ATM 2106 TA Class

April 3, 2019

Department of Atmospheric Sciences, Yonsei University
Air-Sea Modeling Laboratory

Review

Last time

- Climate feedbacks
- Seasonal variability
- Hydrostatic balance
- Practice with python(plot radiation)

Today

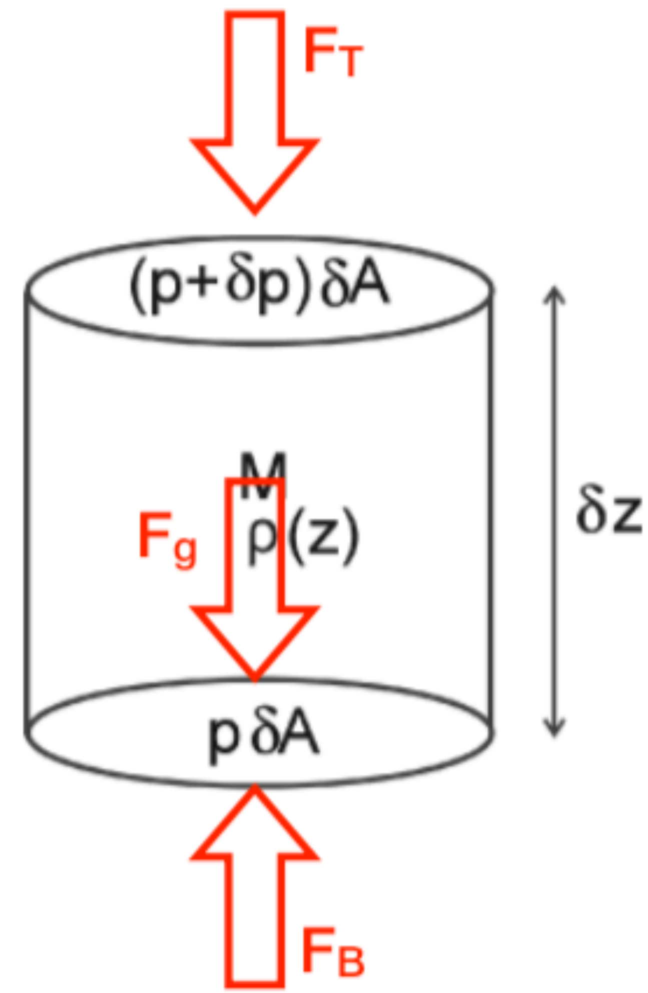
- Stability
- Geopotential height
- Equation of fluid motion

Hydrostatic Balance

- $F_g = -gM = -g\rho\delta A \delta z$
- $F_T = -(p + \delta p)\delta A$
- $F_B = p\delta A$
- $F_g + F_T + F_B = \delta p + g\rho\delta z = 0$
- The equation of hydrostatic balance:

$$\frac{dw}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g = 0.$$

$\frac{dw}{dt} = 0.$ 이 $w=0$을 의미하는 것은 아니다.



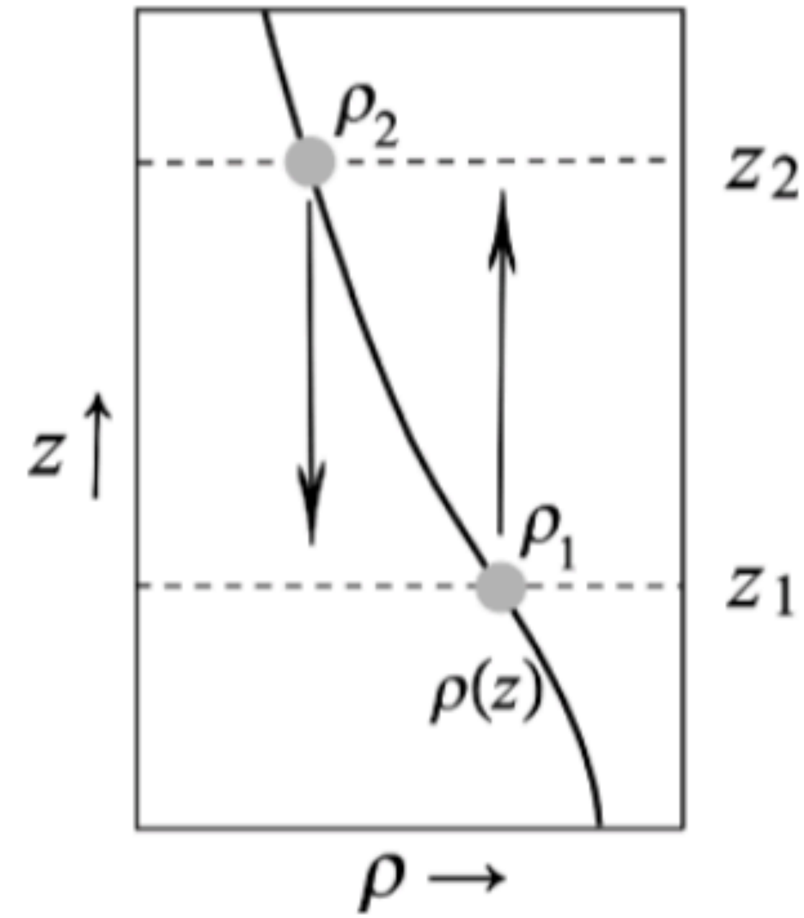
- Let's move the partial at $z=z_1$ to z_2.
- Its density is still $\boldsymbol{\rho}_1$.
- The density of the environment is

$$\rho_E = \rho(z_2) = \rho_1 + \left(\frac{d\rho}{dz}\right)_E \delta z$$

- Then the buoyancy, b, becomes

$$b = \frac{g}{\rho_1}\left(\frac{d\rho}{dz}\right)_E \delta z$$

- b depends on $(d\rho/dz)$!



Stability

- The stability
    - Positively buoyant if $(d\rho/dz)_E > 0 \rightarrow$ unstable
    - Negatively buoyant if $(d\rho/dz)_E < 0 \rightarrow$ stable
    - Neutrally buoyant if $(d\rho/dz)_E = 0 \rightarrow$ neutral
- Since we know that ρ is inversely proportional to T,
    - Unstable if $(dT/dz)_E < 0$
    - Stable if $(dT/dz)_E > 0$
    - Neutral if $(dT/dz)_E = 0$

- The atmosphere is a compressible fluid

$$\rho = \rho(p, T)$$

- For example, from the perfect gas law, $\rho = p/RT$
- Then, from the first law of thermodynamics,

$$\delta Q = \delta U + \delta W$$

ΔHeat = Δ(Internal energy) + Δ(External work done)

- In adiabatic process, $\delta U + \delta W = 0$

- In the textbook, you can find the deviation of the temperature change with height.

$$\frac{dT}{dz} = -\frac{g}{c_p} = \Gamma_d$$

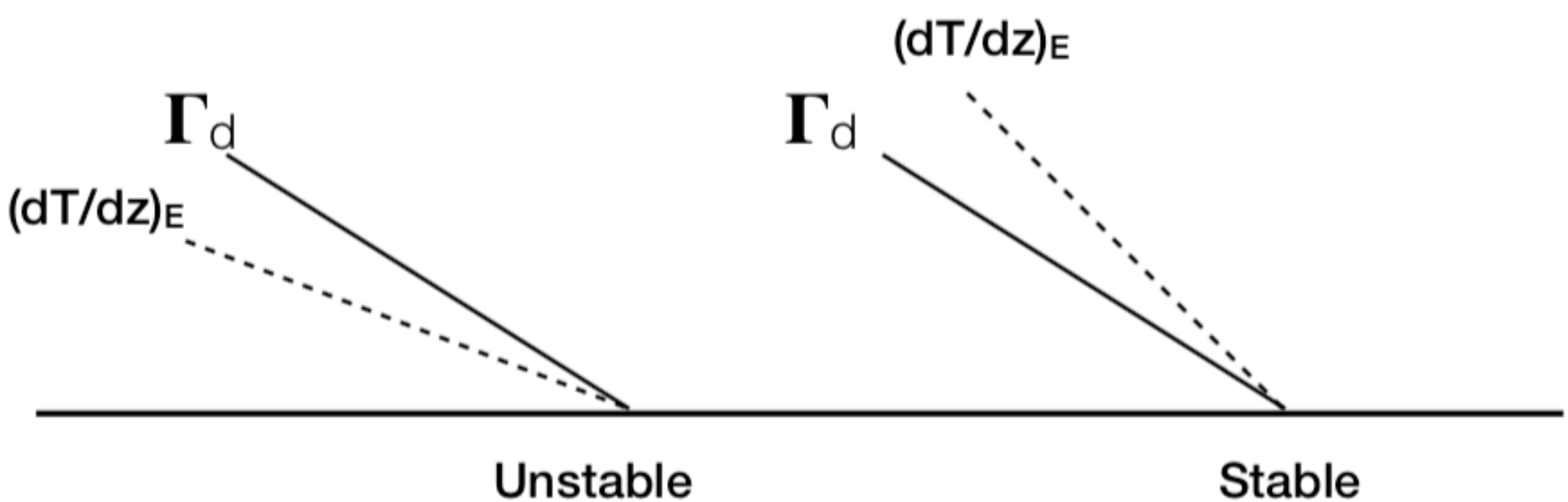
- c_p is specific heat at constant pressure and 1005 J/kg/K
- Then we find

$$\Gamma_d \approx 10 \text{ K km}^{-1}$$

- This is known as the dry adiabatic lapse rate.

Stability

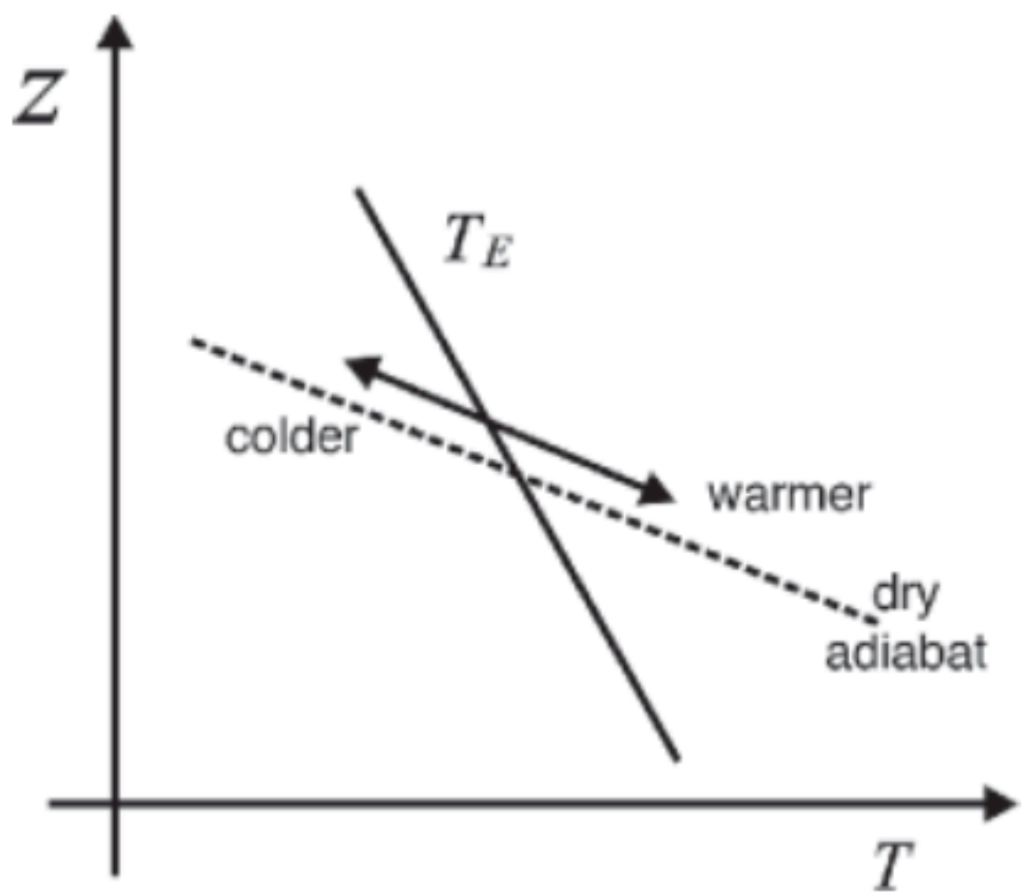
- Unstable if $(dT/dz)_E < -\Gamma_d$
- stable if $(dT/dz)_E > -\Gamma_d$
- Neutral if $(dT/dz)_E = -\Gamma_d$



Stability

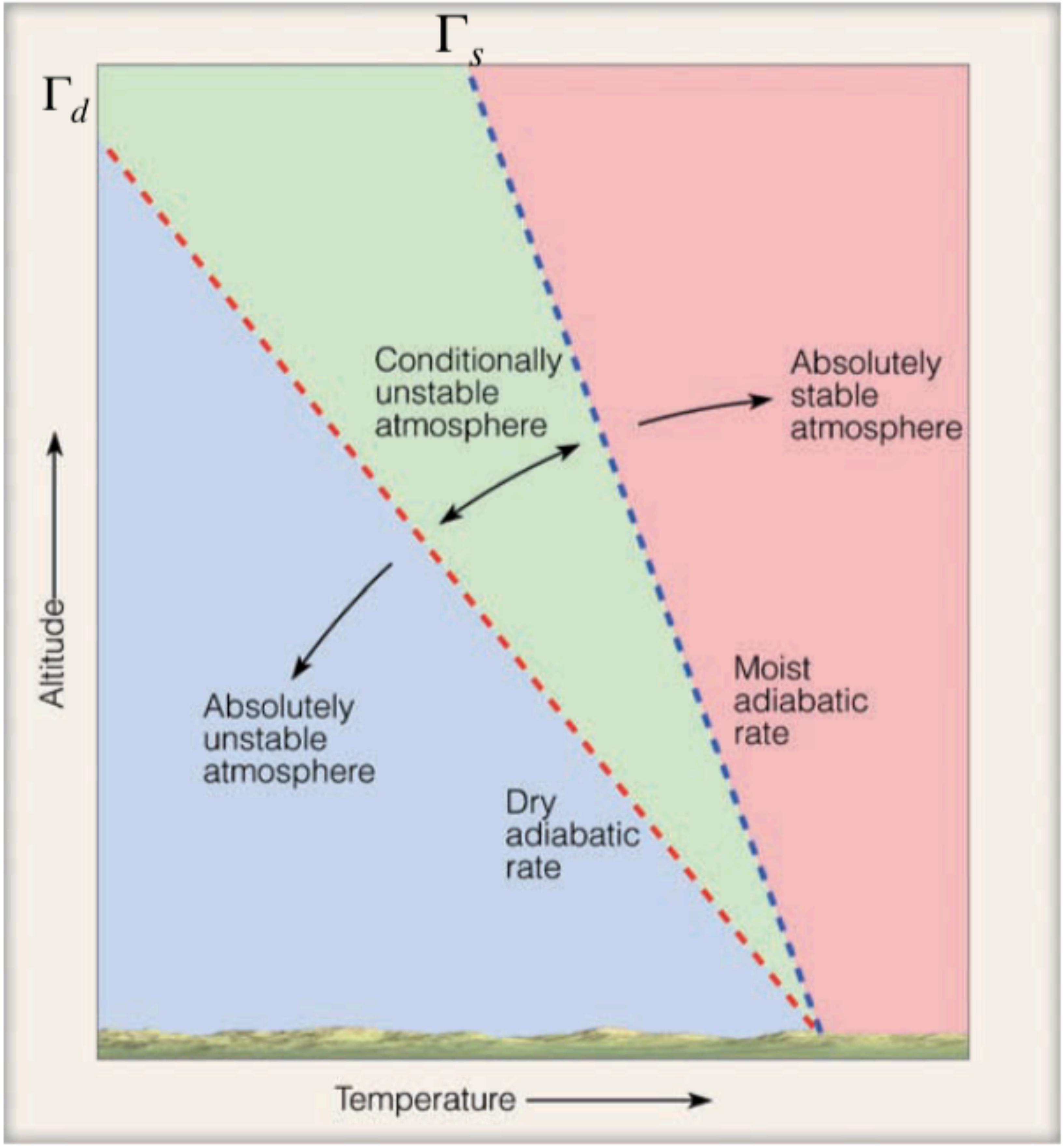
$$\left(\frac{dT}{dz}\right)_E \approx \frac{T(500\ \text{mbar}) - T(1000\ \text{mbar})}{Z(500\ \text{mbar}) - Z(1000\ \text{mbar})}$$

≈ -4.63 K km^{-1}



- $\left(\frac{dT}{dz}\right)_E \approx 0.5 \times \Gamma_d$
- Stable and no (dry) convection
- Why do we see gigantic convection systems?

Stability



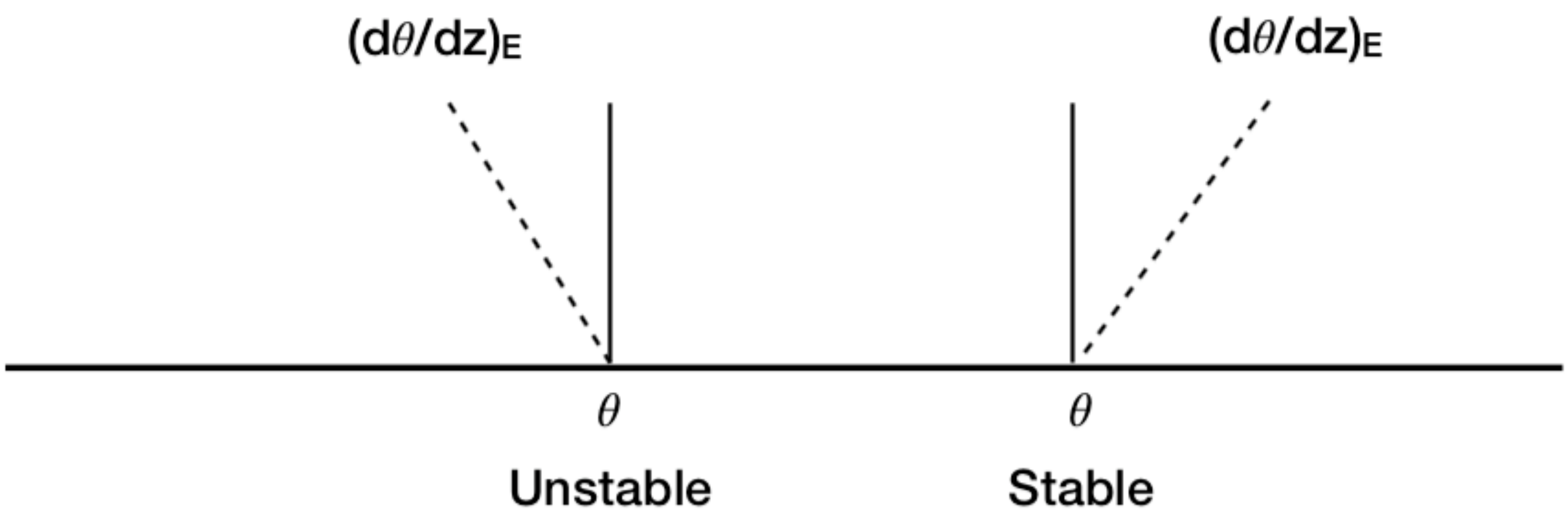
Γ_s
Γ_d
Conditionally unstable atmosphere
Absolutely stable atmosphere
Altitude
Moist adiabatic rate
Absolutely unstable atmosphere
Dry adiabatic rate
Temperature

- Let's replace $T(p_0)$ with θ.

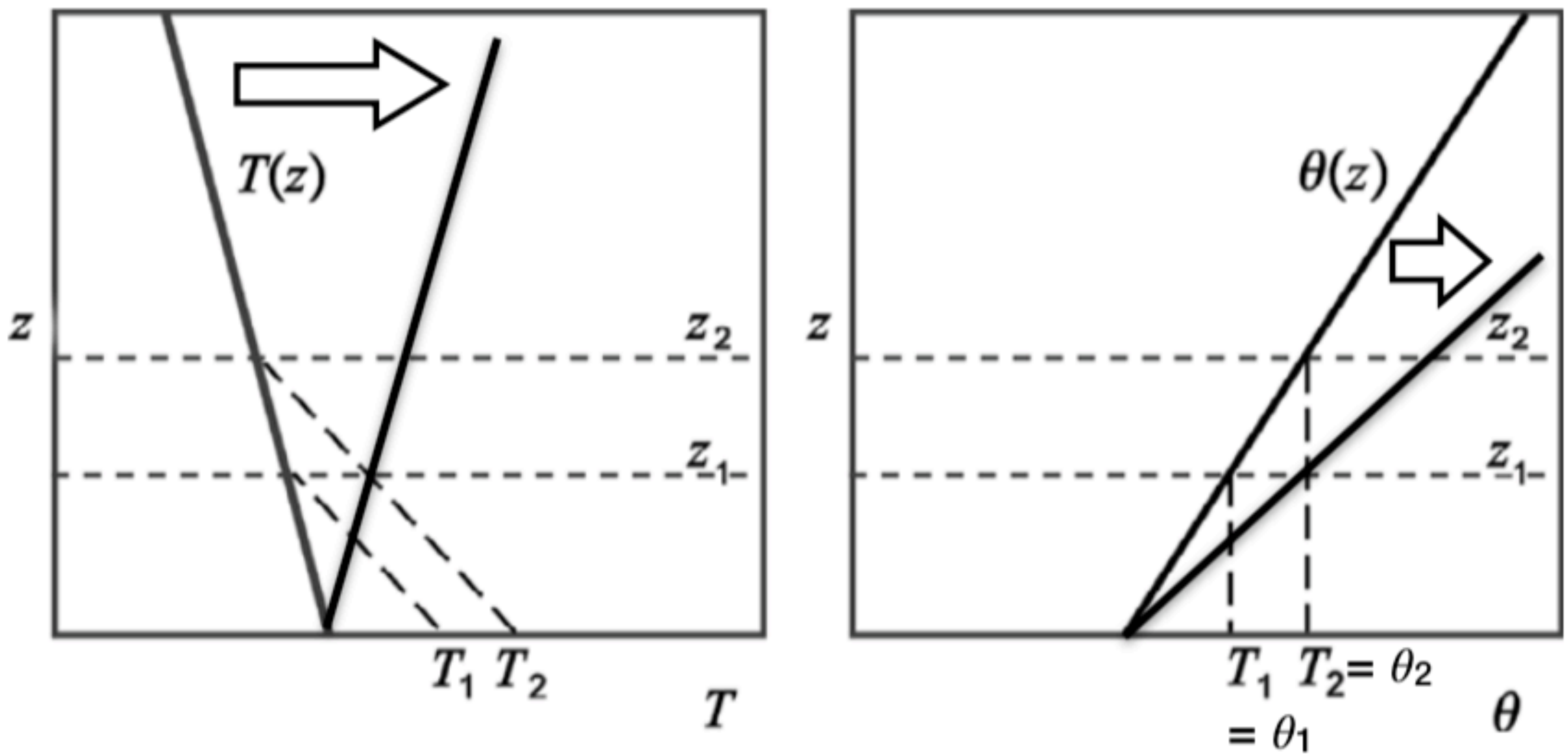
$$\theta = T(p) \left(\frac{p_0}{p} \right)^{\kappa}$$

- θ is called as potential temperature, and it represents the temperature at $p=p_0$. (conventionally, p_0 is 1000 mb.)
- We introduced potential temperature to get a quantity that does not rely on height (or p), but there is p in that equation. So we failed?

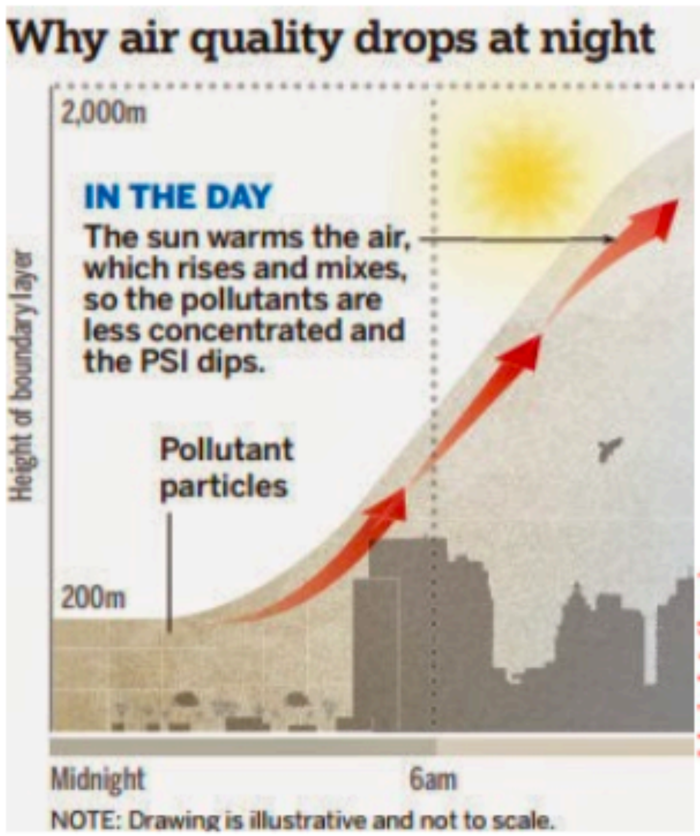
- Stability using potential temperature, θ
  - Unstable if $(d\theta/dz)_E < 0$
  - Neutral if $(d\theta/dz)_E = 0$
  - Stable if $(d\theta/dz)_E > 0$

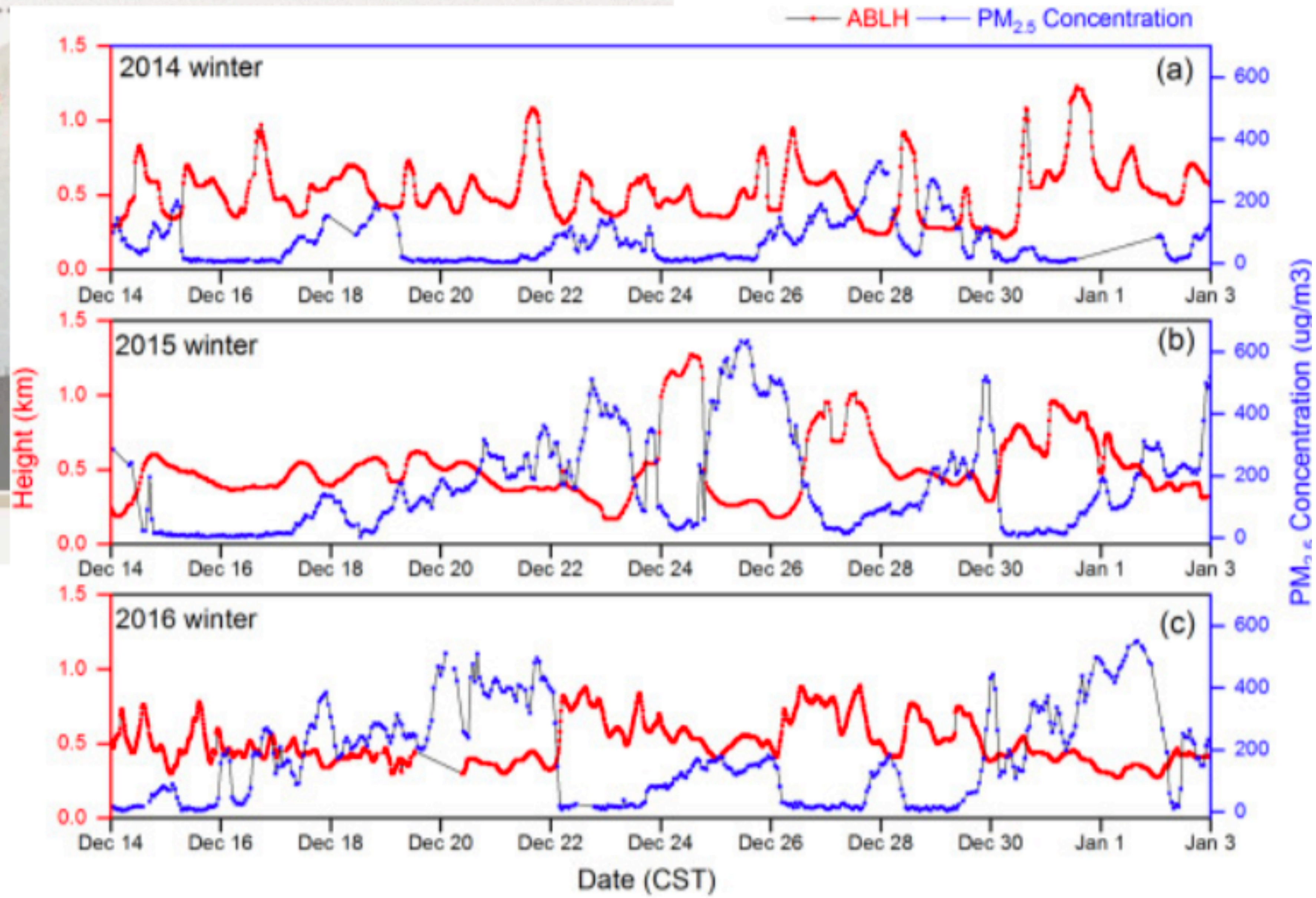


- Temperature inversion : T increases with height
  - very stable and lower boundary layer



- Boundary layer depth and air quality (and dry convection)





- We need a measure for how wet the air is.
- **Specific humidity (q)** : the mass of water vapor to the mass of air per unit volume

The mass of water vapor

$$q = \frac{\rho_v}{\rho} = \frac{\rho_v}{\rho_d + \rho_v}$$

The total mass of air = the mass of water vapor + the mass of dry air

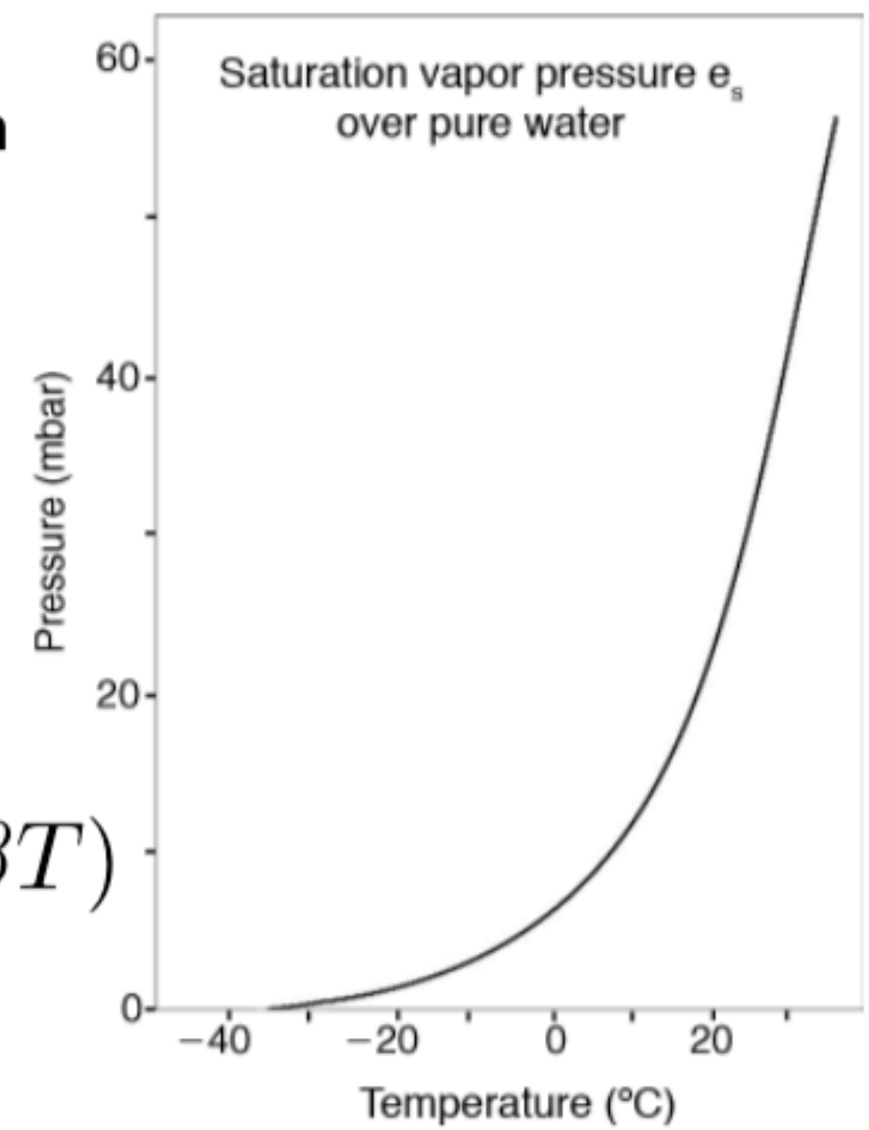
- We need a measure for how wet the air is.
- **Saturation-specific humidity (q_*)** : the specific humidity at which saturation occurs

$$q_* = \underbrace{\rho_{v,*}}_{\text{The mass of water vapor at saturation}} / \rho$$

$$q_* = \frac{\rho_{v,*}}{\rho} = \frac{e_s/R_vT}{p/RT} = \left(\frac{R}{R_v}\right)\frac{e_s}{p}$$

$$q_* = q_*(p, T)$$

$$e_s = A\exp(\beta T)$$



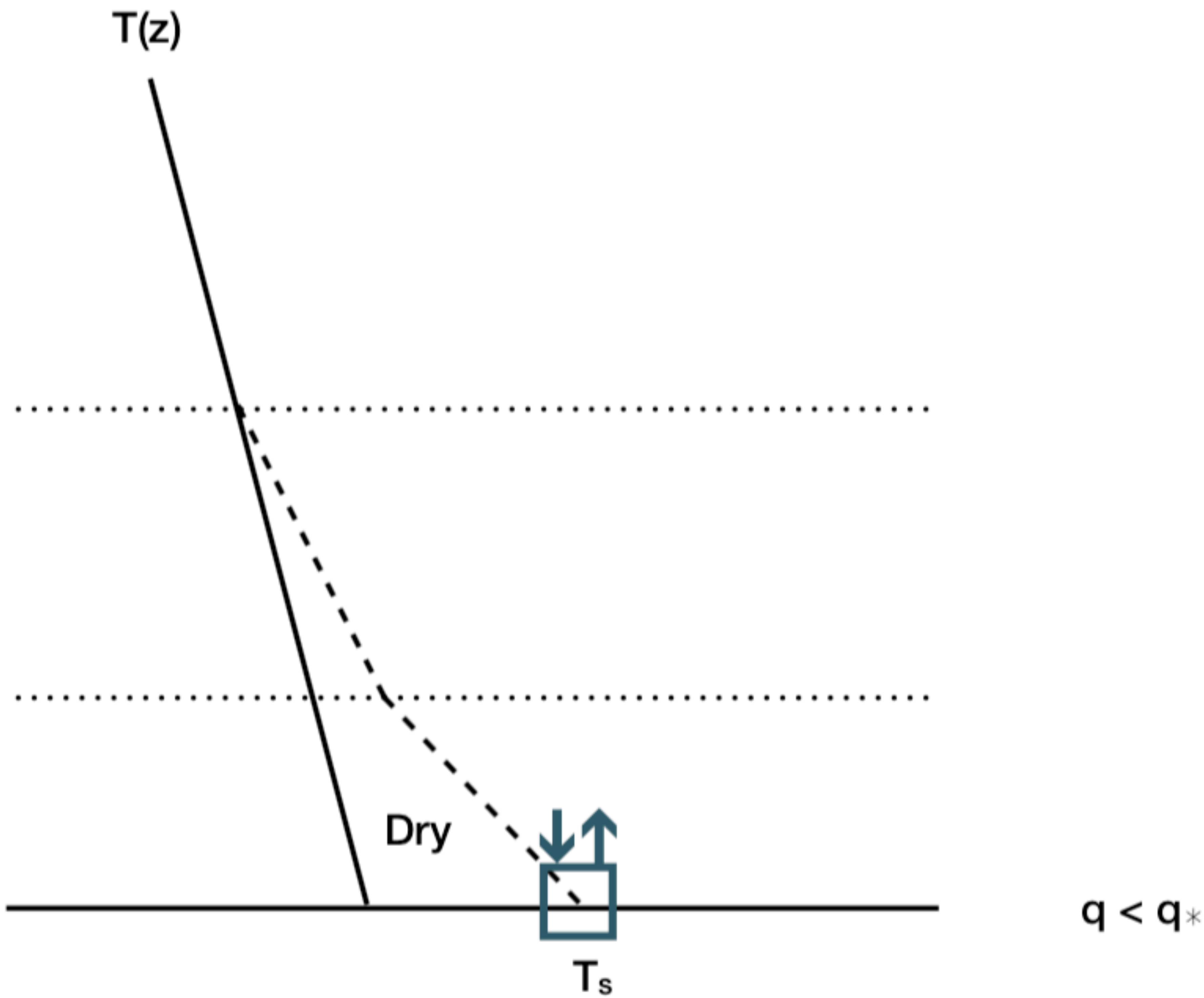
Stability

- **Relative humidity** : the ratio of the specific humidity to the saturation specific humidity

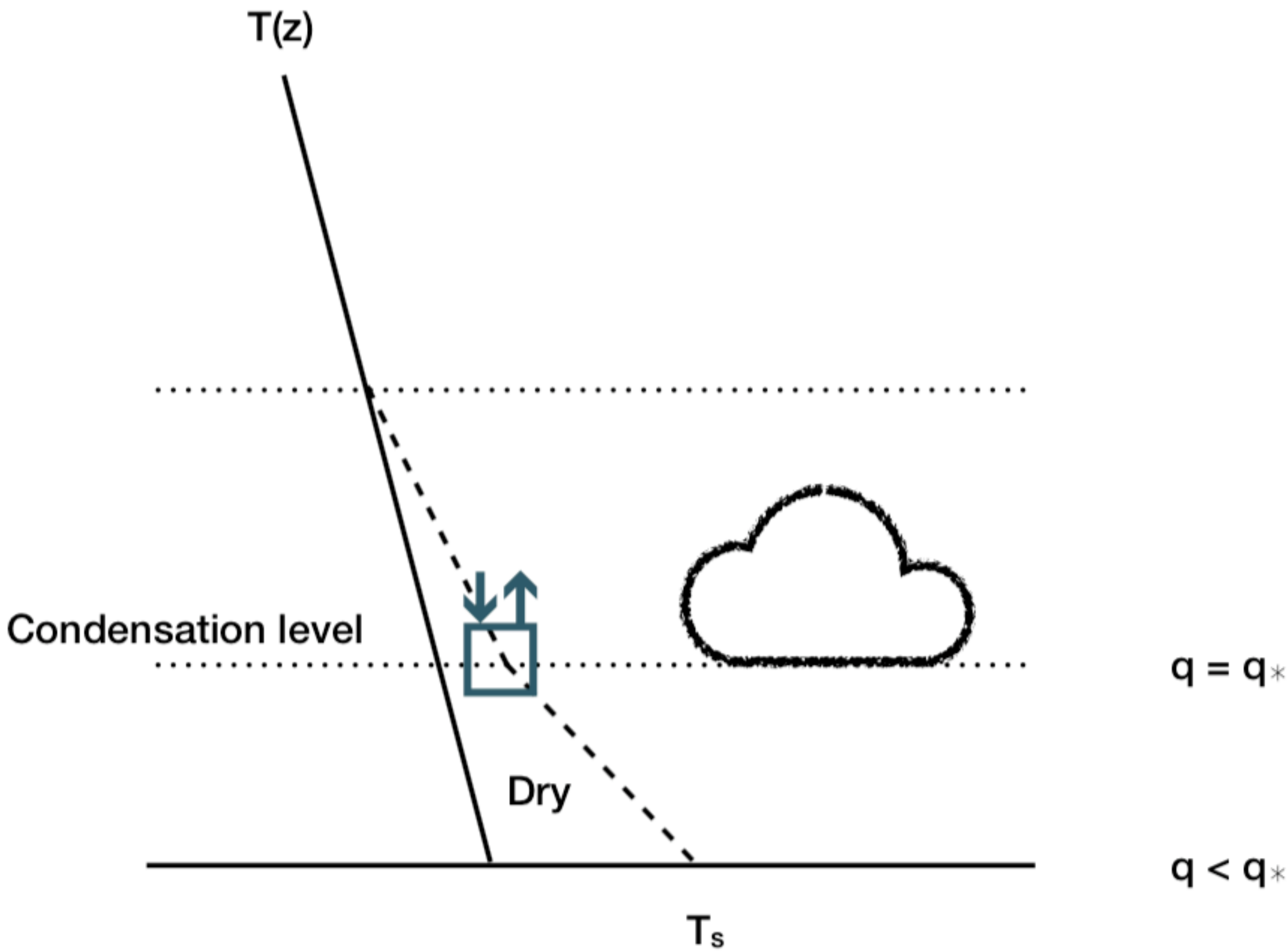
$$U = \frac{q}{q_*} \times 100\%$$

- The surface has higher humidity than aloft (relative humidity is close to 80%).
- Raise humid air..
  - Both p and T decrease, and q_* goes up? Or down?
  - How about q?
  - What happens if $q = q_*$?

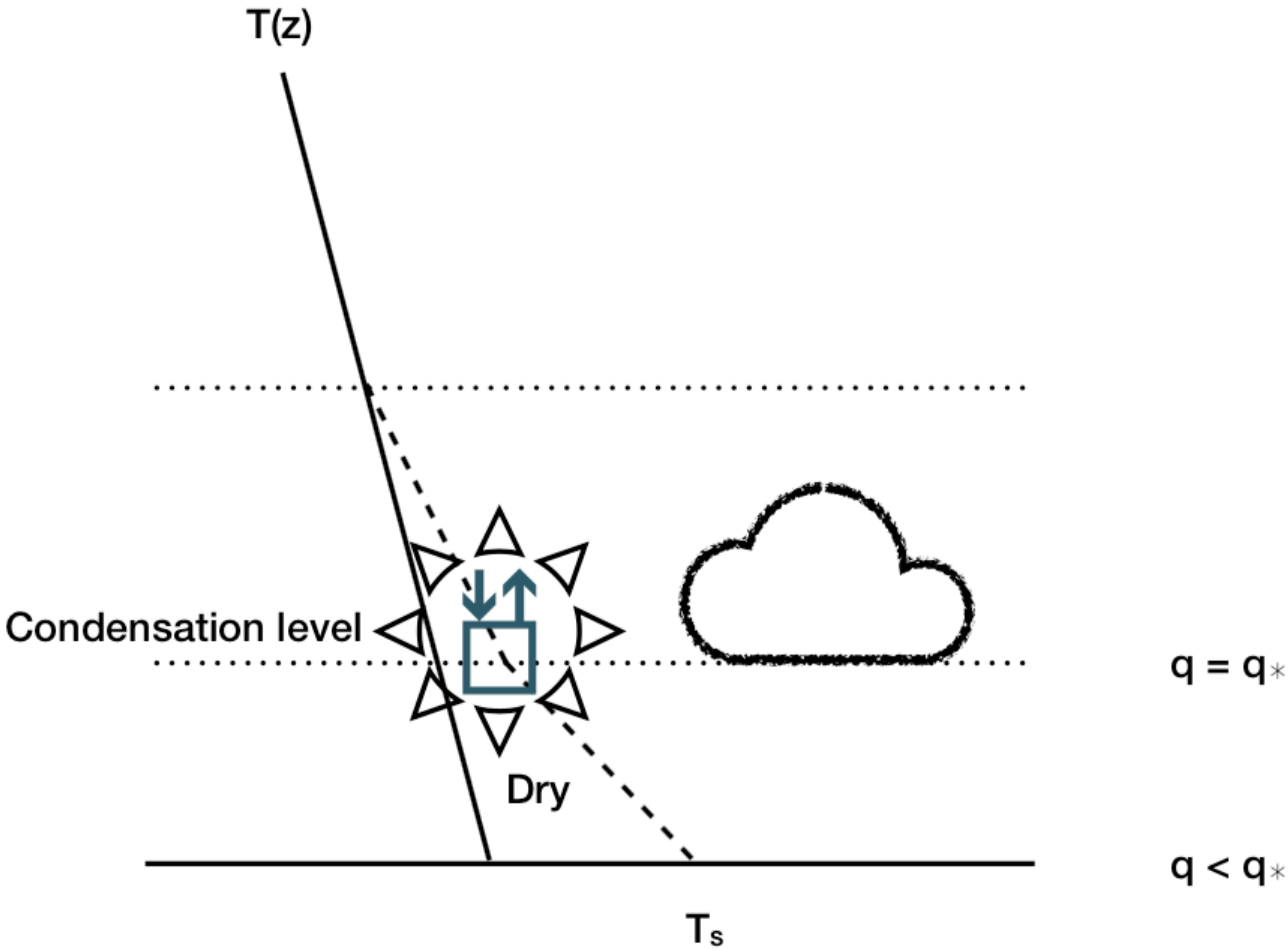
Stability



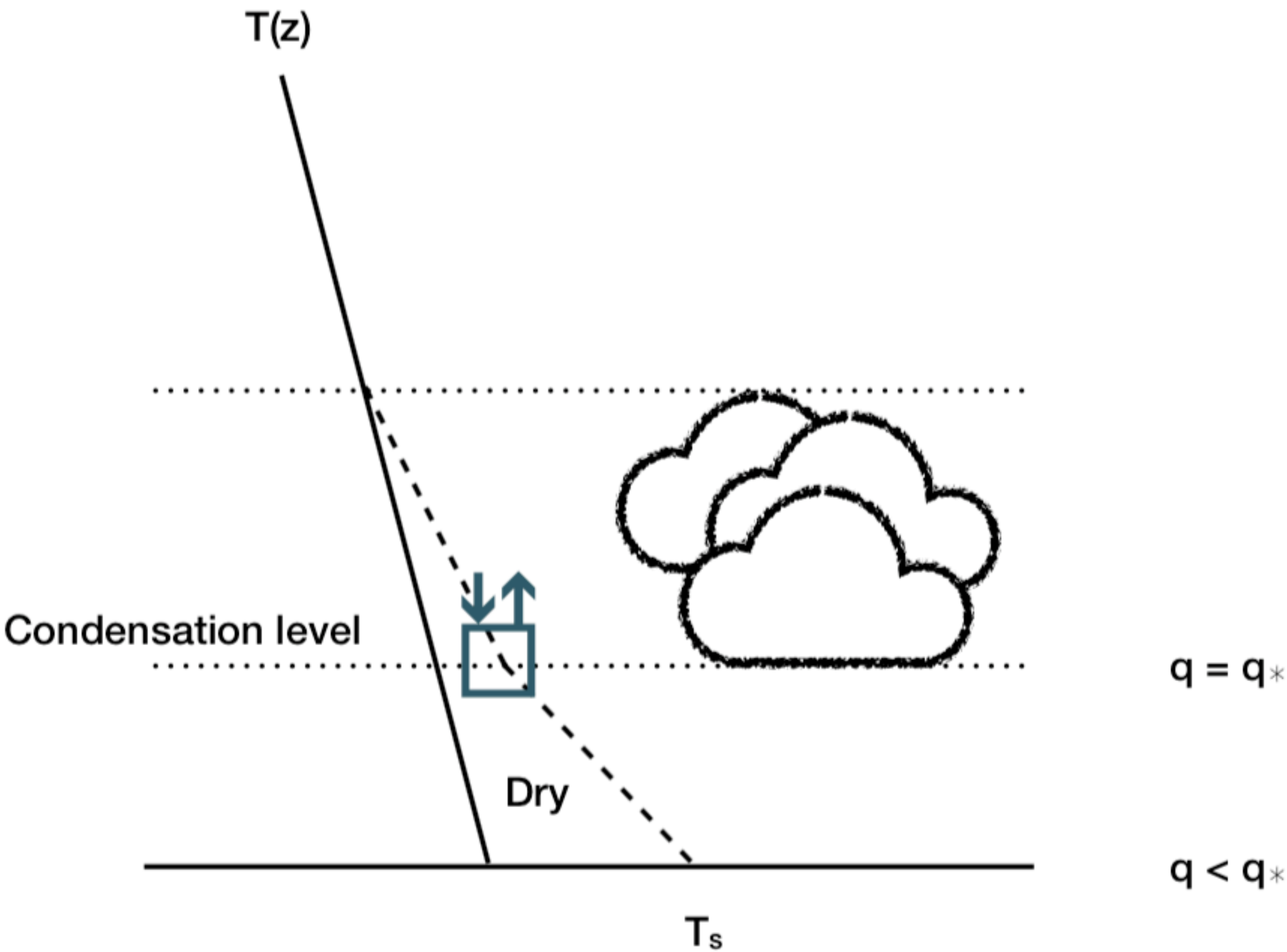
Stability



Stability



Stability



습윤단열감률

- 습윤(포화) 공기가 상승,하강할 때 잠열의 영향으로 기온감률이 감소한다.
- 상승할 땐 수증기가 응결하여 잠열을 방출하여 기온감소가 느려지고
- 반대로 하강할 땐 증발에 의해 잠열을 뺏기면서 기온상승이 느려진다.

온위

- 온위란 기포를 단열적으로 1000hPa 까지 이동시켰을 때 가질 온도로서, 단열과정동안 보존되는 값이다.
- 따라서 단열과정인 기포는 등온위면 상에서 이동하며, 반대로 등온위면을 관통하는 기포는 비단열과정을 겪음을 알 수 있다.

위 단열과정

- 기포 내에서 수증기의 증발 응결이 있는 경우 건조공기를 기준으로 보면 출입하는 잠열이 있으므로 비단열 과정이다. 그러나 수증기와 응결물까지 모두 포함하는 기준으로 보면 단열과정이라 할 수 있는데 이를 위단열과정이라 한다.
- 위 단열 과정에서 수증기는 응결되는 즉시 제거되고, 증발하기 직전에 첨가되는 것으로 가정한다. 또한 위단열과정동안 출입하는 잠열로 인해 온위는 보존되지 않으며 대신 상당온위가 정의되어 이 값이 보존된다.

상당온위

- 상당온위는 기포 내의 모든 수증기가 잠열을 방출하여 응결되었을 때, 기포가 가질 온위이다. 물리적으로는 모든 수증기가 응결할 때까지 고고도로 상승한 기포가 다시 건조단열적으로 1000hPa에 도달했을 때 갖게 될 온도이다.

- From a hydrostatic balance and perfect gas law,

$$\frac{\partial z}{\partial p} = -\frac{RT}{gp}$$

$$z(p) = R\int_{p}^{p_s} \frac{T}{g}\frac{dp}{p}$$

- z(p) is called ***geopotential height.***
- If we assume that T and g does not vary a lot with p, geopotential height is higher when T increases.

Geopotential height

* Hydrostatic balance & perfect gas law

$$\frac{\partial p}{\partial z} + \rho g = 0 \quad , \quad p = \rho RT$$

$$\frac{\partial z}{\partial p} = -\frac{RT}{pg} \qquad dz = -\frac{RT}{g}\frac{dp}{p}$$

$p_s \sim p$ 까지 적분해주면

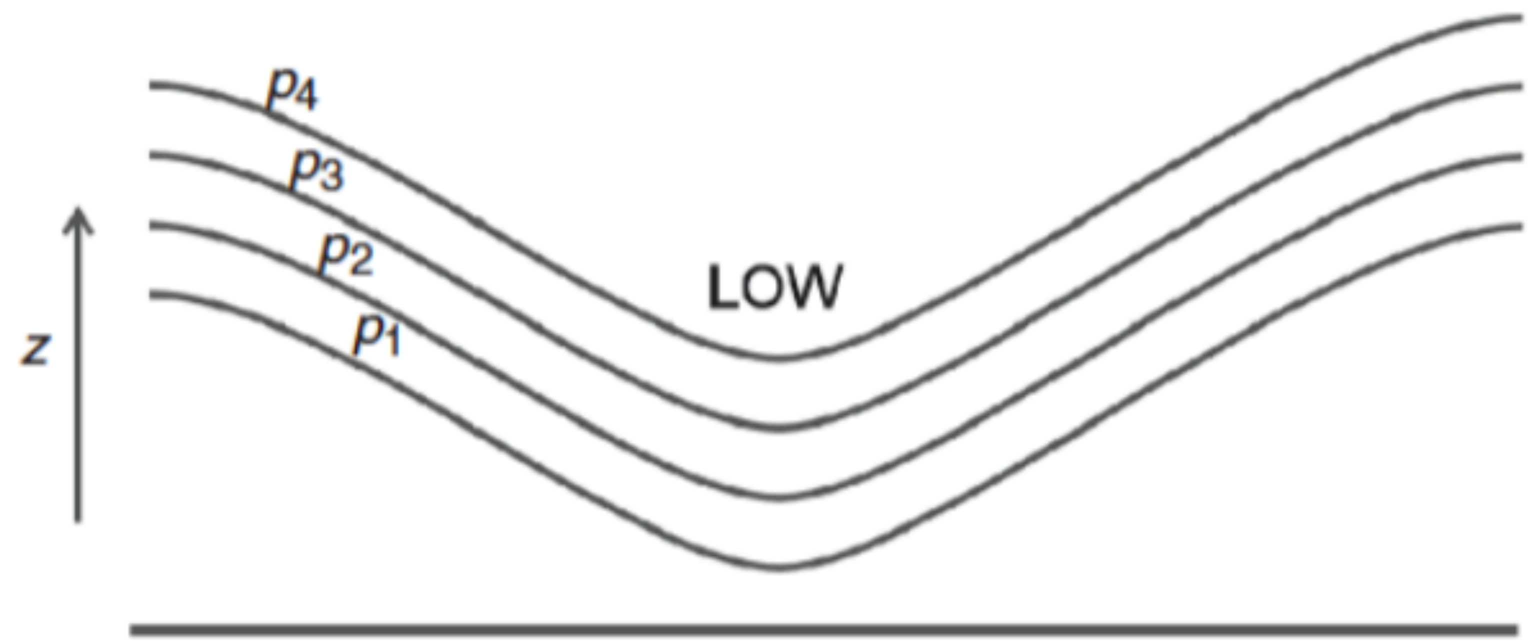
$$z(p) - \cancelto{0}{z(p_s)} = -R\int_{p_s}^{p} \frac{T}{g}\frac{dp}{p}.$$

$$z(p) = \frac{RT}{g}\int_{p}^{p_s}\frac{dp}{p} = \frac{RT}{g}\left(\ln p_s - \ln p\right)$$

- If we assume that g and T do not vary a lot with p,

$$z(p) = \frac{RT}{g}\left(\ln p_s - \ln p\right)$$

- z increases as p decreases.
- Higher T increases geopotential height.



- Geopotential height is lower at the low pressure system.
- Or the high pressure system corresponds to the high geopotential height.
- T tends to be low in the region of low geopotential height.

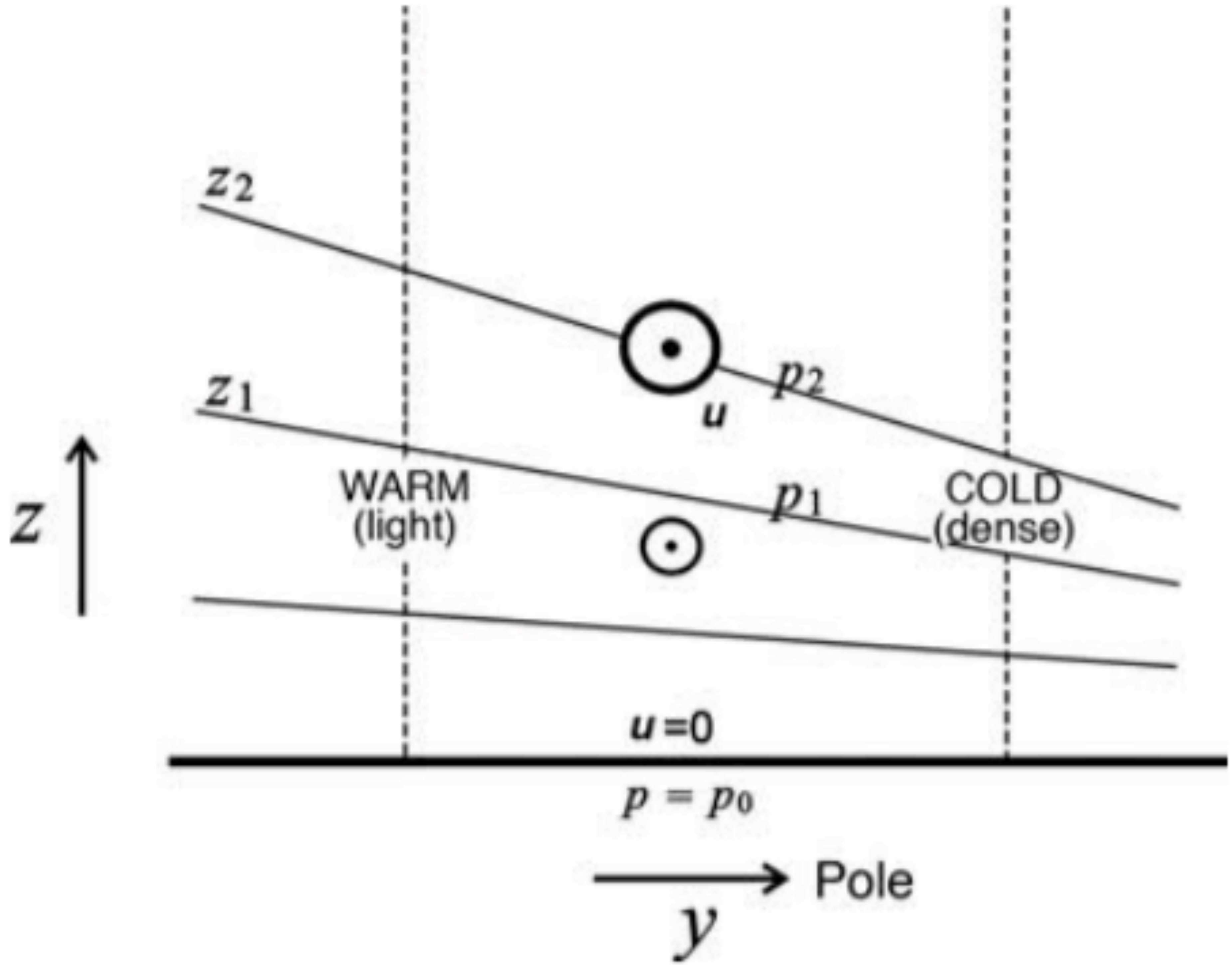
- We can discuss about the slope of the geopotential height if we know the temperature.

$$z_{warm} - z_{cold} = \frac{R}{g}\left(T_{warm} - T_{cold}\right)\left(\ln p_s - \ln p\right)$$

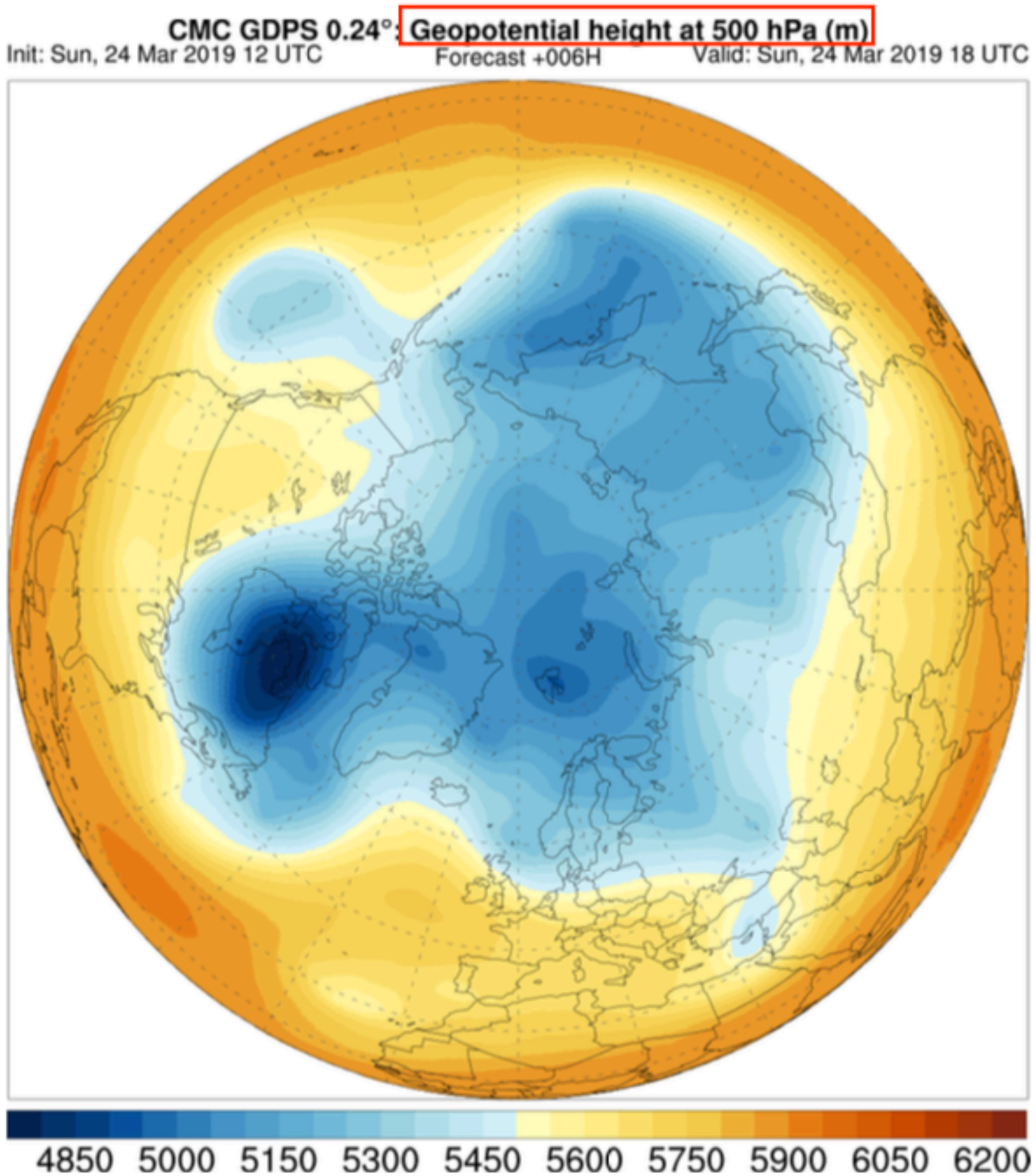
- We can also discuss about the thickness of an atmospheric layer if we know the temperature.

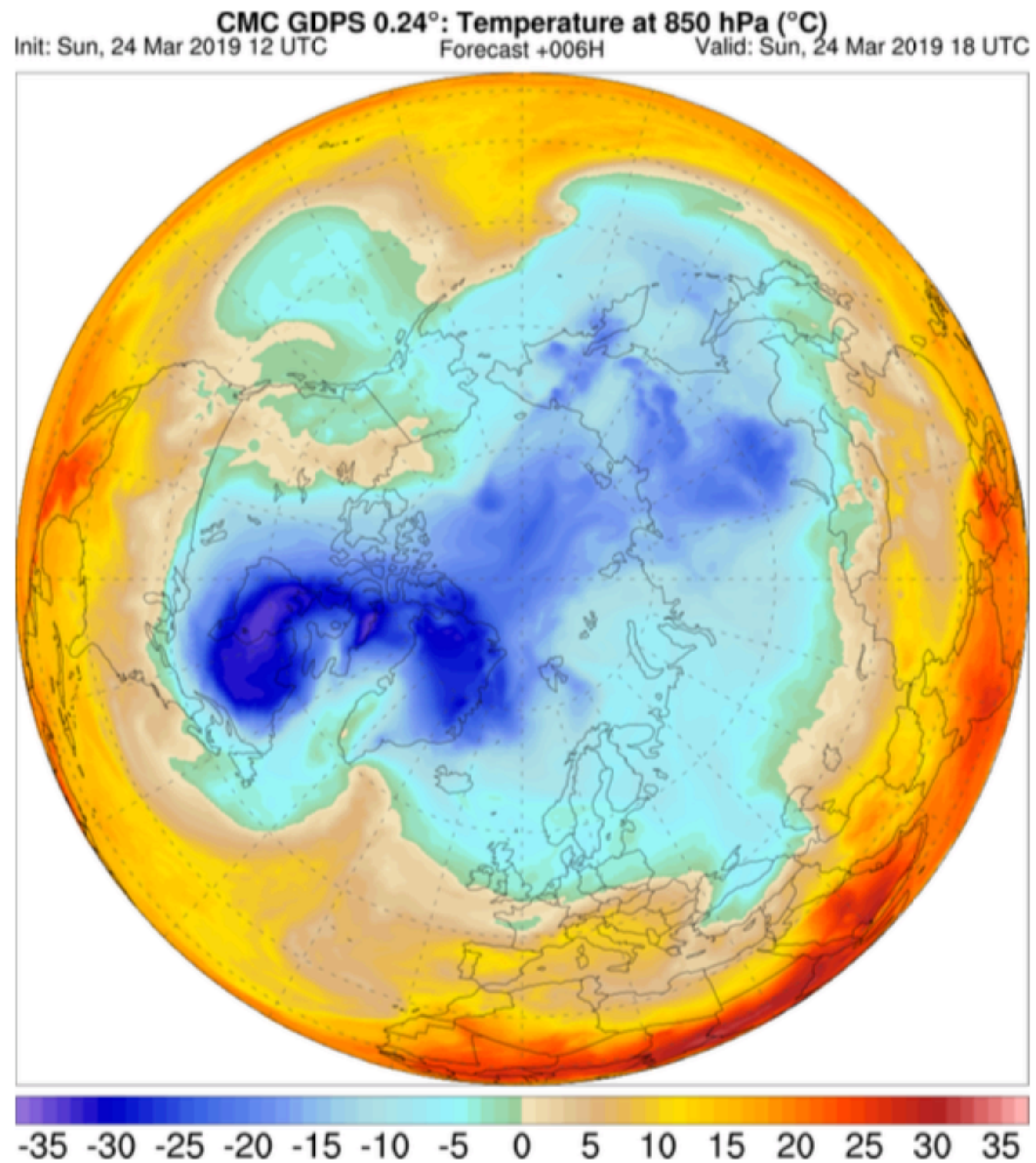
$$z_{p_1} - z_{p_2} = \frac{R\overline{T}}{g}\left(\ln p_2 - \ln p_1\right)$$

Geopotential height



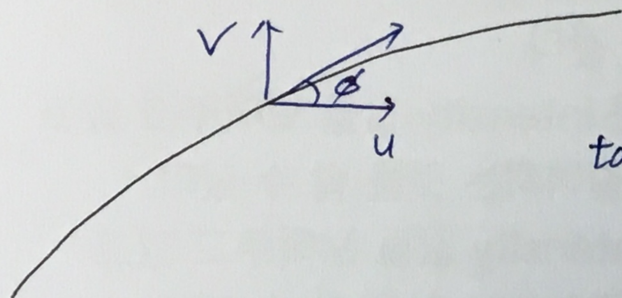
Geopotential height





Stream lines (유선 : 유체가 흐르는 선)

→ a continuous line whose tangent is in the direction of velocity (속도 방향 접선의 연속적인 선)



$\psi = const.$

$$\tan\phi = \frac{v}{u} = \frac{dy}{dx}$$

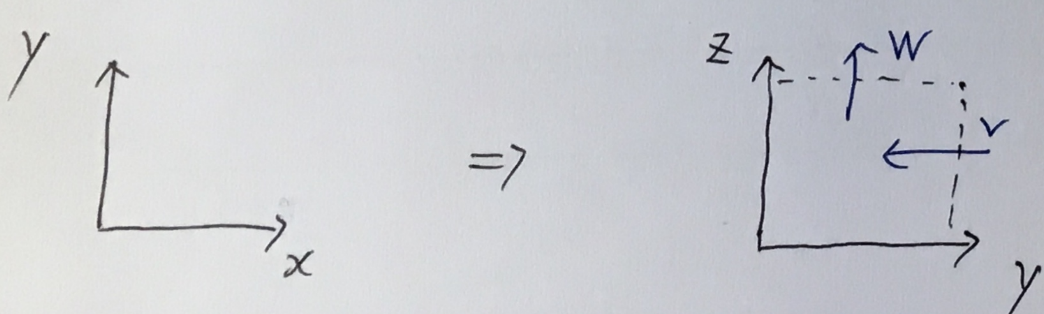
$$\Rightarrow \frac{dx}{u} = \frac{dy}{v}$$

In 2D incompressible flows

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \psi(x, y) \qquad u = \frac{\partial \psi}{\partial y} \qquad \frac{\partial u}{\partial x} = \frac{\partial^2 \psi}{\partial x \partial y}$$

$$v = -\frac{\partial \psi}{\partial x} \qquad \frac{\partial v}{\partial y} = -\frac{\partial^2 \psi}{\partial x \partial y} \qquad \Rightarrow \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

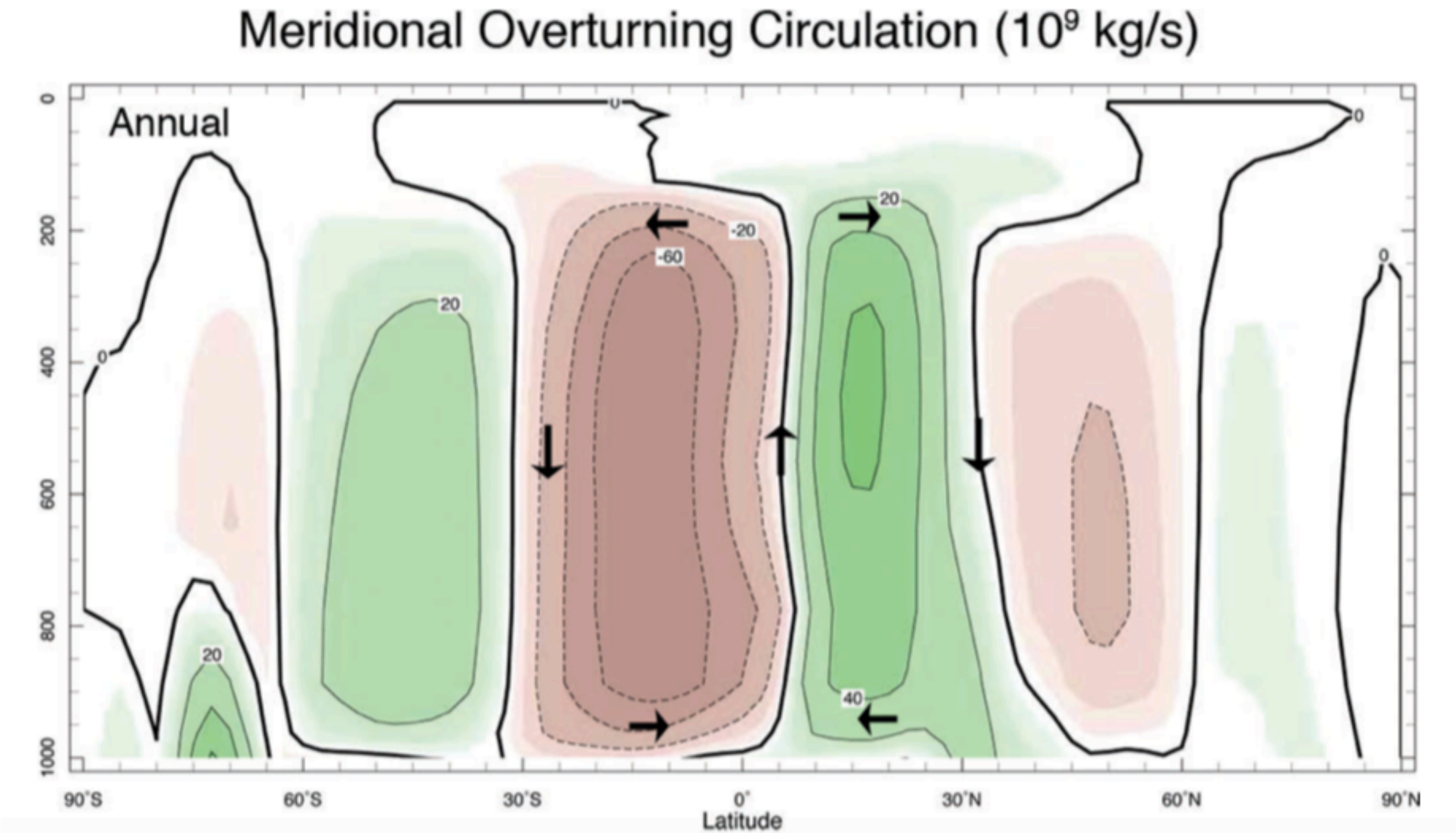
Let's think about below fig.



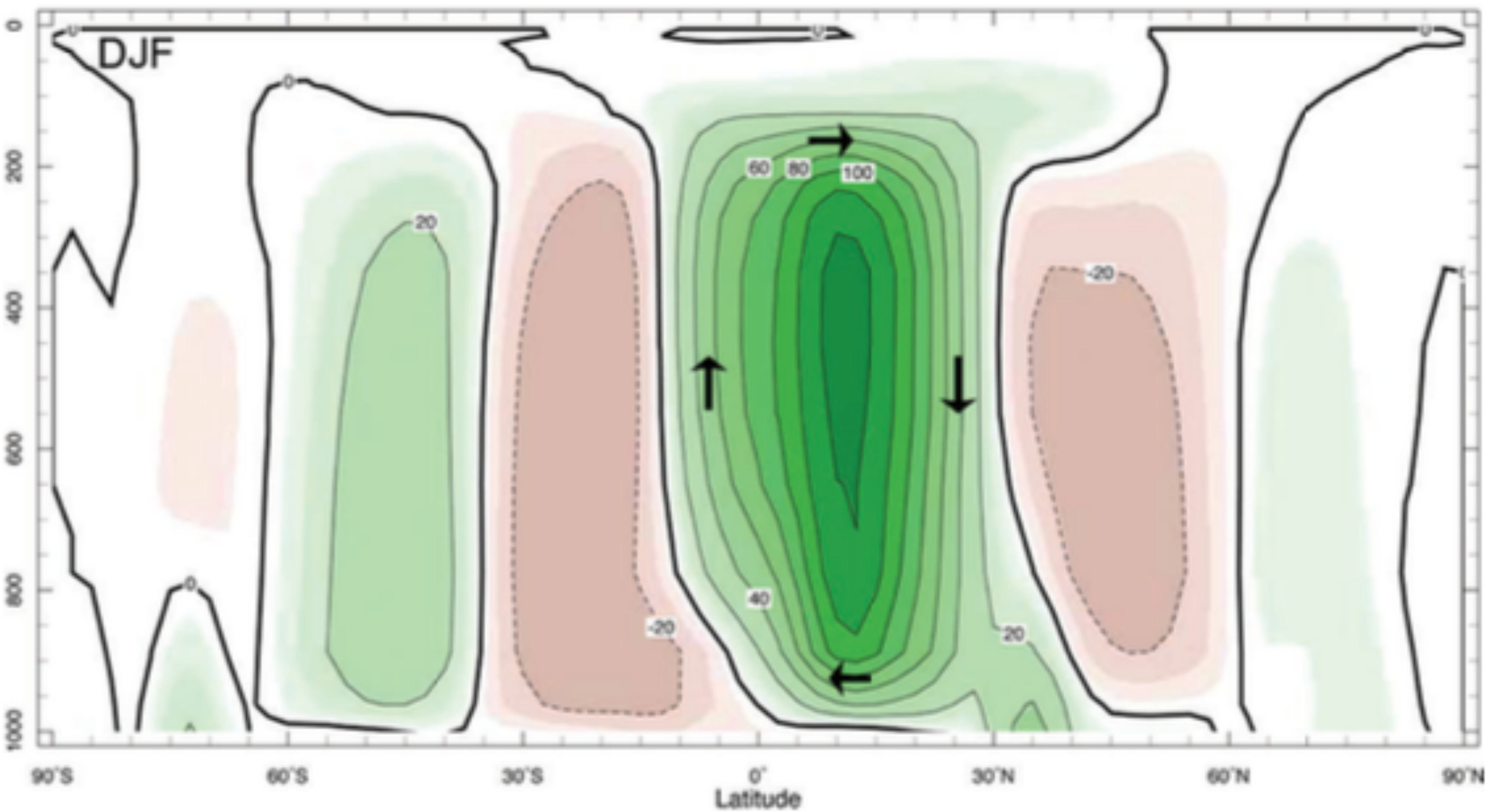
$$\frac{\partial \psi}{\partial z} \equiv -v \qquad \frac{\partial \psi}{\partial y} \equiv w$$

$$\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0.$$

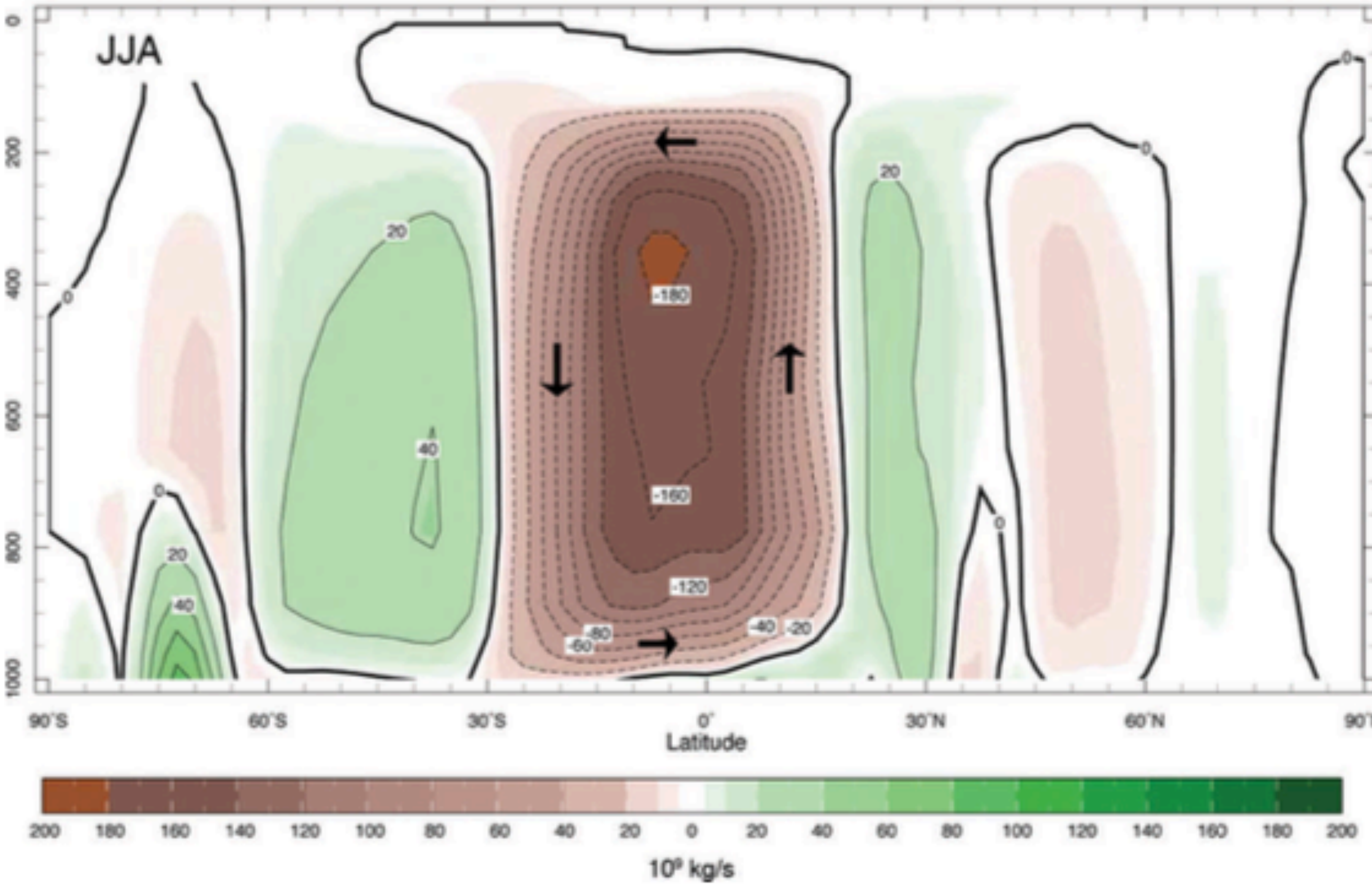
Meridional wind



Meridional wind



Meridional wind



The change of C, δC as follows

$$\delta C = \frac{\partial C}{\partial t}\delta t + \frac{\partial C}{\partial x}\delta x + \frac{\partial C}{\partial y}\delta y + \frac{\partial C}{\partial z}\delta z$$

$$\frac{DC}{Dt} = \lim_{\delta t \to 0} \frac{\delta C}{\delta t} = \frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} + w\frac{\partial C}{\partial z}$$

$$\therefore \frac{DC}{Dt} = \frac{\partial C}{\partial t} + \underbrace{\vec{U}}_{(u,v,w)} \cdot \nabla C$$

Lagrangian
이동점에서의 총 변화율

Eulerian
고정점에서의 변화율

Advection term
이류항 (장의 변화)

- At a given time, the state of the atmosphere or ocean can be defined by five key variables:

$$\mathbf{u} = (u, v, w); \;\; p \textit{ and } T$$

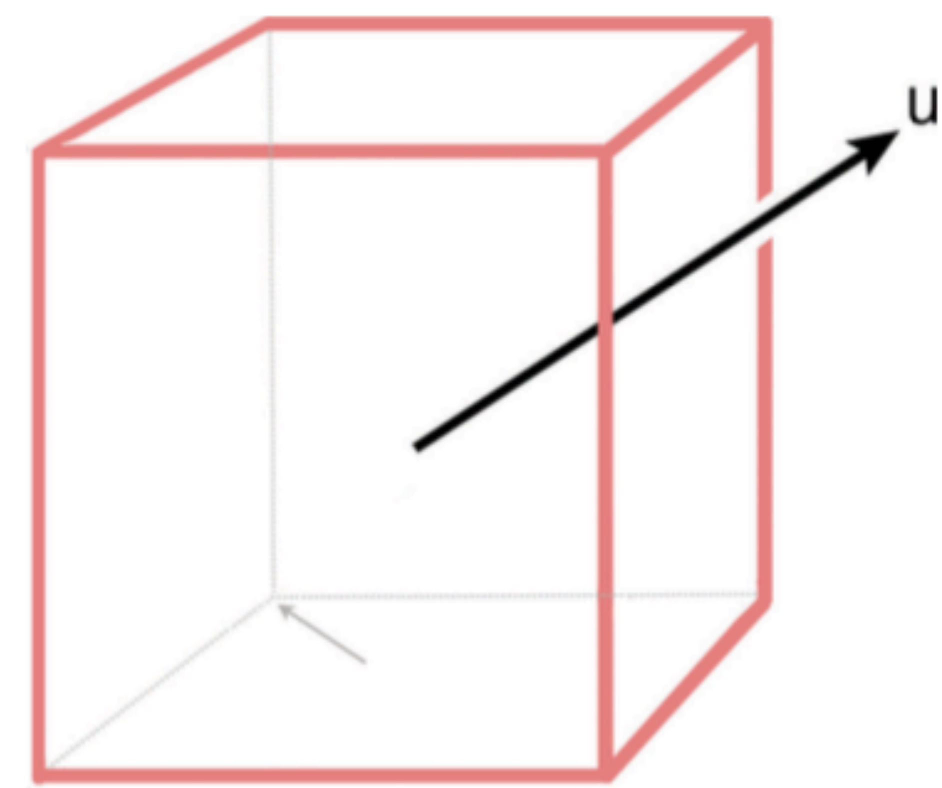
- To find out the values for these five variables, we need five independent equations

    1. Three equations that describe the laws of motion
    2. Conservation of mass
    3. The law of thermodynamics

- Forces on a fluid parcel: **The net force**
  - The mass of the parcel : $\delta M = \rho \delta x \delta y \delta z$
  - Using Newton's Law of Motion, we can write the net force, F as

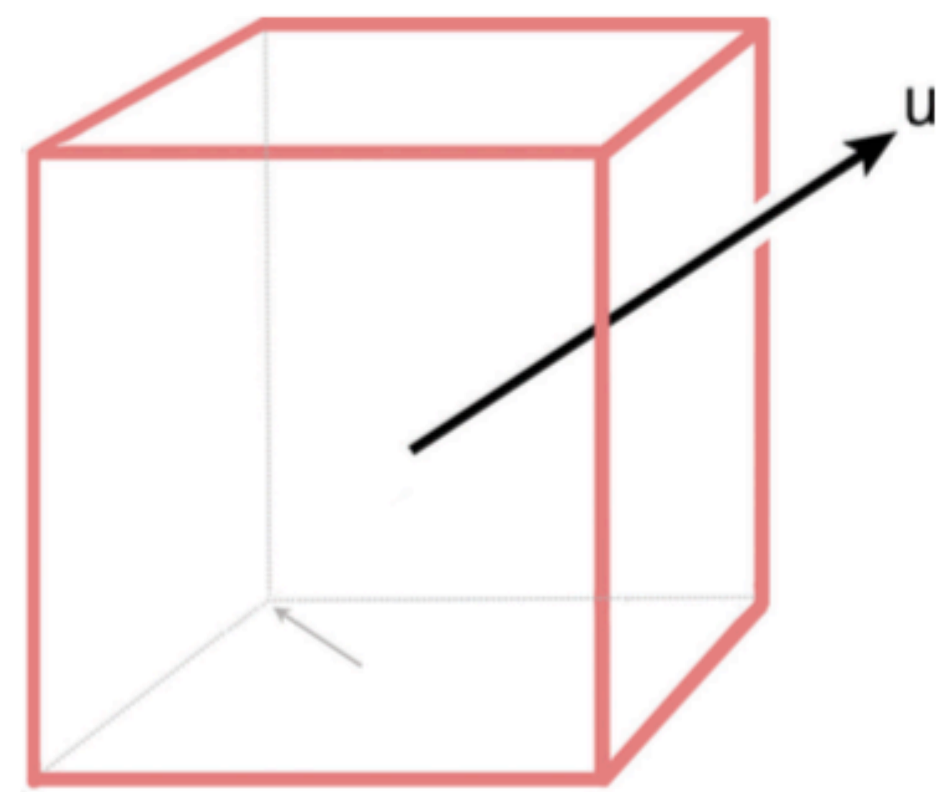
$$F = \rho \delta x \delta y \delta z \frac{D\mathbf{u}}{Dt}$$

where $\frac{D}{Dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$



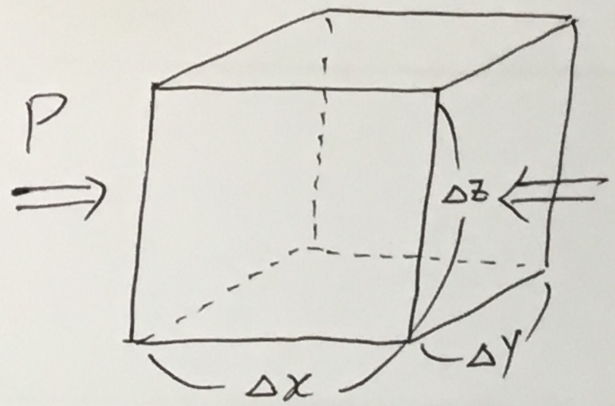
- Forces on a fluid parcel: **gravity**

$$F_{gravity} = -\, g\ \hat{\mathbf{z}}\ \rho\delta x\delta y\delta z$$



Equations of motion for a non-rotating fluid

* 기압경도력 유도



Tayler ~~ster~~ series

$$P+\Delta P = P + \frac{\partial p}{\partial x}\Delta x \left(+ \cdots \right)$$

x 방향으로 작용하는 기압경도력은

$$PGF_x = P\Delta y\Delta z - \left(P + \frac{\partial p}{\partial x}\Delta x\right)\Delta y\Delta z = -\frac{\partial p}{\partial x}\Delta x\Delta y\Delta z$$

단위질량 공기 $\rho\Delta x\Delta y\Delta z = 1$이라 두면

$$PGF_x = -\frac{1}{\rho}\frac{\partial p}{\partial x}$$

마찬가지로 y방향, z 방향으로 구하면

$$PGF_y = -\frac{1}{\rho}\frac{\partial p}{\partial y}, \quad PGF_z = -\frac{1}{\rho}\frac{\partial p}{\partial z}$$

$$\therefore PGF = -\frac{1}{\rho}\nabla P$$

- Forces on a fluid parcel: **pressure gradient**
  - The pressure gradient force in the x-direction:

$$F_x = -\frac{\partial p}{\partial x}\delta x \delta y \delta z$$

  - The pressure gradient force in the y-direction:

$$F_y = -\frac{\partial p}{\partial y}\delta x \delta y \delta z$$

  - The pressure gradient force in the z-direction:

$$F_z = -\frac{\partial p}{\partial z}\delta x \delta y \delta z$$

- Forces on a fluid parcel: **friction**
    - For typical atmospheric and oceanic flows, frictional effects are negligible except close to boundaries.
    - Conceptually, the frictional force on a fluid parcel can be expressed as

$$\mathbf{F}_{fric} = \rho \mathscr{F} \delta x \delta y \delta z$$

where $\mathscr{F}$ is the frictional force per unit mass.

- **The equation of motion**
  - Net force = sum of all forces

$$\rho\ \delta x\ \delta y\ \delta z\ \frac{D\mathbf{u}}{Dt} = \mathbf{F}_{gravity} + \mathbf{F}_{pressure} + \mathbf{F}_{fric}$$

$$\frac{D\mathbf{u}}{Dt} = -g\hat{\mathbf{z}} - \frac{1}{\rho}\nabla p + \mathscr{F}$$

- **The equation of motion**
  - Recall the hydrostatic balance:

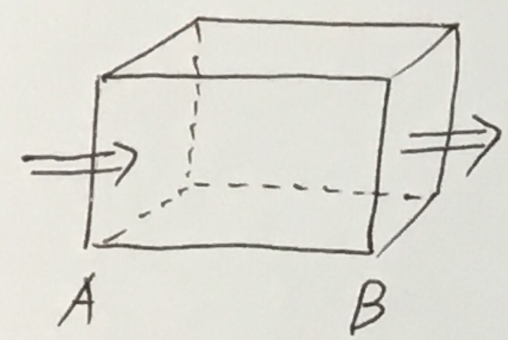
$$\frac{\partial p}{\partial z} + \rho g = 0$$

  - If friction and vertical acceleration (Dw/Dt) are negligible, the equation of motion in the z-direction becomes the equation of hydrostatic balance.

$$\cancel{\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y}} + w\frac{\partial w}{\partial z} + \frac{1}{\rho}\frac{\partial p}{\partial z} + g = \cancel{\mathcal{F}_w}$$

* Conservation of mass

① by Eulerian method



$$\frac{\partial}{\partial t}(\delta M) = (\rho U)_A \delta A - (\rho U)_B \delta A$$

$$\frac{\partial}{\partial t}(\rho \cdot \delta V) = \left[(\rho U)_A - \left[(\rho U)_A + \frac{\partial}{\partial x}(\rho U)\cdot \delta x \right] \right] \delta A$$

$$\delta V \frac{\partial \rho}{\partial t} = -\frac{\partial}{\partial x}(\rho U)\cdot \delta V$$

$$\Rightarrow \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{U}) = 0 \qquad \vec{U} = (u, v, w)$$

$$\frac{\partial \rho}{\partial t} + \vec{U} \cdot \nabla \times \rho + \rho \nabla \cdot \vec{U} = 0$$

$$\frac{d\rho}{dt} + \rho \nabla \cdot \vec{U} = 0$$

Equations of motion for a non-rotating fluid

* Conservation of mass

② by Lagrangian method

$$\delta M = \rho \cdot \delta V = \text{Const} \qquad \therefore \frac{D}{Dt}(\delta M) = 0$$

$$\frac{1}{\delta M}\frac{d}{dt}(\delta M) = \frac{1}{\rho \delta x \delta y \delta z}\frac{d}{dt}(\rho \delta x \delta y \delta z)$$

$$= \frac{1}{\rho}\frac{d\rho}{dt} + \frac{1}{\delta x}\frac{d}{dt}(\delta x) + \frac{1}{\delta y}\frac{d}{dt}(\delta y) + \frac{1}{\delta z}\frac{d}{dt}(\delta z)$$

$$= \frac{1}{\rho}\frac{d\rho}{dt} + \frac{\delta u}{\delta x} + \frac{\delta v}{\delta y} + \frac{\delta w}{\delta z}$$

$$\frac{1}{\rho}\frac{d\rho}{dt} + \nabla \cdot \vec{U} = 0$$

- **Conservation of mass :**
  - For compressible flow, the hydrostatic assumption allows us to write the unit volume as $\delta x\ \delta y\ \delta p$
  - Then the mass of the fluid parcel becomes

$$\delta M = \rho\ \delta x\ \delta y\ \delta z\ = -\frac{1}{g}\delta x\ \delta y\ \delta p$$

  - The mass is conserved in pressure coordinates, and

$$\nabla_p \cdot \mathbf{u}_p = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial z} = 0$$

Equations of motion for a non-rotating fluid

Q. $\nabla \cdot \vec{A}$ 의 의미?

$$\nabla \cdot \vec{A}$$

$$\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \quad (A_x, A_y, A_z)$$

$$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} < 0$$ 수렴

바람이 주변에서 불어오면 Mass ⇑

$$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} > 0$$ 발산

바람이 주변으로 불어나가면 Mass ⇓

$$\boxed{\frac{d\rho}{dt}} = -\rho \nabla \cdot \vec{U}$$

좌측항을 외력으로 보지않고
질량보존으로 살펴봄

* $\nabla \rho \Rightarrow$ 가능 (o)

$\nabla \cdot \rho \Rightarrow$ 불가능 (x)

(ρ는 스칼라이기 때문.)

- **Thermodynamic equation**
  - The first law of thermodynamics we dealt with for the dry adiabatic lapse rate is $\delta Q = c_p dT - \frac{dp}{\rho}$
  - If we consider the first law of thermodynamics applied to a moving parcel of fluid,

$$\frac{DQ}{Dt} = c_p \frac{DT}{Dt} - \frac{1}{\rho}\frac{Dp}{Dt}$$

Diabatic heating rate (e.g. latent heating or cooling)

Temperature changes from the heat and/or expansion or compression

- At a given time, the state of the atmosphere or ocean can be defined by five key variables:

$$\mathbf{u} = (u, v, w);\ \ p\ and\ T$$

- To find out the values for these five variables, we need five independent equations

  1. Three equations that describe the laws of motion
  2. Conservation of mass
  3. The law of thermodynamics

Thank You

- End of the Document -