Pertemuan 17

RUMUS-RUMUS DASAR INTEGRAL

Misal u adalah suatu fungsi yang terintegralkan dan C sebuah konstanta, dengan memperhatikan sifat-sifat operasi Aljabar fungsi (penjumlahan, pengurangan, perkalian dan pembagian) dapat diperikan beberapa sifat Integral tak tentu fungsi yang terintegralkan. Sifat-sifat berikut berlaku untuk syarat yang diberikan.

1. $\int u^n \, du = \frac{u^{n+1}}{n+1} + C$, jika $n \neq -1$
2. $\int [u(x)]^n u'(x) dx = \frac{[u(x)]^{n+1}}{n+1} + C$, jika $n \neq -1$
3. $\int \frac{du}{u} = \ln |u| + C$ atau $\int \frac{f'(x)}{f(x)} dx = \ln |f(x) + C|$
4. $\int e^u \, du = e^u + C$
5. $\int a^u \, du = \frac{a^u}{\ln u} + C$
6. $\int u \, dv = uv - \int v \, du$
7. $\int \sin du = -\cos u + C$
8. $\int \cos u \, du = \sin u + C$
9. $\int \sec^2 u \, du = \tan u + C$
10. $\int \csc^2 u \, du = -\cot u + C$
11. $\int \sec u \tan u \, du = \sec u + C$
12. $\int \csc u \cot u \, du = -\csc u + C$
13. $\int \tan u \, du = \ln |\sec u| + C$
14. $\int \cot u \, du = \ln |\sin u| + C$

15. $\int$ sec u du $= \ln \left|\sec u + \tan u\right| + C$

16. $\int$ csc u du $= \ln \left|c\sec u - \cot u\right| + C$

17. $\int \frac{du}{\sqrt{a^2 - u^2}} = \text{arc sin } \frac{u}{a} + C$

18. $\int \frac{du}{a^2 + u^2} = \frac{1}{a} \text{ arc tan } \frac{u}{a} + C$

19. $\int \frac{du}{a^2 - u^2} = \frac{1}{2a} \ln \left|\frac{u + a}{u - a}\right| + C$

20. $\int \frac{du}{u^2 - a^2} = \frac{1}{2a} \ln \left|\frac{u - a}{u + a}\right| + C$

21. $\int \frac{du}{\sqrt{u^2 + a^2}} = \ln (u + \sqrt{u^2 + a^2}) + C$

22. $\int \frac{du}{\sqrt{u^2 - a^2}} = \ln (u + \sqrt{u^2 - a^2}) + C$

23. $\int \sqrt{a^2 - u^2}$ du = ½ u $\sqrt{u^2 - a^2} - \frac{1}{2} a^2 \arcsin \frac{u}{a} + C$

24. $\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \text{ arc sec } \left|\frac{u}{a}\right| + C$

25. $\int \sqrt{u^2 - a^2}$ du = ½ u $\sqrt{u^2 - a^2} - \frac{1}{2} a^2 \ln \left|u + \sqrt{u^2 - a^2}\right| + C$

26. $\int \sqrt{u^2 + a^2}$ du = ½ u $\sqrt{u^2 + a^2} + \frac{1}{2} a^2 \ln \left|u + \sqrt{u^2 + a^2}\right| + C$

27. $\int \sin^2 u \, du = \frac{1}{2} u - \frac{1}{4} \sin 2u + C$

28. $\int \cos^2 u \, du = \frac{1}{2} u + \frac{1}{4} \sin 2u + C$

29. $\int \tan^2 u \, du = -u + \tan u + C$

30. $\int \cot^2 u \, du = - u - \cot u + C$

31. $\int \sin^3 u \, du = - \frac{1}{3} (2 + \sin^2 u) \cos u + C$

32. $\int \cos^3 u \, du = \frac{1}{3} (2 + \cos^2 u) \sin u + C$

33. $\int \tan^3 u\, du = \frac{1}{2} \tan^2 u + \ln|\cos u| + C$

34. $\int \cot^3 u\, du = -\frac{1}{2} \cot^2 u - \ln|\sin u| + C$

35. $\int \sec^3 u\, du = \frac{1}{2} \sec u \tan u + \frac{1}{2} \ln|\sec u + \tan u| + C$

36. $\int \csc^3 u\, du = -\frac{1}{2} \csc u \cot u + \frac{1}{2} \ln|c\sec u - \cot u| + C$

37. $\int \sin au \sin bu\, du = \frac{\sin(a-b)u}{2(a-b)} - \frac{\sin(a+b)u}{2(a+b)} + C$, jika $a^2 \neq b^2$

38. $\int \cos au \cos bu\, du = \frac{\sin(a-b)u}{2(a-b)} + \frac{\sin(a+b)u}{2(a+b)} + C$, jika $a^2 \neq b^2$

39. $\int \sin au \cos bu\, du = -\frac{\cos(a-b)u}{2(a-b)} - \frac{\cos(a+b)u}{2(a+b)} + C$, jika $a^2 \neq b^2$

40. $\int \sin^n u\, du = -\frac{\sin^{n-1} u \cos u}{n} + \frac{n-1}{n} \int \sin^{n-2} u\, du$

41. $\int \cos^n u\, du = \frac{\cos^{n-1} u \sin u}{n} + \frac{n-1}{n} \int \cos^{n-2} u\, du$

42. $\int \tan^n u\, du = \frac{1}{n-1} \tan^{n-1} u - \int \tan^{n-2} u\, du$ jika $n \neq 1$

43. $\int \cot^n u\, du = -\frac{1}{n-1} \cot^{n-1} u - \int \cot gn^{n-2} u\, du$ jika $n \neq 1$

44. $\int \sec^n u\, du = \frac{1}{n-1} \sec^{n-2} u \tan u + \frac{n-2}{n-1} \int \sec^{n-2} u\, du$, jika $n \neq 1$

45. $\int \csc^n u\, du = -\frac{1}{n-1} \csc^{n-2} u \cot u + \frac{n-2}{n-1} \int \csc^{n-2} u\, du$, $n \neq 1$

46. $\int \sin^n u \cos^m u\, du = -\frac{\sin^{n-1} u \cos^{m+1} u}{n+m} + \frac{n-1}{n+m} \int \sin^{n-2} u \cos^m u\, du$,

$n \neq -m$

47. $\int u \sin u\, du = \sin u - u \cos u + C$

48. $\int u \cos u\, du = \cos u + u \sin u + C$

49. $\int u^n \sin u\, du = -u^n \cos u + n \int u^{n-1} \cos u\, du$

50. $\int u^n \cos u\, du = u^n \sin u + n \int u^{n-1} \sin u\, du$

51. $\int \sin u \; d(\sin u) = \frac{1}{2}\sin^2 u + C$

52. $\int \cos u \; d(\cos u) = \frac{1}{2}\cos^2 u + C$

53. $\int \tan u \; d(\tan u) = \frac{1}{2}\tan^2 u + C$

54. $\int \cot u \, d(\cot u) = \frac{1}{2} \cot^2 u + C$

55. $\int \sec u \, d(\sec u) = \frac{1}{2} \sec^2 u + C$

56. $\int \csc u \, d(\csc u) = \frac{1}{2} \csc^2 u + C$

57. $\int \sqrt{u^2 \pm a^2}\, du = \frac{u}{2}\sqrt{u^2 \pm a^2} \pm \frac{a^2}{2} \ln \left| u + \sqrt{u^2 \pm a^2} \right| + C$

58. $\int \frac{\sqrt{u^2 + a^2}}{u}\, du = \sqrt{u^2 + a^2} - a \ln \left(\frac{a \pm \sqrt{u^2 - u^2}}{u} \right) + C$

59. $\int \frac{du}{\sqrt{u^2 \pm a^2}} = \ln \left| u + \sqrt{u^2 \pm a^2} \right| + C$

60. $\int \frac{\sqrt{u^2 - a^2}}{u}\, du = \sqrt{u^2 - a^2} - a \; \text{arc sec} \frac{u}{a} + C$

61. $\int u^2 \sqrt{a^2 \pm u^2}\, du = \frac{u}{8}(2a^2 \pm u^2)\sqrt{a^2 \pm u^2} - \frac{a^4}{8} \ln \left| u + \sqrt{a^2 \pm u^2} \right| + C$

62. $\int \frac{u^2}{\sqrt{u^2 \pm a^2}}\, du = \frac{u}{2}\sqrt{a^2 \pm u^2} \pm \frac{a^2}{2} \ln \left| u + \sqrt{a^2 \pm u^2} \right| + C$

63. $\int \frac{du}{u^2\sqrt{u^2 \pm a^2}} = \pm \frac{\sqrt{u^2 \pm a^2}}{a^2 u} + C$

64. $\int \frac{\sqrt{u^2 \pm a^2}}{u^2}\, du = - \frac{\sqrt{u^2 \pm a^2}}{u} - \ln \left| u + \sqrt{a^2 \pm u^2} \right| + C$

65. $\int \frac{du}{(u^2 \pm a^2)^{\frac{3}{2}}} = \pm \frac{u}{a^2\sqrt{u^2 \pm a^2}} + C$

66. $\int \frac{udu}{\sqrt{a^2 - u^2}} = - \sqrt{a^2 - u^2} + C$

67. $\int (u^2 \pm a^2)^{3/2}\, du = \frac{u}{8}(2u^2 \pm 5a^2)\sqrt{u^2 \pm a^2} + \frac{3a^4}{8} \ln \left| u + \sqrt{u^2 \pm a^2} \right| + C$

68. $\int \sqrt{a^2-u^2}\ du = \frac{a}{2}\sqrt{a^2-u^2} + \frac{a^2}{u}\arcsin^{-1}\frac{u}{a} + C$

69. $\int \frac{u^2}{\sqrt{a^2-u^2}}\ du = -\frac{a}{2}\sqrt{a^2-u^2} + \frac{a^2}{u}\arcsin^{-1}\frac{u}{a} + C$

70. $\int \frac{\sqrt{a^2-u^2}}{u}\ du = \sqrt{a^2-u^2} - a\ln\left|\frac{a+\sqrt{a^2-u^2}}{u}\right| + C$

71. $\int u^2\sqrt{a^2-u^2}\ du = \frac{u}{8}(2u^2-a^2)\sqrt{a^2-u^2} + \frac{a^4}{8}\arcsin^{-1}\frac{u}{a} + C$

72. $\int \frac{du}{u^2\sqrt{a^2-u^2}} = -\frac{\sqrt{a^2-u^2}}{a^2u} + C$

73. $\int \frac{\sqrt{u^2-a^2}}{u^2}\ du = -\frac{\sqrt{u^2-a^2}}{u} - \arcsin^{-1}\frac{u}{a} + C$

74. $\int \frac{du}{u\sqrt{a^2-u^2}} = -\frac{1}{a}\ln\left|\frac{a+\sqrt{a^2-u^2}}{u}\right| + C$

75. $\int \frac{du}{u\sqrt{1-u}} = \ln\left|\frac{1-\sqrt{1-x}}{1+\sqrt{1-x}}\right| + C$

76. $\int \frac{\sqrt{u}}{1+u}\ du = 2\sqrt{u} - 2\arctan\sqrt{u} + Cl$

77. $\int \frac{du}{\sqrt{u}(1+\sqrt{u})} = 2\ln(1+\sqrt{u})$

78. $\int \frac{du}{(a^2-u^2)^{\frac{3}{2}}} = \frac{u}{a^2\sqrt{a^2-u^2}} + C$

79. $\int (a^2-u^2)^{3/2}\ du = \frac{u}{8}(5a^2-2u^2)\sqrt{a^2-u^2} + \frac{3a^4}{8}\arcsin^{-1}\frac{u}{a} + C$

80. $\int ue^u\ du = (u-1)e^u + C$

81. $\int u^n e^u\ du = u^n e^u - n\int u^{n-1}e^u\ du$

82. $\int \ln u\ du = u\ln u - u + C$

83. $\int u^n \ln u\ du = \frac{u^{n+1}}{n+1}\ln u - \frac{u^{n+1}}{(n+1)^2} + C$

84. $\int$ e^{au} sin bu du = $\frac{e^{au}}{a^2+b^2}$ (a sin bu – b cos bu) + C

85. $\int$ e^{au} cos bu du = $\frac{e^{au}}{a^2+b^2}$ (a cos bu + b sin bu) + C

86. $\int$ arc $\sin^{-1}$ u du = u arc $\sin^{-1}$ u + $\sqrt{1-u^2}$ + C

87. $\int$ arc tan u du = u arc tan u - $\frac{1}{2}$ ln $\left|1+u^2\right|$ + C

88. $\int$ arc sec u du = u arc sin u – ln $\left|u+\sqrt{1+u^2}\right|$ + C

89. $\int$ u arc sin u du = ¼ $(2u^2 - 1)$ arc sin u + $\frac{u}{4}$ $\sqrt{1-u^2}$ + C

90. $\int$ u arc tan u du = ½ $(u^2 + 1)$ arc tan u - $\frac{u}{2}$ + C

91. $\int$ u arc sec u du = $\frac{u^2}{2}$ arc sec u – ½ $\sqrt{u^2-1}$ + C

92. $\int$ u arc sin u du = $\frac{u^{n+1}}{n+1}$ arc sin u - $\frac{1}{n+1}\int\frac{u^{n+1}}{\sqrt{1-u^2}}$ du + C, jika n $\neq$ -1

93. $\int$ u^n arc tan u du = $\frac{u^{n+1}}{n+1}$ arc tan u - $\frac{1}{n+1}\int\frac{u^{n+1}}{1+u^2}$ du + C, jika n $\neq$ -1

94. $\int$ u^n arc sec u du = $\frac{u^{n+1}}{n+1}$ arc sec u - $\frac{1}{n+1}\int\frac{u^{n+1}}{\sqrt{u^2-1}}$ du + C, jika n $\neq$ -1

95. $\int$ sinh u du = cosh u + C

96. $\int$ cosh u du = sinh u + C

97. $\int$ tanh u du = ln (cosh u) + C

98. $\int$ coth u du = ln $\left|\sinh u\right|$ + C

99. $\int$ sech u du = arc tan $\left|\sinh u\right|$ + C

100. $\int$ csch u du = ln $\left|\tanh\frac{u}{2}\right|$ + C

101. $\int$ $\sinh^2$ u du = ¼ sinh u - $\frac{u}{2}$ + C

102. $\int$ $\cosh^2$ u du = ¼ sinh u + $\frac{u}{2}$ + C

103. $\int \tanh^2 u\,du = u - \tanh u + C$

104. $\int \coth^2 u\,du = u - \coth u + C$

105. $\int \operatorname{sech}^2 u\,du = \tanh u + C$

106. $\int \operatorname{csch}^2 u\,du = -\coth u + C$

107. $\int \operatorname{sech} u \tanh u\,du = -\operatorname{sech} u + C$

108. $\int \operatorname{csch} u \coth u\,du = -\operatorname{csch} u + C$

109. $\int u(au+b)^{-1}\,du = \frac{u}{a} - \frac{b}{a^2}\ln|au+b| + C$

110. $\int u(au+b)^{-2}\,du = \frac{1}{a^2}\left[\ln|au+b| + \frac{b}{au+b}\right] + C$

111. $\int u(au+b)^n\,du = \frac{(au+b)^{n+1}}{a^2}\left[\frac{au+b}{n+2} - \frac{b}{n+1}\right] + C$, jika $n \neq -1, -2$

112. $\int \frac{du}{(a^2 \pm u^2)^n} = \frac{1}{2a^2(n-1)}\left[\frac{u}{(a^2 \pm u^2)^{n-1}} + (2n-1)\int \frac{du}{(a^2 \pm u^2)^{n-1}}\right] + C$, $n \neq 1$

113. $\int u\sqrt{au+b}\,du = \frac{2}{15a^2}(3au-2b)(au+b)^{\frac{3}{2}} + C$

114. $\int u^n\sqrt{au+b}\,du = \frac{2}{a(2n+3)}\left(u^n(au+b)^{\frac{3}{2}} - nb\int u^{n-1}\sqrt{au+b}\right) + C$

115. $\int \frac{udu}{\sqrt{au+b}} = \frac{2}{3a^2}(au-2b)\sqrt{au+b} + C$

116. $\int \frac{u^n du}{\sqrt{au+b}} = \frac{2}{a(2n+1)}\left(u^n\sqrt{au+b}\right) - nb\int \frac{u^{n-1}}{\sqrt{au+b}}du$

117. $\int \frac{du}{u\sqrt{au+b}} = \frac{1}{\sqrt{b}}\ln\left|\frac{\sqrt{au+b}-\sqrt{b}}{\sqrt{au+b}+\sqrt{b}}\right| + C$

118. $\int \frac{du}{u^n\sqrt{au+b}} = -\frac{\sqrt{au+b}}{b(n-1)u^{n-1}} - \frac{(2n-3)a}{(2n-2)b}\int \frac{du}{u^{n-1}\sqrt{au+b}} + C$, jika $n \neq 1$

119. $\int \sqrt{2au-u^2} = \frac{u-a}{2}\sqrt{2au-u^2} + \frac{a^2}{n}arc\sin\frac{u-a}{a} + C$

120. $\int \frac{du}{\sqrt{2au-u^2}} = \arcsin\frac{u-a}{a} + C$

121. $\int u^n \sqrt{2au-u^2} = \frac{u^{n-1}(2au-u^2)^{\frac{3}{2}}}{n+2} + \frac{(2n+1)a}{n+2}\int u^{n-1}\sqrt{2au-u^2}\ du$

122. $\int \frac{u^n du}{\sqrt{2au-u^2}} = -\frac{u^{n-1}}{n}\sqrt{2au-u^2} + \frac{(2n-1)a}{n}\int \frac{u^{n-1}du}{\sqrt{2au-u^2}} + C$

123. $\int \frac{\sqrt{2au-u^2}}{u} = \sqrt{2au-u^2} + a \arcsin \frac{u-a}{a} + C$

124. $\int \frac{\sqrt{2au-u^2}}{u^n} = \frac{(2au-u^2)^{\frac{3}{2}}}{(3-2n)au^n} + \frac{n-3}{(2n-3)a}\int \frac{\sqrt{2au-u^2}}{u^{n-1}}du$

125. $\int \frac{du}{u^n(\sqrt{2au-u^2})} = \frac{\sqrt{2au-u^2}}{a(1-2n)u^n} + \frac{n-1}{(2n-1)a}\int \frac{du}{u^{n-1}\sqrt{2u-u^2}}$

126. $\int (\sqrt{2au-u^2})^2 = \frac{na^2}{n+1}\int (\sqrt{2au-u^2})^{n-1}\ du$

127. $\int \frac{du}{(\sqrt{2au-u^2})^4} = \frac{u-a}{(n-2)^2}\left(\sqrt{2au-u^2}\right)^{2-n} + \frac{n-3}{(n-2)a^2}\int \frac{du}{(\sqrt{2au-u^2})^{\frac{3}{2}}}\ du$

128. $\int \frac{du}{\sin u - \cos u - 1} = \ln\left|\tan\frac{1}{2}u - 1\right| + C$

129. $\int \frac{du}{1+\sin u+\cos u} = \ln\left|1+\tan\frac{1}{2}u\right| + C$

130. $\int \frac{\sin u du}{1+\sin^2 u}du = \frac{1}{4}\sqrt{2}\ \ln\left|\frac{\tan^2\frac{u}{2}+3-2\sqrt{2}}{\tan^2\frac{u}{2}+3+2\sqrt{2}}\right| + C$

131. $\int \frac{\sin u\cos u du}{1-\cos u} = \cos u + \ln(1-\cos u) + C$

132. $\int \sin\sqrt{u}\ du = -2\sqrt{u}\cos\sqrt{u} + 2\sin\sqrt{u} + C$

133. $\int \frac{du}{1-2\sin u} = \frac{\sqrt{3}}{3}\ln\left|\frac{\tan\frac{u}{2}-2-\sqrt{3}}{\tan\frac{u}{2}-2+\sqrt{3}}\right| + C$

134. $\int \frac{du}{2+\sin u} = \frac{2}{\sqrt{3}}arctgn\left|\frac{2tgn\frac{u}{2}+1}{\sqrt{3}}\right| + C$

135. $\int \frac{du}{3+5\sin u} = \frac{1}{4}\ln\left|\frac{3\tan\frac{u}{2}+1}{\tan\frac{u}{2}+3}\right| + C$

136. $\int \frac{du}{5+3\sin u} = \frac{1}{2}\arctan\left|\frac{5\tan\frac{u}{2}+3}{4}\right| + C$

137. $\int \frac{du}{1+\sin u-\cos u} = \ln\left|\frac{\tan\frac{u}{2}}{1+\tan\frac{u}{2}}\right| + C$

138. $\int \frac{du}{2-\cos u} = \frac{2}{\sqrt{3}}\arctan(\sqrt{3}\tan\frac{u}{2}) + C$

139. $\int \frac{du}{5+4\sin u} = \frac{2}{3}\arctan\frac{5\tan\frac{u}{2}+4}{3} + C$

140. $\int \frac{du}{2+\cos u} = \frac{2\sqrt{3}}{3}\arctan\left(\frac{\sqrt{3}}{3}\tan\frac{u}{2}\right) + C$

141. $\int \frac{du}{3-2u} = \frac{2\sqrt{5}}{5}\arctan(\sqrt{5}\tan\frac{u}{2}) + C$

142. $\int \frac{\sin u du}{\cos u(1+\cos^2 u)} = \ln\left|\frac{\sqrt{1+\cos^2 u}}{\cos u}\right| + C$

143. $\int \frac{(2+\tan^2 u)\sec^2 u du}{1+\tan^2 u} = \ln|1+\tan u| + \frac{2}{\sqrt{3}}\arctan\frac{2\tan u-1}{\sqrt{3}} + C$

144. $\int \frac{dx}{1-\sin\frac{x}{2}} = 2(\tan\frac{x}{2}+\sec\frac{x}{2}) + C$

145. $\int \frac{dx}{1+\cos 3x} = \frac{1-\cos 3x}{3\sin 3x} + C$

146. $\int \frac{\cos 2x dx}{\sin^2 2x+8} = \frac{\sqrt{2}}{8}\arctan\frac{\sin 2x}{2\sqrt{2}} + C$

147. $\int \frac{\sec^2 x dx}{\sqrt{1-4\tan^2 x}} = \frac{1}{2}\text{arc sin}(2\tan x) + C$

148. $\int \frac{\sin 8x dx}{9+\sin^2 4x} = \frac{1}{12}\arctan\frac{\sin^2 4x}{3} + C$

149. $\int \frac{dx}{1+\sec ax}$ = x + $\frac{1}{a}$ (cot ax-csc ax) + C

150. $\int \sec^2 \frac{x}{a}\tan\frac{x}{a}dx = \frac{1}{2}a\tan^2\frac{x}{a} + C$