Math 1104 Practice Final Exam

Disclaimer: This practice exam does NOT intend to cover all topics that may appear on the final exam. Any topic covered during the course may be tested on the final. Solving these problems alone does not constitute sufficient preparation for the final.

Multiple choice problems

1. Which of the following points is on the line containing the points $(1, 0, 5)$ and $(3, 1, -2)$?

(a) (0, -3, 17)

(b) (9, 4, -23)

(c) (-9, -4, 23)

(d) (0, 3, -17)

(e) (-1, -1, -12)

2. Let P be the plane passing through the point $(1, 1, 0)$ with normal vector $\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$. Which of the following points is in P?

(a) $(-1, 1, 1)$

(b) $(-1, -2, 0)$

(c) $(-1, 1, 3)$

(d) $(3, 4, -1)$

(e) $(1, -2, 1)$

3. Which of the following is NOT a subspace of $\mathbb{R}^3$?

(a) $\left\{ \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \right\}$

(b) The line through $(4, 2, -6)$ in the direction $u = \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix}$.

(c) The plane $x + 2y - 5z = 0$.

(d) The span of $u = \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}$ and $v = \begin{pmatrix} 3 \\ -3 \\ -6 \end{pmatrix}$.

(e) The set of solutions to the linear system $Ax = 0$ where $A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & -1 & -4 \\ -2 & 4 & 10 \end{pmatrix}$.

3. Which one of the following matrices is invertible? (Only one is invertible)

(a) $\begin{pmatrix} 2 & 1 & 0 \\ 1 & 0 & -1 \\ 2 & 3 & 0 \end{pmatrix}$

(b) $\begin{pmatrix} 4 & 1 & -3 \\ 0 & 0 & 8 \\ 0 & 0 & -4 \end{pmatrix}$

(c) $\begin{pmatrix} 3 & -6 & -3 \\ 1 & 1 & 1 \\ 1 & -2 & -1 \end{pmatrix}$

(d) $\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 5 & 0 \end{pmatrix}$

(e) $\begin{pmatrix} 1 & 1 & -3 \\ 2 & -1 & -1 \\ 4 & 1 & -7 \end{pmatrix}$

4. Let A be a 4×4 matrix such that the system of equations $Ax = 0$ has infinitely many solutions. Which of the following statements is true? (Only one is true)

(a) The rows of A span $\mathbb{R}^4$.

(b) The matrix A is invertible.

(c) The rank of A is 4.

(d) The rows of A are linearly dependent.

(e) A can be written as a product of elementary matrices.

5. Suppose that A and B are $4{\times}4$ matrices and that $\det(A) = \det(B) = -\frac{1}{2}$. Then $\det\left(-A^3BA^T(-3B^2)(-A)^{-1}\right)$ is

(a) $-\frac{81}{64}$

(b) $\frac{81}{64}$

(c) $\frac{3}{64}$

(d) $-\frac{3}{64}$

6. Which of the following vectors is orthogonal to both $\begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$?

(a) $\begin{pmatrix} 6 \\ 16 \\ 6 \end{pmatrix}$

(b) $\begin{pmatrix} 3 \\ 8 \\ -3 \end{pmatrix}$

(c) $\begin{pmatrix} 3 \\ -8 \\ 3 \end{pmatrix}$

(d) $\begin{pmatrix} -3 \\ 8 \\ 3 \end{pmatrix}$

7. Let $T : \mathbb{R}^4 \to \mathbb{R}^3$ be a linear transformation such that $\dim(\mathrm{Im}(T)) = 2$. What is the dimension of the kernel (nullspace) of T?

(a) 0

(b) 1

(c) 2

(d) 3

(e) 4

8. Let A be an invertible $n \times n$ matrix, B a $p \times s$ matrix, and C a $q \times t$ matrix. If the matrix expression $AB^T - (CA^{-1})^T$ is defined, then only one of the following is a necessary restriction on the sizes n, p, s, q, t. Which one?

(a) $n = p = q = s = t$

(b) $n = s = q$ and $p = t$

(c) $n = s = t$ and $p = q$

(d) $n = s = p$ and $q = t$

9. What is the value of the variable x_2 in the linear system

$$\begin{aligned} x_1 + x_2 + x_3 &= 1 \\ 3x_1 - 2x_2 + 4x_3 &= 0 \\ -x_1 + 2x_2 - x_3 &= 0 \end{aligned}$$

(Hint: Use Cramer's Rule.)

(a) 3

(b) 0

(c) -1

(d) $\frac{1}{3}$

(e) $\frac{2}{3}$

10. If $z = 3 - 4i$, what is $\frac{1}{z}$?

(a) $\frac{3}{25} + \frac{4}{25}i$

(b) $3 + 4i$

(c) $-3 + 4i$

(d) $-\frac{3}{7} - \frac{4}{7}i$

(e) $\frac{1}{3} - \frac{1}{4}i$

11. Which of the following is an eigenvector of the matrix $\begin{pmatrix} 0 & -2 & 4 \\ -3 & -1 & 8 \\ -2 & -2 & 7 \end{pmatrix}$?

(a) $\begin{pmatrix} 0 \\ -2 \\ 4 \end{pmatrix}$

(b) $\begin{pmatrix} 0 \\ -3 \\ -2 \end{pmatrix}$

(c) $\begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$

(d) $\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$

(e) $\begin{pmatrix} 0 \\ -2 \\ -1 \end{pmatrix}$

12. Let $S = \text{span}\left\{ \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \right\}$. Which of the following sets is a basis of S?

(a) $\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$

(b) $\left\{ \begin{pmatrix} 0 \\ -1 \\ 5 \end{pmatrix}, \begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix} \right\}$

(c) $\left\{ \begin{pmatrix} 0 \\ -1 \\ 2 \end{pmatrix} \right\}$

(d) $\left\{ \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}, \begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix}, \begin{pmatrix} 2 \\ 5 \\ 1 \end{pmatrix} \right\}$

(e) $\left\{ \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} \right\}$

13. Let $v_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \\ 1 \end{pmatrix}$, $v_2 = \begin{pmatrix} -2 \\ 0 \\ 2 \\ -2 \end{pmatrix}$, $v_3 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$, $v_4 = \begin{pmatrix} 1 \\ -1 \\ -3 \\ 1 \end{pmatrix}$, $v_5 = \begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix}$, and $S = \text{span}\{v_1, v_2, v_3, v_4, v_5\}$. What is the dimension of S?

(a) 1

(b) 2

(c) 3

(d) 4

(e) 5

14. Let $A = \begin{pmatrix} 2i & -2+i \\ 1 & 1+i \end{pmatrix}$. Which of the following is A^{-1}?

(a) $\begin{pmatrix} -1+i & 1+2i \\ -i & -2 \end{pmatrix}$

(b) $\begin{pmatrix} 1-i & 1+2i \\ -i & 2 \end{pmatrix}$

(c) $\begin{pmatrix} -1+i & i \\ -1-2i & -2 \end{pmatrix}$

(d) $\begin{pmatrix} 1+i & 2-i \\ -1 & 2i \end{pmatrix}$

(e) $\begin{pmatrix} 1-i & -1-2i \\ i & 2 \end{pmatrix}$

15. For what value(s) of k is the matrix $A = \begin{pmatrix} 1 & 0 & 1 \\ 1 & k & 1 \\ 1 & 1 & k \end{pmatrix}$ singular (non-invertible)?

(a) All real numbers.

(b) All real numbers except 0 and 1.

(c) All real numbers except 1 and -1.

(d) $k = 0, 1$

(e) $k = 1, -1$

Long answer problems

1. Let $T : \mathbb{R}^4 \to \mathbb{R}^4$ be a linear transformation defined by

$$T(x_1, x_2, x_3, x_4) = (x_1 - 2x_2 + x_3, x_2 - 4x_4, x_1 + x_3 - x_4, x_2 - 3x_3 + x_4).$$

(a) Find the standard matrix of T.

(b) Find a basis for the image of T.

(c) Find a basis for the kernel (nullspace) of T.

2. Let $A = \begin{pmatrix} 4 & 2 \\ -1 & 1 \end{pmatrix}$.

(a) Find a diagonal matrix D and an invertible matrix P such that $A = PDP^{-1}$.

(b) Compute A^{2011}. (Hint: Use part (a))

3. Let

$$A = \begin{pmatrix} 1 & 1 & -1 & 4 \\ 2 & 1 & 3 & 0 \\ 0 & 1 & -5 & 8 \end{pmatrix}.$$

(a) Solve the linear system $Ax = b$ where $b = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$.

(b) Solve the homogeneous linear system $Ax = 0$. (Hint: Use (a))

4. Let $u = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$ and $w = \begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix}$. Find the unique vector $v = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$ in $\mathbb{R}^3$ such that $u \cdot v = 1$, v is orthogonal to w, $||v|| = \sqrt{3}$, and $v_2 > 0$.