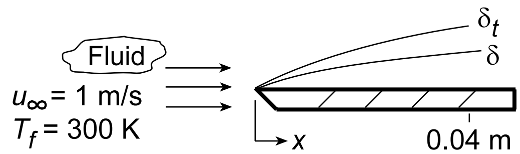
PROBLEM 7.1

KNOWN: Temperature and velocity of fluids in parallel flow over a flat plate.

FIND: (a) Velocity and thermal boundary layer thicknesses at a prescribed distance from the leading edge, and (b) For each fluid plot the boundary layer thicknesses as a function of distance.

SCHEMATIC:



ASSUMPTIONS: (1) Transition Reynolds number is 5×10^5.

PROPERTIES: *Table A.4*, Air (300 K, 1 atm): $\nu = 15.89 \times 10^{-6}$ m^2/s, Pr = 0.707; *Table A.6*, Water (300 K): $\nu = \mu/\rho = 855 \times 10^{-6}$ N·s/m^2/997 kg/m^3 = 0.858×10^{-6} m^2/s, Pr = 5.83; *Table A.5*, Engine Oil (300 K): $\nu = 550 \times 10^{-6}$ m^2/s, Pr = 6400; *Table A.5*, Mercury (300 K): $\nu = 0.113 \times 10^{-6}$ m^2/s, Pr = 0.0248.

ANALYSIS: (a) If the flow is laminar, the following expressions may be used to compute δ and δ_t, respectively,

$$\delta = \frac{5x}{\mathrm{Re}_x^{1/2}} \qquad \delta_t = \frac{\delta}{\mathrm{Pr}^{1/3}}$$

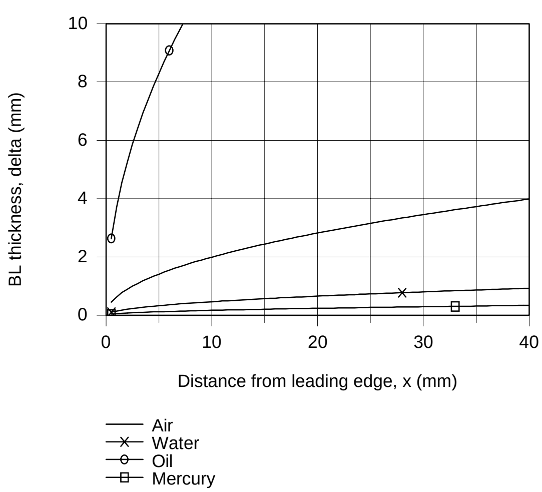
where

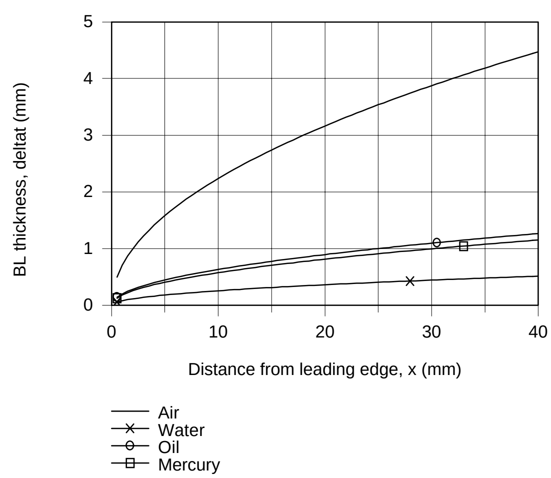
$$\mathrm{Re}_x = \frac{u_\infty x}{\nu} = \frac{1\,\mathrm{m/s}(0.04\,\mathrm{m})}{\nu} = \frac{0.04\,\mathrm{m}^2/\mathrm{s}}{\nu}$$

Fluid	Re_x	δ (mm)	δ_t (mm)
Air	2517	3.99	4.48
Water	4.66×10^4	0.93	0.52
Oil	72.7	23.5	1.27
Mercury	3.54×10^5	0.34	1.17

<

(b) Using IHT with the foregoing equations, the boundary layer thicknesses are plotted as a function of distance from the leading edge, x.





COMMENTS: (1) Note that $\delta \approx \delta_t$ for air, $\delta > \delta_t$ for water, $\delta >> \delta_t$ for oil, and $\delta < \delta_t$ for mercury. As expected, the boundary layer thicknesses increase with increasing distance from the leading edge.

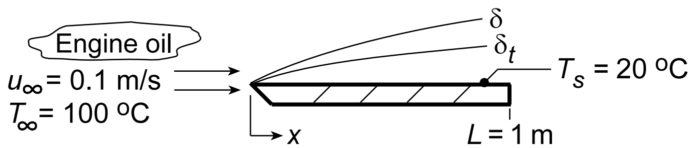
(2) The value of δ_t for mercury should be viewed as a rough approximation since the expression for δ/δ_t was derived subject to the approximation that Pr > 0.6.

PROBLEM 7.2

KNOWN: Temperature and velocity of engine oil. Temperature and length of flat plate.

FIND: (a) Velocity and thermal boundary layer thickness at trailing edge, (b) Heat flux and surface shear stress at trailing edge, (c) Total drag force and heat transfer per unit plate width, and (d) Plot the boundary layer thickness and local values of the shear stress, convection coefficient, and heat flux as a function of x for $0 \le x \le 1$ m.

SCHEMATIC:



ASSUMPTIONS: (1) Critical Reynolds number is 5×10^5, (2) Flow over top and bottom surfaces.

PROPERTIES: *Table A.5*, Engine Oil (T_f = 333 K): ρ = 864 kg/m^3, $\nu = 86.1 \times 10^{-6}$ m^2/s, k = 0.140 W/m·K, Pr = 1081.

ANALYSIS: (a) Calculate the Reynolds number to determine nature of the flow,

$$Re_L = \frac{u_\infty L}{\nu} = \frac{0.1\,\text{m/s} \times 1\,\text{m}}{86.1 \times 10^{-6}\,\text{m}^2/\text{s}} = 1161$$

Hence the flow is laminar at x = L, from Eqs. 7.19 and 7.24, and

$$\delta = 5L\,Re_L^{-1/2} = 5(1\,\text{m})(1161)^{-1/2} = 0.147\,\text{m}$$ <

$$\delta_t = \delta\,Pr^{-1/3} = 0.147\,\text{m}(1081)^{-1/3} = 0.0143\,\text{m}$$ <

(b) The local convection coefficient, Eq. 7.23, and heat flux at x = L are

$$h_L = \frac{k}{L} 0.332\,Re_L^{1/2}\,Pr^{1/3} = \frac{0.140\,\text{W/m}\cdot\text{K}}{1\text{m}} 0.332(1161)^{1/2}(1081)^{1/3} = 16.25\,\text{W/m}^2\cdot\text{K}$$

$$q''_x = h_L(T_s - T_\infty) = 16.25\,\text{W/m}^2\cdot\text{K}(20 - 100)^\circ\text{C} = -1300\,\text{W/m}^2$$ <

Also, the local shear stress is, from Eq. 7.20,

$$\tau_{s,L} = \frac{\rho u_\infty^2}{2} 0.664\,Re_L^{-1/2} = \frac{864\,\text{kg/m}^3}{2}(0.1\,\text{m/s})^2\,0.664(1161)^{-1/2}$$

$$\tau_{s,L} = 0.0842\,\text{kg/m}\cdot\text{s}^2 = 0.0842\,\text{N/m}^2$$ <

(c) With the drag force per unit width given by $D' = 2L\bar{\tau}_{s,L}$ where the factor of 2 is included to account for both sides of the plate, it follows that

$$D' = 2L\left(\rho u_\infty^2/2\right)1.328\,Re_L^{-1/2} = (1\,\text{m})864\,\text{kg/m}^3(0.1\,\text{m/s})^2/2\;1.328(1161)^{-1/2} = 0.337\,\text{N/m}$$ <

For laminar flow, the average value $\bar{h}_L$ over the distance 0 to L is twice the local value, h_L,

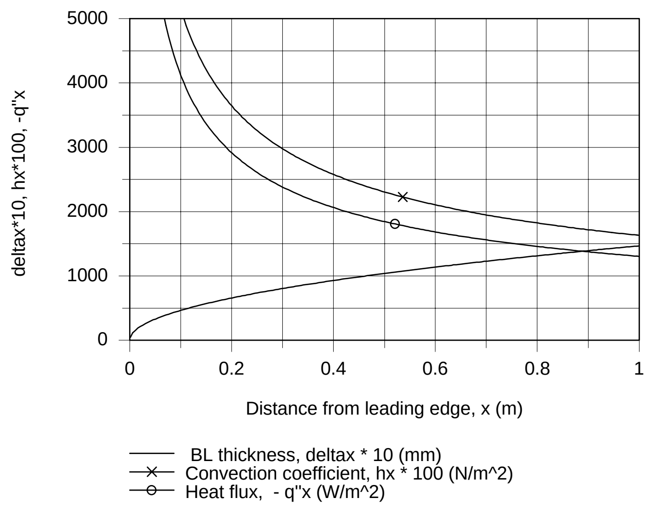
$$\bar{h}_L = 2h_L = 32.5\,\text{W/m}^2\cdot\text{K}$$

The total heat transfer rate per unit width of the plate is

$$q' = 2L\bar{h}_L(T_s - T_\infty) = 2(1\,\text{m})32.5\,\text{W/m}^2\cdot\text{K}(20 - 100)^\circ\text{C} = -5200\,\text{W/m}$$ <

Continued...

(c) Using IHT with the foregoing equations, the boundary layer thickness, and local values of the convection coefficient and heat flux were calculated and plotted as a function of x.



COMMENTS: (1) Note that since Pr >> 1, $\delta \gg \delta_t$. That is, for the high Prandtl liquids, the velocity boundary layer will be much thicker than the thermal boundary layer.

(2) A copy of the *IHT Workspace* used to generate the above plot is shown below.

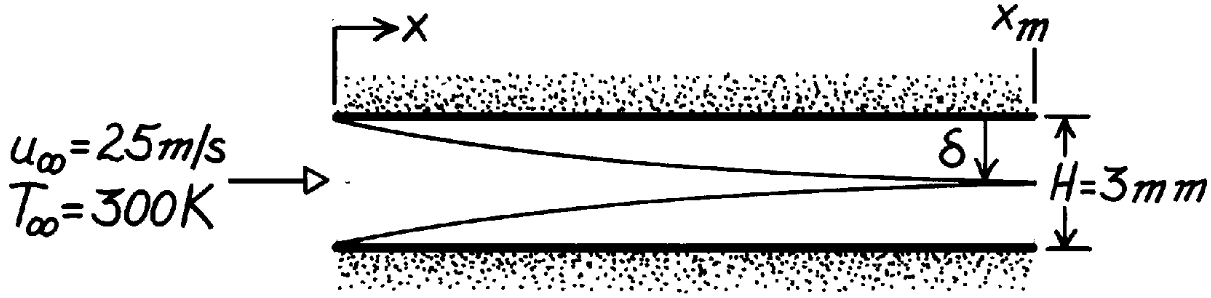
```
// Boundary layer thickness, delta
delta = 5 * x * Rex ^-0.5
delta_mm = delta * 1000
delta_plot = delta_mm * 10           // Scaling parameter for convenience in  plotting

// Convection coefficient and heat flux, q''x
q''x = hx * (Ts - Tinf)
Nux = 0.332 * Rex^0.5 * Pr^(1/3)
Nux = hx * x / k
hx_plot = 100 * hx                   // Scaling parameter for convenience in plotting
q''x_plot = ( -1 ) * q''x            // Scaling parameter for convenience in plotting

// Reynolds number
Rex = uinf * x / nu

// Properties Tool: Engine oil
// Engine Oil property functions : From Table A.5
// Units: T(K)
rho = rho_T("Engine Oil",Tf)         // Density, kg/m^3
cp = cp_T("Engine Oil",Tf)           // Specific heat, J/kg·K
nu = nu_T("Engine Oil",Tf)           // Kinematic viscosity, m^2/s
k = k_T("Engine Oil",Tf)             // Thermal conductivity, W/m·K
Pr = Pr_T("Engine Oil",Tf)           // Prandtl number


// Assigned variables
Tf = (Ts + Tinf) / 2                 // Film temperature, K
Tinf = 100 + 273                     // Freestream temperature, K
Ts = 20 + 273                        // Surface temperature, K
uinf = 0.1                           // Freestream velocity, m/s
x = 1                                // Plate length, m
```

PROBLEM 7.3

KNOWN: Velocity and temperature of air in parallel flow over a flat plate.

FIND: (a) Velocity boundary layer thickness at selected stations. Distance at which boundary layers merge for plates separated by H = 3 mm. (b) Surface shear stress and v(δ) at selected stations.

SCHEMATIC:



ASSUMPTIONS: (1) Steady flow, (2) Boundary layer approximations are valid, (3) Flow is laminar.

PROPERTIES: *Table A-4*, Air (300 K, 1 atm): ρ = 1.161 kg/m^3, ν = 15.89 × 10^{-6} m^2/s.

ANALYSIS: (a) For laminar flow,

$$d = \frac{5x}{Re_x^{1/2}} = \frac{5}{\left(u_\infty / n\right)^{1/2}} x^{1/2} = \frac{5x^{1/2}}{\left(25\ \text{m/s}/15{,}89\times10^{-6}\ \text{m}^2/\text{s}\right)^{1/2}} = 3.99\times10^{-3}\ x^{1/2}.$$

x (m)	0.001	0.01	0.1
d (mm)	0.126	0.399	1.262

Boundary layer merger occurs at x = x_m when δ = 1.5 mm. Hence

$$x_m^{1/2} = \frac{0.0015\ \text{m}}{3.99\times10^{-3}\ \text{m}^{1/2}} = 0.376\ \text{m}^{1/2} \qquad x_m = 141\ \text{mm}.$$ <

(b) The shear stress is

$$t_{s,x} = 0.664\frac{r u_\infty^2/2}{Re_x^{1/2}} = \frac{r u_\infty^2/2}{\left(u_\infty/n\right)^{1/2} x^{1/2}} = \frac{0.664\times1.161\ \text{kg/m}^3\left(25\ \text{m/s}\right)^2/2}{\left(25\ \text{m/s}/15.89\times10^{-6}\ \text{m}^2/\text{s}\right)^{1/2} x^{1/2}} = \frac{0.192}{x^{1/2}}\left(\text{N/m}^2\right).$$

x(m)	0.001	0.01	0.1
$t_{s,x}$ (N/m^2)	6.07	1.92	0.61

The velocity distribution in the boundary layer is v = (1/2) $(\nu u_\infty/x)^{1/2}$ (ηdf/dη - f). At y = δ, η ≈ 5.0, f ≈ 3.24, df/dη ≈ 0.991.

$$v = \frac{0.5}{x^{1/2}}\left(15.89\times10^{-6}\ \text{m}^2/\text{s}\times25\ \text{m/s}\right)^{1/2}\left(5.0\times0.991-3.28\right) = \left(0.0167/x^{1/2}\right)\text{m/s}.$$

x(m)	0.001	0.01	0.1
v(m/s)	0.528	0.167	0.053

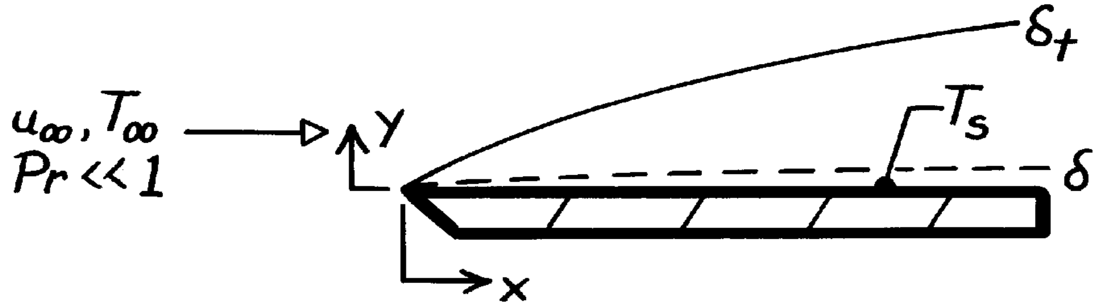
COMMENTS: (1) v << u_∞ and δ << x are consistent with BL approximations. Note, v → ∞ as x → 0 and approximations breakdown very close to the leading edge. (2) Since $Re_{x_m} = 2.22\times10^5$, laminar BL model is valid. (3) Above expressions are approximations for flow between parallel plates, since $du_\infty/dx > 0$ and $dp/dx < 0$.

PROBLEM 7.4

KNOWN: Liquid metal in parallel flow over a flat plate.

FIND: An expression for the local Nusselt number.

SCHEMATIC:



ASSUMPTIONS: (1) Steady, incompressible flow, (2) $\delta << \delta_t$, hence $u(y) \approx u_\infty$, (3) Boundary layer approximations are valid, (4) Constant properties.

ANALYSIS: The boundary layer energy equation is

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}.$$

Assuming $u(y) = u_\infty$, it follows that $v = 0$ and the energy equation becomes

$$u_\infty\frac{\partial T}{\partial x} = \alpha\frac{\partial^2 T}{\partial y^2} \qquad \text{or} \qquad \frac{\partial T}{\partial x} = \frac{\alpha}{u_\infty}\frac{\partial^2 T}{\partial y^2}.$$

Boundary Conditions: $T(x,0) = T_s$, $T(x,\infty) = T_\infty$.

Initial Condition: $T(0,y) = T_\infty$.

The differential equation is analogous to that for transient one-dimensional conduction in a plane wall, and the conditions are analogous to those of Fig. 5.7, Case (1). Hence the solution is given by Eqs. 5.57 and 5.58. Substituting y for x, x for t, T_∞ for T_i, and α/u_∞ for α, the boundary layer temperature and the surface heat flux become

$$\frac{T(x,y) - T_s}{T_\infty - T_s} = \text{erf}\left[\frac{y}{2(\alpha x/u_\infty)^{1/2}}\right]$$

$$q_s'' = \frac{k(T_s - T_\infty)}{(\pi \alpha x/u_\infty)^{1/2}}.$$

Hence, with $$Nu_x \equiv \frac{hx}{k} = \frac{q_s'' x}{(T_s - T_\infty)k}$$

find $$Nu_x = \frac{x}{(\pi \alpha x/u_\infty)^{1/2}} = \frac{(xu_\infty)^{1/2}}{\pi^{1/2}(k/\rho c_p)^{1/2}} = \frac{1}{\pi^{1/2}}\left[\frac{\rho u_\infty x}{\mu}\cdot\frac{c_p \mu}{k}\right]^{1/2}$$

$$Nu_x = 0.564\,(Re_x Pr)^{1/2} = 0.564\,Pe^{1/2}$$ <

where Pe = Re · Pr is the Peclet number.

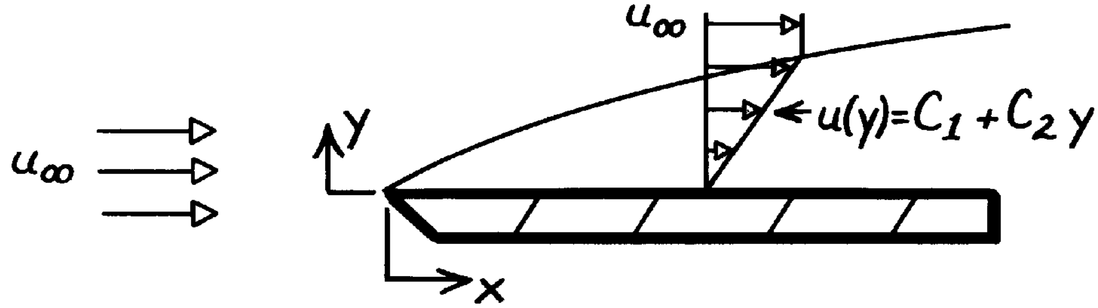
COMMENTS: Because k is very large, axial conduction effects may not be negligible. That is, the $\alpha\,\partial^2 T/\partial x^2$ term of the energy equation may be important.

PROBLEM 7.5

KNOWN: Form of velocity profile for flow over a flat plate.

FIND: (a) Expression for profile in terms of u_∞ and δ, (b) Expression for δ(x), (c) Expression for $C_{f,x}$.

SCHEMATIC:



ASSUMPTIONS: (1) Steady state conditions, (2) Constant properties, (3) Incompressible flow, (4) Boundary layer approximations are valid.

ANALYSIS: (a) From the boundary conditions

$$u(x,0)=0 \rightarrow C_1=0 \qquad \text{and} \qquad u(x,d)=u_\infty \rightarrow C_2=u_\infty/d.$$

Hence, $\quad u=u_\infty(y/d).$ <

(b)From the momentum integral equation for a flat plate

$$\frac{d}{dx}\int_0^d (u_\infty-u)\,u\,dy = t_s/r$$

$$u_\infty^2\frac{d}{dx}\int_0^d\left(1-\frac{u}{u_\infty}\right)\frac{u}{u_\infty}dy = \frac{m}{r}\left.\frac{\P u}{\P y}\right)_{y=0} = \frac{n\,u_\infty}{d}$$

$$u_\infty^2\frac{d}{dx}\int_0^d\left(1-\frac{y}{d}\right)\frac{y}{d}dy = \frac{n\,u_\infty}{d}$$

$$u_\infty^2\frac{d}{dx}\left[\left(\frac{y^2}{2d}-\frac{y^3}{3d^2}\right)\Bigg|_0^d\right] = \frac{m\,u_\infty}{d} \qquad \text{or} \qquad \frac{u_\infty}{6}\frac{dd}{dx}=\frac{n}{d}.$$

Separating and integrating, find

$$\int_0^d d\,d\,d = \frac{6n}{u_\infty}\int_0^x dx \qquad d=\left(\frac{12\,n\,x}{u_\infty}\right)^{1/2}=3.46\,x\left(\frac{n}{u_\infty x}\right)^{1/2}=3.46\,x\,Re_x^{-1/2}.$$ <

(c) The shear stress at the wall is

$$t_s = m\left.\frac{\P u}{\P y}\right)_{y=0} = m\frac{u_\infty}{d} = \frac{m\,u_\infty}{3.46\,x}Re_x^{1/2}$$

and the friction coefficient is

$$C_{f,x}=\frac{t_s}{r\,u_\infty^2/2}=\frac{m}{r\,u_\infty x}\,\frac{2}{3.46}Re_x^{1/2}=0.578\,Re_x^{-1/2}.$$ <

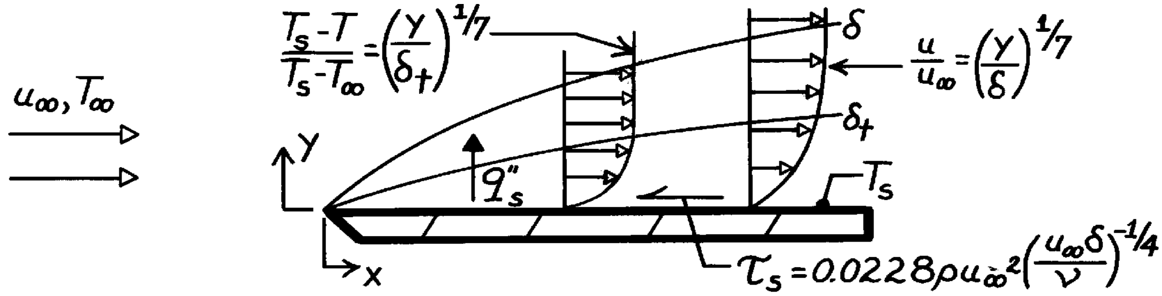
COMMENTS: The foregoing results underpredict those associated with the exact solution $\left(d=4.96\,x\,Re_x^{-1/2},\ C_{f,x}=0.664\,Re_x^{-1/2}\right)$ and the cubic profile $\left(d=4.64\,x\,Re_x^{-1/2},\right.$ $\left.C_{f,x}=0.646\,Re_x^{-1/2}\right).$

PROBLEM 7.6

KNOWN: Velocity and temperature profiles and shear stress-boundary layer thickness relation for turbulent flow over a flat plate.

FIND: (a) Expressions for hydrodynamic boundary layer thickness and average friction coefficient, (b) Expressions for local and average Nusselt numbers.

SCHEMATIC:



ASSUMPTIONS: (1) Steady flow, (2) Constant properties, (3) Fully turbulent boundary layer, (4) Incompressible flow, (5) Isothermal plate, (6) Negligible viscous dissipation, (7) $\delta \approx \delta_t$.

ANALYSIS: (a) The momentum integral equation is

$$r\ u_\infty^2\ \frac{d}{dx}\int_0^{d}\left(1-\frac{u}{u_\infty}\right)\frac{u}{u_\infty}\,dy\ =\ t_s.$$

Substituting the expression for the wall shear stress

$$r\ u_\infty^2\frac{d}{dx}\int_0^{d}\left[1-\left(\frac{y}{d}\right)^{1/7}\right]\left(\frac{y}{d}\right)^{1/7}dy\ =\ 0.0228\ r\ u_\infty^2\left(\frac{u_\infty d}{n}\right)^{-1/4}$$

$$\frac{d}{dx}\int_0^{d}\left[\left(\frac{y}{d}\right)^{1/7}-\left(\frac{y}{d}\right)^{2/7}\right]dy\ =\ \frac{d}{dx}\left(\frac{7}{8}\frac{y^{8/7}}{d^{1/7}}-\frac{7}{9}\frac{y^{9/7}}{d^{2/7}}\right)\Bigg|_0^{d}$$

$$\frac{d}{dx}\left(\frac{7}{8}d-\frac{7}{9}d\right)\ =\ 0.0228\left(\frac{u_\infty d}{n}\right)^{-1/4}$$

$$\frac{7}{72}\frac{d\,d}{dx}=0.0228\left(\frac{n}{u_\infty}\right)^{1/4}d^{-1/4}\qquad\qquad \frac{7}{72}\int_0^{d}d^{1/4}\,dd=0.0228\left(\frac{n}{u_\infty}\right)^{1/4}\int_0^{x}dx$$

$$\frac{7}{72}\times\frac{4}{5}d^{5/4}=0.0228\left(\frac{n}{u_\infty}\right)^{1/4}x,\qquad d=0.376\left(\frac{n}{u_\infty}\right)^{1/5}x^{4/5},\ \ \frac{d}{x}=0.376\,Re_x^{-1/5}.\ <$$

Knowing δ, it follows

$$t_s=0.0228\ r u_\infty^2\left(\frac{u_\infty}{n}\right)^{-1/4}\left[0.376\ x\ Re_x^{-1/5}\right]^{-1/4}$$

$$C_{f,x}=\frac{t_s}{r\ u_\infty^2/2}=0.0456\left[0.376\ \frac{u_\infty}{n}\left(\frac{u_\infty}{n}\right)^{-1/5}x\ x^{-1/5}\right]^{-1/4}=0.0592\ Re_x^{-1/5}.$$

Continued …..

The average friction coefficient is then

$$\overline{C}_{f,x} = \frac{1}{x}\int_0^x C_{f,x}\,dx = \frac{1}{x}0.0592\left(\frac{u_\infty}{\nu}\right)^{-1/5}\int_0^x x^{-1/5}\,dx$$

$$\overline{C}_{f,x} = \frac{1}{x}0.0592\left(\frac{u_\infty}{\nu}\right)^{-1/5} x^{4/5}\left(\frac{5}{4}\right) = 0.074\,Re_x^{-1/5}. \qquad <$$

(b) The energy integral equation for turbulent flow is

$$\frac{d}{dx}\int_0^{\delta_t} u\left(T_\infty - T\right)dy = \frac{q_s''}{\rho\, c_p} = -\frac{h}{\rho\, c_p}\left(T_s - T_\infty\right).$$

Hence,

$$u_\infty \frac{d}{dx}\int_0^{\delta_t}\frac{u}{u_\infty}\,\frac{T - T_\infty}{T_s - T_\infty}\,dy = u_\infty \frac{d}{dx}\int_0^{\delta_t}\left(y/\delta\right)^{1/7}\left[1 - \left(y/\delta_t\right)^{1/7}\right]dy = \frac{h}{\rho\, c_p}$$

$$u_\infty \frac{d}{dx}\left[\frac{7}{8}\,\frac{\delta_t^{8/7}}{\delta^{1/7}} - \frac{7}{9}\,\frac{\delta_t^{8/7}}{\delta^{1/7}}\right] = \frac{h}{\rho\, c_p}$$

or, with $\xi \equiv \delta_t/\delta$,

$$u_\infty \frac{d}{dx}\left[\frac{7}{8}\,\delta\xi^{8/7} - \frac{7}{9}\delta\xi^{8/7}\right) = \frac{h}{\rho\, c_p} \qquad u_\infty \frac{d}{dx}\left\{\frac{7}{72}\delta\xi^{8/7}\right\} = \frac{h}{\rho\, c_p}.$$

Hence, with $\xi \approx 1$ and $\delta/x = 0.376\,Re_x^{-1/5}$,

$$\frac{7}{72}\,u_\infty(0.376)\left(\frac{u_\infty}{\nu}\right)^{-1/5}\frac{d\left(x^{4/5}\right)}{dx} = \frac{h}{\rho\, c_p}$$

$$h = 0.0292\,\rho\, c_p u_\infty\, Re_x^{-1/5} = 0.0292\,\frac{k}{x}\,\frac{\nu}{\alpha}\,\frac{u_\infty x}{\nu}\,Re_x^{-1/5}$$

$$Nu_x = \frac{hx}{k} = 0.0292\,Re_x^{4/5}\,Pr. \qquad <$$

Hence,

$$\overline{h}_x = \frac{1}{x}\int_0^x h\,dx = \frac{0.0292\,Pr}{x}\,k\left(\frac{u_\infty}{\nu}\right)^{4/5}\int_0^x x^{-1/5}\,dx = 0.0292\,\frac{k}{x}\,Pr\left(\frac{u_\infty x}{\nu}\right)^{4/5}\frac{5}{4}$$

$$\overline{Nu}_x = \frac{\overline{h}_x x}{k} = 0.037\,Re_x^{4/5}\,Pr. \qquad <$$

COMMENTS: (1) The foregoing results are in excellent agreement with empirical correlations, except that use of $Pr^{1/3}$ instead of Pr, would be more appropriate.

(2) Note that the 1/7 profile breaks down at the surface. For example,

$$\left.\frac{\partial\left(u/u_\infty\right)}{\partial y}\right)_{y=0} = \frac{1}{7}\,\delta^{-1/7}\,y^{-6/7} = \infty$$

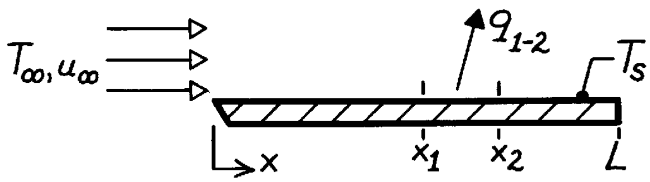
or $\tau_s = \infty$. Despite this unrealistic characteristic of the profile, its use with integral methods provides excellent results.

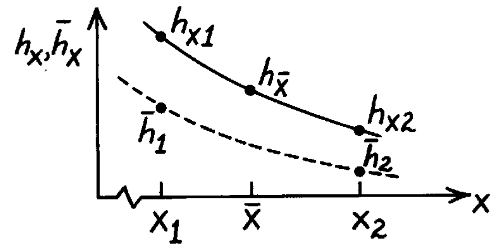
PROBLEM 7.7

KNOWN: Parallel flow over a flat plate and two locations representing a short span x_1 to x_2 where $(x_2 - x_1) << L$.

FIND: Three different expressions for the average heat transfer coefficient over the short span x_1 to x_2, $\overline{h}_{1-2}$.

SCHEMATIC:





ASSUMPTIONS: (1) Parallel flow over a flat plate.

ANALYSIS: The heat rate per unit width for the span can be written as

$$q'_{1-2} = \overline{h}_{1-2}\left(x_2 - x_1\right)\left(T_s - T_\infty\right) \tag{1}$$

where $\overline{h}_{1-2}$ is the average heat transfer coefficient over the span and can be evaluated in either of the following three ways:

(a) *Local coefficient at* $\overline{x} = \left(x_1 + x_2\right)/2$. If the span is very short, it is reasonable to assume that

$$\overline{h}_{1-2} \approx h_{\overline{x}} \tag{2}$$

where $h_{\overline{x}}$ is the local convection coefficient at the mid-point of the span.

(b) *Local coefficients at x_1 and x_2.* If the span is very short it is reasonable to assume that $\overline{h}_{1-2}$ is the average of the local values at the ends of the span,

$$\overline{h}_{1-2} \approx \left[h_{x1} + h_{x2}\right]/2. \tag{3}$$

(c) *Average coefficients for x_1 and x_2.* The heat rate for the span can also be written as

$$q'_{1-2} = q'_{0-2} - q'_{0-1} \tag{4}$$

where the rate q_{0-x} denotes the heat rate for the plate over the distance from 0 to x. In terms of heat transfer coefficients, find

$$\overline{h}_{1-2} \cdot \left(x_2 - x_1\right) = \overline{h}_2 \cdot x_2 - \overline{h}_1 \cdot x_1$$

$$\overline{h}_{1-2} = \overline{h}_2 \frac{x_2}{x_2 - x_1} - \overline{h}_1 \frac{x_1}{x_2 - x_1} \tag{5}$$

where $\overline{h}_1$ and $\overline{h}_2$ are the average coefficients from 0 to x_1 and x_2, respectively.

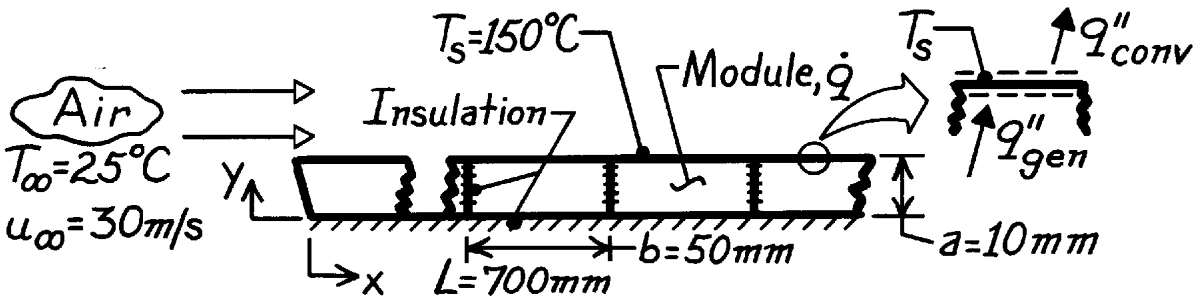
COMMENTS: Eqs. (2) and (3) are approximate and work better when the span is small and the flow is turbulent rather than laminar ($h_x \sim x^{-0.2}$ vs $h_x \sim x^{-0.5}$). Of course, we require that $x_c < x_1$, x_2 or $x_c > x_1$, x_2; that is, the approximations are inappropriate around the transition region. Eq. (5) is an exact relationship, which applies under any conditions.

PROBLEM 7.8

KNOWN: Flat plate comprised of rectangular modules of surface temperature T_s, thickness a and length b cooled by air at 25°C and a velocity of 30 m/s. Prescribed thermophysical properties of the module material.

FIND: (a) Required power generation for the module positioned 700 mm from the leading edge of the plate and (b) Maximum temperature in this module.

SCHEMATIC:



ASSUMPTIONS: (1) Laminar flow at leading edge of plate, (2) Transition Reynolds number of 5×10^5, (3) Heat transfer is one-dimensional in y-direction within each module, (4) $\dot{q}$ is uniform within module, (5) Negligible radiation heat transfer.

PROPERTIES: Module material (given): k = 5.2 W/m·K, c_p = 320 J/kg·K, ρ = 2300 kg/m^3; *Table A-4,* Air $\left(\overline{T}_f = (T_s + T_\infty)/2 = 360\text{ K, 1 atm}\right)$: k = 0.0308 W/m·K, ν = 22.02 × 10^{-6} m^2/s, Pr = 0.698.

ANALYSIS: (a) The module power generation follows from an energy balance on the module surface,

$$q''_{conv} = q''_{gen}$$

$$\overline{h}\left(T_s - T_\infty\right) = \dot{q} \cdot a \qquad \text{or} \qquad \dot{q} = \frac{\overline{h}\left(T_s - T_\infty\right)}{a}.$$

To select a convection correlation for estimating $\overline{h}$, first find the Reynolds numbers at x = L.

$$Re_L = \frac{u_\infty L}{n} = \frac{30\text{ m/s} \times 0.70\text{ m}}{22.02 \times 10^{-6}\text{m}^2/\text{s}} = 9.537 \times 10^5.$$

Since the flow is turbulent over the module, the approximation $\overline{h} \approx h_x\ (L + b/2)$ is appropriate, with

$$Re_{L+b/2} = \frac{30\text{ m/s} \times \left(0.700 + 0.050/2\right)\text{m}}{22.02 \times 10^{-6}\text{m}^2/\text{s}} = 9.877 \times 10^5.$$

Using the turbulent flow correlation with x = L + b/2 = 0.725 m,

$$Nu_x = \frac{h_x x}{k} = 0.0296 Re_x^{4/5} Pr^{1/3}$$

$$Nu_x = 0.0296\left(9.877 \times 10^5\right)^{4/5}\left(0.698\right)^{1/3} = 1640$$

$$\overline{h} \approx h_x = \frac{Nu_x k}{x} = \frac{1640 \times 0.0308\text{ W/m} \cdot \text{K}}{0.725} = 69.7\text{ W/m}^2 \cdot \text{K}.$$

Continued …..

Hence,

$$\dot{q} = \frac{69.7\ \text{W/m}^2 \cdot \text{K}\left(150 - 25\right)\text{K}}{0.010\ \text{m}} = 8.713 \times 10^5\ \text{W/m}^3. \qquad <$$

(b) The maximum temperature within the module occurs at the surface next to the insulation (y = 0). For one-dimensional conduction with thermal energy generation, use Eq. 3.42 to obtain

$$T(0) = \frac{\dot{q}a^2}{2k} + T_s = \frac{8.713 \times 10^5\ \text{W/m}^3 \times (0.010\ \text{m})^2}{2 \times 5.2\ \text{W/m} \cdot \text{K}} + 150^\circ\text{C} = 158.4^\circ\text{C}. \qquad <$$

COMMENTS: An alternative approach for estimating the average heat transfer coefficient for the module follows from the relation

$$q_{\text{module}} = q_{0 \to L+b} - q_{0 \to L}$$

$$\bar{h} \cdot b = \bar{h}_{L+b} \cdot (L + b) - \bar{h}_L \cdot L \qquad \text{or} \qquad \bar{h} = \bar{h}_{L+b} \frac{L+b}{b} - \bar{h}_L \frac{L}{b}.$$

Recognizing that laminar and turbulent flow conditions exist, the appropriate correlation is

$$\overline{Nu}_x = \left(0.037 Re_x^{4/5} - 871\right) Pr^{1/3}$$

With x = L + b and x = L, find

$$\bar{h}_{L+b} = 54.81\ \text{W/m}^2 \cdot \text{K} \qquad \text{and} \qquad \bar{h}_L = 53.73\ \text{W/m}^2 \cdot \text{K}.$$

Hence,

$$\bar{h} = \left[54.81 \frac{0.750}{0.050} - 53.73 \frac{0.700}{0.05}\right] \text{W/m}^2 \cdot \text{K} = 69.9\ \text{W/m}^2 \cdot \text{K}.$$

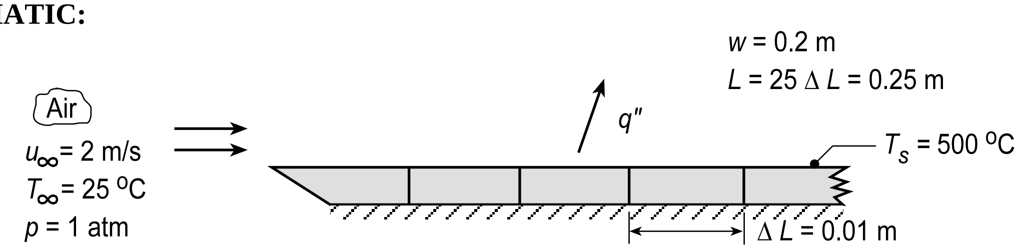
which is in excellent agreement with the approximate result employed in part (a).

PROBLEM 7.9

KNOWN: Dimensions and surface temperature of electrically heated strips. Temperature and velocity of air in parallel flow.

FIND: (a) Rate of convection heat transfer from first, fifth and tenth strips as well as from all the strips, (b) For air velocities of 2, 5 and 10 m/s, determine the convection heat rates for all the locations of part (a), and (c) Repeat the calculations of part (b), but under conditions for which the flow is fully turbulent over the entire array of strips.

SCHEMATIC:



ASSUMPTIONS: (1) Top surface is smooth, (2) Bottom surface is adiabatic, (3) Critical Reynolds number is 5×10^5, (4) Negligible radiation.

PROPERTIES: *Table A.4*, Air (T_f = 535 K, 1 atm): $\nu = 43.54 \times 10^{-6}$ m²/s, k = 0.0429 W/m·K, Pr = 0.683.

ANALYSIS: (a) The location of transition is determined from

$$x_c = 5\times10^5 \frac{\nu}{u_\infty} = 5\times10^5 \frac{43.54\times10^{-6}\,\mathrm{m^2/s}}{2\,\mathrm{m/s}} = 10.9\,\mathrm{m}$$

Since $x_c >> L = 0.25$ m, the air flow is laminar over the entire heater. For the *first* strip, $q_1 = \bar{h}_1(\Delta L \times w)(T_s - T_\infty)$ where $\bar{h}_1$ is obtained from

$$\bar{h}_1 = \frac{k}{\Delta L} 0.664\, Re_x^{1/2}\, Pr^{1/3}$$

$$\bar{h}_1 = \frac{0.0429\,\mathrm{W/m\cdot K}}{0.01\,\mathrm{m}} \times 0.664 \left(\frac{2\,\mathrm{m/s}\times0.01\,\mathrm{m}}{43.54\times10^{-6}\,\mathrm{m^2/s}}\right)^{1/2} (0.683)^{1/3} = 53.8\,\mathrm{W/m^2\cdot K}$$

$$q_1 = 53.8\,\mathrm{W/m^2\cdot K}\,(0.01\,\mathrm{m}\times0.2\,\mathrm{m})(500-25)^\circ\mathrm{C} = 51.1\,\mathrm{W}$$ <

For the *fifth* strip, $q_5 = q_{0-5} - q_{0-4}$,

$$q_5 = h_{0-5}(5\Delta L \times w)(T_s - T_\infty) - \bar{h}_{0-4}(4\Delta L \times w)(T_s - T_\infty)$$

$$q_5 = \left(5\bar{h}_{0-5} - 4\bar{h}_{0-4}\right)(\Delta L \times w)(T_s - T_\infty)$$

Hence, with $x_5 = 5\Delta L = 0.05$ m and $x_4 = 4\Delta L = 0.04$ m, it follows that $\bar{h}_{0-5}$ = 24.1 W/m²·K and $\bar{h}_{0-4}$ = 26.9 W/m²·K and

$$q_5 = (5\times24.1 - 4\times26.9)\,\mathrm{W/m^2\cdot K}\,(0.01\times0.2)\,\mathrm{m^2}\,(500-25)\,\mathrm{K} = 12.2\,\mathrm{W}.$$ <

Similarly, where $\bar{h}_{0-10}$ = 17.00 W/m²·K and $\bar{h}_{0-9}$ = 17.92 W/m²·K.

$$q_{10} = \left(10\bar{h}_{0-10} - 9\bar{h}_{0-9}\right)(\Delta L \times w)(T_s - T_\infty)$$

$$q_{10} = (10\times17.00 - 9\times17.92)\,\mathrm{W/m^2\cdot K}\,(0.01\times0.2)\,\mathrm{m^2}\,(500-25)\,\mathrm{K} = 8.3\,\mathrm{W}$$ <

Continued...

PROBLEM 7.9 (Cont.)

For the entire heater,

$$\bar{h}_{0-25} = \frac{k}{L} 0.664\, Re_L^{1/2}\, Pr^{1/3} = \frac{0.0429}{0.25} \times 0.664 \left(\frac{2\times 0.25}{43.54\times 10^{-6}} \right)^{1/2} (0.683)^{1/3} = 10.75\, W/m^2 \cdot K$$

and the heat rate over all 25 strips is

$$q_{0-25} = \bar{h}_{0-25} (L \times w)(T_s - T_\infty) = 10.75\, W/m^2 \cdot K (0.25 \times 0.2) m^2 (500 - 25)^\circ C = 255.3\, W \quad <$$

(b,c) Using the *IHT Correlations Tool*, *External Flow*, for *Laminar or Mixed Flow Conditions*, and following the same method of solution as above, the heat rates for the first, fifth, tenth and all the strips were calculated for air velocities of 2, 5 and 10 m/s. To evaluate the heat rates for fully turbulent conditions, the analysis was performed setting $Re_{s,c} = 1 \times 10^{-6}$. The results are tabulated below.

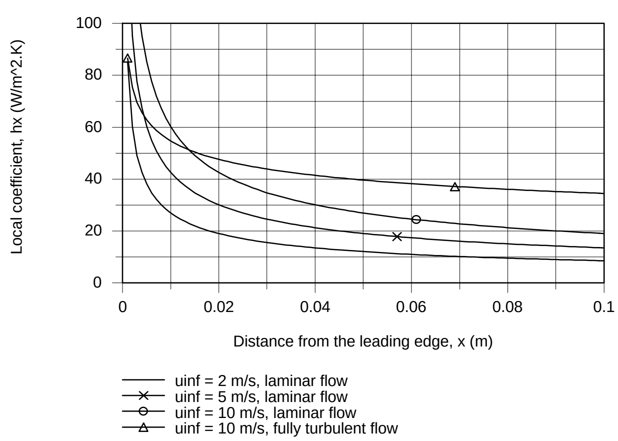
Flow conditions	u_∞ (m/s)	q_1 (W)	q_5 (W)	q_{10} (W)	$q_{0\text{-}25}$ (W)
Laminar	2	51.1	12.1	8.3	256
	5	80.9	19.1	13.1	404
	10	114	27.0	18.6	572
Fully turbulent	2	17.9	10.6	9.1	235
	5	37.3	22.1	19.0	490
	10	64.9	38.5	33.1	853

COMMENTS: (1) An alternative approach to evaluating the heat loss from a single strip, for example, strip 5, would take the form $q_5 = \bar{h}_5 (\Delta L \times w)(T_s - T_\infty)$, where $h_5 \approx h_{x=4.5\Delta L}$ or $\bar{h}_5 \approx (h_{x=5\Delta L} + h_{x=4\Delta L})/2$.

(2) From the tabulated results, note that for both flow conditions, the heat rate for each strip and the entire heater, increases with increasing air velocity. For both flow conditions and for any specified velocity, the strip heat rates decrease with increasing distance from the leading edge.

(3) The effect of flow conditions, laminar vs. fully turbulent flow, on strip heat rates shows some unexpected behavior. For the u_∞ = 5 m/s condition, the effect of turbulent flow is to increase the heat rates for the entire heater and the tenth and fifth strips. For the u_∞ = 10 m/s, the effect of turbulent flow is to increase the heat rates at all locations. This behavior is a consequence of low Reynolds number ($Re_x = 2.3 \times 10^4$) at x = 0.25 m with u_∞ = 10 m/s.

(4) To more fully appreciate the effects due to laminar vs. turbulent flow conditions and air velocity, it is useful to examine the local coefficient as a function of distance from the leading edge. How would you use the results plotted below to explain heat rate behavior evident in the summary table above?

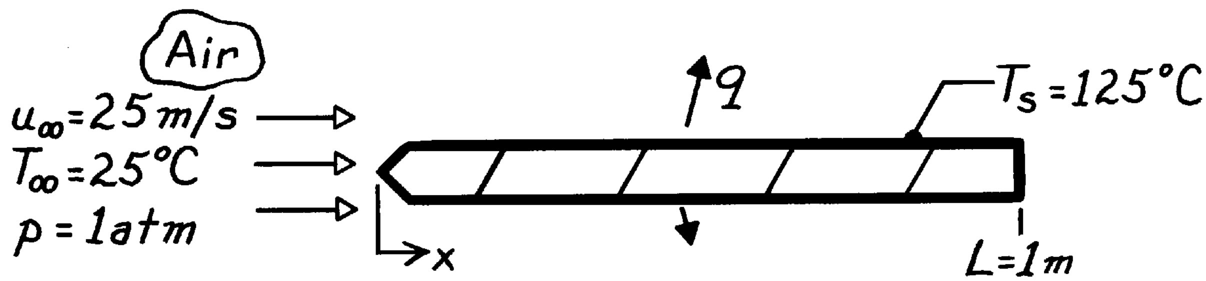


PROBLEM 7.10

KNOWN: Speed and temperature of atmospheric air flowing over a flat plate of prescribed length and temperature.

FIND: Rate of heat transfer corresponding to $Re_{x,c} = 10^5$, 5×10^5 and 10^6.

SCHEMATIC:



ASSUMPTIONS: (1) Flow over top and bottom surfaces.

PROPERTIES: *Table A-4*, Air (T_f = 348K, 1 atm): $\rho = 1.00\ kg/m^3$, $\nu = 20.72 \times 10^{-6}\ m^2/s$, k = 0.0299 W/m·K, Pr = 0.700.

ANALYSIS: With

$$Re_L = \frac{u_\infty L}{n} = \frac{25\ m/s \times 1m}{20.72 \times 10^{-6} m^2/s} = 1.21 \times 10^6$$

the flow becomes turbulent for each of the three values of $Re_{x,c}$. Hence,

$$\overline{Nu}_L = \left(0.037\ Re_L^{4/5} - A\right)\ Pr^{1/3}$$

$$A = 0.037\ Re_{x,c}^{4/5} - 0.664\ Re_{x,c}^{1/2}$$

$Re_{x,c}$	10^5	5×10^5	10^6
A	160	871	1671
$\overline{Nu}_L$	2272	1641	931
$\bar{h}_L\ (W/m^2 \cdot K)$	67.9	49.1	27.8
$q'\ (W/m)$	13,580	9820	5560

where $q' = 2\,\bar{h}_L L\,(T_s - T_\infty)$ is the total heat loss per unit width of plate.

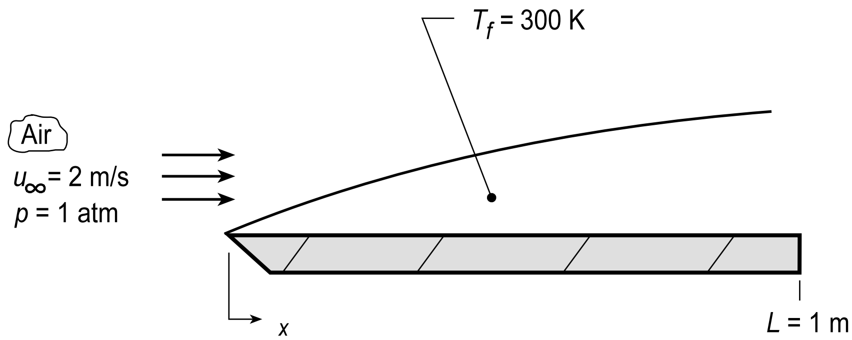
COMMENTS: Note that $\bar{h}_L$ decreases with increasing $Re_{x,c}$, as more of the surface becomes covered with a laminar boundary layer.

PROBLEM 7.11

KNOWN: Velocity and temperature of air in parallel flow over a flat plate of 1-m length.

FIND: (a) Calculate and plot the variation of the local convection coefficient, $h_x(x)$, with distance for flow conditions corresponding to transition Reynolds numbers of 5×10^5, 2.5×10^5 and 0 (fully turbulent), (b) Plot the variation of the average convection coefficient, $\overline{h}_x(x)$, for the three flow conditions of part (a), and (c) Determine the average convection coefficients for the entire plate, $\overline{h}_L$, for the three flow conditions of part (a).

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant surface temperature, and (3) Critical Reynolds depends upon prescribed flow conditions.

PROPERTIES: *Table A.4*, Air (T_f = 300 K, 1 atm): $\nu = 15.89 \times 10^{-6}$ m^2/s, k = 0.0263 W/m·K, Pr = 0.707.

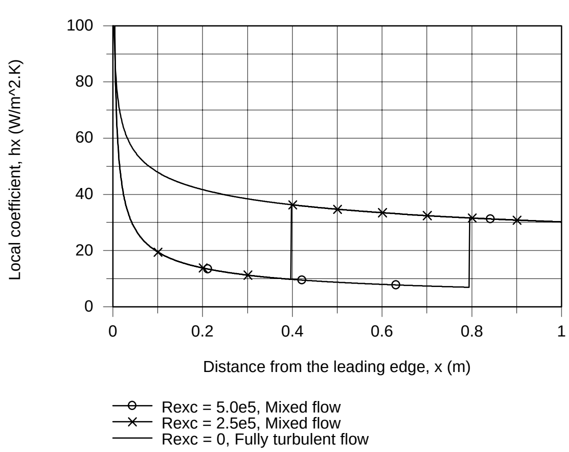
ANALYSIS: (a) The Reynolds number for the plate (L = 1 m) is

$$Re_L = \frac{u_\infty L}{\nu} = \frac{10\,\text{m/s} \times 1\,\text{m}}{15.89 \times 10^{-6}\,\text{m}^2/\text{s}} = 6.29 \times 10^5 .$$

Hence, the boundary layer conditions are mixed with $Re_{x,c} = 5 \times 10^5$,

$$x_c = L\left(Re_{x,c}/Re_L\right) = 1\,\text{m}\,\frac{5\times 10^5}{6.29\times 10^5} = 0.795\,\text{m}$$

Using the *IHT Correlation Tool*, *External Flow*, *Local* coefficients for *Laminar* or *Turbulent Flow*, $h_x(x)$ was evaluated and plotted with critical Reynolds numbers of 5×10^5, 2.5×10^5 and 0 (fully turbulent). Note the location of the laminar-turbulent transition for the first two flow conditions.



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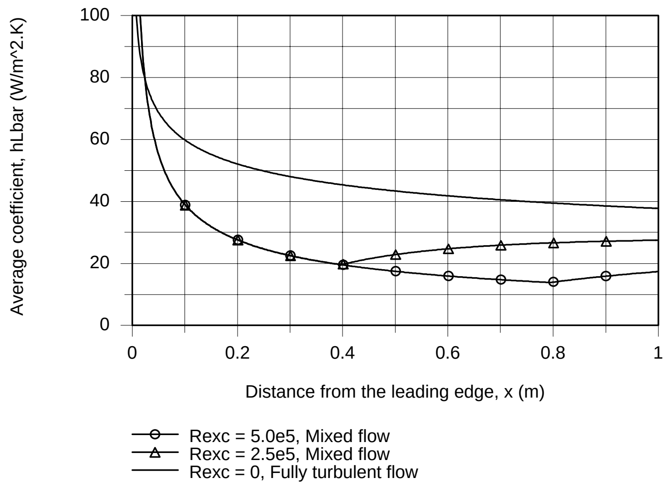
(b) Using the *IHT Correlation Tool*, *External Flow*, *Average* coefficient for *Laminar* or *Mixed Flow*, $\overline{h}_x(x)$ was evaluated and plotted for the three flow conditions. Note that the change in $\overline{h}_x(x)$ at the critical length, x_c, is rather gradual, compared to the abrupt change for the local coefficient, $h_x(x)$.



(c) The average convection coefficients for the plate can be determined from the above plot since $\overline{h}_L = \overline{h}_x(L)$. The values for the three flow conditions are, respectively,

$$\overline{h}_L = 17.4,\ 27.5 \text{ and } 37.8\,\mathrm{W/m^2 \cdot K}$$ <

COMMENTS: A copy of the *IHT Workspace* used to generate the above plots is shown below.

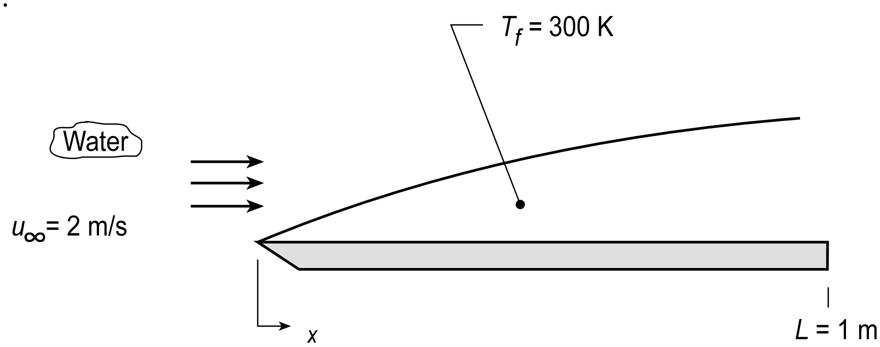
```
// Method of Solution:  Use the Correlation Tools, External Flow, Flat Plate, for (i) Local, laminar or turbulent
flow and (ii) Average, laminar or mixed flow, to evaluate the local and average convection coefficients as a
function of position on the plate.  In each of these tools, the value of the critical Reynolds number, Rexc, can
be set corresponding to the special flow conditions.

// Correlation Tool: External Flow, Plate Plate, Local, laminar or turbulent flow.
Nux = Nux_EF_FP_LT(Rex,Rexc,Pr)     // Eq 7.23,37
Nux = hx * x / k
Rex = uinf * x / nu
Rexc =1e-10
// Evaluate properties at the film temperature, Tf.
//Tf = (Tinf + Ts) / 2

/*  Correlation description: Parallel external flow (EF) over a flat plate (FP), local coefficient; laminar flow (L) for
Rex<Rexc, Eq 7.23; turbulent flow (T) for Rex>Rexc, Eq 7.37; 0.6<=Pr<=60. See Table 7.9.  */

// Correlation Tool: External Flow, Plate Plate, Average, laminar or mixed flow.
NuLbar = NuL_bar_EF_FP_LM(Rex,Rexc,Pr)     // Eq 7.31, 7.39, 7.40
NuLbar = hLbar * x / k                     // Changed variable from L to x
//ReL = uinf * x / nu
//Rexc = 5.0E5

/*  Correlation description: Parallel external flow (EF) over a flat plate (FP), average coefficient; laminar (L) if
ReL<Rexc, Eq 7.31; mixed (M) if ReL>Rexc, Eq 7.39 and 7.40; 0.6<=Pr<=60. See Table 7.9.  */

// Properties Tool - Air:
// Air property functions : From Table A.4
// Units: T(K); 1 atm pressure
nu = nu_T("Air",Tf)            // Kinematic viscosity, m^2/s
k = k_T("Air",Tf)              // Thermal conductivity, W/m·K
Pr = Pr_T("Air",Tf)            // Prandtl number

// Assigned Variables:
x = 1                          // Distance from leading edge; 0 <= x <= 1 m
uinf = 10                      // Freestream velocity, m/s
Tf = 300                       // Film temperature, K
```

PROBLEM 7.12

KNOWN: Velocity and temperature of water in parallel flow over a flat plate of 1-m length.

FIND: (a) Calculate and plot the variation of the local convection coefficient, $h_x(x)$, with distance for flow conditions corresponding to transition Reynolds numbers of 5×10^5, 3×10^5 and 0 (fully turbulent), (b) Plot the variation of the average convection coefficient, $\overline{h}_x(x)$, for the three flow conditions of part (a), and (c) Determine the average convection coefficients for the entire plate, $\overline{h}_L$, for the three flow conditions of part (a).

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant surface temperature, and (3) Critical Reynolds depends upon prescribed flow conditions.

PROPERTIES: *Table A.6*, Water (300 K): $\rho = 997\ kg/m^3$, $\mu = 855 \times 10^{-6}\ N\cdot s/m^2$, $\nu = \mu/\rho = 0.858 \times 10^{-6}\ m^2/s$, k = 0.613 W/m·K, Pr = 583.

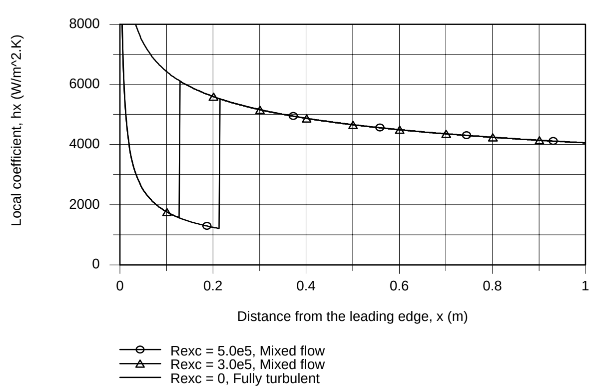
ANALYSIS: (a) The Reynolds number for the plate (L = 1 m) is

$$Re_L = \frac{u_\infty L}{\nu} = \frac{2\ m/s \times 1\ m}{0.858 \times 10^{-6}\ m^2/s} = 2.33 \times 10^6 .$$

and the boundary layer is mixed with $Re_{x,c} = 5 \times 10^5$,

$$x_c = L\left(Re_{x,c}/Re_L\right) = 1\ m \frac{5 \times 10^5}{2.33 \times 10^6} = 0.215\ m$$

Using the *IHT Correlation Tool*, *External Flow*, *Local* coefficients for *Laminar* or *Turbulent Flow*, $h_x(x)$ was evaluated and plotted with critical Reynolds numbers of 5×10^5, 3.0×10^5 and 0 (fully turbulent). Note the location of the laminar-turbulent transition for the first two flow conditions.



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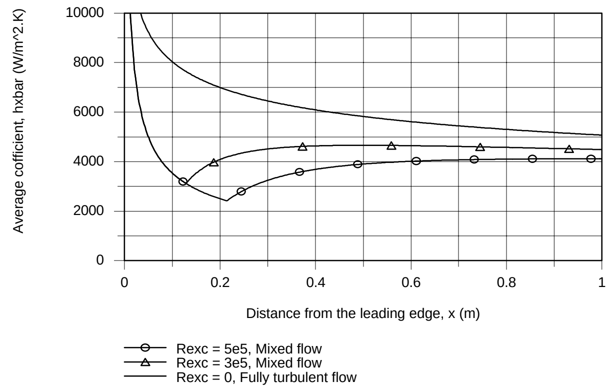
(b) Using the *IHT Correlation Tool*, *External Flow*, *Average* coefficient for *Laminar* or *Mixed Flow*, $\bar{h}_x(x)$ was evaluated and plotted for the three flow conditions. Note that the change in $\bar{h}_x(x)$ at the critical length, x_c, is rather gradual, compared to the abrupt change for the local coefficient, $h_x(x)$.



(c) The average convection coefficients for the plate can be determined from the above plot since $\bar{h}_L = \bar{h}_x(L)$. The values for the three flow conditions are

$$\bar{h}_L = 4110,\ 4490 \text{ and } 5072\ \mathrm{W/m^2 \cdot K} \qquad <$$

COMMENTS: A copy of the *IHT Workspace* used to generate the above plot is shown below.

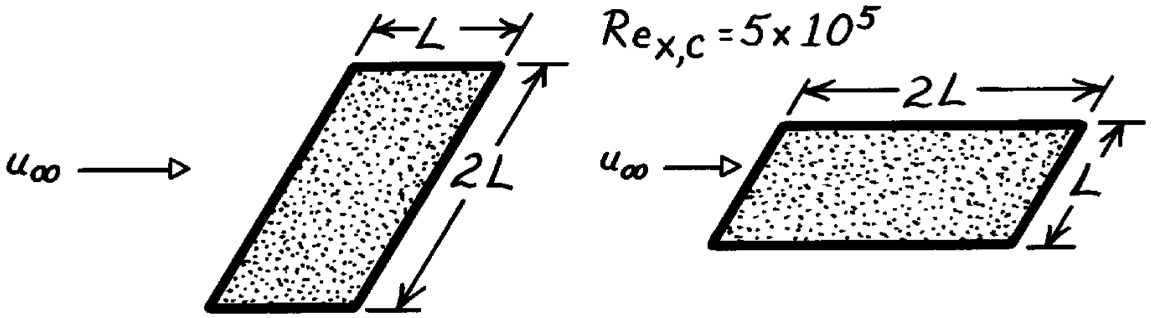
```
/* Method of Solution:  Use the Correlation Tools, External Flow, Flat Plate, for (i) Local, laminar or turbulent
flow and (ii) Average, laminar or mixed flow, to evaluate the local and average convection coefficients as a
function of position on the plate.  In each of these tools, the value of the critical Reynolds number, Rexc, can
be set corresponding to the special flow conditions.   */

// Correlation Tool: External Flow, Plate Plate, Local, laminar or turbulent flow.
Nux = Nux_EF_FP_LT(Rex,Rexc,Pr)     // Eq 7.23,37
Nux = hx * x / k
Rex = uinf * x / nu
Rexc = 1e-10
// Evaluate properties at the film temperature, Tf.
//Tf = (Tinf + Ts) / 2

/*  Correlation description: Parallel external flow (EF) over a flat plate (FP), local coefficient; laminar flow (L) for
Rex<Rexc, Eq 7.23; turbulent flow (T) for Rex>Rexc, Eq 7.37; 0.6<=Pr<=60. See Table 7.9.  */

// Correlation Tool: External Flow, Plate Plate, Average, laminar or mixed flow.
NuLbar = NuL_bar_EF_FP_LM(Rex,Rexc,Pr)     // Eq 7.31, 7.39, 7.40
NuLbar = hLbar * x / k                      // Changed variable from L to x
//ReL = uinf * x / nu
//Rexc = 5.0E5

/*  Correlation description: Parallel external flow (EF) over a flat plate (FP), average coefficient; laminar (L) if
ReL<Rexc, Eq 7.31; mixed (M) if ReL>Rexc, Eq 7.39 and 7.40; 0.6<=Pr<=60. See Table 7.9.  */

// Properties Tool - Water:
// Water property functions :T dependence, From Table A.6
// Units: T(K), p(bars);
xf =  0                    // Quality (0=sat liquid or 1=sat vapor);  "x" is used as spatial coordinate
p = psat_T("Water", Tf)    // Saturation pressure, bar
nu = nu_Tx("Water",Tf,x)   // Kinematic viscosity, m^2/s
k = k_Tx("Water",Tf,x)     // Thermal conductivity, W/m·K
Pr = Pr_Tx("Water",Tf,x)   // Prandtl number

// Assigned Variables:
x = 1                      // Distance from leading edge; 0 <= x <= 1 m
uinf = 2                   // Freestream velocity, m/s
Tf = 300                   // Film temperature, K
```

PROBLEM 7.13

KNOWN: Two plates of length L and 2L experience parallel flow with a critical Reynolds number of 5×10^5.

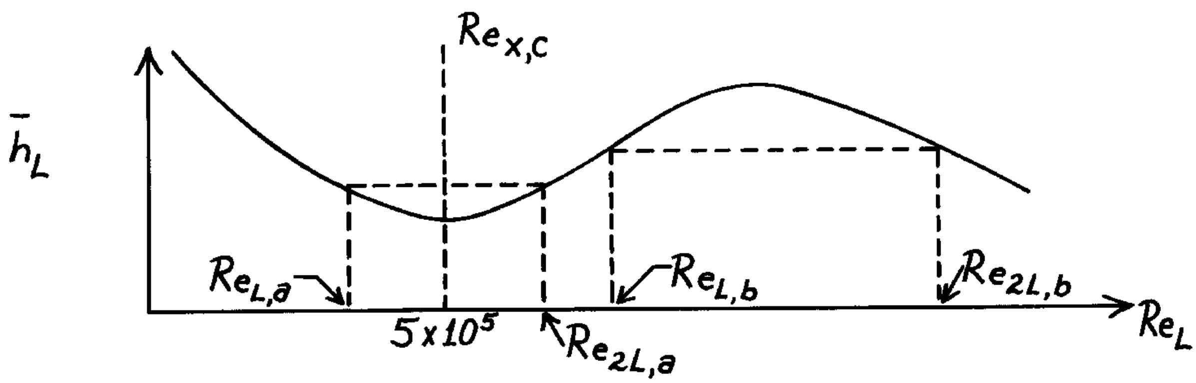
FIND: Reynolds numbers for which the total heat transfer rate is independent of orientation.

SCHEMATIC:



ASSUMPTIONS: (1) Plate temperatures and flow conditions are equivalent.

ANALYSIS: The total heat transfer rate would be the same ($q_L = q_{2L}$), if the convection coefficients were equal, $\bar{h}_L = \bar{h}_{2L}$. Conditions for which such an equality is possible may be inferred from a sketch of $\bar{h}_L$ versus Re_L.



For laminar flow $\left(Re_L < Re_{x,c}\right)$, $\bar{h}_L \; a \; L^{-1/2}$, and for mixed laminar and turbulent flow $\left(Re_L > Re_{x,c}\right)$, $\bar{h}_L = C_1 L^{-1/5} - C_2 L^{-1}$. Hence $\bar{h}_L$ varies with Re_L as shown, and two possibilities are suggested.

Case (a): Laminar flow exists on the shorter plate, while mixed flow conditions exist on the longer plate.

Case (b): Mixed boundary layer conditions exist on both plates.

In both cases, it is required that

$$\bar{h}_L = \bar{h}_{2L} \qquad \text{and} \qquad Re_{2L} = 2\, Re_L .$$

Continued …..

Case (a): From expressions for $\bar{h}_L$ in laminar and mixed flow

$$0.664 \frac{k}{L} Re_L^{1/2} Pr^{1/3} = \frac{k}{2L}\left(0.037\ Re_{2L}^{4/5} - 871\right) Pr^{1/3}$$
$$0.664\ Re_L^{1/2} = 0.032\ Re_L^{4/5} - 435.$$

Since $Re_L < 5 \times 10^5$ and $Re_{2L} = 2\ Re_L > 5 \times 10^5$, the required value of Re_L may be narrowed to the range

$$2.5 \times 10^5 \quad < \quad Re_L \quad < \quad 5 \times 10^5.$$

From a trial-and-error solution, it follows that

$$Re_L \approx 3.2 \times 10^5. \qquad <$$

Case (b): For mixed flow on both plates

$$\frac{k}{L}\left(0.037\ Re_L^{4/5} - 871\right) Pr^{1/3} = \frac{k}{2L}\left(0.037\ Re_{2L}^{4/5} - 871\right) Pr^{1/3}$$

or

$$0.037\ Re_L^{4/5} - 871 = 0.032\ Re_L^{4/5} - 435$$
$$0.005\ Re_L^{4/5} = 436$$

$$Re_L \approx 1.50 \times 10^6. \qquad <$$

COMMENTS: (1) Note that it is impossible to satisfy the requirement that $\bar{h}_L = \bar{h}_{2L}$ if $Re_L < 0.25 \times 10^5$ (laminar flow for both plates).

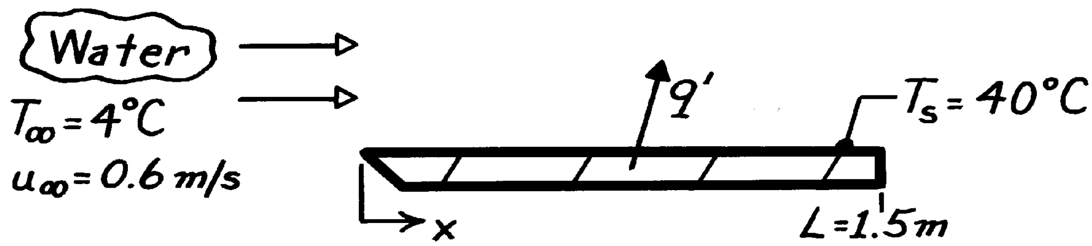
(2) The results are independent of the nature of the fluid.

PROBLEM 7.14

KNOWN: Water flowing over a flat plate under specified conditions.

FIND: (a) Heat transfer rate per unit width, $q'(\mathrm{W/m})$, evaluating properties at $T_f = (T_s + T_\infty)/2$, (b) Error in q' resulting from evaluating properties at T_∞, (c) Heat transfer rate, q', if flow is assumed turbulent at leading edge, x = 0.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions.

PROPERTIES: *Table A-6*, Water (T_∞ = 4°C = 277K): ρ_f = 1000 kg/m^3, $\mu_f = 1560 \times 10^{-6}$ N·s/m^2, $\nu_f = \mu_f/\rho_f = 1.560 \times 10^{-6}$ m^2/s, k_f = 0.577 W/m·K, Pr = 11.44; Water (T_f = 295K): $\nu = 0.961 \times 10^{-6}$ m^2/s, k = 0.606 W/m·K, Pr = 6.62; Water (T_s = 40°C = 313K): $\mu = 657 \times 10^{-6}$ N·s/m^2.

ANALYSIS: (a) The heat rate is given as $q' = \bar{h}L(T_s - T_\infty)$, and $\bar{h}$ must be estimated by the proper correlation. Using properties evaluated at T_f, the Reynolds number is

$$\mathrm{Re}_L = \frac{u_\infty L}{n} = \frac{0.6\ \mathrm{m/s} \times 1.5\mathrm{m}}{0.961 \times 10^{-6}\mathrm{m}^2/\mathrm{s}} = 9.365 \times 10^5.$$

Hence flow is mixed and the appropriate correlation and convection coefficient are

$$\overline{\mathrm{Nu}}_L = \left[0.037\ \mathrm{Re}_L^{4/5} - 871\right]\mathrm{Pr}^{1/3} = \left[0.037\left(9.365 \times 10^5\right)^{4/5} - 871\right] 6.62^{1/3} = 2522$$

$$\bar{h}_L \equiv \frac{\overline{\mathrm{Nu}}_L k}{L} = \frac{2522 \times 0.606\ \mathrm{W/m \cdot K}}{1.5\mathrm{m}} = 1019\ \mathrm{W/m^2 \cdot K}.$$

The heat rate is then

$$q' = 1019\ \mathrm{W/m^2 \cdot K} \times 1.5\mathrm{m}(40-4)^\circ\mathrm{C} = 55.0\ \mathrm{kW/m}. \qquad <$$

(b) Evaluating properties at the free stream temperature, T_∞,

$$\mathrm{Re}_L = \frac{0.6\mathrm{m/s} \times 1.5\mathrm{m}}{1.560 \times 10^{-6}\mathrm{m}^2/\mathrm{s}} = 5.769 \times 10^5$$

The flow is still mixed, giving

$$\overline{\mathrm{Nu}}_L = \left[0.037\left(5.769 \times 10^5\right)^{4/5} - 871\right] 11.44^{1/3} = 1424$$

$$\bar{h}_L = 1424 \times 0.577\ \mathrm{W/m \cdot K}/1.5\mathrm{m} = 575\ \mathrm{W/m \cdot K}$$

$$q' = 575\ \mathrm{W/m \cdot K} \times 1.5\mathrm{m}(40-4)^\circ\mathrm{C} = 31.1\ \mathrm{kW/m}. \qquad <$$

Continued …..

(c) If flow were tripped at the leading edge, the flow would be turbulent over the full length of the plate, in which case,

$$\overline{Nu}_L = 0.037\, Re_L^{4/5}\, Pr^{1/3} = 0.037\left(9.365\times10^5\right)^{4/5} 6.62^{1/3} = 4157$$

$$\bar{h}_L = \overline{Nu}_L\, k/L = 4157\times0.606\ W/m\cdot K/1.5m = 1679\ W/m^2\cdot K$$

$$q' = \bar{h}_L L\left(T_s - T_\infty\right) = 1679\ W/m^2\cdot K\times1.5m\ (40-4)^\circ C = 90.7\ kW/m.$$ <

COMMENTS: Comparing results:

Flow	Part	Property Evaluation	q' (kW/m)	Difference (%)
mixed	(a)	T_f	55.0	--
mixed	(b)	T_∞	31.1	-43
turbulent	(c)	T_f	90.7	--

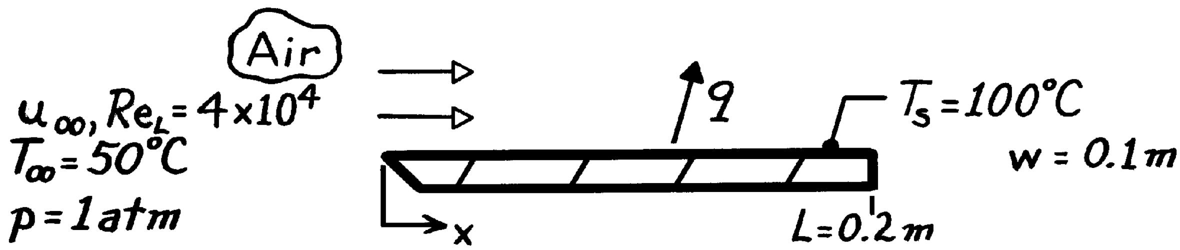
The heat rate is significantly underpredicted if the properties are incorrectly evaluated at T_∞ instead of T_f.

PROBLEM 7.15

KNOWN: Temperature, pressure and Reynolds number for air flow over a flat plate of uniform surface temperature.

FIND: (a) Rate of heat transfer from the plate, (b) Rate of heat transfer if air velocity is doubled and pressure is increased to 10 atm.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform surface temperature, (3) Negligible radiation, (4) $Re_{x_c} = 5\times10^5$.

PROPERTIES: *Table A-4*, Air (T_f = 348K, 1 atm): k = 0.0299 W/m·K, Pr = 0.70.

ANALYSIS: (a) The heat rate is

$$q = \bar{h}_L\left(w\times L\right)\left(T_s - T_\infty\right).$$

Since the flow is laminar over the entire plate for $Re_L = 4\times10^4$, it follows that

$$\overline{Nu}_L = \frac{\bar{h}_L L}{k} = 0.664\ Re_L^{1/2}\ Pr^{1/3} = 0.664\left(40{,}000\right)^{1/2}\left(0.70\right)^{1/3} = 117.9.$$

Hence
$$\bar{h}_L = 117.9\frac{k}{L} = 117.9\frac{0.0299\ W/m\cdot K}{0.2m} = 17.6\ W/m^2\cdot K$$

and
$$q = 17.6\frac{W}{m^2\cdot K}\left(0.1m\times0.2m\right)\left(100-50\right)^\circ C = 17.6\ W.$$ <

(b) With $p_2 = 10\ p_1$, it follows that $\rho_2 = 10\ \rho_1$ and $\nu_2 = \nu_1/10$. Hence

$$Re_{L,2} = \left(\frac{u_\infty L}{n}\right)_2 = 2\times10\left(\frac{u_\infty L}{n}\right)_1 = 20\ Re_{L,1} = 8\times10^5$$

and mixed boundary layer conditions exist on the plate. Hence

$$\overline{Nu}_L = \frac{\bar{h}_L L}{k} = \left(0.037\ Re_L^{4/5} - 871\right)Pr^{1/3} = \left[0.037\times\left(8\times10^5\right)^{4/5} - 871\right]\left(0.70\right)^{1/3}$$

$$\overline{Nu}_L = 961.$$

Hence,
$$\bar{h}_L = 961\frac{0.0299\ W/m\cdot K}{0.2m} = 143.6\ W/m^2\cdot K$$

$$q = 143.6\frac{W}{m^2\cdot K}\left(0.1m\times0.2m\right)\left(100-50\right)^\circ C = 143.6\ W.$$ <

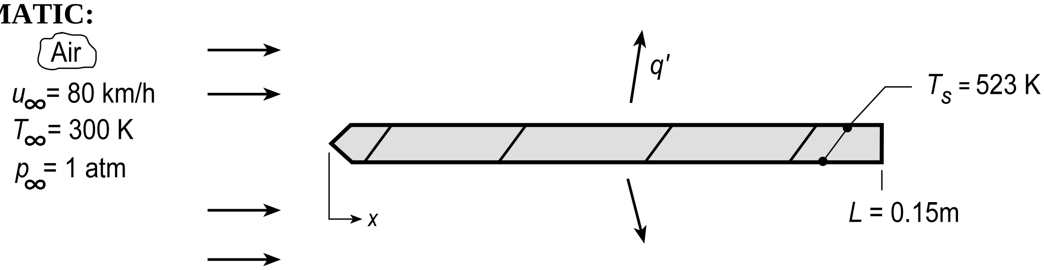
COMMENTS: Note that, in calculating $Re_{L,2}$, ideal gas behavior has been assumed. It has also been assumed that k, μ and Pr are independent of pressure over the range considered.

PROBLEM 7.16

KNOWN: Length and surface temperature of a rectangular fin.

FIND: (a) Heat removal per unit width, q', when air at a prescribed temperature and velocity is in parallel, turbulent flow over the fin, and (b) Calculate and plot q' for motorcycle speeds ranging from 10 to 100 km/h.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible radiation, (3) Turbulent flow over entire surface.

PROPERTIES: *Table A.4*, Air (412 K, 1 atm): $\nu = 27.85 \times 10^{-6}$ m^2/s, k = 0.0346 W/m·K, Pr = 0.69.

ANALYSIS: (a) The heat loss per unit width is

$$q' = 2\times\left[\bar{h}_L L\left(T_s - T_\infty\right)\right]$$

where $\bar{h}$ is obtained from the correlation, Eq. 7.41 but with turbulent flow over the entire surface,

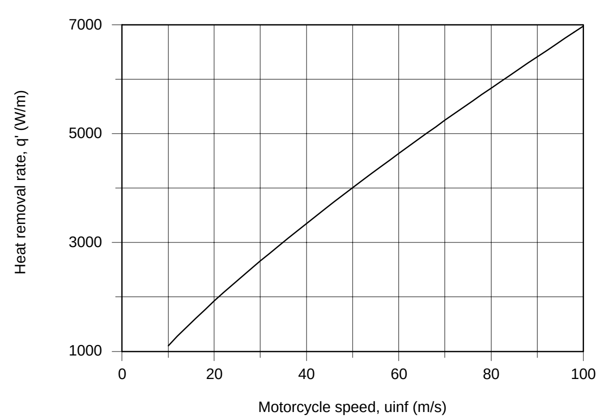
$$\overline{Nu}_L = 0.037\,Re_L^{4/5}\,Pr^{1/3} = 0.037\left[\frac{80\,km/h\times1000\,m/km\times1/3600\,h/s\times0.15\,m}{27.85\times10^{-6}\,m^2/s}\right]^{4/5}\left(0.69\right)^{1/3} = 378$$

Hence,

$$\bar{h}_L = \frac{k}{L}\overline{Nu}_L = \frac{0.0346\,W/m\cdot K}{0.15\,m}378 = 87\,W/m^2\cdot K$$

$$q' = 2\times\left[87\,W/m^2\cdot K\times0.15\,m\left(523-300\right)K\right] = 5826\,W/m\,.$$ <

(b) Using the foregoing equations in the IHT Workspace, q' as a function of speed was calculated and is plotted as shown.



COMMENTS: (1) Radiation emission from the fin is not negligible. With an assumed emissivity of ε = 1, the rate of emission per unit width at 80 km/h would be $q' = \left(\sigma T_s^4\right)2L$ = 1273 W/m. If the fin received negligible radiation from its surroundings, its loss by radiation would then be approximately 20% of that by convection.

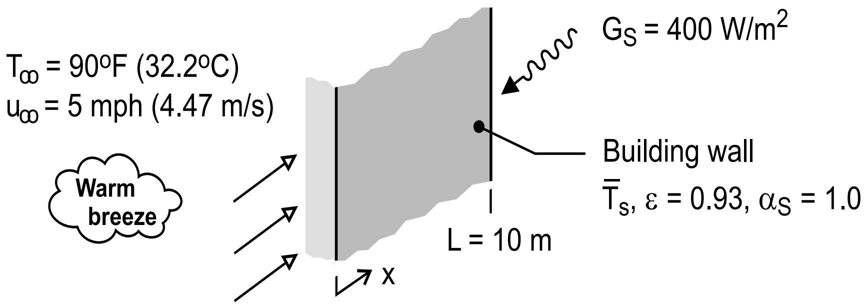
(2) From the correlation and heat rate expression, it follows that $q' \sim u_\infty^{4/5}$. That is, q' vs. u_∞ is nearly linear as evident from the above plot.

PROBLEM 7.17

KNOWN: Wall of a metal building experiences a 10 mph (4.47 m/s) breeze with air temperature of 90°F (32.2°C) and solar insolation of 400 W/m^2. The length of the wall in the wind direction is 10 m and the emissivity is 0.93.

FIND: Estimate the average wall temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) The solar absorptivity of the wall is unity, (3) Sky irradiation is negligible, (4) Wall is isothermal at the average temperature T_s, (5) Flow is fully turbulent over the wall, and (6) Negligible heat transfer into the building.

PROPERTIES: *Table A-4*, Air (assume T_f = 305 K, 1 atm): $\nu = 16.27 \times 10^6$ m^2/s, k = 0.02658 W/m·K, Pr = 0.707.

ANALYSIS: Perform an energy balance on the wall surface considering convection, absorbed irradiation and emission. On a per unit width,

$$\dot{E}'_{in} - \dot{E}'_{out} = 0$$

$$-q'_{cv} + (\alpha_S G_S - E_s) L = 0$$

$$-\overline{h}_L \, L (T_s - T_\infty) + \left(\alpha_S G_S - \varepsilon\sigma T_s^4\right) L = 0 \qquad (1)$$

The average convection coefficient is estimated using Eq. 7.41 assuming fully turbulent flow over the length of the wall in the direction of the breeze.

$$\overline{Nu}_L = \frac{\overline{h}_L \, L}{k} = 0.037 \, Re_L^{4/5} \, Pr^{1/3} \qquad (2)$$

$$Re_L = u_\infty L / \nu = 4.47 \, m/s \times 10 \, m / 16.27 \times 10^6 \, m^2/s = 2.748 \times 10^6$$

$$\overline{h}_L = (0.02658 \; W/m \cdot K / 10 \, m) \times 0.037 \left(2.748 \times 10^6\right)^{4/5} (0.707)^{1/3} = 12.4 \; W/m^2 \cdot K$$

Substituting numerical values into Eq. (1), find T_s.

$$-12.4 \; W/m^2 \times 10 \, m \left[T_s - (32.2 + 273)\right] K$$

$$+\left[1.0 \times 400 \; W/m^2 - 0.93 \times 5.67 \times 10^{-8} W/m^2 \cdot K^4 T_s^4\right] \times 10 \, m = 0$$

$$T_s = 302.2 \; K = 29°C \qquad <$$

COMMENTS: (1) The properties for the correlation should be evaluated at $T_f = (T_s + T_\infty)/2$ = 304 K. The assumption of 305 K was reasonable.

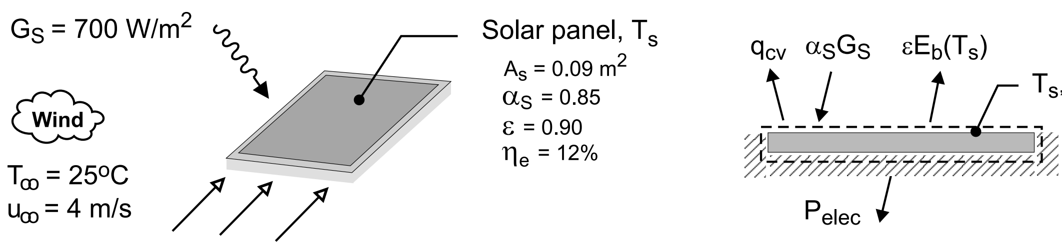
(2) Is the heat transfer by the emission process significant? Would application of a low emissive coating be effective in reducing the wall temperature, assuming α_S remained unchanged? Or, should a low solar absorbing coating be considered?

PROBLEM 7.18

KNOWN: Square solar panel with an area of 0.09 m^2 has solar-to-electrical power conversion efficiency of 12%, solar absorptivity of 0.85, and emissivity of 0.90. Panel experiences a 4 m/s breeze with an air temperature of 25°C and solar insolation of 700 W/m^2.

FIND: Estimate the temperature of the solar panel for: (a) The operating condition *(on)* described above when the panel is producing power, and (b) The *off* condition when the solar array is inoperative. Will the panel temperature increase, remain the same or decrease, all other conditions remaining the same?

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) The backside of the panel experiences no heat transfer, (3) Sky irradiation is negligible, and (4) Wind is in parallel, fully turbulent flow over the panel.

PROPERTIES: *Table A-4*, Air (Assume T_f = 300 K, 1 atm): $\nu = 15.89 \times 10^{-6}$ m^2/s, k = 0.0263 W/m·K, Pr = 0.707.

ANALYSIS: (a) Perform an energy balance on the panel as represented in the schematic above considering convection, absorbed insolation, emission and generated electrical power.

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = 0$$

$$-q_{cv} + \left[\alpha_S G_S - \varepsilon\sigma T_s^4\right] A_s - P_{elec} = 0 \qquad (1)$$

Using the convection rate equation and power conversion efficiency,

$$q_{cv} = \bar{h}_L A_s \left(T_s - T_\infty\right) \qquad P_{elec} = \eta_e \alpha_S G_S A_s \qquad (2,3)$$

The average convection coefficient for fully turbulent conditions is

$$\overline{Nu}_L = \bar{h}L/k = 0.037\ Re_L^{4/5}\ Pr^{1/3}$$

$$Re_L = u_\infty L/\nu = 4\,m/s \times 0.3\,m/15.89\times10^{-6} m^2/s = 7.49\times10^4$$

$$\bar{h}_L = \left(0.0263\ W/m\cdot K/0.3m\right)\times 0.037 \times \left(7.49\times10^4\right)^{4/5}\left(0.707\right)^{1/3}$$

$$\bar{h}_L = 23.0\ W/m^2\cdot K$$

Substituting numerical values in Eq. (1) using Eqs. (2 and 3) and dividing through by A_s, find T_s.

Continued …..

PROBLEM 7.18 (Cont.)

$$23\ \mathrm{W/m^2\cdot K}\left(T_s - 298\right)\mathrm{K} + 0.85\times 700\ \mathrm{W/m^2} - 0.90\times 5.67\times 10^{-8}\,\mathrm{W/m^2\cdot K^4}\,T_s^4$$
$$-0.12\left[0.85\times 700\ \mathrm{W/m^2}\right] = 0 \qquad (4)$$

$$T_s = 302.2\ \mathrm{K} = 29.2°\mathrm{C} \qquad <$$

(b) If the solar array becomes inoperable *(off)* for reason of wire bond failures or the electrical circuit to the battery is opened, the P_{elec} term in the energy balance of Eq. (1) is zero. Using Eq. (4) with η_e = 0, find

$$T_s = 31.7°\mathrm{C} \qquad <$$

COMMENTS: (1) Note how the electrical power P_{elec} is represented by the $\dot{E}_{gen}$ term in the energy balance. Recall from Section 1.2 that $\dot{E}_{gen}$ is associated with conversion *from* some form of energy *to* thermal energy. Hence, the solar-to-electrical power conversion (P_{elec}) will have a negative sign in Eq. (1).

(2) It follows that when the solar array is *on*, a fraction (η_e) of the absorbed solar power (thermal energy) is converted to electrical energy. As such, the array surface temperature will be higher in the *off* condition than in the *on* condition.

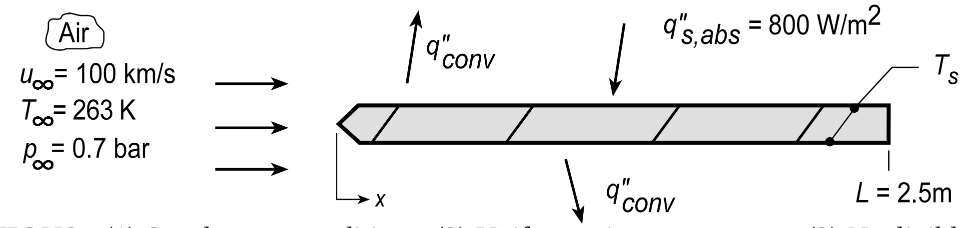
(3) Note that the assumed value for T_f at which to evaluate the properties was reasonable.

PROBLEM 7.19

KNOWN: Ambient air conditions and absorbed solar flux for an aircraft wing of prescribed length and speed.

FIND: (a) Steady-state temperature of wing and (b) Calculate and plot the steady-state temperature for plane speeds 100 to 250 m/s.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform wing temperature, (3) Negligible radiation emission from surface.

PROPERTIES: *Table A.4*, Air ($T_f \approx 270$ K, p = 0.7 bar): k = 0.0239 W/m·K, Pr = 0.715, $\nu = 13.22 \times 10^{-6}$ m²/s (1.0133 bar/0.7 bar) = 19.14×10^{-6} m²/s.

ANALYSIS: From an energy balance on the airfoil

$$q''_{S,abs} A_S = 2q_{conv} = 2\bar{h}_L A_S (T_S - T_\infty) \qquad T_S = T_\infty + q''_{S,abs} / 2\bar{h}_L \qquad (1,2)$$

Since

$$Re_L = u_\infty L/\nu = (100\,m/s)\,2.5\,m / 19.14\times10^{-6}\,m^2/s = 1.31\times10^7 \qquad (3)$$

and $Re_{s,c} = 5 \times 10^5$, the flow may be approximated as turbulent over the entire plate. Hence, from Eq. 7.41,

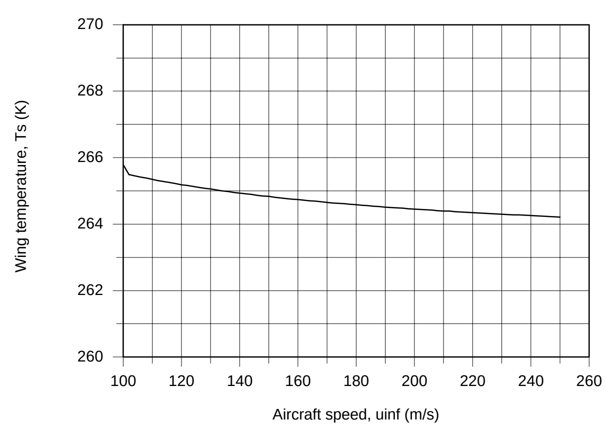
$$\overline{Nu}_L = 0.037\,Re_L^{4/5}\,Pr^{1/3} = 0.037\left(1.31\times10^7\right)^{4/5}(0.715)^{1/3} = 1.63\times10^4 \qquad (4)$$

$$\bar{h}_L = \frac{\overline{Nu}_L k}{L} = \frac{1.63\times10^4\,(0.0239\,W/m\cdot K)}{2.5\,m} = 156\,W/m^2\cdot K \qquad (5)$$

Hence, from the energy balance

$$T_S = 263\,K + 800\,W/m^2 / 2\times156\,W/m^2\cdot K = 266\,K$$ <

(b) Using the energy balance relation for T_s, Eq. (1), and the *IHT Correlations Tool*, *External Flow*, *Average* coefficient for *Laminar* or *Turbulent Flow*, T_s as a function of u_∞ was evaluated.



COMMENTS: (1) Radiation emission from the wing surface would *decrease* T_s, while radiation incident from the earth's surface and the sky would act to *increase* T_s. The net effect on T_s is likely to be small.

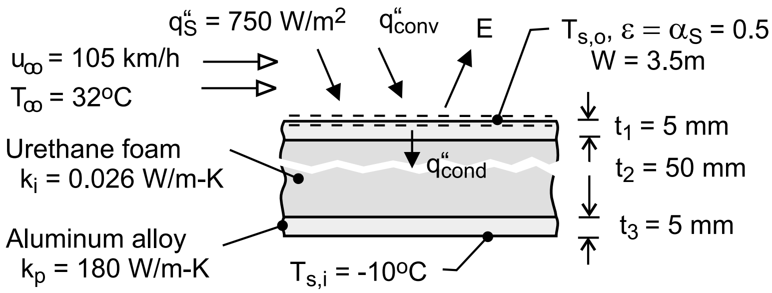
(2) How do you explain that the effect of aircraft speed on T_s appears to be only slight? How does $\bar{h}_L$ dependent upon u_∞? What is the limit of T_s with increasing speed?

PROBLEM 7.20

KNOWN: Material properties, inner surface temperature and dimensions of roof of refrigerated truck compartment. Truck speed and ambient temperature. Solar irradiation.

FIND: (a) Outer surface temperature of roof and rate of heat transfer to compartment, (b) Effect of changing radiative properties of outer surface, (c) Effect of eliminating insulation.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible irradiation from the sky, (2) Turbulent flow over entire outer surface, (3) Average convection coefficient may be used to estimate average surface temperature, (4) Constant properties.

PROPERTIES: *Table A-4*, air (p = 1 atm, $T_f \approx 300K$): $\nu = 15.89 \times 10^{-6}\ m^2/s$, k = 0.0263 W/m·K, Pr = 0.707.

ANALYSIS: (a) From an energy balance for the outer surface,

$$\alpha_S G_S + q''_{conv} - E = q''_{cond} = \frac{T_{s,o} - T_{s,i}}{R''_{tot}}$$

$$\alpha_S G_S + \overline{h}\left(T_\infty - T_{s,o}\right) - \varepsilon\sigma T_{s,o}^4 = \frac{T_{s,o} - T_{s,i}}{2R''_p + R''_i}$$

where $R''_p = \left(t_1 / k_p\right) = 2.78\times10^{-5}\,m^2\cdot K/W$, $R''_i = \left(t_2 / k_i\right) = 1.923\,m^2\cdot K/W$, and with $Re_L = u_\infty L/\nu$ $= 29.2\,m/s\times10m/15.89\times10^{-6}m^2/s = 1.84\times10^7$,

$$\overline{h} = \frac{k}{L}0.037\,Re_L^{4/5}\,Pr^{1/3} = \frac{0.0263\,W/m\cdot K}{10m}0.037\left(1.84\times10^7\right)^{4/5}\left(0.707\right)^{1/3} = 56.2\,W/m^2\cdot K$$

Hence,

$$0.5\left(750\,W/m^2\cdot K\right) + 56.2\,W/m^2\cdot K\left(305 - T_{s,o}\right) - 0.5\times5.67\times10^{-8}\,W/m^2\cdot K^4\,T_{s,o}^4 = \frac{T_{s,o} - 263K}{\left(5.56\times10^{-5} + 1.923\right)m^2\cdot K/W}$$

Solving, we obtain

$$T_{s,o} = 306.8K = 33.8°C \qquad <$$

Hence, the heat load is

$$q = \left(W\cdot L\right)q''_{cnd} = \left(3.5m\times10m\right)\frac{\left(33.8+10\right)°C}{1.923\,m^2\cdot K/W} = 797\,W \qquad <$$

(b) With the special surface finish $\left(\alpha_S = 0.15,\ \varepsilon = 0.8\right)$,

Continued …..

$$T_{s,o} = 301.1\text{K} = 27.1°\text{C}$$ <

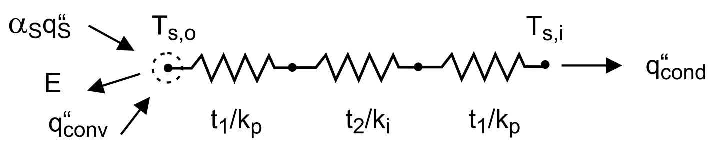
$$q = 675.3\text{W}$$ <

(c) Without the insulation (t_2 = 0) and with $\alpha_S = \varepsilon = 0.5$,

$$T_{s,o} = 263.1\text{K} = -9.9°\text{C}$$ <

$$q = 90,630\text{W}$$ <

COMMENTS: (1) Use of the special surface finish reduces the solar input, while increasing radiation emission from the surface. The cumulative effect is to reduce the heat load by 15%. (2) The thermal resistance of the aluminum panels is negligible, and without the insulation, the heat load is *enormous*.

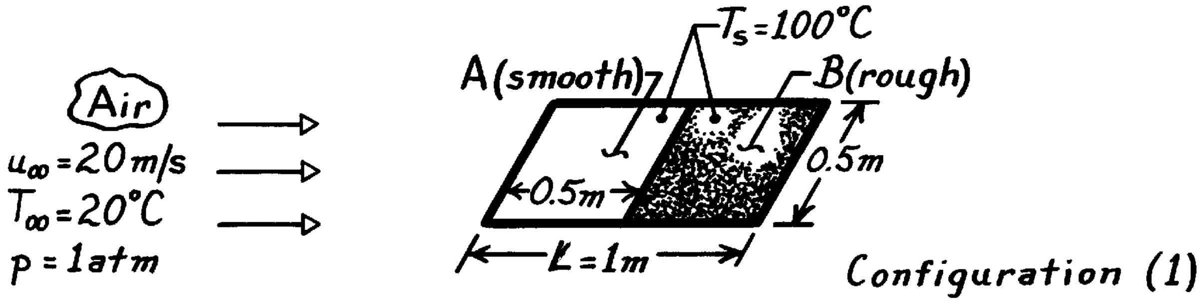


PROBLEM 7.21

KNOWN: Surface characteristics of a flat plate in an air stream.

FIND: Orientation which minimizes convection heat transfer.

SCHEMATIC:



ASSUMPTIONS: (1) Surface B is sufficiently rough to trip the boundary layer when in the upstream position (Configuration 2).

PROPERTIES: *Table A-4*, Air (T_f = 333K, 1 atm): $\nu = 19.2 \times 10^{-6}\ m^2/s$, $k = 28.7 \times 10^{-3}$ W/m·K, Pr = 0.7.

ANALYSIS: Since Configuration (2) results in a turbulent boundary layer over the entire surface, the lowest heat transfer is associated with Configuration (1). Find

$$Re_L = \frac{u_\infty L}{n} = \frac{20\ m/s \times 1m}{19.2 \times 10^{-6} m^2/s} = 1.04 \times 10^6.$$

Hence in Configuration (1), transition will occur just before the rough surface (x_c = 0.48m). Note that

$$\overline{Nu}_{L,1} = \left[0.037\left(1.04 \times 10^6\right)^{4/5} - 871\right] 0.7^{1/3} = 1366$$

$$\overline{Nu}_{L,2} = 0.037\left(1.04 \times 10^6\right)^{4/5} (0.7)^{1/3} = 2139 > \overline{Nu}_{L,1}.$$

For Configuration (1): $$\frac{\bar{h}_{L,1} L}{k} = \overline{Nu}_{L,1} = 1366.$$

Hence

$$\bar{h}_{L,1} = 1366\left(28.7 \times 10^{-3} W/m \cdot K\right)/1m = 39.2\ W/m^2 \cdot K$$

and

$$q_1 = \bar{h}_{L,1} A\left(T_s - T_\infty\right) = 39.2\ W/m^2 \cdot K\left(0.5m \times 1m\right)\left(100 - 20\right)K$$

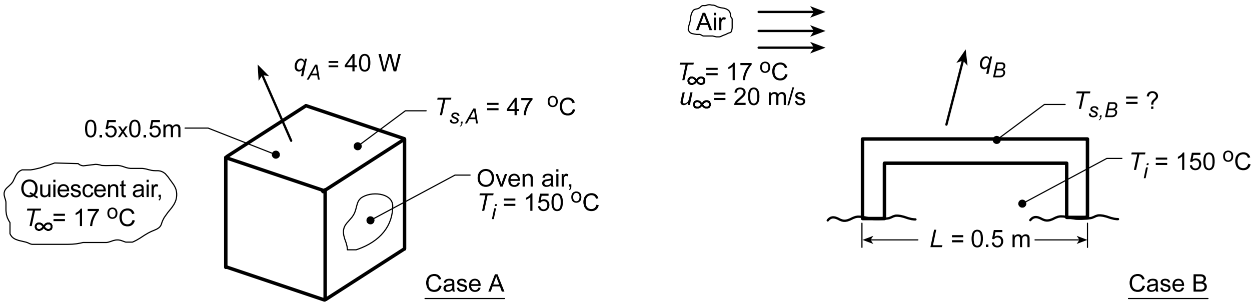
$$q_1 = 1568\ W.$$ <

PROBLEM 7.22

KNOWN: Heat rate from and surface temperature of top surface of an oven under quiescent room air conditions (Case A).

FIND: (a) Heat rate when air at 15 m/s is blown across surface, (b) Surface temperature, T_s, achieved with the forced convection condition, and (c) Calculate and plot T_s as a function of room air velocity for $5 \le u_\infty \le 30$ m/s.

SCHEMATIC:

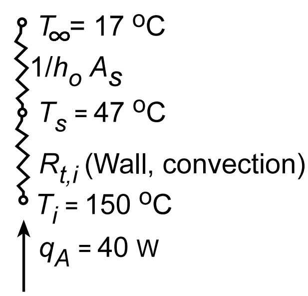


ASSUMPTIONS: (1) Surface has uniform temperature under both conditions, (2) Negligible radiation effects, (3) Air is blown parallel to edge, and (4) Thermal resistance due to oven wall and internal convection are the same for both conditions.

PROPERTIES: *Table A.4*, Air ($\overline{T}_f = \left(T_s + T_\infty\right)/2 \approx (37 + 17)/2 = 27$ °C = 300 K): k = 0.0263 W/m·K, $\nu = 15.89 \times 10^{-6}$ m^2/s, Pr = 0.707.

ANALYSIS: (a) For Case A, we can determine the thermal resistance due to the wall and internal convection as,

$$R_{t,i} = \frac{T_i - T_s}{q_A} = \frac{(150-47)^\circ\,C}{40\,W} = 2.575\,K/W \qquad (1)$$



which remains constant for case B. Hence, for Case B with forced convection, the heat rate is

$$q_B = UA\left(T_i - T_\infty\right) \qquad (2)$$

where

$$(UA)^{-1} = R_{t,i} + \left(1/\overline{h}_o A_s\right) \qquad (3)$$

To estimate $\overline{h}_o$, find

$$Re_L = \frac{u_\infty L}{\nu} = \frac{20\,m/s \times 0.5\,m}{15.89 \times 10^{-6}\,m^2/s} = 6.293 \times 10^5.$$

Assuming $Re_{s,c} = 5 \times 10^5$, flow conditions are mixed; hence

$$\overline{Nu}_L = \frac{\overline{h}_o L}{k} = \left(0.037\,Re_L^{4/5} - 871\right)Pr^{1/3} = \left(0.037\left(6.293 \times 10^5\right)^{0.8} - 871\right)(0.707)^{1/3} = 660.0$$

$$\overline{h}_o = 660.0 \times 0.0263\,W/m \cdot K/0.5\,m = 34.7\,W/m^2 \cdot K.$$

Using Eq. (3) for $(UA)^{-1}$ and Eq. (2) for q_B, find

$$(UA)^{-1} = 2.575\,K/W + \left(1/34.7\,W/m^2 \cdot K\,(0.5\,m)^2\right) = (2.575 + 0.115) = 2.690\,K/W$$

Continued...

$$q_B = (1/2.690\,\mathrm{K/W})(150-17)\,\mathrm{K} = 49.4\,\mathrm{W}.$$ <

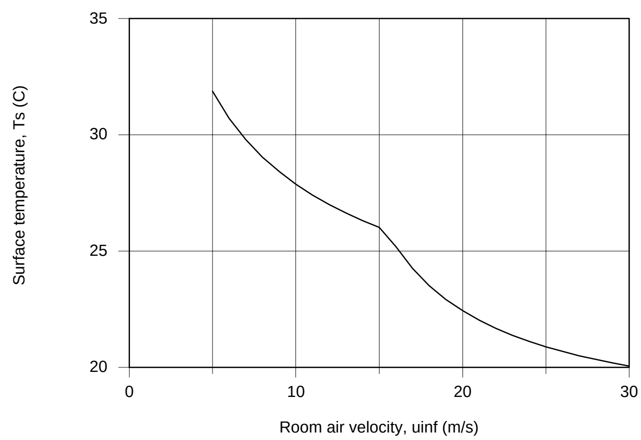
(b) From the rate equation at the surface,

$$T_s = T_\infty + q/\bar{h}_o A_s \tag{4}$$

$$T_s = 17^\circ\mathrm{C} + 49.4\,\mathrm{W}\Big/\left(34.7\,\mathrm{W/m^2\cdot K}\times(0.5\,\mathrm{m})^2\right)$$

$$T_s = (17+5.7)^\circ\,\mathrm{C} = 22.7^\circ\,\mathrm{C}$$ <

(c) Using Eqs. (2), (3) and (4), and evaluating $\bar{h}_o$ using *IHT Correlations Tool*, *External Flow*, *Average* coefficient for *Laminar* or *Mixed Flow*, the surface temperature was evaluated as a function of room air velocity and is plotted below.



COMMENTS: (1) Note that in part (a), $T_f = (T_s + T_\infty)/2 = (22.7 + 17)/2 = 19.8°C = 293$ K compared to the assumed value of 300 K. Performing an iterative solution with IHT, find $T_f = 293$ with $T_s = 22.4°C$ suggesting the approximate value for T_f was satisfactory.

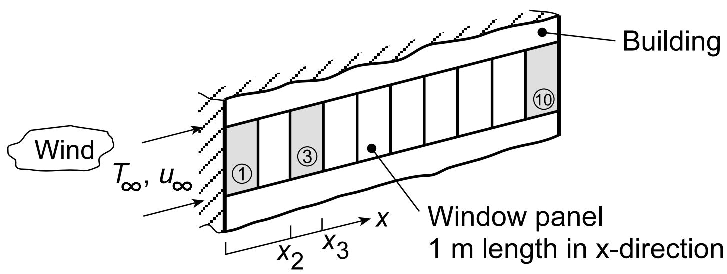
(2) From the plot, as expected, T_s decreases with increasing air velocity. What is the cause of the inflection in the curve at $u_\infty = 15$ m/s? As u_∞ increases, what is the limit for T_s?

PROBLEM 7.23

KNOWN: Prevailing wind with prescribed speed blows past ten window panels, each of 1-m length, on a penthouse tower.

FIND: (a) Average convection coefficient for the first, third and tenth window panels when the wind speed is 5 m/s; evaluate thermophysical properties at 300 K, but determine suitability when ambient air temperature is in the range $-15 \le T_\infty \le 38°C$; (b) Compute and plot the average coefficients for the same panels with wind speeds for the range $5 \le u_\infty \le 100$ km/h; explain features and relative magnitudes.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Wind over panels approximates parallel flow over a smooth flat plate, and (4) Transition Reynolds number is $Re_{s,c} = 5 \times 10^5$.

PROPERTIES: *Table A.4,* Air (T_f = 300 K, 1 atm): $\nu = 15.89 \times 10^{-5}$ m²/s, $k = 26.3 \times 10^{-3}$ W/m·K, Pr = 0.707.

ANALYSIS: (a) The average convection coefficients for the first, third and tenth panels are

$$\overline{h}_1 \qquad \overline{h}_{2-3} = \frac{\overline{h}_3 x_3 - \overline{h}_2 x_2}{x_3 - x_2} \qquad \overline{h}_{9-10} = \frac{\overline{h}_{10} x_{10} - \overline{h}_9 x_9}{x_{10} - x_9} \qquad (1,2,3)$$

where $\overline{h}_2 = \overline{h}_2(x_2)$, etc. If $Re_{x,c} = 5 \times 10^5$, with properties evaluated at T_f = 300 K, transition occurs at

$$x_c = \frac{\nu}{u_\infty} Re_{x,c} = \frac{15.89 \times 10^{-6}\,\mathrm{m^2/s}}{5\,\mathrm{m/s}} \times 5 \times 10^5 = 1.59\,\mathrm{m}$$

The flow over the first panel is laminar, and $\overline{h}_1$ can be estimated using Eq. (7.31).

$$\overline{Nu}_{x1} = \frac{\overline{h}_1 x_1}{k} = 0.664\, Re_x^{1/2}\, Pr^{1/3}$$

$$\overline{h}_1 = (0.0263\,\mathrm{W/m\cdot K} \times 0.664/1\mathrm{m})\left(5\,\mathrm{m/s} \times 1\mathrm{m}/15.89 \times 10^{-6}\,\mathrm{m^2/s}\right)^{1/2} (0.707)^{1/3} = 8.73\,\mathrm{W/m^2\cdot K} \quad <$$

The flow over the third and tenth panels is mixed, and $\overline{h}_2$, $\overline{h}_3$, $\overline{h}_9$ and $\overline{h}_{10}$ can be estimated using Eq. (7.41). For the third panel with x_3 = 3 m and x_2 = 2 m,

$$\overline{Nu}_{x3} = \frac{\overline{h}_3 x_3}{k} = \left(0.037\, Re_x^{4/5} - 871\right) Pr^{1/3}$$

$$\overline{h}_3 = (0.0263\,\mathrm{W/m\cdot K}/3\mathrm{m})$$
$$\times \left[0.037\left(5\,\mathrm{m/s} \times 3\mathrm{m}/15.89 \times 10^{-6}\,\mathrm{m^2/s}\right)^{4/5} - 871\right](0.707)^{1/3} = 10.6\,\mathrm{W/m^2\cdot K}$$

Continued...

$$\bar{h}_2 = (0.0263\,W/m\cdot K/2m)$$
$$\times\left[0.037\left(5\,m/s\times 2m/15.89\times10^{-6}\,m^2/s\right)^{4/5} - 871\right](0.707)^{1/3} = 8.68\,W/m^2\cdot K$$

From Eq. (2),

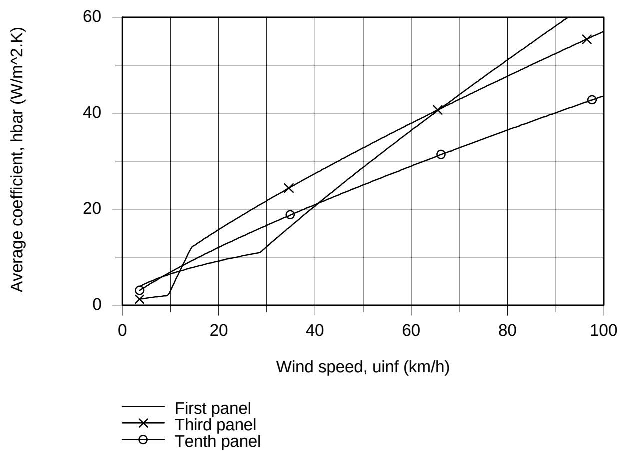
$$\bar{h}_{2-3} = \frac{10.61\,W/m^2\cdot K\times 3m - 8.68\,W/m^2\cdot K\times 2m}{(3-2)m} = 14.5\,W/m^2\cdot K \qquad <$$

Following the same procedure for the tenth panel, find $\bar{h}_{10}$ = 11.64 W/m^2·K and $\bar{h}_9$ = 11.71 W/m^2·K, and

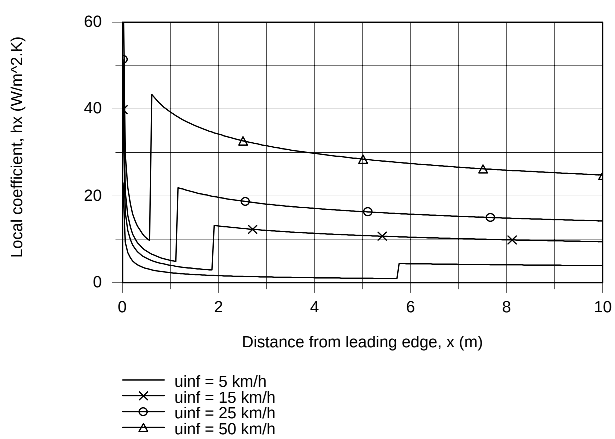
$$\bar{h}_{9-10} = 11.1\,W/m^2\cdot K \qquad <$$

Assuming that the window panel temperature will always be close to room temperature, T_s = 23°C = 296 K. If T_∞ ranges from -15 to 38°C, the film temperature, $T_f = (T_s + T_\infty)/2$, will vary from 275 to 310 K. We'll explore the effect of T_f subsequently.

(b) Using the *IHT Tool*, *Correlations*, *External Flow*, *Flat Plate*, results were obtained for the average coefficients $\bar{h}$. Using Eqs. (2) and (3), average coefficients for the panels as a function of wind speed were computed and plotted.



COMMENTS: (1) The behavior of the panel average coefficients as a function of wind speed can be explained from the behavior of the local coefficient as a function of distance for difference velocities as plotted below.



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PROBLEM 7.23 (Cont.)

For low wind speeds, transition occurs near the mid-panel, making $\bar{h}_1$ and $\bar{h}_{9-10}$ nearly equal and very high because of leading-edge and turbulence effects, respectively. As the wind speed increases, transition occurs closer to the leading edge. Notice how $\bar{h}_{2-3}$ increases rather abruptly, subsequently becoming greater than $\bar{h}_{9-10}$. The abrupt increase in $\bar{h}_1$ around 30 km/h is a consequence of transition occurring with $x < 1$m.

(2) Using the IHT code developed for the foregoing analysis with $u_\infty = 5$ m/s, the effect of T_f is tabulated below

T_f (K)	275	300	310
$\bar{h}_1$ (W/m$^2\cdot$K)	8.72	8.73	8.70
$\bar{h}_{2-3}$ (W/m$^2\cdot$K)	15.1	14.5	14.2
$\bar{h}_{9-10}$ (W/m$^2\cdot$K)	11.6	11.1	10.8

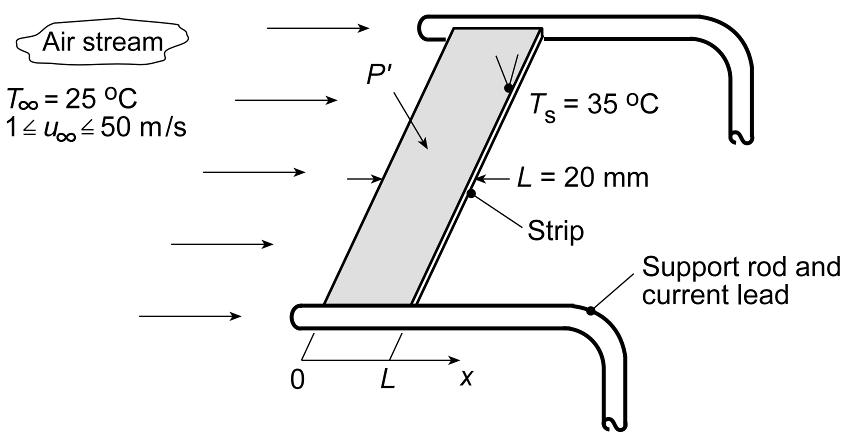
The overall effect of T_f on estimates for the average panel coefficient is slight, less than 5%.

PROBLEM 7.24

KNOWN: Design of an anemometer comprised of a thin metallic strip supported by stiff rods serving as electrodes for passage of heating current. Fine-wire thermocouple on trailing edge of strip.

FIND: (a) Relationship between electrical power dissipation per unit width of the strip in the transverse direction, P' (mW/mm), and airstream velocity u_∞ when maintained at constant strip temperature, T_s; show the relationship graphically; (b) The uncertainty in the airstream velocity if the accuracy with which the strip temperature can be measured and maintained constant is ±0.2°C; (c) Relationship between strip temperature and airstream velocity u_∞ when the strip is provided with a constant power, P' = 30 mW/mm; show the relationship graphically. Also, find the uncertainty in the airstream velocity if the accuracy with which the strip temperature can be measured is ±0.2°C; (d) Compare features associated with each of the operating nodes.

SCHEMATIC:

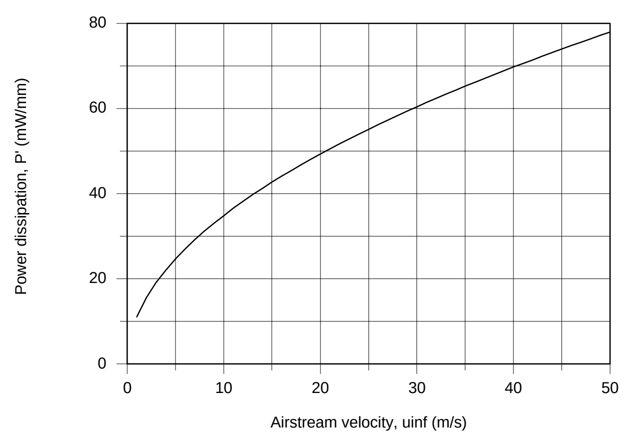


ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Strip has uniform temperature in the midspan region of the strip, (4) Negligible conduction in the transverse direction in the midspan region, and (5) Airstream over strip approximates parallel flow over two sides of a smooth flat plate.

ANALYSIS: (a) In the midspan region of uniform temperature T_s with no conduction in the transverse direction, all the dissipated electrical power is transferred by convection to the airstream,

$$P' = 2\bar{h}_L L\left(T_s - T_\infty\right) \tag{1}$$

where P' is the power per unit width (transverse direction). Using the *IHT Correlation Tool* for *External Flow-Flat Plate* the power as a function of airstream velocity was determined and is plotted below. The IHT tool uses the flat plate correlation, Eq. 7.31 since the flow is laminar over this velocity range.



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PROBLEM 7.24 (Cont.)

(b) By differentiation of Eq. (1), the relative uncertainties of the convection coefficient and strip temperature are, assuming the power remains constant,

$$\frac{\Delta \bar{h}_L}{\bar{h}} = -\frac{\Delta T_s}{T_s - T_\infty} \tag{2}$$

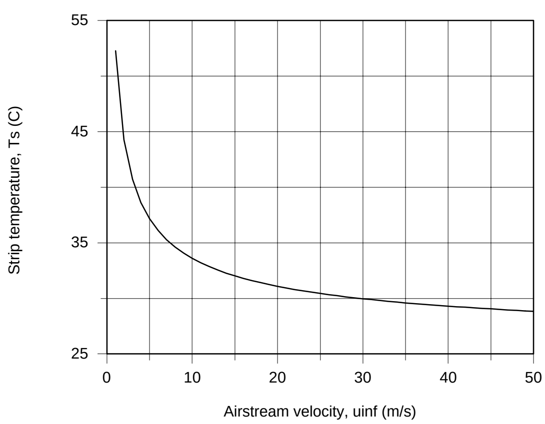
Since the flow was laminar for the range of airstream velocities, Eq. 7.31,

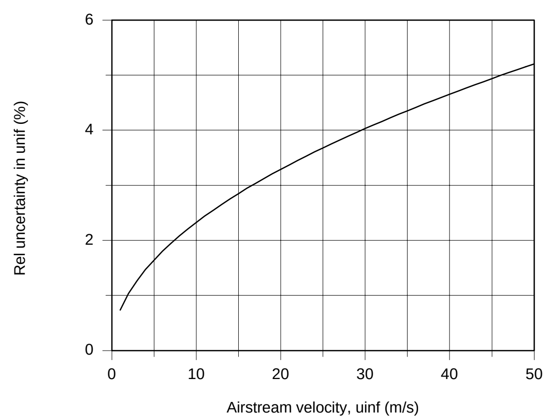
$$\bar{h}_L \sim u_\infty^{1/2} \qquad \text{or} \qquad \frac{\Delta \bar{h}_L}{\bar{h}_L} = 0.5 \frac{\Delta u_\infty}{u_\infty} \tag{3}$$

Hence, the relative uncertainty in the air velocity due to uncertainty in T_s, $\Delta T_s = \pm 0.2^\circ C$

$$\frac{\Delta u_\infty}{u_\infty} = 2 \frac{\Delta T_s}{T_s - T_\infty} = 2 \frac{\pm 0.2^\circ C}{(35 - 25)^\circ C} = \pm 4\% \tag{4}$$ <

(c) Using the IHT workspace setting $P' = 30$ mW/mm, the strip temperature T_s as a function of the airstream velocity was determined and plotted. Note that the slope of the T_s vs. u_∞ curve is steep for low velocities and relatively flat for high velocities. That is, the technique is more sensitive at lower velocities. Using Eq. (4), but with T_s dependent upon u_∞, the relative uncertainty in u_∞ can be determined.





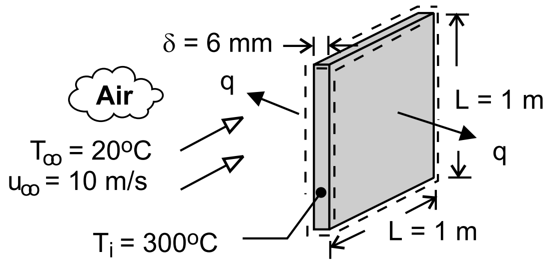
(d) For the constant power mode of operation, part (a), the uncertainty in u_∞ due to uncertainty in temperature measurement was found as 4%, independent of the magnitude u_∞. For the constant-temperature mode of operation, the uncertainty in u_∞ is less than 4% for velocities less than 30 m/s, with a value of 1% around 2 m/s. However, in the upper velocity range, the error increases to 5%.

PROBLEM 7.25

KNOWN: Plate dimensions and initial temperature. Velocity and temperature of air in parallel flow over plates.

FIND: Initial rate of heat transfer from plate. Rate of change of plate temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible radiation, (2) Negligible effect of conveyor velocity on boundary layer development, (3) Plates are isothermal, (4) Negligible heat transfer from sides of plate, (5) $Re_{x,c} = 5\times10^5$, (6) Constant properties.

PROPERTIES: *Table A-1*, AISI 1010 steel (573K): k_p = 49.2 W/m·K, c = 549 J/kg·K, ρ = 7832 kg/m^3. *Table A-4*, Air (p = 1 atm, T_f = 433K): ν = 30.4 × 10^{-6} m^2/s, k = 0.0361 W/m·K, Pr = 0.688.

ANALYSIS: The initial rate of heat transfer from a plate is

$$q = 2\bar{h}A_s\left(T_i - T_\infty\right) = 2\bar{h}L^2\left(T_i - T_\infty\right)$$

With $Re_L = u_\infty L/\nu = 10\,\text{m/s}\times 1\text{m}/30.4\times10^{-6}\text{m}^2/\text{s} = 3.29\times10^5$, flow is laminar over the entire surface and

$$\overline{Nu}_L = 0.664\,Re_L^{1/2}\,Pr^{1/3} = 0.664\left(3.29\times10^5\right)^{1/2}(0.688)^{1/3} = 336$$

$$\bar{h} = (k/L)\overline{Nu}_L = (0.0361\,\text{W/m}\cdot\text{K}/1\text{m})336 = 12.1\,\text{W/m}^2\cdot\text{K}$$

Hence,

$$q = 2\times12.1\,\text{W/m}^2\cdot\text{K}(1\text{m})^2(300-20)°\text{C} = 6780\,\text{W} \qquad <$$

Performing an energy balance at an instant of time for a control surface about the plate, $-\dot{E}_{out} = \dot{E}_{st}$, we obtain (Eq. 5.2),

$$\rho\delta L^2 c\left.\frac{dT}{dt}\right|_i = -\bar{h}\,2L^2\left(T_i - T_\infty\right)$$

$$\left.\frac{dT}{dt}\right|_i = -\frac{2\left(12.1\,\text{W/m}^2\cdot\text{K}\right)(300-20)°\text{C}}{7832\ \text{kg/m}^3\times0.006\text{m}\times549\,\text{J/kg}\cdot\text{K}} = -0.26°\text{C/s} \qquad <$$

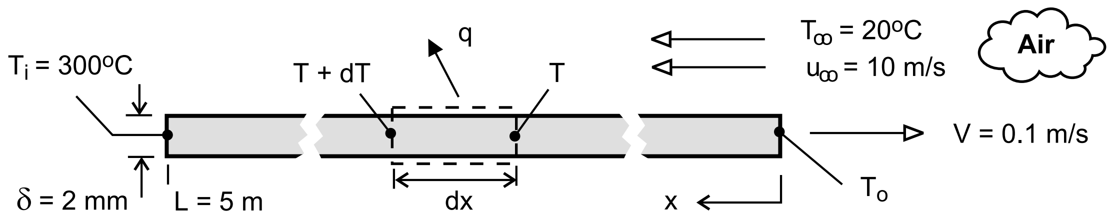
COMMENTS: (1) With $Bi = \bar{h}(\delta/2)/k_p = 7.4\times10^{-4}$, use of the lumped capacitance method is appropriate. (2) Despite the large plate temperature and the small convection coefficient, if adjoining plates are in close proximity, radiation exchange with the surroundings will be small and the assumption of negligible radiation is justifiable.

PROBLEM 7.26

KNOWN: Velocity, initial temperature, and dimensions of aluminum strip on a production line. Velocity and temperature of air in counter flow over top surface of strip.

FIND: (a) Differential equation governing temperature distribution along the strip and expression for outlet temperature, (b) Value of outlet temperature for prescribed conditions.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible variation of sheet temperature across its thickness, (2) Negligible effect of conduction along length (x) of sheet, (3) Negligible radiation, (4) Turbulent flow over entire top surface, (5) Negligible effect of sheet velocity on boundary layer development, (6) Negligible heat transfer from bottom surface and sides, (7) Constant properties.

PROPERTIES: *Table A-1*, Aluminum, 2024-T6 $\left(\overline{T}_{AL} \approx 500\text{K}\right)$: $\rho = 2770\,\text{kg/m}^3$, $c_p = 983\,\text{J/kg}\cdot\text{K}$, k=186 W/m·K. *Table A-4*, Air $\left(p = 1\,\text{atm},\ T_f \approx 400\text{K}\right)$: $\nu = 26.4\times10^{-6}\,\text{m}^2/\text{s}$, $k = 0.0338\,\text{W/m}\cdot\text{K}$, $\text{Pr} = 0.69$

ANALYSIS: (a) Applying conservation of energy to a stationary control surface, through which the sheet moves, steady-state conditions exist and $\dot{E}_{in} - \dot{E}_{out} = 0$. Hence, with *inflow* due to *advection* and outflow due to *advection* and *convection*,

$$\rho V A_c c_p (T + dT) - \rho V A_c c_p T - dq = 0$$

$$+\rho V \delta W c_p dT - h_x (dx \cdot W)(T - T_\infty) = 0$$

$$\frac{dT}{dx} = +\frac{h_x}{\rho V \delta c_p}(T - T_\infty) \qquad (1) \quad <$$

Alternatively, if the control surface is fixed to the sheet, conditions are transient and the energy balance is of the form, $-\dot{E}_{out} = \dot{E}_{st}$, or

$$-h_x (dx \cdot W)(T - T_\infty) = \rho (dx \cdot W \cdot \delta) c_p \frac{dT}{dt}$$

$$\frac{dT}{dt} = -\frac{h_x}{\rho \delta c_p}(T - T_\infty)$$

Dividing the left- and right-hand sides of the equation by dx/dt and dx/dt = - V, respectively, equation (1) is obtained. The equation may be integrated from x = 0 to x = L to obtain

$$\int_{T_o}^{T_i} \frac{dT}{T - T_\infty} = \frac{L}{\rho V \delta c_p}\left[\frac{1}{L}\int_0^L h_x\, dx\right]$$

Continued …..

where $h_x = (k/x)0.0296\,Re_x^{4/5}\,Pr^{1/3}$ and the bracketed term on the right-hand side of the equation reduces to $\bar{h}_L = (k/L)0.037\,Re_L^{4/5}\,Pr^{1/3}$.

Hence,

$$\ln\left(\frac{T_i - T_\infty}{T_o - T_\infty}\right) = \frac{L\,\bar{h}_L}{\rho V \delta c_p}$$

$$\frac{T_o - T_\infty}{T_i - T_\infty} = \exp\left(-\frac{L\,\bar{h}_L}{\rho V \delta c_p}\right) \qquad <$$

(b) For the prescribed conditions, $Re_L \approx u_\infty L/\nu = 20\,m/s \times 5m / 26.4\times10^{-6}\,m^2/s = 3.79\times10^6$ and

$$\bar{h}_L = \left(\frac{0.0338\,W/m\cdot K}{5m}\right)0.037\left(3.79\times10^6\right)^{4/5}(0.69)^{1/3} = 40.5\,W/m^2\cdot K$$

$$T_o = 20°C + (280°C)\exp\left(-\frac{5m\times40.5\,W/m^2\cdot K}{2770\,kg/m^3\times0.1\,m/s\times0.002m\times983\,J/kg\cdot K}\right) = 213°C \qquad <$$

COMMENTS: (1) With $T_o = 213°C$, $\bar{T}_{Al} = 530K$ and $T_f = 411K$ are close to values used to determine the material properties, and iteration is not needed. (2) For a representative emissivity of $\varepsilon = 0.2$ and $T_{sur} = T_\infty$, the maximum value of the radiation coefficient is

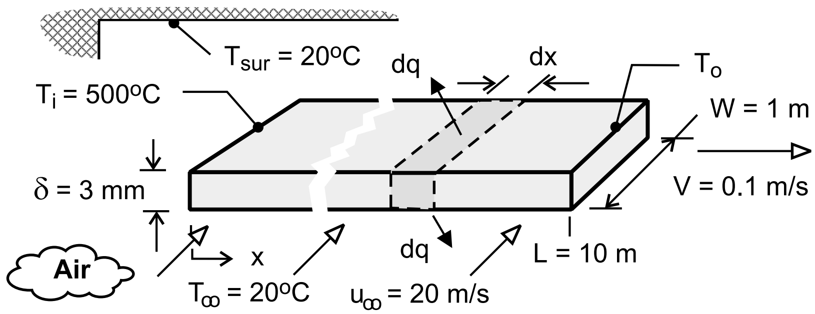
$h_r = \varepsilon\sigma\left(T_i + T_{sur}\right)\left(T_i^2 + T_{sur}^2\right) = 4.1\,W/m^2\cdot K << \bar{h}_L$. Hence, the assumption of negligible radiation is appropriate.

PROBLEM 7.27

KNOWN: Velocity, initial temperature, properties and dimensions of steel strip on a production line. Velocity and temperature of air in cross flow over top and bottom surfaces of strip. Temperature of surroundings.

FIND: (a) Differential equation governing temperature distribution along the strip, (b) Exact solution for negligible radiation and corresponding value of outlet temperature for prescribed conditions, (c) Effect of radiation on outlet temperature, and parametric effect of sheet velocity on temperature distribution.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible variation of sheet temperature across its width and thickness, (2) Negligible effect of conduction along length (x) of sheet, (3) Constant properties, (4) Radiation exchange between small surface (both sides of sheet) and large surroundings, (5) Turbulent flow over top and bottom surfaces of sheet, (6) Motion of sheet has a negligible effect on the convection coefficient, ($V << u_\infty$), (7) Negligible heat transfer from sides of sheet.

PROPERTIES: Prescribed. Steel: $\rho = 7850\,\text{kg/m}^3, c_p = 620\,\text{J/kg}\cdot\text{K}, \varepsilon = 0.70.$ Air: k = 0.044 W/m·K, $\nu = 4.5\times10^{-5}\,\text{m}^2/\text{s}$, Pr = 0.68.

ANALYSIS: (a) Applying conservation of energy to a stationary differential control surface, through which the sheet passes, conditions are steady and $\dot{E}_{in} - \dot{E}_{out} = 0$. Hence, with *inflow* due to *advection* and *outflow* due to *advection*, *convection* and *radiation*

$$\rho V A_c c_p T - \rho V A_c c_p (T + dT) - 2\,dq = 0$$

$$-\rho V \delta W c_p dT - 2(W dx)\left[\bar{h}_W (T - T_\infty) + \varepsilon\sigma\left(T^4 - T_{sur}^4\right)\right] = 0$$

$$\frac{dT}{dx} = -\frac{2}{\rho V \delta c_p}\left[\bar{h}_W (T - T_\infty) + \varepsilon\sigma\left(T^4 - T_{sur}^4\right)\right] \qquad (1) \quad <$$

Alternatively, if the control surface is fixed to the sheet, conditions are transient and the energy balance is of the form, $-\dot{E}_{out} = \dot{E}_{st}$, or

$$-2(W dx)\left[\bar{h}_W (T - T_\infty) + \varepsilon\sigma\left(T^4 - T_{sur}^4\right)\right] = \rho (W \delta\, dx) c_p \frac{dT}{dt}$$

$$\frac{dT}{dt} = -\frac{2}{\rho \delta c_p}\left[\bar{h}_W (T - T_\infty) + \varepsilon\sigma\left(T^4 - T_{sur}^4\right)\right]$$

Dividing the left- and right-hand sides of the equation by dx/dt and V = dx/dt, respectively, Eq. (1) is obtained.

(b) Neglecting radiation, separating variables and integrating, Eq. (1) becomes

$$\int_{T_i}^{T} \frac{dT}{T - T_\infty} = -\frac{2\bar{h}_W}{\rho V \delta c_p}\int_0^x dx$$

Continued …..

$$\ln\left(\frac{T-T_\infty}{T_i-T_\infty}\right)=-\frac{2\bar{h}_W x}{\rho V \delta c_p}$$

$$T=T_\infty+\left(T_i-T_\infty\right)\exp\left(-\frac{2\bar{h}_W x}{\rho V \delta c_p}\right) \qquad (2) \quad <$$

With $Re_W = u_\infty W/\nu = 20\,m/s\times1m/4\times10^{-5}m^2/s = 5\times10^5$, the correlation for turbulent flow over a flat plate yields

$$\overline{Nu}_W = 0.037\,Re_W^{4/5}\,Pr^{1/3} = 0.037\left(5\times10^5\right)^{4/5}\left(0.68\right)^{1/3} = 1179$$

$$\bar{h}_W = \frac{k}{W}\overline{Nu}_W = \frac{0.044\,W/m\cdot K}{1m}1179 = 51.9\,W/m^2\cdot K$$

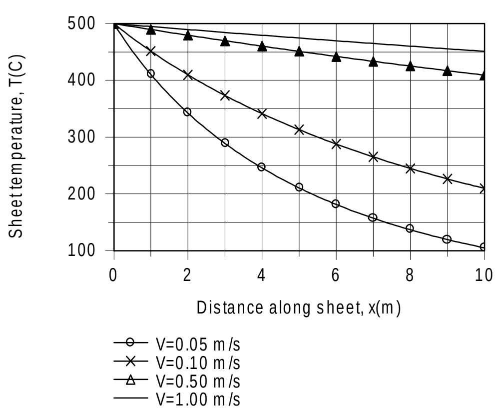
Hence, applying Eq. (2) at x = L = 10m,

$$T_o = 20°C + \left(480°C\right)\exp\left(-\frac{2\times51.9\,W/m^2\cdot K\times10m}{7850\,kg/m^3\times0.1m/s\times0.003m\times620\,J/kg\cdot K}\right) = 256°C \quad <$$

(c) Using the DER function of IHT, Eq. (1) may be numerically integrated from x = 0 to x = L = 10m to obtain

$$T_o = 210°C \quad <$$

Contrasting this result with that of Part (b), it's clear that radiation makes a discernable contribution to cooling of the sheet. IHT was also used to determine the effect of the sheet velocity on the temperature distribution.



The sheet velocity has a significant influence on the temperature distribution. The temperature decay decreases with increasing V due to the increasing effect of advection on energy transfer in the x direction.

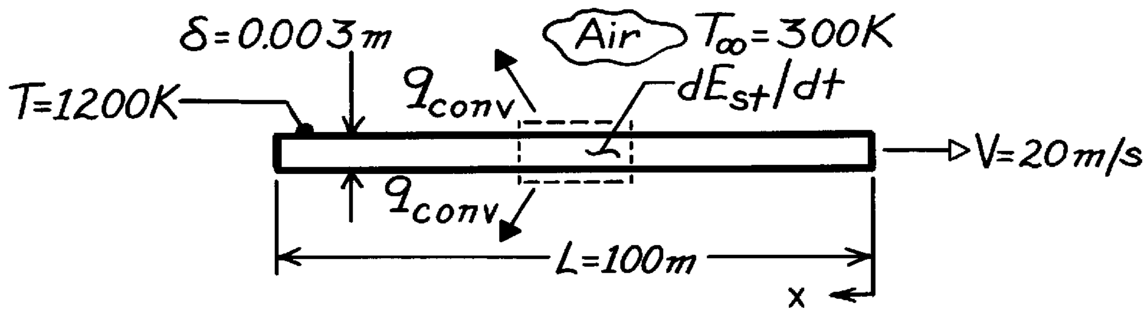
COMMENTS: (1) A critical parameter in the production process is the *coiling temperature*, that is, the temperature at which the wire may be safely coiled for subsequent storage or shipment. The larger the production rate (V), the longer the cooling distance needed to achieve a desired coiling temperature. (2) Cooling may be enhanced by increasing the cross stream velocity u_∞.

PROBLEM 7.28

KNOWN: Length, thickness, speed and temperature of steel strip.

FIND: Rate of change of strip temperature 1 m from leading edge and at trailing edge. Location of minimum cooling rate.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties, (2) Negligible radiation, (3) Negligible longitudinal conduction in strip, (4) Critical Reynolds number is 5×10^5.

PROPERTIES: Steel (given): ρ = 7900 kg/m^3, c_p = 640 J/kg·K. *Table A-4*, Air $(\bar{T} = 750\text{K, 1 atm})$: $\nu = 76.4 \times 10^{-6}$ m^2/s, k = 0.0549 W/m·K, Pr = 0.702.

ANALYSIS: Performing an energy balance for a control mass of unit surface area A_s riding with the strip,

$$-\dot{E}_{out} = dE_{st}/dt$$

$$-2h_x A_s (T - T_\infty) = r d A_s c_p (dT/dt)$$

$$dT/dt) = \frac{-2h_x (T - T_\infty)}{r d c_p} = -\frac{2(900\text{K})h_x}{7900 \text{ kg/m}^3 (0.003 \text{ m}) 640 \text{ J/kg} \cdot \text{K}} = -0.119 h_x \text{ (K/s)}.$$

At x = 1 m, $\quad Re_x = \frac{Vx}{n} = \frac{20 \text{ m/s}(1\text{m})}{76.4 \times 10^{-6} \text{ m}^2/\text{s}} = 2.62 \times 10^5 < Re_{x,c}$. Hence,

$$h_x = (k/x) 0.332 Re_x^{1/2} Pr^{1/3} = \frac{0.0549 \text{ W/m} \cdot \text{K}}{1 \text{ m}} (0.332)\left(2.62 \times 10^5\right)^{1/2} (0.702)^{1/3} = 8.29 \text{W/m}^2 \cdot \text{K}$$

and at x = 1m, $\quad dT/dt) = -0.987$ K/s. <

At the trailing edge, $Re_x = 2.62 \times 10^7 > Re_{x,c}$. Hence

$$h_x = (k/x) 0.0296 Re_x^{4/5} Pr^{1/3} = \frac{0.0549 \text{ W/m} \cdot \text{K}}{100 \text{ m}} (0.0296)\left(2.62 \times 10^7\right)^{4/5} (0.702)^{1/3} = 12.4 \text{W/m}^2 \cdot \text{K}$$

and at x = 100 m, $\quad dT/dt) = -1.47$ K/s. <

The minimum cooling rate occurs just before transition; hence, for $Re_{x,c} = 5 \times 10^5$

$$x_c = 5 \times 10^5 (n/V) = \frac{5 \times 10^5 \times 76.4 \times 10^{-6} \text{m}^2/\text{s}}{20 \text{ m/s}} = 1.91 \text{ m}$$ <

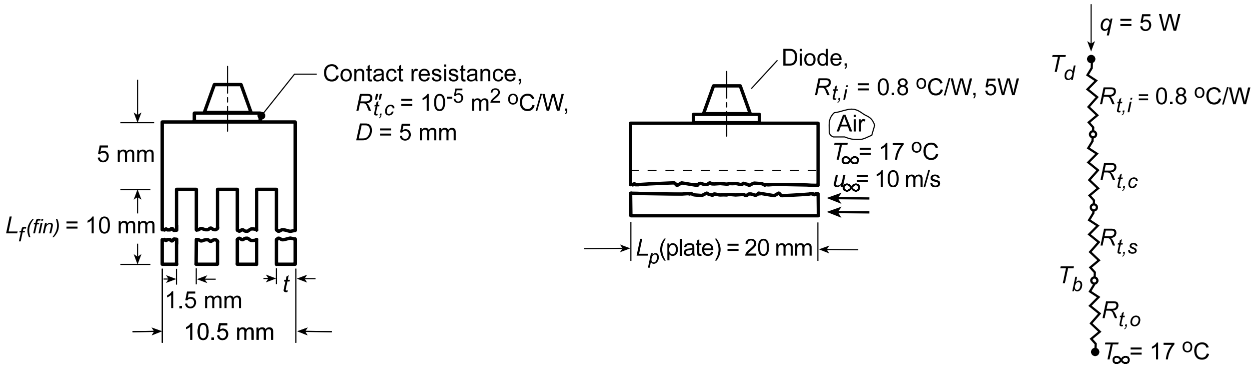
COMMENTS: The cooling rates are very low and would remain low even if radiation were considered. For this reason, hot strip metals are quenched by water and not by air.

PROBLEM 7.29

KNOWN: Finned heat sink used to cool a power diode.

FIND: (a) Operating temperature T_d of the diode for prescribed conditions, (b) Options for reducing T_d.

SCHEMATIC:



ASSUMPTIONS: (1) All diode power is rejected from the four fins, (2) Diode behaves as an isothermal disk on a semi-infinite medium, (3) Fin tips are adiabatic, (4) Fins behave as flat plates with regard to forced convection (boundary layer thickness between fins is less than 1.5 mm/2), (5) Negligible heat loss from fin edges and prime (exposed base) surfaces.

PROPERTIES: *Table A-1*, Aluminum alloy 2024 ($\bar{T} \approx 300$ K): k = 177 W/m·K; *Table A-4*, Air ($T_f = (T_s + T_\infty)/2 \approx 300$ K, 1 atm): $\nu = 15.89 \times 10^{-6}$ m^2/s, k = 0.0263 W/m·K.

ANALYSIS: (a) From the thermal circuit for the system, $T_d = T_\infty + qR_{tot}$ where $R_{tot} = R_{t,i} + R_{t,c} + R_{t,s} + R_{t,o}$.

Thermal contact resistance, $R_{t,c}$:

$$R_{t,c} = R''_{t,c}/A_d = 10^{-5}\,\text{m}^2 \cdot {}^\circ\text{C/W} \big/ (\pi/4)(0.005\,\text{m})^2 = 0.509^\circ\,\text{C/W}.$$

Spreading thermal resistance, $R_{t,s}$: This resistance is due to conduction between the diode (an isothermal disk) and the heat sink (semi-infinite medium). From Table 4.1, the conduction shape factor is S = 2D. Hence,

$$R_{t,s} = 1/k(2D) = 1/177\,\text{W/m}\cdot\text{K}(2\times 0.005\,\text{m}) = 0.565^\circ\,\text{C/W}.$$

Thermal resistance of the fin array, $R_{t,o}$: From Table 3.4 for the fin with insulated tip,

$$R_{t,f} = \frac{\theta_b}{q_f} = \frac{1}{M^* \cdot \tanh(mL_f)}$$

where

$$m^2 = \left(\bar{h}P/kA_c\right) \qquad M^* = \left(\bar{h}PkA_c\right)^{1/2}.$$

To estimate the average heat transfer coefficient, consider the fin as a flat plate in parallel flow along the length, L_p = 20 mm. The Reynolds number is

$$Re_L = \frac{u_\infty L_p}{\nu} = \frac{10\,\text{m/s}\times 0.020\,\text{m}}{15.89\times 10^{-6}\,\text{m}^2/\text{s}} = 1.259\times 10^4.$$

Continued...

The flow is laminar, in which case

$$\overline{Nu}_L = \frac{\bar{h}L}{k} = 0.664\,Re_L^{1/2}\,Pr^{1/3}$$

$$\bar{h} = \frac{0.0263\,W/m\cdot K}{0.020\,m} \times 0.664\left(1.259\times10^4\right)^{1/2}(0.707)^{1/3} = 87.3\,W/m^2\cdot K\,.$$

With $P = (2t + 2L_p) = 0.046$ and $A_c = tL_p = 3\times10^{-5}\ m^2$,

$$m = \left[87.3\,W/m^2\cdot K\times0.046\,m/177\,W/m\cdot K\times3\times10^{-5}\,m^2\right]^{1/2} = 27.52\,m^{-1}$$

$$M^* = \left[87.3\,W/m^2\cdot K\times0.046\,m\times177\,W/m\cdot K\times3\times10^{-5}\right]^{1/2} = 0.146\,W/K\,.$$

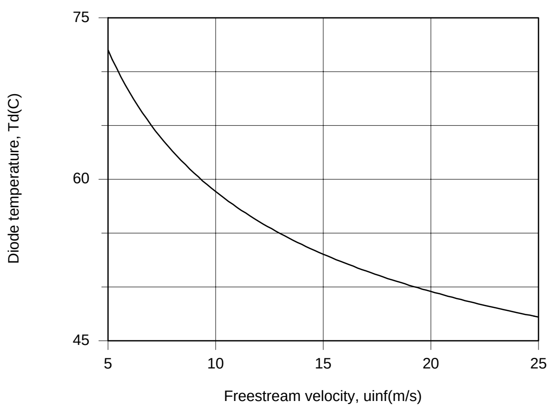
Hence, with L_f = 10 mm,

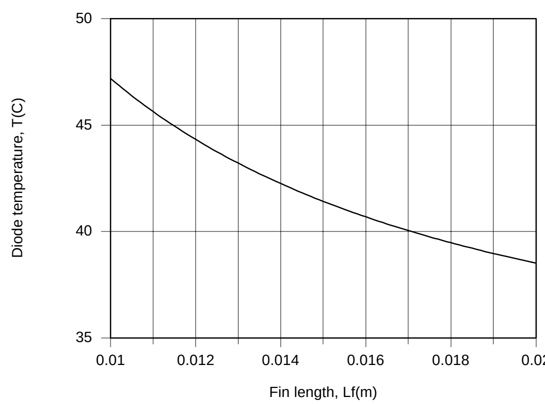
$$R_{t,f} = 1/0.146\,W/K\times\tanh\left(27.52\,m^{-1}\times0.010\,m\right) = 25.51^\circ\,C/W\,.$$

With $R_{t,o} \approx R_{t,f}/4$, the diode temperature is

$$T_d = 17^\circ C + 5\,W\left[0.80 + 0.509 + 0.565 + 0.25(27.3)\right]^\circ C/W \approx 58^\circ C \qquad <$$

(b) The IHT *Extended Surfaces* Model for an *Array of Straight, Rectangular Fins* was used with the *External Flow, Flat Plate* option from the *Correlations* Tool Pad to assess the effects of varying u_∞ and L_f.





Clearly, there are benefits to increasing both quantities, with T_d reduced from approximately 58°C (u_∞ = 10 m/s, L_f = 10 mm) to 47.2°C (u_∞ = 25 m/s, L_f = 10 mm) to 38.5°C (u_∞ = 25 m/s, L_f = 20 mm). For u_∞ = 25 m/s and L_f = 20 mm, the fin efficiency remains large (η_f = 0.87), suggesting that, air flow and space limitations permitting, significant reduction, in T_d could still be gained by going to even larger values of L_f.

Subject to the constraint that the spacing between fins between remain at or larger than 1.5 mm, there is no advantage to reducing the fin thickness. For a thickness of 0.5 mm, it would be possible to add only one more fin (N = 5), yielding T_d = 44.4°C for u_∞ = 25 m/s and L_f = 20 mm.

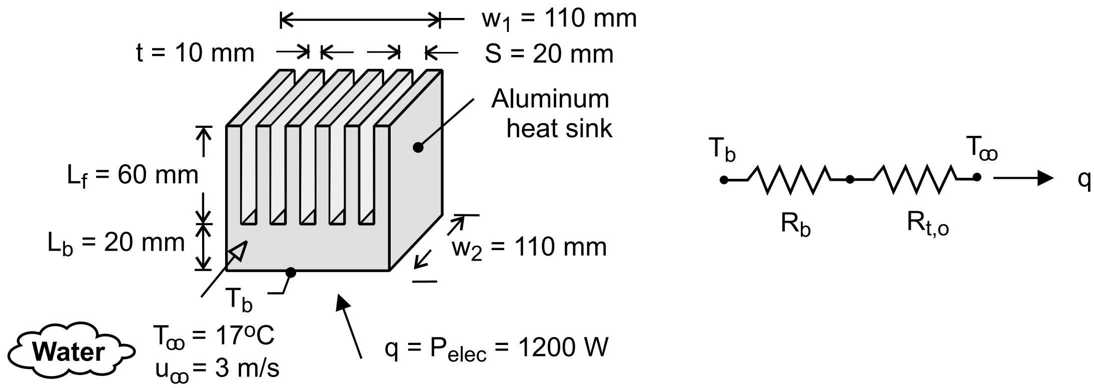
COMMENTS: Note that the fin resistance makes the dominant contribution to the total resistance. Hence, efforts to reduce the total resistance should focus on reducing the fin resistance.

PROBLEM 7.30

KNOWN: Dimensions of aluminum heat sink. Temperature and velocity of coolant (water) flow through the heat sink. Power dissipation of electronic package attached to the heat sink.

FIND: Base temperature of heat sink.

SCHEMATIC:



ASSUMPTIONS: (1) Average convection coefficient association with flow over fin surfaces may be approximated as that for a flat plate in parallel flow, (2) All of the electric power is dissipated by the heat sink, (3) Transition Reynolds number of $Re_{x,c} = 5 \times 10^5$, (4) Constant properties.

PROPERTIES: Given. Aluminum: $k_{hs} = 180$ W/m·K. Water: $k_w = 0.62$ W/m·K, $\nu = 7.73 \times 10^{-7}$ m^2/s, Pr = 5.2.

ANALYSIS: From the thermal circuit,

$$q = P_{elec} = \frac{T_b - T_\infty}{R_b + R_{t,o}}$$

where $R_b = L_b / k_{hs}(w_1 \times w_2) = 0.02\text{m} / 180\,\text{W/m}\cdot\text{K}(0.11\text{m})^2 = 9.18 \times 10^{-3}\,\text{K/W}$ and, from Eqs. 3.102 and 3.103,

$$R_{t,o} = \left\{\bar{h} A_t \left[1 - \frac{NA_f}{A_t}(1 - \eta_f)\right]\right\}^{-1}$$

The fin and total surface area of the array are $A_f = 2w_2(L_f + t/2) = 0.22\text{m}(0.065\text{m}) = 0.0143\text{m}^2$ and $A_t = NA_f + A_b = NA_f + (N-1)(S-t)w_2 = 6\left(0.0143\text{m}^2\right) + 5(0.01\text{m})0.11\text{m} = (0.0858 + 0.0055) = 0.0913\text{m}^2$.

With $Re_{w_2} = u_\infty w_2 / \nu = 3\,\text{m/s} \times 0.11\text{m} / 7.73 \times 10^{-7}\,\text{m}^2/\text{s} = 4.27 \times 10^5$, laminar flow may be assumed over the entire surface. Hence

$$\bar{h} = \left(\frac{k_w}{w_2}\right) 0.664\, Re_{w_2}^{1/2}\, Pr^{1/3} = \left(\frac{0.62\,\text{W/m}\cdot\text{K}}{0.11\text{m}}\right) 0.664 \left(4.27 \times 10^5\right)^{1/2} (5.2)^{1/3} = 4236\,\text{W/m}^2\cdot\text{K}$$

With $m = (2\bar{h}/k_{hs}t)^{1/2} = \left(8472\,\text{W/m}^2\cdot\text{K} / 180\,\text{W/m}\cdot\text{K} \times 0.01\text{m}\right)^{1/2} = 68.6\,\text{m}^{-1}$, $mL_c = 68.6\,\text{m}^{-1}(0.065\text{m}) = 4.46$ and $\tanh mL_c = 0.9997$, Eq. 3.89 yields

$$\eta_f = \frac{\tanh mL_c}{mL_c} = \frac{0.9997}{4.46} = 0.224$$

Continued …..

Hence,

$$R_{t,o} = \left\{4236\,\mathrm{W/m^2\cdot K} \times 0.0913\,\mathrm{m^2}\left[1 - \frac{0.0858\,\mathrm{m^2}}{0.0913\,\mathrm{m^2}}(0.776)\right]\right\}^{-1} = 9.55\times10^{-3}\,\mathrm{K/W} \qquad <$$

and

$$T_b = T_\infty + P_{elec}\left(R_b + R_{t,o}\right) = 17°\mathrm{C} + 1200\,\mathrm{W}\left(0.0187\,\mathrm{K/W}\right) = 39.5°\mathrm{C} \qquad <$$

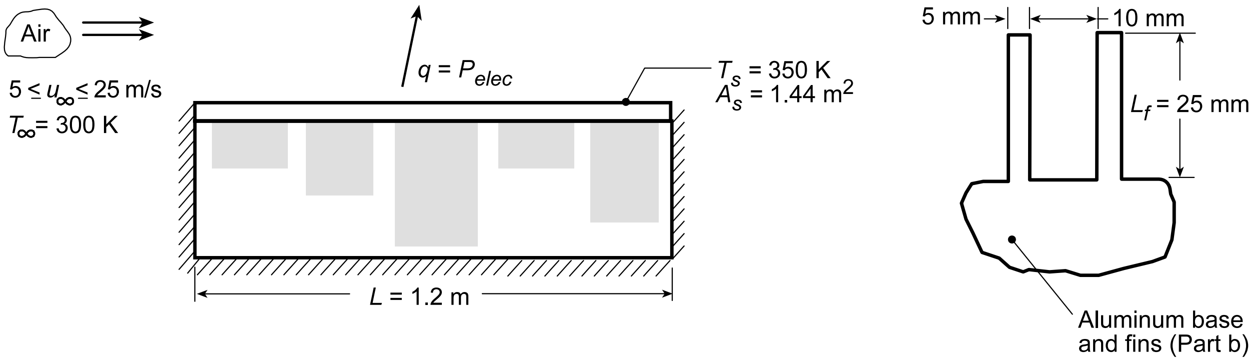
COMMENTS: (1) The boundary layer thickness at the trailing edge of the fin is $\delta = 5w_2/\left(Re_{w_2}\right)^{1/2} = 0.84\ \mathrm{mm} << (S - t)$. Hence, the assumption of parallel flow over a flat plate is reasonable. (2) If a finned heat sink is not employed and heat transfer is simply by convection from the $w_2 \times w_2$ base surface, the corresponding convection resistance would be 0.0195 K/W, which is only twice the resistance associated with the fin array. The small enhancement by the array is attributable to the large value of $\overline{h}$ and the correspondingly small value of η_f. Were a fluid such as air or a dielectric liquid used as the coolant, the much smaller thermal conductivity would yield a smaller $\overline{h}$, a larger η_f and hence a larger effectiveness for the array.

PROBLEM 7.31

KNOWN: Plate dimensions and freestream conditions. Maximum allowable plate temperature.

FIND: (a) Maximum allowable power dissipation for electrical components attached to bottom of plate, (b) Effect of air velocity and fins on maximum allowable power dissipation.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Negligible heat loss form sides and bottom, (4) Transition Reynolds number is 5×10^5, (5) Isothermal plate.

PROPERTIES: *Table A.1*, Aluminum (T ≈ 350 K): k ≈ 240 W/m·K; *Table A.4*, Air (T_f = 325 K, 1 atm): $\nu = 18.4 \times 10^{-6}$ m²/s, k = 0.028 W/m·K, Pr = 0.70.

ANALYSIS: (a) The heat transfer from the plate by convection is

$$P_{elec} = q = \bar{h} A_s \left(T_s - T_\infty\right).$$

For u_∞ = 15 m/s,

$$Re_L = \frac{u_\infty L}{\nu} = \frac{15\,\text{m/s} \times 1.2\,\text{m}}{18.41 \times 10^{-6}\,\text{m}^2/\text{s}} = 9.78 \times 10^5 > Re_{x,c}.$$

Hence, transition occurs on the plate and

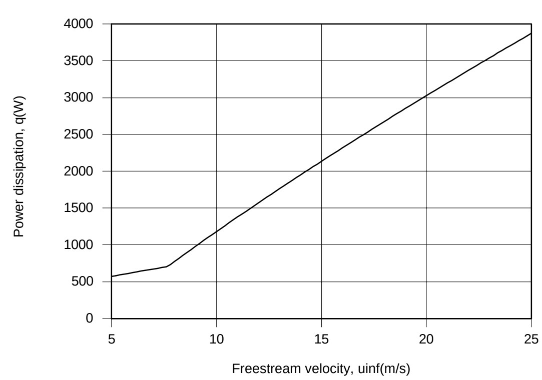
$$\overline{Nu}_L = \left(0.037\,Re_L^{4/5} - 871\right) Pr^{1/3} = \left[0.037\left(9.78 \times 10^5\right)^{4/5} - 871\right]\left(0.70\right)^{1/3} = 1263$$

$$\bar{h} = Nu_L \frac{k}{L} = 1263 \frac{0.028\,\text{W/m}\cdot\text{K}}{1.2\,\text{m}} = 29.7\,\text{W/m}^2\cdot\text{K}$$

The heat rate is

$$q = 29.7\,\text{W/m}^2\cdot\text{K}\left(1.2\,\text{m}\right)^2\left(350 - 300\right)\text{K} = 2137\,\text{W}.$$ $<$

(b) The effect of the freestream velocity was considered by combining the *Correlations* Toolpad for the average coefficient associated with flow over a flat plate with the *Explore* and *Graph* options of IHT.



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The effect of increasing u_∞ is significant, particularly following transition at $u_\infty \approx 7.7$ m/s. A maximum heat rate of q = 3876 W is obtained for u_∞ = 25 m/s, which corresponds to $\bar{h} \approx 54$ W/m$^2\cdot$K and $Re_L = 1.63 \times 10^6$.

The *Extended Surfaces* Model for an *Array* of *Straight Rectangular Fins* was used with the *Correlations* Toolpad to determine the effect of adding fins, and a copy of the program is appended. With L_f = 25 mm, w = 1.2 m, t = 0.005 m, S = 0.015 m, N = 80 and u_∞ = 25 m/s, the solution yields

q = 16,480 W <

which is more than a four-fold increase relative to the unfinned case.

COMMENTS: (1) With a fin efficiency of $\eta_f = 0.978$, there is significant latitude for yet further enhancement in heat transfer, as, for example, by increasing the fin length, L_f.

(2) The *IHT* code below includes the model for the *Extended Surface, Array of Straight Fins* and the *Correlation* for the convection coefficient of a flat plate with mixed flow conditions.

```
/* Fin analysis results, uinf = 25 m/s
Ab      Acb      Af      Ap          At      Aw      etaf    etaoc   m       qt      R"tc
0.96    0.006    0.066   0.0001375           6.24    1.44    0.978   0.9814  9.471
        1.648E4  0       */

/* Correlation results and air thermophysical properties at Tf
NuLbar  Pr       ReL     Tf      hLbar   k        nu        uinf
2294    0.7035   1.63E6  325     53.82   0.02815  1.841E-5  25      */

// IHT Model, Extended Surfaces, Array of Straight Rectangular Fins
/* Model: Fin array with straight fins of rectangular profile, thickness t, width w and length L. Array has N
fins with spacing S. */

/* Find: Array heat rate and performance parameters */

/* Assumptions:(1) Steady-state conditions, (2) One-dimensional conduction along the fin, (3) Constant
properties, (4) Negligible radiation exchange with surroundings, (5) Uniform convection coefficient over
fins and base, (6) Insulated tip, Lc = L + t / 2 */

// The total heat rate for the array
qt = (Tb - Tinf) / (Rtoc)  // Eq 3.104
/* where the fin array thermal resistance, including thermal contact resistance, R"tc,  at the fin base is */
Rtoc = 1 / (etaoc * h * At)

// The overall surface efficiency is
etaoc = 1 - (N * Af / At) * (1 - etaf / C1)          // Eq 3.105
C1 = 1 + etaf * h * Af * (R"tc / Acb)
// where N is the total number of fins, and the surface area of a single fin is
Af = 2 * w * Lc
// where the equivalent length, accounting for the adiabatic tip, is
Lc = Lf + (t / 2)

/* The surface area associated with the fins and the exposed portion of the base (referred to also as the
prime surface, Ab) is */
At = N * Af + Ab
Ab = Aw - N * Acb

// The total area of the base surface follows from the schematic
Aw = w * N * S
// where S is the fin spacing. The base area for a single fin is
Acb = t * w

// The fin efficiency for a single fin is:
etaf = (tanh(m * Lc)) / (m * Lc)                     // Eq 3.89
// where
m = sqrt(2 * h / (kf * t))
```

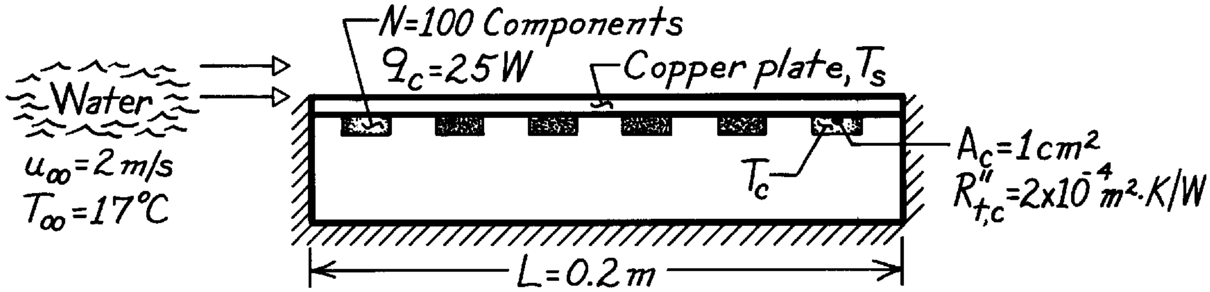
```
/* The input (independent) values for this system are:
Fin characteristics */
Tb =    350                  // base temperature, K
t =     0.005                // thickness, m
w =  1.2                     // spacing width, m
Lf =  0.025                  // length, m
S =    0.015                 // fin spacing, m
N =    80                    // number of fins
kf =    240                  // thermal conductivity, W/m·K

// Convection conditions
Tinf =  300                  // fluid temperature, K
h =    hLbar                 // convection coefficient,W/m^2·K

/* Thermal contact resistance per unit area at fin base. Set equal to zero if not present. */
R''tc =   0             // thermal resistance per unit area, K·m^2/W

// Correlation, External flow, Flate Plate, Laminar or Mixed Flow
NuLbar = NuL_bar_EF_FP_LM(ReL,Rexc,Pr)     // Eq 7.31, 7.39, 7.40
NuLbar = hLbar * L / k
ReL = uinf * L / nu
Rexc = 5.0E5
// Evaluate properties at the film temperature, Tf.
Tf = (Tinf + Tb) / 2
/*  Correlation description: Parallel external flow (EF) over a flat plate (FP), average coefficient; laminar (L)
if ReL<Rexc, Eq 7.31; mixed (M) if ReL>Rexc, Eq 7.39 and 7.40; 0.6<=Pr<=60. See Table 7.9.  */

// Air property functions : From Table A.4
// Units: T(K); 1 atm pressure
nu = nu_T("Air",Tf)          // Kinematic viscosity, m^2/s
k = k_T("Air",Tf)            // Thermal conductivity, W/m·K
Pr = Pr_T("Air",Tf)          // Prandtl number

// Input variables, correlation
uinf = 25                    // freestream velocity, m/s
L = 1.2                      // plate width, m
```

PROBLEM 7.32

KNOWN: Operating power of electrical components attached to one side of copper plate. Contact resistance. Velocity and temperature of water flow on opposite side.

FIND: (a) Plate temperature, (b) Component temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Negligible heat loss from sides and bottom, (4) Turbulent flow throughout.

PROPERTIES: Water (given): $\nu = 0.96 \times 10^{-6}$ m^2/s, k = 0.620 W/m·K, Pr = 5.2.

ANALYSIS: (a) From the convection rate equation,

$$T_s = T_\infty + q/\overline{h}A$$

where q = Nq_c = 2500 W and A = L^2 = 0.04 m^2. The convection coefficient is given by the turbulent flow correlation

$$\overline{h} = \overline{Nu}_L\,(k/L) = 0.037 Re_L^{4/5} Pr^{1/3}(k/L)$$

where

$$Re_L = (u_\infty L/\nu) = (2\text{ m/s} \times 0.2\text{m})/0.96 \times 10^{-6}\text{m}^2/\text{s} = 4.17 \times 10^5$$

and hence

$$\overline{h} = 0.037\left(4.17 \times 10^5\right)^{4/5}(5.2)^{1/3}(0.62\text{ W/m}\cdot\text{K}/0.2\text{ m}) = 6228\text{ W/m}^2\cdot\text{K}.$$

The plate temperature is then

$$T_s = 17^\circ\text{C} + 2500\text{ W}/\left(6228\text{ W/m}^2\cdot\text{K}\right)(0.20\text{ m})^2 = 27^\circ\text{C}. \qquad <$$

(b) For an individual component, a rate equation involving the component's contact resistance can be used to find its temperature,

$$q_c = (T_c - T_s)/R_{t,c} = (T_c - T_s)/\left(R''_{t,c}/A_c\right)$$

$$T_c = T_s + q_c R''_{t,c}/A_c = 27^\circ\text{C} + 25\text{ W}\left(2 \times 10^{-4}\text{ m}^2\cdot\text{K/W}\right)/10^{-4}\text{ m}^2$$

$$T_c = 77^\circ\text{C}. \qquad <$$

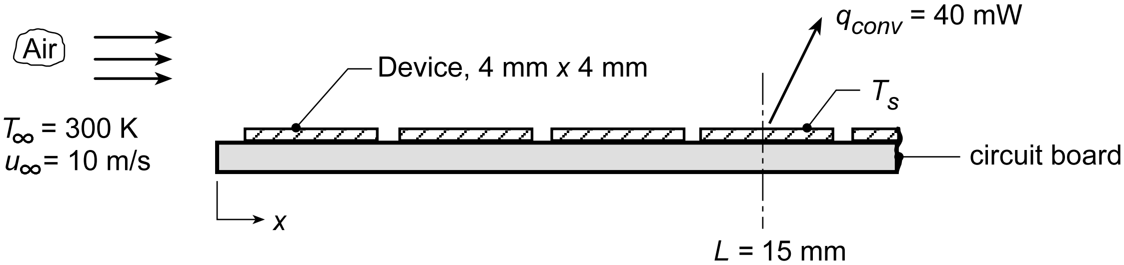
COMMENTS: With $Re_L = 4.17 \times 10^5$, the boundary layer would be laminar over the entire plate without the boundary layer trip, causing T_s and T_c to be appreciably larger.

PROBLEM 7.33

KNOWN: Air at 27°C with velocity of 10 m/s flows turbulently over a series of electronic devices, each having dimensions of 4 mm × 4 mm and dissipating 40 mW.

FIND: (a) Surface temperature T_s of the fourth device located 15 mm from the leading edge, (b) Compute and plot the surface temperatures of the first four devices for the range $5 \le u_\infty \le 15$ m/s, and (c) Minimum free stream velocity u_∞ if the surface temperature of the hottest device is not to exceed 80°C.

SCHEMATIC:



ASSUMPTIONS: (1) Turbulent flow, (2) Heat from devices leaving through top surface by convection only, (3) Device surface is isothermal, and (4) The average coefficient for the devices is equal to the local value at the mid position, i.e. $\bar{h}_4 = h_x(L)$.

PROPERTIES: *Table A.4*, Air (assume T_s = 330 K, $\bar{T} = (T_s + T_\infty)/2$ = 315 K, 1 atm): k = 0.0274 W/m·K, $\nu = 17.40 \times 10^{-6}$ m^2/s, $\alpha = 24.7 \times 10^{-6}$ m^2/s, Pr = 0.705.

ANALYSIS: (a) From Newton's law of cooling,

$$T_s = T_\infty + q_{conv}/\bar{h}_4 A_s \tag{1}$$

where $\bar{h}_4$ is the average heat transfer coefficient over the 4th device. Since flow is turbulent, it is reasonable and convenient to assume that

$$\bar{h}_4 = h_x(L = 15\,\text{mm}). \tag{2}$$

To estimate h_x, use the turbulent correlation evaluating thermophysical properties at $\bar{T}_f$ = 315 K (assume T_s = 330 K),

$$Nu_x = 0.0296\,Re_x^{4/5}\,Pr^{1/3}$$

where

$$Re_x = \frac{u_\infty L}{\nu} = \frac{10\,\text{m/s} \times 0.015\,\text{m}}{17.4 \times 10^{-6}\,\text{m}^2/\text{s}} = 8621$$

giving

$$Nu_x = \frac{h_x L}{k} = 0.0296(8621)^{4/5}(0.705)^{1/3} = 37.1$$

$$\bar{h}_4 = h_x = \frac{Nu_x k}{L} = \frac{37.1 \times 0.0274\,\text{W/m}\cdot\text{K}}{0.015\,\text{m}} = 67.8\,\text{W/m}^2\cdot\text{K}$$

Hence, with A_s = 4 mm × 4 mm, the surface temperature is

$$T_s = 300\,\text{K} + \frac{40 \times 10^{-3}\,\text{W}}{67.8\,\text{W/m}^2\cdot\text{K} \times \left(4 \times 10^{-3}\,\text{m}\right)^2} = 337\,\text{K} = 64^\circ\text{C}. \qquad <$$

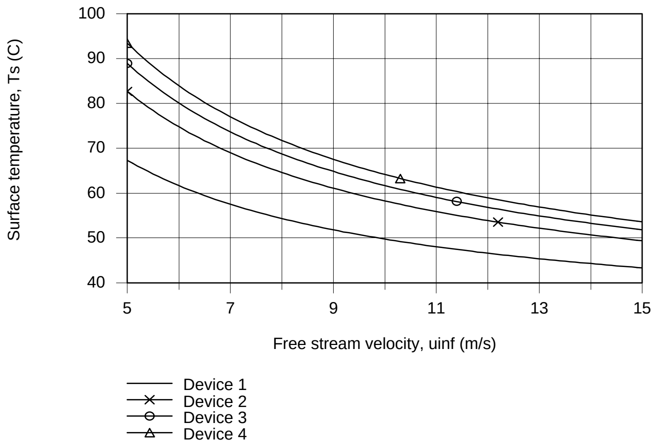
Continued...

PROBLEM 7.33 (Cont.)

(b) The surface temperature for each of the four devices (i = 1, 2, 3 4) follows from Eq. (1),

$$T_{s,i} = T_\infty + q_{conv} / \bar{h}_i A_s \tag{3}$$

For devices 2, 3 and 4, $\bar{h}_i$ is evaluated as the local coefficient at the mid-positions, Eq. (2), x_2 = 6.5 mm, x_3 = 10.75 mm and x_4 = 15 mm. For device 1, $\bar{h}_1$ is the average value 0 to x_1, where evaluated $x_1 = L_1$ = 4.25 mm. Using Eq. (3) in the *IHT Workspace* along with the *Correlations Tool*, *External Flow*, *Local Coefficient* for *Laminar* or *Turbulent Flow*, the surface temperatures $T_{s,i}$ are determined as a function of the free stream velocity.



(c) Using the *Explore* option on the *Plot Window* associated with the IHT code of part (b), the minimum free stream velocity of

$$u_\infty = 6.6 \text{ m/s}$$ <

will maintain device 4, the hottest of the devices, at a temperature $T_{s,4}$ = 80°C.

COMMENTS: (1) Note that the thermophysical properties were evaluated at a reasonable assumed film temperature in part (a).

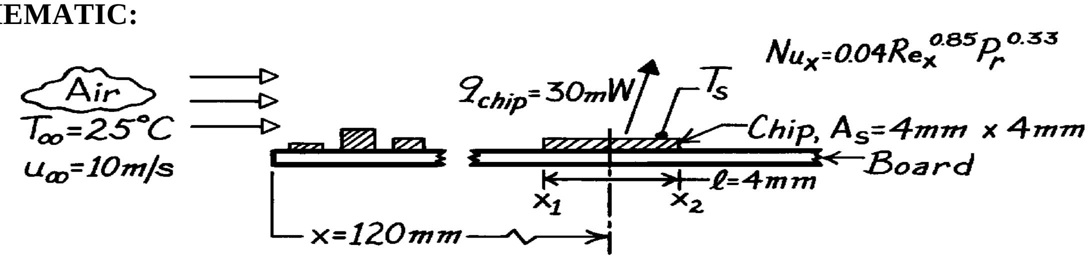
(2) From the $T_{s,i}$ vs. u_∞ plots, note that, as expected, the surface temperatures of the devices increase with distance from the leading edge.

PROBLEM 7.34

KNOWN: Convection correlation for irregular surface due to electronic elements mounted on a circuit board experiencing forced air cooling with prescribed temperature and velocity

FIND: Surface temperature when heat dissipation rate is 30 mW for chip of prescribed area located a specific distance from the leading edge.

SCHEMATIC:



ASSUMPTIONS: (1) Situation approximates parallel flow over a flat plate with prescribed correlation, (2) Heat rate is from top surface of chip.

PROPERTIES: *Table A-4*, Air (assume $T_s \approx 45°C$, then $\overline{T} = (45 + 25)°C/2 \approx 310$ K, 1 atm): k = 0.027 W/m·K, $\nu = 16.90 \times 10^{-6}$ m^2/s, Pr = 0.706.

ANALYSIS: For the chip upper surface, the heat rate is

$$q_{chip} = \overline{h}_{chip} A_s (T_s - T_\infty) \qquad \text{or} \qquad T_s = T_\infty + q_{chip} / \overline{h}_{chip} A_s$$

Assuming the average convection coefficient over the chip length to be equal to the local value at the center of the chip ($x = x_o$), $\overline{h}_{chip} \approx h_x(x_o)$, where

$$Nu_x = 0.04 Re_x^{0.85} Pr^{0.33}$$

$$Nu_x = 0.04\left(10 \text{ m/s} \times 0.120 \text{ m}/16.90 \times 10^{-6} \text{ m}^2/\text{s}\right)^{0.85} (0.706)^{0.33} = 473.4$$

$$h_x = \frac{Nu_x k}{x_o} = \frac{473.4 \times 0.027 \text{ W/m} \cdot \text{K}}{0.120 \text{ m}} = 107 \text{ W/m}^2 \cdot \text{K}$$

Hence,

$$T_s = 25°\text{C} + 30 \times 10^{-3} \text{ W}/107 \text{ W/m}^2 \cdot \text{K} \times \left(4 \times 10^{-3} \text{m}\right)^2 = (25 + 17.5)°\text{C} = 42.5°\text{C}. \quad <$$

COMMENTS: (1) Note that the assumed value of $\overline{T}$ used to evaluate the thermophysical properties was reasonable. (2) We could have evaluated $\overline{h}_{chip}$ by two other approaches. In one case the average coefficient is approximated as the arithmetic mean of local values at the leading and trailing edges of the chip.

$$\overline{h}_{chip} \approx \left[h_{x2}(x_2) + h_{x1}(x_1)\right]/2 = 107 \text{ W/m}^2 \cdot \text{K}.$$

The exact approach is of the form

$$\overline{h}_{chip} \cdot \ell = \overline{h}_{x2} \cdot x_2 - \overline{h}_{x1} \cdot x_1$$

Recognizing that $h_x \sim x^{-0.15}$, it follows that

$$\overline{h}_x = \frac{1}{x}\int_0^x h_x \cdot dx = 1.176 h_x$$

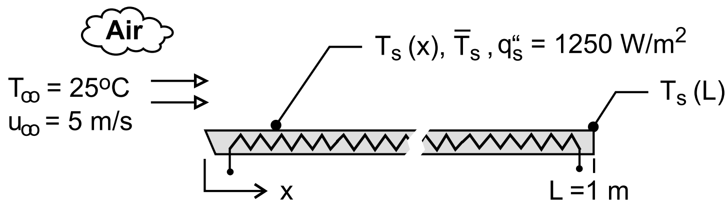
and $\overline{h}_{chip} = 108$ W/m$^2 \cdot$K. Why do results for the two approximate methods and the exact method compare so favorably?

PROBLEM 7.35

KNOWN: Air at atmospheric pressure and a temperature of 25°C in parallel flow at a velocity of 5 m/s over a 1-m long flat plate with a uniform heat flux of 1250 W/m^2.

FIND: (a) Plate surface temperature, $T_s(L)$, and local convection coefficient, $h_x(L)$, at the trailing edge, x = L, (b) Average temperature of the plate surface, $\overline{T}_s$, (c) Plot the variation of the plate surface temperature, $T_s(x)$, and the convection coefficient, $h_x(x)$, with distance on the same graph; explain key features of these distributions.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Flow is fully turbulent, and (3) Constant properties.

PROPERTIES: *Table A-4*, Air (assume T_f = 325 K, 1 atm): $\nu = 18.76 \times 10^{-6}$ m^2/s; k = 0.0284 W/m·K; Pr = 0.703

ANALYSIS: (a) At the trailing edge, x = L, the convection rate equation is

$$q_s'' = q_{cv}'' = h_x(L)\left[T_s(L) - T_\infty\right] \tag{1}$$

where the local convection coefficient, assuming turbulent flow, follows from Eq. 7.51.

$$Nu_x = \frac{h_x x}{k} = 0.0308\ Re_x^{4/5}\ Pr^{1/3} \tag{2}$$

With x = L = 1m, find

$$Re_x = u_\infty L/\nu = 5\ m/s \times 1\ m/18.76\times10^{-6}\ m^2/s = 2.67\times10^5$$

$$h_x(L) = (0.0284\ W/m\cdot K/1m)\times 0.0308\left(2.67\times10^5\right)^{4/5}(0.703)^{1/3} = 17.1\ W/m^2\cdot K$$

Substituting numerical values into Eq. (1),

$$T_s(L) = 25°C + 1250\ W/m^2/17.1\ W/m^2\cdot K = 98.3°C$$ <

(b) The average surface temperature $\overline{T}_s$ follows from the expression

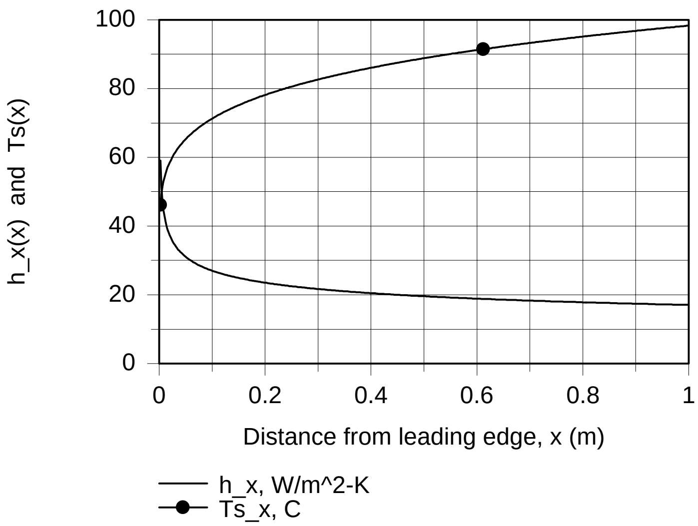
$$\overline{T}_s - T_\infty = \frac{1}{L}\int_0^L (T_s - T_\infty)dx = \frac{q_s''}{L}\int_0^L \frac{x}{k\,Nu_x}dx \tag{3}$$

where Nu_x is given by Eq. (2). Using the *Integral* function in *IHT* as described in Comment (3) find

$$\overline{T}_s = 86.1°C.$$ <

(c) The variation of the plate surface temperature $T_s(x)$ and convection coefficient, $h_x(x)$, shown in the graph are calculated using Eqs. (1) and (2).

Continued …..



COMMENTS: (1) To avoid performing the integration of part (b), it is reasonable to use the approximate, simpler Eqs. 7.53a and integrating Eq. 7.51,

$$\overline{Nu}_L = 0.0385\ Re_L^{4/5}\,Pr^{1/3} = 0.0385\left(2.67\times10^5\right)^{4/5}(0.703)^{1/3} = 751$$

$$\overline{h}_L = \overline{Nu}_L\,k/L = 751\times0.0284\ W/m\cdot K/1\ m = 213\ W/m^2\cdot K$$

$$\overline{T}_s = T_\infty + \frac{q_s'' L}{k\,\overline{Nu}_L} = 83.6°C.$$

(2) The properties for the correlation should be evaluated at $T_f = \left(\overline{T}_s + T_\infty\right)/2$. From the foregoing analyses, T_f = (86.1 + 25)°/2 = 55.5°C = 329 K. Hence, the assumed value of 325 K was reasonable.

(3) The IHT code, excluding the input variables and air property functions, used to evaluate the integral of Eq. (3) and generate the graphs in part (c) is shown below.

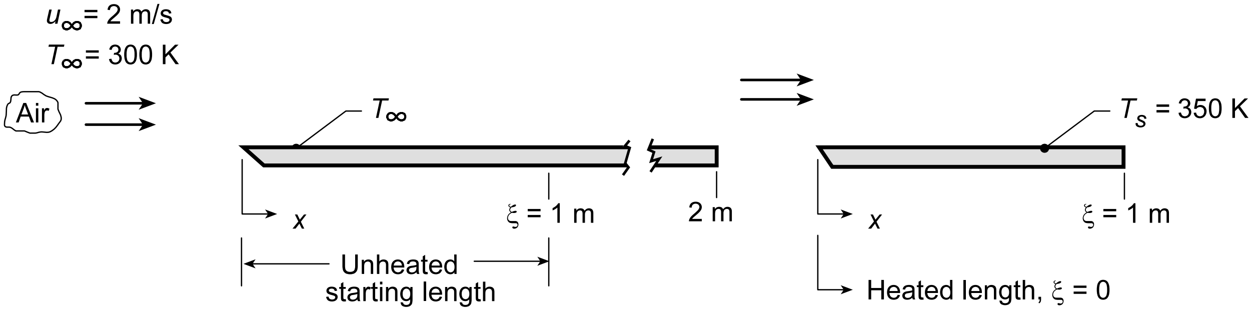
```
/* Programming note: when using the INTEGRAL function, the value of the independent variable
must not be specified as an input variable. If done so, this error message will appear:
"Redefinition of a constant variable." */

// Turbulent flow correlation, Eq. 7.50, local values
Nu_x = 0.0308 * Re_x^0.8 * Pr^0.333
Nu_x = h_x * x / k
Re_x = uinf * x / nu

// Plate temperatures
// Local
Ts_x = Tinf + q''s / h_x
// Average
Ts_avg - Tinf = q''s / L * INTEGRAL (y,x)
delT_avg = Ts_avg - Tinf
y = x / (k * Nu_x)
```

PROBLEM 7.36

KNOWN: Conditions for airflow over isothermal plate with optional unheated starting length.

FIND: (a) local coefficient, h_x, at leading and trailing edges with and without an unheated starting length, ξ = 1 m.

SCHEMATIC:



PROPERTIES: *Table A.4*, Air (T_f = 325 K, 1 atm): $\nu = 18.4 \times 10^{-6}$ m²/s, Pr = 0.703, k = 0.0282 W/m·K.

ANALYSIS: (a) The Reynolds number at ξ = 1 m is

$$Re_\xi = \frac{u_\infty \xi}{\nu} = \frac{2\,\mathrm{m/s} \times 1\,\mathrm{m}}{18.4 \times 10^{-6}\,\mathrm{m^2/s}} = 1.087 \times 10^5$$

If $Re_{x,c} = 5 \times 10^5$, flow is laminar over the entire plate (with or without the starting length). In general,

$$Nu_x = \frac{0.332\,Re_x^{1/2}\,Pr^{1/3}}{\left[1-(\xi/x)^{3/4}\right]^{1/3}} \tag{1}$$

$$h_x = \frac{\left(0.332k\,Pr^{1/3}\right)Re_x^{1/2}}{x\left[1-(\xi/x)^{3/4}\right]^{1/3}} = 0.00832\,\mathrm{W/m\cdot K}\frac{Re_x^{1/2}}{x\left[1-(\xi/x)^{3/4}\right]^{1/3}}.$$

With Unheated Starting Length: Leading edge (x = 1 m): $Re_x = Re_\xi$, $\xi/x = 1$, $h_x = \infty$ <

Trailing Edge (x = 2 m): $Re_x = 2\,Re_\xi = 2.17 \times 10^5$, $\xi/x = 0.5$

$$h_x = 0.00832\,\mathrm{W/m\cdot K}\frac{\left(2.17\times10^5\right)^{1/2}}{2\,\mathrm{m}\left[1-(0.5)^{3/4}\right]^{1/3}} = 2.61\,\mathrm{W/m^2\cdot K}$$ <

Without Unheated Starting Length: Leading edge (x = 0): $h_x = \infty$ <

Trailing edge (x = 1 m): $Re_x = 1.087 \times 10^5$

$$h_x = 0.00832\,\mathrm{W/m\cdot K}\frac{\left(1.087\times10^5\right)^{1/2}}{1\mathrm{m}} = 2.74\,\mathrm{W/m^2\cdot K}$$ <

(b) The average convection coefficient $\bar{h}_L$ for the two cases in the schematic are, from Eq. 6.6,

Continued...

PROBLEM 7.36 (Cont.)

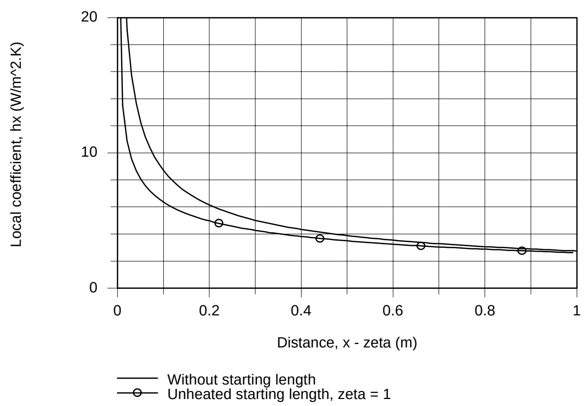
$$\bar{h}_L = \frac{1}{L}\int_{\xi=0}^{L} h_x(x)\,dx \qquad (2)$$

where L is the x location at the end of the heated section. Substituting Eq. (1) into Eq. (2) and numerically integrate, the results are tabulated below:

ξ (m)	h_x (L)(W/m$^2\cdot$K)	$\bar{h}_L$ (W/m$^2\cdot$K)
0	2.74	5.41
1	2.61	4.22

<

(c) The variation of the local convection coefficient over the plate, with and without the unheated starting length, using Eq. (1) is shown below. The abscissa is $x - \xi$.



COMMENTS: (1) When the velocity and thermal boundary layers grow simultaneously (*without starting length*), we expect the local and average coefficients to be larger than when the velocity boundary layer is thicker (*with starting length*).

(2) When $\xi = 0$, $\bar{h}_L = 2h_L$, when $\xi = 1$, $\bar{h}_L < 2h_L$. From Eq. (7.49), $\bar{h}_L = 4.25\ \mathrm{W/m^2\cdot K}$.

(3) The numerical integration of Eq. (2) was performed using the INTEGRAL (f,x) operation in IHT as shown in the Workspace below.

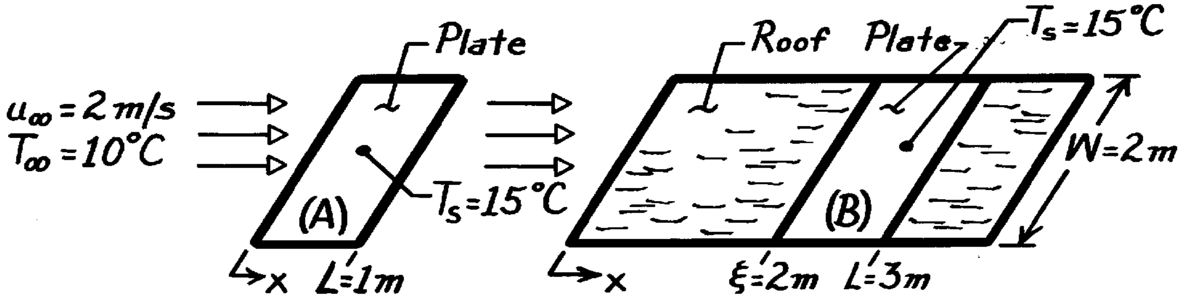
```
// Average Coefficient:
hbarL =  1 / (L - zeta ) * INTEGRAL (hx,x)

// Local Coefficient With Unheated Starting Length:
hx  = ( k / x) *  0.332 *  Rex^0.5 * Pr^0.3333 /  ( 1 - (zeta / x)^(3/4) )^(1/3)
Rex = uinf * x / nu

// Properties Tool - Air:
// Air property functions : From Table A.4
// Units: T(K); 1 atm pressure
nu = nu_T("Air",Tf)                    // Kinematic viscosity, m^2/s
k = k_T("Air",Tf)                      // Thermal conductivity, W/m·K
Pr = Pr_T("Air",Tf)                    // Prandtl number
Tf = 325                               // Film temperature, K

// Assigned Variables:
uinf =  2                              // Airstream velocity, m/s
x = 1                                  // Distance from leading edge, m
L = 2                                  // Full length of plate, m
zeta = 1                               // Starting length, m
xzeta = x - zeta                       // Difference
```

PROBLEM 7.37

KNOWN: Cover plate dimensions and temperature for flat plate solar collector. Air flow conditions.

FIND: (a) Heat loss with simultaneous velocity and thermal boundary layer development, (b) Heat loss with unheated starting length.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible radiation, (3) Boundary layer is not disturbed by roof-plate interface, (4) $Re_{x,c} = 5\times10^5$.

PROPERTIES: *Table A-4*, Air (T_f = 285.5K, 1 atm): $\nu = 14.6 \times 10^{-6}$ m^2/s, k = 0.0251 W/m·K, Pr = 0.71.

ANALYSIS: (a) The Reynolds number for the plate of L = 1m is

$$Re_L = \frac{u_\infty L}{n} = \frac{2\text{ m/s} \times 1\text{m}}{14.6\times10^{-6}\text{ m}^2/\text{s}} = 1.37\times10^5 < Re_{x,c}.$$

For laminar flow

$$\overline{Nu}_L = 0.664\, Re_L^{1/2}\, Pr^{1/3} = 0.664\left(1.37\times10^5\right)^{1/2}(0.71)^{1/3} = 219.2$$

$$q = \frac{k}{L}\overline{Nu}_L\, A_s\left(T_s - T_\infty\right) = \frac{0.0251\text{ W/m}\cdot\text{K}}{1\text{m}} 219.2\left(2\text{m}^2\right)5°\text{C} = 55\text{ W}. \qquad <$$

(b) The Reynolds number for the roof and collector of length L = 3m is

$$Re_L = \frac{2\text{ m/s}\times3\text{m}}{14.6\times10^{-6}\text{ m}^2/\text{s}} = 4.11\times10^5 < Re_{x,c}.$$

Hence, laminar boundary layer conditions exist throughout and the heat rate is

$$q = \int_x^L q''\,dA = \left(T_s - T_\infty\right)0.332\left(\frac{u_\infty}{n}\right)^{1/2} Pr^{1/3} kW \int_x^L \frac{x^{-1/2}dx}{\left[1-\left(x/x\right)^{3/4}\right]^{1/3}}$$

$$q = \left(5°\text{C}\right)0.332\left(\frac{2\text{ m/s}}{14.6\times10^{-6}\text{ m}^2/\text{s}}\right)^{1/2}(0.71)^{1/3}\,0.0251\frac{\text{W}}{\text{m}\cdot\text{K}}2\text{m}\int_x^L \frac{x^{-1/2}dx}{\left[1-\left(x/x\right)^{3/4}\right]^{1/3}}$$

Using a numerical technique to evaluate the integral,

$$q = 27.50\int_2^3 \frac{x^{-1/2}dx}{\left[1-\left(2.0/x\right)^{3/4}\right]^{1/3}} = 27.50\times1.417 = 39\text{ W} \qquad <$$

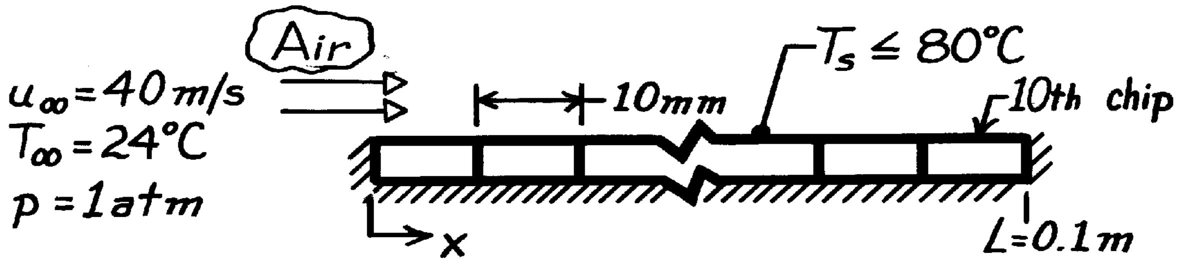
COMMENTS: Values of $\bar{h}$ with and without the unheated starting length are 3.9 and 5.5 W/m^2·K. Prior development of the velocity boundary layer decreases $\bar{h}$.

PROBLEM 7.38

KNOWN: Surface dimensions for an array of 10 silicon chips. Maximum allowable chip temperature. Air flow conditions.

FIND: Maximum allowable chip electrical power (a) without and (b) with a turbulence promoter at the leading edge.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Film temperature of 52°C, (3) Negligible radiation, (4) Negligible heat loss through insulation, (5) Uniform heat flux at chip interface with air, (6) $Re_{x,c} = 5\times10^5$.

PROPERTIES: *Table A-4*, Air (T_f = 325K, 1 atm): $\nu = 18.4 \times 10^{-6}\ m^2/s$, k = 0.0282 W/m·K, Pr = 0.703.

ANALYSIS: $Re_L = u_\infty L/n = 40\ m/s \times 0.1\ m/18.4 \times 10^{-6}\ m^2/s = 2.174\times10^5$. Hence, flow is laminar over all chips without the promoter.

(a) For *laminar flow*, the minimum h_x exists on the last chip. Approximating the average coefficient for Chip 10 as the local coefficient at x = 95 mm, $\bar{h}_{10} = h_{x=0.095m}$.

$$\bar{h}_{10} = 0.453\frac{k}{x}Re_x^{1/2}Pr^{1/3}$$

$$Re_x = \frac{u_\infty x}{n} = \frac{40\ m/s \times 0.095\ m}{18.4\times10^{-6}\ m^2/s} = 2.065\times10^5$$

$$\bar{h}_{10} = 0.453\frac{0.0282\ W/m\cdot K}{0.095}\left(2.065\times10^5\right)^{1/2}(0.703)^{1/3} = 54.3 W/m^2\cdot K$$

$$q_{10} = \bar{h}_{10}A\left(T_s - T_\infty\right) = 54.3\frac{W}{m^2\cdot K}(0.01\ m)^2(80-24)^\circ C = 0.30\ W.$$

Hence, if all chips are to dissipate the same power and T_s is not to exceed 80°C.

$$q_{max} = 0.30\ W. \qquad <$$

(b) For *turbulent flow*,

$$\bar{h}_{10} = 0.0308\frac{k}{x}Re_x^{4/5}Pr^{1/3} = 0.308\frac{0.0282 W/m\cdot K}{0.095\ m}\left(2.065\times10^5\right)^{4/5}(0.703)^{1/3} = 145 W/m^2\cdot K$$

$$q_{10} = \bar{h}_{10}A\left(T_s - T_\infty\right) = 1452\frac{W}{m^2\cdot K}(0.01\ m)^2(80-24)^\circ C = 0.81\ W.$$

Hence, $$q_{max} = 0.81\ W. \qquad <$$

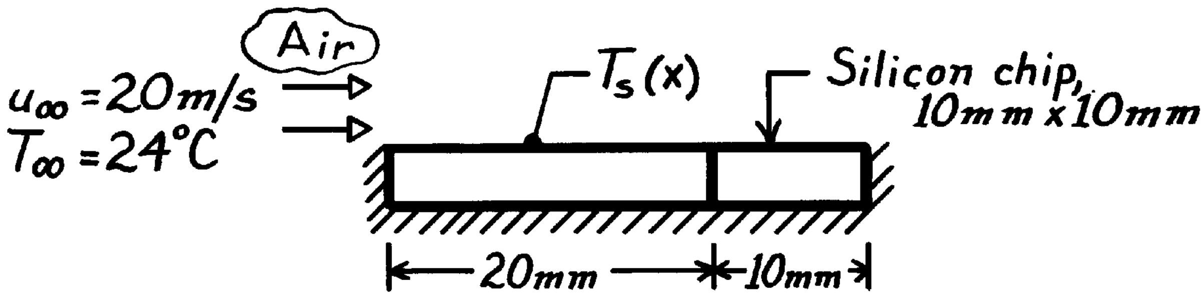
COMMENTS: It is far better to orient array normal to the air flow. Since $\bar{h}_1 > \bar{h}_{10}$, more heat could be dissipated per chip, and the same heat could be dissipated from each chip.

PROBLEM 7.39

KNOWN: Dimensions and maximum allowable temperature of a silicon chip. Air flow conditions.

FIND: Maximum allowable power with or without unheated starting length.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) $T_f = 52°C$, (3) Negligible radiation, (4) Negligible heat loss through insulation, (5) Uniform heat flux at chip-air interface, (6) $Re_{x,c} = 5 \times 10^5$.

PROPERTIES: *Table A-4*, Air (T_f = 325K, 1 atm): $\nu = 18.41 \times 10^{-6}$ m^2/s, k = 0.0282 W/m·K, Pr = 0.703.

ANALYSIS: For uniform heat flux, maximum T_s corresponds to minimum h_x. Without unheated starting length,

$$Re_L = \frac{u_\infty L}{n} = \frac{20\text{ m/s} \times 0.01\text{ m}}{18.41 \times 10^{-6}\text{ m}^2/\text{s}} = 10{,}864.$$

With the unheated starting length, L = 0.03 m, Re_L = 32,591. Hence, the flow is laminar in both cases and the minimum h_x occurs at the trailing edge (x = L).

Without unheated starting length,

$$h_L = \frac{k}{L} 0.453 Re_L^{1/2} Pr^{1/3} = \frac{0.0282\text{ W/m}\cdot\text{K}}{0.01\text{ m}} 0.453 (10{,}864)^{1/2} (0.703)^{1/3} \le$$

$$h_L = 118\text{ W/m}^2\cdot\text{K}$$

$$q''(L) = h_L (T_s - T_\infty) = 118\text{ W/m}^2\cdot\text{K} (80 - 24)^\circ\text{C} = 6630\text{ W/m}^2$$

$$q_{max} = A_s q'' = \left(10^{-2}\text{m}\right)^2 6630\text{ W/m}^2 = 0.66\text{ W.}$$ <

With the unheated starting length,

$$h_L = \frac{k}{L} 0.453 \frac{Re_L^{1/2} Pr^{1/3}}{\left[1 - (x/L)^{3/4}\right]^{1/3}} = \frac{0.0282\text{ W/m}\cdot\text{K}}{0.03\text{ m}} 0.453 \frac{(32{,}951)^{1/2} (0.703)^{1/3}}{\left[1 - (0.02/0.03)^{3/4}\right]^{1/3}}$$

$$h_L = 107\text{ W/m}^2\cdot\text{K}$$

$$q''(L) = h_L (T_s - T_\infty) = 107\text{ W/m}^2\cdot\text{K} \ (80 - 24)^\circ\text{C} = 6013\text{ W/m}^2$$

$$q_{max} = A_s q'' = 10^{-4}\text{m}^2 \times 6013\text{ W/m}^2 = 0.60\text{ W.}$$ <

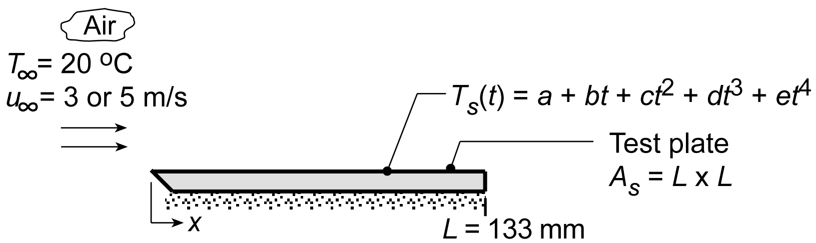
COMMENTS: Prior velocity boundary layer development on the unheated starting section decreases h_x, although the effect diminishes with increasing x.

PROBLEM 7.40

KNOWN: Experimental apparatus providing nearly uniform airstream over a flat *test plate*. Temperature history of the pre-heated plate for airstream velocities of 3 and 9 m/s were fitted to a fourth-order polynomial.

FIND: (a) Convection coefficient for the two cases assuming the plate behaves as a spacewise isothermal object and (b) Coefficients C and m for a correlation of the form $\overline{Nu}_L = C\,Re^m\,Pr^{1/3}$; compare result with a standard-plate correlation and comment on the goodness of the comparison; explain any differences.

SCHEMATIC:



u_∞ (m/s)	3	9
Δt (s)	300	160
a (°C)	56.87	57.00
b (°C/s)	-0.1472	-0.2641
c (°C/s^2)	3×10^{-4}	9×10^{-4}
d (°C/s^3)	-4×10^{-7}	-2×10^{-6}
e (°C/s^4)	2×10^{-10}	1×10^{-9}

ASSUMPTIONS: (1) Airstream over the *test plate* approximates parallel flow over a flat plate, (2) Plate is spacewise isothermal, (3) Negligible radiation exchange between plate and surroundings, (4) Constant properties, and (5) Negligible heat loss from the bottom surface or edges of the test plate.

PROPERTIES: *Table A.4*, Air ($T_f = (T_s - T_\infty)/2 \approx 310$ K, 1atm): $k_a = 0.0269$ W/m·K, $\nu = 1.669 \times 10^{-5}$ m^2/s, Pr = 0.706. Test plate (Given): $\rho = 2770$ kg/m^3, $c_p = 875$ J/kg·K, k = 177 W/m·K.

ANALYSIS: (a) Using the lumped-capacitance method, the energy balance on the plate is

$$-\overline{h}_L A_s \left[T_s(t) - T_\infty\right] = \rho V c_p \frac{dT}{dt} \qquad (1)$$

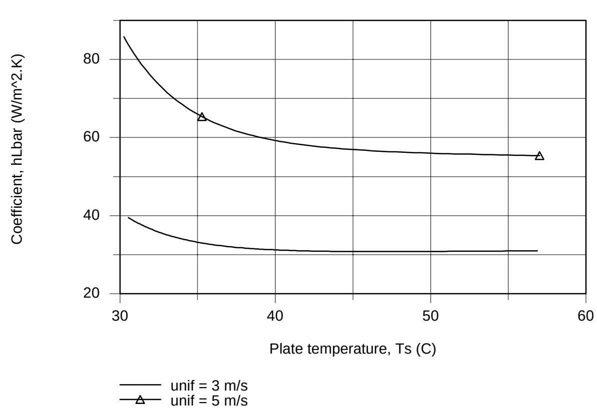
and the average convection coefficient can be determined from the temperature history, $T_s(t)$,

$$\overline{h}_L = \frac{\rho V c_p}{A_s} \frac{(dT/dt)}{T_s(t) - T_\infty} \qquad (2)$$

where the temperature-time derivative is

$$\frac{dT_s}{dt} = b + 2ct + 3dt^2 + 4et^3 \qquad (3)$$

The temperature time history plotted below shows the experimental behavior of the observed data.

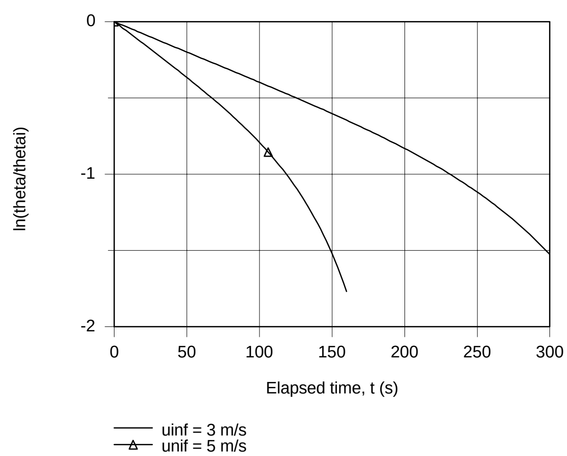


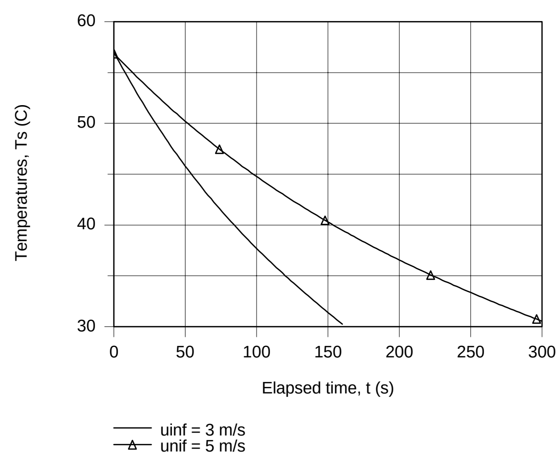
Continued...

Consider now the integrated form of the energy balance, Eq. (5.6), expressed as

$$\ln\frac{T_s(t)-T_\infty}{T_i-T_\infty}=-\left(\frac{\overline{h}_L A_s}{\rho V c}\right)t \tag{4}$$

If we were to plot the LHS vs t, the slope of the curve would be proportional to $\overline{h}_L$. Using IHT, plots were generated of $\overline{h}_L$ vs. T_s, Eq. (1), and $\ln\left[\left(T_s(t)-T_\infty\right)/\left(T_i-T_\infty\right)\right]$ vs. t, Eq. (4). From the latter plot, recognize that the regions where the slope is constant corresponds to early times (≤ 100s when u_∞ = 3 m/s and ≤ 50s when u_∞ = 5 m/s).





Selecting two elapsed times at which to evaluate $\overline{h}_L$, the following results were obtained

u_∞ (m/s)	t (s)	T_s (t), (°C)	$\overline{h}_L$ (W/m^2·K)	$\overline{Nu}_L$	Re_L
3	100	44.77	30.81	152.4	2.39×10^4
9	50	45.80	56.7	280.4	7.17×10^4

where the dimensionless parameters are evaluated as

$$\overline{Nu}_L=\frac{\overline{h}_L L}{k_a} \qquad Re_L=\frac{u_\infty L}{\nu} \tag{5,6}$$

where k_a, ν are thermophysical properties of the airstream.

(b) Using the above pairs of $\overline{Nu}_L$ and Re_L, C and m in the correlation can be evaluated,

$$\overline{Nu}_L = C\,Re_L^m\,Pr^{1/3} \tag{7}$$

$$152.4 = C(2.39\times10^4)^m(0.706)^{1/3}$$

$$280.4 = C(7.17\times10^4)^m(0.706)^{1/3}$$

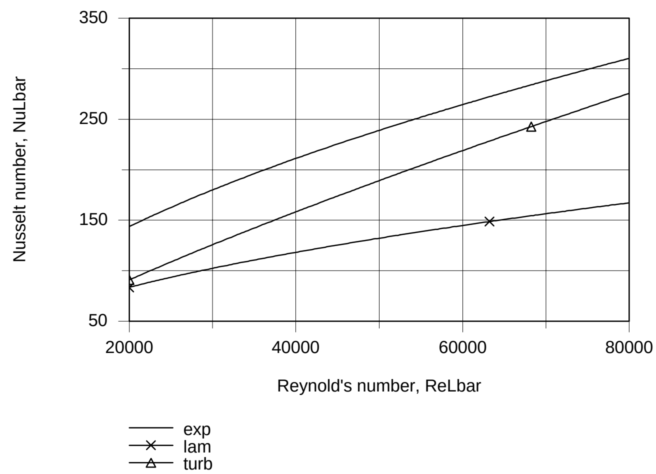
Solving, find

$C = 0.633 \qquad m = 0.555$ (8,9) <

Continued...

PROBLEM 7.40 (Cont.)

The plot below compares the experimental correlation (C = 0.633, m = 0.555) with those for laminar flow (C = 0.664, m = 0.5) and fully turbulent flow (C = 0.037, m = 0.8). The experimental correlation yields $\overline{Nu}_L$ values which are 25% higher than for the correlation. The most likely explanation for this unexpected trend is that the airstream reaching the plate is not parallel, but with a slight impingement effect and/or the flow is very highly turbulent at the leading edge.



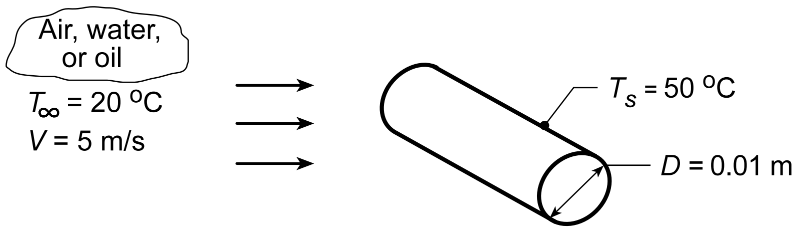
COMMENTS: (1) A more extensive analysis of the experimental observations would involve determining $\overline{Nu}_L$ for the full range of elapsed time (rather than at two selected times) and using a fitting routine to determine values for C and m.

PROBLEM 7.41

KNOWN: Cylinder diameter and surface temperature. Temperature and velocity of fluids in cross flow.

FIND: (a) Rate of heat transfer per unit length for the fluids: atmospheric air and saturated water, and engine oil, for velocity V = 5 m/s, using the Churchill-Bernstein correlation, and (b) Compute and plot q' as a function of the fluid velocity $0.5 \le V \le 10$ m/s.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform cylinder surface temperature.

PROPERTIES: *Table A.4*, Air (T_f = 308 K, 1 atm): $\nu = 16.69 \times 10^{-6}$ m²/s, k = 0.0269 W/m·K, Pr = 0.706; *Table A.6*, Saturated Water (T_f = 308 K): ρ = 994 kg/m³, $\mu = 725 \times 10^{-6}$ N·s/m², k = 0.625 W/m·K, Pr = 4.85; *Table A.5*, Engine Oil (T_f = 308 K): $\nu = 340 \times 10^{-6}$ m²/s, k = 0.145 W/m·K, Pr = 4000.

ANALYSIS: (a) For each fluid, calculate the Reynolds number and use the Churchill-Bernstein correlation, Eq. 7.57,

$$\overline{\mathrm{Nu}}_D = \frac{\bar{h}D}{k} = 0.3 + \frac{0.62\,\mathrm{Re}_D^{1/2}\,\mathrm{Pr}^{1/3}}{\left[1+(0.4/\mathrm{Pr})^{2/3}\right]^{1/4}}\left[1+\left(\frac{\mathrm{Re}_D}{282{,}000}\right)^{5/8}\right]^{4/5}$$

Fluid: Atmospheric Air

$$\mathrm{Re}_D = \frac{VD}{\nu} = \frac{(5\,\mathrm{m/s})\,0.01\,\mathrm{m}}{16.69\times10^{-6}\,\mathrm{m^2/s}} = 2996$$

$$\overline{\mathrm{Nu}}_D = 0.3 + \frac{0.62(2996)^{1/2}(0.706)^{1/3}}{\left[1+(0.4/0.706)^{2/3}\right]^{1/4}}\left[1+\left(\frac{2996}{282{,}000}\right)^{5/8}\right]^{4/5} = 28.1$$

$$\bar{h} = \frac{k}{D}\overline{\mathrm{Nu}}_D = \frac{0.0269\,\mathrm{W/m\cdot K}}{0.01\,\mathrm{m}}\,28.1 = 75.5\,\mathrm{W/m^2\cdot K}$$

$$q' = \bar{h}\pi D(T_s - T_\infty) = 75.5\,\mathrm{W/m^2\cdot K}\,\pi\,(0.01\,\mathrm{m})(50-20)^\circ\mathrm{C} = 71.1\,\mathrm{W/m} \qquad <$$

Fluid: Saturated Water

$$\mathrm{Re}_D = \frac{VD}{\nu} = \frac{(5\,\mathrm{m/s})\,0.01\,\mathrm{m}}{725\times10^{-6}\,\mathrm{N\cdot s/m^2}/994\,\mathrm{kg/m^3}} = 68{,}552$$

$$\overline{\mathrm{Nu}}_D = 0.3 + \frac{0.62(68{,}552)^{1/2}(4.85)^{1/3}}{\left[1+(0.4/4.85)^{2/3}\right]^{1/4}}\left[1+\left(\frac{68{,}552}{282{,}000}\right)^{5/8}\right]^{4/5} = 347$$

Continued...

$$\bar{h} = \frac{k}{D}\overline{Nu}_D = \frac{0.625\,W/m\cdot K}{0.01\,m}347 = 21{,}690\,W/m^2\cdot K \qquad q' = 20{,}438\,W/m \qquad <$$

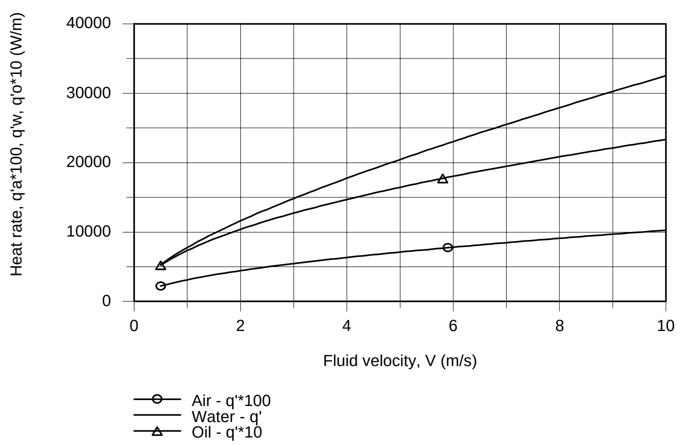
Fluid: Engine Oil

$$Re_D = \frac{VD}{\nu} = \frac{(5\,m/s)\,0.01\,m}{340\times10^{-6}\,m^2/s} = 147$$

$$\overline{Nu}_D = 0.3 + \frac{0.62(147)^{1/2}(4000)^{1/3}}{\left[1+(0.4/4000)^{2/3}\right]^{1/4}}\left[1+\left(\frac{147}{282{,}000}\right)^{5/8}\right]^{4/5} = 120$$

$$\bar{h} = \frac{k}{D}\overline{Nu}_D = \frac{0.145\,W/m\cdot K}{0.01\,m}120 = 1740\,W/m^2\cdot K \qquad q' = 1639\,W/m \qquad <$$

(b) Using the *IHT Correlations Tool*, *External Flow*, *Cylinder*, along with the *Properties Tool* for each of the fluids, the heat rates, q', were calculated for the range $0.5 \le V \le 10$ m/s. Note the q' scale multipliers for the air and oil fluids which permit easy comparison of the three curves.



COMMENTS: (1) Note the inapplicability of the Zhukauskas relation, Eq. 7.56, since $Pr_{oil} > 500$.

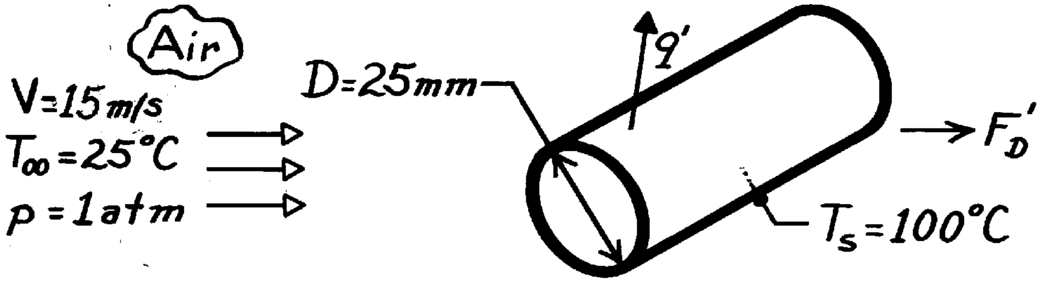
(2) In the plot above, recognize that the heat rate for the water is more than 10 times that with oil and 300 times that with air. How do changes in the velocity affect the heat rates for each of the fluids?

PROBLEM 7.42

KNOWN: Conditions associated with air in cross flow over a pipe.

FIND: (a) Drag force per unit length of pipe, (b) Heat transfer per unit length of pipe.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform cylinder surface temperature, (3) Negligible radiation effects.

PROPERTIES: *Table A-4*, Air (T_f = 335 K, 1 atm): $\nu = 19.31 \times 10^{-6}$ m^2/s, $\rho = 1.048$ kg/m^3, k = 0.0288 W/m·K, Pr = 0.702.

ANALYSIS: (a) From the definition of the drag coefficient with A_f = DL, find

$$F_D = C_D A_f \frac{\rho V^2}{2}$$

$$F_D' = C_D D \frac{\rho V^2}{2}.$$

With

$$Re_D = \frac{VD}{\nu} = \frac{15 \text{ m/s} \times (0.025 \text{ m})}{19.31 \times 10^{-6} \text{ m}^2/\text{s}} = 1.942 \times 10^4$$

from Fig. 7.8, $C_D \approx 1.1$. Hence

$$F_D = 1.1(0.025 \text{ m})\ 1.048 \text{ kg/m}^3\ (15 \text{ m/s})^2/2 = 3.24 \text{ N/m}. \qquad <$$

(b) Using Hilpert's relation, with C = 0.193 and m = 0.618 from Table 7.2,

$$\bar{h} = \frac{k}{D} C\, Re_D^m\, Pr^{1/3} = \frac{0.0288 \text{ W/m} \cdot \text{K}}{0.025 \text{ m}} \times 0.193\left(1.942 \times 10^4\right)^{0.618} (0.702)^{1/3}$$

$$\bar{h} = 88 \text{ W/m}^2 \cdot \text{K}.$$

Hence, the heat rate per unit length is

$$q' = \bar{h}(\pi D)\ (T_s - T_\infty) = 88 \text{ W/m}^2 \cdot \text{K}(\pi \times 0.025 \text{ m})\ (100 - 25)^\circ \text{C} = 520 \text{ W/m}. \qquad <$$

COMMENTS: Using the Zhukauskas correlation and evaluating properties at T_∞ ($\nu = 15.71 \times 10^{-6}$ m^2/s, k = 0.0261 W/m·K, Pr = 0.707), but with Pr_s = 0.695 at T_s,

$$\bar{h} = \frac{0.0261}{0.025} 0.26 \left(\frac{15 \times 0.025}{15.71 \times 10^{-6}}\right)^{0.6} (0.707)^{0.37} (0.707/0.695)^{1/4} = 102 \text{ W/m}^2 \cdot \text{K}.$$

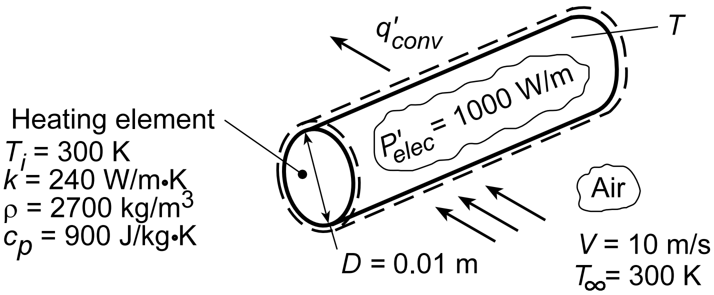
This result agrees with that obtained from Hilpert's relation to within the uncertainty normally associated with convection correlations.

PROBLEM 7.43

KNOWN: Initial temperature, power dissipation, diameter, and properties of heating element. Velocity and temperature of air in cross flow.

FIND: (a) Steady-state temperature, (b) Time to come within 10°C of steady-state temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Uniform heater temperature, (2) Negligible radiation.

PROPERTIES: *Table A.4*, air (assume $T_f \approx 450$ K): $\nu = 32.39 \times 10^{-6}$ m^2/s, k = 0.0373 W/m·K, Pr = 0.686.

ANALYSIS: (a) Performing an energy balance for steady-state conditions, we obtain

$$q'_{conv} = \bar{h}(\pi D)(T - T_\infty) = P'_{elec} = 1000\,\text{W/m}$$

With

$$Re_D = \frac{VD}{\nu} = \frac{(10\,\text{m/s})0.01\,\text{m}}{32.39 \times 10^{-6}\,\text{m}^2/\text{s}} = 3{,}087$$

the Churchill and Bernstein correlation, Eq. 7.57, yields

$$\overline{Nu}_D = 0.3 + \frac{0.62\,Re_D^{1/2}\,Pr^{1/3}}{\left[1+(0.4/Pr)^{2/3}\right]^{1/4}}\left[1+\left(\frac{Re_D}{282{,}000}\right)^{5/8}\right]^{4/5}$$

$$\overline{Nu}_D = 0.3 + \frac{0.62(3087)^{1/2}(0.686)^{1/3}}{\left[1+(0.4/0.686)^{2/3}\right]^{1/4}}\left[1+\left(\frac{3087}{282{,}000}\right)^{5/8}\right]^{4/5} = 28.2$$

$$\bar{h} = \frac{k}{D}\overline{Nu}_D = \frac{0.0373\,\text{W/m}\cdot\text{K}}{0.010\,\text{m}}28.2 = 105.2\,\text{W/m}^2\cdot\text{K}$$

Hence, the steady-state temperature is

$$T = T_\infty + \frac{P'_{elec}}{\pi D \bar{h}} = 300\,\text{K} + \frac{1000\,\text{W/m}}{\pi(0.01\,\text{m})105.2\,\text{W/m}^2\cdot\text{K}} = 603\,\text{K} \quad <$$

(b) With Bi = $\bar{h}r_o/k$ = 105.2 W/m^2·K(0.005 m)/240 W/m·K = 0.0022, a lumped capacitance analysis may be performed. The time response of the heater is given by Eq. 5.25, which, for $T_i = T_\infty$, reduces to

$$T = T_\infty + (b/a)\left[1 - \exp(-at)\right]$$

Continued...

where a = $4\bar{h}/D\rho c_p = \left(4\times105.2\,W/m^2\cdot K\right)/\left(0.01\,m\times2700\,kg/m^3\times900\,J/kg\cdot K\right)$ = 0.0173 s^{-1} and b/a = $P'_{elec}/\pi D\bar{h}$ = $1000\,W/m\Big/\pi\left(0.01\,m\times105.2\,W/m^2\cdot K\right)$ = 302.6 K. Hence,

$$\left[1-\exp\left(-0.0173t\right)\right]=\frac{\left(593-300\right)K}{302.6\,K}=0.968$$

$$t \approx 200s$$ <

COMMENTS: (1) For T = 603 K and a representative emissivity of ε = 0.8, net radiation exchange between the heater and surroundings at $T_{sur} = T_\infty$ = 300 K would be $q'_{rad}=\varepsilon\sigma\left(\pi D\right)\left(T^4-T_{sur}^4\right)$ = 0.8 × 5.67 × 10^{-8} $W/m^2\cdot K^4$ (π × 0.01 m)(603^4 - 300^4)K^4 = 177 W/m. Hence, although small, radiation exchange is not negligible. The effects of radiation are considered in Problem 7.46.

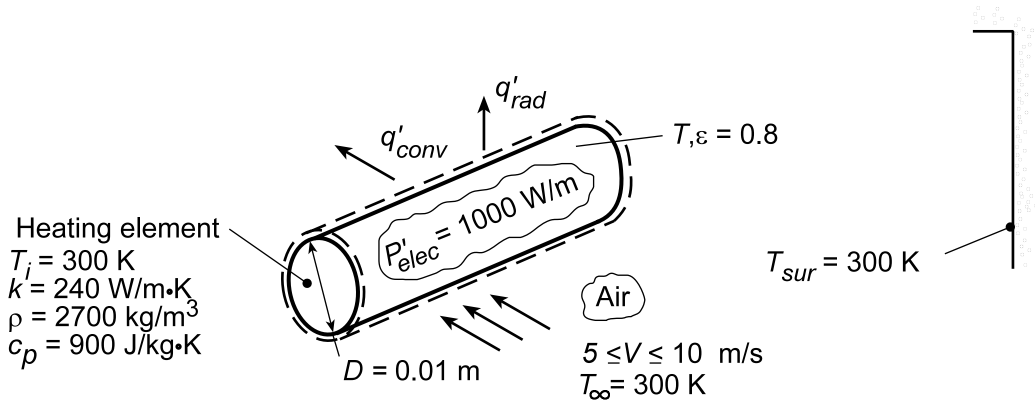
(2) The assumed value of T_f is very close to the actual value, rendering the selected air properties accurate.

PROBLEM 7.44

KNOWN: Initial temperature, power dissipation, diameter, and properties of a heating element. Velocity and temperature of air in cross flow. Temperature of surroundings.

FIND: (a) Steady-state temperature, (b) Time to come within 10°C of steady-state temperature, (c) Variation of power dissipation required to maintain a fixed heater temperature of 275°C over a range of velocities.

SCHEMATIC:



ASSUMPTIONS: Uniform heater surface temperature.

ANALYSIS: (a) Performing an energy balance for steady-state conditions, we obtain

$$q'_{conv} + q'_{rad} = P'_{elec}$$

$$\bar{h}(\pi D)(T - T_\infty) + \varepsilon\sigma(\pi D)\left(T^4 - T_{sur}^4\right) = P'_{elec}$$

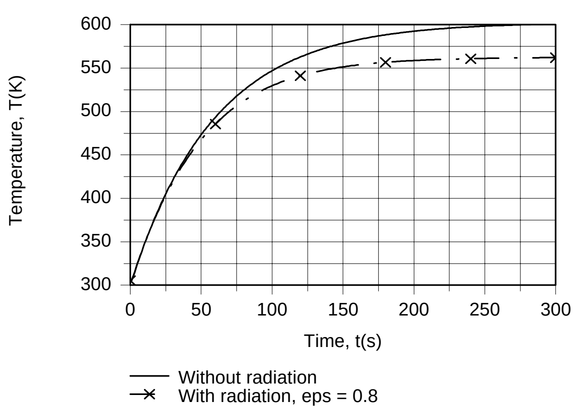
$$\bar{h}(\pi \times 0.01\,\text{m})(T - 300)\text{K} + 0.8\left(5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}\right)\left(T^4 - 300^4\right)\text{K}^4 = 1000\,\text{W/m}$$

Using the *IHT Energy Balance Model* for an *Isothermal Solid Cylinder* with the *Correlations* Tool Pad for a *Cylinder* in *Crossflow* and the *Properties* Tool Pad for Air, we obtain

$$T = 562.4\ \text{K}$$ <

where $\bar{h} = 105.4\,\text{W/m}^2\cdot\text{K}$, $h_r = 15.9\,\text{W/m}^2\cdot\text{K}$, $q'_{conv} = 868.8\,\text{W/m}$, and $q'_{rad} = 131.2\,\text{W/m}$.

(b) With Bi = $\left(\bar{h} + h_r\right)r_o/k$ = (121.3 W/m^2·K)0.005 m/240 W/m·K = 0.0025, the transient behavior may be analyzed using the lumped capacitance method. Using the *IHT Lumped Capacitance Model* to perform the numerical integration, the following temperature histories were obtained.



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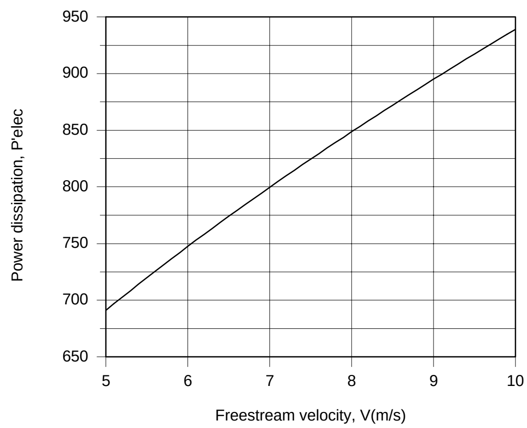
PROBLEM 7.44 (Cont.)

The agreement between predictions with and without radiation for t < 50s implies negligible radiation. However, as the heater temperature increases with time, radiation becomes significant, yielding a reduced heater temperature. Steady-state temperatures correspond to 562.4 K and 602.8 K, with and without radiation, respectively. The time required for the heater to reach 552.4 K (with radiation) is $t \approx 155$s.

(c) If the heater temperature is to be maintained at a fixed value in the face of velocity excursions, provision must be made for adjusting the heater power. Using the *Explore* and *Graph* options of IHT with the model of part (a), the following results were obtained.



For T = 275°C = 548 K, the controller would compensate for velocity reductions from 10 to 5 m/s by reducing the power from approximately 935 to 690 W/m.

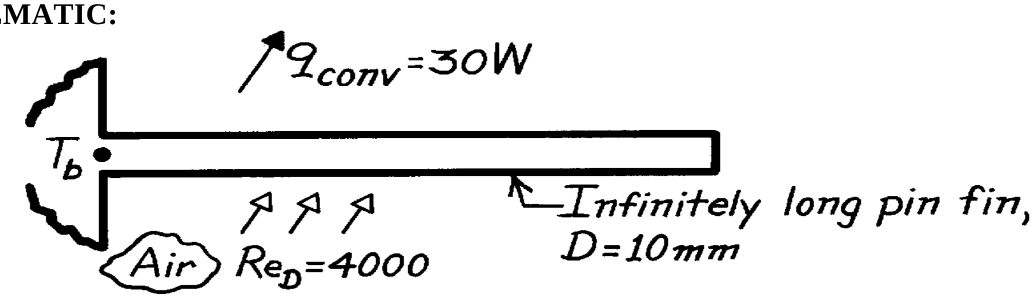
COMMENTS: Although convection heat transfer substantially exceeds radiation heat transfer, radiation is not negligible and should be included in the analysis. If it is neglected, T = 603 K would be predicted for P'_{elec} = 1000 W/m, in contrast 562 K from the results of part (a).

PROBLEM 7.45

KNOWN: Pin fin of 10 mm diameter dissipates 30 W by forced convection in cross-flow of air with $Re_D = 4000$.

FIND: Fin heat rate if diameter is doubled while all conditions remain the same.

SCHEMATIC:



ASSUMPTIONS: (1) Pin behaves as infinitely long fin, (2) Conditions of flow, as well as base and air temperatures, remain the same for both situations, (3) Negligible radiation heat transfer.

ANALYSIS: For an infinitely long pin fin, the fin heat rate is

$$q_f = q_{conv} = \left(\bar{h}PkA_c\right)^{1/2} \theta_b$$

where $P = \pi D$ and $A_c = \pi D^2/4$. Hence,

$$q_{conv} \sim \left(\bar{h} \cdot D \cdot D^2\right)^{1/2}.$$

For forced convection cross-flow over a cylinder, an appropriate correlation for estimating the dependence of $\bar{h}$ on the diameter is

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = C Re_D^m \, Pr^{1/3} = C\left(\frac{VD}{\nu}\right)^m Pr^{1/3}.$$

From Table 7.2 for $Re_D = 4000$, find m = 0.466 and

$$\bar{h} \sim D^{-1}(D)^{0.466} = D^{-0.534}.$$

It follows that

$$q_{conv} \sim \left(D^{-0.534} \cdot D \cdot D^2\right)^{1/2} = D^{1.23}.$$

Hence, with $q_1 \to D_1$ (10 mm) and $q_2 \to D_2$ (20 mm), find

$$q_2 = q_1\left(\frac{D_2}{D_1}\right)^{1.23} = 30\text{ W}\left(\frac{20}{10}\right)^{1.23} = 70.4\text{ W}.$$ <

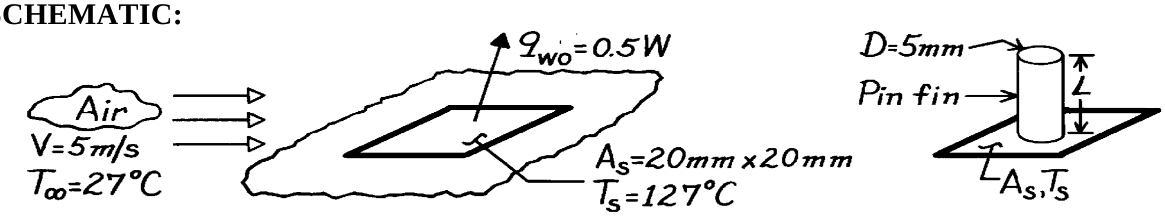
COMMENTS: The effect of doubling the diameter, with all other conditions remaining the same, is to increase the fin heat rate by a factor of 2.35. The effect is nearly linear, with enhancements due to the increase in surface and cross-sectional areas ($D^{1.5}$) exceeding the attenuation due to a decrease in the heat transfer coefficient ($D^{-0.267}$). Note that, with increasing Reynolds number, the exponent m increases and there is greater heat transfer enhancement due to increasing the diameter.

PROBLEM 7.46

KNOWN: Pin fin installed on a surface with prescribed heat rate and temperature.

FIND: (a) Maximum heat removal rate possible, (b) Length of the fin, (c) Effectiveness, ε_f, (d) Percentage increase in heat rate from surface due to fin.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Conditions over A_s are uniform for both situations, (3) Conditions over fin length are uniform, (4) Flow over pin fin approximates cross-flow.

PROPERTIES: *Table A-4*, Air ($T_f = (T_\infty + T_s)/2 = (27 + 127)°C/2 = 350$ K): $\nu = 20.92 \times 10^{-6}$ m^2/s, $k = 30.0 \times 10^{-3}$ W/m·K, Pr = 0.700. *Table A-1*, SS AISI304 ($\overline{T} = T_f = 350$ K): k = 15.8 W/m·K.

ANALYSIS: (a) Maximum heat rate from fin occurs when fin is infinitely long,

$$q_f = M = \left(\overline{h}PkA_c\right)^{1/2} q_b \tag{1}$$

from Eq. 3.80. Estimate convection heat transfer coefficient for cross-flow over cylinder,

$$Re_D = \frac{VD}{n} = 5 \text{ m/s} \times 0.005 \text{ m}/20.92 \times 10^{-6} \text{m}^2/\text{s} = 1195.$$

Using the Hilpert correlation, Eq. 7.55, with Table 7.2,find

$$\overline{h} = \frac{k}{D} C Re_D^m Pr = (0.030 \text{W/m} \cdot \text{K}/0.005\text{m})\, 0.683(1195)^{0.466} (0.700)^{1/3} = 98.9 \text{W/m}^2 \cdot \text{K}$$

From Eq. (1), with P = πD, $A_c = \pi D^2/4$, and $\theta_b = T_s - T_\infty$, find

$$q_f = \left(98.9 \text{ W/m}^2 \cdot \text{K} \times p\,(0.005 \text{ m}) \times 15.8 \text{ W/m} \cdot \text{K} \times p\,(0.005 \text{ m})^2/4\right)^{1/2} (127 - 27)\text{K} = 2.20 \text{ W.} \quad <$$

(b) From Example 3.8, $L \approx L_\infty = 2.65(kA_c/hP)^{1/2}$. Hence,

$$L \approx L_\infty = 2.65\left[15.8 \text{ W/m} \cdot \text{K} \times p\,(0.005 \text{ m})^2/4/98.9 \text{ W/m}^2 \cdot \text{K} \times p\,(0.005 \text{ m})\right]^{1/2} = 37.4 \text{ mm.} \quad <$$

(c) From Eq. 3.81, with h_s used for the base area A_s, the effectiveness is

$$e_f = \frac{q_f}{h_s A_{c,b} q_b} = \frac{q_f}{q_{wo}} \frac{A_s}{A_{c,b}} = \frac{2.2 \text{ W}}{0.5 \text{ W}} \cdot \frac{(0.020 \times 0.020)\text{m}^2}{p\,(0.005 \text{ m})^2/4} = 89.6 \quad <$$

where $\qquad h_s = q_{wo}/A_s q_b.$

(d) The percentage increase in heat rate with the installed fin (w) is

$$\frac{q_w - q_{wo}}{q_{wo}} \times 100 = \left(\left[q_f + h_s\left(A_s - pD^2/4\right)(T_s - T_\infty)\right] - q_{wo}\right) \times 100/q_{wo}$$

$$\Delta q/q = \left\{\left[2.2 \text{ W} + 12.5 \text{ W/m}^2 \cdot \text{K}\left([0.02 \text{ m}]^2 - (p/4)(0.005 \text{ m})^2\right)100 \text{ K} - 0.5 \text{ W}\right\} \times 100/0.5 \text{ W}\right.$$

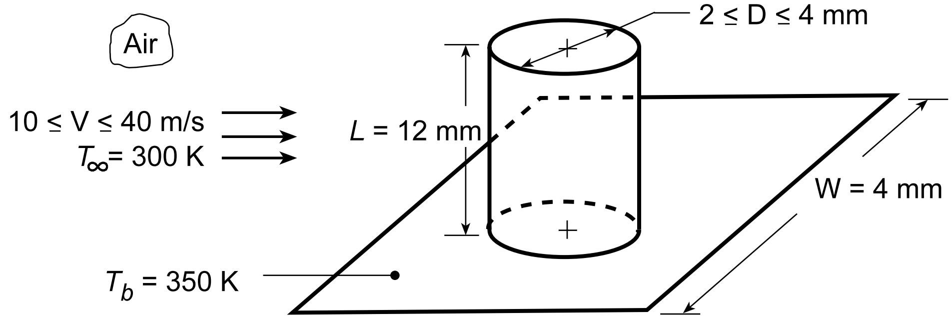
$$\Delta q/q = 435\%. \quad <$$

PROBLEM 7.47

KNOWN: Dimensions of chip and pin fin. Chip temperature. Free stream velocity and temperature of air coolant.

FIND: (a) Average pin convection coefficient, (b) Pin heat transfer rate, (c) Total heat rate, (d) Effect of velocity and pin diameter on total heat rate.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in pin, (3) Constant properties, (4) Convection coefficients on pin surface (tip and side) and chip surface correspond to single cylinder in cross flow, (5) Negligible radiation.

PROPERTIES: *Table A.1*, Copper (350 K): k = 399 W/m·K; *Table A.4*, Air ($T_f \approx$325 K, 1 atm): ν = 18.41×10^{-6} m^2/s, k = 0.0282 W/m·K, Pr = 0.704.

ANALYSIS: (a) With V = 10 m/s and D = 0.002 m,

$$\mathrm{Re_D} = \frac{\mathrm{VD}}{\nu} = \frac{10\,\mathrm{m/s} \times 0.002\,\mathrm{m}}{18.41 \times 10^{-6}\,\mathrm{m^2/s}} = 1087$$

Using the Churchill and Bernstein correlations, Eq. (7.57),

$$\overline{\mathrm{Nu}}_\mathrm{D} = 0.3 + \frac{0.62\,\mathrm{Re_D^{1/2}}\,\mathrm{Pr}^{1/3}}{\left[1+(0.4/\mathrm{Pr})^{2/3}\right]^{1/4}}\left[1+\left(\frac{\mathrm{Re_D}}{282,000}\right)^{5/8}\right]^{4/5} = 16.7$$

$$\bar{\mathrm{h}} = \left(\overline{\mathrm{Nu}}_\mathrm{D}\mathrm{k}/\mathrm{D}\right) = \left(16.7 \times 0.0282\,\mathrm{W/m \cdot K}/0.002\,\mathrm{m}\right) = 235\,\mathrm{W/m^2 \cdot K}$$ <

(b) For the fin with tip convection and

$$\mathrm{M} = \left(\bar{\mathrm{h}}\pi\mathrm{D}\mathrm{k}\pi\mathrm{D}^2/4\right)^{1/2}\theta_\mathrm{b} = (\pi/2)\left[235\,\mathrm{W/m^2 \cdot K}\,(0.002\,\mathrm{m})^3\,399\,\mathrm{W/m \cdot K}\right]^{1/2} 50\,\mathrm{K} = 2.15\,\mathrm{W}$$

$$\mathrm{m} = \left(\bar{\mathrm{h}}\mathrm{P}/\mathrm{kA_c}\right)^{1/2} = \left(4 \times 235\,\mathrm{W/m^2 \cdot K}/399\,\mathrm{W/m \cdot K} \times 0.002\,\mathrm{m}\right)^{1/2} = 34.3\,\mathrm{m^{-1}}$$

$$\mathrm{mL} = 34.3\,\mathrm{m^{-1}}(0.012\,\mathrm{m}) = 0.412$$

$$\left(\bar{\mathrm{h}}/\mathrm{mk}\right) = \left(235\,\mathrm{W/m^2 \cdot K}/34.3\,\mathrm{m^{-1}} \times 399\,\mathrm{W/m \cdot K}\right) = 0.0172.$$

The fin heat rate is

$$\mathrm{q_f} = \mathrm{M}\frac{\sinh \mathrm{mL} + \left(\bar{\mathrm{h}}/\mathrm{mk}\right)\cosh \mathrm{mL}}{\cosh \mathrm{mL} + \left(\bar{\mathrm{h}}/\mathrm{mk}\right)\sinh \mathrm{mL}} = 0.868\,\mathrm{W}\,.$$ <

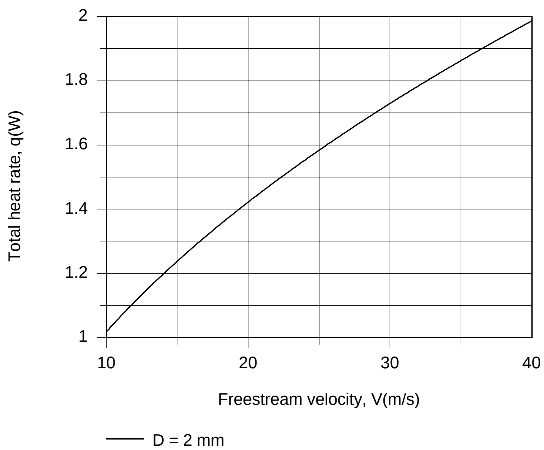
Continued...

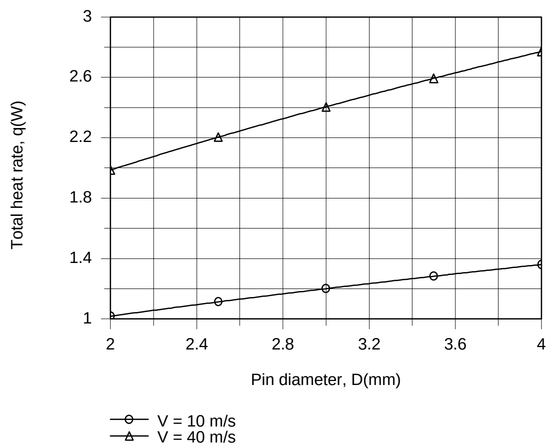
PROBLEM 7.47 (Cont.)

(c) The total heat rate is that from the base and through the fin,

$$q = q_b + q_f = \bar{h}\left(W^2 - \pi D^2/4\right)\theta_b + q_f = (0.151 + 0.868)\,W = 1.019\,W\,. \qquad <$$

(d) Using the IHT Extended Surface Model for a Pin Fin with the Correlations Tool Pad for a Cylinder in crossflow and Properties Tool Pad for Air, the following results were generated.





Clearly, there is significant benefit associated with increasing V which increases the convection coefficient and the total heat rate. Although the convection coefficient decreases with increasing D, the increase in the total heat transfer surface area is sufficient to yield an increase in q with increasing D. The maximum heat rate is q = 2.77 W for V = 40 m/s and D = 4 mm.

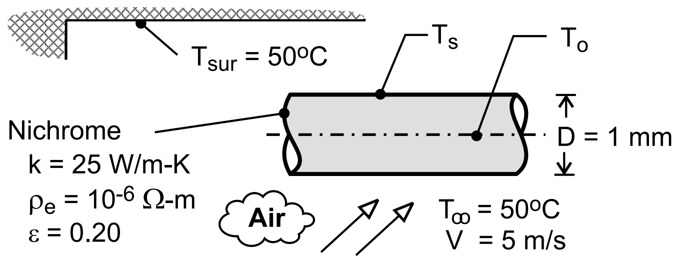
COMMENTS: Radiation effects should be negligible, although tip and base convection coefficients will differ from those calculated in parts (a) and (d).

PROBLEM 7.48

KNOWN: Diameter, resistivity, thermal conductivity and emissivity of Nichrome wire. Electrical current. Temperature of air flow and surroundings. Velocity of air flow.

FIND: (a) Surface and centerline temperatures of the wire, (b) Effect of flow velocity and electric current on temperatures.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) Radiation exchange with large surroundings, (3) Constant Nichrome properties, (4) Uniform surface temperature.

PROPERTIES: Prescribed, Nichrome: k = 25 W/m·K, $\rho_e = 10^{-6}\,\Omega\cdot\text{m}$, $\varepsilon = 0.2$. *Table A-4*, air $\left(T_f \approx 800\text{K}: k_a = 0.057\ \text{W/m}\cdot\text{K},\ \nu = 8.5\times10^{-5}\ \text{m}^2/\text{s},\ \text{Pr} = 0.71\right)$.

ANALYSIS: (a) The surface temperature may be obtained from Eq. 3.55, with $\bar{h} = \bar{h}_c + h_r$ and

$\dot{q} = I^2 R_e / \forall = I^2 \rho_e / A_c^2 = I^2 \rho_e / \left(\pi D^2/4\right)^2 = 1.013\times10^9\ \text{W/m}^3$.

$$T_s = T_\infty + \frac{\dot{q}(D/2)}{2\left(\bar{h}_c + h_r\right)} \tag{1}$$

The convection coefficient is obtained from the Churchill and Bernstein correlation

$$\bar{h}_c = \frac{k_a}{D}\left\{0.3 + \frac{0.62\,Re_D^{1/2}\,Pr^{1/3}}{\left[1+(0.4/Pr)^{2/3}\right]^{1/4}}\left[1+\left(\frac{Re_D}{282{,}000}\right)^{5/8}\right]^{4/5}\right\} = 230\ \text{W/m}^2\cdot\text{K}$$

where $Re_D = VD/\nu = 58.8$, and the radiation coefficient is obtained from Eq. 1.9

$$h_r = \varepsilon\sigma\left(T_s + T_{sur}\right)\left(T_s^2 + T_{sur}^2\right) \tag{2}$$

From an iterative solution of Eqs. (1) and (2), we obtain

$$T_s \approx 1285\text{K} = 1012°\text{C}$$ <

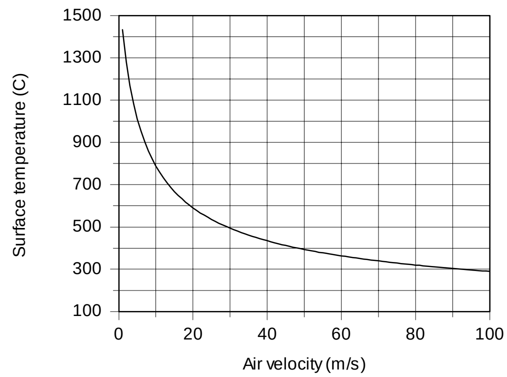
From Eq. 3.53, the centerline temperature is

$$T_o = \frac{\dot{q}(D/2)^2}{4k} + T_s = \frac{1.013\times10^9\ \text{W/m}^3\,(0.0005\text{m})^2}{100\ \text{W/m}\cdot\text{K}} + 1012°\text{C} \approx 1014°\text{C}$$ <

The centerline temperature is only approximately 2°C larger than the surface temperature, and the wire may be assumed to be isothermal.

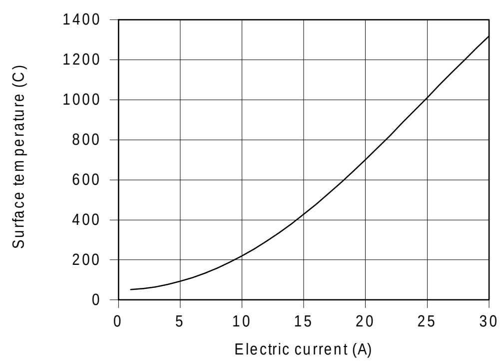
Continued …..

(b) Over the range $1 \le V < 100$ m/s for I = 25A, $\overline{h}_c$ varies from approximately $114\,W/m^2 \cdot K$ to $1050\,W/m^2 \cdot K$, while h_r varies from approximately $69\,W/m^2 \cdot K$ to $4\,W/m^2 \cdot K$. The effect on the surface temperature is shown below.



Maximum and minimum values of $T_s = 1433°C$ and $T_s = 290°C$ are associated with the smallest and largest velocities respectively, while the difference between the centerline and surface temperatures remains at $(T_o - T_s) \approx 2°C$.

For V = 5 m/s, the effect on T_s of varying the current over the range from 1 to 30A is shown below.



From a value of $T_s \approx 52°C$ at 1A, T_s increases to 1320°C at 30A. Over this range the temperature difference $(T_o - T_s)$ increases from approximately 0.01°C to 3°C.

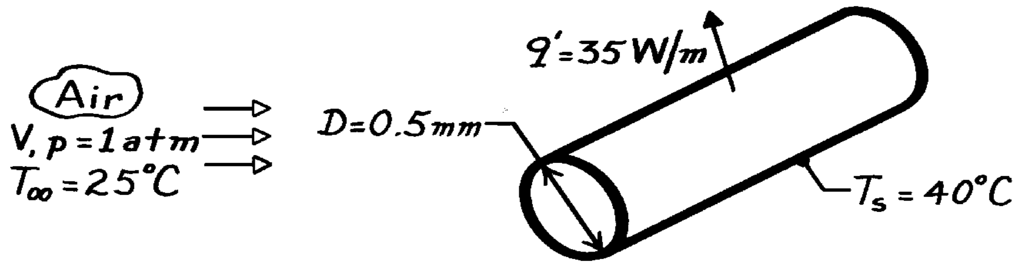
COMMENTS: (1) The radiation coefficient for the conditions of Part (a) is $h_r = 32\,W/m^2 \cdot K$, which is approximately 1/8 of the total coefficient $\overline{h}$. Hence, except for small values of V less than approximately 5 m/s, radiation is negligible compared with convection. (2) The small wire diameter and large thermal conductivity are responsible for maintaining nearly isothermal conditions within the wire. (3) The calculations of Part (b) were performed using the IHT solver with the function $T_f = T_{fluid\_avg}(T_s, T_{inf})$ used to account for the effect of temperature on the air properties.

PROBLEM 7.49

KNOWN: Temperature and heat dissipation in a wire of diameter D.

FIND: (a) Expression for flow velocity over wire, (b) Velocity of airstream for prescribed conditions.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform wire temperature, (3) Negligible radiation.

PROPERTIES: *Table A-4*, Air (T_∞ = 298 K, 1 atm): $\nu = 15.8 \times 10^{-6}$ m^2/s, k = 0.0262 W/m·K, Pr = 0.71; (T_s = 313 K, 1 atm): Pr = 0.705.

ANALYSIS: (a) The rate of heat transfer per unit cylinder length is

$$q' = (q/L) = \bar{h}(\pi D)\ (T_s - T_\infty)$$

where, from the Zhukauskas relation, with Pr ≈ Pr_s,

$$\bar{h} = \frac{k}{D} C\ Re_D^m\ Pr^n = \frac{k}{D}\ C\ \left(\frac{VD}{\nu}\right)^m Pr^n$$

Hence,

$$V = \left[\frac{q'}{(k/D) C\ Pr^n (\pi D)\ (T_s - T_\infty)}\right]^{1/m} \left(\frac{\nu}{D}\right). \qquad <$$

(b) Assuming ($10^3 < Re_D < 2 \times 10^5$), C = 0.26, m = 0.6 from Table 7.3. Hence,

$$V = \left[\frac{35\ \text{W/m}}{0.0262\ \text{W/m}\cdot\text{K} \times 0.26 (0.71)^{0.37}\ \pi\ (40 - 25)^\circ\ \text{C}}\right]^{1/0.6} \left(\frac{15.8\times10^{-6}\text{m}^2/\text{s}}{5\times10^{-4}\text{m}}\right)$$

$$V = 97\ \text{m/s}. \qquad <$$

To verify the assumption of the Reynolds number range, calculate

$$Re_D = \frac{VD}{\nu} = \frac{97\ \text{m/s}\ \left(5\times10^{-4}\text{m}\right)}{15.8\times10^{-6}\text{m}^2/\text{s}} = 3074.$$

Hence the assumption was correct.

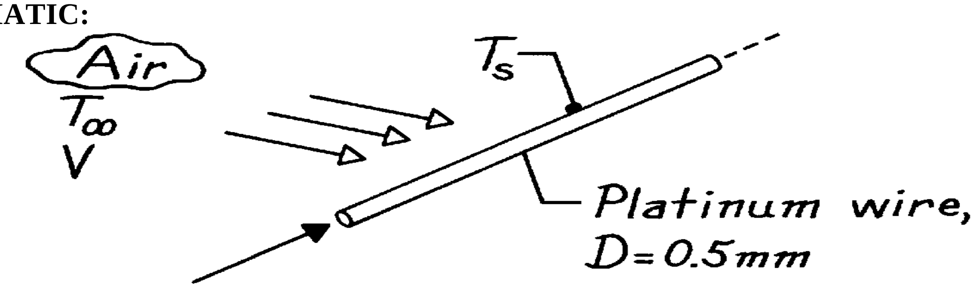
COMMENTS: The major uncertainty associated with using this method to determine V is that associated with use of the correlation for $\overline{Nu}_D$.

PROBLEM 7.50

KNOWN: Platinum wire maintained at a constant temperature in an airstream to be used for determining air velocity changes.

FIND: (a) Relationship between fractional changes in current to maintain constant wire temperature and fractional changes in air velocity and (b) Current required when air velocity is 10 m/s.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Cross-flow of air on wire with $40 < Re_D < 1000$, (3) Radiation effects negligible, (4) Wire is isothermal.

PROPERTIES: Platinum wire (given): Electrical resistivity, $\rho_e = 17.1 \times 10^{-5}$ Ohm·m; *Table A-4*, Air (T_∞ = 27°C = 300 K, 1 atm): $\nu = 15.89 \times 10^{-6}$ m^2/s, k = 0.0263 W/m·K, Pr = 0.707; (T_s = 77°C = 350 K, 1 atm): $Pr_s = 0.700$.

ANALYSIS: (a) From an energy balance on a unit length of the platinum wire,

$$q'_{elec} - q'_{conv} = I^2 R'_e - \bar{h} P \left(T_s - T_\infty\right) = 0 \tag{1}$$

where the electrical resistance per unit length is $R'_e = \rho_e / A_c$, $P = \pi D$, and $A_c = \pi D^2/4$. Hence,

$$I = \left[\frac{\bar{h} P A_c}{\rho_e}\left(T_s - T_\infty\right)\right]^{1/2} = \left[\frac{\pi^2 \bar{h} D^3}{4\rho_e}\left(T_s - T_\infty\right)\right]^{1/2} \tag{2}$$

For the range $40 < Re_D < 1000$, using the Zhukauskas correlation for cross-flow over a cylinder with C = 0.51 and m = 0.5,

$$\overline{Nu}_D = \frac{\bar{h} D}{k} = 0.51\, Re_D^{0.5}\, Pr^{0.37}\left(\frac{Pr}{Pr_s}\right)^{1/4} = 0.51\left(\frac{VD}{\nu}\right)^{0.5} Pr^{0.37}\left(\frac{Pr}{Pr_s}\right)^{1/4} \tag{3}$$

note that $\bar{h} \sim V^{0.5}$, which, when substituted into Eq. (2) yields

$$I \sim \bar{h}^{1/2} = \left(V^{0.5}\right)^{1/2} = V^{1/4}.$$

Differentiating the proportionality and dividing the result by the proportionality, it follows that

$$\frac{\Delta I}{I} \approx \frac{1}{4}\frac{\Delta V}{V}. \tag{4} \quad <$$

(b) For air at T_∞ = 27°C and V = 10 m/s, the current required to maintain the wire of D = 0.5 mm at T_s = 77°C follows from Eq. (2) with $\bar{h}$ evaluated by Eq. (3)

Continued …..

$$\bar{h} = \frac{0.0263\ \text{W/m}\cdot\text{K}}{0.0005\ \text{m}} \times 0.51 \left(\frac{10\ \text{m/s} \times 0.0005\ \text{m}}{15.89 \times 10^{-6}\ \text{m}^2/\text{s}} \right)^{0.5} (0.707)^{0.37} \left(\frac{0.707}{0.700} \right)^{1/4}$$

$$\bar{h} = 420\ \text{W/m}^2 \cdot \text{K}$$

where $Re_D = 315$. Hence the required current is

$$I = \left[\frac{p^2 \times 420\ \text{W/m}^2 \cdot \text{K} (0.0005\ \text{m})^3}{4 \times 17.1 \times 10^{-5}\ \Omega \cdot \text{m}} (77 - 27)\text{K} \right]^{1/2} = 195\ \text{mA}. \qquad (5)$$

COMMENTS: (1) To measure 1% fractional velocity change, a 0.25% fractional change in current must be measured according to Eq. (4). From Eq. (5), this implies that $\Delta I = 0.0025I = 0.0025 \times 195$ mA = 488 μA. An electronic circuit with such measurement sensitivity requires care in its design.

(2) Instruments built on this principle to measure air velocities are called *hot-wire anemometers*. Generally, the wire diameters are much smaller (3 to 30 μm vs 500 μm of this problem) in order to have faster response times.

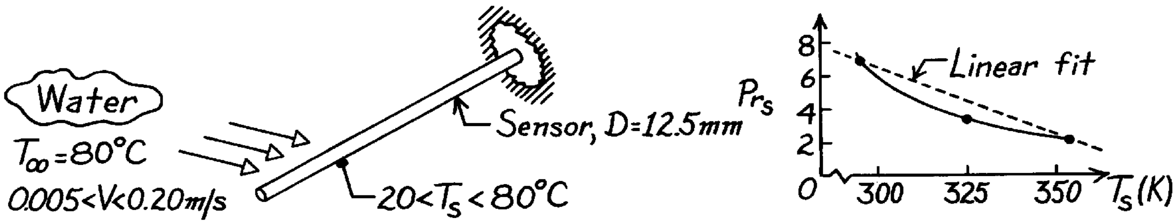
(3) What effect would the presence of radiation exchange between the wire and its surroundings have?

PROBLEM 7.51

KNOWN: Temperature sensor of 12.5 mm diameter experiences cross-flow of water at 80°C and velocity, $0.005 < V < 0.20$ m/s. Sensor temperature may vary over the range $20 < T_s < 80°C$.

FIND: Expression for convection heat transfer coefficient as a function of T_s and V.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Sensor-water flow approximates a cylinder in cross-flow, (3) Prandtl number varies linearly with temperature over the range of interest.

PROPERTIES: *Table A-6*, Sat. water (T_∞ = 80°C = 353 K): k = 0.670 W/m·K, $\nu = \mu v_f = 352 \times 10^{-6}$ N·s/m^2 $\times 1.029 \times 10^{-3}$ m^3/kg $= 3.621 \times 10^{-7}$ m^2/s; Pr_s values for $20 \le T_s \le 80°C$:

T (K)	293	300	325	350	353
Pr	7.00	5.83	3.42	2.29	2.20

ANALYSIS: Using the Zhukauskus correlation for the range $40 < Re_D < 4000$ with C = 0.51 and m = 0.5,

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = 0.51 Re_D^{0.5}\, Pr^{0.37}\left(\frac{Pr}{Pr_s}\right)^{1/4}.$$

with $Re_D = VD/\nu$, the thermophysical properties of interest are k, ν and Pr, which are evaluated at T_∞ = 80°C, and Pr_s which varies markedly with T_s for the range $20 < T_s < 80°C$. Assuming Pr_s to vary linearly with T_s and using the extreme values to find the relation,

$$Pr_s = 7.00 + \frac{(2.20-7.00)}{(353-293)\,K}(T_s - 293)\,K = 7.00 - 0.0800(T_s - 293)$$

where the units of T_s are [K]. Substituting numerical values, find

$$\bar{h}(T_s) = \frac{0.670\ W/m\cdot K}{0.0125\ m} 0.51\left(\frac{V \times 0.0125\ m}{3.621\times10^{-7}\ m^2/s}\right)^{0.5}(2.20)^{0.37}\left(\frac{2.20}{7.00 - 0.080(T_s - 293)}\right)^{1/4}$$

$$\bar{h}(T_s) = 6810 V^{0.5}\left[3.182 - 0.0364(T_s - 293)\right]^{-1/4}. \qquad <$$

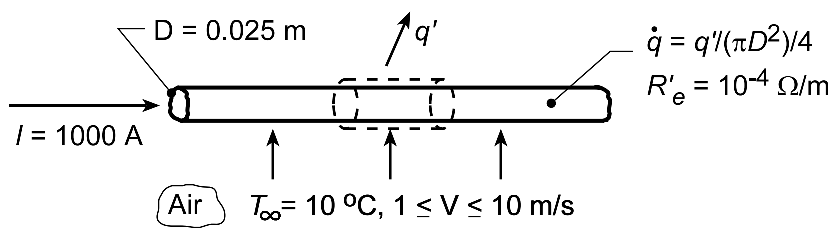
COMMENTS: (1) From the Pr_s vs T_s graph above, a linear fit is seen to be poor for this temperature range. However, because the Pr_s dependence is to the ¼ power, the discrepancy may be acceptable.

PROBLEM 7.52

KNOWN: Diameter, electrical resistance and current for a high tension line. Velocity and temperature of ambient air.

FIND: (a) Surface and (b) Centerline temperatures of the wire, (c) Effect of air velocity on surface temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) One-dimensional radial conduction.

PROPERTIES: *Table A.4*, Air ($T_f \approx 300$ K, 1 atm): $\nu = 15.89 \times 10^{-6}$ m^2/s, k = 0.0263 W/m·K, Pr = 0.707; *Table A.1*, Copper (T ≈ 300 K): k = 400 W/m·K.

ANALYSIS: (a) Applying conservation of energy to a control volume of unit length,

$$\dot{E}_g' = I^2 R_e' = q' = \bar{h}\pi D\left(T_s - T_\infty\right)$$

With

$$Re_D = \frac{VD}{\nu} = \frac{10\,\text{m/s}\left(0.025\,\text{m}\right)}{15.89\times 10^{-6}\,\text{m}^2/\text{s}} = 15,733$$

the Churchill and Bernstein correlation, yields

$$\overline{Nu}_D = 0.3 + \frac{0.62\,Re_D^{1/2}\,Pr^{1/3}}{\left[1+\left(0.4/Pr\right)^{2/3}\right]^{1/4}}\left[1+\left(\frac{Re_D}{282,000}\right)^{5/8}\right]^{4/5} = 69.0$$

Hence,

$$\bar{h} = \overline{Nu}_D\,\frac{k}{D} = 69.0\,\frac{0.0263\,\text{W/m}\cdot\text{K}}{0.025\,\text{m}} = 72.6\,\text{W}/\text{m}^2\cdot\text{K}$$

and

$$T_s = T_\infty + \frac{I^2 R_e'}{\bar{h}\pi D} = 10^\circ\text{C} + \frac{\left(1000\,\text{A}\right)^2 10^{-4}\,\Omega/\text{m}}{\left(72.6\,\text{W}/\text{m}^2\cdot\text{K}\right)\pi\left(0.025\,\text{m}\right)} = 10^\circ\text{C} + 17.6^\circ\text{C} = 27.6^\circ\text{C} \quad <$$

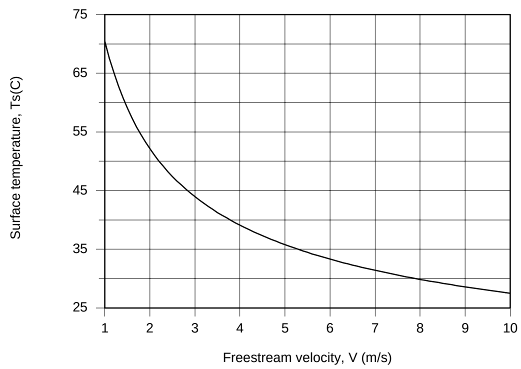
(b) With $\dot{q} = \dot{E}_g'\Big/\left(\pi D^2/4\right) = 4\left(1000\,\text{A}\right)^2\left(10^{-4}\,\Omega/\text{m}\right)\Big/\pi\left(0.025\,\text{m}\right)^2 = 2.04\times 10^5\,\text{W}/\text{m}^3$, Equation 3.53 yields

$$T(0) = \frac{\dot{q}r_o^2}{4k} + T_s = \frac{2.041\times 10^5\,\text{W}/\text{m}^3\left(0.0125\,\text{m}\right)^2}{1600\,\text{W/m}\cdot\text{K}} + 27.6^\circ\text{C} = 0.02^\circ\text{C} + 27.6^\circ\text{C} \approx 27.6^\circ\text{C} \quad <$$

Continued...

PROBLEM 7.52 (Cont.)

(c) The effect of V on the surface temperature was determined using the *Correlations* and *Properties* Tool Pads of IHT.



The effect is significant, with a surface temperature of $T_s \approx 70°C$ corresponding to V = 1 m/s. For velocities of 1 and 10 m/s, respectively, convection coefficients are 21.1 and 72.8 $W/m^2 \cdot K$ and film temperatures are 313.2 and 291.7 K.

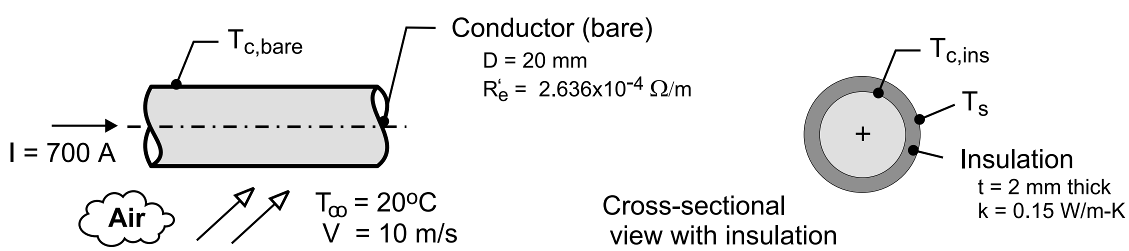
COMMENTS: The small values of $\dot{q}$ and r_o and the large value of k render the wire approximately isothermal.

PROBLEM 7.53

KNOWN: Aluminum transmission line with a diameter of 20 mm having an electrical resistance of $R' = 2.636\times10^{-4}$ ohm/m carrying a current of 700 A subjected to severe cross winds. To reduce potential fire hazard when adjacent lines make contact and spark, insulation is to be applied.

FIND: (a) The bare conductor temperature when the air temperature is 20°C and the line is subjected to cross flow with a velocity of 10 m/s; (b) The conductor temperature for the same conditions, but with an insulation covering of 2 mm thickness and thermal conductivity of 0.15 W/m·K; and (c) Plot the conductor temperatures of the bare and insulated conductors for wind velocities in the range of 2 to 20 m/s. Comment on the features of the curves and the effect that wind velocity has on the conductor operating temperatures.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform surface temperatures, (3) Negligible solar irradiation and radiation exchange, and (4) Constant properties.

PROPERTIES: *Table A-4*, Air ($T_f = (T_s + T_\infty)/2$, 1 atm): evaluated using the *IHT Properties* library with a *Correlation* function; see Comment 2.

ANALYSIS: (a) For the *bare* conductor the energy balance per unit length is

$$\dot{E}'_{in} - \dot{E}'_{out} + \dot{E}'_{gen} = 0$$

$$0 - q'_{cv} + \dot{q}\,A_c = 0 \tag{1}$$

where the crossectional area of the conductor is $A_c = \pi D^2/4$ and the generation rate is

$$\dot{q} = I^2 R'_e / A_c = (700\,A)^2 \times 2.636\times10^{-4}\,\Omega/m \Big/ \left(\pi\,(0.020\,m)^2/4\right) \tag{2}$$

$$\dot{q} = 4.111\times10^5\ W/m^3$$

The convection rate equation can be expressed as

$$q'_{cv} = \left(T_{c,bare} - T_\infty\right)/R'_t \qquad\qquad R'_t = 1/\left(\bar{h}_D \times \pi D\right) \tag{3,4}$$

and the convection coefficient is estimated using the Churchill-Bernstein correlation, Eq. 7.57, with $Re_D = VD/\nu$,

$$\overline{Nu}_L = \frac{\bar{h}_D D}{k} = 0.3 + \frac{0.62\,Re_D^{1/2}\,Pr^{1/3}}{\left[1+(0.4/Pr)^{2/3}\right]^{1/4}}\left[1+\left(\frac{Re_D}{282{,}000}\right)^{5/8}\right]^{4/5} \tag{4}$$

(b) For the conductor *with insulation* thickness t = 2 mm, the energy balance per unit length is

$$\dot{E}'_{in} - \dot{E}'_{out} + \dot{E}'_{gen} = 0$$

$$0 - \left(T_{c,ins} - T_\infty\right)/R'_t + I^2 R'_e / A_c = 0 \tag{5}$$

Continued …..

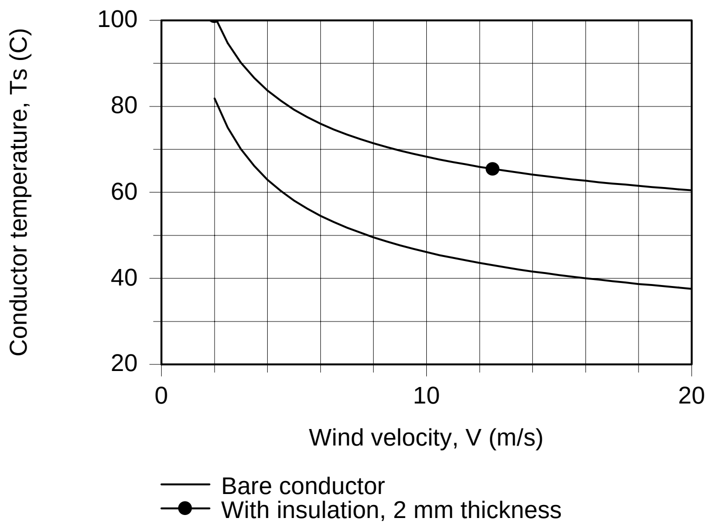
where R'_t is the sum of the insulation conduction and convection process thermal resistances,

$$R'_t = \ell n\left[\left(D+2t\right)/D\right]/\left(2\pi k\right)+1/\left[\bar{h}_{D+2t}\pi\left(0+2t\right)\right]$$
(6)

The results of the analysis using *IHT* are tabulated below.

Condition	V (m/s)	d (mm)	Re_d	$\overline{Nu}_d$	$\bar{h}_d$ ($W/m^2\cdot K$)	R'_t ($m\cdot K/W$)	T_c (°C)
bare	10	20	1.214×10^4	59.6	79.6	0.1998	45.8
insulated	10	24	1.468×10^4	66.3	73.6	0.3736	68.3

(c) Using the *IHT* code with the foregoing relations, the conductor temperatures $T_{c,base}$ and $T_{c,ins}$ for the bare and insulated conditions are calculated and plotted for the wind velocity range of 2 to 20 m/s.



COMMENTS: (1) The effect of the 2-mm thickness insulation is to increase the conductor operating temperature by (68.3 – 46.1)°C = 22°C. While we didn't account for an increase in the electrical resistivity with increasing temperature, the adverse effect is to increase the I^2R loss, which represents a loss of revenue to the power provider. From the graph, note that the conductor temperature increases markedly with decreasing wind velocity, and the effect of insulation is still around +20°C.

(2) Because of the tediousness of hand calculations required in using the convection correlation without fore-knowledge of T_f at which to evaluate properties, we used the *IHT Correlation* function treating T_f as one of the unknowns in the system of equations. Salient portions of the *IHT* code and property values are provided below.

Continued …..

PROBLEM 7.53 (Cont.)

```
// Forced convection, cross flow, cylinder
NuDbar = NuD_bar_EF_CY(ReD,Pr)              // Eq 7.57
NuDbar = hDbar * Do / k
ReD = V * Do / nu                           // Outer diameter; bare or with insulation

// Evaluate properties at the film temperature, Tf.
Tf =  Tfluid_avg (Tinf,Ts)             // Ts is the outer surface temperature
/*  Correlation description: External cross flow (EF) over cylinder (CY), average coefficient,
ReD*Pr>0.2, Churchill-Bernstein correlation, Eq 7.57. See Table 7.9.  */

// Air property functions : From Table A.4
// Units: T(K); 1 atm pressure
nu = nu_T("Air",Tf)                    // Kinematic viscosity, m^2/s
k = k_T("Air",Tf)                      // Thermal conductivity, W/m·K
Pr = Pr_T("Air",Tf)                    // Prandtl number
```

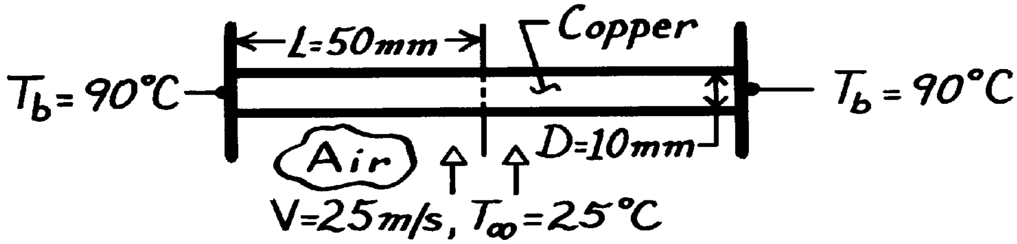
(3) Is the temperature gradient within the conductor significant?

PROBLEM 7.54

KNOWN: Diameter and length of a copper rod, with fixed end temperatures, inserted in an airstream of prescribed velocity and temperature.

FIND: (a) Midplane temperature of rod, (b) Rate of heat transfer from the rod.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in rod, (3) Negligible contact resistance, (4) Negligible radiation, (5) Constant properties.

PROPERTIES: *Table A-1*, Copper ($T \approx 80°C = 353$ K): k = 398 W/m·K; *Table A-4*, Air (T_∞ = 25°C ≈ 300K, 1 atm): $\nu = 15.8 \times 10^{-6}$ m^2/s, k = 0.0263 W/m·K, Pr = 0.707; *Table A-4*, Air ($T_s \approx$ 80°C ≈ 350K, 1 atm): $Pr_s = 0.700$.

ANALYSIS: (a) For case B of Table 3.4, $\frac{q}{q_b} = \frac{\cosh m(L-x)}{\cosh(mL)} = \frac{T - T_\infty}{T_b - T_\infty}$ where

$m = \left(\bar{h}P/kA_c\right)^{1/2} = \left(4\bar{h}/kD\right)^{1/2}$. Using the Zhukauskas correlation with n = 0.37,

$$\overline{Nu}_D = C\, Re_D^m\, Pr^n\, \left(Pr/Pr_s\right)^{1/4}$$

$$Re_D = \frac{VD}{n} = \frac{25 \text{ m/s}\,(0.01 \text{ m})}{15.8\times10^{-6} \text{ m}^2/\text{s}} = 15{,}823$$

and C = 0.26, m = 0.6 from Table 7-4. Hence

$$\overline{Nu}_D = 0.26(15{,}823)^{0.6}\,(0.707)^{0.37}\,(0.707/0.700)^{1/4} = 75.8$$

$$\bar{h} = \frac{k}{D}\overline{Nu}_D = \frac{0.0263 \text{ W/m}\cdot\text{K}}{0.01 \text{ m}}(75.8) = 199 \text{ W/m}^2\cdot\text{K}$$

$$m = \left(\frac{4\times199 \text{ W/m}^2\cdot\text{K}}{398 \text{ W/m}\cdot\text{K}\times0.01 \text{ m}}\right)^{1/2} = 14.2 \text{ m}^{-1}.$$

Hence,

$$\frac{T(L)-T_\infty}{T_b-T_\infty} = \frac{\cosh(0)}{\cosh\left(14.2 \text{ m}^{-1}\times0.05 \text{ m}\right)} = \frac{1}{1.26} = 0.79$$

$$T(L) = 25^\circ\text{C} + 0.79(90-25) = 76.6^\circ\text{C}.$$ <

(b) From Eq. 3.76, $q = 2q_f = 2M \tanh mL$,

$$M = \left(\bar{h}PkA_c\right)^{1/2} q_b = \left[199\frac{\text{W}}{\text{m}^2\cdot\text{K}}(p\times0.01 \text{ m})\left(398\frac{\text{W}}{\text{m}\cdot\text{K}}\right)\frac{p}{4}(0.01 \text{ m})^2\right]^{1/2} 65^\circ\text{C}$$

$$M = 28.7 \text{ W} \qquad q = 2(28.7 \text{ W}) \tanh\left(14.2 \text{ m}^{-1}\times0.05 \text{ m}\right) = 35 \text{ W}.$$ <

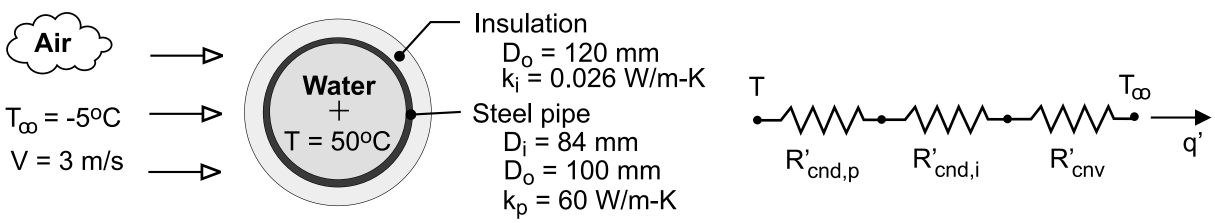
COMMENTS: Note adiabatic condition associated with symmetry about midplane.

PROBLEM 7.55

KNOWN: Diameter, thickness and thermal conductivity of steel pipe. Temperature of water flow in pipe. Temperature and velocity of air in cross flow over pipe. Cost of producing hot water.

FIND: (a) Cost of daily heat loss from an uninsulated pipe, (b) Savings associated with insulating the pipe.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) Negligible convection resistance for water flow, (3) Negligible contact resistance between insulation and pipe, (4) Negligible radiation.

PROPERTIES: *Table A-4*, air $\left(p = 1\,\text{atm},\ T_f \approx 300\text{K}\right)$: $k_a = 0.0263\,\text{W/m}\cdot\text{K}$, $\nu = 15.89\times10^{-6}\,\text{m}^2/\text{s}$, $\text{Pr} = 0.707$.

ANALYSIS: (a) With $\text{Re}_D = VD_o/\nu = 3\,\text{m/s}\times0.1\text{m}/15.89\times10^{-6}\,\text{m}^2/\text{s} = 18,880$, application of the Churchill-Bernstein correlation yields

$$\overline{Nu}_D = 0.3 + \frac{0.62(18,800)^{1/2}(0.707)^{1/3}}{\left[1+(0.4/0.707)^{2/3}\right]^{1/4}}\left[1+\left(\frac{18,880}{282,000}\right)^{5/8}\right]^{4/5} = 76.6$$

$$\bar{h} = \frac{k_a}{D_o}\overline{NU}_D = \frac{0.0263\,\text{W/m}\cdot\text{K}}{0.1\text{m}}76.6 = 20.1\,\text{W/m}^2\cdot\text{K}$$

Without the insulation, the total thermal resistance and heat loss per length of pipe are then

$$R'_{tot(wo)} = \frac{\ln(D_o/D_i)}{2\pi k_p} + \frac{1}{\pi D_o \bar{h}} = \frac{\ln(100/84)}{2\pi\times60\,\text{W/m}\cdot\text{K}} + \frac{1}{\pi(0.1\text{m})20.1\,\text{W/m}^2\cdot\text{K}}$$

$$= \left(4.63\times10^{-4} + 0.158\right)\text{m}\cdot\text{K/W} = 0.159\,\text{m}\cdot\text{K/W}$$

$$q'_{wo} = \frac{T - T_\infty}{R'_{tot(wo)}} = \frac{55°\text{C}}{0.159\,\text{m}\cdot\text{K/W}} = 346\,\text{W/m} = 0.346\,\text{kW/m}$$

The corresponding daily energy loss is

$$Q'_{wo} = 0.346\,\text{kW/m}\times24\,\text{h/d} = 8.3\,\text{kW}\cdot\text{h/m}\cdot\text{d}$$

and the associated cost is

$$C'_{wo} = (8.3\,\text{kW}\cdot\text{h/m}\cdot\text{d})(\$0.05/\text{kW}\cdot\text{h}) = \$0.415/\text{m}\cdot\text{d}$$ <

(b) The conduction resistance of the insulation is

Continued …..

$$R'_{cnd} = \frac{\ln(D_o / D_i)}{2\pi\, k_i} = \frac{\ln(120/100)}{2\pi(0.026\, W/m \cdot K)} = 1.116\, m \cdot K/W$$

Using the Churchill-Bernstein correlation with an outside diameter of $D_o = 0.12m$, $Re_D = 22,660$, $\overline{Nu}_D = 83.9$ and $\bar{h} = 18.4\, W/m^2 \cdot K$. The convection resistance is then

$$R'_{cnv} = \frac{1}{\pi D_o \bar{h}} = \frac{1}{\pi(0.12m)18.4\, W/m^2 \cdot K} = 0.144\, m \cdot K/W$$

and the total resistance is

$$R'_{tot(w)} = \left(4.63\times10^{-4} + 1.116 + 0.144\right) m \cdot K/W = 1.261\, m \cdot K/W$$

The heat loss and cost are then

$$q'_w = \frac{T - T_\infty}{R'_{tot(w)}} = \frac{55°C}{1.261\, m \cdot K/W} = 43.6\, W/m = 0.0436\, kW/m$$

$$C'_w = 0.0436\, kW/m \times 24h/d \times \$0.05/kW \cdot h = \$0.052/m \cdot d$$

The daily savings is then

$$S' = C'_{wo} - C'_w = \$0.363/m \cdot d$$ <

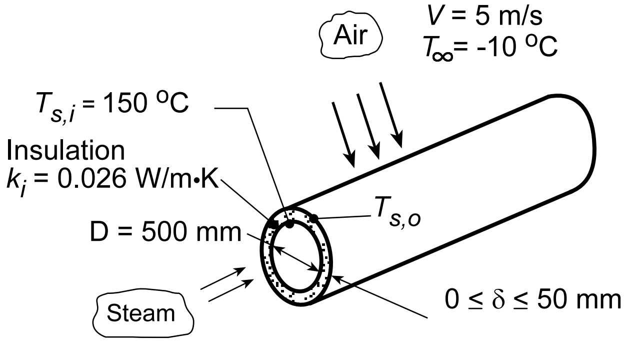
COMMENTS: (1) The savings are significant, and the pipe should be insulated. (2) Assuming a negligible temperature drop across the pipe wall, a pipe emissivity of $\varepsilon_p = 0.6$ and surroundings at $T_{sur} = 268K$, the radiation coefficient associated with the uninsulated pipe is $h_r = \varepsilon\sigma(T + T_{sur})\left(T^2 + T_{sur}^2\right) = 0.6\times5.67\times10^{-8} W/m^2 \cdot K^4 (591K)\left(323^2 + 268^2\right)K^2 = 3.5\, W/m^2 \cdot K$. Accordingly, radiation increases the heat loss estimate of Part (a) by approximately 17%.

PROBLEM 7.56

KNOWN: Diameter and surface temperature of an uninsulated steam pipe. Velocity and temperature of air in cross flow.

FIND: (a) Heat loss per unit length, (b) Effect of insulation thickness on heat loss.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform surface temperature, (3) Negligible radiation.

PROPERTIES: *Table A.4*, Air ($T_f \approx 350$ K, 1 atm): $\nu = 20.9 \times 10^{-6}$ m^2/s, k = 0.030 W/m·K, Pr = 0.70.

ANALYSIS: (a) Without the insulation, the heat loss per unit length is

$$q' = \bar{h}\pi D\left(T_{s,i} - T_\infty\right)$$

where $\bar{h}$ may be obtained from the Churchill-Bernstein relation. With

$$Re_D = \frac{VD}{\nu} = \frac{5\,\text{m/s} \times 0.5\,\text{m}}{20.9 \times 10^{-6}\,\text{m}^2/\text{s}} = 1.196 \times 10^5$$

$$\overline{Nu}_D = 0.3 + \frac{0.62\,Re_D^{1/2}\,Pr^{1/3}}{\left[1 + (0.4/Pr)^{2/3}\right]^{1/4}}\left[1 + \left(\frac{Re_D}{282{,}000}\right)^{5/8}\right]^{4/5} = 242$$

$$\bar{h} = \overline{Nu}_D\,\frac{k}{D} = 242\,\frac{0.030\,\text{W/m}\cdot\text{K}}{0.5\,\text{m}} = 14.5\,\text{W/m}^2\cdot\text{K}$$

The heat rate is then

$$q' = 14.5\,\text{W/m}^2\cdot\text{K}\,\pi\,(0.5\,\text{m})\left(150 - (-10)\right)^\circ\text{C} = 3644\,\text{W/m}.$$ <

(b) With the insulation, the heat loss may be expressed as

$$q' = U_i \pi D\left(T_{s,i} - T_\infty\right)$$

where, from Eq. 3.31,

$$U_i = \left[\frac{(D/2)}{k_i}\ln \bar{r} + \frac{1}{\bar{r}\bar{h}}\right]^{-1}$$

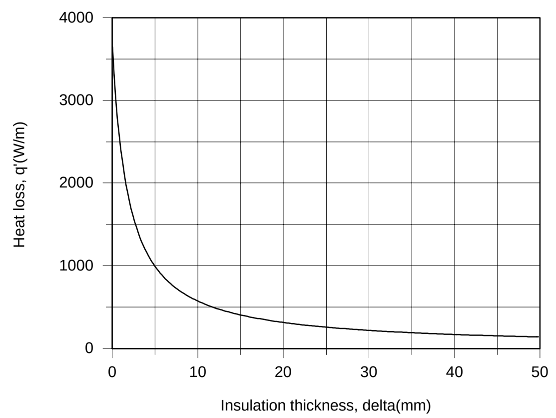
and $\bar{r} \equiv \left(D/2 + \delta\right)/\left(D/2\right)$. The outer diameter, $D_o = D + 2\delta$, as well as the film temperature, $T_f = (T_{s,o} + T_\infty)/2$, must now be used to evaluate the convection coefficient, where

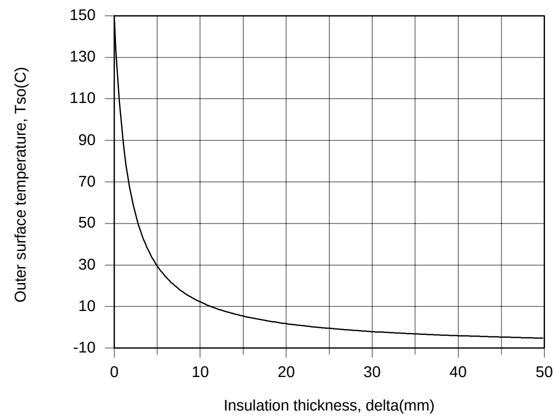
Continued...

PROBLEM 7.56 (Cont.)

$$\frac{T_{s,i} - T_{s,o}}{T_{s,i} - T_{\infty}} = \frac{R'_{cond}}{R'_{tot}} = \frac{(\ln \bar{r})/k_i}{(\ln \bar{r})/k_i + 1/(D/2)\bar{r}\bar{h}}$$

Using the IHT *Correlations* and *Properties* Tool Pads to evaluate $\bar{h}$, the following results were obtained.





The insulation is extremely effective, with a thickness of only 10 mm yielding a 7-fold reduction in heat loss and decreasing the outer surface temperature from 150 to 10°C. For δ = 50 mm, U_i = 0.56 W/m^2·K, q' = 140 W/m and $T_{s,o}$ = -5.2°C.

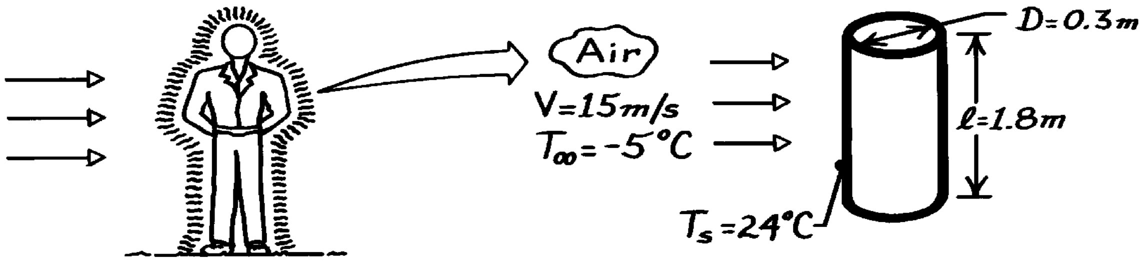
COMMENTS: The dominant contribution to the total thermal resistance is made by the insulation.

PROBLEM 7.57

KNOWN: Person, approximated as a cylinder, is subjected to prescribed convection conditions.

FIND: Heat rate from body for prescribed temperatures.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Person can be approximated by cylindrical form having uniform surface temperature, (3) Negligible heat loss from cylinder top and bottom surfaces, (4) Negligible radiation effects.

PROPERTIES: *Table A-4*, Air (T_∞ = 268 K, 1 atm): $\nu = 13.04 \times 10^{-6}$ m^2/s, $k = 23.74 \times 10^{-3}$ W/m·K, Pr = 0.725; (T_s = 297 K, 1 atm): Pr = 0.707.

ANALYSIS: The heat transfer rate from the cylinder, approximating the person, is given as

$$q = \bar{h} A_s \left(T_s - T_\infty \right)$$

where $A_s = p D \ell$ and $\bar{h}$ must be estimated from a correlation appropriate to cross-flow over a cylinder. Use the Zhukauskas relation,

$$\overline{Nu}_D = \frac{\bar{h} D}{k} = C\, Re_D^m\, Pr^n \left(Pr/Pr_s \right)^{1/4}$$

and calculate the Reynold's number,

$$Re_D = \frac{VD}{n} = \frac{15 m/s \times 0.3\ m}{13.04 \times 10^{-6}\ m^2/s} = 345{,}092.$$

From Table 7-4, find C = 0.076 and m = 0.7. Since Pr < 10, n = 0.37, giving

$$\overline{Nu}_D = 0.076\ (345{,}092)^{0.7}\ 0.725^{0.37} \left(\frac{0.725}{0.707} \right)^{1/4} = 511$$

$$\bar{h} = \overline{Nu}_D \frac{k}{D} = \frac{511 \times 23.74 \times 10^{-3} W/m \cdot K}{0.3\ m} = 40.4\ W/m^2 \cdot K.$$

The heat transfer rate is

$$q = 40.4 W/m^2 \cdot K \left(p \times 0.3\ m \times 1.8\ m \right)\ \left(24 - (-5) \right)^\circ C = 1988\ W. \qquad <$$

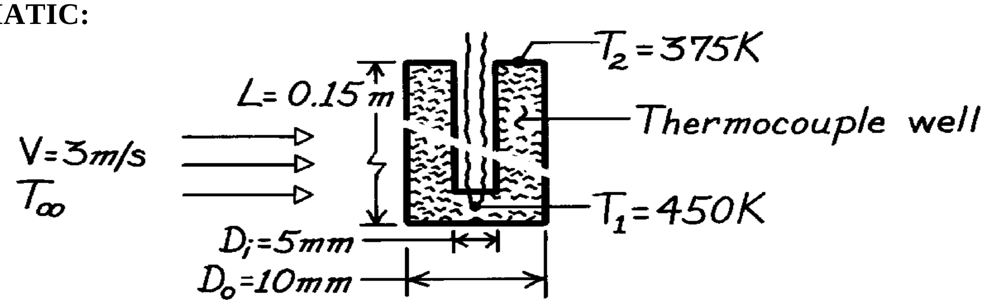
COMMENTS: Note the temperatures at which properties are evaluated for the Zhukauskas correlation.

PROBLEM 7.58

KNOWN: Dimensions and thermal conductivity of a thermocouple well. Temperatures at well tip and base. Air velocity.

FIND: Air temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) One-dimensional conduction along well, (4) Uniform convection coefficient, (5) Negligible radiation.

PROPERTIES: Steel (given): k = 35 W/m·K; Air (given): $\rho = 0.774$ kg/m^3, $\mu = 251 \times 10^{-7}$ N·s/m^2, k = 0.0373 W/m·K, Pr = 0.686.

ANALYSIS: Applying Equation 3.70 at the well tip (x = L), where $T = T_1$,

$$\frac{T_1 - T_\infty}{T_2 - T_\infty} = \left[\cosh mL + \left(\bar{h}/mk\right)\sinh mL\right]^{-1}$$

$$m = \left(\bar{h}P/kA_c\right)^{1/2} \qquad P = \pi D_o = \pi\,(0.010\text{ m}) = 0.0314\text{ m}$$

$$A_c = (\pi/4)\left(D_o^2 - D_i^2\right) = (\pi/4)\left(0.010^2 - 0.005^2\right)\text{m}^2 = 5.89\times10^{-5}\text{ m}^2.$$

With $\quad Re_D = \dfrac{\rho VD}{\mu} = \dfrac{0.744\text{ kg/m}^3(3\text{m/s})\,0.01\text{ m}}{251\times10^{-7}\text{ N}\cdot\text{s/m}^2} = 925$

C = 0.51, m = 0.5, n = 0.37 and the Zhukauskas correlation yields

$$\overline{Nu}_D = 0.51 Re_D^{0.5} Pr^{0.37}\left(Pr/Pr_s\right)^{1/4} \approx 0.51(925)^{0.5}(0.686)^{0.37} \times 1 = 13.5$$

$$\bar{h} = \overline{Nu}_D \frac{k}{D_o} = 13.5\frac{0.0373\text{ W/m}\cdot\text{K}}{0.01\text{ m}} = 50.4\text{ W/m}^2\cdot\text{K}.$$

Hence

$$m = \left[\frac{\left(50.4\text{ W/m}^2\cdot\text{K}\right)0.0314\text{ m}}{(35\text{ W/m}\cdot\text{K})\,5.89\times10^{-5}\text{ m}^2}\right]^{1/2} = 27.7\text{ m}^{-1} \qquad mL = \left(27.7\text{ m}^{-1}\right)0.15\text{ m} = 4.15.$$

With

$$\left(\bar{h}/mk\right) = \left(50.4\text{ W/m}^2\cdot\text{K}\right)/\left(27.7\text{ m}^{-1}\right)(35\text{ W/m}\cdot\text{K}) = 0.0519$$

find $\quad \dfrac{T_1 - T_\infty}{T_2 - T_\infty} = \left[32.62 + (0.0519)\,32.61\right]^{-1} = 0.0291 \qquad T_\infty = 452.2\text{ K.}$ <

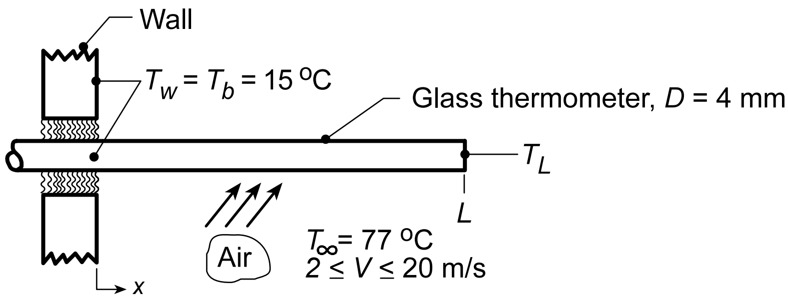
COMMENTS: Heat conduction along the wall to the base at 375 K is balanced by convection from the air.

PROBLEM 7.59

KNOWN: Mercury-in-glass thermometer mounted on duct wall used to measure air temperature.

FIND: (a) Relationship for the immersion error, $\Delta T_i = T(L) - T_\infty$ as a function of air velocity, thermometer diameter and length, (b) Length of insertion if ΔT_i is not to exceed 0.25°C when the air velocity is 10 m/s, (c) For the length of part (b), calculate and plot ΔT_i as a function of air velocity for 2 to 20 m/s, and (d) For a given insertion length, will ΔT_i increase or decrease with thermometer diameter increase; is ΔT_i more sensitive to diameter or velocity changes?

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Thermometer approximates a one-dimensional (glass) fin with an *adiabatic* tip, (3) Convection coefficient is uniform over length of thermometer.

PROPERTIES: *Table A.3*, Glass (300 K): k_g = 1.4 W/m·K; *Table A.4*, Air (T_f = (15 + 77)°C/2 ≈ 320 K, 1 atm): k = 0.0278 W/m·K, $\nu = 17.90 \times 10^{-6}$ m/s^2, Pr = 0.704.

ANALYSIS: (a) From the analysis of a one-dimensional fin, see Table 3.4,

$$\frac{T_L - T_\infty}{T_b - T_\infty} = \frac{1}{\cosh(mL)} \qquad\qquad m^2 = \frac{\bar{h}P}{k_g A_c} = \frac{4\bar{h}}{k_g D} \tag{1}$$

where $P = \pi D$ and $A_c = \pi D^2/4$. Hence, the immersion error is

$$\Delta T_i = T(L) - T_\infty = (T_b - T_\infty)/\cosh(mL). \tag{2}$$

Using the Hilpert correlation for the circular cylinder in cross flow,

$$\bar{h} = \frac{k}{D} C\, Re_D^m\, Pr^{1/3} = \frac{k}{D} C \left(\frac{VD}{\nu}\right)^m Pr^{1/3} = \frac{k\,Pr^{1/3}}{\nu^m} \cdot C \cdot V^m \cdot D^{m-1} \tag{3}$$

$$\bar{h} = N \cdot V^m \cdot D^{m-1} \qquad \text{where} \qquad N = \frac{k\,Pr^{1/3}}{\nu^m} C \tag{4,5}$$

Substituting into Eq. (2), the immersion error is

$$\Delta T_i(V, D, L) = (T_b - T_\infty)/\cosh\left\{\left[(4/k_g) N \cdot V^m \cdot D^{m-2}\right]^{1/2} L\right\} \tag{6}$$ <

where k_g is the thermal conductivity of the glass thermometer.

(b) When the air velocity is 10 m/s, find

$$Re_D = \frac{VD}{\nu} = \frac{10\,m/s \times 0.004\,m}{17.9 \times 10^{-6}\,m/s^2} = 2235$$

Continued...

with C = 0.683 and m = 0.466 from Table 7.2 for the range $40 < Re_D < 4000$. From Eqs. (5) and (6),

$$N = \frac{0.0278\,\mathrm{W/m\cdot K}\,(0.704)^{1/3}}{\left(17.9\times10^{-6}\,\mathrm{m/s^2}\right)^{0.466}}\times 0.683 = 2.753$$

$$\Delta T_i = (15-77)\,\mathrm{K}\Big/\cosh\left\{\left[\frac{4}{1.4\,\mathrm{W/m\cdot K}}\times 2.753\,(10\,\mathrm{m/s})^{0.466}\,(0.004\,\mathrm{m})^{0.466-2}\right]^{1/2} L\right\}$$

and when $\Delta T_i = -0.25°C$, find

$$L = 18.7\ \mathrm{mm}$$ <

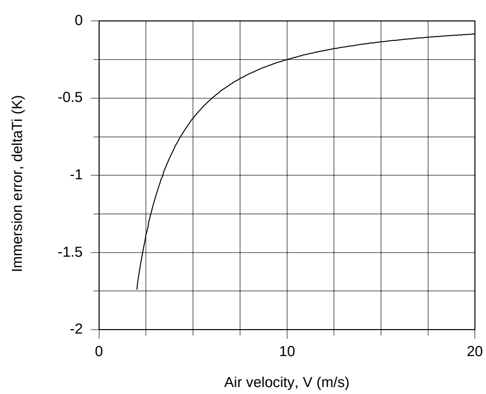
(c) For the air velocity range 2 to 20 m/s, find $447 \le Re_D \le 4470$ for which the previous values of C and m of the Hilpert correlation are appropriate. Hence, the immersion error for an insertion length of L = 18.7 mm, part (b), find

$$\Delta T_i = (15-77)\,\mathrm{K}\Big/\cosh\left\{\left[\frac{4}{1.4\,\mathrm{W/m\cdot K}}\times 2.753\times V^{0.466}\,(0.004\,\mathrm{m})-1.534\right]^{1/2} 0.0187\right\}$$

$$\Delta T_i = -62°\,\mathrm{C}\Big/\cosh\left(3.629V^{0.233}\right)$$ <

where the units of V are [m/s]. Entering the above equation into the IHT Workspace the plot shown below was generated.

V(m/s)	ΔT_i (°C)
2	-1.74
5	-0.63
10	-0.25
15	-0.14
20	-0.08



(d) For a given insertion length, the immersion error will *increase* if the diameter of the thermometer were *increased*. This follows from Eq. (6) written as

$$\Delta T_i \sim 1\Big/\cosh\left(A\cdot D^{(m-2)/2}\right) \tag{7}$$

where A is a constant depending on variables other than D. For a given insertion length and air velocity, from Eq. (6)

$$\Delta T_i \sim 1\Big/\cosh\left(B\cdot V^{m/2}\right) \tag{8}$$

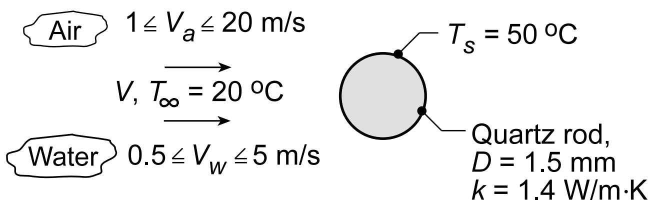
where B is a constant. From Eq. (7) we see ΔT_i relates to change in *diameter* as $D^{-0.767}$ and to change in *velocity* as $V^{0.233}$. That is, to reduce the immersion error decrease D and increase V (both cause $\bar{h}$ to increase!). Based upon the exponents of each parameter, however, diameter change is the more influential.

PROBLEM 7.60

KNOWN: Hot film sensor on a quartz rod maintained at $T_s = 50°C$.

FIND: (a) Compute and plot the convection coefficient as a function of velocity for water, $0.5 \le V_w \le 5$ m/s, and air, $1 \le V_a \le 20$ m/s with $T_\infty = 20°C$ and (b) Suitability of using the hot film sensor for the two fluids based upon Biot number considerations.

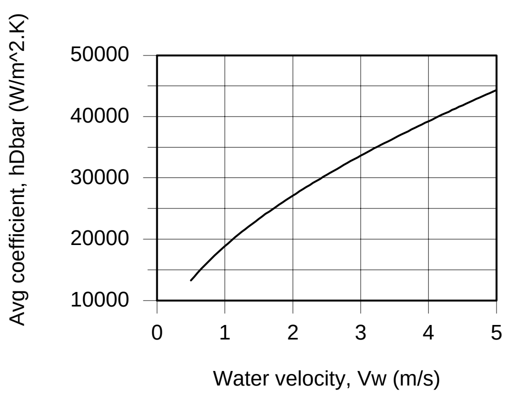
SCHEMATIC:

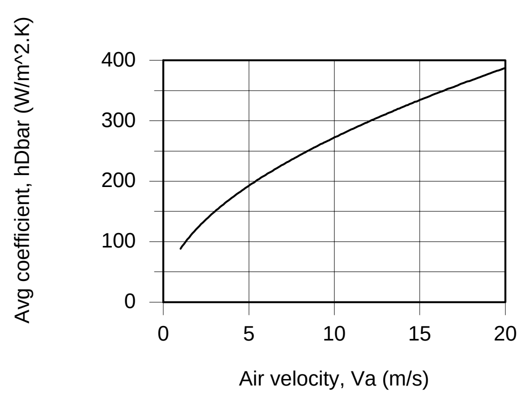


ASSUMPTIONS: (1) Cross-flow over a smooth cylinder, (2) Steady-state conditions, (3) Uniform surface temperature.

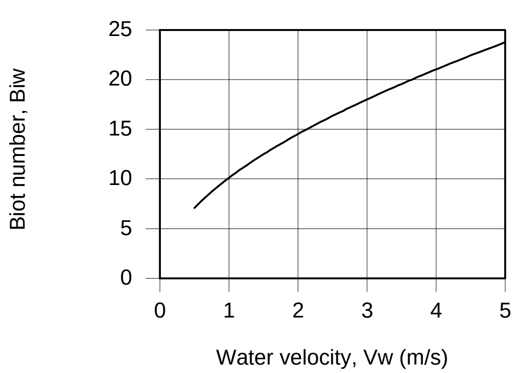
PROPERTIES: *Table A.6*, Water ($T_f = 308$ K, sat liquid); *Table A.4*, Air ($T_f = 308$ K, 1 atm).

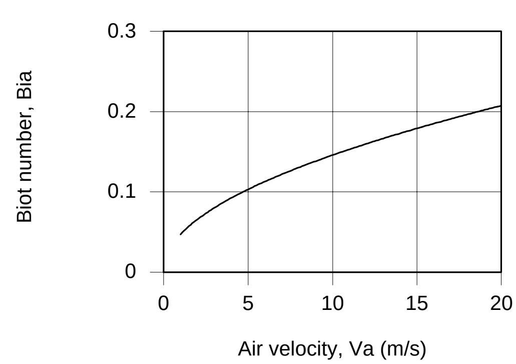
ANALYSIS: (a) Using the *IHT Tool*, *Correlations*, *Cylinder*, along with the *Properties Tool* for *Air* and *Water*, results were obtained for the convection coefficients as a function of velocity.





(b) The Biot number, hD/2k, is the ratio of the internal to external thermal resistances. When Bi >> 1, the thin film is thermally coupled well to the fluid. When Bi ≤ 1, significant power from the heater is dissipated axially by conduction in the rod. The Biot numbers for the fluids as a function of velocity are shown below.





We conclude that the sensor is well suited for use with water, but not so for use with air.

Continued...

COMMENTS: A copy of the IHT workspace developed to generate the above plots is shown below.

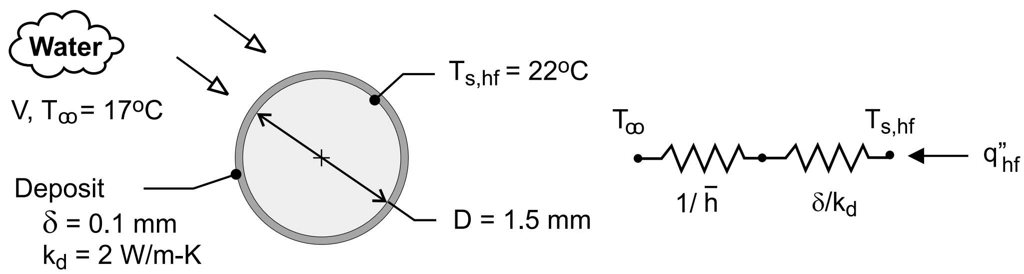
```
// Problem 7.61

// Correlation Tool:  External Flow, Cylinder
/*  Correlation description: External cross flow (EF) over cylinder (CY), average coefficient, ReD*Pr>0.2,
Churchill-Bernstein correlation, Eq 7.57. See Table 7.9.  */
// Air flow  (a)
NuDbara = NuD_bar_EF_CY(ReDa,Pra)               // Eq 7.57
NuDbara = hDbara * D / ka
ReDa = Va * D / nua
// Evaluate properties at the film temperature, Tfa.
Tf = (Tinf + Ts) / 2
Bia = hDbara * D / (2 * k)                     // Biot number
// Properties Tool: Air
// Air property functions : From Table A.4
// Units: T(K); 1 atm pressure
nua = nu_T("Air",Tf)                           // Kinematic viscoty, m^2/s
ka = k_T("Air",Tf)                             // Thermal conductivity, W/m·K
Pra = Pr_T("Air",Tf)                           // Prandtl number

// Water flow (w)
NuDbarw = NuD_bar_EF_CY(ReDw,Prw)              // Eq 7.57
NuDbarw = hDbarw * D / kw
ReDw = Vw * D / nuw
// Evaluate properties at the film temperature, Tfw.
//Tfw = (Tinfw + Tsw) / 2
Biw = hDbarw * D / (2 * k )                    // Biot number
// Properties Tool: Water
// Water property functions :T dependence, From Table A.6
// Units: T(K), p(bars); x = quality (0=sat liquid or 1=sat vapor)
xf = 0
nuw = nu_Tx("Water",Tf,xf)                     // Kinematic viscosity, m^2/s
kw = k_Tx("Water",Tf,xf)                       // Thermal conductivity, W/m·K
Prw = Pr_Tx("Water",Tf,xf)                     // Prandtl number

// Assigned Variables:
Va = 1                                         // Air velocity, m/s; range 1 to 20 m/s
Vw = 0.5                                       // Water velocity, m/s; range 0.5 to 5 m/s
k = 1.4                                        // Thermal conductivity, W/m.K; quartz rod
D = 0.0015                                     // Diameter, m
Ts = 30 + 273                                  // Surface temperature, K
Tinf = 20 + 273                                // Fluid temperature, K

/* Solve, Explore and Graph:  After solving, separate Explore sweeps for 1 <= Va <= 20 and
0.5 <= Vw <= 5 m/s were performed saving results in different Data Sets.  Four separate
plot windows were generated.  */
```

PROBLEM 7.61

KNOWN: Diameter, temperature and heat flux of a hot-film sensor. Fluid temperature. Thickness and thermal conductivity of deposit.

FIND: (a) Fluid velocity, (b) Heat flux if sensor is coated by a deposit.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) Constant properties, (3) Thickness of hot film sensor is negligible, (4) Applicability of Churchill-Bernstein correlation for uniform surface heat flux, (5) $\mathrm{Re_D} \ll 282,000$, (6) Deposit may be approximated as a plane layer.

PROPERTIES: *Table A-6*, water $(T_f = 292.5\mathrm{K})$: $k = 0.602\ \mathrm{W/m\cdot K}$, $\nu = 1.02\times10^{-6}\ \mathrm{m^2/s}$, $\mathrm{Pr} = 7.09$.

ANALYSIS: (a) With $\mathrm{Re_D} << 282,000$ and $\bar{h} = q''_{hf}/\left(T_{s,hf} - T_\infty\right)$, Eq. (7.57) reduces to

$$\overline{Nu}_D = \frac{q''_{hf}D}{k\left(T_{s,hf} - T_\infty\right)} \approx 0.3 + \frac{0.62\,\mathrm{Re}_D^{1/2}\,\mathrm{Pr}^{1/3}}{\left[1+\left(0.4/\mathrm{Pr}\right)^{2/3}\right]^{1/4}} \qquad (1)$$

Substituting for D, $\left(T_{s,hf} - T_\infty\right)$, k and Pr,

$$4.98\times10^{-4}q''_{hf} \approx 0.3 + 1.15\,\mathrm{Re}_D^{1/2}$$

or, with $\mathrm{Re}_D^{1/2} = \left(D/\nu\right)^{1/2}V^{1/2} = 38.3\,V^{1/2}$,

$$4.98\times10^{-4}q''_{hf} \approx 0.3 + 44.1\,V^{1/2} \qquad (2)$$

Substituting for q''_{hf},

$$V = 0.20\ \mathrm{m/s}$$ <

(b) For a fixed value of $T_{s,hf}$, the thermal resistance of the deposit reduces q''_{hf}. From the thermal circuit.

$$q''_{hf} = \frac{T_{s,hf} - T_\infty}{\left(1/\bar{h}\right) + \left(\delta/k_d\right)}$$

Using Eq. (1) to evaluate $\bar{h}$,

Continued …..

$$\bar{h} \approx \frac{k}{(D+\delta)}\left\{0.3+\frac{0.62\,Re_D^{1/2}\,Pr^{1/3}}{\left[1+(0.4/Pr)^{2/3}\right]^{1/4}}\right\}$$

where, with V = 0.20 m/s, $Re_D = V(D+\delta)/\nu = 314$, we obtain

$$\bar{h} \approx \frac{0.602\,W/m\cdot K}{0.0016m}\{20.7\} = 7,780\,W/m^2\cdot K$$

Hence,
$$q''_{hf} = \frac{5°C}{\left(1.285\times10^{-4}+0.5\times10^{-4}\right)m^2\cdot K/W} = 2.80\times10^4\,W/m^2 \quad <$$

With the foregoing heat flux applied to the sensor and use of the model for Part (a), the sensor would indicate a velocity predicted from Eq. (2), or

$$V = \left[\left(4.98\times10^{-4}\times2.80\times10^4-0.3\right)/44.1\right]^2 = 0.096\,m/s$$

The error in the velocity measurement is therefore

$$\%\,Error \equiv \frac{V_{(a)}-V_{(b)}}{V_{(a)}}(100\%) = \frac{0.20-0.096}{0.20}\times100 = 52\% \quad <$$

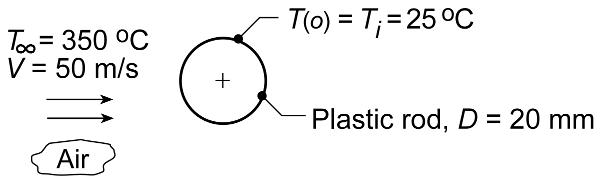
COMMENTS: (1) The accuracy of the hot-film sensor is strongly influenced by the deposit, and in any such application it is important to maintain a clean surface. (2) The Reynolds numbers are much less than 282,000 and assumption 5 is valid.

PROBLEM 7.62

KNOWN: Long coated plastic, 20-mm diameter rod, initially at a uniform temperature of T_i = 25°C, is suddenly exposed to the cross-flow of air at T_∞ = 350°C and V = 50 m/s.

FIND: (a) Time for the surface of the rod to reach 175°C, the temperature above which the special coating cures, and (b) Compute and plot the time-to-reach 175°C as a function of air velocity for 5 ≤ V ≤ 50 m/s.

SCHEMATIC:



ASSUMPTIONS: (a) One-dimensional, transient conduction in the rod, (2) Constant properties, and (3) Evaluate thermophysical properties at $T_f = [(T_s + T_i)/2 + T_\infty] = [(175 + 25)/2 + 350]°C = 225°C = 500$ K.

PROPERTIES: Rod (Given): ρ = 2200 kg/m^3, c = 800 J/kg·K, k = 1 W/m·K, α = k/ρc = 5.68×10^{-7} m^2/s; *Table A.4*, Air ($T_f \approx$ 500 K, 1 atm): ν = 38.79×10^{-6} m^2/s, k = 0.0407 W/m·K, Pr = 0.684.

ANALYSIS: (a) To determine whether the lumped capacitance method is valid, determine the Biot number

$$Bi_{lc} = \frac{\bar{h}(r_o/2)}{k} \tag{1}$$

The convection coefficient can be estimated using the Churchill-Bernstein correlation, Eq. 7.57,

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = 0.3 + \frac{0.63 Re_D^{1/2} Pr^{1/3}}{\left[1+(0.4/Pr)^{2/3}\right]^{1/4}}\left[1+\left(\frac{Re_D}{282,000}\right)^{5/8}\right]^{4/5}$$

$$Re_D = \frac{VD}{\nu} = 50\,m/s \times 0.020\,m \Big/ 38.79\times10^{-6}\,m^2\big/s = 25,780$$

$$\bar{h} = \frac{0.0407\,W/m\cdot K}{0.020\,m}\left\{0.3 + \frac{0.63(25,780)^{1/2}(0.684)^{1/3}}{\left[1+(0.4/0.684)^{2/3}\right]^{1/4}}\left[1+\left(\frac{25,780}{282,000}\right)^{5/8}\right]^{4/5}\right\} = 184\ W/m^2\cdot K \tag{2}$$

Substituting for $\bar{h}$ from Eq. (2) into Eq. (1), find

$$Bi_{lc} = 184\,W\big/m^2\cdot K\,(0.010\,m/2)\big/1\,W/m\cdot K = 0.92 \qquad >> 0.1$$

Hence, the lumped capacitance method is inappropriate. Using the one-term series approximation, Section 5.6.2, Eqs. 5.49 with Table 5.1,

$$\theta^* = C_1 \exp\left(-\zeta_1^2 Fo\right) J_o\left(\zeta_1 r^*\right) \qquad r^* = r/r_o = 1$$

$$\theta^* = \frac{T(r_o,t)-T_\infty}{T_i - T_\infty} = \frac{(175-350)^\circ C}{(25-350)^\circ C} = 0.54$$

$$Bi = \bar{h}r_o/k = 1.84 \qquad \zeta_1 = 1.5308\ rad \qquad C_1 = 1.3384$$

Continued...

PROBLEM 7.62 (Cont.)

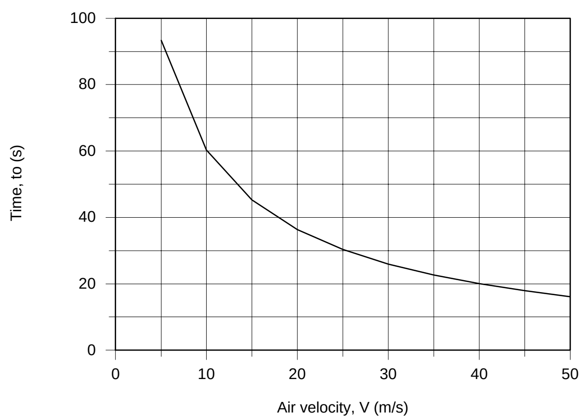
$0.54 = 1.3384\exp[-(1.5308\text{rad})^2 Fo] J_o(1.5308 \times 1)$

Using Table B.4 to evaluate $J_o(1.5308) = 0.4944$, find Fo = 0.0863 where

$$Fo = \frac{\alpha t_o}{r_o^2} = \frac{5.68\times10^{-7}\ m^2/s \times t_o}{(0.010\,m)^2} = 5.68\times10^{-3} t_o \tag{6}$$

$$t_o = 15.2\,s$$ <

(b) Using the *IHT Model*, *Transient Conduction*, *Cylinder*, and the *Tool*, *Correlations*, *External Flow*, *Cylinder*, results for the time-to-reach a surface temperature of 175°C as a function of air velocity V are plotted below.



COMMENTS: (1) Using the *IHT Tool*, *Correlations*, *External Flow*, *Cylinder*, the effect of the film temperature T_f on the estimated convection coefficient with V = 50 m/s can be readily evaluated.

T_f (K)	460	500	623
$\bar{h}$ ($W/m^2 \cdot K$)	187	184	176

At early times, $\bar{h}$ = 184 $W/m^2 \cdot K$ is a good estimate, while as the cylinder temperature approaches the airsteam temperature, the effect starts to be noticeable (10% decrease).

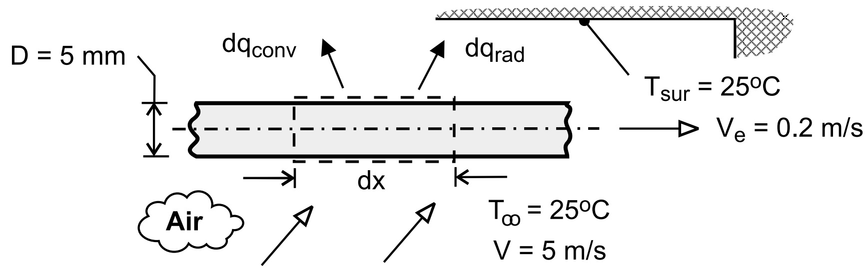
(2) The IHT analysis performed for part (b) was developed in two parts. Using a known value for $\bar{h}$, the *Transient Conduction*, *Cylinder Model* was tested. Separately, the *Correlation Tools* was assembled and tested. Then, the two files were merged to give the workspace for determining the time-to-reach 175°C as a function of velocity V.

PROBLEM 7.63

KNOWN: Velocity, diameter, initial temperature and properties of extruded wire. Temperature and velocity of air. Temperature of surroundings.

FIND: (a) Differential equation for temperature distribution T(x), (b) Exact solution for negligible radiation and corresponding value of temperature at prescribed length of wire, (c) Effect of radiation on temperature of wire at prescribed length. Effect of wire velocity and emissivity on temperature distribution.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible variation of wire temperature in radial direction, (2) Negligible effect of axial conduction along the wire, (3) Constant properties, (4) Radiation exchange between small surface and large enclosure, (5) Motion of wire has a negligible effect on the convection coefficient ($V_e << V$).

PROPERTIES: Prescribed. Copper: $\rho = 8900\,\text{kg/m}^3$, $c_p = 400\,\text{J/kg}\cdot\text{K}$, $\varepsilon = 0.55$. Air: $k = 0.037\,\text{W/m}\cdot\text{K}$, $\nu = 3\times10^{-5}\,\text{m}^2/\text{s}$, $\text{Pr} = 0.69$.

ANALYSIS: (a) Applying conservation of energy to a stationary control surface, through which the wire moves, steady-state conditions exist and $\dot{E}_{in} - \dot{E}_{out} = 0$. Hence, with *inflow* due to *advection* and *outflow* due to *advection*, *convection* and *radiation*,

$$\rho V_e A_c c_p T - \rho V_e A_c c_p (T + dT) - dq_{conv} - dq_{rad} = 0$$

$$-\rho V_e \left(\pi D^2/4\right) c_p dT - \pi D dx \left[\bar{h}(T - T_\infty) + \varepsilon\sigma\left(T^4 - T_{sur}^4\right)\right] = 0$$

$$\frac{dT}{dx} = -\frac{4}{\rho V_e D c_p}\left[\bar{h}(T - T_\infty) + \varepsilon\sigma\left(T^4 - T_{sur}^4\right)\right] \qquad (1) \quad <$$

Alternatively, if the control surface is fixed to the wire, conditions are transient and the energy balance is of the form, $-\dot{E}_{out} = \dot{E}_{st}$, or

$$-\pi D\, dx \left[\bar{h}(T - T_\infty) + \varepsilon\sigma\left(T^4 - T_{sur}^4\right)\right] = \rho\left(\frac{\pi D^2}{4} dx\right) c_p \frac{dT}{dt}$$

$$\frac{dT}{dt} = -\frac{4}{\rho D c_p}\left[\bar{h}(T - T_\infty) + \varepsilon\sigma\left(T^4 - T_{sur}^4\right)\right]$$

Dividing the left- and right-hand sides of the equation by dx/dt and $V_e = dx/dt$, respectively, Eq. (1) is obtained.

(b) Neglecting radiation, separating variables and integrating, Eq. (1) becomes

$$\int_{T_i}^{T} \frac{dT}{T - T_\infty} = -\frac{4\bar{h}}{\rho V_e D c_p}\int_0^x dx$$

Continued …..

PROBLEM 7.63 (Cont.)

$$\ln\left(\frac{T-T_\infty}{T_i-T_\infty}\right)=-\frac{4\bar{h}\,x}{\rho V_e D c_p}$$

$$T=T_\infty+\left(T_i-T_\infty\right)\exp\left(-\frac{4\bar{h}x}{\rho V_e D c_p}\right) \qquad (2) \qquad <$$

With $Re_D = VD/\nu = 5\,m/s\times 0.005\,m/3\times 10^{-5} m^2/s = 833$, the Churchill-Bernstein correlation yields

$$\overline{Nu}_D = 0.3+\frac{0.62(833)^{1/2}(0.69)^{1/3}}{\left[1+(0.4/0.69)^{2/3}\right]^{1/4}}\left[1+\left(\frac{833}{282,000}\right)^{5/8}\right]^{4/5} = 14.4$$

$$\bar{h}=\frac{k}{D}\overline{Nu}_D = \frac{0.037\,W/m\cdot K}{0.005m}14.4 = 107\,W/m^2\cdot K$$

Hence, applying Eq. (2) at x = L,

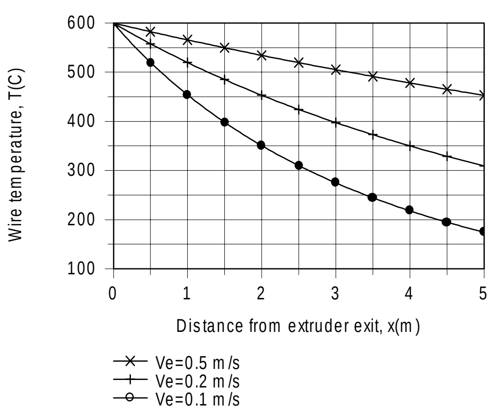
$$T_o = 25°C+\left(575°C\right)\exp\left(-\frac{4\times 107\,W/m^2\cdot K\times 5m}{8900\,kg/m^3\times 0.2\,m/s\times 0.005m\times 400\,J/kg\cdot K}\right)$$

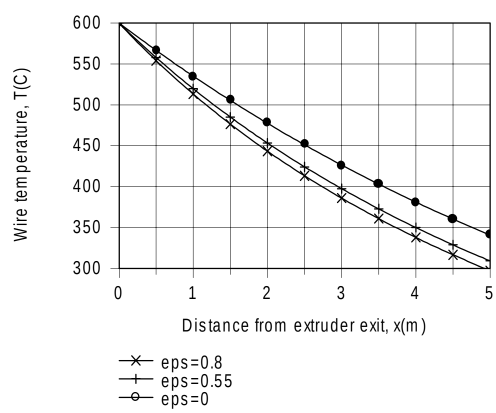
$$T_o = 340°C \qquad <$$

(c) Using the DER function of IHT, Eq. (1) may be numerically integrated from x = 0 to x = L = 5.0m to obtain

$$T_o = 309°C \qquad <$$

Hence, radiation makes a discernable contribution to cooling of the wire. IHT was also used to obtain the following distributions.





The speed with which the wire is drawn from the extruder has a significant influence on the temperature distribution. The temperature decay decreases with increasing V_e due to the increasing effect of advection on energy transfer in the x direction. The effect of the surface emissivity is less pronounced, although, as expected, the temperature decay becomes more pronounced with increasing ε.

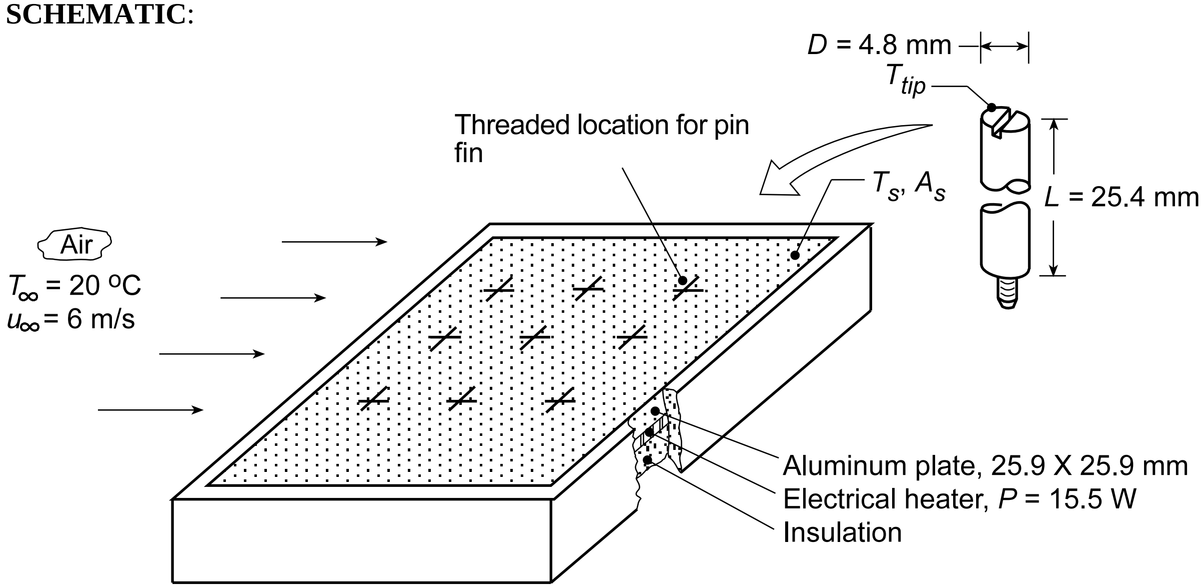
COMMENTS: (1) A critical parameter in wire extrusion processes is the *coiling temperature*, that is, the temperature at which the wire may be safely coiled for subsequent storage or shipment. The larger the production rate (V_e), the longer the cooling distance needed to achieve a desired coiling temperature. (2) Cooling may be enhanced by increasing the cross-flow velocity, and the specific effect of V may also be explored.

PROBLEM 7.64

KNOWN: Experimental apparatus comprised of a flat plate subjected to an airstream in parallel flow. Electrical patch heater on backside dissipates 15.5 W for all conditions. Pin fins fabricated from brass with prescribed diameter and length can be firmly attached to the plate. Fin tip and base temperatures observed for five different configurations (N, number of fins).

FIND: (a) The thermal resistance between the plate and airstream for the five configurations, (b) Model of the plate-fin system using appropriate convection correlations to predict the thermal resistances for the five configurations; compare predictions and observations; explain differences, and (b) Predict thermal resistances when the airstream velocity is doubled.

SCHEMATIC:



Experimental observations:

N	T_{tip} (°C)	T_s (°C)
0	--	70.2
1	40.6	67.4
2	39.5	64.7
5	36.4	57.4
8	34.2	52.1

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible effect of flow interactions between pins, (3) Negligible radiation exchange with surroundings, (4) All heater power is transferred to airstream, and (5) Constant properties.

PROPERTIES: *Table A.4*, Air (T_f = 310 K, 1 atm): k = 0.0270 W/m·K, $\nu = 1.69 \times 10^{-5}$ m^2/s, Pr = 0.706; *Table A.1*, Brass (T = 300 K): k = 110 W/m·K.

ANALYSIS: (a) The thermal resistance between the plate and the airstream is defined as

$$R_{tot} = \frac{T_s - T_\infty}{q} \qquad (1)$$

The heat rate is 15.6 W for all configurations and using T_s values from the above table with T_∞ = 20°C, find

Continued...

N	0	1	2	5	8
R_{tot} (K/W)	3.24	3.06	2.88	2.41	2.07

<

(b) The thermal resistance of the plate-fin system can be expressed as

$$R_{tot} = [1/R_{base} + N/R_{fin}]^{-1} \tag{2}$$

where the thermal resistance of the exposed portion of the base, A_b, is

$$R_{base} = \frac{1}{\bar{h}_b A_b} \tag{3}$$

$$A_b = A_s - NA_c \tag{4}$$

where the A_c is the cross-sectional area of a fin and A_s is the plate surface area. Approximating the airstream over the plate as parallel flow over a plate, use the *IHT Correlation Tool*, *External Flow*, *Flat Plate* assuming the flow is turbulated by the leading edge, to find

$$\bar{h}_b = 51\,\mathrm{W/m^2 \cdot K}.$$

From the experimental observation with no fins (N = 0), the convection coefficient was measured as

$$\bar{h}_{b,cxp} = \frac{q}{A_s(T_s - T_\infty)} = \frac{15.5\,\mathrm{W}}{(0.0259\,\mathrm{m})^2 (70.2-20)^\circ\,\mathrm{C}} = 460\,\mathrm{W/m^2 \cdot K}$$

Since the predicted coefficient is nearly an order of magnitude lower, we chose to use the experimental value in our subsequent analyses to predict overall system thermal resistance.

Approximating the airstream over a pin fin as cross-flow over a cylinder, use the *IHT Correlation Tool*, *External Flow*, *Cylinder* to find

$$\bar{h}_{fin} = 118\,\mathrm{W/m^2 \cdot K}.$$

Using the *IHT Extended Surface Model* for the *Rectangular Pin Fin* (*Temperature Distribution and Heat Rate*) with a convection tip condition, the following fin thermal resistance was found as

$$R_{fin} = 25.4\,\mathrm{K/W}$$

Using the foregoing values for R_{fin} and $\bar{h}_b$, the thermal resistances of the plate-fin system are tabulated below.

N	0	1	2	4	8
R_{base} (K/W)	3.241	3.331	3.426	3.746	4.133
R_{fin} (K/W)	--	25.4	12.7	5.08	3.18
R_{tot} (K/W)	3.24	2.95	2.70	2.16	1.80

<

By comparison with the experimental results of part (a), note that we assured agreement for the N = 0 condition by using the measured rather than estimated (correlation) convection coefficient. The predicted thermal resistances are systematically lower than the experimental values, with the worst case (N = 8) being 13% lower.

Continued...

PROBLEM 7.64 (Cont.)

(c) The effect of doubling the velocity, from u_∞ = 6 to 12 m/s, will cause the fin convection coefficient to increase from $\bar{h}_{fin}$ = 118 to 169 W/m^2·K. For the base convection coefficient, we'll assume the flow is fully turbulent so that $\bar{h} \sim (u_\infty)^{0.8}$ according to Eq. 7.41, hence

$$\bar{h}_b\,(12\,\text{m/s}) = \bar{h}_b\,(6\,\text{m/s})\left(\frac{12}{6}\right)^{0.8} = 460\,\text{W/m}^2\cdot\text{K}\,(2)^{0.8} = 800\,\text{W/m}^2\cdot\text{K}$$

Using the same procedure as above, find

N	0	1	2	4	8
R_{base} (K/W)	1.863	1.915	1.970	2.154	2.376
R_{fin} (K/W)	--	18.96	9.480	4.740	2.370
R_{tot} (K/W)	1.86	1.74	1.63	1.48	1.19

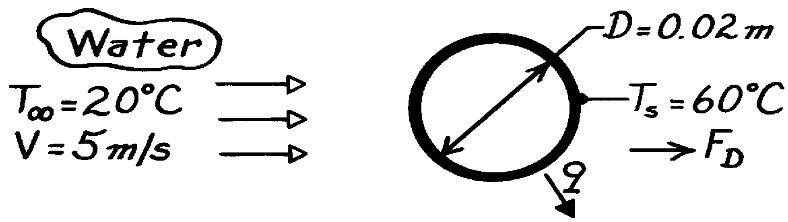
The effect of doubling the airstream velocity is to reduce the thermal resistance by approximately 35%.

PROBLEM 7.65

KNOWN: Temperature and velocity of water flowing over a sphere of prescribed temperature and diameter.

FIND: (a) Drag force, (b) Rate of heat transfer.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform surface temperature.

PROPERTIES: *Table A-6*, Saturated Water (T_∞ = 293K): ρ = 998 kg/m^3, μ = 1007 × 10^{-6} N·s/m^2, k = 0.603 W/m·K, Pr = 7.00; (T_s = 333 K): μ = 467 × 10^{-6} N·s/m^2; (T_f = 313 K): ρ = 992 kg/m^3, μ = 657 × 10^{-6} N·s/m^2.

ANALYSIS: (a) Evaluating μ and ρ at the film temperature,

$$\mathrm{Re_D} = \frac{r\,\mathrm{VD}}{m} = \frac{\left(992\ \mathrm{kg/m^3}\right)5\ \mathrm{m/s}\ (0.02\ \mathrm{m})}{657\times10^{-6}\ \mathrm{N\cdot s/m^2}} = 1.51\times10^5$$

and from Fig. 7.8, C_D = 0.42. Hence

$$F_D = C_D\frac{p\,D^2}{4}r\frac{V^2}{2} = 0.42\frac{p\,(0.02\ \mathrm{m})^2}{4}992\frac{\mathrm{kg}}{\mathrm{m^3}}\frac{(5\ \mathrm{m/s})^2}{2} = 1.64\ \mathrm{N}.$$ <

(b) With the Reynolds number evaluated at the free stream temperature,

$$\mathrm{Re_D} = \frac{r\,\mathrm{VD}}{m} = \frac{998\ \mathrm{kg/m^3}(5\ \mathrm{m/s})\ (0.02\ \mathrm{m})}{1007\times10^{-6}\ \mathrm{N\cdot s/m^2}} = 9.91\times10^4$$

it follows from the Whitaker relation that

$$\overline{\mathrm{Nu}}_D = 2 + \left[0.4\mathrm{Re}_D^{1/2} + 0.06\mathrm{Re}_D^{2/3}\right]\mathrm{Pr}^{0.4}\left(\frac{m}{m_s}\right)^{1/4}$$

$$\overline{\mathrm{Nu}}_D = 2 + \left[0.4\left(9.91\times10^4\right)^{1/2} + 0.06\left(9.91\times10^4\right)^{2/3}\right](7.0)^{0.4}\left(\frac{1007}{467}\right)^{1/4} = 673.$$

Hence, the convection coefficient and heat rate are

$$\bar{h} = \frac{k}{D}\overline{\mathrm{Nu}}_D = \frac{0.603\ \mathrm{W/m\cdot K}}{0.02\ \mathrm{m}}673 = 20{,}300\ \mathrm{W/m^2\cdot K}$$

$$q = \bar{h}\left(p\,D^2\right)\ (T_s - T_\infty) = 20{,}300\frac{\mathrm{W}}{\mathrm{m^2\cdot K}}p\ (0.02\ \mathrm{m})^2\ (60-20)^\circ\mathrm{C} = 1020\ \mathrm{W}.$$ <

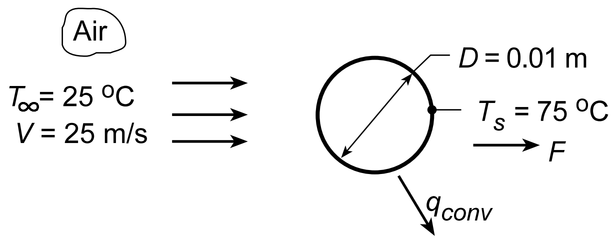
COMMENTS: Compare the foregoing value of $\bar{h}$ with that obtained in the text example under similar conditions. The significant increase in $\bar{h}$ is due to the much larger value of k and smaller value of ν for the water. Note that $\mathrm{Re_D}$ is slightly beyond the range of the correlation.

PROBLEM 7.66

KNOWN: Temperature and velocity of air flow over a sphere of prescribed surface temperature and diameter.

FIND: (a) Drag force, (b) Heat transfer rate with air velocity of 25 m/s; and (c) Compute and plot the heat rate as a function of air velocity for the range $1 \le V \le 25$ m/s.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform surface temperature, (3) Negligible radiation exchange with surroundings.

PROPERTIES: *Table A.4*, Air (T_∞ = 298 K, 1 atm): $\mu = 184 \times 10^{-7}$ N·s/m^2; $\nu = 15.71 \times 10^{-6}$ m^2/s, k = 0.0261 W/m·K, Pr = 0.71; (T_s = 348 K): $\mu = 208 \times 10^{-7}$ N·s/m^2; (T_f = 323 K): $\nu = 18.2 \times 10^{-6}$ m^2/s, ρ = 1.085 kg/m^3.

ANALYSIS: (a) Working with properties evaluated at T_f

$$Re_D = \frac{VD}{\nu} = \frac{25\,\text{m/s}\,(0.01\,\text{m})}{18.2\times10^{-6}\,\text{m}^2/\text{s}} = 1.37\times10^4$$

and from Fig. 7.8, find $C_D \approx 0.4$. Hence

$$F_D = C_D\left(\pi D^2/4\right)\left(\rho V^2/2\right) = 0.4(\pi/4)(0.01\,\text{m})^2\,1.085\,\text{kg/m}^3\,(25\,\text{m/s})^2/2 = 0.011\,\text{N}$$ <

(b) With

$$Re_D = \frac{VD}{\nu} = \frac{25\,\text{m/s}\,(0.01\,\text{m})}{15.71\times10^{-6}\,\text{m}^2/\text{s}} = 1.59\times10^4$$

it follows from the Whitaker relation that

$$\overline{Nu}_D = 2 + \left[0.4\,Re_D^{1/2} + 0.06\,Re_D^{2/3}\right]Pr^{0.4}\left(\frac{\mu}{\mu_s}\right)^{1/4}$$

$$\overline{Nu}_D = 2 + \left[0.4\left(1.59\times10^4\right)^{1/2} + 0.06\left(1.59\times10^4\right)^{2/3}\right](0.71)^{0.4}\left(\frac{184}{208}\right)^{1/4} = 76.7$$

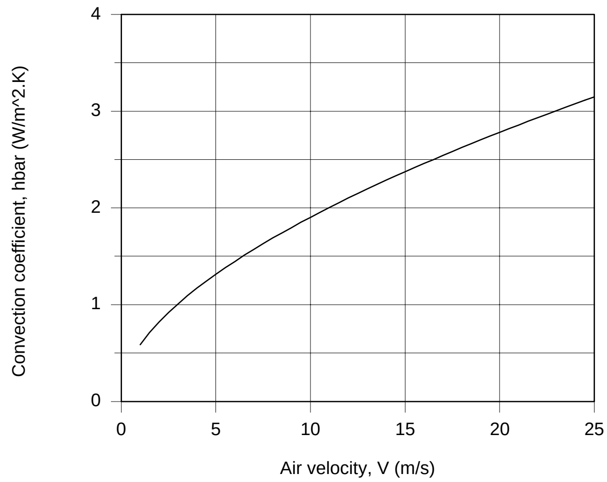
Hence, the convection coefficient and convection heat rate are

$$\bar{h} = \overline{Nu}_D\,\frac{k}{D} = 76.7\,\frac{0.0261\,\text{W/m}\cdot\text{K}}{0.01\,\text{m}} = 200\,\text{W/m}^2\cdot\text{K}$$

$$q = \bar{h}\pi D^2\left(T_s - T_\infty\right) = 200\,\text{W/m}^2\cdot\text{K}\times\pi\,(0.01\,\text{m})^2\,(75-25)^\circ\,\text{C} = 3.14\,\text{W}$$ <

Continued...

(c) Using the *IHT Correlation Tool*, *External Flow*, *Sphere*, the average coefficient and heat rate were calculated and are plotted below.



COMMENTS: (1) A copy of the IHT Workspace used to generate the above plot is shown below.

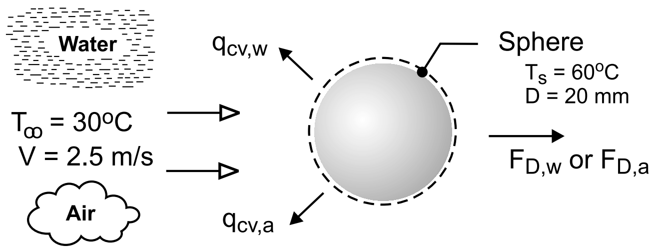
```
// Correlation Tool - External Flow, Sphere:
NuDbar = NuL_bar_EF_SP(ReD,Pr,mu,mus)     // Eq 7.58
NuDbar = hbar * D / k
ReD = V * D / nu
/* Evaluate properties at Tinf and the surface temperature, Ts. */

/*  Correlation description: External flow (EF) over a sphere (SP), average coefficient, 3.5<ReD<7.6x10^4,
0.71<Pr<380, 1.0<(mu/mus)<3.2, Whitaker correlation, Eq 7.59. See Table 7.9.  */

// Properties Tool - Air:
// Air property functions : From Table A.4
// Units: T(K); 1 atm pressure
mu = mu_T("Air",Tinf)              // Viscosity,  N·s/m^2
mus = mu_T("Air",Ts)               // Viscosity,  N·s/m^2
nu = nu_T("Air",Tinf)              // Kinematic viscosity, m^2/s
k = k_T("Air",Tinf)                          // Thermal conductivity, W/m·K
Pr = Pr_T("Air",Tinf)              // Prandtl number

// Heat Rate Equation:
q = hbar * pi * D^2 * (Ts - Tinf)

// Assigned Variables:
D = 0.01                           // Sphere diameter, m
Ts = 75 + 273                      // Surface temperature, K
V = 25                             // Airstream velocity, m/s
Tinf = 25 + 273                    // Airstream temperature, K
```

PROBLEM 7.67

KNOWN: Sphere with a diameter of 20 mm and a surface temperature of 60°C that is immersed in a fluid at a temperature of 30°C with a velocity of 2.5 m/s.

FIND: The drag force and the heat rate when the fluid is (a) water and (b) air at atmospheric pressure. Explain why the results for the two fluids are so different.

SCHEMATIC:



ASSUMPTIONS: (1) Flow over a smooth sphere, (2) Constant properties.

PROPERTIES: *Table A-6*, Water (T_∞ = 30°C = 303 K): $\mu = 8.034 \times 10^{-4}$ N·s/m^2, $\nu = 8.068 \times 10^{-7}$ m^2/s, k = 0.6172 W/m·K, Pr = 5.45; Water (T_s = 333 K): $\mu_s = 4.674 \times 10^{-4}$ N·s/m^2; *Table A-4*, Air (T_∞ = 30°C = 303 K, 1 atm): $\mu = 1.86 \times 10^{-5}$ N·s/m^2, $\nu = 1.619 \times 10^{-5}$ m^2/s, k = 0.0265 W/m·K, Pr = 0.707; Air (T_∞ = 333 K): $\mu_s = 2.002 \times 10^{-5}$ N·s/m^2.

ANALYSIS: The drag force, F_o, for the sphere is determined from the drag coefficient, Eq. 7.54,

$$C_D = \frac{F_D}{A_f\left(\rho V^2/2\right)}$$

where $A_f = \pi D^2/4$ is the frontal area. C_D is a function of the Reynolds number $Re_D = VD/\nu$ as represented in Figure 7.8. For the convection rate equation**,**

$$q = \overline{h}_D A_s\left(T_s - T_\infty\right)$$

where $A_s = \pi D^2$ is the surface area and the convection coefficient is estimated using the Whitaker correlation, Eq. 7.59,

$$\overline{Nu}_D = 2 + \left[0.4\,Re_D^{1/2} + 0.06\,Re_D^{2/3}\right]Pr^{0.4}\left(\mu/\mu_s\right)^{1/4}$$

where all properties except μ_s are evaluated at T_∞. For convenience we will evaluate properties required for the drag force at T_∞. The results of the analyses for the two fluids are tabulated below.

Fluid	Re_D	C_D	F_D (N)	$\overline{Nu}_D$	$\overline{h}_D\left(W/m^2\cdot K\right)$	q(W)
water	6.198×10^4	0.5	0.489	439	13,540	510
air	3.088×10^3	0.4	0.452×10^{-3}	31.9	42.3	1.59

The frontal and surface areas, respectively, are $A_f = 3.142 \times 10^{-4}$ m^2 and $A_s = 1.257 \times 10^{-3}$ m^2.

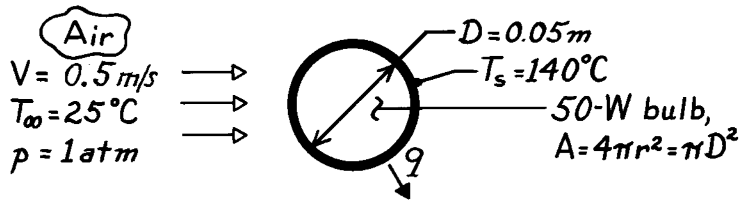
COMMENTS: The Reynolds number is the ratio of inertia to viscous forces. We associate higher viscous shear and heat transfer with larger Reynolds numbers. The drag force also depends upon the fluid density, which further explains why F_D for water is much larger, by a factor of 1000, than for air. Nu_D is dependent upon Re_D^n where n is 1/2 to 2/3, and represents the dimensionless temperature gradient at the surface. Since the thermal conductivity of water is nearly 20 times that of air, we expect a significant difference between $\overline{h}_D$ and q for the two fluids.

PROBLEM 7.68

KNOWN: Conditions associated with airflow over a spherical light bulb of prescribed diameter and surface temperature.

FIND: Heat loss by convection.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform surface temperature.

PROPERTIES: *Table A-4*, Air (T_f = 25°C, 1 atm): $\nu = 15.71 \times 10^{-6}$ m^2/s, k = 0.0261 W/m·K, Pr = 0.71, $\mu = 183.6 \times 10^{-7}$ N·s/m^2; *Table A-4*, Air (T_s = 140°C, 1 atm): $\mu = 235.5 \times 10^{-7}$ N·s/m^2.

ANALYSIS: The heat rate by convection is

$$q = \bar{h}\left(\pi D^2\right)\ \left(T_s - T_\infty\right)$$

where $\bar{h}$ may be estimated from the Whitaker relation

$$\bar{h} = \frac{k}{D}\left[2 + \left(0.4\ Re_D^{1/2} + 0.06\ Re_D^{2/3}\right) Pr^{0.4}\left(\mu/\mu_s\right)^{1/4}\right]$$

where

$$Re_D = \frac{VD}{\nu} = \frac{0.5\ \text{m/s} \times 0.05\ \text{m}}{15.71 \times 10^{-6}\ \text{m}^2/\text{s}} = 1591.$$

Hence,

$$\bar{h} = \frac{0.0261\ \text{W/m}\cdot\text{K}}{0.05\ \text{m}}\ \left\{2 + \left[0.4(1591)^{1/2} + 0.06(1591)^{2/3}\right]\ (0.71)^{0.4}\left(\frac{183.6}{235.5}\right)^{1/4}\right\}$$

$$\bar{h} = 11.4\ \text{W/m}^2\cdot\text{K}$$

and the heat rate is

$$q = 11.4\frac{\text{W}}{\text{m}^2\cdot\text{K}}\pi(0.05\ \text{m})^2\ \ (140 - 25)^\circ\text{C} = 10.3\ \text{W}. \qquad <$$

COMMENTS: (1) The low value of $\bar{h}$ suggests that heat transfer by free convection may be significant and hence that the total loss by convection exceeds 10.3 W.

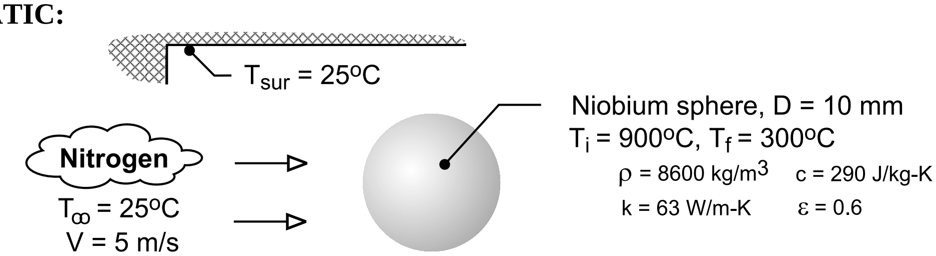
(2) The surface of the bulb also dissipates heat to the surrounding by radiation. Further, in an actual light bulb, there is also heat loss by conduction through the socket.

PROBLEM 7.69

KNOWN: Diameter, properties and initial temperature of niobium sphere. Velocity and temperature of nitrogen. Temperature of surroundings.

FIND: (a) Time for sphere to cool to prescribed temperature if radiation is neglected, (b) Cooling time if radiation is considered. Effect of flow velocity.

SCHEMATIC:



ASSUMPTIONS: (1) Lumped capacitance method is valid, (2) Constant properties, (3) Radiation exchange with large surroundings.

PROPERTIES: *Table A-4*, nitrogen $\left(T_\infty = 298\text{K}\right)$: $\mu = 177\times10^{-7}\ \text{N}\cdot\text{s/m}^2$, $\nu = 15.7\times10^{-6}\,\text{m}^2/\text{s}$, $k = 0.0257\ \text{W/m}\cdot\text{K}$, $\text{Pr} = 0.716$. *Table A-4*, nitrogen $\left(\overline{T}_s = 873\text{K}\right)$: $\mu_s = 368\times10^{-7}\ \text{N}\cdot\text{s/m}^2$.

ANALYSIS: (a) Neglecting radiation, the cooling time may be determined from Eq. (5.5),

$$t = \frac{\rho\left(\pi D^3/6\right)c}{\overline{h}\pi D^2}\ln\frac{\theta_i}{\theta} = \frac{\rho c D}{6\overline{h}}\ln\frac{T_i - T_\infty}{T_f - T_\infty}$$

The convection coefficient is obtained from the Whitaker correlation with $\text{Re}_D = VD/\nu$ $= 5\,\text{m/s}\times0.01\text{m}/15.7\times10^{-6}\,\text{m}^2/\text{s} = 3185$. Hence,

$$\overline{\text{Nu}}_D = \left(\overline{h}D/k\right) = 2 + \left(0.4\,\text{Re}_D^{1/2} + 0.06\,\text{Re}_D^{2/3}\right)\text{Pr}^{0.4}\left(\mu/\mu_s\right)^{1/4}$$

$$\overline{h} = \frac{0.0257\ \text{W/m}\cdot\text{K}}{0.01\text{m}}\left\{2 + \left[0.4\left(3185\right)^{1/2} + 0.06\left(3185\right)^{2/3}\right]\left(0.716\right)^{0.4}\left(\frac{177}{368}\right)^{0.25}\right\} = 71.8\ \text{W/m}^2\cdot\text{K}$$

$$t = \frac{8600\,\text{kg/m}^3\times290\,\text{J/kg}\cdot\text{K}\times0.01\text{m}}{6\times71.8\,\text{W/m}^2\cdot\text{K}}\ln\frac{\left(900-25\right)}{\left(300-25\right)} = 67\,\text{s}$$ <

(b) If the effect of radiation is considered, the cooling time can be obtained by integrating Eq. (5.15). With $A_s/V = \pi D^2/\left(\pi D^3/6\right) = 6/D$, the appropriate form of the equation is

$$\frac{dT}{dt} = -\frac{6}{\rho c D}\left[\overline{h}\left(T - T_\infty\right) + \varepsilon\sigma\left(T^4 - T_{sur}^4\right)\right]$$

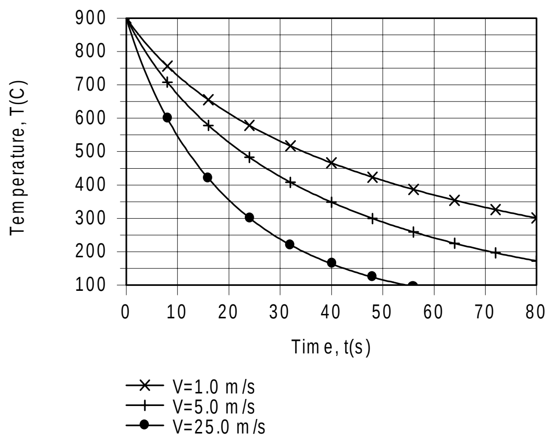
Using the DER function of IHT to integrate this equation over the limits from $T_i = 1173\,\text{K}$ to $T_f = 573\,\text{K}$, we obtain

$$t = 48\,\text{s}$$ <

Continued …..

PROBLEM 7.69 (Cont.)

For V = 1.0 and 25.0 m/s, the cooling times are t ≈ 80 and 24 s, respectively. Temperature histories for the three velocities are shown below.



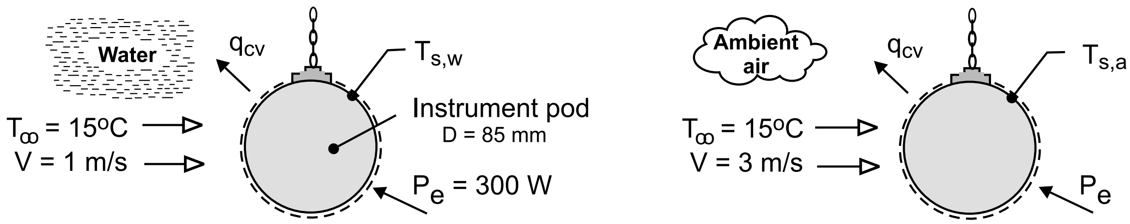
COMMENTS: The cooling time is significantly affected by the flow velocity.

PROBLEM 7.70

KNOWN: An underwater instrument pod having a spherical shape with a diameter of 85 mm dissipating 300 W.

FIND: Estimate the surface temperature of the pod for these conditions: (a) when submersed in a bay where the water temperature is 15°C and the current is 1 m/s, and (b) after being hauled out of the water *without deactivating the power* and suspended in the ambient where the air temperature is 15°C and the wind speed is 3 m/s.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Flow over a smooth sphere, (3) Uniform surface temperatures, (4) Negligible radiation heat transfer for air (a) condition, and (5) Constant properties.

PROPERTIES: *Table A-6*, Water (T_∞ = 15°C = 288 K): $\mu = 0.001053$ N·s/m^2, $\nu = 1.139 \times 10^{-6}$ m^2/s, k = 0.5948 W/m·K, Pr = 8.06; *Table A-4*, Air (T_∞ = 288 K, 1 atm): $\mu = 1.788 \times 10^{-5}$ N·s/m^2, $\nu = 1.482 \times 10^{-5}$ m^2/s, k = 0.02534 W/m·K; Air (T_s = 945 K): $\mu_s = 4.099 \times 10^{-5}$ N·s/m^2, Pr = 0.710.

ANALYSIS: The energy balance for the submersed-in-water (w) and suspended-in-air (a) conditions are represented in the schematics above and have the form

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = -q_{cv} + P_e = 0 \tag{1}$$

$$-\bar{h}_D A_s \left(T_s - T_\infty\right) + P_e = 0$$

where $A_s = \pi D^2$ and $\bar{h}_D$ is estimated using the Whitaker correlation, Eq. 7.59,

$$\overline{Nu}_D = 2 + \left[0.4\, Re_D^{1/2} + 0.06\, Re_D^{2/3}\right] Pr^{0.4} \left(\mu / \mu_s\right)^{1/4} \tag{2}$$

where all properties except μ_s are evaluated at T_∞. The results are tabulated below.

Condition	Re_D	$\overline{Nu}_D$	$\bar{h}_D$ (W/m^2·K)	T_s (°C)
(w) water	7.465×10^4	509	3559	18.7
(a) air	1.72×10^4	67.5	20.1	672

COMMENTS: (1) While submerged and dissipating 300 W, the pod is safely operating at a temperature slightly above that of the water. When hauled from the water and suspended in air, the pod temperature increases to a destruction temperature (672°C). The pod gets smoked!

(2) The assumption that $\mu/\mu_s \approx 1$ is appropriate for the water (w) condition. For the air (a) condition, $\mu/\mu_s = 0.436$ and the final term of the correlation is significant. Recognize that radiation exchange with the surroundings for the air condition should be considered for an improved estimate.

Continued …..

PROBLEM 7.70 (Cont.)

(3) Why such a difference in T_s for the water (w) and air (a) conditions? From the results table note that the Re_D, Nu_D, and $\overline{h}_D$ are, respectively, 4x, 7x and 170x times larger for water compared to air. Water, because of its thermophysical properties which drive the magnitude of $\overline{h}_D$, is a much better coolant than air for similar flow conditions.

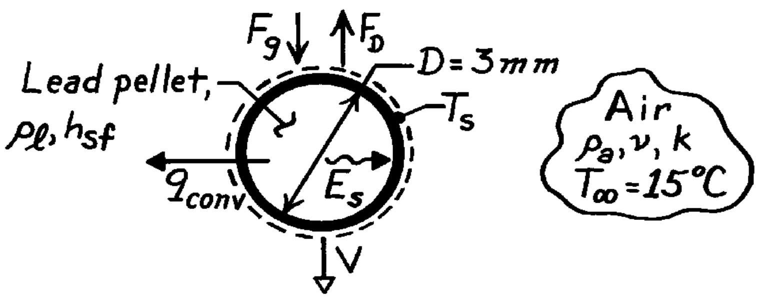
```
/* Comment:  Because Ts is much larger than Tinf for the in-air operation, the ratio of mu / mus
exceeds the limits for the correlation.  Hence, a warning message comes with the IHT solution.  */

/* Results  - operation in air
As      NuDbar   Pr       ReD     Tinf    Ts      Ts_C    hbar    k        mu
        mus      nu       D       Pelec   Tinf_C  V
0.0227  67.5     0.7101   1.72E4  288     944.8   671.8   20.12   0.02534  1.786E-5
        4.099E-5 1.482E-5 0.085   300     15      3  */

// Correlation, sphere
NuDbar = NuL_bar_EF_SP(ReD,Pr,mu,mus)     // Eq 7.59
NuDbar = hbar * D / k
ReD = V * D / nu
/* All properties except mus are evaluated at Tinf. */
/*  Correlation description: External flow (EF) over a sphere (SP), average coefficient,
3.5<ReD<7.6x10^4, 0.71<Pr<380, 1.0<(mu/mus)<3.2, Whitaker correlation, Eq 7.59. See Table 7.9.  */

// Energy balance
Pelec - hbar * As * (Ts - Tinf) = 0
As = pi * D^2

// Input variables
D = 0.085
//V = 1.0                // Water current
V = 3                    // Wind speed
Tinf_C = 15
Pelec = 300

// Conversions
Tinf = Tinf_C + 273
Ts = Ts_C + 273

// Air property functions : From Table A.4
// Units: T(K); 1 atm pressure
mu = mu_T("Air",Tinf)        // Viscosity,  N·s/m^2
mus = mu_T("Air",Ts)         // Viscosity,  N·s/m^2
// mus = mu
nu = nu_T("Air",Tinf)        // Kinematic viscosity, m^2/s
k = k_T("Air",Tinf)          // Thermal conductivity, W/m·K
Pr = Pr_T("Air",Tinf)        // Prandtl number
```

PROBLEM 7.71

KNOWN: Air cooling requirements for lead pellets in the molten slate.

FIND: Height of tower from which pellets must be dropped to convert from liquid to solid state.

SCHEMATIC:



ASSUMPTIONS: (1) Pellet remains at melting point temperature throughout process, (2) Density of lead, r_ℓ, remains constant (at density of molten lead) throughout process, (3) Radiation effects are negligible.

PROPERTIES: *Table A-7*, Lead (M.P. = T_s = 327.2°C): $r_\ell \approx 10{,}600\ \mathrm{kg/m^3}$; *Handbook Chemistry and Physics*: Latent heat of fusion, h_{sf} = 24.5 kJ/kg; *Table A-4*, Air (T_∞ = 15°C): ρ_a = 1.22 kg/m^3, $\nu = 14.8 \times 10^{-6}$ m^2/s, k = 25.3×10^{-3} W/m·K, Pr = 0.71, $\mu = 178.6 \times 10^{-7}$ N·s/m^2; (T_s = 327°C): $\mu_s = 306 \times 10^{-7}$ N·s/m^2.

ANALYSIS: Conservation of energy dictates that the energy released to solidification must be given off to the air by convection. Applying the conservation of energy requirement on a time interval basis,

$$-E_{out} = \Delta E_{st} \qquad \text{where} \qquad E_{out} = q_{conv} \cdot t_s$$

and t_s is the time required to completely solidify a pellet. Hence,

$$-\bar{h}\left(p D^2\right)\ \left(T_s - T_\infty\right) \cdot t_s = -h_{sf}\ r_\ell\left(p D^3/6\right).$$

With the pellet moving at the terminal velocity, V, the height of the tower must be

$$H = V \cdot t_s = \frac{V h_{sf}\ r_\ell D}{6\bar{h}\left(T_s - T_\infty\right)}$$

The terminal velocity may be obtained from a force balance on the pellet. Equating the drag and gravity forces,

$$F_g = F_D$$

where $F_g = r_\ell\left(p D^3/6\right) g$ and F_D is obtained from the drag coefficient

$$r_\ell\left(p D^3/6\right) g = C_D\left(p D^2/4\right)\ \left(r_a V^2/2\right)$$

$$V = \left(\frac{4}{3}\ \frac{r_\ell}{r_a}\ \frac{gD}{C_D}\right)^{1/2} - \left[\frac{4}{3}\frac{10{,}600\ \mathrm{kg/m^3}}{1.22\ \mathrm{kg/m^3}}\frac{\left(9.8\ \mathrm{m/s^2}\right)\ (0.003\ \mathrm{m})}{C_D}\right]^{1/2}$$

$$V\,(\mathrm{m/s}) = 18.5/C_D^{1/2}.$$ <

Continued …..

The drag coefficient may be obtained from Fig. 7.8 and knowledge of the Reynolds number, where

$$\mathrm{Re_D} = \frac{\mathrm{VD}}{n} = \frac{\mathrm{V}\left(0.003\ \mathrm{m}\right)}{14.8\times10^{-6}\ \mathrm{m^2/s}} = 202.7\ \mathrm{V}\left(\mathrm{m/s}\right).$$

In a trial-and-error procedure which involves guessing a value of V, calculating $\mathrm{Re_D}$, obtaining $\mathrm{C_D}$ from Fig. 7.8, and computing V from Eq. (1), it was found that

$$\mathrm{V} \approx 29\ \mathrm{m/s} \qquad \mathrm{Re_D} \approx 5900.$$

From the Whitaker correlation, it follows that

$$\overline{\mathrm{Nu}}_{\mathrm{D}} = 2 + \left[0.4\mathrm{Re_D^{1/2}} + 0.06\mathrm{Re_D^{2/3}}\right]\mathrm{Pr}^{0.4}\left(\frac{m}{m_{\mathrm{s}}}\right)^{1/4}$$

$$\overline{\mathrm{Nu}}_{\mathrm{D}} = 2 + \left[0.4\left(5900\right)^{1/2} + 0.06\left(5900\right)^{2/3}\right]\ \left(0.71\right)^{0.4}\left(\frac{178.6\times10^{-7}}{306\times10^{-7}}\right)^{1/4} = 40.4$$

$$\overline{\mathrm{h}} = \overline{\mathrm{Nu}}_{\mathrm{D}}\left(\frac{\mathrm{k}}{\mathrm{D}}\right) = 40.4\left(\frac{25.3\times10^{-3}\ \mathrm{W/m\cdot K}}{0.003\ \mathrm{m}}\right) = 341\ \mathrm{W/m^2\cdot K}.$$

Accordingly,

$$\mathrm{H} = \frac{29\ \mathrm{m/s}\times24{,}500\ \mathrm{J/kg}\times10{,}600\ \mathrm{kg/m^3}\times0.003\ \mathrm{m}}{6\times341\ \mathrm{W/m^2\cdot K}\times\left(327.2-15\right)^{\circ}\mathrm{C}} = 35\ \mathrm{m}.$$ <

COMMENTS: (1) In a free fall from such a height (H = 35 m), the pellet will not have sufficient time to reach the terminal velocity (its maximum velocity on impacting the water would be 28.7 m/s). Accordingly, V has been overestimated and the required value of H has been overpredicted. A more accurate treatment would involve applying the energy balance at successive times from the initiation of the fall, using the pellet velocity appropriate to each time.

(2) Accounting for radiation effects would further diminish the required value of H.

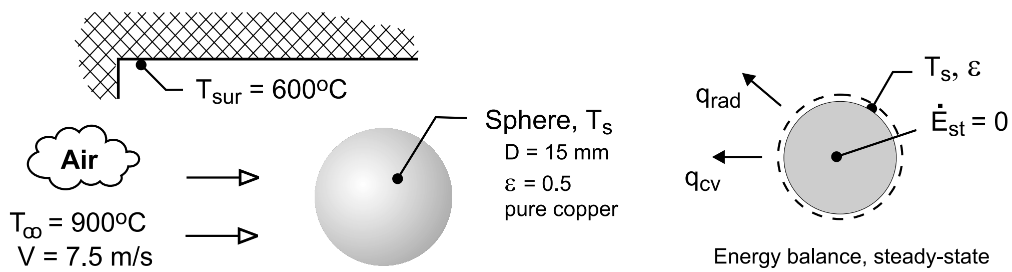
(3) The correlation has been used outside its range of applicability, since $\mu/\mu_s < 1$.

PROBLEM 7.72

KNOWN: A spherical workpiece of pure copper with a diameter of 15 mm and emissivity of 0.5 is suspended in a large furnace with walls at a uniform temperature of 600°C. The air flow over the workpiece has a temperature of 900°C with a velocity of 7.5 m/s.

FIND: (a) The steady-state temperature of the workpiece; (b) Estimate the time required for the workpiece to reach within 5°C of the steady-state temperature if its initial, uniform temperature is 25°C; (c) Estimate the steady-state temperature of the workpiece if the air velocity is doubled with all other conditions remaining the same; also, determine the time required for the workpiece to reach within 5°C of this value. Plot on the same graph the workpiece temperature histories for the two air velocity conditions**.**

SCHEMATIC:



ASSUMPTIONS: (1) Flow over a smooth sphere, (2) Sphere behaves as spacewise isothermal object; lumped capacitance method is valid, (3) Sphere is small object in large, isothermal surroundings, and (4) Constant properties.

PROPERTIES: *Table A-4*, Air (T_∞ = 1173 K, 1 atm): $\mu = 4.665 \times 10^{-5}$ N·s/m^2, ν = 0.0001572 m^2/s, k = 0.075 W/m·K, Pr = 0.728; Air (T_s = 1010 K, 1 atm): $\mu_s = 4.268 \times 10^{-5}$ N·s/m^2.

ANALYSIS: (a) The steady-state temperature is determined from the energy balance on the sphere as represented in the schematic above.

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = 0 \qquad -q_{cv} - q_{rad} + 0 = 0$$

$$-\bar{h}_D A_s \left(T_s - T_\infty\right) - \varepsilon A_s \sigma \left(T_s^4 - T_{sur}^4\right) = 0 \tag{1}$$

where $A_s = \pi D^2/4$. The convection coefficient can be estimated using the Whitaker correlation, Eq. 7.59, where all properties except μ_s are evaluated at T_∞. Assume T_s = 737°C = 1010 K to evaluate μ_s.

$$\overline{Nu}_D = 2 + \left[0.4\, Re_D^{1/2} + 0.06\, Re_D^{2/3}\right] Pr^{0.4} \left(\mu / \mu_s\right)^{1/4} \tag{2}$$

See the table below for results of the correlation calculations. From the energy balance, canceling out A_s, with numerical values, find T_s.

$$-79.8\ \mathrm{W/m^2 \cdot K}\left(T_s - 1173\right)\mathrm{K} - 0.5 \times 5.67 \times 10^{-8}\ \mathrm{W/m^2 \cdot K^4}\left(T_s^4 - 873^4\right)\mathrm{K}^4$$

$$T_s = 1010\ \mathrm{K} = 737°\mathrm{C}.$$ <

(b) The time required for the sphere initially at T_i = 25°C to reach within 5°C of the steady-state temperature can be determined from the energy balance for the transient condition.

Continued …..

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \dot{E}_{st}$$

$$-\overline{h}_D A_s \left(T_s - T_\infty\right) - \varepsilon A_s \sigma \left(T_s^4 - T_{sur}^4\right) = \rho c \left(\pi D^3 / 6\right) \frac{dT}{dt} \qquad (3)$$

Recognize that $\overline{h}_D$ is not constant, but depends upon $T_s(t)$. Using *IHT* to perform the integration, evaluate $\overline{h}_D$, and provide pure copper properties ρ and c as a function of T_s, the time t_o for $T(t_o)$ = (737 – 5)°C = 732°C is

$$t_o = 274 \text{ s} \qquad <$$

See Comments 1 and 2 for details on the *IHT* calculation method.

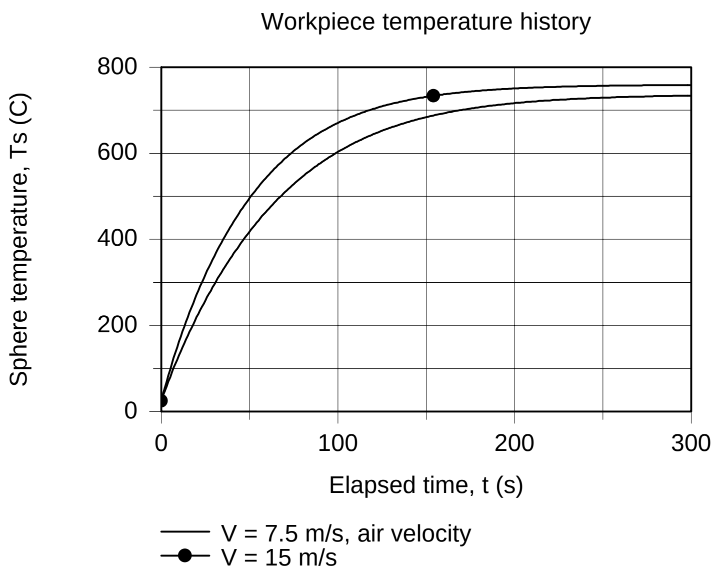
(c) Use Eq. (1) and (2) to find the steady-state temperature when the air velocity is doubled, V = 2 × 7.5 ms = 15 m/s. The results are tabulated below along with those from part (a).

Part	V (m/s)	Re_D	$\overline{Nu}_D$	$\overline{h}_D$ (W/m²·K)	T_s (°C)
a	7.5	715.6	15.96	79.8	737
b	15	1431	22.42	112.1	760

As expected, increasing the air velocity will cause the sphere temperature to increase toward T_∞. Note that $\overline{h}_D$ increases by a factor of 1.4 as the air velocity is doubled. From correlation Eq. (2) note that $\overline{h}_D$ is approximately proportional to V^n where n is in the range 1/2 to 2/3. Using the *IHT* code for the lumped capacitance analysis, the time for $T(t_o)$ = (760 – 5)°C = 755°C is

$$t_o = 230 \text{ s} \qquad <$$

The temperature histories for the two air velocity conditions are calculated using the foregoing transient analyses in the *IHT* workspace.



Continued …..

PROBLEM 7.72 (Cont.)

COMMENTS: **(**1) The portion of the *IHT* code for performing the energy balance and evaluating the convection correlation function using the properties function follows.

```
// Convection correlation, sphere
NuDbar = NuL_bar_EF_SP(ReD,Pr,mu,mus)     // Eq 7.59
NuDbar = hbar * D / k
ReD = V * D / nu
/* All properties except mus are evaluated at Tinf. */
/*  Correlation description: External flow (EF) over a sphere (SP), average coefficient,
3.5<ReD<7.6x10^4, 0.71<Pr<380, 1.0<(mu/mus)<3.2, Whitaker correlation, Eq 7.59. See Table 7.9.  */

// Energy balance, steady-state temperature
-hbar * As * (Ts - Tinf) - eps * sigma * (Ts^4 - Tsur^4) * As = 0
As = pi * D^2
sigma = 5.67e-8

// Air property functions : From Table A.4
// Units: T(K); 1 atm pressure
mu = mu_T("Air",Tinf)          // Viscosity,  N·s/m^2
mus = mu_T("Air",Ts)           // Viscosity,  N·s/m^2
nu = nu_T("Air",Tinf)          // Kinematic viscosity, m^2/s
k = k_T("Air",Tinf)            // Thermal conductivity, W/m·K
Pr = Pr_T("Air",Tinf)          // Prandtl number

// Input variables
D = 0.015
eps = 0.5
V = 7.5
Tinf = 900 + 273
Tsur = 600 + 273
```

(2) Two modifications can be made to the code above to perform the lumped capacitance method for the transient analysis: (a) include the storage term in the energy balance and (b) provide the properties function for copper. The initial condition, T_i = 288 K, is entered as the initial condition when the solver performs the integration.

```
// Energy balance, steady-state; equilibrium temperature
-hbar * As * (Ts - Tinf) - eps * sigma * (Ts^4 - Tfur^4) * As = M * ccu * der(Ts,t)
As = pi * D^2
sigma = 5.67e-8
M = rhocu * pi * D^3 / 6

// Copper (pure) property functions : From Table A.1
// Units: T(K)
rhocu = rho_300K("Copper")             // Density, kg/m^3
kcu = k_T("Copper",Ts)       // Thermal conductivity,W/m·K
ccu = cp_T("Copper",Ts)      // Specific heat, J/kg·K
```

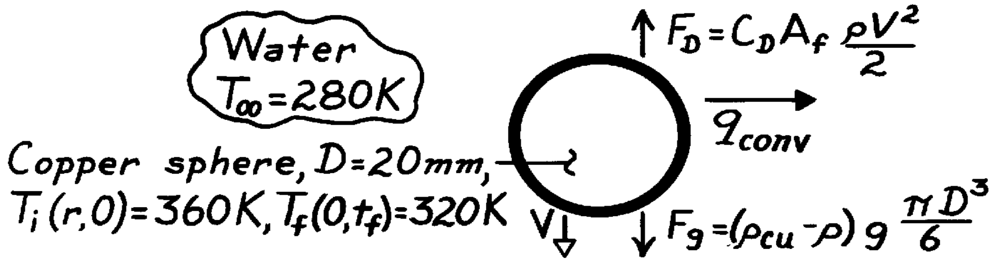
(3) Show that the lumped capacitance method is valid for this application.

PROBLEM 7.73

KNOWN: Diameter and initial and final temperatures of copper spheres quenched in a water bath.

FIND: (a) Terminal velocity in the bath, (b) Tank height.

SCHEMATIC:



ASSUMPTIONS: (1) Sphere descends at terminal velocity, (2) Uniform, but time varying surface, temperature.

PROPERTIES: *Table A-1*, Copper (350K): $\rho = 8933$ kg/m^3, k = 398 W/m·K, $c_p = 387$ J/kg·K; *Table A-6*, Water ($T_\infty = 280$ K): $\rho = 1000$ kg/m^3, $\mu = 1422 \times 10^{-6}$ N·s/m^2, k = 0.582 W/m·K, Pr = 10.26; ($T_s \approx 340$ K): $\mu_s = 420 \times 10^{-5}$ N·s/m^2.

ANALYSIS: A force balance gives $C_D\left(p\,D^2/4\right)\, r\,V^2/2 = \left(r_{cu} - r\right)\, g\, p\, D^3/6$,

$$C_D\ V^2 = \frac{4D}{3}\frac{r_{cu}-r}{r}g = \frac{4 \times 0.02\text{ m}}{3}\cdot\frac{8933-1000}{1000}9.8\text{ m/s}^2 = 2.07\text{ m}^2/\text{s}^2.$$

An iterative solution is needed, where C_D is obtained from Figure 7.8 with $Re_D = VD/\nu = 0.02$ m V/1.42 × 10^{-6} m^2/s = 14,085 V (m/s). Convergence is achieved with

$$V \approx 2.1\text{ m/s}$$ <

for which $Re_D = 29{,}580$ and $C_D \approx 0.46$. Using the Whitaker expression

$$\overline{Nu}_D = 2 + \left(0.4 \times 29{,}850^{1/2} + 0.06 \times 29{,}850^{2/3}\right)\ (10.26)^{0.4}(1422/420)^{1/4} = 439$$

$$\bar{h} = \overline{Nu}_D\ k/D = 439 \times 0.582\text{ W/m}\cdot\text{K}/0.02\text{ m} = 12{,}775\text{ W/m}^2\cdot\text{K}.$$

To determine applicability of lumped capacitance method, find $Bi = \bar{h}\left(r_o/3\right)/\ k_{cu} = 12{,}775$ $\text{W/m}^2\cdot\text{K}\ (0.01\text{ m}/3)/398\text{W/m}\cdot\text{K} = 0.11$. Applicability is marginal. Use Heisler charts,

$$q_o^* = \frac{T_o - T_\infty}{T_i - T_\infty} = \frac{320-280}{360-280} = 0.5, \quad Bi^{-1} = \frac{k}{\bar{h}r_o} = 3.12, \quad Fo \approx 0.88 = \frac{a\,t_f}{r_o^2}.$$

With $\alpha_{cu} = k/\rho c_p$ = 398 W/m·K/(8933 kg/m^3) (387 J/kg·K) = 1.15 × 10^{-4} m^2/s, find

$$t_f = 0.88\ (0.01\text{ m})^2/1.15 \times 10^{-4}\text{m}^2/\text{s} = 0.77\text{ s}.$$

Required tank height is

$$H = t_f \cdot V = 0.77\text{ s} \times 2.1\text{ m/s} = 1.6\text{ m}.$$ <

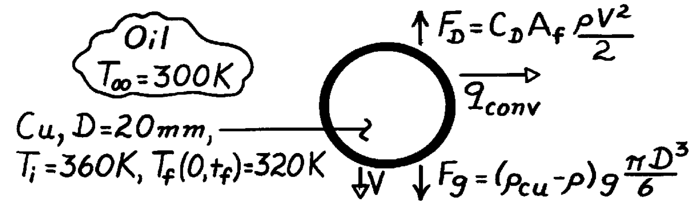
COMMENTS: If t_f is evaluated from the approximate series solution, $q_o^* = C_1 \exp\left(-z_1^2\ Fo\right)$, we obtain $t_f = 0.76$ s. Note that the terminal velocity is not reached immediately. Reduced V implies reduced $\bar{h}$ and increased t_f.

PROBLEM 7.74

KNOWN: Diameter and initial and final temperatures of copper spheres quenched in an oil bath.

FIND: (a) Terminal velocity in bath, (b) Bath height.

SCHEMATIC:



ASSUMPTIONS: (1) Sphere descends at terminal velocity, (2) Uniform, but time varying, surface temperature.

PROPERTIES: *Table A-1*, Copper (350K): ρ_{cu} =8933 kg/m^3, k = 398 W/m·K, c_p = 387 J/kg·K; *Table A-5*, Oil (T_∞ = 300K): ρ = 884 kg/m^3, μ = 0.486 N·s/m^2, k = 0.145 W/m·K, Pr = 6400; ($T_s \approx$ 340K): μ = 0.0531 N·s/m^2.

ANALYSIS: (a) Force balance gives $C_D\left(\pi D^2/4\right)\rho V^2/2 = \left(\rho_{cu} - \rho\right) g\,\pi D^3/6$,

$$C_D\ V^2 = \frac{4D}{3}\frac{\rho_{cu} - \rho}{\rho}\ g = \frac{4 \times 0.02\ \text{m}}{3}\frac{8933 - 884}{884} 9.8\frac{\text{m}}{\text{s}^2} = 2.38\text{m}^2/\text{s}^2.$$

An iterative solution is needed, where C_D is obtained from Fig. 7.8 with

$$Re_D = \frac{VD}{\nu} = \frac{0.02\ \text{m}\ (V)}{(0.486/884)\ \text{m}^2/\text{s}} = 36.4\ V(\text{m/s}).$$

Convergence is achieved for $V \approx 1.1$ m/s <

for which Re_D = 40 and $C_D \approx$ 1.97. Using the Whitaker expression

$$\overline{Nu}_D = 2 + \left(0.4\ Re_D^{1/2} + 0.06\ Re_D^{2/3}\right) Pr^{0.4} \left(\mu/\mu_s\right)^{1/4}$$

$$\overline{Nu}_D = 2 + \left(0.4 \times 40^{1/2} + 0.06 \times 40^{2/3}\right)\ (6400)^{0.4}\left(0.486/0.0531\right)^{1/4} = 189.2$$

$$\overline{h} = \overline{Nu}_D\ k/D = 189.2 \times 0.145/0.02 = 1357\ \text{W/m}^2 \cdot \text{K}.$$

To determine applicability of the lumped capacitance method, find $Bi = \overline{h}\left(r_o/3\right)/k_{cu}$ =

$1357\ \text{W/m}^2 \cdot \text{K}\ (0.01\ \text{m}/3)/398\ \text{W/m} \cdot \text{K} = 0.011$. Hence lumped capacitance method can be used; from Eq. 5.5,

$$t_f = \frac{(\rho\ c)_{cu}\pi D^3/6}{\overline{h}\pi D^2}\ell n\frac{T_i - T_\infty}{T_f - T_\infty}$$

$$t_f = \frac{8933\ \text{kg/m}^3 \times 387\ \text{J/kg} \cdot \text{K}}{1357\ \text{W/m}^2 \cdot \text{K}}\frac{0.02\ \text{m}}{6}\ell n\frac{60}{20} = 9.33\ \text{s}.$$

Required tank height is H = t_f · V = 9.33 s × 1.1 m/s = 10.3 m. <

COMMENTS: (1) Whitaker correlation has been used well beyond its limits (Pr >> 380). Hence estimate of $\overline{h}$ is uncertain. (2) Since terminal velocity is not reached immediately,

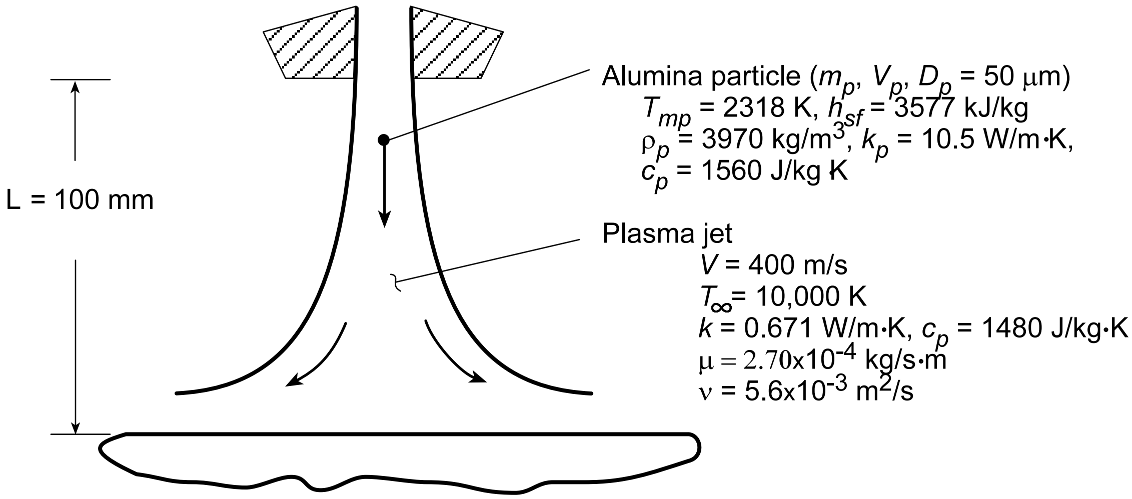
$\overline{h} < 1357\ \text{W/m}^2 \cdot \text{K}$ and $t_f > 9.33$ s.

PROBLEM 7.75

KNOWN: Velocity of plasma jet and initial particle velocity in a plasma spray coating process. Distance from particle injection to impact.

FIND: (a) Particle velocity and distance of travel as a function of time. Time-in-flight and particle impact velocity, (b) Convection heat transfer coefficient and time required to heat particle to melting point and to subsequently melt it.

SCHEMATIC:



ASSUMPTIONS: (1) Applicability of Stokes' law, (2) Constant particle and plasma properties, (3) Negligible influence of viscosity ratio in Whitaker correlation, (4) Negligible radiation effects, (5) Validity of lumped capacitance approximation.

ANALYSIS: (a) From Eqs. 7.54 and 7.58,

$$C_D \equiv \frac{F_D}{A_f\left(\rho \bar{V}^2/2\right)} = \frac{24}{Re_D} = \frac{24}{\rho \bar{V} D_p/\mu}$$

where $\bar{V} \equiv V - V_p$ is the relative velocity and $A_f = \pi D_p^2/4$. Hence, the drag force on the particle is

$$F_D = 3\pi\mu D_p \bar{V} = m_p\left(dV_p/dt\right) = -m_p\left(d\bar{V}/dt\right)$$

Separating variables and integrating from the nozzle exit, where $V_p = 0$, $\bar{V} = V$ and t = 0,

$$\int_V^{\bar{V}} \frac{d\bar{V}}{\bar{V}} = -\frac{3\pi\mu D_p}{m_p}\int_0^t dt$$

$$\ln\frac{\bar{V}}{V} = -\frac{3\pi\mu D_p t}{m_p}$$

$$\bar{V} = V\exp\left(-3\pi\mu D_p t/m_p\right) = V - V_p$$

Hence,

$$V_p(t) = V\left[1-\exp\left(-3\pi\mu D_p t/m_p\right)\right]$$ <

With $V_p = dx_p/dt$, it follows that

$$\int_o^L dx_p = \int_o^{t_f} V\left[1-\exp\left(-3\pi\mu D_p t/m_p\right)\right]dt$$

Continued...

$$L = Vt_f - \frac{Vm_p}{3\pi\mu D_p}\left[1 - \exp\left(-3\pi\mu D_p t_f / m_p\right)\right] \qquad <$$

Substituting the prescribed values of D_p, L, V and the material properties, the foregoing equations yield

$$V_p = 166.7\,\text{m/s} \qquad\qquad t_f = 0.0011\text{ s} \qquad <$$

(b) Assuming an average value of $\bar{V}$ = 315 m/s, the Reynolds number is

$$Re_D = \frac{315\,\text{m/s} \times 50 \times 10^{-6}\,\text{m}}{5.6 \times 10^{-3}\,\text{m}^2/\text{s}} = 2.81$$

From the Whitaker correlation,

$$\overline{Nu}_D = 2 + \left(0.4\,Re_D^{1/2} + 0.06\,Re_D^{2/3}\right)Pr^{0.4}$$

$$\overline{Nu}_D = 2 + \left(0.4 \times 2.81^{1/2} + 0.06 \times 2.81^{2/3}\right)(0.60)^{0.4} = 2.64$$

$$\bar{h} = 2.64k/D_p = 2.64\left(0.671\,\text{W/m}\cdot\text{K}\right)\Big/50 \times 10^{-6}\,\text{m} = 35,400\,\text{W}\big/\text{m}^2\cdot\text{K} \qquad <$$

The two-step melting process involves (i) the time t_1 to heat the particle to its melting point and (ii) the time t_2 required to achieve complete melting. Hence, $t_m = t_1 + t_2$, where from Eq. 5.5,

$$t_1 = \frac{\rho_p D_p c_p}{6\bar{h}} \ln\frac{T_i - T_\infty}{T_{mp} - T_\infty}$$

$$t_1 = \frac{3970\,\text{kg/m}^3\left(50 \times 10^{-6}\,\text{m}\right)1560\,\text{J/kg}\cdot\text{K}}{6\left(35,400\,\text{W/m}^2\cdot\text{K}\right)} \ln\frac{(300 - 10,000)}{(2318 - 10,000)} = 3.4 \times 10^{-4}\text{s}$$

Performing an energy balance for the second step, we obtain

$$\int_{t_1}^{t_m} q_{conv}dt = \Delta E_{st} = \rho_p \forall h_{sf}$$

Hence,

$$t_2 = \frac{\rho_p D_p}{6\bar{h}}\frac{h_{sf}}{\left(T_\infty - T_{mp}\right)} = \frac{3970\,\text{kg/m}^3\left(50 \times 10^{-6}\,\text{m}\right)}{6\left(35,400\,\text{W/m}^2\cdot\text{K}\right)} \times \frac{3.577 \times 10^6\text{ J/kg}}{(10,000 - 2318)\text{K}} = 4.4 \times 10^{-4}\text{s}$$

Hence,

$$t_m = \left(3.4 \times 10^{-4} + 4.4 \times 10^{-4}\right)\text{s} = 7.8 \times 10^{-4}\text{s} \qquad <$$

and the prescribed value of L is sufficient to insure complete melting before impact.

COMMENTS: (1) Since Bi = $\left(\bar{h}\,r_p/3\right)/k_p \approx 0.03$, use of the lumped capacitance approach is appropriate.

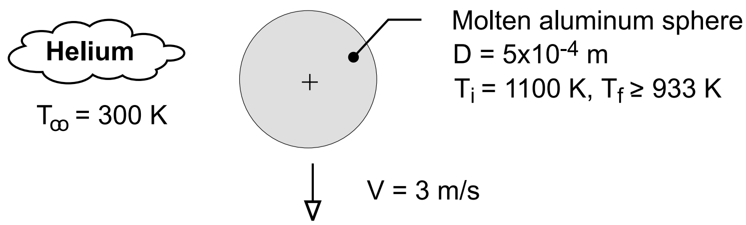
(2) With Re_D = 2.81, conditions are slightly outside the ranges associated with Stokes' law and the Whitaker correlation.

PROBLEM 7.76

KNOWN: Diameter, velocity, initial temperature and melting point of molten aluminum droplets. Temperature of helium atmosphere.

FIND: Maximum allowable separation between droplet injector and substrate.

SCHEMATIC:



ASSUMPTIONS: (1) Lumped capacitance approximation is valid, (2) Constant properties, (3) Negligible radiation.

PROPERTIES: *Table A-4*, Helium $(T_\infty = 300\text{K})$: $\nu = 122\times10^{-6}\,\text{m}^2/\text{s}$, $\mu = 199\times10^{-7}\,\text{N}\cdot\text{s/m}^2$, $k = 0.152\,\text{W/m}\cdot\text{K}$, $\text{Pr} = 0.68$. Helium $(T_s \approx 1000\text{K})$: $\mu_s = 446\times10^{-7}\,\text{N}\cdot\text{s/m}^2$. Given, Aluminum: $\rho = 2500\,\text{kg/m}^3$, $c = 1200\,\text{J/kg}\cdot\text{K}$, $k = 200\,\text{W/m}\cdot\text{K}$.

ANALYSIS: With $\text{Re}_D = VD/\nu = 3\,\text{m/s}\left(5\times10^{-4}\,\text{m}\right)/122\times10^{-6}\,\text{m}^2/\text{s} = 12.3$, the Whitaker correlation yields

$$\bar{h} = \frac{k}{D}\left[2 + \left(0.4\,\text{Re}_D^{1/2} + 0.06\,\text{Re}_D^{2/3}\right)\right]\text{Pr}^{0.4}\left(\mu/\mu_s\right)^{1/4}$$

$$\bar{h} = \frac{0.152\,\text{W/m}\cdot\text{K}}{0.0005\text{m}}\left\{2 + \left[0.4(12.3)^{1/2} + 0.06(12.3)^{2/3}\right](0.68)^{0.4}\left(\frac{199}{446}\right)^{1/4}\right\} = 975\,\text{W/m}^2\cdot\text{K}$$

The *time-of-flight* for the droplet to cool from 1100K to 933K may be obtained from Eq. 5.5.

$$t = \frac{\rho \forall c}{\bar{h} A_s}\ln\frac{\theta_i}{\theta} = \frac{\rho c D}{6\bar{h}}\ln\frac{T_i - T_\infty}{T_f - T_\infty}$$

$$t = \frac{\left(2500\,\text{kg/m}^3\right)1200\,\text{J/kg}\cdot\text{K}\,(0.0005\text{m})}{6\times975\,\text{W/m}^2\cdot\text{K}}\ln\left(\frac{800}{633}\right) = 0.06\,\text{s}$$

The maximum separation is therefore

$$L = V\times t = 3\,\text{m/s}\times0.06\,\text{s} = 0.18\text{m} = 180\ \text{mm}$$ <

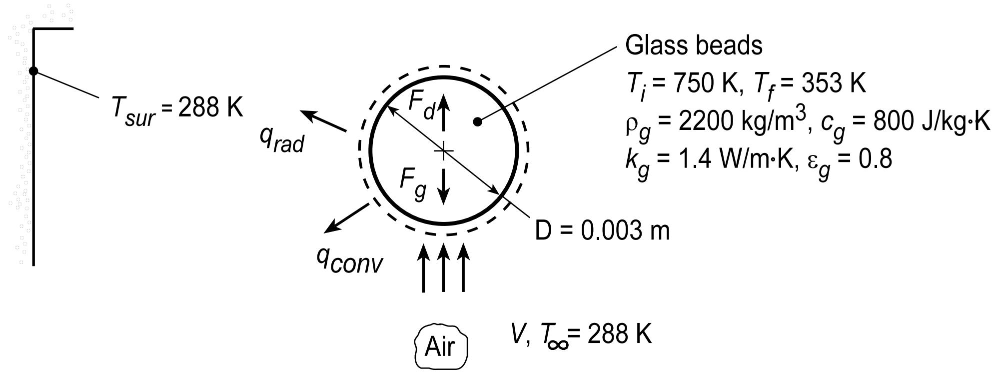
COMMENTS: (1) With $\text{Bi} = \bar{h}(D/6)/k = 4\times10^{-4}$, the lumped capacitance approximation is excellent. (2) With the surroundings assumed to be at $T_{sur} = T_\infty$ and a representative emissivity of ε = 0.1 for molten aluminum, $h_r \leq \varepsilon\sigma\left(T_i + T_\infty\right)\left(T_i^2 + T_\infty^2\right) \approx 10\,\text{W/m}^2\cdot\text{K} << \bar{h} = 975\,\text{W/m}^2\cdot\text{K}$. Hence, radiation is, in fact, negligible.

PROBLEM 7.77

KNOWN: Diameter, initial temperature and properties of glass beads suspended in an airstream of prescribed temperature.

FIND: (a) Velocity of airstream, (b) Time required to cool the beads from 477 to 80°C.

SCHEMATIC:



ASSUMPTIONS: (1) Lumped capacitance approximation may be used, (2) Constant properties, (3) Radiation exchange is with large surroundings at $T_{sur} = T_\infty$.

PROPERTIES: *Table A.4*, Air (T_∞ = 288 K): ρ = 1.21 kg/m^3, $\nu = 14.8 \times 10^{-6}$ m^2/s, $\mu = 179 \times 10^{-7}$ N·s/m^2, k = 0.0253 W/m·K, Pr = 0.71.

ANALYSIS: (a) Using Eq. 7.44 with the force balance, $F_g = F_d$,

$$\rho_g\left(\pi D^3/6\right)g = C_D\left(\pi D^2/4\right)\left(\rho V^2/2\right)$$

$$V = \left(\frac{4}{3}\times\frac{\rho_g}{\rho}\times\frac{gD}{C_D}\right)^{1/2} = \left(\frac{4}{3}\times\frac{2200}{1.21}\times\frac{9.8\,m/s^2\times0.003\,m}{C_D}\right)^{1/2} = \frac{8.44}{C_D^{1/2}}$$

Also,

$$Re_D = \frac{VD}{\nu} = \frac{V(0.003\,m)}{14.8\times10^{-6}\,m^2/s} = 202.7\,V$$

From Fig. 7.8, the foregoing results yield $C_D \approx 0.4$, for which

$$V \approx 13.3 \text{ m/s}$$ <

and $Re_D \approx 2700$.

(b) Applying an energy balance to a control surface about the bead, Eq. 5.15 may be obtained, with $\dot{E}_g$ = 0, $q_s'' = 0$, $A_{s(c,r)} = \pi D^2$, and $\forall = \pi D^3/6$. Hence,

$$\rho_g c_g \frac{dT}{dt} = -(6/D)\left[\bar{h}(T - T_\infty) + \varepsilon_g\sigma\left(T^4 - T_{sur}^4\right)\right]$$

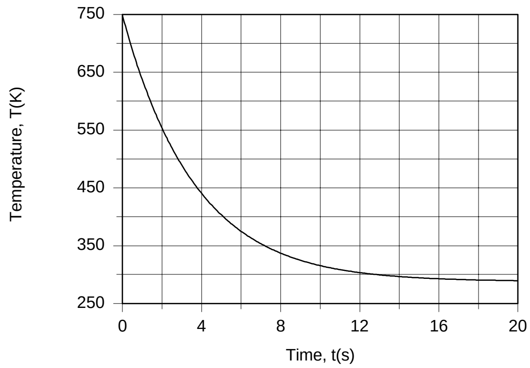
where $\bar{h}$ is given by the Whitaker correlation,

$$\overline{Nu}_D = 2 + \left(0.4\,Re_D^{1/2} + 0.06\,Re_D^{2/3}\right)Pr^{0.4}\left(\mu/\mu_s\right)^{1/4}$$

Using the *IHT Lumped Capacitance Model* with the appropriate *Correlations* and *Properties* Tool Pads, the foregoing integration was evaluated numerically, and the following temperature history was obtained.

Continued...

PROBLEM 7.77 (Cont.)



The desired temperature of T = 80°C = 353 K is obtained at t = 7s, and at t = 20s the temperature is within 1.5°C of ambient conditions.

COMMENTS: (1) With Bi = $\left(\bar{h} + h_{rad}\right) r_o / k$ = (218 + 30) W/m^2·K(0.0005 m)/1.4 W/m·K = 0.089 at T = 750 K, the lumped capacitance assumption is satisfactory and becomes increasingly better as h_{rad} decreases with decreasing T.

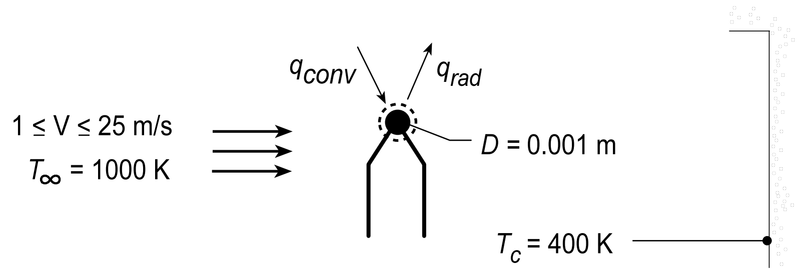
(2) The small bead diameter and large velocity provide a large convection coefficient, which insures rapid cooling to the desired temperature. Even at the maximum temperature (T = 750 K), h_{rad} = 30 W/m^2·K makes a small contribution to the cooling process.

PROBLEM 7.78

KNOWN: Velocity and temperature of combustion gases. Diameter and emissivity of thermocouple junction. Combustor temperature.

FIND: (a) Time to achieve 98% of maximum thermocouple temperature rise, (b) Steady-state thermocouple temperature, (c) Effect of gas velocity and thermocouple emissivity on measurement error.

SCHEMATIC:



ASSUMPTIONS: (1) Validity of lumped capacitance analysis, (2) Constant properties, (3) Negligible conduction through lead wires, (4) Radiation exchange between small surface and a large enclosure (parts b and c).

PROPERTIES: Thermocouple (given): $0.1 \le \varepsilon \le 1.0$, k = 100 W/m·K, c = 385 J/kg·K, $\rho = 8920$ kg/m^3; Gases (given): k = 0.05 W/m·K, $\nu = 50 \times 10^{-6}$ m^2/s, Pr = 0.69.

ANALYSIS: (a) If the lumped capacitance analysis may be used, it follows from Equation 5.5 that

$$t = \frac{\rho V c}{\bar{h} A_s} \ln \frac{T_i - T_\infty}{T - T_\infty} = \frac{D\rho c}{6\bar{h}} \ln(50).$$

Neglecting the viscosity ratio correlation for variable property effects, use of V = 5 m/s with the Whitaker correlation yields

$$\overline{Nu}_D = \left(\bar{h}D/k\right) = 2 + \left(0.4\, Re_D^{1/2} + 0.06\, Re_D^{2/3}\right) Pr^{0.4} \qquad Re_D = \frac{VD}{\nu} = \frac{5\,m/s\,(0.001\,m)}{50\times10^{-6}\,m^2/s} = 100$$

$$\bar{h} = \frac{0.05\,W/m\cdot K}{0.001\,m}\left[2 + \left(0.4(100)^{1/2} + 0.06(100)^{2/3}\right)(0.69)^{0.4}\right] = 328\,W/m^2\cdot K$$

Since Bi = $\bar{h}\left(r_o/3\right)/k$ = 5.5×10^{-4}, the lumped capacitance method may be used. Hence,

$$t = \frac{0.001\,m\left(8920\,kg/m^3\right)385\,J/kg\cdot K}{6\times 328\,W/m^2\cdot K}\ln(50) = 6.83 s \qquad <$$

(b) Performing an energy balance on the junction and evaluating radiation exchange from Equation 1.7, $q_{conv} = q_{rad}$. Hence, with $\varepsilon = 0.5$,

$$\bar{h}A_s\left(T_\infty - T\right) = \varepsilon A_s \sigma\left(T^4 - T_c^4\right)$$

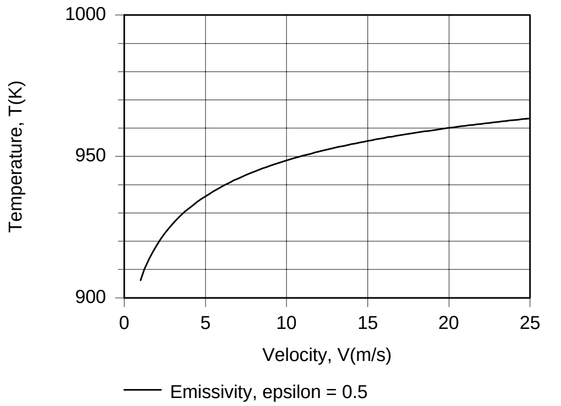
$$(1000 - T)K = \frac{0.5\times5.67\times10^{-8}\,W/m^2\cdot K^4}{328\,W/m^2\cdot K}\left[T^4 - (400)^4\right]K^4.$$

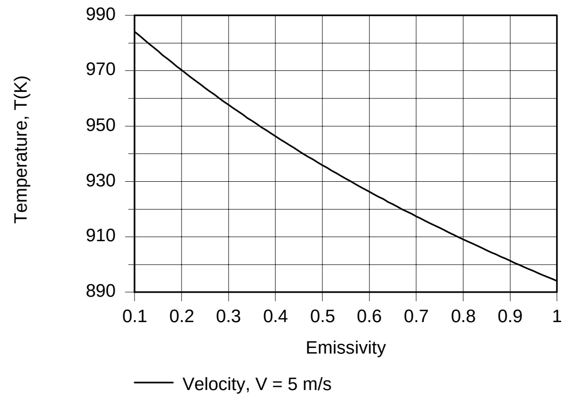
$$T = 936\,K \qquad <$$

(c) Using the *IHT First Law Model* for a *Solid Sphere* with the appropriate *Correlation* for external flow from the Tool Pad, parametric calculations were performed to determine the effects of V and ε_g, and the following results were obtained.

Continued...

PROBLEM 7.78 (Cont.)





Since the temperature recorded by the thermocouple junction increases with increasing V and decreasing ε, the measurement error, T_∞ - T, decreases with increasing V and decreasing ε. The error is due to net radiative transfer from the junction (which depresses T) and hence should decrease with decreasing ε. For a prescribed heat loss, the temperature difference (T_∞ - T) decreases with decreasing convection resistance, and hence with increasing h(V).

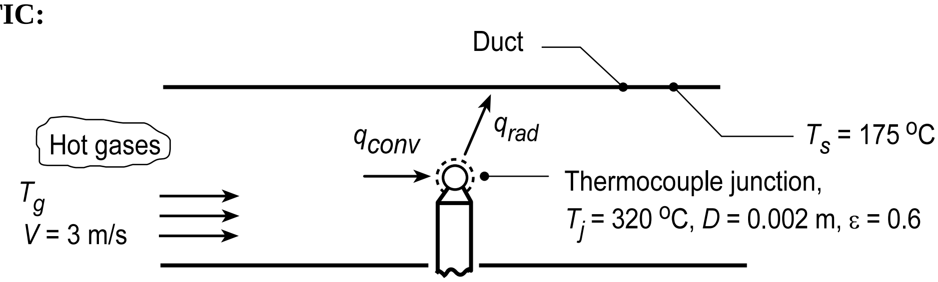
COMMENTS: To infer the actual gas temperature (1000 K) from the measured result (936 K), correction would have to be made for radiation exchange with the cold surroundings.

PROBLEM 7.79

KNOWN: Diameter, emissivity and temperature of a thermocouple junction exposed to hot gases flowing through a duct of prescribed surface temperature.

FIND: (a) Relative magnitudes of gas and thermocouple temperatures if the duct surface temperature is less than the gas temperature, (b) Gas temperature for prescribed conditions, (c) Effect of Velocity and emissivity on measurement error.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Junction is diffuse-gray, (3) Duct forms a large enclosure about the junction, (4) Negligible heat transfer by conduction through the thermocouple leads, (5) Gas properties are those of atmospheric air.

PROPERTIES: *Table A-4*, Air ($T_g \approx 650$ K, 1 atm): $\nu = 60.21 \times 10^{-6}$ m^2/s, k = 0.0497 W/m·K, Pr = 0.690, $\mu = 322.5 \times 10^{-7}$ N·s/m^2; Air ($T_j = 593$ K, 1 atm): $\mu = 304 \times 10^{-7}$ N·s/m^2.

ANALYSIS: (a) From an energy balance on the thermocouple junction, $\underset{(g\to j)}{q_{conv}} = \underset{(j\to s)}{q_{rad}}$. Hence,

$$\bar{h}A\left(T_g - T_j\right) = \varepsilon\sigma A\left(T_j^4 - T_s^4\right) \qquad \text{or} \qquad T_g - T_j = \frac{\varepsilon}{\bar{h}}\sigma\left(T_j^4 - T_s^4\right).$$

If $T_s < T_j$, it follows that $T_j < T_g$. <

(b) Neglecting the variable property correction, $\left(\mu/\mu_s\right)^{1/4} = \left(322.5/304\right)^{1/4}$ = 1.01 ≈ 1.00, and using

$$Re_D = \frac{VD}{\nu} = \frac{3\,\text{m/s}\left(0.002\,\text{m}\right)}{60.21\times10^{-6}\,\text{m}^2/\text{s}} = 100$$

the Whitaker correlation for a sphere gives

$$\bar{h} = \frac{0.0497\,\text{W/m}\cdot\text{K}}{0.002\,\text{m}}\left\{2+\left[0.4\left(100\right)^{1/2}+0.06\left(100\right)^{2/3}\right]\left(0.69\right)^{0.4}\right\} = 163\,\text{W/m}^2\cdot\text{K}.$$

Hence

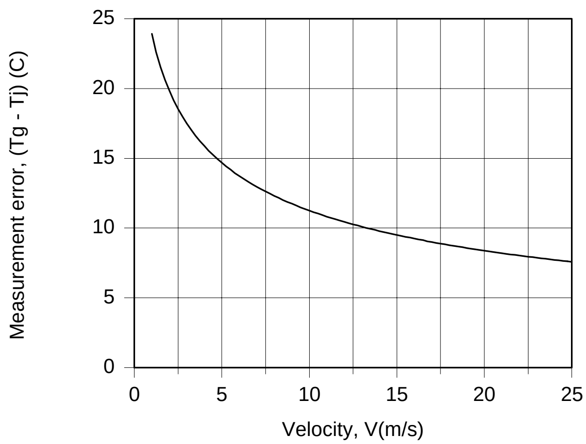
$$\left(T_g - 593\,\text{K}\right) = \frac{0.6}{163\,\text{W/m}^2\cdot\text{K}}5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left[\left(593\,\text{K}\right)^4-\left(448\,\text{K}\right)^4\right] = 17\,\text{K}$$

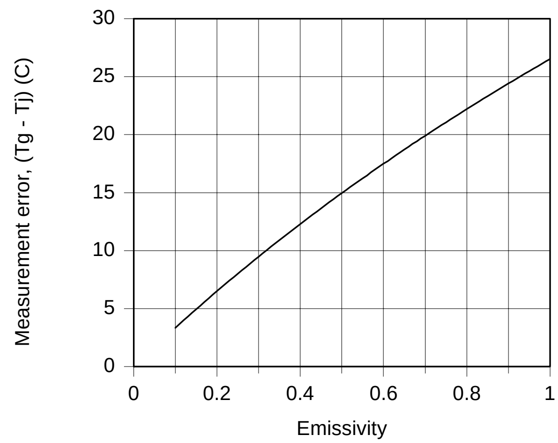
$$T_g = 610\,\text{K} = 337^\circ\text{C}.$$ <

(c) With T_g fixed at 610 K, the IHT *First Law* Model was used with the *Correlations* and *Properties* Tool Pads to compute the measurement error as a function of V and ε.

Continued...

PROBLEM 7.79 (Cont.)





Since the convection resistance decreases with increasing V, the junction temperature will approach the gas temperature and the measurement error will decrease. Since the depression in the junction temperature is due to radiation losses from the junction to the duct wall, a reduction in ε will reduce the measurement error.

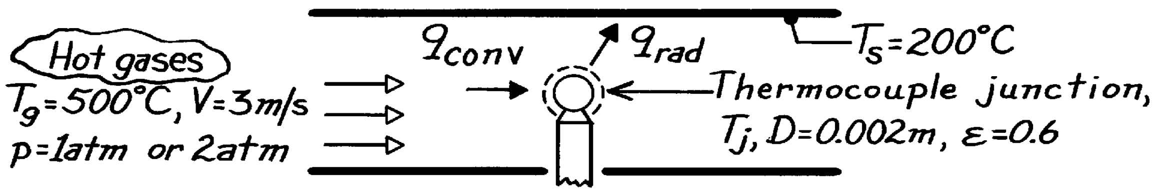
COMMENTS: In part (b), calculations could be improved by evaluating properties at 610 K (instead of 650 K).

PROBLEM 7.80

KNOWN: Diameter and emissivity of a thermocouple junction exposed to hot gases of prescribed velocity and temperature flowing through a duct of prescribed surface temperature.

FIND: (a) Thermocouple reading for gas at atmospheric pressure, (b) Thermocouple reading when gas pressure is doubled.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Junction is diffuse-gray, (3) Duct forms a large enclosure about junction, (4) Negligible heat loss by conduction through thermocouple leads, (5) Gas properties are those of air, (6) Perfect gas behavior.

PROPERTIES: *Table A-4*, Air (T_g = 773 K, 1 atm): $\nu = 80.5 \times 10^{-6}$ m^2/s, k = 0.0561 W/m·K, Pr = 0.705.

ANALYSIS: (a) Performing an energy balance on the junction

$$\underset{(g\to j)}{q_{conv}} = \underset{(j\to s)}{q_{rad}}$$

$$\bar{h}A\left(T_g - T_j\right) = \varepsilon\sigma\, A\left(T_j^4 - T_s^4\right).$$

Neglecting the variable property correction, $(\mu/\mu_s)^{1/4}$, and using

$$Re_D = \frac{VD}{\nu} = \frac{3\text{ m/s}\times 0.002\text{ m}}{80.5\times10^{-6}\text{ m}^2/\text{s}} = 74.5$$

the Whitaker correlation for a sphere gives,

$$\bar{h} = \frac{0.0561\text{ W/m}\cdot\text{K}}{0.002\text{ m}}\left\{2+\left[0.4(74.5)^{1/2}+0.06(74.5)^{2/3}\right](0.705)^{0.4}\right\} = 166\text{ W/m}^2\cdot\text{K}.$$

$$166\left(773-T_j\right) = 0.6\times5.67\times10^{-8}\left[T_j^4-(473)^4\right]$$

and from a trial-and-error solution,

$$T_j \approx 726\text{ K}.$$ <

(b) Assuming all properties other than ν to remain constant with a change in pressure, ↑ p by 2 will ↓ ν by 2 and hence ↑ Re_D by 2, giving Re_D = 149. Hence

$$\bar{h} = \frac{0.0561}{0.002}\left\{2+\left[0.4(149)^{1/2}+0.06(149)^{2/3}\right](0.705)^{0.4}\right\} = 216\text{ W/m}^2\cdot\text{K}.$$

$$216\left(773-T_j\right) = 0.6\times5.67\times10^{-8}\left[T_j^4-(473)^4\right]$$

and from a trial-and-error solution

$$T_j \approx 735\text{ K}.$$ <

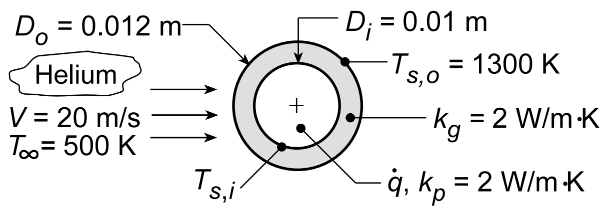
COMMENTS: The thermocouple error will ↓ with ↑ h, which ↑ with ↑p.

PROBLEM 7.81

KNOWN: Velocity and temperature of helium flow over graphite coated uranium oxide pellets. Pellet and coating diameters and thermal conductivity. Surface temperature of coating.

FIND: (a) Rate of heat transfer, (b) Volumetric generation rate in pellet and pellet surface temperature, (c) Radial temperature distribution in pellet, (d) Effect of gas velocity on center and surface temperatures.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady conduction in the radial direction, (2) Uniform generation, (3) Constant properties, (4) Negligible radiation, (5) Negligible contact resistance.

PROPERTIES: *Table A.4*, Helium (T_∞ = 500 K, 1 atm): $\nu = 290 \times 10^{-6}$ m²/s, k = 0.22 W/m·K, Pr = 0.67, $\mu = 283 \times 10^{-7}$ N·s/m²; ($T_{s,o}$ = 1300 K, with extrapolation): $\mu = 592 \times 10^{-7}$ N·s/m².

ANALYSIS: (a) The heat transfer rate is $q = \bar{h} A_s \left(T_{s,o} - T_\infty\right)$, where the convection coefficient can be estimated from $\overline{Nu}_D = 2 + \left(0.4\,Re_D^{1/2} + 0.06\,Re_D^{2/3}\right) Pr^{0.4} \left(\mu_\infty / \mu_s\right)^{1/4}$, where

$$Re_D = \frac{VD_o}{\nu} = \frac{20\,\text{m/s} \times 0.012\,\text{m}}{290 \times 10^{-6}\,\text{m}^2/\text{s}} = 828$$

$$\overline{Nu}_D = 2 + \left[0.4(828)^{1/2} + 0.06(828)^{2/3}\right](0.67)^{0.4}(283/592)^{1/4} = 13.9$$

$$\bar{h} = \frac{k}{D_o}\overline{Nu}_D = \frac{0.22\,\text{W/m}\cdot\text{K}}{0.012\,\text{m}} \times 13.9 = 255\,\text{W/m}^2\cdot\text{K}.$$

Hence, $q = 255\,\text{W/m}^2\cdot\text{K} \times \pi (0.012\,\text{m})^2 (1300 - 500)\,\text{K} = 92.2\,\text{W}$. <

(b) The volumetric heat rate in the pellet is

$$\dot{q} = \frac{q}{\pi D_i^3/6} = \frac{6 \times 92.2\,\text{W}}{\pi (0.01\,\text{m})^3} = 1.76 \times 10^8\,\text{W/m}^3$$ <

The inner surface temperature of the coating is equal to the pellet surface temperature,

$$T_{s,i} - T_{s,o} = q\frac{1}{4\pi k_g}\left(\frac{1}{r_i} - \frac{1}{r_o}\right) = \frac{92.2\,\text{W}}{4\pi(2\,\text{W/m}\cdot\text{K})}\left(\frac{1}{0.005\,\text{m}} - \frac{1}{0.006\,\text{m}}\right) = 122.3\,\text{K}$$

$$T_{s,i} = 1300\,\text{K} + 122.3\,\text{K} = 1422\,\text{K}.$$ <

(c) The heat equation for the spherical pellet reduces to

$$\frac{k_p}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = -\dot{q}$$

Integrating twice,

$$r^2\frac{dT}{dr} = -\frac{\dot{q}}{3k_p}r^3 + C_1 \qquad \frac{dT}{dr} = -\frac{\dot{q}}{3k_p}r + \frac{C_1}{r^2}$$

Continued...

$$T = -\frac{\dot{q}}{6k_p}r^2 - \frac{C_1}{r} + C_2 .$$

Applying boundary conditions,

$r = 0$: $\quad dT/dr)_{r=0} = 0 \quad \rightarrow \quad C_1 = 0$

$r = r_i$: $\quad T(r_i) = T_{s,i} \quad \rightarrow \quad C_2 = T_{s,i} + (\dot{q}/6k_p)r_i^2 .$

Hence the temperature distribution is

$$T(r) = T_{s,i} + (\dot{q}/6k_p)(r_i^2 - r^2) = T(0) - (\dot{q}/6k_p)r^2$$ <

where the temperature at the pellet center is $T(0) = T_{s,i} + (\dot{q}/6k_p)r_i^2$.

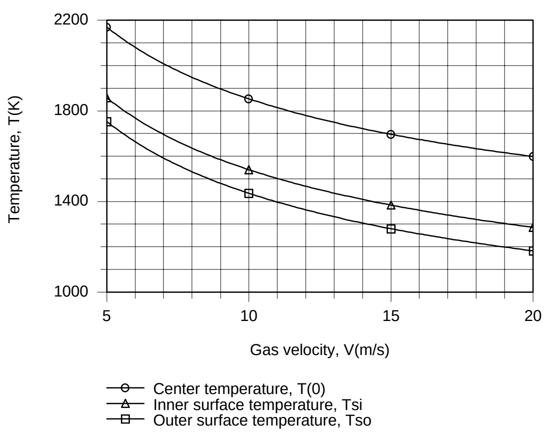
For the prescribed conditions,

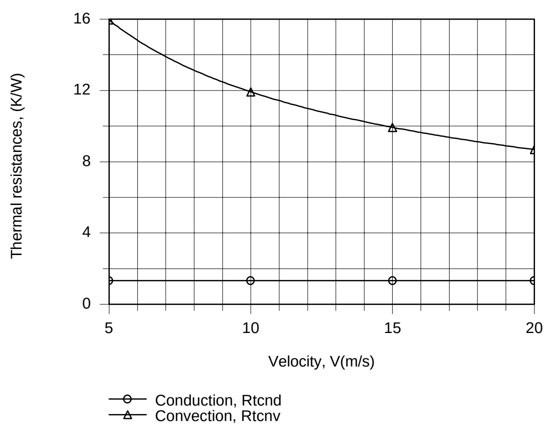
$$T(0) = 1422\,K + \left(1.76\times10^8\,W/m^3 / 6\times2\,W/m\cdot K\right)(0.005\,m)^2 = 1789\,K .$$

(d) With $\dot{q} = 1.5\times10^8\,W/m^3$, parametric calculations were performed using the IHT Model for *One-Dimensional*, *Steady-State Conduction* in a sphere, with the surface condition, $q''(r_i) = (T_{s,i} - T_\infty)/R''_{t,i}$, where the total thermal resistance, $R_{t,i} = R''_{t,i}/4\pi r_i^2$, is

$$R_{t,i} = R_{tcnd} + R_{tcnv} = \frac{(1/r_i) - (1/r_o)}{4\pi k_p} + \frac{1}{4\pi r_o^2 \overline{h}}$$

The *Correlations* and *Properties* Tool Pads were used to evaluate the convection coefficient, and the following results were obtained.





As expected, all temperatures increase with decreasing V, while fixed values of $\dot{q}$, and hence $q(r_i)$, and R_{tcnd} provide fixed values of $(T(0) - T_{s,i})$ and $(T_{s,i} - T_{s,o})$, respectively.

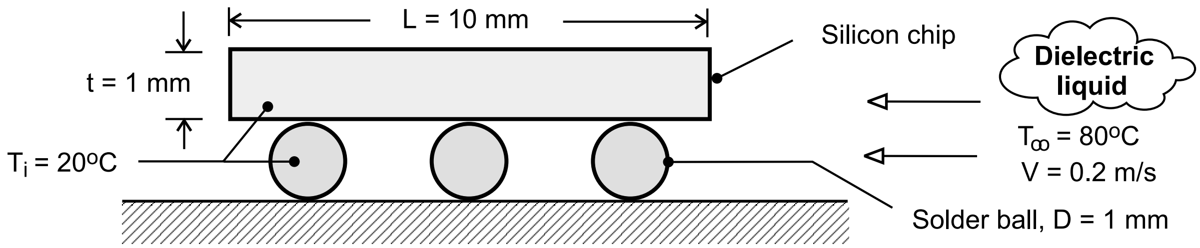
COMMENTS: In a more detailed analysis, radiation heat transfer, which would decrease the temperatures, should be considered.

PROBLEM 7.82

KNOWN: Initial temperature, dimensions and properties of chip and solder connectors. Velocity, temperature and properties of liquid.

FIND: (a) Ratio of time constants (chip-to-solder), (b) Chip-to-solder temperature difference after 0.25s of heating.

SCHEMATIC:



ASSUMPTIONS: (1) Solder balls and chips are spatially isothermal, (2) Negligible heat transfer from sides of chip, (3) Top and bottom surfaces of chip act as flat plates in turbulent parallel flow, (4) Heat transfer from solder balls may be approximated as that from an isolated sphere, (5) Constant properties.

PROPERTIES: Given. Dielectric liquid: $k = 0.064\,W/m\cdot K$, $\nu = 10^{-6}\,m^2/s$, $Pr = 25$; Silicon chip: $k = 150\,W/m\cdot K$, $\rho = 2300\,kg/m^3$, $c_p = 700\,J/kg\cdot K$; Solder ball: $k = 40\,W/m\cdot K$, $\rho = 10,000\,kg/m^3$, $c_p = 150\,J/kg\cdot K$.

ANALYSIS: (a) From Eq. 5.7, the thermal time constant is $\tau_t = (\rho\forall c/\bar{h}A_s)$. Hence,

$$\frac{\tau_{t,ch}}{\tau_{t,sld}} = \frac{(\rho c)_{ch}(L^2 t)}{2\bar{h}_{ch}L^2}\,\frac{\bar{h}_{sld}(\pi D^2)}{(\rho c)_{sld}(\pi D^3/6)} = 3\frac{t}{D}\,\frac{(\rho c)_{ch}}{(\rho c)_{sld}}\,\frac{\bar{h}_{sld}}{\bar{h}_{ch}}$$

The convection coefficient for the chip may be obtained from Eq. 7.44, with $Re_L = VL/\nu = 0.2\,m/s \times 0.01m/10^{-6}\,m^2/s = 2000.$

$$\bar{h}_{ch} = \frac{0.064\,W/m\cdot K}{0.01m}(0.037)(2000)^{4/5}(25)^{1/3} = 302\,W/m^2\cdot K$$

The convection coefficient for the solder may be obtained from Eq. 7.59, with $Re_D = VD/\nu$ $= 0.2\,m/s \times 0.001m/10^{-6}\,m^2/s = 200.$ Neglecting the effect of the viscosity ratio,

$$\bar{h}_{sld} = \frac{0.064\,W/m\cdot K}{0.001m}\left\{2+\left[0.4(200)^{1/2}+0.06(200)^{2/3}\right](25)^{0.4}\right\} = 1916\,W/m^2\cdot K$$

Hence, $$\frac{\tau_{t,ch}}{\tau_{t,sld}} = 3\left(\frac{2300\,kg/m^3 \times 700\,J/kg\cdot K}{10,000\,kg/m^3 \times 150\,J/kg\cdot K}\right)\frac{1916\,W/m^2\cdot K}{302\,W/m^2\cdot K} = 20.4$$ <

Hence, the solder responds much more quickly to the convective heating.

(b) From Eq. 5.6, the chip-to-solder temperature difference may be expressed as

Continued …..

$$T_{ch} - T_{sld} = \left(T_i - T_\infty\right)\left\{\exp\left[-\left(\frac{2\bar{h}}{\rho c t}\right)_{ch} t\right] - \exp\left[-\left(\frac{6\bar{h}}{\rho c D}\right)_{sld} t\right]\right\}$$

$$T_{ch} - T_{sld} = 60°C\left\{\exp\left[-\frac{604\,W/m^2\cdot K}{1610\,J/m^2\cdot K}0.25\,s\right] - \exp\left[-\frac{11,496\,W/m^2\cdot K}{1500\,J/m^2\cdot K}0.25\,s\right]\right\}$$

$$T_{ch} - T_{sld} = 60°C\{0.910 - 0.147\} = 45.8°C \qquad <$$

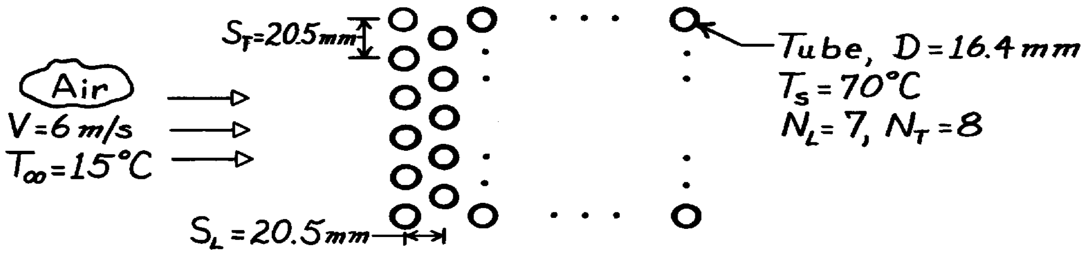
COMMENTS: (1) The foregoing process is used to subject soldered chip connections (a major reliability issue) to rapid and intense thermal stresses. (2) Some heat transfer by conduction will occur between the chip and solder balls, thereby reducing the temperature difference and thermal stress. (3) Constriction of flow between the chip and substrate will reduce $\bar{h}_{sld}$, as well as $\bar{h}_{ch}$ at the lower surface of the chip, relative to values predicted by the correlations. The corresponding time constants would be increased accordingly. (4) With $Bi_{ch} = \bar{h}_{ch}(t/2)/k_{chip} = 0.001 << 1$ and $Bi_{sld} = \bar{h}_{sld}(D/6)/k_{sld} = 0.008 << 1$, the lumped capacitance analysis is appropriate for both components.

PROBLEM 7.83

KNOWN: Conditions associated with Example 7.6, but with reduced longitudinal and transverse pitches.

FIND: (a) Air side convection coefficient, (b) Tube bundle pressure drop, (c) Heat rate.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform tube surface temperature.

PROPERTIES: *Table A-4*, Atmospheric air (T_∞ = 288 K): ρ = 1.217 kg/m^3, ν = 14.82 × 10^{-6} m^2/s, k = 0.0253 W/m·K, Pr = 0.71, c_p = 100.7 J/kg·K; (T_s = 343 K): Pr = 0.701.

ANALYSIS: (a) From the tube pitches, find

$$S_D = \left[S_L^2 + \left(S_T / 2\right)^2\right]^{1/2} = \left[\left(20.5\right)^2 + \left(10.25\right)^2\right]^{1/2} = 22.91 \text{ mm}$$

$$\left(S_T + D\right)/2 = \left(20.5 + 16.4\right)/2 = 18.45 \text{ mm.}$$

Hence, the maximum velocity occurs on the transverse plane, and

$$V_{max} = \frac{S_T}{S_T - D} V = \frac{20.5 \text{ mm}}{\left(20.5 - 16.4\right) \text{ mm}} 6 \text{ m/s} = 30 \text{ m/s.}$$

With $$Re_{D,max} = \frac{V_{max} D}{n} = \frac{30 \text{ m/s}\left(0.0164 \text{ m}\right)}{14.82 \times 10^{-6} \text{ m}^2/\text{s}} = 3.32 \times 10^4$$

and (S_T/S_L) = 1 < 2, it follows from Table 7.7 that

$$C = 0.35 \qquad m = 0.60.$$

Hence, from the Zhukauskas correlation and Table 7.8 (C_2 = 0.95),

$$\overline{Nu}_D = \left(0.95\right) 0.35\, Re_{D,max}^{0.6}\, Pr^{0.36} \left(Pr/Pr_s\right)^{1/4}$$

$$\overline{Nu}_D = \left(0.95\right) 0.35 \left(3.32 \times 10^4\right)^{0.6} \left(0.71\right)^{0.36} \left(0.71/0.701\right)^{1/4} = 152$$

$$\bar{h} = \overline{Nu}_D \frac{k}{D} = 152 \times \frac{0.0253 \text{ W/m} \cdot \text{K}}{0.0164 \text{ m}} = 234 \text{ W/m}^2 \cdot \text{K.}$$ <

(b) From the Zhukauskas relation

$$\Delta p = N_L c \left(\frac{r V_{max}^2}{2}\right) f.$$

With $Re_{D,max}$ = 3.32 × 10^4, P_T = (S_T/D) = 1.25 and (P_T/P_L) = 1, it follows from Fig. 7.14 that

$$\chi \approx 1.02 \qquad f \approx 0.38.$$

Continued …..

Hence

$$\Delta p = 7 \times 1.02 \frac{1.217\ \text{kg/m}^3 (30\ \text{m/s})^2}{2} 0.38 = 1490\ \text{N/m}^2$$

$$\Delta p = 0.0149\ \text{bar.}$$ <

(c) The air outlet temperature is obtained from

$$T_s - T_o = (T_s - T_i) \exp\left(-\frac{\pi D N \bar{h}}{\rho V N_t S_t c_p} \right)$$

$$T_s - T_o = 55^\circ\text{C} \exp\left(\frac{-\pi\ (0.0164\ \text{m}) 56 \left(234\ \text{W/m}^2 \cdot \text{K}\right)}{1.217\ \text{kg/m}^3 \times 6\ \text{m/s} \times 8 \times 0.0205\ \text{m} \times 1007\ \text{J/kg} \cdot \text{K}} \right)$$

$$T_s - T_o = 31.4^\circ\text{C}$$

$$T_o = 38.5^\circ\text{C.}$$ <

The log mean temperature difference is

$$\Delta T_{\ell m} = \frac{\Delta T_i - \Delta T_o}{\ell n\,(\Delta T_i / \Delta T_o)} = \frac{(55 - 31.4)^\circ\ \text{C}}{\ell n\,(55/31.4)} = 42.1^\circ\text{C}$$

$$q' = N \bar{h} \pi D \Delta T_{\ell m} = 56 \left(234\ \text{W/m}^2 \cdot \text{K}\right) \pi\ (0.0164\ \text{m})\ 42.1^\circ\text{C}$$

$$q' = 28.4\ \text{kW/m.}$$ <

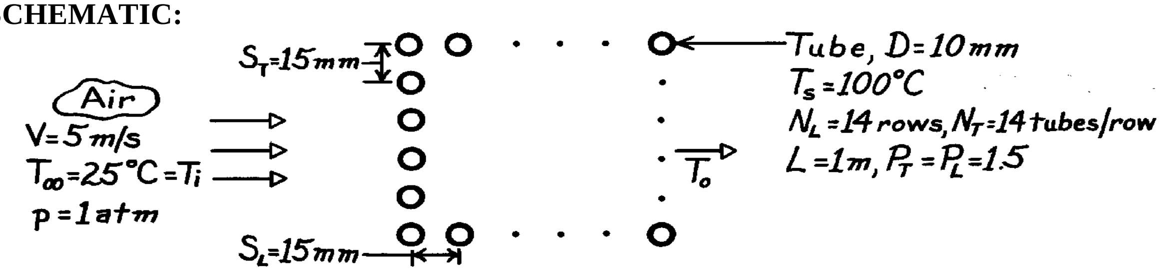
COMMENTS: Making the tube bank more compact has the desired effect of increasing the convection coefficient and therefore the heat transfer rate. However, it has the adverse effect of increasing the pressure drop and hence the fan power requirement. Note that the convection coefficient increases by a factor of (234/135.6) = 1.73, while the pressure drop increases by a factor of (1490/246) = 6.1. This disparity is a consequence of the fact that $\bar{h} \sim V_{max}^{0.6}$, while $\Delta p \sim V_{max}^2$. Hence any increase in V_{max}, which would result from a more closely spaced arrangement, would more adversely affect Δp than favorably affect $\bar{h}$.

PROBLEM 7.84

KNOWN: Surface temperature and geometry of a tube bank. Velocity and temperature of air in cross flow.

FIND: (a) Total heat transfer, (b) Air flow pressure drop.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible radiation, (3) Uniform surface temperature.

PROPERTIES: *Table A-4*, Atmospheric air (T_∞ = 298 K): $\nu = 15.8 \times 10^{-6}$ m^2/s, k = 0.0263 W/m·K, Pr = 0.707, c_p = 1007 J/kg·K, ρ = 1.17 kg/m^3; (T_s = 373 K): Pr = 0.695.

ANALYSIS: (a) The total heat transfer rate is

$$q = \bar{h}N p DL \frac{(T_s - T_i) - (T_s - T_o)}{\ell n\left[(T_s - T_i)/(T_s - T_o)\right]} = \bar{h}N p DL\ \Delta T_{\ell m}.$$

With $V_{max} = \dfrac{S_T}{S_T - D}V = \dfrac{15\text{ mm}}{5\text{ mm}} 5\text{ m/s} = 15\text{ m/s}$, $Re_{D,max} = \dfrac{15\text{ m/s}(0.01\text{ m})}{15.8 \times 10^{-6}\text{ m}^2/\text{s}} = 9494$. Tables 7.7 and 7.8 give C = 0.27, m = 0.63 and $C_2 \approx 0.99$. Hence, from the Zhukauskas correlation

$$\overline{Nu}_D = 0.99 \times 0.27(9494)^{0.63}(0.707)^{0.36}(0.707/0.695)^{1/4} = 75.9$$

$$\bar{h} = \overline{Nu}_D\ k/D = 75.9 \times 0.0263\text{ W/m}\cdot\text{K}/0.01\text{ m} = 200\text{ W/m}^2\cdot\text{K}$$

$$T_s - T_o = (T_s - T_i)\exp\left(-\frac{p DN\bar{h}}{r VN_T S_T c_p}\right) = 75^\circ\text{C}\exp\left(-\frac{p \times 0.01\text{ m} \times 196 \times 200\text{ W/m}^2\cdot\text{K}}{1.17\text{ kg/m}^3 \times 5\text{ m/s} \times 14 \times 0.015\text{ m} \times 1007\text{ J/kg}\cdot\text{K}}\right)$$

$$T_s - T_o = 27.7^\circ\text{C}.$$

Hence

$$q = 200\text{ W/m}^2\cdot\text{K} \times 196 p\ (0.01\text{ m})\ 1\text{ m}\frac{75^\circ\text{C} - 27.7^\circ\text{C}}{\ell n(75/27.7)} = 58.5\text{ kW}.$$ <

(b) With $Re_{D,max}$ = 9494, $(P_T - 1)/(P_L - 1) = 1$, Fig. 7.13 yields f ≈ 0.32 and $\chi = 1$. Hence,

$$\Delta p = N c\left(r V_{max}^2/2\right) f = 14 \times 1\left(\frac{1.17\text{ kg/m}^3(15\text{ m/s})^2}{2}\right)0.32$$

$$\Delta p = 590\text{ N/m}^2 = 5.9 \times 10^{-3}\text{ bar}.$$

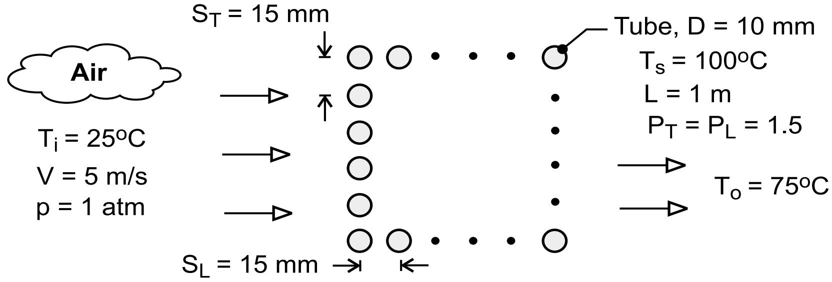
COMMENTS: The heat transfer rate would have been substantially overestimated (93.3 kW) if the inlet temperature difference (T_s - T_i) had been used in lieu of the log-mean temperature difference.

PROBLEM 7.85

KNOWN: Surface temperature and geometry of a tube bank. Inlet velocity and inlet and outlet temperatures of air in cross flow over the tubes.

FIND: Number of tube rows needed to achieve the prescribed outlet temperature and corresponding pressure of drop of air.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) Negligible temperature drop across tube wall and uniform outer surface temperature, (3) Constant properties, (4) $C_2 \approx 1$.

PROPERTIES: *Table A-4*, Atmospheric air. $\left(\overline{T} = (T_i + T_o)/2 = 323\text{K}\right)$: $\rho = 1.085\,\text{kg/m}^3$, $c_p = 1007\,\text{J/kg}\cdot\text{K}$, $\nu = 18.2\times10^{-6}\,\text{m}^2/\text{s}$, $k = 0.028\,\text{W/m}\cdot\text{K}$, $Pr = 0.707$; $(T_i = 298\text{K})$: $\rho = 1.17\,\text{kg/m}^3$; $(T_s = 373\text{K})$: $Pr_s = 0.695$.

ANALYSIS: The temperature difference $(T_s - T)$ decreases exponentially in the flow direction, and at the outlet

$$\frac{T_s - T_o}{T_s - T_i} = \exp\left(-\frac{\pi D N_L \overline{h}}{\rho V S_T c_p}\right)$$

where $N_L = N/N_T$. Hence,

$$N_L = -\frac{\rho V S_T c_p}{\pi D \overline{h}} \ell n\left(\frac{T_s - T_o}{T_s - T_i}\right) \tag{1}$$

With $V_{max} = [S_T/(S_T - D)]V = 15\,\text{m/s}, Re_{D,max} = V_{max}D/\nu = 8240$. Hence, with $S_T/S_L = 1 > 0.7$, C = 0.27 and m = 0.63 from Table 7.7, and the Zhukauskas correlation yields

$$\overline{Nu}_D = C\,C_2\,Re_{D,max}^m\,Pr^{0.36}\left(\frac{Pr}{Pr_s}\right)^{1/4} = 0.27\times1(8240)^{0.63}(0.707)^{0.36}(0.707/0.695)^{1/4} = 70.1$$

$$\overline{h} = \frac{k}{D}\overline{Nu}_D = \frac{0.028\,\text{W/m}\cdot\text{K}}{0.01\text{m}}70.1 = 196.3\,\text{W/m}^2\cdot\text{K}$$

Hence,

$$N_L = -\frac{1.17\,\text{kg/m}^3(5\,\text{m/s})0.015\text{m}(1007\,\text{J/kg}\cdot\text{K})}{\pi(0.01\text{m})196.3\,\text{W/m}^2\cdot\text{K}}\ell n\left(\frac{25}{75}\right) = 15.7$$

and 16 tube rows should be used $\qquad N_L = 16 \qquad$ <

With $Re_{D,max} = 8240$, $P_L = 1.5$ and $(P_T - 1)/(P_L - 1) = 1$, $f \approx 0.35$ and $\chi = 1$ from Fig. 7.13. Hence,

$$\Delta p \approx N_L\chi\left(\frac{\rho V_{max}^2}{2}\right)f = 16\left[\frac{1.085\,\text{kg/m}^3\times(15\,\text{m/s})^2}{2}\right]0.35 = 684\,\text{N/m}^2$$ <

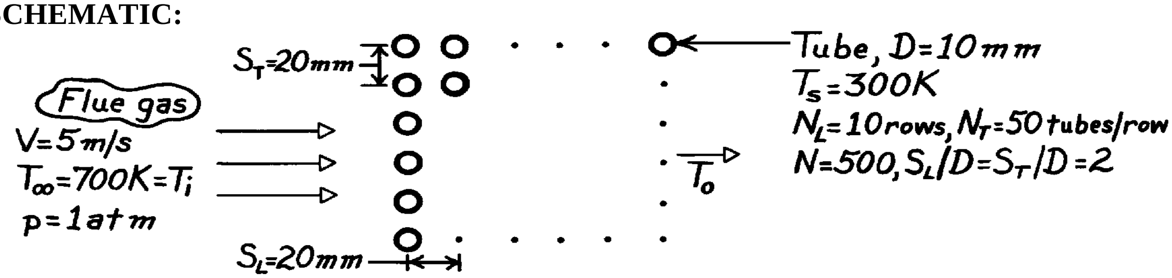
COMMENTS: (1) With $C_2 = 0.99$ for $N_L = 16$ from Table 7.8, assumption 4 is appropriate. (2) Note use of the density evaluated at $T_i = 298\text{K}$ in Eq. (1).

PROBLEM 7.86

KNOWN: Geometry, surface temperature, and air flow conditions associated with a tube bank.

FIND: Rate of heat transfer per unit length.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible radiation effects, (3) Gas properties are approximately those of air.

PROPERTIES: *Table A-4*, Air (300K, 1 atm): Pr = 0.707; *Table A-4*, Air (700K, 1 atm): ν = 68.1 × 10^{-6} m^2/s, k = 0.0524 W/m·K, Pr = 0.695, ρ = 0.498 kg/m^3, c_p = 1075 J/kg·K.

ANALYSIS: The rate of heat transfer per unit length of tubes is

$$q' = \bar{h}N\mathit{p}\,D\,\Delta T_{\ell m} = \bar{h}N\mathit{p}\,D\frac{(T_s - T_i) - (T_s - T_o)}{\ell n\left[(T_s - T_i)/(T_s - T_o)\right]}.$$

With $V_{max} = \frac{S_T}{S_T - D}V = \frac{20}{10}\,5\text{ m/s} = 10\text{ m/s},\ Re_{D,max} = \frac{V_{max}D}{\mathit{n}} = \frac{10\text{ m/s}\times 0.01\text{ m}}{68.1\times 10^{-6}\text{ m}^2/\text{s}} = 1468.$

Tables 7.7 and 7.8 give C = 0.27, m = 0.63 and C_2 = 0.97. Hence from the Zhukauskas correlation,

$$\overline{Nu}_D = CC_2 Re_{D,max}^m Pr^{0.36}\left(Pr/Pr_s\right)^{1/4} = 0.26(1468)^{0.63}(0.695)^{0.36}(0.695/0.707)^{1/4}$$

$$\overline{Nu}_D = 22.4 \qquad \bar{h} = \frac{k}{D}\overline{Nu}_D = 0.0524\text{ W/m}\cdot\text{K}\times 22.4/0.01\text{ m} = 117\text{ W/m}^2\cdot\text{K}.$$

Hence,

$$(T_s - T_o) = (T_s - T_i)\exp\left(-\frac{\mathit{p}\,DN\bar{h}}{\mathit{r}\,VN_T S_T c_p}\right) = -400\text{K}\exp\left(-\frac{\mathit{p}\times 0.01\text{ m}\times 500\times 117\text{ W/m}^2\cdot\text{K}}{0.498\text{ kg/m}^3(5\text{ m/s})50(0.02\text{ m})1075\text{J/kg}\cdot\text{K}}\right)$$

$$T_s - T_o = -201.3\text{K}$$

and the heat rate is

$$q' = \left(117\text{ W/m}^2\cdot\text{K}\right)500\mathit{p}\,(0.01\text{ m})\frac{(-400+201.3)\text{K}}{\ell n\left[(-400)/(-201.3)\right]} = -532\text{ kW/m} \qquad <$$

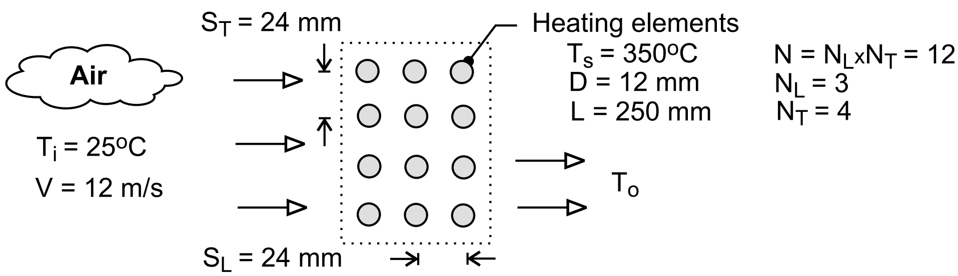
COMMENTS: (1) There is a significant decrease in the gas temperature as it passes through the tube bank. Hence, the heat rate would have been substantially overestimated (- 768 kW) if the inlet temperature difference had been used in lieu of the log-mean temperature difference. (2) The negative sign implies heat transfer to the water. (3) If the temperature of the water increases substantially, the assumption of uniform T_s becomes poor. The extent to which the water temperature increases depends on the water flow rate.

PROBLEM 7.87

KNOWN: An air duct heater consists of an aligned arrangement of electrical heating elements with $S_L = S_T = 24$ mm, $N_L = 3$ and $N_T = 4$. Atmospheric air with an upstream velocity of 12 m/s and temperature of 25°C moves in cross flow over the elements with a diameter of 12 mm and length of 250 mm maintained at a surface temperature of 350°C.

FIND: (a) The total heat transfer to the air and the temperature of the air leaving the duct heater, (b) The pressure drop across the element bank and the fan power requirement, (c) Compare the average convection coefficient obtained in part (a) with the value for an isolated (single) element; explain the relative difference between the results; (d) What effect would increasing the longitudinal and transverse pitches to 30 mm have on the exit temperature of the air, the total heat rate, and the pressure drop?

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible radiation effects, (3) Negligible effect of change in air temperature across tube bank on air properties.

PROPERTIES: *Table A-4*, Air (T_i = 298, 1 atm): $\rho = 1.171$ kg/m^3, $c_p = 1007$ J/kg·K; Air ($T_m = (T_i + T_o)/2 = 309$ K, 1 atm): $\rho = 1.130$ kg/m^3, $c_p = 1007$ J/kg·K, $\mu = 1.89 \times 10^{-5}$ N·s/m^2, k = 0.02699 W/m·K, Pr = 0.7057; Air (T_s = 623 K, 1 atm): $Pr_s = 0.687$; Air ($T_f = (T_i + T_o)/2 = 461$ K, 1 atm): $\nu = 3.373 \times 10^{-5}$ m^2/s, k = 0.03801 W/m·K, Pr = 0.686.

ANALYSIS: (a) The total heat transfer to the air is determined from the rate equation, Eq. 7.71,

$$q = N\left(\bar{h}_D \pi D \Delta T_{\ell m}\right) \qquad (1)$$

where the log mean temperature difference, Eq. 7.69, is

$$\Delta T_{\ell m} = \frac{T_s - T_i}{T_s - T_o} \Big/ \ell m \frac{(T_s - T_i)}{(T_s - T_o)} \qquad (2)$$

and from the overall energy balance, Eq. 7.70,

$$\frac{T_s - T_o}{T_s - T_i} = \exp\left(\frac{\pi D N \bar{h}_D}{\rho V N_T S_T c_p}\right) \qquad (3)$$

The properties ρ and c_p in Eq. (3) are evaluated at the inlet temperature T_i. The average convection coefficient using the Zhukaukus correlation, Eq. 7.67 and 7.68,

$$\overline{Nu}_D = \frac{\bar{h}_D}{k} = C\ Re_{D,max}^m\ Pr^{0.36}\left(Pr/Pr_s\right)^{1/4} \qquad (4)$$

where C = 0.27, m = 0.63 are determined from Table 7.7 for the *aligned* configuration with $S_T/S_L = 1 >$ 0.7 and $10^3 < Re_{D,max} \le 10^5$. All properties except Pr_s are evaluated at the arithmetic mean temperature $T_m = (T_i + T_o)/2$. The maximum Reynolds number, Eq. 7.62, is

Continued …..

$$Re_{D,max} = \rho V_{max} D / \mu \qquad (5)$$

where for the *aligned* arrangement, the maximum velocity occurs at the transverse plane, Eq. 7.65,

$$V_{max} = \frac{S_T}{S_T - D} V \qquad (6)$$

The results of the analyses for $S_T = S_L = 24$ mm are tabulated below.

V_{max} (m/s)	$Re_{D,max}$	$\overline{Nu}_D$	$\overline{h}_D$ ($W/m^2 \cdot K$)	$\Delta T_{\ell m}$ (°C)	q (W)	T_o (°C)	
24	1.723×104	96.2	216	314	7671	47.6	<

(b) The pressure drop across the tube bundle follows from Eq. 7.72,

$$\Delta p = N_L \chi \left(\rho V_{max}^2 / 2 \right) f \qquad (7)$$

where the friction factor, f, and correction factor, χ, are determined from Fig. 7.13 using $Re_{D,max} = 1.723 \times 10^4$,

$f = 0.2 \qquad \chi = 1$

Substituting numerical values,

$$\Delta p = 3 \times 1 \left[1.171 \text{ kg/m}^3 \times (24 \text{ m/s})^2 / 2 \right] \times 0.2$$

$$\Delta p = 195 \text{ N/m}^2 \qquad <$$

The fan power requirement is

$$P = \forall \Delta p = V\, N_T S_T L \Delta p \qquad (8)$$

$$P = 12 \text{ m/s} \times 4 \times 0.024 \text{ m} \times 0.250 \text{ m} \times 195 \text{ N/m}^2$$

$$P = 56 \text{ W} \qquad <$$

where ∀ is the volumetric flow rate. For this calculation, ρ in Eq. (7) was evaluated at T_m.

(c) For a single element in cross flow, the average convection coefficient can be estimated using the Churchill-Bernstein correlation, Eq. 7.57,

$$\overline{Nu}_D = \frac{\overline{h}_D D}{k} = 0.3 + \frac{0.62\, Re_D^{1/2} Pr^{1/3}}{\left[1 + (0.4/Pr)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{Re_D}{282{,}000} \right)^{5/8} \right]^{4/5} \qquad (9)$$

where all properties are evaluated at the film temperature, $T_f = (T_i + T_o)/2$. The results of the calculations are

$Re_D = 4269 \qquad \overline{Nu}_{D,1} = 33.4 \qquad \overline{h}_{D,1} = 106 \text{ W/m}^2 \cdot \text{K} \qquad <$

Continued …..

For the isolated element, $\overline{h}_{D,1} = 106\ \mathrm{W/m^2 \cdot K}$, compared to the average value for the array, $\overline{h}_D = 216\ \mathrm{W/m^2 \cdot K}$. Because the first row of the array acts as a turbulence grid, the heat transfer coefficient for the second and third rows will be larger than for the first row. Here, the array value is twice that for the isolated element.

(d) The effect of increasing the longitudinal and transverse pitches to 30 mm, should be to reduce the outlet temperature, heat rate, and pressure drop. The effect can be explained by recognizing that the maximum Reynolds number will be decreased, which in turn will result in lower values for the convection coefficient and pressure drop. Repeating the calculations of part (a) for $S_L = S_T = 30$ mm, find

V_{max} (m/s)	$Re_{D,max}$	$\overline{Nu}_D$	$\overline{h}_D$ ($\mathrm{W/m^2 \cdot K}$)	$\Delta T_{\ell m}$ (°C)	q (W)	T_o (°C)
12	1.46×10^4	86.7	193	317	6925	41.3

and part (b) for the pressure drop and fan power, find

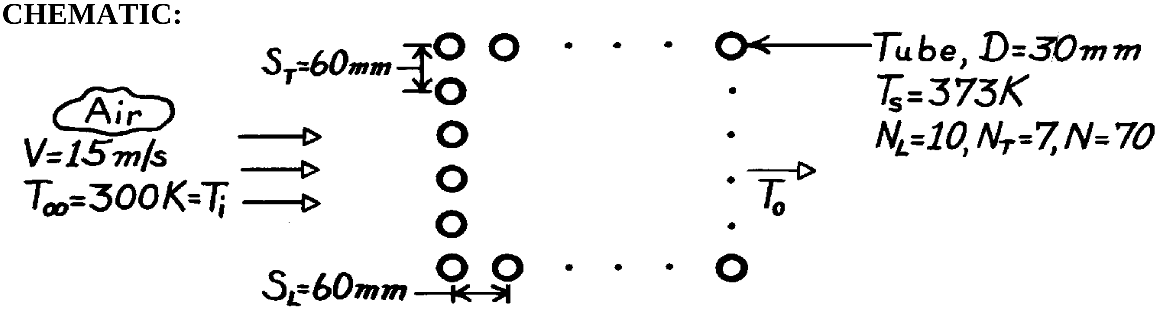
$$f = 0.18 \qquad \chi = 1 \qquad \Delta p = 122\ \mathrm{N/m^2} \qquad P = 44\ \mathrm{W}$$

PROBLEM 7.88

KNOWN: Surface temperature and geometry of a tube bank. Velocity and temperature of air in cross-flow.

FIND: (a) Air outlet temperature, (b) Pressure drop and fan power requirements.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible radiation, (3) Air pressure is approximately one atmosphere, (4) Uniform surface temperature.

PROPERTIES: *Table A-4*, Air (300 K, 1 atm): ρ = 1.1614 kg/m^3, c_p = 1007 J/kg·K, $\nu = 15.89 \times 10^{-6}$ m^2/s, k = 0.0263 W/m·K, Pr = 0.707; (373K): Pr = 0.695.

ANALYSIS: (a) The air temperature increases exponentially, with

$$T_o = T_s - (T_s - T_i)\exp\left(-\frac{p D N \bar{h}}{r V N_T S_T c_p}\right).$$

With $V_{max} = \frac{S_T}{S_T - D}V = \frac{60}{30}15\frac{m}{s} = 30\frac{m}{s}$; $Re_{D,max} = \frac{30\ m/s \times 0.03\ m}{15.89 \times 10^{-6}\ m^2/s} = 56{,}639.$

Tables 7.7 and 7.8 give C = 0.27, m = 0.63 and C_2 = 0.97. Hence from the Zhukauskas correlation,

$$\overline{Nu}_D = 0.27(0.97)(56{,}639)^{0.63}(0.707)^{0.36}(0.707/0.695)^{1/4} = 229$$

$$\bar{h} = \overline{Nu}_D k/D = 229 \times 0.0263\ W/m \cdot K/0.03\ m = 201\ W/m^2 \cdot K.$$

Hence,

$$T_o = 373K - (373 - 300)K \exp\left(-\frac{p \times 0.03\ m \times 70 \times 201\ W/m^2 \cdot K}{1.1614\ kg/m^3 \times 15\ m/s \times 7 \times 0.06\ m \times 1007\ J/kg \cdot K}\right)$$

$$T_o = 373K - 73K \times 0.835 = 312K = 39^\circ C.$$ <

(b) With $Re_{D,max} = 5.66 \times 10^4$, P_L = 2, $(P_T - 1)/(P_L - 1)$ = 1, Fig. 7.13 yields f ≈ 0.19 and χ = 1. Hence,

$$\Delta p = N_L c\left(\frac{r V_{max}^2}{2}\right)f = 10\left(\frac{1.1614\ kg/m^3 \times (30\ m/s)^2}{2}\right)0.19 = 993\ N/m^2 = 0.00993\ bar.$$ <

The fan power requirement is

$$P = \dot{m}_a \Delta p/r = r V N_T S_T L\ \Delta p/r = 15\ m/s \times 7 \times 0.06\ m \times 1m \times 993\ N/m^2 = 6.26\ kW.$$ <

COMMENTS: The heat rate is

$$q = \dot{m}_a c_p(T_o - T_i) = r V N_T S_T L\, c_p(T_o - T_i)$$

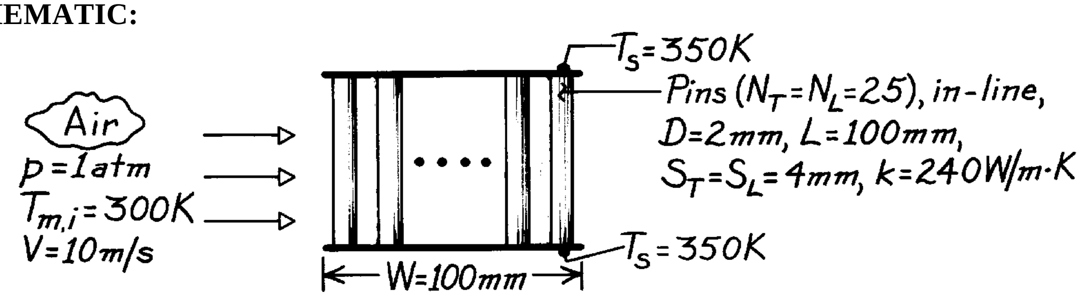
$$q = 1.1614\ kg/m^3 \times 15\ m/s \times 7 \times 0.06m \times 1m \times 1007\ J/kg \cdot K\ (312 - 300)K = 88.4\ kW.$$

PROBLEM 7.89

KNOWN: Characteristics of pin fin array used to enhance cooling of electronic components. Velocity and temperature of coolant air.

FIND: (a) Average convection coefficient for array, (b) Total heat rate and air outlet temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible radiation, (3) One-dimensional conduction in pins, (4) Uniform plate temperature, (5) Plates have a negligible effect on flow over pins, (6) Uniform convection coefficient over all surfaces, corresponding to average coefficient for flow over a tube bank.

PROPERTIES: Air (300 K, 1 atm): ρ = 1.1614 kg/m^3, Pr = 0.707, c_p = 1007 J/kg·K, $\mu = 184.6 \times 10^{-7}$ kg/s·m, k = 0.0263 W/m·K. Aluminum (given): k = 240 W/m·K.

ANALYSIS: (a) From the Zhukauskas relation

$$\overline{Nu}_D = C\,Re_{D,max}^m\,Pr^{0.36}\left(Pr_\infty / Pr_s\right)^{1/4}$$

$$\left(Pr_\infty / Pr_s\right)^{1/4} \approx 1 \qquad V_{max} = \frac{S_T}{S_T - D} V = \frac{4}{4-2} 10\text{ m/s} = 20\text{ m/s}$$

$$Re_{D,max} = \frac{1.164\text{ kg/m}^3 \times 20\text{ m/s} \times 0.002\text{ m}}{184.6 \times 10^{-7}\text{ kg/s}\cdot\text{m}} = 2517$$

From Table 7.7 find C = 0.27 and m = 0.63, hence

$$\overline{Nu}_D = 0.27(2517)^{0.63}(0.707)^{0.36} = 33.1$$

$$\bar{h} = \overline{Nu}_D \frac{k}{D} = 33.1 \times \frac{0.0263\text{ W/m}\cdot\text{K}}{0.002\text{ m}} = 435\text{ W/m}^2\cdot\text{K}. \qquad <$$

(b) If T_s = 350 K is taken to be the temperature of all of the heat transfer surfaces, correction must be made for the actual temperature drop along the pins. This is done by introducing the overall surface efficiency η_o and replacing $\bar{h}A$ by $\bar{h}A_t h_o$. Hence, to obtain the air outlet temperature, we use

$$\frac{T_s - T_o}{T_s - T_i} = \exp\left(-\frac{\bar{h}A_t h_o}{\dot{m}c_p}\right)$$

where

Continued …..

$$A_t = N\,(p DL) + 2W^2 - 2N\left(p D^2/4\right)$$

$$A_t = 625\left(p \times 0.002\text{ m} \times 0.1\text{ m}\right) + 2\left(0.1\text{ m}\right)^2 - 2\times 625p\left(0.002\text{ m}\right)^2/4 = 0.409\text{ m}^2$$

Also $\quad h_o = 1 - \dfrac{A_f}{A_t}\left(1 - h_f\right)$ where h_f is given by Eq. (3.86). With symmetry about the midplane of the pin, q_f = M tanh (mL/2). Hence

$$h_f = \frac{q}{q_{max}} = \frac{\left(\bar{h} p\,Dk p\,D^2/4\right)^{1/2} q_b \tanh\left(mL/2\right)}{\bar{h} p\,D\,(L/2)\,q_b} = \frac{\tanh\left(mL/2\right)}{\left(\bar{h}/kD\right)^{1/2} L}$$

or, with $m = \left[\bar{h} p\,D/\left(k p\,D^2/4\right)\right]^{1/2} = 2\left(\bar{h}/kD\right)^{1/2}$,

$$h_f = \frac{\tanh\left(mL/2\right)}{mL/2}$$

$$m = 2\left(\frac{435\text{ W/m}^2\cdot\text{K}}{240\text{ W/m}\cdot\text{K}\times 0.002\text{ m}}\right)^{1/2} = 60.2\text{ m}^{-1}$$

$$mL/2 = 60.2\text{ m}^{-1}\times 0.05\text{ m} = 3.01 \quad \text{and} \quad \tanh\left(mL/2\right) = 0.995$$

$$h_f = \frac{0.995}{3.01} = 0.331.$$

Hence, $\quad h_o = 1 - \dfrac{625\times p\left(0.002\text{ m}\right)\left(0.1\text{ m}\right)}{0.409\text{ m}^2}\left(1 - 0.331\right) = 0.357$

$$\dot{m} = r\,VLN_T S_T = 1.1614\text{ kg/m}^3\left(10\text{ m/s}\right)0.1\text{ m}\left(25\right)\left(0.004\text{ m}\right) = 0.116\text{ kg/s}.$$

Now evaluating the air outlet temperature,

$$\frac{T_s - T_o}{T_s - T_i} = \exp\left(-\frac{435\text{ W/m}^2\cdot\text{K}\times 0.409\text{ m}^2\times 0.357}{0.116\text{ kg/s}\times 1007\text{ J/kg}\cdot\text{K}}\right) = 0.581$$

$$T_o = T_s - 0.581\left(T_s - T_i\right) = 350\text{ K} - 0.581\left(50\text{ K}\right)$$

$$T_o = 321\text{ K}.$$ <

The total heat rate is

$$q = \dot{m}c_p\left(T_o - T_i\right) = 0.116\text{ kg/s}\left(1007\text{ J/kg}\cdot\text{K}\right)21\text{ K} = 2453\text{ W}.$$ <

COMMENTS: (1) The average surface heat flux which can be dissipated by the electronic components is $q/2W^2$ = 122,650 W/m^2, or 12.3 W/cm^2. (2) To check the numerical results, compute

$$\Delta T_{\ell m} = \frac{\Delta T_o - \Delta T_i}{\ln\left(\Delta T_o/\Delta T_i\right)} = \frac{29\text{ K} - 50\text{ K}}{\ln\left(29/50\right)} = 38.6\text{ K}$$

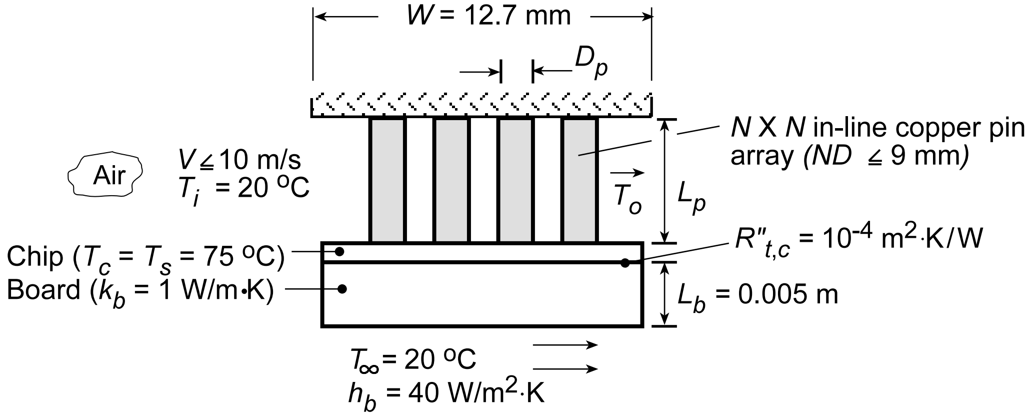
Hence $\quad q = \bar{h}A_t h_o \Delta T_{\ell m} = 435\text{ W/m}^2\cdot\text{K}\times 0.409\text{ m}^2\times 0.357\times 38.6\text{ K} = 2449\text{ W}.$

PROBLEM 7.90

KNOWN: Dimensions and properties of chip, board and pin fin assembly. Convection conditions for chip and board surface. Maximum allowable chip temperature.

FIND: Effect of design and operating conditions on maximum chip power dissipation.

SCHEMATIC:



ASSUMPTIONS: (1) Uniform chip temperature, (2) One-dimensional conduction in pins, (3) Insulated pin tips, (4) Negligible radiation, (5) Uniform convection coefficient over pin and base surfaces.

PROPERTIES: *Table A.1*, copper (T ≈ 340 K): k_p = 397 W/m·K. *Table A.4*, air: properties evaluated using IHT *Properties* Tool Pad.

ANALYSIS: The chip heat rate may be expressed as

$$q_c = \frac{A_c\left(T_c - T_\infty\right)}{\left[R''_{t,c} + \left(L_b/k_b\right) + \left(1/h_b\right)\right]} + q_t$$

where $A_c = W^2$ and q_t is the total heat rate for the fin array. This heat rate must account for the variation of the air temperature across the array. Hence, the appropriate driving potential is $\Delta T_{lm} = \left[\left(T_c - T_i\right) - \left(T_c - T_o\right)\right]/\ln\left[\left(T_c - T_i\right)/\left(T_c - T_o\right)\right]$. However, the total surface area must account for the finite pin length and the exposed base (prime) surface. Hence, from Eqs. 3.101 and 3.102, with ΔT_{lm} replacing θ_b,

$$q_t = \overline{h} A_t \eta_o \Delta T_{lm}$$

where $A_t = N^2 A_f + A_b$, $A_b = A_c - N^2 A_{p,c}$, $A_{p,c} = \pi D_p^2/4$ and

$$\eta_o = 1 - \frac{N^2 A_f}{A_t}\left(1 - \eta_f\right)$$

For an adiabatic tip, Eq. 3.95 yields

$$\eta_f = \frac{\tanh mL_p}{mL_p}$$

where $m = \left(4\overline{h}/k_p D_p\right)^{1/2}$. The air outlet temperature is given by the expression

$$\frac{T_c - T_o}{T_c - T_i} = \exp\left(-\frac{\overline{h} A_t \eta_o}{\dot{m} c_p}\right)$$

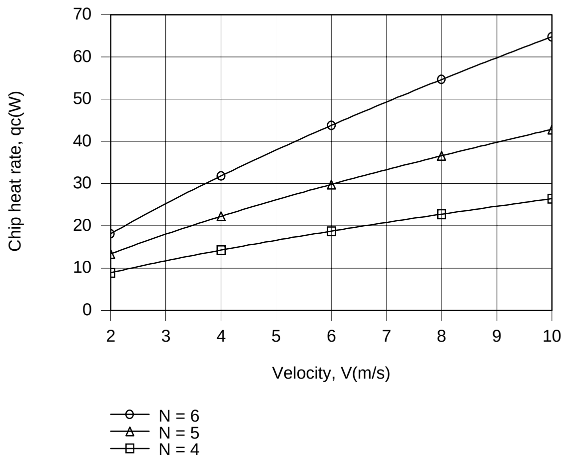
Continued...

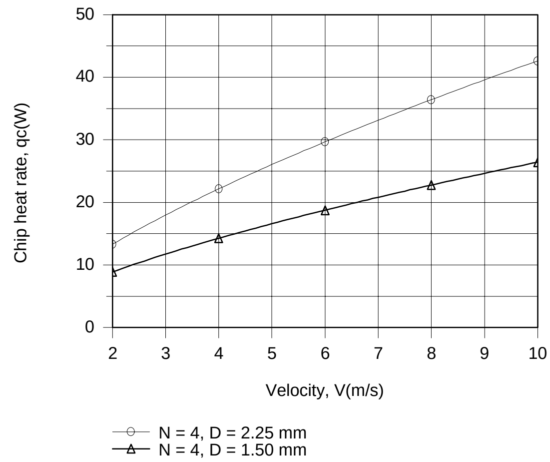
PROBLEM 7.90 (Cont.)

where $\dot{m} = \rho VWL_p$ and $\bar{h}$ is obtained from the Zhukauskas correlation,

$$\overline{Nu}_D = C_2 C\, Re_{D,max}^m\, Pr^{0.36} \left(Pr/Pr_s\right)^{1/4}$$

The foregoing model, including the convection correlation, was entered from the keyboard into the workspace of IHT and used with the *Properties* Tool Pad to perform the following parametric calculations.





Remaining within the limit $ND_p \leq 9$ mm, there is clearly considerable benefit associated with increasing N from 4 to 6 for $D_p = 1.5$ mm or with increasing D_p from 1.5 to 2.25 mm for N = 4. However, the best configuration corresponds to N = 6 and $D_p = 1.5$ mm (a larger number of smaller diameter pins), for which both A_t and $\bar{h}$ are approximately 50% and 20% larger than values associated with N = 4 and D_p = 2.25 mm. The peak heat rate is $q_c = 64.5$ W for V = 10 m/s, N = 6, and $D_p = 1.5$ mm.

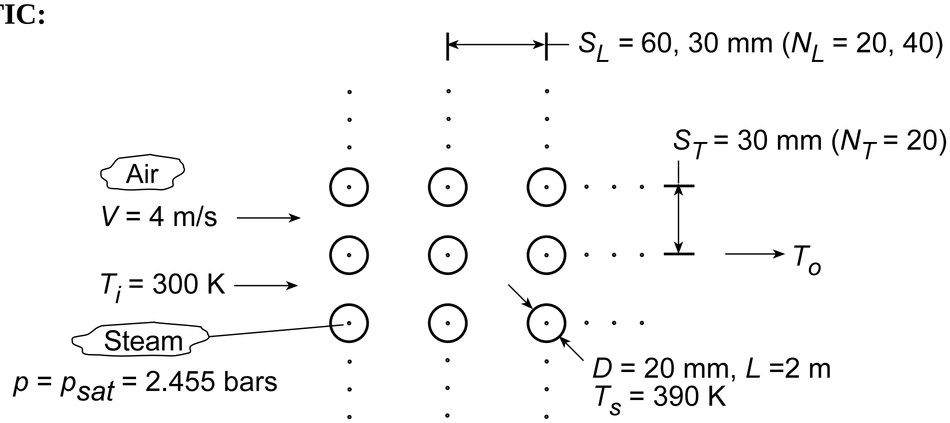
COMMENTS: (1) The heat rate through the board is only $q_b = 0.295$ W and hence a negligible portion of the total heat rate. (2) Values of C = 0.27 and m = 0.63 were used for the entire range of conditions. However, $Re_{D,max}$ was less than 1000 in the mid to low range of V, for which the correlation was therefore used outside its prescribed limits and the results are somewhat approximate. (3) Using the IHT solver, the model was implemented in three stages, beginning with (i) the correlation and the Properties Tool Pad and sequentially adding (ii) expressions for q_t and $(T_c - T_o)/(T_c - T_i)$ without η_o, and (iii) inclusion of η_o in the model. Results computed from one calculation were loaded as initial guesses for the next calculation.

PROBLEM 7.91

KNOWN: Tube geometry and flow conditions for steam condenser. Surface temperature and pressure of saturated steam.

FIND: (a) Coolant outlet temperature, (b) Heat and condensation rates, (c) Effects of reducing longitudinal pitch and change in velocity.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) Negligible radiation, (3) Negligible effect of temperature change on air properties, (parts a and b), (4) Applicability of convection correlation outside designated range.

PROPERTIES: *Table A.4*, air (T_i = 300 K): $\rho = 1.16$ kg/m^3, $c_p = 1007$ J/kg·K, $\nu = 15.89 \times 10^{-6}$ m^2/s, k = 0.0263 W/m·K, Pr = 0.707. (T_s = 390 K): Pr = 0.692. *Table A.6*, saturated water at 2.455 bars: $h_{fg} = 2.183 \times 10^6$ J/kg.

ANALYSIS: (a) From Section 7.6 of the textbook,

$$T_o = T_s - (T_s - T_i)\exp\left(-\frac{\pi D N \bar{h}}{\rho V N_T S_T c_p}\right)$$

With

$$V_{max} = \frac{S_T}{S_T - D} V = \frac{30}{10} 4\,\text{m/s} = 12\,\text{m/s}$$

$$Re_{D,max} = \frac{V_{max} D}{\nu} = \frac{12\,\text{m/s}(0.02\,\text{m})}{15.89 \times 10^{-6}\,\text{m}^2/\text{s}} = 15{,}104$$

Using the Zhukauskas correlation outside its designated range $(S_T/S_L = 0.5)$, Table 7.7 yields C = 0.27 and m = 0.63. Hence, with $C_2 = 1$,

$$\overline{Nu}_D = C\,Re_{D,max}^m\,Pr^{0.36}(Pr/Pr_s)^{1/4} = 0.27(15{,}104)^{0.63}(0.707)^{0.36}\left(\frac{0.707}{0.692}\right)^{1/4} = 103$$

$$\bar{h} = \overline{Nu}_D (k/D) = 103(0.0263\,\text{W/m}\cdot\text{K}/0.02\,\text{m}) = 135\,\text{W/m}^2\cdot\text{K}$$

$$T_o = 390\,\text{K} - (90\,\text{K})\exp\left[-\frac{\pi(0.02\,\text{m})400(135\,\text{W/m}^2\cdot\text{K})}{1.16\,\text{kg/m}^3(4\,\text{m/s})20(0.03\,\text{m})1007\,\text{J/kg}\cdot\text{K}}\right] = 363\,\text{K}$$ <

(b) With $q = q' L$,

Continued...

PROBLEM 7.91 (Cont.)

$$q = N\left(\bar{h}\pi DL\Delta T_{lm}\right)$$

where

$$\Delta T_{lm} = \frac{\left(T_s - T_i\right) - \left(T_s - T_o\right)}{\ln\left(\dfrac{T_s - T_i}{T_s - T_o}\right)} = \frac{\left(90 - 27\right)K}{\ln\left(\dfrac{90}{27}\right)} = 52.3\,K$$

Hence $q = 400\left(135\,W/m^2 \cdot K\right)\pi\left(0.02\,m\right)2\,m\left(52.3\,K\right) = 355\,kW$ <

The condensation rate is

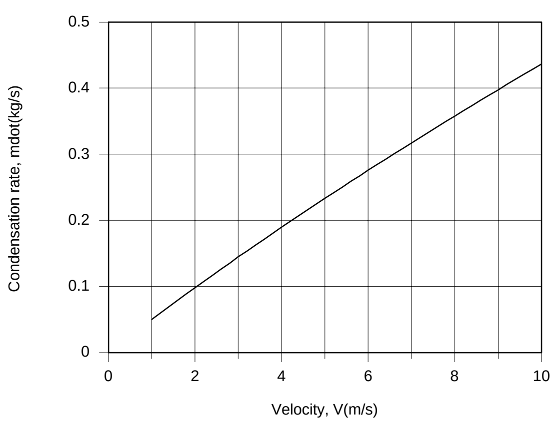
$$\dot{m}_{cond} = \frac{q}{h_{fg}} = \frac{3.55\times10^5\,W}{2.183\times10^6\,J/kg} = 0.163\,kg/s$$ <

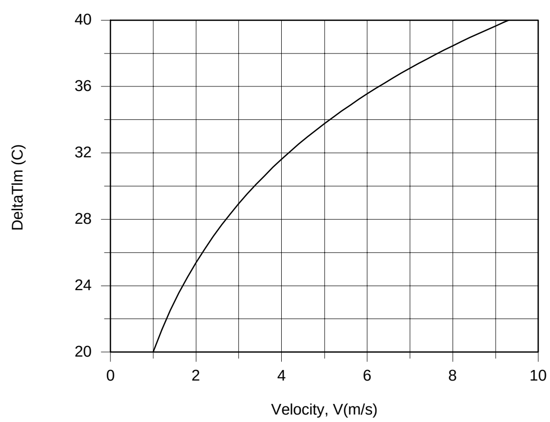
(c) For S_L = 0.03 m, N_L = 40 and N = 800, using IHT with the foregoing model and the Properties Tool Pad to evaluate air properties at $(T_i + T_o)/2$, we obtain

$$T_o = 383.6\,K, \quad \Delta T_{lm} = 31.6\,C, \quad q = 414\,kW, \quad \dot{m}_{cond} = 0.190\,kg/s$$ <

As expected, q and $\dot{m}_{cond}$ increase with increasing N_L. However, due to a corresponding increase in T_o, and hence a reduction in ΔT_{lm}, the increase is not commensurate with the two-fold increase in surface area for the tube bank.

The effect of velocity is shown below.





The heat rate, and hence condensation rate, is strongly affected by velocity, because in addition to increasing $\bar{h}$, an increase in V decreases T_o, and hence increases ΔT_{lm}.

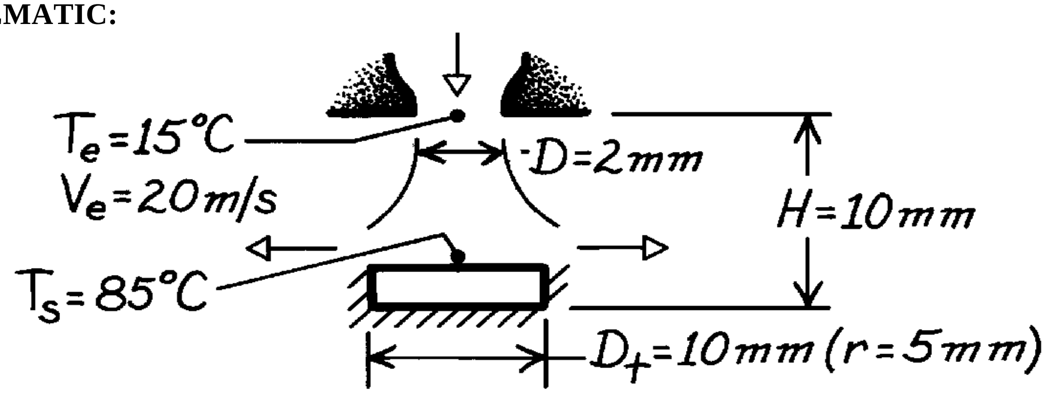
COMMENTS: (1) The calculations of part (a) should be repeated with air properties evaluated at $(T_i + T_o)/2$. (2) the condensation rate could be increased significantly by using a water-cooled (larger $\bar{h}$), rather than an air-cooled, condenser.

PROBLEM 7.92

KNOWN: Geometry of air jet impingement on a transistor. Jet temperature and velocity. Maximum allowable transistor temperature.

FIND: Maximum allowable operating power.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Isothermal surface, (3) Bell-shaped nozzle, (4) All of the transistor power is dissipated to the jet.

PROPERTIES: *Table A-4*, Air (T_f = 323 K, 1 atm): $\nu = 18.2 \times 10^{-6}$ m^2/s, k = 0.028 W/m·K, Pr = 0.704.

ANALYSIS: The maximum power or heat transfer rate by convection is

$$P_{max} = q_{max} = \overline{h}\left(\pi D_t^2 / 4\right)\left(T_s - T_e\right)_{max}.$$

For a single round nozzle,

$$\frac{\overline{Nu}}{Pr^{0.42}} = G\left(r/D, H/D\right) F_1\left(Re\right)$$

where D/r = 0.4 and

$$G = \frac{D}{r}\,\frac{1-1.1\left(D/r\right)}{1+0.1\left(H/D-6\right)\left(D/r\right)} = 0.4\frac{1-0.44}{1+0.1\left(-1\right)0.4} = 0.233.$$

With

$$Re = \frac{V_e D}{\nu} = \frac{\left(20 \text{ m/s}\right)0.002 \text{ m}}{18.2\times 10^{-6} \text{ m}^2/\text{s}} = 2198$$

$$F_1 = 2Re^{1/2}\left(1+0.005Re^{0.55}\right)^{1/2} = 2\left(2198\right)^{1/2}\left[1+0.005\left(2198\right)^{0.55}\right]^{1/2} = 108.7$$

Hence $\overline{h} = \dfrac{k}{D} G F_1 Pr^{0.42} = \dfrac{0.028 \text{ W/m}\cdot\text{K}}{0.002 \text{ m}}\left(0.233\right)\left(108.7\right)\left(0.704\right)^{0.42} = 306 \text{ W/m}^2\cdot\text{K}$

Hence $P_{max} = \left(306 \text{ W/m}^2\cdot\text{K}\right)\left(\pi/4\right)\left(0.01 \text{ m}\right)^2\left(70°\text{C}\right) = 1.68 \text{ W}.$ <

COMMENTS: (1) All conditions required for use of the correlation are satisfied.

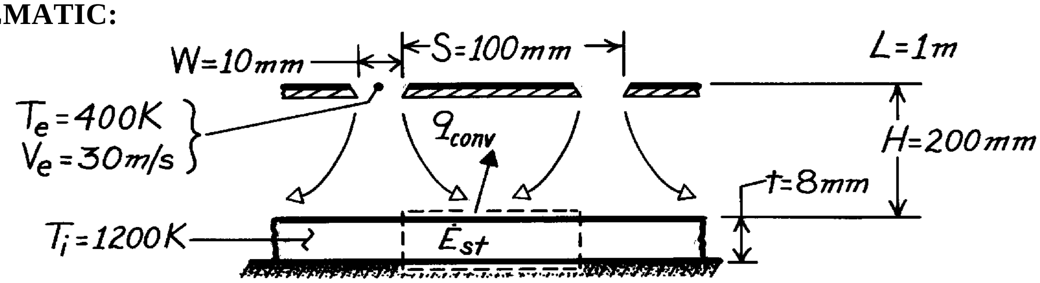
(2) Power dissipation may be enhanced by allowing for heat loss through the side and base of the transistor.

PROBLEM 7.93

KNOWN: Dimensions of heated plate and slot jet array. Jet exit temperature and velocity. Initial plate temperature.

FIND: Initial plate cooling rate.

SCHEMATIC:



ASSUMPTIONS: (a) Negligible variation in h along plate, (b) Negligible heat loss from back surface of plate, (c) Negligible radiation from front surface of plate.

PROPERTIES: *Table A-1*, AISI 304 Stainless steel (1200 K): k = 28.0 W/m·K, c_p = 640 J/kg·K, ρ = 7900 kg/m^3; *Table A-4*, Air ($\overline{T}_f$ = 800 K): ν = 84.9 × 10^{-6} m^2/s, k = 0.0573 W/m·K, Pr = 0.709.

ANALYSIS: Performing an energy balance on a control surface about the plate,

$$-q_{conv} = -\overline{h}A_s\left(T_i - T_e\right) = \dot{E}_{st} = r\left(A_s t\right)c_p\left(dT/dt\right)_i \qquad \left.\frac{dT}{dt}\right)_i = -\frac{\overline{h}\left(T_i - T_e\right)}{r\ c_p t}.$$

For an array of slot nozzles,

$$\frac{\overline{Nu}}{Pr^{0.42}} = \frac{2}{3}A_{r,o}^{3/4}\left[\frac{2Re}{A_r / A_{r,o} + A_{r,o} / A_r}\right]^{2/3}$$

where A_r = W/S = 0.1

$$A_{r,o} = \left\{60 + 4\left[\left(H/2W\right) - 2\right]^2\right\}^{-1/2} = \left\{60 + 4\left(64\right)\right\}^{-1/2} = 0.0563$$

$$Re = \frac{V_e\left(2W\right)}{n} = \frac{30\text{ m/s}\left(0.02\text{ m}\right)}{84.9\times10^{-6}\text{ m}^2/\text{s}} = 7067$$

$$\overline{h} = \frac{0.0573\text{ W/m}\cdot\text{K}}{0.02\text{ m}}\frac{2}{3}\left(0.0563\right)^{3/4}\left[\frac{2\times7067}{1.776 + 0.563}\right]^{2/3} = 73.2\text{ W/m}^2\cdot\text{K}.$$

Hence,

$$\left.\frac{dT}{dt}\right)_i = -\frac{73.2\text{ W/m}^2\cdot\text{K}\left(800\text{ K}\right)}{\left(7900\text{ kg/m}^3\right)\left(640\text{ J/kg}\cdot\text{K}\right)\left(0.008\text{ m}\right)} = 1.45\text{ K/s}.$$ <

COMMENTS: (1) Bi = $\overline{h}t/k$ = (73.2 W/m^2·K) (0.008 m)/28 W/m·K = 0.02 and use of the lumped capacitance method is justified.

(2) Radiation may be significant.

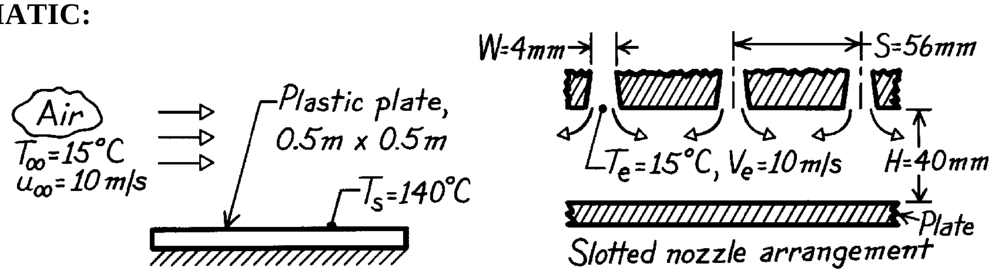
(3) Conditions required for use of the correlation are satisfied.

PROBLEM 7.94

KNOWN: Air at 10 m/s and 15°C is available for cooling hot plastic plate. An array of slotted nozzles with prescribed width, pitch and nozzle-to-plate separation.

FIND: (a) Improvement in cooling rate achieved using the slotted nozzle arrangement in place of turbulent air in parallel flow over the plate, (b) Change in heat rates if air velocities were doubled, (c) Air mass rate requirement for the slotted nozzle arrangement.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) For parallel flow over plate, flow is turbulent, (3) Negligible radiation effects.

PROPERTIES: *Table A-4*, Air (T_f = (140 + 15)°C/2 = 350 K, 1 atm): ρ = 0.995 kg/m^3, $\nu = 20.92 \times 10^{-6}$ m^2/s, k = 30.3×10^{-3} W/m·K, Pr = 0.700.

ANALYSIS: (a) For turbulent flow over the *plate* of length L with

$$\mathrm{Re}_L = \frac{u_\infty L}{n} = \frac{10\ \mathrm{m/s} \times 0.5\ \mathrm{m}}{20.92 \times 10^{-6}\ \mathrm{m^2/s}} = 2.390 \times 10^5$$

using the turbulent flow correlation, find

$$\overline{\mathrm{Nu}}_L = \frac{\bar{h}L}{k} = 0.037\mathrm{Re}_L^{4/5}\,\mathrm{Pr}^{1/3} = 0.037\left(2.390 \times 10^5\right)^{4/5}(0.700)^{1/3} = 659.6$$

$$\bar{h} = \overline{\mathrm{Nu}}_L\, k/L = 659.6 \times 0.030\ \mathrm{W/m \cdot K}/0.5\ \mathrm{m} = 39.6\ \mathrm{W/m^2 \cdot K}.$$

For an array of *slot nozzles*,

$$\overline{\mathrm{Nu}} = \frac{\bar{h}D}{k} = \frac{2}{3}A_{r,o}^{3/4}\left[\frac{2\mathrm{Re}}{A_r/A_{r,o} + A_{r,o}/A_r}\right]^{2/3}\mathrm{Pr}^{0.42}$$

where $$\mathrm{Re} = \frac{V_e D_h}{n} = \frac{10\ \mathrm{m/s}\,(2 \times 0.004\ \mathrm{m})}{20.92 \times 10^{-6}\ \mathrm{m^2/s}} = 3824$$

$$A_{r,o} = \left\{60 + 4\left[(H/2W) - 2\right]^2\right\}^{-1/2} = \left\{60 + 4\left[40/2 \times 4 - 2\right]^2\right\}^{-1/2} = 0.1021$$

$$A_r = W/S = 4\ \mathrm{mm}/56\ \mathrm{mm} = 0.0714$$

$$\overline{\mathrm{Nu}} = \frac{2}{3}(0.1021)^{3/4}\left[\frac{2 \times 3824}{0.0714/0.1021 + 0.1021/0.0714}\right]^{2/3}(0.700)^{0.42} = 24.3$$

$$\bar{h} = \overline{\mathrm{Nu}}k/D_h = 24.3 \times 0.030\ \mathrm{W/m \cdot K}/2 \times 0.004\ \mathrm{m} = 91.1\ \mathrm{W/m^2 \cdot K}.$$

Continued …..

The improvement in heat rate with the slot nozzles (sn) over the flat plate (fp) is

$$\frac{q''_{sn}}{q''_{fp}} = \frac{\overline{h}_{sn}}{\overline{h}_{fp}} = \frac{91.1\ \mathrm{W/m^2 \cdot K}}{39.6\ \mathrm{W/m^2 \cdot K}} = 2.3. \qquad <$$

(b) If the air velocities were doubled for each arrangement in part (a), the heat transfer coefficients are affected as

$$\overline{h}_{sn} \sim Re^{2/3} \qquad \overline{h}_{fp} \sim Re^{4/5}.$$

Hence

$$\frac{\overline{h}_{sn}}{\overline{h}_{fp}} = 2.3\left(\frac{2^{2/3}}{2^{4/5}}\right) = 2.1. \qquad <$$

That is, comparative advantage of the slot nozzle over the flat plate decreases with increasing velocity.

(c) The mass rate of air flow through the array of slot nozzles is

$$\dot{m} = \rho N A_{c,e} = 0.995\,\mathrm{kg/m^3} \times 9\ (0.5\ \mathrm{m} \times 0.004\ \mathrm{m})\,10\,\mathrm{m/s} = 0.179\,\mathrm{kg/s}$$

where the number of slots is determined as

$$N \approx L/S = 0.5\,\mathrm{m}/0.056\ \mathrm{m} = 8.9 \approx 9. \qquad <$$

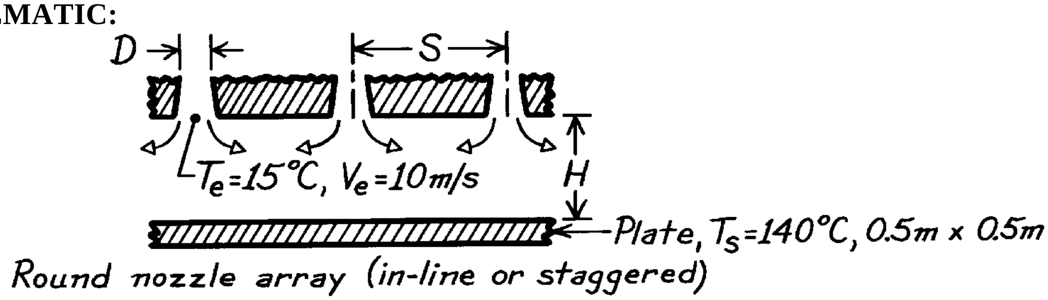
COMMENTS: Note, for the slot nozzle, the hydraulic diameter is $D_h = 2W$ and the relative nozzle area ($A_{c,e}/A_{cell}$) is $A_r = W/S$.

PROBLEM 7.95

KNOWN: Air jet velocity and temperature of 10 m/s and 15°C, respectively, for cooling hot plastic plate..

FIND: Design of optimal round nozzle array. Compare cooling rate with results for a slot nozzle array and flow over a flat plate. Discuss features associated with these three methods relevant to selecting one for this application.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible radiation effects.

PROPERTIES: *Table A-4*, Air (T_f = (140 + 15)°C/2 = 350 K, 1 atm): ρ = 0.995 kg/m^3, ν = 20.92 × 10^{-6} m^2/s, k = 30.0 × 10^{-3} W/m·K, Pr = 0.700.

ANALYSIS: To design an *optimal array* of round nozzles, we require that $D_{h,op} \approx 0.2H$ and $S_{op} \approx 1.4H$. Choose H = 40 mm, the nozzle-to-plate separation, hence

$$D_{h,op} = D = 0.2\times 40\text{ mm} = 8\text{ mm} \qquad S_{op} = 1.4\times 40\text{ mm} = 56\text{ mm.}$$

For an array of round nozzles,

$$\overline{Nu} = K\left(A_r, H/D\right)\cdot G\left(A_r, H/D\right)\cdot F_2\left(Re\right)\cdot Pr^{0.42}$$

where for an *in-line* array, see Fig. 7.17,

$$A_r = \frac{p D^2}{4S^2} = \frac{p\left(8\text{ mm}\right)^2}{4\left(56\text{ mm}\right)^2} = 0.0160$$

$$K = \left[1+\left(\frac{H/D}{0.6/A_r^{1/2}}\right)^6\right]^{-0.05} = \left[1+\left(\frac{40/8}{0.6/0.0160^{1/2}}\right)^6\right]^{-0.05} = 0.9577$$

$$G = 2A_r^{1/2}\frac{1-2.2A_r}{1+0.2\left(H/D-6\right)A_r^{1/2}} = 2\times 0.0160^{1/2}\frac{1-2.2\times 0.0160}{1+0.2\left(40/8-6\right)0.0160^{1/2}}$$

$$G = 0.2504$$

$$F_2 = 0.5Re^{2/3} = 0.5\left(\frac{10\text{ m/s}\times 0.008\text{ m}}{20.92\times 10^{-6}\text{ m}^2/\text{s}}\right)^{2/3} = 122.2.$$

The average heat transfer coefficient for the *optimal in-line* (op, il) array of round nozzles is,

$$\overline{h}_{op,il} = \overline{Nu}\,k/D_{h,op} = \frac{0.030\text{ W/m}\cdot\text{K}}{0.008\text{ m}}\times 0.9577\times 0.2504\times 122.2\left(0.700\right)^{0.42}$$

$$\overline{h}_{op,il} = 94.6\text{ W/m}^2\cdot\text{K}.$$

Continued …..

If an *optimal staggered* (op,s) array were used, see Fig. 7.17, with

$$A_r = \frac{p\,D^2}{2(3)^{1/2}S^2} = \frac{p \times (8\text{ mm})^2}{2(3)^{1/2}(56\text{ mm})^2} = 0.0185$$

find K = 0.9447, G = 0.2632, F_2 = 122.2 and $\overline{h}_{op,s} = 100.0\text{ W/m}^2\cdot\text{K}$.

Using the previous results for *parallel flow* (pf) and the *slot nozzle* (sn) array, the heat rates, which are proportional to the average convection coefficients, can be compared.

Arrangement	Flat plate (fp)	Slot nozzle (sn)	Optimal round nozzle (op) In-line (il)	Staggered (s)
$\overline{h}$, $\text{W/m}^2\cdot\text{K}$	39.6	91.1	94.6	100.0
$\overline{h}/h_{fp}$	1.0	2.30	2.39	2.53
$\dot{m}$, kg/s	---	0.199	0.040	0.046

For these flow conditions, we conclude that there is only slightly improved performance associated with using the round nozzles. As expected, the *staggered* array is better than the *in-line* arrangement, since the former has a higher area ratio (A_r). The air flow requirements for the round nozzle arrays are

$$\dot{m} = r\,NA_{c,e}V_e = r\left(A_s / A_{cell}\right)A_{c,e}V_e = r\,A_r A_s\,V_e$$

where N = A_s/A_{cell} is the number of nozzles and A_s is the area of the plate to be cooled. Substituting numerical values, find

$$\dot{m}_{op,il} = 0.995\text{ kg/m}^3 \times 0.0160\left(0.5\times 0.5\text{ m}^2\right)\times 10\text{ m/s} = 0.040\text{ kg/s}$$

$$\dot{m}_{op,s} = 0.995\text{ kg/m}^3 \times 0.0185\left(0.5\times 0.5\text{ m}^2\right)\times 10\text{ m/s} = 0.046\text{ kg/s}.$$

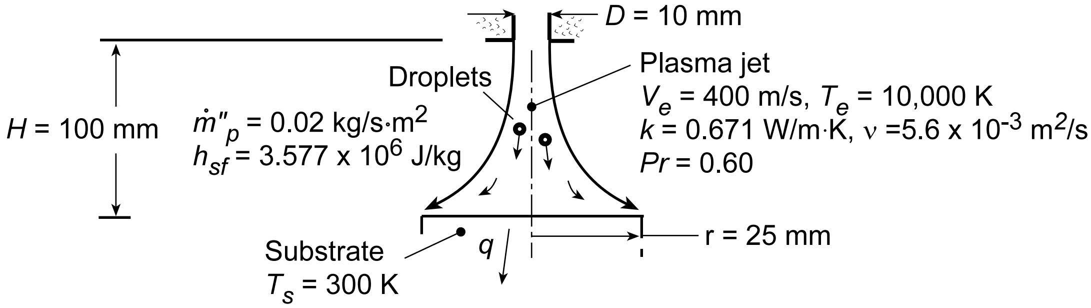
For this application, selection of a nozzle arrangement should be based upon air flow requirements (round nozzles have considerable advantage) and costs associated with fabrication of the arrays (slot nozzle may be easier to form from sheet metal).

PROBLEM 7.96

KNOWN: Exit diameter of plasma generator and radius of jet impingement surface. Temperature and velocity of plasma jet. Temperature of impingement surface. Droplet deposition rate.

FIND: Rate of heat transfer to substrate due to convection and release of latent heat.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible radiation, (3) Negligible sensible energy change due to cooling of droplets to T_s.

ANALYSIS: The total heat rate to the substrate is due to convection from the jet and release of the latent heat of fusion due to solidification, $q = q_{conv} + q_{lat}$. With $Re = V_eD/\nu = (400\text{ m/s})0.01\text{ m}/5.6 \times 10^{-3}\text{ m}^2/\text{s} = 714$, D/r = 0.4, and H/D = 10, $F_1 = 2Re^{1/2}(1 + 0.005\,Re^{0.55})^{1/2} = 58.2$ and $G = (D/r)(1 - 1.1D/r)/[1 + 0.1(H/D - 6)D/r] = 0.193$, the correlation for a single round nozzle (Chapter 7.7) yields

$$\overline{Nu} = GF_1\,Pr^{0.42} = 0.193(58.2)\left(0.60^{0.42}\right) = 9.07$$

$$\bar{h} = \overline{Nu}\,(k/D) = 9.07\,(0.671\,W/m\cdot K/0.01\,m) = 6.09\,W/m^2\cdot K$$

Hence,

$$q = \bar{h}A_s\,(T_e - T_s) = 609\,W/m^2\cdot K \times \pi\,(0.025\,m)^2\,(10{,}000 - 300)\,K = 11{,}600\,W \qquad <$$

The release of latent heat is

$$q_{lat} = A_s\dot{m}''_p h_{sf} = \pi\,(0.025\,m)^2\left(0.02\,kg/s\cdot m^2\right)3.577\times10^6\,J/kg = 140\,W \qquad <$$

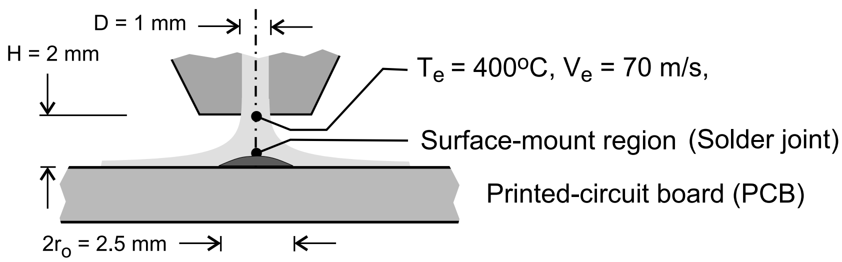
COMMENTS: (1) The large plasma temperature renders heat transfer due to droplet deposition negligible compared to convection from the plasma. (2) Note that Re = 714 is outside the range of applicability of the correlation, which has therefore been used as an approximation to actual conditions.

PROBLEM 7.97

KNOWN: A round nozzle with a diameter of 1 mm located a distance of 2 mm from the surface mount area with a diameter of 2.5 mm; air jet has a velocity of 70 m/s and a temperature of 400°C.

FIND: (a) Estimate the average convection coefficient over the area of the surface mount, (b) Estimate the time required for the surface mount region on the PCB, modeled as a semi-infinite medium initially at 25°C, to reach 183°C; (c) Calculate and plot the surface temperature of the surface mount region for air jet temperatures of 400, 500 and 600°C as a function time for $0 \le t \le 40$ s. Comment on the outcome of your study, the appropriateness of the assumptions, and the feasibility of using the jet for a soldering application.

SCHEMATIC:



ASSUMPTIONS: (1) Air jet is a single round nozzle, (2) Uniform temperature over the PCB surface, and (3) Surface mount region can be modeled as a one-dimensional semiinfinite medium.

PROPERTIES: *Table A-4*, Air (T_f = 486 K, 1 atm): $\nu = 3.693 \times 10^{-6}$ m^2/s, k = 0.03971 W/m·K, Pr = 0.685; Solder (given): ρ = 8333 kg/m^3, c_p = 188 J/kg·K, and k = 51 W/m·K; eutectic temperature, T_{sol} = 183°C; PCB (given): glass transition temperature, T_{gl} = 250°C.

ANALYSIS: For a single round nozzle, from the correlation of Eqs. 7.79, 7.80 and 7.81b, estimate the convection coefficient,

$$\frac{\overline{Nu}}{Pr^{0.42}} = G\left(\frac{r}{D}, \frac{H}{D}\right) F_1(Re) \qquad \overline{Nu} = \frac{\overline{h}D}{k} \tag{1,2}$$

where

$$F_1 = 2\,Re^{1/2}\left(1 + 0.005\,Re^{0.55}\right)^{1/2} \tag{3}$$

$$G = 2\,A_r^{1/2}\,\frac{1 - 2.2\,A_r^{1/2}}{1 + 0.2(H/D - 6)A_r^{1/2}} \tag{4}$$

$$A_r = D^2 / 4r_o^2 \tag{5}$$

The Reynolds number is based on the jet diameter and velocity at the nozzle,

$$Re_D = V_e\,D/\nu \tag{6}$$

and r_o is the radius of the region over which the average coefficient is being evaluated. The thermophysical properties are evaluated at the film temperature, $T_f = (T_e + T_s)/2$. The results of the calculation are tabulated below.

Continued …..

Re	F_1	G	A_r	$\overline{Nu}$	$\overline{h}$ (W/m^2·K)
1895	99.94	0.2667	0.04	22.73	903

<

Consider the surface mount region as a semi-infinite medium, with solder properties, initially at a uniform temperature of 25°C, that experiences sudden exposure to the convection process with the air jet at a temperature T_∞ = 400°C and the convection coefficient as found in part (a). The surface temperature, T(0,t), is determined from Case 3, Fig. 5.7 and Eq. 5.60,

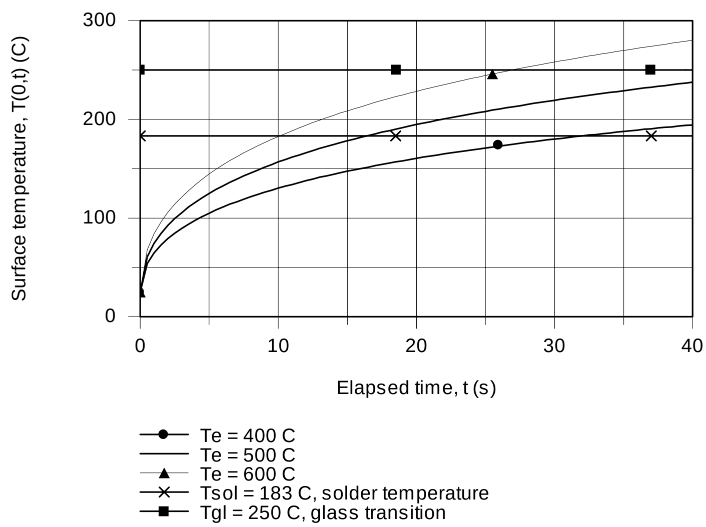
$$\frac{T(0,t)-T_i}{T_\infty - T_i} = -\exp\left(\frac{h^2\alpha t}{k^2}\right)\times \operatorname{erfc}\left(\frac{h(\alpha t)^{1/2}}{k}\right) \tag{7}$$

where $\alpha = k/\rho c_p$. With T_i = 25°C and $T_\infty = T_e$, by trial-and-error, or by using the appropriate *IHT* model, find

$$T(0,t_o) = 183°C \qquad\qquad t_o = 31.9 \text{ s}$$

<

(c) Using the foregoing relations in *IHT*, the surface temperature T(0,t) is calculated and plotted for jet air temperatures of 400, 500 and 600°C for 0 ≤ t ≤ 40 s.



The effect of increasing the jet air temperature is to reduce the time for the surface temperature to reach the solder temperature of 183°C. With the 600°C air jet, it takes about 10 s to reach the solder temperature, and the glass transition temperature is achieved in 27 s. The analysis represents a first-order model giving approximate results only. While the estimates for the average convection coefficients are reasonable, modeling the surface mount region as a semi-infinite medium is an over simplification. The region is of limited extent on the PCB, which is thin and also a poor approximation to an infinite medium. However, the model has provided insight into the conditions under which an air jet could be used for a soldering operation.

COMMENTS: (1) Note that for our application, the round nozzle correlation of part (a) meets the ranges of validity.

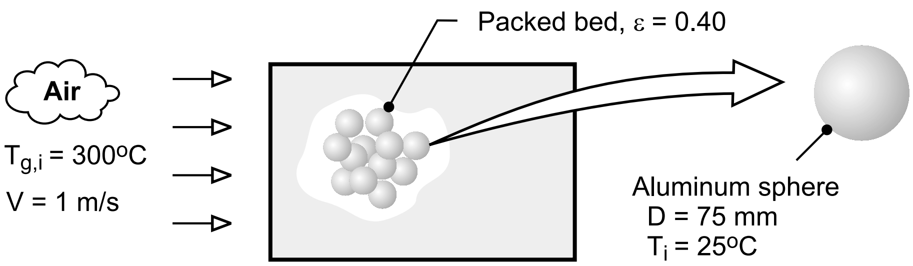
(2) The jet convection coefficient is not strongly dependent upon the air temperature. Values for 400, 500, and 600°C, respectively, are 903, 889, and 876 W/m^2·K.

PROBLEM 7.98

KNOWN: Diameter and properties of aluminum spheres used in packed bed. Porosity of bed and velocity and temperature of inlet air.

FIND: Time for sphere to acquire 90% of maximum possible thermal energy.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat transfer to or from a sphere by radiation or conduction due to contact with other spheres, (2) Validity of lumped capacitance method, (3) Constant properties.

PROPERTIES: Prescribed, Aluminum: $\rho = 2700\,\text{kg/m}^3$, $c = 950\,\text{J/kg}\cdot\text{K}$, $k = 240\,\text{W/m}\cdot\text{K}$. *Table A-4*, Air (573K): $\rho_a = 0.609\,\text{kg/m}^3$, $c_{p,a} = 1045\,\text{J/kg}\cdot\text{K}$, $\nu = 48.8\times10^{-6}\,\text{m}^2/\text{s}$, $k_a = 0.0453\,\text{W/m}\cdot\text{K}$, $\text{Pr} = 0.684$.

ANALYSIS: From Eqs. 5.7 and 5.8a, achievement of 90% of the maximum possible thermal energy storage corresponds to

$$\frac{Q}{\rho c \forall \theta_i} = 0.9 = 1 - \exp\left(-\frac{t}{\tau_t}\right) = 1 - \exp\left(-\frac{\overline{h} A_s t}{\rho \forall c}\right)$$

where the convection coefficient is given by

$$\varepsilon\, j_H = \varepsilon\, \overline{\text{St}}\, \text{Pr}^{2/3} = \varepsilon \frac{\overline{h}}{\rho_a V c_{p,a}} \text{Pr}^{2/3} = \text{Re}_D^{-0.575}$$

With $\text{Re}_D = VD/\nu = 1\,\text{m/s}\times 0.075\,\text{m}/48.8\times10^{-6}\,\text{m}^2/\text{s} = 1537$,

$$\overline{h} = \frac{0.609\,\text{kg/m}^3 \times 1\,\text{m/s} \times 1045\,\text{J/kg}\cdot\text{K}}{0.4(0.684)^{2/3}(1537)^{0.575}} = 30.2\,\text{W/m}^2\cdot\text{K}$$

Hence, with $A_s/\forall = 6/D$,

$$t = -\frac{\rho c D}{6\overline{h}} \ln(0.1) = \frac{2700\,\text{kg/m}^3 \times 950\,\text{J/kg}\cdot\text{K} \times 0.075\,\text{m} \times 2.30}{6\times 30.2\,\text{W/m}^2\cdot\text{K}} = 2445\,\text{s} \quad <$$

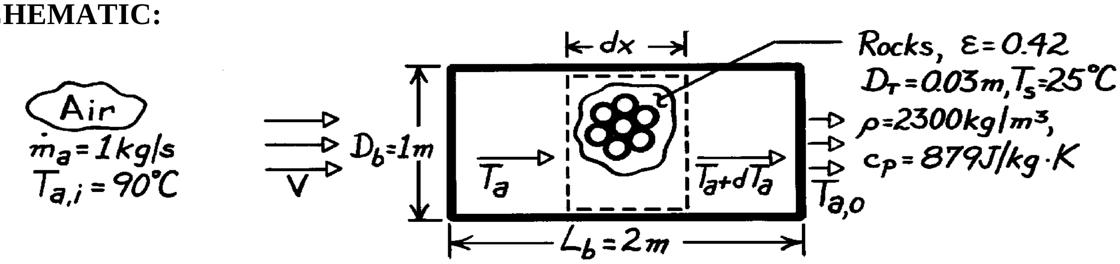
COMMENTS: (1) With $\text{Bi} = \overline{h}(D/6)/k = 0.002$, the spheres are spatially isothermal and the lumped capacitance approximation is excellent. (2) Before the packed bed becomes fully charged, the temperature of the air decreases as it passes through the bed. Hence, the time required for a sphere to reach a prescribed state of thermal energy storage increases with increasing distance from the bed inlet.

PROBLEM 7.99

KNOWN: Overall dimensions of a packed bed of rocks. Rock diameter and thermophysical properties. Initial temperature of rock and bed porosity. Flow rate and upstream temperature of atmospheric air passing through the pile.

FIND: Rate of heat transfer to pile.

SCHEMATIC:



ASSUMPTIONS: (1) Rocks are spherical and at a uniform temperature, (2) Steady-state conditions.

PROPERTIES: *Table A-4*, Atmospheric air (T_∞ = 363K): $\nu = 22.35 \times 10^{-6}\ m^2/s$, k = 0.031 W/m·K, Pr = 0.70, $\rho = 0.963\ kg/m^3$, c_p = 1010 J/kg·K.

ANALYSIS: The heat transfer rate may be expressed as $q = \bar{h} A_{p,t} \Delta T_{\ell m}$ where the total surface area of the rocks is

$$A_{p,t} = V_r \frac{p D_r^2}{p D_r^3/6} = (1-e)\left(\frac{p D_b^2}{4} L_b\right)\frac{6}{D_r} = (1-0.42)\ \left(p 1\ m^2/4\times 2m\right) 6/0.03\ m = 182.2\ m^2.$$

The upstream velocity and Reynolds number are

$$V = \frac{\dot{m}_a}{r p D_b^2/4} = \frac{4\times 1\ kg/s}{\left(0.963\ kg/m^3\right) p\ \ 1m^2} = 1.32\ m/s \quad Re_D = \frac{VD_r}{n} = \frac{1.32\ m/s \times 0.03\ m}{22.35\times 10^{-6}\ m^2/s} = 1772.$$

From Section 7.8, it follows that

$$e \bar{j}_H = e \overline{St} Pr^{2/3} = e \frac{\bar{h}}{r V c_p} Pr^{2/3} = 2.06\ Re_D^{-0.575}$$

$$\bar{h} = \frac{2.06}{e} r V c_p Re_D^{-0.575}\ Pr^{-2/3}$$

$$\bar{h} = \frac{2.06}{0.42} 0.963\ kg/m^3 \times 1.32\ m/s \times 1010 J/kg\cdot K\,(1772)^{-0.575}(0.70)^{-2/3} = 108\ W/m^2\cdot K.$$

The appropriate form of the mean temperature difference, $\Delta T_{\ell m}$, may be obtained by performing an energy balance on a differential control volume about the rock. That is,

$$\dot{m}_a\ c_p\ T_a - \dot{m}_a\ c_p\left(T_a + dT_a\right) - dq_r = 0$$

where $dq_r = \bar{h} A'_{p,t} dx\left(T_a - T_s\right)$ and $A'_{p,t}$ is the rock surface area per unit length of bed. Hence

$$\dot{m}_a\ c_p\ dT_a = -\bar{h} A'_{p,t} dx\left(T_a - T_s\right) \qquad \frac{d\ T_a}{dx} = -\frac{\bar{h} A'_{p,t}}{\dot{m}_a\ c_p}\left(T_a - T_s\right).$$

Continued …..

Integrating between inlet and outlet, it follows that

$$\ell n\left(T_a - T_s\right)\Big|_i^o = -\frac{\overline{h}A'_{p,t}}{\dot{m}_a\ c_p} L_b = -\frac{\overline{h}A_{p,t}}{\dot{m}_a\ cp} \qquad \ell n\frac{T_{a,o} - T_s}{T_{a,i} - T_s} = -\frac{\overline{h}A_{p,t}}{\dot{m}_a\ c_p}.$$

With $$q = \dot{m}_a\ c_p\left(T_{a,i} - T_{a,o}\right) = \dot{m}_a\ c_p\left[\left(T_{a,i} - T_s\right) - \left(T_{a,o} - T_s\right)\right]$$

it follows that

$$q = \overline{h}A_{p,t}\frac{\left(T_{a,i} - T_s\right) - \left(T_{a,o} - T_s\right)}{\ell n\left[\left(T_{a,i} - T_s\right)/\left(T_{a,o} - T_s\right)\right]} = \overline{h}A_{p,t}\Delta T_{\ell m}$$

where $$\Delta T_{lm} = \frac{\left(T_{a,i} - T_s\right) - \left(T_{a,o} - T_s\right)}{\ln\left[\left(T_{a,i} - T_s\right)/\left(T_{a,o} - T_s\right)\right]}.$$

The air outlet temperature may be obtained from the requirement

$$\frac{T_{a,o} - T_s}{T_{a,i} - T_s} = \exp\left(-\frac{\overline{h}A_{p,t}}{\dot{m}_a\ c_p}\right) = \exp\left(-\frac{108\text{W/m}^2\cdot\text{K}\times 182.2\text{ m}^2}{1\text{ kg/s}\times 1010\text{ J/kg}\cdot\text{K}}\right) = 3.46\times 10^{-9}$$

$$T_{a,o} = 25^\circ\text{C} + 65^\circ\text{C}\left(3.46\times 10^{-9}\right) = 25^\circ\text{C} + 2.25\times 10^{-7}\ ^\circ\text{C}$$

$$T_{a,o} \approx T_s = 25^\circ\text{C}.$$

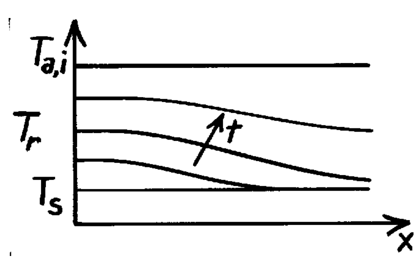
Hence $$\Delta T_{\ell m} = \frac{65^\circ\text{C} - 2.25\times 10^{-7}\ ^\circ\text{C}}{\ell n\left(65^\circ\text{C}/2.25\times 10^{-7}\ ^\circ\text{C}\right)} = 3.34^\circ\text{C}$$

and $$q = 108\text{W/m}^2\cdot\text{K}\left(182.2\text{ m}^2\right)3.34^\circ\text{C} = 65.7\text{ kW}.$$ <

COMMENTS: (1) The above result may be checked from the requirement that $q = \dot{m}_a\ c_p\left(T_{a,i} - T_{a,o}\right) = 1\text{ kg/s}\times 1010\text{ J/kg}\cdot\text{K}\times 65^\circ\text{C} = 65.7\text{ kW}$.

(2) The heat rate would be *grossly* overpredicted by using a rate equation of the form $q = \overline{h}A_{p,t}\left(T_{a,i} - T_s\right)$.

(3) The foregoing results are reasonable during the early stages of the heating process; however q would decrease with increasing time as the temperature of the rock increases. The axial temperature distribution of the rock in the pile would be as shown for different times.

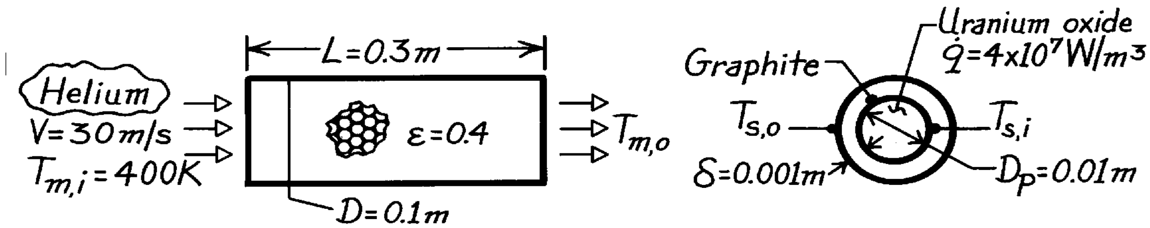


PROBLEM 7.100

KNOWN: Dimensions of packed bed of graphite-coated uranium oxide fuel elements. Volumetric generation rate in uranium oxide and upstream velocity and temperature of helium passing through the bed.

FIND: (a) Mean temperature of helium leaving bed, (b) Maximum temperature of uranium oxide.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Negligible kinetic and potential energy changes, (4) Negligible longitudinal conduction in bed, (5) Bed is insulated from surroundings, (6) One-dimensional conduction in pellets.

PROPERTIES: Helium (given): $\rho = 0.089$ kg/m^3, $c_p = 5193$ J/kg·K, k = 0.236 W/m·K, $\mu = 3 \times 10^{-5}$ kg/s·m, Pr = 0.66; Graphite (given): k = 2 W/m·K; Uranium oxide (given): k = 2 W/m·K.

ANALYSIS: (a) From an energy balance for the entire packed bed

$$q = \dot{m}c_p\left(T_{m,o} - T_{m,i}\right)$$

where the heat rate is due to volumetric generation in the pellets.

$$q = \dot{q}\cdot V_p = \dot{q}\left(\pi D^2/4\right)L\left(1-\varepsilon\right)\left[D_p/\left(D_p+2\delta\right)\right]^3$$

$$q = 4\times10^7\ \text{W/m}^3\left(\pi/4\right)\left(0.1\ \text{m}\right)^2\left(0.3\ \text{m}\right)\left(0.6\right)\left(0.010/0.012\right)^3$$

$$q = 4\times10^7\ \text{W/m}^3\left(8.181\times10^{-4}\ \text{m}^3\right) = 3.272\times10^4\ \text{W}.$$

With $\dot{m} = \rho VA = 0.089\ \text{kg/m}^3\times30\text{m/s}\left(\pi/4\right)\left(0.1\ \text{m}\right)^2 = 0.021\ \text{kg/s}$

the outlet temperature is

$$T_{m,o} = T_{m,i} + \frac{q}{\dot{m}c_p} = 400\ \text{K} + \frac{3.272\times10^4\text{W}}{0.021\ \text{kg/s}\times5193\ \text{J/kg}\cdot\text{K}} = 700\ \text{K}. \qquad <$$

(b) The maximum temperature occurs at the center of the pellet in the exit plane. Beginning with the heat equation for pellet, find

$$\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = -\frac{\dot{q}}{k}r^2$$

$$r^2\frac{dT}{dr} = -\frac{\dot{q}}{3k}r^3 + C_1$$

$$T\left(r\right) = -\frac{\dot{q}}{6k}r^2 - \frac{C_1}{r} + C_2.$$

Continued …..

Applying boundary conditions:

$$\text{at } r = 0 \qquad dT/dr\big|_{r=0} = 0 \qquad \rightarrow \qquad C_1 = 0$$

$$\text{at } r = r_i \qquad T(r_i) = T_{s,i} \qquad \rightarrow \qquad C_2 = T_{s,i} + \frac{\dot{q}}{6k} r_i^2$$

$$T(r) = T_{s,i} + \frac{\dot{q}}{6k}\left(r_i^2 - r^2\right)$$

$$T(0) = T_{s,i} + \frac{\dot{q} D_p^2}{24k}.$$

For one-dimensional conduction in a spherical shell,

$$T_{s,i} = T_{s,o} + \frac{q_p}{4\pi k}\left[\frac{1}{r_i} - \frac{1}{r_o}\right]$$

where $$T_{s,o} = T_o + \frac{q_p}{\bar{h}\pi\left(D_p + 2\delta\right)^2}$$

$$q_p = \dot{q}\left(\pi D_p^3 / 6\right) = 4\times10^7 \text{ W/m}^3 (\pi / 6)(0.010 \text{ m})^3 = 20.9 \text{ W}.$$

The convection coefficient may be obtained from

$$\varepsilon\, \bar{j}_H = 2.06 Re_D^{-0.575}$$

with $$Re_D = VD/\nu = 30 \text{ m/s}(0.012 \text{ m}) \times 0.089 \text{ kg/m}^3 / 3\times10^{-5} \text{ kg/s}\cdot\text{m} = 1068.$$

Hence $$\bar{h} = \frac{\rho V c_p}{\varepsilon}\;\frac{Re_D^{-0.575}}{Pr^{2/3}} \times 2.06$$

$$\bar{h} = \frac{0.089 \text{ kg/m}^3 \times 30 \text{ m/s} \times 5193 \text{ J/kg}\cdot\text{K}}{0.4} \times 2.06(1.068)^{-0.575} / (0.66)^{2/3} = 1709 \text{ W/m}^2\cdot\text{K}.$$

Evaluating the temperatures,

$$T_{s,o} = 700 \text{ K} + \frac{20.9 \text{ W}}{\left(1709 \text{ W/m}^2\cdot\text{K}\right)\pi\,(0.012 \text{ m})^2} = 727 \text{ K}$$

$$T_{s,i} = 727 \text{ K} + \frac{20.9 \text{ W}}{4\pi \times 2 \text{ W/m}\cdot\text{K}}\left(\frac{1}{0.005 \text{ m}} - \frac{1}{0.006 \text{ m}}\right) = 755 \text{ K}$$

$$T(0) = 755 \text{ K} + \frac{4\times10^7 \text{ W/m}^3 (0.01)^2}{24 \times 2 \text{ W/m}\cdot\text{K}} = 838 \text{ K}. \qquad <$$

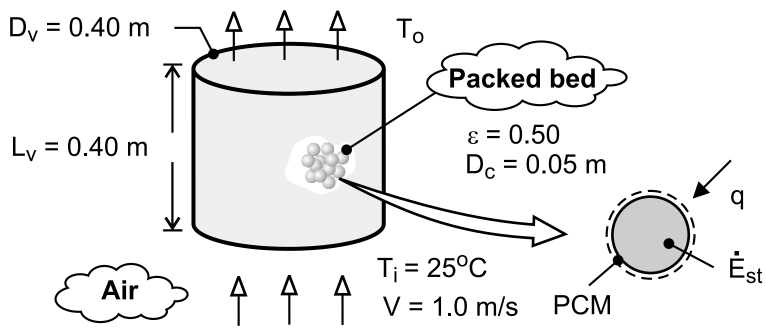
COMMENTS: The prescribed conditions provide for operation well below the melting point of uranium oxide. Hence $\dot{q}$ could be substantially increased to achieve a higher helium outlet temperature.

PROBLEM 7.101

KNOWN: Diameter and properties of phase-change material. Dimensions of cylindrical vessel and porosity of packed bed. Inlet temperature and velocity of air.

FIND: (a) Outlet temperature of air and rate of melting, (b) Effect of inlet velocity and capsule diameter on outlet temperature, (c) Location at which complete melting of PCM is first to occur and subsequent variation of outlet temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible thickness (and thermal resistance) of capsule shell, (2) All capsules are at T_{mp}, (3) Constant properties, (4) Negligible heat transfer from surroundings to vessel.

PROPERTIES: Prescribed, PCM: $T_{mp} = 4°C$, $\rho = 1200\,kg/m^3$, $h_{sf} = 165\,kJ/kg$. *Table A-4*, Air $\left(\text{Assume } (T_i + T_o)/2 = 17°C = 290K\right)$: $\rho_a = 1.208\,kg/m^3$, $c_p = 1007\,J/kg\cdot K$, $\nu = 15.00\times10^{-6}\,m^2/s$, Pr = 0.71.

ANALYSIS: (a) For a packed bed (Section 7.8), the outlet temperature is given by

$$T_o = T_{mp} - (T_{mp} - T_i)\exp\left(-\frac{\bar{h}A_{p,t}}{\rho_a V A_{c,b} c_p}\right)$$

where $A_{c,b} = \pi D_v^2/4 = \pi\left(0.40m^2\right)/4 = 0.126m^2$ and $A_{p,t} = (1-\varepsilon)(\forall_v/\forall_c)\left(\pi D_c^2\right) = (1-\varepsilon)\left(1.5\pi L_v D_v^2/D_c\right) = 0.5\left(1.5\pi\times0.4m^3/0.05m\right) = 3.02m^2$. With $Re_D = VD_c/\nu = 1\,m/s\times0.05m/15.00\times10^{-6}m^2/s = 3333$, the convection correlation for a packed bed yields

$$\varepsilon\bar{j}_H = \varepsilon\overline{St}\,Pr^{2/3} = \varepsilon\frac{\bar{h}}{\rho_a V\,c_p}Pr^{2/3} = 2.06\,Re_D^{-0.575}$$

$$\bar{h} = \frac{2.06\,\rho_a V\,c_p}{\varepsilon\,Pr^{2/3}\,Re_D^{0.575}} = \frac{2.06\times1.208\,kg/m^3\times1\,m/s\times1007\,J/kg\cdot K}{0.5(0.71)^{2/3}(3333)^{0.575}} = 59.4\,W/m^2\cdot K$$

Hence,
$$T_o = 4°C + (21°C)\exp\left(-\frac{59.4\,W/m^2\cdot K\times3.02\,m^2}{1.208\,kg/m^3\times1\,m/s\times0.126\,m^2\times1007\,J/kg\cdot K}\right) = 10.5°C \quad <$$

The rate at which PCM in the vessel changes from the solid to liquid state, $\dot{M}\,(kg/s)$, may be obtained from an energy balance that equates the total rate of heat transfer to the capsules to the rate of increase in latent energy of the PCM. That is

$$q = \frac{d}{dt}(M\,h_{sf}) = h_{sf}\,\dot{M}$$

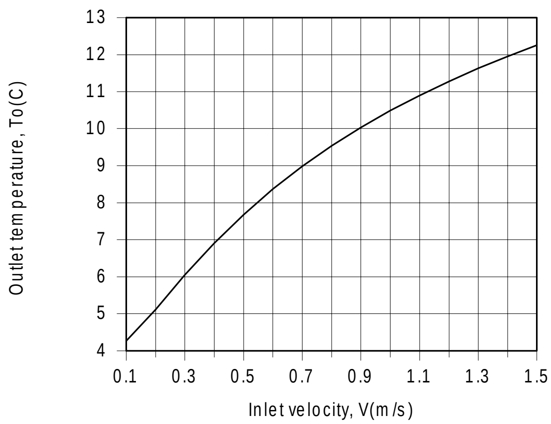
Continued …..

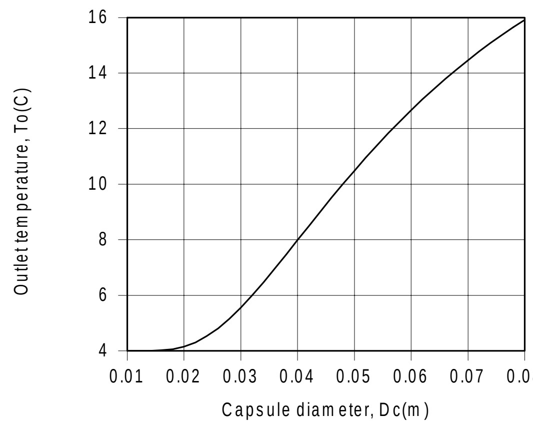
where M is the total mass of PCM and

$$q = -\bar{h}\,A_{p,t}\,\frac{\left(T_{mp}-T_i\right)-\left(T_{mp}-T_o\right)}{\ell n\left(\dfrac{T_{mp}-T_i}{T_{mp}-T_o}\right)} = -59.4\,W/m^2\cdot K \times 3.02\,m^2\,\frac{-14.5°C}{\ell n\left(\dfrac{-21}{-6.5}\right)} = 2220\,W$$

Hence, $\dot{M} = q/h_{sf} = 2220\,W/165{,}000\,J/kg = 0.0134\,kg/s$ <

(b) The effect of the inlet velocity and capsule diameter are shown below.





Despite the reduction in $\bar{h}$ with decreasing V, the reduction in the mass flow rate of air through the vessel and the corresponding increase in the residence time of air in the vessel allow it to more closely achieve thermal equilibrium with the capsules before it leaves the vessel. Hence, T_o decreases with decreasing V, approaching T_{mp} in the limit $V \to 0$. Of course, the production of chilled air in kg/s decreases accordingly. With decreasing capsule diameter, there is an increase in the number of capsules in the vessel and in the total surface area $A_{p,t}$ for heat transfer from the air. Hence, the heat rate increases with decreasing D_c and the outlet temperature of the air decreases.

(c) Because the temperature of the air decreases as it moves through the vessel, heat rates to the capsules are largest and smallest at the entrance and exit, respectively, of the vessel. Hence, complete melting will first occur in capsules at the entrance. After complete melting begins to occur in the capsules, progressing downstream with increasing time, heat transfer from the air will increase the temperatures of the capsules, thereby decreasing the heat rate. With decreasing heat rate, the outlet temperature will increase, approaching the inlet temperature after melting has occurred in all capsules and they achieve thermal equilibrium with the inlet air.

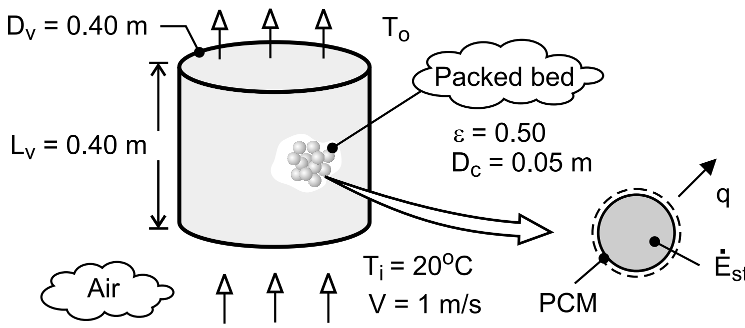
COMMENTS: (1) The estimate of To used to evaluate the properties of air was good, and iteration of the solution is not necessary. (2) The total mass of phase change material in the vessel is $M = N_c\,\rho\,\forall_c = \left[(1-\varepsilon)\forall_v/\forall_c\right]\rho\,\forall_c = (1-\varepsilon)\,\rho\,L_v\left(\pi D_v^2/4\right) = (\pi/4)\,0.5\times 1200\,kg/m^3\,(0.4m)^3 = 30.2\,kg$. At the maximum possible melting rate of $\dot{M} = 0.0134\,kg/s$, it would therefore take 2250s = 37.5 min to melt all of the PCM in the vessel. Why would it, in fact, take longer to melt all of the PCM?

PROBLEM 7.102

KNOWN: Diameter and properties of phase-change material. Dimensions of cylindrical vessel and porosity of packed bed. Inlet temperature and velocity of air.

FIND: (a) Outlet temperature of air and rate of freezing, (b) Effect of inlet velocity and capsule diameter on outlet temperature, (c) Location at which complete melting of PCM is first to occur and subsequent variation of outlet temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible thickness (and thermal resistance) of capsule shell, (2) All capsules are at T_{mp}, (3) Constant properties, (4) Negligible heat transfer from vessel to surroundings.

PROPERTIES: Prescribed, PCM: $T_{mp} = 50°C$, $\rho = 900\,kg/m^3$, $h_{sf} = 200\,kJ/kg$. *Table A-4*, Air (Assume $(T_i + T_o)/2 = 30°C = 303K$): $\rho_a = 1.151\,kg/m^3$, $c_p = 1007\,J/kg \cdot K$, $\nu = 16.2 \times 10^{-6} m^2/s$, $Pr = 0.707$.

ANALYSIS: (a) For a packed bed (Section 7.8), the outlet temperature is given by

$$T_o = T_{mp} - \left(T_{mp} - T_i\right)\exp\left(-\frac{\bar{h}A_{p,t}}{\rho_a V A_{c,b}\, c_p}\right)$$

where $A_{c,b} = \pi D_v^2/4 = 0.126\,m^2$ and $A_{p,t} = (1-\varepsilon)(\forall_v/\forall_c)\pi D_c^2 = 3.02\,m^2$. With $Re_D = VD_c/\nu = 3086$, the convection correlation for a packed bed yields

$$\varepsilon \bar{j}_H = \varepsilon \bar{S}t\, Pr^{2/3} = \varepsilon \frac{\bar{h}}{\rho_a V c_p} Pr^{2/3} = 2.06\, Re_D^{-0.575}$$

$$\bar{h} = \frac{2.06\,\rho_a V c_p}{\varepsilon\, Pr^{2/3}\, Re_D^{0.575}} = \frac{2.06 \times 1.151\,kg/m^3 \times 1\,m/s \times 1007\,J/kg \cdot K}{0.5(0.7)^{2/3}(3086)^{0.575}} = 59.1\,W/m^2 \cdot K$$

Hence,

$$T_o = 50°C - (30°C)\exp\left(-\frac{59.1\,W/m^2 \cdot K \times 3.02\,m^2}{1.151\,kg/m^3 \times 1\,m/s \times 0.126\,m^2 \times 1007\,J/kg \cdot K}\right) = 41.2°C \qquad <$$

The rate at which PCM in the vessel solidifies, $\dot{M}(kg/s)$, may be obtained from an energy balance that equates the total rate of heat transfer from the capsules to the rate at which the latent energy of the PCM decreases. That is,

$$q = \frac{d}{dt}\left(M\, h_{s,f}\right) = h_{sf}\ \dot{M}$$

where M is the total mass of PCM and

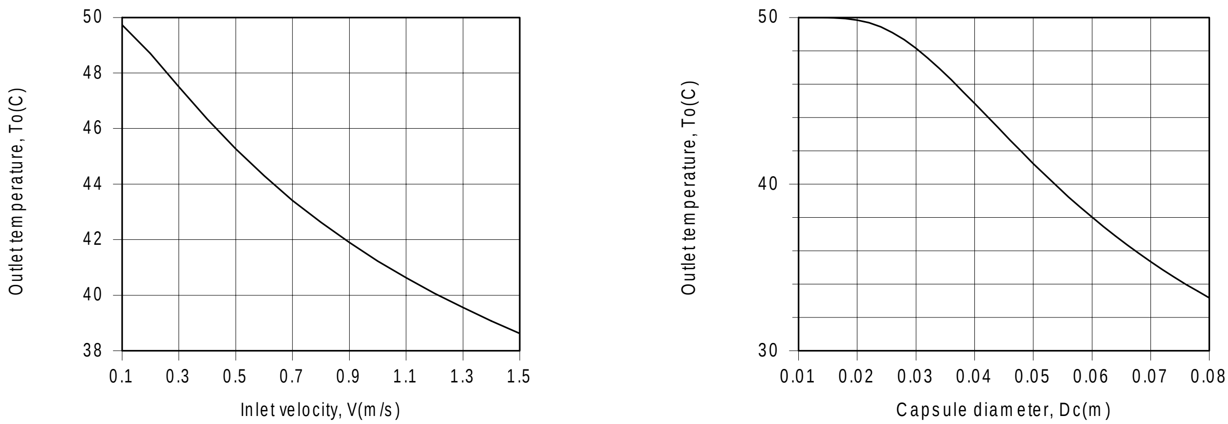
Continued …..

PROBLEM 7.102 (Cont.)

$$q = \bar{h}\, A_{p,t} \frac{\left(T_{mp} - T_i\right) - \left(T_{mp} - T_o\right)}{\ell n\left(\dfrac{T_{mp} - T_i}{T_{mp} - T_o}\right)} = 59.1\,W/m^2 \cdot K \times 3.02\,m^2 \frac{21.2°C}{\ell n\left(\dfrac{30}{8.8}\right)} = 3085\,W$$

Hence, $\dot{M} = q/h_{sf} = 3085\,W/200,000\,J/kg = 0.0154\,kg/s$ <

(b) The effect of V and D_c are shown below



Despite the reduction in $\bar{h}$ with decreasing V, the reduction in the mass flow rate of air in the vessel and the corresponding increase in the residence time of air in the vessel allow it to more closely reach thermal equilibrium with the capsules before it leaves the vessel. Hence, T_o increases with decreasing V, approaching T_{mp} in the limit $V \to 0$. Of course, the production of warm air in kg/s decreases accordingly. With decreasing capsule diameter, there is an increase in the number of capsules in the vessel and in the total surface area $A_{p,t}$ for heat transfer to the air. Hence, the heat rate and the air outlet temperature increase with decreasing D_c.

(c) Because the air temperature increases as it moves through the vessel, heat rates from the capsules are largest and smallest at the entrance and exit, respectively, of the vessel. Hence, complete freezing will first occur in capsules at the entrance. After complete freezing begins to occur in the capsules, progressing downstream with increasing time, heat transfer to the air will decrease the temperatures of the capsules, thereby decreasing the heat rate. With decreasing heat rate, the outlet temperature will decrease, approaching the inlet temperature after freezing has occurred in all capsules and they achieve thermal equilibrium with the inlet air.

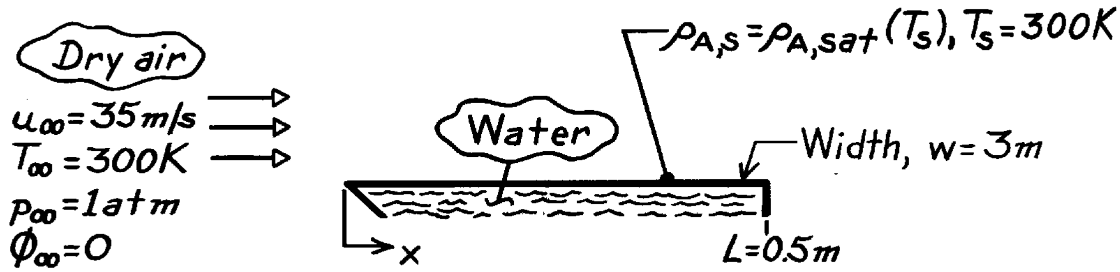
COMMENTS: (1) The estimate of T_o used to evaluate the properties of air was good, and iteration of the solution is not necessary. (2) The total mass of phase change material in the vessel is $M = N_c \rho \forall_c = \left[(1-\varepsilon)\forall_v/\forall_c\right]\rho\forall_c = (1-\varepsilon)\rho L_v\left(\pi D_v^2/4\right) = 22.6\,kg$. At the maximum possible melting rate of $\dot{M} = 0.0154\,kg/s$, it would therefore take 1470s = 24.5 min to freeze all of the PCM in the vessel. Why would it, in fact, take longer to freeze all of the PCM?

PROBLEM 7.103

KNOWN: Flow of air over a flat, smooth wet plate.

FIND: (a) Average mass transfer coefficient, $\overline{h}_m$, (b) Water vapor mass loss rate, n_A (kg/s).

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Heat-mass transfer analogy applies, (3) $Re_{x,c} = 5 \times 10^5$.

PROPERTIES: *Table A-4*, Air (300K): $\nu = 15.89 \times 10^{-6}$ m^2/s, Pr = 0.707; *Table A-8*, Water vapor-air (300K, 1 atm): $D_{AB} = 0.26 \times 10^{-4}$ m^2/s, $Sc = \nu/D_{AB} = 0.611$; *Table A-6*, Water vapor (300K): $\rho_{A,sat} = 1/v_g = 0.0256$ kg/m^3.

ANALYSIS: (a) The Reynolds number for the plate, x = L, is

$$Re_L = \frac{u_\infty L}{n} = \frac{35 \text{ m/s} \times 0.5 \text{ m}}{15.89 \times 10^{-6} \text{ m}^2/\text{s}} = 1.10 \times 10^6.$$

Hence flow is mixed and the appropriate flat plate convection correlation is given by Eq. 7.42,

$$\overline{Sh}_L = \frac{\overline{h}_m L}{D_{AB}} = \left(0.037\, Re_L^{4/5} - 871\right) Sc^{1/3} = \left(0.037\left[1.10 \times 10^6\right]^{0.8} - 871\right) 0.611^{0.33}$$

giving

$$\overline{Sh}_L = 1399 \qquad \overline{h}_m = \frac{1399 \times 0.26 \times 10^{-4} \text{ m}^2/\text{s}}{0.5 \text{ m}} = 0.0728 \text{ m/s}. \qquad <$$

(b) The evaporative mass loss rate is

$$n_A = \overline{h}_m A_s \left(r_{A,s} - r_{A,\infty}\right)$$

where $A_s = L \cdot w$, $r_{A,\infty} = 0$ (dry air) and $r_{A,s} = r_{A,sat}$. Hence,

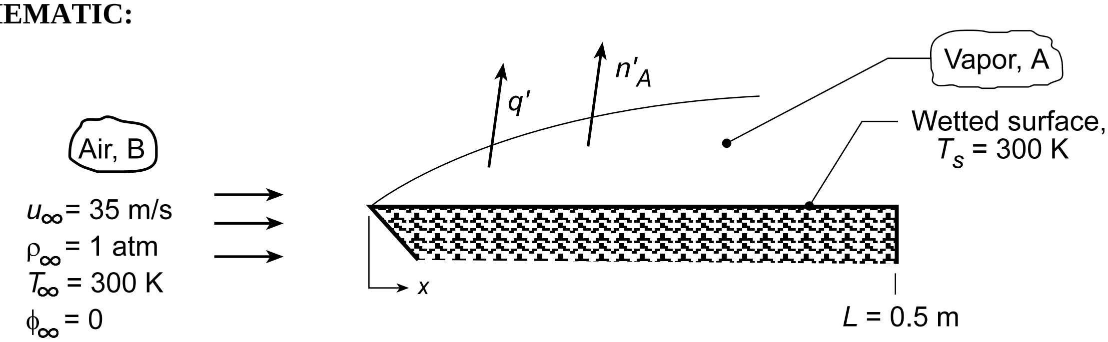
$$n_A = 0.0728 \text{ m/s} \times (0.5 \times 3) \text{ m}^2 \ (0.0256 - 0) \text{ kg/m}^3 = 0.0028 \text{ kg/s}. \qquad <$$

PROBLEM 7.104

KNOWN: Air flow conditions over a wetted flat plate of known length and temperature.

FIND: (a) Heat loss and evaporation rate, per unit plate width, q' and n'_A, respectively, (b) Compute and plot q' and n'_A for a range of water temperatures $300 \le T_s \le 350$ K with air velocities of 10, 20 and 35 m/s, and (c) Water temperature T_s at which the heat loss will be zero for the air velocities and temperatures of part (b).

SCHEMATIC:



ASSUMPTIONS: (1) Heat and mass transfer analogy is applicable, (2) Constant properties, (3) $Re_{x,c} = 5 \times 10^5$.

PROPERTIES: *Table A.4*, Air (T = 300 K, 1 atm): $\nu = 15.89 \times 10^{-6}$ m^2/s; *Table A.6*, Water (300 K): $v_g = 39.13$ m^3/kg, $h_{fg} = 2438$ kJ/kg; *Table A.8*, Water-air (298 K, 1 atm): $D_{AB} = 0.26 \times 10^{-4}$ m^2/s, Sc = 0.61.

ANALYSIS: (a) The heat loss from the plate is due only to the transfer of latent heat. Per unit width of the plate,

$$q' = n'_A h_{fg} \tag{1}$$

$$n'_A = \bar{h}_m L\left[\rho_{A,sat}(T_s) - \rho_{A,\infty}\right] = \bar{h}_m L \rho_{A,sat}(T_s) \tag{2}$$

With

$$Re_L = \frac{u_\infty L}{\nu} = \frac{35\,\text{m/s} \times 0.5\,\text{m}}{15.89 \times 10^{-6}\,\text{m}^2/\text{s}} = 1.10 \times 10^6$$

mixed boundary layer condition exists and the appropriate correlation is Eq. 7.42,

$$\overline{Sh}_L = \left(0.037\,Re_L^{4/5} - 871\right)Sc^{1/3} = \left[0.037\left(1.10 \times 10^6\right)^{4/5} - 871\right](0.61)^{1/3} \tag{3}$$

giving $\overline{Sh}_L = 1398$ and

$$\bar{h}_m = \overline{Sh}_L \frac{D_{AB}}{L} = 1398 \frac{0.26 \times 10^{-4}\,\text{m}^2/\text{s}}{0.5\,\text{m}} = 0.0727\,\text{m/s}\,.$$

with $\rho_{A,sat}(T_s) = v_g^{-1} = 0.0256\,\text{kg/m}^3$,

$$n'_A = 0.0727\,\text{m/s}(0.5\,\text{m})\left(0.0256\,\text{kg/m}^3\right) = 9.29 \times 10^{-4}\,\text{kg/s}\cdot\text{m} \quad <$$

Hence, the evaporative heat loss per unit plate width is

$$q' = n'_A h_{fg} = 9.29 \times 10^{-4}\,\text{kg/s}\cdot\text{m}\left(2.438 \times 10^6\,\text{J/kg}\right) = 2265\,\text{W/m} \quad <$$

Continued...

Heat would have to be applied to the plate in the amount of 2265 W/m to maintain its temperature at 300 K with the evaporative heat loss.

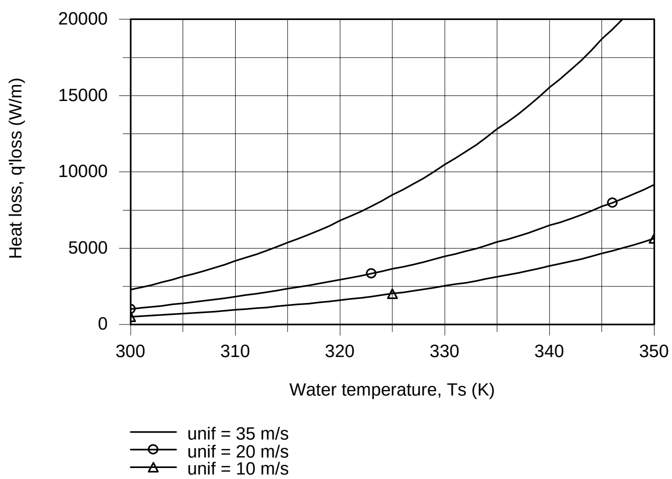
(b) When T_s and T_∞ are different, convection heat transfer will also occur, and the heat loss from the water surface is

$$q'_{loss} = q'_{conv} + q'_{evap} = \overline{h}L\left(T_s - T_\infty\right) + n'_A h_{fg} \quad (4)$$

Invoking the heat-mass analogy, Eq. 6.92 with n = 1/3,

$$\overline{h}/\overline{h}_m = \rho c\left(\alpha/D_{AB}\right)^{2/3} \quad (5)$$

where $\overline{h}_m$ and n'_A are evaluated using Eqs. (3) and (2), respectively. Using the foregoing relations in the *IHT Workspace*, but evaluating $\overline{h}$ (rather than $\overline{h}_m$) with the *Correlations Tool, External Flow*, for the *Average* coefficient for *Laminar* or *Mixed Flow*, q'_{loss} was evaluated as a function of u_∞ with T_∞ = 300 K.



(c) To determine the water temperature T_s at which the heat loss is zero, the foregoing IHT model was run with q'_{loss} = 0 with the result that, for all velocities,

T_s = 281 K $\quad <$

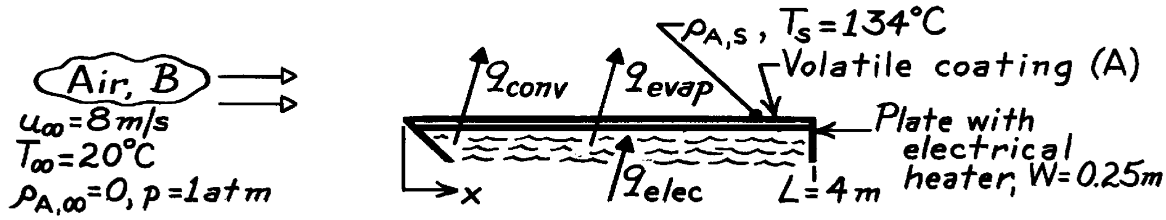
COMMENTS: Why is the result for part (c) independent of the air velocity?

PROBLEM 7.105

KNOWN: Flow over a heated flat plate coated with a volatile substance.

FIND: Electric power required to maintain surface at $T_s = 134°C$.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Heat-mass transfer analogy is applicable, (3) Transition occurs at $Re_{xc} = 5 \times 10^5$, (4) Perfect gas behavior of vapor A, (5) Upstream air is dry, $\rho_{A,\infty} = 0$.

PROPERTIES: *Table A-4*, Air (T_f = (134 + 20)°C/2 = 350 K, 1 atm): $\nu = 20.92 \times 10^{-6}$ m^2/s, k = 0.030 W/m·K, Pr = 0.700; Substance A (given): $\mathcal{M}_A$ = 150 kg/kmol, $p_{A,sat}$ (134°C) = 0.12 atm, $D_{AB} = 7.75 \times 10^{-7}$ m^2/s, $h_{fg} = 5.44 \times 10^6$ J/kg.

ANALYSIS: From an overall energy balance on the plate, the power required to maintain T_s is

$$q_{elec} = q_{conv} + q_{evap} = \overline{h}_L A_s \left(T_s - T_\infty\right) + \overline{h}_{m,L} A_s \left(\rho_{A,s} - \rho_{A,\infty}\right) h_{fg}. \qquad (1)$$

To estimate $\overline{h}_L$, first determine Re_L,

$$Re_L = u_\infty L/\nu = 8\text{ m/s} \times 4\text{ m}/20.92 \times 10^{-6}\text{ m}^2/\text{s} = 1.530 \times 10^6.$$

Hence the flow is mixed and the appropriate correlation:

$$\overline{Nu}_L = \overline{h}_L L/k = \left(0.037\, Re_L^{4/5} - 871\right) Pr^{1/3}$$

$$\overline{h}_L = \left(0.030\text{ W/m}\cdot\text{K}/4\text{m}\right)\left(0.037\left(1.530 \times 10^6\right)^{4/5} - 871\right)\left(0.700\right)^{1/3} = 16.0\text{ W/m}^2\cdot\text{K}.$$

To estimate $\overline{h}_{m,L}$, invoke the heat-mass analogy, with $Sc = \nu_B/D_{AB}$,

$$\overline{h}_{m,L} = \overline{h}_L \frac{D_{AB}}{k}\left(\frac{Sc}{Pr}\right)^{1/3} = 16.0\frac{\text{W}}{\text{m}^2\cdot\text{K}}\left(\frac{7.75 \times 10^{-7}\text{m}^2/\text{s}}{0.030\text{ W/m}\cdot\text{K}}\right)\left(\frac{20.92 \times 10^{-6}\text{m}^2/\text{s}}{7.75 \times 10^{-7}\text{m}^2/\text{s}}/0.700\right)^{1/3} = 0.00140\frac{\text{m}}{\text{s}}.$$

The density of species A at the surface, $\rho_{A,s}(T_s)$, follows from the perfect gas law,

$$\rho_{A,s} = p_{A,s}/\frac{\Re}{\mathcal{M}_A}T_s = 0.12\text{ atm}/\frac{8.205 \times 10^{-2}\text{ m}^3\cdot\text{atm/kmol}\cdot\text{K}}{150\text{ kg/kmol}}\cdot\left(134 + 273\right)\text{K} = 0.539\frac{\text{kg}}{\text{m}^3}.$$

Using values calculated for $\overline{h}_L$, $\overline{h}_{m,L}$ and $\rho_{A,s}$ in Eq. (1), find

$$q_{elec} = \left(4\text{m} \times 0.25\text{m}\right)\left[16.0\frac{\text{W}}{\text{m}^2\cdot\text{K}}\left(134 - 20\right)^\circ\text{C} + 0.00140\frac{\text{m}}{\text{s}}\left(0.539 - 0\right)\frac{\text{kg}}{\text{m}^3} \times 5.44 \times 10^6\frac{\text{J}}{\text{kg}}\right]$$

$$q_{elec} = 1.0\text{ m}^2\left[1{,}824 + 4{,}105\right]\text{ W/m}^2 = 5.93\text{ kW}.$$ <

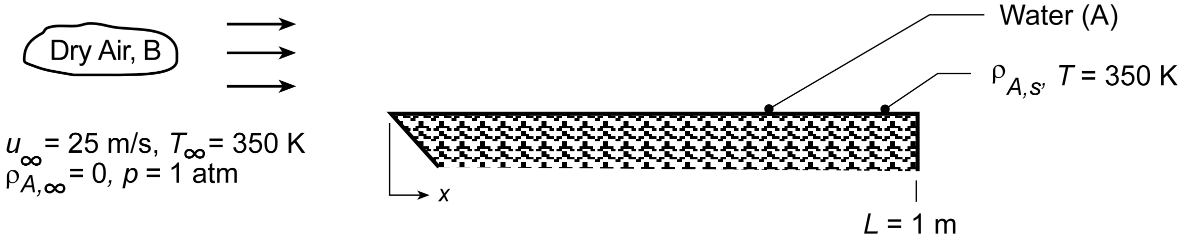
COMMENTS: For these conditions, nearly 70% of the heat loss is by evaporation.

PROBLEM 7.106

KNOWN: Flow of dry air over a water-saturated plate for prescribed flow conditions and mixed temperature.

FIND: (a) Mass rate of evaporation per unit plate width, $n''_A\,(\mathrm{kg/s\cdot m})$, and (b) Calculate and plot n'_A as a function of velocity for the range $1 \le u_\infty \le 25$ m/s for air and water temperatures of $T_s = T_\infty = 300$, 325, and 350 K.

SCHEMATIC:



ASSUMPTIONS: (1) Water surface is smooth, (2) Heat and mass transfer analogy is applicable, (3) $Re_{x,c} = 5 \times 10^5$.

PROPERTIES: *Table A.6*, Water vapor (T_s = 350 K, 1 atm): $\rho_{A,s} = 1/v_g = 1/3.846\ \mathrm{m^3/kg} = 0.2600\ \mathrm{kg/m^3}$; *Table A.4*, Air ($T_f = T_\infty$ = 350 K, 1 atm): $\nu = 20.92 \times 10^{-6}\ \mathrm{m^2/s}$, $\alpha = 29.9 \times 10^{-6}\ \mathrm{m^2/s}$; *Table A.8*, Air-water ($T_f = T_\infty$ = 350 K, 1 atm): $D_{AB} = 0.26 \times 10^{-4}\ \mathrm{m^2/s}\ (350\ \mathrm{K}/293\ \mathrm{K})^{3/2} = 0.339 \times 10^{-4}\ \mathrm{m^2/s}$.

ANALYSIS: (a) Determine the nature of the air flow by calculating Re_L. With L = 1 m,

$$Re_L = \frac{u_\infty L}{\nu} = \frac{25\,\mathrm{m/s}\times 1\,\mathrm{m}}{20.92\times10^{-6}\,\mathrm{m^2/s}} = 1.195\times10^6. \quad (1)$$

Since $Re_L > 5 \times 10^5$, it follows that the flow is mixed, and with Eq. 7.42 using $Sc = \nu/D_{AB}$,

$$\overline{Sh}_L = \frac{\bar{h}_m L}{D_{AB}} = \left(0.037\,Re_L^{4/5} - 871\right)Sc^{1/3}. \quad (2)$$

$$\overline{Sh}_L = \left(0.037\left[1.195\times10^6\right]^{4/5} - 871\right)\left(\frac{20.92\times10^{-6}\,\mathrm{m^2/s}}{0.339\times10^{-4}\,\mathrm{m^2/s}}\right)^{1/3} = 1550$$

The average mass transfer coefficient for the entire plate is

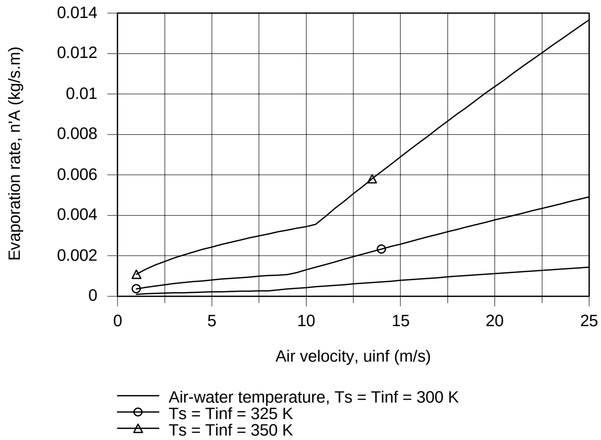
$$\bar{h}_m = \overline{Sh}_L\frac{D_{AB}}{L} = 1550\,\frac{0.339\times10^{-4}\,\mathrm{m^2/s}}{1\,\mathrm{m}} = 0.0526\,\mathrm{m/s}.$$

The mass rate of water evaporation per unit plate width is

$$n'_A = \bar{h}_m L\left(\rho_{A,s} - \rho_{A,\infty}\right) = 0.0526\,\mathrm{m/s}\times1\,\mathrm{m}\,(0.260-0)\,\mathrm{kg/m^3} = 0.0137\,\mathrm{kg/s\cdot m} \quad <$$

(b) Using Eq. (1) and (3) in the IHT Workspace with the *Correlations Tool*, *External Flow*, *Flat Plate*, *Average* coefficient for *Laminar* or *Mixed Flow*, replacing heat transfer with mass transfer parameters, the evaporation rate as a function of a velocity for selected air-water velocities was calculated and is plotted below.

Continued...



COMMENTS: (1) Note carefully the use of the heat-mass transfer analogy, recognizing that air is species B.

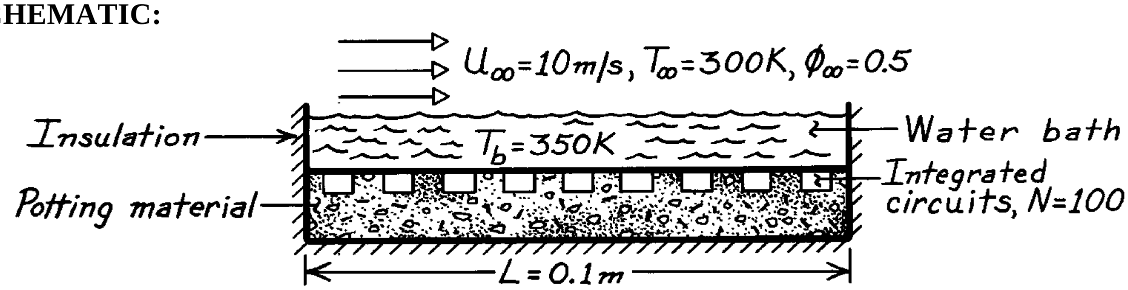
(2) How do you explain the abrupt slope changes in the evaporation rate as a function of velocity in the above plot?

PROBLEM 7.107

KNOWN: Temperature of water bath used to dissipate heat from 100 integrated circuits. Air flow conditions.

FIND: Heat dissipation per circuit.

SCHEMATIC:



ASSUMPTIONS: (1) Heat and mass transfer analogy is applicable, (2) Vapor may be approximated as a perfect gas, (3) Turbulent boundary layer over entire surface, (4) All heat loss is across air-water interface.

PROPERTIES: *Table A-4*, Air (325 K, 1 atm): $\nu = 18.4 \times 10^{-6}$ m^2/s, k = 0.0282 W/m·K, Pr = 0.704; *Table A-8*, Air-vapor (325 K, 1 atm): $D_{AB} = 0.26 \times 10^{-4}$ m^2/s$(325/298)^{3/2} = 0.296 \times 10^{-4}$ m^2/s, Sc = ν/D_{AB} = 0.622; *Table A-6*, Saturated water vapor (T_b = 350 K): ρ_g = 0.260 kg/m^3, $h_{fg} = 2.32 \times 10^6$ J/kg; (T_∞ = 300 K): ρ_g = 0.026 kg/m^3.

ANALYSIS: The heat rate is

$$q_1 = \frac{q}{N} = \frac{L^2}{N}\left[q'' + n''_A h_{fg}\left(T_b\right)\right].$$

Evaluate the heat and mass transfer convection coefficients with

$$Re_L = \frac{u_\infty L}{n} = \frac{10 \text{ m/s} \times 0.1 \text{ m}}{18.4 \times 10^{-6} \text{ m}^2/\text{s}} = 54{,}348$$

$$\bar{h} = (k/L)\, 0.037 Re_L^{4/5} Pr^{1/3} = (0.0282 \text{ W/m}\cdot\text{K}/0.1 \text{ m})\, 0.037 (54{,}348)^{4/5} (0.704)^{1/3} = 57 \text{ W/m}^2\cdot\text{K}$$

$$\bar{h}_m = (D_{AB}/L)\, 0.037 Re_L^{4/5} Sc^{1/3} = \left(0.296 \times 10^{-4} \text{ m}^2\text{/s}/0.1 \text{ m}\right) 0.037 (54{,}348)^{4/5} (0.622)^{1/3} = 0.0574 \text{ m/s}.$$

The convection heat transfer rate is

$$q'' = \bar{h}\left(T_b - T_\infty\right) = 57 \text{ W/m}^2\cdot\text{K}\,(350 - 300)\,\text{K} = 2850 \text{ W/m}^2$$

and the evaporative cooling rate is

$$n''_A h_{fg} = \bar{h}_m\left[r_{A,sat}\left(T_b\right) - f_\infty r_{A,sat}\left(T_\infty\right)\right] h_{fg}\left(T_b\right)$$

$$n''_A h_{fg} = 0.0574 \text{ m/s}\,[0.260 - 0.5 \times 0.026]\,\text{kg/m}^3 \times 2.32 \times 10^6 \text{ J/kg} = 32{,}890 \text{ W/m}^2$$

Hence

$$q_1 = \frac{(0.1 \text{ m})^2}{100}(2850 + 32{,}890) \text{ W/m}^2 = 3.57 \text{ W}. \qquad <$$

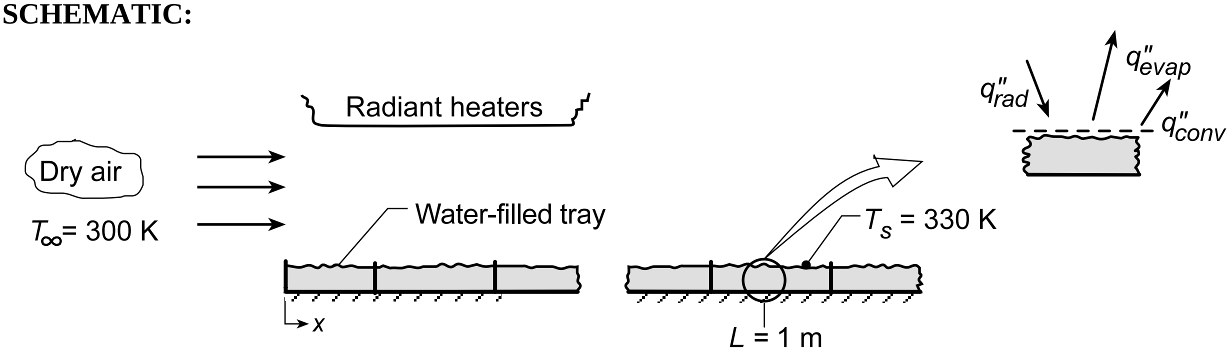
COMMENTS: Heat loss due to evaporative cooling is approximately an order of magnitude larger than that due to the convection of sensible energy.

PROBLEM 7.108

KNOWN: Dry air flows at 300 K over water-filled trays, each 222 mm long, with velocity of 15 m/s while radiant heaters maintain the surface temperature at 330 K.

FIND: (a) Evaporative flux (kg/s·m^2) at a distance 1 m from leading edge, (b) Radiant flux at this distance required to maintain water temperature at 330 K, (c) Evaporation rate from the tray at location L = 1 m, n'_A (kg/s·m) and (d) Irradiation which should be applied to each of the first four trays such that their rates are identical to that found in part (c).

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Heat-mass transfer analogy applicable, (3) Water vapor behaves as perfect gas, (4) All incident radiant power absorbed by water, (5) Critical Reynolds number is 5×10^5.

PROPERTIES: *Table A.4*, Air (T_f = 315 K, 1 atm): $\nu = 17.40 \times 10^{-6}$ m^2/s, k = 0.0274 W/m·K, Pr = 0.705; *Table A.8*, Water vapor-air (T_f = 315 K): $D_{AB} = 0.26 \times 10^{-4}$ m^2/s $(315/298)^{3/2} = 0.28 \times 10^{-4}$ m^2/s, Sc = ν/D_{AB} = 0.616; *Table A.6*, Saturated water vapor (T_s = 330 K): $\rho_{A,sat} = 1/v_g = 0.1134$ kg/m^3, h_{fg} = 2366 kJ/kg.

ANALYSIS: (a) The evaporative flux of water vapor (A) at location x is

$$\dot{n}''_{A,x} = h_{m,x}\left(\rho_{A,s} - \rho_{A,\infty}\right) = h_{m,x}\left[\rho_{A,sat}\left(T_s\right) - \phi_\infty \rho_{A,sat}\left(T_\infty\right)\right] \quad (1)$$

Evaluate Re_x to determine the nature of the flow and then select the proper correlation.

$$Re_x = \frac{u_\infty x}{\nu} = 15\,\text{m/s} \times 1\,\text{m} \big/ 17.40 \times 10^{-6}\,\text{m}^2/\text{s} = 8.621 \times 10^5.$$

Hence, the flow is turbulent, and invoking the heat-mass analogy with Eq. 7.45,

$$Sh_x = \frac{h_m x}{D_{AB}} = 0.0296\, Re_x^{4/5}\, Sc^{1/3}$$

$$h_m = \frac{0.28 \times 10^{-4}\,\text{m}^2/\text{s}}{1\text{m}} \times 0.0296\left(8.621 \times 10^5\right)^{4/5}(0.616)^{1/3} = 3.952 \times 10^{-2}\,\text{m/s}.$$

Hence, the evaporative flux at x = 1 m is

$$\dot{n}''_{A,x} = 3.952 \times 10^{-2}\,\text{m/s}\left(0.1134\,\text{kg/m}^3 - 0\right) = 4.48 \times 10^{-3}\,\text{kg/s}\cdot\text{m}^2 \quad (2) \;<$$

(b) From an energy balance on the differential element at x = 1 m,

$$q''_{rad} = q''_{conv} + q''_{evap} = h_x\left(T_s - T_\infty\right) + \dot{n}''_{A,x} h_{fg} \quad (3)$$

Continued...

To estimate h_x, invoke the heat-mass analogy using the correlation, Eq. 7.45,

$$Nu_x / Sh_x = (Pr/Sc)^{1/3} \qquad \text{or} \qquad h_x = h_{m,x}\, k/D_{AB}\,(Pr/Sc)^{1/3} \qquad (4)$$

$$h_x = 3.95\times10^{-2}\ \text{kg/s}\cdot\text{m}^2 \left(0.0274\ \text{W/m}\cdot\text{K} \big/ 0.28\times10^{-4}\ \text{m}^2/\text{s}\right)(0.705/0.616)^{1/3} = 40.45\ \text{W/m}^2\cdot\text{K}$$

Hence, the required radiant flux is

$$q''_{rad} = 40.45\ \text{W/m}^2\cdot\text{K}\,(330-300)\,\text{K} + 4.48\times10^{-3}\ \text{kg/s}\cdot\text{m}^2 \times 2366\times10^3\ \text{J/kg}$$

$$q''_{rad} = 1,214\ \text{W/m}^2 + 10,600\ \text{W/m}^2 = 11,813\ \text{W/m}^2$$ <

(c) The flow is turbulent over tray 5 having its mid-length at x = 1 m, so that it is reasonable to assume,

$$\bar{h}_5 \approx h_x(1\,\text{m}) \qquad (5)$$

so that the evaporation rate can be determined from the evaporative flux as,

$$n'_A = n''_{A,x}\Delta L = 4.48\times10^{-3}\ \text{kg/s}\cdot\text{m}^2 \times 0.222\ \text{m} = 9.95\times10^{-4}\ \text{kg/s}\cdot\text{m}$$ <

(d) For tray 5, following the form of Eq. (3), the energy balance is

$$q''_{rad,5}\Delta L = \bar{h}_5\Delta L\left(T_{s,5} - T_\infty\right) + n'_{A,5}h_{fg} \qquad (6)$$

and the evaporation rate for the tray is

$$n'_{A,5} = \bar{h}_{m,5}\Delta L\left(\rho_{A,s} - 0\right) \qquad (7)$$

While $\bar{h}_5$ and $\bar{h}_{m,5}$ represent tray averages, Eq. (4) is still applicable. Using the *IHT Correlation Tool*, *External Flow*, *Average* coefficient for *Laminar*, or *Mixed Flow*, $\bar{h}_5$ is evaluated as

$$\bar{h}_5 = \left[\bar{h}_x(1.10\,\text{m})L_5 - \bar{h}_x(0.880\,\text{m})L_4\right]\big/\Delta L \qquad (8)$$

where $\Delta L = L_5 - L_4 = 0.22$ m. The same relations can be applied to trays 2, 3 and 4. For tray 1, $\bar{h}_1 = \bar{h}(0.22\ \text{m})\cdot L_1$, where $L_1 = \Delta L$. With Eqs. (3, 6, 7 and 8) in the IHT Workspace, along with the *Correlations* and *Properties Tools*, the following results were obtained with the requirement that the evaporation rate for each tray is equal at $n'_{A,5} = 10.01\times10^{-4}$ kg/s·m.

Tray	1	2	3	4	5
T_s	342.7	357	348.1	329	330
q''_{rad}	11,920	11,150	11,400	11,950	11,920

COMMENTS: (1) Note carefully at which temperatures the thermophysical properties are evaluated.

(2) Recognize that in part (d), if we require equal evaporation rates for each tray, $n'_{A,5}$, the water temperature, T_s, and radiant flux, q''_{rad}, for each tray must be different since the convection coefficients $\bar{h}_x$ and $\bar{h}_{m,x}$ are different for each of the trays. How do you explain the changes in T_s? Which tray has the highest $\bar{h}$? The lowest $\bar{h}$?

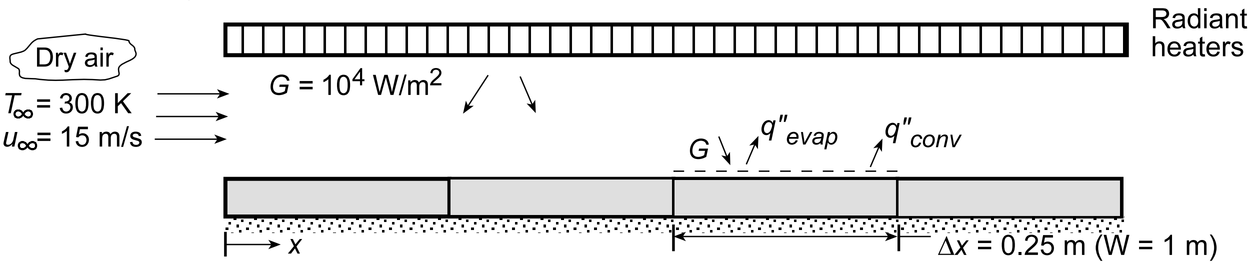
(3) For tray 5, using Eq. (5) we found $\bar{h}_5$ = 40.45 W/m²·K; using the more accurate formulation, Eq. (8), the result is 40.49 W/m²·K. If the flow were laminar or mixed over the tray, Eq. (5) would be inappropriate.

PROBLEM 7.109

KNOWN: Irradiation on sequential water-filled trays of prescribed length and width. Temperature and velocity of airflow over the trays.

FIND: Rate of water loss from first, third and fourth trays and temperature of water in each tray.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform irradiation of each container, (3) Complete absorption of irradiation by water, (4) Negligible heat transfer between containers and from bottom of containers, (5) Validity of heat-mass transfer analogy, (6) Applicability of convection correlations for an isothermal surface, (7) $Re_{x,c} = 5 \times 10^5$.

PROPERTIES: *Table A.4*, air (1 atm, *assume* T_f = 315 K): $\nu = 17.4 \times 10^{-6}$ m^2/s, k = 0.0274 W/m·K, Pr = 0.705. *Table A.8*, vapor/air (1 atm, 315 K): $D_{AB} = 0.26 \times 10^{-4}$ m^2/s $(315/298)^{3/2} = 0.28 \times 10^{-4}$ m^2/s, Sc = ν/D_{AB} = 0.616.

ANALYSIS: The temperature of each tray is determined by a balance between the absorbed radiation and the convection and evaporative losses. Hence,

$$G = q''_{conv} + q''_{evap} = \overline{h}\left(T_s - T_\infty\right) + \overline{h}_m \rho_{A,sat} h_{fg}$$

where, assuming an exponent of n = 1/3, the heat-mass transfer analogy yields

$$\overline{h}_m = \left(D_{AB}/k\right)\left(Sc/Pr\right)^{1/3}\overline{h} = \left(0.26\times10^{-4}\ m^2/s/0.0274\ W/m\cdot K\right)\left(0.616/0.705\right)^{1/3}\overline{h} = \left(9.07\times10^{-4}\ m^3\cdot K/W\cdot s\right)\overline{h}$$

Hence,

$$G = \overline{h}\left[\left(T_s - T_\infty\right) + 9.07\times10^{-4}\rho_{A,sat}h_{fg}\right]$$

With $Re_N = u_\infty N\Delta x/\nu$ = 15 m/s(N × 0.25 m)/17.4 × 10^{-6} m^2/s = (2.155 × 10^5)N, the flow is laminar for N = 1,2 with transition to turbulence occurring for N = 3.

For tray 1,

$$\overline{h} = \left(k/\Delta x\right)0.664\,Re_1^{1/2}\,Pr^{1/3}$$
$$= \left(0.0274\ W/m\cdot K/0.25\ m\right)0.664\left(2.155\times10^5\right)^{1/2}\left(0.705\right)^{1/3} = 30.1\ W/m^2\cdot K$$

For tray 4, with x = 0.875 m (N = 7/2),

$$\overline{h}_4 \approx \left(k/x\right)0.0296\,Re_{7/2}^{4/5}\,Pr^{1/3}$$
$$= \left(0.0274\ W/m\cdot K/0.875\ m\right)0.0296\left(7.543\times10^5\right)^{4/5}\left(0.705\right)^{1/3} = 41.5\ W/m^2\cdot K$$

Continued...

For tray 3, $\bar{h}_3 = \left(\bar{h}_{1-3}L_3 - \bar{h}_{1-2}L_2\right)/\Delta x$, where

$$\bar{h}_{1-3}L_3 = k\left(0.037\,Re_3^{4/5} - 871\right)Pr^{1/3}$$
$$= 0.0274\,W/m\cdot K\,(0.037\times 44,510 - 871)(0.705)^{1/3} = 18.9\,W/m\cdot K$$

$$\bar{h}_{1-2}L_2 = k\left(0.664\,Re_2^{1/2}\,Pr^{1/3}\right)$$
$$= 0.0274\,W/m\cdot K\,(0.664\times 656.5)(0.705)^{1/3} = 10.6\,W/m\cdot K$$

$$\bar{h}_3 = (18.9 - 10.6)\,W/m\cdot K/0.25\,m = 33.1\,W/m^2\cdot K$$

For tray 1, the energy balance yields

$$10^4\,W/m^2 = 30.1\,W/m^2\cdot K\left[(T_s - T_\infty) + 9.07\times 10^{-4}\rho_{A,sat}h_{fg}\right]$$

Since $\rho_{A,sat}$ depends strongly on T_s, the solution to this equation must be obtained by trial-and-error, with $\rho_{A,sat}$ (and h_{fg}) determined from Table A.6. The solution yields

$$T_{s,1} \approx 334.7\,K \qquad <$$

Similarly, for trays 3 and 4

$$T_{s,3} \approx 332.8\,K \qquad T_{s,4} \approx 327.1\,K \qquad <$$

The evaporation rate for tray N is

$$\dot{m}_{evap} = \bar{h}_m \rho_{A,sat}\,(W\Delta x) = 2.27\times 10^{-4}\bar{h}\rho_{A,sat}$$

from which it follows that

$$\dot{m}_{evap,1} \approx 9.5\times 10^{-4}\,kg/s, \qquad \dot{m}_{evap,3} \approx 9.5\times 10^{-4}\,kg/s, \qquad \dot{m}_{evap,4} \approx 9.3\times 10^{-4}\,kg/s \qquad <$$

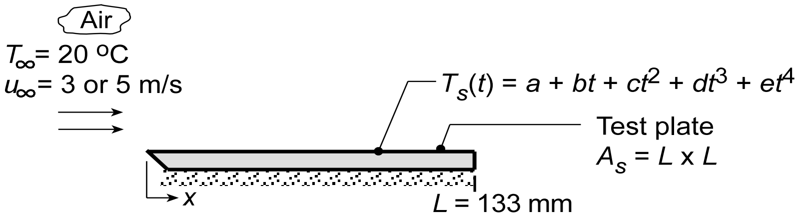
COMMENTS: (1) The largest convection coefficient is associated with the tray for which the entire flow is turbulent. (2) The temperature of the water varies inversely with the average convection coefficient for its tray.

PROBLEM 7.110

KNOWN: Apparatus as described in Problem 7.40 providing a nearly uniform airstream over a flat *test plate* to experimentally determine the heat and mass transfer coefficients. Temperature history of the pre-heated plate for airstream velocities of 3 and 9 m/s were fitted to a fourth-order polynomial for determining the heat transfer coefficient. Water mass loss observations from a water-saturated paper over the plate and its surface temperature for determining the heat transfer coefficient.

FIND: (a) From the temperature-time history, determine the heat transfer coefficients and evaluate the constants C and m for a correlation of the form $\overline{Nu}_L = C\,Re^m\,Pr^{1/3}$; compare results with a standard-plate correlation and comment on the goodness of the comparison; explain any differences; (b) From the water mass loss observations, determine the mass transfer coefficients for the two flow conditions; evaluate the constants C and m for a correlation of the form $\overline{Sh}_L = C\,Re^m\,Sc^{1/3}$; and (c) Using the heat-mass analogy, compare the experimental results with each other and against standard correlations; comment on the goodness of the comparison; explain any differences.

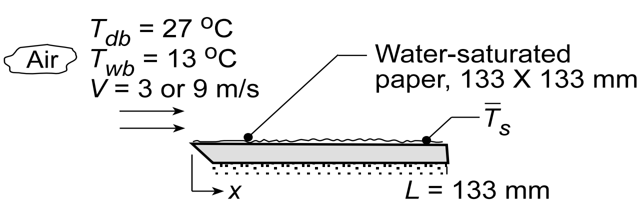
SCHEMATIC:



(a) Heat transfer experiment

Temperature Observations

u_∞ (m/s)	3	9
Δt (s)	300	160
a (°C)	56.87	57.00
b (°C/s)	-0.1472	-0.2641
c (°C/s^2)	3×10^{-4}	9×10^{-4}
d (°C/s^3)	-4×10^{-7}	-2×10^{-6}
e (°C/s^4)	2×10^{-10}	1×10^{-9}



(b) Mass transfer experiment

Mass Loss Observations

V (m/s)	$\overline{T}_s$ (°C)	m (t) (g)	m (t + Δt) (g)	Δt (s)
3	15.3	55.62	54.45	475
9	16.0	55.60	54.50	240

ASSUMPTIONS: (1) Airstream over the test plate approximates parallel flow over a flat plate, (2) Plate is spacewise isothermal, (3) Negligible radiation exchange between plate and surroundings, (4) Constant properties, and (5) Negligible heat loss from the bottom surface or edges of the test plate.

PROPERTIES: *Heat transfer coefficient*, *Table A.4*, Air ($T_f = \left(\overline{T}_s - T_\infty\right)/2$ = 310 K, 1 atm): k_a = 0.0269 W/m·K, $\nu = 1.669 \times 10^{-5}$ m^2/s, Pr = 0.706. *Test plate* (*Given*): ρ = 2770 kg/m^3, c_p = 875 J/kg·K, k = 177 W/m·K. *Mass transfer coefficient*: *Table A.6*, Water vapor ($\overline{T}_s$ = 15.3°C = 288.3 K): $\rho_{A,sat} = 1/v_g$ = 79.81 m^3/kg = 0.01253 kg/m^3; *Table A.6*, Water vapor ($\overline{T}_s$ = 16.0°C = 289 K): $\rho_{A,sat}$ = 0.01322 kg/m^3; *Table A.6*, Water vapor (T_{inf} = 27°C = 300 K): $\rho_{A,sat}$ = 0.02556 kg/m^3; *Table A.8*, Water vapor-air [T_f = $\left(\overline{T}_s + T_\infty\right)/2 \approx$ 295 K]: $D_{AB} = 0.26 \times 10^{-4}$ m^2/s $(295/298)^{1.5} = 0.256 \times 10^{-4}$ m^2/s.

ANALYSIS: (a) Using the lumped-capacitance method, the energy balance on the plate is

$$-\overline{h}_L A_s \left[T_s(t) - T_\infty\right] = \rho V c_p \frac{dT}{dt} \tag{1}$$

Continued...

PROBLEM 7.110 (Cont.)

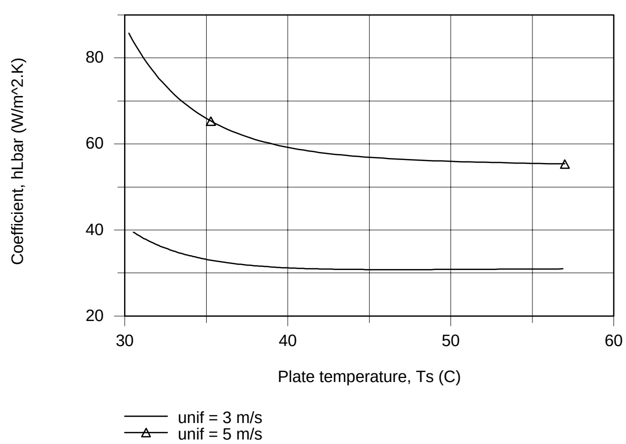
and the average convection coefficient can be determined from the temperature history, $T_s(t)$,

$$\bar{h}_L = \frac{\rho V c_p}{A_s} \frac{(dT/dt)}{T_s(t) - T_\infty} \tag{2}$$

where the temperature-time derivative is

$$\frac{dT_s}{dt} = b + 2ct + 3dt^2 + 4et^3 \tag{3}$$

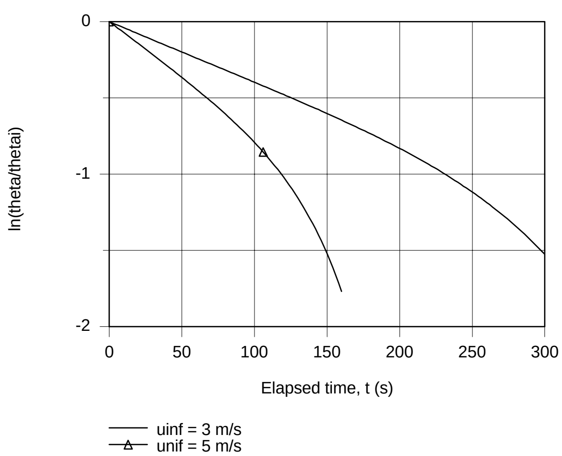
The temperature time history plotted below shows the experimental behavior of the observed data.

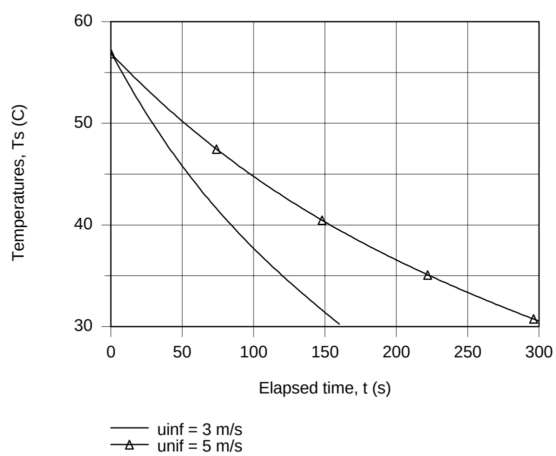


Consider now the integrated form of the energy balance, Eq. (5.6), expressed as

$$\ln \frac{T_s(t) - T_\infty}{T_i - T_\infty} = -\left(\frac{\bar{h}_L A_s}{\rho V c}\right) t \tag{4}$$

If we were to plot the LHS vs t, the slope of the curve would be proportional to $\bar{h}_L$. Using IHT, plots were generated of $\bar{h}_L$ vs. T_s, Eq. (1), and $\ln\left[\left(T_s(t) - T_\infty\right)/\left(T_i - T_\infty\right)\right]$ vs. t, Eq. (4). From the latter plot, recognize that the regions where the slope is constant corresponds to early times (≤ 100s when u_∞ = 3 m/s and ≤ 50s when u_∞ = 5 m/s).





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PROBLEM 7.110 (Cont.)

Selecting two elapsed times at which to evaluate $\overline{h}_L$, the following results were obtained

u_∞ (m/s)	t (s)	T_s (t), (°C)	$\overline{h}_L$ (W/m^2·K)	$\overline{Nu}_L$	Re_L
3	100	44.77	30.81	152.4	2.39×10^4
9	50	45.80	56.7	280.4	7.17×10^4

where the dimensionless parameters are evaluated as

$$\overline{Nu}_L = \frac{\overline{h}_L L}{k_a} \qquad Re_L = \frac{u_\infty L}{\nu} \tag{5,6}$$

where k_a, ν are thermophysical properties of the airstream.

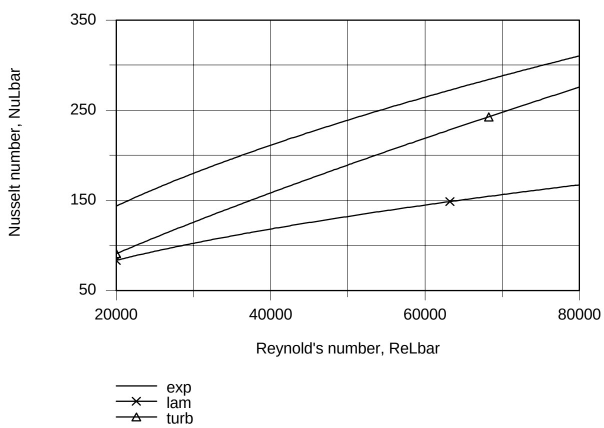
(b) Using the above pairs of $\overline{Nu}_L$ and Re_L, C and m in the correlation can be evaluated,

$$\overline{Nu}_L = C\,Re_L^m\,Pr^{1/3} \tag{7}$$

$$152.4 = C(2.39 \times 10^4)^m(0.706)^{1/3} \qquad 280.4 = C(7.17 \times 10^4)^m(0.706)^{1/3}$$

Solving, find C = 0.633 m = 0.555 (8,9) <

The plot below compares the experimental correlation (C = 0.633, m = 0.555) with those for laminar flow (C = 0.664, m = 0.5) and fully turbulent flow (C = 0.037, m = 0.8). The experimental correlation yields $\overline{Nu}_L$ values which are 25% higher than for the correlation. The most likely explanation for this unexpected trend is that the airstream reaching the plate is not parallel, but with a slight impingement effect and/or the flow is very highly turbulent at the leading edge.



(b) From the convection mass transfer rate equation,

$$n_A = \overline{h}_{m,L} A_s \left(\rho_{A,s} - \rho_{A,\infty}\right) \tag{10}$$

where the evaporation rate can be determined from the paper mass and time interval observations,

$$n_A = \left[m(t + \Delta t) - m(t)\right]/\Delta t \tag{11}$$

and the species densities, $\rho_{A,s}$ and $\rho_{A,\infty}$, correspond to $\rho_{A,sat}\left(\overline{T}_s\right)$ and $\phi_\infty \rho_{A,sat}\left(T_\infty\right)$, respectively.

Using the ASHRAE psychrometric chart (1 atm) with T_{wb} = 13°C and T_{db} = 27°C, find the relative humidity as ϕ_∞ = 0.17. The correlation dimensionless parameters are evaluated as

$$\overline{Sh}_L = \frac{\overline{h}_{m,L} L}{D_{AB}} \qquad Re_L = \frac{u_\infty L}{\nu} \qquad Sc = \frac{\nu}{D_{AB}} \tag{12,13,14}$$

Continued...

PROBLEM 7.110 (Cont.)

where all the properties are evaluated at $T_f = \left(\overline{T}_s + T_\infty\right)/2$. The results of the analyses are summarized in the following table.

u_∞	n_A	$\overline{h}_{m,L}$	$\overline{Sh}_L$	Re_L	Sc
(m/s)	kg/s	(m/s)			
3	2.463×10^{-6}	0.0168	87.58	2.594×10^4	0.603
9	4.583×10^{-6}	0.0288	150	7.767×10^4	0.603

Using the two sets of tabulated values for $\overline{Sh}_L$, Re_L and Sc and the standard correlation of the form,

$$\overline{Sh}_L = C\,Re_L^m\,Sc^{1/3} \tag{15}$$

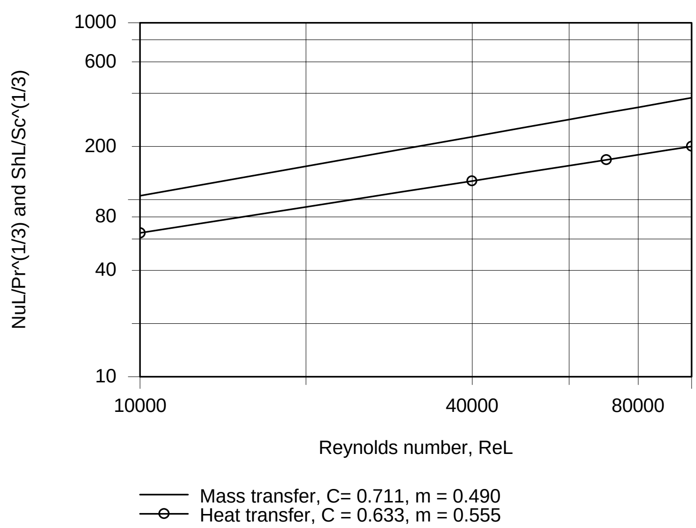
$$87.58 = C\left(2.594\times10^4\right)^m (0.603)^{1/3} \qquad\qquad 150 = C\left(7.767\times10^4\right)^m (0.603)^{1/3}$$

solve simultaneously to find $\quad C = 0.711 \quad\quad m = 0.490 \quad\quad$ (16,17)

From the heat-mass analogy, we expect the constants C and m in Eq. (7) for heat transfer and in Eq. (13) for mass transfer to be the same. From the two experiments, we found

	C	m
Heat transfer	0.633	0.555
Mass transfer	0.711	0.490

In the plot below, the parameters $\overline{Sh}_L/Sc^{1/3}$ or $\overline{Nu}_L/Pr^{1/3}$ are plotted against Re_L using Eq. (15) or (7). Note that the curves are nearly parallel on the log-log axes since their "m" constants are of similar value. The mass transfer results are, however, nearly 50% higher than those for heat transfer. We have no way to explain this systematic difference without more information on the apparatus, observation procedures and repeated observations. However, overall the results support the general form of the heat-mass analogy.

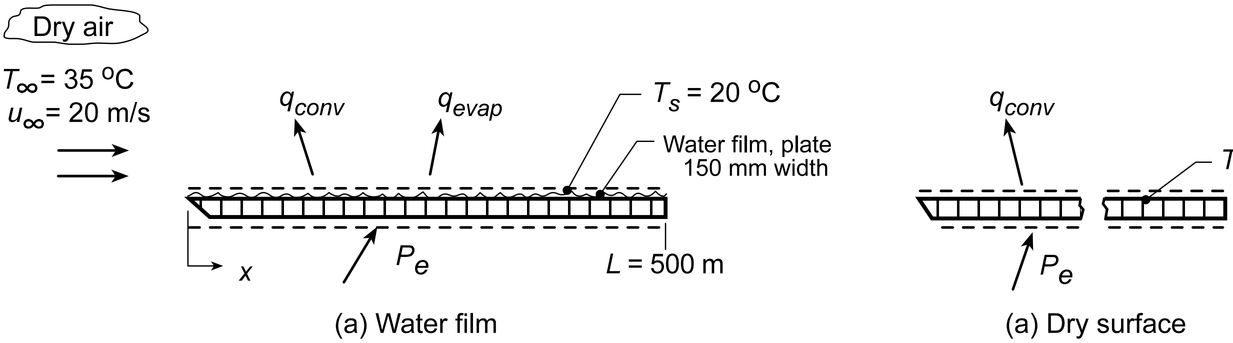


PROBLEM 7.111

KNOWN: Dry air at prescribed temperature and velocity flowing over a wetted plate of length 500 mm and width 150 mm. Imbedded electrical heater maintains the surface at $T_s = 20°C$.

FIND: (a) Water evaporation rate (kg/h) and electrical power P_e (W) required to maintain steady-state conditions, and (b) The temperature of the plate after all the water has evaporated, for the same airstream conditions and heater power of part (a).

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties and (3) Heat-mass transfer analogy is applicable.

PROPERTIES: *Table A.4*, Air ($T_f = (T_s + T_\infty)/2$ = 300 K, 1 atm): ρ = 1.16 kg/m^3, c_p = 1007 J/kg·K, k = 0.0263 W/m·K, ν = 15.94 × 10^{-6} m^2/s, α = 2.257 × 10^{-5} m^2/s, *Table A.6*, Water (T_s = 20°C = 293 K): $\rho_{A,s} = 1/v_g$ = 1/59.04 = 0.0169 kg/m^3, h_{fg} = 2454 kJ/K; *Table A.8*, Water-air (T_f = 300 K): D_{AB} = 0.26 × 10^{-4} m^2/s.

ANALYSIS: (a) Perform an energy balance on the plate,

$$\dot{E}_{in} - \dot{E}_{out} = 0 \qquad\qquad P_e - q_{conv} - q_{evap} = 0 \tag{1}$$

where the convection and evaporation rate equations are,

$$q_{conv} = \bar{h}_L A_s (T_s - T_\infty) \tag{2}$$

$$q_{evap} = n_A h_{fg} = \bar{h}_m A_s (\rho_{A,s} - \rho_{A,\infty}) - h_{fg} \tag{3}$$

The Reynolds number for the plate length L is

$$Re_L = \frac{u_\infty L}{\nu} = \frac{20\,\text{m/s} \times 0.50\,\text{m}}{15.94\times10^{-6}\,\text{m}^2/\text{s}} = 6.274\times10^5$$

so that the flow is mixed and Eq. 7.41 is appropriate to estimate $\bar{h}_L$,

$$\overline{Nu}_L = \frac{\bar{h}_L L}{k} = \left(0.037\,Re_D^{4/5} - 871\right)Pr^{1/3}$$

$$\bar{h}_L = \frac{0.0263\,\text{W/m}\cdot\text{K}}{0.5\,\text{m}}\left(0.037\left[6.274\times10^5\right]^{4/5} - 871\right)(0.707)^{1/3} = 34.5\,\text{W/m}^2\cdot\text{K}$$

Evoking the heat-mass analogy, Eq. 6.92, with n = 1/3

$$\frac{\bar{h}_L}{\bar{h}_m} = \rho c_p \left(\frac{\alpha}{D_{AB}}\right)^{-2/3} = 1.16\,\text{kg/m}^3 \times 1007\,\text{J/kg}\cdot\text{K}\left(\frac{2.257\times10^{-5}\,\text{m}^2/\text{s}}{0.26\times10^{-4}\,\text{m}^2/\text{s}}\right)^{-2/3} = 1284\,\text{J/m}^3\cdot\text{K}$$

Continued...

PROBLEM 7.111 (Cont.)

$$\overline{h}_m = 34.5\,\mathrm{W/m^2 \cdot K} \big/ 1284\,\mathrm{J/m^3 \cdot K} = 0.0269\,\mathrm{m/s}$$

Substituting numerical values, the energy balance, Eq. (1), with $A_s = 0.5\ \mathrm{m} \times 0.15\ \mathrm{m} = 0.075\ \mathrm{m^2}$,

$$\begin{aligned} &P_e - 34.5\,\mathrm{W/m^2 \cdot K} \times 0.075\,\mathrm{m^2}\,(20 - 35)\,\mathrm{K} \\ &\qquad - 0.0269\,\mathrm{m/s} \times 0.075\,\mathrm{m^2}\,(0.0169 - 0)\,\mathrm{kg/m^3} \times 2454 \times 10^3\ \mathrm{J/kg \cdot K} = 0 \end{aligned}$$

$$P_e = -38.8\,\mathrm{W} + 83.7 = 44.9\,\mathrm{W} \qquad <$$

The evaporation rate is

$$n_A = \overline{h}_m A_s \left(\rho_{A,s} - \rho_{A,\infty}\right) = 0.0269\,\mathrm{m/s} \times 0.075\,\mathrm{m^2} \times 0.0169\,\mathrm{kg/m^3} = 0.123\,\mathrm{kg/h} \qquad <$$

(b) When the plate is dry, the energy balance is

$$P_e = \overline{h}_L A_s \left(T_s - T_\infty\right)$$

and with P_e and $\overline{h}_L$ as determined in part (a),

$$T_s = T_\infty + P_e \big/ \overline{h}_L A_s = 35^\circ\mathrm{C} + 44.9\,\mathrm{W} \big/ 34.5\,\mathrm{W/m^2 \cdot K} \times 0.075\,\mathrm{m^2} = 52.3^\circ\mathrm{C} \qquad <$$

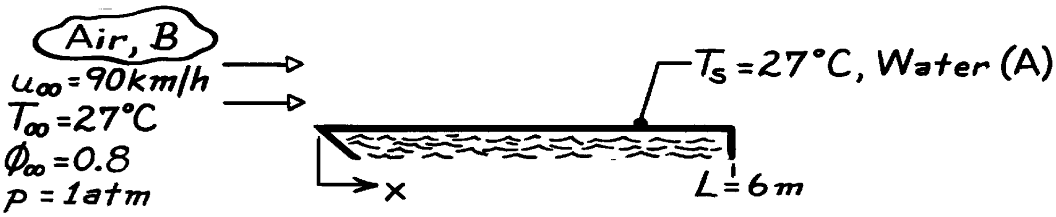
COMMENTS: Using *IHT Correlations Tool*, *External Flow*, *Flat Plate*, the calculation of part (b) was performed using the proper film temperature, $T_f = 318$ K, to find $\overline{h}_L = 32.7\ \mathrm{W/m^2 \cdot K}$ and $T_s = 53.3$°C.

PROBLEM 7.112

KNOWN: Convection mass transfer with turbulent flow over a flat plate (van roof).

FIND: (a) Location on van that will dry last, (b) Evaporation rate at trailing edge, $kg/s\cdot m^2$.

SCHEMATIC:



ASSUMPTIONS: (1) Turbulent flow over entire plate (van top), (2) Heat-mass transfer analogy is applicable, (3) Perfect gas behavior for water vapor (A).

PROPERTIES: *Table A-4*, Air (300 K, 1 atm): $\nu = 15.89 \times 10^{-6}\ m^2/s$, k = 0.0263 W/m·K, Pr = 0.707; *Table A-8*, Air-water vapor (25°C): $D_{AB} = 0.26 \times 10^{-4}\ m^2/s$; *Table A-6*, Saturated water vapor (300K): $\rho_{A,sat} = v_g^{-1} = 0.0256\ kg/m^3$.

ANALYSIS: (a) The mass transfer coefficient, $h_m(x)$, will be largest at x = 0 and smallest at x = L for turbulent flow conditions. Hence, the trailing edge will dry last.

(b) The evaporation rate on a per unit area basis, at the trailing edge where x = L, is given by the rate equation,

$$n''_A = h_{m,L}\left(\rho_{A,s} - \rho_{A,\infty}\right) = h_{m,L}\ \rho_{A,sat}\left(1 - \phi_\infty\right)$$

For turbulent flow the appropriate correlation for estimating $h_{m,L}$ is of the form

$$Sh_x = h_{m,x}x/D_{AB} = 0.0296\,Re_x^{4/5}\ Sc^{1/3}.$$

Substituting numerical values,

$$Re_L = \frac{u_\infty L}{\nu_B} = \frac{90\times10^3\ m/h}{3600\ s/h}\times 6m/15.89\times10^{-6}\ m^2/s = 9.44\times10^6$$

$$Sc = \frac{\nu_B}{D_{AB}} = 15.89\times10^{-6}\ m^2/s/0.26\times10^{-4}\ m^2/s = 0.611$$

$$h_{m,L} = \left(0.26\times10^{-4}\ m^2/s/6m\right)\times 0.0296\ \left(9.44\times10^6\right)^{4/5}\ (0.611)^{1/3} = 0.0414\ m/s.$$

Hence, the evaporation flux (rate per unit area) is

$$n''_A = 0.0414\ m/s \times 0.0256\ kg/m^3\ (1-0.8) = 2.12\times10^{-4}\ kg/s\cdot m^2. \quad <$$

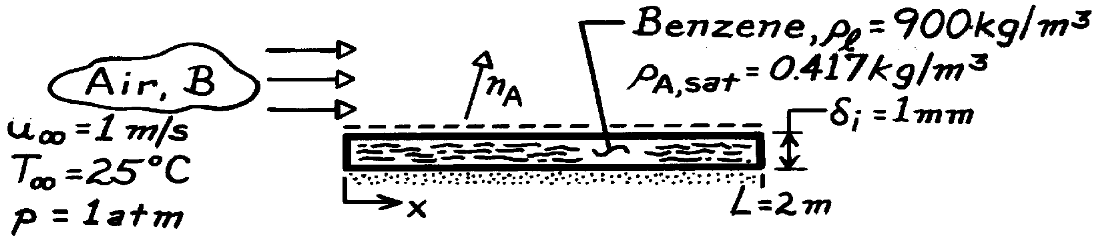
COMMENTS: Recognize how the heat-mass analogy is utilized and the appropriate correlation selected from Table 7.9.

PROBLEM 7.113

KNOWN: Length and thickness of a layer of benzene. Velocity and temperature of air in parallel flow over the layer.

FIND: Time required for complete evaporation.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Smooth liquid surface and negligible free-stream turbulence, (3) Heat and mass transfer analogy is applicable, (4) Negligible benzene vapor concentration in free-stream air, (5) Isothermal conditions at 25°C.

PROPERTIES: *Table A-4*, Air (25°C, 1 atm): $\nu = 15.7 \times 10^{-6}\ m^2/s$; *Table A-8*, Benzene-air, (25°C, 1 atm): $D_{AB} = 0.88 \times 10^{-5}\ m^2/s$, Sc = 1.78.

ANALYSIS: Applying conservation of mass to a control volume about the liquid,

$$\frac{dM}{dt} = \frac{d\left(r_\ell V\right)}{dt} = -n_A.$$

For a unit width, V = L·δ. Hence

$$r_\ell L \frac{d d}{dt} = -n'_A = -\bar{h}_m L\left(r_{A,sat} - r_{A,\infty}\right)$$

and integrating

$$\int_{d_i}^{0} dd = -\frac{\bar{h}_m}{r_\ell} r_{A,sat} \int_0^t dt$$

$$t = \frac{d_i\ r_\ell}{\bar{h}_m r_{A,sat}}.$$

With $\quad Re_L = \dfrac{u_\infty L}{n} = \dfrac{1\text{ m/s}\times 2\text{ m}}{15.7\times 10^{-6}\text{ m}^2/\text{s}} = 1.27\times 10^5,$

the flow is laminar throughout and from Eq. 7.32,

$$\bar{h}_m = \frac{D_{AB}}{L} 0.664\, Re_L^{1/2} Sc^{1/3} = \frac{0.88\times 10^{-5}\text{ m}^2/\text{s}}{2\text{ m}} \times 0.664\left(1.27\times 10^5\right)^{1/2}(1.78)^{1/3}$$

$$\bar{h}_m = 1.26\times 10^{-3}\text{ m/s}$$

and

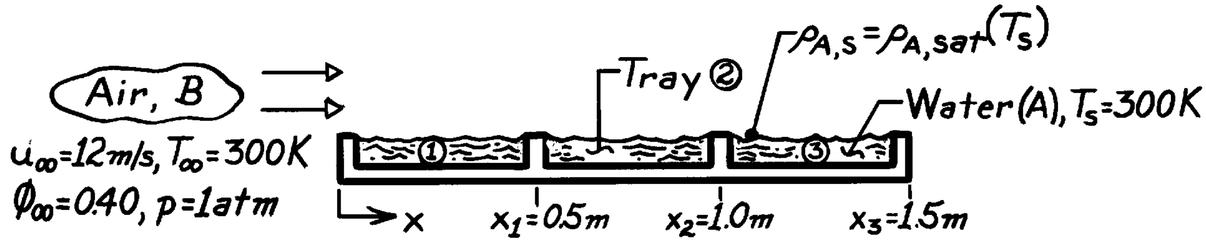
$$t = \frac{0.001\text{ m}\left(900\text{ kg/m}^3\right)}{\left(1.26\times 10^{-3}\text{ m/s}\right)\left(0.417\text{ kg/m}^3\right)} = 1713\text{ s} = 28.6\text{ min.}$$ <

PROBLEM 7.114

KNOWN: Parallel air flow over a series of water-filled trays.

FIND: Power required to maintain each of the first three trays at 300K.

SCHEMATIC:



ASSUMPTIONS: (a) Steady-state conditions, (2) Heat-mass transfer analogy applicable, (3) Perfect gas behavior for water vapor, (4) $Re_{x,c} = 5 \times 10^5$.

PROPERTIES: *Table A-4*, Air (300 K, 1 atm): $\nu = \nu_B = 15.89 \times 10^{-6}$ m^2/s; *Table A-8*, Water vapor-air (300K): $D_{AB} = 0.26 \times 10^{-4}$ m^2/s, $Sc = \nu_B/D_{AB} = 0.611$; *Table A-6*, Saturated water vapor (300K): $\rho_{A,sat} = v_g^{-1} = 0.02556$ kg/m^3, $h_{fg} = 2438$ kJ/kg.

ANALYSIS: Since $T_s = T_\infty$, there is no convective heat transfer, hence,

$$q_{tray} = \dot{m}_{tray} h_{fg} = \overline{h}_m \cdot A_s \cdot r_{A,sat}\left(1 - f_\infty\right) h_{fg} \qquad (1)$$

where

$f_\infty \equiv r_{A,\infty} / r_{A,sat}$ and $r_{A,s} = r_{A,sat}\left(T_s\right)$. Calculate the Reynolds number at x_3,

$$Re_{x3} = u_\infty x_3 / n_B = 12 \text{ m/s} \times 1.5\text{m}/15.89 \times 10^{-6} \text{ m}^2/\text{s} = 1.133 \times 10^6$$

finding that transition occurs at x = 0.662 m, a location on tray 2. The average mass transfer coefficients $\overline{h}_m$ and heat rates for each tray are as follows:

Tray 1: The flow is laminar and the appropriate correlation for $\overline{h}_{m,1}$ and heat rate are

$$\overline{Sh}_{x1} = \overline{h}_{m,1} x_1 / D_{AB} = 0.664\, Re_{x1}^{1/2}\, Sc^{1/3}$$

$$\overline{h}_{m,1} = \left(0.26 \times 10^{-4} \text{ m}^2/\text{s}/0.5 \text{ m}\right) \times 0.664 \left(\frac{12 \text{ m/s} \times 0.5 \text{ m}}{15.89 \times 10^{-6} \text{ m}^2/\text{s}}\right)^{1/2} (0.611)^{1/3} = 1.800 \times 10^{-2} \text{ m/s}$$

$$q_1' = 1.800 \times 10^{-2} \text{ m/s} \times 0.5 \text{ m} \times 0.02556 \text{ kg/m}^3 \left(1 - 0.40\right) \times 2438 \times 10^3 \text{ J/kg} = 337 \text{ W/m.} \qquad <$$

Tray 2: Since transition occurs over the span of tray 2, the rate equation has the form

$$q_2' = \left[x_2 \overline{h}_{m,0-2} - x_1 \overline{h}_{m,0-1}\right] r_{A,sat}\left(1 - f_\infty\right) h_{fg}. \qquad (2)$$

Continued ….

PROBLEM 7.114 (Cont.)

Note that $\overline{h}_{m,0-1} = \overline{h}_{m,1}$ from above and that $\overline{h}_{m,0-2}$ is evaluated using the correlation

$$\overline{Sh}_x = \left(0.037\ Re_x^{4/5} - 871\right)\ Sc^{1/3}$$

$$\overline{h}_{m,0-2} = 2.193\times10^{-2}\ \text{m/s} \qquad q'_2 = 483\ \text{W/m.}$$ <

Tray 3: The rate equation is of the same form as Eq. (2). Alternatively, an approximation can be used,

$$q'_3 = h_m(\overline{x})\ (x_3 - x_2)\ \rho_{A,sat}\ (1 - \phi_\infty)\ h_{fg}$$

where $h_m(\overline{x})$ is the *local* value at the midspan, $\overline{x} = (x_2 + x_3)/2$. Using

$$Sh_x = 0.0296\ Re_x^{4/5}\ Sc^{1/3}$$

and substituting numerical values, find

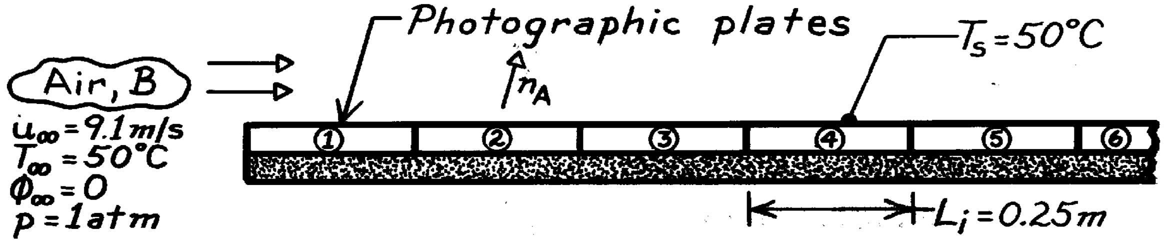
$$h_{m(\overline{x})} = 3.148\times10^{-2}\ \text{m/s} \qquad q'_3 = 589\ \text{W/m.}$$ <

PROBLEM 7.115

KNOWN: Air and surface conditions for a drying process in which photographic plates are aligned in the direction of the air flow.

FIND: (a) Variation of local mass transfer convection coefficient, (b) Drying rate for fastest drying plate, (c) Heat addition needed to maintain the plate temperature.

SCHEMATIC:



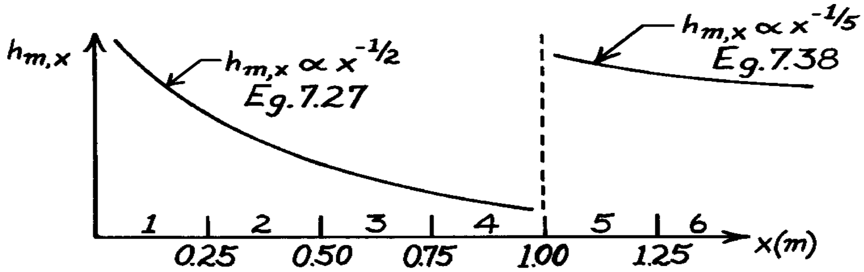
ASSUMPTIONS: (1) Heat and mass transfer analogy is applicable, (2) Critical Reynolds number is $Re_{x,c} = 5 \times 10^5$, (3) Radiation effects are negligible.

PROPERTIES: *Table A-4*, Air (50°C = 323K): $\nu = 18.2 \times 10^{-6}$ m^2/s; *Table A-6*, Water vapor (50°C = 323K): $\rho_{A,sat} = 0.082$ kg/m^3, $h_{fg} = 2383$ kJ/kg; *Table A-8*, Water vapor-air (25°C = 298K) $D_{AB} = 0.26 \times 10^{-4}$ m^2/s; since $D_{AB} \alpha T^{3/2}$, D_{AB}(50°C = 323K) = 0.26×10^{-4} $(323/298)^{3/2} = 0.29 \times 10^{-4}$ m^2/s, $Sc = \nu/D_{AB} = 0.62$.

ANALYSIS: (a) With $Re_{x,c} = u_\infty x_c/\nu = 5 \times 10^5$, the point of transition is

$$x_c = \frac{5\times10^5 \left(18.2\times10^{-6}\,m^2/s\right)}{9.1\ m/s} = 1\ m$$

and the variation of the local mass transfer coefficient is as shown below



<

(b) The largest evaporation will be associated with either the first plate or the fifth plate. For the *first* plate,

$$n_{A,1} = \overline{h}_{m,1} A_{s,1} \left(r_{A,s} - r_{A,\infty} \right)$$

where $\rho_{A,\infty} = 0$ since the upstream air is dry. Since the boundary layer is laminar over the entire plate, with

$$Re_{x,1} = (9.1\ m/s)\ (0.25\ m)/\left(18.2\times10^{-6}\ m^2/s\right) = 1.25\times10^5$$

Continued …..

Eq. 7.32 may be used to obtain

$$\bar{h}_{m,1} = \left(\frac{D_{AB}}{x_1}\right) 0.664\, Re_{x,1}^{1/2}\, Sc^{1/3} = \left(\frac{0.29\times10^{-4}\ m^2/s}{0.25\ m}\right) 0.664\, \left(1.25\times10^5\right)^{1/2} (0.62)^{1/3}$$

$\bar{h}_{m,1} = 0.0232$ m/s.

Hence $\quad n_{A,1} = 0.0232 m/s (0.25\ m\times1\ m)\ \left(0.082 kg/m^3\right) = 4.72\times10^{-4}\ kg/s\cdot m.$

For the *fifth* plate,

$$n_{A,5} = n_{A,0-5} - n_{A,0-4} = \left[\left(\bar{h}_m A_s\right)_{0-5} - \left(\bar{h}_m A_s\right)_{0-4}\right]\ \left(\rho_{A,s} - \rho_{A,\infty}\right).$$

With $Re_{x,5} = 6.25\times10^5$, Eq. 7.42 gives

$$\bar{h}_{m,0-5} = \left(\frac{D_{AB}}{x_5}\right) \left[0.037\, Re_{x,5}^{4/5} - 871\right] Sc^{1/3}$$

$$\bar{h}_{m,0-5} = \left(\frac{0.29\times10^{-4}\ m^2/s}{1.25\ m}\right) \left[0.037\left(6.25\times10^5\right)^{4/5} - 871\right] (0.62)^{1/3}$$

$$\bar{h}_{m,0-5} = 0.0145\ m/s.$$

With $Re_{x,4} = 5\times10^5$, Eq. 7.32 gives

$$\bar{h}_{m,0-4} = \left(\frac{D_{AB}}{x_4}\right) 0.664\, Re_{x,4}^{1/4}\, Sc^{1/3}$$

$$\bar{h}_{m,0-4} = \left(\frac{0.29\times10^{-4}\ m^2/s}{1\ m}\right) \left[0.664\ \left(5\times10^5\right)^{1/2}\ (0.62)^{1/3}\right]$$

$$\bar{h}_{m,0-4} = 0.0116\ m/s.$$

Hence,

$$n_{A,5} = \left[0.0145 m/s\times1.25\ m\times1\ m - 0.0116 m/s\times1\ m\times1\ m\right]\ \left(0.082 kg/m^3\right)$$

$$n_{A,5} = 5.35\times10^{-4}\ kg/s\cdot m.$$ <

Hence the evaporation rate is largest for Plate 5.

(c) Heat would have to be supplied to each plate at a rate which is equal to the evaporative cooling rate in order to maintain the prescribed temperature. Hence

$$q_5 = n_{A,5} h_{fg} = 5.35\times10^{-4} kg/s\cdot m\times2.383\times10^6\ J/kg = 1.275\ kW/m.$$ <

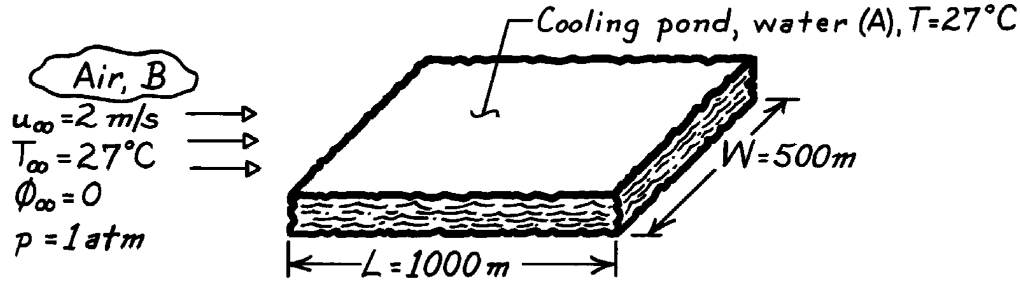
COMMENTS: The large value of q_5 is a consequence of the significant evaporative cooling effect.

PROBLEM 7.116

KNOWN: Dimensions and temperature of a cooling pond. Conditions of air flow.

FIND: Daily make-up water requirement.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Turbulent boundary layer over the entire surface, (3) Heat and mass transfer analogy is applicable.

PROPERTIES: *Table A-4*, Air (T = 300K, 1 atm): $\nu = 15.89 \times 10^{-6}\ m^2/s$, k = 0.0263 W/m·K, Pr = 0.707; *Table A-6*, Water vapor (300K): $r_{A,sat} = v_g^{-1} = 0.0256\ kg/m^3$; *Table A-8*, Water vapor-air (300K): $D_{AB} = 0.26 \times 10^{-4}\ m^2/s$, $Sc = \nu/D_{AB} = 0.61$.

ANALYSIS: The make-up water requirement must equal the daily water loss due to evaporation,

$$\Delta M = \dot{m}_{evap}\Delta t = \overline{h}_m (W \cdot L) \left[r_{A,sat}(T_s) - f_\infty r_{A,sat}(T_\infty) \right] \cdot \Delta t.$$

From Eq. 7.45, $\overline{Sh}_L = 0.037\ Re_L^{4/5}\ Sc^{1/3}$, with

$$Re_L = \frac{u_\infty L}{n} = \frac{2\ m/s \times 1000\ m}{15.89 \times 10^{-6}\ m^2/s} = 1.26 \times 10^8$$

$$\overline{Sh}_L = 0.037 \left(1.26 \times 10^8\right)^{4/5} (0.61)^{1/3} = 9.48 \times 10^4$$

$$\overline{h}_{m,L} = \frac{D_{AB}\overline{Sh}_L}{L} = \frac{0.26 \times 10^{-4}\ m^2/s \times 9.48 \times 10^4}{1000\ m}$$

$$\overline{h}_{m,L} = 2.47 \times 10^{-3}\ m/s.$$

Hence, the make-up water requirement is

$$\Delta M = 2.47 \times 10^{-3}\ m/s\ (500\ m \times 1000\ m)\ 0.0256\ kg/m^3 (24h \times 3600\ s/h)$$

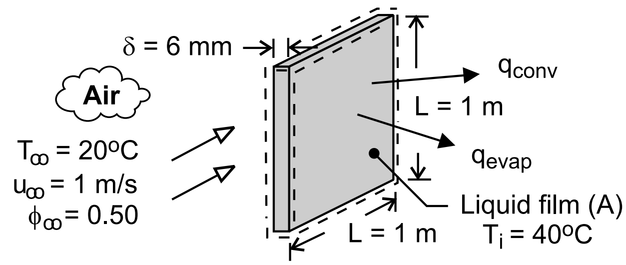
$$\Delta M = 2.73 \times 10^6\ kg/day.$$ <

PROBLEM 7.117

KNOWN: Dimensions and initial temperature of plate covered by liquid film. Properties of liquid. Velocity and temperature of air flow over the plates.

FIND: Initial rate of heat transfer from plate and rate of change of plate temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible effect of conveyor velocity on boundary layer development, (2) Plates are isothermal and at same temperature as liquid film, (3) Negligible heat transfer from sides of plate, (4) Smooth air-liquid interface, (5) Applicability of heat/mass transfer analogy, (6) Negligible solvent vapor in free stream, (7) $Re_{x,c} = 5 \times 10^5$, (8) Constant properties.

PROPERTIES: *Table A-1*, AISI 1010 steel (313K): c = 441 J/kg·K, $\rho = 7832\,kg/m^3$. *Table A-4*, Air (p = 1 atm, T_f = 303K): $\nu = 16.2\times10^{-6} m^2/s$, $k = 0.0265\,W/m\cdot K$, $Pr = 0.707$. Prescribed: Solvent: $\rho_{A,sat} = 0.75\,kg/m^3$, $D_{AB} = 10^{-5} m^2/s$, $h_{fg} = 9\times10^5\,J/kg.$

SOLUTION: The initial rate of heat transfer from the plate is due to both convection and evaporation.

$$q = q_{conv} + q_{evap} = \bar{h}\,A_s\left(T_i - T_\infty\right) + n_A\,h_{fg} = \bar{h}\,A_s\left(T_i - T_\infty\right) + \bar{h}_m A_s\,\rho_{A,sat}\,h_{fg}$$

With $Re_L = u_\infty L/\nu = 1\,m/s\times1m/16.2\times10^{-6}m^2/s = 6.17\times10^4$, flow is laminar over the entire surface. Hence,

$$\overline{Nu}_L = 0.664\,Re_L^{1/2}\,Pr^{1/3} = 0.664\left(6.17\times10^4\right)^{1/2}(0.707)^{1/3} = 147$$

$$\bar{h} = (k/L)\overline{Nu}_L = (0.0265\,W/m\cdot K/1m)147 = 3.9\,W/m^2\cdot K$$

Also, with $Sc = \nu/D_{AB} = 16.2\times10^{-6}m^2/s/10^{-5}m^2/s = 1.62,$

$$\overline{Sh}_L = 0.664\,Re_L^{1/2}\,Sc^{1/3} = 0.664\left(6.17\times10^4\right)^{1/2}(1.62)^{1/3} = 194$$

$$\bar{h}_m = \left(D_{AB}/L\right)\overline{Sh}_L = \left(10^{-5}m^2/s/1m\right)194 = 0.00194\,m/s$$

Hence, with $A_s = 2\,L^2 = 2\,m^2$,

$$q = 2\,m^2\left[3.9\,W/m^2\cdot K\left(20°C\right) + 0.00194\,m/s\times0.75\,kg/m^3\times9\times10^5\,J/kg\right] = 156\,W + 2619\,W = 2775\,W \quad <$$

Performing an energy balance at an instant of time for a control surface about the plate, $-\dot{E}_{out} = \dot{E}_{st}$, we obtain (Eq. 5.2),

$$\left.\frac{dT}{dt}\right|_i = -\frac{q}{\rho\,\delta\,L^2 c} = -\frac{2775\,W}{7832\,kg/m^3\times0.006m\,(1m)^2\,441\,J/kg\cdot K} = -0.13°C/s \quad <$$

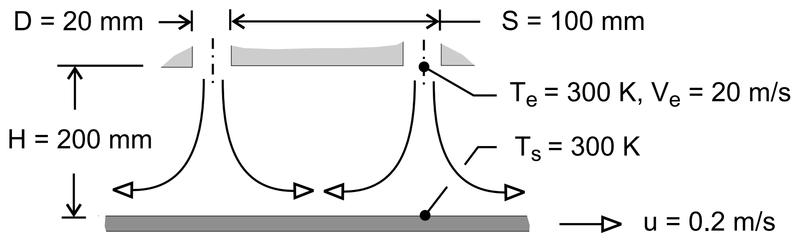
COMMENTS: (1) Heat transfer by evaporation exceeds that due to convection by more than an order of magnitude, (2) The total heat rate is small enough to render the lumped capacitance approximation excellent.

PROBLEM 7.118

KNOWN: Dimensions of round jet array. Jet exit velocity and temperature. Temperature of paper.

FIND: Drying rate per unit surface area.

SCHEMATIC:



ASSUMPTIONS: (1) Applicability of heat and mass transfer analogy. (2) Paper motion has a negligible effect on convection ($u << V_e$), (3) Air is dry.

PROPERTIES: *Table A-4*, Air (300K, 1 atm): $\nu = 15.89 \times 10^{-6}\, m^2/s$; *Table A-6*, Saturated water (300K): $\rho_{A,sat} = v_g^{-1} = 0.0256\, kg/m^3$; *Table A-8*, water vapor-air (300K): $D_{AB} = 0.26 \times 10^{-4} m^2/s$, Sc = 0.61.

ANALYSIS: The average mass evaporation flux is

$$n''_A = \overline{h}_m \left(\rho_{A,s} - \rho_{A,e}\right) = \overline{h}_m\, \rho_{A,s}$$

For an array of round nozzles,

$$\overline{Sh} = 0.5\, K\, G\, Re^{2/3}\, Sc^{0.42}$$

where $Re = V_e D/\nu = 20\, m/s \times 0.02 m / 15.89 \times 10^{-6} m^2/s = 25{,}170$ and, with H/D = 10 and $A_r = \pi D^2/4S^2 = 0.0314$,

$$K = \left[1 + \left(\frac{H/D}{0.6/A_r^{1/2}}\right)^6\right]^{-0.05} = \left[1 + \left(\frac{10}{3.39}\right)^6\right]^{-0.05} = 0.723$$

$$G = 2A_r^{1/2} \frac{1 - 2.2 A_r^{1/2}}{1 + 0.2(H/D - 6) A_r^{1/2}} = 0.354 \frac{1 - 0.390}{1 + 0.2(4) 0.177} = 0.189$$

Hence,

$$\overline{h}_m = \frac{D_{AB}}{D} \overline{Sh} = \frac{0.26 \times 10^{-4} m^2/s}{0.02 m}\left[0.5 \times 0.723 \times 0.189 (25{,}170)^{2/3} (0.61)^{0.42}\right] = 0.062\, m/s$$

The average evaporative flux is then

$$n''_A = 0.062\, m/s \left(0.0256\, kg/m^3\right) = 0.0016\, kg/s \cdot m^2$$ <

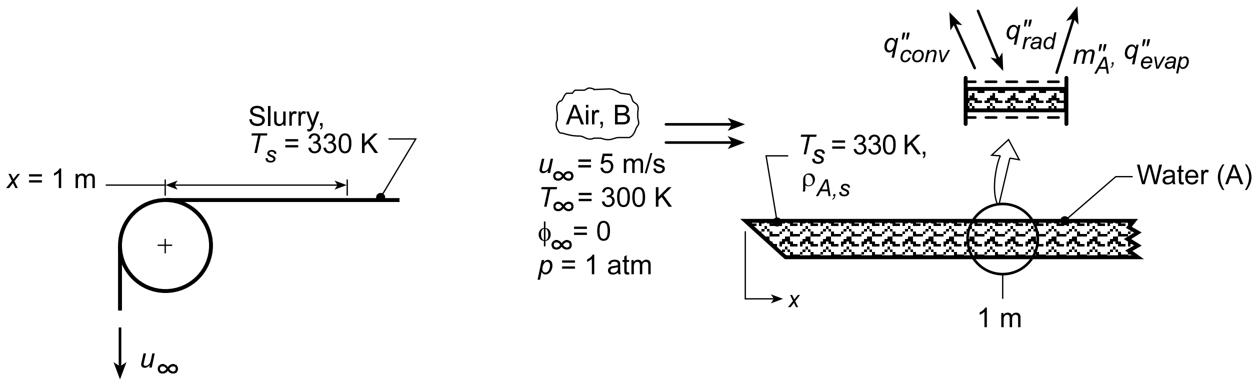
COMMENTS: Note that, for maximum evaporation, the ratio D/H = 0.1 is less than the optimum of $\left.D/H\right)_{op} \approx 0.2$, as is S/H = 0.5 less than $\left.S/H\right)_{op} \approx 1.4$. If H is reduced by a factor of 2 and S is increased by 40%, a near optimal condition could be achieved.

PROBLEM 7.119

KNOWN: Paper mill process using radiant heat for drying.

FIND: (a) Evaporative flux at a distance 1 m from roll edge; corresponding irradiation, G (W/m^2), required to maintain surface at T_s = 300 K, and (b) Compute and plot variations of $h_{m,x}(x)$, N''_A (x), and G(x) for the range $0 \le x \le 1$ m when the velocity and temperature are increased to 10 m/s and 340 K, respectively.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Heat-mass transfer analogy, (3) Paper slurry (water-fiber mixture) has water properties, (4) Water vapor behaves as perfect gas, (5) All irradiation absorbed by slurry, (6) Negligible emission from the slurry, (7) $Re_{x,c} = 5 \times 10^5$.

PROPERTIES: *Table A.4*, Air (T_f = 315 K, 1 atm): $\nu = 17.40 \times 10^{-6}$ m^2/s, k = 0.0274 W/m·K, Pr = 0.705; *Table A.8*, Water vapor-air (T_f = 315 K): $D_{AB} = 0.26 \times 10^{-4}$ m^2/s $(315/298)^{3/2} = 0.28 \times 10^{-4}$ m^2/s, $Sc = \nu_B/D_{AB} = 0.616$; *Table A.6*, Saturated water vapor (T_s = 330 K): $\rho_{A,sat} = 1/v_g = 0.1134$ kg/m^3, h_{fg} = 2366 kJ/kg.

ANALYSIS: (a) Recognize that the drying process can be modeled as flow over a flat plate with heat and mass transfer. For a unit area at x = 1 m,

$$n''_{A,x} = h_{m,x}\left(\rho_{A,s} - \rho_{A,\infty}\right) = h_{m,x}\left[\rho_{A,sat}\left(T_s\right) - \phi_\infty \rho_{A,sat}\left(T_\infty\right)\right] \qquad (1)$$

Evaluate Re_x to determine the nature of flow, select a correlation to estimate $h_{m,x}$,

$$Re_x = u_\infty x/\nu_B = (5\,m/s \times 1\,m)\big/17.40\times10^{-6}\,m^2\big/s = 2.874\times10^5 .$$

Since $Re_x < 5 \times 10^5$, the flow is laminar. Invoking the heat-mass analogy,

$$Sh_x = \frac{h_{m,x}\,x}{D_{AB}} = 0.332\,Re_x^{1/2}\,Sc^{1/3} \qquad (2)$$

$$h_{m,x} = \left(0.28\times10^{-4}\,m^2/s/1\,m\right)\times 0.332\left(2.874\times10^5\right)^{1/2}(0.616)^{1/3} = 4.24\times10^{-3}\,m/s .$$

Hence, the evaporative flux at x = 1 m is

$$n''_{A,x} = 4.24\times10^{-3}\,m/s\left(0.1134\,kg/m^3 - 0\right) = 4.81\times10^{-4}\,kg/s\cdot m^2 \qquad <$$

From an energy balance on the differential element at x = 1 m (see above),

$$G = q''_{conv} + q''_{evap} = h_x\left(T_s - T_\infty\right) + n''_{A,x}h_{fg} . \qquad (3)$$

Continued...

PROBLEM 7.119 (Cont.)

To estimate h_x, invoke the heat-mass transfer analogy using the correlation of Eq. (2),

$$h_x = h_{m,x} \frac{k}{D_{AB}} \left(\frac{Pr}{Sc} \right)^{1/3} = 4.24 \times 10^{-3}\, m/s \left(\frac{0.0274\, W/m \cdot K}{0.28 \times 10^{-4}\, m^2/s} \right) \left(\frac{0.705}{0.616} \right)^{1/3} = 4.34\, W/m^2 \cdot K \qquad (4)$$

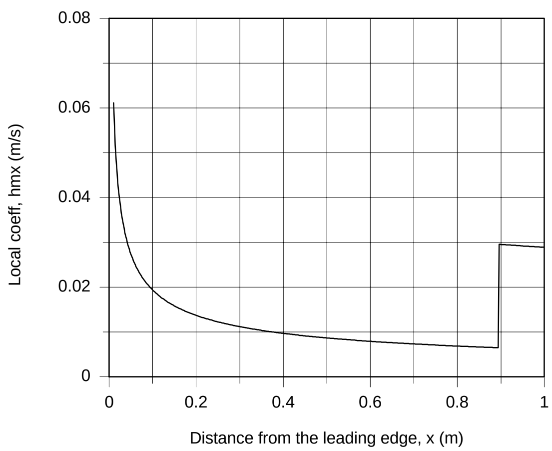
Hence, from Eq. (3), the radiant power required to maintain the slurry at T_s = 330 K is

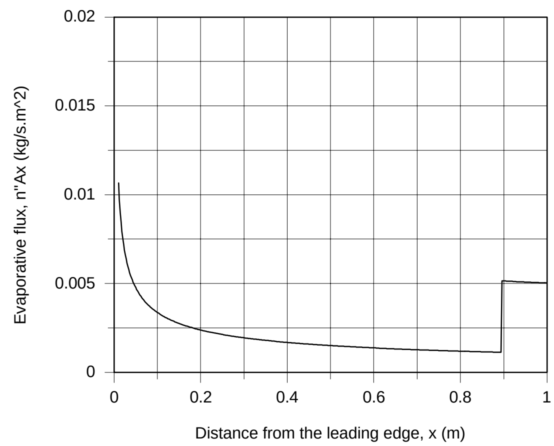
$$G = 4.34\, W/m^2 \cdot K\,(330 - 300)\,K + 4.81 \times 10^{-4}\, kg/s \cdot m^2 \times 2366 \times 10^3\, J/kg$$

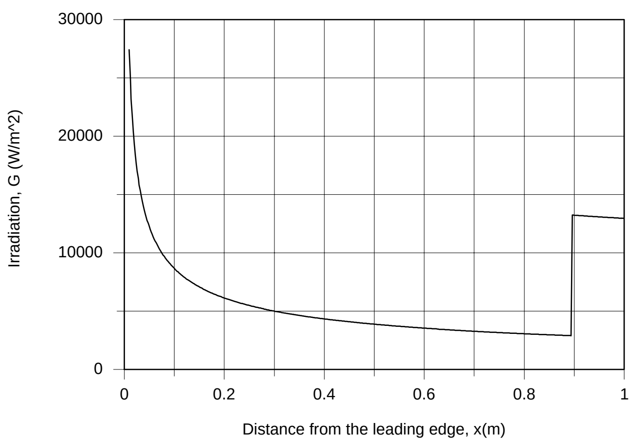
$$G = (130 + 1138)\, W/m^2 = 1268\, W/m^2 .$$

<

(b) Equations (1), (3) and (4) were entered into the *IHT Workspace*. The *Correlations Tool*, *External Flow*, *Local* coefficients for *Laminar* or *Turbulent Flow* was used to estimate the heat transfer convection coefficient. The results for $h_{m,x}(x)$, $n''_{A,x}$ (x) and G(x) were evaluated, and are plotted below for T_s = 340 K and u_∞ = 10 m/s.







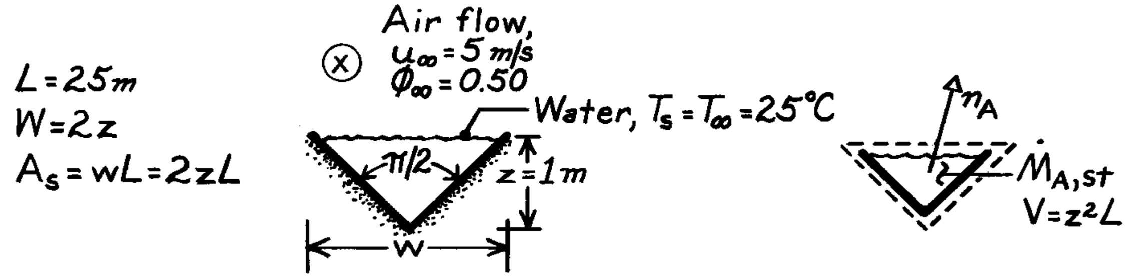
COMMENTS: (1) The abrupt change in the parameter plots occurs at the transition, x_c = 0.9 m.

PROBLEM 7.120

KNOWN: Geometry and air flow conditions for a water storage channel.

FIND: (a) Evaporation rate, (b) Expression for rate of change of water layer depth and time required for complete evaporation.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-stae conditions, (2) Smooth water surface and negligible free stream turbulence, (3) Heat and mass transfer analogy is applicable, (4) $Re_{x,c} = 5 \times 10^5$, (5) Perfect gas behavior for water vapor.

PROPERTIES: *Table A-4* Air (25°C = 298K): $\nu = 15.71 \times 10^{-6}\ m^2/s$; *Table A-6*, Water (25°C = 298K): $\rho_{A,sat} = v_g^{-1} = 0.0226\ kg/m^3$, $\rho_f = v_f^{-1} = 997\ kg/m^3$; *Table A-8*, Water vapor-air (25°C = 298K): $D_{AB} = 0.26 \times 10^{-4}\ m^2/s$, $Sc = \nu/D_{AB} = 0.60$.

ANALYSIS: (a) The evaporation rate is $n_A = \overline{h}_m A_s \left(r_{A,sat} - r_{A,\infty}\right) = \overline{h}_m (w \times L)\, r_{A,sat} \left(1 - f_\infty\right)$. With

$$Re_L = u_\infty L/n = 5\ m/s \times 25\ m/15.71 \times 10^{-6}\ m^2/s = 7.96 \times 10^6$$

Eq. 7.42 yields $\overline{Sh}_L = \left[0.037\left(7.96 \times 10^6\right)^{4/5} - 871\right](0.6)^{1/3} = 9616$

$$\overline{h}_m = 9616\ D_{AB}/L = 9616 \times 0.26 \times 10^{-4}\ m^2/s/(25\ m) = 0.010\ m/s.$$

With w = 2z = 2m,

$$n_A = 0.01\ m/s\ (2m \times 25m)\, 0.0226\ kg/m^3\ (0.5) = 0.00565\ kg/s = 20.3\ kg/h. \qquad <$$

(b) Performing a mass balance on a control volume about the water,

$$-n_A = \dot{m}_{A,st} = \frac{d}{dt}\left(r_f V\right) \qquad -\overline{h}_m (2\,z\,L)\ r_{A,sat}\left(1 - f_\infty\right) = \frac{d}{dt}\left(r_f\ z^2 L\right)$$

$$\frac{dz}{dt} = -\overline{h}_m \frac{r_{A.sat}}{r_f}\left(1 - f_\infty\right).$$

Integrating, $\displaystyle\int_z^0 dz = -\overline{h}_m \frac{r_{A,sat}}{r_f}\left(1 - f_\infty\right) \int_0^t dt$

$$t = \frac{z\, r_f}{\overline{h}_m r_{A,sat}} \frac{1}{1 - f_\infty} = \frac{1\ m \times 997\ kg/m^3}{0.01\ m/s \times 0.0226\ kg/m^3\ (1 - 0.5)} = 8.82 \times 10^6\ s = 2451\ h = 102\ d. \qquad <$$

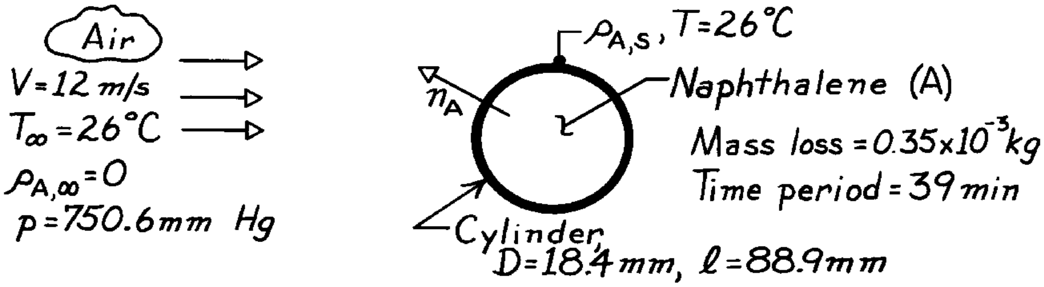
COMMENTS: Although the evaporation rate decreases with increasing time due to decreasing A_s, dz/dt remains constant and the water depth decreases linearly.

PROBLEM 7.121

KNOWN: Mass change for a given time period of a solid naphthalene cylinder subjected to cross flow of air for prescribed conditions.

FIND: (a) Mass transfer coefficient, $\overline{h}_m$, based upon experimental observations and (b) $\overline{h}_m$ based upon appropriate correlation.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible naphthalene vapor in free stream, (3) Heat-mass transfer analogy applies.

PROPERTIES: *Table A-4*, Air (299K, 1 atm): $\nu = 15.80 \times 10^{-6}\ m^2/s$; *Table A-6*, Naphthalene vapor-air (298K, 1 atm): $D_{AB} = 0.62 \times 10^{-5}\ m^2/s$; Naphthalene (given): $\mathcal{M} = 128.16$ kg/kmol, $p_{sat} = p \times 10^E$ where E = 8.67 - (3766/T) with p[bar] and T[K].

ANALYSIS: (a) The rate equation for the sublimation of naphthalene vapor from the solid naphthalene can be written in terms of the mass transfer coefficient.

$$\overline{h}_m = \frac{n_A}{A_s\left(r_{A,s} - r_{A,\infty}\right)} \qquad (1)$$

where $A_s = p D \ell$. From the mass loss and time observations

$$n_A = \frac{\Delta m}{\Delta t} = \frac{0.35\times10^{-3}\,kg}{39\times60\ s} = 1.50\times10^{-7}\ kg/s.$$

The saturation density of the vapor at the solid surface, $\rho_{A,s}$, can be determined from the perfect gas relation,

$$r_{A,s} = C_{A,s}\mathcal{M}_A = \frac{p_{sat}\left(T_s\right)}{\left(\Re/\mathcal{M}_A\right)T_s}. \qquad (2)$$

The saturation pressure, p_{sat}, is given by

$$p_{sat} = p\times10^E \qquad (3)$$

where $E = 8.67 - (3766/T) = 8.67 - (3766/299K) = -3.925$

$$p = 750.6\ mm\ Hg \times \frac{1\ N/m^2}{2.953\times10^{-4}\,in\ Hg} \times \frac{1\ in}{25.4\ mm} \times \frac{1\ bar}{1\times10^5\ N/m^2} = 1.001\ bar$$

or $\quad p_{sat} = 1.001\ bar \times 10^{-3.925} = 1.190\times10^{-4}\,bar.$

Continued …..

Substituting into Eq. (2),

$$\rho_{A,s} = 1.190\times10^{-4}\,\text{bar}\Big/\frac{8.314\times10^{-2}\ \text{m}^3\cdot\text{bar/kmol}\cdot\text{K}}{128.16\ \text{kg/kmol}}\times 299\text{K} = 6.135\times10^{-4}\,\text{kg/m}^3.$$

Using the parameters required for Eq. (1), the mass transfer coefficient is

$$\overline{h}_m = \frac{1.50\times10^{-7}\,\text{kg/s}}{\pi\left(18.4\times10^{-3}\text{m}\right)\left(88.9\times10^{-3}\text{m}\right)\left[6.135\times10^{-4} - 0\right]\ \text{kg/m}^3}$$

$$\overline{h}_m = 4.76\times10^{-2}\,\text{m/s}.$$ <

(b) Invoking the heat-mass transfer analogy and assuming a Prandtl number ratio of unity, Eq. 7.56 can be used to estimate $\overline{h}_m$,

$$\overline{Sh}_D = \frac{\overline{h}_m D}{D_{AB}} = C\,Re_D^m\,Sc^n.$$

With

$$Re_D = \frac{VD}{\nu} = 12\ \text{m/s}\left(18.4\times10^{-3}\right)\text{m}/15.80\times10^{-6}\text{m}^2/\text{s} = 13{,}975$$

it follows from Table 7.4 that C = 0.26 and m = 0.6. With

$$Sc = \nu/D_{AB} = 15.80\times10^{-6}\text{m}^2\ \text{/s}/0.62\times10^{-5}\text{m}^2/\text{s} = 2.55$$

n = 0.37 and

$$\overline{Sh}_D = 0.26(13{,}975)^{0.6}(2.55)^{0.37} = 112.9$$

and

$$\overline{h}_m = \overline{Sh}_D\ \frac{D_{AB}}{D} = 112.9\times\frac{0.62\times10^{-5}\ \text{m}^2/\text{s}}{18.4\times10^{-3}\ \text{m}} = 3.80\times10^{-2}\ \text{m/s}.$$ <

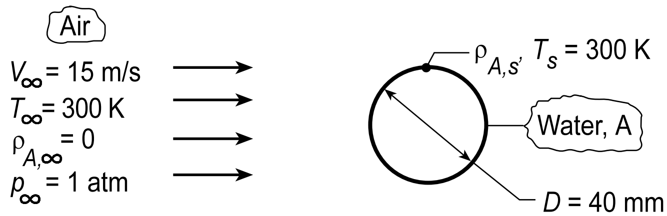
COMMENTS: The result from the correlation is 20% less than the experimental result. This may be considered reasonable in view of the uncertainties associated with the observations and the approximate nature of the correlation.

PROBLEM 7.122

KNOWN: Flow of dry air over a cylindrical medium saturated with water.

FIND: (a) Mass rate of water vapor evaporated per unit length n'_A, when water-air is at 300 K, (b) Briefly explain change in mass rate if temperatures are at 325 K.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Heat-mass transfer analogy.

PROPERTIES: *Table A.4*, Air (300 K, 1 atm): $\nu = 15.89 \times 10^{-6}$ m^2/s, Pr = 0.707; Air (325 K, 1 atm): $\nu = 18.41 \times 10^{-6}$ m^2/s, Pr = 0.703; *Table A.8,* Water vapor-air (300 K): $D_{AB} = 0.26 \times 10^{-4}$ m^2/s; *Table A.6,* Water vapor (300 K, 1 atm): $\rho_{A,sat} = (v_g)^{-1} = (39.13\ m^3/kg)^{-1} = 0.0256\ kg/m^3$; Water vapor (325 K, 1 atm): $\rho_{A,sat} = (v_g)^{-1} = (11.06\ m^3/kg)^{-1} = 0.0904\ kg/m^3$.

ANALYSIS: (a) For cross-flow over a cylinder, Eq. 7.55,

$$\overline{Sh}_D = C\,Re^m\,Sc^{1/3} \tag{1}$$

where m,n are taken from Table 7.2. Calculate the Reynolds number, $Re_D = VD/\nu = 15\ m/s \times 0.04\ m/15.89 \times 10^{-6}\ m^2/s = 37{,}760$. With C = 0.193, m = 0.618, and $Sc \equiv \nu/D_{AB}$,

$$\overline{Sh}_D = \frac{\overline{h}_m D}{D_{AB}} = 0.193(37,760)^{0.618}\left[15.89\times10^{-6}\ m^2/s \big/ 0.26\times10^{-4}\ m^2/s\right]^{1/3} = 110.4 \tag{2}$$

$$\overline{h}_m = \overline{Sh}_D\, D_B/D = 110.4\times0.26\times10^{-4}\ m^2/s \big/ 0.04\ m = 0.0717\ m/s$$

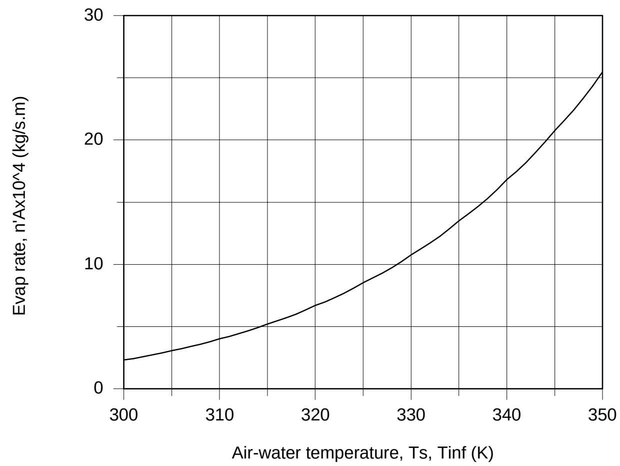
The evaporation rate, with $A_s = \pi D \cdot \ell$, is

$$n_A = \overline{h}_m A_s\left(\rho_{A,s} - \rho_{A,\infty}\right) \qquad n'_A = n_A/\ell = \overline{h}_m \pi D\left(\rho_{A,s} - \rho_{A,\infty}\right) \tag{3}$$

$$n'_A = 0.0717\ m/s(\pi\times0.04\ m)(0.0256-0)\ kg/m^3 = 2.31\times10^{-4}\ kg/s\cdot m \qquad <$$

(b) The foregoing equations were entered into the *IHT Workspace*, and using the *Properties Tools* for air and water vapor thermophysical properties, the evaporation rate n'_A was calculated as a function of air-water temperatures ($T_s = T_{inf}$).

Continued...



As expected, the evaporation rate increased with increasing temperature markedly. For a 50 K increase, the evaporation rate increased by a factor of approximately 12.

COMMENTS: (1) What parameters cause this high sensitivity of n'_A to T_s? From the IHT analysis, we observed only modest changes in D_{AB} (0.26 to 0.33×10^{-4} m^2/s) and $\overline{h}_m$ (0.07273 to 0.0779 m/s) over the range 300 to 350 K. The density of water vapor, $\rho_{A,s}$, however, is highly temperature dependent as can be seen by examining the steam tables, Table A.6. Find $\rho_{A,s}$ (300 K) = 0.02556 kg/m^3 while $\rho_{A,s}$ (350 K) = 0.260 kg/m^3, which accounts for more than a factor of 10 change.

(2) A copy of the IHT Workspace used to perform the analysis is shown below.

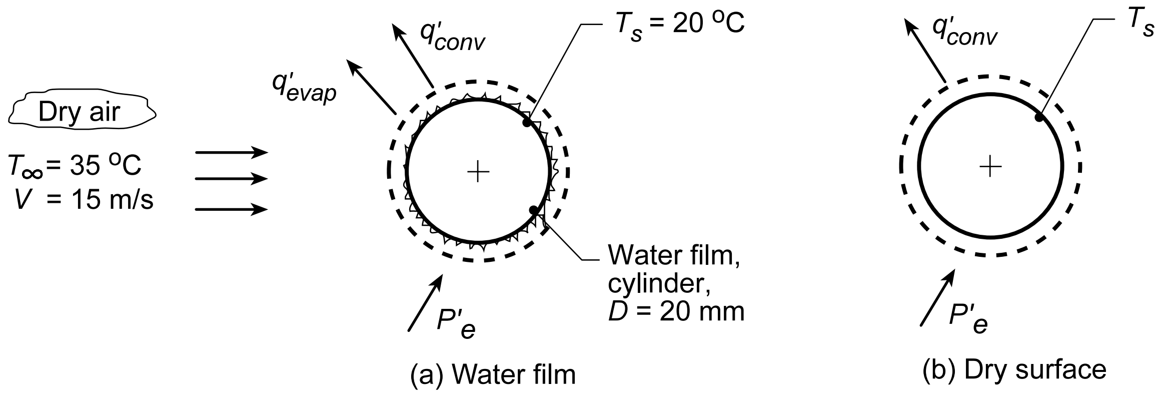
```
// The Mass Transfer Rate Equation:
n'A = hmbar * pi * D * (rhoAs - 0 )       // Eq (3)
n'A_plot = 1e4 * n'A                      // Scale change for plotting

// Mass Transfer Coefficient Correlation:
ShDbar = C * ReD^m * Sc^(1/3)             // Eq (1,2)
ShDbar = hmbar * D / DAB
C = 0.193                      // Table 7.2,  4000 <= ReD <= 40000
m = 0.618
ReD = uinf * D / nu
Sc = nu / DAB

// Properties Tool - Water Vapor:
// Water property functions :T dependence, From Table A.6
// Units: T(K), p(bars);
xs = 1                         // Quality (0=sat liquid or 1=sat vapor)
rhoAs = rho_Tx("Water",Ts,xs)             // Density, kg/m^3

// Properties Tool - Air:
// Air property functions : From Table A.4
// Units: T(K); 1 atm pressure
nu = nu_T("Air",Tf)            // Kinematic viscosity, m^2/s

// Properties, Table A.8, Water Vapor - Air:
DAB = 0.26e-4 * ( Tf / 298 )^1.5          // Table A.8
Tf = (Ts + Tinf ) / 2

// Assigned Variables:
Ts = 300                       // Surface temperature, K
D = 0.040                      // Diameter, m
uinf = 15                      // Airstream velocity, m/s
Tinf = Ts                      // Airstream temperature, K
```

PROBLEM 7.123

KNOWN: Dry air at prescribed temperature and velocity flowing over a long, wetted cylinder of diameter 20 mm. Imbedded electrical heater maintains the surface at $T_s = 20°C$.

FIND: (a) Water evaporation rate per unit length (kg/h·m) and electrical power per unit length P'_e (W/m) required to maintain steady-state conditions, and (b) The temperature of the cylinder after all the water has evaporated for the same airstream conditions and heater power of part (a).

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties and (3) Heat-mass transfer analogy is applicable.

PROPERTIES: *Table A.4*, Air ($T_f = (T_s + T_\infty)/2$ = 300 K, 1 atm): ρ = 1.16 kg/m^3, c_p = 1007 J/kg·K, k = 0.0263 W/m·K, $\nu = 15.94 \times 10^{-6}$ m^2/s, $\alpha = 2.257 \times 10^{-5}$ m^2/s, *Table A.6*, Water (T_s = 20°C = 293 K): $\rho_{A,s} = 1/v_g$ = 1/59.04 = 0.0169 kg/m^3, h_{fg} = 2454 kJ/K; *Table A.8*, Water-air (T_f = 300 K): $D_{AB} = 0.26 \times 10^{-4}$ m^2/s.

ANALYSIS: (a) Perform an energy balance on the cylinder,

$$\dot{E}_{in} - \dot{E}_{out} = 0 \qquad P'_e - q'_{conv} - q'_{evap} = 0 \tag{1}$$

where the convection and evaporation rate equations are,

$$q'_{conv} = \bar{h}_D \pi D (T_s - T_\infty) \tag{2}$$

$$q_{evap} = n_A h_{fg} = \bar{h}_m \pi D (\rho_{A,s} - \rho_{A,\infty}) h_{fg} \tag{3}$$

The convection coefficient can be estimated from the Churchill-Bernstein correlation, Eq. 7.57,

$$\overline{Nu}_D = 0.3 + \frac{0.62 Re_D^{1/2} Pr^{1/3}}{\left[1 + (0.4/Pr)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{Re_D}{282,000}\right)^{3/8}\right]^{4/5}$$

$$Re_D = \frac{VD}{\nu} = \frac{15\,m/s \times 0.020\,m}{15.94 \times 10^{-6}\,m^2/s} = 18,821$$

$$\overline{Nu}_D = 0.3 + \frac{0.62(18,821)^{1/2}(0.707)^{1/3}}{\left[1 + (0.4/0.707)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{18,821}{282,000}\right)^{3/8}\right]^{4/5} = 76.5$$

$$\bar{h}_D = \frac{k}{D}\overline{Nu}_D = \frac{0.0263\,W/m \cdot K}{0.020\,m} \times 76.5 = 101\,W/m^2 \cdot K$$

Continued...

Evoking the heat-mass analogy, Eq. 6.92, with n = 1/3

$$\frac{\overline{h}_D}{\overline{h}_m} = \rho c_p \left(\frac{\alpha}{D_{AB}}\right)^{-2/3} = 1.16\,\mathrm{kg/m^3} \times 1007\,\mathrm{J/kg\cdot K}\left(\frac{2.257\times10^{-5}\,\mathrm{m^2/s}}{0.26\times10^{-4}\,\mathrm{m^2/s}}\right)^{-2/3} = 1284\,\mathrm{J/m^3\cdot K}$$

$$\overline{h}_m = 101\,\mathrm{W/m^2\cdot K}/1284\,\mathrm{J/m^3\cdot K} = 0.0787\,\mathrm{m/s}$$

Substituting numerical values, the energy balance, Eq. (1),

$$P_e - 101\,\mathrm{W/m^2\cdot K}\times\pi\times0.020\,\mathrm{m}(20-35)\mathrm{K}$$
$$-0.0787\,\mathrm{m/s}\times\pi\times0.020\,\mathrm{m}(0.0169-0)\mathrm{kg/m^3}\times2454\times10^3\,\mathrm{J/kg\cdot K} = 0$$

$$P_e = -95.1\,\mathrm{W/m} + 205.1\,\mathrm{W/m} = 110\,\mathrm{W/m} \quad <$$

The evaporation rate is

$$n_A = \overline{h}_m\pi D\left(\rho_{A,s} - \rho_{A,\infty}\right) = 0.0787\,\mathrm{m/s}\,\pi\times0.0020\,\mathrm{m}(0.0169-0)\mathrm{kg/m^3} = 0.301\,\mathrm{kg/h\cdot m} \quad <$$

(b) When the cylinder is dry, the energy balance is

$$P'_e = \overline{h}_D\pi D\left(T_s - T_\infty\right)$$

$$T_s = T_\infty + P'_e/\overline{h}_D\pi D = 35^\circ\mathrm{C} + 110\,\mathrm{W/m}\Big/\left(101\,\mathrm{W/m^2\cdot K}\,\pi\times0.020\,\mathrm{m}\right) = 52.3^\circ\mathrm{C} \quad <$$

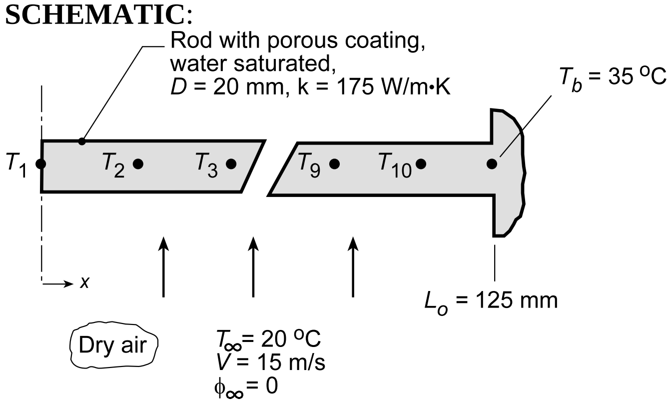
COMMENTS: Using *IHT Correlations Tool, External Flow, Cylinder*, the calculation of part (b) was performed using the proper film temperature, T_f = 316.8 K, to find $\overline{h}_D$ = 99.4 W/m^2·K and T_s = 52.6°C.

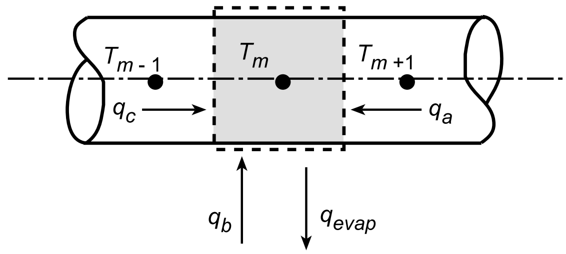
PROBLEM 7.124

KNOWN: Dry air at prescribed temperature and velocity flows over a rod covered with a thin porous coating saturated with water. The ends of the rod are attached to heat sinks maintained at a constant temperature.

FIND: Temperature at the midspan of the rod and evaporation rate from the surface using a steady-state, finite-difference analysis. Validate your code, without the evaporation process, by comparing the temperature distribution with the analytical solution of a fin.

SCHEMATIC:





ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in rod, (3) Constant properties, and (4) Heat-mass transfer analogy is applicable.

PROPERTIES: *Table A.4*, Air ($\overline{T}_f$, see Eq. (2); 1 atm): ρ, c_p, k, α, Pr; *Table A.6*, Water ($T_m = T_{sat,m}$, 1 atm): $\rho_{A,sat} = 1/v_g$, h_{fg}; *Table A.8*, Water Vapor-Air ($\overline{T}_f$, 1 atm): $D_{AB} = D_{AB}(298\text{ K}) \times (\overline{T}_f/298)^{1.5}$.

ANALYSIS: As suggested, the 10-node network shown above represents the half-length of the system. Performing an energy balance on the control volume about the m-th node, the finite-difference equation for the system is derived.

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$q_a - q_{evap} + q_b + q_c = 0$$

$$kA_c \frac{T_{m+1} - T_m}{\Delta x} = n_{A,m} h_{fg,m} + \bar{h} P \Delta x \left(T_\infty - T_m\right) + kA_c \frac{T_{m-1} - T_m}{\Delta x} = 0 \tag{1}$$

where the cross-sectional area and perimeter are $A_c = \pi D^2/4$ and $P = \pi D$, respectively. The average heat transfer coefficient $\bar{h}$ can be evaluated using the Churchill-Bernstein correlation, Eq. 7.57, evaluating thermophysical properties at an average film temperature for the system,

$$\overline{T}_f = \left[\left(T_1 + T_b\right)/2 + T_\infty\right]/2 \tag{2}$$

The evaporation rate from Eq. (1) can be expressed as

$$n_{A,m} = \bar{h}_{D,m} P \Delta x \left(\rho_{A,s,m} - 0\right) \tag{3}$$

where $\bar{h}_{D,m}$ can be determined from the heat-mass analogy, Eq. 6.92, with n = 1/3,

$$\frac{\bar{h}}{\bar{h}_m} = \rho c_p \left(\frac{\alpha}{D_{AB}}\right)^{-2/3} \tag{4}$$

Continued...

where all properties are evaluated at $\overline{T}_f$. The density of water vapor, $\rho_{A,s,m}$, as well as the heat of vaporization, $h_{fg,m}$, must be evaluated at the nodal temperature T_m.

Using the *IHT Correlation Tool*, *External Flow*, *Cylinder*, an estimate of $\overline{h}_D$ = 101 W/m^2·K was obtained with $\overline{T}_f$ = 298.5 K (based upon assumed value of T_1 = 27°C). From the analogy, Eq. (4), find that $\overline{h}_{D,m}$ = 0.0772 m/s. Using the *IHT Workspace*, the finite-difference equations, Eq. (1), were entered and the temperature distribution (K, Case 1) determined as tabulated below. Using this same code with $\overline{h}_{D,m} = 1.0 \times 10^{-10}$ m/s, the temperature distribution (K, Case 2) was obtained. The results compared identically with the analytical solution for a fin with an adiabatic tip using the *IHT Model, Extended Surface, Rectangular Pin Fin*.

Case	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_{10}	T_b
1	287	287.2	287.6	288.3	289.4	290.9	292.9	295.4	298.6	302.7	308
2	300.3	300.4	300.6	300.9	301.4	302.1	302.8	303.8	305	306.4	308

<

The evaporation rate obtained by summing rates from each nodal element including node b is

$$n_{A,tot} = 1.08 \times 10^{-5} \text{ kg/s} \quad <$$

COMMENTS: A copy of the *IHT Workspace* used to perform the above analysis is shown below.

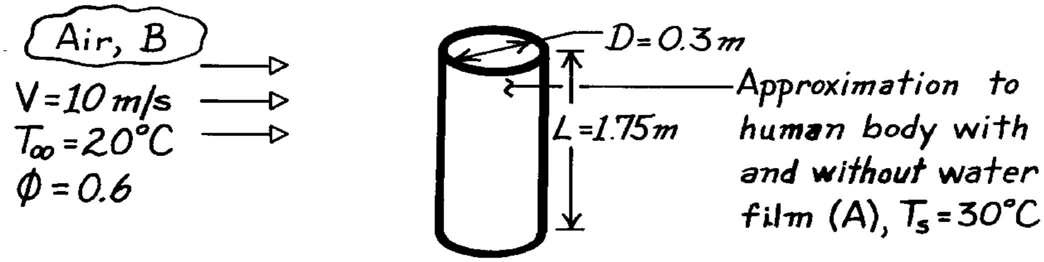
```
// Nodal finite-difference equations (Only Nodes 1, 2 and 10 shown):
k * Ac * (T2 - T1) / delx - mdot1 * hfg1 + hbar * P * delx * (Tinf - T1) + k * Ac * (T2 - T1) / delx = 0
mdot1 = hmbar * P * delx * rhoA1
k * Ac * (T3 - T2) / delx - mdot2 * hfg2 + hbar * P * delx * (Tinf - T2) + k * Ac * (T1 - T2) / delx = 0
mdot2 = hmbar * P * delx * rhoA2
........
........
k * Ac * (Tb - T10) / delx - mdot10 * hfg10 + hbar * P * delx * (Tinf - T10) + k * Ac * (T9 - T10) / delx = 0
mdot10 = hmbar * P * delx * rhoA10

// Evaporation Rate:
mtot = mdot1/2 + mdot2 + mdot3 + mdot4 + mdot5 + mdot6 + mdot7 + mdot8 + mdot9 + mdot10 + mdotb
mdotb = hmbar * P * delx/2 * rhoAb

// Properties Tool - Water Vapor, rhoAm and hfgm
// Water property functions :T dependence, From Table A.6
// Units: T(K), p(bars);
x = 1                              // Quality (0=sat liquid or 1=sat vapor)
rhoA1 = rho_Tx("Water",T1,x)       // Density, kg/m^3
hfg1 = hfg_T("Water",T1)           // Heat of vaporization, J/kg
rhoA2 = rho_Tx("Water",T2,x)       // Density, kg/m^3
hfg2 = hfg_T("Water",T2)           // Heat of vaporization, J/kg
.....
.....
rhoA10 = rho_Tx("Water",T10,x)     // Density, kg/m^3
hfg10 = hfg_T("Water",T10)         // Heat of vaporization, J/kg
rhoAb = rho_Tx("Water",Tb,x)       // Density, kg/m^3
hfgb = hfg_T("Water",Tb)           // Heat of vaporization, J/kg

// Assigned Variables
Ac = pi * D^2 /4                   // Cross-sectional area, m^2
P = pi * D                         // Perimeter, m
D = 0.020                          // Diameter, m
delx = 0.125 /10                   // Spatial increment, m
k = 175                            // Thermal conductivity, W/m.K
Tb = 35 + 273                      // Base temperature, K
Tinf = 20 + 273                    // Fluid temperature, K
hmbar = 0.07719                    // Average mass transfer coefficient, m/s
hbar = 101                         // Average heat transfer coefficient, W/m^2.K
```

PROBLEM 7.125

KNOWN: The dimensions of a cylinder which approximates the human body.

FIND: (a) Heat loss by forced convection to ambient air, (b) Total heat loss when a water film covers the surface.

SCHEMATIC:



ASSUMPTIONS: (1) Direct contact between skin and air (no clothing), (2) Negligible radiation effects, (3) Heat and mass transfer analogy is applicable, (4) Water vapor is an ideal gas.

PROPERTIES: *Table A-6*, Water (30°C = 303 K): $\rho_{A,sat} = v_g^{-1} = 0.0336\ \text{kg/m}^3$, $h_{fg} = 2431$ kJ/kg; Water (20°C = 293K): $\rho_{A,sat} = 0.017\ \text{kg/m}^3$; *Table A-4*, Air: ($T_\infty$ = 20°C = 293K): $\nu = 15.27 \times 10^{-6}\ \text{m}^2/\text{s}$, $k = 25.7 \times 10^{-3}$ W/m·K, Pr = 0.71; *Table A-8*, Water vapor-air (300K): $D_{AB} = 26 \times 10^{-6}\ \text{m}^2/\text{s}$, $Sc = \nu/D_{AB} = 0.59$.

ANALYSIS: (a) The heat rate is

$$q = \bar{h}(\pi DL)\ (T_s - T_\infty).$$

With

$$Re_D = \frac{VD}{\nu} = \frac{(10\ \text{m/s})\ (0.3\ \text{m})}{15.27\times10^{-6}\ \text{m}^2/\text{s}} = 1.96\times10^5$$

obtain $\bar{h}$ from Eq. 7.56, where n = 0.37 and, from Table 7.4, C = 0.26 and m = 0.6,

$$\overline{Nu}_D = 0.6\left(1.96\times10^5\right)^{0.6}\ (0.71)^{0.37}\ (0.71/0.71)^{0.25} = 343.$$

Hence $\bar{h} = \overline{Nu}_D\ \dfrac{k}{D} = 343\times\dfrac{25.7\times10^{-3}\ \text{W/m}\cdot\text{K}}{0.3\ \text{m}} = 29.4\ \text{W/m}^2\cdot\text{K}$

and $q = 29.4\ \text{W/m}^2\cdot\text{K}\ (\pi\times0.3\ \text{m}\times1.75\ \text{m})\ (30-20)^\circ\text{C} = 485\ \text{W}.$ <

(b) The total heat loss with the water film includes latent, as well as sensible, contributions and may be expressed as

$$q = \bar{h}(\pi DL)\ (T_s - T_\infty) + \dot{n}_A\ h_{fg}$$

where $\dot{n}_A = \bar{h}_m\ (\pi DL)\left[\rho_{A,sat}(T_s) - \rho_{A,\infty}\right]$

$$\rho_{A,sat}(T_s) = 0.0336\ \text{kg/m}^3 \qquad \rho_{A,\infty} \approx \phi\rho_{A,sat}(T_\infty) = 0.6(0.017) = 0.010\ \text{kg/m}^3.$$

Continued …..

PROBLEM 7.125 (Cont.)

The convection mass transfer coefficient may be obtained from Eq. 6.92 or by expressing the mass transfer analog of Eq. 7.56. Neglecting the Pr ratio, the analogous form is

$$\overline{\mathrm{Sh}}_D = 0.26\ \mathrm{Re}_D^{0.6}\ \mathrm{Sc}^{0.37}$$

$$\overline{\mathrm{Sh}}_D = 0.26\ \left(1.96\times10^5\right)^{0.6}\left(0.59\right)^{0.37} = 320.$$

Hence

$$\overline{h}_m = 320\frac{D_{AB}}{D} = \frac{320\times0.26\times10^{-4}\ \mathrm{m}^2/\mathrm{s}}{0.3\ \mathrm{m}} = 0.028\ \mathrm{m/s}.$$

The evaporation rate is then

$$\dot{n}_A = 0.028\mathrm{m/s}\ \left(p\times0.3\ \mathrm{m}\times1.75\ \mathrm{m}\right)\ \left[0.0336-0.010\right]\ \mathrm{kg/m}^3$$

$$\dot{n}_A = 1.09\times10^{-3}\ \mathrm{kg/s}.$$

Hence,

$$q = 485\ \mathrm{W} + 1.09\times10^{-3}\mathrm{kg/s}\times2.431\times10^6\ \mathrm{J/kg}$$

$$q = 485\ \mathrm{W} + 2650\ \mathrm{W} = 3135\ \mathrm{W}.$$ $<$

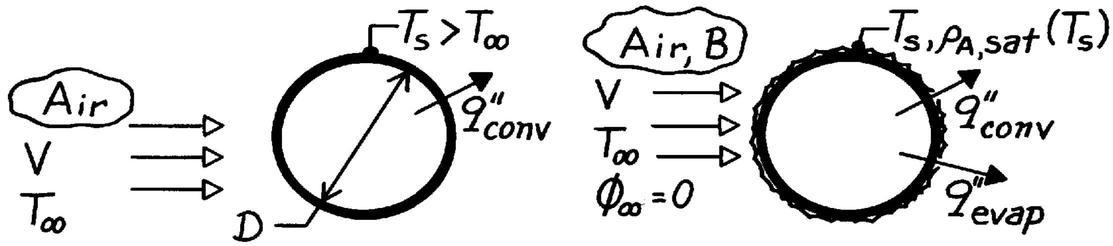
COMMENTS: The evaporative (latent) heat loss dominates over the sensible heat loss. Its effect is often felt when stepping out of a swimming pool or other body of water.

PROBLEM 7.126

KNOWN: Horizontal tube exposed to transverse stream of dry air.

FIND: Equation to determine heat transfer enhancement due to wetting. Evaluate enhancement for prescribed conditions.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Heat-mass transfer analogy applicable, (3) Water vapor behaves as perfect gas.

PROPERTIES: *Table A-4*, Air (310K, 1 atm): $\rho = 1.1281$ kg/m^3, $c_p = 1007.4$ J/kg, $\nu = 16.90 \times 10^{-6}$ m^2/s, Pr = 0.706; *Table A-8*, Air-water vapor mixture (310K): $D_{AB} \approx 0.26 \times 10^{-4}$ m^2/s, Sc = $\nu_B/D_{AB} = 0.650$; *Table A-6*, Saturated water vapor (320K): $\rho_{A,sat} = 1/v_g = 0.07153$ kg/m^3, h_{fg} = 2390 kJ/kg.

ANALYSIS: The enhancement due to wetting can be expressed as the ratio of the wet-to-dry cylinder heat fluxes.

$$\frac{q''_w}{q''_d} = \frac{q''_{conv} + q''_{evap}}{q''_{conv}} = 1 + \frac{q''_{evap}}{q''_{conv}}$$

where

$$q''_{conv} = \overline{h}\left(T_s - T_\infty\right) \qquad q''_{evap} = \dot{m}''_A h_{fg} = \overline{h}_m\left(\rho_{A,s} - \rho_{A,\infty}\right)h_{fg} = \overline{h}_m\,\rho_{A,sat}\,h_{fg}.$$

Invoking the heat-mass transfer analogy, using Eq. 6.92, find

$$\frac{\overline{h}}{\overline{h}_m} = \left(\rho\, c_p\right)_B \,Le^{1-n} = \left(\rho\, c_p\right)_B \,(Sc/Pr)^{2/3}$$

assuming n = 1/3 with $\rho_{A,\infty} = 0$, find

$$\frac{q''_w}{q''_d} = 1 + \left[\left(\rho\, c_p\right)_B (Sc/Pr)^{2/3}\right]^{-1} \frac{\rho_{A,sat}\, h_{fg}}{\left(T_s - T_\infty\right)}. \qquad <$$

Substituting numerical values, the enhancement is

$$\frac{q''_w}{q''_d} = 1 + \left[\left(1.1281\frac{kg}{m^3} \times 1007.4\frac{J}{kg}\right)\left(\frac{0.650}{0.706}\right)^{2/3}\right]^{-1} \frac{0.07153\ kg/m^3 \times 2390 \times 10^3\ J/kg}{\left(320 - 300\right)K} = 9.0. \qquad <$$

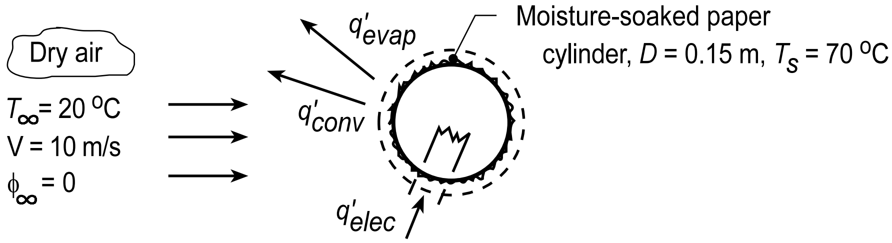
COMMENTS: For the prescribed conditions, the effect of wetting is to enhance the heat transfer by nearly an order of magnitude. Will the enhancement increase or decrease with increasing T_s?

PROBLEM 7.127

KNOWN: Moisture-soaked paper is cylindrical form maintained at given temperature by imbedded heaters. Dry air at prescribed velocity and temperature in cross flow over cylinder.

FIND: (a) Required electrical power and the evaporation rate per unit length, q'_{evap} and n'_A, respectively, and (b) Calculate and plot q' and n'_A as a function of dry air velocity $5 \le V \le 20$ m/s and paper temperatures of 65, 70 and 75°C.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Heat-mass transfer analogy applicable, (3) Negligible radiation effects.

PROPERTIES: *Table A.4*, Air (T_∞ = 20°C = 293 K, 1 atm): ρ = 1.1941 kg/m^3, c_p = 1007 J/kg·K, k = 25.7×10^{-3} W/m·K, $\nu = 15.26 \times 10^{-6}$ m^2/s, Pr = 0.709; (T_s = 70°C = 343 K): Pr_s = 0.701; *Table A.6*, Sat. water vapor (T_s = 70°C = 343 K): $\rho_{A,s} = 1/v_g$ = 0.196 kg/m^3, $h_{fg} = 2334 \times 10^3$ J/kg; *Table A.8*, Air-water vapor mixture ($\overline{T}_f = (T_\infty + T_s)/2$ = 318 K, 1 atm): $D_{AB} = 0.26 \times 10^{-4}$ m^2/s$(318/298)^{3/2} = 0.29 \times 10^{-4}$ m^2/s.

ANALYSIS: (a) From an energy balance on the cylinder on a per unit length basis,

$$q'_{elec} = q'_{conv} + q'_{evap} \qquad q'_{elec} = \pi D\left[\overline{h}\left(T_s - T_\infty\right) + \overline{h}_m\left(\rho_{A,s} - \rho_{A,\infty}\right)h_{fg}\right] \tag{1}$$

where $\rho_{A,\infty}$ = 0, the freestream air is dry, and $\rho_{A,s}$ = $\rho_{A,sat}(T_s)$. To estimate $\overline{h}$, find

$$Re_D = \frac{VD}{\nu} = \frac{10\,m/s \times 0.15\,m}{15.26 \times 10^{-6}\,m^2/s} = 98,296 \tag{2}$$

and using the Zhukauskus correlation, from Table 7.4: C = 0.26, m = 0.6, and n = 0.37,

$$\overline{Nu}_D = \frac{\overline{h}D}{k} = 0.26\,Re^{0.6}\,Pr^{0.37}\left(Pr/Pr_s\right)^{0.25} \tag{3}$$

$$\overline{h} = \frac{0.0257\,W/m\cdot K}{0.15\,m} \times 0.26\left(98,296\right)^{0.6}\left(0.709\right)^{0.37}\left(0.709/0.701\right)^{0.25} = 38.9\,W/m^2\cdot K.$$

Using the heat-mass analogy with n = 1/3, find

$$\overline{h}/\overline{h}_m = \left(\rho c_p\right)_B\left(Sc/Pr\right)^{2/3} = \left(\rho c_p\right)_B\left(\nu/D_{AB}/Pr\right)^{2/3} \tag{4}$$

$$\overline{h}_m = 38.9\,W/m^2\cdot K \Big/ \left(1.1941\,kg/m^3 \times 1007\,J/kg\cdot K\right)\left[\frac{15.26 \times 10^{-6}\,m^2/s / 0.29 \times 10^{-4}\,m^2/s}{0.709}\right]^{2/3}$$

$$\overline{h}_m = 0.03946\,m/s.$$

Hence, the electric power requirement is

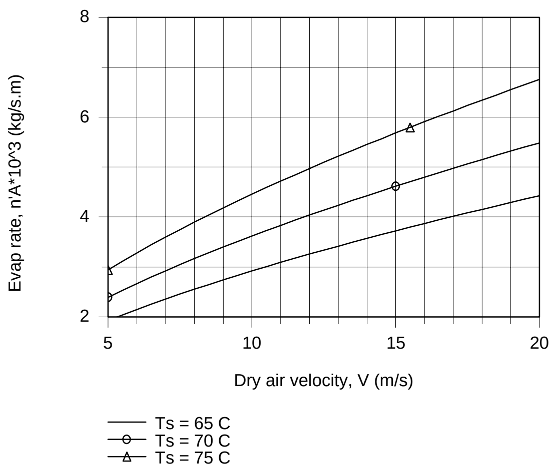
$$q'_{elec} = \pi \times 0.15\,m\left[38.9\,W/m^2\cdot K\left(70 - 20\right)K + 0.03946\,m/s\left(0.196 - 0\right)kg/m^3 \times 2334 \times 10^3\,J/kg\right]$$

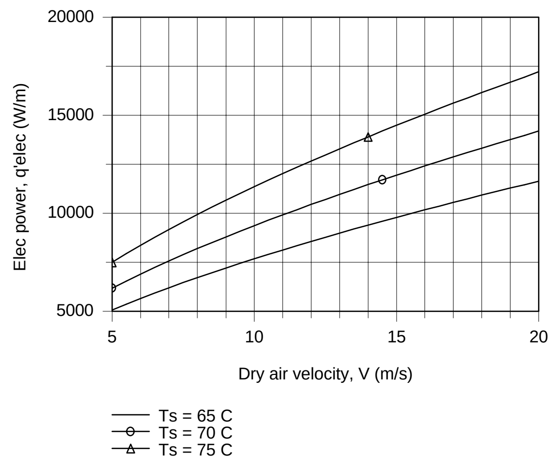
Continued...

PROBLEM 7.127 (Cont.)

$$q'_{elec} = (917 + 8507)\,W/m = 9424\,W/m \qquad (5) <$$

(b) The foregoing equations were entered into the IHT Workspace, and using the *Properties Tools*, for air and water vapor required thermophysical properties, the required electrical power, q', and evaporation rate, n'_A, were calculated as a function of dry air velocity for selected water temperatures.





COMMENTS: (1) Note at which temperatures the thermophysical properties are evaluated.

(2) From Equation (5), note that the evaporation heat rate far exceeds that due to convection.

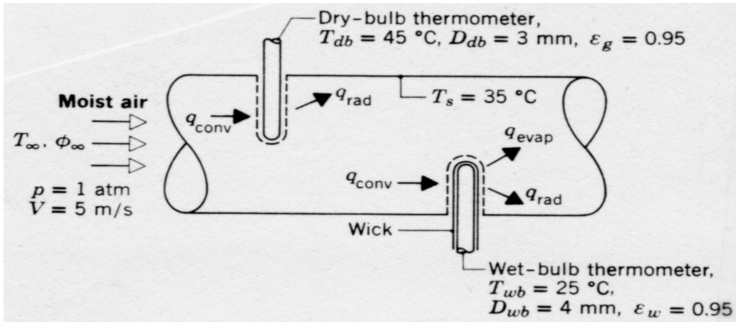
(3) From the plots, note that both q'_{elec} and n'_A are nearly proportional to air velocity, and increase with increasing water temperature.

PROBLEM 7.128

KNOWN: Dry-and wet-bulb temperatures associated with a moist airflow through a large diameter duct of prescribed surface temperature.

FIND: Temperature and relative humidity of airflow.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Conduction along the thermometers is negligible, (3) Duct wall forms a large enclosure about the thermometers.

PROPERTIES: *Table A-4*, Air (318K, 1 atm): $\nu = 17.7 \times 10^{-6}$ m^2/s, k = 0.0276 W/m·K, Pr = 0.70; *Table A-4*, Air (298K, 1 atm): $\nu = 15.7 \times 10^{-6}$ m^2/s, k = 0.0261 W/m·K, Pr = 0.71; *Table A-6*, Saturated water vapor (298K): $v_g = 44.3$ m^3/kg, h_{fg} = 2442 kJ/kg; Saturated water vapor (318.5K): $v_g = 15.5$ m^3/kg; *Table A-8*, Water vapor-air (298K): $D_{AB} = 0.26 \times 10^{-4}$ m^2/s, Sc = 0.60.

ANALYSIS: *Dry-bulb Thermometer*: Since $T_{db} > T_s$, there is net radiation transfer from the surface of the dry-bulb thermometer to the duct wall. Hence to maintain steady-state conditions, the thermometer temperature must be less than that of the air ($T_{db} < T_\infty$) to allow for convection heat transfer from the air. Hence, from application of a surface energy balance to the thermometer, $q_{conv} = q_{rad}$, or, from Eqs. 6.4 and 1.7,

$$\bar{h} A_{db}\left(T_\infty - T_{db}\right) = \varepsilon_g A_{db} \sigma \left(T_{db}^4 - T_s^4\right).$$

The air temperature is then

$$T_\infty = T_{db} + \left(\varepsilon_g \sigma / \bar{h}\right)\left(T_{db}^4 - T_s^4\right) \qquad (1)$$

where $\bar{h}$ may be obtained from Eq. 7.56.

Wet-bulb Temperature: The relative humidity may be obtained by performing an energy balance on the wet-bulb thermometer. In this case convection heat transfer to the wick is balanced by evaporative and radiative heat losses from the wick,

$$q_{conv} = q_{evap} + q_{rad} \qquad q_{evap} = n''_A A_{wb} h_{fg} = \bar{h}_m \left[\rho_{A,sat}\left(T_{wb}\right) - \phi_\infty \rho_{A,sat}\left(T_\infty\right)\right] A_{wb} h_{fg}.$$

$$\bar{h} A_{wb}\left(T_\infty - T_{wb}\right) = \bar{h}_m \left[\rho_{A,sat}\left(T_{wb}\right) - \phi_\infty \rho_{A,sat}\left(T_\infty\right)\right] A_{wb} h_{fg} + \varepsilon_w A_{wb} \sigma \left(T_{wb}^4 - T_s^4\right)$$

$$\phi_\infty = \left\{\rho_{A,sat}\left(T_{wb}\right) + \left[\varepsilon_w \sigma \left(T_{wb}^4 - T_s^4\right) - \bar{h}\left(T_\infty - T_{wb}\right)\right] / h_{fg} \bar{h}_m \right\} / \rho_{A,sat}\left(T_\infty\right) \qquad (2)$$

where $\bar{h}_m$ may be determined from the mass transfer analog of Eq. 7.56.

Continued …..

Convection Calculations: For the prescribed conditions, the Reynolds number associated with the dry-bulb thermometer is

$$Re_{D(db)} = VD_{bd}/\nu = 5\text{ m/s}\times 0.003\text{ m}/17.7\times 10^{-6}\text{ m}^2/\text{s} = 847.$$

Approximating the Prandtl number ratio as unity, from Eq. 7.56 and Table 7.4,

$$\overline{Nu}_{D(db)} = CRe_{D(db)}^{m} Pr^{n} = 0.51(847)^{0.5}(0.70)^{0.37} = 13.01$$

$$\bar{h} = 13.01\frac{k}{D_{db}} = 13.01\frac{0.0276\text{ W/m}\cdot\text{K}}{0.003\text{ m}} = 120\text{ W/m}^2\cdot\text{K}.$$

From Eq. (1) the air temperature is

$$T_\infty = 45^\circ\text{C} + \frac{0.95\times 5.67\times 10^{-8}\text{W/m}^2\cdot\text{K}^4}{120\text{ W/m}^2\cdot\text{K}}\left(318^4 - 308^4\right)\text{K}^4 = 45^\circ\text{C} + 0.55^\circ\text{C} = 45.6^\circ\text{C}. \quad <$$

The relative humidity may now be obtained from Eq. (2). The Reynolds number associated with the wet-bulb thermometer is

$$Re_{D(wb)} = VD_{wb}/\nu = 5\text{ m/s}\times 0.004\text{ m}/15.7\times 10^{-6}\text{ m}^2/\text{s} = 1274.$$

From Eq. 7.56 and Table 7.4, it follows that

$$\overline{Nu}_{D(wb)} = 0.26(1274)^{0.6}(0.71)^{0.37} = 16.71$$

$$\bar{h} = 16.71\frac{k}{D_{wb}} = 16.71\frac{0.0261\text{ W/m}\cdot\text{K}}{0.004\text{ m}} = 109\text{ W/m}^2\cdot\text{K}.$$

Using the mass transfer analog of Eq. 7.56, it also follows that

$$\overline{Sh}_{D(wb)} = 0.26Re_{D(wb)}^{0.6}Sc^{0.37} = 0.26(1274)^{0.6}(0.6)^{0.37} = 15.7$$

$$\bar{h}_m = 15.7\frac{D_{AB}}{D_{wb}} = \frac{15.7\times 0.26\times 10^{-4}\text{ m}^2/\text{s}}{0.004\text{ m}} = 0.102\text{ m/s}.$$

Also, $\rho_{A,sat}(T_{wb}) = v_g(298\text{ K})^{-1} = \left(44.3\text{ m}^3/\text{kg}\right)^{-1} = 0.0226\text{ kg/m}^3$

$$\rho_{A,sat}(T_\infty) = v_g(318.5\text{ K})^{-1} = \left(15.5\text{ m}^3/\text{kg}\right)^{-1} = 0.0645\text{ kg/m}^3.$$

Hence the relative humidity is, from Eq. (2)

$$\phi_\infty = \left(0.0226\text{ kg/m}^3 + \frac{\left[0.95\times 5.67\times 10^{-8}\text{W/m}^2\cdot\text{K}^4\left(298^4 - 308^4\right)\text{K}^4 - 109\text{W/m}^2\cdot\text{K}\left(45.55 - 25\right)\text{K}\right]}{\left(2.442\times 10^6\text{ J/kg}\right)\left(0.102\text{ m/s}\right)}\right)/\,0.0645\text{ kg/m}^3$$

$$\phi_\infty = 0.21 \quad <$$

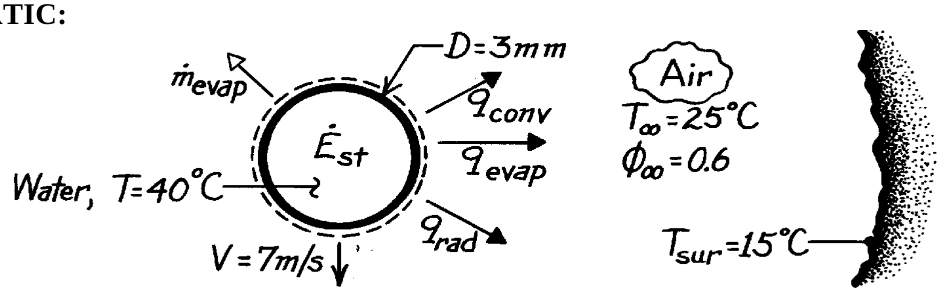
COMMENTS: (1) The effect of radiation exchange between the duct wall and the thermometers is small. For this reason $T_\infty = T_{db}$. (2) The evaporative heat loss is significant due to the small value of ϕ_∞, causing T_{wb} to be significantly less than T_∞.

PROBLEM 7.129

KNOWN: Velocity, diameter and temperature of a spherical droplet. Conditions of surroundings.

FIND: (a) Expressions for droplet evaporation and cooling rates, (b) Evaporation and cooling rates for prescribed conditions.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible temperature gradients in the drop, (2) Heat and mass transfer analogy is applicable, (3) Perfect gas behavior for vapor.

PROPERTIES: *Table A-4*, Air (T_∞ = 298K, 1 atm): $\nu = 15.71 \times 10^{-6}$ m^2/s, k = 0.0261 W/m·K, Pr = 0.71; *Table A-6*, Water (T = 40°C): $\rho_{A,sat}$ = 0.050 kg/m^3, h_{fg} = 2407 kJ/kg, r_ℓ = 992 kg/m^3, $c_{p,\ell}$ = 4179 J/kg·K; (T_∞ = 25°C): $\rho_{A,sat}$ = 0.023 kg/m^3; *Table A-8*, Water vapor-air (298K): $D_{AB} = 0.26 \times 10^{-4}$ m^2/s.

ANALYSIS: (a) The evaporation rate is given by

$$\dot{m}_{evap} = \overline{h}_m A_s \left(r_{A,s} - r_{A,\infty} \right) = \overline{h}_m p D^2 \left[r_{A,sat}(T) - f_\infty r_{A,sat}(T_\infty) \right]. \qquad <$$

The cooling rate is obtained from an energy balance performed for a control surface about the droplet,

$$\dot{E}_{st} = -q_{out} = -\left(q_{conv} + q_{rad} + q_{evap} \right)$$

or
$$\frac{d}{dt}\left(r_\ell \frac{p D^3}{6} c_{p,\ell} T \right) = -A_s \left[\overline{h}\left(T_s - T_\infty\right) + e s \left(T_s^4 - T_{sur}^4 \right) + \dot{m}''_{evap} h_{fg} \right].$$

With $A_s = \pi D^2$, it follows that

$$\frac{dT}{dt} = -\frac{6}{r_\ell c_{p,\ell} D}\left[\overline{h}\left(T_s - T_\infty\right) + e s \left(T_s^4 - T_{sur}^4 \right) + \dot{m}''_{evap} h_{fg} \right]. \qquad <$$

(b) To obtain $\overline{h}_m$, the mass transfer analog of the Ranz-Marshall correlation gives

$$\overline{Sh}_D = 2 + 0.6 Re_D^{1/2} Sc^{1/3}$$

where

$$Re_D = \frac{VD}{n} = \frac{7 \text{ m/s} \times 0.003 \text{ m}}{15.71 \times 10^{-6} \text{ m}^2/\text{s}} = 1337, \qquad Sc = \frac{n}{D_{AB}} = \frac{15.71 \times 10^{-6}}{26 \times 10^{-6}} = 0.60.$$

Continued …..

Hence

$$\overline{Sh}_D = 2 + 0.6(1337)^{1/2}(0.6)^{1/3} = 20.5$$

$$\bar{h}_m = \overline{Sh}_D \frac{D_{AB}}{D} = 20.5 \frac{0.26\times10^{-4}\ m^2/s}{0.003\ m} = 0.18\ m/s$$

$$\dot{m}_{evap} = 0.18\ m/s\ p\ (0.003\ m)^2[0.05 - 0.6\times.023]kg/m^3 = 1.82\times10^{-7} kg/s.$$ <

The evaporative heat flux is then

$$q''_{evap} = \frac{q_{evap}}{A_s} = \frac{\dot{m}_{evap} h_{fg}}{p D^2} = \frac{1.82\times10^{-7}\ kg/s\left(2.407\times10^6\ J/kg\right)}{p\ (0.003\ m)^2}$$

$$q''_{evap} = 15{,}494\ W/m^2.$$

Using the heat transfer correlation, the Nusselt number is

$$\overline{Nu}_D = 2 + 0.6Re_D^{1/2}Pr^{1/3} = 2 + 0.6(1337)^{1/2}(0.71)^{1/3} = 21.58.$$

Hence $$\bar{h} = \overline{Nu}_D \frac{k}{D} = 21.58 \frac{0.0261\ W/m\cdot K}{0.003\ m} = 188\ W/m^2\cdot K$$

and the sensible heat flux is

$$q''_{conv} = \bar{h}(T - T_\infty) = 188\ W/m^2\cdot K\,(40 - 25)^\circ C$$

$$q''_{conv} = 2815\ W/m^2.$$

The net radiative flux is

$$q''_{rad} = es\left(T^4 - T_{sur}^4\right) - 0.95\times5.67\times10^{-8}\ W/m^2\cdot K^4\left[313^4 - 288^4\right]K^4$$

$$q''_{rad} = 146\ W/m^2\cdot K.$$

Hence $$\frac{dT}{dt} = -\frac{6}{992\ kg/m^3\times4179\ J/kg\cdot K(0.003\ m)}(2815 + 146 + 15{,}494)\ W/m^2$$

$$\frac{dT}{dt} = -8.9\ K/s.$$ <

COMMENTS: (1) Evaporative cooling provides the dominant heat loss from the drop. (2) To test the validity of assuming negligible temperature gradients in the drop, calculate

$$Bi \approx \frac{h_{eff}(r_o/3)}{k_\ell}, \quad \text{where} \quad h_{eff} \equiv \frac{q''_{tot}}{T - T_\infty} = \frac{18{,}455}{25} = 738\ W/m^2\cdot K.$$

From Table A-6, $k_\ell = 0.631\ W/m\cdot K$, hence

$$Bi \approx \frac{738\ W/m^2\cdot K(0.0005\ m)}{0.631\ W/m\cdot K} = 0.58.$$

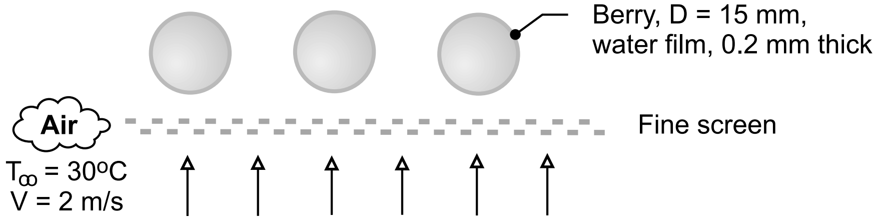
Hence, although suspect, the assumption is not totally unreasonable.

PROBLEM 7.130

KNOWN: Cranberries with an average diameter of 15 mm rolling over a fine screen. Thickness of the water film is 0.2 mm.

FIND: Time required to dry the berries exposed to heated air with a velocity of 2 m/s and temperature of 30°C.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Air stream is dry, (3) Water film on the berries is also at 30°C, (4) Convection process is uniform over the exposed surface, and (5) Heat-mass analogy is applicable.

PROPERTIES: *Table A-6*, Water (T_f = 30°C = 303 K): $\rho_{A,f} = 995.8\ \text{kg/m}^3$, $\rho_{A,g}$ = 0.02985 kg/m^3; *Table A-8*, Water-air (T_f = 303 K, 1 atm): $D_{AB} = 0.26 \times 10^{-4}\ \text{m}^2\text{/s}\ (303/298)^{1.5} = 2.67 \times 10^{-5}\ \text{m}^2\text{/s}$; *Table A-4*, Air ($T_f$ = 303 K, 1 atm): $\mu = \mu_s = 1.86 \times 10^{-5}\ \text{N·s/m}^2$, $\nu = 1.619 \times 10^{-5}\ \text{m}^2\text{/s}$, $\alpha = 2.294 \times 10^{-5}\ \text{m}^2\text{/s}$, k = 0.02652 W/m·K, Pr = 0.861.

ANALYSIS: The evaporation rate of water from the berry surface is given by the rate equation,

$$n = \bar{h}_m A_s \left(\rho_{A,s} - \rho_{A,\infty}\right) \tag{1}$$

where $A_s = \pi D^2$ and $\bar{h}_m$ is determined using the heat-mass analogy, Eq. 6.67,

$$\frac{\bar{h}}{\bar{h}_m} = \frac{k}{D_{AB}} Le^{-n} \tag{2}$$

where $Le = \alpha/D_{AB}$ and typically n = 1/3. The heat transfer coefficient $\bar{h}$ is estimated with the Whitaker correlation, Eq. 7.59,

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = 2 + \left[0.4\ Re_D^{1/2} + 0.06\ Re_D^{2/3}\right] Pr^{0.4} \left(\mu/\mu_s\right)^{1/4} \tag{3}$$

Substituting numerical values, find

$$Re_D = \frac{VD}{\nu} = \frac{2\ \text{m/s} \times 0.015\ \text{m}}{1.86 \times 10^{-5}\ \text{m}^2\text{/s}} = 1853$$

$$Nu_D = 2 + \left[0.4(1853)^{1/2} + 0.06(1853)^{2/3}\right] \times (0.707)^{0.4} \times 1 = 24.9$$

$$\bar{h} = 24.9 \times 0.02652\ \text{W/m} \cdot \text{K} / 0.015\ \text{m} = 43.4\ \text{W/m}^2 \cdot \text{K}$$

and using the heat-mass analogy,

$$\bar{h}_m = 43.4\ \text{W/m}^2 \cdot \text{K} \times \left(2.67 \times 10^{-5}\ \text{m}^2\text{/s} \big/ 0.02652\ \text{W/m} \cdot \text{K}\right) \times (0.861)^{1/3}$$

$$\bar{h}_m = 0.0420\ \text{m/s}$$

Continued …..

PROBLEM 7.130 (Cont.)

where

$$Le = \alpha / D_{AB} = 2.294 \times 10^{-5} m^2/s \Big/ 2.667 \times 10^{-5} m^2/s = 0.861$$

Using Eq. (1), the evaporation rate is

$$n = 0.0420 \text{ m/s} \times \left(\pi (0.015 \text{ m})^2 / 4\right)(0.02985 - 0) \text{kg/m}^3 = 8.87 \times 10^{-7} \text{ kg/s}$$

The time, t_o, required to evaporate the water film of thickness $\delta = 0.2$ mm is

$$n t_o = M_{film} = \rho_{A,\ell} (\pi D) \delta$$

$$t_o = 995.8 \text{ kg/m}^3 (\pi \times 0.015 \text{ m}) \times 0.0002 \text{ m} / 8.87 \times 10^{-7} \text{ kg/s}$$

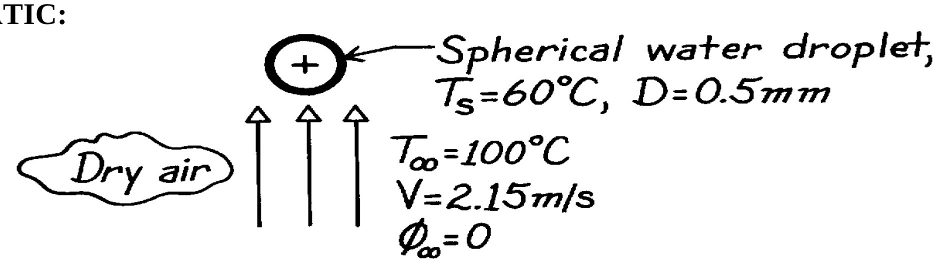
$$t_o = 159 \text{ s}$$ <

PROBLEM 7.131

KNOWN: Spherical droplet at prescribed temperature and velocity falling in still, hotter dry air.

FIND: Instantaneous rate of evaporation.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Heat-mass transfer analogy applicable.

PROPERTIES: *Table A-4*, Air (T_∞ = 100°C = 373K, 1atm): ρ = 0.9380 kg/m^3, c_p = 1011 J/kg·K, k = 0.0317 W/m·K, $\nu = 23.45 \times 10^{-6}$ m^2/s, Pr = 0.695; *Table A-6*, Sat. water (T_s = 60°C = 333 K): $\rho_\ell = 1/v_\ell$ = 983 kg/m^3, $\rho_{A,s} = 1/v_f$ = 0.129 kg/m^3; *Table A-8*, Air-water vapor mixture (T_∞ = 373K, 1 atm): $D_{AB} = 0.267 \times 10^{-4}$ m^2/s $(373/298)^{3/2} = 0.36 \times 10^{-4}$ m^2/s.

ANALYSIS: The instantaneous evaporation rate is

$$\dot{n}_A = \overline{h}_m A_s \left(\rho_{A,s} - \rho_{A,\infty}\right)$$

where $A_s = \pi D^2$, $\rho_{A,\infty} = 0$ and $\rho_{A,s} = \rho_{A,sat}(T_s)$. To estimate $\overline{h}_m$ use the Whitaker correlation, written in terms of mass transfer parameters and with $\mu/\mu_s \approx 1$,

$$\overline{Sh}_D = \frac{\overline{h}_m D}{D_{AB}} = 2 + \left(0.4Re_D^{1/2} + 0.06Re_D^{2/3}\right)Sc^{0.4}$$

$$\overline{h}_m = \frac{0.36\times10^{-4}\ \text{m}^2/\text{s}}{0.0005\ \text{m}}\left[2+\left(0.4(45.8)^{1/2} + 0.06(45.8)^{2/3}\right)\times 0.651^{0.4}\right] = 0.355\ \text{m/s}$$

where $$Re_D = \frac{VD}{\nu} = \frac{2.15\ \text{m/s}\times 0.0005\ \text{m}}{23.45\times10^{-6}\ \text{m}^2/\text{s}} = 45.8$$

$$Sc = \nu/D_{AB} = 23.45\times10^{-6}\ \text{m}^2/\text{s}/0.36\times10^{-4}\ \text{m}^2/\text{s} = 0.651.$$

Hence, the evaporation rate is

$$\dot{n}_A = 0.355\ \text{m/s}\times\pi\,(0.0005\ \text{m})^2\,(0.129-0)\,\text{kg/m}^3 = 3.60\times10^{-8}\ \text{kg/s}.$$ <

COMMENTS: If this evaporation rate were to remain constant with time, the droplet of mass M would be completely evaporated in

$$\Delta t = M/\dot{n}_A = \frac{\rho_\ell\left(4\pi D^3/3\right)}{\dot{n}_A} = \frac{983\ \text{kg/m}^3\left(4\pi\,(0.0005\ \text{m})^3/3\right)}{3.60\times10^{-8}\ \text{kg/s}} = 14.3\ \text{s}.$$

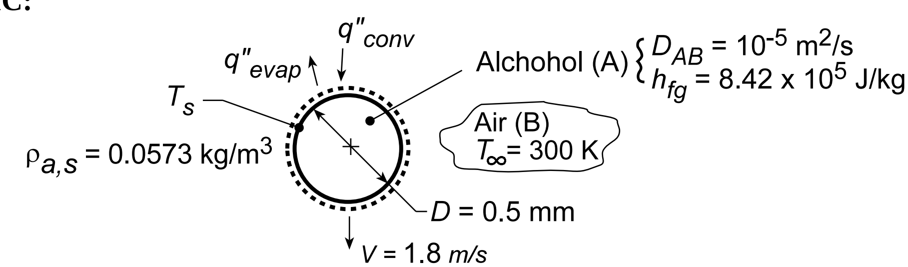
To determine whether the droplet temperature will increase or decrease with time, it is necessary to compare convective heat and evaporation rates. Hence it is not clear whether the time to completely evaporate will be less or greater than 14.3 s.

PROBLEM 7.132

KNOWN: Diameter, velocity and surface vapor concentration of alcohol droplet falling in quiescent air. Latent heat of vaporization and diffusion coefficient. Air temperature.

FIND: Droplet surface temperature

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Applicability of heat and mass transfer analogy, (3) Negligible radiation, (4) Negligible vapor concentration in air ($\rho_{A,\infty}$ = 0).

PROPERTIES: *Table A.4*, air (T_∞ = 300 K): $\nu = 15.89 \times 10^{-6}$ m^2/s, k = 0.0263 W/m·K, Pr = 0.707.

ANALYSIS: Application of a surface energy balance yields

$$q''_{evap} = q''_{conv}$$

$$\bar{h}_m \left(\rho_{A,s} - \rho_{A,\infty}\right) h_{fg} = \bar{h}\left(T_\infty - T_s\right)$$

$$T_s = T_\infty - \frac{\bar{h}_m}{\bar{h}} \rho_{A,s} h_{fg}$$

With $Re_D = VD/\nu = 1.8\,\text{m/s} \times 5 \times 10^{-4}\,\text{m} \big/ 15.89 \times 10^{-6}\,\text{m}^2\big/\text{s}$ = 56.6 and Sc = ν/D_{AB} = 1.59, the Ranz-Marshall correlation yields

$$\overline{Nu}_D = 2 + 0.6 Re_D^{1/2} Pr^{1/3} = 2 + 0.6(56.6)^{1/2}(0.707)^{1/3} = 6.02$$

$$\overline{Sh}_D = 2 + 0.6 Re_D^{1/2} Sc^{1/3} = 2 + 0.6(56.6)^{1/2}\left(1.59^{1/3}\right) = 7.27$$

With $\bar{h}_m / \bar{h}$ = $\overline{Sh}_D \left(D_{AB}/D\right) \big/ \overline{Nu}_D \left(k/D\right)$,

$$\frac{\bar{h}_m}{\bar{h}} = \frac{\overline{Sh}_D \left(D_{AB}\right)}{\overline{Nu}_D \left(k\right)} = \frac{7.27 \times 10^{-5}\,\text{m}^2\big/\text{s}}{6.02 \times 0.0263\,\text{W/m}\cdot\text{K}} = 4.59 \times 10^{-4}\,\text{m}^3 \cdot \text{K}\big/\text{J}$$

Hence,

$$T_s = 300\,\text{K} - 4.59 \times 10^{-4}\,\text{m}^3 \cdot \text{K}\big/\text{J}\left(0.0573\,\text{kg}\big/\text{m}^3\right)\left(8.42 \times 10^5\,\text{J/kg}\right) = 277.9\,\text{K} \qquad <$$

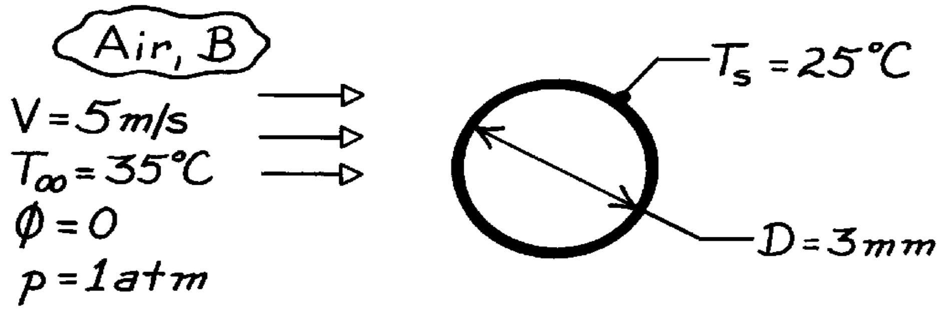
COMMENTS: The large vapor density, $\rho_{A,s}$, renders the *evaporative cooling* effect significant.

PROBLEM 7.133

KNOWN: Diameter, velocity and temperature of water droplets in air of known temperature.

FIND: Evaporation rate for a single drop.

SCHEMATIC:



ASSUMPTIONS: (a) Steady-state conditions, (b) Dry air, (c) Drop oscillations and distortions are negligible.

PROPERTIES: *Table A-4*, Air (35°C = 308K): $\nu = 16.7 \times 10^{-6}\ m^2/s$; *Table A-8*, Water vapor-air (35°C = 308K): $D_{AB} = 0.26 \times 10^{-4}\ m^2/s$; *Table A-6*, Saturated water vapor (25°C = 298K): $v_g = 44.3\ m^3/kg$.

ANALYSIS: The mass evaporation rate is

$$n_A = \overline{h}_m \left(p D^2\right) \left(r_{A,s} - r_{A,\infty}\right)$$

where $r_{A,s} = v_g^{-1} = 0.023\ kg/m^3$ and $r_{A,\infty} = 0$. From Eq. 7.58,

$$\overline{Sh}_D = 2 + \left(0.4\ Re_D^{1/2} + 0.06\ Re_D^{2/3}\right)\ Sc^{0.4}$$

where

$$Re_D = \frac{VD}{n} = \frac{(5\ m/s)\left(3\times10^{-3}\ m\right)}{16.7\times10^{-6}\ m^2/s} = 898 \qquad Sc = \frac{n}{D_{AB}} = 0.64$$

$$\overline{Sh}_D = 2 + \left[0.4(898)^{1/2} + 0.06(898)^{2/3}\right]\ (0.64)^{0.4} = 16.7$$

$$\overline{h}_m = \overline{Sh}_D\ \frac{D_{AB}}{D} = 16.7\frac{0.26\times10^{-4}\ m^2/s}{3\times10^{-3}\ m} = 0.145\ m/s.$$

Hence,

$$n_A = 0.145\ m/s\ p\left(3\times10^{-3}\ m\right)^2 \times 0.023\ kg/m^3 = 9.43\times10^{-8}\ kg/s. \qquad <$$

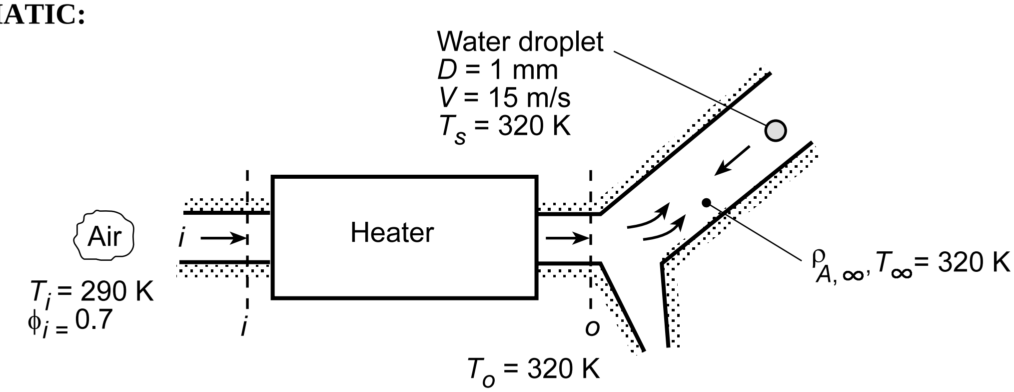
COMMENTS: For the small difference between T_s and T_∞, it is reasonable to neglect the viscosity ratio in Eq. 7.58. Use of Eq. 7.59 gives $\overline{h}_m$ = 0.152 m/s, which is in good agreement with the result from Eq. 7.58.

PROBLEM 7.134

KNOWN: Humidity and temperature of air entering heater; temperature of air leaving heater. Diameter, temperature and relative velocity of injected droplets.

FIND: Droplet evaporation rate.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible change in droplet diameter due to evaporation, (2) Negligible cooling of droplet due to evaporation, (3) Applicability of heat/mass transfer analogy, (4) Ideal gas behavior for vapor.

PROPERTIES: *Table A.4*, air ($T_\infty = T_o = 320$ K): $\nu = 17.90 \times 10^{-6}$ m²/s, k = 0.0278 W/m·K, Pr = 0.705. *Table A.6*, saturated water ($T_i = 290$ K): $p_{sat} = 0.01917$ bars; ($T_o = 320$ K): $p_{sat} = 0.1053$ bars, $v_g = 13.98$ m³/kg. *Table A.8*, H_2O/air (T = 320 K): $D_{AB} = 0.26 \times 10^{-4}$ m²/s $(320/298)^{3/2} = 0.289 \times 10^{-4}$ m²/s.

ANALYSIS: Due to an increase in temperature, the air leaves the heater with a smaller relative humidity. With $\phi_i = 0.7$ and $p_{sat,i} = 0.01917$ bars, the vapor pressure at the heater inlet is $p_i = \phi_i\, p_{sat,i} = 0.7(0.01917 \text{ bars}) = 0.0134$ bars. Since the vapor pressure doesn't change with passage through the heater,

$$\phi_o = \frac{p_i}{p_{sat,o}} = \frac{0.0134\,\text{bars}}{0.1053\,\text{bars}} = 0.127$$

The vapor density associated with air flow around the droplets is therefore

$$\rho_{A,\infty} = \phi_o \rho_{A,sat}(T_o) = \phi_o v_g(T_o)^{-1} = 0.127 \times 0.0715\,\text{kg/m}^3 = 0.0091\,\text{kg/m}^3$$

The droplet evaporation rate is

$$\dot{m}_{evap} = \overline{h}_m A_s \left[\rho_{A,sat}(T_s) - \rho_{A,\infty}\right]$$

where $\overline{h}_m$ may be obtained from the mass transfer analog to the Whitaker correlation. With $Re_D = VD/\nu = 15$ m/s $\times$ 0.001 m/17.9 $\times 10^{-6}$ m²/s = 838, $Sc = \nu/D_{AB} = 17.9 \times 10^{-6}$ m²/s/28.9 $\times 10^{-6}$ m²/s = 0.62, and $\mu/\mu_s = 1$,

$$\overline{Sh}_D = 2 + \left(0.4\,Re_D^{1/2} + 0.06\,Re_D^{2/3}\right)Sc^{0.4} = 2 + \left[0.4(838)^{1/2} + 0.06(838)^{2/3}\right](0.62)^{0.4} = 16.0$$

$$\overline{h}_m = \overline{Sh}_D\,(D_{AB}/D) = 16\left(0.289 \times 10^{-4}\,\text{m}^2/\text{s}/0.001\,\text{m}\right) = 0.462\,\text{m/s}$$

$$\dot{m}_{evap} = (0.462\,\text{m/s})\,\pi\,(0.001\,\text{m})^2\,(0.0715 - 0.0091)\,\text{kg/m}^3 = 9.06 \times 10^{-8}\,\text{kg/s} \qquad <$$

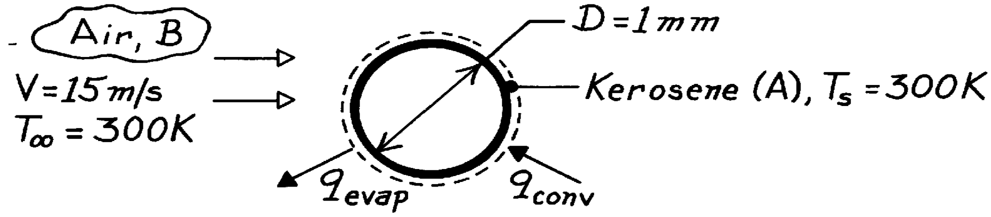
COMMENTS: The energy required for evaporation must be supplied by convection heat transfer from the heated air to the droplet. Hence, in actuality, the droplet temperature T_s must be less than that of the freestream air, T_∞, which in turn will decrease from the value T_o at the heater outlet.

PROBLEM 7.135

KNOWN: Diameter and temperature of sphere wetted with kerosene. Air flow conditions.

FIND: (a) Minimum kerosene flow rate, (b) Air temperature required to maintain wetted surface at 300K.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Sphere mount has a negligible influence on the flow field and hence on $\overline{h}$, (3) Negligible kerosene vapor concentration in free stream.

PROPERTIES: *Table A-4*, Air (300K): $\nu = 15.89 \times 10^{-6}\ m^2/s$, $k = 0.0263\ W/m\cdot K$, $\rho = 1.161\ kg/m^3$, Pr = 0.707; Kerosene (given): $\rho_{A,sat} = 0.015\ kg/m^3$, $h_{fg} = 300\ kJ/kg$; Kerosene vapor-air (given): $D_{AB} = 10^{-5}\ m^2/s$.

ANALYSIS: (a) The kerosene flowrate is $n_A = \overline{h}_m A\left(\rho_{A,sat} - \rho_{A,\infty}\right)$. Using the mass transfer analog of Eq. 7.58 and neglecting the viscosity ratio,

$$\overline{Sh}_D = 2 + \left(0.4\ Re_D^{1/2} + 0.06\ Re_D^{2/3}\right)\ Sc^{0.4}$$

with $$Re_D = \frac{VD}{\nu} = \frac{15\ m/s \times 0.001\ m}{15.89\times10^{-6}\ m^2/s} = 944 \qquad Sc = \frac{\nu}{D_{AB}} = \frac{15.89\times10^{-6}}{10\times10^{-6}} = 1.59$$

$$\overline{Sh}_D = 2 + \left(0.4\times944^{1/2} + 0.06944^{2/3}\right)\ (1.59)^{0.4} = 23.7$$

$$\overline{h}_m = \overline{Sh}_D\ D_{AB}/D = 23.7\times10^{-5}\ m^2/s/0.001\ m = 0.237\ m/s$$

$$n_A = 0.237\ m/s\ \pi\left(10^{-3}\ m\right)^2\ 0.015\ kg/m^3 = 1.12\times10^{-8}\ kg/s. \qquad <$$

(b) An energy balance on the sphere yields $n_A\ h_{fg} = \overline{h}A\ \left(T_\infty - T_s\right)$. Using the Whitaker correlation and neglecting the viscosity ratio,

$$\overline{Nu}_D = 2 + \left(0.4\times944^{1/2} + 0.06\times944^{2/3}\right)\ (0.707)^{0.4} = 17.72$$

$$\overline{h} = \overline{Nu}_D k/D = 17.72\times0.0263\ W/m\cdot K/0.001\ m = 466\ W/m^2\cdot K$$

$$T_\infty = T_s + \frac{n_A\ h_{fg}}{\overline{h}\pi D^2} = 300K + \frac{1.12\times10^{-8}\ kg/s\times3\times10^5\ J/kg}{466\ W/m^2\cdot K\times\pi\left(0.001\ m\right)^2}$$

$$T_\infty = 300K + 2.3K = 302.3K \qquad <$$

or $$T_\infty - T_s = 2.3K.$$

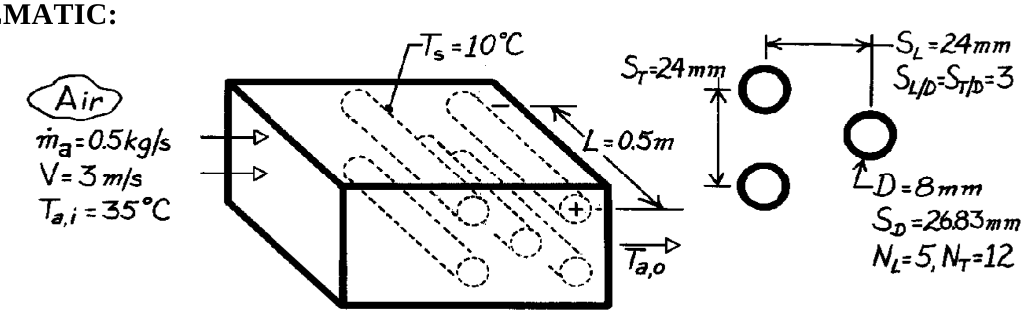
COMMENTS: The small temperature excess (2.3K) is due to comparatively small values of $\rho_{A,sat}$ and h_{fg} for kerosene.

PROBLEM 7.136

KNOWN: Geometry and surface temperature of a tube bank with or without wetted surfaces. Temperature, velocity and flowrate associated with air in cross flow.

FIND: (a) Ratio of air cooling with water film to that without film, (b) Air outlet temperature and specific humidity for prescribed conditions.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Heat and mass transfer analogy is applicable, (3) Air is dry, (4) Heat and mass transfer driving potentials are $T_{a,i} - T_s$ and $\rho_{A,sat}(T_s)$, (5) Vapor has negligible effect on flowrate.

PROPERTIES: *Table A-4*, Air (assume $\overline{T}_a \approx 305K$): $\rho = 1.1448\ kg/m^3$, $c_p = 1007\ J/kg\cdot K$, $\nu = 16.39 \times 10^{-6}\ m^2/s$, $k = 0.0267\ W/m\cdot K$, $Pr = 0.706$, $\alpha = 23.2 \times 10^{-6}\ m^2/s$; *Table A-6*, Water vapor ($T_s = 10°C$): $v_g = 111.8\ m^3/kg$, $\rho_{A,sat} = 8.94 \times 10^{-3}\ kg/m^3$, $h_{fg} = 2.478 \times 10^6\ J/kg$; *Table A-8*, Water vapor-air ($T_f \approx 298K$): $D_{AB} = 0.26 \times 10^{-4}\ m^2/s$, $Sc = (\nu/D_{AB}) = 0.642$.

ANALYSIS: (a) The rate of heat loss from the air may be expressed as

$$q = \dot{m}_a c_{p,a} \left(T_{a,i} - T_{a,o}\right)$$

in which case, the amount of air cooling is

$$\left(T_{a,i} - T_{a,o}\right) = \frac{q}{\dot{m}_a c_{p,a}}. \qquad (1)$$

Without the water film, $q_{wo} \approx \overline{h}A\left(T_{a,i} - T_s\right)$ (2)

With the film,

$$q_w \approx \overline{h}A\left(T_{a,i} - T_s\right) + \dot{m}_{evap}\ h_{fg}$$

$$q_w \approx \overline{h}A\left(T_{a,i} - T_s\right) + \overline{h}_m A\left(\rho_{A,sat} - \rho_{A,\infty}\right) h_{fg} \qquad (3)$$

where $\rho_{A,\infty} = 0$. Hence

$$\frac{\left(T_{a,i} - T_{a,o}\right)_w}{\left(T_{a,i} - T_{a,o}\right)_{wo}} \approx 1 + \frac{\overline{h}_m \rho_{A,sat} h_{fg}}{\overline{h}\left(T_{a,i} - T_s\right)}$$

or substituting from Eq. 6.92, with $Le = \alpha/D_{AB}$ and a value of $n = 0.33$,

$$\frac{\left(T_{a,i} - T_{a,o}\right)_w}{\left(T_{a,i} - T_{a,o}\right)_{wo}} \approx 1 + \frac{\left(D_{AB}/\alpha\right)^{0.67}}{\rho\ c_p}\ \frac{\rho_{A,sat}\ h_{fg}}{\left(T_{a,i} - T_s\right)}$$ <

Continued …..

For the prescribed conditions,

$$\frac{\left(T_{a,i}-T_{a,o}\right)_w}{\left(T_{a,i}-T_{a,o}\right)_{wo}} \approx 1+\frac{\left(\dfrac{0.26\times10^{-4}\text{m}^2/\text{s}}{0.232\times10^{-4}\text{m}^2/\text{s}}\right)^{0.67}}{1.145\ \text{kg/m}^3\times1007\ \text{J/kg}\cdot\text{K}}\times\frac{8.94\times10^{-3}\text{kg/m}^3\times2.478\times10^6\text{J/kg}}{(35-10)^\circ\text{C}} \approx 1.83.$$ <

(b) $T_{a,o}$ may be obtained from Eq. (1), where q is approximated by Eq. (2) or Eq. (3). With S_D = 26.83 mm > $(S_T + D)/2$ = 16, V_{max} is at the transverse plane. Hence

$$V_{max}=\frac{S_T}{S_T-D}V=\frac{24}{16}\times3\text{m/s}=4.5\text{m/s} \qquad Re_{D,max}=\frac{4.5\ \text{m/s}\times0.008\ \text{m}}{16.39\times10^{-6}\ \text{m}^2/\text{s}}=2196.$$

From Tables 7.7 and 7.8, C = 0.35, m = 0.60, C_2 = 0.98 and the Zhukauskas relation gives

$$\overline{Nu}_D=0.35(0.98)(2196)^{0.6}(0.706)^{0.36}=30.6$$

where$(Pr/Pr_s)^{1/4}$ is 1.00. Hence

$$\bar{h}=\overline{Nu}_D\ k/D=30.6(0.0267\ \text{W/m}\cdot\text{K})/0.008\ \text{m}=102\ \text{W/m}^2\cdot\text{K}.$$

Also $$\bar{h}_m=\bar{h}\frac{\left(D_{AB}/a\right)^{0.67}}{r\ c_p}=102\frac{\text{W}}{\text{m}^2\cdot\text{K}}\frac{(0.26/0.232)^{0.67}}{1.145\ \text{kg/m}^3\times1007\ \text{J/kg}\cdot\text{K}}=0.0956\ \text{m/s}.$$

Hence

$$q_{conv}\approx\bar{h}A\left(T_{a,i}-T_s\right)=102\text{W/m}^2\cdot\text{K}\times p\ (0.008\ \text{m})0.5\ \text{m}\times60(35-10)^\circ\text{C}=1923\ \text{W}$$

$$q_{evap}=n_A h_{fg}=\bar{h}_m A\ r_{A,sat}\ h_{fg}$$

$$q_{evap}=0.0956\ \text{m/s}\times p\ (0.008\ \text{m})0.5\ \text{m}\times60\left(8.94\times10^{-3}\text{kg/m}^3\right)2.478\times10^6\ \text{J/kg}$$

$$q_{evap}\approx1597\text{W}.$$

With water film,

$$T_{a,o}=T_{a,i}-\frac{q_{conv}+q_{evap}}{\dot{m}_a\ c_{p,a}}\approx35^\circ\text{C}-\frac{(1923+1597)\text{W}}{0.5\ \text{kg/s}\times1007\ \text{J/kg}\cdot\text{K}}=28.0^\circ\text{C}.$$ <

The specific humidity of the outlet air is

$$w_o=\frac{n_A}{\dot{m}_a}=\frac{\bar{h}_m 60p\,DL\ r_{A,sat}}{\dot{m}_a}$$

$$w_o=\frac{0.0956\ \text{m/s}(60p)\ (0.008\ \text{m})0.5\ \text{m}\left(8.94\times10^{-3}\text{kg/m}^3\right)}{0.5\ \text{kg/s}}=0.00129.$$ <

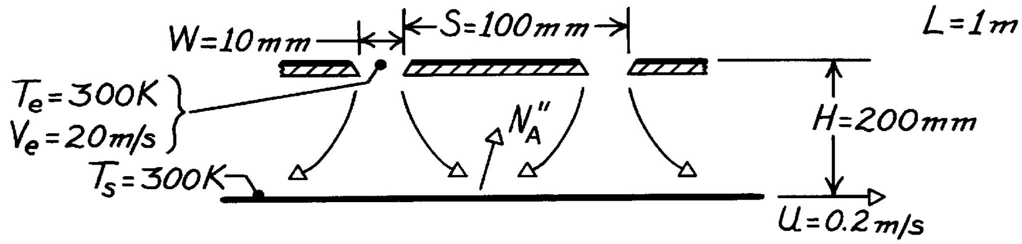
COMMENTS: (1) Enhancement of air cooling by evaporation is significant ($T_{a,o}$ = Ta,i $-q_{conv}/\dot{m}_a\ c_{p,a}\approx31.1^\circ\text{C}$ without the film). (2) Small value of ω_o justifies neglecting effect of evaporation on $\dot{m}_a$. (3) q_{conv} has been overestimated by using $(T_{a,i} - T_s)$ as the driving potential for convection heat transfer. A more accurate determination involves $\Delta T_{\ell m}$ rather than $(T_{a,i} - T_s)$. (4) Apparently the air properties were evaluated at an appropriate $\bar{T}_a$.

PROBLEM 7.137

KNOWN: Dimensions of slot jet array. Jet exit velocity and temperature. Temperature of paper.

FIND: Drying rate per unit surface area.

SCHEMATIC:



ASSUMPTIONS: (1) Applicability of heat and mass transfer analogy, (2) Paper motion has negligible effect on convection ($U << V_e$).

PROPERTIES: *Table A-4*, Air (300 K, 1 atm): $\nu = 15.89 \times 10^{-6}$ m²/s; *Table A-6*, Saturated water (300 K): $\rho_{A,sat} = v_g^{-1} = 0.0256$ kg/m³; *Table A-8*, Water vapor-air (300 K): $D_{AB} = 0.26 \times 10^{-4}$ m²/s, Sc = 0.61.

ANALYSIS: The mass evaporation flux is

$$n''_A = \overline{h}_m \left(r_{A,s} - r_{A,e} \right) = \overline{h}_m \, r_{A,sat}$$

For an array of slot nozzles,

$$\frac{\overline{Sh}}{Sc^{0.42}} = \frac{2}{3} A_{r,o}^{3/4} \left(\frac{2Re}{A_r / A_{r,o} + A_{r,o} / A_r} \right)^{2/3}$$

where

$$A_r = W/S = 0.1$$

$$A_{r,o} = \left\{ 60 + 4\left[\left(H/2W \right) - 2 \right]^2 \right\}^{1/2} = \left\{ 60 + 4(64) \right\}^{-1/2} = 0.0563$$

$$Re = \frac{V_e(2W)}{n} = \frac{20 \text{ m/s} (0.02 \text{ m})}{15.89 \times 10^{-6} \text{m}^2/\text{s}} = 25{,}173.$$

Hence

$$\frac{\overline{Sh}}{Sc^{0.42}} = 0.667 (0.0563)^{3/4} \left(\frac{50{,}346}{1.776 + 0.563} \right)^{2/3} = 59.6$$

$$\overline{h}_m = \frac{D_{AB}}{2W} 59.6 Sc^{0.42} = \frac{0.26 \times 10^{-4} \text{ m}^2/\text{s}}{0.02 \text{ m}} 59.6 (0.61)^{0.42} = 0.063 \text{m/s}.$$

The evaporative flux is then

$$n''_A = 0.063 \text{ m/s} \left(0.0256 \text{ kg/m}^3 \right) = 0.0016 \text{ kg/s} \cdot \text{m}^2.$$ <

COMMENTS: The mass fraction of water vapor to air leaving the sides of the dryer is $n''_A (S \times L) / r_{air} V_e (W \times L) = 7 \times 10^{-4}$. Hence, the assumption of dry air throughout the dryer is reasonable.