University of British Columbia
Department of Mechanical Engineering

MECH366 Modeling of Mechatronic Systems
Homework 7

Solutions

To solve the following questions, if necessary, you can use a calculator.

1. For an unknown system, suppose that we applied a step input with amplitude 10 (i.e., $10u(t)$), and that the response was obtained as shown in Slide 16 of Lecture 17. Derive (roughly) the transfer function. Explain the derivation process in detail.

   Solution: The peak time is about π seconds. So,

   $$\frac{\pi}{|\mathrm{Im}|} \approx \pi \;\Rightarrow\; |\mathrm{Im}| \approx 1$$

   The 2% settling time is about 15 seconds. So,

   $$\frac{4}{|\mathrm{Re}|} \approx 15 \;\Rightarrow\; |\mathrm{Re}| \approx 0.26.$$

   Since the response is converging to 1, the DC gain is 1/10 (because the input amplitude is 10). Thus, the transfer function is

   $$G(s) = \frac{1}{10} \cdot \frac{0.26^2 + 1^2}{(s+0.26)^2 + 1^2} = \frac{0.10676}{s^2 + 0.52s + 1.0676}.$$

2. Let us consider the accelerometer in Slides 20–22 in Lecture 14. In this question, you do not need to re-derive the transfer function from the force $F(s)$ to the displacement $Y(s)$ of the proof-mass; just use

   $$G(s) := \frac{Y(s)}{F(s)} = -\frac{1}{M} \cdot \frac{1}{s^2 + (b/m)s + (k/m)}.$$

   For a vehicle mass $M = 2,000$ [kg] and a proof-mass $m = 0.002$ [kg], design the spring constant k [N/m] and damping constant b [Ns/m], to meet the following requirements. For a unit step input force,

   Requirement 1: the percent overshoot (in this case, undershoot) is about 5%. (Hint: This corresponds to about 45 degree of the pole locations.)

   Requirement 2: the 2% settling time is about 10 [ms] (fast enough, so that the airbag can inflate promptly after the collision).

   (The obtained values might not be realistic, but this is just an exercise question.)

   Solution:

Requirement 1: $|\text{Re}| = |\text{Im}|$
Requirement 2:

$$\frac{4}{|\text{Re}|} = 0.01 \;\Rightarrow\; |\text{Re}| = 400$$

So, the denominator of the transfer function includes

$$(s+400)^2 + 400^2 = s^2 + 800s + 320000 \;\Rightarrow\; \frac{b}{m} = 800,\; \frac{k}{m} = 320000$$

Since $m = 0.002$, we have $b = 1.6$ and $k = 640$.