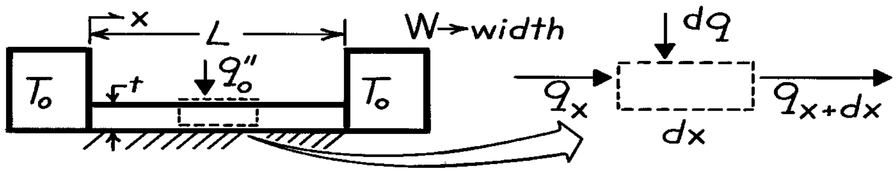
PROBLEM 3.101

KNOWN: Dimensions of a plate insulated on its bottom and thermally joined to heat sinks at its ends. Net heat flux at top surface.

FIND: (a) Differential equation which determines temperature distribution in plate, (b) Temperature distribution and heat loss to heat sinks.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional conduction in x (W,L>>t), (3) Constant properties, (4) Uniform surface heat flux, (5) Adiabatic bottom, (6) Negligible contact resistance.

ANALYSIS: (a) Applying conservation of energy to the differential control volume, $q_x + dq = q_{x+dx}$, where $q_{x+dx} = q_x + (dq_x/dx)\,dx$ and $dq = q_o''\,(W \cdot dx)$. Hence, $(dq_x/dx) - q_o''\,W = 0$. From Fourier's law, $q_x = -k(t \cdot W)\,dT/dx$. Hence, the differential equation for the temperature distribution is

$$-\frac{d}{dx}\left[ktW\,\frac{dT}{dx}\right] - q_o''\,W = 0 \qquad \frac{d^2T}{dx^2} + \frac{q_o''}{kt} = 0. \qquad <$$

(b) Integrating twice, the general solution is,

$$T(x) = -\frac{q_o''}{2kt}x^2 + C_1\,x + C_2$$

and appropriate boundary conditions are $T(0) = T_o$, and $T(L) = T_o$. Hence, $T_o = C_2$, and

$$T_o = -\frac{q_o''}{2kt}L^2 + C_1 L + C_2 \qquad \text{and} \qquad C_1 = \frac{q_o''L}{2kt}.$$

Hence, the temperature distribution is

$$T(x) = -\frac{q_o''L}{2kt}\left(x^2 - Lx\right) + T_o. \qquad <$$

Applying Fourier's law at x = 0, and at x = L,

$$q(0) = -k(Wt)\;dT/dx)_{x=0} = -kWt\left[-\frac{q_o''}{kt}\right]\left[x - \frac{L}{2}\right]\Bigg|_{x=0} = -\frac{q_o''WL}{2}$$

$$q(L) = -k(Wt)dT/dx)_{x=L} = -kWt\left[-\frac{q_o''}{kt}\right]\left[x - \frac{L}{2}\right]\Bigg|_{x=L} = +\frac{q_o''WL}{2}$$

Hence the heat loss from the plates is $q = 2\left(q_o''WL/2\right) = q_o''WL$. <

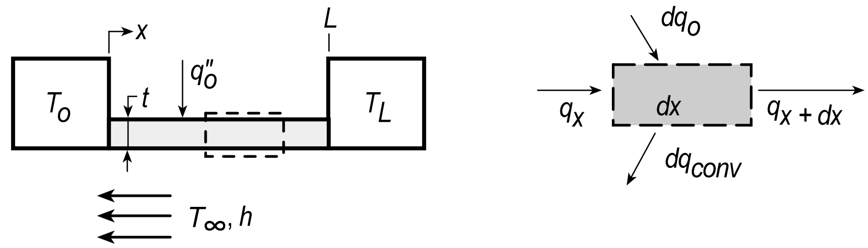
COMMENTS: (1) Note signs associated with q(0) and q(L). (2) Note symmetry about x = L/2. Alternative boundary conditions are $T(0) = T_o$ and $dT/dx)_{x=L/2} = 0$.

PROBLEM 3.102

KNOWN: Dimensions and surface conditions of a plate thermally joined at its ends to heat sinks at different temperatures.

FIND: (a) Differential equation which determines temperature distribution in plate, (b) Temperature distribution and an expression for the heat rate from the plate to the sinks, and (c) Compute and plot temperature distribution and heat rates corresponding to changes in different parameters.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in x (W,L >> t), (3) Constant properties, (4) Uniform surface heat flux and convection coefficient, (5) Negligible contact resistance.

ANALYSIS: (a) Applying conservation of energy to the differential control volume

$$q_x + dq_o = q_{x+dx} + dq_{conv}$$

where

$$q_{x+dx} = q_x + \left(dq_x/dx\right)dx \qquad\qquad dq_{conv} = h\left(T - T_\infty\right)\left(W \cdot dx\right)$$

Hence,

$$q_x + q''_o\left(W \cdot dx\right) = q_x + \left(dq_x/dx\right)dx + h\left(T - T_\infty\right)\left(W \cdot dx\right) \qquad\qquad \frac{dq_x}{dx} + hW\left(T - T_\infty\right) = q''_o W\,.$$

Using Fourier's law, $q_x = -k\left(t \cdot W\right)dT/dx$,

$$-ktW\frac{d^2T}{dx^2} + hW\left(T - T_\infty\right) = q''_o \qquad\qquad \frac{d^2T}{dx^2} - \frac{h}{kt}\left(T - T_\infty\right) + \frac{q''_o}{kt} = 0\,.$$ <

(b) Introducing $\theta \equiv T - T_\infty$, the differential equation becomes

$$\frac{d^2\theta}{dx^2} - \frac{h}{kt}\theta + \frac{q''_o}{kt} = 0\,.$$

This differential equation is of second order with constant coefficients and a source term. With $\lambda^2 \equiv h/kt$ and $S \equiv q''_o/kt$, it follows that the general solution is of the form

$$\theta = C_1 e^{+\lambda x} + C_2 e^{-\lambda x} + S/\lambda^2\,. \tag{1}$$

Appropriate boundary conditions are: $\theta(0) = T_o - T_\infty \equiv \theta_o \qquad \theta(L) = T_L - T_\infty \equiv \theta_L$ (2,3)

Substituting the boundary conditions, Eqs. (2,3) into the general solution, Eq. (1),

$$\theta_o = C_1 e^0 + C_2 e^0 + S/\lambda^2 \qquad\qquad \theta_L = C_1 e^{+\lambda L} + C_2 e^{-\lambda L} + S/\lambda^2 \tag{4,5}$$

To solve for C_2, multiply Eq. (4) by $-e^{+\lambda L}$ and add the result to Eq. (5),

$$-\theta_o e^{+\lambda L} + \theta_L = C_2\left(-e^{+\lambda L} + e^{-\lambda L}\right) + S/\lambda^2\left(-e^{+\lambda L} + 1\right)$$

$$C_2 = \left[\left(\theta_L - \theta_o e^{+\lambda L}\right) - S/\lambda^2\left(-e^{+\lambda L} + 1\right)\right] \Big/ \left(-e^{+\lambda L} + e^{-\lambda L}\right) \tag{6}$$

Continued...

PROBLEM 3.102 (Cont.)

Substituting for C_2 from Eq. (6) into Eq. (4), find

$$C_1 = \theta_o - \left\{ \left[\left(\theta_L - \theta_o e^{+\lambda L} \right) - S/\lambda^2 \left(-e^{+\lambda L} + 1 \right) \right] \Big/ \left(-e^{+\lambda L} + e^{-\lambda L} \right) \right\} - S/\lambda^2 \qquad (7)$$

Using C_1 and C_2 from Eqs. (6,7) and Eq. (1), the temperature distribution can be expressed as

$$\theta(x) = \left[e^{+\lambda x} - \frac{\sinh(\lambda x)}{\sinh(\lambda L)} e^{+\lambda L} \right] \theta_o + \frac{\sinh(\lambda x)}{\sinh(\lambda L)} \theta_L + \left[-\left(1 - e^{+\lambda L}\right) \frac{\sinh(\lambda x)}{\sinh(\lambda L)} + \left(1 - e^{+\lambda L}\right) \right] \frac{S}{\lambda^2} \quad (8)$$

<

The heat rate from the plate is $q_p = -q_x(0) + q_x(L)$ and using Fourier's law, the conduction heat rates, with $A_c = W \cdot t$, are

$$q_x(0) = -kA_c \left.\frac{d\theta}{dx}\right)_{x=0} = -kA_c \left\{ \left[\lambda e^0 - \frac{e^{\lambda L}}{\sinh(\lambda L)} \lambda \right] \theta_o + \frac{\lambda}{\sinh(\lambda L)} \theta_L + \left[-\frac{1 - e^{+\lambda L}}{\sinh(\lambda L)} \lambda - \lambda \right] \frac{S}{\lambda^2} \right\}$$

<

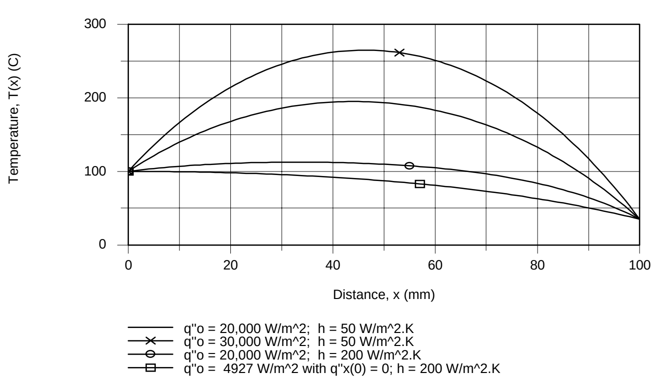
$$q_x(L) = -kA_c \left.\frac{d\theta}{dx}\right)_{x=L} = -kA_c \left\{ \left[\lambda e^{\lambda L} - \frac{e^{\lambda L}}{\sinh(\lambda L)} \lambda \cosh(\lambda L) \right] \theta_o + \frac{\lambda \cosh(\lambda L)}{\sinh(\lambda L)} \theta_L + \left[-\frac{1 - e^{+\lambda L}}{\sinh(\lambda L)} \lambda \cosh(\lambda L) - \lambda e^{+\lambda L} \right] \frac{S}{\lambda^2} \right\}$$

<

(c) For the prescribed base-case conditions listed below, the temperature distribution (solid line) is shown in the accompanying plot. As expected, the maximum temperature does not occur at the midpoint, but slightly toward the x-origin. The sink heat rates are

$$q''_x(0) = -17.22\,\text{W} \qquad\qquad q''_x(L) = 23.62\,\text{W}$$

<



The additional temperature distributions on the plot correspond to changes in the following parameters, with all the remaining parameters unchanged: (i) q''_o = 30,000 W/m^2, (ii) h = 200 W/m^2·K, (iii) the value of q''_o for which $q''_x(0)$ = 0 with h = 200 W/m^2·K. The condition for the last curve is q''_o = 4927 W/m^2 for which the temperature gradient at x = 0 is zero.

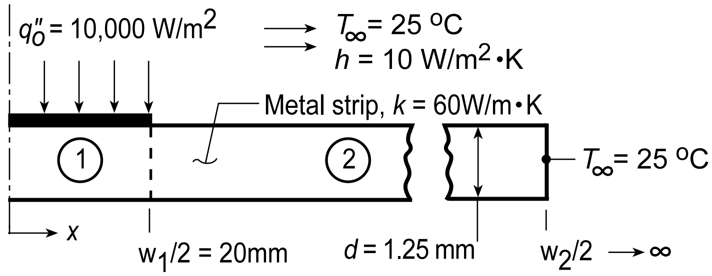
Base case conditions are: q''_o = 20,000 W/m^2, T_o = 100°C, T_L = 35°C, T_∞ = 25°C, k = 25 W/m·K, h = 50 W/m^2·K, L = 100 mm, t = 5 mm, W = 30 mm.

PROBLEM 3.103

KNOWN: Thin plastic film being bonded to a metal strip by laser heating method; strip dimensions and thermophysical properties are prescribed as are laser heating flux and convection conditions.

FIND: (a) Expression for temperature distribution for the region with the plastic strip, $-w_1/2 \le x \le w_1/2$, (b) Temperature at the center (x = 0) and the edge of the plastic strip ($x = \pm w_1/2$) when the laser flux is 10,000 W/m^2; (c) Plot the temperature distribution for the strip and point out special features.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in x-direction only, (3) Plastic film has negligible thermal resistance, (4) Upper and lower surfaces have uniform convection coefficients, (5) Edges of metal strip are at air temperature (T_∞), that is, strip behaves as infinite fin so that $w_2 \to \infty$, (6) All the incident laser heating flux q_o'' is absorbed by the film.

PROPERTIES: Metal strip (given): ρ = 7850 kg/m^3, c_p = 435 J/kg·m^3, k = 60 W/m·K.

ANALYSIS: (a) The strip-plastic film arrangement can be modeled as an infinite fin of uniform cross section a portion of which is exposed to the laser heat flux on the upper surface. The general solutions for the two regions of the strip, in terms of $\theta \equiv T(x) - T_\infty$, are

$$0 \le x \le w_1/2 \qquad \theta_1(x) = C_1 e^{+mx} + C_2 e^{-mx} + M/m^2 \tag{1}$$

$$M = q_o'' P/2kA_c = q_o''/kd \qquad m = (2h/kd)^{1/2} \tag{2,3}$$

$$w_1/2 \le x \le \infty \qquad \theta_2(x) = C_3 e^{+mx} + C_4 e^{-mx}. \tag{4}$$

Four boundary conditions can be identified to evaluate the constants:

At x = 0: $$\frac{d\theta_1}{dx}(0) = 0 = C_1 m e^0 - C_2 m e^{-0} + 0 \quad \to \quad C_1 = C_2 \tag{5}$$

At x = w_1/2: $$\theta(w_1/2) = \theta_2(w_1/2)$$

$$C_1 e^{+mw_1/2} + C_2 e^{-mw_1/2} + M/m^2 = C_3 e^{+mw_1/2} + C_4 e^{-mw_1/2} \tag{6}$$

At x = w_1/2: $$d\theta_1(w_1/2)/dx = d\theta_2(w_1/2)/dx$$

$$mC_1 e^{+mw_1/2} - mC_2 e^{-mw_1/2} + 0 = mC_3 e^{+mw_1/2} - mC_4 e^{-mw_1/2} \tag{7}$$

At x → ∞: $$\theta_2(\infty) = 0 = C_3 e^{\infty} + C_4 e^{-\infty} \quad \to \quad C_3 = 0 \tag{8}$$

With $C_3 = 0$ and $C_1 = C_2$, combine Eqs. (6 and 7) to eliminate C_4 to find

$$C_1 = C_2 = -\frac{M/m^2}{2e^{mw_1/2}}. \tag{9}$$

and using Eq. (6) with Eq. (9) find

$$C_4 = M/m^2 \sinh(mw_1/2) e^{-mx_1/2} \tag{10}$$

Continued...

PROBLEM 3.103 (Cont.)

Hence, the temperature distribution in the region (1) under the plastic film, $0 \le x \le w_1/2$, is

$$\theta_1(x) = -\frac{M/m^2}{2e^{mw_1/w}}\left(e^{+mx} + e^{-mx}\right) + \frac{M}{m^2} = \frac{M}{m^2}\left(1 - e^{-mw_1/2}\cosh mx\right) \qquad (11)$$ <

and for the region (2), $x \ge w_1/2$,

$$\theta_2(x) = \frac{M}{m^2}\sinh(mw_1/2)e^{-mx} \qquad (12)$$

(b) Substituting numerical values into the temperature distribution expression above, $\theta_1(0)$ and $\theta_1(w_1/2)$ can be determined. First evaluate the following parameters:

$$M = 10,000\ \mathrm{W/m^2} / 60\ \mathrm{W/m\cdot K} \times 0.00125\ \mathrm{m} = 133,333\ \mathrm{K/m^2}$$

$$m = \left(2 \times 10\ \mathrm{W/m^2\cdot K} / 60\ \mathrm{W/m\cdot K} \times 0.00125\ \mathrm{m}\right)^{1/2} = 16.33\ \mathrm{m^{-1}}$$

Hence, for the midpoint x = 0,

$$\theta_1(0) = \frac{133,333\ \mathrm{K/m^2}}{\left(16.33\ \mathrm{m^{-1}}\right)^2}\left[1 - \exp\left(-16.33\ \mathrm{m^{-1}} \times 0.020\ \mathrm{m}\right) \times \cosh(0)\right] = 139.3\ \mathrm{K}$$

$$T_1(0) = \theta_1(0) + T_\infty = 139.3\ \mathrm{K} + 25^\circ\mathrm{C} = 164.3^\circ\mathrm{C}.$$ <

For the position $x = w_1/2 = 0.020$ m,

$$\theta_1(w_1/2) = 500.0\left[1 - 0.721\cosh\left(16.33\ \mathrm{m^{-1}} \times 0.020\ \mathrm{m}\right)\right] = 120.1\ \mathrm{K}$$

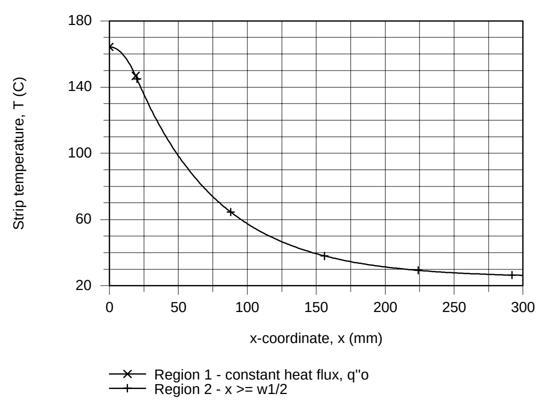
$$T_1(w_1/2) = 120.1\ \mathrm{K} + 25^\circ\mathrm{C} = 145.1^\circ\mathrm{C}.$$ <

(c) The temperature distributions, $\theta_1(x)$ and $\theta_2(x)$, are shown in the plot below. Using IHT, Eqs. (11) and (12) were entered into the workspace and a graph created. The special features are noted:

(1) No gradient at midpoint, x = 0; symmetrical distribution.

(2) No discontinuity of gradient at $w_1/2$ (20 mm).

(3) Temperature excess and gradient approach zero with increasing value of x.



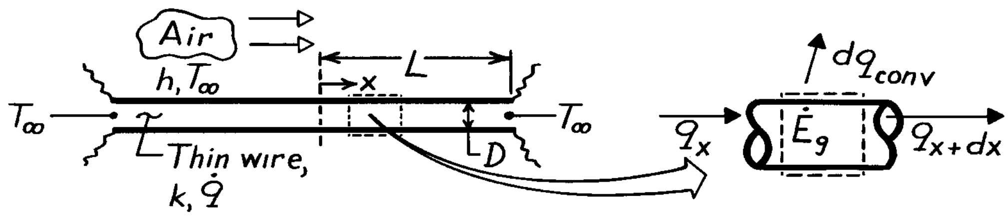
COMMENTS: How wide must the strip be in order to satisfy the infinite fin approximation such that $\theta_2(x \to \infty) = 0$? For x = 200 mm, find θ_2(200 mm) = 6.3°C; this would be a poor approximation. When x = 300 mm, θ_2(300 mm) = 1.2°C; hence when $w_2/2$ = 300 mm, the strip is a reasonable approximation to an infinite fin.

PROBLEM 3.104

KNOWN: Thermal conductivity, diameter and length of a wire which is annealed by passing an electrical current through the wire.

FIND: Steady-state temperature distribution along wire.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction along the wire, (3) Constant properties, (4) Negligible radiation, (5) Uniform convection coefficient h.

ANALYSIS: Applying conservation of energy to a differential control volume,

$$q_x + \dot{E}_g - dq_{conv} - q_{x+dx} = 0$$

$$q_{x+dx} = q_x + \frac{dq_x}{dx}dx \qquad q_x = -k\left(\pi\ D^2/4\right)dT/dx$$

$$dq_{conv} = h\left(\pi\ D\ dx\right)\ \left(T - T_\infty\right) \qquad \dot{E}_g = \dot{q}\left(\pi\ D^2/4\right)dx.$$

Hence,

$$k\left(\pi\ D^2/4\right)\ \frac{d^2T}{dx^2}dx + \dot{q}\left(\pi\ D^2/4\right)dx - h\left(\pi\ Ddx\right)\ \left(T - T_\infty\right) = 0$$

or, with $\theta \equiv T - T_\infty$,
$$\frac{d^2\theta}{dx^2} - \frac{4h}{kD}\theta + \frac{\dot{q}}{k} = 0$$

The solution (general and particular) to this nonhomogeneous equation is of the form

$$\theta = C_1\ e^{mx} + C_2\ e^{-mx} + \frac{\dot{q}}{km^2}$$

where $m^2 = (4h/kD)$. The boundary conditions are:

$$\left.\frac{d\theta}{dx}\right]_{x=0} = 0 = m\ C_1\ e^0 - mC_2\ e^0 \rightarrow \qquad C_1 = C_2$$

$$\theta\left(L\right) = 0 = C_1\left(e^{mL} + e^{-mL}\right) + \frac{\dot{q}}{km^2} \rightarrow \qquad C_1 = \frac{-\dot{q}/km^2}{e^{mL} + e^{-mL}} = C_2$$

The temperature distribution has the form

$$T = T_\infty - \frac{\dot{q}}{km^2}\left[\frac{e^{mx} + e^{-mx}}{e^{mL} + e^{-mL}} - 1\right] = T_\infty - \frac{\dot{q}}{km^2}\left[\frac{\cosh mx}{\cosh mL} - 1\right]. \qquad <$$

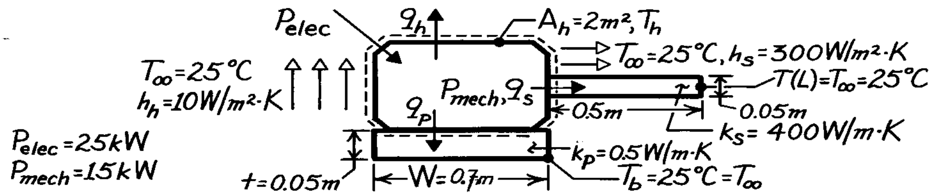
COMMENTS: This process is commonly used to anneal wire and spring products. To check the result, note that T(L) = T(-L) = T_∞.

PROBLEM 3.105

KNOWN: Electric power input and mechanical power output of a motor. Dimensions of housing, mounting pad and connecting shaft needed for heat transfer calculations. Temperature of ambient air, tip of shaft, and base of pad.

FIND: Housing temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in pad and shaft, (3) Constant properties, (4) Negligible radiation.

ANALYSIS: Conservation of energy yields

$$P_{elec} - P_{mech} - q_h - q_p - q_s = 0$$

$$q_h = h_h A_h (T_h - T_\infty), \qquad q_p = k_p W^2 \frac{(T_h - T_\infty)}{t}, \qquad q_s = M \frac{\cosh mL - \theta_L / \theta_b}{\sinh mL}$$

$$\theta_L = 0, \qquad mL = \left(4 h_s L^2 / k_s D\right)^{1/2}, \qquad M = \left(\frac{\pi^2}{4} D^3 h_s k_s\right)^{1/2} (T_h - T_\infty).$$

Hence

$$q_s = \frac{\left(\left[\pi^2/4\right] D^3 h_s k_s\right)^{1/2} (T_h - T_\infty)}{\tanh\left(4 h_s L^2 / k_s D\right)^{1/2}}$$

Substituting, and solving for $(T_h - T_\infty)$,

$$T_h - T_\infty = \frac{P_{elec} - P_{mech}}{h_h A_h + k_p W^2 / t + \left(\left(\pi^2/4\right) D^3 h_s k_s\right)^{1/2} / \tanh\left(4 h_s L^2 / k_s D\right)^{1/2}}$$

$$\left(\left(\pi^2/4\right) D^3 h_s k_s\right)^{1/2} = 6.08 \text{ W/K}, \qquad \left(4 h_s L^2 / k_s D\right)^{1/2} = 3.87, \qquad \tanh mL = 0.999$$

$$T_h - T_\infty = \frac{(25 - 15) \times 10^3 \text{ W}}{\left[10 \times 2 + 0.5 (0.7)^2 / 0.05 + 6.08 / 0.999\right] \text{W/K}} = \frac{10^4 \text{ W}}{(20 + 4.90 + 6.15) \text{W/K}}$$

$$T_h - T_\infty = 322.1 \text{K} \qquad T_h = 347.1°\text{C}$$ <

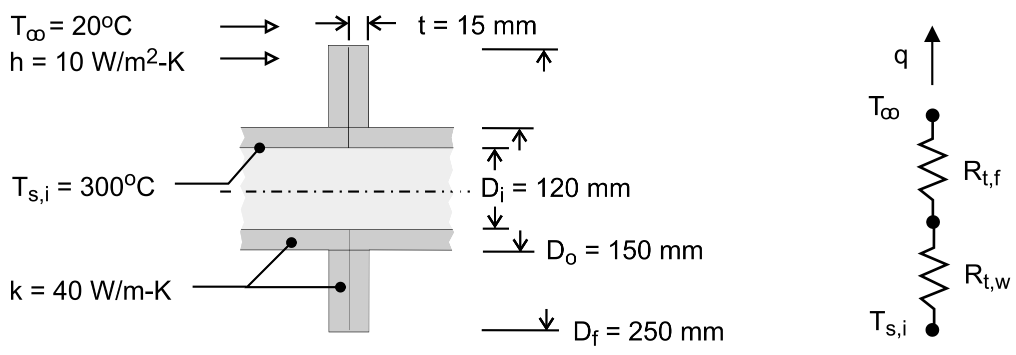
COMMENTS: (1) T_h is large enough to provide significant heat loss by radiation from the housing. Assuming an emissivity of 0.8 and surroundings at 25°C, $q_{rad} = \varepsilon A_h \left(T_h^4 - T_{sur}^4\right)$ = 4347 W, which compares with $q_{conv} = h A_h (T_h - T_\infty) = 5390$ W. Radiation has the effect of decreasing T_h. (2) The infinite fin approximation, $q_s = M$, is excellent.

PROBLEM 3.106

KNOWN: Dimensions and thermal conductivity of pipe and flange. Inner surface temperature of pipe. Ambient temperature and convection coefficient.

FIND: Heat loss through flange.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional radial conduction in pipe and flange, (3) Constant thermal conductivity, (4) Negligible radiation exchange with surroundings.

ANALYSIS: From the thermal circuit, the heat loss through the flanges is

$$q = \frac{T_{s,i} - T_\infty}{R_{t,w} + R_{t,f}} = \frac{T_{s,i} - T_\infty}{\left[\ell n\left(D_o / D_i\right)/4\pi tk\right] + \left(1/hA_f\eta_f\right)}$$

Since convection heat transfer only occurs from one surface of a flange, the connected flanges may be modeled as a single annular fin of thickness $t' = 2t = 30\,\text{mm}$. Hence, $r_{2c} = \left(D_f\,/\,2\right) + t'/2 = 0.140\,\text{m}$, $A_f = 2\pi\left(r_{2c}^2 - r_1^2\right) = 2\pi\left(r_{2c}^2 - D_o/2\right) = 2\pi\left(0.140^2 - 0.06^2\right)\text{m}^2 = 0.101\,\text{m}^2$, $L_c = L + t'/2 =$ $\left(D_f - D_o\right)/2 + t = 0.065\,\text{m}$, $A_p = L_c\,t' = 0.00195\,\text{m}^2$, $L_c^{2/2}\left(h/kA_p\right)^{1/2} = 0.188$. With $r_{2c}/r_1 =$ $r_{2c}/(D_o/2) = 1.87$, Fig. 3.19 yields $\eta_f = 0.94$. Hence,

$$q = \frac{300°\text{C} - 20°\text{C}}{\left[\ell n\left(1.25\right)/4\pi \times 0.03\,\text{m} \times 40\,\text{W/m}\cdot\text{K}\right] + \left(1/10\,\text{W/m}^2\cdot\text{K} \times 0.101\,\text{m}^2 \times 0.94\right)}$$

$$q = \frac{280°\text{C}}{\left(0.0148 + 1.053\right)\text{K/W}} = 262\,\text{W}$$ <

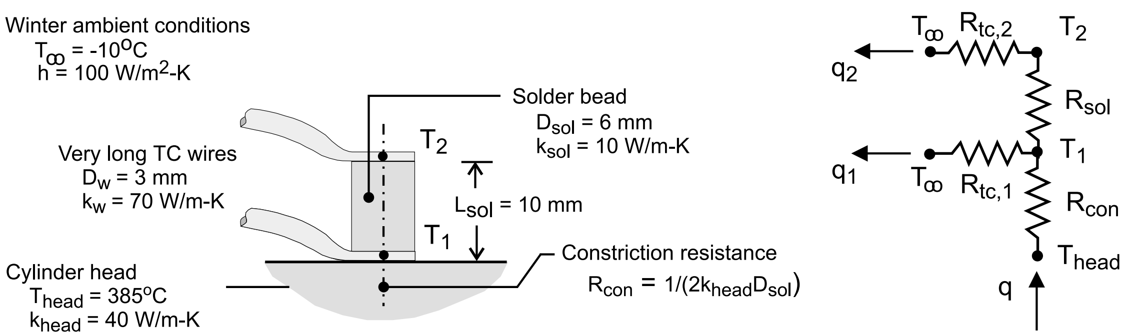
COMMENTS: Without the flange, heat transfer from a section of pipe of width $t' = 2t$ is $q = \left(T_{s,i} - T_\infty\right)/\left(R_{t,w} + R_{t,cnv}\right)$, where $R_{t,cnv} = \left(h \times \pi D_o t'\right)^{-1} = 7.07\,\text{K/W}$. Hence, $q = 39.5\,\text{W}$, and there is significant heat transfer enhancement associated with the extended surfaces afforded by the flanges.

PROBLEM 3.107

KNOWN: TC wire leads attached to the upper and lower surfaces of a cylindrically shaped solder bead. Base of bead attached to cylinder head operating at 350°C. Constriction resistance at base and TC wire convection conditions specified.

FIND: (a) Thermal circuit that can be used to determine the temperature difference between the two intermediate metal TC junctions, ($T_1 - T_2$); label temperatures, thermal resistances and heat rates; and (b) Evaluate ($T_1 - T_2$) for the prescribed conditions. Comment on assumptions made in building the model.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in solder bead; no losses from lateral and top surfaces; (3) TC wires behave as infinite fins, (4) Negligible thermal contact resistance between TC wire terminals and bead.

ANALYSIS: (a) The thermal circuit is shown above. Note labels for the temperatures, thermal resistances and the relevant heat fluxes. The thermal resistances are as follows:

Constriction (con) resistance, see Table 4.1, case 10

$$R_{con} = 1/\left(2k_{bead}D_{sol}\right) = 1/\left(2\times 40\ \text{W/m}\cdot\text{K}\times 0.006\ \text{m}\right) = 2.08\ \text{K/W}$$

TC (tc) wires, infinitely long fins; Eq. 3.80

$$R_{tc,1} = R_{tc,2} = R_{fin} = \left(hPk_wA_c\right)^{-0.5} \qquad P = \pi D_w,\ A_c = \pi D_w^2/4$$

$$R_{tc} = \left(100\ \text{W/m}^2\cdot\text{K}\times\pi^2\times\left(0.003\ \text{m}\right)^3\times 70\ \text{W/m}\cdot\text{K}/4\right)^{-0.5} = 46.31\ \text{K/W}$$

Solder bead (sol), cylinder D_{sol} *and* L_{sol}

$$R_{sol} = L_{sol}/\left(k_{sol}A_{sol}\right) \qquad A_{sol} = \pi D_{sol}^2/4$$

$$R_{sol} = 0.010\ \text{m}/\left(10\ \text{W/m}\cdot\text{K}\times\pi\left(0.006\ \text{m}\right)^2/4\right) = 35.37\ \text{K/W}$$

(b) Perform energy balances on the 1- and 2-nodes, solve the equations simultaneously to find T_1 and T_2, from which ($T_1 - T_2$) can be determined.

Continued …..

Node 1 $$\frac{T_2 - T_1}{R_{sol}} + \frac{T_{head} - T_1}{R_{con}} + \frac{T_\infty - T_1}{R_{tc,1}} = 0$$

Node 2 $$\frac{T_\infty - T_2}{R_{tc,2}} + \frac{T_1 - T_2}{R_{sol}} = 0$$

Substituting numerical values with the equations in the *IHT* Workspace, find

$$T_1 = 359°C \qquad T_2 = 199.2°C \qquad T_1 - T_2 = 160°C$$

COMMENTS: (1) With this arrangement, the TC indicates a systematically low reading of the cylinder head. The size of the solder bead (L_{sol}) needs to be reduced substantially.

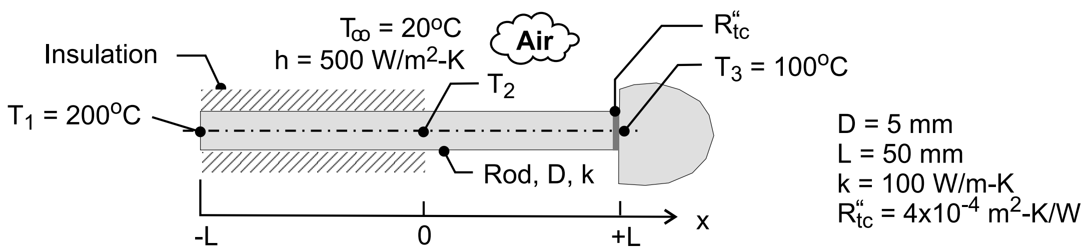
(2) The model neglects heat losses from the top and lateral sides of the solder bead, the effect of which would be to increase our estimate for ($T_1 - T_2$). Constriction resistance is important; note that $T_{head} - T_1$ = 26°C.

PROBLEM 3.108

KNOWN: Rod (D, k, 2L) that is perfectly insulated over the portion of its length $-L \le x \le 0$ and experiences convection (T_∞, h) over the portion $0 \le x \le +L$. One end is maintained at T_1 and the other is separated from a heat sink at T_3 with an interfacial thermal contact resistance R''_{tc}.

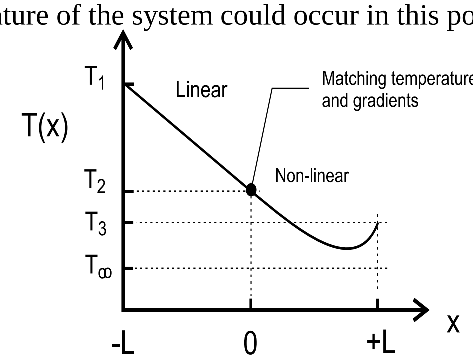
FIND: (a) Sketch the temperature distribution T vs. x and identify key features; assume $T_1 > T_3 > T_2$; (b) Derive an expression for the mid-point temperature T_2 in terms of thermal and geometric parameters of the system, (c) Using, numerical values, calculate T_2 and plot the temperature distribution. Describe key features and compare to your sketch of part (a).

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in rod for $-L \le x \le 0$, (3) Rod behaves as one-dimensional extended surface for $0 \le x \le +L$, (4) Constant properties.

ANALYSIS: (a) The sketch for the temperature distribution is shown below. Over the insulated portion of the rod, the temperature distribution is linear. A temperature drop occurs across the thermal contact resistance at x = +L. The distribution over the exposed portion of the rod is non-linear. The minimum temperature of the system could occur in this portion of the rod.



(b) To derive an expression for T_2, begin with the general solution from the conduction analysis for a fin of uniform cross-sectional area, Eq. 3.66.

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx} \qquad 0 \le x \le +L \tag{1}$$

where $m = (hP/kA_c)^{1/2}$ and $\theta = T(x) - T_\infty$. The arbitrary constants are determined from the boundary conditions.

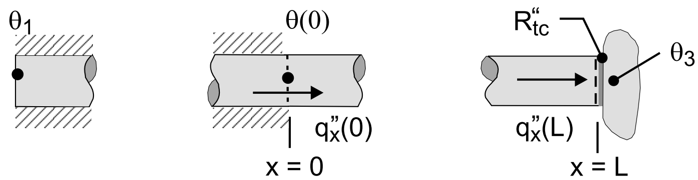
At x = 0, thermal resistance of rod

$$q_x(0) = -kA_c \left.\frac{d\theta}{dx}\right)_{x=0} = kA_c \frac{\theta_1 - \theta(0)}{L} \qquad \theta_1 = T_1 - T_\infty$$

$$m C_1 e^0 - m C_2 e^0 = \frac{1}{L}\left[\theta_1 - \left(C_1 e^0 + C_2 e^0\right)\right] \tag{2}$$

Continued …..

PROBLEM 3.108 (Cont.)



At x=L, thermal contact resistance

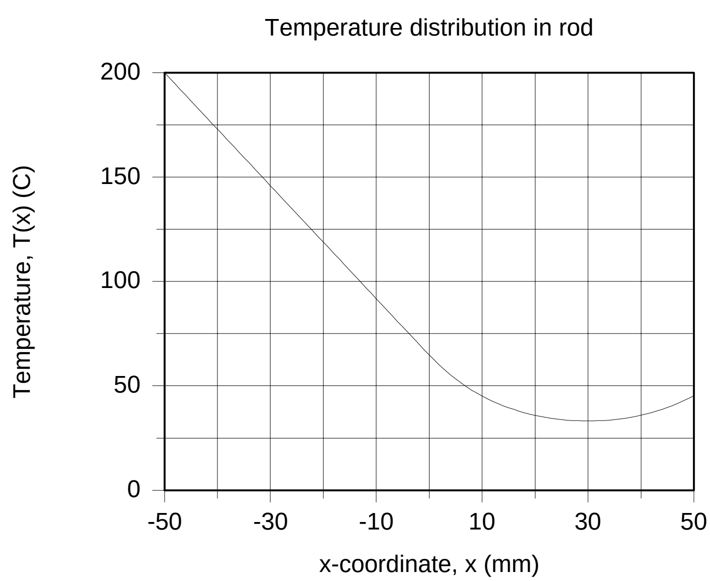
$$q_x(+L) = -kA_c \left.\frac{d\theta}{dx}\right)_{x=L} = \frac{\theta(L)-\theta_3}{R''_{tc}/A_c} \qquad \theta_3 = T_3 - T_\infty$$

$$-k\left[mC_1e^{mL} - mC_2e^{-mL}\right] = \frac{1}{R''_{tc}}\left[C_1e^{mL} + C_2e^{-mL} - \theta_3\right] \tag{3}$$

Eqs. (2) and (3) cannot be rearranged easily to find explicit forms for C_1 and C_2. The constraints will be evaluated numerically in part (c). Knowing C_1 and C_2, Eq. (1) gives

$$\theta_2 = \theta(0) = T_2 - T_\infty = C_1e^0 + C_2e^0 \tag{4}$$

(c) With Eqs. (1-4) in the *IHT Workspace* using numerical values shown in the schematic, find T_2 = 62.1°C. The temperature distribution is shown in the graph below.



COMMENTS: (1) The purpose of asking you to sketch the temperature distribution in part (a) was to give you the opportunity to identify the relevant thermal processes and come to an understanding of the system behavior.

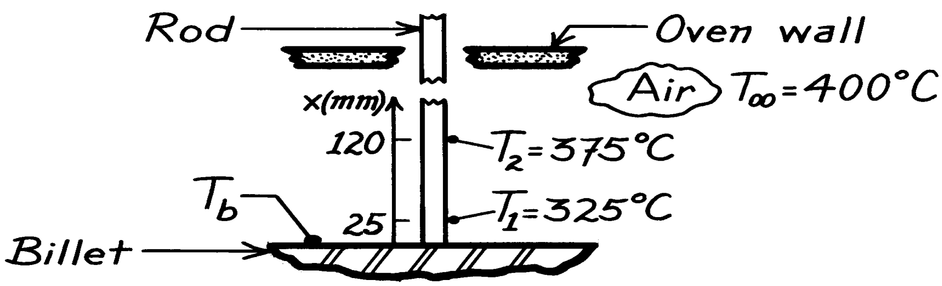
(2) Sketch the temperature distributions for the following conditions and explain their key features: (a) $R''_{tc} = 0$, (b) $R''_{tc} \to \infty$, and (c) the exposed portion of the rod behaves as an infinitely long fin; that is, k is very large.

PROBLEM 3.109

KNOWN: Long rod in oven with air temperature at 400°C has one end firmly pressed against surface of a billet; thermocouples imbedded in rod at locations 25 and 120 mm from the billet indicate 325 and 375°C, respectively.

FIND: The temperature of the billet, T_b.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Rod is infinitely long with uniform cross-sectional area, (3) Uniform convection coefficient along rod.

ANALYSIS: For an infinitely long rod of uniform cross-sectional area, the temperature distribution is

$$\theta(x) = \theta_b e^{-mx} \qquad (1)$$

where

$$\theta(x) = T(x) - T_\infty \qquad \theta_b = T(0) - T_\infty = T_b - T_\infty.$$

Substituting values for T_1 and T_2 at their respective distances, x_1 and x_2, into Eq. (1), it is possible to evaluate m,

$$\frac{\theta(x_1)}{\theta(x_2)} = \frac{\theta_b e^{-mx_1}}{\theta_b e^{-mx_2}} = e^{-m(x_1 - x_2)}$$

$$\frac{(325-400)^\circ C}{(375\text{-}400)^\circ C} = e^{-m(0.025-0.120)m} \qquad m=11.56.$$

Using the value for m with Eq. (1) at location x_1, it is now possible to determine the rod base or billet temperature,

$$\theta(x_1) = T_1 - T_\infty = (T_b - T_\infty)e^{-mx}$$

$$(325-400)^\circ C = (T_b - 400)^\circ C\ e^{-11.56\times 0.025}$$

$$T_b = 300^\circ C. \qquad <$$

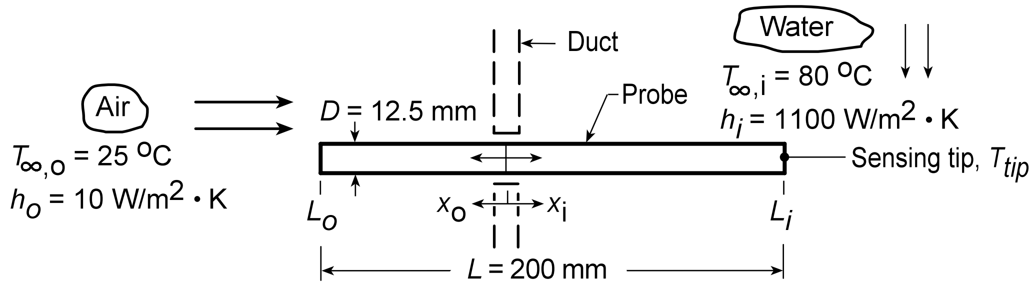
COMMENTS: Using the criteria mL ≥ 2.65 (see Example 3.8) for the infinite fin approximation, the insertion length should be ≥ 229 mm to justify the approximation,

PROBLEM 3.110

KNOWN: Temperature sensing probe of thermal conductivity k, length L and diameter D is mounted on a duct wall; portion of probe L_i is exposed to water stream at $T_{\infty,i}$ while other end is exposed to ambient air at $T_{\infty,o}$; convection coefficients h_i and h_o are prescribed.

FIND: (a) Expression for the measurement error, $\Delta T_{err} = T_{tip} - T_{\infty,i}$, (b) For prescribed $T_{\infty,i}$ and $T_{\infty,o}$, calculate ΔT_{err} for immersion to total length ratios of 0.225, 0.425, and 0.625, (c) Compute and plot the effects of probe thermal conductivity and water velocity (h_i) on ΔT_{err}.

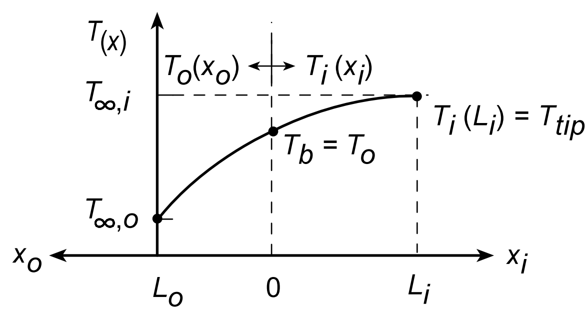
SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in probe, (3) Probe is thermally isolated from the duct, (4) Convection coefficients are uniform over their respective regions.

PROPERTIES: Probe material (given): k = 177 W/m·K.

ANALYSIS: (a) To derive an expression for $\Delta T_{err} = T_{tip} - T_{\infty,i}$, we need to determine the temperature distribution in the immersed length of the probe $T_i(x)$. Consider the probe to consist of two regions: $0 \le x_i \le L_i$, the immersed portion, and $0 \le x_o \le (L - L_i)$, the ambient-air portion where the origin corresponds to the location of the duct wall. Use the results for the temperature distribution and fin heat rate of Case A, Table 3.4:



Temperature distribution in region i:

$$\frac{\theta_i}{\theta_{b,i}} = \frac{T_i(x_i) - T_{\infty,i}}{T_o - T_{\infty,i}} = \frac{\cosh\left(m_i(L_i - x_i)\right) + (h_i/m_i k)\sinh(L_i - x_i)}{\cosh(m_i L_i) + (h_i/m_i k)\sinh(m_i L_i)} \qquad (1)$$

and the tip temperature, $T_{tip} = T_i(L_i)$ at $x_i = L_i$, is

$$\frac{T_{tip} - T_{\infty,i}}{T_o - T_{\infty,i}} = A = \frac{\cosh(0) + (h_i/m_i k)\sinh(0)}{\cosh(m_i L_i) + (h_i/m_i k)\sinh(m_i L_i)} \qquad (2)$$

and hence

$$\Delta T_{err} = T_{tip} - T_{\infty,i} = A\left(T_o - T_{\infty,i}\right) \qquad (3)$$ <

where T_o is the temperature at $x_i = x_o = 0$ which at present is unknown, but can be found by setting the fin heat rates equal, that is,

$$q_{f,o} = -q_{f,i} \qquad (4)$$

Continued...

$$\left(h_o P k A_c\right)^{1/2} \theta_{b,o} \cdot B = -\left(h_i P k A_c\right)^{1/2} \theta_{b,i} \cdot C$$

Solving for T_o, find

$$\frac{\theta_{b,o}}{\theta_{b,i}} = \frac{T_o - T_{\infty,o}}{T_o - T_{\infty,i}} = -\left(h_i P k A_c\right)^{1/2} \theta_{b,i} \cdot C$$

$$T_o = \left[T_{\infty,o} + \left(\frac{h_i}{h_o}\right)^{1/2} \frac{C}{B} T_{\infty,i}\right] \Bigg/ \left[1 + \left(\frac{h_i}{h_o}\right)^{1/2} \frac{C}{B}\right] \tag{5}$$

where the constants B and C are,

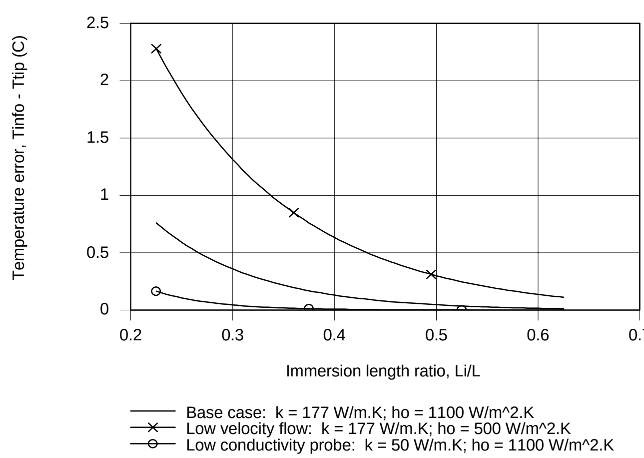
$$B = \frac{\sinh\left(m_o L_o\right) + \left(h_o / m_o k\right)\cosh\left(m_o L_o\right)}{\cosh\left(m_o L_o\right) + \left(h_o / m_o k\right)\sinh\left(m_o L_o\right)} \tag{6}$$

$$C = \frac{\sinh\left(m_i L_i\right) + \left(h_i / m_i k\right)\cosh\left(m_i L_i\right)}{\cosh\left(m_i L_i\right) + \left(h_i / m_i k\right)\sinh\left(m_i L_i\right)} \tag{7}$$

(b) To calculate the immersion error for prescribed immersion lengths, L_i/L = 0.225, 0.425 and 0.625, we use Eq. (3) as well as Eqs. (2, 6, 7 and 5) for A, B, C, and T_o, respectively. Results of these calculations are summarized below.

L_i/L	L_o (mm)	L_i (mm)	A	B	C	T_o (°C)	ΔT_{err} (°C)	
0.225	155	45	0.2328	0.5865	0.9731	76.7	-0.76	$<$
0.425	115	85	0.0396	0.4639	0.992	77.5	-0.10	$<$
0.625	75	125	0.0067	0.3205	0.9999	78.2	-0.01	$<$

(c) The probe behaves as a fin having ends exposed to the cool ambient air and the hot ambient water whose temperature is to be measured. If the thermal conductivity is *decreased*, heat transfer along the probe length is likewise decreased, the tip temperature will be closer to the water temperature. If the velocity of the water *decreases*, the convection coefficient will decrease, and the difference between the tip and water temperatures will increase.

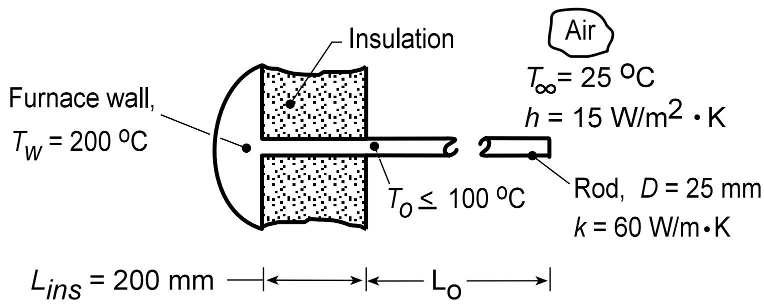


PROBLEM 3.111

KNOWN: Rod protruding normally from a furnace wall covered with insulation of thickness L_{ins} with the length L_o exposed to convection with ambient air.

FIND: (a) An expression for the exposed surface temperature T_o as a function of the prescribed thermal and geometrical parameters. (b) Will a rod of L_o = 100 mm meet the specified operating limit, $T_0 \le 100°C$? If not, what design parameters would you change?

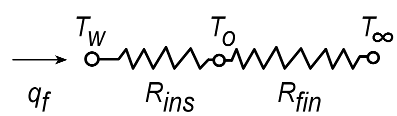
SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in rod, (3) Negligible thermal contact resistance between the rod and hot furnace wall, (4) Insulated section of rod, L_{ins}, experiences no lateral heat losses, (5) Convection coefficient uniform over the exposed portion of the rod, L_o, (6) Adiabatic tip condition for the rod and (7) Negligible radiation exchange between rod and its surroundings.

ANALYSIS: (a) The rod can be modeled as a thermal network comprised of two resistances in series: the portion of the rod, L_{ins}, covered by insulation, R_{ins}, and the portion of the rod, L_o, experiencing convection, and behaving as a fin with an adiabatic tip condition, R_{fin}. For the insulated section:

$$R_{ins} = L_{ins}/kA_c \qquad (1)$$



For the fin, Table 3.4, Case B, Eq. 3.76,

$$R_{fin} = \theta_b/q_f = \frac{1}{\left(hPkA_c\right)^{1/2}\tanh\left(mL_o\right)} \qquad (2)$$

$$m = \left(hP/kA_c\right)^{1/2} \qquad A_c = \pi D^2/4 \qquad P = \pi D \qquad (3,4,5)$$

From the thermal network, by inspection,

$$\frac{T_o - T_\infty}{R_{fin}} = \frac{T_w - T_\infty}{R_{ins} + R_{fin}} \qquad T_o = T_\infty + \frac{R_{fin}}{R_{ins} + R_{fin}}\left(T_w - T_\infty\right) \qquad (6) \quad <$$

(b) Substituting numerical values into Eqs. (1) - (6) with L_o = 200 mm,

$$T_o = 25^\circ C + \frac{6.298}{6.790 + 6.298}\left(200 - 25\right)^\circ C = 109^\circ C \qquad <$$

$$R_{ins} = \frac{0.200\,m}{60\,W/m\cdot K \times 4.909\times10^{-4}\,m^2} = 6.790\,K/W \qquad A_c = \pi\left(0.025\,m\right)^2/4 = 4.909\times10^{-4}\,m^2$$

$$R_{fin} = 1\Big/\left(0.0347\,W^2/K^2\right)^{1/2}\tanh\left(6.324\times0.200\right) = 6.298\,K/W$$

$$\left(hPkA_c\right) = \left(15\,W/m^2\cdot K\times\pi\left(0.025\,m\right)\times60\,W/m\cdot K\times4.909\times10^{-4}\,m^2\right) = 0.0347\,W^2/K^2$$

Continued...

PROBLEM 3.111 (Cont.)

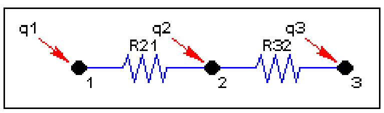
$$m = \left(hP/kA_c\right)^{1/2} = \left(15\,W/m^2\cdot K \times \pi\,(0.025\,m)/60\,W/m\cdot K \times 4.909\times 10^{-4}\,m^2\right)^{1/2} = 6.324\,m^{-1}$$

Consider the following design changes aimed at reducing $T_o \le 100°C$. (1) Increasing length of the fin portions: with L_o = 200 mm, the fin already behaves as an infinitely long fin. Hence, increasing L_o will not result in reducing T_o. (2) Decreasing the thermal conductivity: backsolving the above equation set with T_0 = 100°C, find the required thermal conductivity is k = 14 W/m·K. Hence, we could select a stainless steel alloy; see Table A.1. (3) Increasing the insulation thickness: find that for T_o = 100°C, the required insulation thickness would be L_{ins} = 211 mm. This design solution might be physically and economically unattractive. (4) A very practical solution would be to introduce thermal contact resistance between the rod base and the furnace wall by "tack welding" (rather than a continuous bead around the rod circumference) the rod in two or three places. (5) A less practical solution would be to increase the convection coefficient, since to do so, would require an air handling unit.

COMMENTS: (1) Would replacing the rod by a thick-walled tube provide a practical solution?

(2) The *IHT Thermal Resistance Network Model* and the *Thermal Resistance Tool* for a *fin* with an *adiabatic tip* were used to create a model of the rod. The Workspace is shown below.

```
// Thermal Resistance Network Model:
// The Network:
```



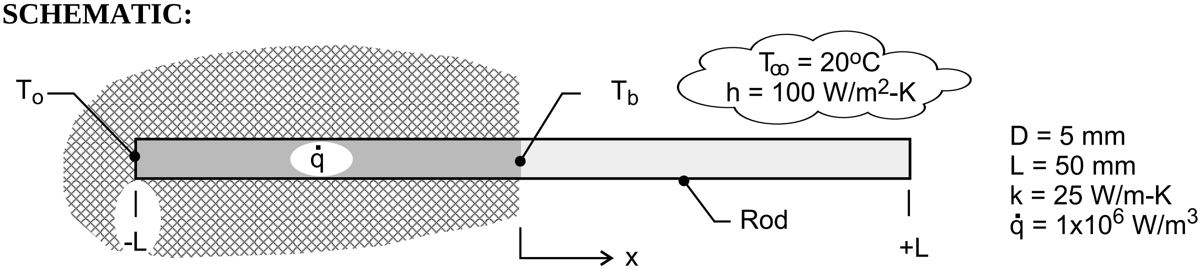

```
// Heat rates into node j,qij, through thermal resistance Rij
q21 = (T2 - T1) / R21
q32 = (T3 - T2) / R32

// Nodal energy balances
q1 + q21 = 0
q2 - q21 + q32 = 0
q3 - q32 = 0

/* Assigned variables list: deselect the qi, Rij and Ti which are unknowns; set qi = 0 for embedded nodal
points at which there is no external source of heat. */
T1 = Tw              // Furnace wall temperature, C
//q1 =               // Heat rate, W
T2 = To              // To, beginning of rod exposed length
q2 =  0              // Heat rate, W; node 2; no external heat source
T3 =  Tinf           // Ambient air temperature, C
//q3 =               // Heat rate, W

// Thermal Resistances:
// Rod - conduction resistance
R21 =  Lins / (k * Ac)          // Conduction resistance, K/W
Ac = pi * D^2 / 4               // Cross sectional area of rod, m^2

// Thermal Resistance Tools - Fin with Adiabatic Tip:
R32 =  Rfin                     //  Resistance of fin, K/W
/* Thermal resistance of a fin of uniform cross sectional area Ac, perimeter P, length L, and thermal
conductivity k with an adiabatic tip condition experiencing convection with a fluid at Tinf and coefficient h, */
Rfin = 1/ ( tanh (m*Lo) * (h * P * k * Ac ) ^ (1/2) )          // Case B, Table 3.4
m = sqrt(h*P / (k*Ac))
P = pi * D                      // Perimeter, m

// Other Assigned Variables:
Tw = 200             // Furnace wall temperature, C
k = 60               // Rod thermal conductivity, W/m.K
Lins = 0.200         // Insulated length, m
D = 0.025            // Rod diameter, m
h = 15               // Convection coefficient, W/m^2.K
Tinf = 25            // Ambient air temperature,C
Lo = 0.200           // Exposed length, m
```

PROBLEM 3.112

KNOWN: Rod (D, k, 2L) inserted into a perfectly insulating wall, exposing one-half of its length to an airstream (T_∞, h). An electromagnetic field induces a uniform volumetric energy generation $(\dot{q})$ in the imbedded portion.

FIND: (a) Derive an expression for T_b at the base of the exposed half of the rod; the exposed region may be approximated as a very long fin; (b) Derive an expression for T_o at the end of the imbedded half of the rod, and (c) Using numerical values, plot the temperature distribution in the rod and describe its key features. Does the rod behave as a very long fin?

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in imbedded portion of rod, (3) Imbedded portion of rod is perfectly insulated, (4) Exposed portion of rod behaves as an infinitely long fin, and (5) Constant properties.

ANALYSIS: (a) Since the exposed portion of the rod ($0 \le x \le +L$) behaves as an infinite fin, the fin heat rate using Eq. 3.80 is

$$q_x(0) = q_f = M = (hPkA_c)^{1/2}(T_b - T_\infty) \qquad (1)$$

From an energy balance on the imbedded portion of the rod,

$$q_f = \dot{q} A_c L \qquad (2)$$

Combining Eqs. (1) and (2), with $P = \pi D$ and $A_c = \pi D^2/4$, find

$$T_b = T_\infty + q_f (hPkA_c)^{-1/2} = T_\infty + \dot{q} A_c^{1/2} L (hPk)^{-1/2} \qquad (3) \quad <$$

(b) The imbedded portion of the rod ($-L \le x \le 0$) experiences one-dimensional heat transfer with uniform $\dot{q}$. From Eq. 3.43,

$$T_o = \frac{\dot{q}L^2}{2k} + T_b \qquad <$$

(c) The temperature distribution T(x) for the rod is piecewise parabolic and exponential,

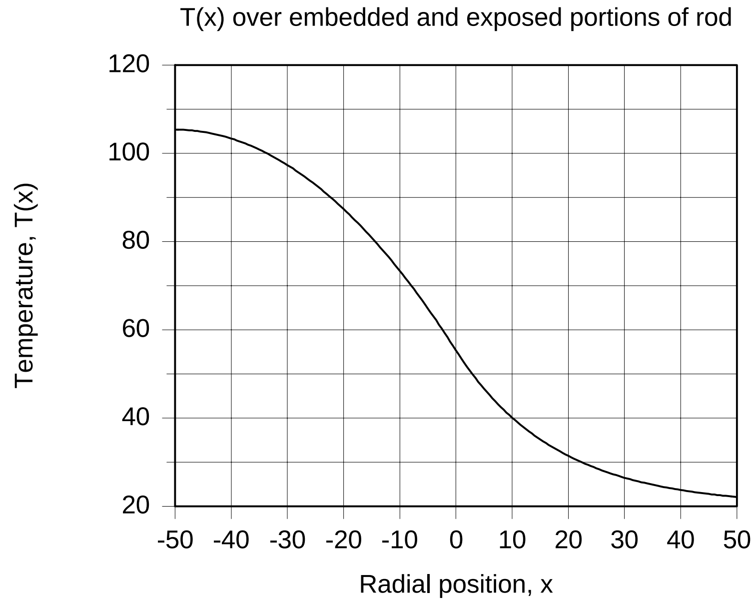
$$T(x) - T_b = \frac{\dot{q}L^2}{2k}\left(\frac{x}{L}\right)^2 \qquad -L \le x \le 0$$

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = \exp(-mx) \qquad 0 \le x \le +L$$

Continued …..

PROBLEM 3.112 (Cont.)

The gradient at x = 0 will be continuous since we used this condition in evaluating T_b. The distribution is shown below with T_o = 105.4°C and T_b = 55.4°C.

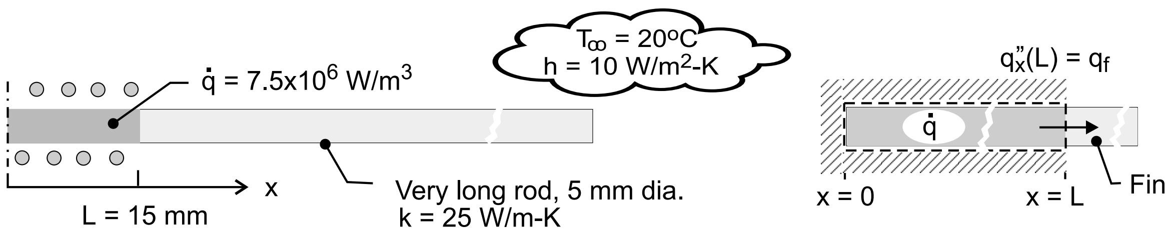


PROBLEM 3.113

KNOWN: Very long rod (D, k) subjected to induction heating experiences uniform volumetric generation $(\dot{q})$ over the center, 30-mm long portion. The unheated portions experience convection (T_∞, h).

FIND: Calculate the temperature of the rod at the mid-point of the heated portion within the coil, T_o, and at the edge of the heated portion, T_b.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction with uniform $\dot{q}$ in portion of rod within the coil; no convection from lateral surface of rod, (3) Exposed portions of rod behave as infinitely long fins, and (4) Constant properties.

ANALYSIS: The portion of the rod within the coil, $0 \le x \le +L$, experiences one-dimensional conduction with uniform generation. From Eq. 3.43,

$$T_o = \frac{\dot{q}L^2}{2k} + T_b \tag{1}$$

The portion of the rod beyond the coil, $L \le x \le \infty$, behaves as an infinitely long fin for which the heat rate from Eq. 3.80 is

$$q_f = q_x(L) = \left(hPkA_c\right)^{1/2}\left(T_b - T_\infty\right) \tag{2}$$

where $P = \pi D$ and $A_c = \pi D^2/4$. From an overall energy balance on the imbedded portion of the rod as illustrated in the schematic above, find the heat rate as

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = 0$$

$$-q_f + \dot{q}A_c L = 0$$

$$q_f = \dot{q}A_c L \tag{3}$$

Combining Eqs. (1-3),

$$T_b = T_\infty + \dot{q}A_c^{1/2}L\left(hPk\right)^{-1/2} \tag{4}$$

$$T_o = T_\infty + \frac{\dot{q}L^2}{2k} + \dot{q}A_c^{1/2}L\left(hPk\right)^{-1/2} \tag{5}$$

and substituting numerical values find

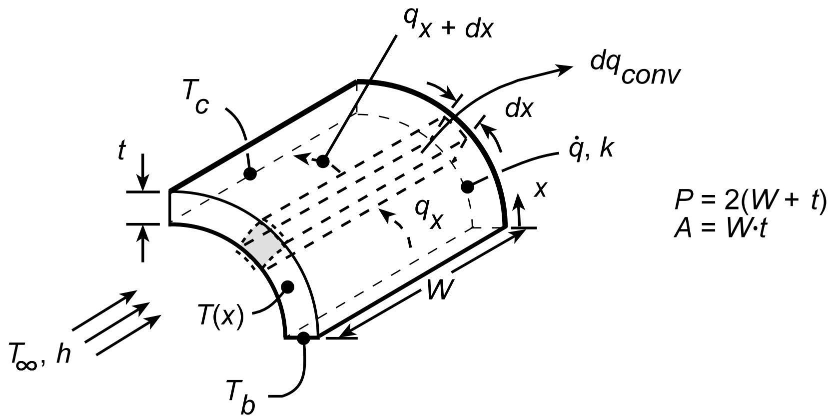
$$T_o = 305°C \qquad\qquad T_b = 272°C$$ <

PROBLEM 3.114

KNOWN: Dimensions, end temperatures and volumetric heating of wire leads. Convection coefficient and ambient temperature.

FIND: (a) Equation governing temperature distribution in the leads, (b) Form of the temperature distribution.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional conduction in x, (3) Uniform volumetric heating, (4) Uniform h (both sides), (5) Negligible radiation.

ANALYSIS: (a) Performing an energy balance for the differential control volume,

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0 \qquad q_x - q_{x+dx} - dq_{conv} + \dot{q}dV = 0$$

$$-kA_c \frac{dT}{dx} - \left[-kA_c \frac{dT}{dx} - \frac{d}{dx}\left(kA_c \frac{dT}{dx}\right)dx\right] - hPdx\left(T - T_\infty\right) + \dot{q}A_c dx = 0$$

$$\frac{d^2T}{dx^2} - \frac{hP}{kA_c}\left(T - T_\infty\right) + \frac{\dot{q}}{k} = 0 \qquad <$$

(b) With a *reduced temperature* defined as $\Theta \equiv T - T_\infty - \left(\dot{q}A_c/hP\right)$ and $m^2 \equiv hP/kA_c$, the differential equation may be rendered homogeneous, with a general solution and boundary conditionsas shown

$$\frac{d^2\Theta}{dx^2} - m^2\Theta = 0 \qquad \Theta(x) = C_1 e^{mx} + C_2 e^{-mx}$$

$$\Theta_b = C_1 + C_2 \qquad \Theta_c = C_1 e^{mL} + C_2 e^{-mL}$$

it follows that

$$C_1 = \frac{\Theta_b e^{-mL} - \Theta_c}{e^{-mL} - e^{mL}} \qquad C_2 = \frac{\Theta_c - \Theta_b e^{mL}}{e^{-mL} - e^{mL}}$$

$$\Theta(x) = \frac{\left(\Theta_b e^{-mL} - \Theta_c\right)e^{mx} + \left(\Theta_c - \Theta_b e^{mL}\right)e^{-mx}}{e^{-mL} - e^{mL}} \qquad <$$

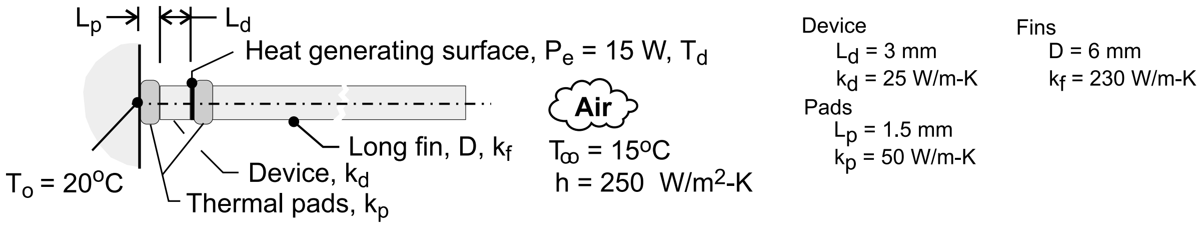
COMMENTS: If $\dot{q}$ is large and h is small, temperatures within the lead may readily exceed the prescribed boundary temperatures.

PROBLEM 3.115

KNOWN: Disk-shaped electronic device (D, L_d, k_d) dissipates electrical power (P_e) at one of its surfaces. Device is bonded to a cooled base (T_o) using a thermal pad (L_p, k_A). Long fin (D, k_f) is bonded to the heat-generating surface using an identical thermal pad. Fin is cooled by convection (T_∞, h).

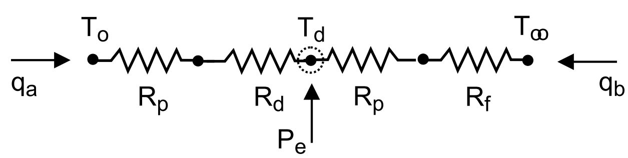
FIND: (a) Construct a thermal circuit of the system, (b) Derive an expression for the temperature of the heat-generating device, T_d, in terms of circuit thermal resistance, T_o and T_∞; write expressions for the thermal resistances; and (c) Calculate T_d for the prescribed conditions.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction through thermal pads and device; no losses from lateral surfaces; (3) Fin is infinitely long, (4) Negligible contact resistance between components of the system, and (5) Constant properties.

ANALYSIS: (a) The thermal circuit is shown below with thermal resistances associated with conduction (pads, R_p; device, R_d) and for the long fin, R_f.



(b) To obtain an expression for T_d, perform an energy balance about the d-node

$$\dot{E}_{in} - \dot{E}_{out} = q_a + q_b + P_e = 0 \tag{1}$$

Using the conduction rate equation with the circuit

$$q_a = \frac{T_o - T_d}{R_f + R_d} \qquad q_b = \frac{T_\infty - T_d}{R_p + R_f} \tag{2,3}$$

Combine with Eq. (1), and solve for T_d,

$$T_d = \frac{P_e + T_o/\left(R_p + R_d\right) + T_\infty/\left(R_p + R_f\right)}{1/\left(R_p + R_d\right) + 1/\left(R_p + R_f\right)} \tag{4}$$

where the thermal resistances with $P = \pi D$ and $A_c = \pi D^2/4$ are

$$R_p = L_p/k_p A_c \qquad R_d = L_d/k_d A_c \qquad R_f = \left(hPk_f A_c\right)^{-1/2} \tag{5,6,7}$$

(c) Substituting numerical values with the foregoing relations, find

$$R_p = 1.061\ \text{K/W} \qquad R_d = 4.244\ \text{K/W} \qquad R_f = 5.712\ \text{K/W}$$

and the device temperature as

$$T_d = 62.4°\text{C}$$ <

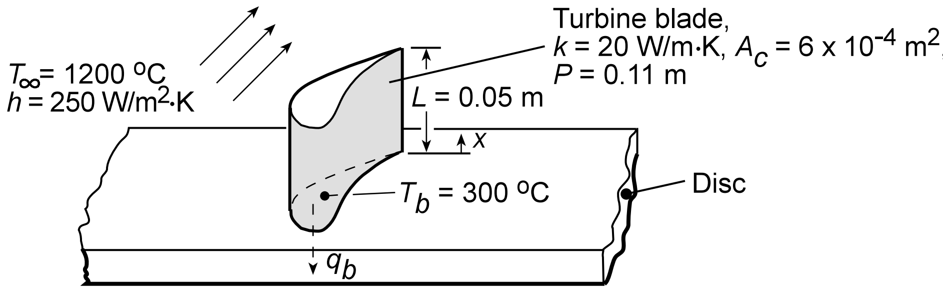
COMMENTS: What fraction of the power dissipated in the device is removed by the fin? Answer: $q_b/P_e = 47\%$.

PROBLEM 3.116

KNOWN: Dimensions and thermal conductivity of a gas turbine blade. Temperature and convection coefficient of gas stream. Temperature of blade base and maximum allowable blade temperature.

FIND: (a) Whether blade operating conditions are acceptable, (b) Heat transfer to blade coolant.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conduction in blade, (2) Constant k, (3) Adiabatic blade tip, (4) Negligible radiation.

ANALYSIS: Conditions in the blade are determined by Case B of Table 3.4.

(a) With the maximum temperature existing at x = L, Eq. 3.75 yields

$$\frac{T(L)-T_\infty}{T_b-T_\infty}=\frac{1}{\cosh mL}$$

$$m=\left(hP/kA_c\right)^{1/2}=\left(250\,W/m^2\cdot K\times 0.11\,m/20\,W/m\cdot K\times 6\times 10^{-4}\,m^2\right)^{1/2}$$

$m = 47.87\ m^{-1}$ and $mL = 47.87\ m^{-1}\times 0.05\ m = 2.39$

From Table B.1, cosh mL = 5.51. Hence,

$$T(L)=1200^\circ C+(300-1200)^\circ C/5.51=1037^\circ C$$ <

and the operating conditions are acceptable.

(b) With $M=\left(hPkA_c\right)^{1/2}\Theta_b=\left(250\,W/m^2\cdot K\times 0.11\,m\times 20\,W/m\cdot K\times 6\times 10^{-4}\,m^2\right)^{1/2}\left(-900^\circ C\right)=-517\,W$,

Eq. 3.76 and Table B.1 yield

$$q_f=M\tanh mL=-517\,W\,(0.983)=-508\,W$$

Hence, $q_b=-q_f=508\,W$ <

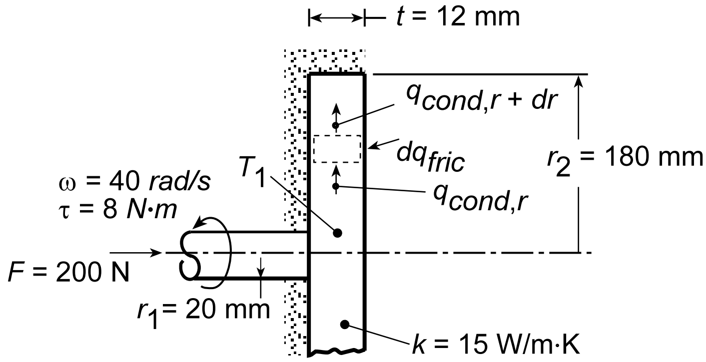
COMMENTS: Radiation losses from the blade surface and convection from the tip will contribute to reducing the blade temperatures.

PROBLEM 3.117

KNOWN: Dimensions of disc/shaft assembly. Applied angular velocity, force, and torque. Thermal conductivity and inner temperature of disc.

FIND: (a) Expression for the friction coefficient μ, (b) Radial temperature distribution in disc, (c) Value of μ for prescribed conditions.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial conduction, (3) Constant k, (4) Uniform disc contact pressure p, (5) All frictional heat dissipation is transferred to shaft from base of disc.

ANALYSIS: (a) The normal force acting on a differential ring extending from r to r+dr on the contact surface of the disc may be expressed as $dF_n = p2\pi r dr$. Hence, the tangential force is $dF_t = \mu p 2\pi r dr$, in which case the torque may be expressed as

$$d\tau = 2\pi\mu p r^2 dr$$

For the entire disc, it follows that

$$\tau = 2\pi\mu p \int_o^{r_2} r^2 dr = \frac{2\pi}{3}\mu p r_2^3$$

where $p = F/\pi r_2^2$. Hence,

$$\mu = \frac{3}{2}\frac{\tau}{F r_2}$$ <

(b) Performing an energy balance on a differential control volume in the disc, it follows that

$$q_{cond,r} + dq_{fric} - q_{cond,r+dr} = 0$$

With $dq_{fric} = \omega d\tau = 2\mu F\omega\left(r^2/r_2^2\right)dr$, $q_{cond,r+dr} = q_{cond,r} + \left(dq_{cond,r}/dr\right)dr$, and

$q_{cond,r} = -k\left(2\pi r t\right)dT/dr$, it follows that

$$2\mu F\omega\left(r^2/r_2^2\right)dr + 2\pi k t\frac{d\left(rdT/dr\right)}{dr}dr = 0$$

or

$$\frac{d\left(rdT/dr\right)}{dr} = -\frac{\mu F\omega}{\pi k t r_2^2}r^2$$

Integrating twice,

Continued...

PROBLEM 3.117 (Cont.)

$$\frac{dT}{dr} = -\frac{\mu F\omega}{3\pi k t r_2^2} r^2 + \frac{C_1}{r}$$

$$T = -\frac{\mu F\omega}{9\pi k t r_2^2} r^3 + C_1 \ell n r + C_2$$

Since the disc is well insulated at $r = r_2$, $dT/dr|_{r_2} = 0$ and

$$C_1 = \frac{\mu F\omega r_2}{3\pi k t}$$

With $T(r_1) = T_1$, it also follows that

$$C_2 = T_1 + \frac{\mu F\omega}{9\pi k t r_2^2} r_1^3 - C_1 \ell n r_1$$

Hence,

$$T(r) = T_1 - \frac{\mu F\omega}{9\pi k t r_2^2}\left(r^3 - r_1^3\right) + \frac{\mu F\omega r_2}{3\pi k t} \ell n \frac{r}{r_1} \qquad <$$

(c) For the prescribed conditions,

$$\mu = \frac{3}{2} \frac{8 N\cdot m}{200 N (0.18 m)} = 0.333 \qquad <$$

Since the maximum temperature occurs at r = r_2,

$$T_{max} = T(r_2) = T_1 - \frac{\mu F\omega r_2}{9\pi k t}\left[1 - \left(\frac{r_1}{r_2}\right)^3\right] + \frac{\mu F\omega r_2}{3\pi k t} \ell n \left(\frac{r_2}{r_1}\right)$$

With $(\mu F\omega r_2 / 3\pi k t) = (0.333 \times 200 N \times 40 rad/s \times 0.18 m / 3\pi \times 15 W/m\cdot K \times 0.012 m) = 282.7^\circ C$,

$$T_{max} = 80^\circ C - \frac{282.7^\circ C}{3}\left[1 - \left(\frac{0.02}{0.18}\right)^3\right] + 282.7^\circ C \ell n \left(\frac{0.18}{0.02}\right)$$

$$T_{max} = 80^\circ C - 94.1^\circ C + 621.1^\circ C = 607^\circ C \qquad <$$

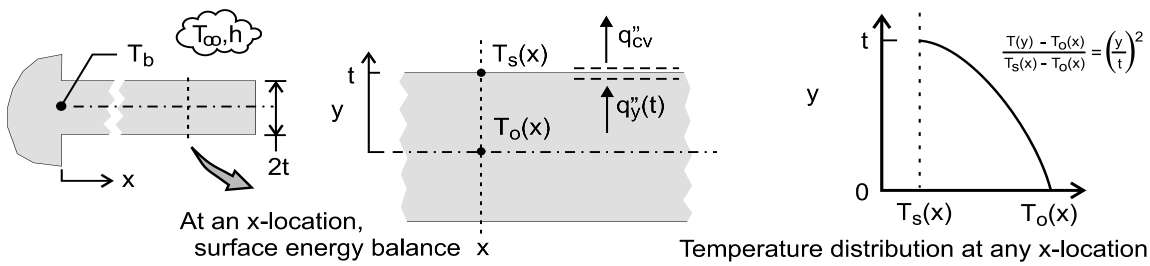
COMMENTS: The maximum temperature is excessive, and the disks should be actively cooled (by convection) at their outer surfaces.

PROBLEM 3.118

KNOWN: Extended surface of rectangular cross-section with heat flow in the longitudinal direction.

FIND: Determine the conditions for which the transverse (y-direction) temperature gradient is negligible compared to the longitudinal gradient, such that the 1-D analysis of Section 3.6.1 is valid by finding: (a) An expression for the conduction heat flux at the surface, $q''_y(t)$, in terms of T_s and T_o, assuming the transverse temperature distribution is parabolic, (b) An expression for the convection heat flux at the surface for the x-location; equate the two expressions, and identify the parameter that determines the ratio $(T_o - T_s)/(T_s - T_\infty)$; and (c) Developing a criterion for establishing the validity of the 1-D assumption used to model an extended surface.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform convection coefficient and (3) Constant properties.

ANALYSIS: (a) Referring to the schematics above, the conduction heat flux at the surface y = t at any x-location follows from Fourier's law using the parabolic transverse temperature distribution.

$$q''_y(t) = -k\left.\frac{\partial T}{\partial y}\right)_{y=t} = -k\left(\left[T_s(x) - T_o(x)\right]\frac{2y}{t^2}\right)_{y=t} = -\frac{2k}{t}\left[T_s(x) - T_o(x)\right] \tag{1}$$

(b) The convection heat flux at the surface of any x-location follows from the rate equation

$$q''_{cv} = h\left[T_s(x) - T_\infty\right] \tag{2}$$

Performing a surface energy balance as represented schematically above, equating Eqs. (1) and (2) provides

$$q''_y(t) = q''_{cv}$$

$$-\frac{2k}{t}\left[T_s(x) - T_o(x)\right] = h\left[T_s(x) - T_\infty\right]$$

$$\frac{T_s(x) - T_o(x)}{T_s(x) - T_\infty(x)} = -0.5\frac{ht}{k} = -0.5\ \text{Bi} \tag{3}$$

where Bi = ht/k, the Biot number, represents the ratio of the convection to the conduction thermal resistances,

$$\text{Bi} = \frac{R''_{cd}}{R''_{cv}} = \frac{t/k}{1/h} \tag{4}$$

(c) The transverse gradient (heat flow) will be negligible compared to the longitudinal gradient when Bi << 1, say, 0.1, an order of magnitude smaller. This is the criterion to validate the one-dimensional assumption used to model extended surfaces.

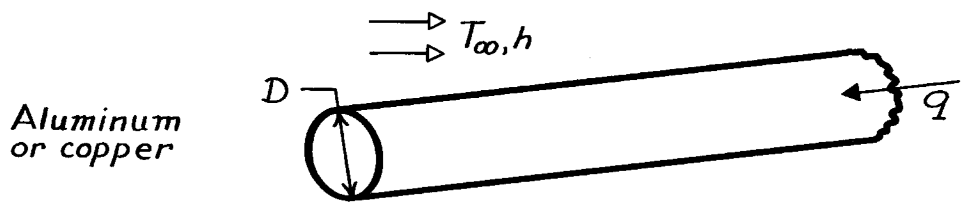
COMMENTS: The coefficient 0.5 in Eq. (3) is a consequence of the parabolic distribution assumption. This distribution represents the simplest polynomial expression that could approximate the real distribution.

PROBLEM 3.119

KNOWN: Long, aluminum cylinder acts as an extended surface.

FIND: (a) Increase in heat transfer if diameter is tripled and (b) Increase in heat transfer if copper is used in place of aluminum.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Uniform convection coefficient, (5) Rod is infinitely long.

PROPERTIES: *Table A-1,* Aluminum (pure): k = 240 W/m·K; *Table A-1,* Copper (pure): k = 400 W/m·K.

ANALYSIS: (a) For an infinitely long fin, the fin heat rate from Table 3.4 is

$$q_f = M = \left(hPkA_c\right)^{1/2}\theta_b$$

$$q_f = \left(h\,\pi\,D\,k\,\pi\,D^2/4\right)^{1/2}\theta_b = \frac{\pi}{2}\left(hk\right)^{1/2}D^{3/2}\;\theta_b.$$

where $P = \pi D$ and $A_c = \pi D^2/4$ for the circular cross-section. Note that $q_f \propto D^{3/2}$. Hence, if the diameter is tripled,

$$\frac{q_f(3D)}{q_f(D)} = 3^{3/2} = 5.2$$

and there is a 420% increase in heat transfer. <

(b) In changing from aluminum to copper, since $q_f \propto k^{1/2}$, it follows that

$$\frac{q_f\,(Cu)}{q_f\,(A1)} = \left[\frac{k_{Cu}}{k_{A1}}\right]^{1/2} = \left[\frac{400}{240}\right]^{1/2} = 1.29$$

and there is a 29% increase in the heat transfer rate. <

COMMENTS: (1) Because fin effectiveness is enhanced by maximizing $P/A_c = 4/D$, the use of a larger number of small diameter fins is preferred to a single large diameter fin.

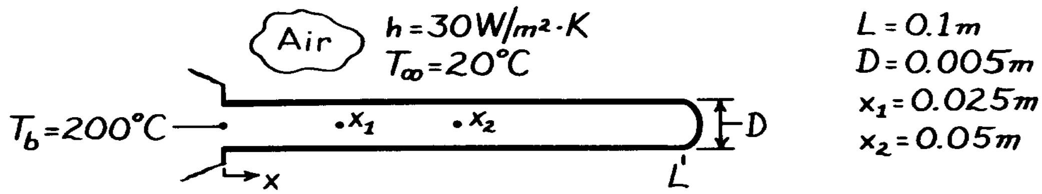
(2) From the standpoint of cost and weight, aluminum is preferred over copper.

PROBLEM 3.120

KNOWN: Length, diameter, base temperature and environmental conditions associated with a brass rod.

FIND: Temperature at specified distances along the rod.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible radiation, (5) Uniform convection coefficient h.

PROPERTIES: *Table A-1*, Brass $\left(\overline{T} = 110^\circ C\right)$: $k = 133\ W/m \cdot K$.

ANALYSIS: Evaluate first the fin parameter

$$m = \left[\frac{hP}{kA_c}\right]^{1/2} = \left[\frac{h\pi\ D}{k\pi\ D^2/4}\right]^{1/2} = \left[\frac{4h}{kD}\right]^{1/2} = \left[\frac{4\times 30\ W/m^2 \cdot K}{133\ W/m \cdot K \times 0.005m}\right]^{1/2}$$

$$m = 13.43\ m^{-1}.$$

Hence, m L = (13.43)×0.1 = 1.34 and from the results of Example 3.8, it is advisable not to make the infinite rod approximation. Thus from Table 3.4, the temperature distribution has the form

$$\theta = \frac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}\theta_b$$

Evaluating the hyperbolic functions, cosh mL = 2.04 and sinh mL = 1.78, and the parameter

$$\frac{h}{mk} = \frac{30\ W/m^2 \cdot K}{13.43m^{-1}\ (133\ W/m \cdot K)} = 0.0168,$$

with θ_b = 180°C the temperature distribution has the form

$$\theta = \frac{\cosh m(L-x) + 0.0168\ \sinh m(L-x)}{2.07}\left(180^\circ C\right).$$

The temperatures at the prescribed location are tabulated below.

x(m)	cosh m(L-x)	sinh m(L-x)	θ	T(°C)	
x_1 = 0.025	1.55	1.19	136.5	156.5	<
x_2 = 0.05	1.24	0.725	108.9	128.9	<
L = 0.10	1.00	0.00	87.0	107.0	<

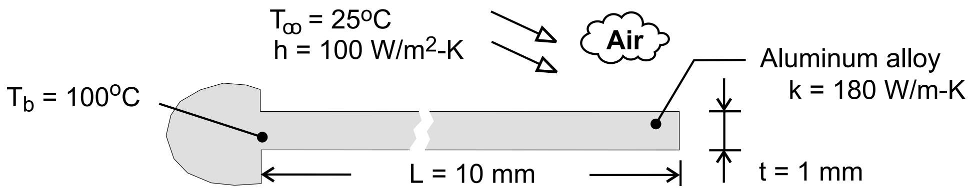
COMMENTS: If the rod were approximated as infinitely long: $T(x_1)$ = 148.7°C, $T(x_2)$ = 112.0°C, and T(L) = 67.0°C. The assumption would therefore result in significant underestimates of the rod temperature.

PROBLEM 3.121

KNOWN: Thickness, length, thermal conductivity, and base temperature of a rectangular fin. Fluid temperature and convection coefficient.

FIND: (a) Heat rate per unit width, efficiency, effectiveness, thermal resistance, and tip temperature for different tip conditions, (b) Effect of convection coefficient and thermal conductivity on the heat rate.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional conduction along fin, (3) Constant properties, (4) Negligible radiation, (5) Uniform convection coefficient, (6) Fin width is much longer than thickness (w >> t).

ANALYSIS: (a) The fin heat transfer rate for Cases A, B and D are given by Eqs. (3.72), (3.76) and (3.80), where $M \approx (2\ hw^2 tk)^{1/2}\ (T_b - T_\infty) = (2 \times 100\ \text{W/m}^2\cdot\text{K} \times 0.001\text{m} \times 180\ \text{W/m}\cdot\text{K})^{1/2}\ (75°\text{C})\ w = 450\ w\ \text{W}$, $m \approx (2h/kt)^{1/2} = (200\ \text{W/m}^2\cdot\text{K}/180\ \text{W/m}\cdot\text{K} \times 0.001\text{m})^{1/2} = 33.3\text{m}^{-1}$, $mL \approx 33.3\text{m}^{-1} \times 0.010\text{m} = 0.333$, and $(h/mk) \approx (100\ \text{W/m}^2\cdot\text{K}/33.3\text{m}^{-1} \times 180\ \text{W/m}\cdot\text{K}) = 0.0167$. From Table B-1, it follows that sinh mL ≈ 0.340, cosh mL ≈ 1.057, and tanh mL ≈ 0.321. From knowledge of q_f, Eqs. (3.86), (3.81) and (3.83) yield

$$\eta_f \approx \frac{q_f'}{h(2L+t)\theta_b}, \quad \varepsilon_f \approx \frac{q_f'}{ht\,\theta_b}, \quad R_{t,f}' = \frac{\theta_b}{q_f'}$$

Case A: From Eq. (3.72), (3.86), (3.81), (3.83) and (3.70),

$$q_f' = \frac{M}{w}\frac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL} = 450\ \text{W/m}\ \frac{0.340 + 0.0167 \times 1.057}{1.057 + 0.0167 \times 0.340} = 151\ \text{W/m}$$
<

$$\eta_f = \frac{151\ \text{W/m}}{100\ \text{W/m}^2 \cdot \text{K}\,(0.021\text{m})\,75°\text{C}} = 0.96$$
<

$$\varepsilon_f = \frac{151\ \text{W/m}}{100\ \text{W/m}^2 \cdot \text{K}\,(0.001\text{m})\,75°\text{C}} = 20.1, \quad R_{t,f}' = \frac{75°\text{C}}{151\ \text{W/m}} = 0.50\ \text{m}\cdot\text{K/W}$$
<

$$T(L) = T_\infty + \frac{\theta_b}{\cosh mL + (h/mk)\sinh mL} = 25°\text{C} + \frac{75°\text{C}}{1.057 + (0.0167)\,0.340} = 95.6°\text{C}$$
<

Case B: From Eqs. (3.76), (3.86), (3.81), (3.83) and (3.75)

$$q_f' = \frac{M}{w}\tanh mL = 450\ \text{W/m}\,(0.321) = 144\ \text{W/m}$$
<

$$\eta_f = 0.92, \quad \varepsilon_f = 19.2, \quad R_{t,f}' = 0.52\ \text{m}\cdot\text{K/W}$$
<

$$T(L) = T_\infty + \frac{\theta_b}{\cosh mL} = 25°\text{C} + \frac{75°\text{C}}{1.057} = 96.0°\text{C}$$
<

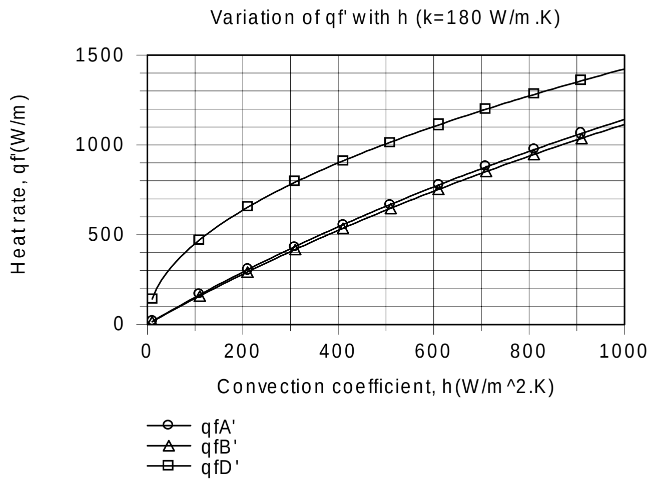
Continued …..

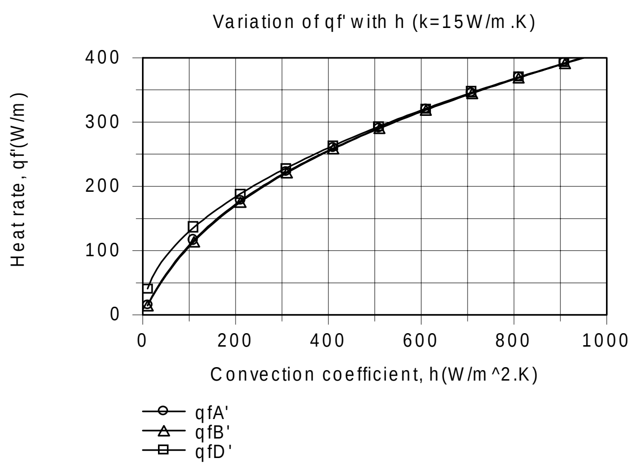
Case D (L $\rightarrow \infty$): From Eqs. (3.80), (3.86), (3.81), (3.83) and (3.79)

$$q_f' = \frac{M}{w} = 450\,\text{W/m}$$ <

$$\eta_f = 0,\ \varepsilon_f = 60.0,\ R_{t,f}' = 0.167\,\text{m}\cdot\text{K/W},\ T(L) = T_\infty = 25°\text{C}$$ <

(b) The effect of h on the heat rate is shown below for the aluminum and stainless steel fins.





For both materials, there is little difference between the Case A and B results over the entire range of h. The difference (percentage) increases with decreasing h and increasing k, but even for the worst case condition (h = 10 W/m^2·K, k = 180 W/m·K), the heat rate for Case A (15.7 W/m) is only slightly larger than that for Case B (14.9 W/m). For aluminum, the heat rate is significantly over-predicted by the infinite fin approximation over the entire range of h. For stainless steel, it is over-predicted for small values of h, but results for all three cases are within 1% for h > 500 W/m^2·K.

COMMENTS: From the results of Part (a), we see there is a slight reduction in performance (smaller values of q_f', η_f and ε_f, as well as a larger value of $R_{t,f}'$) associated with insulating the tip.

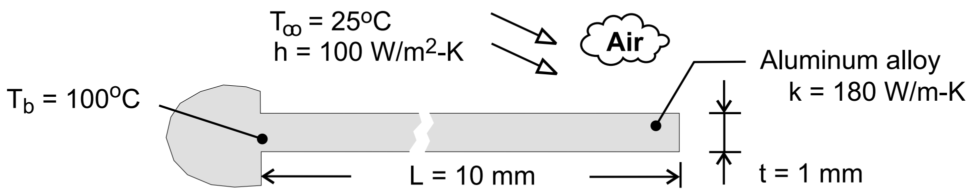
Although $\eta_f = 0$ for the infinite fin, q_f' and ε_f are substantially larger than results for L = 10 mm, indicating that performance may be significantly improved by increasing L.

PROBLEM 3.122

KNOWN: Thickness, length, thermal conductivity, and base temperature of a rectangular fin. Fluid temperature and convection coefficient.

FIND: (a) Heat rate per unit width, efficiency, effectiveness, thermal resistance, and tip temperature for different tip conditions, (b) Effect of fin length and thermal conductivity on the heat rate.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional conduction along fin, (3) Constant properties, (4) Negligible radiation, (5) Uniform convection coefficient, (6) Fin width is much longer than thickness (w >> t).

ANALYSIS: (a) The fin heat transfer rate for Cases A, B and D are given by Eqs. (3.72), (3.76) and (3.80), where $M \approx (2\,hw^2tk)^{1/2}\,(T_b - T_\infty) = (2 \times 100\ \mathrm{W/m^2{\cdot}K} \times 0.001\mathrm{m} \times 180\ \mathrm{W/m{\cdot}K})^{1/2}\,(75°\mathrm{C})\ w = 450\ w\ \mathrm{W}$, $m \approx (2h/kt)^{1/2} = (200\ \mathrm{W/m^2{\cdot}K}/180\ \mathrm{W/m{\cdot}K} \times 0.001\mathrm{m})^{1/2} = 33.3\mathrm{m^{-1}}$, $mL \approx 33.3\mathrm{m^{-1}} \times 0.010\mathrm{m} = 0.333$, and $(h/mk) \approx (100\ \mathrm{W/m^2{\cdot}K}/33.3\mathrm{m^{-1}} \times 180\ \mathrm{W/m{\cdot}K}) = 0.0167$. From Table B-1, it follows that $\sinh mL \approx 0.340$, $\cosh mL \approx 1.057$, and $\tanh mL \approx 0.321$. From knowledge of q_f, Eqs. (3.86), (3.81) and (3.83) yield

$$\eta_f \approx \frac{q'_f}{h(2L+t)\theta_b},\quad \varepsilon_f \approx \frac{q'_f}{ht\,\theta_b},\quad R'_{t,f} = \frac{\theta_b}{q'_f}$$

Case A: From Eq. (3.72), (3.86), (3.81), (3.83) and (3.70),

$$q'_f = \frac{M}{w}\,\frac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL} = 450\ \mathrm{W/m}\,\frac{0.340 + 0.0167 \times 1.057}{1.057 + 0.0167 \times 0.340} = 151\ \mathrm{W/m} \qquad <$$

$$\eta_f = \frac{151\ \mathrm{W/m}}{100\ \mathrm{W/m^2 \cdot K}\,(0.021\mathrm{m})\,75°\mathrm{C}} = 0.96 \qquad <$$

$$\varepsilon_f = \frac{151\ \mathrm{W/m}}{100\ \mathrm{W/m^2 \cdot K}\,(0.001\mathrm{m})\,75°\mathrm{C}} = 20.1,\ R'_{t,f} = \frac{75°\mathrm{C}}{151\ \mathrm{W/m}} = 0.50\ \mathrm{m \cdot K/W} \qquad <$$

$$T(L) = T_\infty + \frac{\theta_b}{\cosh mL + (h/mk)\sinh mL} = 25°\mathrm{C} + \frac{75°\mathrm{C}}{1.057 + (0.0167)\,0.340} = 95.6°\mathrm{C} \qquad <$$

Case B: From Eqs. (3.76), (3.86), (3.81), (3.83) and (3.75)

$$q'_f = \frac{M}{w}\tanh mL = 450\ \mathrm{W/m}\,(0.321) = 144\ \mathrm{W/m} \qquad <$$

$$\eta_f = 0.92,\ \varepsilon_f = 19.2,\ R'_{t,f} = 0.52\ \mathrm{m \cdot K/W} \qquad <$$

$$T(L) = T_\infty + \frac{\theta_b}{\cosh mL} = 25°\mathrm{C} + \frac{75°\mathrm{C}}{1.057} = 96.0°\mathrm{C} \qquad <$$

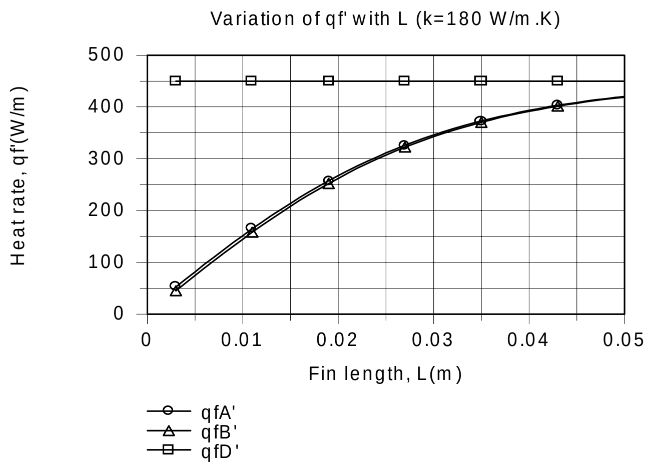
Continued .....

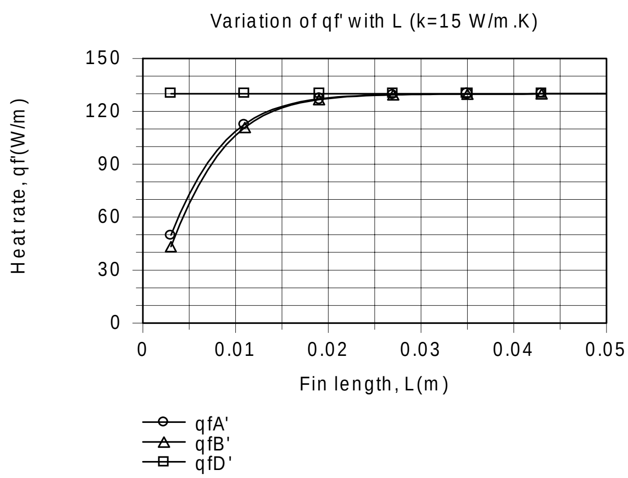
Case D ($L \rightarrow \infty$): From Eqs. (3.80), (3.86), (3.81), (3.83) and (3.79)

$$q'_f = \frac{M}{w} = 450\ \mathrm{W/m}$$ <

$$\eta_f = 0,\ \varepsilon_f = 60.0,\ R'_{t,f} = 0.167\ \mathrm{m \cdot K/W},\ T(L) = T_\infty = 25°\mathrm{C}$$ <

(b) The effect of L on the heat rate is shown below for the aluminum and stainless steel fins.





For both materials, differences between the Case A and B results diminish with increasing L and are within 1% of each other at $L \approx 27$ mm and $L \approx 13$ mm for the aluminum and steel, respectively. At L = 3 mm, results differ by 14% and 13% for the aluminum and steel, respectively. The Case A and B results approach those of the infinite fin approximation more quickly for stainless steel due to the larger temperature gradients, $|dT/dx|$, for the smaller value of k.

COMMENTS: From the results of Part (a), we see there is a slight reduction in performance (smaller values of q'_f, η_f and ε_f, as well as a larger value of $R'_{t,f}$) associated with insulating the tip.

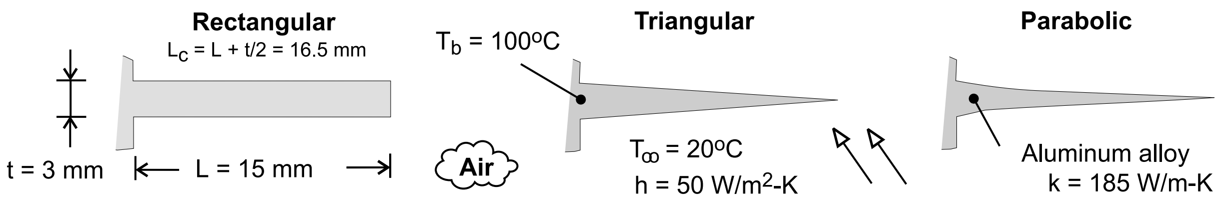
Although $\eta_f = 0$ for the infinite fin, q'_f and ε_f are substantially larger than results for L = 10 mm, indicating that performance may be significantly improved by increasing L.

PROBLEM 3.123

KNOWN: Length, thickness and temperature of straight fins of rectangular, triangular and parabolic profiles. Ambient air temperature and convection coefficient.

FIND: Heat rate per unit width, efficiency and volume of each fin.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible radiation, (5) Uniform convection coefficient.

ANALYSIS: For each fin,

$$q_f' = q_{max}' = \eta_f\, h A_f'\, \theta_b, \qquad V' = A_p$$

where η_f depends on the value of $m = (2h/kt)^{1/2} = (100\ W/m^2{\cdot}K/185\ W/m{\cdot}K \times 0.003m)^{1/2} = 13.4m^{-1}$ and the product $mL = 13.4m^{-1} \times 0.015m = 0.201$ or $mL_c = 0.222$. Expressions for η_f, A_f' and A_p are obtained from Table 3-5.

Rectangular Fin:

$$\eta_f = \frac{\tanh\, mL_c}{mL_c} = \frac{0.218}{0.222} = 0.982,\ \ A_f' = 2L_c = 0.033m \qquad <$$

$$q' = 0.982\left(50\,W/m^2\cdot K\right)0.033m\left(80°C\right) = 129.6\,W/m,\ \ V' = tL = 4.5\times10^{-5}\ m^2 \qquad <$$

Triangular Fin:

$$\eta_f = \frac{1}{mL}\frac{I_1(2mL)}{I_0(2mL)} = \frac{0.205}{(0.201)1.042} = 0.978,\ \ A_f' = 2\left[L^2 + (t/2)^2\right]^{1/2} = 0.030m \qquad <$$

$$q' = 0.978\left(50\,W/m^2\cdot K\right)0.030m\left(80°C\right) = 117.3\,W/m,\ \ V' = (t/2)L = 2.25\times10^{-5}m^2 \qquad <$$

Parabolic Fin:

$$\eta_f = \frac{2}{\left[4(mL)^2+1\right]^{1/2}+1} = 0.963,\ \ A_f' = \left[C_1L + \left(L^2/t\right)\ln\left(t/L + C_1\right)\right] = 0.030m \qquad <$$

$$q_f' = 0.963\left(50\,W/m^2\cdot K\right)0.030m\left(80°C\right) = 115.6\,W/m,\ \ V' = (t/3)L = 1.5\times10^{-5}m^2 \qquad <$$

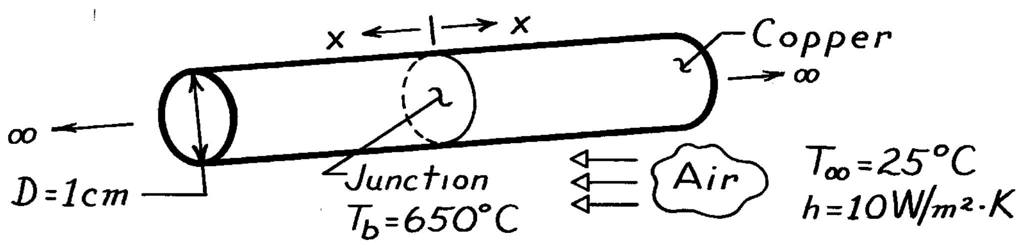
COMMENTS: Although the heat rate is slightly larger (~10%) for the rectangular fin than for the triangular or parabolic fins, the heat rate per unit volume (or mass) is larger and largest for the triangular and parabolic fins, respectively.

PROBLEM 3.124

KNOWN: Melting point of solder used to join two long copper rods.

FIND: Minimum power needed to solder the rods.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction along the rods, (3) Constant properties, (4) No internal heat generation, (5) Negligible radiation exchange with surroundings, (6) Uniform h, and (7) Infinitely long rods.

PROPERTIES: *Table A-1:* Copper $\bar{T} = (650 + 25)^\circ \text{C} \approx 600\text{K}$: $k = 379$ W/m·K.

ANALYSIS: The junction must be maintained at 650°C while energy is transferred by conduction from the junction (along both rods). The minimum power is twice the fin heat rate for an infinitely long fin,

$$q_{min} = 2q_f = 2\left(hPkA_c\right)^{1/2}\left(T_b - T_\infty\right).$$

Substituting numerical values,

$$q_{min} = 2\left[10\frac{\text{W}}{\text{m}^2\cdot\text{K}}(\pi\times 0.01\text{m})\left[379\frac{\text{W}}{\text{m}\cdot\text{K}}\right]\frac{\pi}{4}(0.01\text{m})^2\right]^{1/2}(650-25)^\circ\text{C}.$$

Therefore,

$$q_{min} = 120.9 \text{ W}.$$ <

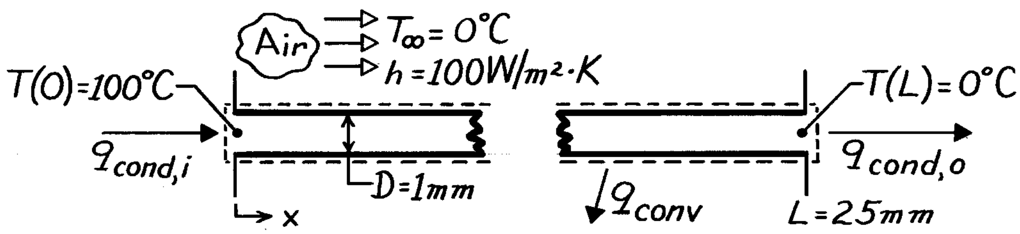
COMMENTS: Radiation losses from the rods may be significant, particularly near the junction, thereby requiring a larger power input to maintain the junction at 650°C.

PROBLEM 3.125

KNOWN: Dimensions and end temperatures of pin fins.

FIND: (a) Heat transfer by convection from a single fin and (b) Total heat transfer from a 1 m^2 surface with fins mounted on 4mm centers.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional conduction along rod, (3) Constant properties, (4) No internal heat generation, (5) Negligible radiation.

PROPERTIES: *Table A-1,* Copper, pure (323K): $k \approx 400$ W/m·K.

ANALYSIS: (a) By applying conservation of energy to the fin, it follows that

$$q_{conv} = q_{cond,i} - q_{cond,o}$$

where the conduction rates may be evaluated from knowledge of the temperature distribution. The general solution for the temperature distribution is

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx} \qquad \theta \equiv T - T_\infty.$$

The boundary conditions are $\theta(0) \equiv \theta_o = 100°C$ and $\theta(L) = 0$. Hence

$$\theta_o = C_1 + C_2$$

$$0 = C_1 e^{mL} + C_2 e^{-mL}$$

Therefore, $$C_2 = C_1 e^{2mL}$$

$$C_1 = \frac{\theta_o}{1-e^{2mL}}, \qquad C_2 = -\frac{\theta_o e^{2mL}}{1-e^{2mL}}$$

and the temperature distribution has the form

$$\theta = \frac{\theta_o}{1-e^{2mL}}\left[e^{mx} - e^{2mL-mx}\right].$$

The conduction heat rate can be evaluated by Fourier's law,

$$q_{cond} = -kA_c \frac{d\theta}{dx} = -\frac{kA_c\theta_o}{1-e^{2mL}} m\left[e^{mx} + e^{2mL-mx}\right]$$

or, with $m = (hP/kA_c)^{1/2}$,

$$q_{cond} = -\frac{\theta_o (hPkA_c)^{1/2}}{1-e^{2mL}}\left[e^{mx} + e^{2mL-mx}\right].$$

Continued …..

Hence at x = 0,

$$q_{cond,i} = -\frac{\theta_o \left(hPkA_c\right)^{1/2}}{1-e^{2mL}}\left(1+e^{2mL}\right)$$

at x = L

$$q_{cond,o} = -\frac{\theta_o \left(hPkA_c\right)^{1/2}}{1-e^{2mL}}\left(2e^{mL}\right)$$

Evaluating the fin parameters:

$$m = \left[\frac{hP}{kA_c}\right]^{1/2} = \left[\frac{4h}{kD}\right]^{1/2} = \left[\frac{4\times 100\ \text{W/m}^2\cdot\text{K}}{400\ \text{W/m}\cdot\text{K}\times 0.001\text{m}}\right]^{1/2} = 31.62\ \text{m}^{-1}$$

$$\left(hPkA_c\right)^{1/2} = \left[\frac{\pi^2}{4}D^3hk\right]^{1/2} = \left[\frac{\pi^2}{4}\times\left(0.001\text{m}\right)^3\times 100\frac{\text{W}}{\text{m}^2\cdot\text{K}}\times 400\frac{\text{W}}{\text{m}\cdot\text{K}}\right]^{1/2} = 9.93\times 10^{-3}\ \frac{\text{W}}{\text{K}}$$

$$mL = 31.62\ \text{m}^{-1}\times 0.025\text{m} = 0.791, \qquad e^{mL} = 2.204, \qquad e^{2mL} = 4.865$$

The conduction heat rates are

$$q_{cond,i} = \frac{-100\text{K}\left(9.93\times 10^{-3}\ \text{W/K}\right)}{-3.865}\times 5.865 = 1.507\ \text{W}$$

$$q_{cond,o} = \frac{-100\text{K}\left(9.93\times 10^{-3}\ \text{W/K}\right)}{-3.865}\times 4.408 = 1.133\ \text{W}$$

and from the conservation relation,

$$q_{conv} = 1.507\ \text{W} - 1.133\ \text{W} = 0.374\ \text{W}.$$ <

(b) The total heat transfer rate is the heat transfer from N = 250×250 = 62,500 rods and the heat transfer from the remaining (bare) surface ($A = 1\text{m}^2 - NA_c$). Hence,

$$q = N\ q_{cond,i} + hA\theta_o = 62,500\ \left(1.507\ \text{W}\right) + 100\text{W/m}^2\cdot\text{K}\left(0.951\ \text{m}^2\right)\ 100\text{K}$$

$$q = 9.42\times 10^4\ \text{W} + 0.95\times 10^4\ \text{W} = 1.037\times 10^5\ \text{W}.$$

COMMENTS: (1) The fins, which cover only 5% of the surface area, provide for more than 90% of the heat transfer from the surface.

(2) The fin effectiveness, $\varepsilon \equiv q_{cond,i}/hA_c\theta_o$, is ε = 192, and the fin efficiency, $\eta \equiv \left(q_{conv}/h\pi\ DL\theta_o\right)$, is $\eta = 0.48$.

(3) The temperature distribution, $\theta(x)/\theta_o$, and the conduction term, $q_{cond,i}$, could have been obtained directly from Eqs. 3.77and 3.78, respectively.

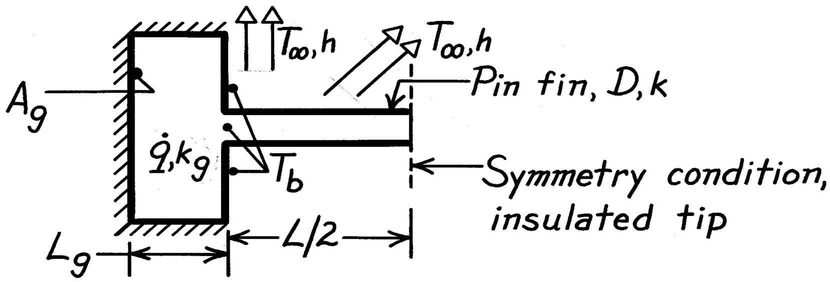
(4) Heat transfer by convection from a single fin could also have been obtained from Eq. 3.73.

PROBLEM 3.126

KNOWN: Pin fin of thermal conductivity k, length L and diameter D connecting two devices (L_g,k_g) experiencing volumetric generation of thermal energy $(\dot{q})$. Convection conditions are prescribed (T_∞, h).

FIND: Expression for the device surface temperature T_b in terms of device, convection and fin parameters.

SCHEMATIC:

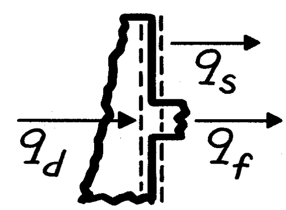


ASSUMPTIONS: (1) Steady-state conditions, (2) Pin fin is of uniform cross-section with constant h, (3) Exposed surface of device is at a uniform temperature T_b, (4) Backside of device is insulated, (5) Device experiences 1-D heat conduction with uniform volumetric generation, (6) Constant properties, and (7) No contact resistance between fin and devices.

ANALYSIS: Recognizing symmetry, the pin fin is modeled as a fin of length L/2 with insulated tip. Perform a surface energy balance,

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$q_d - q_s - q_f = 0 \qquad (1)$$

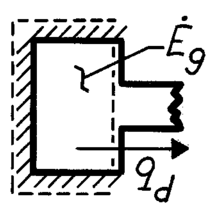


The heat rate q_d can be found from an energy balance on the entire device to find

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0$$

$$-q_d + \dot{q}V = 0$$

$$q_d = \dot{q}A_g L_g \qquad (2)$$



The fin heat rate, q_f, follows from Case B, Table 3.4

$$q_f = M \tanh mL/2 = \left(hPkA_c\right)^{1/2}\left(T_b - T_\infty\right)\tanh\left(mL/2\right), \qquad m = \left(hP/kA_c\right)^{1/2} \qquad (3,4)$$

$$P/A_c = \pi D/\left(\pi D^2/4\right) = 4/D \quad \text{and} \quad PA_c = \pi^2 D^3/4. \qquad (5,6)$$

Hence, the heat rate expression can be written as

$$\dot{q}A_g L_g = h\left(A_g - A_c\right)\left(T_b - T_\infty\right) + \left(hk\left(\pi^2 D^3/4\right)\right)^{1/2} \tanh\left(\left(\frac{4h}{kD}\right)^{1/2} \cdot \frac{L}{2}\right)\left(T_b - T_\infty\right) \qquad (7)$$

Solve now for T_b,

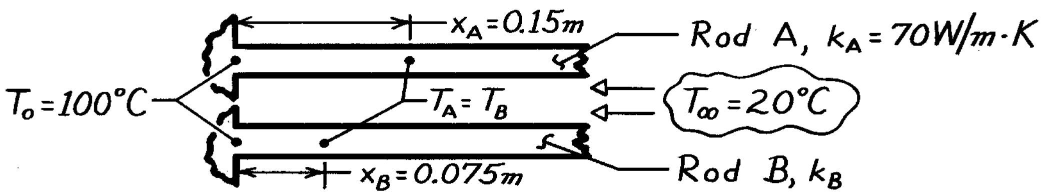
$$T_b = T_\infty + \dot{q}A_g L_g \Big/ \left[h\left(A_g - A_c\right) + \left(hk\left(\pi^2 D^3/4\right)\right)^{1/2} \tanh\left(\left(\frac{4h}{kD}\right)^{1/2} \cdot \frac{L}{2}\right)\right] \qquad (8)$$ <

PROBLEM 3.127

KNOWN: Positions of equal temperature on two long rods of the same diameter, but different thermal conductivity, which are exposed to the same base temperature and ambient air conditions.

FIND: Thermal conductivity of rod B, k_B.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Rods are infinitely long fins of uniform cross-sectional area, (3) Uniform heat transfer coefficient, (4) Constant properties.

ANALYSIS: The temperature distribution for the infinite fin has the form

$$\frac{\theta}{\theta_b} = \frac{T(x) - T_\infty}{T_o - T_\infty} = e^{-mx} \qquad m = \left[\frac{hP}{kA_c}\right]^{1/2}. \tag{1,2}$$

For the two positions prescribed, x_A and x_B, it was observed that

$$T_A(x_A) = T_B(x_B) \qquad \text{or} \qquad \theta_A(x_A) = \theta_B(x_B). \tag{3}$$

Since θ_b is identical for both rods, Eq. (1) with the equality of Eq. (3) requires that

$$m_A x_A = m_B x_B$$

Substituting for m from Eq. (2) gives

$$\left[\frac{hP}{k_A A_c}\right]^{1/2} x_A = \left[\frac{hP}{k_B A_c}\right]^{1/2} x_B.$$

Recognizing that h, P and A_c are identical for each rod and rearranging,

$$k_B = \left[\frac{x_B}{x_A}\right]^2 k_A$$

$$k_B = \left[\frac{0.075\text{m}}{0.15\text{m}}\right]^2 \times 70 \text{ W/m}\cdot\text{K} = 17.5 \text{ W/m}\cdot\text{K}.$$ <

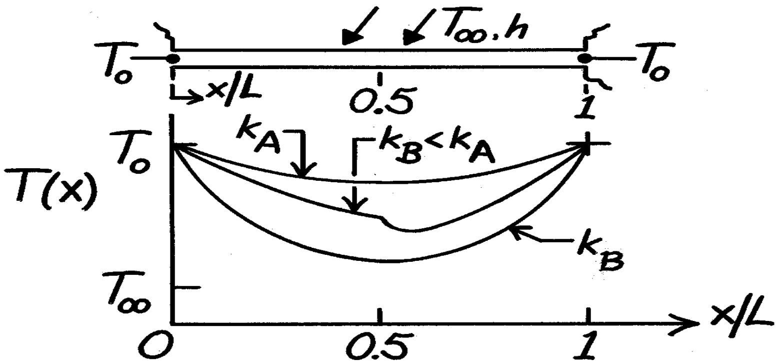
COMMENTS: This approach has been used as a method for determining the thermal conductivity. It has the attractive feature of not requiring power or temperature measurements, assuming of course, a reference material of known thermal conductivity is available.

PROBLEM 3.128

KNOWN: Slender rod of length L with ends maintained at T_o while exposed to convection cooling ($T_\infty < T_o$, h).

FIND: Temperature distribution for three cases, when rod has thermal conductivity (a) k_A, (b) $k_B < k_A$, and (c) k_A for $0 \le x \le L/2$ and k_B for $L/2 \le x \le L$.

SCHEMATIC:



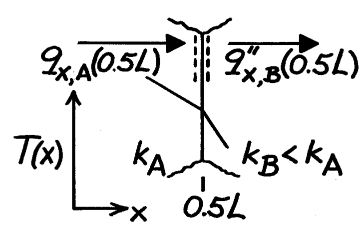
ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, and (4) Negligible thermal resistance between the two materials (A, B) at the mid-span for case (c).

ANALYSIS: (a, b) The effect of thermal conductivity on the temperature distribution when all other conditions (T_o, h, L) remain the same is to reduce the minimum temperature with decreasing thermal conductivity. Hence, as shown in the sketch, the mid-span temperatures are $T_B(0.5L) < T_A(0.5L)$ for $k_B < k_A$. The temperature distribution is, of course, symmetrical about the mid-span.

(c) For the composite rod, the temperature distribution can be reasoned by considering the boundary condition at the mid-span.

$$q''_{x,A}(0.5L) = q''_{x,B}(0.5L)$$

$$-k_A \left.\frac{dT}{dx}\right)_{A,x=0.5L} = -k_B \left.\frac{dT}{dx}\right)_{B,x=0.5L}$$



Since $k_A > k_B$, it follows that

$$\left(\frac{dT}{dx}\right)_{A,x=0.5L} < \left(\frac{dT}{dx}\right)_{B,x=0.5L}.$$

It follows that the minimum temperature in the rod must be in the k_B region, $x > 0.5L$, and the temperature distribution is not symmetrical about the mid-span.

COMMENTS: (1) Recognize that the area under the curve on the T-x coordinates is proportional to the fin heat rate. What conclusions can you draw regarding the relative magnitudes of q_{fin} for cases (a), (b) and (c)?

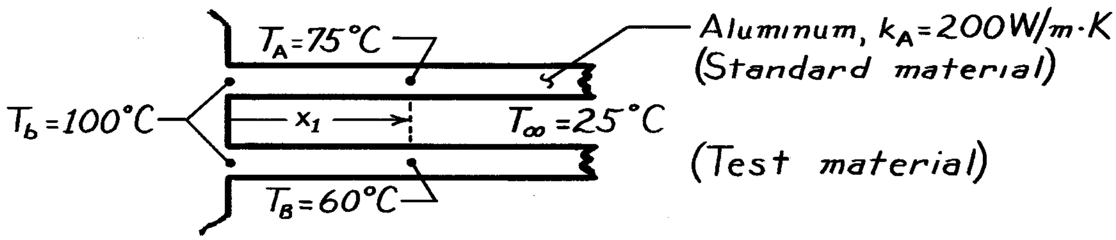
(2) If L is increased substantially, how would the temperature distribution be affected?

PROBLEM 3.129

KNOWN: Base temperature, ambient fluid conditions, and temperatures at a prescribed distance from the base for two long rods, with one of known thermal conductivity.

FIND: Thermal conductivity of other rod.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional conduction along rods, (3) Constant properties, (4) Negligible radiation, (5) Negligible contact resistance at base, (6) Infinitely long rods, (7) Rods are identical except for their thermal conductivity.

ANALYSIS: With the assumption of infinitely long rods, the temperature distribution is

$$\frac{\theta}{\theta_b} = \frac{T - T_\infty}{T_b - T_\infty} = e^{-mx}$$

or

$$\ln \frac{T - T_\infty}{T_b - T_\infty} = -mx = \left[\frac{hP}{kA}\right]^{1/2} x$$

Hence, for the two rods,

$$\frac{\ln\left[\dfrac{T_A - T_\infty}{T_b - T_\infty}\right]}{\ln\left[\dfrac{T_B - T_\infty}{T_b - T_\infty}\right]} = \left[\frac{k_B}{k_A}\right]^{1/2}$$

$$k_B^{1/2} = k_A^{1/2} \frac{\ln\left[\dfrac{T_A - T_\infty}{T_b - T_\infty}\right]}{\ln\left[\dfrac{T_B - T_\infty}{T_b - T_\infty}\right]} = (200)^{1/2} \frac{\ln \dfrac{75-25}{100-25}}{\ln \dfrac{60-25}{100-25}} = 7.524$$

$$k_B = 56.6 \text{ W/m} \cdot \text{K}. \qquad <$$

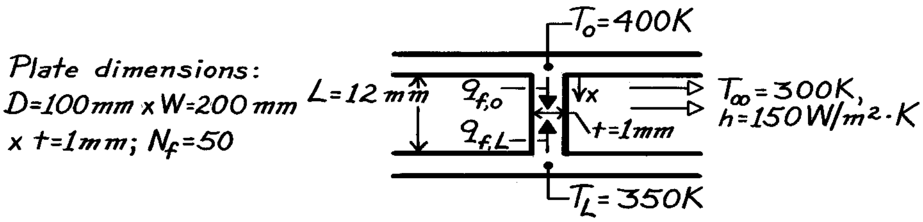
COMMENTS: Providing conditions for the two rods may be maintained nearly identical, the above method provides a convenient means of measuring the thermal conductivity of solids.

PROBLEM 3.130

KNOWN: Arrangement of fins between parallel plates. Temperature and convection coefficient of air flow in finned passages. Maximum allowable plate temperatures.

FIND: (a) Expressions relating fin heat transfer rates to end temperatures, (b) Maximum power dissipation for each plate.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in fins, (3) Constant properties, (4) Negligible radiation, (5) All of the heat is dissipated to the air, (6) Uniform h, (7) Negligible variation in T_∞, (8) Negligible contact resistance.

PROPERTIES: *Table A.1,* Aluminum (pure), 375 K: k = 240 W/m·K.

ANALYSIS: (a) The general solution for the temperature distribution in fin is

$$\theta(x) \equiv T(x) - T_\infty = C_1 e^{mx} + C_2 e^{-mx}$$

Boundary conditions: $\theta(0) = \theta_o = T_o - T_\infty, \qquad \theta(L) = \theta_L = T_L - T_\infty.$

Hence $\quad \theta_o = C_1 + C_2 \qquad \theta_L = C_1 e^{mL} + C_2 e^{-mL}$

$$\theta_L = C_1 e^{mL} + (\theta_o - C_1) e^{-mL}$$

$$C_1 = \frac{\theta_L - \theta_o e^{-mL}}{e^{mL} - e^{-mL}} \qquad C_2 = \theta_o - \frac{\theta_L - \theta_o e^{-mL}}{e^{mL} - e^{-mL}} = \frac{\theta_o e^{mL} - \theta_L}{e^{mL} - e^{-mL}}.$$

Hence $$\theta(x) = \frac{\theta_L e^{mx} - \theta_o e^{m(x-L)} + \theta_o e^{m(L-x)} - \theta_L e^{-mx}}{e^{mL} - e^{-mL}}$$

$$\theta(x) = \frac{\theta_o\left[e^{m(L-x)} - e^{-m(L-x)}\right] + \theta_L\left(e^{mx} - e^{-mx}\right)}{e^{mL} - e^{-mL}}$$

$$\theta(x) = \frac{\theta_o \sinh m(L-x) + \theta_L \sinh mx}{\sinh mL}.$$

The fin heat transfer rate is then

$$q_f = -kA_c\frac{dT}{dx} = -kDt\left[-\frac{\theta_o m}{\sinh mL}\cosh m(L-x) + \frac{\theta_L m}{\sinh mL}\cosh mx\right].$$

Hence $$q_{f,o} = kDt\left(\frac{\theta_o m}{\tanh mL} - \frac{\theta_L m}{\sinh mL}\right) \qquad <$$

$$q_{f,L} = kDt\left(\frac{\theta_o m}{\sinh mL} - \frac{\theta_L m}{\tanh mL}\right). \qquad <$$

Continued …..

(b) $$m=\left(\frac{hP}{kA_c}\right)^{1/2}=\left(\frac{50\ \text{W/m}^2\cdot\text{K}\left(2\times0.1\ \text{m}+2\times0.001\ \text{m}\right)}{240\ \text{W/m}\cdot\text{K}\times0.1\ \text{m}\times0.001\ \text{m}}\right)^{1/2}=35.5\ \text{m}^{-1}$$

$$mL=35.5\ \text{m}^{-1}\times0.012\ \text{m}=0.43$$

$$\sinh\ mL=0.439 \qquad \tanh\ mL=0.401 \qquad \theta_o=100\ \text{K} \qquad \theta_L=50\ \text{K}$$

$$q_{f,o}=240\ \text{W/m}\cdot\text{K}\times0.1\ \text{m}\times0.001\ \text{m}\left(\frac{100\ \text{K}\times35.5\ \text{m}^{-1}}{0.401}-\frac{50\ \text{K}\times35.5\ \text{m}^{-1}}{0.439}\right)$$

$q_{f,o}=115.4\ \text{W}$ (*from* the top plate)

$$q_{f,L}=240\ \text{W/m}\cdot\text{K}\times0.1\ \text{m}\times0.001\ \text{m}\left(\frac{100\ \text{K}\times35.5\ \text{m}^{-1}}{0.439}-\frac{50\ \text{K}\times35.5\ \text{m}^{-1}}{0.401}\right)$$

$q_{f,L}=87.8\ \text{W.}$ (*into* the bottom plate)

Maximum power dissipations are therefore

$$q_{o,max}=N_f q_{f,o}+\left(W-N_f t\right)Dh\theta_o$$

$$q_{o,max}=50\times115.4\ \text{W}+\left(0.200-50\times0.001\right)\text{m}\times0.1\ \text{m}\times150\ \text{W/m}^2\cdot\text{K}\times100\ \text{K}$$

$$q_{o,max}=5770\ \text{W}+225\ \text{W}=5995\ \text{W}$$ <

$$q_{L,max}=-N_f q_{f,L}+\left(W-N_f t\right)Dh\theta_o$$

$$q_{L,max}=-50\times87.8\text{W}+\left(0.200-50\times0.001\right)\text{m}\times0.1\ \text{m}\times150\ \text{W/m}^2\cdot\text{K}\times50\ \text{K}$$

$$q_{L,max}=-4390\ \text{W}+112\text{W}=-4278\ \text{W.}$$ <

COMMENTS: (1) It is of interest to determine the air velocity needed to prevent excessive heating of the air as it passes between the plates. If the air temperature change is restricted to $\Delta T_\infty = 5$ K, its flowrate must be

$$\dot{m}_{air}=\frac{q_{tot}}{c_p\Delta T_\infty}=\frac{1717\ \text{W}}{1007\ \text{J/kg}\cdot\text{K}\times5\ \text{K}}=0.34\ \text{kg/s.}$$

Its mean velocity is then

$$V_{air}=\frac{\dot{m}_{air}}{\rho_{air}A_c}=\frac{0.34\ \text{kg/s}}{1.16\ \text{kg/m}^3\times0.012\ \text{m}\left(0.2-50\times0.001\right)\text{m}}=163\ \text{m/s.}$$

Such a velocity would be impossible to maintain. To reduce it to a reasonable value, e.g. 10 m/s, A_c would have to be increased substantially by increasing W (and hence the space between fins) and by increasing L. The present configuration is impractical from the standpoint that 1717 W could not be transferred to air in such a small volume.

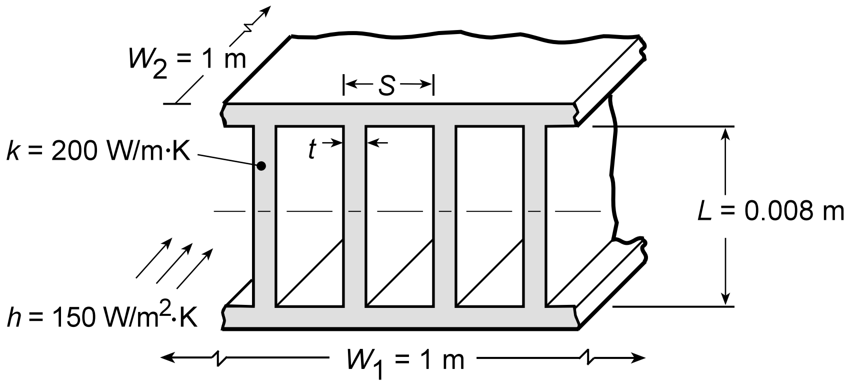
(2) A negative value of $q_{L,max}$ implies that heat must be transferred from the bottom plate to the air to maintain the plate at 350 K.

PROBLEM 3.131

KNOWN: Conditions associated with an array of straight rectangular fins.

FIND: Thermal resistance of the array.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties, (2) Uniform convection coefficient, (3) Symmetry about midplane.

ANALYSIS: (a) Considering a one-half section of the array, the corresponding resistance is

$$R_{t,o} = (\eta_o h A_t)^{-1}$$

where $A_t = NA_f + A_b$. With S = 4 mm and t = 1 mm, it follows that N = W_1/S = 250, A_f = $2(L/2)W_2$ = 0.008 m^2, $A_b = W_2(W_1 - Nt)$ = 0.75 m^2, and A_t = 2.75 m^2.
The overall surface efficiency is

$$\eta_o = 1 - \frac{NA_f}{A_t}(1 - \eta_f)$$

where the fin efficiency is

$$\eta_f = \frac{\tanh m(L/2)}{m(L/2)} \quad \text{and} \quad m = \left(\frac{hP}{kA_c}\right)^{1/2} = \left[\frac{h(2t + 2W_2)}{ktW_2}\right]^{1/2} \approx \left(\frac{2h}{kt}\right)^{1/2} = 38.7\text{m}^{-1}$$

With m(L/2) = 0.155, it follows that η_f = 0.992 and η_o = 0.994. Hence

$$R_{t,o} = \left(0.994 \times 150\text{W/m}^2 \cdot \text{K} \times 2.75\text{m}^2\right)^{-1} = 2.44 \times 10^{-3}\text{K/W}$$ <

(b) The requirements that t ≥ 0.5 m and (S - t) > 2 mm are based on manufacturing and flow passage restriction constraints. Repeating the foregoing calculations for representative values of t and (S - t), we obtain

S (mm)	N	t (mm)	$R_{t,o}$ (K/W)
2.5	400	0.5	0.00169
3	333	0.5	0.00193
3	333	1	0.00202
4	250	0.5	0.00234
4	250	2	0.00268
5	200	0.5	0.00264
5	200	3	0.00334

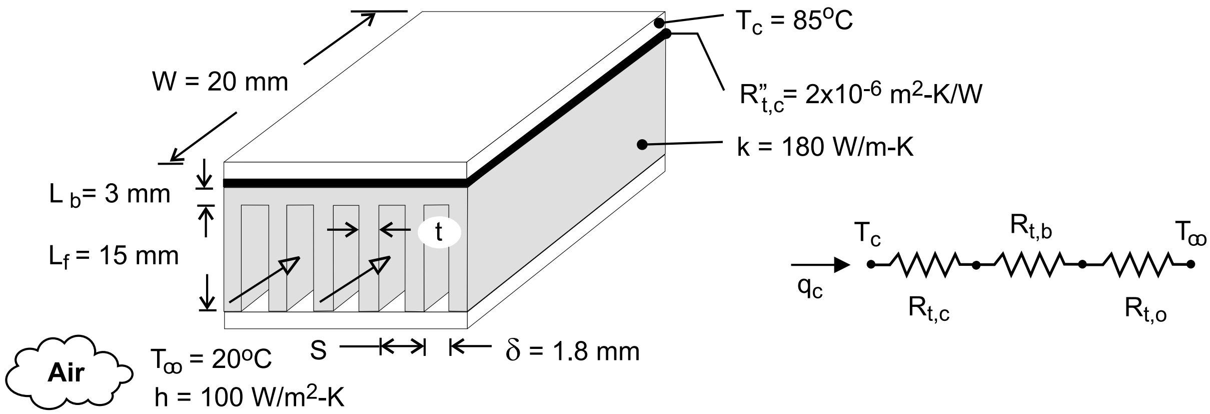
COMMENTS: Clearly, the thermal performance of the fin array improves ($R_{t,o}$ decreases) with increasing N. Because $\eta_f \approx 1$ for the entire range of conditions, there is a slight degradation in performance ($R_{t,o}$ increases) with increasing t and fixed N. The reduced performance is associated with the reduction in surface area of the exposed base. Note that the overall thermal resistance for the entire fin array (top and bottom) is $R_{t,o}/2 = 1.22 \times 10^{-2}$ K/W.

PROBLEM 3.132

KNOWN: Width and maximum allowable temperature of an electronic chip. Thermal contact resistance between chip and heat sink. Dimensions and thermal conductivity of heat sink. Temperature and convection coefficient associated with air flow through the heat sink.

FIND: (a) Maximum allowable chip power for heat sink with prescribed number of fins, fin thickness, and fin pitch, and (b) Effect of fin thickness/number and convection coefficient on performance.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional heat transfer, (3) Isothermal chip, (4) Negligible heat transfer from top surface of chip, (5) Negligible temperature rise for air flow, (6) Uniform convection coefficient associated with air flow through channels and over outer surfaces of heat sink, (7) Negligible radiation.

ANALYSIS: (a) From the thermal circuit,

$$q_c = \frac{T_c - T_\infty}{R_{tot}} = \frac{T_c - T_\infty}{R_{t,c} + R_{t,b} + R_{t,o}}$$

where $R_{t,c} = R''_{t,c} / W^2 = 2\times10^{-6}\text{m}^2 \cdot \text{K/W}/(0.02\text{m})^2 = 0.005 \text{ K/W}$ and $R_{t,b} = L_b / k\left(W^2\right)$

$= 0.003\text{m}/180 \text{ W/m}\cdot\text{K}(0.02\text{m})^2 = 0.042 \text{ K/W}$. From Eqs. (3.103), (3.102), and (3.99)

$$R_{t,o} = \frac{1}{\eta_o h A_t}, \qquad \eta_o = 1 - \frac{N A_f}{A_t}(1-\eta_f), \qquad A_t = N A_f + A_b$$

where $A_f = 2WL_f = 2 \times 0.02\text{m} \times 0.015\text{m} = 6 \times 10^{-4}\text{ m}^2$ and $A_b = W^2 - N(tW) = (0.02\text{m})^2 - 11(0.182 \times 10^{-3}\text{ m} \times 0.02\text{m}) = 3.6 \times 10^{-4}\text{ m}^2$. With $mL_f = (2h/kt)^{1/2} L_f = (200\text{ W/m}^2\cdot\text{K}/180\text{ W/m}\cdot\text{K} \times 0.182 \times 10^{-3}\text{m})^{1/2}(0.015\text{m}) = 1.17$, $\tanh mL_f = 0.824$ and Eq. (3.87) yields

$$\eta_f = \frac{\tanh mL_f}{mL_f} = \frac{0.824}{1.17} = 0.704$$

It follows that $A_t = 6.96 \times 10^{-3}\text{ m}^2$, $\eta_o = 0.719$, $R_{t,o} = 2.00$ K/W, and

$$q_c = \frac{(85-20)°\text{C}}{(0.005+0.042+2.00)\text{K/W}} = 31.8 \text{ W} \qquad <$$

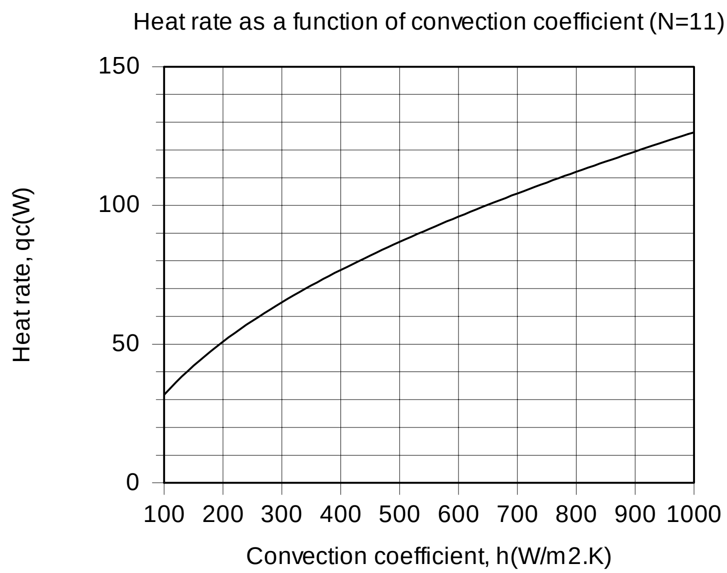
(b) The following results are obtained from parametric calculations performed to explore the effect of decreasing the number of fins and increasing the fin thickness.

Continued …..

PROBLEM 3.132 (Cont.)

N	t(mm)	η_f	$R_{t,o}$ (K/W)	q_c (W)	A_t (m^2)
6	1.833	0.957	2.76	23.2	0.00378
7	1.314	0.941	2.40	26.6	0.00442
8	0.925	0.919	2.15	29.7	0.00505
9	0.622	0.885	1.97	32.2	0.00569
10	0.380	0.826	1.89	33.5	0.00632
11	0.182	0.704	2.00	31.8	0.00696

Although η_f (and η_o) increases with decreasing N (increasing t), there is a reduction in A_t which yields a minimum in $R_{t,o}$, and hence a maximum value of q_c, for N = 10. For N = 11, the effect of h on the performance of the heat sink is shown below.



With increasing h from 100 to 1000 $W/m^2 \cdot K$, $R_{t,o}$ decreases from 2.00 to 0.47 K/W, despite a decrease in η_f (and η_o) from 0.704 (0.719) to 0.269 (0.309). The corresponding increase in q_c is significant.

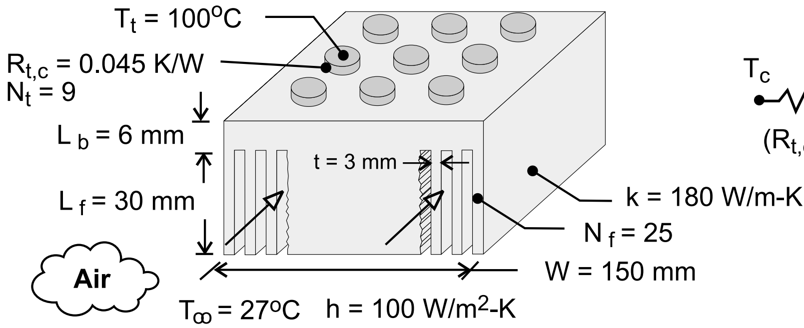
COMMENTS: The heat sink significantly increases the allowable heat dissipation. If it were not used and heat was simply transferred by convection from the surface of the chip with h = 100 $W/m^2 \cdot K$, R_{tot} = 2.05 K/W from Part (a) would be replaced by $R_{cnv} = 1/hW^2$ = 25 K/W, yielding q_c = 2.60 W.

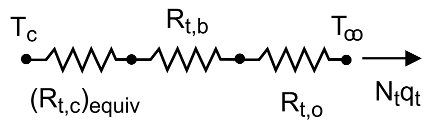
PROBLEM 3.133

KNOWN: Number and maximum allowable temperature of power transistors. Contact resistance between transistors and heat sink. Dimensions and thermal conductivity of heat sink. Temperature and convection coefficient associated with air flow through and along the sides of the heat sink.

FIND: (a) Maximum allowable power dissipation per transistor, (b) Effect of the convection coefficient and fin length on the transistor power.

SCHEMATIC:





ASSUMPTIONS: (1) Steady-state, (2) One-dimensional heat transfer, (3) Isothermal transistors, (4) Negligible heat transfer from top surface of heat sink (all heat transfer is through the heat sink), (5) Negligible temperature rise for the air flow, (6) Uniform convection coefficient, (7) Negligible radiation.

ANALYSIS: (a) From the thermal circuit,

$$N_t\, q_t = \frac{T_t - T_\infty}{\left(R_{t,c}\right)_{equiv} + R_{t,b} + R_{t,o}}$$

For the array of transistors, the corresponding contact resistance is the equivalent resistance associated with the component resistances, in which case,

$$\left(R_{t,c}\right)_{equiv} = \left[N_t\left(1/R_{t,c}\right)\right]^{-1} = \left(9/0.045\,\text{K/W}\right)^{-1} = 5\times10^{-3}\,\text{K/W}$$

The thermal resistance associated with the base of the heat sink is

$$R_{t,b} = \frac{L_b}{k\left(W\right)^2} = \frac{0.006\text{m}}{180\ \text{W/m}\cdot\text{K}\left(0.150\text{m}\right)^2} = 1.48\times10^{-3}\ \text{K/W}$$

From Eqs. (3.103), (3.102) and (3.99), the thermal resistance associated with the fin array and the corresponding overall efficiency and total surface area are

$$R_{t,o} = \frac{1}{\eta_o\, h\, A_t}, \qquad \eta_o = 1 - \frac{N_f A_f}{A_t}\left(1-\eta_f\right), \qquad A_t = N_f A_f + A_b$$

Each fin has a surface area of $A_f \approx 2\,W\,L_f = 2 \times 0.15\text{m} \times 0.03\text{m} = 9 \times 10^{-3}\ \text{m}^2$, and the area of the exposed base is $A_b = W^2 - N_f\,(tW) = (0.15\text{m})^2 - 25\,(0.003\text{m} \times 0.15\text{m}) = 1.13 \times 10^{-2}\ \text{m}^2$. With $mL_f = (2h/kt)^{1/2}\, L_f = (200\ \text{W/m}^2\cdot\text{K}/180\ \text{W/m}\cdot\text{K} \times 0.003\text{m})^{1/2}\,(0.03\text{m}) = 0.577$, $\tanh mL_f = 0.520$ and Eq. (3.87) yields

$$\eta_f = \frac{\tanh\ mL_f}{mL_f} = \frac{0.520}{0.577} = 0.902$$

Hence, with $A_t = [25\,(9 \times 10^{-3}) + 1.13 \times 10^{-2}]\text{m}^2 = 0.236\text{m}^2$,

Continued …..

PROBLEM 3.133 (Cont.)

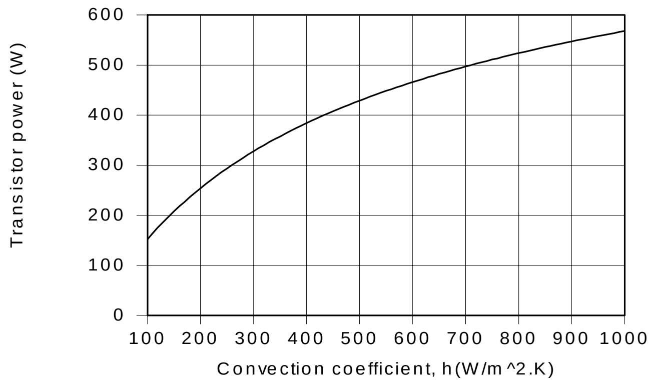
$$\eta_o = 1 - \frac{25\left(0.009\text{m}^2\right)}{0.236\text{m}^2}\left(1-0.901\right) = 0.907$$

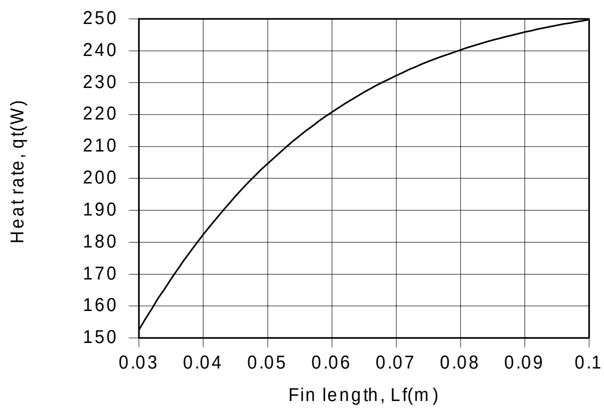
$$R_{t,o} = \left(0.907 \times 100\ \text{W/m}^2 \cdot \text{K} \times 0.236\text{m}^2\right)^{-1} = 0.0467\ \text{K/W}$$

The heat rate per transistor is then

$$q_t = \frac{1}{9}\,\frac{(100-27)^\circ\text{C}}{(0.0050+0.0015+0.0467)\text{K/W}} = 152\ \text{W}$$ <

(b) As shown below, the transistor power dissipation may be enhanced by increasing h and/or L_f.





However, in each case, the effect of the increase diminishes due to an attendant reduction in η_f. For example, as h increases from 100 to 1000 W/m^2·K for L_f = 30 mm, η_f decreases from 0.902 to 0.498.

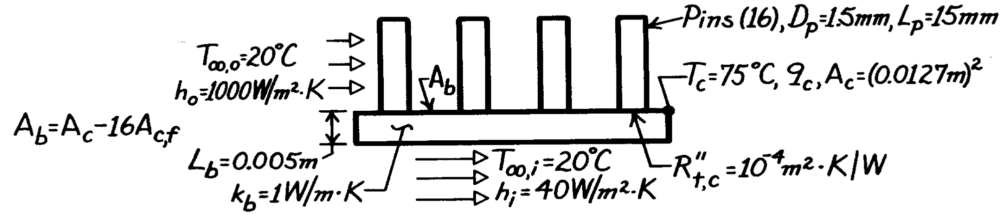
COMMENTS: The heat sink significantly increases the allowable transistor power. If it were not used and heat was simply transferred from a surface of area W^2 = 0.0225 m^2 with h = 100 W/m^2·K, the corresponding thermal resistance would be $R_{t,cnv} = (hW^2)^{-1}$ K/W = 0.44 and the transistor power would be $q_t = (T_t - T_\infty)/N_t\, R_{t,cnv}$ = 18.4 W.

PROBLEM 3.134

KNOWN: Geometry and cooling arrangement for a chip-circuit board arrangement. Maximum chip temperature.

FIND: (a) Equivalent thermal circuit, (b) Maximum chip heat rate.

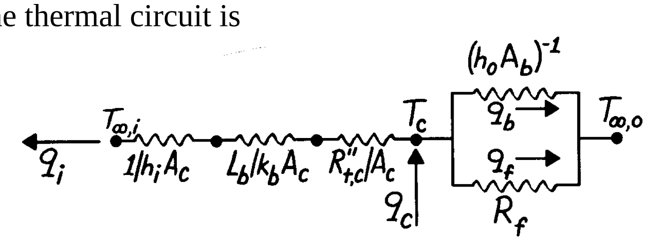
SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat transfer in chip-board assembly, (3) Negligible pin-chip contact resistance, (4) Constant properties, (5) Negligible chip thermal resistance, (6) Uniform chip temperature.

PROPERTIES: *Table A.1,* Copper (300 K): $k \approx 400$ W/m·K.

ANALYSIS: (a) The thermal circuit is



$$R_f = \frac{\theta_b}{16q_f} = \frac{\cosh mL + (h_o/mk)\sinh mL}{16\left(h_oPkA_{c,f}\right)^{1/2}\left[\sinh mL + (h_o/mk)\cosh mL\right]}$$

(b) The maximum chip heat rate is

$$q_c = 16q_f + q_b + q_i.$$

Evaluate these parameters

$$m = \left(\frac{h_oP}{kA_{c,f}}\right)^{1/2} = \left(\frac{4h_o}{kD_p}\right)^{1/2} = \left(\frac{4\times1000\ \text{W/m}^2\cdot\text{K}}{400\ \text{W/m}\cdot\text{K}\times0.0015\ \text{m}}\right)^{1/2} = 81.7\ \text{m}^{-1}$$

$$mL = \left(81.7\ \text{m}^{-1}\times0.015\ \text{m}\right) = 1.23,\qquad \sinh mL = 1.57,\qquad \cosh mL = 1.86$$

$$(h/mk) = \frac{1000\ \text{W/m}^2\cdot\text{K}}{81.7\ \text{m}^{-1}\times400\ \text{W/m}\cdot\text{K}} = 0.0306$$

$$M = \left(h_o\pi\ D_pk\pi\ D_p^2/4\right)^{1/2}\theta_b$$

$$M = \left[1000\ \text{W/m}^2\cdot\text{K}\left(\pi^2/4\right)(0.0015\ \text{m})^3\,400\ \text{W/m}\cdot\text{K}\right]^{1/2}\left(55^\circ\text{C}\right) = 3.17\ \text{W}.$$

Continued …..

The fin heat rate is

$$q_f = M\frac{\sinh mL+(h/mk)\cosh mL}{\cosh mL+(h/mk)\sinh mL} = 3.17\ \text{W}\frac{1.57+0.0306\times1.86}{1.86+0.0306\times1.57}$$

$$q_f = 2.703\ \text{W}.$$

The heat rate from the board by convection is

$$q_b = h_o A_b \theta_b = 1000\ \text{W/m}^2\cdot\text{K}\left[(0.0127\ \text{m})^2 - (16\pi/4)(0.0015\ \text{m})^2\right]55^\circ\text{C}$$

$$q_b = 7.32\ \text{W}.$$

The convection heat rate is

$$q_i = \frac{T_c - T_{\infty,i}}{\left(1/h_i + R''_{t,c} + L_b/k_b\right)(1/A_c)} = \frac{(0.0127\ \text{m})^2\left(55^\circ\text{C}\right)}{\left(1/40+10^{-4}+0.005/1\right)\text{m}^2\cdot\text{K/W}}$$

$$q_i = 0.29\ \text{W}.$$

Hence, the maximum chip heat rate is

$$q_c = \left[16(2.703)+7.32+0.29\right]\text{W} = \left[43.25+7.32+0.29\right]\text{W}$$

$$q_c = 50.9\ \text{W}.$$ <

COMMENTS: (1) The fins are extremely effective in enhancing heat transfer from the chip (assuming negligible contact resistance). Their effectiveness is $\varepsilon = q_f / \left(\pi\ D_p^2/4\right) h_o \theta_b = 2.703$ W/0.097 W = 27.8

(2) Without the fins, q_c = 1000 W/m$^2\cdot$K(0.0127 m)^{2}55°C + 0.29 W = 9.16 W. Hence the fins provide for a (50.9 W/9.16 W) × 100% = 555% enhancement of heat transfer.

(3) With the fins, the chip heat flux is 50.9 W/(0.0127 m)^2 or $q''_c = 3.16\times10^5\ \text{W/m}^2 = 31.6$ W/cm^2.

(4) If the infinite fin approximation is made, q_f = M = 3.17 W, and the actual fin heat transfer is overestimated by 17%.

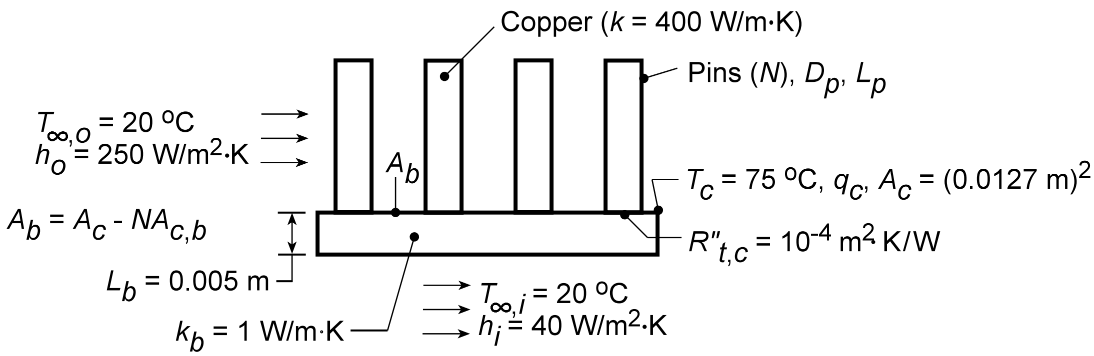
PROBLEM 3.135

KNOWN: Geometry of pin fin array used as heat sink for a computer chip. Array convection and chip substrate conditions.

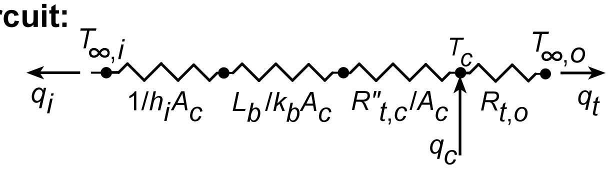
FIND: Effect of pin diameter, spacing and length on maximum allowable chip power dissipation.

SCHEMATIC:

Physical System:



Thermal Circuit:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat transfer in chip-board assembly, (3) Negligible pin-chip contact resistance, (4) Constant properties, (5) Negligible chip thermal resistance, (6) Uniform chip temperature.

ANALYSIS: The total power dissipation is $q_c = q_i + q_t$, where

$$q_i = \frac{T_c - T_{\infty,i}}{\left(1/h_i + R''_{t,c} + L_b/k_b\right)/A_c} = 0.3\text{W}$$

and

$$q_t = \frac{T_c - T_{\infty,o}}{R_{t,o}}$$

The resistance of the pin array is

$$R_{t,o} = \left(\eta_o h_o A_t\right)^{-1}$$

where

$$\eta_o = 1 - \frac{NA_f}{A_t}\left(1-\eta_f\right)$$

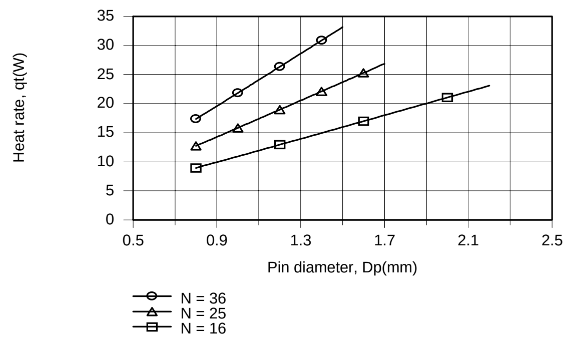
$$A_t = NA_f + A_b$$

$$A_f = \pi D_p L_c = \pi D_p \left(L_p + D_p/4\right)$$

Subject to the constraint that $N^{1/2}D_p \le 9$ mm, the foregoing expressions may be used to compute q_t as a function of D_p for L_p = 15 mm and values of N = 16, 25 and 36. Using the IHT *Performance Calculation, Extended Surface Model* for the *Pin Fin Array*, we obtain

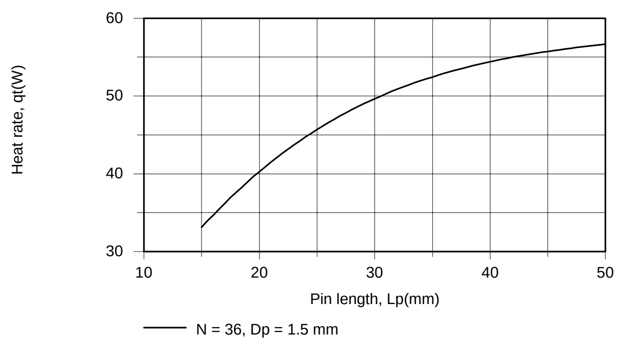
Continued...

PROBLEM 3.135 (CONT.)



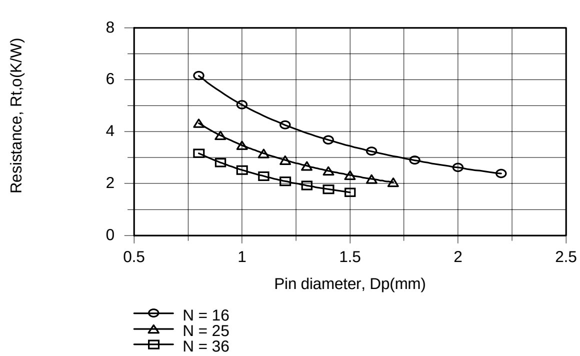
Clearly, it is desirable to maximize the number of pins and the pin diameter, so long as flow passages are not constricted to the point of requiring an excessive pressure drop to maintain the prescribed convection coefficient. The maximum heat rate for the fin array (q_t = 33.1 W) corresponds to N = 36 and D_p = 1.5 mm. Further improvement could be obtained by using N = 49 pins of diameter D_p = 1.286 mm, which yield q_t = 37.7 W.

Exploring the effect of L_p for N = 36 and D_p = 1.5 mm, we obtain



Clearly, there are benefits to increasing L_p, although the effect diminishes due to an attendant reduction in η_f (from η_f = 0.887 for L_p = 15 mm to η_f = 0.471 for L_p = 50 mm). Although a heat dissipation rate of q_t = 56.7 W is obtained for L_p = 50 mm, package volume constraints could preclude such a large fin length.

COMMENTS: By increasing N, D_p and/or L_p, the total surface area of the array, A_t, is increased, thereby reducing the array thermal resistance, $R_{t,o}$. The effects of D_p and N are shown for L_p = 15 mm.

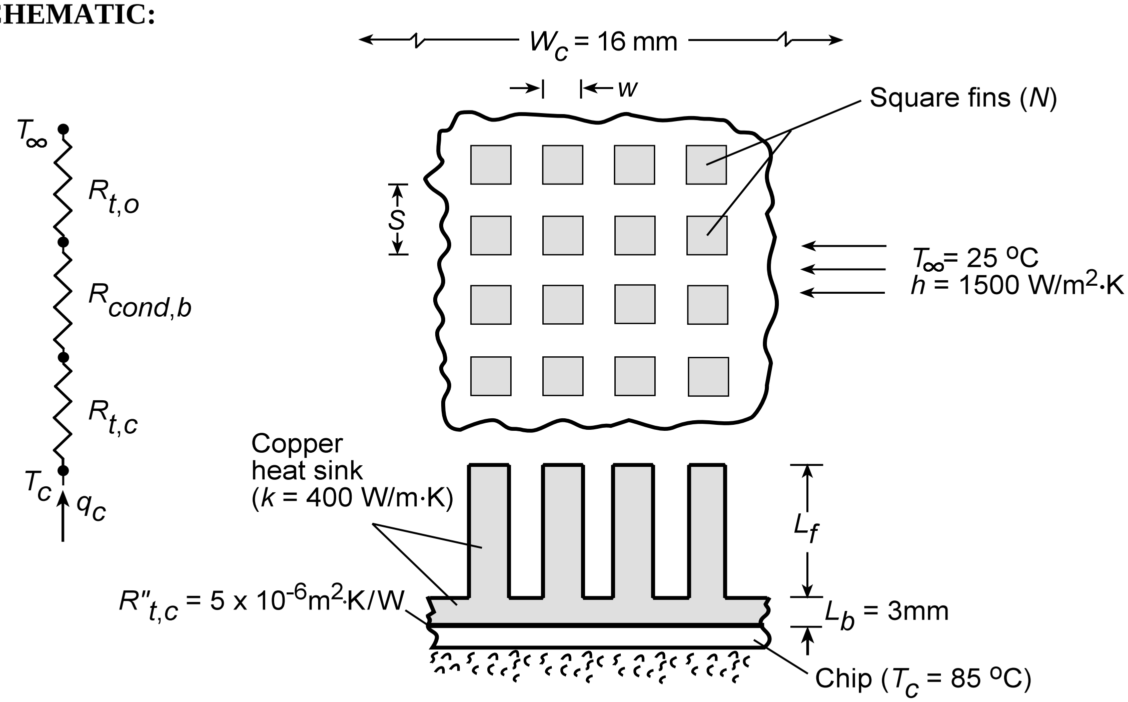


PROBLEM 3.136

KNOWN: Copper heat sink dimensions and convection conditions.

FIND: (a) Maximum allowable heat dissipation for a prescribed chip temperature and interfacial chip/heat-sink contact resistance, (b) Effect of fin length and width on heat dissipation.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat transfer in chip-heat sink assembly, (3) Constant k, (4) Negligible chip thermal resistance, (5) Negligible heat transfer from back of chip, (6) Uniform chip temperature.

ANALYSIS: (a) For the prescribed system, the chip power dissipation may be expressed as

$$q_c = \frac{T_c - T_\infty}{R_{t,c} + R_{cond,b} + R_{t,o}}$$

where $$R_{t,c} = \frac{R''_{t,c}}{W_c^2} = \frac{5\times10^{-6}\,m^2 \cdot K/W}{(0.016m)^2} = 0.0195\,K/W$$

$$R_{cond,b} = \frac{L_b}{kW_c^2} = \frac{0.003m}{400\,W/m\cdot K\,(0.016m)^2} = 0.0293\,K/W$$

The thermal resistance of the fin array is

$$R_{t,o} = (\eta_o h A_t)^{-1}$$

where $$\eta_o = 1 - \frac{NA_f}{A_t}(1-\eta_f)$$

and $$A_t = NA_f + A_b = N(4wL_c) + \left(W_c^2 - Nw^2\right)$$

Continued...

With w = 0.25 mm, S = 0.50 mm, L_f = 6 mm, N = 1024, and $L_c \approx L_f + w/4 = 6.063 \times 10^{-3}$ m, it follows that $A_f = 6.06 \times 10^{-6}\,m^2$ and $A_t = 6.40 \times 10^{-3}\,m^2$. The fin efficiency is

$$\eta_f = \frac{\tanh mL_c}{mL_c}$$

where $m = (hP/kA_c)^{1/2} = (4h/kw)^{1/2}$ = 245 m^{-1} and mL_c = 1.49. It follows that η_f = 0.608 and η_o = 0.619, in which case

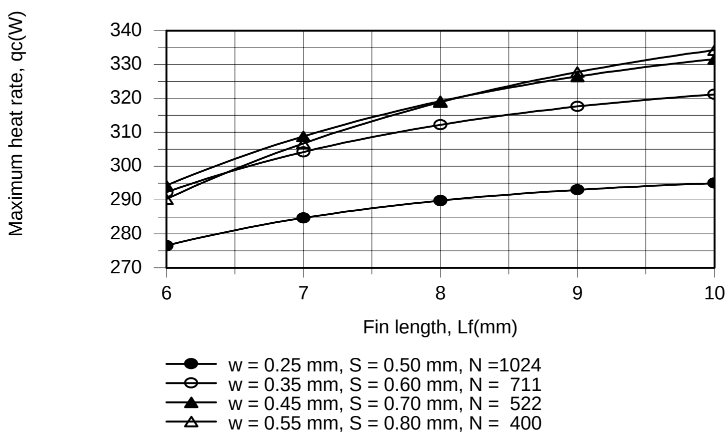
$$R_{t,o} = \left(0.619 \times 1500\,W/m^2 \cdot K \times 6.40 \times 10^{-3}\,m^2\right) = 0.168\,K/W$$

and the maximum allowable heat dissipation is

$$q_c = \frac{(85-25)^\circ C}{(0.0195 + 0.0293 + 0.168)\,K/W} = 276\,W \qquad <$$

(b) The IHT *Performance Calculation, Extended Surface Model* for the *Pin Fin Array* has been used to determine q_c as a function of L_f for four different cases, each of which is characterized by the closest allowable fin spacing of (S - w) = 0.25 mm.

Case	w (mm)	S (mm)	N
A	0.25	0.50	1024
B	0.35	0.60	711
C	0.45	0.70	522
D	0.55	0.80	400



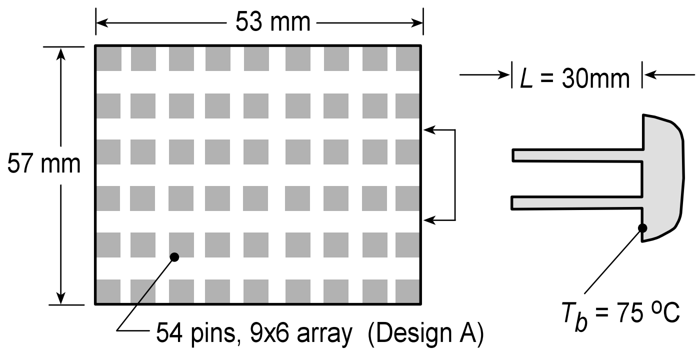
With increasing w and hence decreasing N, there is a reduction in the total area A_t associated with heat transfer from the fin array. However, for Cases A through C, the reduction in A_t is more than balanced by an increase in η_f (and η_o), causing a reduction in $R_{t,o}$ and hence an increase in q_c. As the fin efficiency approaches its limiting value of η_f = 1, reductions in A_t due to increasing w are no longer balanced by increases in η_f, and q_c begins to decrease. Hence there is an optimum value of w, which depends on L_f. For the conditions of this problem, L_f = 10 mm and w = 0.55 mm provide the largest heat dissipation.

Problem 3.137

KNOWN: Two finned heat sinks, Designs A and B, prescribed by the number of fins in the array, N, fin dimensions of square cross-section, w, and length, L, with different convection coefficients, h.

FIND: Determine which fin arrangement is superior. Calculate the heat rate, q_f, efficiency, η_f, and effectiveness, ε_f, of a single fin, as well as, the total heat rate, q_t, and overall efficiency, η_o, of the array. Also, compare the total heat rates per unit volume.

SCHEMATIC:



Design	Fin dimensions Cross section w x w (mm)	Length L (mm)	Number of fins	Convection coefficient (W/m²·K)
A	1 x 1	30	6 x 9	125
B	3 x 3	7	14 x 17	375

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in fins, (3) Convection coefficient is uniform over fin and prime surfaces, (4) Fin tips experience convection, and (5) Constant properties.

ANALYSIS: Following the treatment of Section 3.6.5, the overall efficiency of the array, Eq. (3.98), is

$$\eta_o = \frac{q_t}{q_{max}} = \frac{q_t}{hA_t\theta_b} \tag{1}$$

where A_t is the total surface area, the sum of the exposed portion of the base (prime area) plus the fin surfaces, Eq. 3.99,

$$A_t = N \cdot A_f + A_b \tag{2}$$

where the surface area of a single fin and the prime area are

$$A_f = 4(L \times W) + w^2 \tag{3}$$

$$A_b = b1 \times b2 - N \cdot A_c \tag{4}$$

Combining Eqs. (1) and (2), the total heat rate for the array is

$$q_t = N\eta_f hA_f\theta_b + hA_b\theta_b \tag{5}$$

where η_f is the efficiency of a single fin. From Table 4.3, Case A, for the tip condition with convection, the single fin efficiency based upon Eq. 3.86,

$$\eta_f = \frac{q_f}{hA_f\theta_b} \tag{6}$$

Continued...

where

$$q_f = M\frac{\sinh(mL)+(h/mk)\cosh(mL)}{\cosh(mL)+(h/mk)\sinh(mL)} \qquad (7)$$

$$M=(hPkA_c)^{1/2}\theta_b \qquad m=(hP/kA_c)^{1/2} \qquad P=4w \qquad A_c=w^2 \qquad (8,9,10)$$

The single fin effectiveness, from Eq. 3.81,

$$\varepsilon_f = \frac{q_f}{hA_c\theta_b} \qquad (11)$$

Additionally, we want to compare the performance of the designs with respect to the array volume, vol

$$q_f''' = q_f/\forall = q_f/(b1\cdot b2\cdot L) \qquad (12)$$

The above analysis was organized for easy treatment with equation-solving software. Solving Eqs. (1) through (11) simultaneously with appropriate numerical values, the results are tabulated below.

Design	q_t (W)	q_f (W)	η_o	η_f	ε_f	q_f''' (W/m^3)
A	113	1.80	0.804	0.779	31.9	1.25×10^6
B	165	0.475	0.909	0.873	25.3	7.81×10^6

COMMENTS: (1) Both designs have good efficiencies and effectiveness. Clearly, Design B is superior because the heat rate is nearly 50% larger than Design A for the same board footprint. Further, the space requirement for Design B is four times less ($\forall = 2.12\times10^{-5}$ vs. 9.06×10^{-5} m^3) and the heat rate per unit volume is 6 times greater.

(2) Design A features 54 fins compared to 238 fins for Design B. Also very significant to the performance comparison is the magnitude of the convection coefficient which is 3 times larger for Design B. Estimating convection coefficients for fin arrays (and tube banks) is discussed in Chapter 7.6. Of concern is how the fins alter the flow past the fins and whether the convection coefficient is uniform over the array.

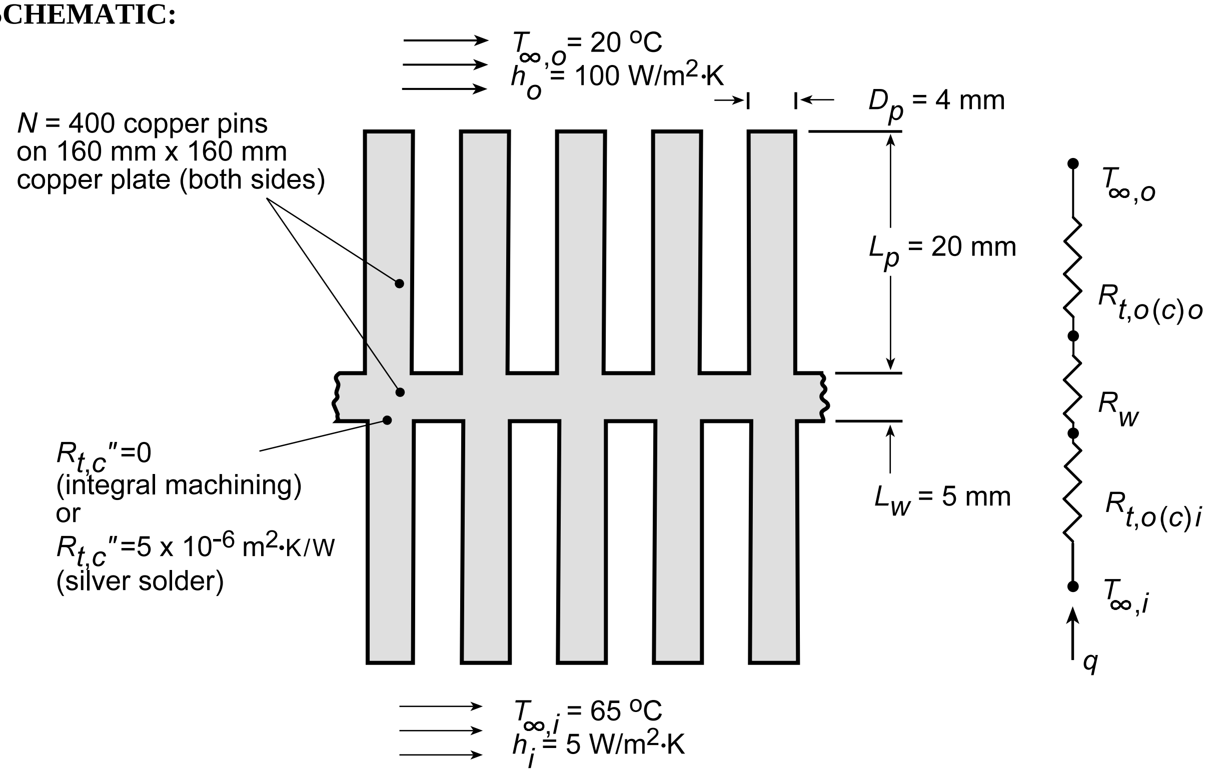
(3) The *IHT Extended Surfaces Model*, for a *Rectangular Pin Fin Array* could have been used to solve this problem.

PROBLEM 3.138

KNOWN: Geometrical characteristics of a plate with pin fin array on both surfaces. Inner and outer convection conditions.

FIND: (a) Heat transfer rate with and without pin fin arrays, (b) Effect of using silver solder to join the pins and the plate.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant k, (3) Negligible radiation.

PROPERTIES: Table A-1: Copper, $\overline{T} \approx 315$ K, k = 400 W/m·K.

ANALYSIS: (a) The heat rate may be expressed as

$$q = \frac{T_{\infty,i} - T_{\infty,o}}{R_{t,o(c),i} + R_w + R_{t,o(c),o}}$$

where

$$R_{t,o(c)} = \left(\eta_{o(c)} h A_t\right)^{-1},$$

$$\eta_{o(c)} = 1 - \frac{NA_f}{A_t}\left(1 - \frac{\eta_f}{C_1}\right),$$

$$A_t = NA_f + A_b,$$

$$A_f = \pi D_p L_c \approx \pi D_p \left(L + D/4\right),$$

$$A_b = W^2 - NA_{c,b} = W^2 - N\left(\pi D_p^2/4\right),$$

$$\eta_f = \frac{\tanh mL_c}{mL_c}, \qquad m = \left(4h/kD_p\right)^{1/2},$$

Continued...

$$C_1 = 1 + \eta_f hA_f \left(R''_{t,c} / A_{c,b}\right),$$

and

$$R_w = \frac{L_w}{W^2 k}.$$

Calculations may be expedited by using the IHT *Performance Calculation, Extended Surface* Model for the *Pin Fin* Array. For $R''_{t,c}$ = 0, C_1 = 1, and with W = 0.160 m, R_w = 0.005 m/(0.160 m)^2 400 W/m·K = 4.88×10^{-4} K/W. For the prescribed array geometry, we also obtain $A_{c,b} = 1.26 \times 10^{-5}$ m^2, $A_f = 2.64 \times 10^{-4}$ m^2, $A_b = 2.06 \times 10^{-2}$ m^2, and A_t = 0.126 m^2.

On the outer surface, where h_o = 100 W/m^2·K, m = 15.8 m^{-1}, η_f = 0.965, η_o = 0.970 and $R_{t,o}$ = 0.0817 K/W. On the inner surface, where h_i = 5 W/m^2·K, m = 3.54 m^{-1}, η_f = 0.998, η_o = 0.999 and $R_{t,o}$ = 1.588 K/W.

Hence, the heat rate is

$$q = \frac{(65-20)^\circ \mathrm{C}}{\left(1.588 + 4.88\times10^{-4} + 0.0817\right)\mathrm{K/W}} = 26.94\,\mathrm{W}$$ <

Without the fins,

$$q = \frac{T_{\infty,i} - T_{\infty,o}}{\left(1/h_i A_w\right) + R_w + \left(1/h_o A_w\right)} = \frac{(65-20)^\circ \mathrm{C}}{\left(7.81 + 4.88\times10^{-4} + 0.39\right)} = 5.49\,\mathrm{W}$$ <

Hence, the fin arrays provide nearly a five-fold increase in heat rate.

(b) With use of the silver solder, $\eta_{o(c),o}$ = 0.962 and $R_{t,o(c),o}$ = 0.0824 K/W. Also, $\eta_{o(c),i}$ = 0.998 and $R_{t,o(c),i}$ = 1.589 K/W. Hence

$$q = \frac{(65-20)^\circ \mathrm{C}}{\left(1.589 + 4.88\times10^{-4} + 0.0824\right)\mathrm{K/W}} = 26.92\,\mathrm{W}$$ <

Hence, the effect of the contact resistance is negligible.

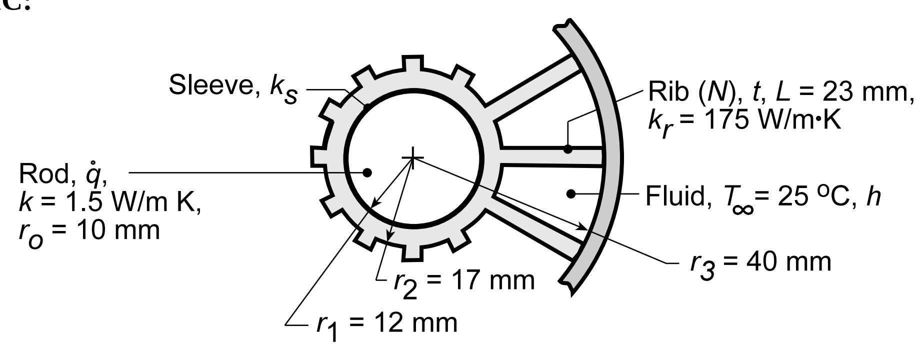
COMMENTS: The dominant contribution to the total thermal resistance is associated with internal conditions. If the heat rate must be increased, it should be done by increasing h_i.

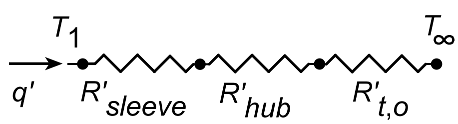
PROBLEM 3.139

KNOWN: Long rod with internal volumetric generation covered by an electrically insulating sleeve and supported with a ribbed spider.

FIND: Combination of convection coefficient, spider design, and sleeve thermal conductivity which enhances volumetric heating subject to a maximum centerline temperature of 100°C.

SCHEMATIC:





ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial heat transfer in rod, sleeve and hub, (3) Negligible interfacial contact resistances, (4) Constant properties, (5) Adiabatic outer surface.

ANALYSIS: The system heat rate per unit length may be expressed as

$$q' = \dot{q}\left(\pi r_o^2\right) = \frac{T_1 - T_\infty}{R'_{sleeve} + R'_{hub} + R'_{t,o}}$$

where

$$R'_{sleeve} = \frac{\ln\left(r_1/r_o\right)}{2\pi k_s}, \quad R'_{hub} = \frac{\ln\left(r_2/r_1\right)}{2\pi k_r} = 3.168\times10^{-4}\,\mathrm{m\cdot K/W}, \quad R'_{t,o} = \frac{1}{\eta_o h A'_t},$$

$$\eta_o = 1 - \frac{NA'_f}{A'_t}\left(1-\eta_f\right), \qquad A'_f = 2\left(r_3 - r_2\right), \qquad A'_t = NA'_f + \left(2\pi r_3 - Nt\right),$$

$$\eta_f = \frac{\tanh m\left(r_3 - r_2\right)}{m\left(r_3 - r_2\right)}, \qquad m = \left(2h/k_r t\right)^{1/2}.$$

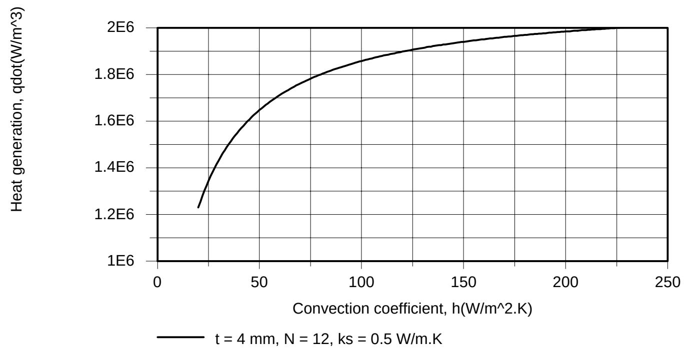
The rod centerline temperature is related to T_1 through

$$T_o = T(0) = T_1 + \frac{\dot{q} r_o^2}{4k}$$

Calculations may be expedited by using the IHT *Performance Calculation, Extended Surface Model* for the *Straight Fin Array*. For base case conditions of k_s = 0.5 W/m·K, h = 20 W/m^2·K, t = 4 mm and N = 12, R'_{sleeve} = 0.0580 m·K/W, $R'_{t,o}$ = 0.0826 m·K/W, η_f = 0.990, q' = 387 W/m, and $\dot{q}$ = 1.23 × 10^6 W/m^3. As shown below, $\dot{q}$ may be increased by increasing h, where h = 250 W/m^2·K represents a reasonable upper limit for airflow. However, a more than 10-fold increase in h yields only a 63% increase in $\dot{q}$.

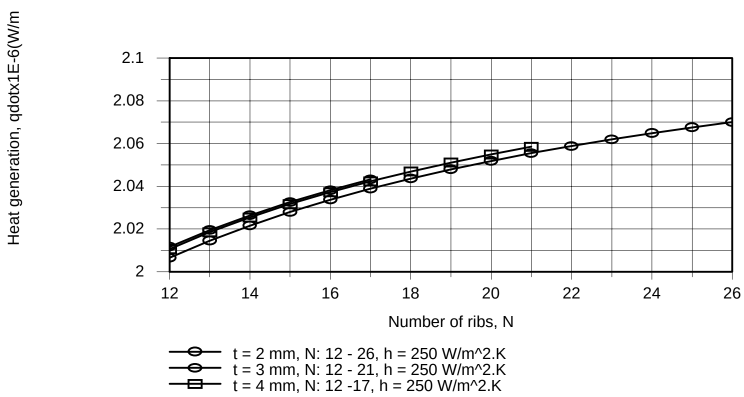
Continued...

PROBLEM 3.139 (Cont.)

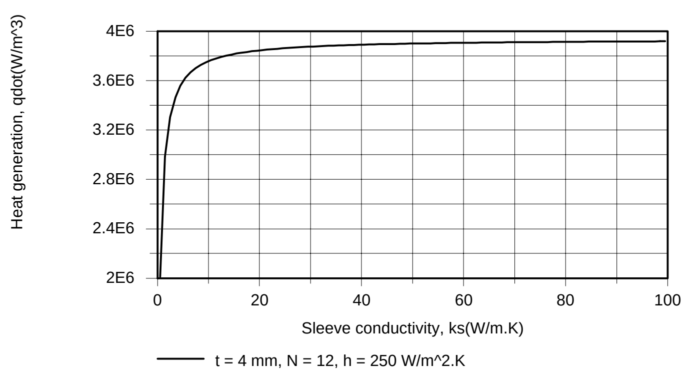


The difficulty is that, by significantly increasing h, the thermal resistance of the fin array is reduced to 0.00727 m·K/W, rendering the sleeve the dominant contributor to the total resistance.

Similar results are obtained when N and t are varied. For values of t = 2, 3 and 4 mm, variations of N in the respective ranges $12 \le N \le 26$, $12 \le N \le 21$ and $12 \le N \le 17$ were considered. The upper limit on N was fixed by requiring that $(S - t) \ge 2$ mm to avoid an excessive resistance to airflow between the ribs. As shown below, the effect of increasing N is small, and there is little difference between results for the three values of t.



In contrast, significant improvement is associated with changing the sleeve material, and it is only necessary to have $k_s \approx 25$ W/m·K (e.g. a boron sleeve) to approach an upper limit to the influence of k_s.



For h = 250 W/m^2·K and k_s = 25 W/m·K, only a slight improvement is obtained by increasing N. Hence, the recommended conditions are:

$$h = 250\,\mathrm{W/m^2\cdot K}, \qquad k_s = 25\,\mathrm{W/m\cdot K}, \qquad N = 12, \qquad t = 4\mathrm{mm} \qquad <$$

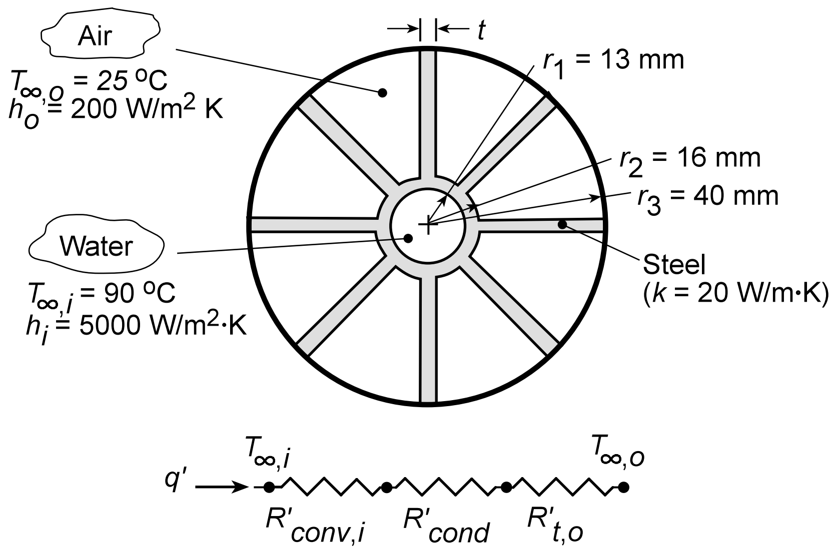
COMMENTS: The upper limit to $\dot{q}$ is reached as the total thermal resistance approaches zero, in which case $T_1 \to T_\infty$. Hence $\dot{q}_{max} = 4k\left(T_o - T_\infty\right)/r_o^2 = 4.5\times10^6\,\mathrm{W/m^3}$.

PROBLEM 3.140

KNOWN: Geometrical and convection conditions of internally finned, concentric tube air heater.

FIND: (a) Thermal circuit, (b) Heat rate per unit tube length, (c) Effect of changes in fin array.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat transfer in radial direction, (3) Constant k, (4) Adiabatic outer surface.

ANALYSIS: (a) For the thermal circuit shown schematically,

$$R'_{conv,i} = (h_i 2\pi r_1)^{-1}, \qquad R'_{cond} = \ln(r_2/r_1)/2\pi k \text{ , and} \qquad R'_{t,o} = (\eta_o h_o A'_t)^{-1},$$

where

$$\eta_o = 1 - \frac{NA'_f}{A'_t}(1-\eta_f), \quad A'_f = 2L = 2(r_3 - r_2), \quad A'_t = NA'_f + (2\pi r_2 - Nt), \text{ and} \quad \eta_f = \frac{\tanh mL}{mL}.$$

(b) $$q' = \frac{(T_{\infty,i} - T_{\infty,o})}{R'_{conv,i} + R'_{cond} + R'_{t,o}}$$

Substituting the known conditions, it follows that

$$R'_{conv,i} = \left(5000\,W/m^2 \cdot K \times 2\pi \times 0.013m\right)^{-1} = 2.45\times10^{-3}\,m\cdot K/W$$

$$R'_{cond} = \ln(0.016m/0.013m)/2\pi\,(20\,W/m\cdot K) = 1.65\times10^{-3}\,m\cdot K/W$$

$$R'_{t,o} = \left(0.575\times200\,W/m^2\cdot K\times0.461m\right)^{-1} = 18.86\times10^{-3}\,m\cdot K/W$$

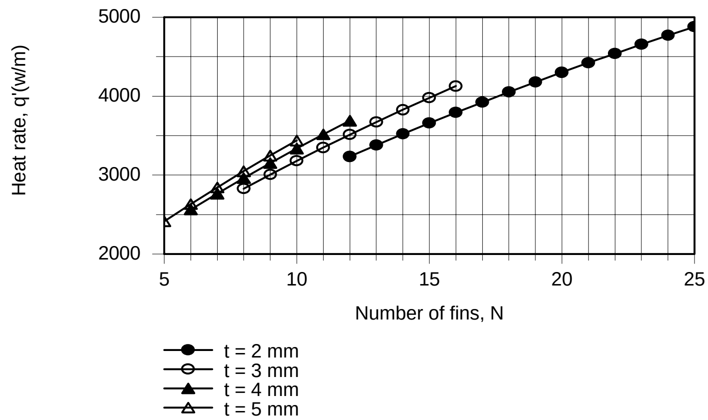
where η_f = 0.490. Hence,

$$q' = \frac{(90-25)^\circ C}{(2.45+1.65+18.86)\times10^{-3}\,m\cdot K/W} = 2831\,W/m$$ <

(c) The small value of η_f suggests that some benefit may be gained by increasing t, as well as by increasing N. With the requirement that Nt ≤ 50 mm, we use the IHT *Performance Calculation, Extended Surface Model* for the *Straight Fin Array* to consider the following range of conditions: t = 2 mm, 12 ≤ N ≤ 25; t = 3 mm, 8 ≤ N ≤ 16; t = 4 mm, 6 ≤ N ≤ 12; t = 5 mm, 5 ≤ N ≤ 10. Calculations based on the foregoing model are plotted as follows.

Continued...

PROBLEM 3.140 (Cont.)



By increasing t from 2 to 5 mm, η_f increases from 0.410 to 0.598. Hence, for fixed N, q' increases with increasing t. However, from the standpoint of maximizing q_t', it is clearly preferable to use the larger number of thinner fins. Hence, subject to the prescribed constraint, we would choose t = 2 mm and N = 25, for which q' = 4880 W/m.

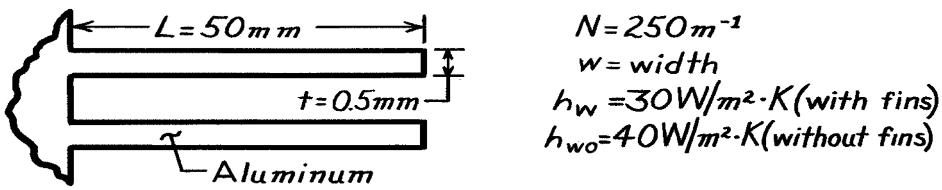
COMMENTS: (1) The air side resistance makes the dominant contribution to the total resistance, and efforts to increase q' by reducing $R_{t,o}'$ are well directed. (2) A fin thickness any smaller than 2 mm would be difficult to manufacture.

PROBLEM 3.141

KNOWN: Dimensions and number of rectangular aluminum fins. Convection coefficient with and without fins.

FIND: Percentage increase in heat transfer resulting from use of fins.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible radiation, (5) Negligible fin contact resistance, (6) Uniform convection coefficient.

PROPERTIES: *Table A-1*, Aluminum, pure: $k \approx 240$ W/m·K.

ANALYSIS: Evaluate the fin parameters

$$L_c = L + t/2 = 0.05025\text{m}$$

$$A_p = L_c t = 0.05025\text{m} \times 0.5 \times 10^{-3}\text{m} = 25.13 \times 10^{-6}\ \text{m}^2$$

$$L_c^{3/2}\left(h_w / kA_p\right)^{1/2} = \left(0.05025\text{m}\right)^{3/2}\left[\frac{30\ \text{W/m}^2 \cdot \text{K}}{240\ \text{W/m} \cdot \text{K} \times 25.13 \times 10^{-6}\text{m}^2}\right]^{1/2}$$

$$L_c^{3/2}\left(h_w / kA_p\right)^{1/2} = 0.794$$

It follows from Fig. 3.18 that $\eta_f \approx 0.72$. Hence,

$$q_f = \eta_f q_{max} = 0.72\ h_w 2wL\ \theta_b$$

$$q_f = 0.72 \times 30\ \text{W/m}^2 \cdot \text{K} \times 2 \times 0.05\text{m} \times \left(w\ \theta_b\right) = 2.16\ \text{W/m} \cdot \text{K}\left(w\ \theta_b\right)$$

With the fins, the heat transfer from the walls is

$$q_w = N\ q_f + \left(1 - Nt\right) w\ h_w\ \theta_b$$

$$q_w = 250 \times 2.16 \frac{\text{W}}{\text{m} \cdot \text{K}}\left(w\ \theta_b\right) + \left(1\text{m} - 250 \times 5 \times 10^{-4}\text{m}\right) \times 30\ \text{W/m}^2 \cdot \text{K}\ \left(w\ \theta_b\right)$$

$$q_w = \left(540 + 26.3\right)\frac{\text{W}}{\text{m} \cdot \text{K}}\left(w\ \theta_b\right) = 566\ w\ \theta_b.$$

Without the fins, $q_{wo} = h_{wo}\ 1\text{m} \times w\ \theta_b = 40\ w\ \theta_b$. Hence the percentage increase in heat transfer is

$$\frac{q_w - q_{wo}}{q_{wo}} = \frac{\left(566 - 40\right) w\ \theta_b}{40\ w\ \theta_b} = 13.15 = 1315\% \qquad <$$

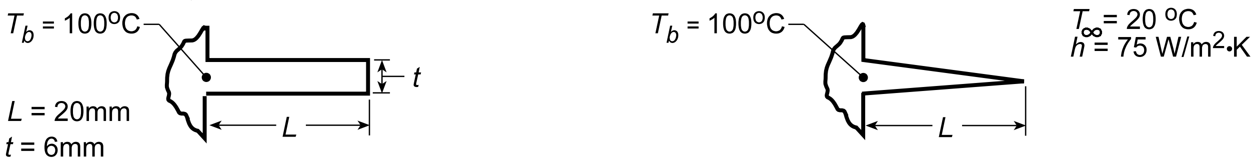
COMMENTS: If the infinite fin approximation is made, it follows that $q_f = (hPkA_c)^{1/2}\ \theta_b = [h_w 2wkwt]^{1/2}\ \theta_b = (30 \times 2 \times 240 \times 5\times10^{-4})^{1/2}\ w\ \theta_b = 2.68\ w\ \theta_b$. Hence, q_f is overestimated.

PROBLEM 3.142

KNOWN: Dimensions, base temperature and environmental conditions associated with rectangular and triangular stainless steel fins.

FIND: Efficiency, heat loss per unit width and effectiveness associated with each fin.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible radiation, (5) Uniform convection coefficient.

PROPERTIES: *Table A-1*, Stainless Steel 304 (T = 333 K): k = 15.3 W/m·K.

ANALYSIS: For the rectangular fin, with $L_c = L + t/2$, evaluate the parameter

$$L_c^{3/2}\left(h/kA_p\right)^{1/2} = (0.023\,\text{m})^{3/2}\left[\frac{75\,\text{W/m}^2\cdot\text{K}}{15.3\,\text{W/m}\cdot\text{K}\,(0.023\,\text{m})(0.006\,\text{m})}\right]^{1/2} = 0.66\,.$$

Hence, from Fig. 3.18, the fin efficiency is

$$\eta_f \approx 0.79$$ <

From Eq. 3.86, the fin heat rate is $q_f = \eta_f hA_f\theta_b = \eta_f hPL_c\theta_b = \eta_f h2wL_c\theta_b$ or, per unit width,

$$q_f' = \frac{q_f}{w} = 0.79\left(75\,\text{W/m}^2\cdot\text{K}\right)2(0.023\,\text{m})80^\circ\text{C} = 218\,\text{W/m}\,.$$ <

From Eq. 3.81, the fin effectiveness is

$$\varepsilon_f = \frac{q_f}{hA_{c,b}\theta_b} = \frac{q_f'\times w}{h(t\times w)\theta_b} = \frac{218\,\text{W/m}}{75\,\text{W/m}^2\cdot\text{K}\,(0.006\,\text{m})80^\circ\text{C}} = 6.06\,.$$ <

For the triangular fin with

$$L_c^{3/2}\left(h/kA_p\right)^{1/2} = (0.02\,\text{m})^{3/2}\left[\frac{75\,\text{W/m}^2\cdot\text{K}}{(15.3\,\text{W/m}\cdot\text{K})(0.020\,\text{m})(0.003\,\text{m})}\right]^{1/2} = 0.81,$$

find from Figure 3.18,

$$\eta_f \approx 0.78,$$ <

From Eq. 3.86 and Table 3.5 find

$$q_f' = \eta_f hA_f'\theta_b = \eta_f h2\left[L^2 + (t/2)^2\right]^{1/2}\theta_b$$

$$q_f' = 0.78\times 75\,\text{W/m}^2\cdot\text{K}\times 2\left[(0.02)^2 + (0.006/2)^2\right]^{1/2}\text{m}\left(80^\circ\text{C}\right) = 187\,\text{W/m}\,.$$ <

and from Eq. 3.81, the fin effectiveness is

$$\varepsilon_f = \frac{q_f'\times w}{h(t\times w)\theta_b} = \frac{187\,\text{W/m}}{75\,\text{W/m}^2\cdot\text{K}\,(0.006\,\text{m})80^\circ\text{C}} = 5.19$$ <

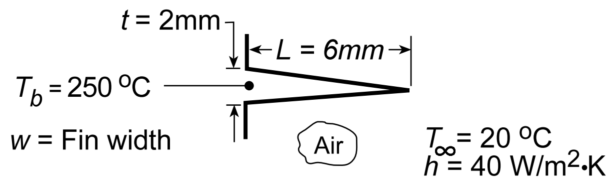
COMMENTS: Although it is 14% less effective, the triangular fin offers a 50% weight savings.

PROBLEM 3.143

KNOWN: Dimensions, base temperature and environmental conditions associated with a triangular, aluminum fin.

FIND: (a) Fin efficiency and effectiveness, (b) Heat dissipation per unit width.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible radiation and base contact resistance, (5) Uniform convection coefficient.

PROPERTIES: *Table A-1*, Aluminum, pure (T ≈ 400 K): k = 240 W/m·K.

ANALYSIS: (a) With $L_c = L = 0.006$ m, find

$$A_p = Lt/2 = (0.006\,\text{m})(0.002\,\text{m})/2 = 6\times10^{-6}\,\text{m}^2,$$

$$L_c^{3/2}\left(h/kA_p\right)^{1/2} = (0.006\,\text{m})^{3/2}\left(\frac{40\,\text{W/m}^2\cdot\text{K}}{240\,\text{W/m}\cdot\text{K}\times6\times10^{-6}\,\text{m}^2}\right)^{1/2} = 0.077$$

and from Fig. 3.18, the fin efficiency is

$$\eta_f \approx 0.99\,.$$ <

From Eq. 3.86 and Table 3.5, the fin heat rate is

$$q_f = \eta_f q_{max} = \eta_f hA_{f\,(tri)}\theta_b = 2\eta_f hw\left[L^2 + (t/2)^2\right]^{1/2}\theta_b\,.$$

From Eq. 3.81, the fin effectiveness is

$$\varepsilon_f = \frac{q_f}{hA_{c,b}\theta_b} = \frac{2\eta_f hw\left[L^2 + (t/2)^2\right]^{1/2}\theta_b}{g(w\cdot t)\theta_b} = \frac{2\eta_f\left[L^2 + (t/2)^2\right]^{1/2}}{t}$$

$$\varepsilon_f = \frac{2\times0.99\left[(0.006)^2 + (0.002/2)^2\right]^{1/2}\text{m}}{0.002\,\text{m}} = 6.02$$ <

(b) The heat dissipation per unit width is

$$q_f' = (q_f/w) = 2\eta_f h\left[L^2 + (t/2)^2\right]^{1/2}\theta_b$$

$$q_f' = 2\times0.99\times40\,\text{W/m}^2\cdot\text{K}\left[(0.006)^2 + (0.002/2)^2\right]^{1/2}\text{m}\times(250-20)^\circ\text{C} = 110.8\,\text{W/m}\,.$$ <

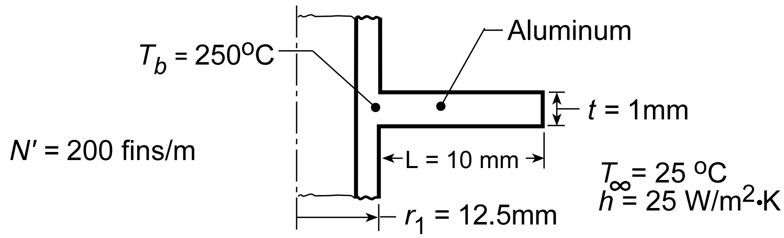
COMMENTS: The triangular profile is known to provide the maximum heat dissipation per unit fin mass.

PROBLEM 3.144

KNOWN: Dimensions and base temperature of an annular, aluminum fin of rectangular profile. Ambient air conditions.

FIND: (a) Fin heat loss, (b) Heat loss per unit length of tube with 200 fins spaced at 5 mm increments.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible radiation and contact resistance, (5) Uniform convection coefficient.

PROPERTIES: *Table A-1*, Aluminum, pure (T ≈ 400 K): k = 240 W/m·K.

ANALYSIS: (a) The fin parameters for use with Figure 3.19 are

$$r_{2c} = r_2 + t/2 = (12.5\,\text{mm} + 10\,\text{mm}) + 0.5\,\text{mm} = 23\,\text{mm} = 0.023\,\text{m}$$

$$r_{2c}/r_1 = 1.84 \qquad L_c = L + t/2 = 10.5\,\text{mm} = 0.0105\,\text{m}$$

$$A_p = L_c t = 0.0105\,\text{m} \times 0.001\,\text{m} = 1.05 \times 10^{-5}\,\text{m}^2$$

$$L_c^{3/2}\left(h/kA_p\right)^{1/2} = (0.0105\,\text{m})^{3/2}\left(\frac{25\,\text{W/m}^2\cdot\text{K}}{240\,\text{W/m}\cdot\text{K} \times 1.05 \times 10^{-5}\,\text{m}^2}\right)^{1/2} = 0.15\,.$$

Hence, the fin effectiveness is $\eta_f \approx 0.97$, and from Eq. 3.86 and Fig. 3.5, the fin heat rate is

$$q_f = \eta_f q_{max} = \eta_f hA_{f\,(ann)}\theta_b = 2\pi\eta_f h\left(r_{2,c}^2 - r_1^2\right)\theta_b$$

$$q_f = 2\pi \times 0.97 \times 25\,\text{W/m}^2\cdot\text{K} \times \left[(0.023\,\text{m})^2 - (0.0125\,\text{m})^2\right] 225^\circ\text{C} = 12.8\,\text{W}\,. \qquad <$$

(b) Recognizing that there are N = 200 fins per meter length of the tube, the total heat rate considering contributions due to the fin and base (unfinned surfaces is

$$q' = N'q_f + h\left(1 - N't\right)2\pi r_1\theta_b$$

$$q' = 200\,\text{m}^{-1} \times 12.8\,\text{W} + 25\,\text{W/m}^2\cdot\text{K}\left(1 - 200\,\text{m}^{-1} \times 0.001\,\text{m}\right) \times 2\pi \times (0.0125\,\text{m})\,225^\circ\text{C}$$

$$q' = (2560\,\text{W} + 353\,\text{W})/\text{m} = 2.91\,\text{kW/m}\,. \qquad <$$

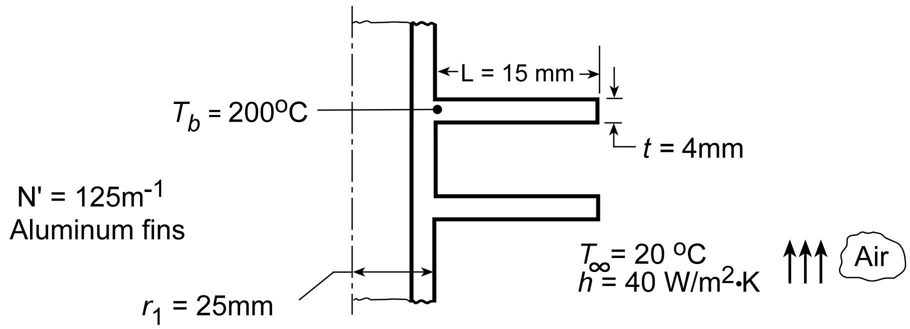
COMMENTS: Note that, while covering only 20% of the tube surface area, the tubes account for more than 85% of the total heat dissipation.

PROBLEM 3.145

KNOWN: Dimensions and base temperature of aluminum fins of rectangular profile. Ambient air conditions.

FIND: (a) Fin efficiency and effectiveness, (b) Rate of heat transfer per unit length of tube.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial conduction in fins, (3) Constant properties, (4) Negligible radiation, (5) Negligible base contact resistance, (6) Uniform convection coefficient.

PROPERTIES: *Table A-1*, Aluminum, pure (T ≈ 400 K): k = 240 W/m·K.

ANALYSIS: (a) The fin parameters for use with Figure 3.19 are

$$r_{2c} = r_2 + t/2 = 40\,\text{mm} + 2\,\text{mm} = 0.042\,\text{m} \qquad L_c = L + t/2 = 15\,\text{mm} + 2\,\text{mm} = 0.017\,\text{m}$$

$$r_{2c}/r_1 = 0.042\,\text{m}/0.025\,\text{m} = 1.68 \qquad A_p = L_c t = 0.017\,\text{m} \times 0.004\,\text{m} = 6.8\times10^{-5}\,\text{m}^2$$

$$L_c^{3/2}\left(h/kA_p\right)^{1/2} = (0.017\,\text{m})^{3/2}\left[40\,\text{W/m}^2\cdot\text{K}/240\,\text{W/m}\cdot\text{K}\times6.8\times10^{-5}\,\text{m}^2\right]^{1/2} = 0.11$$

The fin efficiency is $\eta_f \approx 0.97$. From Eq. 3.86 and Fig. 3.5,

$$q_f = \eta_f q_{max} = \eta_f h A_{f\,(ann)}\theta_b = 2\pi\eta_f h\left[r_{2c}^2 - r_1^2\right]\theta_b$$

$$q_f = 2\pi\times0.97\times40\,\text{W/m}^2\cdot\text{K}\left[(0.042)^2 - (0.025)^2\right]\text{m}^2\times180^\circ\text{C} = 50\,\text{W} \qquad <$$

From Eq. 3.81, the fin effectiveness is

$$\varepsilon_f = \frac{q_f}{hA_{c,b}\theta_b} = \frac{50\,\text{W}}{40\,\text{W/m}^2\cdot\text{K}\;2\pi(0.025\,\text{m})(0.004\,\text{m})180^\circ\text{C}} = 11.05 \qquad <$$

(b) The rate of heat transfer per unit length is

$$q' = N'q_f + h(1 - N't)(2\pi r_1)\theta_b$$

$$q' = 125\times50\,\text{W/m} + 40\,\text{W/m}^2\cdot\text{K}(1 - 125\times0.004)(2\pi\times0.025\,\text{m})\times180^\circ\text{C}$$

$$q' = (6250 + 565)\,\text{W/m} = 6.82\,\text{kW/m} \qquad <$$

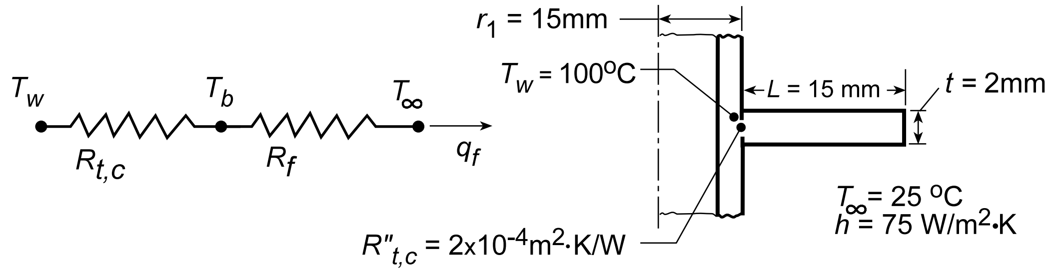
COMMENTS: Note the dominant contribution made by the fins to the total heat transfer.

PROBLEM 3.146

KNOWN: Dimensions, base temperature, and contact resistance for an annular, aluminum fin. Ambient fluid conditions.

FIND: Fin heat transfer with and without base contact resistance.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible radiation, (5) Uniform convection coefficient.

PROPERTIES: *Table A-1*, Aluminum, pure (T ≈ 350 K): k ≈ 240 W/m·K.

ANALYSIS: With the contact resistance, the fin heat loss is $q_f = \dfrac{T_w - T_\infty}{R_{t,c} + R_f}$ where

$$R_{t,c} = R''_{t,c}/A_b = 2\times10^{-4}\,\mathrm{m^2\cdot K/W}\Big/2\pi\left(0.015\,\mathrm{m}\right)\left(0.002\,\mathrm{m}\right) = 1.06\,\mathrm{K/W}\,.$$

From Eqs. 3.83 and 3.86, the fin resistance is

$$R_f = \frac{\theta_b}{q_f} = \frac{\theta_b}{\eta_f q_{max}} = \frac{\theta_b}{\eta_f h A_f \theta_b} = \frac{1}{2\pi h \eta_f\left(r_{2,c}^2 - r_1^2\right)}\,.$$

Evaluating parameters,

$$r_{2,c} = r_2 + t/2 = 30\,\mathrm{mm} + 1\,\mathrm{mm} = 0.031\,\mathrm{m} \qquad L_c = L + t/2 = 0.016\,\mathrm{m}$$

$$r_{2c}/r_1 = 0.031/0.015 = 2.07\ \mathbf{Z} \qquad A_p = L_c t = 3.2\times10^{-5}\,\mathrm{m^2}$$

$$L_c^{3/2}\left(h/kA_p\right)^{1/2} = \left(0.016\,\mathrm{m}\right)^{3/2}\left[75\,\mathrm{W/m^2\cdot K}\Big/240\,\mathrm{W/m\cdot K}\times3.2\times10^{-5}\,\mathrm{m^2}\right]^{1/2} = 0.20$$

find the fin efficiency from Figure 3.19 as η_f = 0.94. Hence,

$$R_f = \frac{1}{2\pi\left(75\,\mathrm{W/m^2\cdot K}\right)0.94\left[\left(0.031\,\mathrm{m}\right)^2 - \left(0.015\,\mathrm{m}\right)^2\right]} = 3.07\,\mathrm{K/W}$$

$$q_f = \frac{\left(100-25\right)^\circ\mathrm{C}}{\left(1.06+3.07\right)\mathrm{K/W}} = 18.2\,\mathrm{W}\,. \qquad <$$

Without the contact resistance, $T_w = T_b$ and

$$q_f = \frac{\theta_b}{R_f} = \frac{75^\circ\mathrm{C}}{3.07\,\mathrm{K/W}} = 24.4\,\mathrm{W}\,. \qquad <$$

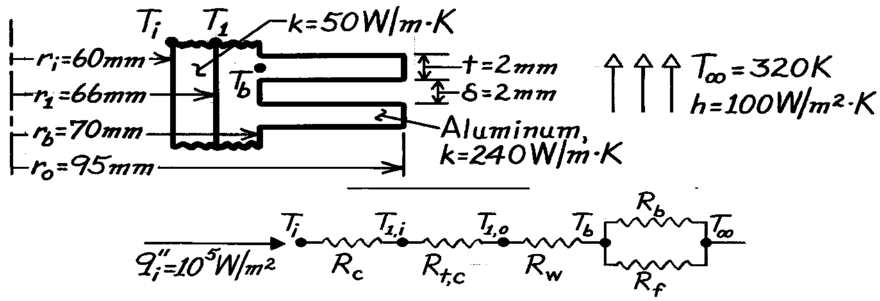
COMMENTS: To maximize fin performance, every effort should be made to minimize contact resistance.

PROBLEM 3.147

KNOWN: Dimensions and materials of a finned (annular) cylinder wall. Heat flux and ambient air conditions. Contact resistance.

FIND: Surface and interface temperatures (a) without and (b) with an interface contact resistance.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conditions, (2) Constant properties, (3) Uniform h over surfaces, (4) Negligible radiation.

ANALYSIS: The analysis may be performed per unit length of cylinder or for a 4 mm long section. The following calculations are based on a unit length. The inner surface temperature may be obtained from

$$q' = \frac{T_i - T_\infty}{R'_{tot}} = q_i''(2\pi\ r_i) = 10^5\ \text{W/m}^2 \times 2\pi \times 0.06\ \text{m} = 37,700\ \text{W/m}$$

where $R'_{tot} = R'_c + R'_{t,c} + R'_w + R'_{equiv};\quad R'_{equiv} = (1/R'_f + 1/R'_b)^{-1}$.

R'_c, *Conduction resistance of cylinder wall:*

$$R'_c = \frac{\ln(r_1/r_i)}{2\pi\ k} = \frac{\ln(66/60)}{2\pi(50\ \text{W/m}\cdot\text{K})} = 3.034\times10^{-4}\ \text{m}\cdot\text{K/W}$$

$R'_{t,c}$, *Contact resistance:*

$$R'_{t,c} = R''_{t,c}/2\pi\ r_1 = 10^{-4}\ \text{m}^2\cdot\text{K/W}/2\pi\times0.066\ \text{m} = 2.411\times10^{-4}\ \text{m}\cdot\text{K/W}$$

R'_w, *Conduction resistance of aluminum base:*

$$R'_w = \frac{\ln(r_b/r_1)}{2\pi\ k} = \frac{\ln(70/66)}{2\pi\times240\ \text{W/m}\cdot\text{K}} = 3.902\times10^{-5}\ \text{m}\cdot\text{K/W}$$

R'_b, *Resistance of prime or unfinned surface:*

$$R'_b = \frac{1}{hA'_b} = \frac{1}{100\ \text{W/m}^2\cdot\text{K}\times0.5\times2\pi(0.07\ \text{m})} = 454.7\times10^{-4}\ \text{m}\cdot\text{K/W}$$

R'_f, *Resistance of fins:* The fin resistance may be determined from

$$R'_f = \frac{T_b - T_\infty}{q'_f} = \frac{1}{\eta_f h A'_f}$$

The fin efficiency may be obtained from Fig. 3.19,

$$r_{2c} = r_o + t/2 = 0.096\ \text{m} \qquad L_c = L + t/2 = 0.026\ \text{m}$$

Continued …..

$$A_p = L_c t = 5.2\times10^{-5}\ \text{m}^2 \qquad r_{2c}/r_1 = 1.45 \qquad L_c^{3/2}\left(h/kA_p\right)^{1/2} = 0.375$$

Fig. 3.19 $\rightarrow \eta_f \approx 0.88$.

The total fin surface area per meter length

$$A_f' = 250\left[\pi\left(r_o^2 - r_b^2\right)\times 2\right] = 250\ \text{m}^{-1}\left[2\pi\left(0.096^2 - 0.07^2\right)\right]\text{m}^2 = 6.78\ \text{m}.$$

Hence $$R_f' = \left[0.88\times100\ \text{W/m}^2\cdot\text{K}\times6.78\ \text{m}\right]^{-1} = 16.8\times10^{-4}\ \text{m}\cdot\text{K/W}$$

$$1/R_{equiv}' = \left(1/16.8\times10^{-4} + 1/454.7\times10^{-4}\right)\text{W/m}\cdot\text{K} = 617.2\ \text{W/m}\cdot\text{K}$$

$$R_{equiv}' = 16.2\times10^{-4}\ \text{m}\cdot\text{K/W}.$$

Neglecting the *contact resistance,*

$$R_{tot}' = \left(3.034 + 0.390 + 16.2\right)10^{-4}\ \text{m}\cdot\text{K/W} = 19.6\times10^{-4}\ \text{m}\cdot\text{K/W}$$

$$T_i = q'R_{tot}' + T_\infty = 37,700\ \text{W/m}\times19.6\times10^{-4}\ \text{m}\cdot\text{K/W}+320\ \text{K} = 393.9\ \text{K} \qquad <$$

$$T_1 = T_i - q'R_w' = 393.9\ \text{K} - 37,700\ \text{W/m}\times3.034\times10^{-4}\ \text{m}\cdot\text{K/W} = 382.5\ \text{K} \qquad <$$

$$T_b = T_1 - q'R_b' = 382.5\ \text{K} - 37,700\ \text{W/m}\times3.902\times10^{-5}\ \text{m}\cdot\text{K/W} = 381.0\ \text{K}. \qquad <$$

Including the *contact resistance,*

$$R_{tot}' = \left(19.6\times10^{-4} + 2.411\times10^{-4}\right)\text{m}\cdot\text{K/W} = 22.0\times10^{-4}\ \text{m}\cdot\text{K/W}$$

$$T_i = 37,700\ \text{W/m}\times22.0\times10^{-4}\ \text{m}\cdot\text{K/W}+320\ \text{K} = 402.9\ \text{K} \qquad <$$

$$T_{1,i} = 402.9\ \text{K} - 37,700\ \text{W/m}\times3.034\times10^{-4}\ \text{m}\cdot\text{K/W} = 391.5\ \text{K} \qquad <$$

$$T_{1,o} = 391.5\ \text{K} - 37,700\ \text{W/m}\times2.411\times10^{-4}\ \text{m}\cdot\text{K/W} = 382.4\ \text{K} \qquad <$$

$$T_b = 382.4\ \text{K} - 37,700\ \text{W/m}\times3.902\times10^{-5}\ \text{m}\cdot\text{K/W} = 380.9\ \text{K}. \qquad <$$

COMMENTS: (1) The effect of the contact resistance is small.

(2) The effect of including the aluminum fins may be determined by computing T_i without the fins. In this case $R_{tot}' = R_c' + R_{conv}'$, where

$$R_{conv}' = \frac{1}{h2\pi\, r_1} = \frac{1}{100\ \text{W/m}^2\cdot\text{K}\ 2\pi\left(0.066\ \text{m}\right)} = 241.1\times10^{-4}\ \text{m}\cdot\text{K/W}.$$

Hence, $R_{tot} = 244.1\times10^{-4}\ \text{m}\cdot\text{K/W}$, and

$$T_i = q'R_{tot}' + T_\infty = 37,700\ \text{W/m}\times244.1\times10^{-4}\ \text{m}\cdot\text{K/W}+320\ \text{K} = 1240\ \text{K}.$$

Hence, the fins have a significant effect on reducing the cylinder temperature.

(3) The overall surface efficiency is

$$\eta_o = 1 - \left(A_f'/A_t'\right)\left(1-\eta_f\right) = 1 - 6.78\ \text{m}/7.00\ \text{m}\left(1-0.88\right) = 0.884.$$

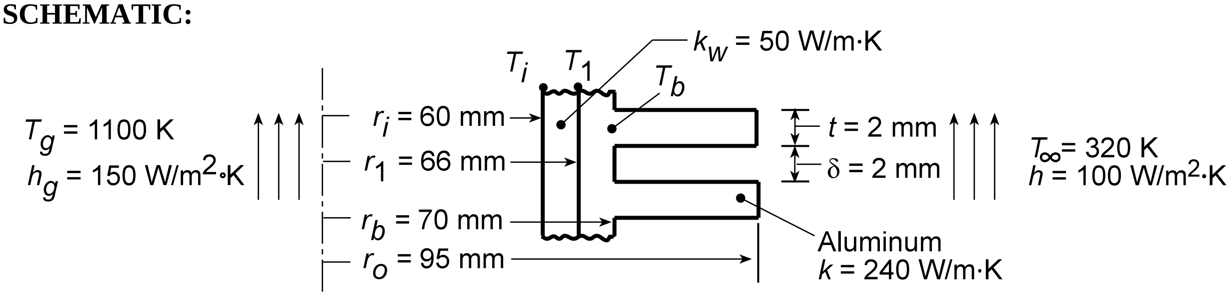
It follows that $q' = \eta_o h_o A_t' \theta_b = 37,700\ \text{W/m}$, which agrees with the prescribed value.

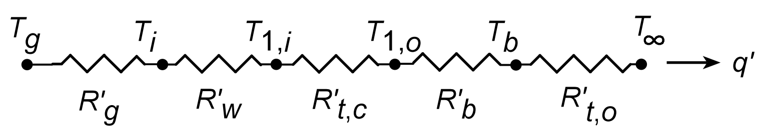
PROBLEM 3.148

KNOWN: Dimensions and materials of a finned (annular) cylinder wall. Combustion gas and ambient air conditions. Contact resistance.

FIND: (a) Heat rate per unit length and surface and interface temperatures, (b) Effect of increasing the fin thickness.

SCHEMATIC:





ASSUMPTIONS: (1) One-dimensional, steady-state conditions, (2) Constant properties, (3) Uniform h over surfaces, (4) Negligible radiation.

ANALYSIS: (a) The heat rate per unit length is

$$q' = \frac{T_g - T_\infty}{R'_{tot}}$$

where $R'_{tot} = R'_g + R'_w + R'_{t,c} + R'_b + R'_{t,o}$, and

$$R'_g = \left(h_g 2\pi r_i\right)^{-1} = \left(150\,\mathrm{W/m^2\cdot K} \times 2\pi \times 0.06\mathrm{m}\right)^{-1} = 0.0177\,\mathrm{m\cdot K/W},$$

$$R'_w = \frac{\ln(r_1/r_i)}{2\pi k_w} = \frac{\ln(66/60)}{2\pi(50\,\mathrm{W/m\cdot K})} = 3.03\times10^{-4}\,\mathrm{m\cdot K/W},$$

$$R'_{t,c} = \left(R''_{t,c}/2\pi r_1\right) = 10^{-4}\,\mathrm{m^4\cdot K/W}\Big/2\pi\times0.066\mathrm{m} = 2.41\times10^{-4}\,\mathrm{m\cdot K/W}$$

$$R'_b = \frac{\ln(r_b/r_1)}{2\pi k} = \frac{\ln(70/66)}{2\pi\times240\,\mathrm{W/m\cdot K}} = 3.90\times10^{-5}\,\mathrm{m\cdot K/W},$$

$$R_{t,o} = \left(\eta_o h A'_t\right)^{-1},$$

$$\eta_o = 1 - \frac{N'A_f}{A'_t}\left(1-\eta_f\right),$$

$$A_f = 2\pi\left(r_{oc}^2 - r_b^2\right)$$

$$A'_t = N'A_f + \left(1-N't\right)2\pi r_b$$

$$\eta_f = \frac{(2r_b/m)}{\left(r_{oc}^2 - r_b^2\right)}\frac{K_1(mr_b)I_1(mr_{oc}) - I_1(mr_b)K_1(mr_{oc})}{I_0(mr_1)K_1(mr_{oc}) + K_0(mr_b)I_1(mr_{oc})}$$

$$r_{oc} = r_o + (t/2), \qquad m = (2h/kt)^{1/2}$$

Continued...

Once the heat rate is determined from the foregoing expressions, the desired interface temperatures may be obtained from

$$T_i = T_g - q'R'_g$$

$$T_{1,i} = T_g - q'\left(R'_g + R'_w\right)$$

$$T_{1,o} = T_g - q'\left(R'_g + R'_w + R'_{t,c}\right)$$

$$T_b = T_g - q'\left(R'_g + R'_w + R'_{t,c} + R'_b\right)$$

For the specified conditions we obtain A'_t = 7.00 m, η_f = 0.902, η_o = 0.906 and $R'_{t,o}$ = 0.00158 m·K/W. It follows that

$$q' = 39,300\,\text{W/m} \qquad <$$

$$T_i = 405\text{K}, \quad T_{1,i} = 393\text{K}, \quad T_{1,o} = 384\text{K}, \quad T_b = 382\text{K} \qquad <$$

(b) The *Performance Calculation, Extended Surface* Model for the *Circular Fin* Array may be used to assess the effects of fin thickness and spacing. Increasing the fin thickness to t = 3 mm, with δ = 2 mm, reduces the number of fins per unit length to 200. Hence, although the fin efficiency increases (η_f = 0.930), the reduction in the total surface area (A'_t = 5.72 m) yields an increase in the resistance of the fin array ($R'_{t,o}$ = 0.00188 m·K/W), and hence a reduction in the heat rate (q' = 38,700 W/m) and an increase in the interface temperatures (T_i = 415 K, $T_{1,i}$ = 404 K, $T_{1,o}$ = 394 K, and T_b = 393 K).

COMMENTS: Because the gas convection resistance exceeds all other resistances by at least an order of magnitude, incremental changes in $R_{t,o}$ will not have a significant effect on q' or the interface temperatures.

PROBLEM 3.149

KNOWN: Dimensions of finned aluminum sleeve inserted over transistor. Contact resistance and convection conditions.

FIND: Measures for increasing heat dissipation.

SCHEMATIC: See Example 3.10.

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible heat transfer from top and bottom of transistor, (3) One-dimensional radial heat transfer, (4) Constant properties, (5) Negligible radiation.

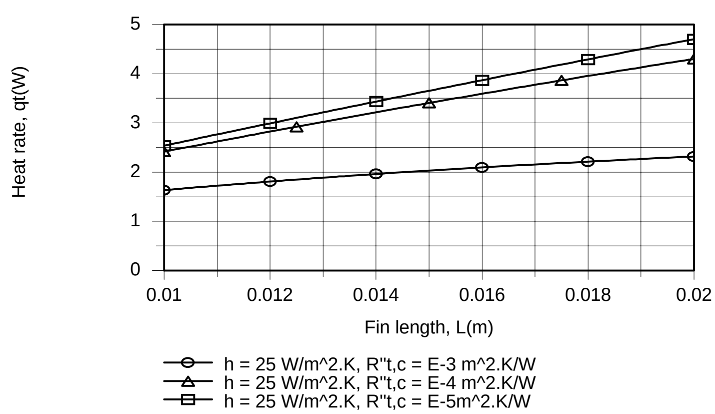
ANALYSIS: With $2\pi r_2 = 0.0188$ m and Nt = 0.0084 m, the existing gap between fins is extremely small (0.87 mm). Hence, by increasing N and/or t, it would become even more difficult to maintain satisfactory airflow between the fins, and this option is not particularly attractive.

Because the fin efficiency for the prescribed conditions is close to unity ($\eta_f = 0.998$), there is little advantage to replacing the aluminum with a material of higher thermal conductivity (e.g. Cu with k ~ 400 W/m·K). However, the large value of η_f suggests that significant benefit could be gained by increasing the fin length, $L = r_3 - r_2$.

It is also evident that the thermal contact resistance is large, and from Table 3.2, it's clear that a significant reduction could be effected by using indium foil or a conducting grease in the contact zone. Specifically, a reduction of $R''_{t,c}$ from 10^{-3} to 10^{-4} or even 10^{-5} $m^2 \cdot K/W$ is certainly feasible.

Table 1.1 suggests that, by increasing the velocity of air flowing over the fins, a larger convection coefficient may be achieved. A value of $h = 100$ $W/m^2 \cdot K$ would not be unreasonable.

As options for enhancing heat transfer, we therefore use the IHT *Performance Calculation, Extended Surface Model* for the *Straight Fin Array* to explore the effect of parameter variations over the ranges $10 \le L \le 20$ mm, $10^{-5} \le R''_{t,c} \le 10^{-3}$ $m^2 \cdot K/W$ and $25 \le h \le 100$ $W/m^2 \cdot K$. As shown below, there is a significant enhancement in heat transfer associated with reducing $R''_{t,c}$ from 10^{-3} to 10^{-4} $m^2 \cdot K/W$, for which $R_{t,c}$ decreases from 13.26 to 1.326 K/W. At this value of $R''_{t,c}$, the reduction in $R_{t,o}$ from 23.45 to 12.57 K/W which accompanies an increase in L from 10 to 20 mm becomes significant, yielding a heat rate of $q_t = 4.30$ W for $R''_{t,c} = 10^{-4}$ $m^2 \cdot K/W$ and L = 20 mm. However, since $R_{t,o} >> R_{t,c}$, little benefit is gained by further reducing $R''_{t,c}$ to 10^{-5} $m^2 \cdot K/W$.



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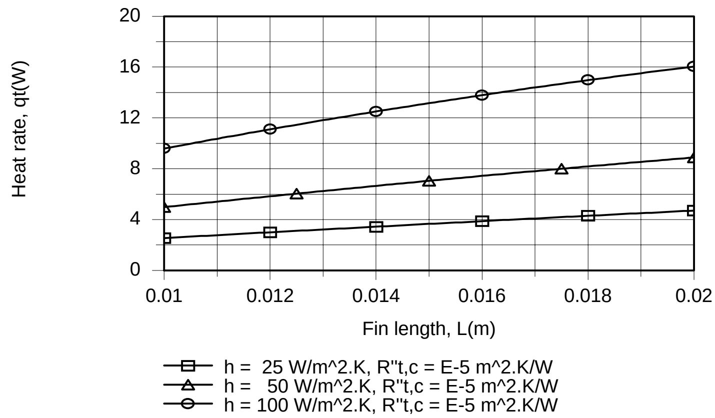
PROBLEM 3.149 (Cont.)

To derive benefit from a reduction in $R''_{t,c}$ to 10^{-5} $m^2 \cdot K/W$, an additional reduction in $R_{t,o}$ must be made. This can be achieved by increasing h, and for L = 20 mm and h = 100 $W/m^2 \cdot K$, $R_{t,o}$ = 3.56 K/W. With $R''_{t,c}$ = 10^{-5} $m^2 \cdot K/W$, a value of q_t = 16.04 W may be achieved.



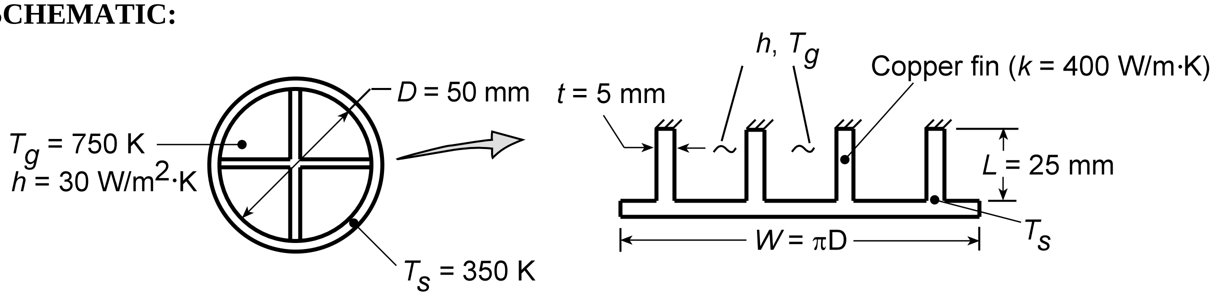
COMMENTS: In assessing options for enhancing heat transfer, the limiting (largest) resistance(s) should be identified and efforts directed at their reduction.

PROBLEM 3.150

KNOWN: Diameter and internal fin configuration of copper tubes submerged in water. Tube wall temperature and temperature and convection coefficient of gas flow through the tube.

FIND: Rate of heat transfer per tube length.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional fin conduction, (3) Constant properties, (4) Negligible radiation, (5) Uniform convection coefficient, (6) Tube wall may be unfolded and represented as a plane wall with four straight, rectangular fins, each with an adiabatic tip.

ANALYSIS: The rate of heat transfer per unit tube length is:

$$q'_t = \eta_o h A'_t \left(T_g - T_s\right)$$

$$\eta_o = 1 - \frac{NA'_f}{A'_t}\left(1 - \eta_f\right)$$

$$NA'_f = 4 \times 2L = 8\left(0.025\text{m}\right) = 0.20\text{m}$$

$$A'_t = NA'_f + A'_b = 0.20\text{m} + \left(\pi D - 4t\right) = 0.20\text{m} + \left(\pi \times 0.05\text{m} - 4 \times 0.005\text{m}\right) = 0.337\text{m}$$

For an adiabatic fin tip,

$$\eta_f = \frac{q_f}{q_{max}} = \frac{M \tanh mL}{h\left(2L \cdot 1\right)\left(T_g - T_s\right)}$$

$$M = \left[h2\left(1\text{m} + t\right)k\left(1\text{m} \times t\right)\right]^{1/2}\left(T_g - T_s\right) \approx \left[30\,\text{W/m}^2 \cdot \text{K}\left(2\text{m}\right)400\,\text{W/m} \cdot \text{K}\left(0.005\text{m}^2\right)\right]^{1/2}\left(400\text{K}\right) = 4382\text{W}$$

$$mL = \left\{\left[h2\left(1\text{m} + t\right)\right]/\left[k\left(1\text{m} \times t\right)\right]\right\}^{1/2} L \approx \left[\frac{30\,\text{W/m}^2 \cdot \text{K}\left(2\text{m}\right)}{400\,\text{W/m} \cdot \text{K}\left(0.005\text{m}^2\right)}\right]^{1/2} 0.025\text{m} = 0.137$$

Hence, tanh mL = 0.136, and

$$\eta_f = \frac{4382\text{W}\left(0.136\right)}{30\,\text{W/m}^2 \cdot \text{K}\left(0.05\text{m}^2\right)\left(400\text{K}\right)} = \frac{595\text{W}}{600\text{W}} = 0.992$$

$$\eta_o = 1 - \frac{0.20}{0.337}\left(1 - 0.992\right) = 0.995$$

$$q'_t = 0.995\left(30\,\text{W/m}^2 \cdot \text{K}\right)0.337\text{m}\left(400\text{K}\right) = 4025\,\text{W/m}$$

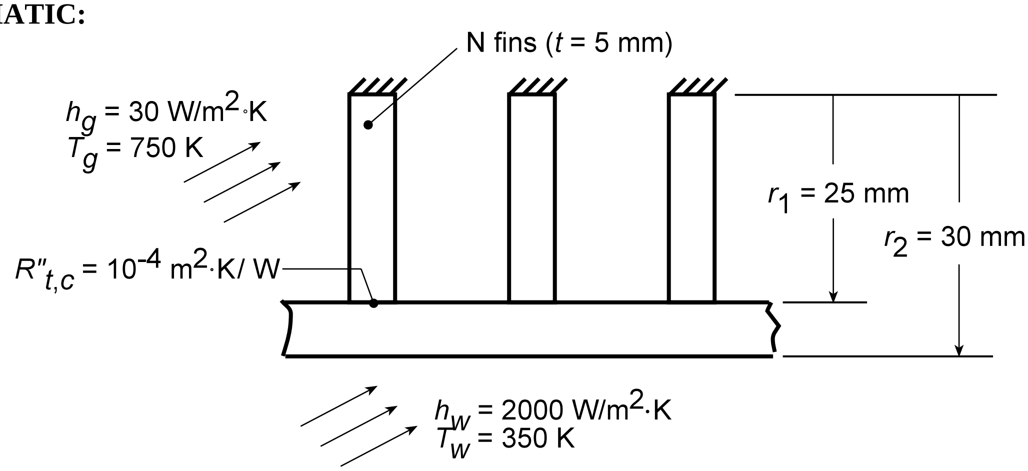
COMMENTS: Alternatively, $q'_t = 4q'_f + h\left(A'_t - A'_f\right)\left(T_g - T_s\right)$. Hence, q' = 4(595 W/m) + 30 W/m^2·K (0.137 m)(400 K) = (2380 + 1644) W/m = 4024 W/m.

PROBLEM 3.151

KNOWN: Internal and external convection conditions for an internally finned tube. Fin/tube dimensions and contact resistance.

FIND: Heat rate per unit tube length and corresponding effects of the contact resistance, number of fins, and fin/tube material.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat transfer, (3) Constant properties, (4) Negligible radiation, (5) Uniform convection coefficient on finned surfaces, (6) Tube wall may be unfolded and approximated as a plane surface with N straight rectangular fins.

PROPERTIES: Copper: k = 400 W/m·K; St.St.: k = 20 W/m·K.

ANALYSIS: The heat rate per unit length may be expressed as

$$q' = \frac{T_g - T_w}{R'_{t,o(c)} + R'_{cond} + R'_{conv,o}}$$

where

$$R_{t,o(c)} = \left(\eta_{o(c)} h_g A'_t\right), \quad \eta_{o(c)} = 1 - \frac{NA'_f}{A'_t}\left(1 - \frac{\eta_f}{C_1}\right), \quad C_1 = 1 + \eta_f h_g A'_f \left(R''_{t,c} / A'_{c,b}\right),$$

$$A'_t = NA'_f + \left(2\pi r_1 - Nt\right), \quad A'_f = 2r_1, \quad \eta_f = \tanh mr_1 / mr_1, \quad m = \left(2h_g / kt\right)^{1/2} \quad A'_{c,b} = t,$$

$$R'_{cond} = \frac{\ln\left(r_2/r_1\right)}{2\pi k}, \quad \text{and} \quad R'_{conv,o} = \left(2\pi r_2 h_w\right)^{-1}.$$

Using the IHT *Performance Calculation, Extended Surface Model* for the *Straight Fin Array*, the following results were obtained. For the *base case*, q' = 3857 W/m, where $R'_{t,o(c)}$ = 0.101 m·K/W, R'_{cond} = 7.25×10^{-5} m·K/W and $R'_{conv,o}$ = 0.00265 m·K/W. If the contact resistance is eliminated ($R''_{t,c}$ = 0), q' = 3922 W/m, where $R'_{t,o}$ = 0.0993 m·K/W. If the number of fins is increased to N = 8, q' = 5799 W/m, with $R'_{t,o(c)}$ = 0.063 m·K/W. If the material is changed to stainless steel, q' = 3591 W/m, with $R'_{t,o(c)}$ = 0.107 m·K/W and R'_{cond} = 0.00145 m·K/W.

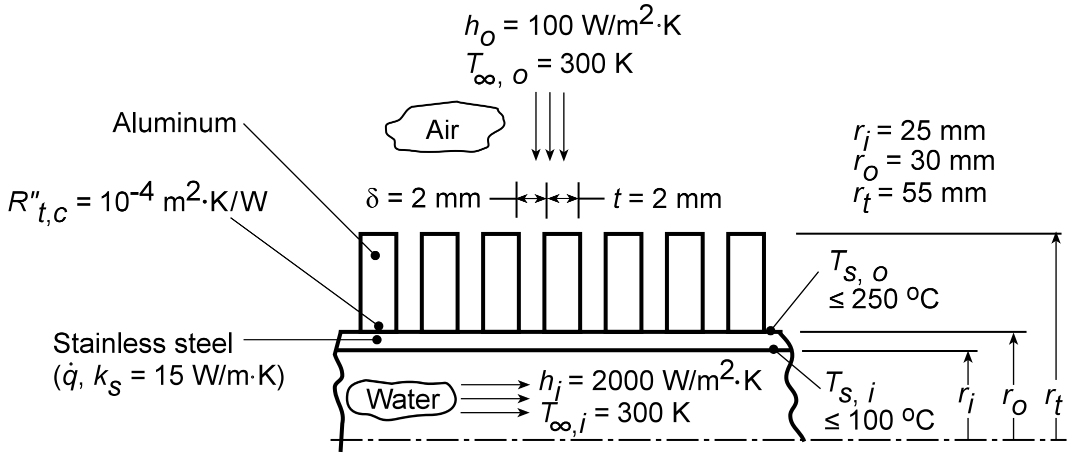
COMMENTS: The small reduction in q' associated with use of stainless steel is perhaps surprising, in view of the large reduction in k. However, because h_g is small, the reduction in k does not significantly reduce the fin efficiency (η_f changes from 0.994 to 0.891). Hence, the heat rate remains large. The influence of k would become more pronounced with increasing h_g.

PROBLEM 3.152

KNOWN: Design and operating conditions of a tubular, air/water heater.

FIND: (a) Expressions for heat rate per unit length at inner and outer surfaces, (b) Expressions for inner and outer surface temperatures, (c) Surface heat rates and temperatures as a function of volumetric heating $\dot{q}$ for prescribed conditions. Upper limit to $\dot{q}$.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) Constant properties, (3) One-dimensional heat transfer.

PROPERTIES: Table A-1: Aluminum, T = 300 K, k_a = 237 W/m·K.

ANALYSIS: (a) Applying Equation C.8 to the inner and outer surfaces, it follows that

$$q'(r_i) = \dot{q}\pi r_i^2 - \frac{2\pi k_s}{\ln(r_o/r_i)}\left[\frac{\dot{q}r_o^2}{4k_s}\left(1-\frac{r_i^2}{r_o^2}\right) + \left(T_{s,o} - T_{s,i}\right)\right]$$ <

$$q'(r_o) = \dot{q}\pi r_o^2 - \frac{2\pi k_s}{\ln(r_o/r_i)}\left[\frac{\dot{q}r_o^2}{4k_s}\left(1-\frac{r_i^2}{r_o^2}\right) + \left(T_{s,o} - T_{s,i}\right)\right]$$ <

(b) From Equations C.16 and C.17, energy balances at the inner and outer surfaces are of the form

$$h_i\left(T_{\infty,i} - T_{s,i}\right) = \frac{\dot{q}r_i}{2} - \frac{k_s\left[\frac{\dot{q}r_o^2}{4k_s}\left(1-\frac{r_i^2}{r_o^2}\right) + \left(T_{s,o} - T_{s,i}\right)\right]}{r_i \ln(r_o/r_i)}$$ <

$$U_o\left(T_{s,o} - T_{\infty,o}\right) = \frac{\dot{q}r_o}{2} - \frac{k_s\left[\frac{\dot{q}r_o^2}{4k_s}\left(1-\frac{r_i^2}{r_o^2}\right) + \left(T_{s,o} - T_{s,i}\right)\right]}{r_o \ln(r_o/r_i)}$$ <

Accounting for the fin array and the contact resistance, Equation 3.104 may be used to cast the overall heat transfer coefficient U_o in the form

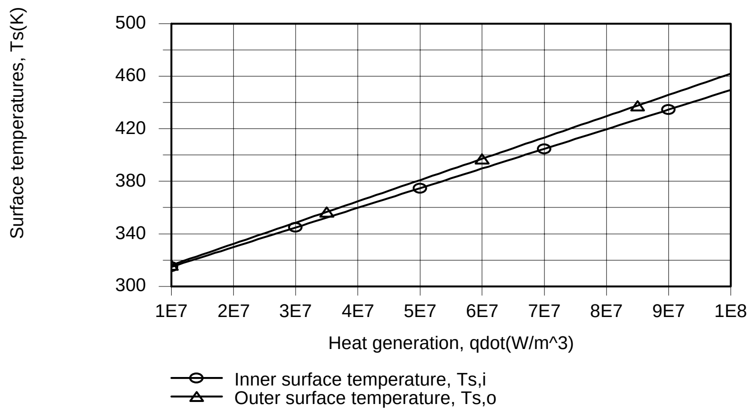
$$U_o = \frac{q'(r_o)}{A'_w\left(T_{s,o} - T_{\infty,o}\right)} = \frac{1}{A'_w R'_{t,o(c)}} = \frac{A'_t}{A'_w}\eta_{o(c)}h_o$$

where $\eta_{o(c)}$ is determined from Equations 3.105a,b and $A'_w = 2\pi r_o$.

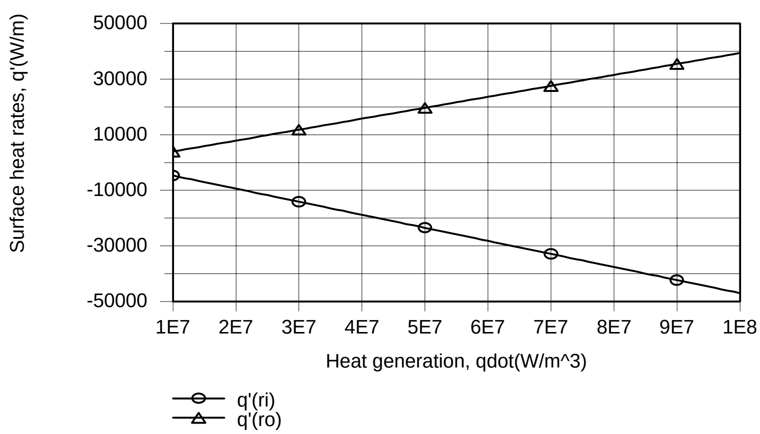
Continued...

PROBLEM 3.152 (Cont.)

(c) For the prescribed conditions and a representative range of $10^7 \le \dot{q} \le 10^8$ W/m^3, use of the relations of part (b) with the capabilities of the IHT *Performance Calculation Extended Surface Model* for a *Circular Fin Array* yields the following graphical results.



It is in this range that the upper limit of $T_{s,i}$ = 373 K is exceeded for $\dot{q}$ = 4.9×10^7 W/m^3, while the corresponding value of $T_{s,o}$ = 379 K is well below the prescribed upper limit. The expressions of part (a) yield the following results for the surface heat rates, where heat transfer in the negative r direction corresponds to $q'(r_i) < 0$.



For $\dot{q}$ = 4.9×10^7 W/m^3, $q'(r_i)$ = -2.30×10^4 W/m and $q'(r_o)$ = 1.93×10^4 W/m.

COMMENTS: The foregoing design provides for comparable heat transfer to the air and water streams. This result is a consequence of the nearly equivalent thermal resistances associated with heat transfer from the inner and outer surfaces. Specifically, $R'_{conv,i} = \left(h_i 2\pi r_i\right)^{-1}$ = 0.00318 m·K/W is slightly smaller than $R'_{t,o(c)}$ = 0.00411 m·K/W, in which case $\left|q'(r_i)\right|$ is slightly larger than $q'(r_o)$, while $T_{s,i}$ is slightly smaller than $T_{s,o}$. Note that the solution must satisfy the energy conservation requirement, $\pi\left(r_o^2 - r_i^2\right)\dot{q} = \left|q'(r_i)\right| + q'(r_o)$.