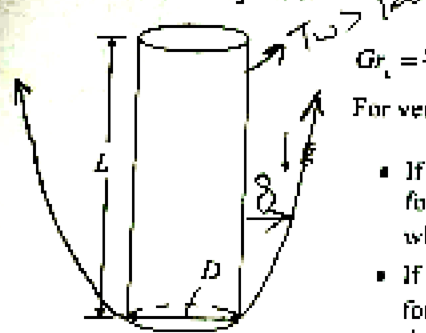
M9

3: Vertical Cylinder



$$Gr_L = \frac{g\beta[T_w - T_\infty]L^3}{\nu^2}$$

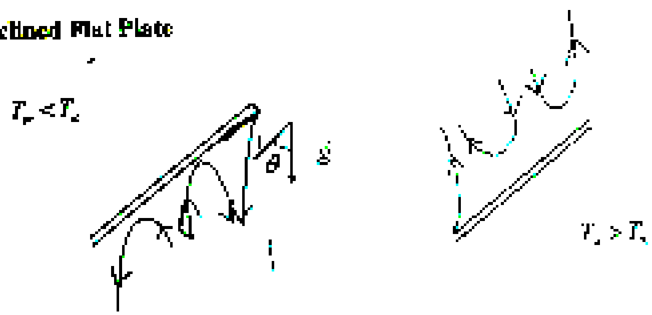
For vertical cylinder:

- If δ << D, use the correlations given for vertical flat plate. The condition is met when: $D/L \geq 35/Gr_L^{1/4}$
- If the $D/L < 35/Gr_L^{1/4}$, the flat plate results for the average heat transfer coefficient should be multiplied by a factor F given below (transverse curvature plays a role):

$$F = 1.3\left[(L/D)/Gr_D\right]^{1/4} + 1.0$$

$$\text{Where: } Gr_D = \frac{g\beta[T_w - T_\infty]D^3}{\nu^2}$$

5: Inclined Flat Plate



- For the ...... surface of cooled plates ($T_w < T_\infty$) and the ...... Surface of heated plates ($T_w > T_\infty$) use the vertical flat plate correlations with g replaced by $g\cos\theta$. This approach is valid for $0 < \theta < 60$.
- For all other cases, look for specialized correlations/data

Check Reference

For the top surface of cooled plates and the bottom surface…

...ommon practice [1st approximation] (Incropera and DeWitt, 1994):

- For Aiding (assisting) and Transverse Flows:

$$(Nu)^n_{mixed} = (Nu)^n_{forced} + (Nu)^n_{Natural}$$

$n = 3$ Assisting flow over vertical plates

$n = 3.5$ Transverse flow over vertical plates

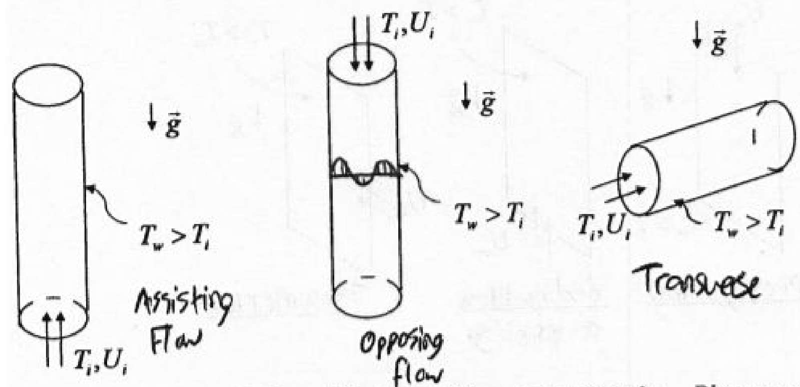
$n = 4$ Transverse flow over cylinders or spheres

- For Opposing Flows:

$$(Nu)^n_{mixed} = (Nu)^n_{forced} - (Nu)^n_{Natural}$$

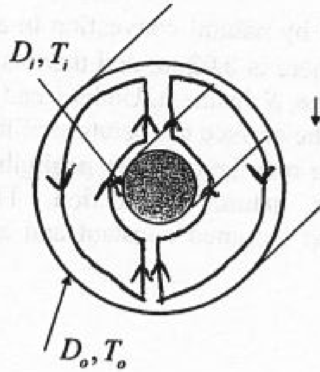
$n = 3$ Opposing flow over vertical plates

...: Internal Mixed Convection in Pipes:



Works of Metais and Eckert; Martinelli, Brawn and Gauvin… Please see Figures 7-13 and 7.14 in J.P. Holman 2002, for summary of available correlations.

3: Horizontal Concentric Cylinders (Isothermal)



$T_i > T_o$

- Rate of heat transfer:

$$q = \frac{(T_i - T_o)}{\ln(D_o / D_i) / 2\pi k_{eff} L} \text{ And,}$$

$$Gr_\delta = \frac{g\beta[T_i - T_o]\delta^3}{\nu^2}; \; Ra_\delta = \Pr Gr_\delta$$

Where $\delta = r_o - r_i$

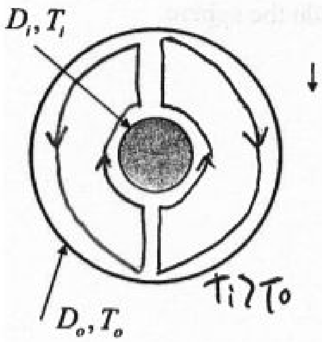
$$\frac{k_{eff}}{k_{fluid}} = 0.11(Gr_\delta \Pr)^{0.29} \quad 6000 \le Gr_\delta \Pr \le 10^6, \; 1 \le \Pr \le 5000$$

$$\frac{k_{eff}}{k} = 0.40(Gr_\delta \Pr)^{0.20} \quad 10^6 \le Gr_\delta \Pr \le 10^8, \; 1 \le \Pr \le 5000$$

f : effective thermal conductivity

$$T_{film} = \frac{T_i + T_o}{2}$$

: Concentric Spheres (Isothermal)



$T_i > T_o$

- Rate of heat transfer:

$$q = \frac{(T_i - T_o)}{(1/r_i - 1/r_o) / 4\pi k_{eff}} \text{ And,}$$

$$Gr_\delta = \frac{g\beta[T_i - T_o]\delta^3}{\nu^2}; \; Ra_\delta = \Pr Gr_\delta$$

Where $\delta = r_o - r_i$

$$T_{film} = \frac{T_i + T_o}{2}$$

$$\frac{k_{eff}}{k} = 0.228(Gr_\delta \Pr)^{0.226} \quad 1.2\times10^2 \le Gr_\delta \Pr \le 1.1\times10^9, \; 0.7 \le \Pr \le 4150, \; 0.25 \le \delta / r_i \le 1.5$$

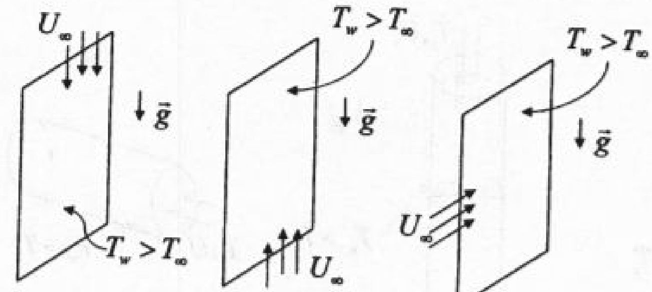
C: Mixed Convection (Combined Forced and Natural Convection)

Reynolds #, $\mathrm{Re}_{L_c} = \dfrac{\rho U_c L_c}{\mu}$; $\; Gr_{L_c} = \dfrac{g\beta(\Delta T)_c L_c^3}{(\mu/\rho)^2}$;

Richardson → Buoyancy Forces / Inertia Forces

$$\frac{Gr_c}{\mathrm{Re}_c^2} = \begin{cases} < 0.1 & \text{Forced convection dominates} \\ 0.1 \le (\;) \le 10 & \text{Mixed convection} \\ > 10 & \text{Natural convection dominate} \end{cases}$$

1: External Mixed Convection:

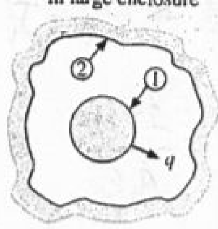


Opposing Flow | Aiding Flow or assisting | Transverse Flow

Examples: $q_{12} = \dfrac{e_{b1} - e_{b2}}{\frac{1-\varepsilon_1}{A_1\varepsilon_1} + \frac{1}{A_1 F_{12}} + \frac{1-\varepsilon_2}{A_2\varepsilon_2}}$

Figure 8-30 | Radiation heat transfer between simple two-body diffuse, gray surfaces. In all cases $F_{12} = 1.0$.

Small convex object in large enclosure

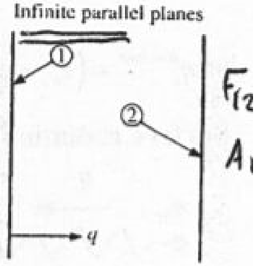


$A_2 \gg A_1$

Please show

$$q = A_1 \epsilon_1 \sigma (T_1^4 - T_2^4)$$
for $A_1/A_2 \to 0$

Infinite parallel planes

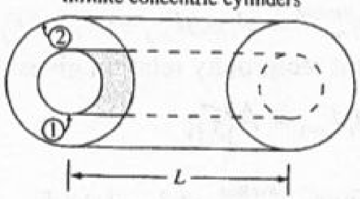


F_{12}

A_1

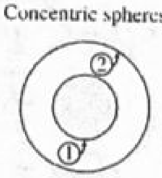
$$(q/A) = \frac{\sigma(T_1^4 - T_2^4)}{1/\epsilon_1 + 1/\epsilon_2 - 1}$$
with $A_1 = A_2$

Infinite concentric cylinders



$$q = \frac{\sigma A_1 (T_1^4 - T_2^4)}{1/\epsilon_1 + (1/\epsilon_2 - 1)(r_1/r_2)}$$
with $A_1/A_2 = r_1/r_2$; $r_1/L \to 0$

Concentric spheres

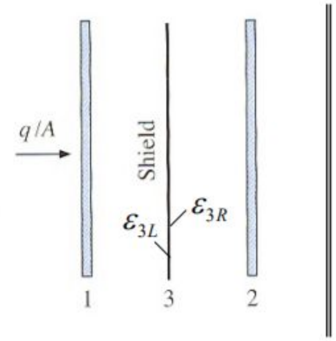


$$q = \frac{\sigma A_1 (T_1^4 - T_2^4)}{1/\epsilon_1 + (1/\epsilon_2 - 1)(r_1/r_2)^2}$$
for $A_1/A_2 = (r_1/r_2)^2$

M10

Radiation Shields

Example 1: Two large parallel flat plates, with a thin-plate radiation shield in between them

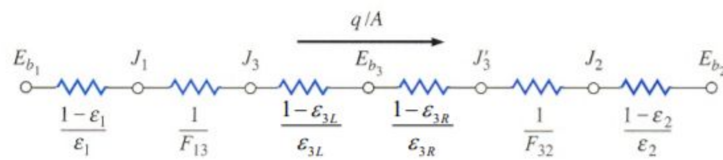


$$A_1 = A_2 = A_3 = A \;\; ; \;\; F_{1-3} = F_{3-2} = 1$$

With no shield:

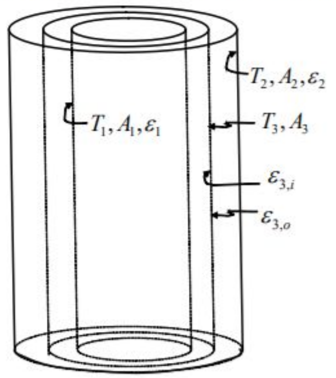
$$q_1^{Net\,Rad} = q_{1-2}^{Net\,Rad} = \frac{A\sigma\left(T_{1,abs}^4 - T_{2,abs}^4\right)}{[(1/\varepsilon_1) + (1/\varepsilon_2) - 1]}$$

Equivalent circuit with shield:



$$\boxed{q_1^{Net\,Rad} = \frac{A\sigma\left(T_{1,abs}^4 - T_{2,abs}^4\right)}{\left[\left(\frac{1}{\varepsilon_1}\right) + \left(\frac{1}{\varepsilon_2}\right) + \left(\frac{1-\varepsilon_{3,L}}{\varepsilon_{3,L}}\right) + \left(\frac{1-\varepsilon_{3,R}}{\varepsilon_{3,R}}\right)\right]}}$$

Example 2: Two long concentric cylindrical shells, with a thin-walled cylindrical radiation shield in between them



$$A_1 = 2\pi r_1 L \;\; ; \;\; A_2 = 2\pi r_2 L \;\; ; \;\; A_3 = 2\pi r_3 L$$

$$F_{1-2} = 1 \;\; ; \;\; F_{1-3} = 1 \;\; ; \;\; F_{3-2} = 1$$

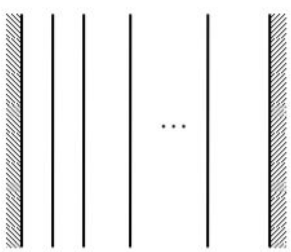
With no shield:

$$q_{1-2}^{Net\,Rad} = \frac{A_1 \sigma\left(T_{1,abs}^4 - T_{2,abs}^4\right)}{\left[\left(\frac{1}{\varepsilon_1}\right) + \left(\frac{1-\varepsilon_2}{\varepsilon_2}\right)\left(\frac{r_1}{r_2}\right)\right]}$$

With shield,

$$q_{1-2}^{Net\,Rad} = \frac{A_1 \sigma\left(T_{1,abs}^4 - T_{2,abs}^4\right)}{\left[\left(\frac{1}{\varepsilon_1}\right) + \left(\frac{1-\varepsilon_{3,i}}{\varepsilon_{3,i}} + \frac{1-\varepsilon_{3,o}}{\varepsilon_{3,o}} + 1\right)\left(\frac{r_1}{r_3}\right) + \left(\frac{1-\varepsilon_2}{\varepsilon_2}\right)\left(\frac{r_1}{r_2}\right)\right]}$$

Example 3: Two large parallel flat plates with multiple thin-walled flat parallel-plate shields in between them



Assumptions:

1) $A_1 = A_2 = \ldots\ldots = A$

2) All shape factors are equal to one ($F_{i-j} = 1$)

3) All surfaces have the same emissivity: ε

- Two end-plates, 1 and 2; N shields (thin, large, flat parallel plates)
- All surface radiation resistances = $(1-\varepsilon)/(A\varepsilon)$
- All space or geometrical radiation resistances = $1/(A_i F_{i-j}) = 1/A$
- There are $(2N + 2)$ surface radiation resistances
- There are $(N + 1)$ space radiation resistances

Therefore,

$$q_{1-2 \atop with\,N\,shields}^{Net\,Rad} = \frac{\sigma\left(T_{1,abs}^4 - T_{2,abs}^4\right)}{(2N+2)\left(\frac{1-\varepsilon}{A\varepsilon}\right) + (N+1)\left(\frac{1}{A}\right)} = \frac{A\sigma\left(T_{1,abs}^4 - T_{2,abs}^4\right)}{(N+1)\left(\frac{2}{\varepsilon} - 1\right)}$$

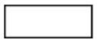
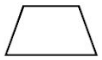
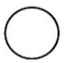
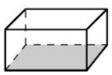
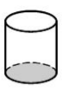
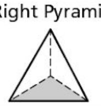
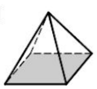
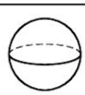
and

$$q_{1-2 \atop with\,no\,shields}^{Net\,Rad} = \frac{A\sigma\left(T_{1,abs}^4 - T_{2,abs}^4\right)}{\left(\frac{2}{\varepsilon} - 1\right)}$$

Thus, $\left[q_{1-2 \atop with\,N\,shields}^{Net\,Rad} \Big/ q_{1-2 \atop with\,no\,shields}^{Net\,Rad} \right] = \left(\frac{1}{N+1}\right)$

Random stuff:

Wisconsin Mathematics Formula Reference Sheet

Shape	Formulas for Area (A) and Circumference (C)
Triangle 	$A = \frac{1}{2}bh = \frac{1}{2} \times$ base $\times$ height
Rectangle 	$A = lw =$ length $\times$ width
Trapezoid 	$A = \frac{1}{2}(b_1 + b_2)h = \frac{1}{2} \times$ sum of bases $\times$ height
Parallelogram 	$A = bh =$ base $\times$ height
Circle 	$A = \pi r^2 = \pi \times$ square of radius $C = 2\pi r = 2 \times \pi \times$ radius
Figure	**Formulas for Volume (V) and Surface Area (SA)**
Rectangular Prism 	$V = lwh =$ length $\times$ width $\times$ height $SA = 2lw + 2hw + 2lh$ $= 2($length $\times$ width$) + 2($height $\times$ width$) + 2($length $\times$ height$)$
General Prisms  	$V = Bh =$ area of base $\times$ height $SA =$ sum of the areas of the faces
Right Circular Cylinder 	$V = Bh =$ area of base $\times$ height $SA = 2B + Ch = (2 \times$ area of base$) + ($circumference $\times$ height$)$
Right Pyramid  	$V = \frac{1}{3}Bh = \frac{1}{3} \times$ area of base $\times$ height $SA = B + \frac{1}{2}P\ell$ $=$ area of base $+ (\frac{1}{2} \times$ perimeter of base $\times$ slant height$)$
Right Circular Cone 	$V = \frac{1}{3}Bh = \frac{1}{3} \times$ area of base $\times$ height $SA = B + \frac{1}{2}C\ell =$ area of base $+ (\frac{1}{2} \times$ circumference $\times$ slant height$)$
Sphere 	$V = \frac{4}{3}\pi r^3 = \frac{4}{3} \times \pi \times$ cube of radius $SA = 4\pi r^2 = 4 \times \pi \times$ square of radius