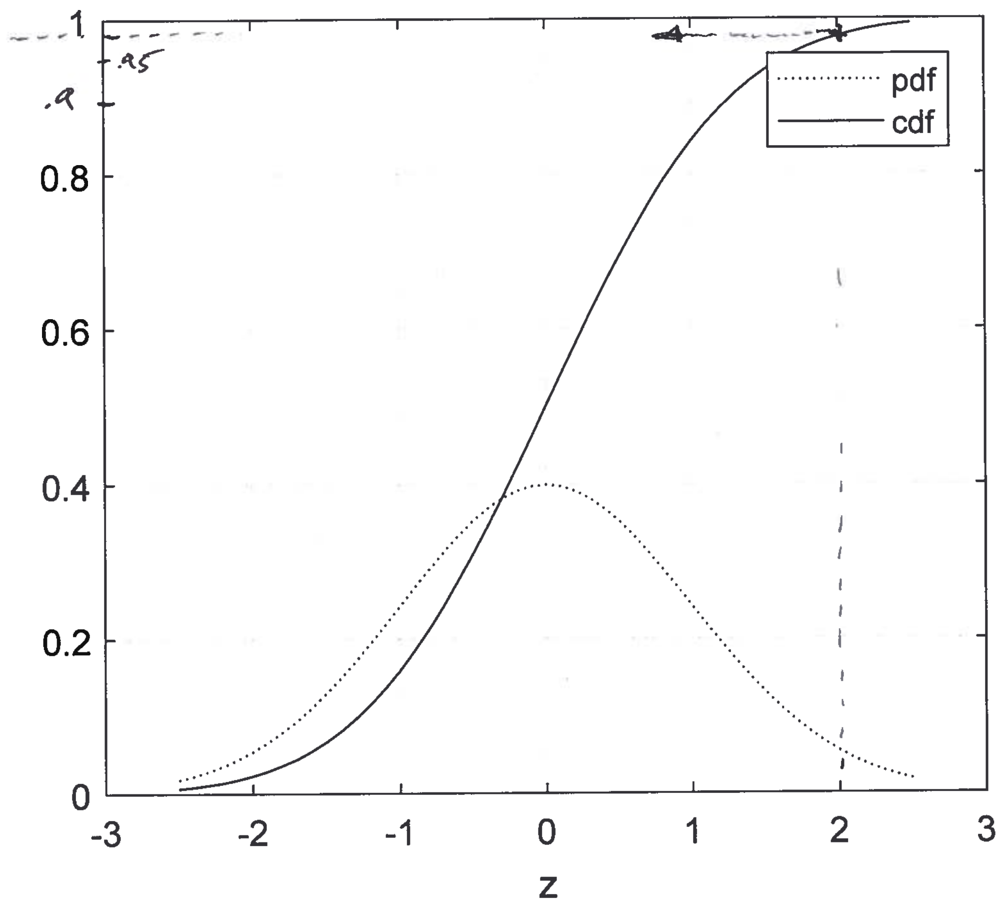
Name: Marking Key for Athena.

1. [*5 marks*] Men's heights h are distributed normally (mean $E(h)$= 70"; standard deviation σ=4"). Using (and marking) the standard-normal plot below, find the fraction of men that are shorter than 78 inches.



$$\underbrace{z = \frac{78-70}{4} = 2}_{\text{3 marks}} \Rightarrow \underbrace{F(2) = 0.98}_{\text{2 marks}} \text{ (approximately)}$$

(marking graph clearly and not giving a number is still ok.)

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2. [*12 marks*] On average, 6 students/hour arrive at Steve's office. Steve would like to take 8 minutes for a coffee. What is the probability that **1 or more** students arrives at his office during this **8-minute period**?

3 This is a Poisson process, $\lambda = \left(\frac{6}{60\text{ min}}\right) 8\text{ min} = 0.8$

3 $P(k) = \frac{e^{-\lambda}\lambda^{k}}{k!}$

3 Want $P(1) + P(2) + P(3) \ldots P(\infty)$

3 But this is just $1 - P(0) = 1 - \frac{e^{-\lambda}\lambda^{0}}{0!} = 1 - e^{-.8}$

$\underline{P(k \geq 1) = 0.55}$

Alternative approach:

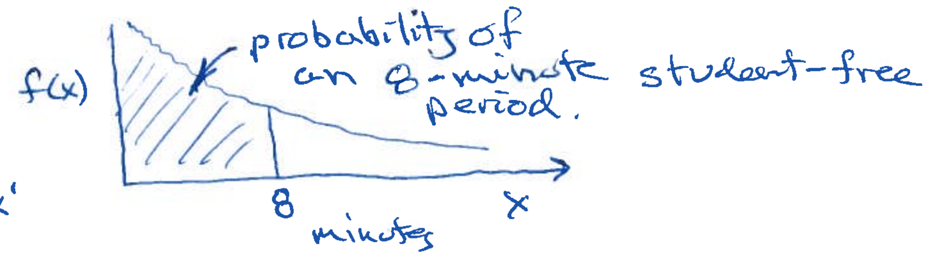
The interval between students is exponentially distributed.

3 

3 $f(x) = \lambda e^{-\lambda x}$

if x is in minutes, want λ in minutes $= \frac{6}{60} = .1$ /min

$f(x) = 0.1 e^{-0.1x}$



3 $F(x) = \int_0^x 0.1 e^{-0.1x'} dx'$

$= 0.1 \left.\frac{e^{-0.1x'}}{-0.1}\right|_0^x = -\left[e^{-0.1x} - 1\right] = 1 - e^{-0.1x}$

3 $\underline{F(8) = 1 - e^{-0.8} = 0.55}$

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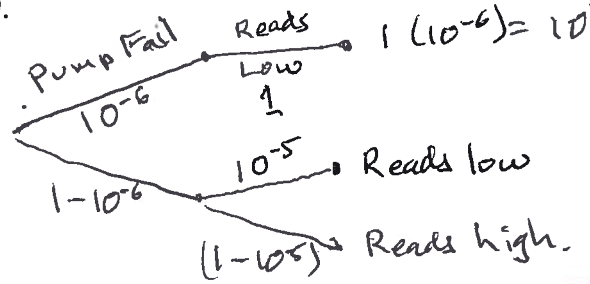
3. [*15 marks*] The probability of a faulty fuel pump (event "F") on a jet airliner is 10^{-6} per flight= P(F). ***If*** the pump is faulty the pressure gauge in the cockpit always reads low (event "L"). However, the pressure gauge can fail, and read low even if the pump is ok. The gauge failure rate is P(GF)=10^{-5} per flight. Before a particular flight, the gauge reads low ("L").

What is the probability that the fuel pump is faulty? *Hint, what we want is P(F given L).*



Some graphical presentation might be considered as a bonus if there are careless errors elsewhere.

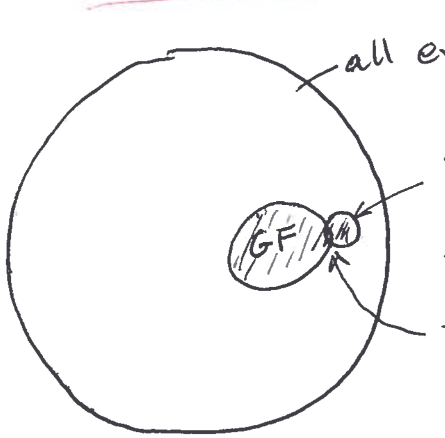
$$\boxed{P(F|L) = \frac{P(L|F)P(F)}{P(L)}} \quad \text{Bayes Thm.}$$ 6

$$\boxed{P(L) = P(F)P(L|F) + P(L|F')P(F')}$$ 5

$$= 10^{-6}(1) + \underbrace{10^{-5}(1-10^{-6})} = 1.1\times10^{-5}$$ 3

This term is approximate ... but a very good approximation!

$$\boxed{P(F|L) = \frac{10^{-6}}{1.1\times10^{-5}} \simeq 0.091 = 9.1\%}$$ 1



Discussion of this might be considered for bonus

shaded area is "L"

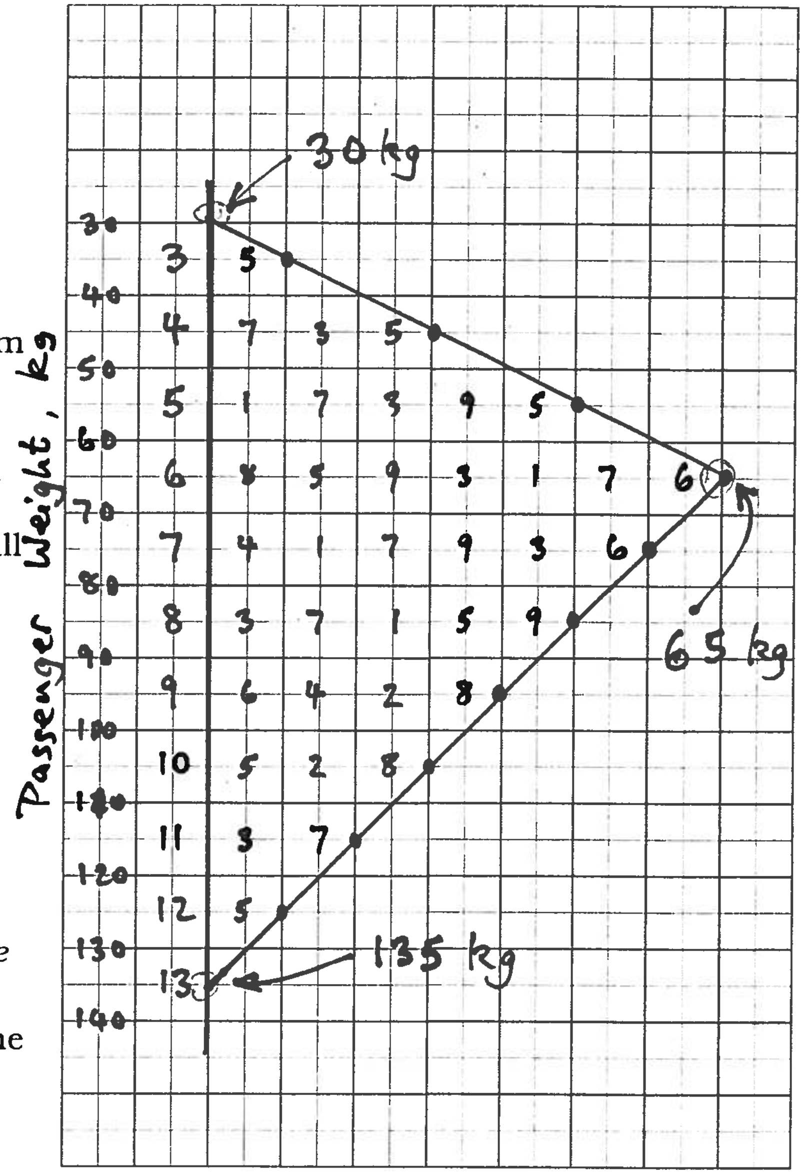
tiny intersection so $P(L) \sim P(F) + P(GF)$

$P(GF \text{ and } F) \sim P(GF)P(F) = 10^{-11}$

Exact: $P(L) = P(F) + P(GF) - P(GF)P(F)$
$= 10^{-6} + 10^{-5} - 10^{-11}$

4. [*28 marks*] An airline company wishes to use a 37-seat plane for a local commuter service. They want to check that the plane will be suitable to carry the total passenger weight. The company collected the following data for the weights in kg of a typical group of 37 passengers:

68	51	65	74	69
83	35	47	87	63
105	71	61	77	81
113	79	125	57	53
73	76	59	117	85
96	55	67	94	89
43	102	62	45	92
108	98			



(a) Draw a stem-and-leaf histogram in the box at the right.

(b) Draw dots at the top of each "leaf" and join them up to form the corresponding probability distribution function (pdf). If all goes well, you should end up with a triangle.

(c) What is the mode (the most common) passenger weight ?

(d) Name the geometrical property of the pdf shape that corresponds to the population mean. Use this property to determine the mean passenger weight. *(Please don't just average all the numbers!)*

(e) Use the geometrical shape of the pdf to determine the median passenger weight.

(f) Name the geometrical property of the pdf shape that corresponds to the population standard deviation.

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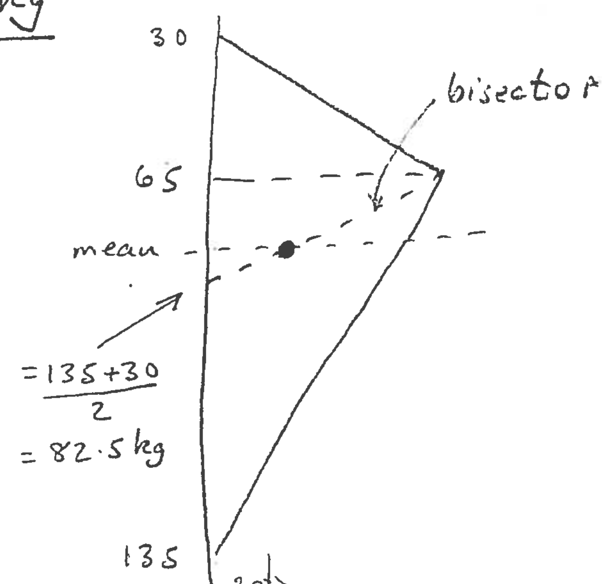
(b) The pdf is a triangle starting at 30 kg, rising to a maximum at 65 kg (centre of interval 60-70 kg) and finishing at 135 kg. If we wish to have a true pdf with area = 1, we would need to scale the height (measured horizontally in the stem and leaf plot) such that $\frac{1}{2}(135-30) \times \text{height} = 1 \rightarrow \text{height} = \frac{2}{105}$

(c) The mode (most common or most likely) weight is at the peak of the pdf = 65 kg

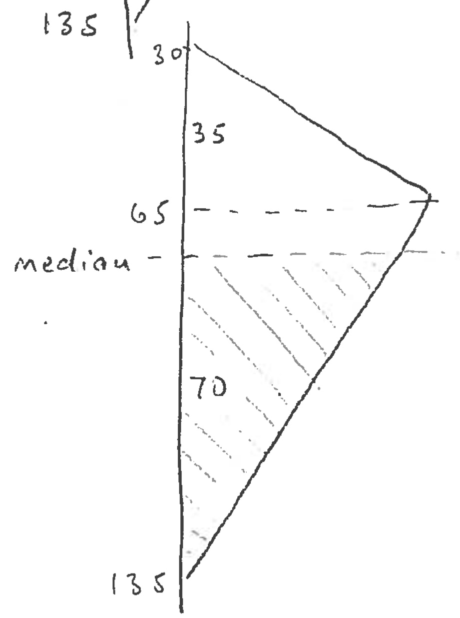
(d) The centroid of the pdf corresponds to the mean. The centroid of a triangle is 1/3 the way up the bisector



$\text{mean} = 82.5 - \frac{1}{3}(82.5 - 65)$

mean = 76.7 kg

(e) The median is the 50% probability line. It therefore divides the pdf into two equal halves. The triangle from 65 to 135 has double the area of the triangle from 30 to 65, therefore it has 2/3 the total pdf area.



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Area of a triangle is proportional to the square of its size.

$\rightarrow$ area ratio $=$ (size ratio)^2

$$\rightarrow \frac{1/2}{2/3} = \left(\frac{135 - \text{median}}{135-65}\right)^2 \rightarrow \frac{135 - \text{median}}{70} = \sqrt{\frac{3}{4}} = 0.866$$

$$\rightarrow \text{median} = 135 - 70 \times 0.866 = \underline{74.4\,\text{kg}}$$

(f) The moment of inertia of the pdf corresponds to the population variance. Multiply by $\frac{N}{N-1}$ to get the sample variance. Take square root to get standard deviation $\rightarrow$ radius of gyration

(moment of inertia $= 475.6$ $\rightarrow$ pop. variance $= 475.6\ \text{kg}^2$
[not asked] std. dev. $= 21.8$ kg)

Actually, we have a sample here, not a population.