Final Exam April Winter 2017, questions

Differential & Integral Calculus II (Concordia University)

CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	**Number**	**Sections**
Mathematics	205	All
Examination	**Date**	**Pages**
Final	April 2017	2
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Special Instructions:	Only approved calculators are allowed. **Show all your work for full marks.**	

MARKS

[12] **1.** **(a)** Sketch the graph of $f(x) = 4 - x^2$ on the interval $[-1, 2]$, and approximate the area between the graph and the x-axis on $[-1, 2]$ by the left Riemann sum L_3 using partitioning of the interval into 3 subintervals of equal length.

(b) For the same $f(x) = 4 - x^2$, write in sigma notation the formula for the left Riemann sum L_n with partitioning of the interval $[-1, 2]$ into n subintervals of equal length, and calculate $\int_{-1}^{2} f(x)\,dx$ as the limit of L_n at $n \to \infty$

NOTE: you may need the formulas $\sum\limits_{k=1}^{n} k = \frac{n(n+1)}{2}$, $\sum\limits_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$.

(c) Calculate the derivative of the function $F(x) = \sec(3x) + \int\limits_{0}^{\tan(3x)} e^{-t^2}\,\mathrm{d}t$

(Hint: use the Fundamental Theorem of Calculus and differentiation rules.)

[12] **2.** Evaluate the following definite integrals (give the exact answers):

(**a**) $\displaystyle\int_0^3 x\sqrt{9 - x^2}\,dx$ (**b**) $\displaystyle\int_1^e \ln^2 x\,dx$

[6] **3.** Find $F(t)$ such that $F'(t) = \sec^4(t)$ and $F\left(\frac{\pi}{4}\right) = 0$.

[10] **4.** Calculate the following indefinite integrals:

(**a**) $\displaystyle\int (x^2 - 2x)\,\sin(2x)\,dx$ (**b**) $\displaystyle\int \frac{x^2 + 3}{x^2 - 3x}\,dx$

[8] **5.** Evaluate the given improper integral or show that it diverges:

(**a**) $\displaystyle\int_0^\infty x^2 e^{-x^3}\,dx$ (**b**) $\displaystyle\int_0^1 \frac{x}{x^2 - 1}\,dx$

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[17] **6.** **(a)** Sketch the curves $y = \sqrt{2x}$ and $y = x$ and find the area enclosed.

(b) Sketch the region enclosed by the parabola $x = y^2 + 1$ and the line $x = 5$ and find the volume of the solid obtained by revolution of this region about the line $x = 5$.

(c) Find the average value of the function $f(x) = x\sqrt{1 + 2x}$ on the interval $[0, 4]$.

[9] **7.** Find the limit of the sequence $\{a_n\}$ or prove that the limit does not exist:

$$(\mathbf{a}) \quad a_n = \frac{3^n - n}{2^{2n}} \qquad (\mathbf{b}) \quad a_n = \frac{\ln(n^3)}{n+1} \qquad (\mathbf{c}) \quad a_n = \sqrt{n+100} - \sqrt{n}$$

[8] **8.** Determine whether the series is divergent or convergent, and if convergent, then absolutely or conditionally :

$$(\mathbf{a}) \quad \sum_{n=2}^{\infty} (-1)^n \frac{\ln n}{n} \qquad (\mathbf{b}) \quad \sum_{n=0}^{\infty} (-1)^{n+1} \frac{n+100}{100n+1}$$

[10] **9.** Find the radius and the interval of convergence of the following series

$$(\mathbf{a}) \quad \sum_{1}^{\infty} \frac{(3x)^n}{n!} \qquad (\mathbf{b}) \quad \sum_{n=1}^{\infty} \frac{(x+1)^{3n}}{n\,8^n}$$

[8] **10.** **(a)** Derive the Maclaurin series of $f(x) = x^3 \ln(1 + 2x^2)$ (HINT: start with the series for $\ln(1 + z)$ where $z = 2x^2$).

(b) Use differentiability of power series to find the sum $F(x) = \sum_{1}^{\infty} \frac{(x-1)^n}{n}$ within its radius of convergence.

[5] **Bonus Question.** A solid is generated by rotating about the x-axis the region enclosed between the curve $y = f(x)$ and x-axis on the interval $[0, b]$, where f is a positive function and $x \geq 0$. For all values of $b \geq 0$ the generated solid has the volume πb^4. Find the function f.