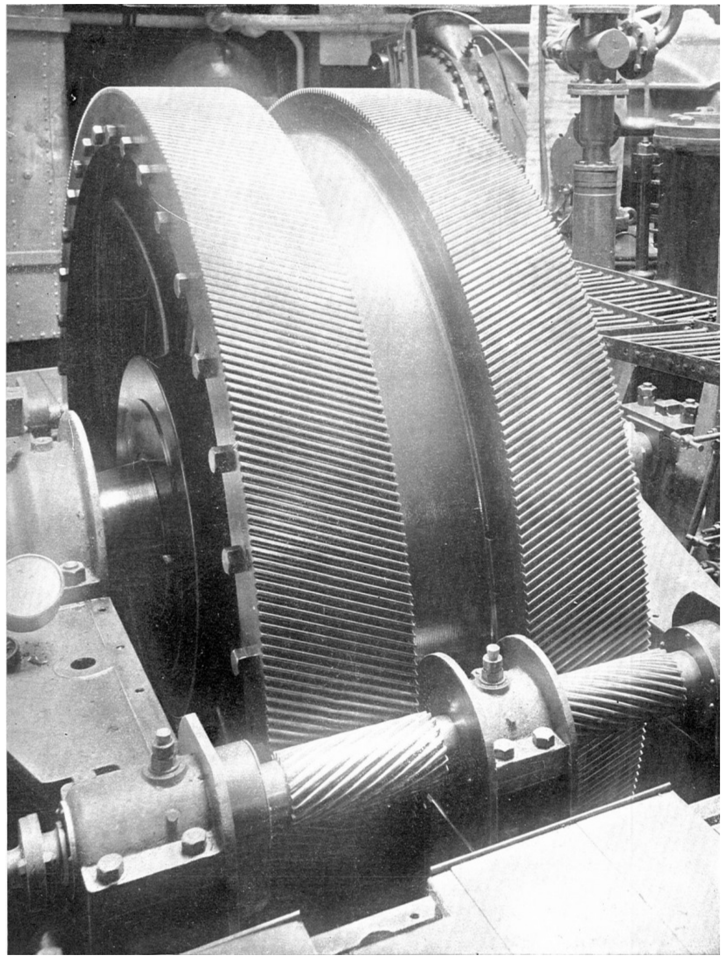
MECH 325
Gear Forces and Kinematics



Objectives

By the end of today's class, you should be able to:

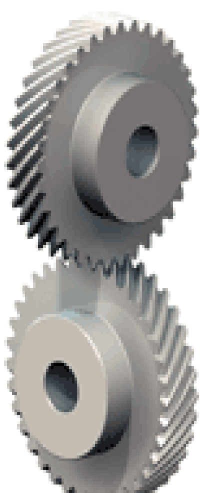
- Analyze gear train kinematics, determining output rotation magnitude and direction
- Analyze gear train power, torque transmission, and reaction forces/moments
- Describe how materials/methods for forming gear teeth influence gear selection and performance

Gear Kinematics - Magnitude

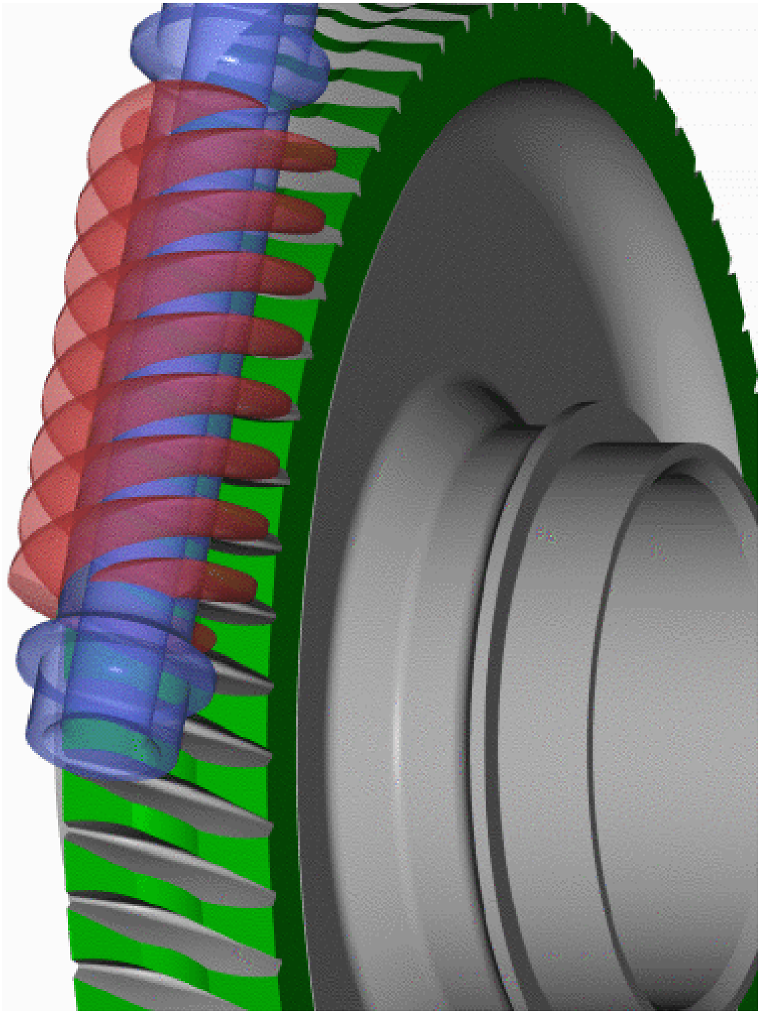
- Need to determine magnitude of input and output velocity
- Train value, *e*
- For simple and compound gear trains
- For planetary gear trains need to work in arm reference frame

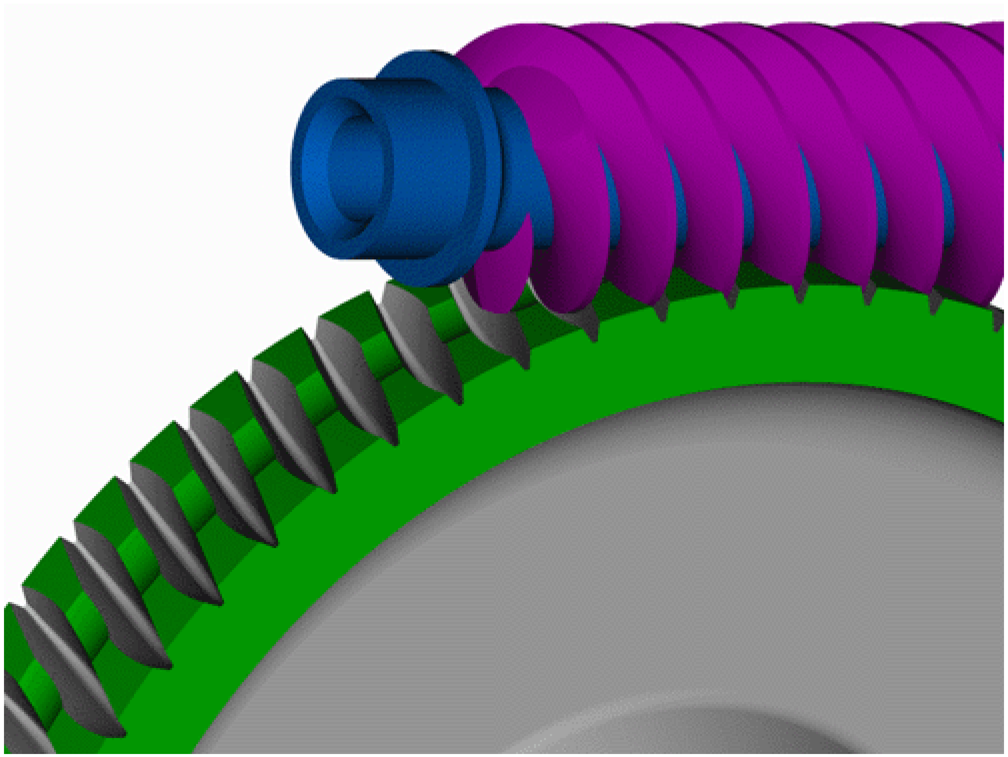
Gear Kinematics - Direction

- To determine direction, better to visualize the problem than memorize relationships
- Remember: mating teeth move same direction



Gear Animation

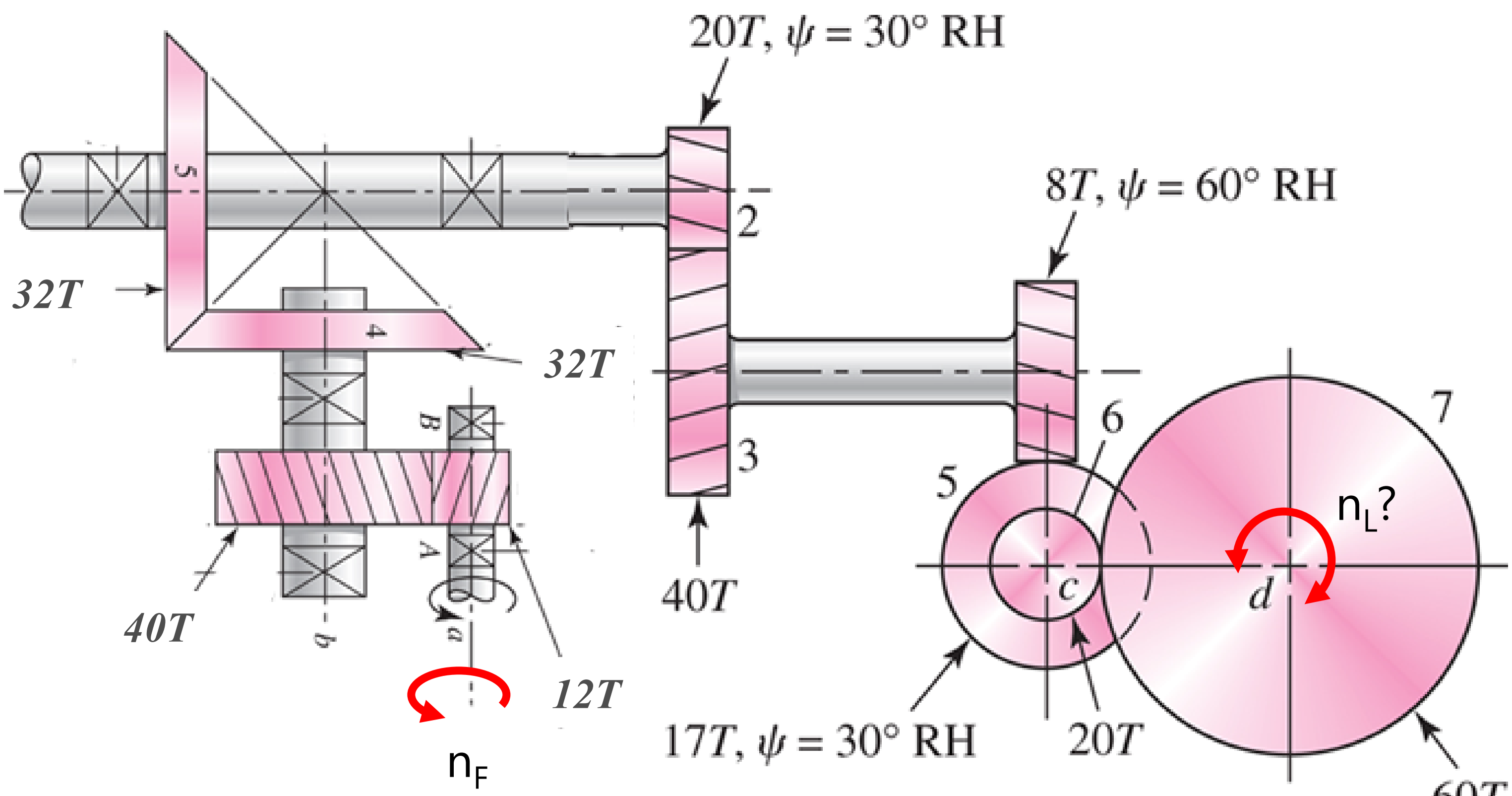




http://www.gearideas.com/tech/warm.htm

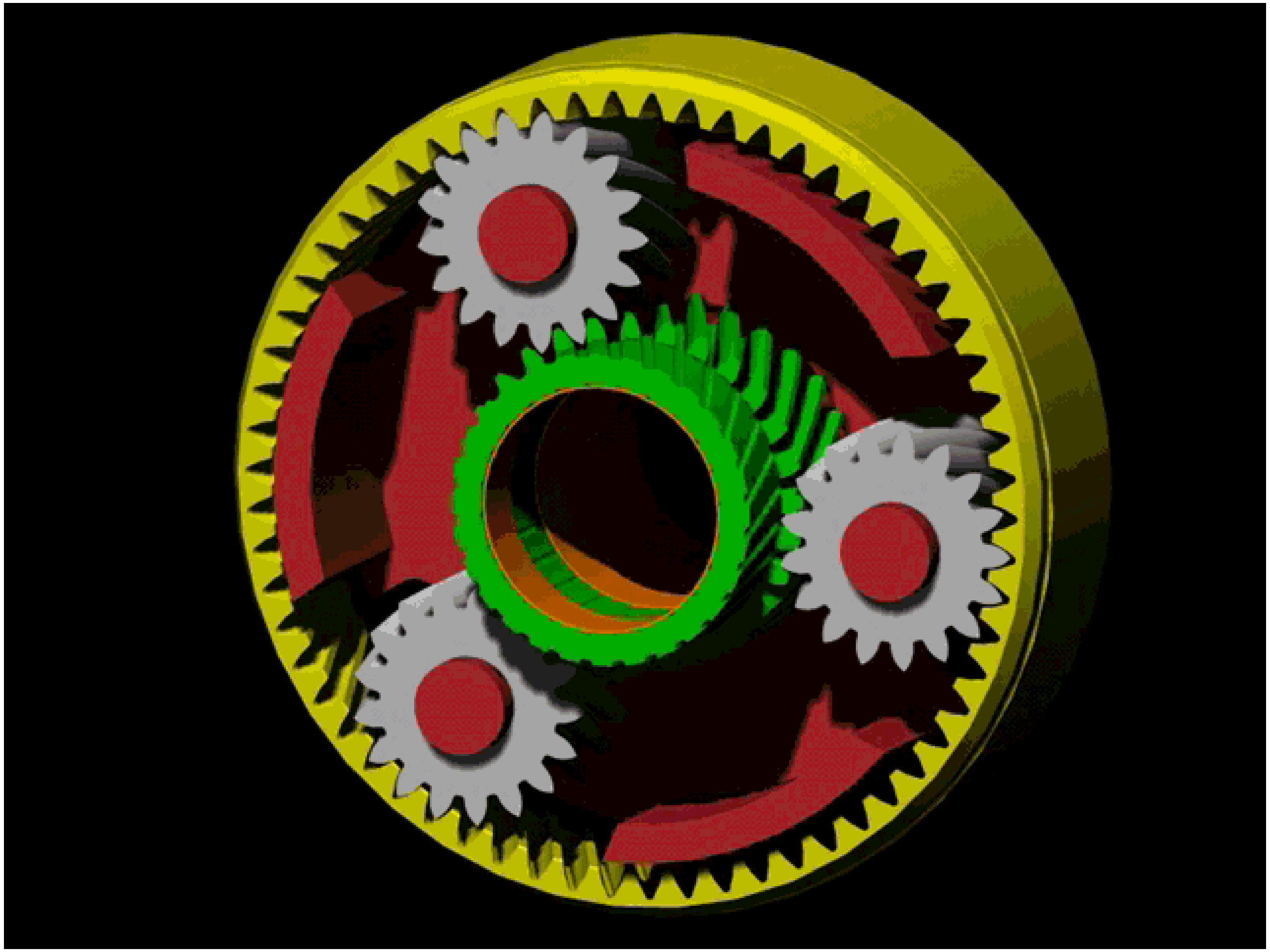
Gear Kinematics - Direction

- Determine speed, n_L , and direction of the output gear if the input gear has a velocity of n_F .



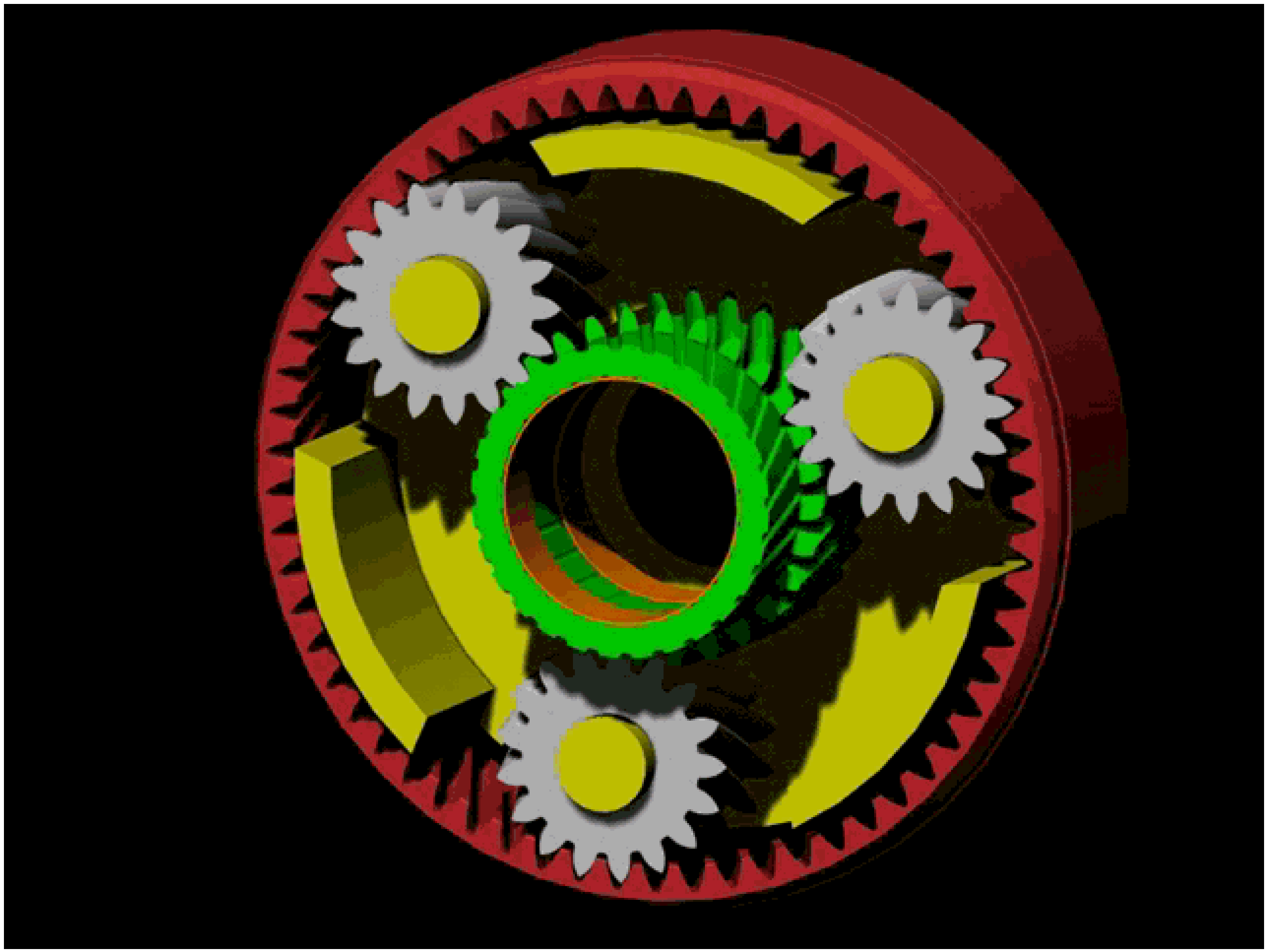
Planetary Gear

- Two degrees of freedom - need to specify two inputs
- Green: Sun = input Red: Carrier = fixed



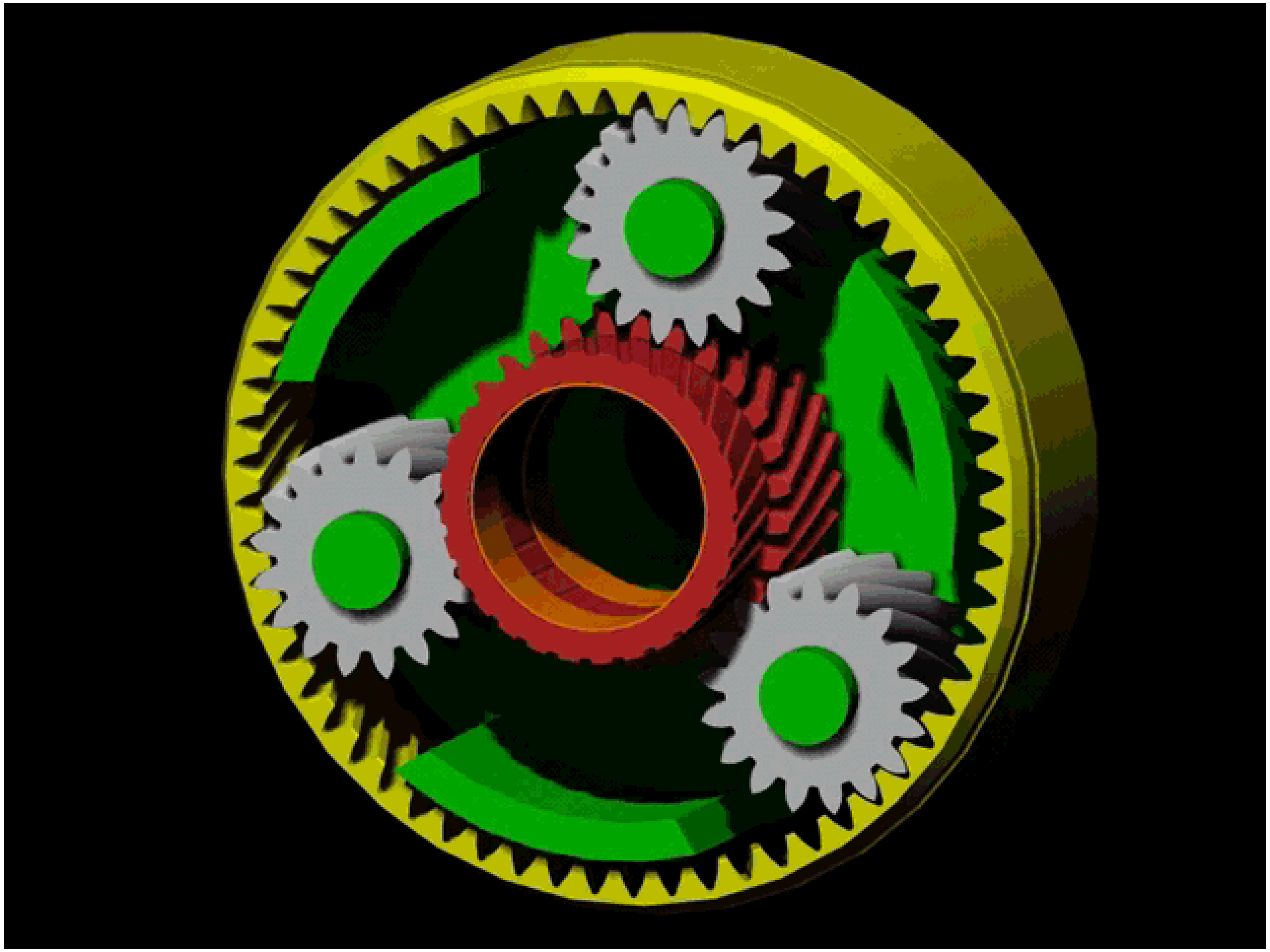
Planetary Gear

Green: Sun = input   Red: Ring = fixed



Planetary Gear

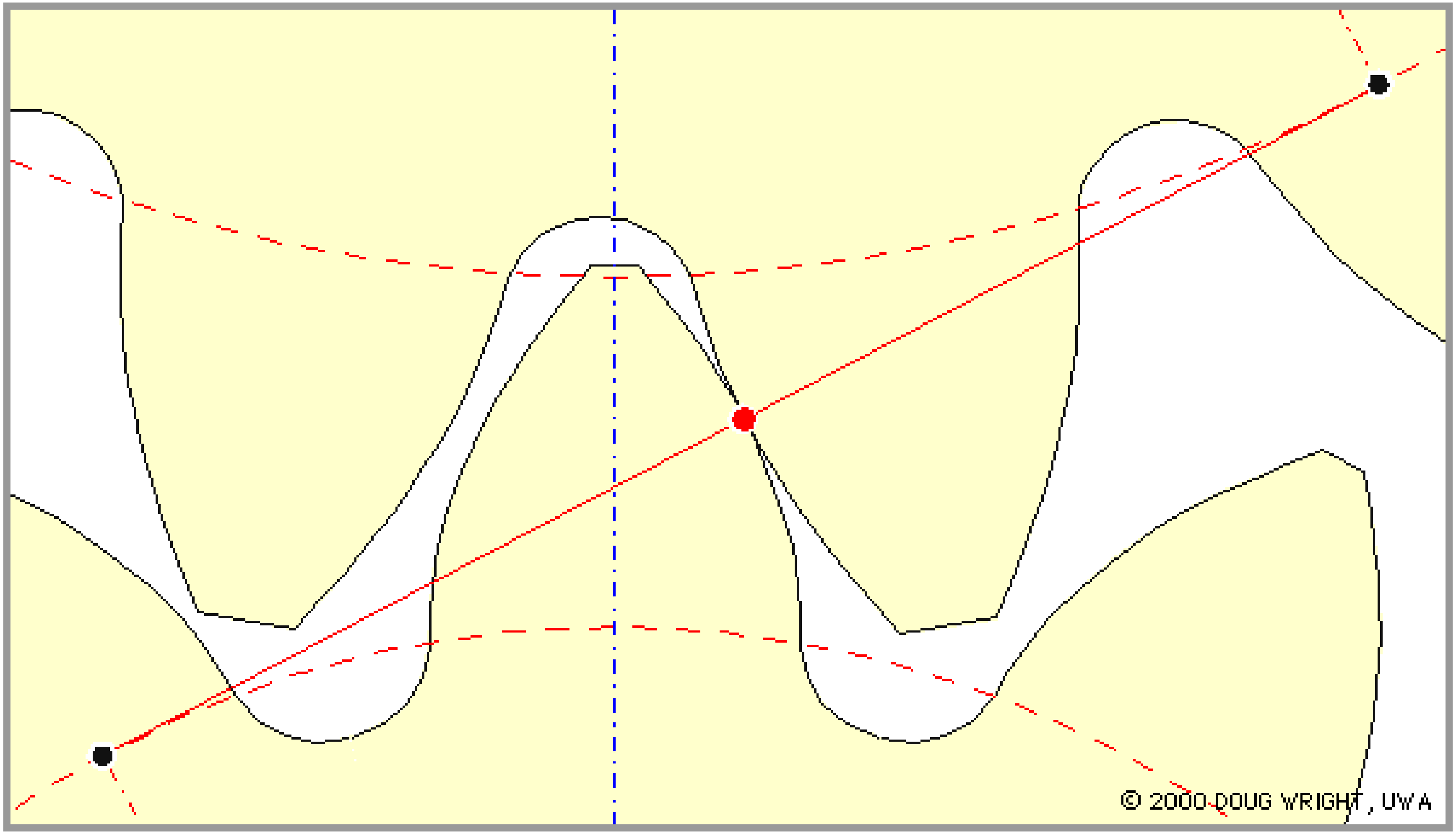
Green: Carrier = input   Red: Sun = fixed



A Note on Units

- All of the familiar equations apply
  - $T = F_t r$ (Torque = Tangential force · radial distance)
  - $H = T\omega$ (Power = Torque · angular velocity)
  - $H = F_t r\omega$
- These equations are only correct in SI units! SI units are:
  - T :
  - r :
  - ω :
  - F_t :
  - H :
- For non-SI units, need a conversion coefficient; e.g.:
  - $T = 63{,}030 H/n$ (if T in lb·in, H in hp, n in rev/min)
  - $F_t = 60{,}000\, H/(\pi d n)$ (if H in W, d in mm, n in rev/min)

Recall: Involute Profile



http://www.mech.uwa.edu.au/DANotes/gears/meshing/gearAnimation.gif

Involute Geometry

- Involute gear teeth are developed from the base circles of the pinion and the gear.
- Creating "conjugate involute profiles" ensure the contact point remains on the pressure line.

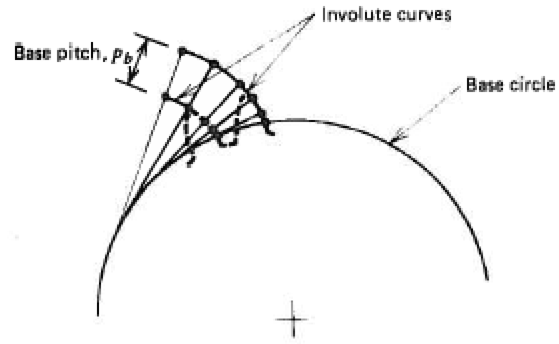


FIGURE 15.4.
Generation of an involute from its base circle.

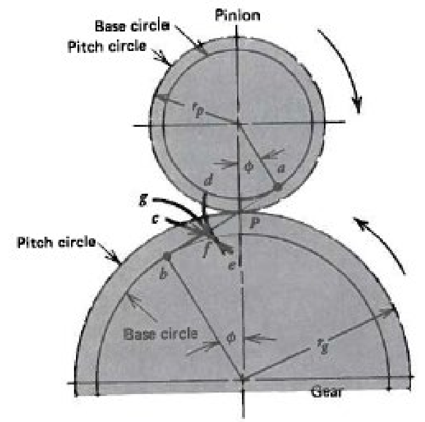


FIGURE 15.7.
Belt cut at *c* to generate conjugate involute profiles.

Involute Geometry

- Selection of the pressure angle defines the shape of the teeth for the

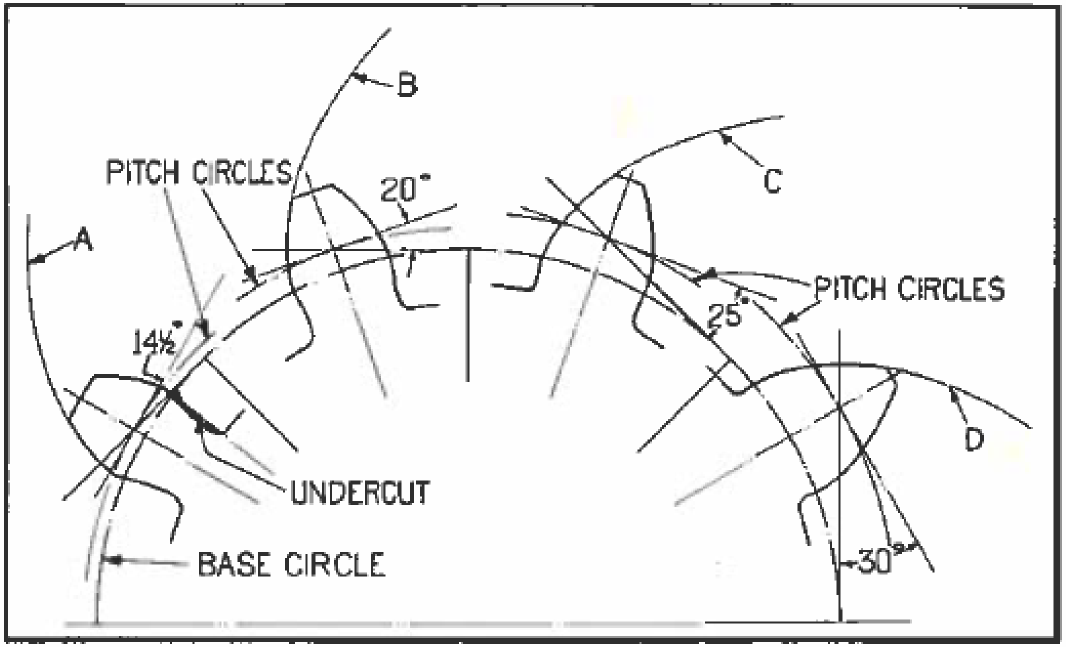


Fig. 22. Diagram Illustrating that Pressure Angle is Governed by Portion of Involute Used for Gear Tooth.

Involute Geometry

- An example of a conjugate “one tooth” gear.
- Contact always remains on the pressure line.
- Conjugate action is often used to develop cam profiles.

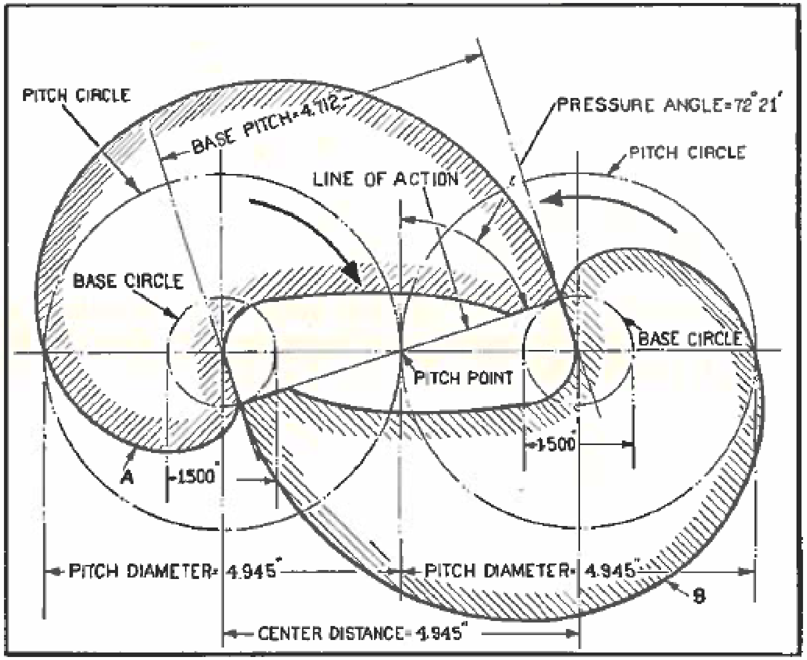


Fig. 20. Involutes Applied to Two One-Tooth Gears, Indicating that the Involute Has Its Origin at the Base Circle, but Is Not Limited in Length.

Diametrical Pitch

- In Imperial Units, the term *pitch* refers to diametrical pitch.

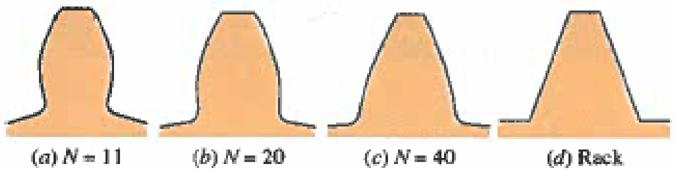


FIGURE 8–14 Involute curve shape for varying numbers of teeth for a diametral pitch of 5

- In metric, the term *pitch* refers to the circular pitch.

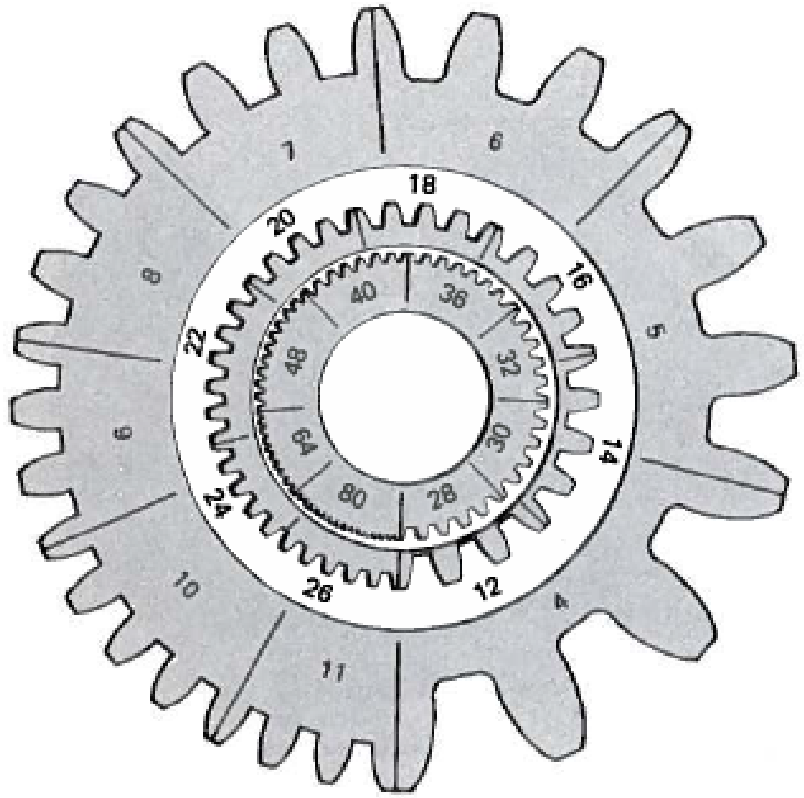


FIGURE 15.10.
Actual sizes of gear teeth of various diametral pitches. (Note: In general, fine-pitch gears have $P \geq 20$; coarse-pitch gears have $P < 20$.) (Courtesy The Barber-Colman Co.)

Interference

- Interference will occur if either of the addendum circles extend beyond the tangent points of the pressure line.
- Interference limits the number of teeth on the pinion (generally 12 teeth for φ =25°

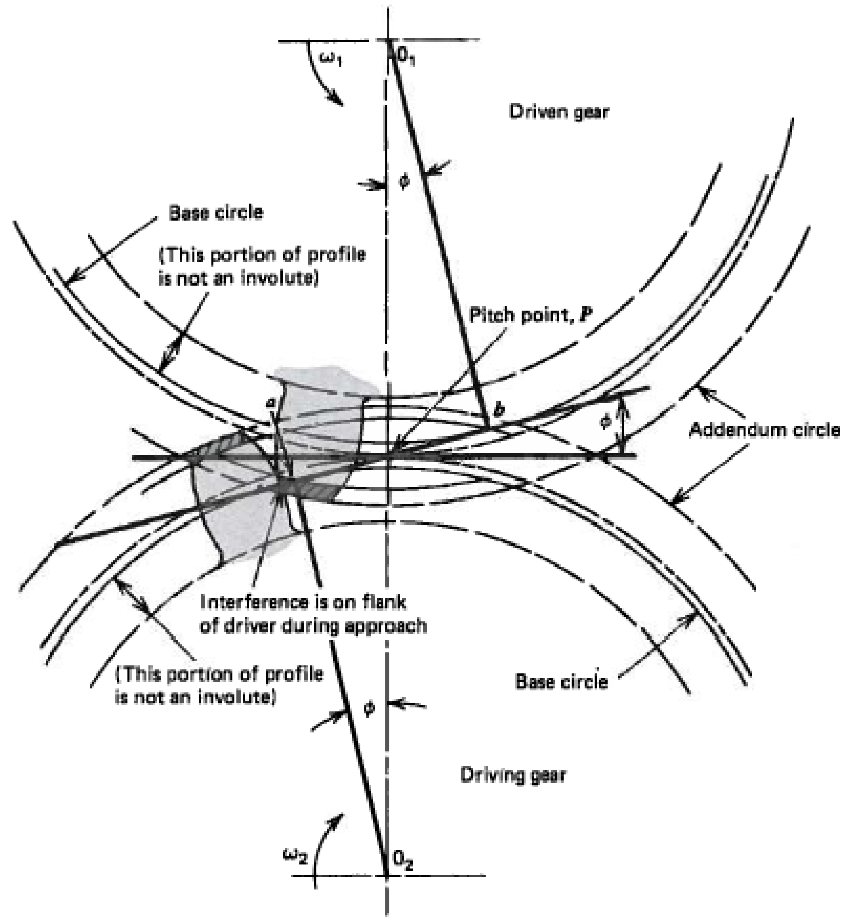


FIGURE 15.15.
Interference of spur gears (eliminated by removing the cross-hatched tooth tips).

Interference

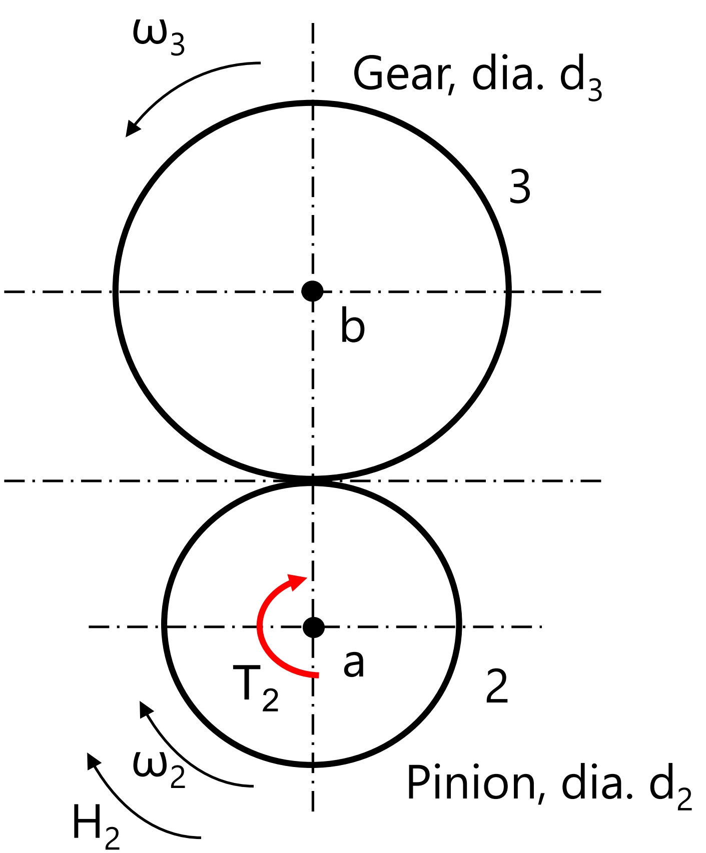
TABLE 8-7 Number of Pinion Teeth to Ensure No Interference

For a pinion meshing with a rack		For a 20°, full-depth pinion meshing with a gear		
Tooth form	**Minimum number of teeth**	**Number of pinion teeth**	**Maximum number of gear teeth**	**Maximum ratio**
$14\frac{1}{2}°$, involute, full-depth	32	17	1309	77.00
20°, involute, full-depth	18	16	101	6.31
25°, involute, full-depth	12	15	45	3.00
		14	26	1.85
		13	16	1.23

Power, Torque, Speed, and Force

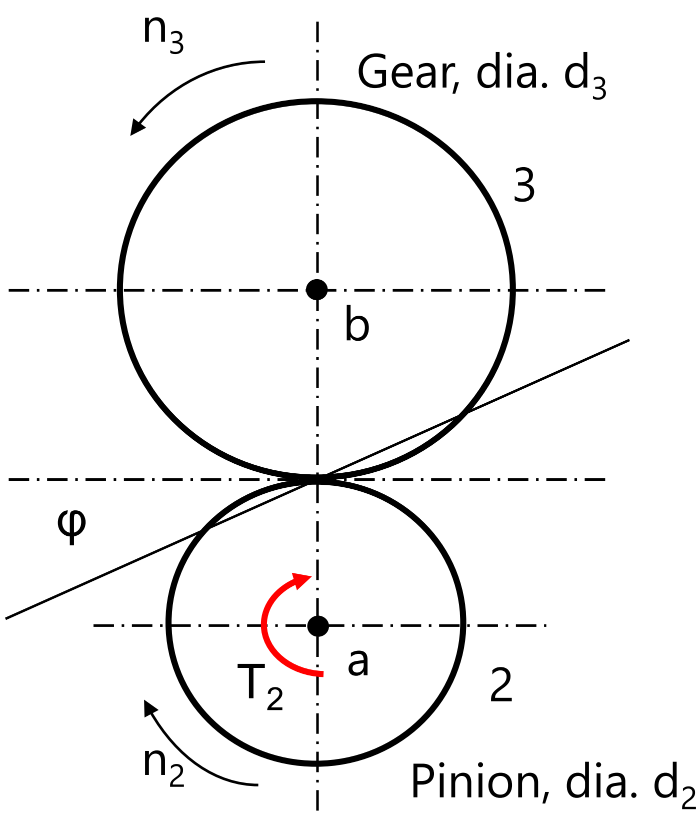
Determine the following (assuming SI units):

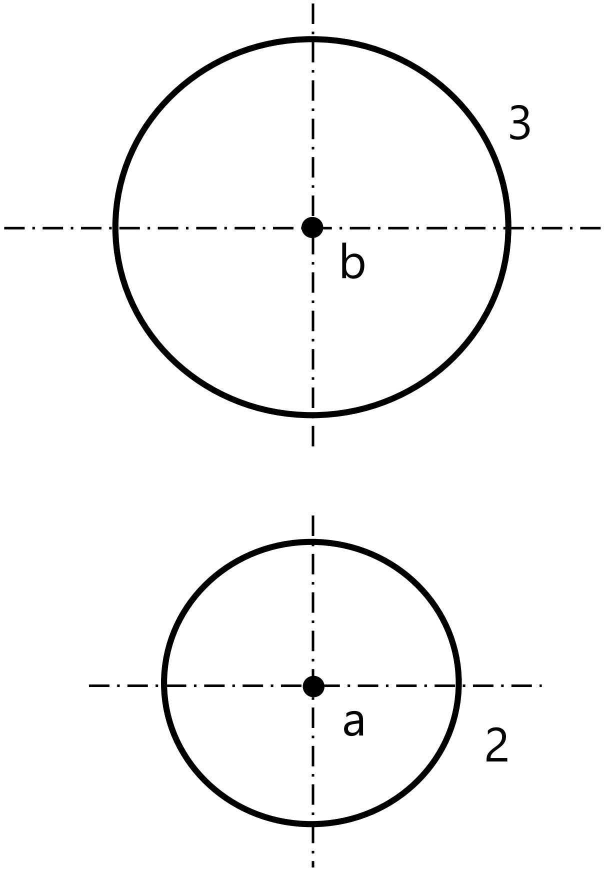
- e in terms of ω_2 and ω_3
- T_2 in terms of H_2 and ω_2 and e
- H_3 in terms of H_2 and e
- Torque T_3 in terms of T_2 and e
- W_t in terms of T_2 and d_2



Spur Gear Forces

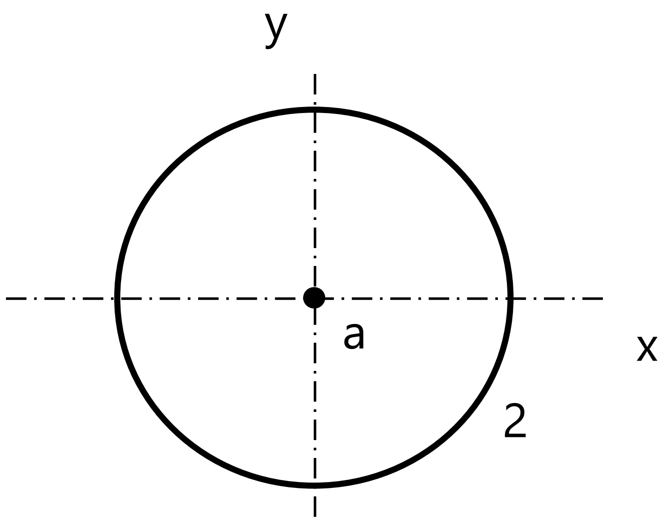
- Draw all forces and torques on the free-body diagrams for the two spur gears; the pinion sees an input torque of T_2





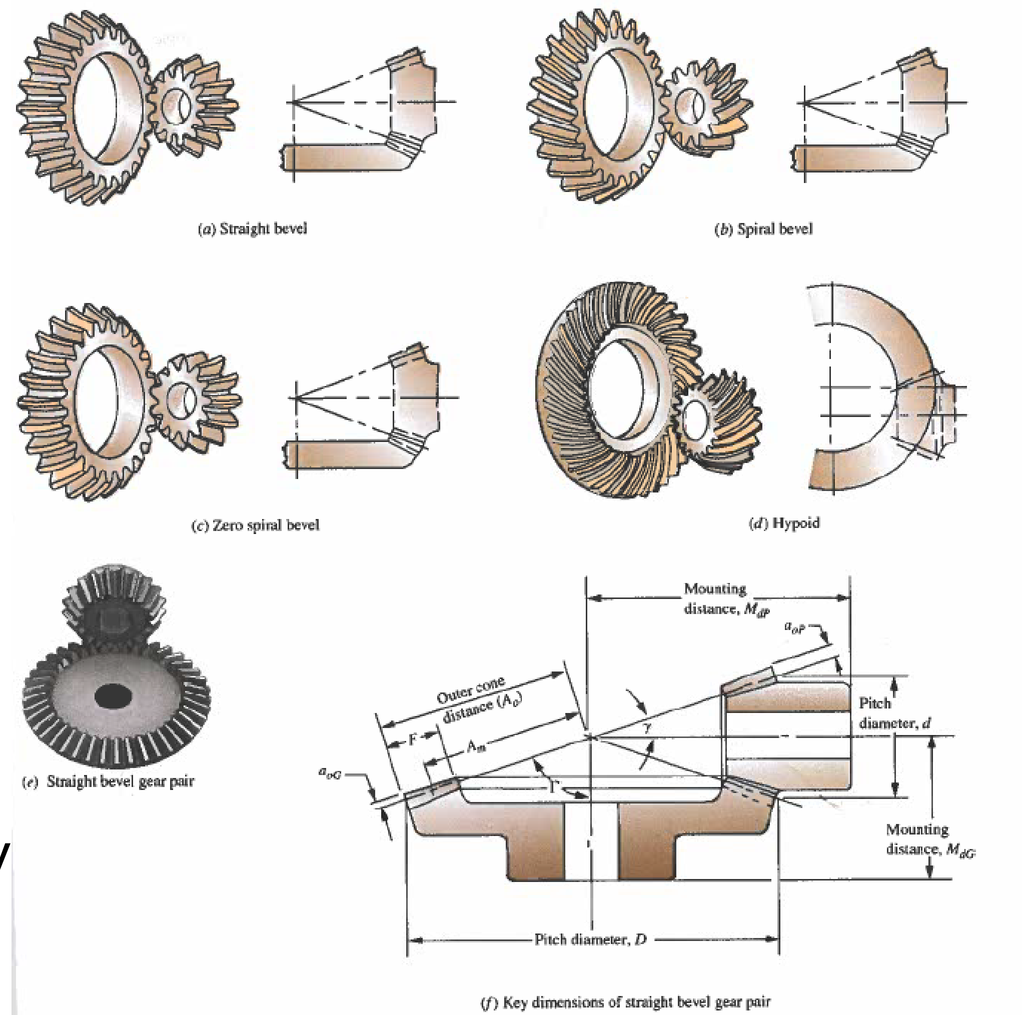
Spur Gear Forces

- Express the forces and torques on gear 2 (the pinion) in the radial and transverse directions



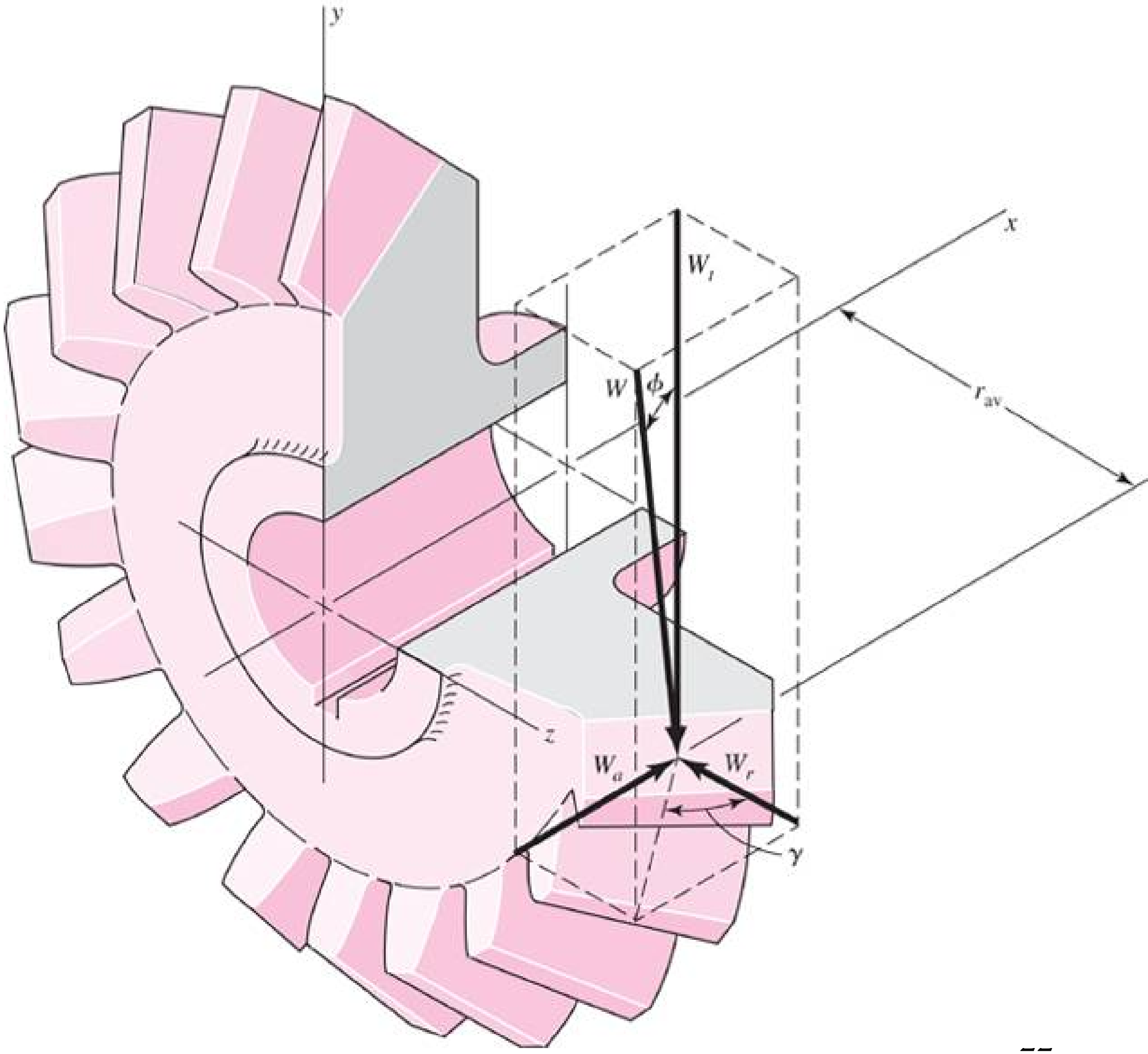
Bevel Gears

- The centerlines of the pinion and gear meet apex of each cone
- Teeth of the bevel gears are tapered towards the center to the cones
- Pressure angles for bevel are typically 20˚
- *Mitre Gears* are simple 1:1 ratio gears used only to change direction



Bevel Gear Forces

- What is W_t as a function of the input torque, T
- What are W_a and W_r as functions of W_t?



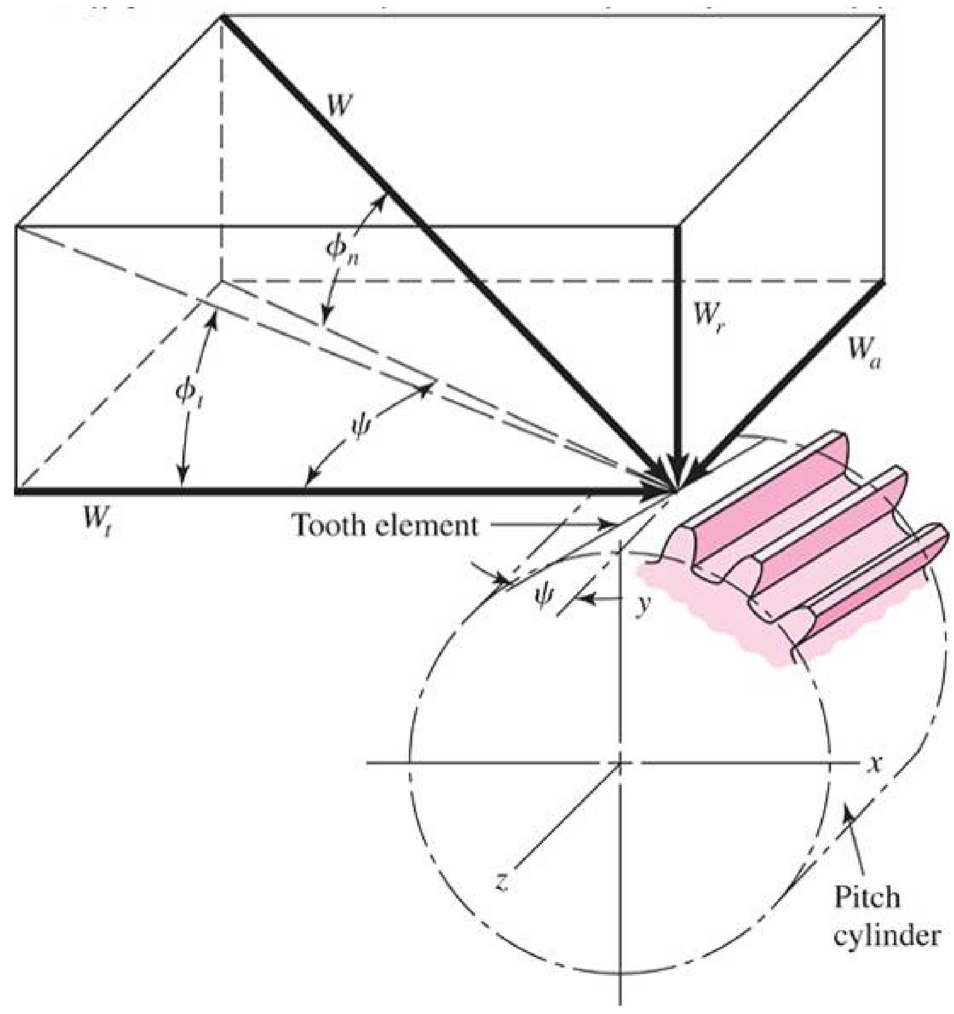
Helical Gear Forces

- Determine components of force in similar manner
- Teeth inclined at helix angle ψ relative to axis
- Similar to spur gear, but axial force introduced

$$W_r = W \sin\phi_n$$

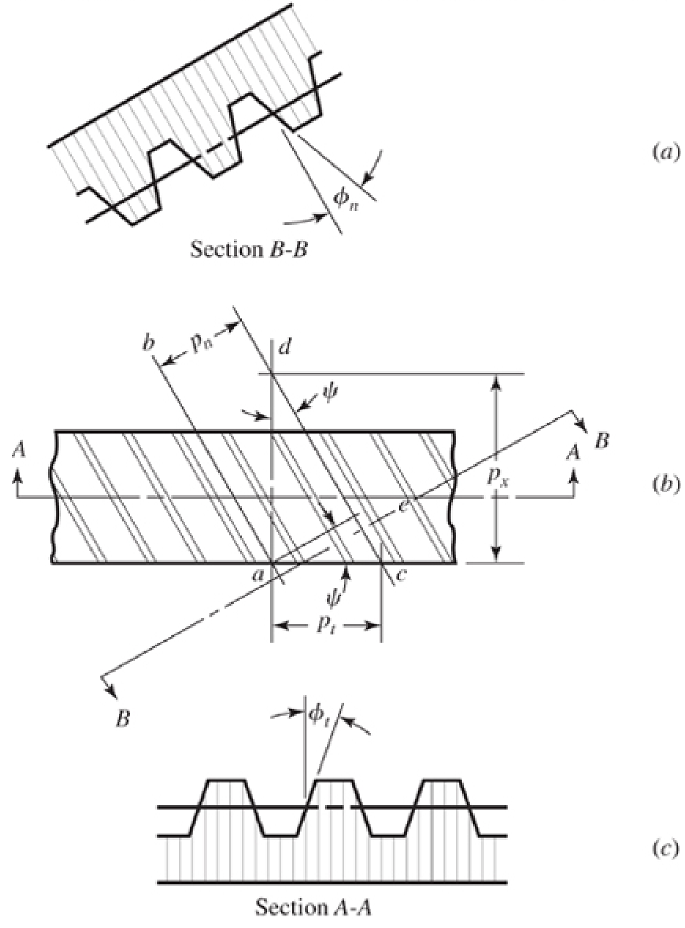
$$W_t = W \cos\phi_n \cos\psi$$

$$W_a = W \cos\phi_n \sin\psi$$



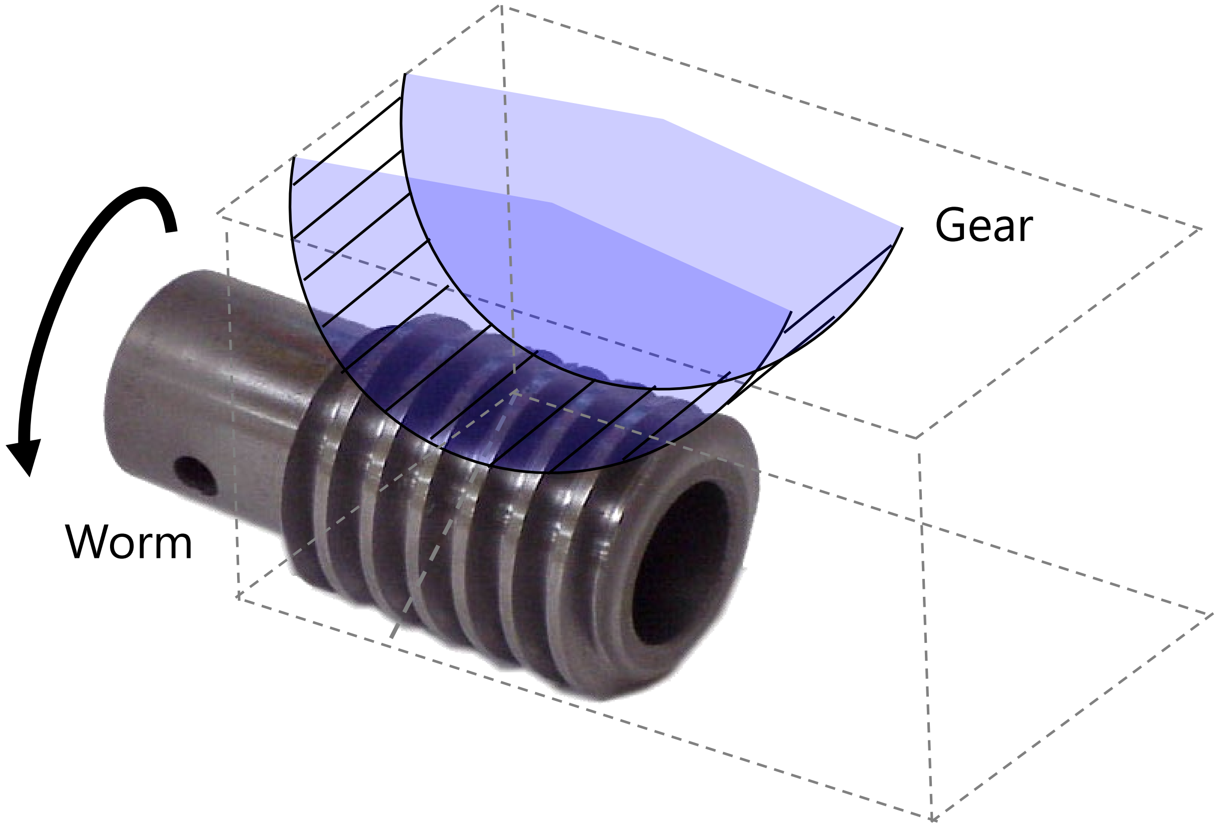
Helical Gear Forces

- Pressure angle and diametral pitch defined in different ways
- What is P_n (normal diametral pitch) in terms of P_t (transverse diametral pitch)?
- What is φ_n (normal pressure angle) in terms of φ_t (transverse pressure angle)?



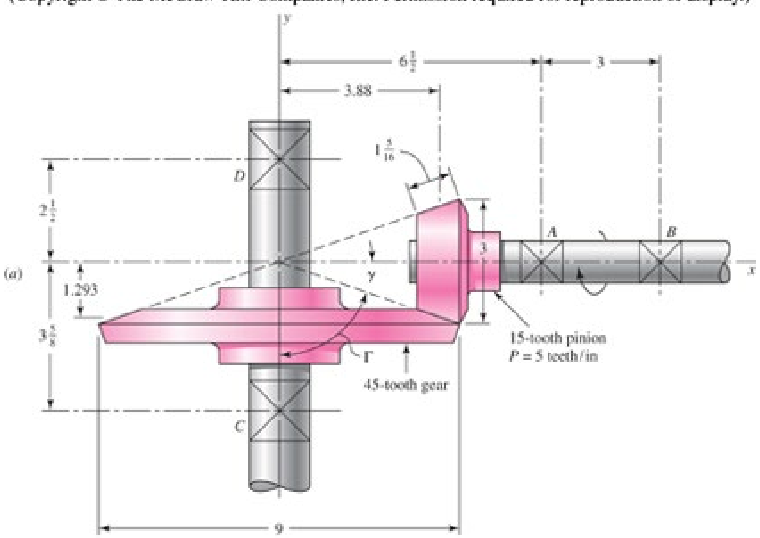
Worm Gear Forces

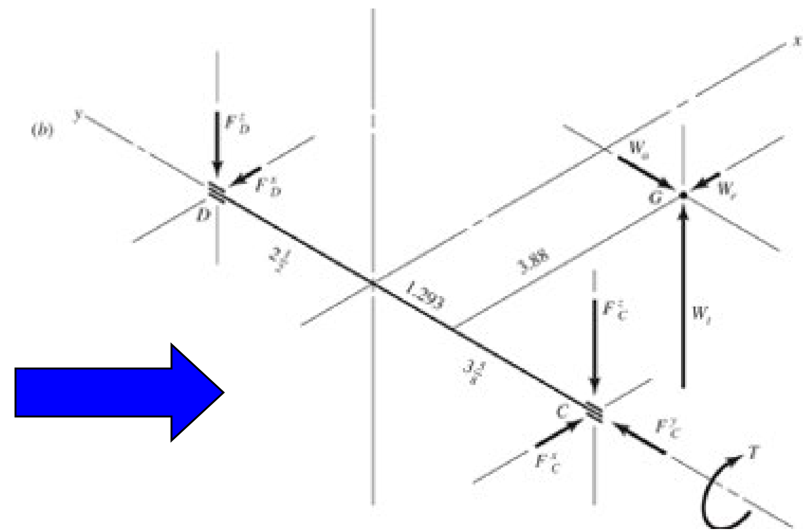
- What are the force names and directions?



Reaction Forces

- Knowing gear contact forces, you can compute reaction forces (e.g. to size bearings)



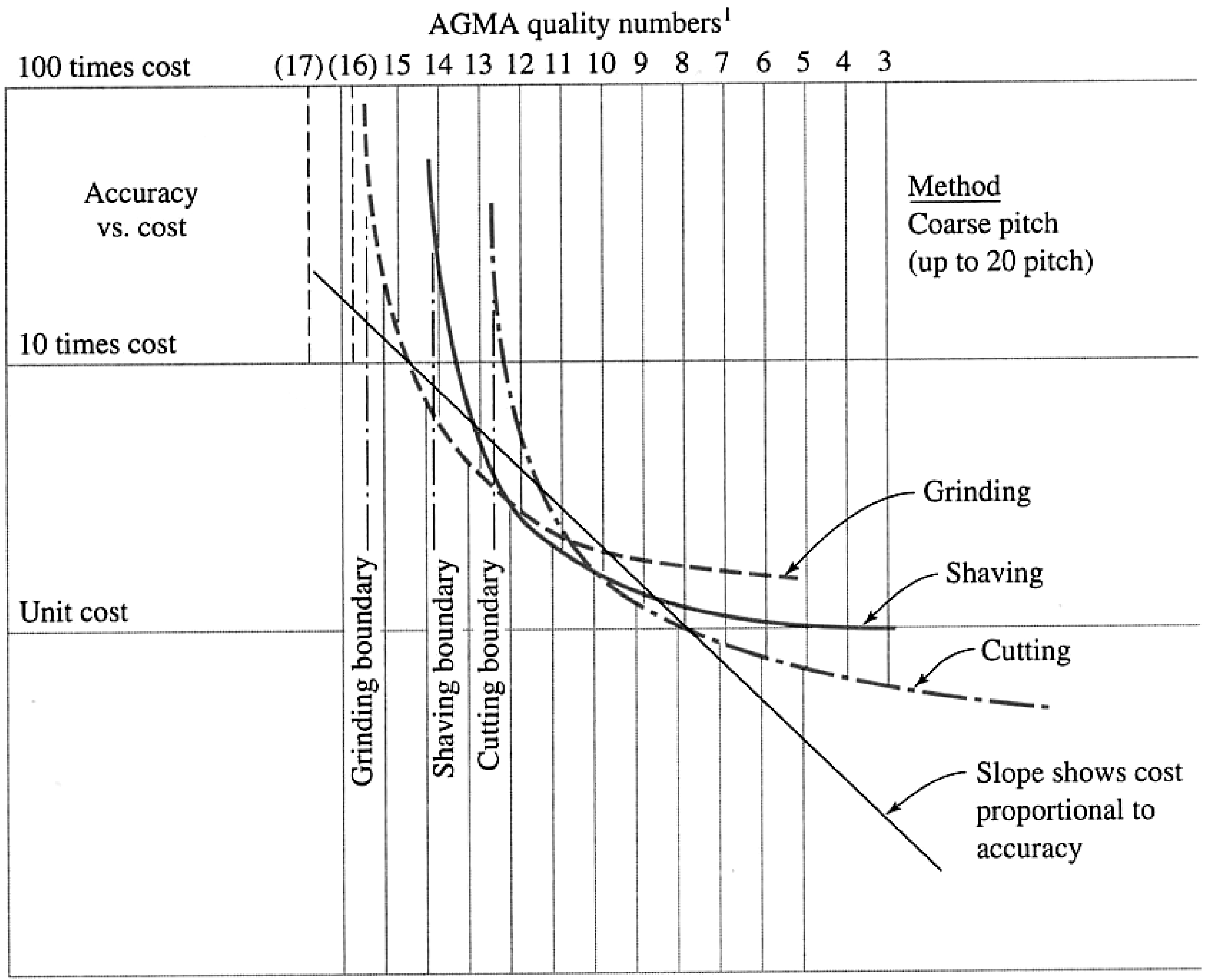


Material, Finishing, and Cost

- How might selecting a more expensive gear (which uses more expensive materials or finishing processes) actually reduce overall cost?

- Are there other advantages that may be gained with a more expensive gear?

Cost versus Accuracy

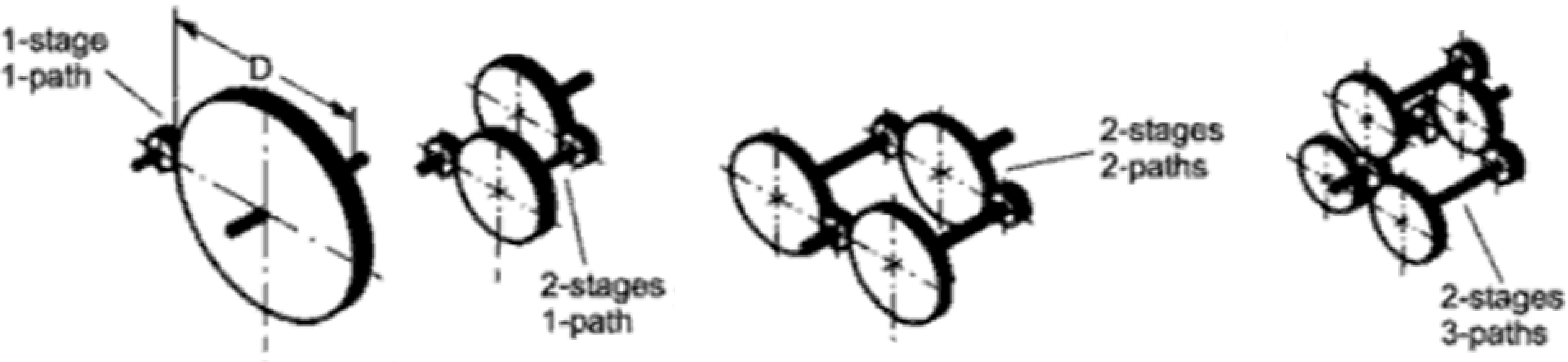


Accuracy Level

Accuracy Level	Approximate *Standard Quality Ranges* AGMA[2] Q_v Value
Highest possible accuracy. Achieved by special toolroom methods. Used for master gears, unusually critical high-speed gears, or when *both* highest load capacity and highest reliability are needed.	14 or 15
High accuracy. Achieved by grinding or shaving with first-rate machine tools, and utilizing skilled operators. Widely used for turbine gearing and aerospace gearing. Sometimes used for critical industrial gears.	12 or 13
Relatively high accuracy. Achieved by grinding or shaving with emphasis on production rate rather than highest quality. May be achieved by hobbing or shaping with best equipment under favorable conditions. Used for medium-speed industrial gears and critical vehicle gears.	10 or 11
Good accuracy. Achieved by hobbing or shaping with first-rate machine tools and skilled operators. May be obtained in high-production grinding or shaving. Typically used for vehicle gears and electric motor industrial gears running at slower speeds.	8 or 9
Nominal accuracy. Can be achieved by hobbing or shaping using older machine tools and less-skilled operators. Typically used for low-speed gears that *wear in* to yield a reasonable fit. (Lower hardness promotes wear-in.)	6 or 7
Minimal accuracy. For gears used at slow speeds and light loads. Teeth may be cast or molded in small sizes. Typically used in toys and gadgets. May be used for low-hardness gears when limited life and lower reliability are acceptable.	4 or 5

Gear Train Weight

- For the same torque transmission, gear train weight (and sometimes cost) goes down as complexity goes up – why?



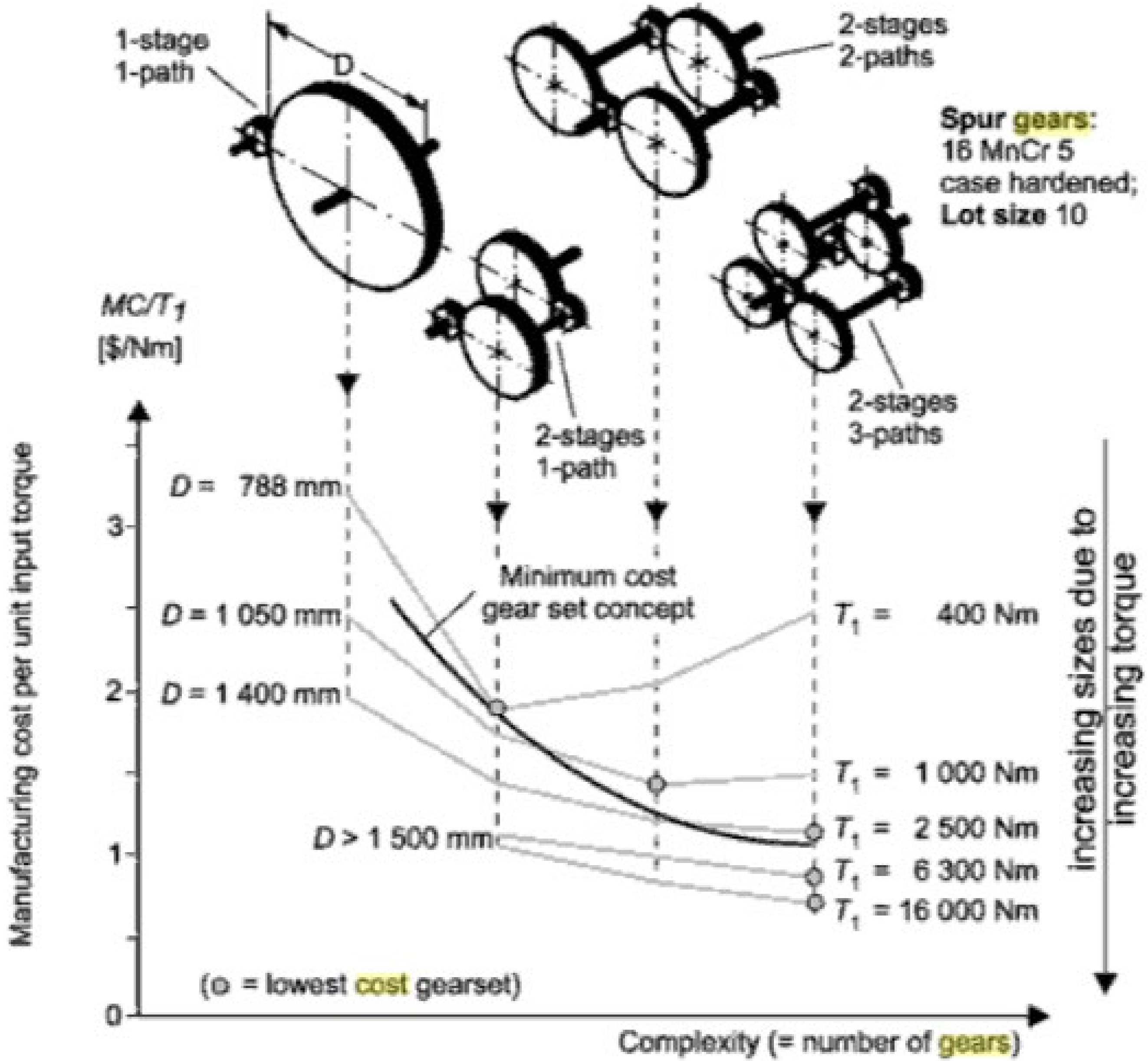


Fig. 7.3-2. Influence of the concept on manufacturing costs of gear trains [Fis83] (*valid only within the parameters shown!*) Gear-ratio $i = 10$