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PETRI NET BASED TRAJECTORY OPTIMIZATION

Ákos Hajdu, Róbert Német,
Szilvia Varró-Gyapay, András Vörös

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OUTLINE OF THE TALK

1. Introduction
2. The CEGAR approach on Petri nets
3. Trajectory optimization using CEGAR
4. Evaluation
5. Conclusions

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1. **Introduction**
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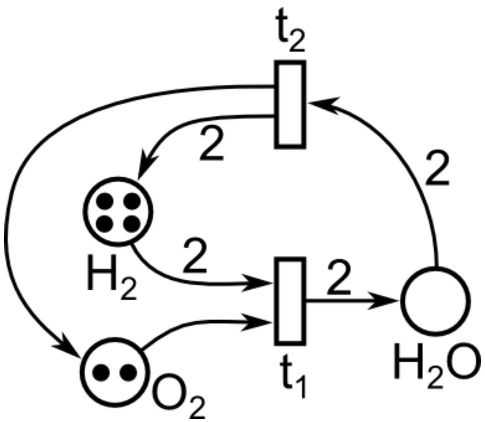
- **Information systems are becoming more complex**
  - Modeling and automatic analysis is important

- **Modeling: Petri Nets**
  - Widely used modeling formalism
    - Asynchronous, distributed, parallel, non-deterministic systems
  - Behavior: possible states and transitions
  - Optimization problems
    - Optimal trajectory from the initial state to a given goal state
    - Reachability analysis



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- **Reachability analysis**
  - Checks, if a given state is reachable from the initial state
  - $m_1 \in R(PN, m_0) \rightarrow$ *„Is m_1 reachable from m_0 in the Petri net PN?”*
  - Drawback: complexity
- **Complexity**
  - State space can be large or infinite
  - Reachability is decidable, but at least EXPSPACE-hard
  - No upper bound is known
  - A possible solution is to use <u>abstraction</u>

INTRODUCTION
The CEGAR approach

- **CounterExample Guided Abstraction Refinement**
  - General approach
    - Can handle large or infinite state spaces
  - Works on an abstraction of the original model
    - Less detailed state space
    - Finite, smaller representation
  - Abstraction refinement is required
    - An action in the abstract model may not be realizable in the original model
    - Refine the abstraction using the information from the explored part of the state space
  - H. Wimmel, K. Wolf
    - Applying CEGAR to the Petri Net State Equation (2011)

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CEGAR APPROACH ON PETRI NETS

Initial abstraction

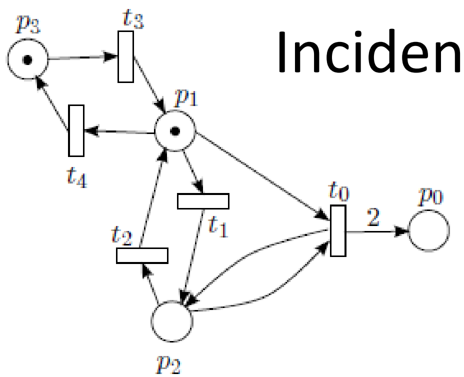
- **Abstraction of Petri nets: state equation**

Initial state

$$\mathbf{m_0 + Cx = m_1}$$

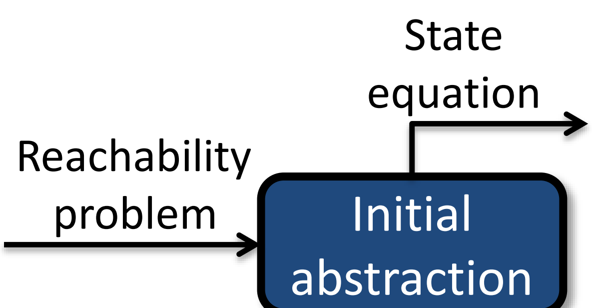
Target state

Incidence matrix



$$\begin{bmatrix} 2 & 0 & 0 & 0 & 0 \\ -1 & -1 & 1 & 1 & -1 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}$$

Firing count of transitions *(unknown)*



CEGAR APPROACH ON PETRI NETS

Analysis of the abstract model

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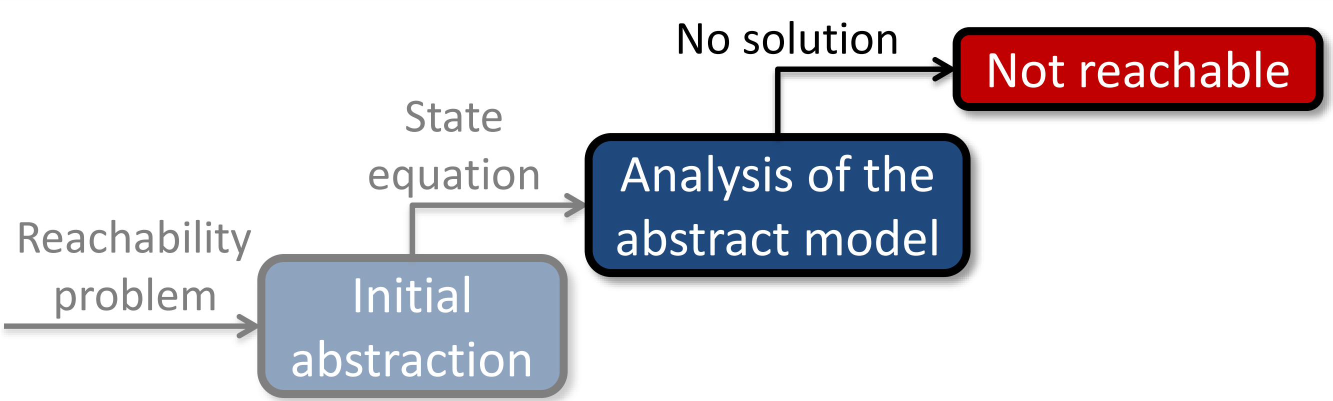
- **Solving the state equation for the firing count of transitions**
- **Integer Linear Programming problem**
- **Necessary, but not sufficient criterion for reachability**

$$m_0 + C\mathbf{x} = m_1$$



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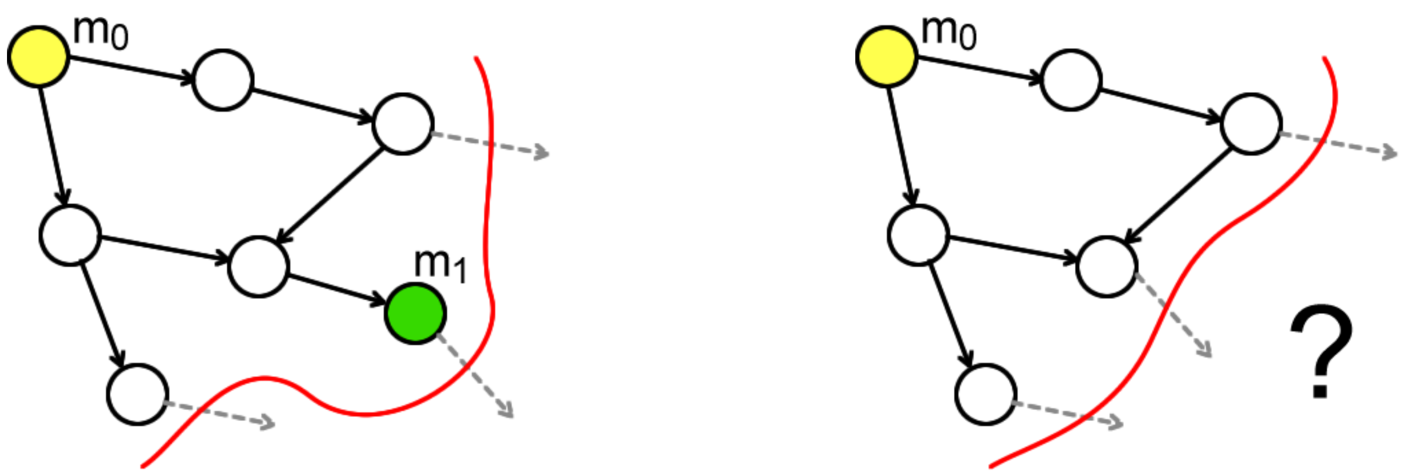
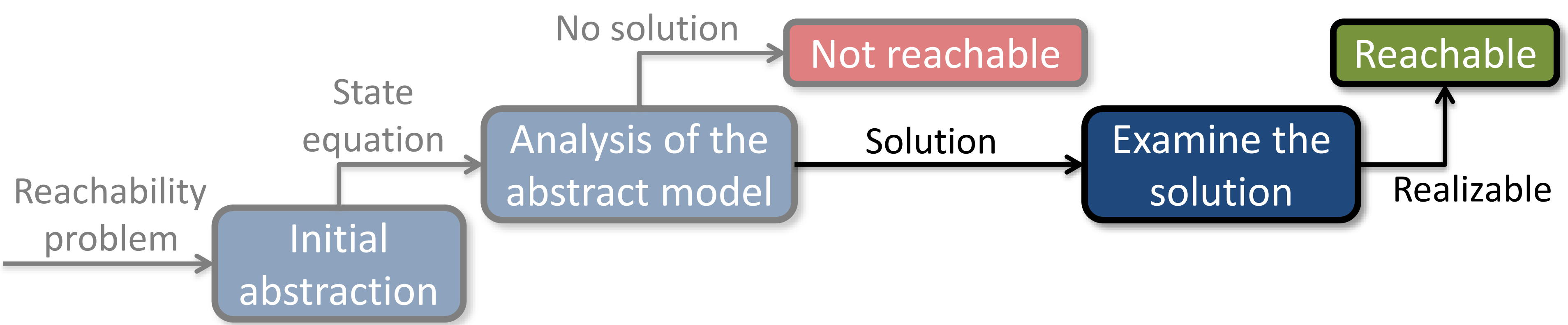
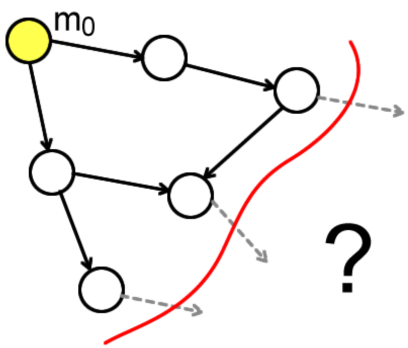
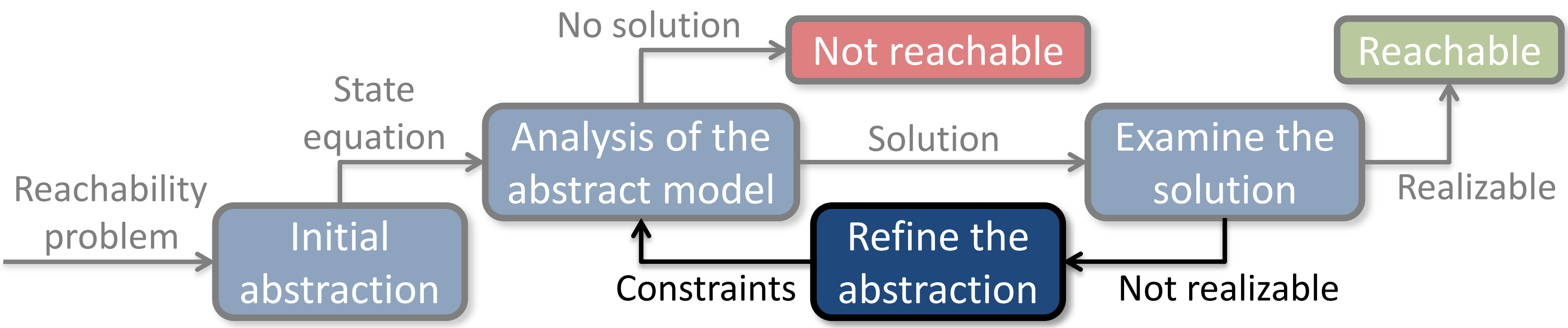
- **Bounded exploration of the state space**
  - Try to fire the transitions in some order





- **Exclude the counterexample without losing any realizable solution**
- **Constraints can be added to the state equation**
  - The state equation may become infeasible
  - A new solution can be obtained
- **Traversing the solution space instead of the state space**





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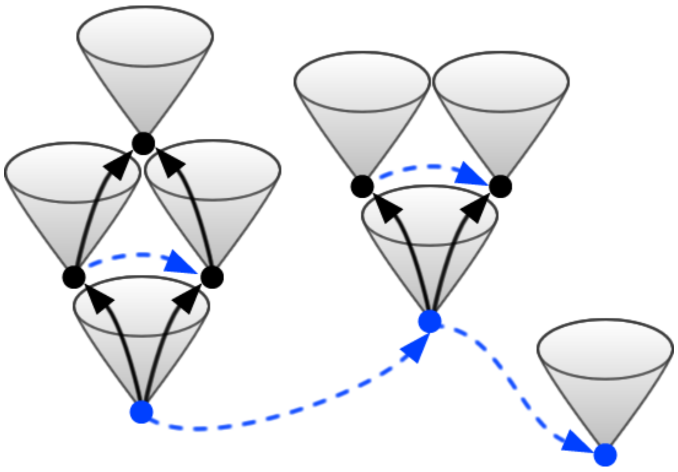
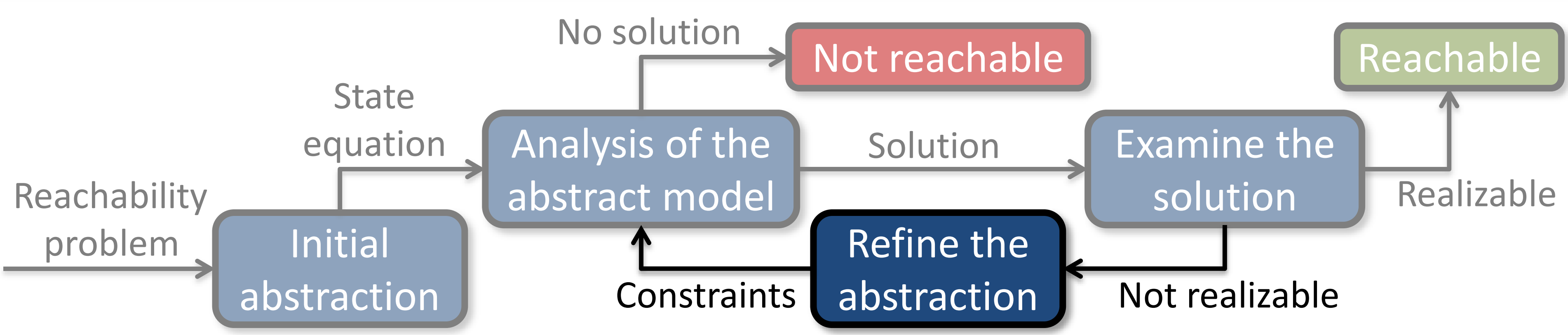
$$m_0 + C\mathbf{x} = m_1$$

- **Semi-linear space**
  - Base solutions
  - T-invariants
    - Solutions of the homogenous part Cy = 0
    - Possible cycles in the Petri Net
- **Two types of constraints**
  - Jump: switch between base solutions
  - Increment: reach non-base solutions





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TRAJECTORY OPTIMIZATION

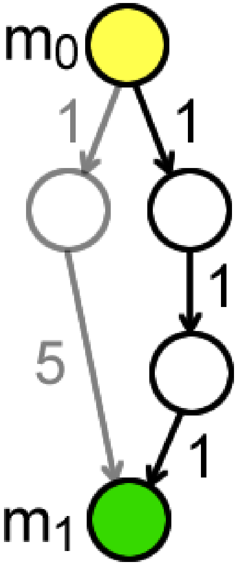
Extensions to the CEGAR approach

- **Our previous work**
  - Analyzing the algorithm
    - Correctness
    - Completeness
  - Extending the set of decidable problems
  - New optimizations
- **Current work**
  - Trajectory optimization using CEGAR
    - Assigning costs to transitions
    - New strategy for the solution space traversal

TRAJECTORY OPTIMIZATION

Assigning costs to transitions

- **Core of the CEGAR approach: state equation**
  - ILP problem
  - ILP solver minimizes a function over the variables
  - Variables are transitions in our case
- **Original algorithm**
  - Verification purpose → *Is there a soluton or not?*
  - Equal cost for each transition → shortest trajectories
- **Our new approach**
  - Optimization purpose → *What is the optimal solution?*
  - Arbitrary cost for transitions
  - ILP solver minimizes using the given cost



TRAJECTORY OPTIMIZATION

New solution space traversal strategy

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- **Traversing the solution space of the state equation**
- **Original algorithm**
  - Verification purpose → *Is there a soluton or not?*
  - Fast convergence → DFS
- **Our new approach**
  - Optimization purpose → *What is the optimal solution?*
  - Store the solutions in a sorted queue
  - Continue with the one with the lowest cost

PSEUDO CODE

Input: Reachability problem $m_1 \in R(PN, m_0)$ and cost function z

Output: Trajectory σ or „*Not reachable*”

$C \leftarrow$ incidence matrix of PN
$Q \leftarrow$ SolveILP(m_0, m_1, C, z, $\emptyset$)
while $Q \neq \emptyset$ do
| $x \leftarrow$ solution from Q minimizing $z \cdot x$
| if x is realizable then stop and output σ for x
| else
| | foreach jump and increment constraint c' do
| | | $Q \leftarrow$ SolveILP(m_0, m_1, C, z, {constraints of x} $\cup$ {c'})
| | end foreach
| end else
end while
Output „*Not reachable*”

Initial abstraction

Analysis of the abstract model

Rechable

Examine solution

Analysis of the abstract model

Refine abstraction

Not reachable

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EVALUATION

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- **Implementation**
  - PetriDotNet framework
    - Modeling and analysis of Petri nets
    - Supports add-ins
- **Measurements**
  - Traveling salesman problem
    - Graph traversal optimization
    - NP-complete

Number of nodes	Runtime (s)
4	0,04
6	0,14
8	0,66
9	0,90
10	1,95
11	9,49
12	24,57
13	1067,00

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CONCLUSIONS

- **New approach for the optimal trajectory problem**
  - Translation to the reachability of Petri nets
  - Solving reachability using CEGAR
    - Handle transition costs
    - New strategy for solution space traversal
- **Implementation and evaluation**
- **Possible future direction**
  - Optimization of continuous systems

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THANK YOU FOR YOUR ATTENTION!

QUESTIONS?

hajduakos182@gmail.com; vori@mit.bme.hu

https://inf.mit.bme.hu/en/research/tools/petridotnet

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