Exploitation des symétries dynamiques pour la résolution des problèmes SAT

Thèse de doctorat de Sorbonne Université

Hakan METIN

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LIP6

Motivation

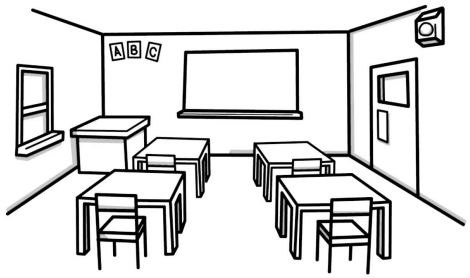
Applications:

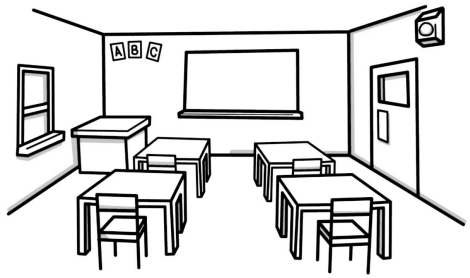
- planning
- scheduling
- combinatorial design
- system analysis (model-checking)
- etc

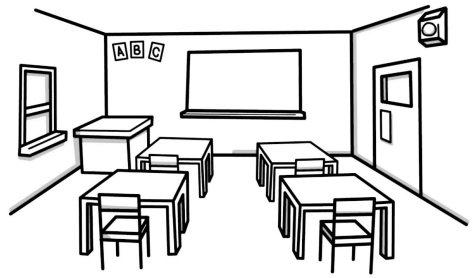
Outline

1. SAT
   - SAT solving
   - Symmetry
   - Challenges
2. Tackling the explosion in the static symmetry breaking approach
   -
   -
   - Experimental results
3. Taking the maximum benefits from static and dynamic approaches
   - a
   - b
   - Experimental results

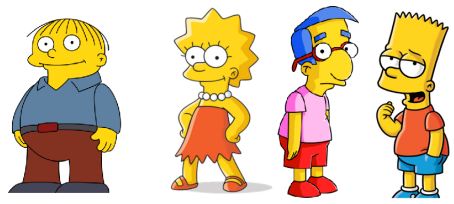
SAT an example

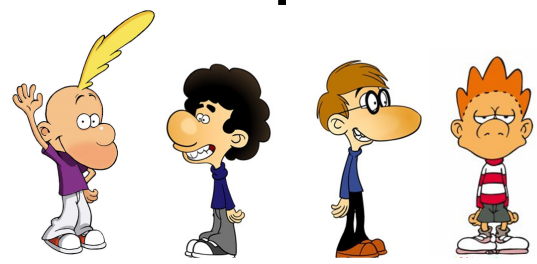






1 2 3

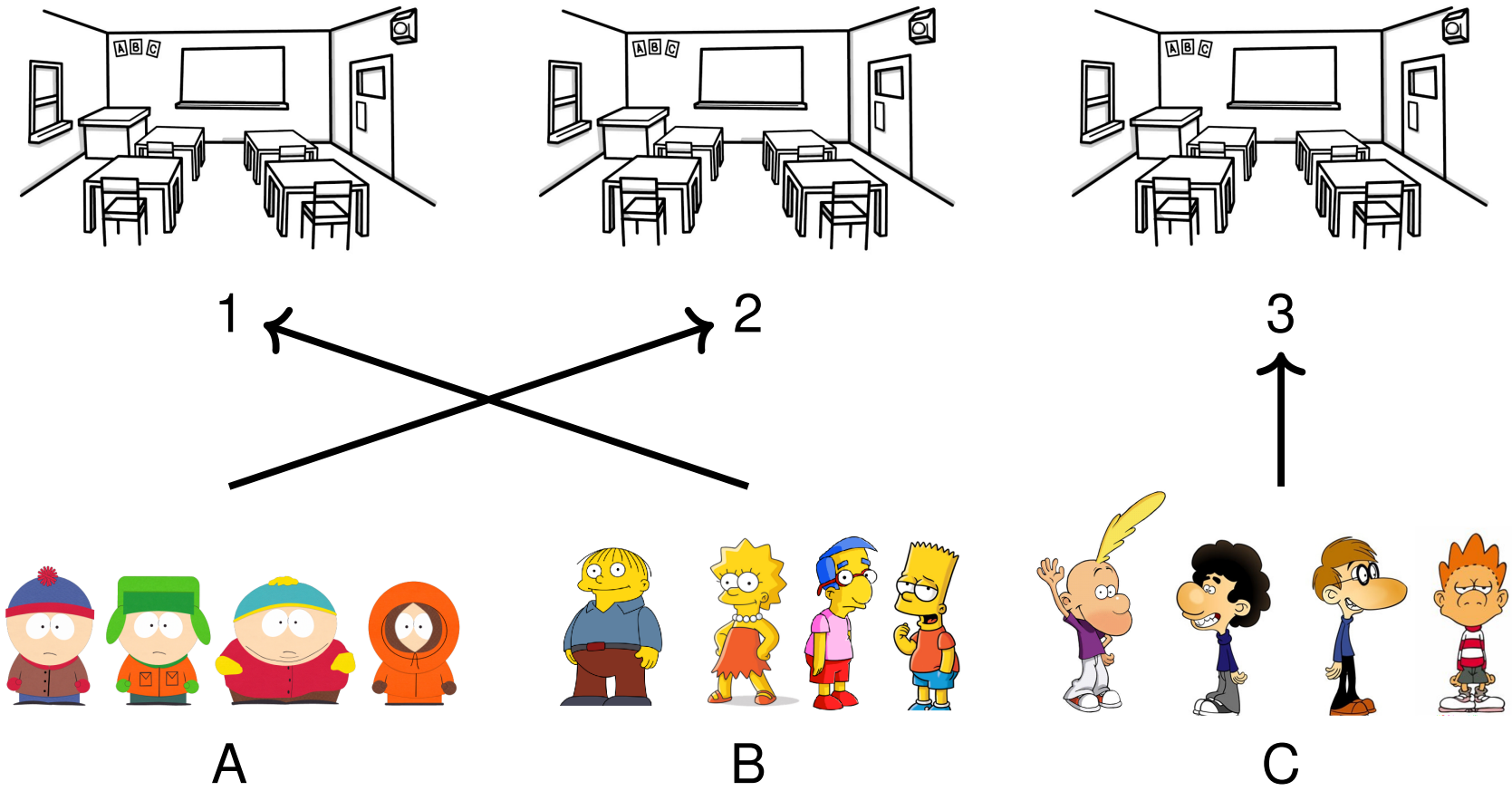




A B C

Attribute each group to a class room

SAT an example



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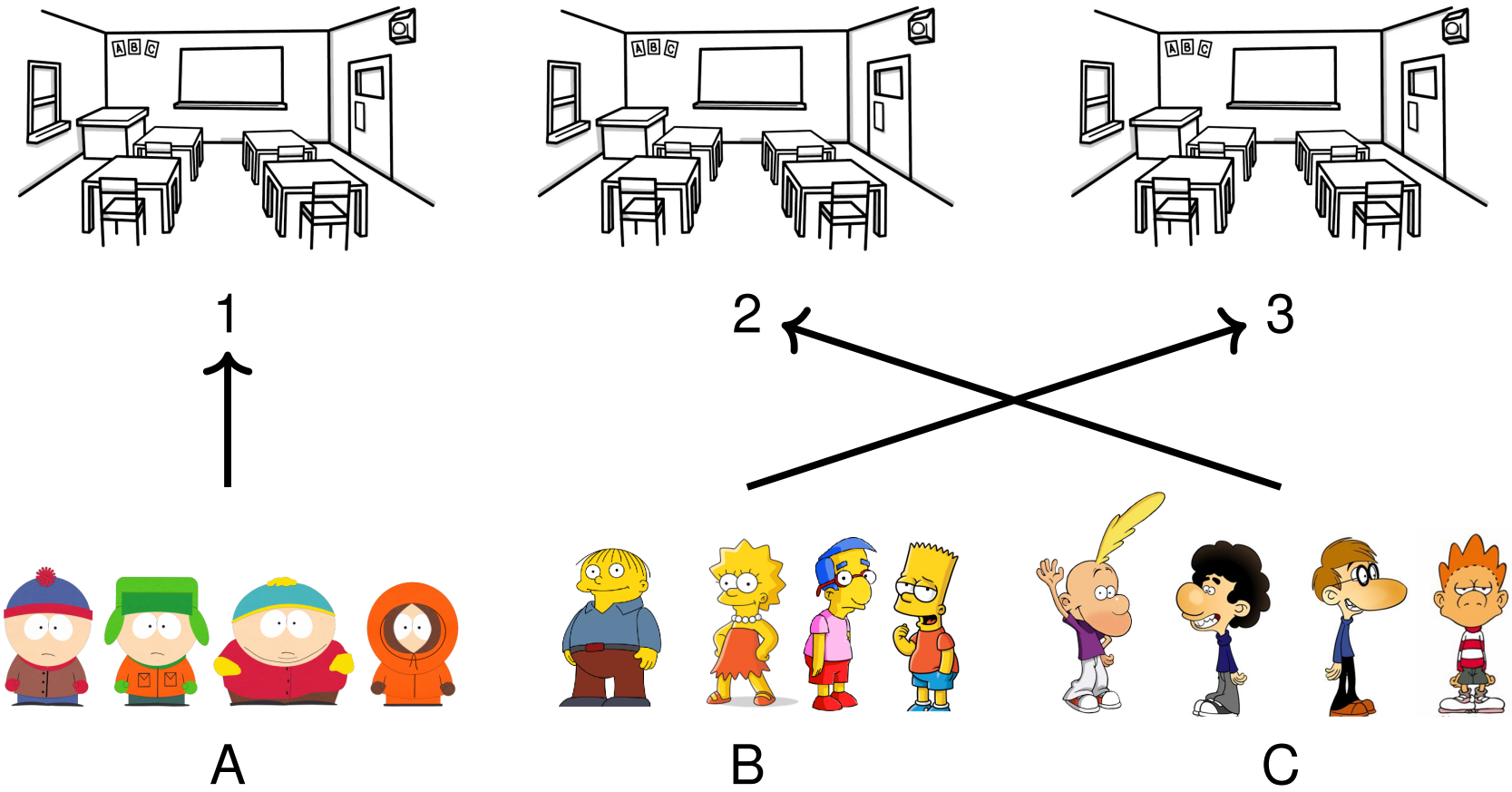

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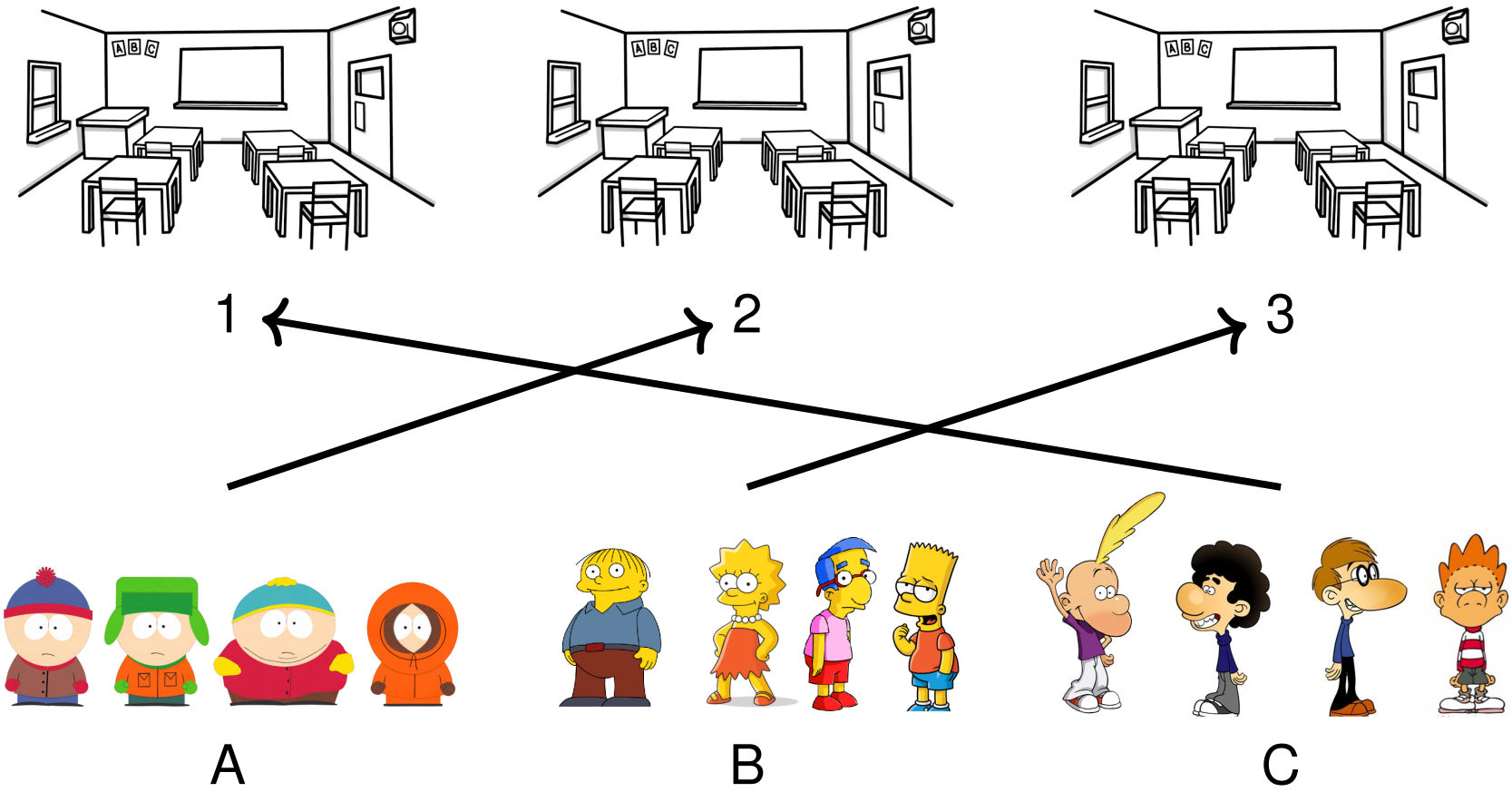

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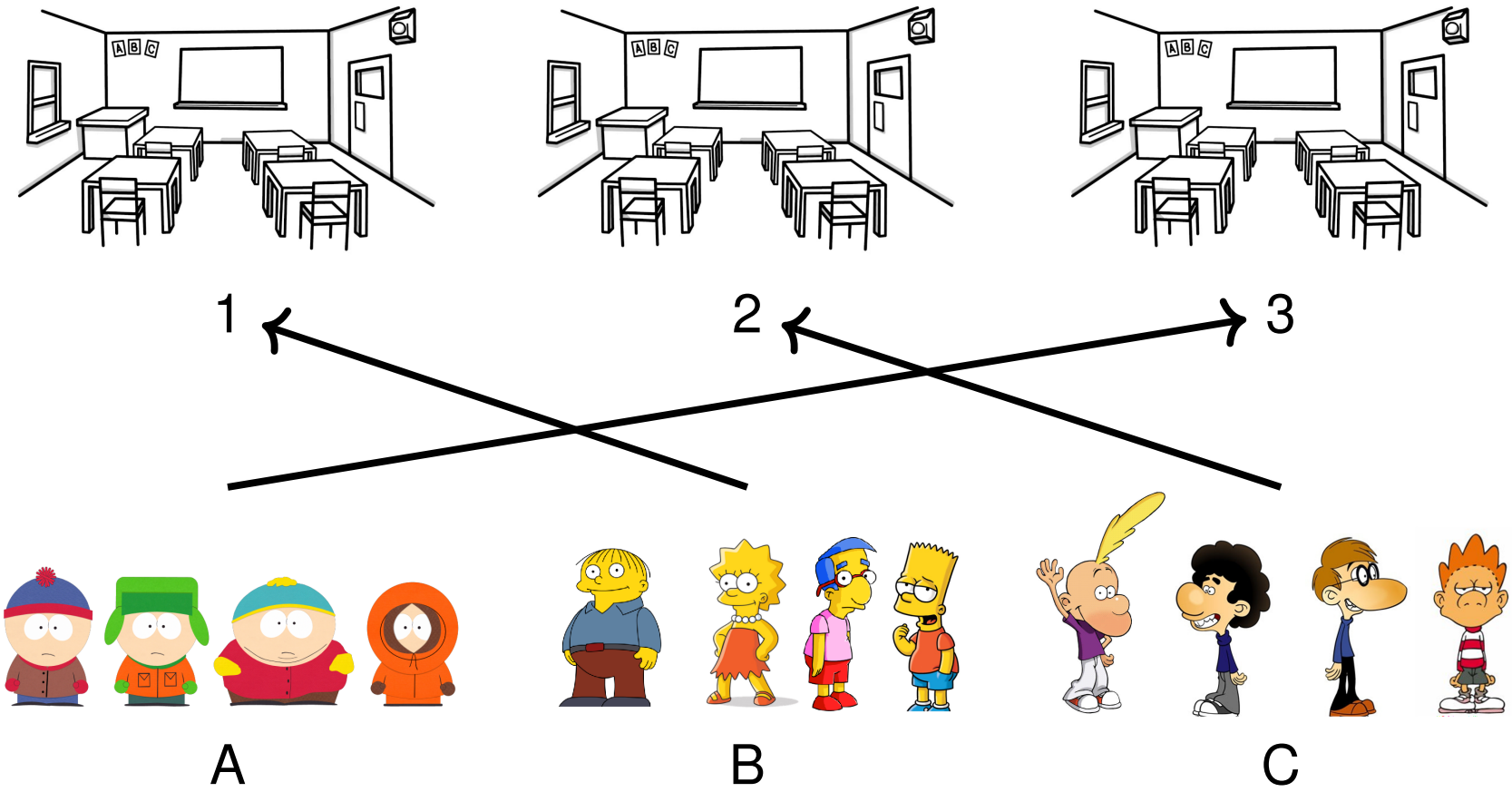

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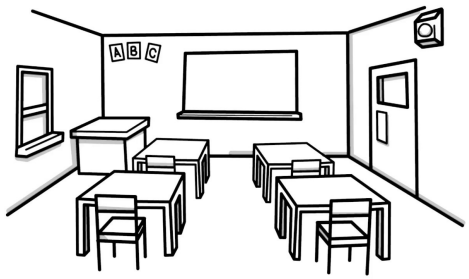
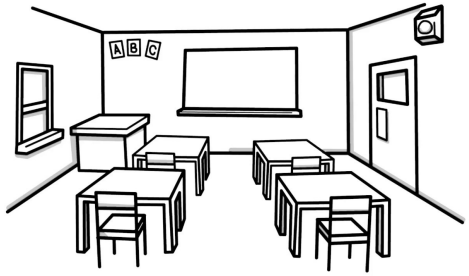
SAT an example 2

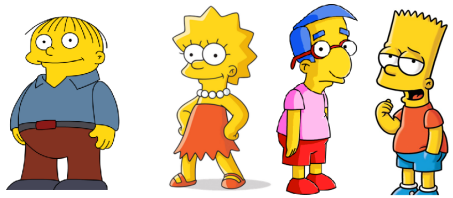


1

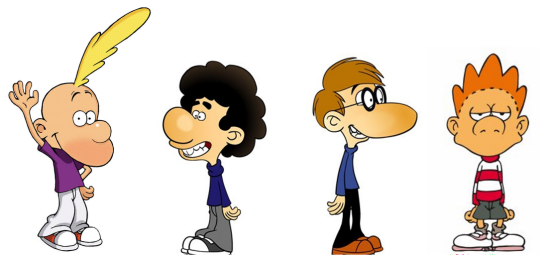


2

A



B



C

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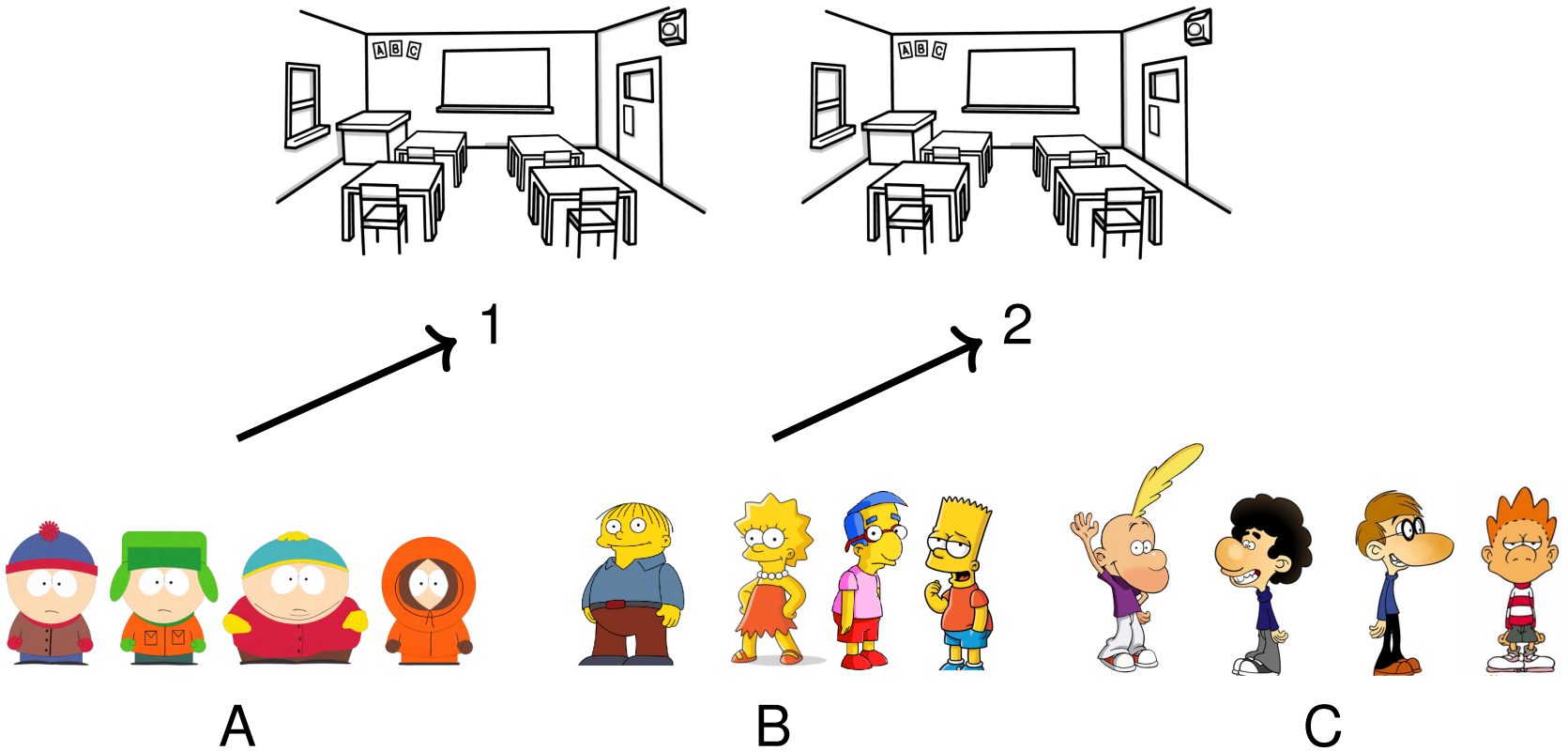
SAT an example 2



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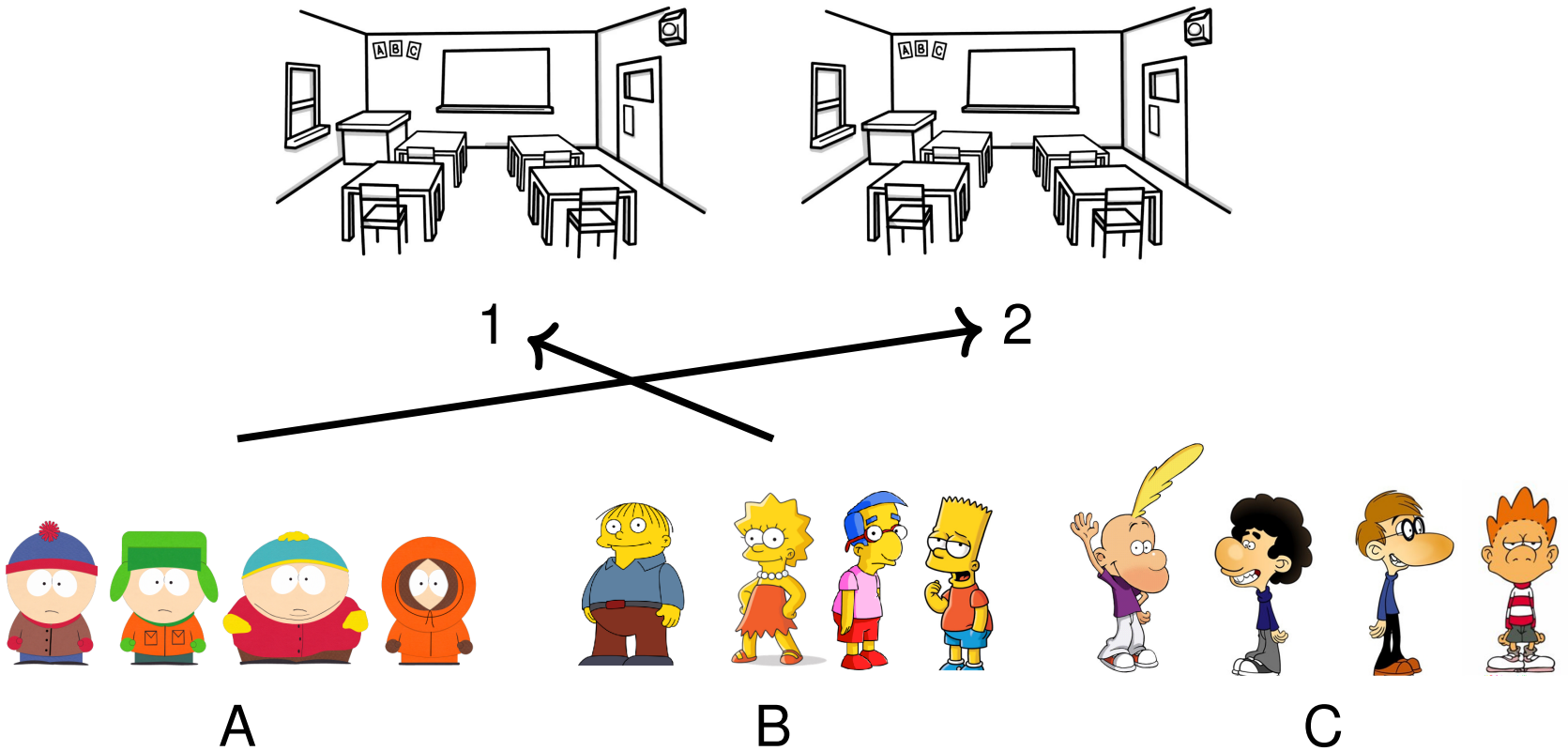

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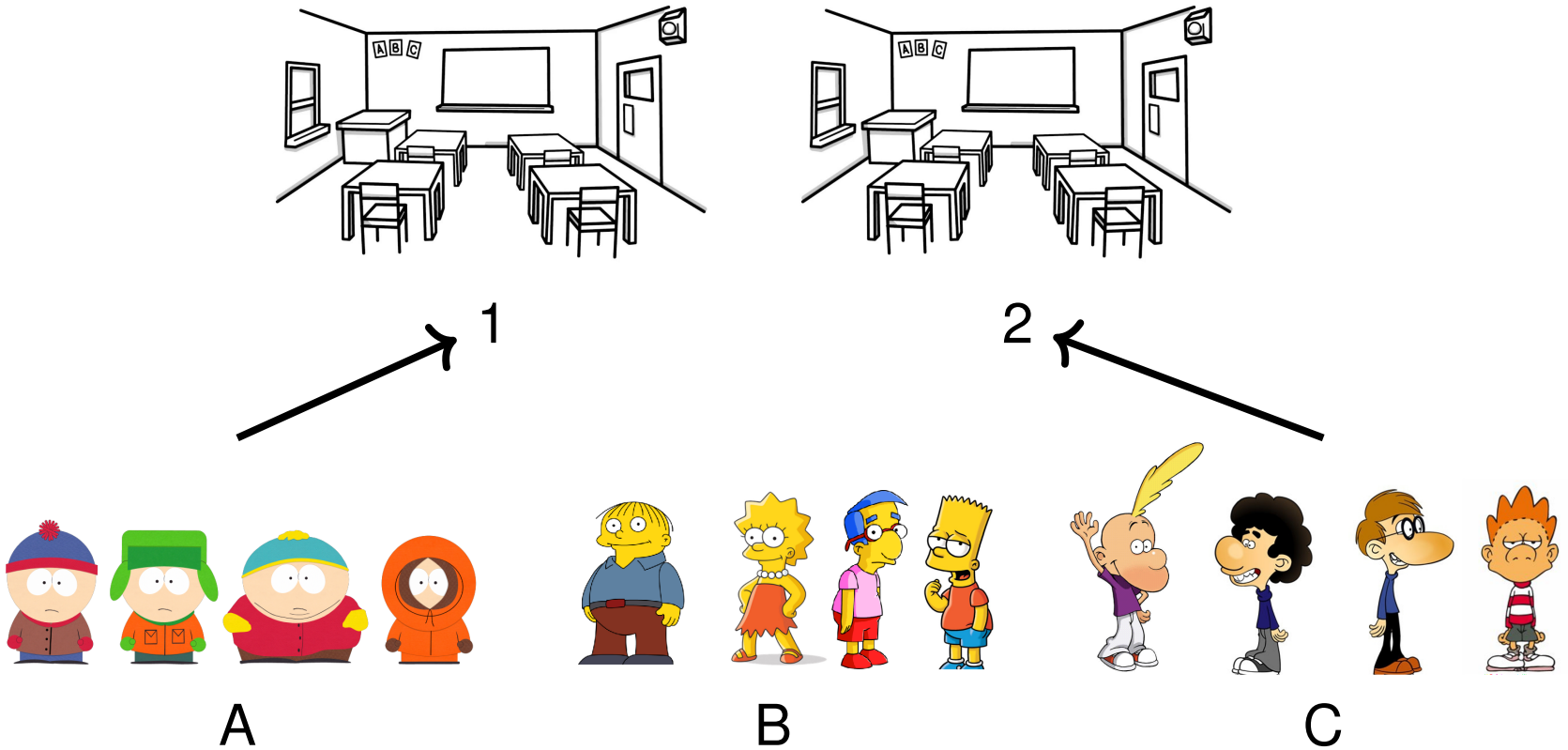

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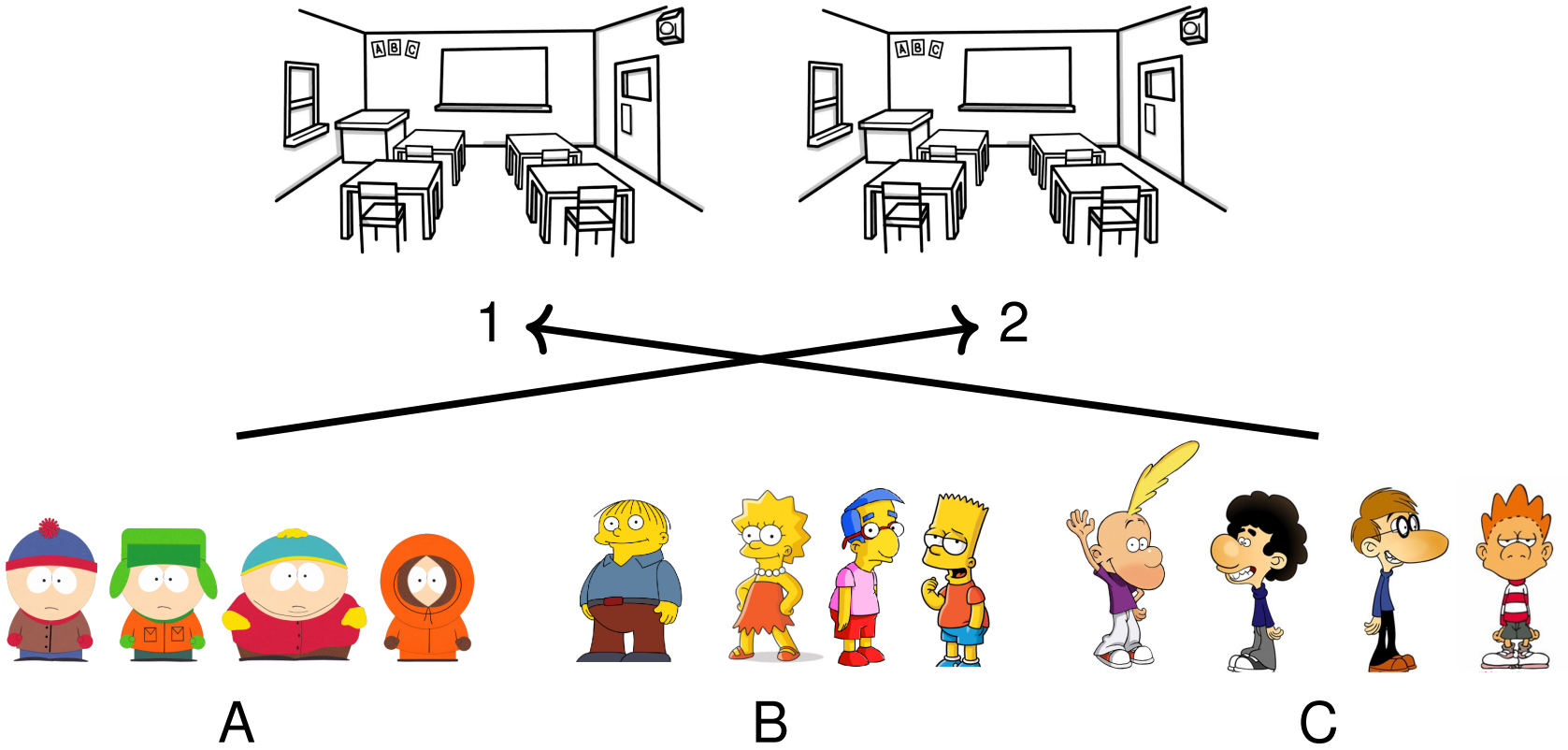

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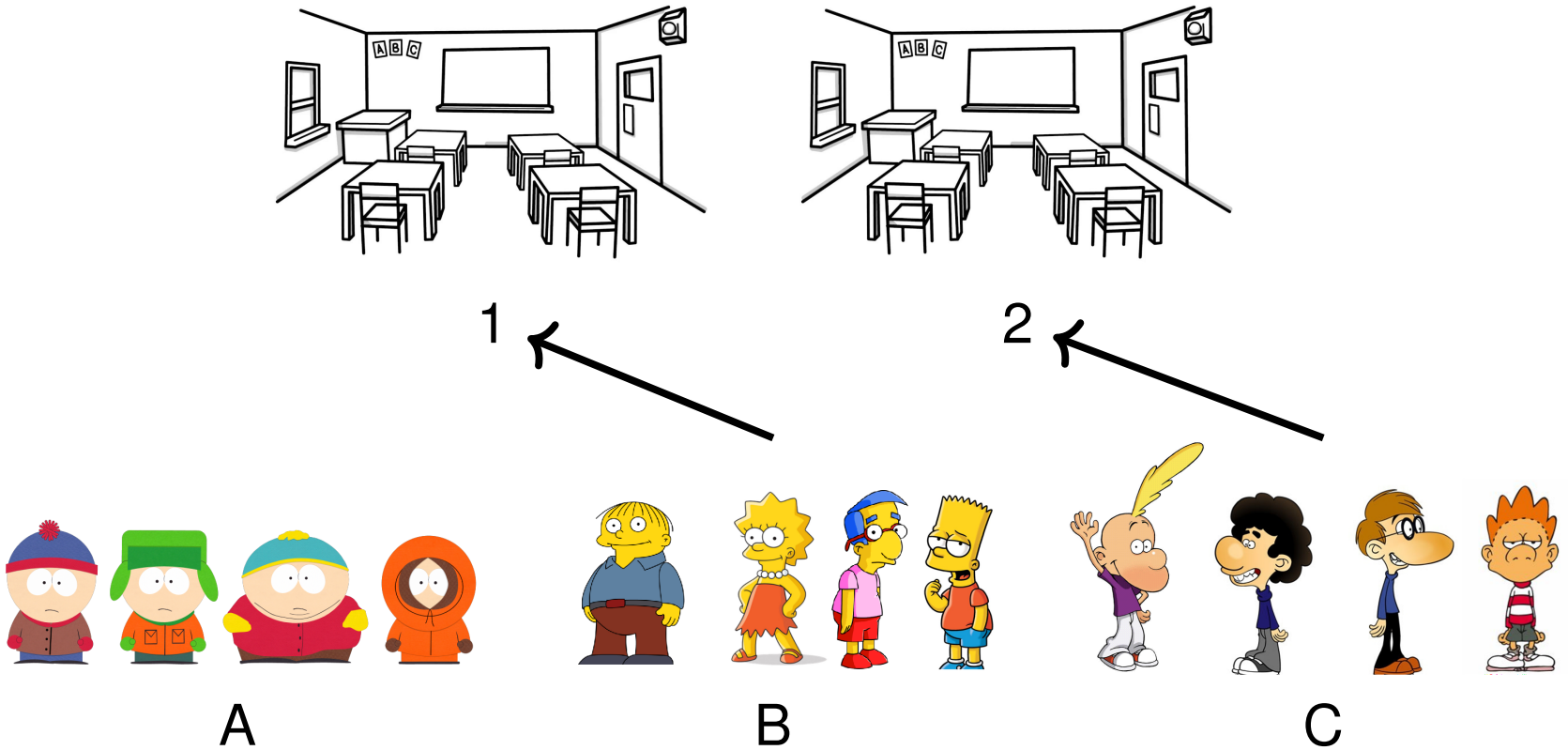

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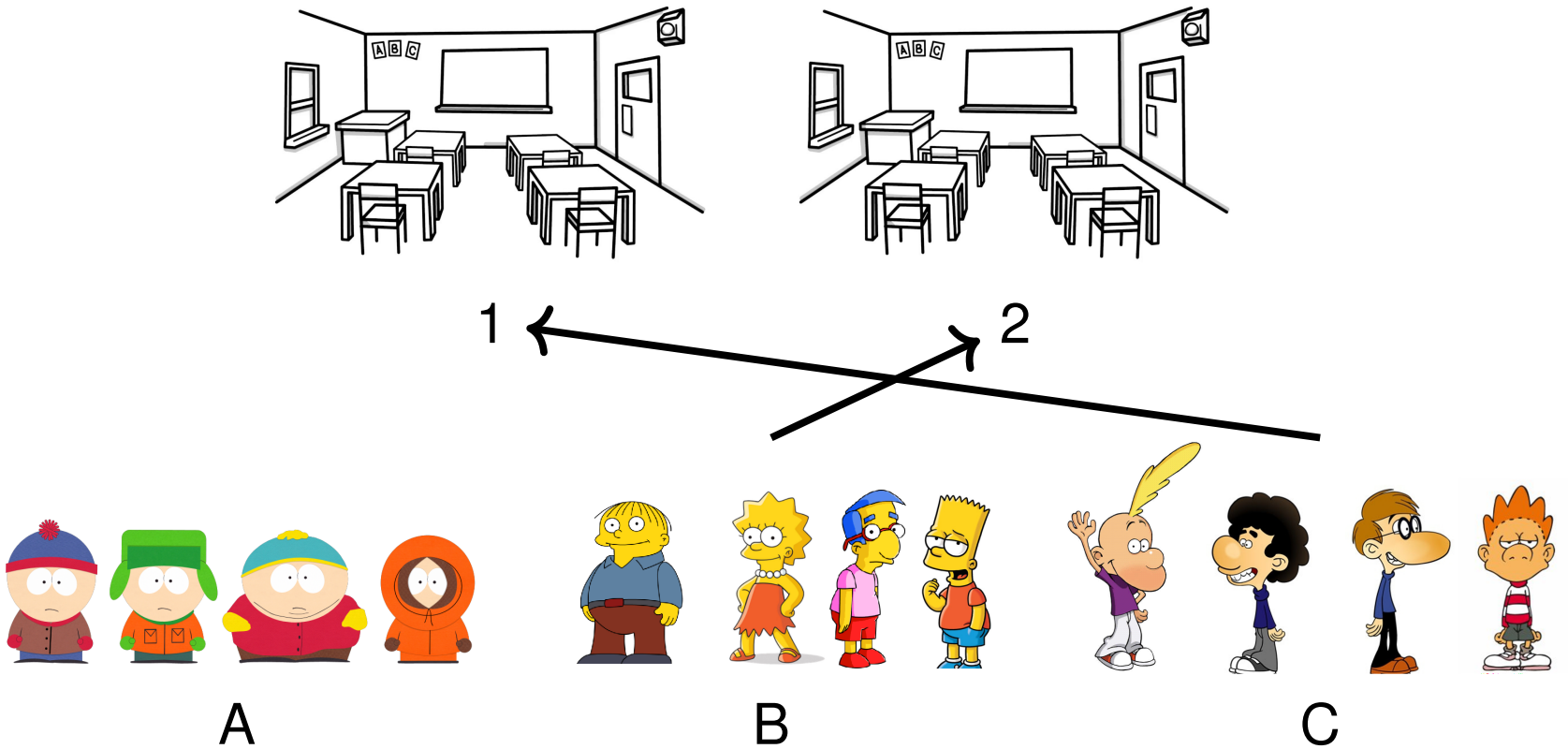

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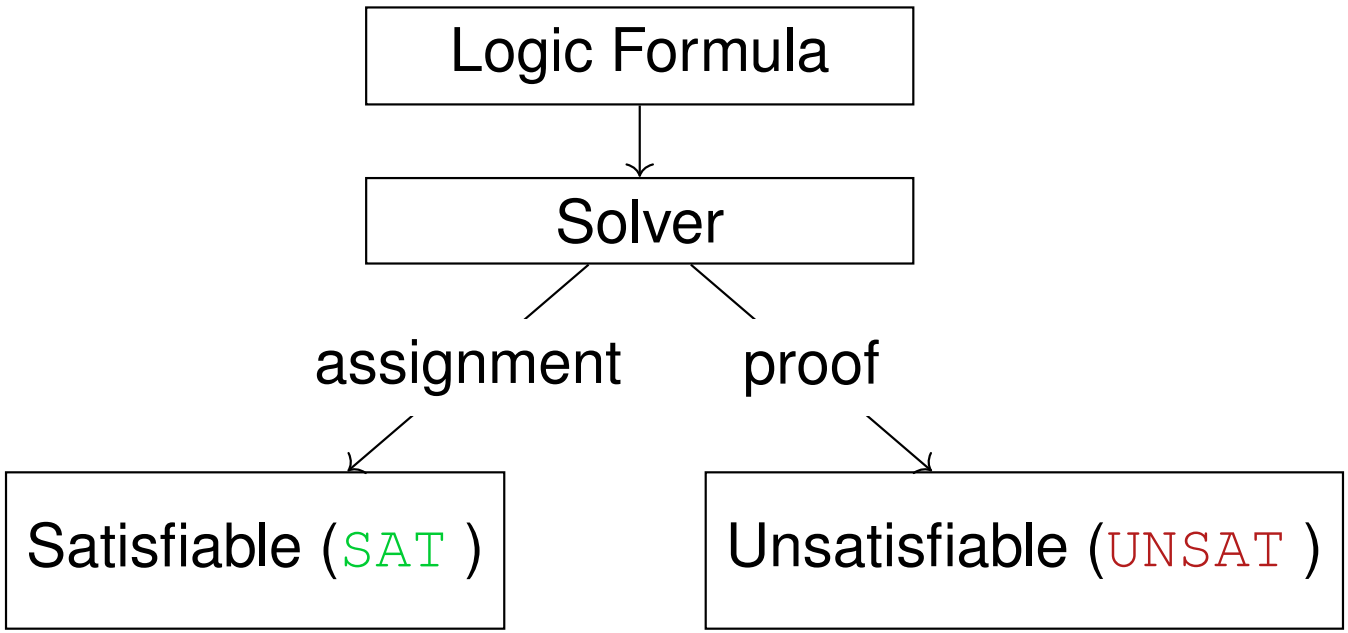
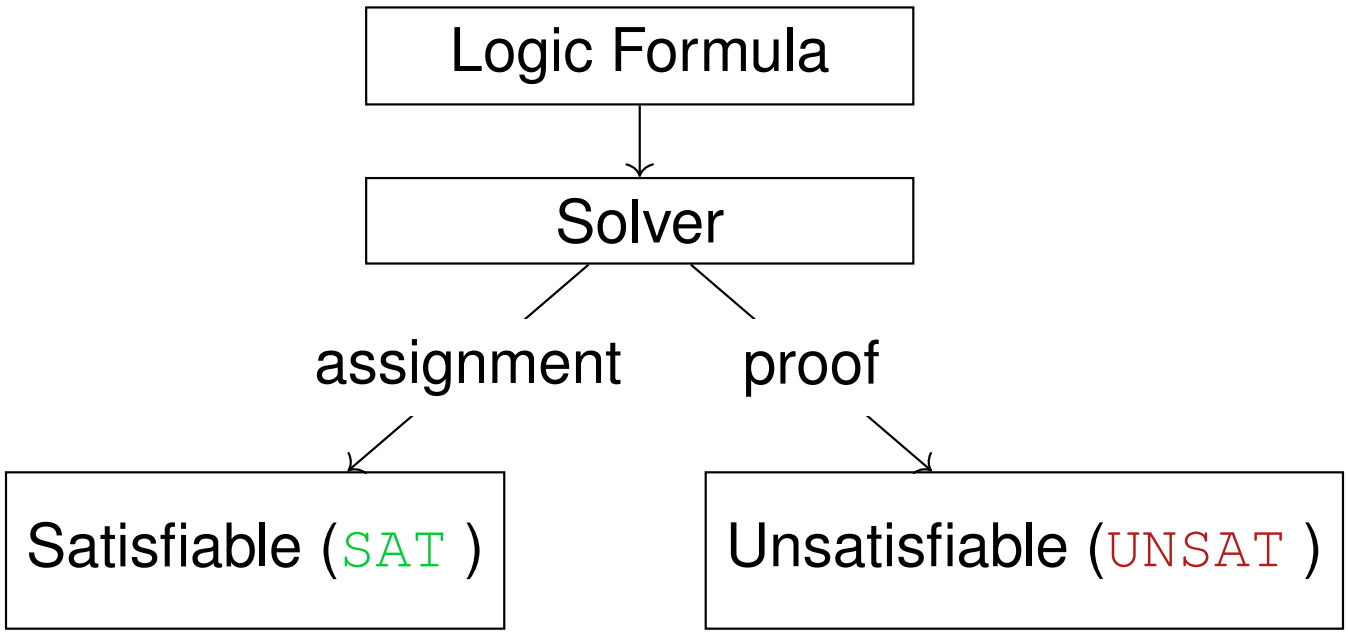
SAT


Logic Formula
Solver
assignment
proof
Satisfiable (SAT)
Unsatisfiable (UNSAT)

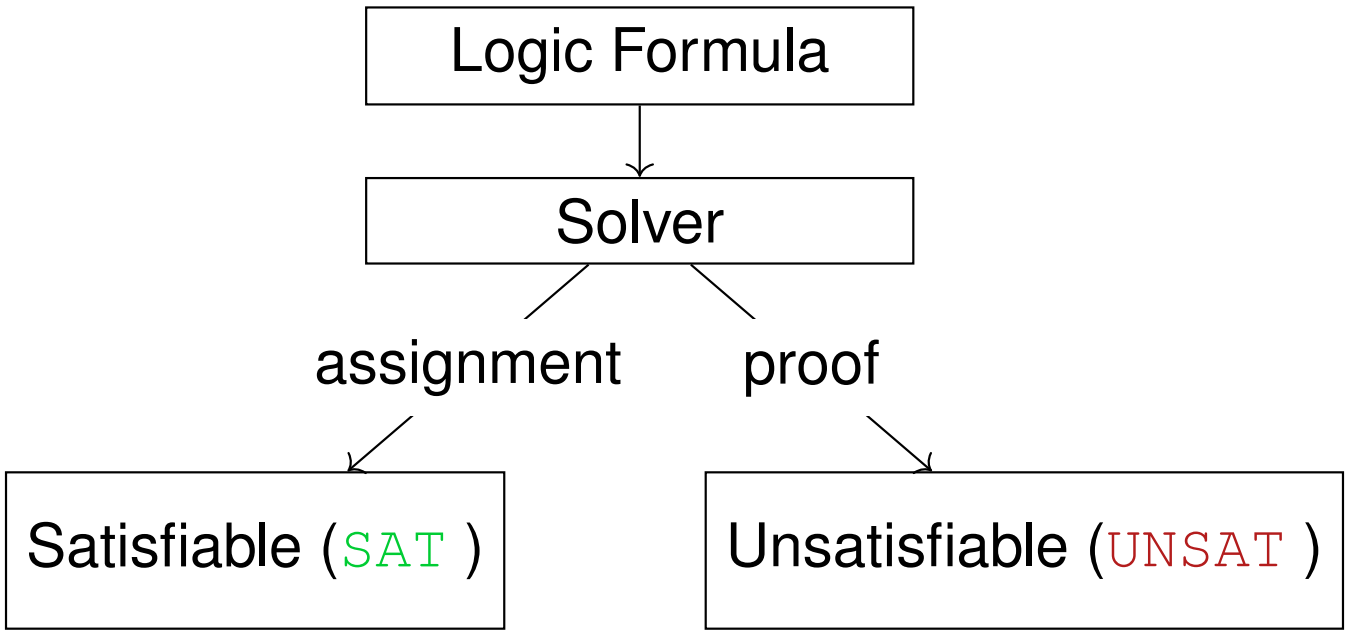
SAT



$$\underbrace{(x_1 \vee x_2 \vee \neg x_3)}$$

Clause with literals $x_1, x_2, \neg x_3$

SAT



Formula (CNF)

$$\overbrace{\underbrace{(x_1 \vee x_2 \vee \neg x_3)}_{\textit{Clause}} \wedge (\neg x_1 \vee \neg x_2) \wedge (x_2 \vee \neg x_4)}$$

Computing symmetries of a SAT problem

CNF formula

$$(x_1 \vee x_2 \vee x_3) \wedge (x_4 \vee x_5 \vee x_6) \wedge (x_7 \vee x_8 \vee x_9)$$
$$\wedge(\neg x_1 \vee \neg x_4) \wedge (\neg x_1 \vee \neg x_7) \wedge (\neg x_4 \vee \neg x_7)$$
$$\wedge(\neg x_2 \vee \neg x_5) \wedge (\neg x_2 \vee \neg x_8) \wedge (\neg x_5 \vee \neg x_8)$$
$$\wedge(\neg x_3 \vee \neg x_6) \wedge (\neg x_3 \vee \neg x_9) \wedge (\neg x_6 \vee \neg x_9)$$

[1] http://www.tcs.hut.fi/Software/bliss/

[2] http://vlsicad.eecs.umich.edu/BK/SAUCY/

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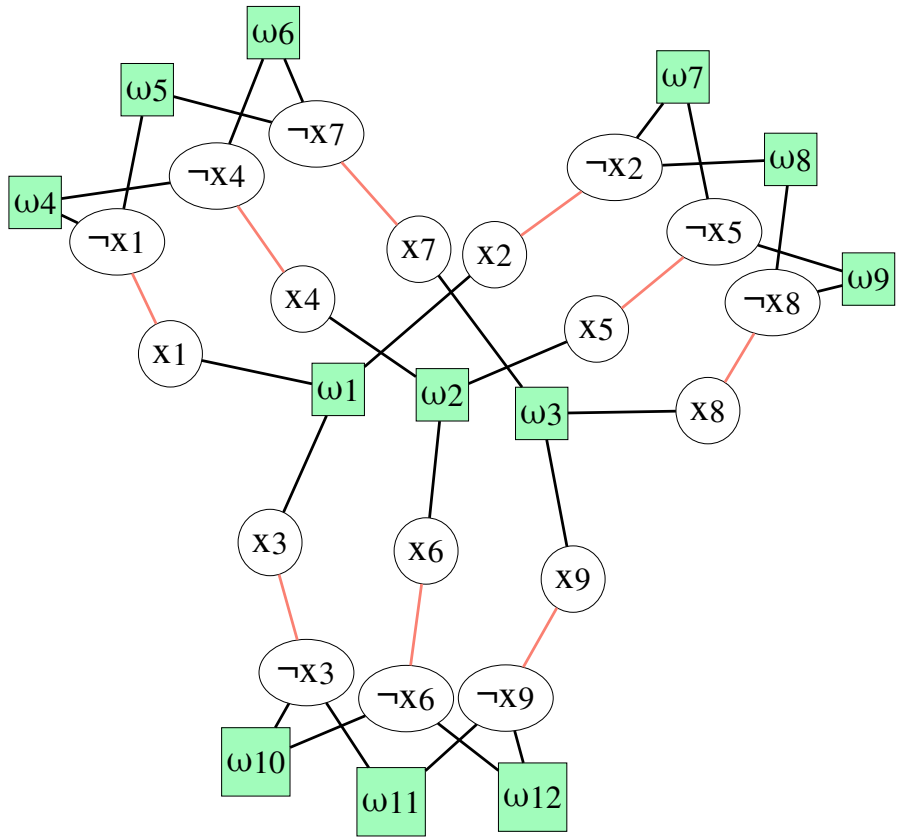
$$\wedge(\neg x_2 \vee \neg x_5) \wedge (\neg x_2 \vee \neg x_8) \wedge (\neg x_5 \vee \neg x_8)$$
$$\wedge(\neg x_3 \vee \neg x_6) \wedge (\neg x_3 \vee \neg x_9) \wedge (\neg x_6 \vee \neg x_9)$$

$\Downarrow$ $\Downarrow$

colored graph



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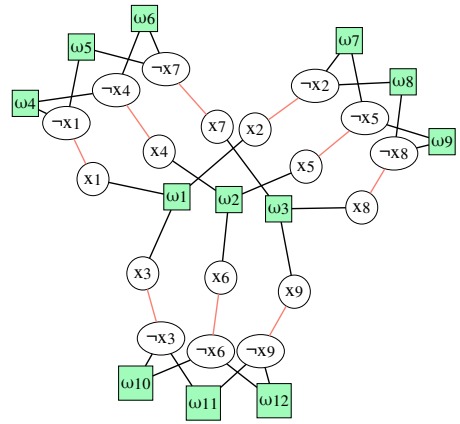
$\Downarrow$ $\qquad$ $\Downarrow$

colored graph



$\|$ $\qquad$ $\|$

graph automorphism $\qquad$ (`bliss` [1] or `saucy` [2])

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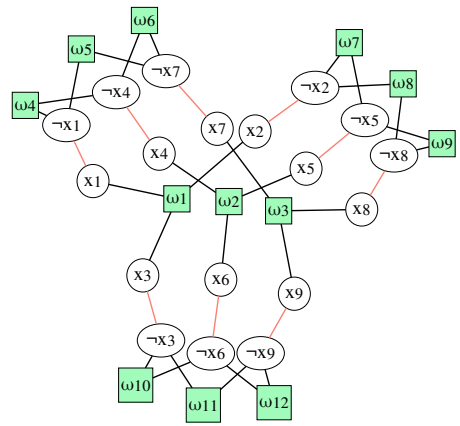
$\Downarrow$

colored graph



$\Downarrow$

graph automorphism

$\Downarrow$

set of symmetries

$\Downarrow$

(`bliss` [1] or `saucy` [2])

$\Downarrow$

$$g_1 = (x_2\ \ x_3)(x_5\ \ x_6)(x_8\ \ x_9)$$
$$g_2 = (x_4\ \ x_7)(x_5\ \ x_8)(x_6\ \ x_9)$$
$$g_3 = (x_1\ \ x_2)(x_4\ \ x_5)(x_7\ \ x_8)$$
$$g_4 = (x_1\ \ x_4)(x_2\ \ x_5)(x_3\ \ x_6)$$

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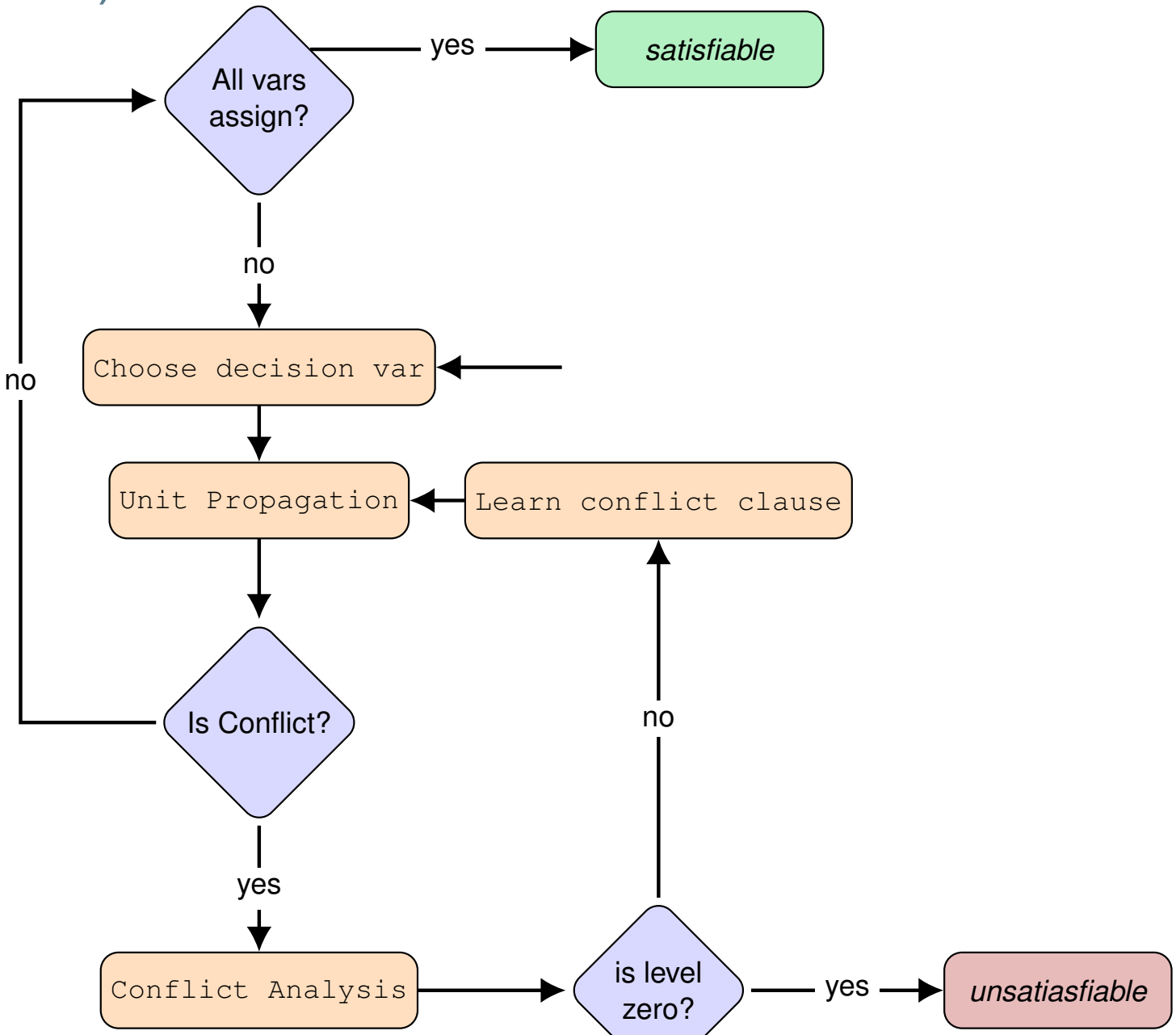
[2] http://vlsicad.eecs.umich.edu/BK/SAUCY/

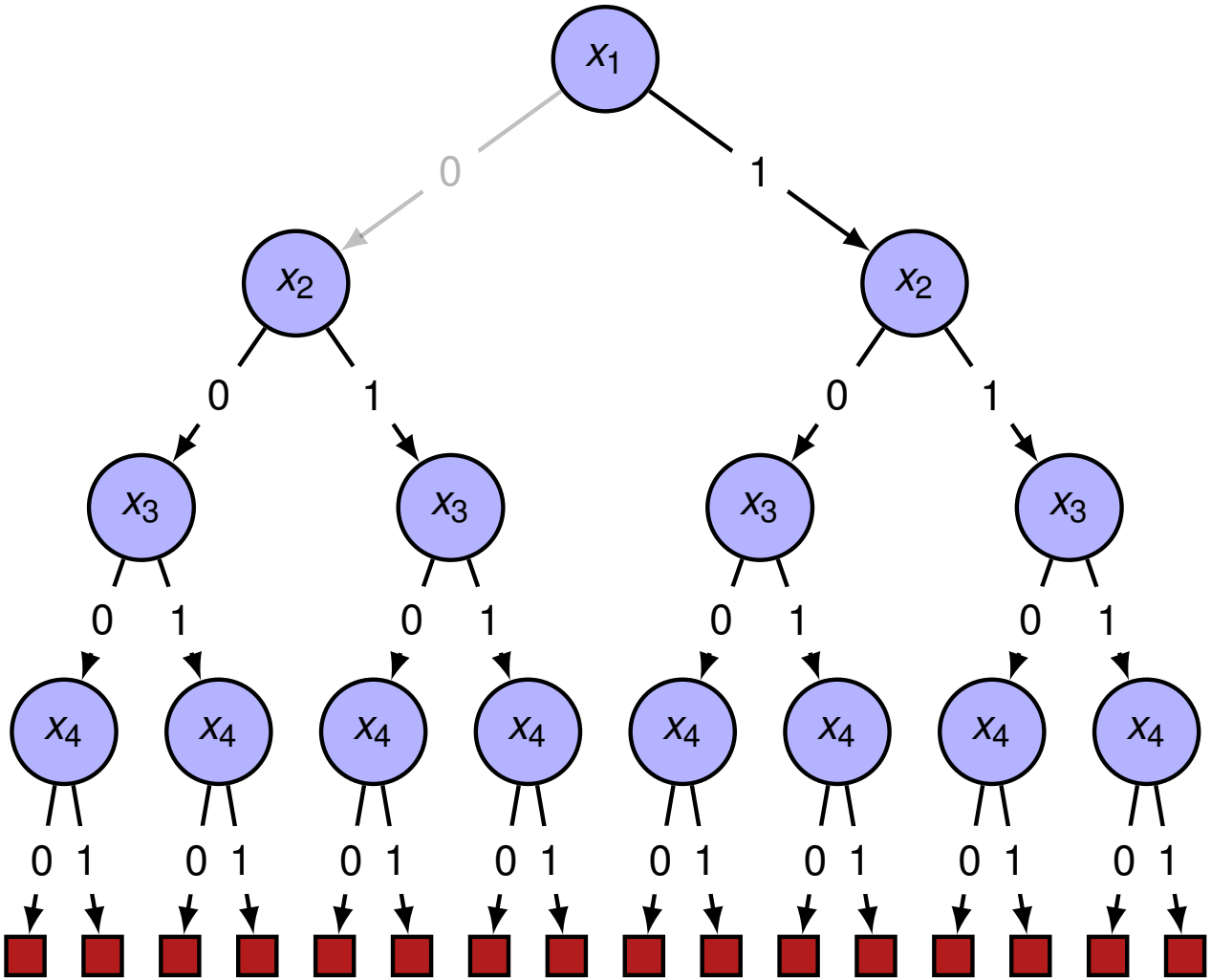
SAT

example solving arbre

Conflict Driven Clause Learning Algorithm (CDCL)



Tree


x_1
0
1
x_2
x_2
0
1
0
1
x_3
x_3
x_3
x_3
0 1
0 1
0 1
0 1
x_4 x_4 x_4 x_4 x_4 x_4 x_4 x_4
0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1