TDT4171 Exercise 1

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I Counting and basic laws of probability

I.1 5-card Poker Hands

a)

The number of 5-card hands is given by how many combinations of 5 there is. This is given by:

$$\text{AtomicEvents} = \binom{52}{5} = \frac{52!}{47! - 5!} = 2598960 \tag{1}$$

b)

Given that the dealer is fair, the probability for each atomic event is given by:

$$P(\text{AtomicEvent}) = \frac{1}{2598960} \tag{2}$$

c)

$$P(\text{RoyalStraightFlush}) = \frac{4}{2598960} \tag{3}$$

$$P(\text{FourOfAKind}) = \frac{13 \times 48}{2598960} = \frac{624}{2598960} \tag{4}$$

The probability for royal straight flush (3) is given by the probability of drawing a random card at first, but thereafter draw on of the needed cards. Since the suit matters, this gives (3). For Four of a kind on the other hand, one needs four of the cards to be of the same suit, but the fifth card does not matter.

I.2 Two cards in a deck

a)

$$P(\text{Pair}) = 1 \times \frac{3}{51} = \frac{3}{51} \tag{5}$$

	diff suit	¬diff suit
pair	$\frac{3}{51}$	0
¬ pair	$\frac{36}{51}$	$\frac{12}{51}$

Table 1: Full joint distribution table

b)

$$\begin{aligned} P(\text{pair}|\text{diff suit}) = \frac{P(\text{pair} \wedge \text{diff suit})}{P(\text{diff suit})} \\ \frac{\frac{3}{51}}{\frac{39}{51}} = \frac{1}{13} \end{aligned} \tag{6}$$

I.3 Conditional probability

$$\begin{aligned} P(A|B) &> P(A) \\ \frac{P(A \wedge B)}{P(B)} &> P(A) \\ P(A \wedge B) &> P(A) \times P(B) \\ \frac{P(B \wedge A)}{P(A)} &> P(B) \\ P(B|A) &> P(B) \end{aligned} \tag{7}$$

II Bayesian Network Construction

I find the following conditional independence properties in the network:

- Household income given by Working parents
- Number of Children given by Working parents, Household income and Religion
- History of illness given by Number of children
- Illness at the moment given by History of illness
- Drinking Habits given by Religion
- Fish- and Fiber-eating habits given Religion

I have created this network using my understanding and interpretation of the world. It is most likely easily to prove me wrong in a lot of my assumptions, by I feel they are valid. Likely one could have added more conditionals, like for instance that the number of children may influence illness at the moment. Or maybe there is a link between eating habits and illness?

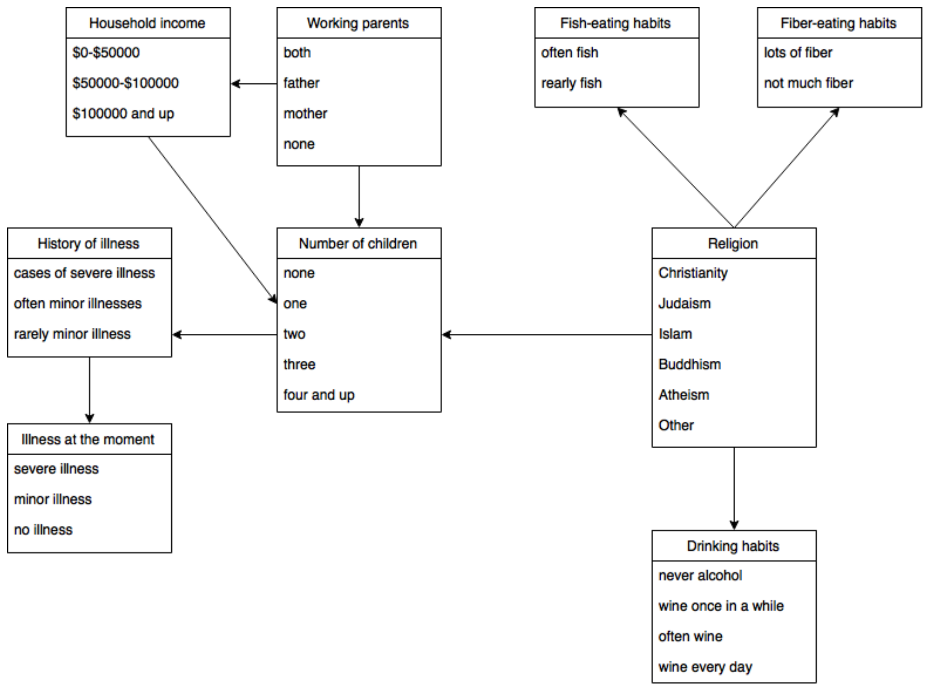
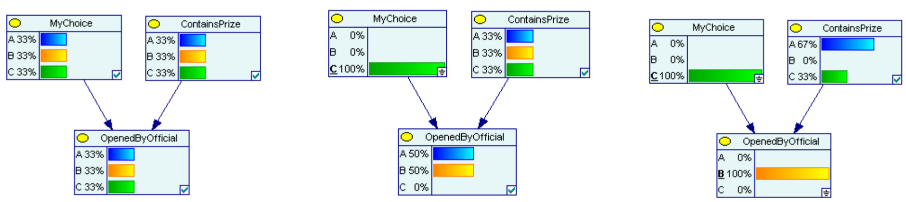


Figure 1: Bayesian Network

III Bayesian Network Application

This problem is also known as the "Monty Hall Problem"". Initially, the probability for each of the available doors is $\frac{1}{3}$. When I, as unknowing choose a certain door, the probability for each door still remains at $\frac{1}{3}$. The key to understanding what happens next is to keep in mind the fact that the official opening a door knows where the prize is. Given my example from GeNIe;

When I in 2b choose C, the official is forced to choose between opening door A and B. If the prize is behind door A, the official would be forced to open door C. In the same manner, should the prize be behind door B, the official would be forced to open door A. Given that C contains the prize, it will be a 50 % probability for each of the doors, A and B. See table 2 for a detailed overview over the conditional probabilities. In the next step, 2c, the official opens the door B. This makes the probability for C increase to $\frac{2}{3}$. This is because of the fact that the official knows which door the prize is located behind. Both my choice, and the officials choice affect the probability for the doors. The initial $\frac{1}{3}$ probability does no longer apply. I do not know anything more about my initial (uninformed) choice, A, but I now know for a fact that B is not the correct door. Therefore the probability for door A doubles to $\frac{2}{3}$.



(a) Initial state (b) Made my choiche (c) Opened door by Offical

Figure 2: Development during the events

MyChoice	A			B			C		
ContainsPrize	A	B	C	A	B	C	A	B	C
A	0	0	0	0	0.5	1	0	1	0.5
B	0.5	0	1	0	0	0	1	0	0.5
C	0.5	1	0	1	0.5	0	0	0	0

Table 2: Conditional Probability Table