Exploitation des symétries dynamiques pour la résolution des problèmes SAT

Thèse de doctorat de Sorbonne Université

Hakan METIN

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LIP6

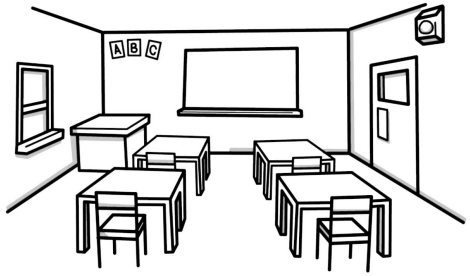
Motivation

SAT is widely used in different domains:

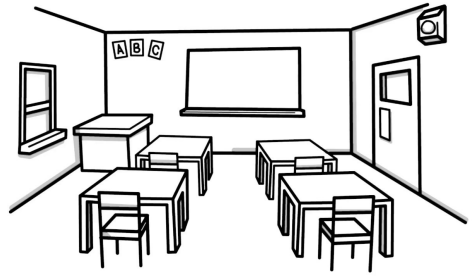
- Artificial intelligence (planning, games, ...)
- Bioinformatics (haplotype inference, ...)
- Security (cryptanalysis, inversion attack on hash function)
- Computationally hard problems (graph coloring, ...)
- Formal Methods (hardware model checking, ...)

Outline

SAT an example



1



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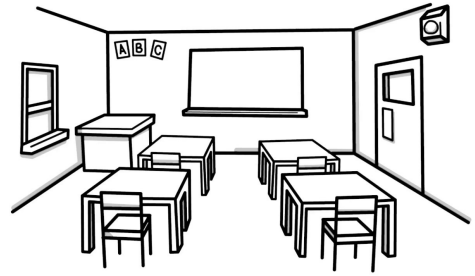

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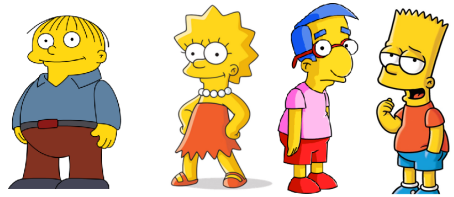
A



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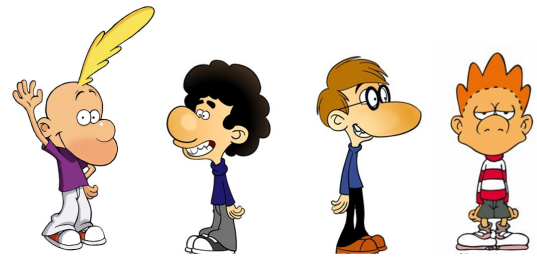
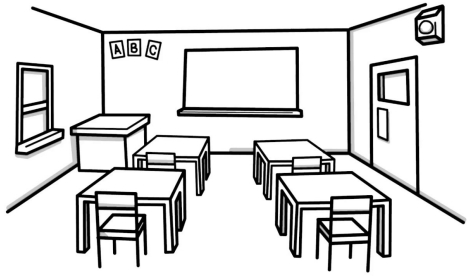
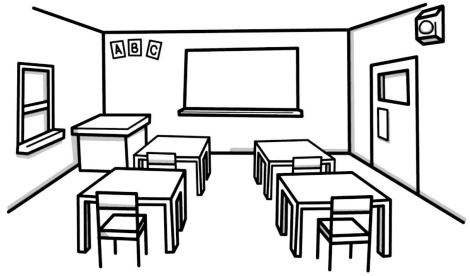


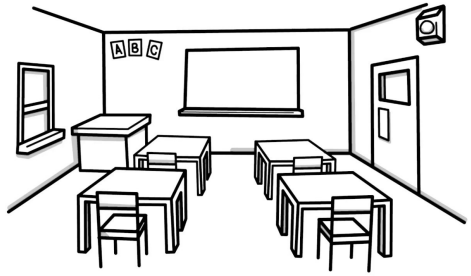
C

Is it possible to attribute each group to a classroom ?

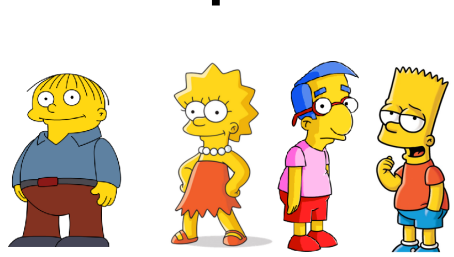
SAT an example

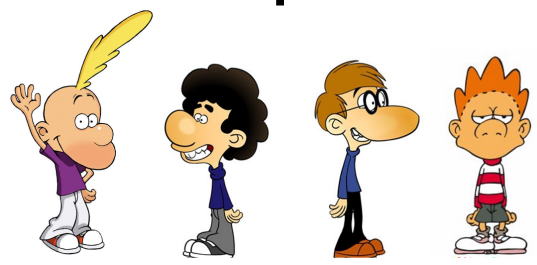






1 2 3



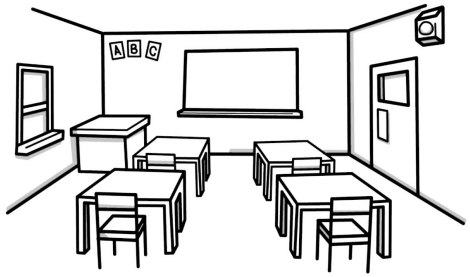
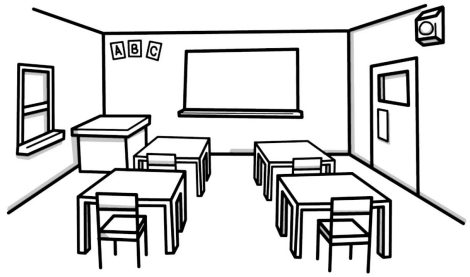
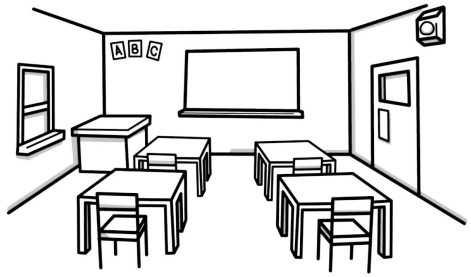


A B C

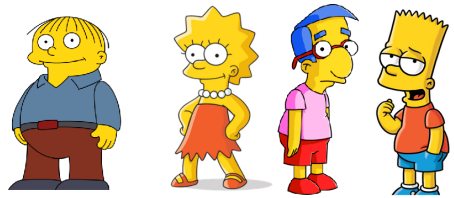
Is it possible to attribute each group to a classroom ?

YES!

SAT an example

1 2 3

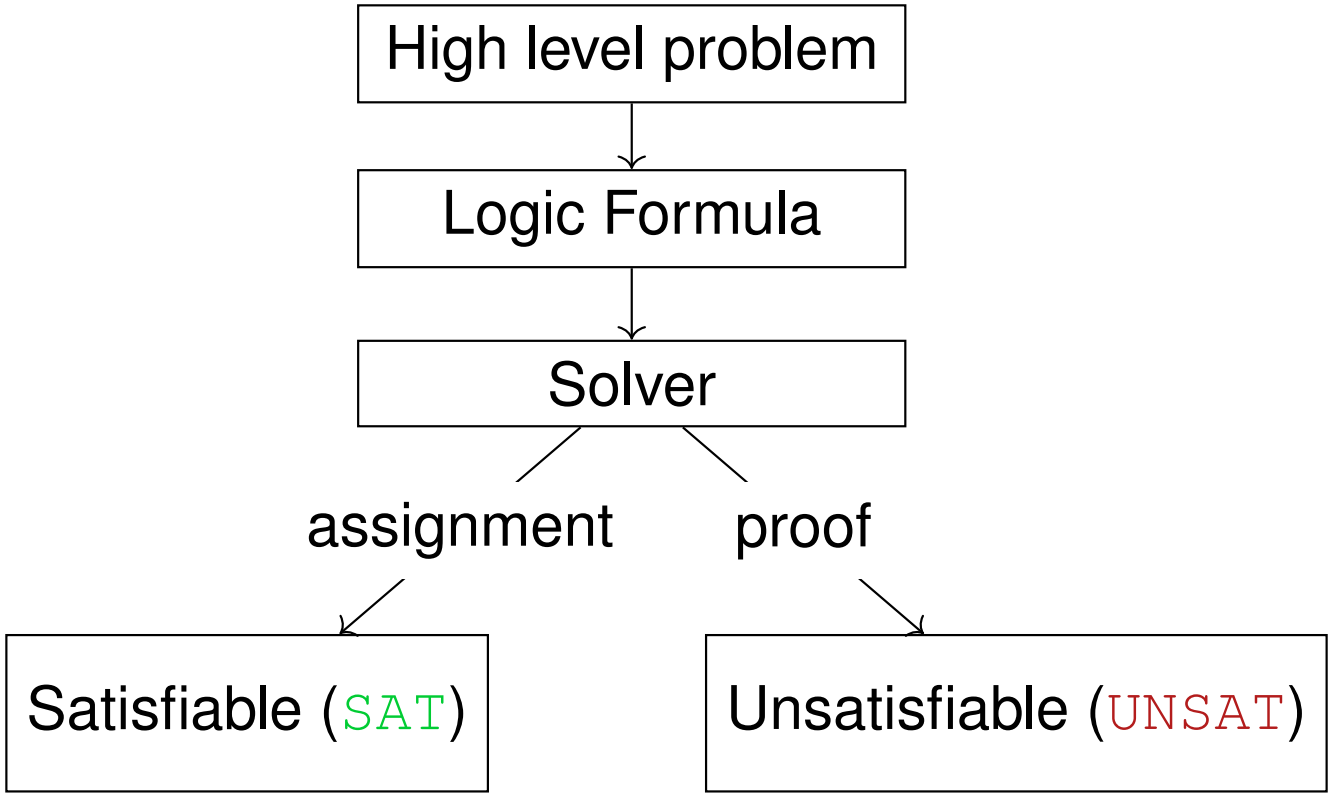




A B C

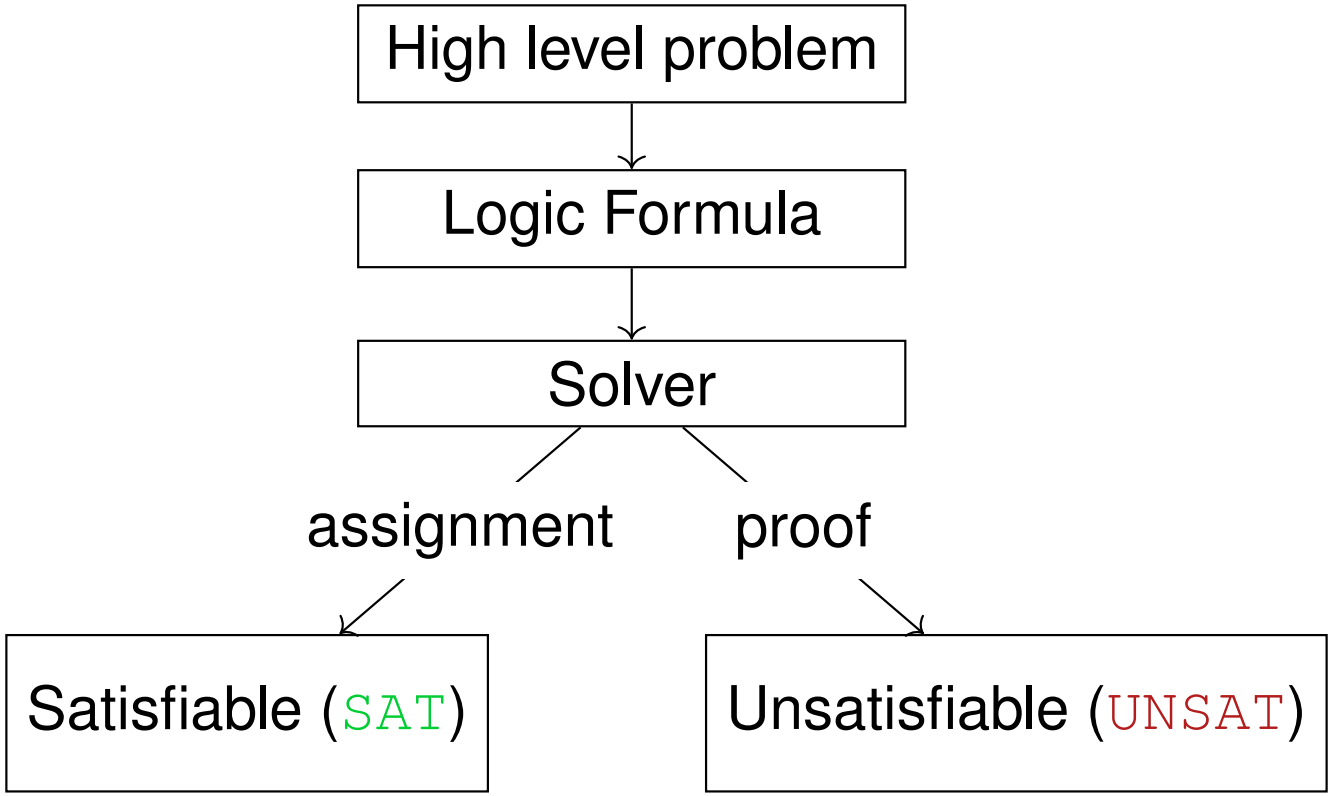
Is it possible to attribute each group to a classroom ?

YES! Many solutions

SAT


High level problem
Logic Formula
Solver
assignment
proof
Satisfiable (SAT)
Unsatisfiable (UNSAT)

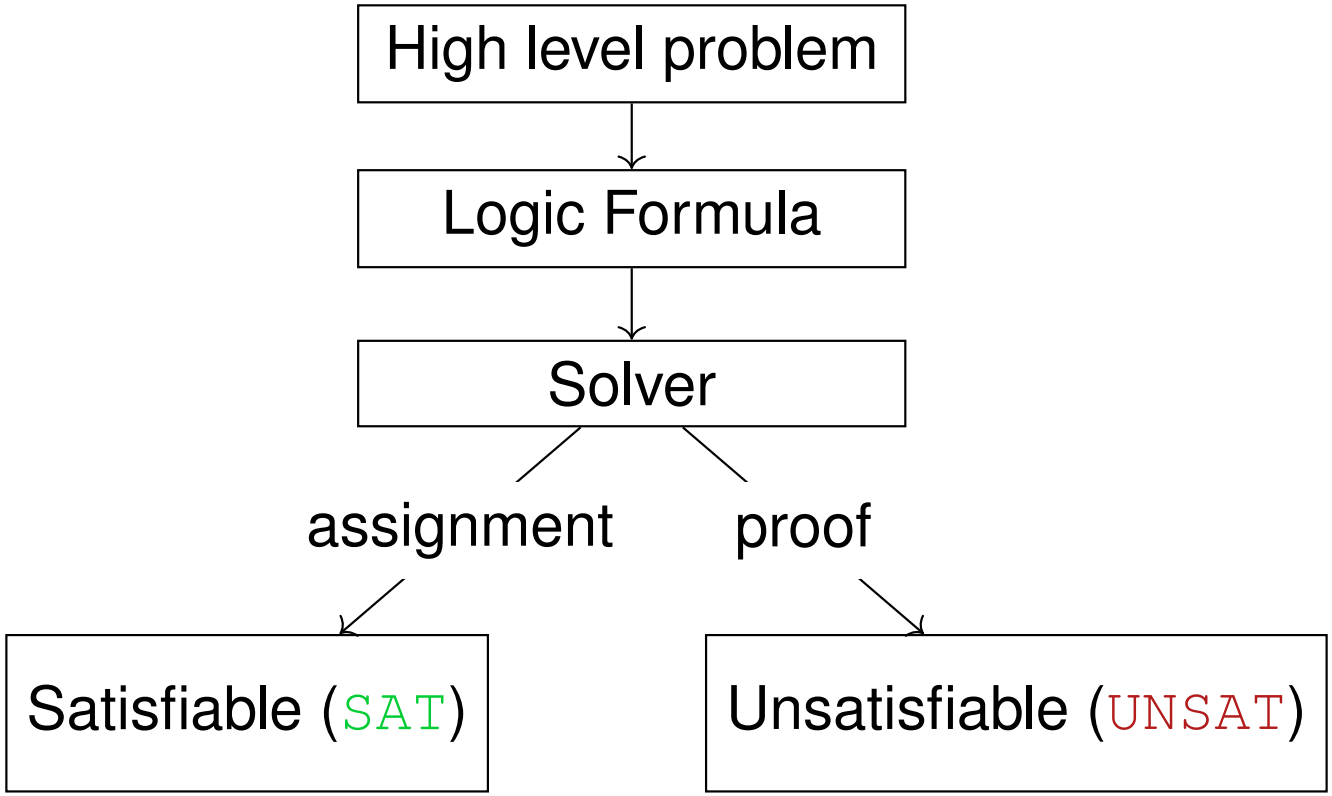
SAT



$$\underbrace{(x_1 \vee x_2 \vee \neg x_3)}$$

Clause with literals $x_1, x_2, \neg x_3$

SAT



$$\overbrace{\underbrace{(x_1 \vee x_2 \vee \neg x_3)}_{\textit{Clause}} \wedge (\neg x_1 \vee \neg x_2) \wedge (x_2 \vee \neg x_4)}^{\text{Formula (CNF)}}$$

Computing symmetries of a SAT problem

CNF formula

$$(x_1 \vee x_2 \vee x_3) \wedge (x_4 \vee x_5 \vee x_6) \wedge (x_7 \vee x_8 \vee x_9)$$
$$\wedge(\neg x_1 \vee \neg x_4) \wedge (\neg x_1 \vee \neg x_7) \wedge (\neg x_4 \vee \neg x_7)$$
$$\wedge(\neg x_2 \vee \neg x_5) \wedge (\neg x_2 \vee \neg x_8) \wedge (\neg x_5 \vee \neg x_8)$$
$$\wedge(\neg x_3 \vee \neg x_6) \wedge (\neg x_3 \vee \neg x_9) \wedge (\neg x_6 \vee \neg x_9)$$

[1] http://www.tcs.hut.fi/Software/bliss/

[2] http://vlsicad.eecs.umich.edu/BK/SAUCY/

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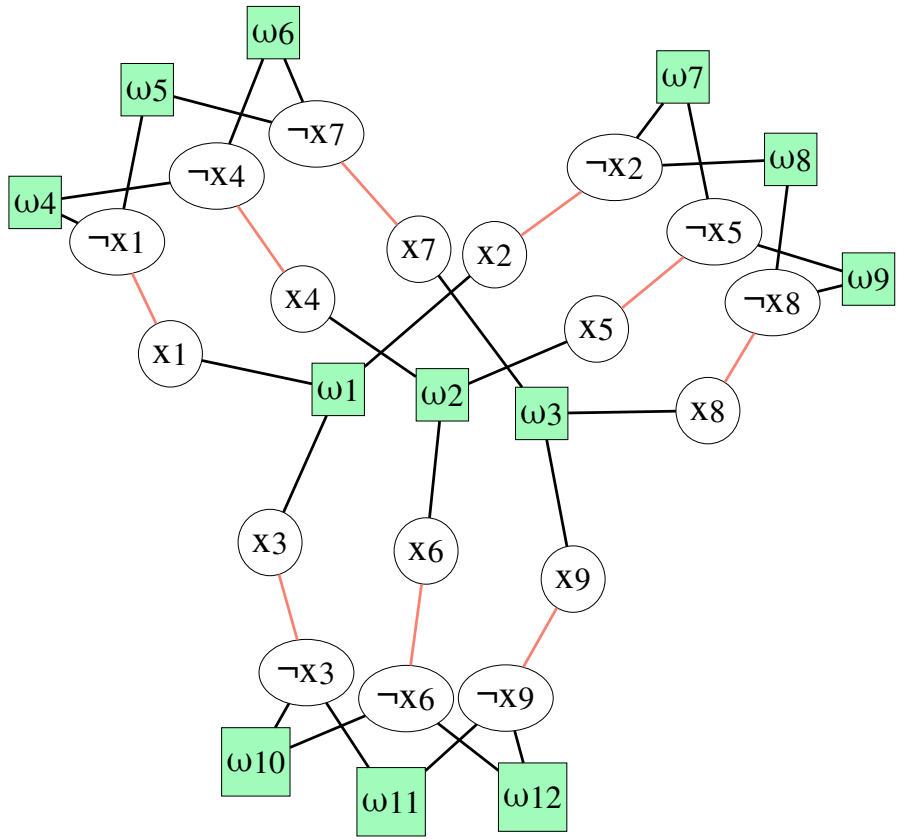
$$\wedge(\neg x_3 \vee \neg x_6) \wedge (\neg x_3 \vee \neg x_9) \wedge (\neg x_6 \vee \neg x_9)$$

$\Downarrow$ $\Downarrow$

colored graph



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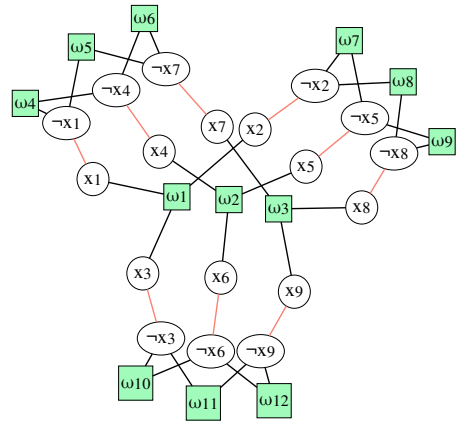
$$\wedge(\neg x_3 \vee \neg x_6) \wedge (\neg x_3 \vee \neg x_9) \wedge (\neg x_6 \vee \neg x_9)$$

$\Downarrow$ $\Downarrow$

colored graph



$\|$ $\|$

graph automorphism (`bliss` [1] or `saucy` [2])

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$$\wedge(\neg x_2 \vee \neg x_5) \wedge (\neg x_2 \vee \neg x_8) \wedge (\neg x_5 \vee \neg x_8)$$
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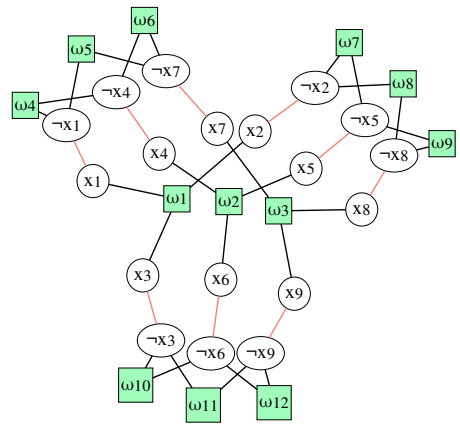
$\Downarrow$ $\qquad$ $\Downarrow$

colored graph



$\Vert$ $\qquad$ $\Vert$

graph automorphism $\qquad$ (`bliss` [1] or `saucy` [2])

$\Downarrow$ $\qquad$ $\Downarrow$

set of symmetries

$$g_1 = (x_2\ \ x_3)(x_5\ \ x_6)(x_8\ \ x_9)$$
$$g_2 = (x_4\ \ x_7)(x_5\ \ x_8)(x_6\ \ x_9)$$
$$g_3 = (x_1\ \ x_2)(x_4\ \ x_5)(x_7\ \ x_8)$$
$$g_4 = (x_1\ \ x_4)(x_2\ \ x_5)(x_3\ \ x_6)$$

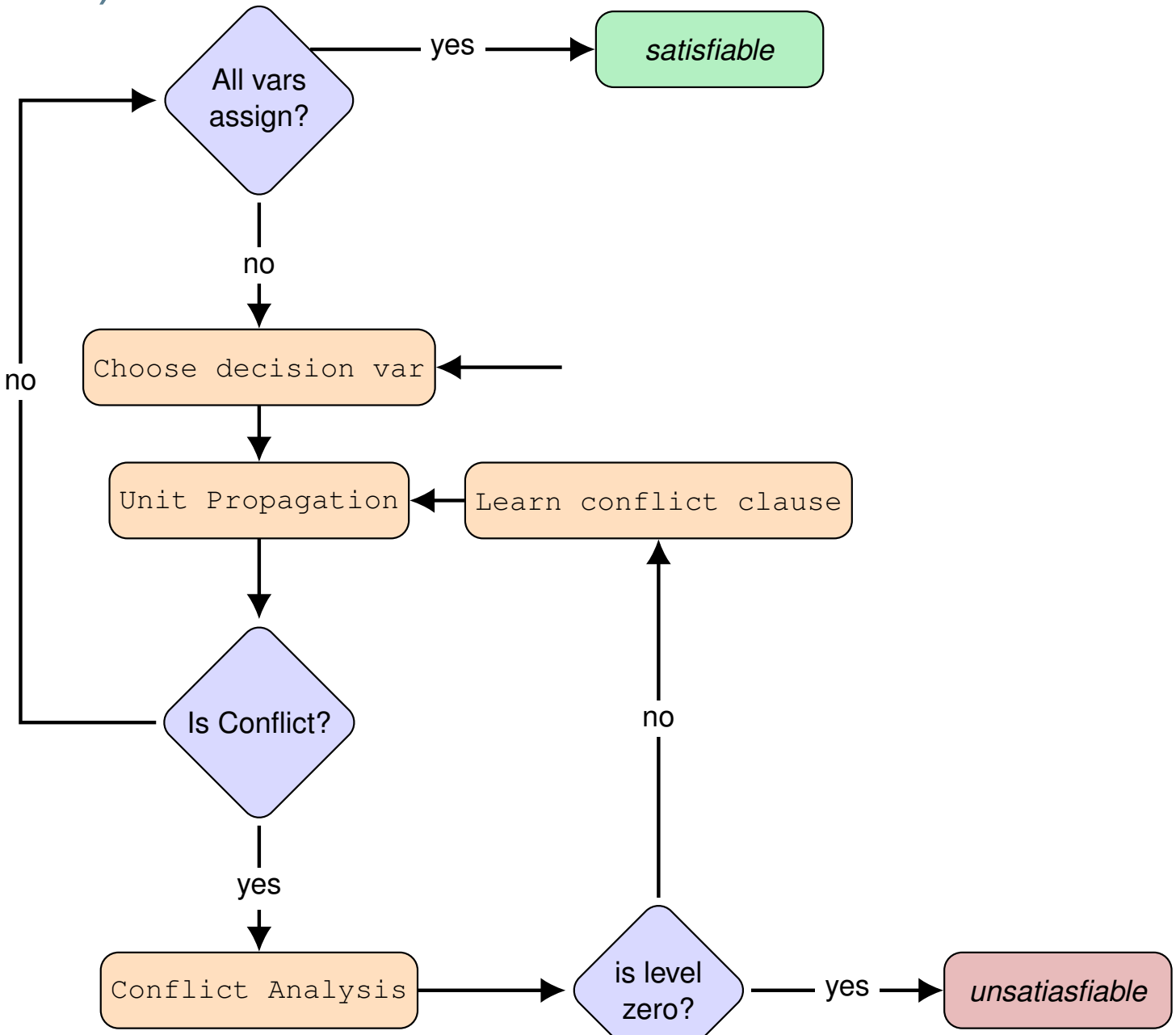
[1] http://www.tcs.hut.fi/Software/bliss/

[2] http://vlsicad.eecs.umich.edu/BK/SAUCY/

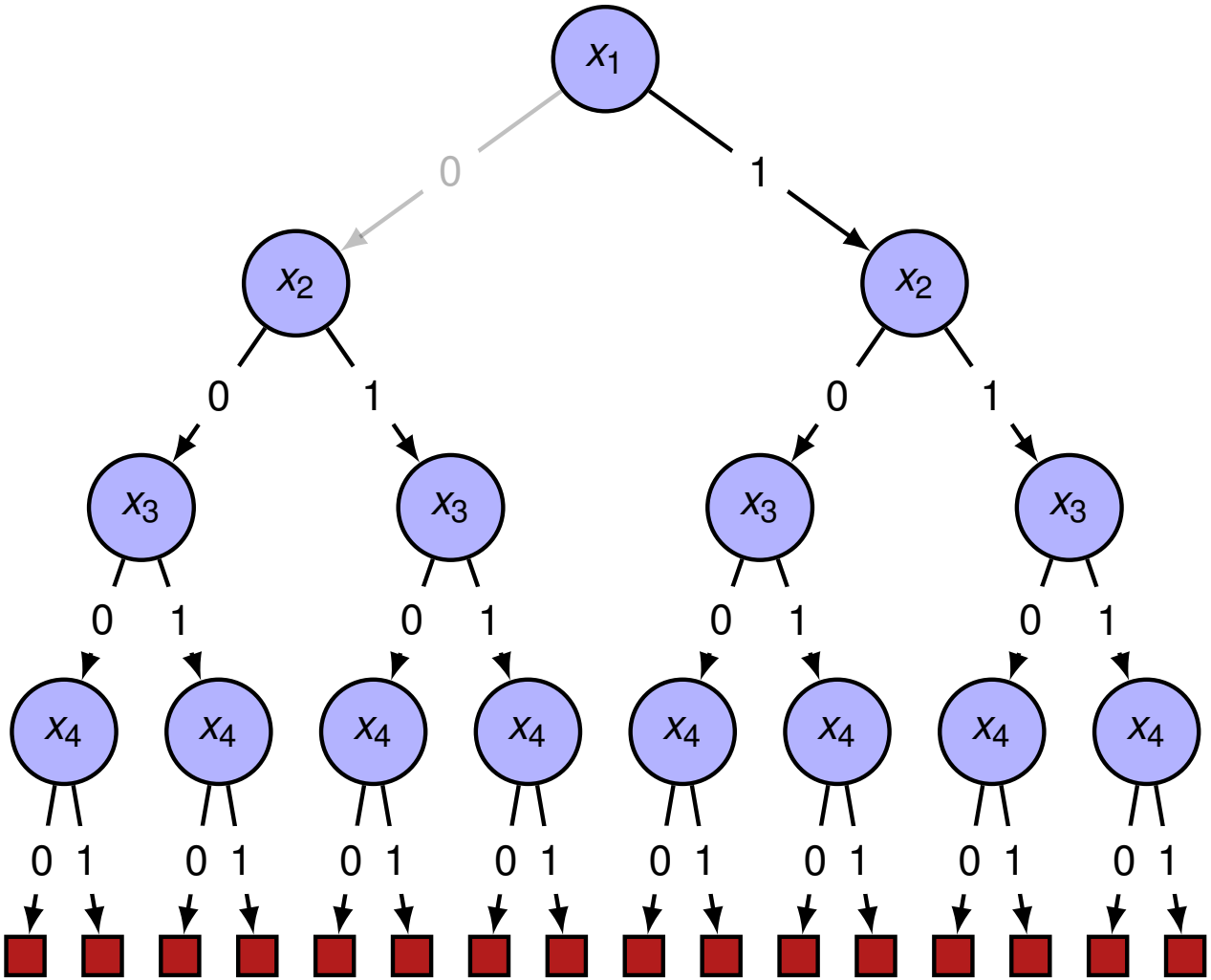
SAT

example solving arbre

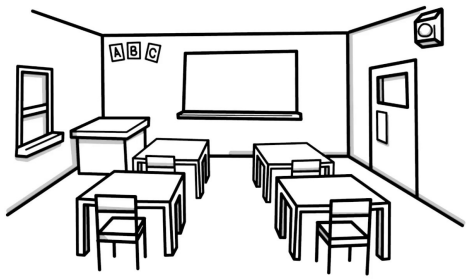
Conflict Driven Clause Learning Algorithm (CDCL)



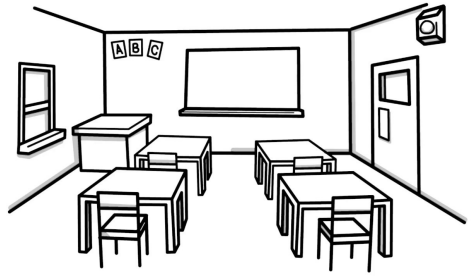
Tree


x_1
0
1
x_2
x_2
0
1
0
1
x_3
x_3
x_3
x_3
0 1
0 1
0 1
0 1
x_4 x_4 x_4 x_4 x_4 x_4 x_4 x_4
0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1

SAT an example 2

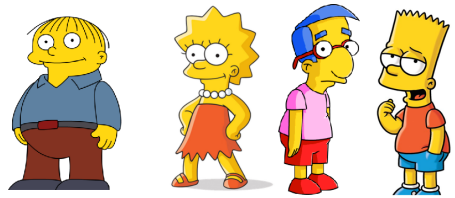


1

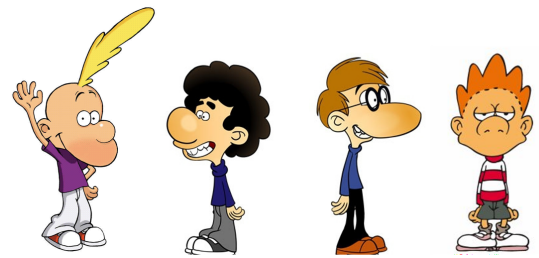


2

A



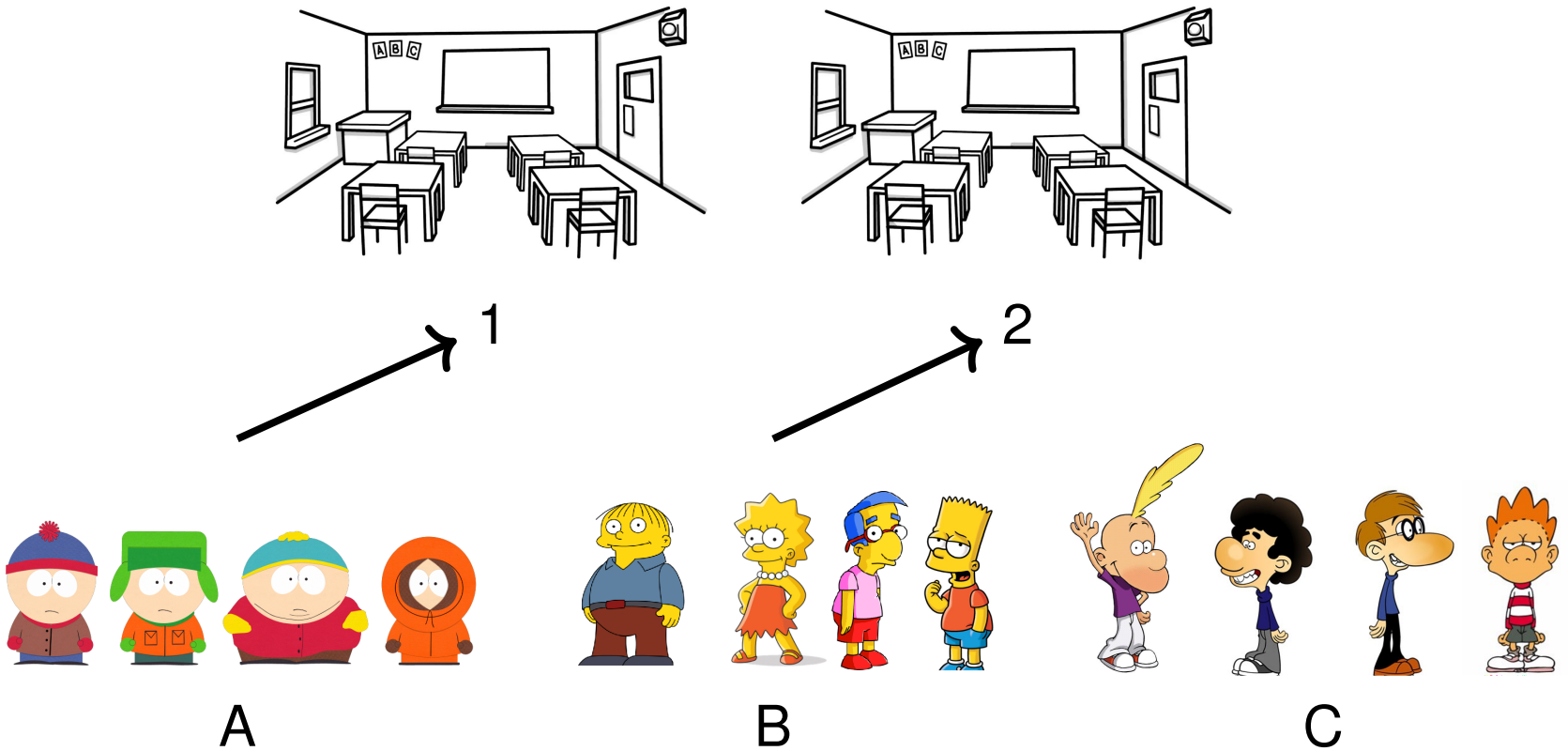
B



C

Attribute each group to a class room

SAT an example 2



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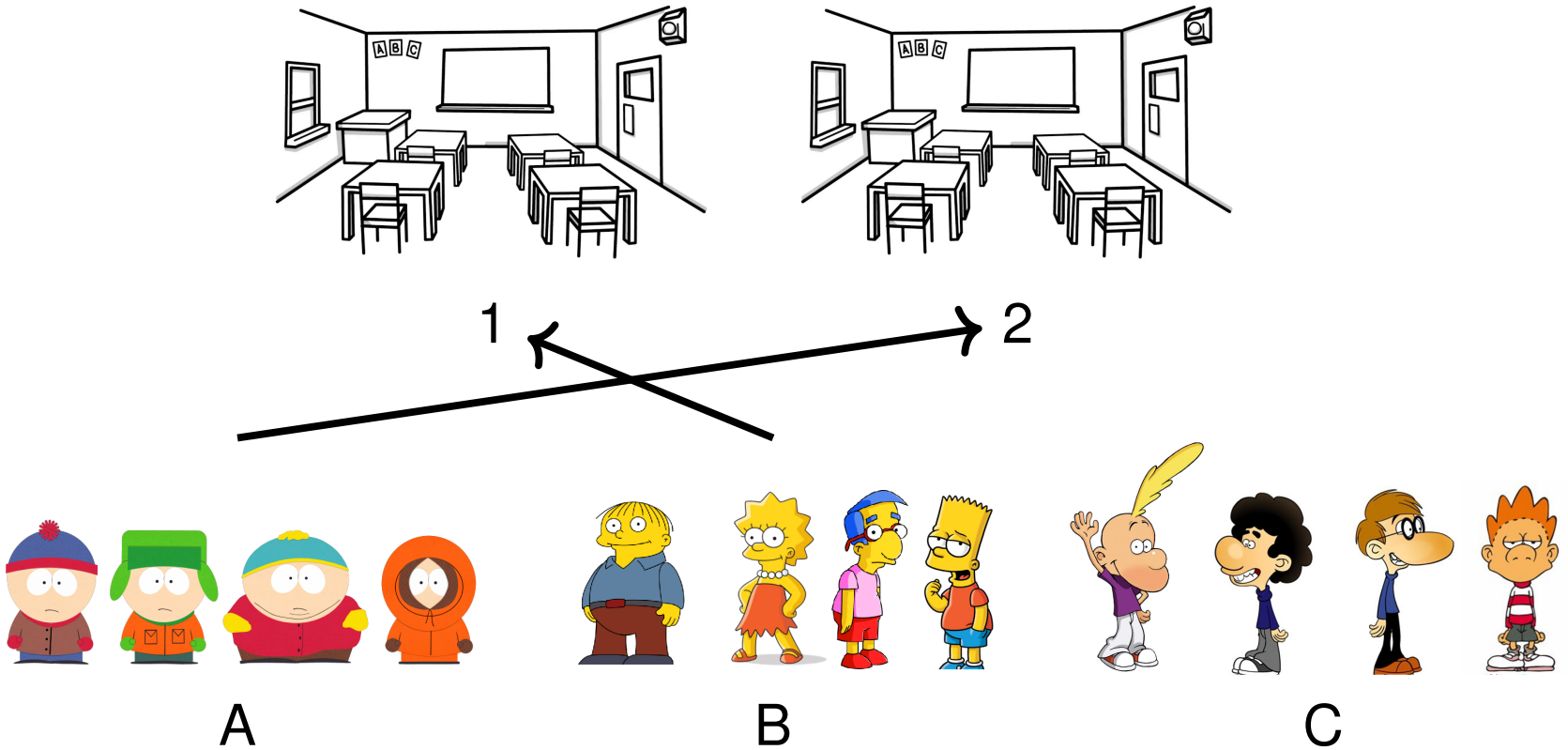

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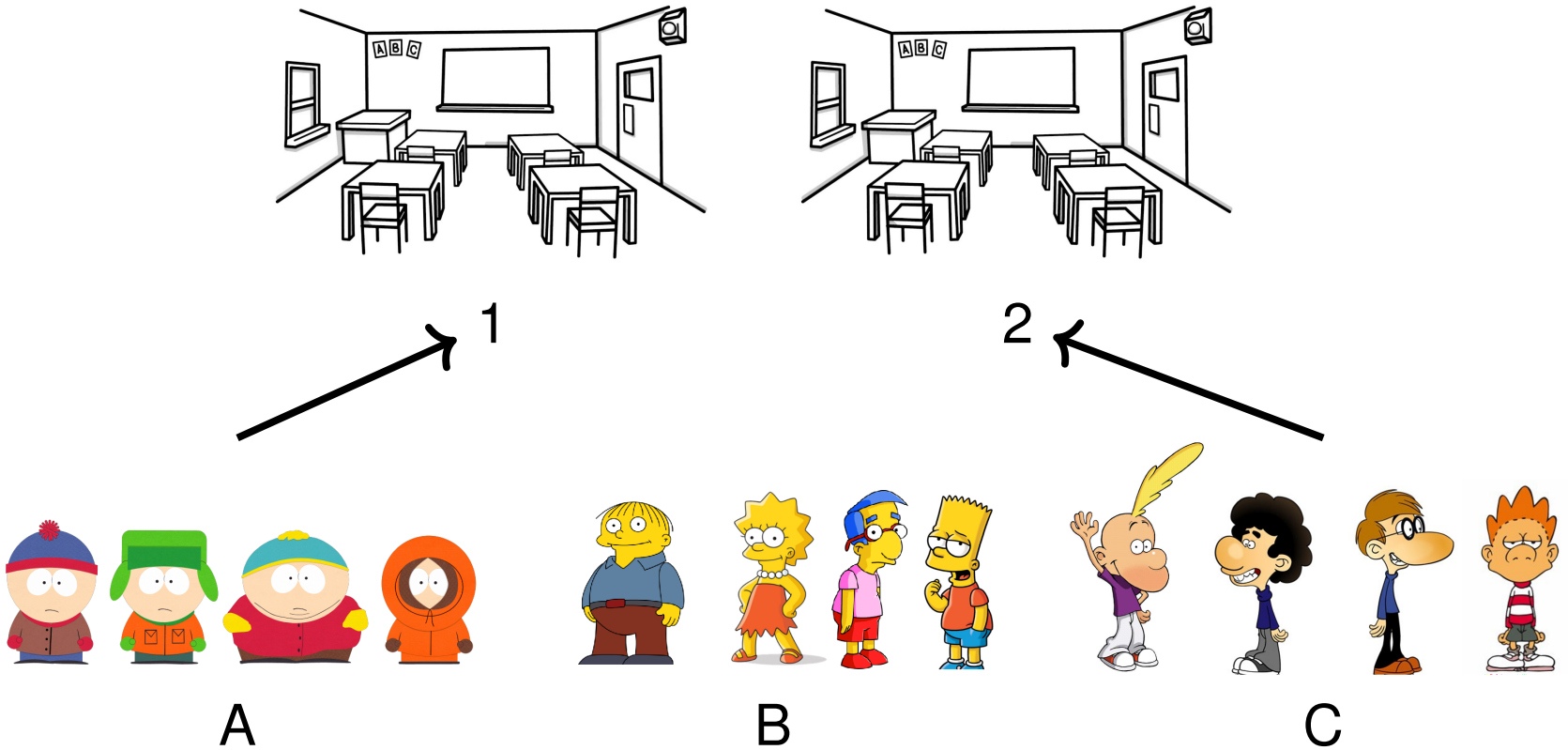

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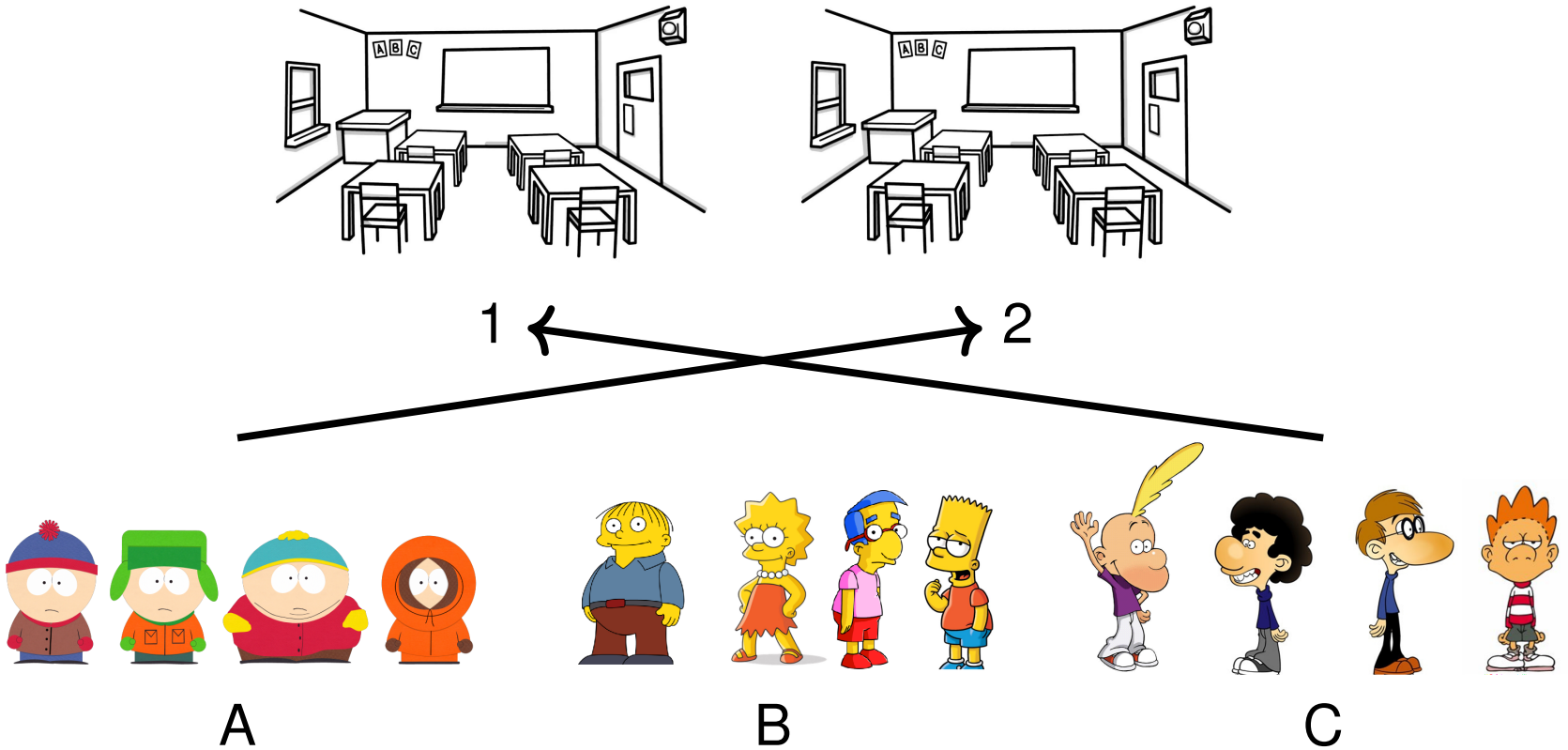

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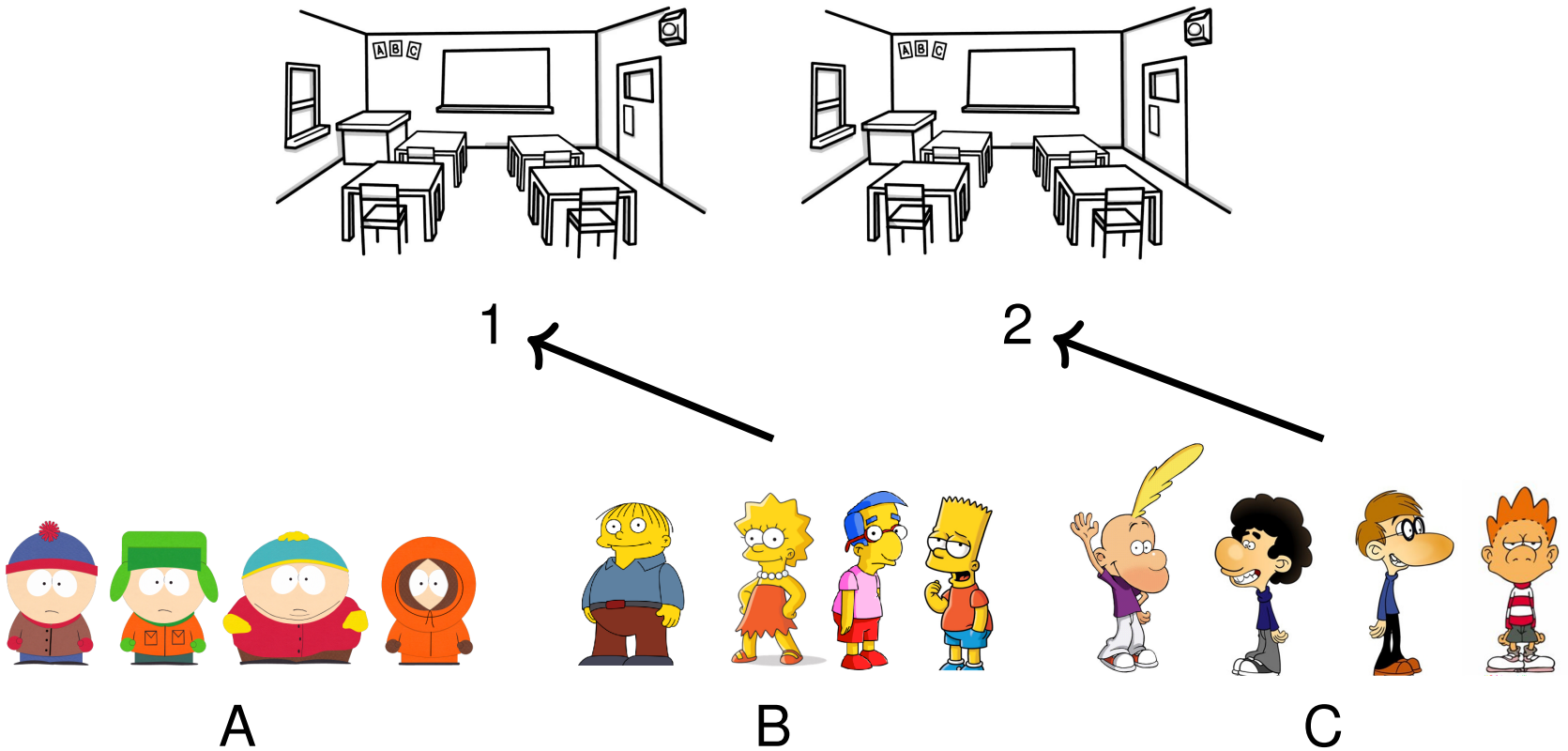

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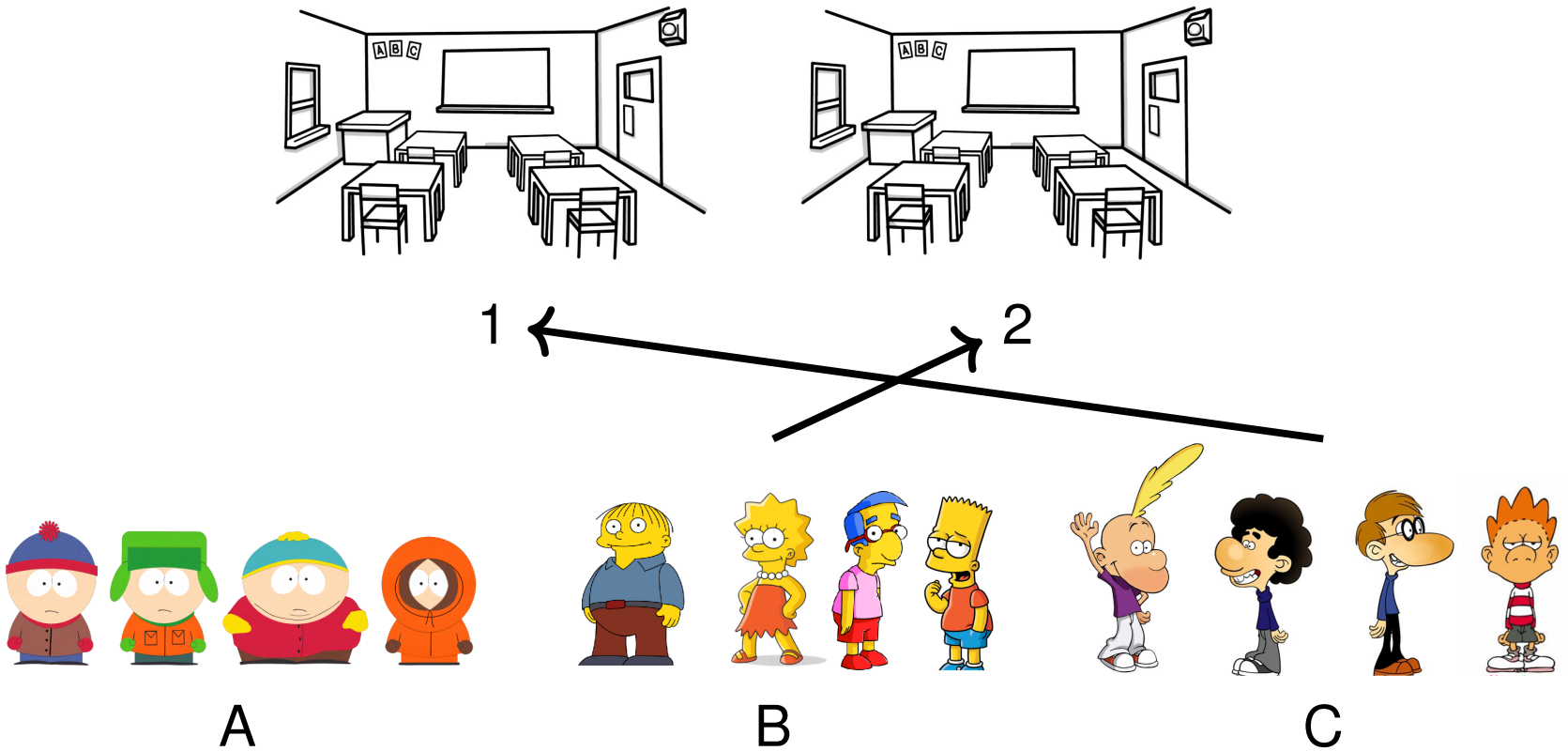

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