Course	Number	Section(s)
Mathematics	204	All

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Problem N1

(a). The system of three linear equations with three variables x, y, z

$$\begin{cases} x + y + z = 1 \\ x - y - z = 0 \\ 3x + ay + bz = 17 \end{cases}$$

has no solutions. The line l passes through the points $(1/2, 0, 1/2)$ and $(1/2, 1/2, 0)$. Find the angle between the line l and the vector $(3, a, b)$.

[Marks= 5]

(b). Write down the parametric equation of the line $l = \alpha \cap \beta$ of intersection of the plane α given by the equation

$$x + 2y - 3z = 0$$

and the plane β given by the equation

$$2x - y + 3z = 4\,.$$

(The parametric equation of a line has the form

$$\vec{x} = \vec{x}_0 + t\vec{v}$$

where t is a real parameter and $\vec{x_0}$, $\vec{v}$ are given vectors.)

Find the coordinates of the point $P = l \cap \gamma$ of intersection of the line l with the plane γ given by the equation

$$x + y + z = 3\,.$$

[Marks= 5]

Problem N2

(a). Compute the determinant

$$\begin{vmatrix} 2 & -5 & 1 & 2 \\ -3 & 7 & -1 & 4 \\ 5 & -9 & 2 & 7 \\ 4 & -6 & 1 & 2 \end{vmatrix}$$

[Marks= 5]

(b). Let a_{ij} be the entry of the matrix

$$A = \begin{pmatrix} 11 & 24 & 35 & 71 \\ 49 & 77 & 87 & 91 \\ 35 & 92 & 22 & 79 \\ 42 & 36 & 14 & 29 \end{pmatrix}$$

standing at the intersection of the i-th row and the j-th column and let C_{ij} be the cofactor of a_{ij}. Compute

$$a_{12}C_{12} + a_{13}C_{13} + a_{14}C_{14} - a_{21}C_{21} - a_{31}C_{31} - a_{41}C_{41}\,.$$

[Marks= 5]

Problem N3

(a). The area of the triangle with vertices $A = (1, 2, 3)$, $B = (2, 3, 4)$ and $C = (1, 1, x)$ is equal to $1/\sqrt{2}$. Find x.

[Marks= 5]

(b). Find a basis of the subspace of $\mathbb{R}^4$ spanned by five vectors

$$(1, 2, 1, -1),$$

$$(10, 15, 15, 0),$$

$$(3, 4, 5, 1),$$

$$(5, 8, 7, -1),$$

and

$$(1, 1, 2, 1)\,.$$

[Marks= 5]

Problem N4

(a). The vectors $\vec{U} = (c^{2/3}, b, 3)$, $\vec{V} = (\sqrt{c}, 2, 1)$ are parallel. Find the norm of the vector $\vec{U}$.

[Marks= 3]

(b). Let a be a real number. Explain why the planes

$$x + 2y + az = 11$$

and

$$ax + 2y + az = 17$$

are not orthogonal.

[Marks= 3]

(c). The plane α passes through the points $A = (1, 2, 3)$, $B = (1, 1, 1)$ and the origin $(0, 0, 0)$. Find the distance $\operatorname{dist}(C, \alpha)$ from the point $C = (3, 5, 7)$ to the plane α.

[Marks= 4]

Problem N5

(a). Find an eigenvector of the matrix

$$\begin{pmatrix} \sqrt{2} & 17 & 2\sqrt{2} \\ \sqrt{3} & 19 & 2\sqrt{3} \\ -\sqrt{11} & 11 & -2\sqrt{11} \end{pmatrix}$$

[**Marks= 4**]

(b). The system of two linear equations with three unknowns x_1, x_2, x_3

$$\begin{cases} ax_1 + bx_2 + cx_3 = 3 \\ dx_1 + ex_2 + fx_3 = 2 \end{cases}$$

has a solution $x_1 = 6, x_2 = 4, x_3 = 12$. Explain why 0 is an eigenvalue of the matrix

$$\begin{pmatrix} a & b & c \\ d & e & f \\ 2a & 2b & 2c \end{pmatrix}$$

and find a non-zero eigenvalue of this matrix.

[**Marks= 6**]

Problem N6

(a). Let

$$A = \begin{pmatrix} 1 & 1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

and let D be the diagonal matrix

$$D = \begin{pmatrix} -1 & 0 & 0 \\ 0 & \sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Find the twelfth power X^{12} of the matrix X that satisfies the equation

$$XA = AD\,.$$

[Marks= 8]

(b). Let $\vec{i} = (1,0)$ and $\vec{j} = (0,1)$. The linear transformation T in $\mathbb{R}^2$ transforms $\vec{i}$ to $\vec{i} + 2\vec{j}$ and $\vec{i} + 3\vec{j}$ to $2\vec{i} + \vec{j}$. Find the vector $\vec{a}$ such that $T(\vec{a}) = 8\vec{i} + 7\vec{j}$.

[Marks= 2]