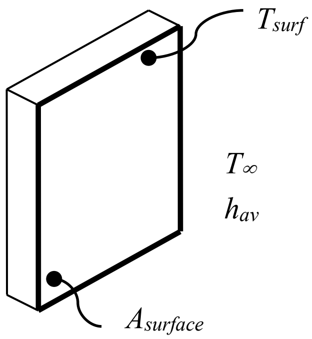
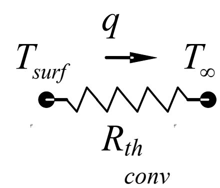
MECH-375: CRIB SHEETS FOR FINAL EXAM

- **Convection** heat loss from an isothermal surface:

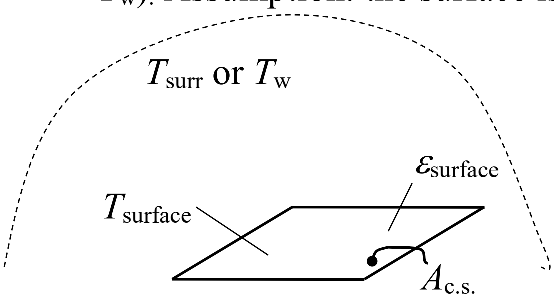


$$q_{conv} = A_{\text{surf}} h_{av}\left(T_{surf} - T_\infty\right) = \frac{\left(T_{surf} - T_\infty\right)}{R_{th,conv}} \quad \text{[W]}$$

$$R_{th,conv} = \frac{1}{A_{\text{surf}} h_{av}} \quad [°\text{C / W}]$$



- Net **Radiation** heat loss from a flat or convex isothermal surface to surroundings at T_{surr} (or a large isothermal wall enclosure, with the walls at T_w). Assumption: the surface is Gray.



$$\underset{Surf \to Surr}{q_{net}} = A_{\text{surf}} \varepsilon_{surf} \sigma \left(\underset{ABS}{T^4_{surf}} - \underset{ABS}{T^4_{surr}} \right) \quad \text{[W]}$$

ε_{surf} : is the surface emissivity

$\sigma = 5.669 \times 10^{-8}$ [W/m^2-K^4]

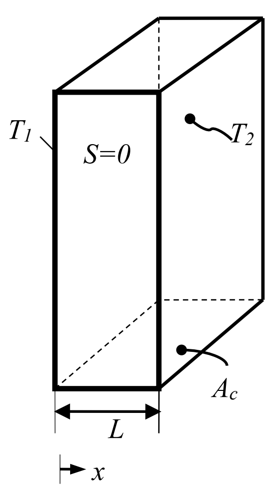
is the Stefan-Boltzmann Constant

Steady state one-dimensional heat conduction

- **Plane Wall**

Gov. Eq.: $\dfrac{d}{dx}\left(k\dfrac{dT}{dx}\right) + S = 0$

E.g., <u>S.S., 1-D, $S = 0$, $k =$ Constant,</u>



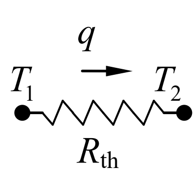
B.Cs.: (i) at $x = 0$ $T = T_1$

(ii) at $x = L$ $T = T_2$

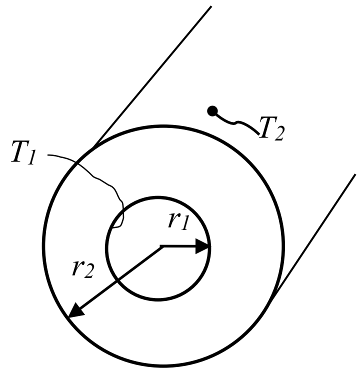
Solution: $\dfrac{T - T_2}{T_1 - T_2} = 1 - \dfrac{x}{L}$

$$q_x = -kA_{c.s} dT/dx = kA_{c.s}\left(T_1 - T_2\right)/L = \left(T_1 - T_2\right)/\underset{\substack{plane\\wall}}{R_{th}} \quad \text{[W]}$$

$$\underset{\substack{plane\\wall}}{R_{th}} = L/\left(kA_{c.s}\right) \quad [°\text{C / W}]$$



- **Long Hollow Cylinder** $[L >> (r_2 - r_1)]$ Gov. Eq.: $\dfrac{1}{r}\dfrac{d}{dr}\left(rk\dfrac{dT}{dr}\right) + S = 0$



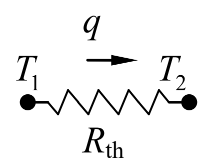
E.g., <u>S.S., 1-D, $S = 0$, k = Constant,</u>

B.Cs.: (i) at $r = r_1$ $\quad T = T_1$ $\qquad$ Solution: $\dfrac{T - T_2}{T_1 - T_2} = \dfrac{\ln(r/r_2)}{\ln(r_1/r_2)}$

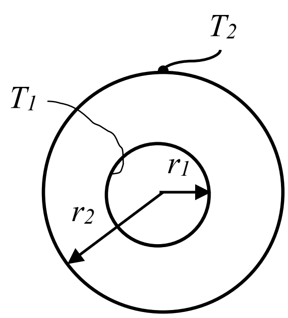
(ii) at $r = r_2$ $\quad T = T_2$

$$q_x = -kA_{\underset{to\ r}{\perp}}\ dT/dr = 2\pi kL(T_1 - T_2)/\ln(r_2/r_1) = (T_1 - T_2)/R_{\underset{\text{long hollow cyl.}}{th}} \quad [\text{W}]$$

$$R_{\underset{\text{long hollow cyl.}}{th}} = \ln(r_2/r_1)/(2\pi kL) \quad [°\text{C}/\text{W}]$$



- **Hollow Sphere** Gov. Eq.: $\dfrac{1}{r^2}\dfrac{d}{dr}\left(r^2 k\dfrac{dT}{dr}\right) + S = 0$



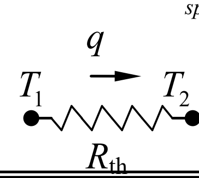
S.S., 1-D, $S = 0$, k = Constant,

B.Cs.: (i) at $r = r_1$ $\quad T = T_1$ $\qquad$ Solution: $\dfrac{T - T_2}{T_1 - T_2} = \dfrac{1/r - 1/r_2}{1/r_1 - 1/r_2}$

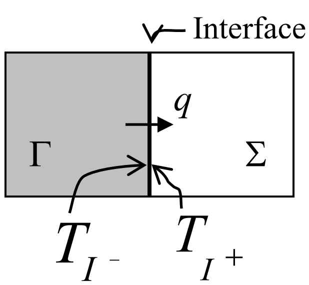
(ii) at $r = r_2$ $\quad T = T_2$

$$q_x = -kA_{\underset{to\ r}{\perp}}\ dT/dr = 4\pi k(T_1 - T_2)/[1/r_1 - 1/r_2] \triangleq (T_1 - T_2)/R_{\underset{\text{hollow sphere}}{th}} \quad [\text{W}]$$

$$R_{\underset{\text{hollow sphere}}{th}} = [1/r_1 - 1/r_2]/(4\pi k) \quad [°\text{C}/\text{W}]$$



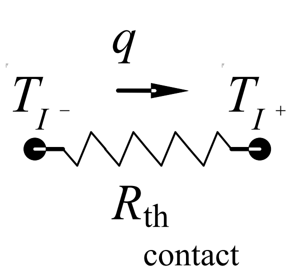
Thermal contact resistance between two solids (solid Γ and solid Σ)



E.g., <u>S.S., 1-D, $S = 0$</u>

$$q = \left(T_{I^-} - T_{I^+}\right)/R_{\underset{\Gamma\leftrightarrow\Sigma}{\underset{contact}{th}}} = h_{\underset{\Gamma\leftrightarrow\Sigma}{contact}} A_{\text{interface}}\left(T_{I^-} - T_{I^+}\right)$$

$$R_{\underset{\Gamma\leftrightarrow\Sigma}{\underset{contact}{th}}} = 1/\left(h_{\underset{\Gamma\leftrightarrow\Sigma}{contact}} A_{\text{interface}}\right) \quad [°\text{C/W}];$$



$h_{\underset{\Gamma\leftrightarrow\Sigma}{contact}}$: thermal contact coefficient $[\text{W/m}^2 - °\text{C}]$

Critical Radius of Insulation/Coating

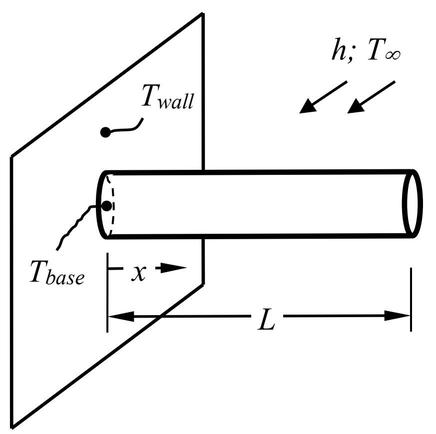
Conduction-convection system; r_1; k_{insul}; T_1; T_∞; h are constant

- Long cylinder (S =0 in insulation)
  When $r_2 = r_{\text{crit}} = k_{\text{insul}}/h \Rightarrow q_{\text{loss}} = q_{\max}$
- Spherical Geometry (S =0 in insulation)
  When $r_2 = r_{\text{crit}} = 2\,k_{\text{insul}}/h \Rightarrow q_{\text{loss}} = q_{\max}$

Note: if r_1; k_{insul}; T_∞; h; q all constant:
When $r_2 = r_{\text{crit}} \Rightarrow T_1 = T_{1,\min}$

Fin Theory

- Classical Fin Theory k_{fin}= constant, h = constant; T_∞ = constant
- S.S., S=0, Quasi 1-D, with $P_{c.s.}$ = const., and $A_{c.s.}$ = const.



$$\boxed{\frac{d^2T}{dx^2} - m^2\left(T - T_\infty\right) = 0;\ \text{where}\ m^2 = \left(hP_{c.s.}\right)/\left(k_{fin}A_{c.s.}\right)}$$

- Fin Base condition [B.C. (i)] :
  At $x = 0, \quad T = T_{x=0} = T_{Base}$
- If thermal contact resistance at the fin base ($x = 0$) is negligible:
  $T_{x=0} = T_{Base} = T_{wall}$

CASE A: (convection from the tip surface)

B.C. (ii): at $x = L, \quad -k_{fin}\left.\frac{dT}{dx}\right|_{x=L} = h\left(T_{x=L} - T_\infty\right),$

$$\frac{\theta}{\theta_{Base}} = \frac{T - T_\infty}{T_{Base} - T_\infty} = \frac{\cosh[m(L-x)] + (h/mk_s)\sinh[m(L-x)]}{\cosh[mL] + (h/mk_s)\sinh[mL]};$$

$$\underset{\substack{Fin\to Fluid\\(lateral+tip)}}{q_{total\ loss}} = -k_{fin}A_{c.s.}\left.\frac{dT}{dx}\right|_{x=0}$$

$$\underset{\substack{Fin\to Fluid\\(lateral+tip)}}{q_{total\ loss}} = \sqrt{k_{fin}A_{c.s.}hP_{c.s.}}\left(T_{Base} - T_\infty\right)\left[\frac{\sinh[mL] + (h/mk_{fin})\cosh[mL]}{\cosh[mL] + (h/mk_{fin})\sinh[mL]}\right]$$

CASE B: (insulated tip) B.C. (ii): at $x = L$, $\left.\dfrac{dT}{dx}\right|_{x=L} = 0$,

$$\frac{\theta}{\theta_{Base}} = \frac{T - T_\infty}{T_{Base} - T_\infty} = \frac{\cosh[m(L-x)]}{\cosh[mL]}$$

$$q_{\substack{total\ loss \\ Fin\to Fluid}} = -k_{fin} A_{c.s.} \left.\frac{dT}{dx}\right|_{x=0} = \sqrt{k_{fin} A_{c.s.} h P_{c.s.}} \left(T_{Base} - T_\infty\right) \tanh\left[mL\right]$$

CASE C: (prescribed temperature) B.C. (ii): at $x = L$, $T_{x=L} = T_L$,

$$\frac{\theta}{\theta_{Base}} = \frac{T - T_\infty}{T_{Base} - T_\infty} = \frac{(\theta_L / \theta_{Base})\sinh[mx] + \sinh[m(L-x)]}{\sinh[mL]}$$

$$q_{\substack{total\ loss \\ from\ Base}} = -k_{fin} A_{c.s.} \left.\frac{dT}{dx}\right|_{x=0} = \sqrt{k_{fin} A_{c.s.} h P_{c.s.}} \left(T_{Base} - T_\infty\right) \left[\frac{\cosh[mL] - \dfrac{\left(T_L - T_\infty\right)}{\left(T_{Base} - T_\infty\right)}}{\sinh[mL]} \right]$$

$$q_{\substack{total\ loss \\ to\ fluid}} = q_{\substack{total\ loss \\ from\ Base}} - q_{\substack{total\ loss \\ from\ the\ Tip}} = \left[-k_{fin} A_{c.s.} \left.\frac{dT}{dx}\right|_{x=0} \right] - \left[-k_{fin} A_{c.s.} \left.\frac{dT}{dx}\right|_{x=L} \right]$$

CASE D: Long fin ($L \to \infty$) B.C. (ii): at $x = L$, $T_{x=L}\left(= T_{Tip}\right) = T_\infty$

$$\frac{\theta}{\theta_{Base}} = \frac{T - T_\infty}{T_{Base} - T_\infty} = e^{-mx};$$

$$q_{\substack{total\ loss \\ Fin\to Fluid}} = -k_{fin} A_{c.s.} \left.\frac{dT}{dx}\right|_{x=0} = \sqrt{k_{fin} A_{c.s.} h P_{c.s.}} \left(T_{Base} - T_\infty\right)$$

Fin efficiency

$$\eta_{Fin} = \frac{\text{[Actual rate of heat loss from the fin]}}{\text{[Rate of heat loss from fin to fluid , if the entire fin were at } T_{\text{Base}}\text{]}_{\substack{\text{Same} \\ \text{tip condition}}}}$$

e.g., Case B: $\eta_{\substack{Fin \\ case-B}} = \tanh(mL) / (mL)$

Fin Thermal Resistance
(Useful for Case A, B, and D)

$$R_{\substack{th\\Fin}} = 1 \Big/ \left(\eta_{Fin} A_{\substack{surface\\total\\heat\ transfer\\for\ Fin}} \; h \right)$$

$$T_{Base} \xrightarrow{q} T_\infty \qquad R_{\substack{th\\Fin}}$$

Fin Effectiveness:

$$\varepsilon_{Fin} = \frac{q_{\substack{actual\\Fin \to Fluid}}}{\text{Area}_{\text{Base}} h \left[T_{wall} - T_\infty \right]}$$

Corrected length: L_c may be used in CASE B solutions to approximate the corresponding CASE A solution:

$$L_c = L + \Delta L \cong L + \frac{A_{c.s.tip}}{P_{c.s.tip}}$$

Fin Design Charts (e.g., see page 6)**:**

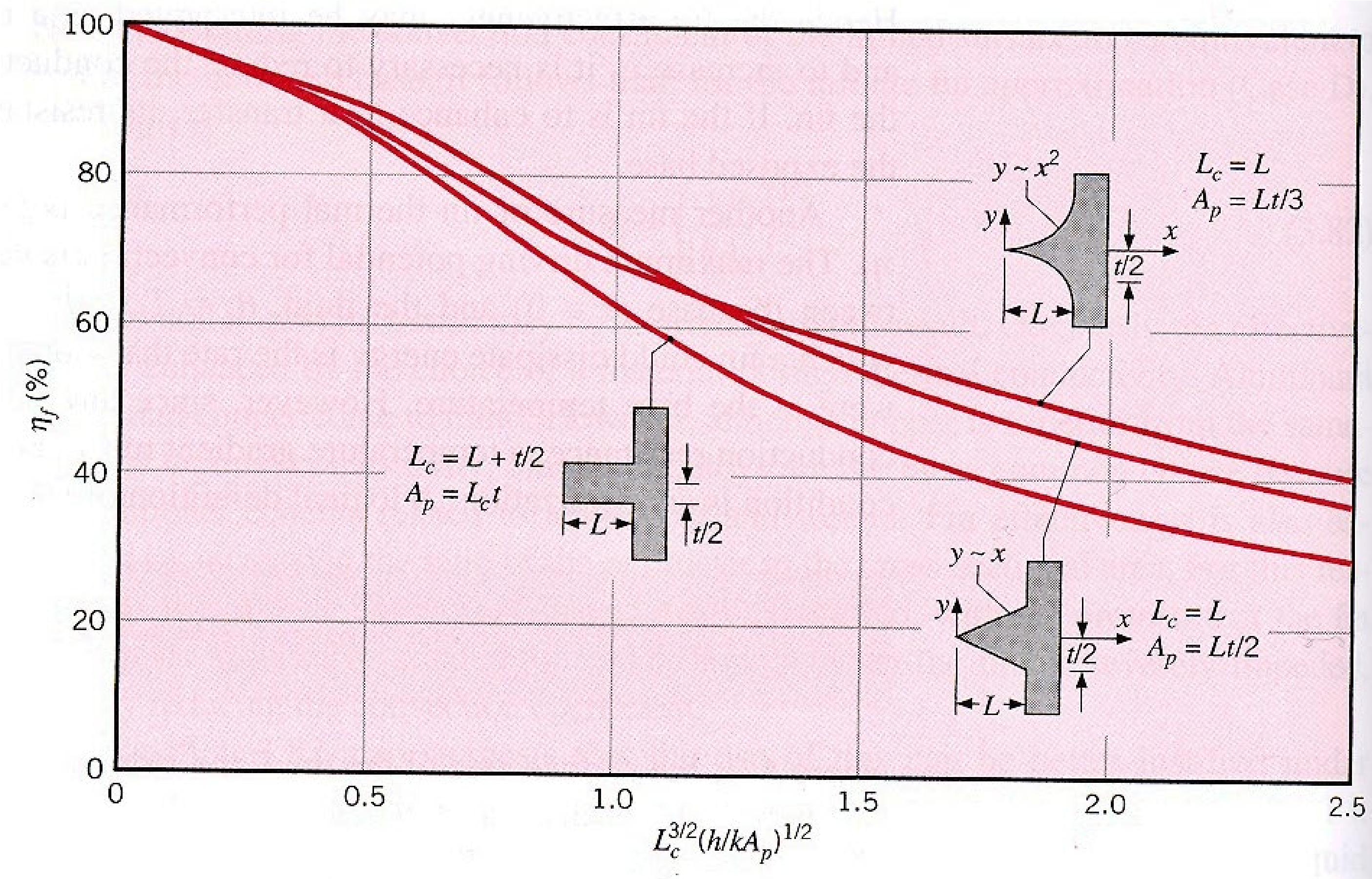
- Based on **CASE B** (insulated tip) solutions
- Use L_c (instead of L) only if tip loses heat by convection
- $\eta_{Fin} = f\left[L_c^{3/2} \left(h / \left[k_{fin} A_m \right] \right)^{1/2}, \text{Geom. Parameters} \right]$
- A_m: Profile Area; $R_{\substack{th\\Fin}} = 1 \Big/ \left(\eta_{Fin} A_{\substack{surface\\total\ lateral\ Fin\\(with\ L_c\ if\ needed)}} \; h \right)$
- $q_{\substack{actual\\total\\fin \to fluid}} = \eta_{Fin} A_{\substack{surface\\total\ lateral\ Fin\\(with\ L_c\ if\ needed)}} h \left(T_{Base} - T_\infty \right) = \left(T_{Base} - T_\infty \right) / R_{\substack{th\\Fin}}$

SHAPE FACTORS (e.g., see pages 7 & 8)
Steady state multi-dimensional heat conduction

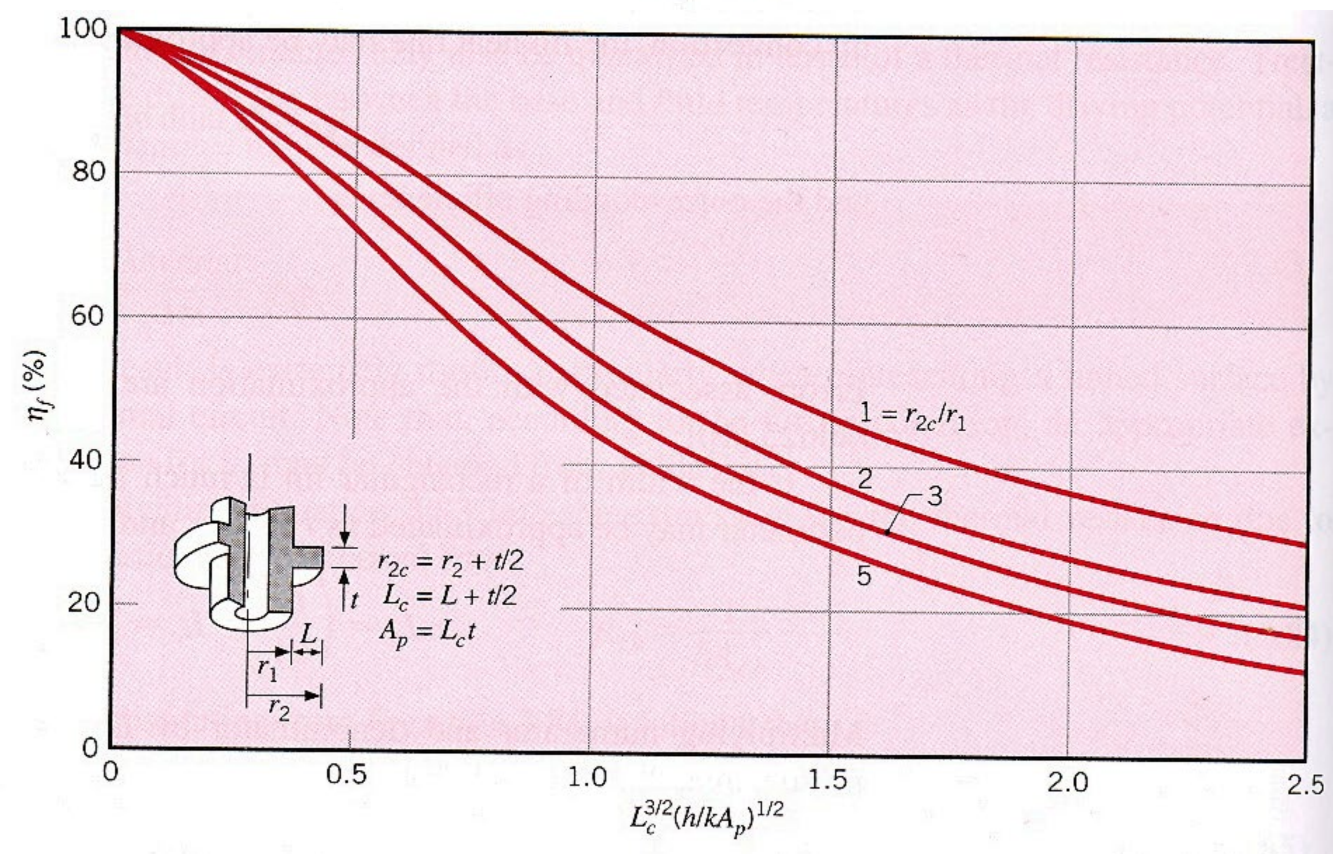
In problems that allow the following approximations/restrictions: Isotropic, homogeneous material; k = constant; S =0 W/m^3; and only two different uniform boundary temperatures:

$$q_{\substack{total\\T_1 \to T_2}} = S\,k\,(T_1 - T_2) = \frac{(T_1 - T_2)}{R_{\substack{th\\cond.}}} ; \Rightarrow R_{\substack{th\\cond.}} = \frac{1}{S\,k} \quad [°\text{C/W}]$$

And S: heat conduction shape factor [m]
For some prescribed geometries these shape factors are tabulated in your text book.



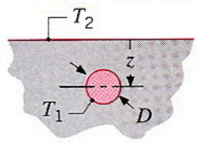
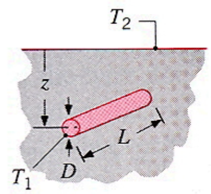
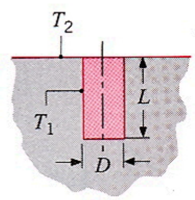
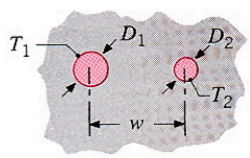
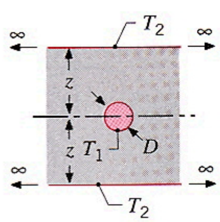
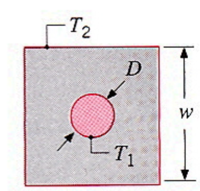
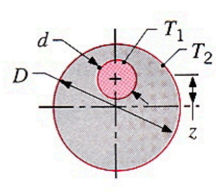
100
80
60
40
20
0
η_f (%)
0
0.5
1.0
1.5
2.0
2.5
$L_c^{3/2}(h/kA_p)^{1/2}$
$y \sim x^2$
$L_c = L$
$A_p = Lt/3$
$t/2$
L
$L_c = L + t/2$
$A_p = L_c t$
$y \sim x$
$L_c = L$
$A_p = Lt/2$



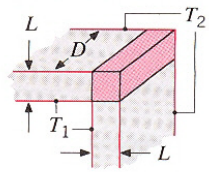
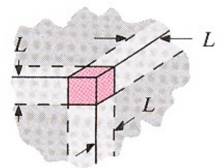
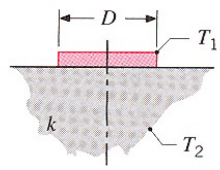
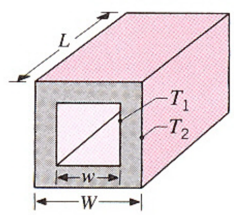
100
80
60
40
20
0
η_f (%)
0
0.5
1.0
1.5
2.0
2.5
$L_c^{3/2}(h/kA_p)^{1/2}$
$1 = r_{2c}/r_1$
2
3
5
$r_{2c} = r_2 + t/2$
$L_c = L + t/2$
$A_p = L_c t$
t
L
r_1
r_2

TABLE 4.1 Conduction shape factors and dimensionless conduction heat rates for selected systems.

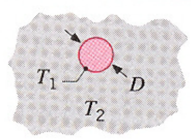
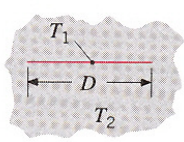
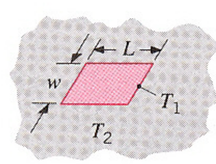
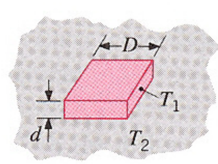
(a) Shape factors $[q = Sk(T_1 - T_2)]$

System	Schematic	Restrictions	Shape Factor
Case 1 Isothermal sphere buried in a semi-infinite medium		$z > D/2$	$\dfrac{2\pi D}{1 - D/4z}$
Case 2 Horizontal isothermal cylinder of length L buried in a semi-infinite medium		$L \gg D$ $L \gg D$ $z > 3D/2$	$\dfrac{2\pi L}{\cosh^{-1}(2z/D)}$ $\dfrac{2\pi L}{\ln(4z/D)}$
Case 3 Vertical cylinder in a semi-infinite medium		$L \gg D$	$\dfrac{2\pi L}{\ln(4L/D)}$
Case 4 Conduction between two cylinders of length L in infinite medium		$L \gg D_1, D_2$ $L \gg w$	$\dfrac{2\pi L}{\cosh^{-1}\left(\dfrac{4w^2 - D_1^2 - D_2^2}{2D_1D_2}\right)}$
Case 5 Horizontal circular cylinder of length L midway between parallel planes of equal length and infinite width		$z \gg D/2$ $L \gg z$	$\dfrac{2\pi L}{\ln(8z/\pi D)}$
Case 6 Circular cylinder of length L centered in a square solid of equal length		$w > D$ $L \gg w$	$\dfrac{2\pi L}{\ln(1.08\, w/D)}$
Case 7 Eccentric circular cylinder of length L in a cylinder of equal length		$D > d$ $L \gg D$	$\dfrac{2\pi L}{\cosh^{-1}\left(\dfrac{D^2 + d^2 - 4z^2}{2Dd}\right)}$

(Table extracted from Incropera et al. 2007)

System	Schematic	Restrictions	Shape Factor
Case 8 Conduction through the edge of adjoining walls		$D > 5L$	$0.54D$
Case 9 Conduction through corner of three walls with a temperature difference ΔT_{1-2} across the walls		$L \ll$ length and width of wall	$0.15L$
Case 10 Disk of diameter D and temperature T_1 on a semi-infinite medium of thermal conductivity k and temperature T_2		None	$2D$
Case 11 Square channel of length L		$\frac{W}{w} < 1.4$	$\frac{2\pi L}{0.785 \ln (W/w)}$
		$\frac{W}{w} > 1.4$	$\frac{2\pi L}{0.930 \ln (W/w) - 0.050}$
		$L \gg W$	

(b) Dimensionless conduction heat rates [$q = q_{ss}^* kA_s(T_1 - T_2)/L_c$; $L_c \equiv (A_s/4\pi)^{1/2}$]

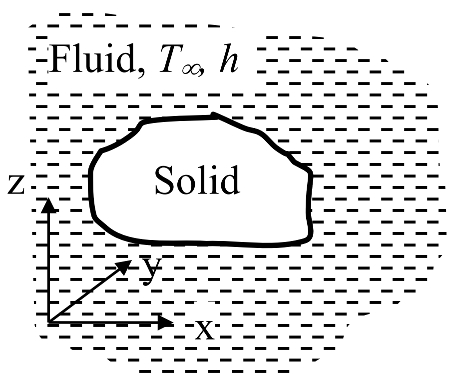
System	Schematic	Active Area, A_s	q_{ss}^*
Case 12 Isothermal sphere of diameter D and temperature T_1 in an infinite medium of temperature T_2		πD^2	1
Case 13 Infinitely thin, isothermal disk of diameter D and temperature T_1 in an infinite medium of temperature T_2		$\frac{\pi D^2}{2}$	$\frac{2\sqrt{2}}{\pi} = 0.900$
Case 14 Infinitely thin rectangle of length L, width w, and temperature T_1 in an infinite medium of temperature T_2		$2wL$	0.932
Case 15 Cuboid shape of height d with a square footprint of width D and temperature T_1 in an infinite medium of temperature T_2		$2D^2 + 4Dd$	d/D — q_{ss}^*: 0.1 — 0.943; 1.0 — 0.956; 2.0 — 0.961; 10 — 1.111

Unsteady Heat Conduction [Isotropic Materials]

- Governing Equation: $div(k\vec{\nabla}T)+S=\rho c_p \dfrac{\partial T}{\partial t}$
- B.C.s and I.C. needed to complete the mathematical model; these are problem specific.
- Biot Number (conduction/convection systems):

$$Bi=\frac{hL_c}{k_{solid}};L_c=\frac{\begin{bmatrix}\text{Volume}\\ \text{of the solid}\end{bmatrix}}{\begin{bmatrix}\text{Surface area exposed}\\ \text{to convection}\end{bmatrix}}$$

Lumped Parameter Analysis (Convection thermal boundary) [valid if $Bi=\dfrac{hL_c}{k_{solid}}\leq 0.1$]



Key Idea: $T(x,y,z,t)=T(t)\quad when\quad Bi=\dfrac{hL_c}{k_{solid}}\leq 0.1$

Gov. Eq.:

$$\underbrace{\int_V SdV}_{q_{\substack{generation\\ total}}}-A_{\substack{surf\\ conv}}h(T-T_\infty)=\rho Vc_p\frac{dT}{dt}$$

V: is the volume of the solid; h: is average heat transfer coeff. on surface exposed to convection.

LPA Solution for ρ = constant, S = 0, and (h, T_∞, c_p) all constant

Governing Equation: $-A_{\substack{surf\\ conv}}h(T-T_\infty)=\rho Vc_p\dfrac{dT}{dt}$; I.C. : at t = 0 $\quad T = T_{ini}$ = constant

$$\frac{\theta}{\theta_{ini}}=\frac{T-T_\infty}{T_{ini}-T_\infty}=\exp\left[-\frac{A_{surf}h}{\rho Vc_p}t\right]$$

V: is the volume of the solid;
A_{surf}: is area exposed to convection.

$\tau=\dfrac{\rho Vc_p}{A_{surf}h}$ is the time constant of the conduction/convection system.

$$\begin{bmatrix}\text{Total heat lost}\\ \text{in time } (t_2 - t_1)\end{bmatrix} = \underset{t_1 \le t \le t_2}{Q_{total}} = \rho V c_p \left[T_{t=t_1} - T_{t=t_2} \right] \quad [\text{J}]$$

$$\underset{t_1 \le t \le t_2}{Q_{total}} = \rho V c_p \left(T_i - T_\infty \right) \left[\exp\left(-t_1/\tau\right) - \exp\left(-t_2/\tau\right) \right]$$

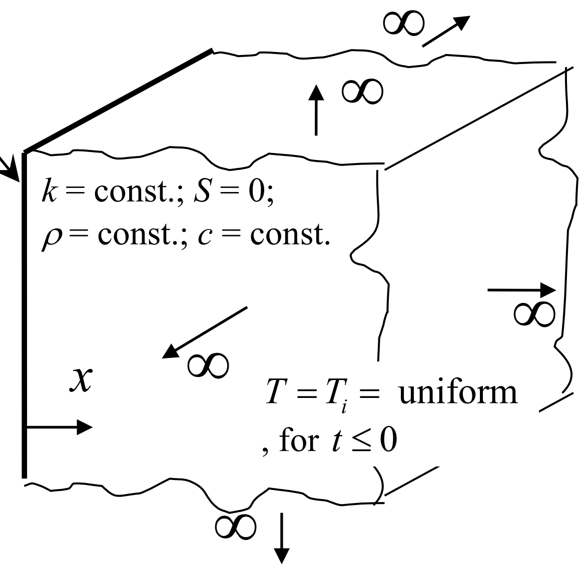
Transient heat conduction in semi-infinite solids

Common B.C.s (imposed) on the surface: for $t > 0$

a) Constant temperature, $T_o \neq T_i$

b) Constant heat flux, q''_0

c) Convection boundary



Mathematical model:
Governing Equation:

$$\frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x} \right) = \rho c \frac{\partial T}{\partial t}$$

or $$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

$$\alpha = \frac{k}{\rho c_p} \qquad [\text{m}^2/\text{s}]$$

I.C.: $at \quad t = 0, \quad T = T_i = const. \quad 0 \le x \le \infty$

a) $$\left\{ \begin{array}{l} at \quad x = 0, \quad T = T_o = const. \\ at \quad x \to \infty, \quad T = T_i = const. \end{array} \right\} \quad for\ all \quad t > 0$$

b) $$\left\{ \begin{array}{l} at \quad x = 0, \quad q''_o = -k \left.\dfrac{\partial T}{\partial x}\right|_{x=0} = const. \\ at \quad x \to \infty, \quad T = T_i = const. \end{array} \right\} \quad for\ all \quad t > 0$$

c) $$\left\{ \begin{array}{l} at \quad x = 0, \quad h(T_\infty - T_{x=0}) = -k \left.\dfrac{\partial T}{\partial x}\right|_{x=0} \\ at \quad x \to \infty, \quad T = T_i = const. \end{array} \right\} \quad for\ all \quad t > 0$$

Solutions:

a) $$\left\{ \frac{T(x,t) - T_o}{T_i - T_o} = \text{erf}\left(\frac{x}{2\sqrt{\alpha t}} \right) \right.$$

Penetration depth: $\delta(t)$

When $x = \delta(t)$,

$$\frac{T(\delta,t) - T_0}{T_i - T_o} = 0.99 = erf\left(\frac{\delta}{2\sqrt{\alpha t}} \right)$$

b) $\left\{ \quad T(x,t)-T_i=\dfrac{2q_o''\sqrt{\alpha t/\pi}}{k}\exp\left(\dfrac{-x^2}{4\alpha t}\right)-\dfrac{q_o''x}{k}\operatorname{erfc}\left(\dfrac{x}{2\sqrt{\alpha t}}\right)\right.$

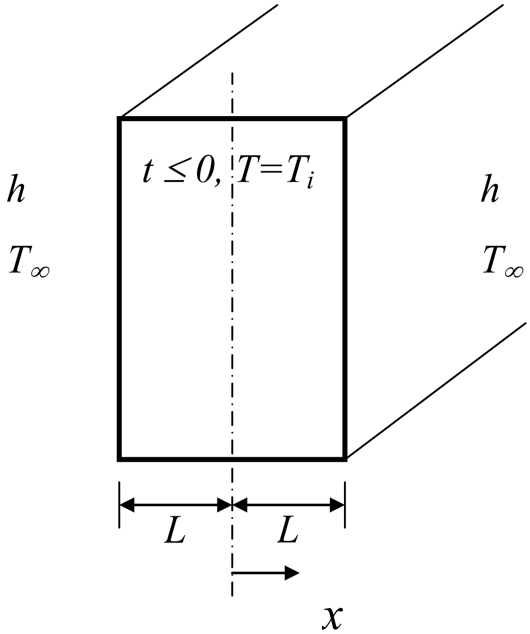
c) $\left\{ \quad \dfrac{T(x,t)-T_i}{T_\infty-T_i}=\operatorname{erfc}\left(\dfrac{x}{2\sqrt{\alpha t}}\right)-exp\left[\dfrac{hx}{k}+\dfrac{h^2\alpha t}{k^2}\right]\operatorname{erfc}\left[\dfrac{x}{2\sqrt{\alpha t}}+\dfrac{h\sqrt{\alpha t}}{k}\right]\right.$

Notes: 1) Error function: $\operatorname{erf}(\eta)=\dfrac{2}{\sqrt{\pi}}\int_0^{\eta}e^{-\eta^2}d\eta$; Values of $\operatorname{erf}(\eta)$ are tabulated below and also in Table A-1, page 593 of Holman, 2002. 2) $\operatorname{erfc}(\eta)\triangleq 1-\operatorname{erf}(\eta)$

x	$erf(x)$	x	$erf(x)$	x	$erf(x)$
0.00	0.00000	0.76	0.71754	1.52	0.96841
0.02	0.02256	0.78	0.73001	1.54	0.97059
0.04	0.04511	0.80	0.74210	1.56	0.97263
0.06	0.06762	0.82	0.75381	1.58	0.97455
0.08	0.09008	0.84	0.76514	1.60	0.97635
0.10	0.11246	0.86	0.77610	1.62	0.97804
0.12	0.13476	0.88	0.78669	1.64	0.97962
0.14	0.15695	0.90	0.79691	1.66	0.98110
0.16	0.17901	0.92	0.80677	1.68	0.98249
0.18	0.20094	0.94	0.81627	1.70	0.98379
0.20	0.22270	0.96	0.82542	1.72	0.98500
0.22	0.24430	0.98	0.83423	1.74	0.98613
0.24	0.26570	1.00	0.84270	1.76	0.98719
0.26	0.28690	1.02	0.85084	1.78	0.98817
0.28	0.30788	1.04	0.85865	1.80	0.98909
0.30	0.32863	1.06	0.86614	1.82	0.98994
0.32	0.34913	1.08	0.87333	1.84	0.99074
0.34	0.36936	1.10	0.88020	1.86	0.99147
0.36	0.38933	1.12	0.88679	1.88	0.99216
0.38	0.40901	1.14	0.89308	1.90	0.99279
0.40	0.42839	1.16	0.89910	1.92	0.99338
0.42	0.44749	1.18	0.90484	1.94	0.99392
0.44	0.46622	1.20	0.91031	1.96	0.99443
0.46	0.48466	1.22	0.91553	1.98	0.99489
0.48	0.50275	1.24	0.92050	2.00	0.99532
0.50	0.52050	1.26	0.92524	2.10	0.997020
0.52	0.53790	1.28	0.92973	2.20	0.998137
0.54	0.55494	1.30	0.93401	2.30	0.998857
0.56	0.57162	1.32	0.93806	2.40	0.999311
0.58	0.58792	1.34	0.94191	2.50	0.999593
0.60	0.60386	1.36	0.94556	2.60	0.999764
0.62	0.61941	1.38	0.94902	2.70	0.999866
0.64	0.63459	1.40	0.95228	2.80	0.999925
0.66	0.64938	1.42	0.95538	2.90	0.999959
0.68	0.66378	1.44	0.95830	3.00	0.999978
0.70	0.67780	1.46	0.96105	3.20	0.999994
0.72	0.69143	1.48	0.96365	3.40	0.999998
0.74	0.70468	1.50	0.96610	3.60	1.000000

One-dimensional unsteady heat conduction in solids with convection B.C. [$S=0$; k, ρ, c_p, h, T_∞ all constant; and $Bi_{L_c} > 0.1$]

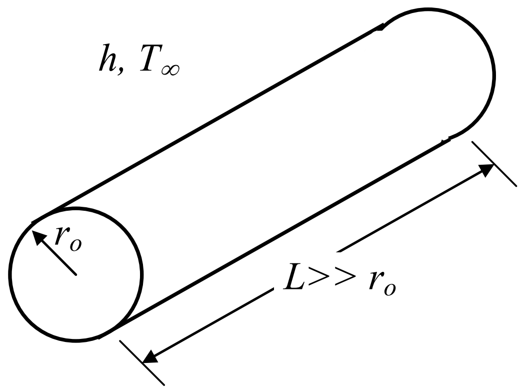
Case A:



Symmetrically cooled/heated plane wall

$$Bi_M = \frac{hL}{k_{solid}};\ x^* = \frac{x}{L};\ t^* = \frac{\alpha t}{L^2}$$

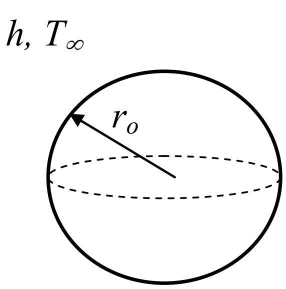
Case B:



Long solid cylinder

$$Bi_M = \frac{hr_o}{k_{solid}};\ r^* = \frac{r}{r_o};\ t^* = \frac{\alpha t}{r_o^2}$$

Case C:



Solid sphere

$$Bi_M = \frac{hr_o}{k_{solid}};\ r^* = \frac{r}{r_o};\ t^* = \frac{\alpha t}{r_o^2}$$

Notes:

- One-dimensional transient heat conduction in these three cases can be predicted analytically: solutions are in the form of infinite series;
- However, these series are rapidly convergent;
- For $t^* \geq 0.2$, one-term approximation of infinite series is excellent: [Error $\lesssim \pm 2\%$]

For $t^* \geq 0.2$, use the one-term approximation presented below

Case A: Symmetrically cooled/heated plane wall [Note: $\theta = (T - T_\infty)$]

$$\theta^*(x^*,t^*) = \frac{\theta}{\theta_i} = \frac{T(x,t)-T_\infty}{T_i - T_\infty} = C_B \exp[-A_B^2 t^*]\cos(A_B x^*)$$

Where $x^* = x/L$; $t^* = \dfrac{\alpha t}{L^2}$

Notes: C_B, A_B are functions of Bi_M and the geometry

$$\theta^*(0,t^*) = \frac{\theta_0}{\theta_i} = \frac{T(x=0,t)-T_\infty}{T_i - T_\infty} = C_B \exp[-A_B^2 t^*]$$

$$\frac{Q}{Q_o} = 1 - [\theta^*(0,t^*)]\left[\frac{\sin(A_B)}{A_B}\right]$$

$$Q = 2\int_0^t q_{\substack{loss\\x=L}}\, dt$$

$$Q_o = m_{\substack{total\\sys}} c_p \left[T_i - T_\infty\right]$$

Case B: Long solid cylinder

$$\theta^*(r^*,t^*) = \frac{\theta}{\theta_i} = \frac{T(r,t)-T_\infty}{T_i - T_\infty} = C_B \exp[-A_B^2 t^*]\mathrm{J}_0(A_B r^*)$$

Where $r^* = r/r_o$; $t^* = \dfrac{\alpha t}{r_o^2}$

$$\theta^*(r^*=0,t^*) = \frac{\theta_0}{\theta_i} = \frac{T(r=0,t)-T_\infty}{T_i - T_\infty} = C_B \exp[-A_B^2 t^*]$$

$$\frac{Q}{Q_o} = 1 - 2[\theta^*(r^*=0,t^*)]\left[\frac{\mathrm{J}_1(A_B)}{A_B}\right]$$

And $Q = \displaystyle\int_0^t q_{\substack{loss\\r=r_o}}\, dt$; and $Q_o = m_{\substack{total\\sys}} c_p \left[T_i - T_\infty\right]$

J_o : Bessel function of the first kind, order zero
J_1 : Bessel function of the first kind, order one
Aside: $\dfrac{d}{dx}\left(J_0(x)\right) = -J_1(x)$
The values of $\mathrm{J}_0(\xi)$ and $\mathrm{J}_1(\xi)$ as functions of ξ are given in Table on the next page

Case C: Solid sphere

$$\theta^*(r^*,t^*) = \frac{\theta}{\theta_i} = \frac{T(r,t)-T_\infty}{T_i - T_\infty} = C_B \exp[-A_B^2 t^*]\frac{\sin(A_B r^*)}{A_B r^*}$$

Where $r^* = r/r_o$; $t^* = \dfrac{\alpha t}{r_o^2}$

$$\theta^*(r^*=0,t^*) = \frac{\theta_0}{\theta_i} = \frac{T(r=0,t)-T_\infty}{T_i - T_\infty} = C_B \exp[-A_B^2 t^*]$$

$$\frac{Q}{Q_o} = 1 - 3[\theta^*(r^*=0,t^*)]\left[\frac{\sin(A_B) - A_B\cos(A_B)}{A_B^3}\right]$$

$$Q = \int_0^t q_{\substack{loss\\r=r_o}}\, dt$$

$$Q_o = m_{\substack{total\\sys}} c_p \left[T_i - T_\infty\right]$$

Notes: C_B, A_B are functions of Bi_M and the geometry (see the Table on the next page)

Values of A_B and C_B for different values of Bi_M and Cases A, B, and C

	Case A		Case B		Case C	
	Plane Wall		Long Cylinder		Sphere	
Bi_M	A_B	C_B	A_B	C_B	A_B	C_B
0.01	0.0998	1.0017	0.1412	1.0025	0.1730	1.0030
0.02	0.1410	1.0033	0.1995	1.0050	0.2445	1.0060
0.04	0.1987	1.0066	0.2814	1.0099	0.3450	1.0120
0.06	0.2425	1.0098	0.3438	1.0148	0.4217	1.0179
0.08	0.2791	1.0130	0.3960	1.0197	0.4860	1.0239
0.1	0.3111	1.0161	0.4417	1.0246	0.5423	1.0298
0.2	0.4328	1.0311	0.6170	1.0483	0.7593	1.0592
0.3	0.5218	1.0451	0.7465	1.0712	0.9208	1.0880
0.4	0.5932	1.0580	0.8516	1.0931	1.0528	1.1164
0.5	0.6533	1.0701	0.9408	1.1143	1.1656	1.1441
0.6	0.7051	1.0814	1.0185	1.1345	1.2644	1.1713
0.7	0.7506	1.0919	1.0873	1.1539	1.3525	1.1978
0.8	0.7910	1.1016	1.1490	1.1724	1.4320	1.2236
0.9	0.8274	1.1107	1.2048	1.1902	1.5044	1.2488
1.0	0.8603	1.1191	1.2558	1.2071	1.5708	1.2732
2.0	1.0769	1.1785	1.5995	1.3384	2.0288	1.4793
3.0	1.1925	1.2102	1.7887	1.4191	2.2889	1.6227
4.0	1.2646	1.2287	1.9081	1.4698	2.4556	1.7202
5.0	1.3138	1.2403	1.9898	1.5029	2.5704	1.7870
6.0	1.3496	1.2479	2.0490	1.5253	2.6537	1.8338
7.0	1.3766	1.2532	2.0937	1.5411	2.7165	1.8674
8.0	1.3978	1.2570	2.1286	1.5526	2.7654	1.8920
9.0	1.4149	1.2598	2.1566	1.5611	2.8044	1.9106
10.0	1.4289	1.2620	2.1795	1.5677	2.8363	1.9249
20.0	1.4961	1.2699	2.2881	1.5919	2.9857	1.9781
30.0	1.5202	1.2717	2.3261	1.5973	3.0372	1.9898
40.0	1.5325	1.2723	2.3455	1.5993	3.0632	1.9942
50.0	1.5400	1.2727	2.3572	1.6002	3.0788	1.9962
100.0	1.5552	1.2731	2.3809	1.6015	3.1102	1.9990

Zeroth- and first-order Bessel functions of the first kind

ξ	$J_0(\xi)$	$J_1(\xi)$
0.0	1.0000	0.0000
0.1	0.9975	0.0499
0.2	0.9900	0.0995
0.3	0.9776	0.1483
0.4	0.9604	0.1960
0.5	0.9385	0.2423
0.6	0.9120	0.2867
0.7	0.8812	0.3290
0.8	0.8463	0.3688
0.9	0.8075	0.4059
1.0	0.7652	0.4400
1.1	0.7196	0.4709
1.2	0.6711	0.4983
1.3	0.6201	0.5220
1.4	0.5669	0.5419
1.5	0.5118	0.5579
1.6	0.4554	0.5699
1.7	0.3980	0.5778
1.8	0.3400	0.5815
1.9	0.2818	0.5812
2.0	0.2239	0.5767
2.1	0.1666	0.5683
2.2	0.1104	0.5560
2.3	0.0555	0.5399
2.4	0.0025	0.5202
2.6	−0.0968	−0.4708
2.8	−0.1850	−0.4097
3.0	−0.2601	−0.3391
3.2	−0.3202	−0.2613

Notes: $(Bi_M)_{PlaneWall} = \dfrac{hL}{k_{solid}}$; $(Bi_M)_{Long\,Solid\,Cylinder} = \dfrac{hr_o}{k_{solid}}$; $(Bi_M)_{Solid\,Sphere} = \dfrac{hr_o}{k_{solid}}$

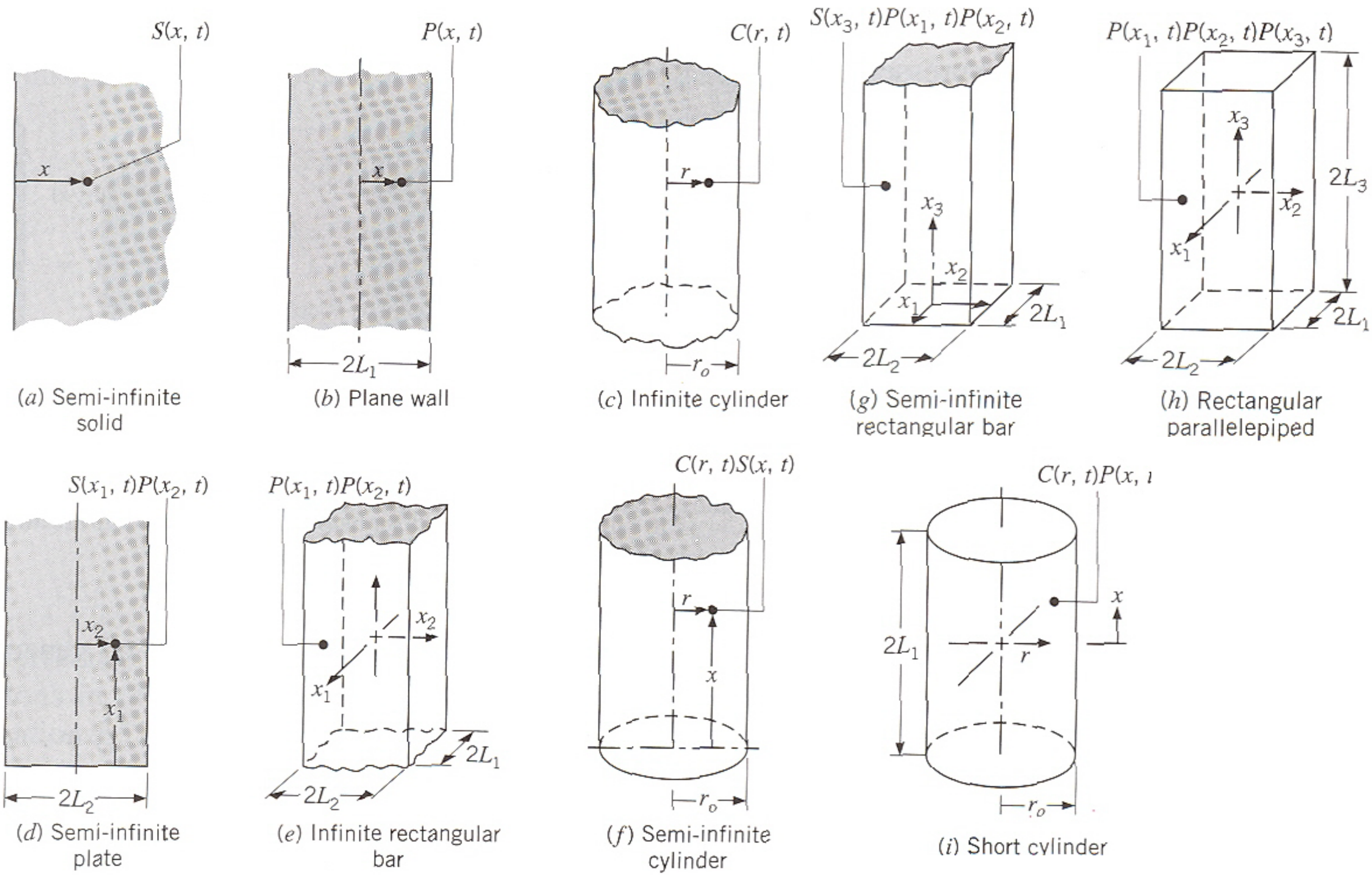
Multi-dimensional unsteady heat conduction in solids with convection B.C. [S =0; k, ρ, c_p, h, T_∞ all constant; and $Bi_{L_c} > 0.1$]

1-D Plane Wall Solution} $P(x,t) = \dfrac{T(x,t) - T_\infty}{T_i - T_\infty}$

1-D Long Cylinder Solution} $C(r,t) = \dfrac{T(r,t) - T_\infty}{T_i - T_\infty}$

Semi-Infinite Solid Solution} $S(x,t) = \dfrac{T(x,t) - T_\infty}{T_i - T_\infty} = 1 - \dfrac{T(x,t) - T_i}{T_\infty - T_i}$

Solution to multidimensional problems may be obtained by:



In general, for three-dimensional unsteady problems:

$$\left(\frac{T-T_\infty}{T_i-T_\infty}\right)_{Full\ 3-D\ solid}=\prod_{j=1,3}\left(\frac{T-T_\infty}{T_i-T_\infty}\right)_{\substack{intersection\\ solid\ j}}=\left(\frac{T-T_\infty}{T_i-T_\infty}\right)_{solid\,1}\times\left(\frac{T-T_\infty}{T_i-T_\infty}\right)_{solid\,2}\times\left(\frac{T-T_\infty}{T_i-T_\infty}\right)_{solid\,3}$$

Total heat transfer in Multi-dimensional systems [Work of Langston]

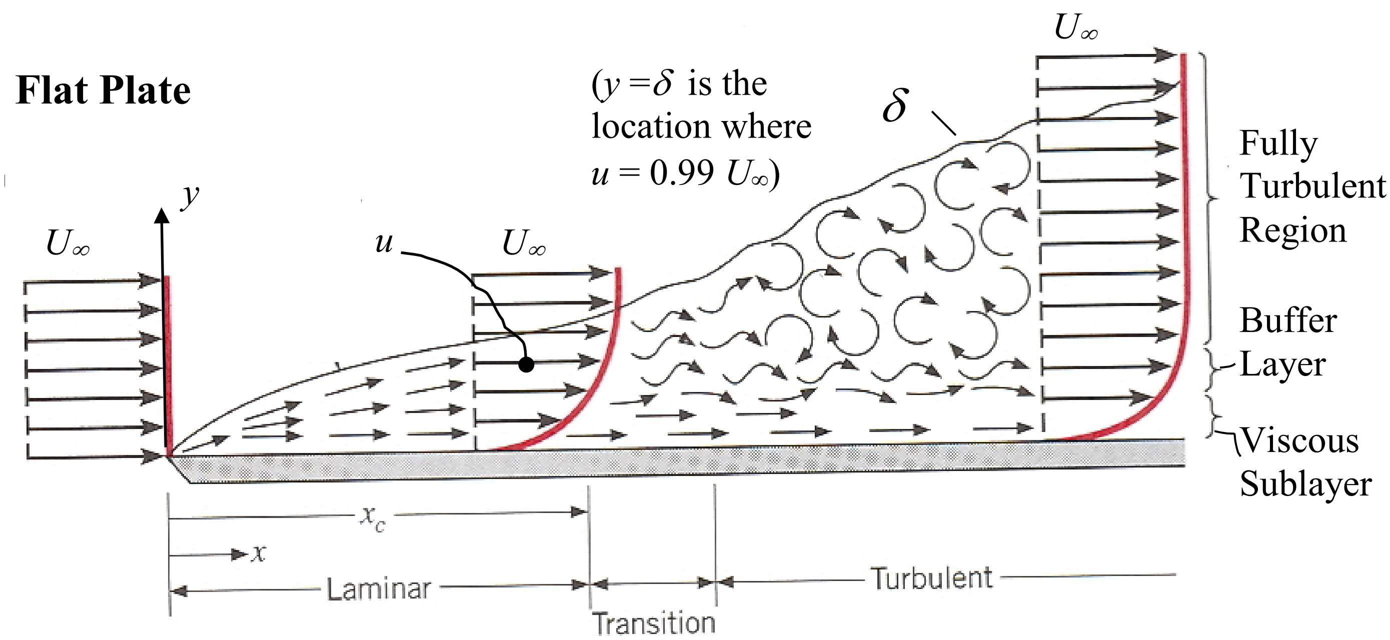
- For solids that can be constructed by the intersection of two objects (1 and 2) for which the 1-D solutions (discussed earlier) apply:

$$\left(\frac{Q}{Q_o}\right)_{total}=\left(\frac{Q}{Q_o}\right)_1+\left(\frac{Q}{Q_o}\right)_2\left[1-\left(\frac{Q}{Q_o}\right)_1\right]$$

- For solids that can be constructed by the intersection of three objects (1, 2, and 3) for which the 1-D solutions (discussed earlier) apply:

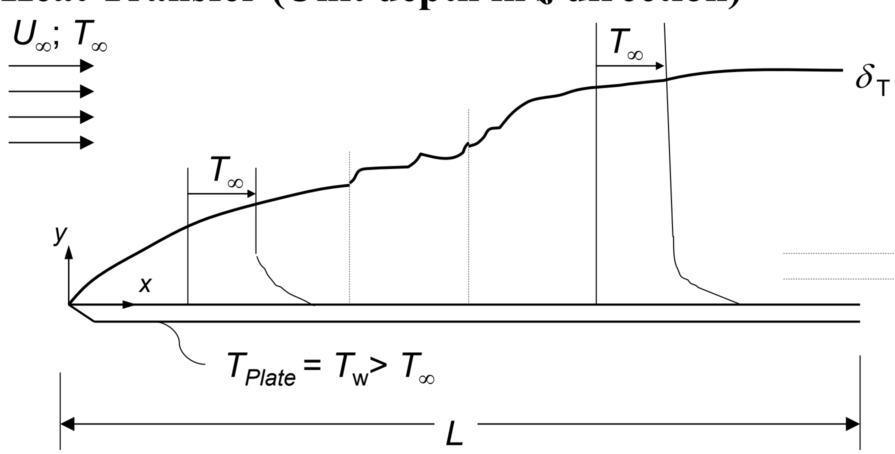
$$\left(\frac{Q}{Q_o}\right)_{total}=\left(\frac{Q}{Q_o}\right)_1+\left(\frac{Q}{Q_o}\right)_2\left[1-\left(\frac{Q}{Q_o}\right)_1\right]+\left(\frac{Q}{Q_o}\right)_3\left[1-\left(\frac{Q}{Q_o}\right)_2\right]\left[1-\left(\frac{Q}{Q_o}\right)_1\right]$$

Forced Convection Heat Transfer: A) External Flow



- Local or "running" Reynolds number, $Re_x = \rho U_\infty x / \mu$
- Transition region: $1 \times 10^5 \le Re_x \le 1 \times 10^6$
- In engineering analyses: $Re_{x_{crit}} = (\rho U_\infty x_{crit} / \mu) = 5 \times 10^5$ (flow over flat plate)

Heat Transfer (Unit depth in *z* direction)



δ_T :Thermal B.L. thickness ($y = \delta_T$ is the location where $\dfrac{T - T_w}{T_\infty - T_w} = 0.99$)

Local heat flux: $q_x'' = -k \left.\dfrac{\partial T}{\partial y}\right|_{y=0} = h_x (T_w - T_\infty)$; $q_{\substack{total \\ top\, surf}} = \int_0^L q_x'' dx = \int_0^L h_x (T_w - T_\infty) dx$

$$q_{\substack{total \\ top\, surf}} = h_{av} A_{\substack{total \\ top\, surf}} (T_w - T_\infty)_{av}; \quad h_{av} = \frac{(q_{\substack{total \\ top\, surf}} / A_{\substack{tota \\ top\, surf}})}{(T_w - T_\infty)_{av}} = \underbrace{\frac{1}{L}\int_0^L h_x dx}_{h_{av}} \left\} \begin{array}{l} note: here \\ (T_w - T_\infty) = const. \end{array} \right.$$

Chilton-Colburn Analogy:

Local form: $\left.\boxed{\frac{1}{2}c_{f,x} = St_x\, Pr^{2/3}}\right\}$ for $0.6 \le Pr \le 60$; where: $St_x = \dfrac{Nu_x}{\mathrm{Re}_x \Pr} = \dfrac{h_x}{\rho U_\infty c_p}$

$$Nu_x = \frac{h_x x}{k_{fluid}}; \Pr = \frac{\upsilon}{\alpha} = \frac{\mu c_p}{k_{fluid}}; \mathrm{Re}_x = \frac{\rho U_\infty x}{\mu}; c_{f,x} = \frac{\tau_{w,x}}{\left(\frac{1}{2}\rho U_\infty^2\right)}$$

Overall or Average form: $\left.\boxed{\frac{1}{2}c_{f,av} = St_{av}\, Pr^{2/3}}\right\}$ for $0.6 \le Pr \le 60$;

$$St_{av} = \frac{Nu_{av}}{\mathrm{Re}_L \Pr} = \frac{h_{av}}{\rho U_\infty c_p};\ Nu_{av} = \frac{h_{av} L}{k_{fluid}}; \Pr = \frac{\upsilon}{\alpha} = \frac{\mu c_p}{k_{fluid}}; \mathrm{Re}_L = \frac{\rho U_\infty L}{\mu}; c_{f,av} = \frac{\underset{0\le x\le L}{\tau_{w,av}}}{\left(\frac{1}{2}\rho U_\infty^2\right)}$$

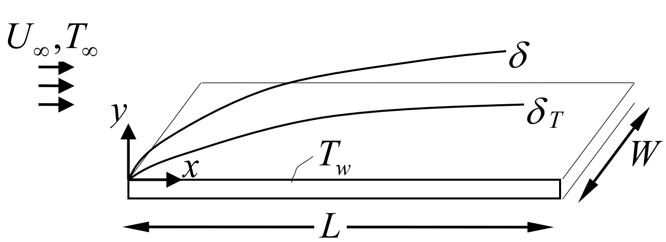
- Laminar and/or turbulent flows over flat plate at zero angle of attack (smooth or rough surface)
- Laminar and/or turbulent flow over streamlined objects, provided there is no flow separation from the surface of the object

Design Correlations for External Forced Convection

In general, for a given geometry: $Nu = C\,\mathrm{Re}^m \Pr^n$ and $c_f = \Im \mathrm{Re}^k$; where C, m, n, $\Im$, and k are constant.

1. Flat Plate

Isothermal surface; Laminar Flow [$Re_x = (\rho U_\infty x/\mu) \le 5\text{x}10^5$; 0.6<Pr<60]



1) $\delta = \dfrac{5.0}{\sqrt{\rho U_\infty/(\mu x)}}$; or $\dfrac{\delta}{x} = \dfrac{5.0}{\sqrt{Re_x}}$; and $(\delta/\delta_T) = Pr^{1/3}$

2) $\underset{@\,any\,x}{\tau_w} = \mu \left.\dfrac{\partial u}{\partial y}\right|_{y=0} = 0.332 U_\infty \sqrt{\rho\mu U_\infty / x}$; thus, $c_{f,x} = \dfrac{\tau_{w,x}}{\frac{1}{2}\rho U_\infty^2} = 0.664(Re_x)^{-1/2}$.

3) $Nu_x = \dfrac{h_x x}{k_{fluid}} = \dfrac{\{q''_{w,x}/(T_w - T_\infty)\}x}{k_{fluid}} = 0.332(Re_x)^{1/2} Pr^{1/3}$ for $0.6 \le Pr \le 60$

4) $$c_{\substack{f,av\\0\le x\le x_1}} = \frac{\tau_{\substack{w,av\\0\le x\le x_1}}}{\frac{1}{2}\rho U_\infty^2} = \frac{(Drag_{Viscous})/(x_1 W)}{\frac{1}{2}\rho U_\infty^2}\Bigg|_{0\le x\le x_1} = 1.328(Re_{x_1})^{-1/2}$$

5) $$Nu_{\substack{av,x_1\\0\le x\le x_1}} = \frac{(h_{\substack{av\\0\le x\le x_1}})x_1}{k_{fluid}} = \left(\frac{\{q_{\substack{w\to fluid\\0\le x\le x_1}}/(x_1 W)\}}{(T_w - T_\infty)}\right)\frac{x_1}{k_{fluid}} = 0.664(Re_{x_1})^{1/2} Pr^{1/3}$$ for $0.6 \le Pr \le 60$.

6) Use fluid properties at the film temperature: $T_{film} = (T_W + T_\infty)/2$

7) For fluid with very low Pr numbers like ***liquid metals*** or very high Pr number fluids use the Churchill-Ozoe correlation:

$$Nu_x = \left.\frac{0.3387\, Re^{1/2}\, Pr^{1/3}}{[1+(0.0468/Pr)^{2/3}]^{1/4}}\right\} \text{ for } Pe_x = (Re_x Pr) \ge 100$$

Constant Surface Heat Flux

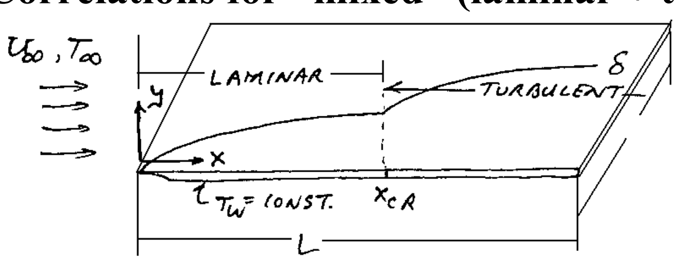
8) For constant surface heat flux and laminar flow use:

$$Nu_x = \frac{h_x x}{k_{fluid}} = \frac{\{q''_{w,x}/(T_w - T_\infty)\}x}{k_{fluid}} = 0.453(Re_x)^{1/2} Pr^{1/3} \text{ for } 0.6 \le Pr \le 60$$

9) For liquid metals and a wide range of Pr numbers for laminar flow over surfaces with constant heat flux the Churchill-Ozoe correlation is given:

$$Nu_x = \left.\frac{0.4637\, Re^{1/2}\, Pr^{1/3}}{[1+(0.0207/Pr)^{2/3}]^{1/4}}\right\} \text{ for } Pe_x = (Re_x Pr) \ge 100$$

Correlations for "mixed" (laminar + turbulent) flow over the plate



It is assumed here that lam.-turb. transition takes place suddenly at $Re_{x_{cr}} = (\rho U_\infty x_{cr}/L) = 5\times 10^5$

Here: $Re_L > 5\times 10^5$

$$h_{\substack{av\\0\le x\le L}} = \left(\frac{q_{plate\to fluid}/A_{surf\ plate}}{T_w - T_\infty}\right) = \frac{1}{(T_w - T_\infty)(WL)}\int_0^L h_x (T_w - T_\infty) W dx = \frac{1}{L}\left[\int_0^{x_{cr}} h_{lam,x} dx + \int_{x_{cr}}^L h_{turb,x} dx\right]$$

Note: Here, $(T_w - T_\infty)$ is const.

1) $$Nu_{\substack{av\\0\le x\le L}} = \frac{(h_{\substack{av\\0\le x\le L}})L}{k_{fluid}} = [0.664 Re_{x_{cr}}^{1/2} + 0.037(Re_L^{4/5} - Re_{x_{cr}}^{4/5})]Pr^{1/3}$$

Valid for $0.6 \le Pr \le 60$ and $5\times10^5 < Re_L < 10^8$

2) With $Re_{x_{cr}} = 5\times10^5$, $Nu_{\substack{av\\0\le x\le L}} = \frac{(h_{\substack{av\\0\le x\le L}})L}{k_{fluid}} = [0.037\,Re_L^{4/5} - 871]Pr^{1/3}$ Valid for $0.6 \le Pr \le 60$ and $5\text{x}10^5 < Re_L < 10^8$

3) $c_{\substack{f,av\\0\le x\le L}} = \frac{\tau_{\substack{w,av\\0\le x\le L}}}{\rho U_\infty^2/2} = \left[\frac{0.074}{Re_L^{1/5}} - \frac{1742}{Re_L}\right]$ Valid for $5\times10^5 < Re_L < 10^8$ and with $Re_{x_{cr}} = 5\times10^5$. Note that $\tau_{\substack{w,av\\0\le x\le L}} = [\text{Viscous Drag}_{0\le x\le L}]/[\text{Surface Area}_{0\le x\le L}]$

4) If $L >> x_{cr}$, then $Re_L >> Re_{x_{cr}}$ and

$$Nu_{\substack{av\\0\le x\le L}} = \frac{(h_{\substack{av\\0\le x\le L}})L}{k_{fluid}} = 0.037\,Re_L^{4/5}\,Pr^{1/3}$$

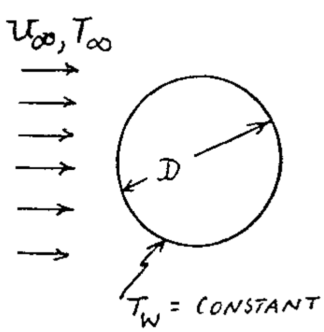
$$c_{\substack{f,av\\0\le x\le L}} = \frac{\tau_{\substack{w,av\\0\le x\le L}}}{\rho U_\infty^2/2} = \frac{0.074}{Re_L^{1/5}}$$

Valid for
$5\times10^5 < Re_L < 10^8$ and
$0.6 \le Pr \le 60$

5) The correlations given above in point 4) also apply if the boundary layer is tripped at the leading edge ($x = 0$) and the flow is turbulent over the whole plate ($0 \le x \le L$)

6) In turbulent heat transfer, generally speaking the correlations obtained for isothermal surfaces can be applied to the surface constant flux case.

2: Cylinder of circular cross-section in uniform cross-flow



(i) Knudsen and Katz correlation

$$Nu_{av} = \frac{h_{av}D}{k_{fluid}} = C\,Re_D^n Pr^{1/3}$$

- All properties are evaluated at T_{film}

- The values of C and n should be obtained from the table:

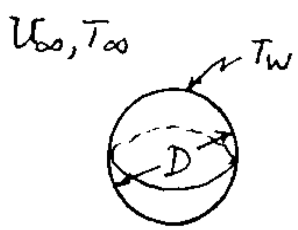
Re_{df}	C	n
0.4–4	0.989	0.330
4–40	0.911	0.385
40–4000	0.683	0.466
4000–40,000	0.193	0.618
40,000–400,000	0.0266	0.805

(ii) Whitaker correlation $Nu_{av} = \dfrac{h_{av}D}{k_{fluid}} = (0.4Re_D^{1/2} + 0.06Re_D^{2/3})Pr^{0.4}\left(\dfrac{\mu_\infty}{\mu_w}\right)^{1/4}$

- $0.65 \le Pr \le 300$; $40 \le Re_D \le 10^5$; and $0.25 \le (\mu_\infty / \mu_w) \le 5.2$ All properties are evaluated at T_∞, except μ_w which is evaluated at T_w

3: Sphere in cross-flow

Whitaker correlation



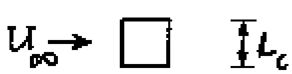
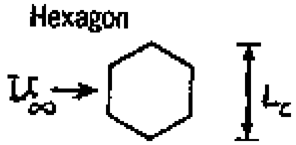
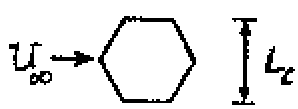
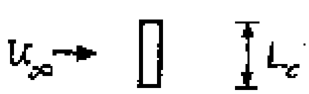
$$Nu_{av} = \frac{h_{av}D}{k_{fluid}} = 2 + [0.4Re_D^{1/2} + 0.06Re_D^{2/3}]Pr^{0.4}\left(\frac{\mu_\infty}{\mu_w}\right)^{1/4}$$

- All properties are evaluated at T_∞, except μ_w which is evaluated at T_w
- $0.7 \le Pr \le 380$; $3.5 \le Re_D \le 7.6 \times 10^4$

4: Cylinders of non-circular cross-section in uniform cross-flow

Jacob correlation: $Nu_{av} = \dfrac{h_{av}L_c}{k_{fluid}} = CRe_{L_c}^m Pr^{1/3}$

L_c = Characteristic length (see table below); All properties at $T_{film} = 0.5(T_w + T_\infty)$

Geometry	Re_{L_c}	C	m
Square 	$5 \times 10^3 - 10^5$	0.246	0.588
	$5 \times 10^3 - 10^5$	0.102	0.675
Hexagon 	$5 \times 10^3 - 1.95 \times 10^4$ $1.95 \times 10^4 - 10^5$	0.160 0.0385	0.638 0.782
	$5 \times 10^3 - 10^5$	0.153	0.638
Vertical plate 	$4 \times 10^3 - 1.5 \times 10^4$	0.228	0.731

5: Heat Transfer across Tube Banks

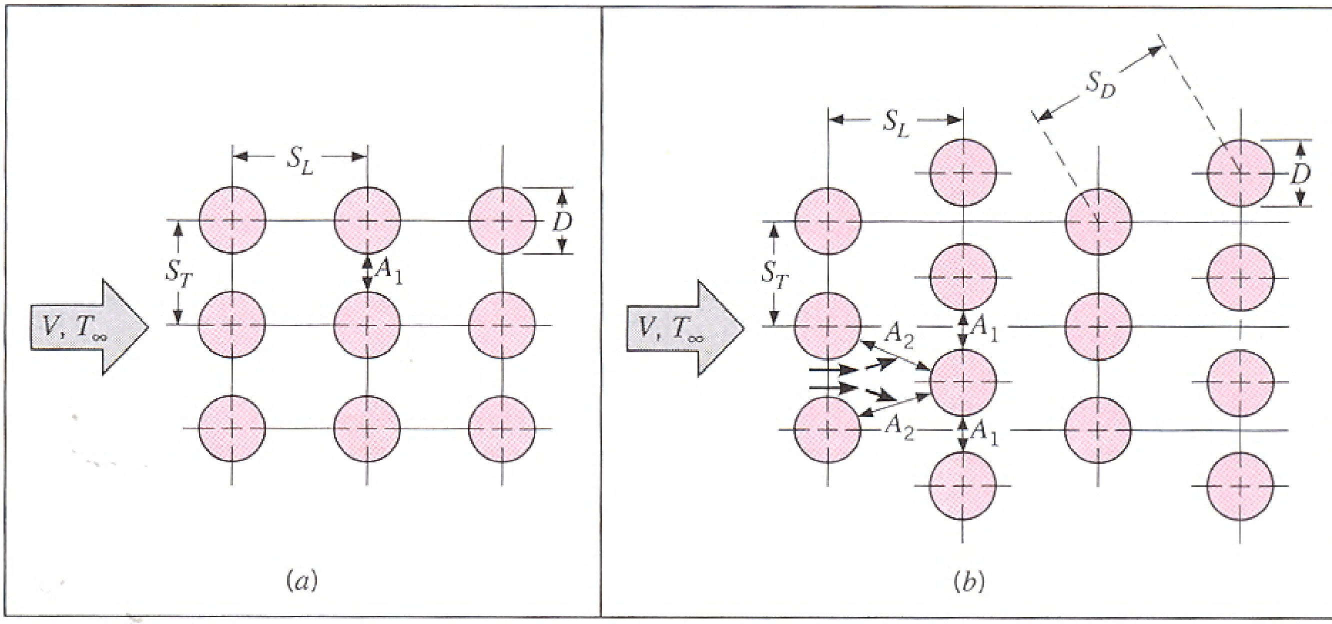


FIGURE 7.11 Tube arrangements in a bank. (a) Aligned. (b) Staggered.

Zukauskas correlation:

$$Nu_{av} = \frac{h_{av} d}{k_{fluid}} = C Re_{D_{Max}}^{n} \, Pr^{0.36} \left(\frac{Pr}{Pr_w} \right)^{1/4}$$

- $Re_{D_{Max}} = \dfrac{\rho V_{\max} D}{\mu}$, where:

  Aligned arrangement

  $V_{\max} = V\left[S_T / \left(S_T - D \right) \right]$

  Staggered arrangement

  $V_{\max} = V\left[S_T / \left(S_T - D \right) \right] \quad if \quad S_D > \left(S_T + D \right)/2$

  $V_{\max} = V\left(S_T / 2 \right) / \left[\left(S_D - D \right) \right] \quad if \ S_D < \left(S_T + D \right)/2$

- Range of validity of this correlation: $0.7 \le Pr \le 500$; $10 \le Re_{D_{Max}} \le 10^6$

- All properties except Pr_w are evaluated at T_∞, and the values of the constants are given below.

TABLE 7.7 Constants of Equation 7.64 for the tube bank in cross flow [15]

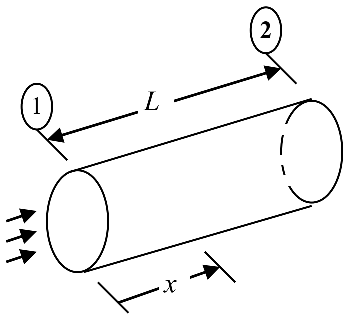
Configuration	$Re_{D,\max}$	C	m
Aligned	10–10^2	0.80	0.40
Staggered	10–10^2	0.90	0.40
Aligned	10^2–10^3	Approximate as a single (isolated) cylinder	
Staggered	10^2–10^3		
Aligned $(S_T/S_L > 0.7)$[a]	10^3–2×10^5	0.27	0.63
Staggered $(S_T/S_L < 2)$	10^3–2×10^5	$0.35(S_T/S_L)^{1/5}$	0.60
Staggered $(S_T/S_L > 2)$	10^3–2×10^5	0.40	0.60
Aligned	2×10^5–2×10^6	0.021	0.84
Staggered	2×10^5–2×10^6	0.022	0.84

[a]For $S_T/S_L < 0.7$, heat transfer is inefficient and aligned tubes should not be used.

TABLE 7.8 Correction factor C_2 of Equation 7.65 for $N_L < 20$ $(Re_{D,\max} \gtrsim 10^3)$ [15]

N_L	1	2	3	4	5	7	10	13	16
Aligned	0.70	0.80	0.86	0.90	0.92	0.95	0.97	0.98	0.99
Staggered	0.64	0.76	0.84	0.89	0.92	0.95	0.97	0.98	0.99

Forced Convection Heat Transfer: B) Internal Flow



Reynolds number based on average velocity and the hydraulic diameter:

$$Re_{D_h} = \frac{\rho u_{av} D_H}{\mu} \;;\; \underbrace{D_h}_{\substack{\text{Hydraulic}\\\text{diameter}}} = \frac{4A_{\substack{c.s.\\ \text{for flow}}}}{\begin{bmatrix}\text{Wetted perimeter}\\ \text{of c.s. for flow}\end{bmatrix}} \;;\; u_{av} = \frac{\dot{m}}{\rho A_{c.s.}}$$

Critical Reynolds number: in engineering analyses

If $Re_{D_h} \le 2300$ then the flow is laminar; If $Re_{D_h} > 2300$ then the flow is turbulent

Darcy Friction Factor: $f_{Darcy} = \dfrac{-(dP/dx)D_H}{\rho u_{av}^2/2}$; (dP/dx) is the axial gradient of the reduced pressure.

Fully Developed Flow: The velocity is invariant with axial distance x, and (dP/dx) is constant. Thus, $f_{Darcy} = \dfrac{[(P_1 - P_2)/L]D_H}{\rho u_{av}^2/2}$.

Bulk Temperature: $T_b = \int_{A_{c.s.}} \rho u c_p T dA_{c.s.} \Big/ \int_{A_{c.s.}} \rho u c_p dA_{c.s.}$

Energy Balance Equations: [Ec<< 1; Pe>>1; incompressible Newtonian fluids]

Overall E-Bal: $\dot{m}c_p(T_{b,2} - T_{b,1}) = q_{\substack{total\\ wall \to fluid}}$

Heat Transfer Coefficient: Local: $h = q''_w/(T_w - T_b)$

$$\text{Average: } h_{av} = \left(q''_w\right)_{av}/(T_w - T_b)_{LMTD} = \frac{q_{\substack{total\\ wall\to fluid}} / (\text{Duct wall surface area})}{(T_w - T_b)_{LMTD}}$$

$$(T_w - T_b)_{LMTD} = \left[(T_w - T_b)_1 - (T_w - T_b)_2\right] / \ln\left[\frac{(T_w - T_b)_1}{(T_w - T_b)_2}\right]$$

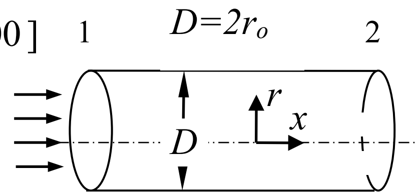
Nusselt Number: Local: $Nu = \dfrac{hD_H}{k_{fluid}}$; Average: $Nu_{av} = \dfrac{h_{av}D_H}{k_{fluid}}$

Prandtl Number: $\Pr = \dfrac{\upsilon}{\alpha} = \dfrac{\mu c_p}{k_{fluid}}$; Peclet Number: $Pe = \Pr \mathrm{Re}$;

Stanton Number: Local: $St = \dfrac{h}{\rho u_{av} c_p}$; Average: $St_{av} = \dfrac{h_{av}}{\rho u_{av} c_p}$

Fully Developed Heat Transfer: The heat transfer coefficient, h is invariant with axial distance x.

Laminar Fully Developed Flow and Heat Transfer in Duct of Circular Cross Section $[D_H = D;\ Re_D = \dfrac{\rho u_{av} D}{\mu} \le 2300]$



Poisseuille Flow:

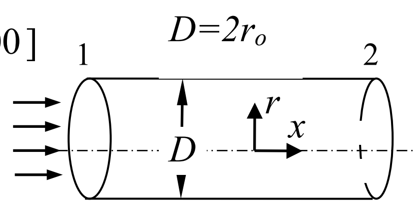
$u = u_c[1-(r/r_o)^2]$; Here, $u_{av} = u_c/2$

(u_c is the centerline velocity)

$$f_{Darcy} = \frac{(P_1 - P_2/L)D}{\rho u_{av}^2/2} = \frac{64}{\mathrm{Re}_D}; \qquad Nu_D = \frac{hD}{k_{fluid}} = \begin{cases} 4.364 & q''_w\text{=constant} \\ 3.658 & T_w\text{=constant} \end{cases}$$

Heat Transfer: $q_{\substack{total \\ wall\to fluid}} = h(\pi DL)(T_w - T_b)_{LMTD} = \dot{m}c_p(T_{b,2} - T_{b,1})$

Turbulent Fully Developed Flow and Heat Transfer in Duct of Circular Cross Section $[D_H = D;\ Re_D = \dfrac{\rho u_{av} D}{\mu} > 2300]$



Use the following explicit approximation of the **Colebrook-White** correlation for Darcy friction factor:

$$f_{Darcy} = \left\{-2.0\log_{10}\left[\frac{(\varepsilon_{rms}/D)}{3.7} - \frac{5.02}{Re_D}\log_{10}\left(\frac{(\varepsilon_{rms}/D)}{3.7} + \frac{14.49}{Re_D}\right)\right]\right\}^{-2}$$

ε_{rms} is the RMS, roughness.

For smooth ducts, use: $\varepsilon_{rms} = 0$ in the above equation or the **Filonenko** equation:

$$f_{darcy} = \frac{[(P_1 - P_2)/L]D}{\rho u_{av}^2/2} = \left[1.82\log_{10}\left(Re_D\right) - 1.64\right]^{-2}$$

Heat Transfer: $q_{\substack{total \\ wall\to fluid}} = h(\pi DL)(T_w - T_b)_{LMTD} = \dot{m}c_p(T_{b,2} - T_{b,1})$

Dittus-Boelter correlation [applies only to smooth pipes]

$$Nu_D = 0.023\,Re_D^{0.8}\,Pr^n \;\Big\} \rightarrow \begin{cases} \text{Restrictions:} \\ 0.6 \le Pr \le 160 \\ Re_D \ge 10{,}000 \end{cases}$$

n = 0.4 for heating of the fluid; n = 0.3 for cooling of the fluid

- All fluid properties, must be evaluated at $(T_b)_{mean} = (T_{b,1} + T_{b,2})/2$

Sieder-Tate correlation [applies only to smooth pipes]

(For flow characterized by large property variations)

$$Nu_D = 0.027\, Re_D^{0.8}\, Pr^{1/3}\left(\frac{\mu}{\mu_w}\right)^{0.14} \Bigg\} \rightarrow \begin{cases} \text{Restrictions:} \\ 0.7 \le Pr \le 16{,}700 \\ Re_D \ge 10{,}000 \\ (L/D) \ge 10 \end{cases}$$

- All fluid properties, except μ_w, must be evaluated at $(T_b)_{mean} = (T_{b,1} + T_{b,2})/2$
- μ_w is the dynamic viscosity of the fluid at $T = T_w$

Petukhov, correlation [applies to both smooth and rough tubes]

$$Nu_D = \left[\frac{(f_{Darcy}/8)\, Re_D\, Pr}{1.07 + 12.7 (f_{Darcy}/8)^{1/2} (Pr^{2/3} - 1)}\right]\left(\frac{\mu_b}{\mu_w}\right)^n \Bigg\} \rightarrow \begin{cases} \text{Restrictions:} \\ 0.5 \le Pr \le 2000 \\ 10^4 \le Re_D \le 5\text{x}10^6 \\ 0.8 \le \dfrac{\mu_b}{\mu_w} \le 40 \end{cases}$$

$n = 0.11$ for heating of the fluid; $n = 0.25$ for cooling of the fluid

$n = 0$ for constant heat flux or for gases.

- All fluid properties, except μ_w, must be evaluated at $(T_b)_{mean} = (T_{b,1} + T_{b,2})/2$

Gnielinski correlation [applies to both smooth and rough tubes]

$$Nu_D = \left[\frac{(f_{Darcy}/8)(Re_D - 1000)\, Pr}{1 + 12.7 (f_{Darcy}/8)^{1/2} (Pr^{2/3} - 1)}\right] \Bigg\} \rightarrow \begin{cases} \text{Restrictions:} \\ 0.5 \le Pr \le 2000 \\ 3000 \le Re_D \le 5\text{x}10^6 \end{cases}$$

- All thermophysical properties of the fluid should be obtained at $(T_b)_{mean} = (T_{b,1} + T_{b,2})/2$

Skupinshi correlation [turbulent flow of **liquid metals** in smooth pipes]

$$Nu_D = 4.82 + 0.0185 (Re_D\, Pr)^{0.827} \Big\} \rightarrow \begin{cases} \text{Restrictions:} \\ q_w^{"} = constant \\ 3.6\times10^3 \le Re_D \le 9.05\times10^5 \\ 10^2 \le (Re_D\, Pr) \le 10^4 \end{cases}$$

All thermophysical properties of the fluid should be obtained at $(T_b)_{mean} = (T_{b,1} + T_{b,2})/2$

Chilton-Colburn analogy for fully-developed turbulent flow and heat transfer in pipes: $\frac{1}{8} f_{Dacry} = St\, Pr^{2/3}$ or $\frac{1}{8}\left[\frac{\{(P_1 - P_2)/L\}D}{\rho u_{av}^2/2}\right] = \left(\frac{h}{\rho u_{av} c_p}\right) Pr^{2/3}$

For Fully-Developed Non-Circular c.s. duct: If specific correlations are not available, use circular c.s. duct correlations with 'D' replaced by 'D_H'.

Natural Convection (Buoyancy-Driven) Heat Transfer

[Note: Ec<<1; $g = 9.81$ m/s^2 = constant]

Thermal volumetric expansion coefficient:

For a perfect gas: $\beta = \frac{1}{T_{ABS}}$ [K^{-1}]

General form of correlations: $Nu_{L_c} = \frac{h_{av} L_c}{k_{fluid}} = C\left(Gr\,\mathrm{Pr}\right)^m = C\left(Ra\right)^m$ (C and m are const.)

NOTE: In the following correlations T_w is the surface temp. and T_∞ is the bulk fluid temperature.

Grashof #: $Gr = \frac{\beta g (T_w - T_\infty) L_c^3}{(\mu/\rho)^2}$; **Prandtl #,** $Pr = \frac{\mu c_p}{k} = \frac{\nu}{\alpha}$;

Rayleigh #, $Ra = Gr\,\mathrm{Pr}$

1. Vertical isothermal plate of length *L* ($L_c = L$)

Churchill-Chu correlations:

$$Nu_{av} = \left(\frac{h_{av} L}{k_{fluid}}\right) = 0.68 + \frac{0.670 Ra_L^{1/4}}{\left[1 + \left(0.492/\mathrm{Pr}\right)^{9/16}\right]^{4/9}} \qquad \text{for } Ra_L \le 10^9$$

$$Nu_{av} = \left(\frac{h_{av} L}{k_{fluid}}\right) = \left[0.825 + \frac{0.387 Ra_L^{1/6}}{\left[1 + \left(0.492/\mathrm{Pr}\right)^{9/16}\right]^{8/27}}\right]^2 \qquad \text{for } Ra_L > 10^9$$

2. Horizontal Flat Plate: Isothermal Surface (T_w = constant)

2-a) Hot plate facing upward; Cold plate facing downward

Use Lloyd –Moran correlation [Property data at $T_{film} = \left(T_w + T_\infty\right)/2$]:

$$Nu_{av} = \left(\frac{h_{av}L_c}{k_{fluid}}\right) = 0.54 Ra_{L_c}^{\ 1/4} \qquad \text{for } 10^4 \le Ra_{L_c} \le 10^7$$

$$Nu_{av} = \left(\frac{h_{av}L_c}{k_{fluid}}\right) = 0.15 Ra_{L_c}^{\ 1/3} \qquad \text{for } 10^7 < Ra_{L_c} \le 10^{11}$$

Notes: $L_c = (\text{Surface area of the plate})/(\text{Perimeter of the plate})$

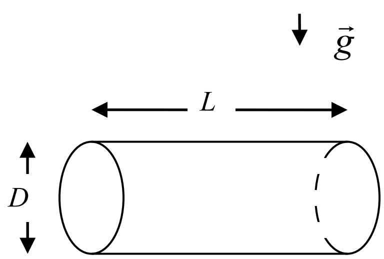
2-b) Hot plate facing downward; Cold plate facing upward

Use McAdams correlation [Property data at $T_{film} = (T_w + T_\infty)/2$]:

$$Nu_{av} = \left(\frac{h_{av}L_c}{k_{fluid}}\right) = 0.27 Ra_{L_c}^{\ 1/4} \qquad \text{for } 10^5 \le Ra_{L_c} \le 10^{11}$$

Notes: $L_c = (\text{Surface area of the plate})/(\text{Perimeter of the plate})$

3. Long Horizontal Cylinder [*L*>>*D*]



Here $L_c = D$

$$Gr_D = \frac{g\beta[T_w - T_\infty]D^3}{\nu^2}; \quad Ra_D = \Pr Gr_D$$

Use Morgan correlation
[Property data at $T_{film} = (T_w + T_\infty)/2$]

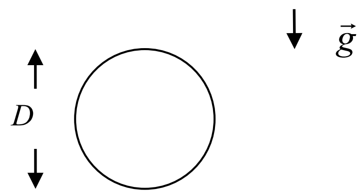
$$Nu_{av} = \left(\frac{h_{av}D}{k_{fluid}}\right) = C.Ra_D^n$$

Ra_D	C	n
10^{-10} to 10^{-2}	0.675	0.058
10^{-2} to 10^{2}	1.02	0.148
10^{2} to 10^{4}	0.850	0.188
10^{4} to 10^{7}	0.480	1/4
10^{7} to 10^{12}	0.125	1/3

Or alternatively use the Churchill-Chu correlation:

$$Nu_{av} = \left(\frac{h_{av}D}{k_{fluid}}\right) = \left\{0.60 + \frac{0.387 Ra_D^{1/6}}{\left[1 + (0.559/\Pr)^{9/16}\right]^{8/27}}\right\}^2 \qquad 10^{-5} < Ra_D < 10^{12}$$

4. Spheres



$$Gr_D = \frac{g\beta[T_w - T_\infty]D^3}{\nu^2}; \quad Ra_D = \Pr Gr_D$$

Use Churchill correlation [Property data at $T_{film} = (T_w + T_\infty)/2$]

$$Nu_{av} = \left(\frac{h_{av}D}{k_{fluid}}\right) = 2 + \frac{0.589 Ra_D^{1/4}}{\left[1 + (0.469/\Pr)^{9/16}\right]^{4/9}} \qquad Ra_D < 10^{11} \text{ and } \Pr \geq 0.5$$

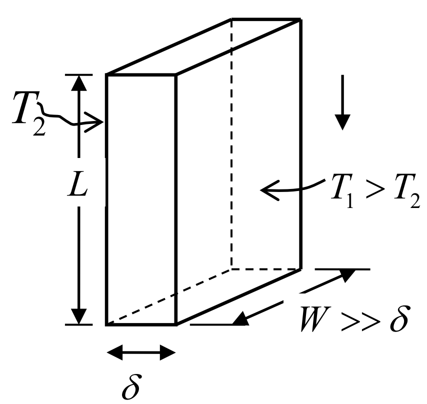
5. Irregular Solids with Isothermal Surfaces in Natural Convection

For the first estimation use Lienhard correlation:

$$Nu_{av} = \left(\frac{h_{av}L_c}{k_{fluid}}\right) = 0.52 Ra_{L_c}^{0.25} \qquad 10^4 \leq Gr_{Lc}\Pr \leq 10^9$$

Where the characteristic length, L_c: Is the distance fluid particle travels in boundary layer

6: Vertical Rectangular Cavity



- Average heat flux: $q''_{av} = \dfrac{q}{WL} = h_{av}(T_1 - T_2)$

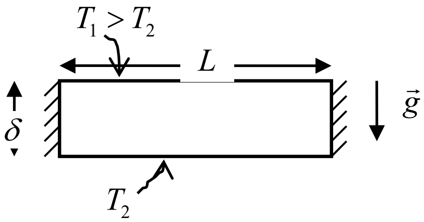
$$Gr_\delta = \frac{g\beta[T_1 - T_2]\delta^3}{\nu^2}; \; Ra_\delta = \Pr Gr_\delta$$

- If $Ra_\delta \leq 1000$, influence of natural convection is very small. Thus, $Nu_{av} = \left(\dfrac{h_{av}\delta}{k_{fluid}}\right) \simeq 1$
- For large Ra_δ use the following correlations:

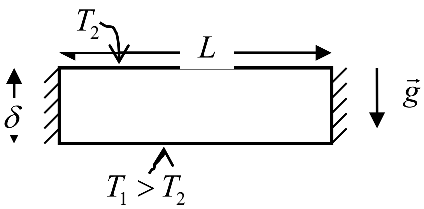
$$Nu_{av} = \left(\frac{h_{av}\delta}{k_{fluid}}\right) = 0.42 Ra_\delta^{1/4} \Pr^{0.012} (L/\delta)^{-0.3} \qquad \begin{matrix} 10^4 \leq Ra_\delta < 10^7 \\ 1 \leq \Pr < 2 \times 10^4 \\ 10 \leq L/\delta < 40 \end{matrix}$$

$$Nu_{av} = \left(\frac{h_{av}\delta}{k_{fluid}}\right) = 0.046 Ra_\delta^{1/3} \qquad \begin{matrix} 10^6 \leq Ra_\delta < 10^9 \\ 1 \leq \Pr < 20 \\ 1 \leq L/\delta < 40 \end{matrix}$$

7: Horizontal Rectangular Cavity



- This situation corresponds to a thermally stable stratification; No flow; Heat Transfer by conduction.
- $Nu_{av} = \left(\frac{h_{av}\delta}{k_{fluid}}\right) = 1$



$$q''_{av} = h_{av}\left(T_1 - T_2\right)$$

$$Gr_\delta = \frac{g\beta[T_1 - T_2]\delta^3}{\nu^2}; \; Ra_\delta = \Pr Gr_\delta$$

If $Ra_\delta \leq 1708$, influence of natural convection is very small. Thus, $Nu_{av} = \left(\frac{h_{av}\delta}{k_{fluid}}\right) = 1$

For large Ra_δ use the following correlations:

$$Nu_{av} \triangleq \left(\frac{h_{av}\delta}{k_{fluid}}\right) = 0.212 Ra_\delta^{1/4} \qquad \begin{array}{l} 7000 \leq Ra_\delta < 3\times10^5 \\ 0.5 \leq \Pr < 2 \end{array}$$

$$Nu_{av} \triangleq \left(\frac{h_{av}\delta}{k_{fluid}}\right) = 0.059 Ra_\delta^{0.4} \qquad \begin{array}{l} 1708 \leq Ra_\delta < 7000 \\ 0.5 \leq \Pr < 2 \end{array}$$

<u>Radiation Heat Transfer</u>

Matter (an intermediate medium) is not necessary for a surface to exchange thermal radiation with another surface

- Thermal radiation: 0.1 μm $\leq \lambda \leq$ 100 μm [λ is the wavelength]
- Visible light 0.4 μm $\leq \lambda \leq$ 0.7 μm [note: 1 μm = 10^{-6} m]

Black body

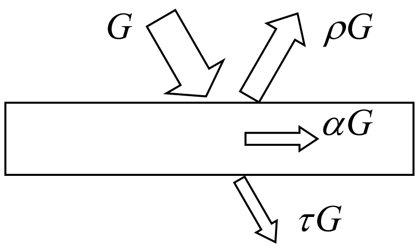
- Diffuse and ideal emitter and absorber;
- Emits radiation according to the **Stefan-Boltzmann law**: $e_b = \sigma T_{abs}^4$ [W/m^2]

σ : Stefan-Boltzmann constant = 5.669 x 10^{-8} W/m^2-K^4; T_{abs} : Absolute temperature [K]

Non-Black body: $e = \varepsilon e_b = \varepsilon \sigma T_{abs}^4$ [W/m²]; ε is the emissivity of the surface, and $0 \le \varepsilon \le 1$.

Basic Radiation Properties

(i) Emission of radiation: emissivity, ε; (ii) Absorption of radiation: absorptivity, α; (iii) Reflection of radiation: reflectivity, ρ; (iv) Transmission of radiation: transmissivity, τ



G: Total hemispherical incident radiation (irradiation) [W/m²]

ρG: Reflected radiation [W/m²]

αG: Absorbed radiation [W/m²]

τG; Transmitted radiation [W/m²]

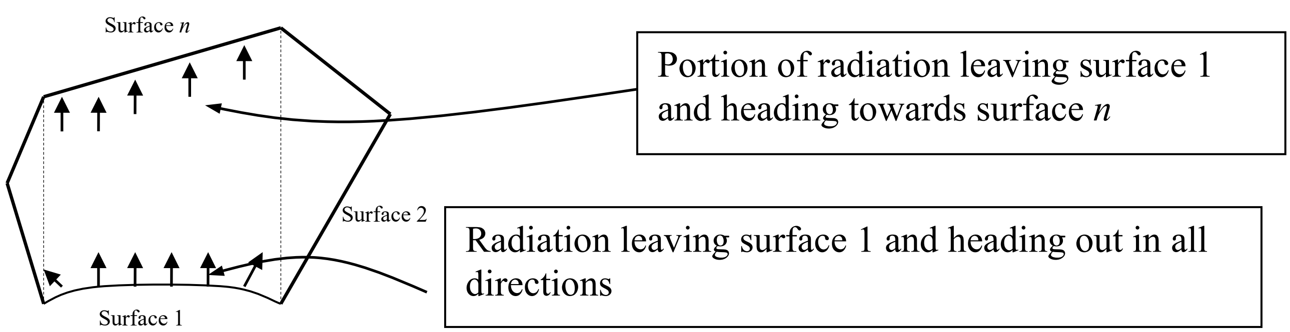
i) In general, $\rho + \alpha + \tau = 1$

Kirchoff's law: $\alpha_\lambda = \varepsilon_\lambda \;\} \Rightarrow \begin{cases} \text{For a gray body:} \\ \alpha = \alpha_\lambda \text{ and } \varepsilon = \varepsilon_\lambda; \text{thus, } \alpha = \varepsilon \end{cases}$

Gray Body: α_λ and ε_λ are independent of λ. Thus, $\alpha = \varepsilon$ (Kirchoff's law)

Radiation Shape Factors[Also called angle, configuration, and view factors]

Consider an enclosure of N isothermal surfaces:



- For a complete enclosure of N diffuse isothermal surfaces:

$$F_{1-1} + F_{1-2} + F_{1-3} + ----- + F_{1-n} + ----- F_{1-N} = 1$$

Thus, $\sum_{n=1}^{N} F_{1-n} = 1$ or $\sum_{n=1}^{N} F_{m-n} = 1 \quad \} \Rightarrow$ $\boxed{\text{Summation Relationship}}$

For any two gray, diffuse, isothermal surfaces:

$A_m F_{m-n} = A_n F_{n-m} \quad \} \Rightarrow$ $\boxed{\text{Reciprocity Relationship}}$

- Sample shape factors (view, configuration, angle factors) are given below:

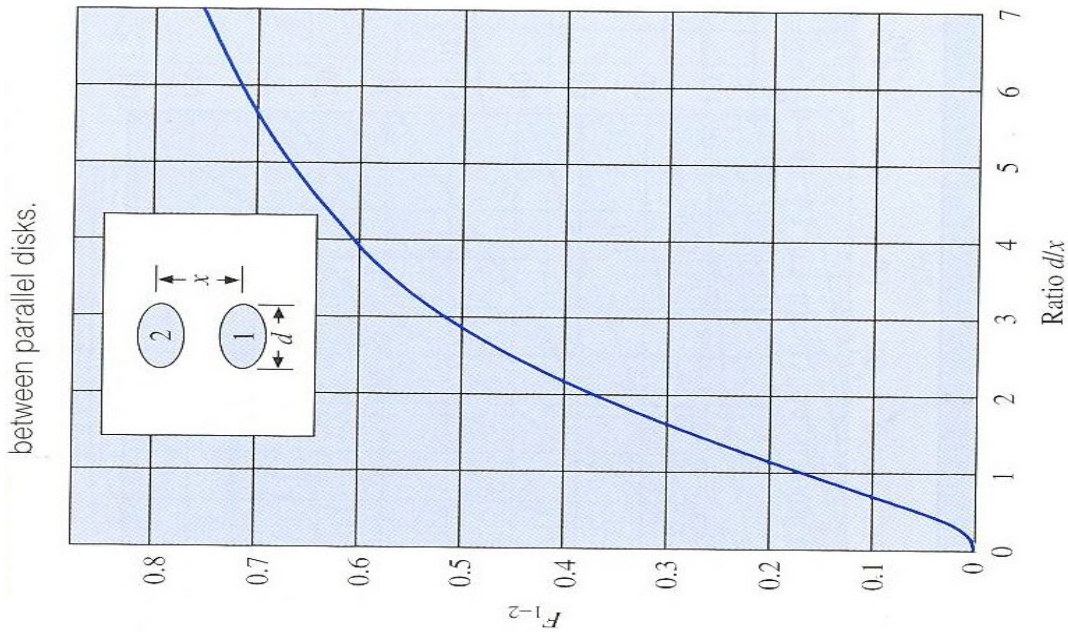
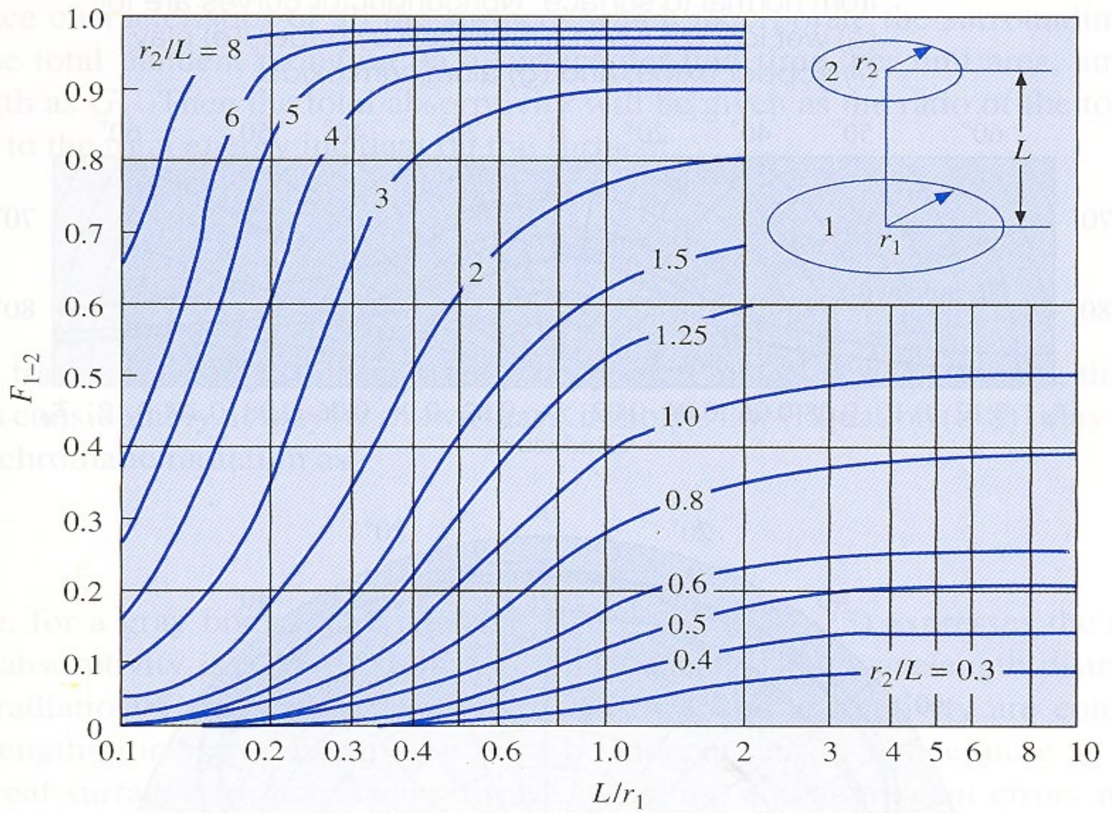
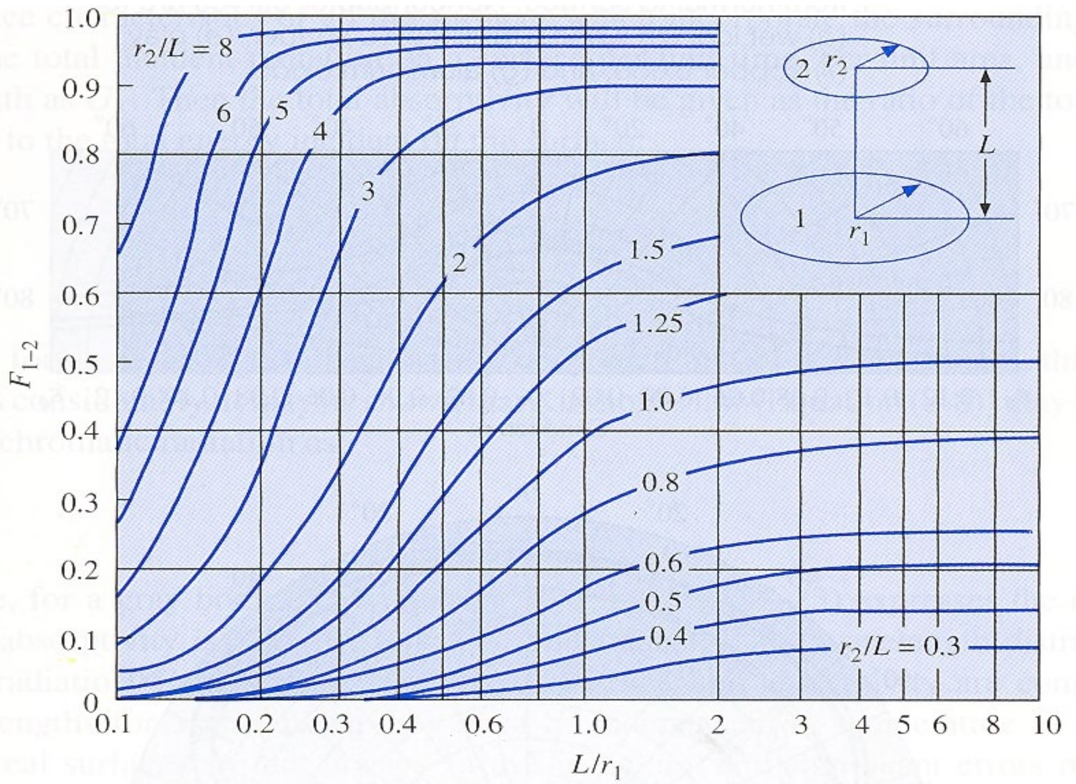
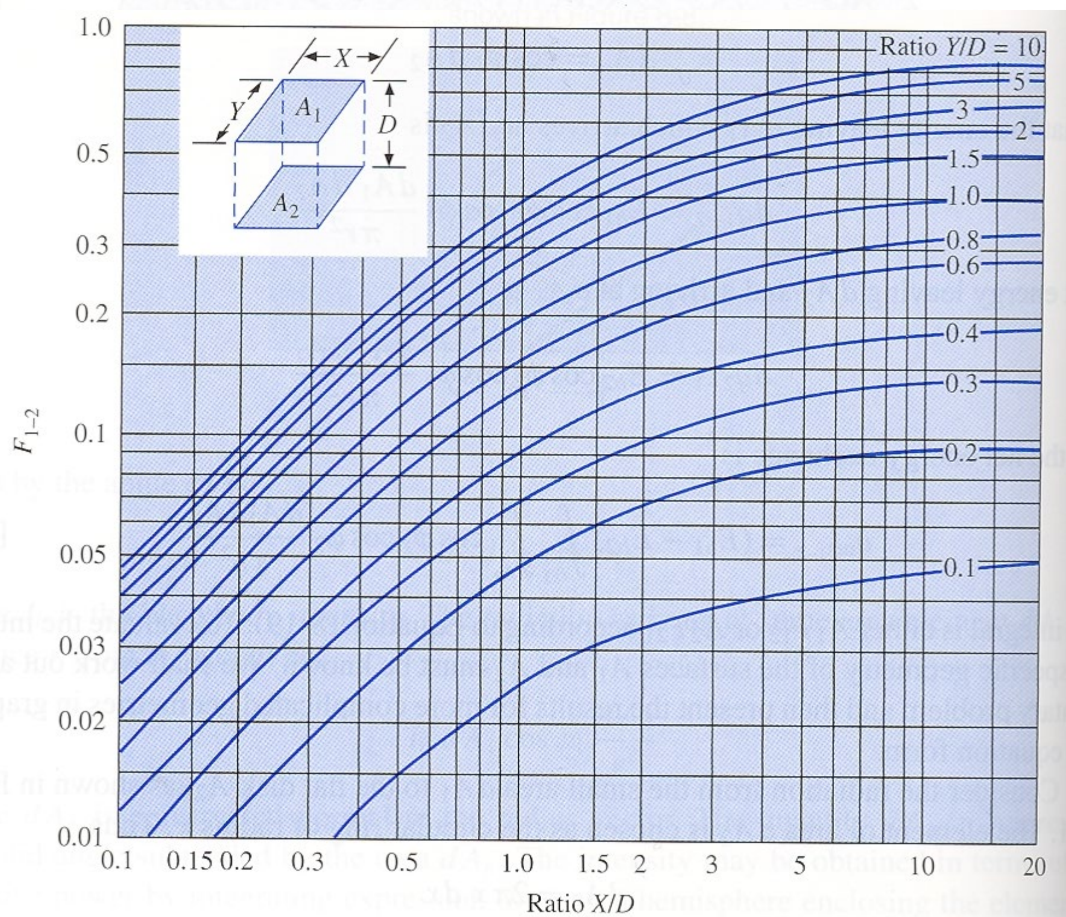


Figure 8-16 | Radiation shape factor for radiation between two parallel coaxial disks.



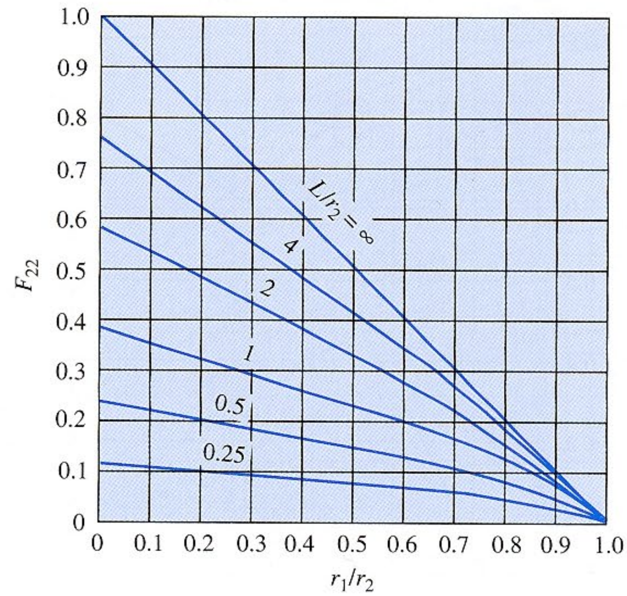


$r_2/L = 8$
6
5
4
3
2
1.5
1.25
1.0
0.8
0.6
0.5
0.4
$r_2/L = 0.3$
F_{1-2}
L/r_1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.1
0.2
0.3
0.4
0.6
1.0
2
3
4
5
6
8
10
2
r_2
L
1
r_1

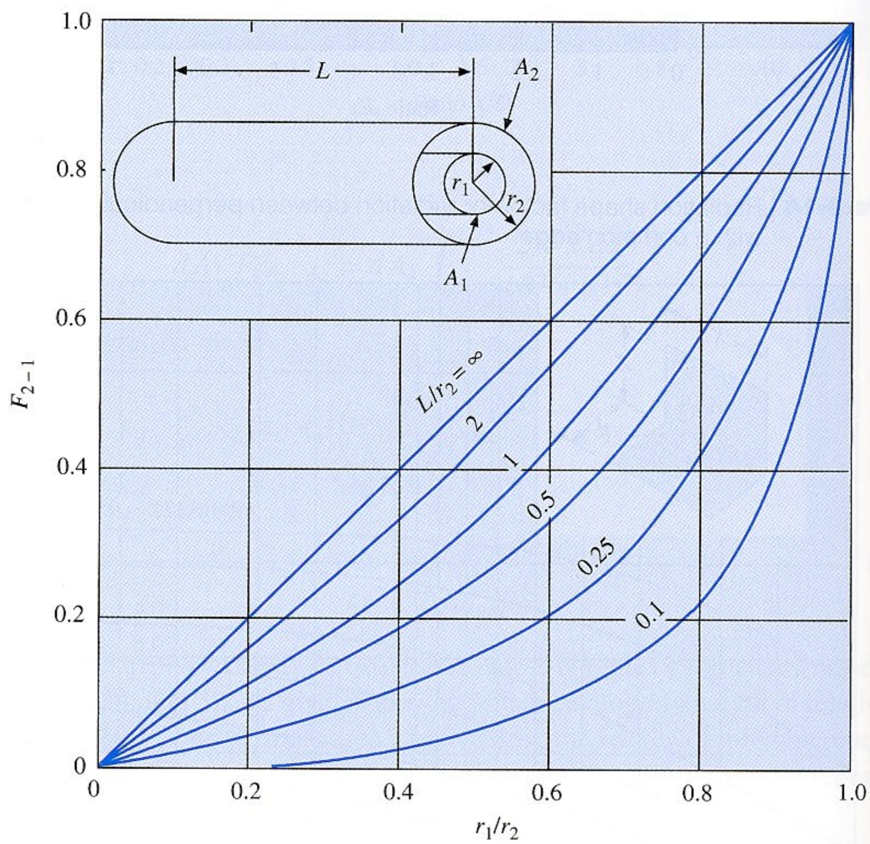


X
Y
A_1
D
A_2
Ratio Y/D = 10
5
3
2
1.5
1.0
0.8
0.6
0.4
0.3
0.2
0.1
F_{1-2}
Ratio X/D
1.0
0.5
0.3
0.2
0.1
0.05
0.03
0.02
0.01
0.1
0.15
0.2
0.3
0.5
1.0
1.5
2
3
5
10
20

Figure 8-15 | Radiation shape factors for two concentric cylinders of finite length. (*a*) Outer cylinder to itself; (*b*) outer cylinder to inner cylinder.



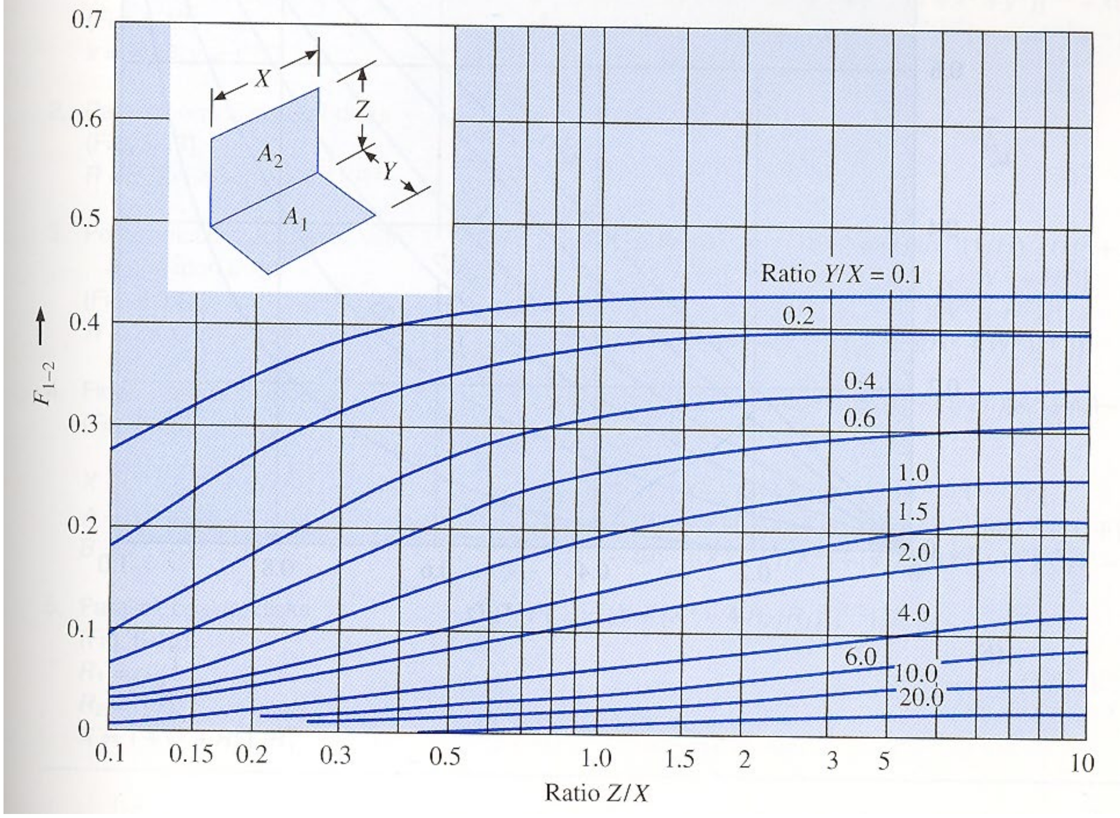
(*a*)



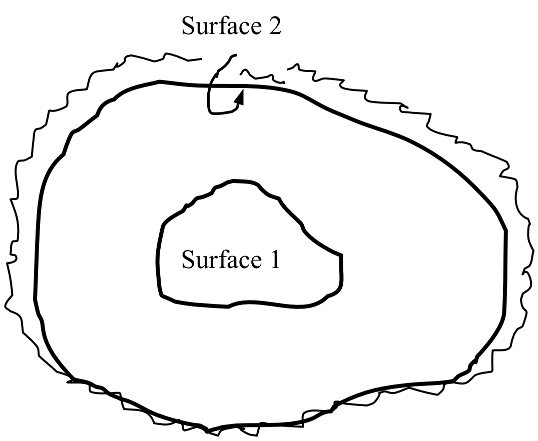
(*b*)

(Figures extracted from Heat Transfer by J.P. Holman, 9th Edition, 2002)

Figure 8-14 | Radiation shape factor for radiation between perpendicular rectangles with a common edge.



Radiation Heat Transfer between Two Black Surfaces Forming an Enclosure



Both black surfaces are isothermal:
$T_1 \neq T_2$

[Amount of radiation energy leaving surface 1 and intercepted, and absorbed, by surface 2] $= F_{1-2}\,(A_1\, e_{b,1})$

$$\underset{\text{net rad loss}}{q_2} = A_2\, F_{2-1}\,(e_{b,2} - e_{b,1}) = A_2\, F_{2-1}\,\sigma(T_{2,abs}^4 - T_{1,abs}^4)$$

$$\underset{\text{net rad loss}}{q_1} = A_1\, F_{1-2}\,(e_{b,1} - e_{b,2}) = A_1\, F_{1-2}\,\sigma(T_{1,abs}^4 - T_{2,abs}^4) = -\underset{\text{net rad loss}}{q_2}$$

Emissive Power of a Blackbody

Planck's radiation relation for a blackbody

Monochromatic emissive power of a blackbody:

$$e_{b,\lambda} = \frac{C_1}{\lambda^5 \left[\exp\left\{ \dfrac{C_2}{\lambda T_{abs}} \right\} - 1 \right]}$$

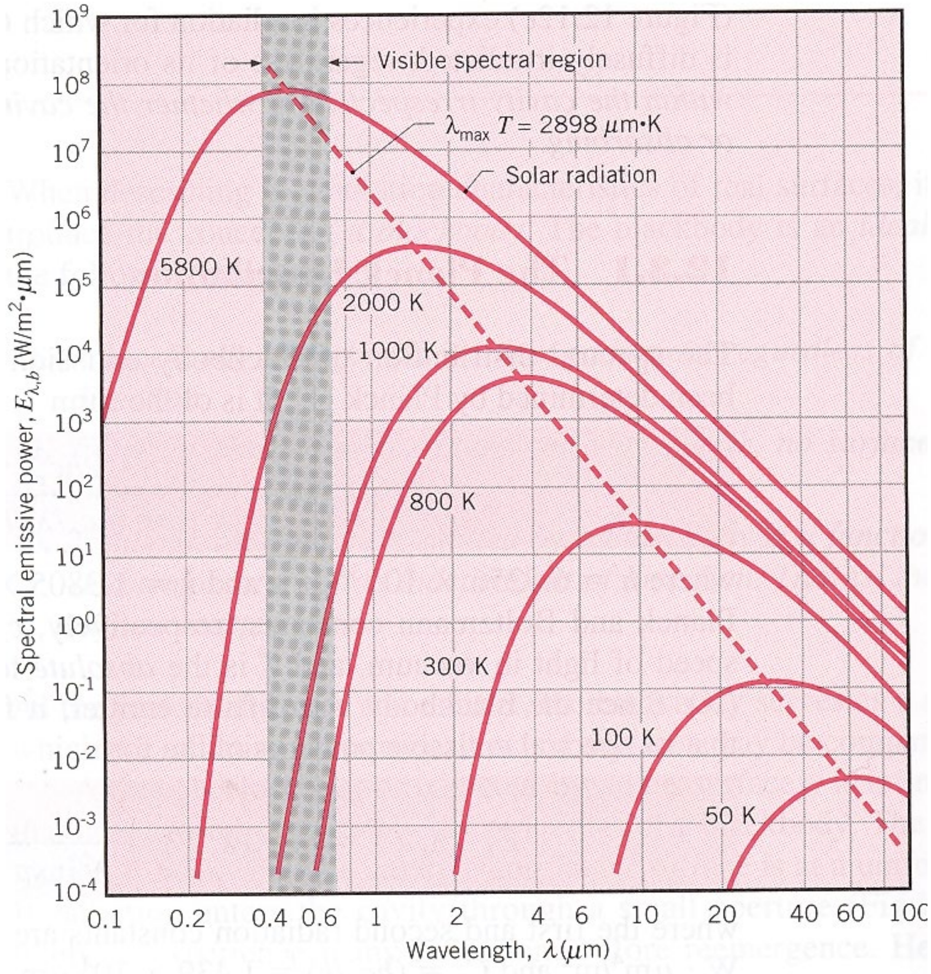
[W/m^3]

$C_1 = 3.7418 \text{ x } 10^{-16}$ W-m^2 : First radiation constant, and

$C_2 = 1.4388 \text{ x } 10^{-2}$ m-K : Second radiation constant

Wien's displacement law:

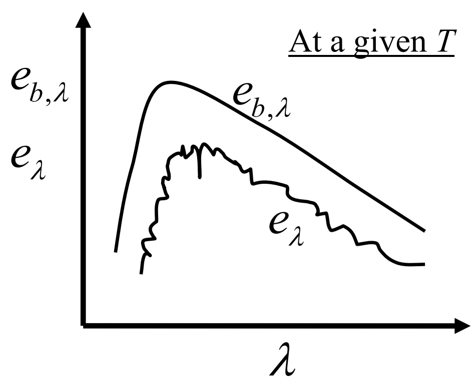
$$\lambda_{\max} T_{abs} = 2.8976 \times 10^{-3} \text{ m-K}$$
$$= 2897.6 \ \mu\text{m-K}$$



Hemispherical total emissive power of a blackbody:

$$e_b = \int_0^\infty e_{b,\lambda} d\lambda = \sigma T_{abs}^4 ; \ \sigma = (\pi / C_2)^4 (C_1/15) \ \text{ or } \sigma = 5.669 \text{ x } 10^{-8} \text{ [W/m}^2\text{-K}^4\text{]}$$

Emissive Power of a Real Body



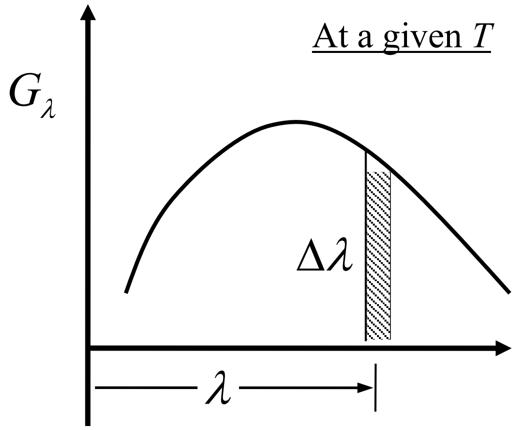
$\varepsilon_\lambda = (e_\lambda / e_{b,\lambda}) \} \Rightarrow$ Monochromatic hemispherical emissivity $0 \le \varepsilon_\lambda \le 1$

ε_λ depends on the wavelength, λ; but it is only a weak function of T

Total emissive power: $e = \int_0^\infty e_\lambda \, d\lambda$

Overall or total emissivity: $$\varepsilon = \frac{e}{e_b} = \frac{\int_0^\infty \varepsilon_\lambda \, e_{b,\lambda} \, d\lambda}{\sigma T_{abs}^4}$$

Absorption



G_λ: Irradiation [W/m^2-m]

G_λ: The rate at which radiation of wavelength λ is incident on a surface per unit area of the surface and per unit wavelength interval Δλ about λ.

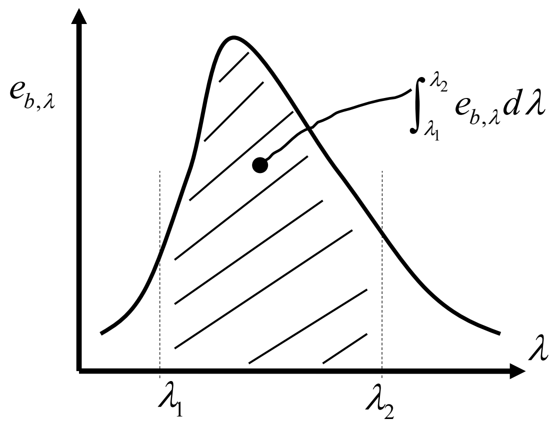
Surface may exhibit selective absorption with respect to the wavelength of the incident radiation.

Hemispherical monochromatic absorptivity } α_λ

For a blackbody: $\alpha_\lambda = \alpha = 1$

Total absorptivity: $\alpha = \int_0^\infty \alpha_\lambda G_\lambda d\lambda \Big/ \int_0^\infty G_\lambda d\lambda$

Blackbody Radiation Functions



$$\int_{\lambda_1}^{\lambda_2} e_{b,\lambda}\, d\lambda = \int_0^{\lambda_2} e_{b,\lambda}\, d\lambda - \int_0^{\lambda_1} e_{b,\lambda}\, d\lambda$$

Note: $\dfrac{\int_0^{\lambda} e_{b,\lambda}\, d\lambda}{\sigma T_{abs}^4} = fnc(\lambda T_{abs})$

Blackbody radiation function: $\dfrac{E_b(0 \to \lambda T_{abs})}{\sigma T_{abs}^4} = \dfrac{\int_0^{\lambda} e_{b,\lambda}\, d\lambda}{\sigma T_{abs}^4}$. Then,

$$\left[\begin{array}{l}\text{Fraction of total blackbody} \\ \text{radiation that lies in } \lambda_1 \le \lambda \le \lambda_2\end{array}\right] = \frac{E_b(0 \to \lambda_2 T_{abs})}{\sigma T_{abs}^4} - \frac{E_b(0 \to \lambda_1 T_{abs})}{\sigma T_{abs}^4}$$

$\left\{ \dfrac{E_b(0 \to \lambda T_{abs})}{\sigma T_{abs}^4} \right\} \to$ Tabulated as a function of (λT_{abs})

Table 8-1 | Radiation functions.

λT $\mu m \cdot K$	$\dfrac{E_{b\lambda}/T^5}{W}$ $m^2 \cdot K^5 \cdot \mu m \times 10^{11}$	$\dfrac{E_{b_{0-\lambda T}}}{\sigma T^4}$	λT $\mu m \cdot K$	$\dfrac{E_{b\lambda}/T^5}{W}$ $m^2 \cdot K^5 \cdot \mu m \times 10^{11}$	$\dfrac{E_{b_{0-\lambda T}}}{\sigma T^4}$
1000	0.02110	0.00032	6300	0.42760	0.76180
1100	0.04846	0.00091	6400	0.41128	0.76920
1200	0.09329	0.00213	6500	0.39564	0.77631
1300	0.15724	0.00432	6600	0.38066	0.78316
1400	0.23932	0.00779	6700	0.36631	0.78975
1500	0.33631	0.01285	6800	0.35256	0.79609
1600	0.44359	0.01972	6900	0.33940	0.80219
1700	0.55603	0.02853	7000	0.32679	0.80807
1800	0.66872	0.03934	7100	0.31471	0.81373
1900	0.77736	0.05210	7200	0.30315	0.81918
2000	0.87858	0.06672	7300	0.29207	0.82443
2100	0.96994	0.08305	7400	0.28146	0.82949
2200	1.04990	0.10088	7500	0.27129	0.83436
2300	1.11768	0.12002	7600	0.26155	0.83906
2400	1.17314	0.14025	7700	0.25221	0.84359
2500	1.21659	0.16135	7800	0.24326	0.84796
2600	1.24868	0.18311	7900	0.23468	0.85218
2700	1.27029	0.20535	8000	0.22646	0.85625
2800	1.28242	0.22788	8100	0.21857	0.86017
2900	1.28612	0.25055	8200	0.21101	0.86396
3000	1.28245	0.27322	8300	0.20375	0.86762
3100	1.27242	0.29576	8400	0.19679	0.87115
3200	1.25702	0.31809	8500	0.19011	0.87456
3300	1.23711	0.34009	8600	0.18370	0.87786
3400	1.21352	0.36172	8700	0.17755	0.88105
3500	1.18695	0.38290	8800	0.17164	0.88413
3600	1.15806	0.40359	8900	0.16596	0.88711
3700	1.12739	0.42375	9000	0.16051	0.88999
3800	1.09544	0.44336	9100	0.15527	0.89277
3900	1.06261	0.46240	9200	0.15024	0.89547
4000	1.02927	0.48085	9300	0.14540	0.89807
4100	0.99571	0.49872	9400	0.14075	0.90060
4200	0.96220	0.51599	9500	0.13627	0.90304
4300	0.92892	0.53267	9600	0.13197	0.90541
4400	0.89607	0.54877	9700	0.12783	0.90770
4500	0.86376	0.56429	9800	0.12384	0.90992
4600	0.83212	0.57925	9900	0.12001	0.91207
4700	0.80124	0.59366	10,000	0.11632	0.91415
4800	0.77117	0.60753	10,200	0.10934	0.91813
4900	0.74197	0.62088	10,400	0.10287	0.92188
5000	0.71366	0.63372	10,600	0.09685	0.92540
5100	0.68628	0.64606	10,800	0.09126	0.92872
5200	0.65983	0.65794	11,000	0.08606	0.93184
5300	0.63432	0.66935	11,200	0.08121	0.93479
5400	0.60974	0.68033	11,400	0.07670	0.93758
5500	0.58608	0.69087	11,600	0.07249	0.94021
5600	0.56332	0.70101	11,800	0.06856	0.94270
5700	0.54146	0.71076	12,000	0.06488	0.94505
5800	0.52046	0.72012	12,200	0.06145	0.94728
5900	0.50030	0.72913	12,400	0.05823	0.94939
6000	0.48096	0.73778	12,600	0.05522	0.95139
6100	0.46242	0.74610	12,800	0.05240	0.95329
6200	0.44464	0.75410	13,000	0.04976	0.95509

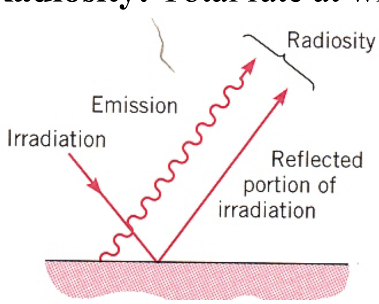
Table 8-1 | (*Continued*).

λT $\mu m \cdot K$	$E_{b\lambda}/T^5$ $\frac{W}{m^2 \cdot K^5 \cdot \mu m \times 10^{11}}$	$\frac{E_{b_0-\lambda T}}{\sigma T^4}$	λT $\mu m \cdot K$	$E_{b\lambda}/T^5$ $\frac{W}{m^2 \cdot K^5 \cdot \mu m \times 10^{11}}$	$\frac{E_{b_0-\lambda T}}{\sigma T^4}$
13,200	0.04728	0.95680	19,800	0.01151	0.98515
13,400	0.04494	0.95843	20,000	0.01110	0.98555
13,600	0.04275	0.95998	21,000	0.00931	0.98735
13,800	0.04069	0.96145	22,000	0.00786	0.98886
14,000	0.03875	0.96285	23,000	0.00669	0.99014
14,200	0.03693	0.96418	24,000	0.00572	0.99123
14,400	0.03520	0.96546	25,000	0.00492	0.99217
14,600	0.03358	0.96667	26,000	0.00426	0.99297
14,800	0.03205	0.96783	27,000	0.00370	0.99367
15,000	0.03060	0.96893	28,000	0.00324	0.99429
15,200	0.02923	0.96999	29,000	0.00284	0.99482
15,400	0.02794	0.97100	30,000	0.00250	0.99529
15,600	0.02672	0.97196	31,000	0.00221	0.99571
15,800	0.02556	0.97288	32,000	0.00196	0.99607
16,000	0.02447	0.97377	33,000	0.00175	0.99640
16,200	0.02343	0.97461	34,000	0.00156	0.99669
16,400	0.02245	0.97542	35,000	0.00140	0.99695
16,600	0.02152	0.97620	36,000	0.00126	0.99719
16,800	0.02063	0.97694	37,000	0.00113	0.99740
17,000	0.01979	0.97765	38,000	0.00103	0.99759
17,200	0.01899	0.97834	39,000	0.00093	0.99776
17,400	0.01823	0.97899	40,000	0.00084	0.99792
17,600	0.01751	0.97962	41,000	0.00077	0.99806
17,800	0.01682	0.98023	42,000	0.00070	0.99819
18,000	0.01617	0.98081	43,000	0.00064	0.99831
18,200	0.01555	0.98137	44,000	0.00059	0.99842
18,400	0.01496	0.98191	45,000	0.00054	0.99851
18,600	0.01439	0.98243	46,000	0.00049	0.99861
18,800	0.01385	0.98293	47,000	0.00046	0.99869
19,000	0.01334	0.98340	48,000	0.00042	0.99877
19,200	0.01285	0.98387	49,000	0.00039	0.99884
19,400	0.01238	0.98431	50,000	0.00036	0.99890
19,600	0.01193	0.98474			

Radiation Heat Transfer between Diffuse-Gray Surfaces in an Enclosure

Irradiation: Total rate of incident radiation on a surface p per unit area $\rightarrow G$ [W/m^2]

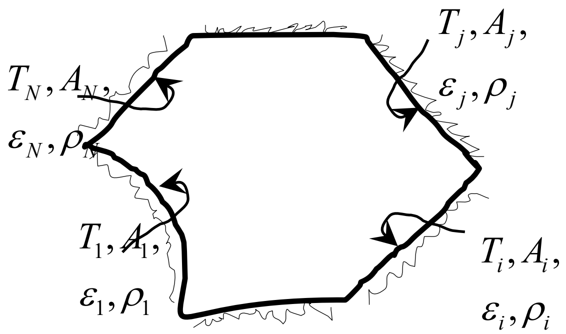
Radiosity: Total rate at which radiation leaves a surface per unit area $\rightarrow J$ [W/m^2]



For a given surface:

$$J = \varepsilon e_b + \rho G$$

Enclosure schematic:

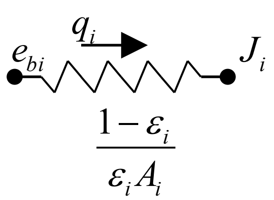


Total number of surfaces: N

Assumptions: [steady-state]

1. All surfaces are diffuse-gray $[\alpha_\lambda = \alpha; \varepsilon_\lambda = \varepsilon; \alpha = \varepsilon]$
2. Each surface is uniform in temperature (isothermal)
3. Reflectivity, ρ, and emissivity, ε, are constant over each surface
4. All surfaces are opaque ($\tau = 0$)
5. G and J are uniform over each surface

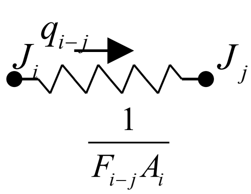
Surface radiation resistance:



$$q_i^{Net\,Rad} = \left(e_{b,i} - J_i\right) \Big/ \left(\frac{1-\varepsilon_i}{\varepsilon_i A_i}\right)$$

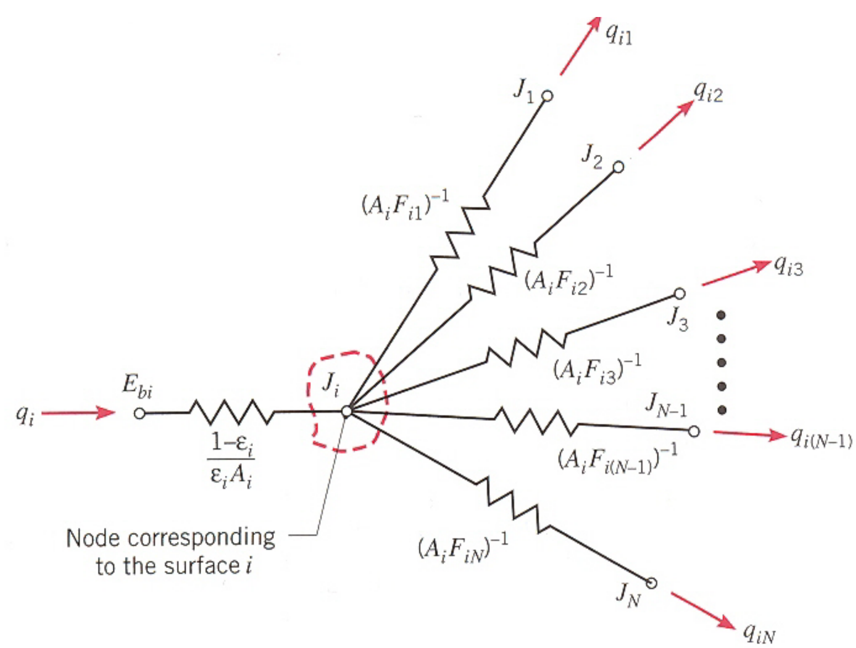
Note: for a Blackbody $\varepsilon_i = 1$ and $e_{b,i} = \sigma T_i^4 = J_i$

Space or geometrical radiation resistance:



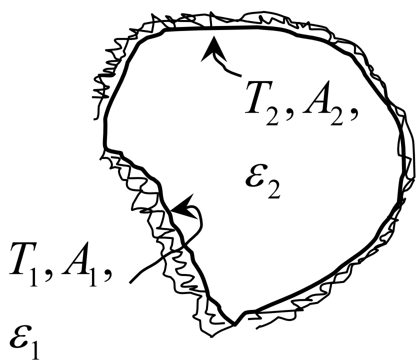
$$q_{i \to j}^{Net\,Rad} = \frac{(J_i - J_j)}{1/(A_i F_{i-j})} = \frac{(J_i - J_j)}{1/(A_j F_{j-i})}$$

Network representation of radiation exchange between surface *i* and all other surfaces of the enclosure:



$$q_i^{Net\,Rad} = \frac{e_{b,i} - J_i}{(1-\varepsilon_i)/(A_i \varepsilon_i)} = \sum_{j=1}^{N} \frac{J_i - J_j}{\{1/(A_i F_{i-j})\}}$$

Two-Surface Diffuse-Gray Enclosure



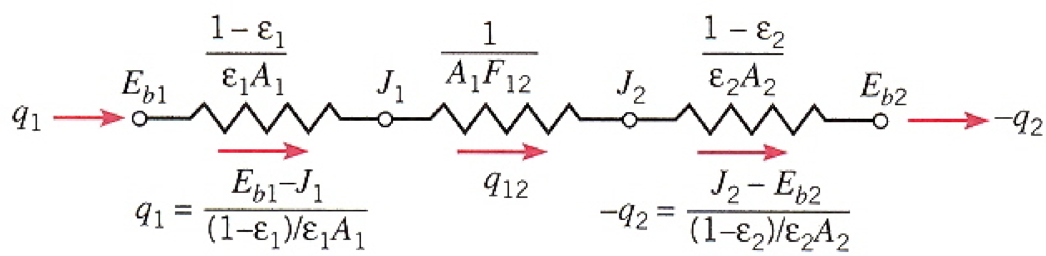
Notes:

1) $q_1^{Net\,Rad} = q_{1-2}^{Net\,Rad} = -q_{2-1}^{Net\,Rad}$

2) $e_{b,1} = \sigma T_{1,abs}^4$

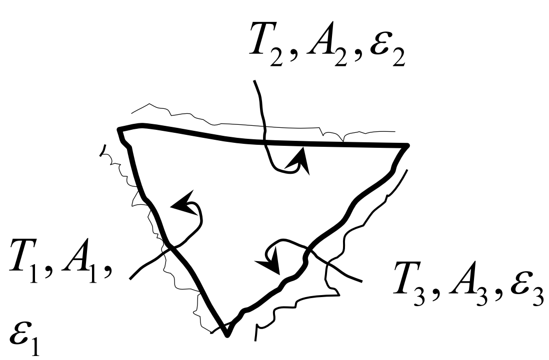
3) $e_{b,2} = \sigma T_{2,abs}^4$

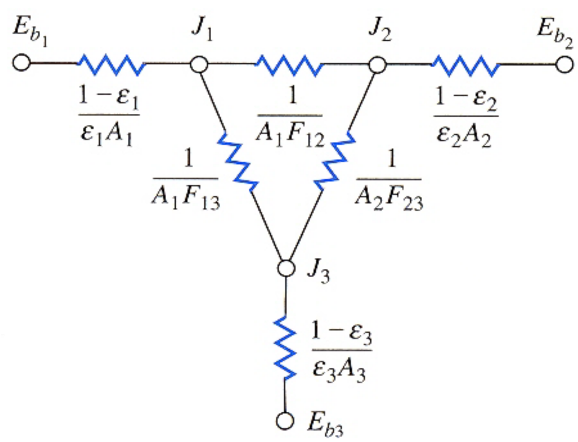
Equivalent circuit:



Here: $q_1^{Net\,Rad} = -q_2^{Net\,Rad} = \dfrac{e_{b,1}-e_{b,2}}{\dfrac{1-\varepsilon_1}{A_1\varepsilon_1}+\dfrac{1}{A_1F_{12}}+\dfrac{1-\varepsilon_2}{A_2\varepsilon_2}}$; Special cases are given in figures below:

Three-Surface Diffuse-Gray Enclosures





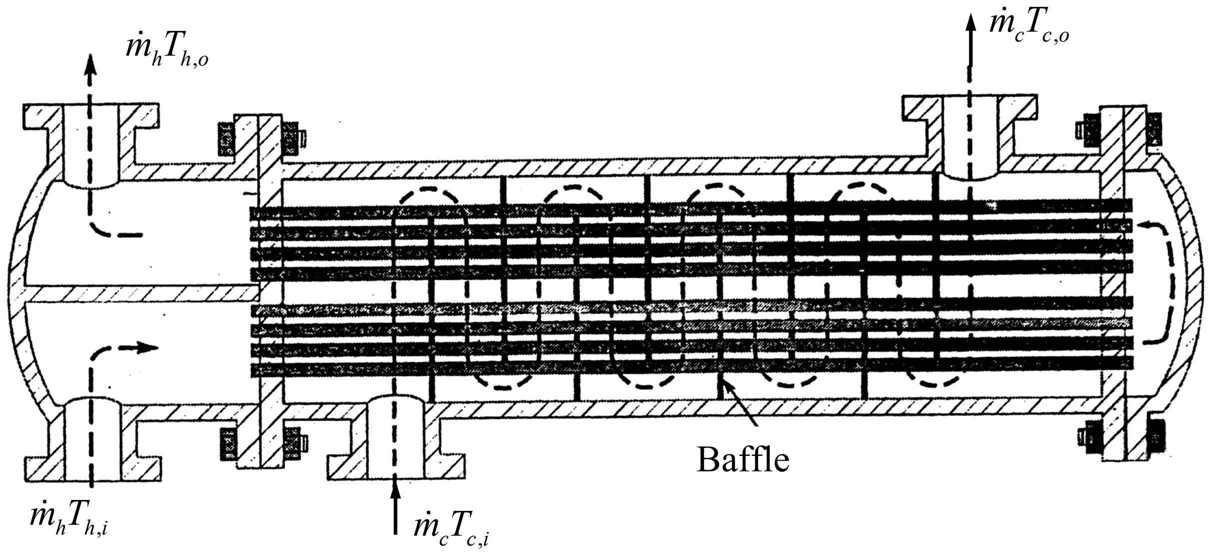
NOTE: i) Net flow of current to any one node = 0. For example, for Node 1:

$$\frac{e_{b,1}-J_1}{\dfrac{1-\varepsilon_1}{A_1\varepsilon_1}}+\frac{J_2-J_1}{\dfrac{1}{A_1F_{12}}}+\frac{J_3-J_1}{\dfrac{1}{A_1F_{13}}}=0;$$

ii) If one of the surfaces be perfectly insulated from one side, and convection effects on the other side be negligible, the surface is considered as *reradiating surface.* In this case, the net radiation transfer for the reradiating surface is zero, i.e., $q^{net}_{\substack{\text{Rerad.}\\\text{surf.}}} = 0$, and

$e_{b,\substack{\text{Rerad.}\\\text{surf.}}} = J_{\substack{\text{Rerad.}\\\text{surf.}}}$.

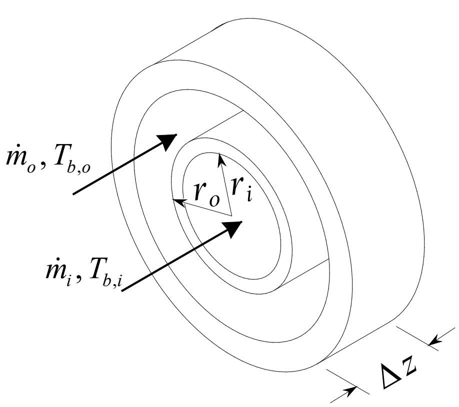
Heat Exchangers



Typical Assumptions: 1) Heat loss to the ambient (outside) negligible; 2) Ec_c<<1; and Ec_h<<1 (viscous dissipation negligible); 3) Enthalpy changes are due essentially to bulk temperature changes; 4) Steady-state conditions.

The Energy Balance: $q_{\substack{hot\to cold\\fluid}} = \dot{m}_c c_c (T_{c,o} - T_{c,i}) = \dot{m}_h c_h (T_{h,i} - T_{h,o})$

Overall Heat Transfer Coefficient:



$$\frac{1}{U_o} = \frac{1}{h_o} + r_o \frac{\ln(r_o / r_i)}{k_{tube}} + \frac{r_o}{r_i}\frac{1}{h_i}$$

$$\frac{1}{U_i} = \frac{r_i}{r_o}\frac{1}{h_o} + r_i \frac{\ln(r_o / r_i)}{k_{tube}} + \frac{1}{h_i}$$

U_o : Overall H.Tr. coef. based on outer surface [W/m^2 – °C]

U_i : Overall H.Tr. coef. based on inner surface [W/m^2 – °C]

Fouling factor:

$$R_{Foul} \triangleq \frac{1}{U_{Foul}} - \frac{1}{U_{Clean}} \qquad [\text{m}^2\text{-°C/W}]$$

Heat Exchanger Analysis [steady-state]

Two main methods are used: (i) the LMTD method; and (ii) the effectiveness-number-of-transfer-units (ε-NTU) method

LMTD method

(a) Parallel-flow (or co-current) heat exchangers

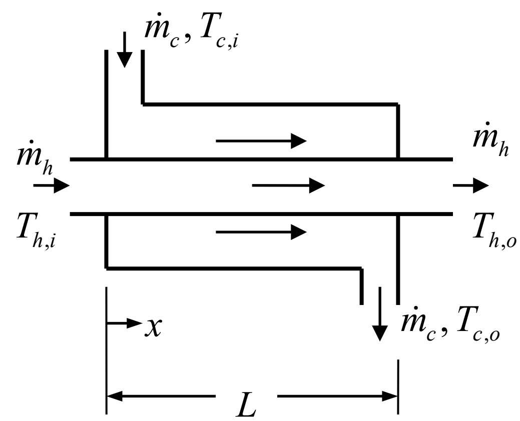
$$\left.\begin{aligned} q_{total} &= \dot{m}_h c_h \left(T_{h,i} - T_{h,o}\right) = \dot{m}_c c_c \left(T_{c,o} - T_{c,i}\right) \\ \text{or} \quad q_{total} &= C_h \left(T_{h,i} - T_{h,o}\right) = C_c \left(T_{c,o} - T_{c,i}\right) \end{aligned}\right\} \rightarrow \text{(A)}$$

$$q_{total} = (UA)(\Delta T)_{LMTD} \; \} \rightarrow \text{(B)}$$

$$(\Delta T)_{LMTD} = \frac{\left(T_{h,i} - T_{c,i}\right) - \left(T_{h,o} - T_{c,o}\right)}{\ln\left[\left(T_{h,i} - T_{c,i}\right) / \left(T_{h,o} - T_{c,o}\right)\right]}$$

here, U: Overall heat transfer coefficient [W/m^2 – °C]

A: Corresponding surface area [m^2]



Heat Capacity Rate

$C_c = \dot{m}_c c_c$ [W/°C]

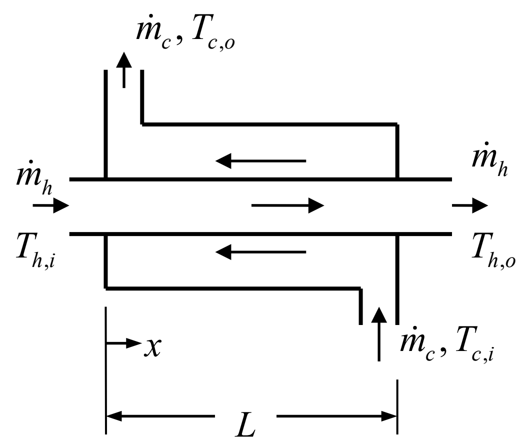
$C_h = \dot{m}_h c_h$

(b) Counter-flow heat exchangers

$$\left.\begin{aligned} q_{total} &= \dot{m}_h c_h \left(T_{h,i} - T_{h,o}\right) = \dot{m}_c c_c \left(T_{c,o} - T_{c,i}\right) \\ \text{or} \quad q_{total} &= C_h \left(T_{h,i} - T_{h,o}\right) = C_c \left(T_{c,o} - T_{c,i}\right) \end{aligned}\right\} \rightarrow \text{(A)}$$

$$q_{total} = (UA)(\Delta T)_{LMTD} \} \rightarrow \text{(B)}$$

$$(\Delta T)_{LMTD} = \frac{\left(T_{h,i} - T_{c,o}\right) - \left(T_{h,o} - T_{c,i}\right)}{\ln\left[\left(T_{h,i} - T_{c,o}\right) / \left(T_{h,o} - T_{c,i}\right)\right]}$$



ε-NTU method
(First proposed by Nusselt; extensively developed by Kays and London)

Effectiveness, $\varepsilon = \dfrac{\text{[Actual rate of heat transfer]}}{\text{[Max. possible rate of heat transfer from the hot fluid stream to the cold one]}}$

$$\varepsilon = \begin{cases} \{(T_{c,o} - T_{c,i})/(T_{h,i} - T_{c,i})\} & when \;\; \dot{m}_c c_c < \dot{m}_h c_h \\ \{(T_{h,i} - T_{h,o})/(T_{h,i} - T_{c,i})\} & when \;\; \dot{m}_h c_h < \dot{m}_c c_c \end{cases} \quad \text{or} \quad \varepsilon = \frac{|\Delta T_b|_{\text{fluid stream with } (\dot{m}c)_{\min}}}{|\Delta T_b|_{\text{Max. in heat exchanger}}}$$

$$\left.\begin{array}{l}\text{Number of}\\ \text{Transfer}\\ \text{Units}\end{array}\right\} \quad \text{NTU} = \left\{ \frac{\text{Heat transfer rate per degree of } \Delta T_{\text{LMTD}}}{\text{Heat transfer rate per degree 'rise' in temperature of the fluid stream with } (\dot{m}c)_{\min}} \right\} = \frac{UA}{(\dot{m}c)_{\min}}$$

Minimum capacity rate ratio} $R_{\min} = \dfrac{(\dot{m}c)_{\min}}{(\dot{m}c)_{\max}} = \dfrac{C_{\min}}{C_{\max}}$

It can be shown that: $\varepsilon = fnc(NTU, R_{\min}, \text{Geometry, Flow Arrangement})$

Notes:

1. For all heat exchangers (regardless of configuration / flow arrangement) when $R_{min} = 0$

$$\varepsilon = 1 - \exp(-NTU)$$

2. ε-NTU design charts for selected heat exchanger geometries and flow arrangements are also shown on the next pages.

Figure 10-13 | Effectiveness for counterflow exchanger performance.

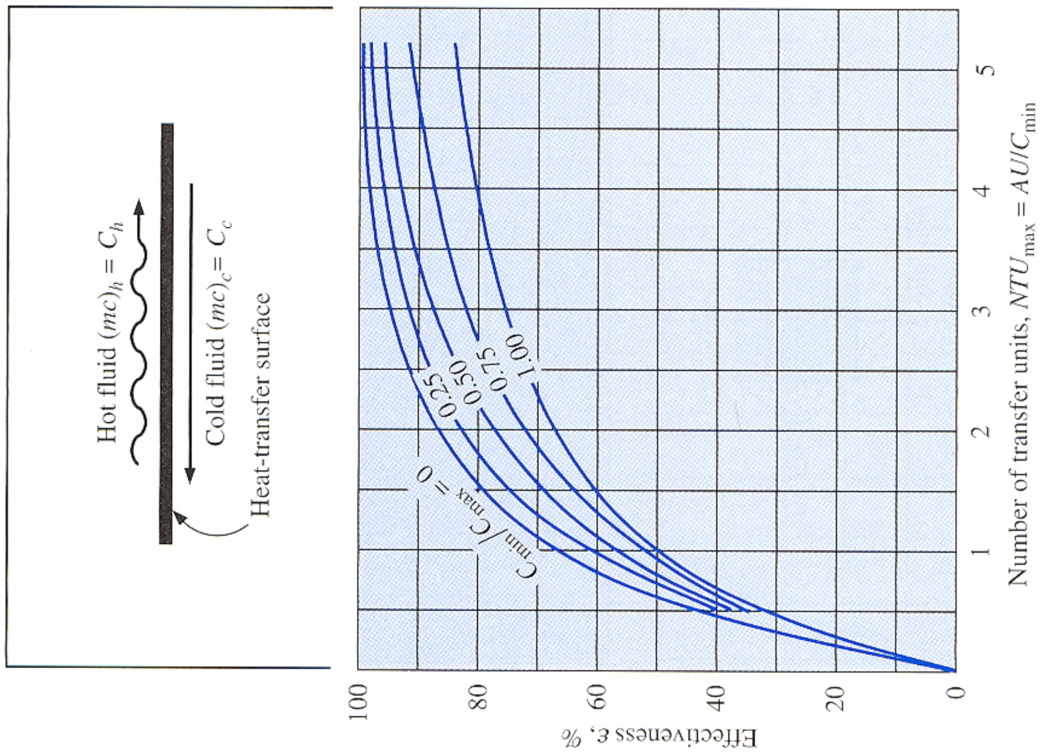
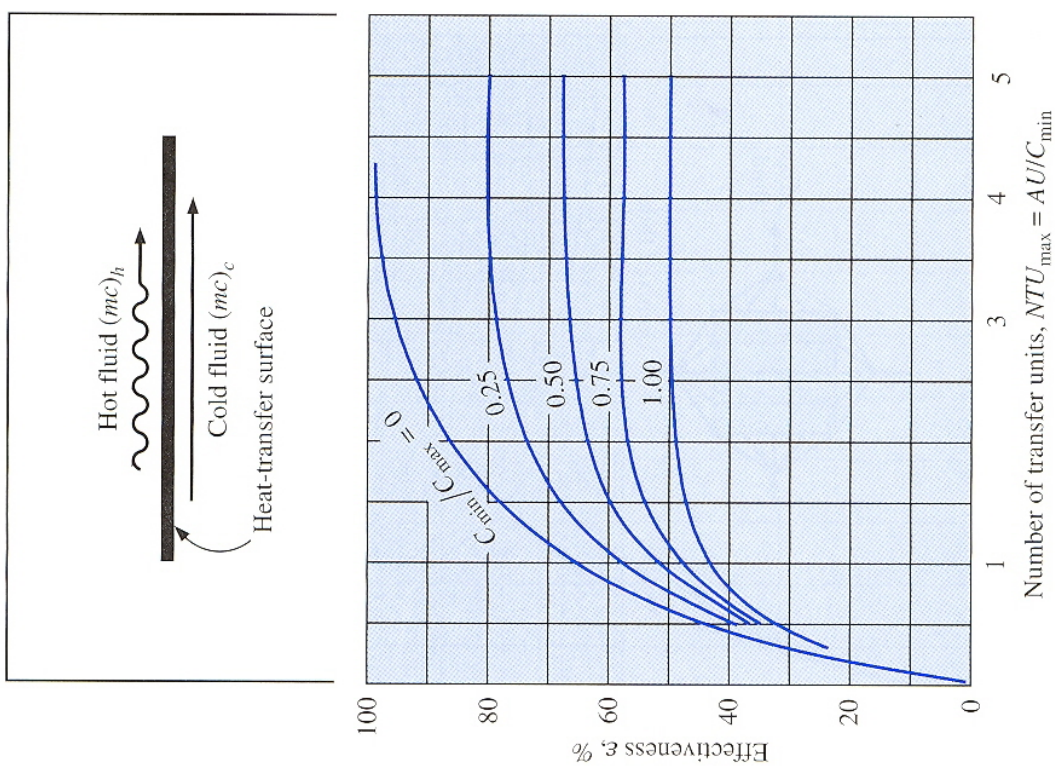


Figure 10-12 | Effectiveness for parallel-flow exchanger performance.



(Figure extracted from Heat Transfer by J.P. Holman, 9th Edition, 2002)

Figure 10-16 | Effectiveness for 1-2 parallel counterflow exchanger performance.

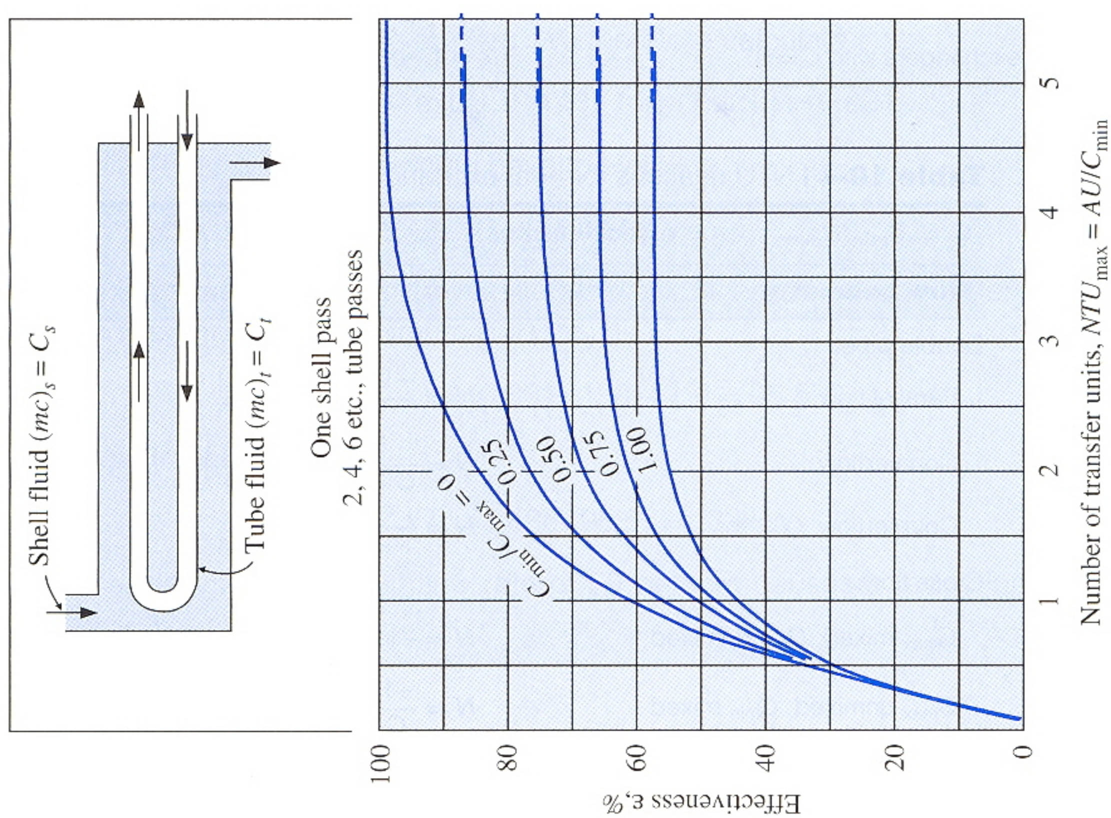


Figure 10-17 | Effectiveness for 2-4 multipass counterflow exchanger performance.

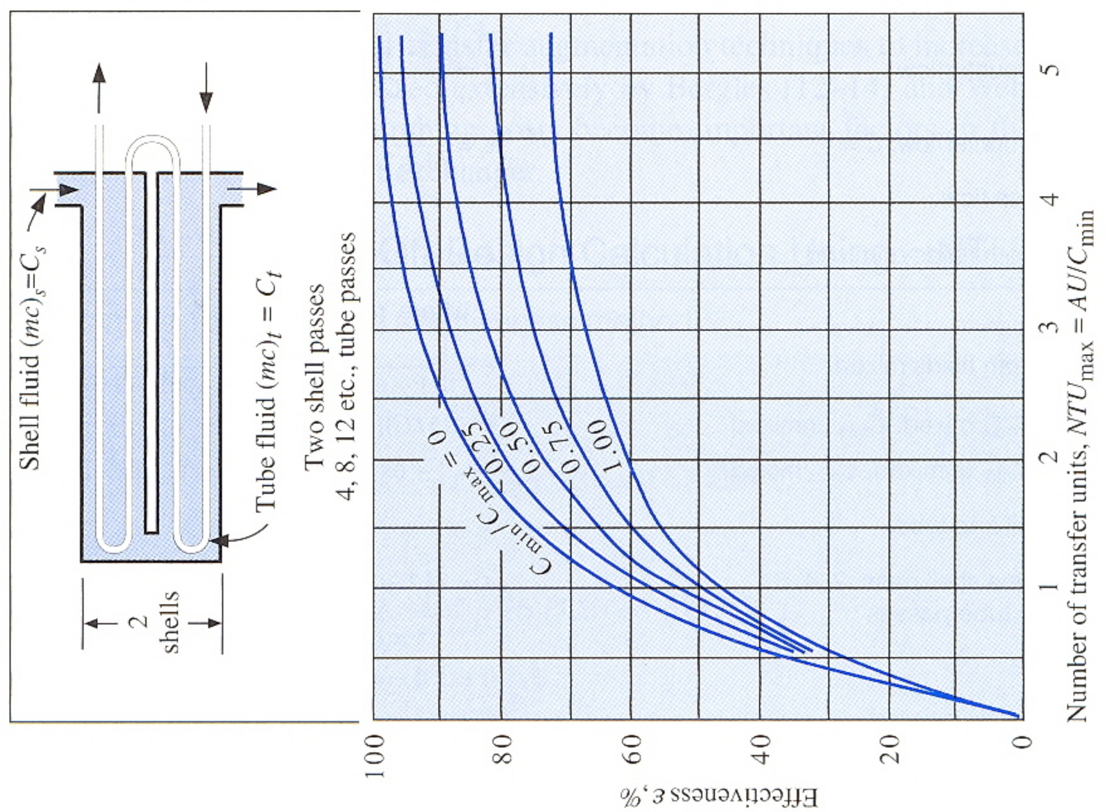


Figure 10-14 | Effectiveness for cross-flow exchanger with one fluid mixed.

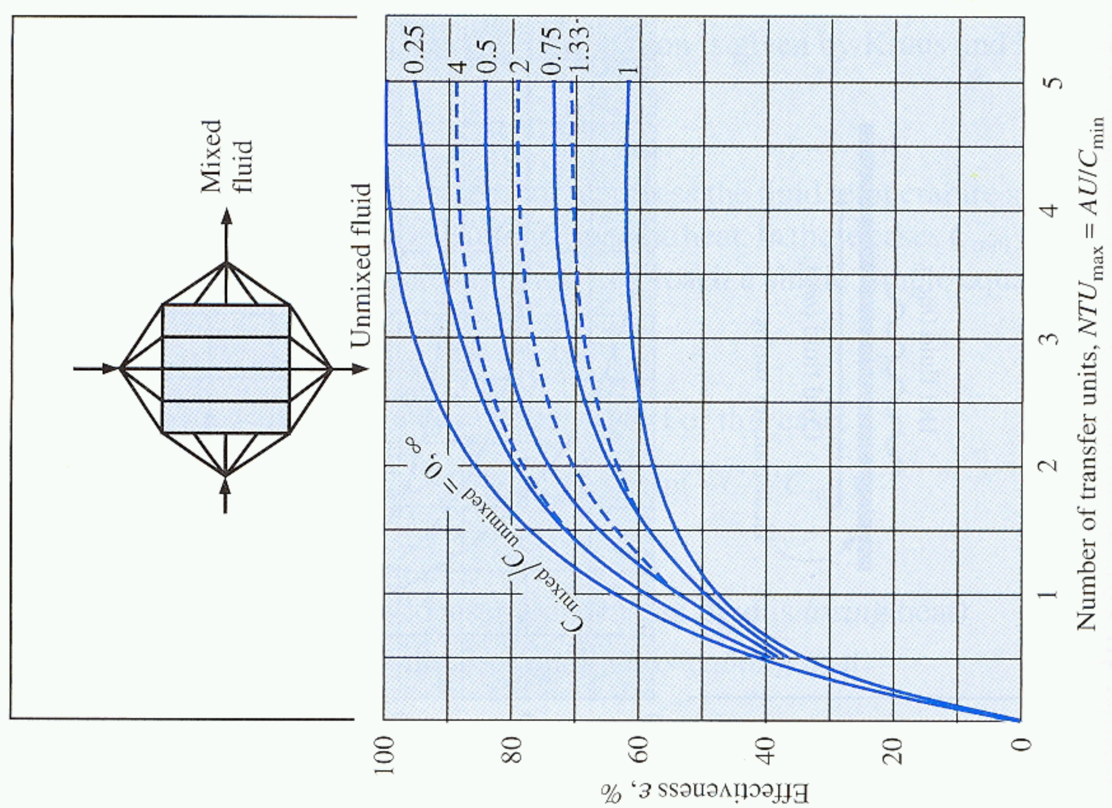
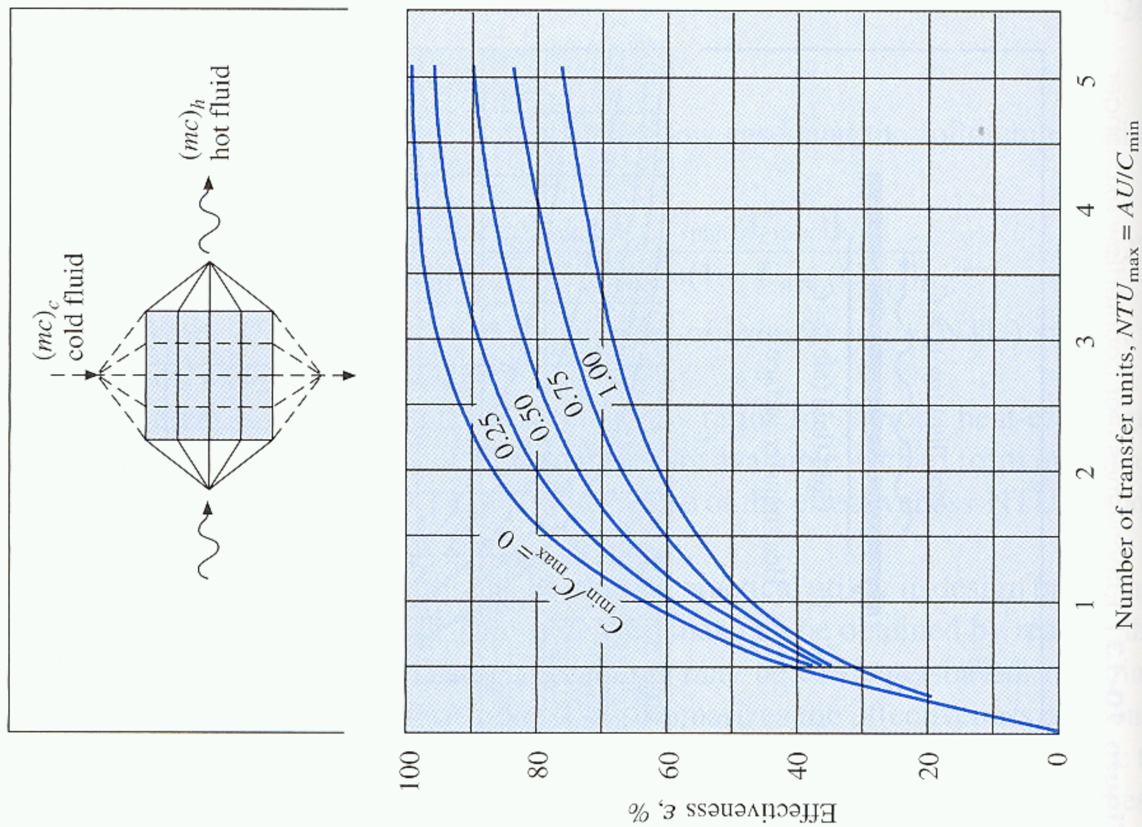


Figure 10-15 | Effectiveness for cross-flow exchanger with fluids unmixed.



Thermophysical properties of saturated water

Temperature, T (K)	Pressure, p (bars)[b]	Specific Volume (m³/kg) $v_f \cdot 10^3$	v_g	Heat of Vaporization, h_{fg} (kJ/kg)	Specific Heat (kJ/kg · K) $c_{p,f}$	$c_{p,g}$	Viscosity (N · s/m²) $\mu_f \cdot 10^6$	$\mu_g \cdot 10^6$	Thermal Conductivity (W/m · K) $k_f \cdot 10^3$	$k_g \cdot 10^3$	Prandtl Number Pr_f	Pr_g	Surface Tension, $\sigma_f \cdot 10^3$ (N/m)	Expansion Coefficient, $\beta_f \cdot 10^6$ (K⁻¹)
273.15	0.00611	1.000	206.3	2502	4.217	1.854	1750	8.02	569	18.2	12.99	0.815	75.5	−68.05
275	0.00697	1.000	181.7	2497	4.211	1.855	1652	8.09	574	18.3	12.22	0.817	75.3	−32.74
280	0.00990	1.000	130.4	2485	4.198	1.858	1422	8.29	582	18.6	10.26	0.825	74.8	46.04
285	0.01387	1.000	99.4	2473	4.189	1.861	1225	8.49	590	18.9	8.81	0.833	74.3	114.1
290	0.01917	1.001	69.7	2461	4.184	1.864	1080	8.69	598	19.3	7.56	0.841	73.7	174.0
295	0.02617	1.002	51.94	2449	4.181	1.868	959	8.89	606	19.5	6.62	0.849	72.7	227.5
300	0.03531	1.003	39.13	2438	4.179	1.872	855	9.09	613	19.6	5.83	0.857	71.7	276.1
305	0.04712	1.005	29.74	2426	4.178	1.877	769	9.29	620	20.1	5.20	0.865	70.9	320.6
310	0.06221	1.007	22.93	2414	4.178	1.882	695	9.49	628	20.4	4.62	0.873	70.0	361.9
315	0.08132	1.009	17.82	2402	4.179	1.888	631	9.69	634	20.7	4.16	0.883	69.2	400.4
320	0.1053	1.011	13.98	2390	4.180	1.895	577	9.89	640	21.0	3.77	0.894	68.3	436.7
325	0.1351	1.013	11.06	2378	4.182	1.903	528	10.09	645	21.3	3.42	0.901	67.5	471.2
330	0.1719	1.016	8.82	2366	4.184	1.911	489	10.29	650	21.7	3.15	0.908	66.6	504.0
335	0.2167	1.018	7.09	2354	4.186	1.920	453	10.49	656	22.0	2.88	0.916	65.8	535.5
340	0.2713	1.021	5.74	2342	4.188	1.930	420	10.69	660	22.3	2.66	0.925	64.9	566.0
345	0.3372	1.024	4.683	2329	4.191	1.941	389	10.89	668	22.6	2.45	0.933	64.1	595.4
350	0.4163	1.027	3.846	2317	4.195	1.954	365	11.09	668	23.0	2.29	0.942	63.2	624.2
355	0.5100	1.030	3.180	2304	4.199	1.968	343	11.29	671	23.3	2.14	0.951	62.3	652.3
360	0.6209	1.034	2.645	2291	4.203	1.983	324	11.49	674	23.7	2.02	0.960	61.4	697.9
365	0.7514	1.038	2.212	2278	4.209	1.999	306	11.69	677	24.1	1.91	0.969	60.5	707.1
370	0.9040	1.041	1.861	2265	4.214	2.017	289	11.89	679	24.5	1.80	0.978	59.5	728.7
373.15	1.0133	1.044	1.679	2257	4.217	2.029	279	12.02	680	24.8	1.76	0.984	58.9	750.1
375	1.0815	1.045	1.574	2252	4.220	2.036	274	12.09	681	24.9	1.70	0.987	58.6	761
380	1.2869	1.049	1.337	2239	4.226	2.057	260	12.29	683	25.4	1.61	0.999	57.6	788
385	1.5233	1.053	1.142	2225	4.232	2.080	248	12.49	685	25.8	1.53	1.004	56.6	814
390	1.794	1.058	0.980	2212	4.239	2.104	237	12.69	686	26.3	1.47	1.013	55.6	841
400	2.455	1.067	0.731	2183	4.256	2.158	217	13.05	688	27.2	1.34	1.033	53.6	896
410	3.302	1.077	0.553	2153	4.278	2.221	200	13.42	688	28.2	1.24	1.054	51.5	952
420	4.370	1.088	0.425	2123	4.302	2.291	185	13.79	688	29.8	1.16	1.075	49.4	1010
430	5.699	1.099	0.331	2091	4.331	2.369	173	14.14	685	30.4	1.09	1.10	47.2	

TABLE A.4 Thermophysical Properties of Gases at Atmospheric Pressure[a]

T (K)	ρ (kg/m^3)	c_p (kJ/kg · K)	$\mu \cdot 10^7$ (N · s/m^2)	$\nu \cdot 10^6$ (m^2/s)	$k \cdot 10^3$ (W/m · K)	$\alpha \cdot 10^6$ (m^2/s)	Pr
Air							
100	3.5562	1.032	71.1	2.00	9.34	2.54	0.786
150	2.3364	1.012	103.4	4.426	13.8	5.84	0.758
200	1.7458	1.007	132.5	7.590	18.1	10.3	0.737
250	1.3947	1.006	159.6	11.44	22.3	15.9	0.720
300	1.1614	1.007	184.6	15.89	26.3	22.5	0.707
350	0.9950	1.009	208.2	20.92	30.0	29.9	0.700
400	0.8711	1.014	230.1	26.41	33.8	38.3	0.690
450	0.7740	1.021	250.7	32.39	37.3	47.2	0.686
500	0.6964	1.030	270.1	38.79	40.7	56.7	0.684
550	0.6329	1.040	288.4	45.57	43.9	66.7	0.683
600	0.5804	1.051	305.8	52.69	46.9	76.9	0.685
650	0.5356	1.063	322.5	60.21	49.7	87.3	0.690
700	0.4975	1.075	338.8	68.10	52.4	98.0	0.695
750	0.4643	1.087	354.6	76.37	54.9	109	0.702
800	0.4354	1.099	369.8	84.93	57.3	120	0.709
850	0.4097	1.110	384.3	93.80	59.6	131	0.716
900	0.3868	1.121	398.1	102.9	62.0	143	0.720
950	0.3666	1.131	411.3	112.2	64.3	155	0.723
1000	0.3482	1.141	424.4	121.9	66.7	168	0.726
1100	0.3166	1.159	449.0	141.8	71.5	195	0.728
1200	0.2902	1.175	473.0	162.9	76.3	224	0.728
1300	0.2679	1.189	496.0	185.1	82	238	0.719
1400	0.2488	1.207	530	213	91	303	0.703
1500	0.2322	1.230	557	240	100	350	0.685
1600	0.2177	1.248	584	268	106	390	0.688
1700	0.2049	1.267	611	298	113	435	0.685
1800	0.1935	1.286	637	329	120	482	0.683
1900	0.1833	1.307	663	362	128	534	0.677
2000	0.1741	1.337	689	396	137	589	0.672
2100	0.1658	1.372	715	431	147	646	0.667
2200	0.1582	1.417	740	468	160	714	0.655
2300	0.1513	1.478	766	506	175	783	0.647
2400	0.1448	1.558	792	547	196	869	0.630
2500	0.1389	1.665	818	589	222	960	0.613
3000	0.1135	2.726	955	841	486	1570	0.536