

Solutions

School of Mathematics and Statistics
Carleton University
Math. 1004A, Fall 2016
TEST 3

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Non programmable calculator ONLY, 1 or more blank sheets permitted for roughs

Print Name :

Student Number:

Tutorial Section (A1, A2, A3, A4, or A5):

PART I: Multiple Choice Questions

(Choose and CIRCLE only ONE answer - No part marks here.)

- [2 marks] If $\log_3(x^2) = y$ and $x > 0$ what is x ?
(a) $x = 3^{y/2}$, (b) $x = y^3$, (c) $x = y^{1/3}$, (d) $x = 3^y$, (e) None of these
- [2 marks] On which one of the following intervals is the function $f(x) = \frac{2x}{x^2 + 4}$ increasing?
(a) $-4 < x < 4$, (b) $-2 < x < 2$, (c) $-4 < x < 2$, (d) $2 < x < 4$, (e) None of these
- [2 marks] Let $f(x) = e^{x^2+1} \ln(\sin x)$. Find $f'(\pi/2)$, that is find the derivative of f at $x = \pi/2$.
(a) 1, (b) -1, (c) 2, (d) 0, (e) None of these
- [2 marks] Which of the following functions is an antiderivative of $f(x) = x \cos(x^2)$?
(a) $-\frac{\sin(x^2)}{2}$, (b) $\frac{\cos(x^2)}{2}$, (c) $\frac{\sin(x^2)}{2}$, (d) $\sin(x^2)$, (e) None of these.

- [2 marks] Evaluate $\int_0^{\pi/2} \sin x \cos x \, dx$.

- (a) $-\frac{1}{2}$, (b) $\frac{1}{2}$, (c) $\frac{3}{2}$, (d) 0, (e) None of these.

PART II: Show all work here and give details.

No additional pages will be accepted

- [5+5 marks] a) Evaluate $\lim_{x \rightarrow \infty} (\sqrt{x+2} - \sqrt{x})$.

- b) Find the area under the curve $y = xe^{x^2}$ between the lines $x = 0$ and $x = 1$.

$$a) \sqrt{x+2} - \sqrt{x} = \frac{(\sqrt{x+2} - \sqrt{x})(\sqrt{x+2} + \sqrt{x})}{\sqrt{x+2} + \sqrt{x}} = \frac{x+2-x}{\sqrt{x+2} + \sqrt{x}} = \frac{2}{\sqrt{x+2} + \sqrt{x}}$$

$$\therefore \lim_{x \rightarrow \infty} (\sqrt{x+2} - \sqrt{x}) = \lim_{x \rightarrow \infty} \frac{2}{\sqrt{x+2} + \sqrt{x}} = \boxed{0}$$

$$b) \text{ Area} = \int_0^1 xe^{x^2} \, dx. \text{ Let } u = x^2, \, du = 2x \, dx, \, \frac{du}{2} = x \, dx$$

$$\text{Then } \int xe^{x^2} \, dx = \frac{1}{2} \int e^u \, du = \frac{1}{2} e^u = \frac{1}{2} e^{x^2}$$

$$\therefore \text{ Area} = \left. \frac{1}{2} e^{x^2} \right|_{x=0}^{x=1} = \boxed{\frac{e-1}{2}}$$

7. [5+5 marks]

a) Find all the horizontal asymptotes of the function defined by $f(x) = \frac{2x}{1+x}$ b) Evaluate $\int x \sqrt{1-2x^2} dx$. = I

$$a) \lim_{x \rightarrow \infty} \frac{2x}{1+x} = \lim_{x \rightarrow \infty} \left(\frac{2}{1} \right) = \boxed{2} \text{ by L'Hospital's Rule.}$$

$\therefore y=2$ is a horizontal asymptote.

$$\lim_{x \rightarrow -\infty} \frac{2x}{1+x} = 2 \text{ (also, so there are no others).}$$

$$b) \text{ Let } u = 1-2x^2 \Rightarrow du = -4x dx \Rightarrow x dx = -\frac{du}{4}$$

$$\begin{aligned} \therefore I &= \int \sqrt{u} \left(-\frac{du}{4} \right) = -\frac{1}{4} \int u^{1/2} du \\ &= -\frac{1}{4} \cdot \left(\frac{2}{3} \right) u^{3/2} = \boxed{-\frac{1}{6} (1-2x^2)^{3/2} + C} \end{aligned}$$