



1910-00321 Supersonic Gas Ejector Design and
Performance Evaluation

Submitted by

Muhammad Hazim Bin Sulaiman (A0155095B)

Department of Mechanical Engineering

In partial fulfilment of the requirements for the
Degree of Bachelor of Engineering National
University of Singapore

Academic Year 2019/2020

Contents

i.	SUMMARY	3
ii.	ACKNOWLEDGEMENT	5
iii.	LIST OF FIGURES	6
1.	INTRODUCTION TO GAS EJECTORS	8
1.1.	SUPERSONIC GAS EJECTORS	10
1.2.	COMPRESSIBLE FLOW CONCEPTS	11
1.3.	PROBLEM STATEMENT	12
2.	LITERATURE REVIEW	13
2.1.	ESDU 84029	16
2.2.	DIAGRAM OF SUPERSONIC GAS EJECTOR	18
3.	METHODOLOGY	20
3.1.	OPERATING CONDITIONS	20
3.2.	FIXED PARAMETERS FOR DIMENSIONS	21
3.3.	OTHER ASSUMPTIONS	23
3.4.	EQUATIONS	23
3.4.1	Secondary flow intake continuity equation	24
3.4.2.	Primary nozzle continuity equation	24
3.4.3.	Mixing duct continuity equation	25
3.4.4.	Diffuser continuity equation	26
3.5.	PROCEDURE	27
4.	RESULTS AND DISCUSSION	28
4.1.	OPTIMUM DIAMETER	31
4.2.	METHANE	32
4.3.	PERFORMANCE EVALUATION WITH EXAMPLE	33
5.	CONCLUSION AND FUTURE WORK	36
5.1.	VARIATIONS OF VALIDATION	36
5.3.	OTHER CONFIGURATIONS	37
6.	REFERENCES	38
7.	APPENDICES	39

i. SUMMARY

The supersonic gas ejector is an appliance that is widely known for its ability to manipulate the pressure value of streams, whether to increase or decrease the pressure using supersonic flows.

It is found that there are currently inadequate papers that discuss the design of ejectors that cater to the methane gas if it was the medium. Typically, the preferred medium of research is air. However, methane has a different set of physical properties to air, mainly specific heat capacity and density. Hence, most calculated values of gas ejectors cannot be used directly to ejectors in the oil and gas industry, since methane, being one of the few major natural gases extracted, is commonly handled.

This paper studies the design condition for gas ejectors with methane as a medium. Design conditions have multiple variations depending on the intended use of the ejector. The objective is stated as the best diffuser parameter, given a fixed set of design parameters and operating conditions.

To aid in the calculation of pressures, the ESDU 84029 is utilised to provide the equations that link the different pressure points of the supersonic gas ejector. The equations are continuity equations and Bernoulli equations that are specific for isentropic flows within the ejector. Therefore, it is a convenient guide to obtain the pressure values.

In addition, performance evaluation can be done using the resulting Excel sheet.

This evaluation can be done to find the right size of diffuser for gas ejectors.

ii. ACKNOWLEDGEMENT

I would like to extend my gratitude to my supervising professor, Associate Professor Loh Wai Lam, for guiding me through the course of the project. His advice and encouragement throughout have ensured an enriching learning experience for me for the past 8 months. Also, thank you to my fellow course mate, Zheng An and Huzaifi, who has provided me with tips and advices on how to handle certain parts of the problem along the way.

iii. LIST OF FIGURES

Fig. 1 Show the increase in flow rate (Q) if there is a decrease in FTHP

Fig. 2 Shows the configuration of extraction process that makes use of an ejector to boost pressure

Fig. 3 At flow regime H, a supersonic flow is experienced, and design conditions are achieved

Fig. 4 A diagram of a typical supersonic nozzle

Fig. 5 Result of Dutton and Carroll's three-dimensional optimisation surface

Fig. 6 Diagram of the three geometries that are experimented

Fig. 7 Graph of optimum performance relationship between pressure ratios and flow ratio. This experimental graph is used for the Quick Design Method

Fig 8 Diagram of the gas ejector that is depicted in the ESDU 84029. A larger picture can be found in Appendix A.

Fig. 9 Closeup view of the entry of the secondary and primary flow

Fig. 10 Diagram of the diffuser of the gas ejector.

Fig. 11 Table of operating conditions

Fig. 12 Table of fixed dimensions, some of which are calculated

Fig. 13 to show the calculation flow to find the relationship of P_{t5} against D_5

Fig. 14 The graph of diffuser exit pressure against diffuser exit diameter for air as a medium

Fig. 15 The extracted value from the optimum relationship graph in ESDU 84029 that was used to validate the result

Fig. 16 Table to tabulate the expected value against the result obtained

Fig. 17 The graph of diffuser exit pressure against diffuser exit diameter for methane as a medium

Fig. 18 Table showing the new set of operating conditions for this performance evaluation example

Fig. 19 Graph of same gas ejector with input pressure of 4 bars for primary flow and 1 bar for secondary flow

1. INTRODUCTION TO GAS EJECTORS

Gas ejectors are used in many appliances throughout the offshore, oil and gas industry. It is an appliance that draws the pressure energy from a high-pressure motive stream to entrain the low pressure stream from a secondary source[1]. This means that it can be used to either increase the pressure of the secondary source or decrease the pressure of the primary flow (motive stream) depending on the configuration of the pipeline. It is widely used because it is convenient to install and environmentally friendly since there are no moving parts. Gas ejectors have to be properly designed to ensure that there will be economical use of the high pressure, which, if design conditions are achieved, will compress the fluid from the low-pressure stream to a higher pressure effectively.

All gas ejectors and jet pumps (gas ejectors handling liquid) have the same working principle. High pressure fluid with low velocity is the motive fluid at the primary inlet. It is then accelerated through a converging-diverging nozzle (for supersonic gas ejectors). The inlet flow now has high velocity and low static pressure which induces the secondary flow that flows in through a separate inlet. Both the inlet flow and secondary flow now combine at the mixing chamber. At the end of the chamber, a diffuser is usually installed to increase the static pressure of the flow.

There are many uses for gas ejectors in the oil and gas industry. One major role it has a part in is the boosting of production in an oil well. This is done by decreasing the Flow Tube Head Pressure (FTHP) of the secondary stream. The corresponding

effect will be the increase of flow rate, evident in the pressure-flow rate (P-Q) curve of a well.

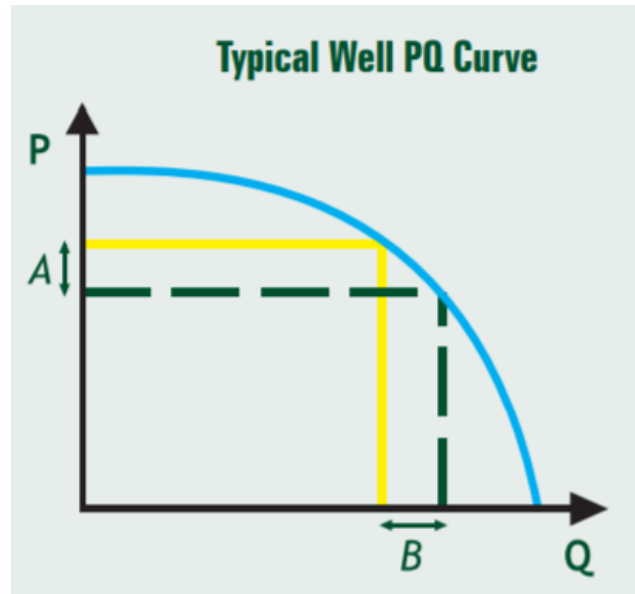


Fig. 1 Show the increase in flow rate (Q) if there is a decrease in FTHP

Another major use is the utilisation of high pressure gas wells to ‘restart’ low pressure gas wells by having the high pressure wells to run as the motive stream and the low pressure wells to be the secondary stream. This will boost the pressure of the low pressure wells and thereby making it possible to extract natural gases from there and also increase production rate[2].

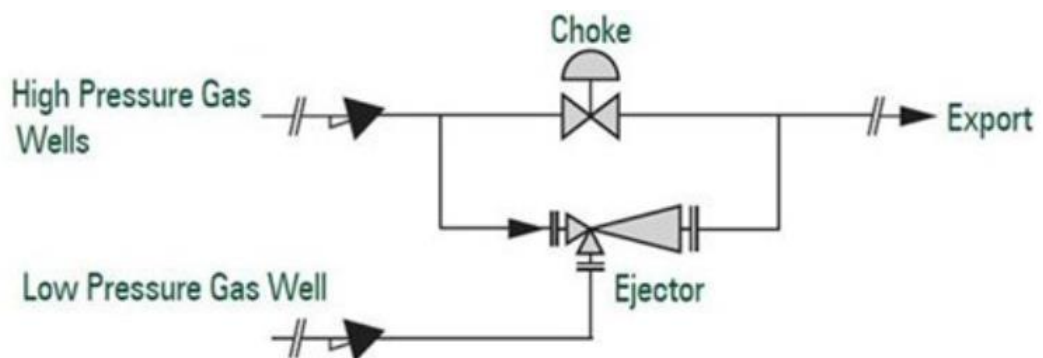


Fig. 2 Shows the configuration of extraction process that makes use of an ejector
to boost pressure

Alongside the stated uses, there are many other applications for the oil and gas sector alone, such as flare gas recovery and sand slurry pumping.

Since, gas ejectors carry many uses that boost oil production, there are many researches that has been done to find the best form and dimensions that provides the optimum efficiency. Albeit the number of papers done on gas ejectors, there are certain conditions that have yet to be explored in this field. This paper is to address the need to have more research done on use of supersonic gas ejectors on methane since it is one of the primary components of natural gasses.

1.1. SUPERSONIC GAS EJECTORS

This research paper is focused on supersonic gas ejectors, which are gas ejectors that has the characteristic converging-diverging nozzle in order achieve a supersonic flow. A supersonic flow is commonly referred to as a fully expanded nozzle flow or a flow running at design conditions. With the converging-diverging nozzle, when the back pressure past the throat exit is low enough, there will be a choked flow. Before the throat, the flow is at subsonic. At the throat the fluid experiences Mach 1 and subsequently a supersonic flow is formed. At this condition, the flow is focused towards the downstream direction only because disturbances will not flow upstream[3].

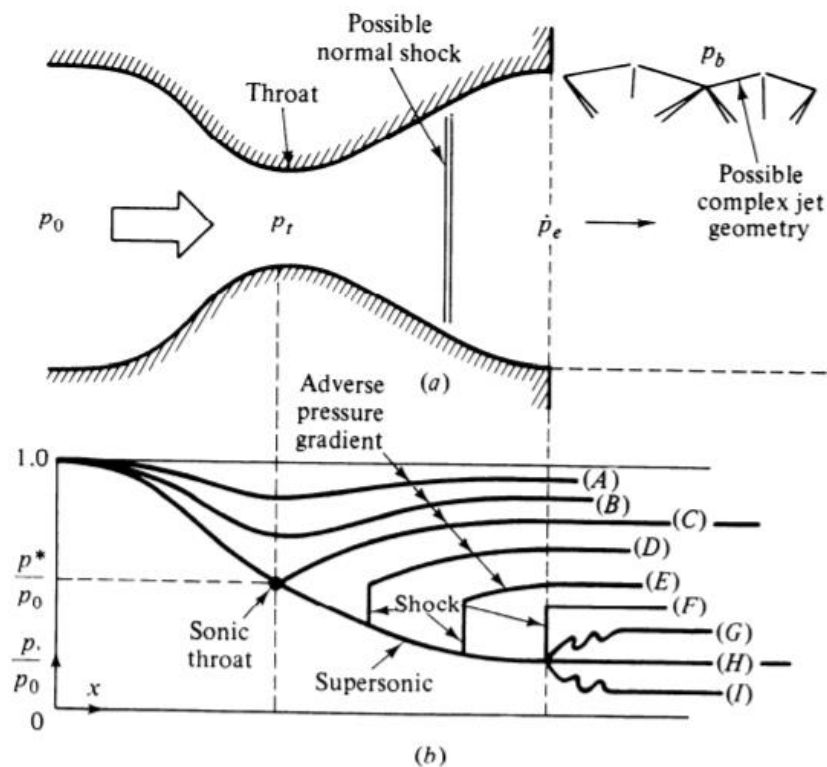


Fig. 3 At flow regime H, a supersonic flow is experienced, and design conditions are achieved

1.2. COMPRESSIBLE FLOW CONCEPTS

There are a few compressible flow concepts that are integral to the calculations of the pressures within the gas ejector. When gas molecules travel at a velocity faster than the speed of sound, there are compressible concepts that need to be followed.

First and foremost, the speed of sound differs for different mediums. This is because speed of sound is dependent on the specific heat ratio (γ), temperature (T) and gas ratio (R). Because of the differing specific heat ratio value, the speed of sound is different for every gas.

$$a = \gamma * R * T$$

Secondly, Mach number (M) is an important value because it is a similarity parameter in compressible equations. Mach number is a dimensionless parameter that is the ratio of the velocity of the flow with the speed of sound of the fluid.

$$M = \frac{V}{a}$$

Another relationship that is critical in understanding the behaviour of compressible fluids, especially in the convergent-divergent nozzle is the velocity change in area equation for isentropic flow. This is obtained from a combination of the continuity equation, the isentropic flow relation, conservation of momentum equation and the ideal gas law equation.

$$(1 - M^2) \frac{dV}{V} = - \frac{dA}{A}$$

At the supersonic phase since Mach number is more than 1, there is an inverse relationship between the change of velocity and change in area. For instance, if the area of the nozzle increases, the velocity increases too. Hence, we expect pressure to decrease through the supersonic nozzle.

1.3. PROBLEM STATEMENT

The aim of the project, as discussed, is to find the design conditions for a supersonic gas ejector with methane as a medium. However, since 'design conditions' can be interpreted in different ways, there is a need to set the problem statement for this project. After much consideration, it is decided that the way the 'design condition' is set for this research is to find the best diffuser diameter that outputs the highest pressure past the diffuser. Additionally, the use can be

translated to different gas medium. However, only use for methane will be explored.

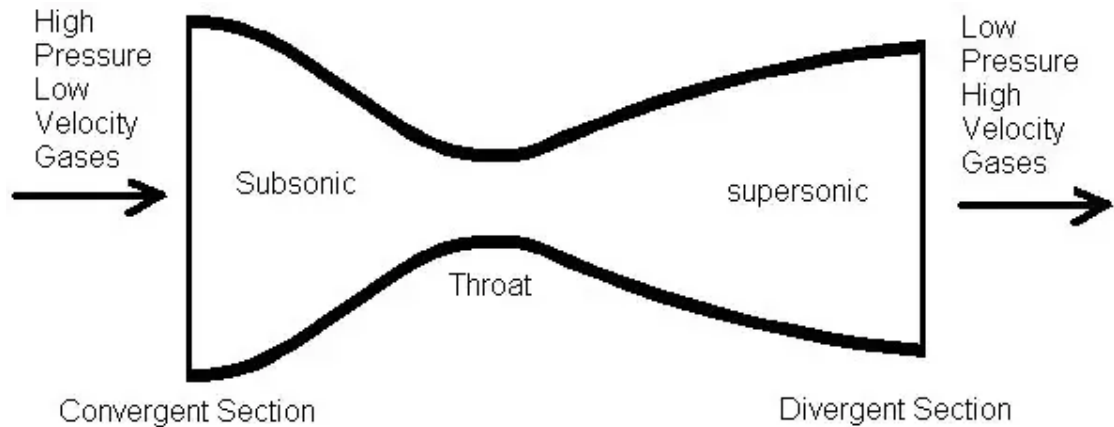


Fig. 4 A diagram of a typical supersonic nozzle

2. LITERATURE REVIEW

The aim of the research of supersonic gas ejector is to find the best parameters for a supersonic gas ejector by fixing certain dimensions and operating conditions. However, there are many approaches to define the 'design condition' for a gas ejector. This literature review will showcase the different methods done up by various researchers to find their take on what the best parameter for a gas ejector is. Also, certain ideas can be attained from the readings to shape up the problem statement of this paper.

Vojta and Dvorak based their most efficient model as one that returns the highest pressure with varying primary and secondary mass flow rate[4]. The investigation is performed numerically and experimentally. Although it is cumbersome to

conduct an experimental procedure, it provided valid data to be cross checked with the numerical analysis.

Dutton and Carrol went even further with a complex three dimensional optimisation surface of secondary to primary mass flowrate ratio against primary-secondary inlet pressure ratio against mixing duct-secondary inlet pressure ratio[5]. These dimensionless variables are used to describe the degree of optimisation of an ejector. This paper also underlines that a one-dimensional model requires a pertinent dimensionless variable to describe the efficiency of a gas ejector.

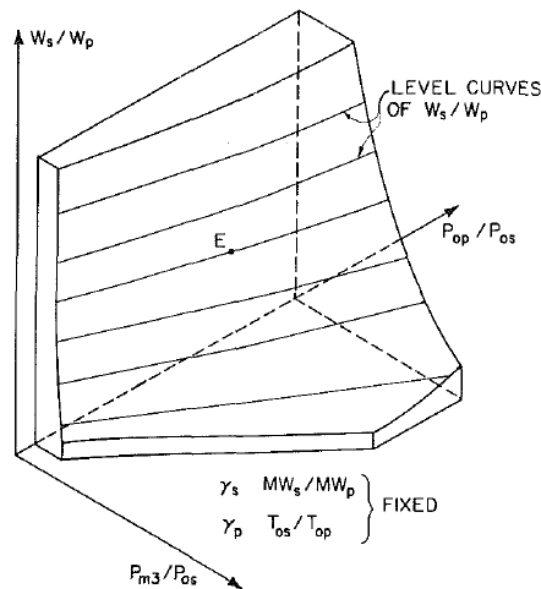


Fig. 5 Result of Dutton and Carrol's three-dimensional optimisation surface

In the study of gas ejector configurations and its performance conducted by El-Zahaby, Hamed, Omara and Eldesoukey, different forms of the gas ejector are subjected to the same input pressure. It is found that a constant area mixing tube is the most effective in mixing the motive and entrained flow, whereas the

constant pressure configuration is the best at retaining the velocity of flow and hence the poorest at recovering the pressure.[6] The figure below displays the three configurations that are experimented with. G1 and G2 are the constant area configuration, with G2 having a diffuser at the end. G3 is the constant pressure mixing configuration.

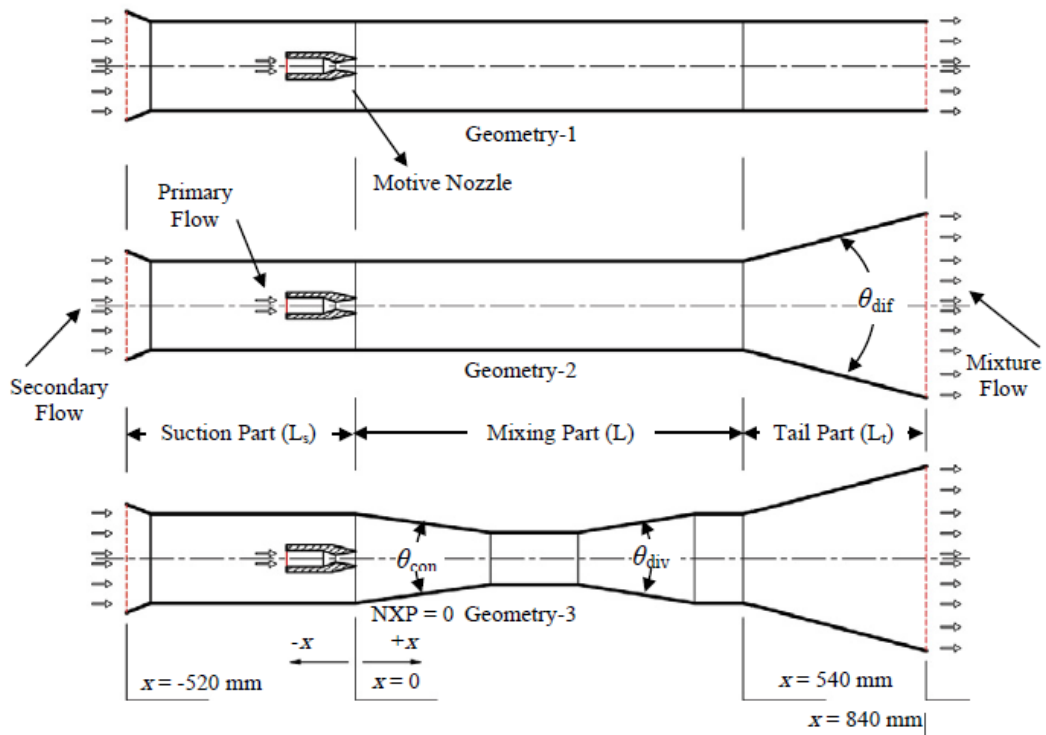


Fig. 6 Diagram of the three geometries that are experimented

From the literature review, most of the methods used are finite element methods based, whereby a virtual control volume is meshed and subjected to pressures and calculation are done using numerical methods software such as Ansys. Although this is a convenient way to investigate the pressure values, numerical analysis can be unreliable because we are not fully in control of the equations that are utilised. Furthermore, the equations are not known to us. Therefore, by means of

theoretical calculation, through manual input of values to the respective equations, the pressure values are more understandable.

Additionally, it is found that the number of researches done on the use of methane gas as a medium is alarmingly low. Hence, this research will largely value-add to the study of gas ejectors.

2.1. ESDU 84029

To initiate the start of this project, a guide for calculation of physical conditions within the isentropic flows of a supersonic gas ejector is needed. Since the calculations involve compressible flows that are supersonic, it would be convenient if there is a guide that provides these equations.

After much search and comparison among other texts, it is apparent that ESDU 84029 (Ejectors and jet pumps: Design and performance for compressible flow) can offer the required equations that would aid in the finding of pressure values and other physical conditions in this project.

Throughout this project, the ESDU booklet, ESDU 84029, will be closely as a guide for the design of gas ejectors in the field. It will be used to find the desired dimensions and boundary conditions, which will be explained in the next few sections. This is also accompanied with findings from other sources.

ESDU, which is short for Engineering Sciences Data Unit, is a branch of engineering advisory organisation that provides design solutions and data for industries in aerospace, oil and gas and more. As stated in the item, the evaluated data is

provided under the supervision of professionally qualified engineers. Hence this is the first step of credibility needed for this research,

The item is a complete guide to designing and predicting performance of a supersonic gas ejector. For design, there are multiple techniques that are stated and thoroughly explained. These techniques include the quick design method and the detailed design method. The quick method is one that makes use of experimental graphs that can be easily referred to at the appendices of the item. This is done after setting up the major non-dimensional parameters of the gas ejector (e.g. the primary ratio, secondary ratio and mass flow ratio).

The Detailed Design method, on the other hand, provides a more thorough approach to obtain an optimal solution for designing a gas ejector. This is done by incorporating losses and other design constraints.

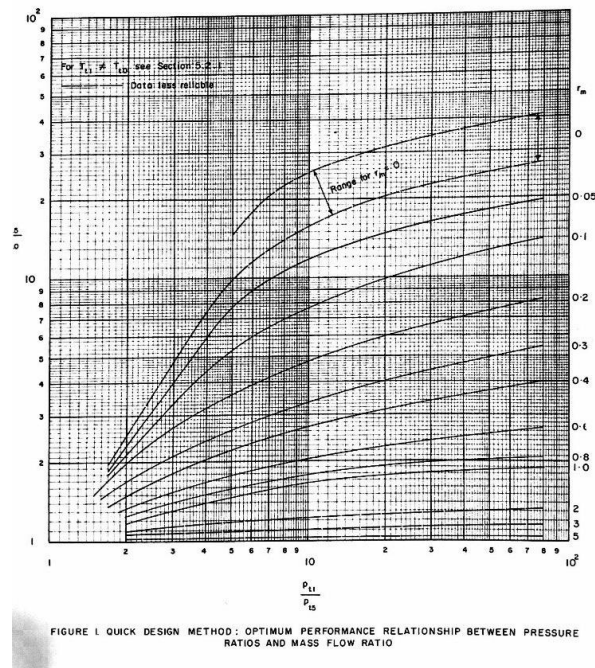


Fig. 7 Graph of optimum performance relationship between pressure ratios and flow ratio. This experimental graph is used for the Quick Design Method

2.2. DIAGRAM OF SUPERSONIC GAS EJECTOR

Before embarking on the calculations by using equations provided in ESDU 84029, it is necessary to familiarise ourselves with the different parts of the supersonic gas ejector. This will ease our understanding before doing the calculations that links the pressures of the different parts of the ejector.

Firstly, there are the inlet tubes. One is for secondary flow and the other is the tube for the primary flow that leads to the nozzle. A convergent-divergent nozzle is used, which is where the high-pressure motive stream will flow through.

Secondly, the mixing duct comes after the nozzle. It is a part that effectively characterises the device. The primary and secondary flow will mix in this section.

Lastly, after the mixing tube, is the diffuser. This is the part where the size will be varied, and the corresponding pressure value is investigated. In the diffuser, we can expect an increase in pressure from the entry to the exit. This is because of the increase in flow area, hence, there will be a decrease in velocity by continuity. Hence, there will be an increase in pressure due to the decrease in velocity, as stated by the Bernoulli's equation.

There are multiple planes that are numbered from 0 to 5 which are the key planes that either describe an entry zone or an exit zone of a certain part of the ejector.

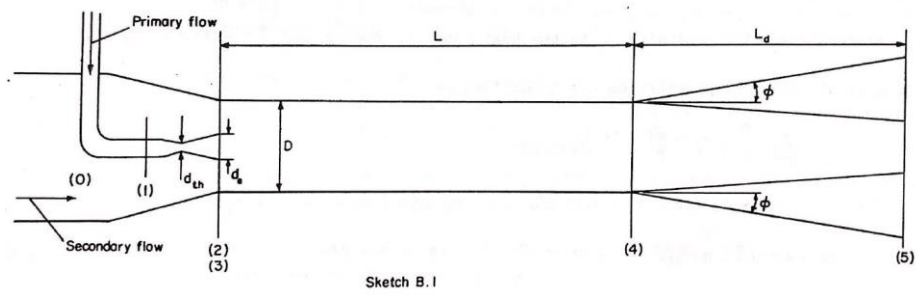


Fig 8 Diagram of the gas ejector that is depicted in the ESDU 84029. A larger picture can be found in Appendix A.

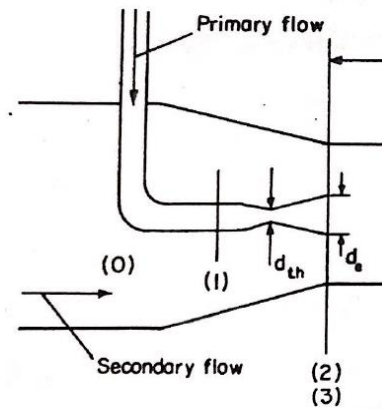


Fig. 9 Closeup view of the entry of the secondary and primary flow

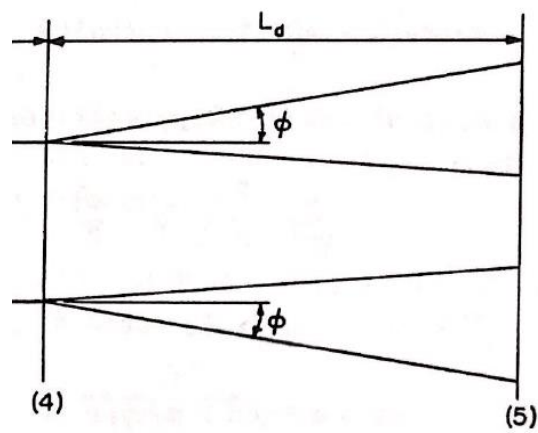


Fig. 10 Closeup view of the diffuser of the gas ejector.

The key planes are indicated by numbers:

0: Entry of secondary flow

1: Entry of primary nozzle

2: Exit plane of primary nozzle

3: Entry of mixing duct

4: Exit plane of mixing duct/Entry plane of diffuser

5: Exit plane of diffuser

th: Throat plane of the primary nozzle

e: Exit plane of the primary nozzle

Superscripts ' and " are used in the calculations which refer to the primary and secondary stream respectively.

3. METHODOLOGY

This section will discuss the process of the calculations. Starting from setting the desired operating conditions to be used followed by fixed dimensions to input into the equations suggested by ESDU 84029. These equations will be then be listed and explained in Section 3.4. Assumptions are also discussed due to the complexity that might arise if some conditions are not stated. Lastly, the procedure of which the calculations are stored and performed is explained in Section 3.5.

3.1. OPERATING CONDITIONS

To initiate the calculations, the operating conditions must be fixed so that they can be input to the continuity and Bernoulli equations. The operating conditions

include the temperature, pressure and mass flow rate of the input primary and secondary flow. These factors are compiled in the table below in Figure 11.

Operating Conditions					
Primary		Secondary		Mixing Duct	
T_p (in Celsius)	25	T_s (in Celsius)	25	T_4 (in Celsius)	25
T_p (in Kelvin)	298	T_s (in Kelvin)	298	T_4 (in Kelvin)	298
P_1 (in Pa)	5.00E+05	P_0 (in Pa)	4.00E+04		
Mass FR (in kg/s)	1	Mass FR (in kg/s)	0.2	Mass FR (in m ³ /s)	1.2
Volumetric FR (in m ³ /s)	1.225	Volumetric FR (in m ³ /s)	0.245		

Fig. 11 Table of operating conditions

For temperature, room temperature is chosen as it provides a convenient value for calculation. For pressure, the stated primary input pressure is set to be at a significantly higher value than the secondary input pressure.

Finally, the mass flow rate is taken from a value from the ESDU 84029 so that it provides a convenient data for us to cross check after the calculations. This convenient value is such that the ratio of the secondary mass flow rate is 0.2. It will be further justified in the results section.

3.2. FIXED PARAMETERS FOR DIMENSIONS

Now, the dimensions of some parameters are fixed such that the only parameter that is varied is the diameter of the diffuser exit (D₅).

As for the diameter of the throat (D_{th}), the throat area (A_{th}) is first calculated through an equation provided from the booklet. This is derived from the compressible flow function of Mach number and inputting M_{th} = 1 for when the flow is sonic. The diameter value is then converted from A_{th} using simple geometrical relations. The corresponding steps are seen below.

$$F = \frac{\dot{m}\sqrt{RT_t}}{AP} = \sqrt{\gamma}M\left[1 + \left(\frac{\gamma - 1}{2}\right)M^2\right]^{0.5}$$

$$A_{th} = \frac{\dot{m}\sqrt{RT_{t1}}}{0.685P_{t1}}$$

Diameters (in mm)		Areas (in m ²)	
D0	200	A0	0.031415
D1	60	A1	0.002827
Dth	23.97252	Ath	0.000451
De	33.90226	Ae	0.000903
D2	100	A2	0.007854
D3	150	A3	0.017671
D3'	33.90226	A3'	0.000903
D3''	116.0977	A3''	0.010586
D4	150	A4	0.017671
D5(to be varied)	-	A5	-

Fig. 12 Table of fixed dimensions, some of which are calculated

The convergent-divergent nozzle, having a A/A_{th} ratio of 2 will correspond to the output Mach number of 2.2. This Mach number is important in the calculation of the pressure in the primary nozzle exit where the compressible flow calculation is

integral. This is obtained from the A/A_{th} ratio function which is a derivation of the compressible flow function.

$$\frac{A}{A_{th}} = \frac{1}{M} \left[\frac{\gamma + 1}{2(1 + \frac{\gamma - 1}{2} M^2)} \right]^{\frac{\gamma + 1}{2(\gamma - 1)}}$$

3.3. OTHER ASSUMPTIONS

Although the gas ejector has a simple enough shape, there are many other forms or configurations that it can take up. For example, the setting up exit plane of the primary nozzle can cause a change in back pressure for the converging-diverging nozzle. However, only one position is considered for this calculation. The exit plane of the diverging part of the supersonic nozzle is coincident to the entry plane of the mixing tube. In addition, a steady, adiabatic flow is assumed. And subsonic, non-compressible flow is assumed at the end of the mixing tube.

3.4. EQUATIONS

Now that the operating conditions are set and the diameter values of certain parameters are fixed, we can start using the equations given in the ESDU 84029 booklet to derive our relationship for the diffuser exit pressure against the diffuser diameter. There are 4 equations that connects the pressure values from the entry of the primary and secondary inlet to the diffuser outlet. The calculations of the different stages will be discussed in the next few subsections.

3.4.1 Secondary flow intake continuity equation

In the continuity equation for secondary flow intake, we can get the pressure of plane 3'' ($P_{t3''}$) by inputting the values of the input secondary pressure (P_{t0}), Mach numbers of the input secondary flow (M_0) and the secondary exit (M_3''), and specific heat ratio of air (γ).

$$A_0 \frac{M_0}{\left[1 + \frac{\gamma'' - 1}{2} M_0^2\right]^{\frac{\gamma'' + 1}{2(\gamma'' - 1)}}} = \frac{P_{t3''}}{P_{t0}} A_3'' \frac{M_3''}{\left[1 + \frac{\gamma'' - 1}{2} M_3''^2\right]^{\frac{\gamma'' + 1}{2(\gamma'' - 1)}}}$$

Mach number at the specific points are found using mass flow rate values. The process is shown in the equations below. The speed of sound (α) is calculated from the equation found in Section 1.2.

$$\dot{m} = \rho AV$$

$$M = \frac{V}{a}$$

$$M = \frac{\dot{m}}{\rho A a}$$

3.4.2. Primary nozzle continuity equation

For the primary nozzle, another continuity equation is used to link the pressures at the primary inlet (P_{t1}) and the primary nozzle outlet ($P_{t3'}$). Similar to the secondary flow continuity equation, the Mach numbers of the corresponding inlet and outlets (M_1 & M_3' respectively), specific heat ratio, and areas of primary inlet (A_1) and outlet (A_3') are substituted into the equation to find the relationship of the pressures at the primary nozzle inlet and outlet. Here, we can just use the input

pressure as mentioned in the operating conditions section to find the pressure, P_{t3}' .

$$P_{t1}A_1 \frac{M_1}{[1 + \frac{\gamma' - 1}{2} M_1^2]^{\frac{\gamma' + 1}{2(\gamma' - 1)}}} = P_{t3}'A_3' \frac{M_3}{[1 + \frac{\gamma'' - 1}{2} M_3'^2]^{\frac{\gamma'' + 1}{2(\gamma'' - 1)}}}$$

At this process the value of Mach number at the nozzle exit is important. Its value can be computed using a variation of the compressible flow function that relates to the area ratios of A/A_{th} . This was previously mentioned in Section 3.2.

$$\frac{A}{A_{th}} = \frac{1}{M} \left[\frac{\gamma + 1}{2(1 + \frac{\gamma + 1}{2} M^2)} \right]^{\frac{-(\gamma + 1)}{2(\gamma - 1)}}$$

3.4.3. Mixing duct continuity equation

The mixing duct continuity equation is lengthier than the previous two continuity equations because it involves the entry of two streams (primary and secondary) that are mixed into one flow. Also, it involves the temperatures of the mixing duct, primary flow outlet and secondary flow outlet.

$$\begin{aligned} \frac{P_{t3}'A_e}{\sqrt{\gamma'R'T_{t3}'}} \frac{M_3}{[1 + \frac{\gamma' - 1}{2} M_3'^2]^{\frac{\gamma' + 1}{2(\gamma' - 1)}}} + \frac{P_{t3}''A_3''}{\sqrt{\gamma''R''T_{t3}''}} \frac{M_3''}{[1 + \frac{\gamma'' - 1}{2} M_3''^2]^{\frac{\gamma'' + 1}{2(\gamma'' - 1)}}} \\ = \frac{P_4A_4}{\sqrt{\gamma RT_{t4}}} [1 + \frac{\gamma - 1}{2} M_4^2]^{\frac{1}{2}} \end{aligned}$$

3.4.4. Diffuser continuity equation

This is the critical continuity equation that will help form the relationship of diffuser pressure against the diffuser diameter at the exit. The diffuser is used to increase the static pressure whilst decreasing the velocity of flow through the exit of the ejector. This is a result of the increase in the flow area as the flow goes along a conical shaped tube. Hence, we can expect the basic relationship to be that as the diameter of the diffuser exit increases, the static pressure will increase.

$$\frac{P_{t4}A_4}{\sqrt{T_{t4}}} \frac{M_4}{\left[1 + \frac{\gamma-1}{2} M_4^2\right]^{\frac{\gamma+1}{2(\gamma-1)}}} = \frac{P_{t5}A_5}{\sqrt{T_{t5}}} \frac{M_5}{\left[1 + \frac{\gamma-1}{2} M_5^2\right]^{\frac{\gamma+1}{2(\gamma-1)}}}$$

For this step, however, the Bernoulli equation is used instead of the continuity equation given. This is because the Bernoulli equation offers a less complex approach to the computation of P_5 .

Nevertheless, is important to take note that the Bernoulli equation must cater for the compressible flow. Therefore, the Bernoulli equation for isentropic ideal gas flow is used. This variation of Bernoulli equation is derived from the basic Euler equation which is the fundamental form of the Bernoulli equation. The derivation can be found in Appendix C.

$$\frac{dP}{\rho} + vdv + gdz = 0$$

$$\left(\frac{\gamma}{\gamma-1}\right) \frac{P}{\rho} + \frac{v^2}{2} + gz = \text{constant}$$

All in all, the calculation process can be summarised in the graphic below. It shows the flow of finding the pressure values from the upstream part of the ejector, to the downstream which is the diffuser. A brief summary of the flow of calculations can be seen in Figure 13.

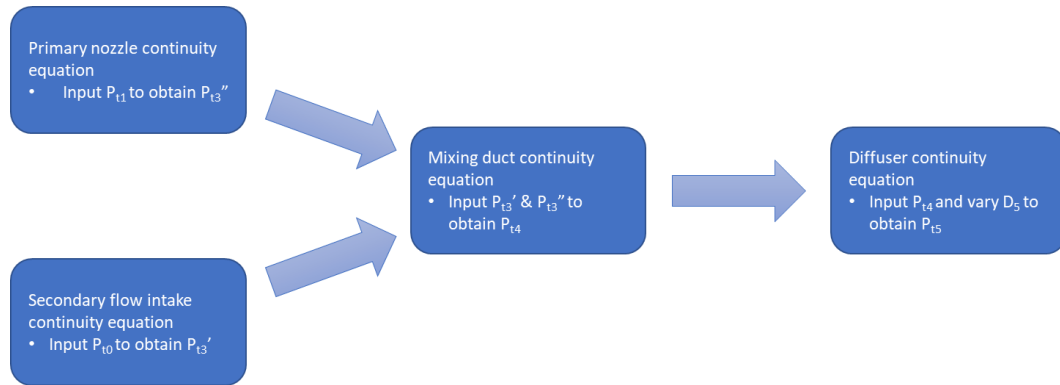


Fig. 13 Shows the calculation flow to find the relationship of P_{t5} against D_5 . A

larger image can be found in Appendix D.

3.5. PROCEDURE

After gathering all the necessary equations, a suitable platform is needed to do these calculations and form necessary graphs. Following that, the platform is also needed to retain the equations so that it can be easily translated to the values of methane by just changing the physical characteristic values, which are density and specific heat ratio.

The chosen program is Microsoft Excel because the choice of values can be easily converted by shifting the cells from one value to another.

4. RESULTS AND DISCUSSION

Once we have obtained the results, there are a few methods to validate the values that we have obtained. The first step is to double check that the changes in pressure agrees with the prediction that was stated previously. The predicted pressure relationship for the diffuser is that as the diffuser exit increases in area, the velocity is expected to decrease by continuity. Hence, the pressure will increase along with the decrease in pressure.



Fig. 14 The graph of diffuser exit pressure against diffuser exit diameter for air as a medium

Through the calculations done on Excel, the graph in Figure 14 is obtained. The vertical axis represents the pressure of the diffuser exit, P_{t5} , whereas the horizontal axis represents the diameter of diffuser exit, D_5 . The general

relationship that can be seen is that there is an inverse relationship between D_5 and P_{t5} . This is what was predicted outcome. Hence it proves the first condition of this experiment.

The next step is to validate the results by cross checking the values with the readily available value found in ESDU 84029. Figure 15 is an optimum performance relationship between pressure ratios P_{t1}/P_{t5} and P_{t5}/P_{t0} with varying mass flow ratios between the primary and secondary flow rate, r_m . This graph is an empirical graph that is used in the Quick Design Method.

$$r_m = \frac{\dot{m}''}{\dot{m}'}$$

To elaborate further, the graph shown in Figure 15 shows different curves that represent varying values of mass flow rate ratios, r_m , that ranges from 0 to 5. The chosen r_m for this project was set to be 0.2, as seen in the table in Figure 11 where the primary mass flow rate is $1 \text{ m}^3/\text{s}$ and the secondary mass flow rate is $0.2 \text{ m}^3/\text{s}$. Therefore, the focus is will be on the curve that represents this value and it is conveniently highlighted in yellow.

Now, we should compare the P_{t1}/P_{t5} and P_{t5}/P_{t0} pressure ratios of the calculated values from the Excel sheet. P_{t1}/P_{t5} is found to be 4.04 ($P_{t1} = 500 \text{ kPa}$ & $P_{t5} = 123.8 \text{ kPa}$) and P_{t5}/P_{t0} is 3.10 ($P_{t5} = 123.8 \text{ kPa}$ & $P_{t0} = 40 \text{ kPa}$).

With the values of $P_{t1}/P_{t5} = 4.04$ and $P_{t5}/P_{t0} = 3.10$, we can then compare to the point on the graph as P_{t1}/P_{t5} represents the horizontal axis and P_{t1}/P_{t5} represents the vertical axis. The point closest to the calculated values, marked by the red cross on Figure 15, do not deviate much. Therefore, we can conclude that the calculation is on track and the next stage of the project can commence.

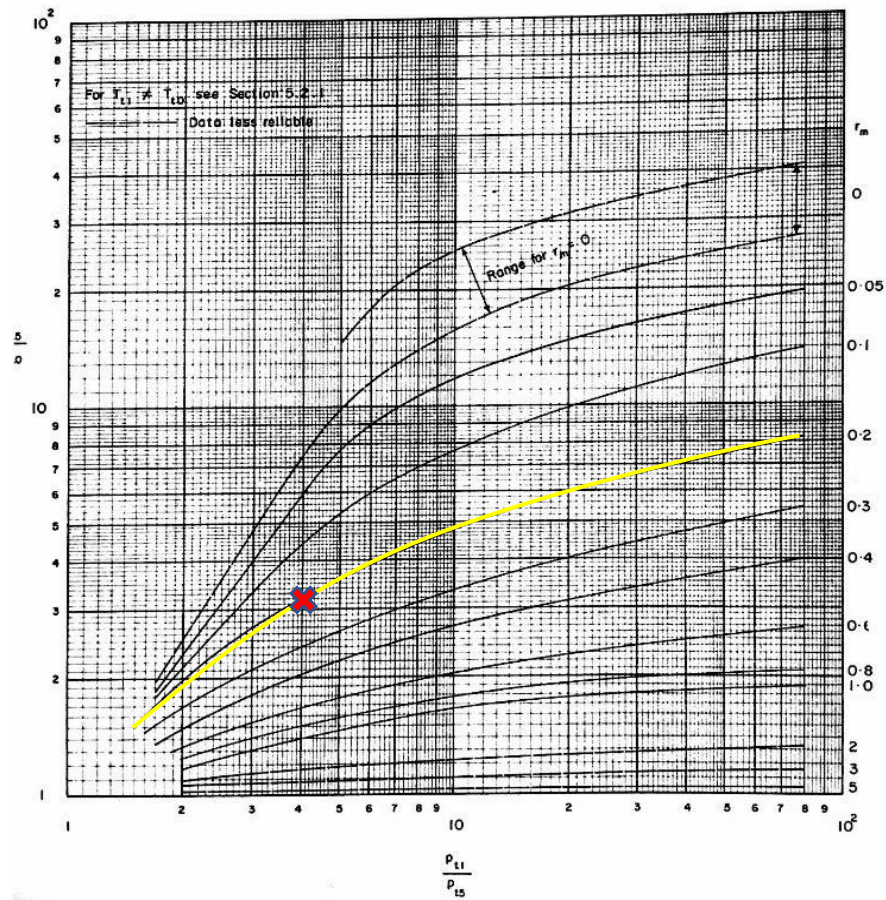


FIGURE 1. QUICK DESIGN METHOD: OPTIMUM PERFORMANCE RELATIONSHIP BETWEEN PRESSURE RATIOS AND MASS FLOW RATIO

Fig. 15 The extracted value from the optimum relationship graph in ESDU 84029

that was used to validate the result

	Expected (Obtained from ESDU 84029)	Calculated
P_{t1}/P_{t5}	4	4.04
P_{t5}/P_{t0}	3.2	3.1

Fig. 16 Table to tabulate the expected value against the result obtained

4.1. OPTIMUM DIAMETER

After ensuring the validity of the values, we can now choose the right diameter for the diffuser. Since we know that the bigger the diameter of the diffuser, the more pressure recovered by the flow, a cut off diameter should be chosen to ensure that we do not choose an excessively large diffuser. This diffuser diameter is chosen to be the 99.8% efficiency value. Therefore, in this case, 99.8% of the max value of 124100 Pa is 124111 Pa which, when extrapolated using the graph obtained, will return a value of around 220mm. And that is the concluded to be the 'design condition' for a supersonic gas ejector for the configuration set for this project.

A point to note is the efficiency is based on the maximum possible value that the diffuser can attain through pressure recovery by increasing the area size of the diffuser exit.

$$Efficiency = \frac{Pressure\ at\ stated\ exit\ diameter}{Maximum\ Pressure} \times 100\%$$

4.2. METHANE

With the calculations validated, the necessary values can then be converted to cater for the calculation of methane. Since the Excel sheet has the calculations done with variables of density and heat ratio incorporated in the equations, we can just swap the values of density and specific heat ratio of methane to obtain the graph of pressure at diffuser exit against its diameter when methane is used. Here, we expect the graph to look like the previous one as it retains the general relationship between the diffuser diameter and the diffuser pressure.

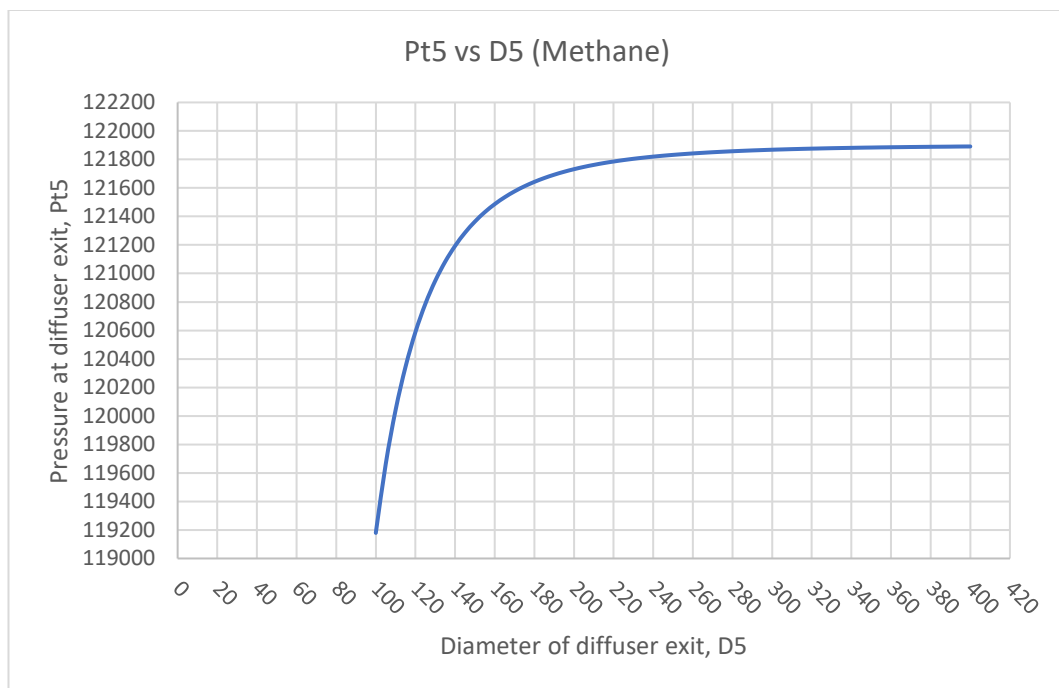


Fig. 17 The graph of diffuser exit pressure against diffuser exit diameter for methane as a medium

As predicted, the graph maintained its shape but it tends to a different maximum value. However a different maximum value (ie. 121900 Pa) is obtained as the physical characteristics were altered for the calculation. With this graph we can

set the preferred 'design condition' for a gas ejector set with the stated dimensions in Section 3.2 and set operating conditions as previously discussed in Section 3.1.

Similar to the final step in finding the preferred diameter for the gas ejector that handles air, we will try to calculate the diameter of diffuser exit is 99.8% efficient compared to the max value of 121900 Pa. This corresponds to the value of 121656.2 Pa. When extrapolated on the graph of Figure 17, the diameter value is about 185 mm. Hence this is set as the diameter of the diffuser.

4.3. PERFORMANCE EVALUATION WITH EXAMPLE

With the Excel sheet laid out with the necessary equations, we then translate the values to cater to different operating conditions, dimensions, or even different gases, if the respective density and specific heat ratio value is input instead of air or methane. From there, the same type of graph would be produced, and the desired diameter can be extrapolated from the graph. This can also be used to find out the efficiency of a gas ejector that is already in use.

For example, if a gas ejector with the following design conditions: similar dimensions as set in Section 3.2 but subjected to different input pressures (4 bar for primary flow and 1 bar for secondary flow), a new graph can be set.

Operating Conditions					
Primary		Secondary		Mixing Duct	
T_p (in Celsius)	25	T_s (in Celsius)	25	T_4 (in Celsius)	25
T_p (in Kelvin)	298	T_s (in Kelvin)	298	T_4 (in Kelvin)	298
P_1 (in Pa)	4.00E+05	P_0 (in Pa)	1.00E+05		
Mass FR (in kg/s)	1	Mass FR (in kg/s)	0.2	Mass FR (in m^3/s)	1.2
Volumetric FR (in m^3/s)	1.225	Volumetric FR (in m^3/s)	0.245		

Fig. 18 Table showing the new set of operating conditions for this performance evaluation example

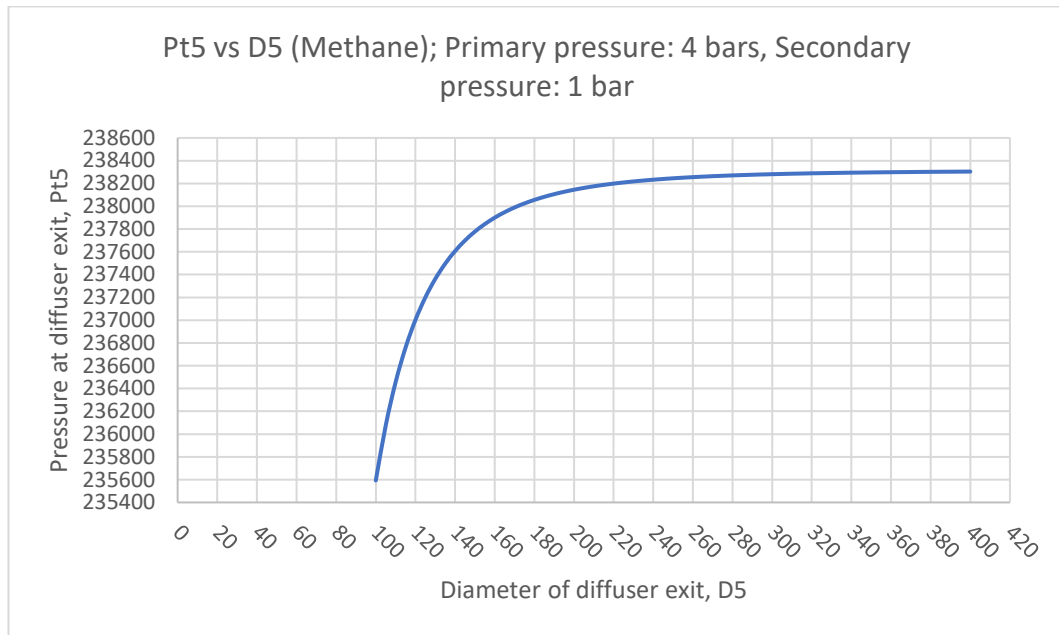


Fig. 19 Graph of same gas ejector with input pressure of 4 bars for primary flow and 1 bar for secondary flow

Figure 19 shows the resulting graph of the changes made to the operating conditions. The asymptote of this graph, which is the maximum possible pressure that this ejector with the stated pressure, is around 238300 Pa. To achieve 99.8% of its efficiency, the diffuser should be able to recover pressure and achieve a total pressure of 237823 Pa at the exit. This corresponds to the value of 155 mm diameter exit. If there is a geometrically similar diffuser with the same operating conditions that is already in use has a different exit diameter, the efficiency of that gas ejector gas be obtained by using the maximum pressure value of the graph in Figure 19.

5. CONCLUSION AND FUTURE WORK

All in all, the outcome of the project has been positive and potentially beneficial to this field of work. The values obtained has been checked with the predicted physical phenomenon at the start and it agrees with one another. However, the validation of results could be further justified through other means for it to be fully considered a success. This is discussed in greater detail in the following section. Also, the possibility of research of other forms of gas ejectors will be analysed.

5.1. VARIATIONS OF VALIDATION

As all calculation-based research, they necessitate the validation through physical experiments. This is to thoroughly cross check the obtained data and hence the model can be used by future references and calculations by other researches or companies working on developing a supersonic gas ejector.

Another layer of validation can be made through computational fluid dynamics (CFD) such as the Ansys Fluent or COMSOL. Computational fluid dynamics, which is a branch of numerical analysis that deals with fluid dynamics, offers a quick approach to finding velocities and pressures. But its usage should not be done without thorough research because there are many different input settings that accompany different types of flow regimes. Hence more knowledge needs to be gathered regarding this method before it can be utilised.

5.3. OTHER CONFIGURATIONS

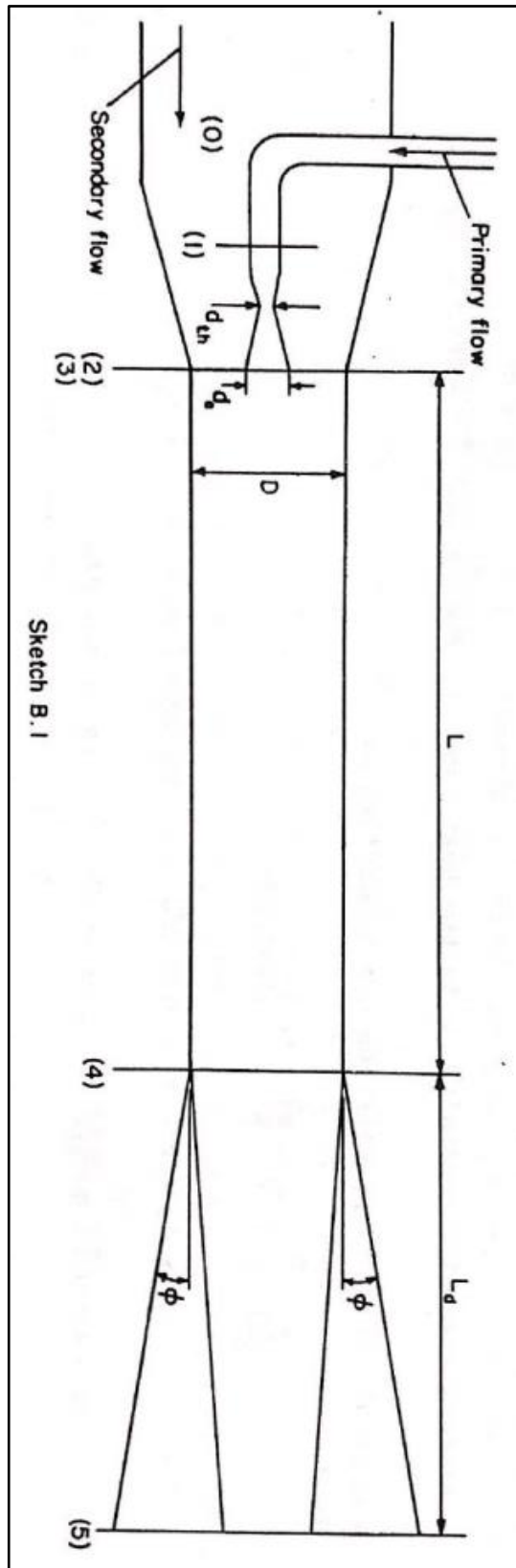
Once the data calculated has been thoroughly evaluated and checked, there are configurations that can be explored within the field of gas ejectors itself. All of which can have unique effects on the flow within the ejector. One such example is the placement of primary nozzle exit. Even altering the horizontal placement will give varying back pressures for the primary nozzle, hence giving different expansion characteristics to the supersonic flow. Furthermore, to reiterate the objective of this project, even though there have been a multitude of findings done by researchers with different combinations, many have yet to experiment their concept on methane as a medium.

6. REFERENCES

- [1] *A-to-Z Guide to Thermodynamics, Heat and Mass Transfer, and Fluids Engineering: AtoZ*. Begellhouse, 2006.
- [2] "Ejectors." International Petroleum Industry Environmental Conservation Association, 10-Apr-2013, [Online]. Available:
<http://www.ipieca.org/resources/energy-efficiency-solutions/efficient-use-of-power/ejectors/>.
- [3] J. M. Cimbala, "Compressible Flow in a Converging-Diverging Nozzle." 19-Jan-2012, [Online]. Available:
https://www.me.psu.edu/cimbala/me325web_Spring_2012/Labs/Compressible/intro.pdf.
- [4] L. Vojta and V. Dvorak, "Measurement and calculating of supersonic ejectors," *EPJ Web Conf.*, vol. 213, p. 02097, 2019, doi: 10.1051/epjconf/201921302097.
- [5] J. C. Dutton and B. F. Carroll, "Optimal Supersonic Ejector Designs," *J. Fluids Eng.*, vol. 108, no. 4, pp. 414–420, Dec. 1986, doi: 10.1115/1.3242597.
- [6] E. Z. Am, H. Mh, O. Zm, and E. Am, "Study of the Configuration and Performance of Air-Air Ejectors based on CFD Simulation," *J. Aeronaut. Aerosp. Eng.*, vol. 06, no. 04, 2017, doi: 10.4172/2168-9792.1000201.

7. APPENDICES

APPENDIX A: DIAGRAM OF SUPERSONIC GAS EJECTOR



APPENDIX B: SAMPLE CALCULATIONS FOR PRIMARY NOZZLE CONTINUITY
EQUATION

$$P_{t1}A_1 \frac{M_1}{[1 + \frac{\gamma' - 1}{2} M_1^2]^{\frac{\gamma'+1}{2(\gamma'-1)}}} = P'_{t3}A'_3 \frac{M'_3}{[1 + \frac{\gamma'' - 1}{2} M'^2_3]^{\frac{\gamma''+1}{2(\gamma''-1)}}}$$

After substituting the value of specific heat ratio for air ($\gamma=1.4$), and rearranging the variables, the following form of the equation is achieved.

$$P'_{t3} = (\frac{A_1}{A'_3})(\frac{M_1}{M'_3})(\frac{1 + 0.2M'^2_3}{1 + 0.2M_1^2})^3 P_{t1}$$

Mach values can be calculated from the mass flow rate, area of pipe, density of fluid and speed of sound in the fluid.

$$M_1 = \frac{\dot{m}_1}{\rho A_1 a}$$

$$M_3 = \frac{\dot{m}_3}{\rho A'_3 a}$$

APPENDIX C: DERIVATION OF BERNOULLI EQUATION FOR ISENTROPIC IDEAL GAS
FLOW FROM EULER EQUATION

$$\frac{dP}{\rho} + vdv + g dz = 0$$

$$\int \frac{dP}{\rho} + \int vdv + \int g dz = c$$

For compressible flow, $\int \frac{dP}{\rho}$ does not equate to $\frac{dP}{\rho}$, which is applicable to non-compressible flow. Therefore, $\frac{dP}{\rho}$ has to be found using the respective compressible flow processes (i.e. Isothermal ($dT = 0$) or adiabatic ($Q = 0$)).

1) Isothermal, $\frac{P}{\rho} = c_1, \rho = \frac{P}{c_1}$

$$\int \frac{dP}{\rho} = c_1 \ln P = \frac{P}{\rho} \ln P$$

$$\frac{P}{\rho} \ln P + \frac{v^2}{2} + gz = \text{constant}$$

2) Adiabatic Process, $\frac{P}{\rho^k} = c_2, \rho = \frac{P^{1/k}}{c_2^{1/k}}$

$$\int \frac{dP}{\rho} = \int c^{1/k} P^{-1/k} dP = \left(\frac{k}{k-1} \right) \frac{P}{\rho}$$

$$\left(\frac{k}{k-1} \right) \frac{P}{\rho} + \frac{v^2}{2} + gz = \text{constant}$$

APPENDIX D: CALCULATION FLOW TO ARRIVE AT THE RELATIONSHIP OF P_{t5}

AGAINST D_5

