MECH366 : Modeling of Mechatronic Systems

L10 : Modeling of DC motors

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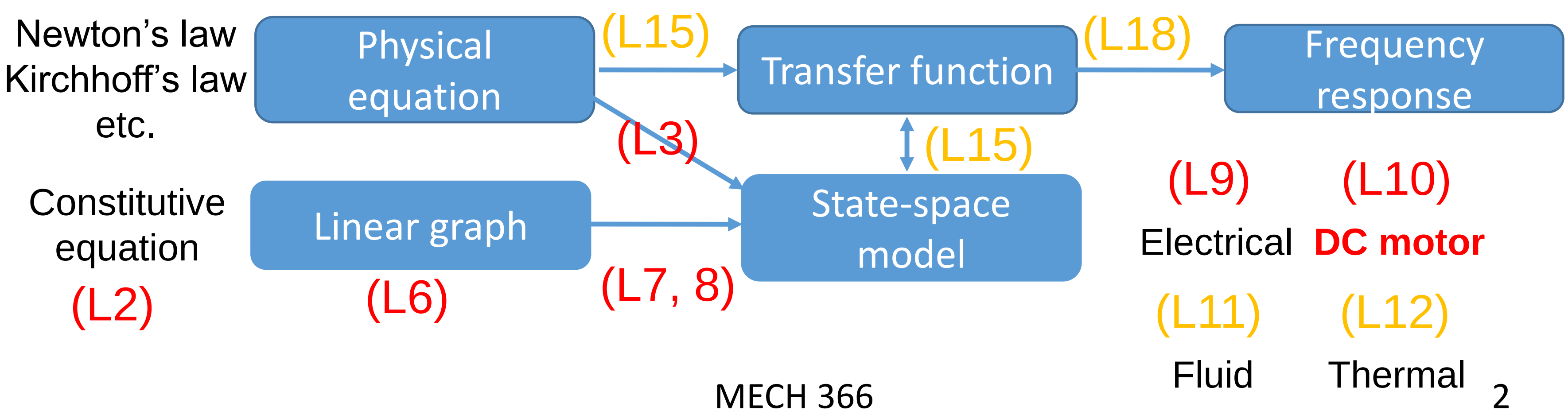
Review and today's topic

- Up to now, we have studied for mechanical and electrical systems
  - How to draw linear graphs
  - How to derive state-space models from linear graphs
- Today, we will study modeling of DC motors.



__ : State variable

	Constitutive relation for			
	Energy storage element		Energy dissipating element	
System type	A-Type	T-Type	D-Type	
Mechanical	Mass	Spring	Viscous Damper	power
(translational)				
v: velocity across var.	$m\underline{\dot{v}} = f$	$\underline{\dot{f}} = kv$	$f = bv$	$\mathcal{P} = fv$
f: force through var.	m: mass	k: stiffness	b: damping const.	
Electrical	Capacitor	Inductor	Resistor	
v: voltage across var.	$C\underline{\dot{v}} = i$	$L\underline{\dot{i}} = v$	$v = Ri$	$\mathcal{P} = iv$
i: current through	C: capacitance	L: inductance	R: resistance	
Thermal	Thermal capacitor	None	Thermal resistor	
T: temperature	$C_t\dot{T} = Q$		$T = R_tQ$	
Q: heat transfer rate	C: thermal capacitance		R_t: thermal resistance	
Fluid	Fluid capacitor	Fluid inertor	Fluid resistor	
P: pressure difference	$C_f\dot{P} = Q$	$I_f\dot{Q} = P$	$P = R_fQ$	
Q: volume flow rate	C_f: fluid capacitance	I_f: fluid inertance	R_f: fluid resistance	

DC motor

- An actuator, converting electrical energy into rotational mechanical energy

Shaft

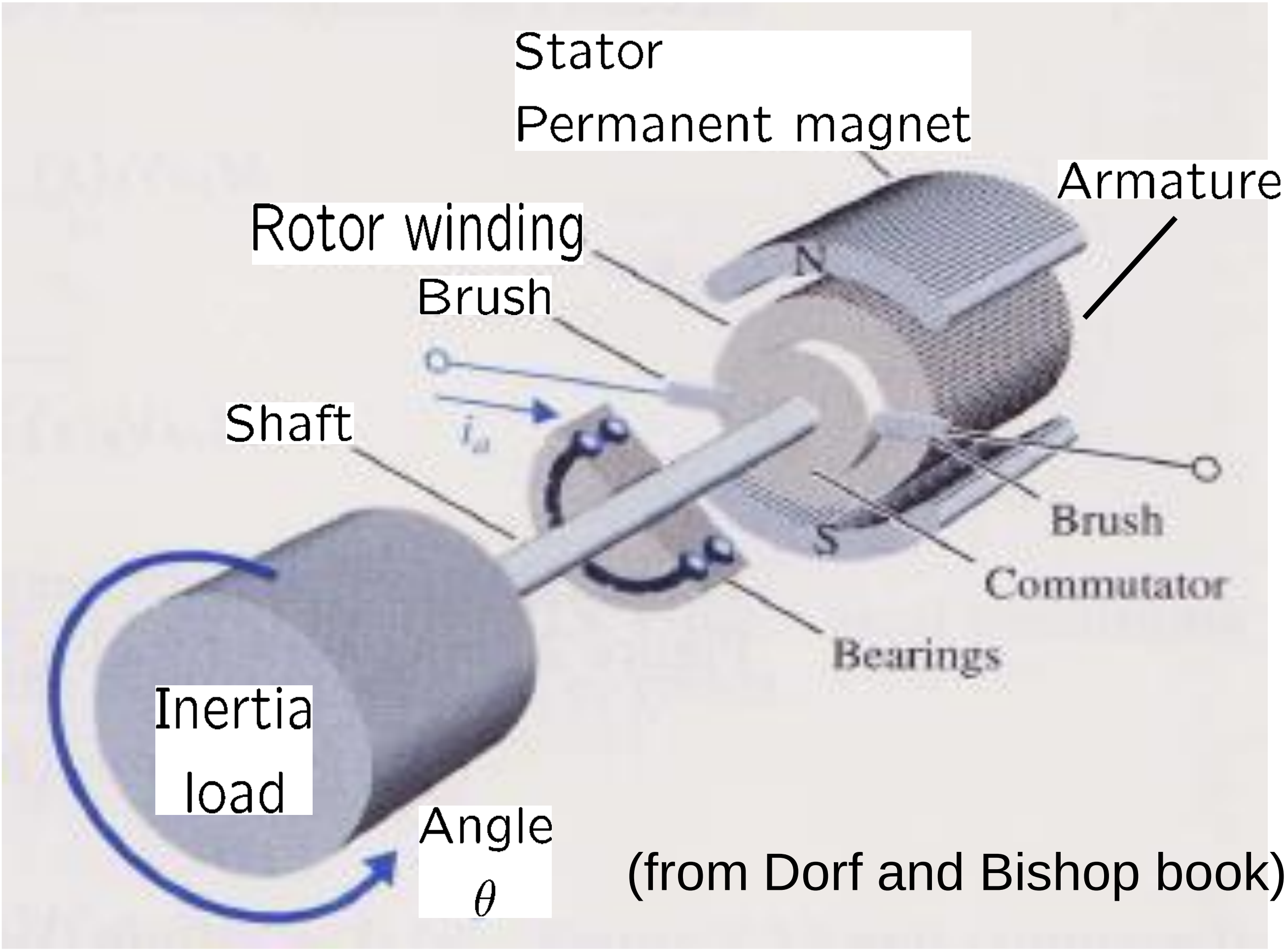
Electrical lead wire

(You will see DC motor in Lab #3-#5.)

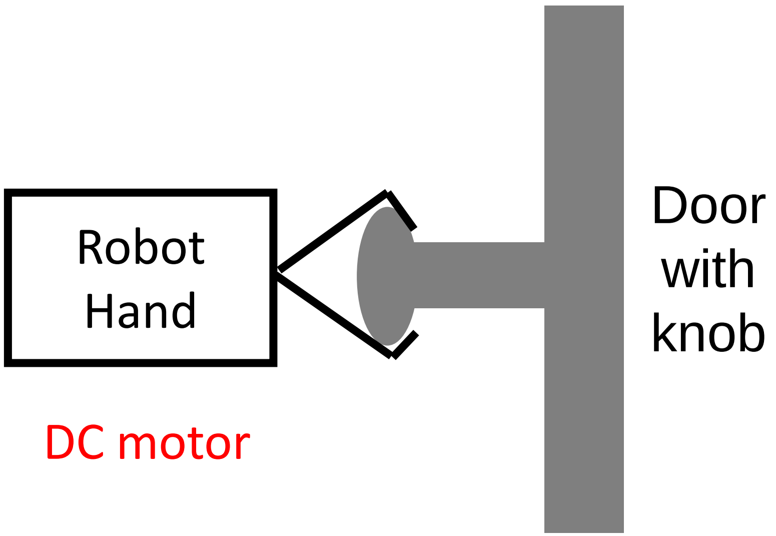
Why DC motor?

- Advantages
  - high torque
  - speed controllability
  - portability, etc.
- Widely used in mechatronics
  - Robot, drone
  - Tape drives
  - Printers
  - Machine tool industries
  - Radar tracking system, etc.

How does DC motor work?



Example (taken from the optional textbook by Dr. de Silva, p. 131) Robotic hand turning a door-knob

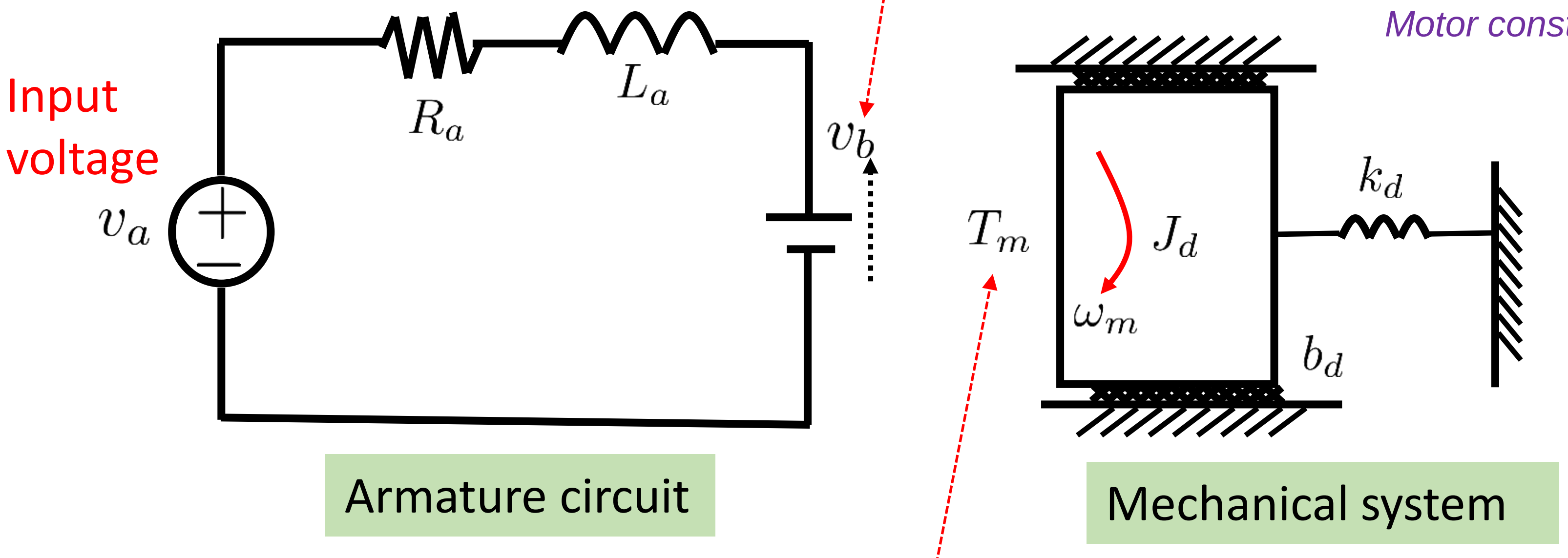


Example
Armature circuit & mechanical system

Back EMF $v_b = k_m \omega_m$

$$\omega_m = \frac{1}{k_m} v_b$$

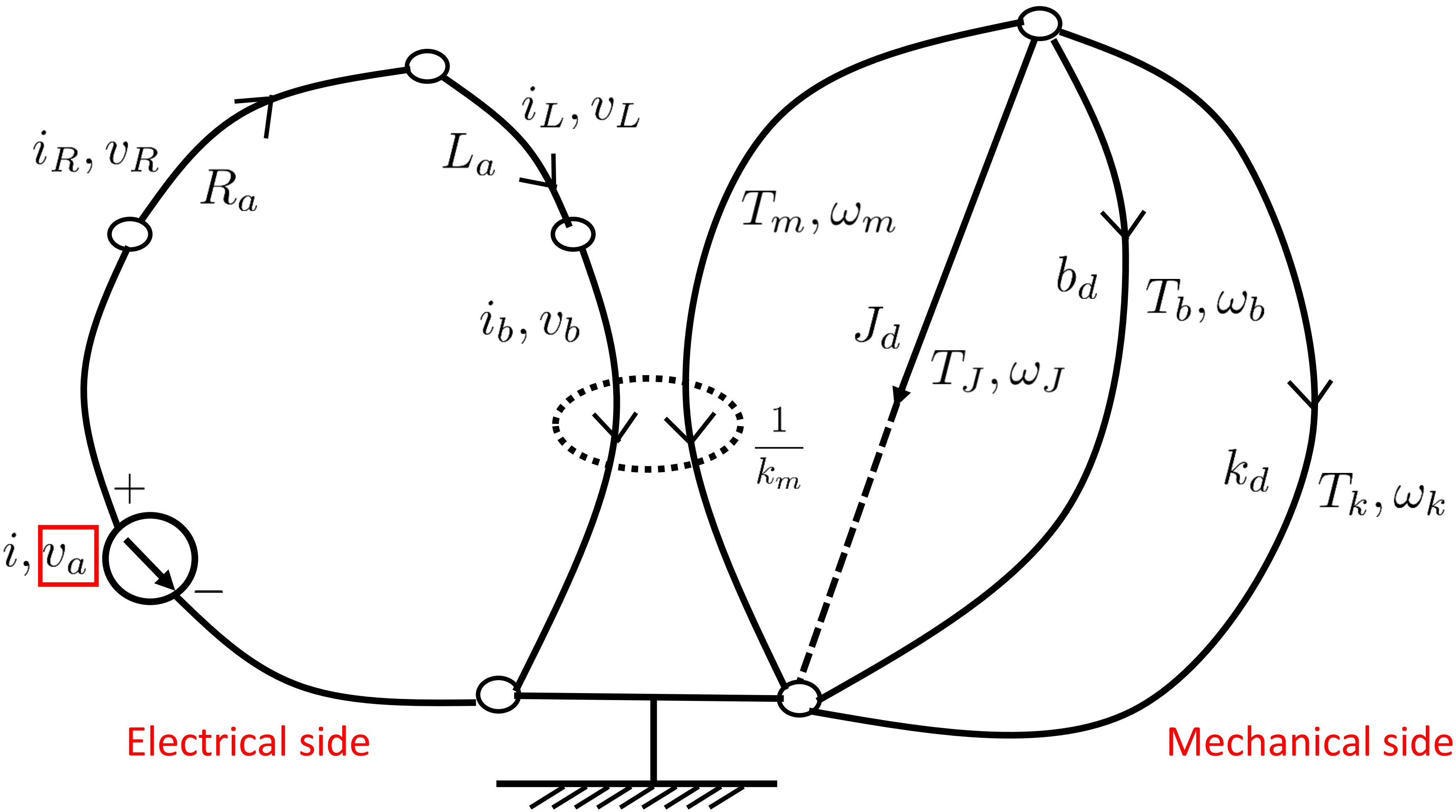
Motor constant



Motor torque $T_m = -k_m i_b$

Due to sign convention of transformer

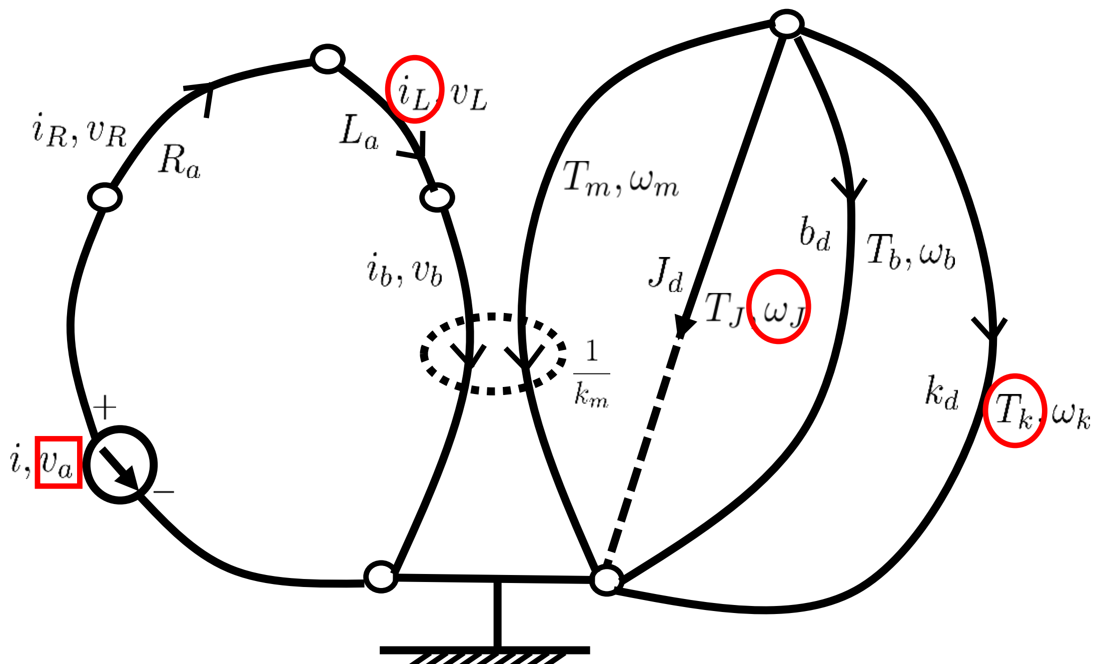
Example
Linear graph drawing



Example
State-variable selection

- Select the following as state variables:
  - Across variables (*v* & ω) for A-type elements (*C* & *J*)
  - Through variables (*i* & *T*)for T-type elements (*L* & *k*)

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} := \begin{bmatrix} i_L \\ \omega_J \\ T_k \end{bmatrix}$$



Example
Constitutive equations

States

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} := \begin{bmatrix} i_L \\ \omega_J \\ T_k \end{bmatrix}$$

- Basic elements

$$v_R = R_a i_R \quad L_a \dot{i}_L = v_L$$

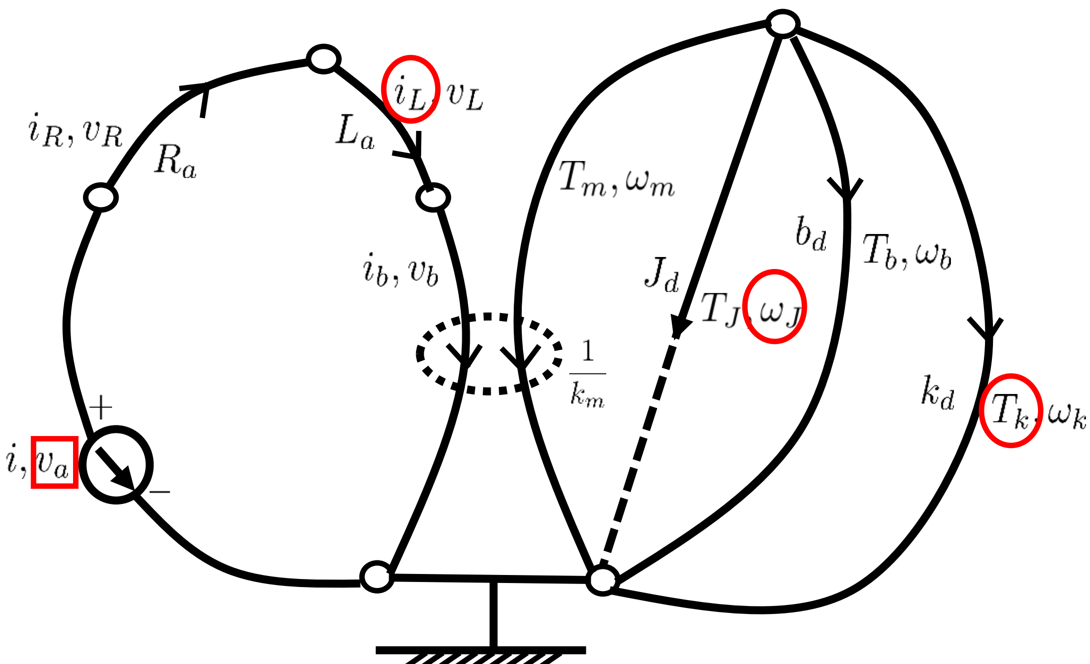
$$\dot{\omega}_J = \frac{1}{J_d} T_J \quad T_d = b_d \omega_d$$

$$\dot{T}_k = k_d \omega_k$$

- Transformer (electromechanical)

$$\omega_m = \frac{1}{k_m} v_b$$

$$T_m = -k_m i_b$$



Example
Loop and node equations

States

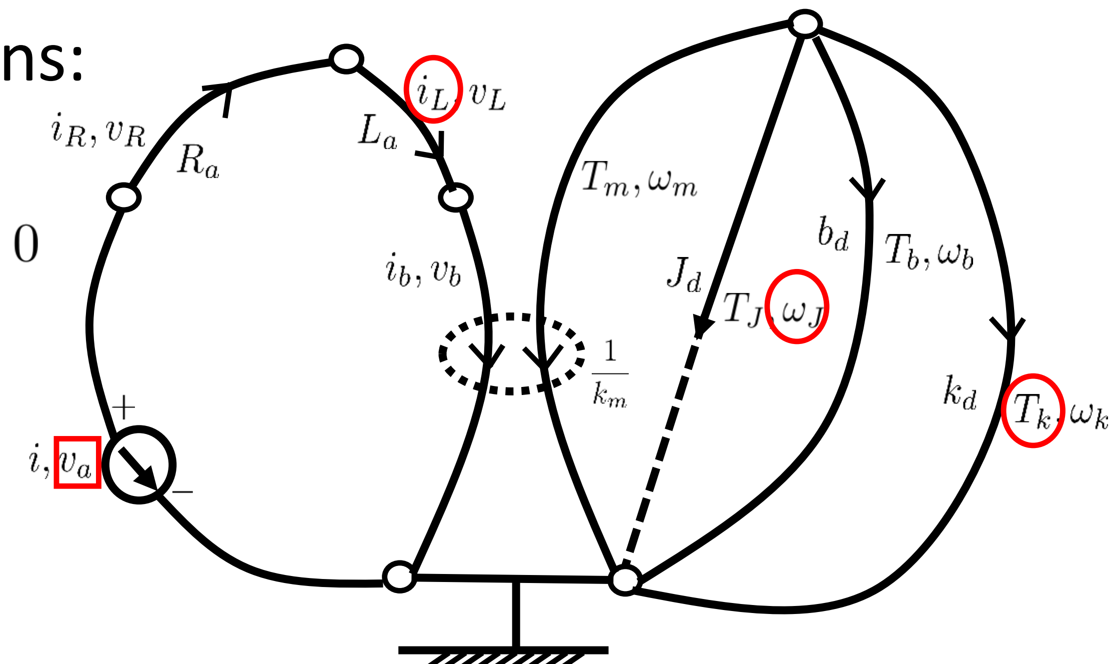
$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} := \begin{bmatrix} i_L \\ \omega_J \\ T_k \end{bmatrix}$$

- From loop equations:

$$\begin{cases} v_a = v_R + v_L + v_b \\ \omega_m = \omega_J = \omega_b = \omega_k \end{cases}$$

- From node equations:

$$\begin{cases} i = i_R = i_L = i_b \\ T_m + T_J + T_b + T_k = 0 \end{cases}$$



Example
State-space model

States

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} := \begin{bmatrix} i_L \\ \omega_J \\ T_k \end{bmatrix}$$

Input $u := v_a$ Outputs $y := \begin{bmatrix} \omega_J \\ T_k \end{bmatrix}$

$$\dot{i}_L = \frac{1}{L_a} v_L = \frac{1}{L_a}(v_a - v_R - v_b) = \frac{1}{L_a}(v_a - R_a i_R - k_m \omega_m) = \frac{1}{L_a}(v_a - R_a i_L - k_m \omega_J)$$

$$\dot{\omega}_J = \frac{1}{J_d} T_J = -\frac{1}{J_d}(T_m + T_k + T_b) = -\frac{1}{J_d}(-k_m i_b + T_k + b_d \omega_b) = -\frac{1}{J_d}(-k_m i_L + T_k + b_d \omega_J)$$

$$\dot{T}_k = k_d \omega_k = k_d \omega_J$$

$$\Rightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -R_a/L_a & -k_m/L_a & 0 \\ k_m/J_d & -b_d/J_d & -1/J_d \\ 0 & k_d & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1/L_a \\ 0 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Example
State-space model

States

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} := \begin{bmatrix} i_a \\ \omega_J \\ \theta_J \end{bmatrix}$$

Input

$$u := v_a$$

Outputs

$$y := \begin{bmatrix} \omega_J \\ \theta_J \end{bmatrix}$$

If the system does not contain rotational spring, and the output contains rotational angle …

$$\dot{i}_L = \frac{1}{L_a} v_L = \frac{1}{L_a}(v_a - v_R - v_b) = \frac{1}{L_a}(v_a - R_a i_R - k_m \omega_m) = \frac{1}{L_a}(v_a - R_a i_L - k_m \omega_J)$$

$$\dot{\omega}_J = \frac{1}{J_d} T_J = -\frac{1}{J_d}(T_m + T_d) = -\frac{1}{J_d}(-k_m i_b + b_d \omega_b) = -\frac{1}{J_d}(-k_m i_a + b_d \omega_J)$$

$\dot{\theta}_J = \omega_J$ (This equation is not from the linear graph.)

$$\Rightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -R_a/L_a & -k_m/L_a & 0 \\ k_m/J_d & -b_d/J_d & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1/L_a \\ 0 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Summary

- Modeling of DC motors
  - Linear graph drawing
  - Derivation of the state-space model
- After midterm,
  - Fluid systems
  - Thermal systems
- No homework, no lab this week

Announcement

- Midterm
  - October 11 (Friday), 3-3:50pm in CEME 1215
  - Covers up to today's material
  - Come to class 5-10 min before 3pm.
  - Policies
    - Closed-book, no calculator/PC
    - One page letter-size hand-written cheat-sheet (both sides)
  - Office hours: Any time! (as long as I am in my office.)
  - Study hard! (Read lecture slides, redo the homework questions, past exams.)