Problem Set # 8

Textbook problems

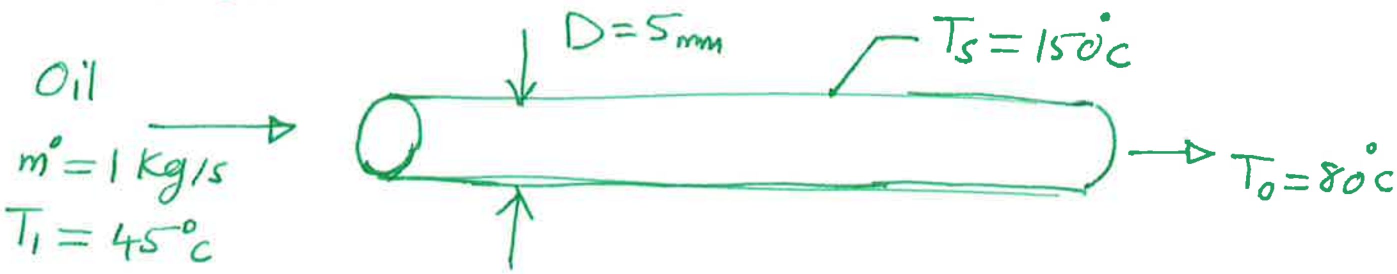
Problem 8.25

Solution:

Known : Oil flow rate. Pipe diameter, outlet and pipe surface temperatures.

Find : Length of tube required to achieve desired outlet temp.

Schematic:



Assumptions: (1) Steady-state (2) Incompressible flow (3) Negligible viscous dissipation

Properties: Table A-5, Engine oil ($T_i = 45°C = 318 K$): $\mu_i = 16.3 \times 10^{-2} N.s/m^2$; ($T_o = 80°C = 353K$): $\mu_o = 3.25 \times 10^{-2} N.s/m^2$.

Analysis: We begin by calculating the Reynolds number at the inlet and outlet, from equation 8.6,

$$Re_{D_i} = \frac{4\dot{m}}{\pi D \mu_i} = \frac{4 \times 1\ kg/s}{\pi \times 0.005m \times 16.3 \times 10^{-2} N.s/m^2} = 1560$$

$$Re_{D_o} = \frac{4 \times 1\ Kg/s}{\pi \times 0.005\,m \times 3.25 \times 10^{-2}\ N.s/m^2} = 7840$$

Therefore the flow is laminar at the inlet and turbulent at the outlet. The transition occurs when $Re_D = 2300$, that is, where

$$\mu = \frac{4m^{\circ}}{\pi D\, 2300} = \frac{4 \times 1\ Kg/s}{\pi \times 0.005\,m \times 2300} = 11.1 \times 10^{-2}\ N.s/m^2$$

From Table A-5, this occurs at a transition temperature of $T_{m,t} = 325\,K = 52^{\circ}C$. Now we proceed to analyze the heat transfer in the laminar and turbulent regions.

<u>Laminar Region</u>. The mean temperature in the laminar region is $\overline{T}_{m1} = (45^{\circ}C + 52^{\circ}C)/2 = 48.5^{\circ}C = 321.5\,K$.

The properties are $C_{p1} = 1999\ J/Kg.K$, $\mu_1 = 13.2 \times 10^{-2}\ N.s/m^2$, $K_1 = 0.143\ W/m.K$, $Pr_1 = 1851$. We recalculate the Reynolds number.

$$Re_{D_1} = \frac{4m^{\circ}}{\pi D \mu_1} = \frac{4 \times 1\ Kg/s}{\pi \times 0.005\,m \times 13.2 \times 10^{-2}\ N.s/m^2} = 1930$$

The hydrodynamic and thermal entry lengths are given by

$$x_{fd,h} = 0.05\, Re_{D_i} = 0.05 \times 1930 \times 0.005\,m = 0.48\,m$$

$$x_{fd,t} = x_{fd,h} \cdot Pr_i = 0.48\,m \times 1851 = 890\,m$$

Based on this information, we assume the flow is hydrodynamically developed but thermally developing, and use equation 8.56 for the Nusselt number (with $Pr > 5$),

$$\overline{Nu}_{D1} = \bar{h}_1 D/k_1 = 3.66 + \frac{0.0668\,(D/L_1)Re_{D1}Pr_1}{1+0.04\left[(D/L_1)Re_{D_1}Pr_1\right]^{2/3}} \quad (1)$$

where L_1 is the length of the laminar region which is as yet unknown. We can also use Equation 8.42

$$\frac{T_s - T_{m,t}}{T_s - T_i} = exp\left[-\frac{\pi D L_1}{\dot{m} c_{p1}}\bar{h}_1\right]$$

Solving for $\bar{h}_1 L_1$, we have

$$\bar{h}_1 L_1 = \frac{-\dot{m}c_{p1}}{\pi D} \ln\left(\frac{T_s - T_{m,t}}{T_s - T_i}\right) = -\frac{1\,Kg/s \times 1999\,J/Kg\cdot K}{\pi \times 0.005m} \times \ln\left(\frac{150-52^\circ C}{150-45^\circ C}\right) \quad (2)$$

$$= 8780\ W/m\text{-}K$$

We can solve by iterating between equations (1) and (2). Begining with the estimate $\overline{Nu}_{D1} = 3.66$, we find $\bar{h}_1 = 3.66\,K_1/D = 105\ W/m^2\cdot K$. From Equation (2), $L_1 = 84m$. Then from Equation (1), $\overline{Nu}_{D_1} = 22.3$ and $\bar{h}_1 = 639\ W/m^2\cdot K$. Continuing the iterations, we find $\overline{Nu}_{D_1} = 16.9$, $\bar{h}_1 = 484\ W/m^2\cdot K$ and $L_1 = 18.1\,m$.

Turbulent Range The mean temperature in the turbulent region is $\bar{T}_{m2} = (52^\circ C + 80^\circ C)/2 = 66^\circ C = 339K$.

The properties are $c_{p2} = 2072 \text{ J/kg}\cdot\text{K}$, $\mu_2 = 5.62 \times 10^{-2} \text{ N}\cdot\text{s/m}^2$, $k_2 = 0.139 \text{ W/m}\cdot\text{K}$, $Pr_2 = 834$.

Thus

$$Re_{D_2} = \frac{4\dot{m}}{\pi D \mu_2} = 4530$$

We assume the flow is fully-developed hydrodynamically and thermally and use Equation 8.62,

$$Nu_{D_2} = \frac{(f/8)(Re_{D_2} - 1000)Pr_2}{1 + 12.7(f/8)^{1/2}(Pr_2^{2/3} - 1)}$$

Where from Equation 8.21

$$f = (0.790 \ln Re_{D_2} - 1.64)^{-2} = (0.790 \ln(4530) - 1.64)^{-2}$$

$$= 0.0398$$

Thus

$$Nu_{D_2} = \frac{(0.0398/8)(4530 - 1000)834}{1 + 12.7(0.0398/8)^{1/2}(834^{2/3} - 1)} = 184$$

and $h_2 = Nu_{D_2} k_2 / D = 5120 \text{ W/m}^2\cdot\text{K}$. Then the required length L_2 can be found from Equation 8.42, expressed between the transition point and the outlet,

$$\frac{T_s - T_o}{T_s - T_{m,t}} = \exp\left(-\frac{\pi D L_2}{\dot{m} c_{p2}} \bar{h}_2\right)$$

$$L_2 = -\frac{\dot{m} c_{p2}}{\pi D h_2} \ln\left(\frac{T_s - T_o}{T_s - T_{m,t}}\right) = -\frac{1\,kg/s \times 2072\,J/kg\cdot k}{\pi \times 0.005 \times 5120\,W/m^2\cdot K} \times \ln\left(\frac{150-80\,^\circ C}{150-52\,^\circ C}\right)$$

$$= 8.7\,m$$

The total length is $L = L_1 + L_2 = 26.8\,m$

Comments: If we had simply calculated the properties based on the mean temperature of $\overline{T}_m = (45^\circ C + 80^\circ C)/2 = 62.5^\circ C = 335.5\,K$, we would have found $Re_D = 3810$.

Assuming the flow to be turbulent throughout would have resulted in a higher average Nusselt number, $\overline{Nu}_D = 159$, and correspondingly lower total length, $L = 11.9\,m$. The variation of properties with temperature can be very important for some fluids such as oils.

If the oil were being cooled by exposure to a cooler wall, the Reynolds number could decrease from a turbulent to a laminar value. The flow would likely not completely "relaminarize" and the heat transfer in the section for which $Re_D < 2300$ would fall between the values calculated using laminar and turbulent Nusselt number correlations.

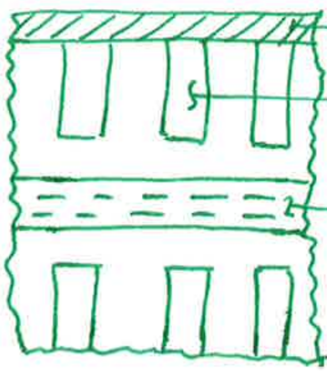
Problem 8.84

Solution:

Known: Heat sink with 24 passages for air flow removes power dissipation from circuit board.

Find: Operating temperature of the board and pressure drop across the sink.

Schematic:



Assumptions: Table A-4, Air ($\bar{T} \sim 310\,K$, 1 atm): $\rho = 1.1281\ Kg/m^3$

$\nu = 16.89 \times 10^{-6}\ m^2/s$, $Pr = 0.706$, $C_p = 1008\ J/Kg\cdot K$

$K = 0.0270\ W/m\cdot K$.

Analysis: The air outlet temperature follows from Eq. 8.41b,

$$\frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(-\frac{PL\bar{h}}{\dot{m} C_p}\right)$$

The mass flow rate for the entire sink is

$$\dot{m} = \rho \dot{\forall} = 1.1281\ Kg/m^3 \times 0.060\ m^3/s = 6.77 \times 10^{-2}\ Kg/s$$

$(\dot{\forall} = Q)$

and the Reynolds number for a rectangular passage is

$$Re_D = \frac{u_m D_h}{\nu}$$

Where $D_h = 4A_c/P = 4(6\,mm \times 25\,mm)/2(6+25\,mm)$

$$= 9.68\,mm$$

$$u_m = \frac{\dot{m}/24}{\rho A_c} = \frac{6.77\times10^{-2}\,kg/s/24}{1.1281\,kg/m^3\,(6\times25)\times10^{-6}\,m^2} = 16.7\,m/s$$

giving $Re_D = \dfrac{16.7\,m/s \times 9.68\times10^{-3}}{16.89\times10^{-6}\,m^2/s} = 9571.$

Assume the flow is turbulent and fully developed and using the Dittus–Boelter correlation (with Re_D close to 10,000) find

$$Nu_D = 0.023 Re^{4/5} Pr^{0.4} = 0.023\,(9571)^{4/5}(0.706)^{0.4} = 30.6$$

$$h = \frac{Nu \cdot k}{D_h} = \frac{30.6 \times 0.027\,W/m.K}{0.00968\,m} = 85.4\,W/m^2.K.$$

From an overall energy balance on the sink,

$$q = \dot{m} c_p (T_{m,o} - T_{m,i}) \rightarrow T_{m,o} = T_{m,i} + q/\dot{m} c_p$$

$$T_{m,o} = 27^\circ C + 50\,W / 6.77\times10^{-2}\,kg/s \times 1008\,J/kg.K = 27.73^\circ C$$

Hence, the operating temperature of the circuit board for these conditions is

$$\frac{T_s - 27.73}{T_s - 27} = \exp\left[-\frac{2(6+25)\times10^{-3}\,m \times 0.150\,m \times 85.4\,W/m^2.K}{(6.77\times10^{-2}\,kg/s/24)\times1008\,J/kg.K}\right]$$

$$T_s = 30^\circ C$$

The pressure drop in the rectangular passage for the smooth surface condition follows from Eqs. 8.22 and 8.20

$$\Delta p = f \frac{\rho u_m^2}{D_h} L$$

where

$$f = 0.316 Re_D^{-1/4} = 0.316(9554)^{-1/4} = 0.0320.$$

$$\Delta p = 0.0320 \frac{1.1281 \text{ kg/m}^3 (16.7 \text{ m/s})^2}{0.00968 \text{ m}} \times 0.150 \text{ m}$$

$$= 156 \text{ N/m}^2$$

Comments: (1) The analysis has been simplified by assuming the channel is rectangular and all four sides experience heat transfer. Since the insulated surface is a small portion of the total passage surface area, the effect can't be very large.

(2) The power required to move the air through the heat sink is $P_{elec} = \dot{\forall} \Delta p = 0.060 \text{ m}^3/\text{s} \times 156 \text{ N/m}^2$

$= 9.4 \text{ W}.$