Solutions - Problem Set # 10

Problem 1:

Assumption: Sun is a black body at 5800K

a) $e_b = \sigma T^4 = 5.669\times10^{-8}\times5800^4 = 6.415\times10^7 \text{ W/m}^2$

b) and c)

$$[\lambda T]_{\max} = 2.8976\times10^{-3}\text{ m-K} \Rightarrow \lambda_{\max} = \frac{2.8976\times10^{-3}}{5800} = 4.996\times10^{-7}\text{ m}$$

$$e_{b,\lambda_{\max}} = \frac{C_1}{\lambda_{\max}^5\left[\exp\left\{\dfrac{C_2}{\lambda_{\max}T_{abs}}\right\}-1\right]} = \frac{3.7418\times10^{-16}}{\left(4.996\times10^{-7}\right)^5\left[\exp\left\{\dfrac{1.4388\times10^{-2}}{2.8976\times10^{-3}}\right\}-1\right]} = 8.4435\times10^{13}\text{ W/m}^3$$

d)

$$\int_{0.4}^{0.7} e_{b,\lambda}d\lambda = \int_{0}^{0.7} e_{b,\lambda}d\lambda - \int_{0}^{0.4} e_{b,\lambda}d\lambda$$

$$\frac{\int_{0.4}^{0.7} e_{b,\lambda}d\lambda}{\int_{0}^{\infty} e_{b,\lambda}d\lambda} = \frac{\int_{0.4}^{0.7} e_{b,\lambda}d\lambda}{\sigma T^4} = \frac{\int_{0}^{0.7} e_{b,\lambda}d\lambda}{\sigma T^4} - \frac{\int_{0}^{0.4} e_{b,\lambda}d\lambda}{\sigma T^4} = \frac{E_b(0\to\lambda T)_{\lambda=0.7}}{\sigma T^4} - \frac{E_b(0\to\lambda T)_{\lambda=0.4}}{\sigma T^4}$$

$$\lambda = 0.7\mu\text{m} \Rightarrow \lambda T = 0.7\times5800 = 4060\ \mu\text{m-K} \xrightarrow{\text{Table 8.1}} \frac{E_b(0\to\lambda T)_{\lambda=0.7}}{\sigma T^4} = 0.49157$$

$$\lambda = 0.4\mu\text{m} \Rightarrow \lambda T = 0.4\times5800 = 2320\ \mu\text{m-K} \xrightarrow{\text{Table 8.1}} \frac{E_b(0\to\lambda T)_{\lambda=0.4}}{\sigma T^4} = 0.12665$$

$$\frac{\int_{0.4}^{0.7} e_{b,\lambda}d\lambda}{\sigma T^4} = \frac{E_b(0\to\lambda T)_{\lambda=0.7}}{\sigma T^4} - \frac{E_b(0\to\lambda T)_{\lambda=0.4}}{\sigma T^4} = 0.49157 - 0.12665 = 0.36492 \simeq 36.49\%$$

Problem 2:

Assumption: Surface temperature is maintained at T_{surf} = 3000K; Irradiation are received from a black surface at 1000 K

Given: $\varepsilon_\lambda = \begin{cases} 0 & \lambda < 0.4\mu \\ 0.6 & 0.4\mu \le \lambda \le 4\mu \\ 0 & \lambda > 4\mu \end{cases}$

a) $$\varepsilon = \frac{\int_0^\infty \varepsilon_\lambda e_{b,\lambda}d\lambda}{\sigma T_{surf}^{\ 4}} = 0\times\frac{\int_0^{0.4} e_{b,\lambda}d\lambda}{\sigma T_{surf}^{\ 4}} + 0.6\times\frac{\int_{0.4}^{4} e_{b,\lambda}d\lambda}{\sigma T_{surf}^{\ 4}} + 0\times\frac{\int_4^\infty e_{b,\lambda}d\lambda}{\sigma T_{surf}^{\ 4}} = 0.6\times\frac{\int_{0.4}^{4} e_{b,\lambda}d\lambda}{\sigma T_{surf}^{\ 4}}$$

$$\frac{\int_{0.4}^{4} e_{b,\lambda} d\lambda}{\sigma T_{surf}^{\ 4}} = \frac{\int_{0}^{4.0} e_{b,\lambda} d\lambda}{\sigma T^4} - \frac{\int_{0}^{0.4} e_{b,\lambda} d\lambda}{\sigma T^4} = \frac{E_b\left(0 \to \lambda T\right)_{\lambda=4.0}}{\sigma T^4} - \frac{E_b\left(0 \to \lambda T\right)_{\lambda=0.4}}{\sigma T^4}$$

$$\lambda = 4.0\mu\text{m} \Rightarrow \lambda T = 4.0 \times 3000 = 12000\ \mu\text{m-K} \xrightarrow{Table\ 8.1} \frac{\underset{0\le\lambda\le 4}{E_b}}{\sigma T^4} = 0.94505$$

$$\lambda = 0.4\mu\text{m} \Rightarrow \lambda T = 0.4 \times 3000 = 1200\ \mu\text{m-K} \xrightarrow{Table\ 8.1} \frac{\underset{0\le\lambda\le 0.4}{E_b}}{\sigma T^4} = 0.00213$$

$$\frac{\int_{0.4}^{4} e_{b,\lambda} d\lambda}{\sigma T_{surf}^{\ 4}} = \frac{E_b\left(0 \to \lambda T\right)_{\lambda=4.0}}{\sigma T^4} - \frac{E_b\left(0 \to \lambda T\right)_{\lambda=0.4}}{\sigma T^4} = 0.94505 - 0.00213 = 0.94292$$

$$\varepsilon = 0.6 \times \frac{\int_{0.4}^{4} e_{b,\lambda} d\lambda}{\sigma T_{surf}^{\ 4}} = 0.6 \times 0.94292 = 0.5657$$

b)

$$\alpha = \frac{\int_0^\infty \alpha_\lambda G_\lambda d\lambda}{\int_0^\infty G_\lambda d\lambda}$$ from Kirchoff's Law: $\boxed{\alpha_\lambda = \varepsilon_\lambda}$; and also $G_\lambda = \left(e_{b,\lambda}\right)_{\text{from a black surface at 1000K}}$, thus,

$$\alpha = 0.6 \times \left(\frac{\int_{0.4}^{4} e_{b,\lambda} d\lambda}{\int_{0}^{\infty} e_{b,\lambda} d\lambda} \right)_{T=1000K} = \left(\frac{\int_{0.4}^{4} e_{b,\lambda} d\lambda}{\sigma T^4} \right)_{T=1000K} = \left[\frac{E_b\left(0 \to \lambda T\right)_{\lambda=4.0}}{\sigma T^4} - \frac{E_b\left(0 \to \lambda T\right)_{\lambda=0.4}}{\sigma T^4} \right]_{T=1000K}$$

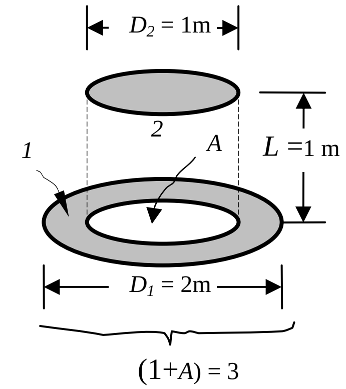
$$\lambda = 4.0\mu\text{m} \Rightarrow \lambda T = 4.0 \times 1000 = 4000\ \mu\text{m-K} \xrightarrow{Table\ 8.1} \frac{\underset{0\le\lambda\le 4}{E_b}}{\sigma T^4} = 0.48085$$

$$\lambda = 0.4\mu\text{m} \Rightarrow \lambda T = 0.4 \times 1000 = 400\ \mu\text{m-K} \xrightarrow{Table\ 8.1} \frac{\underset{0\le\lambda\le 0.4}{E_b}}{\sigma T^4} \simeq 0$$

$$\frac{\int_{0.4}^{4} e_{b,\lambda} d\lambda}{\sigma T^4} = \frac{E_b\left(0 \to \lambda T\right)_{\lambda=4.0}}{\sigma T^4} - \frac{E_b\left(0 \to \lambda T\right)_{\lambda=0.4}}{\sigma T^4} = 0.48085 - 0.0 = 0.48085$$

$$\alpha = 0.6 \times \frac{\int_{0.4}^{4} e_{b,\lambda} d\lambda}{\sigma T^4} = 0.6 \times 0.48085 = 0.2885$$

Problem 3:



Let call surface 1 + surface A as surface 3 which is a large circle with diameter of 2 m.

$$\begin{bmatrix}\text{total radiation from}\\ \text{surface 2 intercepted}\\ \text{by surface 3}\end{bmatrix}=\begin{bmatrix}\text{total radiation from}\\ \text{surface 2 intercepted}\\ \text{by surface A}\end{bmatrix}+\begin{bmatrix}\text{total radiation from}\\ \text{surface 2 intercepted}\\ \text{by surface 1}\end{bmatrix}$$

or

$$\cancel{q_2}F_{2-3}=\cancel{q_2}F_{2-A}+\cancel{q_2}F_{2-1}\Rightarrow F_{2-3}=F_{2-A}+F_{2-1}$$

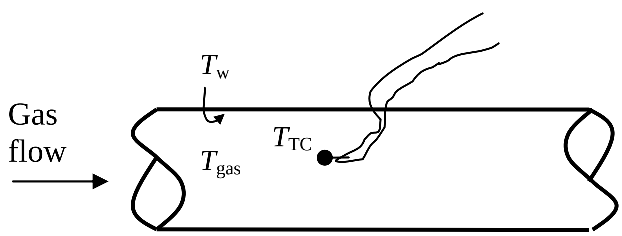
Using Figure 8-16: $L/r_1=1; r_2/L=0.5\rightarrow F_{3-2}\simeq 0.13\Rightarrow F_{2-3}=\dfrac{A_3}{A_2}F_{3-2}=\dfrac{\pi\times 2^2/4}{\pi\times 1^2/4}0.13=0.52$

Using Figure 8-13: $d/x=1\Rightarrow F_{2-A}\simeq 0.17$

$$F_{2-3}=F_{2-A}+F_{2-1}\Rightarrow F_{2-1}=0.52-0.17=0.35$$

$$\Rightarrow F_{1-2}=\frac{A_2}{A_1}F_{2-1}=\frac{\pi\times 1^2/4}{\pi\times\left(2^2-1^2\right)/4}0.35=0.1167$$

Problem 4:



Assumption: With respect to the thermocouple (TC) tip, the pipe wall behaves as a large isothermal enclosure at

T_w = 100°C.

Given: h = 250 W/m^2-°C; T_{TC} = 500°C;

ε_{TC} = 0.5

Steady state E-balance on the TC tip:

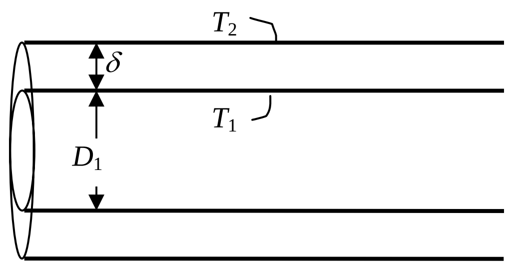
$$q_{\substack{net\ to\\ TC}}=0\Rightarrow q_{\substack{conv\\ TC-gas}}-q_{\substack{rad\\ TC-Wall}}=0$$

$$\rightarrow \cancel{A}_{TC}h\left(T_{gas}-T_{TC}\right)=\cancel{A}_{TC}\varepsilon_{TG}\sigma\left(T_{TC}{}^4-T_{wall}{}^4\right)$$

$$\Rightarrow T_{gas}=T_{TC}+\frac{\varepsilon_{TG}\sigma\left(T_{TC}{}^4-T_{wall}{}^4\right)}{h}$$

$$T_{gas}=773.15+\frac{0.5\times 5.669\times 10^{-8}\times\left(773.15^4-373.15^4\right)}{250}=811.46\text{ K}=538.31°C$$

Problem 5:



Assumption: Long tube; both the radiation shield and the tube surface behaves as gray surfaces; steady-state conditions prevail

Given: D_1 = 100 mm, T_1 = 120°C; gap: δ = 10 mm; T_2 = 35°C; ε_1=0.8, and ε_2=0.1.

Using the relation provided in Figure 8-30 of Holman 2002 (Handout # 10):

$$q = \frac{\sigma A_1\left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \left(\frac{1}{\varepsilon_2} - 1\right)\frac{r_1}{r_2}} = \frac{5.669\times10^{-8}\times\left(\pi\times0.1\times L\right)\left(393.15^4 - 308.15^4\right)}{\frac{1}{0.8} + \left(\frac{1}{0.1} - 1\right)\frac{0.05}{\left(0.05+0.01\right)}}$$

$$\Rightarrow \frac{q}{L} = 30.2 \text{ W/m}$$