Lecture 4 Note - *Vector Dot Product*

Textbook Chapter 2.9

What the dot product is used for:

- In Statics, it's important to know the angle between two forces, and how much of vector ***a*** acts in the direction of vector ***b*** (AKA the components of a force at an angle to another vector). This is what the dot product can solve for.
- In other words, if you projected ***a*** onto ***b***, how long would the shadow of ***a*** be?
- If you see the words "projected component" in a question, dot product is involved.

What *is* the dot product, exactly?:

- The dot product of two vectors is the **product** of the **magnitudes of two vectors and the cosine of the angle between them.**
  - Dot Product = Magnitude of ***A*** times magnitude of ***B*** times cos of $\angle AB$
  - $\mathbf{A} \bullet \mathbf{B} = AB \cos\theta = A_xB_x + A_yB_y + A_zB_z$
    - *Note that the i/j/k disappear here; you can add the latter part to get* $\mathbf{A} \bullet \mathbf{B}$
  - The magnitude of force $\mathbf{F}_b$ projects in direction a is written as $F_a = \mathbf{F}_b \bullet \mathbf{u}_a = \mathbf{F}_a \cos\theta$, where θ is the angle between the two vectors.
  - Note: The dot product of two vectors is a scalar.
  - If you have the magnitude of one vector, and the force it projects along axis *A*, you can find the force perpendicular to *A* with the pythagorean formula (29:45).
  - Note: Check the slides for the proofs of these equations.

~~**Behind the dot product definition (bonus stuff to help understand the formula logic):**~~

- ~~Some laws and definitions for dot products:~~
  - ~~Commutative (switcheroo) law:~~
    - ~~$\mathbf{A} \bullet \mathbf{B} = \mathbf{B} \bullet \mathbf{A} = AB\cos\theta = BA\cos\theta$~~
  - ~~Scalar Multiplication law:~~
    - ~~$a(\mathbf{A} \bullet \mathbf{B}) = (a\mathbf{A}) \bullet B = \mathbf{A} \bullet (a\mathbf{B})$~~
  - ~~Distributive law (like multiplication):~~
    - ~~$\mathbf{A} \bullet (\mathbf{B} \bullet \mathbf{C}) = (\mathbf{A} \bullet \mathbf{B}) + (\mathbf{A} \bullet \mathbf{C})$~~
  - ~~The dot product of two collinear (parallel) UNIT vectors is 1 — they act together~~
    - ~~$(1)(1)\cos(0) = 1$~~
  - ~~The dot product of ANY two perpendicular vectors is 0 — they don't act together~~
    - ~~$(1)(1)\cos(90) = 0$~~
  - ~~These rules show us that $\mathbf{A} \bullet \mathbf{B} = A_xB_x + A_yB_y + A_zB_z$. See slides for full explanation.~~
  - ~~The magnitude of force $\mathbf{F}_b$ projects in direction a is written as $F_a = \mathbf{F}_b \bullet \mathbf{u}_a = F_b\cos\theta$~~
    - ~~Since u_b has a magnitude of 1, the last part of the equation is legal. There are some more proofs in the slides but there's no time for that - skipped.~~