Amplitude ゼミ 第一回

$\underline{\mathcal{N}=4.\ SU(N)\ SYM}$ の amplitude を考える

$\to (A_\mu, \lambda^A, \phi^{[AB]})$

n点関数 $\mathcal{A}^n(\{\underbrace{p_i}_{\text{momentum}}, \underbrace{h_i}_{\text{helicity}}, \underbrace{a_i}_{\text{gauge}}\})$.

single trace
⇒ planar

$$\mathcal{A}^n(p_i, h_i, a_i) = g^{n-2}(2\pi)^4\delta^4(\Sigma p_i = 0)$$
$$\times \sum_{\sigma\in S_n/\mathbb{Z}_n} \mathrm{Tr}[T^{a_{\sigma_1}} \cdots T^{a_{\sigma_n}}] \underbrace{\mathcal{A}(\sigma_1 \cdots \sigma_n)}_{\text{partial amplitude}}$$
$$+ \sum \mathrm{Tr}[T\cdots]\,\mathrm{Tr}[\cdots] \cdots$$
$$+ \sum \mathrm{Tr}\cdots \mathrm{Tr}\cdots \mathrm{Tr}\cdots$$
$$+ \cdots$$

- tree level amplitude は single trace しか含まない。

$$\mathrm{Tr}[T^a X]\,\mathrm{Tr}[T^a Y] = \mathrm{Tr}[XY] - \underbrace{\frac{1}{N}\cdots\cdots}$$

tree なら. gluon 3.4点のたびに trace が立つから. この形しかない.

SU(N) → U(N) にすると 落ちる。U(1) photon は decouple

spinor-helicity formula (in 4 dim.)

(null) vector x. $\longleftrightarrow$ pair of spinor $(\mu.\tilde{\mu})$. / redundancy

$$\sigma^\mu_{\alpha\dot{\alpha}} = (\mathbb{1}. \vec{\sigma})$$

$$p_\mu \sigma^\mu_{\alpha\dot{\alpha}} = \lambda_\alpha\tilde{\lambda}_{\dot{\alpha}} + \mu_\alpha\tilde{\mu}_{\dot{\alpha}} \quad ,\ 一般に.$$

一方. p が null なので. $\det(p\sigma) = 0 \to p\sigma$ は rank 1.

$$\therefore\ p_\mu\sigma^\mu_{\alpha\dot{\alpha}} = \lambda_\alpha\tilde{\lambda}_{\dot{\alpha}} \Rightarrow$$ これを null vec. と spinor pair の変換と思う.

λには "gauge symmetry"　→ λとλ̃はrealで独立ではないと…

$$\lambda \to k\lambda, \quad \tilde\lambda \to k^{-1}\tilde\lambda$$

d.o.f. $\underset{\sim}{4} - 1 = 3$ d.o.f.

λをrealでとると良い.

λをcomplexでとる場合. $\lambda^* = \pm\tilde\lambda$ にできる.

これでも. d.o.f $= 8\times\frac{1}{2} - \underset{\substack{\uparrow \\ \text{phase}}}{1} = 3$ で一致

$p_i = \lambda_i\tilde\lambda_i$ として.

$$\langle ij\rangle \equiv \langle\lambda_i\lambda_j\rangle \equiv \lambda_i^a\lambda_{ja} \equiv \varepsilon_{ab}\lambda_i^a\lambda_j^b \quad (= -\langle ji\rangle)$$

$$[ij] \equiv [\tilde\lambda_i\tilde\lambda_j] \equiv \tilde\lambda_{i\dot a}\lambda_j^{\dot a} \equiv \varepsilon_{\dot a\dot b}\lambda_i^{\dot a}\lambda_j^{\dot b} \quad (= -[ji])$$

polarization vector　　$k^\mu \cdot p_\mu = \frac{1}{2}(k_{a\dot a}p^{a\dot a})$

$\varepsilon^{\pm}(k)_{\alpha\dot\alpha}$　　$k \to \lambda, \tilde\lambda$　　$\eta = (+---)$

$$\varepsilon_{\alpha\dot\alpha} = -\sqrt{2}\,\frac{\lambda_\alpha\tilde\mu_{\dot\alpha}}{[\tilde\lambda\tilde\mu]} \equiv \varepsilon^-_{\alpha\dot\alpha}$$

$\varepsilon^+_\mu\varepsilon^{-\mu} = -1$

1= normalize

$$\tilde\varepsilon_{\alpha\dot\alpha} = -\sqrt{2}\,\frac{\mu_\alpha\tilde\lambda_{\dot\alpha}}{\langle\mu\lambda\rangle} \equiv \varepsilon^+_{\alpha\dot\alpha}$$

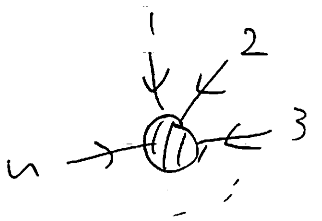
cf. $e(k) = (0,1,0,0)$
$\to e^\mu e_\mu = -1$.

$$\left(\frac{1}{2}\varepsilon_{\alpha\dot\alpha}\tilde\varepsilon^{\alpha\dot\alpha} = \frac{-\langle\lambda\mu\rangle[\tilde\mu\tilde\lambda]}{[\tilde\lambda\tilde\mu]\langle\mu\lambda\rangle} = -1.\right)$$

BCFW recursion relation

gluon tree-level amplitude

$$A(1^{h_1}, 2^{h_2}, \cdots, n^{h_n}) \qquad h_i \in \{1, -1\}$$



($p_2 \cdots p_{j-1}$ まで)

$$A_n = \sum_{j=2}^{n-2} \sum_{\hat{h}=\pm} A_{j+1}\Big((\lambda_1 + s_j \lambda_n, \tilde{\lambda}_1, h_1), \{(\lambda, \tilde{\lambda}, h)\}_{I_{2,j-1}}, (-\hat{P}_{1j}, \hat{h})\Big)$$

(iじゃなくて1)

$$\times \frac{1}{P_{1j}^2}$$

$$\times A_{n-j+1}\Big((\hat{P}_{1j}, \hat{h}), \{(\lambda, \tilde{\lambda}, h)\}_{I_{j+1,n-1}}, (\lambda_n, \tilde{\lambda}_n - s_j \tilde{\lambda}_1, h_n)\Big)$$

$$P_{1j}^{\mu} = \sum_{k \in I_{1j}} p_k^{\mu}$$

$$\hat{P}_{1j}^{\mu} = \sum_{k \in I_{1j}} \hat{p}_k^{\mu}$$

$$\hat{\lambda} = \begin{cases} \lambda_1 + s_j \lambda_n \\ \lambda_i \end{cases}$$

$$s_j = \frac{P_{1j}^2}{\langle n | P_{1j} | 1]}$$

$$\hat{p}_i = \frac{1}{2} \hat{\lambda}_{ia} \tilde{\lambda}_{i\dot{a}} \sigma^{\mu a \dot{a}}$$

($(P_{1j})^2 \neq 0$ だが
$(\hat{P}_{1j})^2 = 0$ なので
Spinor $\mu, \tilde{\mu}$ に分解できる)

これがしたいからずらした.