

## PERTEMUAN 7

### DETERMINAN, MINOR, KOFAKTOR, EKSPANSI BARIS DAN KOLOM

#### A. Tujuan Pembelajaran

Pada akhir pertemuan ini, Mahasiswa mampu menghitung determinan, matriks minor dan kofaktornya serta ekspansi baris dan kolom.

#### B. Uraian Materi

##### 1. Determinan

Determinan adalah suatu nilai matriks yang berbentuk persegi. Determinan matriks hanya dimiliki oleh sebuah matriks yang jumlah kolom dan jumlah barisnya sama. Untuk mencari determinan dari sebuah matriks kita menggunakan metode sarrus untuk ordo  $2 \times 2$  dan metode perkalian untuk ordo  $3 \times 3$ .

Misal  $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$  maka  $\det(A) = ad - bc$

##### a. Ordo $2 \times 2$

###### Contoh:

Tentukanlah determinan dari matriks berikut:

$$1) A = \begin{bmatrix} 3 & -1 \\ 8 & 5 \end{bmatrix}$$

$$4) \begin{bmatrix} -1/4 & -1 \\ 2 & 7 \end{bmatrix}$$

$$2) A = \begin{bmatrix} -1/3 & 1/2 \\ 2 & 3 \end{bmatrix}$$

$$5) \begin{bmatrix} 2 & -1/5 \\ 2/3 & 4 \end{bmatrix}$$

$$3) A = \begin{bmatrix} 4 & 1/5 \\ 2/3 & -1/2 \end{bmatrix}$$

$$6) \begin{bmatrix} 1/3 & -1 \\ 0 & 1/5 \end{bmatrix}$$

###### Jawab:

$$\begin{aligned} 1) A = \begin{bmatrix} 3 & -1 \\ 8 & 5 \end{bmatrix}, \text{ maka } \det(A) &= 3 \times 5 - (-1 \times 8) \\ &= 15 - (-8) \\ &= 15 + 8 \\ &= 23 \end{aligned}$$

$$\begin{aligned} 2) A = \begin{bmatrix} -1/3 & 1/2 \\ 2 & 3 \end{bmatrix}, \text{ maka } \det(A) &= -1/3 \times 3 - 1/2 \times 2 \\ &= -1 - 1 \\ &= -2 \end{aligned}$$

$$3) A = \begin{bmatrix} 4 & 1/5 \\ 2/3 & -1/2 \end{bmatrix}, \text{ maka } \det(A) = (4 \times -1/2) - 1/5 \times 2/3$$

$$= -2 - 2/15$$

$$= -32/15$$

$$4) A = \begin{bmatrix} -1/4 & -1 \\ 2 & 7 \end{bmatrix}, \text{ maka } \det(A) = -1/4 \times 7 - (-1 \times 2)$$

$$= -7/4 + 2$$

$$= 1/4$$

$$5) A = \begin{bmatrix} 2 & -1/5 \\ 2/3 & 4 \end{bmatrix}, \text{ maka } \det(A) = 2 \times 4 - (-1/5 \times 2/3)$$

$$= 8 - (-2/15)$$

$$= 8 + 2/15$$

$$= 122/15$$

$$6) A = \begin{bmatrix} 1/3 & -1 \\ 0 & 1/5 \end{bmatrix}, \text{ maka } \det(A) = 1/3 \times 1/5 - (-1 \times 0)$$

$$= 1/15 + 0$$

$$= 1/15$$

### b. Ordo 3 x 3

Tentukan determinan dari matriks berikut:

$$1) A = \begin{bmatrix} -1 & 2 & 1/2 \\ 3 & -4 & 2/3 \\ 1 & 1/2 & -1/3 \end{bmatrix}$$

**Jawab:**

$$A = \begin{bmatrix} -1 & 2 & 1/2 \\ 3 & -4 & 2/3 \\ 1 & 1/2 & -1/3 \end{bmatrix} \begin{vmatrix} -1 & 2 \\ 3 & -4 \\ 1 & 1/2 \end{vmatrix}$$

Dengan mengalikan dua buah kolom, yaitu kolom pertama dan kolom kedua. Dari kiri atas menuju kanan bawah (penjumlahan) dan dari kiri bawah menuju kanan atas (pengurangan).

$$= (-1 \times -4 \times -1/3) + (2 \times 2/3 \times 1) + (1/2 \times 3 \times 1/2) - (1 \times -4 \times 1/2) - (1/2 \times 2/3 \times -1) - (-1/3 \times 3 \times 2)$$

$$= \frac{-4}{3} + \frac{4}{3} + \frac{3}{4} + 2 + \frac{1}{3} + 2$$

$$= \frac{-16+16+9+24+4+24}{12}$$

$$= \frac{61}{12}$$

$$2) A = \begin{bmatrix} 3 & \frac{1}{5} & \frac{1}{3} \\ -2 & 3 & 1 \\ -1 & -2 & 4 \end{bmatrix}$$

**Jawab:**

maka mencari determinan adalah dengan mengalikan dua buah kolom yaitu kolom pertama dan kolom kedua. Maka:

$$A = \begin{bmatrix} 3 & \frac{1}{5} & \frac{1}{3} \\ -2 & 3 & 1 \\ -1 & -2 & 4 \end{bmatrix} \left| \begin{bmatrix} 3 & 1/5 \\ -2 & 3 \\ -1 & -2 \end{bmatrix} \right|$$

$$\begin{aligned} \text{Det}(A) &= (3 \times 3 \times 4) + (1/5 + 1 \times -1) + (1/3 \times -2 \times -2) - (-1 \times 3 \times 1/3) - (-2 \times 1 \times 3) - (4 \times -2 \times 1/5) \\ &= 36 - 1/5 + 4/3 + 1 + 6 + 8/5 \\ &= 540/15 - 3/15 + 20/15 + 15/15 + 90/15 + 24/15 \\ &= 686/15 \end{aligned}$$

$$3) A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 5 & 6 \\ 7 & 5 & 3 \end{bmatrix}$$

**Jawab:**

$$\begin{aligned} \text{Maka det}(A) &= \begin{bmatrix} 3 & 2 & 1 \\ 4 & 5 & 6 \\ 7 & 5 & 3 \end{bmatrix} \left| \begin{bmatrix} 3 & 2 \\ 4 & 5 \\ 7 & 5 \end{bmatrix} \right| \\ &= (3 \times 5 \times 3) + (2 \times 6 \times 7) + (1 \times 4 \times 5) - (7 \times 5 \times 1) - (5 \times 6 \times 3) - (3 \times 4 \times 2) \\ &= 45 + 84 + 20 - 35 - 90 - 24 \\ &= 0 \end{aligned}$$

$$4) A = \begin{bmatrix} -1 & 3 & -2 \\ 5 & -2 & 4 \\ 8 & 1 & -1 \end{bmatrix}$$

**Jawab:**

$$\text{Maka det}(A) = \begin{bmatrix} -1 & 3 & -2 \\ 5 & -2 & 4 \\ 8 & 1 & -1 \end{bmatrix} \left| \begin{bmatrix} -1 & 3 \\ 5 & -2 \\ 8 & 1 \end{bmatrix} \right|$$

$$\begin{aligned}
&= (-1 \times -2 \times -1) + (3 \times 4 \times 8) + (-2 \times 5 \times 1) - (8 \times -2 \times -2) \\
&\quad - (1 \times 4 \times -1) - (-1 \times 5 \times 3) \\
&= -2 + 96 - 10 - 32 + 4 + 15 \\
&= 71
\end{aligned}$$

## 2. Minor

Apabila A merupakan matriks kuadrat, maka minor  $a_{ij}$  dinyatakan oleh  $M_{ij}$  yang dimana merupakan submatriks A yang diperoleh dengan cara menghilangkan baris ke- i dengan kolom ke- j.

Bentuk operasi minor adalah sebagai berikut:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Maka:

$$M_{11} = \begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix}$$

$$M_{21} = \begin{bmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{bmatrix}$$

$$M_{31} = \begin{bmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{bmatrix}$$

$$M_{12} = \begin{bmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{bmatrix}$$

$$M_{22} = \begin{bmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{bmatrix}$$

$$M_{32} = \begin{bmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{bmatrix}$$

$$M_{13} = \begin{bmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$$

$$M_{23} = \begin{bmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{bmatrix}$$

$$M_{33} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

### Contoh:

1) Tentukanlah  $M_{11}$  sampai  $M_{33}$  dari matriks di bawah ini:

$$A = \begin{bmatrix} 2 & 6 & 8 \\ 1 & -4 & 5 \\ 4 & 3 & 1 \end{bmatrix}$$

### Jawab:

Maka  $M_{11}$  artinya adalah menghilangkan baris pertama kolom pertama. Begitu juga dengan M yang lainnya, bahwa angka pertama setelah M adalah menunjukkan letak baris ke -, dan angka setelahnya adalah menunjukkan kolom ke-, Sehingga didapat:

$$M_{11} = \begin{bmatrix} -4 & 5 \\ 3 & 1 \end{bmatrix}$$

$$M_{21} = \begin{bmatrix} 6 & 8 \\ 3 & 1 \end{bmatrix}$$

$$M_{31} = \begin{bmatrix} 6 & 8 \\ -4 & 5 \end{bmatrix}$$

$$M_{12} = \begin{bmatrix} 1 & 5 \\ 4 & 1 \end{bmatrix}$$

$$M_{22} = \begin{bmatrix} 2 & 8 \\ 4 & 1 \end{bmatrix}$$

$$M_{32} = \begin{bmatrix} 2 & 8 \\ 1 & 5 \end{bmatrix}$$

$$M_{13} = \begin{bmatrix} 1 & -4 \\ 4 & 3 \end{bmatrix}$$

$$M_{23} = \begin{bmatrix} 2 & 4 \\ 4 & 3 \end{bmatrix}$$

$$M_{33} = \begin{bmatrix} 2 & 6 \\ 1 & -4 \end{bmatrix}$$

2) Tentukanlah  $M_{11}$  sampai  $M_{44}$  dari matriks di bawah ini:

$$A = \begin{bmatrix} 1 & 3 & 4 & 2 \\ 5 & -3 & 3 & -4 \\ 4 & 4 & 2 & 1 \\ 2 & 9 & 8 & -6 \end{bmatrix}$$

**Jawab:**

$$M_{11} = \begin{bmatrix} -3 & 3 & -4 \\ 4 & 2 & 1 \\ 9 & 8 & -6 \end{bmatrix}$$

$$M_{21} = \begin{bmatrix} 3 & 4 & 2 \\ 4 & 2 & 1 \\ 9 & 8 & -6 \end{bmatrix}$$

$$M_{31} = \begin{bmatrix} 3 & 4 & 2 \\ -3 & 3 & 1 \\ 9 & 8 & -6 \end{bmatrix}$$

$$M_{12} = \begin{bmatrix} 5 & 3 & -4 \\ 4 & 2 & 1 \\ 2 & 8 & -6 \end{bmatrix}$$

$$M_{22} = \begin{bmatrix} 1 & 4 & 2 \\ 4 & 2 & 1 \\ 2 & 8 & -6 \end{bmatrix}$$

$$M_{32} = \begin{bmatrix} 1 & 4 & 2 \\ 5 & 3 & -4 \\ 2 & 8 & -6 \end{bmatrix}$$

$$M_{13} = \begin{bmatrix} 5 & -3 & -4 \\ 4 & 4 & 1 \\ 2 & 9 & -6 \end{bmatrix}$$

$$M_{23} = \begin{bmatrix} 1 & 3 & 2 \\ 4 & 4 & 1 \\ 2 & 9 & -6 \end{bmatrix}$$

$$M_{33} = \begin{bmatrix} 1 & 3 & 2 \\ 5 & -3 & -4 \\ 2 & 9 & -6 \end{bmatrix}$$

$$M_{14} = \begin{bmatrix} 5 & -3 & 3 \\ 4 & 4 & 2 \\ 2 & 9 & 8 \end{bmatrix}$$

$$M_{24} = \begin{bmatrix} 1 & 3 & 4 \\ 4 & 4 & 2 \\ 2 & 9 & 8 \end{bmatrix}$$

$$M_{34} = \begin{bmatrix} 1 & 3 & 4 \\ 5 & -3 & 3 \\ 2 & 9 & 8 \end{bmatrix}$$

$$M_{41} = \begin{bmatrix} 3 & 4 & 2 \\ -3 & 3 & -4 \\ 4 & 2 & 1 \end{bmatrix}$$

$$M_{42} = \begin{bmatrix} 1 & 4 & 2 \\ 5 & 3 & -4 \\ 4 & 2 & 1 \end{bmatrix}$$

$$M_{43} = \begin{bmatrix} 1 & 3 & 2 \\ 5 & -3 & -4 \\ 4 & 4 & 1 \end{bmatrix}$$

$$M_{44} = \begin{bmatrix} 1 & 3 & 4 \\ 5 & -3 & 2 \\ 4 & 4 & 2 \end{bmatrix}$$

### 3. Kofaktor

Nilai suatu kofaktor matriks diperoleh ketika nilai dari minor diperoleh.

Dimana untuk mencari kofaktor adalah sebagai berikut:

$$C_{ij} = (-1)^{i+j} \cdot \det M_{ij}$$

Keterangan:

Dimana **i = baris** dan **j = kolom**

**Contoh:**

1) Tentukanlah kofaktor dari matriks:

$$A = \begin{bmatrix} 2 & 1 & -3 \\ 4 & 5 & 1 \\ -2 & 3 & 1 \end{bmatrix}$$

**Jawab:**

$$\begin{aligned}
 C_{11} &= (-1)^{1+1} \cdot \det M_{11} \\
 &= (-1)^2 \cdot \det \begin{bmatrix} 5 & 1 \\ 3 & 1 \end{bmatrix} \\
 &= 1 \cdot ( (5 \times 1) - (1 \times 3) ) \\
 &= 1 \cdot (5 - 3) \\
 &= 1 \cdot 2 \\
 &= 2
 \end{aligned}$$

$$\begin{aligned}
 C_{12} &= (-1)^{1+2} \cdot \det M_{12} \\
 &= (-1)^3 \cdot \det \begin{bmatrix} 4 & 1 \\ -2 & 1 \end{bmatrix} \\
 &= -1 \cdot ( (4 \times 1) - (-2 \times 1) ) \\
 &= -1 \cdot (4 + 2) \\
 &= -1 \cdot 6 \\
 &= -6
 \end{aligned}$$

$$\begin{aligned}
 C_{13} &= (-1)^{1+3} \cdot \det M_{13} \\
 &= (-1)^4 \cdot \det \begin{bmatrix} 4 & 5 \\ -2 & 3 \end{bmatrix} \\
 &= 1 \cdot ( (4 \times 3) - (5 \times -2) ) \\
 &= 1 \cdot (12 + 10) \\
 &= 1 \cdot 22 \\
 &= 22
 \end{aligned}$$

$$\begin{aligned}
 C_{21} &= (-1)^{2+1} \cdot \det M_{21} \\
 &= (-1)^3 \cdot \det \begin{bmatrix} 1 & -3 \\ 3 & 1 \end{bmatrix} \\
 &= -1 \cdot ( (1 \times 1) - (-3 \times 3) ) \\
 &= -1 \cdot (1 + 9) \\
 &= -1 \cdot 10 \\
 &= -10
 \end{aligned}$$

$$\begin{aligned}
 C_{22} &= (-1)^{2+2} \cdot \det M_{22} \\
 &= (-1)^4 \cdot \det \begin{bmatrix} 2 & -3 \\ 2 & 1 \end{bmatrix} \\
 &= 1 \cdot ( (2 \times 1) - (-3 \times 2) ) \\
 &= 1 \cdot (2 + 6) \\
 &= 1 \cdot 8 \\
 &= 8
 \end{aligned}$$

$$\begin{aligned}
 C_{23} &= (-1)^{2+3} \cdot \det M_{23} \\
 &= (-1)^5 \cdot \det \begin{bmatrix} 2 & 1 \\ -2 & 3 \end{bmatrix} \\
 &= -1 \cdot ( (2 \times 3) - (1 \times -2) ) \\
 &= -1 \cdot (6 + 2) \\
 &= -1 \cdot 8 \\
 &= -8
 \end{aligned}$$

$$\begin{aligned}
 C_{31} &= (-1)^{3+1} \cdot \det M_{31} \\
 &= (-1)^4 \cdot \det \begin{bmatrix} 1 & -3 \\ 5 & 1 \end{bmatrix} \\
 &= 1 \cdot ( (1 \times 1) - (-3 \times 5) ) \\
 &= 1 \cdot (1 + 15) \\
 &= 1 \cdot 16 = 16
 \end{aligned}$$

$$\begin{aligned}
 C_{32} &= (-1)^{3+2} \cdot \det M_{32} \\
 &= (-1)^5 \cdot \det \begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix} \\
 &= -1 \cdot ( (2 \times 1) - (-3 \times 4) ) \\
 &= -1 \cdot (2 + 12) \\
 &= -1 \cdot 14 = -14
 \end{aligned}$$

$$\begin{aligned}
 C_{33} &= (-1)^{3+3} \cdot \det M_{33} \\
 &= (-1)^6 \cdot \det \begin{bmatrix} 2 & 1 \\ 4 & 5 \end{bmatrix} \\
 &= 1 \cdot ( (2 \times 5) - (1 \times 4) ) \\
 &= 1 \cdot (10 - 4) \\
 &= 1 \cdot 6 = 6
 \end{aligned}$$

2) Tentukan kofaktor dari matriks:

$$A = \begin{bmatrix} -1 & 3 & 4 \\ 5 & 2 & 7 \\ -4 & 2 & 4 \end{bmatrix}$$

**Jawab:**

$$\begin{aligned} C_{11} &= (-1)^{1+1} \cdot \det M_{11} \\ &= (-1)^2 \cdot \det \begin{bmatrix} 2 & 7 \\ 2 & 4 \end{bmatrix} \\ &= 1 (2 \times 4 - 7 \times 2) \\ &= 1 (8 - 14) \\ &= 1 (-6) \\ &= -6 \end{aligned}$$

$$\begin{aligned} C_{12} &= (-1)^{1+2} \cdot \det M_{12} \\ &= (-1)^3 \cdot \det \begin{bmatrix} 5 & 7 \\ -4 & 4 \end{bmatrix} \\ &= -1 (5 \times 4 - 7 \times -4) \\ &= -1 (20 + 28) \\ &= -1 (48) \\ &= -48 \end{aligned}$$

$$\begin{aligned} C_{13} &= (-1)^{1+3} \cdot \det M_{13} \\ &= (-1)^4 \cdot \det \begin{bmatrix} 5 & 2 \\ -4 & 2 \end{bmatrix} \\ &= 1 (5 \times 2 - 2 \times -4) \\ &= 1 (10 + 8) \\ &= 1 (18) \\ &= 18 \end{aligned}$$

$$\begin{aligned} C_{21} &= (-1)^{2+1} \cdot \det M_{21} \\ &= (-1)^3 \cdot \det \begin{bmatrix} 3 & 4 \\ 2 & 4 \end{bmatrix} \\ &= -1 (3 \times 4 - 4 \times 2) \\ &= -1 (12 - 8) \\ &= -1 (4) \\ &= -4 \end{aligned}$$

$$\begin{aligned} C_{22} &= (-1)^{2+2} \cdot \det M_{23} \\ &= (-1)^4 \cdot \det \begin{bmatrix} -1 & 4 \\ -4 & 4 \end{bmatrix} \\ &= 1 \cdot (-1 \times 4 - 4 \times -4) \\ &= 1 \cdot (-4 + 16) \\ &= 1 (12) \\ &= 12 \end{aligned}$$

$$\begin{aligned} C_{23} &= (-1)^{2+3} \cdot \det M_{23} \\ &= (-1)^5 \cdot \det \begin{bmatrix} -1 & 3 \\ -4 & 2 \end{bmatrix} \\ &= -1 (-1 \times 2 - 3 \times -4) \\ &= -1 (-2 + 12) \\ &= -1 (10) \\ &= -10 \end{aligned}$$

#### 4. Ekspansi Baris

Dalam determinan suatu matriks, maka kofaktor yang dihitung hanya tergantung pada baris matriks dan kolom matriks saja. Untuk baris disebut sebagai ekspansi baris dan untuk kolom disebut sebagai ekspansi kolom.

**untuk matriks ordo 3 x 3**

$$\text{Misal } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$\text{Baris 1} = a_{11}c_{11} + a_{12}c_{12} + a_{13}c_{13}$$

$$\text{Baris 2} = a_{21}c_{21} + a_{22}c_{22} + a_{23}c_{23}$$

$$\text{Baris 3} = a_{31}c_{31} + a_{32}c_{32} + a_{33}c_{33}$$

**Contoh:**

1) Tentukanlah ekspansi baris dari matriks  $A = \begin{bmatrix} 1 & -2 & 4 \\ -3 & 1 & 5 \\ 2 & 4 & 3 \end{bmatrix}$

**Jawab:**

Baris 1

$$\begin{aligned} &= a_{11}c_{11} + a_{12}c_{12} + a_{13}c_{13} \\ &= 1. (-1)^{1+1} \cdot \det M_{11}) + (-2. (-1)^{1+2} \cdot \det M_{12}) + (4. (-1)^{1+3} \cdot \det M_{13}) \\ &= 1.(1. \det \begin{bmatrix} 1 & 5 \\ 4 & 3 \end{bmatrix} )) - 2 (-1. \det \begin{bmatrix} -3 & 5 \\ 2 & 3 \end{bmatrix}) + 4. ( 1. \det \begin{bmatrix} -3 & 1 \\ 2 & 4 \end{bmatrix}) \\ &= 1.(1 (1 \times 3 - 5 \times 4)) - 2 (-1. (-3 \times 3 - 5 \times 2) + 4. ( 1. -3 \times 4 - 1 \times 2) \\ &= 1. (1 ( 3 - 20)) - 2 (-1 ( -9 - 10 ) + 4. (1. (-12 - 2)) \\ &= 1.(-17) + 2.(-19) + 4 (-14) \\ &= -17 - 38 - 56 \\ &= -111 \end{aligned}$$

Baris 2

$$\begin{aligned} &= a_{21}c_{21} + a_{22}c_{22} + a_{23}c_{23} \\ &= -3. (-1)^{2+1} \cdot \det M_{21}) + (1. (-1)^{2+2} \cdot \det M_{22}) + (5. (-1)^{2+3} \cdot \det M_{23}) \\ &= -3.(-1. \det \begin{bmatrix} -2 & 4 \\ 4 & 3 \end{bmatrix} )) + 1 (1. \det \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}) + 5. ( -1. \det \begin{bmatrix} 1 & -2 \\ 2 & 4 \end{bmatrix}) \\ &= -3. (-1 (-2 \times 3 - 4 \times 4)) + 1 (1. (1 \times 3 - 4 \times 2) + 5. ( -1.( 1 \times 4 - (-2 \times 2)) \\ &= -3 (-1. (- 6 - 16) + 1 (1. (3 - 8)) + 5. (-1 (4 +4) \\ &= -3 (22) + 1 (-5) + 5 (-8) \\ &= - 66 - 5 - 40 \\ &= - 111 \end{aligned}$$

Baris 3

$$\begin{aligned} &= a_{31}c_{31} + a_{32}c_{32} + a_{33}c_{33} \\ &= 2. (-1)^{3+1} \cdot \det M_{31}) + (4. (-1)^{3+2} \cdot \det M_{32}) + (3. (-1)^{3+3} \cdot \det M_{33}) \\ &= 2.(1. \det \begin{bmatrix} -2 & 4 \\ 1 & 5 \end{bmatrix} )) + 4 (-1. \det \begin{bmatrix} 1 & 4 \\ -3 & 5 \end{bmatrix}) + 3. ( 1. \det \begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix}) \\ &= 2.(1 (-2 \times 5 - 4 \times 1)) + 4 (-1. (1 \times 5 - 4 \times -3) + 3. ( 1.( 1 \times 1 - (-2 \times -3)) \end{aligned}$$



$$\begin{aligned}
&= 2 (1. (-10 - 4) + 4 (-1. (5 + 12)) + 3. (1 (1 - 6))) \\
&= 2 (-14) - 4 (17) + 3 (-5) \\
&= -28 - 68 - 15 \\
&= -111
\end{aligned}$$

## 5. Ekspansi Kolom

Misal  $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

$$\text{Kolom 1} = a_{11}c_{11} + a_{21}c_{21} + a_{31}c_{31}$$

$$\text{Kolom 2} = a_{12}c_{12} + a_{22}c_{22} + a_{32}c_{32}$$

$$\text{Kolom 3} = a_{13}c_{13} + a_{23}c_{23} + a_{33}c_{33}$$

**Contoh:**

1) Tentukanlah ekspansi baris dari matriks  $A = \begin{bmatrix} 1 & -2 & 4 \\ -3 & 1 & 5 \\ 2 & 4 & 3 \end{bmatrix}$

**Jawab:**

Kolom 1

$$\begin{aligned}
&= a_{11}c_{11} + a_{21}c_{21} + a_{31}c_{31} \\
&= 1. (-1)^{1+1} \cdot \det M_{11} + (-3. (-1)^{2+1} \cdot \det M_{21}) + (2. (-1)^{3+1} \cdot \det M_{31}) \\
&= 1. (1. \det \begin{bmatrix} 1 & 5 \\ 4 & 3 \end{bmatrix}) - 3 (-1. \det \begin{bmatrix} -2 & 4 \\ 4 & 3 \end{bmatrix}) + 2. (1. \det \begin{bmatrix} -2 & 4 \\ 1 & 5 \end{bmatrix}) \\
&= 1. (1 (1 \times 3 - 5 \times 4)) - 3 (-1. (-2 \times 3 - 4 \times 4)) + 2. (1. -2 \times 5 - 4 \times 1) \\
&= 1. (1 (3 - 20)) - 3 (-1 (-6 - 16)) + 2. (1. (-10 - 4)) \\
&= 1. (-17) + 3. (-22) + 2 (-14) \\
&= -17 - 66 - 28 \\
&= -111
\end{aligned}$$

Kolom 2

$$\begin{aligned}
&= a_{12}c_{12} + a_{22}c_{22} + a_{32}c_{32} \\
&= -2. (-1)^{1+2} \cdot \det M_{12} + (1. (-1)^{2+2} \cdot \det M_{22}) + (4. (-1)^{3+2} \cdot \det M_{32}) \\
&= -2. (-1. \det \begin{bmatrix} -3 & 5 \\ 2 & 3 \end{bmatrix}) + 1 (1. \det \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}) + 4. (-1. \det \begin{bmatrix} 1 & 4 \\ -3 & 5 \end{bmatrix}) \\
&= -2. (-1 (-3 \times 3 - 5 \times 2)) + 1 (1. (1 \times 3 - 4 \times 2)) + 4. (-1. (1 \times 5 - 4 \times -3)) \\
&= -2. (-1 (-9 - 10)) + 1 (1 (3 - 8)) + 4. (-1. (5 + 12)) \\
&= -2. (19) + 1. (-5) - 4 (17) \\
&= -38 - 5 - 68 \\
&= -111
\end{aligned}$$

Kolom 3

$$\begin{aligned}
 &= a_{13}c_{13} + a_{23}c_{23} + a_{33}c_{33} \\
 &= 4. (-1)^{1+3} \cdot \det M_{13} + (5. (-1)^{2+3} \cdot \det M_{23}) + (3. (-1)^{3+3} \cdot \det M_{33}) \\
 &= 4. (1. \det \begin{bmatrix} -3 & 1 \\ 2 & 4 \end{bmatrix}) + 5 (-1. \det \begin{bmatrix} 1 & -2 \\ 2 & 4 \end{bmatrix}) + 3. (1. \det \begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix}) \\
 &= 4. (1 (-3 \times 4 - 1 \times 2)) + 5 (-1. (1 \times 4 - (-2 \times 2)) + 3. (1. (1 \times 1 - (-2 \times -3))) \\
 &= 4. (1 (-12 - 2)) + 5 (-1 (4 + 4)) + 3. (1. (1 - 6)) \\
 &= 4. (-14) + 5. (-8) + 3 (-5) \\
 &= -56 - 40 - 15 \\
 &= -111
 \end{aligned}$$

2) Tentukanlah ekspansi baris dan ekspansi kolom dari matriks:

$$A = \begin{bmatrix} 4 & 6 & 7 \\ 2 & 3 & 5 \\ -2 & 1 & 4 \end{bmatrix}$$

**Jawab:**

**a) ekspansi baris**

Baris 1

$$\begin{aligned}
 &= a_{11}c_{11} + a_{12}c_{12} + a_{13}c_{13} \\
 &= 4. (-1)^{1+1} \cdot \det M_{11} + (6. (-1)^{1+2} \cdot \det M_{12}) + (7. (-1)^{1+3} \cdot \det M_{13}) \\
 &= 4. (1. \det \begin{bmatrix} 3 & 5 \\ 1 & 4 \end{bmatrix}) + 6 (-1. \det \begin{bmatrix} 2 & 5 \\ -2 & 4 \end{bmatrix}) + 7. (1. \det \begin{bmatrix} 2 & 3 \\ -2 & 1 \end{bmatrix}) \\
 &= 4. (1 (3 \times 4 - 5 \times 1)) + 6 (-1. (2 \times 4 - (5 \times -2)) + 7. (1. (2 \times 1 - (3 \times -2))) \\
 &= 4. (1 (12 - 5)) + 6 (-1 (8 + 10)) + 7. (1. (2 + 6)) \\
 &= 4.(7) - 6.(18) + 7 (8) \\
 &= 28 - 108 + 56 \\
 &= -24
 \end{aligned}$$

Baris 2

$$\begin{aligned}
 &= a_{21}c_{21} + a_{22}c_{22} + a_{23}c_{23} \\
 &= 2. (-1)^{2+1} \cdot \det M_{21} + (3. (-1)^{2+2} \cdot \det M_{22}) + (5. (-1)^{2+3} \cdot \det M_{23}) \\
 &= 2. (-1. \det \begin{bmatrix} 6 & 7 \\ 1 & 4 \end{bmatrix}) + 3 (1. \det \begin{bmatrix} 4 & 7 \\ -2 & 4 \end{bmatrix}) + 5. (-1. \det \begin{bmatrix} 4 & 6 \\ -2 & 1 \end{bmatrix}) \\
 &= 2. (-1 (6 \times 4 - 7 \times 1)) + 3 (1. (4 \times 4 - (7 \times -2)) + 5. (-1. (4 \times 1 - (6 \times -2))) \\
 &= 2. (-1 (24 - 7)) + 3 (1 (16 + 14)) + 5. (-1. (4 + 12)) \\
 &= -2.(17) + 3.(30) - 5 (16) \\
 &= -34 + 90 - 80 = -24
 \end{aligned}$$

Baris 3

$$\begin{aligned}
 &= a_{31}c_{31} + a_{32}c_{32} + a_{33}c_{33} \\
 &= -2. (-1)^{3+1} \cdot \det M_{31} + (1. (-1)^{3+2} \cdot \det M_{32}) + (4. (-1)^{3+3} \cdot \det M_{33}) \\
 &= -2.(1. \det \begin{bmatrix} 6 & 7 \\ 3 & 5 \end{bmatrix}) + 1. (-1. \det \begin{bmatrix} 4 & 7 \\ 2 & 5 \end{bmatrix}) + 4. (1. \det \begin{bmatrix} 4 & 6 \\ 2 & 3 \end{bmatrix}) \\
 &= -2.(1 (6 \times 5 - 7 \times 3)) + 1. (-1. (4 \times 5 - (7 \times 2) + 4. (1.( 4 \times 3 - 6 \times 2)) \\
 &= -2. (1 ( 30 - 21)) + 1. (-1 ( 20 - 14 ) + 4. (1. (12- 12)) \\
 &= -2.(9) - 1.(6) +4 ( 0) \\
 &= -18- 6 - 0 \\
 &= -24
 \end{aligned}$$

### b) ekspansi kolom

Kolom 1

$$\begin{aligned}
 &= a_{11}c_{11} + a_{21}c_{21} + a_{31}c_{31} \\
 &= 4. (-1)^{1+1} \cdot \det M_{11} + (2. (-1)^{2+1} \cdot \det M_{21}) + (-2. (-1)^{3+1} \cdot \det M_{31}) \\
 &= 4.(1. \det \begin{bmatrix} 3 & 5 \\ 1 & 4 \end{bmatrix}) + 2. (-1. \det \begin{bmatrix} 6 & 7 \\ 1 & 4 \end{bmatrix}) -2. (1. \det \begin{bmatrix} 6 & 7 \\ 3 & 5 \end{bmatrix}) \\
 &= 4.(1 (3 \times 4 - 5 \times 1)) + 2. (-1. (6 \times 4 - (7 \times 1) -2. (1.( 6 \times 5 - 7 \times 3)) \\
 &= 4. (1 ( 12 - 5)) + 2. (-1 ( 24 - 7 ) -2. (1. (30 - 21)) \\
 &= 4.(7) - 2.(17) -2 ( 9) \\
 &= 28 - 34 - 18 \\
 &= -24
 \end{aligned}$$

Kolom 2

$$\begin{aligned}
 &= a_{12}c_{12} + a_{22}c_{22} + a_{32}c_{32} \\
 &= 6. (-1)^{1+2} \cdot \det M_{12} + (3. (-1)^{2+2} \cdot \det M_{22}) + (1. (-1)^{3+2} \cdot \det M_{32}) \\
 &= 6.(-1. \det \begin{bmatrix} 2 & 5 \\ -2 & 4 \end{bmatrix}) + 3 (1. \det \begin{bmatrix} 4 & 7 \\ -2 & 4 \end{bmatrix}) + 1. ( -1. \det \begin{bmatrix} 4 & 7 \\ 2 & 5 \end{bmatrix}) \\
 &= 6 .(-1 (2 \times 4 - 5 \times -2)) + 3 (1. (4 \times 4 - (7 \times -2) + 1. ( -1.( 4 \times 5 - 7 \times 2)) \\
 &= 6. (-1 ( 8 + 10)) + 3 (1 ( 16 + 14 ) + 1. (-1. (20 - 14)) \\
 &= -6.(18) + 3.(30) - 1 ( 6) \\
 &= -108 + 90 - 6 \\
 &= -24
 \end{aligned}$$

Kolom 3

$$\begin{aligned}
 &= a_{13}c_{13} + a_{23}c_{23} + a_{33}c_{33} \\
 &= 7. (-1)^{1+3} \cdot \det M_{13} + (5. (-1)^{2+3} \cdot \det M_{23}) + (4. (-1)^{3+3} \cdot \det M_{33}) \\
 &= 7. (1. \det \begin{bmatrix} 2 & 3 \\ -2 & 1 \end{bmatrix}) + 5 (-1. \det \begin{bmatrix} 4 & 6 \\ -2 & 1 \end{bmatrix}) + 4. (1. \det \begin{bmatrix} 4 & 6 \\ 2 & 3 \end{bmatrix}) \\
 &= 7. (1 (2 \times 1 - 3 \times -2)) + 5 (-1. (4 \times 1 - (6 \times -2)) + 4. (1. (4 \times 3 - 6 \times 2)) \\
 &= 7. (1 (2 + 6)) + 5 (-1 (4 + 12)) + 4. (1. (12 - 12)) \\
 &= 7.(8) - 5.(16) + 4 (0) \\
 &= 56 - 90 + 0 \\
 &= -24
 \end{aligned}$$

**C. Latihan Soal/Tugas**

1. Berapakah determinan dari matriks  $A = \begin{bmatrix} 1/2 & 2 \\ 1 & -9 \end{bmatrix}$
2. Berapakah determinan dari matriks  $A = \begin{bmatrix} 1/4 & 3 & 2 \\ 5 & 1/2 & 1 \\ -1 & 2 & 4 \end{bmatrix}$
3. Carilah nilai  $M_{11}$  sampai  $M_{33}$  dari matriks  $A = \begin{bmatrix} 2 & 3 & 2 \\ 6 & -1/5 & -8 \\ 9 & 3 & 4 \end{bmatrix}$
4. Hitunglah nilai  $C_{23}$ ,  $C_{31}$ ,  $C_{33}$  dari matriks  $A = \begin{bmatrix} 3 & 2 & -7 \\ -5 & 6 & 4 \\ -1 & 2 & 8 \end{bmatrix}$
5. Hitunglah nilai ekspansi baris dari matriks  $A = \begin{bmatrix} 3 & 2 & -7 \\ -5 & 6 & 4 \\ -1 & 2 & 8 \end{bmatrix}$
6. Hitunglah nilai ekspansi kolom dari matriks  $A = \begin{bmatrix} 3 & 2 & -7 \\ -5 & 6 & 4 \\ -1 & 2 & 8 \end{bmatrix}$

**D. Daftar Pustaka**

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