Solutions

School of Mathematics and Statistics
Carleton University
Math. 1004A, Fall 2013

TEST 4

Any non-programmable calculator permitted, 1 blank sheet permitted for roughs

Print Name :

Student Number:

Tutorial Section (A1, A4, ...):

PART I: Multiple Choice Questions

(Choose and CIRCLE only ONE answer - No part marks here.)

1. **[3 marks]** Evaluate $\int_0^1 xe^x\,dx$.

   (a) -1, (b) 2, (c) 0, (d) 1, e) none of these.

2. **[3 marks]** Evaluate $\int_1^e 2\ln x\,dx$

   (a) -4, (b) 1, (c) 2, (d) 0, e) none of these

3. **[3 marks]** Evaluate $\int x\sin x\,dx$

   (a) $\sin x - x\cos x + C$, (b) $-2\sin x + x\cos x + C$, (c) $x\cos x + C$, (d) $-x\sin x + \cos x + C$, e) none of these

4. **[3 marks]** Evaluate $\int_0^1 2x^2 e^x\,dx$

   (a) e, (b) $2e-4$, (c) $e-1$, (d) 0, e) none of these

5. **[3 marks]** $\int_{-1}^1 x\,e^{-x}\,dx = -2/e$

   (a) TRUE, (b) FALSE,

PART II: Show all work here and give details. No additional pages will be accepted

6. **[5+5 marks]** a) Find the most general antiderivative, that is, evaluate $\int e^{-x}\sin x\,dx$.

   b) Evaluate the definite integral $\int_1^e 3x^2(\ln x)^2\,dx$.

a)

e^{-x}	$\sin x$	
$-e^{-x}$	$-\cos x$	$+$
e^{-x}	$-\sin x$	$-$

$\therefore I = \int e^{-x}\sin x\,dx = \left(-e^{-x}\cos x - e^{-x}\sin x\right)\frac{1}{2}$

My −1
C +1
A 2
R 1/2

$= \frac{1}{2}\left(-e^{-x}\cos x - e^{-x}\sin x\right) + C$

(1) (1) (1) (1) (1)

2 b) $I = \int_1^e 3x^2(\ln x)^2 \, dx$.

$\ln x = t$ ← (1)

$x = e^t$ ← (1)

$dx = e^t dt$ ← (1)

Now, $\int x^2(\ln x)^2 dx = \int e^{2t} t^2 e^t dt = \int t^2 e^{3t} dt$

t^2	+	e^{3t}
$2t$	−	$e^{3t}/3$
2	+	$e^{3t}/9$
0		$e^{3t}/27$

$= \frac{t^2 e^{3t}}{3} - \frac{2te^{3t}}{9} + \frac{2e^{3t}}{27}$

$= \frac{1}{3}e^{3t}\left(t^2 - \frac{2t}{3} + \frac{2}{9}\right)$

$= \boxed{\frac{1}{3}x^3\left((\ln x)^2 - \frac{2}{3}(\ln x) + \frac{2}{9}\right)}$ ← (1)

$\therefore I = 3\cdot\frac{1}{3}e^3\left(1 - \frac{2}{3} + \frac{2}{9}\right) - 3\cdot\frac{1}{3}\left(0 - 0 + \frac{2}{9}\right) = 3\cdot\frac{5}{27}e^3 - \frac{2\cdot 3}{27} = \boxed{\frac{5e^3 - 2}{9}}$

(1)

7. **[5 marks]** Evaluate the definite integral $\int_0^1 \text{Arctan } x \; dx$.

Arctan x	+	1
$\frac{1}{1+x^2}$	←	x

$I = \int_0^1 \text{Arctan } x \, dx = \underbrace{x\,\text{Arctan } x\Big|_0^1}_{(1)} - \underbrace{\int_0^1 \frac{x}{1+x^2}dx}_{(1)}$

$= \text{Arctan}(1) - \text{Arctan}(0) - \frac{1}{2}\ln(1+x^2)\Big|_0^1$ ← (1) ↑ (1)

$= \underbrace{\frac{\pi}{4} - \frac{1}{2}\ln 2}_{(1)}$