

MECH366 Modeling of Mechatronic Systems

Exercises for ODE solutions

Taken from Appendix B of the Phillips and Parr's textbook.

1. (B.3, B.8) Find the inverse Laplace transform of the following functions. Obtain, if there exists, the final value of each function $f(t)$ by using the final value theorem.

(a) $F(s) = \frac{5}{s(s+1)(s+2)}$

(b) $F(s) = \frac{1}{s^2(s+1)}$

(c) $F(s) = \frac{2s+1}{s^2+2s+10}$

(d) $F(s) = \frac{s-30}{s(s^2+4s+29)}$

2. (B.4) Obtain the inverse Laplace transform of $F(s) = \frac{s+5}{s(s^2+4s+13)}$.

3. (B.9) Solve the differential equation

$$\frac{d^2x(t)}{dt^2} + 5\frac{dx(t)}{dt} + 4x(t) = 10u(t)$$

with the following initial conditions:

(a) $x(0) = x'(0) = 0$.

(b) $x(0) = x'(0) = 1$.

4. (B.10) Solve the differential equation

$$\frac{d^2x(t)}{dt^2} + 2\frac{dx(t)}{dt} + x(t) = 5u(t)$$

with the following initial conditions:

(a) $x(0) = x'(0) = 0$.

(b) $x(0) = 0, x'(0) = 2$.

5. (B.11) For each of the following $G(s)$, transform the relation $C(s) = G(s)R(s)$ into the differential equation.

(a) $G(s) = \frac{60}{s^2+10s+60}$

(b) $G(s) = \frac{3s+20}{s^3+4s^2+8s+20}$

(c) $G(s) = \frac{s+1}{s^2}$

(d) $G(s) = \frac{7e^{-0.2s}}{s^2+5s+32}$