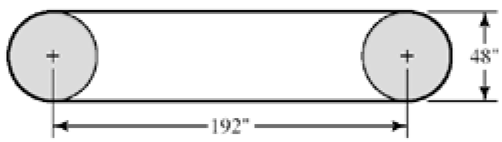
MECH 325
Homework Assignment #2
Due Oct. 18
(Submit to Mech. Office by 4:00pm)

Problem 1 (Question 17-4)

A flat-belt drive is to consist of two 4-ft-diameter cast-iron pulleys spaced 16 ft apart. Select a belt type to transmit 60 hp at a pulley speed of 380 rev/min. Use a service factor of 1.1 and a design factor of 1.0.

17-4



As a design task, the decision set just before Ex. 17-2 is useful.
A priori decisions:
- Function: $H_{\text{nom}} = 60$ hp, $n = 380$ rev/min, $C = 192$ in, $K_s = 1.1$
- Design factor: $n_d = 1$
- Initial tension: Catenary
- Belt material. Table 17-2: Polyamide A-3, $F_a = 100$ lbf/in, $\gamma = 0.042$ lbf/in^3, $f = 0.8$
- Drive geometry: $d = D = 48$ in
- Belt thickness: $t = 0.13$ in

Design variable: Belt width.
Use a method of trials. Initially, choose $b = 6$ in

$$V = \frac{\pi d n}{12} = \frac{\pi(48)(380)}{12} = 4775 \text{ ft/min}$$
$$w = 12\gamma b t = 12(0.042)(6)(0.13) = 0.393 \text{ lbf/ft}$$
$$F_c = \frac{wV^2}{g} = \frac{0.393(4775/60)^2}{32.17} = 77.4 \text{ lbf}$$
$$T = \frac{63\,025 H_{\text{nom}} K_s n_d}{n} = \frac{63\,025(60)(1.1)(1)}{380} = 10\,946 \text{ lbf} \cdot \text{in}$$
$$\Delta F = \frac{2T}{d} = \frac{2(10\,946)}{48} = 456.1 \text{ lbf}$$
$$F_1 = (F_1)_a = bF_a C_p C_v = 6(100)(1)(1) = 600 \text{ lbf}$$
$$F_2 = F_1 - \Delta F = 600 - 456.1 = 143.9 \text{ lbf}$$

Transmitted power H

$$H = \frac{\Delta F(V)}{33\ 000} = \frac{456.1(4775)}{33\ 000} = 66 \text{ hp}$$
$$F_i = \frac{F_1 + F_2}{2} - F_c = \frac{600 + 143.9}{2} - 77.4 = 294.6 \text{ lbf}$$
$$f' = \frac{1}{\theta_d} \ln \frac{F_1 - F_c}{F_2 - F_c} = \frac{1}{\pi} \ln\left(\frac{600 - 77.4}{143.9 - 77.4}\right) = 0.656$$

Eq. (17-2): $L = [4(192)^2 - (48 - 48)^2]^{1/2} + [48(\pi) + 48(\pi)] / 2 = 534.8$ in

Friction is not fully developed, so b_{min} is just a little smaller than 6 in (5.7 in). Not having a figure of merit, we choose the most narrow belt available (6 in). We can improve the design by reducing the initial tension, which reduces F_1 and F_2, thereby increasing belt life (see the result of Prob. 17-8). This will bring f' to 0.80

$$F_1 = \frac{(\Delta F + F_c)\exp(f\theta) - F_c}{\exp(f\theta) - 1}$$

$$\exp(f\theta) = \exp(0.80\pi) = 12.345$$

Therefore

$$F_1 = \frac{(456.1 + 77.4)(12.345) - 77.4}{12.345 - 1} = 573.7 \text{ lbf}$$
$$F_2 = F_1 - \Delta F = 573.7 - 456.1 = 117.6 \text{ lbf}$$
$$F_i = \frac{F_1 + F_2}{2} - F_c = \frac{573.7 + 117.6}{2} - 77.4 = 268.3 \text{ lbf}$$

These are small reductions since f' is close to f, but improvements nevertheless.

$$f' = \frac{1}{\theta_d} \ln \frac{F_1 - F_c}{F_2 - F_c} = \frac{1}{\pi} \ln\left(\frac{573.7 - 77.4}{117.6 - 77.4}\right) = 0.80$$

$$\text{dip} = \frac{3C^2 w}{2F_i} = \frac{3(192 / 12)^2(0.393)}{2(268.3)} = 0.562 \text{ in}$$

Problem 2 (Question 17-17)

A single Gates Rubber V belt is to be selected to deliver engine power to the wheel-drive transmission of a riding tractor. A 5-hp single-cylinder engine is used. The mechanical efficiency of the gearbox is 60% so that only 3 hp is transmitted to the belt. The driving sheave has a diameter of 6.2 in, the driven, 12.0 in. The belt selected should be as close to a 92-in pitch length as possible. The engine speed is governor-controlled to a maximum of 3100 rev/min. Select a satisfactory belt and assess the factor of safety and the belt life in passes.

17-17

Preliminaries: For a single V-belt drive with H_{nom} = 3 hp, n = 3100 rev/min, D = 12 in, and d = 6.2 in, choose a B90 belt, K_s = 1.3 and n_d = 1. From Table 17-10, select a circumference of 90 in. From Table 17-11, add 1.8 in giving

$$L_p = 90 + 1.8 = 91.8 \text{ in}$$

Eq. (17-16*b*):

$$\begin{aligned} C &= 0.25\left\{\left[91.8 - \frac{\pi}{2}(12 + 6.2)\right] + \sqrt{\left[91.8 - \frac{\pi}{2}(12 + 6.2)\right]^2 - 2(12 - 6.2)^2}\right\} \\ &= 31.47 \text{ in} \end{aligned}$$

$$\theta_d = \pi - 2\sin^{-1}\left[\frac{12 - 6.2}{2(31.47)}\right] = 2.9570 \text{ rad}$$

$$\exp(f\theta_d) = \exp[0.5123(2.9570)] = 4.5489$$

$$V = \frac{\pi dn}{12} = \frac{\pi(6.2)(3100)}{12} = 5031.8 \text{ ft/min}$$

Table 17-13:

$$\text{Angle}\,\theta = \theta_d\frac{180°}{\pi} = (2.957 \text{ rad})\left(\frac{180°}{\pi}\right) = 169.42°$$

The footnote regression equation of Table 17-13 gives K_1 without interpolation:

$$K_1 = 0.143\,543 + 0.007\,468(169.42°) - 0.000\,015\,052(169.42°)^2 = 0.9767$$

The design power is

$$H_d = H_{nom}K_s n_d = 3(1.3)(1) = 3.9 \text{ hp}$$

From Table 17-14 for B90, K_2 = 1. From Table 17-12 take a marginal entry of H_{tab} = 4, although extrapolation would give a slightly lower H_{tab}.

Eq. (17-17): $$H_a = K_1K_2H_{tab} = 0.9767(1)(4) = 3.91 \text{ hp}$$

The allowable ΔF_a is given by

$$\Delta F_a = \frac{63\,025 H_a}{n(d/2)} = \frac{63\,025(3.91)}{3100(6.2/2)} = 25.6 \text{ lbf}$$

The allowable torque T_a is

$$T_a = \frac{\Delta F_a d}{2} = \frac{25.6(6.2)}{2} = 79.4 \text{ lbf} \cdot \text{in}$$

From Table 17-16, $K_c = 0.965$. Thus, Eq. (17-21) gives,

$$F_c = K_c \left(\frac{V}{1000}\right)^2 = 0.965\left(\frac{5031.8}{1000}\right)^2 = 24.4 \text{ lbf}$$

At incipient slip, Eq. (17-9) provides:

$$F_i = \left(\frac{T}{d}\right)\left[\frac{\exp(f\theta)+1}{\exp(f\theta)-1}\right] = \left(\frac{79.4}{6.2}\right)\left(\frac{4.5489+1}{4.5489-1}\right) = 20.0 \text{ lbf}$$

Eq. (17-10):

$$F_1 = F_c + F_i\left[\frac{2\exp(f\theta)}{\exp(f\theta)+1}\right] = 24.4 + 20\left[\frac{2(4.5489)}{4.5489+1}\right] = 57.2 \text{ lbf}$$

Thus, $F_2 = F_1 - \Delta F_a = 57.2 - 25.6 = 31.6$ lbf

Eq. (17-26): $$n_{fs} = \frac{H_a N_b}{H_d} = \frac{(3.91)(1)}{3.9} = 1.003 \quad \textit{Ans.}$$

If we had extrapolated for H_{tab}, the factor of safety would have been slightly less than one.

Life Use Table 17-16 to find equivalent tensions T_1 and T_2 .

$$T_1 = F_1 + (F_b)_1 = F_1 + \frac{K_b}{d} = 57.2 + \frac{576}{6.2} = 150.1 \text{ lbf}$$

$$T_2 = F_1 + (F_b)_2 = F_1 + \frac{K_b}{D} = 57.2 + \frac{576}{12} = 105.2 \text{ lbf}$$

From Table 17-17, $K = 1193$, $b = 10.926$, and from Eq. (17-27), the number of belt passes is:

$$N_P = \left[\left(\frac{K}{T_1}\right)^{-b} + \left(\frac{K}{T_2}\right)^{-b}\right]^{-1}$$

$$= \left[\left(\frac{1193}{150.1}\right)^{-10.926} + \left(\frac{1193}{105.2}\right)^{-10.926}\right]^{-1} = 6.72(10^9) \text{ passes}$$

From Eq. (17-28) for $N_P > 10^9$,

$$t = \frac{N_P L_p}{720V} > \frac{10^9(91.8)}{720(5031.8)}$$
$$t > 25\ 340 \text{ h} \qquad \textit{Ans.}$$

Suppose n_{fs} was too small. Compare these results with a 2-belt solution.

$$H_{\text{tab}} = 4 \text{ hp/belt}, \quad T_a = 39.6 \text{ lbf} \cdot \text{in/belt},$$
$$\Delta F_a = 12.8 \text{ lbf/belt}, \quad H_a = 3.91 \text{ hp/belt}$$
$$n_{fs} = \frac{N_b H_a}{H_d} = \frac{N_b H_a}{H_{\text{nom}} K_s} = \frac{2(3.91)}{3(1.3)} = 2.0$$

Also,

$$F_1 = 40.8 \text{ lbf/belt}, \qquad F_2 = 28.0 \text{ lbf/belt}$$
$$F_i = 9.99 \text{ lbf/belt}, \qquad F_c = 24.4 \text{ lbf/belt}$$
$$(F_b)_1 = 92.9 \text{ lbf/belt}, \qquad (F_b)_2 = 48 \text{ lbf/belt}$$
$$T_1 = 133.7 \text{ lbf/belt}, \qquad T_2 = 88.8 \text{ lbf/belt}$$
$$N_P = 2.39(10^{10}) \text{ passes}, \qquad t > 605\ 600 \text{ h}$$

Initial tension of the drive:

$$(F_i)_{\text{drive}} = N_b F_i = 2(9.99) = 20 \text{ lbf}$$

Problem 3 (Question 17-27 modified)

A 700 rev/min 25-hp squirrel-cage induction motor is to drive a two-cylinder reciprocating pump, out-of-doors under a shed. A service factor K_s of 1.5 and a design factor of 1.1 are appropriate. The pump speed is 140 rev/min. Select a suitable number of strands of Number 80 chain and sprocket sizes.

17-27 This is our first design/selection task for chain drives. A possible decision set:

A priori decisions

- Function: H_{nom}, n_1, space, life, K_s
- Design factor: n_d
- Sprockets: Tooth counts N_1 and N_2, factors K_1 and K_2

Decision variables

- Chain number
- Strand count
- Lubrication type
- Chain length in pitches

Function: Motor with $H_{\text{nom}} = 25$ hp at $n = 700$ rev/min; pump at $n = 140$ rev/min;

$m_G = 700/140 = 5$

Design Factor: $n_d = 1.1$

Sprockets: Tooth count $N_2 = m_G N_1 = 5(17) = 85$ teeth–odd and unavailable. Choose 84 teeth. *Decision*: $N_1 = 17$, $N_2 = 84$

Evaluate K_1 and K_2

Eq. (17-38): $H_d = H_{\text{nom}} K_s n_d$

Eq. (17-37): $H_a = K_1 K_2 H_{\text{tab}}$

Equate H_d to H_a and solve for H_{tab} :

$$H_{\text{tab}} = \frac{K_s n_d H_{\text{nom}}}{K_1 K_2}$$

Table 17-22: $K_1 = 1$

Table 17-23: $K_2 = 1, 1.7, 2.5, 3.3$ for 1 through 4 strands

$$H'_{\text{tab}} = \frac{1.5(1.1)(25)}{(1)K_2} = \frac{41.25}{K_2}$$

Prepare a table to help with the design decisions:

Strands	K_2	H'_{tab}	Chain No.	H_{tab}	n_{fs}	Lub. Type
2	1.7	24.3	80	31.0	1.40	B
3	2.5	16.5	80	31.0	2.07	B
4	3.3	12.5	~~60~~ 80	~~13.3~~ 31.0	~~1.17~~ 2.7	B
1	1	41.25	80	31	0.86	B

Design Decisions:

We need a figure of merit to help with the choice. If the best ~~was 4~~ is 2 strands of No. ~~60~~ 80 chain, then

Decision #1 and #2: Choose ~~four~~ two strand No. ~~60~~ 80 roller chain with $n_{fs} = \cancel{1.17}\ 1.40$

$$n_{fs} = \frac{K_1 K_2 H_{\text{tab}}}{K_s H_{\text{nom}}} = \frac{1(\cancel{3.3}\ 1.7)(\cancel{13.3}\ 31.0)}{1.5(25)} = \cancel{1.17}\ 1.40$$

Decision #3: Choose Type B lubrication

Analysis:

Table 17-20: $H_{\text{tab}} = \cancel{13.3}\ 31.0$ hp

Table 17-19: $p = \cancel{0.75}\ 1.0$ in

Try $C = 30$ in in Eq. (17-34):

$$\frac{L}{p} \doteq \frac{2C}{p} + \frac{N_1 + N_2}{2} + \frac{(N_2 - N_1)^2}{4\pi^2 C / p}$$

$$= 2(30 / \cancel{0.75}\ 1.0) + \frac{17 + 84}{2} + \frac{(84 - 17)^2}{4\pi^2(30 / \cancel{0.75}\ 1.0)}$$

$$= \cancel{133.3}\ 114.3$$

$$1.0\ \ 114.3 \simeq 114 \quad \text{(even into odd \# sprocket)}$$

$$L = \cancel{0.75}(\cancel{133.3}) = 100 \text{ in } (\cancel{\text{no need to round}})$$

Eq. (17-36) with $p = \cancel{0.75}\ 1.0$ in: $A = \frac{N_1 + N_2}{2} - \frac{L}{p} = \frac{17 + 84}{2} - \frac{\cancel{100}\ 114}{\cancel{0.75}\ 1} = \cancel{-82.83}\ -63.5$

Eq. (17-35):

$$C = \frac{p}{4}\left[-A + \sqrt{A^2 - 8\left(\frac{N_2 - N_1}{2\pi}\right)^2}\right]$$

$$= \frac{\cancel{0.75}\ 1.0}{4}\left[-(\cancel{-82.83}\ 63.5) + \sqrt{(\cancel{-82.83}\ 65.5)^2 - 8\left(\frac{84 - 17}{2\pi}\right)^2}\right] = \cancel{30.0 \text{ in}}\ 30.4$$

Decision #4: Choose C = 30.0 in.

Problem 4 (Question 17-31)

A 2000-ft mine hoist operates with a 72-in drum using 6 ×19 monitor-steel wire rope. The cage and load weigh 8000 lbf, and the cage is subjected to an acceleration of 2 ft/s^2 when starting.

(*a*) For a single-strand hoist how does the factor of safety $n_f = F_f/F_t$, neglecting bending, vary with the choice of rope diameter?

(*b*) For four supporting strands of wire rope attached to the cage, how does the factor of safety vary with the choice of rope diameter?

17-31 Given: 2000 ft lift, 72 in drum, 6 × 19 MS rope, cage and load 8000 lbf, accel. = 2 ft/s^2.

(a) Table 17-24: $(S_u)_{nom}$ = 106 kpsi; S_u = 240 kpsi (plow steel); Fig. 17-21: $(p/S_u)10^6$ = 0.0014

Eq. (17-44):

$$F_f = \frac{(p / S_u) S_u dD}{2} = \frac{0.0014(240)d(72)}{2} = 12.1d \text{ kip}$$

Table 17-24: $wl = 1.6d^2\, 2000(10^{-3}) = 3.2d^2$ kip

Eq. (17-47):

$$F_t = (W + wl)\left(1 + \frac{a}{g}\right)$$
$$= (8 + 3.2d^2)\left(1 + \frac{2}{32.2}\right)$$
$$= 8.5 + 3.4d^2 \text{ kip}$$

Note that bending is not included.

$$n_f = \frac{F_f}{F_t} = \frac{12.1d}{8.5 + 3.4d^2}$$

d, in	n_f	
0.500	0.650	
1.000	1.020	
1.500	1.124	
1.625	1.125	← maximum n_f *Ans.*
1.750	1.120	
2.000	1.095	

(b) Try m = 4 strands

$$F_t = \left(\frac{8}{4} + 3.2d^2\right)\left(1 + \frac{2}{32.2}\right)$$
$$= 2.12 + 3.4d^2 \text{ kip}$$
$$F_f = 12.1d \text{ kip}$$
$$n_f = \frac{12.1d}{2.12 + 3.4d^2}$$

d, in	n_f	
0.5000	2.037	
0.5625	2.130	
0.6520	2.193	
0.7500	2.250	← maximum n_f *Ans.*
0.8750	2.242	
1.0000	2.192	

Comparing tables, multiple ropes supporting the load increases the factor of safety, and reduces the corresponding wire rope diameter, a useful perspective.