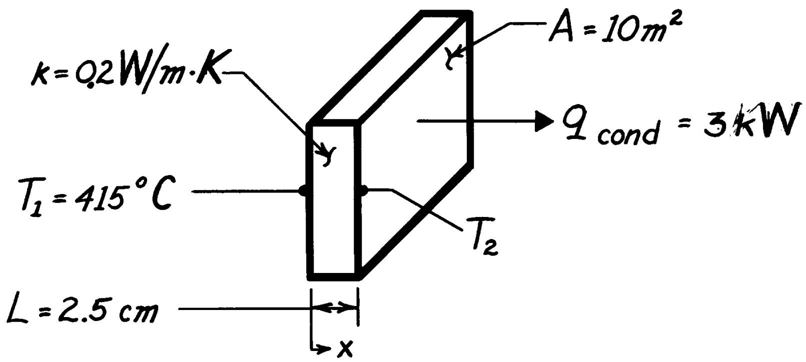
## PROBLEM 1.1

**KNOWN:** Heat rate, q, through one-dimensional wall of area A, thickness L, thermal conductivity k and inner temperature, $T_1$.

**FIND:** The outer temperature of the wall, $T_2$.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) One-dimensional conduction in the x-direction, (2) Steady-state conditions, (3) Constant properties.

**ANALYSIS:** The rate equation for conduction through the wall is given by Fourier's law,

$$q_{cond} = q_x = q''_x \cdot A = -k\frac{dT}{dx}\cdot A = kA\frac{T_1 - T_2}{L}.$$

Solving for $T_2$ gives

$$T_2 = T_1 - \frac{q_{cond}L}{kA}.$$

Substituting numerical values, find

$$T_2 = 415^\circ\text{C} - \frac{3000\text{W}\times 0.025\text{m}}{0.2\text{W/m}\cdot\text{K}\times 10\text{m}^2}$$

$$T_2 = 415^\circ\text{C} - 37.5^\circ\text{C}$$

$$T_2 = 378^\circ\text{C}.$$ <

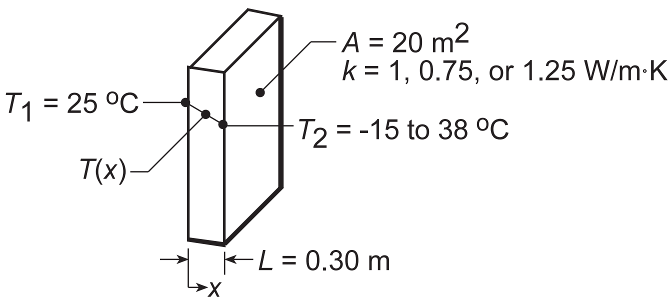
**COMMENTS:** Note direction of heat flow and fact that $T_2$ must be less than $T_1$.

# PROBLEM 1.2

**KNOWN:** Inner surface temperature and thermal conductivity of a concrete wall.

**FIND:** Heat loss by conduction through the wall as a function of ambient air temperatures ranging from -15 to 38°C.

**SCHEMATIC:**



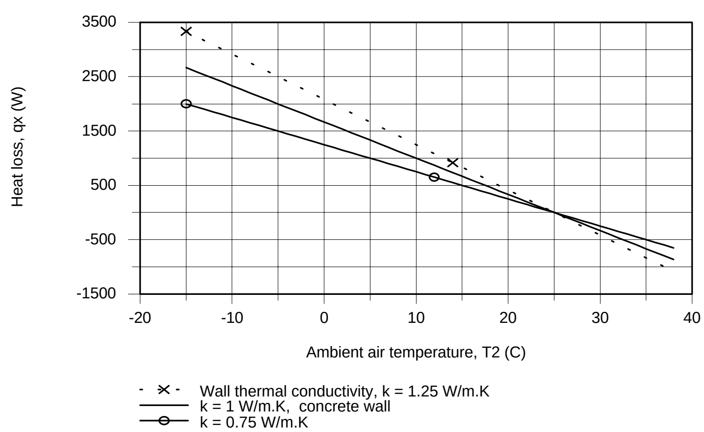

**ASSUMPTIONS:** (1) One-dimensional conduction in the x-direction, (2) Steady-state conditions, (3) Constant properties, (4) Outside wall temperature is that of the ambient air.

**ANALYSIS:** From Fourier's law, it is evident that the gradient, $dT/dx = -q''_x/k$, is a constant, and hence the temperature distribution is linear, if $q''_x$ and k are each constant. The heat flux must be constant under one-dimensional, steady-state conditions; and k is approximately constant if it depends only weakly on temperature. The heat flux and heat rate when the outside wall temperature is $T_2$ = -15°C are

$$q''_x = -k\frac{dT}{dx} = k\frac{T_1 - T_2}{L} = 1\,W/m\cdot K\,\frac{25^\circ C - \left(-15^\circ C\right)}{0.30\,m} = 133.3\,W/m^2 . \tag{1}$$

$$q_x = q''_x \times A = 133.3\,W/m^2 \times 20\,m^2 = 2667\,W . \tag{2}$$ <

Combining Eqs. (1) and (2), the heat rate $q_x$ can be determined for the range of ambient temperature, -15 ≤ $T_2$ ≤ 38°C, with different wall thermal conductivities, k.




For the concrete wall, k = 1 W/m·K, the heat loss varies linearily from +2667 W to -867 W and is zero when the inside and ambient temperatures are the same. The magnitude of the heat rate increases with increasing thermal conductivity.

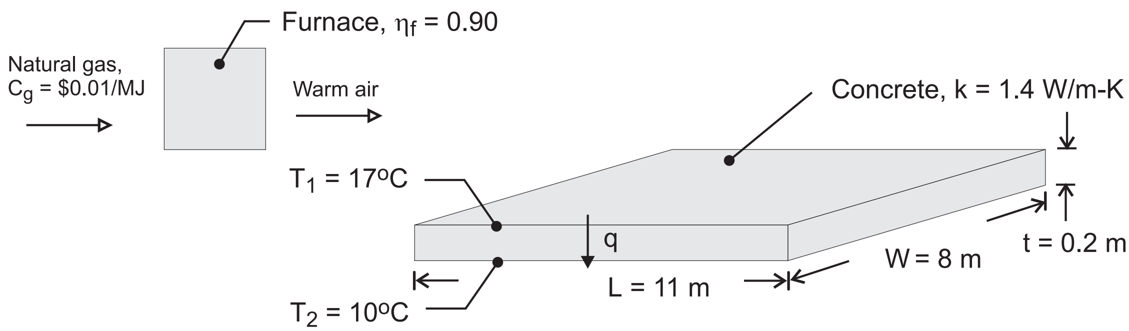
**COMMENTS:** Without steady-state conditions and constant k, the temperature distribution in a plane wall would not be linear.

# PROBLEM 1.3

**KNOWN:** Dimensions, thermal conductivity and surface temperatures of a concrete slab. Efficiency of gas furnace and cost of natural gas.

**FIND:** Daily cost of heat loss.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady state, (2) One-dimensional conduction, (3) Constant properties.

**ANALYSIS:** The rate of heat loss by conduction through the slab is

$$q = k\,(LW)\frac{T_1 - T_2}{t} = 1.4\,\mathrm{W/m\cdot K}\,(11\,\mathrm{m}\times 8\,\mathrm{m})\frac{7^\circ\mathrm{C}}{0.20\,\mathrm{m}} = 4312\ \mathrm{W} \qquad <$$

The daily cost of natural gas that must be combusted to compensate for the heat loss is

$$C_d = \frac{q\,C_g}{\eta_f}(\Delta t) = \frac{4312\,\mathrm{W}\times \$0.01/\mathrm{MJ}}{0.9\times 10^6\,\mathrm{J/MJ}}\left(24\,\mathrm{h/d}\times 3600\,\mathrm{s/h}\right) = \$4.14/\mathrm{d} \qquad <$$

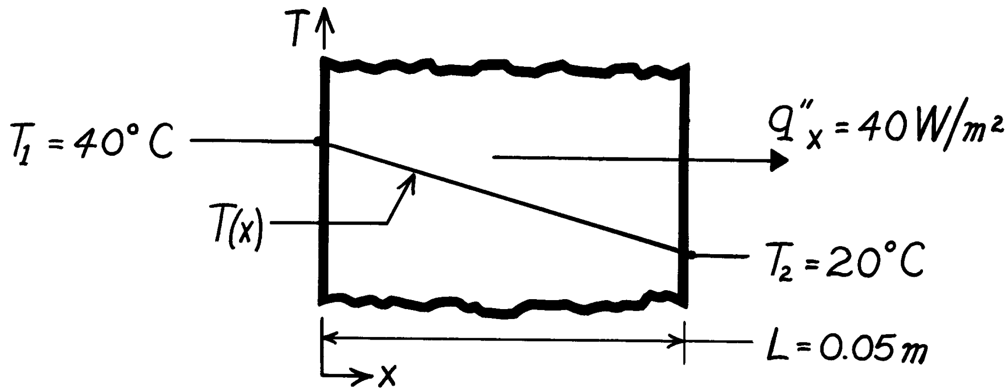
**COMMENTS:** The loss could be reduced by installing a floor covering with a layer of insulation between it and the concrete.

## PROBLEM 1.4

**KNOWN:** Heat flux and surface temperatures associated with a wood slab of prescribed thickness.

**FIND:** Thermal conductivity, k, of the wood.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) One-dimensional conduction in the x-direction, (2) Steady-state conditions, (3) Constant properties.

**ANALYSIS:** Subject to the foregoing assumptions, the thermal conductivity may be determined from Fourier's law, Eq. 1.2. Rearranging,

$$k = q_x'' \frac{L}{T_1 - T_2} = 40 \frac{W}{m^2} \frac{0.05m}{(40\text{-}20)^\circ C}$$

$$k = 0.10 \, W/m \cdot K. \qquad <$$

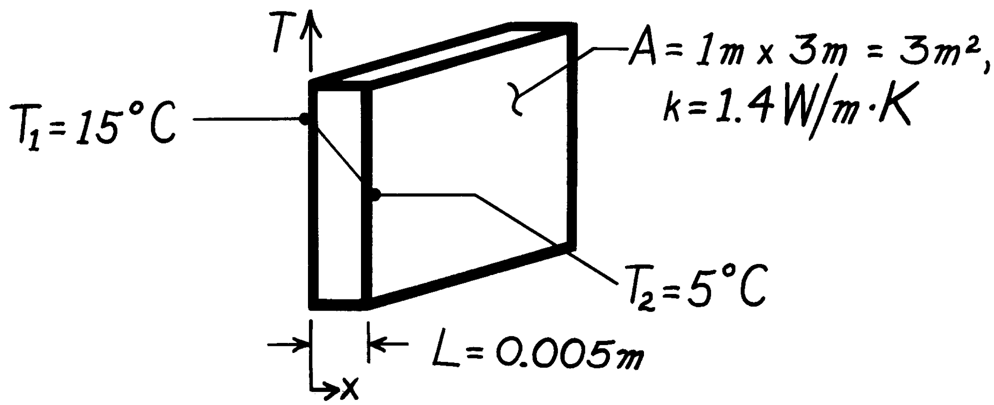
**COMMENTS:** Note that the $^\circ$C or K temperature units may be used interchangeably when evaluating a temperature difference.

**PROBLEM 1.5**

**KNOWN:** Inner and outer surface temperatures of a glass window of prescribed dimensions.

**FIND:** Heat loss through window.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) One-dimensional conduction in the x-direction, (2) Steady-state conditions, (3) Constant properties.

**ANALYSIS:** Subject to the foregoing conditions the heat flux may be computed from Fourier's law, Eq. 1.2.

$$q''_x = k\,\frac{T_1 - T_2}{L}$$

$$q''_x = 1.4\,\frac{W}{m \cdot K}\;\frac{(15\text{-}5)^{\circ}C}{0.005m}$$

$$q''_x = 2800\ W/m^2.$$

Since the heat flux is uniform over the surface, the heat loss (rate) is

$$q = q''_x \times A$$

$$q = 2800\ W/m^2 \times 3m^2$$

$$q = 8400\ W.$$

<

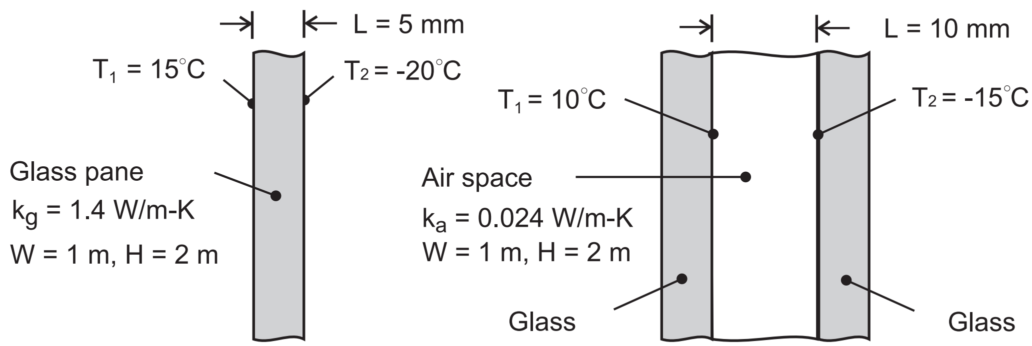
**COMMENTS:** A linear temperature distribution exists in the glass for the prescribed conditions.

## PROBLEM 1.6

**KNOWN:** Width, height, thickness and thermal conductivity of a single pane window and the air space of a double pane window. Representative winter surface temperatures of single pane and air space.

**FIND:** Heat loss through single and double pane windows.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) One-dimensional conduction through glass or air, (2) Steady-state conditions, (3) Enclosed air of double pane window is stagnant (negligible buoyancy induced motion).

**ANALYSIS:** From Fourier's law, the heat losses are

*Single Pane*: $$q_g = k_g A \frac{T_1 - T_2}{L} = 1.4 \text{ W/m}\cdot\text{K}\left(2\text{m}^2\right)\frac{35\ ^\circ\text{C}}{0.005\text{m}} = 19,600 \text{ W}$$

*Double Pane*: $$q_a = k_a A \frac{T_1 - T_2}{L} = 0.024\left(2\text{m}^2\right)\frac{25\ ^\circ\text{C}}{0.010 \text{ m}} = 120 \text{ W}$$

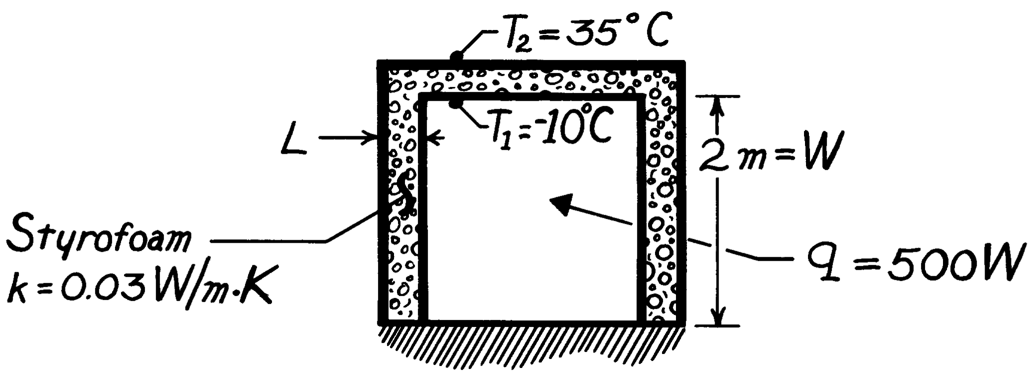
**COMMENTS:** Losses associated with a single pane are unacceptable and would remain excessive, even if the thickness of the glass were doubled to match that of the air space. The principal advantage of the double pane construction resides with the low thermal conductivity of air (~ 60 times smaller than that of glass). For a fixed ambient outside air temperature, use of the double pane construction would also increase the surface temperature of the glass exposed to the room (inside) air.

# PROBLEM 1.7

**KNOWN:** Dimensions of freezer compartment. Inner and outer surface temperatures.

**FIND:** Thickness of styrofoam insulation needed to maintain heat load below prescribed value.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Perfectly insulated bottom, (2) One-dimensional conduction through 5 walls of area A = 4m$^2$, (3) Steady-state conditions, (4) Constant properties.

**ANALYSIS:** Using Fourier's law, Eq. 1.2, the heat rate is

$$q = q'' \cdot A = k \frac{\Delta T}{L} A_{total}$$

Solving for L and recognizing that $A_{total} = 5 \times W^2$, find

$$L = \frac{5\, k\, \Delta\, T\, W^2}{q}$$

$$L = \frac{5 \times 0.03\ \text{W/m}\cdot\text{K}\ \left[35 - (-10)\right]^{\circ}\text{C}\ \left(4\text{m}^2\right)}{500\ \text{W}}$$

$$L = 0.054\text{m} = 54\text{mm}.$$ <

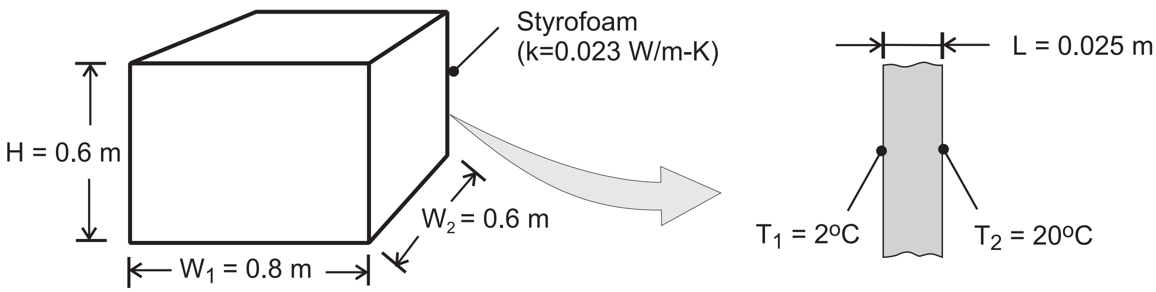
**COMMENTS:** The corners will cause local departures from one-dimensional conduction and a slightly larger heat loss.

## PROBLEM 1.8

**KNOWN:** Dimensions and thermal conductivity of food/beverage container. Inner and outer surface temperatures.

**FIND:** Heat flux through container wall and total heat load.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state conditions, (2) Negligible heat transfer through bottom wall, (3) Uniform surface temperatures and one-dimensional conduction through remaining walls.

**ANALYSIS:** From Fourier's law, Eq. 1.2, the heat flux is

$$q'' = k\frac{T_2 - T_1}{L} = \frac{0.023\ \text{W/m}\cdot\text{K}\ (20-2)^\circ\ \text{C}}{0.025\ \text{m}} = 16.6\ \text{W/m}^2 \qquad <$$

Since the flux is uniform over each of the five walls through which heat is transferred, the heat load is

$$q = q'' \times A_{total} = q''\left[H\left(2W_1 + 2W_2\right) + W_1 \times W_2\right]$$

$$q = 16.6\ \text{W/m}^2\left[0.6\text{m}\left(1.6\text{m} + 1.2\text{m}\right) + \left(0.8\text{m} \times 0.6\text{m}\right)\right] = 35.9\ \text{W} \qquad <$$

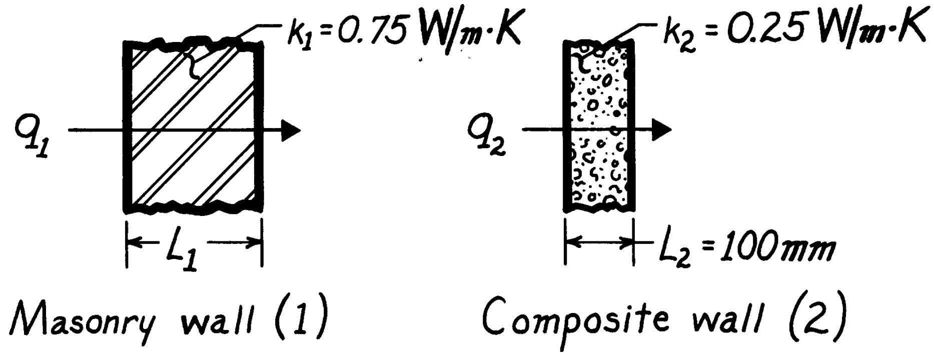
**COMMENTS:** The corners and edges of the container create local departures from one-dimensional conduction, which increase the heat load. However, for H, $W_1$, $W_2$ >> L, the effect is negligible.

**PROBLEM 1.9**

**KNOWN:** Masonry wall of known thermal conductivity has a heat rate which is 80% of that through a composite wall of prescribed thermal conductivity and thickness.

**FIND:** Thickness of masonry wall.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Both walls subjected to same surface temperatures, (2) One-dimensional conduction, (3) Steady-state conditions, (4) Constant properties.

**ANALYSIS:** For steady-state conditions, the conduction heat flux through a one-dimensional wall follows from Fourier's law, Eq. 1.2,

$$q'' = k\frac{\Delta T}{L}$$

where $\Delta T$ represents the difference in surface temperatures. Since $\Delta T$ is the same for both walls, it follows that

$$L_1 = L_2 \frac{k_1}{k_2} \cdot \frac{q_2''}{q_1''}.$$

With the heat fluxes related as

$$q_1'' = 0.8\, q_2''$$

$$L_1 = 100\text{mm}\,\frac{0.75\ \text{W/m}\cdot\text{K}}{0.25\ \text{W/m}\cdot\text{K}} \times \frac{1}{0.8} = 375\text{mm}. \qquad <$$

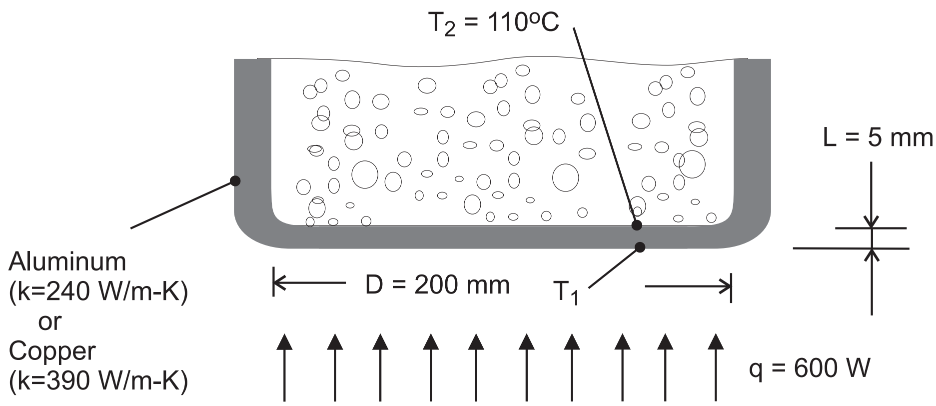
**COMMENTS:** Not knowing the temperature difference across the walls, we cannot find the value of the heat rate.

## PROBLEM 1.10

**KNOWN:** Thickness, diameter and inner surface temperature of bottom of pan used to boil water. Rate of heat transfer to the pan.

**FIND:** Outer surface temperature of pan for an aluminum and a copper bottom.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) One-dimensional, steady-state conduction through bottom of pan.

**ANALYSIS:** From Fourier's law, the rate of heat transfer by conduction through the bottom of the pan is

$$q = kA\frac{T_1 - T_2}{L}$$

Hence,

$$T_1 = T_2 + \frac{qL}{kA}$$

where $A = \pi D^2/4 = \pi(0.2\text{m})^2/4 = 0.0314\ \text{m}^2$.

*Aluminum*: $$T_1 = 110\ ^\circ\text{C} + \frac{600\text{W}(0.005\ \text{m})}{240\ \text{W/m}\cdot\text{K}\left(0.0314\ \text{m}^2\right)} = 110.40\ ^\circ\text{C}$$

*Copper*: $$T_1 = 110\ ^\circ\text{C} + \frac{600\text{W}(0.005\ \text{m})}{390\ \text{W/m}\cdot\text{K}\left(0.0314\ \text{m}^2\right)} = 110.25\ ^\circ\text{C}$$

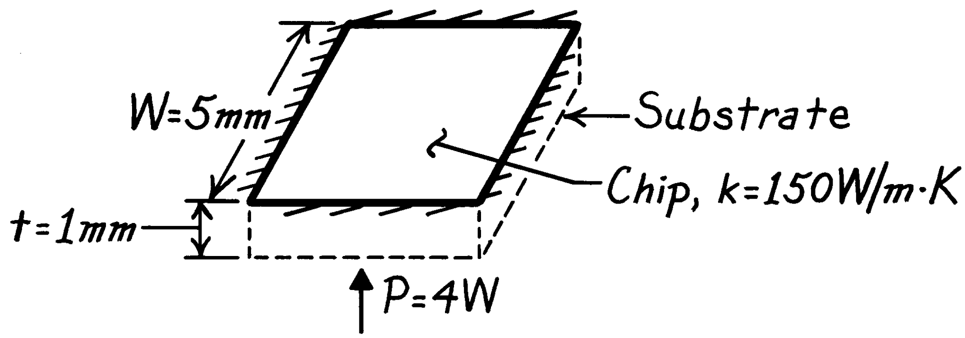
**COMMENTS:** Although the temperature drop across the bottom is slightly larger for aluminum (due to its smaller thermal conductivity), it is sufficiently small to be negligible for both materials. To a good approximation, the bottom may be considered *isothermal* at T ≈ 110 °C, which is a desirable feature of pots and pans.

## PROBLEM 1.11

**KNOWN:** Dimensions and thermal conductivity of a chip. Power dissipated on one surface.

**FIND:** Temperature drop across the chip.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state conditions, (2) Constant properties, (3) Uniform heat dissipation, (4) Negligible heat loss from back and sides, (5) One-dimensional conduction in chip.

**ANALYSIS:** All of the electrical power dissipated at the back surface of the chip is transferred by conduction through the chip. Hence, from Fourier's law,

$$P = q = kA\,\frac{\Delta T}{t}$$

or

$$\Delta T = \frac{t \cdot P}{kW^2} = \frac{0.001\text{ m} \times 4\text{ W}}{150\text{ W/m}\cdot\text{K}\left(0.005\text{ m}\right)^2}$$

$$\Delta T = 1.1^\circ\text{ C.}$$ <

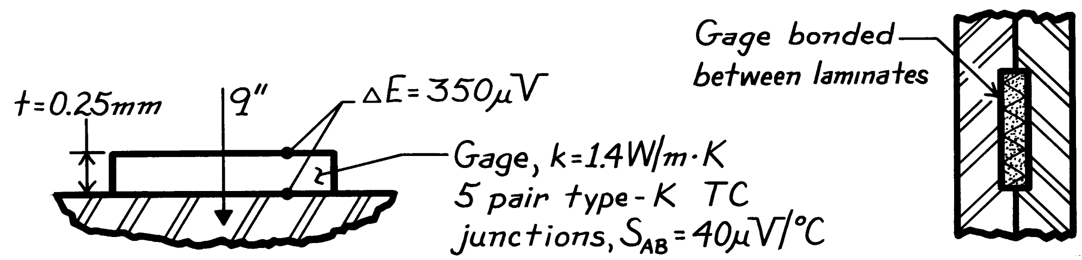
**COMMENTS:** For fixed P, the temperature drop across the chip decreases with increasing k and W, as well as with decreasing t.

**PROBLEM 1.12**

**KNOWN:** Heat flux gage with thin-film thermocouples on upper and lower surfaces; output voltage, calibration constant, thickness and thermal conductivity of gage.

**FIND:** (a) Heat flux, (b) Precaution when sandwiching gage between two materials.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state conditions, (2) One-dimensional heat conduction in gage, (3) Constant properties.

**ANALYSIS:** (a) Fourier's law applied to the gage can be written as

$$q'' = k\frac{\Delta T}{\Delta x}$$

and the gradient can be expressed as

$$\frac{\Delta T}{\Delta x} = \frac{\Delta E / N}{S_{AB} t}$$

where N is the number of differentially connected thermocouple junctions, $S_{AB}$ is the Seebeck coefficient for type K thermocouples (A-chromel and B-alumel), and $\Delta x = t$ is the gage thickness. Hence,

$$q'' = \frac{k\Delta E}{N S_{AB} t}$$

$$q'' = \frac{1.4\ \text{W/m}\cdot\text{K} \times 350 \times 10^{-6}\ \text{V}}{5 \times 40 \times 10^{-6}\ \text{V}/^\circ\text{C} \times 0.25 \times 10^{-3}\ \text{m}} = 9800\ \text{W/m}^2.$$ <

(b) The major precaution to be taken with this type of gage is to match its thermal conductivity with that of the material on which it is installed. If the gage is bonded between laminates (see sketch above) and its thermal conductivity is significantly different from that of the laminates, one dimensional heat flow will be disturbed and the gage will read incorrectly.

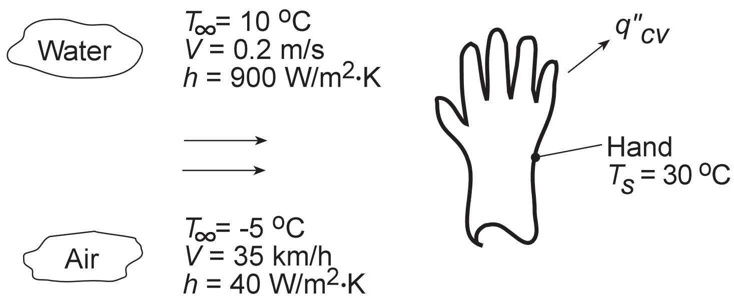
**COMMENTS:** If the thermal conductivity of the gage is lower than that of the laminates, will it indicate heat fluxes that are systematically high or low?

# PROBLEM 1.13

**KNOWN:** Hand experiencing convection heat transfer with moving air and water.

**FIND:** Determine which condition feels colder. Contrast these results with a heat loss of 30 W/m$^2$ under normal room conditions.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Temperature is uniform over the hand's surface, (2) Convection coefficient is uniform over the hand, and (3) Negligible radiation exchange between hand and surroundings in the case of air flow.

**ANALYSIS:** The hand will feel colder for the condition which results in the larger heat loss. The heat loss can be determined from Newton's law of cooling, Eq. 1.3a, written as

$$q'' = h\left(T_s - T_\infty\right)$$

For the air stream:

$$q''_{air} = 40\,W/m^2\cdot K\left[30-(-5)\right]K = 1,400\,W/m^2 \qquad <$$

For the water stream:

$$q''_{water} = 900\,W/m^2\cdot K\left(30-10\right)K = 18,000\,W/m^2 \qquad <$$

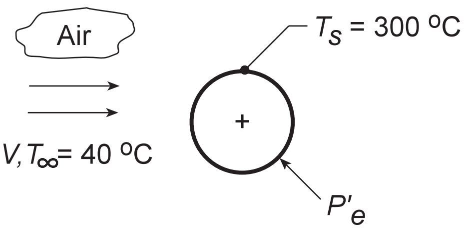
**COMMENTS:** The heat loss for the hand in the water stream is an order of magnitude larger than when in the air stream for the given temperature and convection coefficient conditions. In contrast, the heat loss in a normal room environment is only 30 W/m$^2$ which is a factor of 400 times less than the loss in the air stream. In the room environment, the hand would feel comfortable; in the air and water streams, as you probably know from experience, the hand would feel uncomfortably cold since the heat loss is excessively high.

# PROBLEM 1.14

**KNOWN:** Power required to maintain the surface temperature of a long, 25-mm diameter cylinder with an imbedded electrical heater for different air velocities.

**FIND:** (a) Determine the convection coefficient for each of the air velocity conditions and display the results graphically, and (b) Assuming that the convection coefficient depends upon air velocity as $h = CV^n$, determine the parameters C and n.

**SCHEMATIC:**




| V(m/s) | 1 | 2 | 4 | 8 | 12 |
|---|---|---|---|---|---|
| $P'_e$ (W/m) | 450 | 658 | 983 | 1507 | 1963 |
| h $(W/m^2 \cdot K)$ | 22.0 | 32.2 | 48.1 | 73.8 | 96.1 |

**ASSUMPTIONS:** (1) Temperature is uniform over the cylinder surface, (2) Negligible radiation exchange between the cylinder surface and the surroundings, (3) Steady-state conditions.

**ANALYSIS:** (a) From an overall energy balance on the cylinder, the power dissipated by the electrical heater is transferred by convection to the air stream. Using Newtons law of cooling on a per unit length basis,

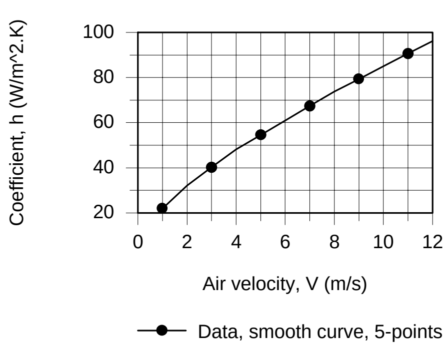
$$P'_e = h(\pi D)(T_s - T_\infty)$$

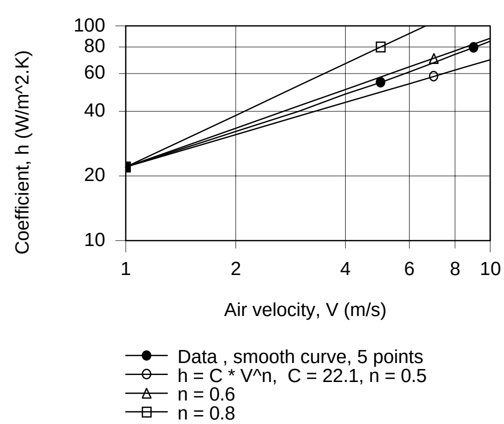
where $P'_e$ is the electrical power dissipated per unit length of the cylinder. For the V = 1 m/s condition, using the data from the table above, find

$$h = 450\,W/m \Big/ \pi \times 0.025\,m\,(300 - 40)^\circ C = 22.0\,W/m^2 \cdot K$$ <

Repeating the calculations, find the convection coefficients for the remaining conditions which are tabulated above and plotted below. Note that h is not linear with respect to the air velocity.

(b) To determine the (C,n) parameters, we plotted h vs. V on log-log coordinates. Choosing C = 22.12 $W/m^2 \cdot K(s/m)^n$, assuring a match at V = 1, we can readily find the exponent n from the slope of the h vs. V curve. From the trials with n = 0.8, 0.6 and 0.5, we recognize that n = 0.6 is a reasonable choice. Hence, C = 22.12 and n = 0.6. <



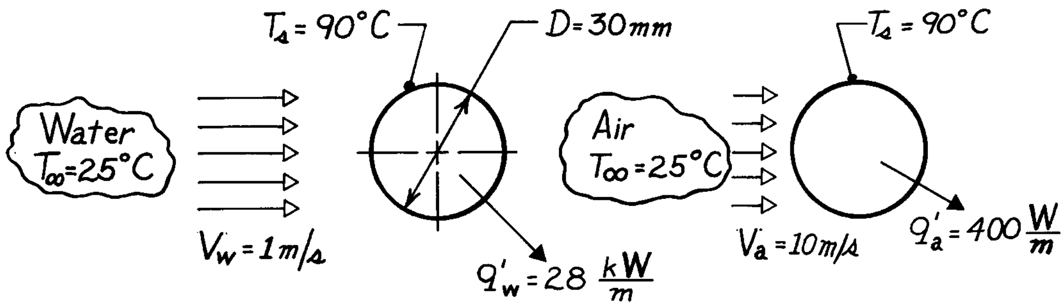



PROBLEM 1.15

**KNOWN:** Long, 30mm-diameter cylinder with embedded electrical heater; power required to maintain a specified surface temperature for water and air flows.

**FIND:** Convection coefficients for the water and air flow convection processes, $h_w$ and $h_a$, respectively.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Flow is cross-wise over cylinder which is very long in the direction normal to flow.

**ANALYSIS:** The convection heat rate from the cylinder per unit length of the cylinder has the form

$$q' = h(\pi D)\ (T_s - T_\infty)$$

and solving for the heat transfer convection coefficient, find

$$h = \frac{q'}{\pi D\ (T_s - T_\infty)}.$$

Substituting numerical values for the water and air situations:

*Water* $$h_w = \frac{28 \times 10^3\ \text{W/m}}{\pi \times 0.030\text{m}\ (90\text{-}25)^\circ\ \text{C}} = 4{,}570\ \text{W/m}^2 \cdot \text{K}$$ <

*Air* $$h_a = \frac{400\ \text{W/m}}{\pi \times 0.030\text{m}\ (90\text{-}25)^\circ\ \text{C}} = 65\ \text{W/m}^2 \cdot \text{K}.$$ <

**COMMENTS:** Note that the air velocity is 10 times that of the water flow, yet

$$h_w \approx 70 \times h_a.$$

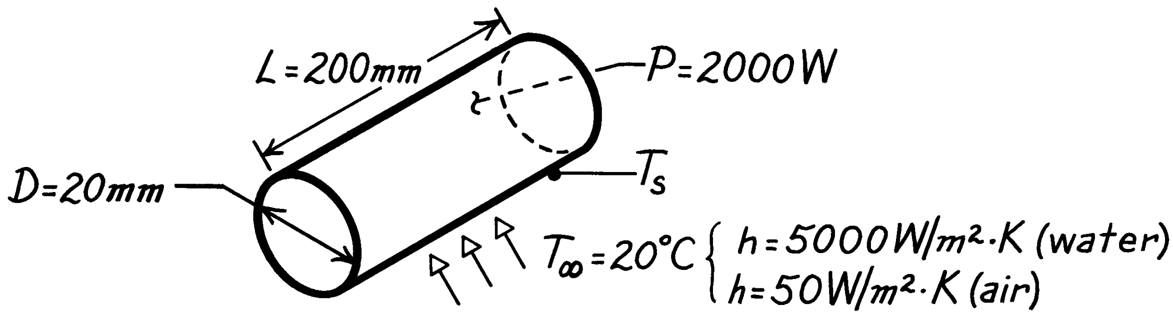
These values for the convection coefficient are typical for forced convection heat transfer with liquids and gases. See Table 1.1.

**PROBLEM 1.16**

**KNOWN:** Dimensions of a cartridge heater. Heater power. Convection coefficients in air and water at a prescribed temperature.

**FIND:** Heater surface temperatures in water and air.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state conditions, (2) All of the electrical power is transferred to the fluid by convection, (3) Negligible heat transfer from ends.

**ANALYSIS:** With P = $q_{conv}$, Newton's law of cooling yields

$$P = hA\left(T_S - T_\infty\right) = h\pi DL\left(T_S - T_\infty\right)$$
$$T_S = T_\infty + \frac{P}{h\pi DL}.$$

In water,

$$T_s = 20^\circ C + \frac{2000\ W}{5000\ W/m^2 \cdot K \times \pi \times 0.02\ m \times 0.200\ m}$$

$$T_s = 20^\circ C + 31.8^\circ C = 51.8^\circ C. \qquad <$$

In air,

$$T_s = 20^\circ C + \frac{2000\ W}{50\ W/m^2 \cdot K \times \pi \times 0.02\ m \times 0.200\ m}$$

$$T_s = 20^\circ C + 3183^\circ C = 3203^\circ C. \qquad <$$

**COMMENTS:** (1) Air is much less effective than water as a heat transfer fluid. Hence, the cartridge temperature is much higher in air, so high, in fact, that the cartridge would melt.

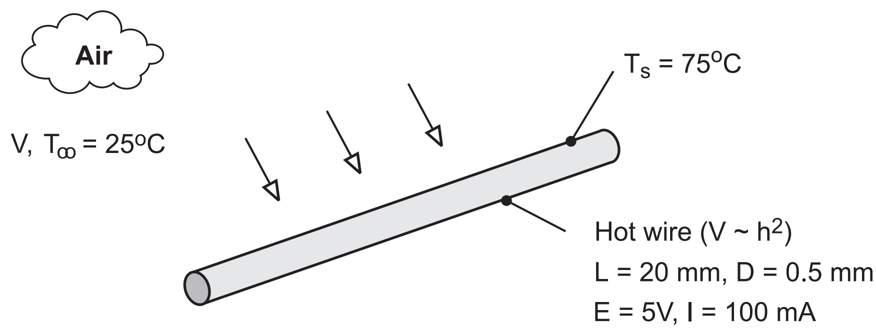
(2) In air, the high cartridge temperature would render radiation significant.

# PROBLEM 1.17

**KNOWN:** Length, diameter and calibration of a hot wire anemometer. Temperature of air stream. Current, voltage drop and surface temperature of wire for a particular application.

**FIND:** Air velocity

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state conditions, (2) Negligible heat transfer from the wire by natural convection or radiation.

**ANALYSIS:** If all of the electric energy is transferred by convection to the air, the following equality must be satisfied

$$P_{elec} = EI = hA\left(T_s - T_\infty\right)$$

where $A = \pi DL = \pi\left(0.0005\text{m} \times 0.02\text{m}\right) = 3.14 \times 10^{-5}\text{m}^2.$

Hence,

$$h = \frac{EI}{A\left(T_s - T_\infty\right)} = \frac{5\text{V} \times 0.1\text{A}}{3.14 \times 10^{-5}\text{m}^2\left(50\ ^\circ\text{C}\right)} = 318\ \text{W/m}^2 \cdot \text{K}$$

$$V = 6.25 \times 10^{-5} h^2 = 6.25 \times 10^{-5}\left(318\ \text{W/m}^2 \cdot \text{K}\right)^2 = 6.3\ \text{m/s} \qquad <$$

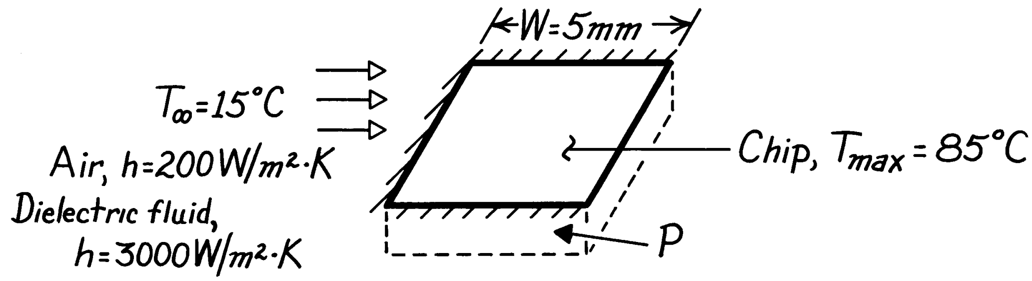
**COMMENTS:** The convection coefficient is sufficiently large to render buoyancy (natural convection) and radiation effects negligible.

## PROBLEM 1.18

**KNOWN:** Chip width and maximum allowable temperature. Coolant conditions.

**FIND:** Maximum allowable chip power for air and liquid coolants.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state conditions, (2) Negligible heat transfer from sides and bottom, (3) Chip is at a uniform temperature (isothermal), (4) Negligible heat transfer by radiation in air.

**ANALYSIS:** All of the electrical power dissipated in the chip is transferred by convection to the coolant. Hence,

$$P = q$$

and from Newton's law of cooling,

$$P = hA(T - T_\infty) = h\,W^2(T - T_\infty).$$

In *air*,

$$P_{max} = 200\ \text{W/m}^2\cdot\text{K}(0.005\ \text{m})^2(85 - 15)\ °\text{C} = 0.35\ \text{W}. \qquad <$$

In the *dielectric liquid*

$$P_{max} = 3000\ \text{W/m}^2\cdot\text{K}(0.005\ \text{m})^2(85\text{-}15)\ °\text{C} = 5.25\ \text{W}. \qquad <$$

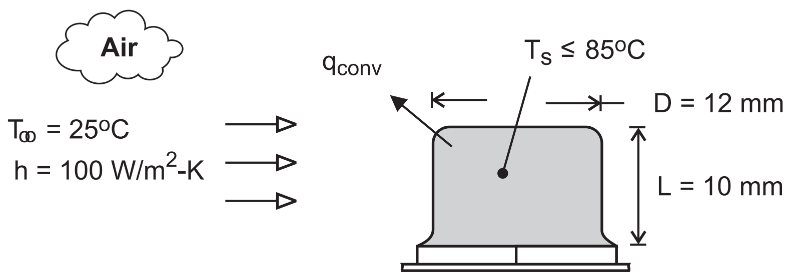
**COMMENTS:** Relative to liquids, air is a poor heat transfer fluid. Hence, in air the chip can dissipate far less energy than in the dielectric liquid.

## PROBLEM 1.19

**KNOWN:** Length, diameter and maximum allowable surface temperature of a power transistor. Temperature and convection coefficient for air cooling.

**FIND:** Maximum allowable power dissipation.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state conditions, (2) Negligible heat transfer through base of transistor, (3) Negligible heat transfer by radiation from surface of transistor.

**ANALYSIS:** Subject to the foregoing assumptions, the power dissipated by the transistor is equivalent to the rate at which heat is transferred by convection to the air. Hence,

$$P_{elec} = q_{conv} = hA\left(T_s - T_\infty\right)$$

where $A = \pi\left(DL + D^2/4\right) = \pi\left[0.012\text{m} \times 0.01\text{m} + \left(0.012\text{m}\right)^2/4\right] = 4.90 \times 10^{-4}\text{m}^2.$

For a maximum allowable surface temperature of 85°C, the power is

$$P_{elec} = 100\ \text{W/m}^2 \cdot \text{K}\left(4.90 \times 10^{-4}\text{m}^2\right)\ \left(85 - 25\right)^\circ \text{C} = 2.94\ \text{W} \qquad <$$

**COMMENTS:** (1) For the prescribed surface temperature and convection coefficient, radiation will be negligible relative to convection. However, conduction through the base could be significant, thereby permitting operation at a larger power.

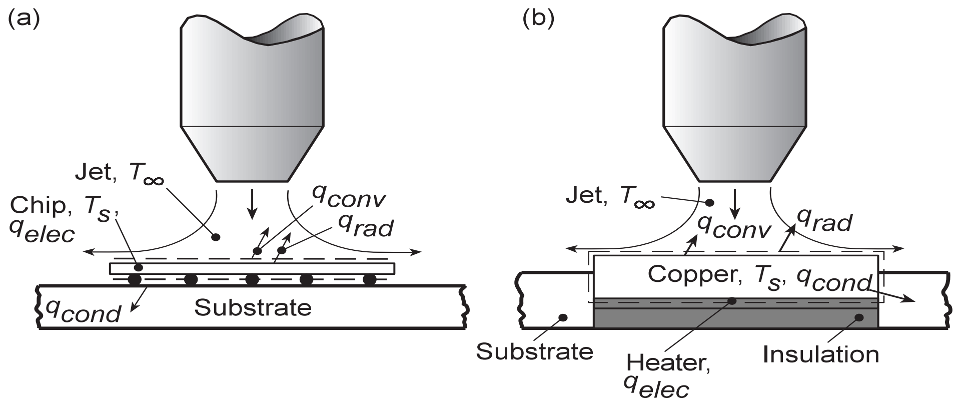
(2) The *local* convection coefficient varies over the surface, and *hot spots* could exist if there are locations at which the local value of *h* is substantially smaller than the prescribed average value.

## PROBLEM 1.20

**KNOWN:** Air jet impingement is an effective means of cooling logic chips.

**FIND:** Procedure for measuring convection coefficients associated with a 10 mm × 10 mm chip.

**SCHEMATIC:**




**ASSUMPTIONS:** Steady-state conditions.

**ANALYSIS:** One approach would be to use the actual chip-substrate system, Case (a), to perform the measurements. In this case, the electric power dissipated in the chip would be transferred from the chip by radiation and conduction (to the substrate), as well as by convection to the jet. An energy balance for the chip yields $q_{elec} = q_{conv} + q_{cond} + q_{rad}$. Hence, with $q_{conv} = hA\left(T_s - T_\infty\right)$, where A = 100 mm$^2$ is the surface area of the chip,

$$h = \frac{q_{elec} - q_{cond} - q_{rad}}{A\left(T_s - T_\infty\right)} \tag{1}$$

While the electric power ($q_{elec}$) and the jet ($T_\infty$) and surface ($T_s$) temperatures may be measured, losses from the chip by conduction and radiation would have to be estimated. Unless the losses are negligible (an unlikely condition), the accuracy of the procedure could be compromised by uncertainties associated with determining the conduction and radiation losses.

A second approach, Case (b), could involve fabrication of a heater assembly for which the conduction and radiation losses are controlled and minimized. A 10 mm × 10 mm copper block (k ~ 400 W/m·K) could be inserted in a poorly conducting substrate (k < 0.1 W/m·K) and a patch heater could be applied to the back of the block and insulated from below. If conduction to both the substrate and insulation could thereby be rendered negligible, heat would be transferred almost exclusively through the block. If radiation were rendered negligible by applying a low emissivity coating ($\varepsilon < 0.1$) to the surface of the copper block, virtually all of the heat would be transferred by convection to the jet. Hence, $q_{cond}$ and $q_{rad}$ may be neglected in equation (1), and the expression may be used to accurately determine h from the known (A) and measured ($q_{elec}$, $T_s$, $T_\infty$) quantities.

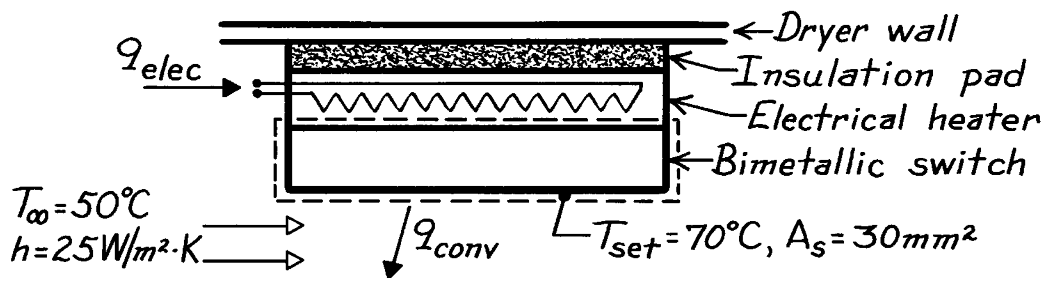
**COMMENTS:** Since convection coefficients associated with gas flows are generally small, concurrent heat transfer by radiation and/or conduction must often be considered. However, jet impingement is one of the more effective means of transferring heat by convection and convection coefficients well in excess of 100 W/m$^2$·K may be achieved.

# PROBLEM 1.21

**KNOWN:** Upper temperature set point, $T_{set}$, of a bimetallic switch and convection heat transfer coefficient between clothes dryer air and exposed surface of switch.

**FIND:** Electrical power for heater to maintain $T_{set}$ when air temperature is $T_\infty = 50°C$.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state conditions, (2) Electrical heater is perfectly insulated from dryer wall, (3) Heater and switch are isothermal at $T_{set}$, (4) Negligible heat transfer from sides of heater or switch, (5) Switch surface, $A_s$, loses heat only by convection.

**ANALYSIS:** Define a control volume around the bimetallic switch which experiences heat input from the heater and convection heat transfer to the dryer air. That is,

$$\dot{E}_{in} - \dot{E}_{out} = 0$$
$$q_{elec} - hA_s\left(T_{set} - T_\infty\right) = 0.$$

The electrical power required is,

$$q_{elec} = hA_s\left(T_{set} - T_\infty\right)$$

$$q_{elec} = 25\ W/m^2 \cdot K \times 30 \times 10^{-6}\ m^2\left(70 - 50\right)K = 15\ mW. \qquad <$$

**COMMENTS:** (1) This type of controller can achieve variable operating air temperatures with a single set-point, inexpensive, bimetallic-thermostatic switch by adjusting power levels to the heater.

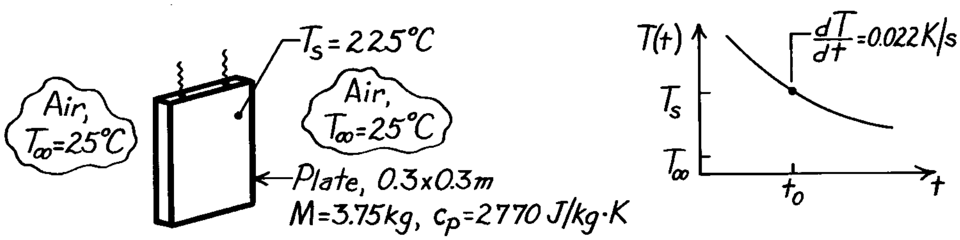
(2) Will the heater power requirement increase or decrease if the insulation pad is other than perfect?

# PROBLEM 1.22

**KNOWN:** Hot vertical plate suspended in cool, still air. Change in plate temperature with time at the instant when the plate temperature is 225°C.

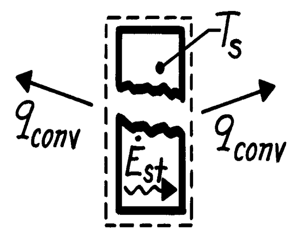
**FIND:** Convection heat transfer coefficient for this condition.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Plate is isothermal and of uniform temperature, (2) Negligible radiation exchange with surroundings, (3) Negligible heat lost through suspension wires.

**ANALYSIS:** As shown in the cooling curve above, the plate temperature decreases with time. The condition of interest is for time $t_o$. For a control surface about the plate, the conservation of energy requirement is




$$\dot{E}_{in} - \dot{E}_{out} = \dot{E}_{st}$$

$$-2hA_s\left(T_s - T_\infty\right) = Mc_p \frac{dT}{dt}$$

where $A_s$ is the surface area of one side of the plate. Solving for h, find

$$h = \frac{Mc_p}{2A_s\left(T_s - T_\infty\right)} \frac{dT}{dt}$$

$$h = \frac{3.75\ \text{kg} \times 2770\ \text{J/kg}\cdot\text{K}}{2 \times (0.3 \times 0.3)\text{m}^2\,(225-25)\text{K}} \times 0.022\ \text{K/s} = 6.4\ \text{W/m}^2\cdot\text{K}$$ <

**COMMENTS:** (1) Assuming the plate is very highly polished with emissivity of 0.08, determine whether radiation exchange with the surroundings at 25°C is negligible compared to convection.

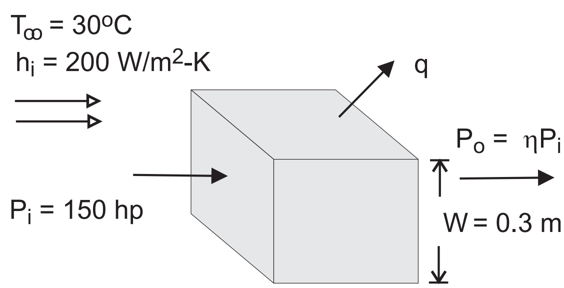
(2) We will later consider the criterion for determining whether the isothermal plate assumption is reasonable. If the thermal conductivity of the present plate were high (such as aluminum or copper), the criterion would be satisfied.

## PROBLEM 1.23

**KNOWN:** Width, input power and efficiency of a transmission. Temperature and convection coefficient associated with air flow over the casing.

**FIND:** Surface temperature of casing.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady state, (2) Uniform convection coefficient and surface temperature, (3) Negligible radiation.

**ANALYSIS:** From Newton's law of cooling,

$$q = hA_s\left(T_s - T_\infty\right) = 6\,hW^2\left(T_s - T_\infty\right)$$

where the output power is $\eta\, P_i$ and the heat rate is

$$q = P_i - P_o = P_i\left(1-\eta\right) = 150\,\text{hp} \times 746\,\text{W/hp} \times 0.07 = 7833\,\text{W}$$

Hence,

$$T_s = T_\infty + \frac{q}{6\ hW^2} = 30°\text{C} + \frac{7833\ \text{W}}{6 \times 200\ \text{W/m}^2 \cdot \text{K} \times \left(0.3\,\text{m}\right)^2} = 102.5°\text{C}$$ <

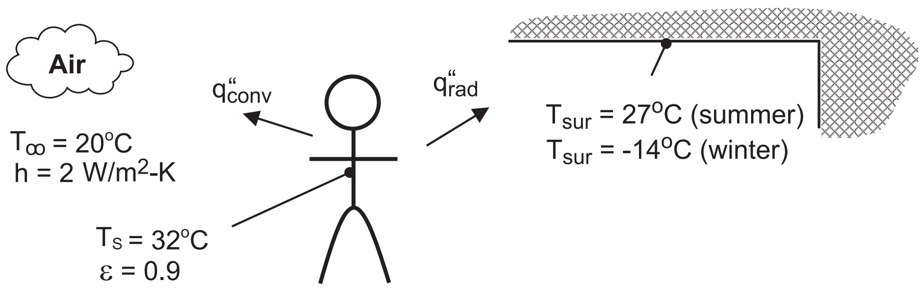
**COMMENTS:** There will, in fact, be considerable variability of the local convection coefficient over the transmission case and the prescribed value represents an average over the surface.

## PROBLEM 1.24

**KNOWN:** Air and wall temperatures of a room. Surface temperature, convection coefficient and emissivity of a person in the room.

**FIND:** Basis for difference in comfort level between summer and winter.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Person may be approximated as a small object in a large enclosure.

**ANALYSIS:** Thermal comfort is linked to heat loss from the human body, and a *chilled* feeling is associated with excessive heat loss. Because the temperature of the room air is fixed, the different summer and winter comfort levels can not be attributed to convection heat transfer from the body. In both cases, the heat flux is

*Summer and Winter*: $$q''_{conv} = h\left(T_s - T_\infty\right) = 2\ W/m^2 \cdot K \times 12\ ^\circ C = 24\ W/m^2$$

However, the heat flux due to radiation will differ, with values of

*Summer*: $$q''_{rad} = \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right) = 0.9 \times 5.67 \times 10^{-8} W/m^2 \cdot K^4 \left(305^4 - 300^4\right) K^4 = 28.3\ W/m^2$$

*Winter*: $$q''_{rad} = \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right) = 0.9 \times 5.67 \times 10^{-8} W/m^2 \cdot K^4 \left(305^4 - 287^4\right) K^4 = 95.4\ W/m^2$$

There is a significant difference between winter and summer radiation fluxes, and the chilled condition is attributable to the effect of the colder walls on radiation.

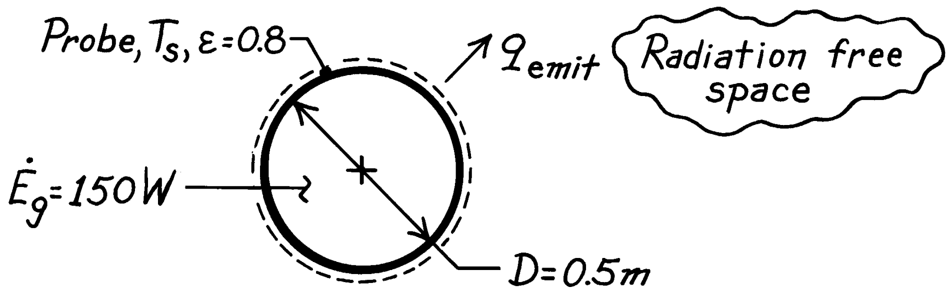
**COMMENTS:** For a representative surface area of A = 1.5 $m^2$, the heat losses are $q_{conv}$ = 36 W, $q_{rad(summer)}$ = 42.5 W and $q_{rad(winter)}$ = 143.1 W. The winter time radiation loss is significant and if maintained over a 24 h period would amount to 2,950 kcal.

## PROBLEM 1.25

**KNOWN:** Diameter and emissivity of spherical interplanetary probe. Power dissipation within probe.

**FIND:** Probe surface temperature.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state conditions, (2) Negligible radiation incident on the probe.

**ANALYSIS:** Conservation of energy dictates a balance between energy generation within the probe and radiation emission from the probe surface. Hence, at any instant

$$-\dot{E}_{out} + \dot{E}_g = 0$$

$$\varepsilon A_s \sigma T_s^4 = \dot{E}_g$$

$$T_S = \left( \frac{\dot{E}g}{\varepsilon \pi D^2 \sigma} \right)^{1/4}$$

$$T_S = \left( \frac{150\text{W}}{0.8\pi \, (0.5 \text{ m})^2 \, 5.67\times10^{-8} \text{ W/m}^2 \cdot \text{K}^4} \right)^{1/4}$$

$$T_s = 254.7 \text{ K.}$$ <

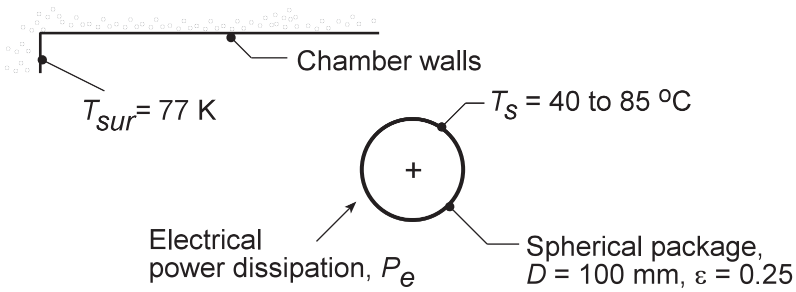
**COMMENTS:** Incident radiation, as, for example, from the sun, would increase the surface temperature.

## PROBLEM 1.26

**KNOWN:** Spherical shaped instrumentation package with prescribed surface emissivity within a large space-simulation chamber having walls at 77 K.

**FIND:** Acceptable power dissipation for operating the package surface temperature in the range $T_s$ = 40 to 85°C. Show graphically the effect of emissivity variations for 0.2 and 0.3.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Uniform surface temperature, (2) Chamber walls are large compared to the spherical package, and (3) Steady-state conditions.

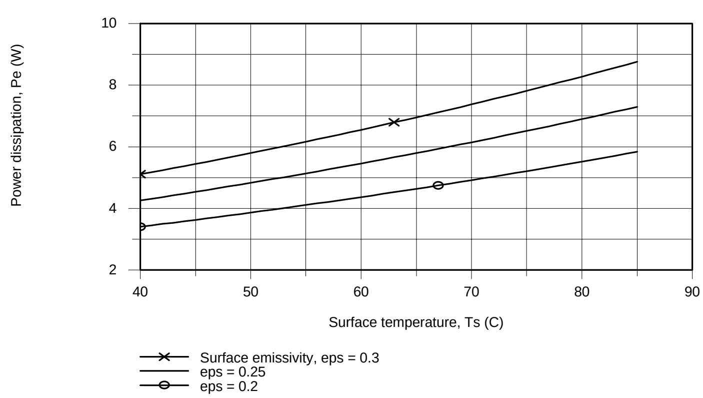
**ANALYSIS:** From an overall energy balance on the package, the internal power dissipation $P_e$ will be transferred by radiation exchange between the package and the chamber walls. From Eq. 1.7,

$$q_{rad} = P_e = \varepsilon A_s \sigma \left(T_s^4 - T_{sur}^4\right)$$

For the condition when $T_s$ = 40°C, with $A_s = \pi D^2$ the power dissipation will be

$$P_e = 0.25(\pi \times 0.10\,\text{m}) \times 5.67 \times 10^{-8}\,\text{W/m}^2 \cdot \text{K}^4 \times \left[(40+273)^4 - 77^4\right]\text{K}^4 = 4.3\,\text{W} \qquad <$$

Repeating this calculation for the range 40 ≤ $T_s$ ≤ 85°C, we can obtain the power dissipation as a function of surface temperature for the ε = 0.25 condition. Similarly, with 0.2 or 0.3, the family of curves shown below has been obtained.




**COMMENTS:** (1) As expected, the internal power dissipation increases with increasing emissivity and surface temperature. Because the radiation rate equation is non-linear with respect to temperature, the power dissipation will likewise not be linear with surface temperature.

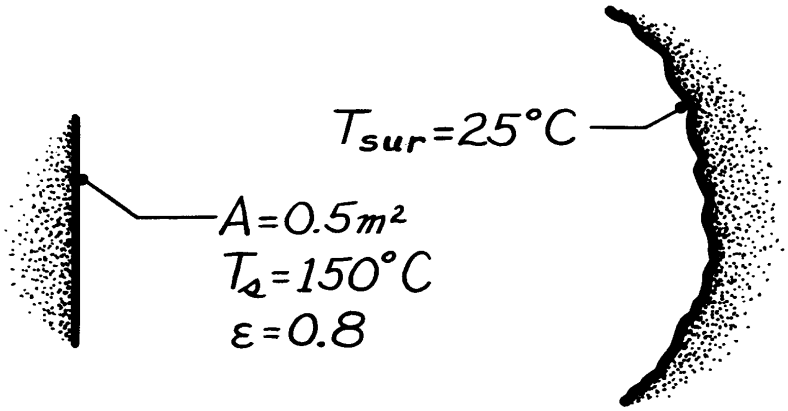
(2) What is the maximum power dissipation that is possible if the surface temperature is not to exceed 85°C? What kind of a coating should be applied to the instrument package in order to approach this limiting condition?

**PROBLEM 1.27**

**KNOWN:** Area, emissivity and temperature of a surface placed in a large, evacuated chamber of prescribed temperature.

**FIND:** (a) Rate of surface radiation emission, (b) Net rate of radiation exchange between surface and chamber walls.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Area of the enclosed surface is much less than that of chamber walls.

**ANALYSIS:** (a) From Eq. 1.5, the rate at which radiation is emitted by the surface is

$$q_{emit} = E \cdot A = \varepsilon A \sigma T_s^4$$

$$q_{emit} = 0.8\left(0.5 \text{ m}^2\right) 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4 \left[(150+273)\text{K}\right]^4$$

$$q_{emit} = 726 \text{ W}. \qquad <$$

(b) From Eq. 1.7, the *net* rate at which radiation is transferred *from* the surface to the chamber walls is

$$q = \varepsilon A \sigma \left(T_s^4 - T_{sur}^4\right)$$

$$q = 0.8\left(0.5 \text{ m}^2\right) 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4 \left[(423\text{K})^4 - (298\text{K})^4\right]$$

$$q = 547 \text{ W}. \qquad <$$

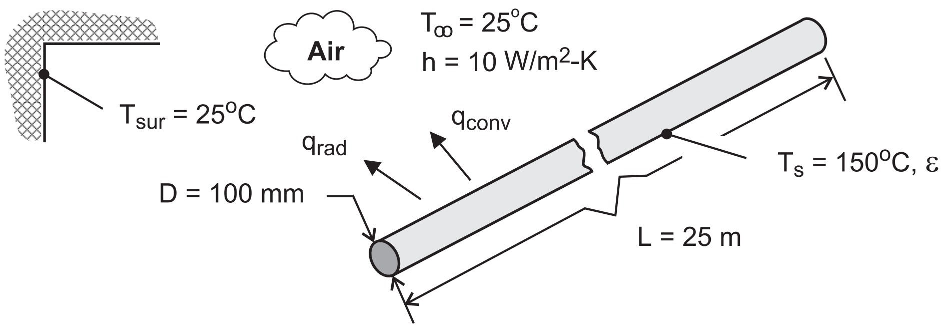
**COMMENTS:** The foregoing result gives the net heat loss from the surface which occurs at the instant the surface is placed in the chamber. The surface would, of course, cool due to this heat loss and its temperature, as well as the heat loss, would decrease with increasing time. Steady-state conditions would eventually be achieved when the temperature of the surface reached that of the surroundings.

## PROBLEM 1.28

**KNOWN:** Length, diameter, surface temperature and emissivity of steam line. Temperature and convection coefficient associated with ambient air. Efficiency and fuel cost for gas fired furnace.

**FIND:** (a) Rate of heat loss, (b) Annual cost of heat loss.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steam line operates continuously throughout year, (2) Net radiation transfer is between small surface (steam line) and large enclosure (plant walls).

**ANALYSIS:** (a) From Eqs. (1.3a) and (1.7), the heat loss is

$$q = q_{conv} + q_{rad} = A\left[h\left(T_s - T_\infty\right) + \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right)\right]$$

where $A = \pi DL = \pi\left(0.1\text{m} \times 25\text{m}\right) = 7.85\text{m}^2$.

Hence,

$$q = 7.85\text{m}^2\left[10\ \text{W/m}^2 \cdot \text{K}\left(150-25\right)\text{K} + 0.8 \times 5.67 \times 10^{-8}\text{W/m}^2 \cdot \text{K}^4\left(423^4 - 298^4\right)\text{K}^4\right]$$

$$q = 7.85\text{m}^2\left(1,250 + 1,095\right)\text{w/m}^2 = \left(9813 + 8592\right)\text{W} = 18,405\ \text{W} \qquad <$$

(b) The annual energy loss is

$$E = qt = 18,405\ \text{W} \times 3600\ \text{s/h} \times 24\text{h/d} \times 365\ \text{d/y} = 5.80 \times 10^{11}\ \text{J}$$

With a furnace energy consumption of $E_f = E/\eta_f = 6.45 \times 10^{11}\ \text{J}$, the annual cost of the loss is

$$C = C_g E_f = 0.01\ \text{\$/MJ} \times 6.45 \times 10^5 \text{MJ} = \$6450 \qquad <$$

**COMMENTS:** The heat loss and related costs are unacceptable and should be reduced by insulating the steam line.

## PROBLEM 1.29

**KNOWN:** Exact and approximate expressions for the linearized radiation coefficient, $h_r$ and $h_{ra}$, respectively.

**FIND:** (a) Comparison of the coefficients with ε = 0.05 and 0.9 and surface temperatures which may exceed that of the surroundings ($T_{sur}$ = 25°C) by 10 to 100°C; also comparison with a free convection coefficient correlation, (b) Plot of the relative error $(h_r - r_{ra})/h_r$ as a function of the furnace temperature associated with a workpiece at $T_s$ = 25°C having ε = 0.05, 0.2 or 0.9.

**ASSUMPTIONS:** (1) Furnace walls are large compared to the workpiece and (2) Steady-state conditions.

**ANALYSIS:** (a) The linearized radiation coefficient, Eq. 1.9, follows from the radiation exchange rate equation,

$$h_r = \varepsilon\sigma\left(T_s + T_{sur}\right)\left(T_s^2 + T_{sur}^2\right)$$

If $T_s \approx T_{sur}$, the coefficient may be approximated by the simpler expression

$$h_{r,a} = 4\varepsilon\sigma\bar{T}^3 \qquad\qquad \bar{T} = \left(T_s + T_{sur}\right)/2$$

For the condition of ε = 0.05, $T_s = T_{sur}$ + 10 = 35°C = 308 K and $T_{sur}$ = 25°C = 298 K, find that

$$h_r = 0.05\times 5.67\times 10^{-8}\,\mathrm{W/m^2\cdot K^4}\,(308+298)\left(308^2+298^2\right)\mathrm{K^3} = 0.32\,\mathrm{W/m^2\cdot K} \qquad <$$

$$h_{r,a} = 4\times 0.05\times 5.67\times 10^{-8}\,\mathrm{W/m^2\cdot K^4}\left((308+298)/2\right)^3\mathrm{K^3} = 0.32\,\mathrm{W/m^2\cdot K} \qquad <$$

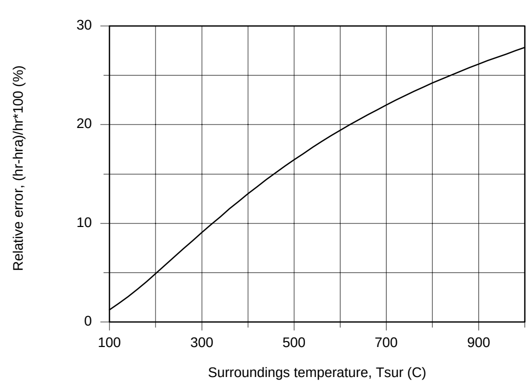
The free convection coefficient with $T_s$ = 35°C and $T_\infty = T_{sur}$ = 25°C, find that

$$h = 0.98\Delta T^{1/3} = 0.98\left(T_s - T_\infty\right)^{1/3} = 0.98\left(308-298\right)^{1/3} = 2.1\,\mathrm{W/m^2\cdot K} \qquad <$$

For the range $T_s - T_{sur}$ = 10 to 100°C with ε = 0.05 and 0.9, the results for the coefficients are tabulated below. For this range of surface and surroundings temperatures, the radiation and free convection coefficients are of comparable magnitude for moderate values of the emissivity, say ε > 0.2. The approximate expression for the linearized radiation coefficient is valid within 2% for these conditions.

(b) The above expressions for the radiation coefficients, $h_r$ and $h_{r,a}$, are used for the workpiece at $T_s$ = 25°C placed inside a furnace with walls which may vary from 100 to 1000°C. The relative error, $(h_r - h_{ra})/h_r$, will be independent of the surface emissivity and is plotted as a function of $T_{sur}$. For $T_{sur}$ > 150°C, the approximate expression provides estimates which are in error more than 5%. The approximate expression should be used with caution, and only for surface and surrounding temperature differences of 50 to 100°C.

| | | Coefficients ($W/m^2\cdot K$) | | |
|---|---|---|---|---|
| $T_s$ (°C) | ε | $h_r$ | $h_{r,a}$ | h |
| 35 | 0.05 | 0.32 | 0.32 | 2.1 |
| | 0.9 | 5.7 | 5.7 | |
| 135 | 0.05 | 0.51 | 0.50 | 4.7 |
| | 0.9 | 9.2 | 9.0 | |

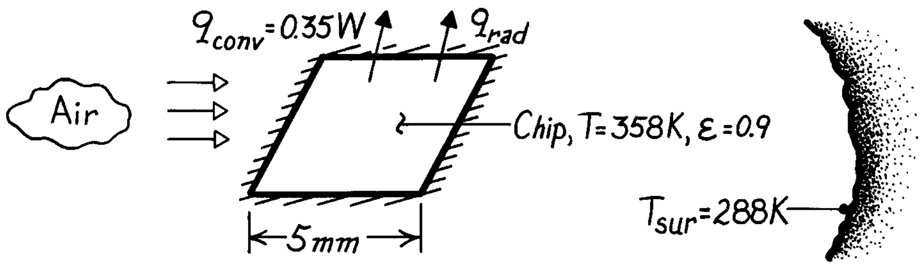


**PROBLEM 1.30**

**KNOWN:** Chip width, temperature, and heat loss by convection in air. Chip emissivity and temperature of large surroundings.

**FIND:** Increase in chip power due to radiation.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state conditions, (2) Radiation exchange between small surface and large enclosure.

**ANALYSIS:** Heat transfer from the chip due to net radiation exchange with the surroundings is

$$q_{rad} = \varepsilon W^2 \sigma \left(T^4 - T_{sur}^4\right)$$

$$q_{rad} = 0.9(0.005\text{ m})^2\, 5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4\left(358^4 - 288^4\right)\text{K}^4$$

$$q_{rad} = 0.0122\text{ W.}$$

The percent increase in chip power is therefore

$$\frac{\Delta P}{P}\times100 = \frac{q_{rad}}{q_{conv}}\times100 = \frac{0.0122\text{ W}}{0.350\text{ W}}\times100 = 3.5\%. \qquad <$$

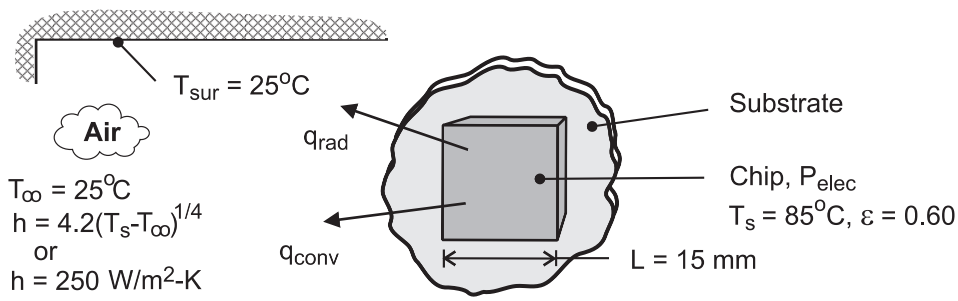
**COMMENTS:** For the prescribed conditions, radiation effects are small. Relative to convection, the effect of radiation would increase with increasing chip temperature and decreasing convection coefficient.

**PROBLEM 1.31**

**KNOWN:** Width, surface emissivity and maximum allowable temperature of an electronic chip. Temperature of air and surroundings. Convection coefficient.

**FIND:** (a) Maximum power dissipation for free convection with $h(W/m^2\cdot K) = 4.2(T - T_\infty)^{1/4}$, (b) Maximum power dissipation for forced convection with $h = 250\ W/m^2\cdot K$.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state conditions, (2) Radiation exchange between a small surface and a large enclosure, (3) Negligible heat transfer from sides of chip or from back of chip by conduction through the substrate.

**ANALYSIS:** Subject to the foregoing assumptions, electric power dissipation by the chip must be balanced by convection and radiation heat transfer from the chip. Hence, from Eq. (1.10),

$$P_{elec} = q_{conv} + q_{rad} = hA\left(T_s - T_\infty\right) + \varepsilon A\sigma\left(T_s^4 - T_{sur}^4\right)$$

where $A = L^2 = \left(0.015m\right)^2 = 2.25\times10^{-4} m^2$.

(a) If heat transfer is by natural convection,

$$q_{conv} = C\ A\left(T_s - T_\infty\right)^{5/4} = 4.2\ W/m^2\cdot K^{5/4}\left(2.25\times10^{-4} m^2\right)\left(60K\right)^{5/4} = 0.158\ W$$

$$q_{rad} = 0.60\left(2.25\times10^{-4} m^2\right)5.67\times10^{-8}\ W/m^2\cdot K^4\left(358^4 - 298^4\right)K^4 = 0.065\ W$$

$$P_{elec} = 0.158\ W + 0.065\ W = 0.223\ W$$ <

(b) If heat transfer is by forced convection,

$$q_{conv} = hA\left(T_s - T_\infty\right) = 250\ W/m^2\cdot K\left(2.25\times10^{-4} m^2\right)\left(60K\right) = 3.375\ W$$

$$P_{elec} = 3.375\ W + 0.065\ W = 3.44\ W$$ <

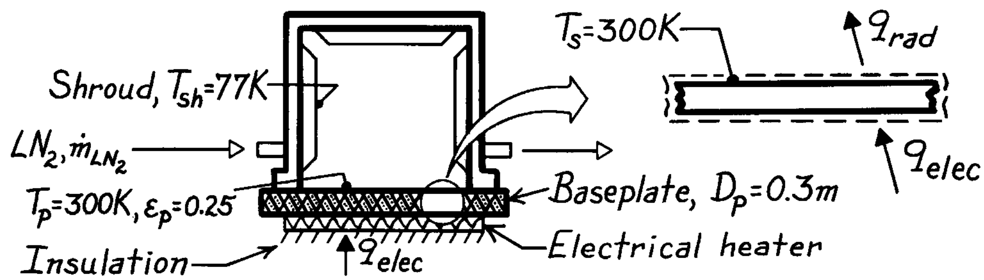
**COMMENTS:** Clearly, radiation and natural convection are inefficient mechanisms for transferring heat from the chip. For $T_s = 85°C$ and $T_\infty = 25°C$, the natural convection coefficient is $11.7\ W/m^2\cdot K$. Even for forced convection with $h = 250\ W/m^2\cdot K$, the power dissipation is well below that associated with many of today's processors. To provide acceptable cooling, it is often necessary to attach the chip to a highly conducting substrate and to thereby provide an additional heat transfer mechanism due to conduction from the back surface.

## PROBLEM 1.32

**KNOWN:** Vacuum enclosure maintained at 77 K by liquid nitrogen shroud while baseplate is maintained at 300 K by an electrical heater.

**FIND:** (a) Electrical power required to maintain baseplate, (b) Liquid nitrogen consumption rate, (c) Effect on consumption rate if aluminum foil ($\varepsilon_p = 0.09$) is bonded to baseplate surface.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state conditions, (2) No heat losses from backside of heater or sides of plate, (3) Vacuum enclosure large compared to baseplate, (4) Enclosure is evacuated with negligible convection, (5) Liquid nitrogen (LN2) is heated only by heat transfer to the shroud, and (6) Foil is intimately bonded to baseplate.

**PROPERTIES:** Heat of vaporization of liquid nitrogen (given): 125 kJ/kg.

**ANALYSIS:** (a) From an energy balance on the baseplate,

$$\dot{E}_{in} - \dot{E}_{out} = 0 \qquad q_{elec} - q_{rad} = 0$$

and using Eq. 1.7 for radiative exchange between the baseplate and shroud,

$$q_{elec} = \varepsilon_p A_p \sigma \left(T_p^4 - T_{sh}^4\right).$$

Substituting numerical values, with $A_p = \left(\pi D_p^2 / 4\right)$, find

$$q_{elec} = 0.25\left(\pi (0.3\text{ m})^2 / 4\right) 5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4 \left(300^4 - 77^4\right)\text{K}^4 = 8.1\text{ W}. \qquad <$$

(b) From an energy balance on the enclosure, radiative transfer heats the liquid nitrogen stream causing evaporation,

$$\dot{E}_{in} - \dot{E}_{out} = 0 \qquad q_{rad} - \dot{m}_{LN2} h_{fg} = 0$$

where $\dot{m}_{LN2}$ is the liquid nitrogen consumption rate. Hence,

$$\dot{m}_{LN2} = q_{rad} / h_{fg} = 8.1\text{ W} / 125\text{ kJ/kg} = 6.48\times10^{-5}\text{ kg/s} = 0.23\text{ kg/h}. \qquad <$$

(c) If aluminum foil ($\varepsilon_p = 0.09$) were bonded to the upper surface of the baseplate,

$$q_{rad,foil} = q_{rad}\left(\varepsilon_f / \varepsilon_p\right) = 8.1\text{ W}\left(0.09/0.25\right) = 2.9\text{ W}$$

and the liquid nitrogen consumption rate would be reduced by

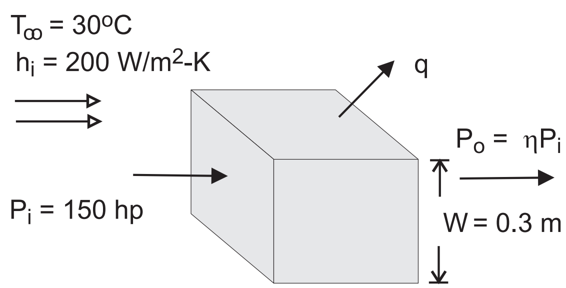
(0.25 - 0.09)/0.25 = 64% to 0.083 kg/h. <

## PROBLEM 1.33

**KNOWN:** Width, input power and efficiency of a transmission. Temperature and convection coefficient for air flow over the casing. Emissivity of casing and temperature of surroundings.

**FIND:** Surface temperature of casing.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady state, (2) Uniform convection coefficient and surface temperature, (3) Radiation exchange with large surroundings.

**ANALYSIS:** Heat transfer from the case must balance heat dissipation in the transmission, which may be expressed as $q = P_i - P_o = P_i (1 - \eta) = 150\ \text{hp} \times 746\ \text{W/hp} \times 0.07 = 7833\ \text{W}$. Heat transfer from the case is by convection and radiation, in which case

$$q = A_s \left[ h\left(T_s - T_\infty\right) + \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right)\right]$$

where $A_s = 6\ W^2$. Hence,

$$7833\,\text{W} = 6\left(0.30\,\text{m}\right)^2 \left[200\,\text{W/m}^2 \cdot \text{K}\left(T_s - 303\text{K}\right) + 0.8 \times 5.67 \times 10^{-8}\,\text{W/m}^2 \cdot \text{K}^4 \left(T_s^4 - 303^4\right)\text{K}^4\right]$$

A trial-and-error solution yields

$$T_s \approx 373\,\text{K} = 100°\text{C}$$ <

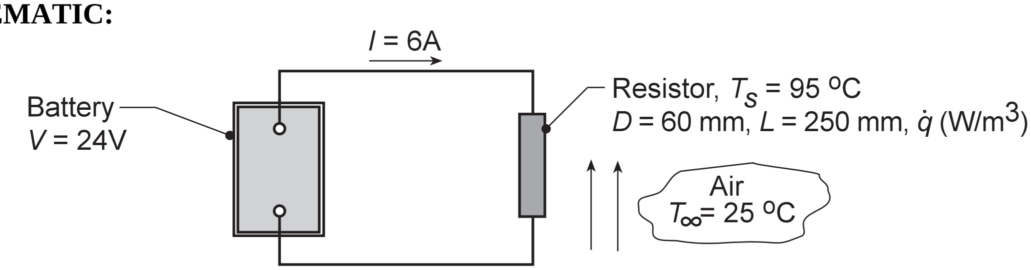
**COMMENTS:** (1) For $T_s \approx 373$ K, $q_{conv} \approx 7{,}560$ W and $q_{rad} \approx 270$ W, in which case heat transfer is dominated by convection, (2) If radiation is neglected, the corresponding surface temperature is $T_s$ = 102.5°C.

# PROBLEM 1.34

**KNOWN:** Resistor connected to a battery operating at a prescribed temperature in air.

**FIND:** (a) Considering the resistor as the system, determine corresponding values for $\dot{E}_{in}\,(W)$, $\dot{E}_g\,(W)$, $\dot{E}_{out}\,(W)$ and $\dot{E}_{st}\,(W)$. If a control surface is placed about the entire system, determine the values for $\dot{E}_{in}$, $\dot{E}_g$, $\dot{E}_{out}$, and $\dot{E}_{st}$. (b) Determine the volumetric heat generation rate within the resistor, $\dot{q}$ (W/m$^3$), (c) Neglecting radiation from the resistor, determine the convection coefficient.

**SCHEMATIC:**



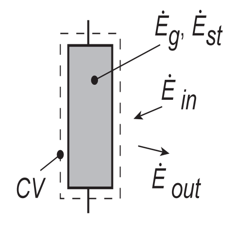

**ASSUMPTIONS:** (1) Electrical power is dissipated uniformly within the resistor, (2) Temperature of the resistor is uniform, (3) Negligible electrical power dissipated in the lead wires, (4) Negligible radiation exchange between the resistor and the surroundings, (5) No heat transfer occurs from the battery, (5) Steady-state conditions.

**ANALYSIS:** (a) Referring to Section 1.3.1, the conservation of energy requirement for a control volume at an instant of time, Eq 1.11a, is

$$\dot{E}_{in} + \dot{E}_g - \dot{E}_{out} = \dot{E}_{st}$$

where $\dot{E}_{in}$, $\dot{E}_{out}$ correspond to *surface* inflow and outflow processes, respectively. The energy generation term $\dot{E}_g$ is associated with conversion of some other energy form (chemical, electrical, electromagnetic or nuclear) to thermal energy. The energy storage term $\dot{E}_{st}$ is associated with changes in the internal, kinetic and/or potential energies of the matter in the control volume. $\dot{E}_g$, $\dot{E}_{st}$ are *volumetric* phenomena. The electrical power delivered by the battery is P = VI = 24V×6A = 144 W.

*Control volume: Resistor.*



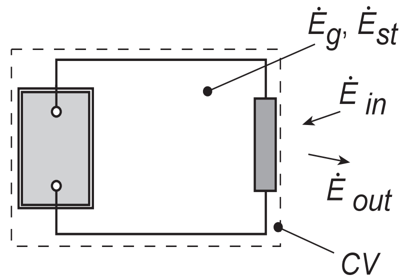

$$\dot{E}_{in} = 0 \qquad \dot{E}_{out} = 144\,\mathrm{W}$$

<

$$\dot{E}_g = 144\,\mathrm{W} \qquad \dot{E}_{st} = 0$$

The $\dot{E}_g$ term is due to conversion of electrical energy to thermal energy. The term $\dot{E}_{out}$ is due to convection from the resistor surface to the air.

Continued...




*Control volume: Battery-Resistor System.*

$$\dot{E}_{in} = 0 \qquad \dot{E}_{out} = 144\,W$$
$$\dot{E}_g = 0 \qquad \dot{E}_{st} = -144\,W$$
<

The $\dot{E}_{st}$ term represents the decrease in the chemical energy within the battery. The conversion of chemical energy to electrical energy and its subsequent conversion to thermal energy are processes internal to the system which are not associated with $\dot{E}_{st}$ or $\dot{E}_g$. The $\dot{E}_{out}$ term is due to convection from the resistor surface to the air.

(b) From the energy balance on the resistor with volume, $\forall = (\pi D^2/4)L$,

$$\dot{E}_g = \dot{q}\forall \qquad 144\,W = \dot{q}\left(\pi\,(0.06\,m)^2/4\right)\times 0.25\,m \qquad \dot{q} = 2.04\times 10^5\,W/m^3$$
<

(c) From the energy balance on the resistor and Newton's law of cooling with $A_s = \pi DL + 2(\pi D^2/4)$,

$$\dot{E}_{out} = q_{cv} = hA_s\left(T_s - T_\infty\right)$$

$$144\,W = h\left[\pi\times 0.06\,m\times 0.25\,m + 2\left(\pi\times 0.06^2\,m^2/4\right)\right](95-25)^\circ C$$

$$144\,W = h\,[0.0471 + 0.0057]\,m^2\,(95-25)^\circ C$$

$$h = 39.0\,W/m^2{\cdot}K$$
<

**COMMENTS:** (1) In using the conservation of energy requirement, Eq. 1.11a, it is important to recognize that $\dot{E}_{in}$ and $\dot{E}_{out}$ will always represent *surface* processes and $\dot{E}_g$ and $\dot{E}_{st}$, *volumetric* processes. The generation term $\dot{E}_g$ is associated with a *conversion* process from some form of energy to *thermal energy*. The storage term $\dot{E}_{st}$ represents the rate of change of *internal energy*.

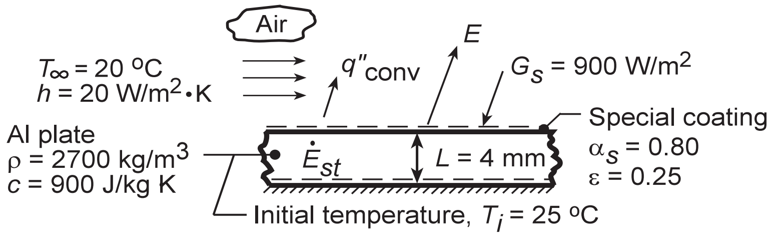
(2) From Table 1.1 and the magnitude of the convection coefficient determined from part (c), we conclude that the resistor is experiencing forced, rather than free, convection.

## PROBLEM 1.35

**KNOWN:** Thickness and initial temperature of an aluminum plate whose thermal environment is changed.

**FIND:** (a) Initial rate of temperature change, (b) Steady-state temperature of plate, (c) Effect of emissivity and absorptivity on steady-state temperature.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Negligible end effects, (2) Uniform plate temperature at any instant, (3) Constant properties, (4) Adiabatic bottom surface, (5) Negligible radiation from surroundings, (6) No internal heat generation.

**ANALYSIS:** (a) Applying an energy balance, Eq. 1.11a, at an instant of time to a control volume about the plate, $\dot{E}_{in} - \dot{E}_{out} = \dot{E}_{st}$, it follows for a unit surface area.

$$\alpha_S G_S \left(1\text{m}^2\right) - E\left(1\text{m}^2\right) - q''_{conv}\left(1\text{m}^2\right) = (d/dt)(McT) = \rho\left(1\text{m}^2 \times L\right)c\,(dT/dt).$$

Rearranging and substituting from Eqs. 1.3 and 1.5, we obtain

$$dT/dt = (1/\rho Lc)\left[\alpha_S G_S - \varepsilon\sigma T_i^4 - h\left(T_i - T_\infty\right)\right].$$

$$dT/dt = \left(2700\,\text{kg/m}^3 \times 0.004\,\text{m} \times 900\,\text{J/kg}\cdot\text{K}\right)^{-1} \times$$

$$\left[0.8 \times 900\,\text{W/m}^2 - 0.25 \times 5.67 \times 10^{-8}\,\text{W/m}^2\cdot\text{K}^4\,(298\,\text{K})^4 - 20\,\text{W/m}^2\cdot\text{K}\,(25-20)^\circ\text{C}\right]$$

$$dT/dt = 0.052^\circ\,\text{C/s}.$$ <

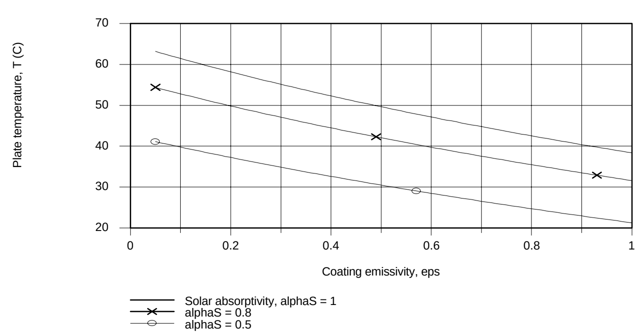
(b) Under steady-state conditions, $\dot{E}_{st} = 0$, and the energy balance reduces to

$$\alpha_S G_S = \varepsilon\sigma T^4 + h\left(T - T_\infty\right) \qquad (2)$$

$$0.8 \times 900\,\text{W/m}^2 = 0.25 \times 5.67 \times 10^{-8}\,\text{W/m}^2\cdot\text{K}^4 \times T^4 + 20\,\text{W/m}^2\cdot\text{K}\left(T - 293\,\text{K}\right)$$

The solution yields T = 321.4 K = 48.4°C. <

(c) Using the IHT *First Law Model* for an *Isothermal Plane Wall*, parametric calculations yield the following results.



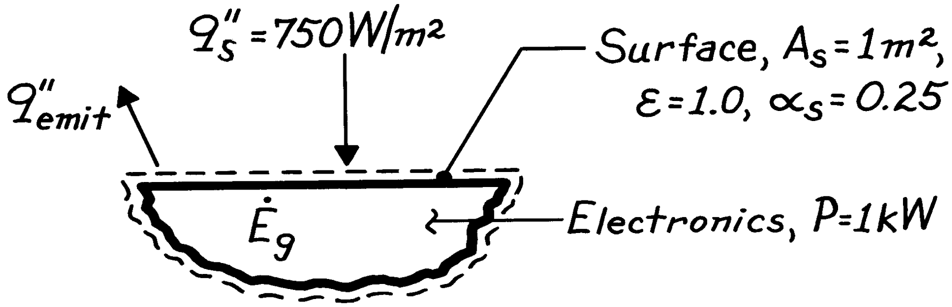

**COMMENTS:** The surface radiative properties have a significant effect on the plate temperature, which decreases with increasing ε and decreasing $\alpha_S$. If a low temperature is desired, the plate coating should be characterized by a large value of $\varepsilon/\alpha_S$. The temperature also decreases with increasing h.

**PROBLEM 1.36**

**KNOWN:** Surface area of electronic package and power dissipation by the electronics. Surface emissivity and absorptivity to solar radiation. Solar flux.

**FIND:** Surface temperature without and with incident solar radiation.

**SCHEMATIC:**




**ASSUMPTIONS:** Steady-state conditions.

**ANALYSIS:** Applying conservation of energy to a control surface about the compartment, at any instant

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0.$$

It follows that, with the solar input,

$$\alpha_S A_S q''_S - A_S E + P = 0$$

$$\alpha_S A_S q''_S - A_S \varepsilon \sigma T_S^4 + P = 0$$

$$T_S = \left( \frac{\alpha_S A_S q''_S + P}{A_S \varepsilon \sigma} \right)^{1/4}.$$

In the shade $\left(q''_S = 0\right)$,

$$T_S = \left( \frac{1000\ \text{W}}{1\ \text{m}^2 \times 1 \times 5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4} \right)^{1/4} = 364\ \text{K}.$$ <

In the sun,

$$T_S = \left( \frac{0.25 \times 1\ \text{m}^2 \times 750\ \text{W/m}^2 + 1000\ \text{W}}{1\ \text{m}^2 \times 1 \times 5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4} \right)^{1/4} = 380\ \text{K}.$$ <

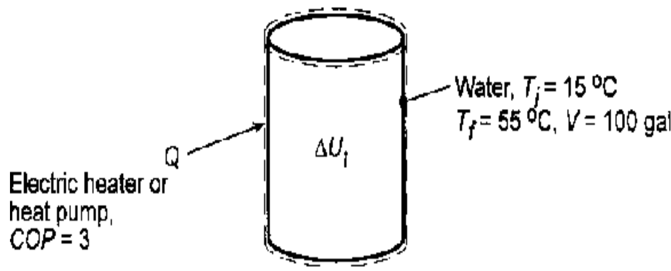
**COMMENTS:** In orbit, the space station would be continuously cycling between shade and sunshine, and a steady-state condition would not exist.

## PROBLEM 1.37

**KNOWN:** Daily hot water consumption for a family of four and temperatures associated with ground water and water storage tank. Unit cost of electric power. Heat pump COP.

**FIND:** Annual heating requirement and costs associated with using electric resistance heating or a heat pump.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Process may be modelled as one involving heat addition in a closed system, (2) Properties of water are constant.

**PROPERTIES:** Table A-6, Water ($T_{ave}$ = 308 K): $\rho = v_f^{-1}$ = 993 kg/m$^3$, $c_{p,f}$ = 4.178 kJ/kg·K.

**ANALYSIS:** From Eq. 1.11c, the daily heating requirement is $Q_{daily} = \Delta U_t = Mc\Delta T$ $= \rho Vc\left(T_f - T_i\right)$. With V = 100 gal/264.17 gal/m$^3$ = 0.379 m$^3$,

$$Q_{daily} = 993\text{kg}/\text{m}^3\left(0.379\,\text{m}^3\right)4.178\text{kJ/kg}\cdot\text{K}\left(40^\circ\text{C}\right) = 62,900\,\text{kJ}$$

The annual heating requirement is then, $Q_{annual} = 365\,\text{days}\left(62,900\,\text{kJ/day}\right) = 2.30\times10^7$ kJ , or, with 1 kWh = 1 kJ/s (3600 s) = 3600 kJ,

$$Q_{annual} = 6380\,\text{kWh}$$ <

With electric resistance heating, $Q_{annual} = Q_{elec}$ and the associated cost, C, is

$$C = 6380\,\text{kWh}\left(\$0.08/\text{kWh}\right) = \$510$$ <

If a heat pump is used, $Q_{annual} = \text{COP}\left(W_{elec}\right)$. Hence,

$$W_{elec} = Q_{annual}/\left(\text{COP}\right) = 6380\text{kWh}/\left(3\right) = 2130\,\text{kWh}$$

The corresponding cost is

$$C = 2130\,\text{kWh}\left(\$0.08/\text{kWh}\right) = \$170$$ <

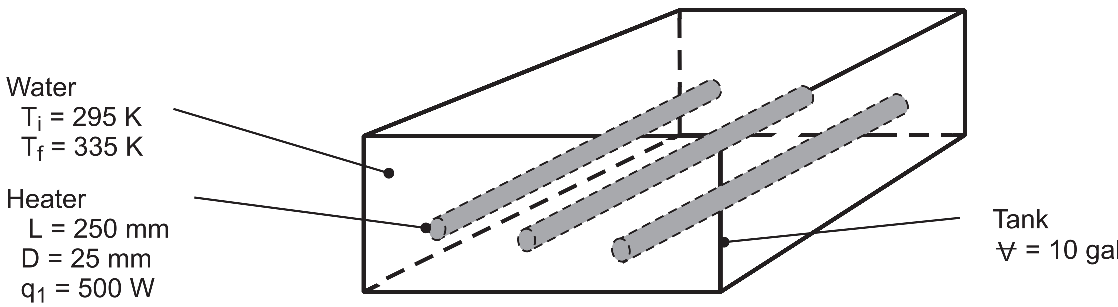
**COMMENTS:** Although annual operating costs are significantly lower for a heat pump, corresponding capital costs are much higher. The feasibility of this approach depends on other factors such as geography and seasonal variations in COP, as well as the time value of money.

## PROBLEM 1.38

**KNOWN:** Initial temperature of water and tank volume. Power dissipation, emissivity, length and diameter of submerged heaters. Expressions for convection coefficient associated with natural convection in water and air.

**FIND:** (a) Time to raise temperature of water to prescribed value, (b) Heater temperature shortly after activation and at conclusion of process, (c) Heater temperature if activated in air.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Negligible heat loss from tank to surroundings, (2) Water is *well-mixed* (at a uniform, but time varying temperature) during heating, (3) Negligible changes in thermal energy storage for heaters, (4) Constant properties, (5) Surroundings afforded by tank wall are large relative to heaters.

**ANALYSIS:** (a) Application of conservation of energy to a closed system (the water) at an instant, Eq. (1.11d), yields

$$\frac{dU}{dt} = Mc\frac{dT}{dt} = \rho \forall c \frac{dT}{dt} = q = 3q_1$$

Hence,

$$\int_0^t dt = \left(\rho \forall c/3q_1\right) \int_{T_i}^{T_f} dT$$

$$t = \frac{990\ \text{kg/m}^3 \times 10\text{gal}\left(3.79\times10^{-3}\text{m}^3/\text{gal}\right)4180\text{J/kg}\cdot\text{K}}{3\times500\ \text{W}}\left(335-295\right)\text{K} = 4180\ \text{s} \qquad <$$

(b) From Eq. (1.3a), the heat rate by convection from each heater is

$$q_1 = Aq_1'' = Ah\left(T_s - T\right) = \left(\pi DL\right)370\left(T_s - T\right)^{4/3}$$

Hence,

$$T_s = T + \left(\frac{q_1}{370\pi DL}\right)^{3/4} = T + \left(\frac{500\ \text{W}}{370\ \text{W/m}^2\cdot\text{K}^{4/3}\times\pi\times0.025\ \text{m}\times0.250\ \text{m}}\right)^{3/4} = \left(T+24\right)\text{K}$$

With water temperatures of $T_i \approx 295$ K and $T_f = 335$ K shortly after the start of heating and at the end of heating, respectively,

$$T_{s,i} = 319\ \text{K} \qquad T_{s,f} = 359\ \text{K} \qquad <$$

Continued …..

(c) From Eq. (1.10), the heat rate in air is

$$q_1 = \pi DL\left[0.70\left(T_s - T_\infty\right)^{4/3} + \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right)\right]$$

Substituting the prescribed values of $q_1$, D, L, $T_\infty = T_{sur}$ and ε, an iterative solution yields

$T_s = 830$ K <

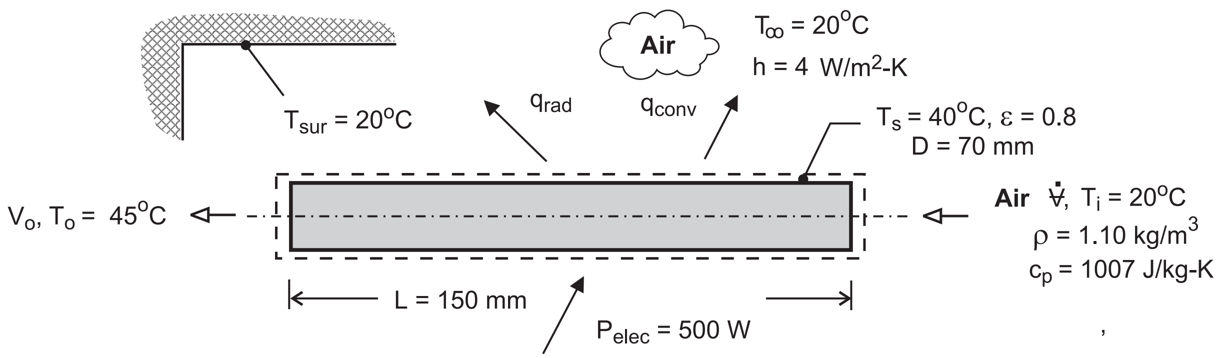
**COMMENTS:** In part (c) it is presumed that the heater can be operated at $T_s = 830$ K without experiencing burnout. The much larger value of $T_s$ for air is due to the smaller convection coefficient. However, with $q_{conv}$ and $q_{rad}$ equal to 59 W and 441 W, respectively, a significant portion of the heat dissipation is effected by radiation.

# PROBLEM 1.39

**KNOWN:** Power consumption, diameter, and inlet and discharge temperatures of a hair dryer.

**FIND:** (a) Volumetric flow rate and discharge velocity of heated air, (b) Heat loss from case.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state, (2) Constant air properties, (3) Negligible potential and kinetic energy changes of air flow, (4) Negligible work done by fan, (5) Negligible heat transfer from casing of dryer to ambient air (Part (a)), (6) Radiation exchange between a small surface and a large enclosure (Part (b)).

**ANALYSIS:** (a) For a control surface about the air flow passage through the dryer, conservation of energy for an open system reduces to

$$\dot{m}(u+pv)_i - \dot{m}(u+pv)_o + q = 0$$

where u + pv = i and q = $P_{elec}$. Hence, with $\dot{m}(i_i - i_o) = \dot{m}c_p(T_i - T_o)$,

$$\dot{m}c_p(T_o - T_i) = P_{elec}$$

$$\dot{m} = \frac{P_{elec}}{c_p(T_o - T_i)} = \frac{500\ \text{W}}{1007\ \text{J/kg}\cdot\text{K}\left(25^\circ\text{C}\right)} = 0.0199\ \text{kg/s}$$

$$\dot{\forall} = \frac{\dot{m}}{\rho} = \frac{0.0199\ \text{kg/s}}{1.10\ \text{kg/m}^3} = 0.0181\ \text{m}^3/\text{s} \qquad <$$

$$V_o = \frac{\dot{\forall}}{A_c} = \frac{4\dot{\forall}}{\pi D^2} = \frac{4 \times 0.0181\ \text{m}^3/\text{s}}{\pi(0.07\ \text{m})^2} = 4.7\ \text{m/s} \qquad <$$

(b) Heat transfer from the casing is by convection and radiation, and from Eq. (1.10)

$$q = hA_s(T_s - T_\infty) + \varepsilon A_s \sigma\left(T_s^4 - T_{sur}^4\right)$$

Continued …..

**PROBLEM 1.39 (Continued)**

where $A_s = \pi DL = \pi(0.07\ \text{m} \times 0.15\ \text{m}) = 0.033\ \text{m}^2$. Hence,

$$q = 4\text{W/m}^2 \cdot \text{K}\left(0.033\ \text{m}^2\right)\left(20^\circ\text{C}\right) + 0.8 \times 0.033\ \text{m}^2 \times 5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4\left(313^4 - 293^4\right)\text{K}^4$$

$$q = 2.64\ \text{W} + 3.33\ \text{W} = 5.97\ \text{W} \qquad <$$

The heat loss is much less than the electrical power, and the assumption of negligible heat loss is justified.

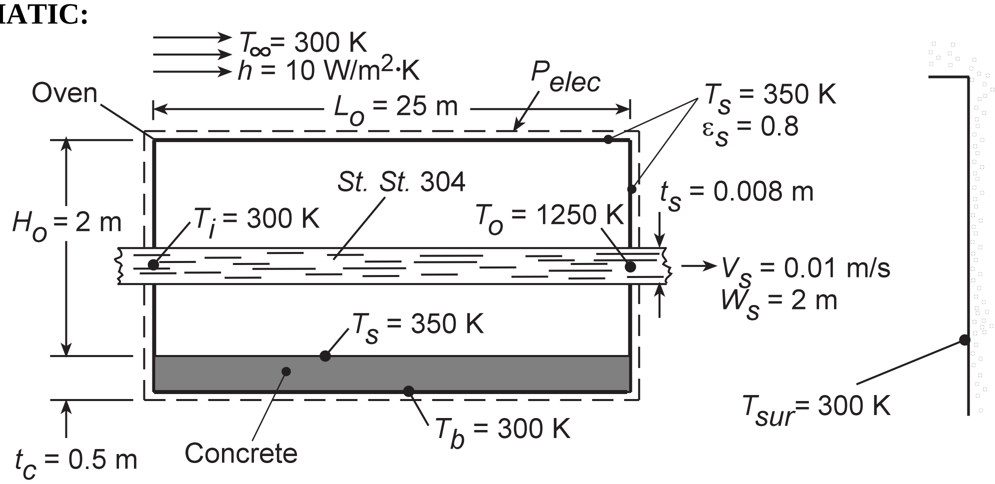
**COMMENTS:** Although the mass flow rate is invariant, the volumetric flow rate increases as the air is heated in its passage through the dryer, causing a reduction in the density. However, for the prescribed temperature rise, the change in $\rho$, and hence the effect on $\dot{\forall}$, is small.

# PROBLEM 1.40

**KNOWN:** Speed, width, thickness and initial and final temperatures of 304 stainless steel in an annealing process. Dimensions of annealing oven and temperature, emissivity and convection coefficient of surfaces exposed to ambient air and large surroundings of equivalent temperatures. Thickness of pad on which oven rests and pad surface temperatures.

**FIND:** Oven operating power.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) steady-state, (2) Constant properties, (3) Negligible changes in kinetic and potential energy.

**PROPERTIES:** Table A.1, St.St.304 $\left(\overline{T} = \left(T_i + T_o\right)/2 = 775\,K\right)$: $\rho$ = 7900 kg/m$^3$, $c_p$ = 578 J/kg·K; Table A.3, Concrete, T = 300 K: $k_c$ = 1.4 W/m·K.

**ANALYSIS:** The rate of energy addition to the oven must balance the rate of energy transfer to the steel sheet and the rate of heat loss from the oven. With $\dot{E}_{in} - \dot{E}_{out} -$ = 0, it follows that

$$P_{elec} + \dot{m}\left(u_i - u_o\right) - q = 0$$

where heat is transferred from the oven. With $\dot{m} = \rho V_s\left(W_s t_s\right)$, $\left(u_i - u_o\right) = c_p\left(T_i - T_o\right)$, and

$$q = \left(2H_oL_o + 2H_oW_o + W_oL_o\right)\times\left[h\left(T_s - T_\infty\right) + \varepsilon_s\sigma\left(T_s^4 - T_{sur}^4\right)\right] + k_c\left(W_oL_o\right)\left(T_s - T_b\right)/t_c\,,$$

it follows that

$$P_{elec} = \rho V_s\left(W_s t_s\right)c_p\left(T_o - T_i\right) + \left(2H_oL_o + 2H_oW_o + W_oL_o\right)\times$$
$$\left[h\left(T_s - T_o\right) + \varepsilon_s\sigma\left(T_s^4 - T_{sur}^4\right)\right] + k_c\left(W_oL_o\right)\left(T_s - T_b\right)/t_c$$

$$P_{elec} = 7900\,kg/m^3 \times 0.01\,m/s\left(2\,m \times 0.008\,m\right)578\,J/kg\cdot K\left(1250 - 300\right)K$$
$$+\left(2\times 2m\times 25m + 2\times 2m \times 2.4m + 2.4m\times 25m\right)[10\,W/m^2\cdot K\left(350 - 300\right)K$$
$$+0.8\times 5.67\times 10^{-8}\,W/m^2\cdot K^4\left(350^4 - 300^4\right)K^4] + 1.4\,W/m\cdot K\left(2.4m\times 25m\right)\left(350 - 300\right)K/0.5m$$

Continued.….

## PROBLEM 1.40 (Cont.)

$$P_{elec} = 694,000\,\text{W} + 169.6\,\text{m}^2\,(500 + 313)\,\text{W/m}^2 + 8400\,\text{W}$$

$$= (694,000 + 84,800 + 53,100 + 8400)\,\text{W} = 840\,\text{kW} \qquad <$$

**COMMENTS:** Of the total energy input, 83% is transferred to the steel while approximately 10%, 6% and 1% are lost by convection, radiation and conduction from the oven. The convection and radiation losses can both be reduced by adding insulation to the side and top surfaces, which would reduce the corresponding value of $T_s$.