Ocean circulation

ATM2016

Last time

- How is the ocean different from the atmosphere?
- Temperature, salinity and density structures
- Ocean mixing
- Ocean currents

Today

- Geostrophic flow

Geostrophic and hydrostatic balance

- On the large-scale, water obeys the same fluid dynamics as air
  - Geostrophic balance
  - Hydrostatic balance
- In the ocean, the density change is rather small, and we can take this advantage in writing the momentum equation and hydrostatic balance equation.

Geostrophic and hydrostatic balance

Momentum equations

$$\frac{Du}{Dt} + \frac{1}{\boxed{\rho_{ref}}}\frac{\partial p}{\partial x} - fv = \mathscr{F}_x$$

$$\frac{Dv}{Dt} + \frac{1}{\boxed{\rho_{ref}}}\frac{\partial p}{\partial y} + fu = \mathscr{F}_y$$

constant reference density

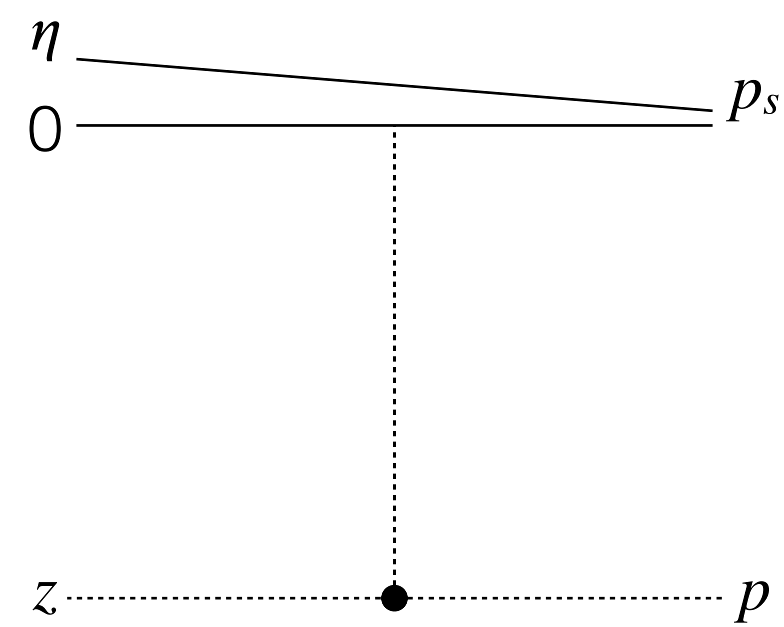
Hydrostatic balance

$$\frac{\partial p}{\partial z} = -g\left(\boxed{\rho_{ref}} + \sigma\right)$$

Geostrophic and hydrostatic balance

Hydrostatic balance in the ocean

$$\frac{\partial p}{\partial z} = -g\left(\rho_{ref} + \sigma\right)$$



$$p(z) = p_s - g\left(\rho_{ref} + \sigma\right)(z - \eta)$$

$$\approx p_s - g\rho_{ref}(z - \eta)$$

- p increases linearly, which is contrasted with the exponential decrease of pressure in the atmosphere.
- Sea level variation can create horizontal pressure gradient at depth.

Geostrophic and hydrostatic balance

Geostrophic balance in the ocean

- Typical ocean flow in the subtropical gyre : U ~ 0.1 m/s
- The size of the gyre in north-south direction : L ~ 2 x 10^6 m
- Coriolis parameter in the midlatitude : f ~ 10^{-4} s^{-1}

$$R_0 = \frac{U}{fL} \sim 10^{-3} \longleftarrow$$

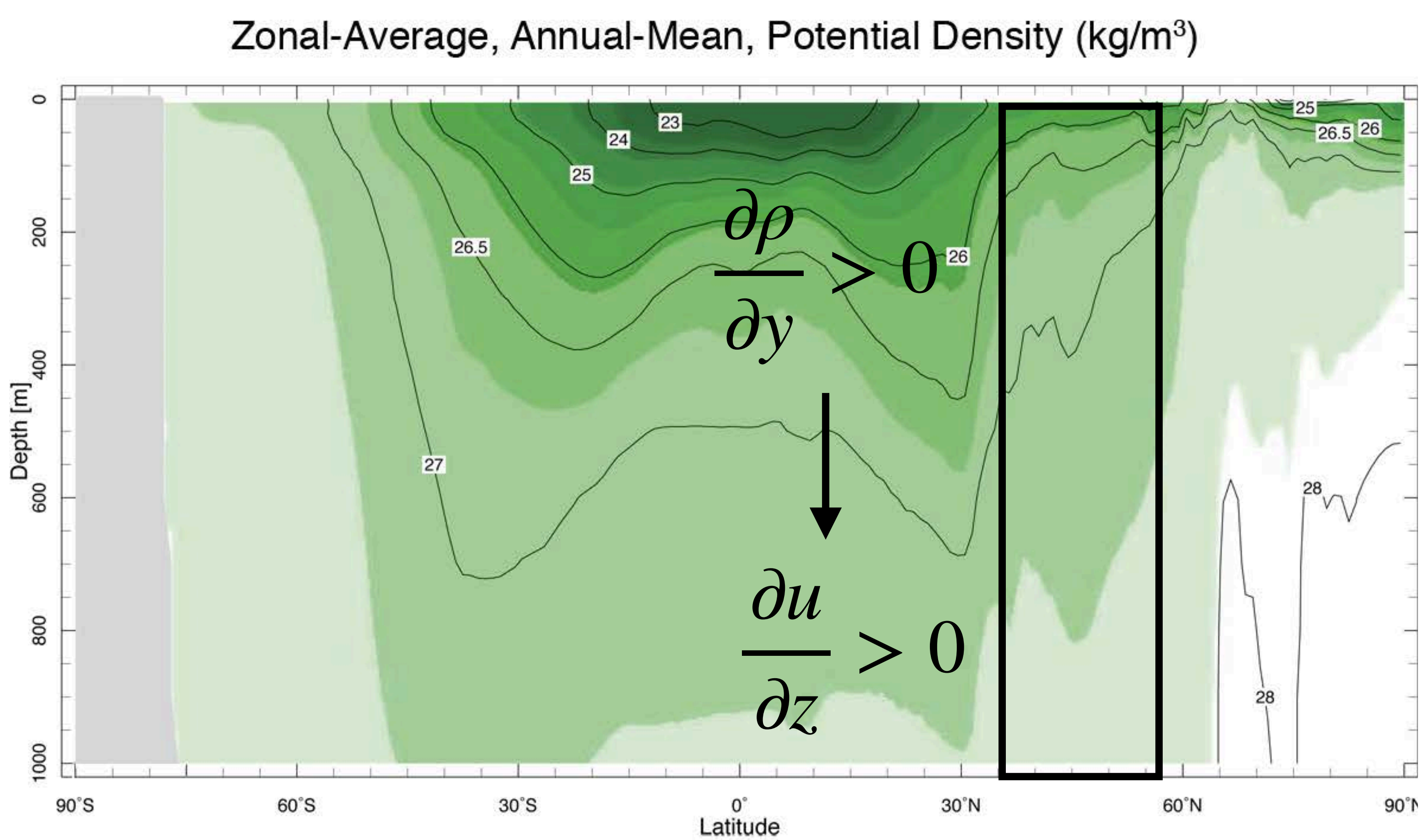
Much smaller than R_0 in the atmosphere ($O(0.1)$)

$\rightarrow$ The geostrophic approximation is valid for the interior of the ocean

Geostrophic and hydrostatic balance

Thermal wind balance in the ocean

$$\frac{\partial u}{\partial z} = \frac{g}{f\rho_{ref}} \frac{\partial \sigma}{\partial y} \qquad \frac{\partial v}{\partial z} = -\frac{g}{f\rho_{ref}} \frac{\partial \sigma}{\partial x}$$



If $u(z) \sim 0$, then what would $u_{surface}$ be?

- u(1000 m) ~ 0 m/s
- g = 10 m/s^2
- ρ_{ref} = 1000 kg/m^3
- $\Delta\sigma$ = 1.5 kg/m^3
- L = 2000 km

Ocean surface structure

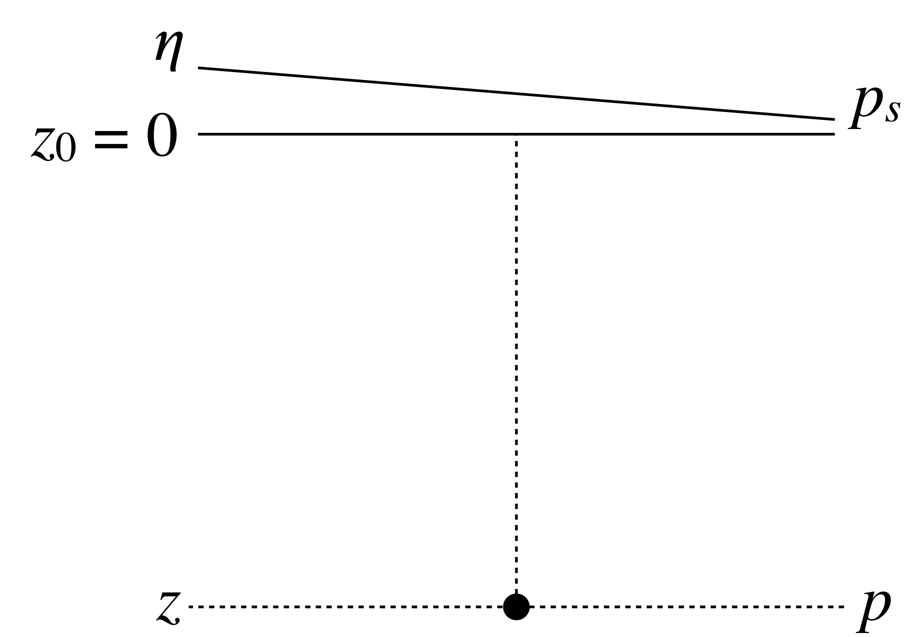
- On the large-scale, we can use geostrophic balance to understand the relationship between the sea level and the current.

$$p(z) = p_s + \int_z^{\eta} g\rho dz = p_s + g\left\langle\rho\right\rangle\left(\eta - z\right)$$

$$\frac{1}{\eta - z}\int_z^{\eta}\rho dz$$

treat it as a constant
near the surface

$$p(z_0) = p_s + g\rho_{ref}\eta$$



$$u_{g,\ surface} = -\frac{g}{f}\frac{\partial\eta}{\partial y},\quad v_{g,\ surface} = \frac{g}{f}\frac{\partial\eta}{\partial x}$$

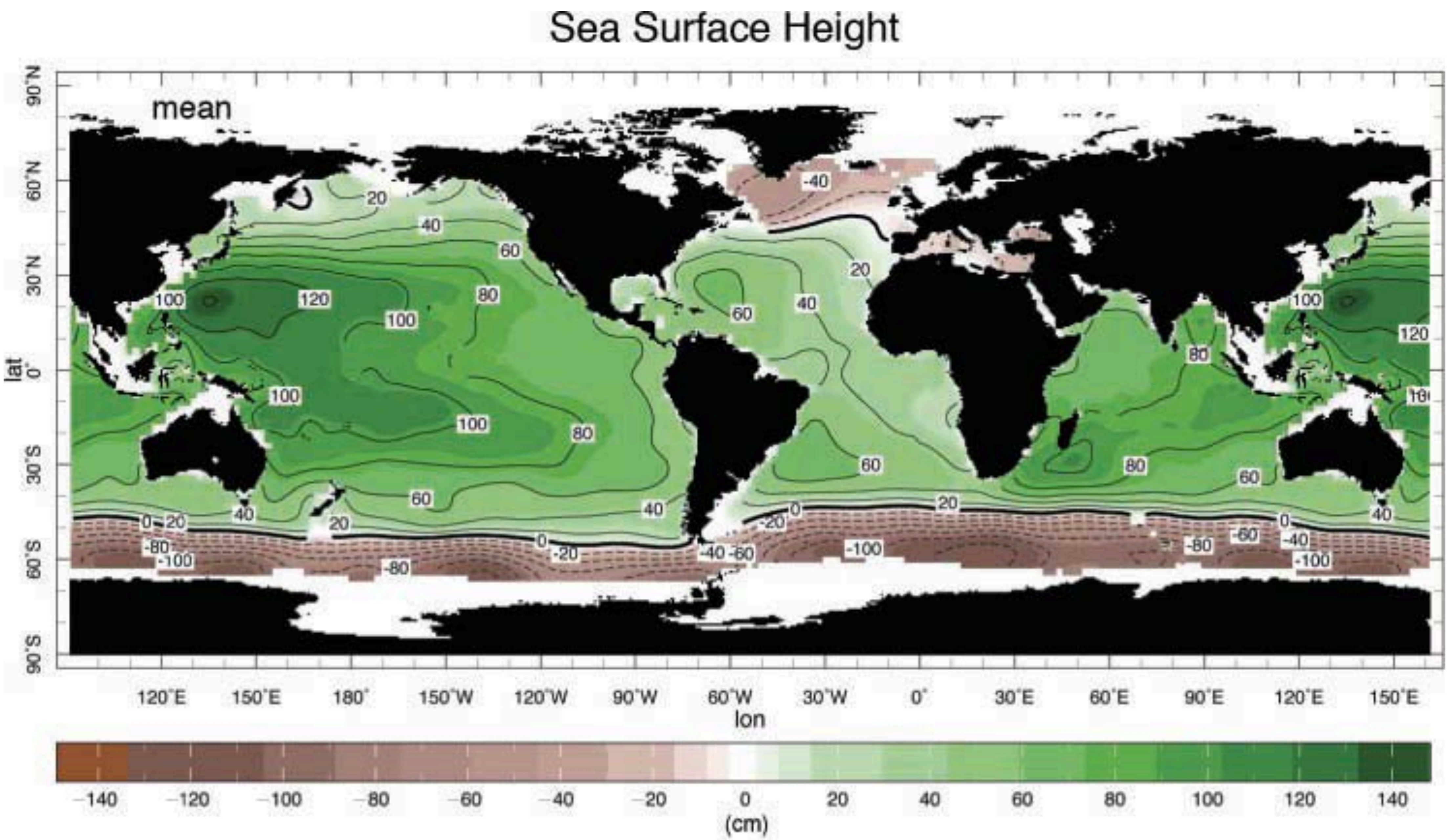
geostrophic current just
beneath the surface

Ocean surface structure

- Estimating sea level changes using the current based on the geostrophic balance

$$\Delta\eta = \frac{\overset{10^{-4}\ \mathrm{s^{-1}}}{f}\ \overset{1000\ \mathrm{km}}{L}\ \overset{10\ \mathrm{cm/s}}{U}}{\underset{10\ \mathrm{m/s^2}}{g}} \longrightarrow \Delta\eta \sim O(1\ m)$$

Ocean surface structure



Geostrophic current at depth

- At depth, we cannot neglect σ (the variation in density)
- It means that $\nabla \langle \rho \rangle \neq 0$

$$\frac{\partial p}{\partial x} = \frac{\partial}{\partial x}\left[p_s + g\langle \rho \rangle (\eta - z)\right]$$

$$= g\left[\frac{\partial \langle \rho \rangle}{\partial x}(\eta - z) + \langle \rho \rangle \frac{\partial \eta}{\partial x}\right]$$

$$\frac{\partial p}{\partial y} = g\left[\frac{\partial \langle \rho \rangle}{\partial y}(\eta - z) + \langle \rho \rangle \frac{\partial \eta}{\partial y}\right]$$

$$\longrightarrow$$

$$u_g = -\frac{g}{f\rho_{ref}}\left[\frac{\partial \langle \rho \rangle}{\partial y}(\eta - z) + \langle \rho \rangle \frac{\partial \eta}{\partial y}\right]$$

$$v_g = \frac{g}{f\rho_{ref}}\left[\frac{\partial \langle \rho \rangle}{\partial x}(\eta - z) + \langle \rho \rangle \frac{\partial \eta}{\partial x}\right]$$

Geostrophic current at depth

$$u_g = -\frac{g}{f\rho_{ref}}\left[\frac{\partial\langle\rho\rangle}{\partial y}(\eta - z) + \langle\rho\rangle\frac{\partial\eta}{\partial y}\right]$$

$$v_g = \frac{g}{f\rho_{ref}}\left[\frac{\partial\langle\rho\rangle}{\partial x}(\eta - z) + \langle\rho\rangle\frac{\partial\eta}{\partial x}\right]$$

If there is no horizontal variation in $\langle\boldsymbol{\rho}\rangle$, the geostrophic current is independent of depth.

⬇

Same current at all depth!

⬇

The ocean moves around as a column.

⬅

No vertical motion (2D flow)

⬇

BUT we observe that the ocean current at depth is slower than the surface flow.

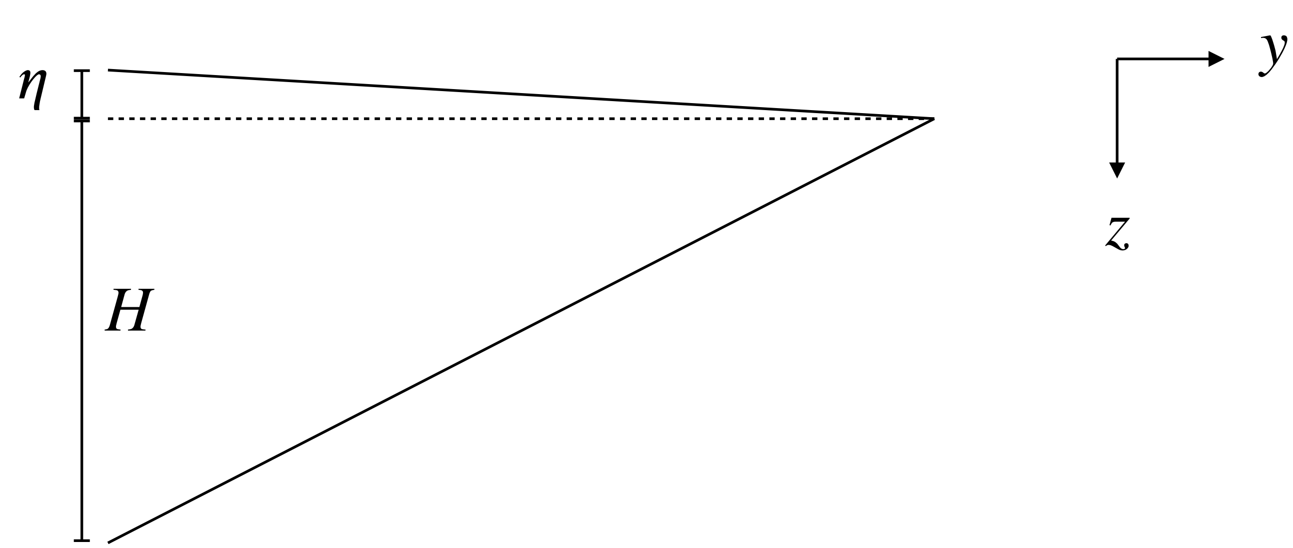
⬇

The first term should reduce the effect from the sea level difference.

Geostrophic current at depth

- How much would interior density surface need to tilt to result in the near-zero geostrophic current at $z = H$?
  - Then the pressure gradient should be close to zero

$$\frac{\partial \langle \rho \rangle}{\partial y} H = - \langle \rho \rangle \frac{\partial \eta}{\partial y}$$



Geostrophic current at depth