Midterm 3 March Winter 2018, questions

Differential & Integral Calculus II (Concordia University)

CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	Number	Sections
Mathematics	205	All

Examination	Date	Duration
Midterm Test	03 March, 2018	1 h 30 min

Special	Only approved calculators are allowed
Instructions:	**Show all your work for full marks**

1. (10 marks): **a.** Sketch the graph of the function

$$f(x) = \begin{cases} \sqrt{9 - x^2} - 1 & -3 \le x \le 0 \\ |2x - 3| - 1 & 0 < x \le 2 \end{cases}$$

on the interval $[-3, 2]$, and find the definite integral $\int\limits_{-3}^{2} f(x)\,dx$ in terms of area (do **not** antidifferentiate).

b. Use the Fundamental Theorem of Calculus to calculate the derivative of $\quad F(x) = \int\limits_{-x^2}^{0} (t-1)\,\cos^4(t+1)\,dt$, and determine whether F is increasing or decreasing at $x = 1/2$.

2. (6 marks): Find $G(x)$ if $\quad G'(x) = \sec^3(x)\tan^3(x)$ and $G(0) = 1$.

3. (10 marks): Find the following indefinite integrals

$$\textbf{(a)} \quad \int \ln(\sqrt{x})\,dx \qquad \textbf{(b)} \quad \int \frac{x+1}{x^3 + x}\,dx$$

4. (12 marks): Find the exact values of the following definite integrals (*do not approximate*):

$$\textbf{(a)} \quad \int\limits_{0}^{a} x\sqrt{a-x}\,dx\,, \text{ where parameter } a > 0 \qquad \textbf{(b)} \quad \int\limits_{0}^{\pi/2} (2 - \sin x)^2\,dx$$

5. (6 marks): Sketch the region bounded by the graph of $f(x) = 4x - x^2$ and the x-axis, and find the volume of the solid obtained by revolution of that region about the axis $y = -2$. .

6. (6 marks): Find the average value of $f(x) = (x-3)^2$ on the interval $[2, 5]$. Sketch the graph of f, and draw a rectangle whose base is the interval $[2, 5]$ and whose height is the average value of f on that interval.

Bonus. (3 marks): Evaluate the limit by recognizing it as a Riemann sum for a function $f(x)$ on the interval $[0, 1]$ and then using integration.

$$\lim_{n \to \infty} \sum_{i=1}^{n} \left(\frac{i^4}{n^5} + \frac{i}{n^2} \right)$$