Lecture 10 Note - *Centroids of Composite Bodies*

Textbook Chapter 9.2

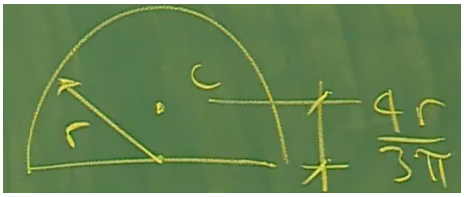
What *are* composite bodies?

- A body made up of several smaller shapes connected together
  - These shapes are usually rectangles, triangles, semi-circles, etc...
  - If the weight and centroid of these smaller shapes are known, we can determine the centroid (centre of gravity) of the whole composite body without integration:

$$\bar{x} = \frac{\sum \tilde{x} W}{\sum W} \quad \bar{y} = \frac{\sum \tilde{y} W}{\sum W} \quad \bar{z} = \frac{\sum \tilde{z} W}{\sum W}$$

  - $\bar{x}\ \bar{y}\ \bar{z}$ are the centroid coordinates of the entire shape.
  - $\tilde{x}\ \tilde{y}\ \tilde{z}$ are the centroid coordinates of EACH piece
  - The way this formula works is by finding the centroid location & area/volume/etc. of one piece, multiplying them together, and then repeating this for all the other pieces and adding them up. Finally, divide this sum by the total area/volume/etc. to get the centroid of the entire shape.

  - In L8 we covered centroid formulas for rectangles & triangles. For semicircles, use this image below. Check the next page for more complicated shape formulas.



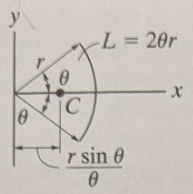
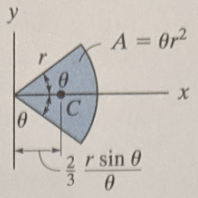
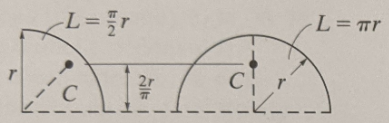
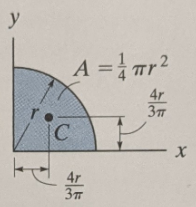
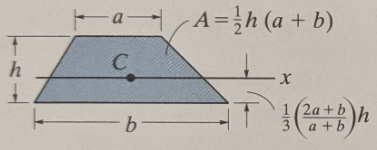
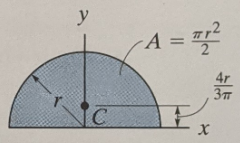
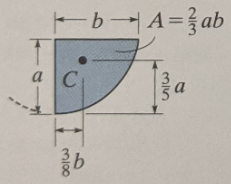
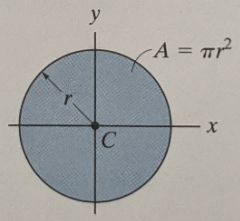
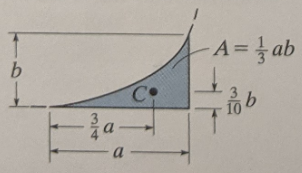
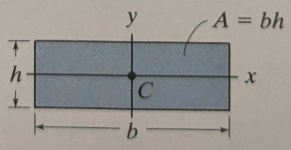
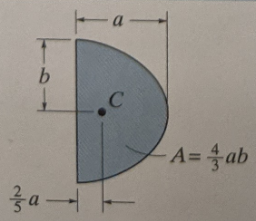
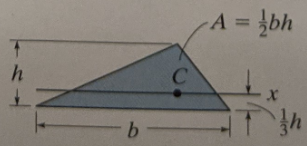
***How* to solve composite bodies?**

- Use the table method combined with the formula shown above.

Component	W (lb)	$\tilde{x}$ (ft)	$\tilde{y}$ (ft)	$\tilde{x}W$ (lbft)	$\tilde{y}W$ (lbft)
1	450	6	7	2700	3150
2	1500	18	16	27000	24000
3	600	26	3	15600	1800
4	280	30	8	8400	2240
$\sum$	2830			53700	31190

- If you're stuck, just visit the slides or watch the lecture. The math is straightforward.
- If you need help calculating the centroids of weird shapes, check out the next page:

Geometric Properties of Line and Area Elements

Centroid Location	Centroid Location	Area Moment of Inertia
 Circular arc segment	 Circular sector area	$I_x = \frac{1}{4}r^4(\theta - \frac{1}{2}\sin 2\theta)$ $I_y = \frac{1}{4}r^4(\theta + \frac{1}{2}\sin 2\theta)$
 Quarter and semicircle arcs	 Quarter circle area	$I_x = \frac{1}{16}\pi r^4$ $I_y = \frac{1}{16}\pi r^4$
 Trapezoidal area	 Semicircular area	$I_x = \frac{1}{8}\pi r^4$ $I_y = \frac{1}{8}\pi r^4$
 Semiparabolic area	 Circular area	$I_x = \frac{1}{4}\pi r^4$ $I_y = \frac{1}{4}\pi r^4$
 Exparabolic area	 Rectangular area	$I_x = \frac{1}{12}bh^3$ $I_y = \frac{1}{12}hb^3$
 Parabolic area	 Triangular area	$I_x = \frac{1}{36}bh^3$