

# Exploitation of dynamic symmetries for solving SAT problems

Thèse de doctorat de Sorbonne Université

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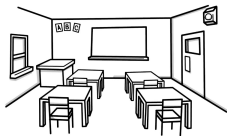


# Motivation

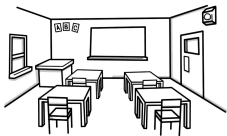
SATisfiability is widely used in different domains

- Artificial intelligence (planning, games, ...)
- Bioinformatics (haplotype inference, ...)
- Security (cryptanalysis, inversion attack on hash , ...)
- Computationally hard problems (graph coloring, ...)
- Formal methods (hardware model checking, ...)

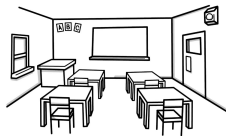
# SAT an example



1



2



3



A



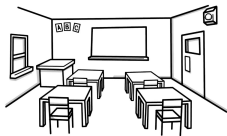
B



C

Is it possible to attribute each group to a classroom?

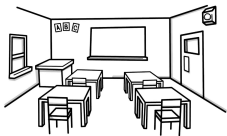
# SAT an example



1  
↑



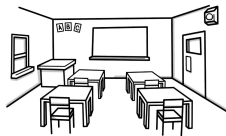
A



2  
↑



B



3  
↑

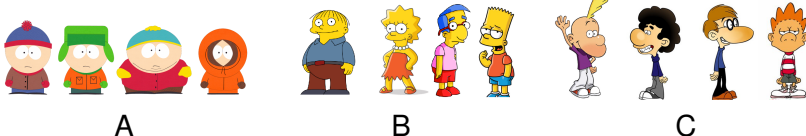
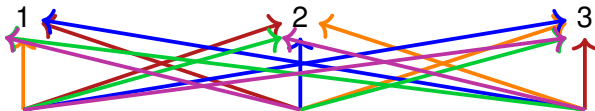
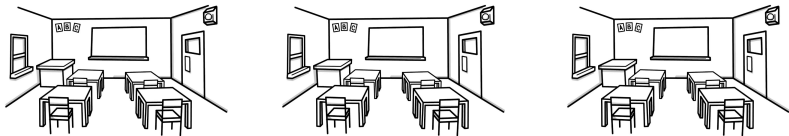


C

Is it possible to attribute each group to a classroom?

YES!

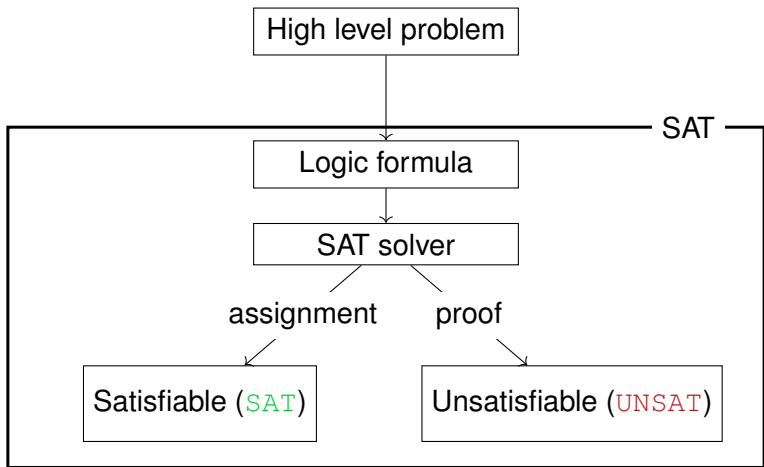
# SAT an example



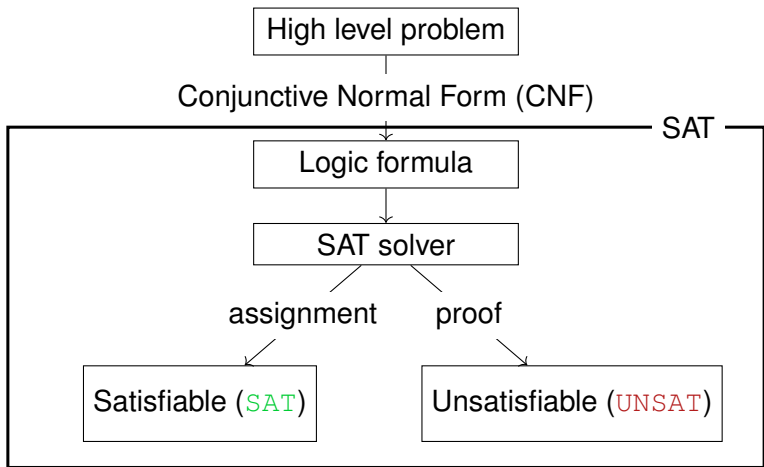
Is it possible to attribute each group to a classroom?

YES! Many solutions

# Boolean SATisfiability problem



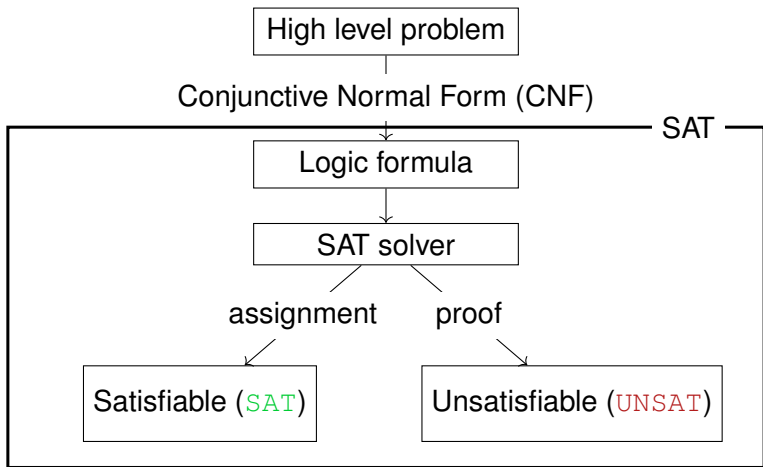
# Boolean SATisfiability problem



CNF representation:

$$\underbrace{(x_1 \vee x_2 \vee \neg x_3)}_{\text{Clause with literals } x_1, x_2, \neg x_3}$$

# Boolean SATisfiability problem

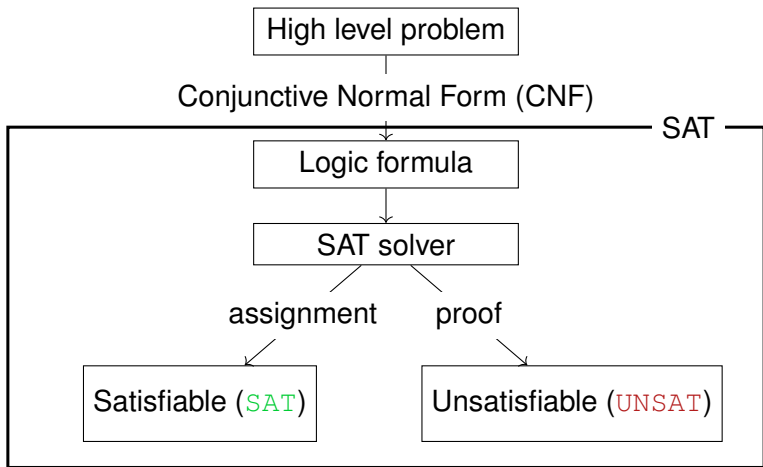


CNF representation:

$$\underbrace{(x_1 \vee x_2 \vee \neg x_3)}_{\text{Clause}} \wedge \underbrace{(\neg x_1 \vee \neg x_2) \wedge (x_2 \vee \neg x_4)}_{\text{Formula (CNF)}}$$



# Boolean SATisfiability problem



Clause representation as a set:

$$(x_1 \vee x_2 \vee \neg x_3) \rightarrow \{x_1, x_2, \neg x_3\}$$

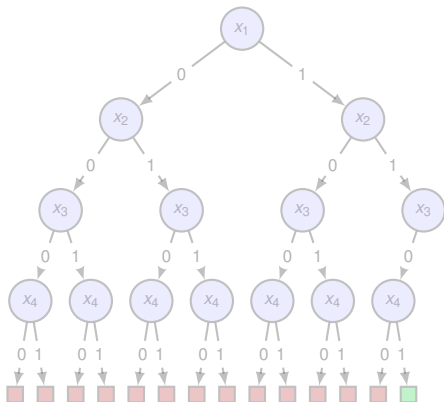
# SAT Solving

Solving SAT formula is known to be **NP-complete** [Coo71]

Enumerative algorithms:

- Davis, Putnam, Logemann, and Loveland (DPLL) [DLL62]
  - Boolean Constraint Propagation (BCP)
- Conflict Driven Clause Learning (CDCL) [MSS99]
  - Derived from DPLL
  - Clause learning

# CDCL in action



$$\omega_1 = \{x_1, x_2, x_3, x_4\}$$

$$\omega_2 = \{x_1, \neg x_4\}$$

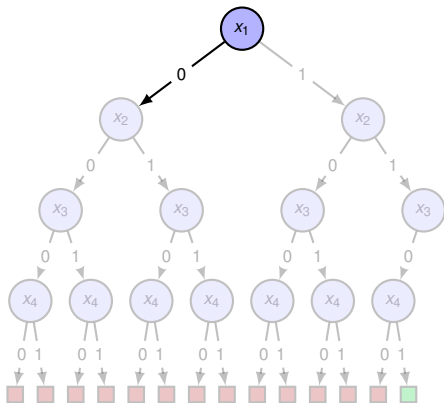
$$\omega_3 = \{x_1, x_4\}$$

$$\omega_4 = \{x_2, \neg x_4\}$$

$$\omega_5 = \{x_2, x_4\}$$

$$\omega_6 = \{x_3, x_4\}$$

# CDCL in action



$$\omega_1 = \{\mathbf{x}_1, x_2, x_3, x_4\}$$

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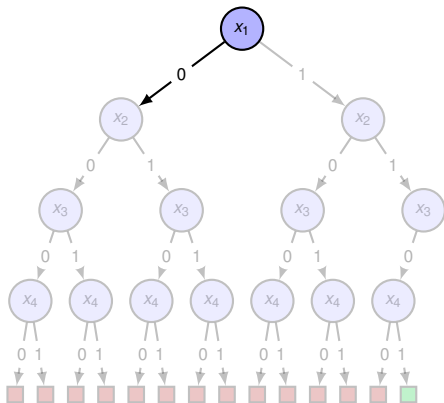
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# CDCL in action



$$\omega_1 = \{\textcolor{red}{x}_1, x_2, x_3, x_4\}$$

$$\omega_2 = \{\textcolor{red}{x}_1, \neg \textcolor{blue}{x}_4\}$$

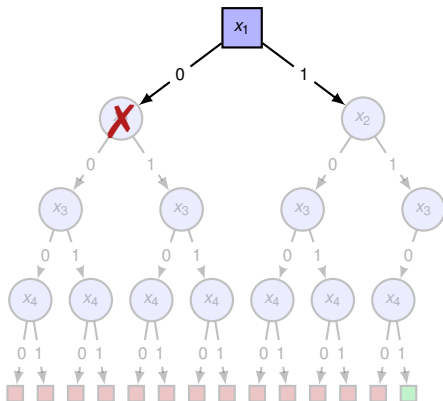
$$\omega_3 = \{\textcolor{red}{x}_1, \textcolor{blue}{x}_4\}$$

$$\omega_4 = \{x_2, \neg x_4\}$$

$$\omega_5 = \{x_2, x_4\}$$

$$\omega_6 = \{x_3, x_4\}$$

# CDCL in action



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$$\omega_2 = \{x_1, \neg x_4\}$$

$$\omega_3 = \{x_1, x_4\}$$

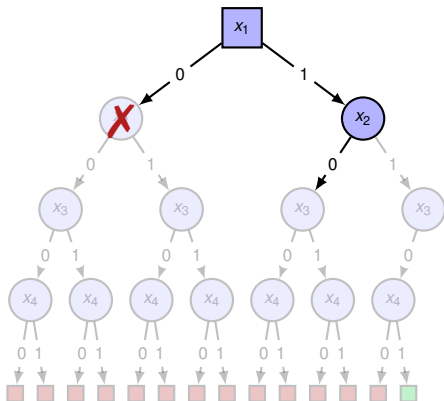
$$\omega_4 = \{x_2, \neg x_4\}$$

$$\omega_5 = \{x_2, x_4\}$$

$$\omega_6 = \{x_3, x_4\}$$

$$\omega_7 = \{x_1\}$$

# CDCL in action



$$\omega_1 = \{x_1, x_2, x_3, x_4\}$$

$$\omega_2 = \{x_1, \neg x_4\}$$

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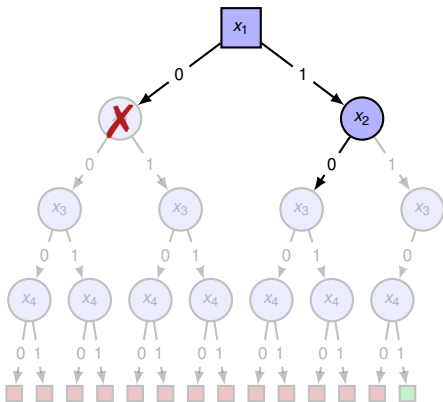
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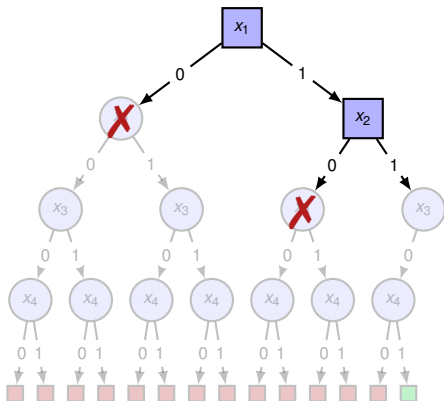
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# CDCL in action



$$\omega_1 = \{x_1, x_2, x_3, x_4\}$$

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$$\omega_3 = \{x_1, x_4\}$$

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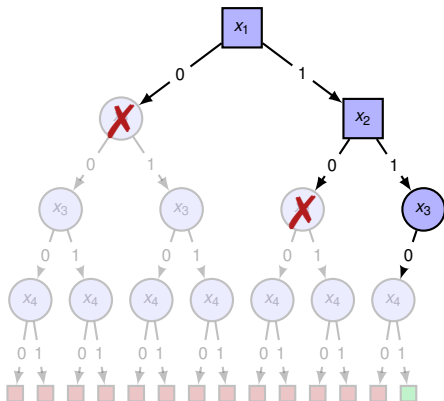
$$\omega_5 = \{x_2, x_4\}$$

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$$\omega_7 = \{x_1\}$$

$$\omega_8 = \{x_2\}$$

# CDCL in action



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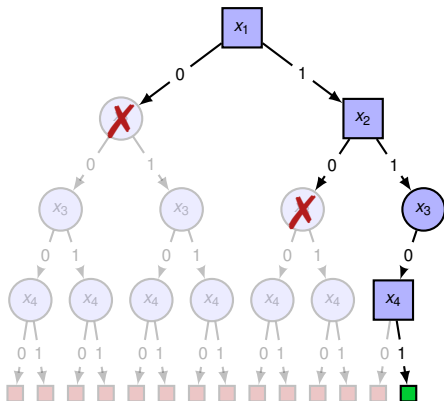
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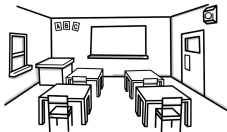
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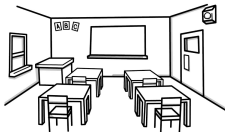
$$\omega_7 = \{x_1\}$$

$$\omega_8 = \{x_2\}$$

# An UNSAT example



1



2



A



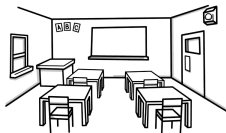
B



C

Is it possible to attribute each group to a classroom?

# An UNSAT example



1

2

?



A



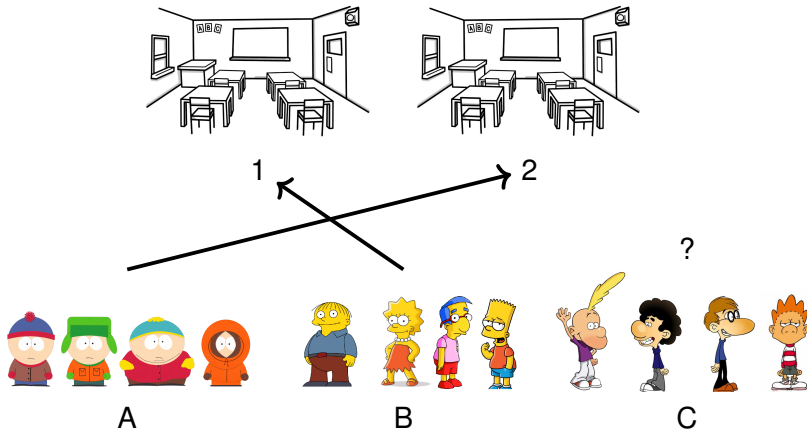
B



C

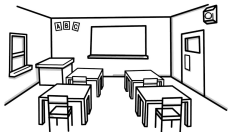
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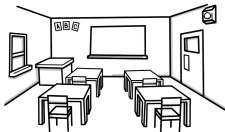


Is it possible to attribute each group to a classroom?

# An UNSAT example



1



2

...



A



B

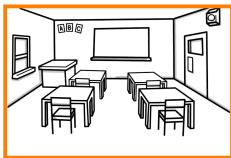


C

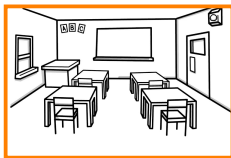
Is it possible to attribute each group to a classroom?

No!

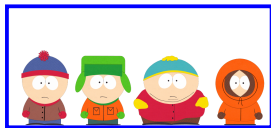
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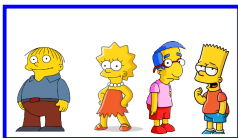
1



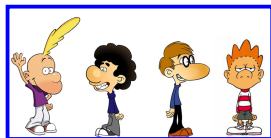
2



A



B



C

Is it possible to attribute each group to a classroom?

No!

Presence of symmetries hinders the performance of the solver



# Outline

## ① SAT overview

- SAT basics

- SAT and symmetries

## ② Existing approaches

- Static symmetry breaking

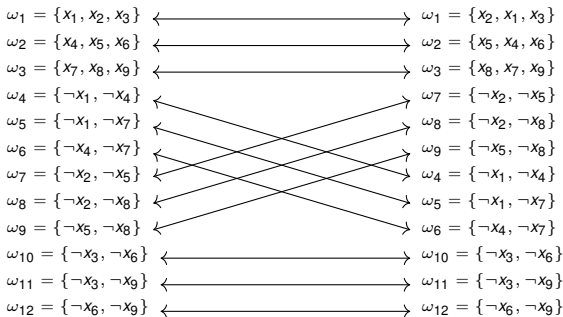
- Dynamic symmetry breaking

## ③ Contribution and results

# Symmetry

A symmetry (permutation)  $g$  is a bijective function (on variables) that leaves the formula invariant.

$$g = \begin{pmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 \\ x_2 & x_1 & x_3 & x_5 & x_4 & x_6 & x_8 & x_7 & x_9 \end{pmatrix} \rightarrow (x_1 \ x_2)(x_4 \ x_5)(x_7 \ x_8)$$



Equi-satisfiability:

$$\alpha \models \varphi \Leftrightarrow g.\alpha \models \varphi$$

# Computing symmetries of a SAT problem

*CNF formula*

$$\begin{aligned} & (x_1 \vee x_2 \vee x_3) \wedge (x_4 \vee x_5 \vee x_6) \wedge (x_7 \vee x_8 \vee x_9) \\ & \wedge (\neg x_1 \vee \neg x_4) \wedge (\neg x_1 \vee \neg x_7) \wedge (\neg x_4 \vee \neg x_7) \\ & \wedge (\neg x_2 \vee \neg x_5) \wedge (\neg x_2 \vee \neg x_8) \wedge (\neg x_5 \vee \neg x_8) \\ & \wedge (\neg x_3 \vee \neg x_6) \wedge (\neg x_3 \vee \neg x_9) \wedge (\neg x_6 \vee \neg x_9) \end{aligned}$$

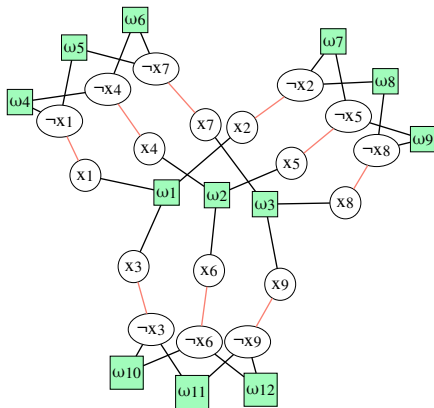
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colored graph



# Computing symmetries of a SAT problem

*CNF formula*

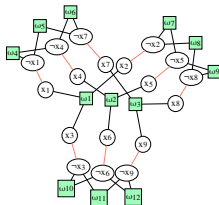


colored graph



graph automorphism

$$\begin{aligned} & (x_1 \vee x_2 \vee x_3) \wedge (x_4 \vee x_5 \vee x_6) \wedge (x_7 \vee x_8 \vee x_9) \\ & \wedge (\neg x_1 \vee \neg x_4) \wedge (\neg x_1 \vee \neg x_7) \wedge (\neg x_4 \vee \neg x_7) \\ & \wedge (\neg x_2 \vee \neg x_5) \wedge (\neg x_2 \vee \neg x_8) \wedge (\neg x_5 \vee \neg x_8) \\ & \wedge (\neg x_3 \vee \neg x_6) \wedge (\neg x_3 \vee \neg x_9) \wedge (\neg x_6 \vee \neg x_9) \end{aligned}$$



(bliss, saucy, ...)

# Computing symmetries of a SAT problem

*CNF formula*



colored graph

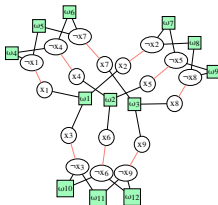


graph automorphism



set of symmetries

$$\begin{aligned} & (x_1 \vee x_2 \vee x_3) \wedge (x_4 \vee x_5 \vee x_6) \wedge (x_7 \vee x_8 \vee x_9) \\ & \wedge (\neg x_1 \vee \neg x_4) \wedge (\neg x_1 \vee \neg x_7) \wedge (\neg x_4 \vee \neg x_7) \\ & \wedge (\neg x_2 \vee \neg x_5) \wedge (\neg x_2 \vee \neg x_8) \wedge (\neg x_5 \vee \neg x_8) \\ & \wedge (\neg x_3 \vee \neg x_6) \wedge (\neg x_3 \vee \neg x_9) \wedge (\neg x_6 \vee \neg x_9) \end{aligned}$$



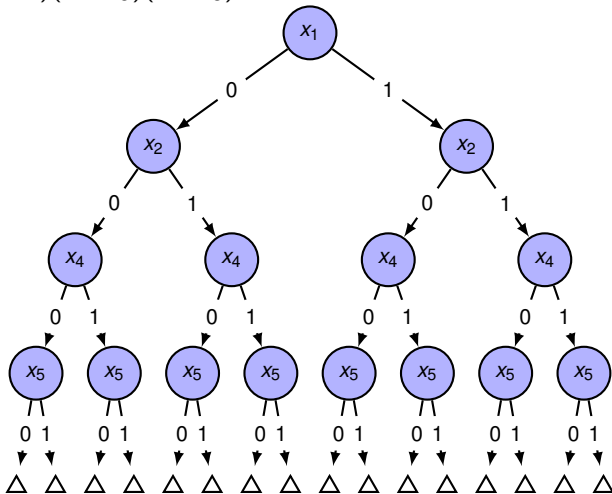
(bliss, saucy, ...)



$$\begin{aligned} g_1 &= (x_2 \ x_3)(x_5 \ x_6)(x_8 \ x_9) \\ g_2 &= (x_4 \ x_7)(x_5 \ x_8)(x_6 \ x_9) \\ g_3 &= (x_1 \ x_2)(x_4 \ x_5)(x_7 \ x_8) \\ g_4 &= (x_1 \ x_4)(x_2 \ x_5)(x_3 \ x_6) \end{aligned}$$

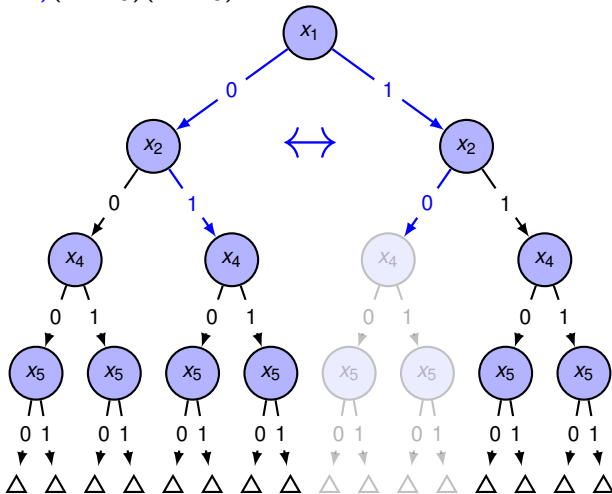
# Using symmetries to prune the search space

$$g = (x_1 \ x_2)(x_4 \ x_5)(x_7 \ x_8)$$



# Using symmetries to prune the search space

$$g = (x_1 \ x_2)(x_4 \ x_5)(x_7 \ x_8)$$



Add symmetry breaking predicates to prune the search space.



# Generates symmetry breaking predicates (SBP)

- Define lexicographic order
  - Define total order on variables
  - Define minimal value
- Forbid non minimal assignment for each orbit

Example:

$$x_1 \leq x_2 \leq x_3 \leq x_4 \leq x_5 \leq x_6 \leq x_7 \leq x_8; \textcolor{red}{F} < \textcolor{green}{T}$$

$$g = (x_1 \ x_2)(x_4 \ x_5)(x_7 \ x_8)$$

$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$\dots$	lex-leader	SBP

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	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$\dots$	lex-leader	SBP
$O_1$	$\textcolor{red}{F}$	$\textcolor{green}{T}$	—	—	—	$\dots$	✓	

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	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$\dots$	lex-leader	SBP
$O_1$	F	T	—	—	—	$\dots$	✓	$\rightarrow \neg x_1 \vee x_2$
	T	F	—	—	—	$\dots$	✗	

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	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$\dots$	lex-leader	SBP
$O_1$	F	T	—	—	—	$\dots$	✓	$\rightarrow \neg x_1 \vee x_2$
	T	F	—	—	—	$\dots$	✗	
$O_2$	F	F	—	F	T	$\dots$	✓	

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$$g = (x_1 \ x_2)(x_4 \ x_5)(x_7 \ x_8)$$

	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$\dots$	lex-leader	SBP
$O_1$	F	T	—	—	—	$\dots$	✓	$\rightarrow \neg x_1 \vee x_2$
	T	F	—	—	—	$\dots$	✗	
$O_2$	F	F	—	F	T	$\dots$	✓	$\rightarrow x_1 \vee x_2 \vee \neg x_4 \vee x_5$
	F	F	—	T	F	$\dots$	✗	

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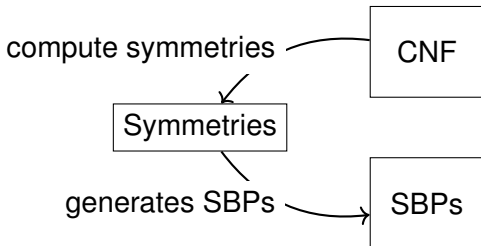
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$$x_1 \leq x_2 \leq x_3 \leq x_4 \leq x_5 \leq x_6 \leq x_7 \leq x_8; \text{F} < \text{T}$$

$$g = (x_1 \ x_2)(x_4 \ x_5)(x_7 \ x_8)$$

	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$\cdots$	lex-leader	SBP
$O_1$	F	T	—	—	—	$\cdots$	✓	$\rightarrow \neg x_1 \vee x_2$
	T	F	—	—	—	$\cdots$	✗	
$O_2$	F	F	—	F	T	$\cdots$	✓	$\rightarrow x_1 \vee x_2 \vee \neg x_4 \vee x_5$
	F	F	—	T	F	$\cdots$	✗	
$\dots$								

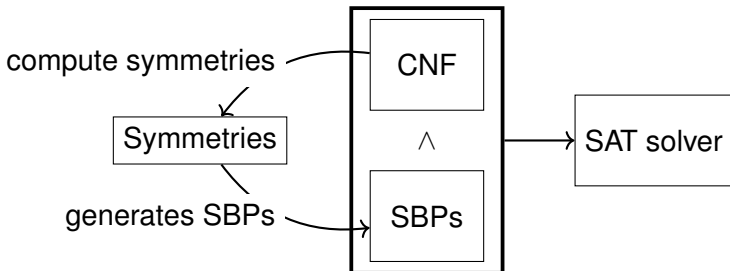
# Static symmetry breaking



Different approaches:

- Shatter [ASM06]
- BreakID [DBBD16]
- ...

# Static symmetry breaking

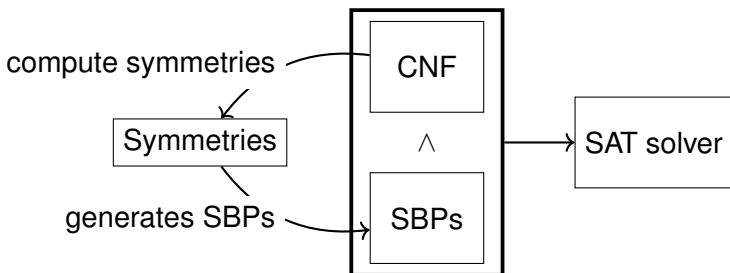


Different approaches:

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# Static symmetry breaking



Different approaches:

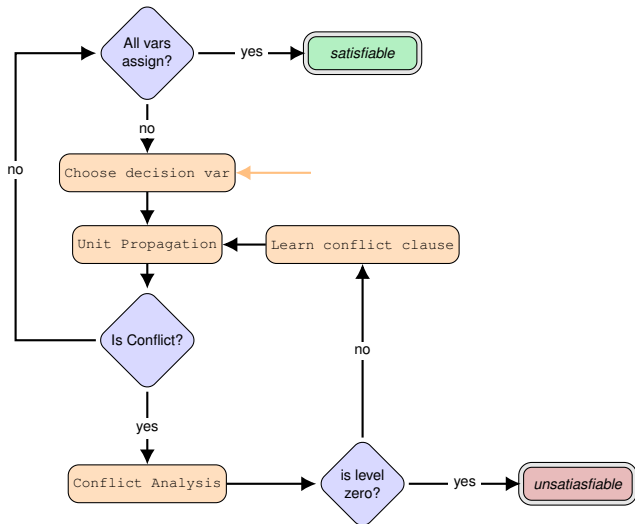
- Shatter [ASM06]
- BreakID [DBBD16]
- ...

Pros/Cons:

- Works well on many symmetric instances
- The solver can "explode" instead of being helped

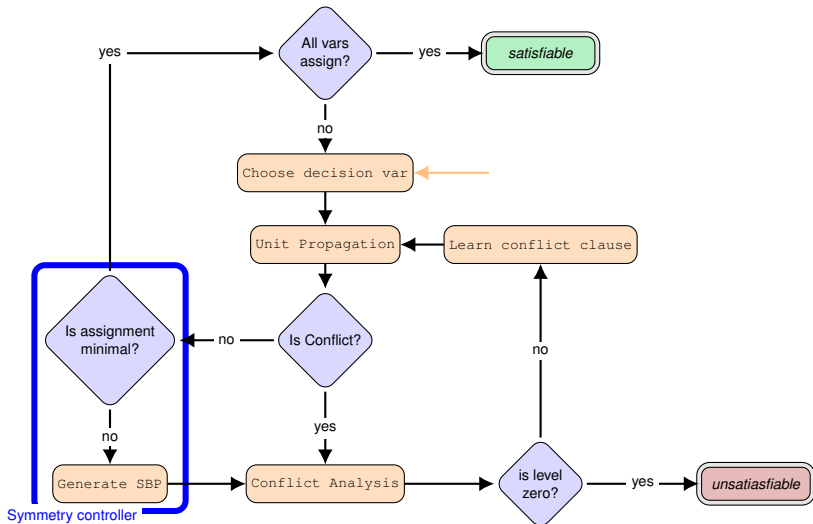
# Our contribution CDCL[Sym]

Compute and inject SBP **opportunistically**, during the solving



# Our contribution CDCL[Sym]

Compute and inject SBP **opportunistically**, during the solving



# Symmetry status

- reducer:  $g.\alpha \prec \alpha$
- inactive:  $\alpha \prec g.\alpha$
- active: *not enough information*

## Efficient implementation of symmetry status

Keep track the smallest unassigned variable  $x$  :

- ①  $\alpha(g.x) \leq \alpha(x)$ , then  $g$  is reducer  $\Rightarrow$  Effective SBP (ESBP)
- ②  $\alpha(x) \leq \alpha(g.x)$ , then  $g$  is inactive  $\Rightarrow g$  cannot reduce  $\alpha$
- ③  $\alpha(g.x)$  or  $\alpha(x)$  is unassigned then  $g$  is active

Update whenever variables are assigned / unassigned

# Example

- 1 reducer:  $\alpha(g.x) \leq \alpha(x)$
- 2 inactive:  $\alpha(x) \leq \alpha(g.x)$
- 3 active:  $\alpha(g.x)$  or  $\alpha(x)$  is unassigned

$$x_1 \leq x_2 \leq x_3 \leq x_4 \leq x_5 \leq x_6 \leq x_7 \leq x_8 ; \text{ F } < \text{ T }$$

$$g_1 = \begin{array}{cc} (x_2 & x_3) & (x_5 & x_6) & (x_8 & x_9) \end{array} \left| \begin{array}{l} x = x_2 \\ g.x = x_3 \\ \text{active} \end{array} \right.$$

↑

$$g_2 = \begin{array}{cc} (x_1 & x_2) & (x_4 & x_5) & (x_7 & x_8) \end{array} \left| \begin{array}{l} x = x_1 \\ g.x = x_2 \\ \text{active} \end{array} \right.$$

↑

...

$$\alpha = \{ \quad \quad \quad \}$$

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$$g_1 = \begin{array}{cc} (\textcolor{red}{x}_2 & x_3) & (x_5 & x_6) & (x_8 & x_9) \end{array} \left| \begin{array}{l} x = \textcolor{red}{x}_2 \\ g.x = x_3 \\ \text{active} \end{array} \right.$$

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↑

...

$$\alpha = \{ \neg x_2 \quad \quad \quad \}$$

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↑

$$g_2 = \begin{array}{cc} (x_1 & x_2) & (x_4 & x_5) & (x_7 & x_8) \end{array} \left| \begin{array}{l} x = x_1 \\ g.x = x_2 \\ \text{reducer} \end{array} \right.$$

↑

...

$$\alpha = \{\neg x_2, \neg x_3, x_1\}$$

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↑

...

$$\alpha = \{\neg x_2, \neg x_3, x_1\}$$

$$g_2 \text{ generates } \omega = \{\neg x_1, x_2\}$$



# CDCL[Sym] Implementation

- Packaged as a library **cosy**<sup>1</sup>, to be combined with your solver  
→ e.g. +3% LOC on MiniSAT.
- Follows symmetry status
- Should work with any enumerative SAT solver

---

<sup>1</sup><https://github.com/lip6/cosy>

# Experiments

## Benchmark:

- from SAT contests 2012 – 2017
- retain only instances for which `bliss` finds significant symmetries in 1000s
- 1350 symmetric instances (out of 3700)

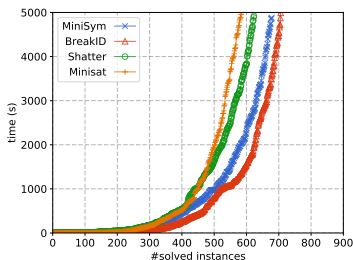
## Setup:

- four tools
  - MiniSat (no symmetry, baseline)
  - MiniSat + BreakID (SOTA SAT solver using symmetries)
  - MiniSat + Shatter (SOTA SAT solver using symmetries)
  - **MiniSym** = MiniSat + CDCL[Sym] (our approach)
- 5000s timeout, 8GB memory
- includes time to compute symmetries (except for MiniSat)

# Experimental results

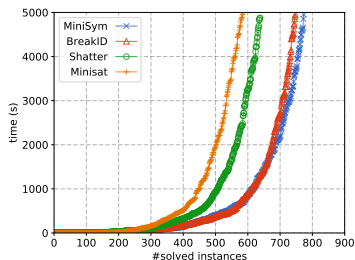
bliss gives more generators than saucy3

Figure: Cactus plot total number of instances



(a) with saucy3

	MiniSAT	Shatter	BreakID	MiniSym
PAR-2 sum	8 074 348	7 770 434	<b>6 909 999</b>	7 229 700
PAR-2 avg	5 981	5 756	<b>5 119</b>	5 355



(b) with bliss

	MiniSAT	Shatter	BreakID	MiniSym
PAR-2 sum	8 074 348	7 517 556	6 444 954	<b>6 245 448</b>
PAR-2 avg	5 981	5 569	4 774	<b>4 626</b>

Table: Time comparison

# Experimental results (UNSAT versus SAT)

	MiniSAT	Shatter	BreakID	MiniSym		MiniSAT	Shatter	BreakID	MiniSym
TOTAL	261	302	<b>371</b>	345	TOTAL	261	324	415	<b>439</b>
(a) With saucy3					(b) With bliss				

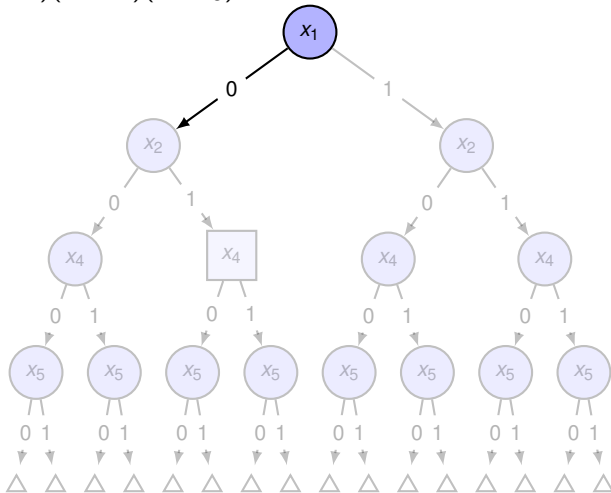
Table: Comparison on UNSAT instances

	MiniSAT	Shatter	BreakID	MiniSym		MiniSAT	Shatter	BreakID	MiniSym
TOTAL	325	323	<b>337</b>	335	TOTAL	325	316	334	<b>336</b>
(a) With saucy3					(b) With bliss				

Table: Comparison on SAT instances

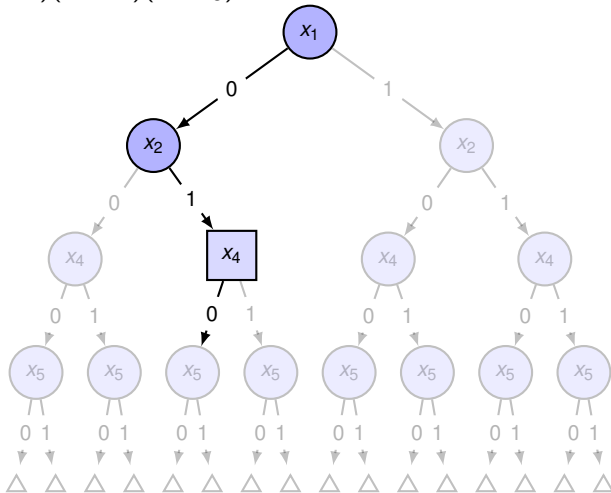
# Using symmetries to accelerate the tree traversal

$$g = (x_1 \ x_2)(x_4 \ x_5)(x_7 \ x_8)$$



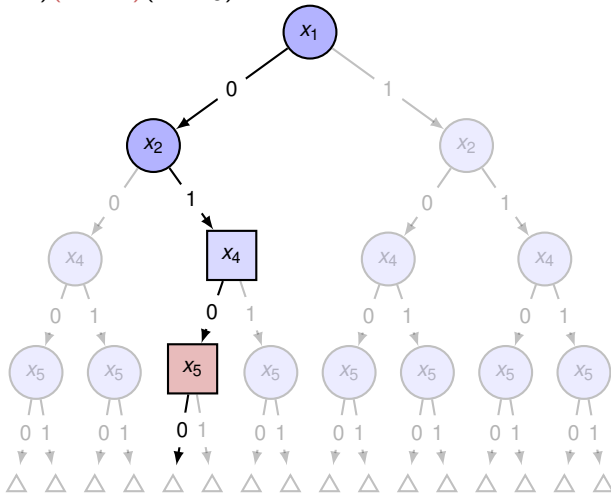
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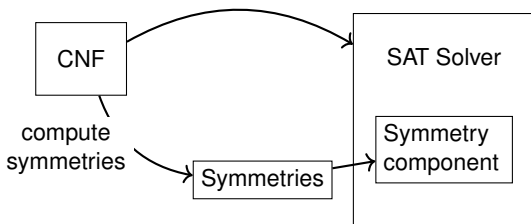
# Using symmetries to accelerate the tree traversal

$$g = (x_1 \ x_2)(x_4 \ x_5)(x_7 \ x_8)$$



Use symmetries to deduce symmetrical facts.

# Dynamic Symmetry Breaking

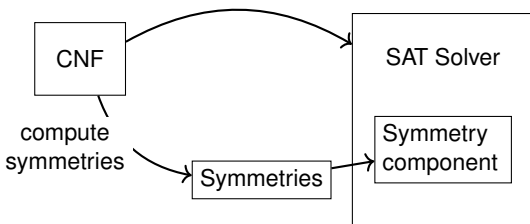


Different approaches:

- Symmchaff [Sab05]
- Symmetry Propagation (SP) [DBdC<sup>+</sup>12]
- Symmetry Learning Scheme (SLS) [BNOS10]
- Symmetry Explanation Learning (SEL) [DBB17]
- ...



# Dynamic Symmetry Breaking



Different approaches:

- Symmchaff [Sab05]
- Symmetry Propagation (SP) [DBdC<sup>+</sup>12]
- Symmetry Learning Scheme (SLS) [BNOS10]
- Symmetry Explanation Learning (SEL) [DBB17]
- ...

Pros/Cons:

- Works well on many symmetric instances
- Cannot handle some instances solved by static approach

# ESBP + SP

Compose the symmetry propagation and the ESBP

*prune the decision tree while accelerating its traversal*

Problems:

- ESBP breaks symmetries (incrementally)
- SP considers the manipulated symmetries valid all time

In a hybrid approach, SP must be able to identify  
**valid symmetries**

# Local symmetries

Formula  $\leftarrow$  (Symmetries)

$\omega_1 \leftarrow$  (Local symmetries)

$\omega_2 \leftarrow$  (Local symmetries)

$\omega_3 \leftarrow$  (Local symmetries)

$\omega_4 \leftarrow$  (Local symmetries)

Macro level

$\rightarrow$

Micro level

# Local symmetries

Formula  $\leftarrow$  (Symmetries)

$\omega_1 \leftarrow$  (Local symmetries)

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$\omega_5$

Macro level

$\rightarrow$

Micro level

# Local symmetries

Formula  $\leftarrow$  (Symmetries)

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$\omega_4 \leftarrow$  (Local symmetries)

$\omega_5 \leftarrow$  (Local symmetries)

Macro level

$\rightarrow$

Micro level

Compute valid local symmetries on-the-fly at a minimal cost.

# Experimental results

## Benchmark:

- from SAT contests 2012 – 2018
- retain only instances for which `bliss` finds significant symmetries in 1000s
- 1400 symmetric instances (out of 4000)

## Setup:

- Three tools
  - MiniSat SP (Minisat with Symmetry Propagation)
  - MiniSat ESBP (Minisat with CDCL[Sym])
  - **Minisat ESBP-SP (our approach)**
- 7200s timeout

## Results:

Solver	PAR-2	ALL	SAT	UNSAT
SP	1674h00	876	406	470
ESBP	1578h30	904	416	488
ESBP-SP	<b>1570h15</b>	<b>911</b>	<b>420</b>	<b>491</b>

# Conclusion

- A new dynamic symmetry breaking approach
  - Generation of SBP on the fly
  - Package as a library cosy usable with any CDCL solver
  - Overcomes drawbacks of the existing approaches
- A new hybrid approach (ESBP-SP)
  - Take advantage of static and dynamic approach
  - Introduce local symmetries

# Perspectives

- Combination of CDCL[Sym] with other dynamic symmetry breaking approach
- Exploitation of partial symmetries



# Perspectives

- Combination of CDCL[Sym] with other dynamic symmetry breaking approach
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**Thanks !**



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Ashish Sabharwal.

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*In AAAI*, volume 5, pages 467–474, 2005.

# CDCL in action TODO



$$\omega_1 = \{x_1, x_2, x_3\}$$

$$\omega_2 = \{x_4, x_5, x_6\}$$

$$\omega_3 = \{\neg x_1, \neg x_5\}$$

$$\omega_4 = \{\neg x_2, \neg x_4\}$$

$$\omega_5 = \{\neg x_3, \neg x_4\}$$

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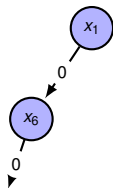
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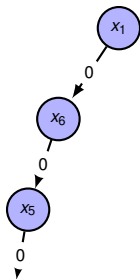
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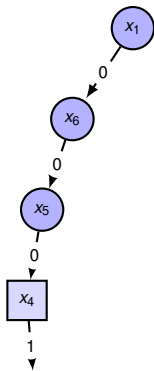
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# CDCL in action TODO



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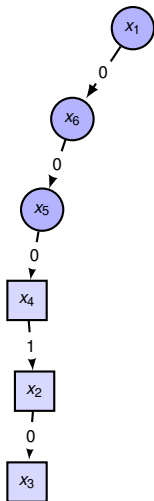
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$$\omega_5 = \{\neg \mathbf{x}_3, \neg \mathbf{x}_4\}$$

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# CDCL in action TODO



$$\omega_1 = \{x_1, x_2, x_3\}$$

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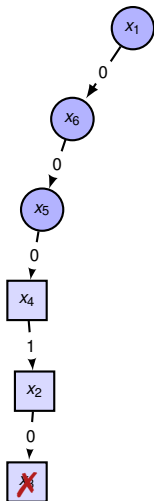
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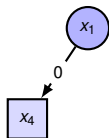
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$$\omega_7 = \{x_1, \neg x_4\}$$

# Weakly active symmetries

## Logical consequence

When  $\omega$  is satisfied in all satisfying assignments of  $\varphi$ , we say that  $\omega$  is a logical consequence of  $\varphi$ , and we denote this by  $\varphi \vdash \omega$ .

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Let a subset  $\delta \subseteq \alpha$ , a symmetry  $\sigma$  of  $\varphi$  such that  $\varphi \cup \delta \vdash \varphi \cup \alpha \wedge \sigma.\delta \subseteq \alpha$  then  $\sigma$  is weakly active symmetry.

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## Symmetry propagation

Let  $\sigma$  a weakly active symmetry, then

$$\varphi \cup \alpha \vdash \{I\} \Leftrightarrow \varphi \cup \alpha \vdash \sigma.\{I\}$$

# Local symmetries

## Logical consequence

When  $\omega$  is satisfied in all satisfying assignments of  $\varphi$ , we say that  $\omega$  is a logical consequence of  $\varphi$ , and we denote this by  $\varphi \vdash \omega$ .

## Local Symmetries

Let  $\varphi$  be a formula. We define  $L_{\omega, \varphi}$ , the set of *local symmetries* for a clause  $\omega$ , and with respect to a formula  $\varphi$ , as follows:

$$L_{\omega, \varphi} = \{\sigma \in \mathfrak{S} \mid \varphi \vdash \sigma.\omega\}$$

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$$L_{\omega, \varphi} = \{\sigma \in \mathfrak{S} \mid \varphi \vdash \sigma.\omega\}$$

We can state that:

$$\bigcap_{\omega \in \varphi} L_{\omega, \varphi} \subseteq G.$$



# Computing local symmetries

Formula can be decomposed as :  $\varphi = \varphi_o \cup \varphi_e \cup \varphi_d$  where

- $\varphi_o$  is the set of the original clauses
- $\varphi_e$  is the set of ESBPs
- $\varphi_d$  is the set of deduced clauses.

## Local symmetries

- $\omega \in \varphi_o, L_{\omega, \varphi} \supseteq G$
- $\omega \in \varphi_e, L_{\omega, \varphi} \supseteq \text{Stab}(\omega) = \{\sigma \in G \mid \omega = \sigma.\omega\}$
- $\omega \in \varphi_d, L_{\omega, \varphi} \supseteq \left( \bigcap_{\omega' \in \varphi_1} L_{\omega', \varphi} \right) \cup \text{Stab}(\omega)$

where  $\varphi_1$  is the set of clauses that derives  $\omega$ .