

Selected Solutions from Tutorials

Adapted from Shigley, Problem 13-16 (8th Ed., 13-14).

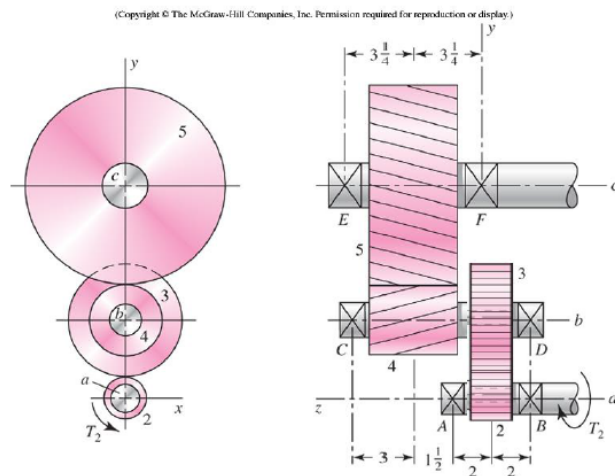
The double-reduction gearset shown in the figure is driven through shaft *a* at a speed of 900 rev/min. Gears 2 and 3 are spur gears and have a diametral pitch of 10 teeth/in and a pressure angle of 20°. The second pair of gears in the train, gears 4 and 5, are helical gears with a normal diametral pitch of 6 teeth/in, a 25° helix angle, and a normal pressure angle of 20° (see Figure 13-22). The tooth numbers for the four gears are: $N_2 = 14$, $N_3 = 54$, $N_4 = 16$, and $N_5 = 36$.

Part 1 (To be demonstrated by the TA). For the first stage (the spur gears) find:

- d. The speed and direction of shaft *b*
- e. The centre distance between shafts *a* and *b*

Part 2 (To be done by students). For the second stage (the helical gears) find:

- f. The speed and direction of shaft *c*
- g. The centre distance between shafts *b* and *c*
- h. The directions of the thrust forces exerted by gears 4 and 5 on shafts *b* and *c*, respectively



2. Adapted from Shigley, Problem 13-16.

a) Find the speed and direction of shaft *b* (same as speed and direction of gear 3)

$$\begin{aligned}
 n_3 &= e_{23} n_2 = -\frac{\text{driving teeth}}{\text{driven teeth}} n_2 \\
 &= -\frac{N_2}{N_3} n_2 = -\frac{14}{54} (-900) \\
 &= 233 \text{ rev/min (CW)}
 \end{aligned}$$

b) Find the centre to centre distance between the shafts *a* and *b*.

When properly spaced, pitch circles of gears 2 and 3 are in contact. The shaft to shaft distance is half of the sum of the two pitch circle diameters.

$$P = \frac{N}{d} \quad (\text{Equation 13-1})$$

where

P = diametral pitch (number of teeth per inch)

N = number of teeth

d = diameter (inch)

$$d_2 = \frac{N_2}{P} = \frac{14 \text{ teeth}}{10 \text{ teeth/in}} = 1.4 \text{ inch}$$

$$d_3 = \frac{N_3}{P} = \frac{54 \text{ teeth}}{10 \text{ teeth/in}} = 5.4 \text{ inch}$$

$$\text{Shaft-to-shaft distance, } y_{23}, = \frac{d_2 + d_3}{2} = \frac{1.4 + 5.4}{2} = 3.4 \text{ inch}$$

c) Find the speed and direction of shaft *c* (same as speed and direction of gear 5).

Note: gear 3 and gear 4 are on common shaft $\rightarrow n_3 = n_4$

$$\begin{aligned} n_5 &= e_{45} n_4 = -\frac{\text{driving teeth}}{\text{driven teeth}} n_4 \\ &= -\frac{N_4}{N_5} n_4 = -\frac{16}{36} (-233) \\ &= 103.6 \text{ rev/min (CCW)} \end{aligned}$$

d) Find the centre to centre distance between the shafts *b* and *c*.

We are given normal diametral pitch (measured perpendicular to teeth). For a helical gear, this needs to be converted to transverse diametral pitch (measured around the circumference). See Fig 13-22 and Eq. 13-18 in text.

$$\begin{aligned} P_{\text{transverse}} &= P_{\text{normal}} \cos \psi \\ &= 6 \cos 25^\circ \\ &= 5.44 \text{ teeth/in} \end{aligned}$$

We expect to cross fewer teeth per inch in the transverse direction so this makes sense.

The rest of the solution is the same as above.

$$P_t = \frac{N}{d}$$

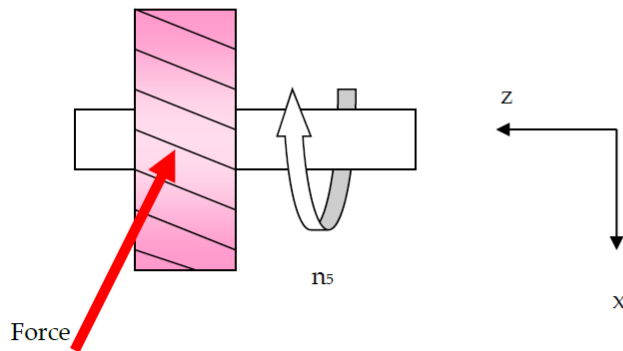
$$d_4 = \frac{N_4}{P_t} = \frac{16 \text{ teeth}}{5.44 \text{ teeth/in}} = 2.94 \text{ inch}$$

$$d_5 = \frac{N_5}{P_t} = \frac{36 \text{ teeth}}{5.44 \text{ teeth/in}} = 6.61 \text{ inch}$$

$$\text{Shaft-to-shaft distance, } y_{45}, = \frac{d_4 + d_5}{2} = \frac{2.94 + 6.61}{2} = 4.78 \text{ inch}$$

e) Find the direction of the thrust force exerted by gears 4 and 5 on shafts b and c, respectively.

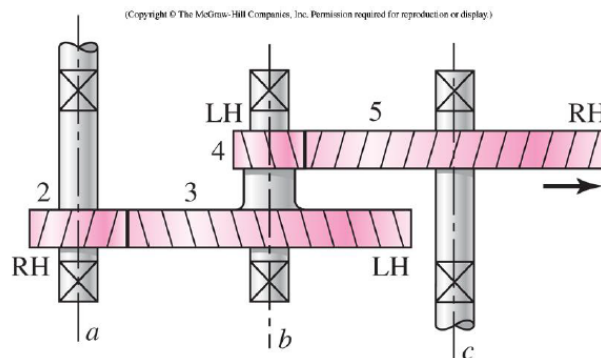
Consider looking at gear 5 and shaft *c* from below. The direction of rotation is as shown. In order to rotate this way, the contact force of gear 4 on gear 5 must be directed as indicated (note: the force is normal to the teeth). This force has a transverse component in the $-z$ direction. Similar arguments can be used for shaft *b*.



Force on shaft *b* is in $+z$ direction; Force on shaft *c* is in $-z$ direction.

Shigley, Problem 13-47 (8th Ed., 13-33). (To be done by students.)

A gear train is composed of four helical gears with the three shaft axes in a single plane, as shown in the figure. The gears have a normal pressure angle of 20° and a 30° helix angle. Shaft *b* is an idler and the transmitted load acting on gear 3 is 500 lbf. The gears on shaft *b* both have a normal diametral pitch of 7 teeth/in and have 54 and 14 teeth, respectively. Find the forces exerted by gears 3 and 4 on shaft *b* (don't worry about determining moments)

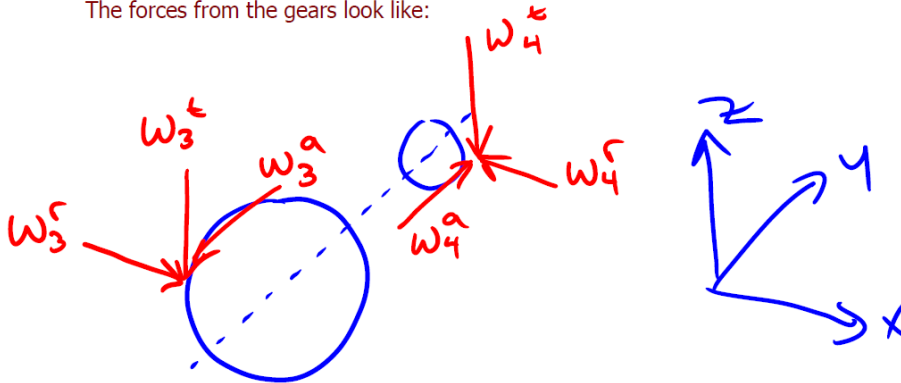


3. **Shigley, Problem 13-47. (To be done by students.)**

Find the forces exerted by gears 3 and 4 on shaft *b*.

Refer to Fig 13-37 (pg. 704)

The forces from the gears look like:



$$W_3^t = 500 \text{ lb (given)}$$

From Eq. 13-39 (pg. 704)

$$W_r = W \sin \phi_n$$

$$W_t = W \cos \phi_n \cos \psi$$

$$W_a = W \cos \phi_n \sin \psi$$

Where $\psi = 30^\circ$ (given) and $\phi_n = 20^\circ$ (given)

Solving for W , W_r , and W_a we find

$$W_3 = 614.4 \text{ lb}$$

$$W_3^r = 210.2 \text{ lb}$$

$$W_3^a = 288.7 \text{ lb}$$

Since gears 3 and 4 are on a common shaft (assumed rotating at constant velocity) the torques must balance. Therefore

$$W_3^t d_3 = W_4^t d_4$$

$$\begin{aligned} W_4^t &= W_3^t d_3 / d_4 \\ &= W_3^t (N_3 / P_{3t}) / (N_4 / P_{4t}) \\ &= W_3^t N_3 / N_4 && \text{(since } P_{3t} = P_{4t}) \\ &= (500 \text{ lb})(54) / (14) \\ &= 1929 \text{ lb} \end{aligned}$$

Solving for W_r and W_a on gear 4, as above, we find

$$W_4^r = 811 \text{ lb}$$

$$W_4^a = 1114 \text{ lb}$$

The total force on shaft *b* is the sum of the forces from both gears. Referring to the diagram at the start for directions:

$$\begin{aligned} \vec{W} &= ((210 - 811)\hat{i} + (-289 + 1114)\hat{j} + (-500 - 1929)\hat{k}) \text{ lb} \\ &= (-601\hat{i} + 825\hat{j} - 2429\hat{k}) \text{ lb} \end{aligned}$$

Adapted from Shigley, Problem 14-19.

A commercial enclosed gear drive consists of a 20° pinion having 16 teeth driving a 48-tooth gear. The pinion speed is 300 rev/min, the face width is 2 in, and the diametral pitch is 6 teeth/in. The gears are grade 1 steel, through-hardened at 200 Brinell, made to No. 6 quality standards, uncrowned, and are to be accurately and rigidly mounted. Assume a pinion life of 10⁸ cycles and a reliability of 0.90. The power to be transmitted is 5 hp.

Part 1: To be demonstrated by the TA.

- a. Determine the AGMA bending stress in the pinion
- b. Determine the corresponding safety factor for the pinion bending stress

Part 2: To be done by students.

- c. Determine the AGMA contact stress in the pinion
- d. Determine the corresponding safety factor for the pinion contact stress

Part 3: Extra practice, to be done by students at home.

- e. Repeat the above bending and contact stress analysis for the gear and show that the stresses are 9.59 ksi and 102.1 ksi, respectively, and that the safety factors in bending and contact are 3.29 and 1.02, respectively.

Part 1: To be demonstrated by the TA.

a. Determine the AGMA bending stress in the pinion

Working from the “roadmap” in Fig 14-17 (p. 758),

$$d_p = \frac{N_p}{P_d} = \frac{16 \text{ teeth}}{6 \text{ teeth/in}} = 2.667 \text{ in} \quad (\text{pinion diameter})$$

$$V = \frac{\pi d n}{12} = \frac{\pi (2.667 \text{ in})(300 \text{ rev/min})}{12} \quad (\text{pitch line velocity, note the units})$$
$$= 209 \text{ ft/min}$$

$$W^t = \frac{33,000H}{V} = \frac{33,000(5 \text{ hp})}{209 \text{ ft/min}} = 789 \text{ lb} \quad (\text{transmitted load})$$

the gear bending stress equation is given by

$$\sigma = W^t K_o K_v K_s \frac{P_d}{F} \frac{K_m K_b}{J} \quad (14-15, \text{ p. 738})$$

We know W^t from above, we are given $P_d = 6 \text{ teeth/in}$, and we are given the face width, $F = 2 \text{ in}$. To compute σ , we need to solve for all of the coefficients.

K_o: overload factor

Since we don't have any other information, assume the power source and the driven machine have uniform loading. From the table on the bottom of p. 758,

$$K_o = 1$$

K_v: velocity factor

Although we could use Figure 14-9 (p. 750) to find K_v, it would be difficult to get an accurate value due to the steepness of the curve and low resolution of the V values on the x-axis.

Instead, use the expressions in eqns. 14-27 and 14-28 on p. 748.

$$K_v = \left(\frac{A + \sqrt{V}}{A} \right)^B$$
$$A = 50 + 56(1 - B)$$
$$B = 0.25(12 - Q_v)^{2/3}$$

With V = 209 ft/min (found above) and Q_v = 6 (given in question) we have

$$B = 0.8255$$

$$A = 59.77$$

$$K_v = 1.196$$

$$K_v = 1.196$$

K_s: size factor

From eq. (a) on p. 751

$$K_s = 1.192 \left(\frac{F \sqrt{Y}}{P_d} \right)^{0.0535}$$

F = 2in (given), Y = 0.296 for the pinion (see Table 14-2, p. 730), P_d = 6 (given)

$$K_s = 1.192 \left(\frac{(2 \text{ in}) \sqrt{0.296}}{6 \text{ teeth/in}} \right)^{0.0535} = 1.088$$

$$K_s = 1.088$$

K_m: Load-distribution factor

From eq. 14-30 (p. 751)

$$K_m = 1 + C_{mc} (C_{pf} C_{pm} + C_{ma} C_e)$$

The "C" terms are all given on pp. 752-3.

$$C_{mc} = 1 \quad (\text{for uncrowned teeth})$$

$$\begin{aligned}
 C_{pf} &= \frac{F}{10d} - 0.0375 + 0.0125F \quad \text{for } 1" < F \leq 17" \\
 &= \frac{2 \text{ in}}{10(2.667 \text{ in})} - 0.0375 + 0.0125(2 \text{ in}) \\
 &= 0.0625
 \end{aligned}$$

$$C_{pm} = 1 \quad (\text{we don't know the pinion mounting but have no reason to assume it isn't centered})$$

$$C_{ma} = 0.16 \quad (\text{from Fig 14-11; could also use eq. 14-34 \& Table 14-9})$$

(note the error in Shigley's solutions; taken for precision enclosed units, not commercial enclosed units)

$$C_e = 1 \quad (\text{eq. 14-35, no information to suggest it should be 0.8})$$

Putting it all together,

$$\begin{aligned}
 K_m &= 1 + 1(0.0625 \cdot 1 + 0.16 \cdot 1) \\
 &= 1.223
 \end{aligned}$$

$$K_m = 1.223$$

K_b: rim thickness factor

From eq. 14-40 and Fig. 14-16 (p. 756), no reason to believe $m_b < 1.2$, so use

$$K_b = 1$$

J: Bending-Strength Geometry Factor

From Fig. 14-6 with $N_p = 16$ = "number of teeth for which geometry factor is desired" and load applied at highest point of single-tooth contact

$$J = 0.27$$

Finally, we can insert all of the coefficients into the stress expression

$$\begin{aligned}
 \sigma &= W^t K_o K_v K_s \frac{P_d}{F} \frac{K_m K_b}{J} \\
 &= (789 \text{ lb})(1)(1.20)(1.088) \frac{6 \text{ teeth/in}}{2 \text{ in}} \frac{(1.223)(1)}{0.27} \\
 &= 14.00 \text{ ksi}
 \end{aligned}$$

The bending stress on the pinion is 14.00 ksi.

b. Determine the corresponding safety factor for the pinion in bending

To find the safety factor in pinion bending, use the allowable stress in eq. 14-41 (p. 757)

$$S_F = \frac{S_t}{\sigma} \frac{Y_N}{K_T K_R}$$

The σ is the stress calculated in part a. The other values are found in a similar manner to above – by looking up the data in the tables and charts as referenced on p. 757).

S_t : bending strength (Fig. 14-2, p. 739)

$$\begin{aligned} S_t &= 77.3H_B + 12,800 \text{ psi} \\ &= 77.3(200) + 12,800 \text{ psi} \quad (H_B = 200 \text{ is the Brinell hardness}) \\ &= 28.3 \text{ ksi} \end{aligned}$$

Y_N : repeated bending stress cycle factor

$$\begin{aligned} \text{With } 10^8 \text{ cycles, } Y_N &\approx 0.95 \quad (\text{read from Fig 14-14, pg. 755}) \\ &\quad (\text{Shigley uses a less conservative value}) \end{aligned}$$

K_T : Temperature factor

$$\text{Since no reason to expect } T > 250^\circ\text{F, } K_T = 1 \quad (\text{Section 14-15, p. 756})$$

K_R : Reliability factor

$$\text{With reliability of 0.90, } K_R = 0.85 \quad (\text{from Table 14-10, p. 756})$$

Combining all terms,

$$S_F = \frac{28.3 \text{ ksi}}{14.0 \text{ ksi}} \frac{0.95}{1 \cdot 0.85} = 2.26$$

The safety factor for the pinion in bending is 2.26.
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c. Determine the AGMA contact stress in the pinion

The process is the same as above. The required equation is eq. 14-16 (p. 759)

$$\sigma_c = C_p \sqrt{W^t K_o K_v K_s \frac{K_m}{d_p F} \frac{C_f}{I}}$$

The terms $W^t = 789 \text{ lb}$, $K_o = 1$, $K_v = 1.196$, $K_s = 1.088$, $K_m = 1.223$, $d_p = 2.667''$, and $F = 2''$ were either given or solved for in the previous part of the problem. It remains to determine C_p , C_f , and I .

C_p : Elastic coefficient

$$C_p = 2300 \text{ psi}^{1/2} \quad (\text{from steel pinion-steel gear in Table 14-8, p. 749})$$

C_f : Surface condition coefficient

$$C_f = 1 \quad (\text{as noted on p. 759})$$

I : Surface-Strength Geometry Factor

$$I = \frac{\cos \phi_t \sin \phi_t}{2m_N} \frac{m_G}{m_G + 1} \quad (\text{eq. 14-23, p. 747, internal gears})$$

As noted below eq. 14-23 (p. 747) $m_N = 1$ for spur gears.

m_G is the speed ratio as given in eq. 14-22 (p. 746)

$$\begin{aligned} m_N &= \frac{N_G}{N_P} = \frac{48}{16} \\ &= 3 \end{aligned}$$

For a spur gear, ϕ_t is just ϕ , the pressure angle. As given in the problem description, $\phi = 20^\circ$ in this case.

Substituting to the expression for I above, we get

$$\begin{aligned} I &= \frac{\cos 20^\circ \sin 20^\circ}{2 \cdot 1} \frac{3}{3 + 1} \\ &= 0.1205 \end{aligned}$$

Plugging everything into the expression for σ_c we get

$$\begin{aligned}\sigma_c &= 2300 \sqrt{(789 \text{ psi}^{1/2})(1)(1.196)(1.088) \frac{1.223}{(2.667 \text{ in})(2 \text{ in})} \frac{1}{0.1205}} \\ &= 101.7 \text{ ksi}\end{aligned}$$

The contact stress is 101.7 ksi

d. Determine the corresponding safety factor for the pinion in terms of contact stress

The safety factor is given by eq. 14-42 (p. 759)

$$S_H = \frac{S_c Z_N C_H}{K_T K_R} \frac{1}{\sigma_c}$$

The same process as before is used. In part a., it was already determined that $K_T = 1$ and $K_R = 0.85$. For σ_c we use our result from part c., $\sigma_c = 101.7 \text{ ksi}$.

S_c : Contact-fatigue strength

$$\begin{aligned}S_c &= 322H_B + 29,100 \text{ psi} \\ &= 322(200) + 29,100\end{aligned} \quad (\text{Fig 14-5, p. 742})$$

$$S_c = 93,500 \text{ psi}$$

Z_N : Stress Cycle Factor

From Figure 14-15 (p. 755) with 10^8 cycles, choose the centre of the shaded region (more conservative than Shigley)

$$Z_N = 0.90$$

C_H : hardness ratio factor

As noted in Sec 14-12 (p. 753), C_H applies to the gear only. We can either neglect the term or use $C_H = 1$, with the same effect.

$$C_H = 1$$

The final expression is

$$S_H = \frac{(93.5 \text{ ksi})(0.90)(1)}{(1)(0.85)} \frac{1}{101.7 \text{ ksi}}$$

$S_H = 0.97$

With a safety factor less than unity, pinion wear is a concern so hardness should be increased. (It won't disintegrate immediately, but it will not last 10^8 cycles with 0.90 reliability.)

Adapted from **Shigley, Problem 17-2** (8th and 9th Eds., 17-1).

A 6-in-wide polyamide F-1 flat belt is used to connect a 2-in-diameter pulley to drive a larger pulley with an angular velocity ratio of 0.5. The centre-to-centre distance is 9 ft. The angular speed of the small pulley is 1750 rev/min as it delivers 2 hp. The application is such that a service factor K_s of 1.25 is appropriate.

Part 1: To be demonstrated by the TA.

a. Find F_c , F_i , F_{1a} , and F_2

$d = 2 \text{ in}$ (drive pulley diameter, given)
 $D = 2 \text{ in} / 0.5 = 4 \text{ in}$ (driven pulley diameter based on 0.5 velocity ratio)
 $b = 6 \text{ in}$ (belt width, given)

From Table 17-2 (p. 869) for a polyamide F-1 flat belt

$t = 0.05 \text{ in}$ (thickness)
 $\gamma = 0.035 \text{ lbf/in}^3$ (specific weight)
 $f = 0.5$ (coefficient of friction)
 $F_a = 35 \text{ lb/in}$ (allowable tension per width at 600 ft/min)

F_c = hoop tension due to centrifugal force

$$F_c = \frac{w}{g} \left(\frac{V}{60} \right)^2 = \frac{w}{32.17} \left(\frac{V}{60} \right)^2 \quad (\text{eq. 17-8e, p. 885})$$

$$\begin{aligned} V &= \pi d n / 12 \\ &= \pi (2 \text{ in}) (1750 \text{ rev/min}) / 12 \quad (\text{belt speed in ft/min, p. 885}) \\ &= 916.3 \text{ ft/min} \end{aligned}$$

$$\begin{aligned}
 w &= 12\gamma bt \\
 &= 12(0.035 \text{ lb/in}^3)(6 \text{ in})(0.05 \text{ in}) \quad (\text{belt weight per foot, p. 885}) \\
 &= 0.126 \text{ lb/ft}
 \end{aligned}$$

$$\begin{aligned}
 F_c &= \frac{0.126 \text{ lb/ft}}{32.17} \left(\frac{916.3 \text{ ft/s}}{60} \right)^2 \\
 &= 0.913 \text{ lb}
 \end{aligned}$$

$F_c = 0.913 \text{ lb}$

F_{1a} = allowable largest tension

$$F_{1a} = F_a b C_p C_v \quad (\text{eq. 17-12, p. 888})$$

$$\begin{aligned}
 F_a &= 35 \text{ lb/in} & (\text{allowable tension per width, Table 17-2, p. 889}) \\
 b &= 6 \text{ in} & (\text{belt width, given})
 \end{aligned}$$

$$C_p = 0.70 \quad (\text{Table 17-4, p. 890, with 2 in small pulley diameter})$$

$$C_v = 1 \quad (\text{velocity correction factor, =1 for polyamide and urethane belts, see note on p. 887})$$

$$\begin{aligned}
 F_{1a} &= (35 \text{ lb/in})(6 \text{ in})(0.70)(1.0) \\
 &= 147 \text{ lb}
 \end{aligned}$$

$F_{1a} = 147 \text{ lb}$

F_2 = loose side tension

$$\begin{aligned}
 F_2 &= F_{1a} - \Delta F \\
 &= F_{1a} - 2T/d
 \end{aligned}$$

Need to find T

$$T = H_d / \omega \quad (\text{torque transmitted} = \text{power supplied} / \text{angular velocity})$$

$$\begin{aligned}
 H_d &= H_{\text{nom}} K_s n_d & (\text{see note on p. 888}) \\
 & & H_d = \text{power delivered} \\
 & & H_{\text{nom}} = \text{nominal power delivered} = 2 \text{ hp in this case, given} \\
 & & K_s = \text{service factor (for deviations from normal load)} \\
 & & \quad = 1.25 \text{ in this case, given} \\
 & & n_d = \text{design safety factor} = 1 \text{ in this case, implied}
 \end{aligned}$$

$T = H_d / \omega$ is in SI units, in units of hp, lb·in, and rev/min the expression is

$$\begin{aligned}
 T &= \frac{63,025 H_d}{n} \\
 &= \frac{63,025 H_{nom} K_s n_d}{n} \\
 &= \frac{63,025 (2 \text{ hp})(1.25)(1)}{1750 \text{ rev/min}} \\
 &= 90.0 \text{ lb}\cdot\text{in}
 \end{aligned}$$

$$\begin{aligned}
 F_2 &= F_{1a} - 2T/d \\
 &= 147 \text{ lb} - 2(90.0 \text{ lb}\cdot\text{in})/(2 \text{ in}) \\
 &= 57 \text{ lb}
 \end{aligned}$$

$$F_2 = 57 \text{ lb}$$

F_i = initial tension

$$\begin{aligned}
 F_i &= 0.5 (F_1 + F_2) - F_c && \text{(eq. 17-8j, p. 886)} \\
 &= 0.5 (147 \text{ lb} + 57 \text{ lb}) - 0.916 \text{ lb} \\
 &= 101.1 \text{ lb}
 \end{aligned}$$

$$F_i = 101.1 \text{ lb}$$

Part 2: To be done by students.

b. Find the allowable power, H_a

$$\begin{aligned}
 H_a &= \text{transmitted power with maximum allowable tension used} \\
 H_a &= (F_{1a} - F_2) \cdot V / 33,000 \text{ (from eq. (j), p. 887, note: the 33,000 is so} \\
 &\quad \text{units work in hp, lb, and ft/min)} \\
 &= (147 \text{ lb} - 57 \text{ lb}) \cdot (916.3 \text{ ft/min}) / 33,000 \\
 &= 2.5 \text{ hp}
 \end{aligned}$$

$$H_a = 2.5 \text{ hp}$$

c. Find the factor of safety, n_{fs}

Note: you may need to explain in words how to compute the factor of safety

$$\begin{aligned}
 n_{fs} &= (\text{allowable power}) / [(\text{nominal power}) \cdot (\text{service factor})] \\
 &= H_a / [H_{nom} \cdot K_s] \\
 &= (2.5 \text{ hp}) / [(2 \text{ hp})(1.25)] \quad \text{(transmitted power found above,)} \\
 &= 1
 \end{aligned}$$

$$n_{fs} = 1$$

d. Find the belt length, L

$$L = \sqrt{4C^2 - (D - d)^2} + \frac{1}{2}(D\theta_D + d\theta_d) \quad (\text{eq. 17-2, p. 883})$$

C = centre distance = 9 ft (given)

d = small pulley diameter = 2 in (given)

D = large pulley diameter = 4 in (found above, $d/0.05$)

$$\begin{aligned}\theta_d &= \pi - 2 \sin^{-1} \frac{D - d}{2C} \\ &= \pi - 2 \sin^{-1} \left(\frac{4 \text{ in} - 2 \text{ in}}{2 \cdot 9 \text{ ft} \cdot 12 \text{ in/ft}} \right) \quad (\text{eq. 17-1, p. 883, must use radians}) \\ &= 3.132 \text{ rad}\end{aligned}$$

$$\begin{aligned}L &= \sqrt{4(9 \text{ ft} \cdot 12 \text{ in/ft})^2 - (4 \text{ in} - 2 \text{ in})^2} + \frac{1}{2}[(4 \text{ in})(3.151) + (2 \text{ in})(3.132)] \\ &= 225.4 \text{ in}\end{aligned}$$

L = 225.4 in

e. Find the dip

$$\begin{aligned}\text{dip} &= \frac{C^2 w}{96 F_t} \\ &= \frac{(9 \text{ ft} \cdot 12 \text{ in/ft})^2 (0.126 \text{ lb/ft})}{96 (101.1 \text{ lb})} \quad (\text{e.q. 17-13, p. 892}) \\ &= 0.151 \text{ in}\end{aligned}$$

(Note: the mismatched units are accounted for in the coefficient 96; see p. 892 for details.)

Dip = 0.151 in

A polyamide A-3 flat belt 6 in wide is used to transmit 15 hp under light shock conditions where $K_s = 1.25$, and a factor of safety equal to or greater than 1.1 is appropriate. The pulley rotational axes are parallel and in the horizontal plane. The shafts are 8 ft apart. The 6-in driving pulley rotates at 1750 rev/min in such a way that the loose side is on top. The driven pulley is 18 in in diameter. See Fig. 17–10. The factor of safety is for unquantifiable exigencies.

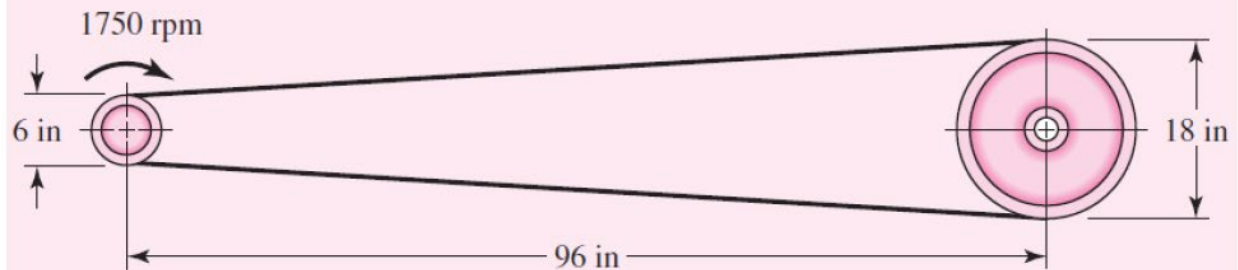
- (a) Estimate the centrifugal tension F_c and the torque T .
 (b) Estimate the allowable F_1 , F_2 , F_i and allowable power H_a .
 (c) Estimate the factor of safety. Is it satisfactory?

Belt 6 in \times 0.130 in

15 hp

$$\gamma = 0.042 \frac{\text{lbf}}{\text{in}^3}$$

$d = 6 \text{ in}$, $D = 18 \text{ in}$



(a) Eq. (17-1):
$$\phi = \theta_d = \pi - 2 \sin^{-1} \left[\frac{18 - 6}{2(8)12} \right] = 3.0165 \text{ rad}$$

Table 17-2:
$$\gamma = 0.042 \text{ lbf/in}^3 \quad f = 0.8 \quad F_a = 100 \text{ lbf/in}$$

$$\exp(f\phi) = \exp[0.8(3.0165)] = 11.17$$

$$V = \pi(6)1750/12 = 2749 \text{ ft/min}$$

$$w = 12\gamma bt = 12(0.042)6(0.130) = 0.393 \text{ lbf/ft}$$

Eq. (e):
$$F_c = \frac{w}{g} \left(\frac{V}{60} \right)^2 = \frac{0.393}{32.17} \left(\frac{2749}{60} \right)^2 = 25.6 \text{ lbf}$$

$$T = \frac{63\,025 H_{\text{nom}} K_s n_d}{n} = \frac{63\,025 (15) 1.25 (1.1)}{1750}$$

$$= 742.8 \text{ lbf} \cdot \text{in}$$

(b) The necessary $(F_1)_a - F_2$ to transmit the torque T , from Eq. (h), is

$$(F_1)_a - F_2 = \frac{2T}{d} = \frac{2(742.8)}{6} = 247.6 \text{ lbf}$$

For polyamide belts $C_v = 1$, and from Table 17-4 $C_p = 0.70$. From Eq. (17-12) the allowable largest belt tension $(F_1)_a$ is

$$(F_1)_a = bF_a C_p C_v = 6(100)0.70(1) = 420 \text{ lbf}$$

then

$$F_2 = (F_1)_a - [(F_1)_a - F_2] = 420 - 247.6 = 172.4 \text{ lbf}$$

and from Eq. (i)

$$F_i = \frac{(F_1)_a + F_2}{2} - F_c = \frac{420 + 172.4}{2} - 25.6 = 270.6 \text{ lbf}$$

The combination $(F_1)_a$, F_2 , and F_i will transmit the design power of $H_a = H_{\text{nom}} K_s n_d = 15(1.25)(1.1) = 20.6 \text{ hp}$ and protect the belt. We check the friction development by solving Eq. (17-7) for f' :

$$f' = \frac{1}{\phi} \ln \frac{(F_1)_a - F_c}{F_2 - F_c} = \frac{1}{3.0165} \ln \frac{420 - 25.6}{172.4 - 25.6} = 0.328$$

As determined earlier, $f = 0.8$. Since $f' < f$, there is no danger of slipping.

(c) From step 9 on p. 880,

$$n_{fs} = \frac{H_a}{H_{\text{nom}} K_s} = \frac{20.6}{15(1.25)} = 1.1 \quad (\text{as expected})$$

The belt is satisfactory and the maximum allowable belt tension exists. If the initial tension is maintained, the capacity is the design power of 20.6 hp.

A mine hoist uses a 2-inch 6×19 monitor-steel wire rope. The rope is used to haul maximum loads of 4 tons from a 480 ft. shaft. The drum has a diameter of 6 feet and the sheaves are of good-quality cast steel, and the smallest is 3 feet in diameter.

Part 1: To be done by the TA.

- Using a maximum hoisting speed of 1200 feet/minute and a maximum acceleration of 2 ft/s^2 , estimate the stresses in the wire rope.

Part 2: To be done by students.

- Estimate the various factors of safety

- a. Using a maximum hoisting speed of 1200 feet/minute and a maximum acceleration of 2 ft/s^2 , estimate the stresses in the wire rope.

The objective of the problem is to explore factors of safety in wire rope. We will express strengths as tensions.

Given: Monitor steel 2-in 6×19 rope, 480 ft long.

From Table 17-27:

Minimum diameter of a sheave is $30d$. To allow a greater factor of safety we can use the “better sheave diameter,” $45d$. preferably $45(2) = 90 \text{ in}$. The hoist abuses the wire when it is bent around a sheave.

Also, from Table 17-24:

The nominal tensile strength as 106 kpsi.

The ultimate load is

$$F_u = (S_u)_{\text{nom}} A_{\text{nom}} = 106 \left[\frac{\pi(2)^2}{4} \right] = 333 \text{ kip} \quad \text{Ans.}$$

The tensile loading of the wire is given by Eq. (17-47)

$$F_t = \left(\frac{W}{m} + wl \right) \left(1 + \frac{a}{g} \right)$$

$$W = 4(2) = 8 \text{ kip}, \quad m = 1$$

From Table (17-24), the weight/foot \times length gives us:

$$wl = 1.60d^2 l = 1.60(2^2)(480) = 3072 \text{ lbf} = 3.072 \text{ kip}$$

Therefore,

$$F_t = (8 + 3.072) \left(1 + \frac{2}{32.2} \right) = 11.76 \text{ kip} \quad \text{Ans.}$$

From Eq. (17-41) and Table 17-27:

$$F_b = \frac{E_r d_w A_m}{D}$$

With a 72-in drum, we get:

$$F_b = 12(10^6)(0.067d)(0.40d^2)/D$$

$$F_b = 12(10^6)(0.067(2))(0.40(2^2))/72 = 35.7 \text{ kip} \quad \text{Ans.}$$

For use in Eq. (17-44), from Fig. 17-21, $S_u = 240 \text{ kpsi}$ from page 912

$$(p / S_u) = 0.0014$$

$$S_u = 240 \text{ kpsi, } p. 920$$

$$F_f = \frac{0.0014(240)(2)(72)}{2} = 24.2 \text{ kip } \textit{Ans.}$$

Tension Load = 11,760 lbs., Bending Load = 35700 lbs, and Fatigue Load = 24200 lbs.
The Ultimate Load is

Part 2: To be done by the students.

b. Estimate the various factors of safety in the wire rope.

Static, no bending:

$$n = \frac{F_u}{F_t} = \frac{333}{11.76} = 28.3 \quad \textit{Ans.}$$

Static, with bending:

Eq. (17-46):
$$n_s = \frac{F_u - F_b}{F_t} = \frac{333 - 39}{11.76} = 25.0 \quad \textit{Ans.}$$

Fatigue without bending:

$$n_f = \frac{F_f}{F_t} = \frac{24.2}{11.76} = 2.06 \quad \textit{Ans.}$$

Fatigue, with bending: For a life of $0.1(10^6)$ cycles, from Fig. 17-21

$$(p / S_u) = 4 / 1000 = 0.004$$

$$F_f = \frac{0.004(240)(2)(72)}{2} = 69.1 \text{ kip}$$

Eq. (17-45):
$$n_f = \frac{69.1 - 39}{11.76} = 2.56 \quad \textit{Ans.}$$

If we were to use the endurance strength at 10^6 cycles ($F_f = 24.2$ kip) the factor of safety would be less than 1 indicating 10^6 cycle life impossible.