Solutions

School of Mathematics and Statistics
Carleton University
Math. 1004A, Fall 2014

TEST 5

STUDIO 56 calculator ONLY permitted, 1 or more blank sheets permitted for roughs

Print Name : _____

Student Number: _____

Tutorial Section (A1, A2, A3, A4, or A5): _____

PART I: Multiple Choice Questions

(Choose and CIRCLE only ONE answer - No part marks here.)

1. **[2 marks]** Evaluate $\int_0^1 xe^x\,dx$.

   (a) e, (b) $e-1$, (c) e^2-1, (d) 1, e) none of these

   $\begin{matrix} x & e^x \\ 1 & e^x \\ 0 & e^x \end{matrix}$ $I = (xe^x - e^x)\Big|_0^1 = (e-e)-(0-1) = 1$

2. **[2 marks]** Evaluate $\int_1^e x\ln x\,dx$.

   (a) e, (b) $\frac{e^2+1}{4}$, (c) 0, (d) $\frac{e-1}{2}$, e) none of these

   $\begin{matrix} \ln x & x \\ \frac{1}{x} & \frac{x^2}{2} \end{matrix}$ $I = \left(\frac{x^2}{2}\ln x - \frac{x^2}{4}\right)\Big|_1^e = \frac{e^2}{2} - \left(-\frac{1}{4}\right) = \frac{e^2+1}{2}$.

3. **[2 marks]** Find an antiderivative of $\frac{e^{\sqrt{x}}}{\sqrt{x}}$.

   (a) $2e^{\sqrt{x}}$, (b) $e^{\sqrt{x}}$, (c) $e^{\sqrt{x}}/2$, (d) $xe^{\sqrt{x}}$

   obvious; find its derivative!

4. **[2 marks]** Evaluate $\int_0^1 x^2 e^{2x}\,dx$

   (a) $\frac{e^2}{4}$, (b) e^2, (c) $\frac{e^2-1}{4}$, (d) 1, e) none of these.

   $\begin{matrix} x^2 & e^{2x} \\ 2x & e^{2x}/2 \\ 2 & e^{2x}/4 \\ 0 & e^{2x}/8 \end{matrix}$ $I = \left(\frac{x^2e^{2x}}{2} - \frac{2xe^{2x}}{4} + \frac{2e^{2x}}{8}\right)\Big|_0^1 = \frac{e^2-1}{4}$.

5. **[2 marks]** $\int_0^\pi \sin^2 x\cos^3 x\,dx = 0$.

   (a) TRUE, (b) FALSE,

   $\sin^2x\cos^3x = \sin^2x\cos^2x\cos x = \sin^2(x)(1-\sin^2x)\cos x$.
   $u = \sin x$, $du = \cos x\,dx$ $I = \int u^2(1-u^2)\,du$.
   $= \frac{u^3}{3} - \frac{u^5}{5} = \frac{\sin^3x}{3} - \frac{\sin^5x}{5}$
   Answer: $= \left(\frac{\sin^3x}{3} - \frac{\sin^5x}{5}\right)\Big|_0^\pi = 0$

PART II: Show all work here and give details.

No additional pages will be accepted

6. **[5+5 marks]** a) Evaluate $\int \frac{dx}{x^2(x-1)}$

   b) Evaluate $\int \frac{x}{(x+2)(x-1)}\,dx$.

a) $\frac{1}{x^2(x-1)} = \underbrace{\frac{A}{x} + \frac{B}{x^2}}_{(1)} + \frac{C}{x-1}$ $\Rightarrow$ $1 = Ax(x-1) + B(x-1) + Cx^2$ (1/2)

$x=0$: $\Rightarrow$ $B=-1$ (1/2)

$x=1$: $\Rightarrow$ $C=1$ (1/2)

$x=2$ $\Rightarrow$ $1 = 2A - 1 + 4C$ $\Rightarrow$ $A=-1$ (1/2)

$\therefore I = \int\left(-\frac{1}{x} - \frac{1}{x^2} + \frac{1}{x-1}\right)dx = -\ln|x| + \frac{1}{x} + \ln|x-1| + C.$

(1/2) (1/2) (1/2) (1/2)

b). $\frac{x}{(x+2)(x-1)} = \frac{A}{x+2} + \frac{B}{x-1}$ ← (1)

$\therefore x = A(x-1) + B(x+2)$

$x=1 \Rightarrow \boxed{B = 1/3}$ ← (1)

$x=-2 \Rightarrow \boxed{A = 2/3}$ ← (1)

} or use "cover up" method is OK too

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$\therefore I = \int \left(\frac{2/3}{x+2} + \frac{1/3}{x-1} \right) dx = \frac{2}{3} \ln|x+2| + \frac{1}{3} \ln|x-1| + C.$

(1) (1)

7. **[5+5 marks]**

a) Evaluate $\int_0^{\pi} \sin 2x \cos 3x \, dx$.

b) Evaluate $\int e^{2x} \sin x \, dx$.

a) (3 row-problem)

$\sin 2x$	(+) $\cos 3x$	
$2\cos 2x$	(−) $\frac{\sin 3x}{3}$	} (1)
$-4\sin 2x$	(+) $-\frac{\cos 3x}{9}$	

$\therefore \int \sin 2x \cos 3x \, dx = \left(\sin 2x \frac{\sin 3x}{3} + \frac{2}{9} \cos 2x \cos 3x \right) \square$

where $\square$ is found using MyCar (or other method is OK too),

My: 4/9, C: −4/9, A: 5/9, R: 9/5 $\therefore \boxed{\square = 9/5}$ ← (1)

$\therefore \int \sin 2x \cos 3x \, dx = \left(\frac{3}{5} \sin 2x \sin 3x + \frac{2}{5} \cos 2x \cos 3x \right) \Big|_0^{\pi}$ (2)

$= -4/5$ (1)

b). (also 3 row problem) $\therefore \int e^{2x} \sin x = \square \left\{ -e^{2x} \cos x + 2e^{2x} \sin x \right\}$.

(1)

e^{2x}	(+) $\sin x$	
$2e^{2x}$	(−) $-\cos x$	
$4e^{2x}$	(+) $-\sin x$	

$\square = 1/5$ ← (1)

(MyCar: −4, C: 4; A: 5, R = 1/5).

$\therefore \int e^{2x} \sin x \, dx = \left(+\frac{2}{5} e^{2x} \sin x - \frac{1}{5} e^{2x} \cos x \right) + C$

(1) (1) (1)