# MATH1104E, Linear Algebra for Engineering or Science, Fall 2014, TEST # 2

Solutions

| Name(print) | Signature | Student Number |
|---|---|---|

**Total Pages: 4** **Total Marks: 40**

INSTRUCTION:
Write your solution in the space provided below the question. If necessary, continue onto the back of the sheet, but remind your marker to look there. Show all your work. Calculator is NOT allowed.

**1.** [**10 marks**] Let $A = \begin{bmatrix} 1 & -2 \\ 2 & 2 \end{bmatrix}$.

(a) [**4 marks**] Find the adjoint of $A$, as well as the inverse of $A$.

$$\operatorname{adj}(A) = \begin{bmatrix} 2 & -2 \\ 2 & 1 \end{bmatrix}^T = \begin{bmatrix} 2 & 2 \\ -2 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{\det(A)} \operatorname{adj} A = \frac{1}{6}\begin{bmatrix} 2 & 2 \\ -2 & 1 \end{bmatrix}$$

(b) [**6 marks**] Find a sequence of elementary matrices whose product is $A^{-1}$.

$$A = \begin{bmatrix} 1 & -2 \\ 2 & 2 \end{bmatrix} \xrightarrow{R_2-2R_1} \begin{bmatrix} 1 & -2 \\ 0 & 6 \end{bmatrix} \xrightarrow{\frac{1}{6}R_2} \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} \xrightarrow{R_1+2R_2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$$

$$E_1 = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} \qquad E_2 = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{6} \end{bmatrix} \qquad E_3 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$E_3 E_2 E_1 A = I_2 \quad \Rightarrow \quad A^{-1} = E_3 E_2 E_1$$

$$= \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{6} \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$$

Answers are not unique!

**2.** [**5 marks**] Use Cramer's rule to find $x_1$ of the solutions of the following system of linear equations.

$$\begin{array}{llll} 2x_1 & +x_2 & +x_3 & = 4 \\ -x_1 & & +2x_3 & = 2 \\ 3x_1 & +x_2 & +3x_3 & = -2 \end{array}$$

$$X_1 = \frac{\begin{vmatrix} 4 & 1 & 1 \\ 2 & 0 & 2 \\ -2 & 1 & 3 \end{vmatrix}}{\begin{vmatrix} 2 & 1 & 1 \\ -1 & 0 & 2 \\ 3 & 1 & 3 \end{vmatrix}} = \frac{0 + (-4) + 2 - 0 - 8 - 6}{0 + 6 + (-1) - 0 - (-3) - 4} = \frac{-16}{4} = -4$$

**3.** **[10 marks]** Find the determinant

$$\begin{array}{rl} \begin{vmatrix} 1 & -1 & -3 & 0 \\ 0 & 1 & 2 & -1 \\ -2 & 2 & 8 & 1 \\ 3 & 1 & -1 & -3 \end{vmatrix} & R_2+2R_1 \end{array}$$

$$\begin{array}{rl} = \begin{vmatrix} 1 & -1 & -3 & 0 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 2 & 1 \\ 0 & 4 & 8 & -3 \end{vmatrix} & \begin{array}{l} \\ \\ R_3+2R_1 \\ R_4-3R_1 \end{array} \end{array}$$

$$\begin{array}{rl} = \begin{vmatrix} 1 & -1 & -3 & 0 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 1 \end{vmatrix} & \begin{array}{l} \\ \\ \\ R_4-4R_2 \end{array} \end{array}$$

$$= 1 \cdot 1 \cdot 2 \cdot 1$$

$$= 2.$$

**4.** [**10 marks**] Determine whether a subset $S = \left\{ \begin{bmatrix} a \\ b \end{bmatrix} : a \geq 0, b \geq 0, a, b \in \mathbb{R} \right\}$ is a subspace of $\mathbb{R}^2$ with the standard operations. Justify your answers.

No, it is not a subspace. because $S$ is not closed with respect to scalar multiplication. for example,

$$(-1) \cdot \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -a \\ -b \end{bmatrix} \notin S \text{ for any } \begin{bmatrix} a \\ b \end{bmatrix} \in S$$

**5.** [**5 marks**] Truth or False questions (1 mark each).

(a) The unit vector of $v = \begin{bmatrix} -1 \\ 2 \\ -4 \\ 2 \end{bmatrix}$ is $\frac{1}{25} \begin{bmatrix} -1 \\ 2 \\ -4 \\ 2 \end{bmatrix}$. T F

(b) The inverse of an elementary matrix is an elementary matrix. T F

(c) Two vectors $\begin{bmatrix} 2 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 4 \\ 8 \end{bmatrix}$ in $\mathbb{R}^2$ are collinear. T F

(d) If $AX = 0$ has only the trivial solution then $det(A) = 0$. T F

(e) If $A$ and $B$ are $3 \times 3$ matrices such that $det(A) = 6$ and $det(B) = 2$, then the determinant of $-3A(B^T)^{-1}A^{-2}B^2$ is $-9$. T F