



School of Mathematics and Statistics  
Carleton University  
Math. 1004A, Fall 2016  
**TEST 4**

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Only calculators are permitted, 1 or more blank sheets permitted for roughs

Print Name :

Student Number:

Tutorial Section (A1, A4, ...):

Show all work here and give details.

1. [5 marks] Evaluate  $\int_0^1 x e^{-x} dx$  using any method

$$u = x \quad dv = e^{-x} dx$$

$$du = dx \quad v = -e^{-x}$$

$$I = -x e^{-x} - \int (-e^{-x}) dx$$

$$= -x e^{-x} + \int e^{-x} dx$$

$$= -x e^{-x} - e^{-x} + C$$

(OR)

$$\begin{array}{rcl} x & + & e^{-x} \\ 1 & \nearrow & -e^{-x} \\ 0 & \nwarrow & e^{-x} \end{array}$$

$$I = -x e^{-x} - e^{-x} + \int 0 \cdot e^{-x} dx$$

$$= -x e^{-x} - e^{-x} + C$$

Answer:  $\frac{e-2}{2}$

2. [5 marks] Evaluate  $\int \ln(2x) dx$  using any method

$$u = \ln 2x \quad dv = dx$$

$$du = \frac{1}{2x} dx \quad v = x$$

$$du = \frac{1}{x} dx$$

$$I = x \ln 2x - \int x \cdot \frac{1}{x} dx$$

$$= x \ln 2x - x + C$$

$$\begin{array}{rcl} \ln 2x & + & 1 \\ \frac{1}{x} & \nwarrow & x \end{array}$$

$$I = x \ln 2x - \int \left(\frac{1}{x}\right) x dx$$

$$= x \ln 2x - x + C$$

(OR)

3. [5 marks] Evaluate  $\int_0^\pi x \sin(2x) dx$  using any method

Easier using Table Method  $\rightarrow$

$$\begin{array}{rcl} x & + & \sin 2x \\ 1 & \nearrow & -\frac{\cos 2x}{2} \\ 0 & \nwarrow & -\frac{\sin 2x}{4} \end{array}$$

$$I = -\frac{x \cos 2x}{2} + \frac{\sin 2x}{4} + C$$

Answer:  $-\pi/2$

4. [5 marks] Evaluate  $\int x^2 e^{2x} dx$  using any method

Easier using Table Method.

$$\begin{array}{rcl} x^2 & + & e^{2x} \\ x & \nearrow & \frac{e^{2x}}{2} \\ 2x & \nearrow & \frac{e^{2x}}{4} \\ 2 & \nearrow & \frac{e^{2x}}{8} \\ 0 & & \end{array}$$

$$I = x^2 \frac{e^{2x}}{2} - x \frac{e^{2x}}{2} + \frac{e^{2x}}{4} + C$$

$$= I$$

5. [5 marks] Evaluate  $\int 4x (\ln x)^2 dx$  using any method

$$\begin{aligned}
 \text{Let } \ln x &= u \\
 x &= e^u \\
 dx &= e^u du
 \end{aligned}
 \quad \left. \begin{array}{l} \\ \\ \end{array} \right\}$$

$$\begin{aligned}
 I &= \int 4e^u u^2 e^u du = 4 \int u^2 e^{2u} du \\
 &= 4 \left( \frac{u^2 e^{2u}}{2} - \frac{ue^{2u}}{2} + \frac{e^{2u}}{4} \right) + C \quad (\text{by \#4}) \\
 &= 2u^2 e^{2u} - 2ue^{2u} + e^{2u} \\
 &= \frac{2x^2 (\ln x)^2 - 2x^2 (\ln x) + x^2 + C}{1}
 \end{aligned}$$

$$= I$$

6. [5 marks] Evaluate  $\int e^{2x} \sin x dx$  using any method

Easier by Table Method

$$I = -e^{2x} \cos x + 2e^{2x} \sin x - \underbrace{\int 4e^{2x} \sin x dx}_{-4I}$$

$$\begin{array}{rcl}
 e^{2x} & \xrightarrow{+} & \sin x \\
 2e^{2x} & \xrightarrow{-} & -\cos x \\
 4e^{2x} & \xleftarrow{+} & -8\sin x
 \end{array}$$

$$\therefore 5I = \dots$$

$$I = \frac{1}{5} (2e^{2x} \sin x - e^{2x} \cos x) + C$$

OR Use "MyCar"

$$I = \boxed{R} (2e^{2x} \sin x - e^{2x} \cos x) + C$$

$$\begin{array}{l}
 \text{My: } 4(-1) = -4 \\
 C: +4 \\
 A: +5 \\
 R: 1/5
 \end{array}$$

$$\therefore I = \frac{1}{5} (2e^{2x} \sin x - e^{2x} \cos x) + C$$

END