ELEC 343
Electromechanics
Module 4: Stepper Motors (Chap. 9)

Spring 2019
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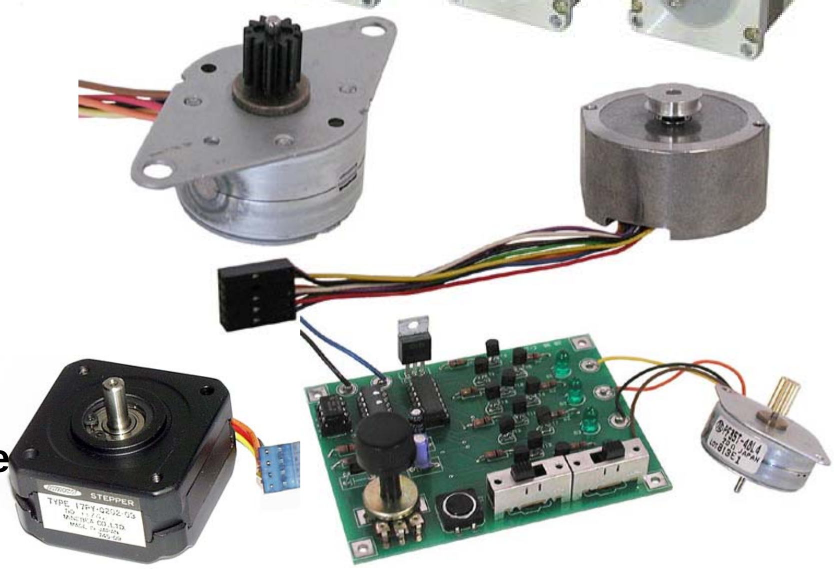
Class Webpage:
http://courses.ece.ubc.ca/elec343

Objectives and Important Concepts
- Types and construction of commonly used Stepper Motors
- Principle of torque production
- Permanent Magnet (PM), Multi-Stack, and Variable Reluctance Motors
- Full-, half-, and micro-stepping operation
- Torque-angle characteristics, static position error
- Driving circuits

Typical Stepper Motors



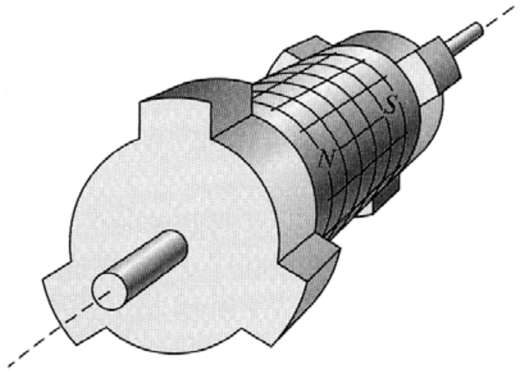
Permanent Magnet
- Bipolar
- Unipolar

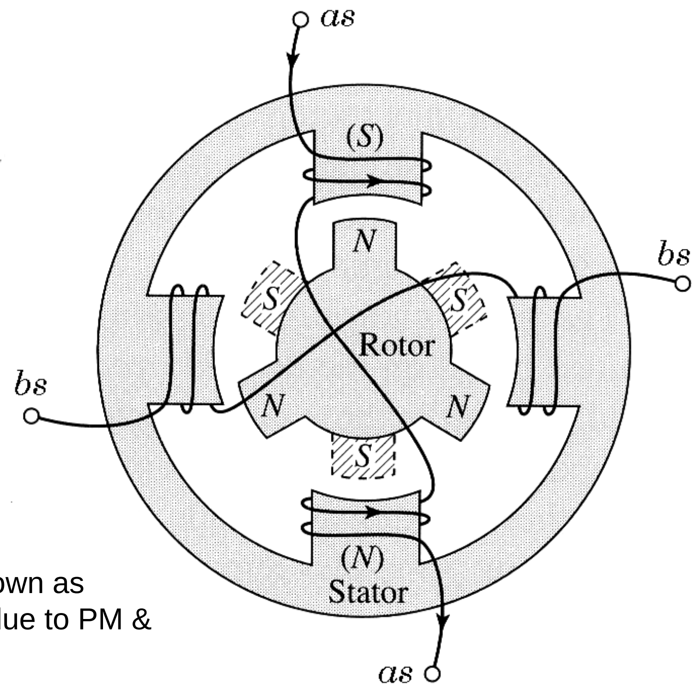
Variable Reluctance
- Multi-Stack
- Single-Stack

Note: Stepper motors are typically used for low speed applications
However, Variable Reluctance, known as Switched Reluctance Motors are used for very high speeds

Permanent Magnet (PM) Stepper Motor

Consider 2-phases $N_\phi =$
2-pole, 2-rotor stack,
3-rotor teeth, 4-stator teeth





This type of stepper motor is also known as **hybrid** because torque is produced due to PM & reluctance effects

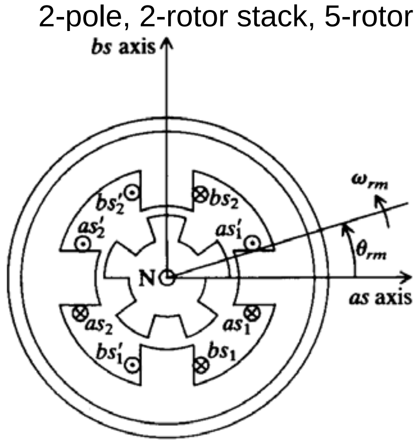
Tooth-Pitch (displacement between teeth) $TP =$

Step Length (SL) $\theta_{SL} =$

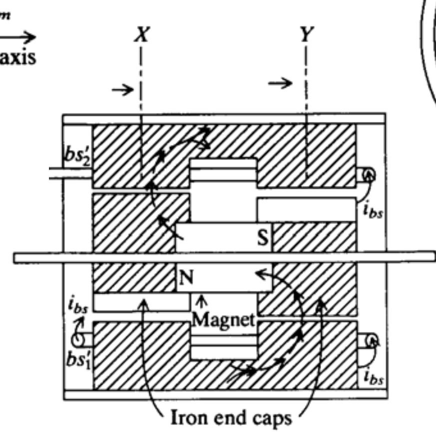
Permanent Magnet (PM) Stepper Motor

Consider 2-phase configurations

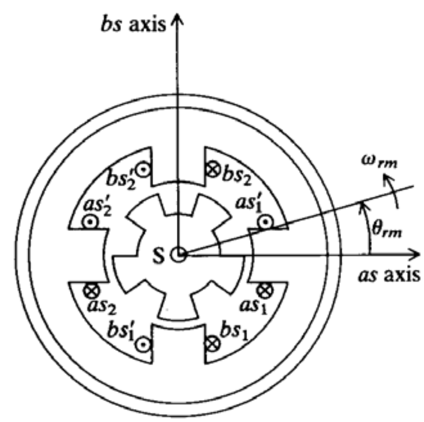
2-pole, 2-rotor stack, 5-rotor teeth, 4-stator teeth



Front



Cross-sectional view



Back

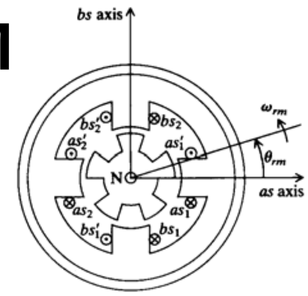
Stepping of PMStM

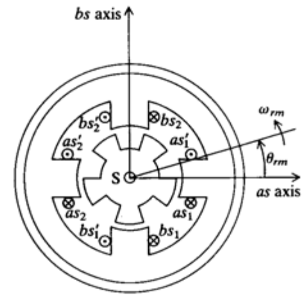
Assume basic commutating circuit

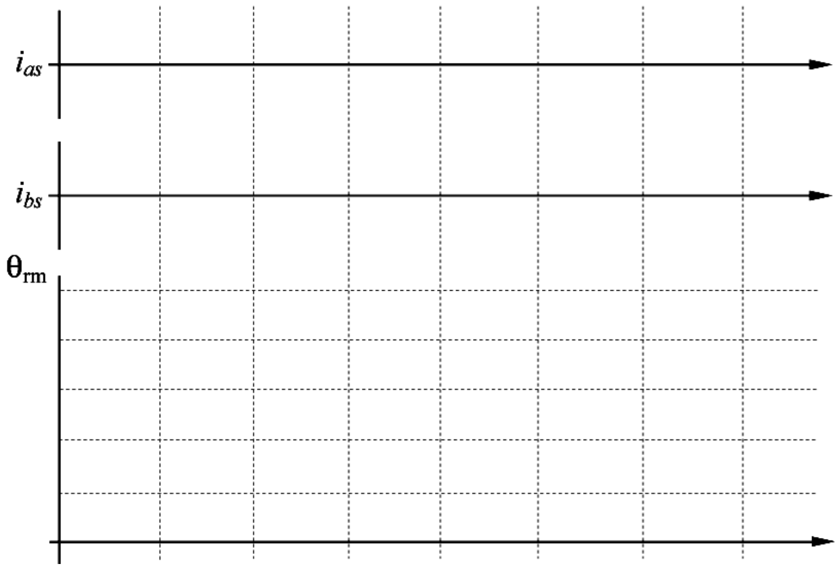
Full Stepping – one phase energized at a time (i_{as}, i_{bs} , -i_{as}, - i_{bs}, i_{as}, … sequence)

Tooth-Pitch

$$TP = \frac{360^\circ}{N_T} =$$







Step Length (SL)

$$\theta_{SL} = \frac{TP}{2N_\phi} =$$

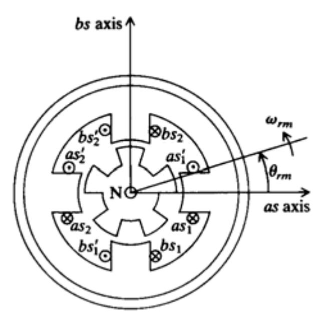
$$= \frac{180^\circ}{N_{RT} N_\phi} =$$

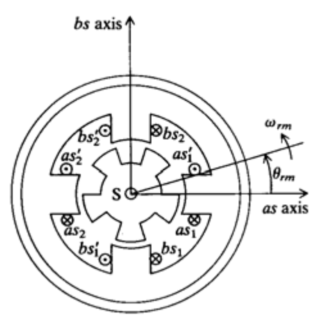
Stepping of PMStM

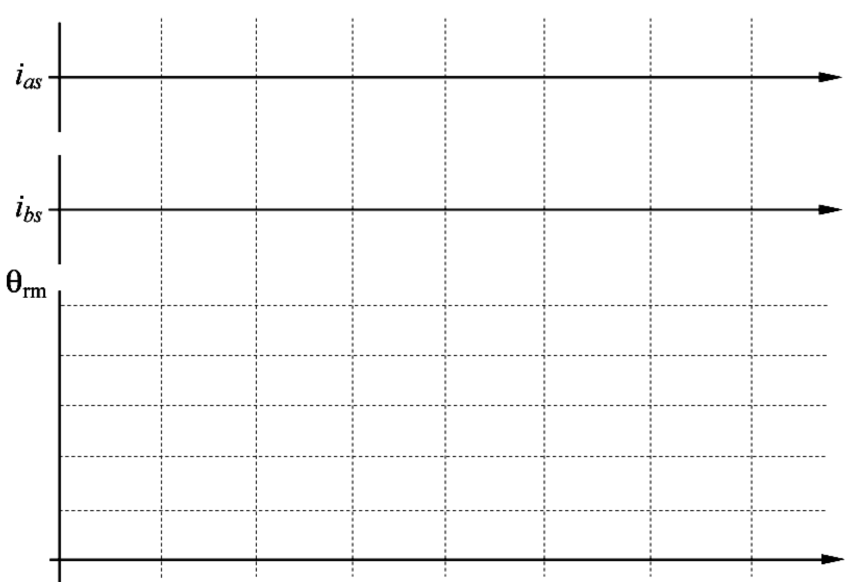
Assume basic commutating circuit

Step Length (SL) $\theta_{SL} =$

Full Stepping – 2 phase energized at all times (a+b, -a+b, -a-b, a-b, … sequence)





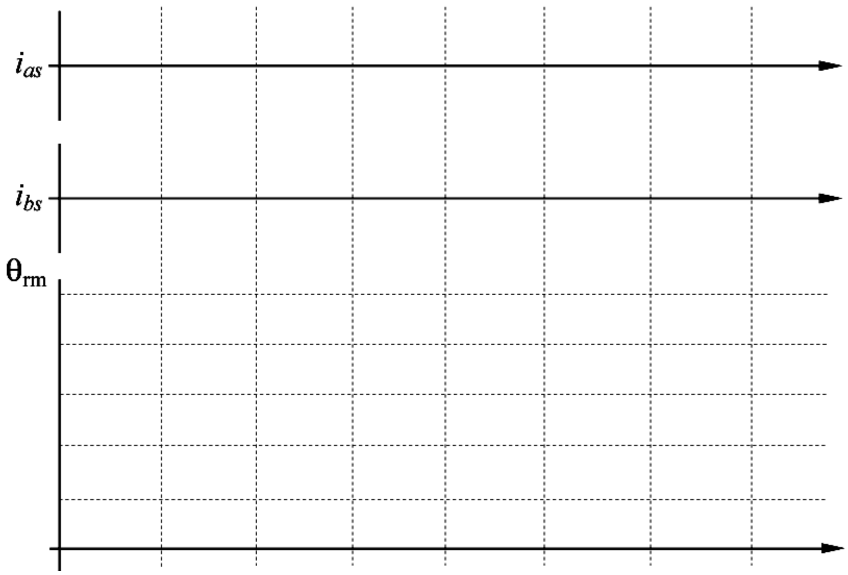


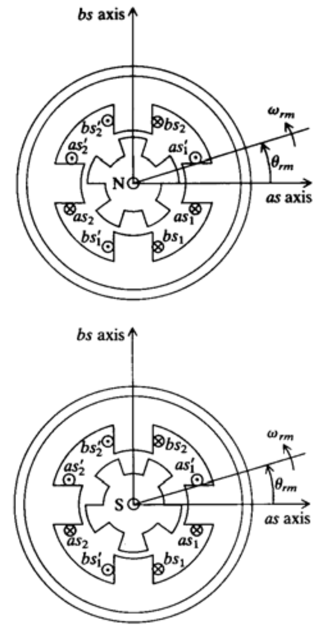
Stepping of PMStM

Assume basic commutating circuit

Step Length (SL) $\theta_{SL,hs} =$

Half Stepping – allow up to 2 phase energized at a time (a, a+b, b, -a+b, -a, -a-b, -b, a-b, … sequence)





Stepper Motors Terminology

Number of phases N_ϕ

Number of Rotor Teeth N_{RT}

Number of Stator Teeth N_{ST}

Tooth-Pitch (displacement between teeth)

$$TP = \frac{360^\circ}{N_T}[\text{deg}] = \frac{2\pi}{N_T}[\text{rad}]$$

Step Length (SL – full step) $$\theta_{SL} = \frac{TP}{2N_\phi} = \frac{\pi}{N_{RT}N_\phi}[\text{rad}] = \frac{180}{N_{RT}N_\phi}[\text{deg}]$$

Resolution (steps / revolution) $$\frac{Steps}{\text{Rev.}} = \frac{360^\circ}{\theta_{SL}[\text{deg}]} = \frac{2\pi}{\theta_{SL}[\text{rad}]}$$

Stepping frequency (pulses per-second per-phase) $f_{step} = [\text{pulse}/\text{sec}]$

Rotor position (for no-load operation) $\theta_{rm} = \theta_{rm}(0) + \theta_{SL} \cdot N_{steps} \pm \varepsilon$

Speed

$$n[\text{rev}/\text{sec}] = \frac{\theta_{SL}[\text{deg}] \cdot f_{step}}{360} = \frac{\theta_{SL}[rad] \cdot f_{step}}{2\pi}$$

$$n[\text{rev}/\text{min}] = n[\text{rpm}] = \frac{\theta_{SL}[\text{deg}] \cdot f_{step}}{6} = 30\frac{\theta_{SL}[rad] \cdot f_{step}}{\pi}$$

2-Pole, 2-Stack, 4/5-Teeth PMStM

Voltage equations

$$v_{as} = r_s i_{as} + \frac{d\lambda_{as}}{dt}$$

$$v_{bs} = r_s i_{bs} + \frac{d\lambda_{bs}}{dt}$$

$$\mathbf{v}_{abs} = \mathbf{r}_s \mathbf{i}_{abs} + \frac{d\boldsymbol{\lambda}_{abs}}{dt}$$

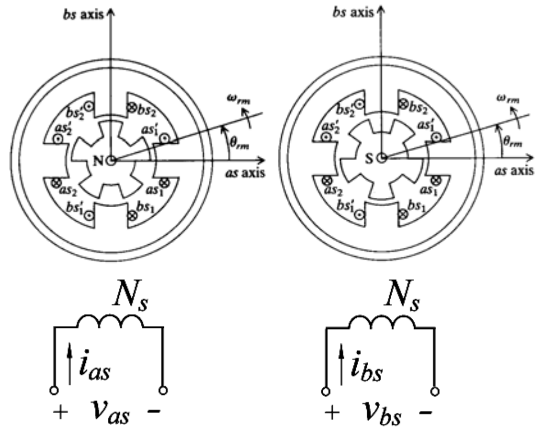
Flux linkage equations

$$\begin{bmatrix} \lambda_{as} \\ \lambda_{bs} \end{bmatrix} = \begin{bmatrix} L_{asas} & L_{asbs} \\ L_{bsas} & L_{bsbs} \end{bmatrix} \cdot \begin{bmatrix} i_{as} \\ i_{bs} \end{bmatrix} + \begin{bmatrix} \lambda_{asm} \\ \lambda_{bsm} \end{bmatrix}$$

$$\boldsymbol{\lambda}_{abs} = \mathbf{L}_s \mathbf{i}_{abs} + \boldsymbol{\lambda}_m$$

Flux linkage due to Permanent Magnet

$$\boldsymbol{\lambda}_m = \lambda_m \begin{bmatrix} \cos(N_{RT}\theta_{rm}) \\ \sin(N_{RT}\theta_{rm}) \end{bmatrix}$$



Neglect mutual inductances =
Neglect reluctance torque

$$\mathbf{L}_s = \begin{bmatrix} L_{asas} & 0 \\ 0 & L_{bsbs} \end{bmatrix}$$

$$= \begin{bmatrix} L_{ls} + L_{ms} & 0 \\ 0 & L_{ls} + L_{ms} \end{bmatrix}$$

2-Pole, 2-Stack, 4/5-Teeth PMStM

Electromagnetic Torque $T_e = \dfrac{\partial W_c(\mathbf{i}, \theta_{rm})}{\partial \theta_{rm}}$

Assume magnetically linear system $W_c = W_f$

$$W_f = \frac{1}{2} L_{asas} i_{as}^2 + \frac{1}{2} L_{bsbs} i_{bs}^2 + \lambda_{asm} i_{as} + \lambda_{bsm} i_{bs}$$

$$T_e = -N_{RT} \lambda_m \left[i_{as} \sin(N_{RT}\theta_{rm}) - i_{bs} \cos(N_{RT}\theta_{rm}) \right]$$

Note: Torque is proportional to the number of teeth & the strength of the PM!

Mechanical System

$$J_{total} \frac{d\omega_{rm}}{dt} + D_m \omega_{rm} = T_e - T_m$$

Rotor position $\theta_{rm} = \theta_{rm}(0) + \int \omega_{rm} dt$

$$J_{total} \frac{d^2\theta_{rm}}{dt^2} + D_m \frac{d\theta_{rm}}{dt} = T_e - T_m$$

Motor Stepping Operation

Electromagnetic Torque $T_e = -N_{RT}\lambda_m\left[i_{as}\sin\left(N_{RT}\theta_{rm}\right)-i_{bs}\cos\left(N_{RT}\theta_{rm}\right)\right]$

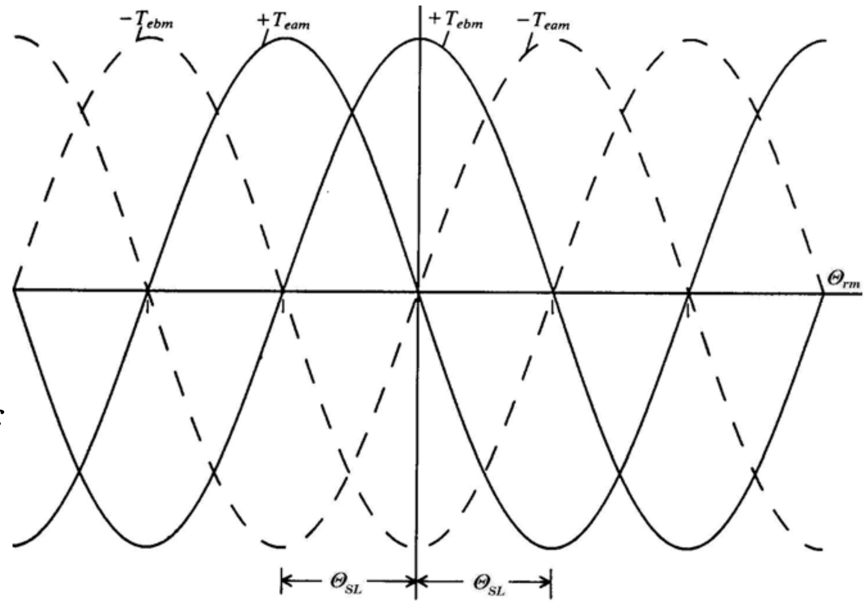
Maximum holding torque

$$T_{\max} = N_{RT}\lambda_m i_{as}$$

Rotor position

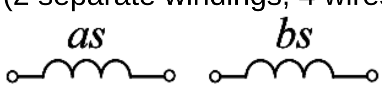
$$\theta_{rm} = \theta_{SL}\cdot N_{steps} \pm \varepsilon$$

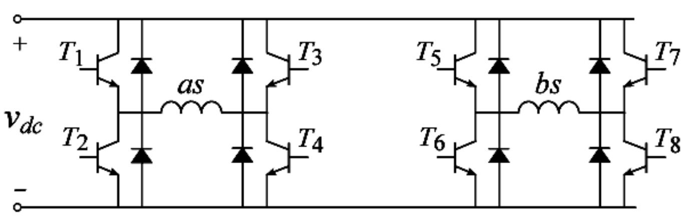
Static position error



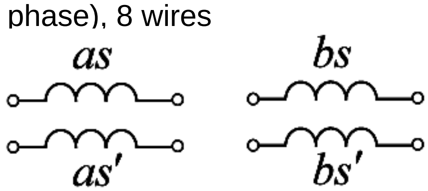
Practical Driver Circuits

Bipolar winding
(2 separate windings, 4 wires)

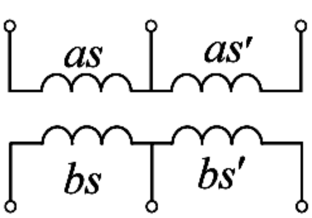


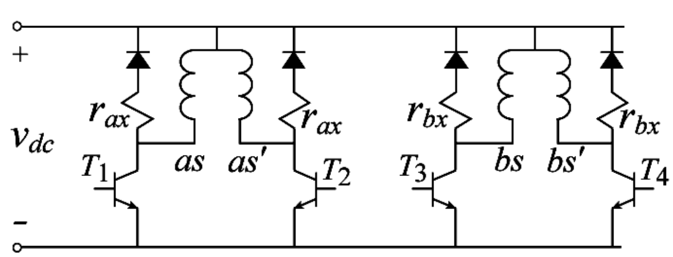


Bifilar winding
(2 separate windings for each phase), 8 wires



Unipolar winding
(each phase has mid-point),
6 or 5 wires

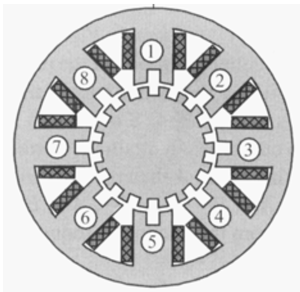


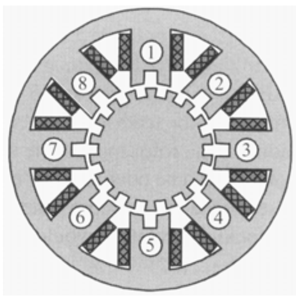


Practical PM Stepper Motor

Consider 2-phase configurations

8-pole, 2-rotor stack,
18-rotor teeth, 16-stator teeth

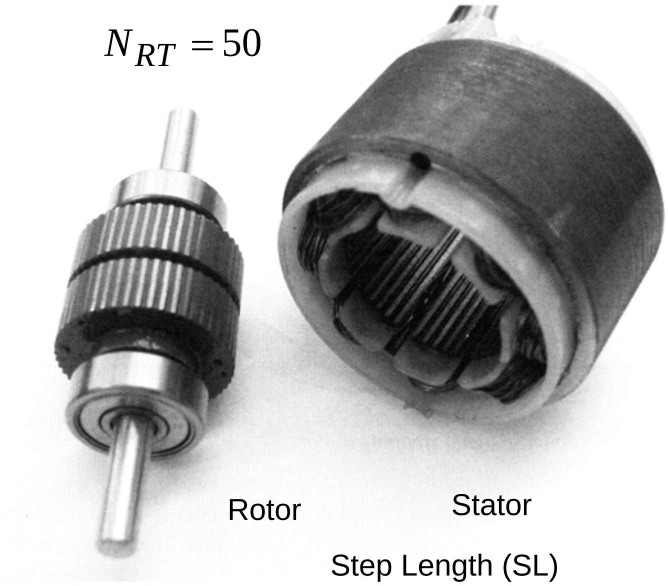




Cross-sectional view

Typical 2-phase motor

$N_{RT} = 50$



Rotor

Stator

Phase A – 1, 3, 5, 7
Phase B – 2, 4, 6, 8

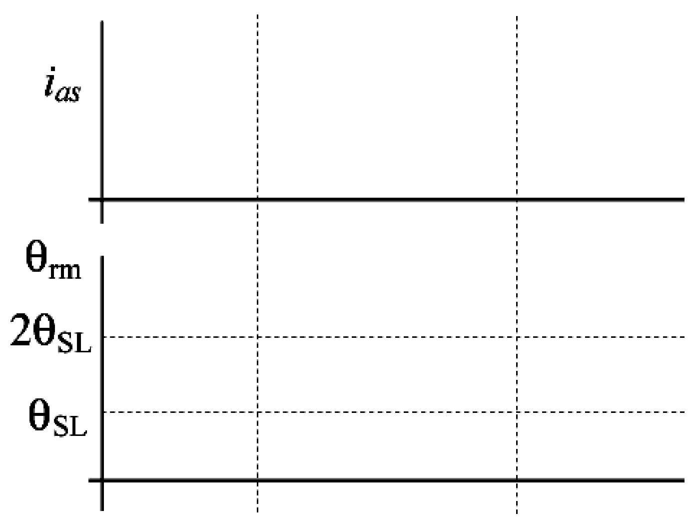
Step Length (SL)

$$\theta_{SL} = \frac{180^\circ}{N_{RT} N_\phi} = 1.8^\circ$$

Practical Issues

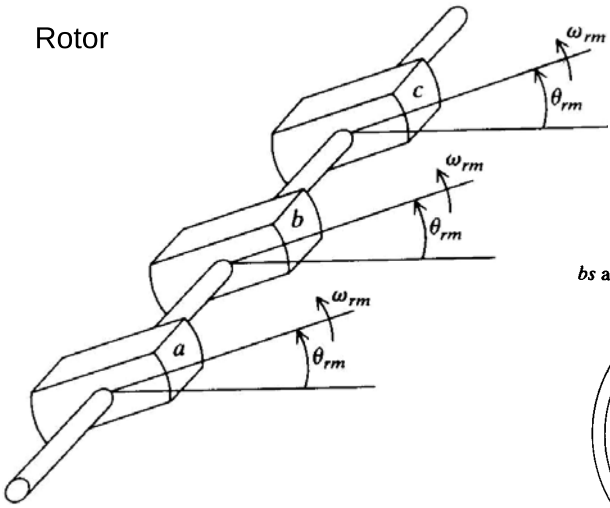
$$J_{total} \frac{d^2\theta_{rm}}{dt^2} + D_m \frac{d\theta_{rm}}{dt} = T_e - T_m$$

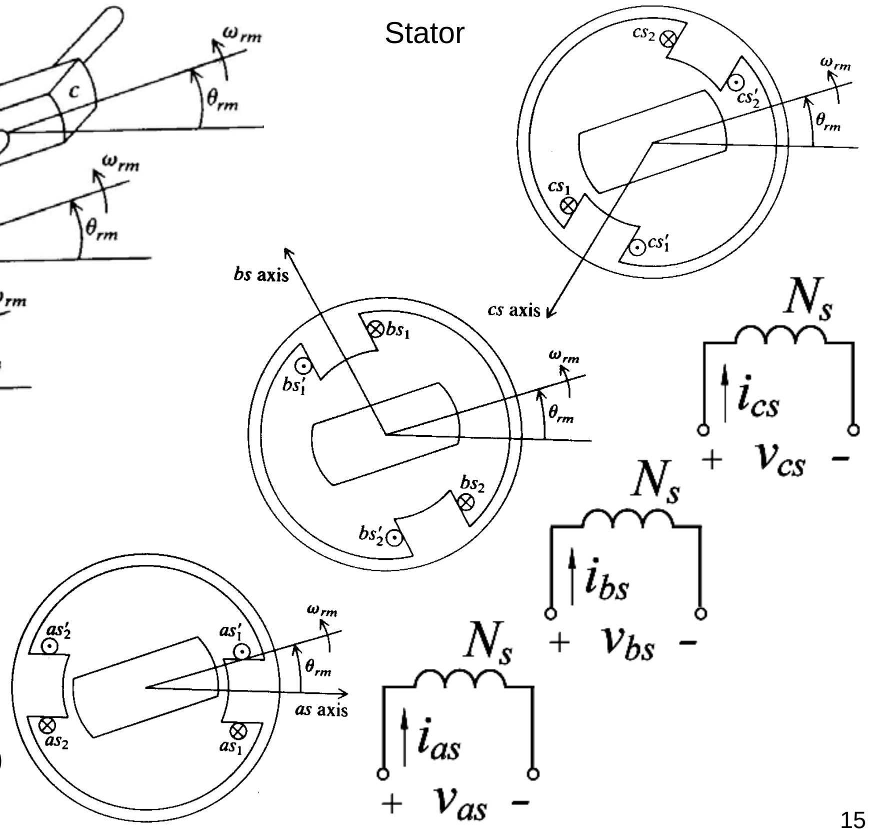
- Electromechanical Resonance !
- Degradation of operation at high speeds !



Multi-Stack VR Stepper Motors (MSVRStM)

Rotor



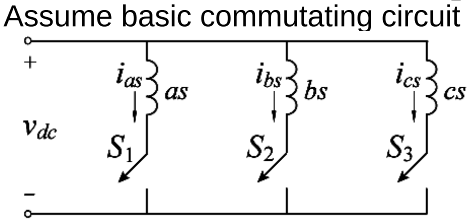


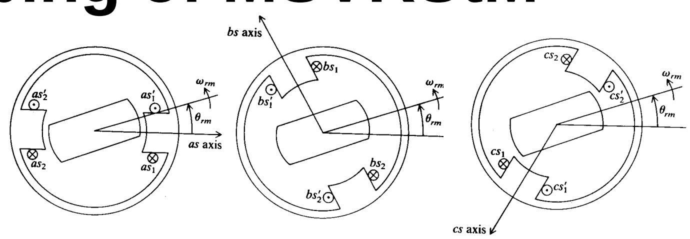
- One stack per electrical "phase"
- "Phases" are magnetically decoupled

2-pole, 3-stack (phases) VR Stepper Motor

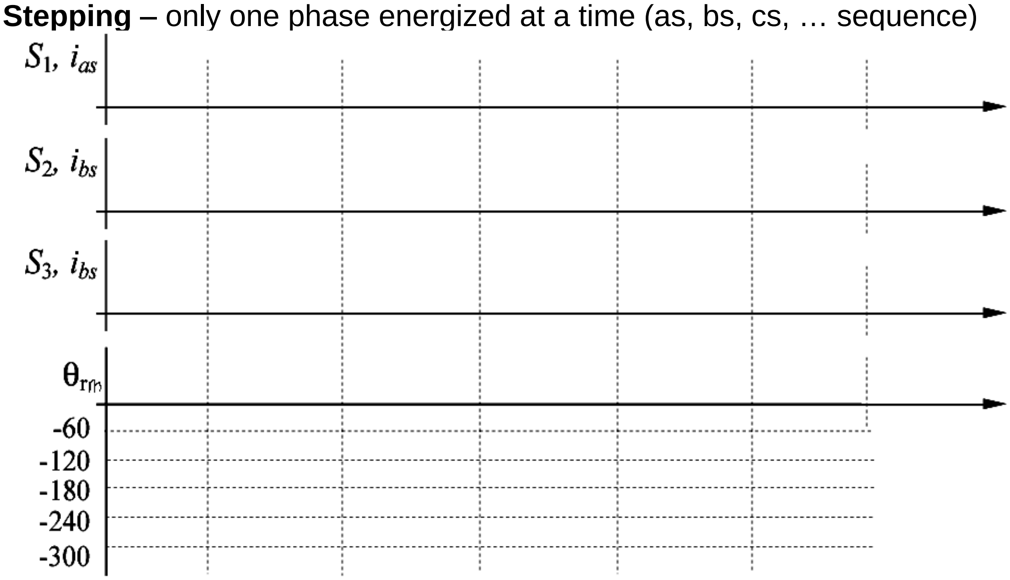
Basic Stepping of MSVRStM

Assume basic commutating circuit



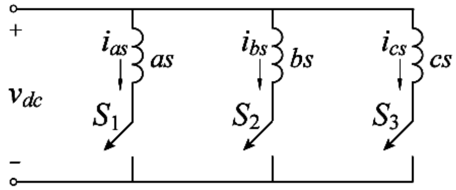


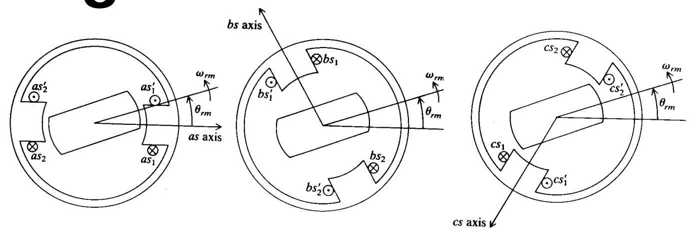
Full Stepping – only one phase energized at a time (as, bs, cs, … sequence)



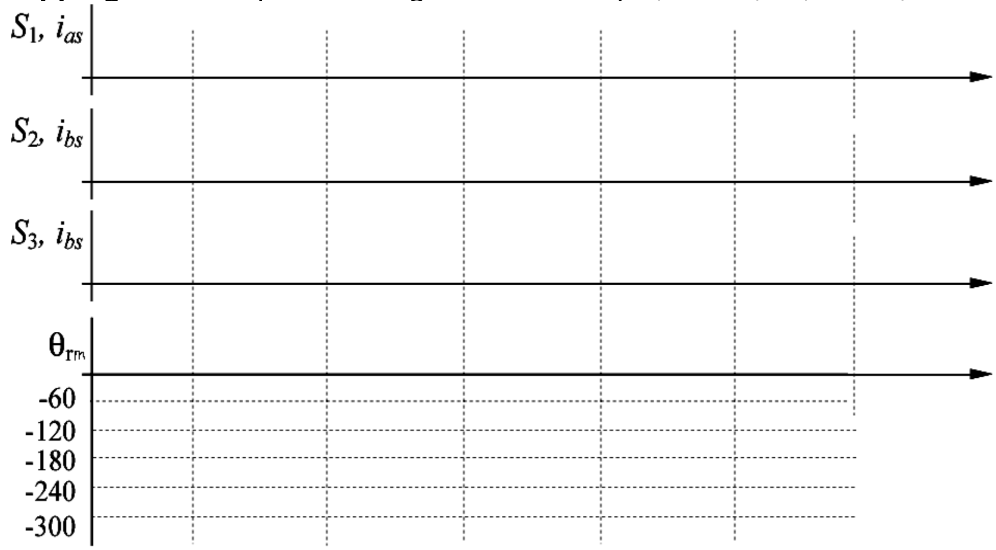
Basic Stepping of MSVRStM

Assume basic commutating circuit



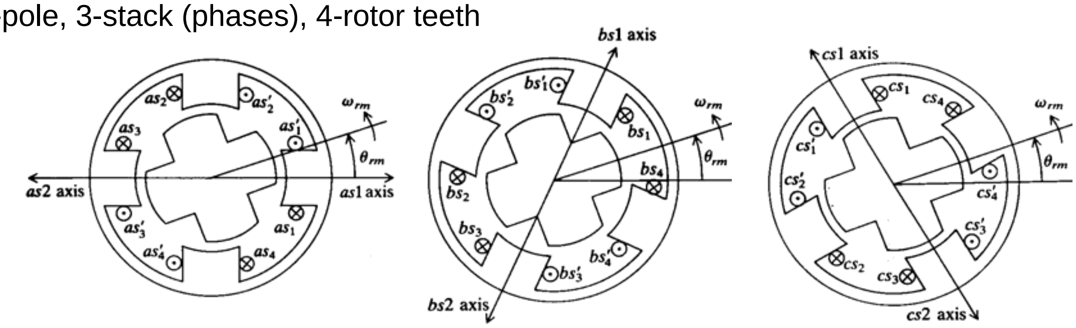


Half Stepping – allow 2 phase energized at a time (as, as+bs, bs, bc+cs, … sequence)

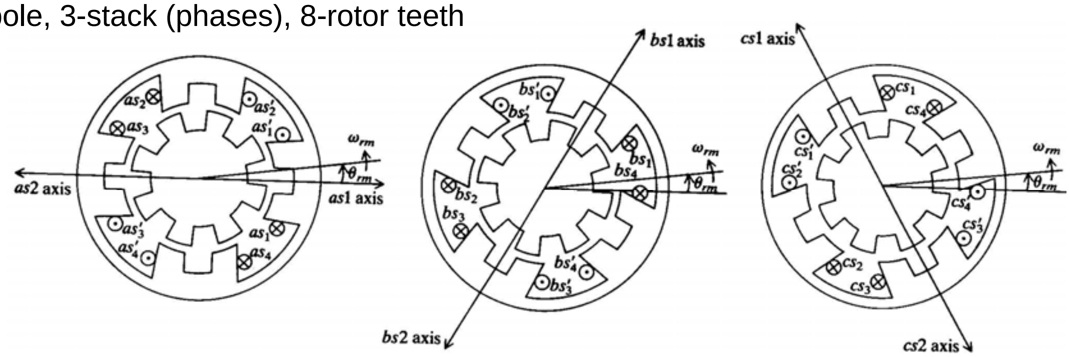


Multi-Stack Stepper Motors

4-pole, 3-stack (phases), 4-rotor teeth

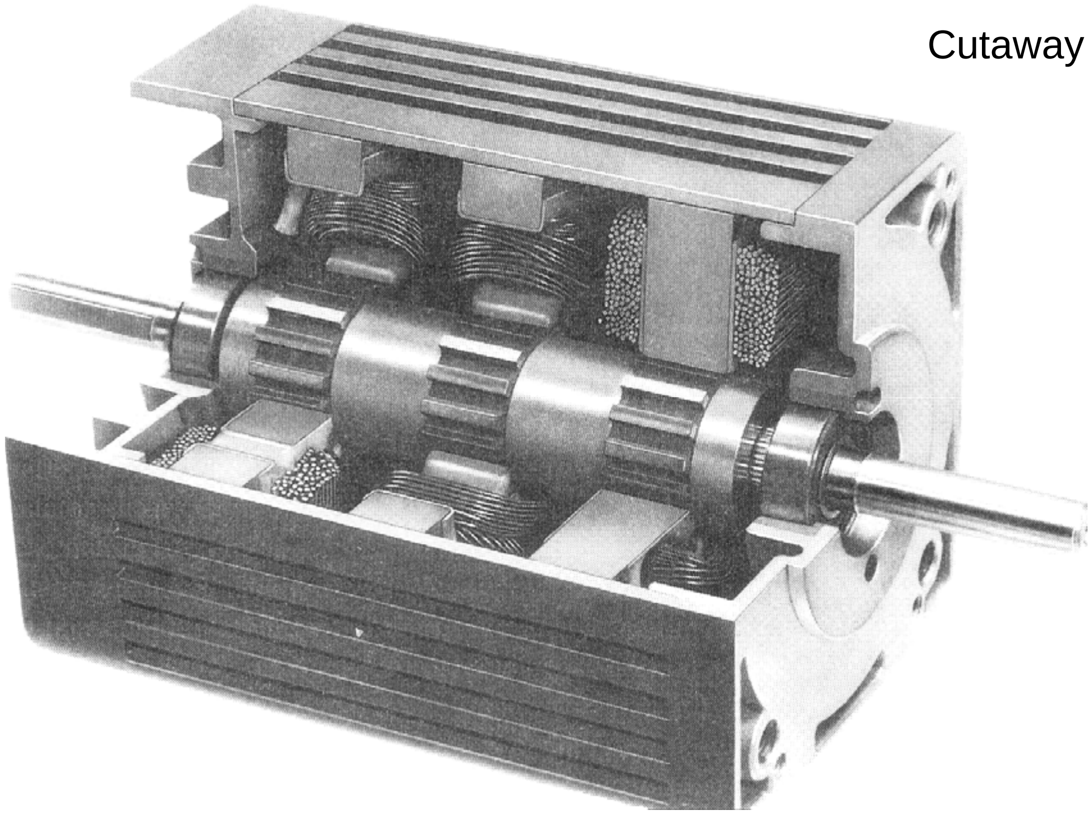


4-pole, 3-stack (phases), 8-rotor teeth



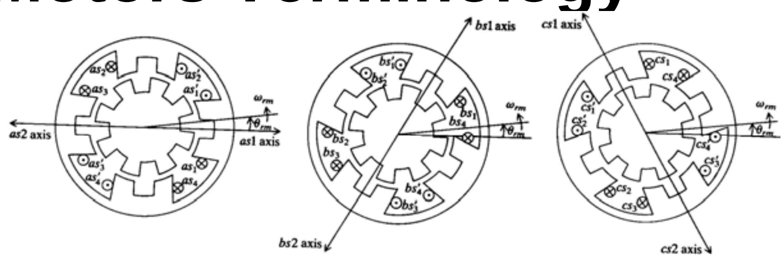
Multi-Stack Stepper Motors

Cutaway view



Stepper Motors Terminology

Multi-Stack



Number of stacks (phases)

$$N_{stack} = N_{\phi}$$

Number of Rotor Teeth N_{RT}

For MSVRStM, $N_{ST} = N_{RT} = N_T$

Number of Rotor Teeth N_{ST}

Tooth-Pitch (displacement between teeth)

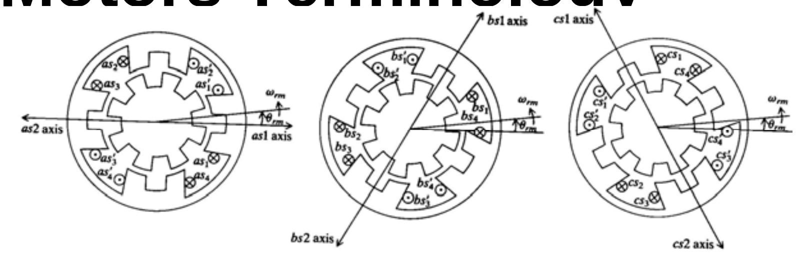
$$TP = \frac{360^{\circ}}{N_T}[\text{deg}] = \frac{2\pi}{N_T}[\text{rad}]$$

Step Length (SL)

$$\theta_{SL} = \frac{TP}{N_{\phi}} = \frac{TP}{N_{stacks}} = \frac{2\pi}{N_T N_{\phi}}[\text{rad}] = \frac{360}{N_T N_{\phi}}[\text{deg}]$$

Stepper Motors Terminology

Multi-Stack



Resolution (steps / revolution) $$\frac{Steps}{\text{Rev.}} = \frac{360^\circ}{\theta_{SL}[\text{deg}]} = \frac{2\pi}{\theta_{SL}[\text{rad}]}$$

Stepping frequency
(pulses per-second per-phase) $$f_{step} = [\text{pulse}/\text{sec}]$$

Rotor position (for no-load operation) $$\theta_{rm} = \theta_{rm}(0) + \theta_{SL} \cdot N_{steps} \pm \varepsilon$$

Speed $$n[\text{rev}/\text{sec}] = \frac{\theta_{SL}[\text{deg}] \cdot f_{step}}{360} = \frac{\theta_{SL}[rad] \cdot f_{step}}{2\pi}$$

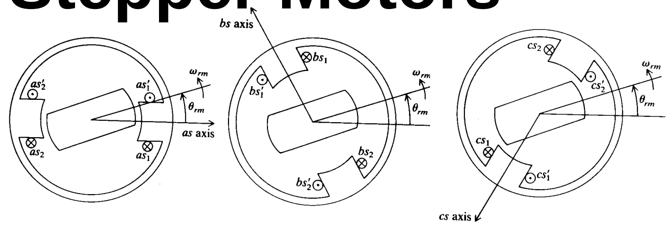
$$n[\text{rev}/\text{min}] = n[\text{rpm}] = \frac{\theta_{SL}[\text{deg}] \cdot f_{step}}{6} = 30\frac{\theta_{SL}[rad] \cdot f_{step}}{\pi}$$

Multi-Stack Stepper Motors

2-pole, 3-stack (phases), 2-rotor teeth

$$TP = \frac{360^\circ}{N_T} = 180^\circ \qquad \theta_{SL} = \frac{TP}{N_\phi} = 60^\circ$$

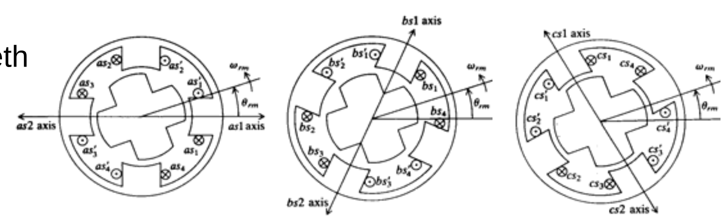
$$\text{Resolution} = \frac{Steps}{\text{Rev.}} = \frac{360^\circ}{\theta_{SL}[\text{deg}]} = 6$$



4-pole, 3-stack (phases), 4-rotor teeth

$$TP = \frac{360^\circ}{N_T} = 90^\circ \qquad \theta_{SL} = \frac{TP}{N_\phi} = 30^\circ$$

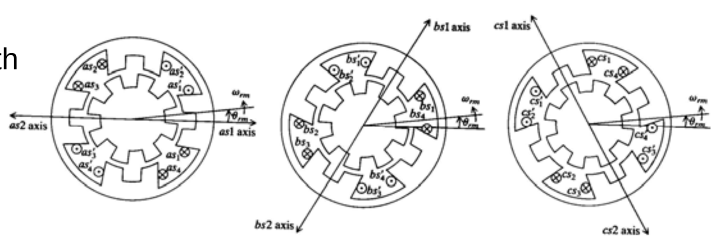
$$\text{Resolution} = \frac{Steps}{\text{Rev.}} = \frac{360^\circ}{\theta_{SL}[\text{deg}]} = 12$$



4-pole, 3-stack (phases), 8-rotor teeth

$$TP = \frac{360^\circ}{N_T} = 45^\circ \qquad \theta_{SL} = \frac{TP}{N_\phi} = 15^\circ$$

$$\text{Resolution} = \frac{Steps}{\text{Rev.}} = \frac{360^\circ}{\theta_{SL}[\text{deg}]} = 24$$



For typical stepper motors one ma have $\text{Resolution} = 24 \cdots 400 \frac{\text{step}}{\text{rev}}$ $\theta_{SL} = 0.9^\circ \cdots 15^\circ$

2-Pole, 3-Stack Stepper Motor

Voltage equations

$$v_{as} = r_s i_{as} + \frac{d\lambda_{as}}{dt}$$

$$v_{bs} = r_s i_{bs} + \frac{d\lambda_{bs}}{dt}$$

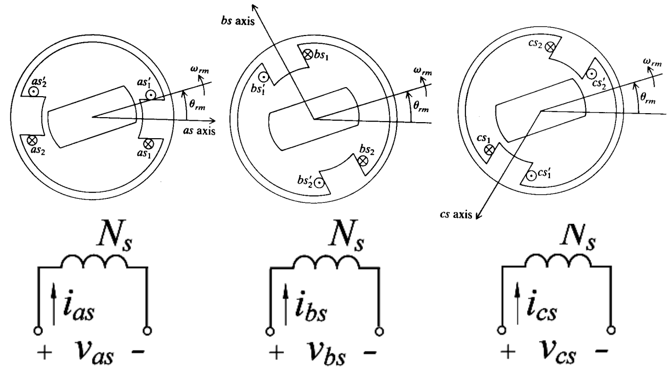
$$v_{cs} = r_s i_{cs} + \frac{d\lambda_{cs}}{dt}$$

$$\mathbf{v}_{abcs} = \mathbf{r}_s \mathbf{i}_{abcs} + \frac{d\boldsymbol{\lambda}_{abcs}}{dt}$$

Resistance matrix

$$\mathbf{r}_s = \begin{bmatrix} r_s & & \\ & r_s & \\ & & r_s \end{bmatrix}$$

For some constant $L_A,\ L_B$



Flux linkage equations

$$\begin{bmatrix} \lambda_{as} \\ \lambda_{bs} \\ \lambda_{sc} \end{bmatrix} = \begin{bmatrix} L_{asas} & 0 & 0 \\ 0 & L_{bsbs} & 0 \\ 0 & 0 & L_{cscs} \end{bmatrix} \cdot \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix} = \mathbf{L}_s \mathbf{i}_{abcs}$$

$$L_{asas} = L_{ls} + L_A + L_B \cos(2[\theta_{rm}])$$

$$L_{bsbs} = L_{ls} + L_A + L_B \cos\left(2\left[\theta_{rm} - 120^\circ\right]\right)$$

$$L_{cscs} = L_{ls} + L_A + L_B \cos\left(2\left[\theta_{rm} + 120^\circ\right]\right)$$

4-Pole, 3-Stack, 4-Teeth Stepper Motor

Voltage equations

$$v_{as} = r_s i_{as} + \frac{d\lambda_{as}}{dt}$$

$$v_{bs} = r_s i_{bs} + \frac{d\lambda_{bs}}{dt}$$

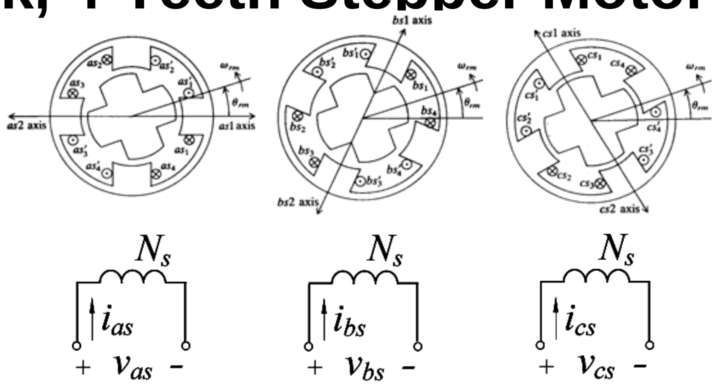
$$v_{cs} = r_s i_{cs} + \frac{d\lambda_{cs}}{dt}$$

$$\mathbf{v}_{abcs} = \mathbf{r}_s \mathbf{i}_{abcs} + \frac{d\boldsymbol{\lambda}_{abcs}}{dt}$$

Flux linkage equations

$$\begin{bmatrix} \lambda_{as} \\ \lambda_{bs} \\ \lambda_{sc} \end{bmatrix} = \begin{bmatrix} L_{asas} & 0 & 0 \\ 0 & L_{bsbs} & 0 \\ 0 & 0 & L_{cscs} \end{bmatrix} \cdot \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix}$$

Resistance matrix $\mathbf{r}_s = \begin{bmatrix} r_s & & \\ & r_s & \\ & & r_s \end{bmatrix}$



Inductances

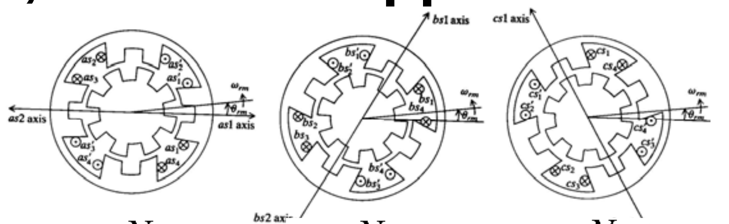
$$L_{asas} = L_{ls} + L_A + L_B \cos(4[\theta_{rm}])$$

$$L_{bsbs} = L_{ls} + L_A + L_B \cos\left(4\left[\theta_{rm} - 60^\circ\right]\right)$$

$$L_{cscs} = L_{ls} + L_A + L_B \cos\left(4\left[\theta_{rm} - 120^\circ\right]\right)$$

For some constant $L_A,\ L_B$

4-Pole, 3-Stack, 8-Teeth Stepper Motor



Voltage equations

$$v_{as} = r_s i_{as} + \frac{d\lambda_{as}}{dt}$$

$$v_{bs} = r_s i_{bs} + \frac{d\lambda_{bs}}{dt}$$

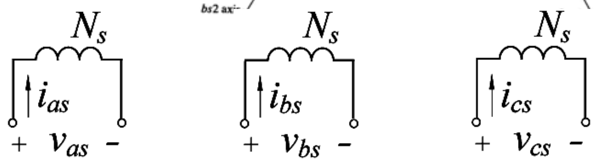
$$v_{cs} = r_s i_{cs} + \frac{d\lambda_{cs}}{dt}$$

$$\mathbf{v}_{abcs} = \mathbf{r}_s \mathbf{i}_{abcs} + \frac{d\boldsymbol{\lambda}_{abcs}}{dt}$$

Flux linkage equations

$$\begin{bmatrix} \lambda_{as} \\ \lambda_{bs} \\ \lambda_{sc} \end{bmatrix} = \begin{bmatrix} L_{asas} & 0 & 0 \\ 0 & L_{bsbs} & 0 \\ 0 & 0 & L_{cscs} \end{bmatrix} \cdot \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix}$$

Resistance matrix $\mathbf{r}_s = \begin{bmatrix} r_s & & \\ & r_s & \\ & & r_s \end{bmatrix}$



Inductances

$$L_{asas} = L_{ls} + L_A + L_B \cos(8[\theta_{rm}])$$

$$L_{bsbs} = L_{ls} + L_A + L_B \cos(8[\theta_{rm} - 60^\circ])$$

$$L_{cscs} = L_{ls} + L_A + L_B \cos(8[\theta_{rm} - 120^\circ])$$

For number of teeth N_T

$$L_{asas} = L_{ls} + L_A + L_B \cos(N_T[\theta_{rm}])$$

$$L_{bsbs} = L_{ls} + L_A + L_B \cos(N_T[\theta_{rm} \pm \theta_{SL}])$$

$$L_{cscs} = L_{ls} + L_A + L_B \cos(N_T[\theta_{rm} \mp \theta_{SL}])$$

$\pm \mp$ takes direction into account

3-Stack, N_T-Teeth Stepper Motor

For Number of Teeth N_T and Step Length (SL) θ_{SL}

Electromagnetic Torque $T_e = \frac{\partial W_c(\mathbf{i}, \theta_{rm})}{\partial \theta_{rm}}$

Assume magnetically linear system $W_c = W_f$

$$W_f = \frac{1}{2} L_{asas} i_{as}^2 + \frac{1}{2} L_{bsbs} i_{bs}^2 + \frac{1}{2} L_{cscs} i_{cs}^2$$

$$T_e = -\frac{N_T}{2} L_B \{ i_{as}^2 \sin(N_T[\theta_{rm}]) + i_{bs}^2 \sin(N_T[\theta_{rm} \pm \theta_{SL}]) + i_{cs}^2 \sin(N_T[\theta_{rm} \mp \theta_{SL}]) \}$$

Note: Torque is proportional to the number of teeth !

Mechanical System

$$J_{total} \frac{d\omega_{rm}}{dt} + D_m \omega_{rm} = T_e - T_m \qquad J_{total} \frac{d^2\theta_{rm}}{dt^2} + D_m \frac{d\theta_{rm}}{dt} = T_e - T_m$$

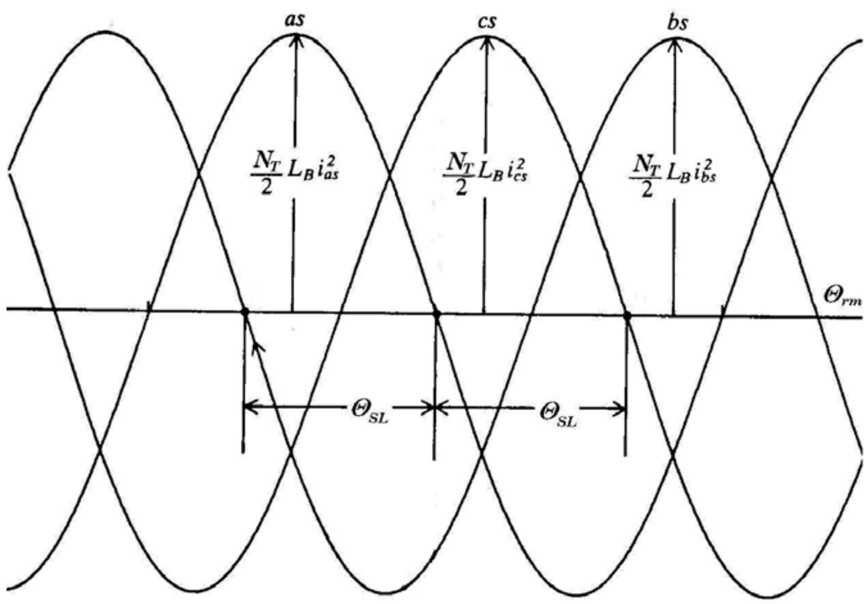
Rotor position $\theta_{rm} = \theta_{rm}(0) + \int \omega_{rm} dt$

Motor Stepping Operation

Electromagnetic Torque $T_e = -\frac{N_T}{2} L_b \left\{ i_{as}^2 \sin\left(N_T\left[\theta_{rm}\right]\right) + i_{bs}^2 \sin\left(N_T\left[\theta_{rm} \pm \theta_{SL}\right]\right) + i_{cs}^2 \sin\left(N_T\left[\theta_{rm} \mp \theta_{SL}\right]\right)\right\}$

Maximum holding torque

$$T_{\max} = \frac{N_T}{2} L_B i_{as}^2$$



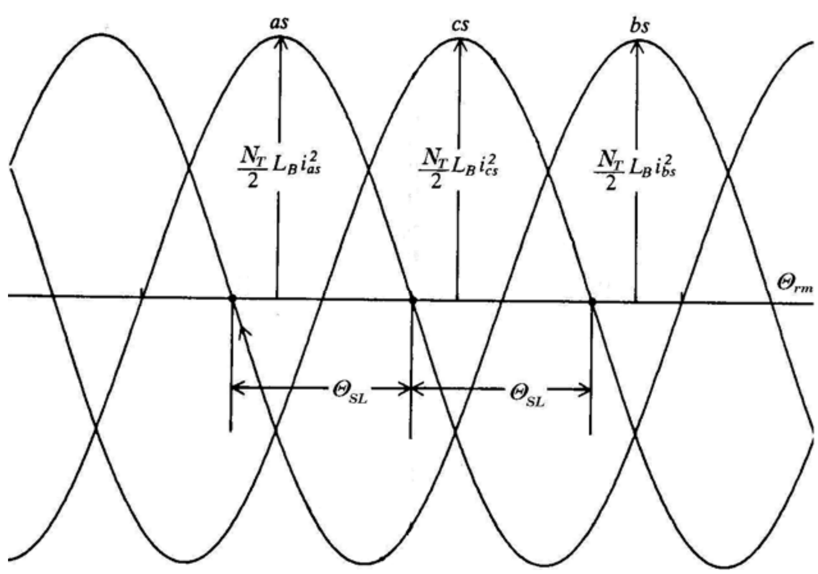
Motor Stepping Under Load

Electromagnetic Torque $T_e = -\frac{N_T}{2} L_b \left\{ i_{as}^2 \sin\left(N_T\left[\theta_{rm}\right]\right) + i_{bs}^2 \sin\left(N_T\left[\theta_{rm} \pm \theta_{SL}\right]\right) + i_{cs}^2 \sin\left(N_T\left[\theta_{rm} \mp \theta_{SL}\right]\right)\right\}$

Rotor position

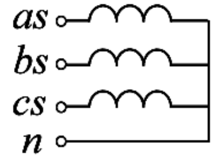
$$\theta_{rm} = \theta_{SL} \cdot N_{steps} \pm \varepsilon$$

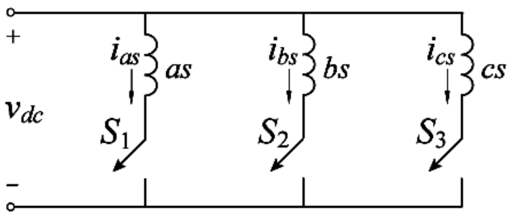
Position error



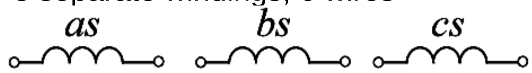
Practical Driver Circuits

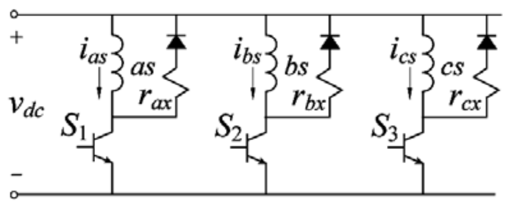
3 wires & 1 common

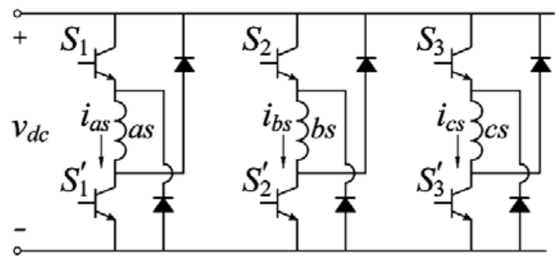




3 separate windings, 6 wires

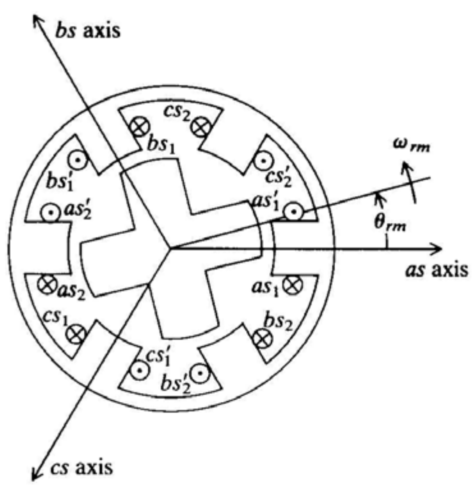






Single-Stack Variable Reluctance Motors

Also known as Switched Reluctance Motors



2-pole, 3-phase, 6/4 - teeth

$$N_{ST} = 6 \quad N_{RT} = 4 \qquad N_{ST} \neq N_{RT}$$

Tooth Pitch

$$TP_{ST} = \frac{360^\circ}{N_{ST}} = 60^\circ$$

$$TP_{RT} = \frac{360^\circ}{N_{RT}} = 90^\circ$$

Step Length (SL)

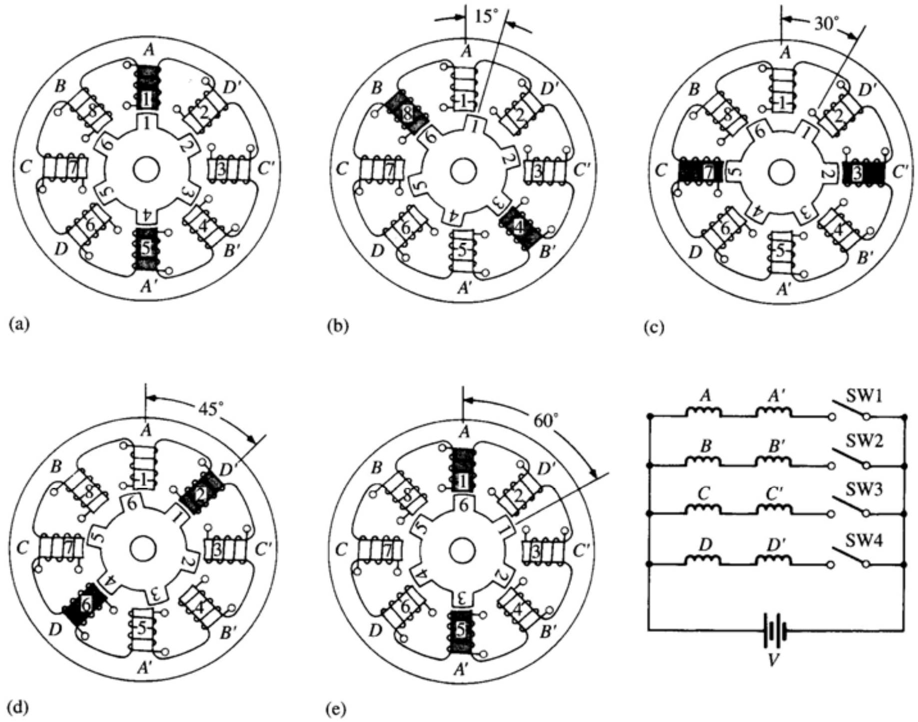
$$\theta_{SL} = \frac{TP_{RT}}{N_\phi} = \frac{360^\circ}{N_{RT} N_\phi} = 30^\circ$$

$$= \left| TP_{RT} - TP_{ST} \right| = 30^\circ$$

What is the direction of rotation for as, bs, cs (CCW) sequence ?

Single-Stack Variable Reluctance Motors

2-pole, 4-phase, 8/6 - teeth



$$TP_{ST} = 45^\circ$$

$$TP_{RT} = 60^\circ$$

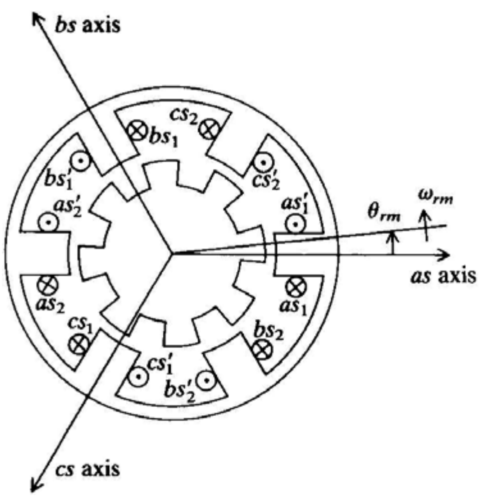
$$\theta_{SL} = \frac{TP_{RT}}{N_\phi}$$

$$= 15^\circ$$

Single-Stack Variable Reluctance Motors

2-pole, 3-phase, 6/8 - teeth

$$N_{ST} = 6 \quad N_{RT} = 8 \qquad N_{ST} \neq N_{RT}$$



What is the direction of rotation for as, bs, cs (CCW) sequence ?

Tooth Pitch

$$TP_{ST} = \frac{360^\circ}{N_{ST}} = 60^\circ$$

$$TP_{RT} = \frac{360^\circ}{N_{RT}} = 45^\circ$$

Step Length (SL)

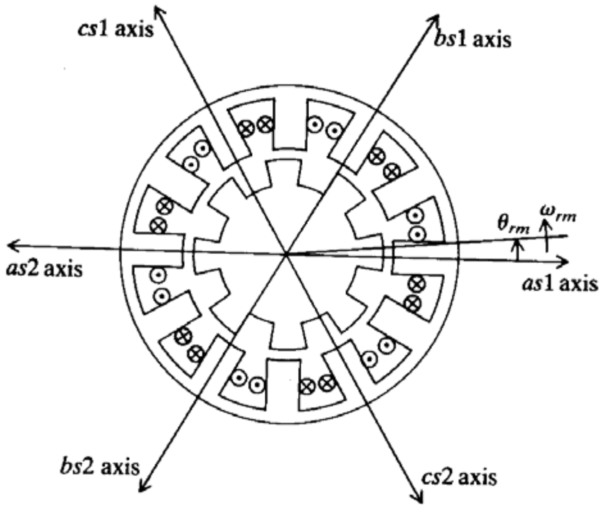
$$\theta_{SL} = \frac{TP_{RT}}{N_\phi} = \frac{360^\circ}{N_{RT} N_\phi} = 15^\circ$$

$$= \left| TP_{RT} - TP_{ST} \right| = 15^\circ$$

Single-Stack Variable Reluctance Motors

4-pole, 3-phase, 12/8 - teeth

$$N_{ST} = 12 \quad N_{RT} = 8 \qquad N_{ST} \neq N_{RT}$$



What is the direction of rotation for as, bs, cs (CCW) sequence ?

Tooth Pitch

$$TP_{ST} = \frac{360^\circ}{N_{ST}} = 30^\circ$$

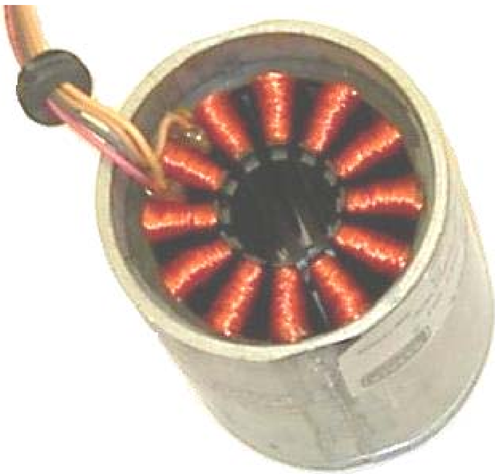
$$TP_{RT} = \frac{360^\circ}{N_{RT}} = 45^\circ$$

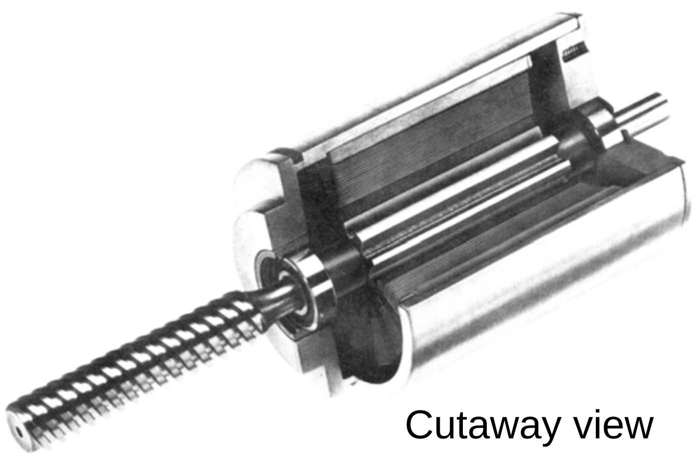
Step Length (SL)

$$\theta_{SL} = \frac{TP_{RT}}{N_\phi} = \frac{360^\circ}{N_{RT} N_\phi} = 15^\circ$$

$$= \left| TP_{RT} - TP_{ST} \right| = 15^\circ$$

12/8 Variable Reluctance Motor

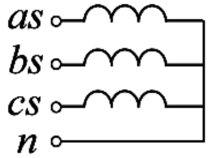


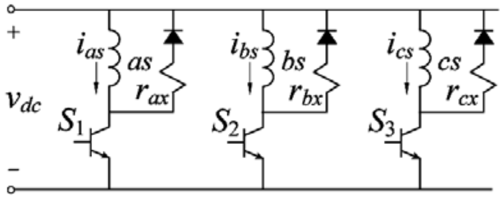


Cutaway view

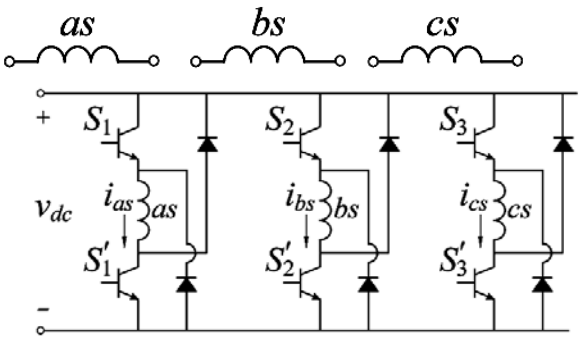
Practical Driver Circuits

3 wires & 1 common

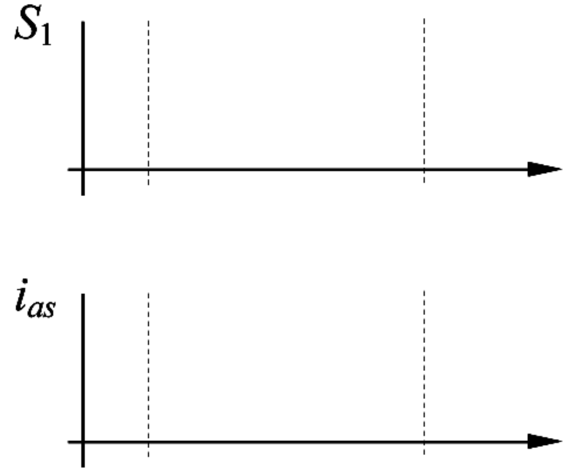




3 separate windings, 6 wires



Use current-control modulation
(recall Hysteresis Modulation)



Stepper Motors & Drives