CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	**Number**	**Sections**
Mathematics	203	All
Examination	**Date**	**Pages**
Final	April 2023	3
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Special	Only approved calculators are allowed	
Instructions:	**Show all your work for full marks.**	

MARKS

[10] **1.** **(a)** Solve for x (find the **exact** value, do not approximate): $2^{2x} = 2^{x+1} + 24$.

(b) Find the inverse function f^{-1} of $f(x) = \log_3(9 + 3^{-x})$.
Determine also the domain and range of $f(x)$ and of $f^{-1}(x)$.

[8] **2.** Find the limit if it exists, otherwise explain why it does not exist:

(a) $\lim\limits_{x\to 2} \dfrac{|x-2|\,(x+3)}{x^2+x-6}$ **(b)** $\lim\limits_{x\to 1} \dfrac{3-\sqrt{x^2+8}}{x^2-1}$

[6] **3.** Find all horizontal and vertical asymptotes of the function

$$f(x) = \frac{(x+1)\sqrt{9x^2+1}}{x^2-25}$$

.

[12] **4.** Find the derivatives of the following functions (**for full marks** you have to show at least **one intermediate step** of your calculations):

(**a**) $f(x) = x^{3/2} \ln x\, e^x$

(**b**) $f(x) = \dfrac{1+\tan x}{1+x^2} + e^3$

(**c**) $f(x) = [\, e^{x \sin x} + \sin(x^2 + x)]^3$

(**d**) $f(x) = (4 + x^2)^{\cos x}$

[6] **5.** Calculate the first derivative $f'(x)$ and the second derivative $f''(x)$ of $f(x) = b^{x^2}$ where the parameter b is a positive real number.
Then find the exact value of $f''(x)$ at $x = 0$ as an expression of b.

[11] **6.** Consider the function $f(x) = \sqrt{25+x}$.

(a) Use the **definition of derivative** to find the formula for df/dx.

(b) Find the linearization $L(x)$ of the function $f(x)$ at $a = 0$

(c) Use this linearization to approximate $\sqrt{30}$ (i.e. approximate $\sqrt{25+5}$) .

[6] **7.** Let $f(x) = x^3 - 2x + 3$.

(a) Find the slope m of the secant line joining the points $(-2, f(-2))$ and $(0, f(0))$.

(b) Find all points $x = c$ (if any) on the interval [-2,0] such that $f'(c) = m$.

[17] **8.** **(a)** Verify that the point (2,1) belongs to the curve defined by the equation $x^2 + 2y^2 + 2 = x^3\, y^3$, and find an equation of the tangent line to the curve at this point.

(b) A box with a square base is to be constructed with a volume of 50 m^3. The material for the bottom and the sides of the box costs \$2/m^2, and the material for the top costs \$5/m^2. Find the length X of the bottom size and the hight Y of the box that minimize its cost.

(c) Use l'Hôpital's rule to evaluate the $\lim\limits_{x\to 0} \dfrac{\sin^2(3x)}{1-\cos(2x)}$.

[11] **9.** **(a)** Find the absolute extrema of $f(x) = \dfrac{x}{x^2 - x + 1}$ on the interval $[0, 3]$.

(b) A rectangle is inscribed with its base on the x-axis and its upper corners on the parabola $y = 12 - x^2$. What is the largest possible area of such rectangle?

[13] **10.** Given the function $f(x) = \frac{2}{3}x^3 - 7x^2 + 12x - 3$.

(a) Calculate $f'(x)$ and use it to determine intervals where the function is increasing, intervals where it is decreasing, and all critical numbers on the x-axis where $f(x)$ has local maximum or local minimum.

(b) Calculate $f''(x)$ and use it to determine intervals where the function is concave upward, intervals where the function is concave downward, and the inflection points (if any).

(c) Sketch the graph of the function $f(x)$ using the information obtained above.

[5] **Bonus Question.** Is it possible to have a function $f(x)$ such that $f(0) = 0$, $f(2) = 4$, **and** $f'(x) < 2$ for all x on the interval $[0, 2]$? Give an example of such function or prove that it is impossible.