



2021 Winter Midterm - Old exam.

Vectors and Matrices (Concordia University)

Math 204, Class test, March 14, 2021

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Time: 75 min for the questions, plus 10 minutes additional for visual registration of the solution pages

TWO PAGES. Answer all questions. Closed book

1. (10 points) Using Gauss-Jordan method solve the following system:

$$\begin{cases} 3x_1 - 2x_2 - 5x_3 + x_4 = 3 \\ 2x_1 - 3x_2 + x_3 + 5x_4 = -3 \\ x_1 + 2x_2 - 4x_4 = -3 \\ x_1 - x_2 - 4x_3 + 9x_4 = 22 \end{cases}$$

2. (10 points)

Find the inverse matrix A^{-1} for

$$A = \begin{pmatrix} 1 & 0 & 0 & 5 \\ 2 & 1 & 0 & 0 \\ 3 & 4 & 5 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

3. (10 points) Solve the following equation for matrix X :

$$\begin{pmatrix} 5 & 7 \\ 4 & 6 \end{pmatrix} X^{-1} \begin{pmatrix} 2 & 2 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} 10 & 11 \\ 9 & 10 \end{pmatrix}$$

4. (10 points) Let A be 1×4 matrix

$$A = (1, 1, 1, 1)$$

and let A^t be its transpose. Find the determinant of the matrix B

$$B = A^t A + I_4,$$

where I_4 is the 4×4 identity matrix.

(To compute the determinant make use of properties of determinants and row operations, expressing determinants of higher order via cofactor expansion is not a good idea.)

5. **(10 points)** Find x, y, z using Cramer's rule:

$$\begin{cases} x + 2y + 3z = 4 \\ 2x + 7y + 7z = 9 \\ 2x + 4y + 8z = 10 \end{cases}$$

6. **(10 points)** For

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 4 & 4 & 5 \\ 1 & 2 & 5 & 5 \\ 1 & 2 & 3 & 8 \end{pmatrix}$$

find

$$(2a_{12} + 7a_{14})C_{12} + (2a_{22} + 7a_{24})C_{22} + (2a_{32} + 7a_{34})C_{32} + (2a_{42} + 7a_{44})C_{42}.$$

without computing the cofactors entering this expression. Here a_{ij} is the entry of matrix A standing in the i -th row and j -th column of the matrix A ; C_{ij} is the cofactor of a_{ij} .