General circulation of the atmosphere

ATM2106

Last time

$$R_o = \frac{U}{fL}$$ → Rossby number ~ 10^{-1}

$$\frac{1}{\rho}\frac{\partial p}{\partial x} - fv = 0$$

$$\frac{1}{\rho}\frac{\partial p}{\partial y} + fu = 0$$

Geostrophic balance

$$u_g = -\frac{1}{f\rho}\frac{\partial p}{\partial y}$$

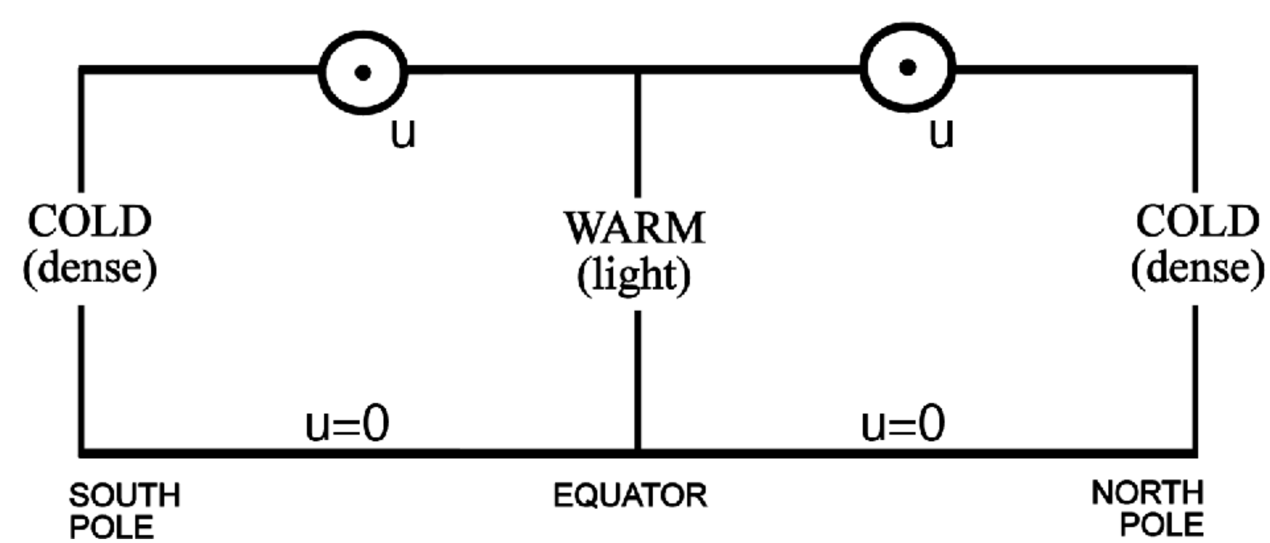
$$v_g = \frac{1}{f\rho}\frac{\partial p}{\partial x}$$

Geostrophic wind

Thermal wind equation

$$\frac{\partial u_g}{\partial p} = \frac{R}{fp}\frac{\partial T}{\partial y}$$

$$\frac{\partial v_g}{\partial p} = -\frac{R}{fp}\frac{\partial T}{\partial x}$$



Last time

Subgeostrophic flow

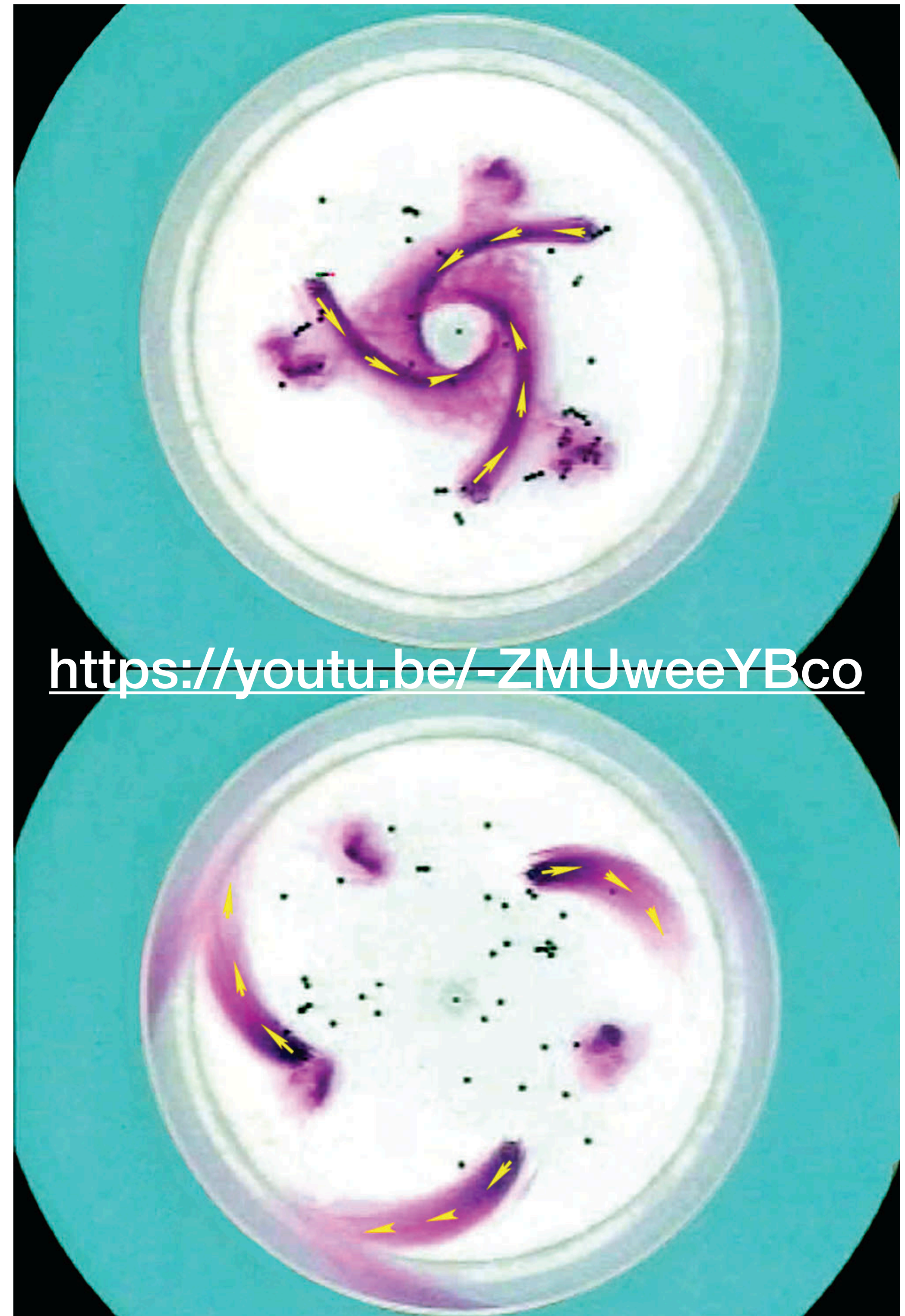
$$\frac{1}{\rho}\frac{\partial p}{\partial x} - f(v_g + v_{ag}) = \mathscr{F}_x$$

$$\frac{1}{\rho}\frac{\partial p}{\partial y} + f(u_g + u_{ag}) = \mathscr{F}_y$$

$$\left.\begin{aligned} -fv_{ag} &= \mathscr{F}_x \\ fu_{ag} &= \mathscr{F}_y \end{aligned}\right] \quad f\hat{\mathbf{z}} \times \mathbf{u}_{ag} = \boldsymbol{\mathscr{F}}$$

$$\downarrow$$

ageostrophic wind is to the right of $\mathscr{F}$ (in NH)

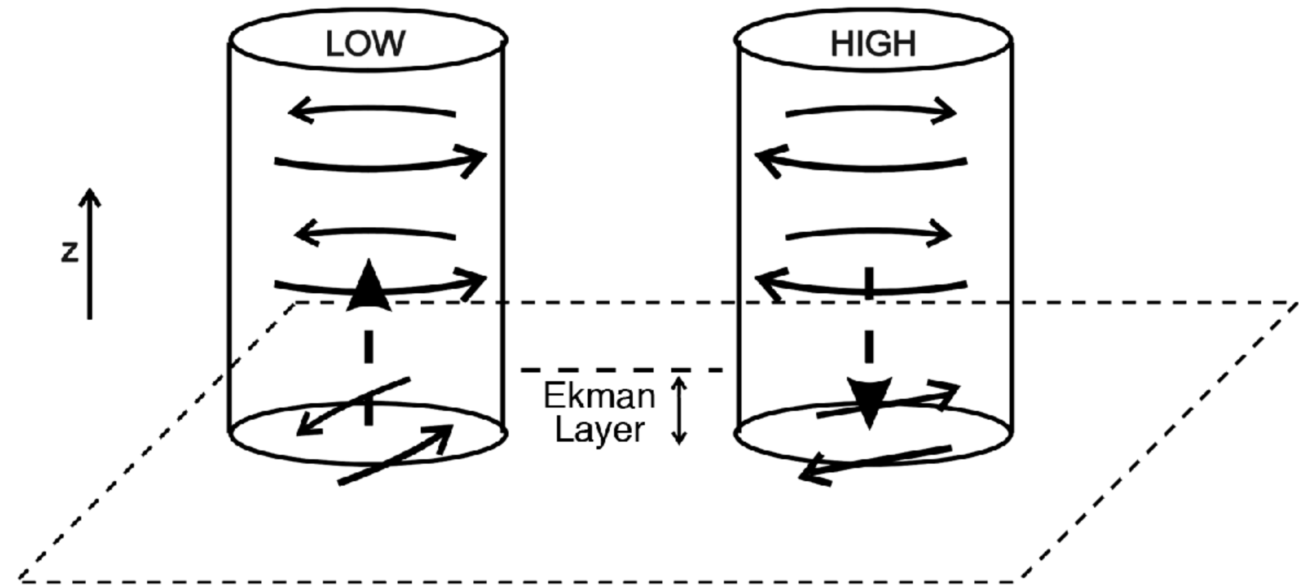


Today's topic

- A little bit more of ageostrophic wind
- General circulation in the atmosphere!

1. Ageostrophy: vertical motion

$$\nabla_p \cdot \mathbf{u}_{ag} + \frac{\partial \omega}{\partial p} = 0$$

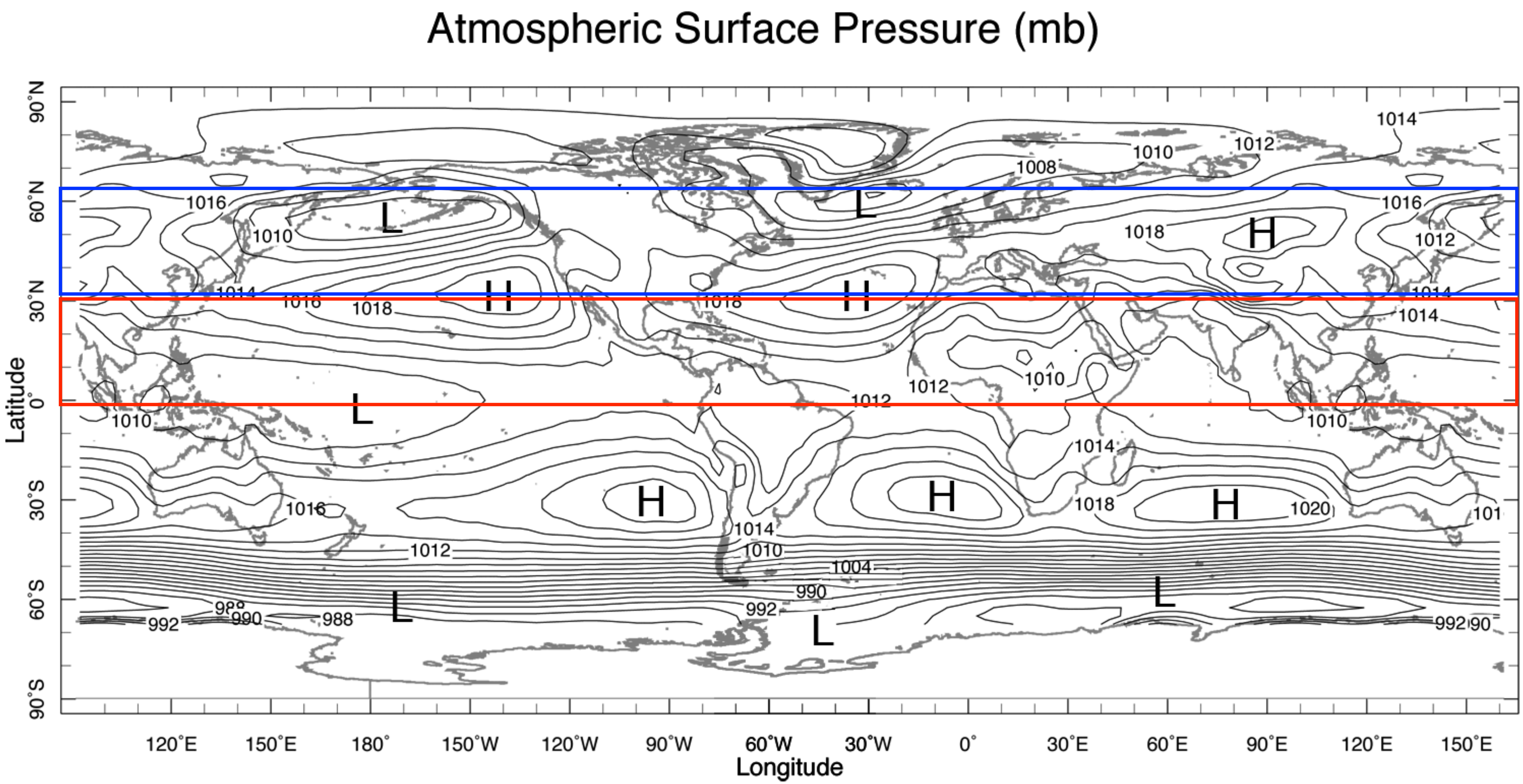


Ekman pumping Ekman suction

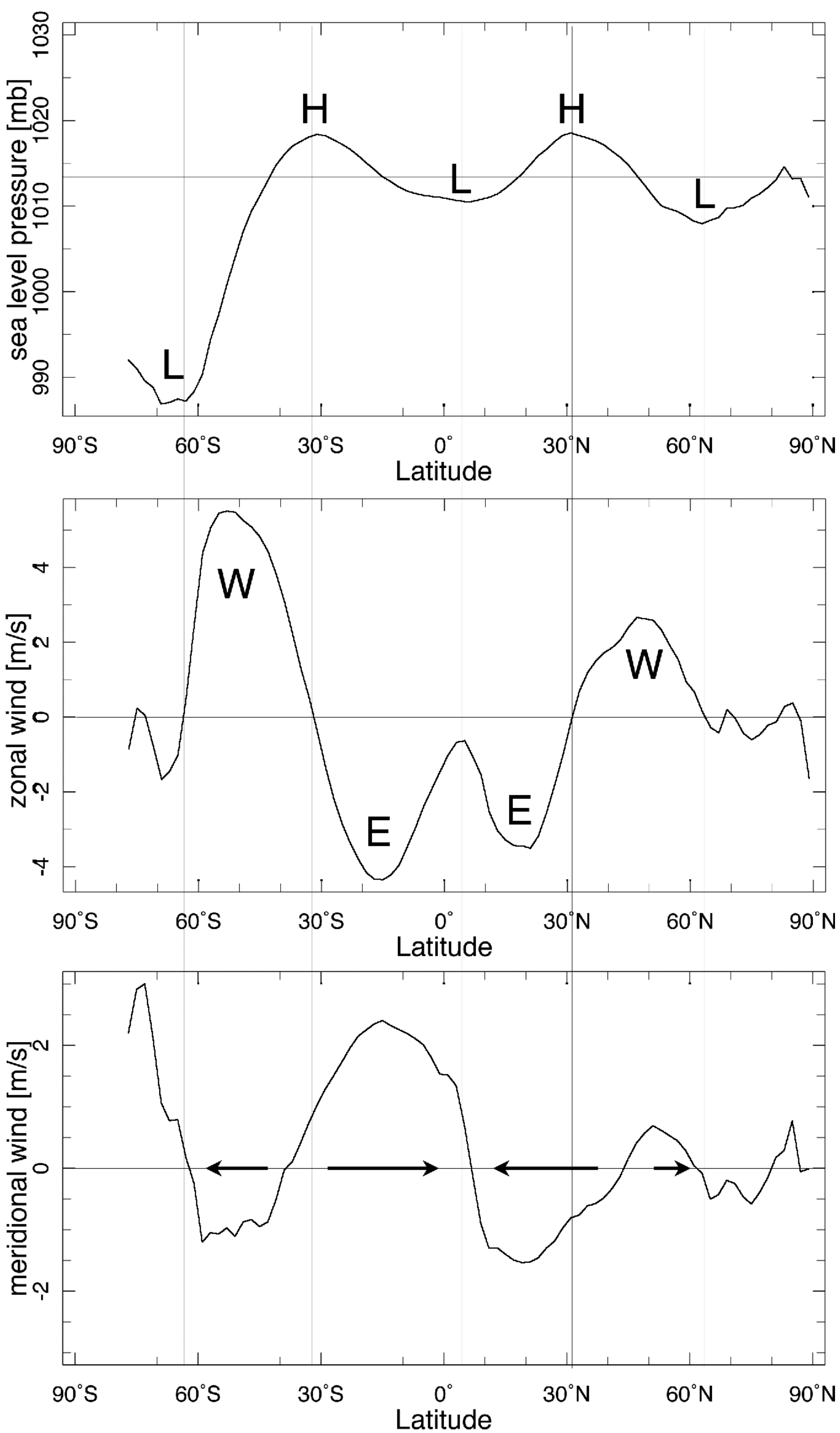
Convergence : $\nabla_p \cdot \mathbf{u}_{ag} < 0 \rightarrow \frac{\partial \omega}{\partial p} > 0 \rightarrow \omega < 0 \rightarrow$ **Upward**

Divergence : $\nabla_p \cdot \mathbf{u}_{ag} > 0 \rightarrow \frac{\partial \omega}{\partial p} < 0 \rightarrow \omega > 0 \rightarrow$ **Downward**

1. Ageostrophy : planetary scale motions



1. Ageostrophy : planetary scale motions



Zonally averaged sea level pressure

Zonally averaged zonal wind

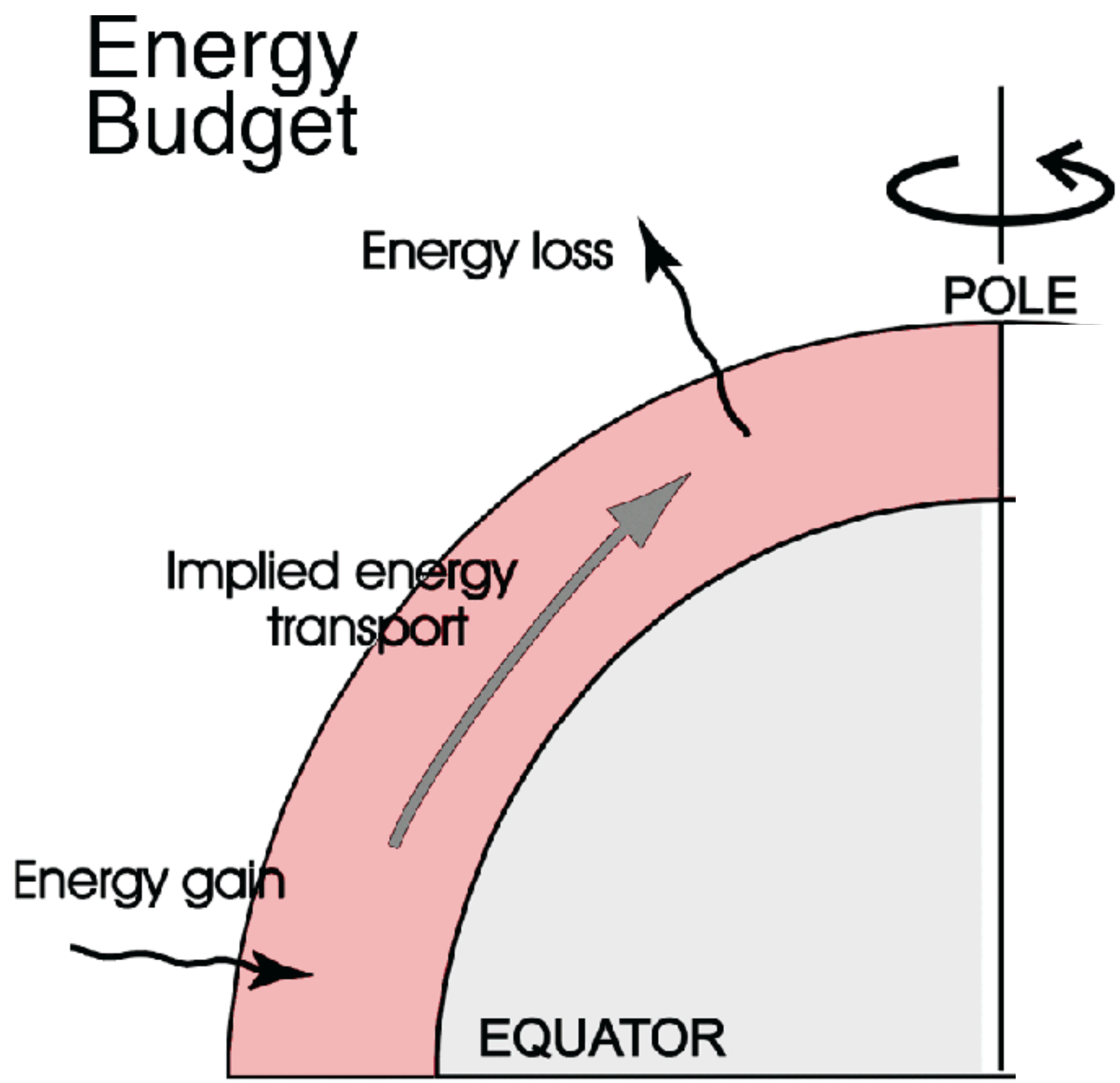
Geostrophic balance

Zonally averaged meridional wind

ageostrophic component

Feeding rising motion

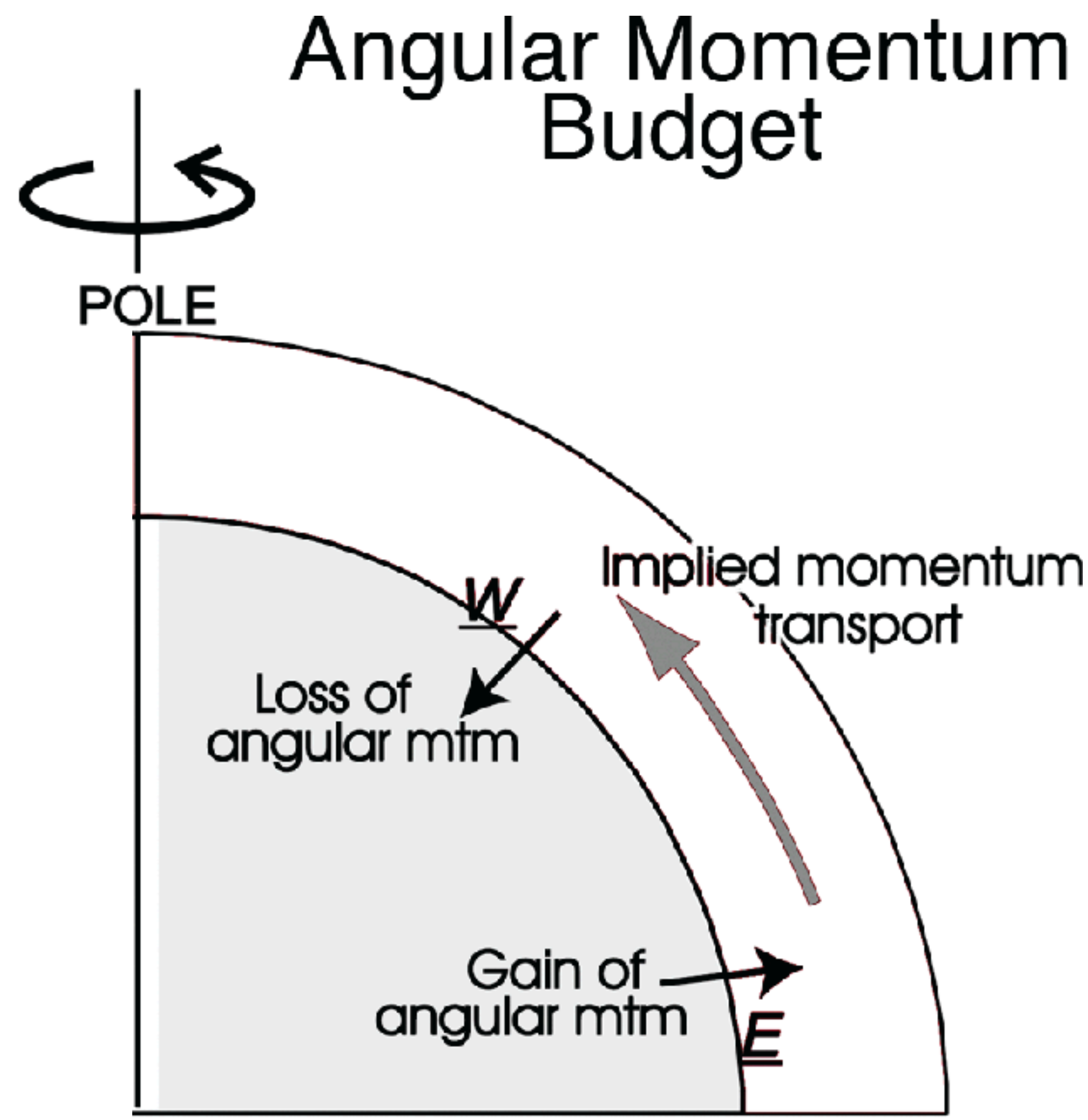
2. General circulation of the atmosphere



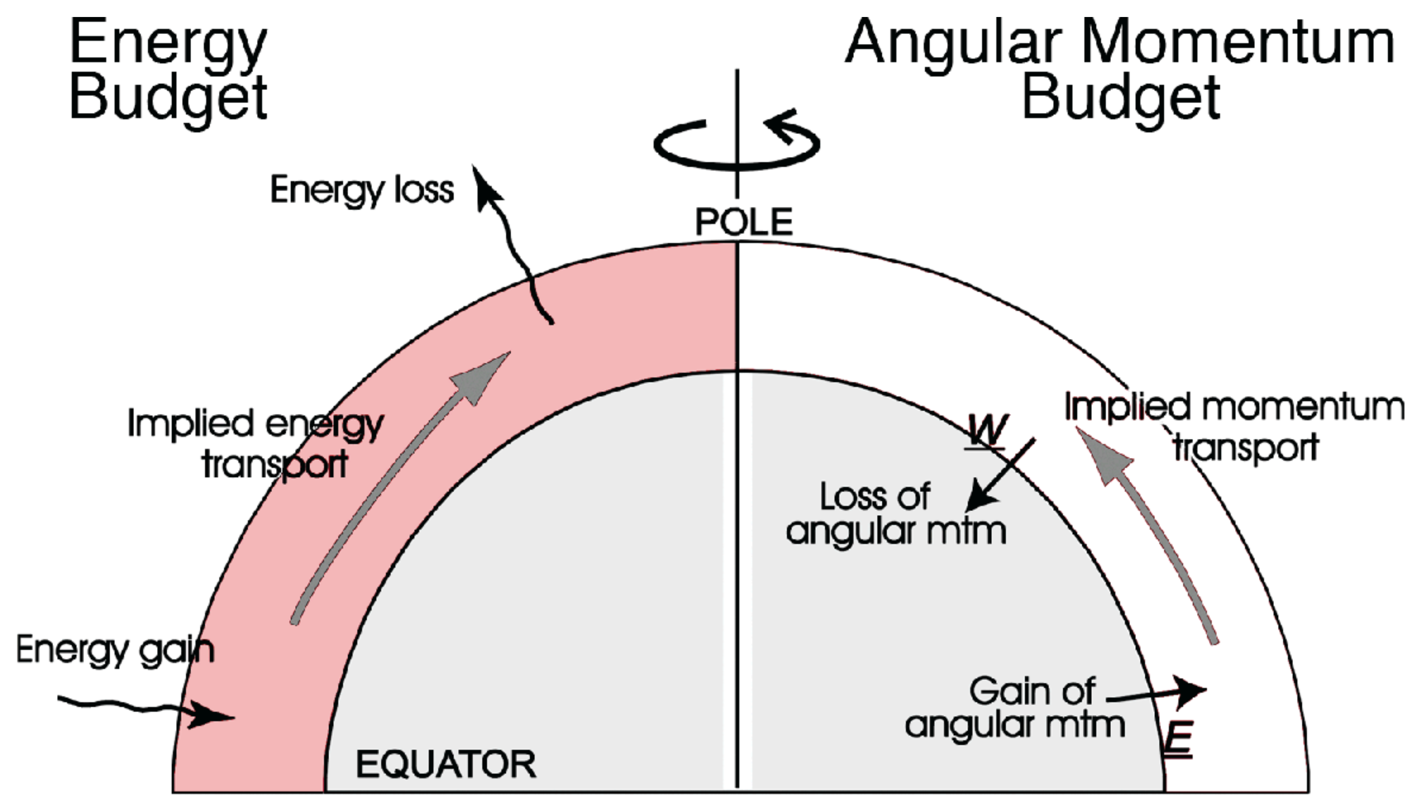
The atmosphere has to transport energy from equator to pole to maintain the temperature gradient

2. General circulation of the atmosphere

The atmosphere has to transport westerly angular momentum from low to middle latitude.



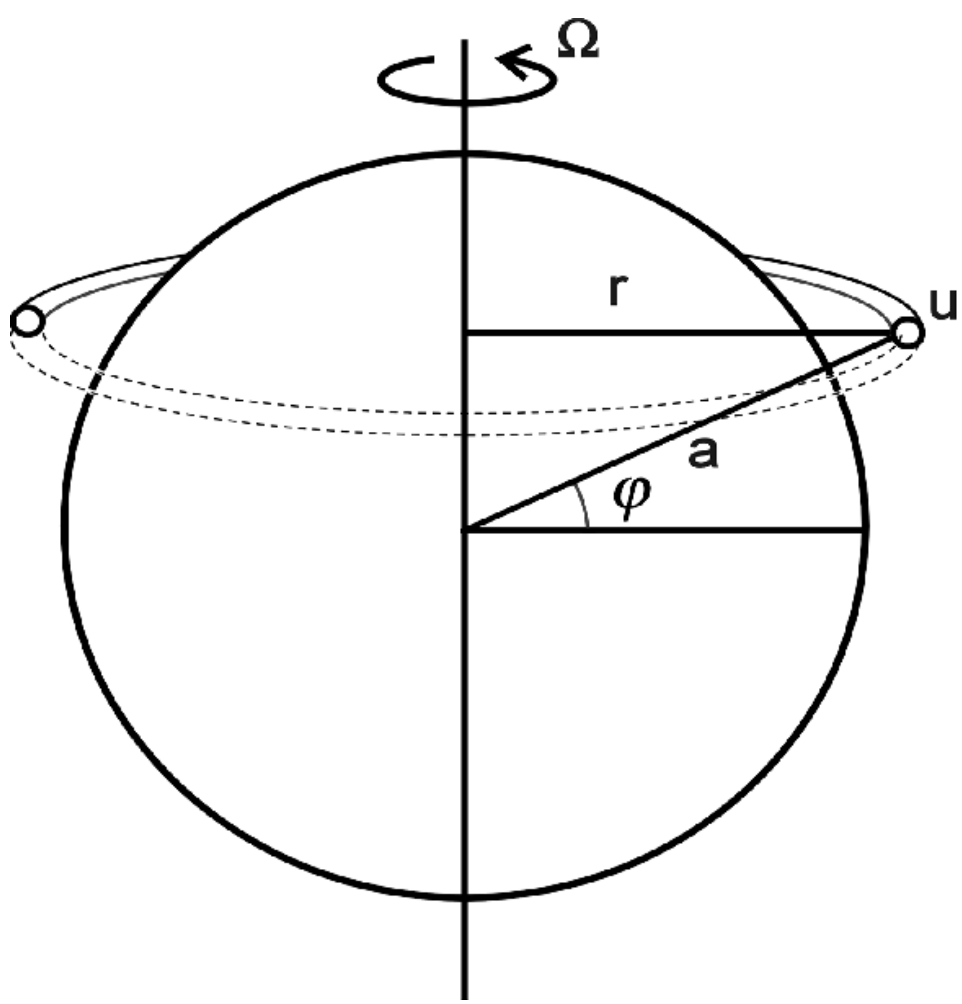
2. General circulation of the atmosphere



- In the upper troposphere, we know that the dominant flow is the west-to-east, which cannot explain the equator-to-pole transport of heat and angular momentum.
- This is why the overturning circulation becomes important.
- But the meridional overturning circulation does not extend all the way to the pole as in the figure. Why?

3. Mechanistic view of the circulation: tropics

- For simplicity, let's assume homogeneous surface with no seasonality.
- So we focus on the temperature gradient in latitudinal direction.

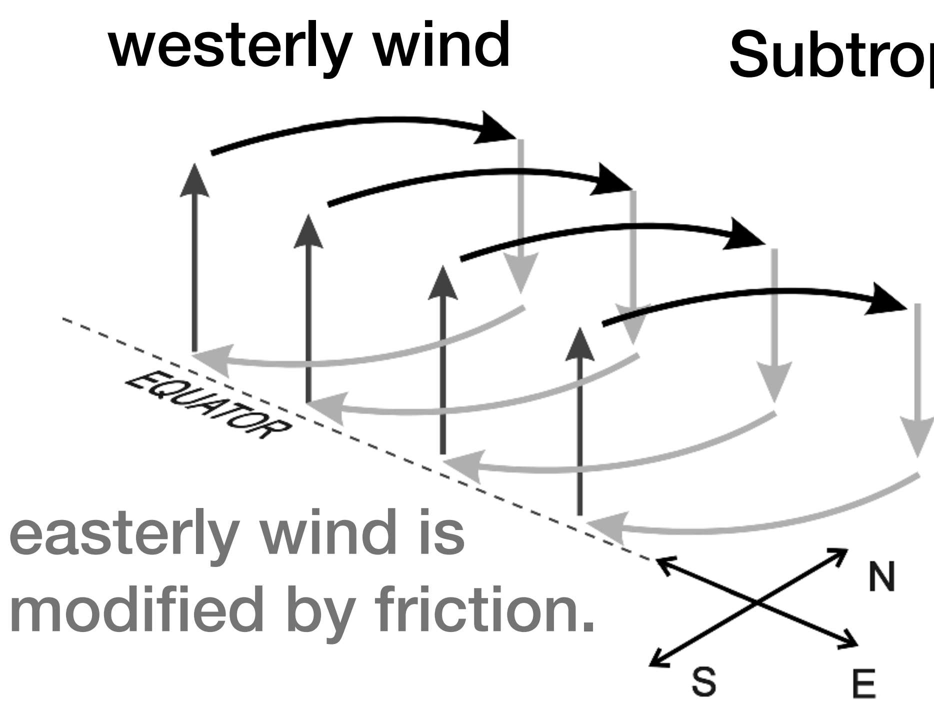


Absolute angular momentum

$$A = \Omega r^2 + ur$$
$$= \Omega a^2 \cos^2\phi + ua\cos\phi$$

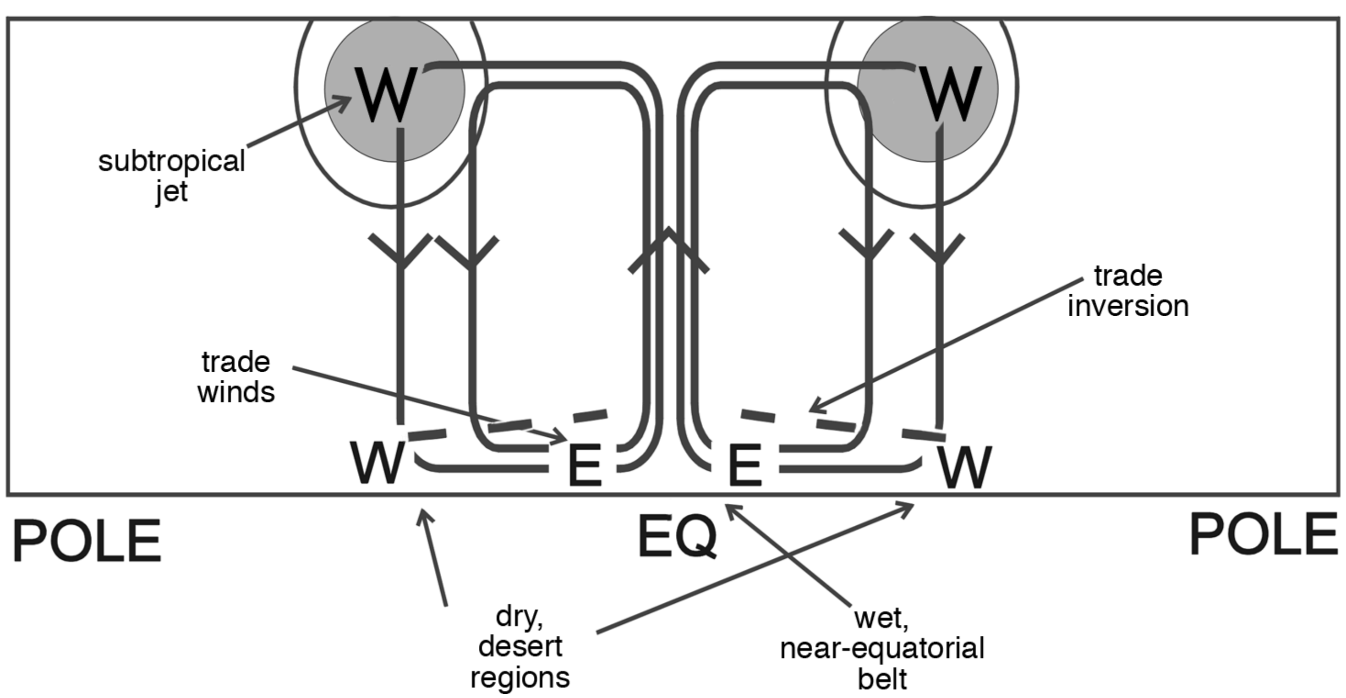
If we start $u=0$ at the equator,
u grows as we go to the north.

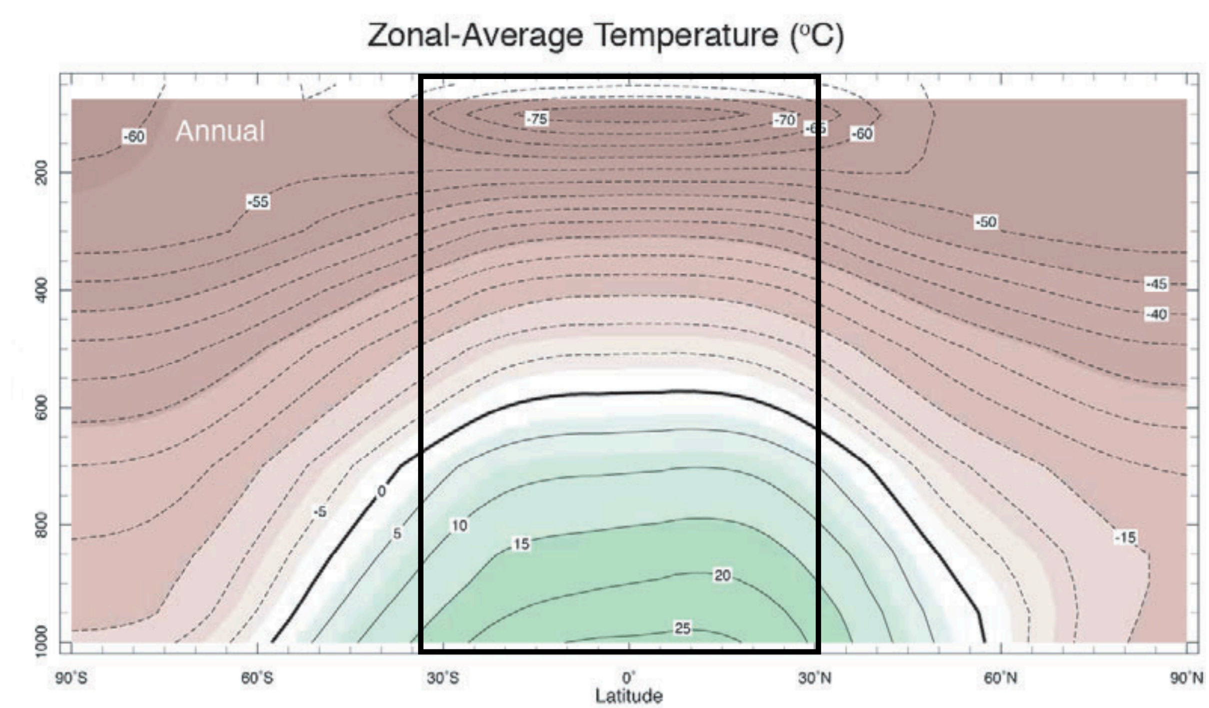
3. Mechanistic view of the circulation: tropics



Subtropical jet is driven in large part by the advection of angular momentum by the Hadley cell.

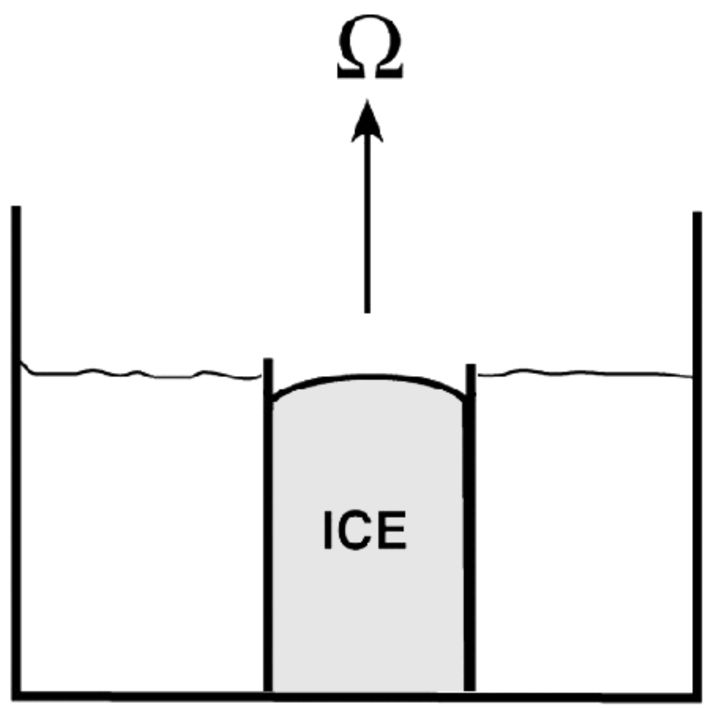
Small T gradient reflects how effective Hadley cell is in transporting the heat.

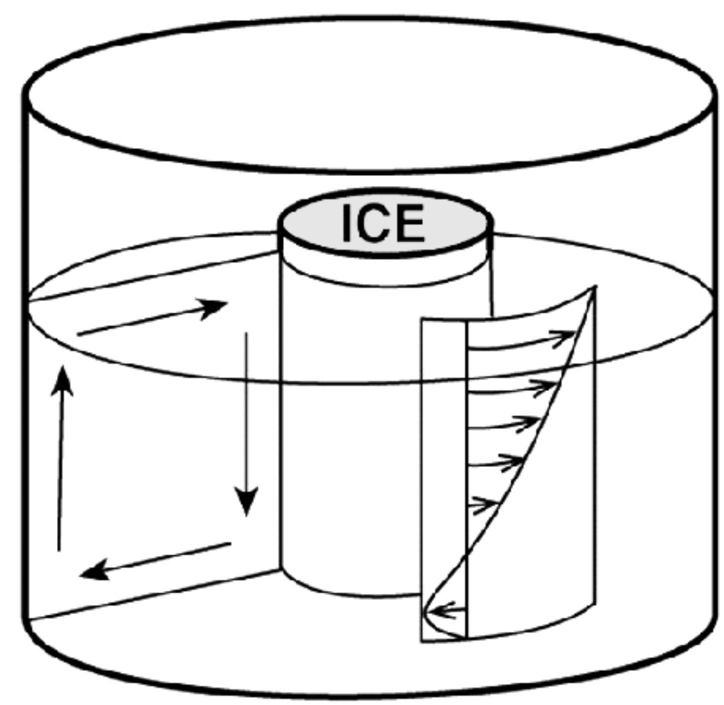




3. Mechanistic view of the circulation: tropics

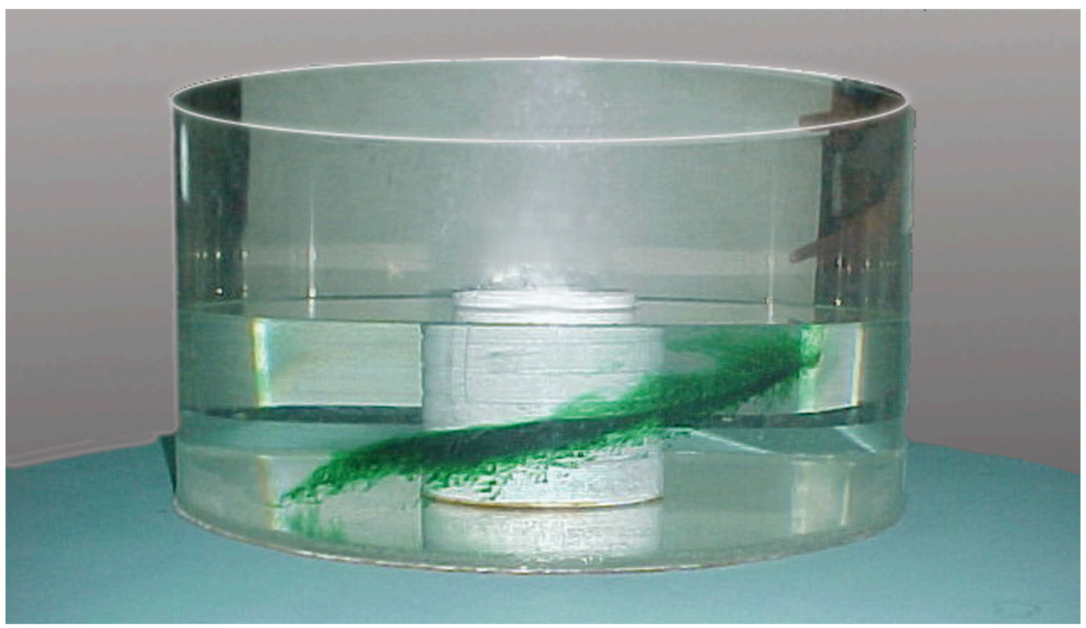
- Visualize the Hadley cell
- Rotation has to be slower (small f) : Ω ~**1** rpm **(**only 1 rotation of the table per minute, which is very slow!)



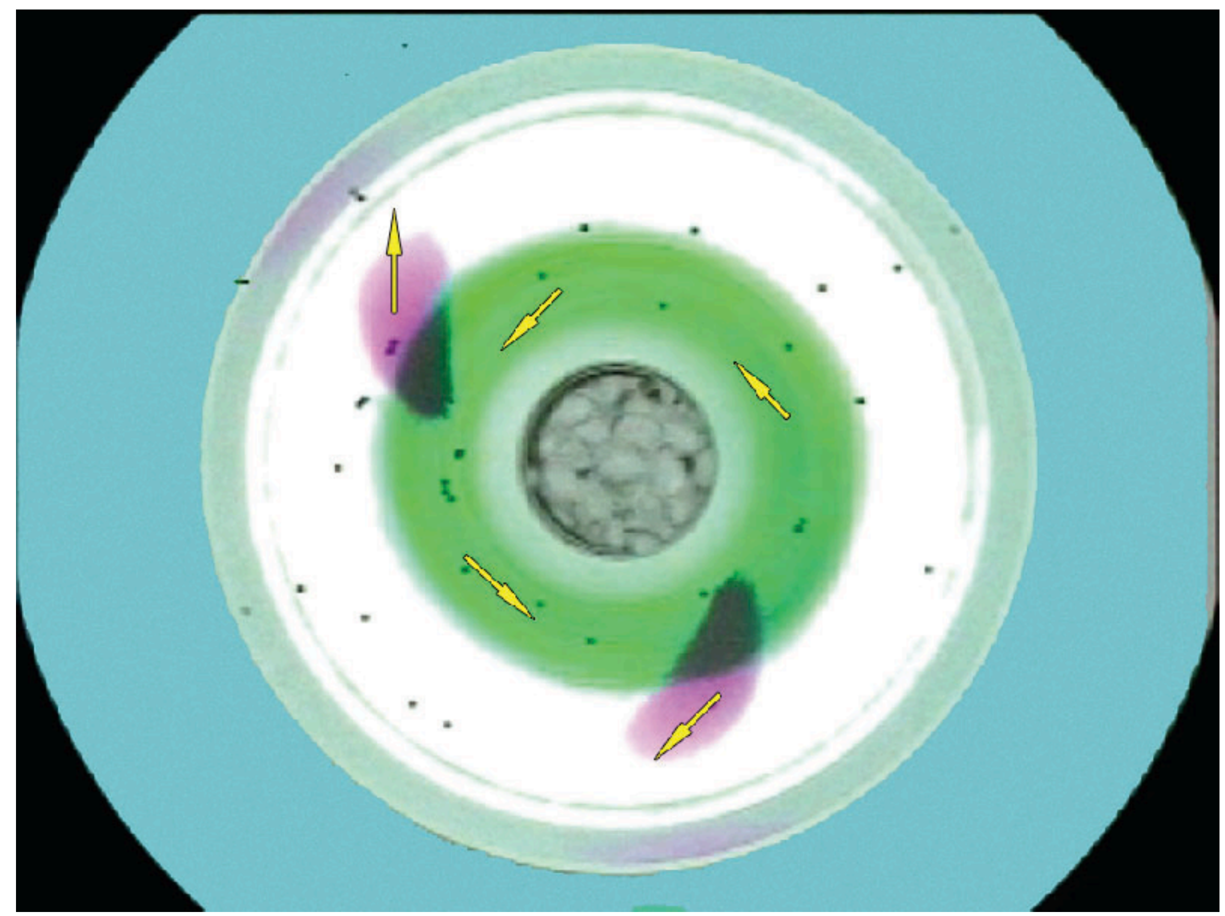


3. Mechanistic view of the circulation: tropics

- Visualize the Hadley cell



https://youtu.be/7BcDOuJRUoo



Angular momentum conservation: $A = \Omega r^2 + ur$

Ω ~ 0.1 s^{-1} (0.95 rpm) give u from 0 to ~ 8 cm/s when moving fluid from 30 cm to 10 cm

3. Mechanistic view of the circulation: extratropics

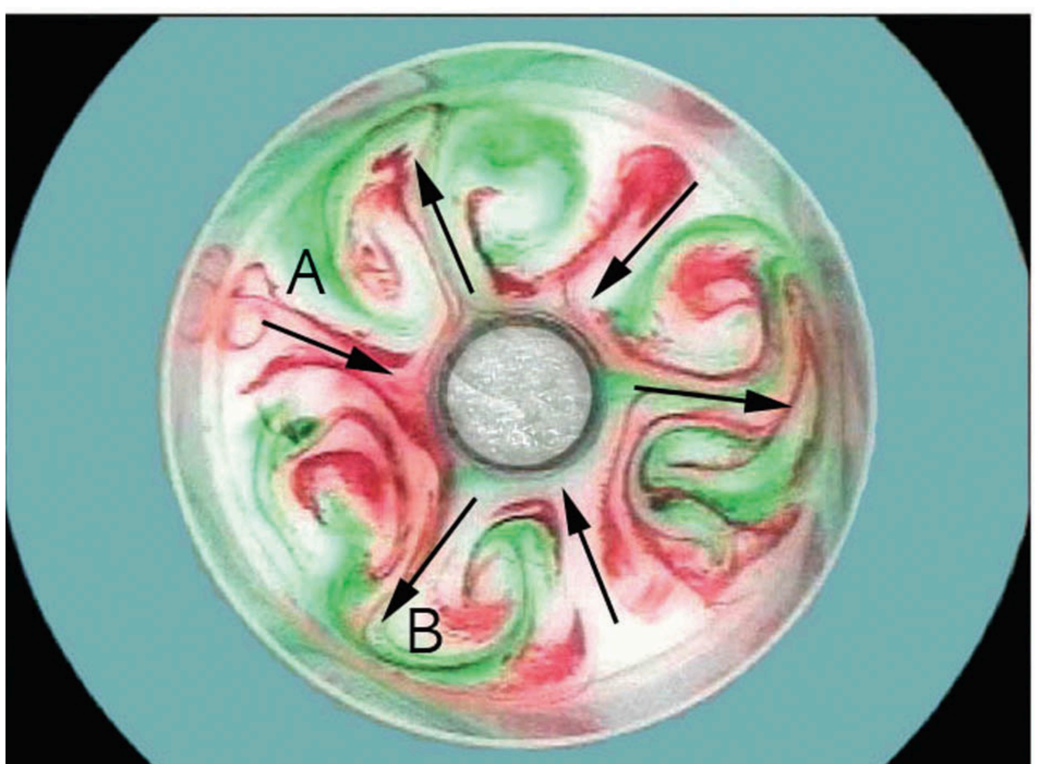
- In the extratropical regions where the effect of rotation is larger than tropics, we should get faster westerly winds that balances with the temperature gradient.
- It means that meridional overturning must be weak.
- Two questions remain
    1. How the heat can be transported further to the north if the meridional circulation is weak?
    2. We do not observe purely zonal wind as predicted. Why?

3. Mechanistic view of the circulation: extratropics

- Let's first evaluate our idea: faster rotation gives us faster zonal flow
- Increase angular velocity in the same experiment setting.
  - Ω from 1 rpm to 10 rpm
- https://youtu.be/bkBG_QokUCY
- Do we see faster zonal flows?
- How is the heat transported?

3. Mechanistic view of the circulation: extratropics

- Thermal wind breaks down.
- Eddies are developed.
- Eddies carry cold water outward, and simultaneously warm water inward.
- Eddies transport heat!



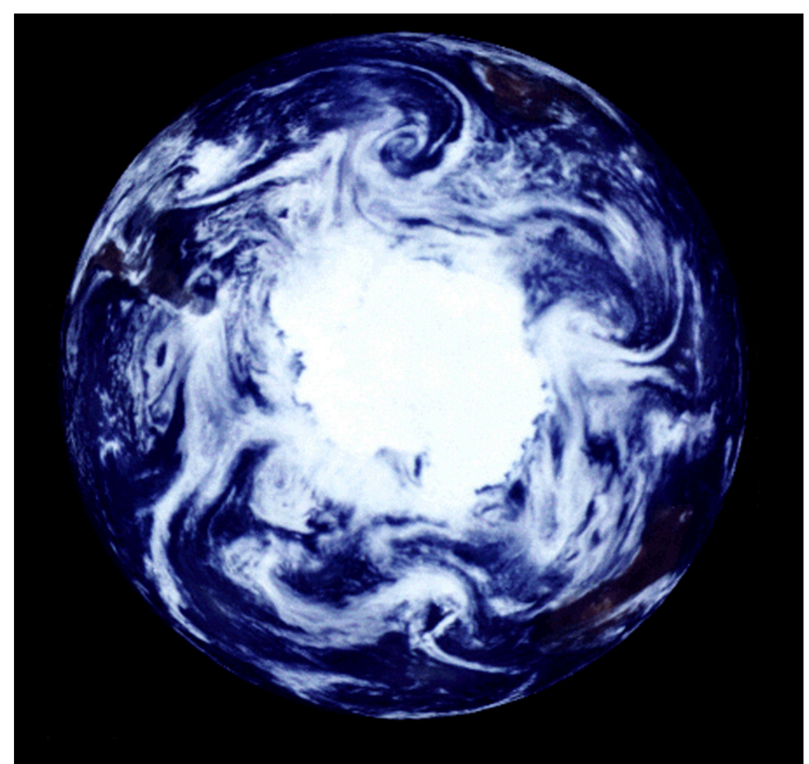
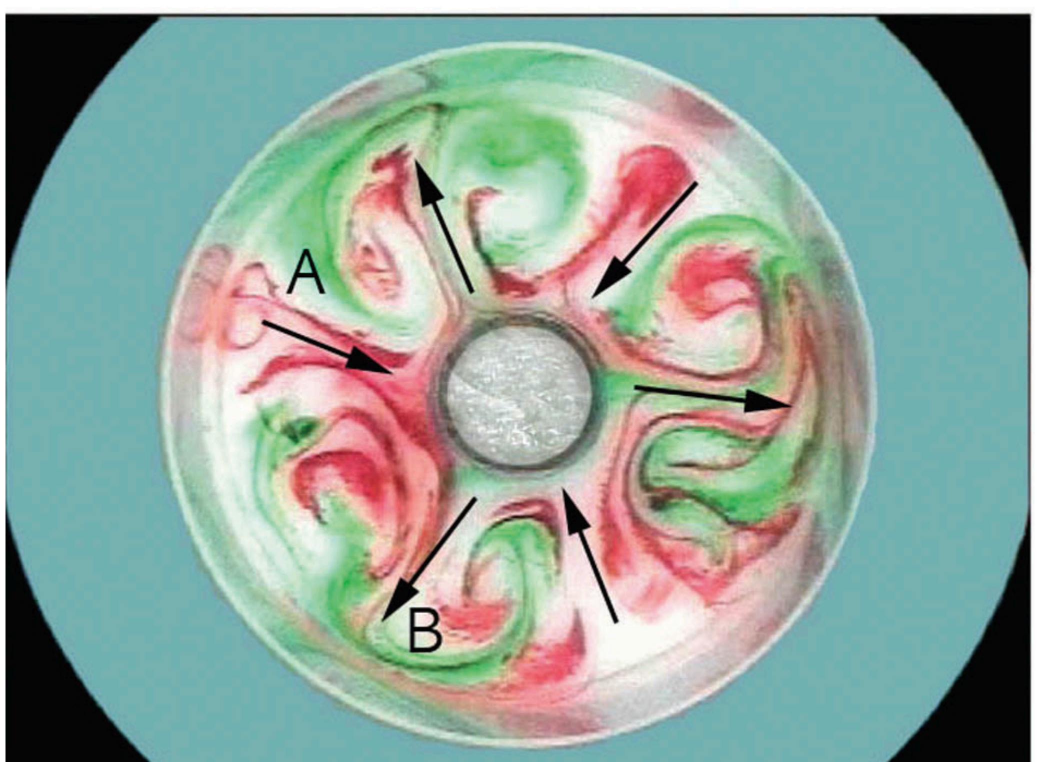


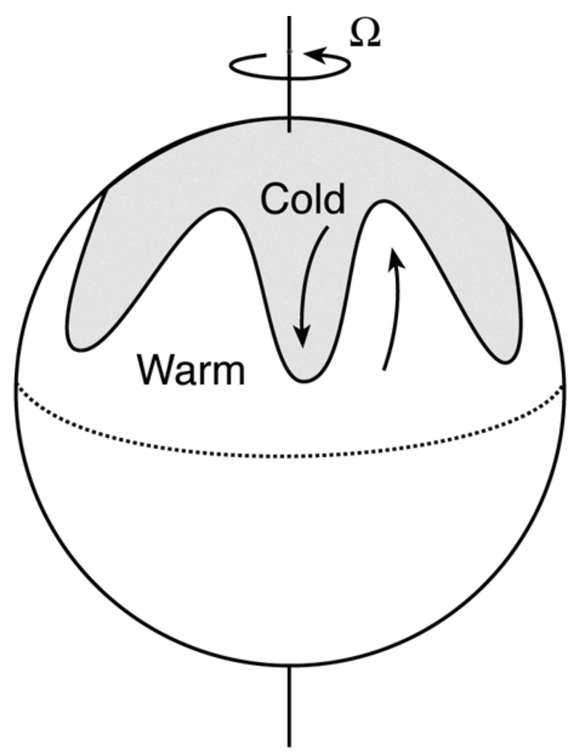
Image from https://www.jpl.nasa.gov/spaceimages/details.php?id=PIA00729

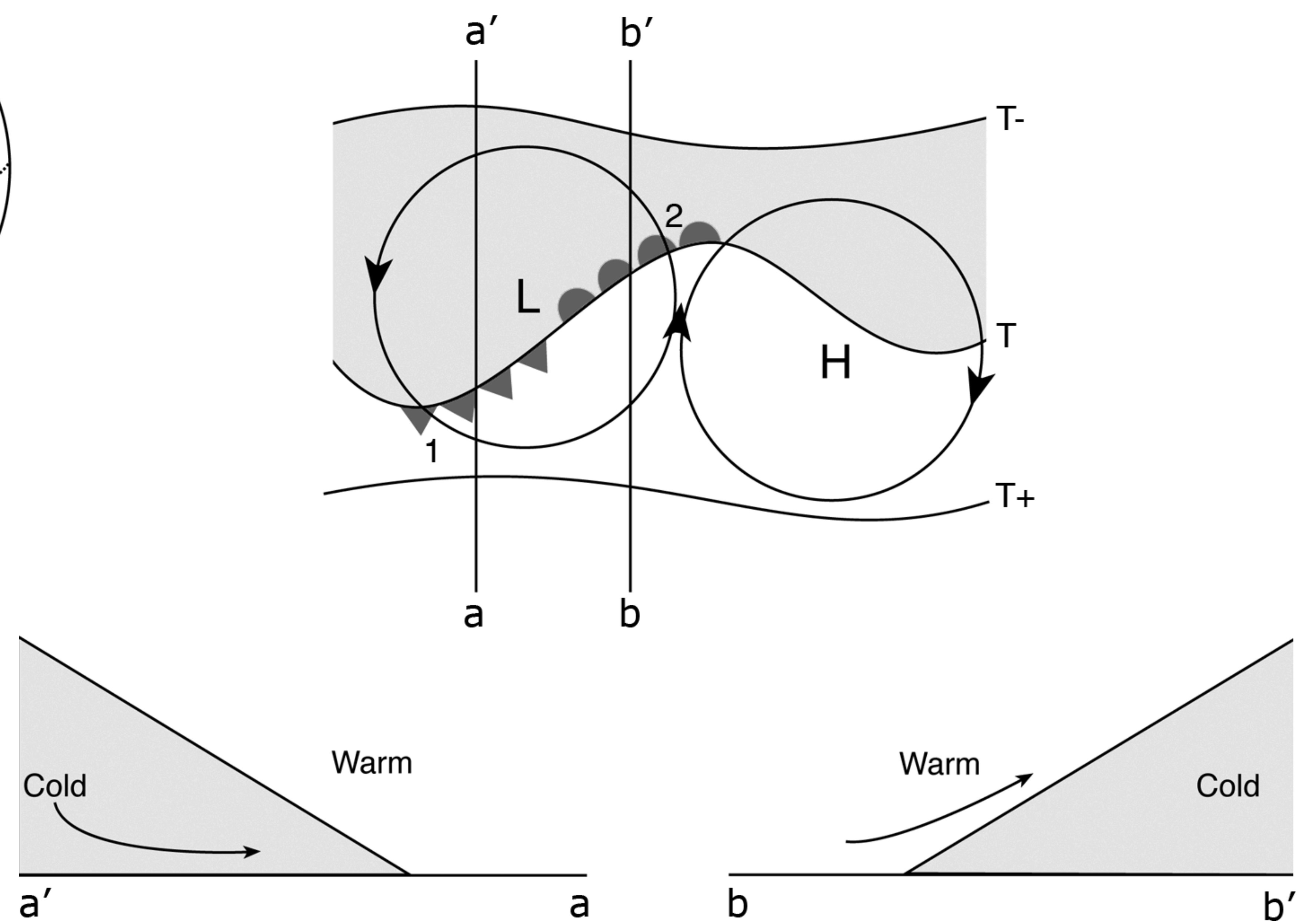
3. Mechanistic view of the circulation: extratropics



- How about the conservation of angular momentum?
- If it does, we should get the flow with close to 100 cm/s!
- In fact, angular momentum is not conserved because of the presence of zonal pressure gradient associated with eddying motion.

3. Mechanistic view of the circulation: extratropics





3. Energetics of the thermal wind equation

Q: Why do we have this energetic flows?

A: Available potential energy that can be released by a redistribution of mass of the system

3. Energetics of the thermal wind equation

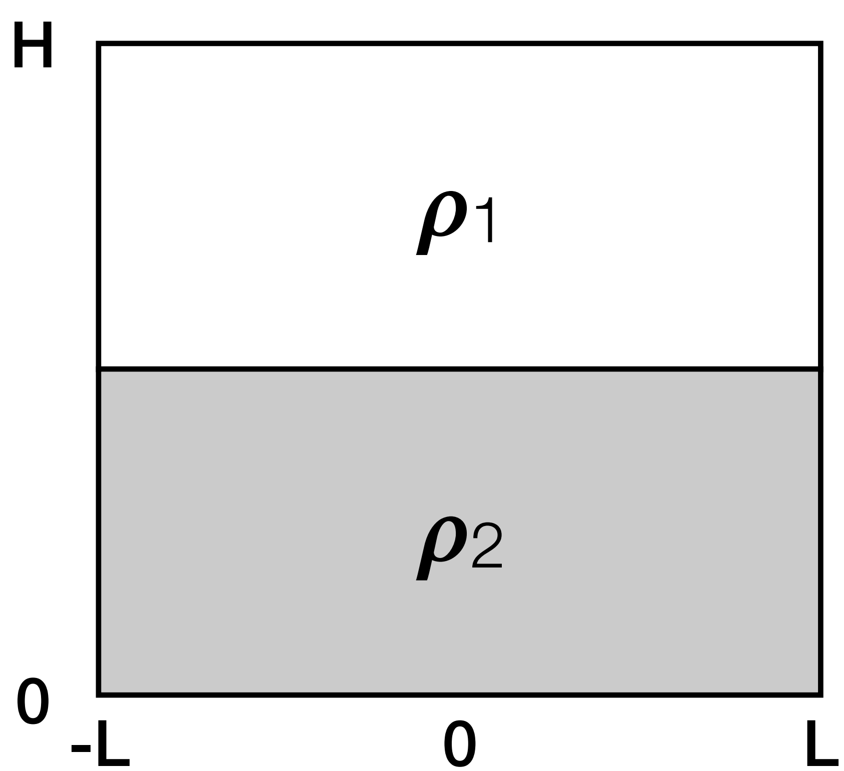
- Let's consider an incompressible fluid, like water, for simplicity.
- A potential energy (PE) of a fluid parcel of volume $dV = dx\, dy\, dz$ and density $\boldsymbol{\rho}$ would be $gz\boldsymbol{\rho}dV$:
- The total potential energy is then

$$PE = g\int z\rho dV = g\int \rho dV \frac{\int z\rho dV}{\int \rho dV} = gM\underbrace{\langle z \rangle}_{\text{The height of the center of mass}}$$

3. Energetics of the thermal wind equation

- Energy can be released and converted to kinetic energy only if some rearrangement of the fluid results in a lower total potential energy

Example:



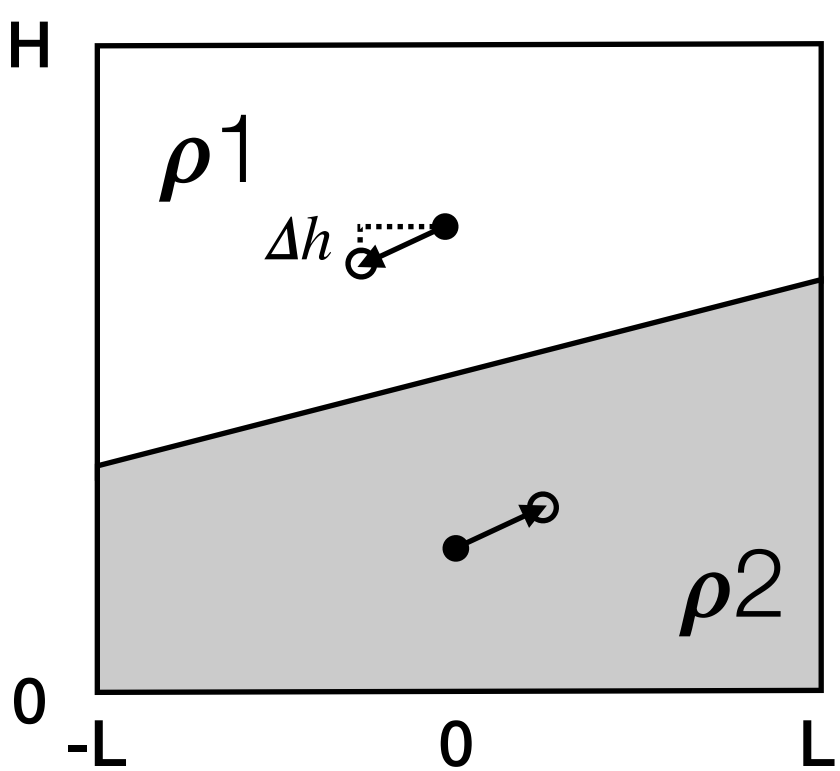
$$PE_1 = \rho_1 g \frac{3}{4} H$$

$$PE_2 = \rho_2 g \frac{1}{4} H$$

$$PE_A = gH\left(\frac{3\rho_1 + \rho_2}{4}\right)$$

3. Energetics of the thermal wind equation

$$PE_B = g\left[\rho_1\left(\frac{3}{4}H - \Delta h\right) + \rho_2\left(\frac{1}{4}H + \Delta h\right)\right]$$



$$PE_1 = \rho_1 g\left(\frac{3}{4}H - \Delta h\right)$$

$$PE_2 = \rho_2 g\left(\frac{1}{4}H + \Delta h\right)$$

3. Energetics of the thermal wind equation

- Which fluid has higher potential energy?

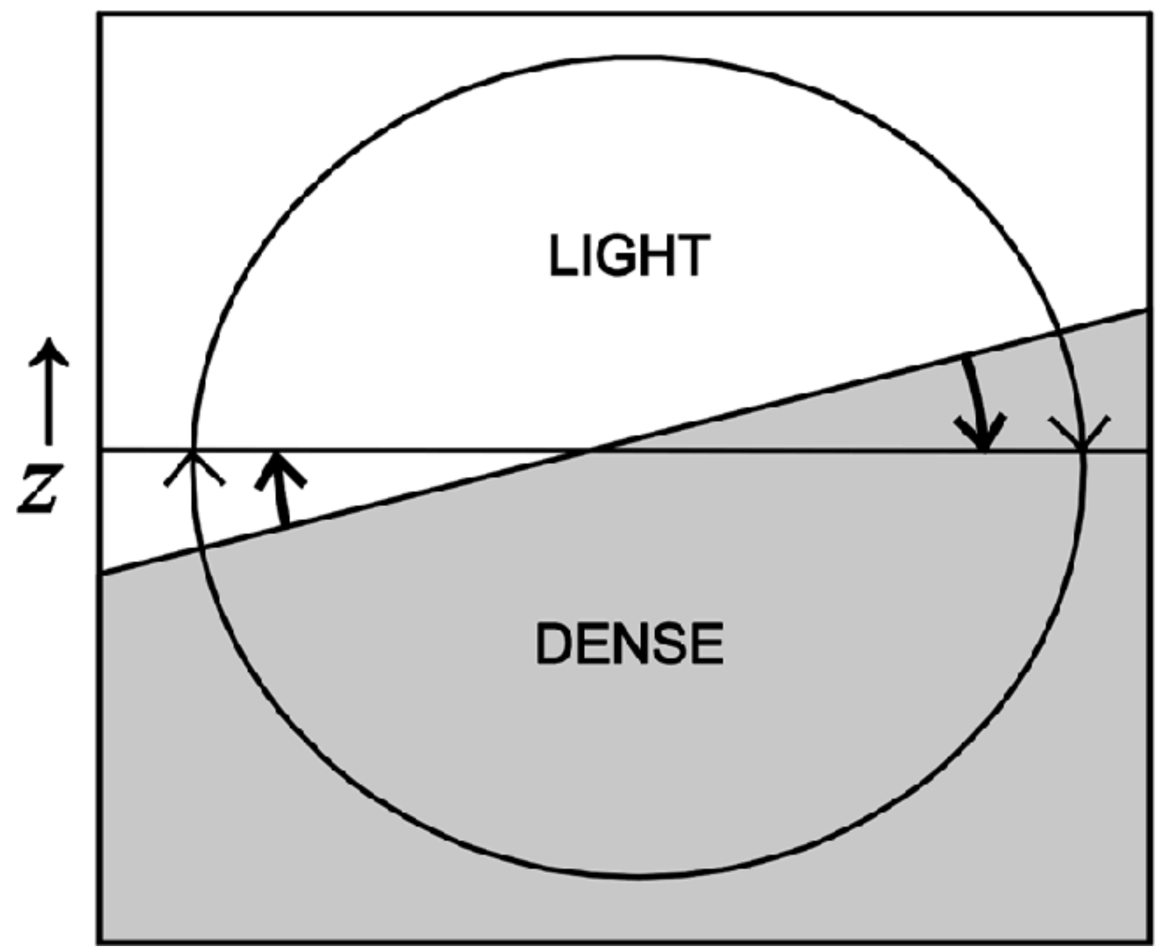
$$PE_A - PE_B = g\left(\rho_1 \frac{3}{4}H + \rho_2 \frac{1}{4}H - \rho_1 \frac{3}{4}H + \Delta h \rho_1 - \rho_2 \frac{1}{4}H - \Delta h \rho_2\right)$$

$$= g\Delta h\left(\rho_1 - \rho_2\right) < 0$$

- The case B has higher potential energy by $g\Delta h\left(\rho_1 - \rho_2\right)$
- This is available potential energy (APE).
- We can expect higher available potential energy when the interface has greater tilt.

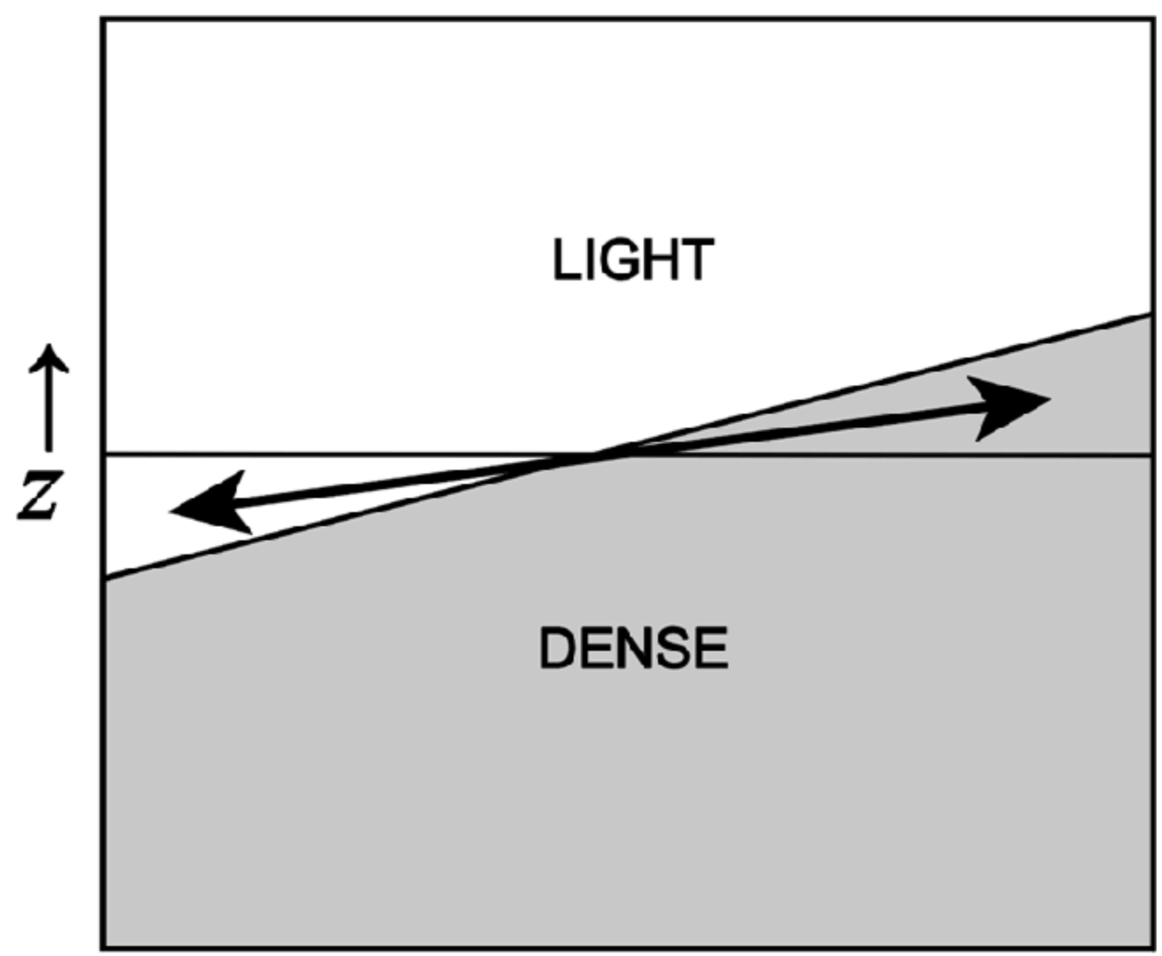
3. Energetics of the thermal wind equation

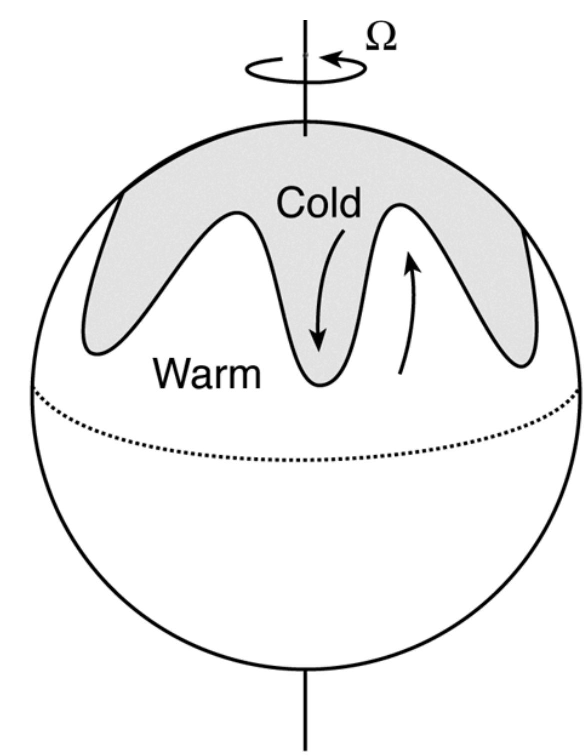
- Release of available potential energy
  - In a non-rotating fluid



3. Energetics of the thermal wind equation

- Release of available potential energy
  - In a rotating fluid, the tilted slope can be maintained in thermal wind balance

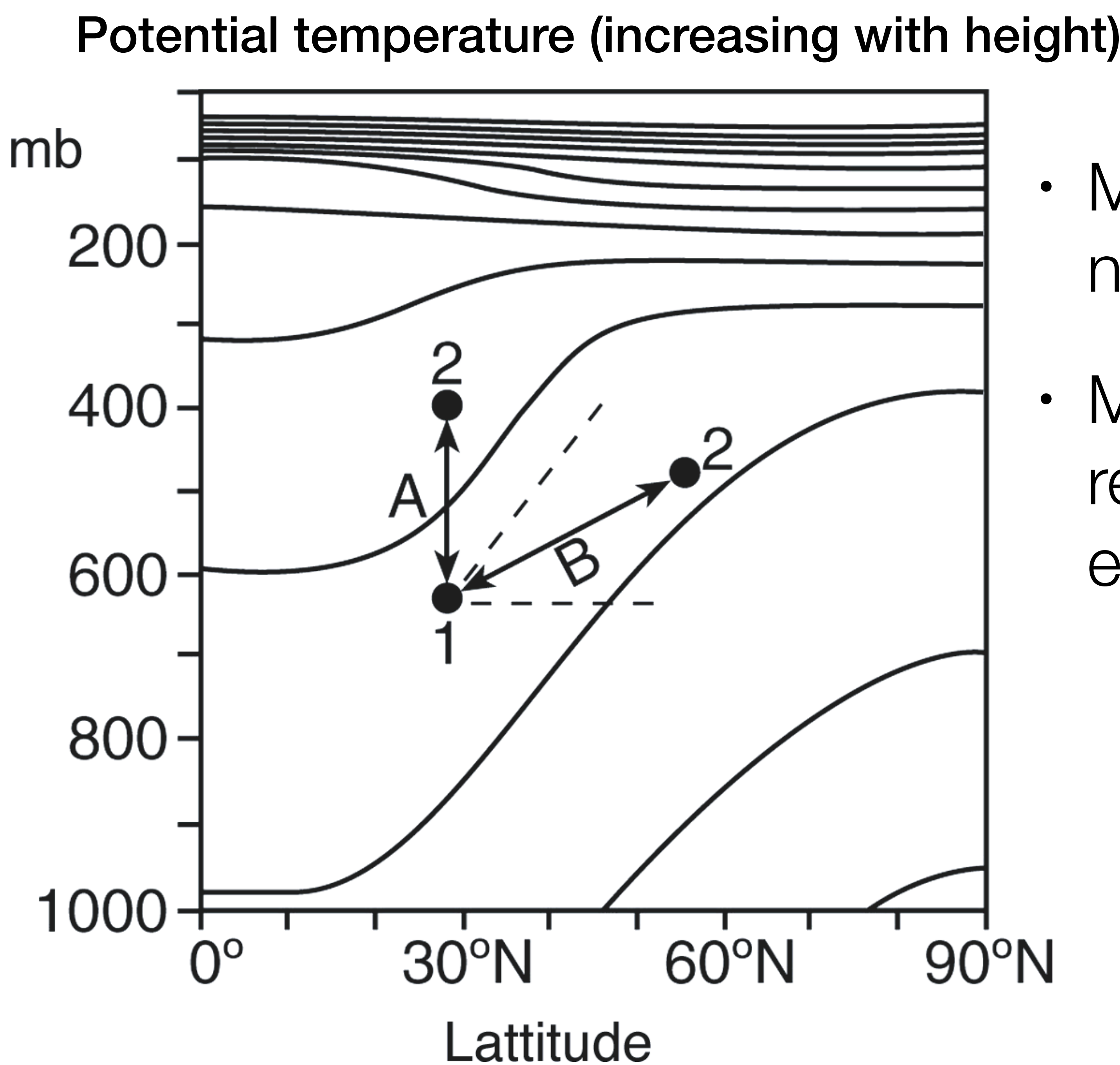




4. Energetics in a compressible atmosphere

- The air is compressible → we need to consider internal energy.
    - Internal energy goes up when compressed
    - Internal energy goes down when expanded
- The air can contain moisture → we need to consider latent heat when condensation occurs.
- Total energy of the atmosphere = potential energy + kinetic energy + internal energy + latent heat content

4. Energetics in a compressible atmosphere



- Moving from 1 to 2 along A : needs energy
- Moving from 1 to 2 along B : release energy → can excite eddies