# Math 204, Class test, October 30, 2022

1. (**10 points**) Solve the following system of equations

$$\begin{cases} 2x_1 + 3x_2 - x_3 + x_4 = 1 \\ 8x_1 + 12x_2 - 9x_3 + 8x_4 = 3 \\ 4x_1 + 6x_2 + 3x_3 - 2x_4 = 3 \\ 2x_1 + 3x_2 + 9x_3 - 7x_4 = 3 \end{cases}$$

2. (**10 points**)

(A) Find the inverse matrix $A^{-1}$ if

$$A = \begin{pmatrix} 2 & 6 & 8 & 5 \\ 0 & 3 & 4 & 5 \\ 0 & 0 & 4 & 5 \\ 0 & 0 & 0 & 5 \end{pmatrix}$$

(B) Solve the following equation for matrix $X$:

$$\begin{pmatrix} 3 & 2 \\ 1 & 3 \end{pmatrix} X = X \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} + \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

3. (**10 points**) Compute the determinant

$$\begin{vmatrix} 1202 & 1000 & 0.1 & 49 \\ 1203 & 1000 & 0.2 & 50 \\ 1204 & 1000 & 0.2 & 51 \\ 1205 & 1000 & 0.2 & 52 \end{vmatrix}$$

4. (**10 points**) Using Cramer's rule, find a number $B$ such that the system

$$\begin{cases} x + 2y + 3z = B \\ x + 3y + 7z = 1 \\ x + 2y + 4z = -1 \end{cases}$$

has a solution $(x, y, z)$ with $x = 2$. (You don't need to find $y$ and $z$.)

5. (**10 points**) Three parallelograms have three common vertices $(1, 2)$, $(2, 3)$ and $(1, 1)$.

1) Find the coordinates of the fourth vertex of each parallelogram.

2) These three parallelograms have 2 diagonals each (6 in total). Find the length of the longest diagonal (among 6).

6. (**10 points**) Find elementary matrices $E_1$, $E_2$, $E_3$ and $E_4$ such that

$$E_4 E_3 E_2 E_1 \begin{pmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 4 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

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