CONCORDIA UNIVERSITY
Department of Mathematics and Statistics

Course	**Number**	**Section(s)**
Mathematics	204	All

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Instructors	**Course Examiner**
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Special Instructions: Only approved calculators are allowed !

1. **[10]** Let

$$A = \begin{pmatrix} 3 & 2 & 3 \\ 4 & 3 & 7 \\ 3 & 2 & 4 \end{pmatrix}$$

Solve the equation

$$AXA^{-1} = \underbrace{\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}}_{C} + A \begin{bmatrix} 3 & 2 & 3 \\ 4 & 3 & 7 \\ 3 & 2 & 4 \end{bmatrix} = \Bigg[\ 4 \quad 3 \quad 4$$

$\cancel{A^{-1}A}XA^{-1}$

$\cancel{XA}^{-1} = A^{-1}C$

for (3×3) matrix X.

$XA^{-1} = A^{-1}C$

$X = A^{-1}CA.$

2. **[10]** Find the determinant of (4×4) matrix X if

$$\det\left[\underbrace{\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \\ 3 & 4 & 1 & 2 \\ 4 & 1 & 2 & 3 \end{pmatrix}}_{A} X \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}}_{B}\right] = \det\left(\underbrace{\begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{C}\right)$$

$\det(ABC) = \det(A)\ \det(B)\ \det(C)$

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3. [10] For which a the system

$$\begin{cases} 3x_1 + 2x_2 + 3x_4 = 0 \\ 11x_1 + 4x_2 + ax_3 + 5x_4 = 0 \\ 4x_1 + 3x_2 + 7x_4 = 0 \\ 3x_1 + 2x_2 + x_3 + 4x_4 = 0 \end{cases}$$

has infinitely many solutions?

4. [10] Three lines l_1, l_2 and l_3 pass through the origin $O = (0,0,0)$ and are parallel to the vectors $(0,4,2)$, $(-1,-3,-2)$ and $(10,2,4)$ respectively. Find the area of the triangle whose vertices are the points of intersection of these three lines with the plane

$$x + 2y - \overset{z}{3x} = 1$$

5. [10] The plane α with equation

$$ax + by + 12z = 2$$

is parallel to the plane

$$x + 2y + 4x = \sqrt{2}.$$

The line l is the intersection of the plane α with the plane

$-14 - 66 + 80$ ✓

$2 - 22 + 20$ ✓

$$x + y + z = 1.$$

Write down the parametric equation of the line l.

6. [10] A vector $\vec{a}$ has norm 2 and is orthogonal to the vectors $\vec{b} = (7,3,4)$ and $\vec{c} = (-1,1,1)$. Find the norm of the vector $2\vec{a} - 0.5\vec{b} + \vec{c}$.

$\vec{a} = (-1, -11, 10)$

$\|\vec{a}\| = \sqrt{1 + 121 + 100}$

$= \sqrt{122 + 100}$

$= \sqrt{222}$

$\dfrac{\vec{a}}{\|a\|} = \dfrac{-1}{\sqrt{222}}, \dfrac{-11}{\sqrt{222}}, \dfrac{10}{\sqrt{222}}$

$2 \cdot \dfrac{\vec{a}}{\|a\|} = \dfrac{-2}{\sqrt{222}}, \dfrac{-22}{\sqrt{222}}, \dfrac{20}{\sqrt{222}}$

$A(A+B)+B(A+B)$

$(A+B)^2$

7. **[10]** For two square (2×2) matrices A and B one has the equalities

$$A^2 = \begin{pmatrix} 3 & 2 \\ 4 & 3 \end{pmatrix}; \quad B^2 = \begin{pmatrix} 2 & 4 \\ 4 & 10 \end{pmatrix};$$

$$AB = \begin{pmatrix} 2 & 4 \\ 3 & 5 \end{pmatrix}; \quad (A+B)^2 = \begin{pmatrix} 10 & 12 \\ 18 & 22 \end{pmatrix}$$

Find the matrix BA.

8. **[10]** Find a basis for the solution space of the homogeneous system

$$\begin{cases} x_1 + x_2 + x_3 + x_4 + x_5 = 0 \\ 2x_1 - x_2 - x_3 - x_4 + x_6 = 0 \\ 4x_1 + x_2 - x_3 + x_4 - 2x_5 + x_6 = 0 \end{cases}$$

9. **[10]** Find the eigenvalue of the matrix

$$A = \begin{pmatrix} 7 & 7 & 1 \\ 13 & 13 & -1 \\ 1 & -3 & 0 \end{pmatrix}$$

corresponding to an eigenvector that is orthogonal to the plane

$$x - y + 2z = -\sqrt{3}$$

10. **[10]** Find an invertible 2×2 matrix P such that the matrix

$$P^{-1} \begin{pmatrix} 17 & -6 \\ 35 & -12 \end{pmatrix} P$$

is diagonal.

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$\frac{4}{2} - \frac{9}{2}$

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