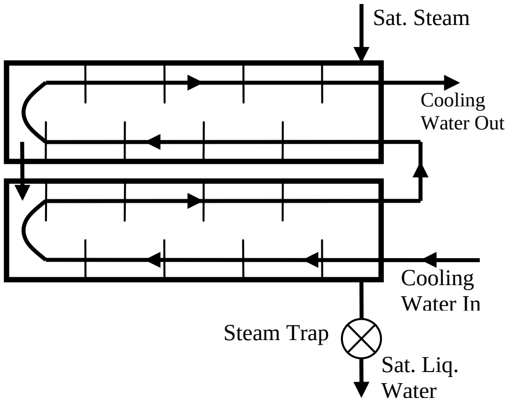
Solutions - Problem Set # 11

Problem 1:



Given: $T_{h,i} = T_{h,o} = T_{sat} = 125°C$; $T_{c,i} = 20°C$;
$\dot{m}_{condensation} = 0.3$ kg/s; $c_c = 4180$ J/kg-°C;
$h_{fg} = 2.2 \times 10^6$ J/kg; A = 4.807 m^2; $\dot{m}_c = 2.5$ kg/s

Assumptions: Steady-state conditions prevail; heat loss to ambient fluid is negligible; Ec<<1

a) We use ε-NTU method. But, let first obtain the cold fluid exit temperature.

$$q_{total} = \dot{m}_{condensation} h_{fg} = \dot{m}_c c_c \left(T_{c,o} - T_{c,i}\right) \Rightarrow T_{c,o} = T_{c,i} + \frac{\dot{m}_{condensation}\ h_{fg}}{\dot{m}_c c_c} = 20 + \frac{0.3 \times 2.2 \times 10^6}{2.5 \times 4180} = 83.16\ °C$$

$$\varepsilon \triangleq \frac{q_{actual}}{q_{\max,possible}} = \frac{\cancel{\dot{m}_c}\cancel{c_c}\left(T_{c,o} - T_{c,i}\right)}{\cancel{\dot{m}_c}\cancel{c_c}\left(T_{h,i} - T_{c,i}\right)} = \frac{(83.16 - 20)}{(125 - 20)} \simeq 0.6015$$; Note: $T_{h,i} = T_{h,o} = T_{sat}$, thus,

$$\left(\dot{m}c\right)_{\min} = \dot{m}_c c_c \text{ and } \left(\dot{m}c\right)_{\max} = \dot{m}_h c_h \rightarrow \infty; \Rightarrow R_{\min} = \frac{C_{\min}}{C_{\max}} = 0$$

$$\text{and } \boxed{\varepsilon = 1 - \exp\left(-NTU\right)} \Rightarrow NTU = -\ln\left(1 - \varepsilon\right) = -\ln\left(1 - 0.6015\right) \simeq 0.92$$

$$NTU \triangleq \frac{UA}{\left(\dot{m}c\right)_{\min}} \Rightarrow U = \frac{\dot{m}_c c_c \times NTU}{A} = \frac{2.5 \times 4180 \times 0.92}{4.807} \simeq 2000 \text{ W/m}^2\text{-°C}$$

b) $$\varepsilon_{foul} \triangleq \frac{q_{\substack{actual\\foul}}}{q_{\max,possible}} = 0.5 \Rightarrow q_{\substack{actual\\foul}} = 0.5 \times q_{\max,possible} = 0.5 \times 2.5 \times 4180 \times \left(125 - 20\right) = 548625 \text{ W}$$

$$q_{\substack{actual\\foul}} = \dot{m}_{\substack{condensation\\foul}} h_{fg} \Rightarrow \dot{m}_{\substack{condensation\\foul}} = \frac{548625}{2.2 \times 10^6} = 0.2494 \text{ kg/s}$$

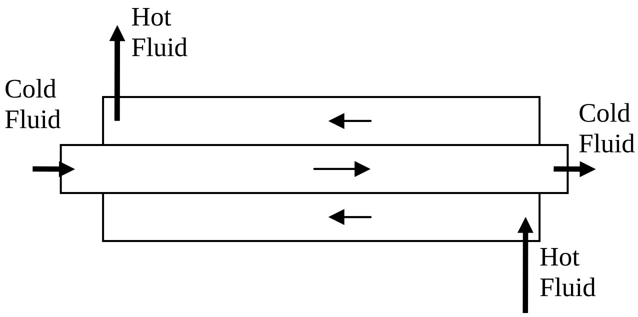
c) as before $$\left(\dot{m}c\right)_{\min} = \dot{m}_c c_c \text{ and } \left(\dot{m}c\right)_{\max} = \dot{m}_h c_h \rightarrow \infty; \Rightarrow R_{\min} = \frac{C_{\min}}{C_{\max}} = 0$$

$$\text{and } \boxed{\varepsilon_{foul} = 1 - \exp\left(-NTU_{foul}\right)} \Rightarrow NTU_{foul} = -\ln\left(1 - \varepsilon_{foul}\right) = -\ln\left(1 - 0.5\right) \simeq 0.693$$

$$NTU_{foul} \triangleq \frac{U_{foul} A}{\left(\dot{m}c\right)_{\min}} \Rightarrow U_{foul} = \frac{\dot{m}_c c_c \times NTU_{foul}}{A} = \frac{2.5 \times 4180 \times 0.693}{4.807} \simeq 1506.52 \text{ W/m}^2\text{-°C}$$

$$R_{foul} = \frac{1}{U_{foul}} - \frac{1}{U_{clean}} = \frac{1}{1506.52} - \frac{1}{2000} \simeq 1.638 \times 10^{-4} \text{ m}^2\text{-°C/W}$$

Problem 2:



Given: $T_{h,i}$ =200°C; $T_{h,o}$ =80°C; $T_{c,i}$ = 20°C; $T_{c,o}$ = 50°C; $\dot{m}_c = 0.5$ kg/s; c_c = 4000 J/kg-°C; A = 1.6 m^2.

Assumptions: Steady-state conditions prevail; heat loss to ambient fluid is negligible; Ec<<1

a) $\varepsilon \triangleq \dfrac{q_{actual}}{q_{\max, possible}} = \dfrac{|\Delta T_b|_{\text{for min. capacity rate fluid}}}{|\Delta T_b|_{\max}}$

$$E-balance: q = \dot{m}_c c_c\left(T_{c,o}-T_{c,i}\right) = \dot{m}_h c_h\left(T_{h,i}-T_{h,o}\right) \Rightarrow \frac{\dot{m}_c c_c}{\dot{m}_h c_h} = \frac{\left(T_{h,i}-T_{h,o}\right)}{\left(T_{c,o}-T_{c,i}\right)} = \frac{(200-80)}{(50-20)} = 4$$

$\Rightarrow \dot{m}_h c_h = \dot{m}_c c_c / 4$ or $\dot{m}_h c_h < \dot{m}_c c_c$; i.e., $(\dot{m}c)_{\min} = \dot{m}_h c_h$ in this problem.

$$\varepsilon = \frac{|\Delta T_b|_{\text{for min. capacity rate fluid}}}{|\Delta T_b|_{\max}} = \frac{\left(T_{h,i}-T_{h,o}\right)}{\left(T_{h,i}-T_{c,i}\right)} = \frac{200-80}{200-20} = \frac{120}{180} = 2/3 \simeq 66.67\%$$

b) $q = \dot{m}_c c_c\left(T_{c,o}-T_{c,i}\right) = \dot{m}_h c_h\left(T_{h,i}-T_{h,o}\right) = (UA)\,\Delta T_{\substack{LMTD\\ counter-flow}}$

$$\Delta T_{\substack{LMTD\\ counter-flow}} = \frac{\left(T_{h,i}-T_{c,o}\right)-\left(T_{h,o}-T_{c,i}\right)}{\ln\left[\dfrac{\left(T_{h,i}-T_{c,o}\right)}{\left(T_{h,o}-T_{c,i}\right)}\right]} = \frac{(200-50)-(80-20)}{\ln\left[\dfrac{(200-50)}{(80-20)}\right]} \simeq 98.22\ ^\circ\text{C}$$

$$U = \frac{\dot{m}_c c_c\left(T_{c,o}-T_{c,i}\right)}{A\,\Delta T_{\substack{LMTD\\ counter-flow}}} = \frac{0.5\times 4000\times(50-20)}{1.6\times 98.22} \simeq 381.8\ \text{W/m}^2\text{-°C}$$

c) $q_{foul} = 0.9 q_{clean} = 0.9\times \underbrace{\dot{m}_c c_c\left(T_{c,o}-T_{c,i}\right)}_{clean} = 0.9\times 0.5\times 4000\times(50-20) = 54000$ W

$(\dot{m}c)_{\min} = \left(\dot{m}_h c_h\right) = \left(\dot{m}_c c_c\right)/4 = (0.5\times 4000/4) = 500$ W/°C

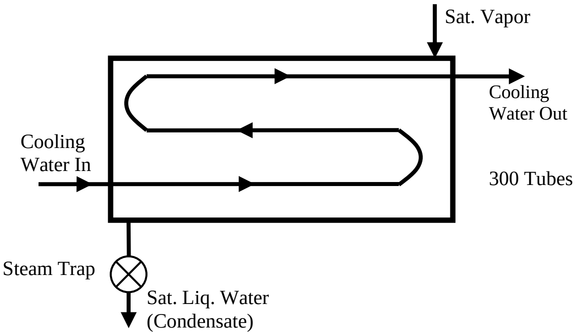
$$\varepsilon_{foul} \triangleq \frac{q_{\substack{actual\\ foul}}}{q_{\max, possible}} = \frac{54000}{(\dot{m}c)_{\min}\left(T_{h,i}-T_{c,i}\right)} = \frac{54000}{\left(\dot{m}_h c_h\right)\left(T_{h,i}-T_{c,i}\right)} = \frac{54000}{500(200-20)} = 0.6$$

d) $R_{\min} = \dfrac{C_{\min}}{C_{\max}} = \dfrac{\left(\dot{m}_h c_h\right)}{\left(\dot{m}_c c_c\right)} = \dfrac{1}{4} = 0.25;\quad \varepsilon_{foul} = 0.6 \xrightarrow[\substack{figure\\ 10-13}]{} NTU_{foul} = 1.0$

$$NTU_{foul} \triangleq \frac{U_{foul}A}{(\dot{m}c)_{\min}} \Rightarrow U_{foul} = \frac{\dot{m}_h c_h \times NTU_{foul}}{A} = \frac{500\times 1.0}{1.6} \simeq 312.5\ \text{W/m}^2\text{-°C}$$

$$R_{foul} = \frac{1}{U_{foul}} - \frac{1}{U_{clean}} = \frac{1}{312.5} - \frac{1}{381.8} \simeq\ 5.81\times 10^{-4}\ \text{m}^2\text{-°C/W}$$

Problem 3:



Given: $T_{h,i} = T_{h,o} = T_{sat} = 40°C$; $T_{c,i} = 20°C$; $T_{c,o} = 30°C$; $h_{fg} = 2.1 \times 10^6$ J/kg; $\dot{m}_{condensation} = 3.0$ kg/s; $h_o = 12500$ W/m^2-°C. Number of tubes: 300. D_i=0.02 m; $D_o = 0.023$ m; $k_{pipe} = 200$ W/m-°C.

Cooling water properties: $\rho = 1000$ kg/m^3; $c_p = 4200$ J/kg-°C; $\mu = 9.0\times 10^{-4}$ kg/m-s; $k = 0.6$W/m-°C.

Assumptions: Steady-state conditions prevail; heat loss to ambient fluid is negligible; Ec<<1

a) $E-balance: q = \dot{m}_{water} c_{water}\left(T_{c,o} - T_{c,i}\right) = \dot{m}_{condensation} h_{fg} = 3.0\times 2.1\times 10^6 = 6.3\times 10^6 \text{ W}$

b) $\dot{m}_{water} = \dfrac{\dot{m}_{condensation} h_{fg}}{c_{water}\left(T_{c,o} - T_{c,i}\right)} = \dfrac{3.0\times 2.1\times 10^6}{4200\times(30-20)} = 150 \text{ kg/s}$

c) $\dot{m}_{\substack{water\\ per\ tube}} = \dfrac{150}{300} = 0.5 \text{ kg/s} \rightarrow u_{av} = \dfrac{\dot{m}_{\substack{water\\ per\ tube}}}{\rho\pi\left(D_i\right)^2/4} = \dfrac{0.5}{1000\times\pi(0.02)^2/4} = 1.592 \text{ m/s}$

d) $\dfrac{1}{UA} = \dfrac{1}{U_o A_o} = \dfrac{1}{U_i A_i} = \dfrac{1}{h_i A_i} + \dfrac{\ln\left(r_o/r_i\right)}{2\pi k_{pipe} L} + \dfrac{1}{h_o A_o}$ where $A_i = \pi D_i L$ and $A_o = \pi D_o L$. we need to calculate first h_i. $\text{Re}_{D_i} = \dfrac{\rho u_{av} D_i}{\mu} = \dfrac{1000\times 1.592\times 0.02}{9.0\times 10^{-4}} = 35377.78 > 2300$ Thus, flow is turbulent

Using Sider-Tate correlation:

$$Nu_{D_i} = \frac{h_i D_i}{k_{water}} = 0.027\,\text{Re}_{D_i}^{0.8}\,\text{Pr}^{1/3}\left(\mu/\mu_{@wall}\right)^{0.14};\ \text{constant props} \Rightarrow \mu/\mu_{@wall} = 1;\ \text{Pr} = \frac{\mu c_p}{k} = 6.3$$

$$h_i = \frac{0.6}{0.02}\times 0.027(35377.78)^{0.8}(6.3)^{1/3}(1)^{0.14} \Rightarrow h_i = 6515.03 \text{ W/m}^2\text{-°C}$$

$$\frac{1}{U_i} = \frac{1}{h_i} + D_i\frac{\ln\left(D_o/D_i\right)}{2k_{pipe}} + \frac{D_i}{D_o}\frac{1}{h_o} = \frac{1}{6515.03} + 0.02\frac{\ln(0.023/0.02)}{2\times 200} + \frac{0.02}{0.023}\frac{1}{12500}$$

$$\frac{1}{U_i} = 1.53485\times 10^{-4} + 6.988097\times 10^{-6} + 6.95652\times 10^{-5} = 2.3\times 10^{-4} \Rightarrow U_i = 4347.1 \text{ W/m}^2\text{-°C}$$

e) For this heat exchanger (condenser), $(\dot{m}c)_{\min} = \dot{m}_c c_c$ and $(\dot{m}c)_{\max} = \dot{m}_h c_h \rightarrow \infty$

$$\Rightarrow R_{\min} = \frac{C_{\min}}{C_{\max}} = 0; \text{ Thus, } \boxed{\varepsilon = 1\text{-}\exp\left(\text{-}NTU\right)}$$

$$\varepsilon = \frac{\left|\Delta T_b\right|_{\substack{\text{for min. capacity}\\ \text{rate fluid}}}}{\left|\Delta T_b\right|_{\max}} = \frac{\left(T_{c,o} - T_{c,i}\right)}{\left(T_{h,i} - T_{c,i}\right)} = \frac{30-20}{40-20} = \frac{10}{20} = 0.5 \text{ and } NTU = -\ln\left(1\text{-}\varepsilon\right) = -\ln\left(1\text{-}0.5\right) = 0.693$$

$$NTU \triangleq \frac{UA}{\left(\dot{m}c\right)_{\min}} \triangleq \frac{U_i A_i}{\dot{m}_c c_c} \Rightarrow A_i = \frac{\dot{m}_c c_c \times NTU}{U_i} = \frac{0.693 \times 150 \times 4200}{4347.1} \simeq 100.43 \text{ m}^2$$

$$A_i = N_{tube} \pi D_i L_{\substack{per\\ tube}} \Rightarrow L_{\substack{per\\ tube}} = \frac{100.43}{300 \times \pi \times 0.02} = 5.33 \text{ m}$$

Please note that because we have three passes for the tube side, the length of the H.E., will be about 5.328/3 ~ 1.8 m.

f) $$\varepsilon_{foul} \triangleq \frac{q_{\substack{actual\\ foul}}}{q_{\max,\, possible}} = \frac{\dot{m}_{\substack{condensation\\ foul}} h_{fg}}{\dot{m}_c c_c \left(T_{h,i} - T_{c,i}\right)} = \frac{2.0 \times 2.1 \times 10^6}{150 \times 4200 \times \left(40-20\right)} = 0.3333$$

$$NTU_{foul} = -\ln\left(1\text{-}\varepsilon_{foul}\right) = -\ln\left(1\text{-}0.3333\right) = 0.405$$

$$NTU_{foul} \triangleq \frac{U_{i,\, foul} A_i}{\left(\dot{m}c\right)_{\min}} \Rightarrow U_{i,\, foul} = \frac{\dot{m}_c c_c \times NTU_{foul}}{A_i} = \frac{150 \times 4200 \times 0.405}{100.43} \simeq 2540.57 \text{ W/m}^2\text{-°C}$$

$$R_{i,\, foul} = \frac{1}{U_{i,\, foul}} - \frac{1}{U_{i,clean}} = \frac{1}{2540.57} - \frac{1}{4347.1} \simeq 1.636 \times 10^{-4} \text{ m}^2\text{-°C/W}$$