VaR.

2.2. Backtesting Methods

The backtesting methods reviewed can be grouped into different categories:

- **Unconditional Coverage test (Kupeic 1995)**:

  Kupiec Unconditional Backtesting compares the actual occurrences of losses against the predicted VaR levels without considering conditional factors. The method assumes that the data generating process remains unchanged over time, allowing for a straightforward assessment of VaR accuracy based on historical observations. Kupiec testing employs statistical tests, often based on binomial or chi-square distribution, to determine if the frequency of exceptions aligns with the specified confidence level.

  The test statistic is

$$LR_{uc} = \frac{\pi_{exp}^{n_1}\left(1 - \pi_{exp}\right)^{n_0}}{\pi_{obs}^{n_1}\left(1 - \pi_{exp}\right)^{n_0}}$$

  Where is the expected proportion of$\pi_{exp}\pi_{obs}$ is the observed proportion of exceedances, n_1 is the number of exceedances and $n_0 = n - n_1$ where n is the sample size.

  The asymptotic distribution of -2ln LRuc is chi-squared with two degrees of freedom.

- **Conditional Coverage test (Christoffersen 1998)**

  Conditional testing, as exemplified by Christoffersen's method, extends the evaluation of Value at Risk (VaR) accuracy by incorporating additional information about market conditions and volatility dynamics. Unlike unconditional testing, which assumes a static data generating process, conditional testing acknowledges the potential for changes in market behavior over time. This section outlines the methodology and formula for Christoffersen Conditional Backtesting.

  The test statistic is

$$LR_{cc} = \frac{\pi_{exp}^{n_1}\left(1 - \pi_{exp}\right)^{n_0}}{\pi_{01}^{n_{01}}\left(1 - \pi_{\exp 01}\right)^{n_0}\pi_{11}^{n_{11}}\left(1 - \pi_{11}\right)^{n_{10}}}$$

  where n_{ij} is the number of indicator i followed by indicator j,
  $\pi_{01} = n_{01}/(n_{00} + n_{01})$ and $\pi_{11} = n_{11}/(n_{10} + n_{11})$

  The asymptotic distribution of -2ln LRcc is chi-squared with two degrees of freedom.

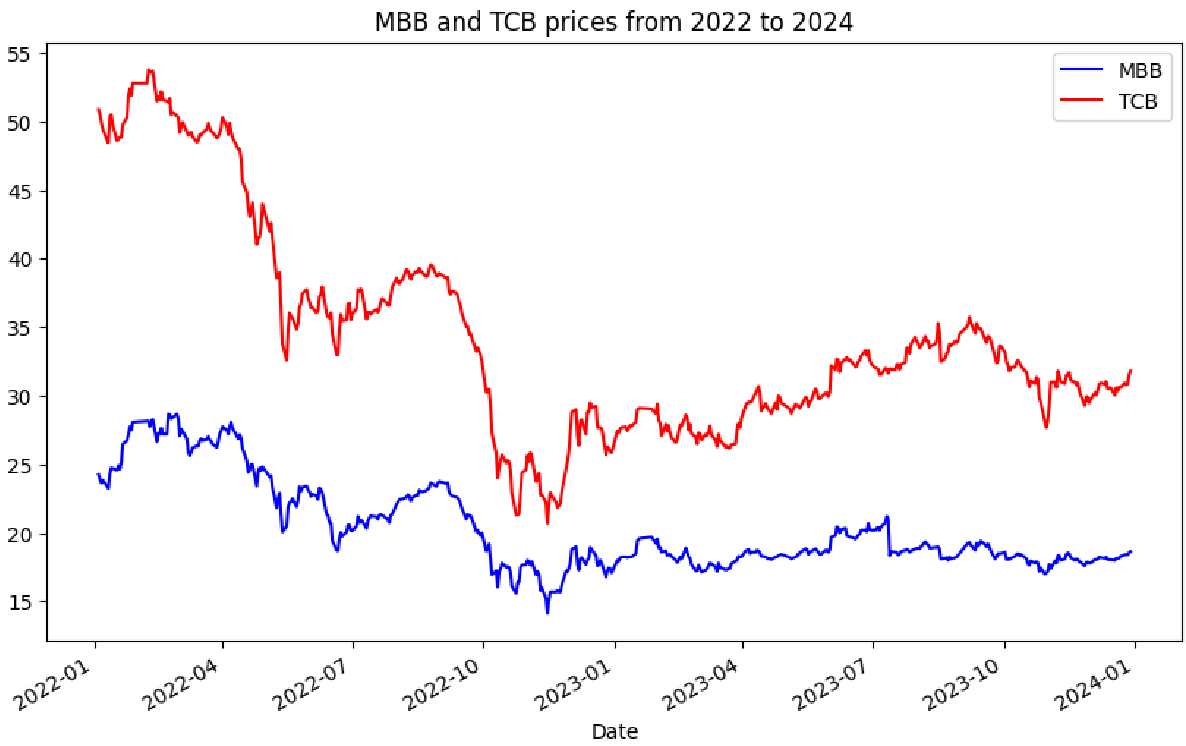
3. Data

Our data comes from vn.investing.com, a well-known trading website trusted for its wealth of financial information. We've focused on two bank stocks, MBB and TCB, to build our portfolio. We've gathered data spanning two years, from 1/1/2022 to 1/1/2024 to capture a broad range of market conditions.

Snapshot of data:

	MBB	TCB
Date		
2023-12-29	18.65	31.80
2023-12-28	18.55	31.50
2023-12-27	18.40	30.85
2023-12-26	18.45	30.80
2023-12-25	18.40	30.95

By analyzing how these stocks performed over this timeframe, we aim to uncover patterns and insights that can guide investment strategies. Through simple yet robust data analysis, we hope to shed light on the behavior of these stocks and help investors make informed decisions.



4. VaR Estimation

4.1. Portfolio Settings

In this project, we establish a portfolio with an initial capital of 100 million VND, with the aim of constructing a diversified investment strategy across two assets: MBB (Millitary Commercial Joint Stock Bank) and TCB (Techcombank). Our portfolio allocation is carefully crafted, with 40% of the total capital allocated to MBB and the remaining 60% to TCB. This strategic allocation is designed to balance risk and return potential across different sectors or asset classes, aiming to optimize portfolio performance while managing risk exposure.

The portfolio information is presented comprehensively below, detailing the allocation of assets, number of shares held, latest prices, and corresponding investment amounts for each asset, with a total initial capital of 100 million VND.

Asset	Shares	Latest Price	Investment Amount
MBB	2,145	18,650	40,000,000 VND
TCB	1,887	31,800	60,000,000 VND
Portfolio			**100,000,000 VND**

Table 1: Portfolio Informations

4.2. VaR Estimation using Parametric Method (Normal Distribution)

To embark on the parametric approach, our initial step involves delving into the descriptive statistics of our portfolio returns. By comprehensively examining the characteristics and behavior of our portfolio's returns, we establish a foundational understanding crucial for statistic analysis.

Table 2: Descriptive Statistic on Assets Returns

Descriptive Statistic	*MBB Returns*	*TCB Returns*	Portfolio Returns
Mean	0.0005	0.0009	0.0008
Standard Error	0.0010	0.0010	0.0009
Median	0.0000	0.0000	-0.0004
Mode	0.0000	0.0000	0.0000
Standard Deviation	0.0219	0.0226	0.0211
Sample Variance	0.0005	0.0005	0.0004
Kurtosis	4.7550	2.4210	2.7247
Skewness	0.8732	0.4125	0.5758
Range	0.2018	0.1398	0.1390
Minimum	-0.0668	-0.0673	-0.0668
Maximum	0.1350	0.0725	0.0722
Sum	0.2625	0.4704	0.3873
Count	497	497	497

The table above presents descriptive statistics for the returns of MBB (Military Commercial Joint Stock Bank), TCB (Techcombank), and the portfolio as a whole. For the Parametric Method, Mean and Standard Deviation will be used to estimate Value at Risk in a day of each asset.

The table below summarizes the VaR values for MBB, TCB, and the overall portfolio using the Parametric Approach:

Table 3: VaR estimation by Parametric (Norm) Method

Asset	**Amount (million VND)**	**VaR (99%, 1 day) (million VND)**
MBB	40	2.014
TCB	60	3.096
Portfolio	100	4.819

MBB's VaR value suggests that with a 99% confidence level, the maximum potential loss for the MBB asset over a one-day period is estimated to be 2.014 million VND. Investors holding MBB can expect losses not to exceed 2.014 million VND 99 times out of 100 under normal market conditions.

Similarly, for TCB, the VaR value indicates that with a 99% confidence level, the maximum potential loss for the TCB asset over a one-day period is estimated to be 3.096 million VND. Investors holding TCB can expect losses not to exceed 3.096 million VND 99 times out of 100 under normal market conditions.

The portfolio's VaR represents the combined risk of holding both MBB and TCB assets together in the specified proportions. With a 99% confidence level, the maximum potential loss for the portfolio over a one-day period is estimated to be 4.82 million VND. This VaR value reflects the diversification effect of combining multiple assets in the portfolio and accounts for the correlation between MBB and TCB.

4.3. VaR Estimation using BHS Method

The table below summarizes the VaR values for MBB, TCB, and the overall portfolio using the Bootstrap Historical Simulation method:

Table 4: VaR estimations by BHS Method

Asset	**Amount (million VND)**	**VaR (99%, 1 day) (million VND)**
MBB	40	2.195
TCB	60	3.926
Portfolio	100	5.821

With a 99% confidence level, the maximum potential loss for the MBB asset over a one-day period is estimated to be 2.195 million VND. This VaR value indicates the level of risk associated with holding the MBB asset, where losses are not expected to exceed 2.195 million VND 99 times out of 100 under normal market conditions.

For TCB, the VaR value suggests that with a 99% confidence level, the maximum potential loss for the TCB asset over a one-day period is estimated to be 3.926 million VND. Investors holding TCB can expect losses not to exceed this amount 99 times out of 100 under normal market conditions.

The portfolio's VaR represents the combined risk of holding both MBB and TCB assets together in the specified proportions. With a 99% confidence level, the maximum potential loss for the portfolio over a one-day period is estimated to be 6.993 million VND. This VaR value accounts for the diversification effect of combining multiple assets in the portfolio and reflects the overall risk exposure of the portfolio to market fluctuations.

In addition, the 99% Boostrap VaR of portfilo confidence interval is [4278.41045631, 6530.39356272].

5. Backtesting

In this section, we perform a backtesting with a sample of 250 data points and apply unconditional and conditional tests to assess the effectiveness of the method in predicting VaR.

Hypothesis for Testing Models:

$$\begin{cases} H_0: Number\ of\ exceptions\ is\ not\ signigicantly\ different\ from\ expected\ number \\ H_1: Number\ of\ exception\ is\ significantly\ different\ from\ expected\ number \end{cases}$$

If the null hypothesis is rejected, it means that alternative hypothesis is accepted then our models passed Backtesting. Otherwise, models did not passed Backtesting.

To perform this hypothesis testing, we can utilize a two-pronged approach involving the Kupiec Unconditional Coverage Test and a separate Christoffersen Conditional Coverage Test. The Kupiec test assesses whether the observed violation rate aligns with the expected violation rate for a chosen VaR (Value-at-Risk) level. The conditional coverage test, often employed in conjunction with the Kupiec test, examines whether violations exhibit any clustering behavior, a potential sign of weaknesses in the model. By combining these two tests, we gain a more comprehensive understanding of the VaR model's effectiveness.

5.1. Unconditional Test

Performance of models under Kupeic Coverage Unconditional testing

5.1.1. Unconditional Testing on Parametric Method (Normal Distribution)

Confidence Level	**VaR**	**Violations**	**LRUC**	**Chi-Square**	**P-Value**	**Test Outcome**
0.99	-0.154605	12	0.08105	5.025168	2.50E-02	Not Reject H0

0.95	-0.066812	22	2.6971E-06	25.646647	4.10E-07	Reject H0
0.9	-0.04373	33	0.00063	14.722006	1.25E-04	Reject H0

Table 5:Unconditional Backtesting on Parametric VaR Estimations

At the 99% confidence level, the model fails to reject the null hypothesis (H0) - indicating no statistically significant difference between the observed violation rate and the expected violation rate. This suggests the model might be performing adequately at this confidence level. For confidence levels of 95% and 90%, the model rejects H0. This implies a statistically significant difference between the observed violations and expected violations, suggesting potential shortcomings in the model's accuracy at these confidence levels.

5.1.2. Unconditional Testing on BHS Method

Confidence Level	VaR	Violations	LRUC	Chi-Squared	P-Value	Test Outcome
0.99	-0.064794	3	0.579842	1.09E+00	0.446821	Not Reject H0
0.95	-0.039839	13	0.600496	1.02E+00	0.438359	Not Reject H0
0.9	-0.025037	25	1	1.40E-14	0.317311	Not Reject H0

Table 6:Unconditional Backtesting on BHS VaR Estimations

The BHS method consistently fails to reject H0 across all confidence levels (99%, 95%, and 90%). This suggests the model might be performing adequately at all tested confidence levels based on this backtesting method.

5.2. Conditional testing

Performance of models under Christoffersen Coverage Conditional testing

5.2.1. Conditional testing on Parametric Method

Confidence Level	VaR	Violations	LRCC	Chi-Square	P-Value	Test Outcome
0.99	-0.154605	12	0.000061	19.398785	0.993752	Not Reject H0
0.95	-0.066812	22	0.013834	8.561248	0.90637	Not Reject H0
0.9	-0.04373	33	0.042134	6.333803	0.837365	Not Reject H0

Table 7: Conditional Backtesting on Parametric VaR Estimations

In the Parametric Method, the VaR estimates are compared against actual losses at different confidence levels. The LRCC (Likelihood Ratio Chi-Square) and Chi-Square statistics are calculated to assess the fit of the VaR estimates to the actual data distribution. The P-Value indicates the probability of observing the test outcome under the null hypothesis (H0), with values above the significance level implying failure to reject H0, suggesting acceptable VaR accuracy.

5.2.2. Conditional testing on BHS Method

Confidence Level	VaR	Violations	LRCC	Chi-Square	P-Value	Test Outcome
0.99	-0.06479	3	4.70E+3	9.652281	0.236387	Not Reject H0
0.95	-0.03984	13	1.12E+3	5.784901	0.222293	Not Reject H0
0.9	-0.02504	25	3.00E+4	4.26099	0.778437	Not Reject H0

Table 8: Conditional Backtesting on BHS VaR Estimations

For the testing on BHS Method, the LRCC and Chi-Square statistics are utilized to determine the goodness-of-fit of VaR estimates, with associated P-Values indicating the likelihood of observing the test outcome under the null hypothesis. In this case, all test outcomes suggest a failure to reject H0, implying satisfactory VaR performance for the BHS Method across different confidence levels.