CONCORDIA UNIVERSITY

Department of Mathematics & Statistics

Course	Number	Sections
Mathematics	205	All
Examination	**Date**	**Pages**
Final	December 2022	2
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Special Instructions:	Only approved calculators are allowed. **Show all your work for full marks.**	

MARKS

[12] **1.** **(a)** Sketch the graph of $f(x) = x^2 - 4$ on the interval $[-1, 2]$, and approximate the definite integral $\int_{-1}^{2} f(x)dx$ by the **left** Riemann sum S_3 using partitioning of the interval into 3 subintervals of equal length.

(b) For the same $f(x) = x^2 - 4$, write in sigma notation the formula for the **right** Riemann sum R_n with partitioning of the interval $[-1, 2]$ into n subintervals of equal length, and calculate $\int_{-1}^{2} f(x)\, dx$ as the limit of S_n at $n \to \infty$

NOTE: you may need the formulas $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$, $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$.

(c) Calculate the derivative of the function $F(x) = \sin(x^2) + \int_{0}^{\cos x} t^2 e^{-t^2}\, dt$

(Hint: use the Fundamental Theorem of Calculus and differentiation rules.)

[10] **2.** Calculate the following indefinite integrals:

(a) $\int e^x \cos(2x)\, dx$ (b) $\int \frac{2x^2+4}{x^2-4}\, dx$

[12] **3.** Evaluate the following definite integrals (give the exact answers):

(a) $\int_{0}^{\pi/4} \sec^4(x) \tan^2(x)\, dx$ (b) $\int_{1}^{e} \ln^2 x\, dx$

[6] **4.** Find $F(t)$ such that $F'(t) = \frac{e^t}{e^t + e^{-t}}$ and $F(0) = 0$.

[8] **5.** Evaluate the given improper integral or show that it diverges:

(a) $\int_{-\infty}^{0} xe^{-x^2}\, dx$ (b) $\int_{0}^{\pi/2} \tan(x)\, dx$

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[17] **6.** **(a)** Sketch the curves $y = x + \dfrac{3}{x}$ and $y = 4$ and find the area enclosed.

(b) Sketch the region enclosed by $f(x) = \sin(x)$ and the $x-$axis on the interval $[0, \pi]$ and find the volume of solid of revolution of this region about the axis $y = -1$.

(c) Find the average value of the function $f(x) = \dfrac{x}{\sqrt{1+2x}}$ on the interval $[0, 4]$.

[9] **7.** Find the limit of the sequence $\{a_n\}$ or prove that the limit does not exist:

(a) $a_n = \dfrac{e^n - n}{(-2)^n}$ (b) $a_n = \dfrac{\ln(n^2)}{n+1}$ (c) $a_n = (\sqrt{n^2 + 10n + 100} - n)$

[8] **8.** Determine whether the series is divergent or convergent, and if convergent, then absolutely or conditionally :

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{2 \ln n}{n^2}$ (b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{(5n+1)^{1/3}}$

[10] **9.** Find the radius of convergence and the interval of convergence of the series

(a) $\displaystyle\sum_{1}^{\infty} \frac{(-4x)^{2n}}{n^2+1}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(x+1)^{3n}}{n\,8^n}$

[9] **10.** **(a)** Derive the Maclaurin series of $f(x) = x^2 e^{2x}$ (HINT: start with the series for e^z where $z = 2x$).

(b) Use differentiability of power series to find the sum $F(x) = \displaystyle\sum_{1}^{\infty} \frac{x^{2n}}{n}$ within its radius of convergence.

[5] **Bonus question.** Assume we know that some power series $S(x)$ about $a = 1$ is convergent at $x = 3$. Is this information sufficient to claim that the series $S(x)$ is also convergent at $x = 0$? Explain why it is sufficient, or give a counter example if it is not.