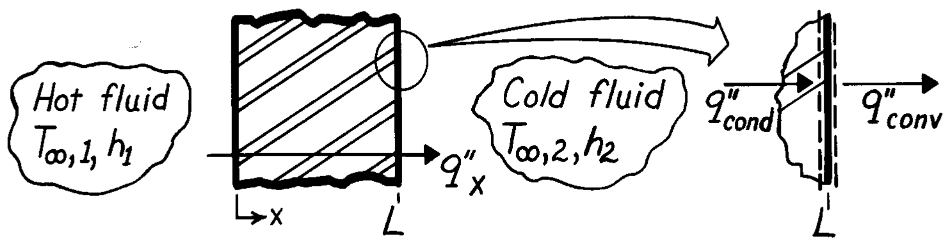
PROBLEM 3.1

KNOWN: One-dimensional, plane wall separating hot and cold fluids at $T_{\infty,1}$ and $T_{\infty,2}$, respectively.

FIND: Temperature distribution, T(x), and heat flux, q''_x, in terms of $T_{\infty,1}$, $T_{\infty,2}$, h_1, h_2, k and L.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) Steady-state conditions, (3) Constant properties, (4) Negligible radiation, (5) No generation.

ANALYSIS: For the foregoing conditions, the general solution to the heat diffusion equation is of the form, Equation 3.2,

$$T(x) = C_1 x + C_2. \tag{1}$$

The constants of integration, C_1 and C_2, are determined by using surface energy balance conditions at x = 0 and x = L, Equation 2.23, and as illustrated above,

$$-k\left.\frac{dT}{dt}\right]_{x=0} = h_1\left[T_{\infty,1} - T(0)\right] \qquad -k\left.\frac{dT}{dx}\right]_{x=L} = h_2\left[T(L) - T_{\infty,2}\right]. \tag{2,3}$$

For the BC at x = 0, Equation (2), use Equation (1) to find

$$-k\left(C_1 + 0\right) = h_1\left[T_{\infty,1} - \left(C_1 \cdot 0 + C_2\right)\right] \tag{4}$$

and for the BC at x = L to find

$$-k\left(C_1 + 0\right) = h_2\left[\left(C_1 L + C_2\right) - T_{\infty,2}\right]. \tag{5}$$

Multiply Eq. (4) by h_2 and Eq. (5) by h_1, and add the equations to obtain C_1. Then substitute C_1 into Eq. (4) to obtain C_2. The results are

$$C_1 = -\frac{\left(T_{\infty,1} - T_{\infty,2}\right)}{k\left[\frac{1}{h_1} + \frac{1}{h_2} + \frac{L}{k}\right]} \qquad C_2 = -\frac{\left(T_{\infty,1} - T_{\infty,2}\right)}{h_1\left[\frac{1}{h_1} + \frac{1}{h_2} + \frac{L}{k}\right]} + T_{\infty,1}$$

$$T(x) = -\frac{\left(T_{\infty,1} - T_{\infty,2}\right)}{\left[\frac{1}{h_1} + \frac{1}{h_2} + \frac{L}{k}\right]}\left[\frac{x}{k} + \frac{1}{h_1}\right] + T_{\infty,1}.$$ <

From Fourier's law, the heat flux is a constant and of the form

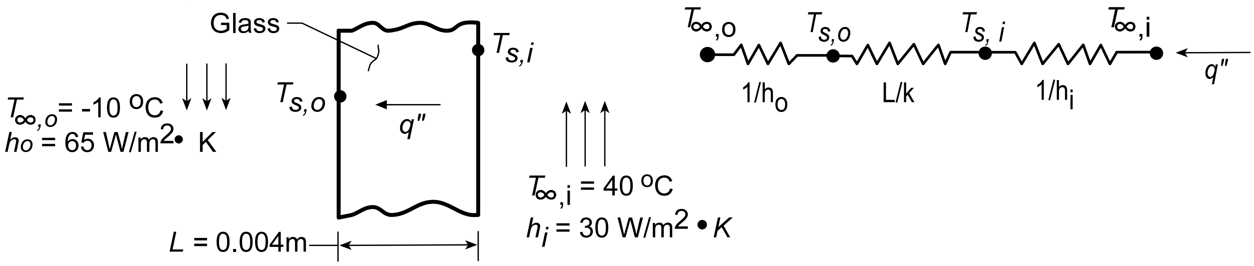
$$q''_x = -k\frac{dT}{dx} = -k\,C_1 = +\frac{\left(T_{\infty,1} - T_{\infty,2}\right)}{\left[\frac{1}{h_1} + \frac{1}{h_2} + \frac{L}{k}\right]}.$$ <

PROBLEM 3.2

KNOWN: Temperatures and convection coefficients associated with air at the inner and outer surfaces of a rear window.

FIND: (a) Inner and outer window surface temperatures, $T_{s,i}$ and $T_{s,o}$, and (b) $T_{s,i}$ and $T_{s,o}$ as a function of the outside air temperature $T_{\infty,o}$ and for selected values of outer convection coefficient, h_o.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Negligible radiation effects, (4) Constant properties.

PROPERTIES: *Table A-3*, Glass (300 K): k = 1.4 W/m·K.

ANALYSIS: (a) The heat flux may be obtained from Eqs. 3.11 and 3.12,

$$q'' = \frac{T_{\infty,i} - T_{\infty,o}}{\frac{1}{h_o} + \frac{L}{k} + \frac{1}{h_i}} = \frac{40^\circ C - \left(-10^\circ C\right)}{\frac{1}{65\,W/m^2\cdot K} + \frac{0.004\,m}{1.4\,W/m\cdot K} + \frac{1}{30\,W/m^2\cdot K}}$$

$$q'' = \frac{50^\circ C}{\left(0.0154 + 0.0029 + 0.0333\right)m^2\cdot K/W} = 968\,W/m^2 .$$

Hence, with $q'' = h_i\left(T_{\infty,i} - T_{\infty,o}\right)$, the inner surface temperature is

$$T_{s,i} = T_{\infty,i} - \frac{q''}{h_i} = 40^\circ C - \frac{968\,W/m^2}{30\,W/m^2\cdot K} = 7.7^\circ C$$ <

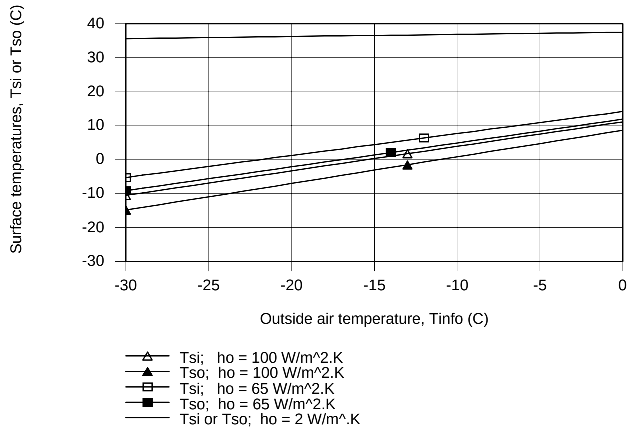
Similarly for the outer surface temperature with $q'' = h_o\left(T_{s,o} - T_{\infty,o}\right)$ find

$$T_{s,o} = T_{\infty,o} - \frac{q''}{h_o} = -10^\circ C - \frac{968\,W/m^2}{65\,W/m^2\cdot K} = 4.9^\circ C$$ <

(b) Using the same analysis, $T_{s,i}$ and $T_{s,o}$ have been computed and plotted as a function of the outside air temperature, $T_{\infty,o}$, for outer convection coefficients of h_o = 2, 65, and 100 W/m^2·K. As expected, $T_{s,i}$ and $T_{s,o}$ are linear with changes in the outside air temperature. The difference between $T_{s,i}$ and $T_{s,o}$ increases with increasing convection coefficient, since the heat flux through the window likewise increases. This difference is larger at lower outside air temperatures for the same reason. Note that with h_o = 2 W/m^2·K, $T_{s,i}$ - $T_{s,o}$, is too small to show on the plot.

Continued …..

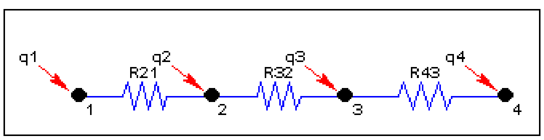
PROBLEM 3.2 (Cont.)



COMMENTS: (1) The largest resistance is that associated with convection at the inner surface. The values of $T_{s,i}$ and $T_{s,o}$ could be increased by increasing the value of h_i.

(2) The *IHT Thermal Resistance Network Model* was used to create a model of the window and generate the above plot. The Workspace is shown below.

// Thermal Resistance Network Model:
// The Network:



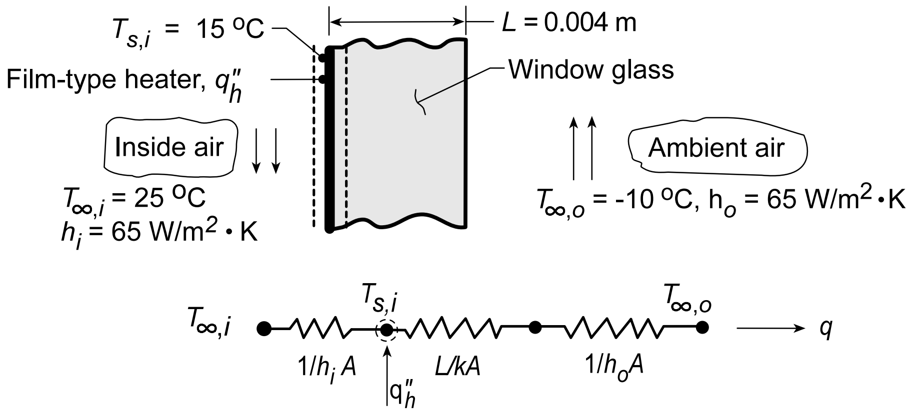

```
// Heat rates into node j,qij, through thermal resistance Rij
q21 = (T2 - T1) / R21
q32 = (T3 - T2) / R32
q43 = (T4 - T3) / R43

// Nodal energy balances
q1 + q21 = 0
q2 - q21 + q32 = 0
q3 - q32 + q43 = 0
q4 - q43 = 0

/* Assigned variables list: deselect the qi, Rij and Ti which are unknowns; set qi = 0 for embedded nodal points
at which there is no external source of heat. */
T1 =  Tinfo            // Outside air temperature, C
//q1 =                 // Heat rate, W
T2 =  Tso              // Outer surface temperature, C
q2 =  0                // Heat rate, W; node 2, no external heat source
T3 =  Tsi              // Inner surface temperature, C
q3 =  0                // Heat rate, W; node 2, no external heat source
T4 = Tinfi             // Inside air temperature, C
//q4 =                 // Heat rate, W

// Thermal Resistances:
R21 =  1 / ( ho * As )          // Convection thermal resistance, K/W; outer surface
R32 =  L / ( k * As )           // Conduction thermal resistance, K/W; glass
R43 =  1 / ( hi * As )          // Convection thermal resistance, K/W; inner surface

// Other Assigned Variables:
Tinfo = -10            // Outside air temperature, C
ho = 65                // Convection coefficient, W/m^2.K; outer surface
L = 0.004              // Thickness, m; glass
k = 1.4                // Thermal conductivity, W/m.K; glass
Tinfi = 40             // Inside air temperature, C
hi = 30                // Convection coefficient, W/m^2.K; inner surface
As = 1                 // Cross-sectional area, m^2; unit area
```

PROBLEM 3.3

KNOWN: Desired inner surface temperature of rear window with prescribed inside and outside air conditions.

FIND: (a) Heater power per unit area required to maintain the desired temperature, and (b) Compute and plot the electrical power requirement as a function of $T_{\infty,o}$ for the range $-30 \le T_{\infty,o} \le 0$°C with h_o of 2, 20, 65 and 100 W/m^2·K. Comment on heater operation needs for low h_o. If $h \sim V^n$, where V is the vehicle speed and n is a positive exponent, how does the vehicle speed affect the need for heater operation?

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat transfer, (3) Uniform heater flux, q''_h, (4) Constant properties, (5) Negligible radiation effects, (6) Negligible film resistance.

PROPERTIES: *Table A-3*, Glass (300 K): k = 1.4 W/m·K.

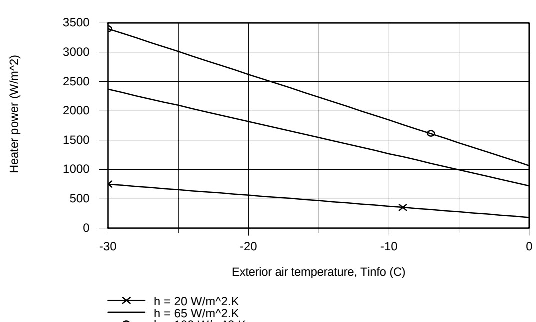
ANALYSIS: (a) From an energy balance at the inner surface and the thermal circuit, it follows that for a unit surface area,

$$\frac{T_{\infty,i} - T_{s,i}}{1/h_i} + q''_h = \frac{T_{s,i} - T_{\infty,o}}{L/k + 1/h_o}$$

$$q''_h = \frac{T_{s,i} - T_{\infty,o}}{L/k + 1/h_o} - \frac{T_{\infty,i} - T_{s,i}}{1/h_i} = \frac{15^\circ\text{C} - \left(-10^\circ\text{C}\right)}{\dfrac{0.004\,\text{m}}{1.4\,\text{W/m}\cdot\text{K}} + \dfrac{1}{65\,\text{W/m}^2\cdot\text{K}}} - \frac{25^\circ\text{C} - 15^\circ\text{C}}{\dfrac{1}{10\,\text{W/m}^2\cdot\text{K}}}$$

$$q''_h = (1370 - 100)\,\text{W/m}^2 = 1270\,\text{W/m}^2$$ <

(b) The heater electrical power requirement as a function of the exterior air temperature for different exterior convection coefficients is shown in the plot. When h_o = 2 W/m^2·K, the heater is unecessary, since the glass is maintained at 15°C by the interior air. If $h \sim V^n$, we conclude that, with higher vehicle speeds, the exterior convection will increase, requiring increased heat power to maintain the 15°C condition.



COMMENTS: With q''_h = 0, the inner surface temperature with $T_{\infty,o}$ = -10°C would be given by

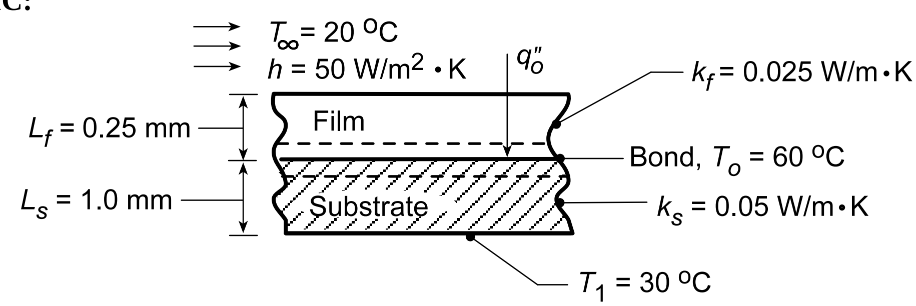
$$\frac{T_{\infty,i} - T_{s,i}}{T_{\infty,i} - T_{\infty,o}} = \frac{1/h_i}{1/h_i + L/k + 1/h_o} = \frac{0.10}{0.118} = 0.846, \qquad \text{or} \qquad T_{s,i} = 25^\circ C - 0.846\left(35^\circ C\right) = -4.6^\circ C .$$

PROBLEM 3.4

KNOWN: Curing of a transparent film by radiant heating with substrate and film surface subjected to known thermal conditions.

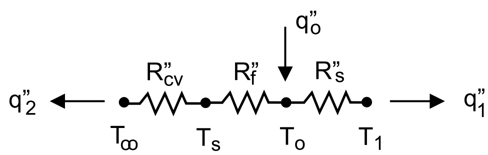
FIND: (a) Thermal circuit for this situation, (b) Radiant heat flux, q''_o (W/m^2), to maintain bond at curing temperature, T_o, (c) Compute and plot q''_o as a function of the film thickness for $0 \le L_f \le 1$ mm, and (d) If the film is not transparent, determine q''_o required to achieve bonding; plot results as a function of L_f.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat flow, (3) All the radiant heat flux q''_o is absorbed at the bond, (4) Negligible contact resistance.

ANALYSIS: (a) The thermal circuit for this situation is shown at the right. Note that terms are written on a per unit area basis.



(b) Using this circuit and performing an energy balance on the film-substrate interface,

$$q''_o = q''_1 + q''_2 \qquad\qquad q''_o = \frac{T_o - T_\infty}{R''_{cv} + R''_f} + \frac{T_o - T_1}{R''_s}$$

where the thermal resistances are

$$R''_{cv} = 1/h = 1/50\,\mathrm{W/m^2\cdot K} = 0.020\,\mathrm{m^2\cdot K/W}$$

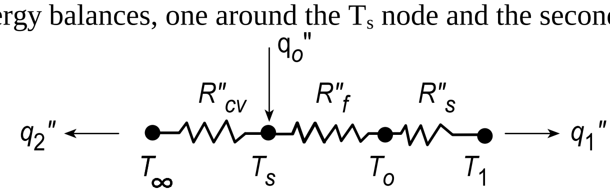
$$R''_f = L_f/k_f = 0.00025\,\mathrm{m}/0.025\,\mathrm{W/m\cdot K} = 0.010\,\mathrm{m^2\cdot K/W}$$

$$R''_s = L_s/k_s = 0.001\,\mathrm{m}/0.05\,\mathrm{W/m\cdot K} = 0.020\,\mathrm{m^2\cdot K/W}$$

$$q''_o = \frac{(60-20)^\circ\mathrm{C}}{[0.020+0.010]\,\mathrm{m^2\cdot K/W}} + \frac{(60-30)^\circ\mathrm{C}}{0.020\,\mathrm{m^2\cdot K/W}} = (133+1500)\,\mathrm{W/m^2} = 2833\,\mathrm{W/m^2} \qquad <$$

(c) For the transparent film, the radiant flux required to achieve bonding as a function of film thickness L_f is shown in the plot below.

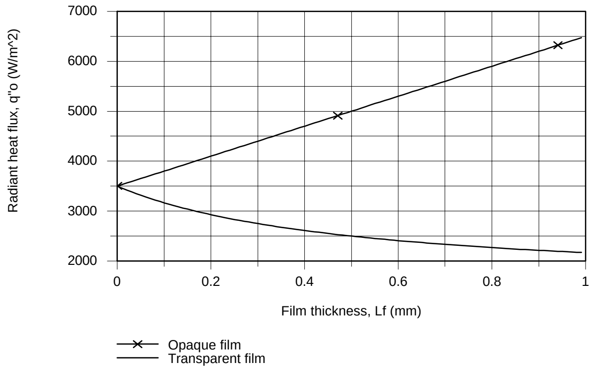
(d) If the film is opaque (not transparent), the thermal circuit is shown below. In order to find q''_o, it is necessary to write two energy balances, one around the T_s node and the second about the T_o node.



.
The results of the analyses are plotted below.

Continued...

PROBLEM 3.4 (Cont.)

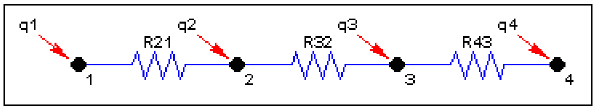


COMMENTS: (1) When the film is transparent, the radiant flux is absorbed on the bond. The flux required decreases with increasing film thickness. Physically, how do you explain this? Why is the relationship not linear?

(2) When the film is opaque, the radiant flux is absorbed on the surface, and the flux required increases with increasing thickness of the film. Physically, how do you explain this? Why is the relationship linear?

(3) The IHT Thermal Resistance Network Model was used to create a model of the film-substrate system and generate the above plot. The Workspace is shown below.

// Thermal Resistance Network Model:
// The Network:



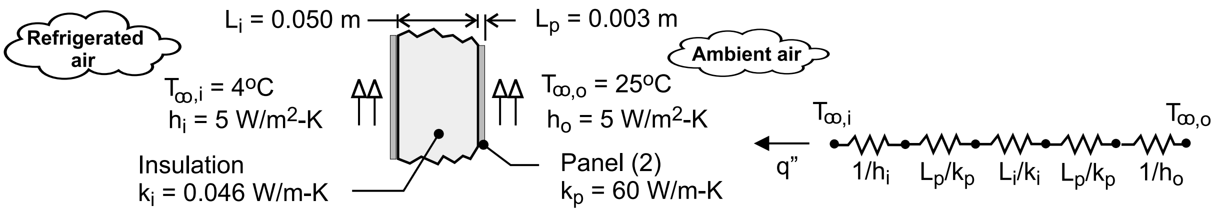

```
// Heat rates into node j,qij, through thermal resistance Rij
q21 = (T2 - T1) / R21
q32 = (T3 - T2) / R32
q43 = (T4 - T3) / R43

// Nodal energy balances
q1 + q21 = 0
q2 - q21 + q32 = 0
q3 - q32 + q43 = 0
q4 - q43 = 0

/* Assigned variables list: deselect the qi, Rij and Ti which are unknowns; set qi = 0 for embedded nodal points
at which there is no external source of heat. */
T1 = Tinf           // Ambient air temperature, C
//q1 =              // Heat rate, W; film side
T2 = Ts             // Film surface temperature, C
q2 = 0              // Radiant flux, W/m^2; zero for part (a)
T3 = To             // Bond temperature, C
q3 = qo             // Radiant flux, W/m^2; part (a)
T4 = Tsub           // Substrate temperature, C
//q4 =              // Heat rate, W; substrate side

// Thermal Resistances:
R21 = 1 / ( h * As )        // Convection resistance, K/W
R32 = Lf / (kf * As)        // Conduction resistance, K/W; film
R43 = Ls / (ks * As)        // Conduction resistance, K/W; substrate

// Other Assigned Variables:
Tinf = 20           // Ambient air temperature, C
h = 50              // Convection coefficient, W/m^2.K
Lf = 0.00025        // Thickness, m; film
kf = 0.025          // Thermal conductivity, W/m.K; film
To = 60             // Cure temperature, C
Ls = 0.001          // Thickness, m; substrate
ks = 0.05           // Thermal conductivity, W/m.K; substrate
Tsub = 30           // Substrate temperature, C
As = 1              // Cross-sectional area, m^2; unit area
```

PROBLEM 3.5

KNOWN: Thicknesses and thermal conductivities of refrigerator wall materials. Inner and outer air temperatures and convection coefficients.

FIND: Heat gain per surface area.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional heat transfer, (2) Steady-state conditions, (3) Negligible contact resistance, (4) Negligible radiation, (5) Constant properties.

ANALYSIS: From the thermal circuit, the heat gain per unit surface area is

$$q'' = \frac{T_{\infty,o} - T_{\infty,i}}{(1/h_i) + (L_p/k_p) + (L_i/k_i) + (L_p/k_p) + (1/h_o)}$$

$$q'' = \frac{(25-4)^\circ C}{2\left(1/5\,W/m^2\cdot K\right) + 2\left(0.003m/60\,W/m\cdot K\right) + \left(0.050m/0.046\,W/m\cdot K\right)}$$

$$q'' = \frac{21^\circ C}{(0.4 + 0.0001 + 1.087)m^2\cdot K/W} = 14.1\ W/m^2 \qquad <$$

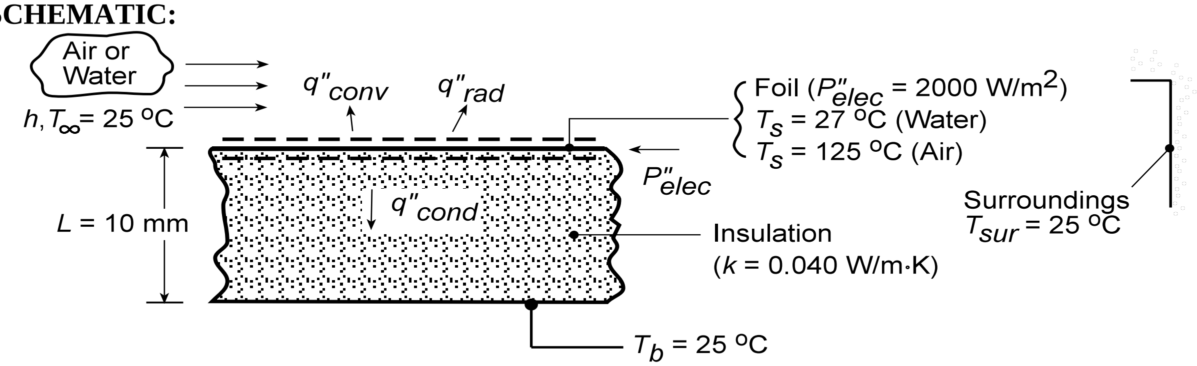
COMMENTS: Although the contribution of the panels to the total thermal resistance is negligible, that due to convection is not inconsequential and is comparable to the thermal resistance of the insulation.

PROBLEM 3.6

KNOWN: Design and operating conditions of a heat flux gage.

FIND: (a) Convection coefficient for water flow (T_s = 27°C) and error associated with neglecting conduction in the insulation, (b) Convection coefficient for air flow (T_s = 125°C) and error associated with neglecting conduction and radiation, (c) Effect of convection coefficient on error associated with neglecting conduction for T_s = 27°C.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional conduction, (3) Constant k.

ANALYSIS: (a) The electric power dissipation is balanced by convection to the water and conduction through the insulation. An energy balance applied to a control surface about the foil therefore yields

$$P''_{elec} = q''_{conv} + q''_{cond} = h\left(T_s - T_\infty\right) + k\left(T_s - T_b\right)/L$$

Hence,

$$h = \frac{P''_{elec} - k\left(T_s - T_b\right)/L}{T_s - T_\infty} = \frac{2000\,W/m^2 - 0.04\,W/m\cdot K\left(2\,K\right)/0.01\,m}{2\,K}$$

$$h = \frac{\left(2000-8\right)W/m^2}{2\,K} = 996\,W/m^2\cdot K \qquad <$$

If conduction is neglected, a value of h = 1000 W/m^2·K is obtained, with an attendant error of (1000 - 996)/996 = 0.40%

(b) In air, energy may also be transferred from the foil surface by radiation, and the energy balance yields

$$P''_{elec} = q''_{conv} + q''_{rad} + q''_{cond} = h\left(T_s - T_\infty\right) + \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right) + k\left(T_s - T_b\right)/L$$

Hence,

$$h = \frac{P''_{elec} - \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right) - k\left(T_s - T_\infty\right)/L}{T_s - T_\infty}$$

$$= \frac{2000\,W/m^2 - 0.15\times5.67\times10^{-8}\,W/m^2\cdot K^4\left(398^4 - 298^4\right)K^4 - 0.04\,W/m\cdot K\,(100\,K)/0.01\,m}{100\,K}$$

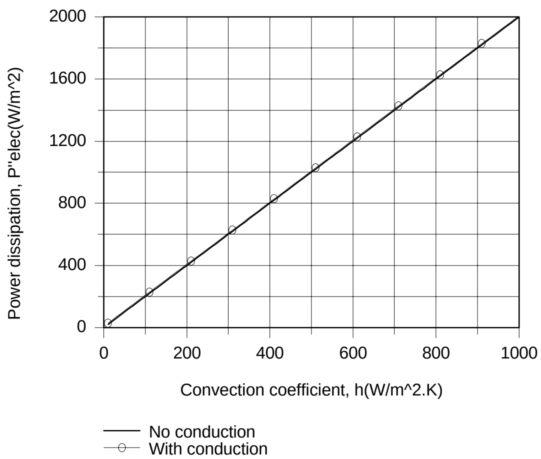
$$= \frac{\left(2000-146-400\right)W/m^2}{100\,K} = 14.5\,W/m^2\cdot K \qquad <$$

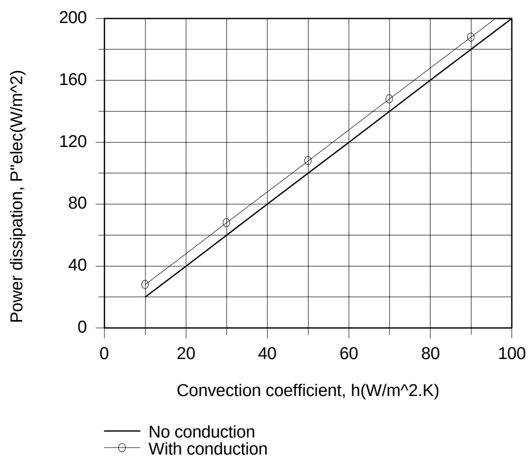
Continued...

PROBLEM 3.6 (Cont.)

If conduction, radiation, or conduction and radiation are neglected, the corresponding values of h and the percentage errors are 18.5 $W/m^2 \cdot K$ (27.6%), 16 $W/m^2 \cdot K$ (10.3%), and 20 $W/m^2 \cdot K$ (37.9%).

(c) For a fixed value of T_s = 27°C, the conduction loss remains at q''_{cond} = 8 W/m^2, which is also the fixed difference between P''_{elec} and q''_{conv}. Although this difference is not clearly shown in the plot for $10 \le h \le 1000$ $W/m^2 \cdot K$, it is revealed in the subplot for $10 \le 100$ $W/m^2 \cdot K$.





Errors associated with neglecting conduction decrease with increasing h from values which are significant for small h ($h < 100$ $W/m^2 \cdot K$) to values which are negligible for large h.

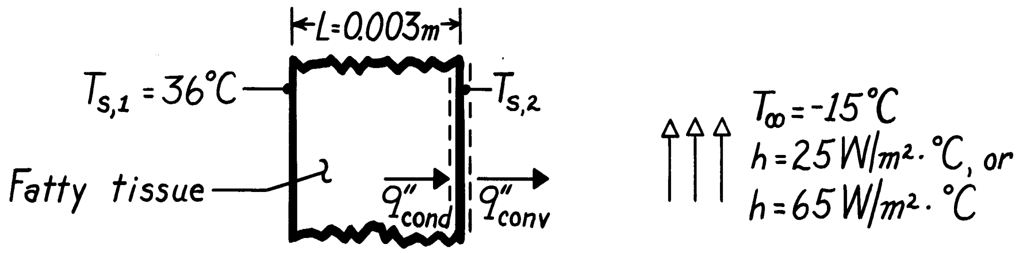
COMMENTS: In liquids (large h), it is an excellent approximation to neglect conduction and assume that all of the dissipated power is transferred to the fluid.

PROBLEM 3.7

KNOWN: A layer of fatty tissue with fixed inside temperature can experience different outside convection conditions.

FIND: (a) Ratio of heat loss for different convection conditions, (b) Outer surface temperature for different convection conditions, and (c) Temperature of still air which achieves same cooling as moving air (*wind chill* effect).

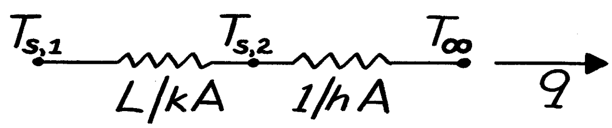
SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction through a plane wall, (2) Steady-state conditions, (3) Homogeneous medium with constant properties, (4) No internal heat generation (metabolic effects are negligible), (5) Negligible radiation effects.

PROPERTIES: *Table A-3*, Tissue, fat layer: k = 0.2 W/m·K.

ANALYSIS: The thermal circuit for this situation is



Hence, the heat rate is

$$q = \frac{T_{s,1} - T_\infty}{R_{tot}} = \frac{T_{s,1} - T_\infty}{L/kA + 1/hA}.$$

Therefore,

$$\frac{q''_{calm}}{q''_{windy}} = \frac{\left[\frac{L}{k} + \frac{1}{h}\right]_{windy}}{\left[\frac{L}{k} + \frac{1}{h}\right]_{calm}}.$$

Applying a surface energy balance to the outer surface, it also follows that

$$q''_{cond} = q''_{conv}.$$

Continued …..

Hence,

$$\frac{k}{L}\left(T_{s,1}-T_{s,2}\right)=h\left(T_{s,2}-T_{\infty}\right)$$

$$T_{s,2}=\frac{T_{\infty}+\dfrac{k}{hL}T_{s,1}}{1+\dfrac{k}{hL}}.$$

To determine the wind chill effect, we must determine the heat loss for the windy day and use it to evaluate the hypothetical ambient air temperature, T'_{∞}, which would provide the same heat loss on a calm day, Hence,

$$q''=\frac{T_{s,1}-T_{\infty}}{\left[\dfrac{L}{k}+\dfrac{1}{h}\right]_{windy}}=\frac{T_{s,1}-T'_{\infty}}{\left[\dfrac{L}{k}+\dfrac{1}{h}\right]_{calm}}$$

From these relations, we can now find the results sought:

(a)
$$\frac{q''_{calm}}{q''_{windy}}=\frac{\dfrac{0.003\text{ m}}{0.2\text{ W/m}\cdot\text{K}}+\dfrac{1}{65\text{ W/m}^2\cdot\text{K}}}{\dfrac{0.003\text{ m}}{0.2\text{ W/m}\cdot\text{K}}+\dfrac{1}{25\text{ W/m}^2\cdot\text{K}}}=\frac{0.015+0.0154}{0.015+0.04}$$

$$\frac{q''_{calm}}{q''_{windy}}=0.553$$ <

(b)
$$\left.T_{s,2}\right]_{calm}=\frac{-15^\circ\text{C}+\dfrac{0.2\text{ W/m}\cdot\text{K}}{\left(25\text{ W/m}^2\cdot\text{K}\right)\left(0.003\text{ m}\right)}36^\circ\text{C}}{1+\dfrac{0.2\text{ W/m}\cdot\text{K}}{\left(25\text{ W/m}^2\cdot\text{K}\right)\left(0.003\text{ m}\right)}}=22.1^\circ\text{C}$$ <

$$\left.T_{s,2}\right]_{windy}=\frac{-15^\circ\text{C}+\dfrac{0.2\text{ W/m}\cdot\text{K}}{\left(65\text{ W/m}^2\cdot\text{K}\right)\left(0.003\text{m}\right)}36^\circ\text{C}}{1+\dfrac{0.2\text{ W/m}\cdot\text{K}}{\left(65\text{ W/m}^2\cdot\text{K}\right)\left(0.003\text{m}\right)}}=10.8^\circ\text{C}$$ <

(c)
$$T'_{\infty}=36^\circ\text{C}-\left(36+15\right)^\circ\text{C}\frac{\left(0.003/0.2+1/25\right)}{\left(0.003/0.2+1/65\right)}=-56.3^\circ\text{C}$$ <

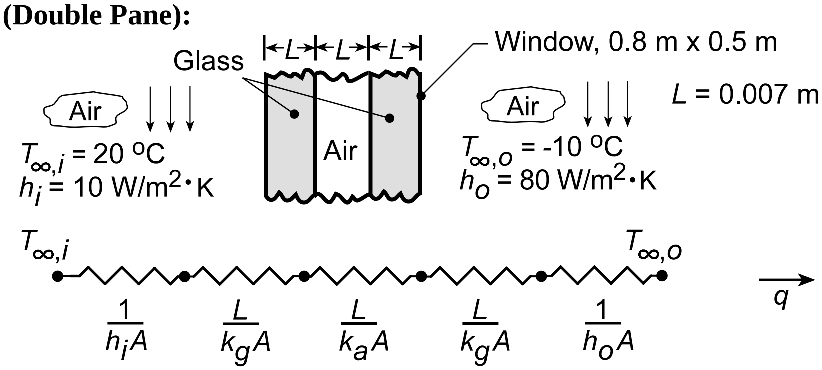
COMMENTS: The wind chill effect is equivalent to a decrease of $T_{s,2}$ by 11.3°C and increase in the heat loss by a factor of $(0.553)^{-1}$ = 1.81.

PROBLEM 3.8

KNOWN: Dimensions of a thermopane window. Room and ambient air conditions.

FIND: (a) Heat loss through window, (b) Effect of variation in outside convection coefficient for double and triple pane construction.

SCHEMATIC (Double Pane):



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat transfer, (3) Constant properties, (4) Negligible radiation effects, (5) Air between glass is stagnant.

PROPERTIES: *Table A-3*, Glass (300 K): k_g = 1.4 W/m·K; *Table A-4*, Air (T = 278 K): k_a = 0.0245 W/m·K.

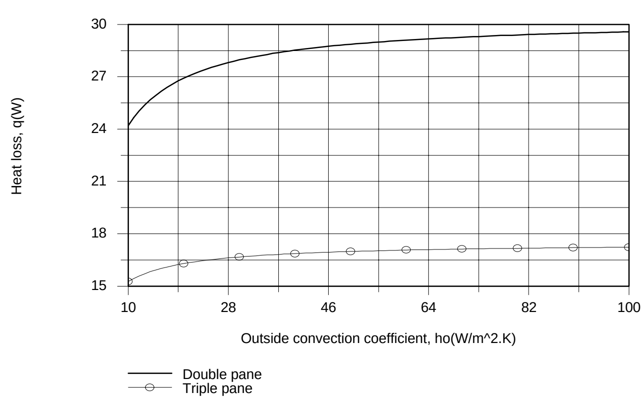
ANALYSIS: (a) From the thermal circuit, the heat loss is

$$q=\frac{T_{\infty,i}-T_{\infty,o}}{\frac{1}{A}\left(\frac{1}{h_i}+\frac{L}{k_g}+\frac{L}{k_a}+\frac{L}{k_g}+\frac{1}{h_o}\right)}$$

$$q=\frac{20^\circ C-\left(-10^\circ C\right)}{\left(\frac{1}{0.4\,m^2}\right)\left(\frac{1}{10\,W/m^2\cdot K}+\frac{0.007\,m}{1.4\,W/m\cdot K}+\frac{0.007\,m}{0.0245\,W/m\cdot K}+\frac{0.007\,m}{1.4\,W/m\cdot K}+\frac{1}{80\,W/m^2\cdot K}\right)}$$

$$q=\frac{30^\circ C}{(0.25+0.0125+0.715+0.0125+0.03125)\,K/W}=\frac{30^\circ C}{1.021\,K/W}=29.4\text{ W}$$ <

(b) For the triple pane window, the additional pane and airspace increase the total resistance from 1.021 K/W to 1.749 K/W, thereby reducing the heat loss from 29.4 to 17.2 W. The effect of h_o on the heat loss is plotted as follows.



Continued...

PROBLEM 3.8 (Cont.)

Changes in h_o influence the heat loss at small values of h_o, for which the outside convection resistance is not negligible relative to the total resistance. However, the resistance becomes negligible with increasing h_o, particularly for the triple pane window, and changes in h_o have little effect on the heat loss.

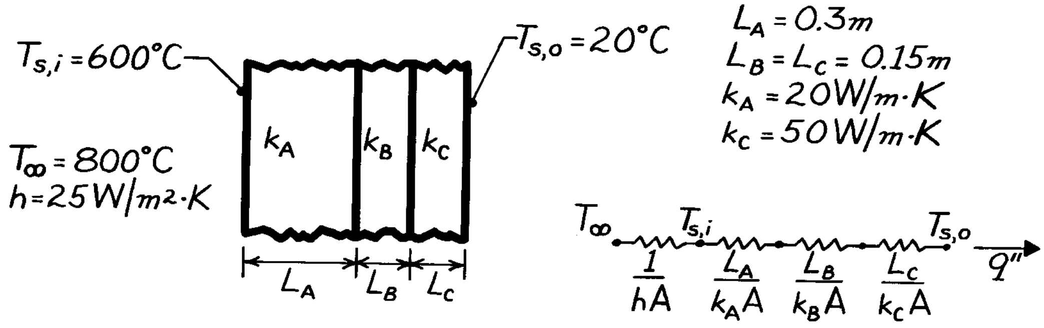
COMMENTS: The largest contribution to the thermal resistance is due to conduction across the enclosed air. Note that this air could be in motion due to free convection currents. If the corresponding convection coefficient exceeded 3.5 $W/m^2 \cdot K$, the thermal resistance would be less than that predicted by assuming conduction across stagnant air.

PROBLEM 3.9

KNOWN: Thicknesses of three materials which form a composite wall and thermal conductivities of two of the materials. Inner and outer surface temperatures of the composite; also, temperature and convection coefficient associated with adjoining gas.

FIND: Value of unknown thermal conductivity, k_B.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible contact resistance, (5) Negligible radiation effects.

ANALYSIS: Referring to the thermal circuit, the heat flux may be expressed as

$$q'' = \frac{T_{s,i} - T_{s,o}}{\frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{L_C}{k_C}} = \frac{(600-20)^\circ \text{C}}{\frac{0.3 \text{ m}}{20 \text{ W/m}\cdot\text{K}} + \frac{0.15 \text{ m}}{k_B} + \frac{0.15 \text{ m}}{50 \text{ W/m}\cdot\text{K}}}$$

$$q'' = \frac{580}{0.018 + 0.15/k_B} \text{W/m}^2. \tag{1}$$

The heat flux may be obtained from

$$q'' = h\left(T_\infty - T_{s,i}\right) = 25 \text{ W/m}^2\cdot\text{K}\,(800\text{-}600)^\circ \text{C} \tag{2}$$

$$q'' = 5000 \text{ W/m}^2.$$

Substituting for the heat flux from Eq. (2) into Eq. (1), find

$$\frac{0.15}{k_B} = \frac{580}{q''} - 0.018 = \frac{580}{5000} - 0.018 = 0.098$$

$$k_B = 1.53 \text{ W/m}\cdot\text{K}. \qquad <$$

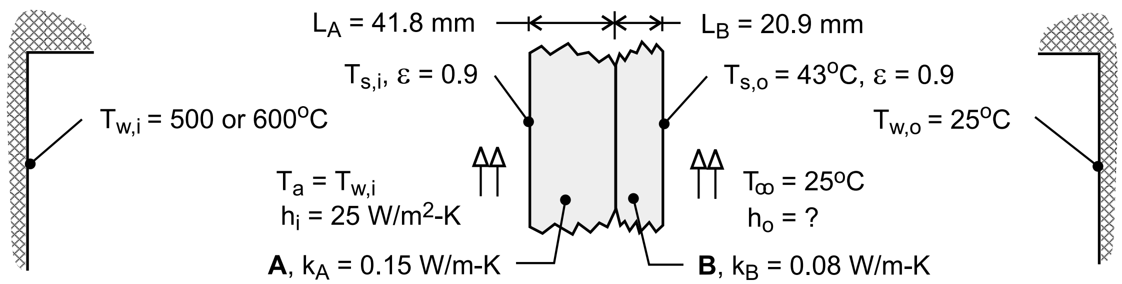
COMMENTS: Radiation effects are likely to have a significant influence on the net heat flux at the inner surface of the oven.

PROBLEM 3.10

KNOWN: Properties and dimensions of a composite oven window providing an outer surface safe-to-touch temperature $T_{s,o}$ = 43°C with outer convection coefficient h_o = 30 W/m^2·K and ε = 0.9 when the oven wall air temperatures are $T_w = T_a$ = 400°C. See Example 3.1.

FIND: Values of the outer convection coefficient h_o required to maintain the safe-to-touch condition when the oven wall-air temperature is raised to 500°C or 600°C.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in window with no contact resistance and constant properties, (3) Negligible absorption in window material, (4) Radiation exchange processes are between small surface and large isothermal surroundings.

ANALYSIS: From the analysis in the Ex. 3.1 Comment 2, the surface energy balances at the inner and outer surfaces are used to determine the required value of h_o when $T_{s,o}$ = 43°C and $T_{w,i} = T_a$ = 500 or 600°C.

$$\varepsilon\sigma\left(T_{w,i}^4 - T_{s,i}^4\right) + h_i\left(T_a - T_{s,i}\right) = \frac{T_{s,i} - T_{s,o}}{\left(L_A / k_A\right) + \left(L_B / k_B\right)}$$

$$\frac{T_{s,i} - T_{s,o}}{\left(L_A / k_A\right) + \left(L_B / k_B\right)} = \varepsilon\sigma\left(T_{s,o}^4 - T_{w,o}^4\right) + h_o\left(T_{s,o} - T_\infty\right)$$

Using these relations in IHT, the following results were calculated:

$T_{w,i}$, T_s(°C)	$T_{s,i}$(°C)	h_o(W/m^2·K)
400	392	30
500	493	40.4
600	594	50.7

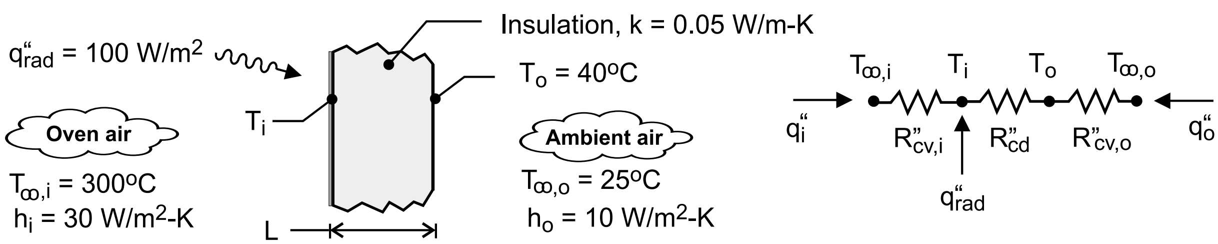
COMMENTS: Note that the window inner surface temperature is closer to the oven air-wall temperature as the outer convection coefficient increases. Why is this so?

PROBLEM 3.11

KNOWN: Drying oven wall having material with known thermal conductivity sandwiched between thin metal sheets. Radiation and convection conditions prescribed on inner surface; convection conditions on outer surface.

FIND: (a) Thermal circuit representing wall and processes and (b) Insulation thickness required to maintain outer wall surface at $T_o = 40°C$.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in wall, (3) Thermal resistance of metal sheets negligible.

ANALYSIS: (a) The thermal circuit is shown above. Note labels for the temperatures, thermal resistances and the relevant heat fluxes.

(b) Perform energy balances on the i- and o- nodes finding

$$\frac{T_{\infty,i} - T_i}{R''_{cv,i}} + \frac{T_o - T_i}{R''_{cd}} + q''_{rad} = 0 \qquad (1)$$

$$\frac{T_i - T_o}{R''_{cd}} + \frac{T_{\infty,o} - T_o}{R''_{cv,o}} = 0 \qquad (2)$$

where the thermal resistances are

$$R''_{cv,i} = 1/h_i = 0.0333\ m^2 \cdot K/W \qquad (3)$$

$$R''_{cd} = L/k = L/0.05\ m^2 \cdot K/W \qquad (4)$$

$$R''_{cv,o} = 1/h_o = 0.0100\ m^2 \cdot K/W \qquad (5)$$

Substituting numerical values, and solving Eqs. (1) and (2) simultaneously, find

$$L = 86\ mm$$ <

COMMENTS: (1) The temperature at the inner surface can be found from an energy balance on the i-node using the value found for L.

$$\frac{T_{\infty,i} - T_i}{R''_{cv,o}} + \frac{T_{\infty,o} - T_i}{R''_{cd} + R''_{cv,i}} + q''_{rad} = 0 \qquad T_i = 298.3°C$$

It follows that T_i is close to $T_{\infty,i}$ since the wall represents the dominant resistance of the system.

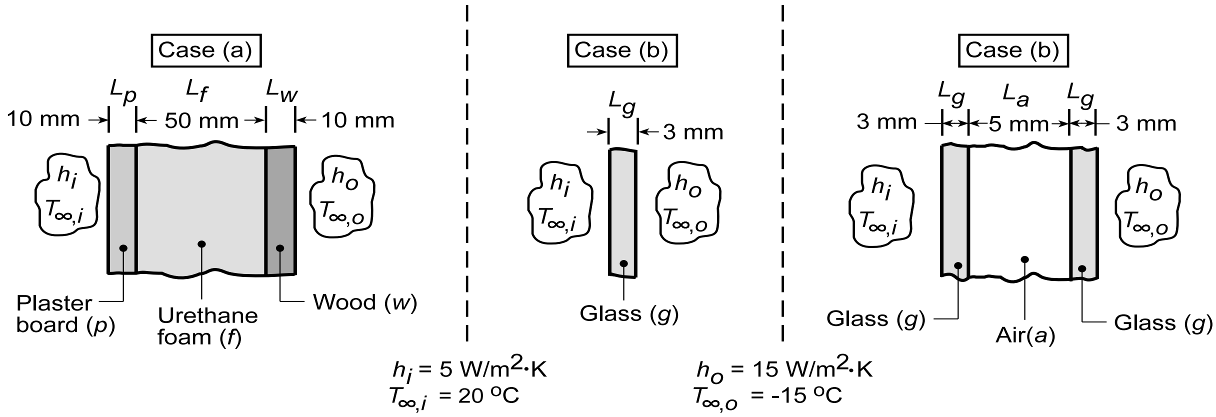
(2) Verify that $q''_i = 50\ W/m^2$ and $q''_o = 150\ W/m^2$. Is the overall energy balance on the system satisfied?

PROBLEM 3.12

KNOWN: Configurations of exterior wall. Inner and outer surface conditions.

FIND: Heating load for each of the three cases.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible radiation effects.

PROPERTIES: (T = 300 K): *Table A.3*: plaster board, $k_p = 0.17$ W/m·K; urethane, $k_f = 0.026$ W/m·K; wood, $k_w = 0.12$ W/m·K; glass, $k_g = 1.4$ W/m·K. *Table A.4*: air, $k_a = 0.0263$ W/m·K.

ANALYSIS: (a) The heat loss may be obtained by dividing the overall temperature difference by the total thermal resistance. For the composite wall of unit surface area, A = 1 m^2,

$$q = \frac{T_{\infty,i} - T_{\infty,o}}{\left[\left(1/h_i\right) + \left(L_p/k_p\right) + \left(L_f/k_f\right) + \left(L_w/k_w\right) + \left(1/h_o\right)\right]/A}$$

$$q = \frac{20^\circ\mathrm{C} - \left(-15^\circ\mathrm{C}\right)}{\left[\left(0.2 + 0.059 + 1.92 + 0.083 + 0.067\right)\mathrm{m}^2 \cdot \mathrm{K/W}\right]/1\,\mathrm{m}^2}$$

$$q = \frac{35^\circ\mathrm{C}}{2.33\,\mathrm{K/W}} = 15.0\,\mathrm{W}$$ <

(b) For the single pane of glass,

$$q = \frac{T_{\infty,i} - T_{\infty,o}}{\left[\left(1/h_i\right) + \left(L_g/k_g\right) + \left(1/h_o\right)\right]/A}$$

$$q = \frac{35^\circ\mathrm{C}}{\left[\left(0.2 + 0.002 + 0.067\right)\mathrm{m}^2 \cdot \mathrm{K/W}\right]/1\,\mathrm{m}^2} = \frac{35^\circ\mathrm{C}}{0.269\,\mathrm{K/W}} = 130.3\,\mathrm{W}$$ <

(c) For the double pane window,

$$q = \frac{T_{\infty,i} - T_{\infty,o}}{\left[\left(1/h_i\right) + 2\left(L_g/k_g\right) + \left(L_a/k_a\right) + \left(1/h_o\right)\right]/A}$$

$$q = \frac{35^\circ\mathrm{C}}{\left[\left(0.2 + 0.004 + 0.190 + 0.067\right)\mathrm{m}^2 \cdot \mathrm{K/W}\right]/1\,\mathrm{m}^2} = \frac{35^\circ\mathrm{C}}{0.461\,\mathrm{K/W}} = 75.9\,\mathrm{W}$$ <

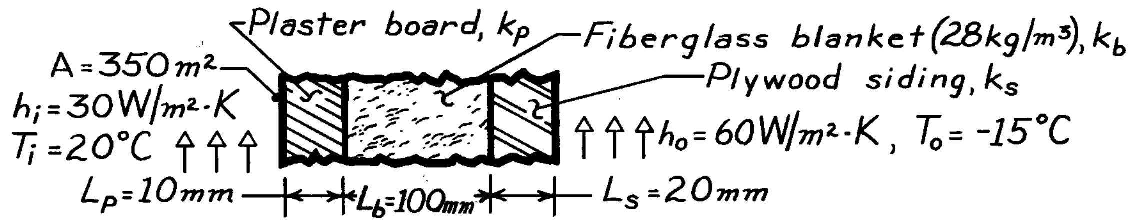
COMMENTS: The composite wall is clearly superior from the standpoint of reducing heat loss, and the dominant contribution to its total thermal resistance (82%) is associated with the foam insulation. Even with double pane construction, heat loss through the window is significantly larger than that for the composite wall.

PROBLEM 3.13

KNOWN: Composite wall of a house with prescribed convection processes at inner and outer surfaces.

FIND: (a) Expression for thermal resistance of house wall, R_{tot}; (b) Total heat loss, q(W); (c) Effect on heat loss due to increase in outside heat transfer convection coefficient, h_o; and (d) Controlling resistance for heat loss from house.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) Steady-state conditions, (3) Negligible contact resistance.

PROPERTIES: *Table A-3,* $\left(\bar{T} = (T_i + T_o)/2 = (20-15)^\circ C/2 = 2.5^\circ C \approx 300K\right)$: Fiberglass blanket, 28 kg/m^3, k_b = 0.038 W/m·K; Plywood siding, k_s = 0.12 W/m·K; Plasterboard, k_p = 0.17 W/m·K.

ANALYSIS: (a) The expression for the total thermal resistance of the house wall follows from Eq. 3.18.

$$R_{tot} = \frac{1}{h_i A} + \frac{L_p}{k_p A} + \frac{L_b}{k_b A} + \frac{L_s}{k_s A} + \frac{1}{h_o A}. \quad <$$

(b) The total heat loss through the house wall is

$$q = \Delta T/R_{tot} = (T_i - T_o)/R_{tot}.$$

Substituting numerical values, find

$$R_{tot} = \frac{1}{30\,W/m^2\cdot K \times 350\,m^2} + \frac{0.01\,m}{0.17\,W/m\cdot K \times 350\,m^2} + \frac{0.10\,m}{0.038\,W/m\cdot K \times 350\,m^2} + \frac{0.02\,m}{0.12\,W/m\cdot K \times 350\,m^2} + \frac{1}{60\,W/m^2\cdot K \times 350\,m^2}$$

$$R_{tot} = [9.52 + 16.8 + 752 + 47.6 + 4.76] \times 10^{-5}\ ^\circ C/W = 831 \times 10^{-5}\ ^\circ C/W$$

The heat loss is then,

$$q = [20-(-15)]^\circ C/831\times 10^{-5}\ ^\circ C/W = 4.21\ kW. \quad <$$

(c) If h_o changes from 60 to 300 W/m^2·K, $R_o = 1/h_oA$ changes from 4.76×10^{-5} °C/W to 0.95×10^{-5} °C/W. This reduces R_{tot} to 826×10^{-5} °C/W, which is a 0.5% decrease and hence a 0.5% increase in q.

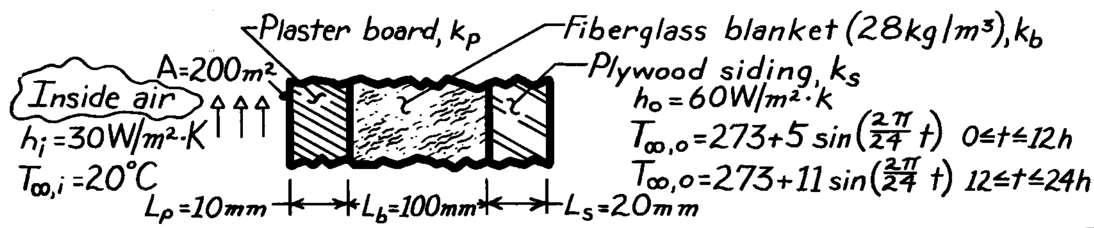
(d) From the expression for R_{tot} in part (b), note that the insulation resistance, L_b/k_bA, is 752/830 ≈ 90% of the total resistance. Hence, this material layer controls the resistance of the wall. From part (c) note that a 5-fold decrease in the outer convection resistance due to an increase in the wind velocity has a negligible effect on the heat loss.

PROBLEM 3.14

KNOWN: Composite wall of a house with prescribed convection processes at inner and outer surfaces.

FIND: Daily heat loss for prescribed diurnal variation in ambient air temperature.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conduction (negligible change in wall thermal energy storage over 24h period), (2) Negligible contact resistance.

PROPERTIES: *Table A-3*, T ≈ 300 K: Fiberglass blanket (28 kg/m^3), k_b = 0.038 W/m·K; Plywood, k_s = 0.12 W/m·K; Plasterboard, k_p = 0.17 W/m·K.

ANALYSIS: The heat loss may be approximated as $Q = \int_0^{24h} \frac{T_{\infty,i} - T_{\infty,o}}{R_{tot}} dt$ where

$$R_{tot} = \frac{1}{A}\left[\frac{1}{h_i} + \frac{L_p}{k_p} + \frac{L_b}{k_b} + \frac{L_s}{k_s} + \frac{1}{h_o}\right]$$

$$R_{tot} = \frac{1}{200\text{m}^2}\left[\frac{1}{30\ \text{W/m}^2\cdot\text{K}} + \frac{0.01\text{m}}{0.17\ \text{W/m}\cdot\text{K}} + \frac{0.1\text{m}}{0.038\ \text{W/m}\cdot\text{K}} + \frac{0.02\text{m}}{0.12\ \text{W/m}\cdot\text{K}} + \frac{1}{60\ \text{W/m}^2\cdot\text{K}}\right]$$

$$R_{tot} = 0.01454\ \text{K/W}.$$

Hence the heat rate is

$$Q = \frac{1}{R_{tot}}\left\{\int_0^{12h}\left[293 - \left[273 + 5\sin\frac{2\pi}{24}t\right]\right]dt + \int_{12}^{24h}\left[293 - \left[273 + 11\sin\frac{2\pi}{24}t\right]\right]dt\right\}$$

$$Q = 68.8\frac{\text{W}}{\text{K}}\left\{\left[20t + 5\left[\frac{24}{2\pi}\right]\cos\frac{2\pi t}{24}\right]\Bigg|_0^{12} + \left[20t + 11\left[\frac{24}{2\pi}\right]\cos\frac{2\pi t}{24}\right]\Bigg|_{12}^{24}\right\}\text{K}\cdot\text{h}$$

$$Q = 68.8\left\{\left[240 + \frac{60}{\pi}(-1-1)\right] + \left[480 - 240 + \frac{132}{\pi}(1+1)\right]\right\}\text{W}\cdot\text{h}$$

$$Q = 68.8\ \{480\text{-}38.2\text{+}84.03\}\ \text{W}\cdot\text{h}$$

$$Q = 36.18\ \text{kW}\cdot\text{h} = 1.302\times10^8\text{J}.$$ <

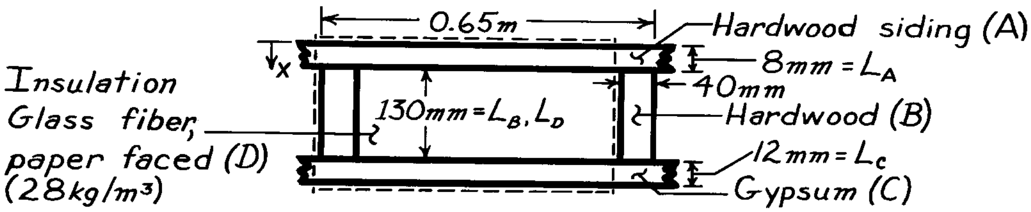
COMMENTS: From knowledge of the fuel cost, the total daily heating bill could be determined. For example, at a cost of 0.10\$/kW·h, the heating bill would be \$3.62/day.

PROBLEM 3.15

KNOWN: Dimensions and materials associated with a composite wall (2.5m × 6.5m, 10 studs each 2.5m high).

FIND: Wall thermal resistance.

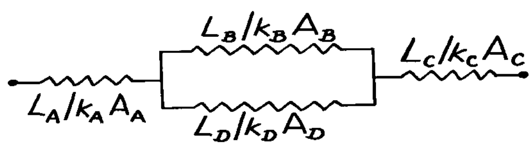
SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Temperature of composite depends only on x (surfaces normal to x are isothermal), (3) Constant properties, (4) Negligible contact resistance.

PROPERTIES: *Table A-3* (T ≈ 300K): Hardwood siding, k_A = 0.094 W/m·K; Hardwood, k_B = 0.16 W/m·K; Gypsum, k_C = 0.17 W/m·K; Insulation (glass fiber paper faced, 28 kg/m^3), k_D = 0.038 W/m·K.

ANALYSIS: Using the isothermal surface assumption, the thermal circuit associated with a single unit (enclosed by dashed lines) of the wall is



$$\left(L_A / k_A A_A\right) = \frac{0.008\text{m}}{0.094\ \text{W/m}\cdot\text{K}\ \left(0.65\text{m}\times 2.5\text{m}\right)} = 0.0524\ \text{K/W}$$

$$\left(L_B / k_B A_B\right) = \frac{0.13\text{m}}{0.16\ \text{W/m}\cdot\text{K}\ \left(0.04\text{m}\times 2.5\text{m}\right)} = 8.125\ \text{K/W}$$

$$\left(L_D / k_D A_D\right) = \frac{0.13\text{m}}{0.038\ \text{W/m}\cdot\text{K}\left(0.61\text{m}\times 2.5\text{m}\right)} = 2.243\ \text{K/W}$$

$$\left(L_C / k_C A_C\right) = \frac{0.012\text{m}}{0.17\ \text{W/m}\cdot\text{K}\left(0.65\text{m}\times 2.5\text{m}\right)} = 0.0434\ \text{K/W.}$$

The equivalent resistance of the core is

$$R_{eq} = \left(1/R_B + 1/R_D\right)^{-1} = \left(1/8.125 + 1/2.243\right)^{-1} = 1.758\ \text{K/W}$$

and the total unit resistance is

$$R_{tot,1} = R_A + R_{eq} + R_C = 1.854\ \text{K/W.}$$

With 10 such units in parallel, the total wall resistance is

$$R_{tot} = \left(10\times 1/R_{tot,1}\right)^{-1} = 0.1854\ \text{K/W.}$$ <

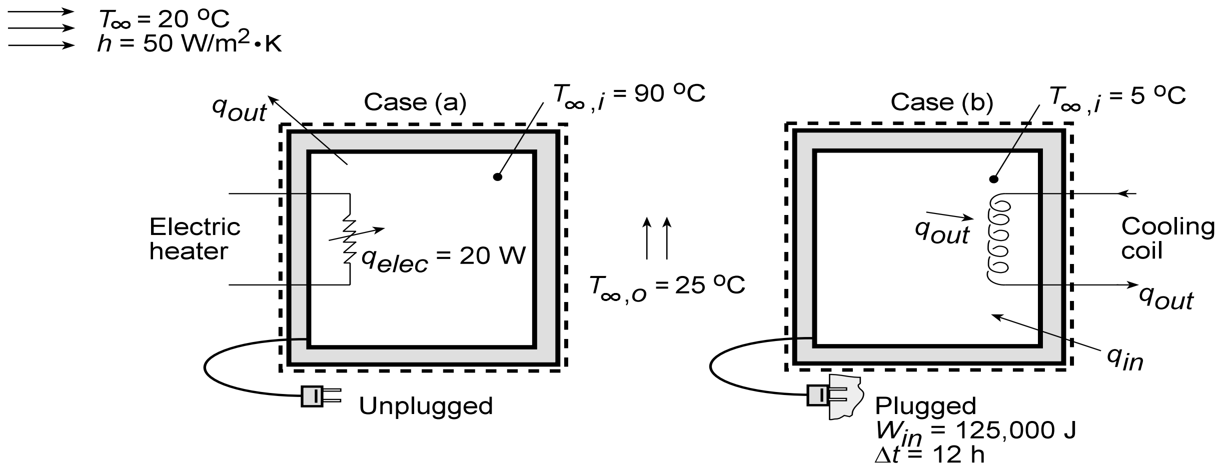
COMMENTS: If surfaces parallel to the heat flow direction are assumed adiabatic, the thermal circuit and the value of R_{tot} will differ.

PROBLEM 3.16

KNOWN: Conditions associated with maintaining heated and cooled conditions within a refrigerator compartment.

FIND: Coefficient of performance (COP).

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state operating conditions, (2) Negligible radiation, (3) Compartment completely sealed from ambient air.

ANALYSIS: The Case (a) experiment is performed to determine the overall thermal resistance to heat transfer between the interior of the refrigerator and the ambient air. Applying an energy balance to a control surface about the refrigerator, it follows from Eq. 1.11a that, at any instant,

$$\dot{E}_g - \dot{E}_{out} = 0$$

Hence,

$$q_{elec} - q_{out} = 0$$

where $q_{out} = \left(T_{\infty,i} - T_{\infty,o}\right)/R_t$. It follows that

$$R_t = \frac{T_{\infty,i} - T_{\infty,o}}{q_{elec}} = \frac{(90-25)^\circ C}{20\ W} = 3.25^\circ C/W$$

For Case (b), heat transfer from the ambient air to the compartment (the heat load) is balanced by heat transfer to the refrigerant ($q_{in} = q_{out}$). Hence, the thermal energy transferred from the refrigerator over the 12 hour period is

$$Q_{out} = q_{out}\Delta t = q_{in}\Delta t = \frac{T_{\infty,i} - T_{\infty,o}}{R_t}\Delta t$$

$$Q_{out} = \frac{(25-5)^\circ C}{3.25^\circ\ C/W}\left(12\ h \times 3600\ s/h\right) = 266,000\ J$$

The coefficient of performance (COP) is therefore

$$COP = \frac{Q_{out}}{W_{in}} = \frac{266,000}{125,000} = 2.13$$ <

COMMENTS: The ideal (Carnot) COP is

$$COP)_{ideal} = \frac{T_c}{T_h - T_c} = \frac{278\ K}{(298-278)\ K} = 13.9$$

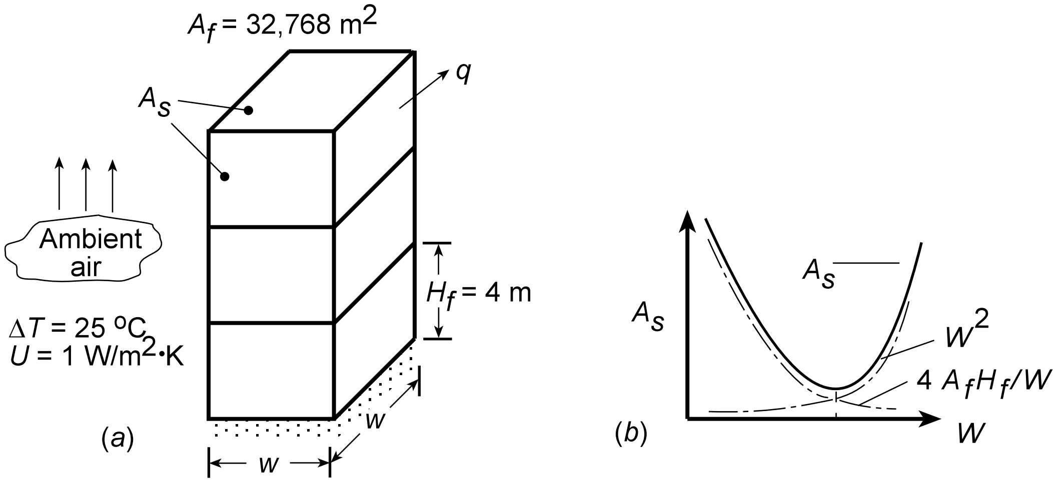
and the system is operating well below its peak possible performance.

PROBLEM 3.17

KNOWN: Total floor space and vertical distance between floors for a square, flat roof building.

FIND: (a) Expression for width of building which minimizes heat loss, (b) Width and number of floors which minimize heat loss for a prescribed floor space and distance between floors. Corresponding heat loss, percent heat loss reduction from 2 floors.

SCHEMATIC:



ASSUMPTIONS: Negligible heat loss to ground.

ANALYSIS: (a) To minimize the heat loss q, the exterior surface area, A_s, must be minimized. From Fig. (a)

$$A_s = W^2 + 4WH = W^2 + 4WN_f H_f$$

where

$$N_f = A_f / W^2$$

Hence,

$$A_s = W^2 + 4WA_f H_f / W^2 = W^2 + 4A_f H_f / W$$

The optimum value of W corresponds to

$$\frac{dA_s}{dW} = 2W - \frac{4A_f H_f}{W^2} = 0$$

or

$$W_{op} = (2A_f H_f)^{1/3} \qquad <$$

The competing effects of W on the areas of the roof and sidewalls, and hence the basis for an optimum, is shown schematically in Fig. (b).

(b) For A_f = 32,768 m² and H_f = 4 m,

$$W_{op} = \left(2 \times 32,768\,\text{m}^2 \times 4\,\text{m}\right)^{1/3} = 64\,\text{m} \qquad <$$

Continued …..

PROBLEM 3.17 (Cont.)

Hence,

$$N_f = \frac{A_f}{W^2} = \frac{32{,}768\,\mathrm{m}^2}{(64\,\mathrm{m})^2} = 8$$ <

and

$$q = UA_s\Delta T = 1\,\mathrm{W/m^2\cdot K}\left[(64\,\mathrm{m})^2 + \frac{4\times 32{,}768\,\mathrm{m}^2\times 4\,\mathrm{m}}{64\,\mathrm{m}}\right]25^\circ\mathrm{C} = 307{,}200\,\mathrm{W}$$ <

For $N_f = 2$,

$W = (A_f/N_f)^{1/2} = (32{,}768\ \mathrm{m}^2/2)^{1/2} = 128\ \mathrm{m}$

$$q = 1\,\mathrm{W/m^2\cdot K}\left[(128\,\mathrm{m})^2 + \frac{4\times 32{,}768\,\mathrm{m}^2\times 4\,\mathrm{m}}{128\,\mathrm{m}}\right]25^\circ\mathrm{C} = 512{,}000\,\mathrm{W}$$

% reduction in q = (512,000 - 307,200)/512,000 = 40% <

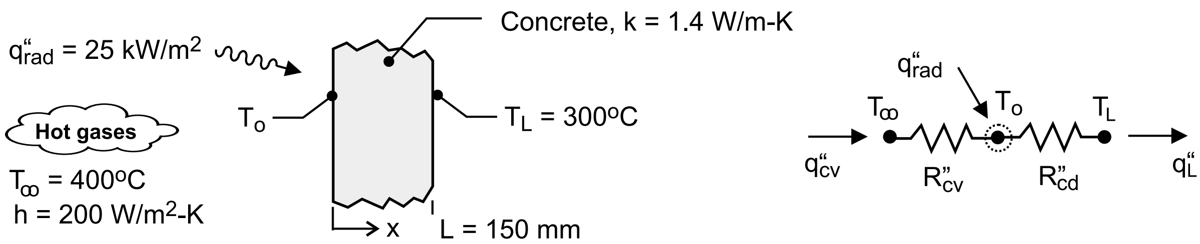
COMMENTS: Even the minimum heat loss is excessive and could be reduced by reducing U.

PROBLEM 3.18

KNOWN: Concrete wall of 150 mm thickness experiences a flash-over fire with prescribed radiant flux and hot-gas convection on the fire-side of the wall. Exterior surface condition is 300°C, typical ignition temperature for most household and office materials.

FIND: (a) Thermal circuit representing wall and processes and (b) Temperature at the fire-side of the wall; comment on whether wall is likely to experience structural collapse for these conditions.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in wall, (3) Constant properties.

PROPERTIES: *Table A-3*, Concrete (stone mix, 300 K): k = 1.4 W/m·K.

ANALYSIS: (a) The thermal cirucit is shown above. Note labels for the temperatures, thermal resistances and the relevant heat fluxes.

(b) To determine the fire-side wall surface temperatures, perform an energy balance on the o-node.

$$\frac{T_\infty - T_o}{R''_{cv}} + q''_{rad} = \frac{T_L - T_o}{R''_{cd}}$$

where the thermal resistances are

$$R''_{cv} = 1/h_i = 1/200\ \text{W/m}^2\cdot\text{K} = 0.00500\ \text{m}^2\cdot\text{K/W}$$

$$R''_{cd} = L/k = 0.150\ \text{m}/1.4\,\text{W/m}\cdot\text{K} = 0.107\ \text{m}^2\cdot\text{K/W}$$

Substituting numerical values,

$$\frac{(400 - T_o)\text{K}}{0.005\ \text{m}^2\cdot\text{K/W}} + 25{,}000\ \text{W/m}^2\ \frac{(300 - T_o)\text{K}}{0.107\ \text{m}^2\cdot\text{K/W}} = 0$$

$$T_o = 515°\text{C}$$ <

COMMENTS: (1) The fire-side wall surface temperature is within the 350 to 600°C range for which explosive spalling could occur. It is likely the wall will experience structural collapse for these conditions.

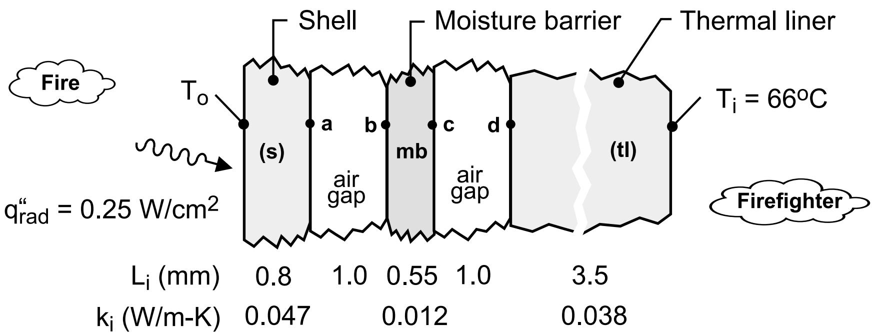
(2) This steady-state condition is an extreme condition, as the wall may fail before near steady-state conditions can be met.

PROBLEM 3.19

KNOWN: Representative dimensions and thermal conductivities for the layers of fire-fighter's protective clothing, a turnout coat.

FIND: (a) Thermal circuit representing the turnout coat; tabulate thermal resistances of the layers and processes; and (b) For a prescribed radiant heat flux on the fire-side surface and temperature of T_i =.60°C at the inner surface, calculate the fire-side surface temperature, T_o.

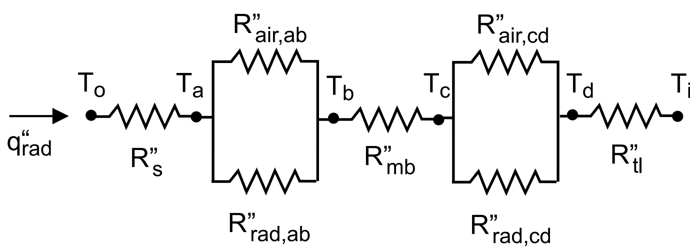
SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction through the layers, (3) Heat is transferred by conduction and radiation exchange across the stagnant air gaps, (3) Constant properties.

PROPERTIES: *Table A-4*, Air (470 K, 1 atm): $k_{ab} = k_{cd} = 0.0387$ W/m·K.

ANALYSIS: (a) The thermal circuit is shown with labels for the temperatures and thermal resistances.



The conduction thermal resistances have the form $R''_{cd} = L/k$ while the radiation thermal resistances across the air gaps have the form

$$R''_{rad} = \frac{1}{h_{rad}} = \frac{1}{4\sigma T_{avg}^3}$$

The linearized radiation coefficient follows from Eqs. 1.8 and 1.9 with ε = 1 where T_{avg} represents the average temperature of the surfaces comprising the gap

$$h_{rad} = \sigma\left(T_1 + T_2\right)\left(T_1^2 + T_2^2\right) \approx 4\sigma T_{avg}^3$$

For the radiation thermal resistances tabulated below, we used $T_{avg} = 470$ K.

Continued …..

PROBLEM 3.19 (Cont.)

	Shell (s)	Air gap (a-b)	Barrier (mb)	Air gap (c-d)	Liner (tl)	*Total* (tot)
$R''_{cd}\left(m^2 \cdot K/W\right)$	0.01702	0.0259	0.04583	0.0259	0.00921	--
$R''_{rad}\left(m^2 \cdot K/W\right)$	--	0.04264	--	0.04264	--	--
$R''_{gap}\left(m^2 \cdot K/W\right)$	--	0.01611	--	0.01611	--	--
R''_{total}	--	--	--	--	--	0.1043

From the thermal circuit, the resistance across the gap for the conduction and radiation processes is

$$\frac{1}{R''_{gap}} = \frac{1}{R''_{cd}} + \frac{1}{R''_{rad}}$$

and the total thermal resistance of the turn coat is

$$R''_{tot} = R''_{cd,s} + R''_{gap,a-b} + R''_{cd,mb} + R''_{gap,c-d} + R''_{cd,tl}$$

(b) If the heat flux through the coat is 0.25 W/cm^2, the fire-side surface temperature T_o can be calculated from the rate equation written in terms of the overall thermal resistance.

$$q'' = \left(T_o - T_i\right)/R''_{tot}$$

$$T_o = 66°C + 0.25\ W/cm^2 \times \left(10^2 cm/m\right)^2 \times 0.1043\ m^2 \cdot K/W$$

$$T_o = 327°C$$

COMMENTS: (1) From the tabulated results, note that the thermal resistance of the moisture barrier (mb) is nearly 3 times larger than that for the shell or air gap layers, and 4.5 times larger than the thermal liner layer.

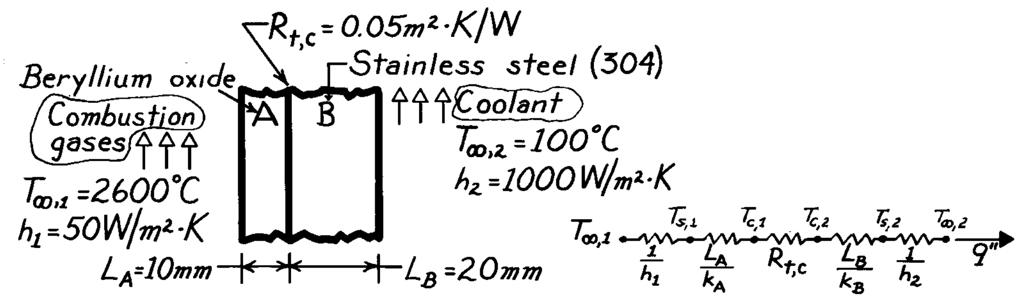
(2) The air gap conduction and radiation resistances were calculated based upon the average temperature of 470 K. This value was determined by setting $T_{avg} = (T_o + T_i)/2$ and solving the equation set using *IHT* with $k_{air} = k_{air}(T_{avg})$.

PROBLEM 3.20

KNOWN: Materials and dimensions of a composite wall separating a combustion gas from a liquid coolant.

FIND: (a) Heat loss per unit area, and (b) Temperature distribution.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional heat transfer, (2) Steady-state conditions, (3) Constant properties, (4) Negligible radiation effects.

PROPERTIES: *Table A-1*, St. St. (304) $\left(\overline{T} \approx 1000K\right)$: k = 25.4 W/m·K; *Table A-2*, Beryllium Oxide (T ≈ 1500K): k = 21.5 W/m·K.

ANALYSIS: (a) The desired heat flux may be expressed as

$$q'' = \frac{T_{\infty,1} - T_{\infty,2}}{\frac{1}{h_1} + \frac{L_A}{k_A} + R_{t,c} + \frac{L_B}{k_B} + \frac{1}{h_2}} = \frac{(2600-100)^\circ C}{\left[\frac{1}{50} + \frac{0.01}{21.5} + 0.05 + \frac{0.02}{25.4} + \frac{1}{1000}\right]\frac{m^2.K}{W}}$$

$$q'' = 34,600 \text{ W/m}^2.$$ <

(b) The composite surface temperatures may be obtained by applying appropriate rate equations. From the fact that $q'' = h_1\left(T_{\infty,1} - T_{s,1}\right)$, it follows that

$$T_{s,1} = T_{\infty,1} - \frac{q''}{h_1} = 2600^\circ C - \frac{34,600 \text{ W/m}^2}{50 \text{ W/m}^2 \cdot K} 1908^\circ C.$$

With $q'' = \left(k_A / L_A\right)\left(T_{s,1} - T_{c,1}\right)$, it also follows that

$$T_{c,1} = T_{s,1} - \frac{L_A q''}{k_A} = 1908^\circ C - \frac{0.01m \times 34,600 \text{ W/m}^2}{21.5 \text{ W/m} \cdot K} = 1892^\circ C.$$

Similarly, with $q'' = \left(T_{c,1} - T_{c,2}\right)/R_{t,c}$

$$T_{c,2} = T_{c,1} - R_{t,c} q'' = 1892^\circ C - 0.05 \frac{m^2 \cdot K}{W} \times 34,600 \frac{W}{m^2} = 162^\circ C$$

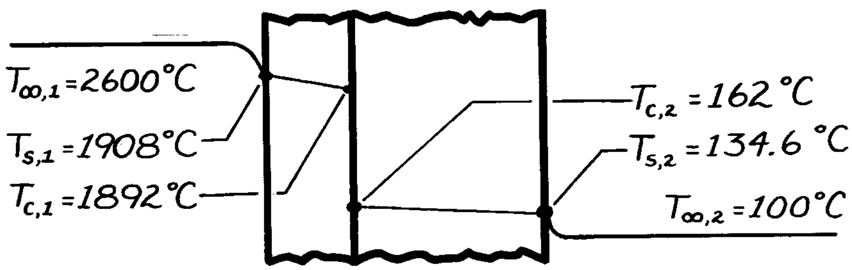
Continued …..

PROBLEM 3.20 (Cont.)

and with $q''=\left(k_B / L_B\right)\left(T_{c,2}-T_{s,2}\right)$,

$$T_{s,2}=T_{c,2}-\frac{L_B q''}{k_B}=162^{\circ}\mathrm{C}-\frac{0.02\,\mathrm{m}\times 34{,}600\ \mathrm{W/m^2}}{25.4\ \mathrm{W/m\cdot K}}=134.6^{\circ}\mathrm{C}.$$

The temperature distribution is therefore of the following form:



<

COMMENTS: (1) The calculations may be checked by recomputing q'' from

$$q''=h_2\left(T_{s,2}-T_{\infty,2}\right)=1000\mathrm{W/m^2\cdot K}\left(134.6\text{-}100\right)^{\circ}\mathrm{C}=34{,}600\mathrm{W/m^2}$$

(2) The initial *estimates* of the mean material temperatures are in error, particularly for the stainless steel. For improved accuracy the calculations should be repeated using k values corresponding to T ≈ 1900°C for the oxide and T ≈ 115°C for the steel.

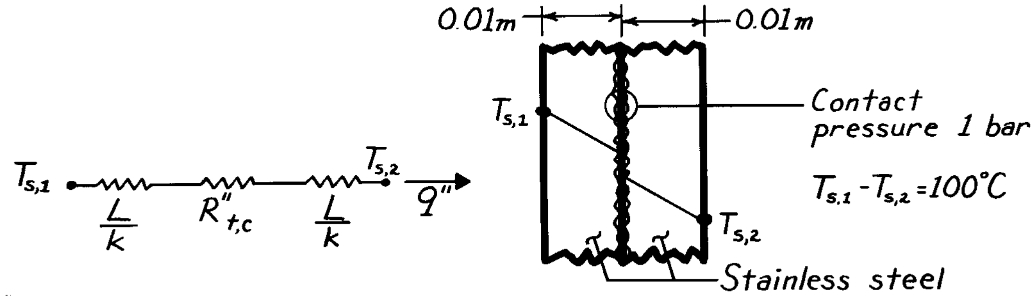
(3) The major contributions to the total resistance are made by the combustion gas boundary layer and the contact, where the temperature drops are largest.

PROBLEM 3.21

KNOWN: Thickness, overall temperature difference, and pressure for two stainless steel plates.

FIND: (a) Heat flux and (b) Contact plane temperature drop.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional heat transfer, (2) Steady-state conditions, (3) Constant properties.

PROPERTIES: *Table A-1*, Stainless Steel (T ≈ 400K): k = 16.6 W/m·K.

ANALYSIS: (a) With $R''_{t,c} \approx 15\times10^{-4}\ \mathrm{m^2\cdot K/W}$ from Table 3.1 and

$$\frac{L}{k} = \frac{0.01\mathrm{m}}{16.6\ \mathrm{W/m\cdot K}} = 6.02\times10^{-4}\mathrm{m^2\cdot K/W},$$

it follows that

$$R''_{tot} = 2(L/k) + R''_{t,c} \approx 27\times10^{-4}\mathrm{m^2\cdot K/W};$$

hence

$$q'' = \frac{\Delta T}{R''_{tot}} = \frac{100^\circ\mathrm{C}}{27\times10^{-4}\mathrm{m^2\cdot K/W}} = 3.70\times10^4\ \mathrm{W/m^2}. \qquad <$$

(b) From the thermal circuit,

$$\frac{\Delta T_c}{T_{s,1}-T_{s,2}} = \frac{R''_{t,c}}{R''_{tot}} = \frac{15\times10^{-4}\mathrm{m^2\cdot K/W}}{27\times10^{-4}\mathrm{m^2\cdot K/W}} = 0.556.$$

Hence,

$$\Delta T_c = 0.556\left(T_{s,1}-T_{s,2}\right) = 0.556\left(100^\circ\mathrm{C}\right) = 55.6^\circ\mathrm{C}. \qquad <$$

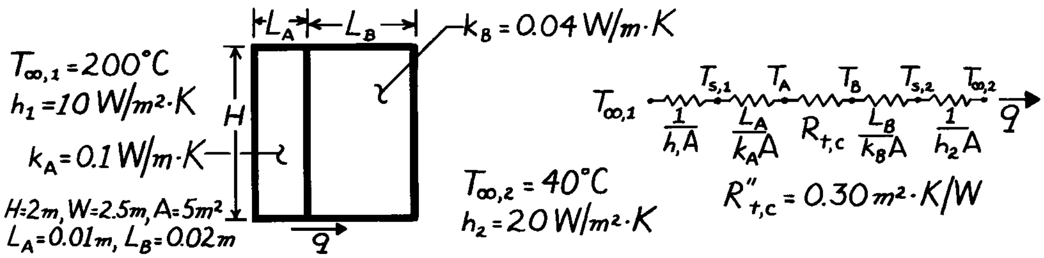
COMMENTS: The contact resistance is significant relative to the conduction resistances. The value of $R''_{t,c}$ would diminish, however, with increasing pressure.

PROBLEM 3.22

KNOWN: Temperatures and convection coefficients associated with fluids at inner and outer surfaces of a composite wall. Contact resistance, dimensions, and thermal conductivities associated with wall materials.

FIND: (a) Rate of heat transfer through the wall, (b) Temperature distribution.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat transfer, (3) Negligible radiation, (4) Constant properties.

ANALYSIS: (a) Calculate the total resistance to find the heat rate,

$$R_{tot} = \frac{1}{h_1A} + \frac{L_A}{k_AA} + R_{t,c} + \frac{L_B}{k_BA} + \frac{1}{h_2A}$$

$$R_{tot} = \left[\frac{1}{10\times5} + \frac{0.01}{0.1\times5} + \frac{0.3}{5} + \frac{0.02}{0.04\times5} + \frac{1}{20\times5}\right]\frac{K}{W}$$

$$R_{tot} = [0.02 + 0.02 + 0.06 + 0.10 + 0.01]\frac{K}{W} = 0.21\frac{K}{W}$$

$$q = \frac{T_{\infty,1} - T_{\infty,2}}{R_{tot}} = \frac{(200-40)^\circ C}{0.21\ K/W} = 762\ W.$$ <

(b) It follows that

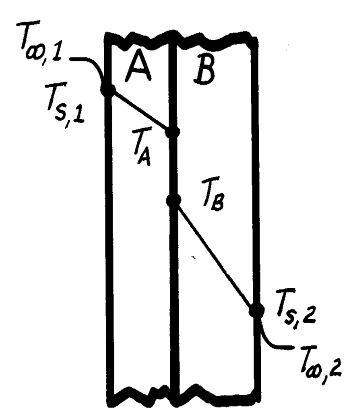
$$T_{s,1} = T_{\infty,1} - \frac{q}{h_1A} = 200^\circ C - \frac{762\ W}{50\ W/K} = 184.8^\circ C$$

$$T_A = T_{s,1} - \frac{qL_A}{k_AA} = 184.8^\circ C - \frac{762W \times 0.01m}{0.1\frac{W}{m\cdot K}\times 5m^2} = 169.6^\circ C$$

$$T_B = T_A - qR_{t,c} = 169.6^\circ C - 762W \times 0.06\frac{K}{W} = 123.8^\circ C$$

$$T_{s,2} = T_B - \frac{qL_B}{k_BA} = 123.8^\circ C - \frac{762W \times 0.02m}{0.04\frac{W}{m\cdot K}\times 5m^2} = 47.6^\circ C$$

$$T_{\infty,2} = T_{s,2} - \frac{q}{h_2A} = 47.6^\circ C - \frac{762W}{100W/K} = 40^\circ C$$

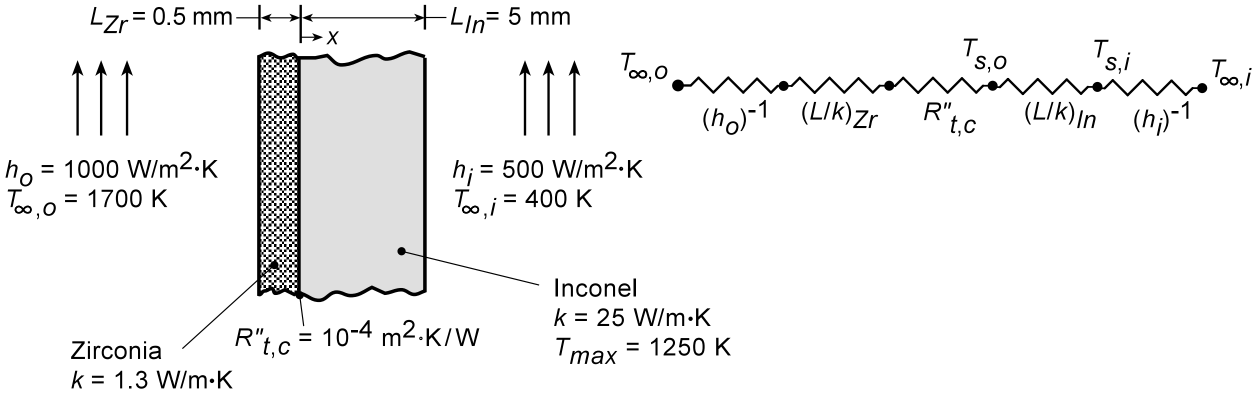


PROBLEM 3.23

KNOWN: Outer and inner surface convection conditions associated with zirconia-coated, Inconel turbine blade. Thicknesses, thermal conductivities, and interfacial resistance of the blade materials. Maximum allowable temperature of Inconel.

FIND: Whether blade operates below maximum temperature. Temperature distribution in blade, with and without the TBC.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conduction in a composite plane wall, (2) Constant properties, (3) Negligible radiation.

ANALYSIS: For a unit area, the total thermal resistance with the TBC is

$$R''_{tot,w} = h_o^{-1} + (L/k)_{Zr} + R''_{t,c} + (L/k)_{In} + h_i^{-1}$$

$$R''_{tot,w} = \left(10^{-3} + 3.85\times10^{-4} + 10^{-4} + 2\times10^{-4} + 2\times10^{-3}\right)m^2\cdot K/W = 3.69\times10^{-3}\,m^2\cdot K/W$$

With a heat flux of

$$q''_w = \frac{T_{\infty,o} - T_{\infty,i}}{R''_{tot,w}} = \frac{1300\,K}{3.69\times10^{-3}\,m^2\cdot K/W} = 3.52\times10^5\,W/m^2$$

the inner and outer surface temperatures of the Inconel are

$$T_{s,i(w)} = T_{\infty,i} + \left(q''_w/h_i\right) = 400\,K + \left(3.52\times10^5\,W/m^2 \big/ 500\,W/m^2\cdot K\right) = 1104\,K$$

$$T_{s,o(w)} = T_{\infty,i} + \left[(1/h_i) + (L/k)_{In}\right]q''_w = 400\,K + \left(2\times10^{-3} + 2\times10^{-4}\right)m^2\cdot K/W\left(3.52\times10^5\,W/m^2\right) = 1174\,K$$

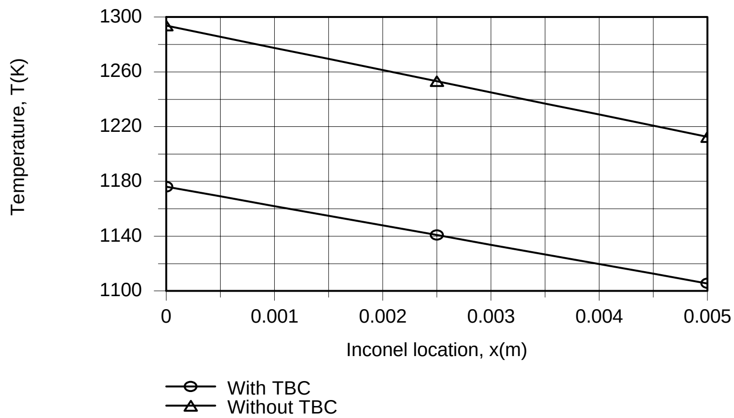
Without the TBC, $R''_{tot,wo} = h_o^{-1} + (L/k)_{In} + h_i^{-1} = 3.20\times10^{-3}\,m^2\cdot K/W$, and $q''_{wo} = \left(T_{\infty,o} - T_{\infty,i}\right)/R''_{tot,wo}$ = (1300 K)/3.20×10^{-3} m^2·K/W = 4.06×10^5 W/m^2. The inner and outer surface temperatures of the Inconel are then

$$T_{s,i(wo)} = T_{\infty,i} + \left(q''_{wo}/h_i\right) = 400\,K + \left(4.06\times10^5\,W/m^2 \big/ 500\,W/m^2\cdot K\right) = 1212\,K$$

$$T_{s,o(wo)} = T_{\infty,i} + \left[(1/h_i) + (L/k)_{In}\right]q''_{wo} = 400\,K + \left(2\times10^{-3} + 2\times10^{-4}\right)m^2\cdot K/W\left(4.06\times10^5\,W/m^2\right) = 1293\,K$$

Continued...

PROBLEM 3.23 (Cont.)



Use of the TBC facilitates operation of the Inconel below T_{max} = 1250 K.

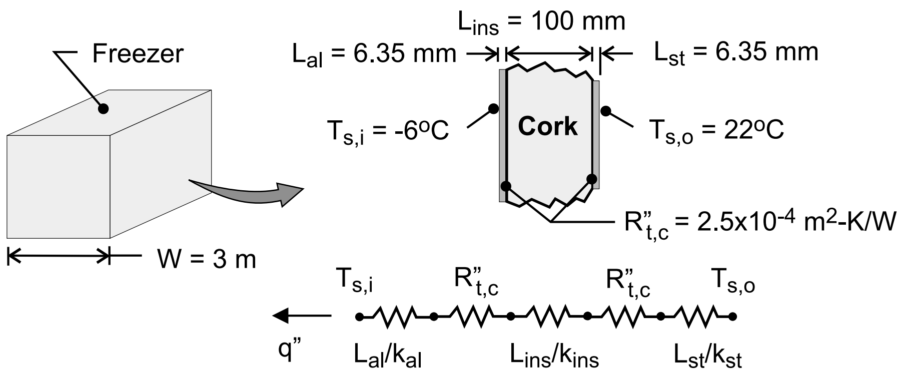
COMMENTS: Since the durability of the TBC decreases with increasing temperature, which increases with increasing thickness, limits to the thickness are associated with reliability considerations.

PROBLEM 3.24

KNOWN: Size and surface temperatures of a cubical freezer. Materials, thicknesses and interface resistances of freezer wall.

FIND: Cooling load.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional conduction, (3) Constant properties.

PROPERTIES: *Table A-1*, Aluminum 2024 (~267K): k_{al} = 173 W/m·K. *Table A-1*, Carbon steel AISI 1010 (~295K): k_{st} = 64 W/m·K. *Table A-3* (~300K): k_{ins} = 0.039 W/m·K.

ANALYSIS: For a unit wall surface area, the total thermal resistance of the composite wall is

$$R''_{tot} = \frac{L_{al}}{k_{al}} + R''_{t,c} + \frac{L_{ins}}{k_{ins}} + R''_{t,c} + \frac{L_{st}}{k_{st}}$$

$$R''_{tot} = \frac{0.00635\text{m}}{173\ \text{W/m}\cdot\text{K}} + 2.5\times10^{-4}\ \frac{\text{m}^2\cdot\text{K}}{\text{W}} + \frac{0.100\text{m}}{0.039\ \text{W/m}\cdot\text{K}} + 2.5\times10^{-4}\ \frac{\text{m}^2\cdot\text{K}}{\text{W}} + \frac{0.00635\text{m}}{64\ \text{W/m}\cdot\text{K}}$$

$$R''_{tot} = \left(3.7\times10^{-5} + 2.5\times10^{-4} + 2.56 + 2.5\times10^{-4} + 9.9\times10^{-5}\right)\text{m}^2\cdot\text{K/W}$$

Hence, the heat flux is

$$q'' = \frac{T_{s,o} - T_{s,i}}{R''_{tot}} = \frac{\left[22-(-6)\right]^\circ\text{C}}{2.56\ \text{m}^2\cdot\text{K/W}} = 10.9\ \frac{\text{W}}{\text{m}^2}$$

and the cooling load is

$$q = A_s\,q'' = 6\,W^2\,q'' = 54\text{m}^2 \times 10.9\ \text{W/m}^2 = 590\,\text{W}$$ <

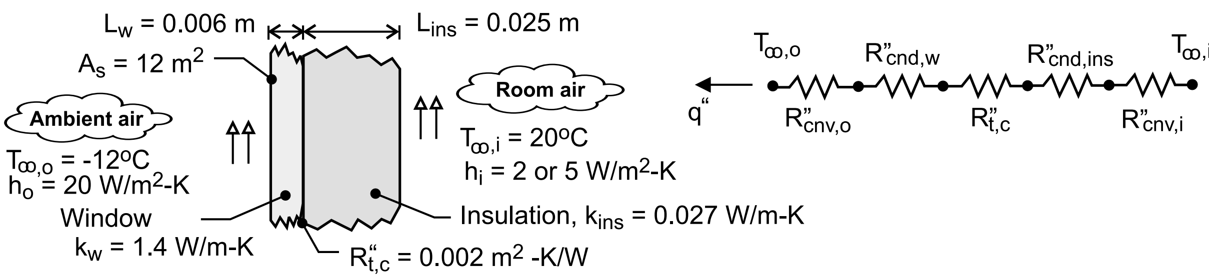
COMMENTS: Thermal resistances associated with the cladding and the adhesive joints are negligible compared to that of the insulation.

PROBLEM 3.25

KNOWN: Thicknesses and thermal conductivity of window glass and insulation. Contact resistance. Environmental temperatures and convection coefficients. Furnace efficiency and fuel cost.

FIND: (a) Reduction in heat loss associated with the insulation, (b) Heat losses for prescribed conditions, (c) Savings in fuel costs for 12 hour period.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional heat transfer, (3) Constant properties.

ANALYSIS: (a) The percentage reduction in heat loss is

$$R_q = \frac{q''_{wo} - q''_{with}}{q''_{wo}} \times 100\% = \left(1 - \frac{q''_{with}}{q''_{wo}}\right) \times 100\% = \left(1 - \frac{R''_{tot,wo}}{R''_{tot,with}}\right) \times 100\%$$

where the total thermal resistances without and with the insulation, respectively, are

$$R''_{tot,wo} = R''_{cnv,o} + R''_{cnd,w} + R''_{cnv,i} = \frac{1}{h_o} + \frac{L_w}{k_w} + \frac{1}{h_i}$$

$$R''_{tot,wo} = (0.050 + 0.004 + 0.200)\,m^2 \cdot K/W = 0.254\ m^2 \cdot K/W$$

$$R''_{tot,with} = R''_{cnv,o} + R''_{cnd,w} + R''_{t,c} + R''_{cnd,ins} + R''_{cnv,i} = \frac{1}{h_o} + \frac{L_w}{k_w} + R''_{t,c} + \frac{L_{ins}}{k_{ins}} + \frac{1}{h_i}$$

$$R''_{tot,with} = (0.050 + 0.004 + 0.002 + 0.926 + 0.500)\,m^2 \cdot K/W = 1.482\ m^2 \cdot K/W$$

$$R_q = (1 - 0.254/1.482) \times 100\% = 82.9\% \qquad <$$

(b) With A_s = 12 m^2, the heat losses without and with the insulation are

$$q_{wo} = A_s\left(T_{\infty,i} - T_{\infty,o}\right)/R''_{tot,wo} = 12\,m^2 \times 32°C/0.254\,m^2 \cdot K/W = 1512\,W \qquad <$$

$$q_{with} = A_s\left(T_{\infty,i} - T_{\infty,o}\right)/R''_{tot,with} = 12\,m^2 \times 32°C/1.482\,m^2 \cdot K/W = 259\ W \qquad <$$

(c) With the windows covered for 12 hours per day, the daily savings are

$$S = \frac{\left(q_{wo} - q_{with}\right)}{\eta_f}\Delta t\ C_g \times 10^{-6}\ MJ/J = \frac{(1512 - 259)\,W}{0.8} 12h \times 3600\,s/h \times \$0.01/MJ \times 10^{-6} MJ/J = \$0.677$$

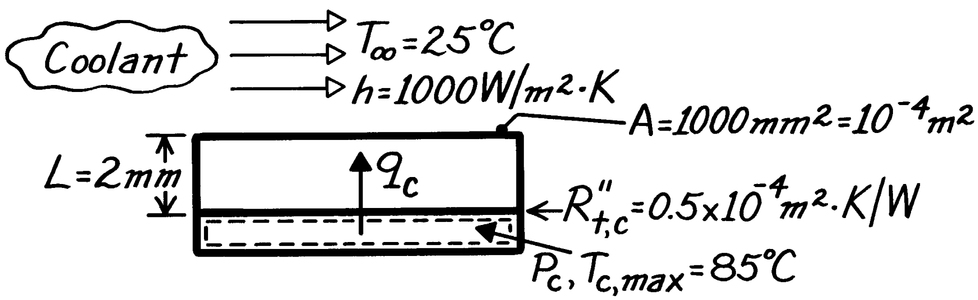
COMMENTS: (1) The savings may be insufficient to justify the cost of the insulation, as well as the daily tedium of applying and removing the insulation. However, the losses are significant and unacceptable. The owner of the building should install double pane windows. (2) The dominant contributions to the total thermal resistance are made by the insulation and convection at the inner surface.

PROBLEM 3.26

KNOWN: Surface area and maximum temperature of a chip. Thickness of aluminum cover and chip/cover contact resistance. Fluid convection conditions.

FIND: Maximum chip power.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat transfer, (3) Negligible heat loss from sides and bottom, (4) Chip is isothermal.

PROPERTIES: *Table A.1*, Aluminum (T ≈ 325 K): k = 238 W/m·K.

ANALYSIS: For a control surface about the chip, conservation of energy yields

$$\dot{E}_g - \dot{E}_{out} = 0$$

or

$$P_c - \frac{(T_c - T_\infty)A}{\left[(L/k) + R''_{t,c} + (1/h)\right]} = 0$$

$$P_{c,max} = \frac{(85-25)^\circ C\left(10^{-4} m^2\right)}{\left[(0.002/238) + 0.5\times10^{-4} + (1/1000)\right] m^2 \cdot K/W}$$

$$P_{c,max} = \frac{60\times10^{-4}\ {}^\circ C \cdot m^2}{\left(8.4\times10^{-6} + 0.5\times10^{-4} + 10^{-3}\right) m^2 \cdot K/W}$$

$$P_{c,max} = 5.7\ W.$$ <

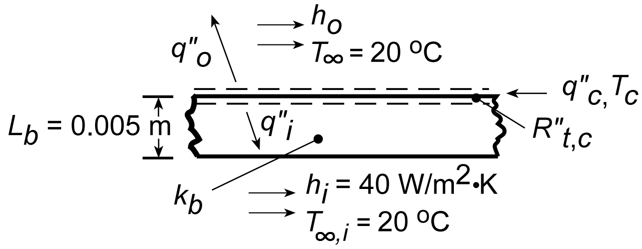
COMMENTS: The dominant resistance is that due to convection $\left(R_{conv} > R_{t,c} >> R_{cond}\right)$.

PROBLEM 3.27

KNOWN: Operating conditions for a board mounted chip.

FIND: (a) Equivalent thermal circuit, (b) Chip temperature, (c) Maximum allowable heat dissipation for dielectric liquid (h_o = 1000 W/m^2·K) and air (h_o = 100 W/m^2·K). Effect of changes in circuit board temperature and contact resistance.

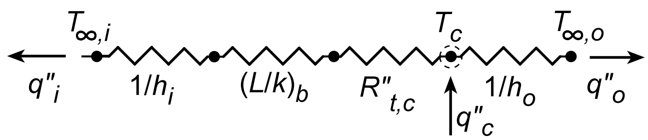
SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Negligible chip thermal resistance, (4) Negligible radiation, (5) Constant properties.

PROPERTIES: *Table A-3*, Aluminum oxide (polycrystalline, 358 K): k_b = 32.4 W/m·K.

ANALYSIS: (a)



(b) Applying conservation of energy to a control surface about the chip $\left(\dot{E}_{in} - \dot{E}_{out} = 0\right)$,

$$q''_c - q''_i - q''_o = 0$$

$$q''_c = \frac{T_c - T_{\infty,i}}{1/h_i + (L/k)_b + R''_{t,c}} + \frac{T_c - T_{\infty,o}}{1/h_o}$$

With $q''_c = 3\times10^4\ \mathrm{W/m^2}$, h_o = 1000 W/m^2·K, k_b = 1 W/m·K and $R''_{t,c} = 10^{-4}\ \mathrm{m^2\cdot K/W}$,

$$3\times10^4\ \mathrm{W/m^2} = \frac{T_c - 20^\circ\mathrm{C}}{\left(1/40 + 0.005/1 + 10^{-4}\right)\mathrm{m^2\cdot K/W}} + \frac{T_c - 20^\circ\mathrm{C}}{(1/1000)\,\mathrm{m^2\cdot K/W}}$$

$$3\times10^4\ \mathrm{W/m^2} = \left(33.2T_c - 664 + 1000T_c - 20{,}000\right)\mathrm{W/m^2\cdot K}$$

$$1003T_c = 50{,}664$$

$$T_c = 49^\circ\mathrm{C}.$$ <

(c) For T_c = 85°C and h_o = 1000 W/m^2·K, the foregoing energy balance yields

$$q''_c = 67{,}160\ \mathrm{W/m^2}$$ <

with q''_o = 65,000 W/m^2 and q''_i = 2160 W/m^2. Replacing the dielectric with air (h_o = 100 W/m^2·K), the following results are obtained for different combinations of k_b and $R''_{t,c}$.

Continued...

PROBLEM 3.27 (Cont.)

k_b (W/m·K)	$R''_{t,c}$ (m^2·K/W)	q''_i (W/m^2)	q''_o (W/m^2)	q''_c (W/m^2)
1	10^{-4}	2159	6500	8659
32.4	10^{-4}	2574	6500	9074
1	10^{-5}	2166	6500	8666
32.4	10^{-5}	2583	6500	9083

$<$

COMMENTS: 1. For the conditions of part (b), the total internal resistance is 0.0301 m^2·K/W, while the outer resistance is 0.001 m^2·K/W. Hence

$$\frac{q''_o}{q''_i} = \frac{\left(T_c - T_{\infty,o}\right)/R''_o}{\left(T_c - T_{\infty,i}\right)/R''_i} = \frac{0.0301}{0.001} = 30\,.$$

and only approximately 3% of the heat is dissipated through the board.

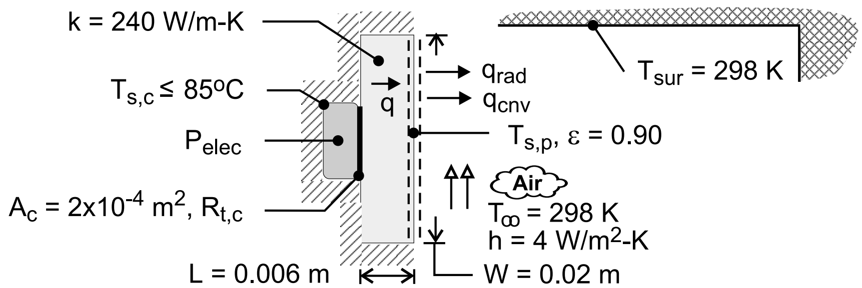
2. With h_o = 100 W/m^2·K, the outer resistance increases to 0.01 m^2·K/W, in which case $q''_o/q''_i = R''_i/R''_o$ = 0.0301/0.01 = 3.1 and now almost 25% of the heat is dissipated through the board. Hence, although measures to reduce R''_i would have a negligible effect on q''_c for the liquid coolant, some improvement may be gained for air-cooled conditions. As shown in the table of part (b), use of an aluminum oxide board increase q''_i by 19% (from 2159 to 2574 W/m^2) by reducing R''_i from 0.0301 to 0.0253 m^2·K/W. Because the initial contact resistance ($R''_{t,c} = 10^{-4} m^2 \cdot K/W$) is already much less than R''_i, any reduction in its value would have a negligible effect on q''_i. The largest gain would be realized by increasing h_i, since the inside convection resistance makes the dominant contribution to the total internal resistance.

PROBLEM 3.28

KNOWN: Dimensions, thermal conductivity and emissivity of base plate. Temperature and convection coefficient of adjoining air. Temperature of surroundings. Maximum allowable temperature of transistor case. Case-plate interface conditions.

FIND: (a) Maximum allowable power dissipation for an air-filled interface, (b) Effect of convection coefficient on maximum allowable power dissipation.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) Negligible heat transfer from the enclosure, to the surroundings. (3) One-dimensional conduction in the base plate, (4) Radiation exchange at surface of base plate is with large surroundings, (5) Constant thermal conductivity.

PROPERTIES: Aluminum-aluminum interface, air-filled, 10 μm roughness, 10^5 N/m^2 contact pressure (Table 3.1): $R''_{t,c} = 2.75 \times 10^{-4}\ m^2 \cdot K/W$.

ANALYSIS: (a) With all of the heat dissipation transferred through the base plate,

$$P_{elec} = q = \frac{T_{s,c} - T_\infty}{R_{tot}} \tag{1}$$

where $R_{tot} = R_{t,c} + R_{cnd} + \left[(1/R_{cnv}) + (1/R_{rad})\right]^{-1}$

$$R_{tot} = \frac{R''_{t,c}}{A_c} + \frac{L}{kW^2} + \frac{1}{W^2}\left(\frac{1}{h + h_r}\right) \tag{2}$$

and $$h_r = \varepsilon\sigma\left(T_{s,p} + T_{sur}\right)\left(T_{s,p}^2 + T_{sur}^2\right) \tag{3}$$

To obtain $T_{s,p}$, the following energy balance must be performed on the plate surface,

$$q = \frac{T_{s,c} - T_{s,p}}{R_{t,c} + R_{cnd}} = q_{cnv} + q_{rad} = hW^2\left(T_{s,p} - T_\infty\right) + h_r\, W^2\left(T_{s,p} - T_{sur}\right) \tag{4}$$

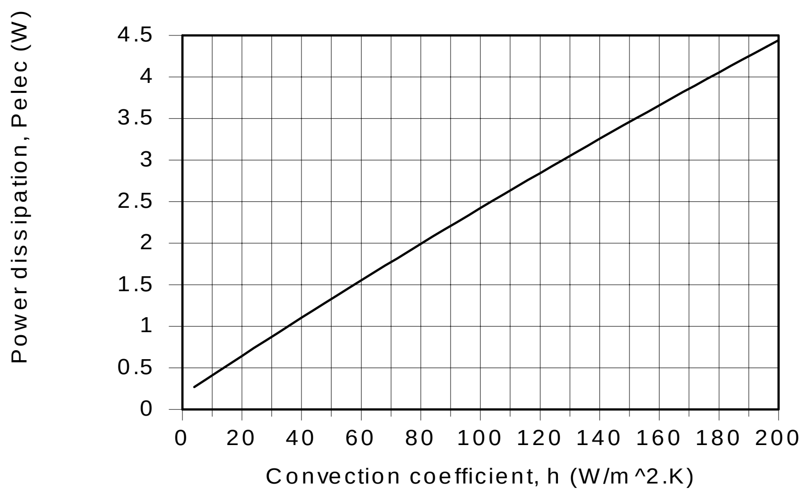
With $R_{t,c} = 2.75 \times 10^{-4}\ m^2 \cdot K/W / 2\times10^{-4}\ m^2 = 1.375$ K/W, $R_{cnd} = 0.006\ m/(240\ W/m\cdot K \times 4 \times 10^{-4}\ m^2)$ = 0.0625 K/W, and the prescribed values of h, W, $T_\infty = T_{sur}$ and ε, Eq. (4) yields a surface temperature of $T_{s,p}$ = 357.6 K = 84.6°C and a power dissipation of

Continued …..

$$P_{elec} = q = 0.268\ W$$ <

The convection and radiation resistances are R_{cnv} = 625 m·K/W and R_{rad} = 345 m·K/W, where h_r = 7.25 W/m^2·K.

(b) With the major contribution to the total resistance made by convection, significant benefit may be derived by increasing the value of h.



For h = 200 W/m^2·K, R_{cnv} = 12.5 m·K/W and $T_{s,p}$ = 351.6 K, yielding R_{rad} = 355 m·K/W. The effect of radiation is then negligible.

COMMENTS: (1) The plate conduction resistance is negligible, and even for h = 200 W/m^2·K, the contact resistance is small relative to the convection resistance. However, $R_{t,c}$ could be rendered negligible by using indium foil, instead of an air gap, at the interface. From Table 3.1, $R''_{t,c} = 0.07\times10^{-4}\ m^2 \cdot K/W$, in which case $R_{t,c}$ = 0.035 m·K/W.

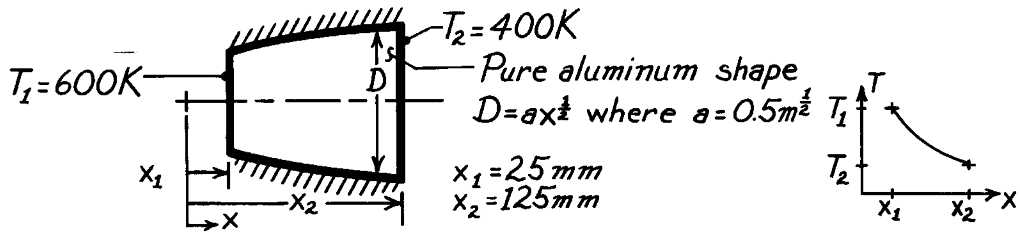
(2) Because $A_c < W^2$, heat transfer by conduction in the plate is actually two-dimensional, rendering the conduction resistance even smaller.

PROBLEM 3.29

KNOWN: Conduction in a conical section with prescribed diameter, D, as a function of x in the form $D = ax^{1/2}$.

FIND: (a) Temperature distribution, T(x), (b) Heat transfer rate, q_x.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in x-direction, (3) No internal heat generation, (4) Constant properties.

PROPERTIES: *Table A-2*, Pure Aluminum (500K): k= 236 W/m·K.

ANALYSIS: (a) Based upon the assumptions, and following the same methodology of Example 3.3, q_x is a constant independent of x. Accordingly,

$$q_x = -kA\frac{dT}{dx} = -k\left[\pi\left(ax^{1/2}\right)^2/4\right]\frac{dT}{dx} \qquad (1)$$

using $A = \pi D^2/4$ where $D = ax^{1/2}$. Separating variables and identifying limits,

$$\frac{4q_x}{\pi a^2 k}\int_{x_1}^{x}\frac{dx}{x} = -\int_{T_1}^{T} dT. \qquad (2)$$

Integrating and solving for T(x) and then for T_2,

$$T(x) = T_1 - \frac{4q_x}{\pi a^2 k}\ln\frac{x}{x_1} \qquad T_2 = T_1 - \frac{4q_x}{\pi a^2 k}\ln\frac{x_2}{x_1}. \qquad (3,4)$$

Solving Eq. (4) for q_x and then substituting into Eq. (3) gives the results,

$$q_x = -\frac{\pi}{4}a^2 k\left(T_1 - T_2\right)/1n\ \left(x_1/x_2\right) \qquad (5)$$

$$T(x) = T_1 + \left(T_1 - T_2\right)\frac{\ln\ \left(x/x_1\right)}{\ln\ \left(x_1/x_2\right)}. \qquad <$$

From Eq. (1) note that (dT/dx)·x = Constant. It follows that T(x) has the distribution shown above.

(b) The heat rate follows from Eq. (5),

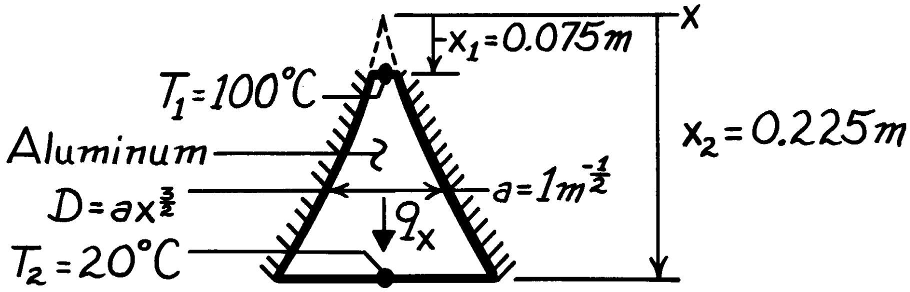
$$q_x = \frac{\pi}{4}\times 0.5^2\,\mathrm{m}\times 236\frac{\mathrm{W}}{\mathrm{m\cdot K}}(600-400)\mathrm{K}/\ln\frac{25}{125} = 5.76\mathrm{kW}. \qquad <$$

PROBLEM 3.30

KNOWN: Geometry and surface conditions of a truncated solid cone.

FIND: (a) Temperature distribution, (b) Rate of heat transfer across the cone.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in x, (3) Constant properties.

PROPERTIES: *Table A-1*, Aluminum (333K): k = 238 W/m·K.

ANALYSIS: (a) From Fourier's law, Eq. (2.1), with $A = \pi D^2/4 = \left(\pi a^2/4\right)x^3$, it follows that

$$\frac{4q_x dx}{\pi a^2 x^3} = -kdT.$$

Hence, since q_x is independent of x,

$$\frac{4q_x}{\pi a^2}\int_{x_1}^{x}\frac{dx}{x^3} = -k\int_{T_1}^{T} dT$$

or

$$\frac{4q_x}{\pi a^2}\left[-\frac{1}{2x^2}\right]\Bigg|_{x_1}^{x} = -k\left(T - T_1\right).$$

Hence

$$T = T_1 + \frac{2q_x}{\pi a^2 k}\left[\frac{1}{x^2} - \frac{1}{x_1^2}\right].$$ <

(b) From the foregoing expression, it also follows that

$$q_x = \frac{\pi a^2 k}{2}\frac{T_2 - T_1}{\left[1/x_2^2 - 1/x_1^2\right]}$$

$$q_x = \frac{\pi\left(1m^{-1}\right)238\ W/m\cdot K}{2}\times\frac{(20-100)^{\circ}C}{\left[(0.225)^{-2} - (0.075)^{-2}\right]m^{-2}}$$

$$q_x = 189\ W.$$ <

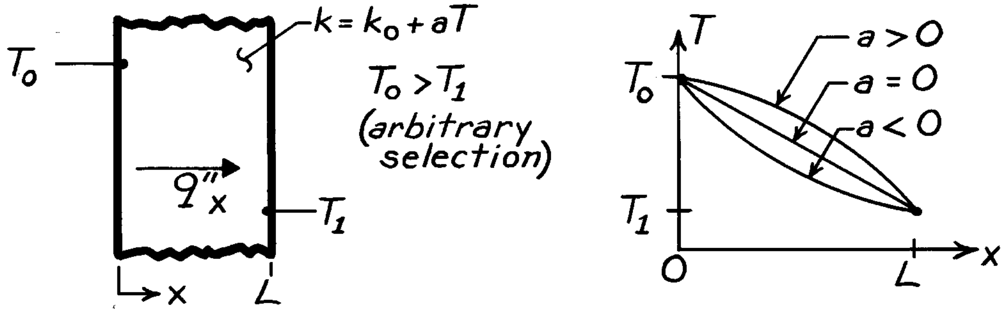
COMMENTS: The foregoing results are approximate due to use of a one-dimensional model in treating what is inherently a two-dimensional problem.

PROBLEM 3.31

KNOWN: Temperature dependence of the thermal conductivity, k.

FIND: Heat flux and form of temperature distribution for a plane wall.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction through a plane wall, (2) Steady-state conditions, (3) No internal heat generation.

ANALYSIS: For the assumed conditions, q_x and A(x) are constant and Eq. 3.21 gives

$$q_x'' \int_0^L dx = -\int_{T_o}^{T_1} (k_o + aT)dT$$

$$q_x'' = \frac{1}{L}\left[k_o (T_o - T_1) + \frac{a}{2}\left(T_o^2 - T_1^2\right)\right].$$

From Fourier's law,

$$q_x'' = -(k_o + aT)\ dT/dx.$$

Hence, since the product of (k_o+aT) and dT/dx) is constant, decreasing T with increasing x implies,

a > 0: decreasing (k_o+aT) and increasing |dT/dx| with increasing x

a = 0: $k = k_o$ => constant (dT/dx)

a < 0: increasing (k_o+aT) and decreasing |dT/dx| with increasing x.

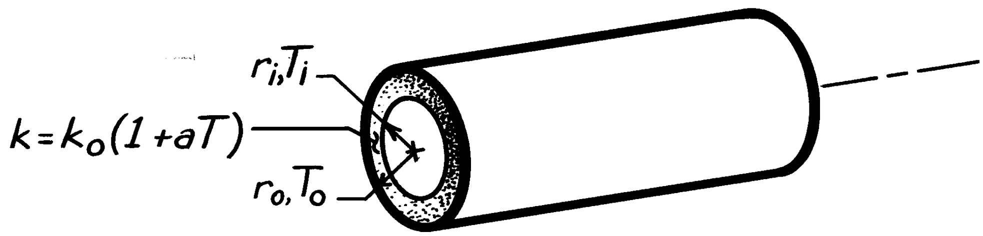
The temperature distributions appear as shown in the above sketch.

PROBLEM 3.32

KNOWN: Temperature dependence of tube wall thermal conductivity.

FIND: Expressions for heat transfer per unit length and tube wall thermal (conduction) resistance.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial conduction, (3) No internal heat generation.

ANALYSIS: From Eq. 3.24, the appropriate form of Fourier's law is

$$q_r = -kA_r \frac{dT}{dr} = -k(2\pi\ rL)\frac{dT}{dr}$$
$$q_r' = -2\pi\ kr\frac{dT}{dr}$$
$$q_r' = -2\pi\ rk_o(1+aT)\frac{dT}{dr}.$$

Separating variables,

$$-\frac{q_r'}{2\pi}\frac{dr}{r} = k_o(1+aT)dT$$

and integrating across the wall, find

$$-\frac{q_r'}{2\pi}\int_{r_i}^{r_o}\frac{dr}{r} = k_o\int_{T_i}^{T_o}(1+aT)dT$$
$$-\frac{q_r'}{2\pi}\ln\frac{r_o}{r_i} = k_o\left[T+\frac{aT^2}{2}\right]\Bigg|_{T_i}^{T_o}$$
$$-\frac{q_r'}{2\pi}\ln\frac{r_o}{r_i} = k_o\left[(T_o-T_i)+\frac{a}{2}\left(T_o^2-T_i^2\right)\right]$$

$$q_r' = -2\pi k_o\left[1+\frac{a}{2}(T_o+T_i)\right]\frac{(T_o-T_i)}{\ln(r_o/r_i)}. \quad <$$

It follows that the overall thermal resistance per unit length is

$$R_t' = \frac{\Delta T}{q_r'} = \frac{\ln(r_o/r_i)}{2\pi k_o\left[1+\frac{a}{2}(T_o+T_i)\right]}. \quad <$$

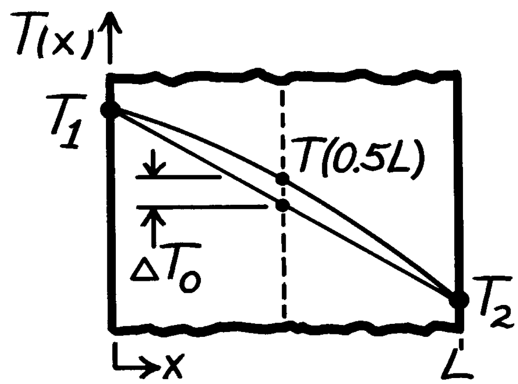
COMMENTS: Note the necessity of the stated assumptions to treating q_r' as independent of r.

PROBLEM 3.33

KNOWN: Steady-state temperature distribution of convex shape for material with $k = k_o(1 + \alpha T)$ where α is a constant and the mid-point temperature is ΔT_o higher than expected for a linear temperature distribution.

FIND: Relationship to evaluate α in terms of ΔT_o and T_1, T_2 (the temperatures at the boundaries).

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) No internal heat generation, (4) α is positive and constant.

ANALYSIS: At any location in the wall, Fourier's law has the form

$$q''_x = -k_o(1+\alpha T)\frac{dT}{dx}. \tag{1}$$

Since q''_x is a constant, we can separate Eq. (1), identify appropriate integration limits, and integrate to obtain

$$\int_0^L q''_x dx = -\int_{T_1}^{T_2} k_o(1+\alpha T)dT \tag{2}$$

$$q''_x = -\frac{k_o}{L}\left[\left(T_2 + \frac{\alpha T_2^2}{2}\right) - \left(T_1 + \frac{\alpha T_1^2}{2}\right)\right]. \tag{3}$$

We could perform the same integration, but with the upper limits at x = L/2, to obtain

$$q''_x = -\frac{2k_o}{L}\left[\left(T_{L/2} + \frac{\alpha T_{L/2}^2}{2}\right) - \left(T_1 + \frac{\alpha T_1^2}{2}\right)\right] \tag{4}$$

where

$$T_{L/2} = T(L/2) = \frac{T_1 + T_2}{2} + \Delta T_o. \tag{5}$$

Setting Eq. (3) equal to Eq. (4), substituting from Eq. (5) for $T_{L/2}$, and solving for α, it follows that

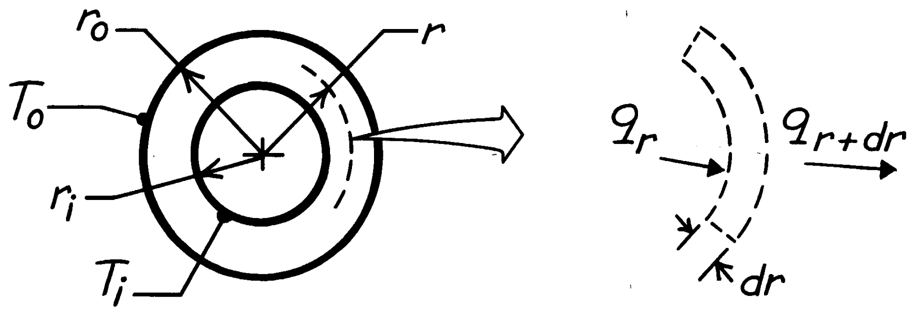
$$\alpha = \frac{2\Delta T_o}{\left(T_2^2 + T_1^2\right)/2 - \left[(T_1 + T_2)/2 + \Delta T_o\right]^2}. \qquad <$$

PROBLEM 3.34

KNOWN: Hollow cylinder of thermal conductivity k, inner and outer radii, r_i and r_o, respectively, and length L.

FIND: Thermal resistance using the alternative conduction analysis method.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial conduction, (3) No internal volumetric generation, (4) Constant properties.

ANALYSIS: For the differential control volume, energy conservation requires that $q_r = q_{r+dr}$ for steady-state, one-dimensional conditions with no heat generation. With Fourier's law,

$$q_r = -kA\frac{dT}{dr} = -k(2\pi\ rL)\frac{dT}{dr} \tag{1}$$

where $A = 2\pi rL$ is the area normal to the direction of heat transfer. Since q_r is constant, Eq. (1) may be separated and expressed in integral form,

$$\frac{q_r}{2\pi\ L}\int_{r_i}^{r_o}\frac{dr}{r} = -\int_{T_i}^{T_o} k(T)\,dT.$$

Assuming k is constant, the heat rate is

$$q_r = \frac{2\pi\ Lk(T_i - T_o)}{\ln(r_o/r_i)}.$$

Remembering that the thermal resistance is defined as

$$R_t \equiv \Delta T/q$$

it follows that for the hollow cylinder,

$$R_t = \frac{\ln(r_o/r_i)}{2\pi\ LK}.$$ <

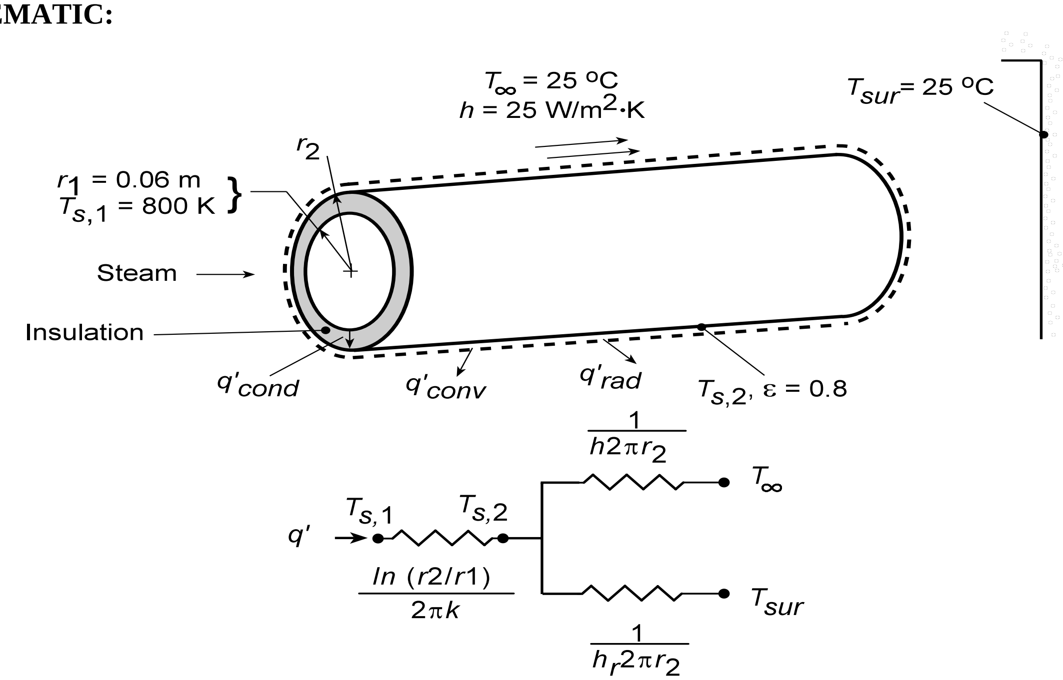
COMMENTS: Compare the *alternative* method used in this analysis with the *standard* method employed in Section 3.3.1 to obtain the same result.

PROBLEM 3.35

KNOWN: Thickness and inner surface temperature of calcium silicate insulation on a steam pipe. Convection and radiation conditions at outer surface.

FIND: (a) Heat loss per unit pipe length for prescribed insulation thickness and outer surface temperature. (b) Heat loss and radial temperature distribution as a function of insulation thickness.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties.

PROPERTIES: *Table A-3*, Calcium Silicate (T = 645 K): k = 0.089 W/m·K.

ANALYSIS: (a) From Eq. 3.27 with $T_{s,2}$ = 490 K, the heat rate per unit length is

$$q' = q_r/L = \frac{2\pi k\left(T_{s,1} - T_{s,2}\right)}{\ln\left(r_2/r_1\right)}$$

$$q' = \frac{2\pi\left(0.089\,W/m\cdot K\right)\left(800-490\right)K}{\ln\left(0.08\,m/0.06\,m\right)}$$

$$q' = 603\,W/m\,.$$ <

(b) Performing an energy for a control surface around the outer surface of the insulation, it follows that

$$q'_{cond} = q'_{conv} + q'_{rad}$$

$$\frac{T_{s,1} - T_{s,2}}{\ln\left(r_2/r_1\right)/2\pi k} = \frac{T_{s,2} - T_\infty}{1/\left(2\pi r_2 h\right)} + \frac{T_{s,2} - T_{sur}}{1/\left(2\pi r_2 h_r\right)}$$

where $h_r = \varepsilon\sigma\left(T_{s,2} + T_{sur}\right)\left(T_{s,2}^2 + T_{sur}^2\right)$. Solving this equation for $T_{s,2}$, the heat rate may be determined from

$$q' = 2\pi r_2\left[h\left(T_{s,2} - T_\infty\right) + h_r\left(T_{s,2} - T_{sur}\right)\right]$$

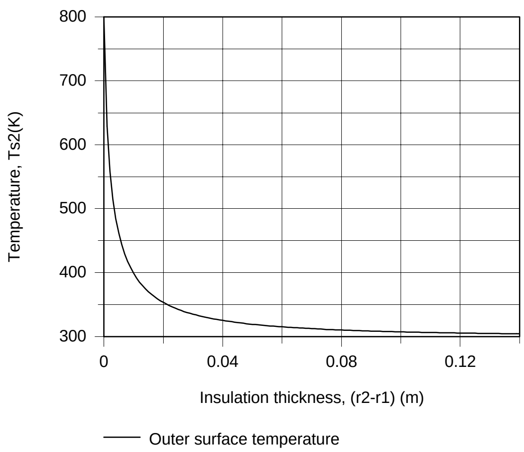
Continued...

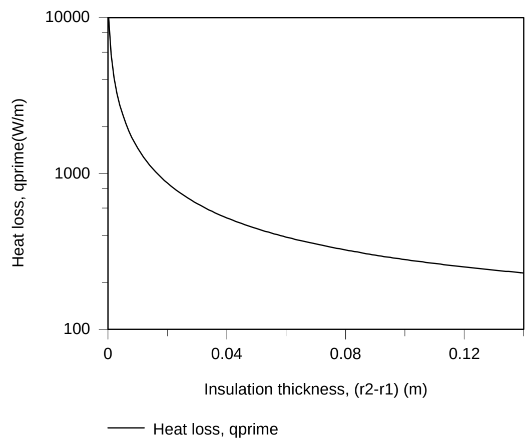
PROBLEM 3.35 (Cont.)

and from Eq. 3.26 the temperature distribution is

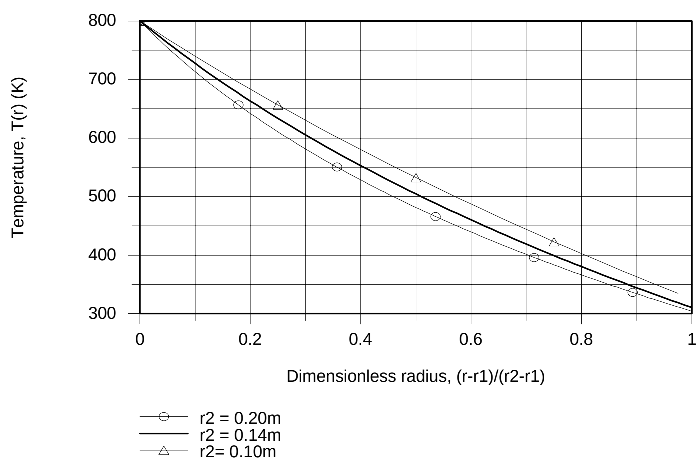
$$T(r) = \frac{T_{s,1} - T_{s,2}}{\ln(r_1/r_2)} \ln\left(\frac{r}{r_2}\right) + T_{s,2}$$

As shown below, the outer surface temperature of the insulation $T_{s,2}$ and the heat loss q' decay precipitously with increasing insulation thickness from values of $T_{s,2} = T_{s,1} = 800$ K and $q' = 11,600$ W/m, respectively, at $r_2 = r_1$ (no insulation).





When plotted as a function of a dimensionless radius, $(r - r_1)/(r_2 - r_1)$, the temperature decay becomes more pronounced with increasing r_2.



Note that $T(r_2) = T_{s,2}$ increases with decreasing r_2 and a linear temperature distribution is approached as r_2 approaches r_1.

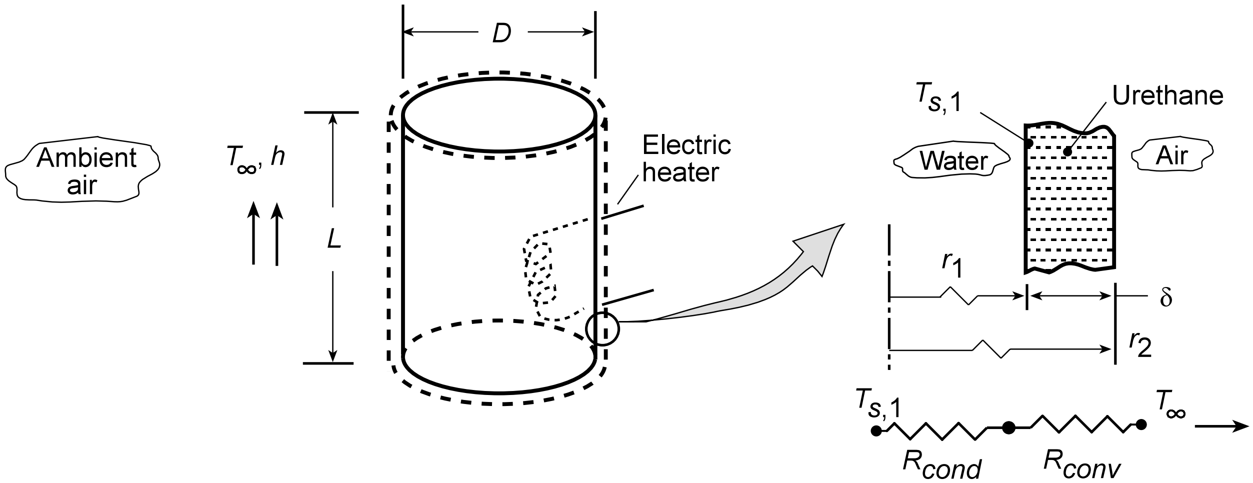
COMMENTS: An insulation layer thickness of 20 mm is sufficient to maintain the outer surface temperature and heat rate below 350 K and 1000 W/m, respectively.

PROBLEM 3.36

KNOWN: Temperature and volume of hot water heater. Nature of heater insulating material. Ambient air temperature and convection coefficient. Unit cost of electric power.

FIND: Heater dimensions and insulation thickness for which annual cost of heat loss is less than $50.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conduction through side and end walls, (2) Conduction resistance dominated by insulation, (3) Inner surface temperature is approximately that of the water ($T_{s,1}$ = 55°C), (4) Constant properties, (5) Negligible radiation.

PROPERTIES: *Table A.3*, Urethane Foam (T = 300 K): k = 0.026 W/m·K.

ANALYSIS: To minimize heat loss, tank dimensions which minimize the total surface area, $A_{s,t}$, should be selected. With $L = 4\forall/\pi D^2$, $A_{s,t} = \pi DL + 2\left(\pi D^2/4\right) = 4\forall/D + \pi D^2/2$, and the tank diameter for which $A_{s,t}$ is an extremum is determined from the requirement

$$dA_{s,t}/dD = -4\forall/D^2 + \pi D = 0$$

It follows that

$$D = \left(4\forall/\pi\right)^{1/3} \qquad \text{and} \qquad L = \left(4\forall/\pi\right)^{1/3}$$

With $d^2A_{s,t}/dD^2 = 8\forall/D^3 + \pi > 0$, the foregoing conditions yield the desired minimum in $A_{s,t}$.
Hence, for $\forall$ = 100 gal × 0.00379 m^3/gal = 0.379 m^3,

$$D_{op} = L_{op} = 0.784\,\text{m}$$ <

The total heat loss through the side and end walls is

$$q = \frac{T_{s,1} - T_\infty}{\dfrac{\ln\left(r_2/r_1\right)}{2\pi k L_{op}} + \dfrac{1}{h 2\pi r_2 L_{op}}} + \frac{2\left(T_{s,1} - T_\infty\right)}{\dfrac{\delta}{k\left(\pi D_{op}^2/4\right)} + \dfrac{1}{h\left(\pi D_{op}^2/4\right)}}$$

We begin by estimating the heat loss associated with a 25 mm thick layer of insulation. With $r_1 = D_{op}/2$ = 0.392 m and $r_2 = r_1 + \delta$ = 0.417 m, it follows that

Continued...

$$q = \frac{(55-20)^\circ C}{\dfrac{\ln(0.417/0.392)}{2\pi(0.026\,W/m\cdot K)0.784\,m} + \dfrac{1}{(2\,W/m^2\cdot K)2\pi(0.417\,m)0.784\,m}}$$

$$+\frac{2(55-20)^\circ C}{\dfrac{0.025\,m}{(0.026\,W/m\cdot K)\pi/4(0.784\,m)^2} + \dfrac{1}{(2\,W/m^2\cdot K)\pi/4(0.784\,m)^2}}$$

$$q = \frac{35^\circ C}{(0.483+0.243)K/W} + \frac{2(35^\circ C)}{(1.992+1.036)K/W} = (48.2+23.1)W = 71.3\,W$$

The annual energy loss is therefore

$$Q_{annual} = 71.3\,W(365\,days)(24\ h/day)\left(10^{-3}\ kW/W\right) = 625\,kWh$$

With a unit electric power cost of \$0.08/kWh, the annual cost of the heat loss is

C = (\$0.08/kWh)625 kWh = \$50.00

Hence, an insulation thickness of

δ = 25 mm <

will satisfy the prescribed cost requirement.

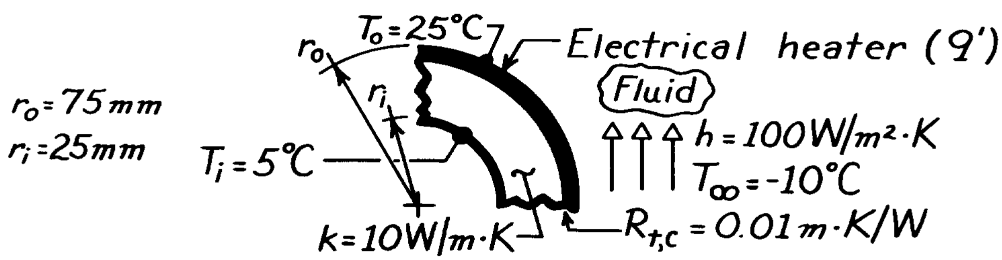
COMMENTS: Cylindrical containers of aspect ratio L/D = 1 are seldom used because of floor space constraints. Choosing L/D = 2, $\forall = \pi D^3/2$ and $D = (2\forall/\pi)^{1/3}$ = 0.623 m. Hence, L = 1.245 m, r_1 = 0.312m and r_2 = 0.337 m. It follows that q = 76.1 W and C = \$53.37. The 6.7% increase in the annual cost of the heat loss is small, providing little justification for using the optimal heater dimensions.

PROBLEM 3.37

KNOWN: Inner and outer radii of a tube wall which is heated electrically at its outer surface and is exposed to a fluid of prescribed h and T_∞. Thermal contact resistance between heater and tube wall and wall inner surface temperature.

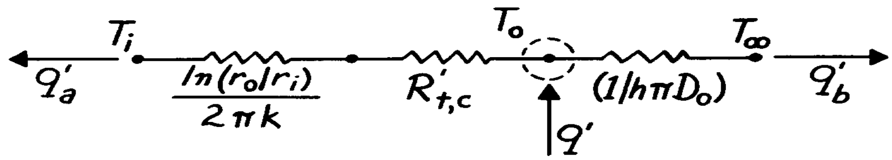
FIND: Heater power per unit length required to maintain a heater temperature of 25°C.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible temperature drop across heater.

ANALYSIS: The thermal circuit has the form



Applying an energy balance to a control surface about the heater,

$$q' = q'_a + q'_b$$

$$q' = \frac{T_o - T_i}{\frac{\ln(r_o/r_i)}{2\pi k} + R'_{t,c}} + \frac{T_o - T_\infty}{(1/h\pi D_o)}$$

$$q' = \frac{(25\text{-}5)^\circ C}{\frac{\ln(75mm/25mm)}{2\pi \times 10\ W/m\cdot K} + 0.01\frac{m\cdot K}{W}} + \frac{[25-(-10)]^\circ C}{\left[1/\left(100\ W/m^2\cdot K \times \pi \times 0.15m\right)\right]}$$

$$q' = (728 + 1649)\ W/m$$

$$q' = 2377\ W/m. \qquad <$$

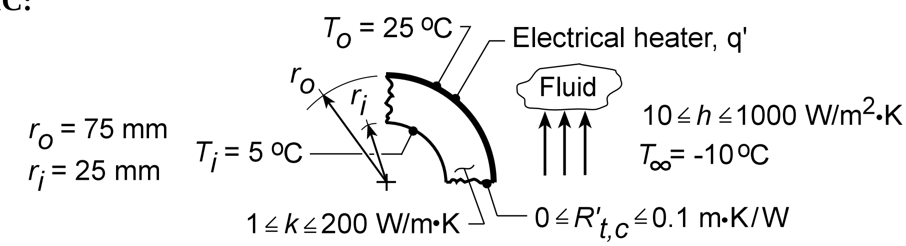
COMMENTS: The conduction, contact and convection resistances are 0.0175, 0.01 and 0.021 m ·K/W, respectively,

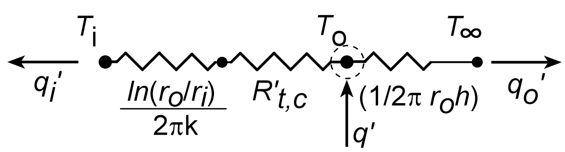
PROBLEM 3.38

KNOWN: Inner and outer radii of a tube wall which is heated electrically at its outer surface. Inner and outer wall temperatures. Temperature of fluid adjoining outer wall.

FIND: Effect of wall thermal conductivity, thermal contact resistance, and convection coefficient on total heater power and heat rates to outer fluid and inner surface.

SCHEMATIC:





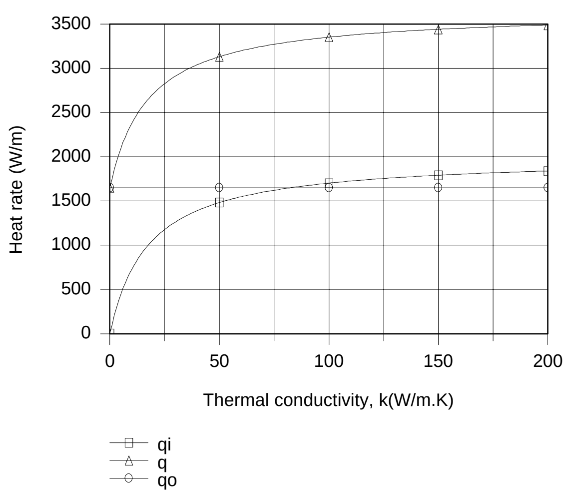
ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible temperature drop across heater, (5) Negligible radiation.

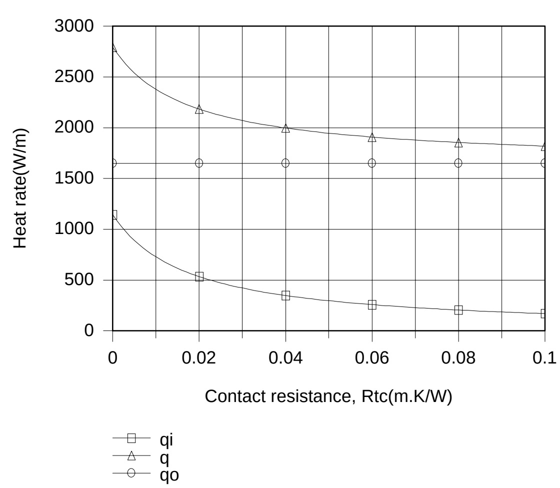
ANALYSIS: Applying an energy balance to a control surface about the heater,

$$q' = q_i' + q_o'$$

$$q' = \frac{T_o - T_i}{\frac{\ln(r_o/r_i)}{2\pi k} + R'_{t,c}} + \frac{T_o - T_\infty}{(1/2\pi r_o h)}$$

Selecting nominal values of k = 10 W/m·K, $R'_{t,c}$ = 0.01 m·K/W and h = 100 W/m²·K, the following parametric variations are obtained





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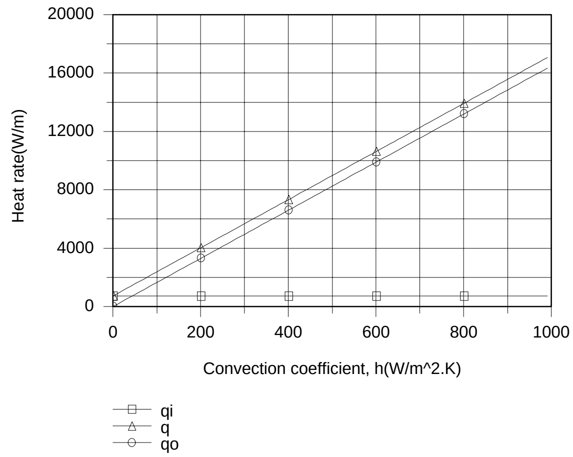
PROBLEM 3.38 (Cont.)



For a prescribed value of h, q'_o is fixed, while q'_i, and hence q', increase and decrease, respectively, with increasing k and $R'_{t,c}$. These trends are attributable to the effects of k and $R'_{t,c}$ on the total (conduction plus contact) resistance separating the heater from the inner surface. For fixed k and $R'_{t,c}$, q'_i is fixed, while q'_o, and hence q', increase with increasing h due to a reduction in the convection resistance.

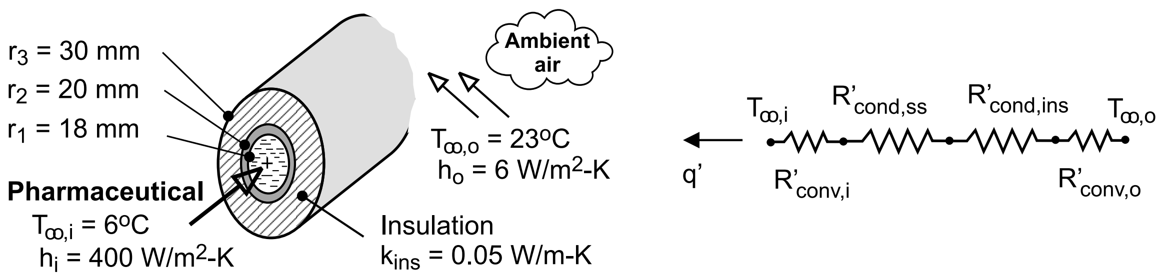
COMMENTS: For the prescribed nominal values of k, $R'_{t,c}$ and h, the electric power requirement is q' = 2377 W/m. To maintain the prescribed heater temperature, q' would increase with any changes which reduce the conduction, contact and/or convection resistances.

PROBLEM 3.39

KNOWN: Wall thickness and diameter of stainless steel tube. Inner and outer fluid temperatures and convection coefficients.

FIND: (a) Heat gain per unit length of tube, (b) Effect of adding a 10 mm thick layer of insulation to outer surface of tube.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial conduction, (3) Constant properties, (4) Negligible contact resistance between tube and insulation, (5) Negligible effect of radiation.

PROPERTIES: *Table A-1*, St. St. 304 (~280K): k_{st} = 14.4 W/m·K.

ANALYSIS: (a) Without the insulation, the total thermal resistance per unit length is

$$R'_{tot} = R'_{conv,i} + R'_{cond,st} + R'_{conv,o} = \frac{1}{2\pi r_i h_i} + \frac{\ln(r_2/r_i)}{2\pi k_{st}} + \frac{1}{2\pi r_2 h_o}$$

$$R'_{tot} = \frac{1}{2\pi(0.018\text{m})400\ \text{W/m}^2\cdot\text{K}} + \frac{\ln(20/18)}{2\pi(14.4\ \text{W/m}\cdot\text{K})} + \frac{1}{2\pi(0.020\text{m})6\ \text{W/m}^2\cdot\text{K}}$$

$$R'_{tot} = \left(0.0221 + 1.16\times10^{-3} + 1.33\right)\text{m}\cdot\text{K/W} = 1.35\ \text{m}\cdot\text{K/W}$$

The heat gain per unit length is then

$$q' = \frac{T_{\infty,o} - T_{\infty,i}}{R'_{tot}} = \frac{(23-6)°\text{C}}{1.35\,\text{m}\cdot\text{K/W}} = 12.6\ \text{W/m} \qquad <$$

(b) With the insulation, the total resistance per unit length is now $R'_{tot} = R'_{conv,i} + R'_{cond,st} + R'_{cond,ins}$ $+ R'_{conv,o}$, where $R'_{conv,i}$ and $R'_{cond,st}$ remain the same. The thermal resistance of the insulation is

$$R'_{cond,ins} = \frac{\ln(r_3/r_2)}{2\pi k_{ins}} = \frac{\ln(30/20)}{2\pi(0.05\ \text{W/m}\cdot\text{K})} = 1.29\ \text{m}\cdot\text{K/W}$$

and the outer convection resistance is now

$$R'_{conv,o} = \frac{1}{2\pi r_3 h_o} = \frac{1}{2\pi(0.03\text{m})6\ \text{W/m}^2\cdot\text{K}} = 0.88\ \text{m}\cdot\text{K/W}$$

The total resistance is now

$$R'_{tot} = \left(0.0221 + 1.16\times10^{-3} + 1.29 + 0.88\right)\text{m}\cdot\text{K/W} = 2.20\,\text{m}\cdot\text{K/W}$$

Continued …..

and the heat gain per unit length is

$$q' = \frac{T_{\infty,o} - T_{\infty,i}}{R'_{tot}} = \frac{17^\circ C}{2.20\ m \cdot K/W} = 7.7\ W/m$$

COMMENTS: (1) The validity of assuming negligible radiation may be assessed for the worst case condition corresponding to the bare tube. Assuming a tube outer surface temperature of $T_s = T_{\infty,i} =$ 279K, large surroundings at $T_{sur} = T_{\infty,o} =$ 296K, and an emissivity of $\varepsilon = 0.7$, the heat gain due to net radiation exchange with the surroundings is $q'_{rad} = \varepsilon\sigma(2\pi r_2)\left(T_{sur}^4 - T_s^4\right) = 7.7\ W/m$. Hence, the net rate of heat transfer by radiation to the tube surface is comparable to that by convection, and the assumption of negligible radiation is inappropriate.

(2) If heat transfer from the air is by natural convection, the value of h_o with the insulation would actually be less than the value for the bare tube, thereby further reducing the heat gain. Use of the insulation would also increase the outer surface temperature, thereby reducing net radiation transfer from the surroundings.

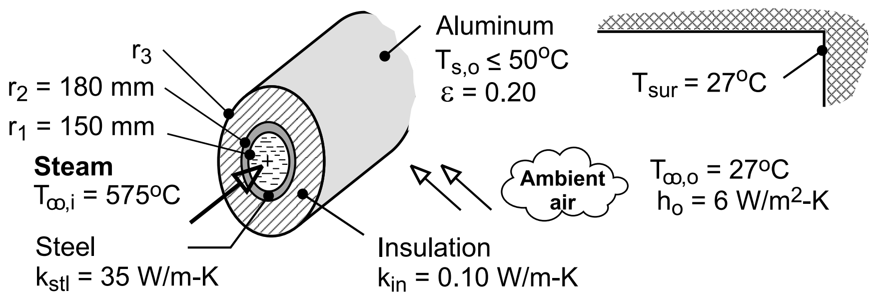
(3) The critical radius is $r_{cr} = k_{ins}/h \approx 8\ mm < r_2$. Hence, as indicated by the calculations, heat transfer is reduced by the insulation.

PROBLEM 3.40

KNOWN: Diameter, wall thickness and thermal conductivity of steel tubes. Temperature of steam flowing through the tubes. Thermal conductivity of insulation and emissivity of aluminum sheath. Temperature of ambient air and surroundings. Convection coefficient at outer surface and maximum allowable surface temperature.

FIND: (a) Minimum required insulation thickness (r3 – r2) and corresponding heat loss per unit length, (b) Effect of insulation thickness on outer surface temperature and heat loss.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional radial conduction, (3) Negligible contact resistances at the material interfaces, (4) Negligible steam side convection resistance ($T_{\infty,i} = T_{s,i}$), (5) Negligible conduction resistance for aluminum sheath, (6) Constant properties, (7) Large surroundings.

ANALYSIS: (a) To determine the insulation thickness, an energy balance must be performed at the outer surface, where $q' = q'_{conv,o} + q'_{rad}$. With $q'_{conv,o} = 2\pi r_3 h_o\left(T_{s,o} - T_{\infty,o}\right)$, $q'_{rad} = 2\pi r_3\ \varepsilon\sigma \left(T_{s,o}^4 - T_{sur}^4\right)$, $q' = \left(T_{s,i} - T_{s,o}\right)/\left(R'_{cond,st} + R'_{cond,ins}\right)$, $R'_{cond,st} = \ell n\left(r_2/r_1\right)/2\pi k_{st}$, and $R'_{cond,ins} = \ell n\left(r_3/r_2\right)/2\pi k_{ins}$, it follows that

$$\frac{2\pi\left(T_{s,i} - T_{s,o}\right)}{\dfrac{\ell n\left(r_2/r_1\right)}{k_{st}} + \dfrac{\ell n\left(r_3/r_2\right)}{k_{ins}}} = 2\pi r_3\left[h_o\left(T_{s,o} - T_{\infty,o}\right) + \varepsilon\sigma\left(T_{s,o}^4 - T_{sur}^4\right)\right]$$

$$\frac{2\pi\left(848 - 323\right)\text{K}}{\dfrac{\ell n\left(0.18/0.15\right)}{35\ \text{W/m}\cdot\text{K}} + \dfrac{\ell n\left(r_3/0.18\right)}{0.10\ \text{W/m}\cdot\text{K}}} = 2\pi r_3\left[6\ \text{W/m}^2\cdot\text{K}\left(323 - 300\right)\text{K} + 0.20\times 5.67\times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4\left(323^4 - 300^4\right)\text{K}^4\right]$$

A trial-and-error solution yields r_3 = 0.394 m = 394 mm, in which case the insulation thickness is

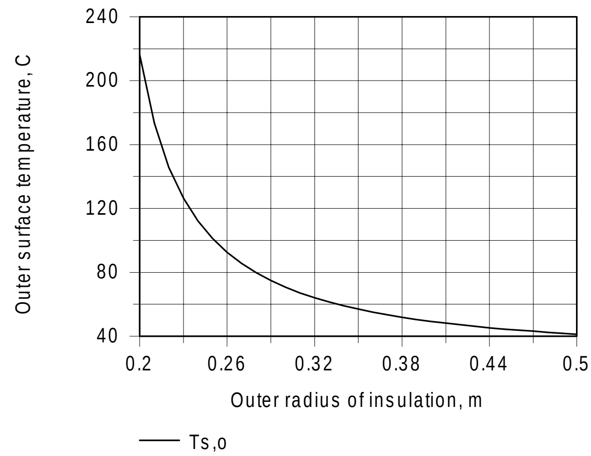
$$t_{ins} = r_3 - r_2 = 214\,\text{mm} \qquad <$$

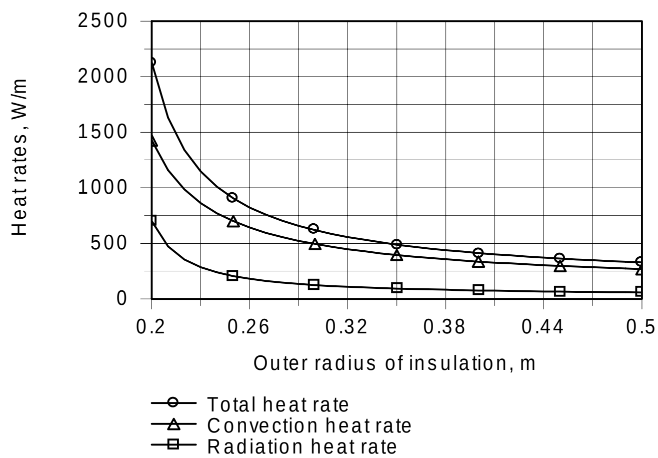
The heat rate is then

$$q' = \frac{2\pi\left(848 - 323\right)\text{K}}{\dfrac{\ell n\left(0.18/0.15\right)}{35\,\text{W/m}\cdot\text{K}} + \dfrac{\ell n\left(0.394/0.18\right)}{0.10\,\text{W/m}\cdot\text{K}}} = 420\,\text{W/m} \qquad <$$

(b) The effects of r_3 on $T_{s,o}$ and q' have been computed and are shown below.

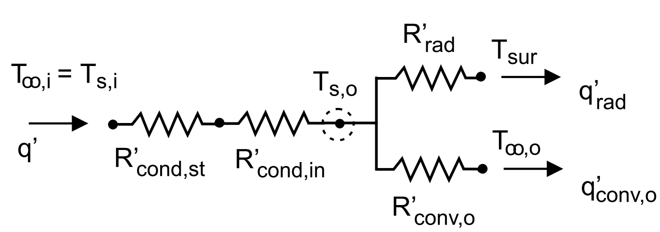
Conditioned …..





Beyond $r_3 \approx 0.40$m, there are rapidly diminishing benefits associated with increasing the insulation thickness.

COMMENTS: Note that the thermal resistance of the insulation is much larger than that for the tube wall. For the conditions of Part (a), the radiation coefficient is $h_r = 1.37$ W/m, and the heat loss by radiation is less than 25% of that due to natural convection $\left(q'_{rad} = 78\,\mathrm{W/m},\ q'_{conv,o} = 342\ \mathrm{W/m}\right)$.

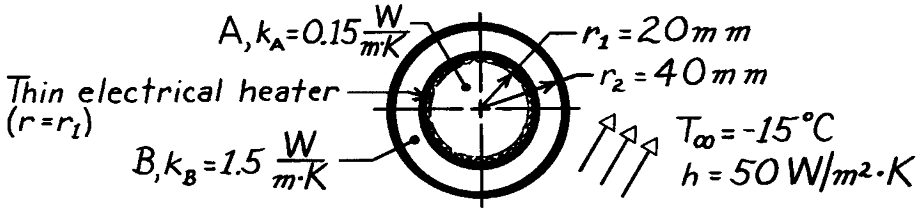


PROBLEM 3.41

KNOWN: Thin electrical heater fitted between two concentric cylinders, the outer surface of which experiences convection.

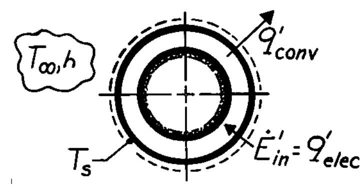
FIND: (a) Electrical power required to maintain outer surface at a specified temperature, (b) Temperature at the center

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, radial conduction, (2) Steady-state conditions, (3) Heater element has negligible thickness, (4) Negligible contact resistance between cylinders and heater, (5) Constant properties, (6) No generation.

ANALYSIS: (a) Perform an energy balance on the composite system to determine the power required to maintain $T(r_2) = T_s = 5°C$.



$$\dot{E}'_{in} - \dot{E}'_{out} + \dot{E}_{gen} = \dot{E}_{st}$$

$$+q'_{elec} - q'_{conv} = 0.$$

Using Newton's law of cooling,

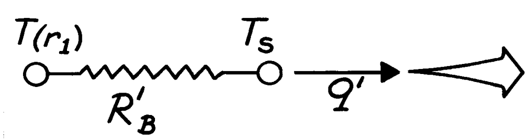
$$q'_{elec} = q'_{conv} = h \cdot 2\pi\, r_2 \left(T_s - T_\infty\right)$$

$$q'_{elec} = 50\frac{\text{W}}{\text{m}^2 \cdot \text{K}} \times 2\pi\left(0.040\text{m}\right)\left[5-\left(-15\right)\right]^{\circ}\text{C} = 251\ \text{W/m}.$$ <

(b) From a control volume about Cylinder A, we recognize that the cylinder must be isothermal, that is,

$$T(0) = T(r_1).$$

Represent Cylinder B by a thermal circuit:



$$q' = \frac{T(r_1) - T_s}{R'_B}$$

For the cylinder, from Eq. 3.28,

$$R'_B = \ln r_2 / r_1 / 2\pi\, k_B$$

giving

$$T(r_1) = T_s + q'R'_B = 5^{\circ}\text{C} + 253.1\frac{\text{W}}{\text{m}}\frac{\ln 40/20}{2\pi \times 1.5\ \text{W/m} \cdot \text{K}} = 23.5^{\circ}\text{C}$$

Hence, $T(0) = T(r_1) = 23.5°C$. <

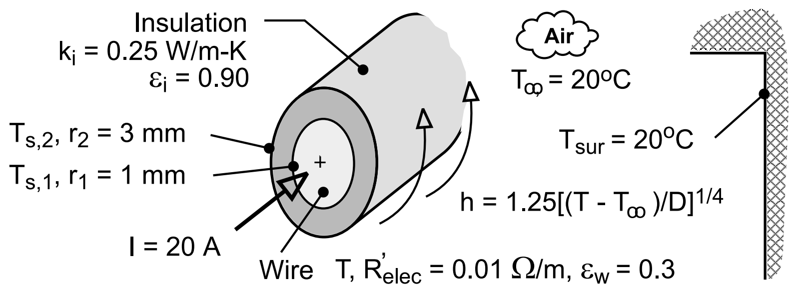
Note that k_A has no influence on the temperature T(0).

PROBLEM 3.42

KNOWN: Electric current and resistance of wire. Wire diameter and emissivity. Thickness, emissivity and thermal conductivity of coating. Temperature of ambient air and surroundings. Expression for heat transfer coefficient at surface of the wire or coating.

FIND: (a) Heat generation per unit length and volume of wire, (b) Temperature of uninsulated wire, (c) Inner and outer surface temperatures of insulation.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional radial conduction through insulation, (3) Constant properties, (4) Negligible contact resistance between insulation and wire, (5) Negligible radial temperature gradients in wire, (6) Large surroundings.

ANALYSIS: (a) The rates of energy generation per unit length and volume are, respectively,

$$\dot{E}'_g = I^2 R'_{elec} = (20\,A)^2 (0.01\Omega/m) = 4\ W/m \qquad <$$

$$\dot{q} = \dot{E}'_g / A_c = 4\dot{E}'_g / \pi D^2 = 16\,W/m/\pi(0.002m)^2 = 1.27\times10^6\ W/m^3 \qquad <$$

(b) Without the insulation, an energy balance at the surface of the wire yields

$$\dot{E}'_g = q' = q'_{conv} + q'_{rad} = \pi D h (T - T_\infty) + \pi D \varepsilon_w \sigma \left(T^4 - T_{sur}^4\right)$$

where $h = 1.25[(T - T_\infty)/D]^{1/4}$. Substituting,

$$4\,W/m = 1.25\pi(0.002m)^{3/4}(T-293)^{5/4} + \pi(0.002m)\,0.3\times5.67\times10^{-8}\ W/m^2\cdot K^4\left(T^4 - 293^4\right)K^4$$

and a trial-and-error solution yields

$$T = 331\,K = 58°C \qquad <$$

(c) Performing an energy balance at the outer surface,

$$\dot{E}'_g = q' = q'_{conv} + q'_{rad} = \pi D h \left(T_{s,2} - T_\infty\right) + \pi D \varepsilon_i \sigma \left(T_{s,2}^4 - T_{sur}^4\right)$$

$$4\,W/m = 1.25\pi(0.006m)^{3/4}\left(T_{s,2}-293\right)^{5/4} + \pi(0.006m)\,0.9\times5.67\times10^{-8}\ W/m^2\cdot K^4\left(T_{s,2}^4 - 293^4\right)K^4$$

and an iterative solution yields the following value of the surface temperature

$$T_{s,2} = 307.8\,K = 34.8°C \qquad <$$

The inner surface temperature may then be obtained from the following expression for heat transfer by conduction in the insulation.

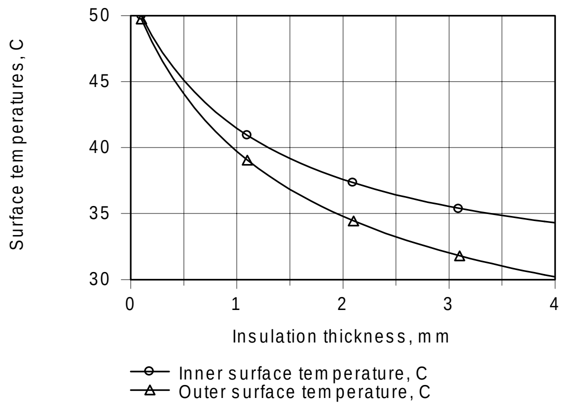
Continued …..

$$q' = \frac{T_{s,i} - T_2}{R'_{cond}} = \frac{T_{s,i} - T_{s,2}}{\ell n(r_2/r_1)/2\pi k_i}$$

$$4\,W = \frac{2\pi(0.25\,W/m\cdot K)(T_{s,i} - 307.8\,K)}{\ell n\,3}$$

$$T_{s,i} = 310.6\,K = 37.6°C \qquad <$$

As shown below, the effect of increasing the insulation thickness is to *reduce*, not increase, the surface temperatures.



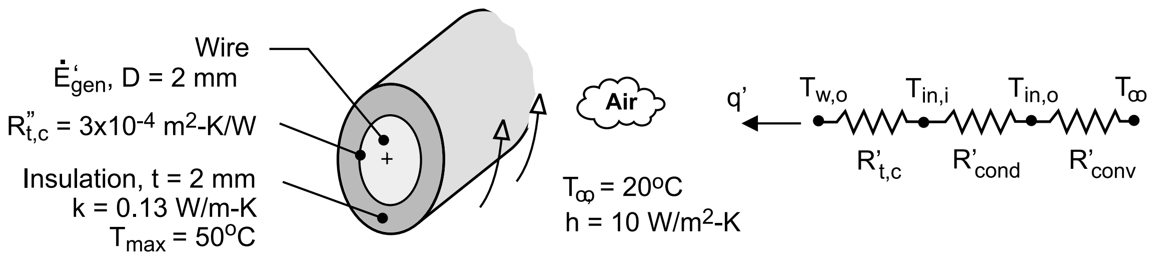
This behavior is due to a reduction in the total resistance to heat transfer with increasing r_2. Although the convection, h, and radiation, $h_r = \varepsilon\sigma(T_{s,2} + T_{sur})(T_{s,2}^2 + T_{sur}^2)$, coefficients decrease with increasing r_2, the corresponding increase in the surface area is more than sufficient to provide for a reduction in the total resistance. Even for an insulation thickness of t = 4 mm, h = h + h_r = (7.1 + 5.4) $W/m^2\cdot K$ = 12.5 $W/m^2\cdot K$, and r_{cr} = k/h = 0.25 W/m·K/12.5 $W/m^2\cdot K$ = 0.020m = 20 mm > r_2 = 5 mm. The outer radius of the insulation is therefore well below the critical radius.

PROBLEM 3.43

KNOWN: Diameter of electrical wire. Thickness and thermal conductivity of rubberized sheath. Contact resistance between sheath and wire. Convection coefficient and ambient air temperature. Maximum allowable sheath temperature.

FIND: Maximum allowable power dissipation per unit length of wire. Critical radius of insulation.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional radial conduction through insulation, (3) Constant properties, (4) Negligible radiation exchange with surroundings.

ANALYSIS: The maximum insulation temperature corresponds to its inner surface and is independent of the contact resistance. From the thermal circuit, we may write

$$\dot{E}'_g = q' = \frac{T_{in,i} - T_\infty}{R'_{cond} + R'_{conv}} = \frac{T_{in,i} - T_\infty}{\left[\ell n\left(r_{in,o}/r_{in,i}\right)/2\pi k\right] + \left(1/2\pi r_{in,o} h\right)}$$

where $r_{in,i} = D/2 = 0.001\text{m}$, $r_{in,o} = r_{in,i} + t = 0.003\text{m}$, and $T_{in,i} = T_{max} = 50°\text{C}$ yields the maximum allowable power dissipation. Hence,

$$\dot{E}'_{g,max} = \frac{(50-20)°\text{C}}{\dfrac{\ell n\, 3}{2\pi \times 0.13\,\text{W/m}\cdot\text{K}} + \dfrac{1}{2\pi\,(0.003\text{m})10\,\text{W/m}^2\cdot\text{K}}} = \frac{30°\text{C}}{(1.35+5.31)\,\text{m}\cdot\text{K/W}} = 4.51\,\text{W/m}$$ <

The critical insulation radius is also unaffected by the contact resistance and is given by

$$r_{cr} = \frac{k}{h} = \frac{0.13\,\text{W/m}\cdot\text{K}}{10\,\text{W/m}^2\cdot\text{K}} = 0.013\text{m} = 13\,\text{mm}$$ <

Hence, $r_{in,o} < r_{cr}$ and $\dot{E}'_{g,max}$ could be increased by increasing $r_{in,o}$ up to a value of 13 mm (t = 12 mm).

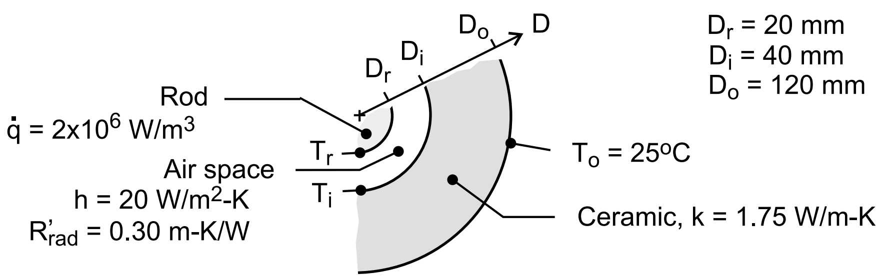
COMMENTS: The contact resistance affects the temperature of the wire, and for $q' = \dot{E}'_{g,max} = 4.51\,\text{W/m}$, the outer surface temperature of the wire is $T_{w,o} = T_{in,i} + q'R'_{t,c} = 50°\text{C} + (4.51\,\text{W/m})\left(3\times10^{-4}\,\text{m}^2\cdot\text{K/W}\right)/\pi\,(0.002\text{m}) = 50.2°\text{C}$. Hence, the temperature change across the contact resistance is negligible.

PROBLEM 3.44

KNOWN: Long rod experiencing uniform volumetric generation of thermal energy, $\dot{q}$, concentric with a hollow ceramic cylinder creating an enclosure filled with air. Thermal resistance per unit length due to radiation exchange between enclosure surfaces is R'_{rad}. The free convection coefficient for the enclosure surfaces is h = 20 W/m^2·K.

FIND: (a) Thermal circuit of the system that can be used to calculate the surface temperature of the rod, T_r; label all temperatures, heat rates and thermal resistances; evaluate the thermal resistances; and (b) Calculate the surface temperature of the rod.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional, radial conduction through the hollow cylinder, (3) The enclosure surfaces experience free convection and radiation exchange.

ANALYSIS: (a) The thermal circuit is shown below. Note labels for the temperatures, thermal resistances and the relevant heat fluxes.

Enclosure, radiation exchange (given):

$$R'_{rad} = 0.30\ \mathrm{m \cdot K/W}$$

Enclosure, free convection:

$$R'_{cv,rod} = \frac{1}{h\pi D_r} = \frac{1}{20\ \mathrm{W/m^2 \cdot K} \times \pi \times 0.020\mathrm{m}} = 0.80\ \mathrm{m \cdot K/W}$$

$$R'_{cv,cer} = \frac{1}{h\pi D_i} = \frac{1}{20\ \mathrm{W/m^2 \cdot K} \times \pi \times 0.040\mathrm{m}} = 0.40\ \mathrm{m \cdot K/W}$$

Ceramic cylinder, conduction:

$$R'_{cd} = \frac{\ell n\left(D_o / D_i\right)}{2\pi k} = \frac{\ell n\left(0.120/0.040\right)}{2\pi \times 1.75\ \mathrm{W/m \cdot K}} = 0.10\ \mathrm{m \cdot K/W}$$

The thermal resistance between the enclosure surfaces (r-i) due to convection and radiation exchange is

$$\frac{1}{R'_{enc}} = \frac{1}{R'_{rad}} + \frac{1}{R'_{cv,rod} + R'_{cv,cer}}$$

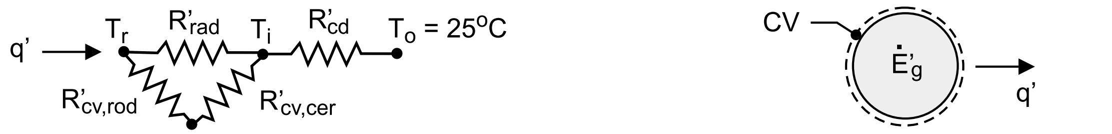
$$R'_{enc} = \left[\frac{1}{0.30} + \frac{1}{0.80 + 0.40}\right]^{-1} \mathrm{m \cdot K/W} = 0.24\ \mathrm{m \cdot K/W}$$

The total resistance between the rod surface (r) and the outer surface of the cylinder (o) is

$$R'_{tot} = R'_{enc} + R'_{cd} = \left(0.24 + 0.1\right)\mathrm{m \cdot K/W} = 0.34\ \mathrm{m \cdot K/W}$$

Continued …..

PROBLEM 3.44 (Cont.)



(a) Thermal circuit

(b) Overall energy balance on rod

(b) From an energy balance on the rod (see schematic) find T_r.

$$\dot{E}'_{in} - \dot{E}'_{out} + \dot{E}'_{gen} = 0$$

$$-q + \dot{q}\forall = 0$$

$$-(T_r - T_i)/R'_{tot} + \dot{q}\left(\pi D_r^2/4\right) = 0$$

$$-(T_r - 25)\text{K}/0.34\ \text{m}\cdot\text{K/W} + 2\times10^6\,\text{W/m}^3\left(\pi\times0.020\text{m}^2/4\right) = 0$$

$$T_r = 239°\text{C}$$ <

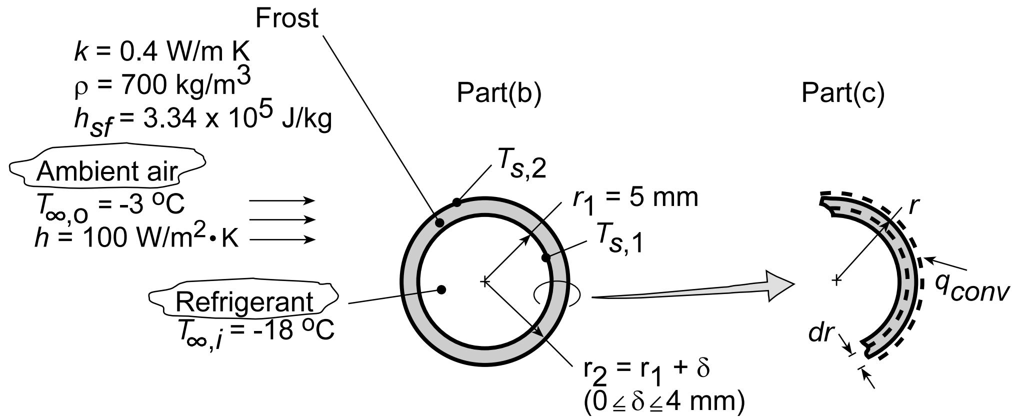
COMMENTS: In evaluating the convection resistance of the air space, it was necessary to define an average air temperature (T_∞) and consider the convection coefficients for each of the space surfaces. As you'll learn later in Chapter 9, correlations are available for directly estimating the convection coefficient (h_{enc}) for the enclosure so that $q_{cv} = h_{enc}\,(T_r - T_1)$.

PROBLEM 3.45

KNOWN: Tube diameter and refrigerant temperature for evaporator of a refrigerant system. Convection coefficient and temperature of outside air.

FIND: (a) Rate of heat extraction without frost formation, (b) Effect of frost formation on heat rate, (c) Time required for a 2 mm thick frost layer to melt in ambient air for which h = 2 W/m^2·K and T_∞ = 20°C.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conditions, (2) Negligible convection resistance for refrigerant flow $\left(T_{\infty,i} = T_{s,1}\right)$, (3) Negligible tube wall conduction resistance, (4) Negligible radiation exchange at outer surface.

ANALYSIS: (a) The cooling capacity in the defrosted condition (δ = 0) corresponds to the rate of heat extraction from the airflow. Hence,

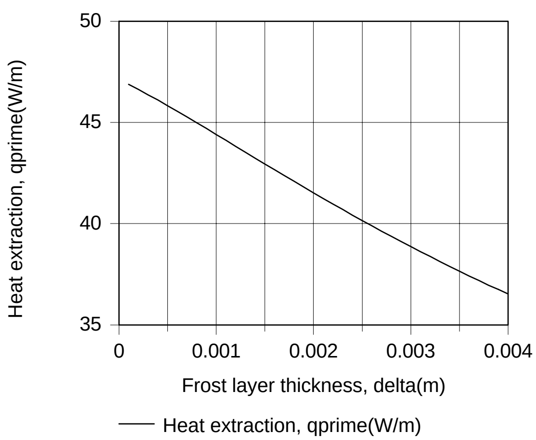
$$q' = h 2\pi r_1 \left(T_{\infty,o} - T_{s,1}\right) = 100\,\mathrm{W/m^2 \cdot K}\left(2\pi \times 0.005\,\mathrm{m}\right)\left(-3+18\right)^\circ \mathrm{C}$$

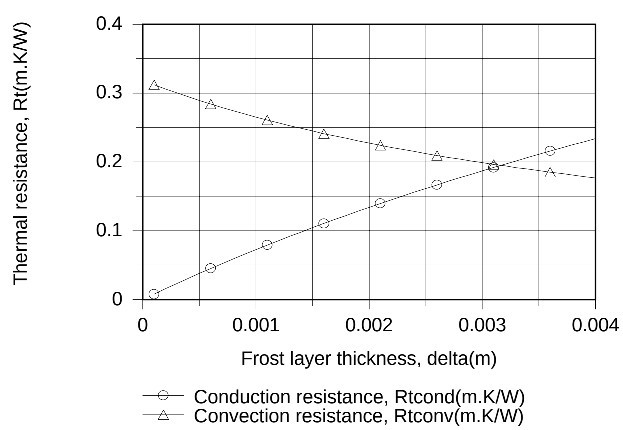
$$q' = 47.1\,\mathrm{W/m}$$ <

(b) With the frost layer, there is an additional (conduction) resistance to heat transfer, and the extraction rate is

$$q' = \frac{T_{\infty,o} - T_{s,1}}{R'_{conv} + R'_{cond}} = \frac{T_{\infty,o} - T_{s,1}}{1/\left(h 2\pi r_2\right) + \ln\left(r_2/r_1\right)/2\pi k}$$

For 5 ≤ r_2 ≤ 9 mm and k = 0.4 W/m·K, this expression yields





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The heat extraction, and hence the performance of the evaporator coil, decreases with increasing frost layer thickness due to an increase in the total resistance to heat transfer. Although the convection resistance decreases with increasing δ, the reduction is exceeded by the increase in the conduction resistance.

(c) The time t_m required to melt a 2 mm thick frost layer may be determined by applying an energy balance, Eq. 1.11b, over the differential time interval dt and to a differential control volume extending inward from the surface of the layer.

$$\dot{E}_{in}dt = dE_{st} = dU_{lat}$$

$$h\left(2\pi rL\right)\left(T_{\infty,o} - T_f\right)dt = -h_{sf}\,\rho d\forall = -h_{sf}\,\rho\left(2\pi rL\right)dr$$

$$h\left(T_{\infty,o} - T_f\right)\int_0^{t_m} dt = -\rho h_{sf}\int_{r_2}^{r_1} dr$$

$$t_m = \frac{\rho h_{sf}\left(r_2 - r_1\right)}{h\left(T_{\infty,o} - T_f\right)} = \frac{700\,\mathrm{kg/m^3}\left(3.34\times10^5\,\mathrm{J/kg}\right)\left(0.002\,\mathrm{m}\right)}{2\,\mathrm{W/m^2\cdot K}\left(20-0\right)^{\circ}\mathrm{C}}$$

$$t_m = 11,690\,\mathrm{s} = 3.25\,\mathrm{h}$$ <

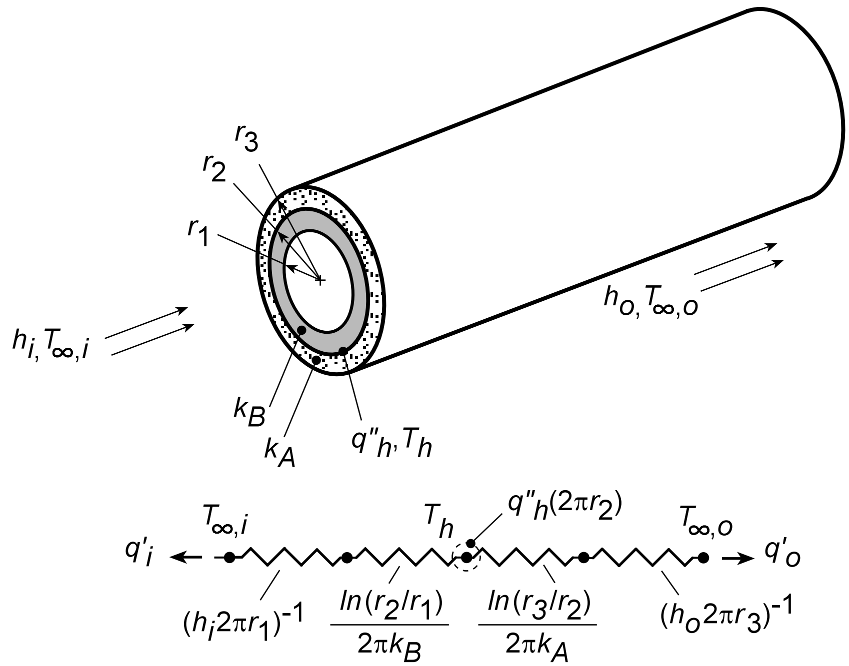
COMMENTS: The tube radius r_1 exceeds the critical radius r_{cr} = k/h = 0.4 W/m·K/100 W/m^2·K = 0.004 m, in which case any frost formation will reduce the performance of the coil.

PROBLEM 3.46

KNOWN: Conditions associated with a composite wall and a thin electric heater.

FIND: (a) Equivalent thermal circuit, (b) Expression for heater temperature, (c) Ratio of outer and inner heat flows and conditions for which ratio is minimized.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conduction, (2) Constant properties, (3) Isothermal heater, (4) Negligible contact resistance(s).

ANALYSIS: (a) On the basis of a unit axial length, the circuit, thermal resistances, and heat rates are as shown in the schematic.

(b) Performing an energy balance for the heater, $\dot{E}_{in} = \dot{E}_{out}$, it follows that

$$q''_h(2\pi r_2) = q'_i + q'_o = \frac{T_h - T_{\infty,i}}{(h_i 2\pi r_1)^{-1} + \frac{\ln(r_2/r_1)}{2\pi k_B}} + \frac{T_h - T_{\infty,o}}{(h_o 2\pi r_3)^{-1} + \frac{\ln(r_3/r_2)}{2\pi k_A}}$$ <

(c) From the circuit,

$$\frac{q'_o}{q'_i} = \frac{(T_h - T_{\infty,o})}{(T_h - T_{\infty,i})} \times \frac{(h_i 2\pi r_1)^{-1} + \frac{\ln(r_2/r_1)}{2\pi k_B}}{(h_o 2\pi r_3)^{-1} + \frac{\ln(r_3/r_2)}{2\pi k_A}}$$ <

To reduce q'_o/q'_i, one could increase k_B, h_i, and r_3/r_2, while reducing k_A, h_o and r_2/r_1.

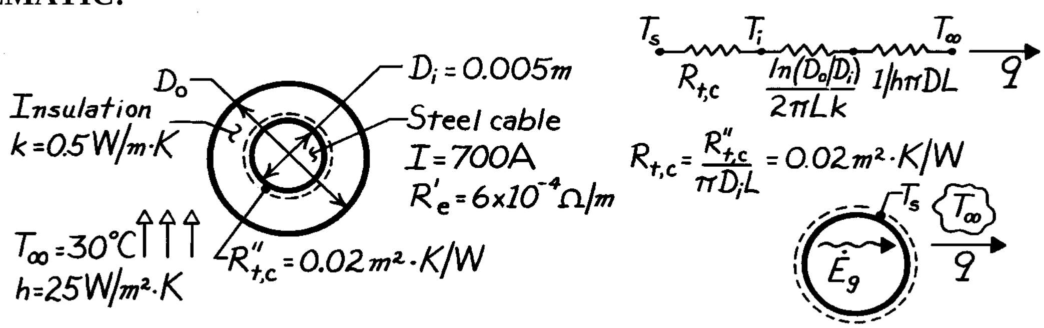
COMMENTS: Contact resistances between the heater and materials A and B could be important.

PROBLEM 3.47

KNOWN: Electric current flow, resistance, diameter and environmental conditions associated with a cable.

FIND: (a) Surface temperature of bare cable, (b) Cable surface and insulation temperatures for a thin coating of insulation, (c) Insulation thickness which provides the lowest value of the maximum insulation temperature. Corresponding value of this temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in r, (3) Constant properties.

ANALYSIS: (a) The rate at which heat is transferred to the surroundings is fixed by the rate of heat generation in the cable. Performing an energy balance for a control surface about the cable, it follows that $\dot{E}_g = q$ or, for the bare cable, $I^2 R'_e L = h(\pi D_i L)(T_s - T_\infty)$. With $q' = I^2 R'_e = (700\text{A})^2 \left(6\times10^{-4}\,\Omega/\text{m}\right) = 294\text{ W/m}$, it follows that

$$T_s = T_\infty + \frac{q'}{h\pi D_i} = 30^\circ\text{C} + \frac{294\text{ W/m}}{\left(25\text{ W/m}^2\cdot\text{K}\right)\pi(0.005\text{m})}$$

$$T_s = 778.7^\circ\text{C}. \qquad <$$

(b) With a thin coating of insulation, there exist contact and convection resistances to heat transfer from the cable. The heat transfer rate is determined by heating within the cable, however, and therefore remains the same.

$$q = \frac{T_s - T_\infty}{R_{t,c} + \dfrac{1}{h\pi D_i L}} = \frac{T_s - T_\infty}{\dfrac{R''_{t,c}}{\pi D_i L} + \dfrac{1}{h\pi D_i L}}$$

$$q' = \frac{\pi D_i (T_s - T_\infty)}{R''_{t,c} + 1/h}$$

and solving for the surface temperature, find

$$T_s = \frac{q'}{\pi D_i}\left[R''_{t,c} + \frac{1}{h}\right] + T_\infty = \frac{294\text{ W/m}}{\pi(0.005\text{m})}\left[0.02\frac{\text{m}^2\cdot\text{K}}{\text{W}} + 0.04\frac{\text{m}^2\cdot\text{K}}{\text{W}}\right] + 30^\circ\text{C}$$

$$T_s = 1153^\circ\text{C}. \qquad <$$

Continued …..

The insulation temperature is then obtained from

$$q=\frac{T_s - T_i}{R_{t,c}}$$

or

$$T_i = T_s - qR_{t,c} = 1153^\circ C - q\frac{R''_{t,c}}{\pi\ D_i L} = 1153^\circ C - \frac{294\frac{W}{m}\times 0.02\frac{m^2\cdot K}{W}}{\pi(0.005m)}$$

$$T_i = 778.7^\circ C.$$ <

(c) The maximum insulation temperature could be reduced by reducing the resistance to heat transfer from the outer surface of the insulation. Such a reduction is possible if $D_i < D_{cr}$. From Example 3.4,

$$r_{cr} = \frac{k}{h} = \frac{0.5\ W/m\cdot K}{25\ W/m^2\cdot K} = 0.02m.$$

Hence, $D_{cr} = 0.04m > D_i = 0.005m$. To minimize the maximum temperature, which exists at the inner surface of the insulation, add insulation in the amount

$$t=\frac{D_o - D_i}{2} = \frac{D_{cr} - D_i}{2} = \frac{(0.04 - 0.005)m}{2}$$

$$t = 0.0175m.$$ <

The cable surface temperature may then be obtained from

$$q'=\frac{T_s - T_\infty}{\frac{R''_{t,c}}{\pi\ D_i} + \frac{\ln(D_{cr}/D_i)}{2\pi\ k} + \frac{1}{h\pi\ D_{cr}}} = \frac{T_s - 30^\circ C}{\frac{0.02\ m^2\cdot K/W}{\pi(0.005m)} + \frac{\ln(0.04/0.005)}{2\pi(0.5\ W/m\cdot K)} + \frac{1}{25\frac{W}{m^2\cdot K}\pi(0.04m)}}$$

Hence,

$$294\frac{W}{m} = \frac{T_s - 30^\circ C}{(1.27+0.66+0.32)m\cdot K/W} = \frac{T_s - 30^\circ C}{2.25\ m\cdot K/W}$$

$$T_s = 692.5^\circ C$$

Recognizing that $q = (T_s - T_i)/R_{t,c}$, find

$$T_i = T_s - qR_{t,c} = T_s - q\frac{R''_{t,c}}{\pi\ D_i L} = 692.5^\circ C - \frac{294\frac{W}{m}\times 0.02\frac{m^2\cdot K}{W}}{\pi(0.005m)}$$

$$T_i = 318.2^\circ C.$$ <

COMMENTS: Use of the critical insulation thickness in lieu of a thin coating has the effect of reducing the maximum insulation temperature from 778.7°C to 318.2°C. Use of the critical insulation thickness also reduces the cable surface temperature to 692.5°C from 778.7°C with no insulation or from 1153°C with a thin coating.

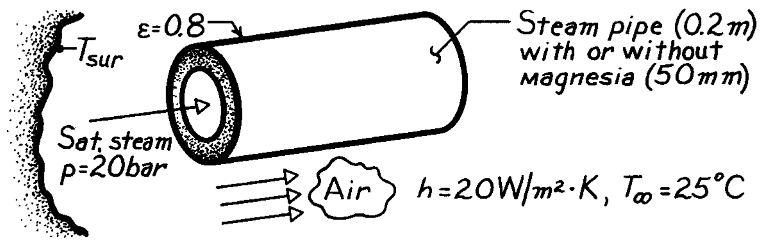
PROBLEM 3.48

KNOWN: Saturated steam conditions in a pipe with prescribed surroundings.

FIND: (a) Heat loss per unit length from bare pipe and from insulated pipe, (b) Pay back period for insulation.

SCHEMATIC:

Steam Costs:
  $4 for 10^9 J
Insulation Cost:
  $100 per meter
Operation time:
  7500 h/yr



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat transfer, (3) Constant properties, (4) Negligible pipe wall resistance, (5) Negligible steam side convection resistance (pipe inner surface temperature is equal to steam temperature), (6) Negligible contact resistance, (7) $T_{sur} = T_\infty$.

PROPERTIES: *Table A-6*, Saturated water (p = 20 bar): $T_{sat} = T_s = 486K$; *Table A-3*, Magnesia, 85% ($T \approx 392K$): k = 0.058 W/m·K.

ANALYSIS: (a) Without the insulation, the heat loss may be expressed in terms of radiation and convection rates,

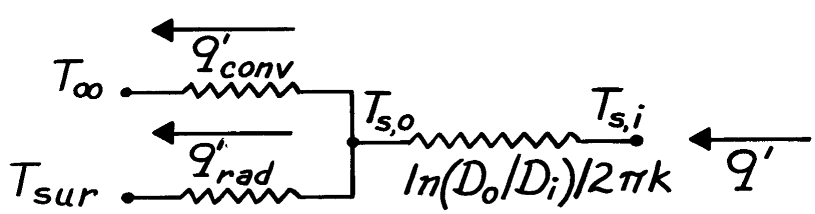
$$q' = \varepsilon\pi D\sigma\left(T_s^4 - T_{sur}^4\right) + h(\pi D)(T_s - T_\infty)$$

$$q' = 0.8\pi(0.2\text{m})5.67\times10^{-8}\ \frac{\text{W}}{\text{m}^2\cdot\text{K}^4}\left(486^4 - 298^4\right)\text{K}^4$$

$$+20\frac{\text{W}}{\text{m}^2\cdot\text{K}}(\pi\times0.2\text{m})\ (486\text{-}298)\text{K}$$

$$q' = (1365+2362)\text{W/m} = 3727\text{ W/m}.$$ <

With the insulation, the thermal circuit is of the form



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From an energy balance at the outer surface of the insulation,

$$q'_{cond} = q'_{conv} + q'_{rad}$$

$$\frac{T_{s,i} - T_{s,o}}{\ln\left(D_o / D_i\right)/2\pi\ k} = h\pi\ D_o\left(T_{s,o} - T_\infty\right) + \varepsilon\sigma\pi\ D_o\left(T_{s,o}^4 - T_{sur}^4\right)$$

$$\frac{\left(486 - T_{s,o}\right)K}{\dfrac{\ln\left(0.3m/0.2m\right)}{2\pi\left(0.058\ W/m\cdot K\right)}} = 20\frac{W}{m^2\cdot K}\pi\left(0.3m\right)\left(T_{s,o} - 298K\right)$$

$$+0.8\times5.67\times10^{-8}\frac{W}{m^2\cdot K^4}\pi\left(0.3m\right)\left(T_{s,o}^4 - 298^4\right)K^4.$$

By trial and error, we obtain

$$T_{s,o} \approx 305K$$

in which case

$$q' = \frac{\left(486\text{-}305\right)K}{\dfrac{\ln\left(0.3m/0.2m\right)}{2\pi\left(0.055\ W/m\cdot K\right)}} = 163\ W/m. \qquad <$$

(b) The yearly energy savings per unit length of pipe due to use of the insulation is

$$\frac{\text{Savings}}{\text{Yr}\cdot\text{m}} = \frac{\text{Energy Savings}}{\text{Yr.}}\times\frac{\text{Cost}}{\text{Energy}}$$

$$\frac{\text{Savings}}{\text{Yr}\cdot\text{m}} = \left(3727 - 163\right)\frac{J}{s\cdot m}\times3600\frac{s}{h}\times7500\frac{h}{Yr}\times\frac{\$4}{10^9 J}$$

$$\frac{\text{Savings}}{\text{Yr}\cdot\text{m}} = \$385/\text{Yr}\cdot\text{m}.$$

The pay back period is then

$$\text{Pay Back Period} = \frac{\text{Insulation Costs}}{\text{Savings/Yr.}\cdot\text{m}} = \frac{\$100/\text{m}}{\$385/\text{Yr}\cdot\text{m}}$$

$$\text{Pay Back Period} = 0.26\ \text{Yr} = 3.1\ \text{mo.} \qquad <$$

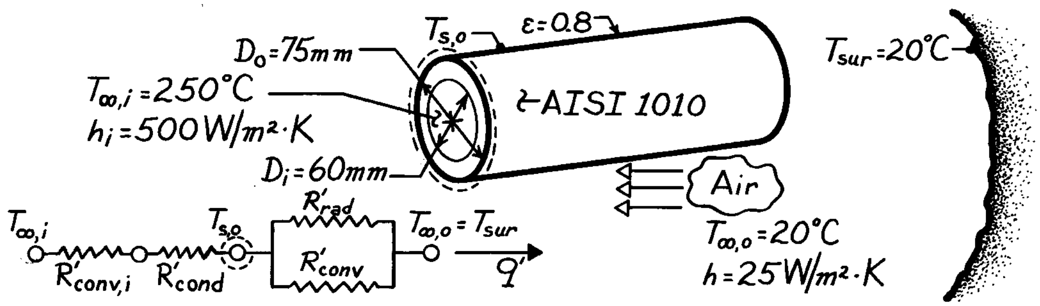
COMMENTS: Such a low pay back period is more than sufficient to justify investing in the insulation.

PROBLEM 3.49

KNOWN: Temperature and convection coefficient associated with steam flow through a pipe of prescribed inner and outer diameters. Outer surface emissivity and convection coefficient. Temperature of ambient air and surroundings.

FIND: Heat loss per unit length.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat transfer, (3) Constant properties, (4) Surroundings form a large enclosure about pipe.

PROPERTIES: *Table A-1*, Steel, AISI 1010 (T ≈ 450 K): k = 56.5 W/m·K.

ANALYSIS: Referring to the thermal circuit, it follows from an energy balance on the outer surface that

$$\frac{T_{\infty,i} - T_{s,o}}{R_{conv,i} + R_{cond}} = \frac{T_{s,o} - T_{\infty,o}}{R_{conv,o}} + \frac{T_{s,o} - T_{sur}}{R_{rad}}$$

or from Eqs. 3.9, 3.28 and 1.7,

$$\frac{T_{\infty,i} - T_{s,o}}{(1/\pi\ D_i h_i) + \ln(D_o/D_i)/2\pi k} = \frac{T_{s,o} - T_{\infty,o}}{(1/\pi\ D_o h_o)} + \varepsilon\pi\ D_o \sigma\left(T_{s,o}^4 - T_{sur}^4\right)$$

$$\frac{523\text{K} - T_{s,o}}{\left(\pi \times 0.6\text{m} \times 500\ \text{W/m}^2\cdot\text{K}\right)^{-1} + \dfrac{\ln(75/60)}{2\pi \times 56.5\ \text{W/m}\cdot\text{K}}} = \frac{T_{s,o} - 293\text{K}}{\left(\pi \times 0.075\text{m} \times 25\ \text{W/m}^2\cdot\text{K}\right)^{-1}}$$

$$+0.8\pi \times (0.075\text{m}) \times 5.67 \times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4\left[T_{s,o}^4 - 293^4\right]\text{K}^4$$

$$\frac{523 - T_{s,o}}{0.0106 + 0.0006} = \frac{T_{s,o} - 293}{0.170} + 1.07 \times 10^{-8}\left[T_{s,o}^4 - 293^4\right].$$

From a trial-and-error solution, $T_{s,o} \approx 502\text{K}$. Hence the heat loss is

$$q' = \pi\ D_o h_o\left(T_{s,o} - T_{\infty,o}\right) + \varepsilon\pi\ D_o \sigma\left(T_{s,o}^4 - T_{sur}^4\right)$$

$$q' = \pi(0.075\text{m})25\ \text{W/m}^2\cdot\text{K}(502\text{-}293) + 0.8\ \pi(0.075\text{m})5.67 \times 10^{-8}\ \frac{\text{W}}{\text{m}^2\cdot\text{K}^4}\left[502^4 - 243^4\right]\text{K}^4$$

$$q' = 1231\ \text{W/m} + 600\ \text{W/m} = 1831\ \text{W/m}.$$ <

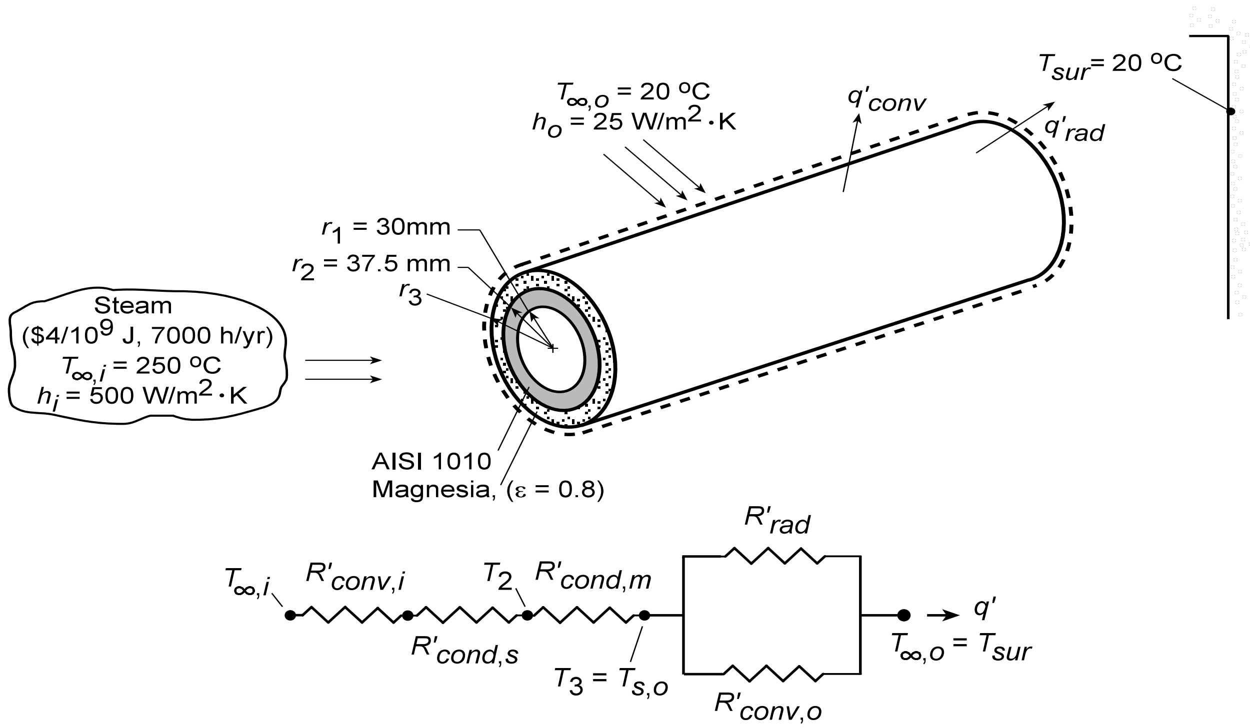
COMMENTS: The thermal resistance between the outer surface and the surroundings is much larger than that between the outer surface and the steam.

PROBLEM 3.50

KNOWN: Temperature and convection coefficient associated with steam flow through a pipe of prescribed inner and outer radii. Emissivity of outer surface magnesia insulation, and convection coefficient. Temperature of ambient air and surroundings.

FIND: Heat loss per unit length q' and outer surface temperature $T_{s,o}$ as a function of insulation thickness. Recommended insulation thickness. Corresponding annual savings and temperature distribution.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat transfer, (3) Constant properties, (4) Surroundings form a large enclosure about pipe.

PROPERTIES: *Table A-1*, Steel, AISI 1010 (T ≈ 450 K): k_s = 56.5 W/m·K. *Table A-3*, Magnesia, 85% (T ≈ 365 K): k_m = 0.055 W/m·K.

ANALYSIS: Referring to the thermal circuit, it follows from an energy balance on the outer surface that

$$\frac{T_{\infty,i} - T_{s,o}}{R'_{conv,i} + R'_{cond,s} + R'_{cond,m}} = \frac{T_{s,o} - T_{\infty,o}}{R'_{conv,o}} + \frac{T_{s,o} - T_{sur}}{R'_{rad}}$$

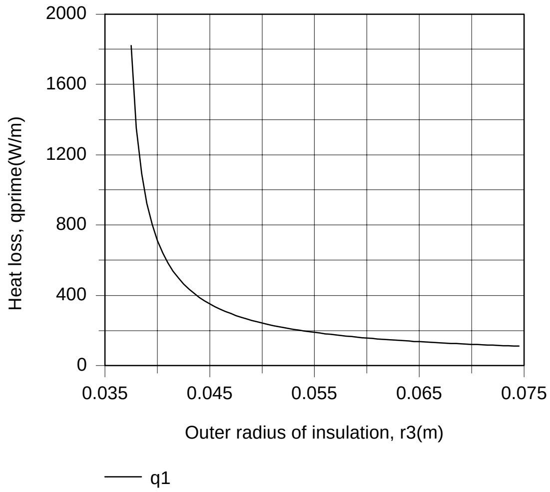
or from Eqs. 3.9, 3.28 and 1.7,

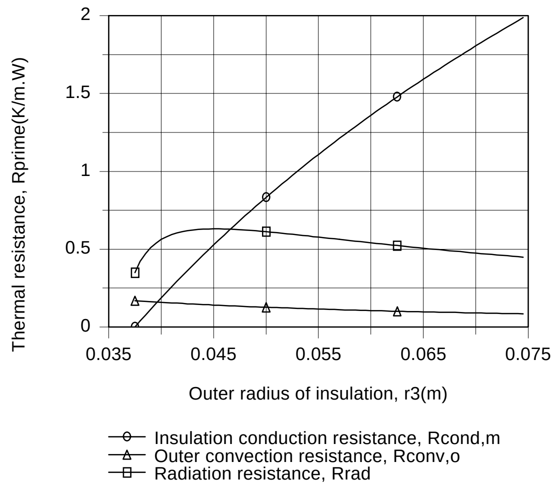
$$\frac{T_{\infty,i} - T_{s,o}}{(1/2\pi r_1 h_i) + \ln(r_2/r_1)/2\pi k_s + \ln(r_3/r_2)/2\pi k_m} = \frac{T_{s,o} - T_{\infty,o}}{(1/2\pi r_3 h_o)} + \frac{T_{s,o} - T_{sur}}{\left[(2\pi r_3)\varepsilon\sigma\left(T_{s,o} + T_{sur}\right)\left(T_{s,o}^2 + T_{sur}^2\right)\right]^{-1}}$$

This expression may be solved for $T_{s,o}$ as a function of r_3, and the heat loss may then be determined by evaluating either the left-or right-hand side of the energy balance equation. The results are plotted as follows.

Continued...

PROBLEM 3.50 (Cont.)



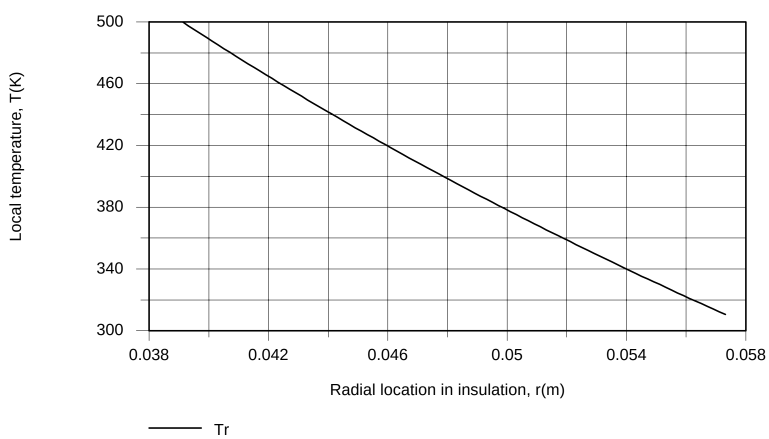


The rapid decay in q' with increasing r_3 is attributable to the dominant contribution which the insulation begins to make to the total thermal resistance. The inside convection and tube wall conduction resistances are fixed at 0.0106 m·K/W and 6.29×10^{-4} m·K/W, respectively, while the resistance of the insulation increases to approximately 2 m·K/W at r_3 = 0.075 m.

The heat loss may be reduced by almost 91% from a value of approximately 1830 W/m at $r_3 = r_2$ = 0.0375 m (no insulation) to 172 W/m at r_3 = 0.0575 m and by only an additional 3% if the insulation thickness is increased to r_3 = 0.0775 m. Hence, an insulation thickness of $(r_3 - r_2)$ = 0.020 m is recommended, for which q' = 172 W/m. The corresponding annual savings (AS) in energy costs is therefore

$$AS = \left[(1830 - 172)\,W/m\right]\frac{\$4}{10^9\,J}\times 7000\frac{h}{y}\times 3600\frac{s}{h} = \$167/m \qquad <$$

The corresponding temperature distribution is



The temperature in the insulation decreases from $T(r) = T_2$ = 521 K at $r = r_2$ = 0.0375 m to $T(r) = T_3$ = 309 K at $r = r_3$ = 0.0575 m.

Continued...

PROBLEM 3.50 (Cont.)

COMMENTS: 1. The annual energy and costs savings associated with insulating the steam line are substantial, as is the reduction in the outer surface temperature (from $T_{s,o} \approx 502$ K for $r_3 = r_2$, to 309 K for $r_3 = 0.0575$ m).

2. The increase in R'_{rad} to a maximum value of 0.63 m·K/W at $r_3 = 0.0455$ m and the subsequent decay is due to the competing effects of h_{rad} and $A'_3 = (1/2\pi r_3)$. Because the initial decay in $T_3 = T_{s,o}$ with increasing r_3, and hence, the reduction in h_{rad}, is more pronounced than the increase in A'_3, R'_{rad} increases with r_3. However, as the decay in $T_{s,o}$, and hence h_{rad}, becomes less pronounced, the increase in A'_3 becomes more pronounced and R'_{rad} decreases with increasing r_3.