Problem 7: Friends and Smokers

Consider the following graph (available as problem-7-friends.csv):



Define a Markov Random Field as follows:

People have a prior probability of smoking of 0.2

All else being equal, it is 3 times as likely that two friends have the same smoking habit than different.

Query: Compute the marginal probability of smoking for each of the unobserved nodes given the six observed nodes:

|  |  |
| --- | --- |
| Node | Observation of Smokes |
| 1 | 1 |
| 2 |  |
| 3 |  |
| 4 | 0 |
| 5 |  |
| 6 | 0 |
| 7 |  |
| 8 | 1 |
| 9 | 1 |
| 10 |  |
| 11 |  |
| 12 |  |
| 13 |  |
| 14 |  |
| 15 | 1 |
| 16 |  |
| 17 |  |
| 18 |  |
| 19 |  |
| 20 |  |

Additional graphs can be generated using the code in problem-7-generator.R. This file also contains a very slow implementation of Gibbs sampling as a baseline method.

Query 1: For each of the unobserved nodes, compute the posterior marginal probability that that person is a smoker.

Metric 1: The sum, over all of the queried nodes, of the absolute difference between the computed and the true posterior probability.