University of British Columbia
Department of Mechanical Engineering

MECH366 Modeling of Mechatronic Systems
Midterm exam

Examiner: Dr. Ryozo Nagamune
October 11 (Friday), 2019, 3pm-3:50pm

Last name, First name

Name: Student #:

Signature:

Exam policies

- Allowed: One-page letter-size hand-written cheat-sheet (both sides).
- Not-allowed: PC, calculators.
- Write all your answers on this booklet. No extra sheet will be provided.
- Motivate your answers properly. (No chance to defend your answers orally.)
- 20 points in total.

Before you start ...

- Use washroom before the exam.
- Turn off your mobile phone.
- No eating.
- Questions are NOT allowed.

If you finish early ...

- Please stay at your seat until the end of exam, i.e., 3:50pm. (You are not allowed to leave the room before the end of exam, except going to washroom.)

To be filled in by the instructor/marker

Problem #	Mark	Full mark
1		5
2		5
3		10
Total		20

1. Answer the following questions **concisely, by a few sentences and/or equations, or even by one-word or two-words if appropriate**.

   (a) For what purposes can a mathematical model of a physical system be used? Give **exactly two** such purposes. (If you write more than two purposes, you will lose some mark.) (1pt)

   Write your answer here.

   (b) Explain why 'voltage' in electrical systems is called 'across variable'. (1pt)

   Write your answer here.

   (c) Explain the 'model validation' step in the modeling procedure. (1pt)

   Write your answer here.

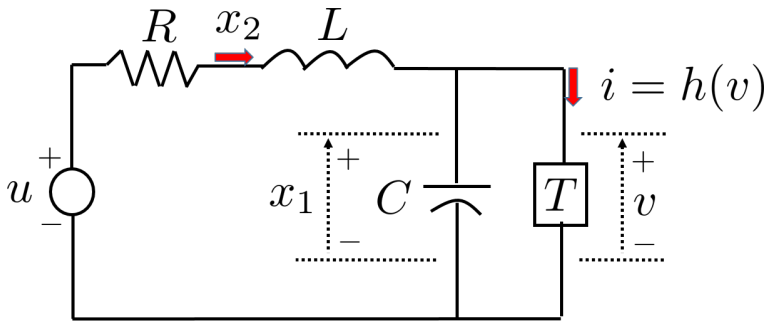
(d) Give the **definition** of a linear system, by using the notations u, x and y as the input, the state and the output of the system, respectively. (**Hint:** 'State-space model' representation is NOT the definition of a linear system.) (1pt)

Write your answer here.

(e) **Using the relation between the energy and the power**, derive the **energy formula** for the electrical **inductor** element. (1pt)

Write your answer here.

2. Consider the electric circuit depicted below. Here, the notations R, L and C respectively denote the resistance, inductance and capacitance, and u is the voltage source. An electrical element T has the characteristic $i = h(v)$, where i is the current through T and v is the voltage across T, and h is a nonlinear function which is differentiable with respect to v (i.e., $h'(v)$ exists).



(a) Let x_1 be the voltage across the capacitance, and x_2 be the current through the inductance. Prove that the state equation for this system is described as follows. (You don't need to use the linear graph.) (2pt)

$$\begin{aligned} \dot{x}_1(t) &= -\frac{1}{C}h(x_1(t)) + \frac{1}{C}x_2(t) \\ \dot{x}_2(t) &= -\frac{1}{L}x_1(t) - \frac{R}{L}x_2(t) + \frac{1}{L}u(t) \end{aligned}$$

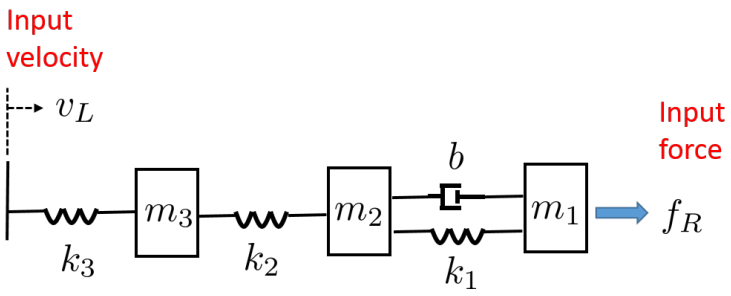
(b) Linearize the state equation above around the operating point $(x_1, x_2, u) = (x_{10}, x_{20}, u_0)$. (2pt)

(c) Express x_{20} and u_0 as functions of x_{10}. (1pt)

Write your answer here.

Write your answer here.

3. Consider a 3-DOF mass-spring-damper system in the figure below. Here, m, b, and k (with subscripts) are respectively mass, viscous damping constant, and spring constant. Two inputs are the velocity v_L and the force f_R as indicated in the figure, and the outputs are the **displacement and acceleration** of m_1 (i.e., right-most mass in the figure).



(a) Draw a linear graph, by introducing notations appropriately. (2pt)

(b) Select the state variables. (1pt)

(c) Write the constitutive equations for the passive elements in the linear graph. (1pt)

Write your answer here.

(d) By using the linear graph, derive a state-space model in a matrix-vector form, i.e., in the form of '$\dot{x} = Ax + Bu$' and '$y = Cx + Du$'. (There is no need to write down the loop and node equations.) (3pt)

——— (Continue to the next page) ———

Write your answer here.

(e) Make an analogous electrical circuit for this mechanical system. (2pt)

(f) Instead of the input velocity v_L, when the input force f_L is applied at the same location as v_L, explain the reason why k_3 is not necessary in the state-space model. (1pt)

——— (End of Midterm Exam) ———

Write your answer here.

Extra page. Write the problem number before writing your answer.

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