TDT4171 - Assignment 2

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1 Part A

Describe the "Umbrella domain" as an HMM:

X_t for a given time-slice t contains $\{\text{Rain}_t\}$ and E_t for a given time-slice t contains $\{\text{Umbrella}_t\}$. The dynamic model (transition model) and the observation model (sensor model) is given by matrices below. $P(X_t|X_{t-1}), T_{ij}$ is given by $P(X_t = j|X_{t-1} = i)$ and in $P(E_t|X_t), T_{ij}$ is given by $P(E_t|X_{t-1} = i)$.

$$P(X_t|X_{t-1}) = \begin{bmatrix} 0.7 & 0.3 \\ 0.3 & 0.7 \end{bmatrix} \quad P(E_t|X_t) = \begin{bmatrix} 0.9 & 0.1 \\ 0.2 & 0.8 \end{bmatrix}$$

In this model we make the assumption that the current state is dependent on the previous state (a Markov assumption). Hence we have a Markov process, in the form of a *first-order Markov process*. Given that the observation U_t is detailed enough, it should be reasonable to assume this assumptions are satisfiable for this particular domain. Stated more clearly, it is reasonable to assume that the probability of rain does not rely on the fact that it rained a couple of days ago. It is much more likely that it relays on the previous day.

2 Part B

Verification of $P(X_2|e_{1:2})$:

```
# FORWARD-MESSAGES:
  Message 1: [ 0.81818182  0.18181818]
  Message 2: [ 0.88335704  0.11664296]
```

Probability of rain at day 5 given the sequence of observations $e_{1:5}$ is $P(X_5|e_{1:5}) \approx 0.867$:

```
# FORWARD-MESSAGES:
  Message 1: [ 0.81818182  0.18181818]
  Message 2: [ 0.88335704  0.11664296]
  Message 3: [ 0.19066794  0.80933206]
  Message 4: [ 0.730794  0.269206]
  Message 5: [ 0.86733889  0.13266111]
```

3 Part C

Verification of $P(X_1|e_{1:2})$:

```
# BACKWARD-MESSAGES:
  Message 2: [1 1]
  Message 1: [ 0.69  0.41]
  Message 0: [ 0.4593  0.2437]
# FORWARD-MESSAGES:
  Message 0: [ 0.5  0.5]
  Message 1: [ 0.81818182  0.18181818]
  Message 2: [ 0.88335704  0.11664296]
# SMOOTHED ESTIMATES:
  Estimate 0: [ 0.65334282  0.34665718]
  Estimate 1: [ 0.88335704  0.11664296]
  Estimate 2: [ 0.88335704  0.11664296]
```

Probability of rain at day 1 given the sequence of observations $e_{1:5}$ is $P(X_1|e_{1:5}) \approx 0.867$

```
# BACKWARD-MESSAGES:
  Message 5: [1 1]
  Message 4: [ 0.69  0.41]
  Message 3: [ 0.4593  0.2437]
  Message 2: [ 0.090639  0.150251]
  Message 1: [ 0.06611763  0.04550767]
  Message 0: [ 0.04438457  0.02422283]
# FORWARD-MESSAGES:
  Message 0: [ 0.5  0.5]
  Message 1: [ 0.81818182  0.18181818]
  Message 2: [ 0.88335704  0.11664296]
  Message 3: [ 0.19066794  0.80933206]
  Message 4: [ 0.730794  0.269206]
  Message 5: [ 0.86733889  0.13266111]
# SMOOTHED ESTIMATES:
  Estimate 0: [ 0.64693556  0.35306444]
  Estimate 1: [ 0.86733889  0.13266111]
  Estimate 2: [ 0.82041905  0.17958095]
  Estimate 3: [ 0.30748358  0.69251642]
  Estimate 4: [ 0.82041905  0.17958095]
  Estimate 5: [ 0.86733889  0.13266111]
```

4 Part D

See the attached code `hmm.py`.