# Queueing network

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# Outline


1. Example
2. Open and closed queueing networks
3. Product form networks
   - Open Jackson networks
4. Queueing network in practice

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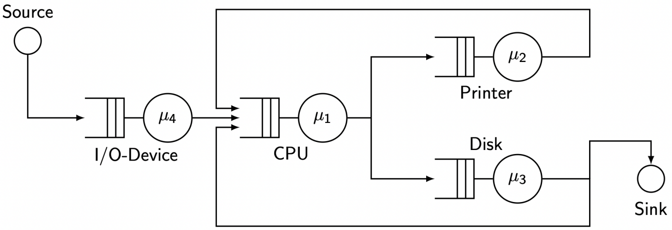
1 Example

2 Open and closed queueing networks

3 Product form networks
- Open Jackson networks

4 Queueing network in practice

# A computer system as 4-node queueing network




- All queues are FCFS
- Interarrival time is exponentially distributed with $\lambda = 4$ jobs/sec.
- The service time at each node is exponentially distributed with $\frac{1}{\mu_1} = 0.04$ sec, $\frac{1}{\mu_2} = 0.03$ sec, $\frac{1}{\mu_3} = 0.06$ sec, and $\frac{1}{\mu_4} = 0.05$ sec.
- The routing probabilities are $p_{12} = p_{13} = 0.5$, $p_{41} = p_{21} = 1$, $p_{31} = 0.6$, $p_{30} = 0.4$.

## Questions

- What is the steady state probability of state (k1, k2, k3, k4) = (3,2,4,1)?
- How many jobs in CPU queue?

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# Open queueing network

## Remarks

A system can be modelled by several networked queues in which a job departing from one queue arrives at another queue (or possibly the same queue)

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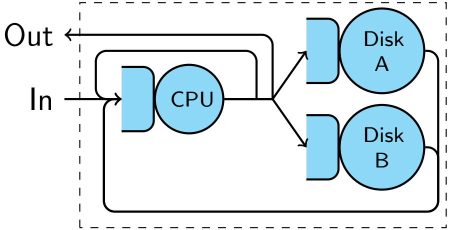
## Open queueing network

A queueing network has external arrivals and departures.



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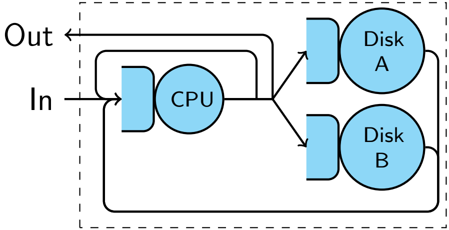
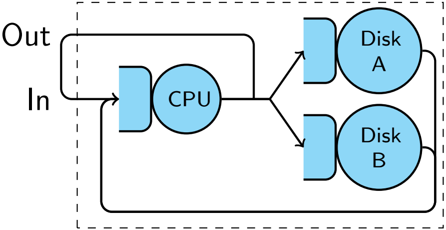



- Number of jobs in system varies with time.
- Assumption: throughput = arrival rate
- Analysis goal: to characterize the distribution of number of jobs in the system.

# Closed queueing network

**Closed queueing network**

A queueing network has no external arrivals and departures.



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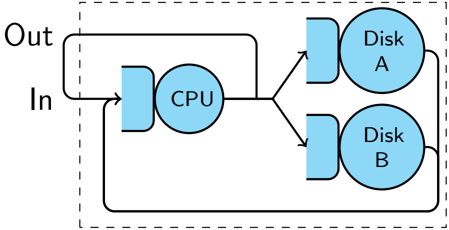


- Total number of jobs in the system is constant.
- Build a closed network from open network: "Out" $\rightarrow$ "In".
- Throughput $=$ flow on Out-to-In link.
- Analysis goal: Given number of jobs, determine throughput.

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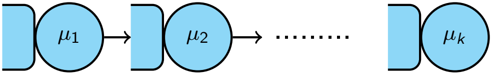
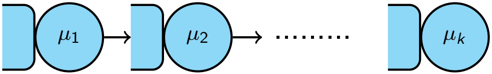
# Series networks




- $k$ M/M/1 queues in series.
- Each individual queue can be analyzed independently of other queues
- Utilization of server $i$: $\rho_i = \lambda/\mu_i$
- Probability of $n_i$ jobs in queue $i$: $P_i(n_i) = (1-\rho_i)\rho_i^{n_i}$
- Joint probability of queue lengths

$$P(n_1, n_2, \ldots, n_k) = \prod_{i=1}^{k} P_i(n_i)$$

# Series networks

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- Joint probability of queue lengths

$$P(n_1, n_2, \ldots, n_k) = \prod_{i=1}^{k} P_i(n_i)$$

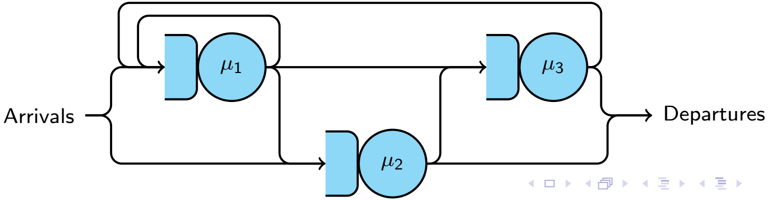
$\Rightarrow$ **Product form network** (i.e., in which the system state probability is a product of device state probabilities).

# Jackson networks (1)

## Definition of Jackson network

Jackson network is an open queueing network which satisfies

- Any external arrivals to node $i$ form a Poisson process.
- Service time = exponential distributed, service discipline = FCFS
- A job after finished at queue $i$, either moves to queue $j$ with probability $p_{ij}$, or leaves the system with probability $1 - \sum_{j=1}^{m} p_{ij}$.
- Utilization of all queues $< 1$.



# Jackson networks (2)

## Jackson theorem

Given a Jackson network of $k$ M/M/1 queues with utilization $\rho_i$. The equilibrium state probability distribution exists and for state $(n_1, n_2, \ldots, n_k)$ is given by the product of the individual queue equilibrium distributions as follows.

$$P(n_1, n_2, \ldots, n_k) = \prod_{i=1}^{k} [\rho_i^{n_i}(1 - \rho_i)].$$

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## Remarks

- Generally, if there is any feedback in the network, the internal flows are not Poisson.
- Queues can be separable and each queue in the network behaves as an M/M/1 system. Calculation of arrival rate of each queue will be shown next.

# Jackson networks (3)

Consider a Jackson network.

- Define

$$a = (\alpha p_{0i})_{i=1}^{k}$$

in which $a$ is arrival rate to the network/

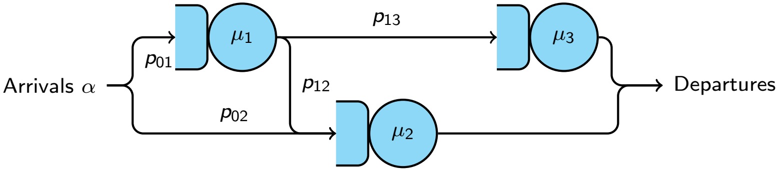
- Denote overall rate to node $i$

$$\lambda_i = \alpha p_{0i} + \sum_{j=1}^{k} \lambda_j p_{ji}, i = 1, \ldots, k$$

- Then,

$$\lambda = (I - P^T)^{-1} a$$

# Jackson networks: example (1)




Suppose

$$\alpha = 5, p_{01} = 0.5, p_{02} = 0.5, p_{03} = 0$$
$$p_{12} = 0.4, p_{13} = 0.6$$

$$P = \begin{bmatrix} 0 & 0.4 & 0.6 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \mu = \begin{bmatrix} 15 \\ 12 \\ 10 \end{bmatrix}$$

$\lambda_i$ for queue $i$ can be calculated as follows.

$$\lambda = (I - P^T)^{-1} a = \begin{bmatrix} 1 & 0 & 0 \\ -0.4 & 1 & 0 \\ -0.6 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0.5 \times 5 \\ 0.5 \times 5 \\ 0 \end{bmatrix} = \begin{bmatrix} 2.5 \\ 3.5 \\ 1.5 \end{bmatrix}$$

# Jackson networks: example (2)

With $\lambda_i$ and $\mu_i$, we can calculate $\rho_i$

$$\rho_1 = 2.5/15$$
$$\rho_2 = 3.5/12$$
$$\rho_3 = 1.5/10$$

Hence the probability that there is one job at each node is

$$P(1,1,1) = \prod_{i=1}^{3}[(1-\rho_i)\rho_i^1] \approx 0.00365$$

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Hence the probability that there is one job at each node is

$$P(1,1,1) = \prod_{i=1}^{3} [(1-\rho_i)\rho_i^1] \approx 0.00365$$

**Remarks**

If there is no feedback in the network, the internal flows are Poisson.

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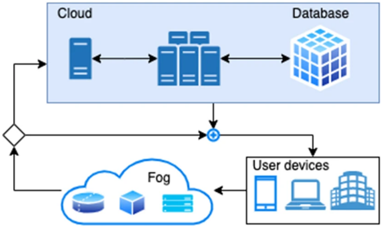
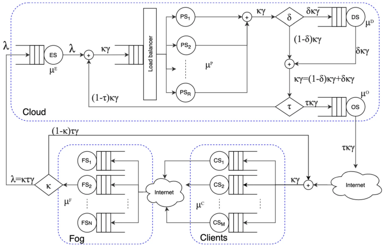
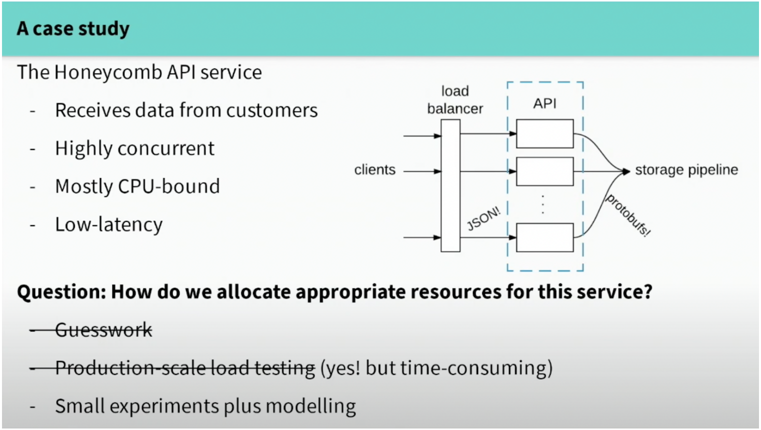
# Fog computing




`https://doi.org/10.1007/s11227-022-04328-3`

Journal of Supercomputing (2022)

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