School of Mathematics and Statistics
Carleton University
Math. 1004A, Fall 2016

SOLUTIONS to TEST 1

PART I: Multiple Choice Questions
(Choose and CIRCLE only ONE answer - No part marks here.)

1. **[2 marks]** Let $f(t) = 3t^2$, $g(x) = x - 1$. Evaluate the composition, $f(g(2))$.

   $\boxed{(a)}$ 3 ,   (b) 1 ,   (c) 0,   (d) 2,   (e) None of these

2. **[2 marks]** Evaluate the following limit, $\lim_{x \to 0} \frac{\sin 2x}{4x}$.

   (a) 1/4,   (b) 1/3,   (c) 0, $\boxed{(d)}$ 1/2   (e) None of these

3. **[2 marks]** Evaluate the following limit, $\lim_{x \to +\infty} \cos\left(\frac{1}{x^2}\right)$.

   (a) 0, $\boxed{(b)}$ 1,   (c) -1,   (d) The limit does not exist,   (e) None of these

4. **[2 marks]** Evaluate $\lim_{t \to \infty} \left(\sqrt{t+2} - \sqrt{t}\right)$

   (a) 2,   (b) 1, $\boxed{(c)}$ 0,   (d) The limit does not exist,   (e) None of these

5. **[2 marks]** Let f be defined by

$$f(t) = \begin{cases} \frac{t}{|t|}, & \text{if} \quad t \neq 0, \\ 1, & \text{if} \quad t = 0. \end{cases}$$

   Is f continuous at $t = 0$?

   (a) YES, $\boxed{(b)}$ NO,

PART II: Show all work here.
No additional pages will be accepted

6. **[5+5 marks]** :

   a) Evaluate the following limit: $\lim_{x \to \infty} \frac{2x^2 + x - 3}{x^2 + 1}$.

$$\frac{2x^2 + x - 3}{x^2 + 1} = \frac{(2x^2 + x - 3)/x^2}{(x^2 + 1)/x^2} = \frac{2 + 1/x - 3/x^2}{1 + 1/x^2}.$$

$$\lim_{x \to \infty} \frac{2x^2 + x - 3}{x^2 + 1} = \lim_{x \to \infty} \frac{2 + 1/x - 3/x^2}{1 + 1/x^2} = \frac{2 + 0 - 0}{1 + 0} = \boxed{2}.$$

b) Evaluate the following limit: $\lim_{x\to 0} \dfrac{1-\cos 3x}{4x}$.

For $x \neq 0$, $\dfrac{1-\cos 3x}{4x} = \dfrac{3}{4}\dfrac{1-\cos 3x}{3x} \Longrightarrow \lim_{x\to 0} \dfrac{1-\cos 3x}{4x} = \lim_{x\to 0} \dfrac{3}{4}\dfrac{1-\cos 3x}{3x} = \dfrac{3}{4}\lim_{x\to 0} \dfrac{1-\cos 3x}{3x}$
$= \dfrac{3}{4}\lim_{3x\to 0} \dfrac{1-\cos 3x}{3x} = \dfrac{3}{4}\lim_{t\to 0} \dfrac{1-\cos t}{t} = \dfrac{3}{4}0 = \boxed{0}$.

7. **[5+5 marks]**

a) Evaluate the following limit: $\lim_{x\to 1} \dfrac{x^3-1}{x-1}$.

Factoring, for $x \neq 1$, we have $\dfrac{x^3-1}{x-1} = x^2+x+1$.

$\lim_{x\to 1} \dfrac{x^3-1}{x-1} = \lim_{x\to 1}(x^2+x+1) = 1^2+1+1 = \boxed{3}$,
by continuity of the polynomial at $x=1$.

b) Let f be defined by

$$f(x) = \begin{cases} x\,|x|, & \text{if} \quad x \neq 0, \\ 1, & \text{if} \quad x = 0, \end{cases}$$

Determine whether f is continuous at $x=0$. Give reasons.

Here, $f(0)=1$.
For $x>0, f(x) = x|x| = x\cdot x = x^2$.
For $x<0, f(x) = x|x| = x\cdot(-x) = -x^2$.

$$\therefore \quad \lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} x^2 = 0.$$

Similarly,

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} (-x^2) = 0.$$
$$\therefore \lim_{x\to 0} f(x) = 0.$$

Since

$$\therefore \lim_{x\to 0} f(x) \neq f(0),$$

f cannot be continuous at $x=0$, by definition.