SOLUTIONS

School of Mathematics and Statistics
Carleton University
Math. 1004A, Fall 2016

TEST 2

ONLY NON-PROGRAMMABLE and NON-GRAPHING Calculators permitted, as well as a few blank sheets but these should NOT be submitted.

Print Name :

Student Number:

Tutorial Section (A1, A4, ...):

PART I: Multiple Choice Questions

(Choose and CIRCLE only ONE answer - No part marks here.)

1. **[2 marks]** Let $f(x) = \frac{x}{\sqrt{1+x}}$. Then the derivative of f at $x = 0$, that is $f'(0)$, is equal to:

   (a) 1, (b) 0, (c) -1, (d) $1/2$, (e) None of these.

2. **[2 marks]** Let f be defined by $f(x) = x^2 + 3$ with domain equal to the interval $[0, 1]$. Using the Mean Value Theorem we can say that there exists a point c between 0 and 1 such that $f'(c)$ equals which one of the following numbers?

   (a) 4, (b) -1, (c) 3, (d) 1 (e) None of these

3. **[2 marks]** Let (x, y) be the set of all points that define the curve $y^5 + x^2y^3 = 10$. Assuming that y is a function of x, find the slope of the tangent line to this curve at the point $(x, y) = (-3, 1)$, that is, find $y'(-3)$.

   (a) $\frac{5}{16}$, (b) $\frac{1}{16}$, (c) $\frac{3}{16}$, (d) $\frac{16}{3}$ (e) None of these

4. **[2 marks]** Let f be defined by $f(t) = \text{Arctan}(\sqrt{t})$. Evaluate its derivative at $t = 4$, that is find $f'(4)$.

   (a) $\frac{1}{5}$, (b) $\frac{1}{20}$, (c) $\frac{1}{4}$, (d) $\frac{1}{8}$. (e) None of these

5. **[2 marks]** The derivative of the function f defined by $f(x) = \frac{x^{2/3}}{1 + 3x^{3/4}}$ evaluated at $x = 1$ is given by $f'(1) = \frac{7}{192}$.

   (a) YES, (b) NO,

PART II: Show all work here.
No additional pages will be accepted

6. **[5+5 marks]** :

   a) Evaluate the following limit using any method (give details for full marks): $\lim_{x\to\infty} \frac{2x+3}{3x-1}$. $= I$

   b) Evaluate the following limit (give details for full marks): $\lim_{x\to 2} \frac{x^2-4}{x^3-8}$.

a) L'Hospital's Rule. $\lim_{x\to\infty} \frac{D(2x+3)}{D(3x-1)} = \lim_{x\to\infty}\left(\frac{2}{3}\right) = \frac{2}{3}$

exists (& is finite) So $\lim_{x\to\infty} \frac{2x+3}{3x-1} = \boxed{\frac{2}{3}}$

OR $\quad I = \lim_{x\to\infty} \dfrac{x\left(2+\frac{3}{x}\right)}{x\left(3-\frac{1}{x}\right)} = \lim_{x\to\infty} \dfrac{2+3/x}{3-1/x} = \boxed{\dfrac{2}{3}}$

b) (i) $\dfrac{x^2-4}{x^3-8} = \dfrac{(x-2)(x+2)}{(x-2)(x^2+2x+4)} = \dfrac{x+2}{x^2+2x+4}$

$\therefore \lim_{x\to 2} \dfrac{x^2-4}{x^3-8} = \lim_{x\to 2} \dfrac{x+2}{x^2+2x+4} = \dfrac{4}{2^2+4+4} = \dfrac{4}{12} = \boxed{\dfrac{1}{3}}$

(ii) L'Hospital's Rule: $\dfrac{D(x^2-4)}{D(x^3-8)} = \dfrac{2x}{3x^2} = \dfrac{2}{3x} \therefore$

$\lim_{x\to 2} \dfrac{D(x^2-4)}{D(x^3-8)} = \dfrac{2}{3\cdot 2} = \boxed{\dfrac{1}{3}}$ & so is the original limit.

$\dfrac{2x}{3x^2} = \dfrac{2}{3x} \to \dfrac{1}{3}$

7. **[5+5 marks]**

a) Evaluate the following limit: $\lim_{x\to 0} \dfrac{x^2 - \sin x}{x^2 - x}$.

b) Let g be defined by $g(t) = t^2 + 1$ for $0 \le t \le 1$. Answer the following questions about g:

1) [1 mark] Show that g is one-to-one or, equivalently, that the graph of g satisfies the Horizontal Line Test.
2) [1 + 1 mark] Let G be the inverse function of g. Find Dom(G) and Ran(G).
3) [2 marks] What is $G(t)$ for any value of t?

a) L'Hospital's Rule; $\dfrac{D(x^2-\sin x)}{D(x^2-x)} = \dfrac{2x-\cos x}{2x-1}$.

$\therefore \lim_{x\to 0} \dfrac{2x-\cos x}{2x-1} = \dfrac{0-1}{0-1} = \boxed{+1}$ exists & is finite

so is the original limit.

b) 1) $g(s)=g(t) \Rightarrow s=t$? Yes, bc. $g(s)=g(t) \iff s^2+1=t^2+1$
$\iff s^2=t^2 \iff s=t$ $\because$ both are non-negative.

2) Dom G = Ran g = $[1,2]$, Ran(G) = Dom(g) = $[0,1]$.

3) Write $t=x$, $y=x^2+1 \Rightarrow x=y^2+1 \Rightarrow y=\sqrt{x-1}$ $\because y\ge 0$

$\therefore G(t) = \sqrt{t-1}$