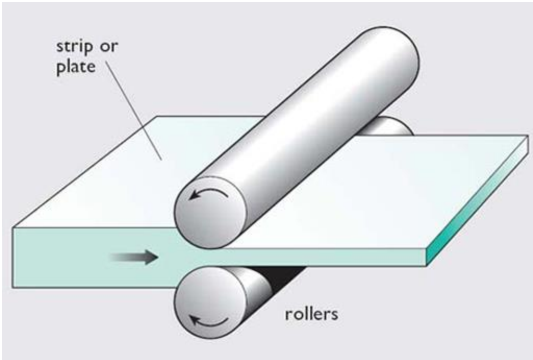
Assignment # 4, Module 8, Metal Bulk Deformation

Please submit on Canvas by Saturday, Nov 9th ,11:59 PM

Q1) An Aluminum alloy strip (width $W = 300mm$, thickness $t_0 = 25mm$) is formed through a rolling mill, each roll radius $R = 250mm$.

The strength coefficient of the material is $K = 275\ MPa$, and the strain hardening exponent $n = 0.15$, the process is a warm rolling and the friction coefficient between each roller and the plate is approximated as $\mu = 0.12$

The rollers speed is $N = 50\ RPM$. In this process, the plate thickness needs to be reduced to $20mm$.

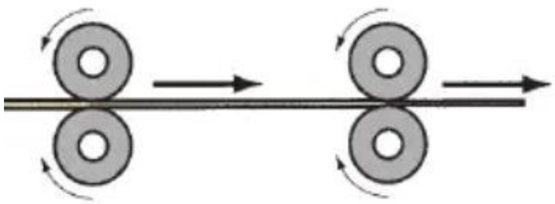


Part 1. check if the rolling process to reduce the thickness to 20mm can be achieved in one path?

$d_{max} = \mu^2 R = 0.12^2 \times 250 = 3.6\ mm$

Therefore, the thickness reduction $25 - 20 = 5mm$ cannot be achieved in one pass.

Part 2. The engineer, in charge of designing the process, decides to conduct the process in two passes; in each pass, the plate thickness is to be reduced 2.5mm. Calculate the required power (for each roller set)? Note that the width of the plate changes after the first pass (Assume 10% slippage between the rollers and the plate)



Required power for step 1, reduction of 2.5mm (from 25 to 22.5):

$$P_1 = \frac{2\pi N}{60} R(t_0 - t_1) W_0 \frac{K}{1+n}\left(ln\frac{t_0}{t_1}\right)^n$$
$$= \frac{2\pi \times 50}{60} \times 0.25 \times \frac{2.5}{1000} \times 0.3 \times \frac{275 \times 10^6}{1+0.15} \times (ln\frac{25}{22.5})^{0.15}$$
$$= 167508 \ Watt$$

Please note: There is a point where workpiece input velocity equals the roller velocity, known as "<u>**no slip point**</u>"; and the forming velocity (exist velocity) is higher than roller (because we are squeezing a material through a narrower channel (the rollers), like when you push a liquid through a narrow nozzle). For simplicity, and initial estimation of power, also assuming higher slippage (10% here is considered very high; usually it is in a range below 4%), in this assignment, it is Okay if someone assume V0 (the enter velocity) to be equal to the roller velocity; This formula is based on many assumptions, and just for preliminary estimation of required power.

Assuming a constant flow rate of material, and 10% slippage, the new width is:

$s = \frac{V_f - V_r}{V_r}$, $0.1 = \frac{V_f - V_r}{V_r}, V_f = 1.1V_r$

$25 \times 300 \times V_r = 22.5 \times W_1 \times 1.1V_r\,,\ W_1 = 303mm$

Required power for step 2, reduction of 2.5mm (from 22.5 to 20):

$$P_2 = \frac{2\pi N}{60} R(t_1 - t_2) W_1 \frac{K}{1+n}\left(ln\frac{t_1}{t_2}\right)^n$$
$$= \frac{2\pi \times 50}{60} \times 0.25 \times \frac{2.5}{1000} \times 0.303 \times \frac{275 \times 10^6}{1+0.15} \times (ln\frac{25}{20})^{0.15}$$
$$= 172035\ Watt$$

Total power

$P_{total} = P_1 + P_2 = 339543\ Watt = \frac{339543}{745.7} = 455\ hp$

Part 3. Optimize the process in terms of power consumption by configuring the thickness reduction in each pass? (i.e. Does the thickness reduction of 2.5mm and 2.5mm in each step requires the minimum power for each roller sets, or the initial thickness reduction must be higher or less than the second step thickness reduction?) (Assume 10% slippage between the rollers and the plate)

Hint: assume a thickness reduction in the first step to be x, then the reduction of next step is 5-x; with 10% slippage, and constant volume flow rate, the width change can be calculated as $25 \times 300 = 1.1 \times (new\ width) \times (25 - x)$. Calculate required power for step 1 (thickness reduction from 25 to 25-x), and required power for step 2 (thickness reduction from 25-x to 20). The total power requirement is sum of each step; find x which minimize the total power.

$$P_{total} = P_1 + P_2 = \frac{2\pi N}{60} R(t_0 - t_1) W_0 \frac{K}{1+n}\left(ln\frac{t_0}{t_1}\right)^n + \frac{2\pi N}{60} R(t_1 - t_2) W_1 \frac{K}{1+n}\left(ln\frac{t_1}{t_2}\right)^n$$

$$= \frac{2\pi N}{60} R \frac{K}{1+n}\left\{\frac{0.3x}{1000}(ln\frac{25}{25-x})^{0.15} + \frac{6.818}{25-x}\frac{(5-x)}{1000}(ln\frac{25-x}{20})^{0.15}\right\} =$$

$\frac{2\pi N}{60} R \frac{K}{1+n}\left\{\frac{0.3x}{1000}(ln\frac{25}{25-x})^{0.15} + \frac{6.818}{25-x}\frac{(5-x)}{1000}(ln\frac{25-x}{20})^{0.15}\right\}$

Assuming a constant flow rate of material, and 10% slippage:

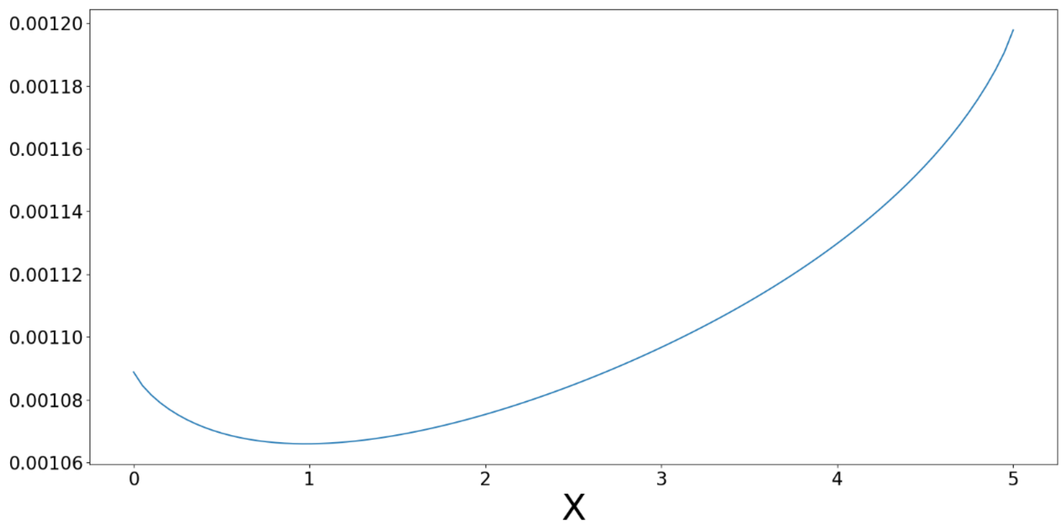
$$s = \frac{V_f - V_r}{V_r}, \quad 0.1 = \frac{V_f - V_r}{V_r}, V_f = 1.1V_r$$

$$25 \times 300 \times V_r = (25 - x) \times W_1 \times 1.1V_r \,, \; W_1 = \frac{6818}{25-x}$$

Now we need to find x to minimize this function:

$$0.3x(ln\frac{25}{25-x})^{0.15} + \frac{6.818}{25-x}(5-x)(ln\frac{25-x}{20})^{0.15}$$

We can plot it, to see where it gets minimum:



It shows, the minimum is around 1 mm, it means the second pass has to reduce the thickness by 4 mm; but we are limited to the maximum thickness reduction of 3.6mm (part 1); therefore, the second choice which will results in minimum power, would be 1.5 mm thickness reduction in the first pass and 3.5mm in the second pass. Now, let's compute the power, based on x=1.5

$$P_{total} = \frac{2\pi N}{60} R \frac{K}{1+n} \left\{ 0.3x(ln\frac{25}{25-x})^{0.15} + \frac{6.818}{25-x}(5-x)(ln\frac{25-x}{20})^{0.15} \right\}$$
$$= \frac{2\pi 50}{60} 0.25 \frac{275 \times 10^6}{1+0.15} \left\{ 0.3(1.5)(ln\frac{25}{25-1.5})^{0.15} + \frac{6.818}{25-1.5}(5-1.5)(ln\frac{25-1.5}{20})^{0.15} \right\} = 334525\ Watt = 448\ hp$$

it is just a small change compared with the previous scenario; therefore, it is safer to go with the initial plan, and use 2.5mm reduction of thickness in each step. As you can see the factor inside bracket does not have significant influence on the power requirement; the main factors which have significant effect on power requirement are: rotation speed (N), roll radius (R), and material properties (K and n).

Q2) In an open forging operation, the starting piece is a cylindrical ingot of 75mm in height and 50mm in diameter. The goal is to reduce the height to 36mm. The ingot material is a low Carbon steel with strength coefficient K=350 MPa, and strain hardening exponent of n=0.17; the coefficient of friction is $\mu = 0.1$

Determine the force as the process begins, at intermediate heights of 62mm, 49mm, and at the final height of 36mm? Using these values, plot the load-stroke curve (a graph of force F as a function of height reduction $h_0 - h$)

Here is the general formula to estimate the required force, in open-die forging:

$$F = K_f.K.A_0.(ln\frac{h_0}{h_1})^n = \left(1 + \frac{0.4\mu D_0}{h_0}\right) K.\left(\frac{\pi {D_0}^2}{4}\right).(ln\frac{h_0}{h_1})^n$$

Here I find the forces for steps of 5 mm starting from 75 to 35 mm, then I plot Force - Height

Step 1: from $\boldsymbol{h_0 = 75mm}$ to $\boldsymbol{h_1 = 70\ mm}$, with $\boldsymbol{D_0 = 50mm}$

$$F_1 = \left(1 + \frac{0.4\times0.1\times50}{75}\right) \times 350 \times 10^6 \times \left(\frac{\pi 0.05^2}{4}\right) \times (ln\frac{75}{70})^{0.17} = 448\ kN$$

Now, let's calculate the new dimeter D_1:

$A_0.h_0 = A_1.h_1 - \Delta V \quad \Delta V = V_0\frac{1-2v}{E} \times \frac{F}{A_0}$ (Please see, Part 2. Class Lecture Notes, on Canvas)

Let's assume, $v = 0.25$, $E = 2 \times 10^5 MPa$ for Steel,

$\frac{\Delta V}{V_0} = \frac{1-2v}{E} \times \frac{F}{A_0} = 0.00057$,

The change in the volume (due to compression) is very small, and for simplicity, we can ignore this change; therefore,

$\frac{\pi 50^2}{4} \times 75 = \frac{\pi {D_1}^2}{4} \times 70$, $D_1 = 51.75mm$

Step 2: from $\boldsymbol{h_1 = 70mm}$ to $\boldsymbol{h_2 = 65\ mm}$ with $\boldsymbol{D_1 = 51.75mm}$

$$F_2 = \left(1 + \frac{0.4\times0.1\times51.75}{70}\right) \times 350 \times 10^6 \times \left(\frac{\pi 0.05175^2}{4}\right) \times (ln\frac{70}{65})^{0.17} = 487\ kN$$

And the new diameter D_2:

$\frac{\pi 51.75^2}{4} \times 70 = \frac{\pi {D_2}^2}{4} \times 65$, $D_2 = 53.7mm$

Step 3: from $\boldsymbol{h_2 = 65mm}$ to $\boldsymbol{h_3 = 60\ mm}$ with $\boldsymbol{D_2 = 53.7mm}$

$$F_3 = \left(1 + \frac{0.4\times0.1\times53.7}{65}\right) \times 350 \times 10^6 \times \left(\frac{\pi 0.0537^2}{4}\right) \times (ln\frac{65}{60})^{0.17} = 533\ kN$$

And the new diameter D_3:

$\frac{\pi 53.7^2}{4} \times 65 = \frac{\pi {D_3}^2}{4} \times 60$, $D_3 = 55.9mm$

Step 4: from $\boldsymbol{h_3 = 60mm}$ to $\boldsymbol{h_4 = 55\ mm}$ with $\boldsymbol{D_3 = 55.9mm}$

$$F_4 = \left(1 + \frac{0.4 \times 0.1 \times 55.9}{60}\right) \times 350 \times 10^6 \times \left(\frac{\pi 0.0559^2}{4}\right) \times (ln\frac{60}{55})^{0.17} = 788\ kN$$

And the new diameter D_3:

$$\frac{\pi 55.9^2}{4} \times 65 = \frac{\pi {D_4}^2}{4} \times 60\ ,\ D_4 = 58.38mm$$

Step 5: from $\boldsymbol{h_4 = 55mm}$ to $\boldsymbol{h_5 = 45\ mm}$ with $\boldsymbol{D_4 = 58.38mm}$ (bigger step here)

$$F_5 = \left(1 + \frac{0.4 \times 0.1 \times 58.38}{55}\right) \times 350 \times 10^6 \times \left(\frac{\pi 0.05838^2}{4}\right) \times (ln\frac{55}{45})^{0.17} = 943\ kN$$

And the new diameter D_3:

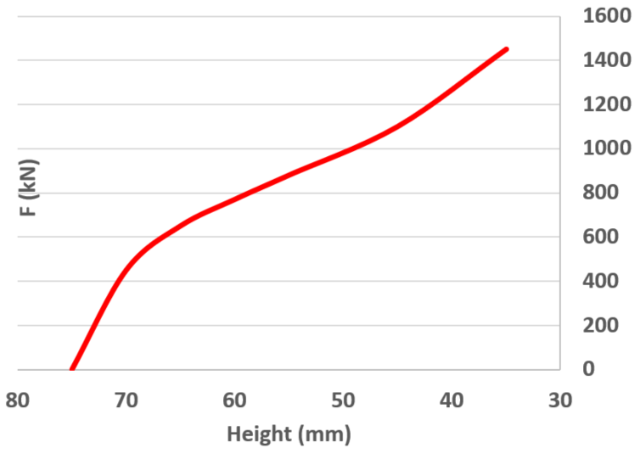
$$\frac{\pi 58.38^2}{4} \times 55 = \frac{\pi {D_5}^2}{4} \times 45\ ,\ D_5 = 64.54mm$$

Step 6: from $\boldsymbol{h_5 = 45mm}$ to $\boldsymbol{h_6 = 35\ mm}$ with $\boldsymbol{D_5 = 64.54\ mm}$ (bigger step here)

$$F_6 = \left(1 + \frac{0.4 \times 0.1 \times 64.54}{45}\right) \times 350 \times 10^6 \times \left(\frac{\pi 0.06454^2}{4}\right) \times (ln\frac{45}{35})^{0.17} = 1360\ kN$$

Now, let's plot these:

Steps	Stroke (mm)	F (kN)
0	Before start at 75	0
1	75-70= 5	448
2	70-65= 5	487
3	65-60= 5	533
4	60-55=5	788
5	55-45=10	943
6	45-35= 10	1360



Note: if you choose a different steps/strokes, let's say higher stroke (i.e. from 75 to 65 then from 65 to 55, and from 55 to 35 mm), then you will get different numbers for force. In other word, the required force in forging, depends on how you design the strokes/steps.