1. Answer the following questions **concisely, within one or two lines (or even by one-word or two-words if appropriate).**

   (a) For what purpose can a mathematical model of a physical system be used? (Giving **only one** such purpose is enough.) (1pt)

   Write your answer here.

   Prediction
   Controller design
   Analysis, Simulation etc.

   (b) For **thermal** systems, what is the **through** variable? (1pt)

   Write your answer here.

   Heat transfer rate

   (c) For **fluid** systems, what is the **across** variable? (1pt)

   Write your answer here.

   Pressure difference

   (d) In **thermal** systems, write the constitutive equation for the **T-type** element. (1pt)

   Write your answer here.

   None

   (e) In **electrical** systems, write the constitutive equation for the **A-type** element. (1pt)

   Write your answer here.

   $C\frac{dV}{dt} = i$

(c) Derive a linearized state equation $\delta\dot{h}(t) = A\delta h(t) + B\delta u(t)$ around the equilibrium point $(h_1, h_2, h_3) = (h_{10}, h_{20}, h_{30})$ and $(u_1, u_2) = (u_{10}, u_{20})$. To answer this question, you do **not** need to use solutions obtained in (a) and (b); just use the notations (h_{10}, h_{20}, h_{30}) and (u_{10}, u_{20}). (2pt)

(d) Define the state vector δh and the input vector δu in the linearized model in (c). (1pt)

Write your answer here.

(a) $\dot{h}_1 = 0 \Rightarrow u_1(t) = k\sqrt{h_1(t)}$. Since $h_1(t) = h_{10}$, $u_{10} = k\sqrt{h_{10}}$.

$\dot{h}_2 = 0 \Rightarrow u_2(t) = k\sqrt{h_2(t)}$. Since $h_2(t) = h_{20}$, $u_{20} = k\sqrt{h_{20}}$.

(b) $\dot{h}_3 = 0 \Rightarrow k\sqrt{h_3(t)} = k\sqrt{h_1(t)} + k\sqrt{h_2(t)}$

Since $h_1(t) = h_{10}$, $h_2(t) = h_{20}$, we have $h_{30} = \left(\sqrt{h_{10}} + \sqrt{h_{20}}\right)^2$.

(c) & (d)

$$\delta\dot{h} = A\delta h + B\delta u \quad \text{where } \delta h = \begin{bmatrix} h_1 - h_{10} \\ h_2 - h_{20} \\ h_3 - h_{30} \end{bmatrix}, \quad \delta u = \begin{bmatrix} u_1 - u_{10} \\ u_2 - u_{20} \end{bmatrix}$$

$$A = \left.\frac{\partial f}{\partial h}\right|_{\substack{h=h_0 \\ u=u_0}} = \begin{bmatrix} -\frac{k}{\rho A_1}\frac{1}{2\sqrt{h_{10}}} & 0 & 0 \\ 0 & -\frac{k}{\rho A_2}\frac{1}{2\sqrt{h_{20}}} & 0 \\ \frac{k}{\rho A_3}\frac{1}{2\sqrt{h_{10}}} & \frac{k}{\rho A_3}\frac{1}{2\sqrt{h_{20}}} & -\frac{k}{\rho A_3}\frac{1}{2\sqrt{h_{30}}} \end{bmatrix}$$

$$B = \left.\frac{\partial f}{\partial u}\right|_{\substack{h=h_0 \\ u=u_0}} = \begin{bmatrix} \frac{1}{\rho A_1} & 0 \\ 0 & \frac{1}{\rho A_2} \\ 0 & 0 \end{bmatrix}$$

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Below, you can **use the notation** r, instead of using R_L and R_H.

(b) Draw a linear graph, by introducing notations appropriately. (4pt)

(c) Select the state variables. (It is fine even if you include redundant state variables.) (1pt)

(d) Write the constitutive equations for the passive elements and the gear (transformer) in the linear graph. (2pt)

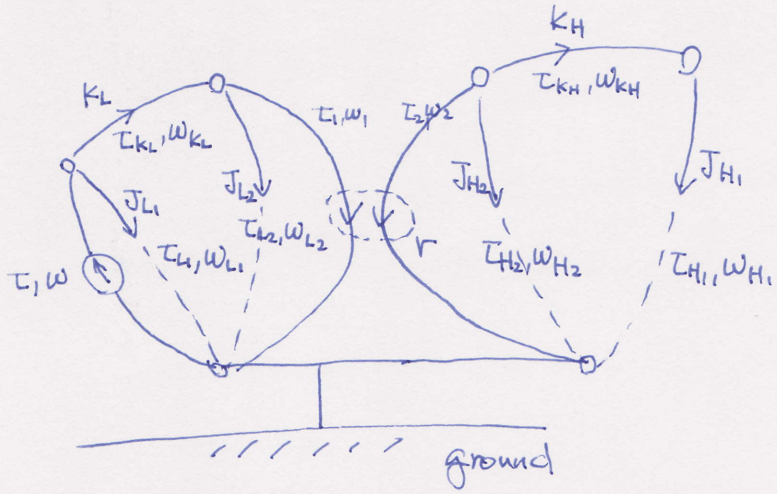
(e) Write loop equations and node equations for the linear graph. (2pt)

——— (End of Midterm Exam) ———

Write your answer here.

(a) $r = \dfrac{R_L}{R_H}$

(b) Linear graph.



(c)
$$x = \begin{bmatrix} \omega_{L_1} \\ \omega_{L_2} \\ \omega_{H_2} \\ \omega_{H_1} \\ \tau_{K_L} \\ \tau_{K_H} \end{bmatrix}$$
(Redundant)

(d)
$$\dot{\tau}_{K_L} = k_L \omega_{K_L}$$
$$\dot{\tau}_{K_H} = k_H \omega_{K_H}$$
$$J_{L_1}\dot{\omega}_{L_1} = \tau_{L_1} \qquad J_{H_1}\dot{\omega}_{H_1} = \tau_{H_1}$$
$$J_{L_2}\dot{\omega}_{L_2} = \tau_{L_2} \qquad J_{H_2}\dot{\omega}_{H_2} = \tau_{H_2}$$
$$\begin{cases} \omega_2 = r\omega_1 \\ \tau_2 = -\frac{1}{r}\tau_1 \end{cases} \quad \text{(gear)}$$

(e) Loop equations
$$\begin{cases} -\omega + \omega_{L_1} = 0 \\ -\omega_{L_1} + \omega_{K_L} + \omega_{L_2} = 0 \\ -\omega_{L_2} + \omega_1 = 0 \\ -\omega_2 + \omega_{H_2} = 0 \\ -\omega_{H_2} + \omega_{K_H} + \omega_{H_1} = 0 \end{cases}$$

Node equations
$$\begin{cases} -\tau + \tau_{K_L} + \tau_{L_1} = 0 \\ -\tau_{K_L} + \tau_{L_2} + \tau_1 = 0 \\ \tau_2 + \tau_{H_2} + \tau_{K_H} = 0 \\ -\tau_{K_H} + \tau_{H_1} = 0 \end{cases}$$