University of British Columbia
Department of Mechanical Engineering

MECH366 Modeling of Mechatronic Systems
Final exam

Examiner: Dr. Ryozo Nagamune
December 13 (Wednesday), 2017, noon-2:30pm

Last name, First name

Name: Student #:

Signature:

Exam policies

- Allowed: One-page letter-size hand-written cheat-sheet (both sides).
- Not-allowed: PC, calculators.
- Write all your answers on this booklet. No extra sheet will be provided.
- Motivate your answers properly. (No chance to defend your answers orally.)
- 50 points in total.

Before you start ...

- Use washroom before the exam.
- Turn off your mobile phone.
- No eating.
- Questions are NOT allowed.

If you finish early ...

- If you would like to leave the room **before 2:20pm**, **raise your hand with this booklet**, and **wait at your seat** until an invigilator comes to you and collects your exam booklet.

To be filled in by the instructor/marker

Problem #	Mark	Full mark
1		10
2		10
3		10
4		10
5		10
Total		50

Extra page. Write the problem number before writing your answer.

Extra page. Write the problem number before writing your answer.

1. Select **only one** correct statement, by **circling one of the numbers i, ii, iii, or iv**, for the following sentences. **No need to motivate your answers**. (1pt each)

   (a) A mathematical model is useful for:

      i. analyzing system properties.
      ii. designing feedback controllers.
      iii. predicting system responses for excitation signals.
      iv. All of i, ii, iii.

   (b) Which of the following can be a through variable?

      i. pressure
      ii. voltage
      iii. temperature
      iv. None of i, ii, iii.

   (c) For thermal systems, which of the following defines power?

      i. heat transfer rate
      ii. temperature
      iii. heat transfer rate times temperature
      iv. None of i, ii, iii.

   (d) For fluid systems, which of the following defines power?

      i. mass flow rate
      ii. pressure
      iii. mass flow rate times pressure
      iv. None of i, ii, iii.

   (e) For fluid systems, which of the following elements stores kinetic energy?

      i. fluid capacitance
      ii. fluid inerter
      iii. fluid resistance
      iv. None of i, ii, iii.

   (f) A loop equation in a linear graph is:

      i. a balance of across variables.
      ii. a balance of through variables.
      iii. a constitutive equation.
      iv. None of i, ii, iii.

(g) Which of the following transfer functions has the largest steady-state value for a unit step input?

i. $G_1(s) = \dfrac{1}{s^2 + s + 1}$

ii. $G_2(s) = \dfrac{1}{s^2 + s + 10}$

iii. $G_3(s) = \dfrac{1}{s^2 + s + 100}$

iv. $G_1(s)$, $G_2(s)$ and $G_3(s)$ has the same steady-state value.

(h) Which of the following transfer functions has the shortest peak time for a unit step input?

i. $G_1(s) = \dfrac{1}{s^2 + s + 1}$

ii. $G_2(s) = \dfrac{1}{s^2 + s + 10}$

iii. $G_3(s) = \dfrac{1}{s^2 + s + 100}$

iv. $G_1(s)$, $G_2(s)$ and $G_3(s)$ has the same peak time.

(i) Which of the following transfer functions has the smallest percent overshoot for a unit step input?

i. $G_1(s) = \dfrac{1}{s^2 + s + 1}$

ii. $G_2(s) = \dfrac{1}{s^2 + s + 10}$

iii. $G_3(s) = \dfrac{1}{s^2 + s + 100}$

iv. $G_1(s)$, $G_2(s)$ and $G_3(s)$ has the same percent overshoot.

(j) Which of the following transfer functions has the shortest settling time for a unit step input?

i. $G_1(s) = \dfrac{1}{s^2 + s + 1}$

ii. $G_2(s) = \dfrac{1}{s^2 + s + 10}$

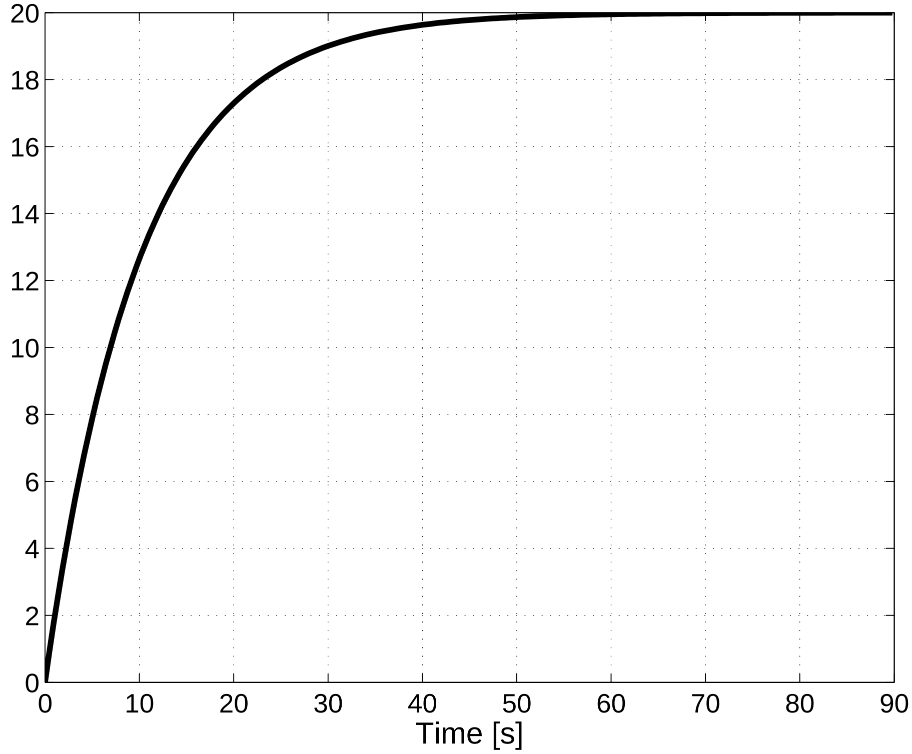
iii. $G_3(s) = \dfrac{1}{s^2 + s + 100}$

iv. $G_1(s)$, $G_2(s)$ and $G_3(s)$ has the same settling time.

2. Answer the following questions on system identification and system analysis.

   (a) For the following response to a step input with amplitude 2, estimate the corresponding first-order transfer function. (2pt)

   (b) For the transfer function estimated in (a), sketch the Bode plot using the straight-line approximation. (3pt)

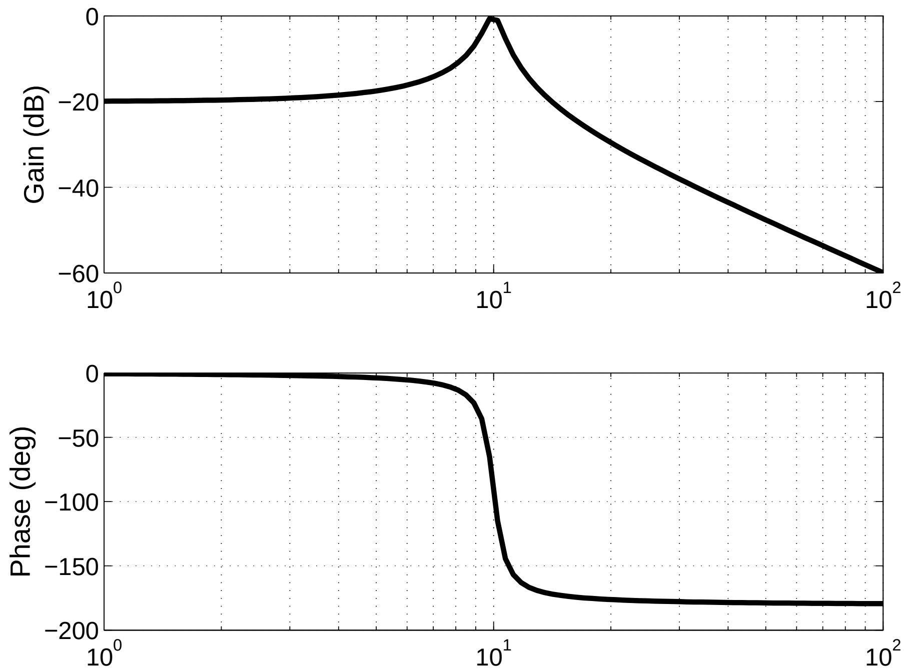


Write your answer here.

(c) For the following Bode plot, estimate the corresponding <u>second-order</u> transfer function, with damping ratio $\zeta = 0.05$. (2pt)

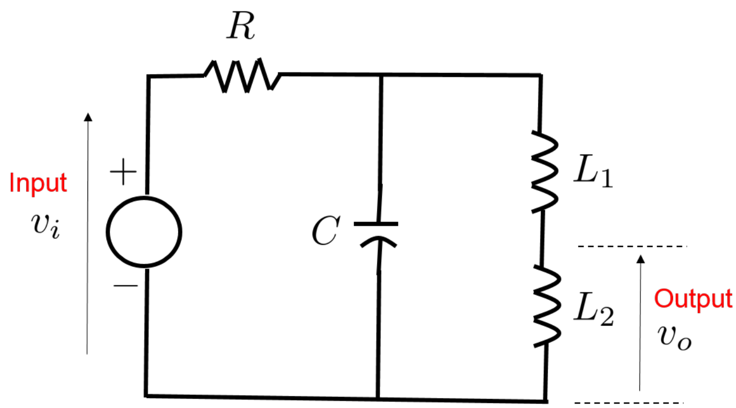
(d) For the transfer function estimated in (c), plot roughly the response to a unit step input. In the plot, indicate the **steady-state value**, **2% settling time**, and **peak time**. There is **no need** to obtain the **percent overshoot**. (3pt)

<u>Hint</u>: Complicated calculations (such as partial fraction expansion or inverse Laplace transform) are NOT necessary for this plotting.



<u>**Write your answer here.**</u>

3. Consider the following electric circuit. Here, R is the resistance, C is the capacitance, and L_1 and L_2 are the inductances. The input is the voltage v_i and the output is the voltage v_o (i.e., voltage across L_2).



(a) Draw the linear graph. (2pt)

(b) Using the linear graph, obtain a state-space model <u>with two states</u>. (3pt)

Write your answer here.

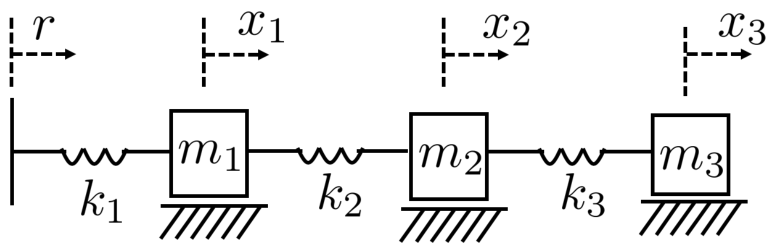
(c) Using any method, obtain a state-space model with two states, which is different from the model obtained in (b). (2pt)

(d) For the state-space model obtained in (b), derive the corresponding transfer function. (3pt)

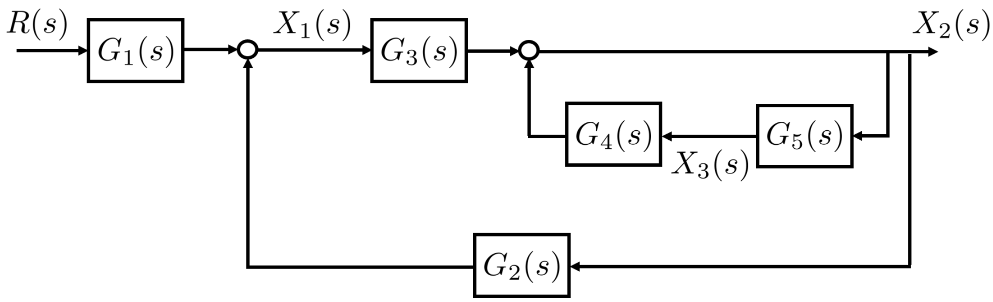
Hint: $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \dfrac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Write your answer here.

4. Consider the three degrees-of-freedom mass-spring system below. Here, m_1, m_2 and m_3 [kg] are masses, and k_1, k_2 and k_3 [N/m] are spring constants. The signals x_1, x_2 and x_3 [m] are displacements. The signal r [m] is the displacement input. Capital letters (for example, $R(s)$) denote the Laplace transform of signals (for example, $r(t)$). Ignore the damping and friction.



(a) The block diagram for this system can be depicted as below. Obtain the transfer functions $G_i(s)$, $i = 1, 2, 3, 4, 5$. (5pt)



<u>**Write your answer here.**</u>

(b) Using the notations $G_i(s)$, $i = 1, 2, 3, 4, 5$, obtain:

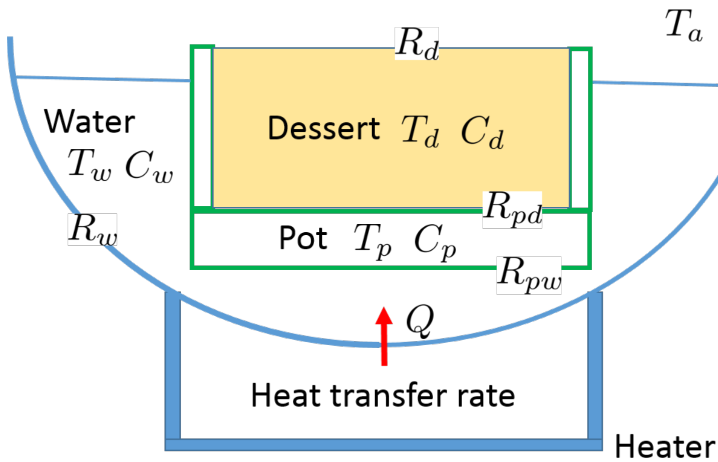
i. the transfer function from $X_1(s)$ to $X_2(s)$. (You can **name this transfer function as $G_6(s)$, and use in questions below**.) (2pt)

ii. the transfer function from $R(s)$ to $X_2(s)$. (2pt)

iii. the transfer function from $R(s)$ to $X_3(s)$. (1pt)

Write your answer here.

5. Consider the thermal system shown below, where the **inputs** are the heat transfer rate Q from the heater to the water and the ambient temperature T_a, and the **output** is the dessert temperature T_d. (This is exactly the dessert making problem that you have seen in class.) Assume that the water and the dessert are well-stirred and have uniform temperatures.



The notations in the figure are given in the table below.

element	temperature	thermal capacitance
water	T_w	C_w
pot	T_p	C_p
dessert	T_d	C_d

R_w : thermal resistance between water and ambient air

R_d : thermal resistance between dessert and ambient air

R_{pw} : thermal resistance between pot and water

R_{pd} : thermal resistance between pot and dessert

(a) Draw a linear graph. (4pt)

(b) Select state variables. (2pt)

(c) Write constitutive equations. (2pt)

(d) Write node equations. (2pt)

——— (End of Final Exam) ———

Write your answer here.

Write your answer here.

Write your answer here.

Extra page. Write the problem number before writing your answer.