

Exploitation of dynamic symmetries for solving SAT problems

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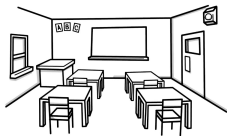


Motivation

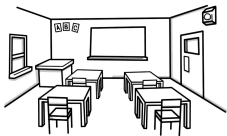
SAT is widely used in different domains

- Artificial intelligence (planning, games, ...)
- Bioinformatics (haplotype inference, ...)
- Security (cryptanalysis, inversion attack on hash , ...)
- Computationally hard problems (graph coloring, ...)
- Formal methods (hardware model checking, ...)

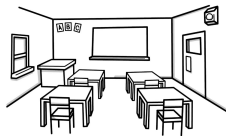
SAT an example



1



2



3



A



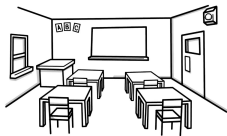
B



C

Is it possible to attribute each group to a classroom?

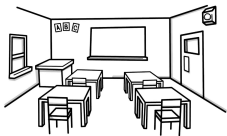
SAT an example



1
↑



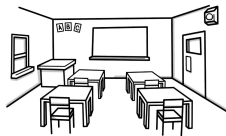
A



2
↑



B



3
↑

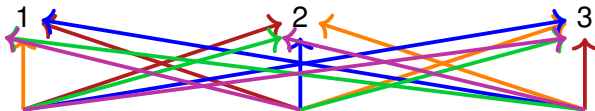
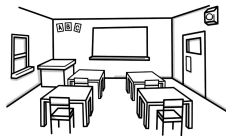
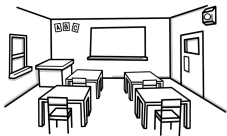
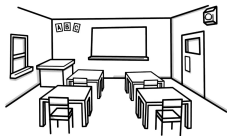


C

Is it possible to attribute each group to a classroom?

YES!

SAT an example



A



B

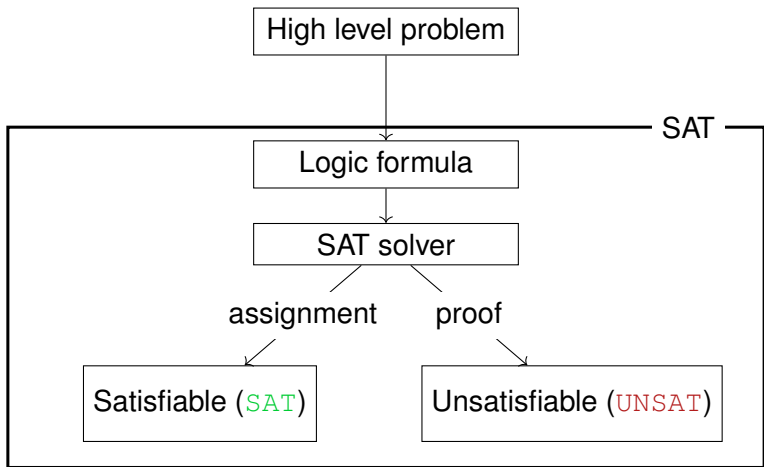


C

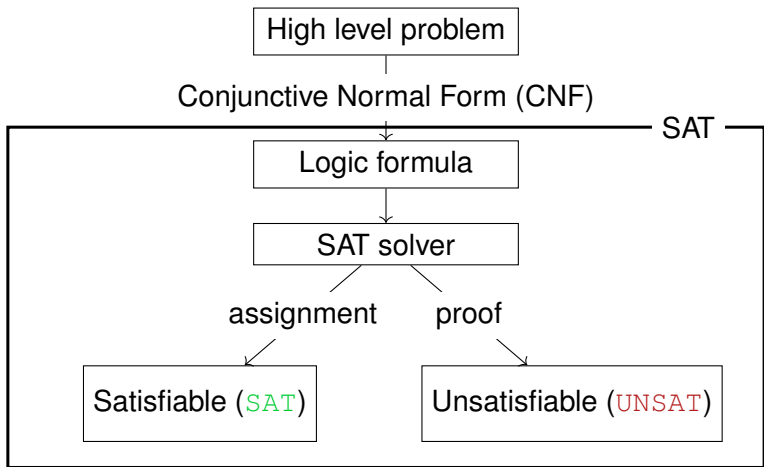
Is it possible to attribute each group to a classroom?

YES! Many solutions

Boolean SATisfiability problem



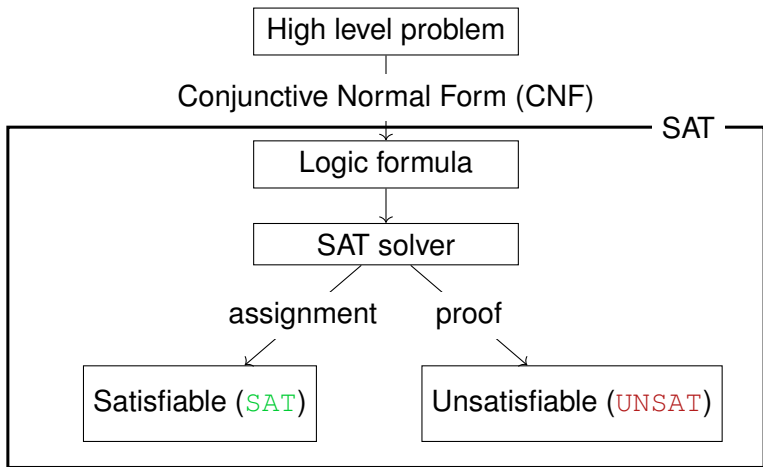
Boolean SATisfiability problem



CNF representation:

$$\underbrace{(x_1 \vee x_2 \vee \neg x_3)}_{\text{Clause with literals } x_1, x_2, \neg x_3}$$

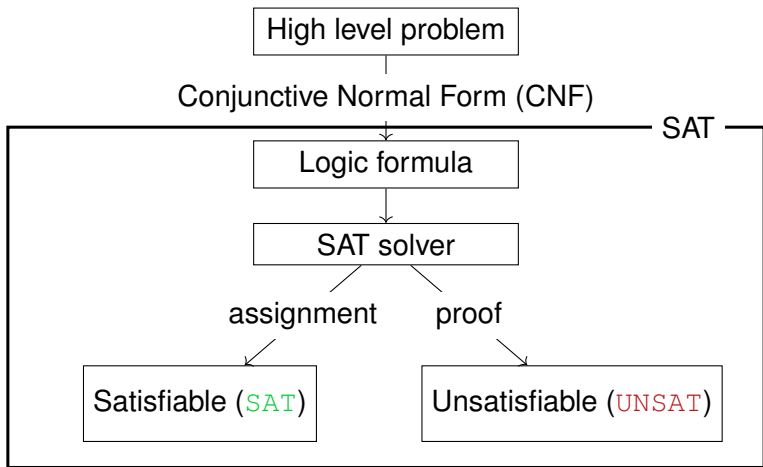
Boolean SATisfiability problem



CNF representation:

$$\underbrace{(x_1 \vee x_2 \vee \neg x_3)}_{\text{Clause}} \wedge \underbrace{(\neg x_1 \vee \neg x_2) \wedge (x_2 \vee \neg x_4)}_{\text{Formula (CNF)}}$$

Boolean SATisfiability problem



Clause representation as a set:

$$(x_1 \vee x_2 \vee \neg x_3) \rightarrow \{x_1, x_2, \neg x_3\}$$

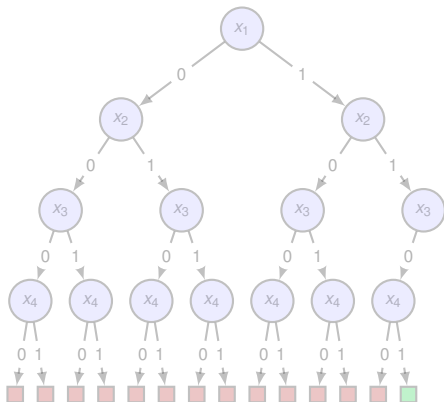
SAT Solving

Solving SAT formula is known to be **NP-complete** [Coo71]

Enumerative algorithm:

- Davis, Putnam, Logemann, and Loveland (DPLL) [DLL62]
 - Boolean Constraint Propagation (BCP)
- Conflict Driven Clause Learning (CDCL) [MSS99]
 - Derived from DPLL
 - Clause learning

CDCL in action



$$\omega_1 = \{x_1, x_2, x_3, x_4\}$$

$$\omega_2 = \{x_1, \neg x_4\}$$

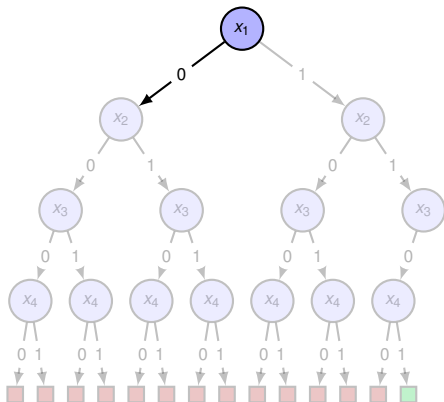
$$\omega_3 = \{x_1, x_4\}$$

$$\omega_4 = \{x_2, \neg x_4\}$$

$$\omega_5 = \{x_2, x_4\}$$

$$\omega_6 = \{x_3, x_4\}$$

CDCL in action



$$\omega_1 = \{\mathbf{x}_1, x_2, x_3, x_4\}$$

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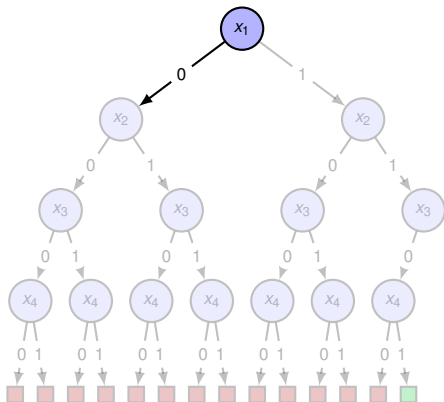
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$$\omega_4 = \{x_2, \neg x_4\}$$

$$\omega_5 = \{x_2, x_4\}$$

$$\omega_6 = \{x_3, x_4\}$$

CDCL in action



$$\omega_1 = \{\textcolor{red}{x}_1, x_2, x_3, x_4\}$$

$$\omega_2 = \{\textcolor{red}{x}_1, \neg \textcolor{blue}{x}_4\}$$

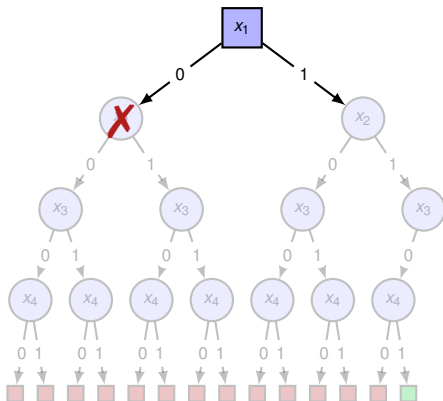
$$\omega_3 = \{\textcolor{red}{x}_1, \textcolor{blue}{x}_4\}$$

$$\omega_4 = \{x_2, \neg x_4\}$$

$$\omega_5 = \{x_2, x_4\}$$

$$\omega_6 = \{x_3, x_4\}$$

CDCL in action



$$\omega_1 = \{x_1, x_2, x_3, x_4\}$$

$$\omega_2 = \{x_1, \neg x_4\}$$

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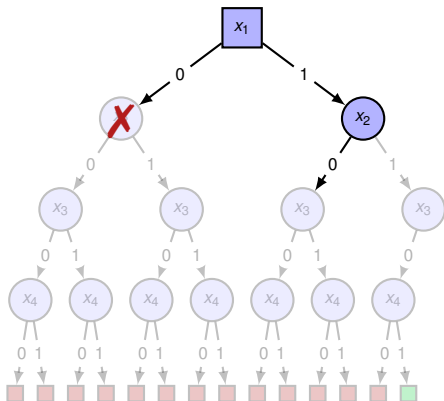
$$\omega_4 = \{x_2, \neg x_4\}$$

$$\omega_5 = \{x_2, x_4\}$$

$$\omega_6 = \{x_3, x_4\}$$

$$\omega_7 = \{x_1\}$$

CDCL in action



$$\omega_1 = \{x_1, x_2, x_3, x_4\}$$

$$\omega_2 = \{x_1, \neg x_4\}$$

$$\omega_3 = \{x_1, x_4\}$$

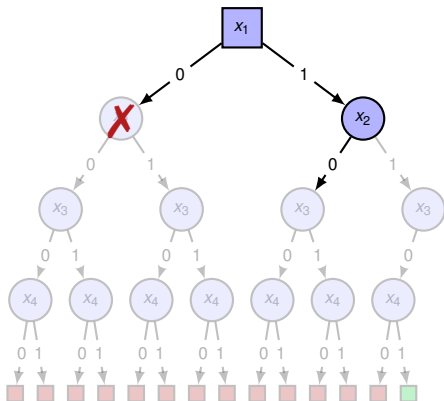
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CDCL in action



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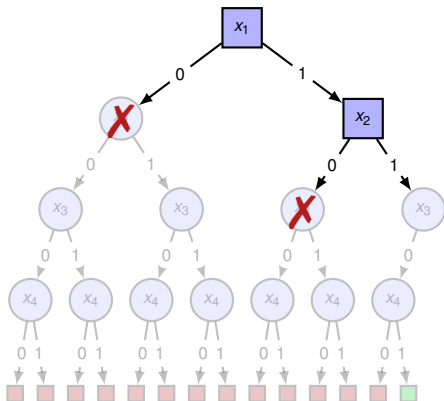
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CDCL in action



$$\omega_1 = \{x_1, x_2, x_3, x_4\}$$

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$$\omega_3 = \{x_1, x_4\}$$

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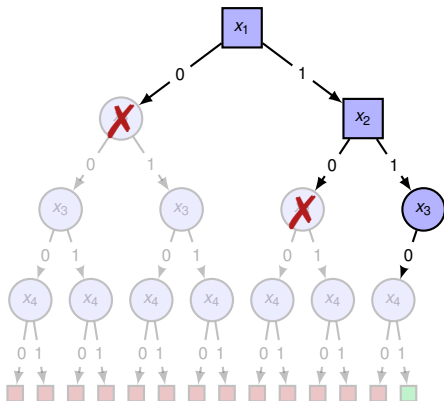
$$\omega_5 = \{x_2, x_4\}$$

$$\omega_6 = \{x_3, x_4\}$$

$$\omega_7 = \{x_1\}$$

$$\omega_8 = \{x_2\}$$

CDCL in action



$$\omega_1 = \{x_1, x_2, x_3, x_4\}$$

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$$\omega_3 = \{x_1, x_4\}$$

$$\omega_4 = \{x_2, \neg x_4\}$$

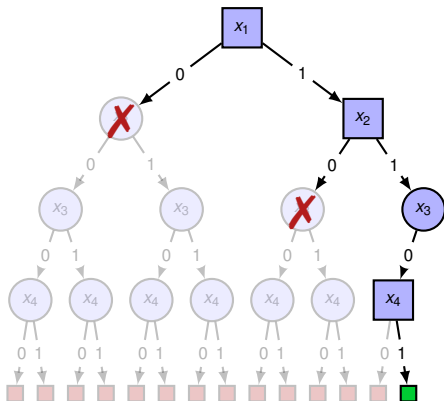
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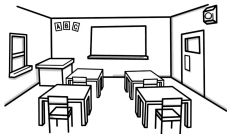
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$$\omega_6 = \{x_3, x_4\}$$

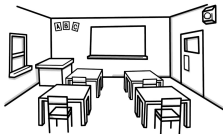
$$\omega_7 = \{x_1\}$$

$$\omega_8 = \{x_2\}$$

An UNSAT example



1



2



A



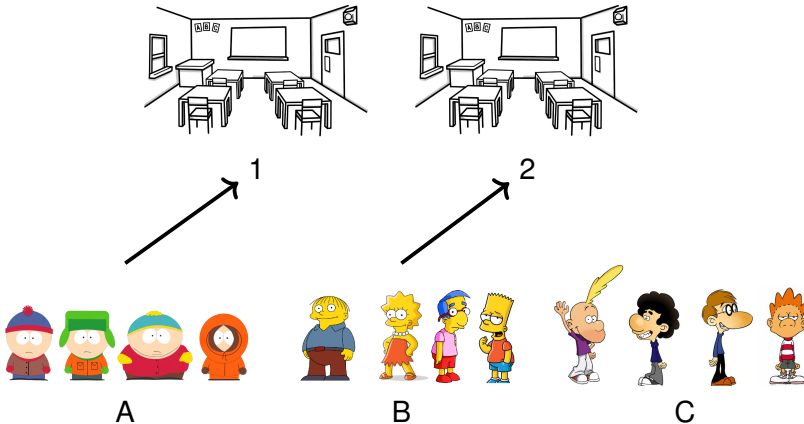
B



C

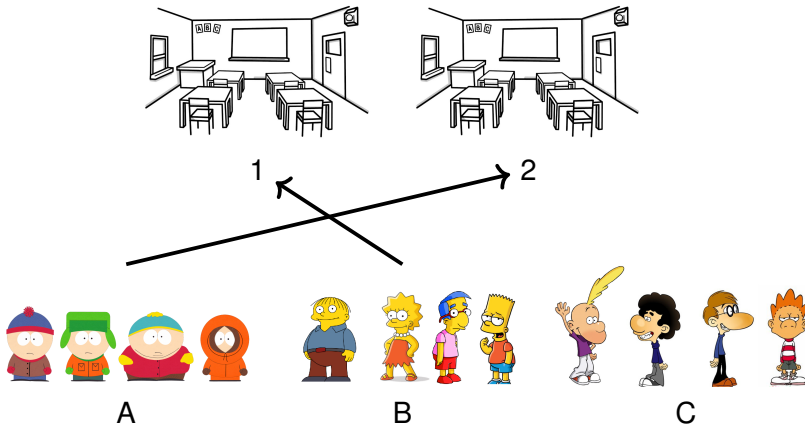
Is it possible to attribute each group to a classroom?

An UNSAT example



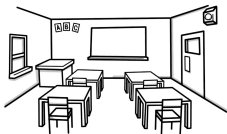
Is it possible to attribute each group to a classroom?

An UNSAT example

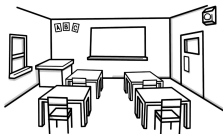


Is it possible to attribute each group to a classroom?

An UNSAT example



1



2

...



A



B

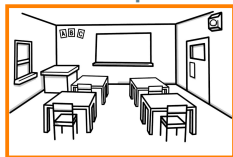


C

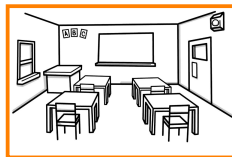
Is it possible to attribute each group to a classroom?

No!

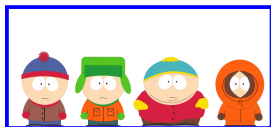
An UNSAT example



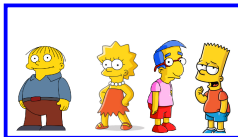
1



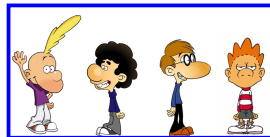
2



A



B



C

Is it possible to attribute each group to a classroom?

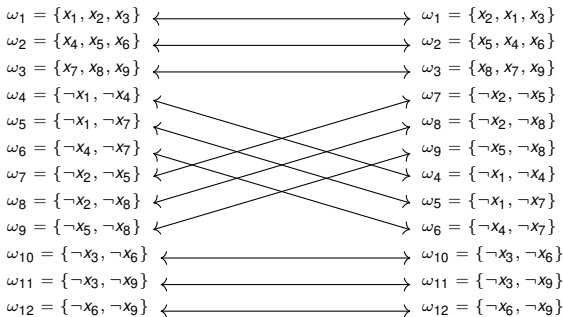
No!

Presence of symmetries hinders the performance of the solver

Symmetry

A symmetry (permutation) g is a bijective function (on variables) that leaves the formula invariant.

$$g = \begin{pmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 \\ x_2 & x_1 & x_3 & x_5 & x_4 & x_6 & x_8 & x_7 & x_9 \end{pmatrix} \rightarrow (x_1 \ x_2)(x_4 \ x_5)(x_7 \ x_8)$$



Computing symmetries of a SAT problem

CNF formula

$$\begin{aligned} & (x_1 \vee x_2 \vee x_3) \wedge (x_4 \vee x_5 \vee x_6) \wedge (x_7 \vee x_8 \vee x_9) \\ & \wedge (\neg x_1 \vee \neg x_4) \wedge (\neg x_1 \vee \neg x_7) \wedge (\neg x_4 \vee \neg x_7) \\ & \wedge (\neg x_2 \vee \neg x_5) \wedge (\neg x_2 \vee \neg x_8) \wedge (\neg x_5 \vee \neg x_8) \\ & \wedge (\neg x_3 \vee \neg x_6) \wedge (\neg x_3 \vee \neg x_9) \wedge (\neg x_6 \vee \neg x_9) \end{aligned}$$

¹<http://www.tcs.hut.fi/Software/bliss/>

²<http://vlsicad.eecs.umich.edu/BK/SAUCY/>

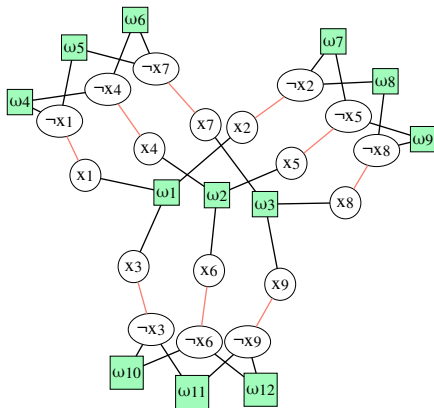
Computing symmetries of a SAT problem

CNF formula

$$\begin{aligned} & (x_1 \vee x_2 \vee x_3) \wedge (x_4 \vee x_5 \vee x_6) \wedge (x_7 \vee x_8 \vee x_9) \\ & \wedge (\neg x_1 \vee \neg x_4) \wedge (\neg x_1 \vee \neg x_7) \wedge (\neg x_4 \vee \neg x_7) \\ & \wedge (\neg x_2 \vee \neg x_5) \wedge (\neg x_2 \vee \neg x_8) \wedge (\neg x_5 \vee \neg x_8) \\ & \wedge (\neg x_3 \vee \neg x_6) \wedge (\neg x_3 \vee \neg x_9) \wedge (\neg x_6 \vee \neg x_9) \end{aligned}$$



colored graph



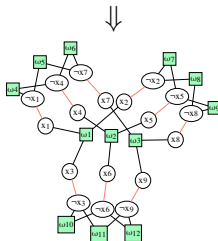
Computing symmetries of a SAT problem

CNF formula

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colored graph



graph automorphism



(bliss¹, saucy², ...)

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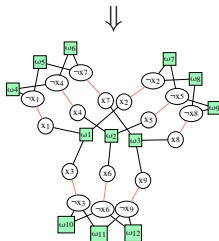
Computing symmetries of a SAT problem

CNF formula

$$\begin{aligned} & (x_1 \vee x_2 \vee x_3) \wedge (x_4 \vee x_5 \vee x_6) \wedge (x_7 \vee x_8 \vee x_9) \\ & \wedge (\neg x_1 \vee \neg x_4) \wedge (\neg x_1 \vee \neg x_7) \wedge (\neg x_4 \vee \neg x_7) \\ & \wedge (\neg x_2 \vee \neg x_5) \wedge (\neg x_2 \vee \neg x_8) \wedge (\neg x_5 \vee \neg x_8) \\ & \wedge (\neg x_3 \vee \neg x_6) \wedge (\neg x_3 \vee \neg x_9) \wedge (\neg x_6 \vee \neg x_9) \end{aligned}$$



colored graph



⇓
graph automorphism

⇓
set of symmetries

⇓
(bliss¹, saucy², ...)

⇓

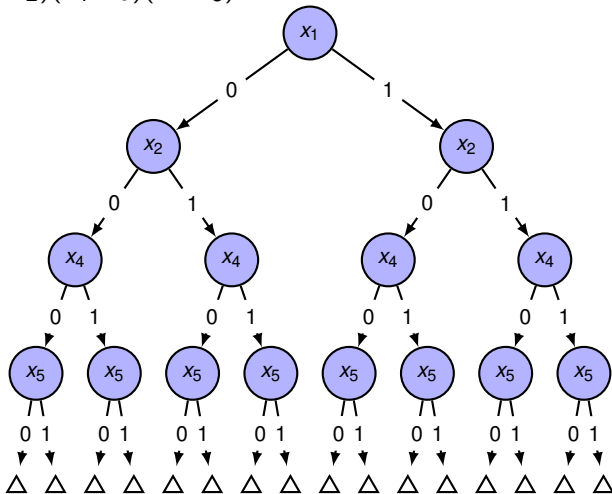
$$\begin{aligned} g_1 &= (x_2 \ x_3)(x_5 \ x_6)(x_8 \ x_9) \\ g_2 &= (x_4 \ x_7)(x_5 \ x_8)(x_6 \ x_9) \\ g_3 &= (x_1 \ x_2)(x_4 \ x_5)(x_7 \ x_8) \\ g_4 &= (x_1 \ x_4)(x_2 \ x_5)(x_3 \ x_6) \end{aligned}$$

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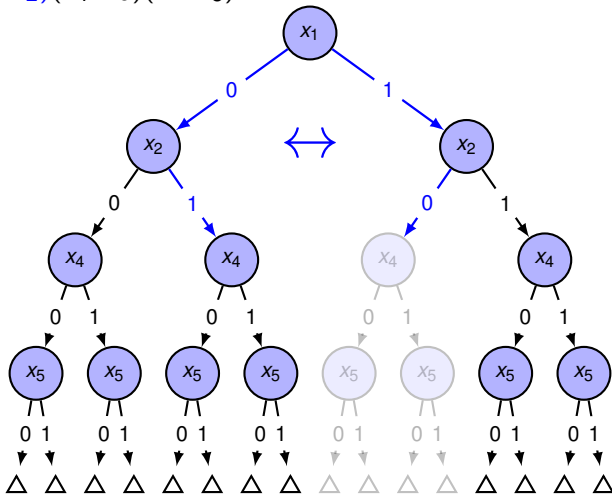
Using symmetries to prune search space

$$g = (x_1 \ x_2)(x_4 \ x_5)(x_7 \ x_8)$$



Using symmetries to prune search space

$$g = (x_1 \ x_2)(x_4 \ x_5)(x_7 \ x_8)$$



Adds additional constraints to prune search space.

Generates symmetry breaking predicates (SBP)

- Define lexicographic order
 - Define total order on variables
 - Define minimal value
- Forbid non minimal assignment with addition of SBP

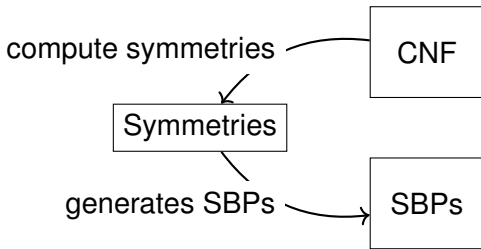
Example:

$$x_1 \leq x_2 \leq x_3 \leq x_4 \leq x_5 \leq x_6 \leq x_7 \leq x_8; \text{false} < \text{true}$$

$$g = (x_1 \ x_2)(x_4 \ x_5)(x_7 \ x_8)$$

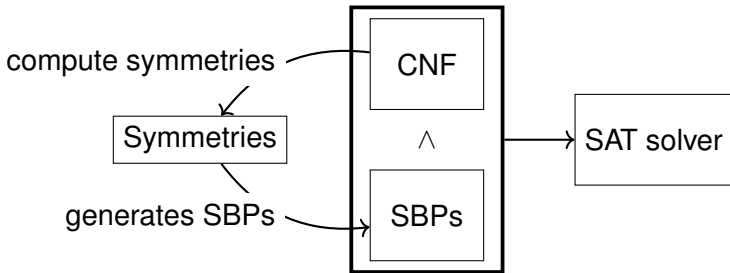
$x_1 \leq x_2$	$x_1 \vee \neg x_2$
$x_1 = x_2 \rightarrow x_4 \leq x_5$	$x_1 \vee x_2 \vee x_4 \vee \neg x_5$ $\neg x_1 \vee \neg x_2 \vee x_4 \vee \neg x_5$
$x_1 = x_2 \wedge x_4 = x_5 \rightarrow x_8 \leq x_3$	$x_1 \vee x_2 \vee x_4 \vee x_5 \vee x_7 \vee \neg x_8$ $\neg x_1 \vee \neg x_2 \vee x_4 \vee x_5 \vee x_7 \vee \neg x_8$...

Static symmetry breaking



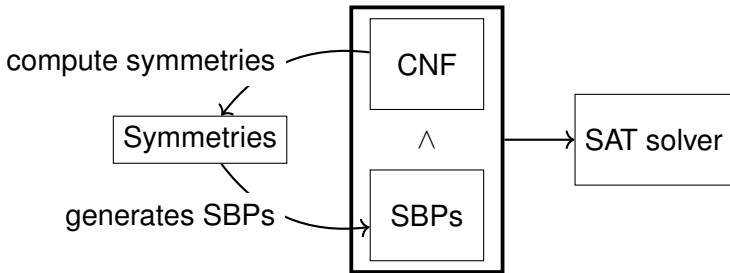
- Works well on many symmetric instances
- The solver can "explode" instead of being helped

Static symmetry breaking



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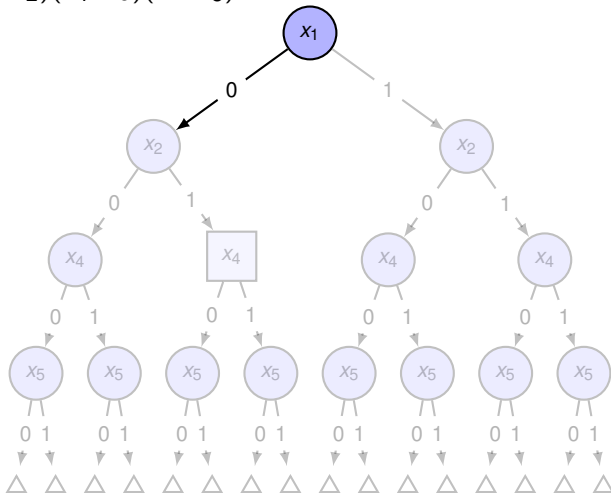
Static symmetry breaking



- Works well on many symmetric instances
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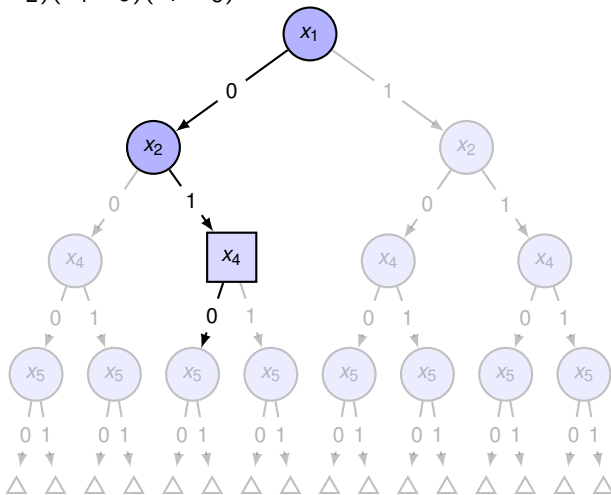
Using symmetries to accelerate tree traversal

$$g = (x_1 \ x_2)(x_4 \ x_5)(x_7 \ x_8)$$



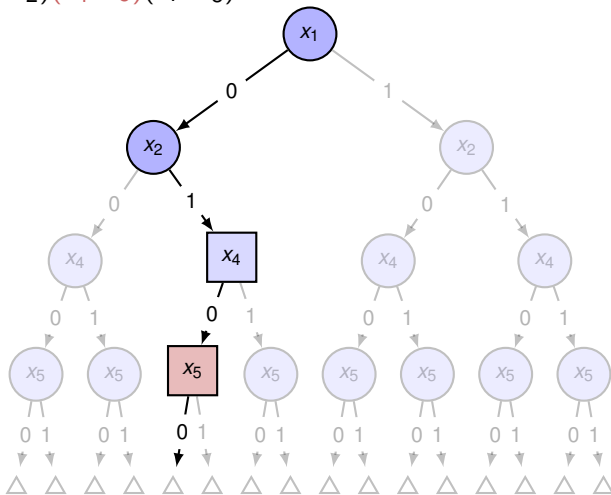
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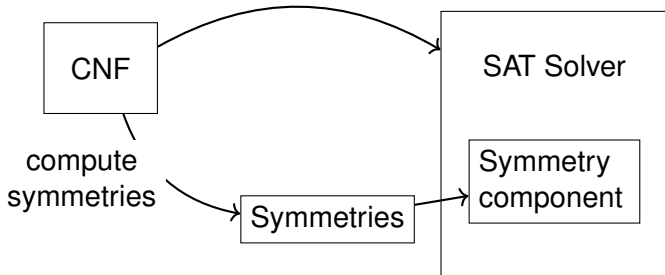
Using symmetries to accelerate tree traversal

$$g = (x_1 \ x_2)(x_4 \ x_5)(x_7 \ x_8)$$



Use symmetries to deduce symmetrical facts.

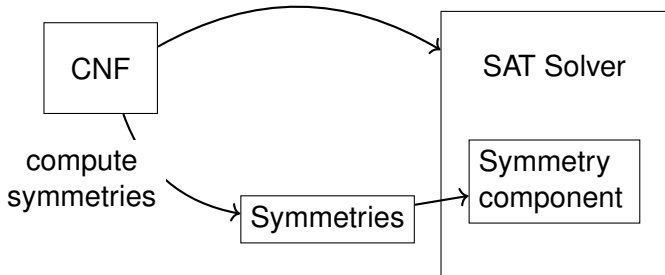
Dynamic Symmetry Breaking



Different approaches:

- Symmchaff [Sab05]
- Symmetry Propagation (SP) [DBdC⁺12]
- Symmetry Learning Scheme (SLS) [BNOS10]
- Symmetry Explanation Learning (SEL) [DBB17]

Dynamic Symmetry Breaking



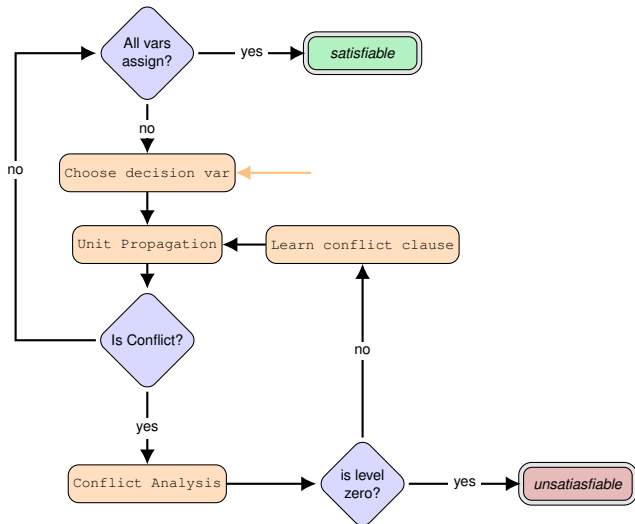
Different approaches:

- Symmchaff [Sab05]
- Symmetry Propagation (SP) [DBdC⁺12]
- Symmetry Learning Scheme (SLS) [BNOS10]
- Symmetry Explanation Learning (SEL) [DBB17]

Works under **some conditions**

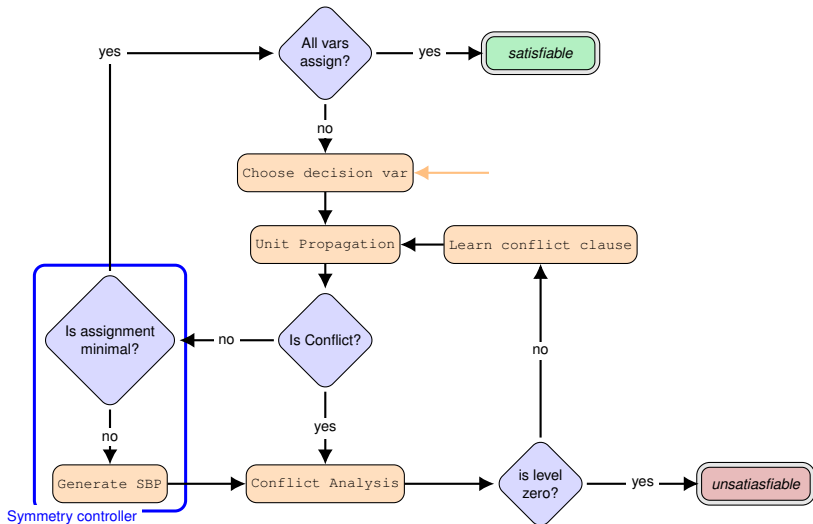
Our contribution CDCL[Sym]

Compute and inject SBP **opportunistically**, during the solving



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Compute and inject SBP **opportunistically**, during the solving



Symmetry status

- reducer: $g.\alpha \prec \alpha$
- inactive: $\alpha \prec g.\alpha$
- active: *not enough information*

Efficient implementation of symmetry status

Keep track the smallest unassigned variable x :

- ① $\alpha(g.x) \leq \alpha(x)$, then g is `reducer` \Rightarrow Effective SBP (ESBP)
- ② $\alpha(x) \leq \alpha(g.x)$, then g is `inactive` $\Rightarrow g$ cannot reduce α
- ③ $\alpha(g.x)$ or $\alpha(x)$ is unassigned then g is `active`

Update whenever variables are assigned / unassigned

Example

- 1 reducer: $\alpha(g.x) \leq \alpha(x)$
- 2 inactive: $\alpha(x) \leq \alpha(g.x)$
- 3 active: $\alpha(g.x)$ or $\alpha(x)$ is unassigned

$x_1 \leq x_2 \leq x_3 \leq x_4 \leq x_5 \leq x_6 \leq x_7 \leq x_8$; false < true

$$g_1 = \begin{array}{cc} (x_2 & x_3) & (x_5 & x_6) & (x_8 & x_9) \end{array} \left| \begin{array}{l} x = x_2 \\ g.x = x_3 \\ \text{active} \end{array} \right.$$

↑

$$g_2 = \begin{array}{cc} (x_1 & x_2) & (x_4 & x_5) & (x_7 & x_8) \end{array} \left| \begin{array}{l} x = x_1 \\ g.x = x_2 \\ \text{active} \end{array} \right.$$

↑

...

$$\alpha = \{ \quad \quad \quad \}$$

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...

$$\alpha = \{ \neg x_2 \quad \}$$

Example

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$$g_1 = \begin{pmatrix} x_2 & x_3 \\ x_5 & x_6 \\ x_8 & x_9 \end{pmatrix} \mid x = x_5 \quad \begin{matrix} g.x = x_6 \\ \text{active} \end{matrix}$$

$$g_2 = \begin{array}{cc|cc|cc} (x_1 & x_2) & (x_4 & x_5) & (x_7 & x_8) \\ \uparrow & & \dots & & & \end{array} \quad \left| \quad x = x_1 \quad g.x = x_2 \right.$$

$$\alpha = \{\neg X_2, \neg X_3, X_1\}$$

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...

$$\alpha = \{\neg x_2, \neg x_3, x_1\}$$

g_2 generates $\omega = \{\neg x_1, x_2\}$

CDCL[Sym] Implementation

- Packaged as a library **cosy**³, to be combined with your solver
→ e.g. +3% LOC on MiniSAT.
- Follows symmetry status
- Should work with any enumerative SAT solver

³<https://github.com/lip6/cosy>

Experiments

Benchmark:

- from SAT contests 2012 – 2017
- retain only instances for which `bliss` finds significant symmetries in 1000s
- 1350 symmetric instances (out of 3700)

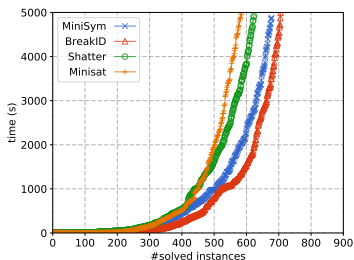
Setup:

- four tools
 - MiniSat (no symmetry, baseline)
 - MiniSat + BreakID (SOTA SAT solver using symmetries)
 - MiniSat + Shatter (SOTA SAT solver using symmetries)
 - **MiniSym** = MiniSat + CDCL[Sym] (our approach)
- 5000s timeout, 8GB memory
- includes time to compute symmetries (except for MiniSat)

Experimental results

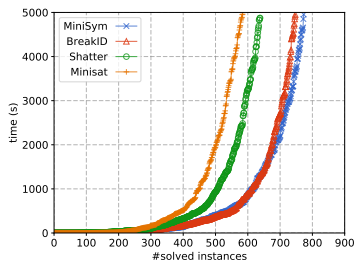
bliss gives more generators than saucy3

Figure: Cactus plot total number of instances



(a) with saucy3

	MiniSAT	Shatter	BreakID	MiniSym
PAR-2 sum	8 074 348	7 770 434	6 909 999	7 229 700
PAR-2 avg	5 981	5 756	5 119	5 355



(b) with bliss

	MiniSAT	Shatter	BreakID	MiniSym
PAR-2 sum	8 074 348	7 517 556	6 444 954	6 245 448
PAR-2 avg	5 981	5 569	4 774	4 626

Figure: Time comparison

Experimental results (UNSAT versus SAT)

	MiniSAT	Shatter	BreakID	MiniSym		MiniSAT	Shatter	BreakID	MiniSym
TOTAL (no dup)	261	302	371	345	TOTAL (no dup)	261	324	415	439
(a) With saucy3					(b) With bliss				

Figure: Comparison on UNSAT instances

	MiniSAT	Shatter	BreakID	MiniSym		MiniSAT	Shatter	BreakID	MiniSym
TOTAL	325	323	337	335	TOTAL	325	316	334	336
(a) With saucy3					(b) With bliss				

Figure: Comparison on SAT instances

ESBP + SP

Compose the symmetry propagation and the ESBP

prune the decision tree while accelerating its traversal

Problems

- ESBP breaks symmetries (incrementally)
- SP considers the manipulated symmetries valid all the time

In hybrid approach, SP must be able to identify
valid symmetries

Local symmetries

Formula \leftarrow (Symmetries)

$\omega_1 \leftarrow$ (Local symmetries)

$\omega_2 \leftarrow$ (Local symmetries)

$\omega_3 \leftarrow$ (Local symmetries)

$\omega_4 \leftarrow$ (Local symmetries)

Macro level

\rightarrow

Micro level

Local symmetries

Formula \leftarrow (Symmetries)

$\omega_1 \leftarrow$ (Local symmetries)

$\omega_2 \leftarrow$ (Local symmetries)

$\omega_3 \leftarrow$ (Local symmetries)

$\omega_4 \leftarrow$ (Local symmetries)

ω_5

Macro level

\rightarrow

Micro level

Local symmetries

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$\omega_4 \leftarrow$ (Local symmetries)

$\omega_5 \leftarrow$ (Local symmetries)

Macro level

\rightarrow

Micro level

Compute valid local symmetries on-the-fly at a minimal cost.

Experimental results

Benchmark:

- from SAT contests 2012 – 2018
- retain only instances for which `bliss` finds significant symmetries in 1000s
- 1400 symmetric instances (out of 4000)

Setup:

- Three tools
 - MiniSat SP (Minisat with Symmetry Propagation)
 - MiniSat ESBP (Minisat with CDCL[Sym])
 - **Minisat ESBP-SP (our approach)**
- 7200s timeout

Results:

Solver	PAR-2	ALL	SAT	UNSAT
SP	1674h00	876	406	470
ESBP	1578h30	904	416	488
ESBP-SP	1570h15	911	420	491

Conclusion

- A new dynamic symmetry breaking approach
 - Generation of SBP on the fly
 - Package as a library cosyusable with any CDCL solver
 - Overcomes drawbacks of the existing approaches
- A new hybrid approach (ESBP-SP)
 - Take advantage of static and dynamic approach

Perspectives

- Combination of ESBP with other dynamic symmetry breaking approach
- Exploitation of partial symmetries

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Thanks !



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Ashish Sabharwal.

Symchaff: A structure-aware satisfiability solver.

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CDCL in action TODO



$$\omega_1 = \{x_1, x_2, x_3\}$$

$$\omega_2 = \{x_4, x_5, x_6\}$$

$$\omega_3 = \{\neg x_1, \neg x_5\}$$

$$\omega_4 = \{\neg x_2, \neg x_4\}$$

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CDCL in action TODO



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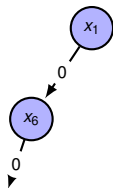
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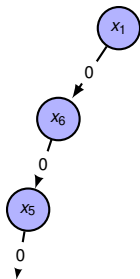
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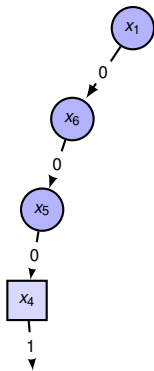
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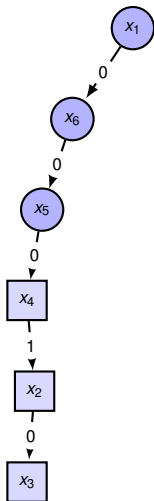
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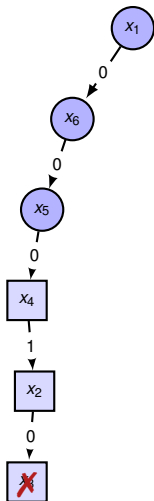
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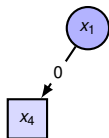
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CDCL in action TODO



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$$\omega_6 = \{\neg x_3, \neg x_6\}$$

$$\omega_7 = \{x_1, \neg x_4\}$$

Weakly active symmetries

Logical consequence

When ω is satisfied in all satisfying assignments of φ , we say that ω is a logical consequence of φ , and we denote this by $\varphi \vdash \omega$.

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Symmetry propagation

Let σ a weakly active symmetry, then

$$\varphi \cup \alpha \vdash \{I\} \Leftrightarrow \varphi \cup \alpha \vdash \sigma.\{I\}$$

Local symmetries

Logical consequence

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Local Symmetries

Let φ be a formula. We define $L_{\omega, \varphi}$, the set of *local symmetries* for a clause ω , and with respect to a formula φ , as follows:

$$L_{\omega, \varphi} = \{\sigma \in \mathfrak{S} \mid \varphi \vdash \sigma.\omega\}$$

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We can state that:

$$\bigcap_{\omega \in \varphi} L_{\omega, \varphi} \subseteq G.$$

Computing local symmetries

Formula can be decomposed as : $\varphi = \varphi_o \cup \varphi_e \cup \varphi_d$ where

- φ_o is the set of the original clauses
- φ_e is the set of ESBPs
- φ_d is the set of deduced clauses.

Local symmetries

- $\omega \in \varphi_o, L_{\omega, \varphi} \supseteq G$
- $\omega \in \varphi_e, L_{\omega, \varphi} \supseteq \text{Stab}(\omega) = \{\sigma \in G \mid \omega = \sigma.\omega\}$
- $\omega \in \varphi_d, L_{\omega, \varphi} \supseteq \left(\bigcap_{\omega' \in \varphi_1} L_{\omega', \varphi} \right) \cup \text{Stab}(\omega)$

where φ_1 is the set of clauses that derives ω .