Presentation of the project : Evolutionary Games Theory

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1 Introduction

This project aims to explore the behavior of four evolutionary games, namely weak prisoner's dilemma, stag hunt, snowdrift, and hawk dove, on different network topologies using various update rules. The analysis focuses on understanding how network structure and update rules influence the evolution of cooperation in complex systems.

To investigate the dynamics of these evolutionary games, several update rules are employed. The "random rule" assigns strategies randomly, while the "stochastic best response rule" selects the best response to a neighbor's strategy with a probability determined by the payoff difference. The "generous tit for tat rule" imitates the strategy of a neighbor with a higher payoff, and the "replicator rule" and "multiple replicator rule" replicate the strategy of a neighbor with a higher payoff based on a normalized probability. The "unconditional imitation rule" adopts the strategy of the neighbor with the highest payoff, while the "moran rule" and "fermi rule" introduce stochasticity based on payoffs and temperature, respectively. The analysis encompasses various network topologies, including complete networks, homogeneous random graphs, Erdos Renyi networks, Barabasi Albert networks, community graphs, Watts-Strogatz graphs a ral twork grap. By exploring these diverse network structures, this project aims to gain insights into how network topology shapes the evolution of cooperation in different evolutionary games. The project also focuses on investigating equilibrium states, dynamics, and the impact of varying values of T (temptation to defect) and S (sucker's payoff) on the evolution of cooperation. Through extensive simulations and analysis, the project aims to uncover the intricate relationship between network topology, update rules, and the emergence of cooperation.

To support the development of this project, three key resources will be thoroughly examined, drawing from existing literature and research. These resources will provide valuable insights into complex networks, evolutionary game theory, and the analysis of network dynamics. By leveraging these resources, the project aims to contribute to the understanding of cooperative behaviors in complex systems and advance the field of network science. Overall, this project combines theoretical analysis, computational simulations, and insights from network science to shed light on the dynamics of cooperation in evolutionary games on diverse network topologies.

2 Work Description

2.1 Logic behind the game theory

The payout matrix W of two-player games can be used to categorize them. A player can be a defector (D) or a collaborator (C).

$$\begin{array}{cc} & \begin{array}{cc} \mathrm{C} & \mathrm{D} \end{array} \\ \begin{array}{c} \mathrm{C} \\ \mathrm{D} \end{array} & \begin{pmatrix} 1 & S \\ T & 0 \end{pmatrix} \end{array}$$

Figure 1: Plot showing the example of the payout matrix

The payoff matrix explains the potential rewards of using a specific strategy (C or D) when playing against a player using a specific strategy. The payment for a C against another C will be 1, and the payoff for a C against a D will be S (also known as the "suckers payoff"). The "temptation payoff" is T when a D plays with a C, but it is 0 when he plays another D.

By examining various combinations of T and S, we can identify distinct games with unique conceptual characteristics. Game theory allows us to classify all 2x2 symmetric games into three distinct categories:

- In the first case, when

$$T < 1, S > 0$$

, or in the second case, when

$$T > 1, S < 0$$

, these games exhibit a unique Nash equilibrium where the dominant strategy is chosen (C in the first case and D in the second case). Examples of such games include the Harmony game and the Prisoner's Dilemma game.

- When

$$T < 0\ , S < 0$$

, these games have multiple Nash equilibria. One of these equilibria is a mixed strategy, which serves as an unstable equilibrium in the replicator dynamics. This mixed strategy acts as a separator between the basins of attraction of two Nash equilibria in pure strategies, which are the stable attractors. An example of such a game is the Stag Hunt.

- When

$$T > 1, S > 0$$

, these games also have multiple Nash equilibria. One of these equilibria is a mixed strategy, and interestingly, it serves as the global attractor in the replicator dynamics. This means that regardless of the initial conditions, the system will converge to this mixed strategy equilibrium. An example of such a game is the Snow Drift.

The main focus of this project revolves around three distinct evolutionary games:

- Weak Prisoner's Dilemma: Our analysis centers on the region where T values range between 1 and 2, while S remains at 0. This game is represented by the color green.

- Stag Hunt: We focus on the diagonal line that extends from (T,S) = (0,0) to (T,S) = (1,-1). The Stag Hunt game is denoted by the color red.

- Snowdrift: Our attention is directed towards the diagonal line spanning from (T,S) = (1,1) to (T,S) = (2,0). The Snowdrift game is distinguished by the color blue.

- Hawk-Dove: The Hawk-Dove game is characterized by T values in the range of [2,3] and S values in the range of [1,2]. In this game, players have two strategies: hawk and dove.

2.2 Inspiration behind the work

This project's core was inspired by the essay suggested to me "Evolutionary game theory: Temporal and spatial effects beyond replicator dynamics." This important essay serves as a lighthouse for navigating the interesting world of evolutionary game theory and venturing beyond the typical replicator dynamics framework.

The article elucidates the complex dynamics that go beyond the conventional understanding of game theory by shedding light on the temporal and spatial aspects of evolutionary games. It underlines how crucial it is to take into account elements like time, place, and network structures when determining how behavior and the evolution of strategies within a population are shaped. In fact, the captivating insights presented in the article have ignited a sense of curiosity and a desire to further investigate the intricacies of evolutionary games. The notion that the fraction of cooperators can be influenced by the random initialization of cooperator and defector nodes has piqued my interest, highlighting the non-trivial nature of studying game dynamics on various network topologies.

The article's findings motivated me a lot to develop this project. My project aims to delve into the behavior of three specific evolutionary games, namely the weak prisoner's dilemma, stag hunt, and snowdrift games but also the hawk dove. By incorporating different network topologies and different update rules (most of them were taken from the article and others were purely the result of my personal research), we seek to uncover the influence of these factors on the emergence and sustenance of cooperation in diverse game scenarios.

Furthermore, inspired by the article's emphasis on temporal effects and the convergence to stationary states, we intend to analyze the equilibrium states and dynamics of the games on various networks. By exploring the different values of T and S, we aim to understand the interplay between network topology, update rules, and the evolution of cooperation. This project is driven by a deep-seated curiosity to expand our understanding of evolutionary game theory beyond replicator dynamics, exploring temporal and spatial effects that have the potential to reshape our insights into strategic interactions and cooperative behavior in complex systems.

2.3 Work Explanation

During the simulation, a thorough analysis was conducted on 50 points along the specified lines. Each Monte Carlo simulation consisted of 50 repetitions to ensure robustness and statistical significance in the results.

Prior to running the full simulations, an investigation was carried out to determine the number of steps required to reach a stationary state for the analyzed graphs, considering different combinations of S and T. We got to the conclusion that the stationary state was almost always attained before reaching the 400th step. And based on this observation, the values of Ttrans = 400 (number of steps in the transitory state) and Tmax = 500 (number of steps in the simulation) were selected as appropriate parameters.

We should note that these values wouldn't be used in all the simulations, different networks, and their respective parameters. With additional time or computational resources, it would be beneficial to increase Ttrans and Tmax for a more comprehensive evaluation in cases where the stationary state is not reached within the initial 400 steps.

For all analyzed networks, the simulations began with an initial configuration where each node had an equal probability of being in strategy C or strategy D (0.5 probability for each). Subsequently, each node engaged in the specified game is defined by the given T and S values with its neighboring nodes, calculating its total payoff based on the outcomes. Once the payoffs were computed, each node had the opportunity to potentially change its strategy by comparing its own payoffs with those of its neighbors, following a chosen update rule. Several update rules were implemented, including the random rule, stochastic best response rule, replicator rule, multiple replicator rule, unconditional imitation rule, generous tit-for-tat rule, Moran rule, and Fermi rule. Detailed descriptions of these update rules will be explained later on.

To manage computational resources effectively, the study focused on networks of limited size, specifically networks with 100 nodes. The following network types were examined:

- **Complete network**
- **homogeneous random graph with degrees equal to 5**
- **Erdos Renyi network with an average degree of 5**
- **Barabasi Albert network with K equal to 5**
- **Lancichinetti-Fortunato-Radicchi (LFR) community graph**
- **Buit in community graph**
- **Watts–Strogatz graph**

- **Real network**

Overall, this experimental setup allowed for a comprehensive exploration of the behavior of the evolutionary games on different network topologies, considering various update rules, and shedding light on the dynamics and evolution of cooperation within the studied systems.

3 Gae Theories ad pdate Rules

3.1 Definition of the game theories

Before discussing the experiments, it's important to define the different game theories we are using.

- **Weak Prisoner's Dilemma** : The Weak Prisoner's Dilemma is a game theory scenario where individuals face a dilemma between cooperating and defecting. In this version, the temptation to defect is relatively low (T values between 1 and 2), and the payoff for mutual cooperation is moderate (S = 0). It is called "weak" because the incentive to defect is not as strong as in the traditional Prisoner's Dilemma. The Weak Prisoner's Dilemma often highlights situations where individuals may still choose to defect despite the potential benefits of cooperation.
- **Stag Hunt** : The Stag Hunt is a game theory concept that focuses on a situation where players must decide between cooperating for a common goal or pursuing individual interests. The payoff structure is characterized by a dilemma between hunting a stag (cooperative action) or settling for a hare (defection). The diagonal line extending from (T, S) = (0, 0) to (T, S) = (1, -1) represents the Stag Hunt game. It represents scenarios where individuals must coordinate and trust each other to achieve the best outcome.
- **Snowdrift** : The Snowdrift game is a game theory scenario that involves a conflict between individual and collective interests. It represents situations where individuals can choose to contribute to a collective effort or "free-ride" and benefit from the contributions of others. The diagonal line spanning from (T, S) = (1, 1) to (T, S) = (2, 0) characterizes the Snowdrift game. It captures scenarios where cooperation can be beneficial, but not without risks or costs, leading to a delicate balance between cooperation and defection.
- **Hawk-Dove** : The Hawk-Dove game is a game theory model that explores the dynamics of aggressive and peaceful strategies in competitive situations. It involves two strategies: hawk and dove. Hawks are aggressive and always fight for resources, while doves are more peaceful and avoid conflicts. The T parameter represents the benefit of winning a fight, and the S parameter represents the cost or risk associated with engaging in a fight. The Hawk-Dove game analyzes the trade-off between aggression and peacefulness in different situations.

3.2 Definition of the update rules

Before discussing the experiment, it's important to define the different update rules we are using in the context of game theory. These update rules govern how individuals in a population update their strategies over time. Here are the definitions of the update rules:

- **Random Rule** : The Random Rule is a simple update rule where individuals randomly choose their actions without considering the actions of others or the game's payoffs. Each individual selects their action independently and randomly, disregarding any strategic considerations.
- **Stochastic Best Response Rule** : The Stochastic Best Response Rule is an update rule in which individuals select their actions based on the probabilities of achieving higher payoffs.

Each individual considers the payoffs associated with different actions and chooses an action probabilistically, favoring actions that have a higher expected payoff.

- **Generous Tit for Tat Rule** : The Generous Tit for Tat Rule is an update rule based on the Tit for Tat strategy. Initially, individuals cooperate, and then they replicate the previous action of their interaction partner. However, the Generous Tit for Tat Rule adds a generous aspect by occasionally cooperating even if the interaction partner defected in the previous round.
- **Replicator Rule** : The Replicator Rule is an update rule based on evolutionary dynamics. It models the idea that successful strategies have a higher probability of being replicated by other individuals in the population. The replicator rule assigns probabilities to different strategies based on their relative success in generating higher payoffs.
- **Multiple Replicator Rule** : The Multiple Replicator Rule extends the Replicator Rule to consider multiple strategies in the population. It assigns probabilities to different strategies based on their relative success, allowing for the coexistence and evolution of multiple strategies over time.
- **Unconditional Imitation Rule** : The Unconditional Imitation Rule is an update rule where individuals directly imitate the strategy of another randomly chosen individual in the population. Individuals observe the strategies of others and adopt the strategy of the observed individual without considering the payoffs or strategic considerations.
- **Moran Rule** : The Moran Rule is an update rule commonly used in evolutionary game theory. It models the dynamics of the population based on the idea of reproduction and selection. In each iteration, an individual is randomly chosen for reproduction, and another individual is randomly chosen for replacement. The probability of reproduction is proportional to the individual's payoff, favoring individuals with higher payoffs.
- **Fermi Rule** : The Fermi Rule is an update rule that introduces stochasticity in the selection process. It models the idea that individuals with lower payoffs still have a small chance of being selected for reproduction. The Fermi Rule incorporates a probabilistic element inspired by the Fermi-Dirac distribution to determine the selection probabilities based on the payoffs of individuals.

4 Results of the experiments

4.1 Complete Graphs

In game theory, complete graphs are often used as a theoretical model to analyze strategic interactions. A complete graph represents a scenario where every player is directly connected to every other player, indicating that they can interact or communicate with any other player in the game. We choose to first work with a complete graph with a number of nodes equal to 100.

We plotted two different figures to compare the computational costs, aiming to identify any variations. Surprisingly, the Moran rule was found to be the most time-consuming, requiring over 2 hours to complete and produce results. For the case of the first game theory : Weak Prisoner´s Dilemma

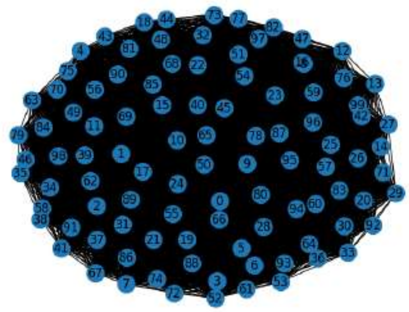

Figure 2: Plot showing the complete graph

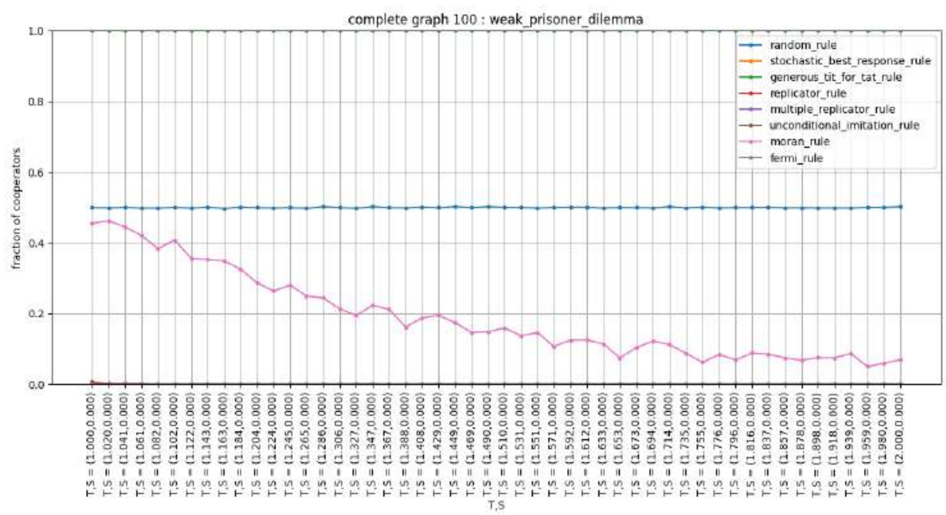

Figure 3: Plot showing the complete graph result of the weak prisoner´s dilemma with all the rules included

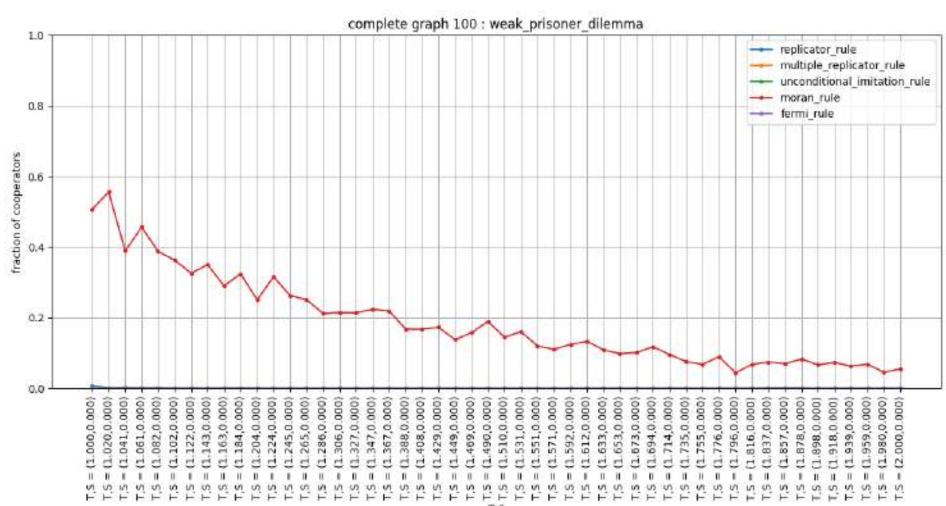

Figure 4: Plot showing the complete graph result of the weak prisoner´s dilemma

For the next game theory, we are going to plot the full plot that includes all the update rules.

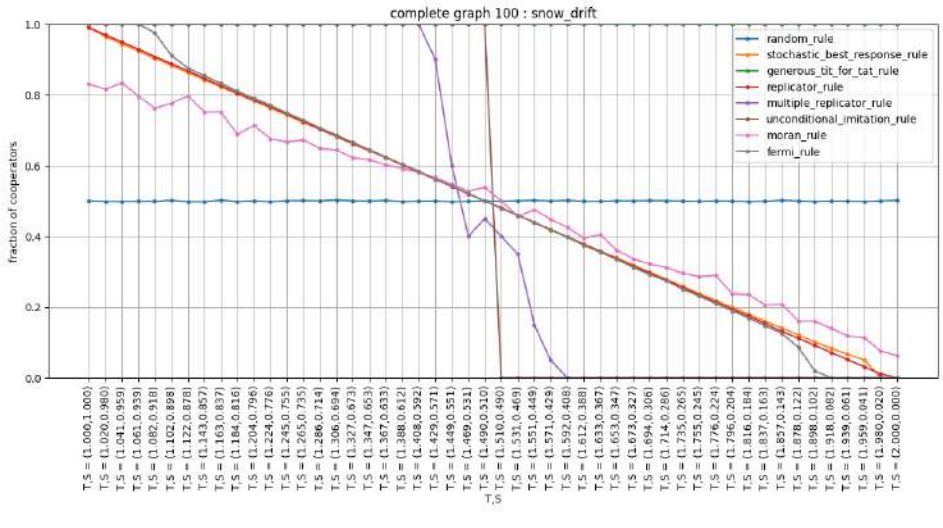

Figure 5: Plot showing the complete graph result of snow drift game

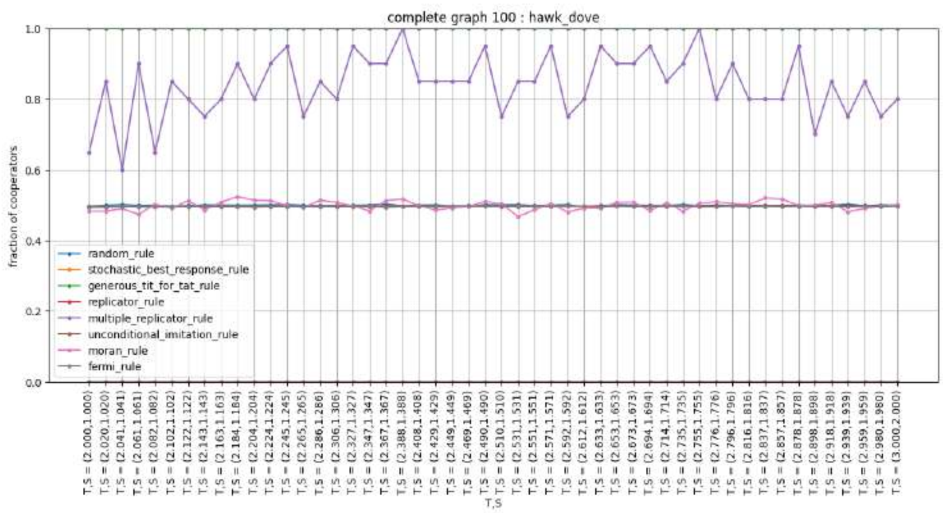

Figure 6: Plot showing the complete graph result of the hawk dove game

For all the rules except the Moran rule, the expected equilibrium involves a fraction of cooperators that is close to zero. This outcome aligns with expectations. However, the behavior with the Moran rule differs, which can be attributed to the fact that, under this rule, a player has a low probability of adopting the strategy of a neighbor who has performed worse.

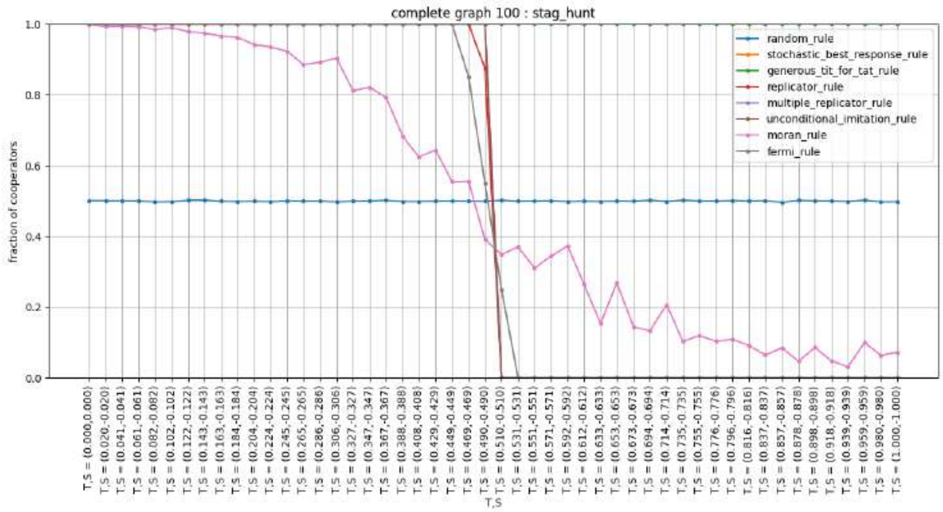

Figure 7: Plot showing the complete graph result of the stag hunt game

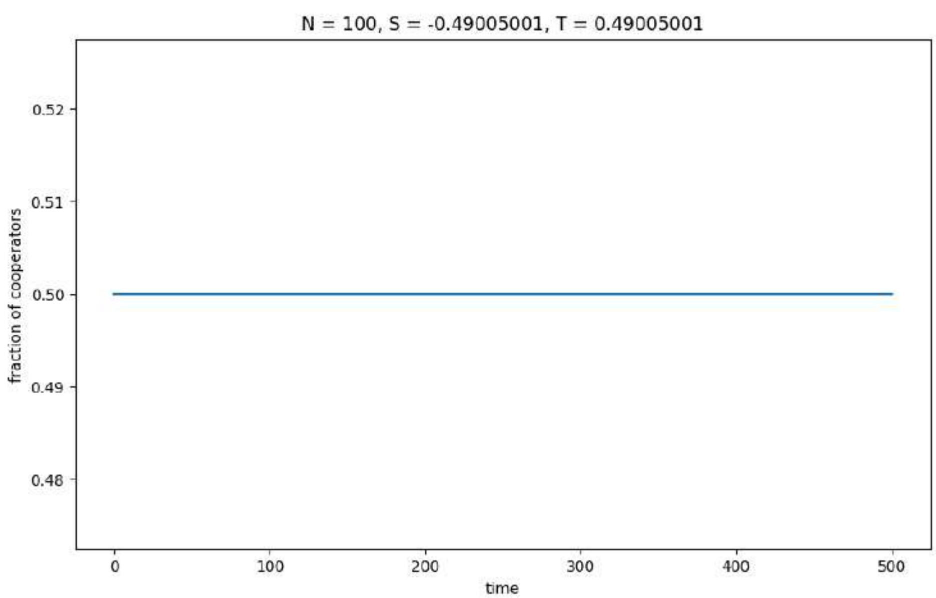

Figure 8: Plot showing the complete graph result of the stag hunt game

By plotting the previous results, we can come along with what the article we got inspired from talked about. According to the paper "Evolutionary game theory: Temporal and spatial effects beyond replicator dynamics" by Roca, Cuesta, and Sánchez, the choice of update rules may not be significant in a complete network, which refers to a well-mixed or unstructured population. In such networks, the differences in update rules generally do not alter the overall evolutionary outcome. However, the paper highlights that these differences in update rules become crucial when the population has some structure, indicating the presence of network connections or spatial arrangements. In other words, when the population is organized in networks other than a complete network, the choice of the update rule has more substantial consequences.

The study contends that the underlying network topology determines how update rules affect a game's evolutionary dynamics. Depending on how the individuals are connected or geographically distributed, different update rules can result in a variety of outcomes, such as the emergence of distinct strategies or the stability of particular strategies. The study thus intends to investigate the impact of the update rule choice on the behavior and results of evolutionary games by looking at various types of networks. It implies that network structure is crucial in establishing the significance and ramifications of choosing particular update rules while attempting to understand the dynamics of population evolution.

4.2 Community Graphs

Community graphs, also known as community structure or modular networks, are a type of complex network where nodes are organized into distinct groups or communities. In fact, in community graphs, nodes within the same community are densely connected to each other, while connections between nodes in different communities are sparser. The goal of community detection is to identify these communities and understand the patterns of connectivity within and between them. In game theory, community graphs play a crucial role in studying and analyzing various aspects of strategic interactions among players. The concept of community graphs helps capture the structure of relationships and interactions within a population of players.

4.2.1 Built In Community Graph

The graph consists of 100 nodes, and the average node degree is 10, indicating that each node is connected to approximately 10 other nodes on average. The graph we did build, is organized into 4 distinct communities, with each community having a higher probability (0.3) of generating edges within itself compared to generating edges between different communities (0.05). This probability distribution reflects the tendency of nodes within the same community to be more strongly connected to each other.

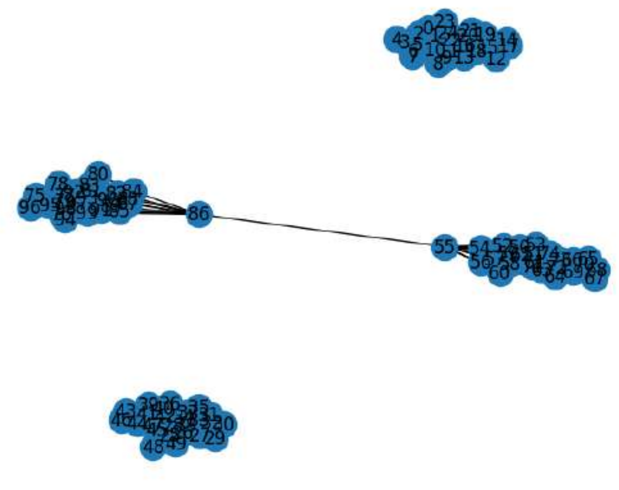

Figure 9: Plot showing the community graph

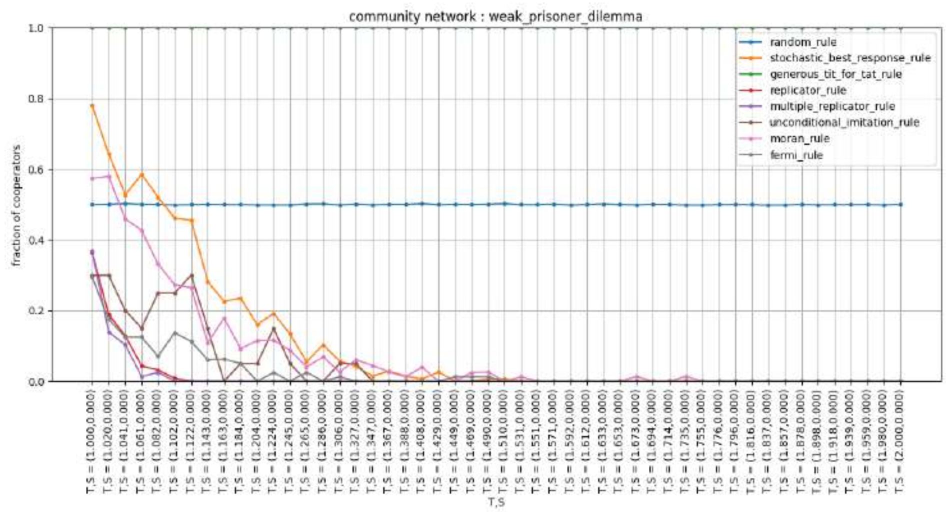

Figure 10: Plot showing the result for the weak prisoner dilemma game

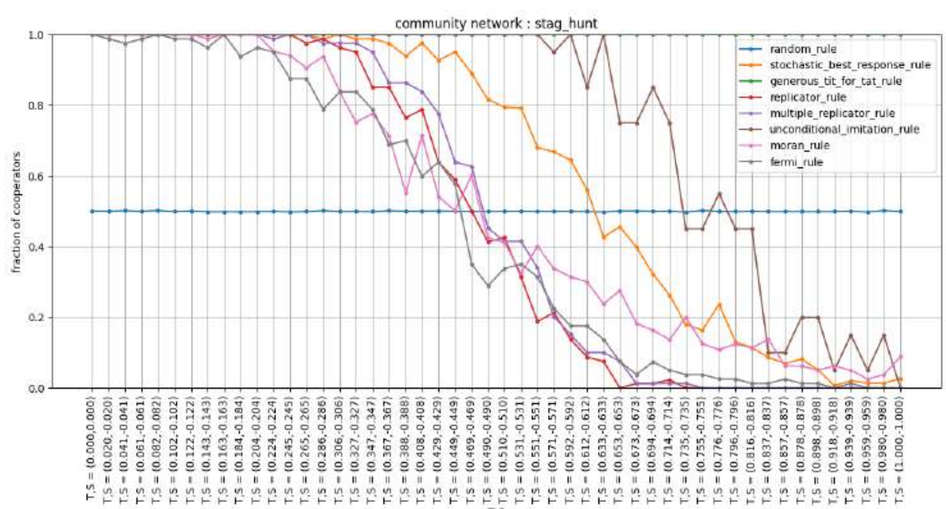

Figure 11: Plot showing the result for stag hunt game

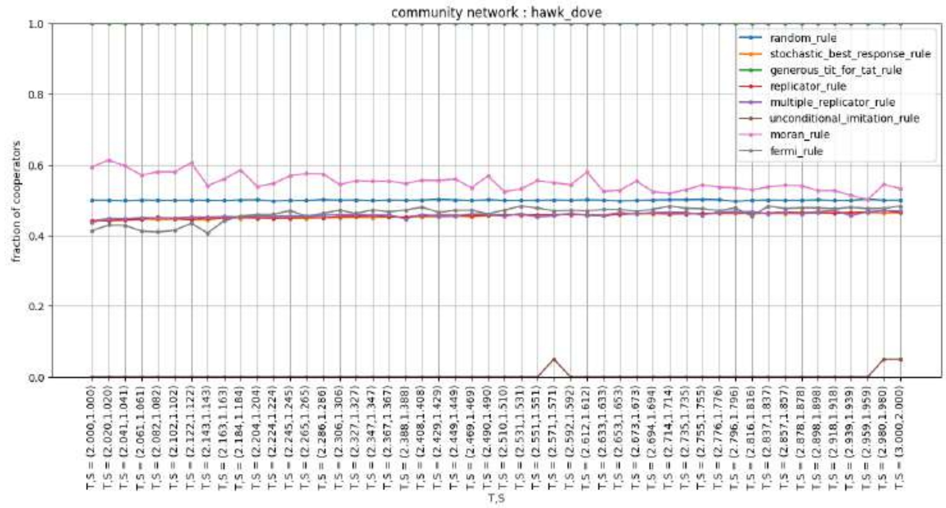

Figure 12: Plot showing the result for the hawk dove game

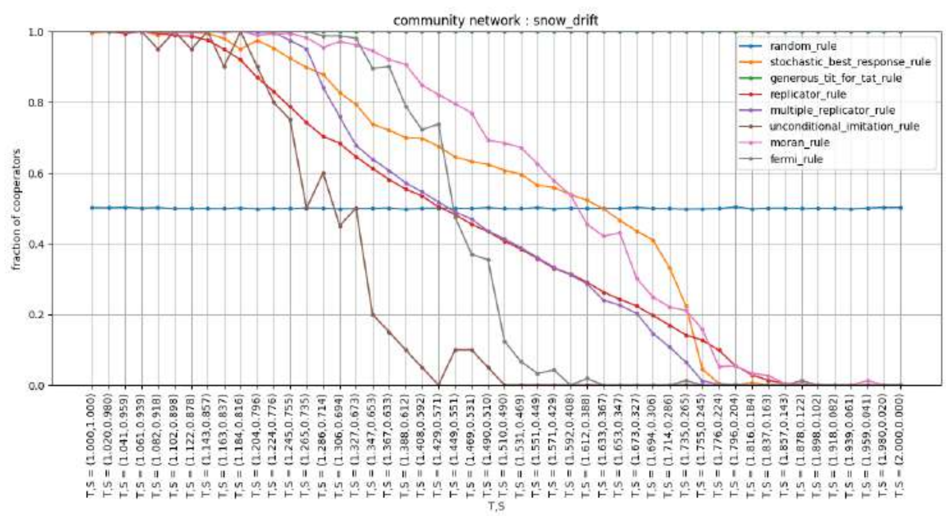

Figure 13: Plot showing the result for the snow drift

4.2.2 LFR Community Graph

LFR (Lancichinetti-Fortunato-Radicchi) community graph is a type of synthetic network model that aims to generate realistic complex networks with community structure. It is based on the principles of hierarchical organization and preferential attachment. In an LFR community graph, nodes are organized into communities or modules, and each node is assigned a community membership. The community structure is characterized by varying community sizes and overlapping memberships, reflecting the heterogeneity and complexity often observed in real-world networks. The model also incorporates the concept of "mixing parameter" that controls the extent of inter-community connections.

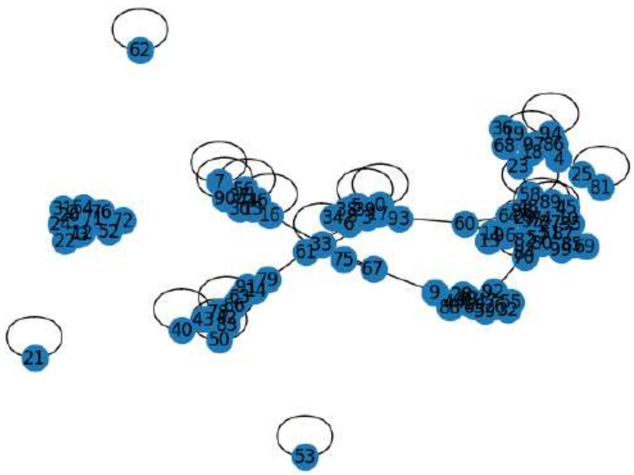

Figure 14: Plot showing the LFR graph

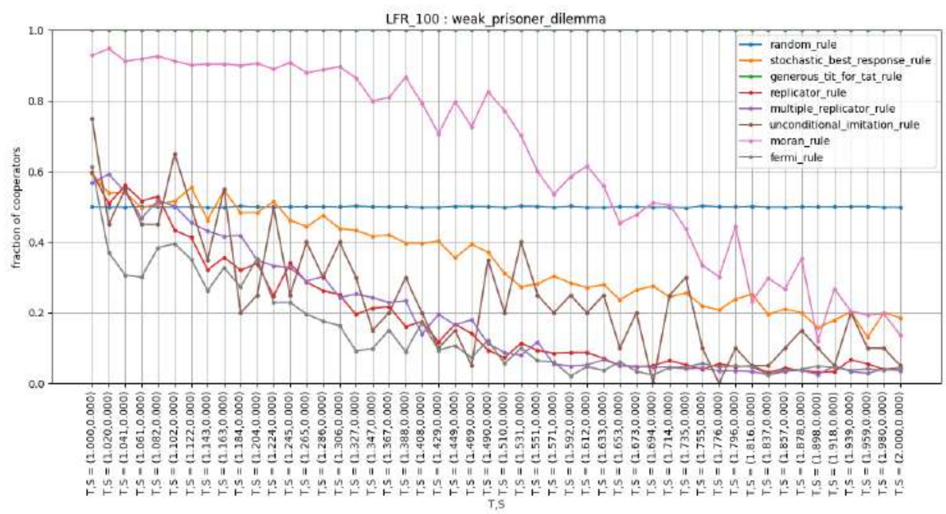

Figure 15: Plot showing the result for the weak prisoner dilemma

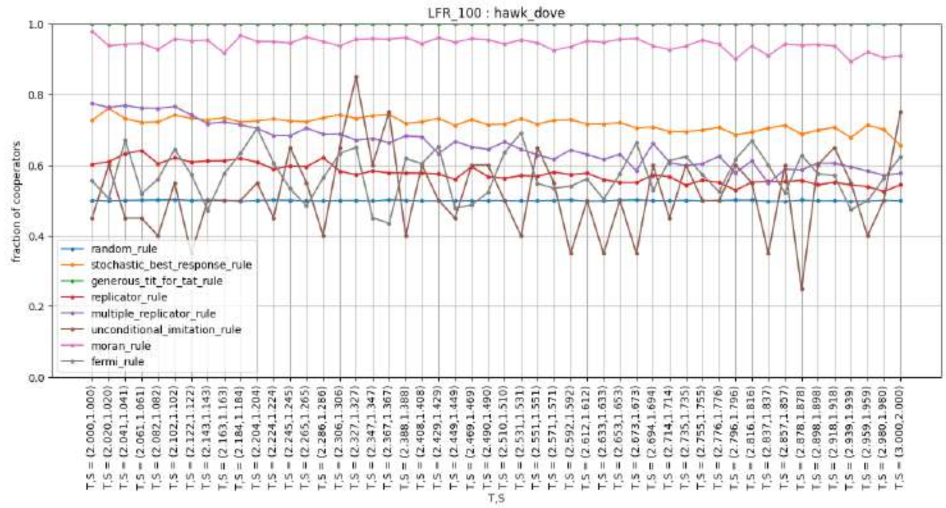

Figure 16: Plot showing the result for the hawk dove

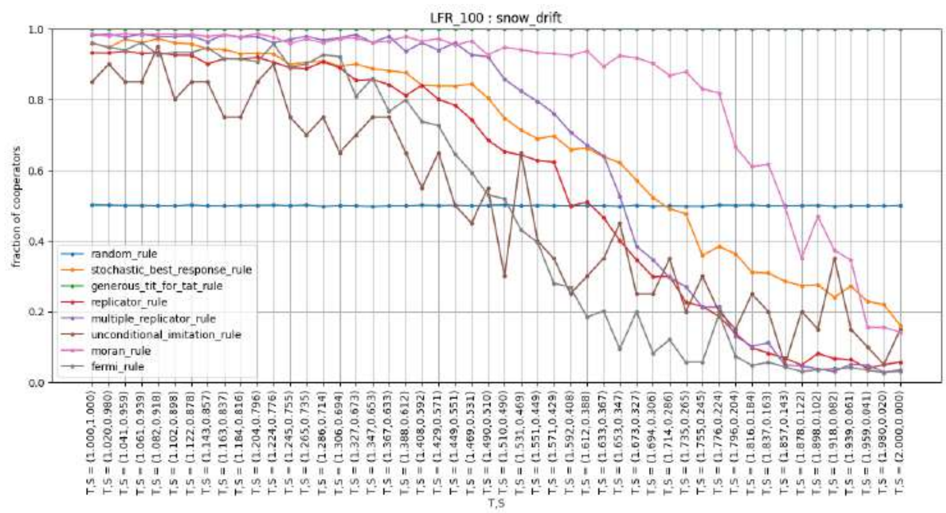

Figure 17: Plot showing the result for the snow drift

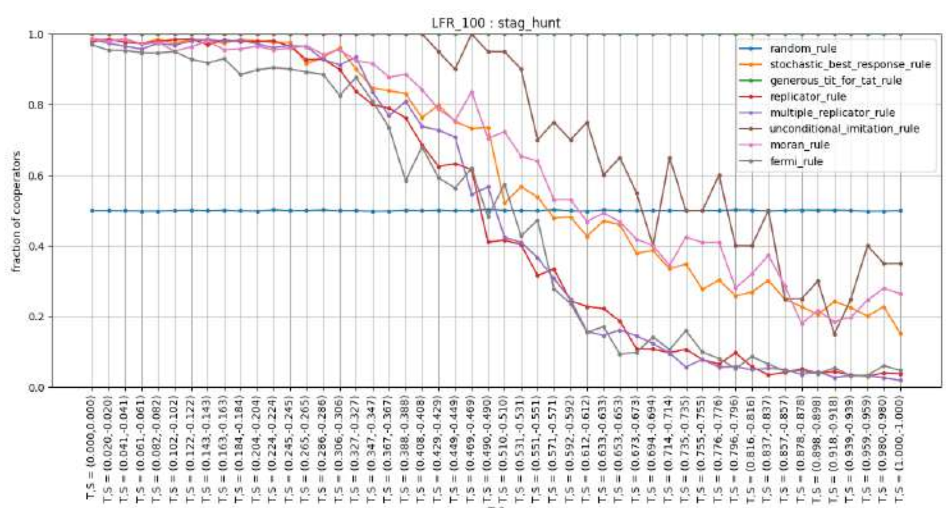

Figure 18: Plot showing the result for the stag hunt

4.2.3 SBM Community Graph

A Stochastic Block Model (SBM) graph is a type of synthetic network model that is generated based on a probabilistic framework. In an SBM graph, the network is partitioned into multiple communities, and the connectivity patterns within and between communities are defined by probabilities. SBM graphs provide a framework for studying networks with community structure, while game theory offers tools to analyze the strategic behavior and interactions of players within and between these communities.

In our case, that´s how we defined the stochastic block model.

- **Node Allocation** : The total number of nodes (n) is divided into the specified number of communities (num-communities). In this case, there are 4 communities, each potentially containing 25 nodes.
- **Edge Generation** : For each pair of nodes, the connectivity between them is determined based on their community membership. If both nodes belong to the same community, an edge is generated with a probability of p-in (0.8 in this case). If the nodes belong to different communities, an edge is generated with a probability of p-out (0.1 in this case).
- **Average Node Degree** : The average node degree (k) specifies the expected number of edges connected to each node on average. It helps ensure that the resulting graph has the desired density of connections.

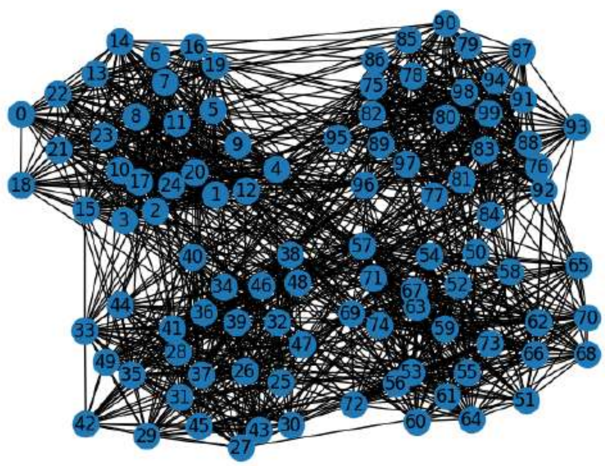

Figure 19: Plot showing the SBM graph

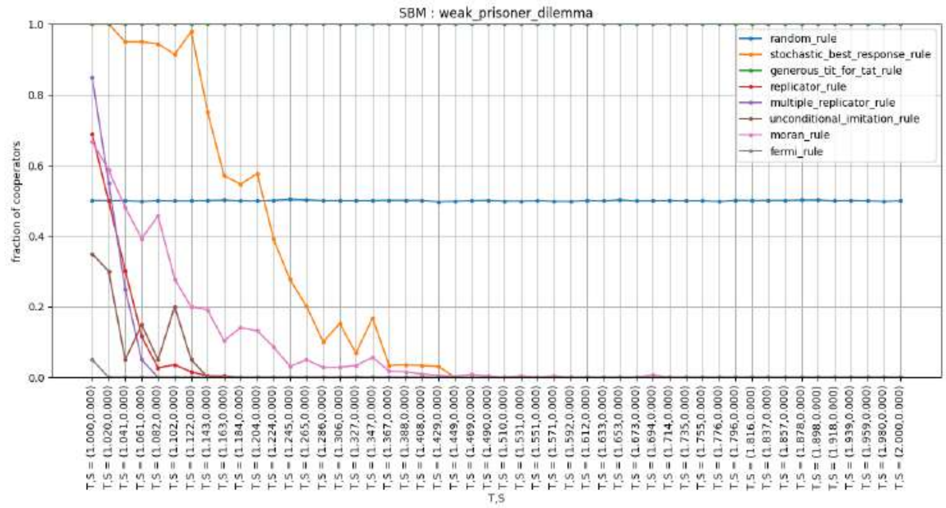

Figure 20: Plot showing the result for the weak prisoner dilemma

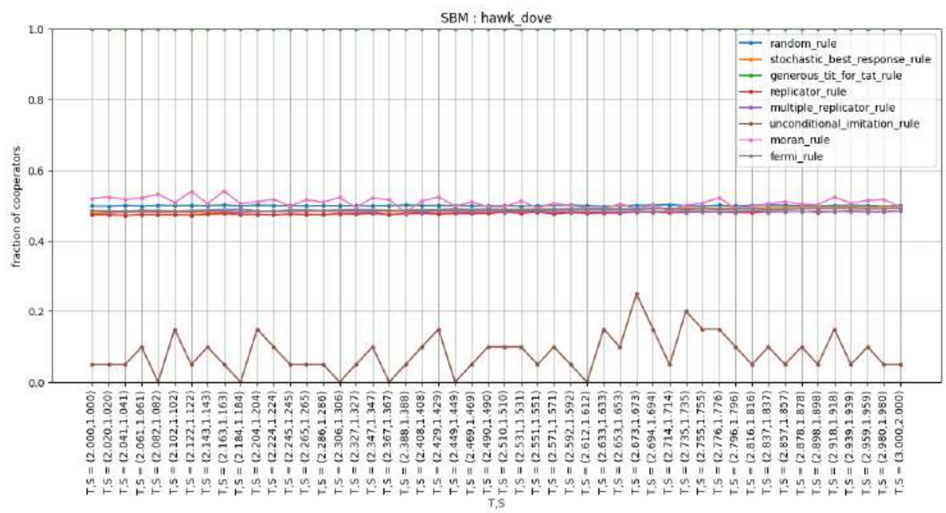

Figure 21: Plot showing the result for the hawk dove

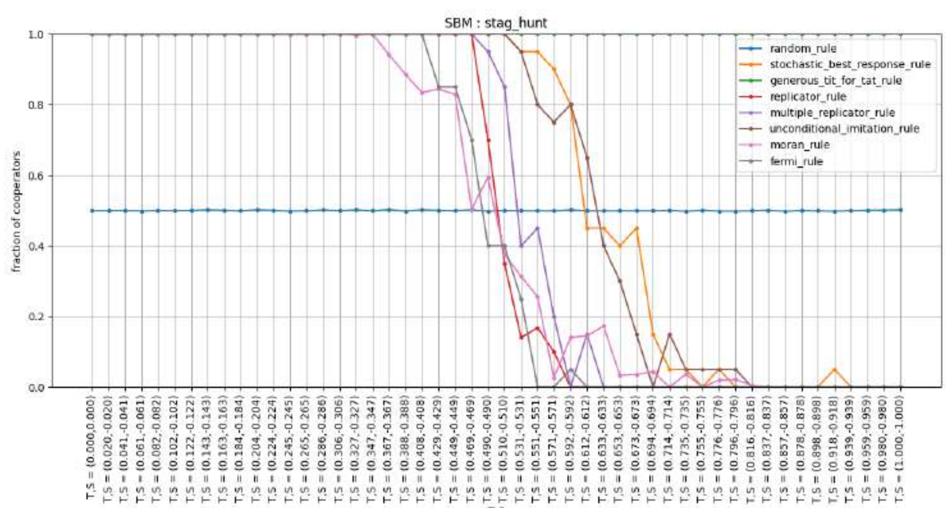

Figure 22: Plot showing the result for the stag hunt

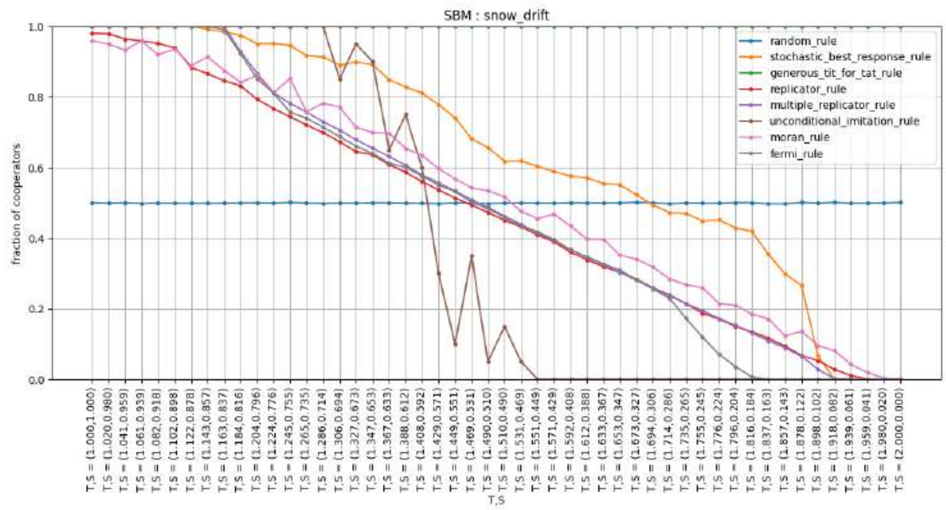

Figure 23: Plot showing the result for the snow drift

4.3 Small World Network

A small-world network refers to a type of network characterized by a high level of clustering, where nodes tend to form tightly connected local groups or clusters, combined with short average path lengths between any pair of nodes. In other words, it exhibits a balance between local clustering and global connectivity. In game theory for instance, small-world networks are significant as they capture the underlying structure of many real-world social, economic, and biological networks. Small-world networks provide a framework for studying how individuals interact and make decisions in a networked environment.

In our case, we are going to use the Watts-Strogatz model. The model was introduced by Duncan J. Watts and Steven H. Strogatz in 1998, is a random graph generation model that aims to capture both the regularity and small-world properties observed in many real-world networks. The construction of the Small World Network involves the following steps:

- **Regular Ring Lattice** : We start with a regular ring lattice, where each node is initially connected to its k nearest neighbors. This regular ring lattice exhibits high local clustering since nodes tend to have connections with their immediate neighbors.
- **Rewiring Process** : We terate through each edge in the regular ring lattice. For each edge, with a probability of p, rewire the edge by randomly choosing another node in the network and connecting the current node to that chosen node. This rewiring process introduces randomness and creates shortcuts or long-range connections.

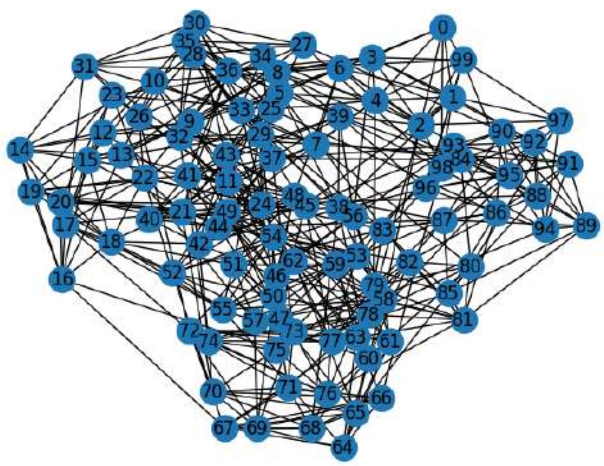

Figure 24: Plot showing the small network graph

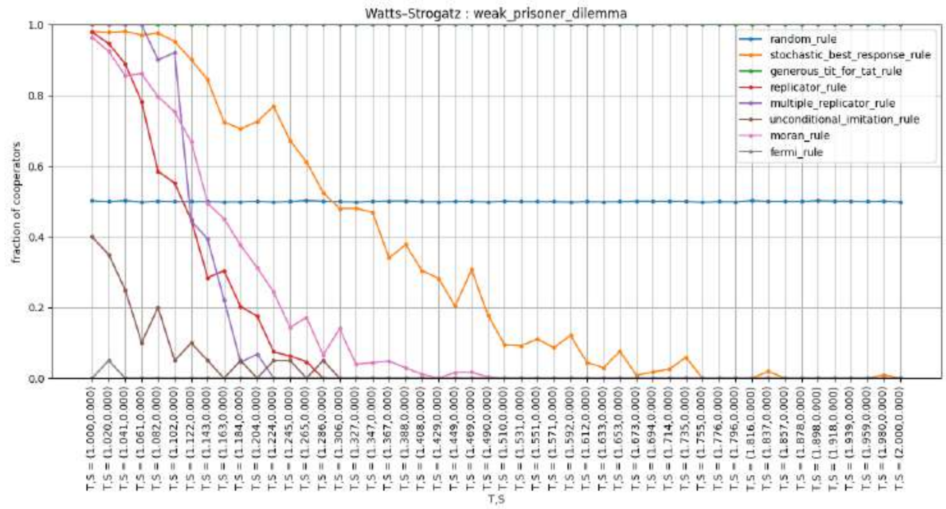

Figure 25: Plot showing the result for the weak prisoner dilemma

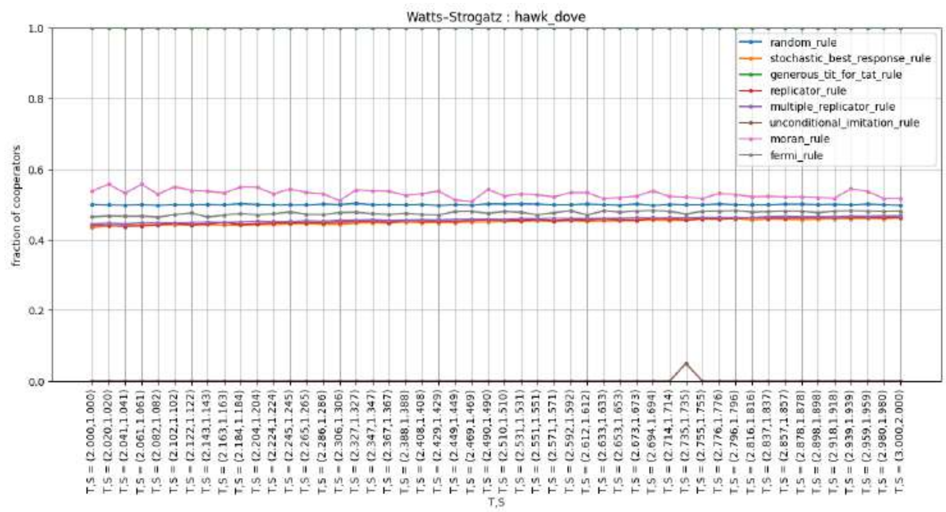

Figure 26: Plot showing the result for the hawk dove

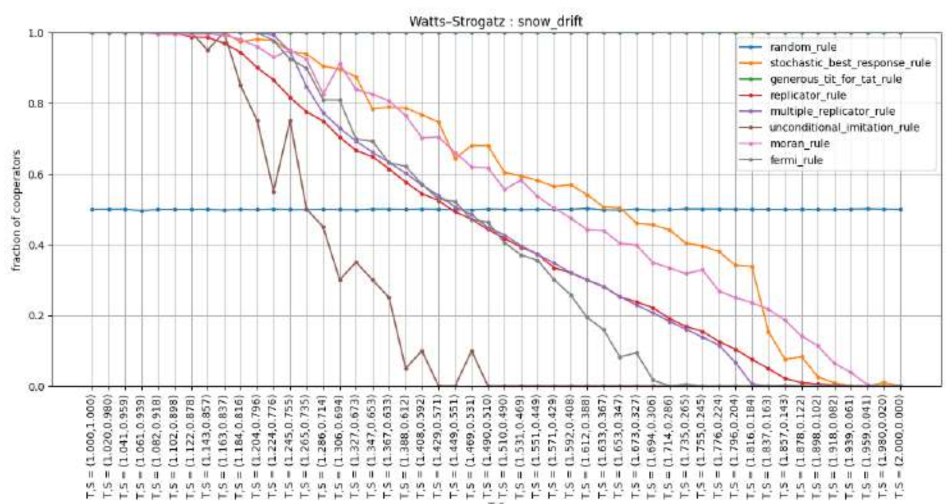

Figure 27: Plot showing the result for the snow drift

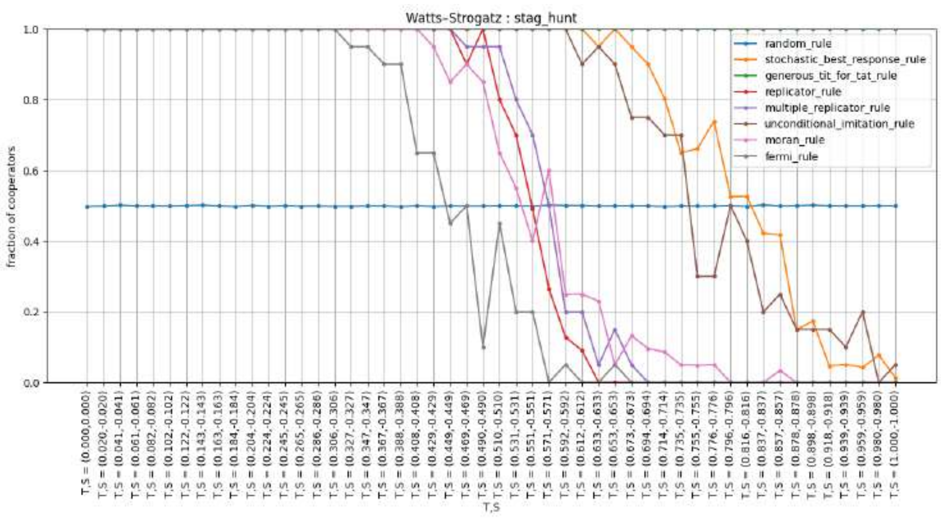

Figure 28: Plot showing the result for the stag hunt

4.4 Real World Network

In this part, and since we were inspired by the article, they often mentioned the use of real world network and their overall impact on the game theory. For our case, we choose the most popular and most used real world case ´Karate Club´ Graph.

The Karate Club graph, also known as the Zachary's Karate Club graph, is a well-known social network graph that represents the social interactions among members of a university karate club. The graph is based on a study conducted by Wayne W. Zachary in the early 1970s, documenting the interactions and subsequent split of the club.

- **Nodes** : he nodes in the graph represent individual members of the karate club. There are 34 nodes in total, each representing a member of the club.
- **Edges** : The edges in the graph represent social connections or interactions between club members. An edge is drawn between two nodes if the corresponding members interacted or had a connection outside of the karate club activities.
- **Weighted Edges** : In the original graph, the edges are unweighted, meaning they do not have specific numerical values or weights associated with them. However, in some versions of the graph, the edges may be assigned weights based on the strength or frequency of interactions between members.

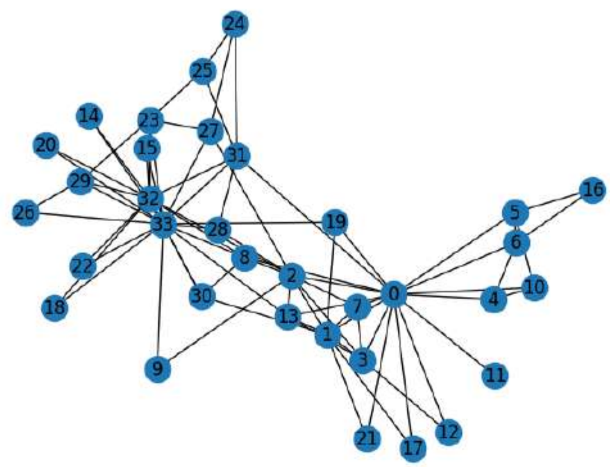

Figure 29: Plot showing the real world graph

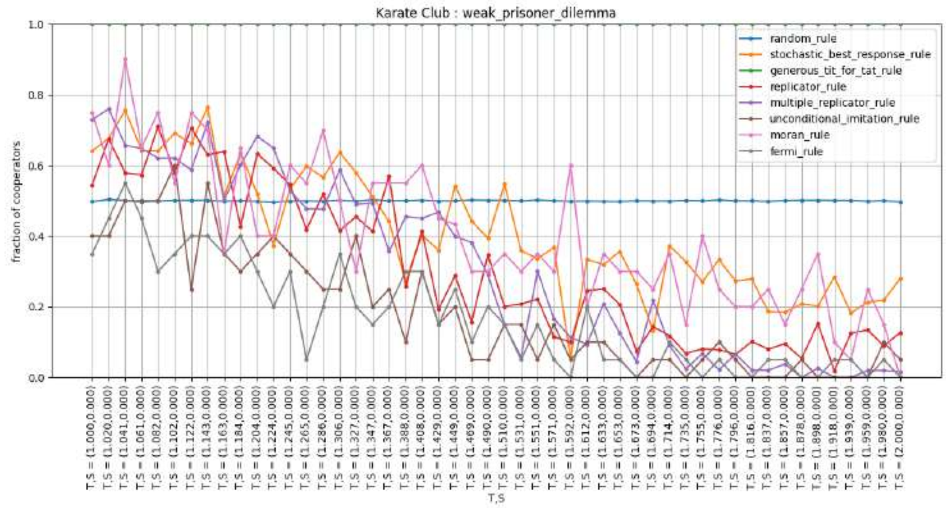

Figure 30: Plot showing the result for the weak prisoner dilemma

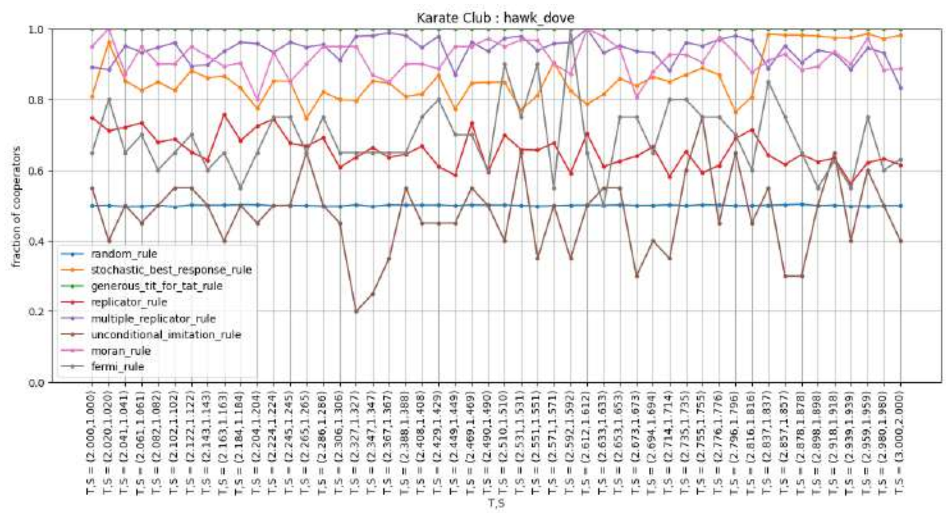

Figure 31: Plot showing the result for the hawk dove

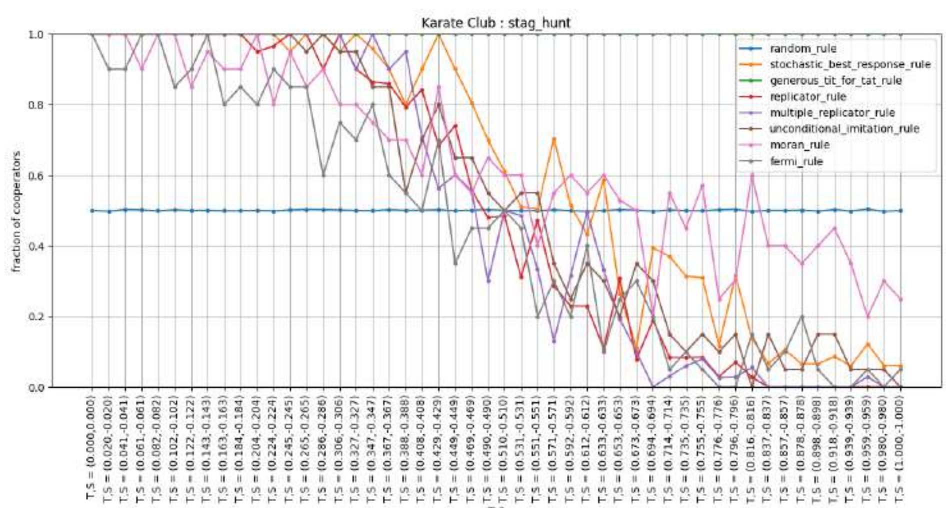

Figure 32: Plot showing the result for the stag hunt

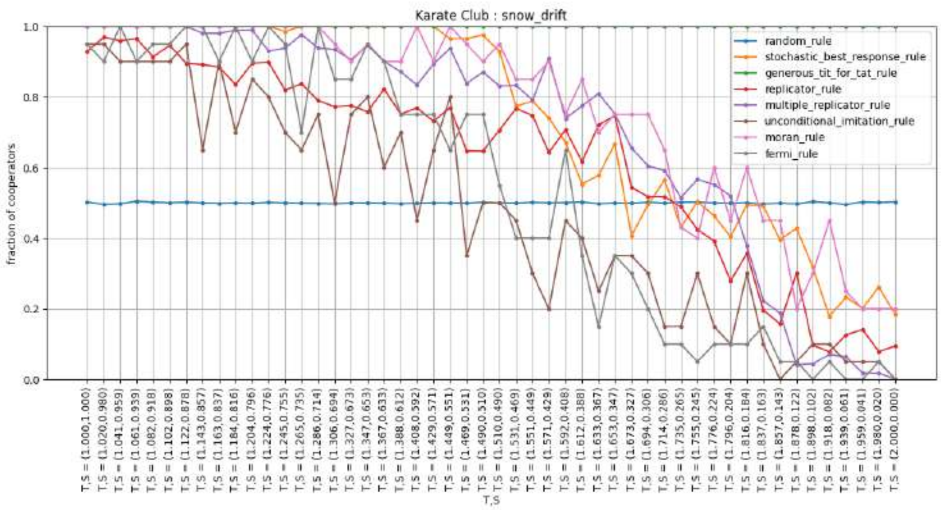

Figure 33: Plot showing the result for the snow drift

4.5 Homogeneous Random Graph

A Homogeneous Random Graph, also known as an Erdős-Rényi random graph, is a simple model for generating random graphs. In our model, the graph is constructed by independently deciding whether each pair of nodes is connected by an edge or not, based on a fixed probability. In fact, in game theory, a Homogeneous Random Graph can be a valuable tool for studying strategic interactions and the dynamics of games among a population of players. We are using networkx to import it and use it, then we define the following parameters : Number of Nodes: 100 and Degree of Nodes: 5.

In this graph, each node is connected to exactly 5 neighbors, resulting in a regular degree distribution throughout the network. The graph is randomly generated, meaning the connections are assigned randomly while ensuring each node has the same degree.

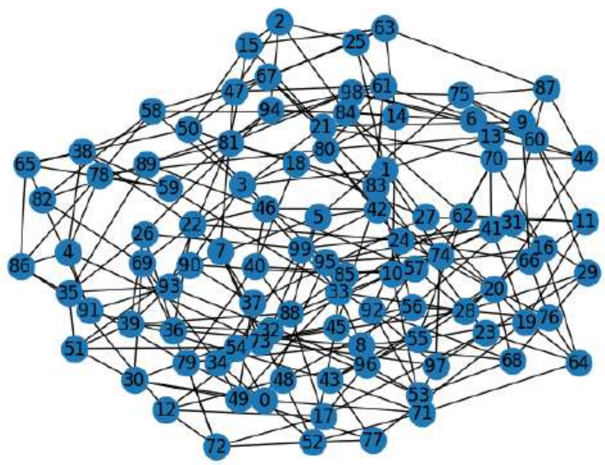

Figure 34: Plot showing the HRG graph

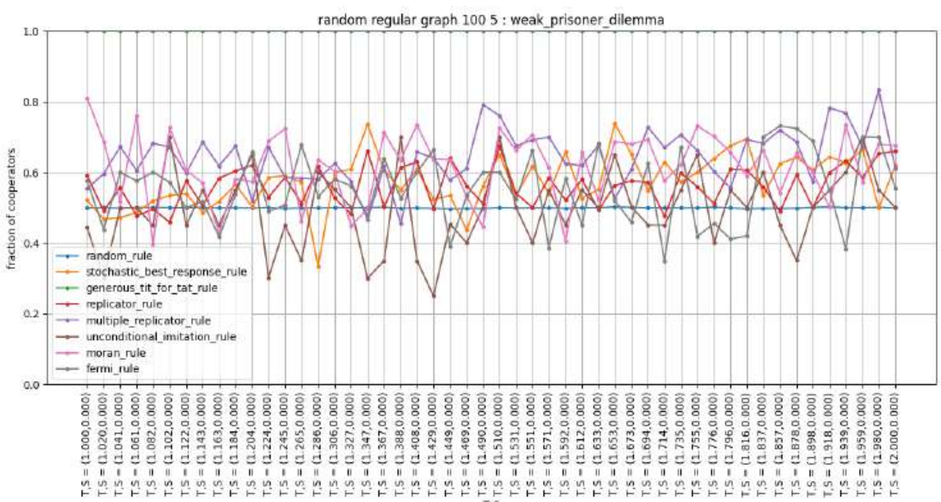

Figure 35: Plot showing the result for the weak prisoner dilemma

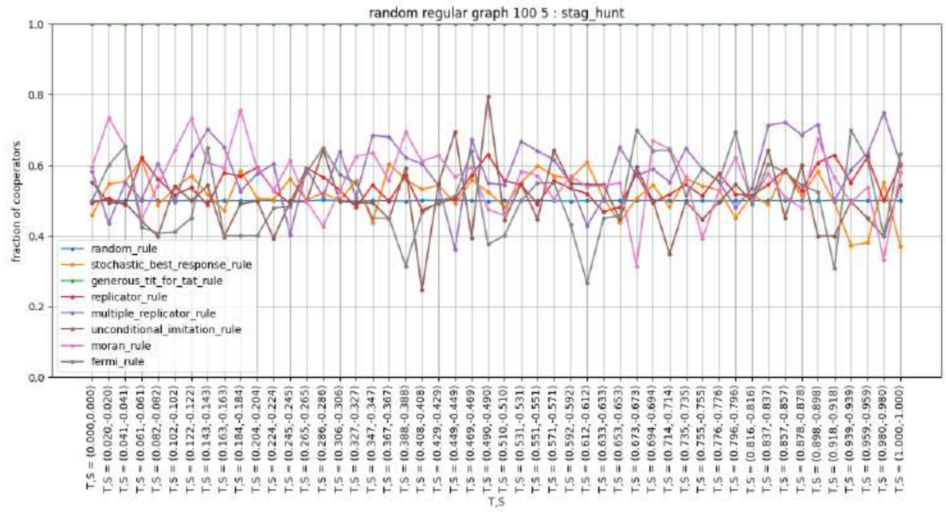

Figure 36: Plot showing the result for the stag hunt

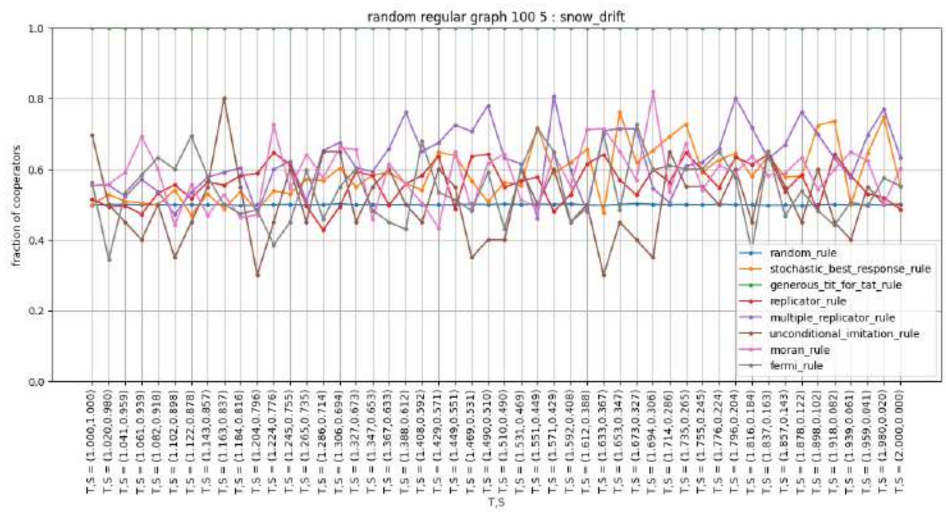

Figure 37: Plot showing the result for the snow drift

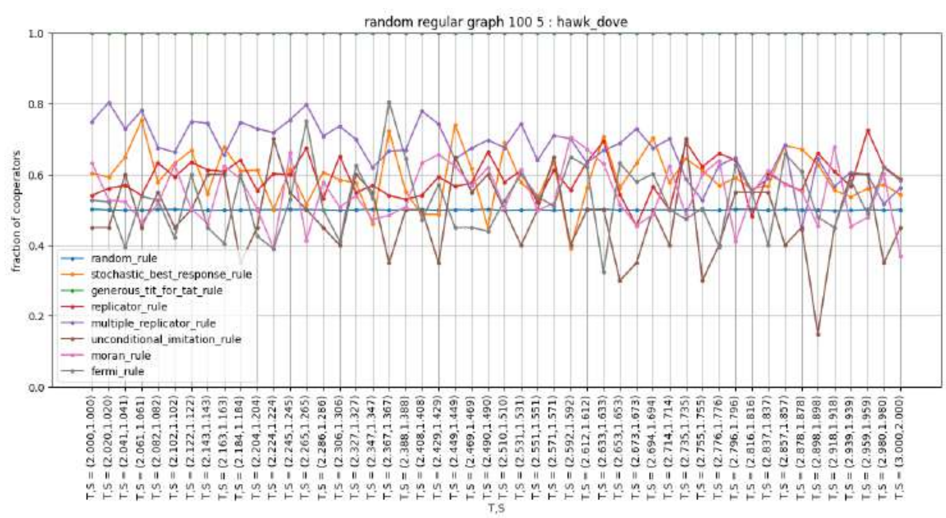

Figure 38: Plot showing the result for the hawk dove

Due to the nature of homogeneous random networks, reaching a stable state takes significantly more time compared to other models. In order to accommodate this, we will extend the simulation time by increasing Tmax and Ttrans values to Ttrans = 9000 and Tmax = 10000, respectively.

As a result, the computational cost substantially increases. We will still be plotting most of the rules since we believe it´s important to get an overview of all the game theories and update rules. And also this selection allows for a comparison of results with subsequent networks that exhibit different degree heterogeneity.

By implementing these changes, we can effectively analyze the behavior and outcomes of all the update rules but especially the replicator and unconditional imitation rules in the context of homogeneous random networks, while also setting the stage for future exploration of networks with varying degrees of heterogeneity.

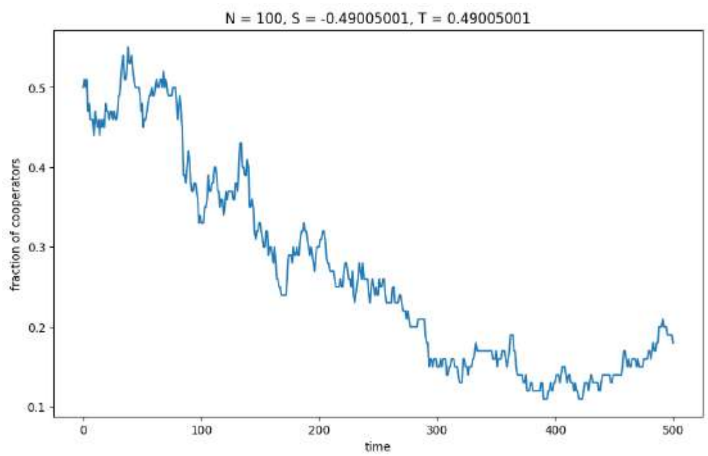

Figure 39: Plot showing the temporal presentation of the simulation

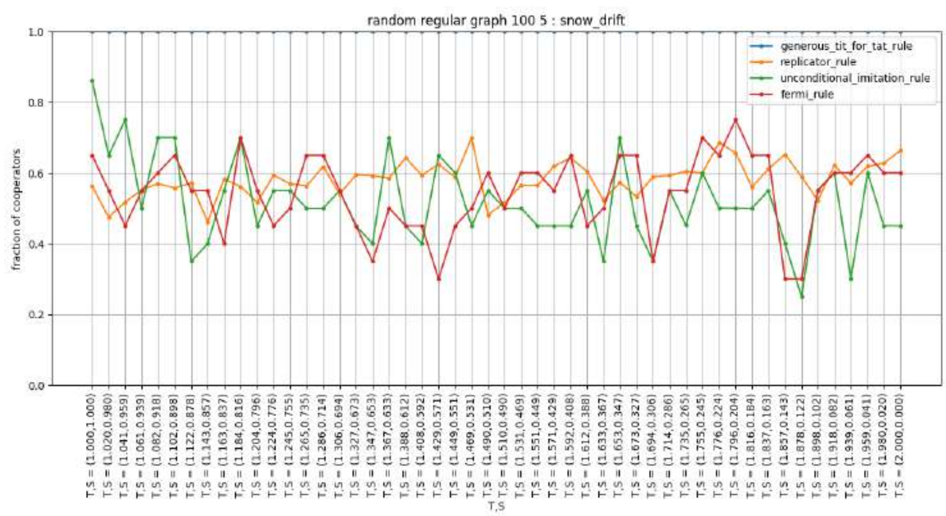

Figure 40: Plot showing the result for the snow drift and Tmax = 1000

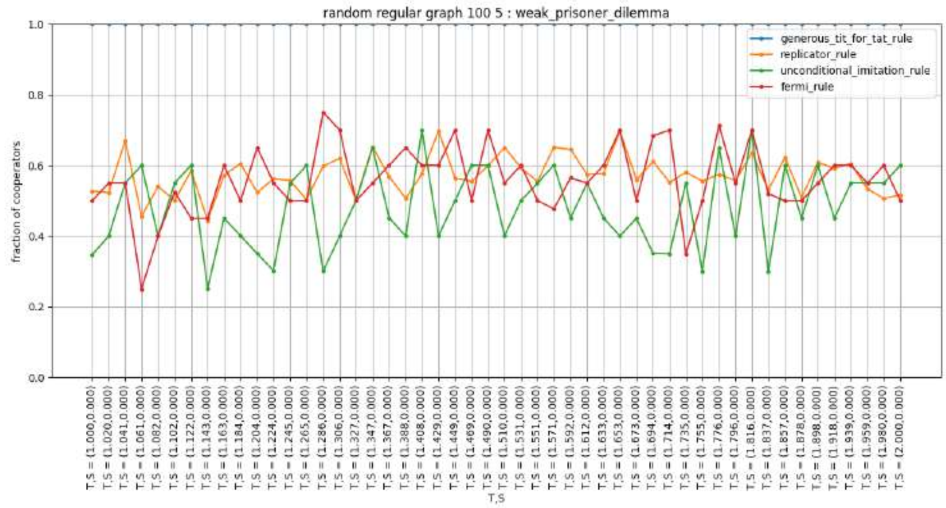

Figure 41: Plot showing the result for the weak prisoner dilemma and Tmax = 1000

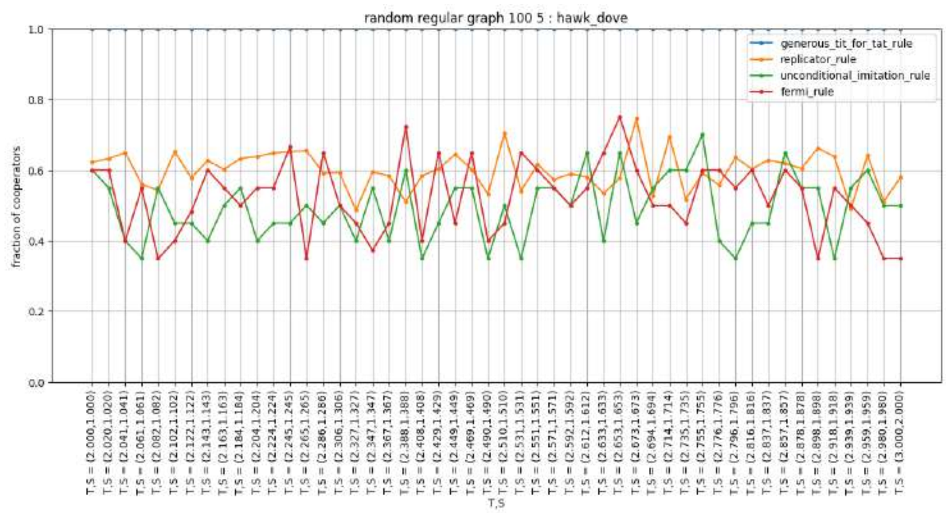

Figure 42: Plot showing the result for the hawk dove and Tmax = 1000

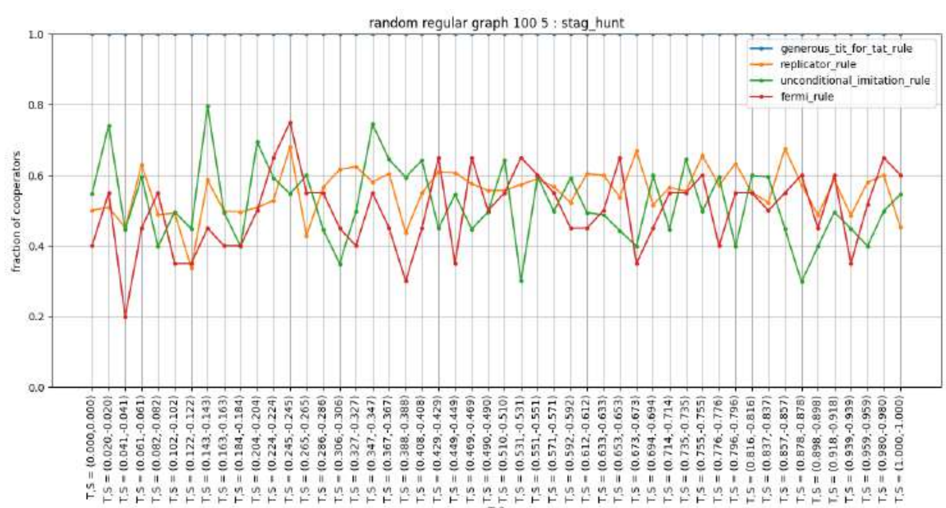

Figure 43: Plot showing the result for the stag hunt and Tmax = 1000

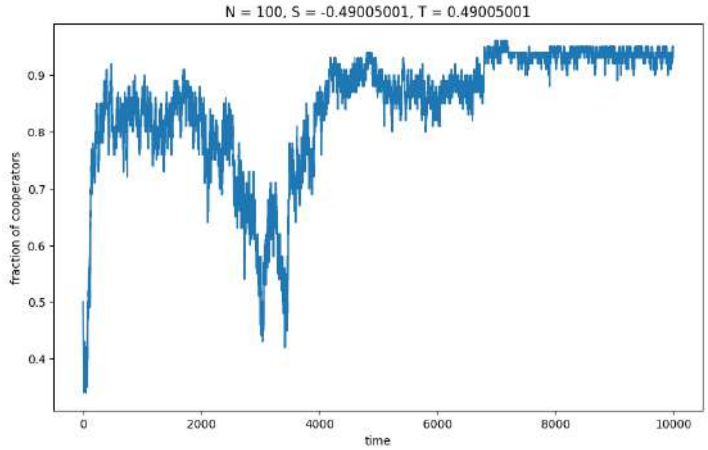

Figure 44: Game Simulation temporal representation with Tmax = 1000 and T = 0.49005001 and S = -0.49005001. Replicator Rule Case.

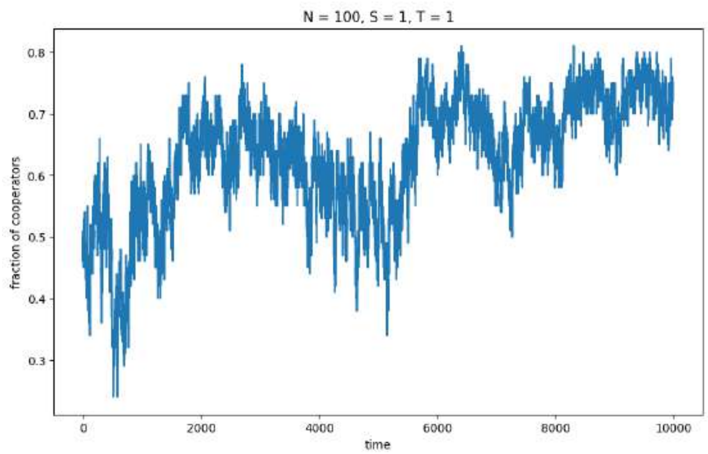

Figure 45: Game Simulation temporal representation with Tmax = 1000 and T = 1 and S = 1. Replicator Rule Case.

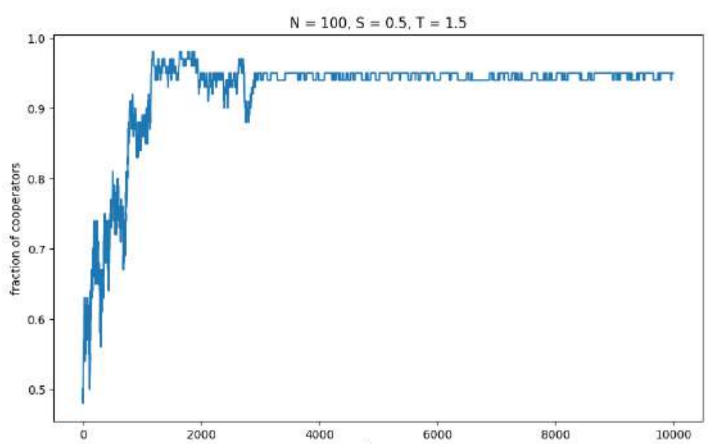

Figure 46: Game Simulation temporal representation with Tmax = 1000 and T = 1,5 and S = 0,5. Replicator Rule Case.

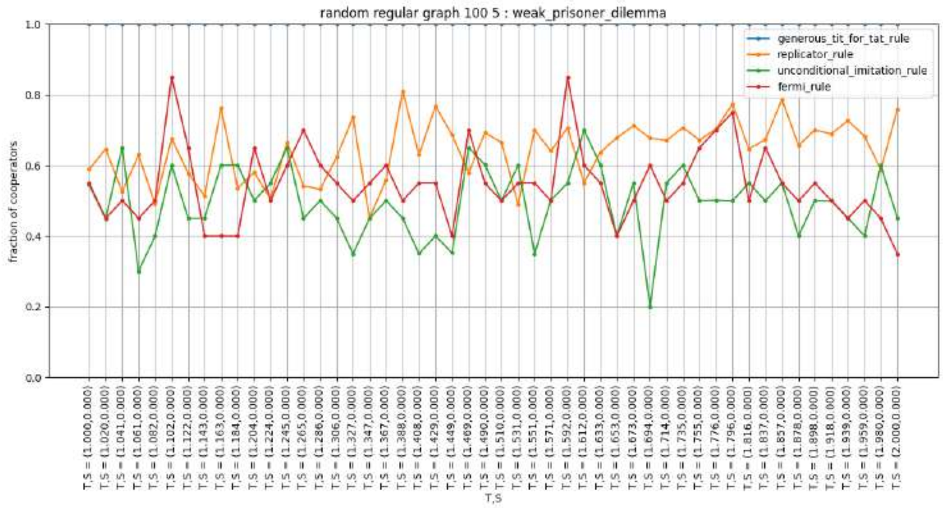

Figure 47: Plot showing the result for the weak prisoner dilemma game and T = 10000

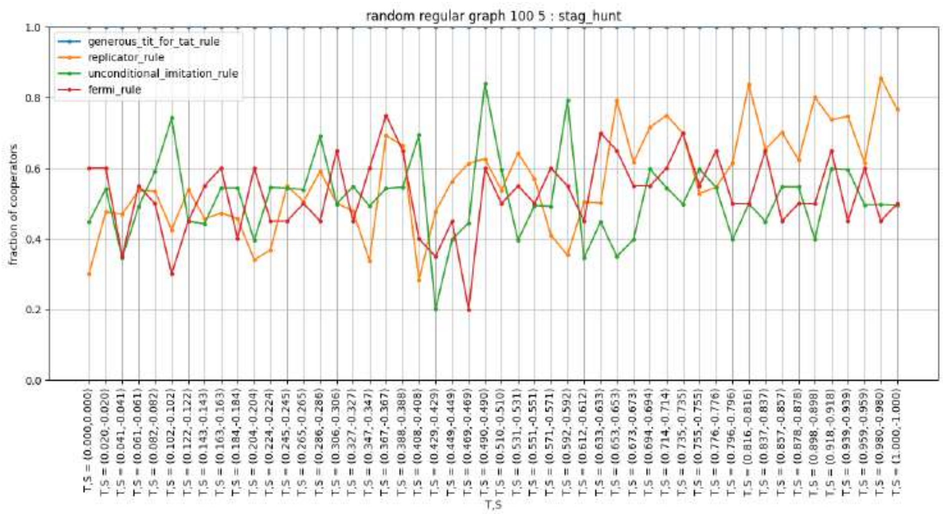

Figure 48: Plot showing the result for the stag hunt game and T = 10000

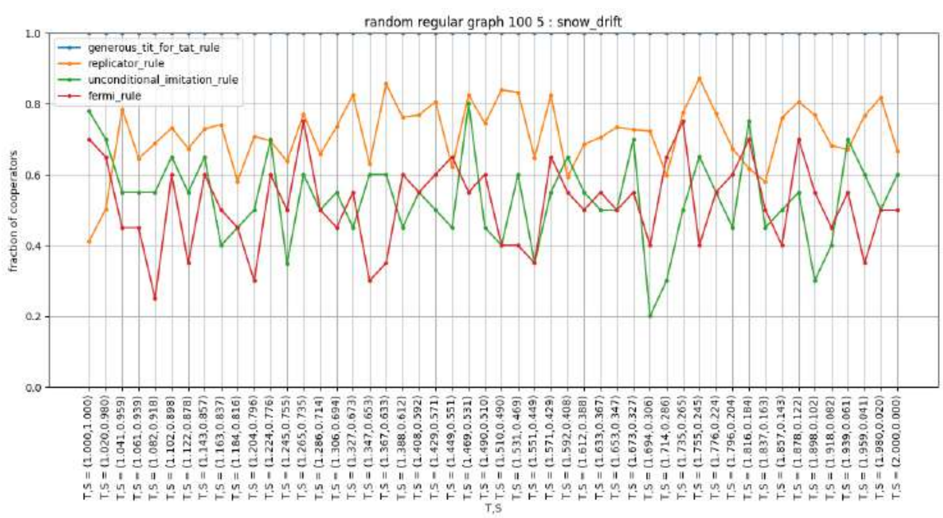

Figure 49: Plot showing the result for the snow drift game and T = 10000

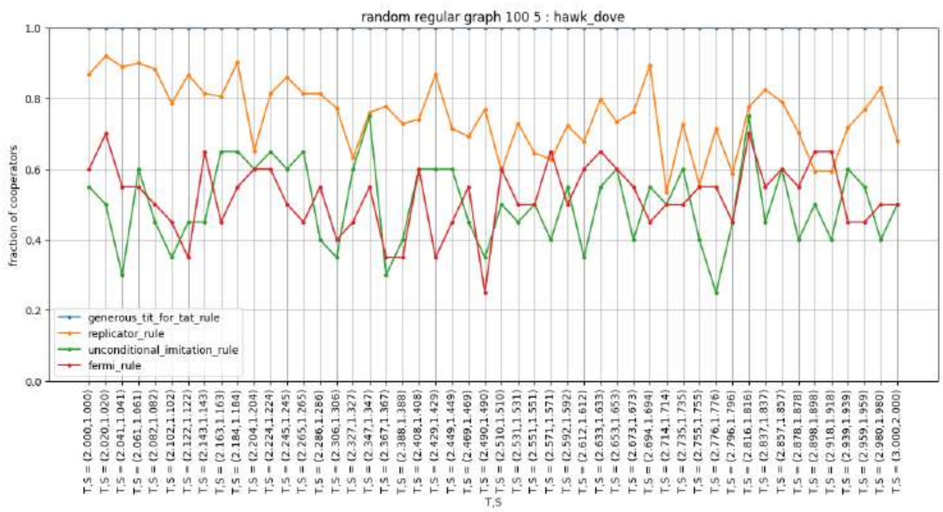

Figure 50: Plot showing the result for the hawk dove game and T = 10000

The line representing the unconditional imitation rule exhibits significant oscillation without a clear trend in the analyzed ST configurations. This rule is entirely deterministic, meaning that the fraction of cooperators reached in the stationary state is solely determined by the initial configuration of cooperators and defector nodes.

Interestingly, even when using the same S and T values and the same network structure, the final fraction of cooperators can vary widely depending on the random initial configuration. In some cases, the fraction may approach 0, indicating a predominantly defection strategy, while in other cases it may reach 1, indicating a majority of cooperation.

This variability in outcomes highlights the sensitivity of the unconditional imitation rule to the initial distribution of strategies. Small changes in the initial setup can lead to significantly different equilibrium states, emphasizing the role of initial conditions in determining the final outcome of the game. It is important to note that the oscillatory behavior and sensitivity to initial conditions make the unconditional imitation rule a complex and dynamic strategy within the context of the game being studied.

4.6 Barabasi-Albert Graph

Barabasi-Albert (BA) graph is a heterogeneous network. It is characterized by preferential attachment, which leads to a highly skewed degree distribution where a few nodes have a significantly higher number of connections compared to the majority of nodes. In the graph, new nodes are added to the network over time, and they preferentially attach themselves to existing nodes that already have a large number of connections.

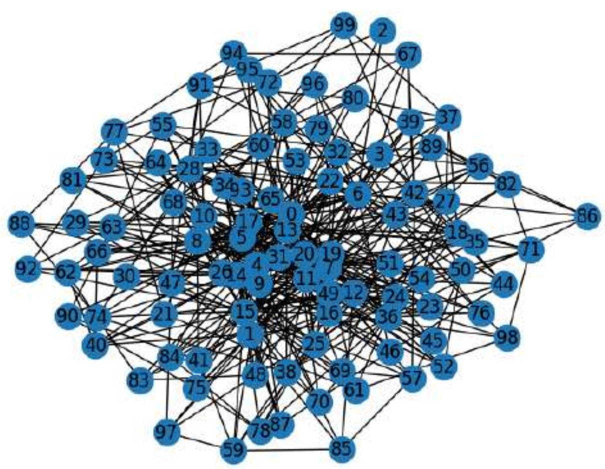

Figure 51: Plot showing the BA graph

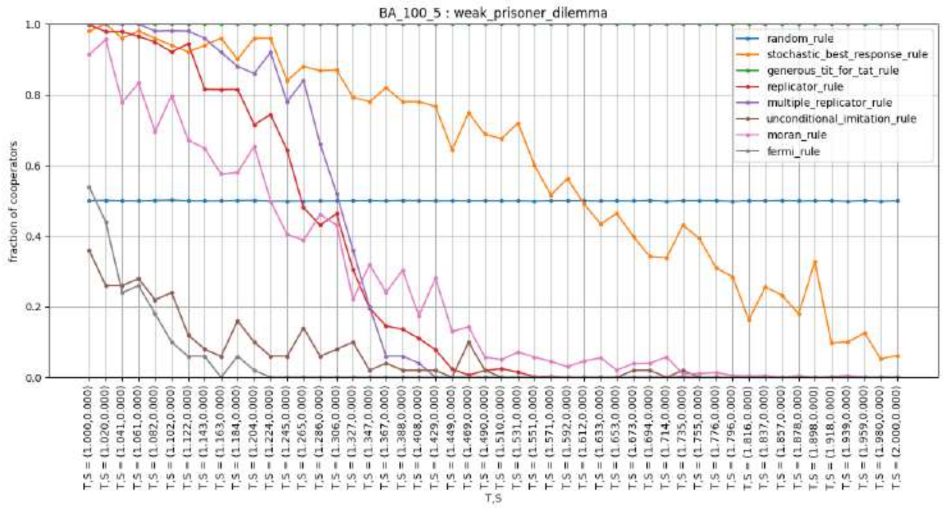

Figure 52: Plot showing the result for the weak prisoner dilemma game for degrees = 5

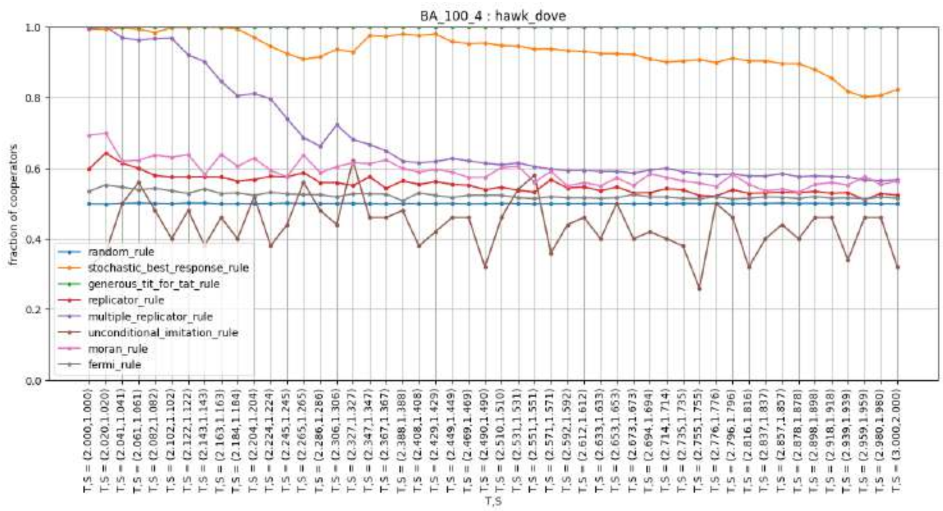

Figure 53: Plot showing the result for the hawk dove game for degrees = 5

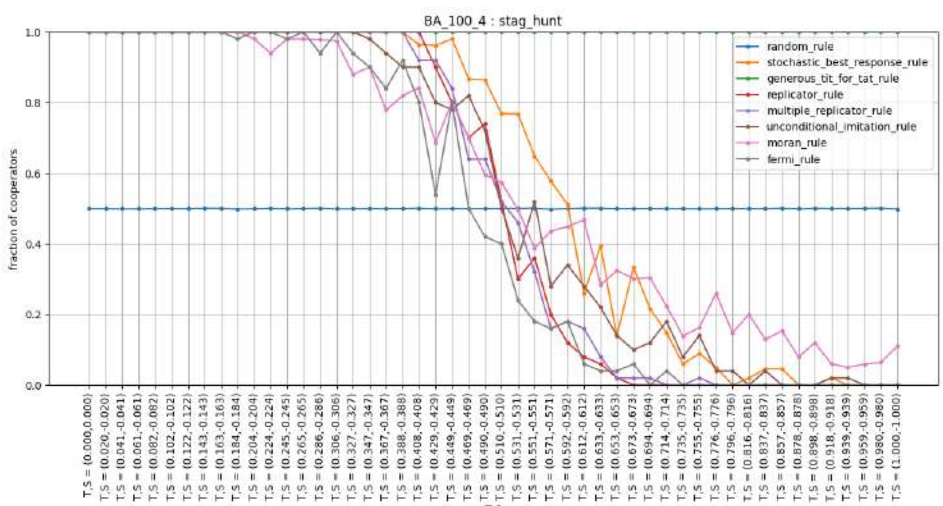

Figure 54: Plot showing the result for the stag hunt game for degrees = 5

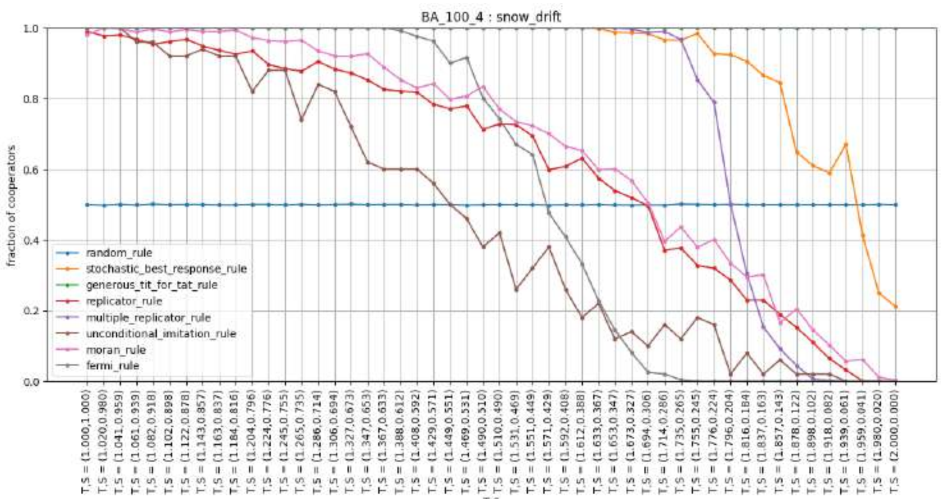

Figure 55: Plot showing the result for the snow drift game for degrees = 5

4.7 Erdos Renyi Graph

To analyse better the difference that connectivity patterns can creates. We are using the Erdos Renyi even if we previously used a random graph but there is a difference between the two. In an Erdos-Renyi graph, the presence of each edge is determined independently, resulting in a relatively uniform distribution of edges among the nodes. As a result, the degree distribution of an Erdos-Renyi graph tends to follow a binomial distribution, where most nodes have a similar number of connections. On the other hand, a homogeneous random network typically refers to a network where each node has the same number of connections or a similar degree. This can be achieved by designing a network with a regular or lattice structure, such as a regular grid or a ring lattice, where all nodes have the same degree.

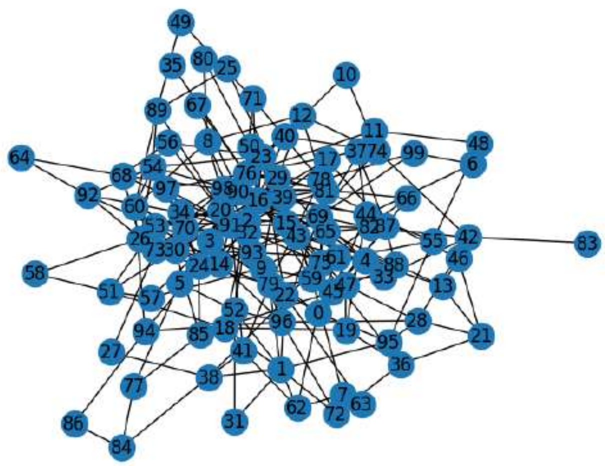

Figure 56: Plot showing the ER graph

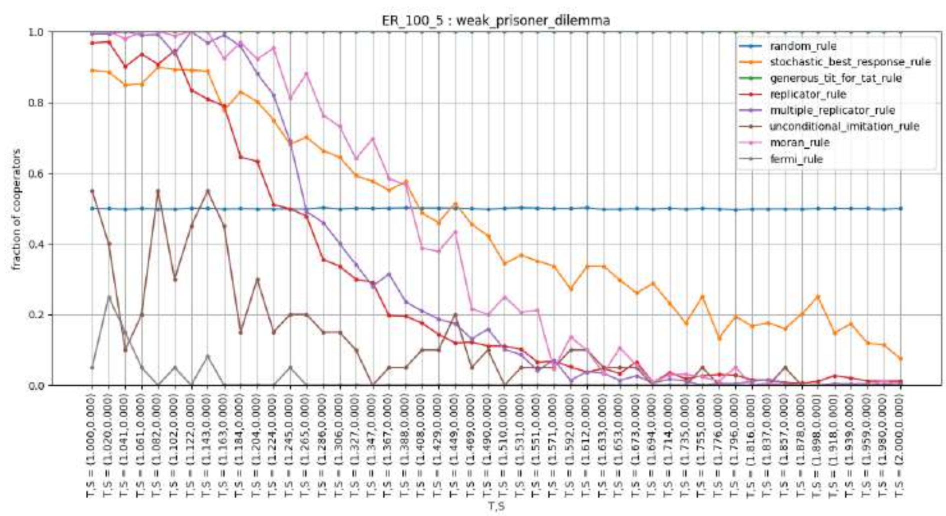

Figure 57: Plot showing the result for the weak prisoner dilemma game for degrees = 5

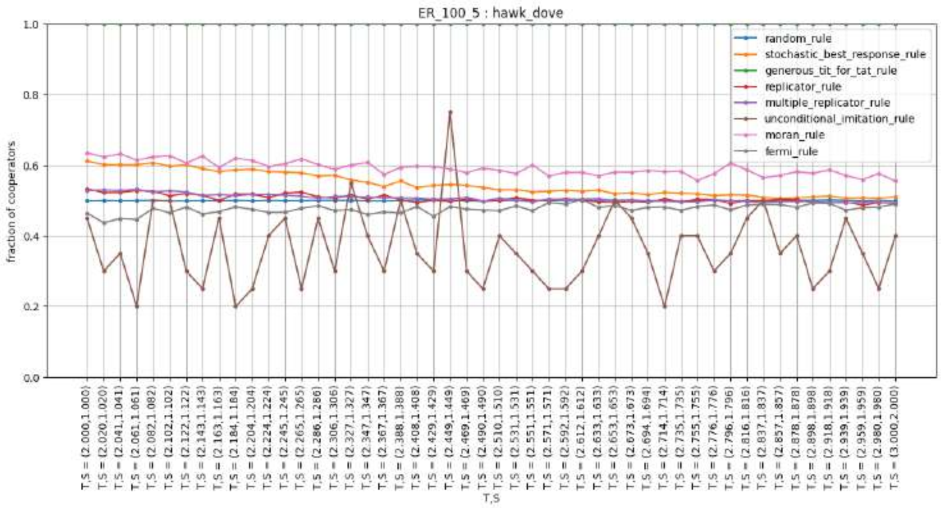

Figure 58: Plot showing the result for the hawk dove game for degrees = 5

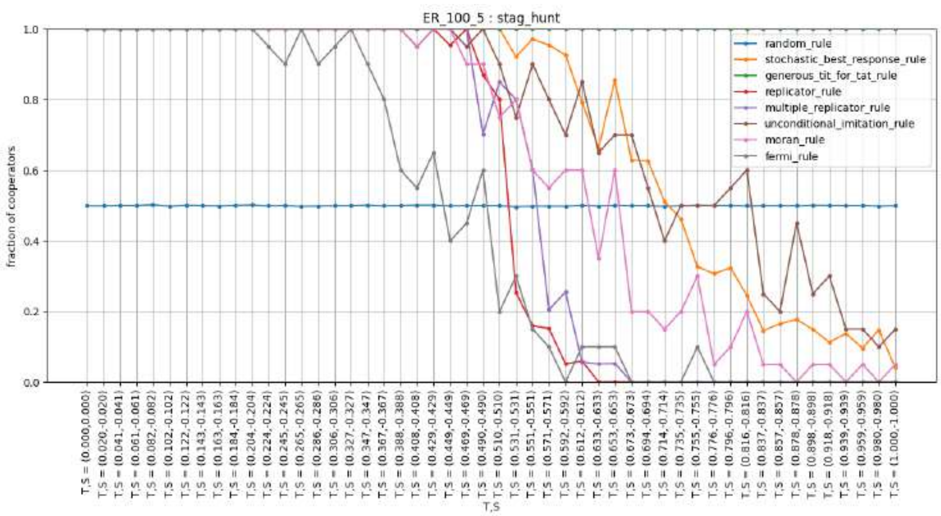

Figure 59: Plot showing the result for the stag hunt game for degrees = 5

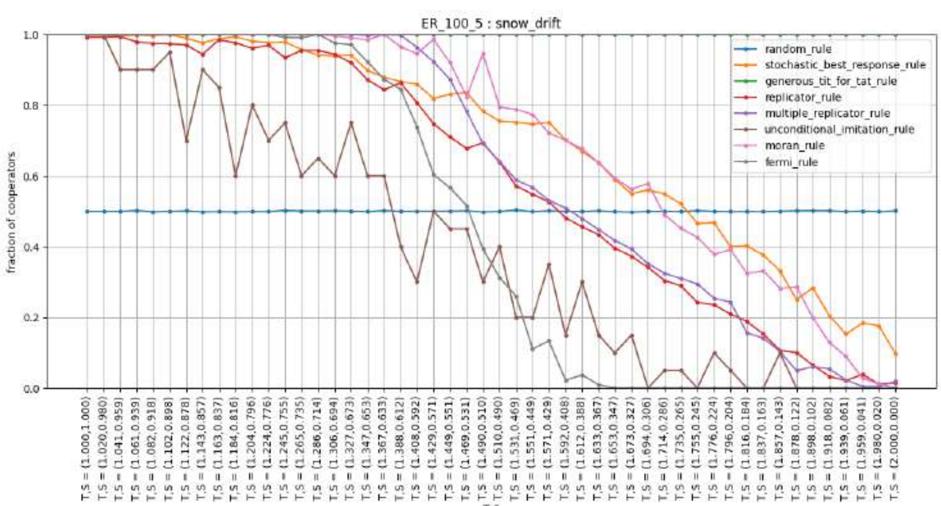

Figure 60: Plot showing the result for the snow drift game for degrees = 5

5 Conclusions of the project

To replicate certain findings from the referenced study, we undertook an investigation involving networks with different levels of heterogeneity in their node connections. The networks of interest included the homogeneous random network, Erdos-Renyi graph, and Barabasi-Albert graph. While other networks mentioned in the article could have been explored, time constraints limited the scope of the investigation.

It is important to note that, unlike the approach employed in the original study, the networks examined in this study had an average degree of 5. This distinction in average degree may have implications for the observed dynamics and outcomes of the simulations.

In our study, we observed a trend where an increase in degree heterogeneity within the networks positively influenced cooperation in snowdrift games. The results revealed that the area under the replicator rule curve, indicative of cooperation, was smaller for the homogeneous random network, larger for the ER network, and only marginally increased for the BA network. This aligns with findings reported in the article, highlighting the impact of network structure on cooperation dynamics.

We also observed a noteworthy trend where an increase in degree heterogeneity within the networks led to a suppression of cooperation in stag hunt games, particularly in the Barabasi-Albert (BA) and Erdos-Renyi (ER) networks. This inhibition of cooperation was evident by the larger area under the curve for the unconditional imitation rule line, with the ER network exhibiting a greater impact. In contrast, the results for the homogeneous random network were heavily influenced by the randomness of the initial configuration, making it challenging to draw conclusive insights. It is possible that conducting a higher number of repetitions for the Monte Carlo simulation could yield more reliable and meaningful results.

In line with findings presented in the article, we also observed an opposite effect in snowdrift games. Specifically, the area under the unconditional imitation rule curve was larger in the BA network, indicating a promotion of cooperation. Regarding the weak prisoner's dilemma, minimal differences were observed between the BA and ER networks analyzed, suggesting similar effects on cooperative behavior. These observations contribute to the growing understanding of how degree heterogeneity in networks can influence cooperative dynamics across different game scenarios. Furthermore, the findings highlight the need for careful consideration of network structures and their impact on game outcomes in future studies.

Personally, this project encompassed a rigorous and time-intensive research effort, involving an in-depth examination of the article and extensive simulations with substantial computation time exceeding 20 hours. Each game theory simulation required approximately 3 hours to execute, highlighting the intricacy and duration of the project. Nevertheless, the endeavor yielded significant findings regarding the influence of network structures and update rules on the overall outcomes of the game. The exploration of diverse network types provided valuable insights into the interplay between network structure and game dynamics. Although the project remains a work in progress, there is ample potential for further development and detailed analysis of each network type, considering the specific characteristics and idiosyncrasies of each network.

In summary, this project represents a significant academic undertaking, with a complex and multifaceted nature, warranting future exploration and refinement to enhance the comprehensive understanding of network effects on game theory outcomes.