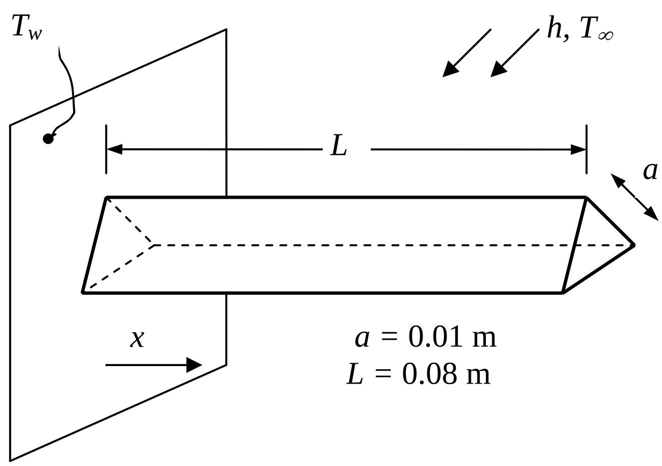
Solutions - Problem Set # 4

Problem 1:



Given: k_{rod} = 138.56 W/m-K; T_w = 118°C; h = 20 W/m^2-K and T_∞ = 18°C

Assumption: i) SS, 1-D heat conduction problem with constant properties: ii) compensated length approach is adequate.

This is case 2 (convection from the tip surface); however, using compensated length approach, the solution to the problem can be approximated using the results for Case 3 (insulated tip).

a) Estimate the tip temperature

We use the corrected length:

$$L_c = L + \Delta L \cong L + \frac{A_{c.s.tip}}{P_{c.s.tip}}$$

$$A_{c.s.tip} = \frac{a^2}{4}\tan(60°) = \frac{0.01^2}{4}\sqrt{3} = 4.33\times10^{-5}\ \text{m}^2$$

$$P_{c.s.tip} = 3a = 0.03\ \text{m}$$

$$L_c = L + \frac{4.33\times10^{-5}}{0.03} = 0.08 + 1.443\times10^{-3} = 0.081443\ \text{m}$$

Solution to Case 3 (insulated tip) using L_c

$$\frac{\theta}{\theta_{Base}} = \frac{T - T_\infty}{T_{Base} - T_\infty} = \frac{\cosh[m(L_c - x)]}{\cosh[mL_c]}$$

$$\rightarrow T_{tip} = T_{x=L_c} = T_\infty + \left(T_{Base} - T_\infty\right)\frac{1}{\cosh[mL_c]}$$

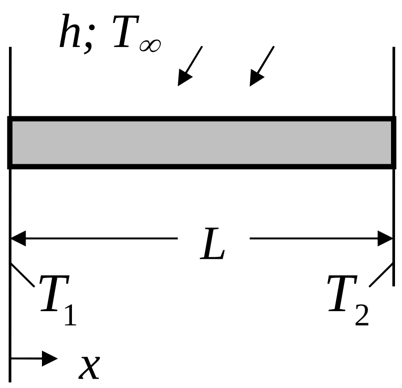
$$m = (hP_{c.s.} / k_s A_{c.s.})^{0.5} = \left(\frac{20\times 0.03}{138.56\times 4.33\times 10^{-5}}\right)^{0.5} = 10\ \mathrm{m^{-1}}$$

$$T_{tip} = T_{x=L_c} = 18 + (118-18)\frac{1}{\cosh[10\times 0.081443]} = 92.05°\mathrm{C}$$

b) Calculate the fin efficiency

$$\eta_{Fin,Case3} = \frac{\tanh[mL_c]}{(mL_c)} = \frac{\tanh[10\times 0.081443]}{(10\times 0.081443)} = 0.825$$

Problem 2:

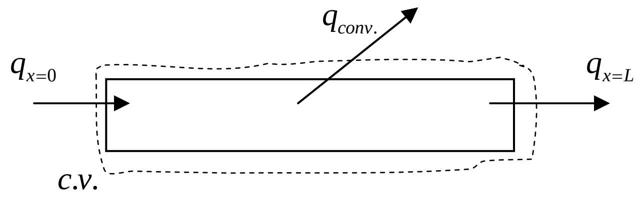


Given:
$T_\infty = 38°\mathrm{C}$; $T_1 = 200°\mathrm{C}$; $T_2 = 93°\mathrm{C}$
$h =$ 17 W/m^2-K; k = 395 W/m-K
$L =$ 30 cm; D =12.5 mm

a)

Assumptions: classical fin theory applies;1-D, SS; excellent thermal contact Wall-Base, $R_{th\ contact} = 0$

Energy balance on the rod:



$$q_{conv.} = q_{x=0} - q_{x=L}$$

$$q_{x=0} = -k_s \left.\frac{dT}{dx}\right|_{x=0} A_{c.s.};\ q_{x=L} = -k_s \left.\frac{dT}{dx}\right|_{x=L} A_{c.s.}$$

Case 4 (prescribed temperature)

The solution for temperature distribution for this case is given in handout #3. You can also drive this by solving the general solution presented and by applying BCs.

$$\frac{\theta}{\theta_{Base}} = \frac{T - T_\infty}{T_{Base} - T_\infty} = \frac{(\theta_L/\theta_{Base})\sinh[mx] + \sinh[m(L-x)]}{\sinh[mL]}$$

$$T = T_\infty + \frac{(T_{Base} - T_\infty)}{\sinh[mL]}\left\{\frac{(T_L - T_\infty)}{(T_{Base} - T_\infty)}\sinh[mx] + \sinh[m(L-x)]\right\}$$

$$\frac{dT}{dx} = \frac{(T_{Base} - T_\infty)}{\sinh[mL]} m \left\{\frac{(T_L - T_\infty)}{(T_{Base} - T_\infty)}\cosh[mx] - \cosh[m(L-x)]\right\}$$

$$\left.\frac{dT}{dx}\right|_{x=0} = \frac{(T_{Base} - T_\infty)}{\sinh[mL]} m \left\{\frac{(T_L - T_\infty)}{(T_{Base} - T_\infty)} - \cosh[mL]\right\}$$

$$\left.\frac{dT}{dx}\right|_{x=L} = \frac{(T_{Base} - T_\infty)}{\sinh[mL]} m \left\{\frac{(T_L - T_\infty)}{(T_{Base} - T_\infty)}\cosh[mL] - 1\right\}$$

$$\left.\frac{dT}{dx}\right|_{x=0} - \left.\frac{dT}{dx}\right|_{x=L} = \frac{(T_{Base} - T_\infty)}{\sinh[mL]} m \left\{\frac{(T_L - T_\infty)}{(T_{Base} - T_\infty)} - \cosh[mL] - \frac{(T_L - T_\infty)}{(T_{Base} - T_\infty)}\cosh[mL] + 1\right\}$$

$$\left.\frac{dT}{dx}\right|_{x=0} - \left.\frac{dT}{dx}\right|_{x=L} = m\frac{(1 - \cosh[mL])}{\sinh[mL]}\{T_L + T_{Base} - 2T_\infty\}$$

$$q_{conv.} = q_{x=0} - q_{x=L} = -kA_{cs}\left[\left.\frac{dT}{dx}\right|_{x=0} - \left.\frac{dT}{dx}\right|_{x=L}\right]$$

after arranging terms:

$$q_{conv.} = -kA_{cs} m \frac{(1 - \cosh[mL])}{\sinh[mL]}\{T_L + T_{Base} - 2T_\infty\}$$

$$R_{th-contact} = 0 \rightarrow T_{Base} = T_1 \text{ and } T_L = T_2$$

$$m = (hP_{c.s.}/k_s A_{c.s.})^{0.5} = \left(\frac{17 \times \pi 12.5 \times 10^{-3}}{395 \times \pi \left(12.5 \times 10^{-3}\right)^2 / 4}\right)^{0.5} = 3.711\,\text{m}^{-1}$$

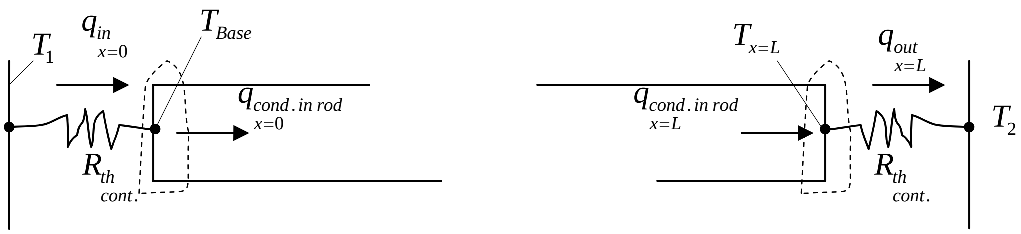
$$q_{conv.} = -395\left(\pi\left(12.5 \times 10^{-3}\right)^2/4\right)3.71\frac{(1 - \cosh[3.711 \times 0.3])}{\sinh[3.711 \times 0.3]}(200 + 93 - 2 \times 38)$$

$$q_{conv.} = 19.73\ \text{W}$$

(b)

$$A_{cs} = \frac{\pi}{4} \times \left(12.5\times10^{-3}\right)^2 = 1.2272\times10^{-4}\ \mathrm{m}^2$$

$$R_{\underset{cont.}{th}} = 1/\left(h_{cont}A_{cs}\right) = 1/\left[1000\times1.2272\times10^{-4}\right] = 8.149 \quad {}^\circ\mathrm{C/W}$$



at $x = 0$ at $x = L$

at $x = 0$

$$q_{\underset{x=0}{in}} = q_{\underset{\substack{rod\\x=0}}{cond.}}$$

$$\frac{T_1 - T_{Base}}{R_{\underset{cont.}{th}}} = -kA_{cs}\left.\frac{dT}{dx}\right|_{x=0}$$

at $x = L$

$$q_{\underset{\substack{rod\\x=L}}{cond.}} = q_{\underset{x=L}{out}}$$

$$\frac{T_{x=L} - T_2}{R_{\underset{cont.}{th}}} = -kA_{cs}\left.\frac{dT}{dx}\right|_{x=L}$$

* at x = 0

$$\frac{T_1 - T_{Base}}{R_{\underset{cont.}{th}}} = -kA_{cs}\frac{\left(T_{Base} - T_\infty\right)}{\sinh[mL]}m\left\{\frac{\left(T_L - T_\infty\right)}{\left(T_{Base} - T_\infty\right)} - \cosh[mL]\right\}$$

$$\frac{200 - T_{Base}}{8.149} = -395\times1.2272\times10^{-4}\frac{\left(T_{Base} - 38\right)}{\sinh[3.711\times0.3]}3.711\left\{\frac{\left(T_L - 38\right)}{\left(T_{Base} - 38\right)} - \cosh[3.711\times0.3]\right\}$$

$$\Rightarrow 2.82T_{Base} - 1.0794T_L = 228.16 \qquad (1)$$

* at x =L

$$\frac{T_L - T_2}{R_{\underset{cont.}{th}}} = -kA_{cs}\frac{\left(T_{Base} - T_\infty\right)}{\sinh[mL]}m\left\{\frac{\left(T_L - T_\infty\right)}{\left(T_{Base} - T_\infty\right)}\cosh[mL] - 1\right\}$$

$$\frac{T_L - 93}{8.149} = -395\times1.2272\times10^{-4}\frac{\left(T_{Base} - 38\right)}{\sinh[3.711\times0.3]}3.711\left\{\frac{\left(T_L - 38\right)}{\left(T_{Base} - 38\right)}\cosh[3.711\times0.3] - 1\right\}$$

$$\Rightarrow -1.0794T_{Base} + 2.82T_L = 121.16 \qquad (2)$$

(1) and (2)
$$\Rightarrow T_{Base} = 114.1^\circ\mathrm{C}$$
$$T_L = 86.62^\circ\mathrm{C}$$

at $x = 0$

$$q_{\substack{in \\ x=0}} = \frac{T_1 - T_{Base}}{R_{\substack{th \\ cont.}}}$$

$$q_{\substack{in \\ x=0}} = \frac{200 - 114.1}{8.149} = 10.54\ \text{W}$$

at $x = L$

$$q_{\substack{out \\ x=L}} = \frac{T_{x=L} - T_2}{R_{\substack{th \\ cont.}}}$$

$$q_{\substack{out \\ x=L}} = \frac{86.62 - 93}{8.149} = -0.783\ \text{W} \quad \text{(thus, heat is conducted in)}$$

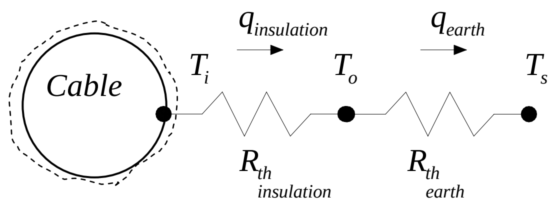
$$q_{conv.} = 10.54 - (-0.783) = 11.3\ \text{W}$$

Problem 3:

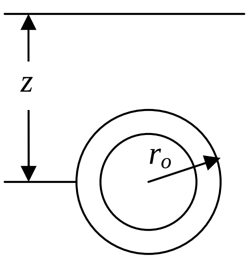
a)

S.S. E-Bal. on the Cable:

$$q_{\substack{generation \\ cable}} = q_{insulation} = q_{earth}$$



$$q_{\substack{generation \\ cable}} = \frac{T_i - T_s}{R_{\substack{th \\ insul.}} + R_{\substack{th \\ Earth}}}$$



$$R_{\substack{th \\ Earth}} = \frac{1}{Sk_{earth}} = \frac{\ln(z/r_o)}{2\pi k_{earth} L} \qquad (z > 3r_o) \quad \text{Table 3-1 in Holman, 2002.}$$

Please note that for this system, other textbooks have provided a different Shape factor: $S = \dfrac{2\pi L}{\ln(2z/r_o)}$ (e.g., Incropera and DeWitt, 1994)

When $I_{cable} = I_{\max}$, $T_i = T_{ins,\max} = 80°\text{C}$

$$q_{\substack{generation \\ cable \\ MAX}} = \frac{80 - 10}{\dfrac{\ln(r_o/r_i)}{2\pi k_{ins} L} + \dfrac{\ln(z/r_o)}{2\pi k_{earth} L}} = \frac{80 - 10}{\dfrac{\ln(0.015/0.01)}{2\pi \times 5} + \dfrac{\ln(0.3/0.015)}{2\pi \times 0.5}} L$$

Thus, $$q_{\substack{generation \\ cable \\ MAX}} = \frac{70}{0.0129 + 0.95} L = 72.4L$$

$$q_{\substack{generation \\ cable \\ MAX}} = I_{MAX}^2 R_e = 72.4L$$

$$\rightarrow I_{MAX}^2 \times \left(5 \times 10^{-4} \times L\right) = 72.4L \Rightarrow I_{Max} = 380.5\ \text{A}$$

b)

under the operating condition in part (a) $S_{\substack{in\\cable}} = \dfrac{q_{generation}}{Volume} = \dfrac{72.4L}{\left[\pi(0.02)^2/4\right]L} = 230456.35\ \text{W/m}^3$

With constant S inside the cable, the temperature profile was developed in handout #2. Thus,

$$T - T_i = \frac{1}{4}\frac{S_{cable}}{k_{cable}} r_i^2 \left[1 - \left(\frac{r}{r_i}\right)^2\right] \text{ and } T_{\max} = T_{r=0}$$

$$T_{\max} = T_i + \frac{1}{4}\frac{S_{cable}}{k_{cable}} r_i^2 = 80 + \frac{1}{4} \times \frac{230456.35}{200}(0.01)^2 = 80.029°\text{C}$$