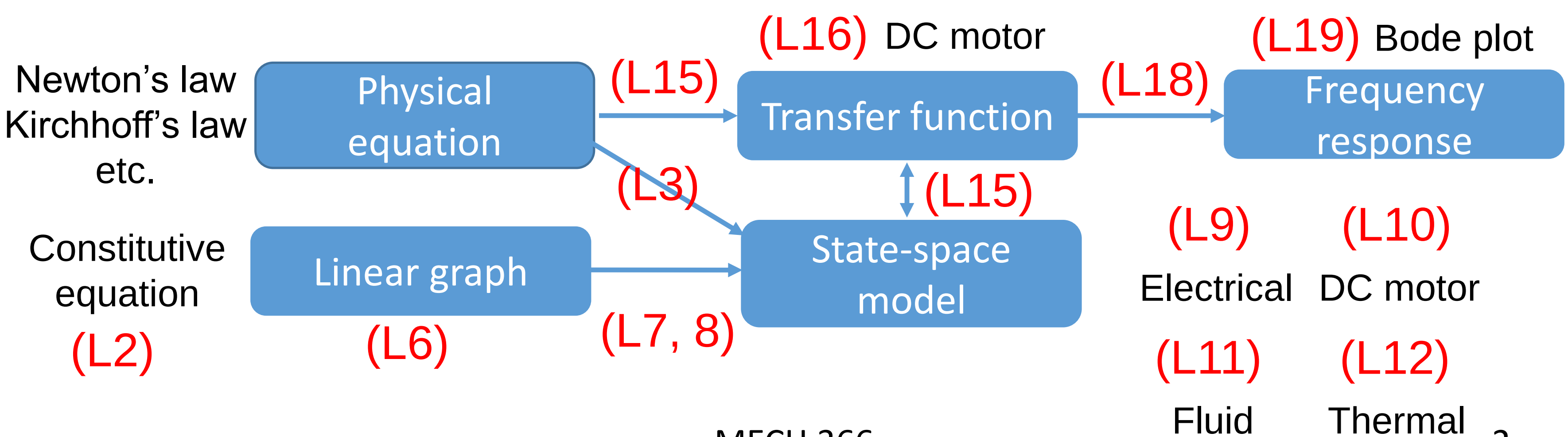
MECH366 : Modeling of Mechatronic Systems

L21 : Stability
Summary of the course

Dr. Ryozo Nagamune
Department of Mechanical Engineering
University of British Columbia

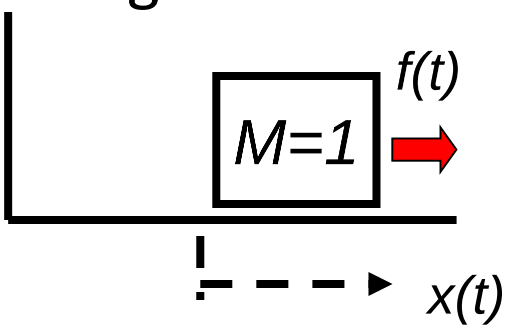
Today's topic & class schedule

- L18: Nov 15 (Fri): Frequency response
- L19: Nov 18 (Mon): Bode diagram (Lab 4 report content, report due Nov 26)
- L20: Nov 22 (Fri): Simulink, overdamped system
- **L21: Nov 25 (Mon): Stability, course summary**

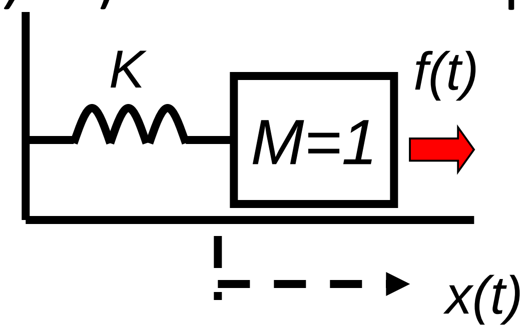


Simple mechanical examples

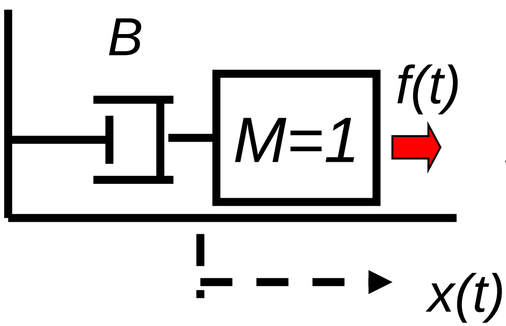
- We want mass to stay at *x=0*, but instantaneous wind gave some initial speed (*f(t)=0*). What will happen?



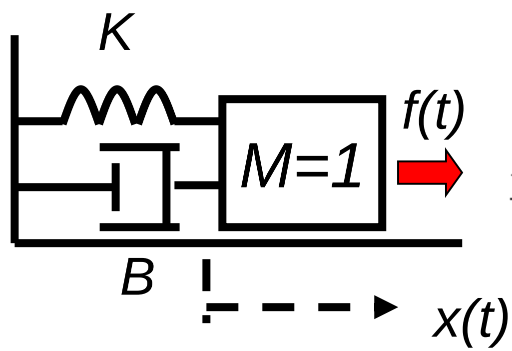
$$\frac{X(s)}{F(s)} = \frac{1}{s^2}$$



$$\frac{X(s)}{F(s)} = \frac{1}{s^2 + K}$$



$$\frac{X(s)}{F(s)} = \frac{1}{s^2 + Bs}$$



$$\frac{X(s)}{F(s)} = \frac{1}{s^2 + Bs + K}$$

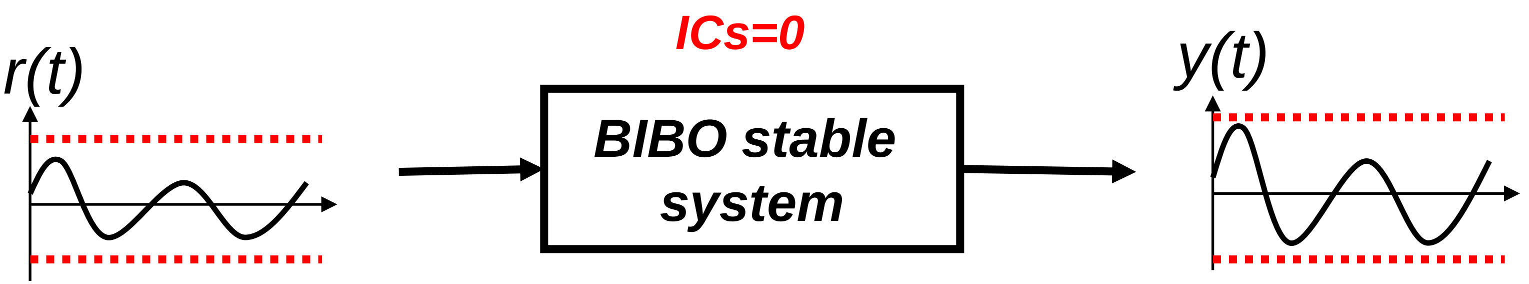
- How to characterize different behaviors with TF?

Stability

- Utmost important specification in system design!
- Unstable systems have to be stabilized by feedback.
- Unstable feedback systems are useless.
- What if a system is unstable? ("out-of-control")
  - It may hit mechanical/electrical "stops" (saturation).
  - It may break down or burn out.
  - Signals diverge.
- Examples of unstable systems
  - Tacoma Narrows Bridge collapse in 1940
  - SAAB Gripen JAS-39 prototype accident in 1989
  - Wind turbine explosion in Denmark in 2008

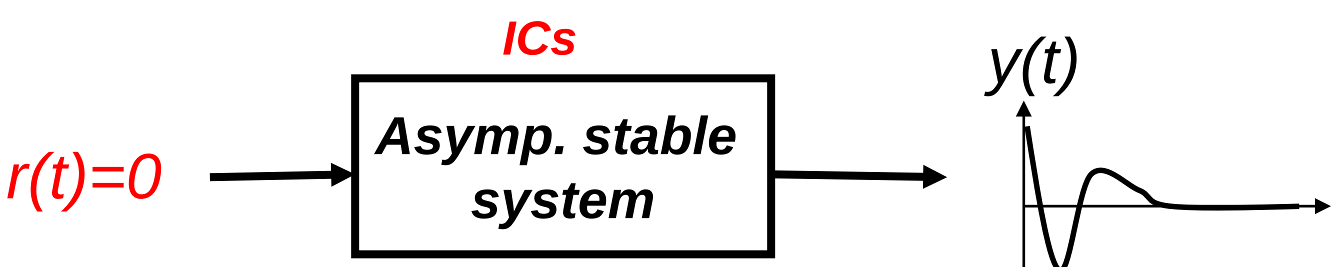
Definitions of stability

- **BIBO** (Bounded-Input-Bounded-Output) **stability**
  Any bounded input generates a bounded output.



- **Asymptotic stability**

  Any ICs generates y(t) converging to zero.



Stability condition in *s*-domain
(Proof omitted, and not required)

- For a system represented by transfer function *G(s)*,

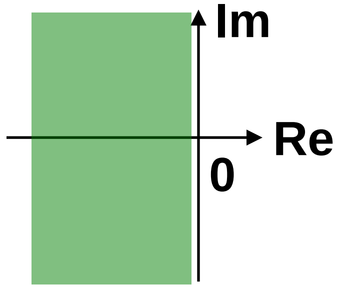
System is BIBO stable

$\Updownarrow$

All the poles of G(s) are in the open left half of the complex plane.

$\Updownarrow$

System is asymptotically stable



Remarks on stability (cont'd)

- Marginally stable if
  - *G(s)* has no pole in the open RHP (Right Half Plane), and
  - *G(s)* has at least one simple pole on *jw*-axis, and
  - *G(s)* has no multiple pole on *jw*-axis.

$$G(s) = \frac{1}{s(s^2+4)(s+1)^2}$$

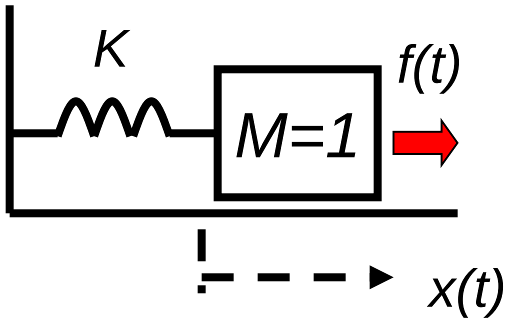
Marginally stable

$$G(s) = \frac{1}{s(s^2+4)^2(s+1)^2}$$

NOT marginally stable

- Unstable if a system is neither stable nor marginally stable.

"Marginally stable" in *t*-domain



$$\frac{X(s)}{F(s)} = \frac{1}{s^2 + K}$$

- For any bounded input, except only special sinusoidal (bounded) inputs, the output is bounded.
  - In the example above, the special inputs are in the form of:

$$f(t) = \alpha \sin\sqrt{K}t + \beta \cos\sqrt{K}t \quad \Rightarrow \quad x(t) \rightarrow \pm\infty$$

- For any nonzero initial condition, the output neither converge to zero nor diverge.

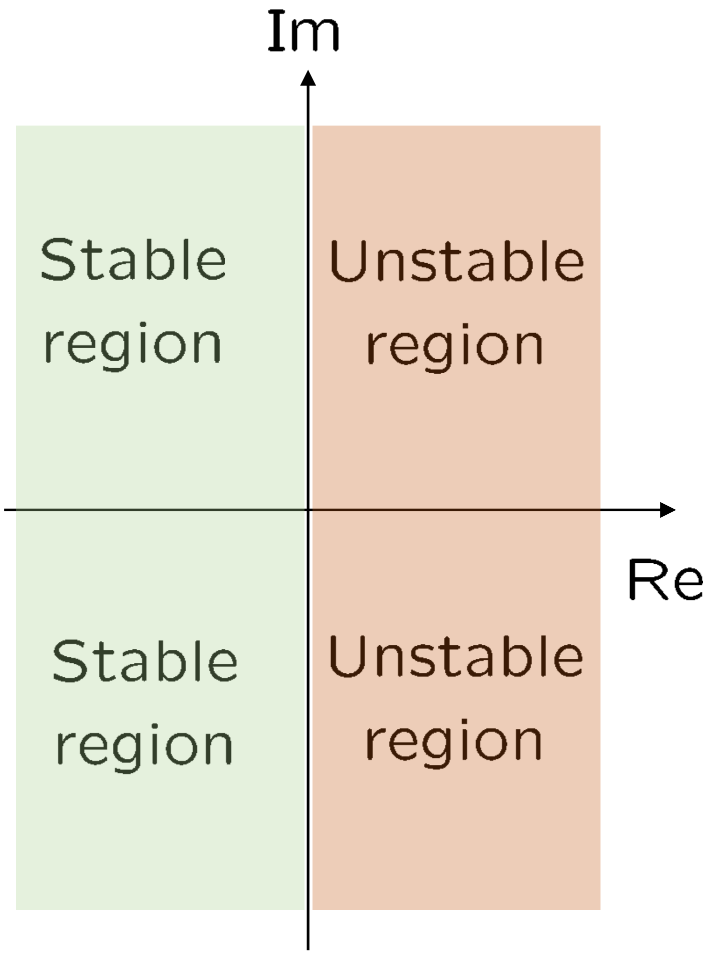
Stability summary

Let s_i be poles of $G(s)$.
Then, $G(s)$ is …

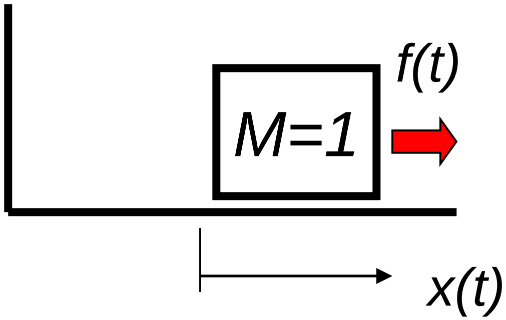
- (BIBO, asymptotically) stable if

  $Re(s_i)<0$ for all i.

- marginally stable if
  - $Re(s_i)<=0$ for all i, and
  - simple pole for $Re(s_i)=0$
- unstable if it is neither stable nor marginally stable.

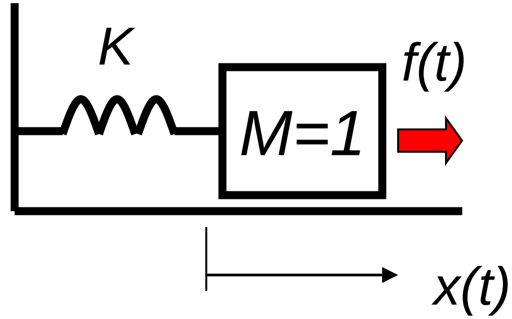


Mechanical examples: revisited



$$\frac{X(s)}{F(s)} = \frac{1}{s^2}$$

Poles=
stable?



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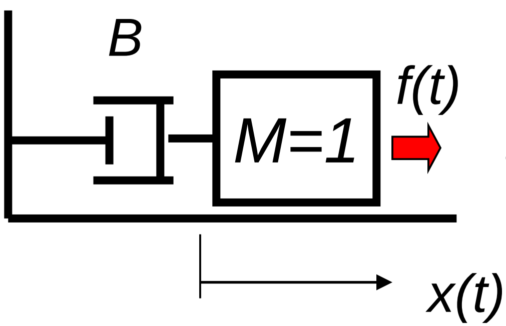
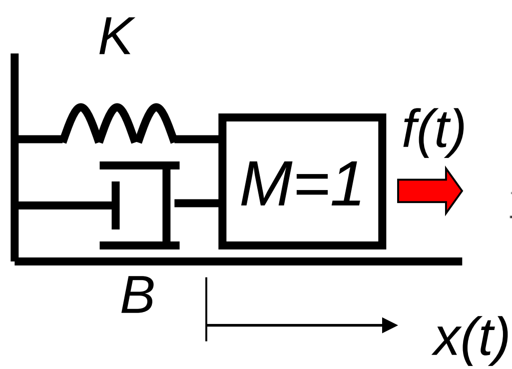


$$\frac{X(s)}{F(s)} = \frac{1}{s^2 + Bs}$$

Poles=
stable?



$$\frac{X(s)}{F(s)} = \frac{1}{s^2 + Bs + K}$$

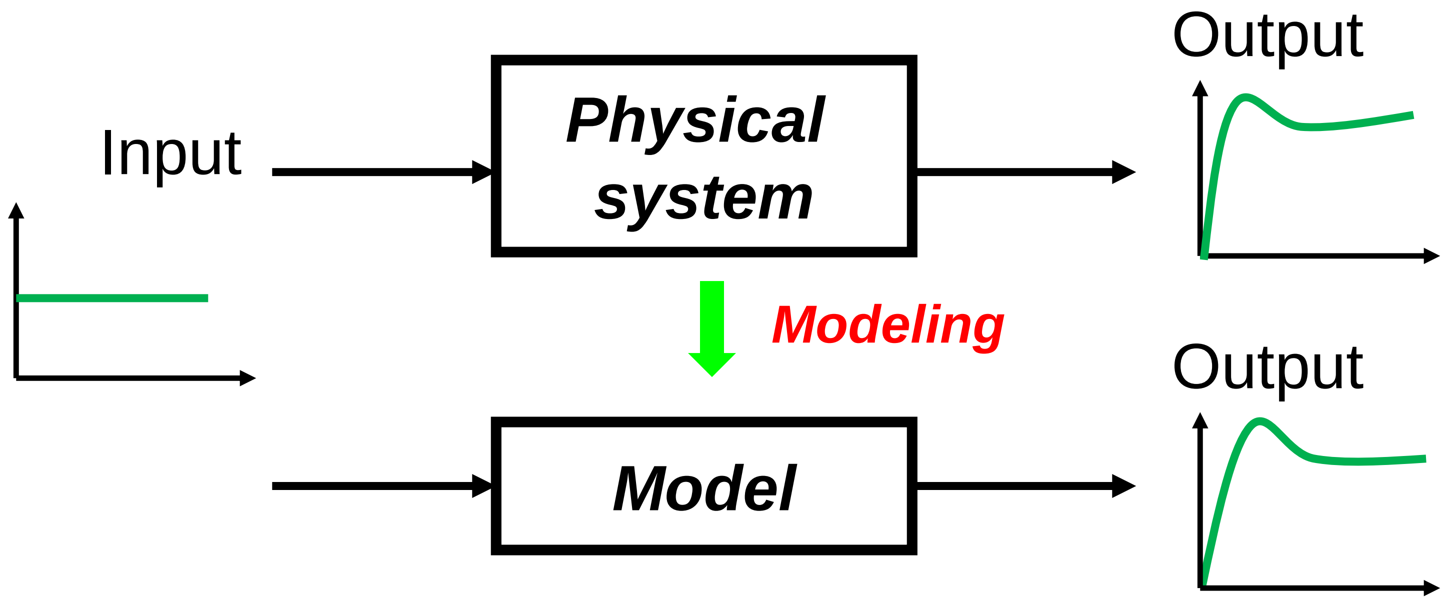
Poles=
stable?

a place of mind

Course summary

Course summary
Model and modeling

- **Model**: Representation of input-output (signal) relationship of a system
- **Modeling**: Process to derive models



Course summary
Goals of this course (in Lecture 1)

- Acquire basic techniques for *modeling of mechatronic systems*! (course title)
- You learned:
  - Modeling of mechanical, electrical, thermal, fluid systems
  - Analogies between different domains
  - Linear graph
  - State-space modeling, linearization
  - Transfer function modeling, block diagram
  - Step response analysis
  - Frequency response analysis, Bode diagram
  - Stability

Course summary
Remark on modeling

- Modeling is NOT a final goal!
  - After modeling, using the model, we want to, e.g.,
    - Predict the response of a system for excitation inputs
    - Analyze the system properties (fast/slow? Oscillatory?)
    - Modify the hardware design (selection of sensors and actuators, modifications of mechanical elements etc.)
    - Design feedback controllers
- Modeling is a very important, but not easy, task.
  - (Very important) If models are incorrect, whatever you do based on the models are all wrong!
  - (Not easy) Real systems are complex.

After studying modeling, what are next?

- MECH467: Computer Control of Mechatronics Systems
  - How to use the transfer function in analysis and design of control/mechatronic systems.
- MECH468: Modern Control Engineering (elective)
  - How to use the state-space model in analysis and design of control/mechatronic systems.
- MECH420: Sensors and Actuators (hardware)
- MECH421: Mechatronic System Instrumentation
- MECH423: Mechatronic Product Design
- MECH458: Mechatronics Design Project (integration)
- Graduate study, research, your future career etc.

Stay tuned!

Project (See “Project Guideline”)

- Project presentation: **FNH (Food, Nutrition, Health) 40**
  - November 29 (Friday)
    - 2-3pm: A1-A6, max 10-minutes/group (including questions)
    - 3:05-3:55pm: B1-B5, max 10-minutes/group (including questions)
    - Every student should say at least one sentence!
- Project report (Template given, max 3 pages)
  - Due December20 (Friday) at 11:59pm
  - Email me with file name e.g. “A1.pdf”, “B2.pdf”.
- Your presentation will be marked by the instructor and TAs individually, and the average will be taken.

Teaching evaluation

- Student Evaluation of Teaching (SEoT) on Canvas
- Please input your evaluation by December 2 (Mon).
- This evaluation is important for me. It will be reviewed carefully both by myself and by the administration.
- It will be used for improving my teaching skill, as well as course contents. Please provide constructive suggestions.

Announcements

- Final exam
  - Date & time: December 9 (Monday), 3:30-6pm
  - Place: CEME 1202
  - Closed book, no calculator, 1-page letter-size hand-written cheat sheet (You can use both-sides of the sheet.)
  - Study lecture slides, homework, exercises, past exams.
- I will neither return your final, nor disclose your score individually.
- No fixed office hour. If you have questions, just drop by my office or make an appointment.
- GOOD LUCK ON YOUR FINAL!

Tips for the final exam

- All questions are solvable! (Of course! ☺)
- "No calculator" means that no complicated numerical calculations are required. Leave $\sqrt{3},\ \log_{10} 2$ as they are, but simplify $\sqrt{4} = 2,\ \log_{10} 10 = 1,\ \sin\frac{\pi}{2} = 1$
- If you find some questions difficult, keep in mind that it may be intentional and that I want to see how you approach to such questions.
- Describe how you are approaching to questions. Correctness of the approach is weighted more than correctness of calculations.