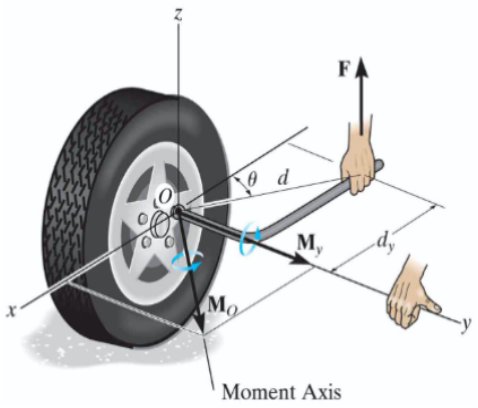
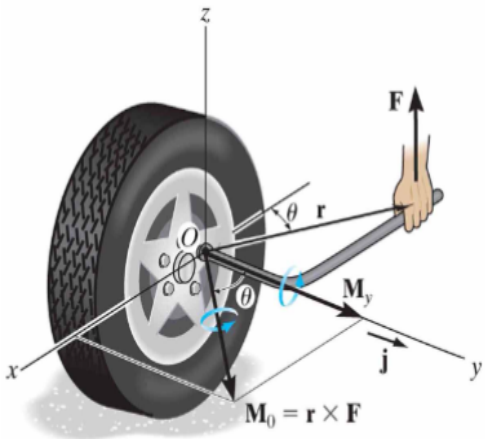
Lecture 7 Note - *Moments II (Moments acting about an axis, couple moments)*

Textbook Chapter 4.5-4.7

Moment of a force about an axis:



- Sometimes we have a moment acting upon multiple axes, but only care about how much of it acts in 1 direction, like the y-axis of the problem on the right.
- The moment about the y-axis in this case can be evaluated by: $M_y = Fd_y = F(d \cos \theta)$.
- It can also be found with a vector approach using $M_y = \boldsymbol{j} \bullet \boldsymbol{M_o} = \boldsymbol{j} \bullet (\boldsymbol{r} \times \boldsymbol{F})$. See image below. *Note that y and **j** can be swapped for x and **i** or z and **k**.*



-In the vector approach, determine moment vector $\boldsymbol{M_o}$ by first crossing ***F*** with its position vector ***r*** relative to *O*. Then dot this moment vector $\boldsymbol{M_o}$ with unit vector $\boldsymbol{u_A}$ to find how much of it acts in direction a.

-The formula for this problem is $M_A = \boldsymbol{u_A} \bullet \boldsymbol{M_o} = \boldsymbol{u_A} \bullet (\boldsymbol{r} \times \boldsymbol{F})$ and is called the *scalar* triple product. It is solved as a cross product with 3 rows; Unit vector, pos. vector, & force vector. Order matters, and M_a can be negative.

- Tip: When solving, pick the position vector with the most 0s to save time. As long as it starts *somewhere* on the axis of a moment and ends on a force's line of action, it's OK.

Couple moments:

- A couple is a pair of two parallel forces with the same magnitude ***F***, but acting in opposite directions separated by distance *d*. The resultant translational force is 0, but they exert a rotational force that we call the couple moment. Think RCS thrusters.
- Can be calculated via $M = Fd$ (scalar approach) OR $\boldsymbol{M = r \times F}$ (vector approach).
- The $\boldsymbol{M_R}$ of multiple couple moments can be calculated via vector addition. Just add the components of all couple moments, or the $(r \times F)$'s.

Not included in these notes: 2D and 3D simplifications of force/couple systems.