CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	**Number**	**Sections**
Mathematics	203	All
Examination	**Date**	**Pages**
Final	December 2022	3
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Special Instructions:	Only approved calculators are allowed **Show all your work for full marks.**	

MARKS

[13] **1.** **(a)** Solve for x: $2\log_2(x-1) - \log_2(x+2) = 2$.

(b) Let $f(x) = 2^{x^2-1}$ and $g(x) = \sqrt{4-x^2}$. Find $h = f \circ g$, and determine the domain and the range of $h(x)$.

(c) Given the function $f(x) = 2\,e^x/(1+e^x)$, find the inverse function f^{-1}, and determine the domain and the range of f^{-1}.

[7] **2.** Find the limit if it exists :

(a) $\displaystyle\lim_{x\to 3} \frac{x^2-2x-3}{|\,x^2-9\,|}$ **(b)** $\displaystyle\lim_{x\to\infty} (2\,x - \sqrt{4x^2-12x+1}\,)$

[6] **3.** Find **(a)** all horizontal, and **(b)** all vertical asymptotes of the function

$$f(x) = \frac{\sqrt{9x^6+3x^3}+3x^3}{(2x+1)^2\,(x+2)}$$

[12] **4.** Find the derivatives of the following functions (**for full marks** you have to show at least **one intermediate step** of your calculations):

(**a**) $f(x) = \dfrac{(1+2x)\,x}{1+\tan x}$

(**b**) $f(x) = (e^3\,x^3 + 3^x)\,\ln\sqrt{1+x}$

(**c**) $f(x) = \cos[x^2 + \cos(x^2 + \cos(x^2))]$

(**d**) $f(x) = (1+x^2)^{\arctan x}$

[6] **5.** Calculate the first derivative $f'(x)$ and the second derivative $f''(x)$ of the function $f(x) = x\,(x + e^{b\,x})$ where b is a parameter (i.e. a real number), and then find the exact value of $f''(1)$ as an expression of b.

[4] **6.** Consider the following piecewise-defined function:

$$f(x) = \begin{cases} -\sin(a\,x) & \text{if} \quad \text{x} \leq 0 \\ x & \text{if} \quad \text{x} > 0 \end{cases}$$

(a) For what values of a, if any, is the function $f(x)$ continuous at $x = 0$?
(b) Is there a value of the parameter a that makes f differentiable at every x?

[6] **7.** Consider the function $y = \arctan(2x)$.

(a) Find the linearization $L(x)$ of the function $y(x)$ at $a = 1/2$.

(b) Find the differential dy and evaluate it for the values $x = 0$ and $dx = 0.1$.

[7] **8.** Let $f(x) = x^3 - 2x + 1$.

(a) Find the slope m of the secant line joining the points $(-2, f(-2))$ and $(0, f(0))$.

(b) Find all points $x = c$ (if any) on the interval [-2,0] such that the rate $f'(c)$ of instantaneous change of f is equal to the slope m of the secant line in **(a)**.

[12] **9.** **(a)** Verify that the point (1,2) belongs to the curve defined by the equation $y^3 - x\,y = 2 + 2\sqrt{3 + x^2}$, and find an equation of the tangent line to the curve at this point.

(b) At 1 PM, ship A is 5 kilometers strictly to the west of ship B. Ship A is sailing west at speed 20 km/hour and ship B is sailing north at 30 km/hour. How fast (in km/hour) is the distance between ships changing at 3 PM?

[14] **10.** **(a)** Find the point (x_0, y_0) on the curve $y = 2\sqrt{x}$ that is closest to the point (3,0).

(b) A box with a square base is to be constructed with a volume of 50 m^3. The material for the bottom and the sides of the box costs $\$2/\text{m}^2$, and the material for the top costs $\$5/\text{m}^2$. Find the dimensions that minimize the cost of the box.

(c) Use l'Hopital's rule to evaluate the $\displaystyle\lim_{x \to 0} \frac{x^2 \cos(x)}{e^{x^2} - 1}$.

[13] **11.** Given the function $f(x) = x^4 - 8\,x^2$.

(a) Calculate $f'(x)$ and use it to determine intervals where the function is increasing, intervals where it is decreasing, and all critical numbers on the $x-$axis where $f(x)$ has local maximum or local minimum.

] **(b)** Calculate $f''(x)$ and use it to determine intervals where the function is concave upward, intervals where the function is concave downward, and the inflection points (if any).

(c) Sketch the graph of the function $f(x)$ using the information obtained above.

[5] **Bonus Question.** Let $p(x) = x^4 + a^2x^2 - 2a^2x$, where a is any real number. Prove that the graph $y = p(x)$ has at least one point of local minimum on the interval $(-1, 1)$.