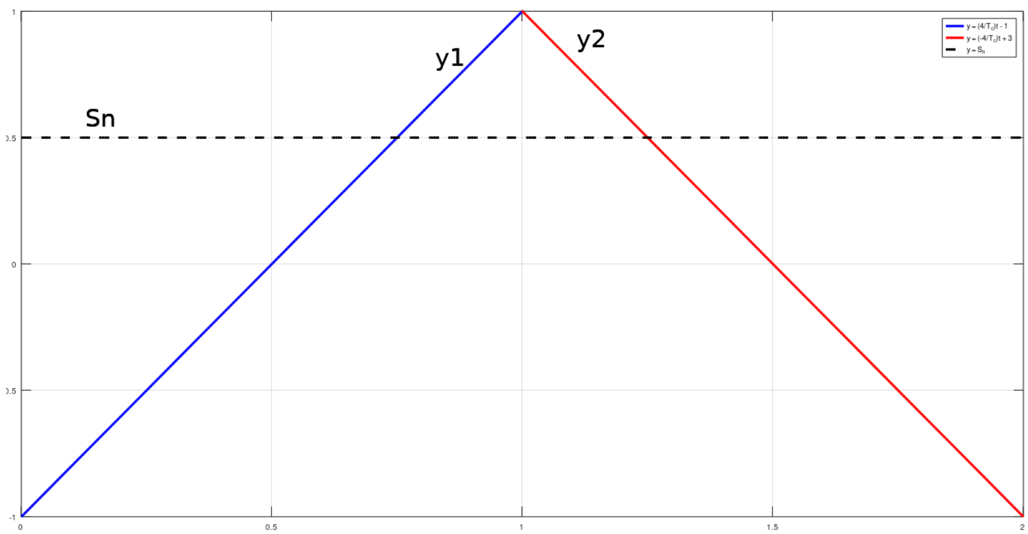
Deriving the Duty Cycle Formula

The goal is to derive an equation for the duty cycle of each point over the sine wave period. The assumption is that the reference signal is constant during the carrier signal switching period.

In this case,

$$S_n = reference\ signal$$

$$y = \text{carrier } signal$$



The carrier signal is a triangular wave which has 2 parts

The first part can be defined with the following equation:

$$y1 \ = \left(\frac{4}{T_{\mathrm{sw}}}\right)t - 1$$

The second part can be defined with the following equation

$$y2 \ = \left(-\frac{4}{T_{\mathrm{sw}}}\right)t + 3$$

The on time will be the total time the reference wave is above the carrier signal

Therefore, the duty cycle is calculated as follows:

$$D(t) = \frac{T_{on}}{T_{sw}}$$

$$D(t) = \frac{t_1 + T_{sw} \ - \ t_2}{T_{sw}}$$

The variable t_1 is calculated as follows using the equation of the first part:

$$S_n = \left(\frac{4}{T_{\mathrm{sw}}}\right)t - 1$$

$$t_1 = \frac{T_{\mathrm{sw}}(S_n + 1)}{4}$$

The variable t_2 is calculated as follows using equation of the second part:

$$S_n = \left(-\frac{4}{T_{\mathrm{sw}}}\right)t + 3$$

$$t_2 = \frac{T_{\mathrm{sw}}(3 - S_n)}{4}$$

The total duty cycle will become the following

$$D(t) = \frac{\frac{T_{\mathrm{sw}}(S_n + 1)}{4} + T_{\mathrm{sw}} - \frac{T_{\mathrm{sw}}(3 - S_n)}{4}}{T_{\mathrm{sw}}}$$

Simplifying the equation leads to:

$$D(t) = \frac{1}{2}(S_n + 1)$$

Where $0 \le S_n < 1$

Therefore

$$D_1(t) = 0.5 + 0.5 \cdot \sin\left(\frac{n}{ARRAY\_SIZE} \times 2\pi\right)$$

$$D_2(t) = 0.5 + 0.5 \cdot \sin\left(\frac{n}{ARRAY\_SIZE} \times 2\pi + \pi\right)$$

Where $0 \le \mathrm{n} < ARRAY\_SIZE$

Including the amplitude modulation index, m_a

$$D_1(t) = 0.5 + 0.5 \cdot m_a \cdot \sin\left(\frac{n}{ARRAY\_SIZE} \times 2\pi\right)$$

$$D_2(t) = 0.5 + 0.5 \cdot m_a \cdot \sin\left(\frac{n}{ARRAY\_SIZE} \times 2\pi + \pi\right)$$

Where $0 \le m_a < 1$

and $0 \le D_1(t) < 1$ and $0 \le D_2(t) < 1$

$$\mathrm{T_{int}} = \frac{\mathrm{T_{ref}}}{ARRAY\_SIZE} = \frac{20 \times 10^{-3}\mathrm{s}}{100} = 200\mu\mathrm{s}$$