University of British Columbia
Department of Mechanical Engineering

MECH366 Modeling of Mechatronic Systems
Homework 5

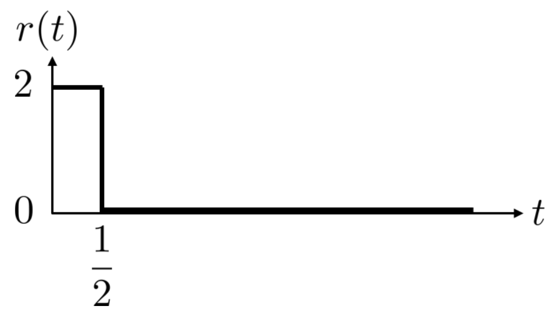
Due: November 4 (Monday), 2019, 3pm

Consider the following ordinary differential equation (ODE):

$$y^{(3)}(t) + 2y^{(2)}(t) + y^{(1)}(t) = r(t),$$

with zero initial conditions $y(0) = y^{(1)}(0) = y^{(2)}(0) = 0$. Here, $y^{(k)}(t)$ denotes the k-th derivative of $y(t)$.

1. Assume that $r(t) = \delta(t)$ (i.e., unit impulse function).
   (a) By using the Laplace transform, solve the ODE (i.e., obtain $y(t)$).
   (b) By using the final value theorem, obtain the final value $\lim_{t\to\infty} y(t)$. (You should verify the applicability of the final value theorem.)
2. Next, assume that $r(t)$ is the function given in the figure below. Note that this is an approximation of the unit impulse function.



By using the final value theorem, obtain the final value $\lim_{t\to\infty} y(t)$. (In this question, you can assume (i.e., you do not need to check) the applicability of the final value theorem.)

Hint: You can use the L'Hospital's Rule:

$$\lim_{s\to 0} \frac{f(s)}{g(s)} = \lim_{s\to 0} \frac{f'(s)}{g'(s)} \quad \text{if } f(0) = g(0) = 0.$$