Given Name (Print)	Family Name (Print)	Student Number

Physics 1004 S2013 Midterm Exam

Answer all questions. Each question is worth 1 mark. Please circle your answer in the box.

Question 1

A 6.0 N force with a fixed orientation does work on a particle, as the particle moved through a displacement **d** = (2.00 m)**i** +(4.00 m)**j** - (3.00 m)**k**. The change in the particle's kinetic energy is +15 J. What is the angle between the force vector and the displacement vector?

(A) 12^{o}	(B) 18^{o}	(C) 22^{o}	(D) 46^{o}	(E) 62^{o}

Solution

The scalar product gives the work done, which is also the change in Kinetic energy, if we apply the Work Energy Theorem

$$\Delta K = W = \vec{F}.\vec{d} = Fd\cos\theta$$

The displacement, d can be calculated using Pythagoras' Theorem

$$d = \sqrt{(2.00\ m)^2 + (4.00\ m)^2 + (3.00\ m)^2} = 5.38\ m$$

and we are told that the magnitude of force F = 6.0 N

$$cos\theta = \frac{\Delta K}{Fd}$$

$$\theta = \cos^{-1}\frac{15.0J}{6.0N \times 5.38m} = 62° \ to\ 2\ s.f.$$

ANSWER (E)

Question 2

A 3.0 kg particle is moving in simple harmonic motion in one dimension and moves according to the equation:

$$x = (5.0\ \mathrm{m})\cos\left[(\pi/3\ \mathrm{rad/s})\mathrm{t} - \pi/4\ \mathrm{rad}\right]$$

where t is measured in seconds. At what value of x is the potential energy of the particle exactly half of the total mechanical energy?

(A) 0.1 m	(B) 2.8 m	(C) 3.5 m	(D) 4.6 m	(E) 4.9 m

Solution

From the equation, if we compare it with the general equation, $x = A\cos(\omega t + \varphi)$, we can see that the angular frequency $\omega = \frac{\pi}{3}\ rad/s$

If we differentiate the equation, we get the speed

$v = -\omega A sin(\omega t + \varphi)$, which has maximum values of $v_{max} = \pm\omega A$

Hence the maximum kinetic energy is

$$K_{max} = \frac{1}{2}mv_{max}^2 = \frac{m\omega^2 A^2}{2} = \frac{3.0kg \times \left(\frac{\pi}{3}\right)^2 \times (5.0\ m)^2}{2} = 41.1J$$

We now need to solve for x, so that $K = \frac{41.1\ J}{2} = 20.6\ J\ to\ 3\ sf$

We can calculate

$$K = \frac{1}{2}mv^2 = \frac{m\omega^2 A^2 sin^2\left(\frac{\pi t}{3} + \frac{\pi}{4}\right)}{2}$$

and solve for t. If we then substitute the value for t in the original equation for x, then we will find the value of x

$$\frac{20.6J}{K_{max}} = sin^2\left(\frac{\pi t}{3} + \frac{\pi}{4}\right)$$

$$\left(\frac{\pi t}{3} + \frac{\pi}{4}\right) = 0.785$$

This gives us t = -0.25 s. We usually take the first positive value for t, which is t = -0.25s + 2π = t=6.03 s

Now we put this value of t into the equation

$$x = (5.0\ \mathrm{m})\cos[(\pi/3\ \mathrm{rad/s})t - \pi/4\ \mathrm{rad}]$$

$$x = (5.0\ m)cos\left[\frac{6.03 \times \pi}{3} + \frac{\pi}{4}\right] = 3.4\ m$$

The closest value is 3.5 m

ANSWER (C)

Question 3

The linear density of a string is 16 g/cm. A transverse wave on the string is described by the equation

$$y = (0.021m)\sin[(2.0m^{-1})x + (32\ s^{-1})t]$$

Calculate the tension in the string

(A) 0.35 N	(B) 2.6 N	(C) 24 N	(D) 410 N	(E) 830 N

The speed of the wave on a string of tension T is

$$v = \sqrt{\frac{T}{\mu}}$$

And the general relationship for the speed of a harmonic wave is

$$v = \frac{\omega}{k}$$

Hence

$$\sqrt{\frac{T}{\mu}} = \frac{\omega}{k}$$

and

$$T = \mu\left(\frac{\omega}{k}\right)^2$$

The value of ω can be read from the equation of the travelling wave, $\omega = 32\ s^{-1}$, and the value of $k = 2.0\ m^{-1}$

$$T = \frac{mass}{length} \times \left(\frac{32}{2}\right)^2 = \frac{16 \times 10^{-3} kg}{1.0 \times 10^{-2} m} \times 16^2 = 410\ N\ to\ 2\ sf$$

Answer (D)

Question 4

If a metal conductor has a charge of -1.45×10^{-7} C, how many excess electrons are there on it?

(A) 1.44×10^{-3}	(B) 2.40×10^{8}	(C) 9.05×10^{11}	(D) 8.31×10^{13}	(E) 1.05×10^{18}

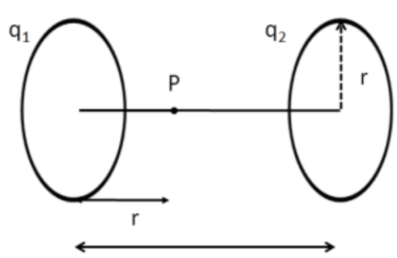
Solution

$$n = \frac{-1.45 \times 10^{-7} C}{-1.6 \times 10^{-19} C} = 9.06 \times 10^{11}$$

Answer (C)

Question 5

The figure shows two parallel non-conducting rings with their central axes along a common line. Ring 1 has a uniform charge q_1 and ring 2 has a uniform charge q_2. Both disks have a radius R, and the separation between the disks is 4R. The net electric field is zero at point P, which is R away from disk 1 and 3R from disk 2. What is the charge ratio q_1/q_2?



(A) 0.268	(B) 0.333	(C) 0.500	(D) 0.750	(E) 1.00

Solution

The equation for the electric field due to a charged ring, along the central axis is

$$E = \frac{kqz}{(z^2 + R^2)^{\frac{3}{2}}}$$

The field due to q_1, at a distance R from point P, is

$$E_1 = \frac{kq_1R}{(R^2 + R^2)^{\frac{3}{2}}} = \frac{kq_1R}{(2R^2)^{\frac{3}{2}}}$$

The field due to q_2, at a distance 3R from point P is

$$E_2 = \frac{kq_2 3R}{((3R)^2 + R^2)^{\frac{3}{2}}} = \frac{kq_2R}{(10R^2)^{\frac{3}{2}}}$$

and is in the opposite direction to field E_1

At point P, the net electric field = 0 so

$$E_1 = \frac{kq_1R}{(2R^2)^{\frac{3}{2}}} = E_2 = \frac{3kq_2R}{(10R^2)^{\frac{3}{2}}}$$

$$\frac{kq_1R}{(2R^2)^{\frac{3}{2}}} = \frac{3kq_2R}{(10R^2)^{\frac{3}{2}}}$$

$$\frac{q_1}{R^3(2)^{\frac{3}{2}}} = \frac{3q_2}{R^3(10)^{\frac{3}{2}}}$$

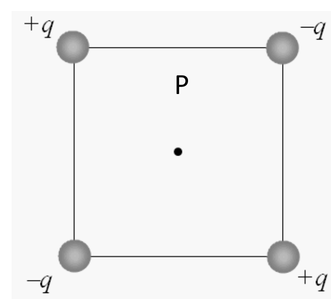
$$\frac{q_1}{q_2} = \frac{3R^3(2)^{\frac{3}{2}}}{R^3(10)^{\frac{3}{2}}}$$

$$\frac{q_1}{q_2} = \frac{3 \times (2)^{\frac{3}{2}}}{(10)^{\frac{3}{2}}} = 3 \times 0.2^{1.5} = 0.268$$

ANSWER (A)

Question 6

Four charges are located on the corners of a square as shown in the drawing. What is the direction of the net electric field at the point labeled P?



(A) Towards the upper left corner of the square
(B) Towards the middle of the right side of the square
(C) Towards the middle of the bottom side of the square
(D) Towards the lower right corner of the square
(E) There is no direction. The electric field at P is zero N/C.

Solution

Each charge is the same distance from the centre of the square, r. The two positive charged produce fields pointing inwards towards P, these are equal magnitudes, so they cancel out. The two negative charges produce fields pointing outwards along the diagonal, away from P. these cancel each other out too. The net field is therefore zero at point P

ANSWER (E)

Question 7

A negatively-charged object is released from rest in a region containing a uniform electric field. Which one of the following statements concerning the subsequent motion of the object is **correct**?

(A) The object will remain motionless.
(B) The object will experience a constant acceleration and move in the direction of the electric field.
(C) The object will experience a constant acceleration and move in the direction opposite that of the electric field.
(D) The object will accelerate to some constant speed and move in the direction of the electric field.
(E) The object will accelerate to some constant speed and move in the direction opposite that of the electric field.

Solution

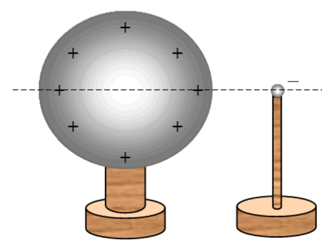
The definition of the electric field is

$$\vec{E} = \frac{\vec{F}}{q}$$

For a negative charge, this means that the force is in the opposite direction to the electric field. If there is a force on the negative charge, by Newton's second law, then there is a constant acceleration on the charge. This acceleration continues until the charge leaves the electric field, at which point a constant velocity will be attained. As we are not told that the charged particle leaves the electric field, we must assume that constant acceleration is the case, so option (C) is correct

ANSWER (C)

Question 8

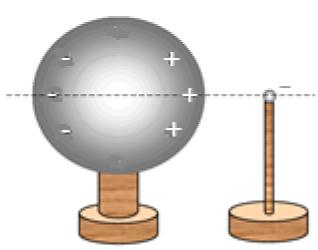


The drawing shows a hollow conducting sphere with a net positive charge uniformly distributed over its surface. A small negatively-charged object is then brought near the sphere as shown. What is the direction of the electric field at the centre of the sphere?

(A) There is no electric field at the centre of the sphere.
(B) to the left
(C) to the right
(D) upward
(E) downward

Solution

The negatively charged object causes the charge on the sphere to become polarized. The positive charge on the sphere will tend to be closest to the charged object, and the side of the sphere on the opposite side of the sphere will tend to be negatively charged as a result.



The electric field within the charged sphere will now tend to be from the positive end, to the negative end, so pointing to the left

ANSWER (B)

Question 9

If a proton is accelerated in a 2.0×10^4 N/C electric field, what is the magnitude of the acceleration? The mass of a proton is 1.67×10^{-27} kg and the charge on the proton (+e) is $+1.60\times10^{-19}$ C.

(A) 1.2×10^{-2} m/s^2	(B) 2.8×10^{2} m/s^2	(C) 5.3×10^{7} m/s^2	(D) 8.4×10^{10} m/s^2	(E) 1.9×10^{12} m/s^2

Solution

$$\vec{E} = \frac{\vec{F}}{q}$$

and using Newton's second Law $\vec{F} = m\vec{a}$ and q = +e

$$\vec{E} = \frac{m\vec{a}}{+e}$$

The acceleration is thus

$$\vec{a} = \frac{e\vec{E}}{m}$$

The magnitude of the acceleration is

$$a = \frac{eE}{m} = \frac{1.60 \times 10^{-19} C \times 2.0 \times 10^{4} N/C}{1.67 \times 10^{-27} kg} = 1.9 \times 10^{12}\ m/s^2$$

ANSWER (E)

Question 10

When a particle with a charge Q is surrounded by a spherical Gaussian surface, the electric flux through the surface is Φ_S. Consider what would happen if the particle was surrounded by a cylindrical Gaussian surface or a Gaussian cube. How would the fluxes through the cylindrical surface, Φ_{Cyl} and cubic surface, Φ_{Cubic}, compare to Φ_S?

(A) $\Phi_{Cyl} > \Phi_{Cubic} = \Phi_S$	(B) $\Phi_{Cyl} > \Phi_{Cubic} > \Phi_S$	(C) $\Phi_{Cyl} = \Phi_{Cubic} > \Phi_S$	(D) $\Phi_{Cyl} = \Phi_{Cubic} = \Phi_S$	(E) $\Phi_{Cubic} > \Phi_{Cyl} > \Phi_S$

Solution

By Gauss' Law, the flux is proportional to the charge enclosed. If the same charge is enclosed by Gaussian surfaces, the net flux through any of the Gaussian surfaces must be identical. Hence option (D) is correct

ANSWER (D)

Answer Key

1. E
2. C
3. D
4. C
5. A
6. E
7. C
8. B

9. E
10. D