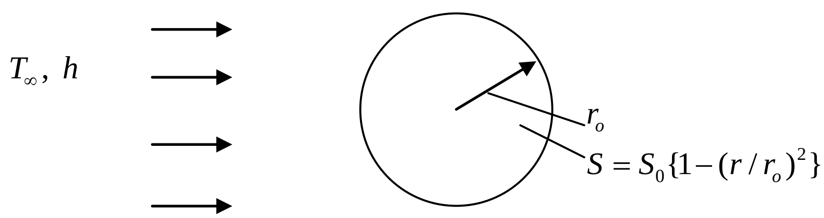
Problem Set # 2

Given: Wed, Sep. 12 **Recommended Completion Date:** Wed., Sep. 19
Do not submit for grading

Problem 1: Radioactive wastes are packed in a thin-walled spherical container. The wastes generate thermal energy according to the relation $S = S_o\{1-(r/r_o)^2\}$, where S is the local rate of heat generation per unit volume, S_o is a constant, and r_o is the radius of the container. Steady-state conditions are maintained by submerging the container in a liquid that is at T_∞ and provides a uniform (for all practical purposes) convection heat transfer coefficient h at the surface of the container. Ignore the thermal conduction resistance of the container wall and also the thermal contact resistance at the interface between the radioactive wastes and the inside surface of the container wall.



(a) Obtain an expression for the total rate of heat generation inside the container.

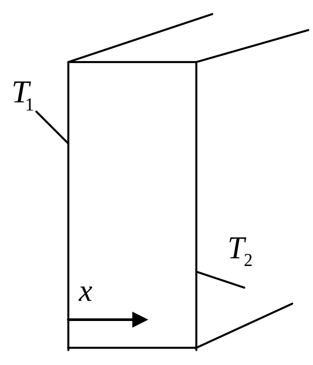
(b) Use the result obtained in part (a) to derive an expression for the temperature of the container wall (T_s).

Ans.: (a) $q_{gen} = \dfrac{8}{15}\pi S_o r_o^3$; (b) $T_s = \dfrac{2}{15}\dfrac{S_o r_o}{h} + T_\infty$

Problem 2: The steady-state temperature distribution inside a certain plane wall is given by the following expression:

$$\frac{(T-T_1)}{(T_2-T_1)} = C_1 + C_2x^2 + C_3x^3$$

where T_1 and T_2 are the temperatures on each side of the wall, as shown in the adjoining figure. The thermal conductivity of the wall, k, is a constant, and its thickness is L. The volumetric rate of heat generation at $x = 0$ is S_o. Derive an expression for the volumetric rate of heat generation inside the wall as a function of $x, S_o, k, L, T_1,$ *and* T_2.



Ans.: $S = S_o - \left(\dfrac{3S_o}{L} - \dfrac{6k(T_1-T_2)}{L^3}\right)x$

Problem 3: A plane wall of thickness of 10 cm is made of an isotropic, constant-density solid that has thermal conductivity k = 22 W/m-K, density ρ = 4500 kg/m^3, and specific heat c = 510 J/kg-K. This wall has an instantaneous temperature distribution of $T = 500 - 2500x + 6000x^2$, with T in K and x in meters; and x = 0 denotes the left surface. There is no volumetric source term inside the wall: S = 0. Determine (i) the net rate of heat transfer to the plane wall by conduction across its boundaries per unit surface area; and (ii) the rate at which the temperature at the centre of the wall is changing with time in [K/s]: $\partial T / \partial t \;_{x=5cm} = ?$

Ans.: (i) 26400 W/m2; (ii) 0.115 K/s

Problem 4: A spherical container has an inner diameter of 40 cm. It wall consists of a 4 cm thick layer of lead, covered by 1 cm thick layer of stainless steel 347, and 3 cm layer of a specialty concrete. The container is filled with nuclear wastes that generate heat at a rate of $S = S_o[1 - (r/r_1)^2]$, where r is the radial coordinate, r_1 = 20 cm, and S_o is a constant (> 0) in [W/m^3]. The container is placed in deep ocean waters, at a location where the far-field temperature is T_∞ = 8 °C and the average heat transfer coefficient at the outer surface of the container is h_{av} = 115 W/m^2-K. The thermal contact coefficients at the interfaces between the nuclear wastes and the lead, the lead and the stainless steel 347, and the stainless steel 347 and the concrete are the following, respectively:
$h_{c,\ \text{nuclear wastes-lead}} = h_{c,\ \text{lead-ss}}$ = 2000 W/m^2-K, and $h_{c,\ \text{ss-concrete}}$ = 1000 W/m^2-K. Design considerations require that the maximum temperature in the lead should remain 100 °C below its melting temperature. For steady-state conditions, and assuming one-dimensional radial heat transfer, (i) sketch a qualitatively accurate temperature profile (T vs. r); (ii) calculate the maximum allowable value of S_o; and (iii) calculate the maximum temperature inside the nuclear wastes for the S_o value calculated in part (ii). Ignore radiation heat transfer from the surface of the spherical container. For the steady-state temperatures expected to prevail in this problem, the following property data may be assumed to apply and remain essentially constant:

Nuclear wastes: $k_{\text{nuclear wastes}}$= 16.0 W/m-K

Lead: k_{lead} = 32.7 W/m-K; $T_{\text{lead, melting}}$= 601 K

347 Stainless Steel: $k_{s.s}$= 17.5 W/m-K

Specialty Concrete: k_{concrete}= 1.1 W/m-K

Ans.: (ii) $S_{o,\ max}$ = 368061.9 W/m^3; (iii) T_{max} = 340.1°C

Selected Problems from the Textbook (Incropera et al.)

Please do the following problems (either from 6th or 7th Editions):

Sixth Edition 2007	Seventh Edition 2011
2. 4	**2. 5**
2.14	**2.19**
2.22	**2.28**
2.39	**2.48**