Lecture 9 Note - *Centre of Gravity and Centroids*

Textbook Chapter 9.1 - [Lecture Notes]

Before we start: What are integrals?

- The derivative of a function gives us its slope. The integral of a function gives us its area.
- If we find the integral (area) of a function, we can find its centroid. Why do this? Well, imagine you want to find the centre of mass of an object. It would be in the centre, but how would you find it if you had a weird curve shape? That's where the integral comes in. It cuts up the curve into slices that are so small we can pretend each is rectangular. It's easy to find the area/centroid of a rectangle, so if you split a curve into rectangles and add them up, you can find its entire area/midpoint.
- The derivative of x^3 is $3x^2$. Likewise, the integral of x^3 is $x^4/4$ +C, using $\int ax^n dx = \frac{ax^{n+1}}{n+1} + C$
- The signs $\int$ and ***dx*** are like a pair of brackets: They go together and enclose something. Once you calculate the integral of the stuff inside, these "brackets" can be removed.
- Leave the proofs to the math profs. For Statics, just know the power rule above ↑

What *is* the centroid, and the Centre of Gravity?

- The centroid and centre of gravity are the same. The centroid is the geographic centre of an area; the midpoint. So it's also the centre of gravity/mass/volume (or area if it's 2D).
- The centroid is located at ($\bar{x}$, $\bar{y}$, $\bar{z}$), a point in an object where $\bar{x}$ is the distance of the centroid from the origin along the *x*-axis, $\bar{y}$ the distance along the *y*-axis, and so on…
- The distance of a particle <u>inside</u> the body from the origin is located at ($\tilde{x}$, $\tilde{y}$, $\tilde{z}$). What particle? Any particle we want to measure — if we break up the shape into smaller pieces for easier calculations, each piece will have a centroid at ($\tilde{x}$, $\tilde{y}$, $\tilde{z}$).

How to solve for the centroid?

Use this: $\boxed{\bar{x} = \frac{\int \tilde{x} dA}{\int dA}}$ (*x* can be switched for *y/z, & dA* can be switched for *dV/dM/etc…*)