Lecture 13: Summary

Pinar Øzturk

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Problem 1. Logic

Somebody stole the Christmas lights of Mr. Olsen in Trondheim. Inspector Harry Hole thinks one of the five famous burglars, Arne, Bernt, Cristoffer, Dina, Edna, in the town must have stolen the lights. He interviews each of them to find out the guilty one.

- The following is the result of the interviews – two statements from each thief. It is well known that exactly one of the two statements of each thief is a lie:

Arne: *It was not Edna. It was Bernt.*

Bernt: *It was not Cristoffer. It was not Edna.*

Cristoffer: *It was Edna. It was not Arne.*

Derek: *It was Cristoffer. It was Bernt.*

Edna: *It was Dina. It was not Arne.*

- Use the following propositional variables:

A=It was Arne. **B**=It was Bernt. **C**=It was Cristoffer. **D**=It was Dina. **E**=It was Edna

Problem 1 . Logic – cont.

1. Translate the evidence from each thief (i.e. from the statements of thieves taking also into account that exactly one of two statements of each statement of each thief is a lie) into propostional logic representation . i.e.,

Arne: ……..

Bernt:….

..

..

Edna: …..

2. It is well known that one thief stole the lights alone. Represent this information as a set of implications.
3. Your data base consists of the statements you wrote in 1) and 2). Convert the statements in the KB to conjunctive normal form.
4. Using this KB, apply resolution refutation in order to infer “It was Cristoffer”. If you cannot, then you must have made a mistake somewhere. Use the following structure to show your proof – add as many lines as you need:

Resolve …………… and …………….. to produce ……………..

Resolve …………… and …………….. to produce ……………..

…………..

Answer to Problem 1

1) Translate the evidence (i.e. from the statements of thieves taking also into account that exactly one of the two statements of each thief is a lie.

Arne: $(E \land B) \lor (\neg E \land \neg B)$

Bernt: $(C \land \neg E) \lor (\neg C \land E)$

Cristoffer: $(\neg E \land \neg A) \lor (E \land A)$

Dina: $(\neg C \land B) \lor (C \land \neg B)$

Edna: $(\neg D \land \neg A) \lor (D \land A)$

Answer to Problem 1

2) Representation of the information that one thief has stolen the lights alone:

$(A \Rightarrow \neg B \land \neg C \land \neg D \land \neg E)$

$(B \Rightarrow \neg A \land \neg C \land \neg D \land \neg E)$

$(C \Rightarrow \neg A \land \neg B \land \neg D \land \neg E)$

$(D \Rightarrow \neg A \land \neg B \land \neg C \land \neg E)$

$(E \Rightarrow \neg A \land \neg B \land \neg C \land \neg D)$

Answer to Problem 1

3) The KB:

1. $(E \land B) \lor (\neg E \land \neg B)$
2. $(C \land \neg E) \lor (\neg C \land E)$
3. $(\neg E \land \neg A) \lor (E \land A)$
4. $(\neg C \land B) \lor (C \land \neg B)$
5. $(\neg D \land \neg A) \lor (D \land A)$
6. $(A \Rightarrow \neg B \land \neg C \land \neg D \land \neg E)$
7. $(B \Rightarrow \neg A \land \neg C \land \neg D \land \neg E)$
8. $(C \Rightarrow \neg A \land \neg B \land \neg D \land \neg E)$
9. $(D \Rightarrow \neg A \land \neg B \land \neg C \land \neg E)$
10. $(E \Rightarrow \neg A \land \neg B \land \neg C \land \neg D)$

Converted to CNF:

$(E \lor \neg B) (\neg E \lor B)$ (using distributivity of $\lor$ over ^ on 1.)
$(C \lor E) (\neg C \lor \neg E)$
$(\neg E \lor A) (E \lor \neg A)$
$(\neg C \lor \neg B) (C \lor B)$
$(\neg D \lor A) (D \lor \neg A)$
$(\neg A \lor \neg B) (\neg A \lor \neg C)$ (implic. elim. +distributivity for 6-10)
$(\neg A \lor \neg D) (\neg A \lor \neg E)$
$(\neg B \lor \neg C) (\neg B \lor \neg D)$
$(\neg B \lor \neg E) (\neg C \lor \neg D)$
$(\neg C \lor \neg E) (\neg D \lor \neg E)$

Answer to Problem 1

4) Add ¬C to the KB and obtain { }

Resolve ¬B ∨ ¬E and C ∨ E to produce ¬B ∨ C
Resolve ¬B ∨ C and C ∨ B to produce C ∨ C = C
Resolve C and ¬C to produce { }

It is possible to do it in other ways as well. For example:

Resolve ¬ C and C ∨ E to give E
Resolve ¬ C and C ∨ B to give B
Resolve E and ¬ B ∨ ¬ E to give ¬ B
Resolve B and ¬ B to give {}

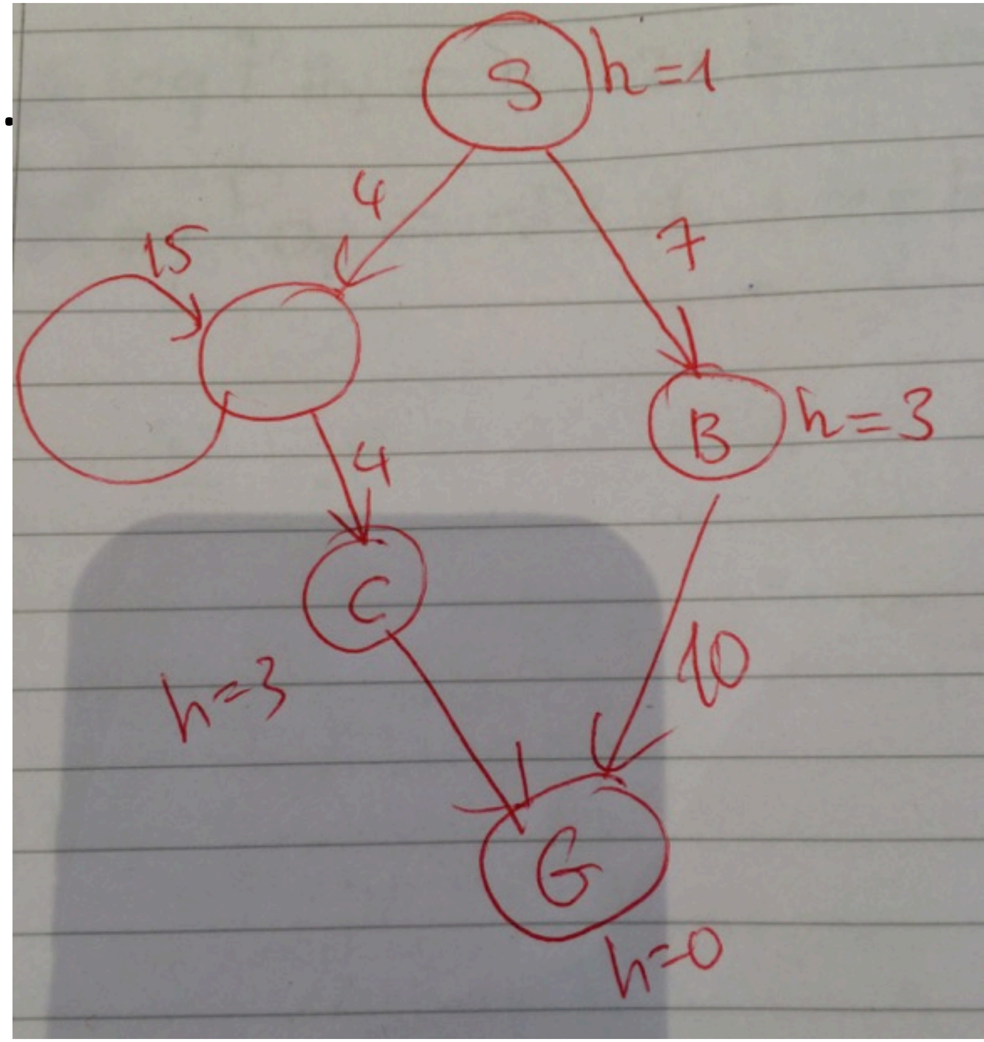
Converted to CNF:

(E ∨ ¬B) (¬E ∨ B)
(C ∨ E) (¬C ∨ ¬E)
(¬E ∨ A) (E ∨ ¬A)
(¬C ∨ ¬B) (C ∨ B)
(¬D ∨ A) (D ∨ ¬A)
(¬A ∨ ¬B) (¬A ∨ ¬C)
(¬A ∨ ¬D) (¬A ∨ ¬E)
(¬B ∨ ¬C) (¬B ∨ ¬D)
(¬B ∨ ¬E) (¬C ∨ ¬D)
(¬C ∨ ¬E) (¬D ∨ ¬E)

Problem 2. Search

Execute Tree Search through the following graph.
Don't remember visited nodes.

Left-to-right order of successors.



Problem 2. Search - cont

For each of the following search strategy, (i) show the order in which nodes are expanded. (ii) Show the path from start to goal, or write “None” if no path to goal is found, and (iii) write the cost of the path found.

1. **Breadth first search**

   Copy the following part into your answer paper and fill it according to the search algorithm)

   Order of expansion: ……………………………….……

   Found Path (from S to G): ……………………………… Path cost: ………………………………

2. **Depth First search**
3. **Uniform cost search**
4. **Greedy(Best-first) search**
5. **A* search**

Answer to Problem 2

1. **Breadth first search**:

Order of expansion: S A B G

Found Path (from S to G): SBG Path cost: 17

2. **Depth First search:**

Order of expansion: S A A A A A A.....

Found Path (from S to G): None Path cost: None

3. **Uniform cost search**

Order of expansion: S A B C G

Found Path (from S to G): SACG Path cost: 12

4. **Greedy(Best-first) search**

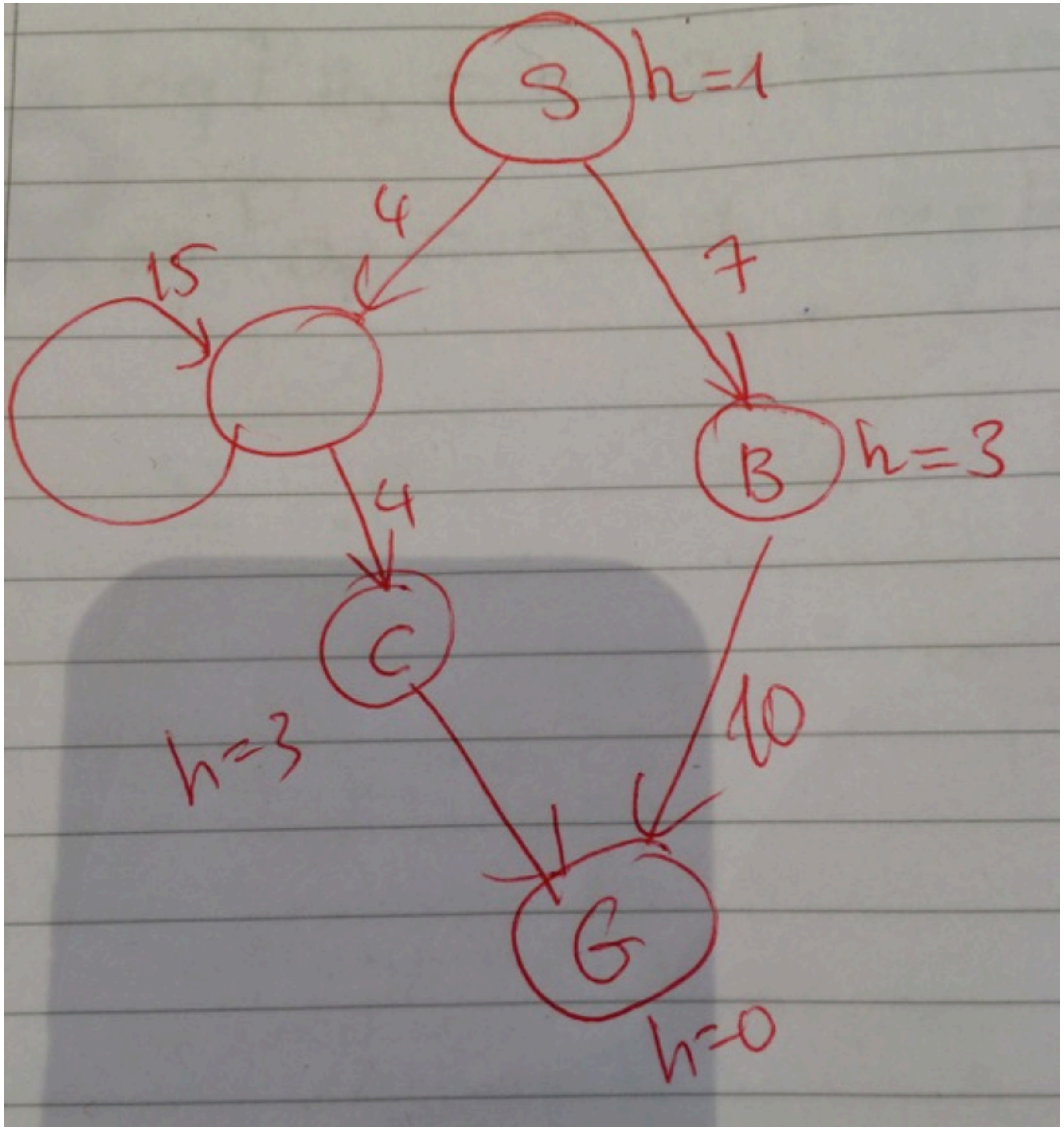
Order of expansion: S A AAAAA…

Found Path (from S to G): None Path cost: None

5. **A* search**

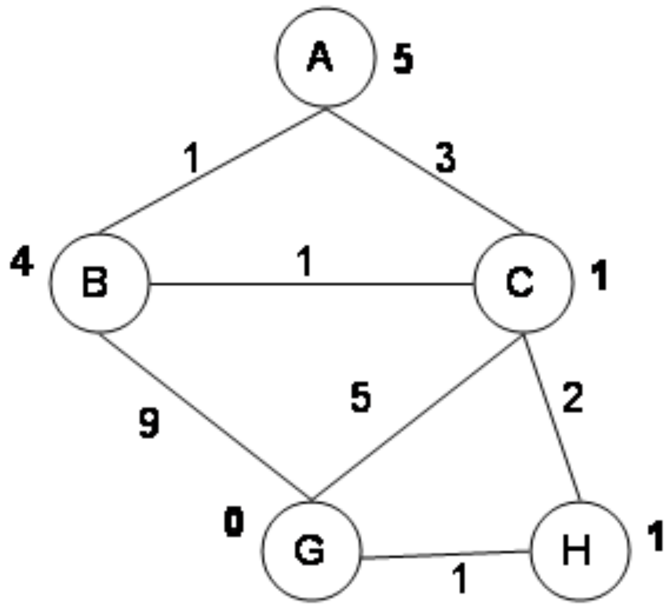
Order of expansion: SABCG

Path (from S to G): SACG Path cost: 12



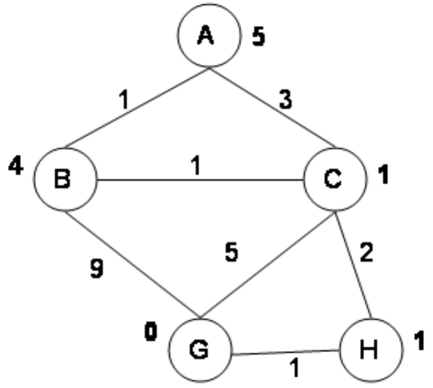
Problem 3: A* alg.

A is the start node, and
G is the goal node



1. What does A* algorithm return (i.e., which path) when the graph search is done?
2. Is this path optimal? Explain why?

Problem 3 (A*) answer



1)

Path to State Expanded	Length of Path	Total Estimated Cost	Expanded List
A	0	5	(A)
C-A	3	4	(C A)
B-A	1	5	(B C A)
H-C-A	5	6	(H B C A)
G-H-C-A	6	6	(G H B C A)

2) Not optimal. This path costs 6 while another one, ABCHG would cost 5

Because the heuristic is not consistent. Change h(C)=3 and try again.

Problem 4: Game Theory

- Camomilla and Daisy are competing sellers. Their price competition can be described by the following game where the payoffs represent sellers' daily profits as thousand NOK.

Daisy

		$1		$2		$3		$4	
Camomilla	$1	2.5	2.5	5	0	5	0	5	0
	$2	0	5	4	4	8	0	8	0
	$3	0	5	0	8	4.5	4.5	9	0
	$4	0	5	0	8	0	9	4	4

Problem 4. Game Theory –cont.

1. Is Camomilla's strategy \$4 stongly dominated? (yes/no)? If 'yes', which strategi(es) strongly dominate \$4?

2. Is Camomilla's strategi \$4 weakly dominated (yes/no)? If 'yes', which strategi(es) dominate it weakly?

3. Is any of Camomilla's strategies not dominated (yes/no)? If 'yes', which one(s)?

4. Does the game have a solution if we use 'Iterated Elimination of dominated Strategies' (yes/no)? What is the solution, if «yes»? You must write the sequence of elimination.

Answer to Problem 4

1. Yes, \$1
2. Yes,\$2 and \$3, and also \$1(strong domination implies weak)
3. Yes,\$1,\$2,\$3
4. Yes,. (\$1,\$1). One possible sequence: (Camomilla \$4 , Daisy \$4, Camomilla \$3, Daisy\$3, Camomilla\$2, Daisy \$2)

Dais y

		\$1		\$2		\$3		\$4	
Camomilla	\$1	2.5	2.5	5	0	5	0	5	0
	\$2	0	5	4	4	8	0	8	0
	\$3	0	5	0	8	4.5	4.5	9	0
	\$4	0	5	0	8	0	9	4	4

Social welfare and pareto optimal

Pareto optimal: There is no other outcome where agents can increase their payoffs without decreasing the payoffs of other agents."

Is a socially optimal outcome also pareto optimal?

		Agent 2	
		A	B
Agent 1	A	9, 9	12, 7
	B	0, 20	8, 8

PROBLEM 5: Game theory

This problem is about watering rice farms in Sumatra. Two farms are located in the riverside, upstream and downstream farms. Water is not sufficient---meaning if both farmers water their rice crops at the same time the upward one gets enough water but the downstream one is not watered sufficiently.

However, if they don’t water their crops at the same time, both farms will suffer from a pest problem.

The game is about the timing of watering. Assume two possible times for watering, A and B.

Problem 5 - cont.

- Let x represent the utility loss for the farmer who gets reduced water and let y represent the loss in utility due to pests. Assume that when there is no crop loss due to lack of water or a pest problem, then the payoff is equal to 1.

- 1. Draw the payoff matrix using A and B as actions. Use x and y to define the payoffs for the upward and downward farms.

- 2. Under which conditions does a strongly dominant strategy equilibrium arise?

Answer to Prob. 5

Farm-down

1.

Farm-up

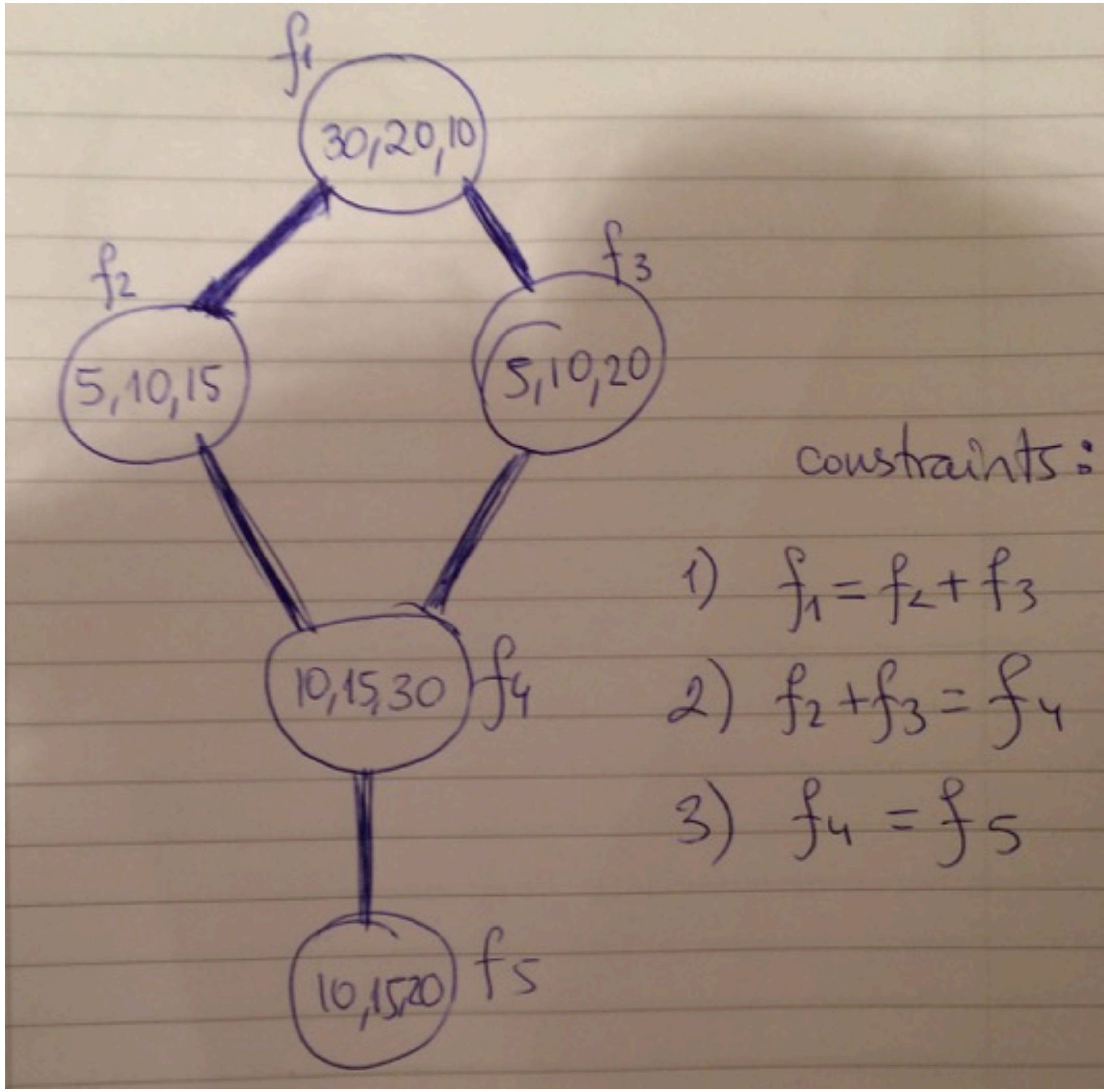
	A	B
A	1, 1 – x	1 – y, 1 – y
B	1 – y, 1 – y	1, 1 – x

Answer to Prob. 5

2. There is no strongly dominant strategy equilibrium in this problem. Assume x,y are positive numbers. Then (B,A) cannot be a dominant strategy equilibrium (DSE) as both agents could get a positive payoff by choosing another action. the same holds for (A,B). For (A,A) to be a DSE, it would have to be the case that $x < y$ (so that A-for the column player- is a strict best response to to A-for the row player). However, for A-column player to be a dominant strategy, it must also do better if row player plays B, which can only be the case when $y < x$. Thus (A,A) cannot be a DSE. The same reasoning holds for (B,B) meaning there is no DSE for any value of x or y.

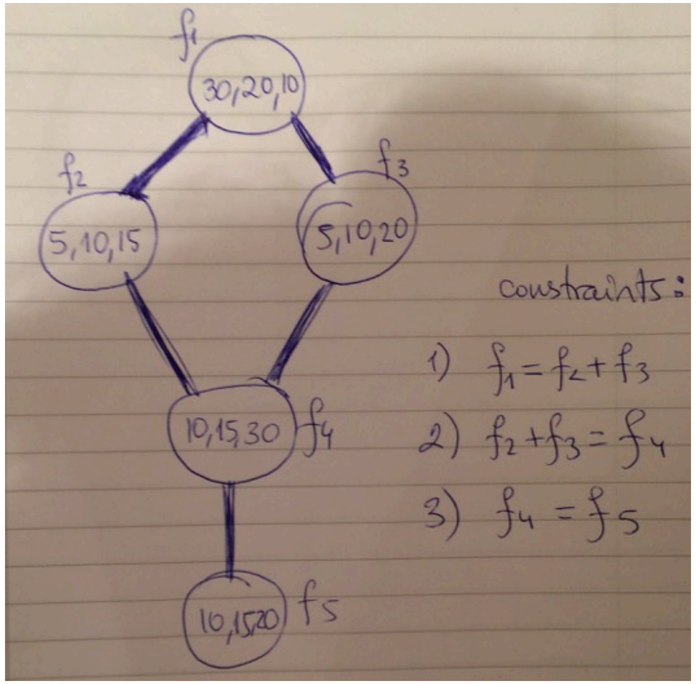
Problem 6: Constraint Satisfaction Problem

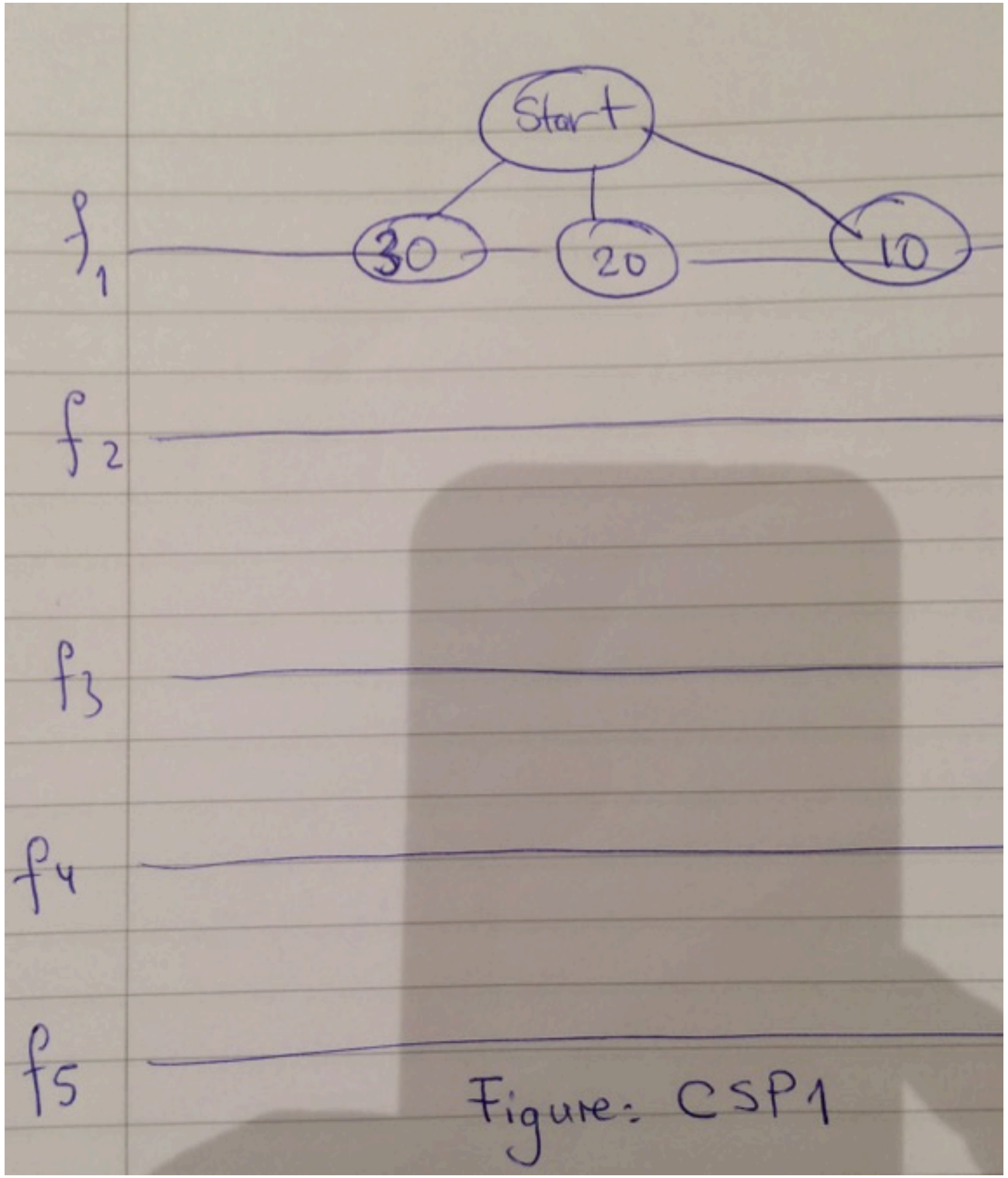
- This problem is about agricultural water management. The network consists of 5 water pipes with various capacities of water flow: f1, f2, f3, f4, f5 .
- You are given the following diagram representing the constraint network. The circles represent the variables and are shown as f1, f2, f3, f4 and f5 while the integers in each circle/node show the possible values of flow for that node.



A) Draw a search tree of the system using Backtracking with forward checking**.** For each node draw only the valid successors at that point.

Select the values to try in the same order as in the nodes of the diagram. Complete the search tree on the right

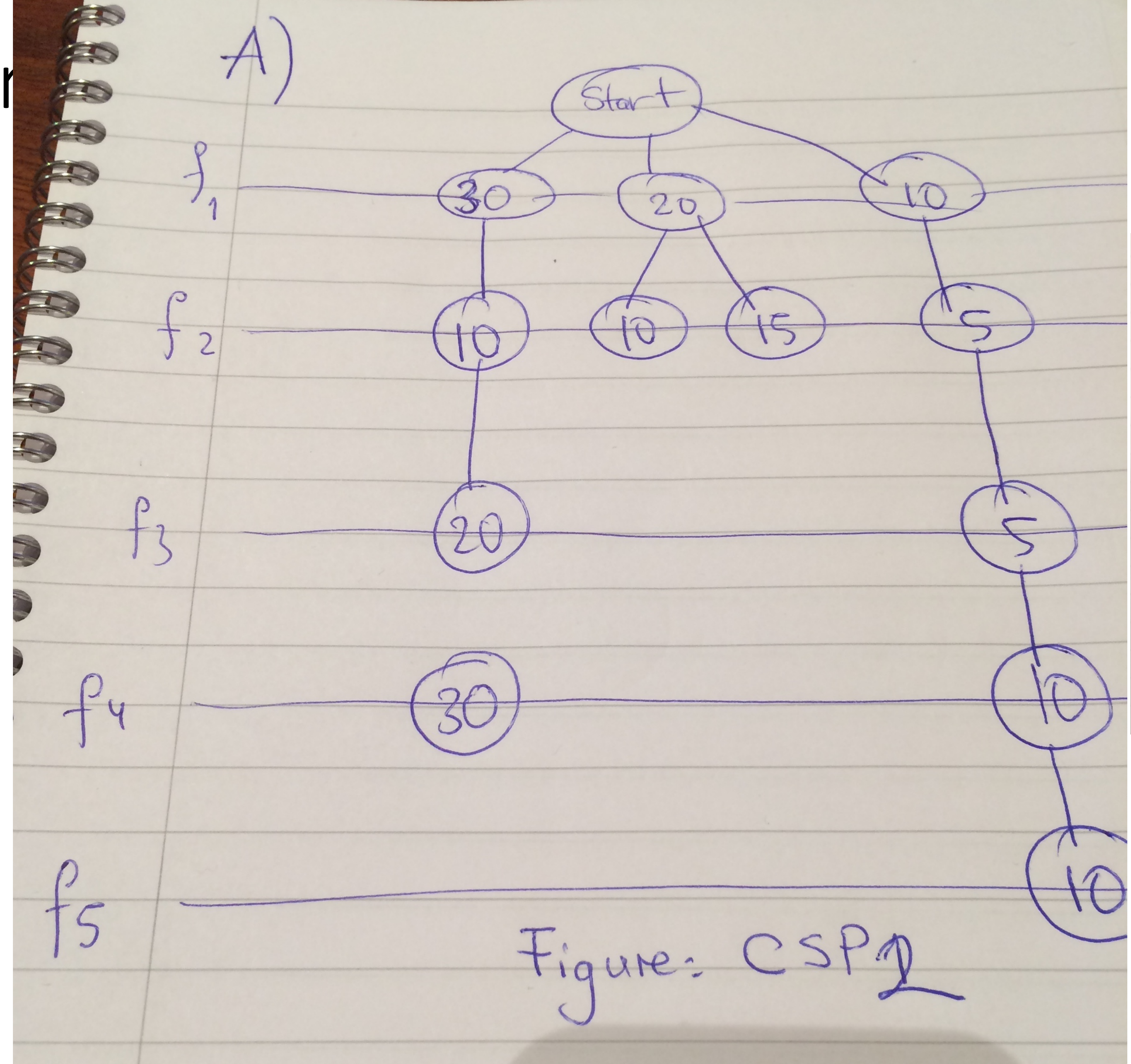


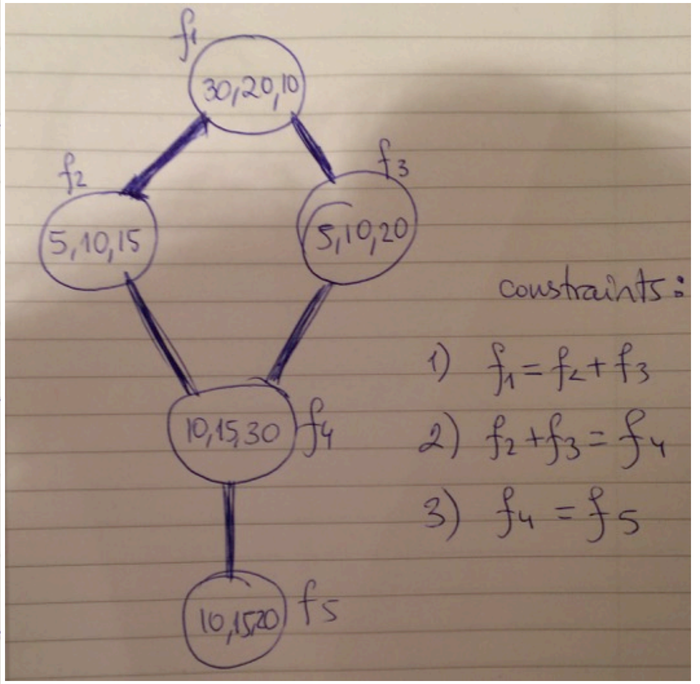


- B) Draw the search tree that results from applying backtracking with forward checking **and propagating through domains that are reduced to** singleton (i.e., only 1 value left) domains.

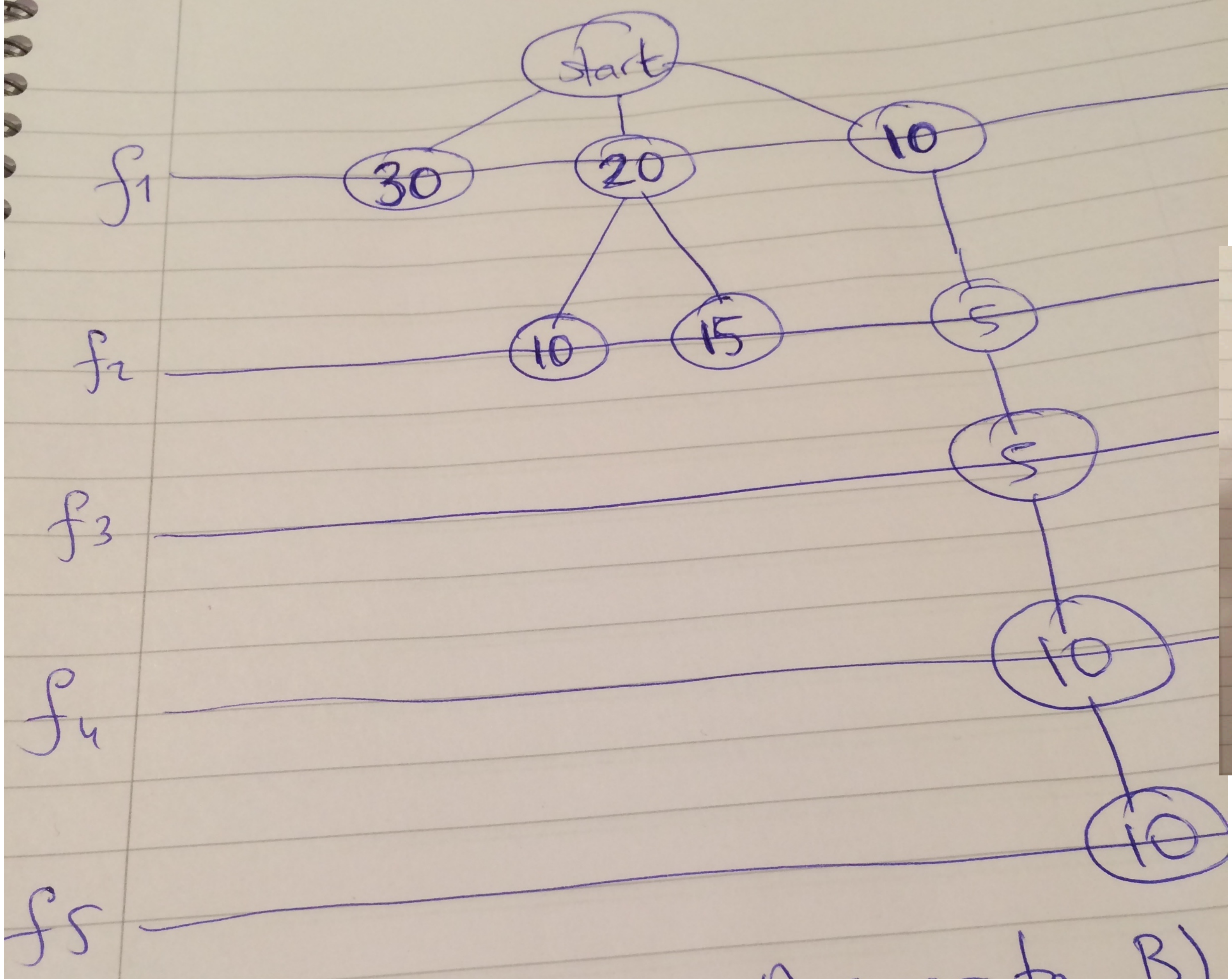
Use the same partial search tree a in Question A and complete the search tree. Again draw only the nodes with valid successors, and try values in the same order as shown in the initial domain representations.

Answer
A)

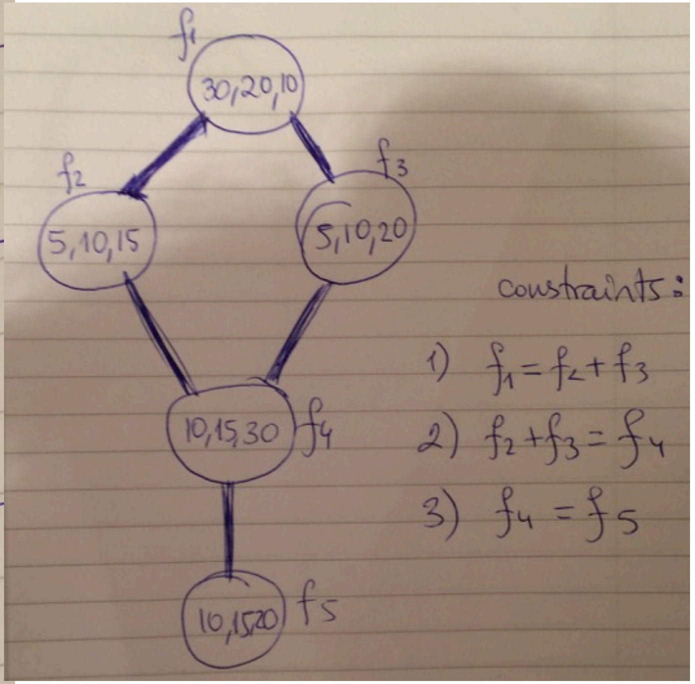

A)
Start
f1
30
20
10
f2
10
10
15
5
f3
20
5
f4
30
10
f5
10
Figure: CSP 2


f1
30,20,10
f2
5,10,15
f3
5,10,20
10,15,30
f4
10,15,20
f5
constraints:
1) f1 = f2 + f3
2) f2 + f3 = f4
3) f4 = f5

B)

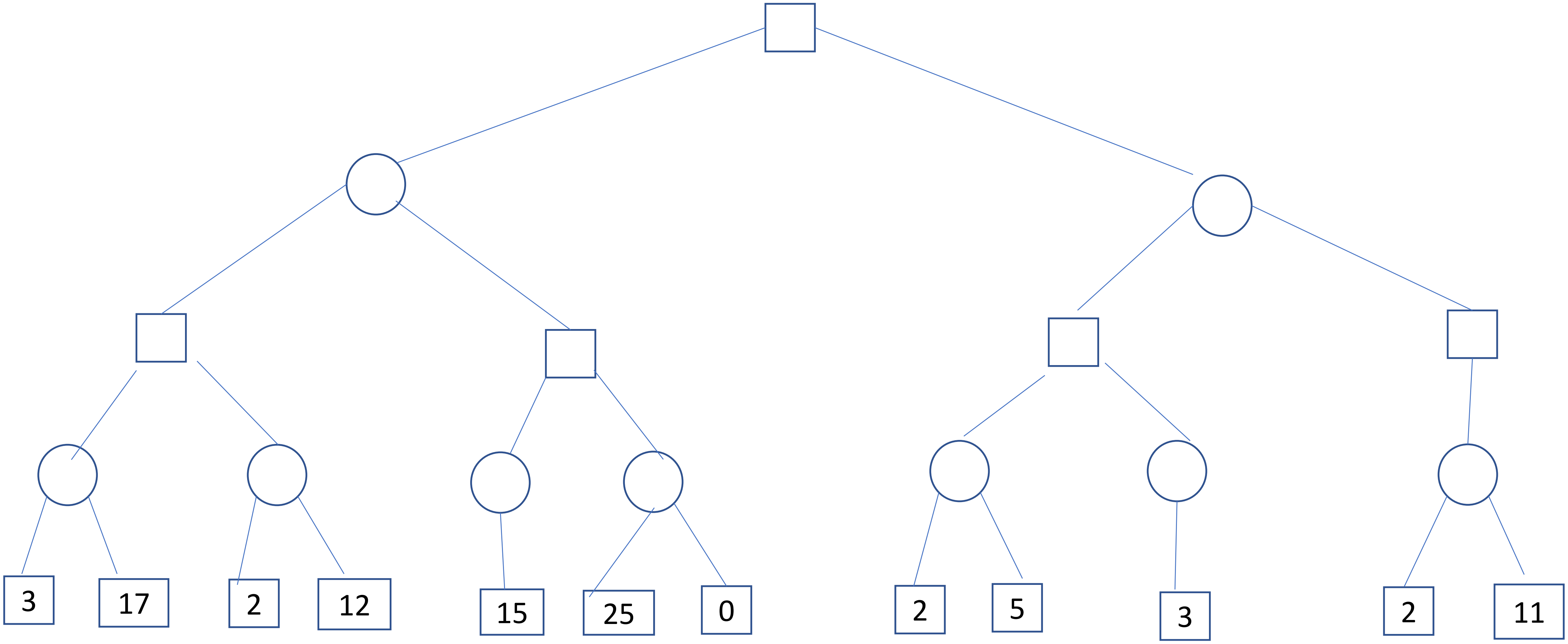


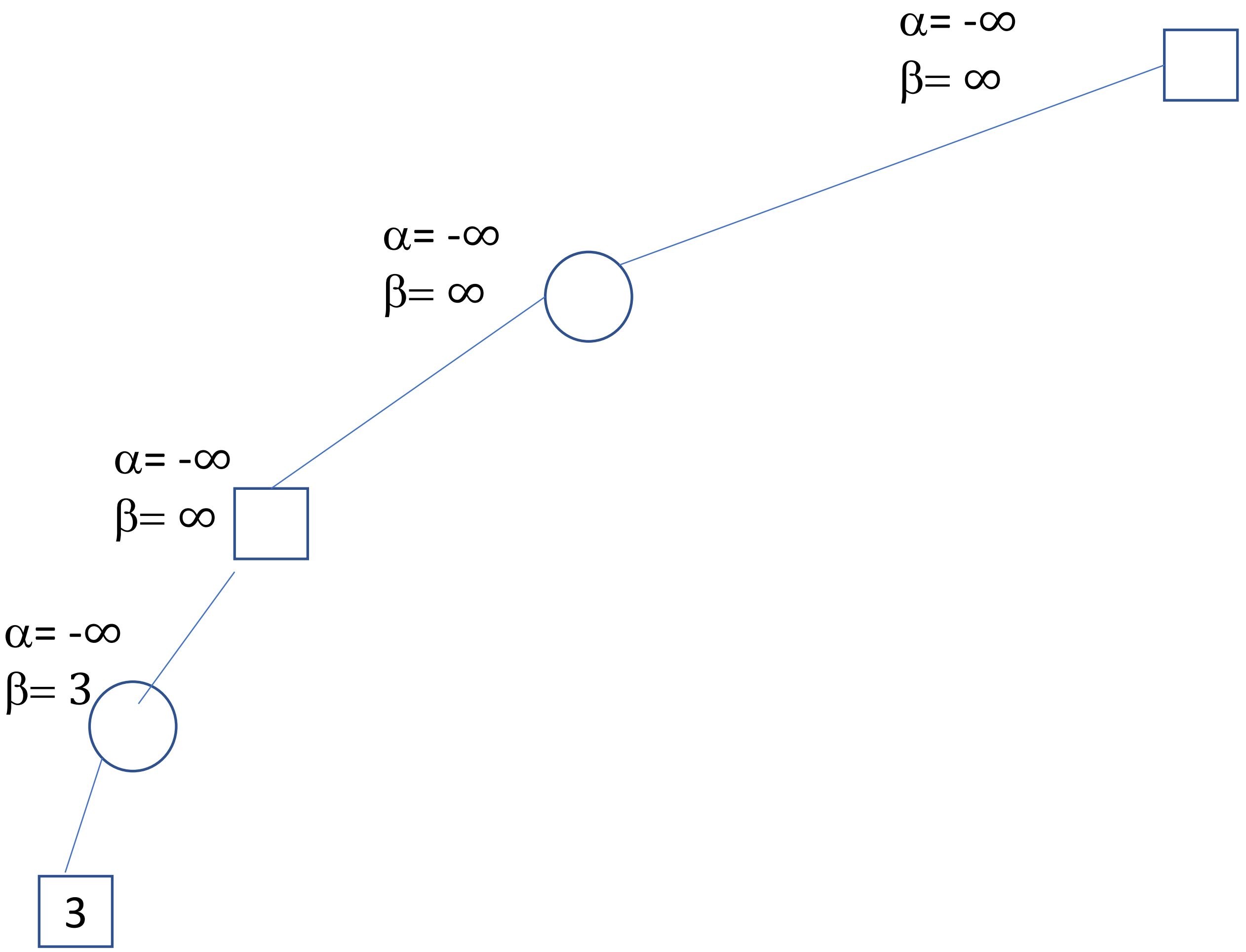
CSP3 : Answer to B)



Problem 7. Alpha-Beta pruning

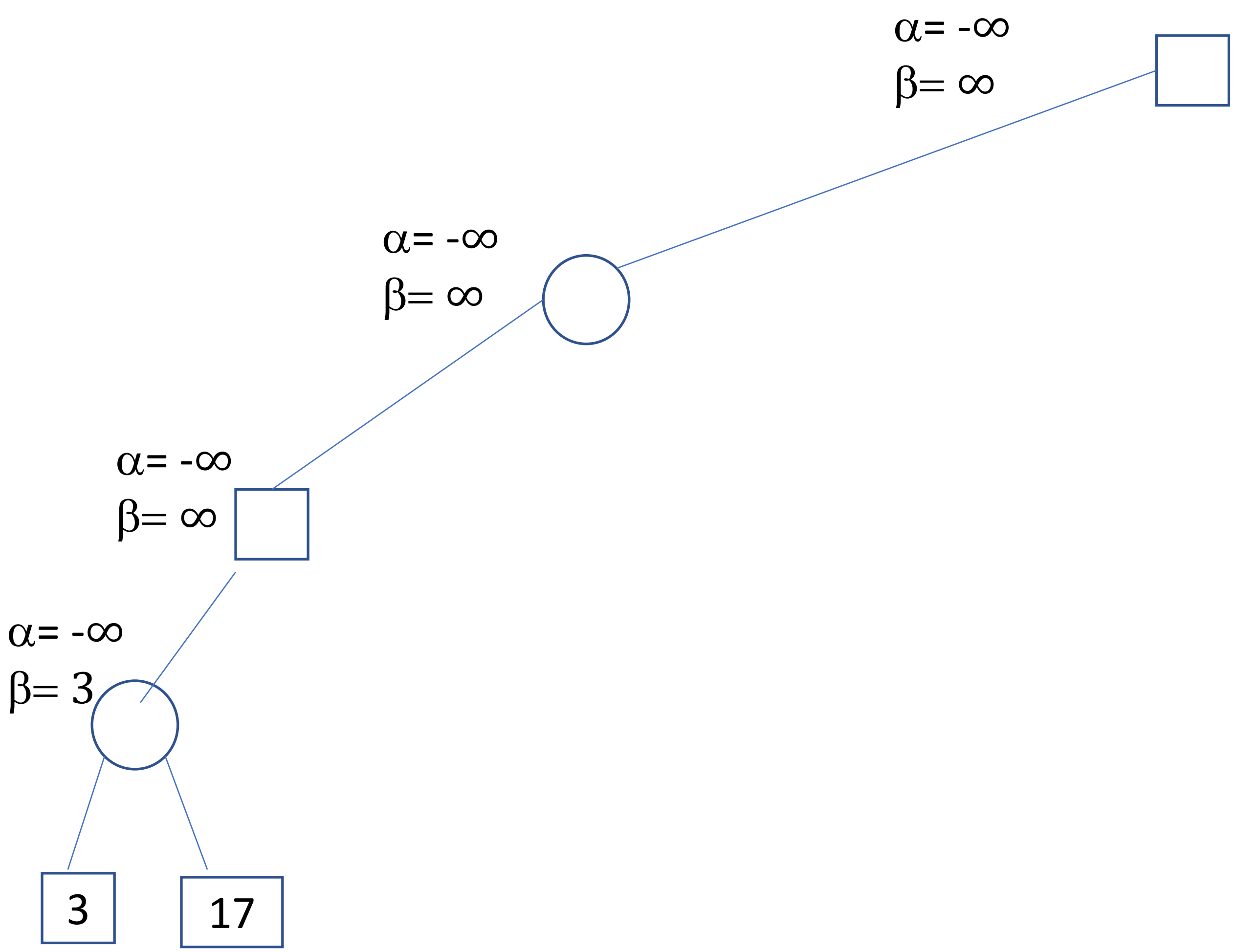
Find the nodes (values) that are pruned by alpha-Beta-pruning alg in the following tree.





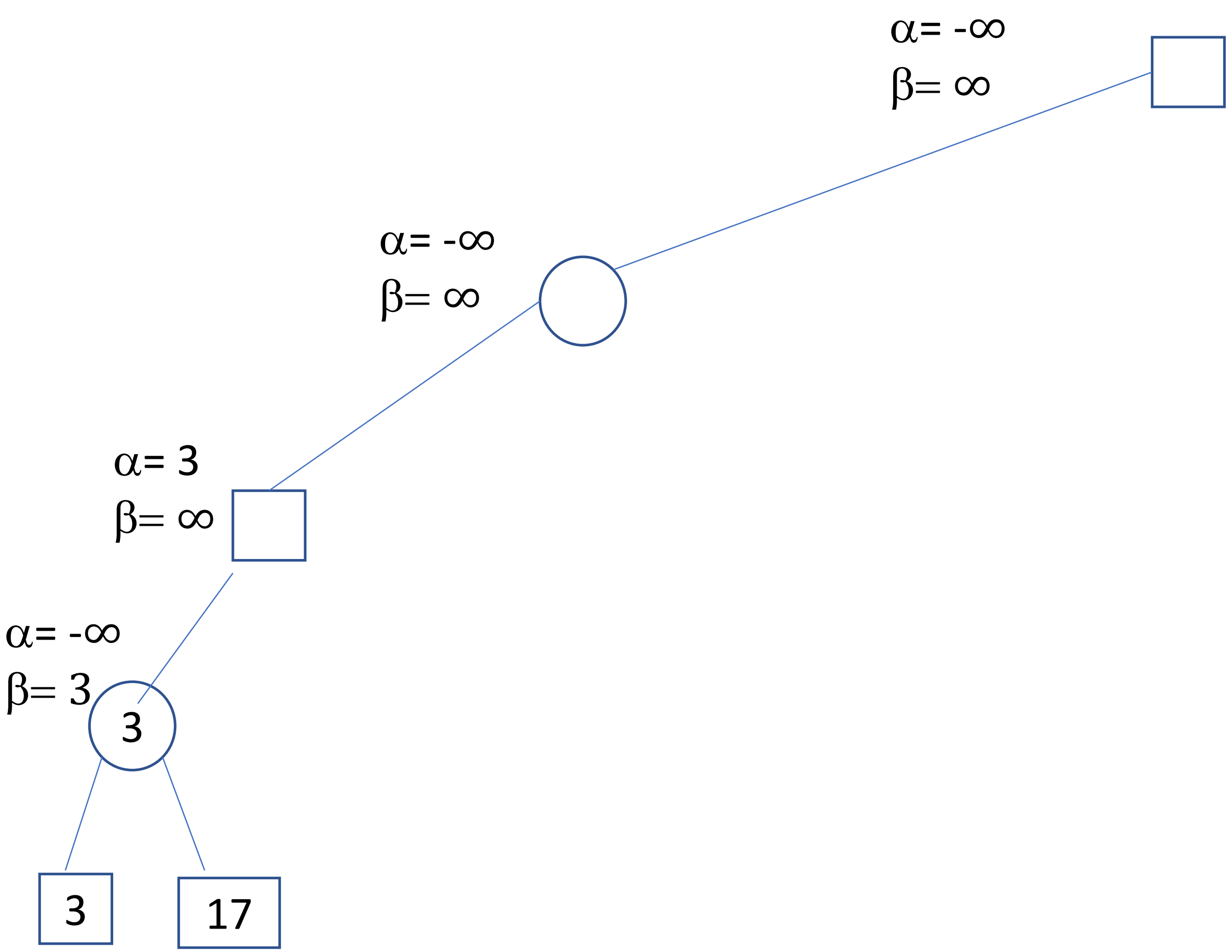
MIN node:
- Prunes if $v \leq \alpha$
- Updates β

MAX node:
- Prunes if $v \geq \beta$
- Updates α



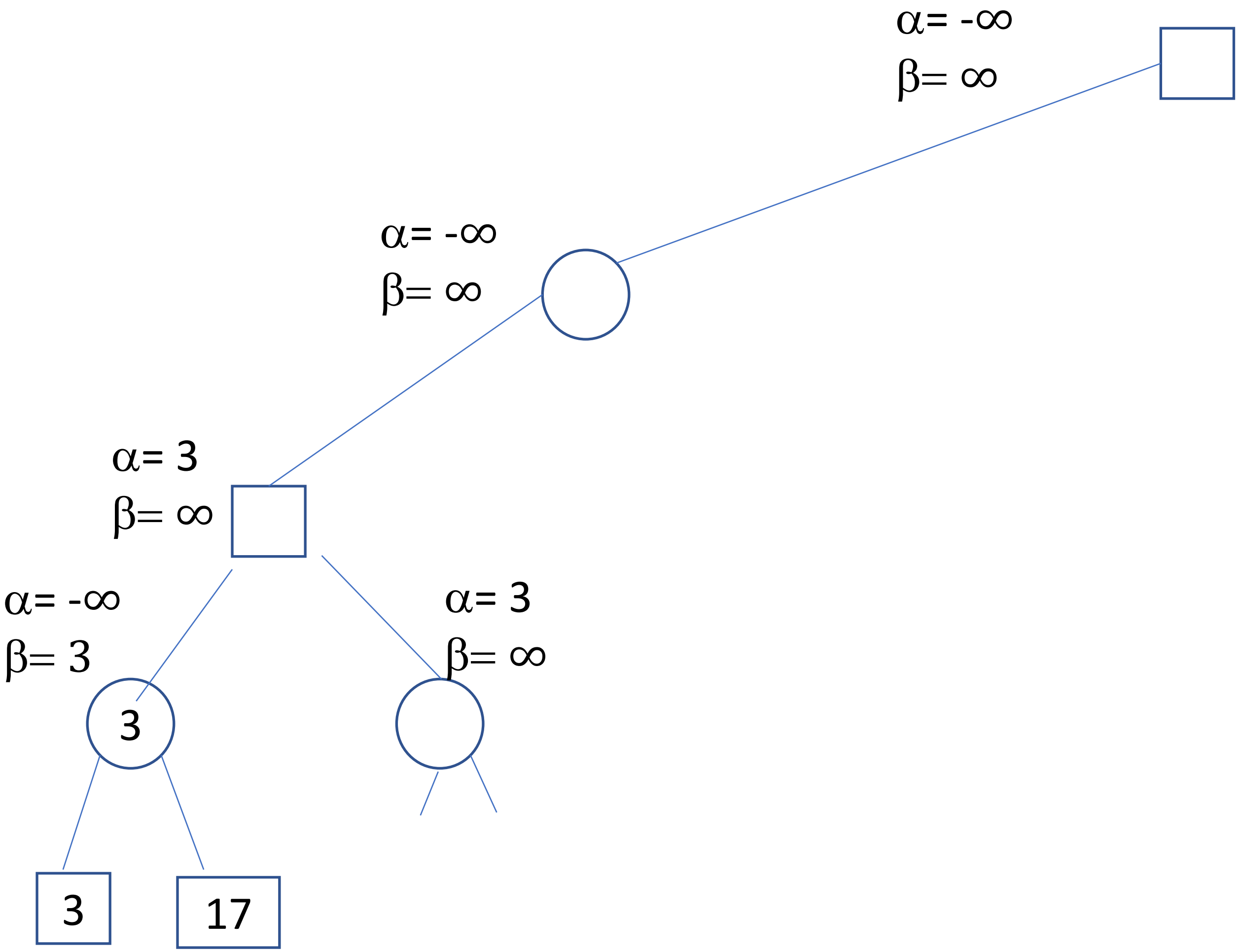
MIN node:
- Prunes if $v \leq \alpha$
- Updates β

MAX node:
- Prunes if $v \geq \beta$
- Updates α



MIN node:
- Prunes if $v \leq \alpha$
- Updates β

MAX node:
- Prunes if $v \geq \beta$
- Updates α



MIN node:
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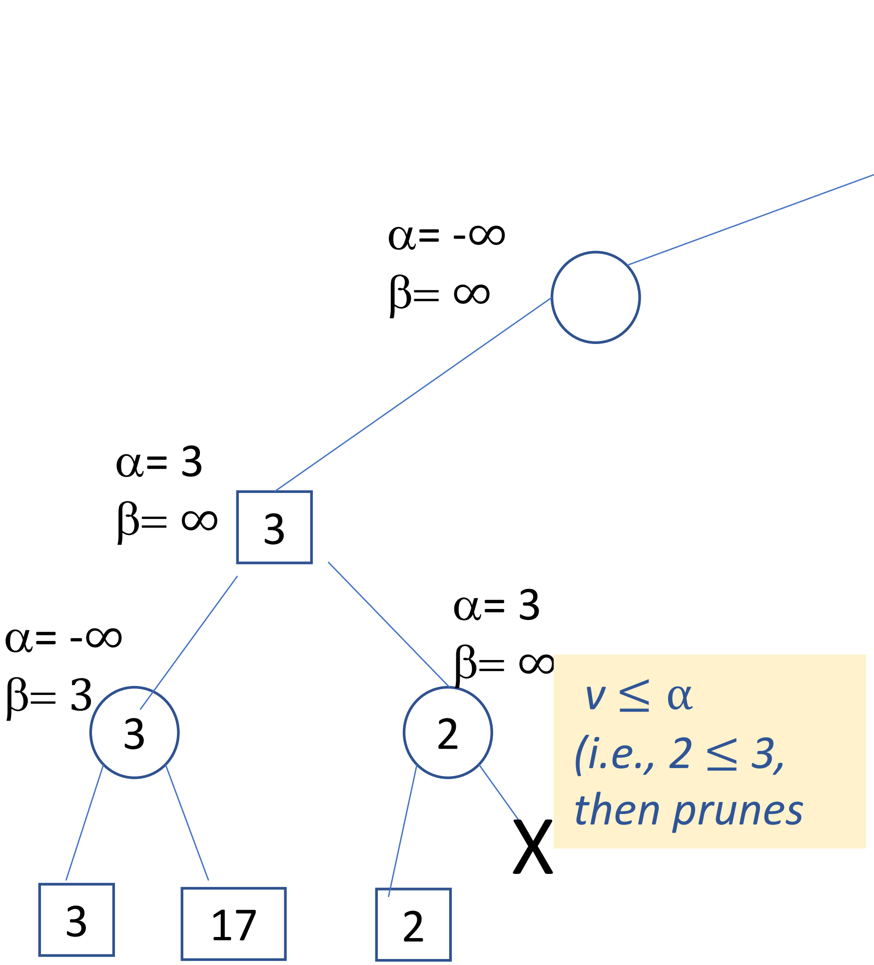
- Updates β

MAX node:
- Prunes if $v \geq \beta$
- Updates α



MIN node:
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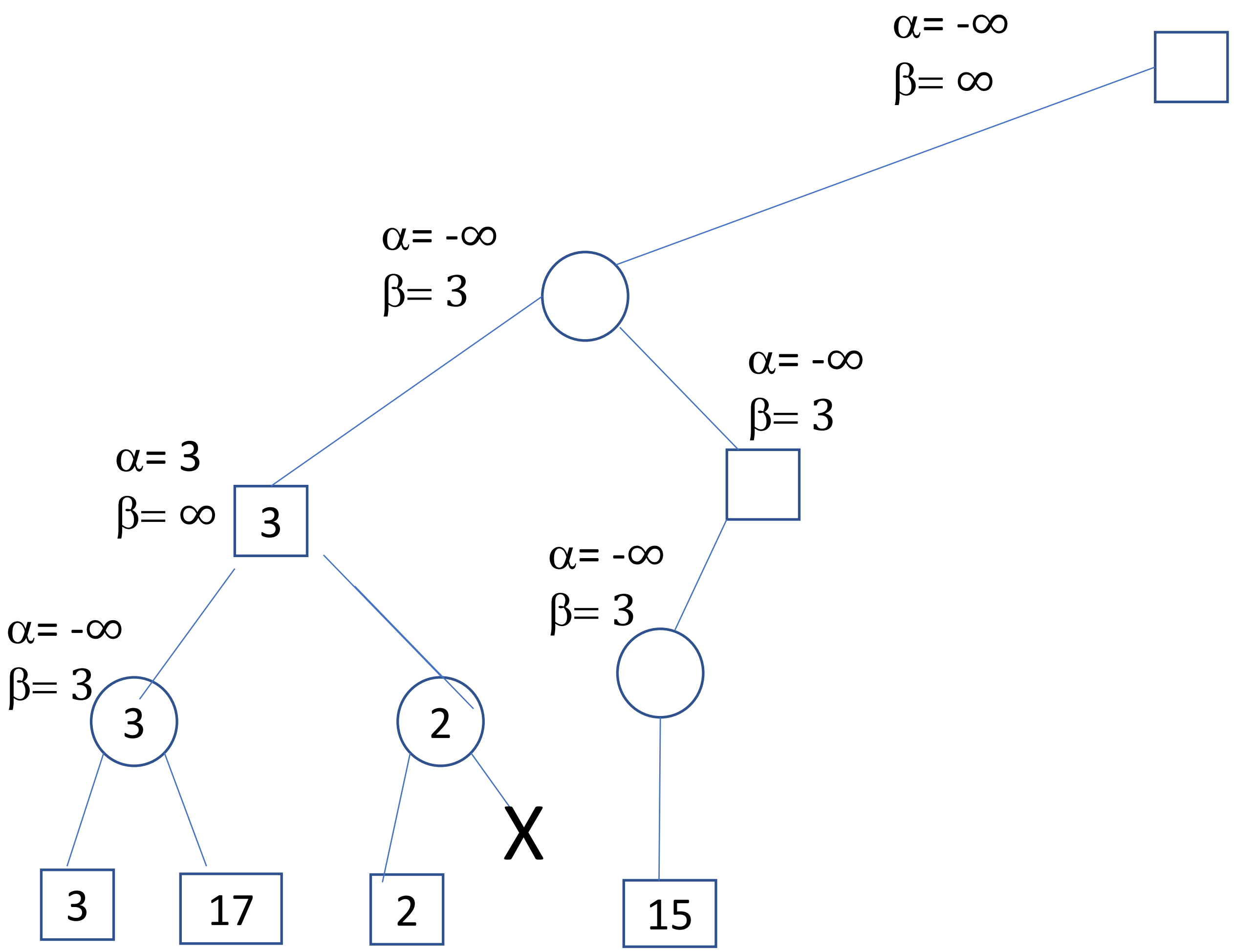
- Updates β

MAX node:
- Prunes if $v \geq \beta$
- Updates α



MIN node:
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- Updates β

MAX node:
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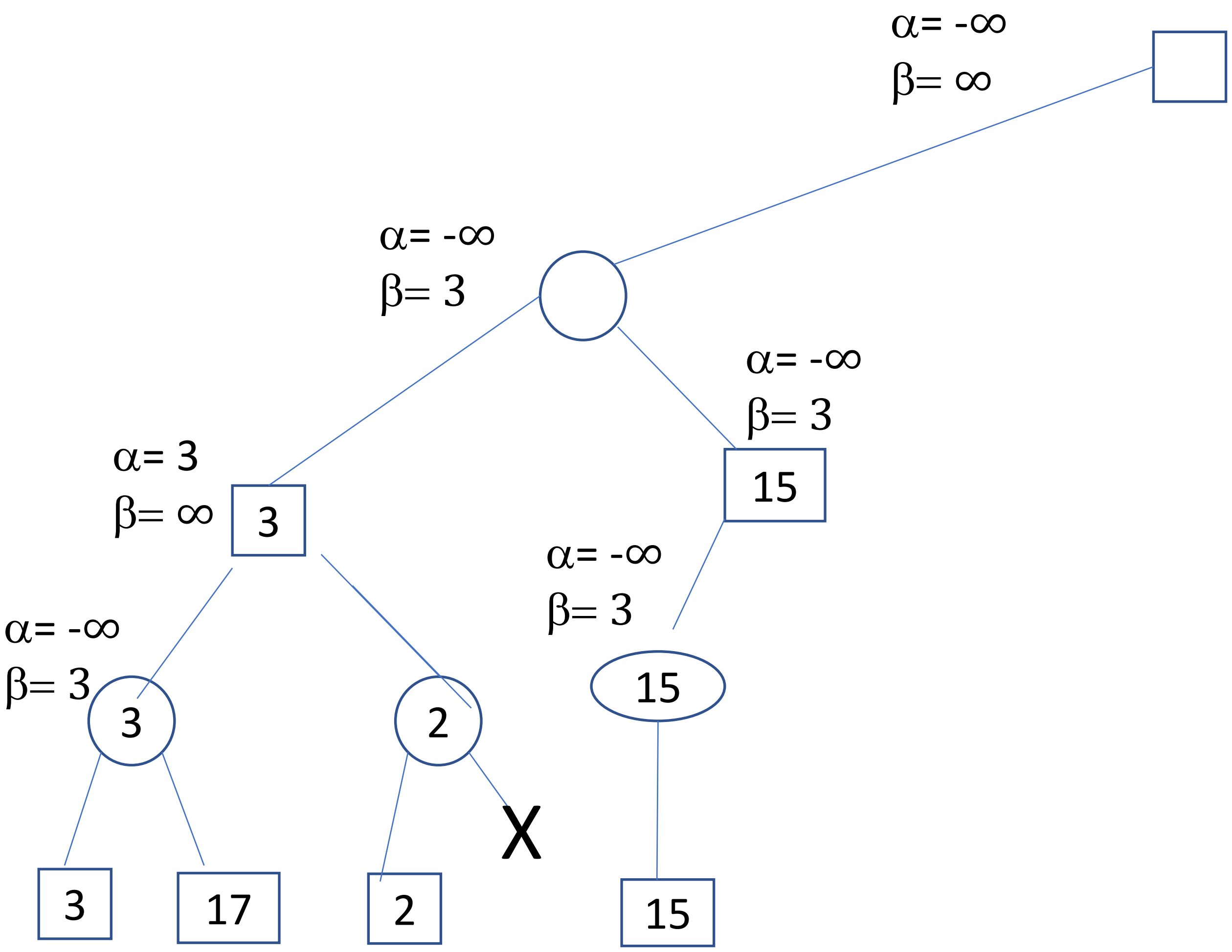
- Updates α



MIN node:
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MAX node:
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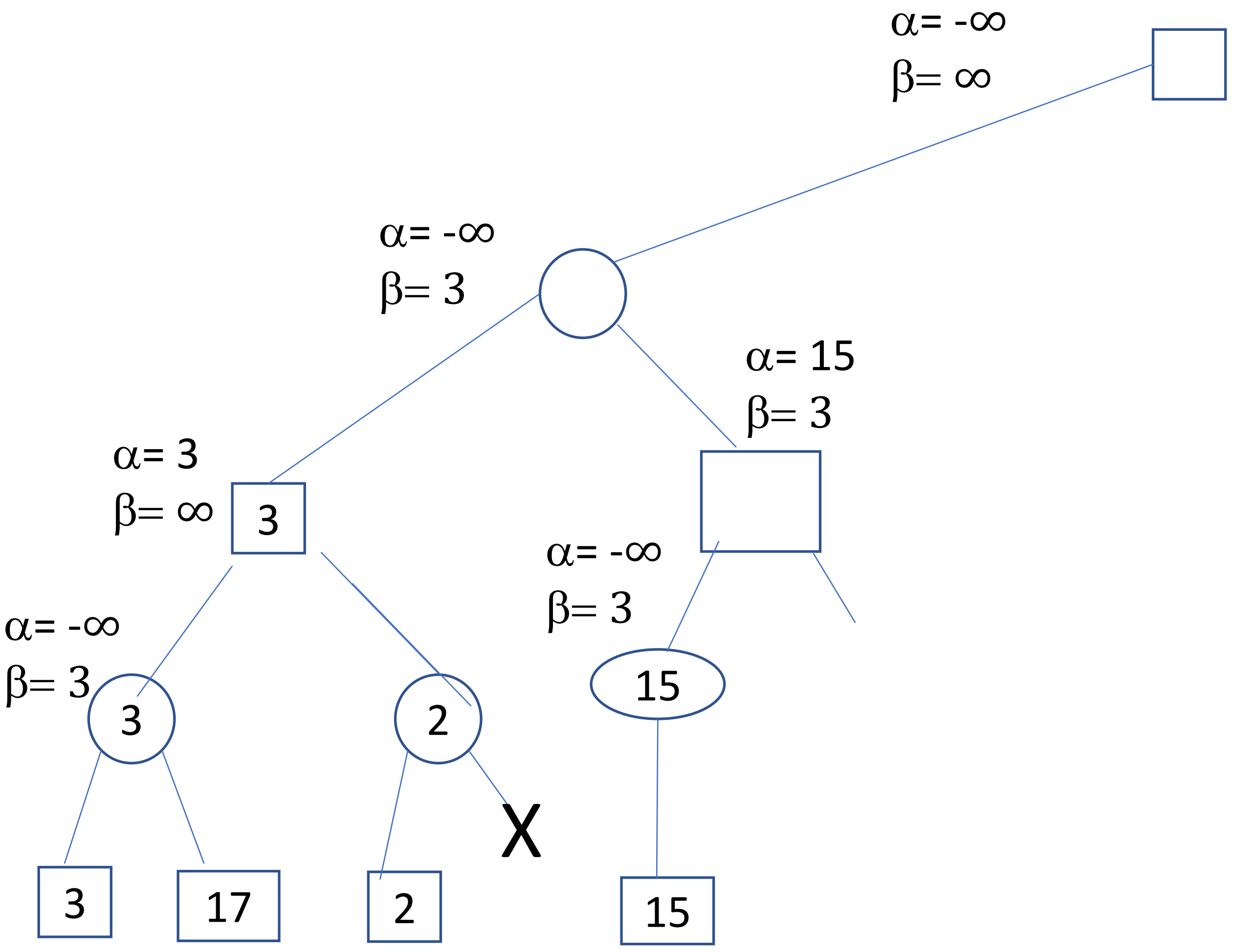
- Updates α



MIN node:
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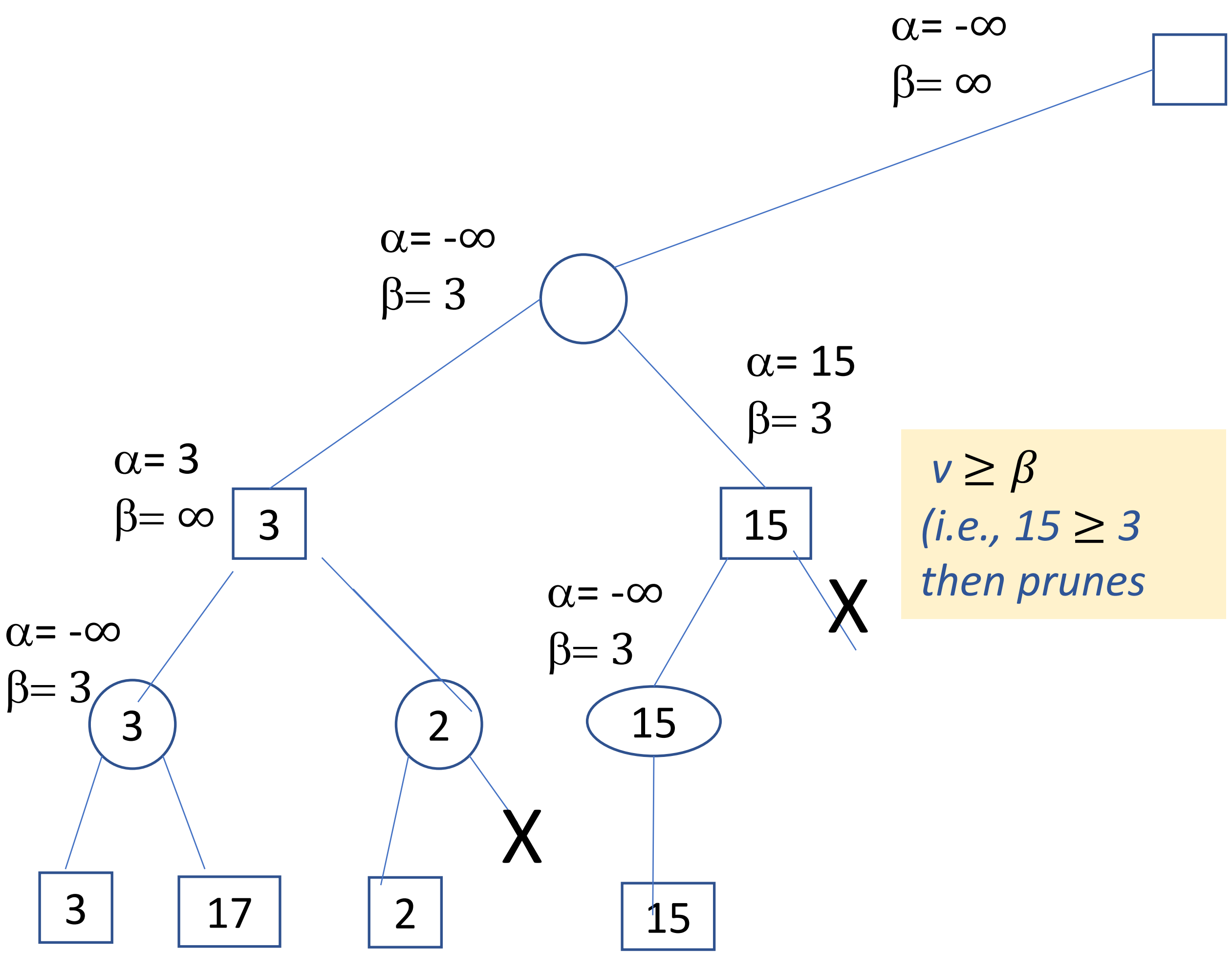
- Updates β

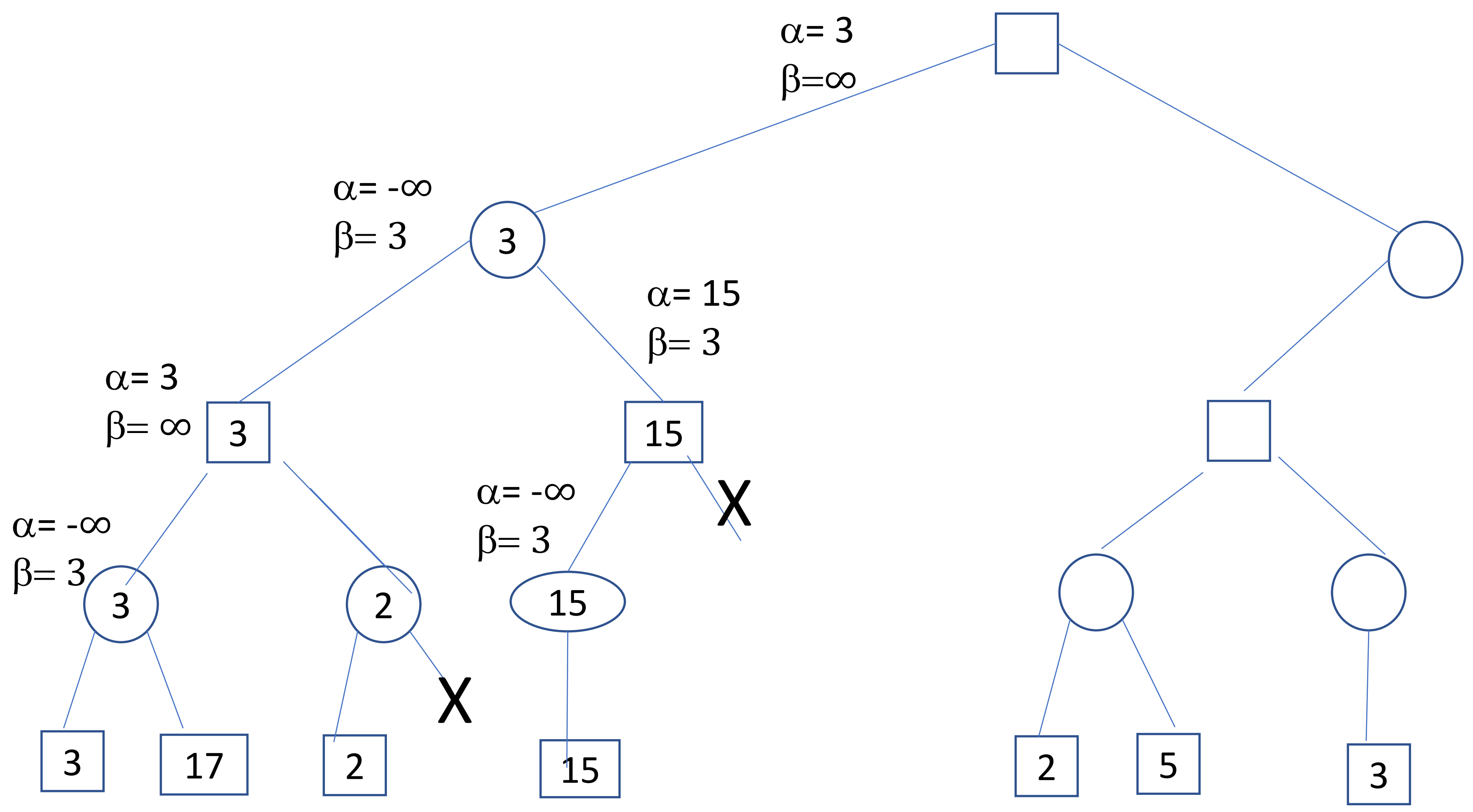
MAX node:
- Prunes if $v \geq \beta$
- Updates α



$v \geq \beta$
(i.e., 15 ≥ 3
then prunes

MIN node:
- Prunes if $v \leq \alpha$
- Updates β

MAX node:
- Prunes if $v \geq \beta$
- Updates α



MIN node:
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MAX node:
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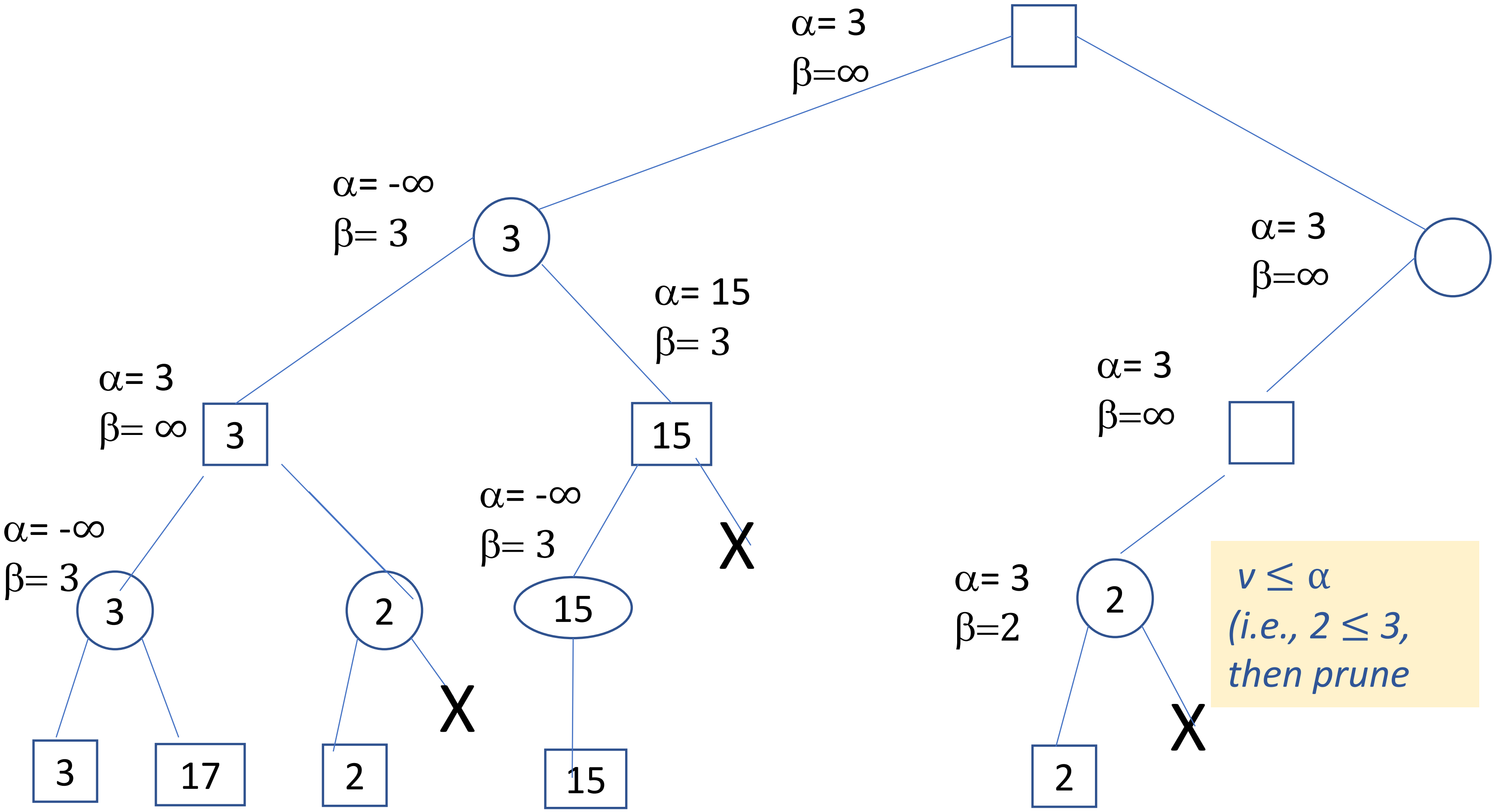

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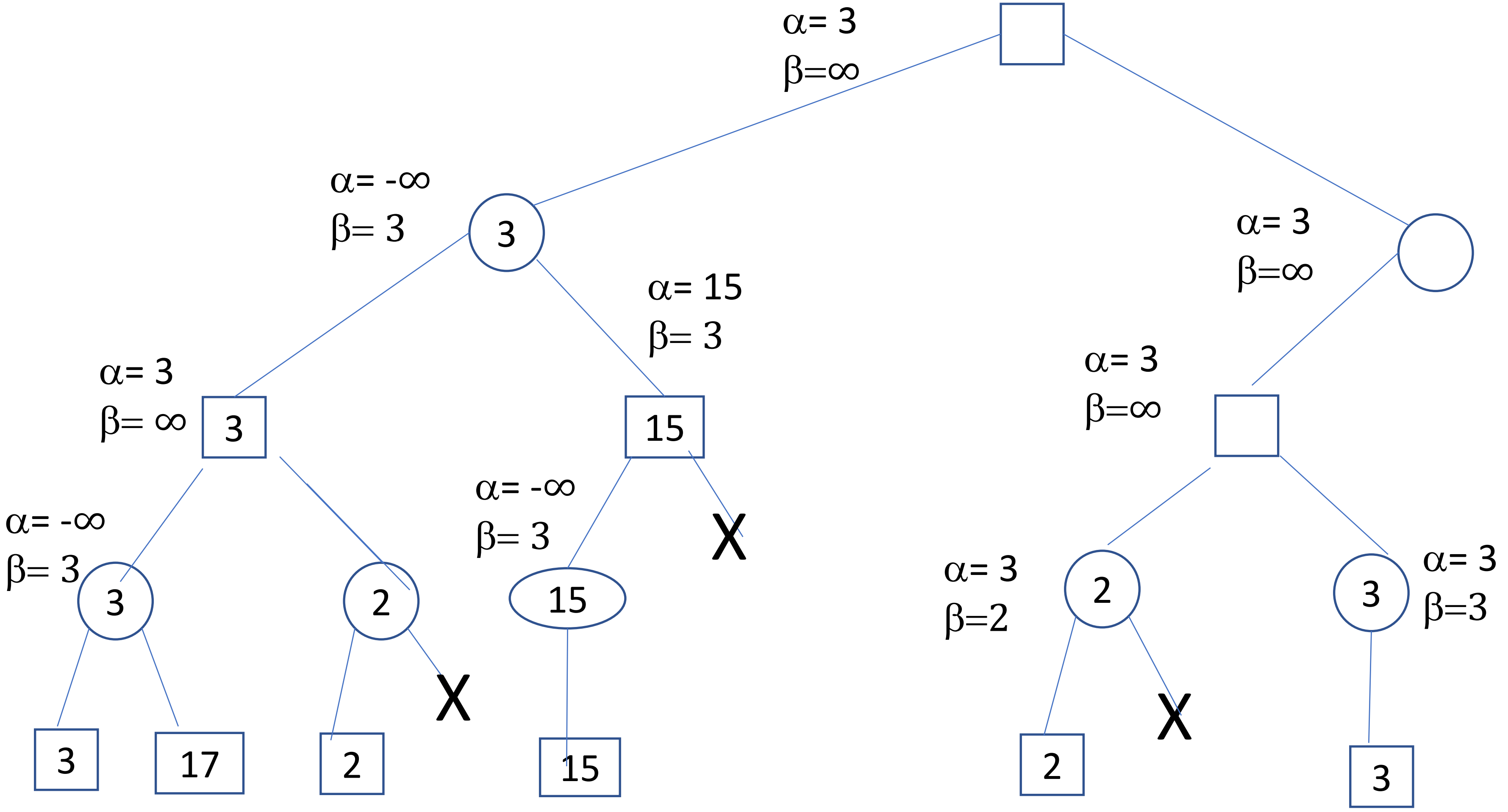

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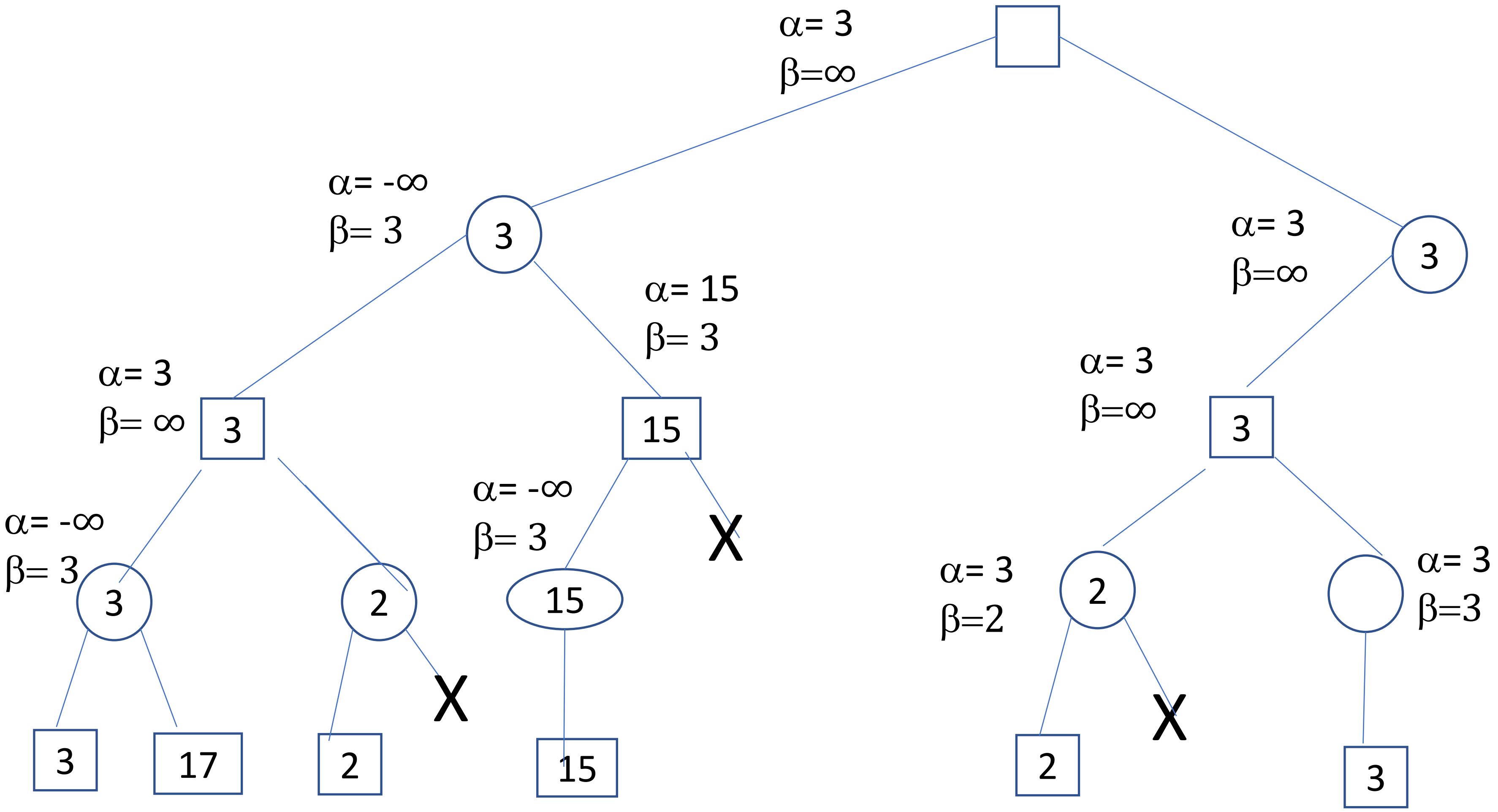

MIN node:
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MAX node:
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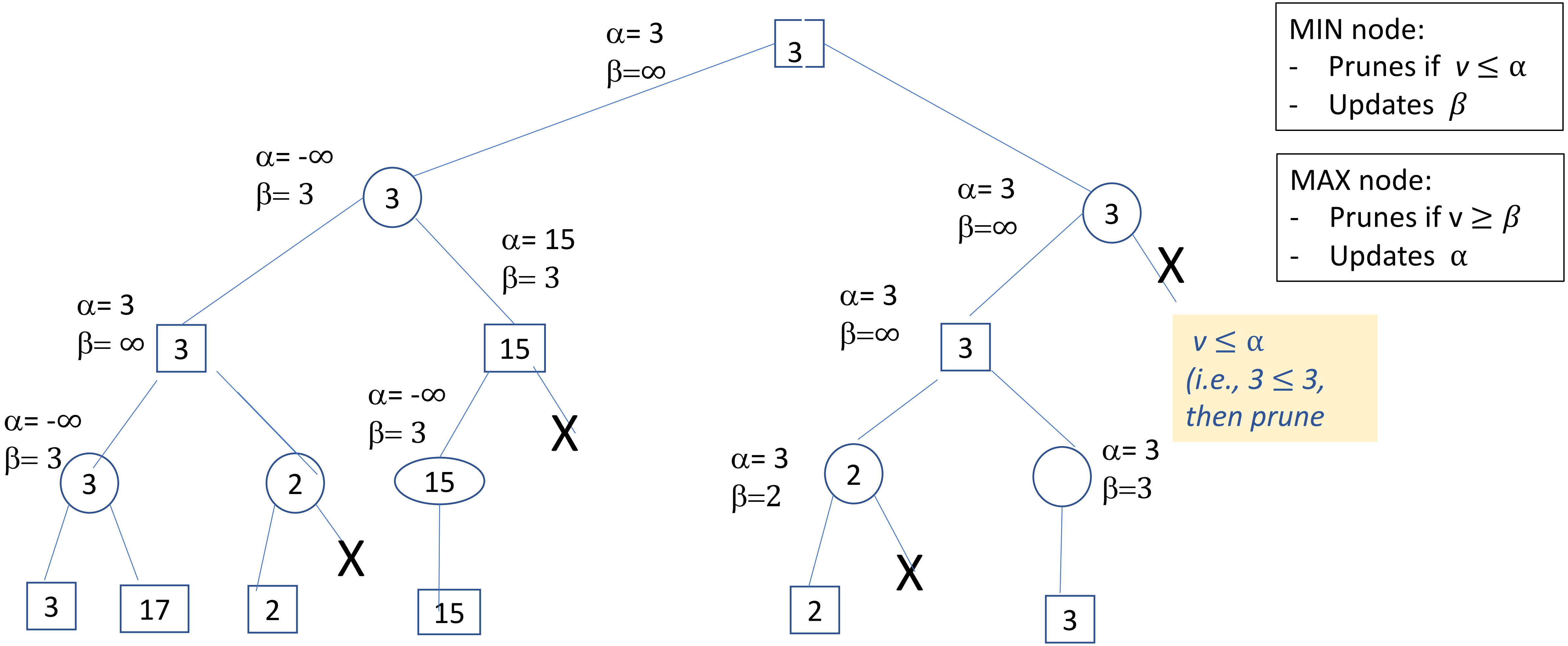
- Updates α


MIN node:
- Prunes if $v \leq \alpha$
- Updates β
MAX node:
- Prunes if v $\geq \beta$
- Updates α
α= 3
β=∞
3
α= -∞
β= 3
3
α= 3
β=∞
3
X
$v \leq \alpha$
(i.e., $3 \leq 3$, then prune
α= 15
β= 3
α= 3
β= ∞
3
15
α= 3
β=∞
3
α= -∞
β= 3
X
α= -∞
β= 3
3
2
15
α= 3
β=2
2
α= 3
β=3
X
X
3
17
2
15
2
3

Extra example: Alpha-beta pruning