Small Problem 3: Discrete-time Discrete-observation HMM

## Discrete time Hidden Markov Model

**Given:**

Discrete time HMM (including transition matrix, initial state distribution, and emission probabilities)

Observed sequence of emissions

**Find:**

Query 1: MAP state sequence.

Query 2: Marginal posterior distribution of states at each time step given all of the observations (smoothing)

Query 3: Marginal posterior distribution of states at each time step given only the observations up to and including that time step (filtering)

**Metrics:**

Metric 1: Hamming distance between the true and the computed MAP state sequence

Metric 2: Smoothing: Total variation distance between the true and computed marginal posteriors at each time step. Mean and variance of this over all time steps.

Metric 3: Filtering: Total variation distance between the true and computed marginal posteriors at each time step. Mean and variance of this over all time steps.

The file “problem-3-generator.R” contains R code to generate a simple discrete-time, discrete-observation HMM. The specific instance for this small problem has 5 states and 5 observations and sequences of length 20. The transition distribution has probability of staying in the same state , of transitioning to state and of transitioning to state . The starting state is , and the states wrap around mod 5.

The observation for state is equal to with probability 0.6 and 0.1 of being equal to one of the other observation values.

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| *Name* | *Description* |
| problem-3-true-state.csv | The true state of the MDP for each time step 1:20 |
| problem-3-outputs.csv | The sequence of observations 1:20 |

Note that this problem involves no learning. The task is just to perform probabilistic inference with a given model and data.

Query 1: The MAP state sequence. Metric: Minimum Hamming distance between the predicted and the true MAP state sequences.

Query 2: Smoothing: For each time step , the marginal distribution , where is the output sequence. Metric: Total variation distance between the true and computed marginal posteriors at each time step. The mean and variance of the per-step total variation distance computed across all time steps.

Query 3: Filtering: For each time step , compute the marginal distribution , where is the vector of outputs from time 1 up to the current time . Metrics: Same as for Query 2.