An Outline of the Siumatgwoon
初論兆物觀

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Definitions

今也，南蠻鴃舌之人，非先王之道。
—— 《孟子．滕文公上》

A Foray into the Axioms of the Siumatgwoon

A Siumatgwoon, or a Metaphysic, is a set S, paired with the binary relations composition $* : S \times S \to S$ and constitution $| : S \times S \to \{\text{True}, \text{False}\}$, such that the following axioms hold.

- Axiom 1 (Reflexivity): For all $a \in S$, $a|a$.
- Axiom 2 (Totality): For all $a, b \in S$, exactly one of the following holds: $a|b$ or not $a \not| b$
- Axiom 3 (Transitivity): For all $a, b, c \in S$, $a|b$ and $b|c$ implies $a|c$.
- Axiom 4: If $a * b = c$ for some $a, b, c \in S$, then $a|c$ and $b|c$.

Some further clarifications:

- $a \not| b$ iff $a|b = \text{false}$

Examples of Siumatgwoons

The Chinese Characters, 字

The Chinese characters, which inspired this whole mathematical exercise, is clearly a Siumatgwoon. If we exercise the synonym exchange of "Siumatgwoon" with "Metaphysic", this is to say, that the Chinese characters is a Metaphysic. Unfortunately we can't really prove that the Chinese characters are indeed a siumatgwoon, given there's an infinite number of them, and we do not have a generating rule for all Chinese characters. However, the fact that any subset of Chinese characters is a siumatgwoon in of itself, lends us confidence - perhaps there's a theorem there waiting to be proved?

The Roman Numerals $\mathfrak{R}$

One can clearly see that Roman Numerals $\mathfrak{R}$ are a Siumatgwoon. However, to appreciate the characteristics that make it a Siumatgwoon, let us consider the subset of Roman Numerals from 1 to 10, which we shall show to also be a Siumatgwoon.

$\mathfrak{R}_{1,10} = \{I, II, III, IV, V, VI, VII, VIII, IX, X\}$

We will say that for two elements $a, b \in \mathfrak{R}_{1,10}$, $a|b$ iff the glyph a appears in b. As such, we can say $I|II$ and $I|III$ as an example, and that $V|IV$ and $X|IX$.

For any elements $a, b \in \mathfrak{R}_{1,10}$, if the glyphs ab so written together forms a glyph that also appears in $\mathfrak{R}_{1,10}$, then we'd say that $a * b \in \mathfrak{R}$.

Now, it's clear that Ax 1 is satisfied trivially.

Ax 2 is also satisfied trivially.

Ax 3 is also satisfied.

Ax 4 is also satisfied. As an example: $I|III, III|VIII$ and we have $I|VIII$.

So therefore, $\mathfrak{R}_{1,10}$ is a Siumatgwoon.

It is also interesting to note that as per the definition of $\mathfrak{R}_{1,10}$, it is not compositionally closed. For example, $II * III$ is not in $\mathfrak{R}_{1,10}$. This makes the Siumatgwoon different from a group, where all compositions are contained inside the group. Intuitively, perhaps this suggests the Siumatgwoon is less rich in structure than the mathematical group? Also, note that what $II * III$ should be in $\mathfrak{R}_{1,10}$ is represented by V. Intuitively, we can feel that in some sense, $II * III = V$ - that they're synonymous, identical, referring to the same referent. This is not unlike the presence of variant characters in the Sinoglyphs, such as 體 (body, object)= 躰 = 躰 = 体, or 信 (trust) = 訫 = 㐰 = 訫 = 㐰…Intuition should hint that this will yield some interesting structures if we pursue the investigation down this path.

Any Numerals System

The fact that the Roman Numerals are a Siumatgwoon should intuitively suggest that any numeral system is a Siumatgwoon. In fact, let us consider the world's many numeral systems, and see if there is one where it is not a siumatgwoon.

	0	1	2	3	4	5	6	7	8	9
唐字數字	〇	一	二	三	四	五	六	七	八	九
唐字數字大寫	零	壹、弌	貳	叄	肆	伍	陸	柒	捌	玖
字喃		𠬠	𠄩	𠀧	𦊚	𠄼	𦒹	𦉱	𠔭	𠃩
蘇州碼子	〇	〡、一	〢、二	〣、三	〤	〥	〦	〧	〨	〩
Roman Numerals		I	II	III	IV	V	VI	VII	VIII	IX
Eastern Arabic	⊠	⊠	⊠	⊠	⊠	⊠	⊠	⊠	⊠	⊠
Persian	⊠	⊠	⊠	⊠	⊠	⊠	⊠	⊠	⊠	⊠
Devanagari	⊠	⊠	⊠	⊠	⊠	⊠	⊠	⊠	⊠	⊠
Gujarati	⊠	⊠	⊠	⊠	⊠	⊠	⊠	⊠	⊠	⊠
Tibetan	⊠	⊠	⊠	⊠	⊠	⊠	⊠	⊠	⊠	⊠
Hebrew		⊠	⊠	⊠	⊠	⊠	⊠	⊠	⊠	⊠
Chinese counting rods		⊠	⊠	⊠	⊠	⊠	⊠	⊠	⊠	⊠
counting 正		⊠	⊠	⊠	⊠	⊠	⊠⊠	⊠⊠	⊠⊠	⊠⊠
Tangut		⍰	⍰	⍰	⍰	⍰	⍰	⍰	⍰	⍰

I don't think there's a single one that's not a siumatgwoon! Most of them are pathological for sure, in the sense that nothing is constituted by anything else, but none of them violate the Siumatgwoon axioms!

The case of the numerals as a Siumatgwoon, or a Metaphysic, is interesting. Numerals all refer to the same referents, the same "things" or "objects", namely, numbers. However, the glyphs in a given numeral system are themselves imbued with a particular set of metaphysical prejudices and judgements. Under the Roman Numeral Metaphysic, the number 3 is composed of 1 and 2, or composed of three 1s. 4 is composed of 1 and 5, but not 3 and 2.

The Polygons $\mathcal{P}$

Consider the following graph. If we take all the polygons, convex and star, as elements in a set called $\mathcal{P}$, we can see that it forms a Siumatgwoon. We state this without formal proof for the infinite set $\mathcal{P}$, but from the subset displayed in the graph below, we can see it is indeed true. A polygon a constitutes polygon b if a appears in b. $\{3\}$, the equilateral triangle, appears in $\{6/2\}$, the star of David, and so $\{3\}|\{6/2\}$

The Schläfli symbol is a recursive description, starting with $\{p\}$ for a p-sided regular polygon that is convex. For example, $\{3\}$ is an equilateral triangle, $\{4\}$ is a square, $\{5\}$ a convex regular pentagon, etc.

Regular star polygons are not convex, and their Schläfli symbols take the form $\{p/q\}$, where p is the number of vertices and q is their turning number. Equivalently, $\{p/q\}$ is created from the vertices of $\{p\}$ by connecting every qth vertex. For example, $\{5/2\}$ is a pentagram, while $\{5\}$ is a pentagon.

Note that p and q must be coprime, or the figure will degenerate, in which case we have the following theorem:

$\{p/q\} = d\{\frac{p}{d}/\frac{q}{d}\}$, where $d = \gcd(p, q)$.

Let us define for any Schläfli symbol $\{p\}|n\{p\}$ for any n. It is intuitively true.

Then clearly axiom 1 is satisfied. Axiom 2 is also satisfied.

Propositional Logic

There are 3 flavors of $*$ in Propositional Logic: $\wedge, \vee, \rightarrow$.

And we define $\phi|\psi$ if ϕ appears in ψ, for any wff ϕ, ψ.

Then clearly all 4 of the core siumatgwoon axioms are satisfied.

1. Trivial that all $\phi|\phi$
2. Also trivial that for any ϕ, ψ, either $\phi|\psi$ or $\phi \not|\psi$.
3. Trivial as well that for any $\phi, \psi|\phi * \psi$.
4. Also trivial that if $\phi|\psi$ and $\psi|\theta$ then $\phi|\theta$.

Propositional Logic as a Siumatgwoon has multiple interesting properties:

1. It is compositionally complete.
2. It is also constitutionally complete.
3. Is it a simple siumatgwoon? Are decompositions finite? Yes. Are decompositions unique? Yes, up to reordering. So yes, it's a simple siumatgwoon.

Consider the old siumatgwoon axioms, and a new relation "synonym", represented by $\sim$, read as "is synonymous with". It satisfies the following axioms.

Synonym Axioms

S1. (reflexivity) for all $x \in S$, $x \sim x$.

S2. (symmetry) For all $a, b \in S$, if $a \sim b$ then $b \sim a$.

S3. (transitivity) for all $a, b, c \in S$, if $a \sim b$, and $b \sim c$, then $a \sim c$.

S4. (Compositional congruence) If $a \sim a'$, then (if they ax or xa exists):

- $ax \sim a'x$,
- $xa \sim xa'$

S5. Composition cancellation

- If $a * b \sim a * c$, then $b \sim c$;
- If $b * a \sim c * a$ then $b \sim c$;

S6. Divisor compatibility: If $a \sim b$, then for all $x \in S$, $x|a$ iff $x|b$.

S7: Upwards divisibility: If $a \sim b$, then for all if $a|x$ then $b|x$.

A Siumatgwoon is Synonym-Closed if in which if $a * b$ exists and $a \sim a'$ and $b \sim b'$ then $a' * b, a * b', a' * b'$ also exist. Most siumatgwoons, including the Sinoglyphs, are not synonym-closed.

Because of S4, Synonym-Closed Siumatgwoons must have $a * b \sim a' * b \sim a * b' \sim a' * b'$.

S8: Synonym Replacement and Existence Preservation

Synonym replacement completion: if $a * b$ exists, and if $b \sim b'$, then $a * b'$ exists in S as well.

Prior to this, we are rather ambiguous and coy as to whether a and b being synonymous means $b * x$ necessarily exists if $a * x$ exists. I feel we should admit closure for synonym substitutional compositions. We can consider other structure where this axiom is not admitted later.

S9. Multidecomposition if $ab \sim cd$ and $cd = e$ then $ab = e$.

From S4 you'd be able to prove that $a \sim a'$ and $b \sim b'$ then $a * b \sim a' * b'$.

The Quotient Siumatgwoon $S/\sim$

Starting Point: Synonymy $\sim$ as an Equivalence Relation

Let S be a Siumatgwoon, and let $\sim$ be a synonymy relation on S satisfying:

- $\sim$ is an equivalence relation: reflexive, symmetric, transitive.
- $\sim$ is compatible with composition:
  If $a \sim a'$, $b \sim b'$, then $a * b \sim a' * b'$.
- $\sim$ is compatible with divisibility:
  If $a \sim b$, then $x|a \iff x|b$ for all $x \in S$.

These give us a solid foundation to define a quotient structure.

The Set $S/\sim$

Let:

$S/\sim = \{[a] : a \in S\}$

where $[a] = \{x \in S \mid x \sim a\}$ is the equivalence class of a.

Defining the Operations on $S/\sim$

Multiplication

We define:

$[a] * [b] := [a * b]$, if $a * b$ exists.

This is well-defined because of compositional congruence (axiom S4).

It is also because that we required if $a * b$ exists, so must $a' * b'$.

That is: if $a \sim a'$, and $b \sim b'$, then:

$a * b \sim a' * b' \Rightarrow [a * b] = [a' * b']$

So the result is independent of the representative.

Divisibility

We define:

$[a]|[b] \iff a|b$

Again, this is well-defined thanks to the divisibility equivalence axiom (S6):

If $a \sim a', b \sim b'$, given we have the axiom that $a|b \iff a'|b'$ then:

$[a]|[b] \iff [a']|[b']$

Thus, $|$ descends to the quotient.

Verifying Siumatgwoon Axioms in $S/\sim$

Let's verify that the quotient structure inherits the original Siumatgwoon axioms:

- Axiom 1 (Reflexivity of $|$):
  Since $a|a \Rightarrow [a]|[a]$
- Axiom 2 (Totality):
  For all $[a], [b]$, either $[a]|[b]$ or not.
  This follows since $|$ on S has totality, and divisibility is preserved under $\sim$.

- Axiom 3 (Transitivity):

  Suppose $[a]|[b]$ and $[b]|[c]$. Recall the definition that $[a]|[b]$ then $a|b$. It's straightforward.

- Axiom 4 (Composition and divisibility):

  We want to prove that if $[a]*[b]=[c]$, then $[a],[b]|[c]$. By definition $[a]*[b]=[c]$ then $a*b \sim c$, and so $a|c$. By the definition of constitutiveness over synonym classes, $[a]|[c]$. The argument same goes for $[b]|[c]$.

Now come axioms 5 and axioms 6. Do they hold in $S/\sim$, even if they might not hold in S?

- Axiom 5 (Complete Constructibility and Generators): There exists a non-empty subset $G_S \subseteq S$ such that:

  - (Generation) Every element $x \in S$ is a product of a finite sequence of elements from G_S, i.e., $x = g_1 * g_2 * \cdots * g_n$, where $g_1, \ldots, g_n \in G_S$

  And a Simple Siumatgwoon is one where:

  1. E is a generating set, and
  2. that decompositions into elementals are unique.

- Axiom 6. (Finite constitution). For any $x \in S$, there are only finitely many objects $y_1, y_2, \ldots, y_n \in S$ such that $y_1, y_2, \ldots, y_n|x$.

Let's prove that axiom 5 holds first. And then axiom 6 holds. Then we prove that $S/\sim$ is a simple siumatgwoon.

Consider some element $[x] \in S/\sim$. Given S is a siumatgwoon, we know that any x has a finite decomposition $g_1 * g_2 * g_3 * \cdots g_n$. Given $x = g_1 * g_2 * g_3 * \cdots g_n$, we have $[x] = [g_1 * g_2 * g_3 * \cdots g_n]$, and by the definition of $[*]$ we have $[x] = [g_1 * g_2 * g_3 * \cdots g_n] = [g_1] * [g_2] * [g_3] * \cdots [g_n]$. Given this applies to every x in S, it is true of every $[x] \in S/\sim$.

Note that some of the $[g_i]$ might be equal to some other $[g_j]$ if $g_i \sim g_j$.

Now for axiom 6.

Consider $[x] \in S/\sim$. By axiom 6, there only finitely many $y_1, y_2, \ldots, y_n|x$. Therefore there are only finitely many $[y_1], [y_2], \ldots, [y_n]|[x]$. Note again, that because there could be $y_i \sim y_j$, then the most reduced list of constituents for $[x]$ might be shorter than n.

Axiom 5 (Generators):

If $G \subseteq S$ is a generating set, then $[G] = \{[g] \mid g \in G\} \subseteq S/\sim$ generates all of $S/\sim$

Hence, $S/\sim$ is a well-defined quotient Siumatgwoon.

Interpretation of $S/\sim$

- Elements of $S/\sim$ are semantic classes: bundles of expressions that mean the same thing.
- Operations and structure are all inherited from how the parts combine.
- This is essentially a move from syntax to semantics —a form of canonical simplification.

Identity and Atoms in $S/\sim$

- The set of atomic classes would be:
  $A/\sim = \{[a] \mid a \in A\}$

  If synonymy is strong (e.g., if all synonyms of atoms are equal), then this is a true set of atomic meanings.
- The quotient set removes redundancy: if two characters are written differently but function identically, they're collapsed.

A Hierarchy of Synonym Relations

Intuitively speaking, the $\sim$ of 言 $\sim$ 文 that allows us write 信 $\sim$ 㕻 seems different from the $\sim$ in 女 $\sim$ 巫, as it does not seem the case that 女 can be exchanged for 巫 in all Chinese characters. This suggests that there is a hierarchy, or rather, a family of synonymous relations, perhaps context dependent.

Some synonyms are more closely clustered together than others. This suggests there is a hierarchy of synonyms.