ii) $\sqrt{x^2+1} - x = \dfrac{(\sqrt{x^2+1} - x)(\sqrt{x^2+1} + x)}{\sqrt{x^2+1} + x} = \dfrac{x^2+1-x^2}{\sqrt{x^2+1} + x} = \dfrac{1}{\sqrt{x^2+1} + x}$ 1 1 1 1

As $x \to \infty$, $\sqrt{x^2+1} + x \to \infty$ $\therefore$ $\dfrac{1}{\sqrt{x^2+1} + x} \to 0.$ 1

Answer: 0

7. [5+5 marks]

a) Evaluate the following limit: $\lim_{x \to 0} \dfrac{\sin 3x}{\sin 5x}$. .

b) Let $f(x) = \cos x$, $g(x) = \sin 2x$. Calculate the value of the derivative of the ordinary product of these two functions, $D(f(x)g(x))$.

a) $\dfrac{\sin 3x}{\sin 5x} = \dfrac{\sin 3x}{3x} \cdot \dfrac{3x}{5x} \cdot \dfrac{5x}{\sin 5x} = \dfrac{3}{5} \cdot \dfrac{\sin 3x}{3x} \cdot \dfrac{1}{\frac{\sin 5x}{5x}}$. 1 1 1

But $\lim_{x \to 0} \dfrac{\sin 3x}{3x} = 1$, $\lim_{x \to 0} \dfrac{\sin 5x}{5x} = 1.$ 1/2 1/2

$\therefore$ $\lim_{x \to 0} \dfrac{\sin 3x}{\sin 5x} = \dfrac{3}{5}$ 1

b). Product Rule 1

$D(f(x)g(x)) = (Df(x))g(x) + f(x)Dg(x)$ (or $D(fg) = Dfg + fDg$ is OK.)

$= D(\cos x) \cdot \sin 2x + \cos x \, D(\sin 2x)$

$= -(\sin x)(\sin 2x) + (\cos x)(2\cos 2x)$

1 1 1 1

Solutions

School of Mathematics and Statistics
Carleton University
Math. 1004A, Fall 2014

TEST 1

STUDIO 56 Calculator ONLY permitted as well as a few blank sheets but these should NOT be submitted.

Print Name :

Student Number:

Tutorial Section (A1, A4, . . .):

PART I: Multiple Choice Questions

(Choose and CIRCLE only ONE answer - No part marks here.)

1. [2 **marks**] Evaluate the limit: $\lim_{t\to 0} \dfrac{t+\sin t}{t}$.
   (a) 1 , (b) 0 , (c) −1, (d) 2.

2. [2 **marks**] Let $f(x) = \dfrac{1}{1+\sqrt{x}}$. Then $f'(1)$ is equal to:
   (a) −1/4, (b) −1/8, (c) 0, (d) 1/2.

3. [2 **marks**] Let f be defined by

$$f(t) = \begin{cases} t^2, & \text{if } t \le 0, \\ \dfrac{1-\cos 3t}{3t}, & \text{if } t > 0. \end{cases}$$

   Is f continuous at $t = 0$?
   (a) YES, (b) NO,

4. [2 **marks**] Evaluate the following limit, $\lim_{x\to-\infty} \dfrac{\cos x}{x^2}$.
   (a) −1, (b) 1, (c) 0, (d) The limit does not exist.

5. [2 **marks**] Let f, g be defined by $f(x) = \cos x$ and $g(z) = \sqrt{z}$. Evaluate their composition $(f \circ g)(0)$ at the point $x = 0$.
   (a) −1, (b) 1, (c) 0, (d) $\pi/2$.

PART II: Show all work here.
No additional pages will be accepted

6. [5+5 **marks**] :
   a) $f(t) = \sin(\sin\,(t^2 + 5t - 6))$. Find the derivative, $f'(t)$ (there is no need to simplify your answer).
   b) Evaluate the following limit: $\lim_{x\to\infty} (\sqrt{x^2+1} - x)$.

1/2 1/2

a) Form is $\sin\square$, (or $\sin(u)$) or whatever...

$D(\sin\square) = \cos\square \cdot D\square$ with $\boxed{\sin(t^2+5t-6)}$ or $u = \sin(t^2+5t-6)$

$= \underline{\cos(\sin(t^2+5t-6))} \cdot D(\sin\square)$, now $\underline{\square = t^2+5t-6}$.

$= \cos(\sin(t^2+5t-6)) \cdot \underline{\cos\square} \cdot \underline{D\square}$

$= \cos(\sin(t^2+5t-6)) \cdot \underbrace{\cos(t^2+5t-6)}_{1/2} \cdot \underbrace{(2t+5)}_{1/2}$

1 1/2 1/2 1/2 1/2