ELEC 343
Electromechanics

Spring 2019

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Class Webpage:
http://courses.ece.ubc.ca/elec343/

Credit: 3-lecture-hour/week + 5 labs

Major Topics Covered

- Principles of electromagnetics, inductance and reluctance (2 lectures)
- Magnetic circuits & magnetically coupled systems (3 lectures)
- Linear and rotating electromechanical devices (4 lectures)
- Electromechanical energy conversion, concept of co-energy, developed forces and torques (5 lectures)
- Brushed dc motors, operation, equivalent circuit (4 lectures)
- DC motor dynamics and drive circuits, one-, two-, and four-quadrant operation (1 lectures)
- Stepper motors, principle of operation, full-step, micro-stepping, driver circuits (4 lectures)
- Rotating magnetic field, poly-phase systems (1 lectures)
- Synchronous motor, operation, dynamic & steady-state equivalent circuit, modelling (5 lectures)
- Brushless dc motors, operation, steady-state and dynamics, modelling (4 lectures)
- Induction motor, operation, equivalent circuit (5 lectures)
- Single phase AC motors (1 lecture)

Laboratory Experiments

Lab-0: Lab Equipment and Safety Rules
- There will be a safety instruction and a quiz!

Lab-1: Linear Solenoid Actuators
- 24V dc solenoid, principles of electromechanical energy conversion
- inductance and force vs. plunger positions and current values
- magnetic nonlinearities, hysteresis

Lab-2: Brushed DC Motors
- ¼ HP 48V Permanent Magnet DC Motor
- resistance, inductance, friction, torque constant, moment of inertia, etc
- performance & efficiency under load

Lab-3: Permanent Magnet Stepper Motors
- torque & inductance vs. position
- full, half, and micro-step operation
- effect of inductance on speed
- electromechanical resonance

Lab-4: Permanent Magnet Synchronous and Brushless DC Motors
- 210W 36V permanent magnet synchronous motor
- brushless dc motors, drive circuits, etc.

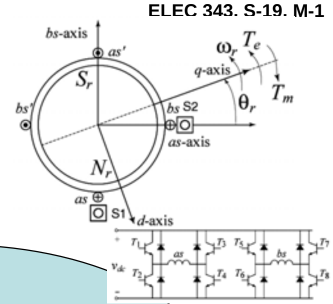
Lab-5: Induction Motors & Variable Frequency Drives
- industrial ¼ HP 34V Induction Motor,
- resistance, inductance, developed torque, performance & efficiency under load

Key Approach

$$J_t \frac{d\omega_{rm}}{dt} = T_e - T_{fric} - T_m$$

$$\mathbf{v}_{abcs} = \mathbf{r}_s \mathbf{i}_{abcs} + \frac{d\lambda_{abcs}}{dt}$$

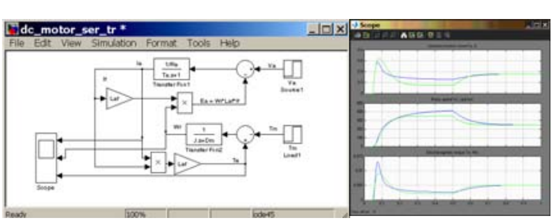
$$T_e = \frac{P}{2} \cdot \frac{\partial W_c(\mathbf{i}, \theta_r)}{\partial \theta_r}$$

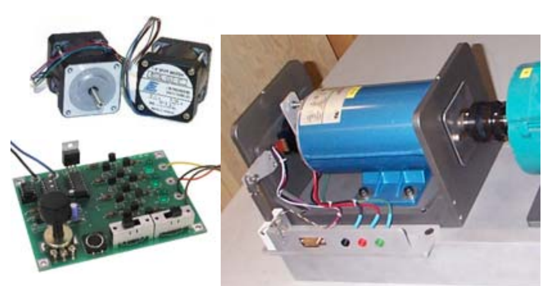


Theoretical fundamentals of electromechanical Actuators and Motors

Modelling as a tool for analysis and predicting the system behavior – modern design approach !

Lab Experiments – interaction with real devices, verification of the theory and models, hands-on engineering





Modelling and simulation is part of design/synthesis process!

Electromechanics Applications:

- Generation of Electricity – Big Scale

Thermal Power Plant

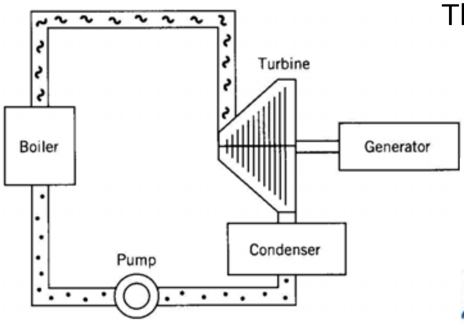
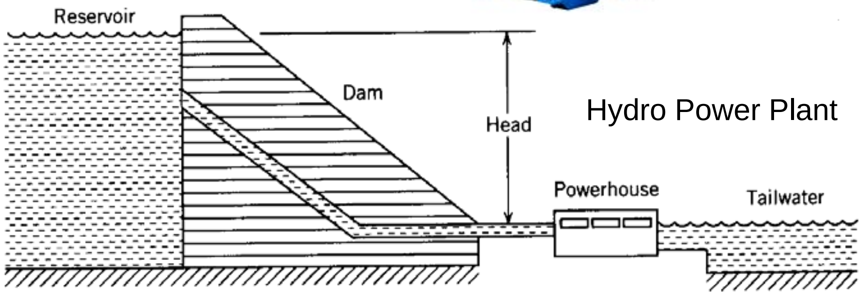


figure. King Mountain Wind Range, consisting of 214 1.3-MW wind turbines in Upton County, West Texas. (Courtesy: Bonus Energy Systems A/S.)

Wind Power



Hydro Power Plant

Electromechanical Devices

Manufacturing
Automotive
Aircraft
Ships
Computers
Office
Household
…

Electromechanical Applications:

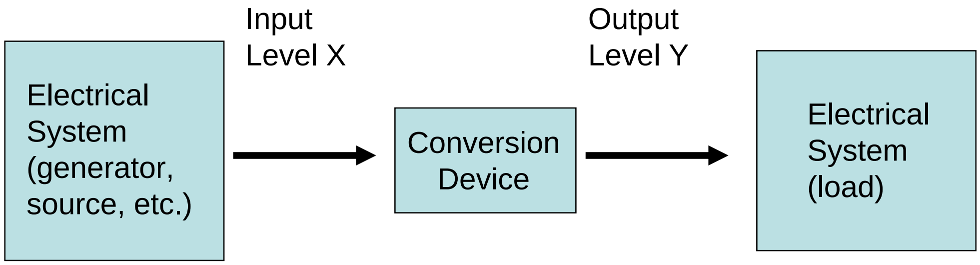
Electromechanical Energy Conversion

Electrical System (generator, source, etc.) → Input Electrical Energy → Conversion Device → Output Mechanical Energy → Mechanical System (machine tool, plant, etc.)

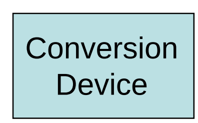
Mechanical System (Prime Mover, Turbine, etc.) → Input Mechanical Energy → Conversion Device → Output Electrical Energy → Electrical System (transmission, distribution, load, etc.)

Electromechanical Energy Conversion

- Transformation of Electrical Energy



Electromechanical Energy Conversion



- Electrical Machines
  - Stationary
    - Transformers
  - Rotating
    - Motors, generators
  - Linear Devices
    - Solenoids, linear motors, other actuators

- Power Electronics (Switched Mode , SMPSs, Motor & Actuator Drivers, …)
  - Rectifiers
    - AC to DC
  - Converters
    - DC to DC
  - Inverters
    - DC to AC

Very broad & interesting area, requires its own course!

Module 1, Part 1

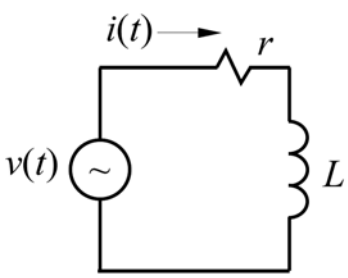
Review of AC Circuits, Phasors, 3-Phase (Read Chap. 1.1 and Appendix C)

Most Important Topics and Concepts

- Concept of phasors & notations
- RMS value
- Phasor diagrams for basic RLC circuits
- Balanced 3-phase system, Y / Δ connections
- Real, reactive, and apparent power in 1-phase and 3-phase systems, power factor

Review of AC Circuits

- Consider linear inductive circuit



Steady-state Solution

$$i(t) = I_m \cos(\omega t + \varphi_i)$$

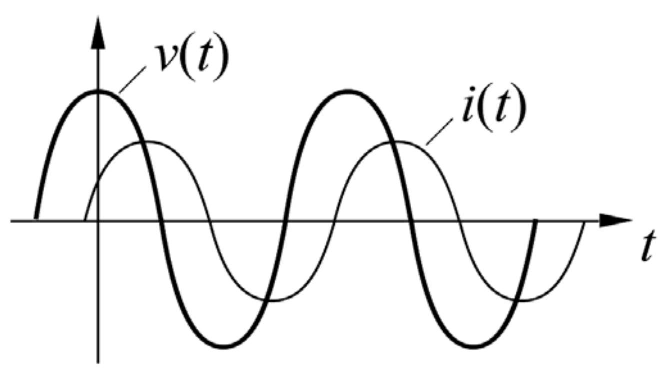
$$e(t) = E_m \cos(\omega t + \varphi_e)$$

$$v(t) = V_m \cos(\omega t)$$

KVL

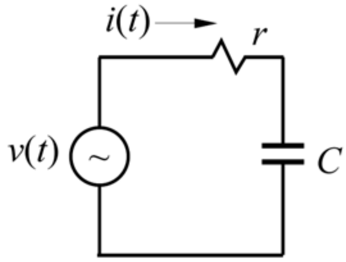
$$v(t) = ri + e = ri + \frac{d\lambda}{dt}$$

$$= ri + L\frac{di}{dt}$$



Review of AC Circuits

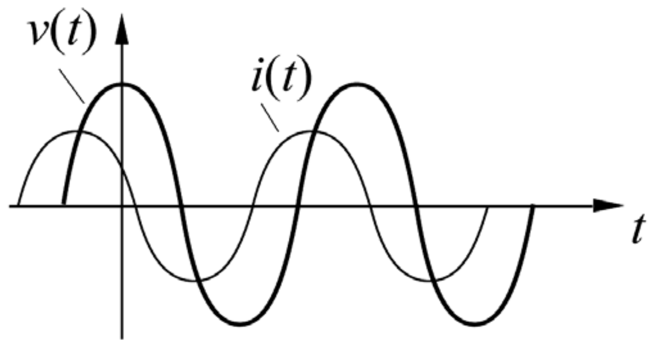
- Consider linear capacitive circuit



Steady-state Solution

$$v_c(t) = V_{c,m}\cos(\omega t + \varphi_v)$$

$$i(t) = I_m \cos(\omega t + \varphi_i)$$



$$v(t) = V_m \cos(\omega t)$$

KVL $\quad v(t) = ri + v_c$

$$i = \frac{v - v_c}{r} = C\frac{dv_c}{dt}$$

$$C\frac{dv_c}{dt} + \frac{1}{r}v_c = \frac{1}{r}v(t)$$

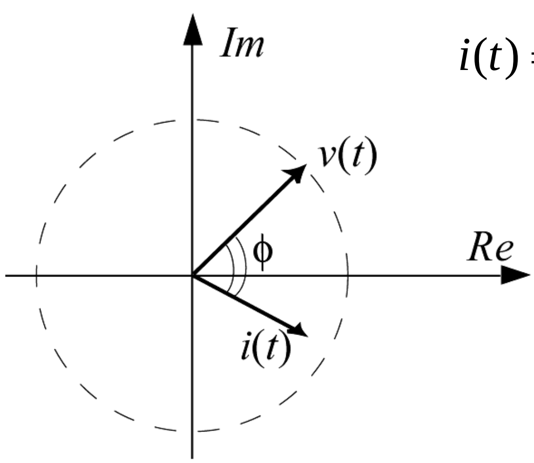
Note: In both cases we need to know only amplitude & phase !

Review of Phasors

- Complex Plane

$$v(t) = V_m\cos(\omega t) + j\cdot V_m \sin(\omega t)$$

$$i(t) = I_m\cos(\omega t - \phi) + j\cdot I_m \sin(\omega t - \phi)$$



Euler's Identity

$$e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$$

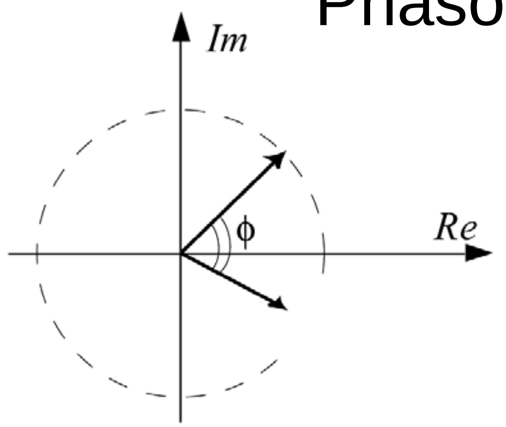
$$v(t) = V_m e^{j\omega t}$$

$$i(t) = I_m e^{j(\omega t - \phi)}$$

Note:

1. All vectors rotate at the same speed ω !
2. Only the amplitudes and their phase differences are important

Phasor Notations



$$i(t) = I_m e^{j(\omega t - \phi)} \equiv I_m \angle -\phi$$

$$v(t) = V_m e^{j\omega t} \equiv V_m \angle 0$$

Time Domain	Phasor Representation
$A\cos(\omega t \pm \theta)$	$A\angle \pm\theta$
$A\sin(\omega t \pm \theta)$	$A\angle \pm\theta - 90^\circ$

Review of Phasors

Linear Passive Elements		Complex Impedance
	R	$Z = R$
	L	$Z = j\omega L = (\omega L)\angle 90^\circ$
	C	$Z = \dfrac{1}{j\omega C} = \left(\dfrac{1}{\omega C}\right)\angle -90^\circ$

$$Z = \frac{V}{I} = \frac{V_m \angle \theta_v}{I_m \angle \theta_i} = \frac{V_m}{I_m} \angle \theta_v - \theta_i = Z_m \angle \theta_z$$

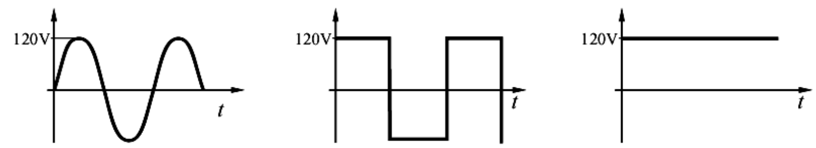
$$Z = R + jX, \quad Z_m = \sqrt{R^2 + X^2}, \quad \theta_z = \arctan\left(\frac{X}{R}\right)$$

Root-Mean-Square (RMS) Value

* Value equivalent to the DC voltage (current) that when applied to a resistor dissipates the same average amount of power as the given AC

$$P(t) = vi = v\frac{v}{r} = \frac{1}{r}v^2 \qquad P_{ave} = \frac{1}{T}\int_0^T P(t)dt = \frac{1}{r}\cdot\frac{1}{T}\int_0^T v^2(t)dt$$

$$V_{(rms)} = \sqrt{\frac{1}{T}\int_0^T v^2(t)dt}$$



$V_{rms} =$ $\qquad V_{rms} =$ $\qquad V_{rms} =$

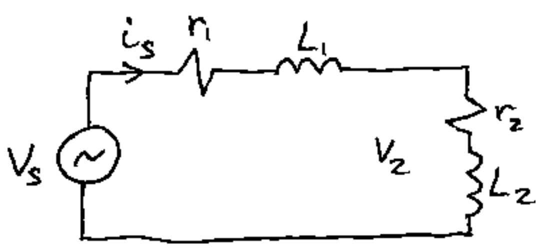
Given sinusoidal voltage (current), RMS values are often used with Phasors

$$v(t) = \sqrt{2}\cdot V_{rms}\cos(\omega t + \varphi_v) \qquad \tilde{V} = V_{rms}\angle\varphi_v$$

$$i(t) = \sqrt{2}\cdot I_{rms}\cos(\omega t + \varphi_i) \qquad \tilde{I} = I_{rms}\angle\varphi_i$$

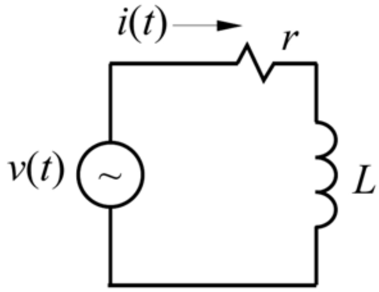
Qualitative Phasor Diagrams

Consider RLC Circuit



Power in AC Circuits

Given inductive load $v(t)=\sqrt{2}\cdot V\cos(\omega t)$ $\quad i(t)=\sqrt{2}\cdot I\cos(\omega t+\varphi)$



Instantiations power is always this

$$p(t)=vi=2VI\cos(\omega t)\cos(\omega t+\varphi)$$
$$=VI\cos(\varphi)+VI\cos(2\omega t+\varphi)$$

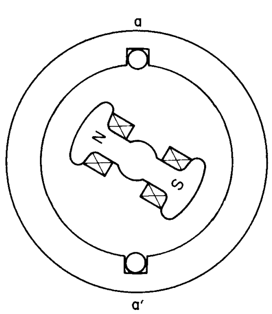
Average (real) power over one cycle $P=VI\cos(\varphi)$ $\quad [W],[kW],[MW]$

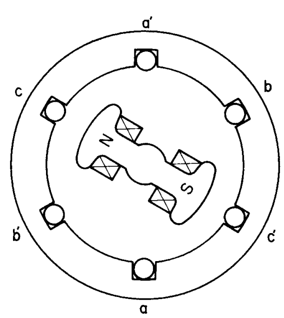
Apparent power $S=VI=\sqrt{P^2+Q^2}$ $\quad [VA],[kVA],[MVA]$

Reactive power $Q=VI\sin(\varphi)$ $\quad [VAR],[kVAR],[MVAR]$

Power Factor (pf) $\cos(\varphi)=\dfrac{P}{VI}$

Single vs. Three-Phase Systems





For balanced system we have $v_a+v_b+v_c=0$

$v(t)=V_m\cos(\omega t)$

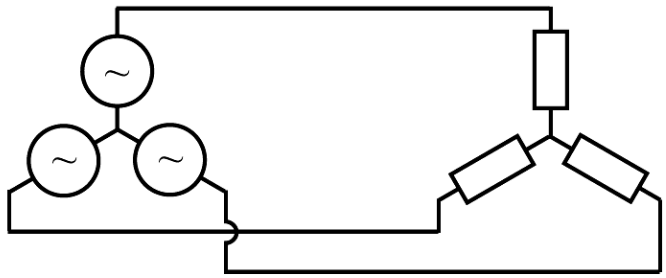
Easy to produce !

$$v_a(t)=V_m\cos(\omega t)$$
$$v_b(t)=V_m\cos(\omega t-120^\circ)$$
$$v_c(t)=V_m\cos(\omega t+120^\circ)$$

Just as easy to produce !

Balanced Three-Phase Systems

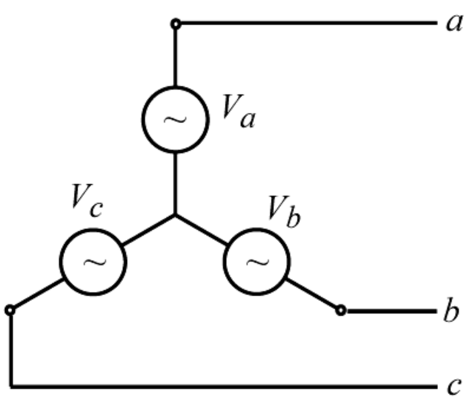
Single vs. Three Phases



Efficient transmission of power
– just one more conductor
= 3 times more power

Three-Phase Source

Wye (Y) - Connected

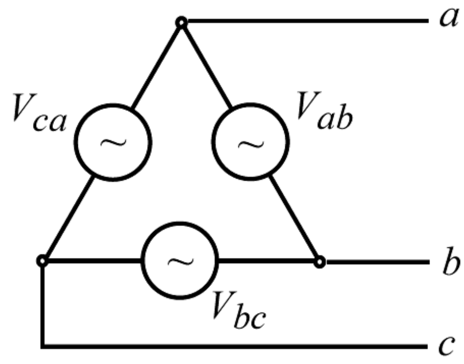


Line Voltages

$$V_{ab} = V_a - V_b$$

Three-Phase Source

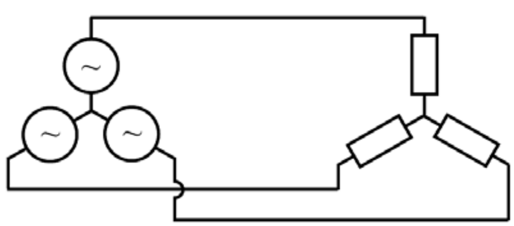
Delta (Δ) - Connected



Line Currents

$$I_a = I_{ab} - I_{ca}$$

Power in Three-Phase Systems



$$v_a(t) = \sqrt{2}V_{ph}\cos(\omega t)$$
$$v_b(t) = \sqrt{2}V_{ph}\cos(\omega t - 120^\circ)$$
$$v_c(t) = \sqrt{2}V_{ph}\cos(\omega t + 120^\circ)$$

$$i_a(t) = \sqrt{2}I_{ph}\cos(\omega t - \varphi)$$
$$i_b(t) = \sqrt{2}I_{ph}\cos(\omega t - 120^\circ - \varphi)$$
$$i_c(t) = \sqrt{2}I_{ph}\cos(\omega t + 120^\circ - \varphi)$$

Instantaneous Power

$$P_{3\phi}(t) = P_a + P_b + P_c$$
$$= i_a v_a + i_b v_b + i_c v_c$$

Power in Balanced Three-Phase Systems

In terms of phase quantities (Y - connection)

$$P_{3\phi} = 3P_{ph} = 3V_{ph} I_{ph} \cos(\varphi_{ph})$$
$$Q_{3\phi} = 3Q_{ph} = 3V_{ph} I_{ph} \sin(\varphi_{ph})$$
$$S_{3\phi} = 3S_{ph} = 3V_{ph} I_{ph}$$

In terms of line-to-line quantities

Y - connection

$$I_{ph} = I_L,\ V_{ph} = V_L / \sqrt{3}$$

Δ - connection

$$I_{ph} = I_L / \sqrt{3},\ V_{ph} = V_L$$

$$P_{3\phi} = \sqrt{3} V_L I_L \cos(\varphi_{ph})$$
$$Q_{3\phi} = \sqrt{3} V_L I_L \sin(\varphi_{ph})$$
$$S_{3\phi} = \sqrt{3} V_L I_L$$

Module 1, Part 2

Introduction & Magnetic Circuits

(Read Chapter 1.3 – 1.4)

Most Important Topics

- Fundamentals of Electromagnetics, Maxwell’s Equations
- Sign & direction conventions
- Basic magnetic circuits, concepts, analogies, calculations
- Flux, flux linkage, inductance
- Magnetic materials, saturation, hysteresis loop
- Coil under ac excitation, type of core losses

Review of Basic Quantities and Units

E – electric field intensity $\left[\frac{V}{m}\right]$

B – magnetic flux density $\left[Tesla = \frac{Weber}{meter^2}\right]$ $\left[T = \frac{Wb}{m^2}\right]$

H – magnetic field intensity $\left[\frac{A}{m}\right]$

Φ – magnetic flux $\left[Wb = T \cdot m^2\right]$

B-H Relation

- Current produces the H field (see Ampere's law)
- H is related to B

$$B = \mu H = \mu_0 \mu_r H$$

μ – permeability (characteristic of the medium) $\left[\frac{T \cdot m}{A} = \frac{Henry}{meter} = \frac{H}{m}\right]$

μ_0 – permeability of vacuum $= 4 \cdot \pi \cdot 10^{-7} [H/m]$

μ_r – relative permeability of material

magnetic materials $\mu_r = 100 \cdots 100{,}000$

Fundamentals

Summarized in Maxwell's Equations (1870s)

1) Gauss's Law for Electric Field

$$\oint_s \mathbf{E} \cdot d\mathbf{a} = \frac{q}{\varepsilon_0} = \Phi_e = \int E \cos\theta da$$

Electric flux out of any closed surface is proportional to the total charge enclosed

Fundamentals

Summarized in Maxwell's Equations (1870s)

2) Gauss's Law for Magnetic Field

$$\oint_s \mathbf{B} \cdot d\mathbf{a} = \Phi_m = 0$$

Magnetic flux out of any closed surface is zero

There are no magnetic charges

Fundamentals

Summarized in Maxwell's Equations (1870s)

3) Faraday's Law

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt}\int_S \mathbf{B} \cdot d\mathbf{a} = -\frac{d\Phi}{dt} = emf$$

Electro**M**otive **F**orce (emf)

The line integral of the electric field around a closed loop/contour C is equal to the negative of the rate of change of the magnetic flux through that loop/contour

Fundamentals

Summarized in Maxwell's Equations (1870s)

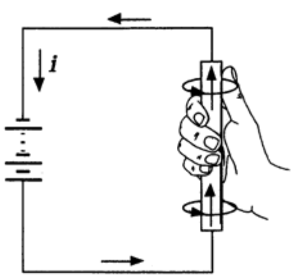
4) Ampere's Law (for static electric field)

$$\oint_C \mathbf{B}\cdot d\mathbf{l} = \mu_0 \int_S J\cdot da = \mu_0 I_{net}$$

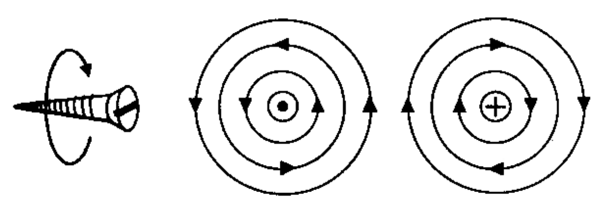
The line integral of the magnetic field B around a closed loop C is proportional to the net electric current flowing through that loop/contour C

Conventions

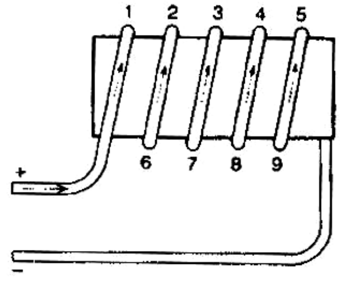
Right hand rule

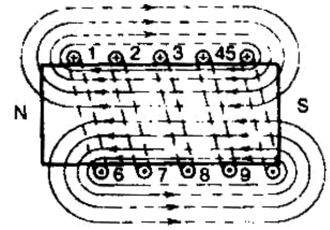


Right-screw rule
Dot and cross notations



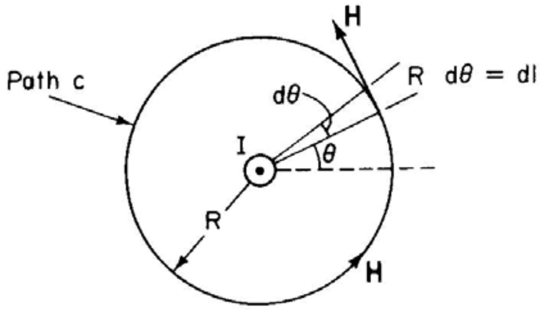
Magnetic field produced by coil (solenoid)





Flux Lines:
- form a closed loop/path
- Lines do not cut across or merge
- Go from North to South magnetic poles

Magnetic Field of an Infinite Conductor



Apply Ampere's Law

$$\oint_C \mathbf{H}\cdot\mathbf{dl} = I_{enclosed}$$

Incremental length $\mathbf{dl} = Rd\theta$

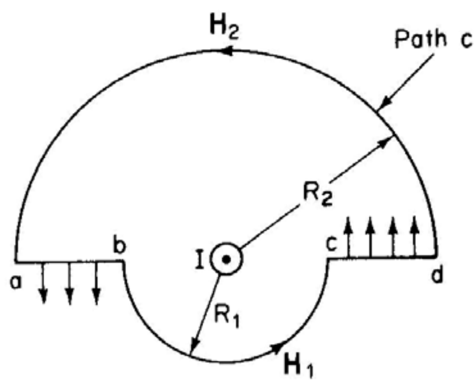
$$H\cdot 2\pi R = I$$

H and **dl** have the same direction

$$H = \frac{I}{2\pi R}$$

$$B = \mu H = \frac{\mu I}{2\pi R}$$

Magnetic Field of an Infinite Conductor



Apply Ampere's Law

$$\oint_C \mathbf{H}\cdot\mathbf{dl} = I_{enclosed}$$

$$\oint_C \mathbf{H}\cdot\mathbf{dl} = \int_a^b H\cdot dl + \int_b^c H_1\cdot dl + \int_c^d H\cdot dl + \int_d^a H_2\cdot dl$$

$$\int_b^c H_1\cdot dl = \int_{-\pi}^{0} \frac{I}{2\pi R_1} R_1 d\theta = \frac{I}{2}$$

$$\int_d^a H_2\cdot dl = \int_0^{\pi} \frac{I}{2\pi R_2} R_2 d\theta = \frac{I}{2}$$

Some Definitions

Magnetic Flux

$$\Phi = \int_S \mathbf{B} \cdot \mathbf{da} = B_c A_c$$

Flux is always continuous

Recall Faraday's Law - **E**lectro**m**otive **F**orce (emf)

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt} \int_S \mathbf{B} \cdot d\mathbf{a} = -\frac{d\Phi}{dt}$$

- voltage induced in one turn due to the changing magnetic flux

For coil with *N* turns $e = N \cdot \dfrac{d\Phi}{dt}$

Flux Linkage

$$\lambda = N \cdot \Phi \quad [Wb \cdot t]$$

flux scaled by the number of turns

Total induced emf $e = \dfrac{d\lambda}{dt} \quad [V]$

Some Definitions

Inductance

Need a function that relates Flux Linkage to the Current

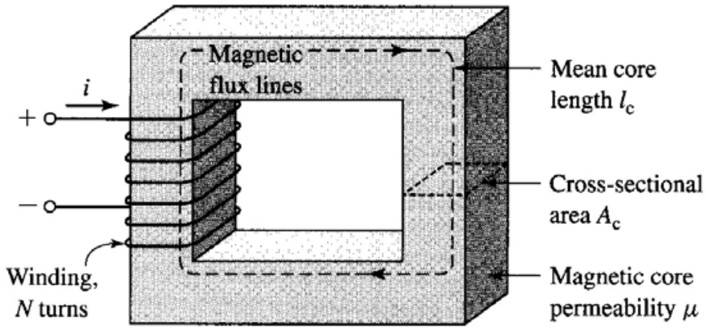
Consider $\lambda = f(i) = L(\cdot) \cdot i$ $\qquad L = \dfrac{\lambda}{i} \quad \left[\dfrac{Wb \cdot t}{A} = H\right]$

Recall

$$L = \frac{\lambda}{i} = \frac{N \cdot \Phi}{i}$$

Magnetic Circuits

- Consider basic magnetic circuit



Assume $\mu >> \mu_0$

⇒All magnetic field is concentrated inside the core

Recall Ampere's Law

$$\oint_{l_c} \mathbf{H}_c \cdot \mathbf{dl} = I_{net}$$

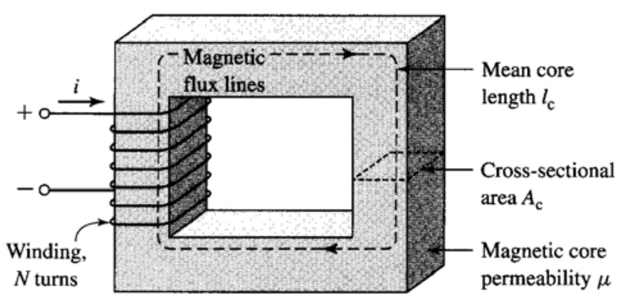
Magneto**m**otive **f**orce (mmf) $F = Ni$

Source of magnetic field is ampere-turn product

Assume uniform core $F = Ni = I_{net} = \oint_{l_c} H_c \cdot dl = H_c l_c \quad [Ampere \cdot turn]$

Magnetic Circuits

- Consider basic magnetic circuit



Assume all magnetic field is confined inside the core

Define Magnetic Flux

$$\Phi = \int_S \mathbf{B} \cdot \mathbf{da} = B_c A_c$$

Flux is always continuous $[Wb]$

Consider mmf $F = Ni = H_c l_c = \frac{B_c l_c}{\mu} = \Phi \frac{l_c}{\mu A_c} = \Phi \Re_c$

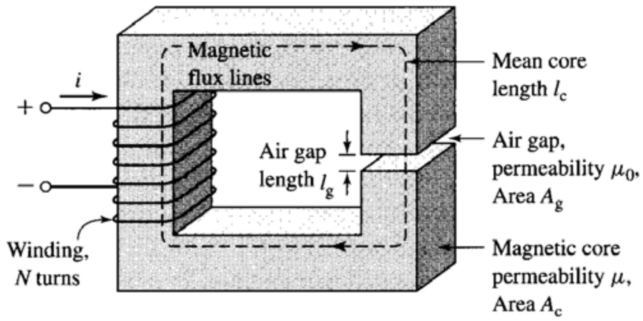
Define Reluctance (of the given magnetic path) $\Re_c = \frac{l_c}{\mu A_c} \quad \left[\frac{A}{Wb}\right]$

Recall Inductance

$$L = \frac{\lambda}{i} = \frac{N \cdot \Phi}{i} = \frac{N \cdot N \cdot i}{i \cdot \Re_c} = \frac{N^2}{\Re_c}$$

Magnetic Circuits

- Magnetic circuit with air gap



Consider mmf

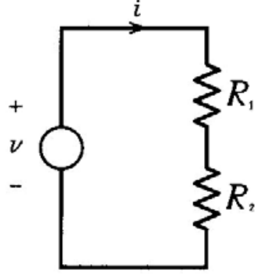
$$F = Ni = \oint_C \mathbf{H} \cdot \mathbf{dl}$$
$$= H_c l_c + H_g l_g$$
$$= \frac{B_c l_c}{\mu} + \frac{B_g l_g}{\mu_0}$$

Assuming all magnetic flux is confined inside the core $B_c = \frac{\Phi}{A_c}$ and $B_g = \frac{\Phi}{A_g}$

$$F = \Phi\left(\frac{l_c}{\mu A_c} + \frac{l_g}{\mu_0 A_g}\right) = \Phi(\Re_c + \Re_g) = \Phi \sum \Re_i = \Phi \Re_{total}$$

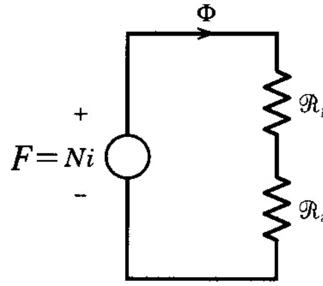
Magnetic and Electric Circuits Analogy

Electric Circuit



$$i = \frac{v}{R_1 + R_2}$$

Magnetic Circuit



$$\Phi = \frac{F}{\Re_1 + \Re_2}$$

Magnetic and Electric Circuits Analogy

Electric Circuit

- Voltage (emf), $V, [Volt]$
- Current, $I, [Amps]$
- Resistance, $R = \frac{l}{\sigma A}, [\Omega]$
- Conductance, $G = \frac{1}{R}, [Siemens]$
- Conductivity, $\sigma, \left[\frac{Siemens}{m}\right]$

For loop $v = \sum R_n i_n$

For node $\sum_N i_n = 0$

Magnetic Circuit

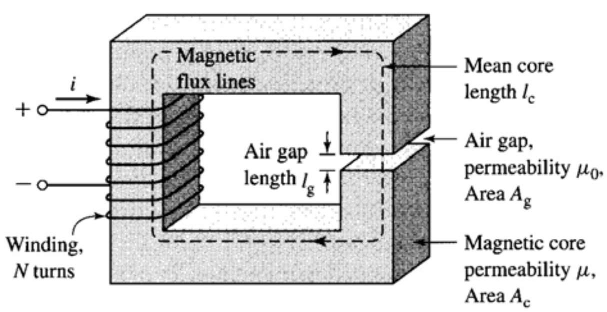
- mmf, $F, [A \cdot t]$
- Flux $\Phi, [Wb]$
- Reluctance, $\Re = \frac{l}{\mu A}, \left[\frac{A}{Wb}\right]$
- Permeance, $\rho = \frac{1}{\Re}, \left[\frac{Wb}{A}\right]$
- Permeability, $\mu, \left[\frac{H}{m}\right]$

For loop $F = \sum H_n l_n$

For node $\sum_N \Phi_n = 0$

Inductance: Example 1

Consider the following electromagnetic system (device)



Equivalent Magnetic Circuit

Equivalent Electric Circuit

Inductance: Example 1

Consider the following electromagnetic system

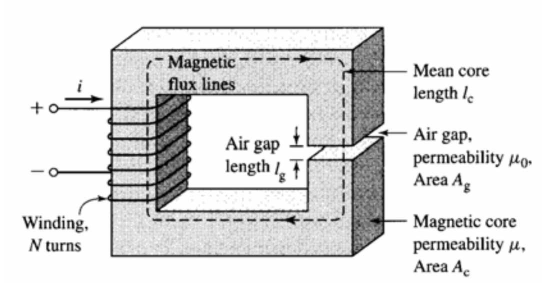
$i = 1\,A,\ N = 400$

$l_c = 50\,cm,\ l_g = 1\,mm$

$A_c = A_g = 15\,cm^2$

$\mu_r = 3000$

Find inductance $L = \dfrac{N^2}{\mathcal{R}_c + \mathcal{R}_g}$



$$\mathcal{R}_c = \frac{l_c}{\mu_r \mu_0 A_c} = \frac{50e{-2}}{3000 \cdot 4\pi \cdot 1e{-7} \cdot 15e{-4}} \approx 88.42e{+3}\ A{\cdot}t/Wb$$

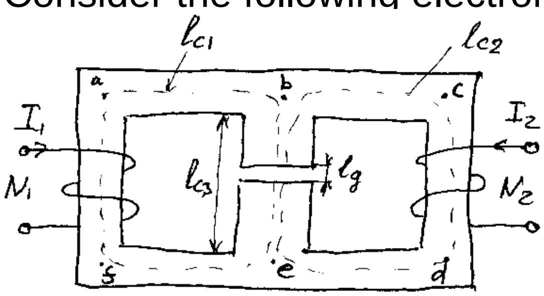
$$\mathcal{R}_g = \frac{l_g}{\mu_0 A_g} = \frac{1e{-3}}{4\pi \cdot 1e{-7} \cdot 15e{-4}} \approx 530.515e{+3}\ At/Wb$$

$$L = \frac{400^2}{(88.42 + 530.515) \cdot e{+3}} = 258.52e{-3}\ H$$

$$= 258.52\ mH$$

Magnetic Circuits: Example 2

Consider the following electromagnetic system



Path l_{c1} = bafe

Path l_{c2} = bcde

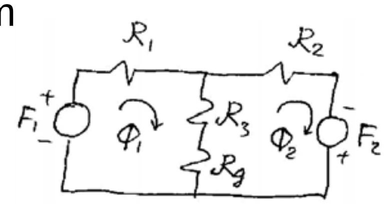
Path l_{c3} = be − l_g

l_{c3} includes only core

Equivalent Magnetic Circuit

Magnetic Circuits: Example 2

Consider the following electromagnetic system



Loop equations

$$\Phi_1(\mathcal{R}_1 + \mathcal{R}_3 + \mathcal{R}_g) - \Phi_2(\mathcal{R}_3 + \mathcal{R}_g) = F_1$$

$$-\Phi_1(\mathcal{R}_3 + \mathcal{R}_g) + \Phi_2(\mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_g) = F_2$$

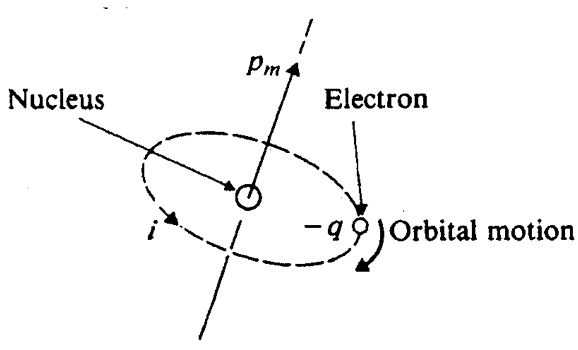
Solve for fluxes

$$\begin{bmatrix} \mathcal{R}_1 + \mathcal{R}_3 + \mathcal{R}_g & -(\mathcal{R}_3 + \mathcal{R}_g) \\ -(\mathcal{R}_3 + \mathcal{R}_g) & \mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_g \end{bmatrix} \begin{bmatrix} \Phi_1 \\ \Phi_2 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \begin{bmatrix} N_1 \cdot I_1 \\ N_2 \cdot I_2 \end{bmatrix}$$

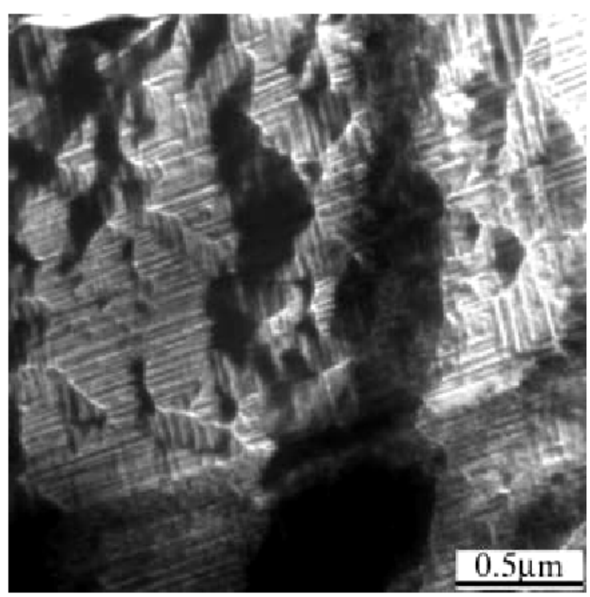
$$A \cdot x = b \Rightarrow x = A^{-1} \cdot b$$

Magnetic Materials

Magnetic moment of an atom

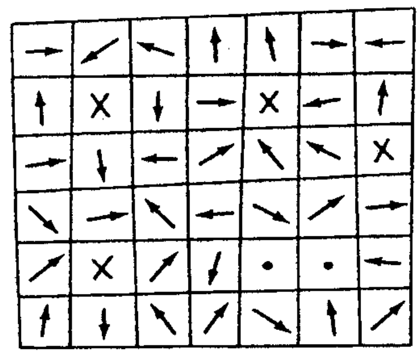


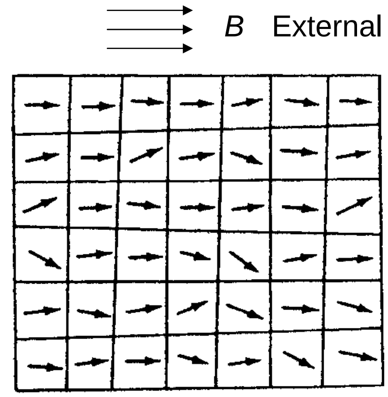
Magnetic Domain Structure



Magnetic Material Domain Model

B External field

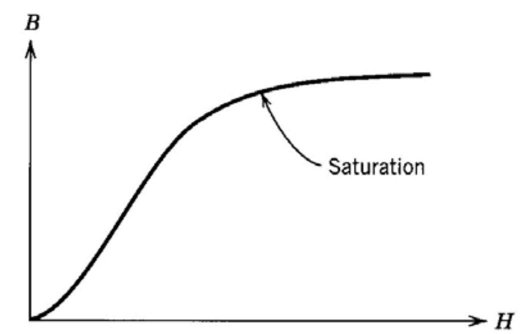


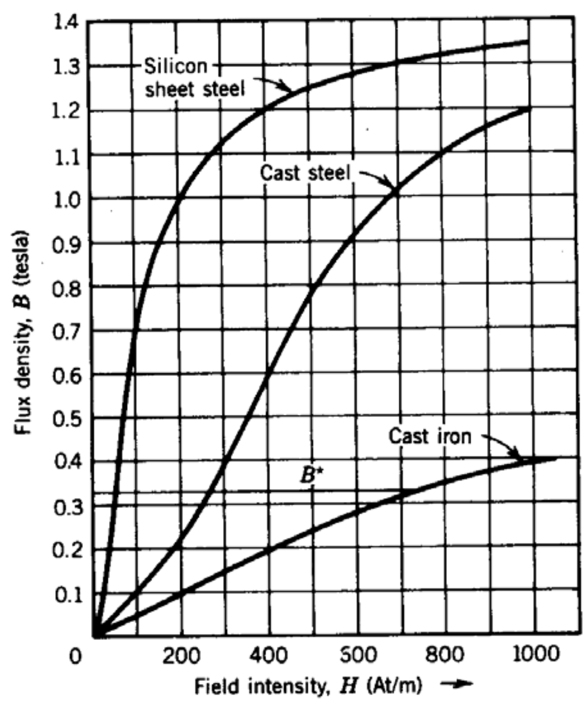


demagnetized

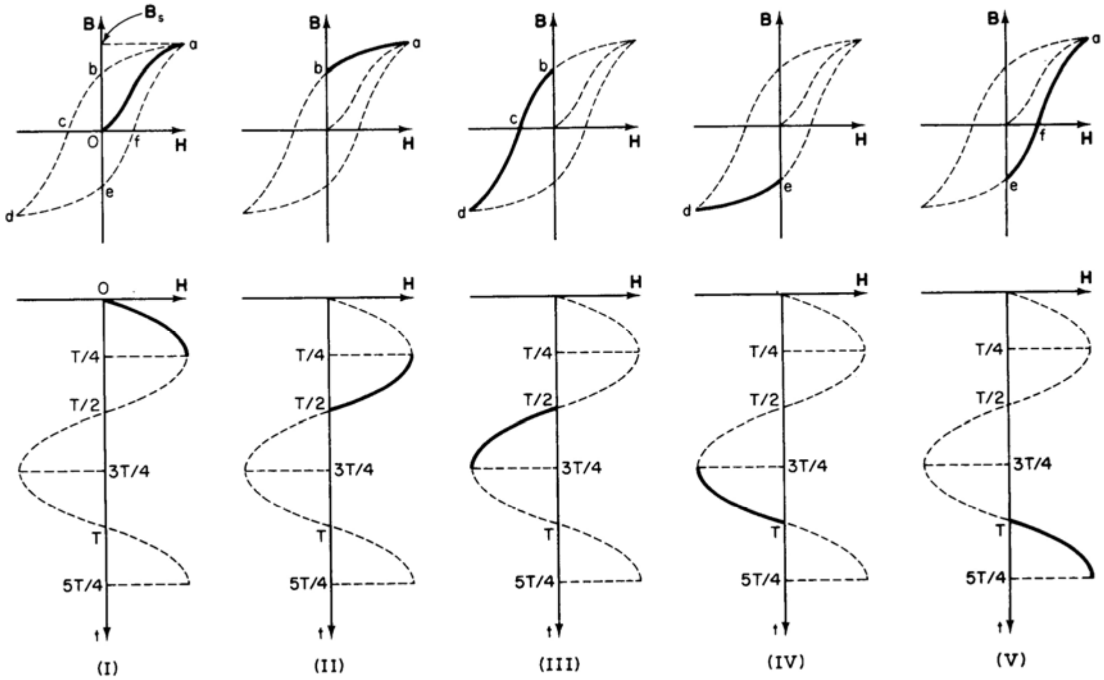
magnetized

Magnetic Saturation

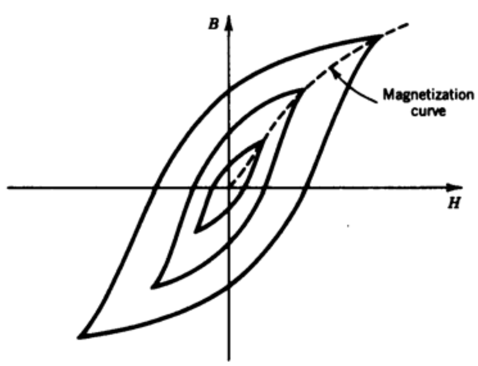




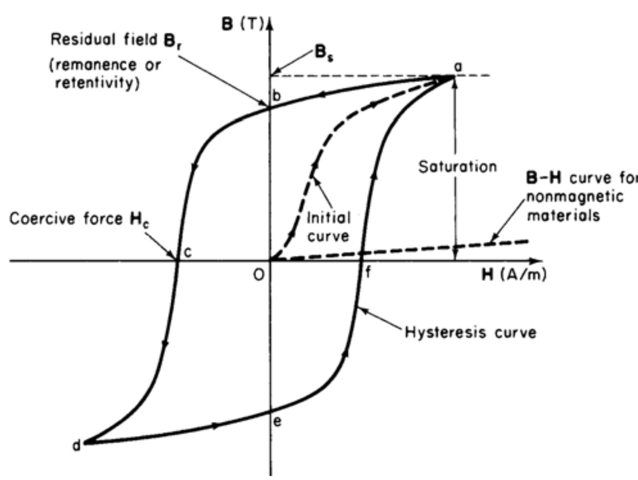
Hysteresis Loop



Hysteresis Loop



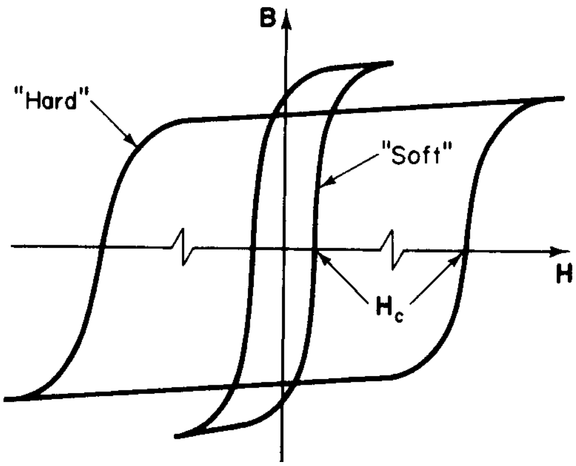
Hysteresis loops for different excitation levels



B_r – residual magnetism
H_c – coercivity force, external field required to demagnetize the material

Magnetic Materials

Classes of Magnetic Materials



Soft mag. materials

$$H_c \sim 0.1 \cdots 100 \, [A/m]$$

Hard mag. materials

$$H_c > 100 \, [A/m]$$

Permanent magnets (PM)

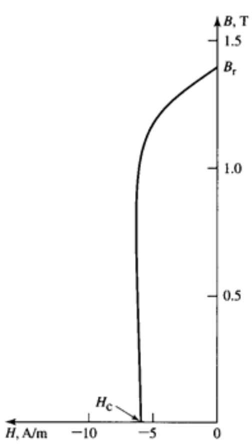
$$H_c \sim 10^4 \cdots 10^6 \, [A/m]$$

Types of PMs

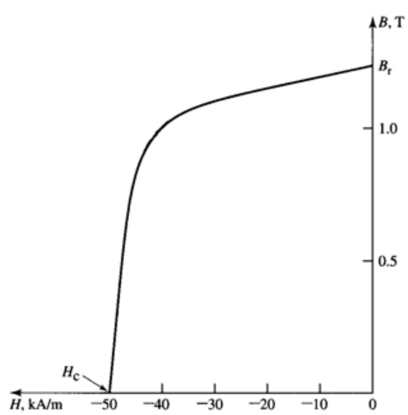
- Neodymium Iron Boron (NdFeB or NIB)
- Samarium Cobalt (SmCo)
- Aluminum Nickel Cobalt (Alnico)
- Ceramic or Ferrite, very popular
  Iron-oxide, barium, etc. compressed powder

Magnetic Materials

Second quadrant hysteresis curve for M-5 steel and Alnico 5



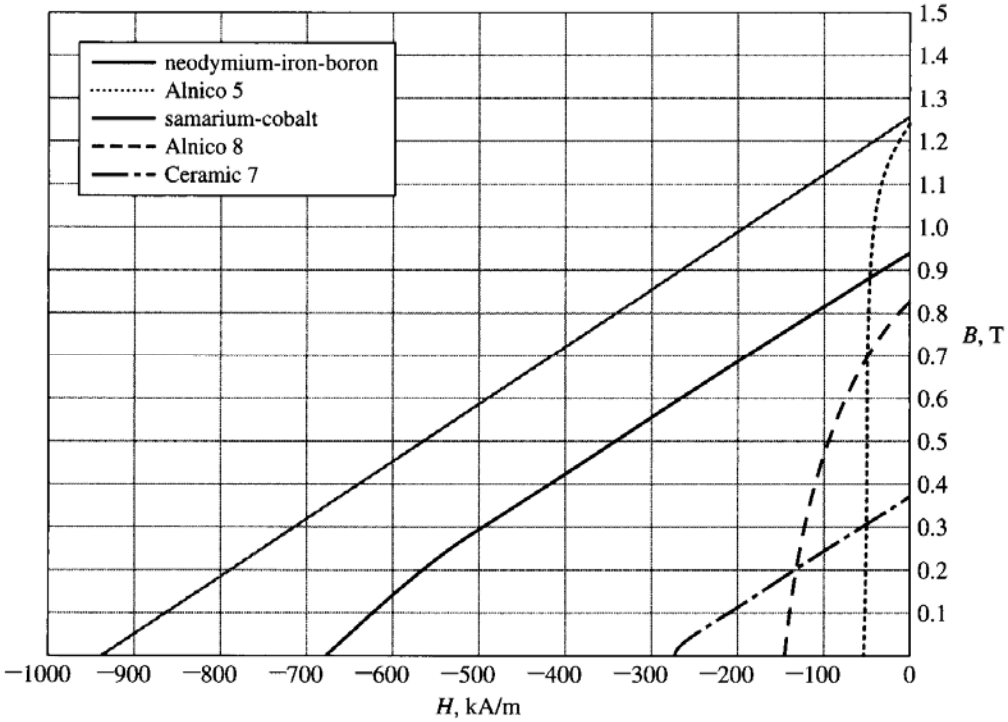
M-5 Steel



Alnico 5

Magnetic Materials

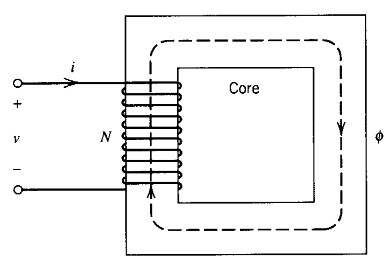
Second quadrant hysteresis curve for some common PM materials

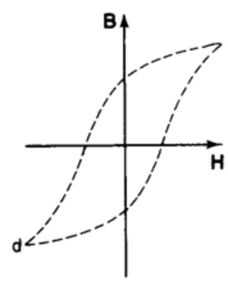


Core Losses

Hysteresis Losses

Consider AC excitation





$$\Delta W_{h,cycle} = \oint i d\lambda = \oint \left(\frac{H_c l_c}{N} \right) \left(N A_c dB_c \right) = l_c A_c \oint H_c dB_c$$

Power loss can be approximated as

$$P_h = K_h \cdot f \cdot \left(B_{c,\max} \right)^n \qquad n \sim 1.5 \cdots 2.5$$

Where the constants K_h and n determined experimentally

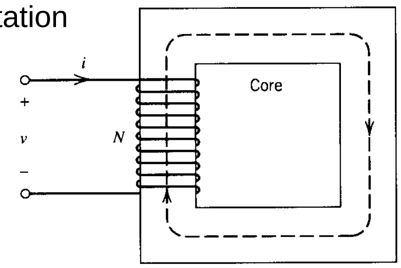
Core Losses

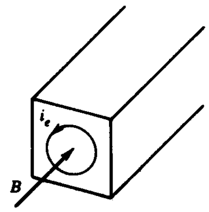
Faraday's law

$$\oint_C \overline{E} \cdot dl = -\frac{d}{dt}\int_S B \cdot da$$

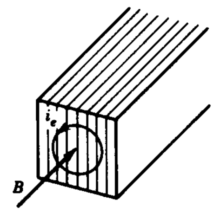
Eddy Current Losses

Consider AC excitation





Solid-iron core



Laminated core

Power loss can be approximated as

$$P_e = K_e \cdot f^2 \cdot \left(B_{c,\max}\right)^2$$

Where the constant K_e depends on lamination thickness and is determined experimentally

Equivalent circuit including core losses ?

Module 1, Part 3

Stationary Magnetically Coupled Systems (Chap. 1.5)

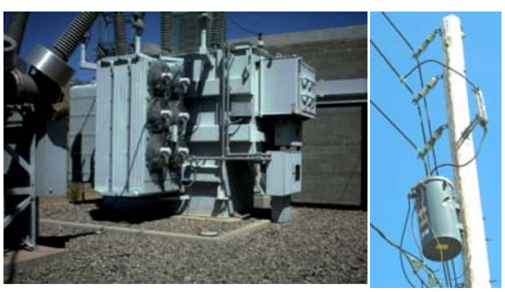
Most Important Topics and Concepts

- Types and construction of typical transformers
- Terminology, winding polarity
- Ideal transformer, turns ratio, referring quantities
- Non-ideal transformer, equivalent magnetic and electric circuits
- 3-phase transformers, winding connections

Transformer

- Stationary electromagnetic device
  - Is not an energy conversion device
  - It allows to scale voltage/current levels
  - Inverting polarity of signals
  - Galvanic decoupling
  - A transformer may have multiple windings for different voltage levels

Transformer Applications



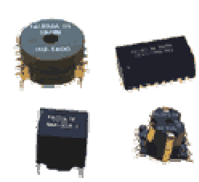
Power systems

Small power supplies

Toroids

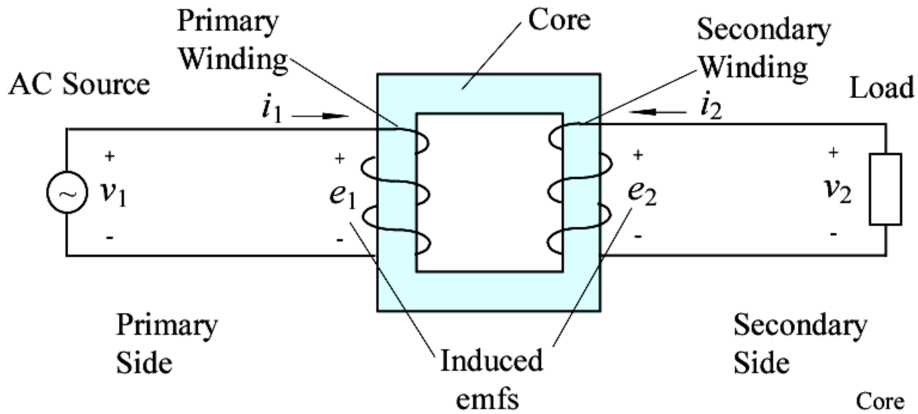
Variacs

instrumentation



communication

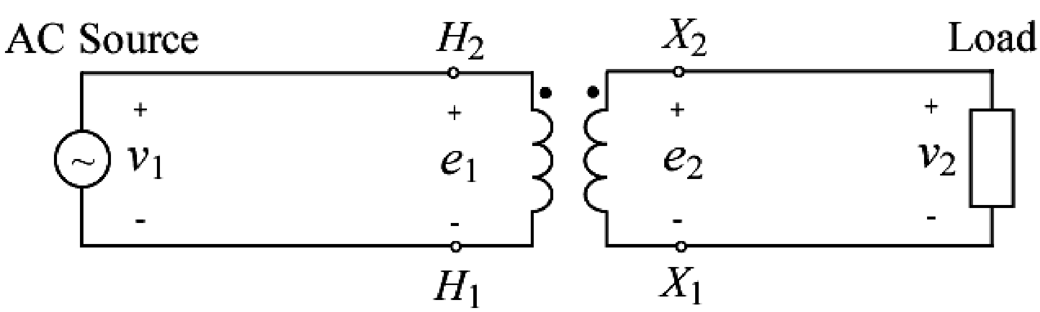
Basic Transformer Terminology



- Step-down
  - **Primary** (Source) – high voltage and number of turns (low current)
  - **Secondary** (Load) – low voltage and number of turns (high current)
- Step-up
  - **Primary** (Source) – low voltage and number of turns (high current)
  - **Secondary** (Load) –high voltage and number of turns (low current)

Transformer Polarity

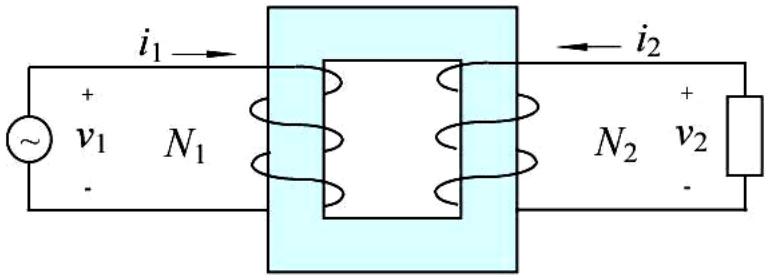
- Dot convention (most common) – same polarity terminals
- *H-X* convention (*H* – high voltage, *X* – low voltage)



Ideal Transformer

Assumptions

- Winding resistance is negligible $r_1, r_2 \to 0$
- Infinite core permeability $\mu_{core} \to \infty$
- All flux is confined inside the core
- No core losses



Properties of Ideal Transformer

Primary Side

$$v_1 = \sqrt{2} V_{1,rms} \cos(\omega t)$$

$$v_1 = e_1 = \frac{d\lambda_1}{dt} = N_1 \frac{d\Phi}{dt}$$

$$\Phi = \frac{\sqrt{2} V_{1,rms}}{N_1 \omega} \sin(\omega t) = \Phi_{peak} \sin(\omega t)$$

$$\Phi_{peak} = \frac{\sqrt{2} V_{1,rms}}{N_1 \omega} = \frac{V_{1,rms}}{\sqrt{2} N_1 \pi \cdot f}$$

Secondary Side

$$v_2 = e_2 = \frac{d\lambda_2}{dt} = N_2 \frac{d\Phi}{dt}$$

$$\frac{d\Phi}{dt} = \frac{v_1}{N_1} = \frac{v_2}{N_2}$$

$$\frac{v_1}{v_2} = \frac{N_1}{N_2} = a$$ Turns ratio

$$B_{peak} = \frac{\Phi_{peak}}{A_c} \approx 1 \ldots 2\,\mathrm{T}$$ Maximum Flux Density

Properties of Ideal Transformer

MMF $F = N_1 i_1 + N_2 i_2 = \Phi \Re_{total} = 0$

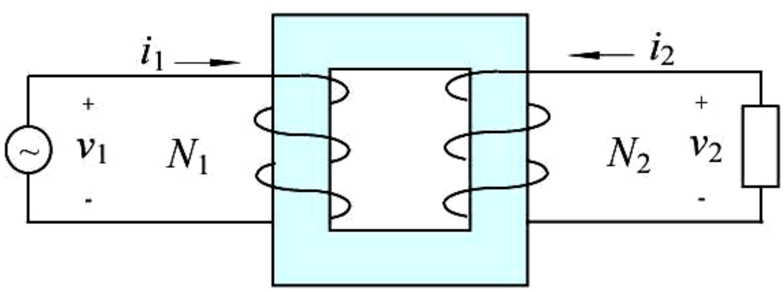
=> $N_1 i_1 = -N_2 i_2$ => $\frac{i_1}{i_2} = \frac{-N_2}{N_1} = \frac{-1}{a}$

Power $P(t) = i_1 v_1 = -i_2 v_2$ $P_1 = -P_2$ $P_{in} = P_{out}$ No losses

Equivalent Electric Circuit

Equivalent Magnetic Circuit

Practical (Non-Ideal) Transformer

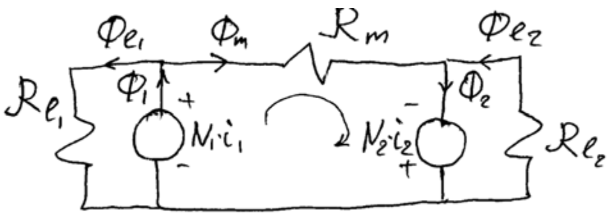


Equivalent Magnetic Circuit ?

Φ_m - mutual or magnetizing flux (goes inside the core and links both windings)

Φ_{l1}, Φ_{l2} - flux leakage (goes in the air and links only one winding)

Transformer



Consider Fluxes

$$\Phi_1 = \Phi_{l1} + \Phi_m = \frac{N_1 i_1}{\Re_{l1}} + \frac{N_1 i_1 + N_2 i_2}{\Re_m}$$

$$\Phi_2 = \Phi_{l2} + \Phi_m = \frac{N_2 i_2}{\Re_{l2}} + \frac{N_1 i_1 + N_2 i_2}{\Re_m}$$

Voltage Equations

$$v_1 = r_1 i_1 + \frac{d\lambda_1}{dt}$$

$$v_2 = r_2 i_2 + \frac{d\lambda_2}{dt}$$

Transformer

Recall inductance $L = \frac{N^2}{\Re}$

Consider Flux Linkages

$$\lambda_1 = N_1 \Phi_1 = \frac{N_1^2}{\Re_{l1}} i_1 + \frac{N_1^2}{\Re_m} i_1 + \frac{N_1 N_2}{\Re_m} i_2 = L_{l1} i_1 + L_{m1} i_1 + L_{12} i_2$$

$$\lambda_2 = N_2 \Phi_2 = \frac{N_2^2}{\Re_{l2}} i_2 + \frac{N_2^2}{\Re_m} i_2 + \frac{N_2 N_1}{\Re_m} i_1 = L_{l2} i_2 + L_{m2} i_2 + L_{21} i_1$$

Define self-inductances (always positive)

$$L_{11} = L_{l1} + L_{m1}$$

$$L_{22} = L_{l2} + L_{m2}$$

Define mutual inductances

$$L_{12} = \frac{N_1 N_2}{\Re_m} \qquad L_{21} = \frac{N_2 N_1}{\Re_m}$$

$$L_{12} = \frac{N_2}{N_1} L_{m1} = \frac{N_1}{N_2} L_{m2}$$

(could be positive or negative)

Flux Linkages

$$\lambda_1 = L_{11} i_1 + L_{12} i_2$$

$$\lambda_2 = L_{21} i_1 + L_{22} i_2$$

Referring Parameters

Re-write Flux Linkages

$$\lambda_1 = L_{l1} i_1 + L_{m1}\left(i_1 + \frac{N_2}{N_1} i_2 \right)$$

$$\lambda_2 = L_{l2} i_2 + L_{m2}\left(\frac{N_1}{N_2} i_1 + i_2 \right)$$

Substitute (referred) variables

$$i_2' = \frac{N_2}{N_1} i_2$$

$$p(t) = v_2 i_2 = v_2' i_2'$$ Power the same

$$v_2' = \frac{N_1}{N_2} v_2$$ and $$\lambda_2' = \frac{N_1}{N_2} \lambda_2$$

Re-write Flux Linkages

$$\lambda_1 = L_{l1} i_1 + L_{m1}(i_1 + i_2')$$

$$\lambda_2' = L_{l2}' i_2' + L_{m1}(i_1 + i_2')$$

where

$$L_{l2}' = \left(\frac{N_1}{N_2} \right)^2 L_{l2}$$

$$\lambda_1 = L_{11} i_1 + L_{m1} i_2'$$

$$\lambda_2' = L_{m1} i_1 + L_{22}' i_2'$$

where

$$L_{22}' = \left(\frac{N_1}{N_2} \right)^2 L_{22}$$

T – Equivalent Circuit

Voltage Equation

$$\begin{bmatrix} v_1 \\ v_2' \end{bmatrix} = \begin{bmatrix} r_1 & 0 \\ 0 & r_2' \end{bmatrix} \begin{bmatrix} i_1 \\ i_2' \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \lambda_1 \\ \lambda_2' \end{bmatrix}$$

where $r_2' = \left(\frac{N_1}{N_2} \right)^2 r_2$

Flux Linkages

$$\lambda_1 = L_{l1} i_1 + L_{m1}(i_1 + i_2')$$

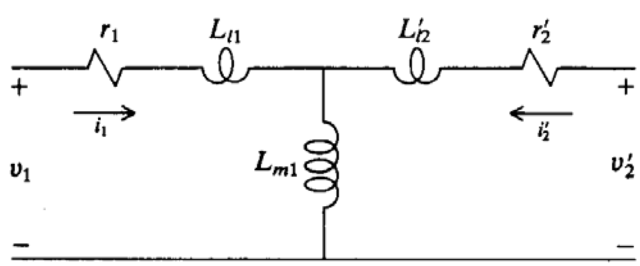
$$\lambda_2' = L_{l2}' i_2' + L_{m1}(i_1 + i_2')$$

Define Reactances

$$X_{l1} = L_{l1}\omega_e$$

$$X_{l2}' = L_{l2}'\omega_e$$

$$X_{m1} = L_{m1}\omega_e$$



Transformer Example

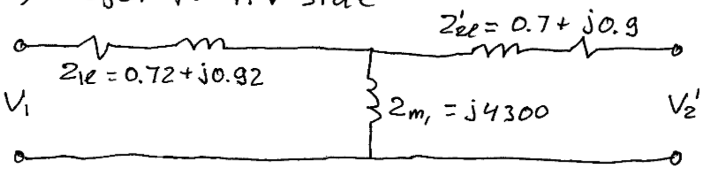
Consider a 50kVA, 2400/240V transformer with leakage impedances $Z_1 = 0.72 + j0.92$ and $Z_2 = 0.007 + j0.009$, and the magnetizing shunt impedance $Z_{m2} = j43$.

a) Draw eq. circuit referred to high voltage side, label impedances

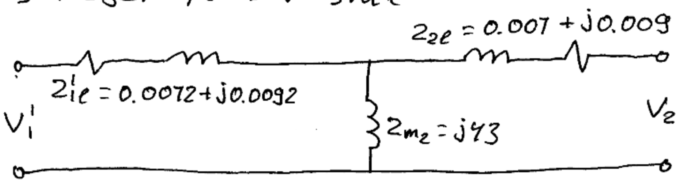
b) Draw eq. circuit referred to low voltage side, label impedances

Turns-ration $a = \frac{2400}{240} = 10$

a) Refer to HV side

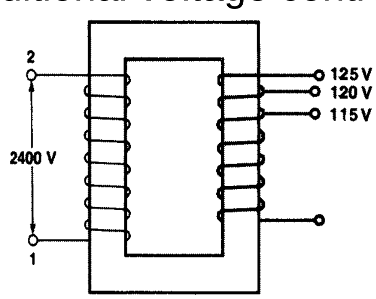


b) Refer to LV side

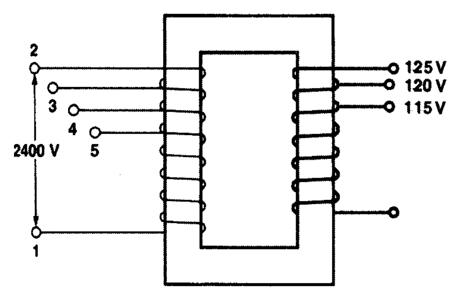


Tap-Changing Multi-winding Transformers

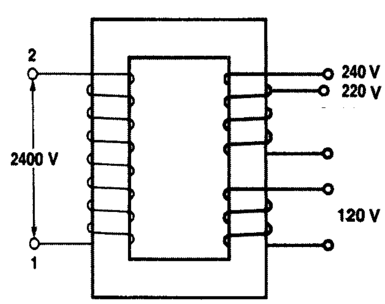
Additional voltage control / regulation



Changing N_2



Changing N_1 and N_2

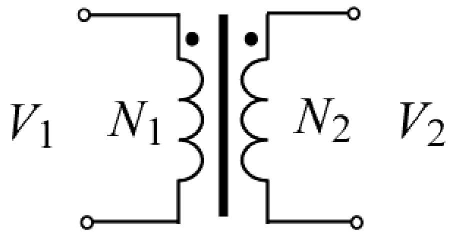


High voltage loads

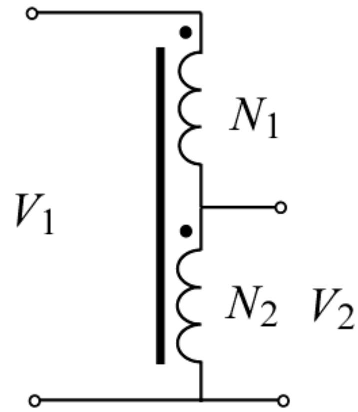
Regular voltage loads

Two loads are galvanically decoupled !

Autotransformers



Regular transformer

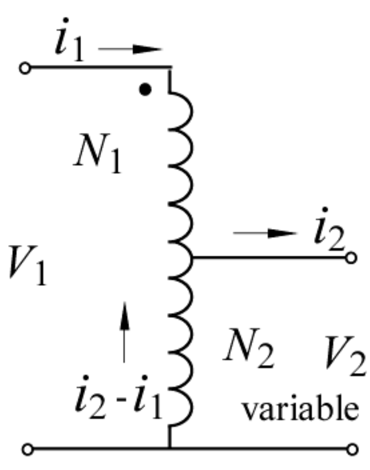


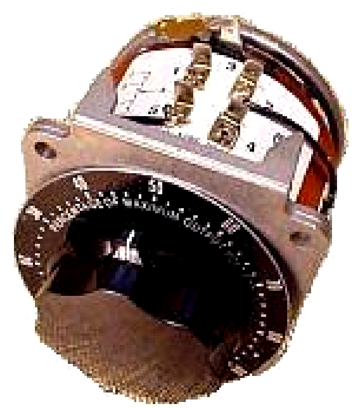
Autotransformer

- No electric isolation between primary & secondary
- Safety is a concern
  120/110 or 240/120 is OK
  12kV/240 or 2.4kV/120 is not safe !
- Smaller than regular transformers
- Economical (material, losses, etc.)

Variable Autotransformer - Variac

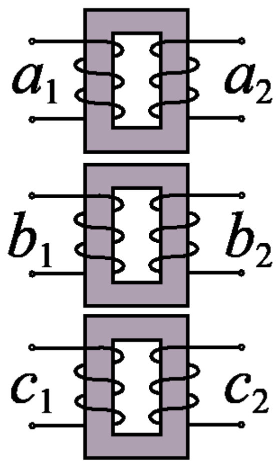
Often used in Labs as a source of variable AC voltage



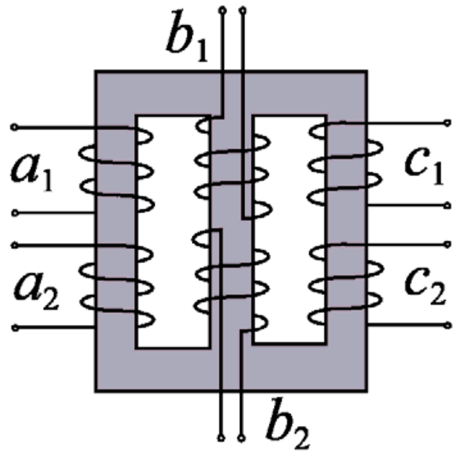


Three Phase Transformers

Construction



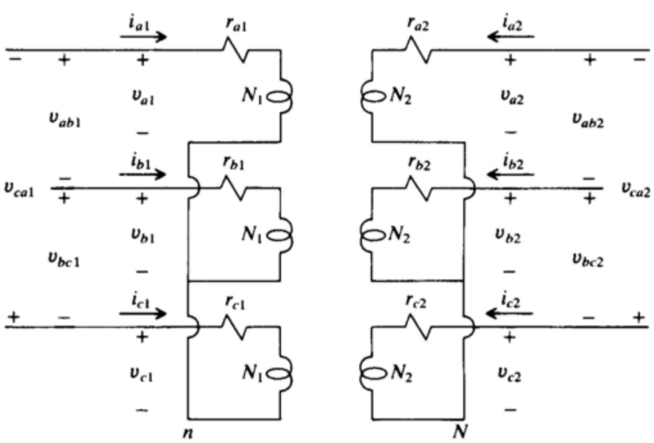
Use 3 transformers
-no magnetic coupling
 between phases

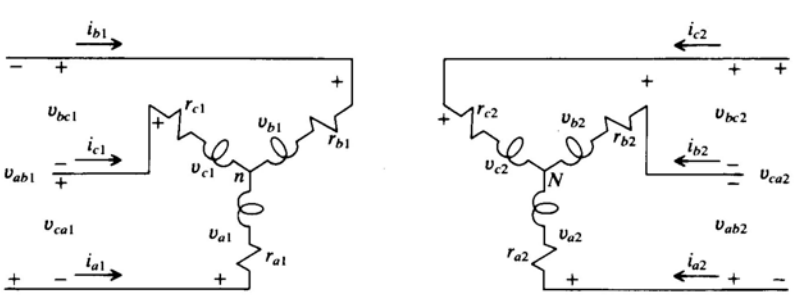


Use 3-phase core
- efficient use of materials

Wye-Wye Connections

ELEC 343, S-19, M-1





Phase voltages

Line-to-neutral voltages

$$v_{a1}, v_{b1}, v_{c1}$$

Line-to-line voltages

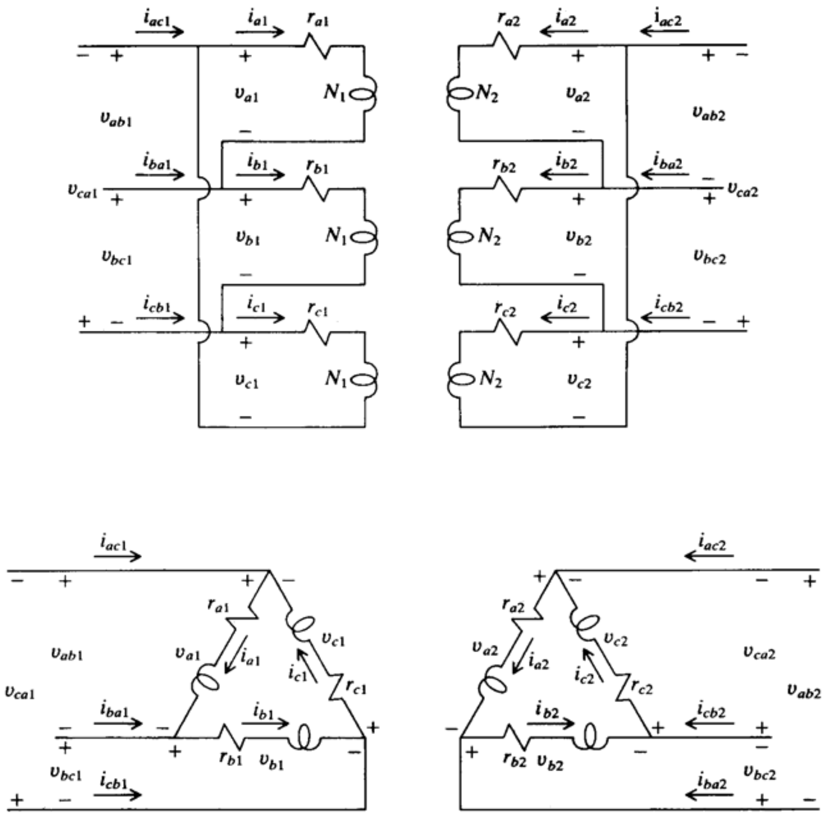
$$v_{ab1} = v_{a1} - v_{b1}$$

$$v_{bc1} = v_{b1} - v_{c1}$$

$$v_{ca1} = v_{c1} - v_{a1}$$

Line and phase
currents are the same

Delta-Delta Connections



Phase currents

$$i_{a1}, i_{b1}, i_{c1}$$

Line currents

$$i_{ab1}, i_{bc1}, i_{ca1}$$

Line currents

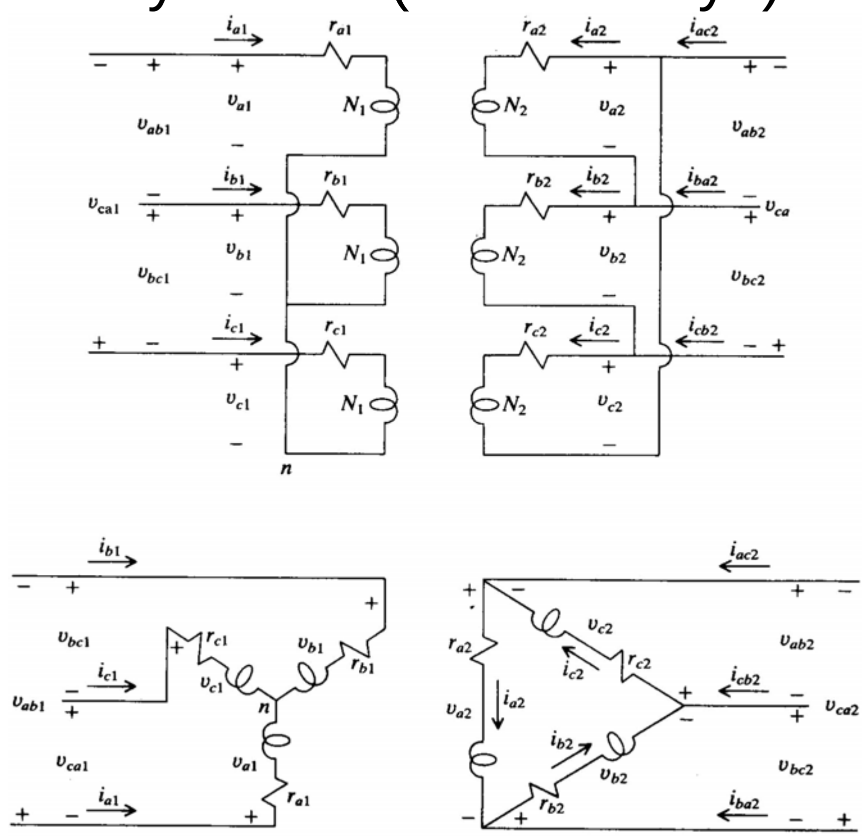
$$i_{ac1} = i_{a1} - i_{c1}$$

$$i_{ba1} = i_{b1} - i_{a1}$$

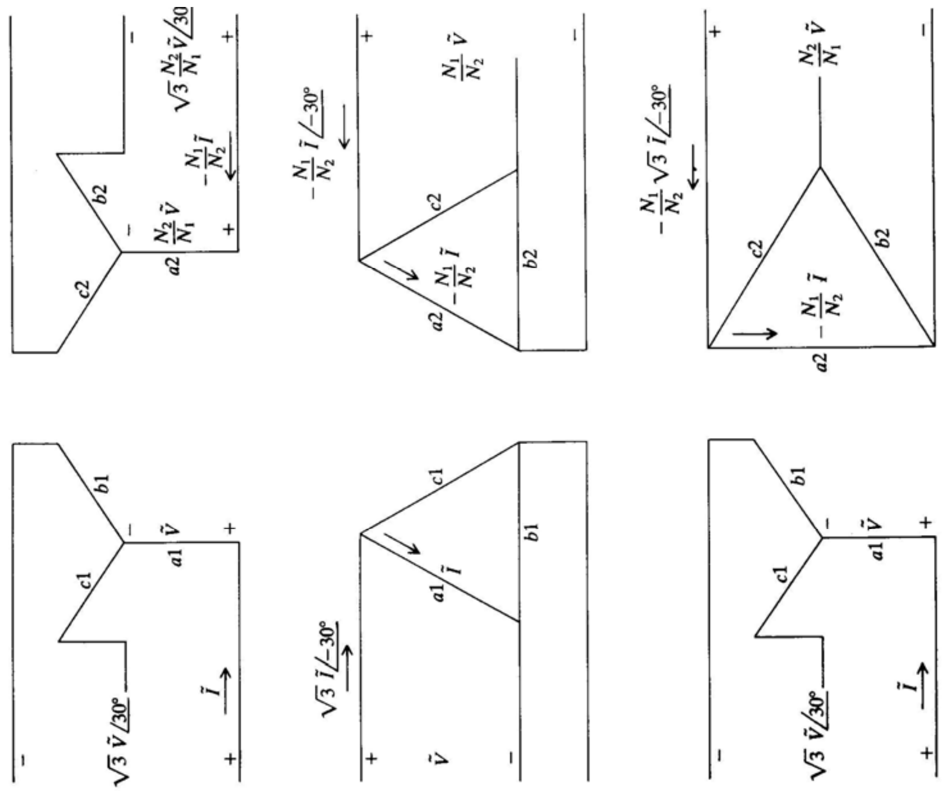
$$i_{cb1} = i_{c1} - i_{b1}$$

Line and phase voltages are the same

Wye-Delta (or Delta-Wye) Connections



For Ideal Transformer



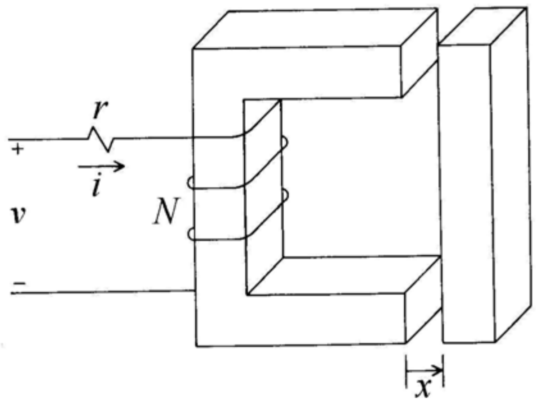
Module 1, Part 4

Basic Magnetic Systems with Motion (Chap. 1.7)

Most Important Topics and Concepts

- Basic linear devices
- Concept of position-dependent reluctances & inductances in linear devices
- Mechanical and electrical inputs/outputs
- Basic rotating devices, windings in relative motion
- Magnetic axes
- Concept of position-dependent reluctances & inductances in rotating devices

Elementary Electromagnet



Voltage equation (Faraday's law + KVL) $v = ri + \dfrac{d\lambda}{dt}$

Flux linkage

$$\lambda = N\Phi = N\left(\Phi_m + \Phi_l\right)$$

$\Phi_l = Ni/\Re_l$ - Flux leakage

$\Phi_m = Ni/\Re_m$ - Magnetizing flux

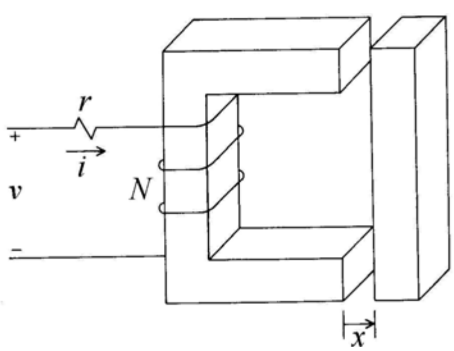
Flux linkage & inductances

$$\lambda = \left(\frac{N^2}{\Re_l} + \frac{N^2}{\Re_m}\right) i = \left(L_l + L_m\right) i$$

L_l - Leakage inductance (assume constant)

L_m - Magnetizing inductance (depends of position x)

Elementary Electromagnet



Consider Magnetizing Path

$$\Re_m = \Re_c + 2\Re_g$$

$\Re_c = \dfrac{l_c}{\mu_r \mu_0 A_c}$ - Reluctance of the stationary + movable core

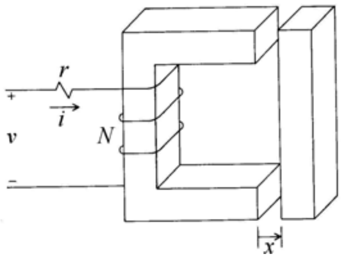
$\Re_g(x) = \dfrac{x}{\mu_0 A_g}$ - Reluctance of the air-gap

Assume $A_c = A_g = A$ we get $\Re_m(x) = \dfrac{1}{\mu_0 A}\left(\dfrac{l_c}{\mu_c} + 2x\right)$

Magnetizing inductance

$$L_m = \frac{N^2}{\Re_m} = N^2 \mu_0 A \frac{1}{\left(l_c/\mu_c + 2x\right)} = \frac{k_1}{k_2 + x}$$

Elementary Electromagnet



Magnetizing Inductance

$$L_m = N^2 \mu_0 A \frac{1}{(l_c/\mu_c + 2x)} = \frac{k_1}{k_2 + x}$$

where $k_1 = \dfrac{N^2 \mu_0 A}{2}$ and $k_2 = \dfrac{l_c}{2\mu_c}$

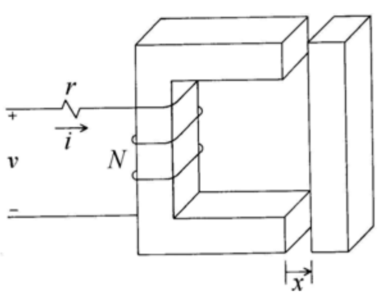
Voltage Equation

$$v = ri + \frac{d\lambda}{dt}$$ where $\lambda(i,x) = L(x)i = [L_l + L_m(x)]i$

$$\lambda' = L'i + Li' = \frac{dL}{dt} i + L\frac{di}{dt} = \frac{\partial L}{\partial x}\frac{dx}{dt} i + L\frac{di}{dt}$$

And finally we get $v = ri + \left[\dfrac{\partial L_m(x)}{\partial x}\dfrac{dx}{dt}\right] i + [L_l + L_m(x)]\dfrac{di}{dt}$

Elementary Electromagnet



The elementary electromagnet is very similar to a plunger solenoid (Lab-1)

How do we solve Voltage Equation ?

$$v = ri + \left[\frac{dL_m(x)}{dx}\frac{dx}{dt}\right] i + [L_l + L_m(x)]\frac{di}{dt}$$

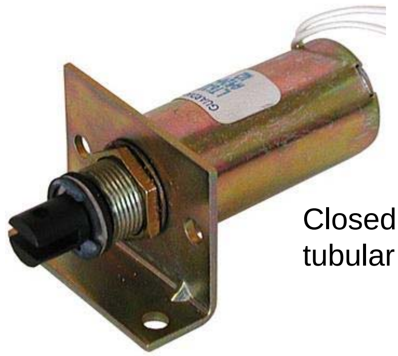
What is $\dfrac{dx}{dt}$

$$\frac{di}{dt} = [L_l + L_m(x)]^{-1}\left\{v - \left[r + \frac{\partial L_m(x)}{\partial x}\frac{dx}{dt}\right] i\right\}$$

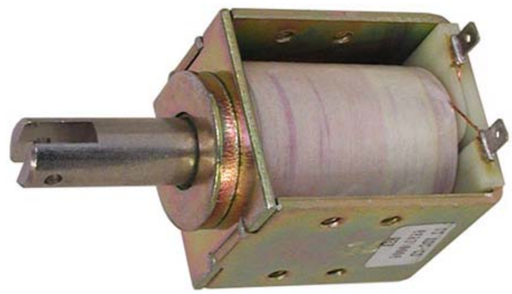
Need dynamic equation of mechanical system !

Other Reluctance Devices

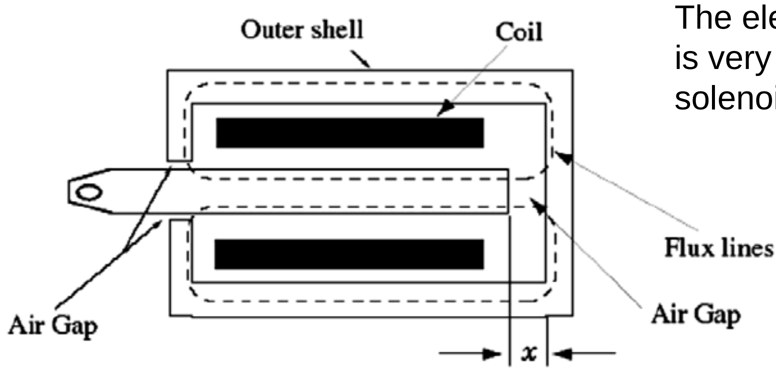
Plunger solenoid (Lab-1)



Closed-frame tubular solenoid



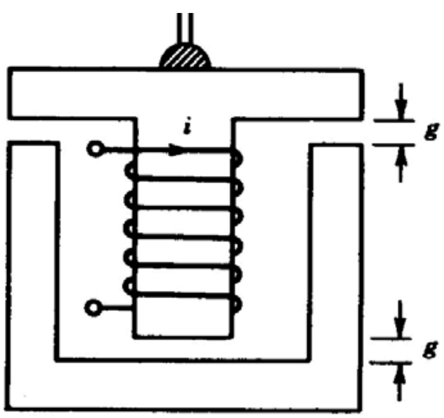
Open-frame solenoid

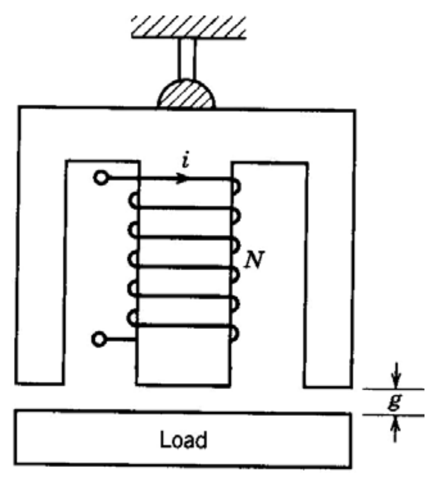


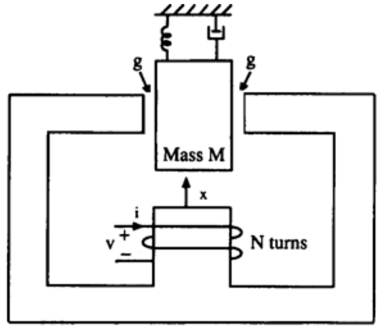
The elementary electromagnet is very similar to a plunger solenoid (Lab-1)

Other Reluctance Devices

Plunger solenoid

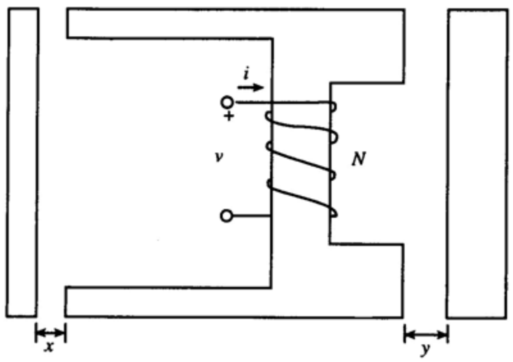


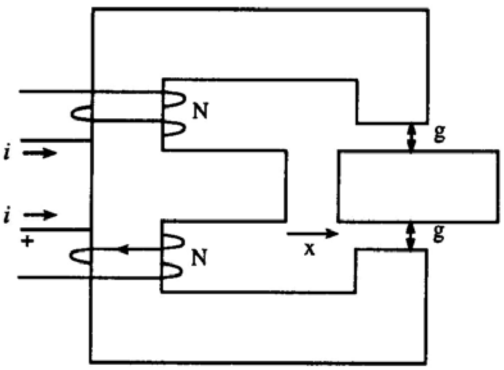




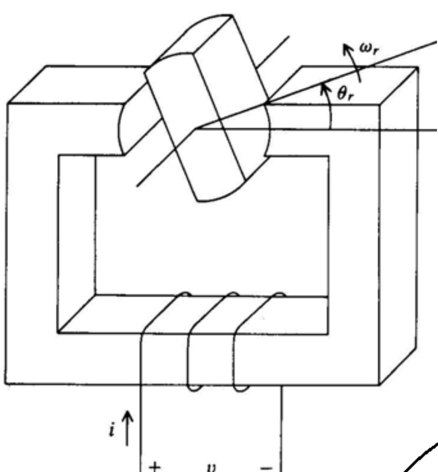
Other Reluctance Devices

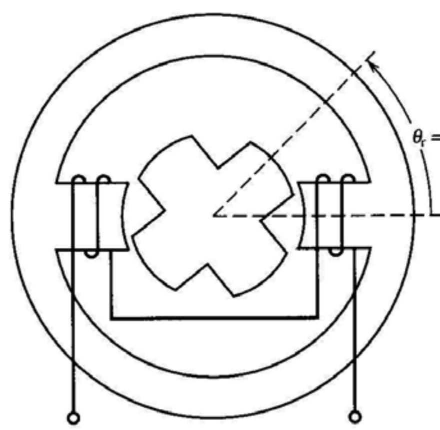
Multi-input/output

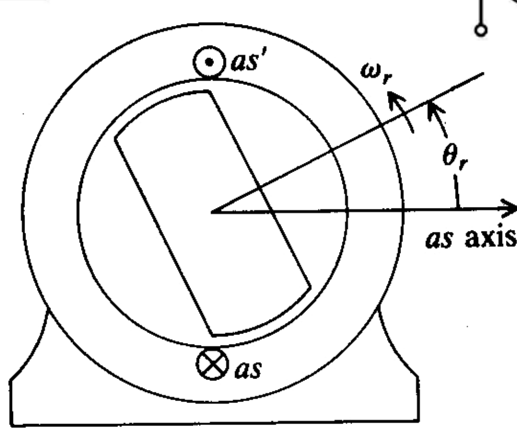




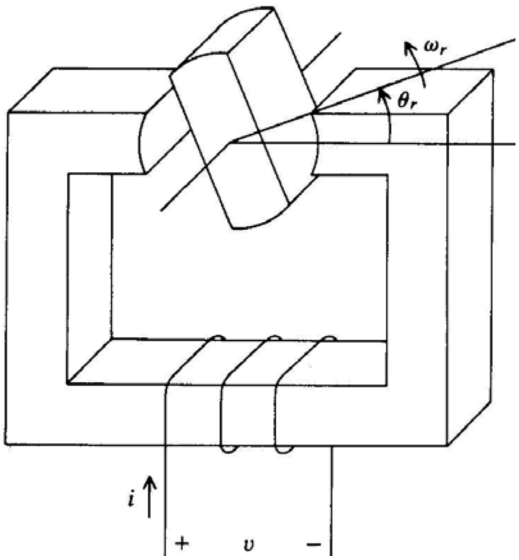
Rotating Reluctance Devices







Rotating Reluctance Devices



Flux linkage & inductances

$$\lambda = \left(\frac{N^2}{\Re_l} + \frac{N^2}{\Re_m} \right) i = \left(L_l + L_m \right) i$$

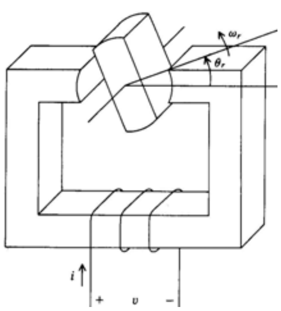
Magnetizing inductance

$$L_m = L_m(\theta_r) = \frac{N^2}{\Re_m(\theta_r)}$$

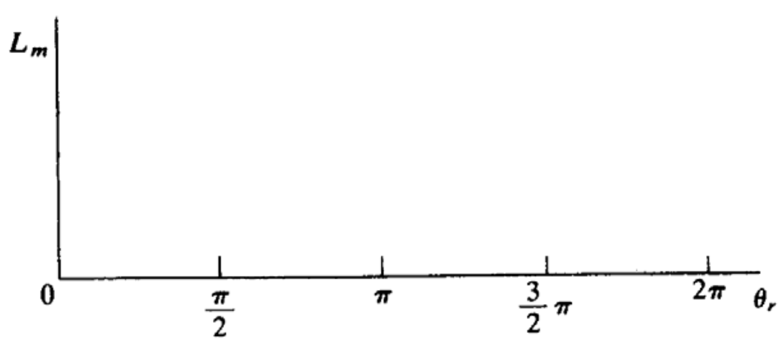
$\Re_m(0)$ - Maximum reluctance

$\Re_m(\pi/2)$ - Minimum reluctance

Rotating Reluctance Devices



Approximation of Magnetizing Inductance



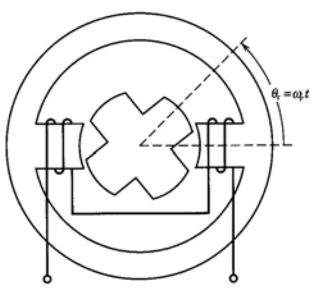
$$L_m(\theta_r) = L_A - L_B \cos(2\theta_r)$$

Resulted Self-Inductance

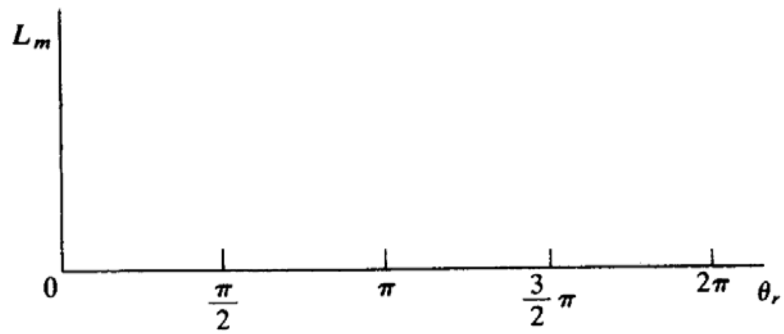
$$L_m(0) = L_A - L_B$$

$$L_m(\pi/2) = L_A + L_B$$

Rotating Reluctance Devices



Approximation of Magnetizing Inductance



$L_m(\theta_r) =$

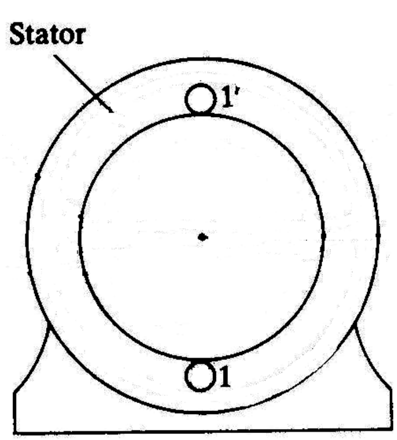
Resulted Self-Inductance

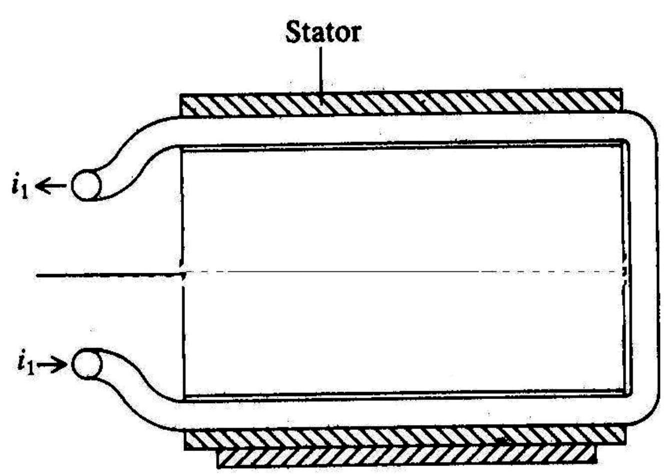
$L_m(0) =$

L_m

Windings in Relative Motion

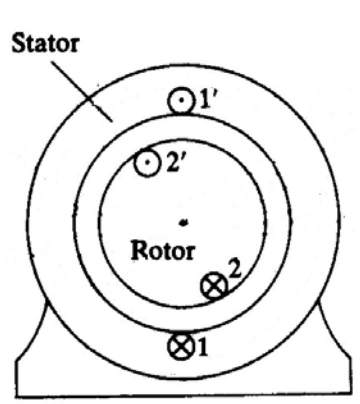
Stator Winding Magnetic Axis

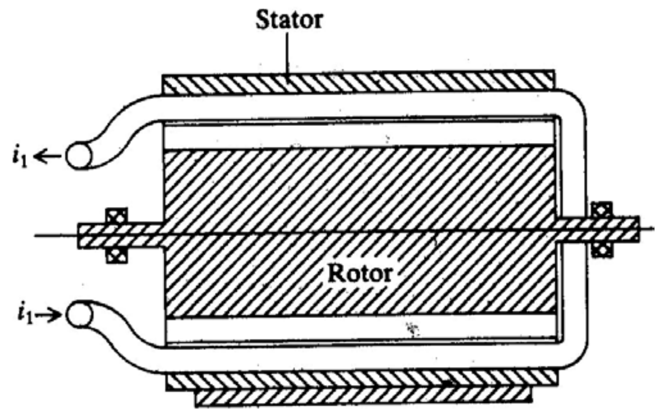




Windings in Relative Motion

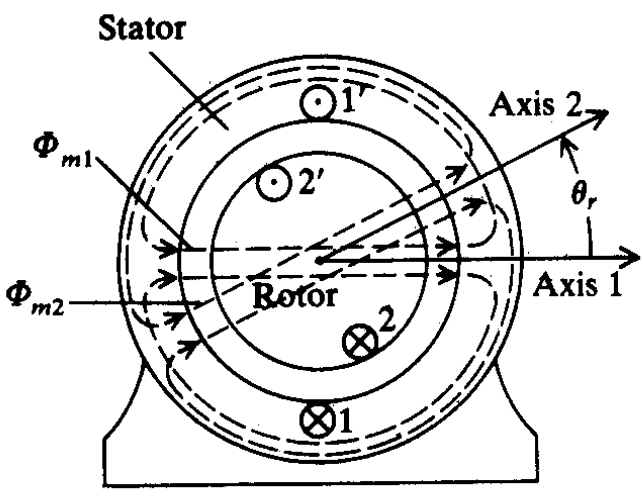
Stator & Rotor Winding Magnetic Axes





Windings in Relative Motion

Stator & Rotor Winding Magnetic Axes



Voltage Equations

- similar to transformer

$$v_1 = r_1 i_1 + \frac{d\lambda_1}{dt}$$

$$v_2 = r_2 i_2 + \frac{d\lambda_2}{dt}$$

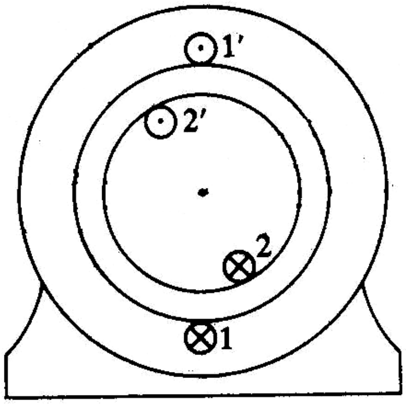
Flux linkages

$$\lambda_1 = L_{11} i_1 + L_{12} i_2$$

$$\lambda_2 = L_{22} i_2 + L_{21} i_1$$

Windings in Relative Motion

Stator & Rotor Inductances



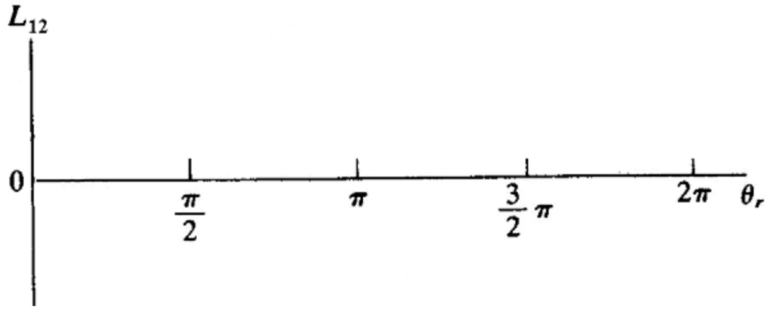
Self-inductances

$$L_{11} = L_{l1} + L_{m1} \text{ and } L_{22} = L_{l2} + L_{m2}$$

Mutual-inductances

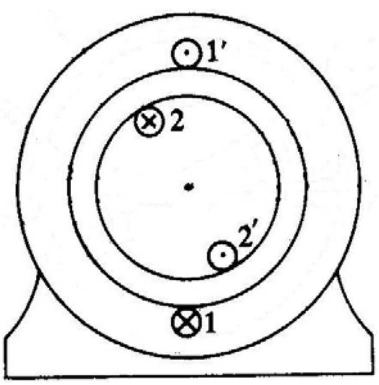
$$L_{12} = L_{21} = L_{12}(\theta_r)$$

$$L_{12}(0) \qquad\qquad L_{12}(\pi/2)$$



Example SP1.7-3

Rotor current changed direction



Self-inductances

Mutual-inductances

$$L_{12} = L_{21} = L_{12}(\theta_r)$$

$$L_{12}(0) \qquad\qquad L_{12}(\pi/2)$$