On level crossing in conformal field theories

1512.05362

1. Intro

~~Assume a CFT has a large-N limit.~~

and a "coupling const" g

e.g. $\mathcal{N}=4$: N, λ.

large-spin twist-2 sector : N, $\frac{1}{S}$.

QCD. Veneziano lim. : N_c, $\frac{N_f}{N_c}$

ある operatorの次元

$\downarrow$

so that $\Delta_i = \Delta_i^{(0)}(g) + \frac{1}{N^2}\Delta_i^{(1)}(g) + \frac{1}{N^4}\Delta_i^{(2)}(g) + \cdots$

~~(satisfy this)~~

$\mathcal{O}_1$, $\mathcal{O}_2$: scalar primary op.s

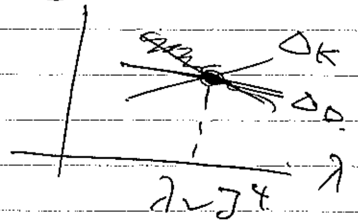
Suppose $\Delta_1^{(0)} = \Delta_2^{(0)}$ at some $g = g_*$

e.g. $\mathcal{N}=4$.

$\mathcal{O}_K = \frac{1}{N}\mathrm{Tr}(\phi^A\phi^A)$ $\quad \Delta_K = 2\lambda^{1/4} - 2 + \cdots$

$\mathcal{O}_D = \frac{1}{N^3}\mathrm{Tr}(\phi^{(A_1}\cdots\phi^{A_j)})\,\mathrm{Tr}(\phi^{(A_1}\cdots\phi^{A_j)})$

$\Delta_D = 2J + \cdots$

$\rightsquigarrow$  $\rightsquigarrow \lambda^{1/4} \sim J$

von Neumann - Wigner thm

~~It is generally hard that $\mathcal{O}_1$ and $\mathcal{O}_2$ have the same dilatation eigenvalue.~~

Suppose that at some (generic) (N, g), $\Delta_1(N, g) \neq \Delta_2(N, g)$

at $(N + \delta N, g + \delta g)$, consider dilatation matrix

$$D = \begin{pmatrix} \Delta_1 & \\ & \Delta_2 \end{pmatrix} + \begin{pmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{pmatrix}$$

$$D \propto \mathbb{1} \Rightarrow \begin{cases} \delta_{11} - \delta_{22} = -\Delta_1 + \Delta_2 \\ \delta_{12} (= \delta_{21}) = 0 \end{cases}$$

2 variables vs. ~~2 eqns.~~ 3 (real) equations.

→ generically, solutions are isolated points in (N, g)-space

(任意のNに対しては、$\Delta_1 = \Delta_2$ を満たす g は多分 genericには存在しない)

Remember that in QM, the perturbation theory is valid only when $|\Delta_1^{(0)} - \Delta_2^{(0)}|$ is much larger than the leading energy correlation.

$$\delta\Delta_n \sim \sum_i \frac{|\langle n | \delta V | i \rangle|^2}{\Delta_n - \Delta_i}$$

When $\Delta_n - \Delta_i \sim \mathcal{O}(1/N)$, $1/N$-perturbation breaks down & resummation is needed.

(さっき Δ_K と Δ_D が同じなように見えたのはまちがい)

2. resummed scaling dimensions

$$\langle O_i(x)\, O_j(y)\rangle \propto \frac{\delta_{ij}}{|x-y|^{2\Delta_i}}$$

Assume at $g=g_*$, $\Delta_1^{(0)} = \Delta_2^{(0)}$,

& other scalar primaries satisfy

$$|\Delta_{\text{other}} - \Delta_{1,2}^{(0)}| \sim O(1)$$

At $N \to \infty$, $\langle O_1^{(0)}(x)\, O_2^{(0)}(y)\rangle = 0$

At $N < \infty$, $\langle O_1^{(0)}(x)\, O_2^{(0)}(y)\rangle = \frac{1}{N}\, \underset{\uparrow}{\mathcal{C}}\,(|x-y|^2)$

op. mixing

e.g. $N<\infty$ では Δ_K と Δ_D は dilatation eigenstate ではないので混ざる

Assume $\langle O_{1,2}^{(0)}(x)\, O_{1,2}^{(0)}(y)\rangle\Big|_{\text{non-planar}} = O(1/N^2)$

At $N<\infty$ we expect

$$\left.\begin{aligned} O_+ &= O_1^{(0)} + c_2\, O_2^{(0)} \\ O_- &= O_2^{(0)} + c_1\, O_1^{(0)} \end{aligned}\right\} \text{dilatation eigenstates}$$

↓

対角化される.

$\langle O_+ O_+\rangle \neq 0$, $\langle O_- O_-\rangle \neq 0$, $\langle O_+ O_-\rangle = 0$

$$0 = \langle O_+(x)\, O_-(0)\rangle$$

$$\to \frac{1+c_1c_2}{N}\,\mathcal{C}(|x|) = -(x^2)^{-\Delta_1^{(0)}}\Big[c_1 + c_2 + \underbrace{c_2\,\epsilon \log|x|^2}\Big]$$

$\epsilon = \Delta_1^{(0)} - \Delta_2^{(0)}$

$$|x|^{-2\Delta_2} = |x|^{-2\Delta_1}(1 + \cdots \log|x|^2 \cdots)$$

At $|\epsilon| \sim 1/N$.

$$c_1 + c_2 = 0 + \frac{\beta}{N} + \mathcal{O}(1/N^2)$$

$$\rightarrow \varphi(x) = (x^2)^{-\Delta_1} \frac{\beta + c_1 \epsilon N \log x^2}{1 - c_1^2}$$

$$\langle O_-(t)\, O_-(0) \rangle = (\text{const.}) \times \frac{1}{|x|^{2\Delta_-}}$$

$$\sim \Delta_- = \Delta_2 + \frac{\gamma}{N} c_1 \qquad \gamma = \frac{\epsilon N c_1}{1 - c_1^2}$$

$$\Delta_\pm = \frac{\Delta_1 + \Delta_2}{2} \pm \sqrt{\frac{\epsilon^2}{4} + \frac{\gamma^2}{N^2}} + \mathcal{O}(1/N^2)$$

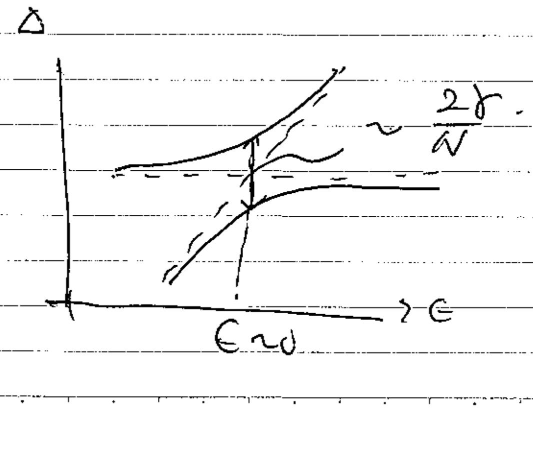
Expanding in $\frac{1}{N}$ leads to

$$\Delta_- = \Delta_2 - \frac{\gamma^2}{N^2 \epsilon} + \frac{\gamma^4}{N^4 \epsilon^3} + \cdots$$

$\rightarrow$ each term diverges at $\epsilon \sim 0$.

around $\epsilon \sim 0$,

$$\triangleright \; |\Delta_+ - \Delta_-| \geq \frac{2\gamma}{N}$$



3. Resummed OPE coefficients

ϕ: another scalar primary.

$|\Delta_\phi - \Delta_I| = O(1)$

$$\langle \phi(x_1)\phi(x_2)\cdots\phi(x_4)\rangle = \frac{1}{(x_{12}^2 x_{34}^2)^{\Delta_\phi}}$$

$$\times\Big[F_{O_+}(u,v) + F_{O_-}(u,v) + \sum_i F_{\Delta_i}(u,v)\Big]$$

$O_\pm$ の λ_i 依存

conf. block

$F_O(u,v)$ u, v: cross ratios.

$$u = \left(\frac{|x_{12}||x_{34}|}{|x_{13}||x_{24}|}\right)^2 \quad v = u\big|_{2\leftrightarrow 4}$$

In the limit $x_{12}\to 0$.

$$F_{O_I}(u,v) \sim C^2_{\phi\phi O_2}\, u^{\frac{1}{2}\Delta_I} + (x_{12}\text{-suppressed})$$

$$C^2_{\phi\phi O_\pm} \sim \frac{\langle\phi\phi O_\pm\rangle\langle O_I\phi\phi\rangle}{\langle\phi\phi\rangle^2\langle O_\pm O_1\rangle}$$

$$C_{\phi\phi O_\pm} \sim \frac{(C_{\phi\phi O_{1,2}} \mp C_1 C_{\phi\phi O_{2,1}})^2}{1+C_1^2}$$

$\langle\phi\phi\phi\phi\rangle$ is a well-def func. in CFT

↳ must be non-singular in the limit $\epsilon\to 0$

F_{ϕ_i} = non-singular

F_{O_I} : the large-N perturbation becomes invalid at $\epsilon\to 0$.

$$F_{O_+} + F_{O_-} \underset{x_{12}\to 0}{\sim} u^{\frac{\Delta_1+\Delta_2}{2}} \Big[C^2_{\phi\phi O_1} u^{\frac{\epsilon}{4}} + C^2_{\phi\phi O_2} u^{-\frac{\epsilon}{4}} - 2\gamma C_{\phi\phi O_1} C_{\phi\phi O_2} \frac{u^{\frac{\epsilon}{4}} - u^{-\frac{\epsilon}{4}}}{N\epsilon} \Big] + O(1/N^2)$$

$O(\epsilon^0)$

By the assumption, $O(1/N)$ term does not exist

$$\to \frac{C_{\phi\phi O_1}}{C_{\phi\phi O_2}} = O\left(\frac{1}{N}\right) \text{ or } O(N)$$

$$C^2_{\phi\phi O_\pm} = \frac{1}{2} C^2_{\phi\phi O_1} \left(1 \pm \frac{\epsilon}{\sqrt{\epsilon^2 + \frac{4\gamma^2}{N^2}}} \right)$$

resumする
$\epsilon \to 0$ でok

$$\sim \begin{cases} \dfrac{C_{\phi\phi O_+}}{C_{\phi\phi O_1}} = 1 - \dfrac{\gamma^2}{2N^2\epsilon^2} + \cdots \\[2ex] \dfrac{C_{\phi\phi O_-}}{C_{\phi\phi O_1}} = \dfrac{\gamma}{N\epsilon}\left(1 - \dfrac{3\gamma^2}{2N^2\epsilon^2} + \cdots \right) \end{cases}$$

$$|C_{\phi\phi O_+}| \sim 1$$

$$C_{\phi\phi O_-} \sim 0, \quad (1 \gg \epsilon \gg 1/N)$$

$$F_{O_+} + F_{O_-} \sim \frac{1}{2}\left(u^{\frac{1}{2}(\Delta_1 + \frac{\gamma}{N})} + u^{\frac{1}{2}(\Delta_1 - \frac{\gamma}{N})} \right)$$

$$\to (\Delta_+ - \Delta_1) \geq \frac{2\gamma}{N}$$