Mandatory Assignment 1

HÅKON SILSETH

UiT - Norges Arktiske Universitet

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Exercise 1: Particle in an electric field

a)

See the attached python file for the code (task 1.py), the file contains a 2d Boris mover program for 1 particle with the magnetic field set to zero. Another thing to note is that both fields are one dimensional as we only deal with an electric field in the x-axis and a magnetic field in the z-axis. Two separate functions are included in the code, one for a linear field (task 1b) and one for a sinusoidal field (task 1c).

b) Linear electric field

The plot for a linear field $E(x) = -\epsilon x$ looks like this:

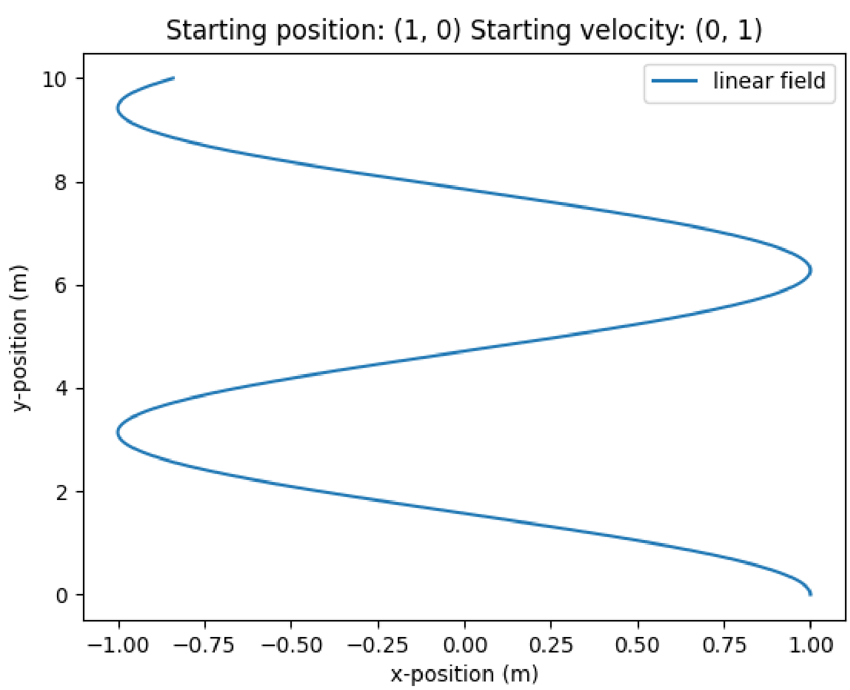


Figure 1: Plot of a positively charged particle in a linear electric field ($E(x) = -x$)

(Just to make everything simpler I set $\epsilon = 1$, but it does not really matter, the only thing that changes with ϵ is the scaling, the plot would look the same.)
The plot looks as expected, the way the electric field is defined it will point towards a line x=0 getting stronger the farther away from x=0 we get. Meaning we will get a particle that oscillates back and forth over the line. The initial velocity here is set as positive in the y-direction so we can see the oscillations clearly, with $v_{y,0} = 0$ we will get a plot that is a straight line so we wont see the oscillations.

c) Sinusoidal electric field

The plot for a particle an a sinusoidal electric field $E(x) = -\epsilon sin(x)$(again with $\epsilon = 1$) looks like this:

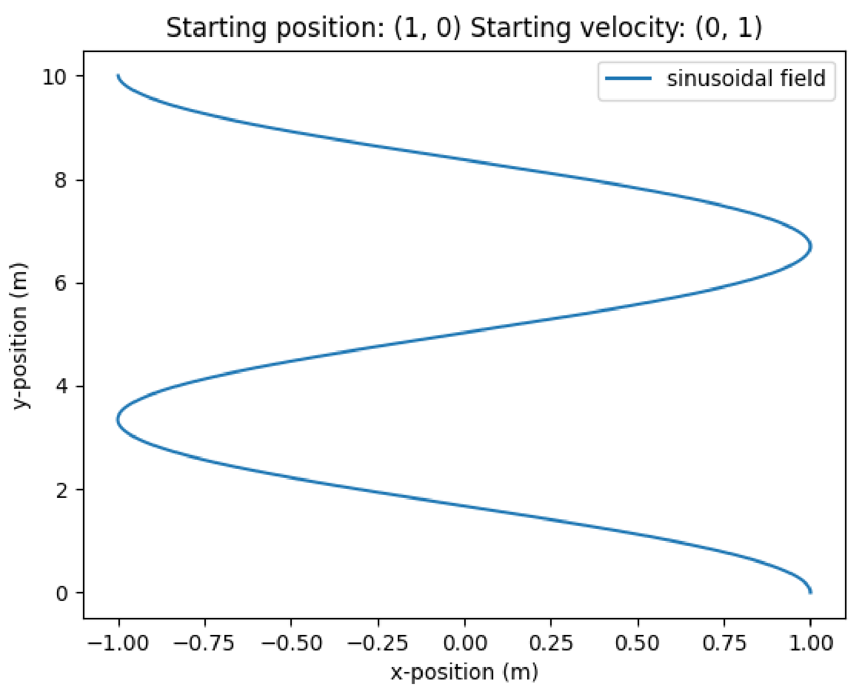
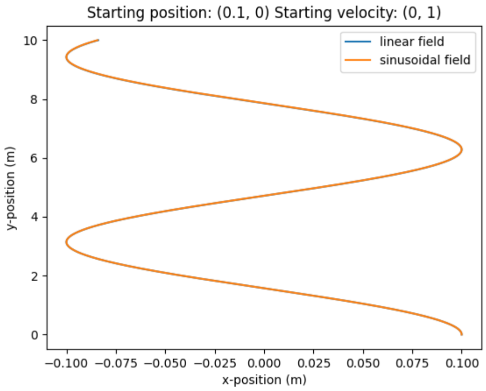
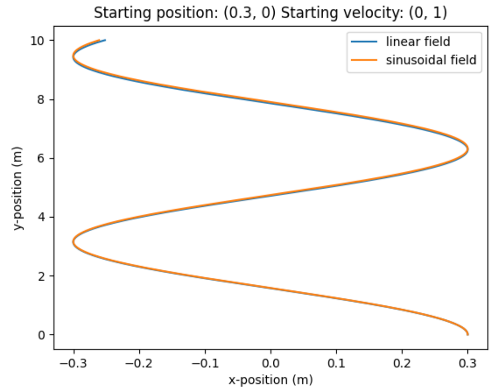


Figure 2: Plot of a positively charged particle in a sinusoidal electric field $E(x) = -sin(x)$

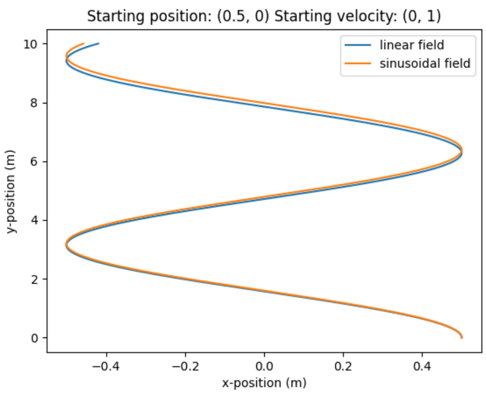
As we can see here it more or less looks the same as it oscillates over x=0, but if we choose a different starting position we get a different plot. This happens because when x is close to zero we get $sin(x) \approx x$ which we can see if we plot both the particle in the linear field and particle in sinusoidal field in the same plot for different values of x:



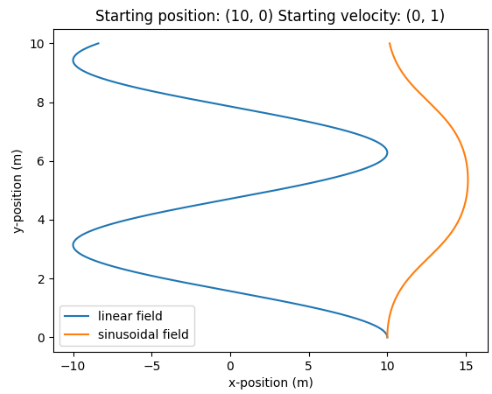
(a) $x_0 = 0.1$



(b) $x_0 = 0.3$



(c) $x_0 = 0.5$



(d) $x_0 = 10$

Figure 3: Comparison of the two electric fields for different starting positions.

As we can see in Figure 3a the two plots are almost identical. In Figure 3b we can see that they are still really close but they differ a little, especially towards the end. In figure 3c we can see that they differ a bit more. And in figure 3d we see that they are completely different. This is completely expected, as mentioned earlier we have for values of x very close to 0 we have $sin(x) \approx x$ and as we can see in this plot this holds true until around x=0.4.

Exercise 2: Many particles in an electric field

a) Extend the particle mover to include N particles

See the attached python file to see the code (task 2.py). For this task I took a bit of a sneaky approach, instead of changing the code the way the task suggested, I made a function containing a for-loop that calls upon the previous function (that creates a particle in an electric field) N times. The way the code is set up in the file is with random x-positions, but changing this to define them manually is very quick and simple.

b) Using E(x) = -Ex, start several particles from rest with different x-positions. Check that the amplitude increases with x-position while the oscillation frequency remains the same.

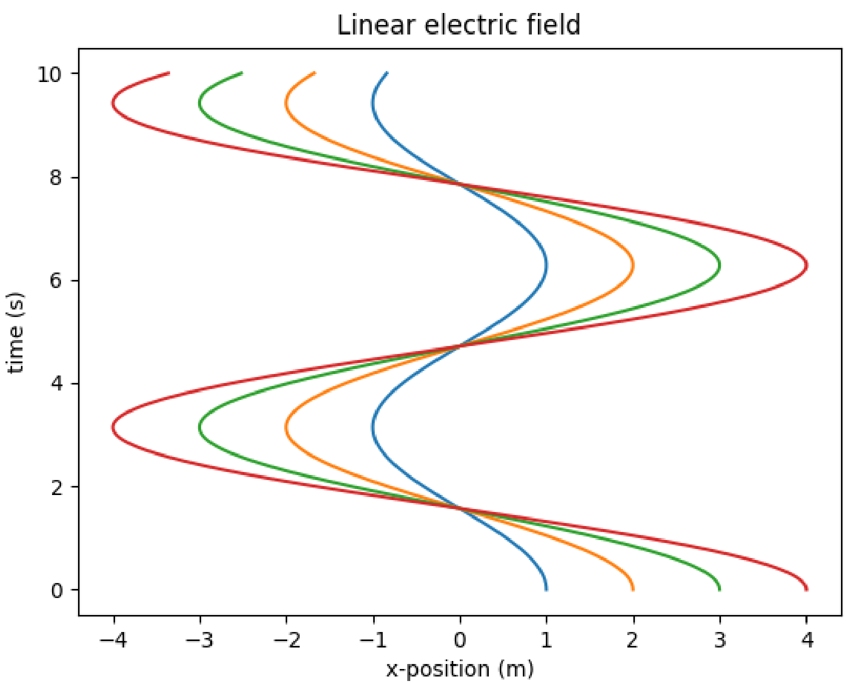


Figure 4: x-position plotted against time for 4 particles with different initial position

In figure 4 we can see 4 particles plotted from rest with increasing starting position. The initial velocity is set to **0** and the x-position is plotted against time so we can see the frequency. As we can clearly see in the plot the amplitude is directly related to the initial position, it is in fact the same as the initial position, which is to be expected as there is no way for the particles to loose energy in this system. And we can easily see that the frequency is the same for all particles no matter the starting position, this is exactly what we expect to happen as it is a known result for most oscillating systems.

c) Using E(x) = Esin(x), start several particles from the trough of the sine function with different velocities. What happens to the particles as the initial velocity increases?

The trough of the sine function is at $x = \frac{\pi}{2}$ so the particles will be released from here with different initial velocities.

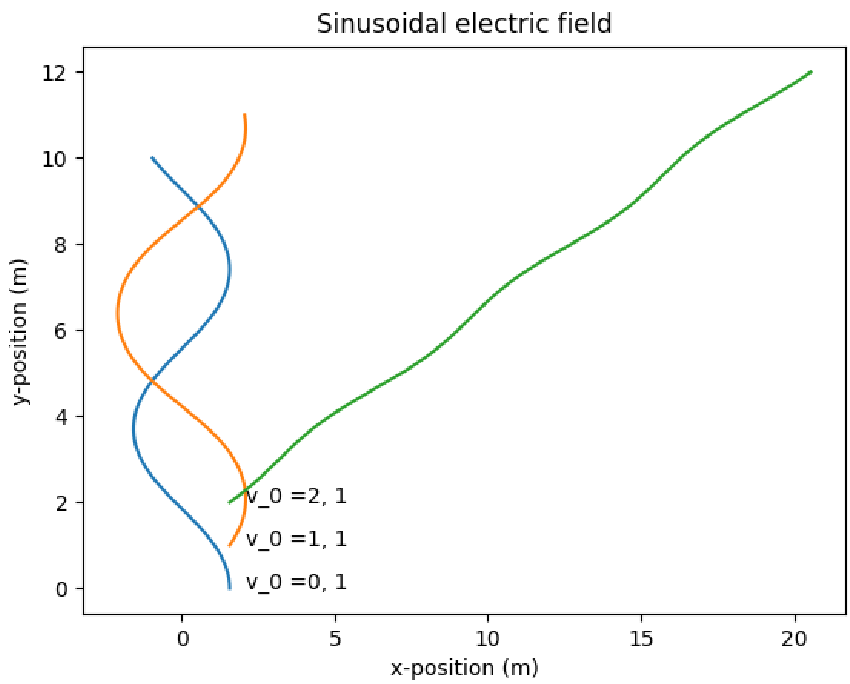


Figure 5: 3 particles released in the trough of the sine function with different initial velocities.

The interesting thing we can see in this plot is that one particle escapes the electric field and flies off into infinity. We can actually find the minimal initial velocity a particle needs to escape the electric field. For a particle to escape the electric field it needs to have enough kinetic energy to reach $x = \pi$. Beyond $x = \pi$ the field is periodic so the kinetic energy will vary, but never reach 0 so it will continue on. The kinetic energy needed to escape the field will be the same as the amount of work needed to move the particle from $\frac{\pi}{2}$ to π:

$$W = -\int_{\frac{\pi}{2}}^{\pi} \mathbf{E}(x) \cdot d\mathbf{l}$$

We can then set this equal to the kinetic energy and solve for v:

$$W = -\int_{\frac{\pi}{2}}^{\pi} \mathbf{E}(x) \cdot d\mathbf{l} = E_k$$
$$\frac{1}{2}mv^2 = -(cos(\pi) - cos(\pi/2))$$
$$v = \sqrt{1 * 2/m}$$
$$v = \sqrt{2}$$

So to escape the electric field a particle needs an initial velocity (in the x-direction) of at least $v = \sqrt{2}$. We can double check this by plotting particles with initial velocities close to $\sqrt{2}$

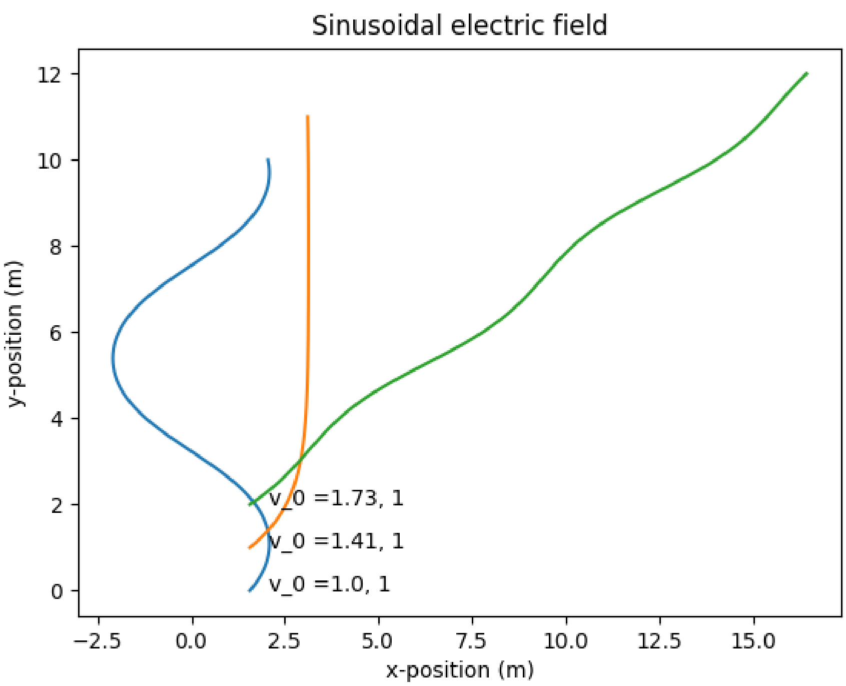


Figure 6: 3 particles with initial velocities of 1, $\sqrt{2}$ and $\sqrt{3}$ showing the minimal velocity needed to escape the electric field

In theory a particle with initial velocity $v_{x,0} = \sqrt{2}$ would not escape the electric field but rather barely reach $x = \pi$ and remain there. (all the particles in the plots here have an initial velocity in the y-direction of 1, this makes no difference in the behaviour of the particles, but helps show the motion of the particles better.)

Exercise 3: Electric field given on a grid

For this task I attempted to implement the Electric field on the grid. But unfortunately I was unsuccessful in my attempt. The code (Task 3.py) has some errors and the plots therefore come out looking odd. See Figure 7

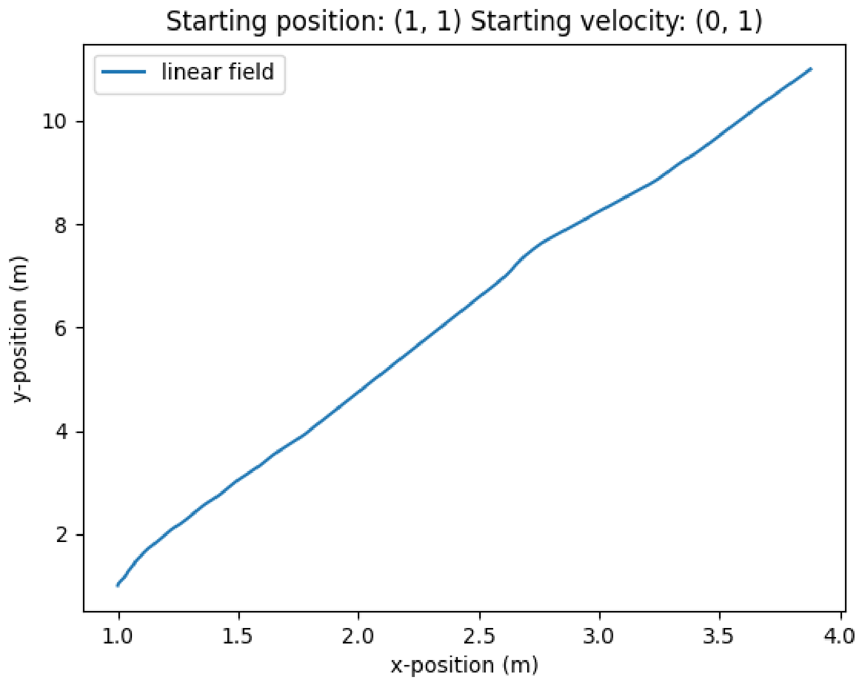


Figure 7

Exercise 4: Particle motion in electromagnetic fields

a)

The code used here is the exact same as in Task 2, except the electric field is set to 0 and the magnetic field is constant.

b) Using E = 0, B = B_0, start several particles from the same position with different initial velocities (some positive, some negative). Make sure only the gyro-radius and gyro-centre changes while the angular frequency remains the same. Check that energy is conserved for each particle.

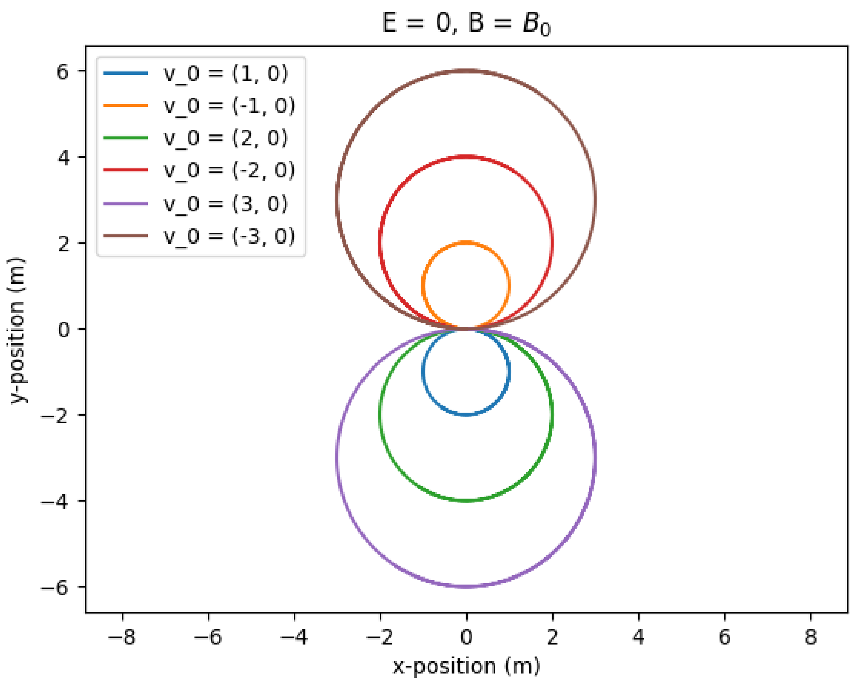


Figure 8: 6 particles released from the origin with different initial velocities

As we can clearly see in figure 8 all the particles move in a circular orbit, as is expected, the radius of the orbit also increases with the magnitude of the initial velocity. And the sign of the initial velocity decides which way it orbits, clockwise or anti-clockwise.
The angular frequency of each particle will be equal as it is not related to the initial velocity. This can be proven, but this plot illustrates it very well.
Here we can see the motion of the particles after pi seconds, as we can see each particle has travelled 180° around their gyro-centre, meaning they have the same angular frequency.

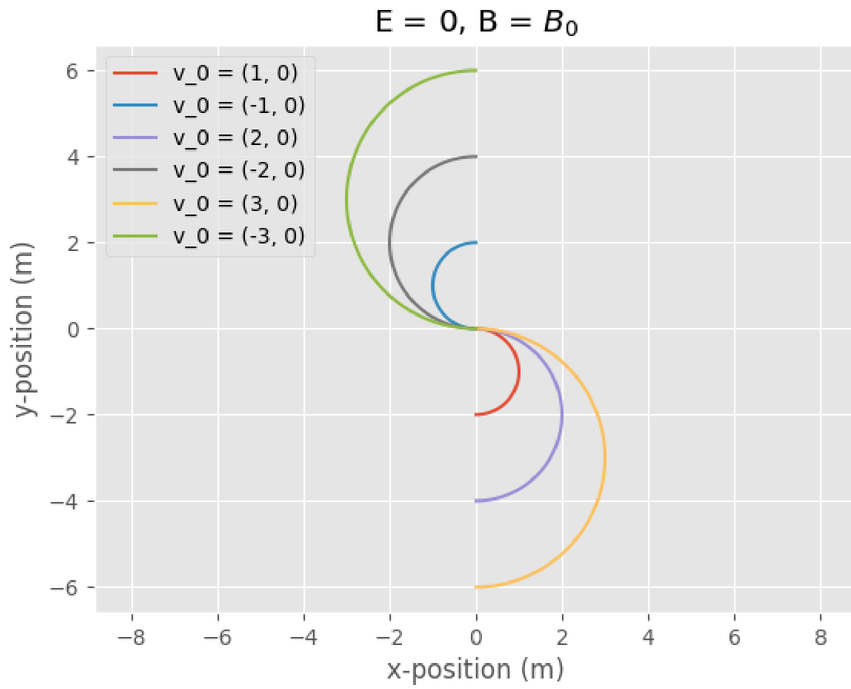


Figure 9: 6 particles released from the origin with different initial velocities after π seconds.

By definition the work done on the particles by the magnetic field is zero, this is because the direction of the magnetic force is perpendicular to the velocity. Since the work done on the particles are zero their energy remains constant. This can also be seen in the plots as they remain moving in the same orbit indefinitely with the same energy, the potential in the field also remains the same as the field is constant. Thus the energy is conserved.

c) Using constant electric and magnetic fields, compare your results to the analytic results for the E×B-drift calculated in the exercise sessions. Remember to check a few different combinations of initial positions and velocities.

The E cross B drift can be calculated using:

$$x(t) = x_{c0} + r_c \sin(\Omega_c t + \theta) + \frac{E_y}{B} t$$
$$y(t) = y_{c0} + r_c \cos(\Omega_c t + \theta) - \frac{E_x}{B} t$$
$$z(t) = z_0 + v_{z0} t + \frac{qE_z}{2m} t^2$$

z(t) will always be 0 (in our case where $z_0 = 0, v_{z,0} = 0, E_z = 0$) so we only have to consider the x, and y directions.
E_y is also zero so the equations of motion become:

$$x(t) = x_{c0} + r_c \sin(\Omega_c t + \theta)$$
$$y(t) = y_{c0} + r_c \cos(\Omega_c t + \theta) - \frac{E_x}{B} t$$

Where: $r_c = \frac{m}{qB}\sqrt{v_{x0}^2 + (v_{y0} + E/B)^2}$, $\Omega_c =$ Using the initial conditions $x(0) = x_0$ and $y(0) = y_0$ we can rearrange the equations to get expressions for x_{c0} and y_{c0}:

$$x_{c0} = x_0 - r_c sin(\theta)$$
$$y_{c0} = y_0 - r_c cos(\theta)$$

And we have $\theta = \frac{v_{y,0} - E/B}{v_{x,0}}$
Plotting this analytic solution in the same plot as the numerical solution yields:

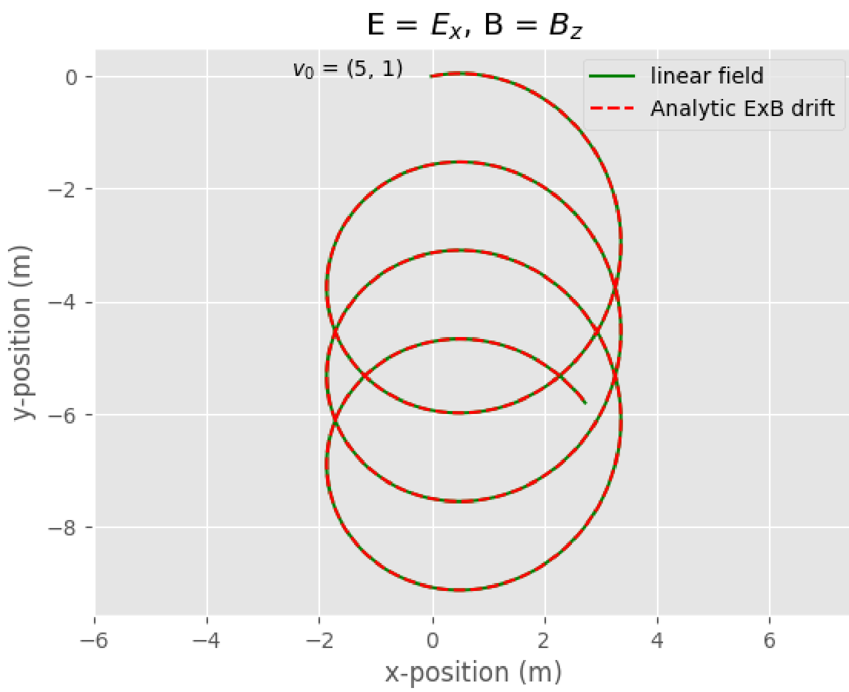


Figure 10: Numerical and analytic solution plotted in the same plot

As we can see here the plots line up perfectly, meaning that both simulations are most likely correct. (There could of course be an error in both programs which make both wrong, but still agree with each other, but this is very unlikely). To make sure they agree for all values of initial velocity and initial position we could plot both simulations for many different initial parameters:

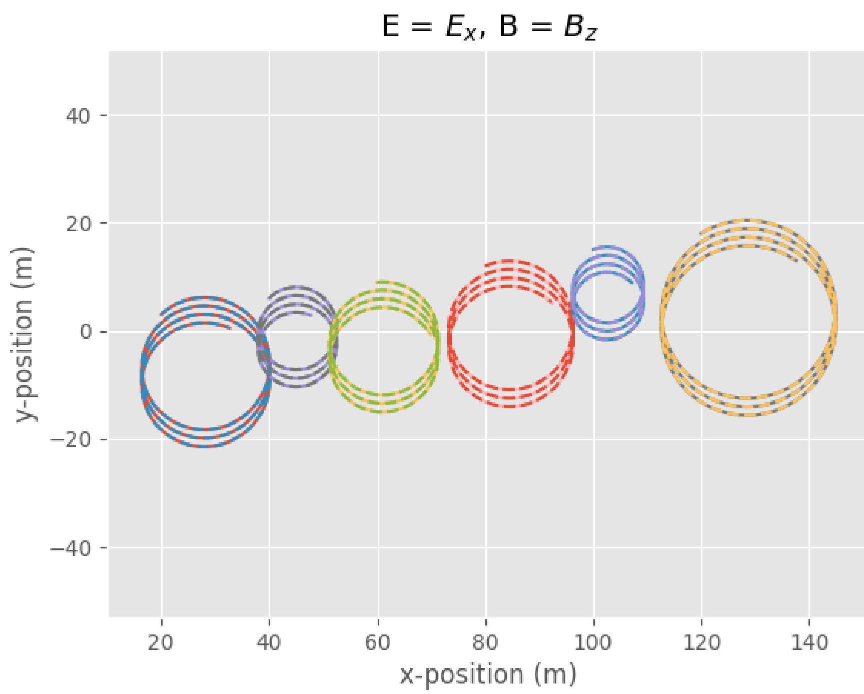


Figure 11: analytic and numerical simulation plotted for 6 different sets of initial values.

Figure 11 shows that the numerical and analytic ExB drift agree for many different initial conditions. (One can see that they agree as the full line and dashed line always overlap).
If we zoom in close we can see that they dont actually overlap, but they are really close:
In figure 12 we see the disagreement between the two solutions. And as we can see it is very very small. (Take note of the scaling on the axes). This is more or less to be expected, the two solutions will never be 100% be exactly the same, but for it is more than close enough.

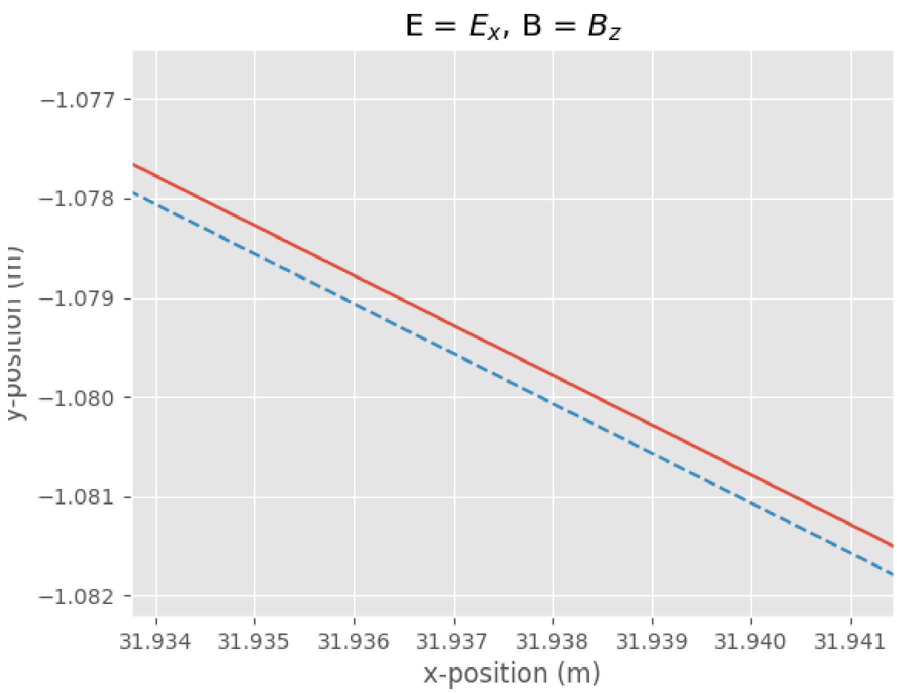


Figure 12: Close up of the analytic and numerical solution.