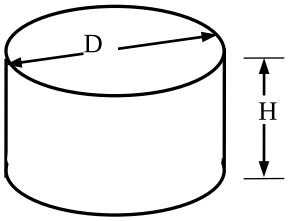
Problem 1 (50%)



Find the time for which $T_{\min} = 90°\mathrm{C}$

Given: $T_i = 40^o\mathrm{C}$; $T_\infty = 110^o\mathrm{C}$;
$h = 470\ \mathrm{W/m^2\text{-}K}$
Vegetable properties approximate water
D =12cm and H =10 cm

Assumptions: Unsteady heat conduction (heating) with constant properties; Radiation may be considered negligible. No natural convection within the can (pure conduction)

Solution:
Check for LPA:
Guess average temperature of (40+110)/2=75 °C and read the thermophysical properties of water:

$$k = 0.671\ \mathrm{W/m\text{-}°C};\ \rho = 974.9\ \mathrm{kg/m^3};\ c_p = 4190\ \mathrm{J/kg\text{-}°C};\ \alpha = \frac{k}{\rho c_p} = \frac{0.671}{974.9\times 4190} = 1.64\times 10^{-7}\,\mathrm{m^2/s}$$

$$Bi = \frac{hL_c}{k};\quad L_c = \frac{\text{volume}}{\text{Surf. total exposed conv.}} = \frac{\pi D^2/4\times H}{2\ \ \pi D^2/4\ \ +\pi DH} = \frac{D/4\times H}{D/2\ \ +H} = \frac{0.12/4\times 0.10}{0.12/2+0.10} = 0.01875\ \mathrm{m}$$

$$Bi = \frac{470\ \ 0.01875}{0.671} = 13.13 > 0.1 \Rightarrow LPA \text{ not valid}$$

The $T_{\min} = 90°\mathrm{C}$ **is at the center of the can**, i.e., farthest point from the hot thermal boundaries. The can geometry can be constructed by intersecting a long cylinder of diameter $D=2r_o$ and an infinite plate of thickness H. t is not known at this stage.

Assume: Plane wall: $t_1{*} = \alpha t/\ \ H/2\ \ ^2 > 0.2$, LongCylinder: $t_2{*} = \alpha t/r_o^2 > 0.2$
thus, 1-term approximation of infinite series solution is adequate

$$\varphi_{r,x,t} = \frac{T_{r,x,t} - T_\infty}{T_i - T_\infty} = C_{r,t}\times P_{x,t};\quad \varphi_{0,0,t} = \frac{T_{0,0,t} - T_\infty}{T_i - T_\infty} = C_{0,t}\times P_{0,t}$$

$$CASE: A \Rightarrow P_{0,t} = C_{B1}\exp\ -A_{B1}^2 t_1{*}\qquad CASE: B \Rightarrow C_{0,t} = C_{B2}\exp\ -A_{B2}^2 t_2{*}$$

$$\varphi_{0,0,t} = \frac{T_{0,0,t} - T_\infty}{T_i - T_\infty} = C_{0,t}\times P_{0,t} = C_{B1}C_{B2}\exp\left(-[\frac{A_{B1}^2}{H/2\ \ ^2} + \frac{A_{B2}^2}{r_o^2}]\alpha t\right)$$

$$Bi_{M1} = \frac{h\ \ H/2}{k} = \frac{470\ \ 0.05}{0.671} = 35 \xrightarrow[Table]{} A_{B1} = 1.5264;\ C_{B1} = 1.272$$

$$Bi_{M2} = \frac{hr_o}{k} = \frac{470\ \ 0.06}{0.671} = 42 \xrightarrow[\substack{Table\\ Bi_M\,\square\,40}]{} A_{B2} = 2.3455;\ C_{B2} = 1.5993$$

$$\frac{90-110}{40-110} = 1.272\times 1.5993\times\exp\left(-[\frac{1.5264^2}{0.05^2} + \frac{2.3455^2}{0.06^2}]1.64\times 10^{-7}t\right)$$

$$0.1404 = \exp\ -0.000403459t$$

$$\Rightarrow t = 4865.2\ \mathrm{s}$$

Quick check:

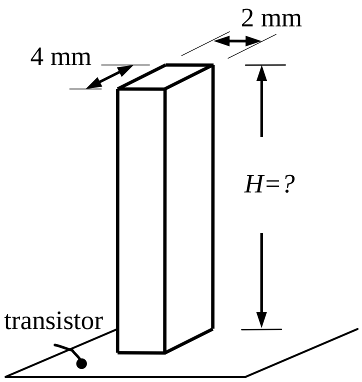
$$t_1{*} = \alpha t/\ \ H/2\ \ ^2$$

$$t_1{*} = \frac{1.64\times 10^{-7}\times 4865.2}{0.05^2} = 0.319$$

$$t_2{*} = \alpha t/r_o^2$$

$$t_2{*} = \frac{1.64\times 10^{-7}\times 4865.2}{0.06^2} = 0.2216$$

Problem 2 (50%)



Given: T_{base} = 80°C; T_{∞} = 25°C;
$h = 14$ W/m^2-K
k_{fin} = 165 W/m-K
Fin effectiveness ε_{fin} =20,
12 identical fins
Transistor area 30 mm × 20 mm

Assumptions: Quasi 1D steady heat conduction in the fin. Using compensated length, Case 3 solution (insulated tip) approximation is adequate. Radiation may be considered negligible.

a) Find *H*.

$$A_{cs} = \left(4\times10^{-3}\right)\times\left(2\times10^{-3}\right) = 8\times10^{-6}\ \text{m}^2$$

$$P_{cs} = 2\times\left(4\times10^{-3}+2\times10^{-3}\right) = 12\times10^{-3}\ \text{m}$$

$$\text{m}=\sqrt{\frac{hP_{cs}}{k_{fin}A_{cs}}} = \sqrt{\frac{14\times12\times10^{-3}}{165\times8\times10^{-6}}} = 11.28\quad \text{m}^{-1}$$

$$\varepsilon_{Fin} \equiv \frac{\underset{\text{Fin}\rightarrow\text{Fluid}}{\text{q}_{\text{actual}}}}{\text{Area}_{\text{Base}}h\left(T_{wall}-T_{\infty}\right)} \rightarrow \varepsilon_{\substack{Fin\\ Case\,3}} = \frac{\sqrt{k_{fin}A_{c.s.}hP_{c.s.}}\left(T_{Base}-T_{\infty}\right)\tanh\left(mL_c\right)}{\underset{Base}{A_{c.s.}}\,h\left(T_{Base}-T_{\infty}\right)} = \sqrt{\frac{k_{fin}P_{c.s.}}{A_{c.s.}h}}\tanh\left(mL_c\right)$$

$$\varepsilon_{Fin} = 20 = \sqrt{\frac{165\times12\times10^{-3}}{8\times10^{-6}\times14}}\tanh\left(11.28L_c\right) \Rightarrow 0.15 = \tanh\left(11.28L_c\right)$$

$$\Rightarrow 11.28L_c = 0.151 \Rightarrow L_c = 0.0134\ \text{m}\ \text{ or } 13.4\ \text{mm}$$

$$L_c = H + \frac{A_{cs}}{P_{cs}} \Rightarrow H = 0.0134 - \frac{8\times10^{-6}}{12\times10^{-3}} = 0.0127\ \text{m or } 12.7\ \text{mm}$$

b)

$$q_{tot} = q_{unfinned} + q_{finned} = \left(A_{tot}-A_{finned}\right)h\left(T_{Base}-T_{\infty}\right) + N\times q_{one-fin}$$

$$q_{tot} = \left(A_{tot}-A_{finned}\right)h\left(T_{Base}-T_{\infty}\right) + N\times\varepsilon_{fin}\underset{fin}{A_{cs}}\,h\left(T_{Base}-T_{\infty}\right)$$

$$\frac{q_{finned}}{q_{tot}} = \frac{N\times\varepsilon_{fin}\underset{fin}{A_{cs}}\,h\left(T_{Base}-T_{\infty}\right)}{\left(A_{tot}-A_{finned}\right)h\left(T_{Base}-T_{\infty}\right) + N\times\varepsilon_{fin}\underset{fin}{A_{cs}}\,h\left(T_{Base}-T_{\infty}\right)}$$

$$\frac{q_{finned}}{q_{tot}} = \frac{N\times\varepsilon_{fin}\underset{fin}{A_{cs}}}{\left(A_{tot}-N\underset{fin}{A_{cs}}\right)+N\times\varepsilon_{fin}\underset{fin}{A_{cs}}} = \frac{\varepsilon_{fin}}{\left(A_{tot}/N\underset{fin}{A_{cs}}-1\right)+\varepsilon_{fin}} = \frac{20}{\left[30\times20/\left(2\times4\times2\right)-1\right]+20} = 79.2\%$$