October 2021Mdt - Exam

Vectors and Matrices (Concordia University)

Math 204, Class test, October 24, 2021

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Time: 75 min for the questions, plus 10 minutes additional for visual registration of the solution pages

TWO PAGES. Answer all questions. Closed book

1. **(10 points)** Solve the following system (mind that specific features of the matrix of the system can provide short-cuts to Gauss-Jordan method):

$$\begin{cases} x_1 + x_2 - 6x_3 - 4x_4 = 6 \\ 3x_1 - x_2 - 6x_3 - 4x_4 = 2 \\ 2x_1 + 3x_2 + 9x_3 + 2x_4 = 6 \\ 3x_1 + 2x_2 + 3x_3 + 8x_4 = -7 \end{cases}$$

2. **(10 points)**

   Find the inverse matrix A^{-1} for

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 1 & 2 & 3 & 5 \end{pmatrix}$$

3. **(10 points)** Solve the following equation for matrix X:

$$\begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix} (X + I_2)^{-1} \begin{pmatrix} 2 & 2 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} 55 & 27 \\ 2 & 1 \end{pmatrix},$$

   where I_2 is the 2×2 identity matrix.

4. **(10 points)**

   Compute the determinant

$$\begin{vmatrix} 6 & -5 & 8 & 4 \\ 9 & 7 & 5 & 2 \\ 7 & 5 & 3 & 7 \\ -4 & 8 & -8 & -3 \end{vmatrix}$$

5. **(10 points)** Find x, y, z using Cramer's rule:

$$\begin{cases} 2x + 3y + 5z = 10 \\ 3x + 7y + 4z = 3 \\ x + 2y + 2z = 3 \end{cases}$$

6. **(10 points)** Find the determinant

$$\begin{vmatrix} C_{11} & C_{12} & C_{13} & C_{14} \\ C_{21} & C_{22} & C_{23} & C_{24} \\ C_{31} & C_{32} & C_{33} & C_{34} \\ C_{41} & C_{42} & C_{43} & C_{44} \end{vmatrix}$$

where C_{ij} is the cofactor of the entry a_{ij} of the matrix A

$$A = \begin{pmatrix} 2 & 3 & 4 & 5 \\ 2 & 7 & 8 & 9 \\ 2 & 3 & 3 & 8 \\ 2 & 3 & 4 & 7 \end{pmatrix}$$

standing in the i-th row and j-th column.