MECH 325
Homework Assignment #4
Due Nov. 29
(Hand in to Box in MECH Office, CEME 2054)

Problem 1 (Question 10-6)

A helical compression spring is to be made of oil-tempered wire of 4-mm diameter with a spring index of $C = 10$. The spring is to operate inside a hole, so buckling is not a problem and the ends can be left plain. The free length of the spring should be 80 mm. A force of 50 N should deflect the spring 15 mm.

(*a*) Determine the spring rate.

(*b*) Determine the minimum hole diameter for the spring to operate in.

(c) Determine the total number of coils needed.

(*d*) Determine the solid length.

(*e*) Determine a static factor of safety based on the yielding of the spring if it is compressed to its solid length.

Solution 10-6

Given: Oil-tempered wire, $d = 4$ mm, $C = 10$, plain ends, $L_0 = 80$ mm, and at $F = 50$ N, $y = 15$ mm.

(**a**) $k = F/y = 50/15 = 3.333$ N/mm *Ans.*

(**b**) $D = Cd = 10(4) = 40$ mm

OD $= D + d = 40 + 4 = 44$ mm *Ans.*

(**c**) From Table 10-5, $G = 77.2$ GPa

Eq. (10-9): $$N_a = \frac{d^4 G}{8kD^3} = \frac{4^4(77.2)10^3}{8(3.333)40^3} = 11.6 \text{ coils}$$

Table 10-1: $N_t = N_a = 11.6$ coils *Ans.*

(**d**) Table 10-1: $L_s = d(N_t + 1) = 4(11.6 + 1) = 50.4$ mm *Ans.*

(**e**) Table 10-4: $m = 0.187$, $A = 1855$ MPa·mm^m

Eq. (10-14): $S_{ut} = \dfrac{A}{d^m} = \dfrac{1855}{4^{0.187}} = 1431$ MPa

Table 10-6: $S_{sy} = 0.50\ S_{ut} = 0.50(1431) = 715.5$ MPa

$y_s = L_0 - L_s = 80 - 50.4 = 29.6$ mm

$F_s = k\ y_s = 3.333(29.6) = 98.66$ N

Eq. (10-5): $K_B = \dfrac{4C+2}{4C-3} = \dfrac{4(10)+2}{4(10)-3} = 1.135$

Eq. (10-7): $\tau_s = K_B \dfrac{8F_s D}{\pi d^3} = 1.135 \dfrac{8(98.66)40}{\pi(4^3)} = 178.2$ MPa

$n_s = \dfrac{S_{sy}}{\tau_s} = \dfrac{715.5}{178.2} = 4.02$ *Ans.*

Problem 2 (Question 10-31)

A compression spring is needed to fit over a 0.5-in diameter rod. To allow for some clearance, the inside diameter of the spring is to be 0.6 in. To ensure a reasonable coil, use a spring index of 10. The spring is to be used in a machine by compressing it from a free length of 5 in through a stroke of 3 in to its solid length. The spring should have squared and ground ends, unpeened, and is to be made from cold-drawn wire.

(*a*) Determine a suitable wire diameter.

(*b*) Determine a suitable total number of coils.

(*c*) Determine the spring constant.

(*d*) Determine the static factor of safety when compressed to solid length.

(*e*) Determine the fatigue factor of safety when repeatedly cycled from free length to solid length. Use the Gerber-Zimmerli fatigue-failure criterion.

10-31 Given: ID = 0.6 in, $C = 10$, $L_0 = 5$ in, $L_s = 5 - 3 = 2$ in, sq. & grd ends, unpeened, HD A227 wire.

(**a**) With ID = $D - d = 0.6$ in and $C = D/d = 10 \Rightarrow 10\,d - d = 0.6 \Rightarrow d = 0.0667$ in *Ans.*, and $D = 0.667$ in.

(**b**) Table 10-1: $L_s = dN_t = 2$ in $\Rightarrow N_t = 2/0.0667 = 30$ coils *Ans.*

(**c**) Table 10-1: $N_a = N_t - 2 = 30 - 2 = 28$ coils

Table 10-5: $G = 11.5$ Mpsi

Eq. (10-9):
$$k = \frac{d^4 G}{8D^3 N_a} = \frac{0.0667^4 (11.5)10^6}{8(0.667^3)28} = 3.424 \text{ lbf/in} \qquad \textit{Ans.}$$

(**d**) Table 10-4: $A = 140$ kpsi·in^m, $m = 0.190$

Eq. (10-14):
$$S_{ut} = \frac{A}{d^m} = \frac{140}{0.0667^{0.190}} = 234.2 \text{ kpsi}$$

Table 10-6: $S_{sy} = 0.45\, S_{ut} = 0.45\,(234.2) = 105.4$ kpsi

$$F_s = ky_s = 3.424(3) = 10.27 \text{ lbf}$$

Eq. (10-5):
$$K_B = \frac{4C+2}{4C-3} = \frac{4(10)+2}{4(10)-3} = 1.135$$

Eq. (10-7):
$$\tau_s = K_B \frac{8F_s D}{\pi d^3} = 1.135 \frac{8(10.27)0.667}{\pi(0.0667^3)}$$
$$= 66.72(10^3) \text{ psi} = 66.72 \text{ kpsi}$$
$$n_s = \frac{S_{sy}}{\tau_s} = \frac{105.4}{66.72} = 1.58 \qquad \textit{Ans.}$$

(**e**) $\tau_a = \tau_m = 0.5\,\tau_s = 0.5(66.72) = 33.36$ kpsi, $r = \tau_a / \tau_m = 1$. Using the Gerber fatigue failure criterion with Zimmerli data,

Eq. (10-30): $S_{su} = 0.67\, S_{ut} = 0.67(234.2) = 156.9$ kpsi

The Gerber ordinate intercept for the Zimmerli data is obtained using Eqs. (10-28) and (10-29b).
$$S_{se} = \frac{S_{sa}}{1-(S_{sm}/S_{su})^2} = \frac{35}{1-(55/156.9)^2} = 39.9 \text{ kpsi}$$

The Gerber fatigue criterion from Eq. (6-48), adapted for shear,
$$n_f = \frac{1}{2}\left(\frac{S_{su}}{\tau_m}\right)^2 \frac{\tau_a}{S_{se}}\left[-1+\sqrt{1+\left(\frac{2\tau_m S_{se}}{S_{su}\tau_a}\right)^2}\right]$$
$$= \frac{1}{2}\left(\frac{156.9}{33.36}\right)^2 \frac{33.36}{39.9}\left[-1+\sqrt{1+\left(\frac{2(33.36)39.9}{156.9(33.36)}\right)^2}\right]$$
$$= 1.13 \qquad \textit{Ans.}$$

Problem 3 (Question 8-17)

Two identical aluminum plates are each 2 in thick, and are compressed with one bolt and nut. Washers are used under the head of the bolt and under the nut.
Washer properties: steel; ID = 0.531 in; OD = 1.062 in; thickness = 0.095 in Nut properties: steel; height = $\frac{7}{16}$ in Bolt properties: $\frac{1}{2}$ in-13 UNC grade 8
Plate properties: aluminum; E = 10.3 Mpsi; $S_u = 47$ kpsi; $S_y = 25$ kpsi
(*a*) Determine a suitable length for the bolt, rounded up to the nearest $\frac{1}{4}$ in.
(*b*) Determine the bolt stiffness.
(*c*) Determine the stiffness of the members.

8-17 **a**) Grip, $l = 2(2 + 0.095) = 4.19$ in. $L \geq 4.19 + 7/16 = 4.628$ in.
Rounding up, $L = 4.75$ in *Ans.*

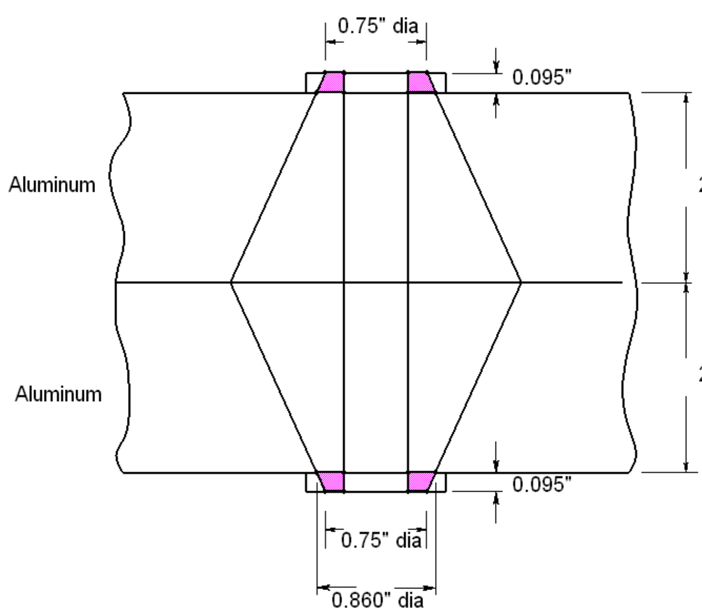
(**b**) From Eq. (8-13), $L_T = 2d + 1/4 = 2(0.5) + 0.25 = 1.25$ in

From Table 8-7, $l_d = L - L_T = 4.75 - 1.25 = 3.5$ in, $l_t = l - l_d = 4.19 - 3.5 = 0.69$ in

$A_d = \pi(0.5^2)/4 = 0.1963$ in^2. From Table 8-2, $A_t = 0.1419$ in^2. From Eq. (8-17)

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{0.1963(0.1419)30}{0.1963(0.69) + 0.1419(3.5)} = 1.322 \text{ Mlbf/in} \qquad Ans.$$

(**c**)



Upper and lower halves are the same. For the upper half, Steel frustum: $t = 0.095$ in, $d = 0.531$ in, $D = 0.75$ in, and $E = 30$ Mpsi. From Eq. (8-20)

$$k_1 = \frac{0.5774\pi(30)0.531}{\ln\dfrac{[1.155(0.095)+0.75-0.531](0.75+0.531)}{[1.155(0.095)+0.75+0.531](0.75-0.531)}} = 89.20 \text{ Mlbf/in}$$

Aluminum: $t = 2$ in, $d = 0.5$ in, $D = 0.75 + 2(0.095)\tan 30° = 0.860$ in, and $E = 10.3$ Mpsi. Eq. (8-20) $\Rightarrow$ $k_2 = 9.24$ Mlbf/in

For the top half, $k'_m = (1/k_1 + 1/k_2)^{-1} = (1/89.20 + 1/9.24)^{-1} = 8.373$ Mlbf/in

Since the bottom half is the same, the overall stiffness is given by

$$k_m = (1/k'_m + 1/k'_m)^{-1} = k'_m/2 = 8.373/2 = 4.19 \text{ Mlbf/in} \qquad \textit{Ans}$$

Problem 4 (Question 8-49)

For a bolted assembly with eight bolts, the stiffness of each bolt is $k_b = 1.0$ MN/mm and the stiffness of the members is $k_m = 2.6$ MN/mm per bolt. The bolts are preloaded to 75 percent of proof strength. Assume the external load is equally distributed to all the bolts. The bolts are M6 × 1 Class 5.8 with rolled threads. A fluctuating external load is applied to the entire joint with $P_{\max} = 60$ kN and $P_{\min} = 20$ kN. Note the fully corrected endurance strength for Class 5.8 bolts is estimated as $S_e = 78.7$ MPa.

(*a*) Determine the yielding factor of safety.
(*b*) Determine the overload factor of safety.
(*c*) Determine the factor of safety based on joint separation.
(*d*) Determine the fatigue factor of safety using the Goodman criterion.
(e) Comment on the results.

8-49

Per bolt, $P_{b\max} = 60/8 = 7.5$ kN, $P_{b\min} = 20/8 = 2.5$ kN

$$C = \frac{k_b}{k_b + k_m} = \frac{1}{1+2.6} = 0.278$$

(**a**) Table 8-1, $A_t = 20.1$ mm^2; Table 8-11, $S_p = 380$ MPa

Eqs. (8-31) and (8-32), $F_i = 0.75\, A_t\, S_p = 0.75(20.1)380(10^{-3}) = 5.73$ kN

Yield, Eq. (8-28), $$n_p = \frac{S_p A_t}{CP + F_i} = \frac{380(20.1)10^{-3}}{0.278(7.5)+5.73} = 0.98 \qquad \textit{Ans.}$$

(**b**) Overload, Eq. (8-29), $$n_L = \frac{S_p A_t - F_i}{CP} = \frac{380(20.1)10^{-3} - 5.73}{0.278(7.5)} = 0.915 \qquad \textit{Ans.}$$

(**c**) Separation, Eq. (8-30), $$n_0 = \frac{F_i}{P(1-C)} = \frac{5.73}{7.5(1-0.278)} = 1.06 \quad \textit{Ans.}$$

(**d**) Goodman, Eq. (8-35), $$\sigma_a = \frac{C(P_{b\max} - P_{b\min})}{2A_t} = \frac{0.278(7.5-2.5)10^3}{2(20.1)} = 34.6 \text{ MPa}$$

Eq. (8-36), $$\sigma_m = \frac{C(P_{b\max} + P_{b\min})}{2A_t} + \frac{F_i}{A_t} = \frac{0.278(7.5+2.5)10^3}{2(20.1)} + \frac{5.73(10^3)}{20.1} = 354.2 \text{ MPa}$$

Table 8-11, S_{ut} = 520 MPa, $\sigma_i = F_i / A_t = 5.73(10^3)/20.1 = 285$ MPa

S_e = 78.7 MPa

Eq. (8-38),

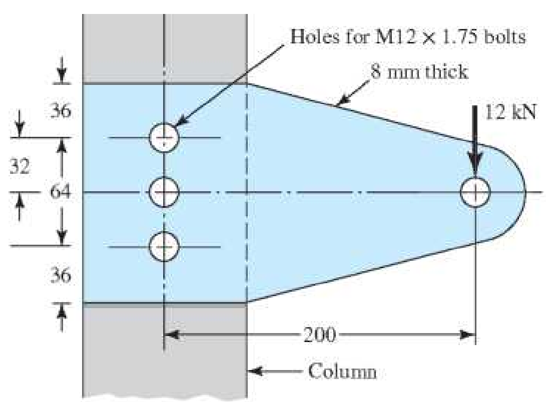
$$n_f = \frac{S_e(S_{ut} - \sigma_i)}{S_{ut}\sigma_a + S_e(\sigma_m - \sigma_i)} = \frac{78.7(520-285)}{520(34.6) + 78.7(354.2-285)} = 0.789 \quad \textit{Ans.}$$

It is obvious from the various answers obtained, the bolted assembly is undersized. This can be rectified by a one or more of the following: more bolts, larger bolts, higher class bolts.

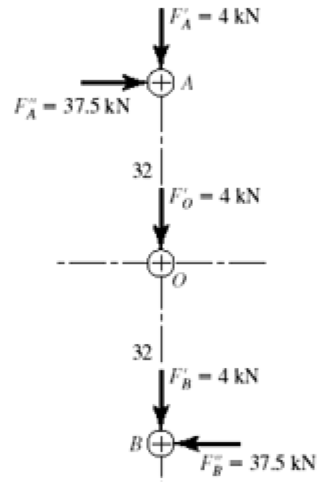
Problem 5 (Question 8-77)

The cantilever bracket is bolted to a column with three M12 × 1.75 ISO 5.8 bolts. The bracket is made from AISI 1020 hot-rolled steel. Assume the bolt threads do not extend into the joint. Find the factors of safety for the following failure modes: shear of bolts, bearing of bolts, bearing of bracket, and bending of bracket (top of the bracket at the bolt centerline). Comment on the results

Problem 8–77
Dimensions in millimeters.



8-77 Bolts, from Table 8-11, $S_y = 420$ MPa
Bracket, from Table A-20, $S_y = 210$ MPa



$$F' = \frac{12}{3} = 4 \text{ kN}; \; M = 12(200) = 2400 \text{ N} \cdot \text{m}$$
$$F''_A = F''_B = \frac{2400}{64} = 37.5 \text{ kN}$$
$$F_A = F_B = \sqrt{(4)^2 + (37.5)^2} = 37.7 \text{ kN}$$
$$F_O = 4 \text{ kN}$$

Bolt shear:
The shoulder bolt shear area, $A_s = \pi(12^2) / 4 = 113.1$ mm^2

$S_{sy} = 0.577(420) = 242.3$ KPa

$$\tau = \frac{37.7(10)^3}{113} = 333 \text{ MPa}$$
$$n = \frac{S_{sy}}{\tau} = \frac{242.3}{333} = 0.728 \qquad \textit{Ans.}$$

Bearing on bolts:
$$A_b = 12(8) = 96 \text{ mm}^2$$
$$\sigma_b = -\frac{37.7(10)^3}{96} = -393 \text{ MPa}$$
$$n = \frac{S_{yc}}{|\sigma_b|} = \frac{420}{393} = 1.07 \qquad \textit{Ans.}$$

Bearing on member:
$$\sigma_b = -393 \text{ MPa}$$
$$n = \frac{S_{yc}}{|\sigma_b|} = \frac{210}{393} = 0.534 \qquad \textit{Ans.}$$

Bending stress in plate:

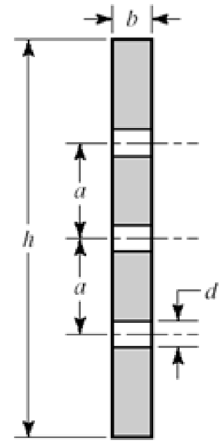
$$I = \frac{bh^3}{12} - \frac{bd^3}{12} - 2\left(\frac{bd^3}{12} + a^2bd\right)$$

$$= \frac{8(136)^3}{12} - \frac{8(12)^3}{12} - 2\left[\frac{8(12)^3}{12} + (32)^2(8)(12)\right]$$

$$= 1.48(10)^6 \text{ mm}^4 \quad \textit{Ans.}$$

$$\sigma = \frac{Mc}{I} = \frac{2400(68)}{1.48(10)^6}(10)^3 = 110 \text{ MPa}$$

$$n = \frac{S_y}{\sigma} = \frac{210}{110} = 1.91 \quad \textit{Ans.}$$



Failure is predicted for bolt shear and bearing on member.

Problem 6 (Fluid Power Question)

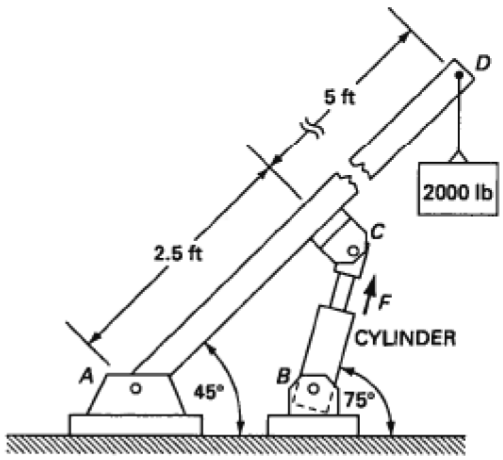


Figure 6-27. System for Exercise 6-27.

a) Determine the required force, F, to support the 2000 lb. load in the position shown above.

b) With a working pressure of 1000psi, select an appropriate cylinder size for system (for the position shown).

c) If the extension for the cylinder is 12 inches, determine the required rod diameter for the system (for the position shown).

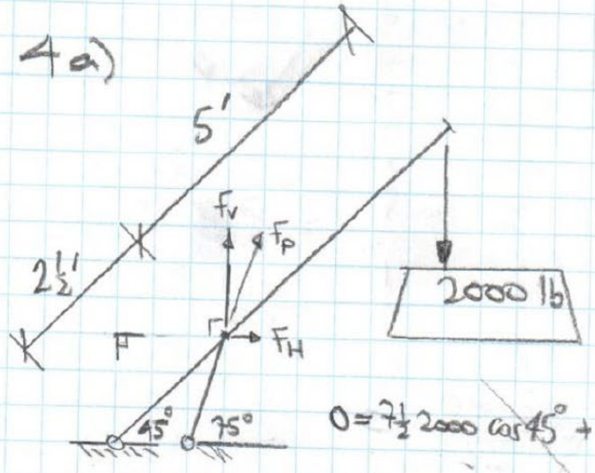
d) If we wish to lift the load at a rate of 1 foot/sec (at the position shown), what size of pump, piping system, and electric motor are required?

e) If we include the fact that we wish to accelerate the cylinder to its required speed within ¾", with this change the results above (at the position shown)?

f) Looking at the system, do you expect the required thrust to be higher at the starting position (fully retracted rod) or at the fully extended position? Let's assume the position above is for the 50% point.

4a)

HYD cylinder (6)



$\sum M_A = \emptyset$

F_H F_V

$0 = 7\frac{1}{2}\, 2000 \cos 45° + F_P \sin 15° \sin 45°\, 2\frac{1}{2}' - F_P \cos 15 \cos 45$

$F_P(\cos 15 - \sin 15) = 2000 \cdot \dfrac{7\frac{1}{2}}{2\frac{1}{2}}$

a) $\boxed{F_P = \dfrac{2000 \cdot 3}{(\cos 15 - \sin 15)}\ \text{lb} = 8485\ \text{lb}}$

b) WORKING PRESSURE 1000 psi $3\frac{1}{4}$" 8300 lb too small

PARKER TABLE B-1 TRY 4" BORE

$\boxed{\text{RATING } 12{,}570\ \text{lb}}$ TABLE B8 is Available.

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c) ROD BUCKLING Assume K = 2 (TRUNNION + CLEVIS CAP)

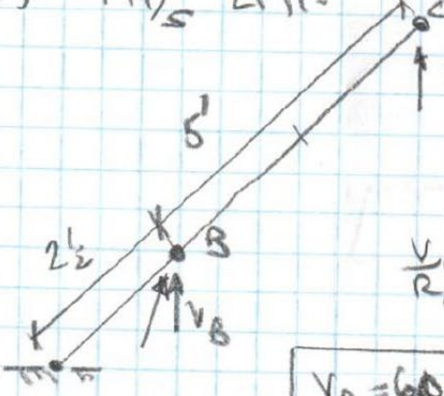
EXTENSION 12 inches Effective L = 24"

@ 8485 lb $\boxed{\text{ROD DIA} = 1\,3/8"}$

available in 4" BORE ☑

7

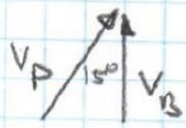
d) 1 ft/s Lift. 60 ft/min



Assume constant ω

$$\frac{V_C}{R_C} = \frac{V_B}{R_B} \Rightarrow V_B = \frac{V_C}{R_C} R_B$$

$$\boxed{V_B = \frac{60 \frac{ft}{min} \cdot 2\frac{1}{2}'}{7\frac{1}{2}'} = 20 \text{ ft/min}}$$



$$V_P = \frac{V_B}{\cos\theta} = \frac{20 \text{ ft/min}}{\cos 15°} = 20.7 \frac{ft}{min}$$

check if Power conserved $F_C \cdot V_C = F_P \cdot V_P$

$$V_P = \frac{F_C V_C}{F_B} = \frac{2000 lb \cdot 60 \frac{ft}{min}}{8435 lb} = 14 \text{ ft/min} ?$$

FOR Pump use TABLE B-5 using Rod ϕ = 0 inch

$$Q = \frac{6.53 \text{ gpm}}{10 \text{ ft/min}} \times 20.7 \frac{ft}{min} = \boxed{13.52 \text{ gpm} = Q}$$

Pipe supply flow in supply < 15 $\frac{ft}{s}$ for 10 $\frac{ft}{min}$ ∴ 7.77 ft/s

Select $\frac{3}{4}$" supply $\left(\text{gives } \frac{4.84 \text{ ft/s}}{10 \text{ ft/min}} \times 20.7 \frac{ft}{min} = 10 \frac{ft}{s}\right)$ OK < 15 ft/s

PART D cont'd

SELECT Pump 13.52 gpm

$$\text{POWER} = Q \cdot P = \frac{13.52\text{gpm} \; 1000\text{ psi}}{1714 \times 0.85}$$

POWER = 9.28 HP

must use 3 phase

10 HP.	220V	40 AMP
	or 440V	20 AMP

could reduce operating pressure to reduce power

e) Accelerate cylinder to speed within 3/4"

USING BLOCK WEIGHT INERTIA — V = 20.7 ft/min PISTON — S = 3/4"

OR Weight 60 ft/min S = ?

PISTON acc DISTANCE = 3/4"

ASSUME CONSTANT ENERGY

$$\boxed{F_p \cdot S_p = F_W \cdot S_W} \quad \therefore S_W = \frac{F_P}{F_W} \cdot S_P = \frac{3}{4}'' \cdot \frac{8485\text{ lb}}{2000\text{ lb}} = 3.18''$$

$$g = \frac{V^2}{S} \cdot 0.0000517 \quad \text{PARKER FORMULA}$$

@ WEIGHT $$g = \frac{\left(60\frac{ft}{min}\right)^2 \cdot 0.0000517}{3.18\text{ inch}} = 0.0585$$

on 2000 lb.

= 117 lb @ weight

$$\therefore @\text{ cylinder } \Delta F = 117\text{b} \cdot \frac{8485}{2000\text{ lb}}$$

$$= 497\text{ lb}$$

$$\Sigma\; 8485\text{ lb} + 497\text{ lb} = 8982\text{ lb}$$

NO CHANGE IN PISTON.

f) <u>Thrust is more starting than extended.</u>

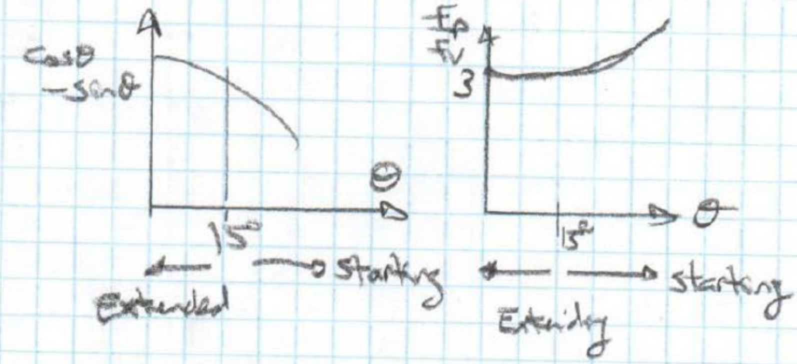
from a)

$$F_P \simeq \frac{2000\,\text{lb} \cdot 7\tfrac{1}{2}/2\tfrac{1}{2}}{(\cos\theta - \sin\theta)}$$

	θ	$\frac{F_P}{F_V \cdot 3}$
EXTENDED	0	6000 lb
HALF	15	8485 lb
STARTING	30	16,392 lb.

STARTING WORSE.

$\therefore$ L



USE RATE OF CHANGE = DERIVATIVE

$$\frac{\partial F}{\partial \theta} = \frac{\partial}{\partial \theta}\left(\frac{1}{\cos\theta - \sin\theta}\right) \quad \text{chain Rule} \quad = \frac{-\frac{\partial}{\partial\theta}(\cos\theta - \sin\theta)}{(\cos\theta - \sin\theta)^2}$$

$$\frac{\partial F}{\partial \theta} = \frac{-(-\sin\theta - \cos\theta)}{(\cos\theta - \sin\theta)^2} = \left.\frac{\sin\theta + \cos\theta}{(\cos\theta - \sin\theta)^2}\right|_{15^\circ}$$

$$\frac{\partial F}{\partial \theta} = \frac{\sin 15 + \cos 15}{(\cos 15 - \sin 15)^2} = 2.44$$

positive F increase with θ

$\therefore$ STARTING WORSE.