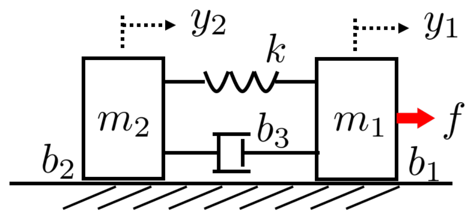
University of British Columbia
Department of Mechanical Engineering

MECH366 Modeling of Mechatronic Systems
Homework 1

Due: September 23 (Monday), 2019, 3pm

1. Consider 2-DOF mass-spring-damper system in the figure below, where k [N/m] is the linear spring constant, b_1 [Ns/m] and b_2 [Ns/m] are viscous friction coefficients (between masses and ground), and b_3 [Ns/m] is the damping coefficient. f [N] is the force input and outputs are displacements y_1 [m] and y_2 [m].



   (a) By selecting the following states, obtain the state-space model.
      i. $x_1 := y_1$, $x_2 := \dot{y}_1$, $x_3 := y_2$, $x_4 := \dot{y}_2$.
      ii. $x_1 := \dot{y}_2$, $x_2 := y_2$, $x_3 := \dot{y}_1$, $x_4 := y_1$.

   Hint: Once you get a state-space model for (a), you do not need to go through the derivation process for (b) and (c). (See Lecture 3, Slide 8.)

   (b) (In this question, use the states in (a)-i.) Instead of the linear spring, we assume the nonlinear spring which has the force-displacement relation of the spring as $f_z = k_1 z + k_2 z^3$ (instead of the relation $f_z = kz$ for the linear spring), where f_z is the force and z is the displacement. Obtain the linearized state-space model for the equilibrium point $y_{10} = 0$ [m], $y_{20} = 0$ [m] and $\dot{y}_{10} = \dot{y}_{20} = 0$ [m/s].
   Also, explain the reason why the linearization around $y_{10} = 1$ [m], $y_{20} = 0$ [m] and $\dot{y}_{10} = \dot{y}_{20} = 0$ [m/s] is impossible.

2. Obtain a state-space model for the RLC circuit depicted below. Here, the system has two input voltages u_1 and u_2 [V] and one output voltage [V] across the rightmost capacitor, as indicated in the figure. R, L and C are some given constants. (Hint: You may have lengthy calculations.)