a place of mind

MECH366 : Modeling of Mechatronic Systems

L20 : Simulink
Step response of overdamped systems

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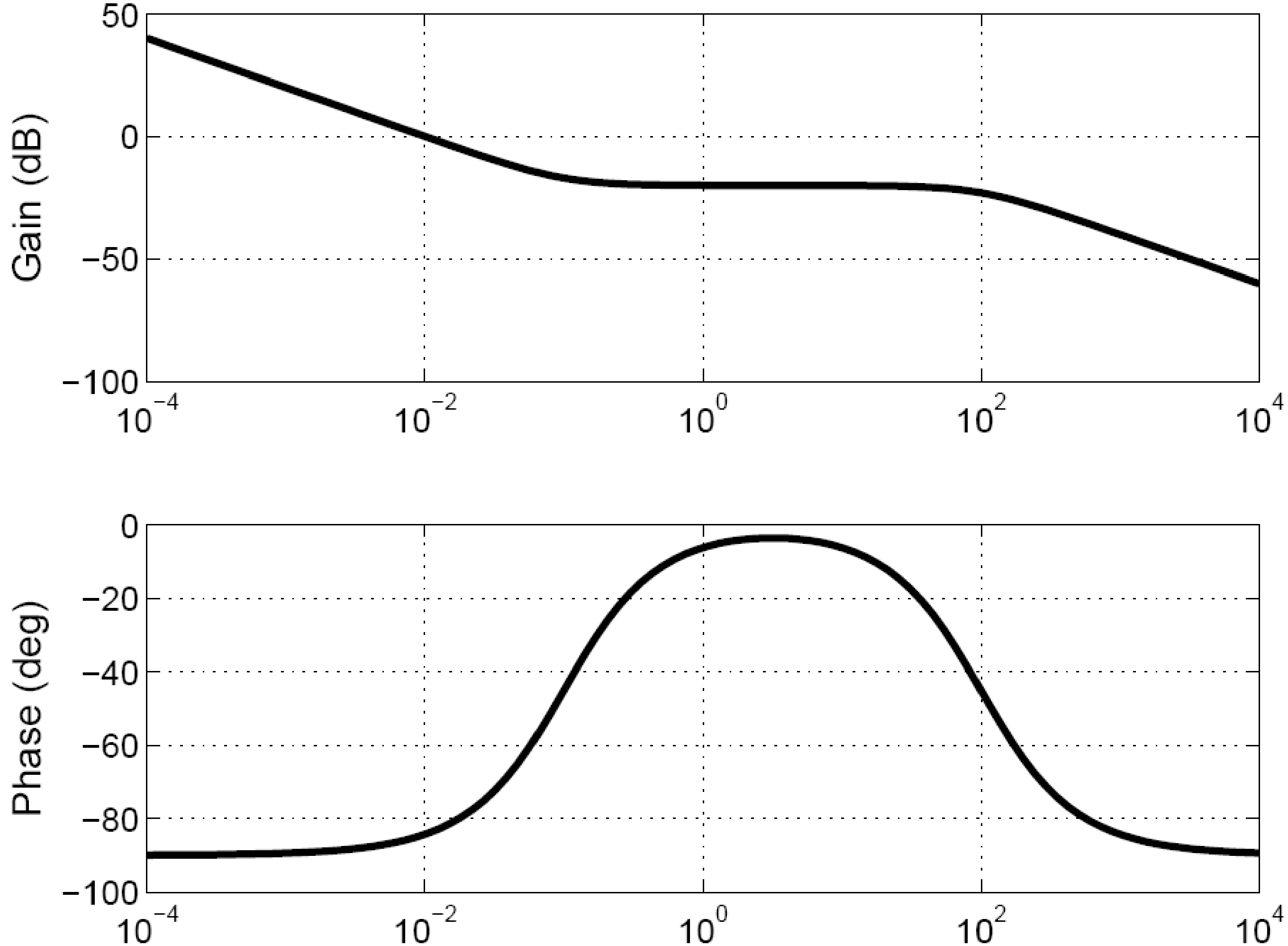
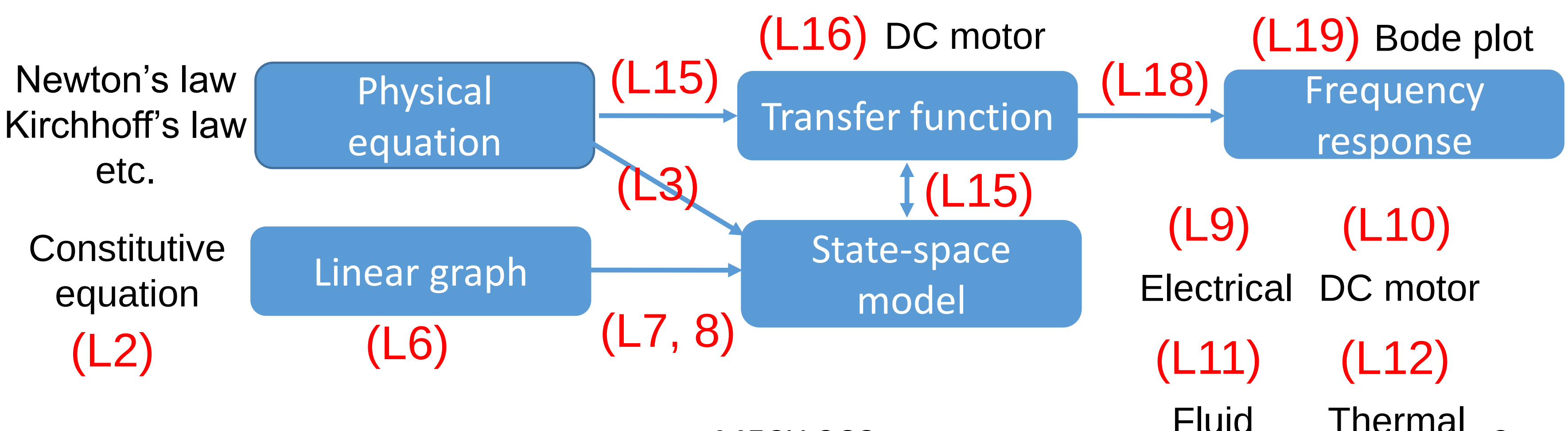
Modeling example based on FRF



Today's topic & class schedule

- L18: Nov 15 (Fri): Frequency response
- L19: Nov 18 (Mon): Bode diagram (Lab 4 report content, report due Nov 26)
- L20: Nov 22 (Fri): Simulink, overdamped system
- L21: Nov 25 (Mon): Stability, course summary

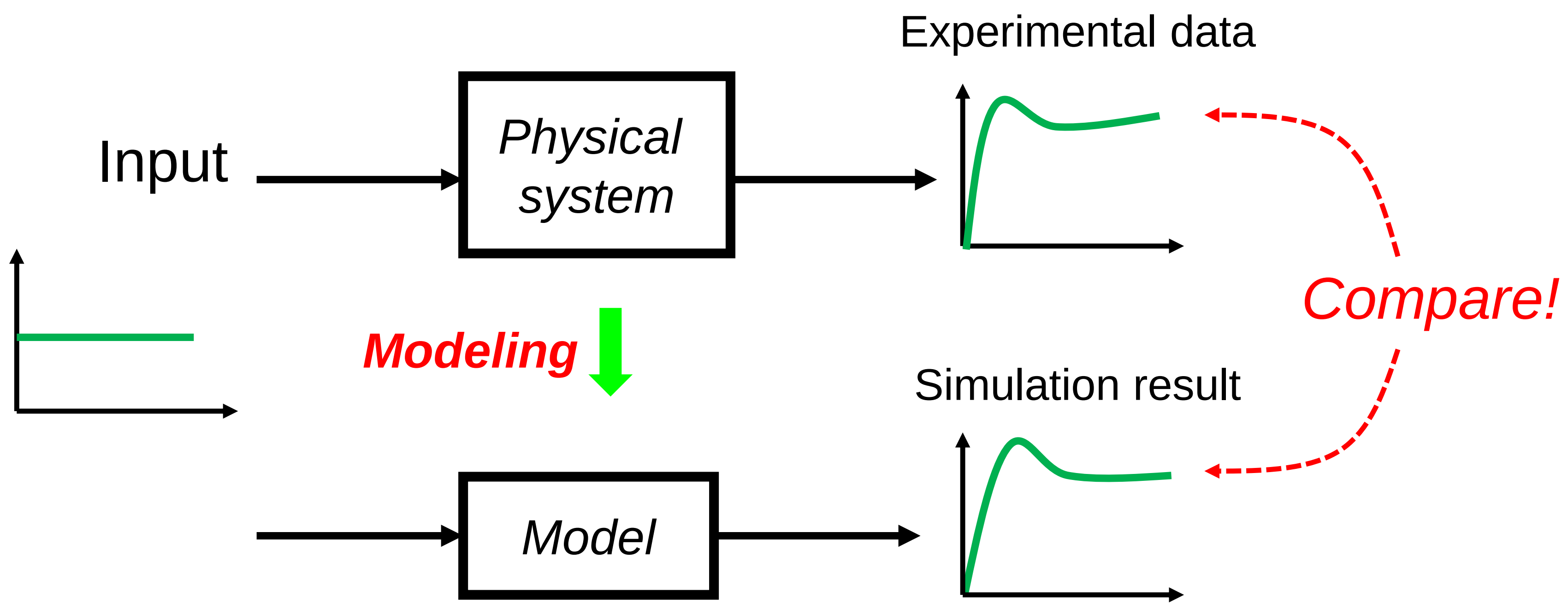


Simulation of models

- Throughout the course, we have studies how to derive mathematical models for dynamic systems:
  - State-space model $$\begin{cases} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{cases}$$
  - Transfer function $$Y(s) = G(s)U(s)$$
  - Block diagram (indicating the system connections)
- How can we conduct time-domain simulations?

Model validation

- After modeling, we need to check how good the model is approximating the physical system.



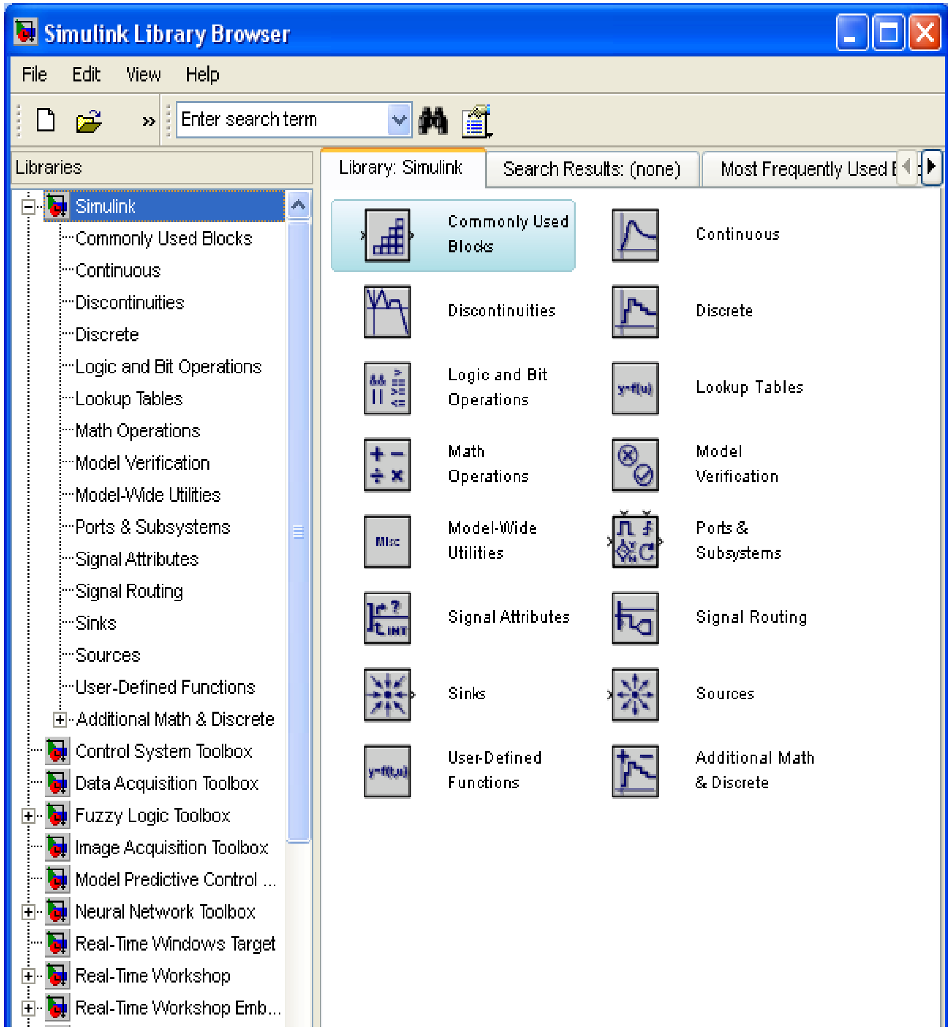
How to get the simulation result?

Simulink

- Software to model and simulate a system, as well as to realize/program controllers in microcontrollers (Arduino, dSPACE etc.)
- Engineers use Simulink to solve engineering problems in many industries.
  - Automotive
  - Aerospace
  - Process industries
  - Communications
  - Industrial automation
  - Electronics

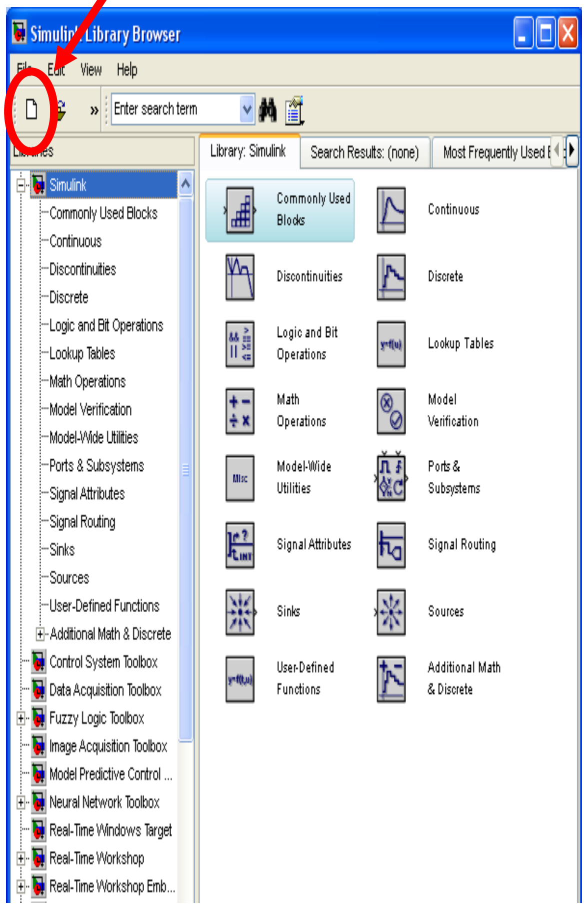
Start Simulink

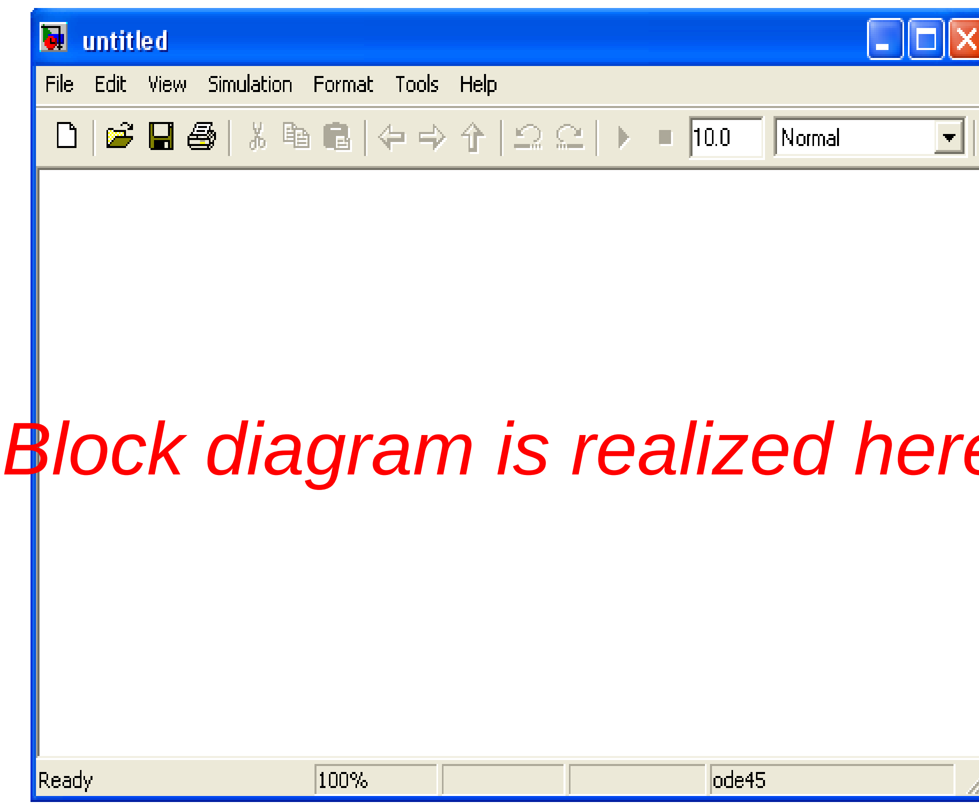
- In MATLAB prompt, type “simulink”.
- Then, Simulink Library Browser pops up.



Create a model

- Click here. Then, a new blank model pops up.

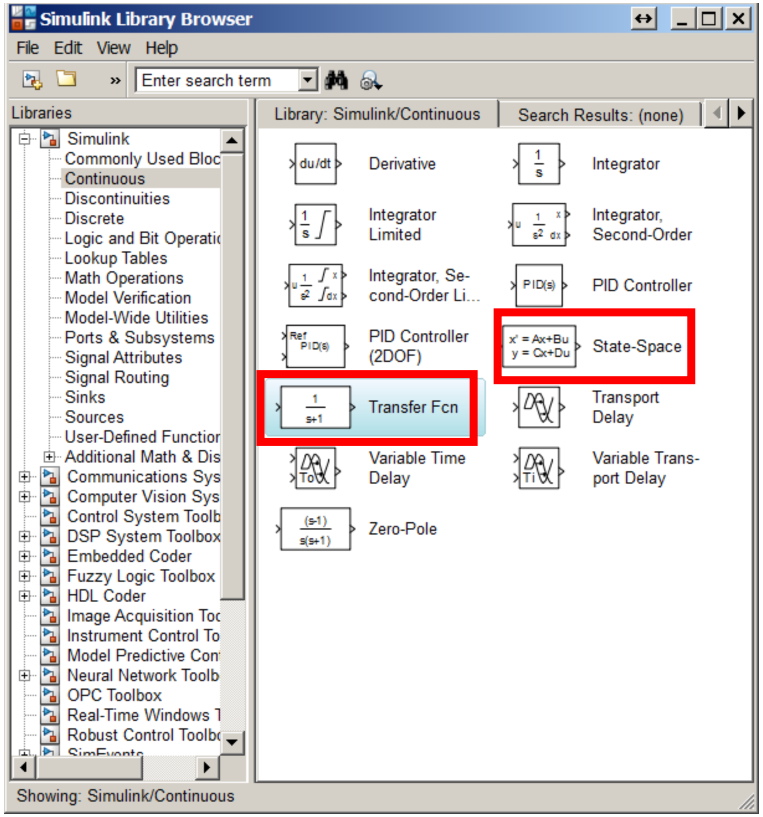




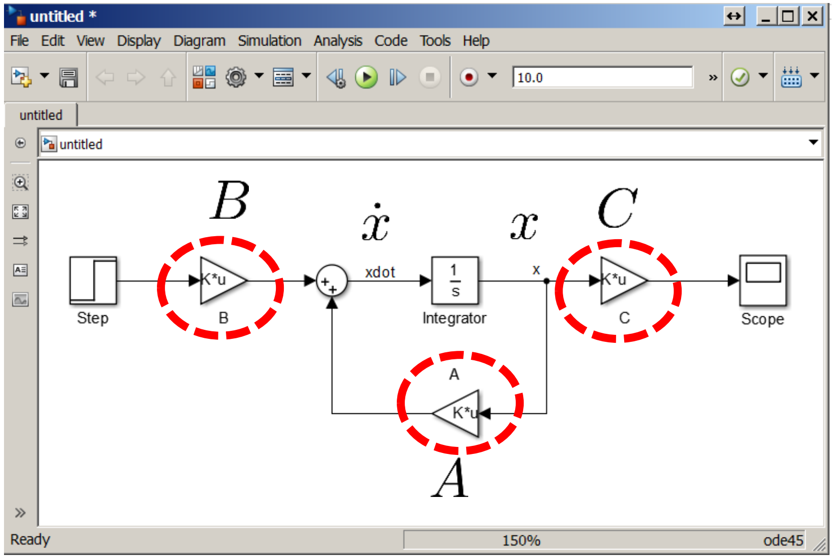
Block diagram is realized here

Transfer function & state-space model

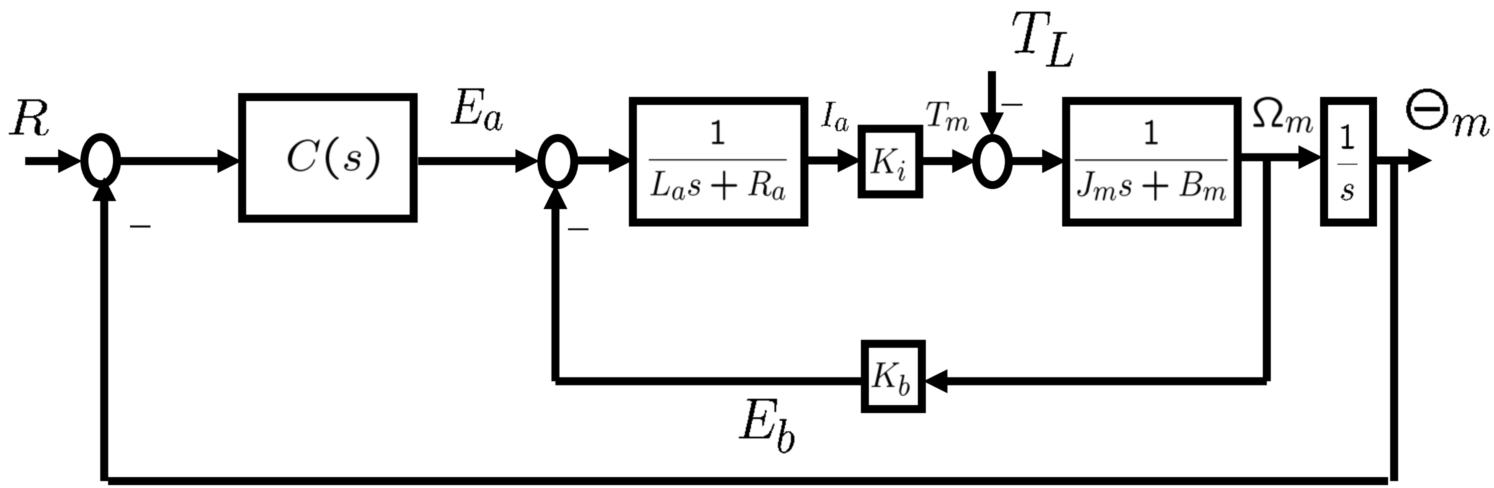
- In the “Library”, ”Continuous”, you can find blocks.



Realization of $\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases}$



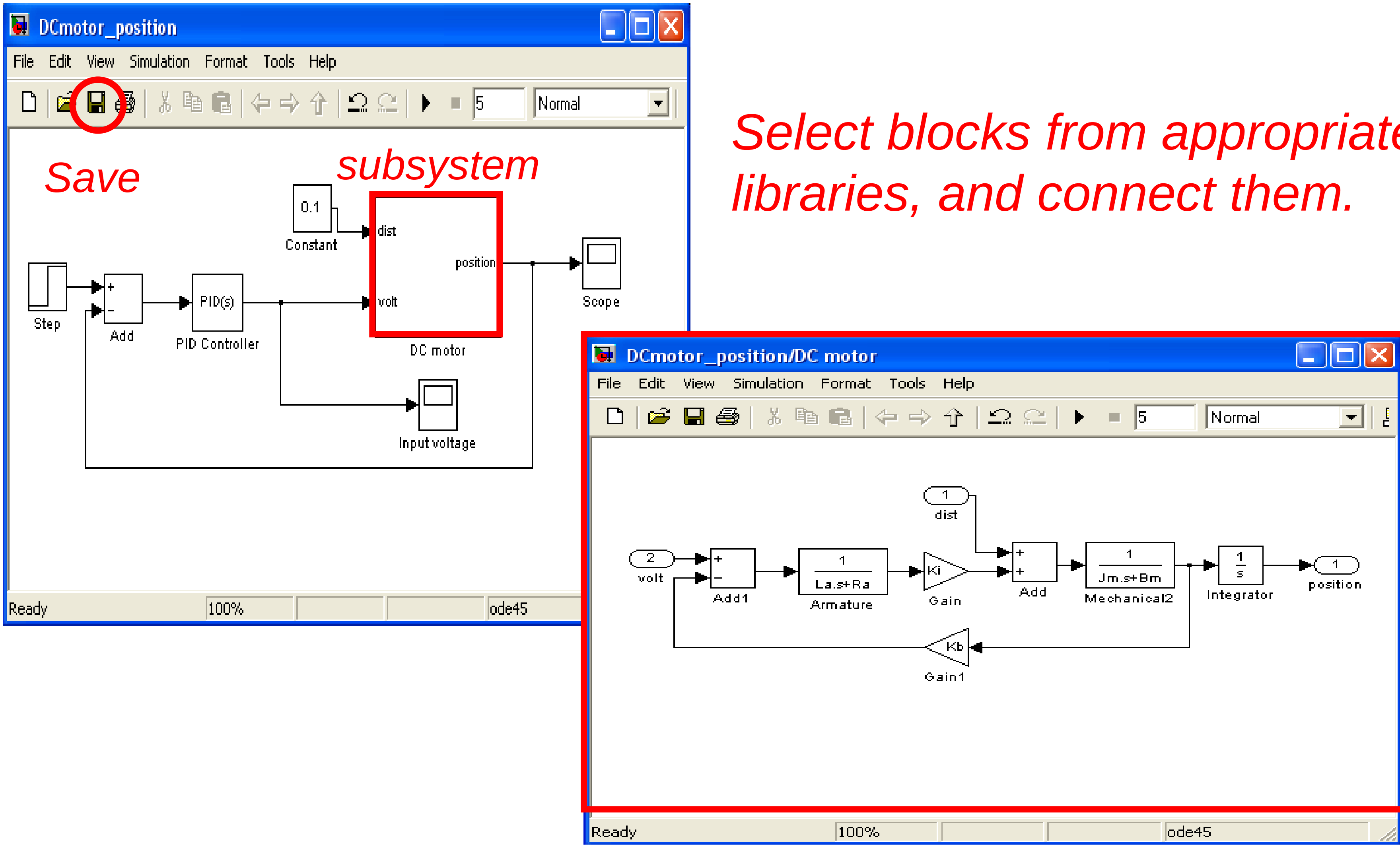
DC motor position control




```
>> Jm = 3.672e-5;
>> Bm = 6.744e-6;
>> Kb = 0.068;
>> Ki = 0.068;
>> La = 3e-3;
>> Ra = 1.63;
```

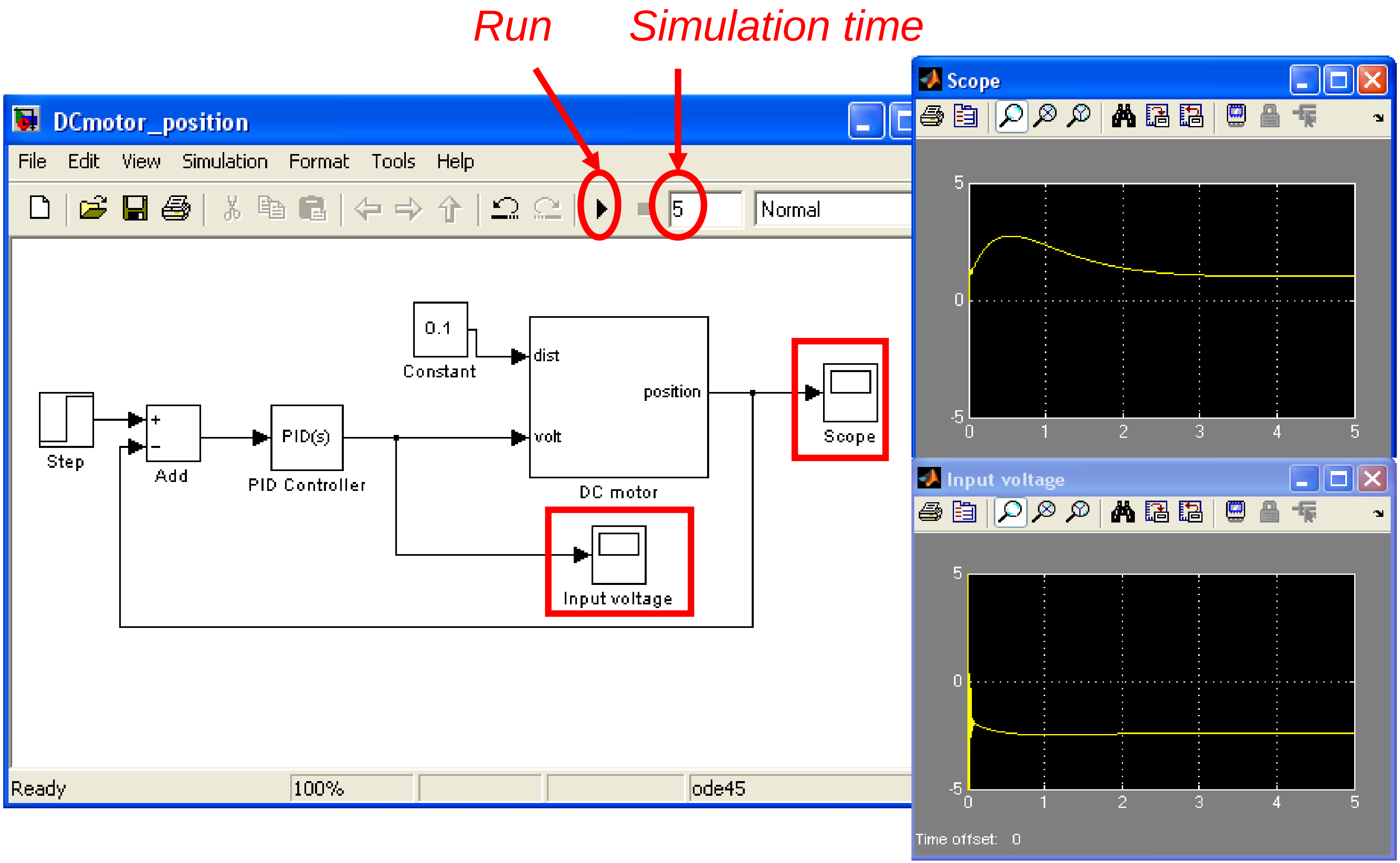
$kg \cdot m^2$
$Nm/[rad/sec]$
$Volt/[rad/sec]$
Nm/A
H
Ω

Create a model (cont'd)

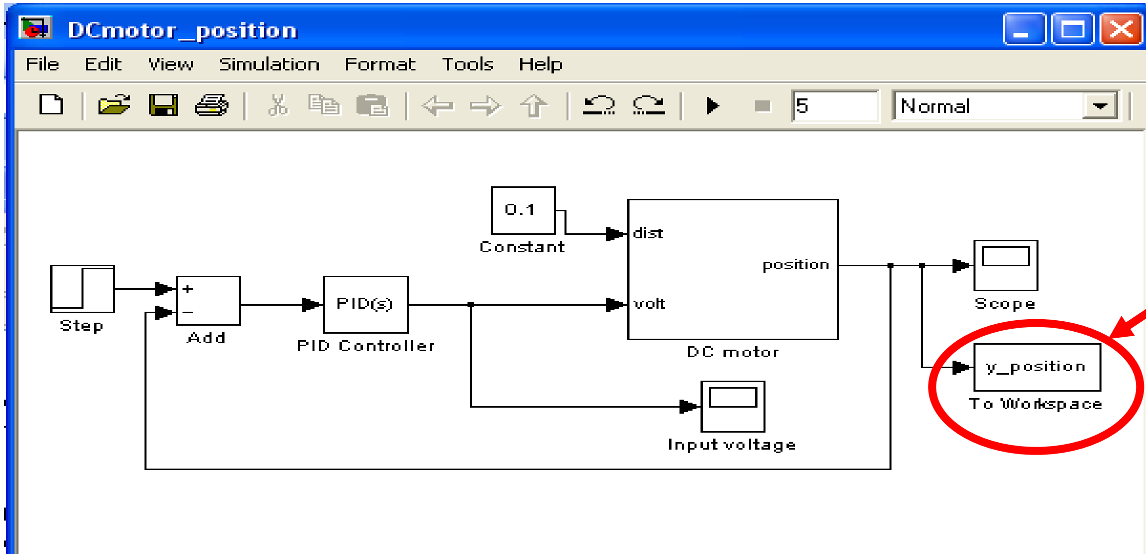


Select blocks from appropriate libraries, and connect them.

Simulate a model



Export data in workspace



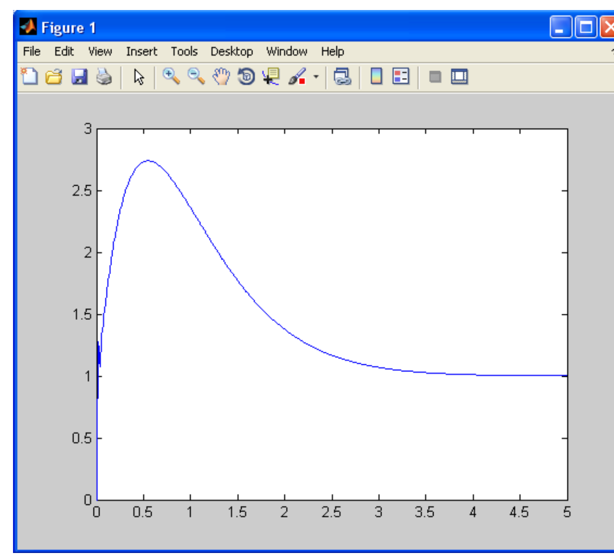
Export to workspace

```
>> who

Your variables are:

Bm          Kb          La          tout
Jm          Ki          Ra          y_position

>> plot(tout,y_position)
```

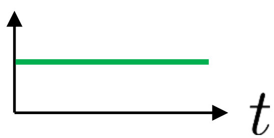
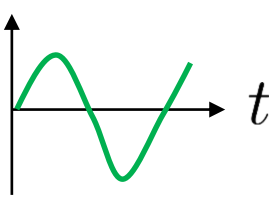
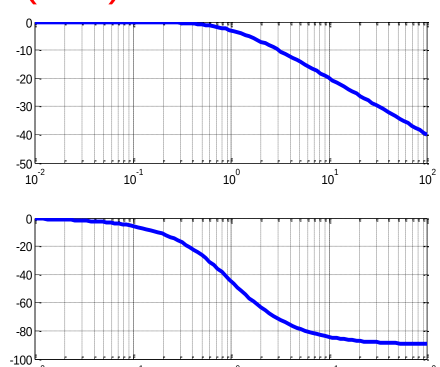
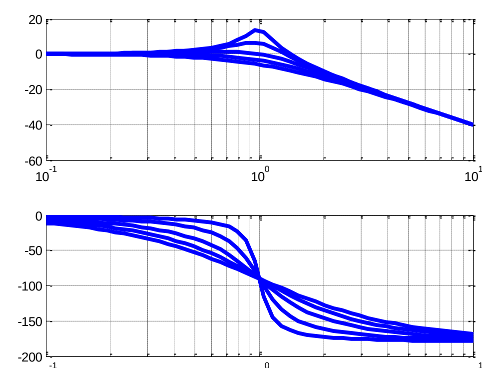


MATLAB & Simulink Tutorials (They are all free!)

- Interactive MATLAB & Simulink Based Tutorials (by Mathworks Inc.)

  http://www.mathworks.com/academia/student_center/tutorials/
  - MATLAB Tutorial
  - Simulink Tutorial
  - Signal Processing Tutorial
  - Control Systems Tutorial
  - Computational Mathematics Tutorial
- Control Tutorials for MATLAB & Simulink (by University of Michigan) ctms.engin.umich.edu

Response analyses (useful for modeling and controller design)

$G(s)$	$\dfrac{K}{Ts+1}$	$\dfrac{K\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$	
Step response 	(L16) 	(L17) underdamped 	(L20) overdamped 
Frequency response (L18) 	(L19) Slide 13 	(L19) Slide 16 	

Step response of 2nd-order system for various damping ratios (review)

- Undamped

$$\zeta = 0$$

- Underdamped

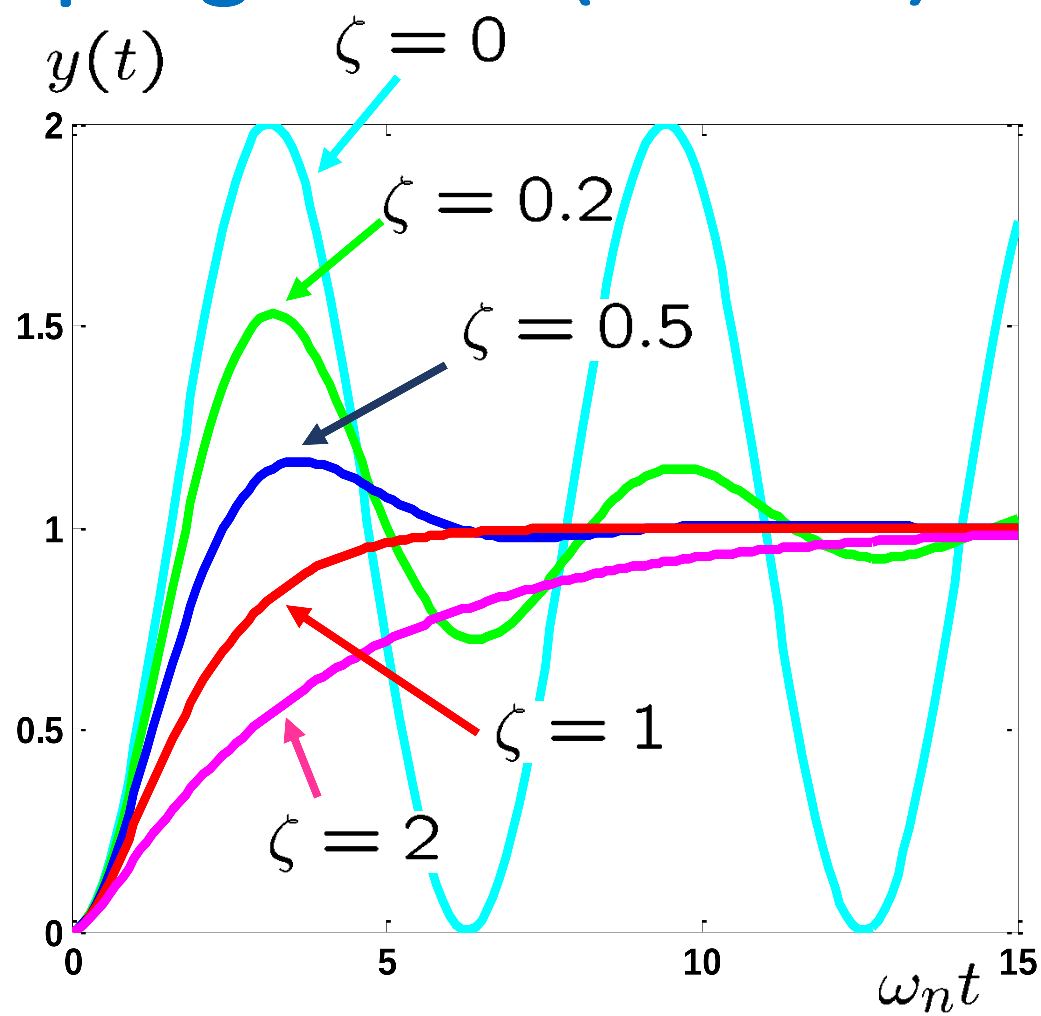
$$0 < \zeta < 1$$

- Critically damped

$$\zeta = 1$$

- Overdamped

$$\zeta > 1$$



Step response of 2nd-order system

Underdamped case $0 < \zeta < 1$

- Math expression of *y(t)*

$$Y(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \cdot \frac{1}{s}$$

$$\xrightarrow{\mathcal{L}^{-1}} \boxed{y(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin\left(\omega_d t + \cos^{-1}\zeta\right)}$$

Damped natural frequency → $\omega_d := \omega_n\sqrt{1-\zeta^2}$

Step response of 2nd-order system
Critically damped case $\zeta = 1$
Overdamped case $\zeta > 1$

- Math expression of *y(t)*

$$Y(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \cdot \frac{1}{s}$$

$$\xrightarrow{\mathcal{L}^{-1}}$$

$$\boxed{y(t) = 1 - (\omega_n t + 1)e^{-\omega_n t}} \quad (\zeta = 1)$$

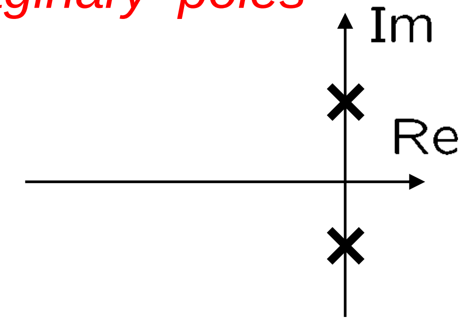
$$\boxed{y(t) = 1 - \frac{1}{2\omega_n\sqrt{\zeta^2 - 1}}(\lambda_1 e^{\lambda_2 t} - \lambda_2 e^{\lambda_1 t})} \quad (\zeta > 1)$$

$$\lambda_1 := -\zeta\omega_n + \sqrt{\zeta^2 - 1}$$
$$\lambda_2 := -\zeta\omega_n - \sqrt{\zeta^2 - 1}$$

Pole locations & damping

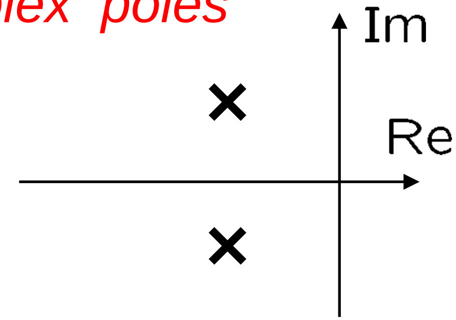
- Undamped

Imaginary poles



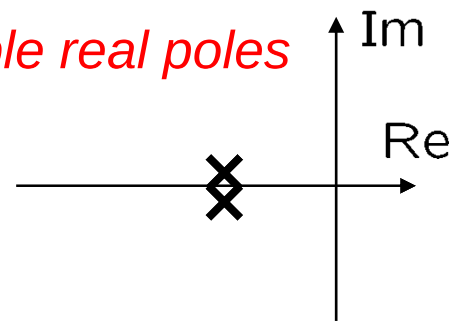
- Underdamped

Complex poles



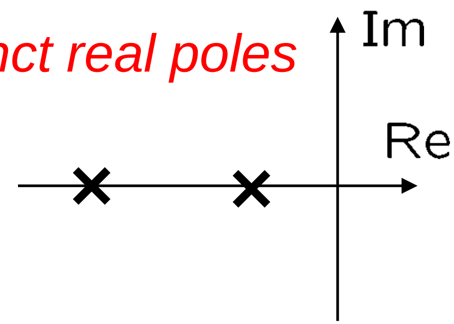
- Critically damped

Double real poles



- Overdamped

Distinct real poles



Example of overdamped system

- Consider a 2nd order overdamped system

$$G(s) = \frac{10}{s^2 + 11s + 10} = \frac{10}{(s+1)(s+10)}$$

For unit step input, obtain (or estimate)

  - steady state value,
  - 2% settling time, and
  - percent overshoot.

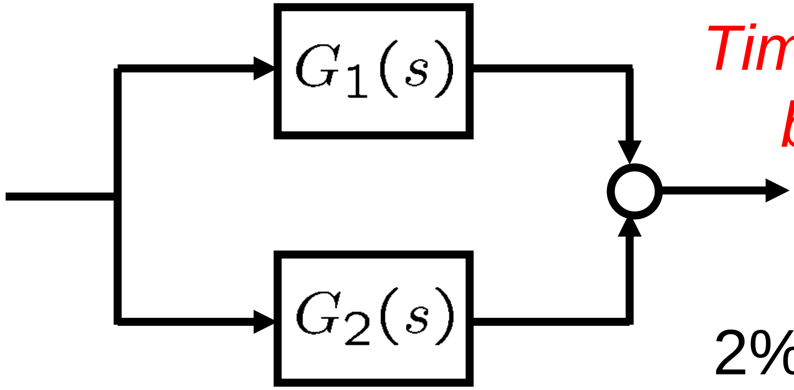
- Steady state value is the DC gain *G(0)=1*.
- No overshoot (overdamped!)

Example (cont'd)

- Poles are -1 and -10.

$$G(s) = \frac{10}{s^2 + 11s + 10} = \underbrace{\frac{A}{s+1}}_{G_1(s)} + \underbrace{\frac{B}{s+10}}_{G_2(s)}$$

Time constant = 1



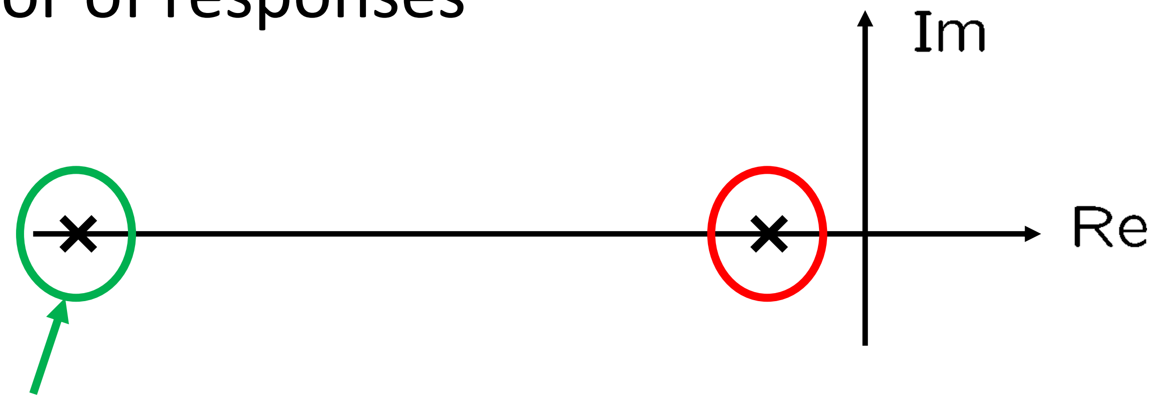
Time constant = 0.1

Time constant of G(s) is estimated by a slower subsystem $G_1(s)$

2% settling time : about 4 seconds

Example (cont'd)

- Dominant poles: Poles closest to the imaginary axis and far away from remaining poles dominate the behavior of responses

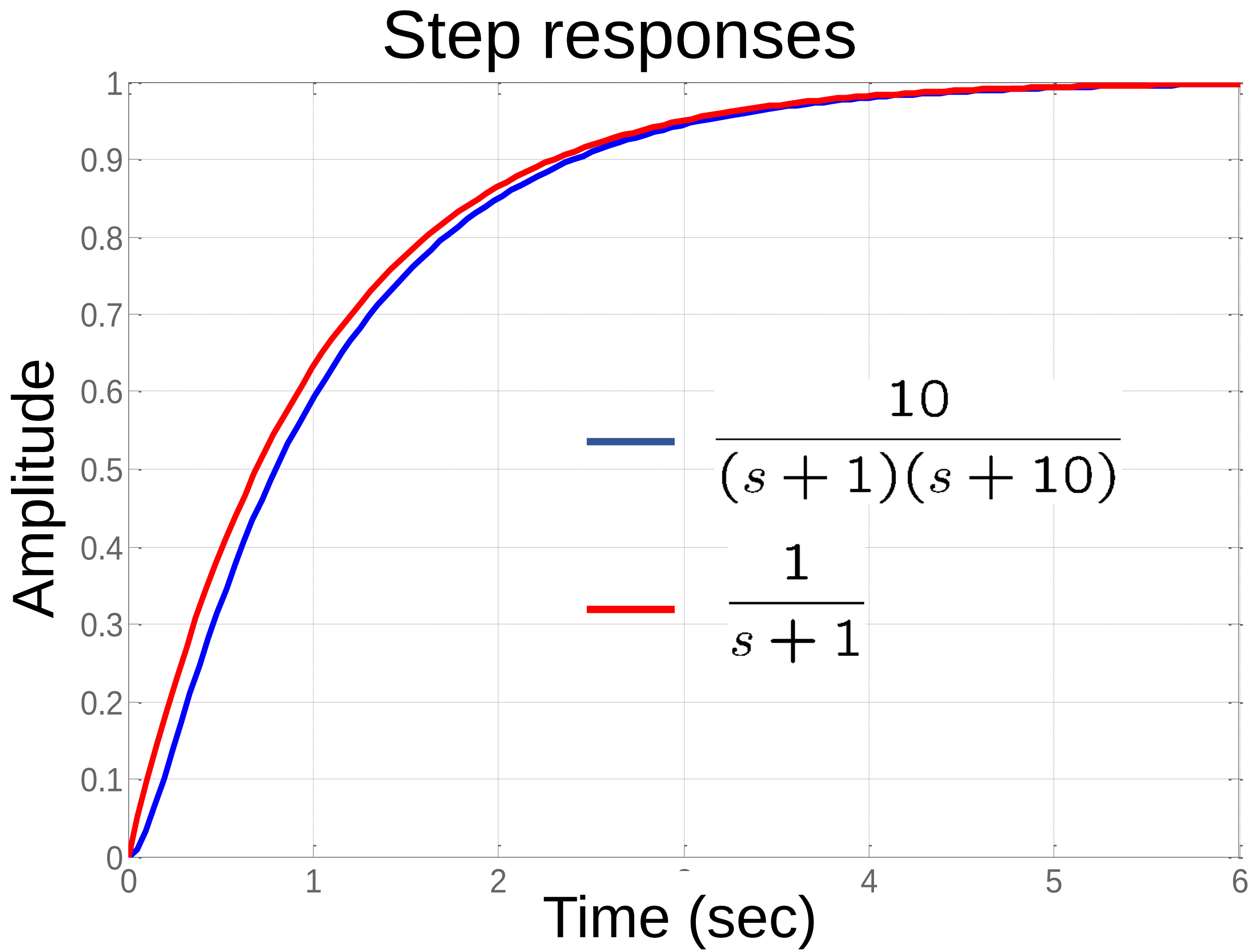


Poles far left (5-10 times) from dominant poles may be ignored.

$$G(s) = \frac{10}{(s+1)(s+10)} \approx \frac{1}{s+1}$$

Same DC gain

Example (cont’d)



Summary

- Today's topics
  - Simulink
  - Step response of second-order overdamped systems
- Next is the last class!
  - Stability
  - Course summary
- Project: Friday Nov 29 (presentation)
- Lab 4 report: Due Nov 25 (Monday), 6pm