Final 2018 math 205

MATH (Concordia University)

CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	**Number**	**Sections**
Mathematics	205	All
Examination	**Date**	**Pages**
Final	April 2018	2
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Special Instructions:	Only calculators approved by the Department are allowed. **For full marks show all your work.**	

[12] **1.** **(a)** Sketch a graph of the function

$$f(x) = \begin{cases} 4 + x & \text{for } -4 \le x < -2 \\ 2 - \sqrt{4 - x^2} & \text{for } -2 \le x \le 2 \end{cases}$$

on the interval $-4 \le x \le 2$ and calculate the definite integral $\int_{-4}^{2} f(x)\,dx$ in terms of signed area (*do not* antidifferentiate).

(b) Now graph $g(x) = f(x) - 1$. What is the value of $\int_{-4}^{2} g(x)\,dx$?

(c) Find the derivative $F'(x)$ of $F(x) = \sin x + \int_{0}^{3x} \sqrt{a^2 + u^2}\,\cos u\,du$, where a is a parameter, and use it to determine whether $F(x)$ is increasing or decreasing at $x = \pi$.

[10] **2.** Find the following indefinite integrals:

(**a**) $\displaystyle\int x \arcsin(x^2)\,dx$ (**b**) $\displaystyle\int \frac{x}{x^2 - 3x - 4}\,dx$

[12] **3.** Evaluate the following definite integrals (give the exact answers):

(**a**) $\displaystyle\int_{e}^{e^4} \frac{1}{x\sqrt{\ln x}}\,dx$ (**b**) $\displaystyle\int_{0}^{\pi/6} \sqrt{1 + \cos(2x)}\,dx$

[6] **4.** Find $F(t)$ such that $F'(t) = \left(\dfrac{\sec t}{\tan t}\right)^4$ and $F\left(\dfrac{\pi}{4}\right) = -1$

[8] **5.** Evaluate the given improper integral or show that it diverges:

(**a**) $\displaystyle\int_{1}^{\infty} \frac{\sqrt{x}}{x + \sqrt{x}}\,dx$ (**b**) $\displaystyle\int_{0}^{\pi/2} \frac{\cos x}{\sqrt{1 - \sin x}}\,dx$

[17] **6.** **(a)** Sketch the curves $y = 6x - x^2$ and $y = x^2 - 2x$, and find the area enclosed by these curves.

(b) Find the volume of the solid obtained by rotating the region bounded by the curve $y = x^3$ and the lines $x = 0$ and $y = 8$ about the axis $x = 2$.

(c) A cup of coffee cools from 95°C to about 61°C in 30 minutes according to the equation $T(t) = 20 + 75e^{-0.02t}$. Find the average temperature T_{av} of the coffee during the interval $[0, 30]$.

[9] **7.** Find the limit of the sequence $\{a_n\}$ at $n \to \infty$ or prove that it does not exist:

(**a**) $a_n = \dfrac{n\cos(n\pi)}{\sqrt{9+n^3}}$ (**b**) $a_n = n\sin\left(\dfrac{\pi}{n}\right)$ (**c**) $a_n = 2\ln(1+3n) - \ln(4+n^2)$

[8] **8.** Determine whether the series is divergent or convergent, and if convergent, then absolutely or conditionally :

(**a**) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n\, n!}{(n+100)!}$ (**b**) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{3n+1}$

[10] **9.** Find the radius and the interval of convergence of the following series

(**a**) $\displaystyle\sum_{n=1}^{\infty} \frac{(3x-2)^n}{5^n}$ (**b**) $\displaystyle\sum_{n=1}^{\infty} \frac{(x-4)^n}{n^2}$

[8] **10.** **(a)** Find the Maclaurin series for $f(x) = x\, e^{x^2}$
(Hint: start with the series for e^t, then replace t with x^2 . . .)

(b) Use differentiability of the power series to find the sum
$F(x) = \displaystyle\sum_{n=0}^{\infty} \frac{(x+1)^{n+1}}{n+1}$ within its radius of convergence.

[5] **Bonus Question.** Find all continuous functions $f(x)$ other than $f(x) = 0$ satisfying the equation

$$\int_0^x f(t)\,dt = (f(x))^2 - (f(0))^2$$

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