Selected Solutions from Tutorials Part 2

Adapted from **Shigley, Example 7-6** (8^{th} and 9^{th} Eds.) A steel shaft has a diameter of 2 in. The shaft rotates at 600 rev/min and transmits 5 hp through a gear. (to be done by students)

a. What size cup-point setscrew would be required to hold the gear. Note: a safety factor of 4 is a typical minimum for a setscrew subjected to dynamic loading.

b. What size dowel pin would be required for the same gear, if the pin passes through both sides of the shaft? (select a pin from the class notes) Assume a safety factor of 3 is required.

c. What size taper pin would be required for the same gear, assuming the pin is loaded in single shear? (select a pin from the class notes) Assume a safety factor of 3 is required.

d. Select an appropriate size gib-head key to hold the gear if the power transmitted is increased to 40 hp (assume the key is made of steel with yield strength of 70 kpsi). A safety factor of 2.5 is desired. Use the distortion-energy theory to select a key (note that under distortion-energy, the shear yield strength is $S_{sy} = 0.577S_y$).

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1. Adapted from Shigley, Example 7-6 (8th and 9th Eds.)

A steel shaft has a diameter of 2 in. The shaft rotates at 600 rev/min and transmits 5 hp through a gear.

Part 1 – To be done by the students

a. **What size cup-point setscrew would be required to hold the gear. Note: a safety factor of 4 is a typical minimum for a setscrew subject to dynamic loading.**

The torque transmitted by the shaft, T, is related to power transmitted, H, and rotation rate, ω

$$T = H/\omega \qquad \text{(in SI units)}$$

When converted to torque in lbf·in, power in hp, and rotation rate in rev/min (where ω is replaced by n to acknowledge the change in units)

$$\begin{aligned} T &= 63{,}025\ H/n \\ &= 63{,}025\ (5\ \text{hp}) / (600\ \text{rev/min}) \\ &= 525\ \text{lbf·in} \end{aligned}$$

For a 2” diameter shaft, the force at the surface is:

$$
\begin{aligned}
F &= T / (d/2) \\
&= 2T / d \\
&= 2\,(525 \text{ lbf}\cdot\text{in})/(2 \text{ in}) \\
&= 525 \text{ lbf}
\end{aligned}
$$

Applying a safety factor of 4, the setscrew must support a load of 4 times the surface force = 2100 lbf. From Table 7-4, a 7/16” setscrew would be sufficient (a 7/16” setscrew provides a holding power of 2500 lbf). As a rule of thumb, set screw diameter should normally be about ¼ of the mating shaft diameter - this is the case here.

Select a 7/16” setscrew

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b. What size dowel pin would be required for the same gear, if the pin passes through both sides of the shaft? (select a pin from the class notes) Assume a safety factor of 3 is required.

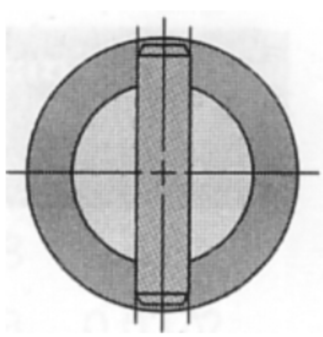
From part a, the torque to be transmitted is

$$T = 525 \text{ lbf}\cdot\text{in}$$

and the force at the shaft surface (for a 2" diameter shaft) is:

$$F = 525 \text{ lbf}$$

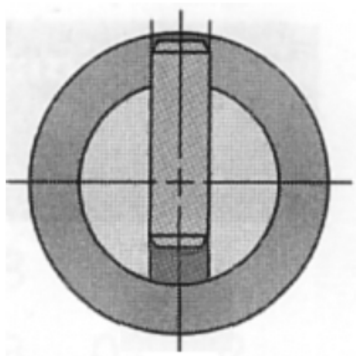
Applying a safety factor of 3, the pin must support a load of 3 times the surface force = 1575 lbf. The pin passes through the shaft, so it is loaded in double shear (see image below). From the table, a 0.125” diameter dowel pin in double shear can handle 3200 lbf.

Double shear: 

Select 0.125" = 1/8" diameter dowel pin

c. What size taper pin would be required for the same gear, assuming the pin is loaded in single shear? (select a pin from the class notes) Assume a safety factor of 3 is required.

This is the same as part b, except now the taper pin is loaded in single shear. The numbers from the table must be reduced by two (or the required load doubled).

Single shear: 

The force at the shaft surface (for a 2" diameter shaft) is:

$$F = 525 \text{ lbf}$$

Applying a safety factor of 3, the pin must support a load of 3 times the surface force = 1575 lbf. From the table, a 0.125" diameter taper pin in double shear can handle 2900 lbf. In single shear, the pin an handle approximately 1450 lbf (it will actually be a bit more, since the wide end of the pin will be taking the load). Therefore, select a 0.188" diameter taper pin (6600 lbf in double shear, approx. 3300 lbf in single shear).

Select a 0.188" = 3/16" diameter taper pin

d. Select an appropriate size gib-head key to hold the gear if the power transmitted is increased to 40 hp (assume the key is made of steel with yield strength of 70 kpsi). A safety factor of 2.5 is desired. Use the distortion-energy theory to select a key (note that under distortion-energy, the shear yield strength is $S_{sy} = 0.577S_y$).

The new torque transmitted to the shaft is

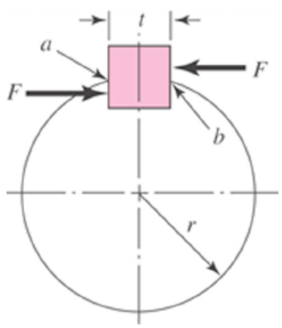
$$\begin{aligned} T &= 63{,}025\, H/n \\ &= 63{,}025\,(40 \text{ hp}) / (600 \text{ rev/min}) \\ &= 4200 \text{ lbf}\cdot\text{in} \end{aligned}$$

For a 2” diameter shaft, the force at the surface is:

$$
\begin{aligned}
F &= T / (d/2) \\
&= 2T / d \\
&= 2\,(4200\ \text{lbf}\cdot\text{in})/(2\ \text{in}) \\
&= 4200\ \text{lbf}
\end{aligned}
$$

(Unlike the previous part, the convention for this case is to apply the safety factor to the shear stress prior to comparison with material strength. The answer would still work out correctly if the safety factor were applied to the above force, as was done in the previous part.)

This force places the key in direct shear as well as in compression.



The shear stress is $\tau = F/A_s$ where A_s is the area of the key between points a and b in the figure above. Specifically, $A_s = w\cdot l$ where w is the thickness of the key (shown as t in the figure) and l is the depth into the page. Both of these quantities need to be determined by selecting a key from Table 7-6.

$$\tau = F/(w\cdot l)$$

This stress must remain below the shear yield strength, adjusted by the safety factor.

$$\tau \le S_{sy} / n$$

$$F/(w\cdot l) \le S_{sy} / n$$

$$(w\cdot l) \ge F\cdot n/S_{sy}$$

The shear yield strength (as given in the question) is:

$$
\begin{aligned}
S_{sy} &= 0.577\ S_y \\
&= 0.577\ (70\ \text{kpsi}) \\
&= 40.4\ \text{kpsi}
\end{aligned}
$$

$$(l\cdot w) \ge (4200\ \text{lbf})(2.5) / (40.4\ \text{kpsi})$$

$$(l\cdot w) \ge 0.2600$$

In addition, the key must resist crushing by compression. The area, A_c, to be used in this case is the product of the depth of the key into the shaft with the shaft length, l. Standard practice (and as shown by comparing the last two columns of Table 7-6) is to have the key depth equal to one-half of the key height, h/2.

$\sigma = F / A_c$

$F/(l \cdot h/2) \le S_y / n$

$l \cdot h \ge 2F \cdot n / S_y$

The compression yield strength is:

$S_y = 70.0$ kpsi

$(l \cdot h) \ge 2(4200 \text{ lbf})(2.5)/(70 \text{ kpsi})$

$(l \cdot h) \ge 0.300$

From Table 7-6, the recommended key size for a 2 in shaft (i.e. between 1-3/4" and 2-1/4") is either:

w×h = ½" × 3/8"   or   ½" × ½"

Either key size would work, as shown below.

<u>For w×h = ½" × 3/8"</u>

$l \cdot w \ge 0.26$
$l \ge 0.26 / 0.5$ in
≥ 0.52 in

$l \cdot h \ge 0.30$
$l \ge 0.30 / 0.375$ in
≥ 0.80 in

Minimum length, 0.80 in

<u>For w×h = ½" × ½"</u>

$l \cdot w \ge 0.26$
$l \ge 0.26 / 0.5$ in
≥ 0.52 in

$l \cdot h \ge 0.30$
$l \ge 0.30 / 0.5$ in
≥ 0.60 in

Minimum length, 0.6 in

Choose either a 0.80 in long ½"×3/8" key or a 0.60 in long ½"×½" key.

Adapted from **Shigley, Problems 10-3 and 10-5** (8th and 9th Eds.)

A helical compression spring is wound using 0.105-in diameter music wire. The spring has an outside diameter of 1.225 in with plain ground ends, and 12 total coils.

Part 1 – To be demonstrated by the TA

a. What force is needed to compress this spring to closure if the spring is to just reach the yield point upon closure?
b. What is the approximate spring rate for this spring?
c. What should the free length be to ensure that when the spring is compressed solid, the torsional stress does not exceed the yield strength (that is, it is solid safe)?
d. What is the pitch of this spring?
e. Is there a possibility that this spring might buckle in service? (use a safety factor of 5)

Part 2 – To be done by the students

Repeat the analysis above for a helical compression spring with 8-1/2 total coils made of A227 hard-drawn spring steel wire 2 mm diameter and outside diameter of 22 mm. At full compression, the material should remain below yield with a safety factor of 2. The ends are squared and ground and are held in an assembly that allows them to pivot.

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1. Adapted from Shigley, Problems 10-3 and 10-5 (8th and 9th Eds.)

A helical compression spring is wound using 0.105-in diameter music wire. The spring has an outside diameter of 1.225 in with plain ground ends, and 12 total coils. The spring ends are to be fixed and the spring compressed between flat parallel plates.

Part 1 – To be demonstrated by the TA

a. What force is needed to compress this spring to closure if the spring is to just reach the yield point upon closure?

Some preliminary values:

d	$= 0.105$ in	spring wire diameter
D_o	$= 1.225$ in	spring outside diameter
D	$= D_o - 2 \cdot d/2$	mean diameter (distance between coil centres)
	$= D_o - d$	
	$= 1.225$ in $- 0.105$ in	
	$= 1.120$ in	

The relationship between applied force and stress is

$$\tau = K_s \frac{8FD}{\pi d^3} \qquad \text{eq. 10-2 (p. 519)}$$

Where

τ	= shear stress = S_{sy} (shear yield strength) in this case
K_s	= shear stress correction factor (K_W or K_B, see below)
F	= applied force (to be solved for)
D	= mean diameter (above)
d	= wire diameter (given)

Need to find the shear yield strength, S_{sy}

S_{sy} = $0.45S_{ut}$ See Table 10-6 (p. 526) for music wire

S_{ut} = A/d^m Table 10-4 (p. 525)

A = 201 kpsi·in^m Table 10-4 (p. 525)
m = 0.145
S_{ut} = (201 kpsi·in$^{0.145}$)/(0.105 in)$^{0.145}$
= 279 kpsi

S_{sy} = 0.45(279 kpsi)
= 125.4 kpsi

K_s shear stress concentration factor
Can use either $K_s = K_W$ or $K_s = K_B$
As noted in Shigley (bottom of p. 519), K_W and K_B differ by only 1%

so as done in Shigley, use K_B
K_B Bergstrasser factor (eq. 10-5, p. 519)

$$K_B = \frac{4C+2}{4C-3}$$

C = D/d eq. 10-1 (p. 519)
= (1.120 in)/(0.105 in)
= 10.67

$$K_B = \frac{4(10.67)+2}{4(10.67)-3}$$

$$= 1.126$$

Rearranging 10-2 with $K_s = K_B$ and $\tau = S_{sy}$ in terms of F gives

$$F = \frac{\pi d^3 S_{sy}}{8K_B D}$$
$$= \frac{\pi(0.105 \text{ in})^3(125.4 \text{ kpsi})}{8(1.126)(1.120 \text{ in})}$$
$$= 0.0452 \text{ klbf}$$
$$= 45.2 \text{ lbf}$$

A force of 45.2 lbf is required to compress the spring to yield at closure

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b. What is the approximate spring rate for this spring?

The approximate spring rate, k, is given by eq. 10-9 (p. 520)

$$k \cong \frac{d^4 G}{8D^3 N} \qquad \text{eq. 10-9 (p. 520)}$$

Where

d & D	= diameters as defined above	
G	= modulus of rigidity	
	= 11.75 Mpsi	from Table 10-5 (p. 526) for d = 0.105 in music wire
N	= N_a = number of active coils	

To find N_a, use Table 10-1 (p. 521)

N_a	= N_t -1	for plain and ground ends
N_t	= 12	total number of coils, given
N_a	= 12 - 1	
	= 11	

$$k \cong \frac{(0.105 \text{ in})^4(11.75\times10^6 \text{ psi})}{8(1.120 \text{ in})^3(11)}$$
$$= 11.55 \text{ lbf/in}$$

The approximate spring rate is k = 11.55 lbf/in

c. What should the free length be to ensure that when the spring is compressed solid, the torsional stress does not exceed the yield strength (that is, it is solid safe)?

Knowing the spring rate and the applied force, we can determine how much the spring compresses

$\Delta y = F/k$ F was found in Part a. and k in b.
$= (45.2\ \text{lbf})/(11.55\ \text{lbf/in})$
$= 3.91\ \text{in}$

At maximum compression, the spring is at its solid length, L_s

$L_s = dN_t$ from inspection (or Table 10-1, p. 521)
$= (0.105\ \text{in})(12)$
$= 1.26\ \text{in}$

The free length is just the solid length plus the deflection required to bring the spring to its solid length

$L_0 = L_s + \Delta y$
$= 1.26\ \text{in} + 3.91\ \text{in}$
$= 5.17\ \text{in}$

The free length should be 5.17 in

d. What is the pitch of this spring?

Pitch, p, is distance between coils, paying attention to end conditions. For a plain and ground spring, Table 10-1 (p. 521) gives the pitch

$p = L_o/(N_a + 1)$
$= L_o/N_t$
$= (5.17\ \text{in}) / 12$
$= 0.431\ \text{in}$

The pitch is 0.431 in

e. Is there a possibility that this spring might buckle in service (using a safety factor of 5)?

Buckling is a concern if the free length exceeds a critical value. (The more slender a spring is, the more likely it is to buckle.)

To avoid buckling, $L_0 < L_{o|crit}$. With a safety factor of n, $L_o < L_{o|crit}/n$

$L_{o|crit} = 2.63\ D/\alpha$ eq. 10-13 (p. 523)

Where

$L_{o|crit}$ = the critical length for buckling

α = a constant related to the end condition

= 0.5 (fixed ends, From Table 10-2, p. 522)

D = the mean diameter, found above

= 1.120 in

$L_{o|crit}$ = 2.63 (1.120 in)/0.5

= 5.89 in

$L_{o|cirt}/n$ = 5.89 in/5

= 1.18 in

L_o = 5.17 in From Part c.

$L_o > L_{o|crit}/n$ therefore the design should not be considered safe from buckling

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Part 2 – To be done by the students

Repeat the analysis above for a helical compression spring made of A227 hard-drawn spring steel wire 2 mm in diameter and with an outside diameter of 22 mm. At full compression, the material should remain below yield with a safety factor of 2. The ends are squared and ground and are held in an assembly that allows them to pivot. There are 8-1/2 total coils.

a. What force is needed to compress this spring to closure?

In this case, the safety factor of 2 requires that the shear stress in the spring at maximum compression be half of the shear yield strength ($\tau = S_{sy}/2$)

Some preliminary values:

d	= 2 mm	spring wire diameter
D_o	= 22 mm	spring outside diameter
D	$= D_o - 2 \cdot d/2$	mean diameter (distance between coil centres)
	$= D_o - d$	
	= 22 mm - 2 mm	
	= 20 mm	

The relationship between applied force and stress is

$$\tau = K_s \frac{8FD}{\pi d^3}$$ eq. 10-2 (p. 519)

Where

τ = shear stress = S_{sy} (shear yield strength) in this case

K_s = shear stress correction factor (K_W or K_B, see below)

F = applied force (to be solved for)

D = mean diameter (above)

d = wire diameter (given)

Need to find the shear yield strength, S_{sy}

S_{sy} = $0.45S_{ut}$ See Table 10-6 (p. 526) for cold-drawn carbon steel

S_{ut} = A/d^m Table 10-4 (p. 525)

A = 1.783 GPa·mm^m Table 10-4 (p. 525)
m = 0.190
S_{ut} = (1.783 GPa·mm$^{0.190}$)/(2 mm)$^{0.190}$
= 1.563 GPa

S_{sy} = 0.45(1.563 GPa)
= 703 MPa

K_s shear stress concentration factor
Can use either $K_s = K_W$ or $K_s = K_B$
As noted in Shigley (bottom of p. 519), K_W and K_B differ by only 1%
so as done in Shigley, use K_B

K_B Bergstrasser factor (eq. 10-5, p. 519)

$$K_B = \frac{4C+2}{4C-3}$$

C = D/d eq. 10-1 (p. 519)
= (20 mm)/(2 mm)
= 10.0

$$K_B = \frac{4(10)+2}{4(10)-3}$$
$$= 1.135$$

Rearranging 10-2 in terms of F with $K_s = K_B$ and $\tau = S_{sy}/2$ (the 2 comes from the stated safety factor of 2 required on stresses) gives

$$F = \frac{\pi d^3 S_{sy}/2}{8K_B D}$$
$$= \frac{\pi (0.002\text{ m})^3 (703\text{ MPa})/2}{8(1.135)(0.020\text{ m})}$$
$$= 48.6\text{ N}$$

A force of 48.6 N is required to compress the spring to closure while maintaining maximum stress below yield with a safety factor of 2

b. What is the approximate spring rate for this spring?

The approximate spring rate, k, is given by eq. 10-9 (p. 520)

$$k \cong \frac{d^4 G}{8D^3 N}$$ eq. 10-9 (p. 520)

Where

d & D	= diameters as defined above (d = 2 mm = 0.0787 in)	
G	= modulus of rigidity	
	= 79.3 GPa	from Table 10-5 (p. 526) for d = 0.0787 A227 steel
N	= N_a = number of active coils	

To find N_a, use Table 10-1 (p. 521)

N_a	= N_t -2	for squared and ground ends
N_t	= 8.5	total number of coils, given
N_a	= 8.5 - 2	
	= 6.5	

$$k \cong \frac{(0.002\ \text{m})^4 (79.3 \times 10^9\ \text{Pa})}{8(0.020\ \text{m})^3 (6.5)}$$

$$= 3050\ \text{N/m}$$

The approximate spring rate is k = 3.05 kN/m

c. What should the free length be to ensure that when the spring is compressed solid, the torsional stress does not exceed the yield strength (that is, it is solid safe)?

Knowing the spring rate and the applied force, we can determine how much the spring compresses

Δy	= F/k	F was found in Part a. and k in b.
	= (48.6 N)/(3050 N/m)	
	= 0.0159 m	

At maximum compression, the spring is at its solid length, L_s

L_s	= dN_t	from Table 10-1 (p. 521)
	= (0.002 m)(8.5)	
	= 0.017 m	

The free length is just the solid length plus the deflection required to bring the spring to its solid length

L_0	= L_s + Δy
	= 0.017 m + 0.0159 m
	= 0.0329 m
	= 32.9 mm

The free length should be 32.9 mm

d. What is the pitch of this spring?

Pitch, p, is distance between coils, paying attention to end conditions. For a plain and ground spring, Table 10-1 (p. 521) gives the pitch

$p = (L_o - 2d)/N_a$
$= (32.9\text{ mm} - 2\cdot 2\text{mm})/6.5$
$= 4.45\text{ mm}$

The pitch is 4.45 mm

e. Is there a possibility that this spring might buckle in service (using a safety factor of 5)?

Buckling is a concern if the free length exceeds a critical value. (The more slender a spring is, the more likely it is to buckle.)

To avoid buckling, $L_0 < L_{o|crit}$. With a safety factor of n, $L_o < L_{o|crit}/n$

$L_{o|crit} = 2.63\ D/\alpha$ eq. 10-13 (p. 523)

Where

$L_{o|crit}$ = the critical length for buckling
α = a constant related to the end condition
$= 1$ (pivoting ends, From Table 10-2, p. 522)
D = the mean diameter, found above
$= 20$ mm

$L_{o|crit} = 2.63\ (20\text{ mm})/1.0$
$= 52.6\text{ mm}$
$L_{o|cirt}/n = 52.6\text{ mm}/5$
$= 10.52\text{ mm}$

$L_o = 32.9\text{ mm}$ From Part c.

$L_o > L_{o|crit}/n$ therefore the design should not be considered safe from buckling

Adapted from **Shigley, 9th Ed. Problem 10-30** (8th Ed., 10-23)

A 16-coil helical compression spring with plain and ground ends is needed for an application where load varies from a maximum of 4 lbf to a maximum of 18 lbf. The spring rate k is to be 9.5 lbf/in. Wire diameters of 0.080 in and 0.0915 in are available (wires are unpeened A313 Stainless Steel, G = 10 Mpsi, $\gamma = 0.283$ lbf/in³).

Part 1 – To be demonstrated by the TA

a. Using the Sines fatigue-failure criterion with Zimmerli data, determine the mean spring diameter for the 0.080 in wire such that the fatigue design factor is $n_f = 1.5$.
b. Determine the critical frequency of the spring (the spring ends are always in contact with the mounting plates).

Part 2 – To be done by the students

c. Using the Gerber fatigue-failure criterion with Zimmerli data, determine the fatigue design factor, nf, if the mean diameter is 0.558 in and the 0.0915 in wire is used.
d. Determine the critical frequency of the spring (the spring ends are always in contact with the mounting plates).

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1. Adapted from Shigley, 9th Ed., Problem 10-30 (8th Ed, 10-23)

A 16-coil helical coil compression spring with plain and ground ends is needed for an application where load varies from a maximum of 4 lbf to a maximum of 18 lbf. The spring rate k is to be 9.5 lbf/in. Wire diameters of 0.080 in and 0.0915 in are available (wires are unpeened A313 Stainless Steel, G = 10 Mpsi, γ = 0.283 lbf/in^3).

Part 1 - To be demonstrated by the TA

a. Using the Sines fatigue-failure criterion with Zimmerli data, determine the mean spring diameter for the 0.080 in wire such that the fatigue design factor is n_f = 1.5.

$d = 0.080$ in

$S_{ut} = A/d^m$ — Table 10-4 (p. 525)

$A = 169$ kspi·in^m — Table 10-4 (p. 525)

$m = 0.146$ — Table 10-4 (p. 525)

$S_{ut} = (169\ \text{kspi·in}^m)/(0.080\ \text{in})^{0.146}$
$= 244$ kpsi

Strengths for unpeened steel (note: not all are needed for this part)

$S_{sa} = 35$ kpsi — Alternating (shear) strength, eq. 10-28 (p. 536)

$S_{sm} = 55$ kpsi — Midrange (shear) strength, eq. 10-28 (p. 536)

$S_{su} = 0.67S_{ut}$ — Torsional modulus of rupture, eq. 10-30 (p. 537)
$= 0.67(244\ \text{kpsi})$
$= 163.5$ kpsi

$S_{sy} = 0.35(S_{ut})$ — Table 10-6 (p. 526) for Austenitic stainless steel
$= 0.35(244\ \text{kpsi})$
$= 85.4$ kpsi

Shear endurance limit from Gerber-Zimmerli fatigue-failure criterion (to be used in part 1c.)

$$\begin{aligned} S_{se} &= \frac{S_{sa}}{1-\left(\frac{S_{sm}}{S_{su}}\right)^2} \\ &= \frac{(35 \text{ kpsi})}{1-\left(\frac{55 \text{ kpsi}}{163.5 \text{ kpsi}}\right)^2} \\ &= 39.5 \text{ kpsi} \end{aligned}$$

eq. 6-42 (p. 306) or below 10-29 (p. 536)

The Sines Failure Criterion is described at the bottom of p. 536. Specifically, the maximum alternating stress that may be imposed without causing fatigue failure for springs in torsion is independent of mean stress. The safety factor is the ratio of shear strength (S_{sa}) to shear stress (τ)

$n_f = S_{se}/\tau_a = S_{sa}/\tau$

The stress is found the same way as in Tutorial 12

$$\tau = K_B \frac{8FD}{\pi d^3}$$ eq. 10-2 (p. 519)

Where

K_B is the stress concentration factor

$$K_B = \frac{4C+2}{4C-3}$$ eq. 10-4 (p. 519)

F is the applied force
D is the mean diameter (to be determined)
d is the wire diameter (0.080 in, given)

For this particular case, we are interested in the alternating stress and therefore the alternating component of force, F_a

$$\begin{aligned} F_a &= (F_{max} - F_{min})/2 \\ &= (18 \text{ lbf} - 4 \text{ lbf})/2 \\ &= 7 \text{ lbf} \end{aligned}$$

Combining the expression for τ_a with the expression for the fatigue safety factor

$n_f = S_{sa}/\tau_a$

$$n_f \frac{4C+2}{4C-3}\frac{8F_a C}{\pi d^2} = S_{se}$$

$$n_f (4C+2) 8F_a C = S_{se}(4C-3)\pi d^2$$

This gives a quadratic in terms of C (everything else is known)

$$32n_f F_a C^2 + \left(16n_f F_a - 4S_{se}\pi d^2\right)C + 3S_{se}\pi d^2 = 0$$

$$32(1.5)(7 \text{ lbf})C^2 + \left(16(1.5)(7 \text{ lbf}) - 4(35 \text{ kpsi})\pi(0.080 \text{ in})^2\right)C$$

$$+3(35 \text{ kpsi})\pi(0.080 \text{ in})^2 = 0$$

$$336C^2 - 2647C + 2111 = 0$$

From which C = 6.98 or 0.90. We can immediately reject the 0.90 spring index since it corresponds to a physically impossible spring with the wire diameter larger than the spring diameter.

C = 6.98

By definition, C = D/d

D = d·C
= (0.080 in)(6.98)
= 0.558 in

The mean diameter should be 0.558 in

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b. Determine the critical frequency of the spring.

The critical frequency is give by

$$f = \frac{1}{2}\sqrt{\frac{kg}{W}}$$ eq. 10-25 (p. 535)

Where
- f = frequency in Hz
- k = spring rate (lbf/in)
- g = acceleration due to gravity (in/s^2) = 386 in/s^2
- W = weight of the active part of the spring

W = ALγ

- A = cross sectional area = $\pi d^2/4$
- L = length of active portion of spring wire = $\pi D N_a$
- N_a = number of active coils
- γ = specific weight of spring material = 0.283 lbf/in^3 (given)

For a plain and ground spring,

N_a = N_t - 1 Table 10-1 (p. 521)
= 16 - 1
= 15

W = $(\pi d^2/4)(\pi D N_a)\gamma$
= $\pi^2(0.080 \text{ in})^2(0.558 \text{ in})(15)(0.283 \text{ lbf/in}^3)/4$
= 0.0374 lbf

$$f = \frac{1}{2}\sqrt{\frac{(9.5\text{ lbf/in})(386\text{ in/s}^2)}{0.0374\text{ lbf}}}$$
$$= 156.6\text{ Hz}$$

The critical frequency is 156.6 Hz

Part 2 – To be done by the students

c. Using the Gerber fatigue-failure criterion with Zimmerli data, determine the fatigue design factor, n_f, if the mean diameter is 0.558 in and the 0.0915 in wire is used.

The Gerber fatigue failure criterion for shafts is given by

$$\frac{n_f\sigma_a}{S_e} + \left(\frac{n_f\sigma_m}{S_{ut}}\right)^2 = 1 \qquad \text{eq. 6-42 (p. 306)}$$

Re-written for spring torsion, the equation has the form

$$\frac{n_f\tau_a}{S_{se}} + \left(\frac{n_f\tau_m}{S_{su}}\right)^2 = 1$$

The torsional modulus of rupture and shear endurance strength can be found as above

S_{ut} = (169 kspi·in^m)/(0.0915 in)$^{0.146}$
= 240 kpsi

S_{su} = 0.67S_{ut}
= 160.5 kpsi

$$S_{se} = \frac{S_{sa}}{1-\left(\frac{S_{sm}}{S_{su}}\right)^2}$$
$$= \frac{(35\text{ kpsi})}{1-\left(\frac{55\text{ kpsi}}{160.5\text{ kpsi}}\right)^2}$$
$$= 39.7\text{ kpsi}$$

S_{se} = 39.7 kspi

The spring index is

C = D/d
= (0.558 in)/(0.0915 in)
= 6.10

The stress concentration factor, K_B, is

$$K_B = \frac{4C+2}{4C-3}$$
$$= \frac{4(6.1)+2}{4(6.1)-3} \qquad \text{eq. 10-4 (p. 519)}$$
$$= 1.234$$

The alternating and mean torsional stresses are

$$\tau_a = K_B \frac{8F_a D}{\pi d^3}$$
$$\tau_m = K_B \frac{8F_m D}{\pi d^3}$$

eq. 10-2 (p. 519) with stress concentration K_B

The forces F_a and F_m are given by

F_a = 7 lbf   found above
F_m = $(F_{max} + F_{min})/2$
= (18 lbf + 4 lbf)/2
= 11 lbf

$$\tau_a = K_B \frac{8F_a D}{\pi d^3}$$
$$= 1.234 \frac{8(7 \text{ lbf})(0.558 \text{ in})}{\pi (0.0915 \text{ in})^3}$$
$$= 16.02 \text{ kpsi}$$

$$\tau_m = K_B \frac{8F_m D}{\pi d^3}$$
$$= 1.234 \frac{8(11 \text{ lbf})(0.558 \text{ in})}{\pi (0.0915 \text{ in})^3}$$
$$= 25.2 \text{ kpsi}$$

Substituting into the Gerber fatigue failure expression

$$\frac{n_f (16.02 \text{ kpsi})}{39.7 \text{ kpsi}} + \left(\frac{n_f (25.2 \text{ kpsi})}{160.5 \text{ kpsi}} \right)^2 = 1$$

Which reduces to

$0.0247n_f^2 + 0.404n_f - 1 = 0$

Solving this quadratic gives solutions n_f = 2.18 and -18.54. Ignoring the negative value, the safety factor is 2.18.

The safety factor is 2.18

d. Determine the critical frequency of the spring.

The critical frequency is as given above:

$$f = \frac{1}{2}\sqrt{\frac{kg}{W}}$$ eq. 10-25 (p. 535)

Where

- f = frequency in Hz
- k = spring rate (lbf/in) = 9.5 lbf/in (given)
- g = acceleration due to gravity (in/s^2) = 386 in/s^2

W = ALγ

N_a = 15 Table 10-1 (p. 521)

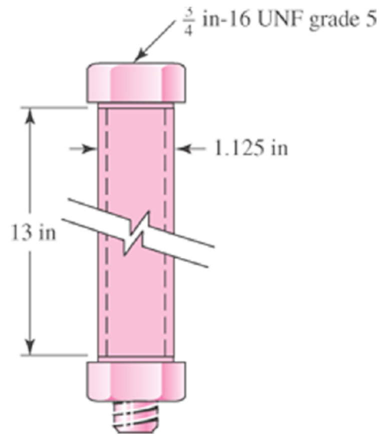
W = $(\pi d^2/4)(\pi D N_a)\gamma$
= $\pi^2(0.0915\text{ in})^2(0.558\text{ in})(15)(0.283\text{ lbf/in}^3)/4$
= 0.0489 lbf

$$f = \frac{1}{2}\sqrt{\frac{(9.5\text{ lbf/in})(386\text{ in/s}^2)}{0.0489\text{ lbf}}}$$
$$= 136.9\text{ Hz}$$

The critical frequency is 136.9 Hz

Adapted from **Shigley, 9th Ed. Problem 8-26 (8th Ed., 8-15)**

A ¾ in-16 UNF series SAE grade 5 bolt is fitted through a ¾-in ID steel tube, 13 in long. The tube is clamped between washer faces of the bolt and nut by turning the nut snug and adding one-third of a turn. The tube OD is the washer-face diameter d_w = 1.5d = 1.125 in OD.



Part 1 – To be done by the students

a. What is the spring rate of the bolt? (Assume the shank extends the full length of the grip.)

b. What is the spring rate of the tube?

Part 2 – To be demonstrated by the TA

c. What is the joint constant C?

d. When the one-third turn-of-nut is applied, what is the initial tension F_i in the bolt?

Part 3 – To be done by the students

e. What is the bolt tension at opening if addition tension is applied to the bolt external to the joint?

Part 4 – To be done at home

f. If a UNC bolt is used instead of UNF and one-quarter of a turn is used for preload instead of one-third of a turn, show the resulting initial tension is 13.16 klbf.

g. Show the tension at opening, if applied to the bolt external to the joint, is 25.5klbf.

Part 1 - To be done by the students

a. What is the spring rate of the bolt? (Assume the shank extends the full length of the grip.)

The expression for k_d from eq. 8-16 (p. 427) can be used where the area is based on the major diameter and the length is the entire grip length in this case

$$k = \frac{A_d E}{l}$$

Where

A_d	= major diameter area	
	= $\pi/4d^2$	
	= $\pi/4\ (0.75\text{ in})^2$	
	= 0.442 in^2	
E	= 30×10^6 psi	(Table 8-8 for steel, given)
l	= length of grip	
	= 13 in	(given)

$$k = \frac{(0.442\text{ in}^2)(30\times10^6\text{ psi})}{13.0\text{ in}}$$
$$= 1.020\times10^6\text{ lbf/in}$$

The spring rate of the bolt is 1.020×10^6 lbf/in.

b. What is the spring rate of the tube?

The same expression as above can be used for the spring rake, k_m, of the member (i.e. the tube).

$$k_m = \frac{AE}{l}$$

Where

A	= tube cross sectional area	
	$= \pi/4(d_o^2 - d_i^2)$	
	$= \pi/4\,[(1.125\text{ in})^2 - (0.75\text{ in})^2]$	
	$= 0.552\text{ in}^2$	
E	$= 30\times10^6$ psi	(Table 8-8 for steel, given)
l	= length of tube	
	= 13 in	(given)

$$k_m = \frac{\left(0.552\text{ in}^2\right)\left(30\times10^6\text{ psi}\right)}{13.0\text{ in}}$$

$$= 1.274\times10^6\text{ lbf/in}$$

The spring rate of the tube is 1.274×10^6 lbf/in.

Part 2 – To be demonstrated by the TA

c. What is the joint constant C?

The joint constant, C, describes the fraction of external load carried by the bolt and is given by eq. (f) on p. 436

$$C = \frac{k_b}{k_b + k_m}$$

$$= \frac{1.020\times10^6\text{ lbf/in}}{1.020\times10^6\text{ lbf/in} + 1.274\times10^6\text{ lbf/in}}$$

$$= 0.445$$

d. When the one-third turn-of-nut is applied, what is the initial tension F_i in the bolt?

We have three conditions to apply:

Geometry: with known thread geometry, we know the displacement for 1/3 of a turn;

Equilibrium: we know the initial tension in the bolt (F_i) is the same as the initial compressive force (P) on the tube;

Compatibility: the displacement in compression of the tube, δ_t, plus the extension in tension of the bolt, δ_b, must equal the total displacement, δ

Geometry

From Table 8-2 (p. 413) a ¾" UNF bolt has 16 threads per inch (0.0625 in per turn). For 1/3 of a turn:

δ = (1/3 turn)(0.0625 in/turn)
δ = 0.0208 in

Note: this does not consider the change in length of the bolt; since k_b is large the change in length is small and can be neglected.

Equilibrium

$F_i = P$

Compatibility

$$
\begin{aligned}
\delta &= \delta_{bolt} + \delta_{tube} \\
&= F_i/k_b + P/k_m \\
&= F_i/k_b + F_i/k_m \qquad \text{(since } F_i = P \text{ from compatibility)} \\
&= F_i(1/k_b + 1/k_m)
\end{aligned}
$$

$$
\begin{aligned}
F_i &= \delta / (1/k_b + 1/k_m) \\
&= (0.0208 \text{ in}) / (1/1.020\times10^6 \text{ lbf/in} + 1/1.274\times10^6 \text{ lbf/in}) \\
&= 11.78 \text{ klbf}
\end{aligned}
$$

The bolt preload is 11.78 klbf.

Note: there is no external load in this particular case so the joint constant C is not of use.

Part 3 – To be done by the students

e. **What is the bolt tension at opening if addition tension is applied to the bolt external to the joint?**

At opening, the resultant load on the members (the tube in this case) is $F_m = 0$. We can work from equation 8-25 (p. 436) to relate the force in the members, F_m, the joint stiffness constant, C, the external force, P, and the bolt preload, F_i

$F_m = (1 - C)P - F_i$ eq. 8-25 (p. 436)

$$
\begin{aligned}
P &= (F_m + F_i)/(1 - C) \\
&= (0 + 11.78 \text{ klbf})/(1 - 0.445) \\
&= 21.2 \text{ klbf}
\end{aligned}
$$

The bolt tension is 21.2 klbf

Note: Since the load is applied directly to the bolt, this is the bolt tension. This result is consistent with eq. 8-24 (p. 436):

$F_b = CP + F_i$ eq. 8-24 (p. 436)
$= (0.445)(21.2 \text{ klbf}) + (11.78 \text{ klbf})$
$= 21.2$ klbf (i.e. unchanged)

Part 4 – To be done at home

f. If a UNC bolt is used instead of UNF and one-quarter of a turn is used for preload instead of one-third of a turn, what is the resulting initial tension in the bolt?

We have three conditions to apply:

Geometry: with known thread geometry, we know the displacement for 1/4 of a turn;

Equilibrium: we know the initial tension in the bolt (F_i) is the same as the initial compressive force (P) on the tube;

Compatibility: the displacement in compression of the tube, δ_t, plus the extension in tension of the bolt, δ_b, must equal the total displacement, δ

Geometry

From Table 8-2 (p. 413) a ¾" UNC bolt has 10 threads per inch (0.100 in per turn). For 1/4 of a turn:

$$\delta = (1/4 \text{ turn})(0.100 \text{ in/turn})$$
$$\delta = 0.0250 \text{ in}$$

Note: this does not consider the change in length of the bolt; since k_b is large the change in length is small and can be neglected.

Equilibrium

$$F_i = P$$

Compatibility

$$\begin{aligned}
\delta &= \delta_{bolt} + \delta_{tube} \\
&= F_i/k_b + P/k_m \\
&= F_i/k_b + F_i/k_m \qquad \text{(since } F_i = P \text{ from compatibility)} \\
&= F_i(1/k_b + 1/k_m)
\end{aligned}$$

$$\begin{aligned}
F_i &= \delta\ /(1/k_b + 1/k_m) \\
&= (0.0250 \text{ in}) \ / \ (1/1.020\times10^6 \text{ lbf/in} + 1/1.274\times10^6 \text{ lbf/in}) \\
&= 14.16 \text{ klbf}
\end{aligned}$$

The bolt preload is 14.16 klbf.

g. What is the new tension at opening, if applied to the bolt external to the joint?

At opening, the resultant load on the members (the tube in this case) is $F_m = 0$. We can work from equation 8-25 (p. 436) to relate the force in the members, F_m, the joint stiffness constant, C, the external force, P, and the bolt preload, F_i

$$F_m = (1 - C)P - F_i \qquad \text{eq. 8-25 (p. 436)}$$

$$\begin{aligned}
P &= (F_m + F_i)/(1 - C) \\
&= (0 + 14.16 \text{ klbf})/(1 - 0.445) \\
&= 25.5 \text{ klbf}
\end{aligned}$$

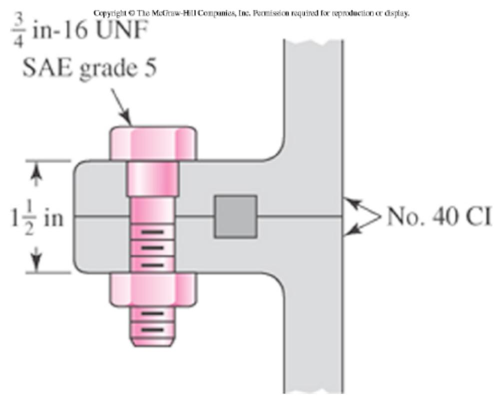
The bolt tension is 25.5 klbf

Adapted from **Shigley, Problem 8-34** (8th Ed.).

The section of the sealed joint shown in the figure is loaded by a repeated force P = 6 kip. The cast iron members have E = 16 Mpsi. Hardened steel washers 0.134 in thick are used under the head and nut. The bolt has been preloaded to a force of F_i = 25 kip each.

Part 1 – To be done at home

a. Show that a 2-½ in bolt has sufficient length.
b. Show that the spring rate of the bolt is 6.78×10^6 lbf/in.
c. Show that the spring rate of the member (including washers) is 14.41×10^6 lbf/in.
d. Show that the stiffness constant of the joint is C = 0.320



Part 2 – To be done by the students

e. Determine the fatigue factor of safety using the Goodman criterion
f. Determine the fatigue factor of safety using the Gerber criterion
g. Determine the load factor guarding against overproof loading.

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2. Adapted from Shigley, Problem 8-34 (8th Ed.)

The section of the sealed joint shown in the figure is loaded by a repeated force P = 6 kip. The cast iron members have E = 16 Mpsi. Hardened steel washers 0.134 in thick are used under the head and nut. The bolt has been preloaded to a force of F_i = 25 kip each.

Part 1 - To be done at home

a. Show that a 2-½ in bolt has sufficient length.

The bolt must be at least as long as the joint thickness plus the two washers plus the nut.

Joint thickness	= 1.5 in	(given)
Washer height	= 0.134 in	(given)
Nut height	= 41/64 in = 0.641 in	(given)

The required length is

1.5 in + 2(0.134 in) + 0.641 = 2.41 in

A 2.5 in bolt is sufficiently long.

---

b. Show that the spring rate of the bolt is 6.78×10^6 lbf/in.

The spring rate of the bolt is based on the spring rate of the threaded portion and the spring rate of the unthreaded portion. Referring to tutorial 15, the bolt spring rate is

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d}$$

Where

A_t = tensile stress area (Table 8-2, p. 413, ¾"UNF fastener)
= 0.373 in^2

A_d = major diameter area
= $\pi/4 d^2$
= $\pi/4 (0.75\ \text{in})^2$
= 0.442 in^2

E = 30 Mpsi (Table 8-8 for steel)

l_t = threaded length of grip

l_d = unthreaded length of grip

l_t and l_d must still be determined.

l is the grip length = material thickness plus thickness of two washers

l = 1.5 in + 2(0.134 in)
= 1.768 in

From Table 8-7 (p. 412)

$$L_T = \begin{cases} 2d + \frac{1}{4}\ \text{in} & \text{L} \le 6\ \text{in} \\ 2d + \frac{1}{2}\ \text{in} & L > 6\ \text{in} \end{cases}$$

In this case, L = 2.5 in so

L_T = 2d + ¼ in
= 2(0.75 in) + 0.25 in
= 1.75 in

Referring to Table 8-7, the length of the unthreaded portion, l_d, is

l_d = $L - L_T$ (L = 2.5 in given; L_T = 1.75 in from above.)
= 2.5 in - 1.75 in
= 0.75 in

The threaded length in the grip, l, is also shown in Table 8-7,

l_t = $l - l_d$ (l = 1.768 in from above; l_d = 0.75 in from above.)
= 1.768 in - 0.75 in
= 1.018 in

$$k_b = \frac{\left(0.442\ \text{in}^2\right)\left(0.373\ \text{in}^2\right)\left(30\times 10^6\ \text{psi}\right)}{\left(0.442\ \text{in}^2\right)\left(1.018\ \text{in}\right) + \left(0.373\ \text{in}^2\right)\left(0.75\ \text{in}\right)}$$

$$= 6.78\times 10^6\ \text{lbf/in}$$

The spring rate of the bolts is 6.78×10^6 lbf/in.

c. Show that the spring rate of the member (including washers) is 14.41×10^{6} lbf/in.

The spring rate is given by the eq. 8-20 (p. 429). Note: the exponential curve fit (eq. 8-23, p. 429) cannot be used in this case because the washer material is different from the other material.

$$k = \frac{0.577\pi E d}{\ln\dfrac{(1.155t + D - d)(D + d)}{(1.155t + D + d)(D - d)}} \qquad \text{eq. 8-20 (p. 429)}$$

For the washers

E	= washer elastic modulus (steel)	
	= 30.0 MPsi (Table 8-8, p. 430)	
D	= width of the bolt head	(Table A-29)
	= 1.125 in	

Note: the washer diameter is larger than that of the bolt head, but the full washer diameter is not compressed.

d	= bolt major diameter	(given)
	= 0.75 in	
t	= washer thickness	
	= 0.134 in	

$$k_w = \frac{0.577\pi(30\text{ Mpsi})(0.75\text{ in})}{\ln\dfrac{(1.155(0.134\text{ in}) + 1.125\text{ in} - 0.75\text{ in})(1.125\text{ in} + 0.75\text{ in})}{(1.155(0.134\text{ in}) + 1.125\text{ in} + 0.75\text{ in})(1.125\text{ in} - 0.75\text{ in})}}$$

$$= 153.2\times10^{6}\text{ lbf/in}$$

For the flange

E	= material elastic modulus (cast iron)	
	= 14.5 MPsi (Table 8-8, p. 430)	
D	= this diameter is larger than that used with the washer above due to the spreading of the load (see Fig 8-15, p. 428). The angle of spread is 30° (as noted on p. 428)	
	= D_{bolt} + 2t·tan(α)	
	= 1.125 in + 0.134 tan(30°)	
	= 1.280 in	
d	= bolt major diameter	(given)
	= 0.75 in	
t	= joint thickness (see Figure 8-15, p. 428)	
	= 0.75 in	

$$k_f = \frac{0.577\pi(16.0\text{ Mpsi})(0.75\text{ in})}{\ln\dfrac{(1.155(0.75\text{ in}) + 1.280\text{ in} - 0.75\text{ in})(1.280\text{ in} + 0.75\text{ in})}{(1.155(0.75\text{ in}) + 1.280\text{ in} + 0.75\text{ in})(1.280\text{ in} - 0.75\text{ in})}}$$

$$= 35.5\times10^{6}\text{ lbf/in}$$

There are two washers and two sections of material all in series. The net material spring rate is

$$\frac{1}{k_m} = 2\frac{1}{k_w} + 2\frac{1}{k_f}$$
$$= 2\frac{1}{153.3\ \text{Mlbf/in}} + 2\frac{1}{35.5\ \text{Mlbf/in}}$$

The final spring rate is $k_m = 14.40\times10^6$ lbf/in

---

d. Show that the stiffness constant of the joint is C = 0.320

The joint constant, C, describes the fraction of external load carried by the bolt and is given by eq. (f) on p. 436

$$C = \frac{k_b}{k_b + k_m}$$
$$= \frac{6.78\times10^6\ \text{lbf/in}}{6.78\times10^6\ \text{lbf/in} + 14.40\times10^6\ \text{lbf/in}}$$
$$= 0.320$$

The joint constant is 0.320.

Part 2 – To be done by the students

e. Determine the fatigue factor of safety using the Goodman criterion

For the Goodman criterion without safety factor looks like

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = 1 \qquad \text{(eq. 6-40, p. 306)}$$

Substituting the relationship for σ_m and noting that the fatigue safety factor, n_f, applies to the alternating component of stress (not the preload)

$$\frac{n_f\sigma_a}{S_e} + \frac{\sigma_i + n_f\sigma_a}{S_{ut}} = 1$$

Rearranging this expression for n gives

$$n_f = \frac{S_e\left(S_{ut} - \sigma_i\right)}{\sigma_a\left(S_{ut} + S_e\right)}$$

As shown in the figure for the problem, the bolt is an SAE grade 5. From Table 8-17 (p. 445), the endurance strength is

S_e = 18.6 kpsi

From Table 8-9 (p. 433) the minimum ultimate tensile strength is

S_{ut} = 120 kpsi

The bolt preload is 25 kip (given) and thus the initial stress in the bolts is

$$\sigma_i = F_i/A_t = (25 \text{ kip}) / (0.373 \text{ in}^2) = 67.0 \text{ kpsi}$$

The alternating force magnitude is half of the applied 6 kip force. The bolt experiences a fraction, C, of this load. (C = 0.320, given in Part d.)

$$\sigma_a = CP/(2\,A_t) = (0.320)(6000 \text{ lbf})/(2 \cdot 0.373 \text{ in}^2) = 2.57 \text{ kpsi}$$

$$n_f = \frac{S_e\left(S_{ut}-\sigma_i\right)}{\sigma_a\left(S_{ut}+S_e\right)} = \frac{(18.6 \text{ kpsi})(120 \text{ kpsi} - 67.0 \text{ kpsi})}{(2.57 \text{ kpsi})(120 \text{ kpsi} + 18.6 \text{ kpsi})} = 2.77$$

The fatigue safety factor for the Goodman criterion is 2.77.

---

f. Determine the fatigue factor of safety using the Gerber criterion

The Gerber criterion safety factor looks can be taken from above

$$\frac{n_f\sigma_a}{S_e} + \left(\frac{\sigma_i + n_f\sigma_a}{S_{ut}}\right)^2 = 1$$

Expanding this expression into a quadratic for n_f gives

$$\sigma_a^2 S_e n_f^2 + \sigma_a\left(2\sigma_i S_e + S_{ut}^2\right)n_f + S_e\left(\sigma_i^2 - S_{ut}^2\right) = 0$$

Solving this quadratic expression for n_f, taking the positive term, and simplifying gives

$$n_f = \frac{-\left(2\sigma_i S_e + S_{ut}^2\right) + \sqrt{\left(2\sigma_i S_e + S_{ut}^2\right)^2 - 4S_e\left(\sigma_i^2 S_e - S_e S_{ut}^2\right)}}{2\sigma_a S_e}$$

The same values of S_e, S_{ut}, σ_i, and σ_a from above can be used (with the units of kpsi omitted)

$$n_f = \frac{-\left(2(67.0)(18.6)+(120)^2\right)+\sqrt{\left(2(18.6)(67.0)+(120)^2\right)^2-4(18.6)^2\left((67.0)^2-(120)^2\right)}}{2(2.57)(18.6)}$$
$$= 4.19$$

The fatigue safety factor using the Gerber Criterion is 4.19

---

g. Determine the load factor guarding against overproof loading.

The proof-strength line on a fatigue diagram is similar to the Goodman line except it intersects both the midrange and alternating stress axes at the proof strength. (see Figure 8-22, p. 449).

The equation describing this line is

$$\frac{n\sigma_a}{S_p} + \frac{\sigma_i + n\sigma_a}{S_p} = 1$$

Rearranging this expression for n gives

$$n = \frac{S_p - \sigma_i}{2\sigma_a}$$

From Table 8-9 (p. 433) the proof strength is

S_p = 85 kpsi

The initial and alternating stresses were found above

σ_i = 67.0 kpsi

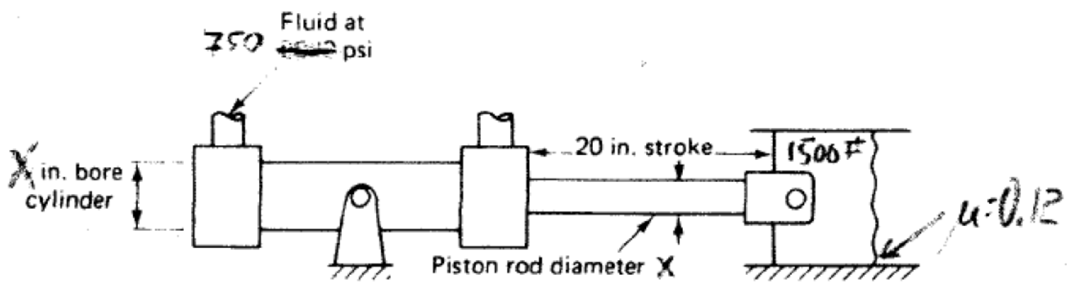
σ_a = 2.57 kpsi

$$n = \frac{S_p - \sigma_i}{2\sigma_a}$$
$$= \frac{85 \text{ kpsi} - 67 \text{ kpsi}}{2 \cdot 2.57 \text{ kpsi}}$$
$$= 3.50$$

The safety factor for overproof loading is 3.50.

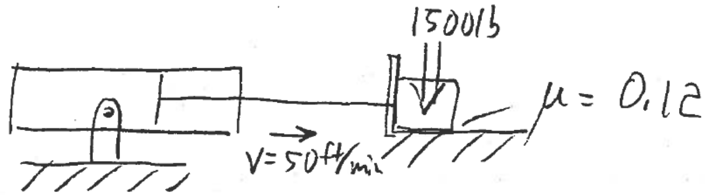
2. **Problem related to Fluid Power Component Sizing**

The following horizontal cylinder is required to move a load of 1500 lbf at a speed of 50 ft/min. over a distance of 20 inches. The friction coefficient has been determined as 0.12 between the load and the floor. In addition, the cylinder must accelerate the load within a distance of 0.75 inches.



a) Including acceleration and friction, determine the required cylinder bore for the system. What is the maximum thrust developed in our cylinder?
b) Select a rod size for the cylinder. Does this fit within the bore selected in part a)? If not, select the minimum required bore for the rod.
c) Size a pump for the system. What is the required flow rate in GPM? What is the required HP for the electric motor?
d) What suggestions can you make in order to reduce the overall cost of the system?

Solution to Problem 2



relief pressure $= 750$ psi
acceleration distance $= 0.75$ inch
stroke length $= 20$ inches

a) total force $= W \times$ force factor $+$ friction

$V = 50$ ft/min $\quad S = 0.75$

from graph b-2 $\quad g \doteq 0.18$

acceleration (deceleration) force $= 1500$ lbs $\times 0.18 = 270$ lbs

friction force $= \mu W = 0.12(1500 \text{ lbs}) = 180$ lbs

$\therefore$ total max force $= 270 + 180 = \underline{450 \text{ lbs}}$

From Table b-1 @ 750psi bore $= 1''$ gives 588 lbs max.

b) Determine Rod Size

Use stroke selection table Table b-4 to determine stroke factor

[illegible] Intermediate Trunnion & load is supported

$\therefore$ Stroke Factor $= 1.5$

Basic Length $=$ Actual Stroke $\times$ Stroke Factor

$= 20'' \times 1.5 = 30$ inches

From Piston Rod - Stroke Selection Guide

Thrust = 450 lbs

Basic Length = 30"

value just > 5/8" ∴ use 1" rod diameter

Hence a 1" bore will not work.

Use Table b-5 to select minimum bore with 1" rod size

Bore : $1\frac{1}{2}$ with 1" dia rod.

∴ Cylinder Bore : $1\frac{1}{2}$

Rod Dia : 1"

Stroke : 20"

From Table b-1 max thrust is 1325 lbs at 750 psi

c) Size pump to move 1" rod in $1\frac{1}{2}$ bore at a rate of 50 ft/min

From Table b-5 Cylinder Port Size and Piston Speed

Cylinder End $\varnothing = 1\frac{1}{2}$ rod $\varnothing = 0"$ GPM = 0.92 per 10 ft/min speed

For 50 ft/min speed $Q = 5(0.92) = 4.6$ GPM

<u>double check</u>

$$\text{vol gpm} = \frac{A \times L \times 60}{231 \times \text{time in sec}} \quad (\text{page } i-9)$$

$$A = \pi r^2 = \pi(1.5/2)^2 = 1.767 \text{ in}^2$$

$$L = 20"$$

$$v = 50 \text{ ft/min} = 50 \times 12/60 = 10"/\text{sec}$$

∴ total time for stroke = 2 sec.

$$\text{vol. (gpm)} = \frac{1.77 \times 20 \times 60}{231 \times 2} = 4.60 \text{ GPM}$$

Pump HP (page i-3)

$$Hp = \text{GPM} \times \text{psi} / 1714 \times \eta_{pump}$$

Select $\eta = 0.85$ (typical value)

$HP = (4.6\,gpm)(750\,psi)/1714 \times 0.85$

$= 2.36\ hp.$

From table j (c-3) or Table i-5

for $n = 1800\,rpm$ motor we need to select a motor of 3 HP

d) To reduce costs, it is recommended to reduce working pressure to 500 psi

@ 500 psi max thrust = 885 lbs (table b-1)

$\therefore$ 885 lbs > 450 lbs working load ✓

$HP = (4.6)(500)/1714 \times 0.85$

$= \frac{2}{3}(2.36) = 1.57\ HP.$

@ 400 psi working pressure

max thrust $= \frac{4}{5}(885) = 708$ lbs > 450 ✓

$HP = (4.6)(400)/1714 \times 0.85$

$= \frac{4}{5}(1.57) = 1.256$

by reducing pressure to 400 psi we can select a $1\frac{1}{2}$ H.P motor rather than 3 H.P