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Milestone I:
Computing the Hubble parameter and conformal time.

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1. INTRODUCTION

Milestone I is the first part of a project consisting of four Milestones. The main goal of the project is to calculate the CMB power spectrum. In Milestone I, we will study the background evolution of the universe. To do this we will compute the Hubble parameter from early times to today as to describe the expansion history of the universe. We will also study the evolution of the density parameters for baryons, cold dark matter, photons and cosmological constant as well as the conformal time.

2. METHOD

When calculating the conformal time we start with the differential equation

$$\frac{d\eta}{dt} = \frac{c}{a}, \tag{1}$$

where η is the conformal time, c is the speed of light, a is the scale factor and t is time. By performing a change of variables by multiplying with $\frac{dx}{dx}$ we get

$$\frac{d\eta}{dx}\frac{dx}{dt} = \frac{c}{a}, \tag{2}$$

where we introduce the coordinate change $x = log(a)$. Using that $\frac{dx}{dt} = H$ (Callin (2006)), where H is the hubble paramater, we get an expression for η

$$\eta = \int_0^x \frac{c}{\mathcal{H}(x)} dx, \tag{3}$$

where $\mathcal{H}(x) = e^x H(x)$ in our new coordinate system. The next step is to derive an expression for $H(x)$. By neglecting density parameters for neutrinos and curvature, we have and expression for the Hubble parameter given by

$$H(x) = H_0\sqrt{(\Omega_{b,0} + \Omega_{CDM,0})e^{-3x} + \Omega_{r,0}e^{-4x} + \Omega_{\Lambda,0}} \tag{4}$$

(Winther (2020)), where $\Omega_{b,0}$ is the density parameter for baryons, $\Omega_{CDM,0}$ is for cold dark matter, $\Omega_{r,0}$ is for radiation and $\Omega_{\Lambda,0}$ is for cosmological constant. The sub index 0 represents the value for the parameters today. The values for the density parameters today are given as follows

$$\Omega_{CDM,0} = 0.25 \tag{5}$$
$$\Omega_{b,0} = 0.05 \tag{6}$$
$$\Omega_{\Lambda,0} = 0.7 \tag{7}$$
$$\tag{8}$$

Respectively. The parameter for radiation today is given as

$$\Omega_{r,0} = 2\frac{\pi^2(k_b T_{cmb}^4)}{30\hbar^3 c^5}\frac{8\pi G}{3H_0^2}, \tag{9}$$

where G is the gravitational constant, H_0 is the Hubble parameter today and $T_{cmb} = 2.7255$K is the temperature of the CMB measured today (Winther (2020)).

We also want to study how the density parameters evolve with time. For the density parameters, we have that

$$\Omega_i = \frac{\rho_i}{\rho_c} \tag{10}$$
$$= \frac{8\pi G}{3H(x)^2}\rho_i \tag{11}$$
$$= \frac{8\pi G}{3H(x)^2}\rho_{i,0}e^{-3x(1+w_i)}, \tag{12}$$

where $\rho_{i,0}$ is the critical density today and w is the equation of state parameter defined as $w_i \equiv \frac{P_i}{\rho_i}$, where P_i is pressure and ρ_i is density for a given contribution to energy component i (Spør om dette). By using the relation $\rho_{i,0} = \rho_{c,0}\Omega_{i,0}$, we get the expression for a given density parameter as

$$\Omega_i(x) = \frac{H_0^2}{H(x)^2}e^{-3x(1+w_i)}\Omega_{i,0}. \tag{13}$$

Using that $w_{CDM} = 0$, $w_b = 0$, $w_\Lambda = -1$ and $w_r = 1/3$ (Winther (2020)) we can get calculate the density parameters for a given x allowing us to evaluate the Hubble parameter given by equation (4) for a given x. This allows us to finally calculate the integral for η given by equation (3).

3. RESULTS

REFERENCES

Callin, P. 2006, How to calculate the CMB spectrum, , , arXiv:astro-ph/0606683

Winther, H. A. 2020, Overview: Milestone I, , , http://folk.uio.no/hansw/AST5220/notes/milestone1.html, Visited: 10.02.2020