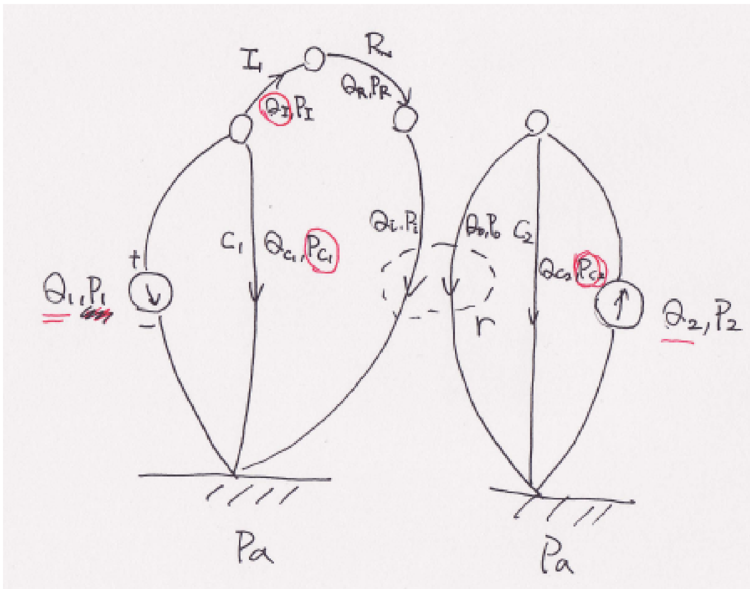
University of British Columbia
Department of Mechanical Engineering

MECH366 Modeling of Mechatronic Systems
Homework 4

Due: October 28 (Monday), 2019, 3pm

1. **Solution:** Linear graph is shown below, where $r := A_1/A_2$.



State variables are $x := [P_{C_1}, P_{C_2}, Q_I]^T$. Then, the state equation can be derived as follows.

$$
\begin{aligned}
\dot{P}_{C_1} &= \frac{1}{C_1}Q_{C_1} = \frac{1}{C_1}(Q_1 - Q_I) \\
\dot{P}_{C_2} &= \frac{1}{C_2}Q_{C_2} = \frac{1}{C_1}(Q_2 - Q_o) = \frac{1}{C_1}(Q_2 + \frac{1}{r}Q_i) \\
&= \frac{1}{C_1}(Q_2 + \frac{1}{r}Q_I) \\
\dot{Q}_I &= \frac{1}{I}P_I = \frac{1}{I}(P_{C_1} - P_R - P_i) = \frac{1}{I}(P_{C_1} - RQ_R - \frac{1}{r}P_o) \\
&= \frac{1}{I}(P_{C_1} - RQ_I - \frac{1}{r}P_{C_2})
\end{aligned}
$$

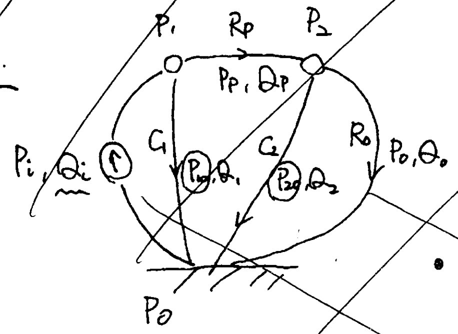
We can rewrite this in a matrix form as

$$
\begin{bmatrix} \dot{P}_{C_1} \\ \dot{P}_{C_2} \\ \dot{Q}_I \end{bmatrix} = \begin{bmatrix} 0 & 0 & -\frac{1}{C_1} \\ 0 & 0 & \frac{1}{rC_1} \\ \frac{1}{I} & -\frac{1}{rI} & -\frac{R}{I} \end{bmatrix} \begin{bmatrix} P_{C_1} \\ P_{C_2} \\ Q_I \end{bmatrix} + \begin{bmatrix} \frac{1}{C_1} & 0 \\ 0 & \frac{1}{C_2} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix}
$$

$$
\begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{\rho g} & 0 & 0 \\ 0 & \frac{1}{\rho g} & 0 \end{bmatrix} \begin{bmatrix} P_{C_1} \\ P_{C_2} \\ Q_I \end{bmatrix}
$$

HW4 Solutions

Q1.



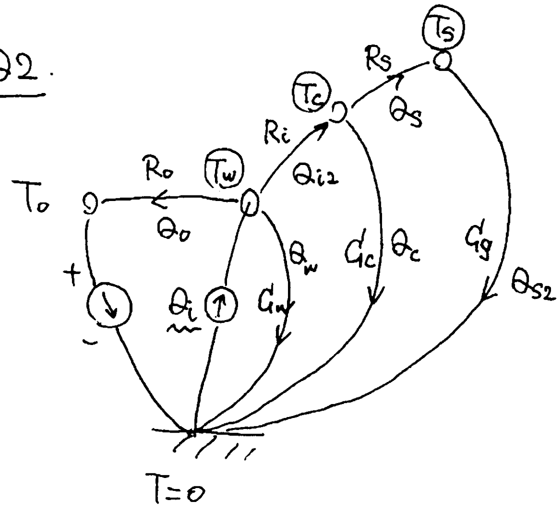
$$\dot{P}_{10} = \frac{1}{C_1} Q_1 = \frac{1}{C_1}\left(Q_i - \frac{1}{R_P} P_P\right) = \frac{1}{C_1}\left(Q_i - \frac{1}{R_P}(P_{10} - P_{20})\right)$$

$$\dot{P}_{20} = \frac{1}{C_2} Q_2 = \frac{1}{C_2}(Q_P - Q_o) = \frac{1}{C_2}\left(\frac{1}{R_P}(P_{10} - P_{20}) - \frac{1}{R_o} \underbrace{P_o}_{= P_{20}}\right)$$

- State-space model

$$\therefore \left\{ \begin{aligned} \begin{bmatrix} \dot{P}_{10} \\ \dot{P}_{20} \end{bmatrix} &= \begin{bmatrix} -\frac{1}{C_1 R_P} & \frac{1}{C_1 R_P} \\ \frac{1}{C_2 R_P} & -\frac{1}{C_2}\left(\frac{1}{R_P} + \frac{1}{R_O}\right) \end{bmatrix} \begin{bmatrix} P_{10} \\ P_{20} \end{bmatrix} + \begin{bmatrix} \frac{1}{C_1} \\ 0 \end{bmatrix} Q_i \\ \begin{bmatrix} H_1 \\ H_2 \\ Q_o \end{bmatrix} &= \begin{bmatrix} \frac{1}{\rho g} & 0 \\ 0 & \frac{1}{\rho g} \\ 0 & \frac{1}{R_o} \end{bmatrix} \begin{bmatrix} P_{10} \\ P_{20} \end{bmatrix} \end{aligned} \right.$$

Q2.



$$\dot{T}_s = \frac{1}{C_s} Q_{s2} = \frac{1}{C_s} Q_s = \frac{1}{C_s}\frac{1}{R_s}(T_c - T_s)$$

$$\dot{T}_c = \frac{1}{C_c} \underbrace{Q_c}_{= Q_{i2} - Q_s} = \frac{1}{C_c}\left\{\frac{1}{R_i}(T_w - T_c) - \frac{1}{R_s}(T_c - T_s)\right\}$$

$$\dot{T}_w = \frac{1}{C_w} \underbrace{Q_w}_{= Q_i - Q_{i2} - Q_o} = \frac{1}{C_w}\left\{Q_i - \frac{1}{R_i}(T_w - T_c) - \frac{1}{R_o}(T_w - T_o)\right\}$$

$$R_s = \frac{1}{A_s h_s}, \quad R_i = \frac{1}{A_i h_i}, \quad R_o = \frac{1}{A_o h_o}$$

$$C_s = m_s c_s, \quad C_c = m_c c_c, \quad C_w = m_w c_w$$

- state-space model

$$\left\{ \begin{aligned} \begin{bmatrix} \dot{T}_s \\ \dot{T}_c \\ \dot{T}_w \end{bmatrix} &= \begin{bmatrix} -\frac{1}{C_s R_s} & \frac{1}{C_s R_s} & 0 \\ \frac{1}{C_c R_s} & -\frac{1}{C_c}\left(\frac{1}{R_i} + \frac{1}{R_s}\right) & \frac{1}{C_c R_i} \\ 0 & \frac{1}{C_w R_i} & -\frac{1}{C_w}\left(\frac{1}{R_i} + \frac{1}{R_o}\right) \end{bmatrix} \begin{bmatrix} T_s \\ T_c \\ T_w \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{C_w} & \frac{1}{C_w R_o} \end{bmatrix} \begin{bmatrix} Q_i \\ T_o \end{bmatrix} \\ T_s &= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} T_s \\ T_c \\ T_w \end{bmatrix} \end{aligned} \right.$$