



Zewail City of Science, Technology, and Innovation
University of Science and Technology
Communications and Information Engineering

“Report 2”

Math 404

Prepared By

“Hajar Al-Shafai 201900537”

Supervised By

“Ahmed Abdelsamea”

Problem 1:

$$\text{Min } f = -1.1x_1 - x_2$$

$$\text{s.t.} \quad x_1 + x_2 + x_3 = 6$$
$$x_1, \quad x_2, \quad x_3 \geq 0$$

Solution:**Merhotra:**

Starting point x:

3
3
3

Starting point y:

-0.7000

Starting point s:

0.5000
0.6000
1.6000

Optimal solution x:

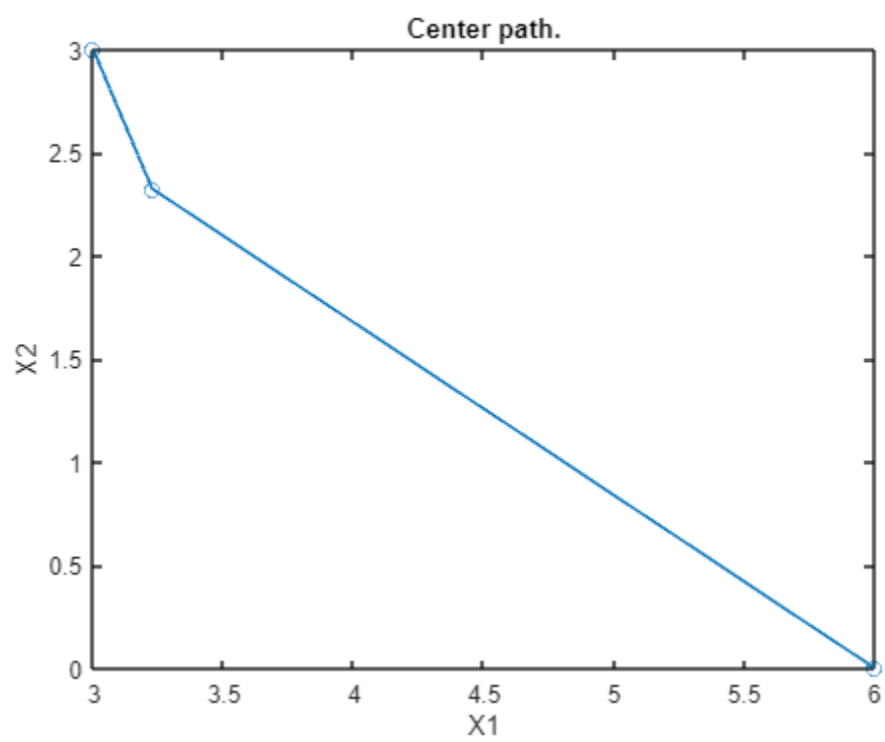
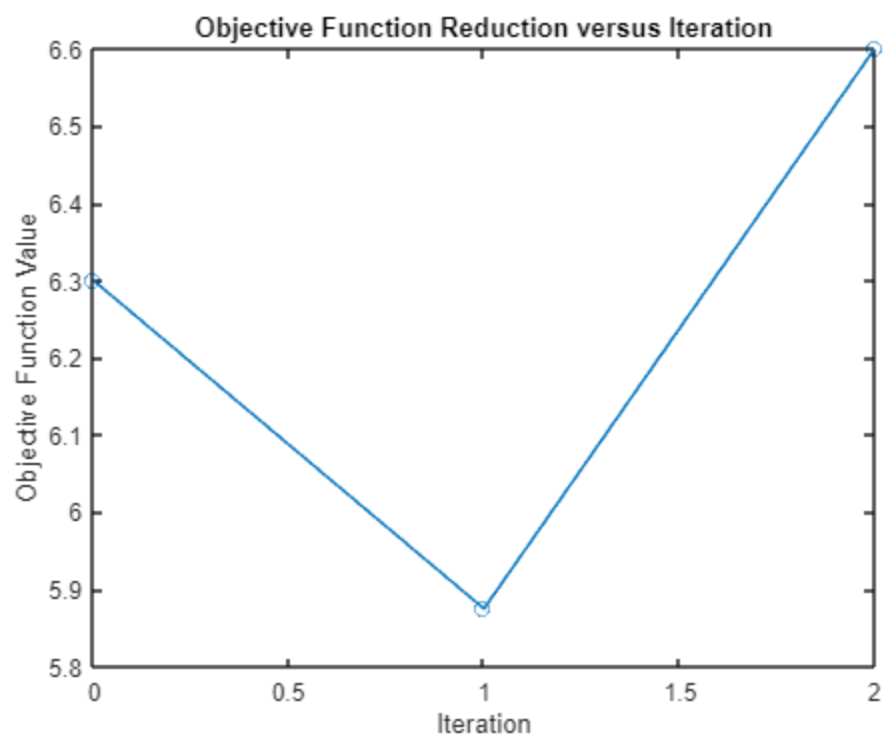
3.0000	3.2257	6.0000
3.0000	2.3261	0
3.0000	0.4481	0

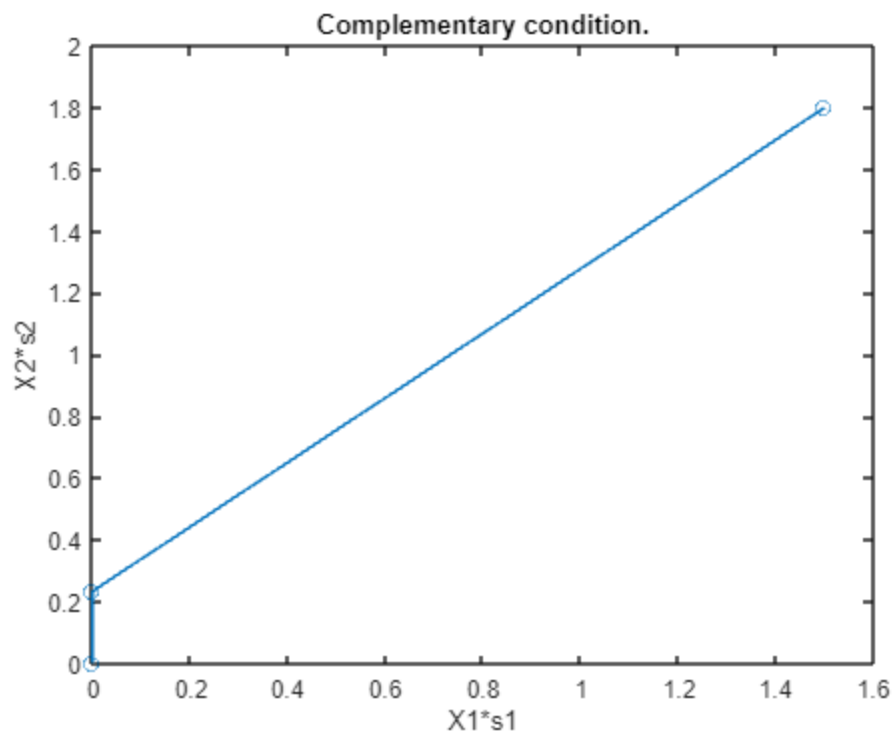
Optimal slack variables s:

0.5000	0	0.0000
0.6000	0.1000	0.1000
1.6000	1.1000	1.1000

Number of iterations :

2





Fixed:

Starting point x:

3
3
3

Starting point y:

-0.7000

Starting point s:

0.5000
0.6000
1.6000

Optimal solution x:

Columns 1 through 18

3.0000	2.8981	2.8827	2.9521	3.0020	3.2573	3.4720	3.7233	4.0055	4.3058	4.6042	4.8803	5.1204	5.3193	5.4786	5.6035	5.6998	5.7735
3.0000	2.6650	2.4971	2.4056	2.3346	2.2491	2.1282	1.9618	1.7509	1.5080	1.2546	1.0132	0.7995	0.6205	0.4761	0.3626	0.2747	0.2074
3.0000	1.9369	1.3702	1.0173	0.7709	0.5873	0.4467	0.3384	0.2553	0.1920	0.1441	0.1080	0.0809	0.0606	0.0454	0.0340	0.0255	0.0191

Columns 19 through 36

5.8294	5.8717	5.9036	5.9276	5.9457	5.9592	5.9694	5.9771	5.9828	5.9871	5.9903	5.9927	5.9946	5.9959	5.9969	5.9977	5.9983	5.9987
0.1563	0.1175	0.0883	0.0663	0.0498	0.0374	0.0280	0.0210	0.0158	0.0118	0.0089	0.0067	0.0050	0.0037	0.0028	0.0021	0.0016	0.0012
0.0143	0.0108	0.0081	0.0060	0.0045	0.0034	0.0026	0.0019	0.0014	0.0011	0.0008	0.0006	0.0005	0.0003	0.0003	0.0002	0.0001	0.0001

Columns 37 through 54

5.9990	5.9993	5.9995	5.9996	5.9997	5.9998	5.9998	5.9999	5.9999	5.9999	5.9999	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000
0.0009	0.0007	0.0005	0.0004	0.0003	0.0002	0.0002	0.0001	0.0001	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0001	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Columns 55 through 57

6.0000	6.0000	6.0000
0.0000	0.0000	0.0000
0.0000	0.0000	0.0000

Optimal slack variables s:

Columns 1 through 18

0.5000	0.4920	0.4236	0.3351	0.2515	0.1827	0.1301	0.0914	0.0638	0.0446	0.0314	0.0224	0.0161	0.0118	0.0087	0.0064	0.0048	0.0035
0.6000	0.5920	0.5236	0.4351	0.3515	0.2827	0.2301	0.1914	0.1638	0.1446	0.1314	0.1224	0.1161	0.1118	0.1087	0.1064	0.1048	0.1035
1.6000	1.5920	1.5236	1.4351	1.3515	1.2827	1.2301	1.1914	1.1638	1.1446	1.1314	1.1224	1.1161	1.1118	1.1087	1.1064	1.1048	1.1035

Columns 19 through 36

0.0026	0.0020	0.0015	0.0011	0.0008	0.0006	0.0005	0.0004	0.0003	0.0002	0.0001	0.0001	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000
0.1026	0.1020	0.1015	0.1011	0.1008	0.1006	0.1005	0.1004	0.1003	0.1002	0.1001	0.1001	0.1001	0.1001	0.1000	0.1000	0.1000	0.1000
1.1026	1.1020	1.1015	1.1011	1.1008	1.1006	1.1005	1.1004	1.1003	1.1002	1.1001	1.1001	1.1001	1.1001	1.1000	1.1000	1.1000	1.1000

Columns 37 through 54

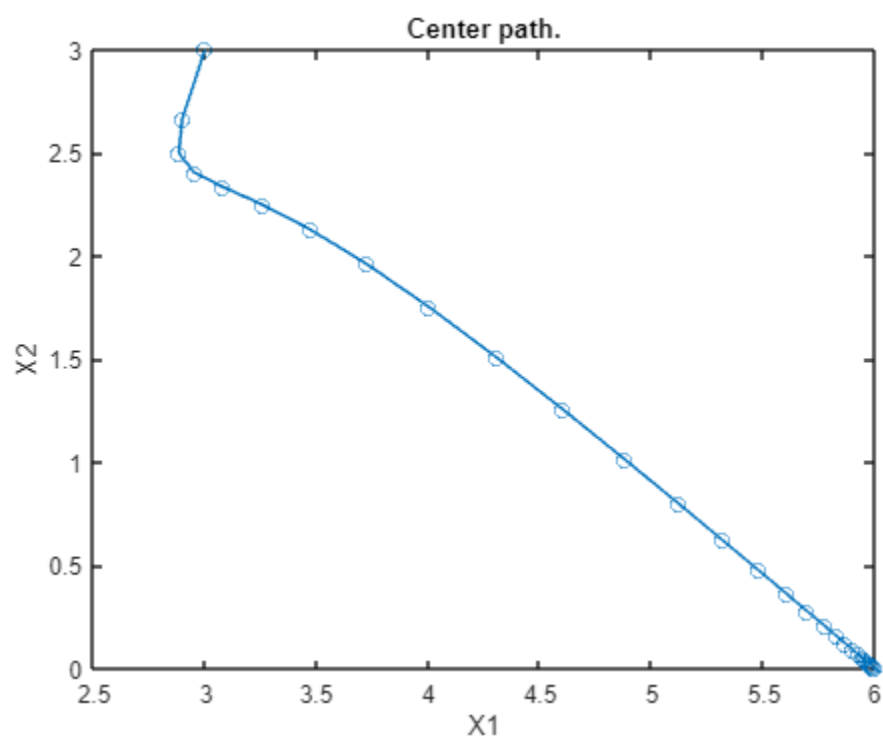
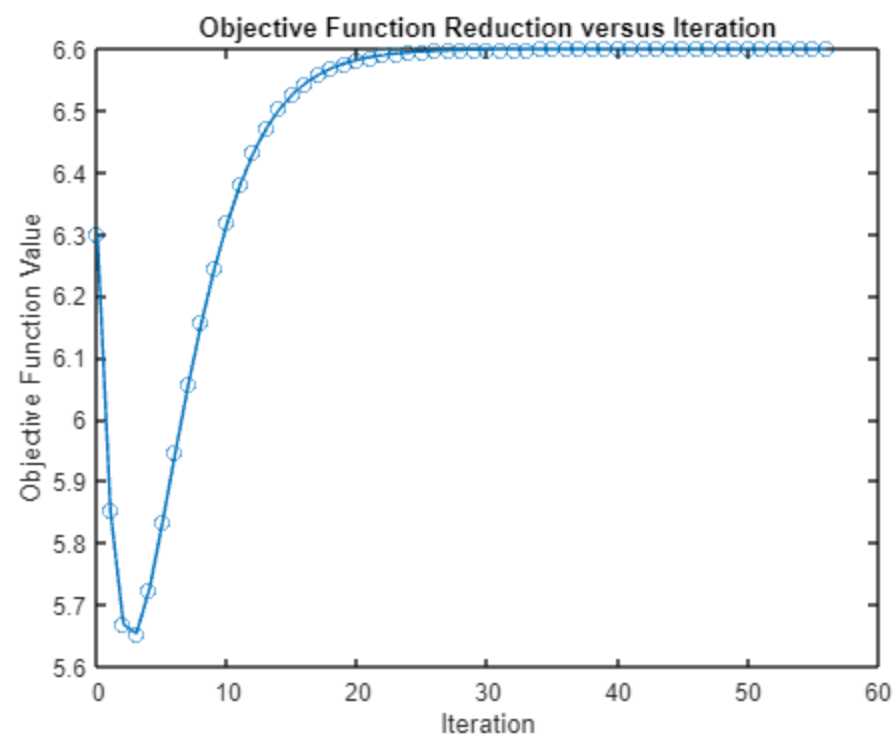
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000
1.1000	1.1000	1.1000	1.1000	1.1000	1.1000	1.1000	1.1000	1.1000	1.1000	1.1000	1.1000	1.1000	1.1000	1.1000	1.1000	1.1000	1.1000

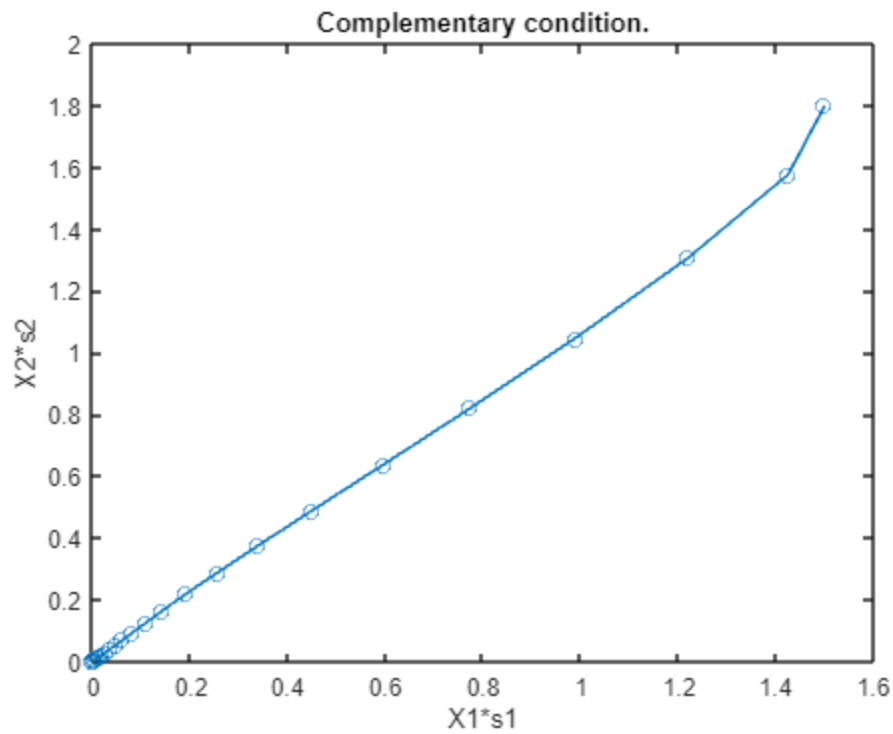
Columns 55 through 57

0.0000	0.0000	0.0000
0.1000	0.1000	0.1000
1.1000	1.1000	1.1000

Number of iterations :

56





Adaptive

Starting point x:

3
3
3

Starting point y:

-0.7000

Starting point s:

0.5000
0.6000
1.6000

Optimal solution x:

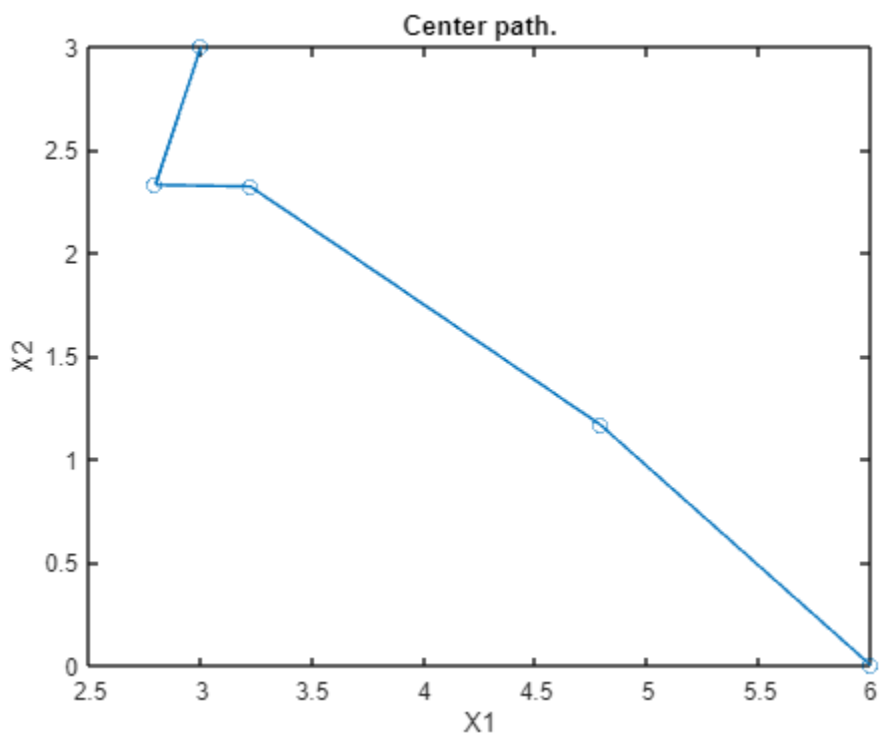
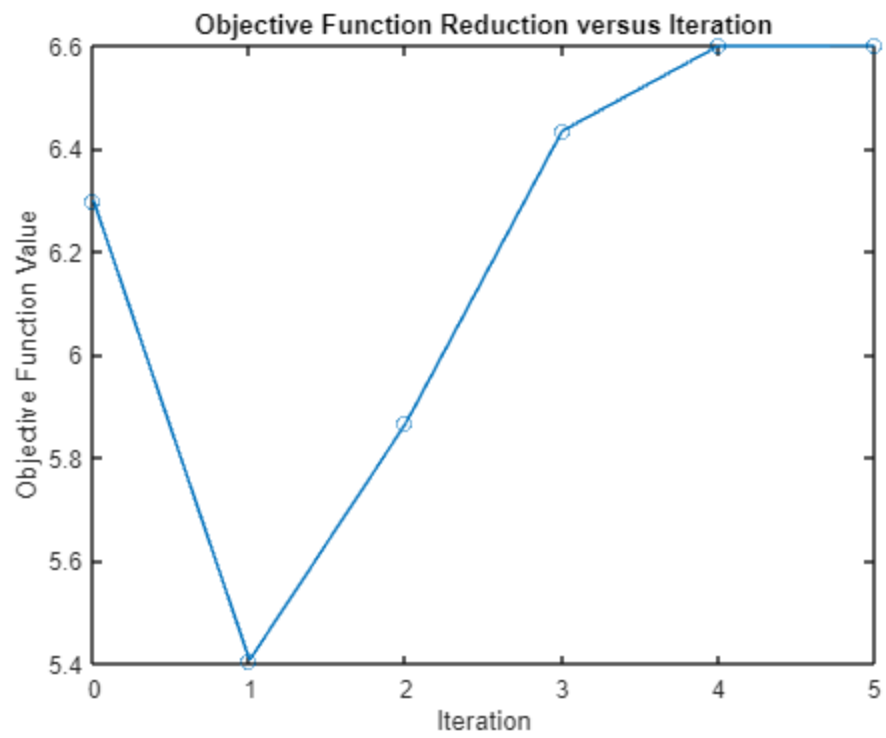
3.0000	2.7961	3.2217	4.7884	5.9982	6.0000
3.0000	2.3301	2.3225	1.1671	0.0016	0.0000
3.0000	0.8738	0.4558	0.0446	0.0002	0.0000

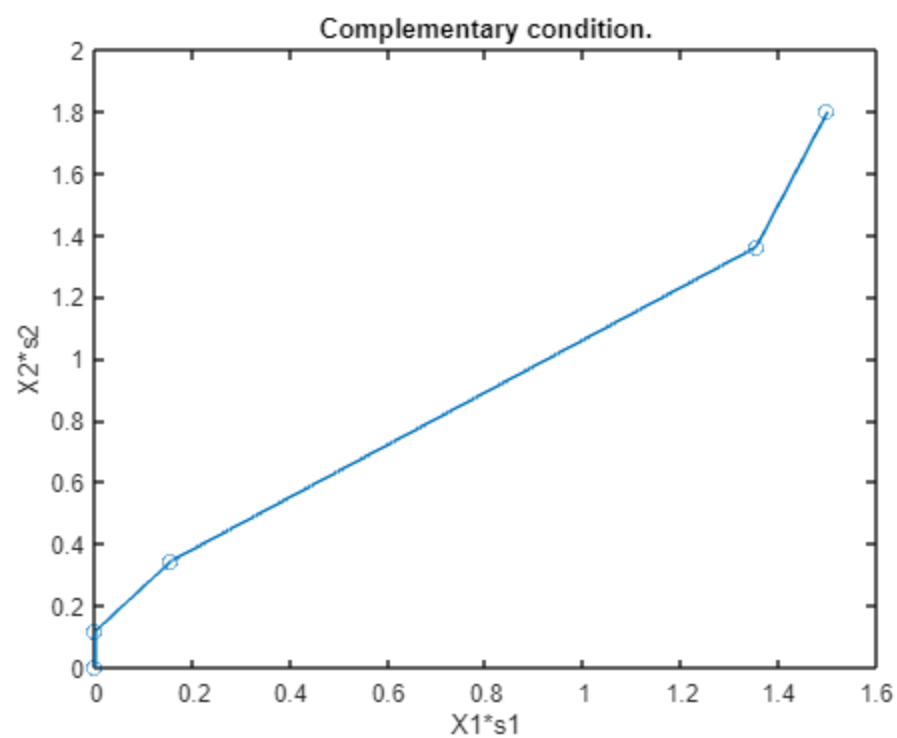
Optimal slack variables s:

0.5000	0.4840	0.0485	0	0.0000	0
0.6000	0.5840	0.1485	0.1000	0.1000	0.1000
1.6000	1.5840	1.1485	1.1000	1.1000	1.1000

Number of iterations :

5





Matlab built-in function:

LP preprocessing removed 0 inequalities, 0 equalities,
2 variables, and 2 non-zero elements.

Iter	Fval	Primal Infeas	Dual Infeas	Complementarity
0	-3.905000e+00	0.000000e+00	5.100000e+00	2.550000e+00
1	-6.588890e+00	0.000000e+00	6.066773e-02	1.009987e-02
2	-6.599988e+00	5.115908e-13	3.033387e-05	1.100707e-05
3	-6.600000e+00	2.025047e-13	1.516693e-08	5.503538e-09

```
real_x = 3×1
```

```
6.0000
```

```
0
```

```
0
```

```
real_obj_func = -6.6000
```

```
exitflag = 1
```

```
output = struct with fields:
```

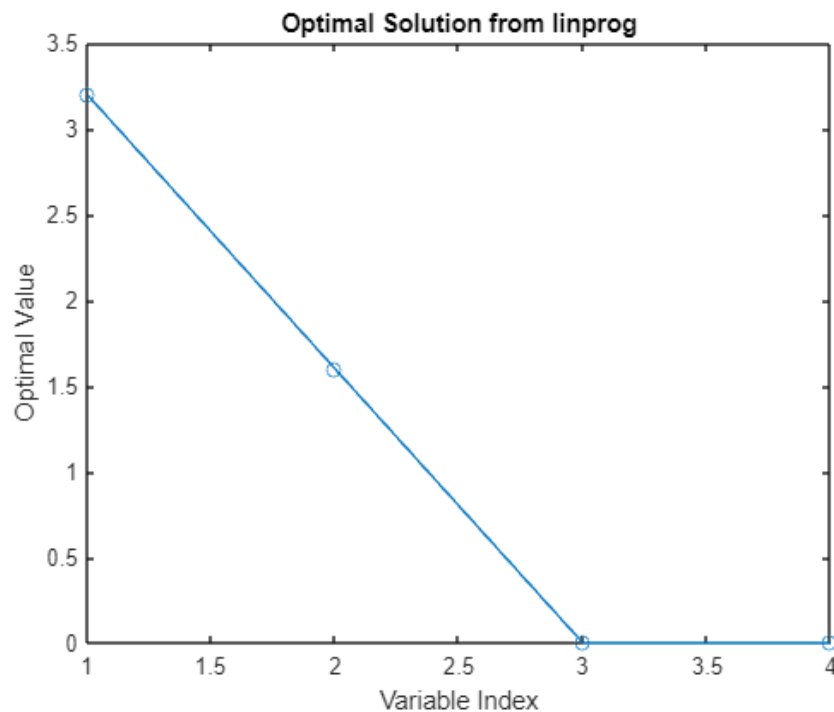
```
iterations: 3
```

```
message: 'Minimum found that satisfies the constraints. #Optimization
```

```
algorithm: 'interior-point'
```

```
constrviolation: 0
```

```
firstorderopt: 1.5167e-08
```



Problem 2:

$$\text{Min } f = -30x_1 - 20x_2$$

$$\begin{aligned} \text{s.t.} \quad & 2x_1 + x_2 + x_3 = 8 \\ & x_1 + 3x_2 + x_4 = 8 \\ & x_1, x_2, x_3, x_4 \geq 0 \end{aligned}$$

Solution:

Merhotra:

Starting point x:

3.0944
2.3139
1.7285
0.7529

Starting point y:

-10.4878
-3.4146

Starting point s:

7.3312
13.6727
23.4288
16.3556

Optimal solution x:

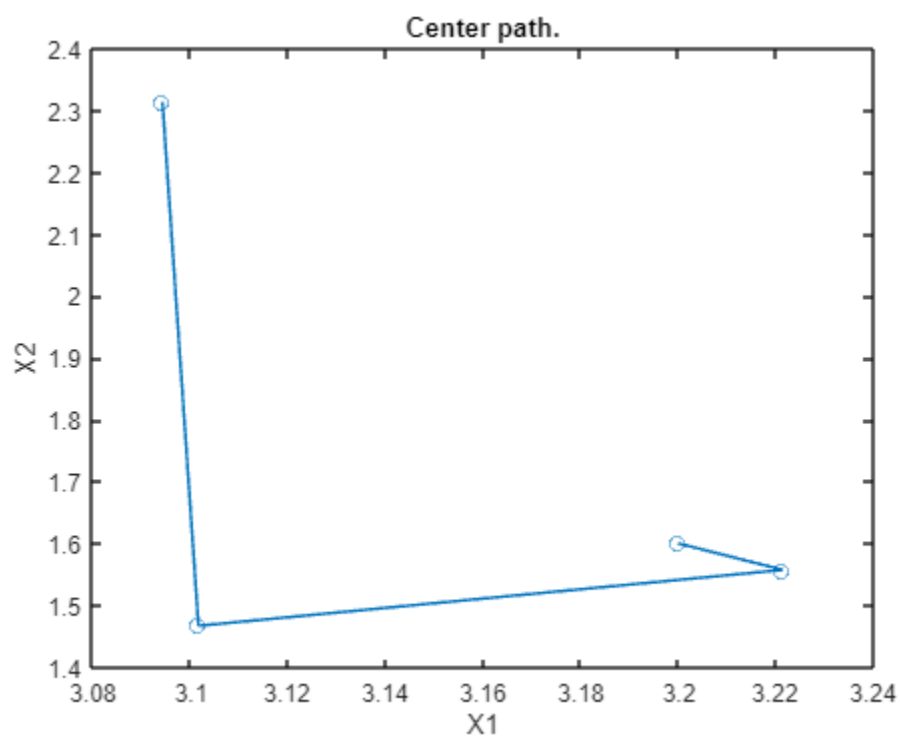
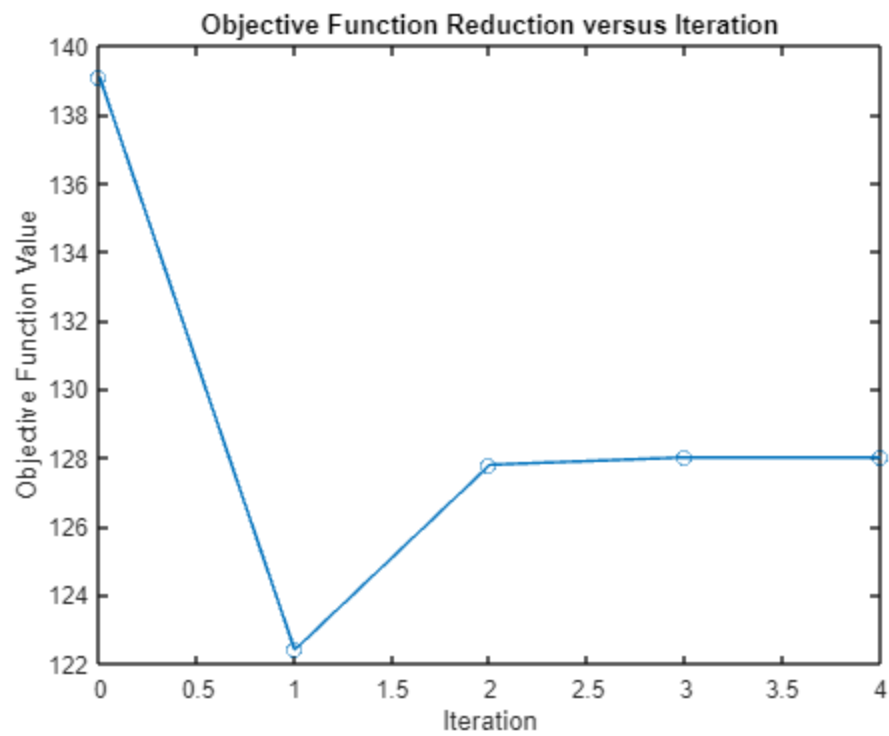
3.0944	3.1017	3.2214	3.2000	3.2000
2.3139	1.4667	1.5571	1.6000	1.6000
1.7285	0.3298	0	0.0000	0.0000
0.7529	0.4980	0.1072	0.0000	0

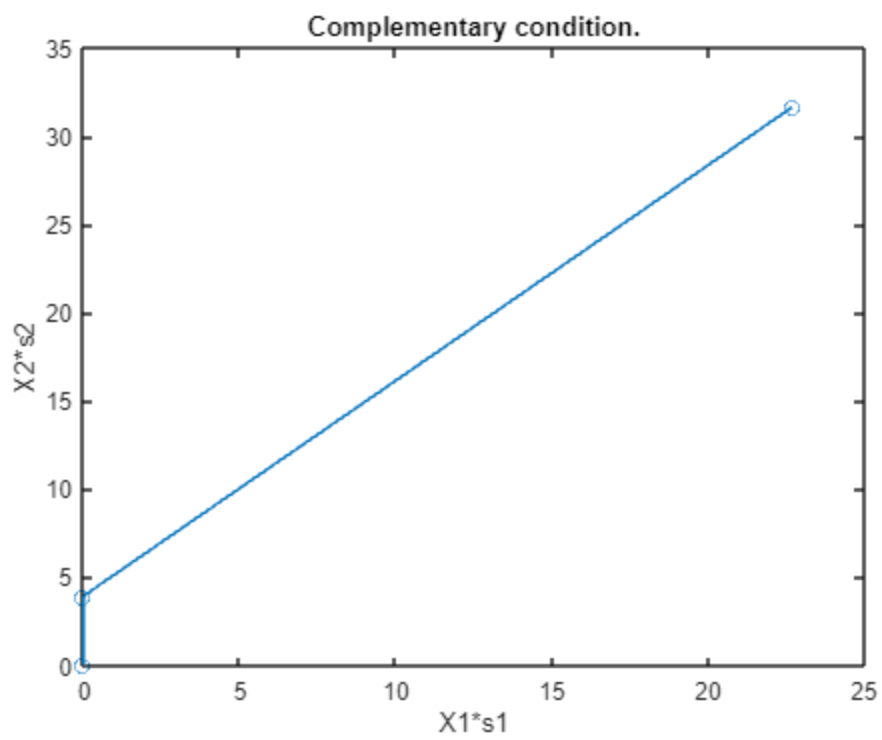
Optimal slack variables s:

7.3312	0	0.0001	0.0000	0.0000
13.6727	2.6714	0	0.0000	0.0000
23.4288	13.9108	14.0143	14.0000	14.0000
16.3556	3.6620	2.0190	2.0000	2.0000

Number of iterations :

4





Adaptive:

Starting point x:

3.0944
2.3139
1.7285
0.7529

Starting point y:

-10.4878
-3.4146

Starting point s:

7.3312
13.6727
23.4288
16.3556

Optimal solution x:

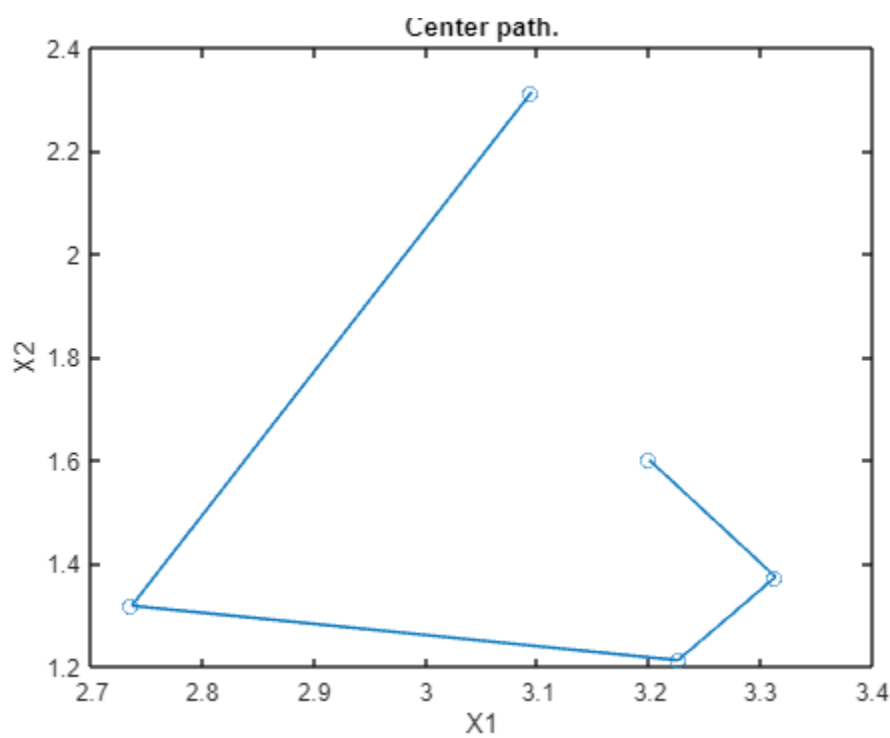
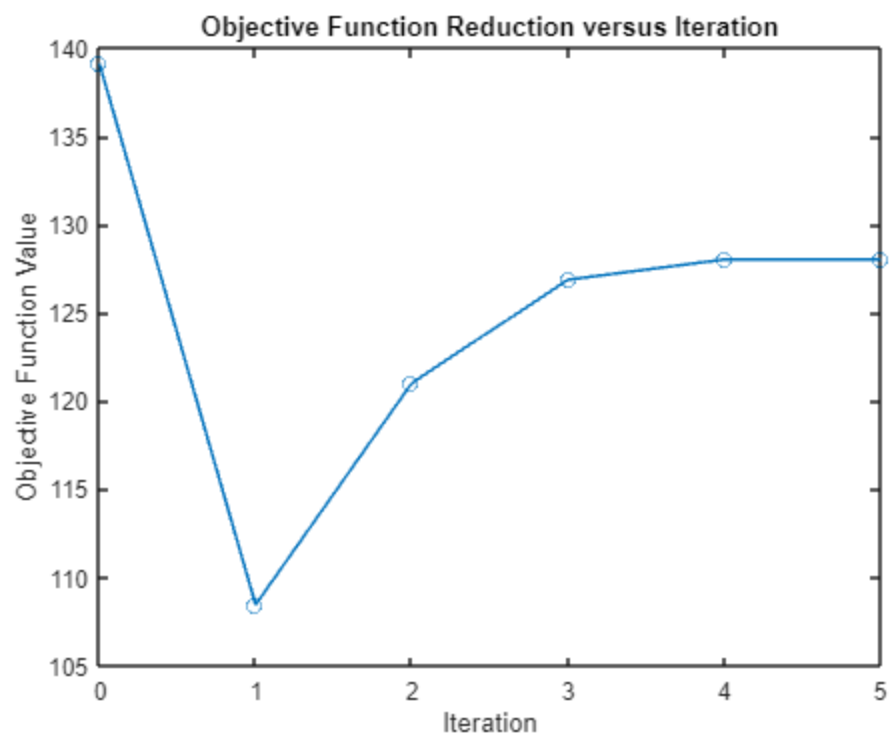
3.0944	2.7362	3.2258	3.3131	3.1999	3.2000
2.3139	1.3172	1.2114	1.3731	1.6000	1.6000
1.7285	1.2104	0.3371	0.0007	0.0001	0.0000
0.7529	1.3122	1.1401	0.5676	0	0.0000

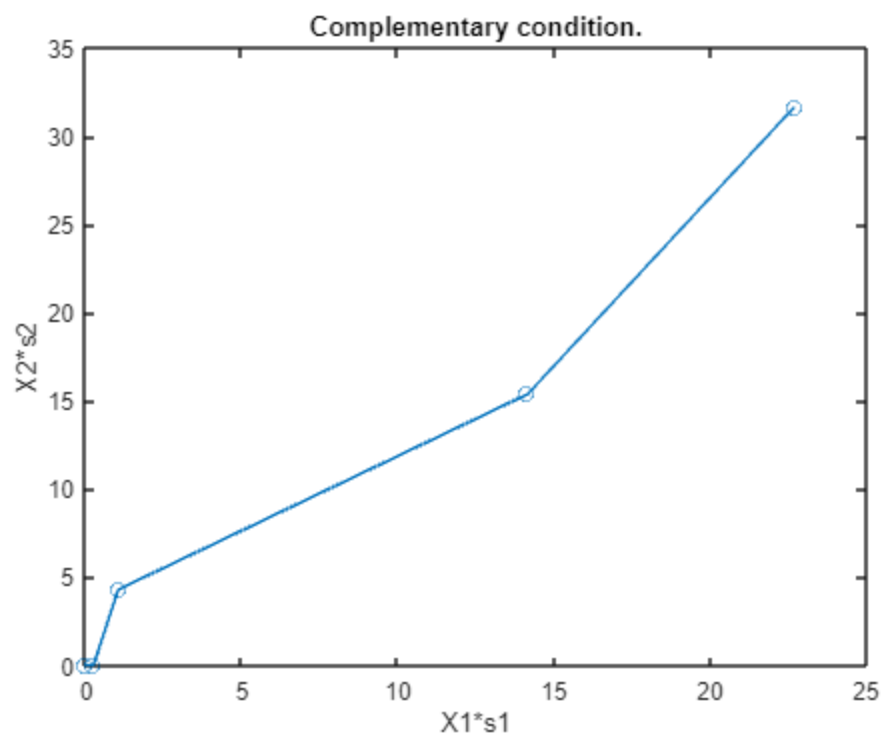
Optimal slack variables s:

7.3312	5.1762	0.3260	0.0783	0.0031	0
13.6727	11.6770	3.5389	0	0.0010	0.0000
23.4288	14.7703	13.4878	14.0470	14.0017	14.0000
16.3556	5.6355	3.3503	1.9843	1.9998	2.0000

Number of iterations :

5





Fixed:

Starting point x:

3.0410
2.2605
1.6752
0.6996

Starting point y:

-10.4878
-3.4146

Starting point s:

3.1014
9.4429
19.1990
12.1258

Optimal solution x:

Columns 1 through 18

3.0944	2.9153	2.9288	3.0143	3.1102	3.1888	3.2420	3.2712	3.2813	3.2787	3.2691	3.2571	3.2455	3.2354	3.2272	3.2207	3.2157	3.2118
2.3139	1.8155	1.5081	1.3256	1.2259	1.1817	1.1766	1.1998	1.2425	1.2958	1.3513	1.4026	1.4465	1.4822	1.5104	1.5322	1.5488	1.5615
1.7285	1.4694	1.1921	0.9247	0.6931	0.5105	0.3744	0.2753	0.2036	0.1512	0.1128	0.0843	0.0631	0.0473	0.0354	0.0266	0.0199	0.0149
0.7529	1.0326	1.2442	1.3575	1.3864	1.3532	1.2719	1.1513	1.0021	0.8393	0.6797	0.5364	0.4157	0.3183	0.2419	0.1830	0.1380	0.1038

Columns 19 through 36

3.2089	3.2067	3.2050	3.2038	3.2028	3.2021	3.2016	3.2012	3.2009	3.2007	3.2005	3.2004	3.2003	3.2002	3.2002	3.2001	3.2001	3.2001
1.5710	1.5782	1.5837	1.5877	1.5908	1.5931	1.5948	1.5961	1.5971	1.5978	1.5984	1.5988	1.5991	1.5993	1.5995	1.5996	1.5997	1.5998
0.0112	0.0084	0.0063	0.0047	0.0035	0.0027	0.0020	0.0015	0.0011	0.0008	0.0006	0.0005	0.0004	0.0003	0.0002	0.0001	0.0001	0.0001
0.0781	0.0586	0.0440	0.0330	0.0248	0.0186	0.0139	0.0105	0.0078	0.0059	0.0044	0.0033	0.0025	0.0019	0.0014	0.0010	0.0008	0.0006

Columns 37 through 54

3.2001	3.2000	3.2000	3.2000	3.2000	3.2000	3.2000	3.2000	3.2000	3.2000	3.2000	3.2000	3.2000	3.2000	3.2000	3.2000	3.2000	3.2000
1.5998	1.5999	1.5999	1.5999	1.5999	1.6000	1.6000	1.6000	1.6000	1.6000	1.6000	1.6000	1.6000	1.6000	1.6000	1.6000	1.6000	1.6000
0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0004	0.0003	0.0002	0.0002	0.0001	0.0001	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Columns 55 through 66

3.2000	3.2000	3.2000	3.2000	3.2000	3.2000	3.2000	3.2000	3.2000	3.2000	3.2000	3.2000
1.6000	1.6000	1.6000	1.6000	1.6000	1.6000	1.6000	1.6000	1.6000	1.6000	1.6000	1.6000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Optimal slack variables s:

Columns 1 through 18

7.3312	6.2537	4.8272	3.5745	2.6300	1.9482	1.4556	1.0948	0.8270	0.6260	0.4739	0.3584	0.2706	0.2040	0.1536	0.1155	0.0868	0.0652
13.6727	12.6748	11.2605	9.5209	7.6517	5.8767	4.3507	3.1370	2.2285	1.5780	1.1240	0.8089	0.5885	0.4319	0.3192	0.2370	0.1766	0.1318
23.4288	19.0995	16.5853	15.2111	14.5330	14.2363	14.1245	14.0901	14.0808	14.0752	14.0671	14.0571	14.0466	14.0370	14.0288	14.0221	14.0169	14.0128
16.3556	10.9956	8.1270	6.3876	5.1817	4.2845	3.6109	3.1167	2.7664	2.5262	2.3649	2.2569	2.1838	2.1332	2.0976	2.0720	2.0534	2.0398

Columns 19 through 36

0.0489	0.0367	0.0275	0.0207	0.0155	0.0116	0.0087	0.0065	0.0049	0.0037	0.0028	0.0021	0.0016	0.0012	0.0009	0.0007	0.0005	0.0004
0.0986	0.0738	0.0553	0.0414	0.0310	0.0233	0.0175	0.0131	0.0098	0.0074	0.0055	0.0041	0.0031	0.0023	0.0017	0.0013	0.0010	0.0007
14.0097	14.0073	14.0055	14.0041	14.0031	14.0023	14.0017	14.0013	14.0010	14.0007	14.0006	14.0004	14.0003	14.0002	14.0002	14.0001	14.0001	14.0001
2.0297	2.0222	2.0166	2.0124	2.0093	2.0070	2.0052	2.0039	2.0029	2.0022	2.0017	2.0012	2.0009	2.0007	2.0005	2.0004	2.0003	2.0002

Columns 37 through 54

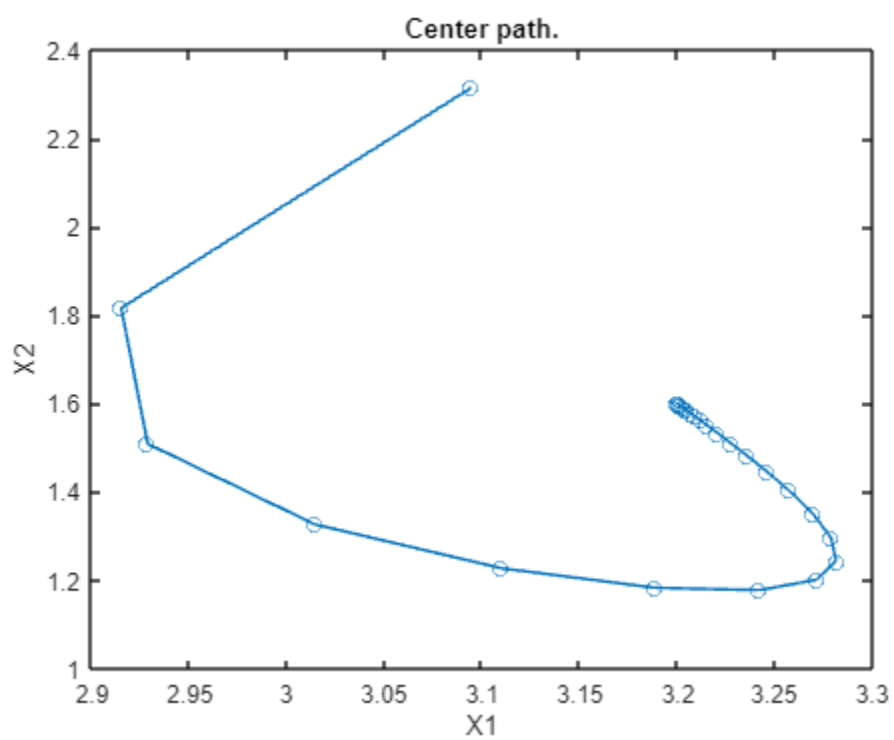
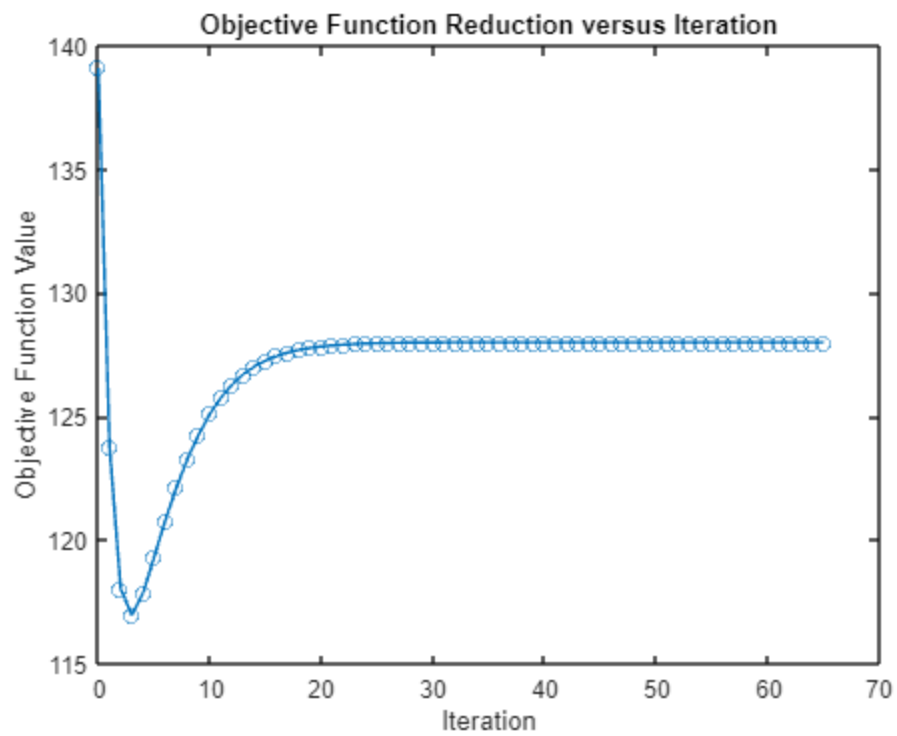
0.0003	0.0002	0.0002	0.0001	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0006	0.0004	0.0003	0.0002	0.0002	0.0001	0.0001	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
14.0001	14.0000	14.0000	14.0000	14.0000	14.0000	14.0000	14.0000	14.0000	14.0000	14.0000	14.0000	14.0000	14.0000	14.0000	14.0000	14.0000	14.0000
2.0002	2.0001	2.0001	2.0001	2.0001	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000

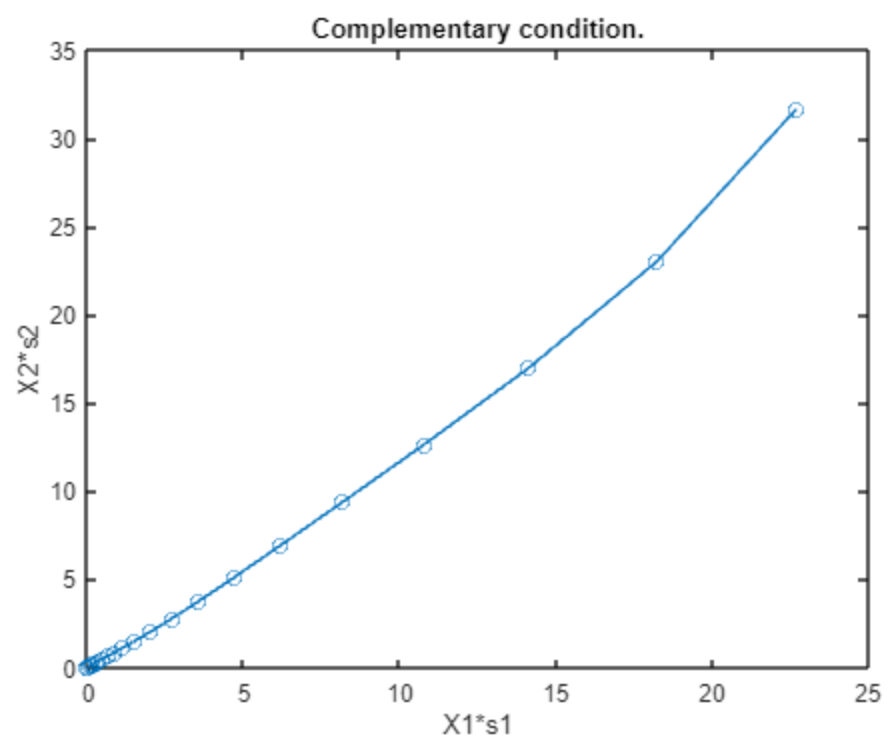
Columns 55 through 66

0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
14.0000	14.0000	14.0000	14.0000	14.0000	14.0000	14.0000	14.0000	14.0000	14.0000	14.0000	14.0000
2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000

Number of iterations :

65





Matlab:

LP preprocessing removed 0 inequalities, 0 equalities,
2 variables, and 2 non-zero elements.

Iter	Fval	Primal Infeas	Dual Infeas	Complementarity
0	-1.663528e+02	8.209557e+00	1.762126e+01	8.810631e+00
1	-1.068929e+02	4.104779e-03	8.810631e-03	5.043920e+00
2	-1.235720e+02	7.275760e-04	4.405315e-06	3.503437e-01
3	-1.279999e+02	5.643485e-07	2.202658e-09	4.674868e-04
4	-1.280000e+02	2.821743e-10	1.776357e-15	1.242561e-08

Minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in feasible direction to within the selected value of the constraint tolerance.

```
real_x = 4x1
    3.2000
    1.6000
         0
         0
```

```
real_obj_func = -128.0000
```

```
exitflag = 1
```

```
output = struct with fields:
```

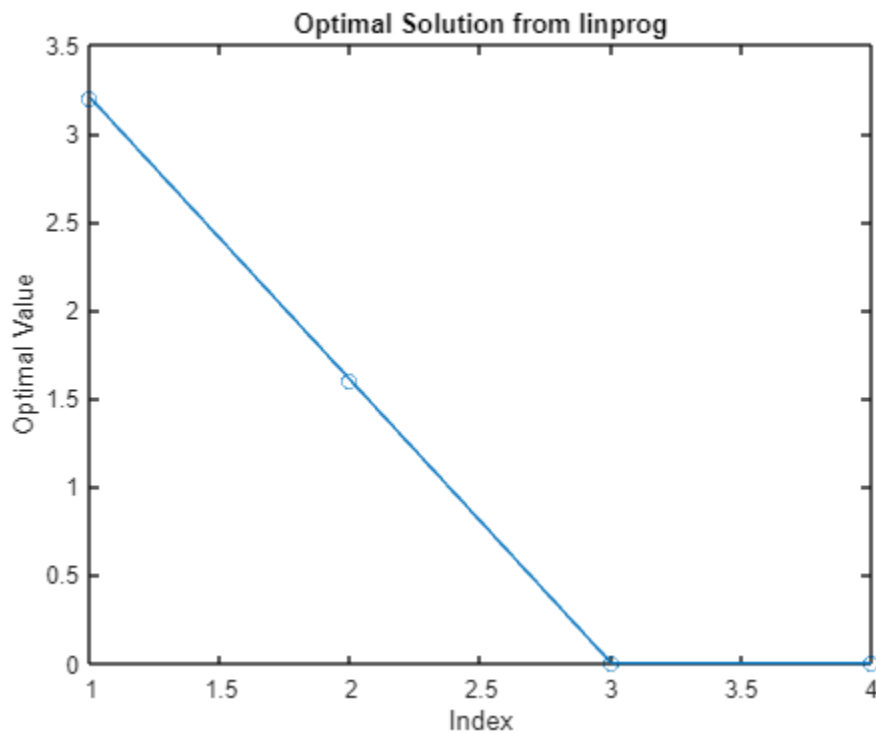
```
    iterations: 4
```

```
    message: 'Minimum found that satisfies the constraints.##Optimization completed
```

```
    algorithm: 'interior-point'
```

```
    constrviolation: 0
```

```
    firstorderopt: 3.9925e-08
```



Problem 3:

$$\text{Minimize } f = -x_1 - 2x_2 - x_3$$

subject to

$$2x_1 + x_2 - x_3 \leq 2$$

$$2x_1 - x_2 + 5x_3 \leq 6$$

$$4x_1 + x_2 + x_3 \leq 6$$

$$x_i \geq 0; \quad i = 1, 2, 3$$

Solution:**Fixed:**

Starting point x:

1.4579

0.3998

0.9482

0.3223

0.3521

0.4296

Starting point y:

-0.1505

-0.0700

-0.2608

Starting point s:

4.1802

2.0372

3.1564

3.8464

3.7659

3.9567

Optimal solution x:

Columns 37 through 54

0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3.9999	3.9999	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000
2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0001	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0001	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Columns 55 through 57

0.0000	0.0000	0.0000
4.0000	4.0000	4.0000
2.0000	2.0000	2.0000
0.0000	0.0000	0.0000
0.0000	0.0000	0.0000
0.0000	0.0000	0.0000

Optimal slack variables s:

Columns 37 through 54

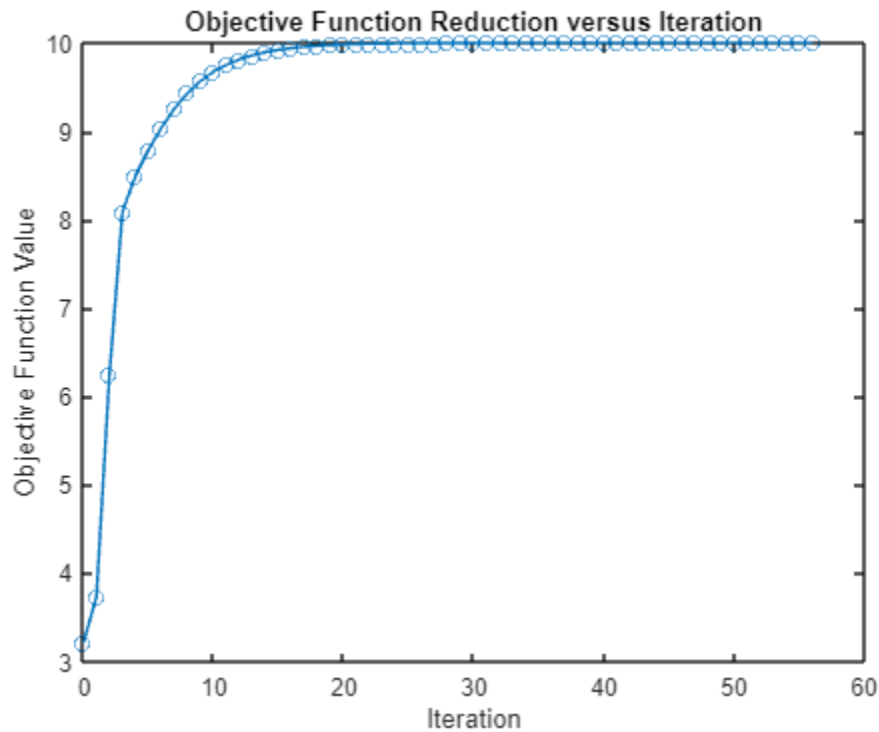
6.0001	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1.9291	1.9291	1.9291	1.9291	1.9291	1.9291	1.9291	1.9291	1.9291	1.9291	1.9291	1.9291	1.9291	1.9291	1.9291	1.9291	1.9291	1.9291
0.4764	0.4764	0.4764	0.4764	0.4764	0.4764	0.4764	0.4764	0.4764	0.4764	0.4764	0.4764	0.4764	0.4764	0.4764	0.4764	0.4764	0.4764
0.5473	0.5473	0.5473	0.5473	0.5473	0.5473	0.5473	0.5473	0.5473	0.5473	0.5473	0.5473	0.5473	0.5473	0.5473	0.5473	0.5473	0.5473

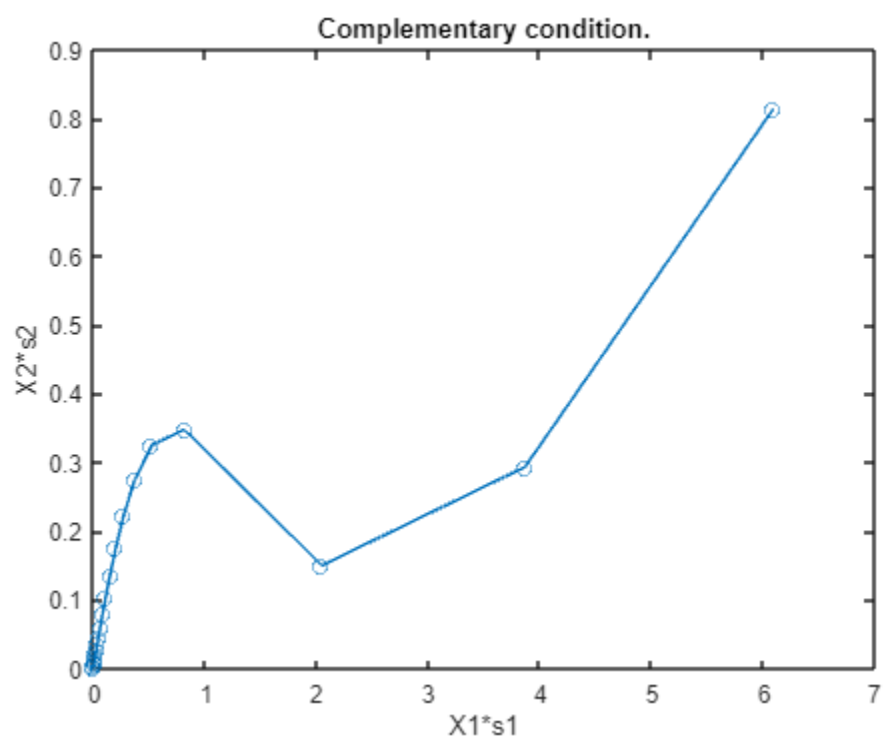
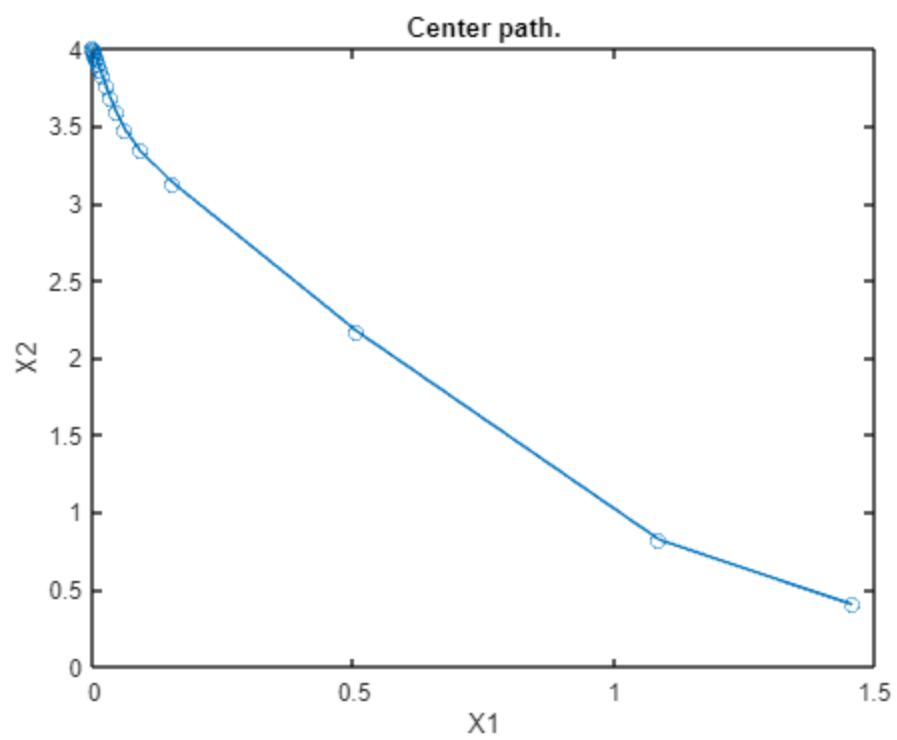
Columns 55 through 57

6.0000	6.0000	6.0000
0.0000	0.0000	0.0000
0.0000	0.0000	0.0000
1.9291	1.9291	1.9291
0.4764	0.4764	0.4764
0.5473	0.5473	0.5473

Number of iterations :

56





Adaptive

starting point x:

1.4579
0.3998
0.9482
0.3223
0.3521
0.4296

starting point y:

-0.1505
-0.0700
-0.2608

starting point s:

4.1802
2.0372
3.1564
3.8464
3.7659
3.9567

Optimal solution x:

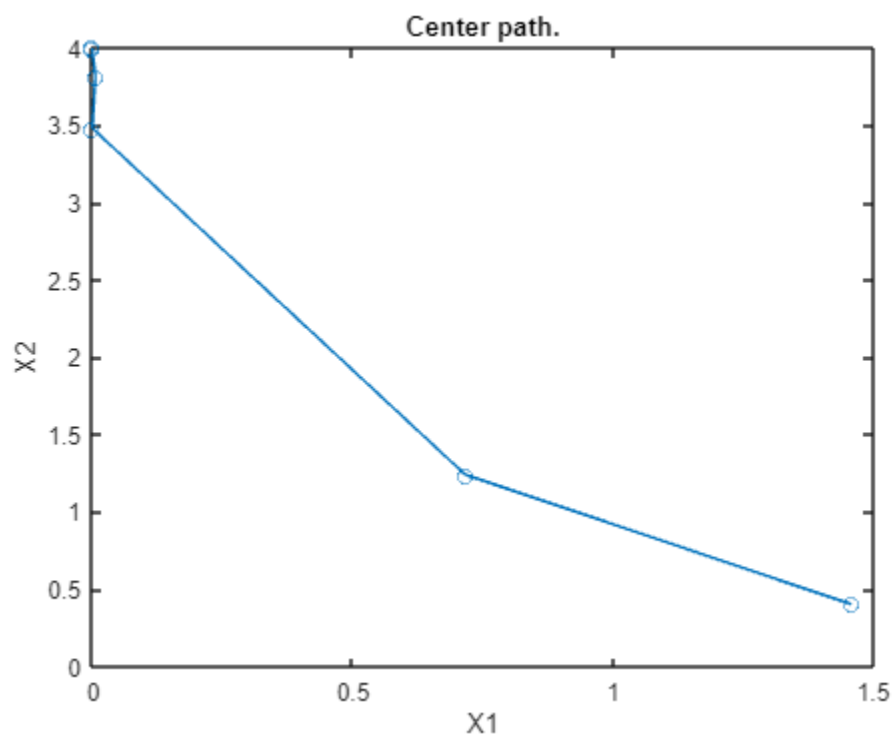
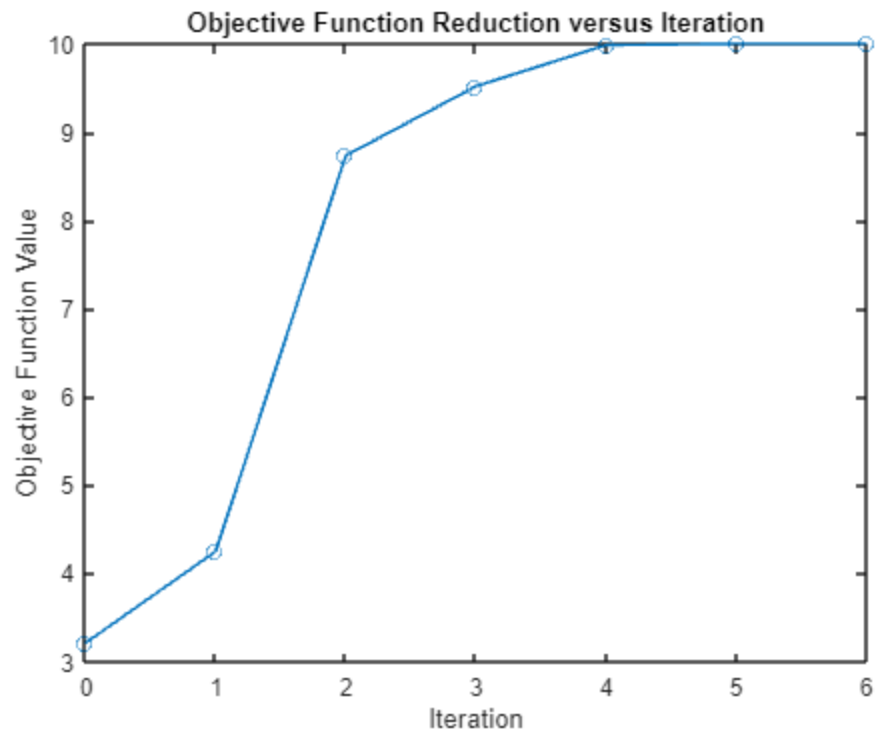
1.4579	0.7159	0	0.0056	0	0.0000	0
0.3998	1.2387	3.4800	3.8115	3.9952	4.0000	4.0000
0.9482	1.0506	1.7719	1.8902	1.9958	2.0000	2.0000
0.3223	0.3802	0.2919	0.0674	0.0006	0.0000	0.0000
0.3521	0.5538	0.6206	0.3492	0.0164	0.0000	0.0000
0.4296	0.8472	0.7482	0.2758	0.0091	0.0000	0.0000

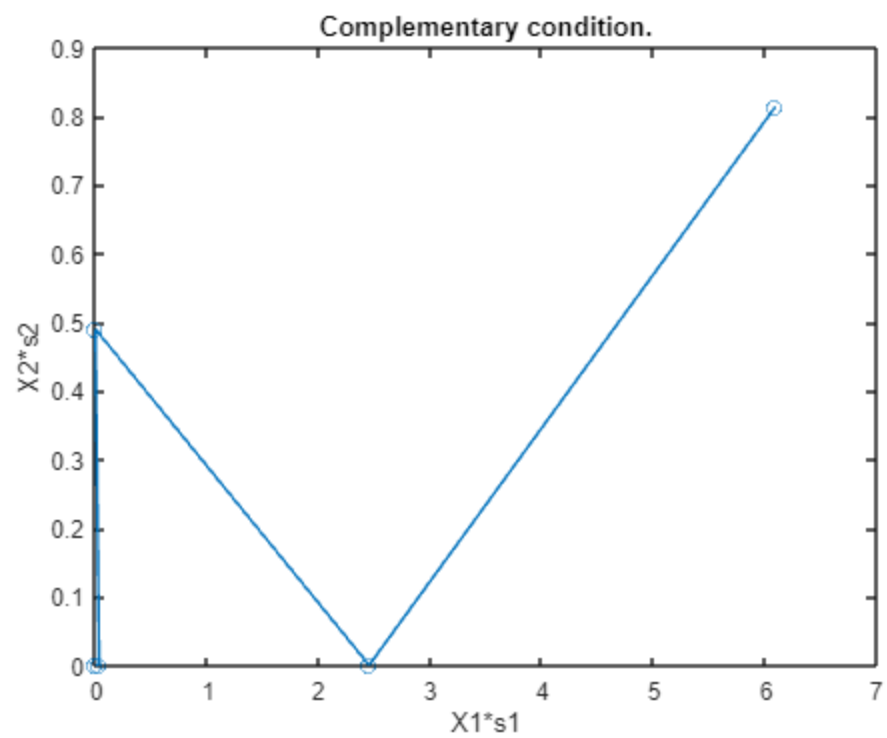
Optimal slack variables s:

4.1802	3.4266	4.4709	5.9319	5.9969	6.0000	6.0000
2.0372	0	0.1408	0	0.0001	0	0.0000
3.1564	1.7905	0	0.0137	0	0.0000	0
3.8464	3.3159	2.5472	2.0663	2.0568	2.0566	2.0566
3.7659	2.2068	1.0926	0.5425	0.5197	0.5189	0.5189
3.9567	0.8909	0.6862	0.4762	0.4630	0.4623	0.4623

Number of iterations :

6





Merhotra:

Starting point x:

1.4579
0.3998
0.9482
0.3223
0.3521
0.4296

Starting point y:

-0.1505
-0.0700
-0.2608

Starting point s:

4.1802
2.0372
3.1564
3.8464
3.7659
3.9567

Optimal solution x:

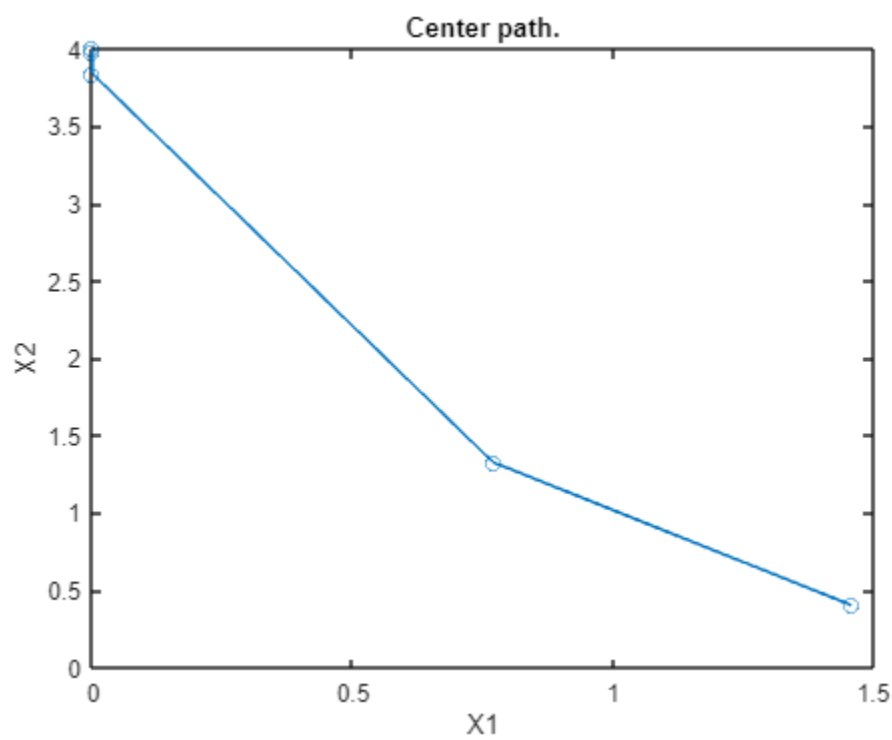
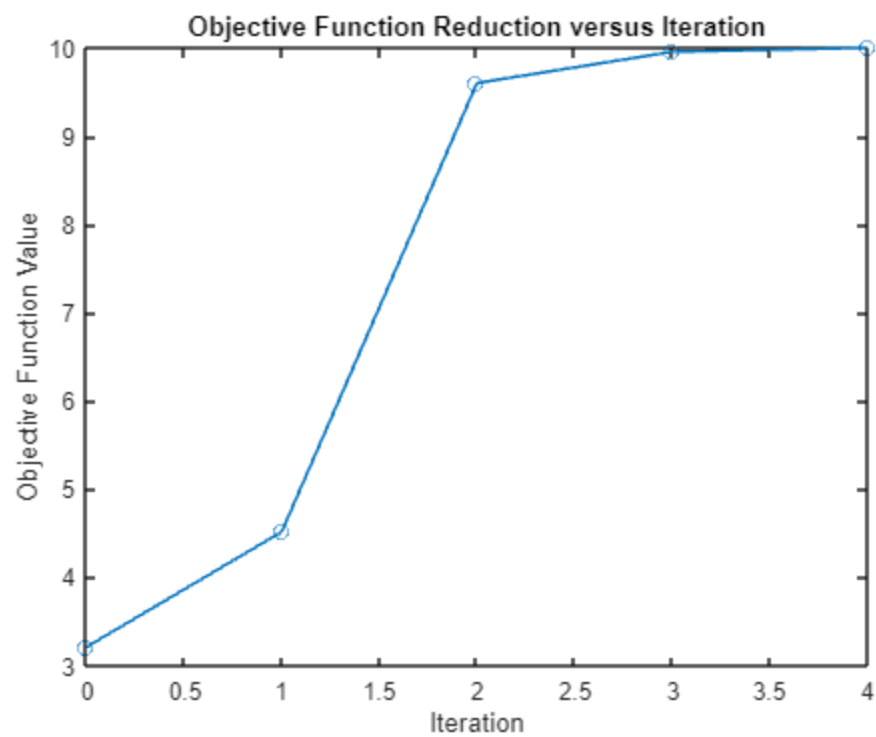
1.4579	0.7694	0.0000	0.0000	0
0.3998	1.3253	3.8399	3.9849	4.0000
0.9482	1.0922	1.9205	1.9849	2.0000
0.3223	0.2281	0.0806	0	0.0000
0.3521	0.3258	0.2375	0.0605	0.0000
0.4296	0.5051	0.2396	0.0303	0.0000

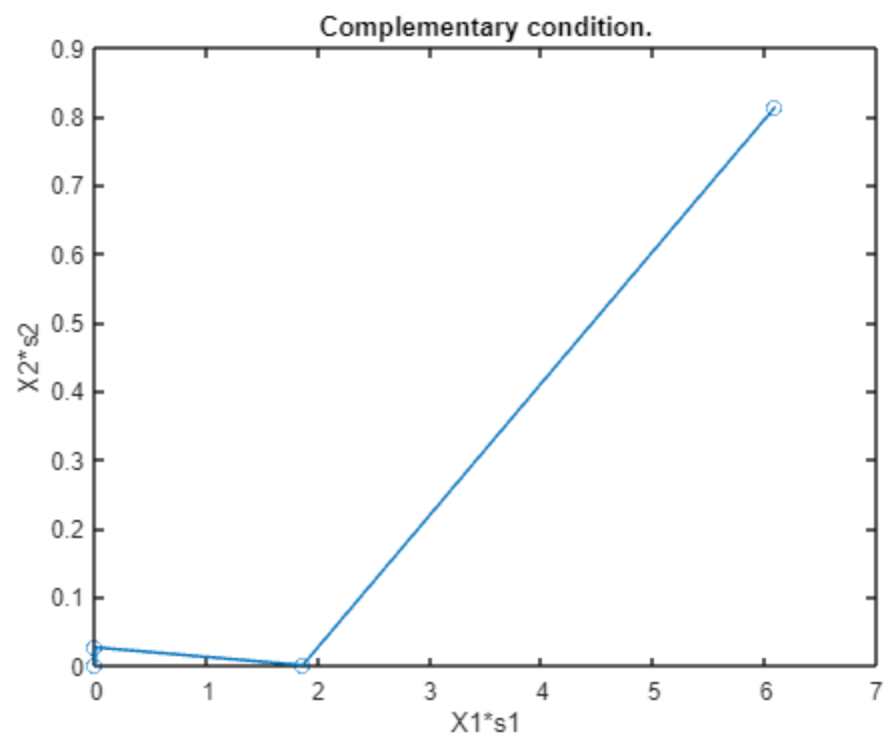
Optimal slack variables s:

4.1802	2.4158	8.0996	6.0001	6.0000
2.0372	0	0.0068	0.0000	0.0000
3.1564	1.0223	2.0793	0	0.0000
3.8464	2.0321	1.7163	1.6659	1.6658
3.7659	1.7048	0.7508	0.3886	0.3886
3.9567	1.6727	1.0413	0.7228	0.7228

Number of iterations :

4





Matlab:

```
real_x = 6x1
    0.0000
    4.0000
    2.0000
     0
     0
     0

real_obj_func = -10.0000
exitflag = 1
output = struct with fields:
    iterations: 4
    message: 'Minimum found that satisfies the constraints.##Optimization completed'
    algorithm: 'interior-point'
    constrviolation: 0
    firstorderopt: 1.1952e-15
```

