

# THE UNIVERSITY OF BRITISH COLUMBIA

## Department of Electrical and Computer Engineering

### EECE 373: Electromechanical Energy Conversion and Transmission

**Final Exam**  
**April 21, 2004**

Magnetic Circuits, Transformers, DC Machines, Induction Machines, Synchronous Machines, Stepper Motors

Surname: \_\_\_\_\_

First Name: \_\_\_\_\_

Student ID: \_\_\_\_\_

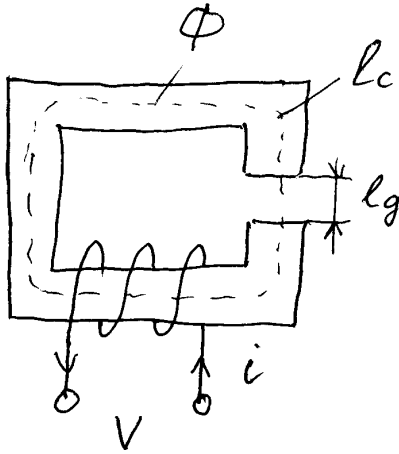
Signature: \_\_\_\_\_

- **Close notes and books.**
- You are allowed to have only a **calculator** and a **pen/pencil**.
- Show your work including **derivations, comments, assumptions**, and **units** where ever appropriate.
- Use back side of each page or ask for additional pages if you need extra space to write your answers.
- Exams suspected of cheating and/or turned in late will not be marked – **failed exam**.
- You have about 90 minutes to answer the following questions:

Problem	Points	Max.
1		16
2		17
3		17
4		17
5		17
6		16
Total		100

**Problem 1 (16pts)**

Consider the magnetic system shown below. The length of the air-gap is such that the air-gap reluctance is  $\mathfrak{R}_g = 8.75 \times 10^5 \text{ At/Wb}$ . The effective length and material of the core are such that the effective reluctance of the magnetic path inside the core is  $\mathfrak{R}_c = 1.25 \times 10^5 \text{ At/Wb}$ . The cross-sectional area of the core and air-gap is  $A_c = A_g = 10 \text{ cm}^2$ . The coil has 100 turns and dc resistance of  $3 \Omega$ . Neglect flux leakage, fringing, and core losses. Assume that an external voltage source is connected to the coil.



- (a) (8pts) Draw an equivalent **magnetic** circuit, show the direction of mmf  $F$  and the flux  $\Phi$
- (b) (8pts) Calculate the rms value of ac voltage required to produce the flux density in the air-gap  $B_g = 1 \text{ T rms}$

**Problem 2 (17pts)**

Consider a single-phase 60-Hz transformer with the following parameters:  $R_1 = 1\ \Omega$ ,  $X_1 = 3\ \Omega$ ,  $X_m = 30\ \Omega$ ,  $R_2 = 1.2\ \Omega$ , and  $X_2 = 5\ \Omega$ , respectively. Assume that both windings have the same number of turns and that the primary side is connected to a 240-V ac source.

- (a) (8pts) Based on the given information, sketch the T-equivalent circuit and label all its elements
- (b) (9pts) Assume a resistor load  $R_{load} = 40\ \Omega$  is connected to the secondary side. Calculate the voltage regulation VR expressed in percent.

Recall: 
$$VR = \frac{V_{no\_load} - V_{load}}{V_{load}} \cdot 100\%$$

**Problem 3 (17pts)**

Consider a **Permanent-Magnet DC Machine** with the armature resistance  $R_a = 1\ \Omega$ . When the machine is connected to a 12-V battery it draws the armature current  $I_a = 0.2\text{-A}$  and the shaft speed is  $n = 1800\text{ rpm}$  CCW at no load. The battery voltage and friction torque can be assumed constant.

- (a) (8pts) Calculate the torque constant  $K_t$  and the friction torque  $T_{fric}$
- (b) (9pts) Assume that you need to use this DC Machine as a generator to charge the battery. Calculate the external mechanical torque  $T_m$  required to produce the charging current of 1A. In what direction, CW or CCW, should this torque be applied? What is the resulting speed  $n$ ?

**Problem 4 (17pts)**

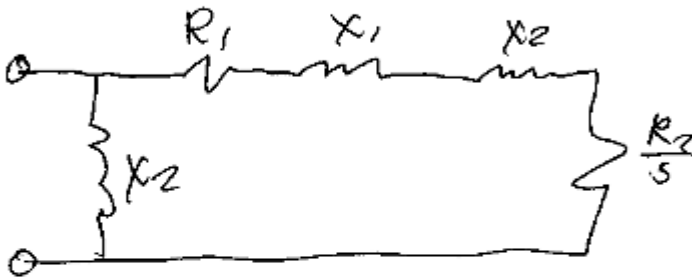
Consider a 60Hz, 208V (line-to-line), Y-connected, NEMA Class B (Squirrel-Cage) Induction Motor with the following per-phase parameters:  $R_1 = 1 \Omega$ ,  $R_2 = 1.5 \Omega$ ,  $X_1 = X_2 = 3 \Omega$ , and  $X_m = 40 \Omega$  (all referred to the stator). The motor is supplied with the nominal (rated) voltage and is driving a mechanical load. The speed of the motor shaft is  $n = 840$  rpm. You can neglect core and friction losses and use an approximate equivalent circuit. Determine the following:

- (a) (4pts) Number of poles  $p$   
 (b) (4pts) Slip  $s$   
 (c) (5pts) Stator current  $I_1$  and power factor PF  
 (d) (4pts) Total (3-phase) input real power

$$\begin{aligned} 3600 &- 2 \\ 1800 &- 4 \\ 1200 &- 6 \\ 900 &- 8 \end{aligned}$$

$$\underline{p = 8}$$

$$s = \frac{900 - 840}{900} = 0.0667 = 6.67\%$$



$$Z = \frac{jX_m(R_1 + \frac{R_2}{s} + jX_1 + jX_2)}{R_1 + \frac{R_2}{s} + j(X_1 + X_2 + X_m)} = 14.09 + j12.4$$

$$I_1 = \frac{V_1}{Z} = \frac{208}{\sqrt{3}(14.09 + j12.4)} = 4.79 - j4.22$$

$$pf = \cos \phi = \cos(\text{angle}(I_1)) = 0.75$$

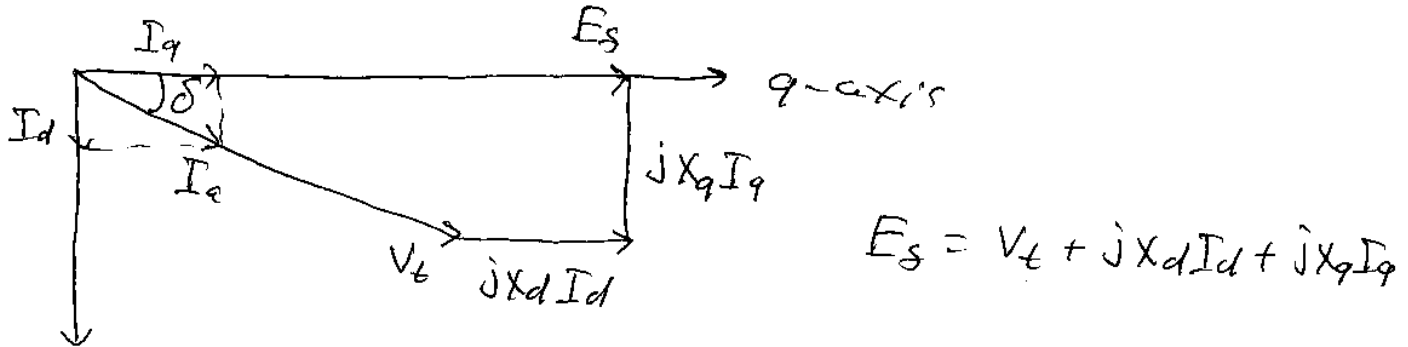
$$P_{3\phi} = 3I_1 V_1 \cos \phi = 1.728 \text{ kW}$$

**Problem 5 (17pts)**

Consider a Salient-Pole Synchronous Generator with the following parameters:  $X_q = 0.5 pu$ ,  $X_d = 0.9 pu$  (the stator winding resistance is negligible). The excitation is adjusted so that the open-circuit voltage is  $E_f = 1.2969 pu$ . The generator is supplying a resistive load that draws rated (nominal) current  $I_a = 1 pu$  and the terminal voltage is  $V_t = 1 pu$ .

(a) (8pts) Sketch a phasor diagram and label all relevant axes, angles, and phasors

(b) (9pts) Calculate the rotor angle  $\delta$  in degrees



$$V_t \cdot \sin \delta = X_q \cdot I_q = X_q \cdot I_a \cdot \cos \delta$$

$$\tan \delta = \frac{X_q I_a}{V_t} = X_q = 0.5; \quad \delta = \tan^{-1}(X_q) = 26.6^\circ$$

$$E_f = V_t \cdot \cos \delta + X_d \cdot I_a \cdot \sin \delta$$

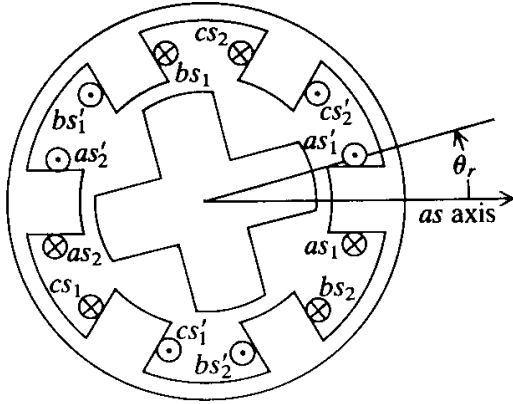
$$\sin \delta = \frac{X_q I_a \cdot \cos \delta}{V_t}$$

$$E_f = V_t \cdot \cos \delta + X_d I_a \cdot \frac{X_q I_a \cdot \cos \delta}{V_t} = \left[ V_t + \frac{X_d X_q I_a^2}{V_t} \right] \cos \delta$$

$$\delta = \cos^{-1} \left[ \frac{E_f}{V_t + \frac{X_d X_q I_a^2}{V_t}} \right] = 26.6^\circ$$

### Problem 6 (16pts)

Consider a single-stack stepper motor shown below. Complete the following table:



2pts	Stator tooth pitch (STP)	60 degrees
2pts	Rotor tooth pitch (RTP)	90 degrees
3pts	Step length $\Delta\Theta$	-30 degrees
3pts	Number of steps per revolution (Resolution)	12
3pts	Assume sequence of pulses as B – C – A, determine the direction of rotation (CW or CCW)	CCW
3pts	Assume you have a 3-phase pulse generator to supply this motor. The generator produces 6 pulses per second. What is the motor speed in rpm?	30 rpm