Math 204 midterm 2023 winter

Vectors and Matrices (Concordia University)

Math 204, Class test, March 12, 2023

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Time: 1 Hour and 30 minutes

Answer all questions. **Only approved calculators are allowed**

1. **(10 points)** Solve the following system of equations

$$\begin{cases} x_1 + x_2 - 6x_3 - 4x_4 = 6 \\ 3x_1 - x_2 - 6x_3 - 4x_4 = 2 \\ 2x_1 + 3x_2 + 9x_3 + 2x_4 = 6 \\ 3x_1 + 2x_2 + 3x_3 + 8x_4 = -7 \end{cases}$$

2. **(10 points)**

(A) Find the inverse matrix A^{-1} if

$$A = \begin{pmatrix} 3 & 8 & 9 & 6 \\ 0 & 2 & 6 & 4 \\ 0 & 0 & 3 & 6 \\ 3 & 8 & 9 & 8 \end{pmatrix}$$

(B) Solve the following equation for matrix X:

$$\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} X = X \begin{pmatrix} 2 & 2 \\ 1 & 2 \end{pmatrix} + \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

3. **(10 points)** Compute the determinant

$$\begin{vmatrix} 1303 & 100 & 0.01 & 39 \\ 1304 & 100 & 0.02 & 40 \\ 1305 & 100 & 0.01 & 41 \\ 1306 & 100 & 0.02 & 42 \end{vmatrix}$$

4. **(10 points)** Using Cramer's rule, find a number A such that the system

$$\begin{cases} x + 5y + 6z = 1 \\ x + 4y + 7z = A \\ x + 5y + 4z = -1 \end{cases}$$

has a solution (x, y, z) with $y = 2$. (You don't need to find x and z.)

5. **(10 points)** The sum of squares

$$|AB|^2 + |AD|^2$$

of the two sides of a parallelogram $ABCD$ is equal to 12. Find the sum of squares

$$|AC|^2 + |BD|^2$$

of the diagonals of this parallelogram. (Hint: express all the quantities through vectors $\vec{b} = \vec{AB}$ and $\vec{d} = \vec{AD}$ and make use of the properties of the dot product.)

6. **(10 points)** Find elementary matrices E_1, E_2, E_3 and E_4 such that

$$E_4E_3E_2E_1 \begin{pmatrix} 1 & 0 & 0 \\ 7 & 2 & 0 \\ 5 & 8 & 3 \end{pmatrix} = \begin{pmatrix} 0 & 2 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$