Mech 366
Modeling of Mechatronic Systems

Lab #4
System Identification of a First Order System

LEARNING OBJECTIVES

The objectives of the experiments in this lab are:

- to identify the system parameters of a first order system.
- to illustrate ***time domain*** and ***frequency domain*** analysis.
- to illustrate ***bode plots*** representation.

BACKGROUND

This lab is similar to Lab #3 Experiment A (open-loop case) in which the objective was to control the velocity of a DC motor. However, in this experiment you must identify the dynamic parameters of this control system in both time domain and frequency domain and compare your results.

In order to perform the experiment, one must understand a number of fundamentals in control systems. The rotational dynamics of a DC motor is governed by the following second order differential equation:

$$J_m \ddot{\theta} + b\dot{\theta} = K_t\, i$$

where b is friction (damping) coefficient and J is moment of inertia of the rotor.

The KVL (**K**irchhoff's **V**oltage **L**aw) loop analysis of the armature windings shows the electrical equation to be:

$$L_a \frac{di}{dt} + R_a\, i = v - K_e \dot{\theta}$$

where L and R are inductance and resistance of the armature respectively. Figure 1 shows a block diagram for the DC motor.

The variable that you are controlling in this lab (***control variable***) is the rotational speed of the motor $\dot{\theta}$. The sensor that measures the rotational speed of the motor is the ***tachometer***. A tachometer is an electromechanical device that converts mechanical energy into electrical energy. The device works essentially as a generator, with the output voltage proportional to the magnitude of the rotational speed. The dynamics of the tachometer can be represented by the equation:

$$e_t(t) = K_{Tach} \frac{d\theta(t)}{dt} = K_{Tach} . \omega(t)$$

where $e_t(t)$ is the output voltage, $\theta(t)$ the rotor displacement in radians, $\omega(t)$ the rotor angular velocity in rad/s, and K_{Tach} the ***tachometer voltage constant*** in Volt / (rad/s).

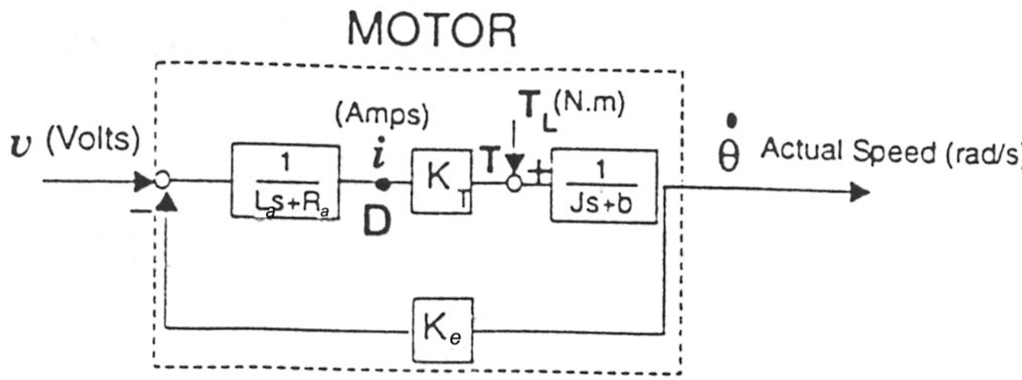

Figure 1. Block Diagram of the DC Motor

The transfer function of a tachometer is obtained by taking Laplace transform on both sides of the above equation.

$$\frac{E_t(s)}{\Theta(s)} = K_{Tach} \; s$$

PRELAB

If you have trouble with this section, please consult TA.

As a means to help understand these concepts, a number of questions which must be completed as part of the experiment.

1. Using the equations provided above, write the general form of the transfer function for the velocity control of a DC motor in the open loop configuration. *Hint: it should match the graphical description given in Figure 1.*

2. You will find that the velocity control of a DC motor has been considered as a second order system. However, velocity control is often considered as a first order system by assuming the inductance of the armature is relatively small. Show that this assumption results in a first order system.

3. Write the first order form of the velocity control of a DC motor in general form and clearly show the ***gain*** and ***time constant.*** I.e. use only two constants: $K_{motor\text{-}gain}$ and τ_{motion} for the gain and time constant respectively***.***
   Hint: Use equation 6 from section B

4. Give the definition of ***time constant*** for a first order system and how it can be measured with a step input.

5. Derive the mechanical and electrical time constants of this control system and compare with the values given in the DC motor specifications (Appendix). Discuss any differences in the values.

In this lab, you are going to treat this system as a first order system and determine the ***gain*** and the ***time constant*** (parameters of a first order system) using two methods: the time domain method (step response) and the frequency domain method (by means of Bode plot). These experimentally derived parameters will be compared to the theoretical values of the first order system.

NOTE: These questions must be answered and included in your laboratory report.

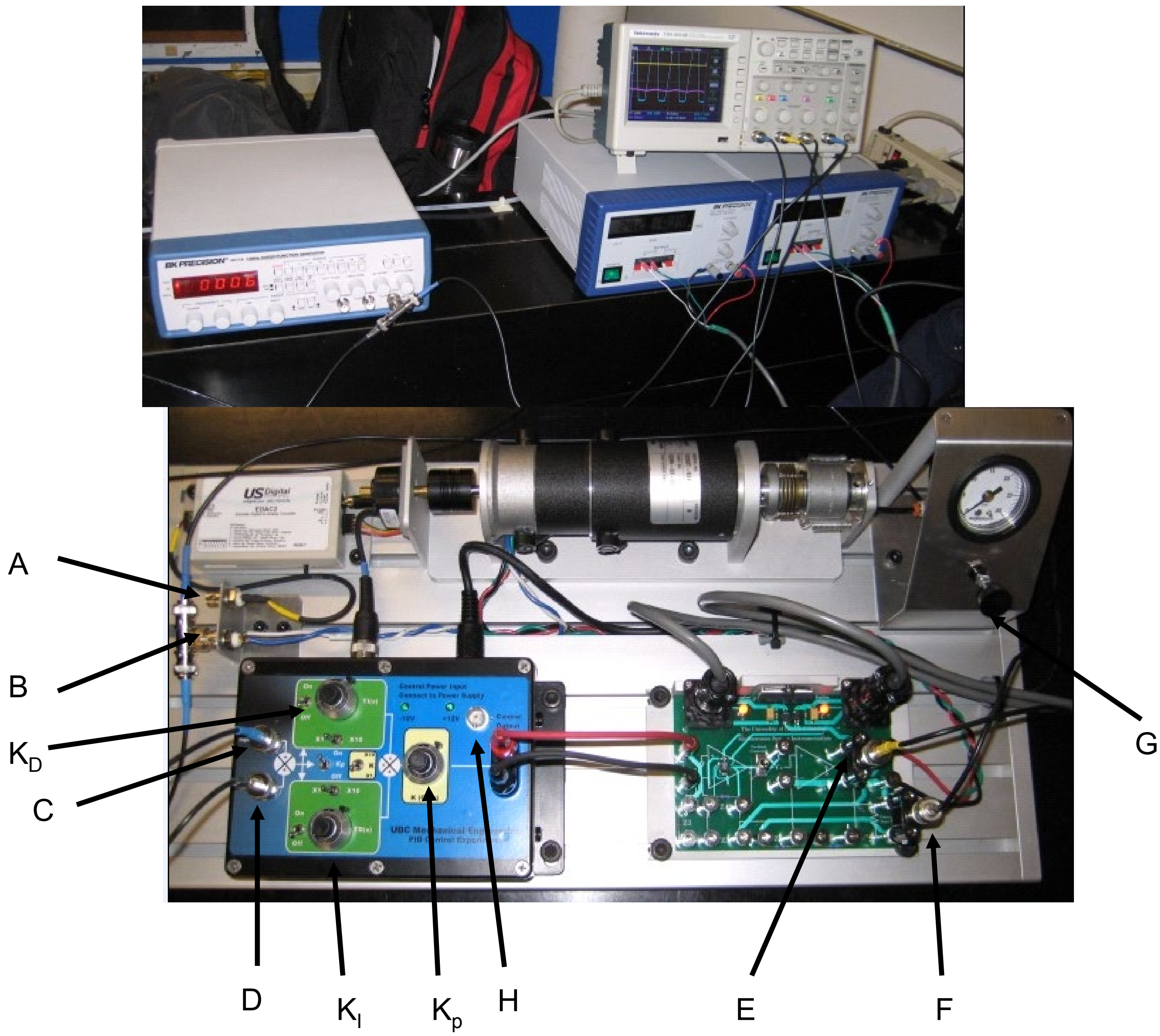

Figure 2. View of the Front Panel (same as Figure 5 in Lab 3)

A = Position Output Signal
B = Velocity / Tachometer Output Signal (connect to oscilloscope channel 2)
C = Feedback Input (connect to B/velocity signal ONLY IF PERFORMING FEEDBACK EXPERIMENTS)
D = Controller Input (connect to oscilloscope channel 1)
E = Motor Voltage Output Signal (connect to oscilloscope channel 4)
F = Motor Current Output Signal (connect to oscilloscope channel 3)
G = Pneumatic Clutch Controller
H = Controller Output
K_p = Proportional Gain Control
K_I = Integral Gain Control
K_D = Derivative Gain Control

EXPERIMENT A
Time Domain Analysis of Velocity Control (Step Response)

BACKGROUND

The experimental set-up is the same as the one used for Lab 3 Experiment A. The front panel of the apparatus is shown on Figure 2. Open loop response will be analyzed in this experiment. The block diagram for the open loop system is shown in Figure 1.

PROCEDURE

For this experiment you will input a step input (square wave) and determine the governing parameters for the velocity control system.

Open Loop Step Input

1. Build an open loop system and connect the cables for monitoring the signals according to Figure 2.

2. Set-up the parameters of the signal generator as follows:

   INPUT FROM SIGNAL GENERATOR:
   - Square Wave
   - Frequency: 0.7 Hz.
   - Pk-pk: 7 V_{PP}

3. Set-up the parameters of the controller in the following fashion.

   CONTROLLER PARAMETERS:
   - Proportional Control (K) - ON - x1
   - Derivative Control (Kd) – OFF
   - Integral Control (1/Ki) -OFF

4. Set-up the oscilloscope in order to monitor the operation of the plant. Refer to Figure 2 to connect the required leads to the oscilloscope. Using the oscilloscope, you will want to measure the input signal (Channel 1) amplitude of motor voltage (Channel 4), motor current (Ch. 3), and motor speed (tachometer voltage connected to Ch. 2).

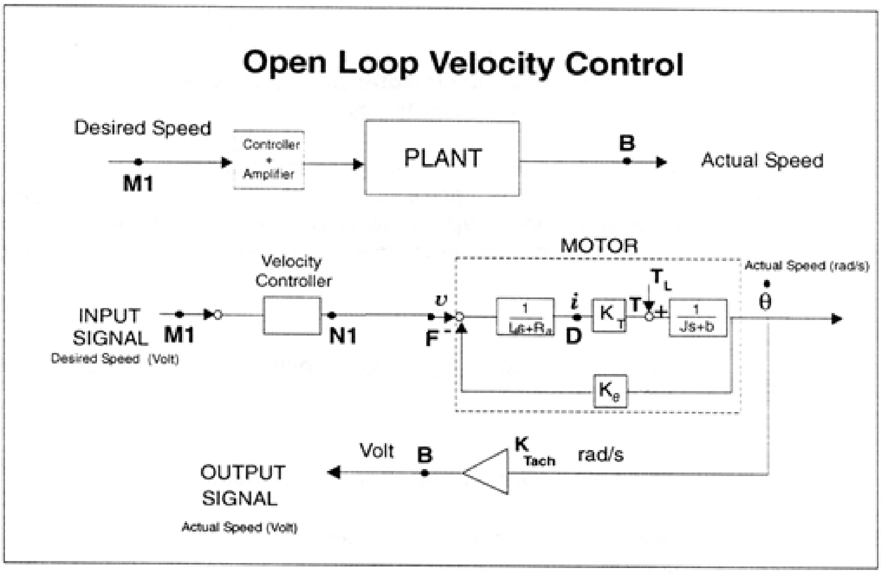

Figure 3. Open Loop Velocity Control Block Diagram

5. Be sure that the pneumatic clutch coupling is backed up fully (counter-clockwise) so as to provide minimum friction on the motor.

6. Turn on the motor. Determine the ***time constant, and gain (between motor voltage and motor speed)*** of the system.
   To measure the <u>time constant</u>:
   Use RUN/STOP to capture a trace, and use Select to measure time between 0% to 63% of maximum
   OR
   Measure the time from 0V to $0.63 * V_{Peak}$ of the tach voltage.
   Ask the TA for help, if necessary.
   Note: <u>Gain</u> = output/input, here motor speed/input voltage. Use the tach voltage and appendix to get the motor speed in rad/s.

7. Adjust the pneumatic clutch to provide to higher load settings and repeat Step 6. Complete Table 1. To determine the max load, adjust the pneumatic clutch **until the tachometer voltage trace no longer has the characteristic exponential shape**.

Table 1. Open Loop Step Input Results

Motor Load in Amps (nominal)	**Measure**			**Calculate**	**Measure**	**Calculate**	
	Actual Motor Load (amps)	Motor Voltage (V_{PP})	Tach Voltage (V_{PP})	63% of response (V_P)	Time Constant (τ_{motion})	Motor Speed (rad/s)	Gain [(rad/s)/V]
No load							
Halfway to Max Load		Do not write in laboratory manuals					
Max Load							

REPORT REQUIREMENTS

Summarize your observations in this experiment and answer the following questions:

1. Answer all questions contained in the introduction section of Experiment A.

2. Compare the measured time constant, and gain results determined from your measurements with values derived theoretically. Discuss any differences and the difficulty in measuring system parameters in the time domain.

EXPERIMENT B
Frequency Domain Analysis of Velocity Control (Bode plots)

BACKGROUND

The frequency domain analysis of a system can be conducted from the frequency-domain plots of $G(s)$ with s replaced with $j\omega$. The function $G(j\omega)$ is generally a complex function of the frequency ω and can be written as:

$$G(j\omega) = |G(j\omega)|\angle G(j\omega) \tag{4}$$

with

$|G(j\omega)|$: Magnitude of $G(j\omega)$

$\angle G(j\omega)$: Phase of $G(j\omega)$

One method for illustrating the frequency domain is the use of a Bode plot. A Bode plot of the control function $G(j\omega)$ is composed of two plots. The first plot shows the magnitude of $G(j\omega)$ plotted in decibels (dB) versus ω or $\log_{10}\omega$, while the other plot shows the phase of $G(j\omega)$ in degrees as a function of ω or $\log_{10}\omega$. Conventionally these two plots are plotted on the same page, the top one being the magnitude (dB) versus ω, and the bottom one being phase shift (in degrees) versus ω.

Recall that the decibel (dB) is a multiplication of the logarithm to the base 10 of a ratio by 20. That is,

$$dB = 20\log_{10}(ratio) \tag{5}$$

where $G(j\omega)$ and $|G(j\omega)|$ are both ratios.

The Bode plot can be constructed by using straight-line approximations that are asymptotic to the actual plot. In simple terms, the Bode plot has the following features:

1. Since the magnitude of $G(j\omega)$ in the Bode plot is expressed in dB (i.e. the $\log_{10}$ scale), any product or division factors in $G(j\omega)$ become simple additions and subtractions, respectively. The phase relations are also added and subtracted from each other algebraically.
2. The magnitude plot of the Bode plots of $G(j\omega)$ can be approximated by straight-line segments, which allow the simple sketching of the Bode plot without detailed computation.

In this experiment, you consider the system as a first order system (velocity control of a DC motor) which has the following transfer function:

$$G(s) = \frac{K}{\tau s + 1} \text{ (in Laplace domain)} \tag{6}$$

$$G(j\omega) = \frac{K}{\tau j\omega + 1} \text{ (in frequency domain)} \tag{7}$$

where K = $K_{motorgain}$ and τ = τ_{motion} from the Prelab.

In order to determine the **magnitude** of $G(j\omega)$ one can re-arrange the frequency domain equations.

$$\left|G(j\omega)\right|_{dB} = 20.\log_{10}\left|\frac{K}{\tau j\omega+1}\right| = 20.\log_{10}\left|K\right| - 20.\log_{10}\left|\tau j\omega+1\right|$$

$$K_{dB} = 20.\log_{10} K = Cte$$

$$\left|\frac{1}{\tau j\omega+1}\right| = -20.\log_{10}\left|\tau j\omega+1\right| = -20.\log_{10}\sqrt{1+\omega^2\tau^2} \quad (8)$$

In order to determine the phase of $G(j\omega)$, one must rearrange equation (6).

$$\angle G(j\omega) = \angle K - \angle(\tau j\omega+1) \quad (9)$$

$$\angle K = \begin{cases} 0^\circ & K>0 \\ 180^\circ & K<0 \end{cases}$$

$$\angle G(j\omega) = -\tan^{-1}(\omega\tau) \quad (10)$$

These two equations verify one of the unique characteristics of the Bode plot, in that each factor can be considered as a separate plot and the individual plots can be added or subtracted accordingly to yield the total magnitude in dB and the phase plot of $G(j\omega)$. The curves can easily be plotted on semilog graph paper or linear rectangular-coordinate graph paper, depending on whether ω or $\log_{10}\omega$ is used as the abscissa.

Creation of a Bode Plot

A Bode plot can be created though the use of Equations (8) through (10). Four procedures should be followed in order to create the plot. These steps are:

a. How to plot $-20.\log_{10}\left|\tau j\omega+1\right|$ (Equation 8)

Firstly, one must obtain asymptotic approximations by considering both very large and very small values of ω. At **very low frequencies** we have;

$$\omega\tau \ll 1 \Rightarrow \omega^2\tau^2 \to 0 \Rightarrow -20\log_{10}\left|\tau j\omega+1\right| \approx -20\log_{10} 1 = 0\,\text{dB} \quad (11)$$

At **very high frequencies** we have;

$$\omega\tau \gg 1 \Rightarrow (1+\omega^2\tau^2) \approx \omega^2\tau^2 \Rightarrow -20\log_{10}\left|\tau j\omega+1\right| \approx -20\log_{10}\sqrt{\omega^2\tau^2} = -20\log\omega\tau \quad (12)$$

Equation (12) represents a straight line with a slope of –20dB/decade of frequency. By intersecting the line (0 dB) and ($-20\log\omega\tau$), we can find the term $\omega = \frac{1}{\tau}$. This is the frequency where the high-frequency approximation plot and low-frequency approximation plot intersect in 0dB axis. This particular frequency is known as ***Corner Frequency*** (since the asymptotic plot forms the shape of a corner at this frequency).

The actual frequency plot is a smooth curve and deviates only slightly from the straight-line approximation. The error between the actual magnitude curve and the straight-line asymptotes is symmetric with respect to the corner frequency. It is useful to remember that the error is 3dB at the Corner Frequency, and 1dB at 1 octave above ($\omega = \frac{2}{\tau}$)and one octave below ($\omega = \frac{1}{2\tau}$) the corner frequency. At 1 decade above and below the corner frequency, the error is dropped to approximately 0.3dB.

Based on these facts, the procedure of making a sketch of $-20.\log_{10}|\tau j\omega + 1|$ is as follows;

1. Locate the corner frequency $\omega = \frac{1}{\tau}$ on the frequency axis
2. Draw the –20dB/decade line and the horizontal line at 0dB, with two lines intersecting at $\omega = \frac{1}{\tau}$.
3. The actual magnitude curve is obtained by adding the errors to the asymptotic plot at the strategic frequency. Usually a smooth curve can be sketched simply by locating the 3dB point at the Corner Frequency and the 1dB points at 1octave above and below the corner frequency.

b. How to plot $|G(j\omega)|_{dB} = \left|\frac{K}{\tau j\omega + 1}\right|$

Plot the constant line $K_{dB} = 20.\log_{10} K = Cte$ and also plot $-20.\log_{10}|\tau j\omega + 1|$ as shown in the last section. Therefore, $|G(j\omega)|_{dB}$ will be the addition of these two plots.

c. How to plot $-\angle(\tau j\omega + 1) = -\tan^{-1}(\omega\tau)$

Just as in the procedure to create the magnitude curve, a straight-line approximation can also be created for the phase curve. Since the phase of $-\angle(\tau j\omega + 1)$ varies from 0 to –90 degrees, we can draw a line from 0 degree at 1 decade below the corner frequency to –90degree at 1 decade above the corner frequency. The maximum deviation between the straight-line approximation and the actual curve is less than 6 degrees.

d. How to plot $\angle G(j\omega)$

Plot the constant phase since $\angle K = \begin{cases} 0^\circ & K > 0 \\ 180^\circ & K < 0 \end{cases}$ and plot $-\angle(\tau j\omega + 1)$, as shown in the last section. Hence, $\angle G(j\omega)$ is the addition of these two plots.

As a means to better understand the frequency response of a control system, review your course notes and textbook and answer the following questions:

1. Explain briefly the characteristics of a Bode plot.

2. Knowing that you are estimating this system as a first order system, what shape would you expect for the bode plot of your response, both in magnitude and phase (in degrees)? Illustrate this by a hand sketch.

3. What is the *corner frequency* (break point) and what is the *lag* (in degrees) of a first order system at this particular frequency?

4. What is the maximum lag you could expect for a first order system? Explain according to your transfer function.

5. How do you find the *Time Constant* of the system on a Bode plot?

PROCEDURE

For this experiment, you will build an **open loop** control circuit and you will input a sine wave input (of variable frequency) and measure the amplitude and phase angle of the output (velocity) signal (volts).

1. Build an open loop system as done in the previous experiments.

2. Set the parameters of the signal generator to:
   - Sine wave (frequency response)
   - Frequency: Variable **(Table 2)**
   - Pk-Pk : 7 volts

   Set the controller parameters to the following:
   - Proportional Control (K) - ON - x1
   - Derivative Control (Kd) – OFF
   - Integral Control (1/Ki) – OFF

The clutch should be on the lowest setting (no load) so it can be compared to the first test of Experiment A.

3. Begin with lowest frequency input and use the oscilloscope to measure the phase lag and amplitude of the output (velocity). Measure the phase lag (degrees) between the input (motor voltage) and output (velocity) signals. Amplitude should

get derived by dividing velocity (rad/sec) by input motor voltage (Vpp). Vary the frequency to complete Table 2.

Table 2. Frequency Response Data

Frequency of **input signal** (Hz)	**OUTPUT SIGNAL** Motor Speed (from TACH)			
	Phase Lag (ms)	Tach Voltage (V_{PP})	Speed (rad/sec)	Mo
2				
3				
4				
5				
6				
7				
8				
9				
10				
15				
20				
25				
30				
35				

Note: Don't forget to convert your phase lag to degrees and your tach voltage to rad/s. Use the appendix.

REPORT REQUIREMENTS

Summarize your observations in this experiment and answer the following questions:

1. Answer all questions contained in the introduction section of Experiment B.
2. Plot a Bode diagram with the experimentally obtained measurements and determine the time constant and gain of the system.
3. Compare the time constant and gain derived from the Bode plot with the values obtained from the step response for the open loop circuits and discuss any differences.
4. Compare the values determined from your Bode plot measurements with values derived theoretically and those published in the Appendix. Discuss any differences.

GENERAL REPORT REQUIREMENTS

As part of your formal write-up, include an objective for each experiment followed by a brief background related to that experiment section. Include the required data for each section and answer the questions. Analyze your results and write a conclusion for each of the experiment sections.

APPENDIX: Specification of the DC Motor

(same as Lab 3)

Specifications of the DC motor		
J_m	Inertia of the motor	5.62 x 10^{-5} $kg.m^2$
b	friction coefficient	2.61 x 10^{-4} $N.m.\sec$
K_e	electric "emf" constant	0.068 $Volt/[rad/s]$
K_t	torque constant	0.068 Nm/A
L_a	armature inductance	3.0×10^{-3} H
R_a	armature resistance	1.63 Ω
τ_m	mechanical time constant	0.216 sec
τ_e	electrical time constant	1.84×10^{-3} sec
K_{Tach}	Tachometer Voltage Constant	0.134 $Volt/[rad/s]$
K_i	Current Sensor Constant	0.2 $Volt/A$