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CONTENTS

PART-A

1. INTRODUCTION

Decision Makers	1
Engineering and Economics	2
Problem solving and Decision making	3
Intuition and Analysis	4
Tactics and Strategy	4
Engineering Economic Decision Maze	5
Law of Demand and Supply	5
Law of Returns	13

INTEREST AND INTEREST FACTORS

Introduction	15
Simple Interest	15
Compound Interest	16
Cash Flow Diagram	17
Types of Compound Interest Problems	18
Personal Loan &EMI Payment	27
Worked Problems	30
Summary	47
Exercise Problems	50

2. PRESENT WORTH COMPARISON

Introduction	52
Present Worth Method	52
Worked Examples	54
Conditions for PW Comparison	72
Basic PW Comparisons	72
Present Worth Equivalence	73
Net Present Worth	73
Assets with unequal lives	85
Assets having infinite lives	87
Future Worth Comparison	88
Worked Examples	96
Pay-back Comparison	99
Summary	100
Exercise Problems	105

3. EQUIVALENT ANNUAL WORTH COMPARISON

Introduction	105
EAW Comparison method	105
Worked Examples	106

Situations for EAW Comparison	125
Consideration of Asset Life	126
Comparison of assets with equal and unequal lives	127
Use of Sinking Fund Method	127
Annuity contract for guaranteed income	128
Summary	129
Exercise Problems	130
4 RATE OF RETURN CALCULATIONS & DEPRECIATION	134
Introduction	134
Rate of Return	134
Minimum Acceptable Rate of Return	134
Internal Rate of Return	134
External Rate of Return	135
Worked Examples	136
Detailed Concept of IRR	146
IRR Misconceptions	147
Cost of Capital Concepts	149
5 DEPRECIATION	150
Causes of depreciation	150
Basic Methods of computing Depreciation	151
Worked Examples	163
Tax Concepts	168
Corporate Income Tax	169
Summary	171
Exercise Problems	174
PART-B	
6 ESTIMATING AND COSTING	179
Introduction	179
Elements of Costs	180
Determination of Selling Price	183
Estimation for Simple components	186
Worked Examples	186
Summary	186
Exercise Problems	193
7 FINANCIAL MANAGEMENT	195
Introduction	196
Objectives of Financial Management	196
Scope of Finance	196
Sources of Financial Information	197
Accounting Process	199
Book-Keeping	200
	205

Books-of Account	208
Financial Statements	210
Profit & Loss Statement	212
Balance Sheet	213
Relationship between Balance Sheet and P & L Account	213
Worked Problems	214
Summary	219
8 FINANCIAL RATIO ANALYSIS	222
Introduction	222
Nature of Ratio Analysis	223
Liquidity Ratios	228
Leverage Ratios	230
Activity Ratios or Turnover Ratios	232
Profitability Ratios	234
Other Ratios	238
Evaluation of firms earning power	240
Comparative Statement Analysis	240
Worked Problems	242
Summary	246
Exercise Problems	249
9 FINANCIAL AND PROFIT PLANNING	251
Introduction	251
Financial Planning	251
Profit Planning	251
Objectives of Profit Planning	253
Essentials of Profit Planning	253
Budget Administration	255
Types of Budgets	256
Preparation of Budgets	259
Advantages of Budgeting	261
Dangers of Budgeting	262
Introduction to Benchmarking	262
Summary	266
Appendix: Syllabus of 'Engineering Economics'	269
Index	271



CHAPTER
1

INTRODUCTION

INTRODUCTION

Students of engineering in all disciplines often consider themselves as strict disciples of technology and usually concentrate only on the technology side of any business. They often think "*Money matters will be handled by some one else. It is not something that I need to worry about*". But in reality, engineers should be as conversant with the economics of products as much as their technology.

Economics play an important role in all engineering decisions. Many of the decisions which have to be taken concern costs as much as performances. Economic criteria such as interest, depreciation, taxes and profit have to be considered as much as Young's modulus, Fatigue strength or Toughness of materials.

In broad terms, for any engineering product to be successful, it must not only be technically sound but also produce some economic benefits. It means that revenues must exceed the costs associated with the product so that the value is positive. The discipline, therefore, concerned with the economic aspects of engineering is known as **Engineering economics**. Engineering economics is nothing but the systematic evaluation of the costs and benefits associated with the design, manufacture and sale of engineering products. In other words, engineering economy quantifies the benefits and costs of engineering products to determine whether they make (or save) enough money to justify their capital investments. Thus the study of Engineering Economics is of utmost importance to engineering students and in the modern world where managers have to understand and execute all facets of business, economics and engineering have become inseparable. It is economics embedded engineering which is the foundation for success in any business, run on technology.

ENGINEERING DECISION - MAKERS

Engineers are basically decision makers. The practice of engineering involves decision making to choose among alternative designs, procedures, plans and methods. Since each available alternate source of action involves different amounts of investments and different operating costs and incomes, the question that is almost always asked is "*Is it worth it?*" In fact, the three most important questions that can be asked to all engineering proposals that come up for review are :

1. Why do this at all ?

2. Why do it now ?

3. Why do it this way ?

1. Why do this at all ?

This is the fundamental of all questions. The *raison d'être* (reason for existence) has to be established first before thinking any further. Additional questions asked in this domain are : Why should the proposed new activity be undertaken ? Why not an existing activity be expanded, contracted or abandoned ? Why not modify existing Standards and Operating Procedures ?

2. Why do it now ?

Questions asked here would be : Shall we build now a plant of excess capacity in expectation of higher demand or with only sufficient capacity to satisfy current demand ? Are the costs of capital and other business conditions favorable to a present development ? Shall we postpone all decisions so that things become more clear and decision making easier ?

3. Why do it this way ?

Shall we abandon the old method and look for a totally different approach ? Shall we adopt traditional methods or go for modern techniques ?

Engineering economics deals with principles and techniques that are useful in securing rational answers to all the above questions. The following general questions are representative of those that an engineer might encounter.

- Which body design to be selected for the new car among the several proposed?
- Is it time to replace the UTM with a new one in the M.T. Lab ?
- With limited capital available, which production expenditures should be given preference ?
- Would it be preferable to pursue a conservative course of action to get less returns or follow a riskier one that offers higher potential returns ?
- What is the minimum volume of production needed to cover costs and begin profit ?
- Are the benefits expected from the river networking project large enough to make its implementation costs acceptable ?
- Is it better to sell on credit basis and generate larger demand or sell against cash payment only and have reduced demand ?

Two characteristics stand out among all the above questions. First, each deals with a choice among alternatives ; Second, all involve economic considerations. These and other Economists.

- (i) Identify alternative uses for limited resources and obtain appropriate data.
- (ii) Analyze the data to determine the preferred alternative.

✓ PROBLEM SOLVING AND DECISION MAKING

The fundamental approach to solve problems in economics is through scientific methods. But scientific methods use both theoretical and practical knowledge to solve the same. In other words, it takes both the real world of facts and figures and the symbolic world of theories and hypothesis to solve problems, through an iterative process. The following steps gives a general problem - solving process involving both worlds.

- (i) Problems in engineering and managerial economy originate in the real world of economic planning, management and control.

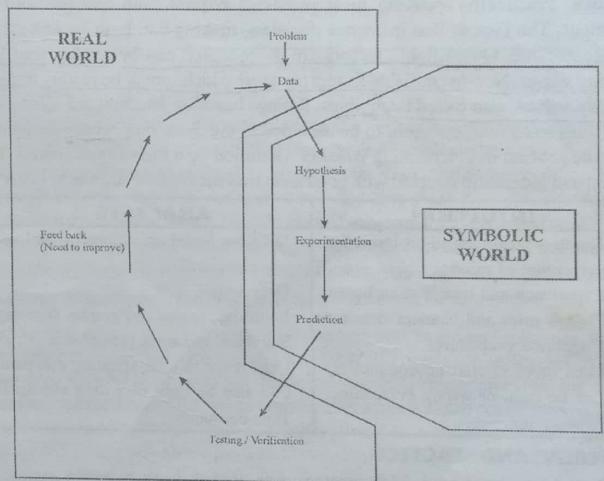


Fig. 1.1 Problem solving iteration

- (ii) The problem is defined and clarified by data from the real world.
- (iii) This information is subjected to analysis based on scientific principles to formulate a hypothesis in symbolic terms. This is like postulation of a theory or deriving a formula from fundamental principles.
- (iv) By manipulating and experimenting, an analyst can simulate and project reality in multiple configurations so as to understand all outcomes. Imagine a CAD software which can alter product design in many ways.
- (v) From these activities, usually a prediction (or a forecast) emerges. This can be considered a possible solution to the decision problem.
- (vi) This prediction is subjected to verification in the real world for its practical usage. If it gives desired results, then the problem is solved.

- (vii) If not, the cycle is repeated with valuable feedback from previous approach adding to data.

INTUITION AND ANALYSIS

Intuition can be defined as the immediate perception by the mind without reasoning. In other words, any decision taken on the basis of personal judgement without standard scientific procedures is said to be an intuitive decision. On the other hand decisions can be taken on the basis of scientific reasoning and analysis. Decisions based on intuition and analysis can both be independently right or wrong, once when the outcome of the decision is known. Practically speaking most problems require both analysis and personal judgement. The factors that influence decision-making can both be quantifiable and non-quantifiable. Quantifiable factors are those which can be readily translated into monetary values. Non-quantifiable factors are those which cannot be readily translated into monetary values, also called intangibles. Ratings based on intuition are often assigned to intangibles so as to allow them to be included in the decision process. Judgement also enters the process in determining whether a solution be a logically accepted. Therefore intuition and judgement coupled with good analytical methods contribute to better decision.

INTUITION	ANALYSIS
1. Quick decision based on immediate perception of mind.	Decision based on rationality and reasoning.
2. Experience and imagination based.	Data based.
3. Thumb rules and chances dominate.	Formulae, tables and graphs dominate.
4. Undefined procedures.	Standard operating procedures.
5. Used more in start-up companies.	Used more in long-standing companies.
6. Can be right or wrong eventually.	Can also be right or wrong eventually.
7. Time-saving.	Time-consuming.

STRATEGY AND TACTICS

Somebody once said, "My strategy has always been to make money, but my tactics have been to avoid work"! Strategy here refers to broad plans that are set and major objectives that are to be achieved whereas tactics refer to the maneuvers and manipulations of the resources to attain the same objectives.

Strategy and tactics are historically military terms associated with broad plans set by the high command and specific schedules proposed by the lower cadres, respectively. Strategy sets ultimate objectives and also the associated tactics required to achieve these objectives. But for economic studies, strategic and tactical consideration essentially have the same meaning.

There are usually several strategies available to an organization. But all strategies aim to make the best use of the organization's resources in accordance with its long-range objectives. The measure of merit for various strategies would be their effectiveness, which is nothing but the degree to which a plan meets economic targets.

As an example for strategy and tactics, let us consider the case of an entrepreneur who wants to invest in garments. The following table gives the information on strategies and its associated tactics along with their expected returns.

	Strategy	Tactics	Investment (Lakhs)	Expected Returns Rs. (in lakhs/Year)
1.	Buy and sell (Trading only)	(a) High-end user	50	10
		(b) Low-end user	36	7
2.	Manufacture Large Scale	(a) High-end user	200	30
		(b) Low-end user	100	12
3.	Manufacture-Small Scale	(a) High-end user	75	15
		(b) Low-end user	30	10

The strategies in front of the entrepreneur is to either buy and sell as a trader or manufacture. If he chooses to manufacture he further can select the strategy of either producing large scale or small scale. Among all the three strategies, he could pursue the tactics of targeting either the high-end user or low-end user. Pure economic consideration only can resolve this issue and help take decision. Sometimes the expected result can be the same for different considerations of strategies and tactics. In such cases, non-economic objectives (intangibles) may help in resolving the issue.

Engineering Economic Decision Maze

The decision-making process in any business situation, based on economics, is a long drawn-out procedure. It takes into account a number of criteria while several analytical tools have to be applied in order to get the best decision. The decision-making route in front of an Engineering Economist can be visualized as a maze which contains of all relevant criteria and factors required to reach the destination i.e., a stable, scientific decision. Fig.1.2 in the following page shows the well-known Engineering-Economic-Decision Maze.

LAW OF DEMAND AND SUPPLY

Demand may be defined as the desire to possess a thing coupled with the means of purchasing it and the willingness to use the means for buying. In other words, demand refers to the amount of a commodity which people are willing to buy at that price. Demand cannot be defined without a price attached to it.

Supply refers to the quantity of a commodity which is offered for sale at a particular price. Like demand, supply always comes at a price. Here supply has to be distinguished from Stock. The stock is the quantity of goods that could be sold whereas the supply is the quantity of goods that would be sold at a given price. It is obvious that supply cannot exceed stock.

It is an interesting aspect of economy to note that the demand and supply of a product are dependent on each other and are sensitive to the price of that product. The interrelationship between them are shown in figure.

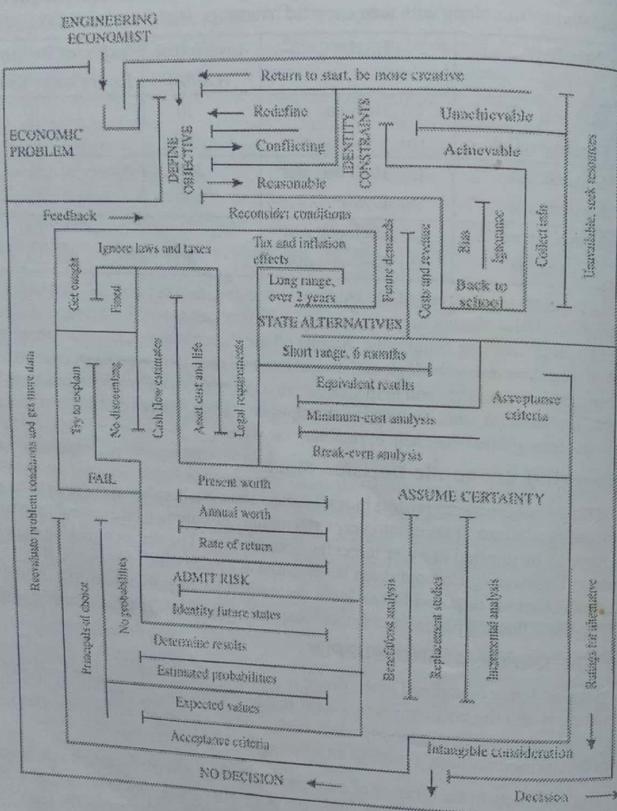


Fig. 1.2 Engineering Economic Decision Maze

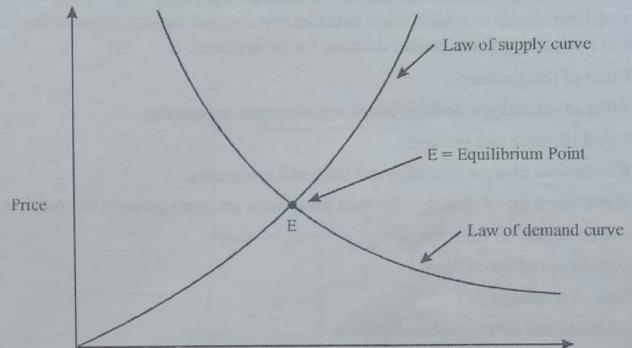
Law of demand

Fig. 1.3 Price Vs Demand / supply

The Law of demand states that "the demand for a commodity increases when its price decreases and vice-versa, all other things remaining the same". When we observe the demand curve it is clear that as prices fall, demand increases. Conversely, for the same curve, we can understand that as prices increase, demand falls. Thus, demand is inversely proportional to its price.

Law of supply

The law of supply states that, "the supply of a commodity increases when its price increases and vice-versa, all other things remaining the same". When we observe the supply curve, it is clear that as prices increase, supply also increases. Conversely, for the same curve, we can understand that as prices fall, the supply also falls. Thus, supply is directly proportional to its price.

When it is said "all other things remaining the same", it means that the curves hold good only when other factors which govern the demand and supply such as cost of production, consumer's income, change in technology, climate etc., remain constant.

The point of intersection of the supply curve and the demand curve is known as the equilibrium point (E). At the price corresponding to this point, the quantity of supply is equal to the quantity of demand. Hence, this point is called the equilibrium point.

Factors influencing demand or demand determinants

Various factors influence the market demand for a product. The knowledge of the factors and the nature of relationship between the demand and its determinants is very helpful in analyzing and estimating demand for the products.

1. Price of the product.
2. Price of related goods-substitutes, supplements, accessories.
3. Level of consumer income.
4. Continuous change in consumers taste and preference.
5. Advertisement / Publicity / Product promotion schemes given to the product.
6. Credit facility to consumers.
7. Population of the country.
8. Ease of availability.
9. Climatic and demographic condition.
10. General economic climate in the country as well as distribution pattern of national income.

Elasticity of Demand

Elasticity is nothing but the measure of the sensitiveness of one variable to change in some other variable. In this case, elasticity of demand is the rate at which the demand for a product changes with respect to changes in its price. It is the responsiveness or sensitiveness of demand for a commodity to the changes in its price. To be precise elasticity of demand is the percentage change in demand as a result of one percent change in the price of the commodity.

$$\therefore e_p = \frac{\text{Percentage change in demand}}{\text{Percentage change in price}}$$

Demand elasticity (e_p) is a ratio and hence without any units. When $e_p > 1$ the demand for the product is said to be elastic, which means that for a small fall in price, there is a large increase in demand. When $e_p < 1$, the demand is said to be inelastic which means that for any fall in price, there is only a marginal increase in demand. When $e_p = 1$, the demand has unit elasticity, being neither elastic nor inelastic which means that demand has grown or fallen proportionate to its price.

Let P = original price of a product

Q = original demand for the product

ΔP = change in price of the product

ΔQ = change in demand for the product

$$\therefore e_p = \frac{\frac{\Delta Q}{Q}}{\frac{\Delta P}{P}} = \frac{\Delta Q}{\Delta P} \times \frac{P}{Q}$$

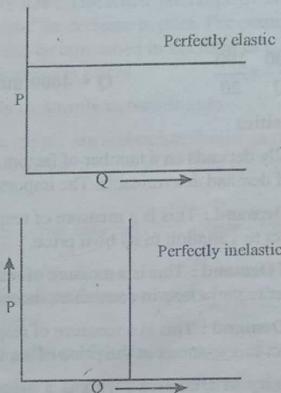


Fig. 1.4 Price vs Demand

WORKED EXAMPLE

1. A product has a demand of 3000 units when priced at Rs. 100/unit. When the price is reduced at Rs. 80/unit the sales increases to 3800 units.
 - (i) Find whether the demand is elastic or inelastic.
 - (ii) At what quantity of sales can the demand be called elastic or inelastic.

(VTU June '08)

Given : $P = \text{Rs. } 100/-$

$Q = 3000$ copies

$\Delta P = \text{Rs. } 100 - 80 = 20$

$\Delta Q = 3800 - 3000 = 800$

$$(i) \quad e_p = \frac{\Delta Q/Q}{\Delta P/P} = \frac{800/3000}{20/100}$$

$$e_p = 1.333$$

Since $e_p > 1$ the demand is said to be very elastic. That is, for a small fall in the price of the textbook there is more than proportionate increase in the sales of the book.

- (ii) What is Q when $e_p = 1$

$$e_p = \frac{\Delta Q/Q}{\Delta P/P}$$

$$1 = \frac{800/Q}{20/100} = \frac{800}{Q} \times \frac{100}{20} \quad Q = 4000 \text{ units}$$

Types of Demand Elasticities

Since demand basically depends on a number of factors, there are as many demand elasticities as the number of demand determinants. The important ones are the following:

- (a) **Price Elasticity of Demand** : This is a measure of responsiveness of demand for a product with respect to variation in its own price.
- (b) **Income Elasticity of Demand** : This is a measure of responsiveness of demand for a product with respect to variations in consumers income.
- (c) **Cross Elasticity of Demand** : This is a measure of responsiveness of demand for a product with respect to variations in the price of another related product.
- (d) **Promotional Elasticity of Demand** : This is a measure of responsiveness of demand for a product to variations in advertisement budget for the product.

Factors influencing the Elasticity of Demand

Following are a few important factors which influence the elasticity of demand

- (i) **Availability of Substitutes** : The demand of a commodity is said to be elastic if it has substitutes. Substitutes are those alternative products / services which are available as choices to the consumer. For example, a small increase in the prices of railway tickets will result in a fairly higher-than-a-proportionate fall in the demand, because of the presence of alternative mode of transport like buses. Alternatively, a small fall in the prices of railway tickets may result in a fairly higher-than-a-proportionate increase in people travelling by train. In both cases, $e_p > 1$ where demand is said to be elastic.
- (ii) **Nature of commodity** : The demand for luxurious items is said to be elastic whereas the demand for essential items of daily need is said to be inelastic. In other words, small changes in prices of luxury items can bring in large fluctuations in the demand, making it elastic. On the other hand minor deviations in prices of essential commodities (e.g., salt) will not cause major fluctuation in its demand, making it inelastic. This is because, essential commodities are bought anyway by almost all people keeping the demand fluctuation to a minimum, even with variations in its price.

(iii) **Price of commodity** : The demand is said to be elastic for products of higher prices whereas it is inelastic for products which are priced low. Products like TV, Refrigerator, Personal Computers, Silk Sarees etc., are elastic whereas sugar, match boxes, toothpastes, pens are inelastic in nature.

(iv) **Range of commodity use** : The wider the range of the uses of a product higher is the elasticity of demand for decrease in price. For example, a multi-use commodity like milk which can also be consumed in the form of curd, ghee, cheese etc. has a elastic demand for decrease in prices.

Factors influencing supply or supply determinants

Factors that influence supply are somewhat similar to demand determinants, with some exceptions. Some of them are :

- (i) Price of the product.
- (ii) Availability and prices of raw materials that go into production.
- (iii) Production technology.
- (iv) Company's expectation about future prospects for prices, costs, sales and the state of the economy in general.
- (v) Goals, objectives and policies of the company.
- (vi) Weather, strikes and other forces which affect production and distribution.
- (vii) Number of competitive firms in the industry.

Elasticity of supply

Elasticity of supply is the rate at which the supply of a commodity changes with respect to changes in its price. It is the responsiveness or sensitiveness of demand for a commodity to the changes in its price. To be precise, elasticity of supply is the percentage change in supply as a result of one percent change in the price of the commodity.

$$e_p = \frac{\text{Percentage change in quantity supplied}}{\text{Percentage change in price}}$$

$$e_p = \frac{\Delta Q/Q}{\Delta P/P}$$

Where

Q = quantity supplied

P = Original Price

ΔQ = Change in quantity supplied

ΔP = Change in price

Supply elasticity (e_p) is a ratio and hence without any units. Note that the formula for measuring the elasticity of supply is the same as for the elasticity of demand. But the difference is that demand elasticity is considered -ve while supply elasticity is considered +ve.

When $e_p > 1$, the supply is said to be elastic, which means that for a small increase in price there is a larger-than-proportionate increase in supply.

When $e_p < 1$, the supply is said to be inelastic which means that for any increase in price there is a less-than-proportionate increase in supply. When $e_p = 1$, the demand has unit elasticity which means that supply varies just proportionately to the prices.

Factors influencing elasticity of supply

1. Nature of commodities

Perishable commodities - Less elastic
Durable commodities - More elastic

2. Capital supply

in the short run - Inelastic
in the long run - Elastic

3. Supply of labour

Skilled labour - Elastic
Unskilled labour - Inelastic

4. Scale of production

Small scale production - Inelastic
Large scale production - Elastic

5. Time Period

Short periods : Supply quantity is almost fixed and hence inelastic

Long periods : supply quantity can vary and hence elastic

(Please note that short and long periods are terms very relative to each product.)

6. Production Technique

Advanced technology (Capital investment) - Elastic
Lower-end technology (Labour intensive) - Inelastic

LAW OF RETURNS

The law of returns relates the behaviour of output as a response to changes in inputs. In other words, this law explains the relationship between the various factors of production with the output quantity. Consider an expansion in the scale of operations in an industry i.e., increasing inputs proportionately and simultaneously. In such a case there are three technical possibilities -

- Total output may increase more than proportionately which is called the **Law of Increasing Returns**.
- Total output may increase proportionately which is called the **Law of Constant Returns**.
- Total output may increase less than proportionately, which is called the **Law of Diminishing Returns**.

Most of the production systems show normally a sequence of increasing returns first followed by the law of constant returns and then by the law of diminishing returns. Figure shows a typical graph of output Vs input where the I phase of the graph represents increasing returns, II phase represents somewhat constant returns and the III phase shows decline i.e., diminishing returns. Law of return is also known as law of variable returns.

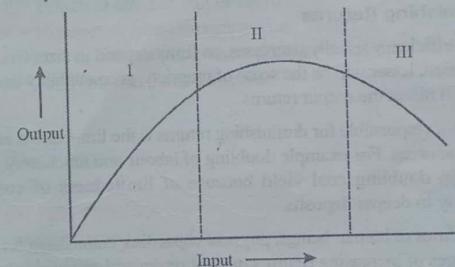


Fig. 1.5 Law of Returns

Example

When a farmer cultivates fresh farm land, normally he gets a high yield for his first crop, average yield for his second crop and lower yield for his third crop. This is because the productive capacity of fresh virgin land is higher for his first crop than for successive crops.

Law of returns is also applicable to many areas such as mines, fisheries, quarries, forest resources, manufacturing etc.

Let us now look at some causes for the law of increasing returns, constant returns and diminishing returns, respectively.

Causes of Increasing Returns

1. In the beginning of any business, both machinery and labour would be working at less than optimal conditions. When both move towards optimality, output increases without much increase in inputs. Hence increasing returns are possible.
2. Same logic can be applied to other resources like space, building, communication networks which all are on the road to maximization during period of increasing returns.
3. Our country India can be considered to be in the period of increasing returns because of immense potential to reach optimality.

Causes of Constant Returns

1. When all the resources are nearing optimality the rate of returns is proportionate to the inputs.

In other words, during the period of constant returns, production just matches the inputs.

Causes of Diminishing Returns

1. Managerial efficiency usually decreases, as firms expand in size. Greater the size of the organization, lesser will be the sense of propriety, accountability and achievement, which all will affect the output returns.
2. Another factor responsible for diminishing returns is the limitedness or exhaustibility of natural resources. For example doubling of labour and machinery in a coal mine will not help doubling coal yield because of limitedness of coal deposits or inaccessibility to deeper deposits.

Learning abilities of human beings, physical capacities, enthusiasm levels of workers all undergo the stages of increasing return, constant return and diminishing return.

✓ INTEREST AND INTEREST FACTORS

INTRODUCTION

Paying interest for money borrowed or alternately earning interest for money lent has been a practice that exists since ages. Records reveal the existence of the concept of 'Interest' in Babylon in 2000 BC as well as in other old civilizations. Various interest rates ranging from 5 to 40% have been recorded throughout the development of mankind all over the world.

Interest is nothing but the rent that is to be paid for the money borrowed. It is the cost of using capital and represents time value of money. Interest represents the growth of capital per unit period. The period may be a week, month quarter, bi-annual or a year. Rate of interest is normally expressed as a percent with the period of interest being one year.

There are two main rates of interest which are commonly practiced.

- (i) Simple Interest
- (ii) Compound Interest

I. SIMPLE INTEREST

When the total interest earned or charged is linearly proportional to the initial amount of loan (principal) the interest and interest rate are said to be simple. Under simple interest the interest earned is the same for all the interest periods.

If P = Principal amount lent or borrowed
 n = Number of interest periods (i.e., Years)
 i = Rate of interest per interest period (in fraction)
 I = Total interest earned or paid.

Then

$$\begin{aligned} I &= P \cdot n \cdot i \\ \text{if } P &= 200000/- \\ n &= 2 \text{ years} \\ i &= 12\% \\ \therefore & \text{Interest earned or paid for 2 years} \\ I &= 200000 \times 2 \times 0.12 \\ I &= \text{Rs. } 48000/- \text{ (for 2 years)} \\ \therefore & \text{Interest paid per year, } I = \text{Rs. } 24,000/- \\ \therefore & \text{Interest paid per month, } I = \text{Rs. } 2,000/- \end{aligned}$$

Transaction ends if principal amount is paid along with interest at the end of any interest period. In the above case,

Interest paid along with principal amount at the end of
 Year 1 = $200000 + 24000 = \text{Rs. } 2,24,000/-$
 Year 2 = $200000 + 48000 = \text{Rs. } 2,48,000/-$
 Year 3 = $200000 + 72000 = \text{Rs. } 2,72,000/-$

COMPOUND INTEREST

If the borrower does not pay the interest earned at the end of each period, interest amount is added to the principal amount for future interest calculation. In other words, interest charged, for example at the end of second year would be based on initial principal amount plus interest amount for the period ending first year. Therefore the interest owed in the previous year becomes part of the total amount owed for this year. This is known as compound interest, where interest is calculated on interest. Modern commercial practice makes use of compound interest more than simple interest.

Example

- If an investor invests a sum of Rs. 100/- in a fixed deposit for five years with an interest rate of 15% compounded annually, the accumulated amount at the end of each interest period (i.e., year) is as shown in the table.

Year end	Interest (Rs.)	Compound Amount (Rs.)
0	-	100.00
1	15.00	115.00
2	17.25	132.25
3	19.84	152.09
4	22.81	174.90
5	26.24	201.14
6	30.17	231.31

The formula to find the compound amount in the third column at the end of an interest period is

$$F = P \times (1 + i)^n$$

Where P = Principal amount invested at time zero

F = Future amount (or compound amount)

i = Rate of interest compounded annually (fraction)

n = number of periods of deposit

For example at the end of 6th year

$$F = 100 \times (1 + 0.15)^6 = \text{Rs. } 231.30$$

Graphical Comparison of Simple and Compound Interests

Taking the example of previous problem if we consider an initial amount of Rs. 100/-, future amounts based on simple and compound interest (both at 15% rate of interest) would grow as shown in the following figure :

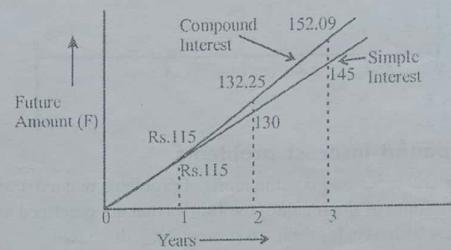
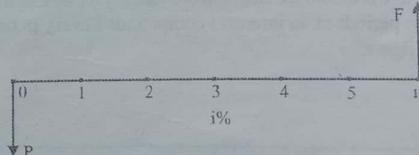


Fig. 1.6 Simple Interest Vs Compound Interest

This means that an initial amount of Rs. 100/- invested at time zero would grow to Rs. 145/- if simple interest is calculated or to Rs. 152.09 at the end of third year if compound interest is calculated and compounded annually.

CASH FLOW DIAGRAM

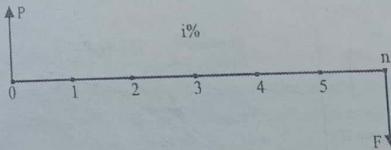
A cash flow diagram (CFD) is a pictorial representation of all the quantities used in computation of compound interest such as present amount (P), future amount (F), rate of interest (i), number of interest periods (n) and so on. A typical CFD would be



Following are some of the conventions used while drawing cash flow Diagrams :

- The horizontal line is a time scale, with time progressing from left to right. Intervals marked 0, 1, 2, 3, ... represent the end of those periods (or even the beginning of next period). Periods can be years, months, days, etc.
- Arrows signify cash flows and are placed at the end of the period. Downward arrows represent costs (cash outflows) whereas upward arrows represent revenues (cash inflows).

3. The cash flow diagram is dependent on the point of view. Problem stated would normally specify whether it is from a lender's point of view or from a borrower's point of view. For example, if the earlier diagram represents the lender's point of view, the CFD from the borrower's point of view would become



Types of compound interest problems

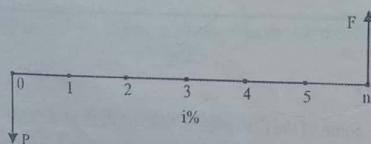
There are several types and combinations of problems that exist when calculating compound interest. Some of the regular possibilities that are practiced in businesses are given below. Interest formulas for each are subsequently discussed.

1. Single-payment compound amount
2. Single-payment present worth amount
3. Equal-payment series compound amount
4. Equal-payment series sinking fund
5. Equal-payment series present worth amount
6. Equal-payment series capital recovery amount
7. Uniform gradient series annual equivalent amount.

1. SINGLE-PAYMENT COMPOUND AMOUNT $P(F/P, i, n)$

(Finding F when given P)

Here, the objective is to find the single future sum (F) for an initial payment (P) made at time 0, i.e., after n periods at an interest i compounded every period. The cash flow diagram is shown in figure.



Formula

$$F = P(1 + i)^n$$

WORKED EXAMPLES

1. A worked example on single payment compound amount is already shown in page 16.
2. A person deposits a sum of Rs. 10,000/- in a bank at a nominal rate of interest of 12% for 10 years. Find the maturity amount of the deposit after 10 years, if the compounding is done quarterly.

Solution:

Given $P = \text{Rs. } 10,000/-$

$n = 10$ years

$i = 12\%$ (Compounded quarterly)

$F = ?$

When compounding is done for any period other than annual, either of the following two methods can be used.

Method 1 : Altering 'i' and 'n' in $F = P(1 + i)^n$

In this method, rate of interest 'i' is broken into as many interest periods for which compounding is to take place in a year. 'n' becomes the total number of interest periods. In this case,

No. of interest periods per year = 4

No. of interest periods for 10 years = $10 \times 4 = 40$

Revised number of periods (Quarters) $n = 40$

Rate of interest per quarter $i = 12/4 = 3\% = 0.03$

$$\therefore F = 10000(1 + 0.03)^{40}$$

$$F = \text{Rs. } 32620.37$$

Method 2 : Find effective rate of interest (Changing 'i' to 'R')

In this, only 'i' is changed to find effective rate of interest to suit the interest period which is less than a year. In order to do so, an effective rate of interest (R) is found using the formula

$$R = (1 + i/c)^c - 1$$

where i = Annual rate of interest (given)

c = no. of interest periods / year

$$\therefore R = (1 + 0.12 / 4)^4 - 1$$

$$R = 0.1255 \text{ or } 12.55\%$$

Here 'R' replaces 'i' in the formula $F = P(1+i)^n$

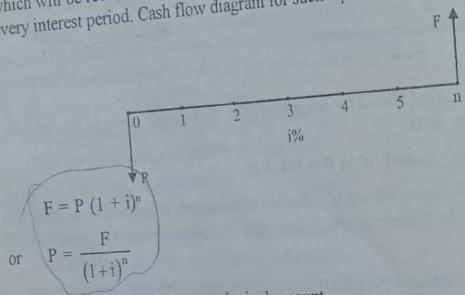
$$\therefore F = P(1+R)^n$$

$$F = 10000(1 + 0.1255)^{10}$$

$$F = \text{Rs. } 32,620/-$$

2. SINGLE-PAYMENT PRESENT WORTH AMOUNT, F (P/F, I, N)

Here, the objective is to find the present worth amount (P) of a single future sum (F) which will be received after n periods at an interest rate of i compounded at the end of every interest period. Cash flow diagram for such a problem is as shown below.



Where P = present amount or principal amount

F = Future sum

i = rate of interest (Annual)

n = number of interest periods (i.e., Years)

WORKED EXAMPLES

- A person wishes to have a future sum of Rs. 20 lakhs for his son's engineering education 10 years from now. What is the single payment that he should deposit now so that he gets the desired amount after 10 years? The bank gives 12% rate of interest compounded annually.

Solution:

Given : $F = 2000000/-$

$i = 12\%$ (Compounded annually)

$n = 10$ years

$$P = ?$$

$$P = \frac{F}{(1+i)^n}$$

$$P = \frac{20,00,000}{(1+0.12)^{10}}$$

$$P = \text{Rs. } 6,43,946/-$$

Therefore a sum of Rs. 6,43,946/- has to be deposited now to realize Rs. 20 lakhs ten years from now at a rate of interest of 12% compounded annually.

- If the same person above has the choice of investing in a private bank which pays a rate of interest of 11% but compounded quarterly, should he go for it? His desire is to receive 20 Lakhs 10 years from now.

Solution:

Given : $F = 2000000$

$i = 11\%$ (Compounded quarterly)

$n = \text{no. of interest periods}$

No. of interest periods per year = 4

Total no. of interest periods for 10 years = $4 \times 10 = 40$

Interest rate for one year = $11\% = 0.1$

Interest rate quarterly = $0.11 / 4$

$$\therefore P = \frac{F}{(1+i)^n} = \frac{2000000}{(1+0.11/4)^{40}}$$

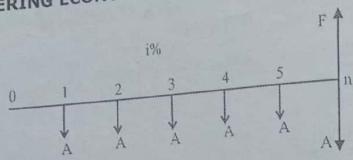
$$P = \text{Rs. } 6,75,704/-$$

This is the amount he has to invest in the private bank at rate of interest 11% compounded quarterly in order to realize Rs. 20 lakhs after 10 years.

Clearly the earlier option is better, so the person should not go to the private bank.

3. EQUAL PAYMENT SERIES COMPOUND AMOUNT, A (F/A, I, N)

In this, the objective is to find the future compound amount for n equal payments which are made at the end of every interest period till the end of n^{th} interest period at an interest rate i compounded at the end of each interest period. The corresponding cash flow diagram is shown in figure.



The above diagram means an amount A is deposited at the end of each interest period 1, 2, 3, ... n . F is the future compound amount realized for ' n ' installments of ' A '.

If A = Equal amount deposited at the end of each interest period.

n = number of interest periods

i = rate of interest

F = Future single compound amount.

Formula to get F would be

$$F = A \left[\frac{(1+i)^n - 1}{i} \right]$$

$$100 \left[\frac{(1+0.05)^{11} - 1}{0.05} \right]$$

Please note that the above formula is derived assuming that the first payment A starts from the end of first year. The derivation is shown at the end of this chapter.

WORKED EXAMPLES

1. A 45 year old person is planning for his retired life. He plans to divert Rs. 30,000/- from his bonus as investment every year for the next 15 years. The bank gives 12% interest rate compounded annually. Find the maturity value of his account when he is 60 years old.

Solution:

Given: A = Rs. 30,000

n = 15 years (compounded annually)

i = 20%

F = ?

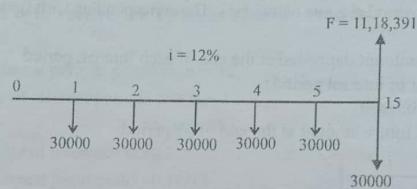
$$\therefore F = A \left[\frac{(1+i)^n - 1}{i} \right]$$

$$F = 30000 \left[\frac{(1+0.12)^{15} - 1}{0.12} \right]$$

$$F = \text{Rs. } 11,18,391/-$$

The person will receive the above amount at the age of 60 years if he invests Rs. 30,000 every year for the next 15 years.

Cash flow diagram for the above solution would be



2. If the person in the above problem has a choice of investing Rs. 2500/- every month in a private chit fund which assures him a rate of interest 11% compounded monthly, should he go for it? He is planning to receive the compound amount 15 years from now like in previous case.

Solution:

$$\text{In the formula } F = A \left[\frac{(1+i)^n - 1}{i} \right]$$

$$\begin{aligned} i &= 11\% \\ &\text{and } n \text{ have to be altered} \\ &\text{Annual Rate of interest} = 11\% \\ &n = 12 \times 15 = 180 \end{aligned}$$

$$\begin{aligned} \text{Monthly rate of interest} &= 11/12 = 0.916\% \\ &= 0.00916 \end{aligned}$$

$$\text{Number of interest periods per year} = 12$$

$$\text{Number of interest periods for 15 years, } n = 12 \times 15$$

$$= 180$$

$$\therefore F = 2500 \left[\frac{(1+0.00916)^{180} - 1}{0.00916} \right]$$

$$F = \text{Rs. } 11,35,875$$

This is the amount the person receives if he saves Rs. 2500/- every month and invests at a rate of interest 11% compounded monthly for the next 15 years. The person may go for it.

24 ENGINEERING ECONOMICS

4. EQUAL PAYMENT SERIES SINKING FUND F (A / F, i, n)

In this, the objective is to find the equivalent amount (A) that should be deposited at the end of every interest period for n interest periods to realize a future sum (F) at the end of nth interest period at a rate of interest i. The corresponding cash flow diagram would be

If A = equal amount deposited at the end of each interest period

n = number of interest periods

i = rate of interest

F = Single future amount at the end of nth period

$$\therefore A = F \left[\frac{i}{(1+i)^n - 1} \right]$$

WORKED EXAMPLES

1. A person estimates an expenditure of Rs. 10,00,000 for his daughter's wedding about 8 years from now. He plans to deposit an equal amount at the end of every year for the next 8 years at a rate of interest of 11% compounded annually. Find the equivalent amount that must be deposited at the end of every year for the next 8 years.

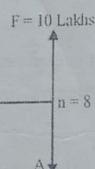
Solution:

Given : F = 10,00,000/-

A = ?

i = 11% (compounded annually)

n = 8 years



$$A = F \left[\frac{i}{(1+i)^n - 1} \right]$$

$$A = 1000000 \left[\frac{0.11}{(1+0.11)^8 - 1} \right]$$

A = Rs. 84,321/-

INTEREST AND INTEREST FACTOR 25

This is amount to be paid at the end of each year to realize Rs. 10 Lakhs at the end of eight years.

2. If the same person has the choice of going to a private bank which assures him 10% rate of interest compounded monthly, should he go for it? His intention is to receive Rs. 10 lakhs after 8 years.

Solution:

No. of interest periods per year, n = 12

Total No. of interest periods n = 8 × 12

n = 96

Annual Rate of interest = 0.10

Rate of interest per month = 0.10/12

$$\therefore A = 1000000 \left[\frac{0.10/12}{(1+0.10/12)^{96} - 1} \right]$$

A = Rs. 6,840/- per month.

This is amount he has to pay per month. At this rate he would pay $6840 \times 12 = 82,089$ per year to realize Rs. 10 Lakhs after 8 years. So he might go for it.

5. EQUAL PAYMENT SERIES PRESENT WORTH AMOUNT, A (P/A, i, n)

In this, the objective is to find the present worth of equal payments which are to be made at the end of every interest period for n terms at an interest rate of i compounded at the end of every interest period. The corresponding cash flow diagram is shown in figure.

If P = Present worth

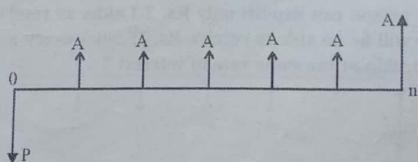
A = Annual equivalent payment

i = interest rate

n = number of interest periods

The formula to find P would be

$$\therefore P = A \left[\frac{(1+i)^n - 1}{i(1+i)^n} \right]$$



WORKED EXAMPLES

1. A person wants to give scholarships to poor students to the tune of Rs. 25,000/- every year, in memory of his late father. He wants to deposit a lumpsum in the bank which makes him receive the required amount every year for the next 20 years. The reserve is assumed to grow annually at the rate of 9%. Find the single payment that must be made now as the reserve amount?

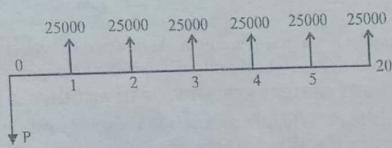
Solution:

Given : $A = 25,000/-$

$i = 9\%$

$n = 20$ years

$P = ?$



$$P = A \left[\frac{(1+i)^n - 1}{i(1+i)^n} \right]$$

$$P = 25,000 \left[\frac{(1+0.09)^{20} - 1}{0.09(1+0.09)^{20}} \right]$$

$$P = \text{Rs. } 2,28,213$$

This is the amount the person has to keep as reserve in order to receive interest of Rs. 25,000/- every year for the next 20 years.

2. If the same person can deposit only Rs. 2 Lakhs as reserve now for how many years will he be able to receive Rs. 25,000/- every year so as to give away scholarship at the same rate of interest?

Solution:

Given : $A = 2,00,000/-$

$i = 9\%$

$n = 20$ years

$$P = ?$$

$$\therefore P = A \left[\frac{(1+i)^n - 1}{i(1+i)^n} \right]$$

$$200000 = 25000 \left[\frac{(1+0.09)^n - 1}{0.09(1+0.09)^n} \right]$$

$$18000 (1 + 0.09)^n = 25000 (1 + 0.09)^n - 25000$$

$$25000 = 25000 (1 + 0.09)^n - 18000 (1 + 0.09)^n$$

$$25000 = 7000 (1 + 0.09)^n$$

$$(1 + 0.09)^n = \frac{25000}{7000} = 3.571$$

Taking log on both sides

$$n \log (1 + 0.09) = \log 3.571$$

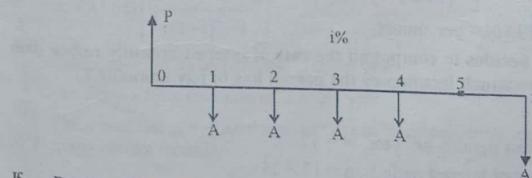
$$n = \frac{0.5528}{0.0374}$$

$$n = 14.77 \text{ Years}$$

The person will be able to give scholarships of Rs. 25,000/- every year for the next 14.77 years only if he sets up a reserve of 2 lakhs.

6. EQUAL PAYMENT SERIES CAPITAL RECOVERY AMOUNT, P (A / P, i, n) OR PERSONAL LOANS OR EMI PAYMENT CALCULATIONS

In this, the objective is to find the annual equivalent amount (A) which is to be recovered at the end of every interest period for n terms for a loan amount (P) which is sanctioned now at a rate of interest of i compounded at the end of every interest period.



$$\text{If } P = \text{Rs. } 30,00,000/- \\ A = ?$$

i = rate of interest
 n = number of interest periods.

$$A = P \left[\frac{i(1+i)^n}{(1+i)^n - 1} \right]$$

WORKED EXAMPLES

1. ICICI bank is offering a Rs. 30 Lakhs home loan to a person to buy a new apartment at a interest rate of 7.5% compounded annually. This amount should be repaid in 15 yearly equal installments. Find the annual instalment amount the person has to pay to the bank.

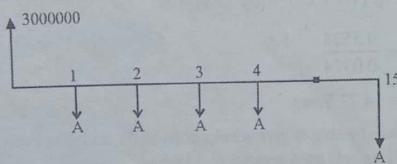
Solution:

Given : $P = \text{Rs. } 30,00,000/-$

$A = ?$

$i = 7.5\%$ (Compounded annually)

$n = 15$ years



$$A = P \left[\frac{i(1+i)^n}{(1+i)^n - 1} \right]$$

$$A = 30,00,000/- \left[\frac{0.075(1+0.075)^{15}}{(1+0.075)^{15} - 1} \right]$$

$A = \text{Rs. } 3,39,861/- \text{ per annum.}$

2. If the bank decides to compound the rate of interest monthly rather than annually, how much less money the person has to pay annually?

Solution:

No. of interest periods per year, $N = 12$

Total number of interest periods, $n = 12 \times 15$

$= 180$

Annual Rate of interest = 7.5%

\therefore Rate of interest per month = $0.075 / 12$

$$A = 30,00,000 \left[\frac{0.075/12(1+0.075/12)^{180}}{(1+0.075/12)^{180} - 1} \right]$$

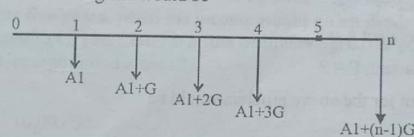
$A = \text{Rs. } 27810.37$

\therefore He has to pay $(339861 - 27810 \times 12) = \text{Rs. } 6141$ less per year.

Note : Please note here that on the EMI that he pays, if interest is calculated every month, he ends up paying more than Rs. 3,39,861 per annum

7. UNIFORM GRADIENT SERIES ANNUAL EQUIVALENT AMOUNT

In this, the objective is to find the annual equivalent amount of a series of payments with an amount A_1 at the end of first year and with an equal increment (G) at the end of each of the following $n - 1$ years, with an interest rate i compounded annually. The corresponding cash flow diagram would be



If, A = Annual equivalent amount of a series of increasing or decreasing installments

A_1 = First instalment

G = Increment for successive installments

i = rate of interest (Compound annually)

n = number of interest periods

The formula to compute 'A' is

$$A = A_1 \pm G \left[\frac{(1+i)^n - in - 1}{i(1+i)^n - i} \right]$$

+ve sign \rightarrow For an incremental G every year

-ve sign \rightarrow for a decrease in G every year.

The annualized or averaged amount 'A' can be subsequently used to find either 'P' or 'F' using relevant formula.

WORKED EXAMPLES

1. A person wants to gift a car to his daughter when she would turn 18 years, six years from now. He decides to put away money in her name during her next six birthdays. He wants to deposit Rs. 25,000/- in the first year and go on increasing it by Rs. 5,000/- every year for the next 6 years. If he estimates that a car would cost Rs. 5 Lakhs when he wants to buy one, how much more money should be added to the maturity amount that he receives from the bank, if it is assumed to grow at 11.5% compounded annually?

Solution:

Given: $A_1 = \text{Rs. } 25,000/-$

$$G = \text{Rs. } 5,000/-$$

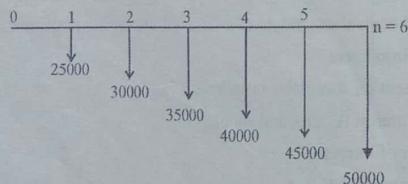
$$i = 11.5\%$$

$$n = 6 \text{ years}$$

$$\text{Annual equivalent } A = ?$$

$$\text{Maturity amount, } F = ?$$

Cash flow diagram for the above problem would be



$$\text{Annual equivalent amount } A = A_1 + G \left[\frac{(1+i)^n - i - 1}{i(1+i)^n - i} \right]$$

$$A = 25000 + 5000 \left[\frac{(1+0.115)^6 - (0.115 \times 6) - 1}{0.115(1+0.115)^6 - 0.115} \right]$$

$$A = 25000 + 5000 (2.184)$$

$$A = 35924$$

$$F = A \left[\frac{(1+i)^n - 1}{i} \right]$$

$$F = 35924 \left[\frac{(1+0.115)^6 - 1}{0.115} \right]$$

$$F = \text{Rs. } 2,87,872/-$$

The maturity value of the incremental installments of the person after 6 years would be Rs. 2,87,872. In order to buy a car worth Rs. 5 Lakhs he has to additionally put in Rs. 5,00,000 - Rs. 2,87,872 = Rs. 2,12,127/-

Note : Strictly speaking, the person pays Rs. 50,000/- at the end of sixth year and immediately receives the maturity amount, that is assuming that he invests from the beginning of the next year. This is because of the nature of the formula employed.

2. A film star is at the height of his career. He wants to invest Rs. 10 Lakhs from the end of this year and follow it up with 9 Lakhs, 8 Lakhs and so on for the next five years, when his income would go on diminishing. Find the maturity amount 6 years later if a film producer agrees to pay him 15% rate of interest, compounded annually.

Solution:

Given: $A_1 = \text{Rs. } 10,00,000/-$

$$G = \text{Rs. } 1,00,000/-$$

$$i = 15\%$$

$$n = 6 \text{ years}$$

$$A = ?$$

$$F = ?$$

$$A = A_1 - G \left[\frac{(1+i)^n - in - 1}{i(1+i)^n - i} \right]$$

Note : -ve sign is used because of diminishing installments

$$A = 1000000 - 100000 \left[\frac{(1+0.15)^6 - (0.15 \times 6) - 1}{0.15(1+0.15)^6 - 0.15} \right]$$

$$A = 1000000 - 100000 [2.097]$$

$$A = \text{Rs. } 7,90,280$$

This is the annual equivalent amount for the series of payments spread over six years.

$$\text{Maturity amount, } F = A \left[\frac{(1+i)^n - 1}{i} \right]$$

$$F = 790280 \left[\frac{(1+0.15)^5 - 1}{0.15} \right]$$

F = Rs.53,28,369.

8. GEOMETRIC SERIES PRESENT AMOUNT

In this, the objective is to find the Present Value of a series of payments or incomes, with an amount A at the end of first year, and an annual increment of g% over the previous year's value.

- If, A_1 = First year cost or revenue
- g = rate of increase every year
- i = rate of interest
- n = number of years
- P_g = Present Value

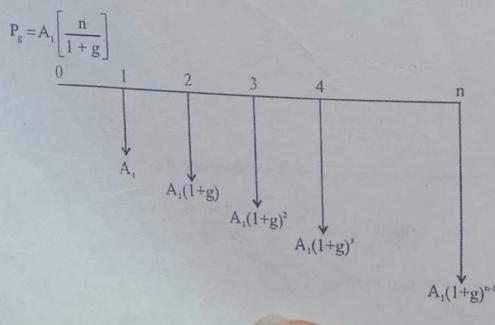
For such a geometric series, the Present Amount is calculated for two cases i.e., when $g \neq i$ and when $g = i$

Case 1: When $g \neq i$

$$P_g = \frac{A_1}{(1+g)} \left[\frac{(1+g^1)^n - 1}{g^1(1+g^1)^n} \right] \text{ Where } g^1 = \frac{1+i}{1+g} - 1$$

Case 2:

When $g = i$



WORKED EXAMPLE

If the annual maintenance and operating costs of a dam are expected to be Rs.4,00,000 for the first year and increasing at a rate of 10% per year, determine the Present Value of all operating costs spent over a 30 year life. Assume rate of interest as 12%. If the rate of increase per year would be 12% instead of 10%, what difference would it make to the answer?

Solution:

Given,

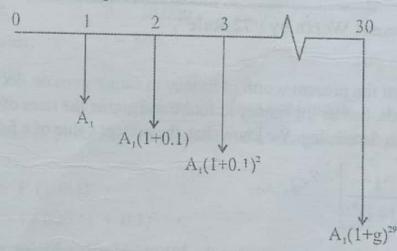
$$A_1 = 4,00,000$$

$$g = 10\%$$

$$i = 12\%$$

$$n = 30 \text{ years}$$

The CFD for the above problem would be



$$\text{We have, } P_g = \frac{A_1}{1+g} \left[\frac{(1+g^1)^n - 1}{g^1(1+g^1)^n} \right]$$

$$\text{When } g^1 = \frac{1+i}{1+g} - 1$$

$$g^1 = \frac{1+0.12}{1+0.1} - 1$$

$$g^1 = 0.01818$$

$$\therefore P_g = \frac{4,00,000}{(1+0.1)} \left[\frac{(1+0.01818)^{30} - 1}{0.01818(1+0.01818)^{30}} \right]$$

$$\therefore P_g = \text{Rs.} 83,51,505$$

This is the present amount for the geometric series.

If $i = g$, then

$$\text{we have } P_g = A_1 \left[\frac{n}{1+g} \right]$$

$$= 4,00,000 \left[\frac{30}{1+0.12} \right]$$

$$\therefore P_g = \text{Rs.} 107,14,285.$$

SOME MORE WORKED PROBLEMS

- Explain Present Worth by "72 Rule".

Solution:

We know that the present worth of money in future goes on decreasing with time. In other words, further the money in future and higher the rates of interest, its present value goes on decreasing. We know that, the present value of a future sum F is given

$$\text{by } P = F \left[\frac{1}{(1+i)^n} \right]$$

From the formula, we can see that the PW decreases with increase in i and n. But the pattern of decrease in the value of PW for all future sums is shown by '72 Rule' which is a thumb-rule. The '72-Rule' indicates the approximate number of years n^* at which the PW is half of a future sum at an annually compounded rate of interest i.

$$\therefore 72 \text{ Rule : } n^* = \frac{72}{(i)(100)}$$

$$\begin{aligned} \text{Here, } n^* &= 14 \text{ years when } i = 5\% & \dots & (1) \\ n^* &= 7 \text{ years when } i = 10\% & \dots & (2) \\ n^* &= 4 \text{ years when } i = 20\% & \dots & (3) \end{aligned}$$

The first value means that the PW of a future sum of say Rs. 1,000 considered 14 years from now, is now Rs.500/- at a low rate of interest of 5%.

The second value means that the PW of a future sum of say Rs. 1,000 considered 7 years from now, is now Rs.500/- at a rate of interest of 10%.

The third value means that the PW of a future sum of say Rs. 1,000 considered 4 years from now, is now Rs. 500 at a high rate of interest of 20%.

- A person takes a loan of Rs 10,000 from a Bank at interest of 10% P.A. Find the amount if

- (i) Interest is compounded annually.
- (ii) Interest is compounded half yearly.
- (iii) Interest is compounded quarterly.
- (iv) Interest is compounded monthly.

(VTU June 2008)

Solution:

Given,

$$\text{loan amount, } P = 10,000$$

$$\text{Rate of interest, } i = 10\%$$

$$\text{Future amount, } F = ?$$

$$\text{No.of years, } n = 1 \text{ (assumed, since not given)}$$

- When interest is compounded annually, the compound amount at the end of one year is

$$\begin{aligned} F &= P (1 + i)^n \\ &= 10,000 (1 + 0.1)^1 \end{aligned}$$

$$\text{Future Amount, } F = \text{Rs.} 11,000$$

- When interest is compounded half yearly, the compound amount at the end of one year is

$$\begin{aligned} F &= P (1 + i)^n \\ &= P \left(1 + \frac{i}{2}\right)^{n \times 2} \\ &= 10,000 \left(1 + \frac{0.1}{2}\right)^2 \\ &= \text{Rs.} 11,025 \end{aligned}$$

36 ENGINEERING ECONOMICS

(iii) When interest is compounded quarterly, the compound amount at the end of 1 year is

$$F = P \left(1 + \frac{i}{4}\right)^{n \times 4}$$

$$F = 10,000 \left(1 + \frac{0.1}{4}\right)^4$$

$$F = \text{Rs.}11,038$$

(iv) When interest is compounded monthly, the compound amount at the end of the 1 year is,

$$F = P \left(1 + \frac{i}{12}\right)^{n \times 12}$$

$$= 10,000 \left(1 + \frac{0.1}{12}\right)^{12}$$

$$F = \text{Rs.}11,047$$

3. An utility is purchased for an account of Rs. 17,500/- and it is expected to serve for four years. If the Interest rates are 6% for simple interest case and 10% for compound interest case. Calculate the interest earned in both the cases and offer your comments. (VTU Jan 2008)

Solution:

Given,

Principal $P = \text{Rs. } 17,500$

Period $A = 4$ years

Simple interest $i_s = 6\%$

Compound interest $i_c = 10\%$

Interest earned through Simple Interest

$$S.I. = Pni$$

$$= 17,500 \times 4 \times 0.06$$

$$S.I. = \text{Rs.}4,200.$$

Interest earned through Compound Interest

$$F = P(1+i)^n$$

$$= 17,500 (1+0.1)^4$$

$$F = \text{Rs.}25,621.75$$

INTEREST AND INTEREST FACTOR 37

F represents the Compound amount i.e., Principal + Interest

$$\therefore \text{Compound Interest} = F - P$$

$$= 25,621.75 - 17,500$$

$$C.I. = \text{Rs.}8,121.75$$

4. Determine the effective interest rate in the following cases

(i) Nominal rate of 12% compounded monthly with time interval of one year.

(ii) Nominal rate of 18% compounded weekly with a time interval of one year.

(iii) Nominal rate of 13% compounded monthly with time interval of two years.

(iv) Nominal rate of 9% compounded semiannually with time interval of two years.

(VTU, July 2007)

Solution:

$$(i) i_{\text{eff}} = \left(1 + \frac{i}{c}\right)^c - 1 \quad \text{Where } i = \text{annual rate of interest}$$

$c = \text{no. of interest periods/year}$

$$= \left(1 + \frac{0.12}{12}\right)^{12} - 1$$

$$i_{\text{eff}} = 12.68\%$$

$$(ii) i_{\text{eff}} = \left(1 + \frac{i}{c}\right)^c - 1$$

$$= \left(1 + \frac{0.18}{52}\right)^{52} - 1$$

$$i_{\text{eff}} = 19.68\%$$

$$(iii) i_{\text{eff}} = \left(1 + \frac{0.13}{12}\right)^{12} - 1$$

$$i_{\text{eff}} = 13.80\%$$

(Please note that i_{eff} is always found on the Compounding interest period and not on trial duration)

\therefore Time interval of 2 years given is redundant information

$$(iv) i_{\text{eff}} = \left(1 + \frac{0.09}{2}\right)^2 - 1$$

$i_{\text{eff}} = 9.20\%$

5. A loan of Rs. 5,000 is to be repaid in equal monthly installments over $2\frac{1}{2}$ years. The nominal interest rate is 6 percent. How much is each installment? (VTU July 2006)

Solution:

Given : $P = \text{Rs. } 5,000$

$$\begin{aligned} n &= 2\frac{1}{2} \text{ years} = 30 \text{ months} \\ i &= 6\% \end{aligned}$$

$$\therefore \text{Monthly instalment, } A = P \left[\frac{i(1+i)^n}{(1+i)^n - 1} \right] = 5,000 \left[\frac{0.06 \left(1 + \frac{0.06}{12}\right)^{30}}{\left(1 + \frac{0.06}{12}\right)^{30} - 1} \right]$$

$A = \text{Rs. } 179.89 \text{ Per month}$

6. An amount of Rs. 1,200 per year is to be paid into an account each of the next five years. Using a nominal interest of 12% determine the total amount the account will have at the end of 5th year under the following conditions.

- (i) Deposit made at the end of each year with interest compounded monthly.
- (ii) Deposit made at the end of each year with interest compounded continuously. (VTU MQP)

$$\text{Solution: (i)} i_{\text{eff}} = \left(1 + \frac{i}{c}\right)^c - 1 = \left(1 + \frac{0.12}{12}\right)^{12} - 1 = 12.68\%$$

$$F = A \left[\frac{(1+i_{\text{eff}})^n - 1}{i_{\text{eff}}} \right] = 1200 \left[\frac{(1+0.1268)^5 - 1}{0.1268} \right] = \text{Rs. } 7727.08/-.$$

(ii) Continuous Compounding

Explanation: In calculating compound interest, compounding is done annually or quarterly or monthly and so on. But the ultimate limit for the number of compounding periods in 1 year is called continuous compounding. Under this Accrual pattern, m (number of interest periods in a year) approaches infinity as interest compounds continuously, moment by

INTEREST AND INTEREST FACTOR 39

moment. The effective rate of interest (i_{∞}) for continuous compounding for a nominal rate of interest i is found as follows:

When the interest rate is made infinitesimally small, applying the effective rate of interest formula $R = \left(1 + \frac{i}{c}\right)^c - 1$

$$\text{RHS is rearranged to include } i \text{ in the exponent } i_{\infty} = \lim_{c \rightarrow \infty} \left[\left(1 + \frac{i}{c}\right)^c \right] - 1$$

$$\therefore i_{\infty} = e^i - 1 \rightarrow \text{because by definition } \lim_{c \rightarrow \infty} \left[\left(1 + \frac{i}{c}\right)^{\frac{c}{i}} \right] = e$$

In the given problem

$$A = 1,200$$

$i = 12\%$ (Compounded annually)

$$n = 5 \text{ years.}$$

for continuous compounding

$$i_{\infty} = e^i - 1$$

$$i_{\infty} = e^{0.12} - 1$$

$$i_{\infty} = 0.1275$$

$$\therefore F = A \left[\frac{(1+i_{\infty})^5 - 1}{i_{\infty}} \right]$$

$$F = 1,200 \left[\frac{(1+0.1275)^5 - 1}{0.1275} \right]$$

$F = \text{Rs. } 7,737 \rightarrow$ This is the compounding amount.

7. A person borrows Rs. 1,000 on loan at a simple interest rate of 10% per annum for 8 years. What will be the compound interest rate if he has to pay the same amount after 5 years? (VTU MQP)

Solution:

Given:

$$P = \text{Rs. } 1,000, \quad i = 10\%, \quad n = 8 \text{ years}$$

$$S.I = P \times n \times i$$

$$= 1,000 \times 8 \times 0.1 = 800$$

$$\begin{aligned}
 F &= P + 800 \\
 &= 1,800 \\
 C.I., F &= P \times (1+i)^n \\
 1,800 &= 1,000 (1+i)^5 \\
 (1+i) &= \left(\frac{1,800}{1,000} \right)^{\frac{1}{5}} \\
 i &= 12.47\%
 \end{aligned}$$

8. The rights to a patent have been sold under an agreement in which annual year end payments of Rs. 100,000 are to be made for the next 10 years. What is the future sum of this annuity? What is the present worth of the annuity at an interest rate of 7 percent? (VTU July 2005)

Solution:

Given,
 $A = 100,000$ $F = ?$
 $n = 10$ years $P = ?$
 $i = 7\%$

This is equal payment series compound amount ($F/A, i, n$)

$$\begin{aligned}
 \text{Future sum } F &= A \left[\frac{(1+i)^n - 1}{i} \right] \\
 \therefore F &= A [F/A, i, n] \\
 &= A [F/A, 7\%, 10] \\
 F &= 100,000 \times 13.81645
 \end{aligned}$$

$$\therefore F = \text{Rs.} 13,81,645$$

$$\begin{aligned}
 \text{Present Worth, } P &= F \left[\frac{1}{(1+i)^n} \right] \quad \text{or} \quad P = A \left[\frac{(1+i)^n - 1}{i(1+i)^n} \right] \\
 P &= F (P/F, i\%, n) \quad \text{or} \quad A (P/A, i\%, n) \\
 P &= F (P/F, 7\%, 10) \quad \text{or} \quad A (P/A, 7\%, 10) \\
 P &= 13,81,645 \times 0.50835 \quad \text{or} \quad 1,00,000 \times 7.0236 \\
 P &= \text{Rs.} 7,02,360
 \end{aligned}$$

9. How much interest is earned on a principal of Rs.750 for 5 years 9 months at 6 percent compounded monthly? (VTU January 2006)

Solution:

Given,
Principle, $P = \text{Rs.} 750$
Duration, $n = 5$ years 9 months
 $= 69$ months
Nominal Rate of interest, $i = 6\%$
Compound Interest, $C.I. = ?$

$$\text{Compound Interest earned} = \text{Compound amount} - \text{Principle}$$

$$\begin{aligned}
 &= F - P \\
 &= P (1+i)^n - P \\
 &= 750 \left(1 + \frac{0.06}{12} \right)^{69} - 750 \\
 C.I. \text{ earned} &= \text{Rs.} 308.08
 \end{aligned}$$

10. Find the effective rate of interest for an actual rate of interest of 10% when compounded -

- (i) Yearly - 1
- (ii) Biannually - 2
- (iii) Quarterly - 4
- (iv) Monthly - 12
- (v) Daily - 365
- (vi) Hourly - 365¹²
- (vii) Continuously (infinite Compounding)

Solution:

We have, $i_{\text{eff}} = \left(1 + \frac{i}{c} \right)^c - 1$
where $i = \text{annual rate of interest}$
 $c = \text{no. of interest periods per year}$

(i) Yearly, $i_{\text{eff}} = \left(1 + \frac{0.1}{1} \right)^1 - 1$
 $i_{\text{eff}} = 10\%$

(ii) Bi-annually, $i_{\text{eff}} = \left(1 + \frac{0.1}{2}\right)^2 - 1 = 10.25\%$

(iii) Quarterly, $i_{\text{eff}} = \left(1 + \frac{0.1}{4}\right)^4 - 1 = 10.38\%$

(iv) Monthly, $i_{\text{eff}} = \left(1 + \frac{0.1}{12}\right)^{12} - 1 = 10.47\%$

(v) Daily, $i_{\text{eff}} = \left(1 + \frac{0.1}{365}\right)^{365} - 1 = 10.5155\%$

(vi) Hourly $i_{\text{eff}} = \left(1 + \frac{0.1}{365 \times 24}\right)^{365 \times 24} - 1 = 10.51703\%$

(vii) Infinitely $i_{\infty} = e^i - 1 = 10.51709$

Note: Students are directed to observe the fact that there is only a half percent difference between yearly compounding and infinite compounding. For small principal amounts, this may not make a big difference but for large principal amounts this may become a huge difference.

11. Derive the following Compound Interest formulae from first principles with usual notation.

(i) Single Payment Compound amount i.e., $F = P(1+i)^n$

(ii) Equal Payment Series Compound amount, $F = A \left[\frac{(1+i)^n - 1}{i} \right]$

(iii) Equal Payment Series Present worth amount, $P = A \left[\frac{1 - (1+i)^{-n}}{i} \right]$

Solution:

(i) To derive $F = P(1+i)^n$

Let P = Principal amount

i = rate of interest

n = no. of years

F_1 = Compound amount in 1st year

F_2 = Compound amount in 2nd year

F_n = Compound amount in n^{th} year

Wkt Compound Amount = Principal + Principal \times Rate of interest

$$F_1 = P + Pi$$

$$F_1 = P(1+i)$$

$$F_2 = F_1 + F_1 i$$

$$F_2 = F_1(1+i)$$

$$F_2 = P(1+i)(1+i)$$

$$F_2 = P(1+i)^2$$

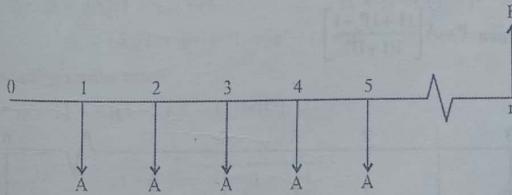
Similarly,

$$F_3 = F_2(1+i)$$

$$F_3 = P(1+i)^3$$

$$\therefore F_n = P(1+i)^n$$

(ii) To prove $F = A \left[\frac{(1+i)^n - 1}{i} \right]$



In the above CFD, let A = annual equal payment

n = no. of years

i = Rate of interest

F_1 = Compound amount of payment A in 2nd year

F_2 = Compound amount of payment A in 3rd year

F = Compound amount for all annual payment

We have, $F_1 = A(1+i)^{n-1}$ [Using formula $F = P(1+i)^n$]

$$F_2 = A(1+i)^{n-2}$$

$$F_3 = A(1+i)^{n-3}$$

$$\therefore F_n = A(1+i)^{n-n}$$

$$F_n = A$$

$$\therefore F_1 + F_2 + F_3 + \dots + F_n = A(1+i)^{n-1} + A(1+i)^{n-2} + A(1+i)^{n-3} + \dots + A \\ F = A[(1+i)^{n-1} + (1+i)^{n-2} + (1+i)^{n-3} + \dots + 1] \quad \dots \dots \dots (1)$$

Multiplying throughout by $(1+i)$

$$F(1+i) = A[(1+i)^n + (1+i)^{n-1} + (1+i)^{n-2} + \dots + (1+i)] \quad \dots \dots \dots (2)$$

Subtracting (1) from (2), i.e., (2) - (1)

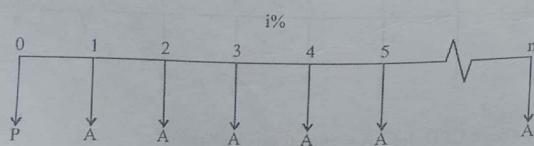
$$F(1+i) - F = \{A[(1+i)^n + (1+i)^{n-1} + (1+i)^{n-2} + \dots + (1+i)]\} - \\ \{A[(1+i)^{n-1} + (1+i)^{n-2} + (1+i)^{n-3} + \dots + 1]\}$$

$$\therefore F(1+i-1) = A(1+i)^n - 1$$

or

$$F = A \left[\frac{(1+i)^n - 1}{i} \right]$$

$$(iii) \text{ To prove } P = A \left[\frac{(1+i)^n - 1}{i(1+i)^n} \right]$$



Let, A = Equal annual payment

i = Rate of interest

n = No. of years

P_1 = PW of payment A in 1st year

P_2 = PW of payment A in 2nd year

P_n = PW of payment A in nth year

P = PW of all payments A for n years

From CFD

$$P_1 = \frac{A}{(1+i)^1} \quad \left[\text{From formula } F = P(1+i)^n \right]$$

$$P_1 = A(1+i)^{-1}$$

$$P_2 = \frac{A}{(1+i)^2} = A(1+i)^{-2}$$

$$P_n = A(1+i)^{-n}$$

$$\therefore P_1 + P_2 + P_3 + \dots + P_n = A(1+i)^{-1} + A(1+i)^{-2} + A(1+i)^{-3} + \dots + A(1+i)^{-n}$$

$$\therefore P = A[(1+i)^{-1} + (1+i)^{-2} + (1+i)^{-3} + \dots + (1+i)^{-n}] \quad \dots \dots \dots (1)$$

Multiplying equation (1) by $(1+i)^{-1}$ through out,

$$P(1+i)^{-1} = A[(1+i)^2 + (1+i)^3 + \dots + (1+i)^n + (1+i)^{-n+1}] \quad \dots \dots \dots (2)$$

Subtracting eq. (1) by eq. (2) i.e., (2) - (1)

Expanding and cancelling

$$P(1+i)^{-1} - P = \{A[(1+i)^2 + (1+i)^3 + \dots + (1+i)^n + (1+i)^{-n+1}]\} - \{A[(1+i)^1 + (1+i)^2 + (1+i)^3 + \dots + (1+i)^n]\}$$

By cancelling similar terms

$$P[(1+i)^{-1} - 1] = A[(1+i)^{-n+1} - (1+i)^{-1}]$$

$$\textcircled{2} \text{ But } (1+i)^{-1} - 1 = \frac{1}{(1+i)} - 1 = \frac{1-(1+i)}{(1+i)} = \frac{-i}{1+i}$$

$$\therefore P \left[\frac{-i}{1+i} \right] = A[(1+i)^{-n+1} - (1+i)^{-1}] \quad \dots \dots \dots (3)$$

Multiplying eq. (3) through out by $-(1+i)$

$$\therefore P[i] = A[-(1+i)(1+i)^{-n+1} + (1+i)(1+i)^{-1}]$$

$$P = \frac{A}{i} \left[- (1+i)^{-n-1} + \frac{(1+i)}{(1+i)^n} \right]$$

$$P = \frac{A}{i} \left[- (1+i)^{-n} + 1 \right]$$

or

$$P = \frac{A}{i} \left[1 - \frac{1}{(1+i)^n} \right]$$

$$P = A \left[\frac{(1+i)^n - 1}{i(1+i)^n} \right]$$

1. SUMMARY

- Engineering Decision-Makers :** In any industry, engineers take decisions regarding design, procedures, products, plans and methods. Often, they have to choose among many alternatives. Questions asked to review business proposals are,
 - Why do this at all?
 - Why do it now?
 - Why do it this way?
- Problem Solving and Decision Making :** Both theoretical and practical knowledge is used to solve problems in engineering economy. In other words, it takes both the real world of facts and figures and the symbolic world of theories and hypothesis to solve problem, through an iterative process.
- Intuition and Analysis :** Intuition is nothing but an immediate perception of mind. Reasoning based on rationality & scientific principles in Analysis. Both are needed for solving problems on engineering economy.
- Strategy and Tactics :** Strategy refers to broad plans that are set and major objectives that are to be achieved whereas tactics refers to the manipulation of the resources to attain the same objectives.
- Engineering Economic Decision Maze :** This is the inter-relationship of all economic criteria the study of which help arriving at solution. Refer page no.6.
- Law of Demand :** "The demand for a commodity increases, when its prices decreases and vice-versa, all other things remaining constant". Thus, demand is inversely proportional to its price.
- Law of Supply :** "The supply for a commodity increases, when its prices increases and vice-versa, all other things remaining constant". Thus, supply is directly proportional to its price.
- Elasticity of Demand(e_p) :** It is the responsiveness of demand of a product to changes in its prices.

$$e_p = \frac{\text{Percentage change in demand}}{\text{Percentage change in price}}$$

When $e_p > 1$, the demand is elastic

$e_p < 1$, the demand is inelastic

$e_p = 1$, neither elastic nor inelastic or demand with unit elasticity.

- Supply Elasticity :** It is the responsiveness of supply of a product to changes in its price.

$$e_p = \frac{\text{Percentage change in supply}}{\text{Percentage change in price}}$$

48 ENGINEERING ECONOMICS

10. Law of Returns : In any business, when there is an increase in the inputs,
- Total output may increase more than proportionately which is called the Law of Increasing Returns.
 - Total output may increase just proportionately which is called the Law of Constant Returns.
 - Total output may increase less than proportionately, which is called the Law of Diminishing Returns.

11. Simple Interest

When a constant interest amount is earned every year for a principle amount, it is said to be simple interest.

Interest earned or paid, $I = Pni$.

12. Compound Interest

When the interest amount is not collected for a deposit of principle amount, interest amount for future periods is calculated on principle amount plus interest of previous period. This is known as compound interest.

13. Cash Flow Diagram

A CFD is a pictorial representation of all revenues and expenditures incurred over the lifetime of a business project or for any plan period.

If P = Principle Amount / present worth

i = Rate of interest (compounded annually)

n = number of interest periods

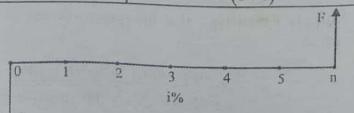
F = Future compound amount

A = annual amount deposited or recovered

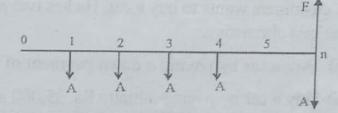
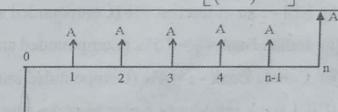
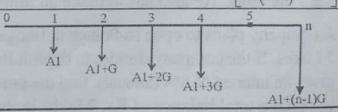
G = Increment for successive instalment

A_1 = First instalment

Formulae for all Standard Compound Interest calculations

Sl. No.	To find	Notation	Formula
1.	Single Future amount (F)	$F = P (F/P, i, n)$	$F = P (1 + i)^n$
2.	Single Present Amount (P)	$P = F (P/F, i, n)$	$P = \frac{F}{(1+i)^n}$
	Cash Flow diagram for 1 & 2		

INTEREST AND INTEREST FACTOR 49

Sl. No.	To find	Notation	Formula
3.	Equal Payment compound Amount (F)	$F = A (F/A, i, n)$	$F = A \left[\frac{(1+i)^n - 1}{i} \right]$
4.	Equal Payment Annual Amount (A)	$A = F (A/F, i, n)$	$A = F \left[\frac{i}{(1+i)^n - 1} \right]$
	Cash Flow diagram for 3 and 4		
5.	Equal Payment Present Amount (P)	$P = A (P/A, i, n)$	$P = A \left[\frac{1 - (1+i)^{-n}}{i(1+i)^n} \right]$
6.	Equal Payment Capital Recovery (A)	$A = P (A/P, i, n)$	$A = P \left[\frac{i(1+i)^n}{(1+i)^n - 1} \right]$
	Cash Flow diagram for 5 and 6		
7.	Uniform Gradient Series Annual equivalent amount (A)	$A = G (A/G, i, n)$	$A = A_1 + G \left[\frac{(1+i)^n - 1 - ni - 1}{i(1+i)^n - i} \right]$
	Cash Flow diagram for 7		
8.	Geometric Series Present Amount		<p>Case 1: When $g \neq i$</p> $P_g = \frac{A_1}{(1+g)} \left[\frac{(1+g^1)^n - 1}{g^1(1+g^1)^n} \right] \text{ Where } g^1 = \frac{1+i}{1+g} - 1$ <p>Case 2: When $g = i$</p> $P_g = A_1 \left[\frac{n}{1+g} \right]$

EXERCISE PROBLEMS

1. A business magnate wants to start a new business on textiles, and the expected money required for investment is Rs. 7 crores seven years from now. He can invest Rs. 70 lacs at the end of every year for the next 7 years. If he gets a rate of return of 12% on this investment, check whether the accumulated sum in the account would be sufficient to meet the fund for the textile business. If not, by how much does he fall short?
2. A customer wants to buy a car. He has two alternatives to choose from. Suggest the best alternative.
 - (i) Buy a car by paying a down payment of Rs. **12.6** lacs.
 - (ii) Buy a car by paying initially Rs. 35,000 and Rs. 20,000 annually for 10 years at an interest rate of 9%.
3. A person wants to save money for his daughter's marriage, which he plans to do 10 years from now. He initially makes a deposit of Rs. 30,000 in a bank. He plans to add Rs. 8,000 at the end of 1st year. He also wishes to deposit the same amount (Rs. 8,000) every year with an annual increase of Rs. 1000 for the next 9 years. What will be the total amount in his account at the end of 10 years if he deposits in,
 - (i) SBI - rate of interest 7% (Compounded annually)
 - (ii) Indian Bank - $i = 7.5\%$ (Compounded annually)
 - (iii) Canara Bank - $i = 4\%$ (Compounded annually)
4. ICICI Bank introduces a plan to pay a sum of 25 lacs after 15 years at a rate of 8% compounded annually. Find the annual equivalent amount that a person should invest at the end of every year for the next 10 years to receive 25 lacs after 15 years from the bank. He also has to make an initial investment of Rs. 20,000.
5. A company plans to open its branch in Bangalore. It estimates a requirements of Rs. 7 Lakhs. If the company decides to deposit Rs. 25,000 every month in a bank which gives an interest of 8% (annual), find the time period, the company needs to deposit to reach a total balance of Rs. 7 lakhs in its account if,
 - (i) Compounded monthly.
 - (ii) Compounded yearly.
6. A person takes a loan to build a house from the Home Loan Scheme. He takes a loan of Rs. 15 Lakhs. This amount is to be repaid in 144 monthly equal installments at 8% interest rate compounded monthly. Find the instalment amount to be paid by the person to clear up his loan.

7. A person who is 30 years old is planning for his retired life. He plans to invest an equal sum of Rs. 12,000 at the end of every year, till he is 60 years old. He has two options (i) to deposit in SBI giving an annual rate of interest of 6%, compounded monthly. (ii) to deposit in ICICI Bank giving an annual rate of interest of 7%, compounded annually. Suggest the best option.



PRESENT WORTH COMPARISONS

INTRODUCTION

The most important work of top management in any company is to take decisions. Many of these decision-making activities would involve the selection of best alternative from a set of competing alternatives. For example, if there are three designs of a particular car to be manufactured, with each car having different initial outlays and different expected annual revenues, which design to go for? The C.E.O may have to select the best alternative among the three projects. In order to do so, there are several bases for comparing the worthiness of the projects. In other words, the comparison among the alternatives could be made based on any of the following methods which make use of different economic criteria:

1. Present worth comparison
2. Future worth comparison
3. Equivalent annual worth comparison
4. Rate of return calculation.

We shall be studying the first two methods in this chapter and the remaining two in subsequent chapters.

PRESENT WORTH METHOD OF COMPARISON

In this method of comparison, the cash flows of all the alternatives will be reduced to time zero (i.e., now), by assuming a rate of interest 'i'. The best alternative is then selected by comparing the present worths of all the alternatives.

Any given business alternative will always have cash inflows (revenues/income/etc.) and cash outflows (all types of costs) associated with it. The net present worth of the business alternative is then calculated by

$$\text{Net PW} = \text{PW}(\text{all revenues}) - \text{PW}(\text{all costs})$$

When net present worths of all business alternatives are calculated, the one with the highest net PW is chosen. Obviously if the net PW's of all projects have -ve value, the alternative with the least negative value is chosen.

A general procedure in solving problems in present worth comparison would be as follows:

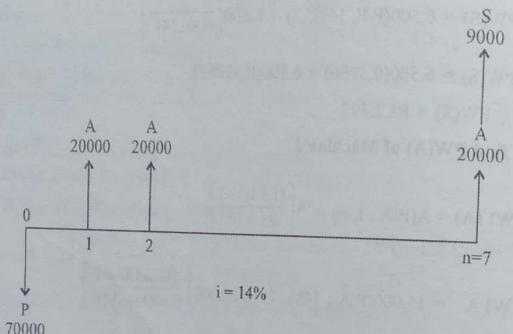
PRESENT WORTH COMPARISONS 53

1. List all data given. Distinguish all monetary values as either cash inflow or cash outflow i.e., as either revenue or as cost.
2. Draw cash flow diagram indicating all cash inflows in upward direction and cash outflows in downward direction. Cash inflows are generally - *annual revenue, salvage value, annual savings etc.* While cash outflows are generally - *initial cost, annual costs, maintenance costs, operational costs etc.*
3. Calculate the Present Worth of each component on the CFD using appropriate compound interest formula.
4. Add the Present Worths of all cash inflows on one hand and add the present worths of all cash outflows on the other hand.
5. The net present worth of a business alternative is then obtained by

$$\text{Net Present Worth} = \text{PW}(\text{all cash inflows}) - \text{PW}(\text{all cash outflows}) \text{ or}$$

$$\text{NPW} = \text{PW}(\text{all revenues}) - \text{PW}(\text{all costs})$$
6. Like wise, the net PW of all business alternatives are found.
7. That alternative with the highest algebraic PW value is chosen since Revenue minus Costs indicate profit which has to be maximized. If all net PW values are negative, then the least -Ve value is chosen which indicates the best among the worst i.e., the one with the least cost.
8. The Present Worths can be calculated either by using formulae or interest tables, although the latter is much easier. But in certain cases where the values of 'i' and 'n' are not included in the Tables book, it becomes imperative to use the formulae. The student, therefore, should be well versed in both methods.
9. The general steps involved in using Compound Interest tables would be:
 - (i) Go to that page which has the appropriate rate of interest 'i' as is given in the problem (i.e., $i = 10\%, 11\% \dots$)
 - (ii) The first column in every page indicates years 'n'. Select the service life 'n' as is given in the problem.
 - (iii) Now, select the value along the given 'n' under the type of compound interest problem type.
 - (iv) Please note that the value represents only the corresponding formulae. This value should be multiplied with the appropriate quantity P or A or F.

The CFD for the above problem would be



To find PW(S) of M/c 3

$$PW(S) = .S(P/F, i, n) = S \left[\frac{1}{(1+i)^n} \right]$$

$$PW(S) = 9,000(P/F, 14\%, 7) = 9000 \left[\frac{1}{(1+0.14)^7} \right]$$

$$PW(S) = 9,000(0.3996) = 9,000(0.39963)$$

$$PW(S) = Rs.3,596$$

To find PW(A) of M/c 3

$$PW(A) = A(P/A, i, n) = A \left[\frac{(1+i)^n - 1}{i(1+i)^n} \right]$$

$$PW(A) = 20,000(P/A, 14\%, 7) = 20,000 \left[\frac{(1+0.14)^7 - 1}{0.14(1+0.14)^7} \right]$$

$$PW(A) = 20,000(4.2883) = 20,000(4.2883)$$

$$PW(A) = Rs.85,766$$

To find PW(P) of M/c 3

$$PW(P) = Rs.70,000$$

Substituting the values of PW(S), PW(A) and PW(P) in equation (3), we have,

$$PW(M/c 3) = 3,596 + 85,766 - 70,000$$

$$PW(M/C 3) = Rs.19,362$$

Answer: From the above calculations, it is clear that the present worth of machine 2 is the highest among all the machines. Therefore it is suggested to the entrepreneur to buy machine 2.

2. A businessman has two investment proposals P and Q in front of him to help him expand his operations. The net cash flows of the proposals are as follows:

Proposal	End of years				
	0	1	2	3	4
P (Rs)	-20,000	7,000	9,000	7,000	8,000
Q (Rs)	-20,000	10,000	6,000	7,000	6,000

Compare the Present Worth of proposal P with that of Q at $i = 13\%$. Which proposal should be selected?

Solution: The Present Worths of both proposals should be calculated. Please note that the values in the first column denote investment (cash outflow) since it has -Ve mark.

Given data: Initial cost, $P = Rs.20,000 \rightarrow$ (outflow)

Annual Revenue for 1 year, $A_1 = Rs.7,000 \rightarrow$ (inflow)

$A_2 = Rs.9,000 \rightarrow$ (inflow)

$A_3 = Rs.7,000 \rightarrow$ (inflow)

$A_4 = Rs.8,000 \rightarrow$ (inflow)

$$i = 13.5\%$$

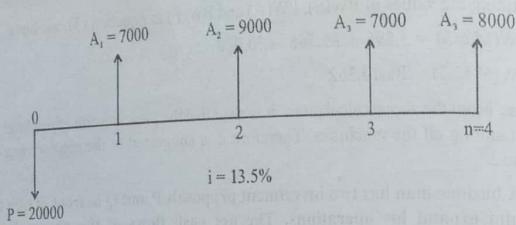
Salvage value, $S = 0$ (not given)

$$PW(P) = PW(\text{all inflows}) - PW(\text{all outflows})$$

or

$$PW(P) = PW(\text{all revenues}) - PW(\text{all costs})$$

$$PW(P) = PW(A_1) + PW(A_2) + PW(A_3) + PW(A_4) - PW(P)$$

**To find PW of Proposal 'P'**

Since the revenue for each year is different, we cannot use $(P/A, i, n)$ but should use $(P/F, i, n)$ for each revenue. In other words, PW of each year-end revenue should be found and then added. Therefore A_1, A_2, A_3 and A_4 are nothing but future amounts whose PW has to be calculated. Note that $n = 1$ for A_1 , $n = 2$ for A_2 and so on.

Using Tables

$$\therefore PW(P) = A_1(P/F, i, n) + A_2(P/F, i, n) + A_3(P/F, i, n) + A_4(P/F, i, n) - 20,000$$

$$PW(P) = 7,000(P/F, 13.5\%, 1) + 9,000(P/F, 13.5\%, 2) + 7,000(P/F, 13.5\%, 3) + 8,000(P/F, 13.5\%, 4) - 20,000$$

From the compound interest tables,

$$PW(P) = 7,000(0.8811) + 9,000(0.7763) + 7,000(0.6839) + 8,000(0.6026) - 20,000$$

$$PW(P) = \text{Rs.} 2762.5$$

Using Formulae

$$PW = A_1 \left[\frac{1}{(1+i)^n} \right] + A_2 \left[\frac{1}{(1+i)^n} \right] + A_3 \left[\frac{1}{(1+i)^n} \right] + A_4 \left[\frac{1}{(1+i)^n} \right] - PW(P)$$

$$PW = 7,000 \left[\frac{1}{(1+0.135)^1} \right] + 9,000 \left[\frac{1}{(1+0.135)^2} \right] + 7,000 \left[\frac{1}{(1+0.135)^3} \right] \\ + 8,000 \left[\frac{1}{(1+0.135)^4} \right] - 20,000$$

$$PW = 7,000(0.8810) + 9,000(0.7762) + 7,000(0.6839) + 8,000(0.6025) - 20,000$$

$$\therefore PW = \text{Rs.} 2,761 \rightarrow (\text{Small changes in value between tables and formulae are expected})$$

To find PW of Proposal Q

Given, $P = 20,000 \rightarrow$ (Outflow)

$$A_1 = 10,000 \rightarrow$$
 (Inflow)

$$A_2 = 6,000 \rightarrow$$
 (Inflow)

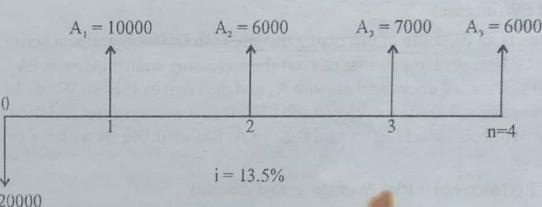
$$A_3 = 7,000 \rightarrow$$
 (Inflow)

$$A_4 = 6,000 \rightarrow$$
 (Inflow)

$$i = 13.5\%$$

$$n = 4 \text{ years}$$

The CFD for the above problem would be



Solution: This is similar to previous proposal.

Using Tables

$$PW(\text{Proposal Q}) = PW(\text{inflows}) - PW(\text{outflow})$$

$$= PW(A_1) + PW(A_2) + PW(A_3) + PW(A_4) - PW(P)$$

$$= A_1(P/F, i, n) + A_2(P/F, i, n) + A_3(P/F, i, n) + A_4(P/F, i, n) - 20,000$$

$$= 10,000(P/F, 13.5\%, 1) + 6,000(P/F, 13.5\%, 2) +$$

$$7,000(P/F, 13.5\%, 3) + 6,000(P/F, 13.5\%, 4) - 20,000$$

$$PW(Q) = 10,000(0.8811) + 6,000(0.7763) + 7,000(0.6839) + 6,000(0.6026) - 20,000$$

$$PW(Q) = \text{Rs.} 1871.7$$

Answer: Since the present worth of Proposal P is higher than that of Q, the businessman should select P.

3. A Xerox machine in a new locality with an initial outlay of Rs. 1,00,000 yields Rs.80,000 during first year of its operations and the yield increases by Rs.10,000 from its second year up to the 8th year of operation. At the end of the life of business, the machine becomes scrap and has zero salvage value. Find the present worth of the business assuming a rate of interest of 13%, compounded annually.

Solution.

$$PW = PW(\text{inflows}) - PW(\text{outflow})$$

$$PW = PW(\text{income}) - PW(\text{initial investment})$$

$$PW = PW(\text{income}) - 10,000Q \quad \dots \dots \dots (1)$$

$$PW = PW(\text{income}) - PW(P)$$

To find PW(income)

This is an example of increasing income with Uniform gradient series. In Uniform gradient series, we have to first convert the increasing annual incomes ($A_1, A_1 + G, A_1 + 2G, \dots$) to an annual equivalent amount A , and then find its Present Worth. In other words, the increasing values of income on the CFD is first averaged out and then its collective Present Worth is found i.e., to find P given A . Before doing so we have to find A given A_1 and G .

But according to Uniform gradient series

$$A = A_1 + G \left(A/G, i, n \right) \dots \dots \dots (3)$$

Using Tables

Substituting (3) in (2)

$$\begin{aligned} \text{PW(income)} &= [A_1 + G(A/G, i, n)] [(P/A, i, n)] \\ &= [80,000 + 10,000(A/G, 13\%, 8)] [(P/A, 13\%, 8)] \\ &= [80,000 + 10,000 (2.8685)] [4.7988] \end{aligned}$$

$$PW(\text{income}) = 5,21,557$$

Substituting the above value in (1)

$$PW = 5,21,557 - 1,00,000$$

∴ Present Worth of the business PW = Rs.4,21,557

Using Compound Interest Formulae

$$PW(\text{income}) = A \left[\frac{(1+i)^n - 1}{i(1+i)^n} \right]$$

$$\text{But } A = A_1 + G \left[\frac{(1+i)^n - 1}{i(1+i)^n - i} \right]$$

$$\therefore \text{PW(income)} = \left\{ A_1 + G \left[\frac{(1+i)^n - 1}{i(1+i)^n - i} \right] \right\} \left[\frac{(1+i)^n - 1}{i(1+i)^n} \right]$$

$$\text{PW(income)} = \left\{ 80,000 + 10,000 \left[\frac{(1+0.13)^8 - 1}{0.13(1+0.13)^8 - 1} \right] \right\} \left[\frac{(1+0.13)^8 - 1}{0.13(1+0.13)^8} \right]$$

$$\therefore \text{PW(income)} = [80,000 + 10,000(2.8685)] / 4.7988]$$

$$\therefore \text{PW(income)} = 5,21,557$$

$$\therefore \text{PW(income)} = 5,21,557 - 100000$$

$$\text{PW(business)} = \text{Rs.} 4,21,557$$

4. A small dam and an irrigation system are expected to cost Rs.300,000.00. Annual maintenance and operating costs are expected to be Rs.4,00,000 for the first year and will increase at a rate of 10 percent per year. Determine the equivalent present worth of building dam and operating the system with interest of 10 percent over a 30 year life. (VTU July '05)

Solution: This problem is an example of geometric series.

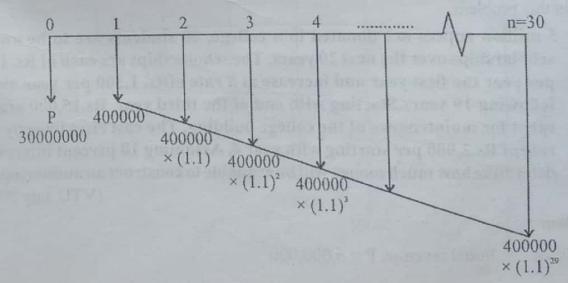
Given: Initial cost, $P = 300,000,00$

Rate of interest, $j = 10\%$

Service life, n = 30 years

First year maintenance cost, $A_1 \equiv 400,000$

Maintenance cost increase every year, $g = 10\%$



The equivalent Present Worth (EPW) of the dam and irrigation system can be found by
 $EPW(\text{dam}) = PW(\text{all revenues}) - PW(\text{all costs}) \dots\dots\dots(1)$
 $EPW(\text{dam}) = 0 - PW(\text{all costs}) \quad (\because \text{there are no revenues})$
 $PW(\text{Costs}) = PW(\text{Initial cost}) + PW(\text{Maintenance costs}) \dots\dots\dots(2)$

To find $PW(\text{Maintenance cost})$

We know that maintenance costs is a geometric series where rate of increase

$$\begin{aligned} g &= 0.1 \\ i &= 0.1 \\ A_1 &= 4,00,000 \end{aligned}$$

Here $g = i$, therefore

$$PW(\text{MC}) = A_1(P/A, g, i, n)$$

$$\begin{aligned} &= A_1 \left[\frac{n}{1+i} \right] \\ &= 4,00,000 \left[\frac{30}{1+0.1} \right] \end{aligned}$$

$$PW(\text{MC}) = 109,09,090$$

Substituting this in Eq (2), we get

$$\begin{aligned} PW(\text{Costs}) &= 300,00,000 + 109,09,090 \\ &= 409,09,090 \end{aligned}$$

Substituting this in Eq (1), we get

$$EPW(\text{dam}) = 0 - 409,09,090$$

$$\therefore EPW(\text{dam}) = -409,09,090$$

This is the equivalent Present Worth of the dam. Please note that tables cannot be used in this problem.

5. 5 million rupees are donated to a college, 20 students are to be awarded scholarships over the next 20 years. The scholarships are each of Rs. 12,000 per year the first year and increase at a rate of Rs. 1,500 per year over the following 19 years. Starting with end of the third year, Rs. 15,000 are to be spent for maintenance of the college building. The cost rises linearly at the rate of Rs. 2,000 per year starting with year 4. Assuming 10 percent interest rate determine how much money will be available to construct an auditorium now.

(VTU July 2006)

Solution:

Given: Initial revenue, $P = 5,000,000$

Scholarship in first year, $A_1 = 12,000$

Subsequent increase per year, $G = 1,500$

Maintenance Cost in third year, $A_{m3} = 15,000$

Subsequent increase in M.Cost per year, $G_m = 2,000$

Rate of interest, $i = 10\%$

No. of years, $n = 20$ years

If 5 million rupees is the revenue and if all the costs (both scholarship and maintenance) are deducted by it, the money available to construct an auditorium how is given by the Present Worth of the whole cash flow diagram. Therefore

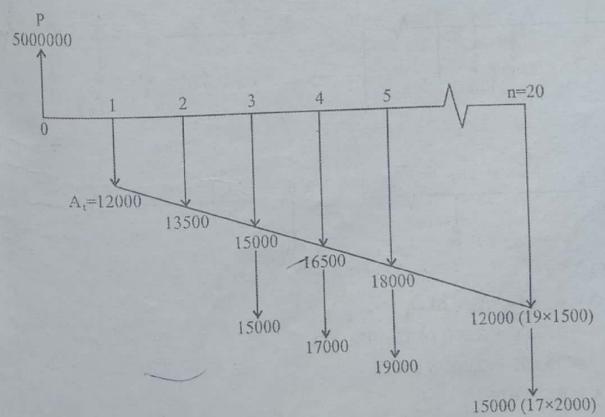
Net Present Worth = $PW(\text{all revenues}) - PW(\text{all costs})$

$$\begin{aligned} &= PW(\text{initial revenue}) - PW(\text{scholarship costs}) \\ &\quad - PW(\text{maintenance cost}) \dots\dots\dots(1) \end{aligned}$$

To find $PW(\text{Scholarship Costs})$

P.S: If we read the problem, there is a slight confusion. Whether 20 students should be given scholarship each year or one student per year in the question. But since the problem says 20 students over 20 years, we assume that it is one student per year.

$$\therefore PW(\text{Scholarship Costs}) = A(P/A, i, n)$$



But, according to Uniform gradient series,

$$\text{Annual Cost, } A = A_1 + G(A/G, i, n)$$

$$= 12,000 + 1,500(A/G, 10\%, 20)$$

$$\therefore \text{PW(Scholarship Cost)} = [A_1 + G(A/G, i, n)] [P/A, i, n]$$

$$= [12,000 + 1,500(A/G, 10\%, 20)] [P/A, 10\%, 20]$$

$$= [12,000 + 1,500(6.5081)] [8.5136]$$

$$\text{PW(S.Costs)} = \text{Rs.}185274 \quad \dots \dots \dots (2)$$

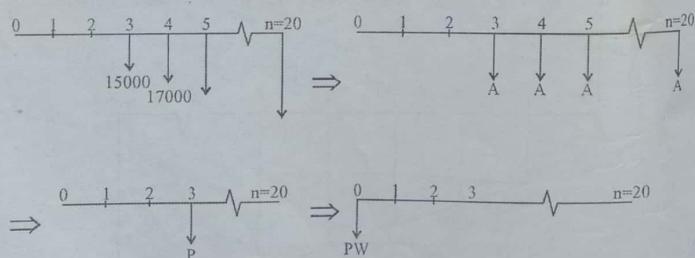
To find PW(Maintenance Costs)

The following steps are followed to arrive at the Present Worth of maintenance Costs.

(i) The first maintenance cost is in year 3 which is Rs.15,000. Subsequent increase from 4th year is Rs.2,000 per year. Hence this is a Uniform gradient series. This Uniform gradient series is first converted to annual equivalent using formula $A = A_1 + G(A/G, i, n)$

(ii) The annual (average) maintenance cost is then converted to the present cost i.e., at the third year using formula $P = A(P/A, i, n)$, as shown in the figure.

(iii) The present value at the end of third year is brought to time zero (i.e., now) considering it as a future sum F, using formula $P = F(P/F, i, n)$



Step 1:

$$A = A_1 + G(A/G, i, n)$$

$$= 15,000 + 2,000 (A/G, 10\%, 17)$$

$$= 15,000 + 2,000(5.8071)$$

$$A = \text{Rs.}26,614$$

Step 2:

$$P = A(P/A, i, n)$$

$$P = 26,614(P/A, 10\%, 17)$$

$$= 26,614(8.0216)$$

$$P = \text{Rs.}213,488$$

→ This is considered F for next step.

Step 3:

$$PW(M.C) = F(P/F, i, n)$$

$$= 213,488(P/F, 10\%, 3)$$

$$= 213,488(0.7513)$$

$$PW(M.Cost) = \text{Rs.}1,60,393.$$

Substituting the values of Eq (2) and (3) in Eq (1), we get

$$\text{Net Present Worth} = 50,00,000 - 1,85,274 - 1,60,393$$

$$NPW = \text{Rs.}4,654,333$$

This is the amount available from the initial revenue to build an auditorium now after subtracting scholarship and maintenance costs.

6. A person wanted to buy a motorbike to his son. Upon down payment the motorbike would cost him Rs.55,000/- while in the instalment scheme, he had to pay an EMI of Rs.900/- for 10 years at the rate of interest of 14%, compounded monthly. Suggest the person which scheme is more economical.

Solution:

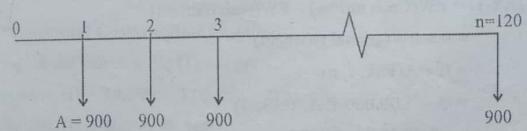
Given P = Zero

A = Rs.900/month

n = 10 years = $10 \times 12 = 120$ months

$$i = 14\% \text{ (annual)} = \left(\frac{0.14}{12}\right)\% \text{ per month}$$

$$i = 14\%$$



Note: Please note that in this problem 'A' stands for monthly instalment and is for 120 months. Also rate of interest $i = 14\%$ is annual and it becomes $\left(\frac{0.14}{12}\right)\%$ per month. Also please also note here that interest tables cannot be used since $i = 1.16\%$ and not given in the table.

Present Worth = PW(all revenues) - PW(all costs)

$$\begin{aligned} &= -A(P/A, i, n) \\ &= -A \left[\frac{(1+i)^n - 1}{i(1+i)^n} \right] \\ &= -900 \left[\frac{\left(1 + \frac{0.14}{12}\right)^{120} - 1}{\frac{0.14}{12} \left(1 + \frac{0.14}{12}\right)^{120}} \right] \end{aligned}$$

PW = Rs.57964

7. A company needs a mini-bus to bring its employees to work and take them home. It has two alternatives:

- (i) To rent a vehicle for a payment of Rs.3 Lakhs per year for the next 5 years.
- (ii) To buy a second hand vehicle for Rs.3Lakhs with an operating and maintenance cost of Rs.2 Lakhs per year. The salvage value of the vehicle after the five year would be about 1 lakh.

Select the best alternative based on Present Worth method of comparison using an interest rate of 13% compounded annually.

Solution:

To find PW (Alternative I)

$$\begin{aligned} PW(\text{Alt-I}) &= PW(\text{Cash inflow}) - PW(\text{cash outflow}) \\ &= 0 - FV(\text{annual payment}) \\ &= 0 - A(P/A, i, n) \\ &= 0 - 3,00,000(P/A, 0.13, 5) \\ &= -3,00,000 \times 3.5172 \end{aligned}$$

PW(Alt 1) = Rs. - 10,55,160

To find PW (Alternative II)

Given,

$$\begin{aligned} \text{Initial Cost}, P &= 3,00,000 \rightarrow (\text{outflow}) \\ \text{Annual Maintenance Cost}, A &= 2,00,000/\text{year} \rightarrow (\text{outflow}) \end{aligned}$$

$$\text{Salvage value}, S = 1,00,000 \rightarrow (\text{inflow})$$

$$i = 13\%$$

$$n = 5 \text{ years}$$

$$PW(\text{Alternative II}) = PW(\text{inflow}) - PW(\text{outflow})$$

$$= PW(\text{Salvage}) - PW(\text{Initial Cost}) - PW(\text{Annual maintenance cost}) \quad \dots\dots\dots(1)$$

To find PW (Salvage)

$$\begin{aligned} PW(\text{Salvage}) &= \text{To find } P \text{ given } F \text{ (or } S) \\ &= F(P/F, i, n) \\ &= 1,00,000(P/F, 13\%, 5) \\ &= 1,00,000(0.5428) \end{aligned}$$

$$PW(\text{Salvage}) = Rs.54280$$

To find PW (Initial Cost)

Since the initial cost is increased now (i.e., time zero) its PW is the same. Therefore

$$PW(\text{Initial Cost}) = Rs.3,00,000$$

To find PW (Annual Maintenance Cost)

$$\begin{aligned} PW(\text{Annual Cost}) &= \text{To find } P \text{ given } A \\ &= A(P/A, i, n) \\ &= 2,00,000(P/A, 13\%, 5) \\ &= 2,00,000(3.5172) \end{aligned}$$

$$PW(\text{Annual Costs}) = Rs.7,03,440$$

Substituting all the values in Eq (1), we get

$$PW(\text{Alternative II}) = 54,280 - 3,00,000 - 7,03,440$$

$$PW(\text{Alt-II}) = Rs. -9,49,160$$

Answer: Since the present worth of Alternative II is lower, it is suggested to buy a second hand vehicle.

8. (a) An aircraft assembly fixture has a purchase price of Rs.9,00,000 and classed as a 5 year property. Use of the fixture is expected to result in an annual before tax savings of Rs.3,00,000 for a period of 6 years, at the end of which time it will be obsolete and virtually worthless. Applying the appropriate accelerated schedule, determine.

- (i) The before tax Present Worth of the investment at an interest rate of 40 percent.
- (ii) The after tax Present Worth of the investment with an effective tax rate of 40 percent and an interest rate of 20 percent. (VTU July '05)

Given data:

Purchase price, $P = \text{Rs.}9,00,000$ (Cost)

Annual savings before tax, $A = \text{Rs.}3,00,000$ (Revenue)

Service life, $n = 6$ years

Salvage value, $S = \text{zero}$

Before tax interest rate, $i = 40\%$

After tax interest rate, $i = 20\%$

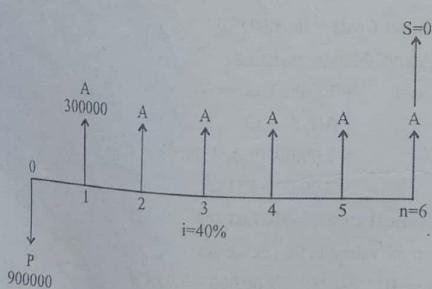
Tax rate, $t = 40\%$

$PW(\text{before tax}) = ?$

$PW(\text{after tax}) = ?$

To find PW of investment(before tax)

The CFD for the above case would be



Note: The Present Worth of the equipment before tax means that we have to find the PW for the CFD as shown above.

$$\begin{aligned} \therefore PW(\text{before tax}) &= PW(\text{Revenues}) - PW(\text{Costs}) \\ &= A(P/A, i, n) - 9,00,000 \\ &= 3,00,000(P/A, 40\%, 6) - 9,00,000 \\ &= 3,00,000(2.1680) - 9,00,000 \end{aligned}$$

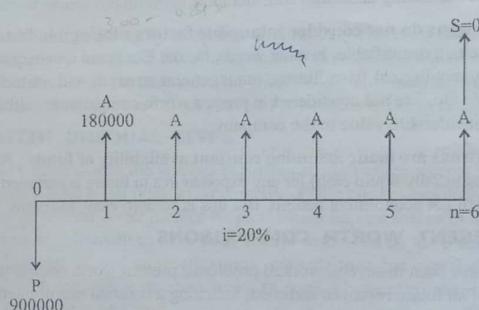
$$PW(\text{before tax}) = \text{Rs.} - 2,49,600$$

To find PW of the investment (after tax)

Here we have to first find the income after tax. In other words 40% tax is deducted from annual savings. Convert it into income after tax. PW of this is found but taking an interest of 20% as is given in the problem.

$$\begin{aligned} \text{Annual Savings per year} &= 3,00,000 - 0.4 \times 3,00,000 \\ &= \text{Rs.}1,80,000 \end{aligned}$$

The CFD for the above case would be,



$$\begin{aligned} PW(\text{after tax}) &= PW(\text{Revenues}) - PW(\text{Costs}) \\ &= A(P/A, i, n) - 9,00,000 \\ &= 1,80,000(P/A, 20\%, 6) - 9,00,000 \\ &= 1,80,000(3.3255) - 9,00,000 \end{aligned}$$

$$PW(\text{after tax}) = \text{Rs.} - 3,01,410$$

Conditions for Present Worth Comparison

All present worth comparisons that are made have a common background. In other words, before making present worth comparison, it is necessary to follow some basic assumptions as follows :

1. **Cash flows are known :** While doing present worth calculation we assume that all future estimates of revenues and costs are known.
2. **Inflation is not considered :** Present worth calculations do not consider inflation which brings down the value of money with time.
3. **Interest rates are known :** Interest rates at any given point of time is a complex function of the economy of the country. Present worth calculations are made assuming that interest rates do not change with time.
4. **Comparisons are made prior to tax :** All revenues and expenditures in any business are subject to various taxes of the state. These taxes vary with magnitude of revenue or expenditure. Present worth comparisons, at an elementary level, are made without taking taxes into consideration.
5. **Comparisons do not consider intangible factors :** Intangible factors are those which are non quantifiable. In other words, factors like brand image, good will of the company, ecological friendliness, management strength and attitude, quality of workforce etc., are not considered in present worth comparisons, although they all are of considerable value to the company.
6. **Comparisons are made assuming constant availability of funds :** Availability of capital (especially liquid cash) for any expenditures in future is assumed as and when required to get economical benefit. But this is seldom easy, in reality.

BASIC PRESENT WORTH COMPARISONS

As we have seen in several worked problems, present worth comparisons give the current value of all future revenues and costs, assuming a constant rate of interest i . In other words, the present worth of any business can be estimated when all future investments and returns are well known. When comparing alternatives, present worth calculations done so far have assumed the same life for all alternatives. Present worth comparisons can also be made between alternatives which have unequal lives, which will be dealt later. When making basic present worth comparison, two general patterns emerge :

1. PRESENT - WORTH EQUIVALENCE

In this, the present worth of a series of future transactions are made at time zero. The purpose is to secure one figure that represents all the future transactions. This figure can then be compared with a corresponding figure that represents transactions from a competing alternative, or it can be compared with the option

of doing nothing. A do-nothing option is always an alternative, even if it results only in postponement.

As we have seen in the worked examples, the present worth comparisons are made both on revenue - dominated cash flow problems and cost-dominated cash flow problems.

2. NET PRESENT WORTH

In this, there is always an investment at time zero followed by possible receipts and expenditures at future time periods. In other words, there is an initial outlay followed by a series of revenue and disbursements. This is the most frequently encountered pattern, which leads to the fundamental relation.

$$\text{Net present worth} = P_w (\text{revenues}) - P_w (\text{costs})$$

The criteria for choosing between mutually exclusive alternatives is to select the one that maximizes the net present worth or simply the one that yields the larger positive P_w . A negative P_w means that the alternative does not satisfy the rate-of-return requirement.

In addition to mutually exclusive alternatives where we can select only one alternative, we could have independent alternatives where more than one alternative may be selected. The criterion for consideration of independent alternatives is that they should have a P_w which is equal to or greater than zero.

ASSETS WITH UNEQUAL LIVES

All cases discussed in the worked examples section are problems which concern alternatives which have equal lives. In other words, all the present-worth comparisons that were made so far was for co-terminated projects which implies that lives involved have a common end point. Alternatives have been always compared on the basis of equivalent outcomes. However in real time situations, comparisons have to be made among alternatives which may have unequal lives. In such a case two prominent methods, described as under, are used.

1. Common multiple method
2. Study period method

1. COMMON MULTIPLE METHOD

In this method, a common multiple of all the different service lives of various alternatives is chosen. The alternatives are all then compared against this common service life, and the appropriate alternative is chosen, depending on whether it is cost dominated or revenue dominated. In other words, all the alternatives are made to coterminate by selecting an analysis period that spans a common multiple of the lives of the involved assets. For example, if four alternatives have

lives of 2, 3, 4 and 6 years, the least common multiple which is 12 years, is chosen. Then the asset with a life of two years would be replaced 6 times during the analysis period. The assets with 2, 4 and 6 years lives would be replaced 4, 3, 2 times, respectively.

The method of least common multiple is used under the assumption that assets will be repeatedly replaced by successors having identical cost characteristics. In other words, it is assumed that neither inflation nor interest rates is going to change, with passage of time.

This method is more applicable to such alternatives whose least common multiple life is not a large number.

2. STUDY PERIOD METHOD

In this method, a common study-period for all the alternatives which is proportionate to the length of the project or the period of time the assets are expected to be in service, is selected. An appropriate study period reflects the replacement circumstances. Setting up of a study-period could depend upon the length of :

- (i) The shortest life of all competing alternatives, or
- (ii) The known duration of required services, or
- (iii) The time before which a better replacement becomes available.

A study period comparison based on the shortest life of all competing alternatives gives protection against technological obsolescence. A study period comparison usually presumes that all assets will be disposed of at the end of the period. Sometimes it therefore becomes necessary to estimate the income that can be realized from the sale of an asset which can still provide useful service.

Step by step procedure to solve Assets with unequal lives

1. The least common multiple (LCM) of the various service lives of different alternatives is calculated. For example, if four alternatives are having service lives 2, 3, 4 and 6 years, the LCM, which is 12, is chosen.
2. The Present Worth of each alternative is now calculated taking service life of $n = 12$ years.
3. For example, if the service life of the alternative actually 2 years, its CFD is first drawn. In order to find the PW of this alternative for 12 years, 6 CFD's are drawn and connected end to end, to make one big CFD.
4. This multiple CFD is now considered as one and its Present Worth may be calculated, by using appropriate formulae.

5. In the combined CFD, there could be multiple first costs and salvage costs which are considered as future amounts (F) whose $PW(P)$ has to be found.
6. Similarly, multiple CFD's are connected in case of other alternatives and their PW's are calculated.
7. Now the PW of all alternatives, calculated for 12 years, can be compared and the answer suggested.

WORKED EXAMPLES

Example:

1. Two holiday cottages are under consideration. Compare the Present Worth of the cost of 24 years service, at an interest rate of 5 percent, when neither cottage has a realizable salvage value.

	Cottage 1	Cottage 2
First cost	Rs.4,500	Rs.10,000
Estimated life	12 years	24 years
Annual maintenance cost	Rs.1,000	Rs.720

(VTU Jan 2006)

Solution: In this problem, the service life of cottage 1 is 24 years while the life of cottage 2 is 12 years. The LCM of 24 and 12 happens to be 24. Hence the Present Worth of both cottages have to be found for $n = 24$ years.

To find $PW(\text{Cottage } 2)$

Given data:

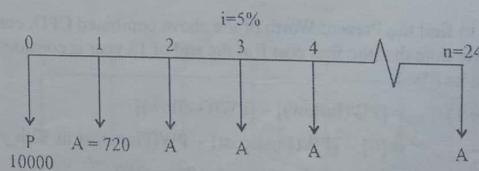
First Cost, $P = 10,000 \rightarrow (\text{outflow})$

Annual Maintenance Cost, $A = Rs.720 \rightarrow (\text{outflow})$

Estimate life, $n = 24$ years

Rate of interest, $i = 5\%$

The cash flow diagram for the above problem would be,



$$\begin{aligned}
 PW(\text{Cottage 2}) &= PW(\text{Inflow}) - PW(\text{Outflow}) \\
 &= 0 - PW(\text{First cost}) - PW(\text{Annual Cost}) \\
 &= 0 - 10,000 - A(P/A, i, n) \\
 &= 0 - 10,000 - 720(P/A, 5\%, 24) \\
 &= 0 - 10,000 - 720(13.7987) \\
 PW(\text{Cottage 2}) &= \text{Rs.} - 19,935
 \end{aligned}$$

To find PW(Cottage 1)

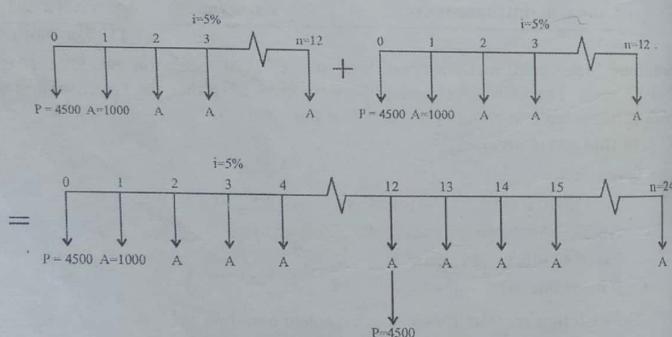
Given data,

$$P = \text{Rs.} 4500 \rightarrow (\text{outflow})$$

$$A = \text{Rs.} 1000 \rightarrow (\text{outflow})$$

$$n = 12 \text{ years (to be doubled)}$$

$$i = 5\%$$



We have to find the Present Worth of the above combined CFD, considering it as one. Also please note that the first cost P at the end of 12 year is considered as a future amount to find its PW.

$$\begin{aligned}
 \therefore PW(\text{Cottage 1}) &= [PW(\text{Inflow})] - [PW(\text{Outflow})] \\
 &= [0] - [PW(\text{First cost}) + PW(\text{First cost in 12th year})] \\
 &\quad + PW(\text{Annual cost for 24 years})
 \end{aligned}$$

$$\begin{aligned}
 &= -4,500 - F(P/F, i, n) - 1000(P/A, i, n) \\
 &= -4,500 - F(P/F, 5\%, 12) - 1,000(P/A, 5\%, 24) \\
 &= -4,500 - 4,500 (0.5568) - 1,000(13.7986)
 \end{aligned}$$

$$PW(\text{Cottage 1}) = \text{Rs.} - 20,804$$

Since cottages 1 and 2 are being compared for the same service life i.e., $n = 24$ years, their Present Worths can be compared. Since the algebraic value PW of cottage 2 is higher, it is selected. It means that cottage 2 has lower costs than cottage 1.

Example:

2. The following alternatives are available to accomplish an objective of 12 year duration.

	Plan A	Plan B	Plan C
Life cycle(yrs)	6	3	4
First cost (Rs.)	2,000	8,000	10,000
Annual cost (Rs.)	3,200	700	500

Compare the present worth of the alternatives using an interest rate of 7 percent.
(VTU July 2006)

Solution: In the given problem, service lives of the 3 plans are 6, 3 and 4 years respectively. The LCM here is 12 years. Each plan therefore has to be evaluated for 12 years with their CFD's suitably multiplied.

To find Present Worth (Plan A)

Given data:

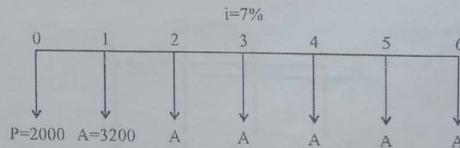
$$\text{First Cost, } P = \text{Rs.} 2,000 \rightarrow (\text{outflow})$$

$$\text{Annual cost, } A = \text{Rs.} 3,200 \rightarrow (\text{outflow})$$

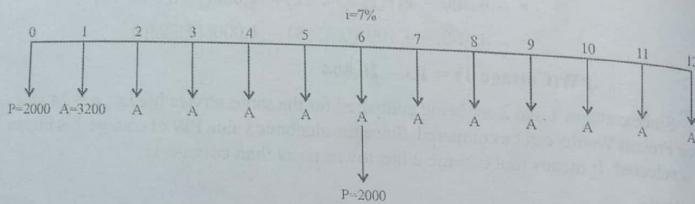
$$\text{Life cycle, } n = 6 \text{ years} \rightarrow (\text{to be doubled})$$

$$\text{Rate of interest, } i = 7\%$$

Single CFD for the above problem would be:



If we double the above CFD, then it would become



Now, Present Worth for the combined CFD for Plan A would be found as follows. Please note that first cost P at $n=6$ is considered as F whose PW has to be found.

$$\begin{aligned} \text{PW(Plan A)} &= [\text{PW(all inflow)}] - [\text{PW(all outflow)}] \\ &= [0] - [\text{PW(First cost at year 0)} + \text{PW(First cost considered at year 6)} + \text{PW(Annual cost for 12 years)}] \\ &= 0 - 2000 - F(P/F, i, n) - A(P/A, i, n) \\ &= -2000 - 2000(P/F, 7\%, 6) - 3200(P/A, 7\%, 12) \\ &= -2000 - 2000(0.6664) - 3200(7.9427) \\ &\therefore \text{PW(Plan A)} = \text{Rs.}-28,749 \end{aligned}$$

To find Present Worth (Plan B)

Given data,

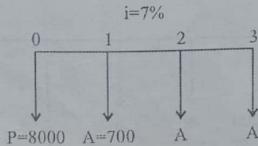
$$P = \text{Rs.}8,000 \rightarrow (\text{outflow})$$

$$A = \text{Rs.}7,00 \rightarrow (\text{outflow})$$

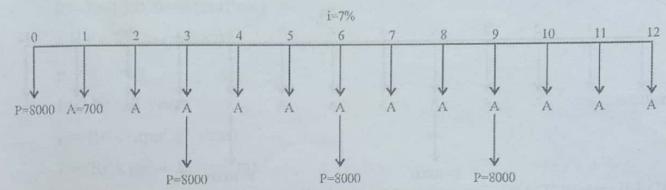
$$n = 3 \text{ years} \rightarrow (\text{to be quadrupled})$$

$$i = 7\%$$

Single CFD for the above problem would be,



Four times the above CFD would become,



Now, Present Worth of the above combined CFD of Plan B would be,

$$\begin{aligned} \text{PW(Plan B)} &= \text{PW(inflow)} - \text{PW(outflow)} \\ &= 0 - \text{PW(First cost at } n=0) - \text{PW(First cost at } n=3) - \text{PW(First cost at } n=6) - \text{PW(First cost at } n=9) - \text{PW(Common annual cost for 12 years)} \\ &= -F(P/F, i, n) - F(P/F, i, n) - F(P/F, i, n) - F(P/F, i, n) \\ &\quad - A(P/A, i, n) \\ &= -8,000 - 8,000(P/F, 7\%, 3) - 8,000(P/F, 7\%, 6) \\ &\quad - 8,000(P/F, 7\%, 9) - 700(P/A, 7\%, 12) \\ &= -8,000 - 8,000(0.8163) - 8,000(0.6664) - 8,000(0.5439) \\ &\quad - 700(7.9427) \\ &\therefore \text{PW(Plan B)} = -29,772 \end{aligned}$$

To find Present Worth (Plan C)

Given data:

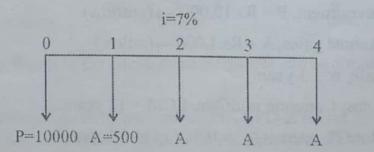
$$P = \text{Rs.}10,000 \rightarrow (\text{outflow})$$

$$A = \text{Rs.}500 \rightarrow (\text{outflow})$$

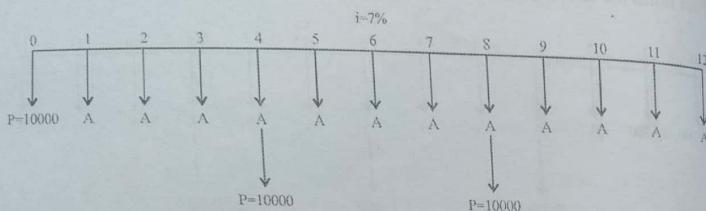
$$n = 4 \text{ years} \rightarrow (\text{to be trebled})$$

$$i = 7\%$$

Single CFD for the above problem would be



If the above CFD is trebled, then it would become



Now, the Present Worth for the above combined CFD of Plan C would be,

$$\begin{aligned} \text{PW(Plan C)} &= \text{PW(Inflow)} - \text{PW(Outflow)} \\ &= 0 - P - \text{PW(First cost at } n=0) - \text{PW(First cost at } n=4) - \text{PW(First} \\ \text{cost} &\quad \text{at } n=8) - \text{PW(Common annual cost for 12 years)} \\ &= -P - F(P/F, 7\%, 4) - F(P/F, 7\%, 8) - A(P/A, 7\%, 12) \\ &= -10,000 - 10,000(0.7629) - 10,000(0.5820) - 500(7.9427) \\ \text{PW(Plan C)} &= \text{Rs.} - 27,420 \end{aligned}$$

Answer: Comparing the Present Worth of Plans A, B and C, we see that Plan C has the highest PW (or the least -Ve value). Hence Plan C is the best.

Example:

3. Analyse and take a decision on the following proposals based on engineering economy analysis. Proposal one has a life of three years with an investment of Rs. 10,000/- and the second proposal has a life of 4 years at an investment of Rs. 12,000/-. Annual gain in operating expenses in both the cases are Rs.1,000 and Rs. 500 respectively. Assume an interest rate of 10% which gets doubled at LCM midway. Comment on the proposal using correct method? (VTU Jan 2008)

Solution

Proposal I

Given, Investment, $P = \text{Rs.} 10,000 \rightarrow (\text{outflow})$

Annual gains, $A = \text{Rs.} 1,000 \rightarrow (\text{inflow})$

Life, $n = 3$ years

Least Common multiple, $\text{LCM} = 12$ years

Rate of interest, $i = 10\%$ (up to 6th year)

$i = 20\%$ (6 – 12 years)

Proposal II

$P = \text{Rs.} 12,000 \rightarrow (\text{outflow})$

$A = \text{Rs.} 500 \rightarrow (\text{inflow})$

$n = 4$ years

$\text{LCM} = 12$ years

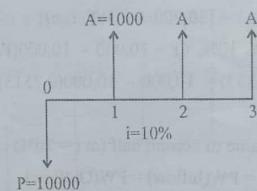
$i = 10\%$ (upto 6th year)

$i = 20\%$ (6th – 12th years)

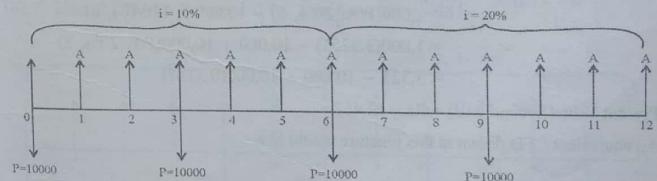
Note: Please note that rate of interest 'i' doubles at midway of LCM. Therefore if LCM is 12 years, i becomes 20% half way through i.e., from end of 6th year.

To find Present Worth of Proposal I

The CFD for proposal I for a single cycle would be



The combined CFD for proposal I considered for 12 years would be



The net Present Worth of the above combined CFD can be found by the following steps.

Step 1: The Present Worth of the first half of CFD is found with $i = 10\%$, $n = 6$ years, $A = 1,000$ (for 6 years) and the first three values of P

Step 2: The PW of the second half of CFD is found independently considering end of 6th year as time zero. In other words, the combined present value of the second half of CFD is found at n = 6 years, and i = 20%.

Step 3: This present value at the end of 6th year is now considered as a future sum F, and its PW (i.e., its value now) is found at i = 10%, n = 6 years.

Step 4: The Present Worth of the whole CFD is nothing but the sum of PW of first half and the Present Worth of the present value of the second half.
(Note: Please note that there is no need to write any procedure in the exam)

Step 1: To find PW of the first half (at i = 10%)

$$\begin{aligned} \text{PW(first half)} &= \text{PW(Inflow)} - \text{PW(Outflow)} \\ &= \{\text{PW(annual gain for 6 years)}\} - \{\text{PW(investment at time zero)} \\ &\quad + \text{PW(investment at } n = 3)\} \\ &= A(P/A, i, n) - [10,000 + F(P/F, i, n)] \\ &= 1,000(P/A, 10\%, 6) - 10,000 - 10,000(P/F, 10\%, 3) \\ &= 1,000(4.3553) - 10,000 - 10,000(0.7513) \end{aligned}$$

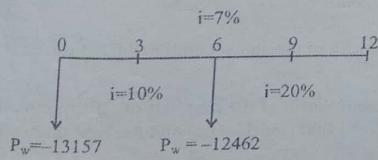
$$\text{PW(first half)} = -13,157$$

Step 2: To find Present Value of second half (at i = 20%)

$$\begin{aligned} \text{Present Value(Second half)} &= \text{PW(Inflow)} - \text{PW(Outflow)} \\ &= A(P/A, i, n) - \{\text{PW(investment P at time zero)} \\ &\quad + \text{PW(investment P at } n = 3)\} \\ &= 1,000(P/A, 20\%, 6) - 10,000 - F(P/F, i, n) \\ &= 1,000(3.3255) - 10,000 - 10,000(P/F, 20\%, 3) \\ &= 3,325 - 10,000 - 10,000(0.5787) \end{aligned}$$

$$\text{Present Value(Second half)} = \text{Rs.} - 12,462$$

An equivalent CFD drawn at this juncture would like



$$P_w = -13157$$

$$P_w = -12462$$

PRESENT WORTH COMPARISONS

Step 3: To find the Present Worth of Present value(of second half)

$$\begin{aligned} \text{PW(Second half)} &= \text{To find P given F} \rightarrow \text{here F is Present Value at } n = 6 \\ &= F(P/F, i, n) \\ &= -12,462 (P/F, 10\%, 6) \\ &= -12,462 (0.5645) \end{aligned}$$

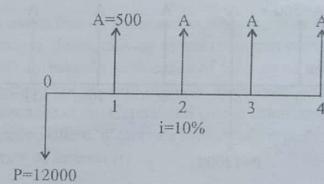
$$\text{PW(Second half)} = \text{Rs.} - 7,034.$$

Step 4: To find PW(of Proposal I)

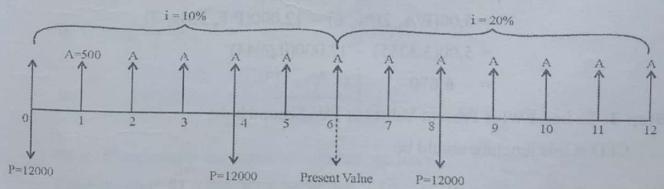
$$\begin{aligned} \therefore \text{Present Worth(Proposal I)} &= \text{PW(First half)} + \text{PW(Second half)} \\ &= -13,157 + (-7,037) \\ \text{PW(Proposal I)} &= -20,191 \end{aligned}$$

To find the Present Worth of Proposal II

The CFD for proposal II for a Single cycle would be



The combined CFD for Proposal II for 3 such cycles would be



Steps to find the Present Worth of Proposal II are as follows:

Step 1: Find the PW of the first half (i.e., for 6 years) of annual gain A (for 6 years) and investment P (at n = 0 and n = 4), where i = 10%

ENGINEERING ECONOMICS

- Step 2:** Find the Present Value of second half assuming end of 6th year as time zero.
This is calculated at $i = 20\%$
- Step 3:** Find the PW of this Present Value of the second half. The sum of PW(first half) and PW(second half) would then give the PW(Proposal II)

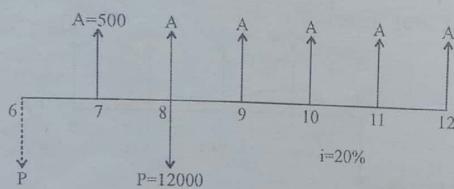
Step 1: To find PW(First half)

$$\begin{aligned} \text{PW(First half)} &= \text{PW(annual gain)} - \text{PW(investments)} \\ &= A(P/A, i, n) - \text{PW}(P \text{ at time zero}) - \text{PW}(P \text{ at } n = 4) \\ &= 5,00(P/A, 10\%, 6) - 12,000 - 12,000(P/F, 10\%, 4) \\ &= 5,00(4.3553) - 12,000 - 12,000(0.6830) \end{aligned}$$

$$\text{PW(First half)} = -18,018$$

Step 2: To find Present Value of second half

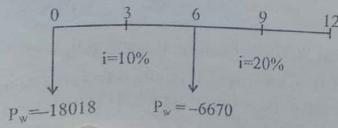
The CFD for this is



$$\begin{aligned} \text{Present Value(at } n=6) &= \text{PW(annual gain for 6 years)} - \text{PW(Initial investment at } n=2) \\ &= A(P/A, i, n) - F(P/F, i, n) \\ &= 5,00(P/A, 20\%, 6) - 12,000(P/F, 20\%, 2) \\ &= 5,00(3.3255) - 12,000(0.6944) \\ &= -6,670 \end{aligned}$$

Step 3: To find PW(of Present Value) or PW(Second half)

CFD at this juncture would be



PRESENT WORTH COMPARISONS

$\text{PW}(\text{of Present Value})$ = To find P given $F(\text{or PV})$

$$\begin{aligned} &= F(P/F, i, n) \\ &= -6,670(P/F, 10\%, 6) \\ &= -6,670(0.5645) \end{aligned}$$

$$\text{PW}(\text{Second half}) = -3,765$$

Step 4: To find PW of Proposal II

$$\begin{aligned} \text{PW}(\text{Proposal II}) &= \text{PW(First half)} + \text{PW}(\text{Second half}) \\ &= -18,018 + (-3,765) \end{aligned}$$

$$\text{PW}(\text{Proposal II}) = \text{Rs.} - 21,783$$

Answer: Comparing Present Worth of Proposal I and II, we can see that PW of Proposal I has higher value (or lesser -Ve value). Therefore Proposal I is better because it has lesser costs involved.

ASSETS HAVING INFINITE LIVES

There are many assets that we come across which may be considered to have infinite lives. For example bridges, dams, railway tunnels etc., can provide extended service and can be assumed to last forever. In such cases of long lived assets, Capitalized cost is calculated which is nothing but the sum of initial investment and the present worth of all operational and maintenance costs. Capitalized cost is calculated in the same way as in a present-worth comparison, where 'n' (service life) equals infinity. This makes the analysis very sensitive to the rate of interest (i)

$$\text{w.k.t. } (P/A, i, n) = \left[\frac{(1+i)^n - 1}{i(1+i)^n} \right]$$

If $n \rightarrow \infty$

$$\text{i.e., } \lim_{n \rightarrow \infty} (P/A, i, n) = \left[\frac{(1+i)^n - 1}{i(1+i)^n} \right]$$

So if 'P' represents the initial investment, we have

$$\text{Capitalized Cost} = P + A (P / A, i, \infty)$$

$$= P + A \times \frac{1}{i} = P + \frac{A}{i}$$

Where A = Uniform annual costs (disbursements)

i = rate of interest (forever)

P = initial out lay (first cost)

If revenues are not considered, then

$$\text{Capitalized Cost} = P + \frac{\text{annual disbursements}}{i}$$

WORKED EXAMPLE

1. Rs. 10 crores was granted by the management of an engineering college for the construction of its new Mechanical Sciences Block. Annual Maintenance for the block is estimated to be Rs. 10 Lakhs. In addition, Rs. 12 lakhs will be needed every 10 years for painting and major repairs.

If the budget granted has to take care of perpetual maintenance, how much of the amount can be used for initial construction costs? Deposited funds can earn 6% rate of interest, compounded annually. Assume that taxes and inflation do not come into picture.

Solution

Given Capitalized cost = Rs. 10 Crores

Annual Maintenance Costs = Rs. 10 Lakhs Per Year

Painting costs = Rs. 12 Lakhs / 10 years

(to be annualized)

Rate of interest, i = 6% (Compounded annually)

Service life, $n = \infty$

Initial investment or first cost P = ?

$$\text{Capitalized cost} = \text{Initial investment} + \frac{\text{Total Annual costs}}{\text{rate of interest}}$$

$$\text{or Initial construction cost} = \text{Capitalized cost} - \frac{\text{TAC}}{i}$$

We have,

Total annual costs = Annual Maintenance cost + Annual repair & Painting cost

But repair and painting costs are given for every 10 years which have to be annualized. For that, equal payment series sinking fund formula, i.e., $F(A/F, i, n)$ is used.

$$P + \frac{1}{i}$$

Therefore, in this problem

$$A = F \left[\frac{i}{(1+i)^n - 1} \right]$$

Where A = Annual amount = ?

F = Sum at the n^{th} period = 1200000

n = number of years = 10 years

i = Rate of interest (Compounded annually) = 6%

$$A = 1200000 \left[\frac{0.06}{(1+0.06)^{10} - 1} \right]$$

$$A = \text{Rs. } 91041$$

$$\therefore \text{Total annual costs} = 1000000 + 91041$$

$$\text{TAC} = 1091041$$

$$\begin{aligned} \therefore \text{Initial cost} &= \text{capitalized cost} - \frac{\text{TAC}}{i} \\ &= 100000000 - \frac{1091041}{0.06} \\ &= 100000000 - 18184025 \end{aligned}$$

$$\text{Initial construction cost} = \text{Rs. } 8,18,15,974$$

Funds to be deposited to earn interest for the sake of maintenance = Rs. 1814025.

FUTURE WORTH COMPARISONS

Future worth comparison is the opposite of present worth comparison. While present worth comparison gives the current worth of all future transactions, future worth project all transactions during a certain period to a time in future. In other words, in future worth method of comparison of alternatives, the future value of various alternatives will be computed. Then the alternative with the maximum future worth of revenue or with minimum future worth of cost will be selected as the best alternative for implementation.

WORKED PROBLEMS

1. Consider the following two mutually exclusive alternatives.

Alterations	0	1	2	3	4
X(Rs.)	- 2,50,000	1,00,000	1,00,000	1,00,000	1,00,000
Y(Rs.)	- 3,00,000	1,40,000	1,10,000	90,000	1,00,000

Select the better alternative based on the future-worth method of comparison. If rate of interest is 9.75%, compounded annually.

Solution: The negative values given in the box represent investments or cash outflow while all others are revenues or inflow. In this problem the net future worth of both alternatives are found and then compared to select the best one. Also please note that, compound interest tables cannot be used in this problem since rate of interest ($i = 9.75\%$) is not accommodated in the interest tables. Therefore only formula have to be used.

To find Future Worth of Alternative X

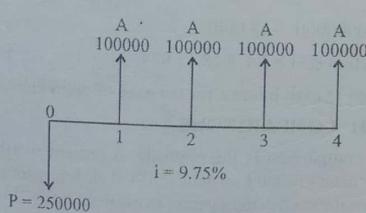
Given data, Initial investment, $P = \text{Rs.}2,50,000 \rightarrow (\text{Cost})$

Uniform revenue for all four years, $A = \text{Rs.}1,00,000 \rightarrow (\text{Revenue})$

Rate of interest, $i = 9.75\%$

Service life, $n = 4$ years.

The CFD for the above problem would be



$$\begin{aligned} FW(\text{Alternative X}) &= FW(\text{All revenues}) - FW(\text{All Costs}) \\ &= FW(\text{uniform annual revenue } A) - FW(\text{initial cost}) \\ &= (\text{To find } F \text{ given } A) - (\text{To find } F \text{ given } P) \\ &= A(F/A, i, n) - P(F/P, i, n) \end{aligned}$$

$$\begin{aligned} &= A \left[\frac{(1+i)^n - 1}{i} \right] - P(1+i)^n \\ &= 1,00,000 \left[\frac{(1+0.0975)^4 - 1}{0.0975} \right] - 2,50,000(1+0.0975)^4 \\ &= 4,62,395 - 3,62,708 \end{aligned}$$

$$FW(\text{Alt X}) = \text{Rs.}99,686.$$

To find Future Worth of Alternative Y

Given Data,

$$P = 3,00,000 \text{ (Cost)}$$

Annual revenue for First year, $A_1 = 1,40,000 \rightarrow (\text{Revenue})$

$$A_2 = 1,10,000 \rightarrow (\text{Revenue})$$

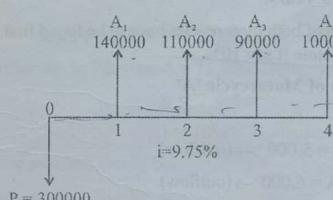
$$A_3 = 90,000 \rightarrow (\text{Revenue})$$

$$A_4 = 1,00,000 \rightarrow (\text{Revenue})$$

$$i = 9.75\%$$

$$n = 4 \text{ years}$$

The CFD for the above case would be



In this case, the revenue for each year is different. Hence we have to find the future worth of each year's revenue individually.

$$\begin{aligned} FW(\text{Alternative Y}) &= FW(\text{Revenues}) - FW(\text{Costs}) \\ &= \{FW(\text{Revenue in 1st year}) + FW(\text{Revenue in 2nd year}) \\ &\quad + FW(\text{Revenue in 3rd year}) + FW(\text{Revenue in 4th year})\} \\ &\quad - \{FW(\text{Initial cost})\} \end{aligned}$$

$$\begin{aligned}
 &= \{(To \text{ find } F \text{ given } P(A_1) \text{ at } n = 3) + (To \text{ find } F \text{ given } P(A_2) \text{ at } n = 2) + (To \text{ find } F \text{ given } P(A_3) \text{ at } n = 1) + (To \text{ find } F \text{ given } P(A_4) \text{ at } n = 0)\} - (To \text{ find } F \text{ given } P \text{ at } n = 4) \\
 &= A_1(F/P, i, n) + A_2(F/P, i, n) + A_3(F/P, i, n) + A_4 - P(F/P, i, n) \\
 &= A_1(1+i)^n + A_2(1+i)^n + A_3(1+i)^n + A_4 - P(1+i)^n \\
 &= 1,40,000(1 + 0.0975)^3 + 1,10,000(1 + 0.0975)^2 + \\
 &\quad 90,000(1 + 0.0975)^1 + 1,00,000 - 3,00,000(1 + 0.0975)^0 \\
 &= 5,16,343 - 4,35,250
 \end{aligned}$$

$FW(\text{Alt Y}) = \text{Rs.}81,093$

Answer: Comparing the future worths of Alternatives X and Y, it is evident that Alternative X makes more profit than alternative Y. Therefore alternative X is suggested.

Example:

2. Two motorcycles of brand 'A' and 'B' are available on the following terms:
- (i) Motorcycle 'A' - Make a down payment of Rs.5,000 and then Rs.6,000 at the end of each year for 7 years.
 - (ii) Motorcycle 'B' - Make a down payment of Rs.15,000 and no payment for the next 3 years. From end of the 4th year annual payments of Rs.12,000 for the next 3 years. (VTU Jan 2006)

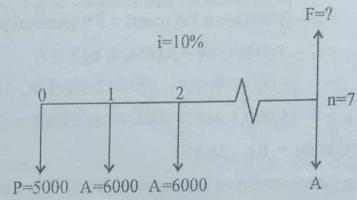
Solution: Future Worths of both motorcycles have to be found first. Since rate of interest is not given, we can assume it say 10%.

To find Future Worth of Motorcycle 'A'

Given:

$$\begin{aligned}
 \text{Down Payment, } P &= 5,000 \rightarrow (\text{outflow}) \\
 \text{Annual payment, } A &= 6,000 \rightarrow (\text{outflow}) \\
 \text{No. of years, } n &= 7 \text{ years} \\
 \text{Rate of interest, } i &= 10\% \rightarrow (\text{assumed})
 \end{aligned}$$

The CFD for the above problem would be:



$$\begin{aligned}
 FW(\text{Motorcycle A}) &= FW(\text{Inflow}) - FW(\text{Outflow}) \\
 &= 0 - [FW(\text{Down Payment}) + FW(\text{Annual payment})] \\
 &= 0 - [(To \text{ find } F \text{ given } P) + (To \text{ find } F \text{ given } A)] \\
 &= 0 - P(F/P, i, n) - A(F/A, i, n) \\
 &= -5,000(F/P, 10\%, 7) - 6,000(F/A, 10\%, 7) \\
 &= -5,000(1.9487) - 6,000(9.4872)
 \end{aligned}$$

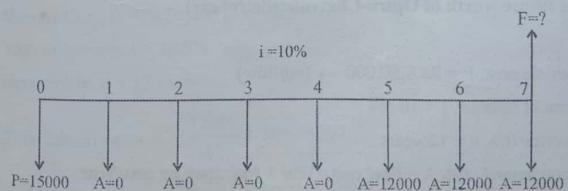
$FW(\text{Motorcycle A}) = \text{Rs.} - 66,666$

To find Future Worth of Motorcycle B

Given,

$$\begin{aligned}
 \text{Down Payment, } P &= 15,000 \rightarrow (\text{outflow}) \\
 \text{Annual payment for 4th to 7th year, } A &= 12,000 \rightarrow (\text{outflow}) \\
 n &= 7 \text{ years} \\
 i &= 10\% \rightarrow (\text{assumed})
 \end{aligned}$$

The CFD for the above problem would be:



$$\begin{aligned}
 FW(\text{Motorcycle B}) &= FW(\text{Inflow}) - FW(\text{Outflow}) \\
 &= 0 - [FW(\text{Down Payment}) + FW(\text{Annual payment for 3 years})] \\
 &= 0 - P(F/P, i, n) - A(F/A, i, n) \\
 &= -15,000(F/P, 10\%, 7) - 12,000(F/A, 10\%, 3) \\
 &= -15,000(1.9487) - 12,000(3.3100)
 \end{aligned}$$

FW(Motorcycle B) = Rs. -68,950

Answer: Comparing future worths of Motorcycles A and B, we see that Motorcycle A has a lower cost. Therefore it is selected.

Example:

3. A software engineer wanting to buy a car has two models in his mind, a petrol run Optra Chevrolet or a diesel run Skoda Octavia. The comparative data are as follows.

Particulars	Optra-Chevrolet (Petrol) (Rs.)	Skoda Octavia (Diesel) (Rs.)
1. Vehicle cost	8,50,000	10,00,000
2. Fuel cost per litre	58	36
3. Mileage in km/litre	8 km	11 km
4. Distance travelled per year	12000	12000
5. Annual maintenance cost	18000	36000
6. Expected life	12	12
7. Salvage value	400000	400000

If the software engineer considers 10.5% as good value for money, Suggest him the better option.

Solution:

Note: To solve this problem, all costs have to be first classified as initial cost or annual costs and then grouped together.

To find future worth of Optra-Chevrolet(Petrol car)

Given:

Vehicle cost, $P = \text{Rs. } 8,50,000 \rightarrow (\text{outflow})$

Rate of interest, $i = 10.5\%$

Service life, $n = 12$ years

Total annual cost, $A = \text{Fuel costs/Year} + \text{Maintenance cost/Year}$

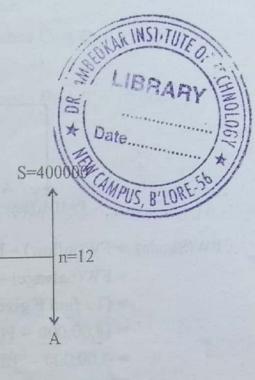
$$A = \frac{\text{Distance travelled}}{\text{Mileage}} \times \text{Litres of petrol} + M.Cost/Year$$

$$A = \frac{12000}{8} \times 58 + 18000$$

$A = \text{Rs. } 1,05,000 \rightarrow (\text{outflow})$

Salvage Value, $S = \text{Rs. } 4,00,000 \rightarrow (\text{inflow})$

The CFD for the above problem would be



$$\begin{aligned}
 \therefore FW(\text{Optra}) &= FW(\text{Inflow}) - FW(\text{Outflow}) \\
 &= FW(\text{Salvage}) - [FW(\text{initial cost}) + FW(\text{Annual cost})] \\
 &= 4,00,000 - [\text{To find F given P} + \text{To find F given A}] \\
 &= 4,00,000 - P(F/P, i, n) - A(F/A, i, n) \\
 &= 4,00,000 - 8,50,000(F/P, 10.5\%, 12) - 1,05,000(F/A, 10.5\%, 12) \\
 &= 4,00,000 - 8,50,000(3.3140) - 1,05,000(22.0377) \\
 &= 4,00,000 - 28,570 - 22,794 \\
 &= \text{Rs. } -47,30,858
 \end{aligned}$$

To find Future Worth of Skoda Octavia(Diesel Car)

Given:

Vehicle Cost, $P = 10,50,000$ (outflow)

Rate of interest, $i = 10.5\%$

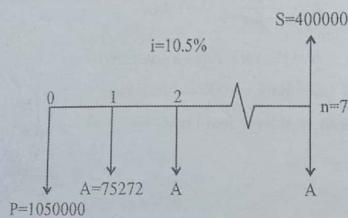
Service life, $n = 12$ years

$$\text{Total annual costs, } A = \frac{12000}{11} \times 36 + 36000$$

$A = \text{Rs. } 75,272 \rightarrow (\text{outflow})$

Salvage Value, $S = \text{Rs. } 4,00,000 \rightarrow (\text{inflow})$

CFD for the above problem would be



$$\begin{aligned} \therefore FW(\text{Skoda}) &= FW(\text{inflow}) - Fw(\text{outflow}) \\ &= FW(\text{Salvage}) - [FW(\text{Vehicle cost}) + FW(\text{Annual cost})] \\ &= (\text{To find } F \text{ given } F) - (\text{To find } F \text{ given } P) - (\text{To find } F \text{ given } A) \\ &= [4,00,000 - P(F/P, i, n)] - A(F/P, i, n) \\ &= 4,00,000 - 10,50,000(3.3140) - 75,272(22.0377) \\ \text{FW(Skoda)} &= \text{Rs. } -47,38,521 \end{aligned}$$

Answer: Comparing the FW of the 2 cars Optra and Skoda, the cost of Optra is less. Hence it is suggested to buy Optra.

Example:

4. A crane can be taken on lease for a project for 3 years for Rs.180,000 payable now, maintenance included. It can also be purchased for Rs.240,000 and be sold at the end of 3 years for Rs.100,000. Maintenance costs are expected to be Rs.5,000 per year for the first two years and Rs.10,000 for the third year payable at the end of each year. At what interest rates would the two alternatives be equivalent? (VTU July 2006)

Explanation:- The problem describes the cash flows of a crane which can either be purchased or taken on a lease. But here, we have to find the rate of interest (i) at which both purchase and lease proposals become equal. To find i we can equate either their present worths or future worths or equivalent worths. Let us consider future worth in this case.

Given:

Lease Proposal

Lease amount, $P = 1,80,000$

Lease Period, $n = 3$ years

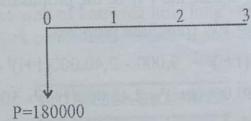
Purchase proposal

Purchase price, $P = 2,40,000$

Salvage value, $S = 1,00,000$

Maintenance Cost $C_1 = 5,000$
in first year,
Maintenance Cost $C_2 = 5,000$
2nd year
Maintenance Cost $C_3 = 10,000$
3rd year

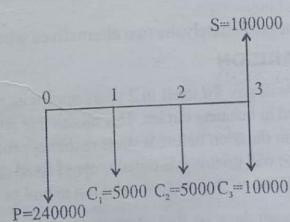
The CFD for the lease proposal would be



Future worth of Lease proposal

$$\begin{aligned} FW(\text{lease Proposal}) &= Fw(\text{revenues}) - Fw(\text{costs}) \\ &= 0 - (\text{To find } F \text{ given } p) \\ &= -p(F/P, i, n) \\ &= -1,80,000(1+i)^3 \quad (1) \end{aligned}$$

The CFD for the purchase proposal would be



Future worth of Purchase proposal

$$\begin{aligned} FW(\text{purchase Proposal}) &= Fw(\text{revenues}) - Fw(\text{costs}) \\ &= \{Fw(\text{salvage value})\} - \{Fw(\text{purchase price}) \\ &\quad + Fw(\text{1st year Maint. Cost}) + Fw(\text{2nd year M.Cost}) \\ &\quad + Fw(\text{3rd year Maint. Cost})\} \end{aligned}$$

$$\begin{aligned}
 &= \{(\text{To find } F \text{ given } F)\} - \{(\text{To find } F \text{ given } P) \\
 &\quad + (\text{To find } F \text{ given } P) + (\text{To find } F \text{ given } p) \\
 &\quad + (\text{To find } F \text{ given } F)\} \\
 &= 1,00,000 - 2,40,000(1+i)^3 - 5,000(1+i)^2 - 5000(1+i) \\
 &\quad - 10,000 \\
 F_w (\text{Purchase proposal}) &= 90,000 - 2,40,000(1+i)^3 - 5,000(1+i)^2 - \\
 &\quad 5,000(1+i) \quad (2)
 \end{aligned}$$

To find i we have to equate the F_w of the proposals,

$$\begin{aligned}
 F_w (\text{lease Proposal}) &= F_w (\text{purchase proposal}) \\
 -1,80,000(1+i)^3 &= 9,000 - 2,40,000(1+i)^3 - 5000(1+i)^2 - 5000(1+i) \\
 90,000 + 1,80,000(1+i)^3 &- 2,40,000(1+i)^3 - 5000(1+i)^2 - 5000(1+i) = 0
 \end{aligned}$$

We can solve for i only by trial and error method

Let $i = 0.1$, LHS = -1410

Let $i = 0.09$, LHS = 907.76

By applying interpolation

$$0.09 + \left[\frac{907 - 0}{907 - (-1410)} \right] \times (1 - 0.09) \\
 i \approx 0.094$$

Answer: At $i = 9.4\%$ approximately, the two alternatives would be equal.

PAY-BACK COMPARISON

"This investment will pay for itself in 2 years approximately / $2\frac{1}{2}$ years / n years" is a typical comment heard in business circles. This means that any investment made in any business will take a certain duration before it starts realizing profit. In other words, in any business, the initial income will go towards initial costs of fixed capital and working capital as well as interests calculated on this, before enjoying actual profit.

In order to find a rough duration that an investment takes to pay for itself is known as payback period. Payback period is nothing but an estimate of the elapsed time before a proposed investment begins to show profit. A simple formula to find an approximate payback period is

$$\text{Payback period} = \frac{\text{required investment}}{\text{annual receipts} - \text{annual expenditure}}$$

$$\text{Payback period} = \frac{P}{R - C}$$

$$\text{or Payback period} = \frac{\text{First cost}}{\text{Net annual savings}}$$

Payback periods could be used to compare alternatives. Such a comparison would be a very simple and direct way of knowing how long will it take before the amount invested is recovered. Payback comparisons usually ignore effects of both depreciation as well as interests.

WORKED EXAMPLE

- Following are the estimates of three alternative investments made on 3 different machines in an industry. Find out which machine has the fastest payback period.

	Particulars	Machine A	Machine B	Machine C
1.	Initial Investment	30,000	38,000	42,000
2.	Annual receipts	20,000	23,500	26,000
3.	Annual expenditures	5,500	6,500	7,000
4.	Economic life	4 years	4 years	4 years

Solution:

Machine A

Given

$$\text{Required Investment } P = 30000$$

$$\text{Annual receipts } R = 20000$$

$$\text{Annual expenditure } C = 5500$$

Economic life, $n = 4$ years (redundant information)

$$\therefore \text{Payback period} = \frac{\text{Required investment}}{\text{Annual receipts} - \text{Annual expenditure}} = \frac{P}{R - C} \\
 = \frac{30000}{20000 - 5500} \\
 = 2.06 \text{ years}$$

Machine B

$$P = \text{Rs.}38000$$

$$R = \text{Rs.}23500$$

$$C = \text{Rs.}6500$$

$$\text{Payback period} = \frac{P}{R-C} = \frac{38000}{23500-6500}$$

Payback period = 2.23 years

Machine C

$$P = \text{Rs.}42000$$

$$R = \text{Rs.}26000$$

$$C = \text{Rs.}6500$$

$$\text{Payback period} = \frac{P}{R-C} = \frac{42000}{26000-7000} = 2.21 \text{ years}$$

Answer:

Payback period of machine A is the least

**SUMMARY**

1. In PW Comparison, the present worths of all business alternatives are reduced to time zero for comparison. The net PW of that alternative which has a highest algebraic value is selected over the others.

2. Conditions for Present Worth Comparison

- i) All cash flows are known
- ii) Inflation is not considered
- iii) Interest rates are known
- iv) Comparisons are made prior to tax
- v) Intangible factors not considered
- vi) Constant availability of funds assumed

3. Present Worth Equivalence

It is the same as present worth. PWE is nothing but finding an equivalent worth of all future transactions of various business alternatives.

4. Net present worth

When costs and revenues are both involved in cash flows, the net present worth is $\text{NPW} = \text{PW}\{\text{revenues}\} - \text{PW}\{\text{cost Rs}\}$. Obviously NPW has to be a +ve value for the business alternative to be viable.

5. Assets with Unequal lives

Comparison of alternatives having varying business lives are done by

i. **Common Multiple Method** : In this method, the cash flows of all alternatives are projected over a common time frame, which is nothing but the least common multiple of the different business lives.

ii. **Study Period Method** : In this, the alternative with the shortest life is chosen and all other alternatives are compared against this.

6. Assets having infinite lives

Assets like bridges, tunnels, roads are considered to have infinite lives. In such long-lived assets, Capitalized costs are calculated which is given by

$$\text{Capitalized cost} = \text{Initial cost (P)} + \frac{\text{Annual Maintenance costs (TAC)}}{\text{Rate of Interest (i)}}$$

9. Future Worth Comparison

In this method, cash flows of various business alternatives are projected to the end of the business lives of alternatives for comparison.

12. Pay-back comparison

Each business alternative, when started, has to overcome initial costs before realizing a profit. In other words, business projects take a certain time, known as pay-back period, to first break-even and then to register profits. This pay-back period is roughly calculated as

$$\text{Payback Period} = \frac{\text{Initial Cost}}{\text{Annual Revenues} - \text{Annual expenditures}}$$

EXERCISE PROBLEMS

1. A person plans to build a hospital. The plan involves an initial investment of Rs. 3 crores with the following transactions for the next five years. If he decides to stop the work and sell the building to start a new business, the salvage value he gets is Rs. 40 lacs. Draw the cash flow diagram and find the present worth by assuming $i = 8\%$ compounded annually.

End of year	Maintenance & operating expenses(Rs.)	Revenue (Rs.)
1	5,00,000	50,00,000
2	6,50,000	55,00,000
3	6,00,000	60,00,000
4	7,00,000	65,00,000

2. Find the PW of the given cash flow series at $i = 9\%$ compounded annually.

End of year	0	1	2	3	4	5	6	7
Cash flow Rs.	-5000	15000	15000	15000	15000	15000	15000	15000

3. Maruti Udyog has come out with an offer of taking home ZEN at a down payment of Rs. 1,75,000. Alternatively the car can be taken home by giving an initial amount of Rs. 20,000 and an equal yearly amount of Rs. 15,000 for 15 years at a rate of interest of 5%, compounded annually. Suggest the best alternative for a customer to buy the car by PW method.

4. Merindus Consultancy has two alternatives to satisfy its travelling requirements.

Alternative (1) Renting a vehicle at cost of Rs. 30,00,000 per year.

Alternative (2) Buying a vehicle at Rs. 8,00,000 with an operating cost of Rs. 5,00,000 per year. The salvage value of the vehicle after five years is Rs. 3,00,000.

Suggest best alternative based on present worth method of comparison using an interest rate of 10% compounded annually.

5. A new piece of material handling equipment costs Rs. 1,25,000 and is expected to save Rs. 20,000 at the end of first year of operation. Maintenance and operating cost increases are expected to reduce the net savings by Rs. 5,000 per year for each additional year of operation until the equipment is worn out at the end of 8 years. Determine the net present worth of the equipment at an interest rate of 7%.

6. Two machines are under consideration by a metal fabricating company. Machine A will have a first cost of Rs. 1,50,000/- an annual maintenance and operation cost of Rs. 30,000/- and a Rs. 30,000/- salvage value. Machine B will have a first cost of Rs. 2,20,000/- an annual cost of Rs. 15,000/- and a Rs. 50,000/- salvage value.

i) If both the machines are expected to last for 10 years, determine which machine should be selected on the basis of present-worth comparison using an interest rate of 12% per year?

ii) If machine A has an expected life of 6 years and machine B an expected life of 9 years which machine should be selected? Use a interest rate of 12% per year.

7. The machines A & B have same service life of 5 years. The other expenses are given below. If the money is worth 10% p.a, which machine is more economical ?

Cash flows	M/c A	M/c B
First cost	2,50,000	1,50,000
Uniform end of year maintenance	2,000	4,000
Overhaul at the end of 3rd year	-	3,000
Salvage value	10,000	-
Yearly savings on account of less inspection	1,000	-

8. Machine A costs Rs. 12,000, no salvage value at the end of 10 years of useful life and annual expenses of Rs. 2,200. The Machine B costs Rs. 40,000 now and has an expected salvage value of Rs. 10,000 at the end of 25 years. Annual expenses of Machine B are Rs. 1,000. Compare the two alternatives on the basis of their present worth using repeated project assumption at 15% annual interest. Use CFD for your analysis.

9. Two type of trucks are available for transportation use. They are needed for 10 years. The details are :

	Truck A	Truck B
First cost	Rs. 10,00,000	Rs. 15,00,000
Estimated annual maintenance cost	Rs. 20,000	Rs. 15,000
Estimated life	5 years	10 years
Estimated salvage value	Rs. 2,00,000	Rs. 5,00,000

10. A company wishes to install a small plastic moulding press. Two different presses are available with the details given :

	Presses	
	A	B
Investment (Rs.)	6,000	7,600
Economic life (years)	5	5
Annual disbursements :	Rs.	Rs.
Power	680	680
Labour	6,600	6,000
Maintenance	400	450
Property taxes	120	152
Total annual disbursement	7800	7282

11. Data for 3 alternate plans are as follows :

Alternative	Investment	Salvage value	Life	Annual net cash flow
X	6,000	0	3	2,600
Y	12,000	3,000	6	2,500
Z	18,000	0	6	4,000

If MARR is 10% which plan is to be selected based on present worth method ?

12. A company borrowed Rs. 1,00,000 to finance a new product. The loan was for 20 years at a nominal interest rate of 8 percent compounded semiannually. It was to be repaid in 40 equal payments. After one-half the payments were made, the company decided to pay the remaining balance in one final payment at the end of the 10 year. How much was owed ?
13. A proposed improvement in an assembly line will have an initial purchase and installation cost of Rs. 1,75,000. The annual maintenance cost will be Rs. 6,000; periodic overhauls once every 3 years, excluding the last year of use, will cost Rs. 11,500 each. The improvement will have a useful life of 9 years, at which time it will

have no salvage value. What is the present worth of the 9-year costs of the improvement at $i = 8$ percent ?

14. A small dam and an irrigation system are expected to cost Rs. 3,00,000. Annual maintenance and operating costs are expected to be Rs. 40,000 the first year and will increase at a rate of 10 percent over a 30-year life.
15. It is estimated that additional reinforcement of the dam in Prob. 3.9 in the first year at a cost of Rs. 65,000 will reduce annual maintenance costs to Rs. 25,000 in the first year with an increase of 5 percent per year over the rest of the 30-year life. Should the civil engineer in charge of the project seriously consider the proposal to do the reinforcing ?
16. A newly developed electric car will cost Rs. 21,000 to purchase. Operating and maintenance costs, including home charging of the batteries, are estimated to be Rs. 350 for the first year with annual increases thereafter of Rs. 50 per year. Salvage value after 5 years is estimated to be Rs. 6,500. A new gasoline runabout will cost Rs. 16,000 and will average 30 miles per gallon. Gasoline costs Rs. 1.26 per gallon and is expected to increase at a rate of Rs. 0.05 per year for each of the next 4 years. Maintenance costs are estimated to be Rs. 300 per year including warranty coverage. Salvage value is estimated to be Rs. 1,500 after 5 years of service. If the vehicles are expected to be driven for 20,000 miles per year, determine which option will have the lower cost over 5 years. Use present-worth analysis with a 10 percent rate of interest.
17. A commercial rental property is for sale at Rs. 1,00,000. A prospective buyer estimates that the property would be held for 12 years, at the end of which it could be sold for Rs. 90,000. During the ownership period, annual receipts from rentals would be Rs. 15,000 and average disbursements for all purposes in connection with ownership would be Rs. 6,000. If a rate of 9 percent is expected, what is the maximum bid that the prospective purchaser should make to buy the property ?
18. A company is considering the purchase of a new piece of testing equipment that is expected to produce Rs. 8,000 additional income during the first year of operation; this amount will probably decrease by Rs. 500 per year for each subsequent year of ownership. The equipment costs Rs. 20,000 and will have an estimated salvage value of Rs. 3,000 after 8 years of use. For an interest rate of 15 percent, determine the net present worth of this investment. Neglect taxes in your computations.
19. A refining company entered into a contract for raw materials with an agreement to pay Rs. 6,00,000 now and Rs. 1,50,000 per year beginning at the end of the fifth year. The contract was made for 10 years. At the end of the third year, because of unexpected profits, the company requested that it be allowed to make a lump-sum payment in advance for the rest of the contract. Both parties agreed that 7

- percent compounded annually was a fair interest rate. What was the amount of the lump sum?
20. A machine can be repaired today for Rs. 2,000. If repairs are not made, the operating expenses will increase by Rs. 200 each year for the next 5 years. Assume that the expenses will occur at the end of each year and that the machine will have no value under either alternative at the end of the 5-year period. the minimum acceptable rate of return is 12 percent. Compare the present worths of the two alternatives.
 21. The following alternatives are available to accomplish an objective of 12 years duration.

	Plan A	Plan B	Plan C
Life cycle	6 years	3 years	4 years
First cost	Rs. 2,000	Rs. 8,000	Rs. 10,000
Annual cost	Rs. 3,200	Rs. 700	Rs. 500

Compare the present worth of the alternatives, using an interest rate of 7 percent.

22. The lining of a chemical tank must be replaced every 3 years at a cost of Rs. 3,500. a new type of lining is available that is more resistant to corrosion. The new lining costs Rs. 5,100. If the minimum rate of return required is 12 percent and taxes and insurance are 4 percent of the first cost annually, how long must the improved lining last to be more economical than the present lining ?
23. A wealthy industrial economist dies, and her will specifies that Rs. 5 million of her estate will go to Arizona State University (ASU) to fund a small engineering economy building as well as 20 graduate scholarships per year for the first year and should increase at a rate of Rs. 1,500 per year over the following 19 years. ASU requires that Rs. 15,000, starting with the third year of the bequest be reserved for building maintenance and operating costs. These costs are to have a linear increase of Rs. 2,000 per year, starting with year 4. Assuming that a 10 percent interest rate is used for such analyses, determine how much will be available for building first costs.
24. A bond with a face value of Rs. 5,000 pays quarterly interest of 1.5 percent each period. Twenty-six interest payments remain before the bond matures. How much would you be willing to pay for this bond today, if the next interest payment is due now and you want to earn 8 percent compounded quarterly on your money ?

□ □ □

CHAPTER 3

EQUIVALENT ANNUAL WORTH COMPARISONS

INTRODUCTION

Any business would involve an initial investment followed by a series of transactions in the form of revenues and expenditures. One of the methods of comparing the worthiness of various businesses and their transactions is to find their equivalent annual worth. As we have already seen in chapter 1, equivalent annual worth represents the average annual value of any business with respect to its entire business life. In other words, an annual worth is representative of all the receipts and disbursements occurring over a period of time and converted into an equivalent uniform yearly amount.

Annual worth comparison is a popular method because of the wide-spread inclination to view a year's gains and losses as a yardstick of progress. Cost accounting procedures, depreciation expenses, tax calculations, and other summary reports are usually annual in nature . These yearly costs tabulations make use of annual worth method of comparison since it is the best method to understand and present facts and figures, than other comparison methods.

The assumption that are made for present worth method of comparison also apply to basic annual worth comparison, i.e.,

- i Cash flows and interest rates are known
- ii Cash flows are before taxes
- iii Free flow of finance is assumed
- iv Intangible factors are not considered

In annual equivalent method of comparison, first annual equivalent cost of the revenue of each alternative will be computed. Then the alternative with the maximum annual equivalent worth is selected as the best alternative, as in the previous chapter.

COMPARISON BY EQUIVALENT ANNUAL WORTH

The following points may be noted while solving problems involving equivalent annual worth comparison.

1. The terms - equivalent annual worth (EAW) or annual equivalent worth (AEW) or simply annual worth, (AW) are all one and the same.
2. Equivalent annual worth comparison is just another way of comparing project alternatives, like Present Worth or future worth comparisons. In fact, all three methods suggest the same alternative, if cash flows remains the same.

3. The only difference in EAW comparison is that all the values of costs and revenues should be converted into annual values before being grouped together.
4. Please note that certain costs and revenues are already given as annual values and there is no need for annualization and can be retained as they are.
5. When comparing business alternatives, that alternative which has a higher algebraic value of Annual Worth is chosen, as before. If all values are -ve, the least is chosen, again as before.
6. Lastly, in EAW comparison, we need not use LCM method while comparing alternatives with varying service lives. In other words, we can compare the annual worths of various projects even if they have different service lives even without taking LCM. This is the biggest advantage of this method. (See page no. 116 for an explanation).

WORKED EXAMPLES**Example 1:**

A publishing house wants to purchase an offset printing press. Three dealers have responded to the tender called whose particulars are given in the table. Determine the best alternative based on the annual equivalent method by assuming $i = 12.5\%$, compounded annually.

Manufacturer	Down payment (Rs.)	Yearly equal instalment(Rs.)	Salvage Value (Rs.)	No. of years of payment and life
X	8,00,000	2,25,000	0	10
Y	7,00,000	2,00,000	0	10
Z	5,00,000	2,50,000	0	10

Solution: We have to find the annual equivalent worth of each alternative and then compare them.

To find Annual equivalent worth of Manufacturer X

Given data:

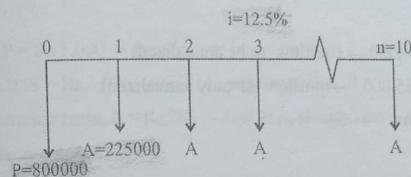
Down payment, $P = \text{Rs.} 8,00,000 \rightarrow$ (outflow, to be annualized)

Yearly instalment, $A = \text{Rs.} 2,25,000 \rightarrow$ (outflow, already annualized)

Service life, $n = 10$ years

Rate of interest, $i = 12.5\%$

The CFD for the above problem would be



$$\text{AEW(Manf. X)} = \text{AE(all inflow)} - \text{AE(all outflow)}$$

$$\begin{aligned} &= \text{Zero} - [\text{AE(down payment)} + \text{AE(yearly instalments)}] \\ &= 0 - (\text{To find } A \text{ given } P) - (\text{To find } A \text{ given } A) \\ &= -P(A/P, i, n) - A \\ &= -8,00,000(A/P, 12.5\%, 10) - 2,25,000 \\ &= -8,00,000(0.1806) - 2,25,000 \end{aligned}$$

$$\text{AEW(Manf. X)} = \text{Rs.} - 3,69,480$$

To find AEW of Manufacturer Y

Given:

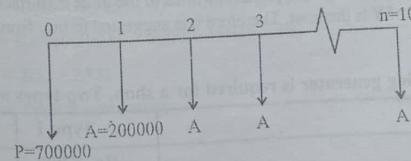
$$P = 7,00,000 \rightarrow (\text{outflow-to be annualized})$$

$$A = 2,00,000 \rightarrow (\text{outflow-already annualized})$$

$$n = 10 \text{ years}$$

$$i = 12.5\%$$

The CFD for the above problem would be



$$\text{AEW(Manf. Y)} = \text{AE(inflow)} - \text{AE(outflow)}$$

$$\begin{aligned} &= 0 - \{P(A/P, i, n) + A\} \\ &= 0 - 7,00,000(A/P, 12.5\%, 10) - 2,00,000 \\ &= -7,00,000(0.1806) - 2,00,000 \end{aligned}$$

$$\text{AEW(Manf. Y)} = \text{Rs.} - 3,26,420$$

To find AEW of Manufacturer Z

Given:

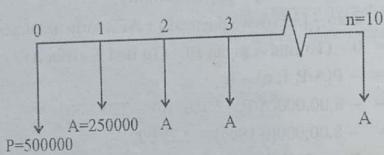
$$P = \text{Rs. } 5,00,000 \rightarrow (\text{outflow - to be annualized})$$

$$A = \text{Rs. } 2,25,000 \rightarrow (\text{outflow - already annualized})$$

$$n = 10 \text{ years}$$

$$i = 12.5\%$$

The CFD for the above problem is



$$\text{AEW(Manf.Z)} = \text{AE(inflow)} - \text{AE(outflow)}$$

$$= 0 - \text{AE(of } P) - \text{AE(of } A)$$

$$= -P(A/P, i, n) - A$$

$$= -5,00,000(A/P, 12.5\%, 10) - 2,50,000$$

$$= -5,00,000(0.1806) - 2,50,000$$

$$\text{AEW(Manf.Z)} = \text{Rs.} - 3,40,300$$

Answer: Comparing the Annual equivalent worths of the three manufacturers X, Y and Z, we see that AEW of Y is the least. Therefore it is suggested to buy from manufacturer Y.

Example 2

A Standby lighting generator is required for a shop. Two types are available.

	Type 1	Type 2
First cost	Rs. 5,000	Rs. 3,200
Salvage value	Rs. 1,000	Nil
Annual Operating costs	Rs. 780	Rs. 950

If both generators have a life of 4 years and the interest rate is 15 percent per year, which offers the lowest equivalent annual cost. (VTU Jan 2006)

To find AEW of Type 1 generator

Given,

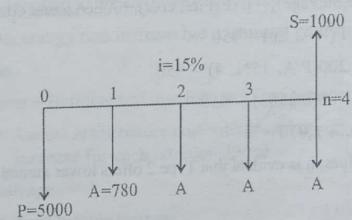
$$\text{First cost, } P = \text{Rs. } 5,000 \rightarrow (\text{outflow, to be annualized})$$

$$\text{Salvage cost, } S = \text{Rs. } 1,000 \rightarrow (\text{inflow, to be annualized})$$

$$\text{Annual operating costs, } A = \text{Rs. } 780 \rightarrow (\text{outflow, already annualized})$$

$$\text{Service life, } n = 4 \text{ years}$$

$$\text{Rate of interest, } i = 15\%$$



$$\text{AEW(Type 1)} = \text{AE(all inflow)} - \text{AE(all outflow)}$$

$$= [\text{AE(Salvage)}] - [\text{AE(First cost)} + \text{AE(Annual costs)}]$$

$$= [(To \text{ find } A \text{ given } F)] - [(To \text{ find } A \text{ given } P) + (To \text{ find } A \text{ given } A)]$$

$$= F(A/F, i, n) - P(A/P, i, n) - A$$

$$= 1000(A/F, 15\%, 4) - 5000(A/P, 15\%, 4) - 780$$

$$= 1000(0.2003) - 5000(0.3503) - 780$$

$$\text{AEW(Type 1)} = \text{Rs.} - 2331$$

To find AEW of Type 2 generator

Given,

$$P = \text{Rs. } 3200 \rightarrow (\text{outflow, to be annualized})$$

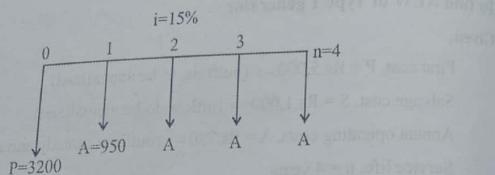
$$S = \text{Nil}$$

$$A = \text{Rs. } 950 \rightarrow (\text{outflow, already averaged})$$

$$n = 4 \text{ years}$$

$$i = 15\%$$

$$\text{Salvage Value, } S = 0 \text{ (Nil)}$$



$$\begin{aligned}AEW(\text{Type 2}) &= AE(\text{inflow}) - AE(\text{outflow}) \\&= [AE(\text{Salvage})] - [AE(\text{First cost}) + AE(\text{Annual costs})] \\&= 0 - P(P/A, i, n) - 950 \\&= -3,200(P/A, 15\%, 4) - 950 \\&= -3,200(0.3503) - 950\end{aligned}$$

$$AEW(\text{Type 2}) = \text{Rs. } -2,070$$

Answer: From the values, it is evident that Type 2 offers lower annual equivalent cost than Type 1.

Example 3

A food and beverage company is planning expansion of its cold storage facility. Three alternate site design proposals are being considered that uses MARR at 10%. Plans A and B require an expenditure of Rs. 35,00,000 for land which retain its value for 10 years, While plan C requires Rs. 42,50,000 for land which will also retain its value for 10 years. The estimated income increase due to facility available is annualized at Rs. 24,80,000 per year. The company requires that a life of 10 years be used for analysis. Data pertaining to the project are given in the table below.

	Proposal A	Proposal B	Proposal C
Building and Installation	60,00,000	70,00,000	40,00,000
Compressors	10,00,000	13,50,000	8,50,000
Expected energy cost first year	6,50,000	4,80,000	6,50,000
Energy cost increase for each additional year	30,000	20,000	35,000
Annual Maintenance cost	2,00,000	1,50,000	5,00,000
Estimated salvage value	3,50,000	4,30,000	1,80,000

Evaluate which proposal to recommend using equated annual worth analysis.

(VTU July 2005)

Solution: The following points may be noted while solving the above problem:

1. MARR stands for minimum acceptable Rate of return. 10% of MARR simply means the rate of return or rate of interest is 10%
2. Plans A, B, C have their expenditure on land which is considered initial cost. The information that the land retains its value for 10 years is ignored.
3. The estimated income increase due to facility indicates revenue or cash inflow.
4. All the values that are given for the 3 proposals are first grouped as costs and revenues. Among them, costs are classified as initial costs or annual costs while revenues are classified as annual revenues or fixed revenues and then grouped.
5. Please note that energy cost increase has a Uniform gradient series.

In this problem given,

Initial cost = Land cost + Building and installation + Compressors

Total Annual costs = Annual maintenance cost + Expected energy Cost(1st year) + Cost increase for each additional year

Fixed Revenues = Salvage

Annual Revenue = Annual Income increase

To find Equated Annual Worth of Proposal A

Given,

$$\begin{aligned}\text{Initial Cost, } P &= 35,00,000 + 60,00,000 + 10,00,000 \\P &= 105,00,000 \rightarrow (\text{outflow, to be annualized})\end{aligned}$$

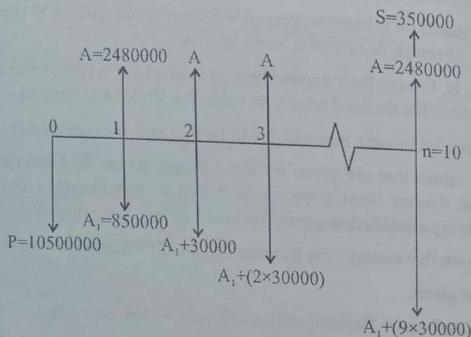
$$\begin{aligned}\text{Total Annual Costs in first year, } A_1 &= 2,00,000 + 6,50,000 \\A_1 &= 8,50,000\end{aligned}$$

Energy Cost increase per year, $G = 30,000 \rightarrow (\text{out flow, to be annualized})$

Income increase per year, $A = 24,80,000 \rightarrow (\text{inflow, already annualized})$

Salvage, $S = 3,50,000 \rightarrow (\text{inflow, to be annualized})$

The CFD for the above problem would be



$$\begin{aligned} \therefore \text{EAW(Proposal A)} &= \{\text{AE(all revenues)}\} - \{\text{AE(all costs)}\} \\ &= \{\text{AE(annual revenue } A\text{)} + \text{AE(Salvage value } S\text{)}\} \\ &\quad - \{\text{AE(initial cost, } P\text{)} + \text{AE(Maintenance cost + Energy cost)}\} \\ &= \{\text{(To find A given A)} + \text{(To find A given F)}\} \\ &\quad - \{\text{(To find A given P)} + \text{(To find A given G)}\} \\ &= A + S(A/F, i, n) - P(A/P, i, n) - [A_i + G(A/G, i, n)] \\ &= 24,80,000 + 3,50,000(A/F, 10\%, 10) - 105,00,000 \\ &\quad (A/P, 10\%, 10) - 8,50,000 - 30,000(A/G, 10\%, 10) \\ &= 24,80,000 + 3,50,000(0.0627) - 105,00,000(0.1627) \\ &\quad - 85,000 - 30,000(3.7255) \end{aligned}$$

$$\therefore \text{EAW(Proposal A)} = \text{Rs.} - 1,68,170$$

To find EAW of proposal B

Given,

$$\text{Initial cost, } P = 70,00,000 + 35,00,000 + 13,50,000$$

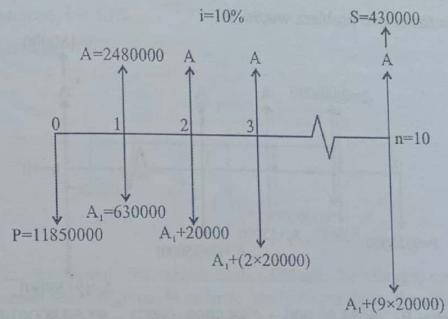
$$P = 1,18,50,000 \rightarrow (\text{outflow, to be annualized})$$

$$\begin{aligned} \text{Total annual costs(Maintenance + Energy), } A_i &= 1,50,000 + 4,80,000 \\ A_i &= 6,30,000 \end{aligned}$$

Energy cost increase per year, $G = 20,000 \rightarrow (\text{outflow, to be annualized})$

Income increase per year, $A = 24,80,000 \rightarrow (\text{inflow, already annualized})$

Salvage value, $S = 4,30,000 \rightarrow (\text{inflow, to be annualized})$



$$\begin{aligned} \therefore \text{EAW(Proposal B)} &= \{\text{AE(inflow)}\} - \{\text{AE(outflow)}\} \\ &= \{\text{AE(annual income)} + \text{AE(Salvage value } S\text{)}\} \\ &\quad - \{\text{AE(initial cost)} + \text{AE(Maintenance cost + Energy cost)}\} \\ &= \{\text{(To find A given A)} + \text{(To find A given F)}\} \\ &\quad - \{\text{(To find A given P)} + \text{(To find A given G)}\} \\ &= A + S(A/F, i, n) - P(A/P, i, n) - A_i - G(A/G, i, n) \\ &= 24,80,000 + 4,30,000(A/F, 10\%, 10) - 11,85,000 \\ &\quad (A/P, 10\%, 10) - 6,30,000 - 20,000(A/G, 10\%, 10) \\ &= 24,80,000 + 4,30,000(0.0627) - 11,85,000(0.1627) \\ &\quad - 6,30,000 - 20,000(3.7255) \end{aligned}$$

$$\text{EAW(Proposal B)} = \text{Rs.} - 1,25,544$$

To find EAW of proposal C

Given,

$$\begin{aligned} \text{Initial cost, } P &= 45,00,000 + 40,00,000 + 8,50,000 \\ P &= 93,50,000 \rightarrow (\text{outflow}) \end{aligned}$$

$$\begin{aligned} \text{Total annual cost (maintenance + Energy) in first year, } A_i &= 5,00,000 + 6,50,000 \\ A_i &= 11,50,000 \rightarrow (\text{outflow}) \end{aligned}$$

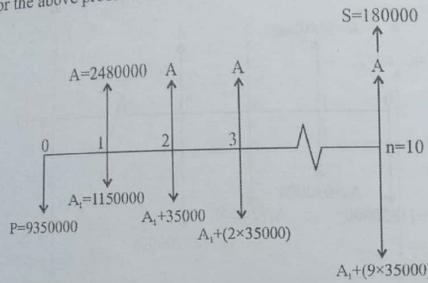
Energy cost increase per year, $G = 35,000$

Annual income increase, $A = 24,80,000 \rightarrow (\text{inflow})$

Salvage value, $S = 1,80,000 \rightarrow (\text{inflow})$

114 ENGINEERING ECONOMICS

The CFD for the above problem would be



$$\therefore \text{EAW(Proposal B)} = 24,80,000 + 1,80,000(0.0627) - 93,50,000(0.1627) \\ - 11,50,000 - 35,000(3.7255)$$

$$\text{EAW(Proposal C)} = \text{Rs.} - 3,10,351$$

Answer:

Comparing the equivalent worths of the three proposals, it is seen that the cost involved in proposal B is the least. Therefore B is selected.

Example 4:

'Megha' electronics is considering the purchase of a new programmable circuit to improve its product quality. The equipment has a first cost of Rs. 85,000 and the salvage value is predicted to be Rs 6,000 after a service life of 5 years. Maintenance and operating costs are expected to be Rs. 8,000 for the first year of operation and increased by Rs. 1,500 per year for each additional year of use. Using an interest rate of 10% determine what annual savings must be obtained through the use of this equipment to make it economically justifiable.

(VTU July 2005)

Given

First cost, $P = 85,000 \rightarrow$ (outflow, to be annualized)

Salvage value, $S = 6,000 \rightarrow$ (inflow, to be annualized)

Service life, $n = 5$ years

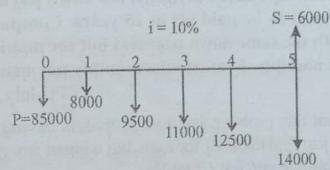
1st year maintenance cost, $A_1 = 8,000 \rightarrow$ (outflow, to be annualized)

Annual increase, $G = 1,500$

EQUIVALENT ANNUAL WORTH COMPARISONS 115

Rate of interest, $i = 10\%$

Annual Savings, $A = ?$



Note:- In the above problem, the various costs (and also the salvage) involved in the purchase of a circuit tester is given. In order to justify its purchase, the circuit tester has to earn at least the amount that it takes to buy it. In other words, the annual savings that must be obtained through the use of this equipment is nothing but the net costs that are involved in buying it.

Therefore we have to find AE of the circuit tester which is nothing but the differences between its costs and revenues.

$$\text{Net Annual equipment} = \{\text{AE (revenues)}\} - \{\text{AE (costs)}\}$$

$$\begin{aligned} &= \{\text{AE}(\text{Salvage cost})\} - \{\text{AE}(\text{First cost}) + \text{AE}(\text{maint. cost})\} \\ &= (\text{To find } A \text{ given } F) - (\text{To find } A \text{ given } p) - (\text{To find } A \text{ given } G) \\ &= S(A/F, i, n) - P(A/P, i, n) - [A_1 + G(A/G, i, n)] \\ &= S(A/F, 10\%, 5) - P(A/P, 10\%, 5) - [A_1 + G(A/G, 10\%, 5)] \\ &= 6,000(0.1638) - 85,000(0.2638) \\ &\quad - 8,000 - 1,500(1.8101) \end{aligned}$$

$$\text{AEW (tester)} = \text{Rs.} - 32,155$$

Ans:- Rs. 32,155 is the net costs which are involved in the maintenance of the circuit tester. Therefore the tester has to help earn a minimum of Rs. 32,155 through its usage in order to justify its purchase.

This is nothing but the annual savings that must be obtained with the help of the tester.

Example 5:

A plot can be purchased for Rs.13,80,000. Company A offers a loan at 7.5 percent nominal interest to be compounded monthly, if a down payment of Rs.25,000, is paid initially. The loan is to be paid off in 15 years. Company B offers 20 year repayment period with the same down payment but the nominal interest rate is 9 percent compounded monthly. Evaluate the monthly payments for the above two alternatives.

(VTU July 2006)

Note : Please note that this problem has two proposals having different repayment periods(n). We need to just find the EMI for each, but without any comparison. Also note that we have to use formulae and not CI tables.

To find EMI for Company A**Given :**

$$\text{Loan received, } P = 13,80,000$$

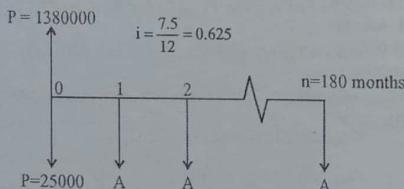
$$\text{Down payment, } P = 25,000$$

$$\text{Rate of interest, } i = 7.5\%$$

$$\text{Repayment period, } n = 15 \text{ years}$$

$$\text{or } n = (15 \times 12) = 180 \text{ months.}$$

The CFD for the above problem would be,



P.S.:- The above CFD basically means that Company A gives a loan of 13,80,000 at time zero while at the same time a down payment of Rs. 25,000 is made at time zero. Therefore the net loan obtained at time zero is $(13,80,000 - 25,000) = 13,55,000$. We have to find the monthly payment to be made for this net loan taken.

$$\text{EMI} = (\text{To find } A \text{ given } P)$$

$$= P (A/P, i, n)$$

$$= P \left[\frac{i(1+i)^n}{(1+i)^n - 1} \right]$$

$$= 13,55,000 \left[\frac{\left(\frac{0.075}{12}\right)\left(1 + \frac{0.075}{12}\right)^{15 \times 12}}{\left(1 + \frac{0.075}{12}\right)^{15 \times 12} - 1} \right]$$

$$\text{EMI (Company A)} = \text{Rs. } 12,563$$

To find EMI of Company B**Given :-**

$$\text{Loan received, } P = 13,80,000$$

$$\text{Down payment, } P = 25,000$$

$$\text{Net loan taken, } P = 13,80,000 - 25,000$$

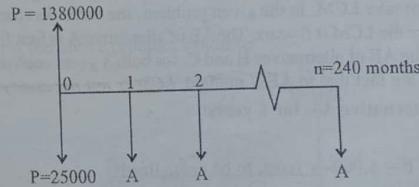
$$P = 13,55,000$$

$$\text{Repayment period, } n = 20 \text{ years}$$

$$n = (20 \times 12) \text{ months}$$

$$i = 9\%.$$

The CFD for the above problem would be



$$\text{EMI} = (\text{To find } A \text{ given } P)$$

$$= P (A/P, i, n)$$

$$= P \left[\frac{i(1+i)^n}{(1+i)^n - 1} \right]$$

$$= 13,55,000 \left[\frac{0.09 \left(1 + \frac{0.09}{12} \right)^{240}}{\left(1 + \frac{0.09}{12} \right)^{240} - 1} \right]$$

EMI (Company B) = Rs.12,191

Ans: Therefore the monthly payments are,

Company A = Rs.12,563

Company B = Rs.12,191

Example 6:

The following alternatives can perform the same function.

Alternative	First Cost (Rs.)	Life (years)	Salvage Value (Rs.)	Annual Cost (Rs.)
A	6,000	6	2,000	800
B	3,000	3	1,000	1,000
C	2,000	3	NIL	1,200

At an annual rate of 12 percent, rank the alternatives as per the equivalent annual cost. (VTU Jan 2006)

Solution: The given problem has 3 alternatives and have different service lives. But there is no need to take LCM, if the cash flows are to be annualized. Finding the annual equivalent is nothing but finding the average, over the service life of an alternative. Even if we were to take LCM for alternatives with unequal lives, it would give the same answer as when we do not take LCM. In the given problem, the three service lives are 6, 3, 3 respectively, where the LCM is 6 years. The AE of alternative A is first found for 6 years. Then let us find the AE of alternatives B and C, for both 3 years each and 6 years each, so as to establish the fact that in AEW method, LCM is not necessary.

To find AE of alternative A – for 6 years

Given,

First Cost, P = 6,000 → (cost, to be annualized)

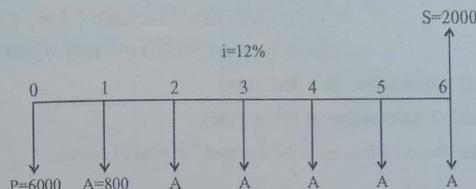
Annual Cost, A = 800 → (cost, already annualized)

Salvage value, S = 2000 → (revenues, to be annualized)

Service life, n = 6 years.

Rate of interest, i = 12%

The CFD of the above problem would be



$$AE(\text{alternative A}) = \{AE(\text{revenues})\} - \{AE(\text{costs})\}$$

$$\begin{aligned} &= \{AE(\text{salvage value})\} - \{AE(\text{First cost}) + AE(\text{Annual cost})\} \\ &= S \left(\frac{A}{F, i, n} \right) - P \left(\frac{A}{P, i, n} \right) - A \\ &= 2,000 \left(\frac{1}{0.1232} \right) - 6,000 \left(\frac{0.2432}{0.1232} \right) - 800 \end{aligned}$$

$$AE(\text{Alt. A}) = \text{Rs.} - 2012$$

To find AE of alternative B – for 3 years

Given,

First Cost, P = 3,000

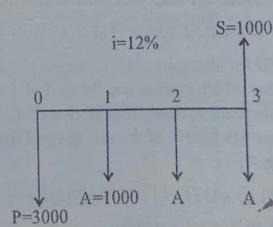
Annual Cost, A = 1,000

Salvage value, S = 1,000

n = 3 years

i = 12 %

The CFD for the above problem would be



$$AE(\text{Alternative B}) = \{AE(\text{Revenues})\} - \{AE(\text{costs})\}$$

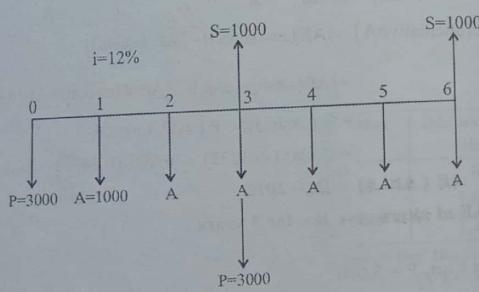
120 ENGINEERING ECONOMICS

$$\begin{aligned}
 &= S(A/p, i, n) - P(A/p, i, n) - A \\
 &= 1,000 (A/F, 12\%, 3) - 3,000 (A/p, 12\%, 3) - 1,000 \\
 &= 1,000 (0.2963) - 3,000 (0.4163) - 1,000
 \end{aligned}$$

AE (alternative B) = Rs. -1952

To find AE of Alternative B for 6 years

If the above CFD were to be doubled, it would become



AE (alternative B) = {AE (Revenues)} - AE (costs)}

$$\begin{aligned}
 &= \{AE (\text{Salvage at } n=3) + AE (\text{salvage at } n=6)\} \\
 &\quad - \{AE (\text{First cost at } n=0) + AE (\text{First cost at } n=3) \\
 &\quad + AE (\text{annual cost at } n=0)\} \\
 &\quad \{[S(P/F, 12\%, 3) (A/P, 12\%, 6) + S(A/F, 12\%, 6)] - \\
 &\quad \{P(A/P, 12\%, 6) + [F(P/F, 12\%, 3) (A/P, 12\%, 6)] + A\}
 \end{aligned}$$

(P.S = From the CFD for alternative B for 6 years please note that in order to find AE of Salvage and first cost which is shown at the end of 3 years, its present worth has to be found out first, and then annualized. Therefore $S(P/F, 12\%, 3)$ and $F(P/F, 12\%, 3)$ in the above equation represents the P_w of the salvage and first cost and are immediately considered as P for finding A)

$$\begin{aligned}
 &= \{[1,000 (0.7118) (0.2432)] + 1,000 (0.1232)\} - \{3,000 (0.2432) + \\
 &\quad [3,000 (0.7118) (0.2432)] + 1000\}
 \end{aligned}$$

AE (alternative B) = Rs. -1952

EQUIVALENT ANNUAL WORTH COMPARISONS 121

As we can see from the above calculations, the AE of alternative B for both 6 years as well as for 3 years is the same i.e. $B = -1952$

To find AE of Alternative C - for 3 years

Given

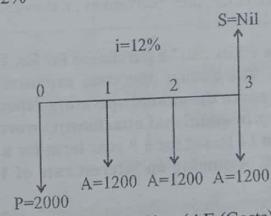
First Cost, $P = 2,000$

Annual Cost $A = 1,200$

Salvage Value $S = \text{nil}$.

$n = 3$ years

$i = 12\%$

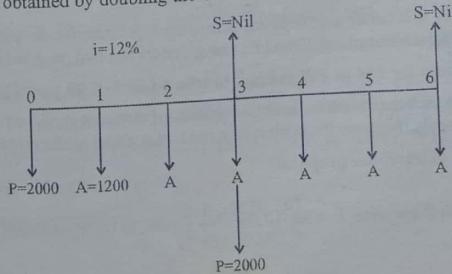


$$\begin{aligned}
 AE (\text{alternative C}) &= \{AE (\text{Revenues})\} - \{AE (\text{Costs})\} \\
 &= 0 - P(A/P, i, n) - A \\
 &= -2000 (0.4163) - 1,200
 \end{aligned}$$

AE (alternative) = Rs. -2032

To find AE of Alternative C - for 6 years

The CFD is obtained by doubling the CFD drawn for 3 years.



$$\begin{aligned}
 AE(\text{alternative C}) &= \{\text{AE(Revenues)}\} - \{\text{AE(Costs)}\} \\
 &= 0 - P(A/P, i, n) - \{[F(P/F, i, n)] [(A/P, i, n)] + A\} \\
 &\quad (\text{Please note in the above equation, } F(P/F, i, n) \text{ represents the } P_w \text{ of } p = 2,000 \\
 &\quad \text{at } 3. \text{ This value is multiplied to } (A/P, i, n) \text{ which represents the } PW \text{ annualized}) \\
 &= [-2,000(A/P, 12\%, 6)] - [2,000(P/F, 12\%, 3)(A/P, 12\%, 6)] + A \\
 &= -2,000 (0.2432) - [2,000 (0.7118) (0.2432)] + 1,200
 \end{aligned}$$

AE (alternative C) = Rs. - 2032

∴ The AE of alternative C is the same whether $n = 3$ or $n = 6$ years

Answer: Comparing the AE of the three alternatives, it is evident that Alternative B is the best because of least cost.

Example 8: A production utility can be purchased for Rs. 10,000 which is supposed to serve a 5 year term. The annual operating expenses is estimated to be Rs. 2,500. It is intended to reduce the annual operating expenses to Rs. 1,000 for the next 5 years by investing in additional attachments worth Rs. 13,000 in the 6th year. A similar utility can be leased for a 8 year term for a one time payment with no returns of Rs. 50,000. Assuming an interest rate of 10% comment on these proposals.

(VTU Jan 2008)

Please note the following in the process of solving the above problem:

- The production utility in question has two ownership proposals purchase and lease.
- The purchase proposal has a cash flow given for 10 years while the lease proposal has a cash flow for 8 years.
- The problem requires us to comment on these proposals. But whether the comparisons are to be made based on present worth, or future worth or annual equivalent worth is not specified in the problem.
- Therefore it is better to choose AE comparison because we do not need to take LCM for the service lives 10 and 8 years given for the two proposals respectively. Had it been for PW or FW, we had to take a LCM of 40 years!
- Students are here advised to select AE method if different services lives are given and also only if no specific comparison method is asked in the problem.

To find AEW of purchase proposal

Given,

Initial Purchase price, $P = \text{Rs. } 10,000 \rightarrow (\text{costs, to be annualized})$

Additional attachments in 6th year, $= \text{Rs. } 13,000 \rightarrow (\text{costs, to find } p \text{ and then annualized})$

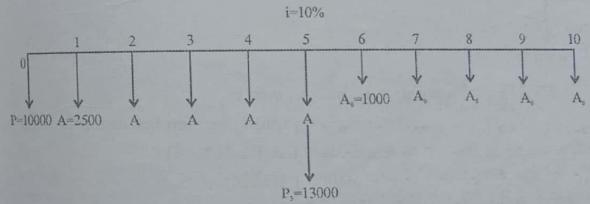
Annual operating expenses for first 5 years $A = 2,500 \rightarrow (\text{cost, already annualized})$

Annual operating expenses from 6th year to 10th year, $A = 1,000$

Total service life, $n = 10$ years

Rate of interest, $i = 10\%$

The CFD for the above problem would be



(PS : Please note that if annual operating expenses reduce to Rs. 1,000 from 6th year onwards due to investment in additional attachments, that investment has to be made at the beginning of 6th year which is also the end of 5th year. The AE of the above CFD can be found by the following steps:

- The annual operating costs are different for the first 5 years and then for the next 5 years. Therefore it is better to find the net P_w of all the values and then this P_w can be annualized.
- The P_w of A for the first five years has to be found first.
- The P_w of A_6 for years 6 – 10 is then found which gets added to P_w .
- The P_w of cumulative P_6 is then found, and subsequently added to P to give net P_w .
- The annual worth A of this net P_w is calculated.

$$\begin{aligned}
 \text{Net } P_w &= -P - A (P/A, 10\%, 5) - \{[P_5 + A_6 (P/A, 10\%, 5)] \{ P/F, 10\%, 5 \}\} \\
 &= -10,000 - 2500 (3.7908) - [13,000 + 1,000 (3.708)] [0.6209]
 \end{aligned}$$

$$P = \text{Rs. } -29,851$$

$$\text{AEW (purchase proposal)} = P (A/P, 10\%, 10)$$

$$= -29,851 (0.1627)$$

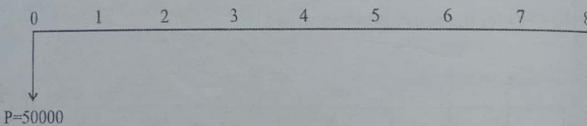
$$\text{AEW (purchase Proposal)} = \text{Rs. } -4,856$$

To find AEW of Lease Proposal

Given:-

$$\begin{aligned} \text{One time payment, } P &= 50,000 \\ n &= 8 \text{ years} \\ i &= 10\% \end{aligned}$$

The CFD for this would be.



AEW (Lease Proposal) = To find A given P

$$\begin{aligned} &= -P(A/P, i, n) \quad (-\text{ve sign because it is cost}) \\ &= -50,000(A/P, 10\%, 8) \\ &= -50,000(0.1874) \end{aligned}$$

AEW (Lease Proposal) = Rs. - 9,370

Answer : From the AEW of purchase and lease proposals, it is clear that purchase proposal has far lower costs, and hence the same is suggested.

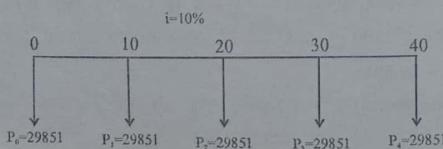
If LCM were to be taken in the Purchase proposal for $n = 40$ years

$$\begin{aligned} \text{Net Pw} &= -P_{w_0} - P_{w_1}(P/F, 10\%, 10) - P_{w_2}(P/F, 10\%, 20) \\ &\quad - P_{w_3}(P/F, 10\%, 30) - P_{w_4}(P/F, 10\%, 40) \\ &= -29,851 - 29851(0.3855) - 29,851(0.1486) - 29,851(0.0573) \\ &\quad - 29,851(0.0221) \end{aligned}$$

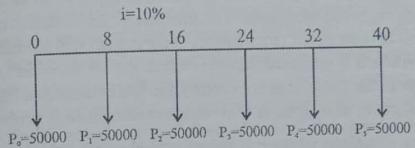
$$\text{Net Pw} = -48,164$$

$$\begin{aligned} \text{AE (purchase proposal)} &= -48,164(A/P, 10\%, 50) \\ &= -48,164 \times 0.1009 \end{aligned}$$

AE (purchase at $n = 50$) = -4,860 → This is approx. the same as when $n = 10$ years.



AEW of Lease proposal for $n = 40$ years



$$\begin{aligned} \text{Net PW} &= -P_{w_0} - P_{w_1}(P/F, 10\%, 8) - P_{w_2}(P/F, 10\%, 16) - P_{w_3}(P/F, 10\%, 24) - \\ &\quad P_{w_4}(P/F, 10\%, 32) - P_{w_5}(P/F, 10\%, 40) \\ &= -50,000 - 50,000(0.4665) - 50,000(0.2176) - 50,000(0.1015) - \\ &\quad 50,000(0.0474) - 50,000(0.0221) \end{aligned}$$

$$\text{Net PW} = \text{Rs. } -92,755$$

$$\begin{aligned} \text{AE (Lease Proposal)} &= \text{Net PW}(A/P, 10\%, 40) \\ &= 92755(0.1023) \end{aligned}$$

AE(lease Proposal) = Rs. - 9488

(This is approximately equal to the AE(lease) at $n = 8$ years. The difference in value is due to the lease value of Rs.50,000 being multiplied by only four decimal places and not more)

SITUATIONS FOR EQUIVALENT ANNUAL WORTH COMPARISON

Annual equivalent worth comparison is a very well known and important method of economic analysis. Alternatives can be compared with objective of either minimization of costs or maximization of profits. The term **equivalent annual cost** is designated for comparison of alternatives involving only costs whereas the term **equivalent annual worth** is used when costs and revenues are both present.

The term equivalent annual worth or just annual worth suggests a +ve value but the calculation is actually can produce a -ve value. This indicates a situation of loss where costs exceed the revenues. Negative worths usually mean that the alternative is unacceptable, but with some exceptions. For example, when there are projects involving security or infrastructure or some vital necessities, or when there are projects involving security or infrastructure or some vital necessities, or when a "do nothing" option does not exist, alternatives with negative worth may have to be accommodated.

It is usually a business as a whole which is considered for economic analysis. In other words, it is often very difficult, and not worth the required study time, to discover the income derived from only one component especially in a complex system having many different components. For example, the income produced by a copier machine is

troublesome to derive exactly since its output is utilized by many people, often from different departments, working on many projects. In this type of situation, alternatives which satisfy copying needs are evaluated on the basis of their relative costs, because each alternative capable of meeting the requirements of the system will produce the same income to the system. When it is apparent that only costs are involved in an evaluation, it is convenient to ignore the minus sign convention and let comparison figures represent the absolute value of costs, as is done in cost-dominated cash flow problems.

Some more situations where equivalent annual worth calculation are applied are mentioned below.

1. CONSOLIDATION OF CASH FLOW

What's it worth? This is a typical situation in which annual worth may give a good answer for any given business alternative. It can be a decisive query in the appraisal of a proposal. However it is often difficult associated with its conduct are collectively analyzed.

2. RECOVERY OF INVESTED CAPITAL

Will it pay off? This is another question that investors need answer for. An adequate pay off recovers the invested capital plus the desired rate of return. Since returns are spread over the life of the investment, it is convenient to convert capital recovery costs to the same annual pattern. The net result of combining uniform cost and revenue flows could be positive, zero or negative which means that the business alternative in question could be profitable, adequate or loss-making, respectively.

3. NET CASH FLOW COMPARISON

Which one is better? Is another often asked question when there are alternatives present. If the criterion is strictly economics, the alternative with the highest net worth is preferred, if the worth is measured by revenues. But when alternatives have only costs and no income, a low annual worth is preferred.

CONSIDERATION OF ASSET LIFE

For economic analysis of any kind, a proper assessment of the life of the asset is necessary. In other words, selecting an appropriate study period for the alternatives in question is a key factor in annual equivalent worth comparisons.

In AEW comparisons, 'n' is the number of compounding periods (annual or otherwise) selected which is appropriate for the analysis of cash flows. But 'n' take on special meaning when it represents the life of an asset that loses value as function of use or time. The more frequently applied to describe the life of an asset are listed and defined as follows:

(I) OWNERSHIP LIFE OR SERVICE LIFE

This is the period of time an asset is kept in service by the owners. This implies a period of useful service from the time of purchase until disposal. Sentimental attachment to equipment that is being used for long times exists, especially in our country. People always tend to retain old equipment even beyond a point where it is capable of satisfying its intended function, under the expectation that it might some how again prove useful. A machine can have a physical life longer than its service life; the machine is still physically sound, but there is no useful function for it to perform.

(II) ACCOUNTING LIFE

This is nothing but the life expectancy based primarily on book keeping and tax considerations. It may or may not correspond to the period of usefulness and economic desirability.

(III) ECONOMIC LIFE

This is the time period that minimizes the asset's total annual costs or maximizes its net annual incomes. At the end of this period, the asset would be displaced by a more profitable replacement if service were still required. Economic life is also referred to as the optimal replacement interval and is the condition appropriate for many engineering economic studies.

COMPARISON OF ASSETS WITH EQUAL AND UNEQUAL LIVES

Business alternatives are easily compared when their service lives (n) are equal or assumed to be equal. However in practice, alternatives of unequal business lives are have to be compared too.

Note : Students are directed to go through a similar topic dealt in the previous chapter on page 73-85.

Use Of a sinking fund

A sinking fund is an equivalent amount that is deposited at the end of every interest period for 'n' interest periods to realize a future sum (F) at the end of it at a rate of interest i . Many organizations are sometimes obligated by legislated or contractual agreements to establish a sinking fund, separate from their internal operations, to accumulate a specified amount by a specified time.

Provision for a sinking fund requires that an organization set aside a portion of the income derived from sales or taxes each year in order to retire a bond issue. Failure to meet the sinking fund payments forces the bond issue to be thrown into default, causing serious credit and credibility problems. The consequential payments made can be a direct cash drain on the organization. That is exactly the purpose of the sinking fund – to protect the investors by enforcing an orderly retirement of debt from current income.

When a sinking fund is established to accumulate sufficient money to meet the bond cost at maturity, annual payments are normally required to be invested in a saving institution. However, the interest rates for these 'savings' is typically lower than what the organization earns on its own capital. But the sinking fund is nevertheless made not considering it as an earning device but a way to ensure that funds will not be diverted to other ventures. "Save it for the rainy day", is the use of a sinking fund.

ANNUITY CONTRACT FOR A GUARANTEED INCOME

An annuity is a interest payment received each year as a result of having invested a large sum or a series of sums over the years. An annuity contract is similar to the pension scheme where one receives pension from the age of retirement until death (or even continued to be given to dependents) after having made savings when in service.

Therefore annuity contract is an after retirement plan for a guaranteed income.

An annuity contract can also be considered as essentially the reverse of a life insurance policy. In the insurance policy , the company pays a stipulated sum to heirs based on the amount of payments made during the policy holders lifetime. In annuity contracts, an individual pay a stipulated sum, or a number of payments, to the company and then receives regular income payments, starting at designated time and continuing for life.

Varieties of annuity contracts are available. A straight life annuity is one where, income payments terminate with the death of the annuitant (receiver). An extended annuity is one where, companies offer to continue to make pension payments even in case of early death of the annuitant. There are annuity plans which may have variable rates that are linked to indexes, gold prices, or foreign currencies. Annuity plans are indexed to any one or several of these quantities in order to conserve the buying power of annuity income. The major objective of any annuity buyer would be to balance risks against returns.

SUMMARY

1. Equivalent annual worth is nothing but the average annual value of uneven cash flows throughout the business lives of alternatives.
2. When comparing business alternatives, that alternative which has a higher algebraic value of Annual Worth is chosen. If all values are -ve, the least is chosen.
3. In EAW comparison, we need not use LCM method while comparing alternatives with varying service lives. In other words, we can compare the annual worths of various projects even if they have different service lives even without taking LCM.
4. Situation for Equivalent Annual Worth Comparison

AEW Comparison is an important method of comparing business alternatives.

Typical business situations when AEW comparison is used are :

- i **Consolidation of cash flows** : A consolidated annual worth is calculated for business alternatives when they are faced with the question " What is its worth"?
- ii **Recovery of invested Capital** : "Will it pay off?" is another typical situation that AEW answers.
- iii **Net Cash flow comparison** : "Which one is better?". Here AEW can compare business alternatives.

5. Consideration of Asset Life

Life of assets vary widely in their usage. But generally each asset is said to have three kinds of lives based on who is estimating it.

i **Ownership Life** : Estimated or used by owner.

ii **Accounting life** : Estimated by Accountant for book keeping and tax considerations.

iii **Economic life** : Estimated by Engineering Economist based on its profitable life.

6. Use of a Sinking Fund

Many organizations set aside a certain sum from their profits and make a sinking fund

to take care of any future financial contingencies. Use of sinking fund may be voluntary in some cases and in some more case, they may be obligatory.

7. Annuity Contract for a Guaranteed Income

This is nothing but the pension scheme. In this, deductions are made from monthly or annual income and invested appropriately in such a way that after retirement the investor would get a guaranteed annuity (pension) as long as he lives. Or in some cases pension is paid to the spouse if the investor dies.

EXERCISE PROBLEMS

- Amjay Company is currently renting a parking lot for employee and visitor use at an annual cost of Rs. 9,000, payable on the first of each year. The company has an opportunity to buy the lot for Rs. 50,000. Maintenance and taxes on the property are expected to cost Rs. 2,500 annually. Given that property will be needed for 10 more years, determine what sales price must be obtained at the end on that period in order for Amjay to break even, when the interest rate is 12 percent.
- A bakery is thinking of purchasing a small delivery truck that has a first cost of Rs. 18,000 and is to be kept in service for 6 years, at which time the salvage value is expected to be Rs. 2,500. Maintenance and operating costs are estimated at Rs. 2,500 the first year and will increase at a rate of Rs. 200 per year. Determine the annual worth of this vehicle, using an interest rate of 12 percent.
- Machine A has a first cost of Rs. 9,000 no salvage value at the end of its 6-year useful life, and annual operating costs of Rs. 5,000. Machine B costs Rs. 16,000 new and has an expected resale value of 4,000 per year. Compare the two alternatives on the basis of their annual worths, using the repeated - projects assumption at 10 percent annual interest.
- To attract industry, a city has made an offer to corporation. The city will install all roads and service for the plant site at no immediate cost to the corporation, but the corporation will be expected to bear the cost of operation and maintenance on a cost-sharing schedule where by Rs. 65,000 is paid the first year of the project (2 years from now), each subsequent yearly payment will be Rs. 5,000 less, and the corporation's obligation will end with last Rs. 5,000 payment. What is the annual worth today of this agreement, if the annual interest rate is 9 Percent?
- A marina has two alternative plans for constructing a small- boat landing on a lake behind the sales building; one is a wooden dock, and the other is a metal – and – concrete wharf. Date for the two plans are as shown.

	Wood	Metal and Concrete
First cost	Rs.35,000	Rs.55,000
Period before replacement	10 years	15 years
Salvage value	Rs. 5,000	000000
Annual maintenance	Rs. 6,000	Rs.3,200

using a minimum attractive rate of return of 10 percent, compare the present worths of the two plans. Assume that both will provide adequate service and that replacement costs will be the same as the original cost.

- Autocon company is evaluating three robots for possible use in its assembly operations. (Only one robot will be purchased). Data associated with these robots are as follows :

	Robot A	Robot B	Robot C
First Cost	55,000	58,000	53,000
Operating and maintenance Costs Rs.	3,000/year	4,5000/years	4,000 years
Expected income, Rs.	Rs 40,000/yr	44,000/yr	38,000/yr
Estimated salvage value, Rs	Rs.4,000	6,000	4,000

Assuming a technological life of 3 years and a desired interest rate of 12 percent, which robot seems to be preferable, assuming all other factors are equal? Use a net present – worth evaluation.

- Two abrasive wheel cutters A and B have both a life of 4 years and the minimum rate of return is 15 percent. Which cutter has the lowest equivalent annual cost?

	Cutter A	Cutter B
First cost	Rs.5,000	Rs.3,200
Annual operating cost	Rs.780	Rs.950
Salvage Value	Rs.1,000	Nil

- Determine the equivalent present worth and the annual worth of the following series of year and cash flows extending over 8 years using annual interest, rate of 20%. The amounts are Rs. 100 for 1st Year, Rs. 200 for 2nd year, Rs.500 for 3rd years, Rs.400 for 4th, 5th, 6th, 7th & 8th years. Draw CFD for above series of payments.

- A manufacturing company is considering the purchase of new equipment in order to improve its product quality. The equipment has a first cost of Rs. 85,000 and the salvage value is expected to be Rs. 6,000 after a service life of 5 years. Maintenance and operating costs are expected to be Rs.8,000/- during the first year of operation

and it increases by Rs. 1,500 per year for each additional year of use. Using an interest rate of 10% determine what annual saving must be obtained through the use of this equipment to make it economically justifiable.

10. Mr. X takes a loan Rs. 8,000 with an assurance of making equivalent annual payment at a rate of interest of 15% for 5 years. How much MR.. X has to pay? Draw the cash flow diagram for the same.

11. In the design of new facility, the following mutually exclusive are under consideration.

	Alternative 1	Alternative 2	Alternative 3
First Cost (Rs)	28,000	16,000	23,500
Net	5,500	3,300	4,800
Cashflow/yr (Rs.)	1,500	000	000
Salvage Value (Rs)	10	10	10
Useful life (Yrs)			

Assume MARR of 15% using AW method choose the best of these three feasible alternatives. Also draw the CFD for the same.

12. Two models of small machines perform the same function. Machine A has an initial cost of Rs. 9,500 operating costs Rs. 1,900 per year and a life of 4 years. Machine B has an initial cost of Rs. 25,100, and can be kept in service economically without operating costs for 8 years. There is no salvage value in both the cases which machine is preferred with a MARR of 8%. Draw the conclusion based on equal annual cost method.
13. An investment of Rs. 10,000 can be made in a project (no salvage value) that will produce a uniform annual revenue of Rs. 4,838 for 5 year. From this revenue Rs. 2,000 per year will have to be paid for operation and maintenance cost and Rs.200 per year for taxes. Show whether this is a desirable investment if the minimum return to be earned is 9%.
14. A food processing unit is considering two equal service tomato peeling machines to be used in a food canning plant. The following costs are estimated.

Cost Details	Machine A Rs.	Machine B Rs.
First cost	2,60,000/-	3,60,000/-
Annual Maintenance cost	8,000/-	3,000/-
Annual labour cost	11,000/-	7,000/-
Additional income taxes	—	—
Salvage value	20,000/-	30,000/-
Life years	6	10

If the minimum required rate of return is 15% per year, which machine should be selected?

15. A sea food company is planning an expansion to a cold storage facility . Three alternative site design proposals are being considered that use a MARR of 10 percent plans A and B require, an expenditure of Rs. 3,50,000/- for land which will retain its value for 10 years, while plan C requires Rs. 4,25,000/- for land which will also retain its value in 10 years. The estimated income increase due to facility available is annualized at Rs. 2,48,000 per year the company requires that a life of 10 years be used for analysis. Data pertaining to the project are given in the table below :

ITEM	Proposal A Rs.	Proposal B Rs.	Proposal C Rs.
Building and Installation	60,000	70,000	85,000
Compressors	1,00,000	1,35,000	85,000
Expected energy costs with first costs	65,000	4,800	65,000
Increase for each additional year	3,000	2,000	3,500
Annual maintenance costs	20,000	15,000	50,000
Estimated salvage value	35,000	43,000	18,000

If the minimum required rate of return is 15% per year, which meaning should be selected?

□ □ □ □

INTRODUCTION

Rate of Return is a general associated with rate of interest. It is nothing but a percentage that indicates the relative yield on different uses of capital. In other words, the rate of return is the rate at which investments made gives returns to the investors.

Three rates of return appear frequently in engineering economic studies:

- i. The Minimum Acceptable Rate of Return (MARR)
- ii. The Internal Rate of Return (IRR)
- iii. The External Rate of Return (ERR)

I. MINIMUM ACCEPTABLE RATE OF RETURN (MARR)

This is rate of interest set by an organization to designate the lowest level of return that makes an investment acceptable. In other words, a minimum acceptable rate of return is the lowest rate of interest at which an independent business alternative is still attractive. For example, a businessman may decide 20% returns is minimum that he expects from his business, Otherwise he may withdraw. Here MARR is 20%.

Also known as the minimum attractive rate of return, MARR is the lower limit for investment acceptability set by organizations or individuals. It is a device designed to make the best possible use of a limited resource, for example money. Minimum acceptable rates of return very widely according to the type of organization and may even vary within the organization. For example, the rate return required for cost reduction proposals may be lower than that required for R&D Projects in which there is less certainty about prospective cash flows.

II. Internal Rate of Return

The Internal Rate of Return is nothing but the rate of interest i at which all costs of a business is equal to all its revenues. In other words, IRR represents the rate of interest at which all outflows equals all inflows in a cash flow diagram. This basically means that for a given CFD there exists a value of i at which there is no loss or no profit, which is nothing but IRR. Internal Rate of Return is also known as *Time-Rate of Return*.

In order to find the IRR of a CFD we have to equate the PW or FW or AE of its revenues with the PW or FW or AE of its costs respectively. In this chapter, Present Worths have been equated, although the students could use FW or AE too.

$$\text{i.e. } \text{PW(Revenues)} = \text{PW(Costs)}$$

$$\text{PW(Revenues)} - \text{PW(Costs)} = 0$$

In a given problem which asks for IRR to be found, all values of costs and revenues would be given along with service life. We may make use of compound interest formulae or tables in order to solve for i , which is done by trial and error. Interpolation is normally used to get the value of i .

III. EXTERNAL RATE OF RETURN (ERR)

This is rate of interest external to a project at which net cash flows generated by a project over its life can be reinvested. In other words, ERR represents that rate of interest available outside the project in the real world such that it becomes attractive to withdraw revenues from the business and invest it elsewhere to enjoy higher returns. In principle, it is not fair, but it exists as an economic option, because it does not make sense to continue with a business alternative which earns less, than a possible investment outside. Usually ERR would be equated to MARR, so that money invested in business is acceptable. In other words, a businessman would not withdraw his money if the returns from the project is at least equal to ERR which would be then called MARR.

Following are the general steps to find IRR using PW values

Step 1: Since i is not given, assume i . Let the value of i be a low value, say 5%. Find the PW of the alternative taking $i = 5\%$. At lower values of i , PW is usually +ve.

Step 2: Assume a higher value of i say 10%. Find the PW of the alternative. If PW value is -ve, then stop. Else, assume a even higher value of i , say 15% and then find PW again. Repeat this until PW is -ve.

Step 3: We now have a value of i at which PW is +ve and another value of i at which PW is -ve. By interpolation we can therefore find the value of i at which PW = 0. This i is nothing but the internal rate of return.

P.S: Please note that i_{IRR} obtained this way is approximate because of the very nature of interpolation. A more accurate value of i_{IRR} can be obtained if the difference between $i(+Ve)$ and $i(-Ve)$ value is as low as possible, while carrying out interpolation.

Also please note that i_{IRR} can be found only if a given CFD has both revenues and costs. If a CFD has only revenues or only costs, i_{IRR} cannot be found. This is because for any given i , the net PW would always remain either +ve or -ve respectively, never becoming zero.

WORKED EXAMPLES

Example 1:

A fresh mechanical engineer wants to become an entrepreneur. He starts a (CAD) Centre to give design solutions to small-scale industries. He expects that his software and business would last for 5 years. The initial outlay and cash flow pattern for the new business are as listed below. Find the Internal rate of return for the new business if he estimates a salvage value of Rs. 1,00,000 at the end of 5 years.

Period	0	1	2	3	4	5
Cash flow	-700000	180000	190000	210000	225000	200000

Given,

Initial Investment, $P = \text{Rs. } 7,00,000$ \longrightarrow Cost

$$\begin{aligned} \text{Revenues for years 1 to 5, } R_1 &= \text{Rs. } 1,80,000 \\ R_2 &= \text{Rs. } 1,90,000 \\ R_3 &= \text{Rs. } 2,10,000 \\ R_4 &= \text{Rs. } 2,25,000 \\ R_5 &= \text{Rs. } 2,00,000 \end{aligned}$$

Revenues

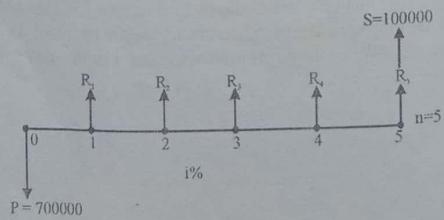
Salvage value, $S = \text{Rs. } 1,00,000$

Life of business, $n = 5$ years

Present worth, $PW = ?$

Internal rate of return, $i = ?$

CFD for the above problem would be



Step 1: Assume a low value for i say $i = 5\%$ and find PW

$$\begin{aligned} PW(\text{at } i = 5\%) &= PW(\text{Revenues}) - PW(\text{Costs}) \\ &= R_1(P/F, 5\%, 1) + R_2(P/F, 5\%, 2) + R_3(P/F, 5\%, 3) \\ &\quad + R_4(P/F, 5\%, 4) + R_5(P/F, 5\%, 5) + S(P/F, 5\%, 5) - P \\ &= 1,80,000(P/F, 5\%, 1) + 1,90,000(P/F, 5\%, 2) + 2,10,000(P/F, 5\%, 3) \\ &\quad + 2,25,000(P/F, 5\%, 4) + 2,00,000(P/F, 5\%, 5) + 1,00,000 \\ &\quad (P/F, 5\%, 5) - 7,00,000 \\ &= 1,80,000(0.9524) + 1,90,000(0.9070) + 2,10,000(0.8638) + \\ &\quad 2,25,00(0.8227) + 2,00,000(0.7835) + 1,00,000(0.7835) - 7,00,000 \\ PW(\text{at } i = 5\%) &= \text{Rs. } 2,45,317 \end{aligned}$$

Step 2: Now, let $i = 10\%$ and find the PW

$$\begin{aligned} PW(\text{at } i = 10\%) &= PW(\text{Revenue}) - PW(\text{Costs}) \\ &= 1,80,000(P/F, 10\%, 1) + 1,90,000(P/F, 10\%, 2) + 2,10,000 \\ &\quad (P/F, 10\%, 3) + 2,25,000(P/F, 10\%, 4) + 2,00,000 \\ &\quad (P/F, 10\%, 5) + 1,00,000(P/F, 10\%, 5) - 7,00,000 \\ PW(\text{at } i = 10\%) &= \text{Rs. } 1,18,372 \end{aligned}$$

Step 3: Since PW is still +ve, increase value of i say $i = 15\%$ and find PW

$$\begin{aligned} PW(\text{at } i = 15\%) &= 1,80,000(0.8696) + 1,90,000(0.7561) + 2,10,000(0.6575) + \\ &\quad 2,25,000(0.5718) + 2,00,000(0.4972) + 1,00,000(0.4972) - \\ &\quad 7,00,000 \\ PW(\text{at } i = 15\%) &= \text{Rs. } 16,077. \end{aligned}$$

Step 4: Since PW is again slightly +ve, assume $i = 18\%$ and find PW

$$\begin{aligned} PW(\text{at } i = 18\%) &= 1,80,000(0.8475) + 1,90,000(0.7182) + 2,10,000(0.6086) + \\ &\quad 2,25,000(0.5158) + 2,00,000(0.4371) + 1,00,000(0.4371) - \\ &\quad 7,00,000 \\ PW(\text{at } i = 18\%) &= \text{Rs. } -35,947. \end{aligned}$$

Step 5: Therefore, we have, $PW = 16,077$ when $i = 15\%$ and

$PW = -35,947$ when $i = 18\%$

Therefore we can find i at which $PW = 0$ by interpolation

$$\therefore i_{IRR} = i_{(PW+Ve)} + \frac{PW(+Ve) - 0}{PW(+Ve) - PW(-Ve)} \times \text{increment in } i \text{ from +Ve to -Ve}$$

$$i_{IRR} = 0.15 + \left[\frac{16,077 - 0}{16,077 - (-35,947)} \times (0.18 - 0.15) \right]$$

$$i_{IRR} = 0.1592$$

or

$i_{IRR} = 15.92\%$ → This is the internal Rate of Return.

An IRR of 15.92% means that PW has +Ve value until this particular rate of interest and then becomes -Ve beyond this value.

This again means that the Present Worth of all revenues is equal to PW of costs at this value of interest rate. We can also infer here that if the mechanical engineer in this problem borrows Rs. 7,00,000 from the bank, at less than 15.92% rate of interest, he will be making profit. If he has to pay a higher rate of interest than that, he would be under loss. Therefore 15.92% represents the rate at which he makes no loss—no profit, for the given cash flow diagram.

Verification

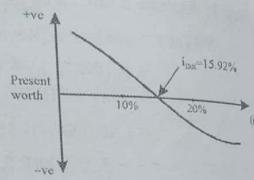
We can verify whether 15.92% is really the internal rate of return by substituting $i = 15.92\%$ and finding PW. But please note here that tables cannot be used and hence we use formula

$$\therefore PW(at i = 15.92\%) = PW(Revenues) - PW(Costs)$$

$$\begin{aligned} &= R_1 \left[\frac{1}{(1+i)^1} \right] + R_2 \left[\frac{1}{(1+i)^2} \right] + R_3 \left[\frac{1}{(1+i)^3} \right] + R_4 \left[\frac{1}{(1+i)^4} \right] \\ &\quad + R_5 \left[\frac{1}{(1+i)^5} \right] + S \left[\frac{1}{(1+i)^5} \right] - P \\ &= \frac{1,80,000}{(1+0.1592)^1} + \frac{1,90,000}{(1+0.1592)^2} + \frac{2,10,000}{(1+0.1592)^3} \\ &\quad + \frac{2,25,000}{(1+0.1592)^4} + \frac{2,00,000}{(1+0.1592)^5} + \frac{2,00,000}{(1+0.1592)^5} - 7,00,000 \end{aligned}$$

$$PW(at i = 15.92\%) = -571 \approx 0$$

This is permissible as deviation given the high values in the CFD.



Revised PW vs P/F

Example 2: A device has developed a unique prototype and spent Rs. 5 lakhs. The return of Rs. 7 lakhs is expected at the year end and it is expected to fetch Rs. 3 lakhs for the next three years. Calculate the rate of return for his prototype. (VTU Jan. 2008)

Given data,

Initial cost $P = \text{Rs. } 5,00,000$

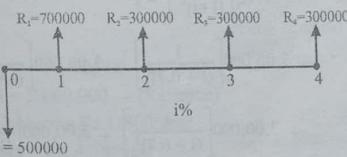
Return in 1st year, $R_1 = 7$ lakhs

Return in 2nd, 3rd and 4th years = 3 lakhs

$$i_{IRR} = ?$$

Solution:

Step 1: Assume $i = 5\%$ and find PW after drawing CFD



$$\begin{aligned} PW(at i = 5\%) &= PW(\text{Revenues}) - PW(\text{Costs}) \\ &= PW(R_1) + PW(R_2) + PW(R_3) + PW(R_4) - PW(P) \\ &= R_1(P/F, 5\%, 1) + R_2(P/F, 5\%, 2) + R_3(P/F, 5\%, 3) + R_4(P/F, 5\%, 4) - 5,00,000 \\ &= 7,00,000(0.9524) + 3,00,000(0.9070) + 3,00,000(0.8638) \\ &\quad + 3,00,000(0.8227) - 5,00,000 \\ PW(at i = 5\%) &= 9,44,730 \end{aligned}$$

Step 2: Since PW is highly +Ve, assume $i = 20\%$ and find PW

$$\begin{aligned} PW(at i = 20\%) &= PW(R_1) + PW(R_2) + PW(R_3) + PW(R_4) - PW(P) \\ &= R_1(P/F, 20\%, 1) + R_2(P/F, 20\%, 2) + R_3(P/F, 20\%, 3) + R_4(P/F, 20\%, 4) - P \\ &= 7,00,000(0.8333) + 3,00,000(0.6944) + 3,00,000(0.5787) \\ &\quad + 3,00,000(0.4823) - 5,00,000 \\ PW(at i = 20\%) &= 6,09,930 \end{aligned}$$

Step 3: Since PW is not -Ve, we have to continue the iteration. Let $i = 50\%$ and find PW

$$\begin{aligned} \text{PW(at } i = 50\%) &= R_1(P/F, 50\%, 1) + R_2(P/F, 50\%, 2) + R_3(P/F, 50\%, 3) + \\ &\quad R_4(P/F, 50\%, 4) - P \\ &= 7,00,000(0.6667) + 3,00,000(0.4444) + 3,00,000(0.2963) + \\ &\quad 3,00,000(0.1963) - 5,00,000 \\ \text{PW(at } i = 50\%) &= 2,47,790 \end{aligned}$$

Step 4: Since PW is still not -Ve, we have to continue the iteration. Let $i = 70\%$ and find PW. Since table values are not available for $i = 70\%$. We have to use compound interest formulae.

$$\begin{aligned} \therefore \text{PW(at } i = 70\%) &= R_1 \left[\frac{1}{(1+i)^n} \right] + R_2 \left[\frac{1}{(1+i)^n} \right] + R_3 \left[\frac{1}{(1+i)^n} \right] + R_4 \left[\frac{1}{(1+i)^n} \right] \\ &\quad + R_5 \left[\frac{1}{(1+i)^n} \right] - P \\ &= 7,00,000 \left[\frac{1}{(1+0.7)^1} \right] + 3,00,000 \left[\frac{1}{(1+0.7)^2} \right] \\ &\quad + 3,00,000 \left[\frac{1}{(1+0.7)^3} \right] + 3,00,000 \left[\frac{1}{(1+0.7)^4} \right] - 5,00,000 \\ &= 4,11,764 + 1,03,806 + 61,062 + 35,919 - 5,00,000 \end{aligned}$$

$$\text{PW(at } i = 70\%) = 1,12,551$$

Step 5: PW is still not +Ve at $i = 70\%$. Now let $i = 90\%$ and find PW

$$\text{PW(at } i = 90\%) = \frac{7,00,000}{(1+0.9)^1} + \frac{3,00,000}{(1+0.9)^2} + \frac{3,00,000}{(1+0.9)^3} + \frac{3,00,000}{(1+0.9)^4} - 5,00,000$$

$$\text{PW(at } i = 90\%) = 18,281$$

Step 6: We have no other choice but to continue the iteration! Let $i = 100\%$ and find PW

$$\text{PW(at } i = 100\%) = \frac{7,00,000}{(1+1)^1} + \frac{3,00,000}{(1+1)^2} + \frac{3,00,000}{(1+1)^3} + \frac{3,00,000}{(1+1)^4} - 5,00,000$$

$$\text{PW(at } i = 100\%) = -18,750$$

Step 7: At last PW is a -Ve value at $i = 100\%$!

We now have $\text{PW} = 18,281$ at $i = 90\%$ and $\text{PW} = -18,750$ at $i = 100\%$.

By interpolation

$$i_{IRR} = i_{(PW+VE)} + \left[\frac{\text{PW}(+VE)}{\text{PW}(+VE) - (\text{PW}-VE)} \right] \times |\text{increment in } i \text{ from } +VE \text{ to } -VE|$$

$$\begin{aligned} \therefore i_{IRR} &= 0.9 + \left[\frac{18,281}{18,281 - (-18,750)} \right] (1 - 0.9) \\ \therefore i_{IRR} &= 0.9494 \\ \text{or} \\ i_{IRR} &= 94.94\% \end{aligned}$$

Therefore the internal rate of return is 94.94%. This means that the business will continue to give profits until the rate of interest touches 94.94% because it is only beyond this % that $\text{PW}(\text{Revenue}) - \text{PW}(\text{Costs})$ becomes -Ve which indicate loss. This means that the business is extremely profitable.

Example 3:

A farm house can be purchased for Rs. 90,000 and the expected resale value after 20 years is Rs. 60,000. If the annual rental income is Rs. 11,800 and expenses Rs. 4,700 what will be the rate of return earned on this farm house?

(VTU Jan 2006)

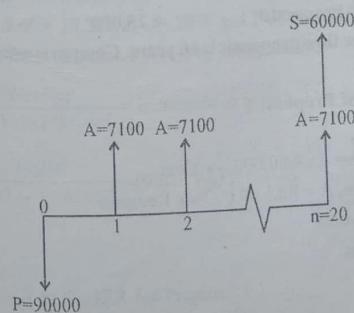
Given,

$$\begin{aligned} \text{Purchase price, } P &= \text{Rs.} 90,000 \rightarrow (\text{Cost}) \\ \text{Resale value, } S &= \text{Rs.} 60,000 \rightarrow (\text{Revenue}) \\ \text{Annual Rental income} &= \text{Rs.} 11800/\text{year} \\ \text{Annual Expenses} &= \text{Rs.} 4700/\text{year} \\ \therefore \text{Net Annual income, } A &= 11800 - 4700 \\ &= \text{Rs.} 7100 \rightarrow (\text{Revenue}) \end{aligned}$$

No. of years, $n = 20$ years

$$i_{IRR} = ?$$

CFD for the above problem is



Step 1: Assume $i = 5\%$ and find PW

$$\begin{aligned} \text{PW(at } i = 5\%) &= \text{PW(Revenue)} - \text{PW(Costs)} \\ &= \text{PW}(S) + \text{PW}(A) - \text{PW}(P) \\ &= S(P/F, 5\%, 20) + A(P/A, 5\%, 20) - P \\ &= 60,000(0.3769) + 7,100(12.4622) - 90,000 \\ \text{PW(at } i = 5\%) &= 21,095.62 \end{aligned}$$

Step 2: Now assume $i = 10\%$ and find PW

$$\begin{aligned} \text{PW(at } i = 10\%) &= S(P/F, 10\%, 20) + A(P/A, 10\%, 20) - P \\ &= 60,000(0.1486) + 7,100(8.5136) - 90,000 \\ \text{PW(at } i = 10\%) &= -20,637 \end{aligned}$$

Step 3: Now, we have, PW = 21,095.62 at $i = 5\%$ and PW = -20,637.44 at $i = 10\%$
Applying interpolation,

$$0.05 + \left[\frac{21,095.62}{21,095.62 - (-20,637.44)} \right] \times (0.1 - 0.05)$$

$$i_{IRR} = 0.07527$$

$i_{IRR} = 7.52\%$ → This is the internal rate of return.

A low value of i_{IRR} indicates that in order to make profit the borrowing rate should be less than 7.52% which when only makes the net PW +Ve.

Example 4: Compare the three investment proposals given below, if the firm's MARR is 15%.

Investment Proposal	Initial cost	Annual return
Proposal 1	4,00,000	1,00,000
Proposal 2	5,50,000	1,40,000
Proposal 3	6,25,000	1,60,000

Life of all the three proposals is 10 years. Compare using IRR.

(VTU June 2008)

To find the IRR of Proposal 1

Given,

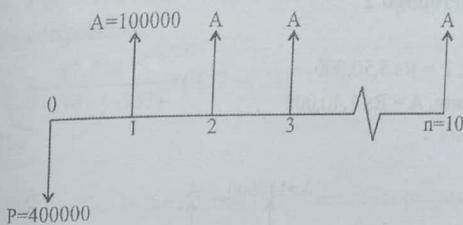
Initial cost, P = Rs.4,00,000 → Cost

Annual Return, A = Rs.1,00,000 → Revenue

Life, n = 10 years

MARR = 15%

$i_{IRR} = ?$



Note: Since MARR is given as 15%, it means that the minimum acceptable rate of return is 15%. Hence i can be assumed as 15% to start the trial and error process.

Step 1: Assume $i = 15\%$

$$\begin{aligned} \text{PW(at } i = 15\%) &= \text{PW(Revenues)} - \text{PW(Costs)} \\ &= A(P/A, 15\%, 10) - P \\ &= 1,00,000(5.0188) - 4,00,000 \end{aligned}$$

$$\text{PW(at } i = 15\%) = 1,01,880$$

Step 2: Since PW is a high +Ve no increase value of i more.

Assume $i = 20\%$ and find PW

$$\begin{aligned} \text{PW(at } i = 20\%) &= A(P/A, 20\%, 10) - P \\ &= 1,00,000(4.1925) - 4,00,000 \end{aligned}$$

$$\text{PW(at } i = 20\%) = 19,250$$

Step 3: Since PW is still +Ve, assume $i = 25\%$ and find PW

$$\text{PW(at } i = 25\%) = 1,00,000(3.5705) - 4,00,000$$

$$\therefore \text{PW(at } i = 25\%) = -42,950$$

Step 4: Now we have PW = 19,250 at $i = 20\%$ and PW = -42,950 at $i = 25\%$

Therefore we can find i at which PW = 0 by interpolation

$$i_{IRR} = i_{+Ve} + \frac{\text{PW(+Ve)}}{\text{PW(+Ve)} - \text{PW(-Ve)}} \times \text{increment in } i \text{ from +Ve to -Ve}$$

$$= 0.2 + \left[\frac{19,250}{19,250 - (-42,950)} \times (0.25 - 0.2) \right]$$

$$i_{IRR} = 0.2154$$

or

$$i_{IRR} = 21.54\% \rightarrow \text{is the IRR for Proposal 1}$$

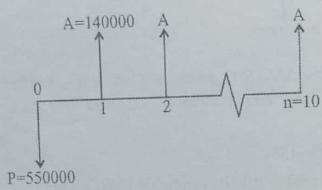
To find IRR of Proposal 2

Given,

Initial cost, $P = \text{Rs. } 5,50,000$

Annual return, $A = \text{Rs. } 1,40,000$

$n = 10$ years



Step 1: Assume $i = 15\%$ and find PW

$$\begin{aligned} \text{PW}(\text{at } i = 15\%) &= \text{PW}(\text{Revenues}) - \text{PW}(\text{Costs}) \\ &= A(P/A, 15\%, 10) - P \\ &= 1,40,000(5.0188) - 5,55,000 \end{aligned}$$

$$\text{PW}(\text{at } i = 15\%) = 1,47,632$$

Step 2: If we observe the equation for PW we can see that for PW to be equal to zero the value of $(P/A, i, n)$ should be exactly 3.9285. In other words we can search the tables book along the row $n = 10$ and under $(P/A, i, n)$ and identify the i values which fall on either side of 3.9285. When we do this, we see that $(P/A, 21\%, 10) = 4.0541$ while $(P/A, 22\%, 10) = 3.9232$. If we therefore find PW for these rates i.e., 21% and 22%. We get the +Ve and -Ve values of PW closest to zero. Subsequently by interpolation we can find i_{IRR} which is a precise value falling somewhere between 21% and 22%.

∴ Assume $i = 21\%$ and find PW

$$\begin{aligned} \text{PW}(\text{at } i = 21\%) &= \text{PW}(\text{Revenues}) - \text{PW}(\text{Costs}) \\ &= A(P/A, 21\%, 10) - P \\ &= 1,40,000(4.0541) - 5,50,000 \end{aligned}$$

$$\text{PW}(\text{at } i = 21\%) = 17,574$$

Step 3: Assume $i = 22\%$ and find PW

$$\begin{aligned} \text{PW}(\text{at } i = 22\%) &= 1,40,000(3.9232) - 5,50,000 \\ &= -752 \end{aligned}$$

Step 4: We have, $\text{PW} = 17,574$ at $i = 21\%$ and $\text{PW} = -752$ at $i = 22\%$

Therefore by interpolation

$$i_{IRR} = 0.21 + \frac{17,574}{17,574 - (-752)} \times (0.22 - 0.21)$$

$$i_{IRR} = 0.2195$$

or

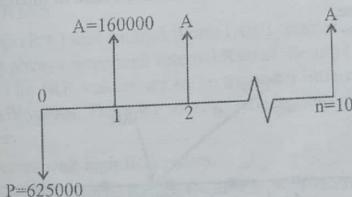
$i_{IRR} = 21.95\% \rightarrow$ This is the Internal rate of return for proposal 2.

To find IRR of Proposal 3

Given, $P = 6,25,000$

$A = 1,60,000$

$n = 10$ years



Step 1: Assume $i = 15\%$ and find PW

$$\begin{aligned} \text{PW}(\text{at } i = 15\%) &= \text{PW}(\text{Revenues}) - \text{PW}(\text{Costs}) \\ &= A(P/A, 15\%, 10) - P \\ &= 1,60,000(5.0188) - 6,25,000 \\ &= 1,78,008 \end{aligned}$$

Step 2: Assume $i = 23\%$ and find PW

$$\begin{aligned} \text{PW}(\text{at } i = 23\%) &= A(P/A, 23\%, 10) - P \\ &= 1,60,000(3.7993) - 6,25,000 \\ &= -17,112 \end{aligned}$$

Step 3: By interpolation

$$i_{IRR} = 0.15 + \frac{1,78,008}{1,78,008 + 17,112} \times (0.23 - 0.15)$$

$$i_{IRR} = 0.2229$$

or

$i_{IRR} = 22.29\% \rightarrow$ This is the IRR of proposal 3

Solution: Following is the ranking of IRR of the 3 proposals

Proposal	IRR (%)
1	21.54
2	21.95
3	22.29

Comparing the IRR of the 3 proposals it is advised to go for proposal 3. This is because under normal circumstances as is in this problem, the best proposal is the one with the highest IRR. It is also to be noted here that the IRR values of all proposals are more than MARR which is given in the problem as 15%.

DETAILED CONCEPT OF IRR

If we plot a graph of Present Worth of a business Vs Rate of interest, the following general relationship might arise.

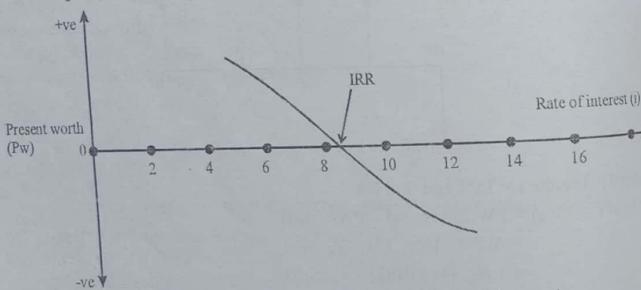


Figure 4.1 : Present Worth V/s Rate of Interest

We can observe the following points w.r.t above graph:

- For a regular cash flow diagram as seen in worked examples, the PW is +Ve at lower values of i and becomes -Ve beyond some higher value of i . This is evident from fig. 4.1. The IRR here therefore is the point at which the curve intersects the horizontal line.
- IRR can also be defined as the interest rate earned on the unrecovered balance over an investments life so that the unrecovered balance at the end of that time is zero. Here Unrecovered balance refers to the costs invested in the business which have to be recovered through revenues. If all costs are recovered at the end of service life with the help of revenues, then that particular rate of interest is called the Internal rate of return.

- IRR is also the rate of interest i at which costs equals revenues leading to a no loss-no profit situation.
- For CFD's where the PW Vs i relationship is as shown in fig(1), IRR represents that rate beyond which the business is non-profitable. In other words, the rate of interest applicable over the service life of a business should be less than IRR for revenues to be more than costs.
- Alternately if the IRR of a business alternatives is high, it means that the alternative is in a better position to be profitable.
- While comparing alternatives with different values of IRR, it is preferable to select the alternative with highest IRR, but with exceptions.
- Exceptions are those where the CFD's are complex with mixed revenues and costs and having multiple service lives.

IRR MISCONCEPTIONS

The present worth (PW) and Annual Worth (AW) comparisons give consistently good results and almost always agree with Internal Rate of Return (IRR) calculations, but with rare exceptions. The IRR values are to be treated a little more carefully, when compared to PW and AW values. There are some misconceptions about IRR which needs to be clarified, as under:

- Choosing alternatives with high IRR values.
- More than one possible Rate of Return (Non-simple investment).
- Alternatives with unequal lives.

1. CHOOSING ALTERNATIVES WITH HIGH IRR VALUES

In case of comparison of alternatives having different Cash Flows, the alternatives having the highest PW, AW or IRR is selected. In most of the cases higher values of PW or AW corresponds with higher values of IRR, but with some exceptions. For example, consider two business alternatives X & Y, having an equal initial investment P. If X has increasing revenues over the years, and Y decreasing revenues over the years, their PW values indicate one decision where as IRR indicates the opposite, for a certain range of rates of interest, as shown in figure;

Observing the fig. 4.2, we can see that, PW of project X is greater than PW of project Y until $i = 10\%$. But for $i > 10\%$, IRR of project Y is higher than that of X. Therefore, one has to be careful in comparing the decision based on PW & IRR, since they also are associated with range of values, as in the above case.

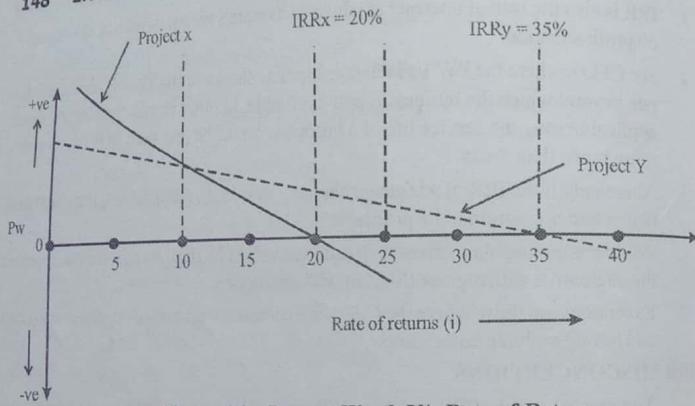


Figure 4.2 : Present Worth V/s Rate of Returns

So, the misconception is that ranking IRR values from higher to lower, for decision-making, is correct all the time.

2. MORE THAN ONE POSSIBLE RATE OF RETURNS

When the Cash Flow of Project switches from +ve to -ve (or the reverse) more than once, the project may have more than one value of IRR, as shown in the fig. 4.3 Such cases may occur when the investments are non-simple in nature.

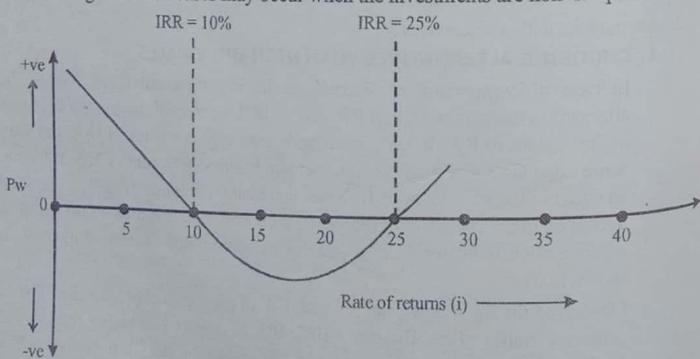


Figure 4.3 : PW V/s Rate of Returns

In such cases, we have to determine which root is, in fact, the true IRR value and which one to be considered for decision-making.
Some methods which help in either overcoming more than one rate of return or converting the Cash Flow into a single direction flow are:

- i. Explicit Reinvestment rate method
- ii. Historical External Rate-of-Return method (HERR).
- iii. Project Balance method.

3. ALTERNATIVES WITH UNEQUAL LIVES

Another potential difficulty with IRR computation relates to comparison of business alternatives with unequal lives. One can use the Least Common Multiple (LCM) method which involves business life cycles of alternatives which have to be replicated many times. Manual IRR determinations in such cases can get very messy. It is therefore, recommended to use PW, FW or AW comparisons for alternatives with unequal lives rather than IRR Comparisons.

COST-OF-CAPITAL CONCEPTS

To run any business, capital is needed—both fixed and working capital. Capital pooled, for any reason, can often come from several sources having different rates of interest. The proportion of capital from different sources obtained at different costs is represented by Weighted Cost-of-Capital. This can be calculated by the formula:

$$i_w = p_1 i_1 + p_2 i_2 + p_3 i_3 \dots p_n i_n$$

Where i_w = Weighted cost of Capital

$i_1, i_2 \dots i_n$ = Rates of interest for various sources of finances.

$p_1, p_2, \dots p_n$ = Proportions of total capital from various sources.

Example : If a firm is financed from source X, Y, Z with rates of interest 7.9 and 11 percent and in proportions 20, 30, and 50 percent, respectively, then Weighted Cost of Capital.

$$i_w = 0.07 (0.2) + 0.09 (0.3) + 0.11(0.5)$$

$$i_w = 0.096$$

or

$$i_w = 9.6\%$$

The cost of Capital is not an easy quantity to estimate despite the apparent simplicity of the formula. When sources of finance is from bonds, stocks, banks, depreciation funds, unpaid taxes, bank overdrafts, hand loans, profits re-invested etc., it becomes troublesome to determine proportions and rates of interests.

The weighted cost of Capital is used to determine minimum acceptable rate of return (MARR) of business alternatives. However, the actual rate of return expected from new investments is normally greater than Weighted cost of capital. In other words, the overriding criterion of any business is to see that the actual rate of return is comfortably higher than a well-estimated weighted cost of capital. If not, it is better to invest capital elsewhere to get a higher return!

In a business organization, the cost of capital is seldom determined by the engineering economist. It will be determined by the Chief Financial Officer (or someone reporting to the CEO) in conjunction with the accounting department.

DEPRECIATION

Introduction

Depreciation basically means a decrease in worth. It is nothing but the reduction in value of any physical asset with the passage of time. Depreciation is the opposite of Appreciation. Except for some commodities like real estate, art objects etc., whose value appreciates with time, all physical resources that we come across loses its value with time. This is because equipment may wear and tear or become obsolete with respect to technology or with time, which all reduces its value.

In business, depreciation of all equipment is calculated and considered as operating costs, which will also help reduce taxes. Strictly speaking, if an amount equal to depreciated value of any equipment is saved every year, the sum total could be used in future to buy new equipment, when old equipment becomes useless. This is the use and importance of calculating depreciation.

In any industry, new machinery needs to be bought continuously to replace old ones, and this requires money. This money has to be generated from the earnings of the old equipment. Money which is set aside from the earnings of an equipment for its replacement purpose is called **Depreciation Fund**. But seldom is a depreciation fund actually established to accumulate money earmarked for the replacement of any specific asset. Instead, **Depreciation Reserves**, are used to fund proposals to improve operations, by way of replacing worn-out equipment or by way of improving working conditions to improve life of assets. Recovered capital is thus reinvested in a general way to maintain the company's physical resources and to pursue new ventures.

Cause of Depreciation

Assets may depreciate in value for several reasons. Their decreasing worth may be attributed to any of the following reasons:

1. Physical Depreciation

The everyday wear and tear of operation gradually reduces the physical ability of any equipment to perform its intended function. A good maintenance program may probably improve the life of the equipment, but they can seldom be compared to new machines. In addition to normal wear, accidental physical damage can impair ability. Wear and tear is an obvious addition to cost of production.

2. Functional Depreciation

Performance of any equipment continuously decreases with time, unable to meet the demand. In other words, demands made on an asset may increase beyond its capacity to produce. At the other extreme, the demand for services may cease to exist. In other words, a machine may be producing a product well but the product may well be no longer in demand.

3. Technological Depreciation

Development of new machines and process may render the current ones uneconomical. This is especially true in the present world with respect to telecommunication products, software, electronic goods etc... Along with machines, the technology associated with them also depreciates.

4. Sudden Failure

This refers to sudden or catastrophic loss in value of an equipment due to the presence of inherent technological flaws. However, this does not include loss due to accident or misuse. Light bulbs burn out as a natural consequence of regular use and with little loss in its brightness up to the point of failure. Generally, this category of assets include items used in large numbers with a relatively low unit cost.

5. Depletion

Consumption of an exhaustible natural resource like petroleum, coal etc., continuously leads to a depletion of sources. This can reduce the worth of sites from where these sources are exploited. In other words, depletion of natural resources leads to a reduction in earnings derived from the resources

6. Monetary Depreciations

Depreciation calculations always consider original prices of assets. But these prices can vary because of inflation and other market forces, which can make replacements worrisome. If prices rise during the life of an asset, then a comparable replacement becomes more expensive. This means that depreciation fund set aside will be insufficient to provide an adequate substitute for the worn-out machine. It also suggests that the selling price of the product being produced by the machine does not accurately reflect the cost of production. Because the depreciation is actually happening to the invested capital representing the equipment, instead of the equipment itself, monetary depreciation is very difficult to accommodate. Monetary depreciation cannot be even considered as operating expense for tax benefits.

Basic Methods of Computing Depreciation Charges

There are several methods for computing depreciation charges and for accounting depreciation funds. Those are as follows:

1. Straight-Line (SL) Method
2. Declining-Balance (DB) Method
3. Sum-of-the-Years-Digits (SYD) Method
4. Sinking Fund (SF) Method
5. Service Output (SO) or Units production Method

PFTSDM

Each of the above methods will be discussed Step by step with the help of a worked example.

1. Straight-Line (SL) or Fixed Percentage Method of Depreciation

In this method of depreciation, an equipment is considered to be depreciating uniformly throughout its service life. In such a case, a fixed sum is charged as depreciation amount throughout the lifetime of the equipment. The accumulated sum at the end of the life of the equipment is exactly equal to the purchase value of the equipment minus its salvage value. In other words, with depreciation fund plus salvage value, one can buy a new piece of the same equipment, provided its price has not changed! This is known as the SL Method of depreciation.

Let P = First cost of the equipment

S = Salvage value of the equipment

n = Life of the equipment

B_t = Book value of the asset at the end of period t

D_t = Depreciation amount for the period t

$P =$

Some Definitions**i. Book Value (BV)**

This is the worth of a depreciable property as shown on the accounting records of the company at any given point of time (t) during its life. It is based on the original cost of the property. It thus represents the amount of capital that remains invested in the property and must be recovered in the future through the accounting process. Book value at the end of time ' t ' can also be thought of as the amount recovered (salvaged) if the asset is sold at that point. But most times, book value of a property may not be a useful measure of its market value, practically speaking.

ii. Depreciation Amount (DE)

Money invested on machines bring revenues. Machines also depreciates on a continuous basis. Therefore, a portion of money, is deducted from revenue and set aside as depreciation amount. This amount may vary during the service life of an asset and hence it may be calculated for any time period t . Depreciation fund is the sum of depreciation amounts up to a time period, t . Depreciation amount is also simply known as depreciation.

Formulae for Depreciation Amount (D_t) and Book Value (B_t)

$$D_t = \frac{P - S}{n}$$

$B_t = B_{t-1} - D_t \rightarrow$ for successive periods

$$\text{or } B_t = P - t \left[\frac{P - S}{n} \right] \rightarrow \text{for any given period}$$

WORKED EXAMPLE

1. A student has bought a moped whose first cost is Rs. 10,000 with an estimated life of 8 Years. The estimated Salvage value of the moped at the end of its lifetime is Rs 2,000. Determine the depreciation amount and the book value at the end of various years using Straight-Line (SL) method of depreciation. Also find the book value at end of 7th year as a specific period.

Solution

Given Initial cost, $P = \text{Rs.} 10,000$

Salvage Value, $S = \text{Rs.} 2,000$

Life of Moped, $n = 8$ years

Depreciation amount, $D_t = ?$

Book Value $B_t = ?$

$$D_t = \frac{P - S}{n}$$

$$D_t = \frac{10000 - 2000}{8}$$

$D_t = \text{Rs.} 1000 \rightarrow$ This is uniform and independent of time period.

This means that the moped depreciates every year by Rs.1,000/- uniformly.

Book value at time zero, $B_0 = \text{Rs.} 10,000$

Book value at time period, $B_t = B_{t-1} - D_t$

$$B_t = B_{t-1} - D_t$$

$$\therefore B_t = B_0 - D_t (\because D_1 = D_2 = \dots D_8)$$

$$B_t = 10000 - 1000$$

$$= \text{Rs.} 9000$$

Table showing the values of D_t and B_t for all years

End of year (t)	Depreciation (D_t) Amount	Book value $B_t = B_{t-1} - D_t$
0	—	10000
1	1000	9000
2	1000	8000
3	1000	7000

4	1000	6000
5	1000	5000
6	1000	4000
7	1000	3000
8	1000	2000

To find Book value for Specific period of 7th year

When $t = 7$

$$B_t = P - t \left[\frac{P-S}{n} \right]$$

$$B_7 = 10000 - 7 \left[\frac{10000 - 2000}{8} \right]$$

$B_7 = \text{Rs.}3000 \rightarrow$ This is the book value at the end of 7th year

II. Declining-Balance (DB) Method of Depreciation

In this method, depreciation amount for any period is a percentage of the book value of the previous period. In other words, depreciation amount decreases for each successive period unlike SL method where it is constant for every period. This approach is more realistic since, depreciation is more dependent on recent book values rather than the original one. But the sum of all depreciation amounts plus the Salvage value of the asset will not be equal to the first cost of asset as in SL method. This is a major limitation of DB method.

Let P = First cost of asset

S = Salvage value of the asset

n = Life of asset

D_t = Depreciation amount for period t

B_t = Book value of asset at the end of period t

$$k = \text{A fixed percentage (to find } D_t) = \left[1 - \left(\frac{S}{P} \right)^{1/n} \right] \quad (\text{If not given in problem})$$

Formulae to find D_t and B_t

$$D_t = k \times B_{t-1}$$

$B_t = B_{t-1} - D_t \rightarrow$ To find book value for successive periods.

$B_t = B_{t-1}(1 - k) \rightarrow$ To find book value for any specific time period t when B_{t-1} is known.

Formulae to find D_t and B_t in terms of P

$$D_t = k(1 - k)^{t-1} \times P$$

$$B_t = (1 - k)^t \times P$$

WORKED EXAMPLE

1. Let us take the example of the same problem as in SL method

Where

$$P = \text{Rs.}10,000$$

$$S = \text{Rs.}2,000$$

$$N = 8 \text{ years}$$

Let $k = 10\%$ or $k = 0.10$

To find Depreciation amount when $t = 1$

$$D_t = k \times B_{t-1}$$

$$D_1 = k \times B_0$$

$$D_1 = 0.1 \times 10000$$

$$D_1 = 1000$$

To find book value when $t = 1$

$$B_t = B_{t-1} - D_t$$

$$B_1 = B_0 - D_1 = 10000 - 1000$$

$$B_1 = 9000$$

Table Showing values of D_t and B_t for all years

End of year t	Depreciation (D_t) = $k \times B_{t-1}$	Book value $B_t = B_{t-1} - D_t$
0	—	10000
1	1000.00	9000.00
2	900.00	8100.00
3	810.00	7290.00
4	729.00	6561.00
5	656.10	5904.90
6	590.49	5314.4

7	531.44	4782.97
8	478.29	4304.68

From the above values it is observed that depreciation amount is not constant for each period. It is also observed that book value decreases at a decreasing rate.

To find D_t and B_t at the end of specific period,

When $t = 7$

$$D_t = k (1 - k)^{t-1} \times P \\ = 0.1 (1 - 0.1)^{7-1} \times 10000$$

$D_t = 531.441 \rightarrow$ is the depreciation amount for period $t = 7$

$$B_t = (1 - k)^t \times P \\ = (1 - 0.1)^7 \times 10000$$

$B_t = 4782.97 \rightarrow$ is the book value at the end of period $t = 7$

Note : When $k = \frac{200\%}{n}$, then it is known as Double-declining balance method.

It has the same characteristics as the declining-balance method.

III. Sum-of-the-year-Digits (SYD) method

Following is a step-by-step description of SYD method.

- i. In this method also, it is assumed that the book value of the asset decreases at a decreasing rate.
- ii. If the asset has a life of 10 years, first the sum of the years is calculated as.
Sum of the years = $1+2+3+4+5+6+7+8+9+10 = 55$

$$\text{or Sum of years} = \frac{n(n+1)}{2} = \frac{10(10+1)}{2}$$

$$\text{Sum of years} = 55$$

- iii. Next, the rate of depreciation for the first year is assumed as the highest and then it decreases. The rates of depreciation for the years 1-10, respectively are as follows:

$$\text{Year 1} = 10/55$$

$$\text{Year 2} = 9/55$$

$$\text{Year 3} = 8/55$$

$$\text{Year 6} = 5/55$$

$$\text{Year 7} = 4/55$$

$$\text{Year 8} = 3/55$$

Year 4 = 7/55

Year 9 = 2/55

Year 5 = 6/55

Year 10 = 1/55

iv. For any year, the depreciation is then calculated by

$$Dt = \text{Yearly Rate} \times (P - S)$$

where P = initial cost

S = Salvage value

$$B_t = B_{t-1} - D_t$$

Formulae to find D_t and B_t for a Specific year t

$$D_t = \frac{n-t+1}{n(n+1)/2} (P - S)$$

$$B_t = \left[(P - S) \left(\frac{n-t}{n} \right) \left(\frac{n-t+1}{n+1} \right) \right] + S$$

WORKED EXAMPLE

1. Let us consider the same example as in SL method

Where $P = \text{Rs. } 10,000$

$S = \text{Rs. } 2,000$

but $n = 10$ Years

To find D_t and B_t for Successive periods

When $t = 1$, $B_0 = 10,000$

D_t = Yearly rate $(P - S)$

$$D_1 = \frac{10}{55} \times (10000 - 2000)$$

$D_1 = \text{Rs. } 1454.54 \rightarrow$ depreciation for 1 year.

$$B_t = B_{t-1} - D_t$$

$$= 10000 - 1454.54$$

$B_1 = \text{Rs. } 8545.45 \rightarrow$ Book value at the end of 1 year

Table Showing values of D_t and B_t for all years

End of year n	Depreciation $D_t = \text{year rate} \times (P - S)$	Book Value $B_t = B_{t-1} - D_t$
0	—	10000.00
1	1454.54	8545.45
2	1309.09	7236.36
3	1163.63	6072.73
4	1018.18	5054.55
5	872.72	4181.83
6	727.27	3454.56
7	581.81	2872.75
8	436.36	2436.39
9	290.90	2145.49
10	145.45	2000.00

Observing the values in the table we can see that depreciation amount varies and decreases for successive years, while book value decreases at a decreasing rate. Book value at the end of 10th year is same as salvage value. Book value is nothing but the amount salvaged out of the asset when it is sold at any point of time 't'.

Formulae to find D_t and B_t for Specific period

Let $t = 7, n = 10$ years

$$D_t = \frac{n-t+1}{n(n+1)/2} \times (P-S)$$

$$D_7 = \frac{10-7+1}{10(10+1)/2} \times (10000 - 2000)$$

$$D_7 = 581.81$$

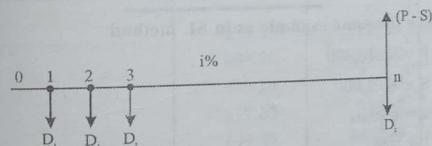
$$B_t = \left[(P-S) \left(\frac{n-t}{n} \right) \left(\frac{n-t+1}{n+1} \right) \right] + S$$

$$B_7 = \left[(10000 - 2000) \left(\frac{10-7}{10} \right) \left(\frac{10-7+1}{10+1} \right) \right] + 2000$$

$$B_7 = 2872.75$$

iv. Sinking Fund (SF) Method of Depreciation

- In This method, the book values decreases at an increasing rate with respect to life of the asset.
- First, the loss of asset value ($P-S$) over its life time is found out.
- The shortfall is made up by setting aside a depreciation amount every year which is equivalent to an annual deposited in a bank at a rate of interest i , compounded annually.
- In other words, a constant sum is deposited in a bank which grows at ' i ' rate of interest and the maturity amount at the end of asset's life (n) is equal ($P-S$). It is nothing but equivalent to Equal Payment Series sinking fund where $A = D_t$ and $F = P - S$. It would have a CFD of



- Hence, although a fixed depreciation amount is set aside every year, the interests accumulated makes the depreciation fund increase more than normally every year.

Let P = First cost of asset

S = Salvage value of the asset

n = Life of asset

i = Rate of interest, compounded annually

A = Annual equivalent amount

D_t = Depreciation amount for period t

B_t = Book value of asset at the end of period t

Formulae to find D_t and B_t for successive periods

$$D_t = (P - S) \times (A/F, i, n)$$

$$\text{or } D_t = (P - S) \left[\frac{i}{(1+i)^n - 1} \right]$$

$$B_t = B_{t-1} - D_t$$

Formulae to find D_t and B_t Specific period t

$$D_t = (P - S) \times (A/F, i, n) (F/P, i, t - 1)$$

$$\text{or } D_t = (P - S) \left[\frac{i}{(1+i)^n - 1} \right] (1+i)^{t-1}$$

$$B_t = P - [(P - S) (A/F, i, n) (F/A, i, t)]$$

$$B_t = P - D_t$$

$$B_t = P - \left[(P - S) \left(\frac{i}{(1+i)^n - 1} \right) \left(\frac{(1+i)^t - 1}{i} \right) \right]$$

WORKED EXAMPLE

1. Let us take the same example as in SL method

Where $P = \text{Rs.}10,000$

$S = \text{Rs.}2,000$

$n = 8$ years

$i = 10\%$

To find D_t and B_t for successive periods

$$D_t = (P - S) (A/F, i, n)$$

$$D_1 = (10000 - 2000) (A/F, 10\%, 8)$$

$$D_1 = (8000) (0.0874) \text{ (from tables)}$$

$D_1 = \text{Rs.}699.20$ Depreciation amount per year.

Therefore, a constant amount of Rs. 699.20 is collected as depreciation amount and invested in a bank. If rate of interest is 10 %, compounded annually.

Depreciation amount D_t for year 1, $D_1 = \text{Rs.}699.20$.

But, $D_2 = 699.20$ (current year) + 699.20×0.10 (interest from previous year)

$$D_2 = 769.12$$

D_t for year 3 = 699.20 (this year) + 699.20×0.1 (interest from D_2) + 769.12×0.1 (interest from D_1)

$$\therefore D_3 = 846.03$$

$$D_4 = 699.20 + (699.20 + 769.12 + 846.03) \times 0.1$$

$$D_4 = 930.63$$

$$D_5 = 699.20 + (699.20 + 769.12 + 846.03 + 930.63) \times 0.1$$

$$D_5 = 1023.69$$

$$D_6 = 699.20 + (699.20 + 769.12 + 846.03 + 930.63 + 1023.69) \times 0.1$$

$$D_6 = 1126.06$$

$$D_7 = 699.20 + (699.20 + 769.12 + 846.03 + 930.63 + 1023.69 + 1126.06) \times 0.1$$

$$D_7 = 1238.67$$

$$D_8 = 699.20 + (699.20 + 769.12 + 846.03 + 930.63 + 1023.69 + 1126.06 + 1238.67) \times 0.1$$

$$D_8 = 1362.54$$

Table Showing value of D_t and B_t for all years

End of year t	D_t	D_n	B_t
0	—	—	10000.00
1	699.20	699.20	9300.00
2	699.20	769.12	8531.68
3	699.20	846.03	7685.65
4	699.20	930.63	6755.02
5	699.20	1023.69	5731.33
6	699.20	1126.06	4605.27
7	699.20	1238.67	3366.60
8	699.20	1362.54	2004.00

Observing the values in the table, we can see that although depreciation amount set aside each year is constant, interest accrued on it makes net depreciation (D_t) each year to be progressively higher. Therefore, Book value decreases at an increasing rate to come down to salvage value of Rs.2,000 (approx) at the end of life of the asset after 8 years.

To find D_t and B_t for specific periods

Let $t = 7$, $B_0 = \text{Rs.}10000$, $n = 8$, $i = 10\%$

$$D_t = (P - S) \left(\frac{i}{(1+i)^n - 1} \right) (1+i)^{t-1}$$

$$D_t = (10000 - 2000) \left(\frac{0.1}{(1+0.1)^8 - 1} \right) (1+0.1)^{7-1}$$

$$D_t = 1239.29$$

$$B_i = P - \left[(P-S) \left(\frac{i}{(1+i)^n - 1} \right) \left(\frac{(1+i)^i - 1}{i} \right) \right]$$

$$B_i = 10000 - \left[(10000 - 2000) \left(\frac{0.1}{(1+0.1)^8 - 1} \right) \left(\frac{(1+0.1)^8 - 1}{0.1} \right) \right]$$

$$B_i = 3363.22$$

Hence values of D_i and B_i are approximately the same as in the table allowing for fractional errors.

*v. Service Output or Units of production method of Depreciation

Depreciation calculated based on only passage of time does not always make sense. The wear and tear of equipment depends on the extent of usage at any given time period rather than just passage of time. Therefore, it becomes prudent sometimes to calculate depreciation on the extent of service rendered or on quantities of output produced.

Let P = First cost of the asset

S = Salvage value of the asset

C = Maximum capacity of service of the asset

q = Quantity of service rendered in a period

D_u = Depreciation per unit of service rendered

D_q = Depreciation for q units of service rendered

$$\therefore D_u = \left(\frac{P-S}{C} \right)$$

$$D_q = \left(\frac{P-S}{C} \right) \times q$$

WORKED EXAMPLE

- A hi-tech bus was initially bought for Rs.50 lakhs. Its salvage value after 8 years of service would be 10 lakhs. In its lifetime it can be driven for a distance of 10 lakh kms. In its 5th year of operation, if it has already traversed a total distance of 8 lakh kms, find the depreciation of the bus at the point.

Solution

Given $P = \text{Rs.}50,000$

$n = 8$ years

$$S = \text{Rs.}10,00,000$$

$$C = 10,00,000 \text{ Kms}$$

$$q = 8,00,000 \text{ Kms}$$

Unit Depreciation,

$$D_u = \frac{P-S}{C} = \frac{5000000-1000000}{1000000}$$

$$D_u = \text{Rs.}4 \text{ per km driven}$$

Depreciation after driving q kms,

$$D_q = \left(\frac{P-S}{C} \right) \times q = 4 \times 800000$$

$$Dq = \text{Rs.}32 \text{ Lakhs}$$

The bus would have depreciated by this amount at the end year 5.

Note: Please note that in the above problem ' n ' is inconsequential. In other words, 8 years of service or 5th year of operation has no effect on the depreciation of the vehicle. It is only the distance travelled which depreciates the vehicle.

SOME MORE WORKED EXAMPLES

- A CNC machine costing Rs.22,00,000/- is estimated to serve for 5 years after which its salvage value is estimated to be Rs.2,00,000/- Find

(i) Depreciation during third year by fixed percentage method.

(ii) Book value of the machine after two years by sum of years digits method.

(VTU MQP)

Solution

Given,

Initial cost, $P = 22,00,000$

Salvage value, $S = 2,00,000$

Life of machine, $n = 5$ years

- Depreciation during third year by fixed percentage method (or straight-line method)

In this method, depreciation amount is same for all years. Therefore

$$D_3 = D_u = \frac{P-S}{n} = \frac{22,00,000 - 2,00,000}{5}$$

$D_3 = 4,00,000 \rightarrow$ This is the depreciation amount during third year.

- (ii) Book value of the machine after two years by sum-of-years-digits method

Book value for any specific 't' in this method is

$$B_t = \left[(P-S) \left(\frac{n-t}{n} \right) \left(\frac{n-t+1}{n+1} \right) \right] + S$$

$$B_2 = \left[(22,00,000 - 2,00,000) \left(\frac{5-2}{5} \right) \left(\frac{5-2+1}{5+1} \right) \right] + 2,00,000$$

$B_2 = \text{Rs.}10,00,000 \rightarrow$ This is the book value of the machine after 2 years.

2. A CNC machine costs Rs.30,00,000 is estimated to serve for 8 years after which its salvage value is estimated to be Rs.2,50,000. Find

- (i) Depreciation fund at the end of the 5th year by fixed percentage method and declining balance method.
(ii) Book value of the machine after 4th year and 6th year by declining balance method.

(VTU July 2005)

Solution Cost of CNC Machine, $P = \text{Rs.}30,00,000$
Salvage value, $S = \text{Rs.}2,50,000$
Life of machine, $n = 8$ years

- (i) Fixed Percentage (or straight line) method of depreciation

$$\text{Depreciation Amount per year, } D_t = \frac{P-S}{n} = \frac{30,00,000 - 2,50,000}{8}$$

$$D_t = \text{Rs.}3,43,750$$

Depreciation fund at the end of 5 years = $3,43,750 \times 5 = \text{Rs.}17,18,750$

- (ii) Declining Balance Method

Book value after 4th year, $B_4 = k(1-k)^{t-1} \times P$

$$B_t = \left(1 - \left(\frac{S}{P} \right)^{\frac{1}{n}} \right) \left[1 - \left(1 - \left(\frac{S}{P} \right)^{\frac{1}{n}} \right)^{t-1} \right] \times P$$

$$B_t = P \left(\frac{S}{P} \right)^{\frac{1}{n}}$$

$$B_4 = 30,00,000 \left(\frac{2,50,000}{30,00,000} \right)^{\frac{4}{8}}$$

Book value at the end of 4 years $B_4 = \text{Rs.}8,66,025$

$$B_6 = P \left(\frac{S}{P} \right)^{\frac{1}{n}} = 30,00,000 \left(\frac{2,50,000}{30,00,000} \right)^{\frac{6}{8}}$$

$$B_6 = \text{Rs.}4,65,302$$

3. The original assets of a company are Rs.5,80,000. The life of the plant is 9 years. If the scrap value at the time is expected to be Rs.80,000 calculate the depreciation at the end of each year by sum of the years digit method.

Solution Sum of years digits method

(VTU Jan 2006)

$$P = \text{Rs.}5,80,000$$

$$S = \text{Rs.}80,000$$

$$n = 9 \text{ years}$$

$$\text{Sum of years} = \frac{n(n+1)}{2} = \frac{9(9+1)}{2} = 45$$

For any year, the depreciation is calculated by, $D_t = \text{yearly rate} \times (P - S)$

When yearly rate = $9/45$ (1st year), $8/45$ (2nd year) and so on.

Table shows the depreciation at the end of each year

End of year t	Depreciation (D_t) amount
1	$9/45(5,80,000 - 80,000) = 1,00,000$
2	$8/45(5,80,000 - 80,000) = 88,888.88$
3	$7/45(5,00,000) = 77,777.77$
4	$6/45(5,00,000) = 66,666.66$
5	$5/45(5,00,000) = 55,555.55$
6	$4/45(5,00,000) = 44,444.44$
7	$3/45(5,00,000) = 33,333.33$
8	$2/45(5,00,000) = 22,222.22$
9	$1/45(5,00,000) = 11,111.11$

4. A device is purchase for Rs. 1,000/- and has no salvage value. It is expected to serve for 5 years. Calculate the book value by sum of years digit method and double declining balance method.

(VTU Jan.2008)

Solution,

Given data,

Purchase Price, $P = 1000$

Life of device, $n = 5$ years

Salvage value = zero

(i) Book value by SYD method

In SYD method we can find the book value by using any of the two formulae:

$$B_t = B_{t-1} - D_t \quad \text{Where } D_t = \text{Depreciation}$$

$$\text{or } B_t = \left[(P-S) \frac{(n-t)}{n} \frac{(n-t+1)}{n+1} \right] + S = \text{Yearly Rate}(P-S)$$

Using the direct formulae we get

$$\therefore B_1 = \left[(10000-0) \left(\frac{5-1}{5} \right) \left(\frac{5-1+1}{5+1} \right) \right] + 0$$

$$B_1 = 6666.66$$

Similarly B_2 to B_5 are calculated

Year	Book Value
1	6666.66
2	4000
3	2000
4	666.66
5	0

(ii) Book value by Double declining balance method

In this method, book value is given by

$$B_t = (1-k)^t \times P$$

$$\text{When } k = \frac{200\%}{n}$$

$$\therefore k = \frac{200\%}{5}$$

$$\therefore k = 40\%$$

$$B_1 = (1 - 0.4)^1 \times 10,000 \\ = 6,000$$

$$B_2 = (1 - 0.4)^2 \times 10,000 \\ = 3,600$$

$$B_3 = (1 - 0.4)^3 \times 10,000 \\ = 2,160$$

$$B_4 = (1 - 0.4)^4 \times 10,000$$

$$= 1,296$$

$$B_5 = (1 - 0.4)^5 \times 10,000$$

$$= 777.6$$

Year	Book Value
1	6000
2	3600
3	2160
4	1296
5	777.6

5. An asset cost was Rs. 400 when purchased 4 years ago. A scrap value of Rs. 50 was expected at the end of the 7 year useful life. Determine the depreciation charge during the coming year and the assets current book value by.

(i) Straight line method of depreciation.

(ii) Declining - balance depreciation (using the salvage value to determine the depreciation rate.)

(VTU July 2006)

Sol.

Given data,

$$\text{Asset cost, } P = \text{Rs. } 4,00$$

$$\text{Salvage value, } S = \text{Rs. } 50$$

$$\text{Life of asset, } n = 7 \text{ years}$$

$$\text{Depreciation in 5th year, } D_5 = ?$$

$$\text{Book value at the end of 4th year, } B_4 = ?$$

(i) To find D_5 and B_4 by St-line method

$$D_5 = \frac{P - S}{n} \quad B_4 = P - t \left[\frac{P - S}{n} \right]$$

$$= \frac{400 - 50}{7} \quad 400 - 4 \left[\frac{400 - 50}{7} \right]$$

$$D_5 = 50 \quad B_4 = 200$$

(ii) To find D_3 and B_4 by declining balance method

$$D_i = k \times (1 - k)^{i-1} \times P$$

$$\text{When } k = \left[1 - \left(\frac{S}{P} \right)^{\frac{1}{n}} \right] \rightarrow \text{If } k \text{ is not given}$$

$$\therefore k = \left[1 - \left(\frac{50}{400} \right)^{\frac{1}{5}} \right]$$

$$k = 0.257$$

$$\therefore D_3 = 0.257(1 - 0.257)^{3-1} \times 400$$

$$D_3 = \text{Rs.}31.33$$

$$B_1 = (1 - k)^4 \times P$$

$$B_4 = (1 - 0.257)^4 \times 400$$

$$B_4 = \text{Rs.}121.90$$

TAX CONCEPTS

A tax is a compulsory contribution levied on an individual or a corporation by the Government of a country without any reference to any benefit. The aim of taxation is to raise funds to build infrastructure, to defend the country from enemies, to prohibit or regulate certain activities, and for social equalization.

Kinds of Taxes : Taxes may be direct or indirect.

A **Direct Tax** is a tax, the burden of which is mainly borne by the person on whom it is imposed. The income tax, inheritance tax, property tax are direct taxes.

An **Indirect Tax** is a tax, the burden of which is passed on by the person on whom it is imposed, to others. Indirect taxes are generally levied on commodities. The customs, sales tax, and the excise duties etc., are indirect taxes.

Following are the principle types of taxes and their relevance to engineering economic studies:

1. Property Taxes : These are charged by state Governments on land, buildings, machinery and equipment, inventory etc.. The amount of tax is dependent on the estimated value of assets and the prevalent tax rate. Property taxes are usually not

a significant factor in an engineering economics study because of their small magnitude compared with income taxes and their similar effect on competing proposals.

2. Excise Taxes : These are imposed on the production of certain products such as tobacco and alcohol. These do not have a big effect on engineering and its economics.

3. Sales Tax : Whenever a commodity exchanges hands, money is paid and a bill is raised, sales tax comes into the picture. This plays a major role in engineering economics.

4. Value-added Tax (VAT) : Value-added tax is tax levied on a company only on the value addition that it does. In other words, for a company producing any product, tax calculated is on the difference of value of outgoing product and the value of incoming material. A company is liable to be taxed only on the portion of the value that it adds to the product.

Value-added tax depends on both costs and profits, and not on profits alone. VAT provides incentives for higher efficiencies of companies and encourages reduction of wasteful expenditures. But VAT is also considered inflationary and regressive with respect to income.

5. Income Taxes : This is imposed on personal and corporate incomes. Income tax rates are usually higher for higher incomes. They are based on net income after deduction allowed for permissible expenses.

The following chart shows the sources of national income of India during 2004-2005:

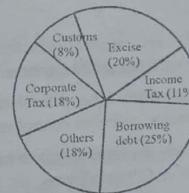


Fig. Sources of Indian National Income

CORPORATE INCOME TAX

Just like individual, companies are subjected to income tax. This is known as Corporate Income Tax which is imposed on the net profits of the company.

Corporate Tax is payable by all forms of business associations, joint-stock companies, insurance companies, and trusts and partnerships that operates as corporations. Organizations of doctors, lawyers, engineers and other professionals are generally recognized as corporations. Such organization have the following characteristics :

1. Organized association to carry on business.
2. Profits from business that are divided.
3. Continuity of life and centralized management.
4. Limited liability and free transferability of interests.

Corporate tax in India is imposed on the net profits made by the company. Generally net profits are obtained by deducting all kinds of expenses, interest on debts, depreciation and so on from Gross income.

$$\text{Taxable Income} = \text{Gross Income} - (\text{Deduction})$$

$$\text{Taxable Income} = \text{Gross Income} - \text{expenses} - \text{interest on debt} - \text{depreciation} - \text{other allowable deductions.}$$

Gross Income: Includes revenues from sale of goods and services to customers; dividends received on stocks; interest from loans and securities; rents, royalties and other gains from ownership of capital or property.

Expenses: Include wages, salaries, rents, repairs, taxes, materials, employee benefits, advertising etc.

Interest: Interest paid on debts and borrowings are also deductible from taxable income. When interest is paid on a loan borrowed to buy a house, the interest payable is deducted from taxable income.

Depreciation: It is the cost equivalent of reduction in the worth of machinery and equipment, which is also deducted from taxable income.

Other deductions: Include many other provisions such as losses from fire theft, insurances paid R & D expenditures, pollution control expenditures etc.

There are many ways in which corporate taxes in India are levied in the form of surtax, Penal tax, Capital Gains tax, Commodity tax, tax on capital stock, income tax, tax on excess profits, tax on undistributed profits, value-added-tax etc.

□ □ □

SUMMARY

1. Rate of Return

Rate of return is the rate at which investments return profit to the investor. From an investors point of view, rate of interest is the interest paid towards sources of finance whereas rate of return is the return from the business on which money has been invested.

2. Minimum Acceptable Rate of Return (MARR)

This is minimum rate at which investment made should fetch profit. This may vary from business to business. But MARR has to be more than the interest rate that is available in the real world.

3. Internal Rate of Return (IRR)

This is the rate of interest at which costs and revenues of any business is equal. In other words, it is the rate of interest at which businesses break even.

4. External Rate of Return (ERR)

This is the rate of interest available in the outside world (outside to business) like post offices, banks, mutual funds, other business etc. If ERR is more than the return actually obtained from the business, it is better to close the business and enjoy higher rates of return outside.

5. Producer to find IRR

IRR is the rate of interest at which PW of FW values becomes zero.

Step 1 By trial and error, find two successive values of i at which PW changes from +ve to -ve.

Step 2 By interpolation, find the exact value of i at which PW is zero.

Step 3 Repeat the steps for all the alternatives and select that alternatives with highest IRR.

6. IRR Misconceptions

Some misconceptions about calculating and interpreting IRR are:

i. **Selecting alternative with higher IRR value:** Selecting an alternative with a higher IRR value alternatives with lower IRR values is not always correct. Decision based on PW, FW or AEW comparison methods need not always be consistent with decision based on IRR comparison.

ii. **Multiple IRRs :** In some business projects there could be more than one value of IRR which makes decision-making difficult.

iii. Cost-of-Capital Concepts: To run any business finance is gathered from several sources to which of which different rates of interest are due. Therefore a resultant rate of interest is calculated which is known as weighted cost of capital.

7. Depreciation

Depreciation is the reduction in value of any equipment with passage of time. Causes for depreciation would be:

- Physical depreciation : Depreciation due to physical wear and tear of machinery.
- Functional depreciation : Depreciation due to reduction in machines utilization value.
- Technological depreciation : Development of new technology may render the current machines *hors de combat* (out of contest).
- Sudden failure : Failures of machinery due to inherent technological flaws may drastically reduce or annul in values.
- Depletion : Assets with limited resources may depreciate due depletion of resources.

8. Book Value (B)

It is nothing but the value of an asset at any given point of time, and is based on depreciation calculation.

9. Depreciation Amount (D)

Reduction in value of any machine in any given year is known as the depreciation amount. This could be constant for each successive year or could be varying.

10. Methods of computing depreciation.

See in the next page box.

11. Tax concepts

Taxes are levied by the state on its people for maintenance of the state. Taxes are several kinds—property tax, Excise tax, Sales tax, Value-added-tax (VAT), Income tax, etc.

12. Corporate Income Tax

Tax levied on the income of a company is known as corporate income tax. Corporate income tax is calculated after subtracting all kinds of deduction from the gross income, i.e.,

$$\text{Taxable income} = \text{Gross Income} - \{\text{deductions}\}$$

$$\frac{1}{10} \times 50 = \frac{n(n+1)}{2} = 55$$

15

Taxable income = Gross Income – Expenses – Interest on debt – depreciation – other allowable deductions.

Formulas to find Depreciation amount (D_t) & Book value (B_t) for various methods of depreciation

Sl. No.	Method of Computing Depreciation	D_t for Successive years	B_t for successive years	D_t for any specific period	B_t for any specific period
1.	Straight line Method	$D_t = \frac{P-S}{n}$	$B_t = B_{t-1} - D_t$	$D_t = \frac{P-S}{n}$	$B_t = P - t \left(\frac{P-S}{n} \right)$
2.	Declining Balance method	$D_t = K \times B_{t-1}$	$B_t = B_{t-1} - D_t$	$D_t = k(1-k)^{t-1} \times P$	$B_t = (1-k)^t P$
3.	Sum of years Digits method	$D_t = \text{Yearly rate} \times (P-S)$	$B_t = B_{t-1} - D_t$	$D_t = \left(\frac{n-t+1}{n(n+1)/2} \right) \times (P-S)$	$B_t = \left[(P-S) \left(\frac{n-t}{n} \right) \times \left(\frac{n-t+1}{n+1} \right) \right] + S$
4.	Sinking fund method	$D_t = (P-S) \left(\frac{i}{(1+i)^n - 1} \right) \times \frac{1}{P}$	$B_t = B_{t-1} - D_t$	$D_t = (P-S) \times \left[\frac{i}{(1+i)^n - 1} \right] \times \frac{1}{P}$	$B_t = P - \left[(P-S) \times \left(\frac{i}{(1+i)^n - 1} \right) \times \frac{1}{P} \right]$
5.	Service output method	Unit depreciation		Depreciation for q	$D_q = \left(\frac{P-S}{C} \right) \times q$

EXERCISE PROBLEMS

1. A manufacturing company is planning to buy a CNC milling machine. It has identified 2 different companies for the supply of the machine. The two CNC milling machines have both lives of 4 years and the minimum rate of return is 10 % Find the best alternative based on the rate of return method.

Machine	A	B
Initial Investment (Rs)	50 lakhs	60 lakhs
Annual income (Rs)	12,02,800	16,00,000

2. A transportation unit required 2 trucks for transportation use. The details are.

	Truck 1	Truck 2
First cost (Rs)	15,00,000	20,00,000
Estimated annual maintenance cost(Rs)	15,000	30,000
Estimated life	10 yrs	10 yrs
Salvage value	5,00,000	7,50,000

Both trucks deliver the same amount of work. Take rate of interest 7%. Select the best alternative based on rate of return method.

3. A Lathe is required in the machine shop, the owner of the machine shop is in the process of selecting the best alternative among the following.

Alternative	Initial Investments (Rs)	Annual Revenue (Rs)	Life (yrs)
X	6,00,000	1,00,000	8
Y	9,00,000	1,60,000	8
Z	4,00,000	90,000	8

Use rate return method of comparison.

4. A Power plant requires a turbine for a particular application. It has 2 different companies for the supply of turbine.

	Company A	Company B
Initial cost (Rs)	20,000	35,000
Estimated service life	6 yrs	9 yrs
Salvage value (Rs)	6,000	9,000
Incremental revenue (Rs)	5,000	2,400

The MARR for the plant is 12%. Suggest the best company based on the rate of return method.

5. Data for 3 alternate projects plans are as follows

Alternative	Initial investment	Salvage value	Life	Annual net Cash flow
A	12,000	0	6	5,200
B	24,000	6000	12	5,000
C	36,000	0	12	8,000

If MARR is 12 % which plan is to be selected based on rate of return method.

6. A finance company introduced 2 different investment schemes whose details are as follows. Find the best investment alternative from the investor's point of view.

	Scheme A	Scheme B
Amount of deposit (Rs)	2,00,000	4,00,000
Period of deposit (Yrs)	10 Yrs	6 Yrs
Maturity amount (Rs)	6,00,000	9,00,000

7. Two plans are under consideration. Plan 1 requires an immediate investment of Rs.60 lakhs and first year expenditure of Rs.2 lakhs. This amount is expected to rise at rate of Rs.50,000 per year. Plan 2 would have first cost of Rs.85 lakhs and the total first year expenses of Rs.9 lakhs with an increase of Rs.1 Lakh per year. The economic life of each plan is 10 yrs. After 10 yrs, during the life the plan is expected to produce Rs.21 lakhs and plan 2 of Rs.25 lakhs. Determine the rate of return for each plan using MARR of 12%.

8. A house is for sale for Rs.1,00,000. A prospective buyer wants to keep it for 15 years and estimates its selling price then to be Rs.90,000. The annual estimated rent is Rs.15,000 and maintenance cost could be Rs.6,000. If the MARR is to be 9 per cent, what is the maximum bid the purchaser should make to buy the house?

Problems on Depreciation

9. A small scale industry has purchased a CNC machine for Rs.10,00,000. The production engineer estimates that the machine has a useful life of 8 years and a salvage value of Rs.50,000 at the end of its useful life. Compute the depreciation schedule for the machine using the following depreciation methods.

- (a) Straight line method of depreciation
- (b) sum-of-the-years-digits method of depreciation
- (c) Double declining balance method of depreciation

$$\text{Hint : take } K = \frac{200}{N} \%$$



10. A company has recently purchased a vehicle for Rs.15,00,000. Its expected life is 12 years and the salvage value at the end of its life is Rs.2 lakhs. Find the depreciation amount and the book value at the end of 4 th and 5 th year of the purchase of vehicle by,
- Straight line method
 - Declining balance method
 - Sinking fund method
11. For the above problem find the depreciation amount and book value for all the years using S-Y-D method.
12. A Maruti car service centre has bought a wheel alignment device for Rs.8 lakhs. The device can be used for 12 years at the end of which the salvage value would roughly be equal to 10% of its purchase value. Find the following:
- Depreciation at the end of 3rd year using SYD method.
 - Depreciation at the end of 5th year using SF method.
 - Depreciation at the end of 7 th year using declining balance method.
 - Depreciation at the end of 7 th year using double declining balance method.
- Take $K = \frac{n}{2}$
- Depreciation at end of 9 th year using straight line method.
 - Book value at the end of 4 th year using SYD method.
 - Book value at the end of 6 th year using SF method.
 - Book values at the end of 8th year using both declining balance as well as double-declining balance method.
 - Book value at the end of 20 th year using straight line method.
13. A college has purchased a Xerox machine for Rs.1,50,000 whose salvage value at the end of its expected life would be negligible. The maximum number of copies that can be taken during its lifetime is about 50 lakhs. During its 5 th year of operation, if the number of copies taken out is already 30 lakhs, find the depreciation and book value at that point using output method of depreciation.
14. Calculate the depreciation fund at the end of each year if the first cost of the machine is Rs.10,000 salvage value is Rs.500 and life is 5 years. Use sum of year digits method.

- An asset cost Rs.400 when purchased before 4 years. Expected salvage value after seven years life is Rs.50. Determine the assets depreciation charge after the current year and its present book value by
 - Straight line method
 - Declining balance method (use the salvage value to determine the depreciation rate)
 - Sinking fund method
- An asset purchased for Rs.120 and estimated to have a life of 10 years and a salvage value of Rs.20. What will be the annual depreciation cost, total depreciation amount at the end of sixth year and the book value at the end of sixth year by (i) Straight line method and (ii) Declining balance method with interest rate of 3%?
- A Drilling machine is purchased for 20,000 Rs. value of the machine reduces every year by 10%. Find depreciation amount and book value at the end of various years using declining balance method of depreciation.
- In the above problem if the initial cost of the machine is 20,000 & the salvage value of the machine after 8 years is 8609.3. Calculate the percentage by which the value of the machine reduces every year.
- A person purchases a refrigerator worth Rs. 12,390. The person estimates that the refrigerator will have a salvage value of Rs. 5,000 at the end of 5 years. Estimate the depreciation amount, Book value at the end of various years using
 - Straight line method of depreciation
 - Sum of the years Digits methods of depreciation.
- A person has purchased a sewing machine worth Rs. 20,000. The expected life of the machine at the end of its life is Rs. 6,000. Determine the following using sinking fund method of depreciation.
 - Depreciation at the end of 2nd and 3rd year.
 - Book value at the end of 4th & 5th year.
- Consider the above problem and find the following using sum of years digits method of depreciation.
 - Depreciation at the end of 3rd & 4th year.
 - Book value at the end of 4th & 5th year.
- The first cost of a Dot matrix printer is Rs. 12,000 the machine as an insignificant scrap value at the end of its useful life time. A maximum of 3,00,00,000 copies can be taken during the life time of the machine. The number of copies taken during its 5th year of its operation is 15,00,000. Use units of production method & find the depreciation for the 4th year of operation of the machine.
- The initial cost of a motor is Rs. 15,000. Its salvage value after 6 years is Rs. 3,000. The number of rotations made by the motor during its lifetime is 10 million. In its 4th

- year of operation, the number of rotations is 8 million. Find the depreciation of the equipment for that year.
24. An asset has a first cost of Rs. 45,000 with an estimated life of 10 years. The salvage value at that time is estimated to be Rs. 5,000. What is the total accumulated depreciation charged during the first 4 years of the asset's life for SYD depreciation and sinking fund depreciation for an interest rate of 12%.
25. If an asset has a first cost of Rs. 50,000 with a Rs. 10,000 salvage value after 5 years. Determine the following:
- Depreciation charge using straight line and SYD method.
 - Book value of machine/asset at the end of year.
 - Plot the graph of time versus Book value.

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CHAPTER

5

ESTIMATING
AND COSTING

INTRODUCTION

An important problem that is faced by many industries is fixing the selling prices of its products or services. The selling price have to take care of the various costs that are involved in the manufacture of a product apart from being competitive in the market and at the same time bring profit to the organization.

In such a scenario, costs involved in the manufacture of a product have to be estimated as accurately as possible so as to be a strong basis for fixing the selling price. It is estimating & costing which analyses the expenditure involved in production so as to ascertain the various costs of all products manufactured by the firm and fix the prices so as to earn a profit. It also helps in controlling the costs and providing necessary costing information to management for decision making. In short, costing (or also Cost-Accountancy) can be called as a specialized branch of accounting which deals with classification, recording, allocation and control of costs etc. With the help of Estimating & Costing, a manufacturer finds out the total cost of each article he makes and fixes the selling price of the article in order to make a definite profit. He would know which group of products are most profitable and those which are least profitable. He can detect losses of time and wastage in the use of materials. He knows which process is expensive and can compare the cost of manufacture by one process with that of another, or the cost of the same process at different periods but with different materials, or labour.

Need for Estimating & Costing

- For determining the cost of production :** Estimating & Costing provides reliable data regarding expenditure on materials, wages and other things which helps in determining the cost of production precisely.
- For Controlling the costs :** Costing provides the cost for each product, process, department, job, contract etc, element by element. Thus the profitable and unprofitable areas are identified in the organization. This information guides the management in taking corrective measures of their unprofitable activities.
- For fixing selling price :** Costing provides information for fixing selling price of the product. The cost of production, volume of production, profit analysis, break-even analysis etc. serves as a basis for determining the selling price of the product.
- For Preparing Quotations and submitting tenders :** Cost Accounting principles help immensely in preparing quotations and submitting tenders. A quotation is the

information regarding the selling price of a product or service offered to a prospective buyer. A tender also is information regarding the selling price given to a prospective buyer, but given in a sealed envelop.

5. For specific Managerial Decisions : Costing provides invaluable information for taking the following managerial decisions :

- i) Make or Buy
- ii) Whether to own fixed assets or hire them.
- iii) Whether to replace the existing plant or machinery before its useful life.
- iv) Whether to replace the existing plant or machinery before its useful life.
- v) Determining the expansion or diversification pSolution.

6. For other Reasons : Costing provides information for wage Incentive plans, cost control measures for materials and supplies, budgeting and budgetary control, standard costing, changes in design of products etc.

Elements of Product Cost

The total cost of a product is the sum of several elementary costs that are involved in its manufacture. The major costs in manufacturing a product consists of :

1. Material cost ✓
 - (a) Direct Material cost
 - (b) Indirect Material cost
2. Labour cost ✓
 - (a) Direct labour cost
 - (b) Indirect labour cost
3. Expenses ✓
 - (a) Direct Expenses
 - (b) Indirect Expenses or Overheads.

1. Material cost : It is the cost of raw materials and additional materials required for the manufacture of the product.

- (a) **Direct material cost :** It is the cost of materials with which the product is made of. In other words it is the cost of materials which are processed through various stages to form a part of the product or the whole product itself.
e.g. Aluminium ingots for casting, Mild Steel rods for making shafts, sheet metal for making almirahs etc.

Direct material costs include the purchase price as well as the incidental (associated) charges like transportation, insurance, loading and unloading charges, import duties etc.

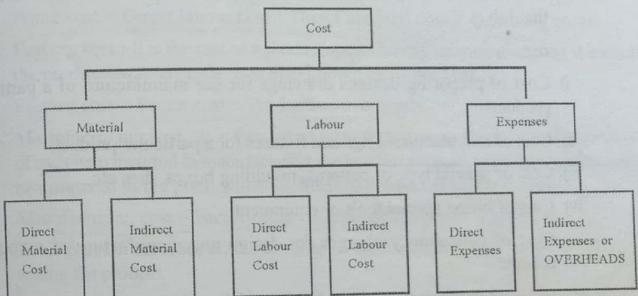


Fig. 5.1 : Elements of Product Cost

(b) Indirect Material Cost : It is the cost of materials necessary for the production process but is not directly used in the product itself. In other words, it is the cost of materials which are essentially needed for helping the direct materials to be converted into finished product.

e.g. Grease, lubricating oil, coolants, cotton waste etc.

2. Labour cost : It is the cost of remuneration paid to the employees of an organization. It includes wages, incentives, bonus, compensation, commissions etc.

(a) **Direct Labour cost :** It consists of wages paid to the workers directly engaged in the manufacturing of product. It also includes the wages paid to the workers engaged in handling the product inside the department.

e.g. Wages paid to workers like machinist, turner, fitter, moulder, welder, wire binder etc.

(b) **Indirect Labour cost :** It consists of wages paid to the worker who are indirectly helpful for the production. In other words it is the wages paid to the labour who help the direct labour in performing their duties. Indirect labour cost cannot be associated directly to a particular job but are charged on the whole log of products produced in the plant during a particular period.

e.g. Wages paid to workers like supervisor, inspector, sweeper, watchman, helper, loader etc.

3. Expenses : Apart from Material cost and Labour cost, there are several other costs involved in the manufacture of a product. They are known as Expenses. Sometimes the indirect material cost and indirect labour cost are included in the expenses.

(a) **Direct Expenses** : It is those which can be charged directly to a particular job. In other words direct expenses are done for that specific job only and can be identified and allocated to persons and materials involved in that job.

e.g.

- i) Cost of preparing designs drawings for the manufacture of a particular product.
- ii) Cost of manufacturing jigs and fixtures for a particular product.
- iii) Cost of special type of patterns, moulding boxes, dies etc.
- iv) Cost of hiring special tools or equipment.
- v) Cost of consultancy charges for design and manufacture of a specific product.

(b) **Indirect Expenses or Overheads** : Indirect expenses are those which cannot be charged directly to a particular product manufactured. All expenses other than the direct material cost, direct labour cost and direct expenses are considered as Indirect Expenses. Indirect Expenses are also called Overheads, On costs, or Burden.

Indirect Expenses can be classified as :

- i) **Production or Factory Overheads** : Eg. Costs of indirect materials such as grease, coolants, cotton waste etc.; wages paid to indirect labour such as inspectors, sweeper, helpers etc.; Labour welfare activities expenses, cost of fuel, power, internal transport, maintenance; Rent of building, depreciation on machinery; are some of the production overheads.
- ii) **Administrative Overheads** : Eg. Salaries of MD, GM, Clerks, typists, Personnel Manager, Medical Officer, Security staff etc.; Expenses incurred on legal charges, audit fees; Expenses on stationery and communication; etc., are some of the administrative overheads.
- iii) **Selling Overheads** : Eg. Salaries of Sales Manager, sales representatives, agents; cost of advertisement and publicity, Expenses incurred for the preparation of tenders and quotation; etc. are some selling overheads.
- iv) **Distribution Overheads** : Eg. Packing expenses; transportation expenses; salaries of stores officers, store-keepers and their assistants; Loading and unloading expenses; are some distribution overheads.
- v) **R & D Overheads** : Salaries of R & D staff; costs of R & D equipment; costs of R & D activities; are some of the R & D overheads.

determination of Selling Price

1. **Prime cost** : It is the sum of all direct costs. It is the cost resulting from the costs spent within the organization to produce it. It is also known as direct costs.
 $\text{Prime cost} = \text{Direct labour cost} + \text{Direct material cost} + \text{Direct expenses}$.

2. **Factory cost** : It is the cost of a product spent directly on manufacturing it including the production overheads. It is also known as works cost.
 $\text{Factory cost} = \text{Prime cost} + \text{Production overheads}$.

3. **Manufacturing cost** : It is the cost of manufacturing an article. It includes the cost of each item incurred in manufacturing the finished product, right from purchasing the raw material to the point when the finished product is ready for sale.
 $\text{Manufacturing cost} = \text{Factory cost} + \text{Administrative overheads}$

4. **Total cost** : It includes all the costs up to the last rupee spent towards producing, selling the product
 $\text{Total cost} = \text{Manufacturing cost} + \text{Selling overheads} + \text{Distribution overheads}$

5. **Selling Price** : The customers buy the product by paying the price which is called as selling price.
 $\text{Selling price} = \text{Total cost} + \text{profit}$

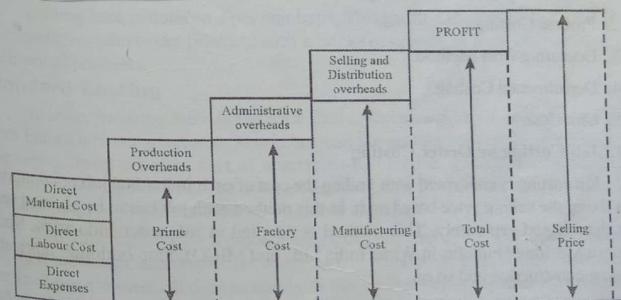


Fig. 5.2: Determination of Selling Price

6. **First cost** : It is the same as Prime cost.

7. **Marginal Cost** : It is nothing but the extra cost incurred for every unit increase in production. In other words, it is the rate of change of cost with output of production. If it is assumed that fixed costs remain unchanged by increasing output by one more unit, the marginal cost of a product will consist of the variable cost only.

OTHER RELATED COSTS**Fixed costs and Variable costs**

1. **Fixed costs :** These costs remain fixed or constant irrespective of the volume of production. They remain the same whether the production is smaller, larger or Nil.
Eg. Costs on land, building, salaries to top management, insurance, depreciation, taxes on property, equipment etc.
2. **Variable costs :** These costs vary with the volume of production. Higher the production, higher will be the variable costs. In other words, variable costs are the function of output. Variable costs become zero when production is stopped. Prime costs are also known as variable cost.
Eg. Cost of raw materials, labour, transportation of finished goods, packing costs etc.

Methods of Costing

The method of Costing adopted differs according to the nature of business and types of products manufactured. Generally the following methods of costing are more commonly used to help the determination of the selling price of a product or service.

1. Job Costing or Order Costing.
2. Process Costing.
3. Operating Cost Method.
4. Departmental Costing.
5. Unit Cost.

1. Job Costing or Order Costing

Job costing is concerned with finding the cost of each individual job or contract and then fixing the selling price based on it. In this method each job has to be planned and its cost determined separately. This method is adopted in job order industries, Special Purpose Machine Units (as in Widia India Ltd., and MICO), Ship building, fabrication, building construction and so on.

2. Process Costing

This method is employed when a standard product is made which involves a sequence of a number of processes. This method is used in industries such as chemical, paper making, oil refining, paint and cement manufacturing etc. By-products and their cost of disposal should be also taken into account while calculating the cost of each process of manufacture and the subsequent selling price.

This method indicates the cost of a product at different stages as it passes through various operation or processes or departments. For example, in the case of manufacture of portland cement, the operation of mixing, grinding the raw material, burning, cooling and grinding the clinker are readily separable and the cost of each of these stages can be quite accurately calculated. The cost calculation of each stage also takes into account the time spent and materials used on each process, as well as services such as power, lighting and heating.

3. Operating Cost

This method is used in firms providing utility services like transport service, water works, electricity boards, railways, airways etc. Here the cost is determined on the basis of operating expenses and charges are made in terms of per km, per litre, per kilowatt-hour etc.

4. Departmental Costing

In big industries like steel industries or automobile industry each department produces independently one or more components. Departmental costing method is used in such industries and the actual expenditures of each department on various products is entered on a separate cost sheet and the costing of each departments is separately undertaken.

5. Unit Cost

Costing here is done on a per unit basis. This method is adopted by single product manufacturers who make products such as bricks or cement or milk or cheese etc., than a variety of products.

Standard Costing

In every industry, there is always a need to estimate the total cost of every product even before it is manufactured. This is because the actual total cost incurred may not be the best or most efficient cost of operations involved in manufacturing the product. Therefore, the right cost or the most efficient cost of the product is calculated before production through a method called 'Standard Costing'.

Thus, standard cost is a predetermined or budgeted cost which is calculated from managements standards of efficient operation and economical expenditures. Standard costs are built upon theoretical desired standards that an industry is capable of attaining under practical and professional operational conditions. The standards are decided by using past experiences and by taking the help of concepts like Time study, Motion study, Therbligs, Process charts etc.

Standard cost represents the best estimate that can be made of what cost should be for material, labour and overheads after eliminating inefficiencies and waste. The actual cost is compared with the standard cost to find the differences which is commonly known as 'Variance'. If the actual cost is more than the standard cost corrective measures are taken

to reduce the cost of production. Standard costing is an important activity to determine the efficiency of cost controlling in an industry.

Procedure for Standard Costing

Step 1: Standard costs are calculated separately for each cost element like material, labour, overheads etc.

Step 2: To find out the actual costs incurred.

Step 3: Comparison of standard costs with actual costs so as to determine the difference called as "Variance".

Step 4: Analyze these variances to find out the reasons.

Step 5: Report the analysis to management. Suggest further action.

Advantages of Standard Costing

1. Standard costing provides a check on various expenses and helps in cost control.
2. It detects wastage of time, material, labour etc.
3. It helps in formulating policies and budgeting.
4. It helps in fixing the selling price.
5. It helps in establishing efficiency of each department as well as that of entire organization.

Disadvantages

1. Standard costing depends on a number of assumptions.
2. It depends more on theory than on practicalities.
3. It cannot accurately take into account miscellaneous expenditure.
4. It is not suited to industries that produce non-standardized products.
5. It is difficult to choose appropriate standards for each cost centre.

Estimation for Simple components

Estimation is the assessment of the total cost in manufacturing a product even before it is manufactured. A sound knowledge of material cost, processing costs, labour costs, etc., are necessary in order to do a proper estimation. Estimation of costs is required in case of new products to ascertain their viability, saleability and profitability. Following are the few problems which deal with estimation for simple components.

WORKED PROBLEMS

1. MICO factory produces 500 Spark plugs a day involving direct material costs of Rs. 40,000, direct labour cost of Rs. 35,000 and factory overheads of Rs. 10,000. Assuming a profit of 15% of the selling price and selling overheads to be 30% of the factory cost, calculate the selling price of one spark plug.

$$SP = 110500 + 0.15 SP$$

$$\Rightarrow SP =$$

$$130000 \\ - 260$$

$$TC/10 \text{ 500}$$

$$TC = 80000 + 0.385000$$

ESTIMATING AND COSTING 187

Note : Costs given are for the whole batch of 500 Spark plugs.

Solution.

$$\text{Selling Price} = \text{Total cost} + \text{Profit}$$

$$\text{Selling Price} = \text{Prime cost} + \text{Total cost}$$

$$\text{Selling Price} = \text{Prime cost} + \text{Factory overheads} + \text{Administrative overheads} + \text{Selling overheads}$$

Here profit is given as 0.15 times selling price, Also here selling overheads is 0.3 times factory cost which is (Prime cost + factory overheads). Since Administrative overheads and direct expenses are not mentioned they are assumed to be zero.

$$\therefore S.P. - 0.15 S.P. = \text{Direct labour cost} + \text{Direct material cost}$$

$$+ \text{Direct expenses} + \text{Production overheads} \\ + \text{Administrative overheads} + \text{Selling overheads}$$

$$0.85 S.P. = 35000 + 40000 + 0 + 10000 + 0 + (35000 + 40000 + 10000) 30\%$$

$$0.85 S.P. = 110500 \text{ for 500 spark plugs}$$

$$\therefore \text{Selling price} = 130000 \text{ for 500 spark plugs}$$

$$\therefore \text{Selling price of one spark plug} = \frac{130000}{500} = 260$$

$$\therefore \text{Selling Price} = \text{Rs. 260}$$

2. Determine the selling price of a gear wheel from the following data

$$\text{i) No. of gear wheels produced 200}$$

$$\text{ii) Labour cost Rs. 2500}$$

$$\text{iii) Material cost Rs. 3800}$$

$$\text{iv) Factory overheads 40% of Direct cost}$$

$$\text{v) Administrative and selling overheads 25% of factory cost}$$

$$\text{vi) Profit 30% of total cost}$$

(April, 99, Bangalore University)

Solution.

$$\text{Selling Price} = \text{Prime cost} + \text{Production overhead} \\ + \text{Administration overhead} + \text{Selling overhead} + \text{Profit}$$

Also

$$\text{i) Direct costs} = \text{Prime cost}$$

$$\text{ii) Factory cost} = \text{Prime cost} + \text{Production overhead}$$

$$\text{iii) Total cost} = \text{Factory cost} + \text{Administrative overheads} + \text{Selling overheads}$$

$$\text{iv) Prime costs} = 2500 + 3800$$

$$\text{or Direct cost} = 6300$$

$$\text{ii) Factory cost} = \text{Prime cost} + 0.4 \times \text{Prime cost}$$

$$= 6300 + 0.4 \times 6300 = 8820$$

$$\text{iii) Total cost} = \text{Factory cost} + 0.25 \times \text{Factory cost}$$

$$= 8820 + 0.25 \times 8820 = 11025$$

Selling price = Total cost + profit

$$\text{Selling price} = 11025 + 0.30 \times 11025$$

$$\text{Selling price for 200 gears} = 14332.50$$

$$\text{Selling price of 1 gear} = \text{Rs. } 71.66$$

3. 'Pizza corner' employed 75 workers in a particular month to work in the outlets as well for home delivery. The following are the details of expenditure.

- i) Cost of material = Rs. 80,000
- ii) Rate of wages for each workers = Rs. 20 per hour of normal duty, Rs. 40 per hour of overtime duty.
- iii) Man hours per day of normal duty = 8 hours
- iv) No. of holidays per month (without wages) = 5 days
- v) Total overhead expenses = Rs. 20,000
- vi) Total overtime availed by workers = 200 hours
- vii) Profit = 20% of Total cost

Determine

- i) Total cost for the month
- ii) Profit for the month
- iii) Man hour rate of overheads

Solution.

$$\text{i) Total cost} = \text{Direct labour cost} + \text{Direct material cost} + \text{Direct expenses} \\ + \text{Total overhead expenses}$$

$$\text{Direct labour cost} = \text{Cost of Labour during normal working hours} \\ + \text{cost of labour in overtime} \\ = 20 \times 8(30 - 5) 75 + 200 \times 40 \\ = \text{Rs. } 3,08,000$$

$$\text{Total cost} = 3,08,000 + 20,000 = 328000$$

$$\text{ii) Profit} = 0.20 \times 328000 \\ = \text{Rs. } 65,600/-$$

$$\text{iii) Man hour rate overheads} = \frac{\text{Total overhead costs for the month}}{\text{No. of man hours for the month}}$$

$$\text{Man hour Rate} = \frac{2000}{75 \times 8(30 - 5) + 200}$$

$$\text{Man Hour Rate} = \text{Rs. } 6.25$$

4. Find the factory cost of a forge hammer made from solid cast Iron press of circular cross section of 30 cm dia and 160 cm length. The casting ad machining time taken to make the press in 150 minutes and the labour rate is Rs. 22 per hour. Factory overheads are 40% of direct labour cost. The density of the material is 6.8 gm/cm³ and the cost of the material is Rs. 12/kg.

Solution.

$$\text{Density of CI} = 6.8 \text{ gm / cm}^3 = 0.0068 \text{ kg/cc}$$

$$\text{Weight of Iron press} = \text{volume} \times \text{density}$$

$$= \frac{\pi d^2}{4} \times L \times \rho$$

$$= \frac{\pi \times 30^2}{4} \times 160 \times 0.0068$$

$$\text{Weight of cast Iron} = \text{Rs. } 51.27 \text{ kg}$$

$$\text{Cost of material} = \text{weight of CI} \times \text{cost of CI per kg}$$

$$= 51.27 \times 12$$

$$= \text{Rs. } 615.25$$

$$\text{Labour cost} = \text{No. of hours worked} \times \text{labour rate per hour}$$

$$= 150 \text{ Minutes} \times 60 \text{ per hour}$$

$$= 2.5 \times 60$$

$$= \text{Rs. } 150$$

$$\text{Factory overheads} = 0.4 \times 150$$

$$= \text{Rs. } 60$$

$$\text{Factory cost} = \text{Material cost} + \text{Labour cost} + \text{Factory overheads}$$

$$= 615.25 + 150 + 60$$

$$\text{Factory cost of CI press} = \text{Rs. } 825.25$$

5. A factory manufactures steel bolts and nuts whose cost estimates are made on the basis of lots of 2000 nuts and bolts as shown below.

Rs. 1000

- i) Direct material cost
- ii) Direct labour cost

(a) Forging shop	
Cutting to length	Rs. 40
Setting up	Rs. 150
(b) Machine shop	
Milling heads	Rs. 150
Threading	Rs. 90
Drilling	Rs. 75
Chamfering	Rs. 55
iii) Direct Expenses	
Cost of Tools	Rs. 110
Jigs & Fixtures	Rs. 150
iv) On costs	
(a) Forging shop	120% of labour cost
(b) Machine shop	100% of labour cost
(c) Office establishment	20% of factory cost
(d) Packing and transporting	Rs. 120

If the selling price of each nut and bolt is Rs. 2.50 determine whether the factory is making a profit or loss. If so by what amount per piece.

Solution.

$$\text{Prime cost} = \text{Direct material cost} + \text{Direct labour cost} \\ + \text{Direct expenses}$$

$$\therefore \text{Prime cost} = 1000 + [40 + 150 + 150 + 90 + 75 + 55] \\ + [110 \text{ (for 2000 nuts & bolts)} + 150 + 1000 + 560 + 260]$$

$$\text{Prime cost} = 1820$$

$$\text{Factory costs} = \text{Prime cost} + \text{Factory overheads}$$

$$= 1820 + \text{Forging shop overheads} + \text{Machine shop overheads} \\ = 1820 + [1.2 \times 560 + 1 \times 560] \\ = 3500$$

$$\text{Manufacturing cost} = \text{Factory cost} + \text{Administrative overheads}$$

$$= 3500 + 0.2 \times 3500 \\ = 4200$$

$$\text{Total cost} = \text{Manufacturing cost} + \text{Selling overheads}$$

$$= 4200 + 120$$

$$\text{Total cost} = 4320$$

$$\quad \quad \quad (\text{for 2000 nuts & bolts})$$

$$\text{Selling price of 2000 Nuts & bolts} = 2.5 \times 2000$$

$$= 5000$$

$$\text{Profit} = \text{Selling price} - \text{Factory cost}$$

$$= 5000 - 4320$$

$$\text{Profit} = \text{Rs. } 680/-$$

Therefore profit is made by the factory on each lot of 2000 nuts and bolts and is $\text{Rs. } 680/-$

6. The catalogue price of a washing machine is Rs. 7500 and the commission allowed to the proprietor of the showroom is 20%. The administrative and the selling expenses are 60% of the factory cost and the material cost, labour cost and factory overheads are in the ratio of 2 : 3 : 1. If the cost of the labour on the manufacture of machine is Rs. 1650, determine the profit on each washing machine.

Solution.

$$\text{Catalogue price of each washing machine} = \text{Rs. } 9000.$$

$$\text{If commission earned by distributor is } 20\%, \text{ the selling price of the machine to the distributor} = 9000 - 0.2 \times 9000 = 7200/-$$

$$\text{Direct Material cost} : \text{Direct Labour cost} : \text{Factory overheads}$$

$$2 : 3 : 1$$

$$\text{If Direct labour cost} \text{ Rs. } 1650$$

$$\text{Direct Material cost} = \frac{1650}{3} \times 2 = 1100$$

$$\text{Factory overhead} = \frac{1650}{3} \times 1 = 550$$

$$\text{PC} = 1100 + 550 = 1650$$

$$\text{Factory cost} = \text{Prime cost} + \text{Factory overheads}$$

$$= 1650 + 1100 + 550$$

$$= 3300$$

Total cost of each machine = Factory cost + Administrative and selling overheads
 $= 3300 + 0.6 \times 3300$

$T_{\text{Total}} = 5280$

Therefore profit on each washing machine

$= \text{Selling price} - \text{Total cost}$
 $= 7500 - 5280$

Profit = Rs. 2200/-

7. A company produces 30,000 units per annum. The various cost components are as follows :

Direct materials Rs. 6/- per unit

Direct labor Rs. 5/- per unit

Fixed overheads Rs. 60,000

Variable overheads Rs. 2.50 per unit

Prepare the fixed budget for the above.

(VTU Jan'06)

Solution.

Given : No. of units = 30,000

Direct materials = Rs. 6/unit

Direct labor = Rs. 5/unit

Fixed overheads = Rs. 60,000

Variable overheads = Rs. 2.50/unit

Total Costs = ?

Note : To prepare fixed budget is to find the total expenses to be incurred in producing all the units required.

$\begin{aligned} \text{Total costs} &= \text{Direct material cost} + \text{Direct labor cost} + \text{Fixed overheads} + \text{Total variable overheads} \\ &= 6 \times 30000 + 5 \times 30000 + 60000 + 2.50 \times 30000 \end{aligned}$

$\therefore \text{Total costs} = 465000$

$\text{Total cost / unit} = \frac{465000}{30000} = \text{Rs. } 15.50/\text{unit}$

□ □ □

SUMMARY

1. Costing

With the practice of Cost Accounting, a manufacturer finds out the total cost of each product he makes and fixes the selling price of the product in order to make a definite profit.

Costing is needed for determining the cost of production, for controlling the costs, for fixing the selling price, for preparing quotation and tenders, for helping management to take specific decisions.

2. Elements of Product Cost

Material Cost : It is the cost of raw materials and additional materials required for the manufacture of the product. Material costs can be direct or indirect costs.

Labour Cost : It is the cost of remuneration paid to the employees. They can be direct or indirect labour costs.

Expenses : All costs other than material costs and labour costs are known as expenses. Expenses can also be direct or indirect.

3. Indirect Expenses or Overheads

Indirect expenses are those which cannot be charged directly to a particular product manufactured. All expenses other than the direct material cost, direct labour cost and direct expenses are considered as indirect expenses. They can be classified as production or factory overheads, Administrative overheads, Selling overheads, Distribution overheads, R & D overheads.

4. Determination of Selling Price in terms of Elements of Product Cost

Prime Cost = Direct Material Cost + Direct Labour Cost + Direct Expenses

Factory Cost = Prime Cost + Production or Factory Overheads

Manufacturing Cost = Factory Cost + Administrative Overheads

Total Cost = Manufacturing Cost + Selling & Distribution Overheads

Selling Price = Total Cost + Profit

First Cost = Same as Prime Cost

Marginal Cost = Increase in costs for unit increase in Production.

Different methods of overhead allocation are :

5. Standard Costing

Standard costing is the method of calculating the right cost or the most efficient cost of a product even before its manufacture. Standard costs are calculated based on past

experiences of most economical expenditures as well as using concepts like Time study, Motion study, therbligs, Process charts etc. After the actual production, the actual cost is then compared to the standard cost, to find the 'Variance'. This helps in taking corrective actions and cost controlling measures.

EXERCISE PROBLEMS

- A factory produces 150 electric bulbs per day. It involves direct material of Rs. 250/- and direct labour of Rs. 225/-. Assuming a profit of 20% of the selling price and the selling and distribution overheads as 30% of the total cost, the factory and administrative overheads being Rs. 275/-, calculate the selling price of an electric bulb. (Aug' 96' Bangalore University)

- Calculate the selling price of a PEN from the following information.

- i) No. of pens produced Rs. 150
- ii) Labour wages Rs. 200
- iii) Material cost Rs. 160
- iv) Factory overheads 35% of direct costs
- v) Administrative & selling overheads 20% of Factory cost
- vi) Profit 25% of Total cost

- A certain product is manufactured in batches of 100. The direct material cost is Rs. 600/-, direct labour cost is Rs. 750 and the factory overheads are 50% of the prime cost. If the selling expenses are 30% of the factory cost, what would be the selling price of each product so that the profit is 15% of the total cost.

- Sharp company manufactures pocket calculators. The details of its monthly expenditure are as follows :

Direct material	Rs. 10000
Direct labour	200 hours at the rate of Rs. 5/- per hour
Factory overheads	10% of prime cost
Other overheads	10% of works cost
Profit	20% of total cost

Number of units manufactured per month = 100

Estimate the selling price of each pocket calculator.

- In a factory, two types of Sand Mullers for foundry are prepared - Grade I and Grade II. From the following data, prepare a statement showing the cost per sand muller of each model and profit on each model sold.

	Grade I	Grade II
Material cost per machine	Rs. 1500	Rs. 1250
Labour cost per machine	Rs. 900	Rs. 750
Factory on cost	20% of Prime cost	20% of Prime cost
Office on cost	10% of Factory cost	10% of Factory cost
Selling price	Rs. 6000	Rs. 5000

- Two moulders, can cast twenty-five gears in a day. Each gear weighs 3 kg and the gear material costs Rs. 12.50 per kg. If the overhead expenses are 150% of direct labour cost and the two moulders are paid Rs. 70/- per day, Calculate the cost of producing one gear. [Solution. Rs. 44.50].
- A wet grinder costs Rs. 6000. A discount of 25% of this price is given to the distributor. If labour cost, material cost and factory overheads are as 4 : 1 : 2 and selling expenses are 25% of the factory cost, calculate the profit of the factory for one wet grinder. Assume factory overheads of Rs. 800 [Solution. Rs. 1000].
- A non-ferrous foundry employees 25 persons. It consumes material worth Rs. 35,000 and pays workers at the rate of Rs. 5 per hour and incurs total overheads of Rs. 20,000. In a particular month (25 days), workers had an overtime of 150 hours and were paid double than the normal rate. Find (i) Total cost and (ii) the Man-Hour Rate of overheads. Assume a 8-hour working day.
- Find the factory cost of a component made from solid steel bar 5 cm long and 2 cm in diameter. The machining operation requires 1.5 hours. Assume the following data :
 - i) Density of mild steel 18 gm/cc
 - ii) Cost of mild steel Rs. 2 per kg
 - iii) Labour charges Rs. 10 per hour
 - iv) Overhead charges 50% of direct labour cost
- An ancillary unit in BEL Industrial Area is producing 5000 resistors per month. The fixed overheads for the month are Rs. 9000 and the variable cost of each component is Rs. 10. If the selling price of each component is Rs. 12, estimate the minimum monthly production that may not cause any loss to the factory owner.

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CHAPTER
6

FINANCIAL MANAGEMENT

INTRODUCTION

Every business enterprise in this world irrespective of its nature and scale of operations needs finance to carry out its activities and accomplish its goals. Money is a continuous necessity in running any organization and without it very few opportunities can be taken advantage of. Finance is actually the life-blood of any business.

Financial management basically deals with procurement of funds needed for business and its efficient utilization. In other words management of finance is the anticipation of financial needs, acquiring financial resources and allocating funds to different departments.

Financial management plays a key role in any business and in fact, will be a difference between success and failure. The important objectives of any organization are profitability, growth and survival. The attainment of these objectives largely depend upon the efficient management of finance.

Financial management has become one of the most sought-after streams in management studies in the world because it offers some of the greatest challenges to budding MBA's trying to make a name for themselves in the world of finance. Financial Management is also very popular because it is an inter-disciplinary subject and students from non-engineering background can also pursue it. The best financial management talent in the world cross swords around the world in places like Stock-Exchanges, Finance Companies, Mutual Fund Companies, Insurance Companies, International Banking etc., Financial Management is for those who want to be in the forefront of the business world!

Objectives of Financial Management (Also Role of a Finance Manager)

1. To maximize profits and minimize losses.
2. To determine the financial needs of the company.
3. To raise funds for both fixed and working capital needs.
4. To pay salaries and wages to the employees.
5. To control all financial activities of the company through standard costing, Budgetary control, Financial Analysis, Break-Even Analysis etc.
6. To prepare financial statement like profit and loss accounts and balance sheet.
7. To forecast the economic trends of the market well in advance.

SCOPE OF FINANCE

What is finance? What are the financial activities of a firm? How are they related to the firm's other activities? Firms establish manufacturing facilities for production of goods while some provide services to customers. They sell their goods or services to earn profit. They raise funds to acquire production and other various facilities. Thus, the three most important activities of a business firm are :

- (i) Production
- (ii) Marketing
- (iii) Finance

The broad objective of any firm would be to raise the finance it needs and employ it in production and marketing activities in order to generate returns on the invested capital. Therefore, there exists an inseparable relationship between finance on one hand and production and marketing on the other hand. A regular supply of finance in any firm ensures efficient production and marketing activities, which in turn helps returns, which again improves the flow of finance. A company in a tight financial position may compromise on production and marketing activities which in turn leads to poor returns and poorer flow of funds. Hence, the entire business of managing the finance depends heavily on the sources of finance.

Sources of Finance

All Business enterprises have to raise funds from various sources in order to invest in their business and earn profits out of them. Funds raised are used for purchasing fixed assets and for working capital. Fixed assets are those like land, building, machinery, office equipment, furniture etc., which have to be bought even before beginning the operations of the company.

Working capital is that which is required to meet the expenditure for day-to-day working of the business. It includes the cost of maintenance and service activities, cost of sales activities etc.

Following are the different methods of raising capital :

I. External Sources

1. Raising money through partnership
2. Loans from banks, financial institutions, pawn brokers etc.
3. Shares - preference, equity and deferred shares.
4. Debentures - convertible and non-convertible
5. Public Deposits
6. Hire Purchase
7. Trade Credit

II. Internal Sources

1. Personal savings and assets inherited.
2. Ploughing back of earnings
3. Depreciation Money
4. Deferred taxation.

FINANCE FUNCTIONS

The most important work of a Finance Manager in a company is to raise money from various sources, allocate them wisely and distribute the returns to all the shareholders appropriately. These are known as Finance functions. While performing these functions, a Finance Manager attempts to balance cash inflows and outflows. This is called liquidity decision and can be added to the list of important Finance function. Therefore, the Finance Function or decisions include.

- (i) Investment or long-term asset-mix decision.
- (ii) Financing or capital-mix decision.
- (iii) Dividend or profit allocation decision.
- (iv) Liquidity or short-term asset-mix decision.

I. Investment Decision

Investment decision or capital budgeting involves the decision of allocation of funds to long-term assets that would yield benefits in future. Two important aspects of the investment decision are :

- (a) the evaluation of prospective profitability of new investments, and
- (b) The measurement of a cut-off rate against which the prospective return of new investments could be compared.

Because it is very difficult to measure future benefits of investments and cannot be predicted with certainty. Because of uncertain future, investment decisions involve risk. Investment proposals should, therefore, be evaluated in terms of both expected return and risk. Usually, greater the risk, higher will be the expected return, in any kind of business. Therefore, one should know the maximum risk that one can take in order to maximize returns.

II. Financing Decision

This is the second most important function to be performed by the Finance Manager. A Finance Manager should broadly determine when, where, why, and how much of funds to acquire in order to meet the firm's investment needs. The most important issue in front of a Finance Manager is to determine the proportion of equity and debt. The mix of debt

and equity is known as the firm's Capital Structure. A finance manager must strive to obtain the best financing mix or the optimum Capital Structure according to the policies of the company. A company's Capital Structure is considered to be optimum when the market value of its shares is maximized.

III. Dividend Decision

This is the third major financial decision. The Finance Manager must decide, in conjunction with the top management, whether to distribute all profits, or retain them, or distribute a portion and retain the balance. The dividend policy should be determined knowing the impact on the Shareholders value. An optimum dividend policy is one that maximizes the market value of the firm's shares. A Finance Manager should also decide on issues such as cash dividends, dividend stability, bonus shares, pay-out ratio etc.

IV. Liquidity Decision

This is the fourth important financial decision. Liquidity generally refers to the ability of a firm to meet its financial obligations in the short run, usually one year. Liquidity literally mean flow of cash. A business which deals only in cash i.e. sells in cash and buys in cash, is said to be having high liquidity. But it might lose profitability because idle current assets (or cash) would not earn any thing against being invested. On the other hand, businesses which buys and sells more on credit will be low on liquidity, which means there is always a shortage of cash. It is a situation of always having money and making profits, with always a probability of losing it. Therefore, a conflict exists between profitability and liquidity while managing current assets, and a proper trade-off must be achieved.

In order to ensure that neither insufficient nor unnecessary funds are invested in current assets, a Finance Manager should develop sound techniques of managing current assets. He should anticipate and estimate correctly a firm's need for current assets and make sure that funds would be made available when needed.

SOURCES OF FINANCIAL INFORMATION

In any organization information regarding finance has to be systematically presented to properly understand, interpret and act upon it. Following is a list of all sources of financial information in any organized business.

I. Accounting Process

1. Financial Accounting
2. Cost Accounting
3. Tax Accounting
4. Management Accounting

II. Book-Keeping

1. Single entry
2. Double Entry

III. Books of Account

1. Journal
2. Ledger

IV. Financial Statements

1. Trial Balance
2. Profit & Loss Account
3. Balance Sheet

ACCOUNTING PROCESS

Any business usually involves a large number of transactions. Buying, selling, paying and receiving are very frequent. It is humanly impossible to remember all transactions. Hence it becomes necessary for us to record all the transactions in a note-book. Accounting is the art of recording, classifying, summarizing and reporting all business transactions. Accounting is the language of business and its main purpose is to produce business data. Accounting is also the science of recording business transactions in a methodical manner so as to show:

- (a) the true state of affairs of a business at any instant of time.
- (b) loss or profit occurred during a specific accounting period.

Objectives of Accounting

The objectives of accounting can be stated as follows:

- (a) **To maintain systematic records :** Accounting is used to maintain systematic records of all financial transactions like purchase and sale of goods, cash receipts and cash payments etc. It is also used for recording various assets and liabilities of the business.
- (b) **To ascertain net profit or net loss of the business :** A business man would be interested in periodically finding the net result of his business operations i.e., whether the business has earned profit or incurred some loss. A proper record of all income and expenses helps in preparing a profit and loss account and find out the net result of business operations during a particular period.
- (c) **To ascertain the financial position of the business :** The businessman is also interested in ascertaining the financial position of his business at a end of a particular period i.e., how much it owns and how much it owes to others. He would like to know what happened to his capital, whether it has increased or decreased or remained constant. A systematic record of assets and liabilities facilitates the preparation of a position statement called Balance Sheet which provides the necessary information.

- (d) **To provide accounting information to interested parties :** Apart from owners, there are various other parties who are interested in the accounting information. These are : bankers, creditors, share holders, tax authorities, prospective investors etc. They need such information to assess the profitability and the financial soundness of the business. The accounting information is communicated to them in the form of an annual report.

Limitations of Accountancy

- Accounting does not reflect non-financial factors:** Accounting does not record transactions and events which are non-financial in character. Hence they do not reveal a complete picture because facts like quality of human resources, locational advantage, business contacts, licences, patents and trademarks possessed etc., do not find any place in the books of account.
- Data is historical in nature:** The accountants adopt historical cost as the basis for giving values and reporting all assets and liabilities. They do not reflect current values. It is quite possible that items like land and buildings may have much more value than what is stated in the balance sheet.
- Estimation and personal judgements are utilized:** Factors recorded in financial statements are greatly influenced by accounting conventions and personal judgements. Hence, they do not reveal the true picture. In many cases, estimations are used to determine the value of various items like inventories, fixed assets, goodwill etc.
- Insufficient data:** Data provided in the financial statements is insufficient for proper analysis and decision making. It only provides information about the overall profitability of the business. No information is given about the cost and profitability of different activities.

Classification of Accounting

Accounting can be broadly classified into:

1. Financial Accounting
2. Cost-Accounting
3. Tax Accounting
4. Management Accounting

1. Financial Accounting

Financial Accounting deals with the recording, summarizing and reporting of the financial transactions in the important areas of sales, service, purchase etc. The transactions taking place could either be through cash or credit. Financial Accounting reflects the general financial position of the company and hence is also known as 'General Accounting'.

Financial Accounting covers :	
i. Cash Accounts	Cash receipts and payments
ii. Personal Accounts	Credit given and taken
iii. Real Accounts	Property bought and sold
iv. Nominal Accounts	Expenses incurred

Financial Accounting is a legal necessity and is vital to show the indebtedness and creditworthiness of the company.

Objectives of Financial Accounting

- (i) To take care of Stock Exchange requirements.
- (ii) To meet tax and other legal requirements
- (iii) To safeguard the share holders.
- (iv) To prepare the dividend policy
- (v) To prepare financial statements such as Balance sheet, Profit and Loss account etc.

2. Cost Accountancy

Cost Accountancy deals with the classifying and recording of all expenditures incurred on the manufacture of a product. In other words, cost accountancy can be simply defined as the process of determining the actual cost of an article so as to be the basis for fixing its selling price and make a profit out of it.

While Financial Accounting is mainly concerned with external transactions or an organization, Cost Accountancy is concerned with internal matters like expenditure incurred on production, profitability of work centres and so on.

3. Tax Accounting

Tax Accounting deals with the reporting and filing of tax returns to Government agencies like Income Tax department, Commercial Tax department, Sales Tax department, Excise department etc., so as to keep them at bay !

4. Management Accounting

The purpose of management accounting is to assist the management in taking rational policy decisions and to evaluate the impact of its decisions and actions. Examples of such decisions are : Pricing decisions, make or buy decisions, capital expenditure decisions etc. This branch of accounting is primarily concerned with providing the necessary accounting information about funds, costs, profits etc. to the management which may help them in such decisions and also in planning and controlling business operations.

Basic concepts in accounting or accounting terminology

1. **Assets:** Assets are the properties that are owned and that which have monetary value. Assets are in fact, the resources of the business enterprise.
e.g.. Cash, machinery, vehicles, land, building, inventory etc.
Assets are classified as :
 1. Current Assets
 2. Fixed Assets
 3. Intangible Assets
 - i. **Current Assets:** Current assets are those which can be converted into cash in a short period of time.
e.g.. Cash in hand, cash in bank, inventories, accounts receivable etc.
 - ii. **Fixed Assets:** Fixed assets are those which have relatively long existence which cannot be readily converted into cash.
e.g.. Land, building, furniture, machinery etc.
 - iii. **Intangible Assets:** There are long-term assets that have no physical existence but nevertheless have certain rights and values. e.g.. Patents, trademarks, goodwill, copyright.
2. **Liabilities:** Liabilities are the claims that can be made by the share holders, banks and other Financial lenders against a business enterprise in case it goes to losses. Different liabilities are :
 - i. Fixed liabilities
 - ii. Current liabilities
 - iii. Contingent liabilities
- i. **Fixed or long-term liabilities:** They are the long term obligations of the company and which cannot be liquidated suddenly from the current assets. Generally payable in more than one year.
e.g.. Long term loans, debentures, bonds, mortgage payable, owners investment etc.*
- ii. **Current liabilities:** The debts of the company which have to be repaid within a short period of time are known as current liabilities. Generally payable in an year or less.
e.g.. Wages, short-term loans, income-tax, rent, trade-credit, bank over-drafts etc.,
- iii. **Contingency Liabilities:** A contingency liability arises when a firm becomes a party to a bill of exchange. It becomes a real liability if the debtor of the bill fails to pay.
3. **Transaction:** Transaction is the monetary dealing between two parties or two persons.

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 ii. Personal Accc
 iii. Real Account
 iv. Nominal Acc
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- (ii) To meet tax
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- (iv) To prepare
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204 ENGINEERING ECONOMICS

4. **Creditor:** Creditor is a person who supplies finance to others i.e. he has to get back his money.
5. **Debtor:** Debtor is the person who owes money to others i.e. he has to pay money.
6. **Bad Debt :** A debt which becomes irrecoverable for some reasons becomes a bad debt. If the debtor becomes a pauper (claims insolvency) or dies without leaving any money behind, it becomes a bad debt.
7. **Turnover :** Turnover is the total sales income of a business either through credit cash during a given period.
8. **Cash and Credit Trade :** When goods are purchased or sold on cash only, it is known as cash trade. When the goods are purchased or sold on credit it is called credit trade.
9. **Commission :** The percentage remuneration given by a firm to a person for services rendered by him is called commission.
10. **Over draft :** It is the money drawn from the bank by the company over and above the permissible credit limit or the bank balance.
11. **Drawing :** It is the withdrawal of money from the business by the owner or of the partners for his personal use.
12. **Sales Returns :** When a customer who has purchased certain goods, returns some of them (due to some reason) to the firm, it is called sales return.
13. **Purchase Returns :** If the firm returns to the supplier the goods purchased from him (due to some reason), it is known as purchase returns.
14. **Bill :** Bill is a written statement received by the buyer from the seller after purchasing goods from him (due to some reason), it is known as purchase bill.
15. **Receipt :** Receipt is another statement received by the buyer from the seller having paid for the goods.
16. **Invoice :** A statement issued by the seller to the buyer giving details of the goods sold.
17. **Voucher :** Any written evidence in support of a business transaction is called a voucher. Bill and Receipt are also types of vouchers. If on the basis of these vouchers entries are made in the books of accounts.

Classification of Accounts:

All business accounts are broadly classified into three classes :

1. **Personal Accounts :** Those relating to persons and particular firms.
2. **Real Accounts :** Those relating to property (assets)
3. **Nominal Accounts :** Those relating to incomes and expenses.

Real and Nominal accounts together are called as 'Impersonal accounts'

FINANCIAL MANAGEMENT 205

1. Personal Accounts : Accounts which show transactions with persons are called 'Personal Accounts'. A separate account is kept in the name of each person for recording the benefits given to or received from the person in the course of dealings with him. In other words, personal accounts are those that record dealings with persons or even firms who may be debtors or creditors.

E.g., Vineet's account, Thejaswi's account, Bose's Account, Tabrez & Sons Account, Canara Bank Account etc.

2. Real Accounts : Accounts related to properties or assets are known as 'Real Accounts'. Every business needs assets such as machinery, furniture etc. for running its activities. A separate account is maintained for each asset owned by the business. All transactions relating to a particular asset are recorded in the concerned asset account.

E.g., Cash account, Furniture account, Machinery account, Building account etc. They are known as real accounts because they represent real things of value owned by the business.

3. Nominal Accounts : Accounts relating to expenses, losses, incomes and gains are known as 'Nominal Accounts'. A separate account is maintained for each item of expense, loss, income or gain.

E.g., Wages Account, Salaries Account, Commission Received Account and Interest received account etc.

Rules of Debit and Credit for different Accounts for Book-keeping

- | | |
|----------------------------|---|
| 1. For Personal Accounts - | Debit the receiver and credit the 'giver' |
| 2. For Real Accounts - | Debit what comes in and credit what goes out. |
| 3. For Nominal Accounts - | Debit all expenses and losses and credit all incomes and gains. |

BOOK-KEEPING

To run a business well, it is necessary to know the amount of money received, amount of money spent and how the money is spent. A book-keeping system can provide information in a systematic manner.

Book keeping helps in showing :

- i. All purchases, sales and returns in the financial year.
- ii. Transactions with creditors and debtors.
- iii. Quantitative value of the goods available with the firm.
- iv. Information about assets and liabilities of the firm.

- i. Financial Accounts
 - ii. Cash Accounts
 - iii. Personal Accounts
 - iv. Real Accounts
 - v. Nominal Accounts
 - vi. Financial Accounts
- creditworthiness of t

Objectives of Finance

- (i) To take care of financial resources
- (ii) To meet tax requirements
- (iii) To safeguard assets
- (iv) To prepare financial statements
- (v) To prepare financial budgets

2. Cost Accounting

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3. Tax Accounting

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4. Management

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206 ENGINEERING ECONOMICS

- v. Profit and loss accounts.
- vi. Cash available in the company.

Systems of Book-keeping

1. Double Entry system
2. Single Entry system

1. Double Entry system

Every business transaction has 2 aspects (i) the aspect of receiving and (ii) the aspect of giving. For example when a company purchases goods for cash from a supplier, the stock of goods increases and cash decreases. Both should be recorded which involves two entries, one in Goods Account and the other in Cash Account. This method of recording every business transaction is called Double Entry system which recognizes and records both aspects of every transaction. The Double Entry system uses rules of Debit and Credit. According to this, the account which involves the receiving aspect (Purchase Account) is debited and the account which involves giving aspect (Account) is credited. Thus for every debit there will be an equivalent credit.

The advantages of Double Entry system are :

- (i) It provides complete and reliable record of all business transactions, it records both the aspects.
- (ii) It supplies full information about the incomes, expenses, assets and liabilities of the business. This helps the management in taking appropriate decisions.
- (iii) The arithmetical accuracy of the books of account can be easily verified by preparing a trial balance.
- (iv) The financial result of a business organization i.e., profit or loss can be correctly ascertained.
- (v) The financial position of the business can also be ascertained at any time.

2. Single Entry system

Single entry system does not mean that only one aspect of a transaction is recorded. It simply refers to incomplete records. Under this system neither all the transactions are recorded nor all the accounts maintained. In certain cases the two aspects of information about the amount of profit made or the loss incurred during the year are recorded while in others only one aspect is recorded. The Single Entry system is a mixture of double entry, single entry and no entry. The accounts maintained under this system are incomplete and non-systematic and hence not reliable. The main缺点 of this system is that the arithmetical accuracy of the books of account cannot be guaranteed.

because a trial balance cannot be prepared. It also becomes difficult to ascertain the correct amount of profit or loss. This system is followed by small business firms, individuals recording their personal expenditure, etc.

II. ACCOUNTING PROCESS

The accounting process consists of the following four stages :

- i. Recording the transactions
- ii. Classifying the transactions
- iii. Summarizing the transactions
- iv. Interpreting the results.

i. Recording the transactions : The accounting process begins with the recording of all transactions in the book of original entry. This book is called 'Journal'. All transactions are recorded in the Journal in a chronological order (date-wise) with the help of various vouchers such as cash, receipts, invoices, cash memos etc.

ii. Classifying the transactions : The second stage consists of grouping the transactions of similar nature and posting them to the concerned accounts in another book called 'Ledger'. For example, all transactions related to cash are posted to Cash Account, transactions related to different persons are entered separately in the account of each person etc. The objective of classifying the transactions in this manner is to ascertain the combined effect of all transactions of a given period with respect to each account. For this purpose, all accounts are balanced periodically.

Summarizing the transactions : The next step is to prepare a year-end summary known as 'Final Accounts'. But before preparing the final accounts, a statement called 'Trial Balance' is prepared to check the arithmetical accuracy of the journal and ledger. Thus, with the help of trial balance and some other relevant information, the final accounts are prepared. The objectives of preparing the final accounts are

- (a) to know the net result of business activities
- (b) to ascertain the financial position of the business

Interpreting the results : The last stage consists of analyzing and interpreting the results shown by the final accounts. This involves computation of various financial ratios and trends.

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accountancy ratios to assess the liquidity, solvency, and profitability of the business. Such analysis is meant for interested parties like management, investors, bankers, creditors etc. The balances on various accounts appearing in the Balance sheet will then be transferred to the new books of account for the next year. Thereafter the process of recording the transactions for the next year begins again.

III. BOOKS OF ACCOUNT**1. Journal**

Journal is a daily record of business transactions. It is also called a 'Day Book' and is used for recording all day-to-day transactions in the order in which they occur. It is also called the book of prime entry or the book of original entry because all transactions are recorded first in this book. In other words, a journal contains the chronological listing of all transactions and constitutes the original record of all transactions. The process of recording a transaction in the journal is called 'journalizing' and the entries made in this book are called 'Journal Entries'.

Examples of a Journal and Journal entries (see figure)

Journal

Date Amount	Particulars	LF*	Dr Rs.	Dr Amount Rs.
June 01	Cash Account To Capital Account	Dr. 2 1	100000 2000	10000 2000
June 02	Furniture Account To Bank Account (Being furniture purchased and paid by cheque)	Dr. 16 3		
June 03	Cash Account To sales Account (Being Goods sold for cash)	Dr. 2 4	3000	3000
June 04	Drawings account to cash account (Being cash withdrawn for personal use)	Dr. 10 2	500	500

* Imaginary Ledger Folio Numbers

As is seen above a journal is divided into 5 columns. The explanation of each column of the journal and its entries are given below :

- Column 1 : Used for writing the date of the transaction
- Column 2 : This is the Particulars column. The name of the account to be debited is written first and the abbreviation 'Dr' is written against it. In the next line, the name of the amount to be credited is written and is always preceded by the word 'To'. It is not necessary to write 'Cr' against the name of the account to be credited. In the next line, a brief description of the transaction called 'Narration' is given in the bracket.
- Column 3 : This gives the LF (Ledger Folio) No. Which is nothing but the page number in the ledger where the concerned account appears.
- Column 4 and 5 : These columns are meant for recording the amounts with which the two accounts have been affected. The amount to be debited is entered in the 4th column and the amount to be credited is entered in the 5th column against respective accounts. Both the amounts will always be equal.

2. Ledger

We know that the journal is a chronological record of all business transactions. But it does not provide all the information regarding a particular person or item at one place. This makes it difficult to know the net effect of various transactions with a particular person. Suppose you want to know the amount due to a particular supplier or the amount due from a particular customer, you will have to go through the whole journal. To overcome this difficulty, another book called 'Ledger' is maintained. In this book, separate accounts, are opened for each person and each item in separate pages. Then all transactions related to that particular person or item as recorded in the journal are posted in the concerned account. For example, all transactions related to a particular supplier, say Neil, are posted to Neil's account. Similarly, all cash payments and cash receipts can be posted to Cash Account, and so on.

Thus, ledger is a book where all accounts relating to different items are maintained and into which all journal entries must be posted. In fact, ledger is the principle book of entry which provides complete information about various transactions relating to all people and all items of asset, incomes and expenses.

Example of a ledger :

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- i. Cash
 - ii. Perso
 - iii. Real
 - iv. Nom

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From the format of the ledger above, we can notice that both sides of the Account have date, particulars, folio and amount columns. The example shown above is one of the pages in the ledger where Cash Account is kept. The entries that are posted in this ledger page are from the journal shown in the previous page.

IV. FINANCIAL STATEMENTS

The preparation of final accounts namely the profit and loss statement and the Balance sheet are prepared after the Trial Balance of the Books of account.

- 1. Trial Balance :** After posting all journal entries into the ledger, a statement called trial balance is prepared to check the arithmetical accuracy of the books of account. It is usually prepared just before preparing the final accounts.

A Trial Balance basically shows the balances of all the accounts which appear in the ledger. We know that every transaction has two aspects under double entry system i.e. credit and debit. The debit balances are shown in one column and the credit balances in the other in the trial balance. We know that for every debit there is an equal and corresponding credit, under the Double Entry system. So, the total of debits given to different accounts must be equal to the total of the credits given to different accounts. Similarly, the total of the debit balances in different accounts must be equal to the total of credit balances in different accounts. Now if the Trial Balance tallies i.e., the total of its debit balances column, is equal to the total of its credit balances column, it would mean that both the aspects of each transaction have been correctly recorded in the ledger. However, if the two totals do not tally it implies that some errors have been committed while posting the transactions into the ledger.

Methods of Preparing Trial Bal-

- i. **Totals Method** : In this, the totals of each side of an account is shown in the Trial Balance. The debit side total of an account is shown in the debit column of the Trial Balance and the credit side total of the account is the credit column.

- ii. **Balances Method** : In this, only the balances of each account is shown in the Trial Balance. This method is more convenient and commonly used because it eliminates all those accounts which have nil balance.

Figure shows an example of Trial Balance prepared by Balances Method.

Trial Balance As On December 31, 2000 (Example)

Sl.No.	Name of the Account	LF	Dr. Balance	Cr. Balance
1.	Cash Account	2	12500	
2.	Capital Account	1		16000
3.	Furniture Account	16	2000	
4.	Bank Account	3		2000
5.	Sales Account	4		3000
6.	Drawings Account	10	500	
7.	Salaries Account	5	5000	
8.	Bad Debts Account	20	1000	
			21,000	21,000

2. Profit and Loss Statement

It is a statement which shows the details of income and expenditure of the organization for a particular period, generally one year. The profit and loss statement gives the detailed list of revenues on one side and the detailed list of expenditures on the other side. Finally, it shows the net income for the period indicated, which could be profit or loss. It is for this reason that a profit and loss statement is sometimes referred to as 'Income Statement' or 'Revenue Statement'. A typical profit and loss statement is shown in the following figure.

XYZ COMPANY
PROFIT & LOSS ACCOUNT

For the Financial year 2000-2001 (1st April 2000 - 31st March 2001)

Revenues		Expenditures	
Sales	Rs.		
Product X	- 20000	Raw materials	- 15000
Product Y	- 15000	Production costs	- 10000
Product Z	- 20000	Administration Expenses	- 5000
Scraps	- 5000	Depreciation costs	- 2000
Old Machines	- 5000	New Machinery	- 8000
TOTAL	- 65000	Travelling Expenses	- 1000
		Salaries & Wages	- 3000
		Advertisements	- 1000
		TOTAL	- 50000
Total Revenues	- 65000		
Total Expenses	- 50000		
Total Income	- 15000		
Income Tax	- 3000		
Net income (Profit)	- 12000		

Figure : Profit & Loss Account

3. Balance Sheet

A Balance sheet is a statement which shows the financial status of the company at any given point of time. It is a statement of assets, liabilities and capital (or net worth) of a specified date. In other words, a Balance Sheet shows the summary of all the resources of the business and how they are locked up in various assets and liabilities. Since the Balance Sheet reflects the financial position of the company, it is also sometimes referred to as the 'Position Statement'.

Balance sheets are normally prepared once in a year, but may be prepared more often whenever it is asked for. The need to ask for a Balance sheet arises on specific occasions like when the company wants to take decisions regarding diversification, large investments, dividend, distribution etc. In general, a Balance Sheet is helpful for investors on the business, creditors and the organization to prepare its annual budgets. A typical Balance sheet is shown in the following figure.

ABC COMPANY
BALANCE SHEET
(As on 31st March 2001)

Liabilities		Assets	
Fixed Liabilities		Fixed Assets	
Shares	-	Land	- 50000
Debentures	-	Building	- 100000
Bonds	-	Equipment	- 50000
Total	- 150000	Total	- 200000
Current Liabilities		Current Assets	
Bank Loans	-	Cash	- 10000
Bills payable	-	Receivables	- 20000
Over drafts	-	Inventories	- 20000
Trade creditors	-	Total	- 50000
Total	- 200000	Investments	
		Bonds	- 50000
		Securities	- 5000
		Total	- 100000
Total Liabilities	Rs. 3,50,000	Total Assets	Rs. 3,50,000

Figure : Balance Sheet

Relation between Balance Sheet and Profit & Loss Account

The Balance Sheet and the Profit and Loss Account are not two separate and independent statements, but they are related to each other. The Profit & Loss Statement is a link between the Balance Sheet at the beginning of the period and the Balance Sheet at the end of the period. We can easily understand the impact of profit & loss account if we remember that revenue is an inflow of assets (or outflow of liabilities) and expenditure is an outflow of assets (or inflow of liabilities).

Generally the Profit and Loss Account is prepared to calculate net profit. Net profit can also be calculated by comparing the Balance Sheet at the beginning and end of the financial period. This fact demonstrates the role of the profit and loss account as a link between consecutive statements of financial position. Net Profit (or net loss) for a financial period is equal to the change in the equity held by the owners, during that period. In other words, the difference in the beginning and ending of the owners equity is the net profit (or net loss).

Parted liabilities → share, Equity

214 ENGINEERING ECONOMICS

WORKED PROBLEMS

1. The company X having certain reserves and surplus has the following details as on 31st Dec. 2000

Dividend payable	72,000	Debtors	1,60,000
Bank Balance	10,000	Bills payable	20,000
Equity shares	2,00,000 ✓	Plant & Equipment ✓	80,000
Provision for taxes	40,000 ✓	Bills receivable	20,000
Stock	77,000	Creditors	55,000
8% Preference Shares	1,35,000 ✓	General Reserve	40,000
Land & Building	2,00,000 ✓	Cash in Hand	15,000

Prepare a balance sheet as on 31st December 2000

Solution.

(VTU Model Q.Paper)

From the data given in the problem and comparing the same with the various entries in a standard Balance Sheet, we can prepare the Balance Sheet for the company X as on 31st December 2000, as follows.

'X' Company

BALANCE SHEET

(As on 31st December - 2000)

Liabilities		Assets	
Fixed liabilities		Fixed Assets	
Equity Shares	2,00,000	Land & Building	2,00,000
8% Preference shares	1,35,000	Plant & equipment	80,000
Total	3,35,000	Total	2,80,000
Current Liabilities		Current Assets	
General Reserve	40,000	Cash in hand	15,000
Dividend Payable	72,000	Bank balance	10,000
Provision for taxes	40,000	Bills receivable	20,000
Bills Payable	20,000	Debtors	1,60,000
Creditors	55,000	Stock	77,000
Total	2,27,000	Total	2,82,000
Total Liabilities	5,62,000	Total Assets	5,62,000

FINANCIAL MANAGEMENT 215

2. The following are the items of the profit and loss account for VOLTAS limited for the year ended on 31st March 2003. You are required to arrange them systematically and indicate

- (i) Profit before taxation and
(ii) Profit after taxation.

VOLTAS Limited
Profit and Loss account
for the year ended 31st March 2003

	Rs.(Lakh)
Operating and administrative expenses	10,440.60
Depreciation	1,382.80
Provision for income tax	210.00
Interest	2595.30
Cost of sales and services	54,773.90
Sales and service ✓	69,552.90
Provision for wealth tax	3.50
Other income ✓	517.60
Excess provision of tax in previous years ✓	143.00
Proposed dividend	643.80

Solution.

(VTU, July 2005)

VOLTAS LIMITED
PROFIT & LOSS ACCOUNT
(For the financial year ending 31st March 2003)

Revenue	Rs.(Lakh)	Expenditure	Rs.(Lakh)
Sales & Service	69,552.90	Operating & Administration	
Other income	517.60	Expenses	10440.60
Excess provision of tax in previous year	143.00	Depreciation	1382.80
		Interest	2595.30
		Cost of sales & Services	54,773.90
		Proposed Dividend	643.80
Total	70213.50	Total	69,836.40
Total Revenues	70213.50		
Total Expenses	69836.40		
Profit Before tax	377.10		
Provision for Income tax	210.00		
Provision Wealth tax	3.50		
Profit after tax	163.60		

216 ENGINEERING ECONOMICS

i. Cash A	= Rs. 377.10 lakh
ii. Person	= Rs. 163.60 lakh
iii. Real A	
iv. Nomina	
Financi	
creditworthin	
Objectives	
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3. Following is the financial status of a company as on 31st March 2005.

Sundry Debtors	Rs.10,000
Cash in Hand	Rs.22,000
Bank loans	Rs.40,000
Bills payable	Rs.20,000
Equity shares	Rs.1,13,000
Land & Buildings	Rs.50,000
Plant & Machinery	Rs.90,000
Inventories	Rs.15,000
Creditors	Rs.30,000
Bank balance	Rs.1,60,000

Prepare a balance sheet as on 31st March 2005

(VTU Jan'06)

Solution.

XYZ Company

BALANCE SHEET

(as on 31st March 2005)

Liabilities		Assets	
Fixed Liabilities		Fixed Assets	
Equity shares	1,13,000	Land & Building	50,000
		Plant & Machinery	90,000
Total	1,13,000	Total	1,40,000
Current Liabilities		Current Assets	
Bank Loans	40,000	Cash in hand	22,000
Bills payable	20,000	Bank balance	16,000
Creditors	30,000	Inventories	15,000
Total	90,000	Sundry debtors	10,000
Total Liabilities	2,03,000	Total	63,000
		Total Assets	2,03,000

FINANCIAL MANAGEMENT 217

4. A company having reserves and surplus has following year end details.

Equity shares	2,00,000
Bank balance	10,000
Dividend payable	72,000
Provision for tax	40,000
Preference shares	1,35,000
Land and Building	2,00,000
Debtors	1,60,000
Bills payable	20,000
Plant and equipment	80,000
Bills receivable	20,000
Creditors	55,000
General Reserves	40,000
Cash in hand	15,000
Stock	77,000

Prepare the Balance Sheet.

(VTU June 2008)

Solution.

The balance sheet for company XXX is as under:

Company XXX

Balance Sheet as on 31st Dec

(As on 31st March 2005)

Liabilities		Assets	
Fixed Liabilities		Fixed Assets	
Equity shares	2,00,000	Land & Buildings	2,00,000
8% Preference shares	1,35,000	Plant & Equipment	80,000
Total	3,35,000	Total	2,80,000
Current Liabilities		Current Assets	
General Reserve	40,000	Cash in hand	15,000
Dividend payable	72,000	Bank balance	10,000
Provision for taxes	40,000	Bills receivable	20,000
Bills payable	20,000	Debtors	1,60,000
Creditors	55,000	Stock	77,000
Total	2,27,000	Total	2,82,000
Total Liabilities	5,62,000	Total Assets	5,62,000

5. Following are the items of the profit and loss account of ABC Company for the year ended 31st March 2004. Arrange them systematically and indicate

(i) profit before taxation and (ii) profit after taxation

(Rs. Lakhs)

Operating and administrative expenses	10,440.6
Depreciation	1,382.8
Provision for income tax	0.0
Interest	2,595.3
Cost of sales and services	54,773.9
Sales and services	69,552.9
Provision for wealth tax	3.5
Other income	517.6
Excess provision of tax in previous years	143.0
Proposed dividend	643.8

Solution.

ABC Co.,
PROFIT & LOSS ACCOUNT
as on 31st March 2004

Revenues	Rs.(Lakh)	Expenditure	Rs.(Lakh)
Sales & Services	69,552.90	Operating	
Other income	517.60	Administrative Ex.	10440.60
Excess provision of tax in previous year	143.00	Depreciation	1382.80
		Interest paid	2595.30
		Cost of sales & service	54773.90
		Proposed dividend	643.80
Total	70,213.50	Total	69,836.40
Total Revenues	70,213.50		
Total Expenditure	69,836.40		
Profit before tax	377.10		
Provision for Income tax	0.0		
Provision for Wealth tax	3.50		
Profit after tax	373.60		

∴ (i) Profit before tax = Rs. 377.10 lakhs.

(ii) Profit after tax = Rs. 373.60 lakhs.

SUMMARY

1. Role of a Finance Manager

To maximize profits and minimize losses, to know the financial needs of the company and to raise appropriate funds, to pay salaries, to control all financial activities to help prepare financial statements.

2. Scope of Finance

Finance is the life-blood of any business. Finance has to be raised through various sources to find the production, marketing and other activities. Money earned through sale of goods and services generates returns on the invested capital.

3. Finance Function

A Finance manager of a company has to raise money from various sources, allocate them wisely and distribute the profits to all the shareholders appropriately. These are known as finance functions and are all accomplished through.

- (a) **Investment decisions** : A Finance manager has to allocate funds to long-term assets that would yield benefits in future.
- (b) **Financing decision** : A Finance manager has to determine when, where, why, and how much of the funds to acquire in order to meet the firm's investment needs.
- (c) **Dividend decision** : A Finance manager has to decide how and in what proportion should the profit be distributed among the various stakeholders.
- (d) **Liquidity decision** : A Finance manager has to correctly estimate the firm's need for current assets and make sure that funds are made available when needed.

4. Sources of Financial Information

- Accounting Process
- Book-keeping
- Books-of-account
- Financial Statements

5. Basic concepts in Accounting

Accounting is the art of recording, classifying, summarizing and reporting all business transactions. Basic concepts in Accounting are Assets, Liabilities, Transaction, Creditor, Debtor, Bed-debt, Turnover, Cash & Credit trade, Commission, Overdraft, Drawing, Sales, Returns, Purchase Returns, Bill, Receipt, Invoice, Voucher.

6. Classification of Accounts

Personal Accounts : Those relating to persons, particular firms.
 Real Accounts : Those relating to property (Assets)
 Nominal Accounts : Those relating to Income and Expenses

7. Book-keeping

Book-keeping is the art of recording business transactions in a systematic manner for the sake of reference.

Double Entry system - This system records and recognizes the aspect of receiving as well as the aspect of giving of every transaction. Rules of Debit & credit for different types of accounts are made use for Double Entry System.

Single Entry System : Single Entry system refers to incomplete records. Single entry system is a mixture of single entry, double entry and no entry. Personal expenditures are usually recorded this way.

8. Books of Account

Journal : Journal is a daily record of business transactions and is used to record day-to-day transactions in the order in which they occur. The process of recording a transaction in the journal is called journalizing and the entries made in the book are called journal entries.

Ledger : In a ledger, separate accounts are opened in different pages for each person and each item. Then all transactions relating to that particular person or item are recorded in the journal and posted in the concerned account. Therefore Ledger is the principle book of entry which provides complete information about various transactions relating to all people and all items of asset, incomes & expenses.

9. Trial Balance

It is a statement prepared to check the arithmetical accuracy of the books of account. It is usually prepared just before preparing the final accounts i.e., Profit & Loss statement and Balance Sheet.

Trial Balance can be prepared by either the Totals method or Balances Method.

10. Profit & Loss Statement

It is a statement which shows the details of Revenue and Expenditure of the organization for a particular period. Finally it shows the net income for the period indicated, which could be profit or loss. Also see Figure for an example of profit & loss statement.

11. Balance Sheet

A Balance Sheet is a statement which shows the financial status of the company at any given point of time. It is a list of current values of assets on one side and liabilities on the other side. A balance sheet generally reflects the financial position of the company. Also see Figure for example of Balance Sheet.

12. Relation between Balance sheet and Profit & Loss Account

The Profit & Loss Statement is a link between the balance sheet at the beginning of a period and the Balance Sheet at the end of the period. The net period of a company can be calculated by comparing its balance sheet at the beginning and end of the financial periods.



CHAPTER
7

FINANCIAL RATIO ANALYSIS

INTRODUCTION

Information contained in Financial Statements like Balance Sheet and Profit & Loss Account are highly useful to various stakeholders of a company such as management, creditors, shareholders, SEBI etc. A study, observation and interpretation of these statements help in understanding the operating performance, financial position, financial strengths and weaknesses etc. of the firm. This is known as Financial Analysis.

Financial Analysis

It is defined as the process of identifying the financial strengths and weaknesses of a company by properly establishing relationship between the various entries on the Balance Sheet and the Profit & Loss Statement. Financial analysis can be undertaken by the management, creditors, owners, shareholders etc. The nature of analysis will differ depending on the purpose of the analyst. Some interests of parties involved in a company would be:

i. Trade Creditors

They are interested in the company's ability to meet its claims over a very short period of time. In other words, the trade creditors, which mean day-to-day buyers and sellers associated with the company, will be interested in the liquidity position of the company.

ii. Shareholders

They would be most concerned about the company's revenues. Shareholders or investors would concentrate on the company's present and future profitability. They will also be interested in the firm's structure to the extent it influences the firm's ability to earn and the risks thereof.

iii. Suppliers

Various suppliers, who supply material, machinery etc., will be concerned with the firm's long term solvency and survival. They analyze the firm's profitability over time, its ability to generate cash and pay interest or repay principal amounts. Long-term creditors also analyze past financial statements of the company to ascertain their creditworthiness.

iv. Management

It would be interested in every aspect of the financial analysis. It is the overall responsibility of the management to see that the resources of the firm are used most effectively and efficiently, and that the firm's financial condition is sound.

NATURE OF RATIO ANALYSIS

In the financial world, several ratios are used to study and interpret the financial statements and other accounting information. A financial ratio is a relationship between two accounting figures, expressed mathematically. Financial ratios help in summarizing large quantities of financial data and to make qualitative judgement about the firm's financial performance. Various performance measures such as company liquidity, profitability, creditworthiness, efficiency, productivity, etc., can all be understood by calculating and interpreting various financial ratios.

Types of Financial Ratios

Following is the classification of all financial ratios. Each one of them reflects the health of various aspects of the company.

I. Liquidity Ratios

- Current ratio
- Quick ratio or Acid Test ratio
- Internal measure

II. Leverage Ratios

- Debt ratio
- Debt-equity ratio
- Capital-equity ratio
- Interest coverage

III. Activity Ratios

- Inventory Turnover
- No. of days, inventory
- Debtors Turnover
- Collection period
- Assets Turnover
- Working Capital Turnover

IV. Profitability Ratios

- Gross Profit Margin
- Net Profit Margin
- PAT to EBIT ratio

- Return on Investment (ROI) before tax
- Return on Investment (ROI) after tax
- Return on Equity (ROE)

V. Other Ratios

- Earnings Per Share (EPS)
- Dividend Per Share (DPS)
- Dividend Payout Ratio
- Price-earnings (PE) ratio
- Market-value Book Value ratio

VI. Evaluation of a Firm's Earning Power (RONA)

Ratio analysis is a very useful tool to answer a number of questions regarding managerial issues. It provides clues to investigate those issues in detail. However, one should be cautious in applying and interpreting these ratios as they are calculated from accounting data. Accounting numbers are sensitive to changes in accounting policies, allocations and budgets and of course inflation.

Let us study the definition and interpretation of all the financial ratios with the help of the financial statements of a company.

WORKED EXAMPLE

1. FANUC Motors Ltd, manufactures all types of stepper motors. The company started with a share capital of 50 lakhs 20 years ago, which has now increased to 450 lakhs. The number of shares outstanding is 45 lakhs. The average market price of the Company's share has been Rs.52.76 during 2003, Rs.69.00 during 2004 and Rs.58.50 during 2005. The financial data for the company are given in the following tables. Find the values of all financial ratios and interpret them.

**TABLE 1: FANUC MOTORS
PROFIT AND LOSS ACCOUNT
for the year ending 31st March 2005**

	Particulars	2005 (All values in Rs. Lakhs)
A.	Net Sales*	7434.46
B.	Cost of goods sold**	6107.32
C.	Gross Profit (A - B)	1327.14
D.	Less : Selling & administrative expenses	715.74
E.	Operating income (C-D)	611.40
F.	Add : Other income	73.82
G.	Earning before interest & tax (EBIT) (E + F)	685.22
H.	Less : Interest	286.92
I.	Profit before tax (PBT) (G - H)	398.30
J.	Provision for Tax	128.58
K.	Profit after tax (PAT) (I-J)	269.72
L.	Effective tax rate***	32%
M.	Dividend distributed	90.00
N.	Retained earnings	179.72
	* Net excise duty	
	** Depreciation included	
	*** Provision for tax dividend by profit before tax.	

TABLE 2 : FANUC MOTORS
Statement showing Cost of goods sold
for the year ending on March 2005

	Particulars	2005 (Rs. Lakh)
A.	Raw Materials	5503.04
B.	Direct Labour	457.88
C.	Depreciation	83.18
D.	Other production expenses	658.88
		6702.98
	Add : Opening stock in process	301.10
		7004.08
E.	Less : Closing stock in process	461.66
	Cost of production	6542.42
	Add : Opening finished stock	488.52
		7030.94
	Less : Closing finished stock	926.62
F.	Cost of Goods sold	6107.32

TABLE 3 : FANUC MOTORS
Balance Sheet (As on 31st March 2005)

	Particulars	2005 (Rs. Lakh)
A.	Net worth	
	Share Capital	450.00
	Reserve	895.74
	Net Worth	1345.74
B.	Borrowings	
	Long Term : Debentures	152.92
	Others	625.46
	Long term debt	778.38
	Short-term : Bank borrowings	1679.74
	Borrowings	2458.12
C.	Capital Employed (E + B)	3803.74
D.	Fixed Assets	
	Gross Block	1843.10
	Less : Depreciation	470.88
	Net Block	1372.22
	Other non-current assets	121.44
	Net Fixed assets	1493.66
E.	Current Assets	
	Inventories	
	Raw Materials	915.48
	Stock in process	461.68
	Finished goods	923.62
	Inventories	
	Debtors	2300.78
	Cash and Bank balance	966.36
	Others	52.16
	Current assets	422.54
F.	Less : Current liabilities	3741.84
	Trade Creditors	
	Provision & Others	678.70
	Current liabilities	753.06
G.	Net Current Assets (E-F)	1431.76
H.	Net Assets (D + G)	2310.08
		3803.74

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Solution**I. Liquidity Ratios**

These ratios generally measure the company's ability to meet current financial obligations.

$$1. \text{ Current Ratio: } \text{Current Ratio} = \frac{\text{Current assets}}{\text{Current liabilities}}$$

Current Assets = Item E on Balance Sheet = 3741.84

$$\text{Current Liabilities} = \text{Item F on B.S.} + \text{Item B(iv) on B.S.} \\ = 1431.76 + 1679.44 = 3111.50$$

$$\therefore \text{Current Ratio} = \frac{3741.84}{3111.50} = 1.20$$

$\therefore \text{Current Ratio} : 1.2 : 1$

Interpretation of Current ratio

Current ratio is a measure of a company's short-term liquidity. As a convention rule, a current ratio of 2 : 1 or more is considered satisfactory. Fanuc Motors has current ratio of 1 : 2 : 1, which indicates insufficient liquidity.

2. Quick Ratio or Acid test Ratio

$$\text{Quick Ratio} = \frac{\text{Current assets} - \text{Inventories}}{\text{Current liabilities}}$$

Inventories = Item E (iv) on Balance Sheet = 2300.78

$$\therefore \text{Quick Ratio} = \frac{3741.84 - 2300.78}{3111.50} = 0.46$$

$\therefore \text{Quick Ratio} = 0.46 : 1$

Interpretation of Quick Ratio

Quick ratio is a measure of a company's extreme short-term liquidity. Inventories normally require sometime for realizing into cash, they are subtracted. Generally a Quick ratio of 1 : 1 would be satisfactory. But Fanuc Motors has Quick ratio of 0.46 : 1 which would be insufficient.

3. Cash Ratio :

$$\text{Cash Ratio} = \frac{\text{Cash} + \text{Bank Balance}}{\text{Current liabilities}}$$

we have,

$$\begin{aligned} \text{Cash} + \text{Bank Balance} &= \text{Item E (vi) on Balance Sheet} \\ &= 52.16 \text{ lakhs} \end{aligned}$$

$$\therefore \text{Cash Ratio} = \frac{52.16}{3111.50} = 0.016$$

$\therefore \text{Cash Ratio} = 1.6\%$

Interpretation of Cash Ratio

Cash ratio is nothing but cash held to satisfy its short-term or current liabilities. Fanuc Motors has only 1.6% cash against outstanding liabilities of 3111 lakhs, which is very less. If companies have Overdraft (OD) facility from the bank, they can borrow if needed.

4. Internal Measure

$$\text{Internal Measure} = \frac{\text{Current assets} - \text{Inventory}}{\text{Average daily operating expenses}}$$

Total operating expenses per year = Cost of goods sold + Selling & administration expenses - Depreciation

$$\begin{aligned} &= \text{Item B on Table 1} + \text{Item D on Table 1} - \text{Item C on Table 2} \\ &= 6107.32 + 715.74 - 83.18 \end{aligned}$$

T.O.E per year = 6739.88

$$\therefore \text{Average daily operating expenses} = \frac{6739.88}{365} = 18.465$$

$$\therefore \text{Internal Measure} = \left(\frac{3741.84 - 2300.78}{6739.88/365} \right) \text{days}$$

Internal Measure = 78.04 days

Interpretation of Internal Measure

This is a measure of a company's liquidity with respect to company's day-to-day operating expenditures. An internal measure of 78.04 days means that the company

Financial
 i. Cash
 ii. Person
 iii. Real Assets
 iv. Nominal

Financial
 creditworthiness

Objectives
 (i) To take
 (ii) To meet
 (iii) To safeguard
 (iv) To prepare
 (v) To prepare

2. Cost A

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II. Leverage Ratios

They generally show the capital structure i.e., proportion of debt and equity, of the company.

1. Debt Ratio

$$\text{Debt Ratio} = \frac{\text{Total Debt(TD)}}{\text{NetAssets(NA)}} \\ = \frac{\text{Total debt(TD)}}{\text{Total debt+Networth(NW)}}$$

$$\text{Total Debt} = \text{Long term debt} + \text{Short term} - \text{Bank borrowings}$$

$$= \text{Item B(iii) on table 3} + \text{Item B(iv) on table 4}$$

$$= 778.38 + 1679.74$$

$$= 2458.12$$

$$\text{Net worth(NW)} = \text{Item A(iii) on table 3}$$

$$= 1345.74$$

$$\therefore \text{Debt Ratio} = \frac{2458.12}{2458.12 + 1345.74} = 0.646$$

$$\text{Debt Ratio} = 0.646.$$

Interpretation of Debt Ratio

Debt ratio indicates the extent of net assets financed by the lenders and shareholders and banks. In this case, Fanuc Motors has a debt ratio of 0.646 which means that lenders have financed 64.6% (about 2/3rds) of Fanuc Motors net assets while the remaining 35.4% has been contributed by the owners themselves.

2. Debt-Equity Ratio

$$\text{Debt-Equity ratio} = \frac{\text{Total Debt(TD)}}{\text{NetWorth(NW)}} \\ = \frac{2458.12}{1345.74} = 1.83$$

$$\therefore \text{D/E Ratio} = 1.83$$

Interpretation of Debt Equity Ratio

Debt equity ratio indicates contribution w.r.t. owners contribution. In this case, debt equity ratio of 1.83 means, lenders to Fanuc Motors have contributed Rs. 1.83 for every rupee invested by the owners, or simply 1.83 times the owners's contribution.

3. Capital-Equity Ratio

$$\text{Capital equity ratio} = \frac{\text{Capital Employed(CE)}}{\text{Net Worth(NW)}}$$

$$\text{Capital employed (CE)} = \text{Item C on Table 3}$$

$$= 3803.74$$

$$\therefore \text{Capital Equity Ratio} = \frac{3803.74}{1345.74} = 2.82$$

$$\therefore \text{Capital Equity ratio} = 2.82$$

Interpretation of Capital-Equity Ratio

Capital-equity ratio indicates how much of funds are being contributed together by lenders and owners for each rupee of owners contribution. A value of 2.82 indicates that lenders and owners together contribute 2.82 times the contribution of owners alone.

4. Interest coverage

$$\text{Interest coverage} = \frac{\text{Earnings before interest and taxes (EBIT)}}{\text{Interest}}$$

$$\text{EBIT} = \text{Item G on table 1}$$

$$\text{Interest} = \text{Item H on table 1}$$

$$\therefore \text{Interest coverage} = \frac{685.22}{286.92} = 2.38 \text{ times}$$

$$\text{Interest coverage} = 2.38 \text{ times}$$

Interpretation of Interest Coverage

Interest coverage indicates the company's ability to meet interest obligations. The ratio indicates the number of times the availability of funds w.r.t interest charges. In other words, a value of 2.38 times means that the company has funds equivalent to 2.38 times at any time with which it can pay its interest. This is the confidence level estimated by lenders to the company.

III. Activity Ratios or Turnover Ratios

They generally reflect company's efficiency in utilizing its assets.

1. Inventory Turnover Ratio

$$\text{Inventory Turnover} = \frac{\text{Cost of goods sold}}{\text{Average inventory}}$$

We have,

cost of goods sold = Item B on table 1 = 6107.32

$$\text{Average Inventory} = \frac{\text{Opening finished stock} + \text{Closing finished stock}}{2}$$

= Items E on Table 2

$$= \frac{488.52 + 923.62}{2} = 706.07$$

$$\therefore \text{Inventory turnover} = \frac{6107.32}{706.07} = 8.65$$

Interpretation of Inventory turnover ratio

This indicates the efficiency of the firm in producing and selling its products. In other words, the value 8.65 indicates the number of times in a year the inventory of finished goods is sold away.

2. No. of days, Inventory

$$\text{No. of days, Inventory} = \frac{365}{\text{Inventory Turnover}}$$

$$= \frac{365}{8.65} = 42 \text{ days}$$

\therefore Number of days, Inventory = 42 days

Interpretation of No. of days, Inventory

It indicates the average number of days the finished stock is held as inventory before it is sold. The value of 42 days indicates hold-up of finished goods for a little long, holding up investment along with it.

3. Debtors Turnover

$$\text{No. of days, Inventory} = \frac{365}{\text{Inventory Turnover}}$$

$$\text{Debtors turnover} = \frac{\text{Net sales}}{\text{Debtors}}$$

We have,

Net sale = Item A on table 1 = 7434.46

Debtors = Item E (v) on table 3 = 966.36

$$\therefore \text{Debtors turnover} = \frac{7434.46}{966.36} = 7.7 \text{ times}$$

Interpretation of Debtor's turnover

Debtor's turnover indicates the number of times debtors turnover each year. In other words, the ratio indicates the number of times debtors (people who owe money to the company) pay-off their dues, on an average, in a year. Generally, higher this value, the more efficient is the management of credit.

4. Collection Period

$$\text{Collection period} = \frac{365}{\text{Debtors turnover}}$$

$$= \frac{365}{7.7} = 47.5 \text{ days}$$

Collection Period = 47.4 days

Interpretation of collection period

The collection period indicates the average no. of days taken by the debtors to pay-off their dues. It gives us a measure of the quality of debtors since it indicates the speed of their collection. A short period implies prompt payment of debtors. But the terms long and short-duration depends on the type of business and clientele.

5. Net Assets Turnover

$$\text{Net sales Turnover} = \frac{\text{Net Sales}}{\text{Net Assets}}$$

We have

$$\text{Net sales} = \text{Item A in table 1} = 7434.46$$

$$\text{Net Assets} = \text{Item H on table 3} = 3803.74$$

$$\therefore \text{Net Assets Turnover} = \frac{7434.46}{3803.74} = 1.95$$

$$\therefore \text{Net Assets Turnover} = 1.95 \text{ times}$$

Interpretation of Net Assets Turnover

This value indicates the ability of the company to produce a large volume of sales for a given amount of net assets. In other words, net assets turnover means the number of times the net assets have been turned over to produce saleable goods. A value of 1.95 times implies that Fanuc Motors is producing Rs. 1.95 of sales for one rupee capital employed in net assets.

6. Working Capital Turnover

$$\text{Working Capital Turnover} = \frac{\text{Net Sales}}{\text{Net Current assets}}$$

$$\text{Net Sales} = 7434.46$$

$$\text{Net current assets} = \text{Item G on table 3} = 2310.08$$

$$\therefore \text{Working capital Turnover} = \frac{7434.46}{2310.08} = 3.2 \text{ times}$$

$$\therefore \text{Working capital Turnover} = 3.2 \text{ times}$$

Interpretation of working capital turnover

This value indicates the ability of the company to produce and sell for a given amount of working capital. In other words, working capital turnover means the extent and no. of times to which working capital has been effectively utilized to produce saleable goods. A value of 3.2 times implies the Fanuc Motors is producing Rs. 3.2 worth saleable goods for every one rupee of working capital available in the system.

IV. PROFITABILITY RATIOS

They are measures of company's overall performance and effectiveness.

1. Gross Profit Margin

$$\begin{aligned} \text{Gross Profit Margin} &= \frac{\text{Net sales} - \text{Cost of goods sold}}{\text{Net Sales}} \\ &= \frac{\text{Gross Profit}}{\text{Net sales}} \end{aligned}$$

Where,

$$\text{Net Sales} = 7434.46$$

$$\text{Cost of goods sold} = 6107.32$$

$$\therefore \text{Gross Profit Margin} = \frac{7434.46 - 6107.32}{7434.46} = 0.18$$

$$\text{or Gross Profit Margin} = 18\%$$

Interpretation of Gross Profit Margin

The gross profit margin reflects the efficiency with which management produces each unit of the product. In other words, higher the value of gross profit margin, higher is the ability of the company to produce at low costs. For example, the value of GPM will increase if the company is able to (i) produce higher quantities at same net costs, or (ii) produce the same quantity at net lower costs, or (iii) produces same quantity at same costs but selling at higher prices! A value of 18% is good only if it is consistent.

2. Net Profit Margin

$$\text{Net Profit Margin} = \frac{\text{Profit after tax}}{\text{Net sales}}$$

Where,

$$\text{Profit after tax} = \text{Item K on table 1} = 269.72$$

$$\text{Net sales} = 7434.46$$

$$\therefore \text{Net profit margin} = \frac{269.72}{7434.46} = 0.036$$

$$\therefore \text{Net profit margin} = 3.6\%$$

Interpretation of Net Profit Margin

Between Gross Profit and Net profit, a company may lose a lot towards taxes, overhead and other expenses. Therefore, net profit is more important because it is the profit that is realized. Hence, net profit margin is computed which indicates the firm's ability to turn each rupee of sales into net profit. Net profit margin ratio establishes a relationship between net profit and sales which indicates management's efficiency in production, administration and sales of the product. A value of 3.6% means the percentage value of net sales which has been realized as net profit, which is low from industry standards.

3. PAT to EBIT Ratio

$$\text{PAT to EBIT Ratio} = \frac{\text{PAT}}{\text{EBIT}} \times \frac{(\text{Profit after tax})}{(\text{Earnings before interest and tax})}$$

where

PAT = Item K on table 1 = 269.72

EBIT = Item G on table 1 = 685.22

$$\therefore \text{PAT to EBIT ratio} = \frac{269.72}{685.22} = 0.393$$

$$\therefore \text{PAT to EBIT ratio} = 39.3\%$$

Interpretation of PAT to EBIT Ratio

This value indicates the extent to which interest payments and taxes take off the profit. In other words PAT to EBIT ratio indicates the portion of deduction that is made in the earnings for the sake of interests and taxes. Higher value of this ratio indicates high taxes and usage of funds bought at high rates of interest which are reflected into profits. A value of 39.3% is fairly acceptable.

4. Return on Investment (ROI) before tax

$$\text{ROI} = \frac{\text{EBIT}}{\text{Net Assets}}$$

where

EBIT = 685.22

Net Assets = 1901.87

$$\therefore \text{ROI (before tax)} =$$

$$\frac{685.22}{3803.74} = 0.18$$

$$\text{ROI (before tax)} = 18\%$$

Interpretation of ROI (before tax)

This value indicates the average return from the business on the investment made, but before taxes. A value of 18% is not considered to be very high.

5. Return on Investment (ROI) after tax

$$\text{ROI (after tax)} = \frac{\text{EBIT}(1 - \text{Tax rate})}{\text{Net asset}}$$

$$\text{EBIT} = 685.22$$

$$\text{Tax Rate} = \text{Item L on Table 1} = 32\%$$

$$\text{Net asset} = 3803.74$$

$$\therefore \text{ROI (after tax)} = \frac{685.22(1 - 0.32)}{3803.74} = 0.122$$

$$\text{ROI (after tax before interest)} = 12.25\%$$

Interpretation of ROI (after tax)

This value indicates the average return from the business on the investments made, after taxes, but before interests. In other words, for the money invested the business is paying back at this rate. If one is borrowing at this rate to run business, then the actual return would be at this rate. If one is borrowing at a higher rate of interest, one has to pay for it which effectively brings down the net return.

6. Return on Equity (ROE)

$$\text{ROI (after tax)} = \frac{\text{EBIT}(1 - \text{Tax rate})}{\text{Net asset}}$$

Where,

$$\text{PAT} = 269.72$$

$$\text{NW} = 1345.74$$

$$\therefore \text{ROE} = \frac{269.72}{1345.74} = 0.2$$

$$\text{ROE} = 20\%$$

Interpretation of ROE

ROE indicates how well the firm has used the resources of the owners. It is one of the most important ratios in financial analysis. It is this percentage which money invested in business is fetching, after all deductions. This ratio represents the extent to which satisfactory return are achieved from business. This ratio is of great interest to the present as well as prospective shareholders and also of great concern to management, which has the responsibility of maximizing benefits to the owners.

V. OTHER RATIOS

These are measures that interest stock market traders.

1. Earnings Per Share

$$\text{EPS} = \frac{\text{PAT}}{\text{Number of shares}}$$

where PAT = 269.72 lakhs

No. of shares = 45 lakhs

$$\therefore \text{EPS} = \frac{269.72}{45} = 6$$

$$\therefore \text{EPS} = \text{Rs. } 6$$

Interpretation of EPS

EPS indicates the earnings that each share makes in the business. In this every share having a face value of Rs. 10 (225 lakh / 22.5 lakh) has earned Rs. 6/- per share in the year ending March 2005. In other words, EPS shows the profitability of the company on a per share basis. But it does not reflect how much is retained in the business. EPS is a valuable and widely used ratio which indicates whether the firm's earnings power has changed over years. EPS of a firm is generally compared with industry averages as well as with EPS of competitive industries.

2. Dividends per share (DPS)

$$\text{DPS} = \frac{\text{Dividend distributed to share holders}}{\text{Number of Shares}}$$

where Dividend distributed = Item M on table 1 = Rs. 90 lakhs

No. of shares = 45 lakhs

$$\therefore \text{DPS} = \frac{90}{45} = 2$$

$$\text{DPS} = \text{Rs. } 2$$

Interpretation of DPS

EPS represents all the earnings that is made out of a share. But all these earnings are not transferred to the shareholders. Only a portion of the profit, known as dividends, is distributed among the shareholders. EPS may cause a raise in the market value of the share, but immediate profit sharing is through dividends. Therefore, a large number of present and potential investors may be interested in DPS, rather than EPS.

3. Dividend Payout Ratio

$$\text{DPO} = \frac{\text{DPS}}{\text{EPS}} = \frac{2}{6} = 0.33$$

$$\therefore \text{Dividend Payment ratio} = 33.33\%$$

Interpretation of DPO

This value indicates the percentage of profits distributed to shareholders. Obviously, (1 - DPO) represents the portion retained in business, for further expansion.

4. Price-Earnings (PE) Ratio

$$\text{P/E Ratio} = \frac{\text{Market value of shares}}{\text{EPS}}$$

$$\text{P/E Ratio} = \frac{29.25}{6} = 4.88 \text{ times}$$

$$\text{P/E Ratio} = 4.88 \text{ times}$$

Interpretation of P/E Ratio

The P/E ratio is widely used by stock market analysts to value the firm's performance as expected by investors. P/E ratio reflects investors' expectations about the growth in firm's earnings. But P/E ratio's vary widely according to the industry.

5. Market Value - Book Value Ratio

$$\text{MV / BV Ratio} = \frac{\text{Market Value per Share}}{\text{Book Value per Share}}$$

where

$$\text{Market value per share} = \text{Rs. } 29.25$$

$$\text{Book value per share} = \frac{\text{Net Worth}}{\text{Outstanding shares}} = \frac{1345.74}{4.5}$$

$$\text{BV per share} = 29.905$$

$$\therefore \text{MV/BV Ratio} = \frac{29.25}{29.905} = 0.978$$

$$\therefore \text{MV/BV Ratio} = 0.98$$

Interpretation of MV/BV Ratio

MV/BV Ratio indicates the company's worth in terms of funds invested by shareholders. In other words MV/BV ratio of 0.98 means that the company is worth less than the funds which shareholders have invested.

VI. Evaluation of A Firm's Earnings Power or Return of Net Assets (RONA)

$$\text{RONA} = \frac{\text{Net Sales}}{\text{Net Assets}} \times \frac{\text{GP}}{\text{Net Sales}} \times \frac{\text{EBIT}}{\text{GP}}$$

$$\therefore \text{RONA} = \frac{\text{EBIT}}{\text{Net Assets}}$$

where

$$\text{EBIT} = 685.22$$

$$\text{Net Asset} = 3803.74$$

$$\therefore \text{RONA} = \frac{685.22}{3803.74} = 0.178$$

$$\text{RONA} = 17.8\%$$

Interpretation of RONA

RONA is a measure of the firm's total operating performance. It indicates firm's earnings power and is a product of asset turnover, gross profit margin and operating leverage. RONA can improve with improvement in any of these three values, with the remaining constant.

All firms would like to improve their RONA. However, in practice, price restrictions and competitions put a limit on RONA. Also there is always a trade-off between turnover and GPM, to be considered. When one is increased the other is decreased to improve profit margin, some firms resort to vertical integration for cost reduction synergistic benefits.

COMPARATIVE STATEMENT ANALYSIS

Financial statements such as Balance Sheet and Profit and Loss accounts present actual values of any business. The volume of information is high and comparison is difficult, especially in the absence of some standard values. Therefore, in order to help compare each quantity with the other, a common base is established. For example, in the case of account, Net sales figure is assumed to be common base (and therefore, equal to 100%) and other items are expressed as percentage of total assets or total funds, considered as common base respectively.

The financial statements prepared in terms of common base percentages are called common-size statements. And this kind of analysis is called Comparative Statement Analysis or Vertical Analysis.

A common-size profit and loss account and Balance Sheet is presented below with fictitious values.

Common - size Profit and Loss Statement

	2004	2005
A. Net Sales	100.00	100.00
B. Cost of goods sold	79.00	81.00
C. Gross Profit	21.00	19.00
D. Less : Selling & admin expenses	11.00	10.00
E. Operating Income	10.00	9.00
F. Add : Other Income	0.09	1.00
G. EBIT	10.9	10.00
H. Less : Interest	4.5	4.00
I. PBT	6.40	6.00
J. Taxes	2.20	2.20
K. PAT	4.20	3.80

Common Size Balance Sheet

	2004	2005
A. Current Liabilities	55.0	58.0
B. Long term Liabilities	18.0	15.0
C. Total Liabilities	72.0	75.0
D. Net worth		
Shares Capital	11.0	9.0
Reserve	18.0	17.0
Total	29.0	26.0
E. Total Funds	100.0	100.0
F. Current Assets		
Cash & Bank Balance	5.0	1.0
Trade Debtors	17.0	19.0
Inventory	40.0	42.0
Others	10.0	9.0
Total	72.0	71.0
G. Fixed Assets		
Gross Block	40.0	36.0

Less : Accumulated Depreciation	10.0	9.0
Net Block	30.0	25.0
Other Non-Current assets	1.0	2.0
Total	31.0	27.0
H. Total Assets	100	100

WORKED EXAMPLES

1. The total sales call credit of a firm is Rs. 6,40,000. It has a gross profit margin of 15% and current ratio is 2.5. The firm's current liability is 96,000. Inventories Rs. 48,000 and cash at Rs. 16,000.
- (i) Determine the average inventory to be carried by the firm, if an inventory turnover of 5 times is expected. (Assume a 360 day year).
- (ii) Determine the average collection period if the opening balance of debtors intended to be Rs. 80,000. (Assume a 360 day year). (VTU July 2008)

Solution.

$$\text{Given : Total sales credit} = \text{Rs. } 6,40,000$$

$$\text{Gross profit margin} = 15\%$$

$$\text{Current ratio} = 2.5$$

$$\text{Current liability} = \text{Rs. } 96,000$$

$$\text{Inventories} = \text{Rs. } 48,000$$

$$\text{Cash on hand} = \text{Rs. } 16,000$$

$$\text{Inventory turnover} = 5 \text{ times}$$

$$\text{Opening debtors} = \text{Rs. } 80,000$$

$$\text{No. of working days per year} = 360 \text{ days}$$

(i) To find Average Inventory

$$\text{w.k.t} \quad \text{Inventory turnover} = \frac{\text{cost of goods sold}}{\text{Average Inventory}}$$

$$\text{Average Inventory} = \frac{\text{Cost of goods Sold}}{\text{Inventory turnover}}$$

$$\text{Costs of goods sold} = \text{total sales credit} - \text{profit}$$

$$= 640000 - 640000 \times 0.15$$

$$= 544000$$

$$\text{Average Inventory} = \frac{544000}{5} = 108800$$

(ii) To find Average Collection Period

$$\text{Average collection period} = \frac{360}{\text{Debtors turnover}}$$

$$\text{Debtors turnover} = \frac{\text{Net sales}}{\text{Average debtors}}$$

$$\text{Average Debtors} = \frac{\text{Opening debtors} + \text{closing debtors}}{2}$$

$$\text{Closing debtors} = \text{Current assets} - [\text{Inventories} + \text{cash}]$$

$$\text{Current assets} = 2.5 \times \text{current liabilities}$$

$$= 2.5 \times 96000$$

$$= 240000$$

$$\text{Closing debtors} = 240000 - 48000 - 16000$$

$$= 176000$$

$$\text{Average debtors} = \frac{80000 + 176000}{2}$$

$$= 128000$$

$$\text{Debtors turnover} = \frac{640000}{128000} = 5$$

$$\therefore \text{Average collection period} = \frac{360}{5} = 72 \text{ days.}$$

2. From the following balance sheet of ABC company as on 31.12.2006 work out the various turn over ratios

Liabilities	Amount	Assets	Amount
Share capital	8.1	Goodwill	0.8 L
Capital reserve	2.1 L	Land and building	2 L
Mortgage	4 L	Machinery	5 L
Current liabilities	2 L	Patents	0.2 L
		Debtors	1.5 L
		Cash in hand	2 L
		Bills receivables	0.5 L
		Prepaid expenditures	0.4 L
		Stock of raw material	1 L
		Stock of finished goods	3 L
Total	16.1 L	Total	16.1 L

Adjustments :

$$\text{Gross profit} = 33 \frac{1}{3}\% \text{ of sales}$$

$$\text{Net sales} = 30 \text{ L}$$

$$\text{Opening stock of raw material} = 1.4 \text{ L}$$

$$\text{Opening stock of finished goods} = 3.4 \text{ L}$$

(VTU Jan)

Solution.

To find Turnover Ratios or Activity Ratios

$$(1) \text{ Inventory turnover} = \frac{\text{Cost of goods sold}}{\text{Average inventory}}$$

$$= \frac{\text{Net sales} - \text{Gross profit}}{\frac{\text{Opening stock} + \text{Closing stock}}{2}}$$

$$= \frac{3000000 - 0.3333 \times 3000000}{\frac{140000 + 340000}{2}}$$

$$= \frac{2000000}{240000} = 8.33$$

$$(2) \text{ No. of days, Inventory} = \frac{365}{\text{Inventory Turnover}}$$

$$= \frac{365}{8.33} = 43.81 \text{ days}$$

$$(3) \text{ Debtors turnover} = \frac{\text{Net sales}}{\text{Debtors}}$$

$$= \frac{3000000}{150000} = 20$$

$$(4) \text{ Collection period} = \frac{365}{\text{Debtors Turnover}}$$

$$= \frac{365}{20} = 18.25 \text{ days}$$

$$(5) \text{ Net assets turnover} = \frac{\text{Net Sales}}{\text{Net Assets}}$$

$$\text{Net Assets} = \text{Fixed Assets} + \text{Net Current Assets}$$

$$\text{Net Current Assets} = \text{Current Assets} - \text{Current Liabilities}$$

$$\text{Fixed Assets} = \text{Land \& Building} + \text{Machinery} + \text{Patents}$$

$$= 2,00,000 + 5,00,000 + 20,000$$

$$= 7,20,000.$$

$$\text{Current Assets} = \text{Goodwill} + \text{Cash in Hand} + \text{Prepaid Expenditures}$$

$$= 80,000 + 2,00,000 + 40,000$$

$$= 3,20,000.$$

$$\text{Current Liabilities} = 2,00,000$$

$$\therefore \text{Net Current Assets} = 3,20,000 - 2,00,000$$

$$= 1,20,000.$$

$$\therefore \text{Net Assets Turnover} = \frac{3000000}{720000 + 120000} = 3.57$$

$$(6) \text{ Working Capital Turnover} = \frac{\text{Net Sales}}{\text{Net Current Assets}}$$

$$= \frac{3000000}{120000} = 25.$$

□ □ □

SUMMARY

1. Analysis of Financial statements like Balance sheet and profit & loss account with the help of several mathematical ratios is known as Financial Ratio Analysis.

2. Financial Analysis

This is done to identify the financial strengths and weaknesses of a company associated with any company such as bankers, creditors, shareholders, suppliers etc. would carry out financial analysis.

3. Nature of Ratio Analysis

There are several ratios such as liquidity ratios, leverage ratios, activity ratios, Profitability ratios and other ratios which help in financial analysis.

4. Formulae to calculate different ratios**i) Liquidity ratios**

$$(a) \text{Current Ratio} = \frac{\text{current assets}}{\text{current liabilities}}$$

$$(b) \text{Quick ratio} = \frac{\text{current assets} - \text{inventories}}{\text{current liabilities}}$$

$$(c) \text{Cash ratio} = \frac{\text{cash} + \text{bank balance}}{\text{current liabilities}}$$

$$(d) \text{Internal measure} = \frac{\text{current assets} - \text{Inventory}}{\text{Average daily operating expenses}}$$

ii) Leverage Ratios

$$(a) \text{Debt ratio} = \frac{\text{Total debt}}{\text{Net assets}}$$

$$(b) \text{Debt equity ratio} = \frac{\text{Total debt}}{\text{net worth}}$$

$$(c) \text{Capital equity ratio} = \frac{\text{Capital employed}}{\text{Net worths}}$$

$$(d) \text{Interest Coverage} = \frac{\text{Earnings before interests and taxes (EBIT)}}{\text{Interest}}$$

iii) Activity Ratios

$$(a) \text{Inventory turnover} = \frac{\text{Cost of goods sold}}{\text{Average inventory}}$$

$$(b) \text{No. of days, Inventory} = \frac{365}{\text{Inventory turnover}}$$

$$(c) \text{Debtors turnover} = \frac{\text{Net sales}}{\text{Average Debtors}}$$

$$(d) \text{Average Collection period} = \frac{365}{\text{Debtors turnover}}$$

$$(e) \text{Net assets turnover} = \frac{\text{Net sales}}{\text{Net assets}}$$

iv) Profitability Ratios

$$(a) \text{Gross Profit Margin} = \frac{\text{Net sales} - \text{Cost of goods sold}}{\text{Net sales}}$$

$$(b) \text{Net Profit Margin} = \frac{\text{Profit after tax}}{\text{Net Sales}}$$

$$(c) \text{PAT / EBIT ratio} = \frac{\text{Profit after tax (PAT)}}{\text{Earnings before interest and tax (EBIT)}}$$

$$(d) \text{Return on investment (Roi) (before tax)} = \frac{\text{EBIT}}{\text{Net Assets}}$$

$$(e) \text{Roi (after tax)} = \frac{\text{EBIT} (1 - \text{TAX rates})}{\text{Net asset}}$$

$$(f) \text{Return on equity (ROE)} = \frac{\text{Profit after tax (PAT)}}{\text{Net worth}}$$

v) Other Ratios

$$(a) \text{Earnings per share (EPS)} = \frac{\text{PAT}}{\text{Number of Shares}}$$

$$(b) \text{Dividends per share (DPS)} = \frac{\text{Dividend distributed to share holders}}{\text{Number of shares (EPS)}}$$

$$(c) \text{Dividend Payout ratio} = \frac{\text{Dividend per share (DPS)}}{\text{Earnings per share (EPS)}}$$

$$(d) \text{ Price-Earnings (PE) Ratio} = \frac{\text{Market Value of Shares}}{\text{EPS}}$$

$$(e) \text{ Market value - Book value ratio} = \frac{\text{Market value per share}}{\text{Book value per share}}$$

vi) Evaluation of a firm's earning power or

$$\text{Return on Net assets (RONA)} = \frac{\text{EBIT}}{\text{Net Assets}}$$

5. Comparative Statement Analysis

Balance Sheet and Profit & Loss Statements of any Company present values costs, incomes, assets and liabilities. When all these values are converted relative values compared against a common base, then it is known as Comparative Statement analysis.

EXERCISE PROBLEMS

- Find the P/E ratio of a firm that has a net profit after taxes of Rs. 3,00,000 and 60,000 shares outstanding, selling at a market price of Rs. 10 per share. What rate of return do shareholders expect ?
 - A firm has shareholders equity of Rs. 3,00,000. total assets are 150% of the shareholders equity, while the asset turnover is 5. If the firm has an inventory turnover of 6, determine the amount of inventory.
 - If the sales of a company are Rs. 5,00,000, cost of goods sold is Rs. 2,50,000 and inventory is Rs. 1,00,000 what is its turnover ? Also, calculate the firms gross margin.
 - A company has a cost of Rs. 4,00,000 sales of Rs. 5,00,000 and an asset turnover of 4. What is the rate of return on asset ?
 - A firm has a net profit after taxes of Rs. 2,40,000 and pays a cash dividend of Rs. 90,000 on its 40,000 shares outstanding at a time where the share is selling at Rs. 18. What is the yield and the dividend payout ?
 - A company has profit before interest and taxes of Rs. 1,00,000 interest charges of Rs. 10,000, taxes of Rs. 35,000, total assets of Rs. 6,00,000 and total liabilities of Rs. 3,50,000. What is its Return on equity and Interest Coverage ?
 - A firm possesses the following current assets-cash Rs. 1,50,000, inventories Rs. 6,00,000 and debtors Rs. 5,00,000. If the current ratio for the firm is 2:1, determine its current liabilities. Also calculate the firms Acid-test ratio (Quick ratio). If the cost of goods sold for Rs. 30,00,000 and the company's turnover ratio is 5, determine the opening balance of inventory.
 - If a firm has an inventory of Rs. 2,00,000, debtors of Rs. 1,25,000, inventory turnover of 6, gross profit margin is 10% credit sales as 20% of the total sales, calculate the average collection period.
 - The total sales of a company is Rs. 5,00,000. It has a gross profit margin of 20%. If the company has an average turnover of Rs. 60,000, determine the average inventory.
 - The Balance Sheet and Profit & Loss Account for the year ending 31st march 2005 of two companies X Ltd, and Y Ltd, are given in tables. It is assumed that stocks have increased evenly throughout the year. Compute, compare and Interpret all the financial ratios possible with the given data for the two companies.

Balance Sheet as on 31st March, 2005

	X Ltd.	Y Ltd.
Fixed assets at cost	120000	60000
Less : Provision for depreciation	4000	20000
	<u>80000</u>	<u>40000</u>
Current assets		
Stock	104000	60000
Debtors	44000	40000
Cash	22000	20000
	<u>180000</u>	<u>120000</u>
Less : current liabilities	60000	60000
Net current assets	120000	60000
Net assets	20000	10000
Paid up share capital	190000	90000
Revenue reserve	10000	10000
	<u>200000</u>	<u>100000</u>

Profit and Loss Account
for the year ended 31st March, 2005

	X Ltd.	Y Ltd.
Less : current liabilities	320000	240000
Stock at April 1st, 2004	78000	40000
Add : Purchases	<u>128000</u>	<u>170000</u>
Current assets	306000	210000
Less : Stock at 31st March, 2005	104000	60000
Cost of goods sold	192000	150000
Gross profit	128000	90000
Less General expenses	112000	78000
Net profit for the year	16000	12000
Add : Balance brought forward	6000	10000
	<u>22000</u>	<u>10000</u>
Less : Dividend paid	12000	10000
Balance carried forward		

CHAPTER

8

FINANCIAL AND
PROFIT PLANNING

INTRODUCTION

Any organization should be managed effectively and efficiently so as to be in business first and then to sustain it. In this world of stiff competition in all spheres of activities, it takes a lot of planning, co-ordination and control of efforts to achieve the objectives of the organization.

Planning is one of the most important functions of management, in any industry. Planning has to take place for all major activities like purchase, production, sales, etc.. In any business, **Financial Planning and Profit Planning** (budgeting) plays important roles in deciding the fortunes of the company.

FINANCIAL PLANNING

Financial planning is the process of identifying the firm's investments and financial needs, given its growth objectives. It estimates a firm's growth, performance, investments and requirements of funds during a given period of time, usually three to five years. A financial plan also projects for this period - Profit and Loss Account, Balance Sheet and Fund Flow Statement.

Need for Financial Planning

1. To help have an optimum utilization of the available resources.
2. To prepare for raising of funds by appropriate methods.
3. To formulate programs to provide the most effective cost-volume-profit relationships.
4. To establish and maintain a system of control, governing the allocation and use of funds.
5. To decide what, when, where, why, who and how each financial activity be done.

Steps in Financial Planning

According to Ernest W. Walker and William H. Banhan, there are four steps in Financial Planning:

1. Establishing Objectives

The financial objectives of any business enterprise should be firmly established. Major objectives of all organizations would be to maximize the profits and/ or minimize the losses. Both long-term and short-term objectives should be clearly specified.

2. Policy Formulation

Financial policies are guides to all action which deals with procuring, administering and disbursing the funds of business. These policies may be classified into several broad categories:

- Policies governing the amount of capital required to achieve financial objectives.
- Policies which determine the control by the parties who furnish the capital.
- Policies which act as a guide in the use of debt or equity capital.
- Policies which guide the management in the selection of sources of funds.
- Policies which govern credit and collection activities of the business enterprise.

3. Forecasting

Forecasting is the estimation of future events. In business, forecasting is done to estimate future sales, so that production needs and consequently financial needs can be determined. Financial forecasting involves the estimation of all financial needs for planning periods.

4. Formulation of procedures

Procedures have to be formulated to practice all the policies already stated. In other words, for the financial policies to be executed properly, detailed procedures have to be evolved for better organization.

PROFIT PLANNING

Profit Planning is the preparation of a detailed plan of action for the flow of finances during a period of one year or less. Profit planning is popularly known as Budgeting while a profit plan is called a Budget.

Profit Plan or Budget

A budget is a written plan or programme of the future activities expressed in monetary terms. Examples for budgets are sales budget, capital budget, production budget etc., Budgets set out the targets to be achieved in their respective areas. Budgets also specifies the expenditure within which the targets have to be achieved. Therefore budget is a managerial tool for planning, programming and controlling business activities. It provides pre-determined standards of performance for all the efforts and activities of the business.

Budgets are prepared with the help of scientific estimation and also taking previous statistical data. But before fixing the budget various departmental heads should be consulted for finalizing the figures. Budgets should not be too rigid and should be flexible enough to meet the changes in business conditions.

Budgets should show definite proposals and clear-cut estimates of business activities. For example, a sales budget should clearly show the proposed volume and the value of sales for definite period. Similarly, a production budget should give the estimate of types and quantity of products to be manufactured for a definite budgetary period.

Uses of budget

- It helps in planning
- It helps in programming and controlling business activities.
- It is a written plan of action.
- It helps in anticipation and control of financial requirements of different branches of business.
- It gives an overall view of the business in terms of sales, production and expenditure.

BUDGETING

Budgeting is the art of budget making. It is the process of preparing a budget. In other words Budgeting implies forecasting and pre-planning for the budget period, based upon past experience, statistical data and present conditions, budgeting always uses the accounting language.

OBJECTIVES OF PROFIT PLANNING (OR BUDGETING)

The major objectives of Budgeting are:

- To establish and state the goals and objectives clearly to avoid confusion and to facilitate their attainability.
- To communicate clearly the expectations of the management to everyone in the organization so that they are understood, supported and implemented.
- To provide a detailed plan of action for reducing uncertainty and for the proper co-ordination and uniform direction of individual and group efforts.
- To achieve optimum use of all resources Viz., men, material, machinery and money.
- To employ a systematic method of measuring and controlling the performance of individuals and that of teams and to take corrective action.
- To provide for continuity and flexibility in the event of uncertainty and sudden changes.

ESSENTIALS OF PROFIT PLANNING

The following are some of the important essentials or requirements for the success of profit planning or budgeting:

1. Top Management Support

Top Management (GM and above) should fully understand and support the budgeting system so that the lower cadres follow them. Any business culture usually trickles down from the top. Top Management should necessarily:

- (a) understand the nature and characteristics of budgeting;
 (b) be willing to devote the time and efforts to make it operative;
 (c) support the program in all its consequences.

2. Have clear & Realistic Goals

Budgeting is an important tool to achieve goals and objectives. This tool will work if goals are not clear and realistic. The efforts of employees and management get wasted, if there is a lack of clarity in goals.

3. Assignment of Authority & Responsibility

Authorities and responsibilities of each manager should be clearly identified established for the proper implementation of budget plans.

4. Creation of Responsibility Centres

A large firm may be divided into departments, divisions or various units, each headed by a manager. These managers are given specific authorities and are held responsible for their decision affecting those activities. In other words, each manager is accountable for the centre he heads.

5. Cost Centres

Some units within an organization are treated only as cost centres. The managers of these centers continuously strive to reduce costs. They will generally be looking at estimated cash flow diagrams, production centres, purchase departments, maintenance and all considered cost centres.

6. Profit Centres

Profit Centres are those departments which bring in revenues and profits to the organization. Sales or marketing department is the best example. Managers of these centres continuously strive to improve profits. They will generally be looking at revenue dominated Cash Flow diagrams.

7. Investment Centre

This is a centre where both cost and profit are looked into simultaneously. The manager continuously strives to reduce costs and improve profits, as they are dependent on each other. Managers of small businesses or heads of independent large organization may be evaluated on both costs as well as profits.

8. Adaptation of the Accounting System

Budgeting is generally based on the data generated by the accounting system. The accounting system needs to be well adapted to make budgeting and budgetary control a success.

9. Full Participation

No system can work if the people who work for it are not committed. Full participation of managers and their subordinates at all levels should be sought in developing the budgeting system.

10. Effective Communication

Communication is fundamental and vital to all decisions. Budgeting is nothing but a way of communicating plans, objectives and goals to various responsibility Centres. The success of budgeting lies in the practice of proper communication at all levels of the organization.

BUDGET ADMINISTRATION

The process of preparation of Budgets becomes more complicated in case of large organizations. There has to be a systematic way in which budgets are prepared, presented and presided upon. Usually members of top management form a Budget Committee under the stewardship of a Budget director come together to prepare a detail budget and present it in the Budget Manual. It must be noted here that preparation of a budget is a line function whereas the organization and administration of budgeting is a staff function.

Following are the responsibilities and salient features of Budget Committee, Budget director and budget Manual.

Budget Committee

1. The members of a budget committee usually consist of top executives from production, sales and finance departments.
2. The budget director is the overall in charge of the budget committee.
3. The budget committee is basically a management committee which attempts to bring together activities of all departments in a co-ordinated way.
4. The Committee is also responsible for effectively controlling the activities of all departments to ensure respect and implementation of budget.
5. The budget committee ensures that all departments prepare budgets, provides them technical advice whenever needed and reviews individual budget estimates.
6. It is the budget committee which reviews departmental budgets, suggests modifications and finally approves them.
7. The committee also ensures that the budget goals of the department are not in conflict with the objectives of the organization.
8. The committee co-ordinates the budgeting activities of various departments and pulls-up departments for any deviation.

9. The budget committee also enforces control by scrutinizing the budget reports and determines the prizes and penalties for favourable and unfavourable results.

Budget Director

1. The Budget Director may be the Chief Financial Officer (or Director of Finance General Manager Of Finance) or the Controller or the Chief Accountant.
2. The overall responsibility for the functions of the budget committee lies on the Budget Director.
3. He is responsible for drawing up a detailed time-table for the preparation of budgets and making efforts to synchronize all individual budgets into a Master Budget.
4. The Budget Director should be objective and unbiased in his approach.
5. He is a staff expert who provides technical assistance to line executives, He should also listen to them and should not impose his ideas on them.

Budget Manual

1. A budget manual expresses in writing the objectives, goals, organizational structure, authority and responsibility all with respect to the budget.
2. The budget manual is a written set of instructions and relevant information that serve as a rule book and a reference for the implementation of a budget program.
3. With respect to budget activities, the manual informs when, what, where, why, and how of all activities.

TYPES OF BUDGETS

The different types of budgets can be classified as follows:

1. Fixed Budget
2. Flexible Budget
3. Functional Budgets
 - (a) Sales Budget
 - (b) Production Budget
 - (c) Financial Budget
 - (d) Material Purchase Budget
 - (e) Labour Budget
 - (f) Selling and Distribution Budget
4. Master Budget

S P
VAP
SD

1. Fixed Budget

A Fixed budget or Static budget gives a solitary plan of action. It is prepared on the basis of certain fixed pre-determined level of activity. In the fixed budget monetary allowances are fixed and do not vary with any changes in the conditions. Fixed budgets are suitable only for such enterprises whose quality and quantity of production as well as sales during the budget period can be pre-determined with reasonable accuracy. E.g., Government Budgets, R & D budgets, School Budgets.

2. Flexible Budget or Variable Budget

A flexible budget makes provisions in advance for variations in production and expenditures in accordance with variation in sales. A flexible budget reflects the fluctuating characteristics of the costs. Flexible costs segregate the cost into two parts:

- i. Those costs which vary with passage of time
e.g., Salaries, depreciations, interests etc.,
- ii. Those costs which vary with volume of production
e.g., Raw material, direct labour, transportation etc.,

Unlike fixed budget, a flexible budget assumes that the cost will vary with the volume of activity. The flexible budget makes allowances for different levels of operating condition in a factory, say budget at 50%, 60%, 75%, 90%, 100% level of operating conditions.

3. Functional Budget or Operating Budget

A functional budget is one which relates to any of the functions of the organization such as sales, production, materials, expenditure etc.,

(a) Sales Budget

This budget is prepared prior to all other budgets and all other budgets are prepared based on sales budget. The sales budget is a plan of future sales. The quantity of production is naturally determined by the estimates of sales in future. Therefore, the sales budget is prepared first. This budget gives the income from the products likely to be sold and the expenditure involved in sales.

All activities in an industry revolve around the likely volume of sales and hence all other budgets follow sales budget. It is for this reason that, the sales budget has to be prepared with as much care and accuracy as possible. If not, it leads to a chain of inaccurate estimates and likely losses.

Sales budget is prepared on the basis of past sales and the reports received from every salesman, who will indicate the approximate quantity of products likely to be sold in his area, forecast of the market and general business condition. Sales budget

takes into account the expenses on advertisement, analysis, salesmen's salaries, office expenses etc., for the budget period.

Figure shows an example of sales budget which gives the actual sales volume of the previous quarter along with the estimate of likely sales for the next quarter (3 months). This gives the management a knowledge of how the salesmen are doing and can prepare any corrective actions, if necessary.

SALES BUDGET											
				Third Quarter Year - 2001							
Name of the company		Sales Area		Sales Maxi							
Product Model		Previously Quarterly		Quarterly Estimates		Monthly Estimate					
				June		July		August			
		Volume	Value	Volume	Value	Volume	Value	Volume	Value	Volume	Value
X											
Y											
Z											

(b) Production Budget

The production budget broadly shows the estimated quantities of the products to be manufactured for a particular future period eg. For the next month, for the next quarter.

It basically shows the schedule of production to meet the demand in sales. The production budget is prepared based on the sales budget and the producing capacity of the plant. It is usually the production manager who prepares the production budget.

A comparison between the production budget estimates and actual production rate gives the management an idea about the relative performance of the production department. If the actual production falls short of the budgeted production, the management finds out the reasons, fixes the responsibilities for shortfalls and takes corrective actions.

development of a Production Budget

The production budget of an industry is developed based on the following factors.

i. Sales Budgets

The quantity of production always depends on the likely future sales. And all activities of production revolve around the quantity to be produced. Therefore the sales budget is the principle basis for the production budget. But the production budget may also include for a extra production to take care of wastage, damage, testing and inspection, complements etc.,

ii. Production capacity

Sometimes the capacity of the plant to produce may be less than the sales forecast. In such cases, the excess demand may be met through sub-contracting. Sometimes the capacity of the plant could be more than the sales forecast. In such cases, the options are to produce as much that is required or produce more upto the plants capacity and try to sell by giving off-season discounts, complementary products etc., The production budget has to have provisions for all these because one of the principle objectives of any budget is to make optimum use of all resources.

iii. Seasonal Fluctuations in Demand

There could always be seasonal and unpredictable fluctuations in sales demand in such a case the sales department and the production department should come together and devise methods for optimum results. In such cases either the capacity is modified or the sales is modified. Some methods of modifying capacity are:

- Vary the work force. In other words hire more workers when demand is high and fire them when demand is low.
- Vary the working hours. In other words, the constant work force works overtime during high demand and idles during low demand.
- Vary the inventory levels. In other words a constant work force produces a constant production rate and the inventory is built up during low demand periods to take care of higher demands later.

The production budget should be flexible enough to allow for all such provisions since the main aim of any organization is to maximize the profit & minimize the losses.

iv. Inventory & Safety stock Requirements

Sometimes it is the business policy of any organization to have a surplus of production either to have as inventory or as safety stocks. Production budget should take care of these needs.

v. Economic Lot Size (or EOQ)

The production budget should always be planned so to carry out production in certain lots known as Economic lot Size. This is done in order to keep the production cost to a minimum. For example a batch of 100 products produced at a time could bring down the overall costs and thus the cost per product.

vi. Availability of Raw Material and Labour

Quality and quantity of goods produced depends upon the availability of raw material and labour. Thus, the production budget should also consider this factor before planning its production.

Figure gives an example of Production budget.

PRODUCTION BUDGET						
Name of the company.....	For the year.....					
	Units of output					
	Jan	Feb	March	Quarter 2	Quarter 3	Quarter 4
Product X						
Product Y						
Product Z						
Total of X + Y + Z						

(c) Financial Budget

Figure: Production budget

Financial budget gives the estimates of expected income and expenditure. The financial budget forecasts the profit and loss statement and the financial position of the concern at the end of the budget period. The budget is prepared for the use of top management to know the profitability of a budgetary programme.

In view of this budget, the management has to arrange for cash or credit for meeting the production and sales requirements.

(d) Material Purchase Budget

Material and purchase budget gives the estimates of quantity, cost and timing of different material, parts, tools and supplies required as per the production budget.

(e) Labour Budget

It indicates the quality and quantity of labour (skilled, unskilled, semiskilled etc.,) requirements and their costs of wages, maintenance, welfare etc.,

(f) Selling and Distribution Budget

Once the goods are produced as per the sales and production budgets, distribution channels take over. Selling and distribution budget estimates the selling and distribution costs of the products.

Master Budget

Any business consists of a number of activities. Separate budgets are prepared normally for different activities which are then co-ordinated into one 'Master Budget'.

Master budget is the budget which gives an overall idea of the company activities for a budgeted period. Thus it is prepared after the preparation of all the functional budgets. A master budget emphasizes the relationship between best outputs and optimum level of inputs.

A master budget is an integration of the following types of budgets.

- (a) Sales budget
- (b) Production budget
- (c) Financial budget
- (d) Material purchase budget
- (e) Labour budget
- (f) Selling and distribution budget

Based on the master budget, top executives control the production, financial and other related activities in the organization. A master budget is generally used as a management tool to project balance sheet and profit and loss statement to show the anticipated financial position of the company, at the end of the financial year.

ADVANTAGES OF BUDGETING

1. Budgetary control ensures tight and professional performances by the employees.
2. It ensures co-ordination and harmony of all departments.
3. Budgeting ensures that the organization plans for its own future performance. It then, through the control process tries to ensure adherence to the plans.
4. It targets, goals and policies of a business enterprise are clearly defined.
5. It provides information to top management of the time - to - time progress made towards achieving goals of the organization.

6. It facilitates financial control by estimating the capital required to produce a product, in advance.
7. If budgeting estimates the sales and production targets fairly accurately, man power requirement can also be planned accurately. This can lead to job security among current employees which helps in building motivation levels.
8. Budgetary control serves as basis for determining the efficiency of various departments and also the efficiency of the entire organization.

DANGERS OF BUDGETING

1. Budgetary control requires forecasting which is prone to errors.
2. Frequent revisions of budget and the activities of the company thereof can be a costly affair.
3. Conflicts may arise between departmental heads because of the tendency of each of one of them is to get a larger share of budgetary allocation.
4. If budget estimates are not fair to the capabilities of all departments, it is bound to encounter trouble.
5. Budgeting is difficult to apply in the following cases:
 - (a) when the products manufactured are subjected to rapid changes
 - (b) when the company deals with job orders (made to order)
 - (c) when the enterprise is subjected to uncertain market conditions e.g. Recession in industry, U.S. economy slow down, technological break through etc.,

Benchmarking of Manufacturing Operations

Benchmarking is a systematic method by which organizations can measure themselves against the best in the industry. In other words, benchmarking is the search for best practices, which could be prevalent in other organizations, and incorporating the same into its own operations. Benchmarking helps a company therefore to learn the strengths and weaknesses vis-a-vis the industrial leaders, so as to move towards a superior performance.

Benchmarking can thus be defined in one sentence as, "Measuring our performance against that of best-in-class companies, determining how the best-in-class achieve those performance levels and using the information as a basis for our own company's targets, strategies and operations".

The essence of benchmarking is not only about reaching out to match the best-in-class industry but also trying to surpass it. It is about 'moving from where we are to where we want to be'.

Although Benchmarking existed as a concept since early 1800's, modern benchmarking was initiated by Xerox in the 1980's. They studied their competitors in detail and identified many of their own shortfalls with respect to production costs, rejection rate, product lead times, supplier efficiency, inventory costs etc., as compared to other organizations. This led to a re-haul of Xerox and it soon became the best-in-class.

e.g. 1 A car manufacturer, for e.g., TATA Motors can benchmark TOYOTA to improve its operations and sell on a world wide scale. Benchmarking of manufacturing operations would be in the areas of:

1. Forecasting techniques used.
2. Manufacturing operations and processes used.
3. Materials technology prevalent.
4. Inventory control techniques employed.
5. Extent of quality control.
6. Testing procedures.
7. Scheduling practices etc.

e.g. 2 An engineering college can also benchmark another top engineering college and compare itself in the following areas so as to attempt to become the best in the State.

1. Input student quality.
2. Infrastructure.
3. Placement opportunities.
4. Faculty competence.
5. Value-added courses.
6. Projects and R & D activities.
7. Scope for cultural activities and Sports etc.

The Seven-Step Benchmarking process

Benchmarking is an ongoing process that requires data gathering, goal-setting and analysis. These are accomplished by a seven step model. The seven steps are:

Step 1: Identify what to benchmark. For e.g., performance, process, strategies, brand image or simply profit.

Step 2: Determine what to measure. For e.g., various parameters within the benchmarked area.

Step 3: Identify who to benchmark. For e.g., which company, which competitor. Benchmarking in one area is usually only one company.

Step 4: Collect all relevant data.

Step 5: Analyze data and determine the existing gap. In other words where and how much are we falling short.

Step 6: Set goals and develop action plan.

Step 7: Monitor the process and generate feedback.

WORKED EXAMPLE

- For Philips India, prepare a production budget for 3 months ending 31.3.2004 for four products on the basis of following information.

Product	Estimated Stock on 01.01.2004	Estimated Sales during Jan - March 2004	Desired closing stock on 31.03.2004
A	20,000	1,00,000	30,000
B	30,000	1,50,000	50,000
C	40,000	1,30,000	30,000
D	30,000	1,20,000	20,000

If the cost of product is Rs. 150 per unit for A and B, mention the production units and costs during the three months period.

Also cost of product is Rs. 200 per unit for product C and D, mention the production units and costs during the three month period.

Solution.

Considering the data given in the problem, the production budget for 3 months can be prepared as follows :

Product	Production Budget		
	Name of the company = XYZ	Period : 3 months ending 31.03.2004	Units of output
	Estimated stock on 01.01.04	Production in Quarter 1 Jan-March '04	Desired closing stock on 31.03.04
A	20,000	110000	30,000
B	30,000	170000	50,000
C	40,000	120000	30,000
D	30,000	110000	20,000

Specimen Calculation

Product A :

No. of units of product A to be produced

$$= \text{Expected sales} + \text{Desired closing Stock} - \text{Available stock}$$

$$= 100000 + 30000 - 20000$$

$$= 110000$$

□ □ □

SUMMARY**1. Financial Planning**

It is the process of identifying a company's financial needs for a certain planning horizon for its successful operation. A financial plan also projects various financial statements.

2. Steps in Financial Planning

- Establishing of Objectives
- Policy Formulation
- Forecasting
- Formulation of Procedures

3. Profit Planning (Budgeting)

It is the preparation of a detailed plan of action for the flow of finances for a period of one year or less.

4. Objectives of Profit Planning

- to clearly establish goals and objectives
- ensure that management's expectations are understood by everyone
- to provide a detailed plan of action
- to achieve optimum use of all resources
- to measure all performances systematically
- to provide for continuity and flexibility

5. Essentials of profit planning

- Top management support should always be there
- Have clear and realistic goals.
- Authorities and responsibilities to be clearly delegated
- People and departments to be appropriately made responsible
- Cost centres to be identified
- Profit centres to be identified
- Investment centers to be identified
- Good accounting system to be used
- Participation of all employees to be ensured
- Practice of effective communication

6. Budget Administration

This is carried out by forming a budget committee under the leadership of a budget director. A budget manual is eventually prepared which expresses in writing the objectives, goals, organizational structure, authority and responsibility with respect to the budget.

7. Types of budgets

Fixed, flexible, functional and master budgets are different types of budgets. Among these functional budgets are several types such as sales budget, production budget, financial budget and so on.

8. Sales Budget

This budget is prepared prior to all budgets. The sales budget is plan of future sales. All activities in an industry revolve around the likely volume of sales and hence all other budgets follow sales budgets. Figure shows an example of a sales budget which gives the actual sales volume of the previous quarter along with the estimate of likely sales for the next quarter.

9. Production Budget And Its Development

The production budget shows the estimated quantity of products to be manufactured for a particular future period such as the coming quarter etc. It basically shows the schedule of production for various periods. The production budget is prepared based on the sales budget and the producing capacity of the plant. See figure for an example of production budget.

A production budget is developed based on the following factors:

Sales Budget : Sales budget determines the demand in sales and production budget depends on the likely demand.

Production Capacity : It may be less or more than the likely sales demand. If it is less, excess demand could be met by sub-contracting. If it is more, produce more and try to sell by giving discounts or by other methods.

Seasonal Fluctuations : Seasonal fluctuation can be offset by modifying either capacity or sales. One can modify capacity by three well known strategies.

- Hire and fire employees when demand is high and slow respectively
- Have a constant work force working less or more as per demand
- Produce a constant quantity and build up inventory for high seasons.

Inventory and Safety Stock : Requirements should be met by production budget.

Economic Lot Size : Production budget should be planned as per lot size (Economic order quantities) which consumes least expenditure.

10. Master Budget

Master budget is a budget which gives an overall idea of the company activities for the budgeted period.

It is an integration of all the functional budgets like Sales budget, Production budget, Financial budget, material Purchase budget, Labour budget and Selling and distribution budget.

11. Advantages of budgeting

- ensures better performance by employees
- ensures co-ordination & harmony
- ensures better planning and control
- goals and targets are clearly defined
- information on progress provided to management
- facilitates better financial control
- estimates labour and raw material needs
- prepares a basis for efficiency measurement

12. Dangers of Budgeting

- Budgets uses forecasts which are error - prone
- Budget administration adds costs to the company
- Conflicts may arise among departments
- To estimate exact capability of each dept. is difficult
- Budgeting is difficult in cases of companies making diverse products, jobbing industries and products subjected to rapid changes.

13. Benchmarking

- identifying the best-in-class industries.
- study their processes and understand their formula for success.
- Compare our processes and performance against the best-in-class.
- identify areas for improvement and adopt them.
- match the best-in-class or even surpass them.

APPENDIX

ENGINEERING ECONOMICS

IA Marks: 25

Exam Hours : 03

Exam Marks:100

PART - A

Sub Code: 06 ME55
Hrs/Week: 04
Total Hrs: 52

Unit - 1

Introduction: Engineering Decision - Makers, Engineering and Economics, problem solving and Decision making, Intuition and Analysis. Tactics and Strategy, Engineering Economic Decision Maze, Law of demand and supply, Law of returns. Interest and interest factors, Interest rate, Simple interest, Compound interest, Cash - flow diagrams, personnel loans and EMI Payment Exercises and Discussion - 08 Hrs.

Unit - 2

Present Worth Comparisons: Conditions for present worth comparisons, Basic present worth comparison, Present worth equivalence, Net present worth, Assets with unequal lives, infinite lives, Future worth comparison, pay back comparison, Exercises, Discussions and problems - 06 Hrs.

Unit - 3

Equivalent Annual Worth Comparisons: Equivalent Annual Worth Comparison Methods, Situations for Equivalent Annual worth Comparisons, Consideration of asset life, Comparison of assets with equal and unequal lives, Use of sinking fund method, Annuity contract for guaranteed income, Exercises, Problems - 06 Hrs.

Unit - 4

Rate of Returns Calculations and Depreciation: Rate of return, Minimum acceptable rate of return, IRR, IRR misconceptions, Cost of capital concepts, Causes of Depreciation, Basic methods of computing depreciation charges, Tax concepts, corporate income tax - 06 Hrs.

PART - B

Unit - 5

Estimating and Costing: Components of costs such as Direct Material Costs, Direct Labor Costs, Fixed Over- Heads, Factory cost, Administrative Over- Heads, First cost, Marginal cost, Selling price, Estimation for simple components 07 Hrs.

Unit - 6

Introduction, Scope of finance, Finance functions: Statements of Financial information, introduction, Source of financial information, Financial statements, Balance Sheet, Profit and Loss account, relation between Balance Sheet and Profit and Loss account - 06 Hrs.

Unit-7 Financial Ratio Analysis: Introduction, Nature of ratio analysis, Liquidity ratios, Leverage ratios, Activity ratios, Profitability ratios, Evaluation of a firm's earning power, Comparative statements analysis - 06 Hrs.

Unit-8 Financial and Profit Planning: Introduction, Financial planning, Profit planning Objectives of profit planning, Essentials of profit planning, Budget administration type of budgets, preparation of budgets, advantages, problems and dangers of budgeting, Introduction to Bench Marking of Manufacturing Operation-07 Hrs

TEXT BOOKS:

- (1) Engineering Economy, Riggs J.L.. McGraw Hill. 2002.
- (2) Engineering Economy, Thuesen H.G PHI. 2002.

REFERENCE BOOKS:

- (1) Engineering Economy, Tarachand. 2000
- (2) Industrial Engineering and Management, O.P Khanna, Dhanpat Rai & Sons.2000
- (3) Financial Management, I M Panday, Vikas Publishing House 2002
- (4) Engineering Economy, Paul Degarmo, Macmillan, Co. 2001

SCHEME OF EXAMINATION

One question to be set from each chapter. Students have to answer any FIVE full questions out of EIGHT questions, choosing at least TWO questions from Part A and TWO questions from Part B.

INDEX

A	Accounting Process - 183 Accounting Terminology - 186 Acid-test Ratio - 210 Activity Ratios - 215 Analysis - 4 Annuity Contract - 128 Assets - 186 Assets having infinite lives - 85 Assets with unequal lives - 73
B	Balance Sheet - 195 Basic PW Comparison - 136 Benchmarking - 262 Book Value - 136 Book-Keeping - 188 Budget administration - 238 Budget Committee - 238 Budget Director - 239 Budget Manual - 239 Budgeting - 236
C	Capital- Equity ratio - 24 Capitalized Cost - 85 Cash Flow Diagram - 17 Cash Ratio - 211 Collection Period - 216 Common Multiple Method - 73 Comparative Statement analysis - 223 Compound Interest - 16 Condition for PW comparison - 72 Copsideration of asset life - 126
D	Continuous Compounding - 38 Corporate Income Tax - 153 Cost Accounting - 185 Costing - 162,167 Cost-of-capital Concepts - 133 Current Ratio - 211
E	Debtors Turnover - 216 Debt-equity Ratio - 213 Debt-ratio - 213 Decision Maze - 6 Decision-making - 1,3 Declining-Balance Depreciation - 136 Depreciation Fund - 136 Depreciation Amount - 136 Depreciation - 134 – Depletion - 135 – Monetary - 135 – Physical - 134 – Technological - 135 Direct Tax - 152 Dividend Payout ratio - 221 Double declining balance method - 38,147 Double-entry System - 189
F	EMI Calculation - 27 Earnings per share - 221 Economic lot size (EOQ) - 242 Effective Rate of Interest - 19 Elasticity of Demand - 8 Elasticity of Supply - 11