Ocean: Wind-driven circulation #2

ATM2106

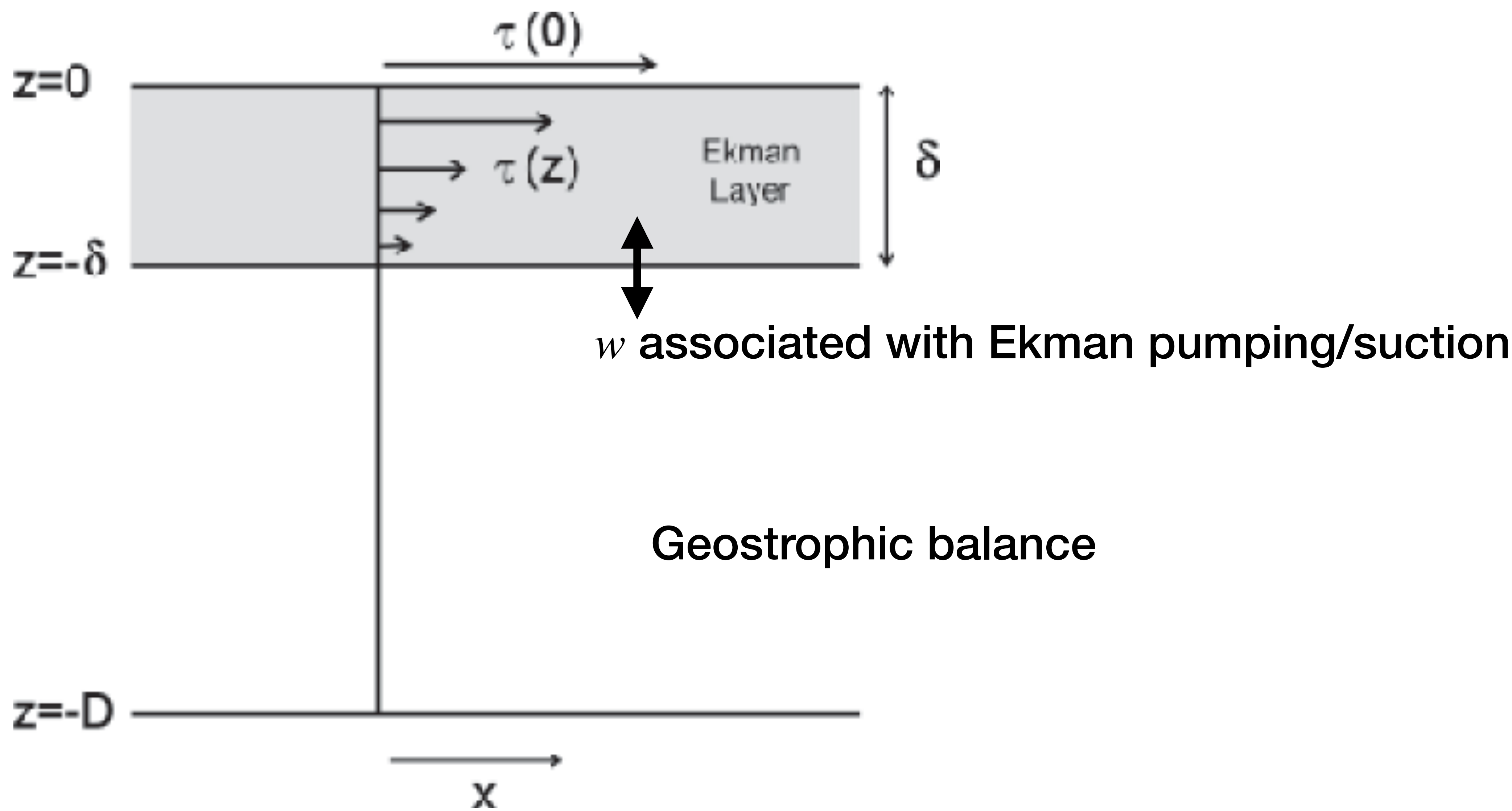
Last time…

- **Ekman layer** : The layer near the surface that feels the wind stress
- **Ekman transport** : Wind drives ocean currents
  - Ocean transport to the right of the wind in NH.
  - Ocean transport to the left of the wind in SH.
- **Ekman pumping & suction**:
  - Convergence : Ekman pumping (downward)
  - Divergence : Ekman suction (upward)

What does Ekman pumping do to the interior of the ocean?

Remember that…

- Ekman pumping in the Ekman layer.
- Beneath the Ekman layer, the flow is in the geostrophic balance.



Using momentum equations…

$$-fv = -\frac{1}{\rho_{ref}}\frac{\partial p}{\partial x}$$

$$fu = -\frac{1}{\rho_{ref}}\frac{\partial p}{\partial y}$$

$$\longrightarrow$$

$$\frac{\partial}{\partial y}(-fv) = -\frac{1}{\rho_{ref}}\frac{\partial}{\partial y}\left(\frac{\partial p}{\partial x}\right)$$

$$\frac{\partial}{\partial x}(fu) = -\frac{1}{\rho_{ref}}\frac{\partial}{\partial x}\left(\frac{\partial p}{\partial y}\right)$$

$$\longrightarrow \quad \frac{\partial}{\partial y}(-fv) = \frac{\partial}{\partial x}(fu)$$

Using momentum equations…

$$\frac{\partial}{\partial y}(-fv) = \frac{\partial}{\partial x}(fu)$$

If we assume that

$$f = f_0 + \beta y$$

$$-f\frac{\partial v}{\partial y} - \beta v = f\frac{\partial u}{\partial x}$$

$$-f\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = \beta v$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$f\frac{\partial w}{\partial z} = \beta v$$

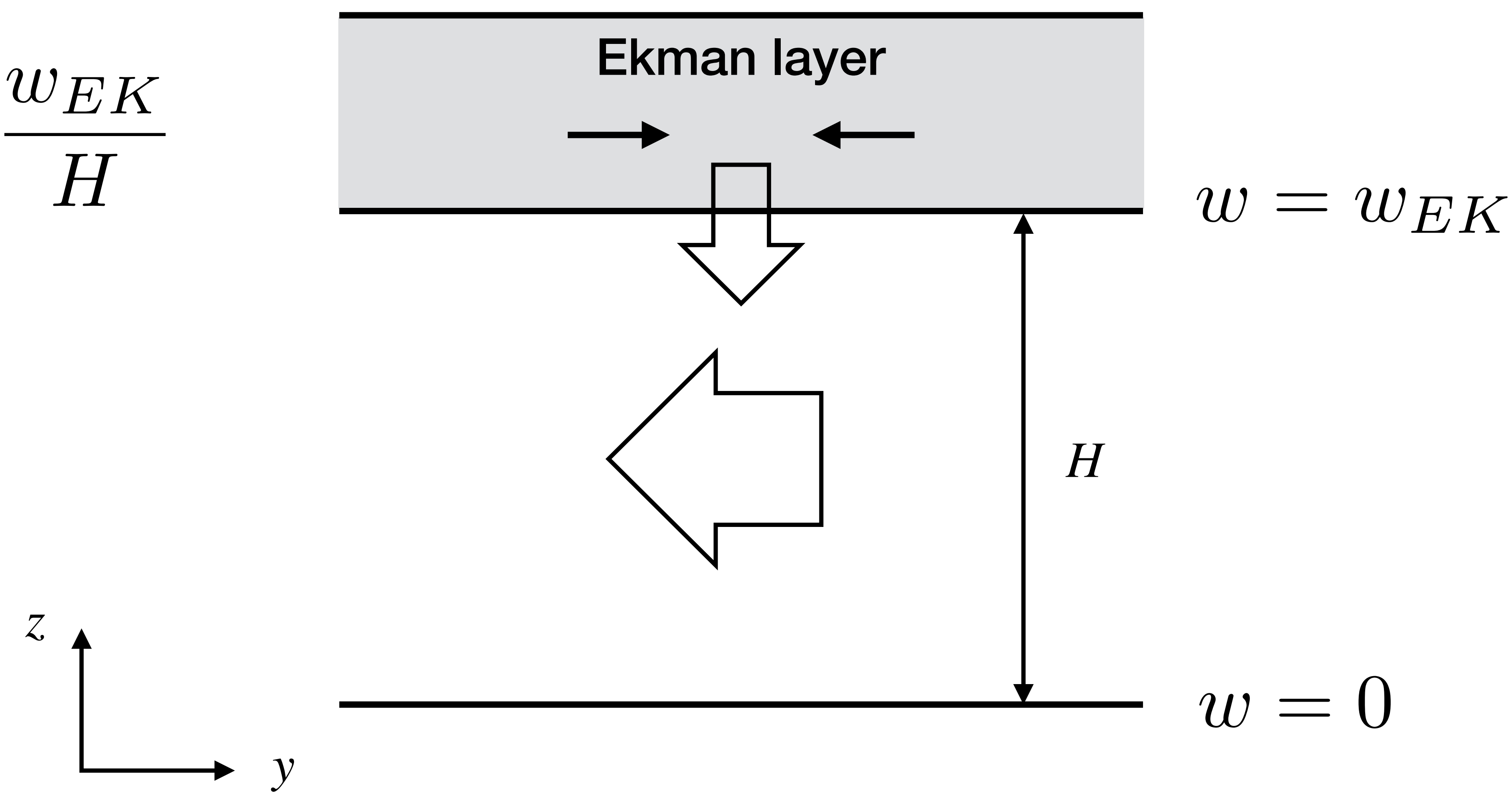
1. Ekman pumping $(w_{EK} < 0)$

$$f\frac{\partial w}{\partial z} = \beta v$$

$$\downarrow$$

$$v = \frac{f}{\beta}\frac{w_{EK}}{H}$$

Ekman pumping leads to v < 0 in the NH (f > 0).



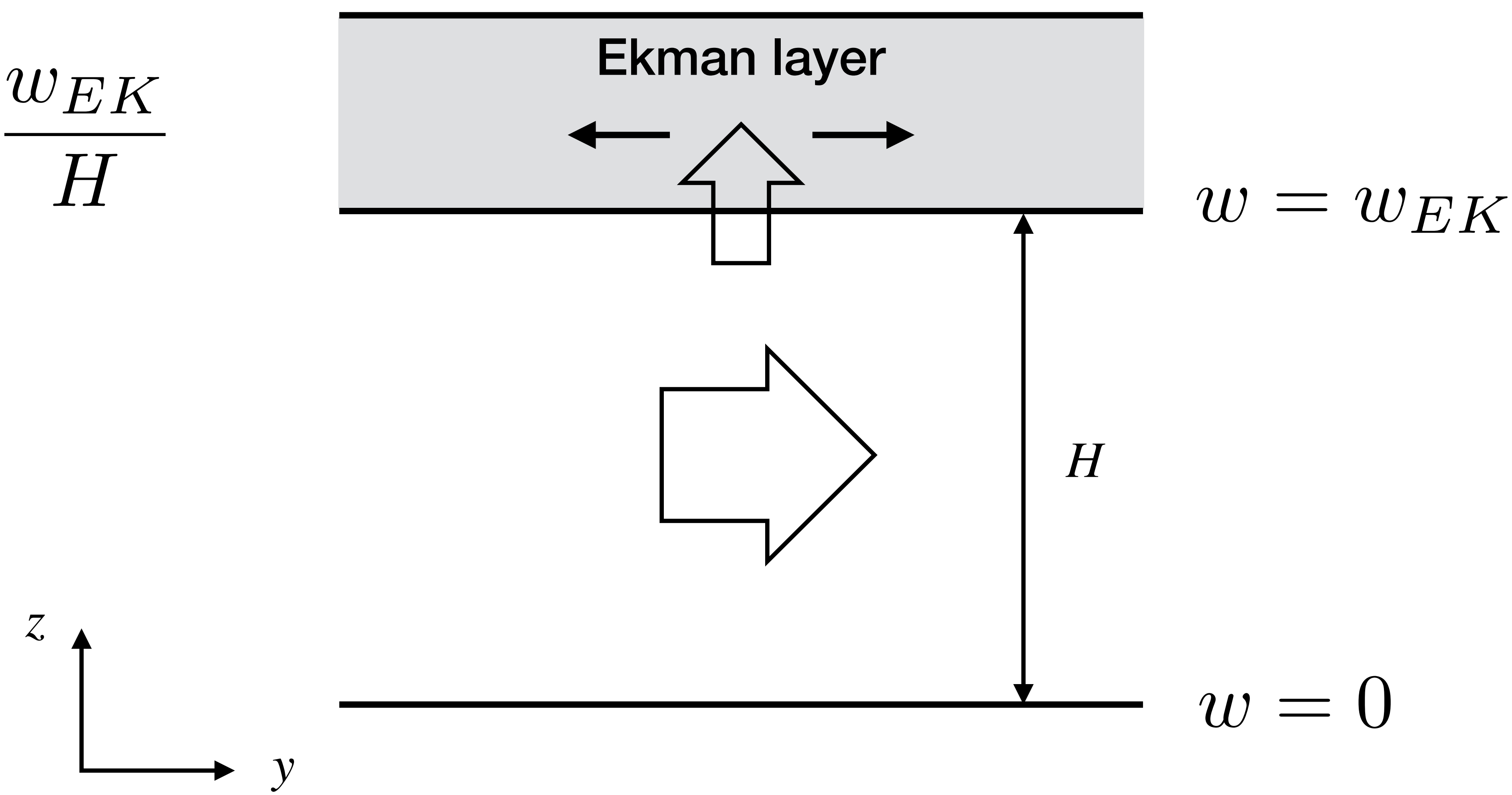
1. Ekman suction $\quad (w_{EK} > 0)$

$$f\frac{\partial w}{\partial z} = \beta v$$

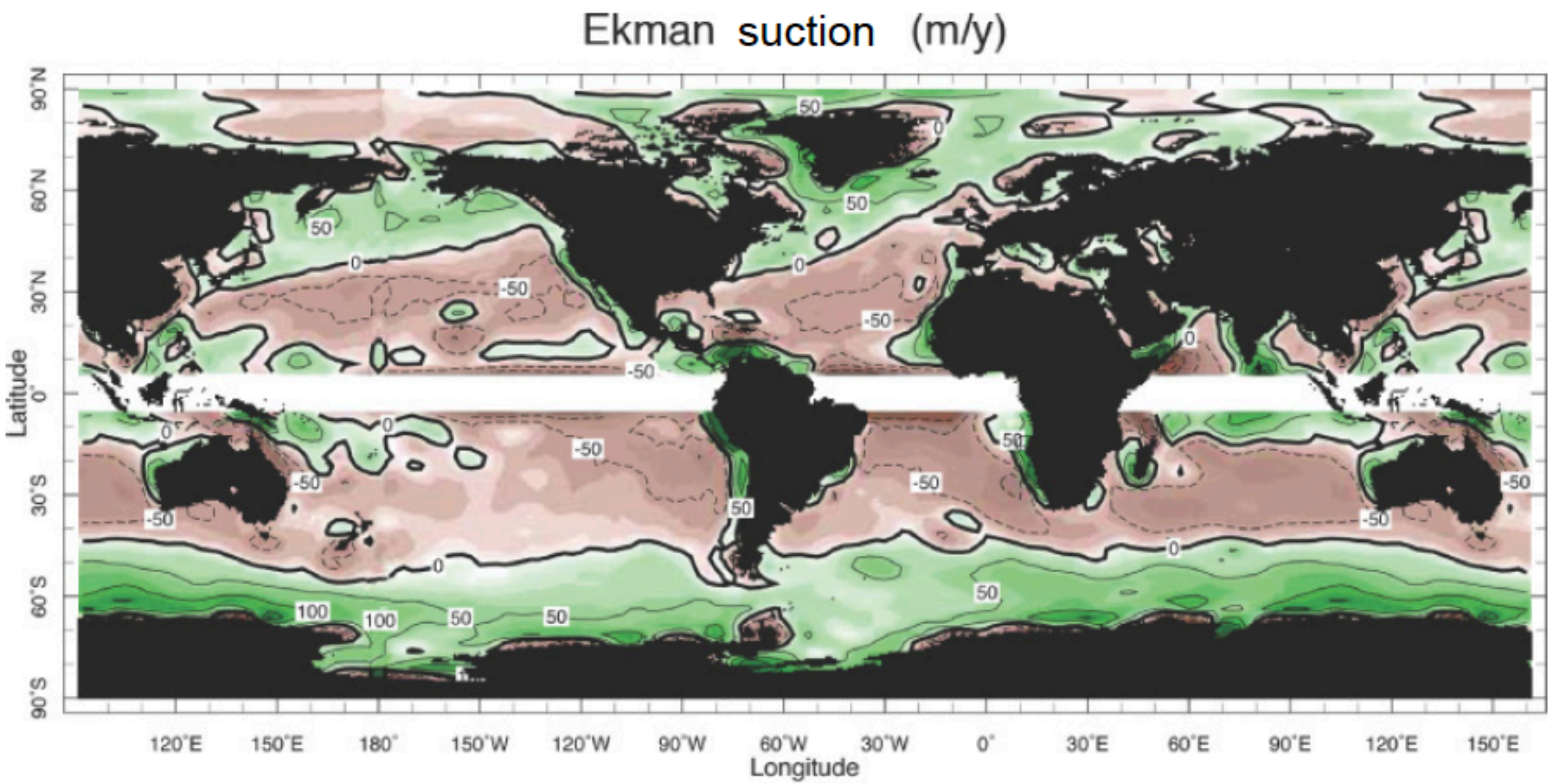
$$\downarrow$$

$$v = \frac{f}{\beta}\frac{w_{EK}}{H}$$

Ekman pumping leads to v > 0 in the NH (f > 0).



Ekman pumping / suction



How fast is v driven by Ekman pumping/suction?

$1 \times 10^{-4}\ s^{-1}$ $\quad$ $30\ m\ yr^{-1}$

$$v = \frac{f}{\beta}\frac{w_{EK}}{H}$$

$2 \times 10^{-11}\ m^{-1}\ s^{-1}$ $\quad$ $1000\ m$

(Thermocline depth)

$$\boxed{v = O(1cm/s)}$$

The depth-integrated circulation

- To simplify the ocean circulation, let's consider the vertical integration of the momentum equation.

$$-fv + \frac{1}{\rho_{ref}}\frac{\partial p}{\partial x} = \frac{1}{\rho_{ref}}\frac{\partial \tau_x}{\partial z} \longrightarrow -f\frac{\partial v}{\partial y} - \beta v + \frac{1}{\rho_{ref}}\frac{\partial^2 p}{\partial x \partial y} = \frac{1}{\rho_{ref}}\frac{\partial}{\partial z}\left(\frac{\partial \tau_x}{\partial y}\right)$$

$$fu + \frac{1}{\rho_{ref}}\frac{\partial p}{\partial y} = \frac{1}{\rho_{ref}}\frac{\partial \tau_y}{\partial z} \longrightarrow f\frac{\partial u}{\partial x} + \frac{1}{\rho_{ref}}\frac{\partial^2 p}{\partial x \partial y} = \frac{1}{\rho_{ref}}\frac{\partial}{\partial z}\left(\frac{\partial \tau_y}{\partial x}\right)$$

$$\Downarrow$$

$$\beta v = f\frac{\partial w}{\partial z} + \frac{1}{\rho_{ref}}\frac{\partial}{\partial z}\left(\frac{\partial \tau_y}{\partial x} - \frac{\partial \tau_x}{\partial y}\right)$$

The depth-integrated circulation

$$\beta v = f\frac{\partial w}{\partial z} + \frac{1}{\rho_{ref}}\frac{\partial}{\partial z}\left(\frac{\partial \tau_y}{\partial x} - \frac{\partial \tau_x}{\partial y}\right)$$

Vertical integration

$$\int_{-D}^{0} \beta v dz = \int_{-D}^{0} f\frac{\partial w}{\partial z}dz + \int_{-D}^{0} \frac{1}{\rho_{ref}}\frac{\partial}{\partial z}\left(\frac{\partial \tau_y}{\partial x} - \frac{\partial \tau_x}{\partial y}\right)dz$$

$$\beta V = \frac{1}{\rho_{ref}}\left(\frac{\partial \tau_{wind,y}}{\partial x} - \frac{\partial \tau_{wind,x}}{\partial y}\right)$$

w(z=0) = w(z=-D) = 0

$\tau(z = -D) = 0$

Sverdrup relation

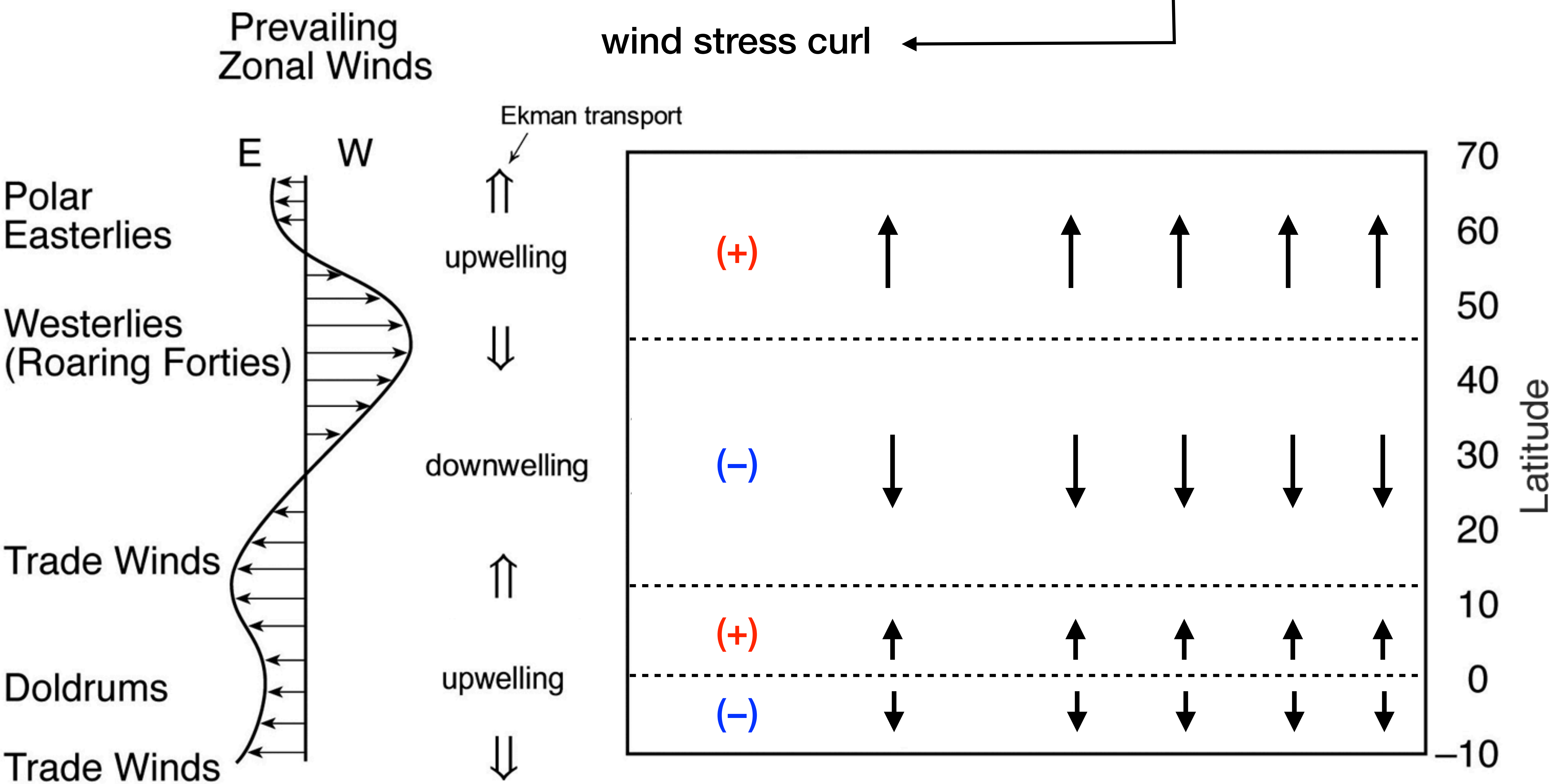
$$\beta V = \frac{1}{\rho_{ref}} \left(\frac{\partial \tau_{wind,y}}{\partial x} - \frac{\partial \tau_{wind,x}}{\partial y} \right)$$

The depth-integrated meridional transport is determined by wind stress when we assume

1. Coriolis force, pressure gradient force and stress are balanced.
2. Homogeneous fluid (so density is constant)
3. No vertical flow at the surface and the bottom
4. Stress is zero at the bottom

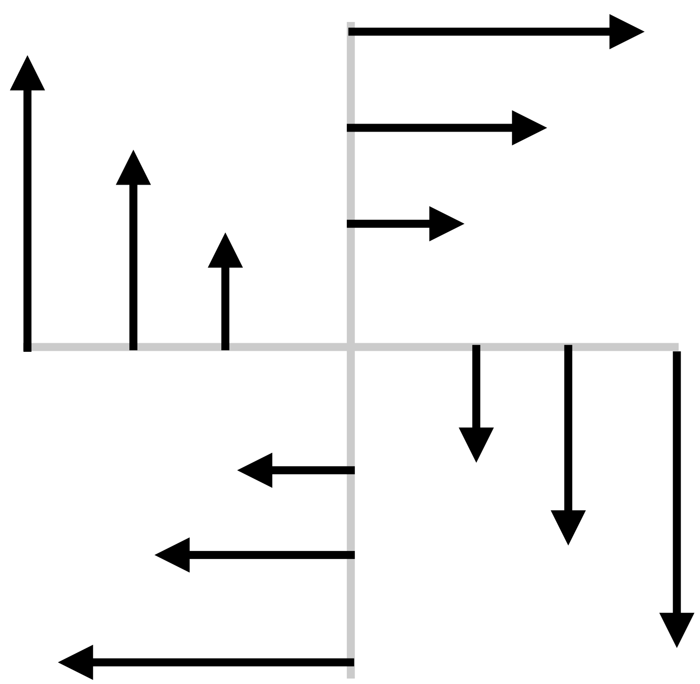
Sverdrup relation

$$\beta V = \frac{1}{\rho_{ref}} \underbrace{\left(\frac{\partial \tau_{wind,y}}{\partial x} - \frac{\partial \tau_{wind,x}}{\partial y} \right)}_{\text{wind stress curl}}$$



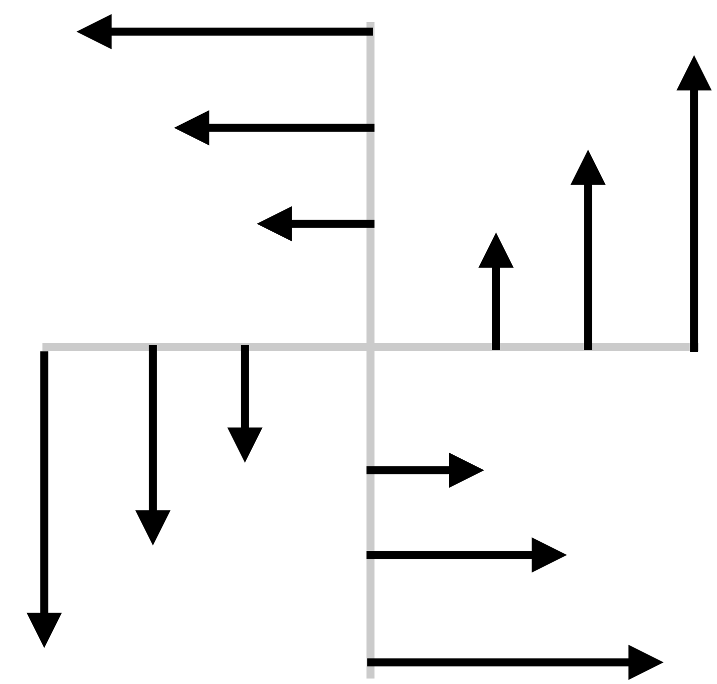
Wind stress curl, geostrophic current, Sverdrup relation

$$\frac{\partial \tau_y}{\partial x} - \frac{\partial \tau_x}{\partial y} < 0$$



- **Northern H. : equatorward**
- **Southern H. : poleward**

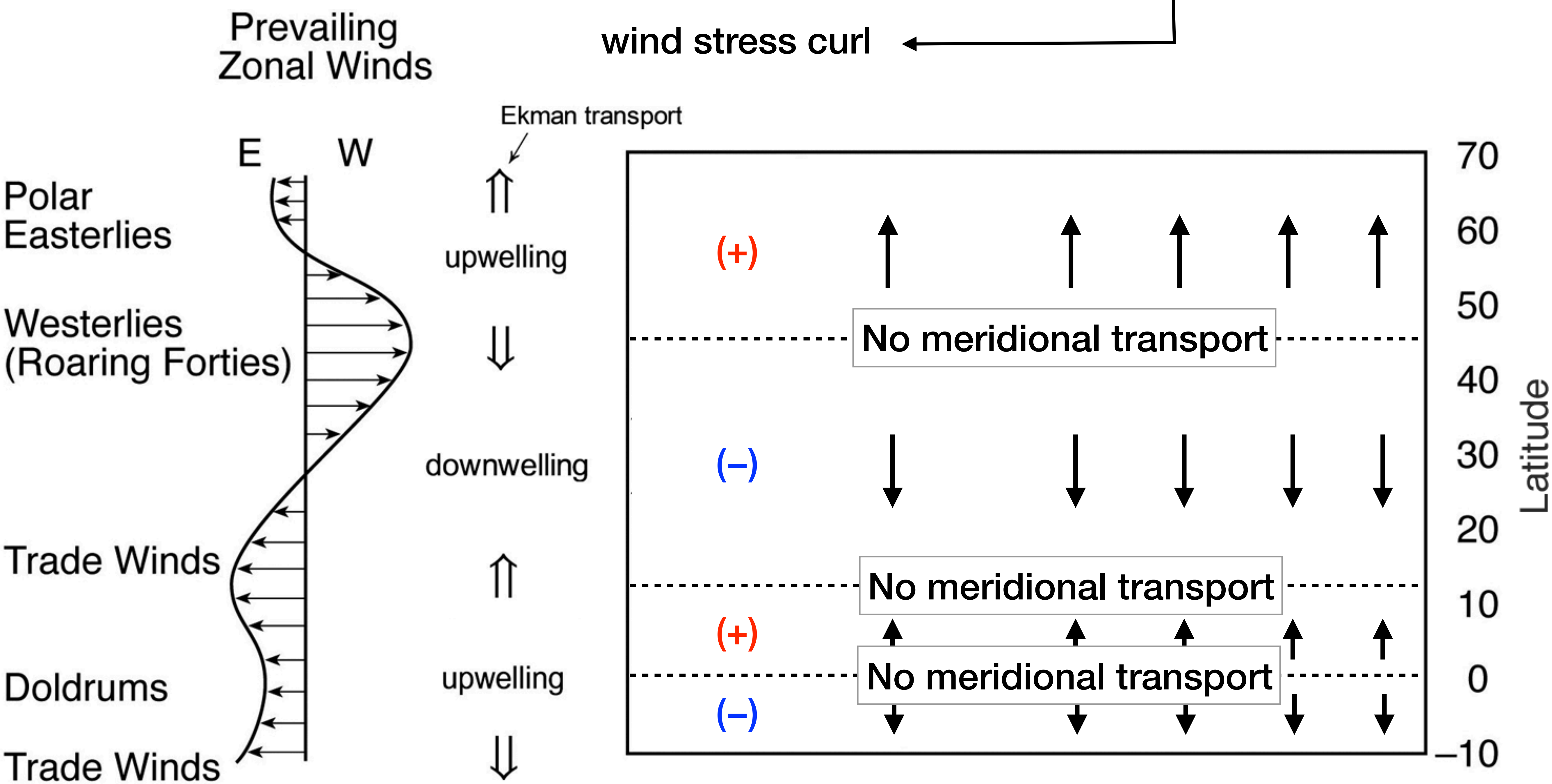
$$\frac{\partial \tau_y}{\partial x} - \frac{\partial \tau_x}{\partial y} > 0$$



- **Northern H. : poleward**
- **Southern H. : equatorward**

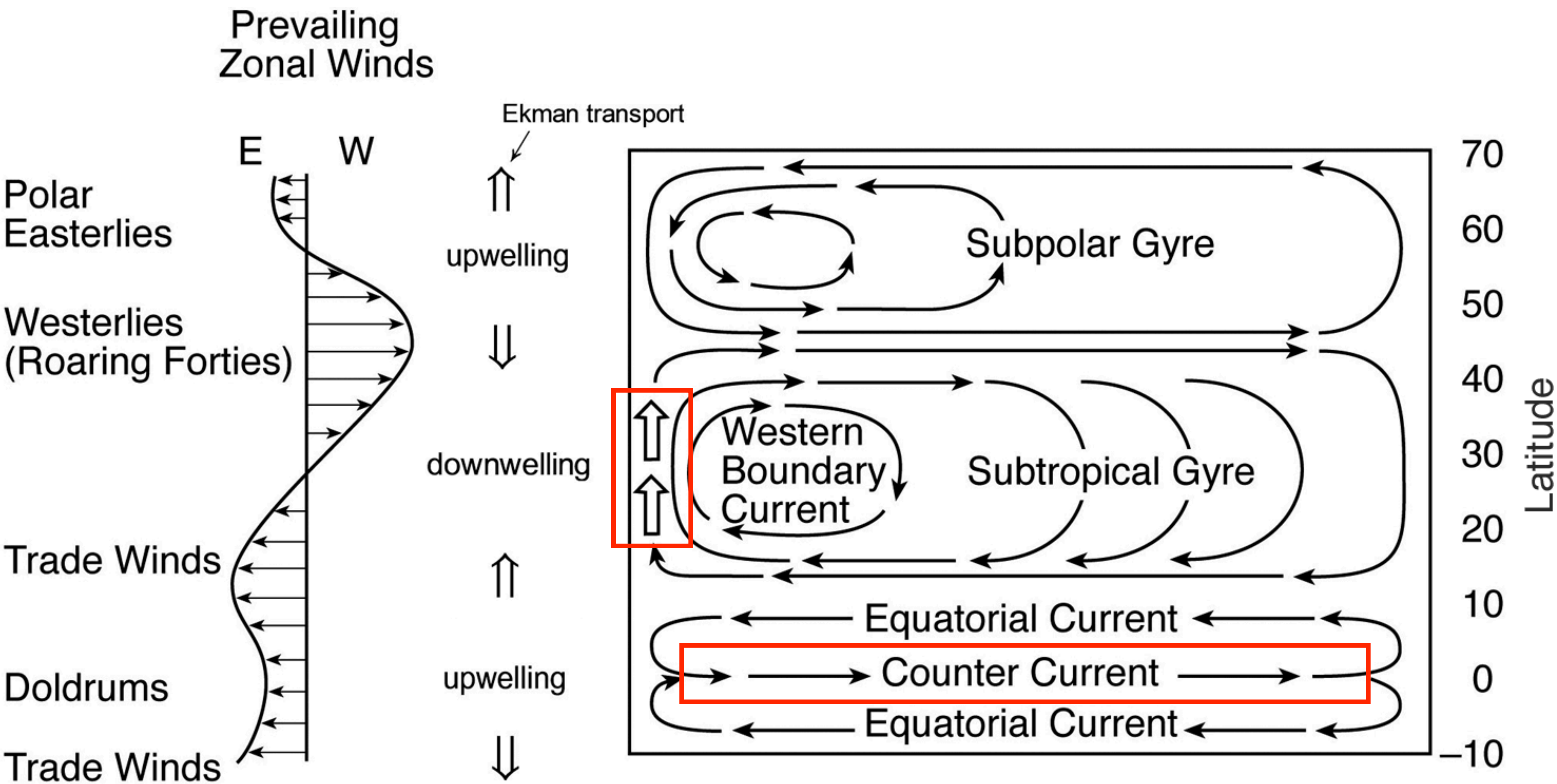
Sverdrup relation

$$\beta V = \frac{1}{\rho_{ref}} \underbrace{\left(\frac{\partial \tau_{wind,y}}{\partial x} - \frac{\partial \tau_{wind,x}}{\partial y} \right)}_{\text{wind stress curl}}$$

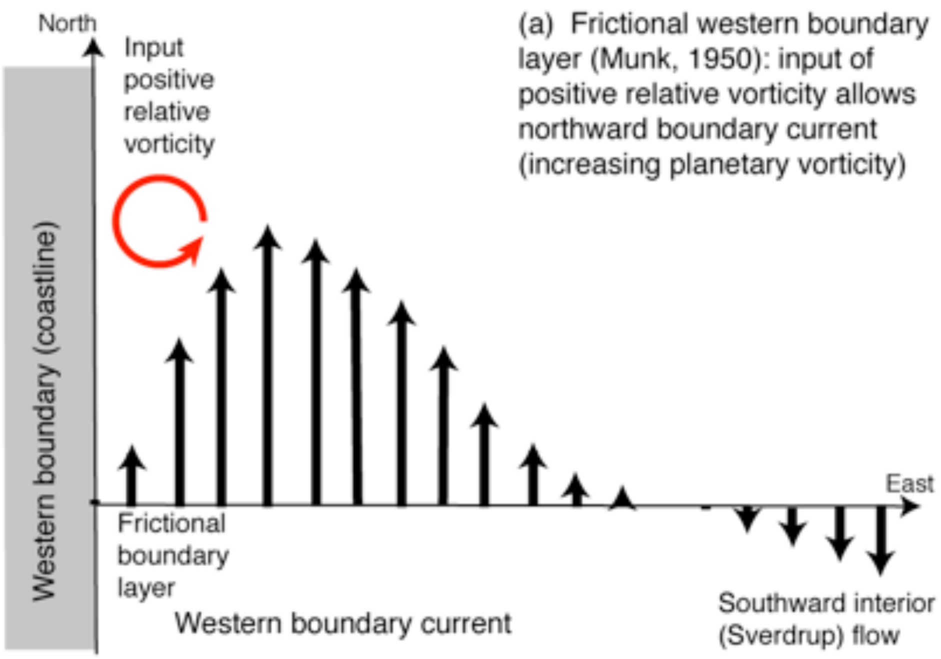


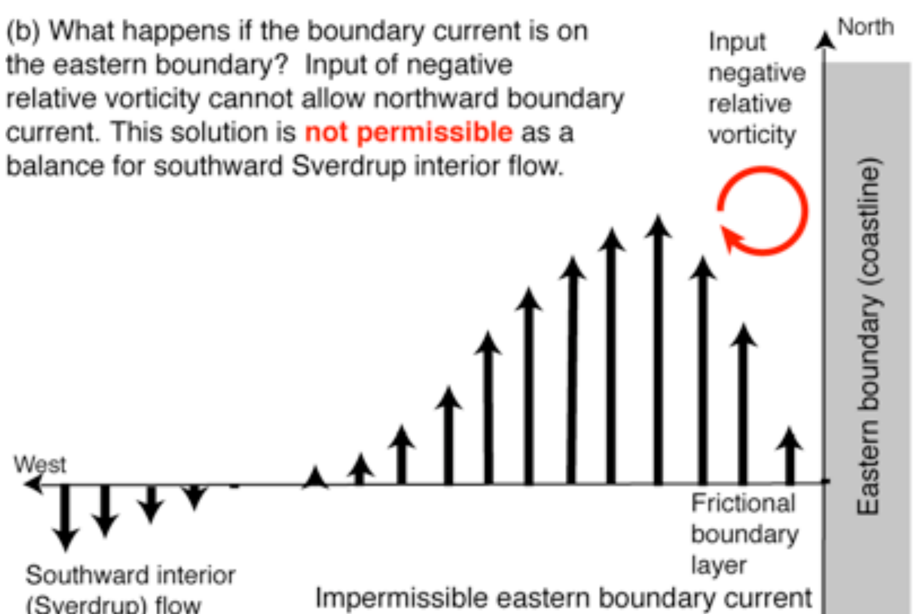
Sverdrup relation

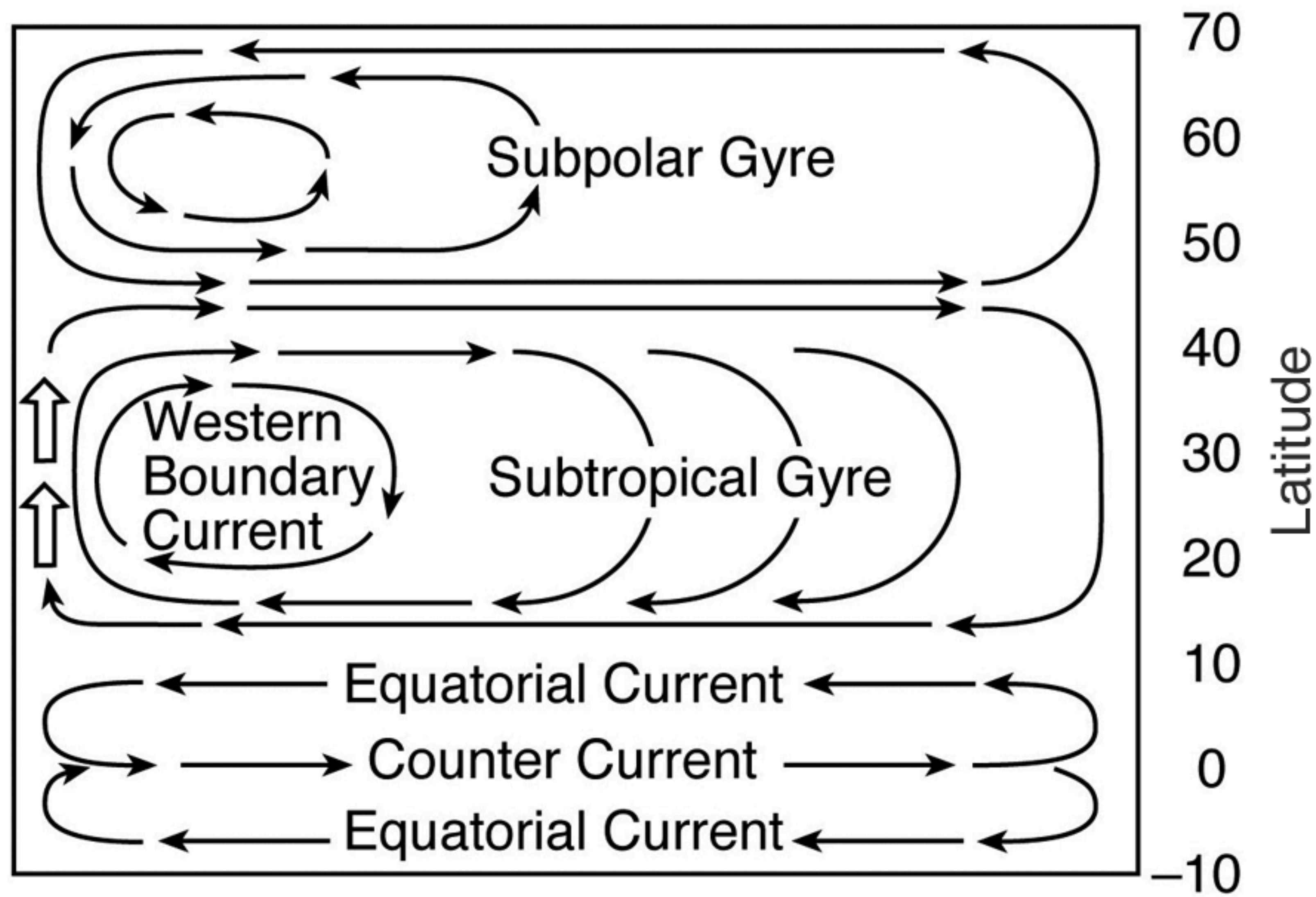
$$\beta V = \frac{1}{\rho_{ref}} \left(\frac{\partial \tau_{wind,y}}{\partial x} - \frac{\partial \tau_{wind,x}}{\partial y} \right)$$

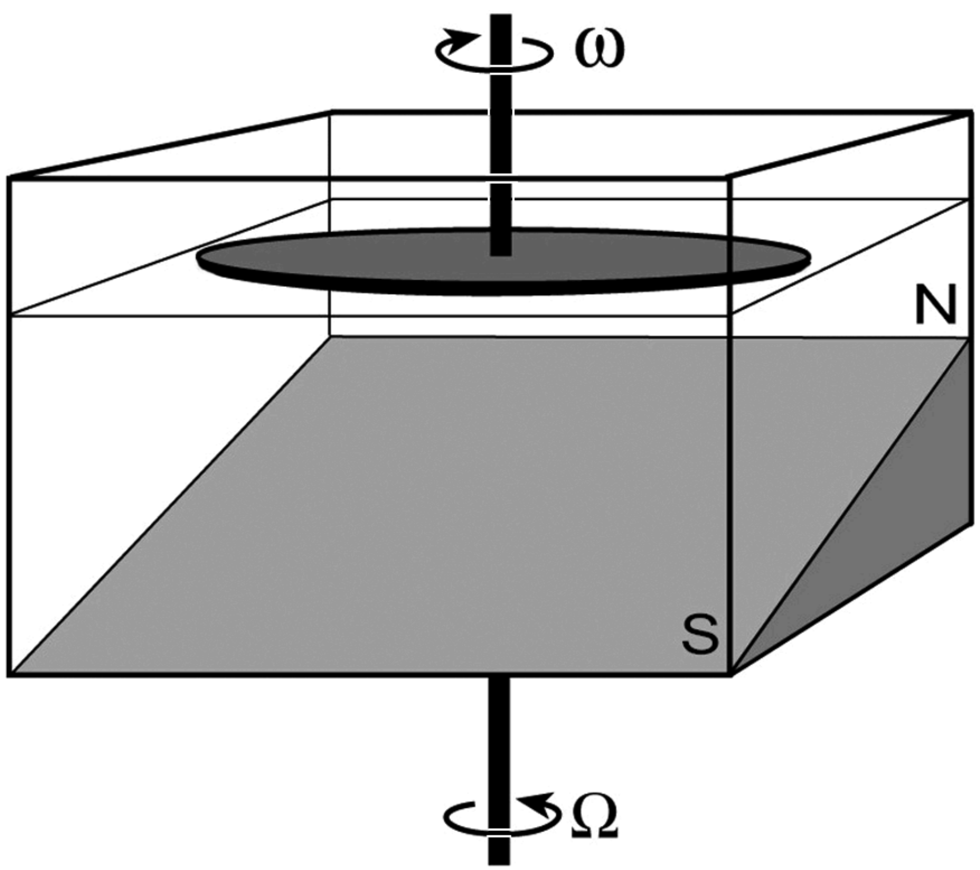


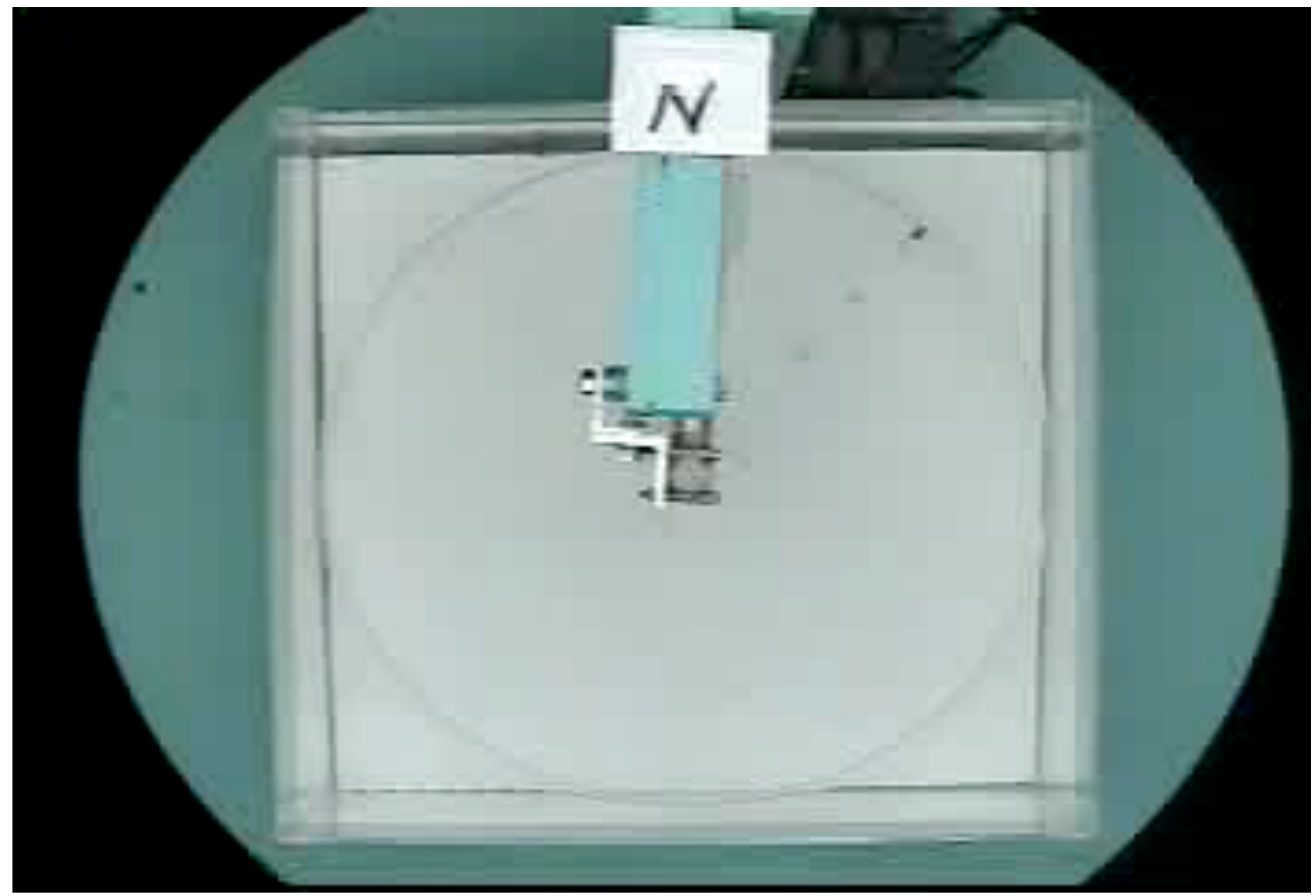
Why does it have to be on the west side?

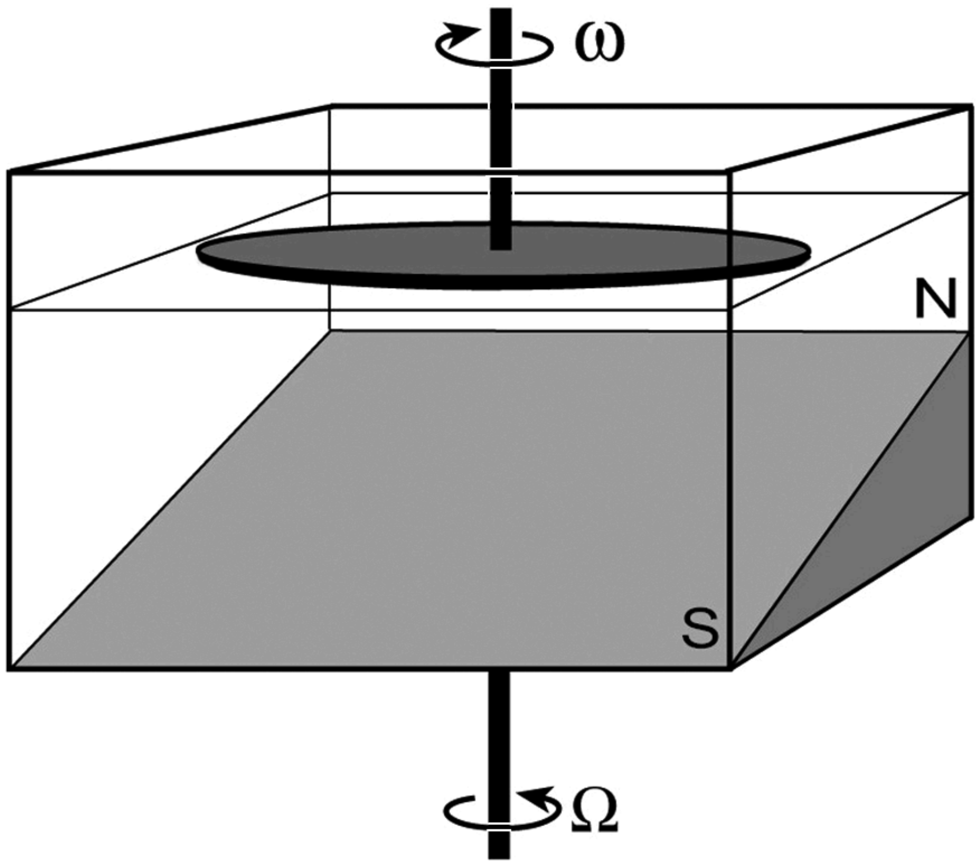







ω
N
S
Ω


N


ω
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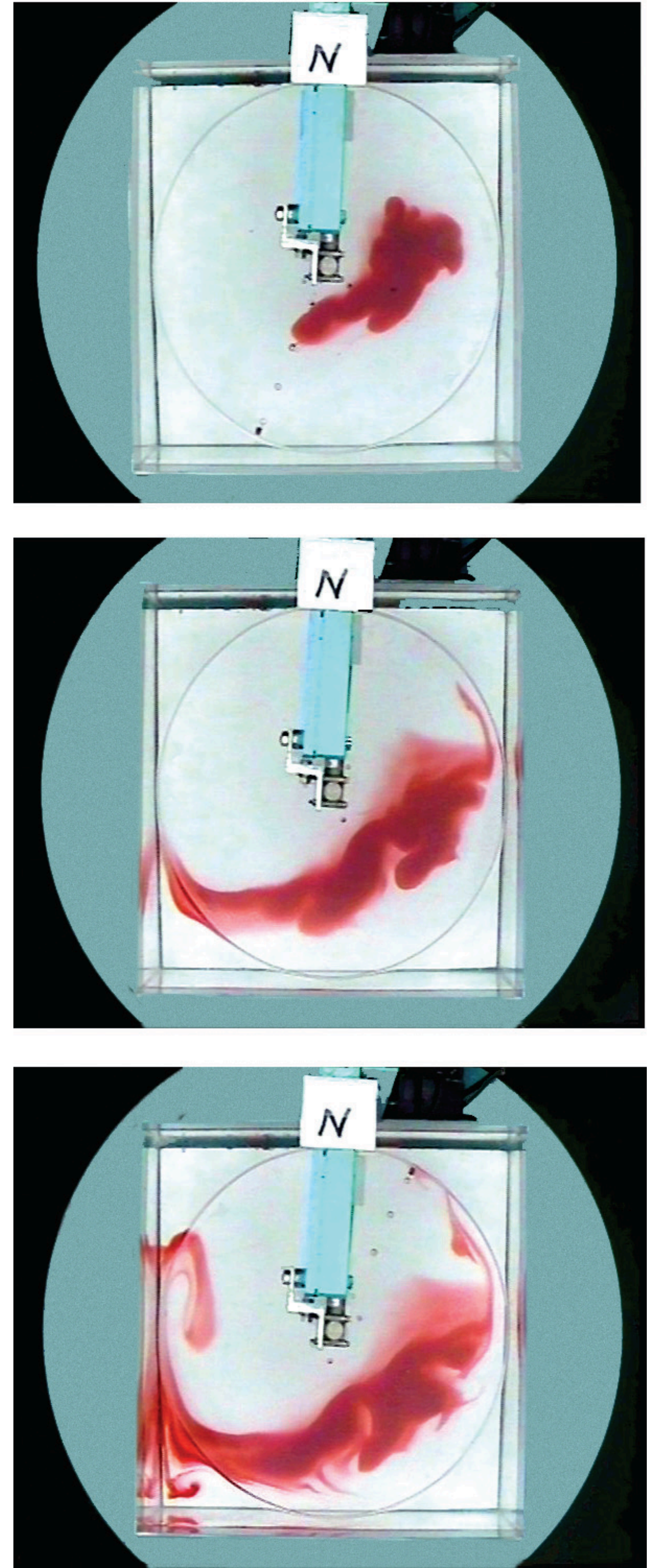
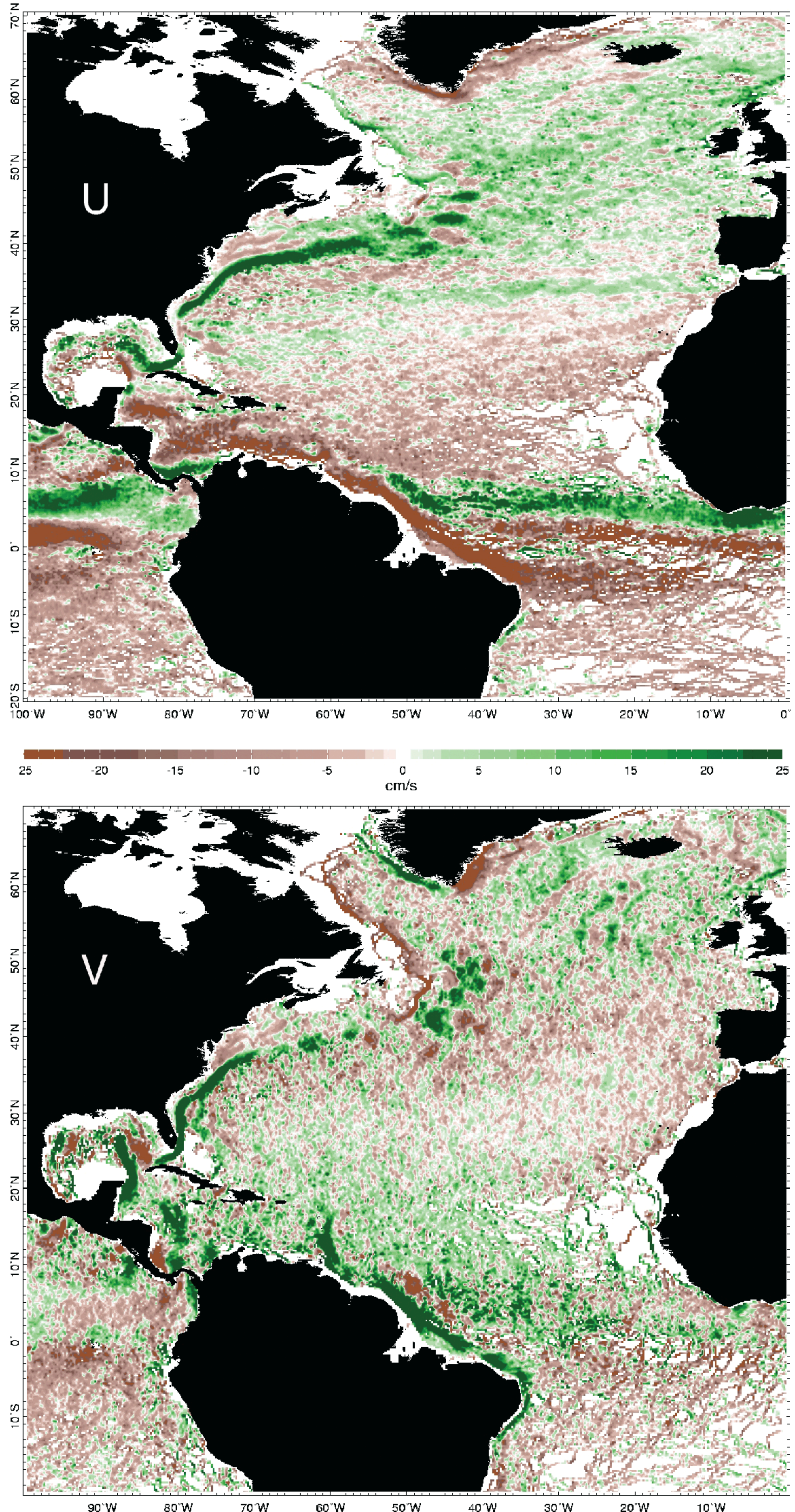

N
N
N

Surface Current Components

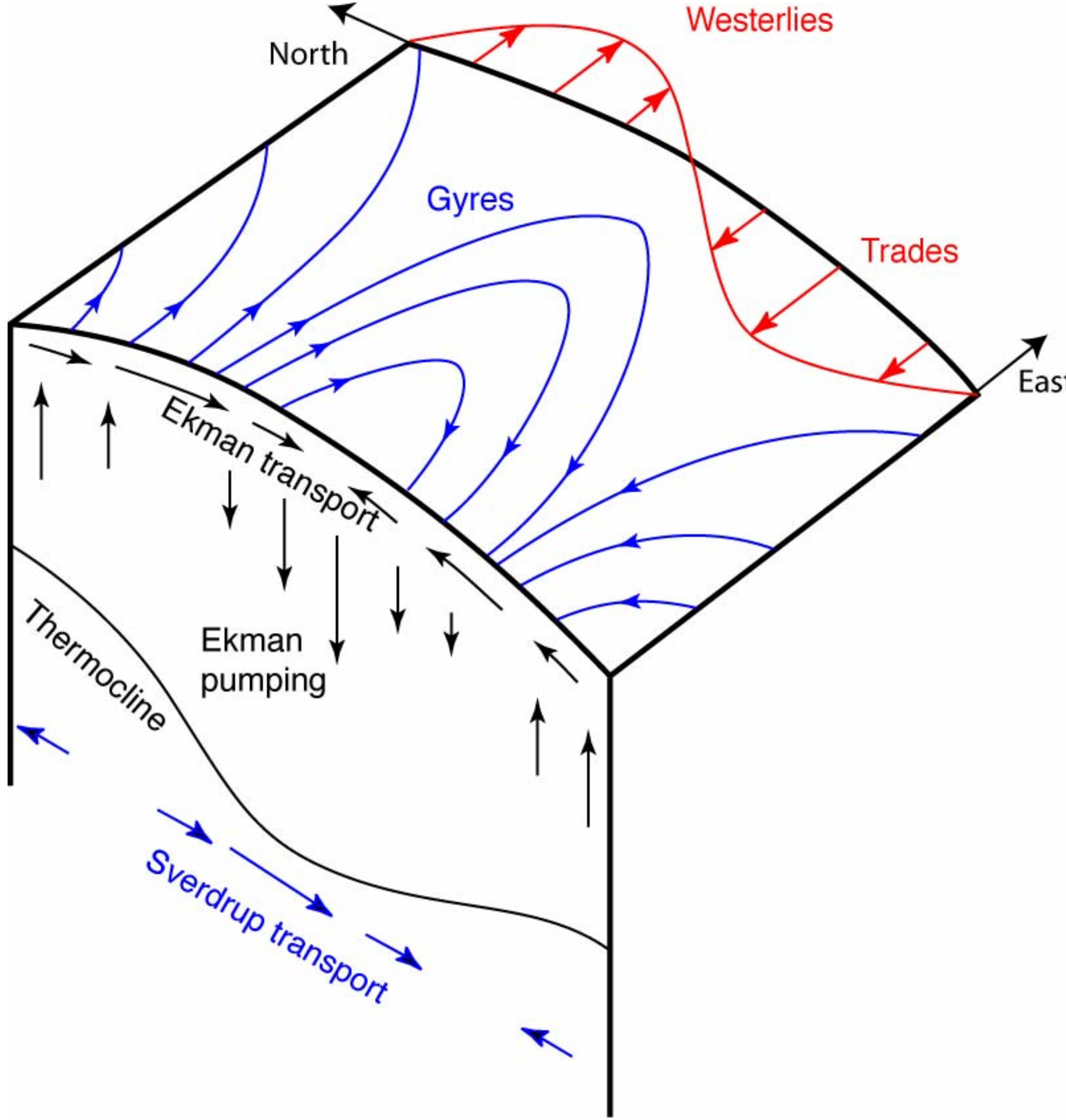


Green: eastward

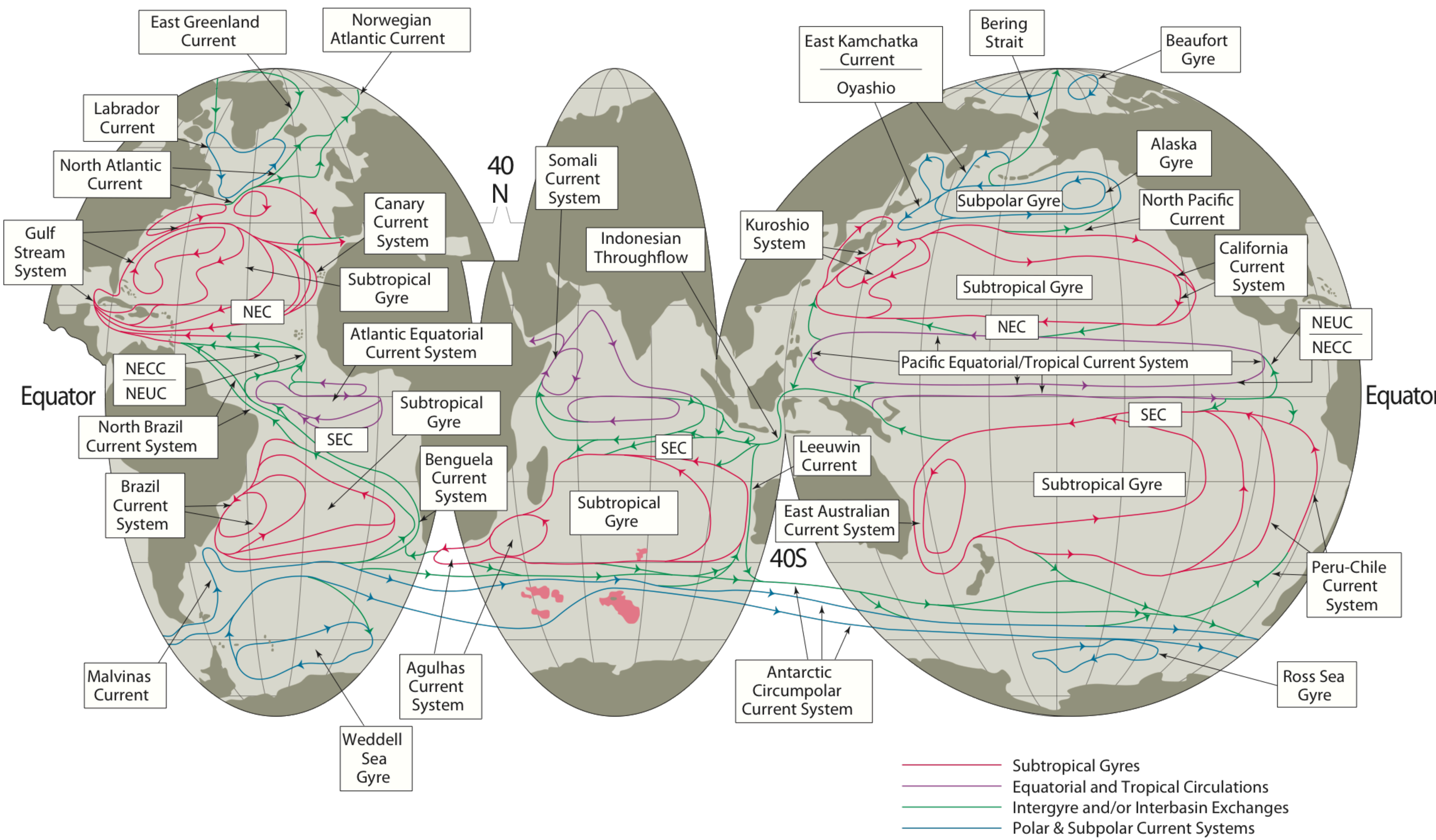
brown: westward

Green: northward

brown: southward

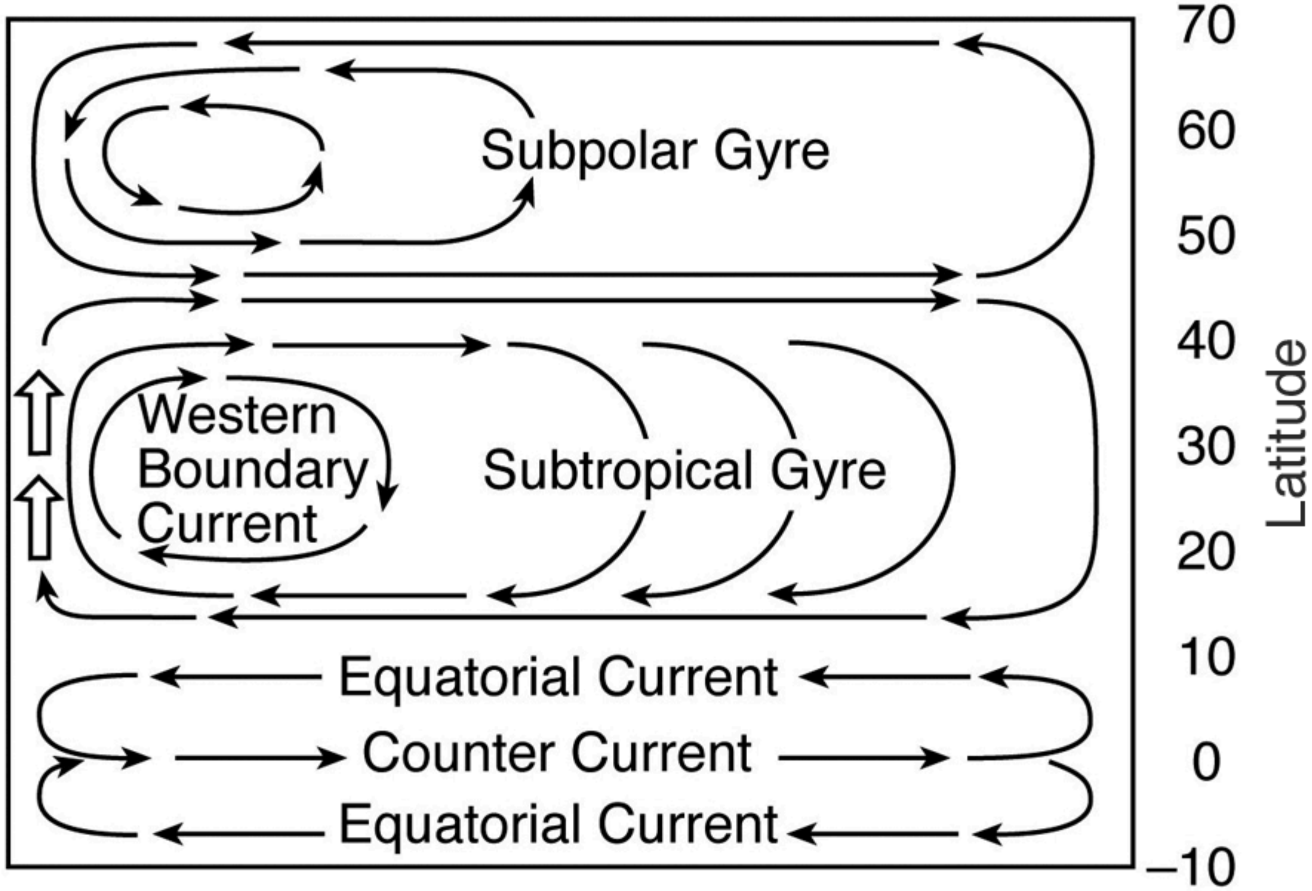

North
Westerlies
Gyres
Trades
East
Ekman transport
Ekman
pumping
Thermocline
Sverdrup transport

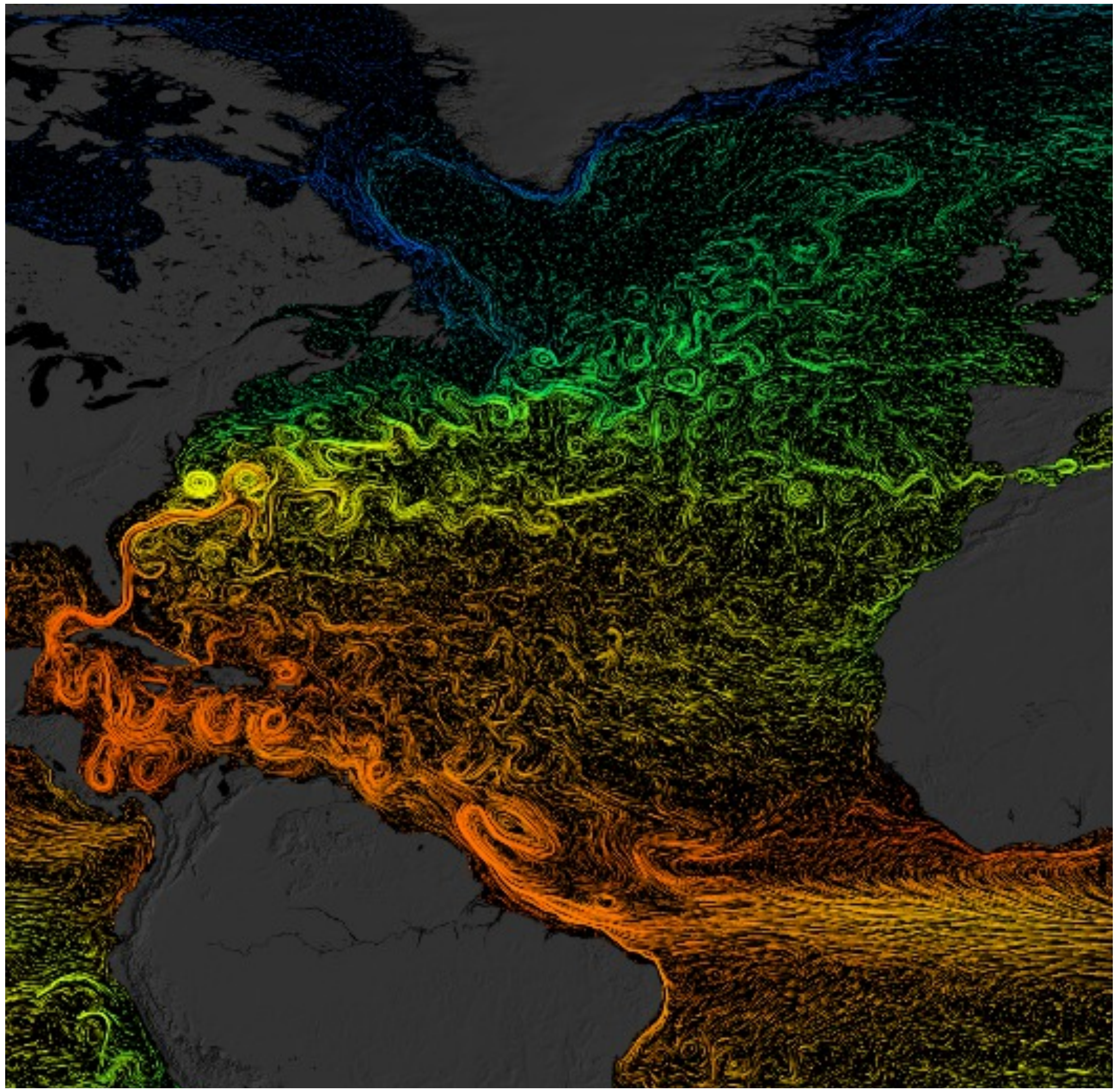
Schematic of surface circulation (modified from Schmitz, 1995)



Stratified ocean and eddies

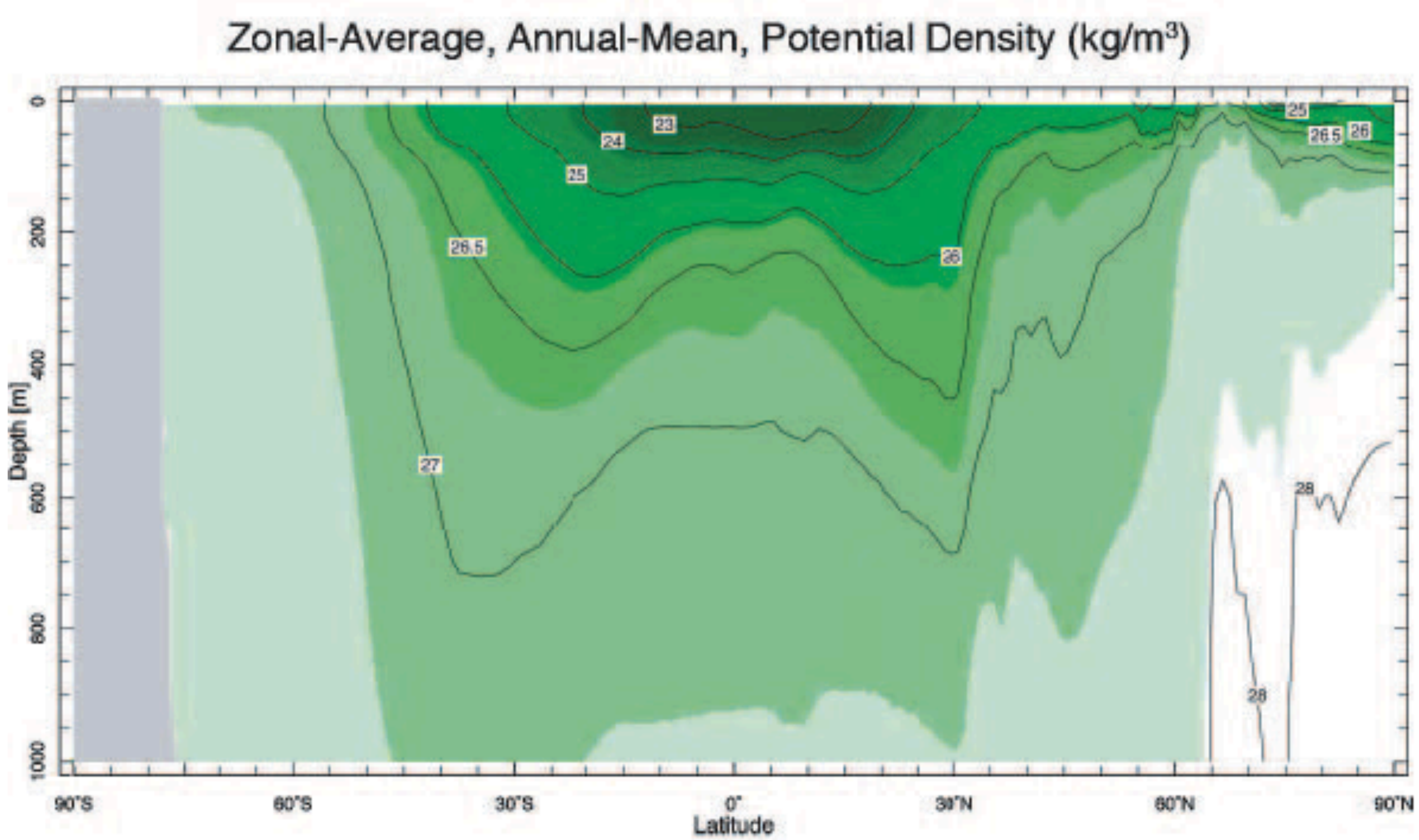
Homogeneous ocean v.s. ocean with eddies





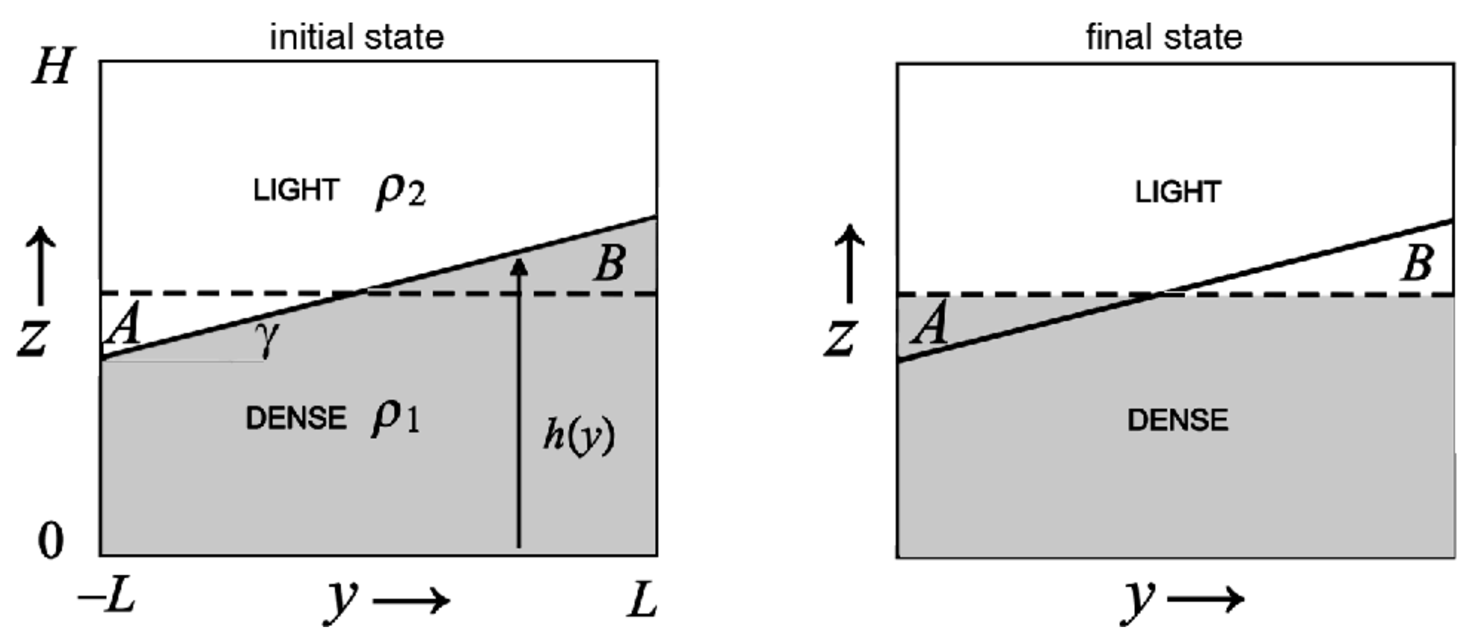
Baroclinic instability

- Stratified ocean has tilted isopycnal (same density line).
- It means that there is **available potential energy (APE).**

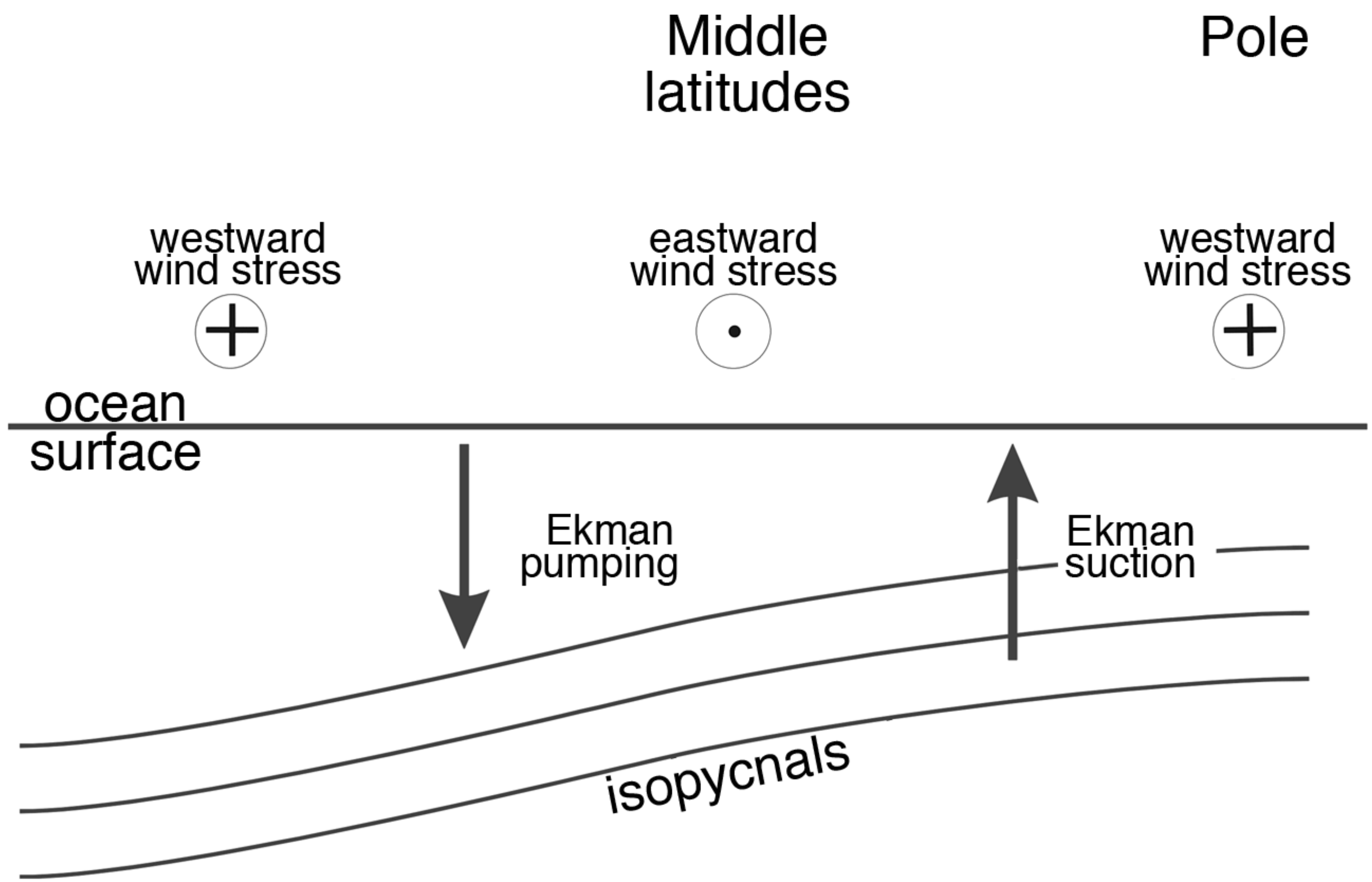


Available potential energy

- It is responsible for the atmospheric eddies.
- It is also responsible for the oceanic eddies.
- The stratified fluid with tilted isopycnal has higher potential energy than that with flat isopycnal.



A schematic of the mechanism maintaining the tilt isopycnal



Atmospheric eddies v.s. oceanic eddies

	Atmospheric eddies	Oceanic eddies
Length scale	700 km	50 km
Time scale	1 day	A week
Source	APE by radiative imbalance	Mechanically produced APE
Role	Heat and momentum transport	Heat and momentum transport but less crucial than those in the atmosphere