ELEC 343

Electromechanics

Module 5 (Start Reading Chap. 4 & 5):

Rotating Magnetic Field & AC Motors

Spring 2019
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Class Webpage:
http://courses.ece.ubc.ca/elec343

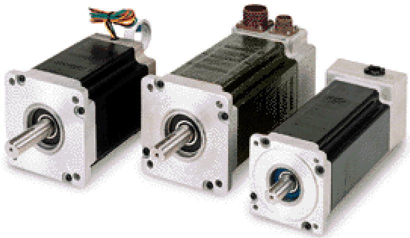
Learning Objectives & Important Concepts / Topics

- 2 and 3 phase stator systems and windings
- Concept of rotating magnetic field
- Multi-pole (*P*-pole) stator system
- Types of basic multi-phase ac rotating devices
- Concept of rotating reference frame – *qd*-coordinates

AC Motors

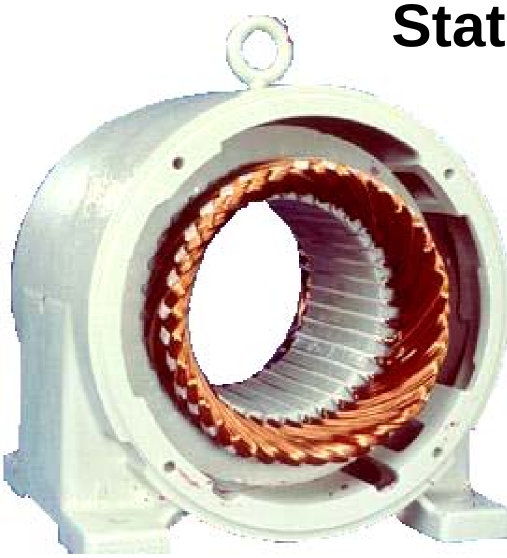
Asynchronous

Synchronous

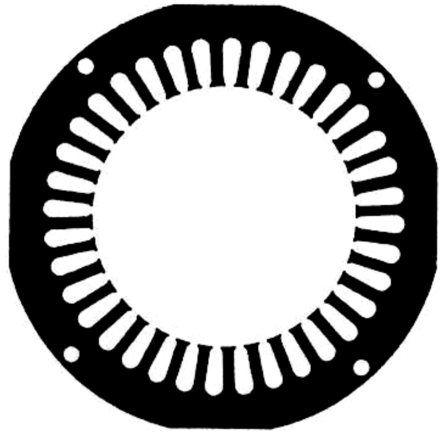


Brushless DC

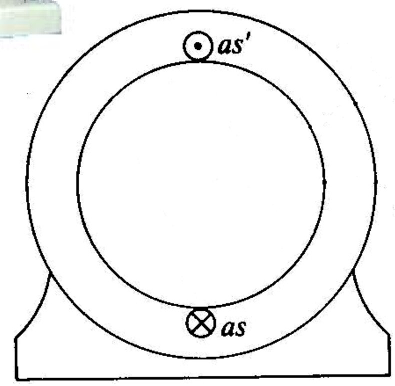
Stator Winding



Real winding

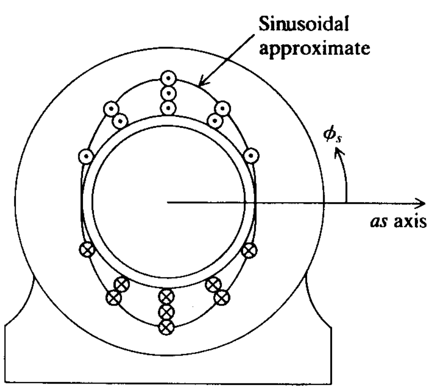


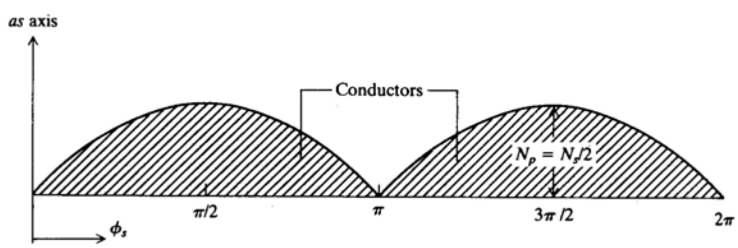
Stator Lamination



Equivalent concentrated winding

Sinusoidally Distributed Stator Winding





$$N_{as} = N_p \sin(\phi_s) \qquad N_{as} = -N_p \sin(\phi_s)$$

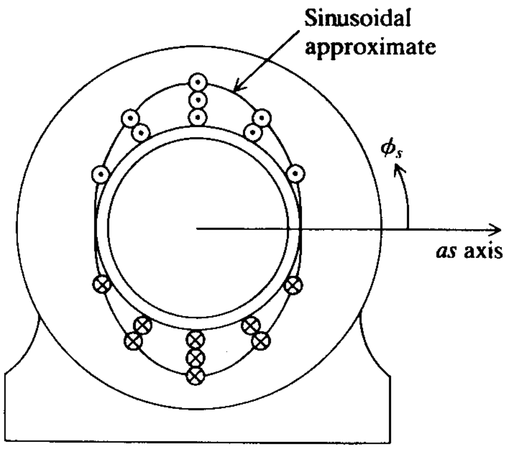
Total number of turns of the equivalent sinusoidal winding

$$N_s = \int_0^{\pi} N_p \sin(\phi_s) d\phi_s = 2N_p$$

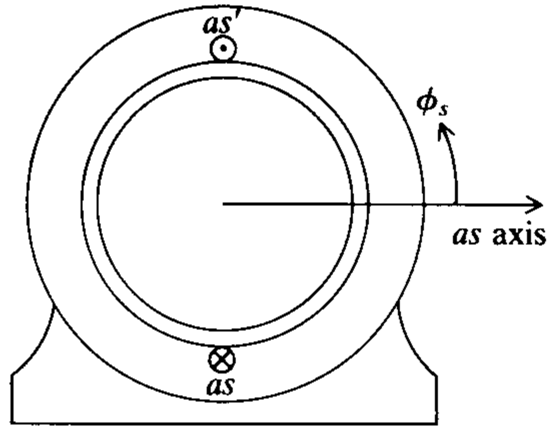
This winding will produce MMF distributed in the air-gap

$$mmf_{as} = F_{as} = \frac{N_s}{2} i_{as} \cos(\phi_s)$$

Sinusoidally Distributed Stator Winding



Distributed winding



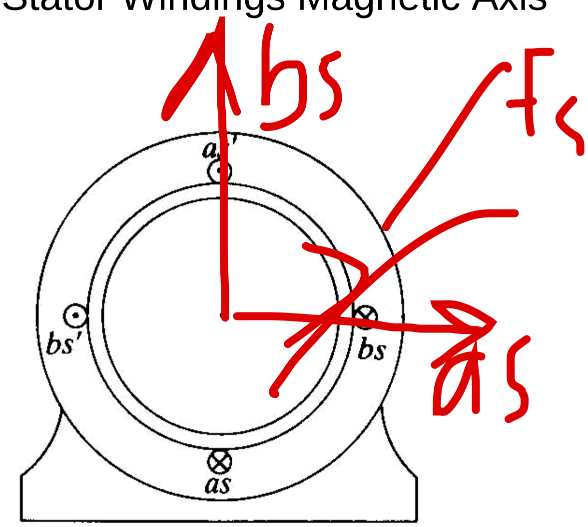
Distributed winding represented by $as - as'$ at the maximum turn density

Assume mmf distributed in the air-gap

$$mmf_{as} = F_{as} = \frac{N_s}{2} i_{as} \cos(\phi_s)$$

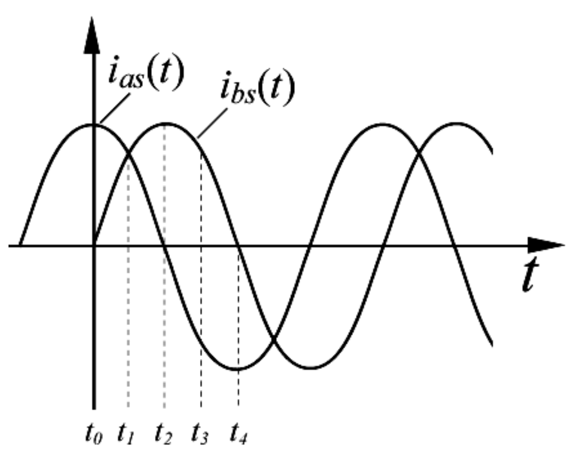
Rotating Magnetic Field

Stator Windings Magnetic Axis



Apply balanced set of currents

$$i_{as} = I_m \cos(\omega_e t)$$
$$i_{bs} = I_m \cos(\omega_e t - 90^\circ)$$



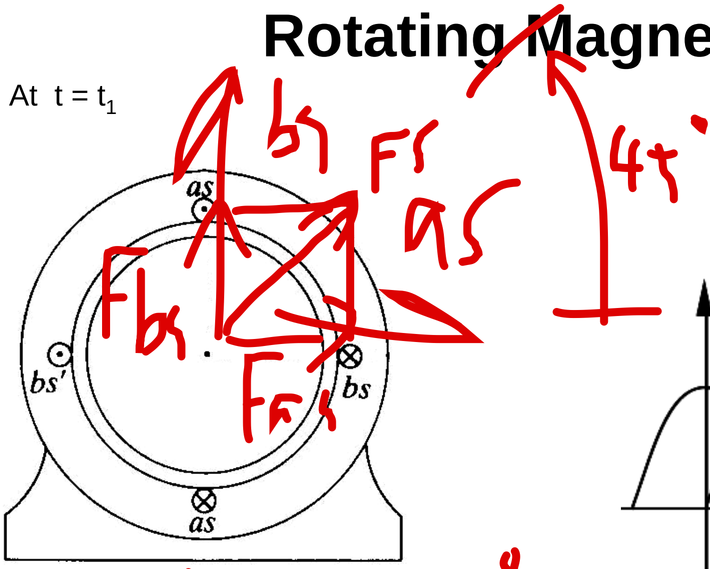
Resulting MMF $\mathbf{F}_s = \mathbf{F}_{as} + \mathbf{F}_{bs}$

At $t = t_0$ $F_{as} = F_m$ $F_{bs} = 0$

$\mathbf{F}_s = F_m \angle 0^{e}$

Rotating Magnetic Field

At $t = t_1$

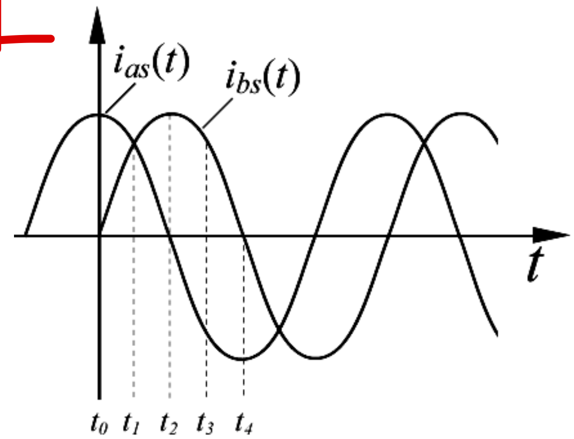


$\theta = 45°$

Apply balanced set of currents

$$i_{as} = I_m \cos(\omega_e t)$$

$$i_{bs} = I_m \cos(\omega_e t - 90^\circ)$$

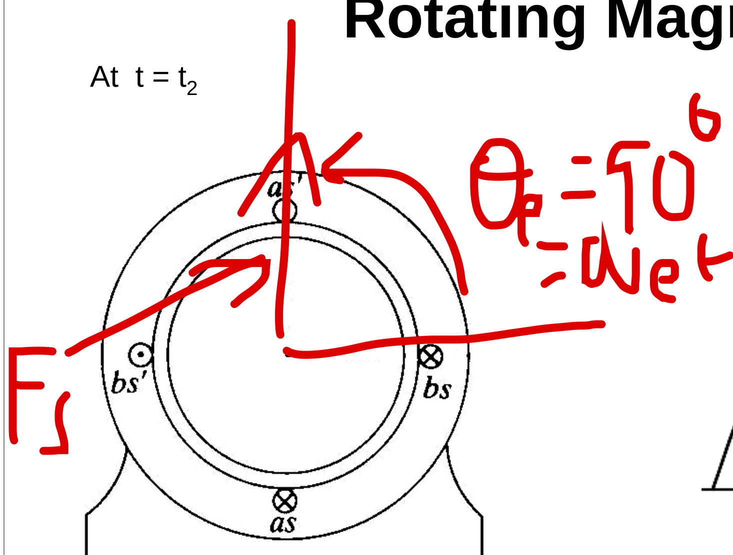


Resulting MMF $\quad \mathbf{F}_s = \mathbf{F}_{as} + \mathbf{F}_{bs}$

$F_{as} = \dfrac{F_m}{\sqrt{2}} \quad F_{bs} = \dfrac{F_m}{\sqrt{2}} \quad \mathbf{F}_s = F_m \angle 45°$

Rotating Magnetic Field

At $t = t_2$

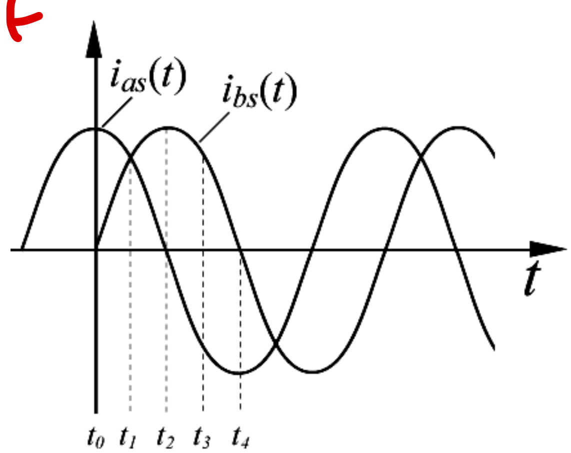


$\theta_e = 90° = \omega_e t$

Apply balanced set of currents

$$i_{as} = I_m \cos(\omega_e t)$$

$$i_{bs} = I_m \cos(\omega_e t - 90^\circ)$$



Resulting MMF $\quad \mathbf{F}_s = \mathbf{F}_{as} + \mathbf{F}_{bs}$

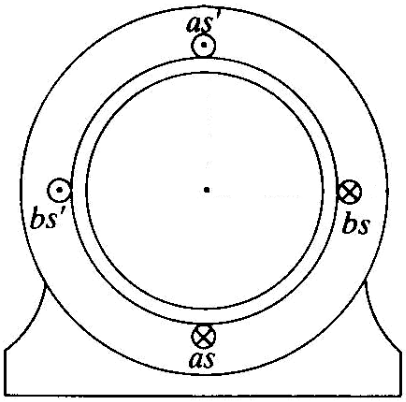
$F_{as} = 0 \quad F_{bs} = F_M \quad \mathbf{F}_s = F_m \angle$

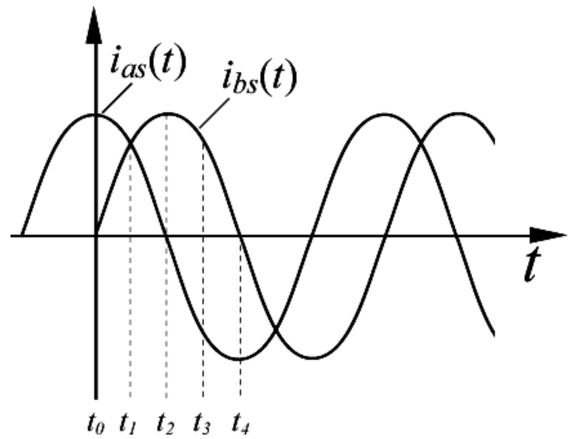
Rotating Magnetic Field

At $t = t_4$

Apply balanced set of currents

$$i_{as} = I_m \cos(\omega_e t)$$
$$i_{bs} = I_m \cos(\omega_e t - 90^\circ)$$





Resulting MMF $\mathbf{F}_s = \mathbf{F}_{as} + \mathbf{F}_{bs}$

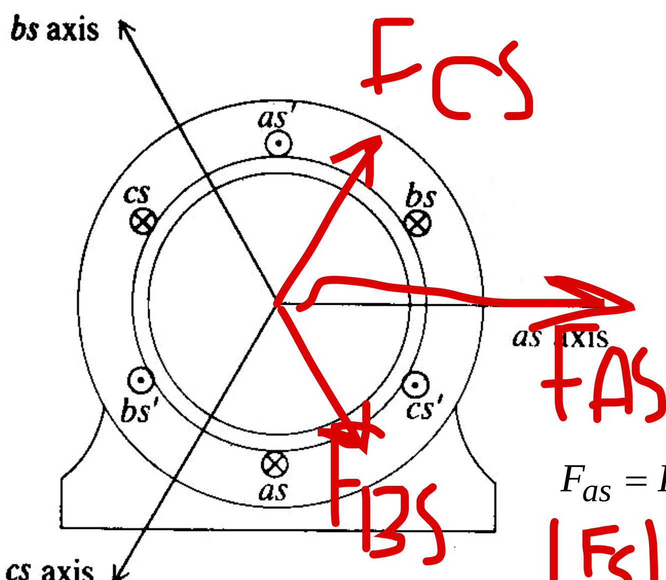
$F_{as} =$ $F_{bs} =$ $\mathbf{F}_s = F_m \angle$

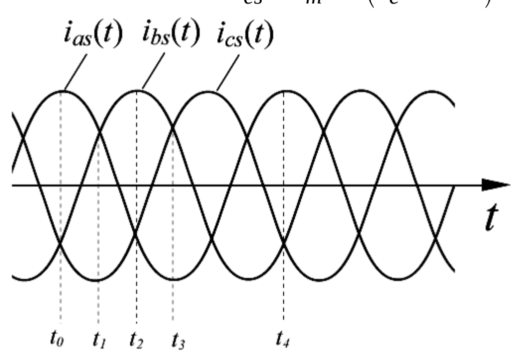
3-Phase Rotating Magnetic Field

At $t = t_0$

Apply balanced set of currents

$$i_{as} = I_m \cos(\omega_e t)$$
$$i_{bs} = I_m \cos(\omega_e t - 120^\circ)$$
$$i_{cs} = I_m \cos(\omega_e t + 120^\circ)$$





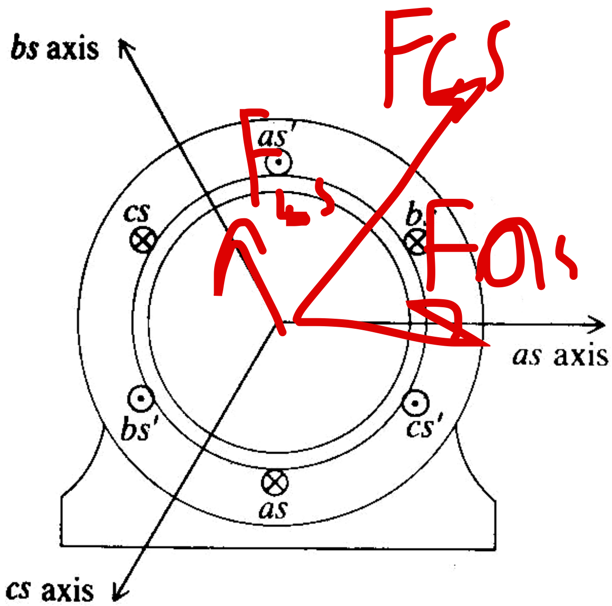
$F_{as} = F_m$ $F_{bs} = -\frac{1}{2}F_m$ $F_{cs} = -\frac{1}{2}F_m$

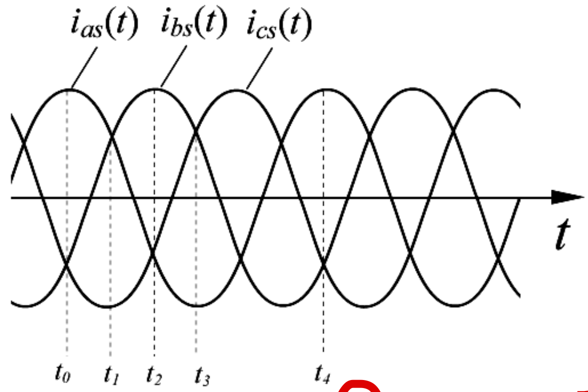
$|F_s| = F_m + 2\left(\frac{1}{2}F_m \cos 60^\circ\right)$
$= \frac{3}{2}F_m$

Resulting MMF vector $\mathbf{F}_s = \frac{3}{2}F_m \angle 0$

3-Phase Rotating Magnetic Field

At $t = t_1$



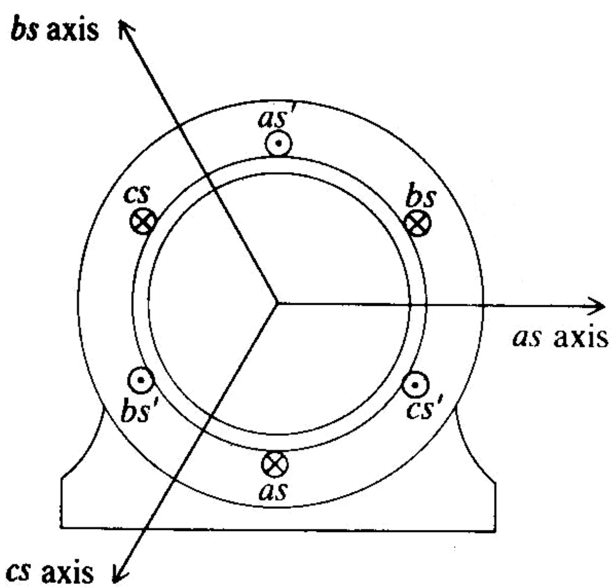


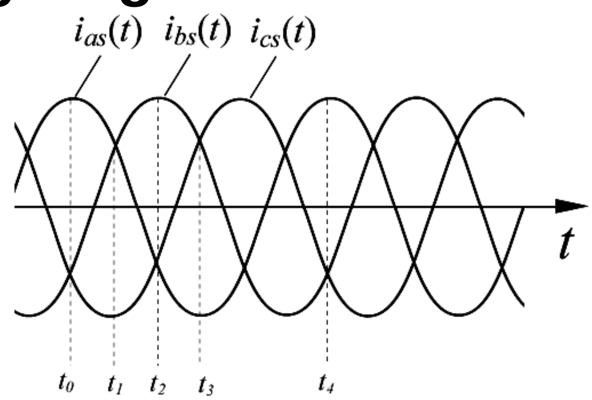
$F_{as} = \frac{F_m}{2}$ $F_{bs} = \frac{F_m}{2}$ $F_{cs} = -F_M$

Resulting MMF vector $\mathbf{F}_s = \frac{3}{2} F_m \angle$

3-Phase Rotating Magnetic Field

At $t = t_2$



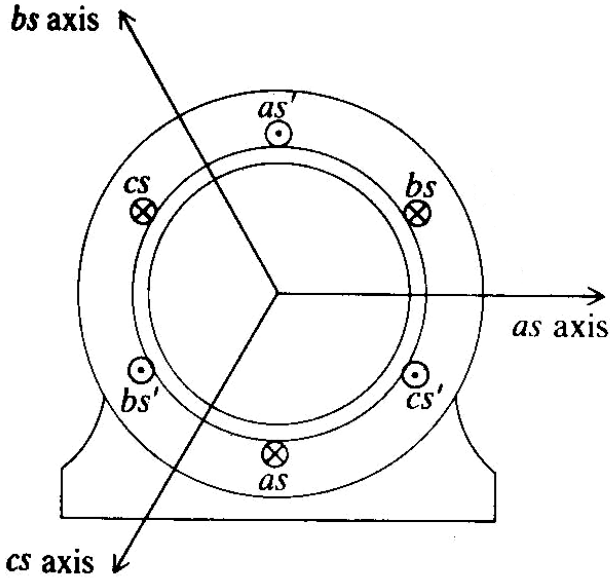


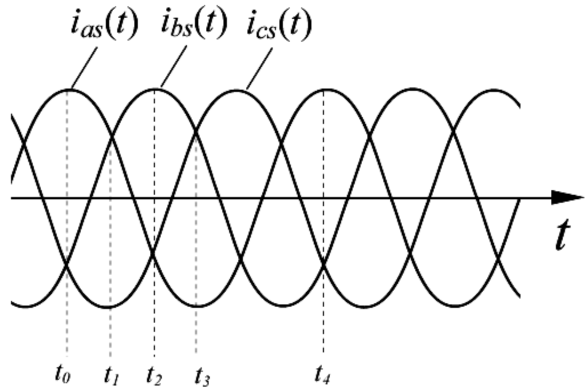
$F_{as} =$ $F_{bs} =$ $F_{cs} =$

Resulting MMF vector $\mathbf{F}_s = \frac{3}{2} F_m \angle$

3-Phase Rotating Magnetic Field

At $t = t_4$

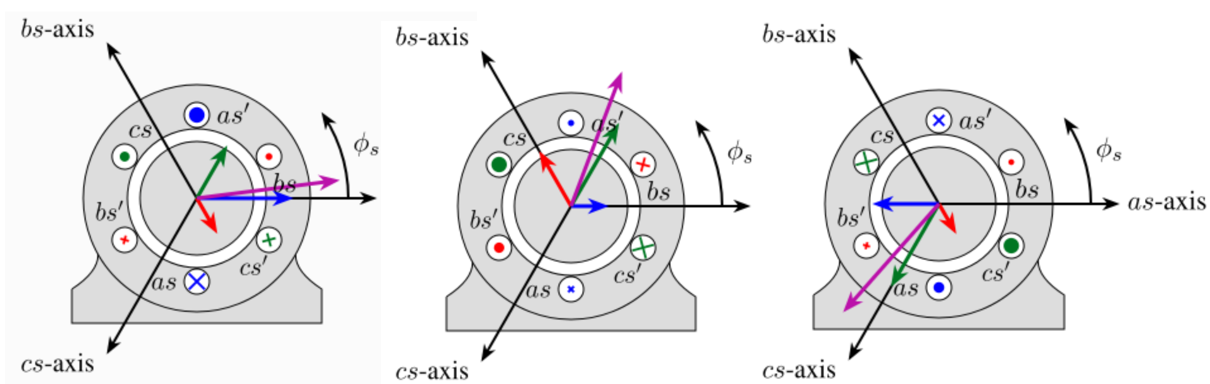


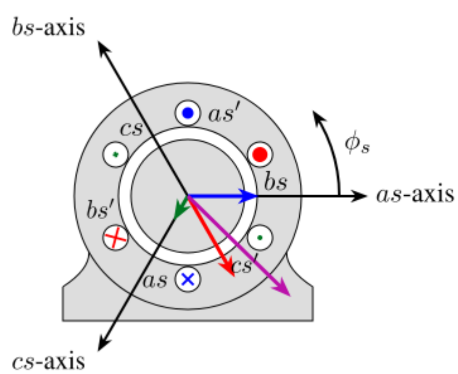


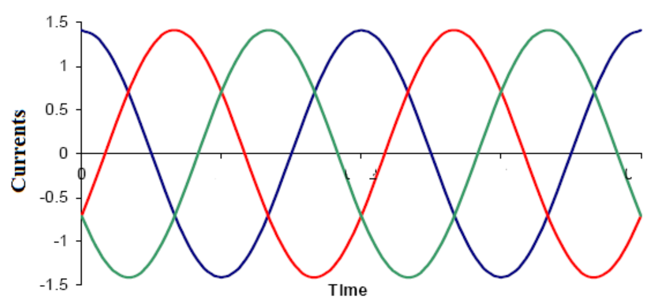
$F_{as} =$ $F_{bs} =$ $F_{cs} =$

Resulting MMF vector $\mathbf{F}_s = \frac{3}{2} F_m \angle$ 360°

Rotating Magnetic Field







Producing Rotating Magnetic Field

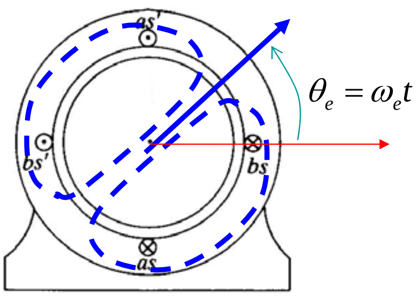
1. Given a set of ac currents shifted in time
2. Apply these currents to shifted in space stator windings

=> Produce vector $\mathbf{F}_s$ that

- Has constant magnitude
- Rotates in space

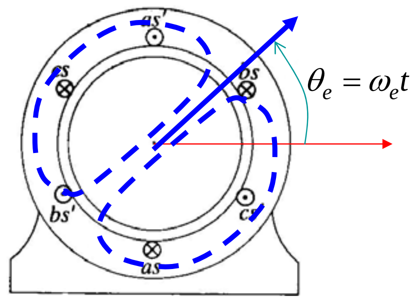
How many phases can you have ?
2, **3**, 5, 6, 9, 12, 15 …

$$i_{as} = I_m \cos(\omega_e t)$$
$$i_{bs} = I_m \cos(\omega_e t - 90^\circ)$$



$$F_s = F_m \angle \theta_e$$

$$i_{as} = I_m \cos(\omega_e t)$$
$$i_{bs} = I_m \cos(\omega_e t - 120^\circ)$$
$$i_{cs} = I_m \cos(\omega_e t + 120^\circ)$$



$$F_s = (3/2) F_m \angle \theta_e$$

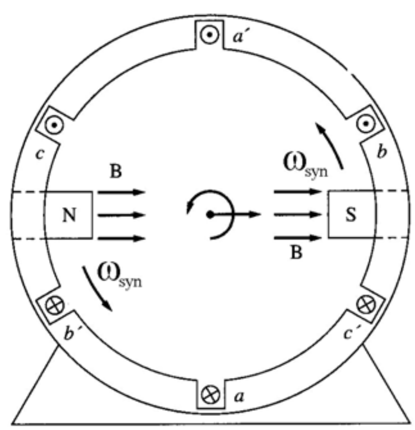
How do you change direction of rotation ?
Reverse the sequence of phases

2-Pole Rotating Magnetic Field

For 2-pole Stator System
Let p – represent number of poles (p = 2)

$$\theta_e = \omega_e t$$ Electrical displacement

$$\theta_e = 0 \rightarrow 2\pi$$ One cycle of currents = one complete revolution of magnetic poles

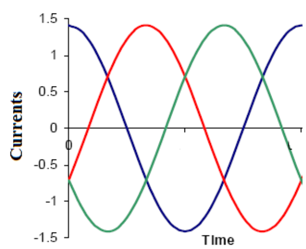


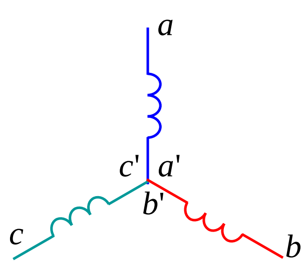
Synchronous speed is the speed at which stator magnetic poles rotate
In this case, this is the same as electrical frequency / speed

$$\omega_{syn} = \omega_e$$, 60Hz

$$n_{syn} = 60 \cdot f_e \quad [\text{rpm}]$$

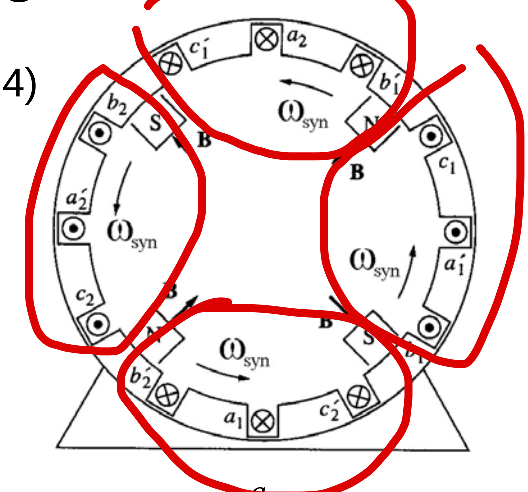
= 3600





4-Pole Rotating Magnetic Field

For 4-pole Stator System
Let p – represent number of poles (p = 4)



$\theta_e = \omega_e t$ Electrical displacement

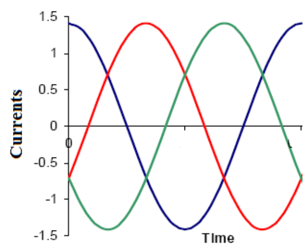
$\theta_e = 0 \rightarrow 2\pi$ One cycle of currents = half revolution of magnetic poles

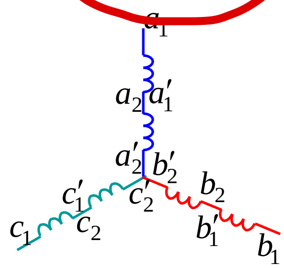
Synchronous speed is the speed at which stator magnetic poles rotate
In this case, this is half of the electrical frequency / speed

$$\omega_{syn} = \frac{1}{2}\omega_e$$

$$n_{syn} = 30 \cdot f_e \quad [\text{rpm}]$$

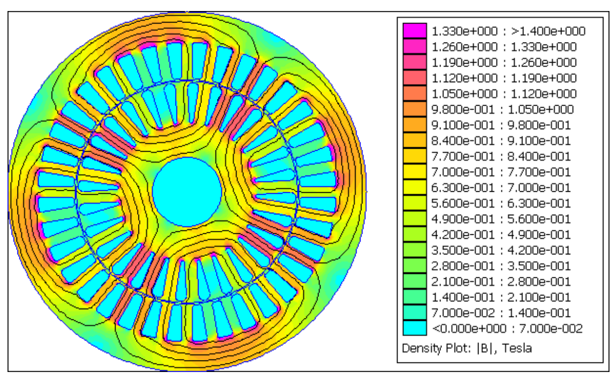
$= 1800$





4-Pole Rotating Magnetic Field

For 4-pole Stator System
Let p – represent number of poles (p = 4)



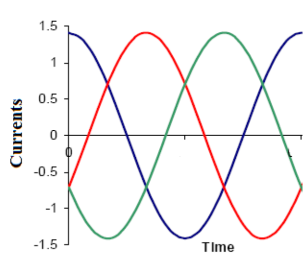
$\theta_e = \omega_e t$ Electrical displacement

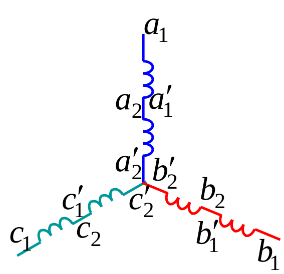
$\theta_e = 0 \rightarrow 2\pi$ One cycle of currents = half revolution of magnetic poles

Synchronous speed is the speed at which stator magnetic poles rotate
In this case, this is half of the electrical frequency / speed

$$\omega_{syn} = \frac{1}{2}\omega_e$$

$$n_{syn} = 30 \cdot f_e \quad [\text{rpm}]$$





P-Pole Rotating Magnetic Field

For *P*-pole Stator System
P – number of poles

$\theta_e = \omega_e t$ Electrical displacement

$\theta_e = 0 \rightarrow 2\pi$ One cycle of currents = 2/*P* revolution of magnetic poles

$$\omega_{syn} = \frac{2}{p}\omega_e$$

Synchronous speed of magnetic poles is 2/*P* of the electrical speed

$$n_{syn} = \frac{120}{p} \cdot f_e \quad [\text{rpm}]$$

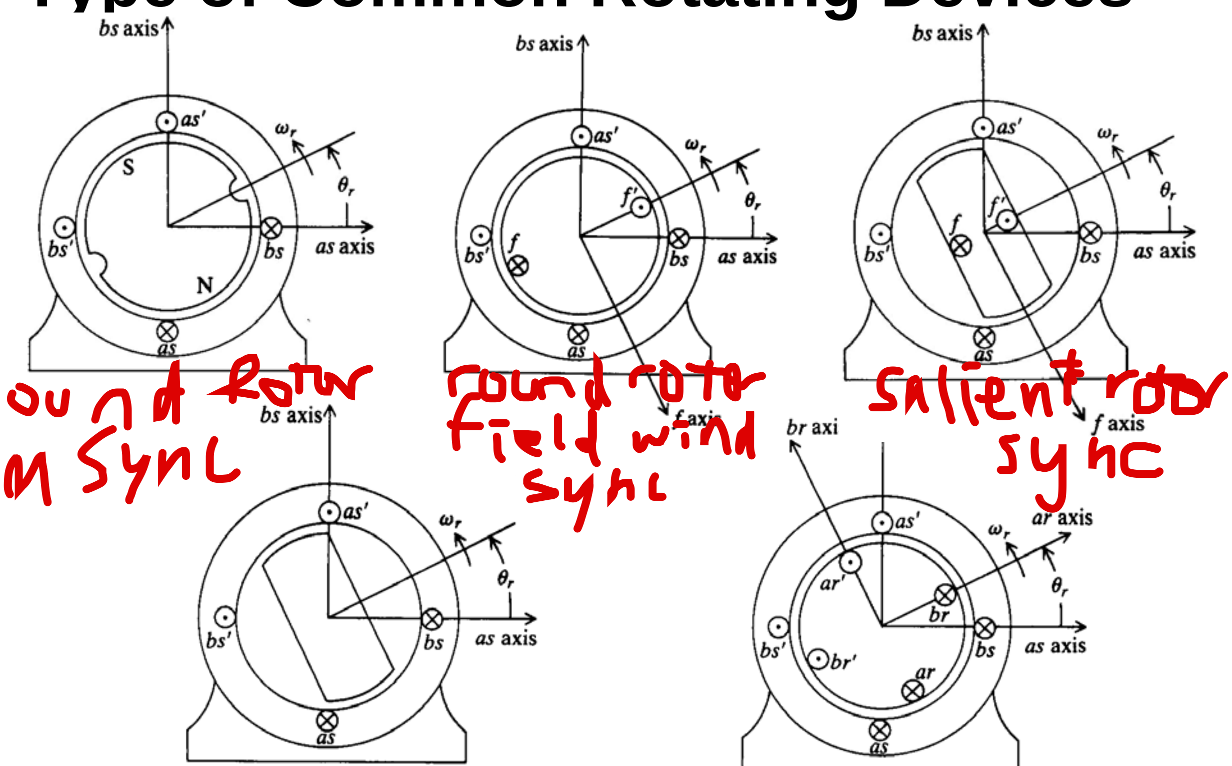
For f_e = 60 Hz

p	n_{syn}	ω_{syn}
2	3600	120π
4	1800	60π
6	1200	40π
8	900	

in pair

↑P ↓ω ↑T

Type of Common Rotating Devices



Round Rotor PM Sync

round rotor field wind sync

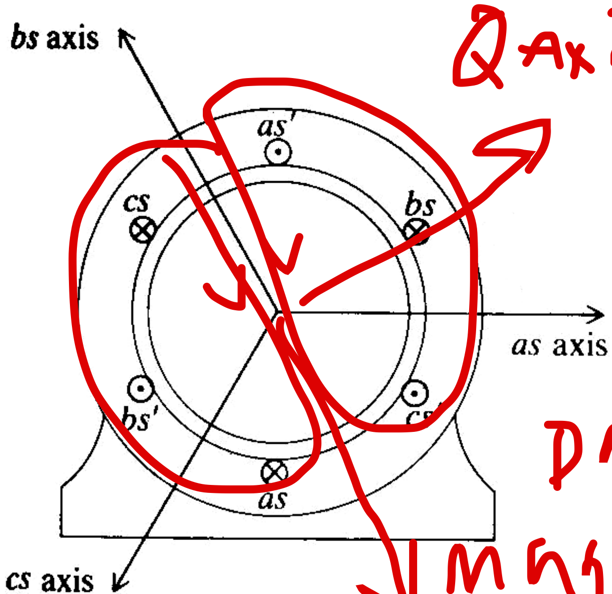
salient rotor sync

sync reluctance

Induction

Rotating Frame of Reference: Chap. 5

$$i_{as} = I_m \cos(\omega_e t)$$
$$i_{bs} = I_m \cos(\omega_e t - 120^\circ)$$
$$i_{cs} = I_m \cos(\omega_e t + 120^\circ)$$



Q Axis

D axis

mag flux axis

Assume rotating MMF

$$\mathbf{F}_s = (3/2) F_m \angle \theta_e$$

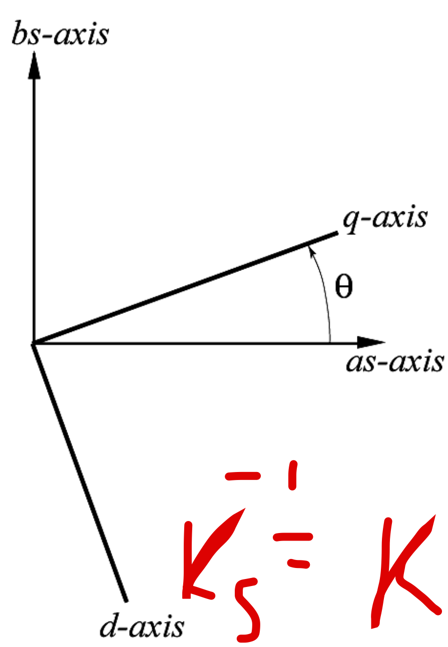
where $\theta_e = \omega_e t$

Is there a system of coordinates in which the magnetic field appears stationary ?

How does the number of phases come into play ?

Rotating Frame of Reference: Chap. 5

2-phase stator system



$K_s^{-1} = K_s$

Consider that each phase has variables

$$f = i, v, \lambda, \ldots$$

Define a vector $\mathbf{f}_{abs} = [f_{as} \quad f_{bs}]^T$

How would these variables look if we view them in *qd*-coordinate system ?

$$\mathbf{f}_{qds} = [f_{qs} \quad f_{ds}]^T$$

$$\begin{bmatrix} f_{qs} \\ f_{ds} \end{bmatrix} = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ \sin(\theta) & -\cos(\theta) \end{bmatrix} \cdot \begin{bmatrix} f_{as} \\ f_{bs} \end{bmatrix}$$

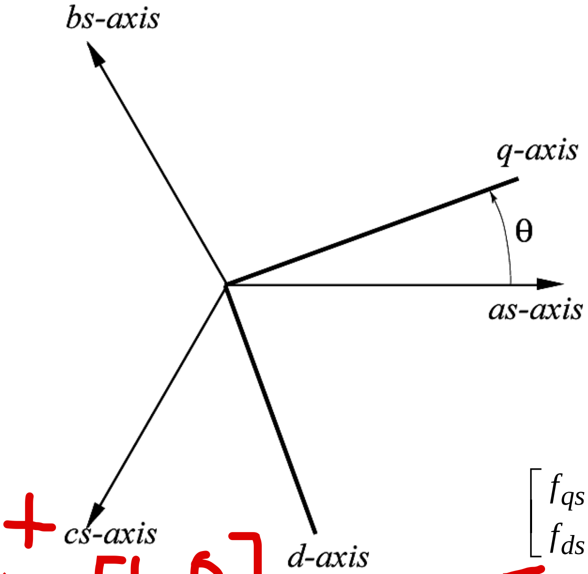
DC ← AC

$$\mathbf{f}_{qds} = \mathbf{K}_s \mathbf{f}_{abs}$$

$f_{abs} = K^{-1} f_{qds}$

Rotating Frame of Reference: Chap. 5

3-phase stator system



Consider that each phase has variables

$$f = i, v, \lambda, \ldots$$

Define a vector $\quad \mathbf{f}_{abcs} = \begin{bmatrix} f_{as} & f_{bs} & f_{cs} \end{bmatrix}^T$

How would these variables look if we view them in *qd*-coordinate system ?

$$\mathbf{f}_{qds} = \begin{bmatrix} f_{qs} & f_{ds} \end{bmatrix}^T$$

$$\mathbf{f}_{qds} = \mathbf{K}_s \mathbf{f}_{abcs}$$

$$\begin{bmatrix} f_{qs} \\ f_{ds} \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \cos(\theta) & \cos(\theta - 120^\circ) & \cos(\theta + 120^\circ) \\ \sin(\theta) & \sin(\theta - 120^\circ) & \sin(\theta + 120^\circ) \end{bmatrix} \cdot \begin{bmatrix} f_{as} \\ f_{bs} \\ f_{cs} \end{bmatrix}$$

$$K_s K = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad K^T = \begin{bmatrix} \cos\theta & \sin\theta \\ \cos\theta-120 & \sin\theta-120 \\ \cos\theta+120 & \sin\theta+120 \end{bmatrix}$$

Rotating Frame of Reference: Chap. 5

Trigonometric Identities

$e^{j\alpha} = \cos\alpha + j\sin\alpha$

$a\cos x + b\sin x = \sqrt{a^2+b^2}\cos(x+\phi) \qquad \phi = \tan^{-1}(-b/a)$

$\cos^2 x + \sin^2 x = 1$

$\sin 2x = 2\sin x\cos x$

$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$

$\cos x\cos y = \frac{1}{2}\cos(x+y) + \frac{1}{2}\cos(x-y)$

$\sin x\sin y = \frac{1}{2}\cos(x-y) - \frac{1}{2}\cos(x+y)$

$\sin x\cos y = \frac{1}{2}\sin(x+y) + \frac{1}{2}\sin(x-y)$

$\cos(x\pm y) = \cos x\cos y \mp \sin x\sin y$

$\sin(x\pm y) = \sin x\cos y \pm \cos x\sin y$

$\cos^2 x + \cos^2(x - \frac{2}{3}\pi) + \cos^2(x + \frac{2}{3}\pi) = \frac{3}{2}$

$\sin^2 x + \sin^2(x - \frac{2}{3}\pi) + \sin^2(x + \frac{2}{3}\pi) = \frac{3}{2}$

$\sin x\cos x + \sin(x - \frac{2}{3}\pi)\cos(x - \frac{2}{3}\pi) + \sin(x + \frac{2}{3}\pi)\cos(x + \frac{2}{3}\pi) = 0$

$\cos x + \cos(x - \frac{2}{3}\pi) + \cos(x + \frac{2}{3}\pi) = 0$

$\sin x + \sin(x - \frac{2}{3}\pi) + \sin(x + \frac{2}{3}\pi) = 0$

$\sin x\cos y + \sin(x - \frac{2}{3}\pi)\cos(y - \frac{2}{3}\pi) + \sin(x + \frac{2}{3}\pi)\cos(y + \frac{2}{3}\pi)$
$\quad = \frac{3}{2}\sin(x-y)$

$\sin x\sin y + \sin(x - \frac{2}{3}\pi)\sin(y - \frac{2}{3}\pi) + \sin(x + \frac{2}{3}\pi)\sin(y + \frac{2}{3}\pi)$
$\quad = \frac{3}{2}\cos(x-y)$

$\cos x\sin y + \cos(x - \frac{2}{3}\pi)\sin(y - \frac{2}{3}\pi) + \cos(x + \frac{2}{3}\pi)\sin(y + \frac{2}{3}\pi)$
$\quad = -\frac{3}{2}\sin(x-y)$

$\cos x\cos y + \cos(x - \frac{2}{3}\pi)\cos(y - \frac{2}{3}\pi) + \cos(x + \frac{2}{3}\pi)\cos(y + \frac{2}{3}\pi)$
$\quad = \frac{3}{2}\cos(x-y)$

$\sin x\cos y + \sin(x + \frac{2}{3}\pi)\cos(y - \frac{2}{3}\pi) + \sin(x - \frac{2}{3}\pi)\cos(y + \frac{2}{3}\pi)$
$\quad = \frac{3}{2}\sin(x+y)$

$\sin x\sin y + \sin(x + \frac{2}{3}\pi)\sin(y - \frac{2}{3}\pi) + \sin(x - \frac{2}{3}\pi)\sin(y + \frac{2}{3}\pi)$
$\quad = -\frac{3}{2}\cos(x+y)$

$\cos x\sin y + \cos(x + \frac{2}{3}\pi)\sin(y - \frac{2}{3}\pi) + \cos(x - \frac{2}{3}\pi)\sin(y + \frac{2}{3}\pi)$
$\quad = \frac{3}{2}\sin(x+y)$