hw4-q2-logistic

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0.1 Q2 Logistic Regression

```
In [1]: import numpy as np
        import matplotlib.pyplot as plt
        import matplotlib as mpl
        from mpl_toolkits.mplot3d import Axes3D
        import math
        %config InlineBackend.figure_format = 'retina'
        %matplotlib inline
        from sklearn.utils import shuffle
        import scipy.io as sio
        plt.rcParams['figure.figsize'] = 8,8
```

0.1.1 Original Data

```
In [2]: X_and_Y_train = np.load('./hw4-q2-logistic-train.npy')
        X_train = X_and_Y_train[:, :2]     # Shape: (70,2)
        X_train = np.matrix(np.hstack((np.ones((len(X_train), 1)), X_train))) # Shape: (70, 3)
        Y_train = X_and_Y_train[:, 2]     # Shape: (70,)
```

```
In [3]: X_and_Y_test = np.load('./hw4-q2-logistic-test.npy')
        X_test = X_and_Y_test[:, :2]    # Shape: (30,2)
        X_test = np.matrix(np.hstack((np.ones((len(X_test), 1)), X_test))) # Shape: (30, 3)
        Y_test = X_and_Y_test[:, 2]    # Shape: (70,)
```

```
In [4]: mpl.style.use('seaborn')

        fig = plt.figure()
        plt.scatter([X_train[Y_train==0, 1]], [X_train[Y_train==0, 2]], marker='x', color='b', a
        plt.scatter([X_train[Y_train==1, 1]], [X_train[Y_train==1, 2]], marker='o', color='r', a
        plt.xlabel('X1')
        plt.ylabel('X2')
        plt.legend(loc='upper right', fontsize=10)
        plt.title('Training data')
        plt.show()
        #fig.savefig('scatter_1.png', format='png', dpi=400)
```

$$\mathcal{L}(\theta) = -\sum_{i} \left[y^{(i)} \ln h(x^{(i)};\theta) + (1-y^{(i)}) \ln\left(1-h(x^{(i)};\theta)\right) \right]$$

Let $z = f(x^{(i)};\theta) = -\sum_{k=0}^{K} \theta_k x_k$

By the Chain Rule,

$$\frac{\partial \mathcal{L}(\theta)}{\partial \theta} = -\sum \left[y^{(i)} \frac{1}{h(z)} + (1-y^{(i)}) \frac{1}{(1-h(z))} \right] \frac{\partial}{\partial \theta} h(z)$$

Again by the Chain Rule,

$$= -\sum \left[y^{(i)} \frac{1}{h(z)} + (1-y^{(i)}) \frac{1}{(1-h(z))} \right] h(z) \cdot (1-h(z)) \frac{\partial}{\partial \theta} z$$

$$= -\sum \left[y^{(i)}(1-h(z)) + (1-y^{(i)}) h(z) \right] x^{(i)}$$

$$= -\sum \left[y^{(i)} - h(z) \right] x^{(i)}$$

$$\therefore = -\sum \left[y^{(i)} - h(x^{(i)};\theta) \right] x^{(i)}$$

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0.1.2 Gradient Descent

```
In [5]: def sigmoid(z):
            return 1.0/(1.0+np.exp(-z))
```

```
In [6]: # gradient of loss function L(theta)
        def L_prime_theta(X, Y, theta):
            Y = Y.reshape(-1, 1)
            retVal = -((Y - sigmoid(X.dot(theta))).T.dot(X)).T
            return retVal
```

```
In [7]: def L_theta(X, Y, theta):
            return -(Y.dot(np.log(sigmoid(X.dot(theta)))) + (1 - Y).dot(np.log(1 - sigmoid(X.dot
```

```
In [8]: learning_rate = 0.001
        n_iter = 10000
        theta = np.zeros((X_train.shape[1], 1))
        # We will keep track of training loss over iterations
        iterations = [0]
        L_theta_list = [L_theta(X_train, Y_train, theta)]
        for i in range(n_iter):
            gradient = L_prime_theta(X_train, Y_train, theta)
```

```
            theta_new = theta - learning_rate * gradient
            iterations.append(i+1)
            L_theta_list.append(L_theta(X_train, Y_train, theta_new))

            if np.linalg.norm(theta_new - theta, ord = 1) < 0.001:
                print("gradient descent has converged after " + str(i) + " iterations")
                break
            theta = theta_new

        print ("theta vector: \n" + str(theta))
```

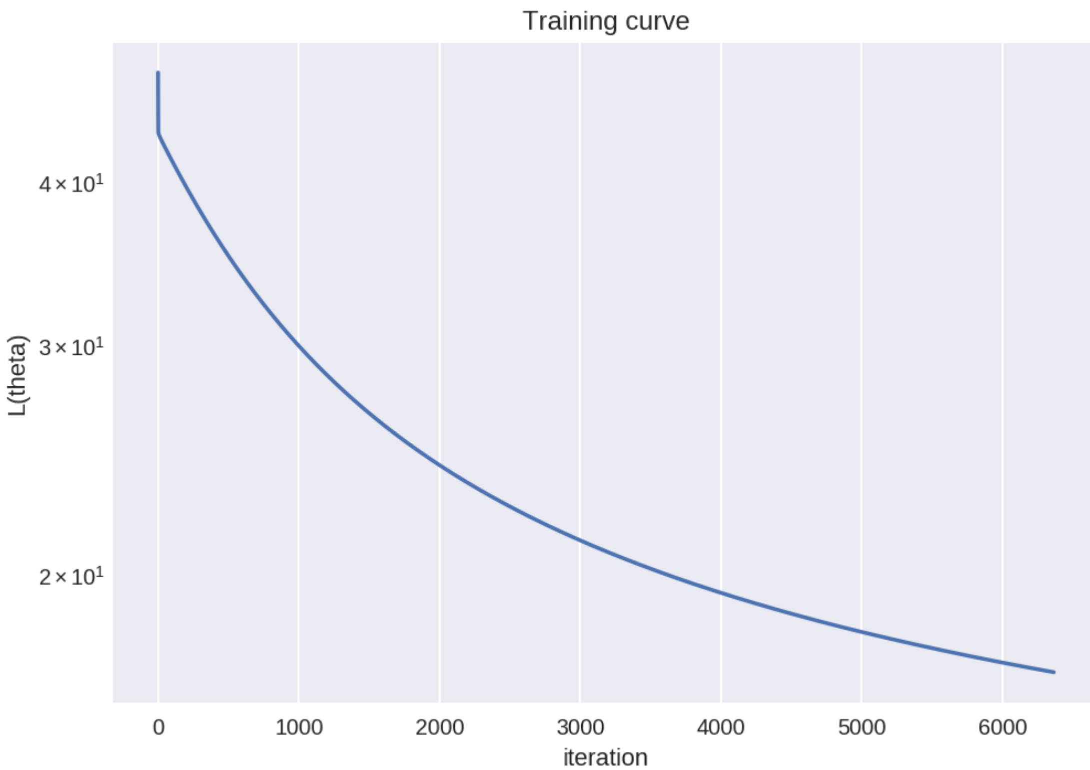
```
gradient descent has converged after 6365 iterations
theta vector:
[[-11.48322099]
 [  0.97466299]
 [  0.88907048]]
```

Equation of decision boundary corresponding to optimal θ^*:

$$y = \begin{cases} 1 & -11.4832 + 0.9747x_1 + 0.8891x_2 \geq 0 \\ 0 & -11.4832 + 0.9747x_1 + 0.8891x_2 < 0 \end{cases}$$

0.1.3 Training curve

```
In [9]: plt.title('Training curve')
        plt.xlabel('iteration')
        plt.ylabel('L(theta)')
        plt.semilogy(iterations, np.array(L_theta_list).reshape(-1, 1))
        plt.show()
```



0.1.4 Results on Training data

```
In [10]: prediction = sigmoid(np.dot(X_train, theta)) >= 0.5
         testing_accuracy = np.sum(prediction == Y_train.reshape(-1, 1))*1.0/X_train.shape[0]
         print(prediction.shape, Y_test.shape)
         print ("training accuracy: " + str(testing_accuracy))

         x = np.arange(np.min(X_train[:,1])-1,np.max(X_train[:,1])+1,1.0)
         y = (-theta[0][0]-theta[1][0]*x)/theta[2][0]
         plt.scatter([X_train[Y_train==0, 1]], [X_train[Y_train==0, 2]], marker='x', color='b',
         plt.scatter([X_train[Y_train==1, 1]], [X_train[Y_train==1, 2]], marker='o', color='r',


         plt.xlabel('X1')
         plt.ylabel('X2')
         plt.plot(x,y.T, 'dodgerblue', label='decision boundary')
         plt.title('Training data and decision boundary')

         plt.legend(loc='upper right', fontsize=10)
```

```
(70, 1) (30,)
training accuracy: 0.914285714286
```

```
Out[10]: <matplotlib.legend.Legend at 0x7f512a5ed9e8>
```

0.1.5 Results on Testing data

```
In [11]: prediction = sigmoid(np.dot(X_test, theta)) >= 0.5
         testing_accuracy = np.sum(prediction == Y_test.reshape(-1, 1))*1.0/X_test.shape[0]
         print ("testing accuracy: " + str(testing_accuracy))

         x = np.arange(np.min(X_train[:,1])-1,np.max(X_train[:,1])+1,1.0)
         y = (-theta[0][0]-theta[1][0]*x)/theta[2][0]
         plt.scatter([X_test[Y_test==0, 1]], [X_test[Y_test==0, 2]], marker='x', color='b', alph
         plt.scatter([X_test[Y_test==1, 1]], [X_test[Y_test==1, 2]], marker='o', color='r', alph


         plt.xlabel('X1')
         plt.ylabel('X2')
         plt.plot(x,y.T, 'dodgerblue', label='decision boundary')
         plt.title('Testing data and decision boundary')

         plt.legend(loc='upper right', fontsize=10)

testing accuracy: 0.833333333333
```

Out[11]: <matplotlib.legend.Legend at 0x7f5128d59898>