# Stability

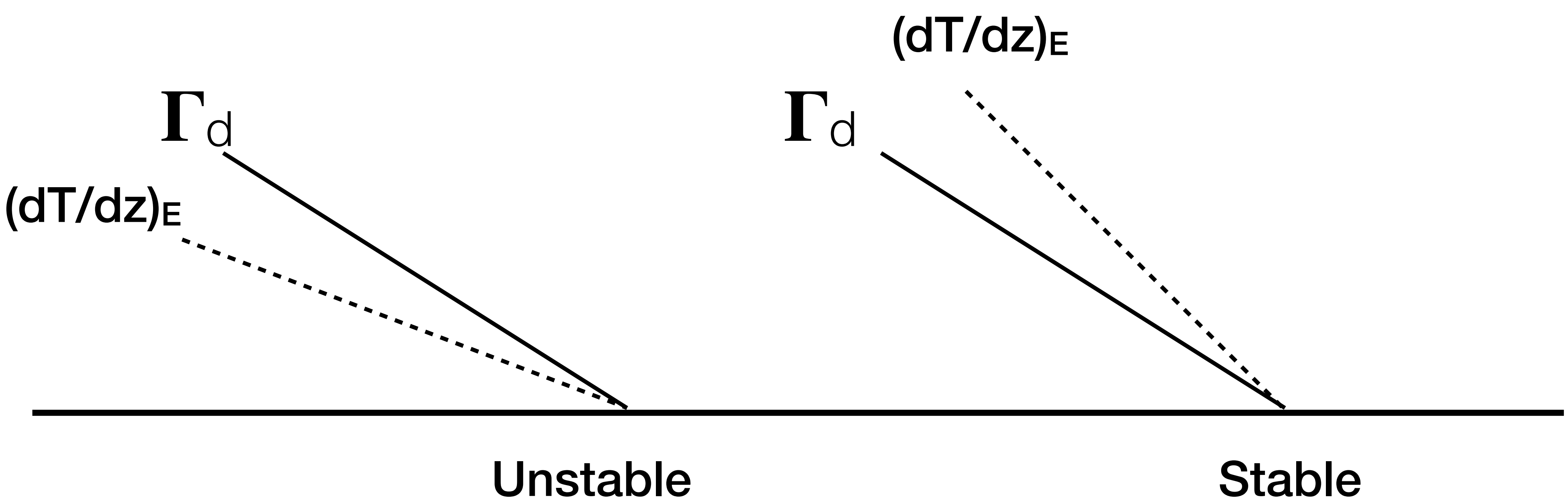
ATM2106

# Last time

- Dry adiabatic lapse rate

$$\frac{dT}{dz} = -\frac{g}{c_p} = \boxed{-}\Gamma_d$$

- We find $\Gamma_d \approx 10 \text{ K km}^{-1}$

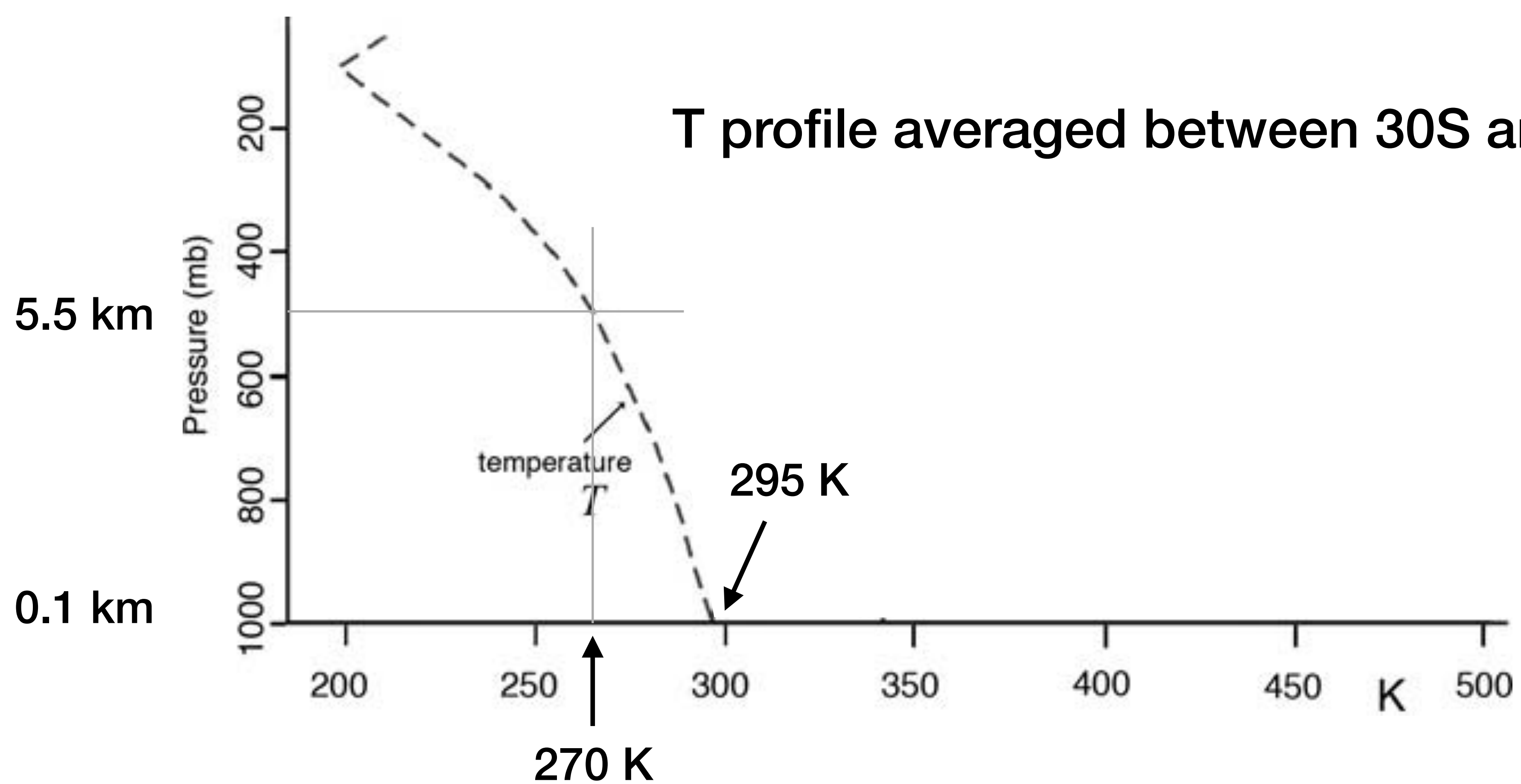


# What is the lower troposphere lapse rate?

$$\left(\frac{dT}{dz}\right)_E \approx \frac{T(500\ \mathrm{mbar}) - T(1000\ \mathrm{mbar})}{Z(500\ \mathrm{mbar}) - Z(1000\ \mathrm{mbar})}$$

**$\approx$ -4.63 K km$^{-1}$**

T profile averaged between 30S and 30N



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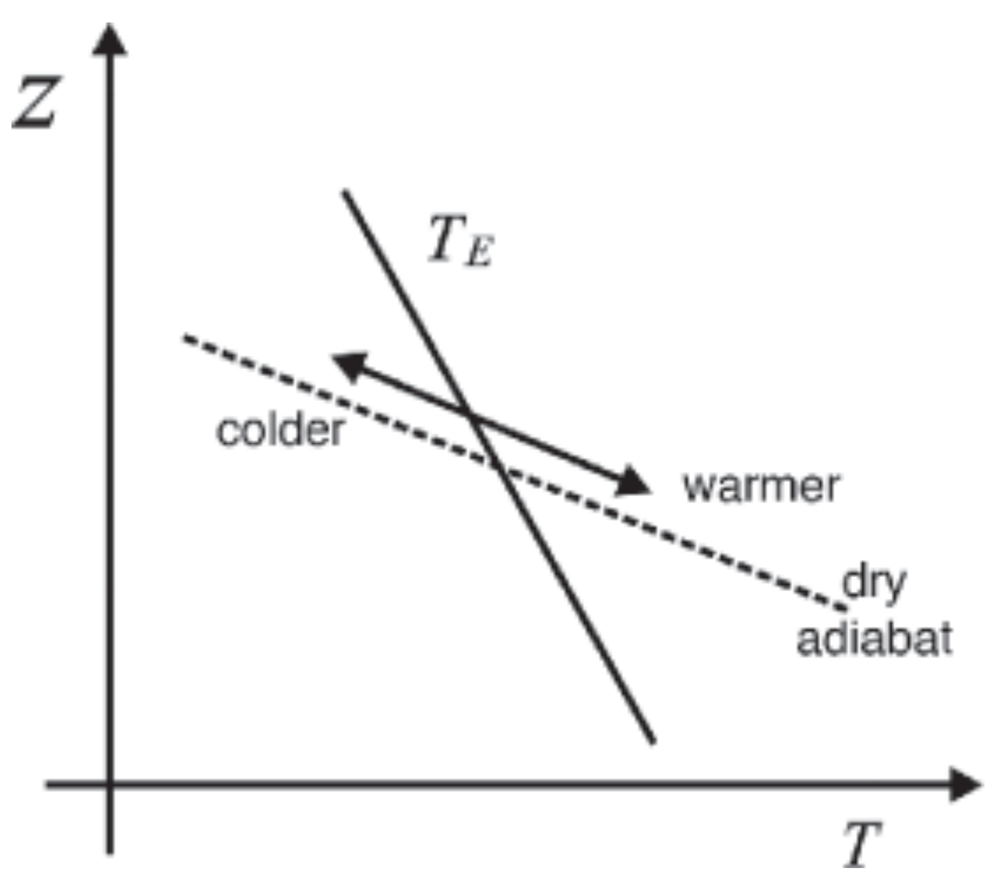



- $\left(\frac{dT}{dz}\right)_E \approx 0.5 \times \Gamma_d$
- Stable and no (dry) convection
- Why do we see gigantic convection systems?

# Today's topic

- Potential temperature
- Moist convection

# 1. Potential temperature

- Let's go back to the dry adiabatic lapse rate

$$\frac{dT}{dz} = -\frac{g}{c_p} = -\Gamma_d$$

- Using hydrostatic balance, we can rewrite this as

$$c_p dT = -g dz = \frac{1}{\rho} dp$$

- Then, using the perfect gas law, this equation becomes

$$\frac{dT}{T} = \frac{R}{c_p}\frac{dp}{p} = \kappa\frac{dp}{p}$$

# 1. Potential temperature

- Further, this PDE can be arranged as

$$d \ln T = \kappa d \ln p$$

- And we get this relationship: $$\frac{T}{p^\kappa} = const.$$

- It means that T has to go down as p decreases, or vice versa.

- If we integrate the first equation from $p=p_0$ to $p=p$,

$$T(p_0) = T(p)\left(\frac{p_0}{p}\right)^\kappa$$

# 1. Potential temperature

- Let's replace $T(p_0)$ with $\theta$.

$$\theta = T(p)\left(\frac{p_0}{p}\right)^{\kappa}$$

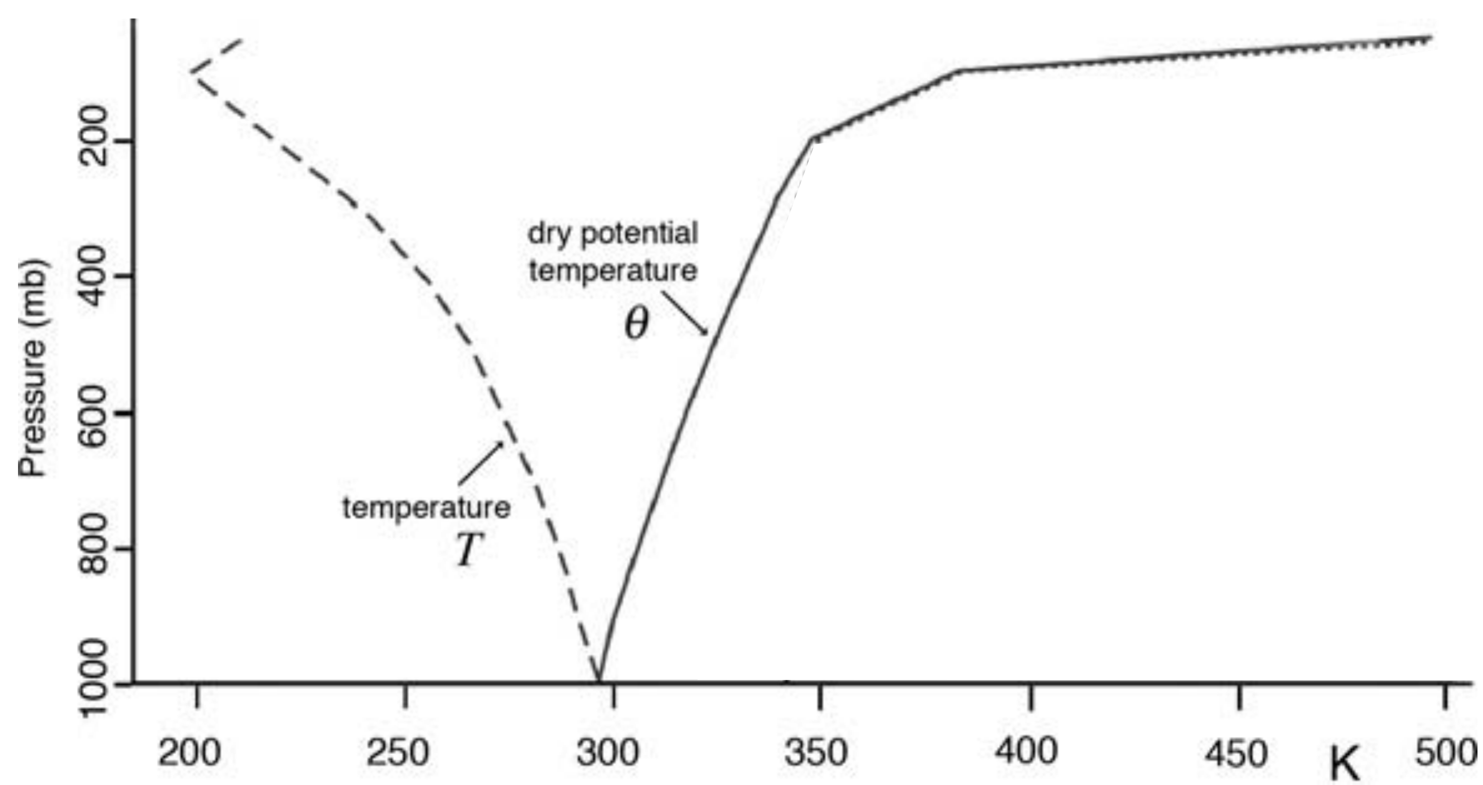
- $\theta$ is called as potential temperature, and it represents the temperature at $p=p_0$. (conventionally, $p_0$ is 1000 mb.)
- We introduced potential temperature to get a quantity that does not rely on height (or p), but there is p in that equation. So we failed?

# 1. Potential temperature

- If $\theta$ does not depend on p, then d$\theta$/dp should be zero.

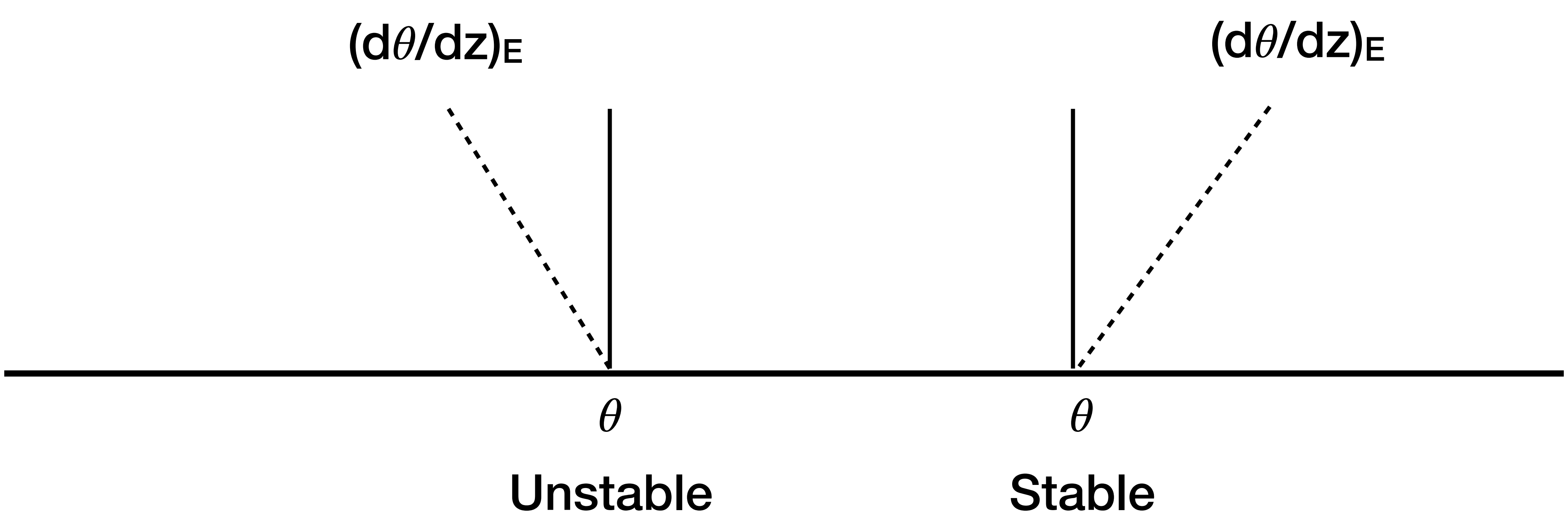
$$\frac{d\theta}{dp} = \frac{dT}{dp}\left(\frac{p_0}{p}\right)^{\kappa} - \kappa\frac{T}{p}\left(\frac{p_0}{p}\right)^{\kappa} = 0$$

- T and $\theta$ have to converge at p=1000 mb.



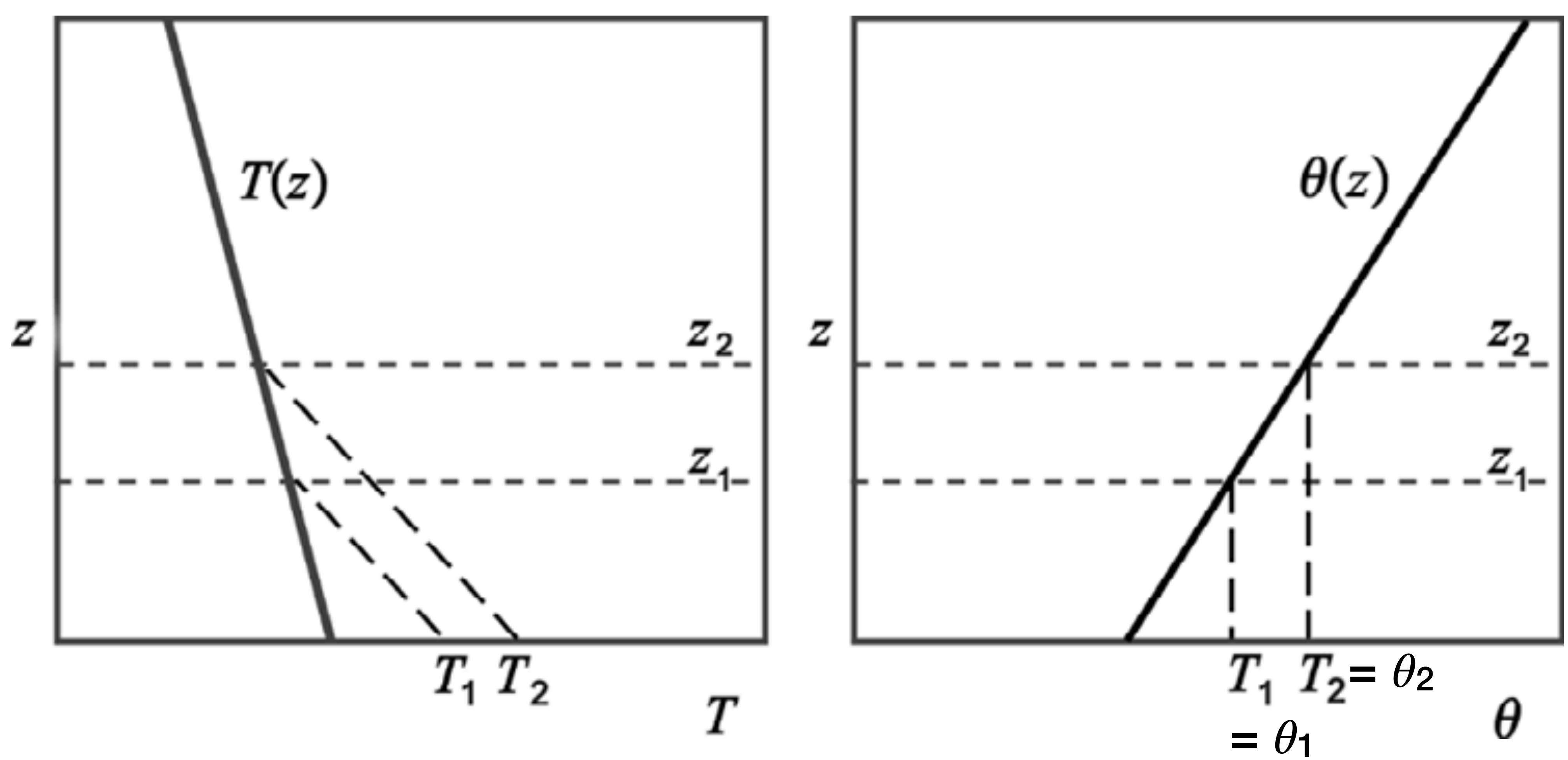
# 2. Potential temperature and stability

- Stability using potential temperature, $\theta$
  - Unstable if $(d\theta/dz)_E < 0$
  - Neutral if $(d\theta/dz)_E = 0$
  - Stable if $(d\theta/dz)_E > 0$



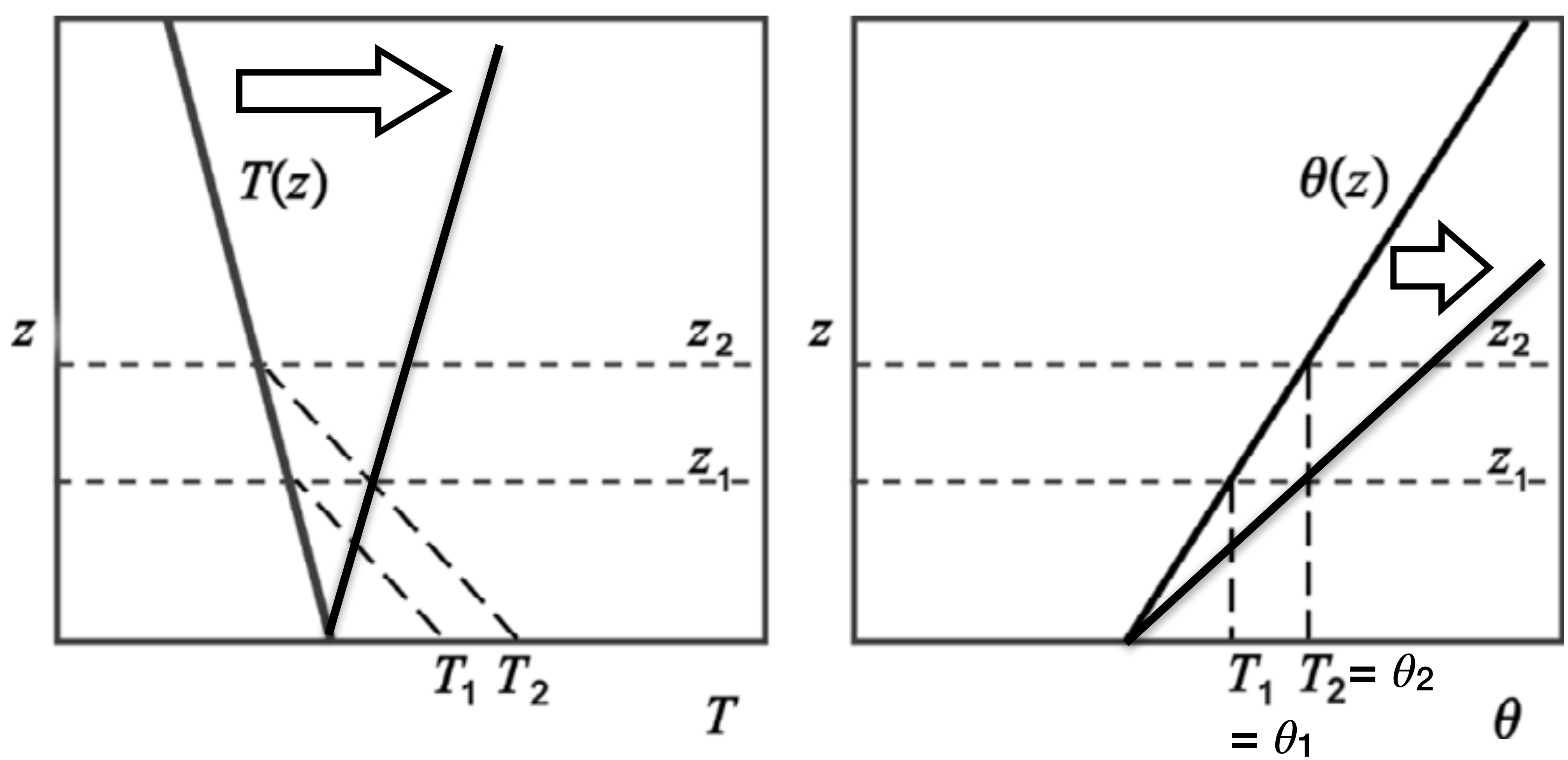
# 2. Potential temperature and stability

- Height of dry convection



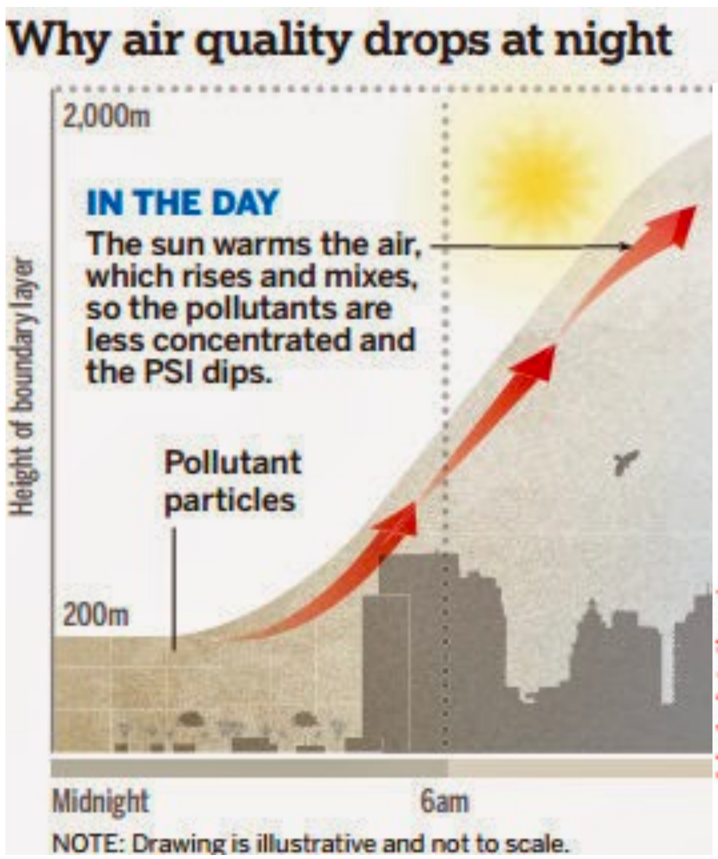
# 2. Potential temperature and stability

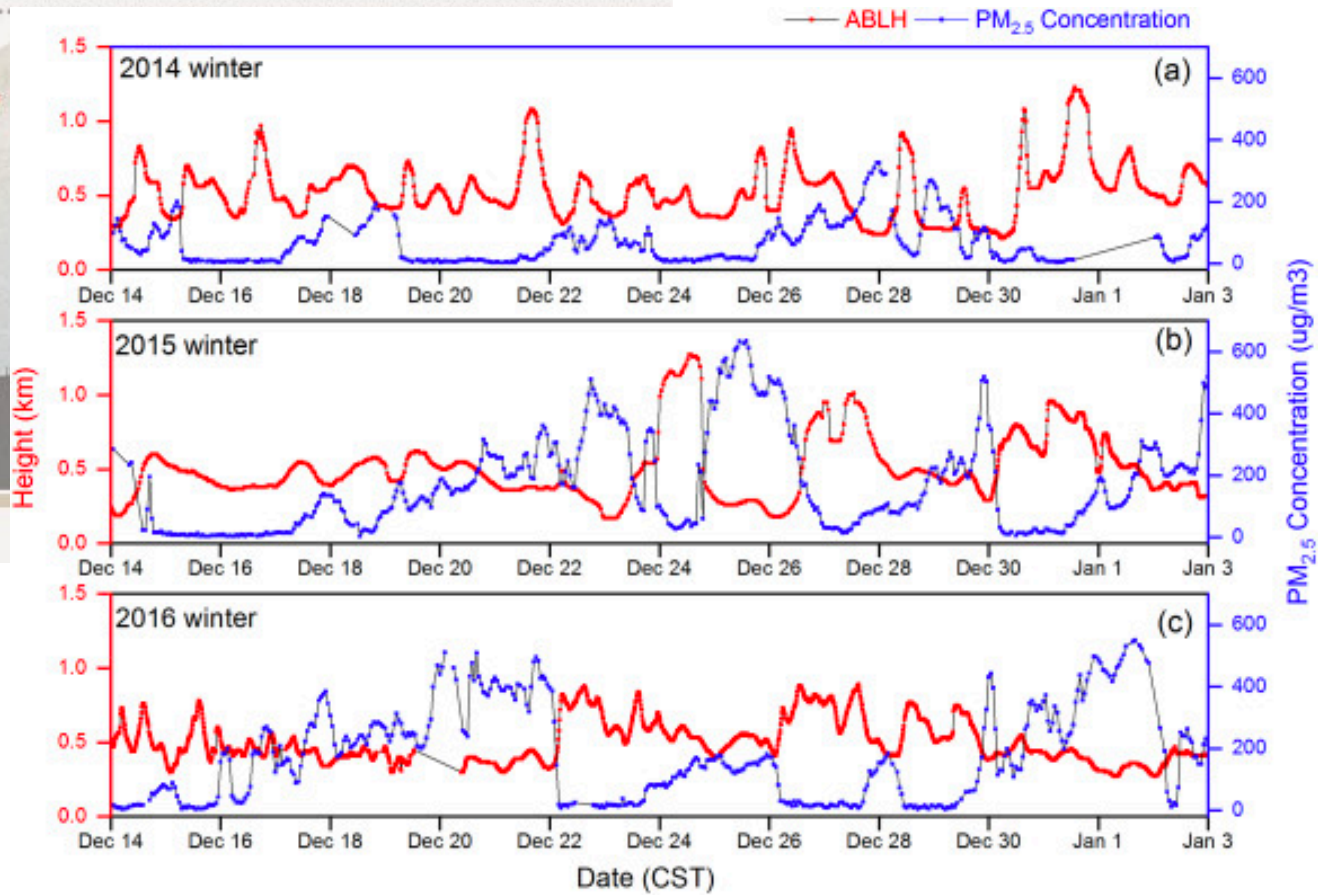
- Temperature inversion : T increases with height
    - very stable and lower boundary layer



# 2. Potential temperature and stability

- Boundary layer depth and air quality (and dry convection)



# 3. Moist convection

- The troposphere is mostly stable to dry convection.
- If so, why do we see gigantic convection systems?
- Air is ***moist***!

# 3. Moist convection : humidity

- We need a measure for how wet the air is.
- **Specific humidity (q)** : the mass of water vapor to the mass of air per unit volume

**The mass of water vapor**

$$q = \frac{\rho_v}{\rho} = \frac{\rho_v}{\rho_d + \rho_v}$$

**The total mass of air = the mass of water vapor + the mass of dry air**

# 3. Moist convection : humidity

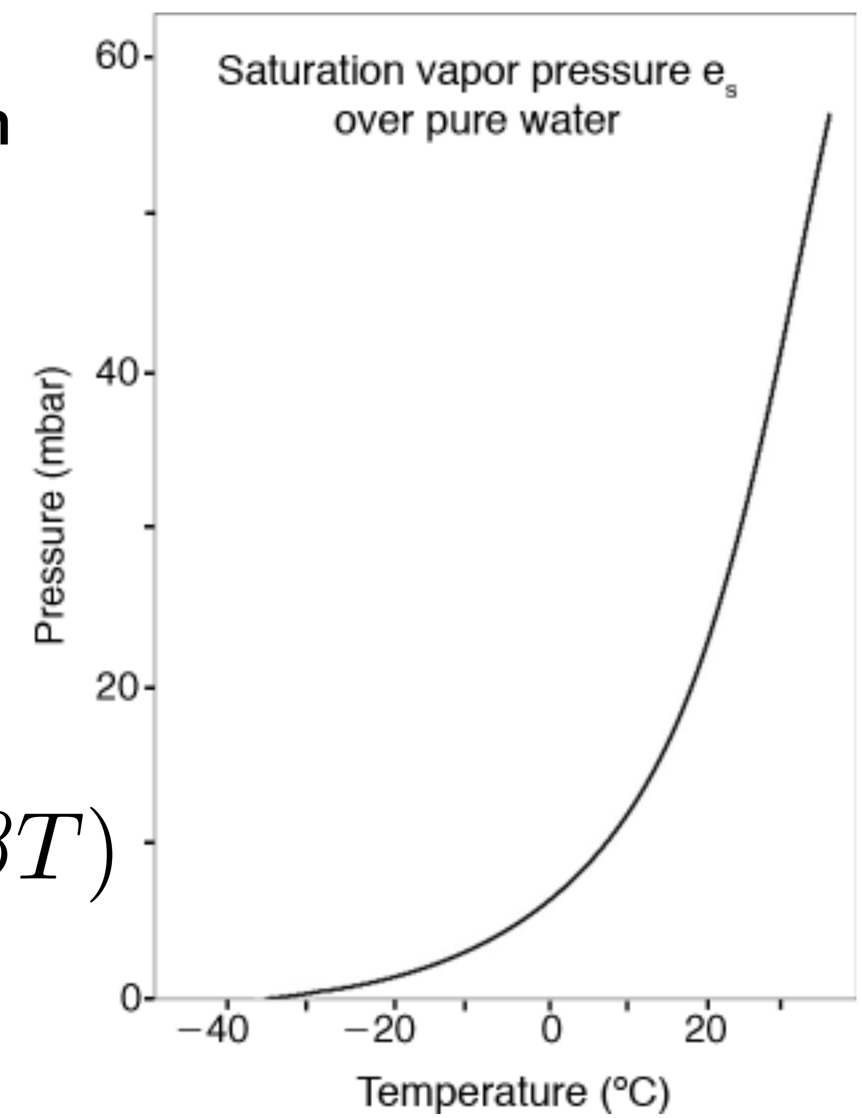
- We need a measure for how wet the air is.
- **Saturation-specific humidity ($q_*$)** : the specific humidity at which saturation occurs

The mass of water vapor at saturation

$$q_* = \frac{\rho_{v,*}}{\rho} = \frac{e_s/R_vT}{p/RT} = \left(\frac{R}{R_v}\right)\frac{e_s}{p}$$

$$q_* = q_*(p, T)$$

$$e_s = A\exp(\beta T)$$



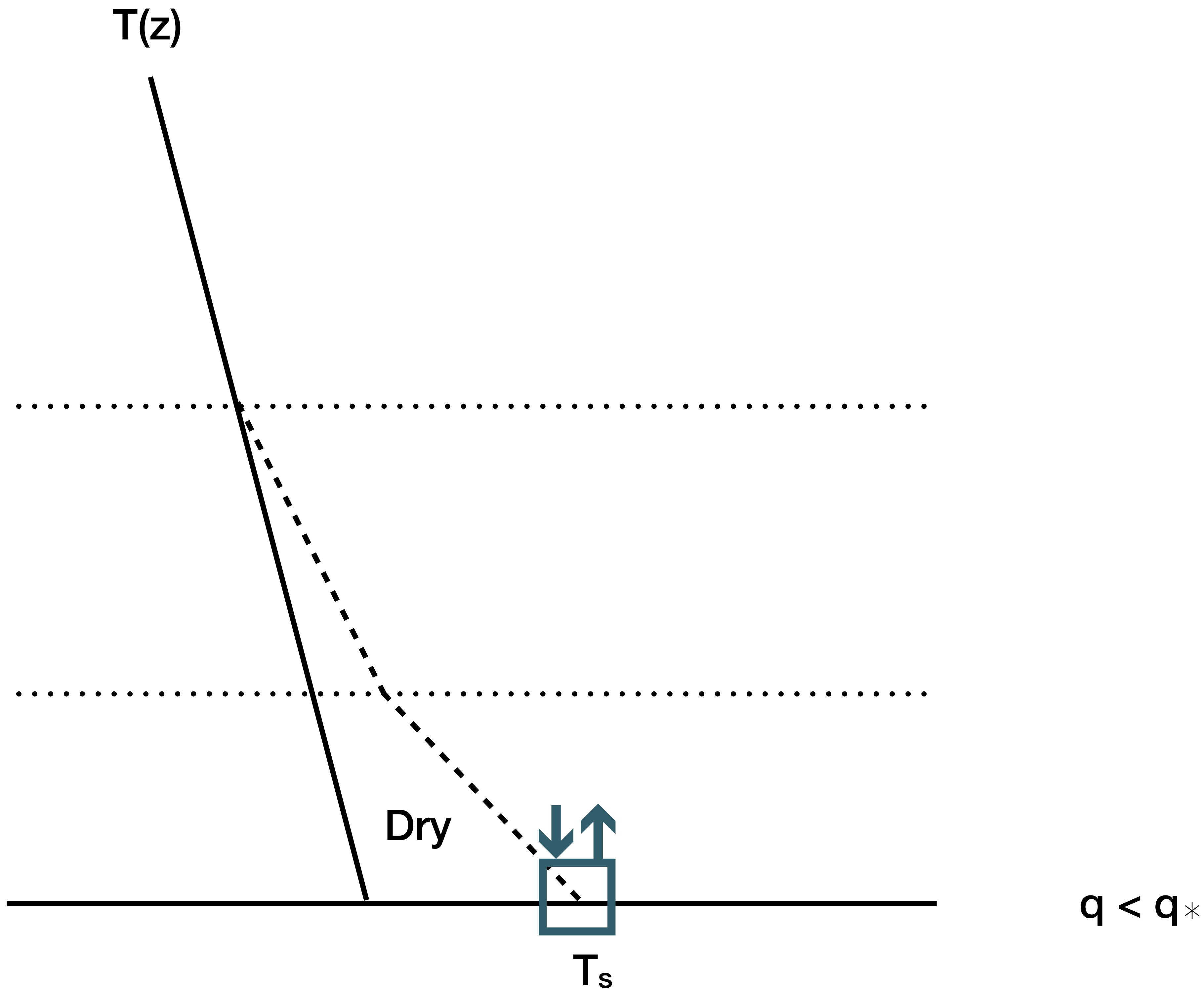
# 3. Moist convection : humidity

- **Relative humidity** : the ratio of the specific humidity to the saturation specific humidity

$$U = \frac{q}{q_*} \times 100\%$$

- The surface has higher humidity than aloft (relative humidity is close to 80%).
- Raise humid air..
  - Both p and T decrease, and $q_*$ goes up? Or down?
  - How about q?
  - What happens if $q = q_*$?

# 3. Moist convection : humidity



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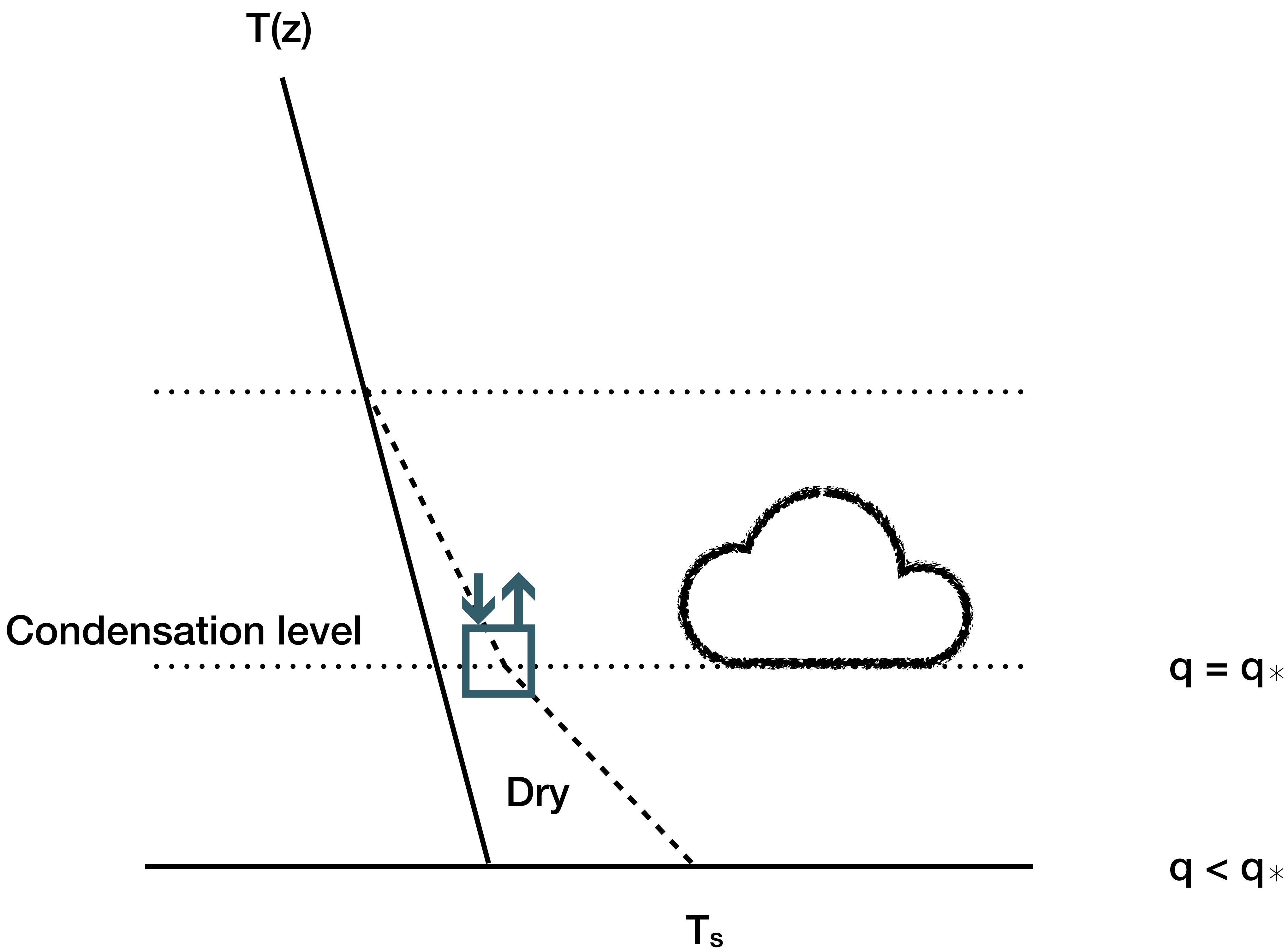

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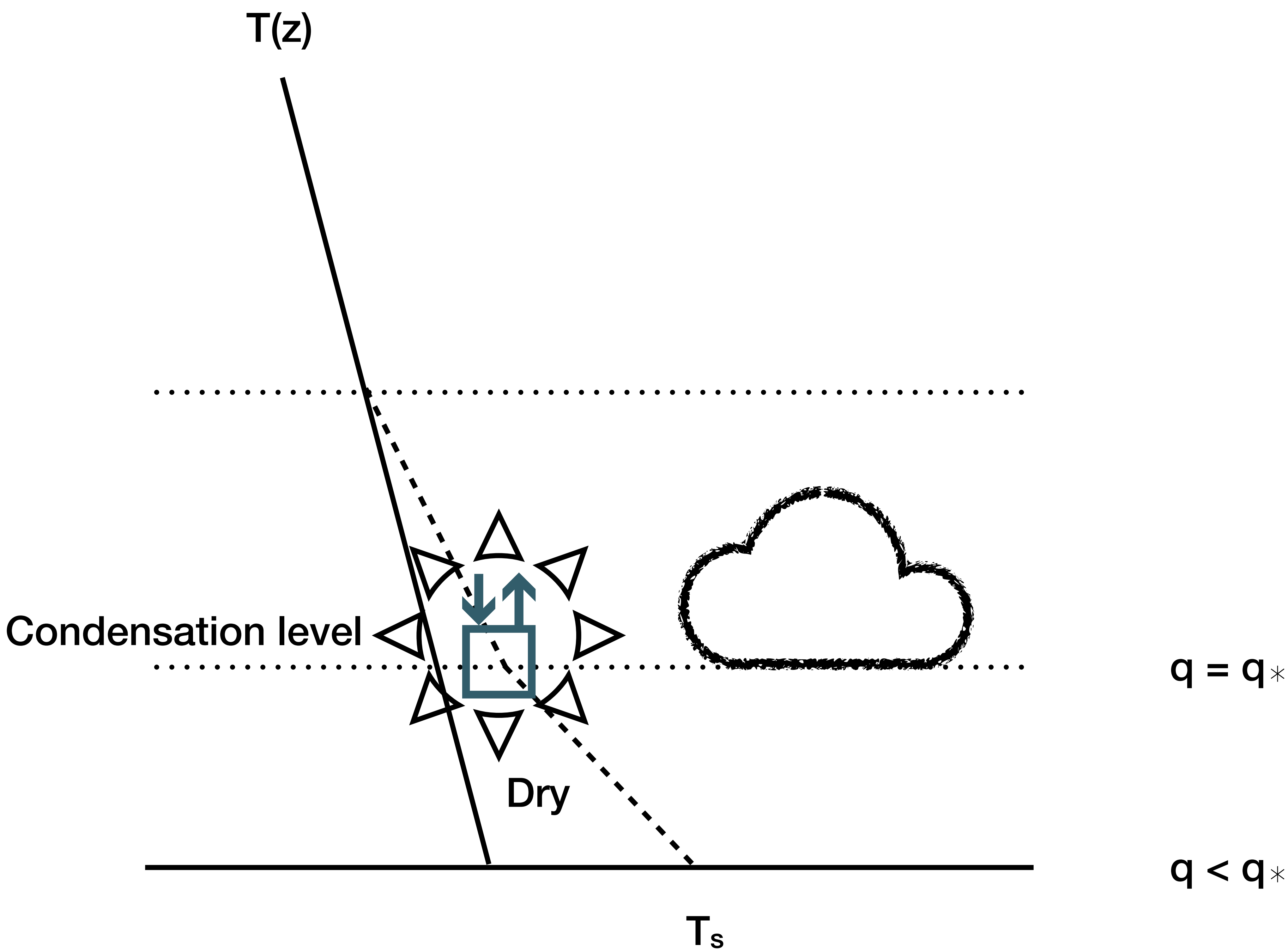

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