Mech 366
Modeling of Mechatronic Systems

Lab #3
Fundamentals and Speed Control of a DC Motor

LEARNING OBJECTIVES

The objectives of the experiments in this lab are:
- to become familiar with the DC Motor and its related performance parameters
- to illustrate fundamentals of pulse-width modulation, open-loop control and closed-loop control

BACKGROUND

This is the first of three experiments involving a custom designed experimental control system. The laboratory set-up utilizes common industrial motors and positioning assembly (UniSlide Motor Driven Positioning System manufactured by VELMEX Inc.). Research staff from the UBC Mechanical Engineering Department have customized the control system in order to highlight several of the concepts taught with MECH 366. The unit utilizes a DC motor coupled directly to a precision lead screw and slider. A tachometer and rotary encoder supply velocity and position signals to custom built controllers that control the motor speed, rotational direction and position. A pneumatic actuated friction clutch controller provides an adjustable load for the system.

Referring to the standard terms used in modeling of mechatronics systems, the components found in this experiment can be grouped as follows: the ***plant*** is the assembly of a DC motor, amplifier, lead screw and slider. The speed and position of the slider is controlled by the servomechanism. A ***servomechanism*** is an electromechanical system comprised of devices designed specifically for accurate positioning or controlling of motion. The ***actuator*** that drives this system is a DC motor (servomotor). A ***servomotor*** is a precision electric motor whose function is to cause motion in the form of rotation or linear motion in response to a supplied electrical command signal.

DC Motor Fundamentals

A DC motor is often a fundamental component for a modern mechatronic device. In general, DC motors have predictable torque to speed relationships that allows for smooth control of a device. In addition, DC motors have a high torque to rotor inertia that provides for quick response to a control signal. DC motors are relatively inexpensive when compared to other types servomotors and can be safer to operate due to the lower voltage requirements for the device. The basic components of a simple DC motor (permanent magnet DC motor) are shown on Figure 1. Also see section 2.7.8 in the text.

The stationary portion of the motor (stator) has magnets that establish a magnetic field across the rotating portion of the motor (rotor). The brushes conduct current through the armature (wire wound around the rotor) which is rotating through a magnetic field that produces the required mechanical torque. The (rotating) commutator is designed so that the current is always sent through the windings to produce the maximum torque in the desired direction. If the direction of the current is reversed, the direction of the torque is reversed.

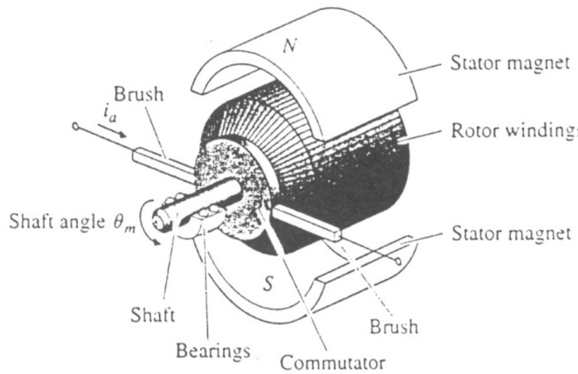


Figure 1. Components of a DC Motor

Figure 2 illustrates a simplified circuit for a DC motor. A supply source supplies the necessary motor voltage V_s, and current, *i,* to the motor. Due to the interaction of the current passing through the stator magnetic field, a motor torque is produced. This mechanical torque is directly related to the armature current;

$$T = K_t \ i$$

where K_t is the torque constant (a motor specification).

As the armature moves through the field, a voltage is induced in the armature windings. This voltage, called *back emf*, is opposing the supply motor voltage. The back emf, *e*, is directly proportional to the rotational speed $\dot{\theta}$ or ω of the armature.

$$e = K_e \ \omega$$

where K_e is electric "emf" constant. (*emf* stands for **e**lectro**m**otive **f**orce). Again, since the generated electromotive force works against the applied armature voltage, it is normally called *back emf***.**

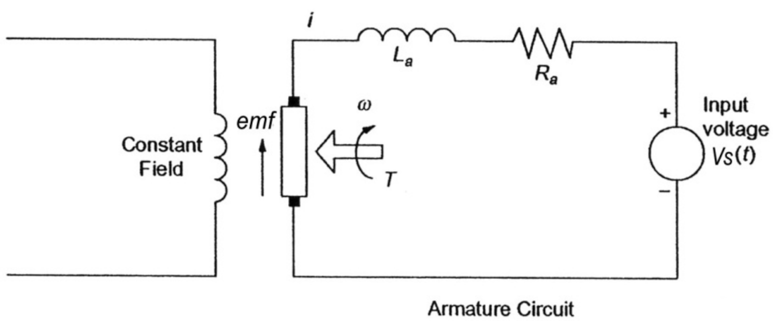


Figure 2. Simplified circuit diagram of the DC motor

The KVL (**K**irchhoff's **V**oltage **L**aw) loop analysis of the armature windings shows the electrical equation to be:

$$V_s = emf + L_a \frac{di}{dt} + R_a i \qquad 1$$

where L_a and R_a are inductance and resistance of the armature respectively.

At steady state, equation 1, simplifies to,

$$V_s = K_e \omega + R_a i \qquad 2$$

We can rewrite equation 2 in terms of motor speed and torque,

$$V_s = K_e \omega + \frac{R_a}{K_t} T \qquad 3$$

$$T = \frac{K_t}{R_a} V_s - \frac{K_e K_t}{R_a} \omega \qquad 4$$

Equation 4 predicts a linear torque-speed relationship for a permanent magnet DC motor at a fixed supply voltage (Figure 3). The maximum torque occurs at zero speed, called *starting or stall torque* T_s while maximum speed occurs at zero torque, called *no-load speed,* ω_{max}. The torque-speed relationship can be expressed simply in terms of the stall torque and no-load speed,

$$T(\omega) = T_s \left(1 - \frac{\omega}{\omega_{\max}} \right) \qquad 5$$

Since the mechanical output power is torque times rotational speed, we can write;

$$P(\omega) = \omega T_s \left(1 - \frac{\omega}{\omega_{\max}} \right) \qquad 6$$

It can be shown through differentiation of equation 6 that the maximum power output occurs at exactly one-half of the no-load speed. It can also be shown that by equating the electrical input power (Vi) with the mechanical output power ($T\omega$) we can show,

$$K_t = K_e \quad \text{(true for SI units)} \qquad 7$$

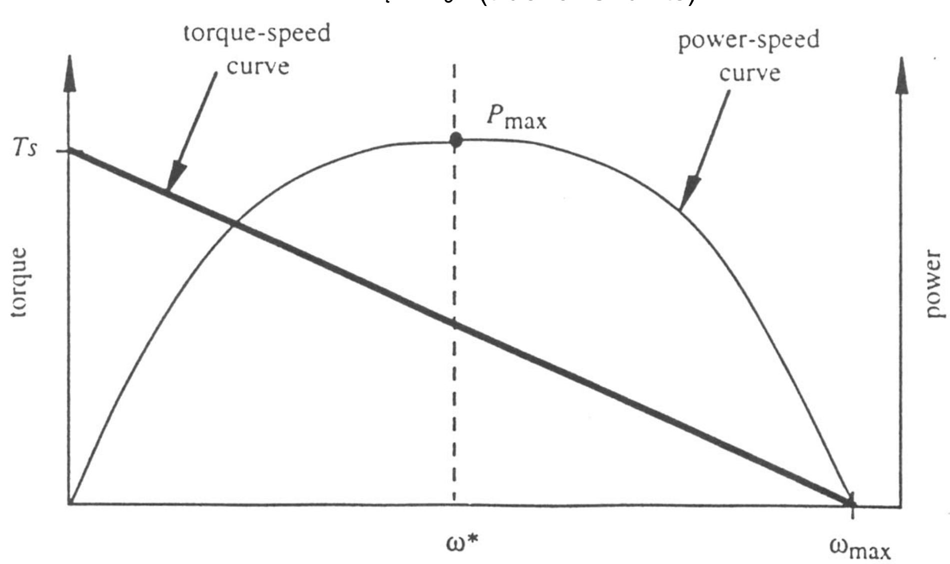


Figure 3.Permanent Magnet DC Motor Characteristics (Ideal)

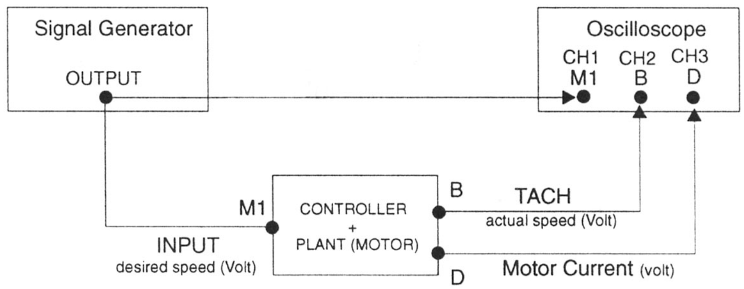


Figure 4. Simplified Overview of the Set-up

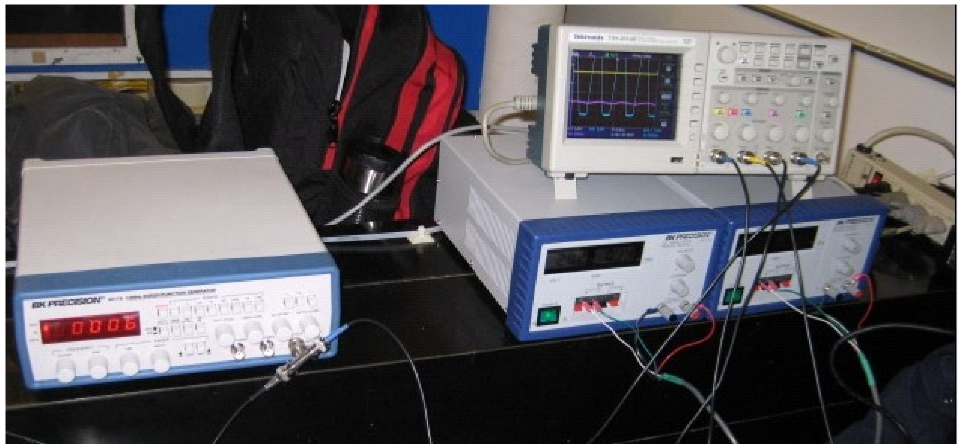
Figure 4 is a simplified illustration of the experimental set-up. The experiment requires an adjustable input generated by the signal generator (also called *function generator*) to be the input to the plant. The behaviour of the plant and controller can be viewed on the Oscilloscope (*Tectronix 420A).* Figure 5 shows the view of the front panel of the experimental setup. Using cables with BNC connectors, you will wire various portions of the plant together and view the results on the oscilloscope. Figure 5 is also used to illustrate the various connections you need to complete this experiment.

In this first experiment, you will be using only one of the controllers. Within this circuit, you will only be using the proportional gain controller. The proportional gain generates a signal proportional to the input voltage. In Experiment B, we will see it is used to generate a signal proportional to the error signal (*error being defined as the difference between the desired speed (reference input) and the actual speed)*. Load on the motor is developed through a use of a pneumatic pressure controller with adjustable torque range form 3.4 to 76.0 oz.-in.).

The velocity of the motor is measured through the use of a tachometer directly coupled to the motor. A tachometer is an electromechanical device that converts mechanical energy into electrical energy. The device works essentially as a generator, with the output voltage proportional to the magnitude of the rotational speed. The dynamics of the tachometer can be represented by the equation:

$$V_{tach}(t) = K_{Tach}\frac{d\theta}{dt} = K_{Tach}.\omega(t) \qquad 8$$

where $V_{tach}(t)$ is the output voltage, $\theta(t)$ the rotor displacement in radians, $\omega(t)$ the rotor angular velocity in rad/s, and K_{Tach} the *tachometer voltage constant* in Volt / (rad/s). It must be noted that K_{Tach} is often given in Volt/rpm. The tachometer constant for this motor is provided in the appendix.



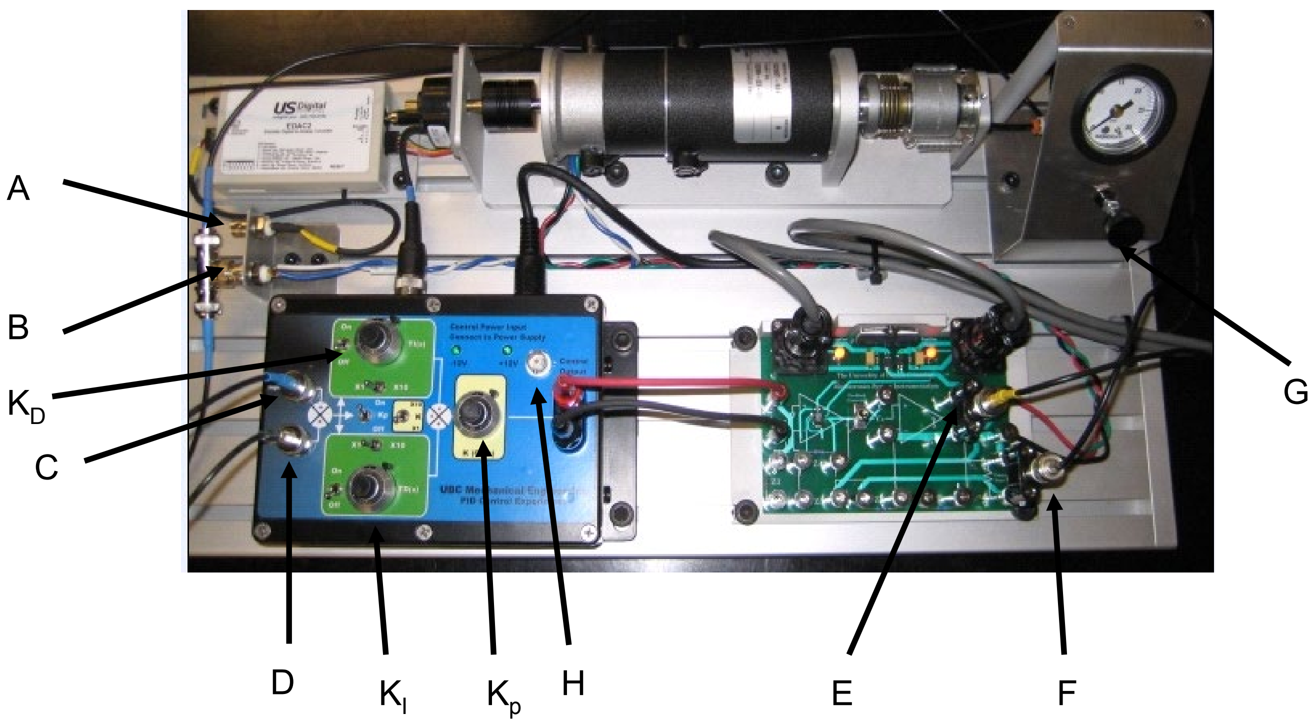


Figure 5. View of Front Panel for Linear Actuator Experiment

A = Position Output Signal
B = Velocity / Tachometer Output Signal (connect to oscilloscope channel 2)
C = Feedback Input (connect to B/velocity signal **ONLY IF PERFORMING FEEDBACK EXPERIMENTS**)
D = Controller Input (connect to oscilloscope channel 1)
E = Motor Voltage Output Signal (connect to oscilloscope channel 4)
F = Motor Current Output Signal (connect to oscilloscope channel 3)
G = Pneumatic Clutch Controller
H = Controller Output
K_p = Proportional Gain Control
K_I = Integral Gain Control
K_D = Derivative Gain Control

EXPERIMENT A
Steady State DC Motor Parameter Identification

The objective of this experiment is to determine through experimentation, the various motor parameters and compare them with the published values shown in the appendix. You vary the voltage on the motor while keeping the motor current constant (i.e. load) and measure the speed and motor voltage.

PROCEDURE

In this experiment, you will want the motor to oscillate back and forth between two set points (i.e. the function generator will input a square wave that will switch from a positive voltage to a negative voltage thereby causing a reciprocating motion of the shaft).
This is done to average positive and negative readings, thereby eliminating any zero offset.

Display Voltages: **The motor current (output F - oscilloscope Ch. 3) is measured over a 0.2Ω resistor. The voltage measured on the oscilloscope must therefore be converted to amps (use V=RI).**

1. Create an **open loop circuit** for the plant using the BNC cables. Refer to Figure 5.

2. Setup the parameters of the signal generator for your desired speed as follows:

   INPUT FROM SIGNAL GENERATOR:
   - Square Wave
   - Frequency : 0.7 Hz
   - Peak to peak : 5 volts

3. Set-up the parameters of the controller in the following fashion.

   CONTROLLER PARAMETERS:
   - Proportional Control (Kp) - ON - x1
   - Derivative Control (Kd) – OFF
   - Integral Control (1/Ki) – OFF

4. Set-up the oscilloscope in order to monitor the operation of the plant. Refer to Figure 5 to connect the required leads to the oscilloscope (you should use the figure to interpret the different components in the system, not as a guide to the exact wiring connections to make as it changes from experiment to experiment – make sure you are in open loop control and measuring the tachometer signal). Using the oscilloscope, connect the input signal to channel 1, the tachometer voltage to channel 2, the motor current to channel 3, and the motor voltage to channel 4.

5. Measure the **actual speed of the motor in volts** as outputs from the tachometer at point B (Figure 5). **Note:** again, you measure peak-to-peak voltage **using the cursors**.

6. Be sure pneumatic clutch coupling is backed up fully (counter-clockwise) so as to provide minimum load on the motor.

7. Turn on the motor and adjust the motor load by adjusting the pneumatic load knob clockwise until a motor current of approximately 1.5-2.5 amps is measured (before uneven back and forth motion of the motor). Note that you will want to measure the steady state current so use the cursors and ignore the spikes. Record signal generator voltage, motor voltage and tachometer voltage. Use Table 1.

8. Keeping the load constant, repeat for signal generator voltage of 5 volts to 10 volts. Complete Table 1.

Table 1. Constant Motor Current Results

Signal Generator Voltage (peak to peak)	Signal Voltage (actual) (V_{PP})	Motor Voltage (terminal E) (V_{PP})	Motor Current (terminal F) (V_{PP})	Tach. Voltage (terminal B) (V_{PP})
5				
6				
7				
8				
9				
10				

Note: Don't forget to convert the measured motor current to amps and the tach voltage to rad/s for your calculations (use the appendix).

Note: Please be advised to maintain the current value constant while changing the signal generator's voltage. Since the external force is constant, changes in current should not be so steep; however, slight adjustment of pneumatic clutch is necessary to keep the current consistent through experiment.

9. Adjust signal generator to approximately 7 volts peak-to-peak on the oscilloscope. Starting from the no-load case (pneumatic clutch backed off fully), record motor voltage, motor current, and tachometer voltage. Use Table 2.

10. Now adjust pneumatic clutch to a high value just before uneven back and forth motion of the motor. Record motor parameters.

11. Adjust pneumatic clutch to provide seven intermediate motor current points between the ones measured in steps 9 and 10 in order to complete Table 2. Note: you may not be able to fill in entire table, but you want enough points to draw the graph. (Don't forget to include the no-load and max-load condition in your table)

Table 2. Constant Motor Voltage Results

Signal Generator Dial (peak-to-peak)	Signal Generator Voltage (V_{PP})	Monitored Motor Voltage (actual) (V_{PP})	Motor Current (terminal F) (V_{PP})	Tach. Voltage (terminal B) (V_{PP})	Torque (N.m) $T = K_t i$
7					
7					
7					
7					
7					
7					
7					
7					
7					

Do not write in laboratory manuals

Note: Don't forget to convert the measured motor current to amps and the tach voltage to rad/s for your calculations (use the appendix).

Note: Don't forget to include the no-load condition in the first row of this table.

Note: In derivation of $T - \omega$ curve please be advised to find actual $T(N.m)$ by $T = K_t i$, which is derived from conservation of power and more accurate. (K_t used here should be the experimental value not the value provided by the manufacturer)
Please Do Not use theoretical modelling formulation (eq. 4 in prelab), not including un-modelled terms.

REPORT REQUIREMENTS

Using the given equations from the background materials, determine the motor constants by plotting motor voltage versus speed at a constant current. How do your values of K_t, K_e, and R_a compare with the manufacturers specifications given in the Appendix? Comment on the results. Using your experimental motor parameters and the given equations, develop a torque-speed curve for the DC motor. In addition develop a power curve for the motor. Compare your experimental curves with the ideal case. Comment on the results.

1) From the first table filled in part A, derive experimental $V_s - \omega$ curve, find experimental values for K_e and R_a using this curve (Current is maintained constant and thus R_a can get found easily). How much do these values match with those provided by the manufacturer? Any clues for the deviations?

2) From data collected in the second table of this section and using experimental values derived previously, develop a torque versus speed curve

3) Discuss on experimental values found on T_{stall} and ω_{max} and compare them with values provided by the manufacturer.

4) Develop an experimental $P(\omega)$ versus ω curve and compare it with the ideal curve. Discuss on ω^* and P_{max} value. How much did they get changed from their theoretical values?

Note: $P(\omega)$ versus ω curve should get derived through finding the best parabolic curve passing through data points.

EXPERIMENT B
Introduction to Feedback

The objective of this experiment is to observe how a feedback loop can be used to control the motor better.

BACKGROUND

Control system using DC Motors normally require set points for their desired operation. These set points are normally either a position or speed parameter. It is the role of the control system to maintain these targets under a variety of loading conditions. In this experiment, we will concentrate only on velocity control for the DC motor.

Figures 6 shows the simplified closed loop diagram for the system. An input signal (voltage signal) from signal generator is fed into our plant. This input signal corresponds to the *desired speed* (*Reference input*) we wish to achieve. In the open loop, no error signal is developed (i.e. the junction point is open) while in the closed loop, a feedback signal (from the tachometer) is input to the junction point creating an error signal to the controller (difference between desired speed and actual speed in volts). The controller actively changes the motor output to move closer to the set point.

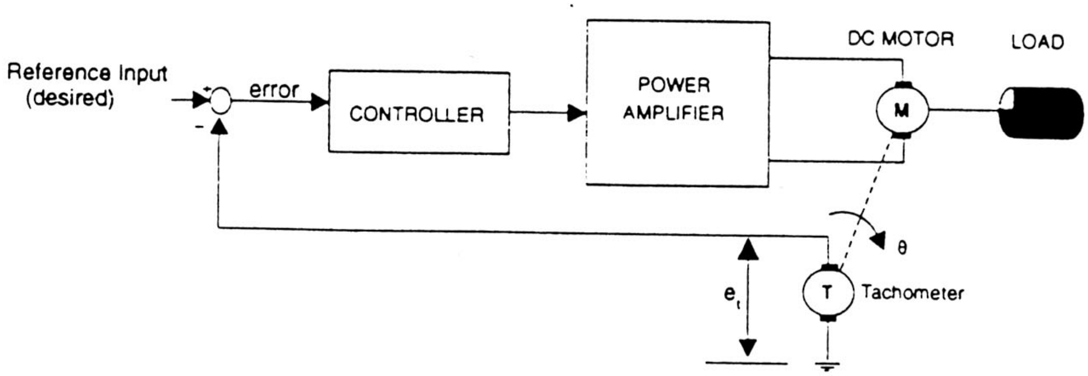


Figure 6. Speed Control of a DC Motor with Tachometer Feedback.

One of the characteristics of any servomechanism is that there is always an upper limit to the output. This is referred to as *Saturation.* Saturation is a nonlinear phenomenon in actuators and amplifiers. When the input signal magnitude in the amplifier exceeds a certain level, the output magnitude no longer increases with the input. Similarly, in the DC motor (actuator), when the magnetic field of the motor is saturated, increasing the armature current will no longer produce additional torque.

PROCEDURE

In this experiment, you will want the motor to oscillate back and forth between two set points (i.e. the function generator will input a square wave that will switch from a positive voltage to a negative voltage thereby causing a reciprocating motion of the slider). As the desired speed we choose an input of 7 volts peak to peak. You will now want to introduce feedback by inputting the tachometer output voltage into the control circuit.

1. Establish a closed loop system by applying the reference signal (7 volts amplitude) to the controller (D). Connect the feedback signal from the tachometer to the feedback input (C). Use Figure 5 to assist with the completion of the control loop.

2. Set-up the parameters of the controller and signal generator in the following fashion.

3. For the following signal find the maximum gain value for proportional controller for the device you are working with to be stable (K_max) (in the vicinity of instability)

   INPUT FROM SIGNAL GENERATOR:
   - Square Wave
   - Frequency : 0.7 Hz
   - Peak-to-peak : 7 v

   CONTROLLER PARAMETERS:
   - Proportional Control (Kp) - ON - x K_max, K_max/2
   - Derivative Control (Kd) – OFF
   - Integral Control (1/Ki) – OFF

4. Back off the pneumatic clutch to minimize motor load.

5. Measure the tachometer voltage and motor current. Enter values on Table 3.

6. Apply motor load by turning the pneumatic clutch in order to generate motor current values of approximately 1.5 to 5.0 amps (or until motor moves unevenly – use your own judgment for range and increment to get a good plot). Record values on Table 3.

Table 3. Closed Loop Results Proportional Gain, $K = K_{max}/2$

K = K_max/2		
Motor Current (terminal F) (VPP)	Tach. Voltage (terminal B) (VPP)	Torque (N.m) $T = K_t i$
Do not write in laboratory manuals		

Note: Don't forget to convert the measured motor current to amps and the tach voltage to rad/s for your calculations (use the appendix).

Note: In derivation of $T - \omega$ curve please be advised to find actual $T(N.m)$ by $T = K_t i$, which is derived from conservation of power and more accurate.
Please Do Not use theoretical modelling formulation (eq. 4 in prelab), not including un-modelled terms.

6. Repeat for K =K_max in Table 4.

Table 4. Closed Loop Results Proportional Gain, $K = K_{max}$

K = K_max		
Motor Current (terminal F) (V_{PP})	Tach. Voltage (terminal B) (V_{PP})	Torque (N.m) $T = K_t i$

Note: Don't forget to convert the measured motor current to amps and the tach voltage to rad/s for your calculations (use the appendix).

Note: the slope and intercept of these graphs do not give the same motor properties as in Experiment A because the feedback loop changes the characteristics of the system.

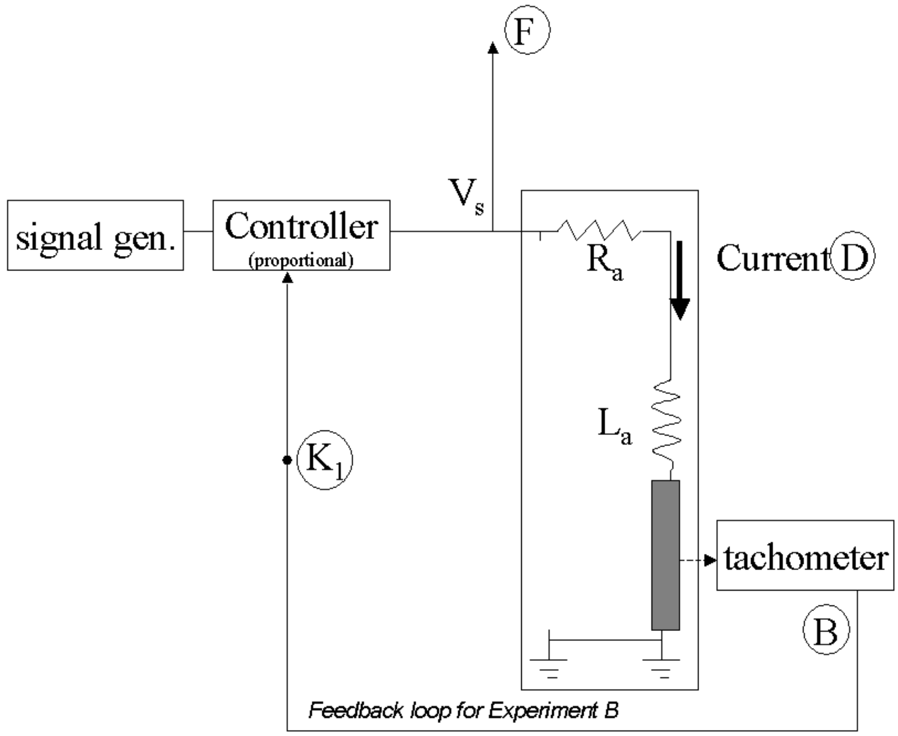
REPORT REQUIREMENTS

Use your results to plot motor speed versus motor torque for the closed loop conditions you completed in this experiment. Use the published motor parameters given in the Appendix. Comment on the results of the open versus closed system. Comment on any speed differences you saw in the closed-loop versus open-loop cases

1) Comparison of $T - \omega$ graphs in both closed-loop cases with different gains, how does the slope and intercept get changed by the rise in gain?
2) Comparison of $T - \omega$ graphs in closed and open loop cases, how does the slope and intercept get changed by changing from open loop to closed loop (Theoretical lines with the same gain for proportional controller in both open-loop and closed-loop cases can get derived)
3) Steady state error analysis on both closed-loop systems, how does the steady state error get changed by increasing proportional controller gain?
4) Steady state error analysis of closed-loop system versus open-loop system. How does the steady state error get changed by changing from open-loop to closed-loop circuit?

GENERAL REPORT REQUIREMENTS

As part of your formal write-up, include an objective for each experiment followed by a brief background related to that experiment section. Complete the tables in each section and answer the questions. Analyze your results and write a conclusion for each of the experiment sections. Also explain the relationship between the steady-state error and the use of proportional feedback control. What happens to steady state error when the proportional gain is increased?


F
V_s
signal gen.
Controller
(proportional)
R_a
Current D
L_a
K_1
tachometer
B
Feedback loop for Experiment B

APPENDIX: Specification of the DC Motor

Specifications of the DC motor			
J_m	Inertia of the motor	5.62 x 10^{-5}	$kg.m^2$
b	friction coefficient	2.61 x 10^{-4}	$N.m.\sec$
K_e	electric "emf" constant	0.068	$Volt/[rad/s]$
K_t	torque constant	0.068	Nm/A
L_a	armature inductance	3.0×10^{-3}	H
R_a	armature resistance	1.63	Ω
τ_m	mechanical time constant	0.216	sec
τ_e	electrical time constant	1.84×10^{-3}	sec
K_{Tach}	Tachometer Voltage Constant	0.134	$Volt/[rad/s]$
K_i	Current Sensor Constant	0.2	$Volt/A$

*Note: The textbook uses the following symbols instead: J_d, b_d, k_m, …see Section 2.7.8 to compare symbols.

*Aside: In many other textbooks, K_e is labeled as K_b where the subscript 'b' indicates 'back emf' (not viscosity).