Layout Planning Optimization Model

Subscripts

M $Number\ of\ machines$

i $Index\ for\ machine\ (m = 1, 2, \dots, M)$

j $Index\ for\ machine\ (m = 1, 2, \dots, M)$

Input parameters

w_i $The\ half\ width\ of\ machine\ i$

h_i $The\ half\ height\ of\ machine\ i$

W $The\ width\ of\ production\ area$

H $The\ height\ of\ production\ area$

s_{ij} $The\ number\ of\ successor\ operations\ between\ machine\ i\ and\ machine\ j$

Decision variables

x_i $The\ horizontal\ position\ of\ machine\ i$

y_i $The\ vertical\ position\ of\ machine\ i$

d_{ij} $The\ distance\ between\ machine\ i\ and\ machine\ j\ (Manhattan\ distance)$

α_{ij} $1, if\ the\ machine\ i\ is\ to\ the\ left\ or\ behind\ the\ machine\ j;\ 0, otherwise$

β_{ij} $1, machine\ i\ and\ machine\ j\ don't\ overlap\ on\ the\ vertical\ axis;\ 0, otherwise$

γ_i $1, if\ machine\ i\ is\ placed\ parallel\ to\ the\ production\ area;\ 0, otherwise$

Mathematical Model

$$Minimize\ \sum_i \sum_j (d_{ij} \times s_{ij}) \tag{1}$$

$Subject\ to;$

$$d_{ij} \geq x_i - x_j + y_i - y_j \qquad \forall\ i, j \tag{2}$$

$$d_{ij} \geq x_i - x_j + y_j - y_i \qquad \forall\ i, j \tag{3}$$

$$d_{ij} \geq x_j - x_i + y_i - y_j \qquad \forall\ i, j \tag{4}$$

$$d_{ij} \geq x_j - x_i + y_j - y_i \qquad \forall\ i, j \tag{5}$$

$$x_i - x_j \geq w_i + w_j - M \times \alpha_{ij} - M \times \beta_{ij} - M \times \gamma_i - M \times \gamma_j \quad \forall\ i, j \tag{6}$$

$$x_j - x_i \geq w_i + w_j - M \times (1 - \alpha_{ij}) - M \times \beta_{ij} - M \times \gamma_i - M \times \gamma_j \quad \forall\ i, j \tag{7}$$

$$y_i - y_j \geq h_i + h_j - M \times \alpha_{ij} - M \times (1 - \beta_{ij}) - M \times \gamma_i - M \times \gamma_j \quad \forall\ i, j \tag{8}$$

$$y_j - y_i \geq h_i + h_j - M \times (1 - \alpha_{ij}) - M \times (1 - \beta_{ij}) - M \times \gamma_i - M \times \gamma_j \quad \forall\, i,j \tag{9}$$

$$x_i - x_j \geq w_i + w_j - M \times \alpha_{ij} - M \times \beta_{ij} - M \times (1 - \gamma_i) - M \times \gamma_j \quad \forall\, i,j \tag{10}$$

$$x_j - x_i \geq w_i + w_j - M \times (1 - \alpha_{ij}) - M \times \beta_{ij} - M \times (1 - \gamma_i) - M \times \gamma_j \quad \forall\, i,j \tag{11}$$

$$y_i - y_j \geq h_i + h_j - M \times \alpha_{ij} - M \times (1 - \beta_{ij}) - M \times (1 - \gamma_i) - M \times \gamma_j \quad \forall\, i,j \tag{12}$$

$$y_j - y_i \geq h_i + h_j - M \times (1 - \alpha_{ij}) - M \times (1 - \beta_{ij}) - M \times (1 - \gamma_i) - M \times \gamma_j \quad \forall\, i,j \tag{13}$$

$$x_i - x_j \geq w_i + w_j - M \times \alpha_{ij} - M \times \beta_{ij} - M \times \gamma_i - M \times (1 - \gamma_j) \quad \forall\, i,j \tag{14}$$

$$x_j - x_i \geq w_i + w_j - M \times (1 - \alpha_{ij}) - M \times \beta_{ij} - M \times \gamma_i - M \times (1 - \gamma_j) \quad \forall\, i,j \tag{15}$$

$$y_i - y_j \geq h_i + h_j - M \times \alpha_{ij} - M \times (1 - \beta_{ij}) - M \times \gamma_i - M \times (1 - \gamma_j) \quad \forall\, i,j \tag{16}$$

$$y_j - y_i \geq h_i + h_j - M \times (1 - \alpha_{ij}) - M \times (1 - \beta_{ij}) - M \times \gamma_i - M \times (1 - \gamma_j) \quad \forall\, i,j \tag{17}$$

$$x_i - x_j \geq w_i + w_j - M \times \alpha_{ij} - M \times \beta_{ij} - M \times (1 - \gamma_i) - M \times (1 - \gamma_j) \quad \forall\, i,j \tag{18}$$

$$x_j - x_i \geq w_i + w_j - M \times (1 - \alpha_{ij}) - M \times \beta_{ij} - M \times (1 - \gamma_i) - M \times (1 - \gamma_j) \quad \forall\, i,j \tag{19}$$

$$y_i - y_j \geq h_i + h_j - M \times \alpha_{ij} - M \times (1 - \beta_{ij}) - M \times (1 - \gamma_i) - M \times (1 - \gamma_j) \quad \forall\, i,j \tag{20}$$

$$y_j - y_i \geq h_i + h_j - M \times (1 - \alpha_{ij}) - M \times (1 - \beta_{ij}) - M \times (1 - \gamma_i) - M \times (1 - \gamma_j) \quad \forall\, i,j \tag{21}$$

$$x_i \geq w_i - M \times \gamma_i \quad \forall\, i \tag{22}$$

$$x_i \geq h_i - M \times (1 - \gamma_i) \quad \forall\, i \tag{23}$$

$$y_i \geq h_i - M \times \gamma_i \quad \forall\, i \tag{24}$$

$$y_i \geq w_i - M \times (1 - \gamma_i) \quad \forall\, i \tag{25}$$

$$x_i \leq W - w_i - M \times \gamma_i \quad \forall\, i \tag{26}$$

$$x_i \leq W - h_i - M \times (1 - \gamma_i) \quad \forall\, i \tag{27}$$

$$y_i \leq H - h_i - M \times \gamma_i \quad \forall\, i \tag{28}$$

$$y_i \leq H - w_i - M \times (1 - \gamma_i) \quad \forall\, i \tag{29}$$

$$x_i \geq 0 \quad \forall\, i \tag{30}$$

$$y_i \geq 0 \quad \forall\, i \tag{32}$$

$$\gamma_{ij} \geq 0 \quad \forall\, i,j \tag{33}$$

$$\alpha_{ij} \in \{0,1\} \quad \forall\, i,j \tag{34}$$

$$\beta_{ij} \in \{0,1\} \quad \forall\, i,j \tag{35}$$

$$\gamma_i \in \{0,1\} \quad \forall\, i \tag{36}$$

The objective (1) is to minimize the total product transport distance. There are three types of basic constraints in layout planning optimization. These constraints ensure that the distance between the machines (constraint 2to 5) is calculated correctly, the machines don't overlap (constraint 6 to 21) and the machines are not positioned outside the production area (constraint 22 to 21). Extra constraints can be added based on property of the production area.

$|x_i - x_j| + |y_i - y_j|$ formula is used to calculate the distance between the two machines. The absolute value can be calculated with four different formulas based on the machine positions. These formulas are ensured by constraints 2, 3, 4 and 5. It is clear that the absolute distance is equal to the maximum of these four formulas. Thus, when the distance is calculated, these four constraints will be met. Because the objective (1) is to minimize the distance, the distance will never be greater than the maximum of the four constraints.

It is enough that only one of sixteen constraints (6 to 21) is met because machines don't overlap. When one of the constraints is met, other constraints are overridden by the Big M method. The met constraint depends on the placement of the machines (α_{ij}, β_{ij} and γ_i).

Machines cannot be placed outside of the production area on the horizontal and vertical axis. For this, it is enough to meet four of the constraints between 22 and 29.