Solutions for the textbook problems

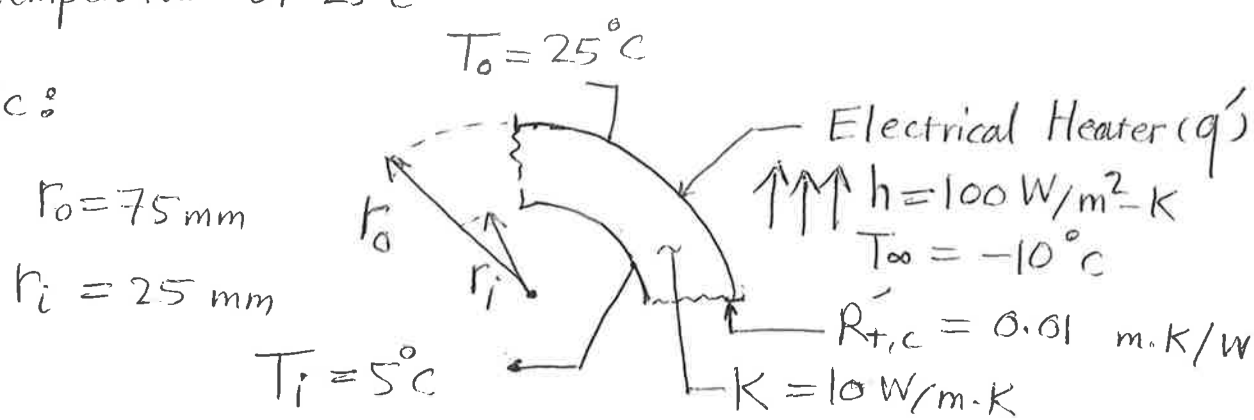
PS #3

Problem 3.37

Known: Inner and outer radii of a tube wall which is heated electronically at its outer surface and is exposed to a fluid of prescribed h and T_∞. Thermal contact resistance between heater and tube wall and wall inner surface temperature.

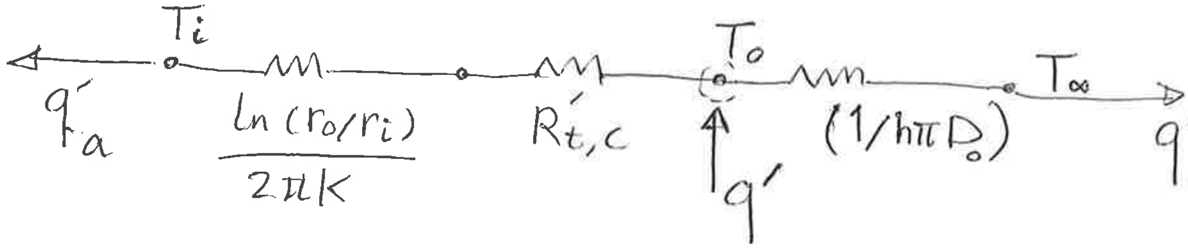
Find: Heater power per unit length required to maintain a heater temperature of 25°c

Schematic:



Assumptions: (1) Steady-State conditions, (2) One-dimensional conduction, (3) Constant properties (4) Negligible temperature drop across heater

Analysis: The thermal circuit has the form



(#1)

Applying an energy balance to a control surface about the heater.

$$q' = q'_a + q'_b$$

$$q' = \frac{T_o - T_i}{\frac{\ln(r_o/r_i)}{2\pi K} + R'_{t,c}} + \frac{T_o - T_\infty}{(1/h\pi D_o)}$$

$$q' = \frac{(25-5)^\circ C}{\frac{\ln(75\,mm/25\,mm)}{2\pi \times 16\,W/m\cdot K} + 0.01\,\frac{m\cdot K}{W}} + \frac{[25-(-10)]^\circ C}{[1/(100\,\frac{W}{m^2\cdot K} \times \pi \times 0.15\,m)]}$$

$$q' = (728 + 1649)\,W/m = 2377\,W/m$$

Comments: The conduction, contact and convection resistances are 0.0175, 0.01 and 0.021 m.K/W respectively.

(#2)

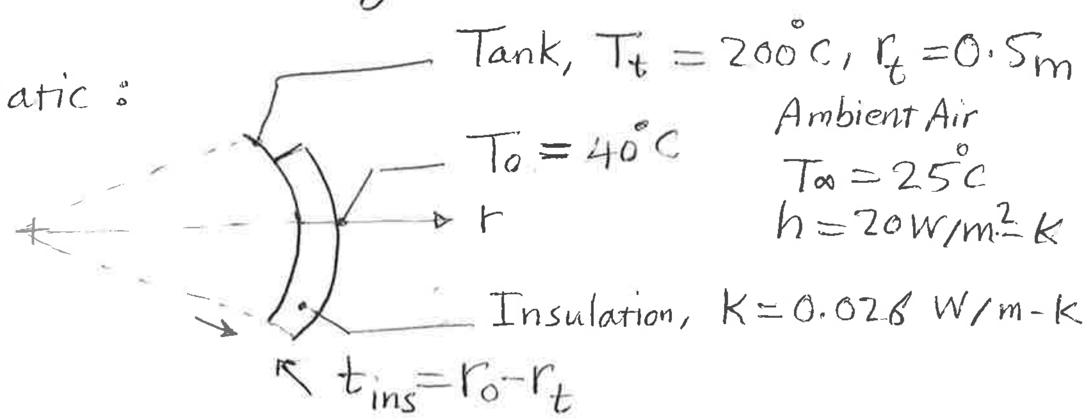
Problem 3. 61

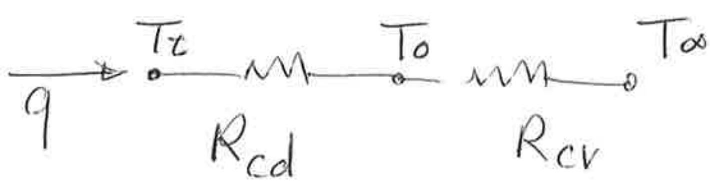
Solution:

Known: Spherical tank of 1-m diameter containing an exothermic reaction and is at 200°C when the ambient is at 25°C. Convection coefficient on outer surface is 20 $W/m^2 \cdot K$.

Find: Determine the thickness of urethane foam required to reduce the exterior temperature to 40 °C.
Determine the percentage reduction in the heat rate achieved using the insulation.

Schematic:





Assumptions: (1) Steady-State Conditions, (2) One-dimensional, radial (spherical) conduction and (3) Negligible radiation exchange between the insulation outer surface and the ambient surroundings.

Properties: Table A-3, urethane, rigid foam (300K):
$K = 0.026$ W/m.k.

(#3)

Analysis: (a) The heat transfer situation for the heat rate from the tank can be represented by the thermal circuit shown above. The heat rate from the tank is

$$q = \frac{T_t - T_\infty}{R_{cd} + R_{cv}}$$

where the thermal resistances associated with conduction within the insulation (Eq. 3.36) and convection for the exterior surface, respectively are

$$R_{cd} = \frac{(1/r_t - 1/r_o)}{4\pi k} = \frac{(1/0.5 - 1/r_o)}{4\pi \times 0.026\ W/m\cdot k}$$

$$= \frac{(1/0.5 - 1/r_o)}{0.3267}\ K/W$$

$$R_{cv} = \frac{1}{hA_s} = \frac{1}{4\pi h r_o^2} = \frac{1}{4\pi \times 20\ W/m^2\cdot k \times r_o^2}$$

$$= 3.979 \times 10^{-3}\ r_o^{-2}\ K/W$$

To determine the required insulation thickness so that $T_o = 40^\circ c$, perform an energy balance on the o-node.

$$\frac{T_t - T_o}{R_{cd}} + \frac{T_\infty - T_o}{R_{cv}} = 0$$

$$\frac{(200-40)K}{(1/0.5 - 1/r_o)/0.3267\ K/W} + \frac{(25-40)K}{3.979 \times 10^{-3}\ r_o^{-2}\ K/W} = 0$$

(#4)

$r_o = 0.5135 \quad t = r_o - r_i = (0.5135 - 0.5) = 13.5 mm$

From the rate equation, for the bare and insulated surfaces, respectively.

$$q_o = \frac{T_t - T_\infty}{1/4\pi h r_t^2} = \frac{(200-25)K}{0.01592\,K/W} = 10.99\,KW$$

$$q_{ins} = \frac{T_t - T_\infty}{R_{cd} + R_{cv}} = \frac{(200-25)}{(0.161 + 0.01592)K/W} = 0.994\,kW$$

Hence, the percentage reduction in heat loss achieved with the insulation is,

$$\frac{q_{ins} - q_o}{q_o} \times 100 = - \frac{0.994 - 10.99}{10.99} = 91\%$$

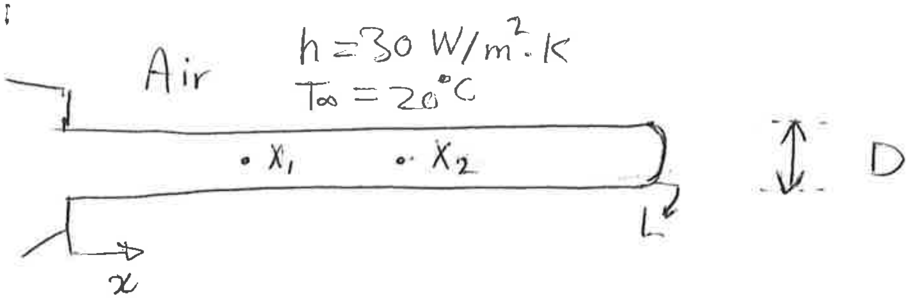
Problem 3.120

Solution:

Known: Length, diameter, base temperature and environmental conditions associated with a brass rod.

Find: Temperature at specified distances along the rod.

Schematic:



$L = 0.1\,m$

$D = 0.005\,m$

$X_1 = 0.025\,m$

$X_2 = 0.05\,m$

Assumptions: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant Properties, (4) Negligible radiation, (5) Uniform convection coefficient h.

Properties: Table A-1 Brass ($\bar{T} = 110°C$): $K = 133\ W/m.k$.

Analaysis: Evaluate the fin parameter

$$m = \left[\frac{hP}{KA_c}\right]^{1/2} = \left[\frac{h\pi D}{K\pi D^2/4}\right]^{1/2} = \left[\frac{4h}{KD}\right]^{1/2}$$

$$= \left[\frac{4 \times 30\ W/m^2.K}{133\ W/m.k \times 0.005\,m}\right]^{1/2} = 13.43\ m^{-1}$$

Hence $mL = 13.43 \times 0.1 = 1.34$ and from the results of Example 3.9. it is advisable not to make the infinite rod approximation.

Thus from Table 3.4, the temperature distribution has the form

$$\theta = \frac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}\,\theta_b$$

Evaluating the hyperbolic functions, $\cosh mL = 2.04$ and $\sinh mL = 1.78$ and the paramete

$$\frac{h}{mk} = \frac{30\ W/m^2\cdot K}{13.43\ (133\ W/m.K)} = 0.0168$$

with $\theta_b = 180°C$ the temperature distribution has the form

$$\theta = \frac{\cosh m(L-x) + 0.0168\sinh m(L-x)}{2.07}\,(180°C)$$

The temperatures at the prescribed locations are tabulated below.

X (m)	Cosh m(L−x)	Sinh m(L−x)	θ	T (°C)
$X_1 = 0.025$	1.55	1.19	136.5	156.5
$X_2 = 0.05$	1.24	0.725	108.9	128.9
L = 0.10	1.00	0.00	87.0	107.0

Comments: If the rod were approximated as infinitely long:

$T(x_1) = 148.7°C$, $T(x_2) = 112.0°C$ and $T(L) = 67.0°C$

The assumption would therefore result in significant underestimates of the rod temperature.

(#7)

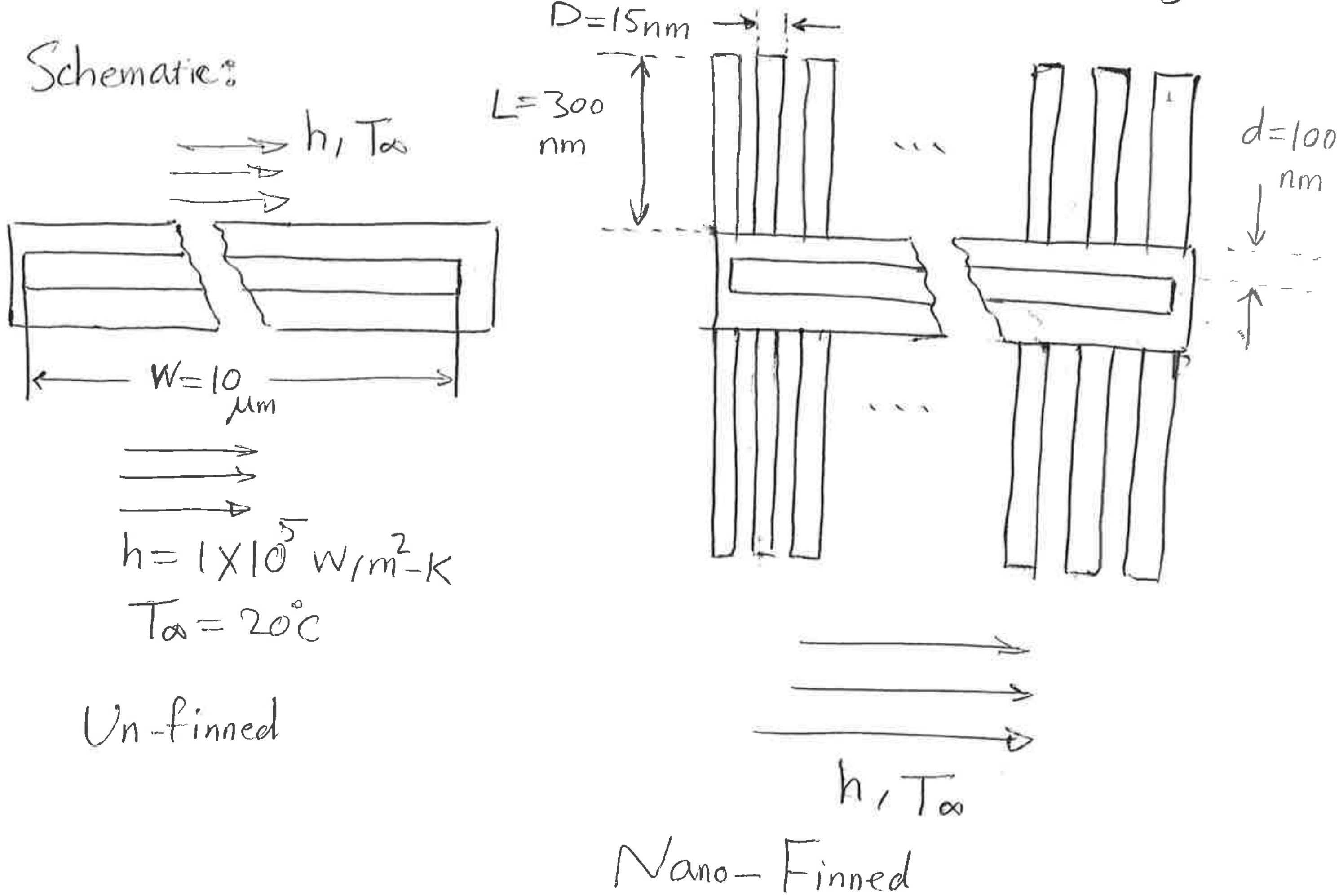
Problem 3.133

Solution:

Known: Dimensions of electronics package and finned nano-heat sink. Temperature and heat transfer coefficient of coolant.

Find: Maximum heat rate to maintain temperature below 85°C for finned and un-finned packages.

Schematic:



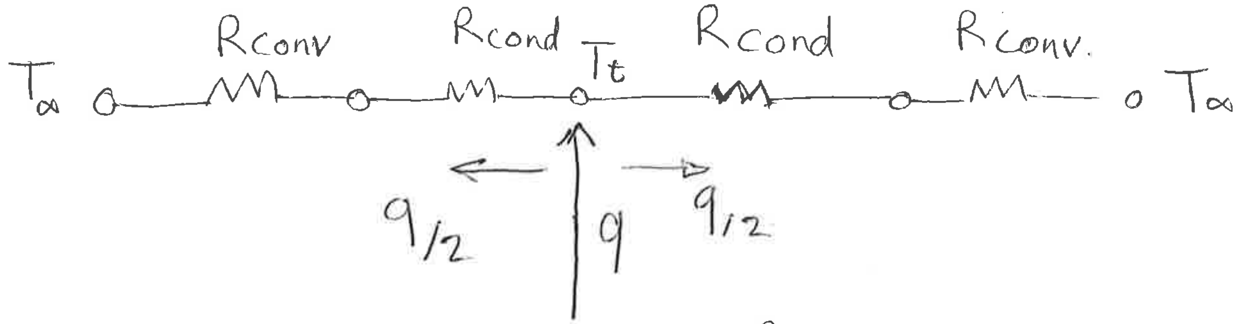
Assumptions: (1) Steady-state (2) Negligible temperature variation across fin thickness. (3) Constant properties, (4) Uniform heat transfer coefficient (5) Negligible contact resistance

(#8)

(6) Negligible heat loss from edges of package

Properties: Table A.2, Silicon carbide ($T \approx 300K$): $K = 490 \frac{W}{m.K}$

Analysis: (a) The thermal circuit for the un-finned package is



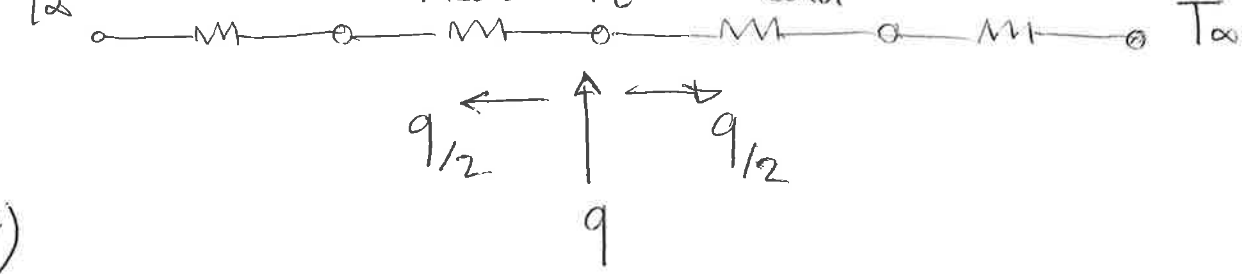
where $R_{Cond} = \frac{d}{KA} = \frac{100 \times 10^{-9} m}{490 \frac{W}{m.K} \times (10 \times 10^{-6} m)^2} = 2.04 \ K/W$

$$R_{Conv.} = \frac{1}{hA} = \frac{1}{10^5 \frac{W}{m^2.K} \times (10 \times 10^{-6} m)^2} = 1 \times 10^5 \ K/W$$

$$\text{Thus } q = 2 \frac{(T_t - T_\infty)}{R_{Conv} + R_{Cond}} = \frac{2(85-20)^\circ C}{(2.04 + 10^5) \ K/W}$$

$$= 1.30 \times 10^{-3} W$$

for the finned nano-heat sink, the convection resistance is replaced by a fin array thermal resistance:



From Equations 3.103, 3.102 and 3.99.

$$R_{t,o} = \frac{1}{\eta_o h A_t}, \quad \eta_o = 1 - \frac{NA_f}{A_t}(1-\eta_f), \quad A_t = NA_f + A_b$$

where $A_f = \pi D L_c = \pi D (L + D/4) = \pi \times 15 \times 10^{-9} m \times (300 + \frac{15}{4}) \times 10^{-9} m$

$$= 1.43 \times 10^{-14} m^2,$$

$$A_b = W^2 - N\pi D^2/4 = (10 \times 10^{-6} m)^2 - 40{,}000 \times \pi \times (15 \times 10^{-9} m)^2/4$$

$$= 9.29 \times 10^{-11} m^2, \text{ and}$$

$$A_t = 40{,}000 \times 1.43 \times 10^{-14} m^2 + 9.29 \times 10^{-11} m^2 = 6.65 \times 10^{-10} m^2$$

Then with

$$mL_c = (4h/kD)^{1/2} L_c = \left(4 \times 10^5 \frac{W}{m^2 - K} / 490 \frac{W}{m \cdot K} \times 15 \times 10^{-9} m\right)^{\frac{1}{2}} \times 304 \times 10^{-9} m = 7.09 \times 10^{-2},$$

$$\eta_f = \frac{\tanh(7.09 \times 10^{-2})}{7.09 \times 10^{-2}} = 0.998$$

It follows that

$$\eta_o = 1 - \frac{40{,}000 \times 1.43 \times 10^{-14} m^2}{6.65 \times 10^{-10} m^2}(1 - 0.998) = 0.999$$

and

$$R_{t,o} = \frac{1}{0.999 \times 10^5 \frac{W}{m^2 - K} \times 6.65 \times 10^{-10} m^2} = 1.50 \times 10^4 \frac{K}{W}$$

Therefore

$$q = 2\frac{(T-T_\infty)}{R_{cond}+R_{t,o}} = 2\frac{(85^\circ C - 20^\circ C)}{2.04\,K/W + 1.50\times10^{4}\,K/W}$$

$$= 8.64\times10^{-3}\,W$$

Comments: (1) The conduction resistance of the silicon carbide sheets is negligible (2) The fins increas the allowable heat rate significantly. (3) We have neglected the contact resistance between the electronics and the silicon carbide sheets. If it dominates, the fins will not be effective in increasing the allowable heat rate. Little is known about contact resistance at nanoscale.