Midterm Exam Stage 3: Collaborative Exam
(LS 30B - Spring 2022)

- We strongly encourage you to collaborate closely with your teammates on these problems by talking synchronously via Zoom or in person, rather than dividing up the work, because these problems are challenging and we want you to learn from each other! Teams who do that also tend to earn higher scores!
- You may use your study guide, other notes, our textbook, our course's BruinLearn site, CoCalc, and the Internet (not including Chegg, CourseHero, or similar services), but you may **only discuss exam content with your learning team** (and our instructional team via Campuswire) until the end of the exam period (i.e., Friday May 6 @ 5 pm).
- If you have any questions or concerns about this exam, including difficulties with your learning team, please post a private question on Campuswire ("To Instructor and TAs").
- We want to make sure everyone contributes, so the first page should include the **names and contributions** of all members of the learning team (worth 5 points, while each problem is worth the number of points indicated below).
- Your response can be handwritten or typed and should **include your work** for each problem. If typed, you should use a font of at least 12pt. Hand-written text must be clear and legible. If you use CoCalc, please include a screenshot of your code and its result. If you use Google Docs, you may want to use its built-in equation editor or the Hypatia add-on (which was used below).
- You can use this template Google Doc, but, if you use it, check before submitting that your answers are consistent with the questions in the original PDF since you can edit the Google Doc.
- This exam should be submitted to **Gradescope by Friday May 6 @ 5 pm**. **Only one member from each team should submit the collaborative stage on Gradescope**, as a group assignment including their team members. **Make sure to properly select each problem in Gradescope.**

Learning Team Contributions - *list team member names & describe who did what*

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Problem 1: Predicting Oscillations

Consider the following system of differential equations:

$$W'(t) = \frac{8}{1 + X(t-5)^{12}} - 0.4[W(t)]^2$$
$$X'(t) = 5Z(t) - 0.3X(t)$$
$$Y'(t) = \frac{0.4}{1 + [X(t)]^7} - 0.1Y(t)$$
$$Z'(t) = \frac{[Y(t)]^4}{1 + [Y(t)]^4} - 0.15Z(t)$$

A. (4 points) Draw a feedback diagram (also known as a cause/effect diagram) for this system.

B. (4 points) Would you expect this model to have oscillatory behavior? Explain why, using the specific features of the model that helped you reach this conclusion.

C. (4 points) Suppose that the model above does show oscillations, but it is a model of a biological system for which oscillations are harmful. What parameters in the model could be changed to make it not oscillate, and how should they most likely be changed?

Problem 2: Programming Discrete-Time Exponential Growth

A. (6 points) Write a Python function that uses iteration to simulate discrete-time exponential growth $X_{t+1} = RX_t$. The function's inputs are the initial value X_0, the number of time steps t_{max}, and the constant R. The function should output a list of values of X_t. Write a comment for each line of your code explaining what it does.

B. (6 points) Test your function by using it to simulate and plot the time series for the following problem (as always, plots should have appropriate axes labels). Check your function's final answer using the solution to discrete-time exponential growth.

An elephant population is growing at 3% per year. If the population starts with 1000 elephants, how many elephants will there be in 15 years?

Problem 3: Modeling Gorillas

(12 points) Gorillas can be classified as immature (I), mature (M), or senior (S). Write a discrete-time matrix model of a gorilla population in a typical year using the following assumptions.

- A gorilla reaches maturity at 12 years of age, so about 8% of immature gorillas reach maturity each year.
- About 7% of mature gorillas will give birth to one offspring each year.
- A gorilla is considered a senior when it reaches about 28 years, or about 16 years after reaching maturity. Thus, about 6% of mature gorillas become seniors each year.
- Rarely, a senior gorilla will give birth. Assume that on average only 0.7% of seniors produce offspring each year.
- About 92% of mature gorillas survive and remain mature gorillas each year.
- Approximately 15% of immature gorillas and 11% of seniors die each year.

Use the assumptions above to draw a compartment/flow diagram and write a discrete-time model. If this model is linear, write the matrix that represents this model. If the model is not linear, explain which term fails the properties of linear functions.

Problem 4: Linear Algebra

A. (6 points) Suppose $f : \mathbb{R}^2 \to \mathbb{R}^3$ such that

$$f\left(\begin{bmatrix}6\\5\end{bmatrix}\right) = \begin{bmatrix}20\\-13\\84\end{bmatrix},\ f\left(\begin{bmatrix}0\\-3\end{bmatrix}\right) = \begin{bmatrix}6\\-3\\-18\end{bmatrix}.$$

Find the matrix that represents f. Check your work using matrix-vector multiplication.

(4 points each) Based on the information given for each function below, state if each function is a linear function and use the abstract definition of linear functions to explain your decision. If you can't determine if the function is linear, describe what additional information you need. If you can't use the abstract definition, explain using another definition of linear functions for partial credit. Show any work necessary.

B. $$g\left(\begin{bmatrix}X\\Y\end{bmatrix}\right) = \begin{bmatrix}XY\\2Y\end{bmatrix}$$

C. $$h\left(\begin{bmatrix}1\\-2\end{bmatrix}\right) = \begin{bmatrix}3\\1\\2\end{bmatrix},\ h\left(\begin{bmatrix}-3\\4\end{bmatrix}\right) = \begin{bmatrix}5\\-3\\7\end{bmatrix},\ h\left(\begin{bmatrix}10\\-14\end{bmatrix}\right) = \begin{bmatrix}12\\10\\-19\end{bmatrix}$$