Draft version February 19, 2020
Typeset using LaTeX default style in AASTeX62

Milestone I:
Computing the Hubble parameter and conformal time.

Håkon Tansem

1. INTRODUCTION

Milestone I is the first part of a project consisting of four Milestones. The main goal of the project is to calculate the CMB power spectrum. In Milestone I, we will study the background evolution of the universe. To do this we will compute the Hubble parameter from early times to today as to describe the expansion history of the universe. We will also study the evolution of the density parameters for baryons, cold dark matter, photons and cosmological constant as well as the conformal time.

2. METHOD

When calculating the conformal time we start with the differential equation

$$\frac{d\eta}{dt} = \frac{c}{a}, \tag{1}$$

where η is the conformal time, c is the speed of light, a is the scale factor and t is time. By performing a change of variables by multiplying with $\frac{dx}{dx}$ we get

$$\frac{d\eta}{dx}\frac{dx}{dt} = \frac{c}{a}, \tag{2}$$

where we introduce the coordinate change $x = log(a)$. Using that $\frac{dx}{dt} = H$ (Callin (2006)), where H is the hubble paramater, we get an ODE we can solve for η as

$$\frac{d\eta}{dx} = \frac{c}{\mathcal{H}(x)}, \tag{3}$$

where $\mathcal{H}(x) = e^x H(x)$ is the scaled Hubble parameter given in our new coordinate system. By neglecting density parameters for neutrinos and curvature, we have and expression for the Hubble parameter given by

$$H(x) = H_0\sqrt{(\Omega_{b,0} + \Omega_{CDM,0})e^{-3x} + \Omega_{r,0}e^{-4x} + \Omega_{\Lambda,0}}, \text{ (Winther (2020))} \tag{4}$$

where H_0 is the Hubble parameter today, $\Omega_{b,0}$ is the density parameter for baryons, $\Omega_{CDM,0}$ is for cold dark matter, $\Omega_{r,0}$ is for radiation and $\Omega_{\Lambda,0}$ is for cosmological constant. The sub index 0 for the density parameters represents the value for the parameters today. The values for the density parameters today are given as follows

$$\Omega_{CDM,0} = 0.25 \tag{5}$$

$$\Omega_{b,0} = 0.05 \tag{6}$$

$$\Omega_{\Lambda,0} = 0.7. \tag{7}$$

The parameter for radiation today is given as

$$\Omega_{r,0} = 2\frac{\pi^2(k_b T_{cmb}^4)}{30\hbar^3 c^5}\frac{8\pi G}{3H_0^2}, \tag{8}$$

where G is the gravitational constant, k_b is the Boltzmann constant and $T_{cmb} = 2.7255\text{K}$ is the temperature of the CMB measured today (Winther (2020)).

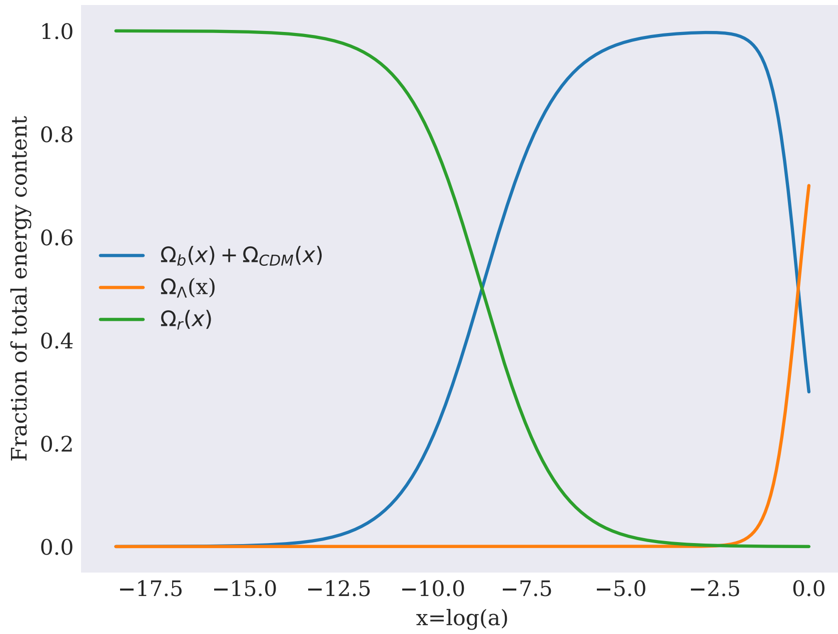

Figure 1. Figure showing the evolution of the density parameters Ω for CDM, radiation and cosmological constant as a function of $x = log(a)$.

We also want to study how the density parameters evolve with time. For the density parameters, we have that

$$\Omega_i = \frac{\rho_i}{\rho_c} \tag{9}$$

$$= \frac{8\pi G}{3H(x)^2}\rho_i \tag{10}$$

$$= \frac{8\pi G}{3H(x)^2}\rho_{i,0}e^{-3x(1+w_i)}, \tag{11}$$

where $\rho_{i,0}$ is the density today and w is the equation of state parameter defined as $w_i \equiv \frac{P_i}{\rho_i}$, where P_i is pressure and ρ_i is density for a given contribution to energy component i. By using the relation $\rho_{i,0} = \rho_{c,0}\Omega_{i,0}$, we get the expression for a given density parameter as

$$\Omega_i(x) = \frac{H_0^2}{H(x)^2}e^{-3x(1+w_i)}\Omega_{i,0}. \tag{12}$$

Using that $w_{CDM} = 0$, $w_b = 0$, $w_\Lambda = -1$ and $w_r = 1/3$ (Winther (2020)), we can now calculate the evolution of the density parameters for a given x. We also want to study the evolution of the Hubble parameter as a function of redshift z. Using the relation $a(t) = \frac{1}{1+z}$, we get, by substituting $a = e^x$, that the redshift as a function of our new coordinate system is given as

$$z = e^{-x} - 1. \tag{13}$$

3. RESULTS/DISCUSSION

Figure (1) shows the evolution of the density parameters Ω as a function of $x = log(a)$. Here we can see that in early times the universe was heavily dominated by radiation. At around $x = -8.0$ matter-radiation equality occurs and at $x \approx -5.0$ the universe is matter dominated. At around $x = 0.0$ the universe starts to become dominated by a cosmological constant.

Figure (2) shows the evolution of the conformal time, the Hubble parameter and the scaled Hubble parameter as functions of x as well as the Hubble parameter as a function of redshift z. For early times we see that the Hubble

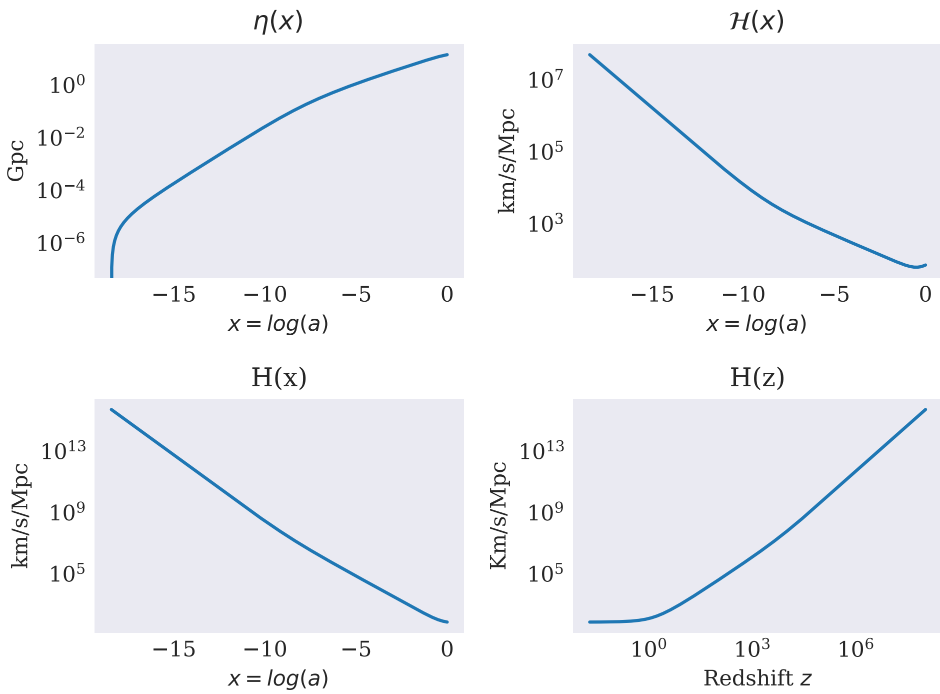

Figure 2. Starting clockwise from the top left, the figure shows the evolution of the conformal time η as a function of x, the scaled Hubble parameter as a function of x, the hubble paramater as a function of redshift z and the Hubble parameter as a function of x.

parameter is very large making the universe expand fast. As time goes on the Hubble parameter decreases. By looking at the plot for the Hubble parameter as a function of redshift z, we can see that by the time the CMB is released, at $z \approx 1100$, the Hubble parameter has decreased by a factor of around $10^7 \sim 10^8$. This shows that the early universe expanded very fast before it slowed down during the onset of matter domination. From the plots for $H(x)$ and $\mathcal{H}(x)$, we can also see a slight change in slope which coincides with the same value for x as matter-radiation equality in Figure 1. One can also see, while studying the plot for the conformal time η, that it is almost inversely proportional to the expansion rate. When the expansion rate is at its highest, the horizon grows the fastest. From the scaled Hubble parameter, which can be interpreted as $\dot{a}$, we can see that the expansion is decelerating throughout the radiation dominated and matter dominated eras. The expansion rate seems to accelerate again when the universe enters cosmological constant dominated era at around $x = 0$, which is indicated by $\mathcal{H}(x)$ gaining a positive slope, but the equations were not solved going into the future.

4. BENCHMARK

The time elapsed for calculating eta turned out to be 0.000474332 sec measured using the C++ chrono library.

REFERENCES

Callin, P. 2006, How to calculate the CMB spectrum, , ,

arXiv:astro-ph/0606683

Winther, H. A. 2020, Overview: Milestone I, Universitet i Oslo, last accessed: 18.02.2020. http://folk.uio.no/hansw/AST5220/notes/milestone1.html