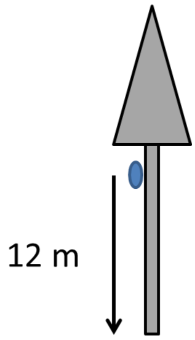
Physics 1004 S2012 Deferred Final Exam Solutions Section B

Section B

Answer **5** questions out of **6**. Use the check boxes on the front page to indicate which questions you want marked. If the check boxes are not filled in, the first five questions encountered will be marked. All questions are worth 10 marks. **Show all work**. Equations not on the formula sheet must be derived from first principles. The appropriate number of significant figures must be used in the final answer.

B1

A 28 kg bear slides 12 m down a tree, from rest, moving with a speed of 5.6 m/s just before hitting the ground.



(a) What is the change in gravitational potential energy of the bear? *(3 marks)*

(b) What is the kinetic energy of the bear just as it hits the ground? *(3 marks)*

(c) What is the magnitude of the average friction force operating on the bear as it slides down the tree? *(4 marks)*

Solution:

(a) We take the initial gravitational potential energy to be U_i = 0. Then the final gravitational potential energy is U_f = –mgL, where L is the length of the tree. The change is

U_f = –mgL=(28 kg)(9.8 m/s^2)(12 m) = -3.30×10^3 J

(b) The kinetic energy is $K = \frac{1}{2}mv^2 = 0.5 \times (28kg)(5.6\text{m/s})^2 = 4.40 \times 10^2 \, J$.

(c) The work done by the non-conservative force is

$$W_{nc} = \Delta U + \Delta K = -3.29 \times 10^3 \, J + 4.39 \times 10^2 \, J = -2.9 \times 10^3 \, J$$

It's negative because the friction force is in the opposite direction to the displacement (down the tree)

$$W_{nc} = -2.9 \times 10^3 \, J = Fd \cos\theta$$

Since ϑ=180^o

$$W_{nc} = -2.85 \times 10^3 \, J = -F \times (12m)$$
$$F = 240N \text{ to } 2 \text{ s.f.}$$

B2

23.31 Two long, charged, thin-walled, concentric, cylindrical shells have radii of 3.0 cm and 6.0 cm. The charge per unit length is 5.0×10^{-6} C/m^2 on the inner cylinder, and -7.0×10^{-6} C/m^2 on the outer cylinder.

(a) Calculate the magnitude and direction of the electric field at radial distance r = 4.0 cm (5 marks)

(a) Calculate the magnitude and direction of the electric field at radial distance r = 8.0 cm (5 marks)

Solution

. We denote the inner and outer cylinders with subscripts *i* and *o*, respectively.

(a) Since $r_i < r = 4.0$ cm $< r_o$,

$$E(r) = \frac{\lambda_i}{2\pi\varepsilon_0 r} = \frac{5.0\times10^{-6}\ \text{C/m}}{2\pi(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(4.0\times10^{-2}\ \text{m})} = 2.3\times10^{6}\ \text{N/C}.$$

(b) The electric field $\vec{E}(r)$ points radially outward.

(c) Since $r > r_o$,

$$E(r = 8.0\ \text{cm}) = \frac{\lambda_i + \lambda_o}{2\pi\varepsilon_0 r} = \frac{5.0\times10^{-6}\ \text{C/m} - 7.0\times10^{-6}\ \text{C/m}}{2\pi(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(8.0\times10^{-2}\ \text{m})} = -4.5\times10^{5}\ \text{N/C},$$

or $|E(r = 8.0\ \text{cm})| = 4.5\times10^{5}\ \text{N/C}.$

(d) The minus sign indicates that $\vec{E}(r)$ points radially inward.

B3

The electric potential in a region of space is given by the equation:

$$V = (2.0\ \text{V/m}^2)x^2 + (1.5\ \text{V/m})x - (3.0\ \text{V/m})y + (4.0\ \text{V/m}^2)z^2$$

(a) Find the equation for the electric field in this region of space (7 marks)

(b) Find the electric field at the point (3.0 m, 2.0 m, 1.5 m). (3marks)

Solution:

$$E_x = -\frac{\partial V}{\partial x} \qquad E_y = -\frac{\partial V}{\partial y} \qquad E_z = -\frac{\partial V}{\partial z} \qquad \mathbf{E} = -\left(\frac{\partial V}{\partial x}\mathbf{i} + \frac{\partial V}{\partial y}\mathbf{j} + \frac{\partial V}{\partial z}\mathbf{k}\right)$$

Differentiate the potential function w.r.t. x to get the x-component of E $E_x = \frac{\partial V}{\partial x} = 2(2.0\ \text{V/m}^2)x + (1.5\ \text{V/m})$

Differentiate the potential function w.r.t. y to get the y-component of E $E_y = \frac{\partial V}{\partial y} = -(3.0\ \text{V/m})$

Differentiate the potential function w.r.t. z to get the z-component of E $E_z = \frac{\partial V}{\partial z} = 2(4.0\ \text{V/m}^2)z$

At x = 3.0 m, $E_x = \frac{\partial V}{\partial x} = 2(2.0\ \text{V/m}^2)(3.0\ \text{m}) + (1.5\ \text{V/m}) = 13.5\ \text{V/m}$

At y = 2.0 m $E_y = \frac{\partial V}{\partial y} = -(3.0\ \text{V/m})$

At z = 1.5 m $E_z = \frac{\partial V}{\partial z} = 12\ \text{V/m}$

$$\mathbf{E} = (13.5\ \text{V/m})\,\mathbf{i} - (3.0\ \text{V/m})\,\mathbf{j} + (12\ \text{V/m})\,\mathbf{k}$$

B4

A 2.0 µF capacitor and a 4.0 µF capacitor are connected in parallel across a 240 V potential difference. The 4.0 µF capacitor has a parallel plate configuration, with a surface area of 110 cm^2, and a dielectric material (paper) between the plates with κ = 3.5

(a) Calculate the total charge stored on the capacitors (3 marks)

(b) Calculate the total energy stored in the capacitors (3 marks)

(c) Calculate the spacing between the plates in the 4.0 µF capacitor. (4 marks)

Solution:

(a) The capacitors are in parallel, so the equivalent capacitance is the sum = 6.0 µF

$$C = \frac{q}{V}$$

$$q = CV = (6.0 \times 10^{-6} F)(240\ \mathrm{V}) = 1.44 \times 10^{-3} C$$

(b) $U = \frac{1}{2}CV^2 = \frac{1}{2}(6.0 \times 10^{-6} F)(240\ \mathrm{V})^2 = 0.17\ \mathrm{J}$

(c) The capacitance is related to the capacitor dimensions by

$$C = \frac{\kappa \varepsilon_0 A}{d}$$

$$d = \frac{\kappa \varepsilon_0 A}{C} = \frac{3.5 \times (8.85 \times 10^{-12}\,\mathrm{F/m}) \times (110 \times 10^{-4}\ \mathrm{m}^2)}{6.0 \times 10^{-6}\,\mathrm{C}} = 5.7 \times 10^{-8} m$$

B5

An electrical cable consists of 125 identical strands of copper wire, each with a resistance of 265 mΩ. The same potential difference is applied between the ends of all the strands and results in a total current of 65.0 mA

(a) What is the current in each strand of wire?

(b) What is the applied potential difference?

(c) What is the resistance of the cable?

(d) How much power is dissipated in the cable?

Solution

(a) The current flows equally through each strand so

$$i = \frac{65.0\times10^{-3}A}{125} = 5.2\times10^{-4}A$$

(b) The Potential difference can be calculated using Ohm's law applied to one strand

$$V = 5.2\times10^{-4}A\times2.65\times10^{-3}\ \Omega = 1.38\times10^{-6}\ \mathrm{V}$$

(c) The total resistance can be calculated from the equivalent resistance for parallel resistors

$$\frac{1}{R_{eq}} = \frac{125}{R}$$

$$R_{eq} = \frac{2.65\times10^{-3}\Omega}{125} = 2.12\times10^{-5}\Omega$$

(d) The power lost in the cable is $P = IV = (65.0\times10^{-3}A)(1.38\times10^{-6}V) = 8.9\times10^{-6}\ \mathrm{W}$

Question B6

A proton (mass 1.67×10^{-27} kg) is travelling through uniform magnetic and electric fields. The electric field is (4.00 V/m)**k** and the magnetic field is **B** = (-2.50 mT)**i**. If the velocity of the proton is (3450 m/s)**k** , then using unit vector notation, calculate

(a) The electric force on the proton *(3marks)*

(b) The magnetic force on the proton *(4 marks)*

(c) The net acceleration on the proton *(3 marks)*

Solution

(a) The electric force is defined as $\mathbf{F_E} = e\mathbf{E}$

$$\mathbf{F_E} = e\mathbf{E} = (1.602\times10^{-19}C)(4.00\text{ V/m})\mathbf{k} = (6.41\times10^{-19}\text{ N})\mathbf{k}$$

(b) The magnetic force is defined as

$$\mathbf{F} = \mathrm{q}(\mathbf{v}\times\mathbf{B}) = \mathrm{e}(4.00\text{ V/m})\mathbf{k}\times(-2.50\times10^{-3}T)\mathbf{i}$$

$$\vec{c} = \vec{a}\times\vec{b} = (a_y b_z - b_y a_z)\mathbf{i} + (a_z b_x - b_z a_x)\mathbf{j} + (a_x b_y - b_x a_y)\mathbf{k}$$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 0 & 0 & 3450\text{ m/s} \\ -2.50mT & 0 & 0 \end{vmatrix}$$

$$\mathbf{F} = \mathrm{q}(\mathbf{v}\times\mathbf{B}) = \mathrm{e}(3450\text{ m/s})(2.50\times10^{-3}T)\mathbf{j} = (1.38\times10^{-19}\text{ N})\mathbf{j}$$

(c) **a** = **F**/m = $\mathbf{a} = \frac{\mathbf{F}}{\mathrm{m}} = \frac{1}{1.67\times10^{-27}kg}\left[(1.38\times10^{-19}\text{ N})\mathbf{j} + (6.41\times10^{-19}\text{ N})\mathbf{k}\right]$

$$\mathbf{a} = \frac{\mathbf{F}}{\mathrm{m}} = \left[8.26\times10^{7}\text{ m/s}^2)\mathbf{j} + (3.84\times10^{8}\text{ m/s}^2)\mathbf{k}\right]$$