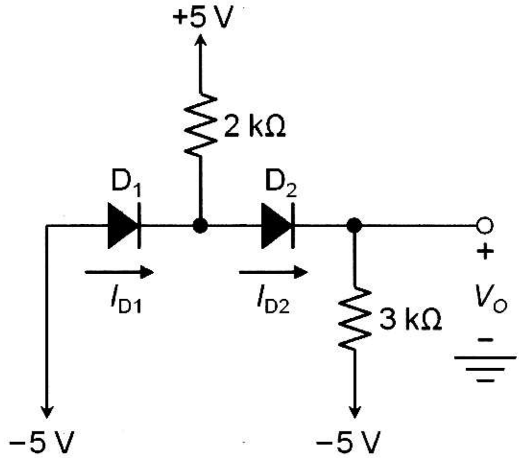
1) The diodes D_1 and D_2 in the circuit shown below are identical and characterized by a constant voltage drop of 0.7 V when forward biased. Find the output voltage V_O and the diode currents I_{D1} and I_{D2} labeled in the figure.

(5 points)



D_1 is REV biased, D_2 is FWD biased.

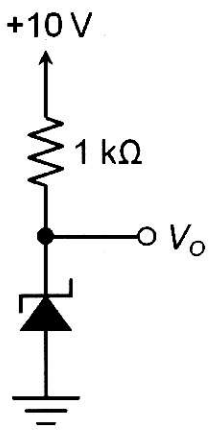
$I_{D1} = 0$ A

$I_{D2} = [5\text{ V} - 0.7\text{ V} - (-5\text{ V})]/(2\text{ k}\Omega + 3\text{ k}\Omega)$
$= 1.86$ mA

$V_O = -5\text{ V} + (3\text{ k}\Omega)(1.86\text{ mA}) = 0.58$ V

2) The Zener diode in the circuit below is characterized by $V_Z = 5.0$ V at $I_Z = 5$ mA, $V_{ZK} = V_{Z0}$, and $r_Z = 10\ \Omega$. Find the output voltage V_O. If the +10V supply voltage varies ±1 V, what voltage variation will appear at the output?

(5 points)



$V_{Z0} = V_Z - r_Z I_Z = 5.0\text{ V} - (10\ \Omega)(5\text{ mA}) = 4.95$ V

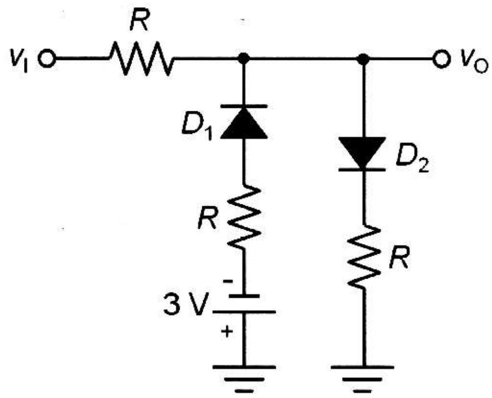
$I = (10\text{ V} - 4.95\text{ V})/(1\text{ k}\Omega + 50\ \Omega) = 4.8$ mA

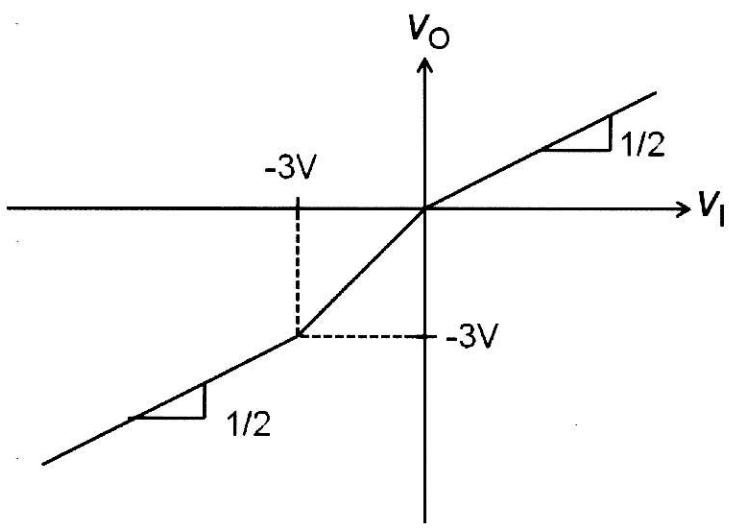
$V_O = 10\text{ V} - (4.8\text{ mA})(1\text{ k}\Omega) = 5.2$ V

$\Delta V = (\pm 1\text{ V})[10\ \Omega/(1\text{ k}\Omega + 10\ \Omega)] = \pm 9.9$ mV

3) For the circuit below, the diodes D_1 and D_2 are identical and assumed to be ideal diodes. All the three resistances (denoted by R) in the circuit are identical. Sketch the transfer characteristic (v_O vs. v_I) indicating all the important points and slopes. Give expressions for v_O as a function of v_I for all the sections of the transfer characteristic.

(7 points)





For $0\text{ V} \le v_I$ and $v_I \le -3\text{ V}$, $v_O = (1/2)v_I$

For $-3\text{ V} \le v_I \le 0\text{ V}$, $v_O = v_I$

4) Each of the following questions refers to the physical structure and operating principles of *Bipolar Junction Transistors* (BJTs).

a) Regarding the three main modes of operation, i.e., *Active*, *Saturation* and *Cutoff*, which mode(s) should be used for the following applications: i) Amplification, ii) Switching.

(3 points)

Amplification: Active
Switching: Saturation and cutoff

b) For a BJT, the *Emitter* is more doped than the *Base*. State the reason.

(3 points)

The current is mainly generated by injecting carriers from the Emitter to the Base. For this purpose, the Emitter is more doped to create a large gradient of carrier concentration in the Base. Also, since the Emitter is typically created within the Base, the dopant concentration for the Emitter needs to be higher than that for the Base to convert the type of Si.

c) Briefly explain how a built-in voltage is generated at a *pn* junction.

(3 points)

The diffusion of the carriers across the junction ionizes the dopant atoms on both sides of the junction due to the recombination. The fixed charges attributed to these ions cause an electric field hence a built-in voltage at the junction.

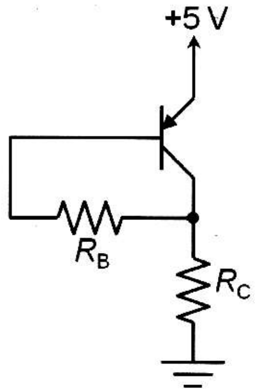
d) What is the name of the effect that gives rise to the finite output resistance r_O of a BJT? Briefly describe the cause of this effect.

(3 points)

The Early effect. This effect is induced by the modulation of depletion region width of the reverse biased CBJ, i.e., the effective width of the Base.

5) The BJT in the circuit below is characterized by the following parameters: $\beta = 80$ (in the *Active Mode*), $V_{EB} = 0.7$ V, $V_A = \infty$. Assume that the BJT is operating in the *Active Mode*. Find the values for R_B and R_C to give $I_C = 0.05$ mA and $V_{EC} = 3$ V.

(5 points)



$R_C = (5\text{ V} - V_{EC})/I_E = (5\text{ V} - V_{EC})(\alpha/I_C)$

$= (5\text{ V} - 3\text{ V})(80/81)/(0.05\text{ mA})$

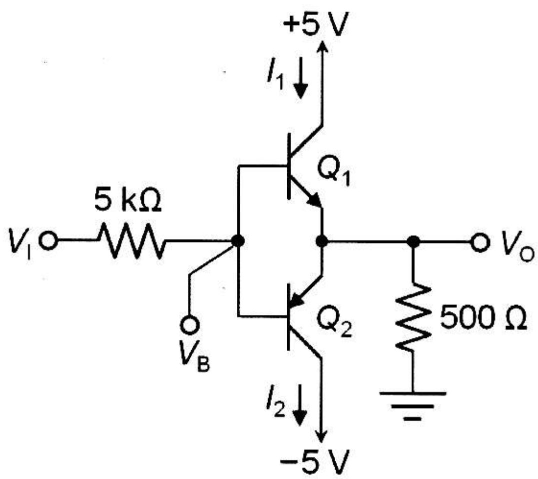
$= 39.5\text{ k}\Omega$

$R_B = V_{BC}/I_B = (V_{EC} - V_{EB})(\beta/I_C)$

$= (3\text{ V} - 0.7\text{ V})(80)/(0.05\text{ mA})$

$= 3.68\text{ M}\Omega$

6) The BJTs Q_1 and Q_2 in the circuit below are characterized by the following parameters: $\beta =$ 50 (in the *Active Mode*), $|V_{BE}| = 0.7$ V, $V_A = \infty$. Assume that the input voltage V_I is –5 V. Answer the following questions.



a) What is the operating mode of each BJT?

(2 points)

Q_1: cutoff, Q_2: Active

b) Find the values of I_1, I_2, V_B, and V_O as indicated in the circuit.

(7 points)

$I_1 = 0$ A

$V_I = -(0.5\text{ k}\Omega)(\beta+1)I_B - V_{BE} - (5\text{ k}\Omega)I_B$

Since $V_I = -5$V and $V_{BE} = 0.7$ V, $I_B = (5\text{ V} - 0.7\text{ V})/(5\text{ k}\Omega + 51(0.5\text{ k}\Omega)) = 0.14$ mA

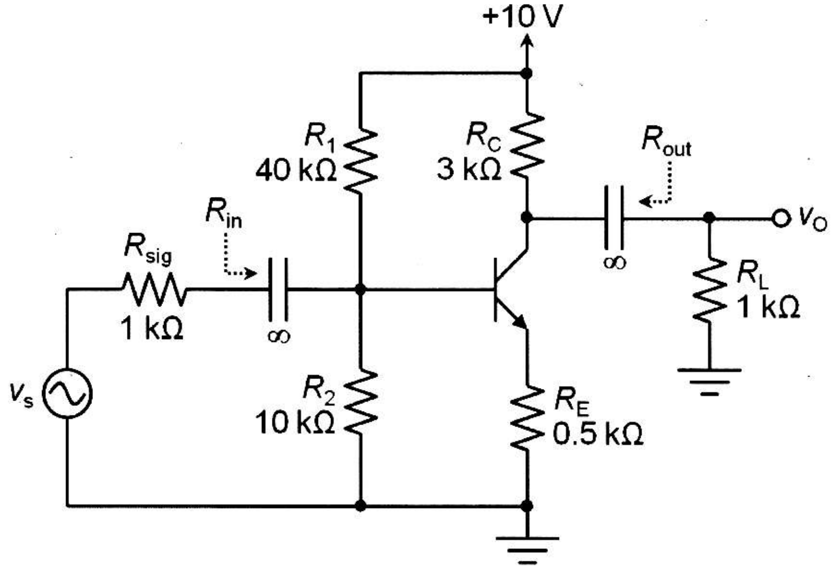
$I_2 = I_C = \beta I_B = 50(0.14\text{ mA}) = 7.0$ mA

$I_E = 51(0.14\text{ mA}) = 7.14$ mA

$V_B = V_I + (5\text{ k}\Omega)I_B = -5\text{ V} + (5\text{ k}\Omega)(0.14\text{ mA}) = -4.3$ V

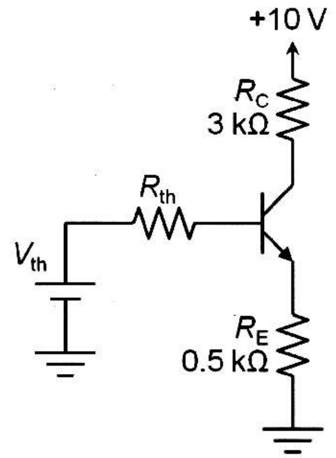
$V_O = -(0.5\text{ k}\Omega)(7.14\text{ mA}) = -3.6$ V

7) This problem deals with the BJT amplifier below, where the BJT is characterized by the following parameters: $\beta = 100$ (in the *Active* mode), $V_{BE} = 0.7$ V, $V_A = \infty$. All the capacitors in the circuit are very large. Answer the following questions. Assume $V_T = 25$ mV.



a) Find the value of I_C. Show that the BJT is operating in the *Active Mode*.

(5 points)



DC equivalent

Thévenin equivalent:

$V_{th} = V_{CC}R_2/(R_1 + R_2) = (10\text{ V})(10\text{ k}\Omega)/(50\text{ k}\Omega) = 2\text{ V}$

$R_{th} = R_1//R_2 = 8\text{ k}\Omega$

Operating point:

$V_{th} = R_{th}I_B + V_{BE} + R_E I_E = R_{th}I_B + V_{BE} + R_E(\beta+1)I_B$

$$I_B = \frac{V_{th} - V_{BE}}{R_{th} + R_E(\beta+1)} = \frac{1.3\text{ V}}{(8\text{ k}\Omega) + 101(0.5\text{ k}\Omega)} = 22.2\ \mu\text{A}$$

$I_C = \beta I_B = 100(22.2\ \mu\text{A}) = 2.22\text{ mA}$

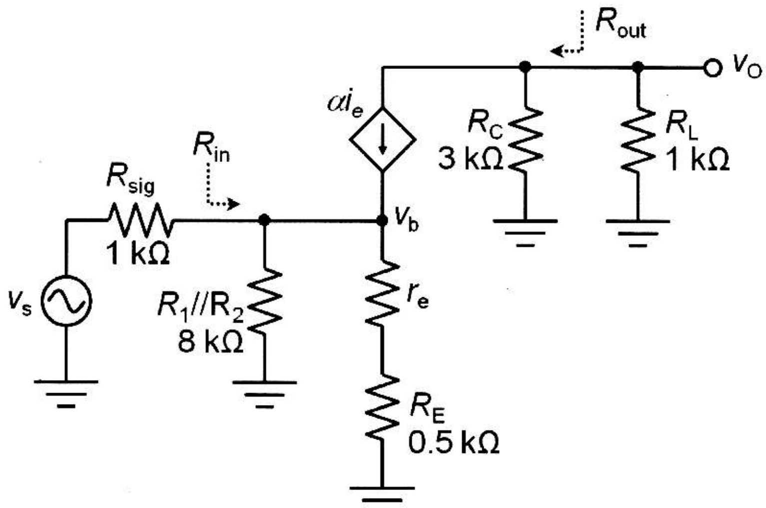
$V_B = V_{th} - R_{th}I_B = 2\text{ V} - (8\text{ k}\Omega)(22.2\ \mu\text{A}) = 1.82\text{ V}$
$V_C = 10\text{ V} - R_C I_C = 10\text{ V} - (3\text{ k}\Omega)(2.22\text{ mA}) = 3.34\text{ V}$

$V_C > V_B \quad \therefore$ Active

Question 7, *Cont'd.*

b) Sketch and label a small-signal equivalent circuit of the amplifier using the "*T*" model.

(4 points)



c) Find expressions for the input resistance R_{in} and the output resistance R_{out} as indicated in the circuit. Calculate their numerical values.

(6 points)

$g_m = I_C/V_T = (2.22\text{ mA})/0.025\text{ V} = 88.8\text{ mA/V}$
$r_e = \alpha/g_m = 100/101/(88.8\text{ mA/V}) = 11.1\text{ k}\Omega$

$R_{in} = R_{th}//[(r_e+R_E)(\beta+1)] = (8\text{ k}\Omega)//[(11.1\ \Omega + 500\ \Omega)101] = (8\text{ k}\Omega)//\ (51.6\text{ k}\Omega) = 6.93\text{ k}\Omega$

$R_{out} = R_C = 3\text{ k}\Omega$

d) Find an expression for the voltage gain v_o/v_s and calculate the numerical value.

(5 points)

$$v_o = -\alpha i_e(R_C // R_L) = -\alpha\frac{v_b}{r_e+R_E}(R_C // R_L) = -g_m\frac{r_e}{r_e+R_E}\left(\frac{R_{in}}{R_s+R_{in}}v_s\right)(R_C // R_L)$$

Then,

$$\frac{v_o}{v_s} = -g_m\frac{r_e}{r_e+R_E}\frac{R_{in}}{R_s+R_{in}}(R_C // R_L)$$

$$= -(88.8\text{ mA/V})\frac{11.1\,\Omega}{11.1\,\Omega+500\,\Omega}\frac{6.93\,\Omega}{1\text{ k}\Omega+6.93\,\Omega}(0.75\text{ k}\Omega)$$

$$= -1.26\text{ V/V}$$

8) Each of the following questions refers to the physical structure and operating principles of *Metal-Oxide-Semiconductor Field-Effect Transistors* (MOSFETs).

a) The BJT uses two *pn* junctions to control the current through the device. In the MOSFET, how many *pn* junctions are used to control the current through the device?

(2 points)

Zero.

b) How does the structure of a <u>depletion-type</u> PMOS device differ from that of the enhancement type? What is the effect of this difference in terms of V_t?

(3 points)

A depletion-type PMOS has a built-in channel that is physically doped to be p-type. Due to the presence of this physical channel, the device conducts a current if a V_{DS} is applied even the gate voltage is zero. To turn off the device, a positive voltage needs to be applied to the gate to deplete the channel, i.e., the threshold voltage V_t is positive for this device.

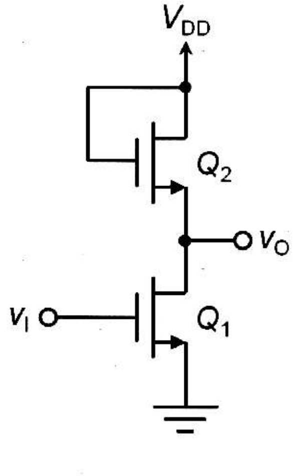
c) Briefly explain the cause of the *Early Effect* in the MOSFET.

(3 points)

Modulation of the channel length caused by the extension of pinched-off region (due to $V_{DS} > V_{GS} - V_t$).

d) Assume that the circuit below is used for integrated circuit applications. The connections to the bodies of the MOSFETs Q_1 and Q_2 in the circuit are not shown. Does Q_1 exhibit the *Body Effect*? What about Q_2? Explain the reasons of your answers.

(4 points)



In IC applications, the body of an NMOS is connected the most negative voltage source in the circuit, which is the ground in this case.

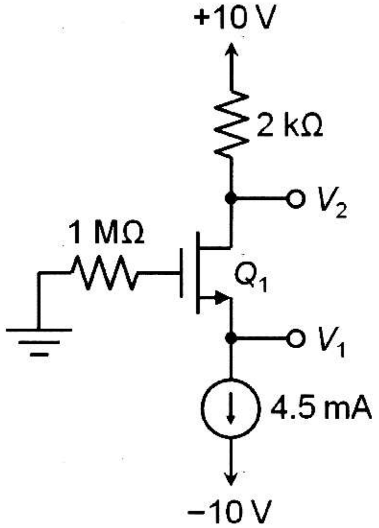
Given that, the source and body of Q_1 will be short circuited (to the ground) i.e. $v_{BS} = 0$ V hence no body effect involves.

The body of Q_2 is also grounded but the source is not so the body effect should be taken into account here.

9) Answer the following questions.

a) In the circuit below, the NMOS Q_1 is characterized by $k_n'\left(\frac{W}{L}\right) = 1$ mA/V^2, $V_t = 3.5$ V, and $V_A = \infty$. Find V_1 and V_2 indicated in the circuit assuming that the NMOS is operating in the *Saturation Mode*. Verify that the assumption is correct.

(5 points)



$V_2 = 10$ V - (2 kΩ)(4.5 mA) = 1 V

$I_{D1} = (0.5\ \text{mA/V}^2)(V_{GS1} - 3.5\ \text{V})^2 = 4.5$ mA

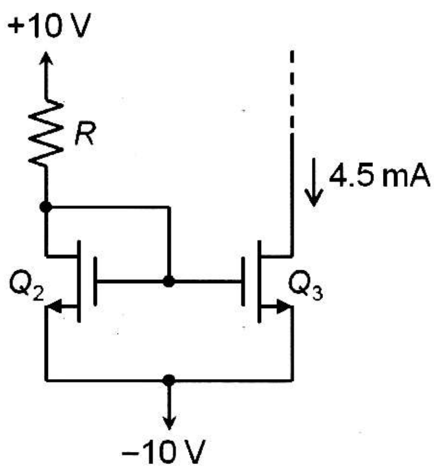
Then $V_{GS1} = 6.5$ V $\quad \therefore V_1 = 0 - V_{GS1} = -6.5$ V

$V_{DS1} = V_2 - V_1 = 1 - (-6.5) = 7.5$ V

$V_{DS1} = 7.5\ \text{V} > V_{GS1} - V_t = 3$ V $\quad \therefore$Saturation

b) Now, the constant current source of 4.5 mA in the above circuit is implemented by the circuit below. Find the value of R if Q_1, Q_2 and Q_3 are all matched and assumed to have $V_A = \infty$.

(3 points)



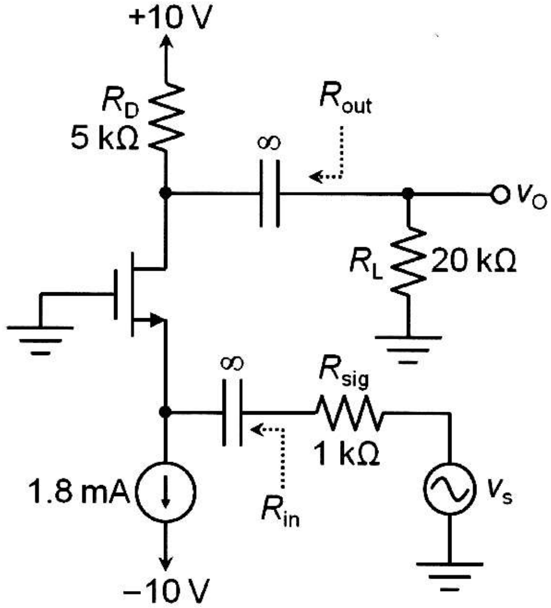
$I_{D1} = I_{D3} = I_{D2} = 4.5$ mA

$V_{GS2} = V_{GS3} = V_{GS1} = 6.5$ V

$V_{D2} = V_{G2} = -10\text{V} + V_{GS2} = -3.5$ V

$R = (10\text{V} - V_{D2})/I_{D2} = (13.5\ \text{V})/(4.5\ \text{mA}) = 3$ kΩ

10) This problem deals with the MOSFET amplifier below, where the NMOS is characterized by the following parameters: $k_n'\left(\frac{W}{L}\right) = 0.4$ mA/V^2, $V_t = 1$ V, and $V_A = \infty$. All the capacitors in the circuit are very large. Answer the following questions.



a) Quantitatively justify that the NMOS is operating in saturation.

(5 points)

$K_n = (1/2)k_n'(W/L) = 0.2$ mA/V^2

$V_{GS} = \pm(I_D/K_n)^{1/2} + V_t = \pm[(1.8 \text{ mA})/(0.2 \text{ mA/V}^2)]^{1/2} + 1 \text{ V} = \pm 3 \text{ V} + 1 \text{ V} = 4 \text{ V or -2 V}$

$V_{GS} = 4$V for $V_{GS} > V_t$

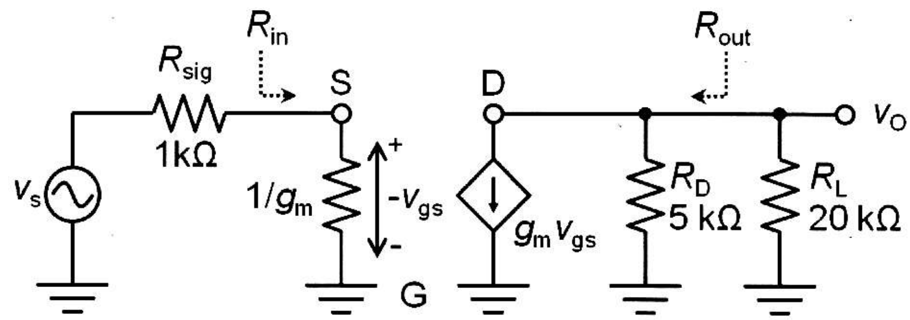
$V_D = 10 \text{ V} - I_D R_D = 10 \text{ V} - (1.8 \text{ mA})(5 \text{ k}\Omega) = 1 \text{ V}$
$V_S = V_G - V_{GS} = 0 \text{ V} - 4 \text{ V} = -4 \text{ V}$

Then V_{DS} (= 5 V) > $V_{GS} - V_t$ (= 4 V - 1 V = 3 V) $\therefore$ Saturation

Question 10, *Cont'd.*

b) Sketch and label a small-signal equivalent circuit of the amplifier.

(4 points)



c) Find expressions for the input resistance R_{in} and the output resistance R_{out} as indicated in the circuit.

(3 points)

$g_m = (4K_nI_D)^{1/2} = [4(0.2\text{ mA/V}^2)(1.8\text{ mA})]^{1/2} = 1.2\text{ mA/V}$

$R_{in} = 1/g_m = 833\ \Omega$

For R_{out}, short circuiting v_s results in $v_{gs} = 0$ V, i.e., $i_d = g_m v_{gs} = 0$ A or D is open circuit.
$\therefore R_{out} = R_D = 5\text{ k}\Omega$

d) Find an expression for the voltage gain v_o/v_s and calculate the numerical value.

(5 points)

$$
\begin{aligned}
v_o &= -g_m v_{gs}(R_D \,//\, R_L) \\
&= g_m(-v_{gs})(R_D \,//\, R_L) \\
&= g_m\left[\frac{1/g_m}{(1/g_m)+R_{sig}} v_s\right](R_D \,//\, R_L) \\
&= \frac{R_D \,//\, R_L}{(1/g_m)+R_{sig}} v_s
\end{aligned}
$$

$$\therefore A_v = \frac{v_o}{v_s} = \frac{R_D \,//\, R_L}{(1/g_m)+R_{sig}} = -\frac{4\text{ k}\Omega}{833\,\Omega + 1\text{ k}\Omega} = 2.18\text{ V/V}$$