Vertical structure of the atmosphere

ATM2106

Last time

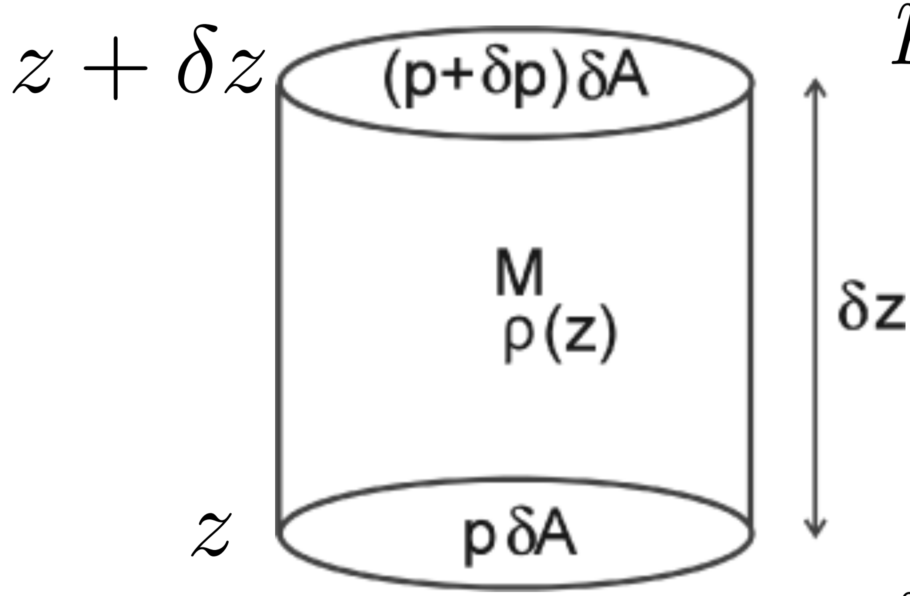
- Climate feedbacks
- Variability

Today's topic

- Hydrostatic balance
- Vertical structure of pressure and density

1. Hydrostatic Balance

- If the atmosphere were at rest, pressure at any level would depend on the weight of the fluid above that level.
- This is called **hydrostatic balance.**
- Pressure and density are functions of height z.



$$
\begin{aligned}
p_T &= p(z + \delta z) \\
&= p(z) + \delta p \\
&= p(z) + \frac{\partial p}{\partial z}\delta z
\end{aligned}
$$

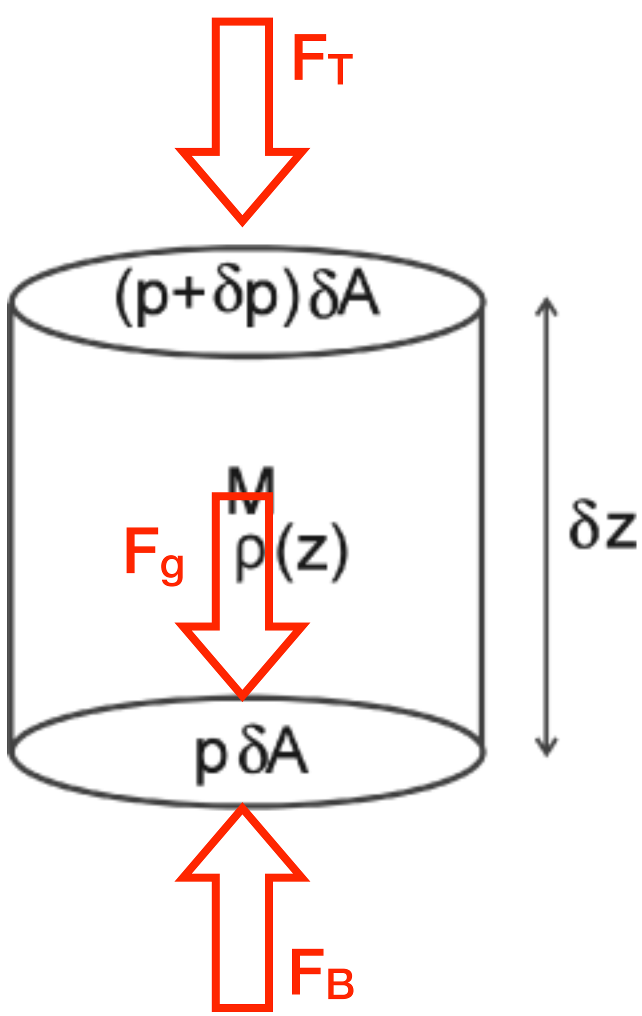
$$p_B = p(z)$$

1. Hydrostatic Balance

- Now, the mass of the cylinder is

$$M = \rho \delta A \delta z$$

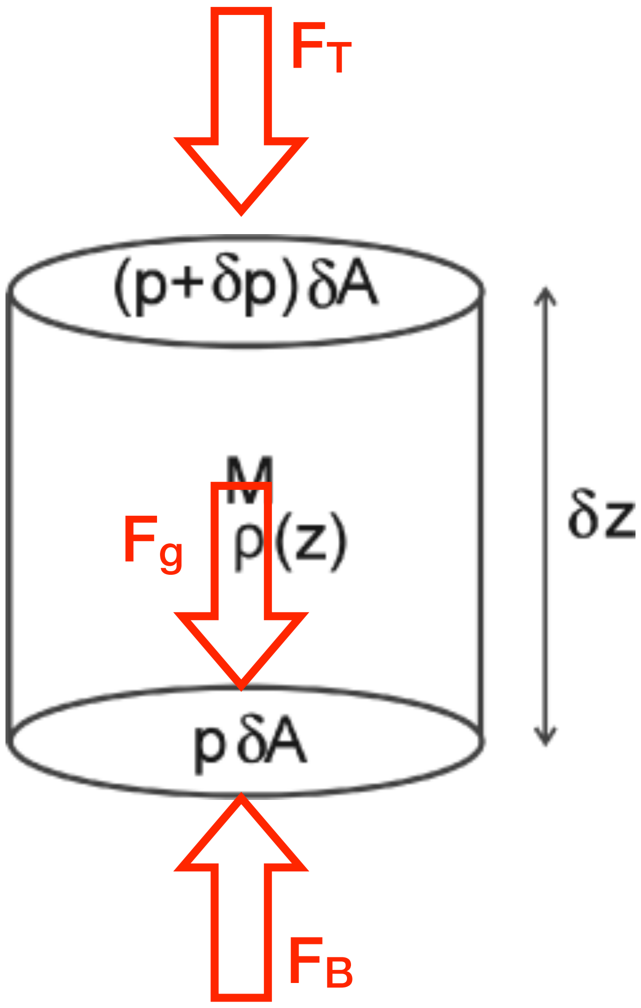
- If this cylinder is not accelerating, the net force should be zero!
    - Gravitational force (F_g)
    - Pressure force at the top (F_T)
    - Pressure force at the bottom (F_B)



1. Hydrostatic Balance

- $F_g = -gM = -g\rho\delta A\delta z$
- $F_T = -(p + \delta p)\delta A$
- $F_B = p\delta A$
- $F_g + F_T + F_B = \delta p + g\rho\delta z = 0$
- The equation of hydrostatic balance:

$$\frac{\partial p}{\partial z} + g\rho = 0$$



1. Hydrostatic Balance

- Since p must vanish as z goes infinity,

$$p(z) = g \int_z^{\infty} \rho dz$$

- This simply means that the pressure is the mass per unit area of atmospheric column above z times g.
- Keep in mind that hydrostatic balance works well when the net force is (close to) zero.
- To actually compute p(z), we need to know ρ(z).

2. Vertical structure of pressure and density

- $$\frac{\partial p}{\partial z} + g\rho = 0$$

$$p = \rho R T$$

$$\frac{\partial p}{\partial z} = -\frac{gp}{RT}$$

$$\frac{\partial p}{\partial z} = -\frac{gp}{RT} = -p\frac{g}{RT} = -\frac{p}{H}$$

$$H = \frac{RT}{g}$$

- R = 287 J/kg/K
- 1 J = 1 kg m^2 s^{-2}

2. Vertical structure of pressure and density

- When we assume T is constant with height ($T=T_0$), and $p=p_s$ at $z=0$,

$$p(z) = p_s \exp\left(-\frac{z}{H}\right)$$

- Pressure decreases exponentially with height.
- And the density becomes

$$\rho(z) = \frac{p_s}{RT_0} \exp\left(-\frac{z}{H}\right)$$

- Density also decreases exponentially with height.

2. Vertical structure of pressure and density

- When we assume T is NOT constant with height (T=T(z)), and p=p_s at z=0,

$$H(z) = \frac{RT(z)}{g}$$

$$\frac{\partial p}{\partial z} = -\frac{p}{H(z)} \longrightarrow \frac{1}{p}\frac{\partial p}{\partial z} = \frac{\partial \ln p}{\partial z} = -\frac{1}{H(z)}$$

$$p(z) = p_s \exp\left(-\int_0^z \frac{dz'}{H(z')}\right)$$

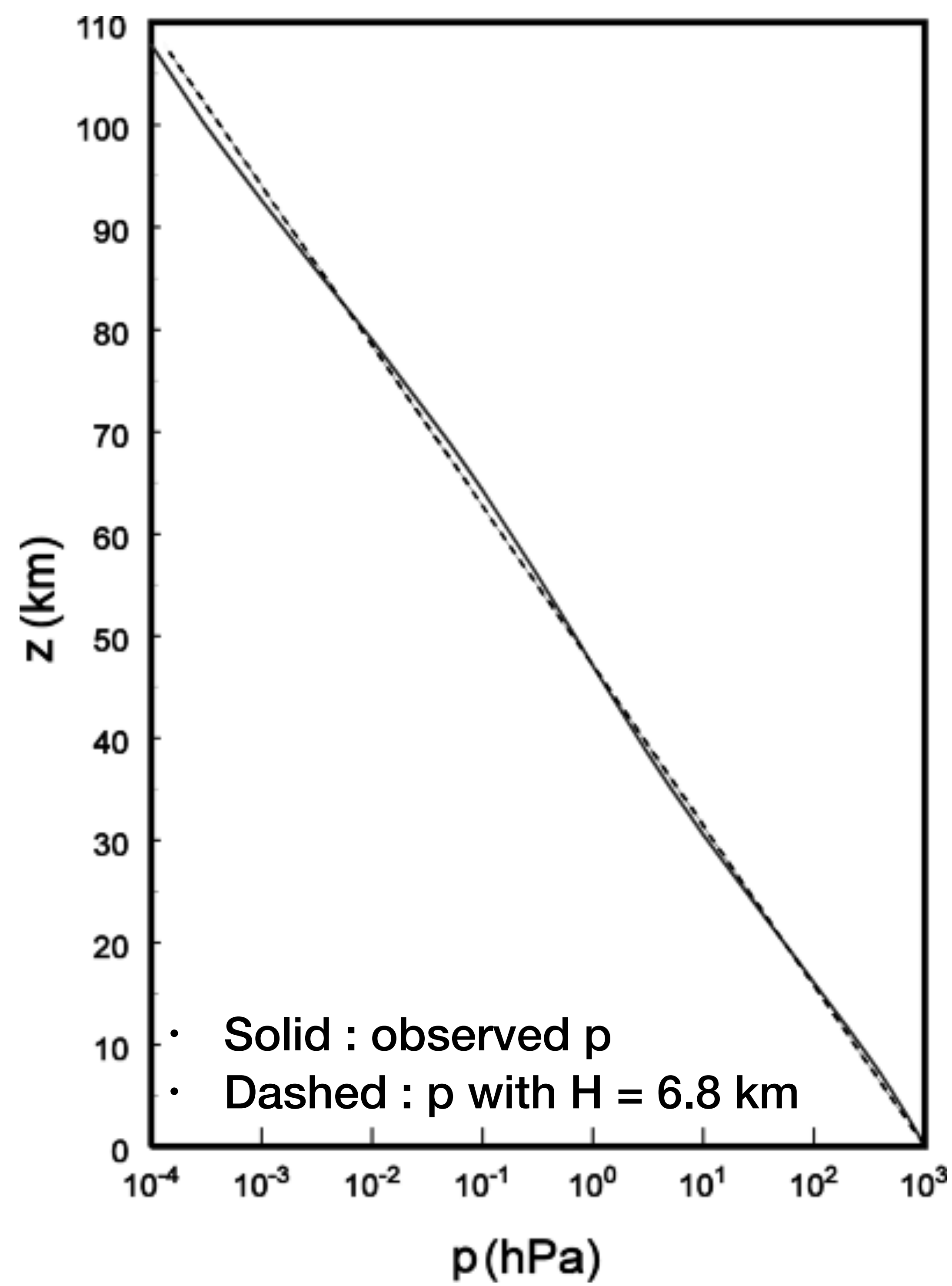
$$\rho(z) = \frac{p_s}{RT(z)} \exp\left(-\int_0^z \frac{dz'}{H(z')}\right)$$

2. Vertical structure of pressure and density

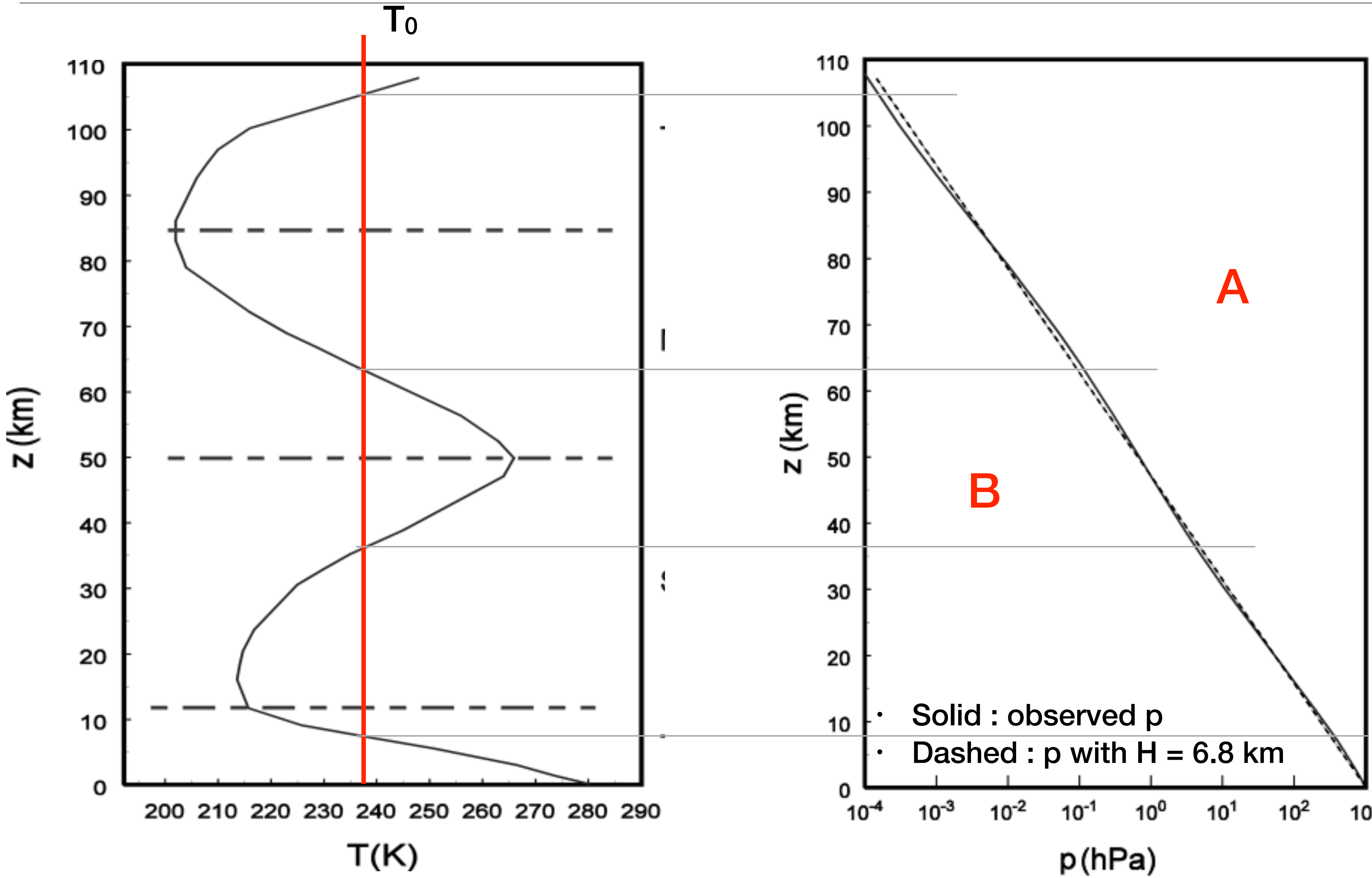
- $T = T_0$ can be a good approximation
- $T_0 = gH_0/R = 237.08$ K with $H_0 = 6.8$ km
- What determines the rate of p decrease?

$$p(z) = p_s \exp\left(-\frac{z}{H}\right)$$

- The greater H is (or the warmer T_0 is), the slower the decrease of p.



2. Vertical structure of pressure and density



Convection

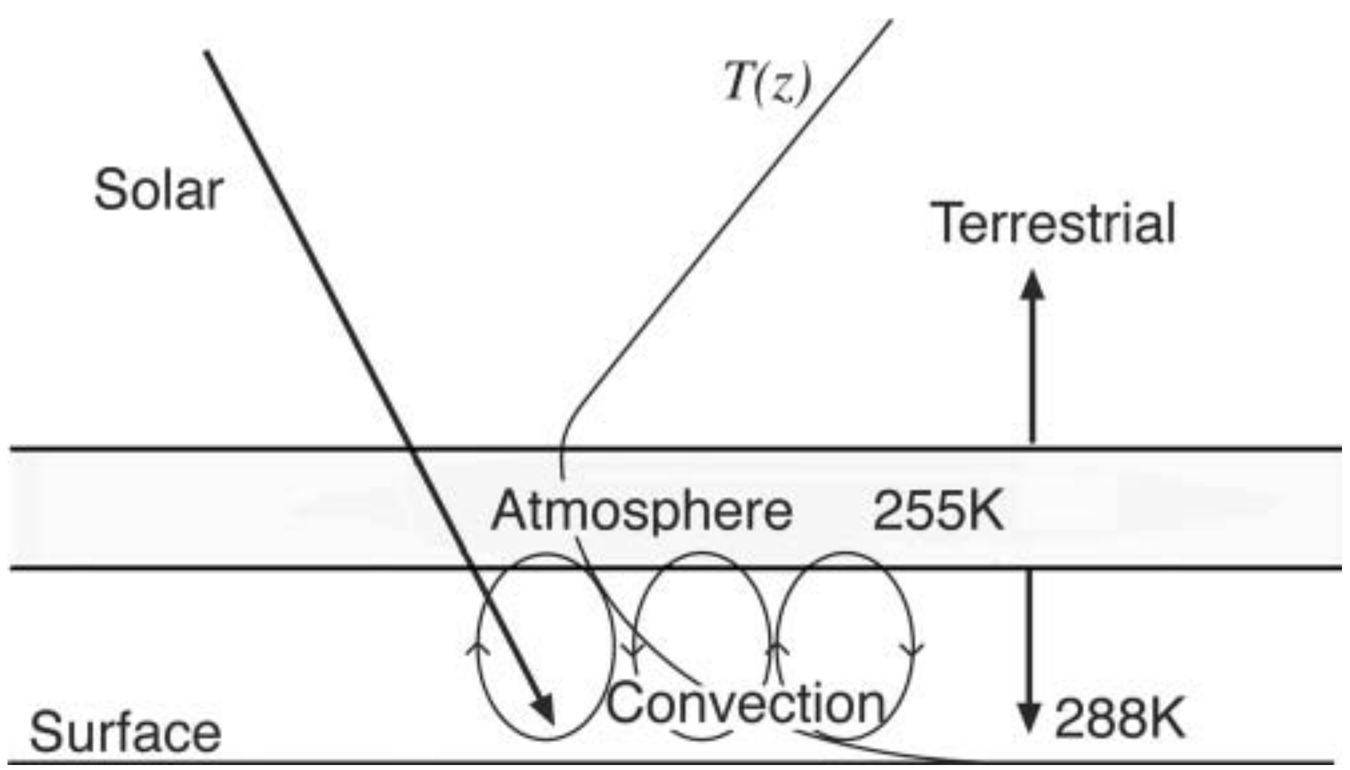
ATM2106

Today’s topic

- Convection
- Buoyancy and stability
- Convection in the ocean
- Dry convection

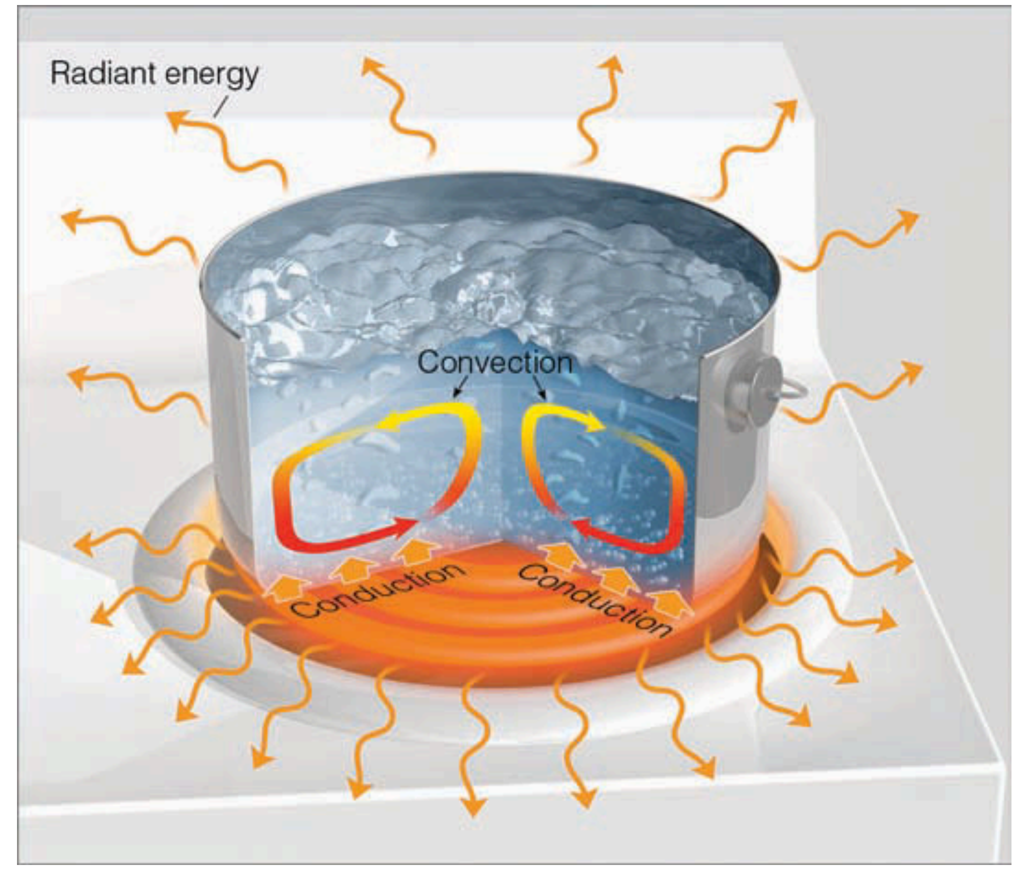
The nature of convection

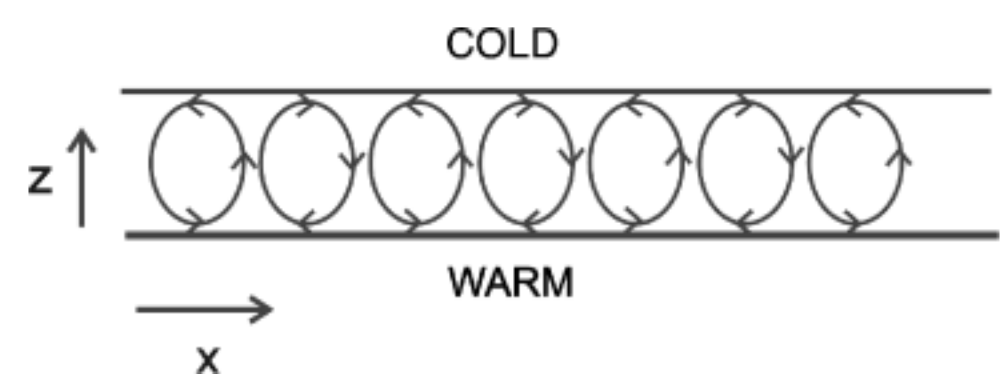
- In radiative equilibrium, the surface is water than the overlying atmosphere.
- This is unstable and we see convection occurs.
- What is convection?



Convection in a shallow flood

- In atmosphere and ocean: convection refers to motions that are driven by density differences in presence of gravity.
- The motions driven by convection are horizontally inhomogeneous even with uniform heating.
- Convection is not directly forced motion but from **instability**.





Instability

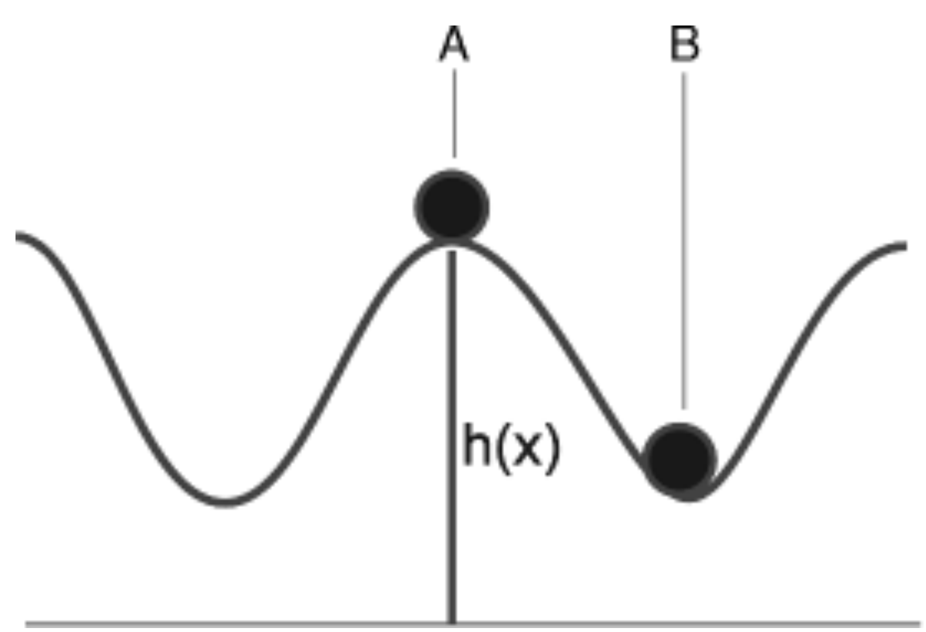
- Instability arises if, in response to a perturbation, the system tends to drive the perturbation further from the equilibrium state.

A: Kinetic energy will increase in exchange for a loss of potential energy

└→ Unstable

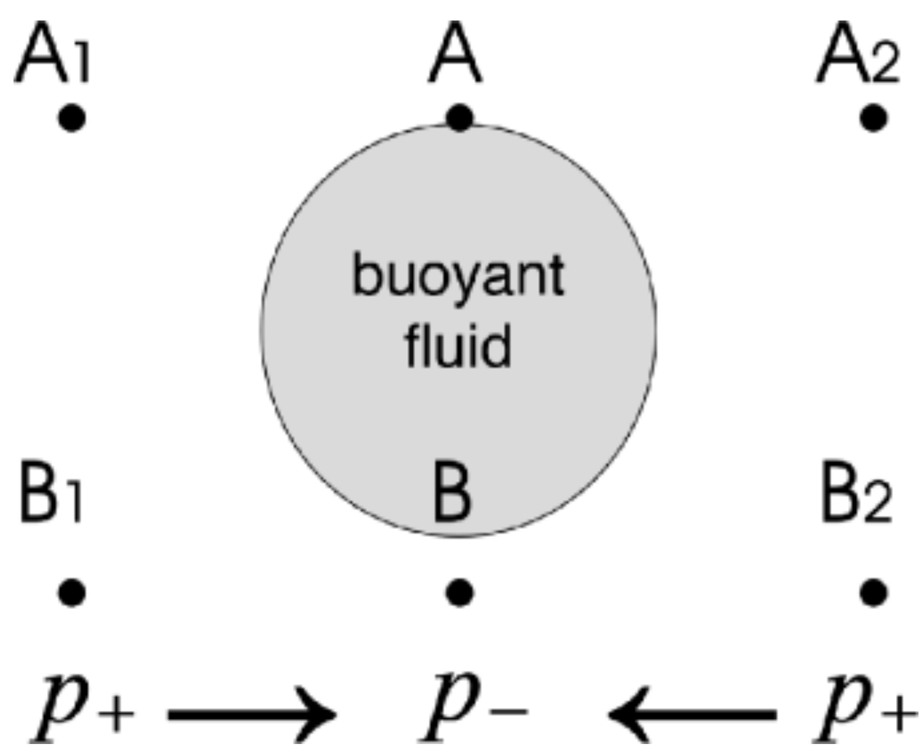
B: Needs external energy to move

└→ Stable



Convection in water: 1. Buoyancy

- Let's suppose that density of the fluid (water in this case) depends only on temperature.



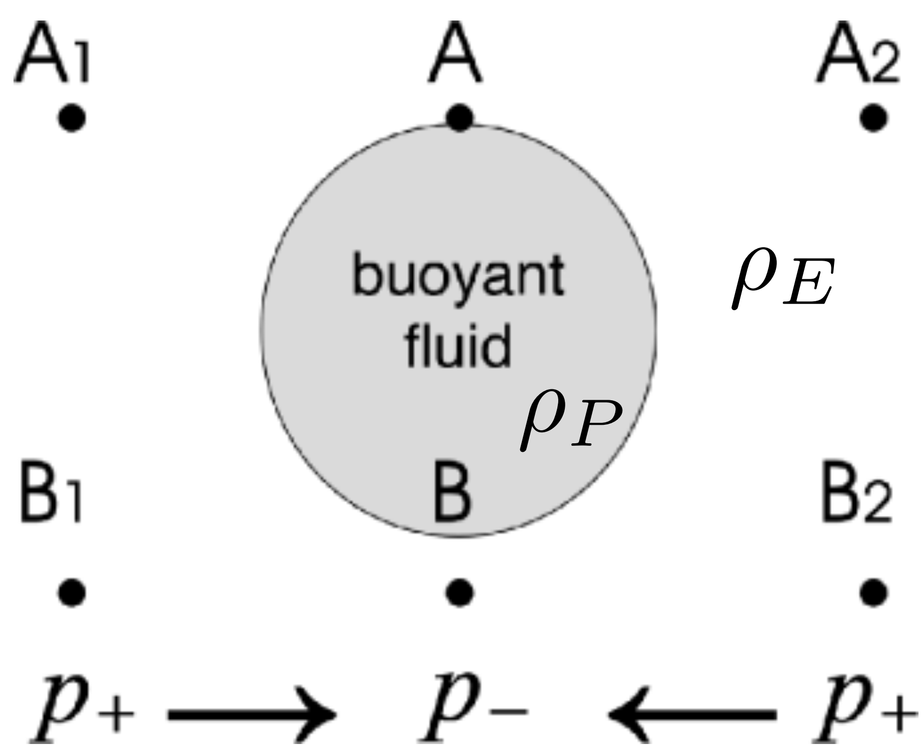
$P_{A1} = P_A = P_{A2}$

$P_{B1} = P_{B2} > P_B$

$P_B > P_A \longrightarrow$ **The light fluid will rise.**

Convection in water: 1. Buoyancy

- The acceleration of the parcel of fluid is



$$-g\frac{(\rho_P - \rho_E)}{\rho_P} \to b$$

Buoyancy

- **Positively buoyant : $\rho_P < \rho_E$**
- **Negatively buoyant : $\rho_P > \rho_E$**
- **Neutrally buoyant : $\rho_P = \rho_E$**

Convection in water: 2. Stability

- Let's consider the incompressible fluid parcel again.
- Suppose $\rho = \rho_{ref} \left(1 - \alpha[T - T_{ref}]\right)$
- Also, assume that we move the parcel fast enough that there is no time for the heat exchange between the parcel and the environment → adiabatic
- Since the fluid is incompressible, T is conserved without heating or cooling.

Convection in water: 2. Stability

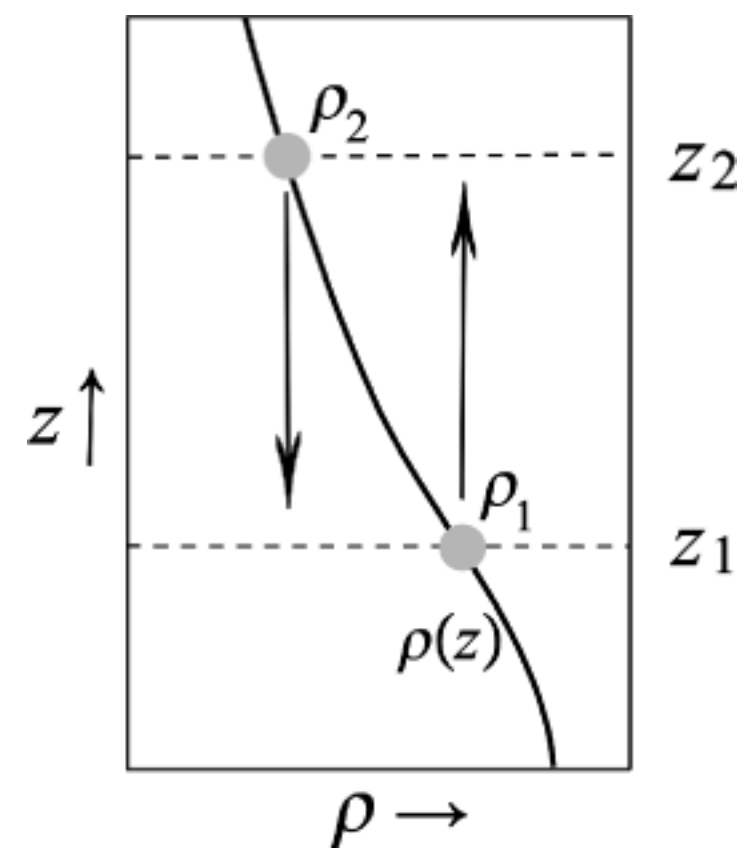
- Let's move the partial at $z=z_1$ to z_2.
- Its density is still $\boldsymbol{\rho}_1$.
- The density of the environment is

$$\rho_E = \rho(z_2) = \rho_1 + \left(\frac{d\rho}{dz}\right)_E \delta z$$

- Then the buoyancy, *b*, becomes

$$b = \frac{g}{\rho_1}\left(\frac{d\rho}{dz}\right)_E \delta z$$

- *b* depends on $(d\rho/dz)$!

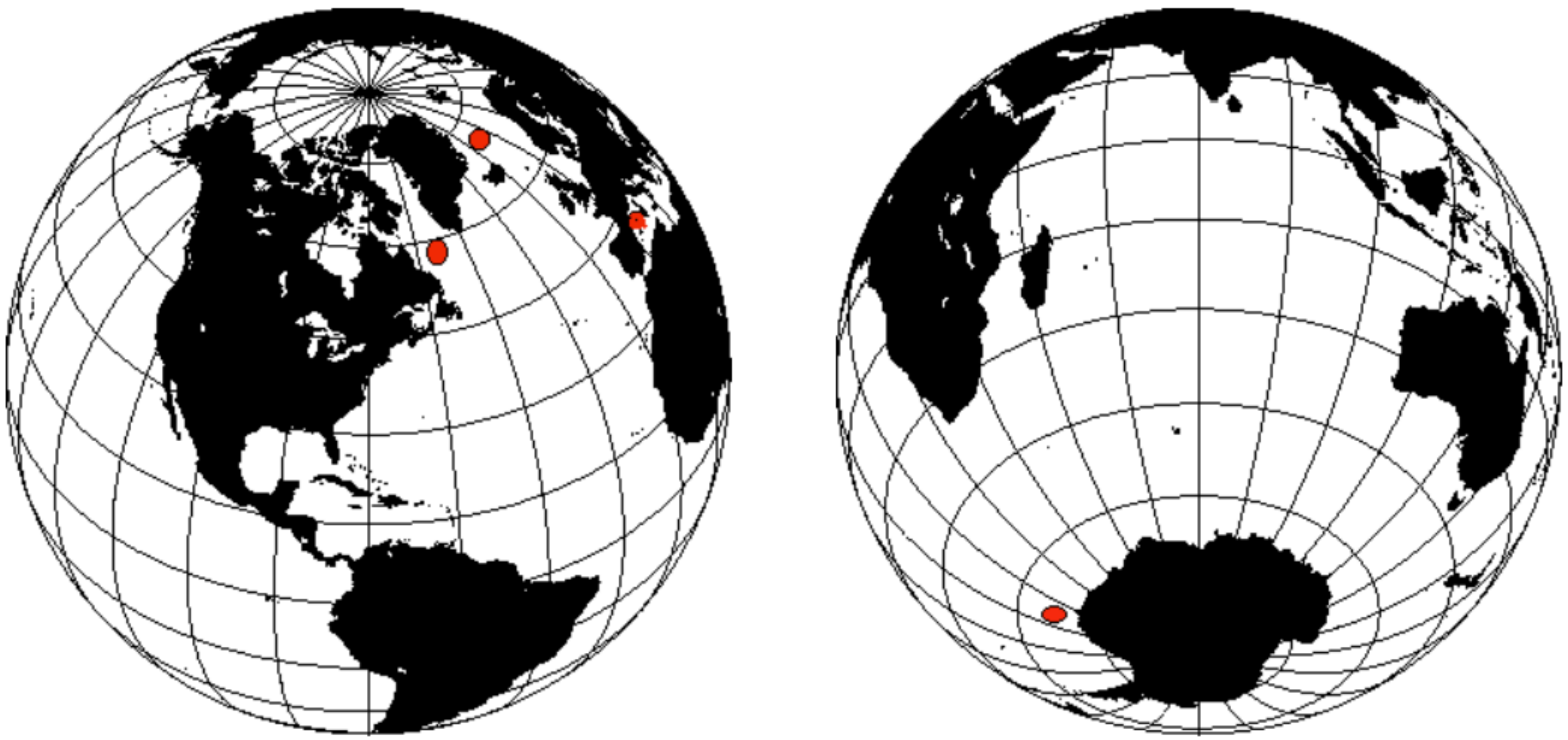


Convection in water: 2. Stability

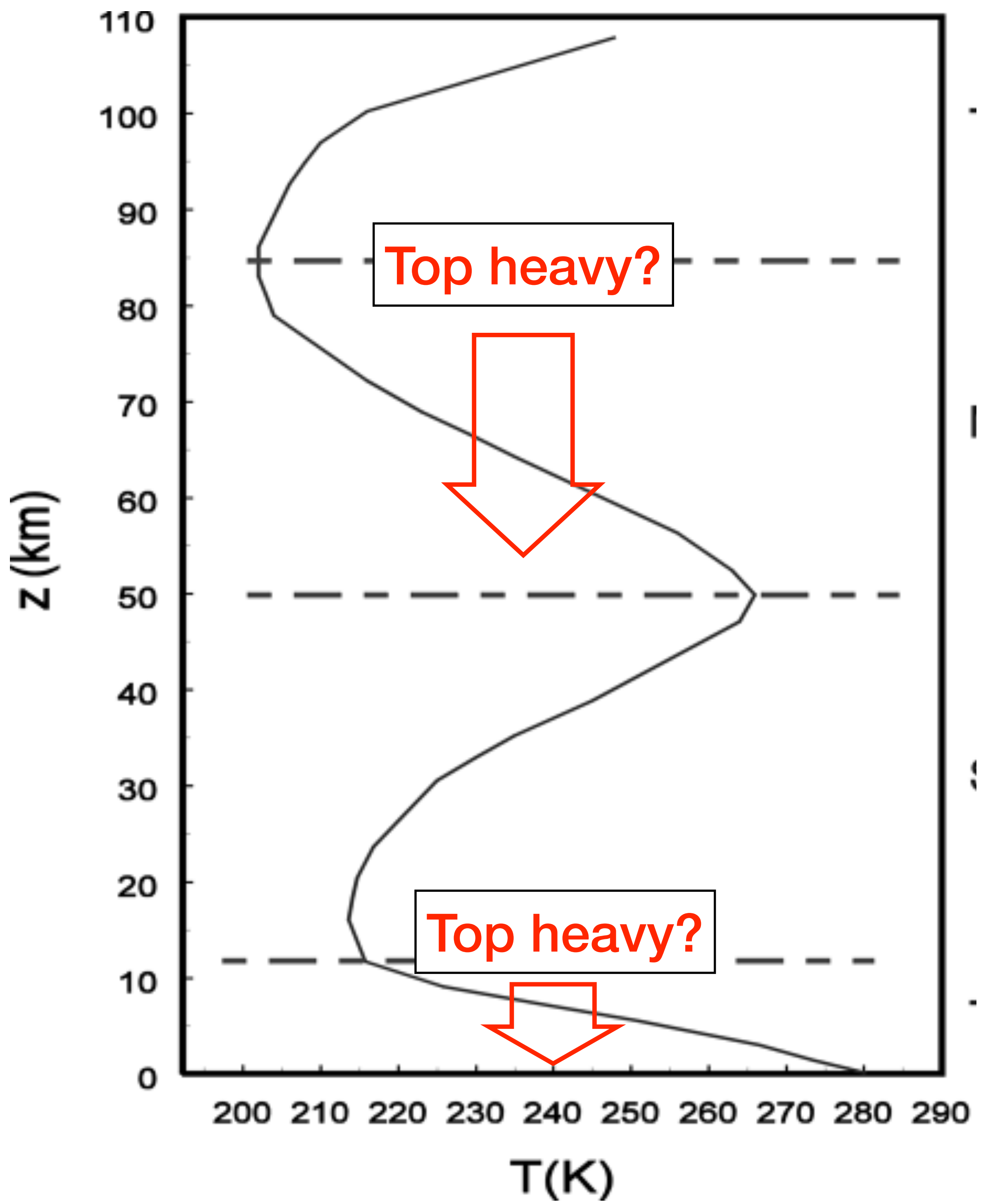
- The stability
  - Positively buoyant if $(d\rho/dz)_E > 0$ → unstable
  - Negatively buoyant if $(d\rho/dz)_E < 0$ → stable
  - Neutrally buoyant if $(d\rho/dz)_E = 0$ → neutral
- Since we know that ρ is inversely proportional to T,
  - Unstable if $(dT/dz)_E < 0$
  - Stable if $(dT/dz)_E > 0$
  - Neutral if $(dT/dz)_E = 0$

Convection in the ocean

- Convective motions develop as a result of surface cooling
- Convection develops
  - Even night in the upper 10-100 m of the oceans
  - In winter at high latitudes thought the whole water column



Dry convection in a compressible atmosphere



Dry convection in a compressible atmosphere

- The atmosphere is a compressible fluid

$$\rho = \rho(p, T)$$

- For example, from the perfect gas law, $\rho = p/RT$
- Then, from the first law of thermodynamics,

$$\delta Q = \delta U + \delta W$$

ΔHeat = Δ(Internal energy) + Δ(External work done)

- In adiabatic process, $\delta U + \delta W = 0$

Dry convection in a compressible atmosphere

- In the textbook, you can find the deviation of the temperature change with height.

$$\frac{dT}{dz} = -\frac{g}{c_p} = \Gamma_d$$

- c_p is specific heat at constant pressure and 1005 J/kg/K
- Then we find

$$\Gamma_d \approx 10 \text{ K km}^{-1}$$

- This is known as the dry adiabatic lapse rate.

Dry convection in a compressible atmosphere

- Unstable if $(dT/dz)_E < -\mathbf{\Gamma}_d$
- stable if $(dT/dz)_E > -\mathbf{\Gamma}_d$
- Neutral if $(dT/dz)_E = -\mathbf{\Gamma}_d$