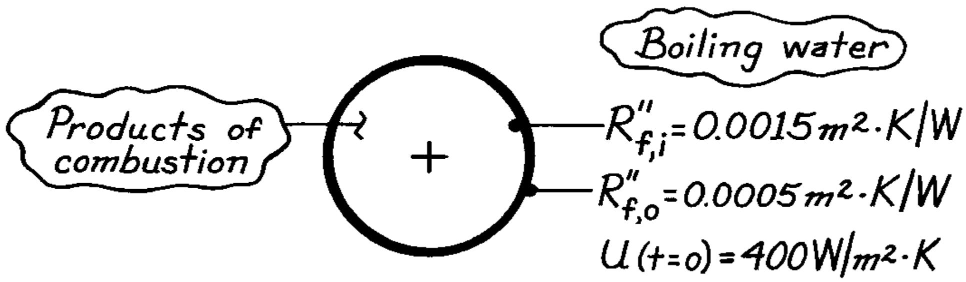
PROBLEM 11.1

KNOWN: Initial overall heat transfer coefficient of a fire-tube boiler. Fouling factors following one year's application.

FIND: Whether cleaning should be scheduled.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible tube wall conduction resistance, (2) Negligible changes in h_c and h_h.

ANALYSIS: From Equation 11.1, the overall heat transfer coefficient after one year is

$$\frac{1}{U} = \frac{1}{h_i} + \frac{1}{h_o} + R''_{f,i} + R''_{f,o}.$$

Since the first two terms on the right-hand side correspond to the reciprocal of the initial overall coefficient,

$$\frac{1}{U} = \frac{1}{400\ W/m^2 \cdot K} + (0.0015 + 0.0005)\ m^2 \cdot K/W = 0.0045\ m^2 \cdot K/W$$

$$U = 222\ W/m^2 \cdot K.$$

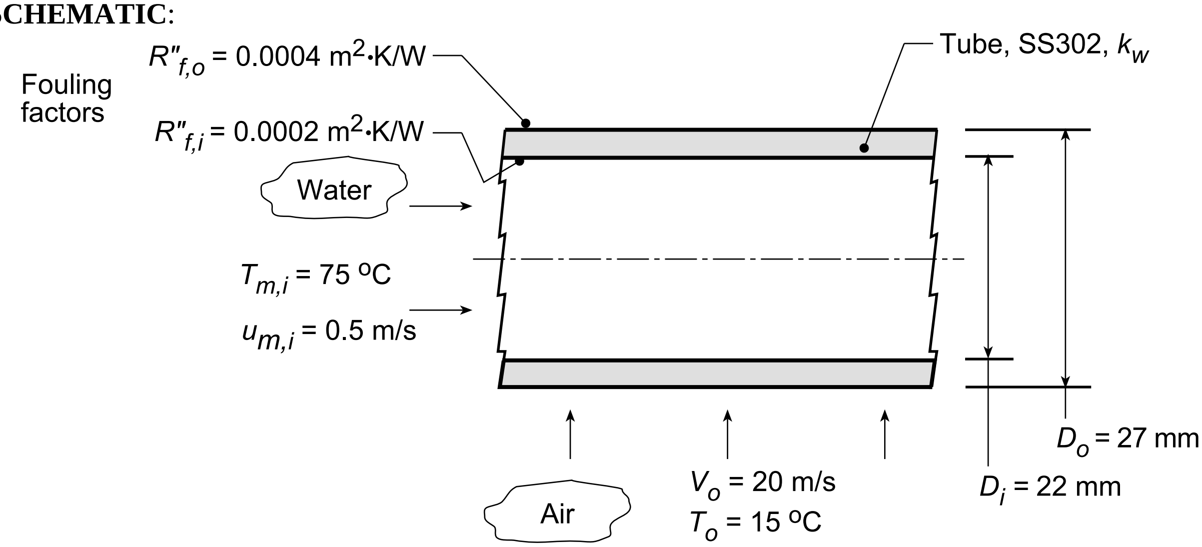
COMMENTS: Periodic cleaning of the tube inner surfaces is essential to maintaining efficient fire-tube boiler operations.

PROBLEM 11.2

KNOWN: Type-302 stainless tube with prescribed inner and outer diameters used in a cross-flow heat exchanger. Prescribed fouling factors and internal water flow conditions.

FIND: (a) Overall coefficient based upon the outer surface, U_o, with air at T_o =15°C and velocity V_o = 20 m/s in cross-flow; compare thermal resistances due to convection, tube wall conduction and fouling; (b) Overall coefficient, U_o, with water (rather than air) at T_o = 15°C and velocity V_o = 1 m/s in cross-flow; compare thermal resistances due to convection, tube wall conduction and fouling; (c) For the water-air conditions of part (a), compute and plot U_o as a function of the air cross-flow velocity for $5 \le V_o \le 30$ m/s for water mean velocities of $u_{m,i}$ = 0.2, 0.5 and 1.0 m/s; and (d) For the water-water conditions of part (b), compute and plot U_o as a function of the water mean velocity for $0.5 \le u_{m,i} \le 2.5$ m/s for air cross-flow velocities of V_o = 1, 3 and 8 m/s.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Fully developed internal flow,

PROPERTIES: *Table A.1*, Stainless steel, AISI 302 (300 K): k_w = 15.1 W/m·K; *Table A.6*, Water ($\overline{T}_{m,i}$ = 348 K): ρ_i = 974.8 kg/m^3, $\mu_i = 3.746 \times 10^{-4}$ N·s/m^2, k_i = 0.668 W/m·K, Pr_i = 2.354; *Table A.4*, Air (assume $\overline{T}_{f,o}$ = 315K, 1 atm): k_o = 0.02737 W/m·K, $\nu_o = 17.35 \times 10^{-6}$ m^2/s, Pr_o = 0.705.

ANALYSIS: (a) For the water-air condition, the overall coefficient, Eq. 11.1, based upon the outer area can be expressed as the sum of the thermal resistances due to convection (cv), tube wall conduction (w) and fouling (f):

$$1/U_o A_o = R_{tot} = R_{cv,i} + R_{f,i} + R_w + R_{f,o} + R_{cv,o}$$

$$R_{cv,i} = 1/\overline{h}_i A_i \qquad R_{cv,o} = 1/\overline{h}_o A_o$$

$$R_{f,i} = R''_{f,i}/A_i \qquad R_{f,o} = R''_{f,o}/A_o$$

and from Eq. 3.28,

$$R_w = \ln(D_o/D_i)/(2\pi L k_w)$$

The convection coefficients can be estimated from appropriate correlations.

Continued...

PROBLEM 11.2 (Cont.)

Estimating $\bar{h}_i$: For internal flow, characterize the flow evaluating thermophysical properties at $T_{m,i}$ with

$$Re_{D,i} = \frac{u_{m,i} D_i}{\nu_i} = \frac{0.5\,m/s \times 0.022\,m}{3.746 \times 10^{-4}\,N \cdot s/m^2 / 974.8\,kg/m^3} = 28,625$$

For the turbulent flow, use the Dittus-Boelter correlation, Eq. 8.60,

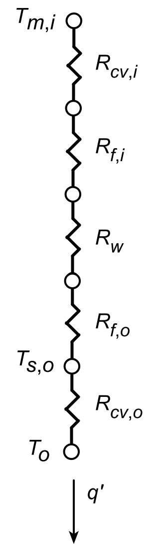
$$Nu_{D,i} = 0.023\, Re_{D,i}^{0.8}\, Pr_i^{0.4}$$

$$Nu_{D,i} = 0.023(28,625)^{0.8}(2.354)^{0.4} = 119.1$$

$$\bar{h}_i = Nu_{D,i}\, k_i / D_i = 119.1 \times 0.668\,W/m^2 \cdot K / 0.022\,m = 3616\,W/m^2 \cdot K$$

Estimating $\bar{h}_o$: For external flow, characterize the flow with

$$Re_{D,o} = \frac{V_o D_o}{\nu_o} = \frac{20\,m/s \times 0.027\,m}{17.35 \times 10^{-6}\,m^2/s} = 31,124$$



evaluating thermophysical properties at $T_{f,o} = (T_{s,o} + T_o)/2$ when the surface temperature is determined from the thermal circuit analysis result,

$$(T_{m,i} - T_o)/R_{tot} = (T_{s,o} - T_o)/R_{cv,o}$$

Assume $T_{f,o}$ = 315 K, and check later. Using the Churchill-Bernstein correlation, Eq. 7.57, find

$$\overline{Nu}_{D,o} = 0.3 + \frac{0.62\, Re_{D,o}^{1/2}\, Pr_o^{1/3}}{\left[1 + (0.4/Pr_o)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{Re_{D,o}}{282,000}\right)^{5/8}\right]^{4/5}$$

$$\overline{Nu}_{D,o} = 0.3 + \frac{0.62(31,124)^{1/2}(0.705)^{1/3}}{\left[1 + (0.4/0.705)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{31,124}{282,000}\right)^{5/8}\right]^{4/5}$$

$$\overline{Nu}_{D,o} = 102.6$$

$$\bar{h}_o = \overline{Nu}_{D,o}\, k_o / D_o = 102.6 \times 0.02737\,W/m \cdot K / 0.027\,m = 104.0\,W/m \cdot K$$

Using the above values for $\bar{h}_i$, and $\bar{h}_o$, and other prescribed values, the thermal resistances and overall coefficient can be evaluated and are tabulated below.

$R_{cv,i}$ (K/W)	$R_{f,i}$ (K/W)	R_w (K/W)	$R_{f,o}$ (K/W)	$R_{cv,o}$ (K/W)	U_o ($W/m^2 \cdot K$)	R_{tot} (K/W)
0.00436	0.00578	0.00216	0.00236	0.1134	92.1	0.128

The major thermal resistance is due to outside (air) convection, accounting for 89% of the total resistance. The other thermal resistances are of similar magnitude, nearly 50 times smaller than $R_{cv,o}$.

(b) For the water-water condition, the method of analysis follows that of part (a). For the internal flow, the estimated convection coefficient is the same as part (a). For an assumed outer film coefficient, $\overline{T}_{f,o}$ = 292 K, the convection correlation for the outer water flow condition V_o = 1 m/s and T_o = 15°C, find

PROBLEM 11.2 (Cont.)

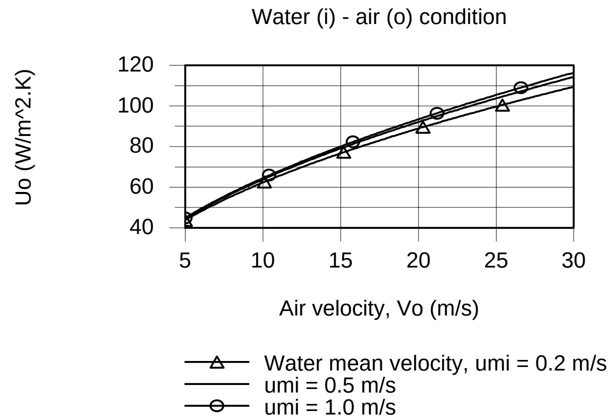
$$Re_{D,o} = 26,260 \qquad Nu_{D,o} = 220.6 \qquad \bar{h}_o = 4914\ W/m^2 \cdot K$$

The thermal resistances and overall coefficient are tabulated below.

$R_{cv,i}$ (K/W)	$R_{f,i}$ (K/W)	R_w (K/W)	$R_{f,o}$ (K/W)	$R_{cv,o}$ (K/W)	R_{tot} (K/W)	U_o ($W/m^2 \cdot K$)
0.00436	0.00579	0.00216	0.00236	0.00240	0.0171	691

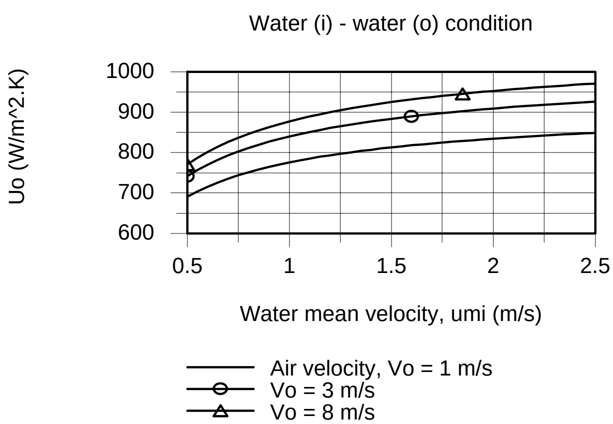
Note that the thermal resistances are of similar magnitude. In contrast with the results for the water-air condition of part (a), the thermal resistance of the outside convection process, $R_{cv,o}$, is nearly 50 times smaller. The overall coefficient for the water-water condition is 7.5 times greater than that for the water-air condition.

(c) For the water-air condition, using the IHT workspace with the analysis of part (a), U_o was calculated as a function of the air cross-flow velocity for selected mean water velocities.



The effect of increasing the cross-flow air velocity is to increase U_o since the $R_{cv,o}$ is the dominant thermal resistance for the system. While increasing the water mean velocity will increase $\bar{h}_i$, because $R_{cv,i} << R_{cv,o}$, this increase has only a small effect on U_o.

(d) For the water-water condition, using the IHT workplace with the analysis of part (b), U_o was calculated as a function of the mean water velocity for selected air cross-flow velocities.



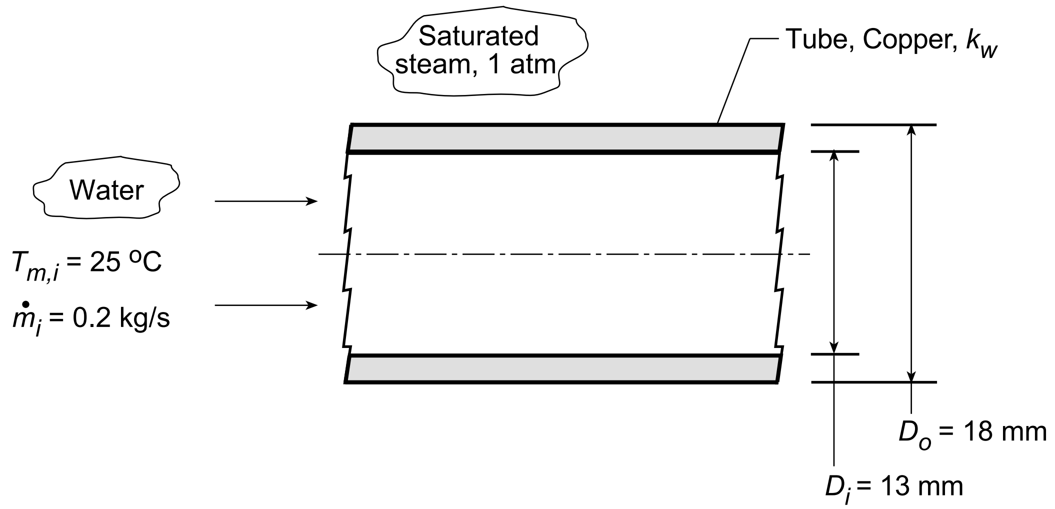
Because the thermal resistances for the convection processes, $R_{cv,i}$ and $R_{cv,o}$, are of similar magnitude according to the results of part (b), we expect to see U_o significantly increase with increasing water mean velocity and air cross-flow velocity.

PROBLEM 11.3

KNOWN: Copper tube with prescribed inner and outer diameters used in a shell-and-tube heat exchanger. Conditions prescribed for internal water flow and steam condensation on external surface.

FIND: (a) Overall heat transfer coefficient based upon the outer surface area, U_o; compare thermal resistances due to convection, tube wall conduction and condensation, and (b) Compute and plot U_o, water-side convection coefficient, h_i, and steam-side convection coefficient, h_o, as a function of the water flow rate for the range $0.2 \le \dot{m}_i \le 0.8$ kg/s.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Fully developed internal flow.

PROPERTIES: *Table A.1*, Copper, pure (300 K): k_w = 401 W/m·K; *Table A.6*, Water ($T_{m,i}$ = 298 K): $\mu_i = 8.966 \times 10^{-4}$ N·s/m^2, k_i = 0.6102 W/m·K, Pr_i = 6.146. *Table A.6*, Water, (assume $T_{s,o}$ = 351 K, $T_{f,o}$ = 362 K): ρ_ℓ = 965.7 kg/m^3, $c_{p,\ell}$ = 4205 J/kg·K, $\mu_\ell = 3.172 \times 10^{-4}$ N·s/m^2, k_ℓ = 0.6751 W/m·K; *Table A.6* Water (T_{sat} = 373 K, 1 atm): ρ_v = 0.5909 kg/m^3, h_{fg} = 2257 kJ/kg.

ANALYSIS: (a) The overall coefficient, Eq 11.1, based upon the outer surface area can be expressed as the sum of the thermal resistances due to convection (cv), tube wall conduction (w, see Eq. 3.28) and condensation (cnd):

$$1/U_o A_o = R_{tot} = R_{cv} + R_w + R_{cnd}$$

$$R_{cv} = 1/h_i A_i \qquad R_w = \ln(D_o/D_i)/(2\pi L k_w) \qquad R_{cnd} = 1/h_o A_o$$

The convection coefficients can be estimated from appropriate correlations.

Estimating h_i: For internal flow, characterize the flow using thermophysical properties evaluated at $T_{m,i}$ with

$$Re_{D,i} = \frac{4\dot{m}_i}{\pi D_i \mu_i} = \frac{4 \times 0.2\,\text{kg/s}}{\pi \times 0.013\text{m} \times 8.966 \times 10^{-4}\,\text{N/s}\cdot\text{m}^2} = 21{,}847$$

For turbulent flow, use the Dittus-Boelter correlation, Eq. 8.60,

$$Nu_{D,i} = 0.023 Re_{D,i}^{0.8} Pr_i^{0.4} = 0.023(21{,}847)^{0.8}(6.146)^{0.4} = 140.8$$

$$h_i = Nu_{D,i}\, k_i / D_i = 140.8 \times 0.6102\,\text{W/m}\cdot\text{K}/0.013\text{m} = 6610\,\text{W/m}^2\cdot\text{K}$$

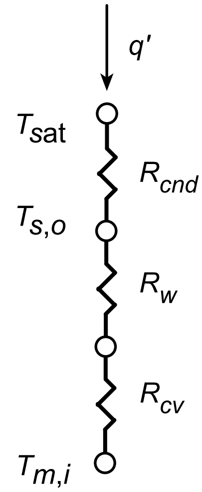
Continued...

PROBLEM 11.3 (Cont.)

Estimating h_o : For the horizontal tube, average convection coefficient for film condensation, Eq. 10.40, is

$$h_o = 0.729\left[\frac{g\rho_\ell(\rho_\ell - \rho_v)k_\ell^3 h'_{fg}}{\mu_\ell(T_{sat} - T_{s,o})D_o}\right]^{1/4}$$

$$h'_{fg} = h_{fg} + 0.68c_{p,\ell}(T_{sat} - T_{s,o})$$



The vapor (v) properties and h_{fg} are evaluated at T_{sat} , while the liquid properties (ℓ) are evaluated at the film temperature $T_{f,o} = (T_{s,o} - T_{sat})$ where the surface temperature is determined from the thermal circuit analysis result,

$$(T_{m,i} - T_{sat})/R_{tot} = (T_{s,o} - T_{sat})/R_{cnd}$$

Assume $T_{s,o}$ = 351 K so that $T_{f,o}$ = 362 K, and check later. Hence,

$$h_o = 0.729\left[\frac{9.8\,m/s^2 \times 965.7\,kg/m^3 \times (965.7 - 0.5909)\,kg/m^3 \times (0.6751\,W/m\cdot K)^3 \times 2321\,kJ/kg}{3.172\times10^{-4}\,N\cdot s/m^2\,(373-351)\,K \times 0.018\,m}\right]^{1/4}$$

$$h_o = 11{,}005\ W/m^2\cdot K$$

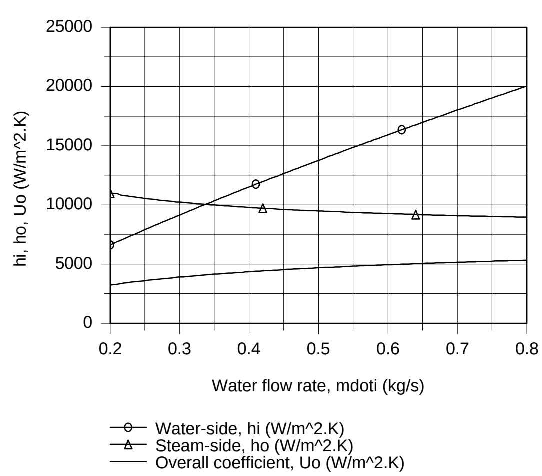
Using the above values for h_i, h_o and other prescribed values, the thermal resistances and overall coefficient can be evaluated and are tabulated below.

$R_{cv}\times10^3$ (K/W)	$R_w\times10^3$ (K/W)	$R_{cnd}\times10^3$ (K/W)	$R_{tot}\times10^3$ (K/W)	U_o ($W/m^2\cdot K$)
3.704	1.292	1.610	5.444	3249

The largest resistance is that due to convection on the water-side. Interestingly, the wall thermal resistance for the pure copper, while the smallest for all the process, is still significant relative to that for the condensation process.

(b) The foregoing relations were entered into the IHT workspace along with the *Correlations Tools* for *Forced Convection*, *Internal Flow*, *Turbulent Flow* and for *Film Condensation*, *Horizontal Cylinder* with the appropriate *Properties Tools* for *Air* and *Water*. The coefficients U_o, h_i and h_o were computed and plotted as a function of the water flow rate.

Note that the overall coefficient increases nearly 50% over the range of the water flow rate. The water-side coefficient increases markedly, by nearly a factor of 4, with increasing flow rate. The steam-side coefficient, h_o , is larger than h_i by a factor of 2 at the lowest flow rate. However, h_o decreases with increasing water flow rate since the tube wall temperature, $T_{s,o}$, decreases causing the water film thickness to increase with the net effect of reducing h_o.

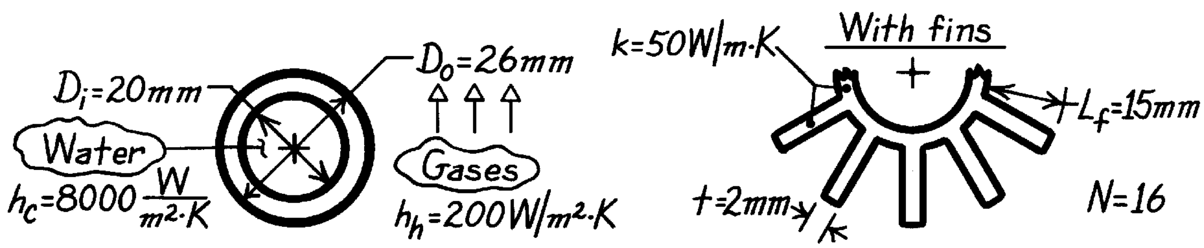


PROBLEM 11.4

KNOWN: Dimensions of heat exchanger tube with or without fins. Cold and hot side convection coefficients.

FIND: Cold side overall heat transfer coefficient without and with fins.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible fouling, (2) Negligible contact resistance between fins and tube wall, (3) h_h is not affected by fins, (4) One-dimensional conduction in fins, (5) Adiabatic fin tip.

ANALYSIS: From Eq. 11.1,

$$\frac{1}{U_c} = \frac{1}{(h_o h)_c} + \frac{D_i \ln(D_o / D_i)}{2k} + \frac{A_c}{(h_o hA)_h}$$

Without fins: $h_{o,c} = h_{o,h} = 1$

$$\frac{1}{U_c} = \frac{1}{8000\ W/m^2\cdot K} + \frac{(0.02m)\ln(26/20)}{100\ W/m\cdot K} + \frac{1}{200\ W/m^2\cdot K}\frac{20}{26}$$

$$1/U_c = \left(1.25\times10^{-4} + 5.25\times10^{-5} + 3.85\times10^{-3}\right) m^2\cdot K/W = 4.02\times10^{-3}\ m^2\cdot K/W$$

$$U_c = 249\ W/m^2\cdot K. \qquad <$$

With fins: $h_{o,c} = 1,\ h_{o,h} = 1-(A_f / A)(1-h_f)$ Per unit length along the tube axis,

$$A_f = N(2L_f + t) = 16(30+2)mm = 512\ mm$$

$$A_h = A_f + (p\, D_o - 16t) = (512 + 81.7 - 32)mm = 561.7\ mm$$

With $m = (2h/kt)^{1/2} = \left(400\ W/m^2\cdot K / 50\ W/m\cdot K\times0.002m\right)^{1/2} = 63.3m^{-1}$

$$mL_f = \left(63.3m^{-1}\right)(0.015m) = 0.95$$

and Eq. 11.4 yields

$$h_f = \tanh(mL_f)/mL_f = 0.739/0.95 = 0.778.$$

The overall surface efficiency is then

$$h_o = 1-(A_f / A_h)(1-h_f) = 1-(512/561.7)(1-0.778) = 0.798.$$

Hence

$$\frac{1}{U_c} = \left(1.25\times10^{-4} + 5.25\times10^{-5} + \frac{p(20)}{0.798(200)561.7}\right) m^2\cdot K/W = 8.78\times10^{-4} m^2\cdot K/W$$

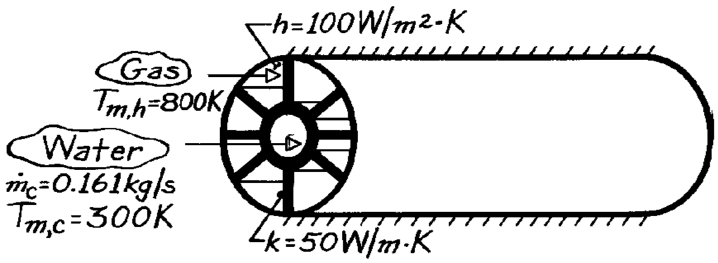
$$U_c = 1138\ W/m^2\cdot K. \qquad <$$

PROBLEM 11.5

KNOWN: Geometry of finned, annular heat exchanger. Gas-side temperature and convection coefficient. Water-side flowrate and temperature.

FIND: Heat rate per unit length.

SCHEMATIC:



$D_o = 60$ mm
$D_{i,1} = 24$ mm
$D_{i,2} = 30$ mm
t = 3 mm = 0.003m
L = (60-30)/2 mm = 0.015m

ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) One-dimensional conduction in strut, (4) Adiabatic outer surface conditions, (5) Negligible gas-side radiation, (6) Fully-developed internal flow, (7) Negligible fouling.

PROPERTIES: *Table A-6*, Water (300 K): k = 0.613 W/m·K, Pr = 5.83, $\mu = 855 \times 10^{-6}$ N·s/m^2.

ANALYSIS: The heat rate is

$$q = (UA)_c \left(T_{m,h} - T_{m,c}\right)$$

where

$$1/(UA)_c = 1/(hA)_c + R_w + 1/(\eta_o hA)_h$$

$$R_w = \frac{\ln\left(D_{i,2}/D_{i,1}\right)}{2\pi kL} = \frac{\ln(30/24)}{2\pi\,(50\ \mathrm{W/m\ K})\,1\mathrm{m}} = 7.10\times10^{-4}\,\mathrm{K/W}.$$

With

$$Re_D = \frac{4\dot{m}}{\pi D_{i,1}\mu} = \frac{4\times0.161\,\mathrm{kg/s}}{\pi\,(0.024\mathrm{m})\,855\times10^{-6}\,\mathrm{N\cdot s/m^2}} = 9990$$

internal flow is turbulent and the Dittus-Boelter correlation gives

$$h_c = \left(k/D_{i,1}\right)0.023Re_D^{4/5}\,Pr^{0.4} = \left(\frac{0.613\,\mathrm{W/m\cdot K}}{0.024\mathrm{m}}\right)0.023(9990)^{4/5}\,(5.83)^{0.4} = 1883\,\mathrm{W/m^2\cdot K}$$

$$(hA)_c^{-1} = \left(1883\,\mathrm{W/m^2\cdot K}\times\pi\times0.024\mathrm{m}\right)^{-1} = 7.043\times10^{-3}\,\mathrm{K/W}.$$

Find the fin efficiency as

$$\eta_o = 1 - \left(A_f/A\right)\left(1-\eta_f\right)$$

$$A_f = 8\times2\,(L\cdot w) = 8\times2\left(0.015\mathrm{m}\times1\mathrm{m}\right) = 0.24\mathrm{m}^2$$

$$A = A_f + \left(\pi D_{i,2} - 8t\right)w = 0.24\mathrm{m}^2 + \left(\pi\times0.03\mathrm{m} - 8\times0.003\mathrm{m}\right) = 0.31\mathrm{m}^2.$$

From Eq. 11.4,

$$h_f = \frac{\tanh(mL)}{mL}$$

where

$$m = [2h/kt]^{1/2} = \left[2\times 100\ W/m^2\cdot K/50\ W/m\cdot K(0.003m)^{1/2}\right] = 36.5m^{-1}$$

$$mL = (2h/kt)^{1/2}L = 36.5m^{-1}\times 0.015m = 0.55$$

$$\tanh\left[(2h/kt)^{1/2}L\right] = 0.499.$$

Hence

$$h_f = 0.800/1.10 = 0.907$$

$$h_o = 1-(A_f\ /A)(1-h_f\ ) = 1-(0.24/0.31)(1-0.907) = 0.928$$

$$(h_o hA)_h^{-1} = \left(0.928\times 100\ W/m^2\cdot K\times 0.31m^2\right)^{-1} = 0.0347\ K/W.$$

Hence

$$(UA)_c^{-1} = \left(7.043\times 10^{-3} + 7.1\times 10^{-4} + 0.0347\right)K/W$$

$$(UA)_c = 23.6\ W/K$$

and

$$q = 23.6\ W/K(800-300)K = 11,800\ W \qquad <$$

for a 1m long section.

COMMENTS: (1) The gas-side resistance is substantially decreased by using the fins ($A_f >> pD_{i,2}$) and q is increased.

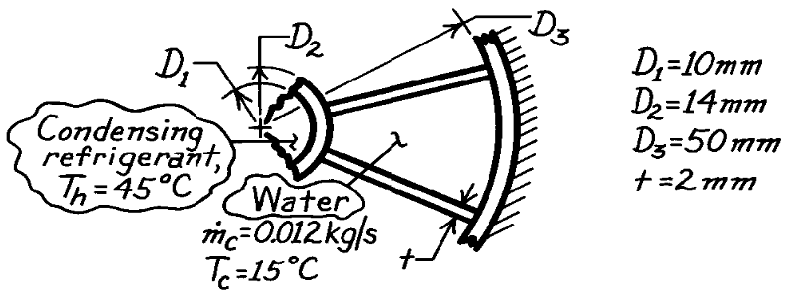
(2) Heat transfer enhancement by the fins could be increased further by using a material of larger k, but material selection would be limited by the large $T_{m,h}$.

PROBLEM 11.6

KNOWN: Condenser arrangement of tube with six longitudinal fins (k = 200 W/m·K). Condensing refrigerant temperature at 45°C flows axially through inner tube while water flows at 0.012 kg/s and 15°C through the six channels formed by the splines.

FIND: Heat removal rate per unit length of the exchanger.

SCHEMATIC:



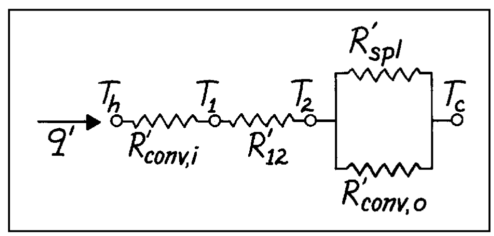
ASSUMPTIONS: (1) No heat loss/gain to the surroundings, (2) Negligible kinetic and potential energy changes, (3) Negligible thermal resistance on condensing refrigerant side, $h_i \to \infty$, (4) Water flow is fully developed, (5) Negligible thermal contact between splines and inner tube, (6) Heat transfer from outer tube negligible.

PROPERTIES: *Table A-6*, Water ($\overline{T}_c$ = 15°C = 288 K): ρ = 1000 kg/m^3, k = 0.595 W/m·K, $\nu = \mu/\rho$ = 1138 × 10^{-6} N·s/m^2/1000 kg/m^3 = 1.138 × 10^{-6} m^2/s, Pr = 8.06; Tube fins (given): k = 200 W/m·K.

ANALYSIS: Following the discussion of Section 11.2,

$$q' = UA'(T_h - T_c)$$

$$\frac{1}{UA'} = R'_h + R'_w + R'_c = R'_w + \frac{1}{(\eta_o hA')_c}$$



where $R'_h = 0$, due to the negligible thermal resistance on the refrigerant side (h), and

$$R'_w = \frac{\ln(D_2/D_1)}{2\pi k} = \frac{\ln(14/10)}{2\pi(200\ \mathrm{W/m\cdot K})} = 2.678\times10^{-4}\,\mathrm{m\cdot K/W}.$$

To estimate the thermal resistance on the water side (c), first evaluate the convection coefficient. The hydraulic diameter for a passage, where A_c is the cross-sectional area of the passage is

$$D_{h,c} = \frac{4A_c}{P} = \frac{4\left[\pi\left(D_3^2 - D_2^2\right)/4 - 6(D_3 - D_2)t/2\right]/6}{(\pi D_2 - 6t)/6 + (\pi D_3 - 6t)/6 + 2(D_3 - D_2)/2}$$

$$D_{h,c} = \frac{4\left[\pi\left(50^2 - 14^2\right)/4 - 6(50-14)\right]\times10^{-6}\mathrm{m}^2/6}{\left[(14\pi - 6\times2)/6 + (50\pi - 6\times2)/6 + (50-14)\right]\times10^{-3}\mathrm{m}}$$

$$D_{h,c} = \frac{4\times2.656\times10^{-4}\mathrm{m}^2}{6.551\times10^{-2}\mathrm{m}} = 0.01622\,\mathrm{m}.$$

Hence the Reynolds number is

Continued …..

$$Re_{D,c} = \frac{\left[(0.012\ kg/s/6)/\left(1000\ kg/m^3 \times 2.656\times10^{-4} m^2\right)\right]\times 0.01622m}{1.138\times10^{-6} m^2/s} = 107$$

and assuming the flow is fully developed,

$$Nu_{D,c} = \frac{h_c D_{h,c}}{k} = 3.66$$

$$h_c = 3.66\times 0.595\ W/m\cdot K/0.01622 = 134\ W/m^2\cdot K.$$

The temperature effectiveness of the splines (fins) on the cold side is

$$\eta_o = 1 - \frac{A_{f,c}}{A_c}(1-\eta_f)$$

where $A_{f,c}$ and A_c are, respectively, the finned and total (fin plus prime) surface areas, while

$$\eta_f = \frac{\tanh(mL)}{mL}$$

$$m = (2h_c/kt)^{1/2} = \left[\left(2\times 134\ W/m^2\cdot K\right)/\left(200\ W/m\cdot K\times 0.002m\right)\right]^{1/2} = 25.88\,m^{-1}$$

$$\eta_f = \frac{\tanh\left(25.88\,m^{-1}\times 0.018m\right)}{25.88\,m^{-1}\times 0.018m} = \frac{0.4348}{0.4658} = 0.934.$$

Hence

$$\eta_o = 1 - \frac{6(D_3 - D_2)}{6(D_3 - D_2) + (\pi D_2 - 6t)}[1-\eta_f]$$

$$\eta_o = 1 - \frac{6(50-14)}{6(50-14)+(14\pi - 6\times 2)}(1-0.934) = 0.943$$

$$\frac{1}{\eta_o h A_c'} = \frac{1}{0.943\times 134\ W/m^2\cdot K\left[6(50-14)+(14\pi - 6\times 2)\right]\times 10^{-3} m} = 3.22\times 10^{-2}\ m\cdot K/W$$

and the heat rate is

$$q' = \frac{T_h - T_c}{R_w' + 1/(\eta_o h A')_c}$$

$$q' = \frac{(45-15)K}{2.678\times 10^{-4} m\cdot K/W + 3.22\times 10^{-2} m\cdot K/W} = 924\ W/m.$$ <

COMMENTS: (1) The effective length of the fin representing the splines was conservatively estimated. The heat transfer by conduction through the splines to the outer tube and then by convection to the water was ignored.

(2) Without the splines, find $D_h = (D_3 - D_2) = 36$ mm so that $h_c = 60.5\ W/m^2\cdot K$. The heat rate with $A_c' = \pi D_2$ is

$$q' = (hA_c')(T_h - T_c) = 60.5\ W/m^2\cdot K(0.014\pi\ m)(45-15)K = 79\ W/m.$$

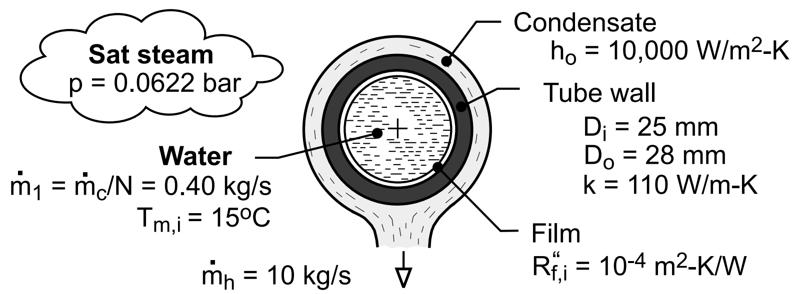
The splines enhance the heat transfer rate by a factor of 924/79 = 11.7.

PROBLEM 11.7

KNOWN: Number, inner-and outer diameters, and thermal conductivity of condenser tubes. Convection coefficient at outer surface. Overall flow rate, inlet temperature and properties of water flow through the tubes. Flow rate and pressure of condensing steam. Fouling factor for inner surface.

FIND: (a) Overall coefficient based on outer surface area, U_o, without fouling, (b) Overall coefficient with fouling, (c) Temperature of water leaving the condenser.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible flow work and kinetic and potential energy changes for water flow, (2) Fully-developed flow in tubes, (3) Negligible effect of fouling on D_i.

PROPERTIES: Water (Given): c_p = 4180 J/kg·K, $\mu = 9.6 \times 10^{-4}$ N·s/m^2, k = 0.60 W/m·K, Pr = 6.6. *Table A-6*, Water, saturated vapor (p = 0.0622 bars): T_{sat} = 310 K, $h_{fg} = 2.414 \times 10^6$ J/kg.

ANALYSIS: (a) Without fouling, Eq. 11.5 yields

$$\frac{1}{U_o} = \frac{1}{h_i}\left(\frac{D_o}{D_i}\right) + \frac{D_o \ln(Do/Di)}{2k_t} + \frac{1}{h_o}$$

With $Re_{D_i} = 4\dot{m}_1/\pi D_i \mu = 1.60\,kg/s/\left(\pi \times 0.025m \times 9.6 \times 10^{-4}\,N \cdot s/m^2\right) = 21,220,$ flow in the tubes is turbulent, and from Eq. 8.60

$$h_i = \left(\frac{k}{D_i}\right) 0.023 Re_{D_i}^{4/5} Pr^{0.4} = \left(\frac{0.60\,W/m \cdot K}{0.025m}\right) 0.023(21,200)^{4/5}(6.6)^{0.4} = 3400\,W/m^2 \cdot K$$

$$U_o = \left[\frac{1}{3400}\left(\frac{28}{25}\right) + \frac{0.028\ln(28/25)}{2 \times 110} + \frac{1}{10,000}\right]^{-1} W/m^2 \cdot K =$$

$$\left(3.29 \times 10^{-4} + 1.44 \times 10^{-5} + 10^{-4}\right)^{-1} W/m^2 \cdot K = 2255\,W/m^2 \cdot K \qquad <$$

(b) With fouling, Eq. 11.5 yields

$$U_o = \left[4.43 \times 10^{-4} + (Do/Di)R''_{f,i}\right]^{-1} = \left(5.55 \times 10^{-4}\right)^{-1} = 1800\,W/m^2 \cdot K \qquad <$$

(c) The rate at which energy is extracted from the steam equals the rate of heat transfer to the water, $\dot{m}_h h_{fg} = \dot{m}_c c_p (T_{m,o} - T_{m,i}),$ in which case

$$T_{m,o} = T_{m,i} + \frac{\dot{m}_h h_{fg}}{\dot{m}_c c_p} = 15°C + \frac{10\,kg/s \times 2.414 \times 10^6\,J/kg}{400\,kg/s \times 4180\,J/kg \cdot K} = 29.4°C \qquad <$$

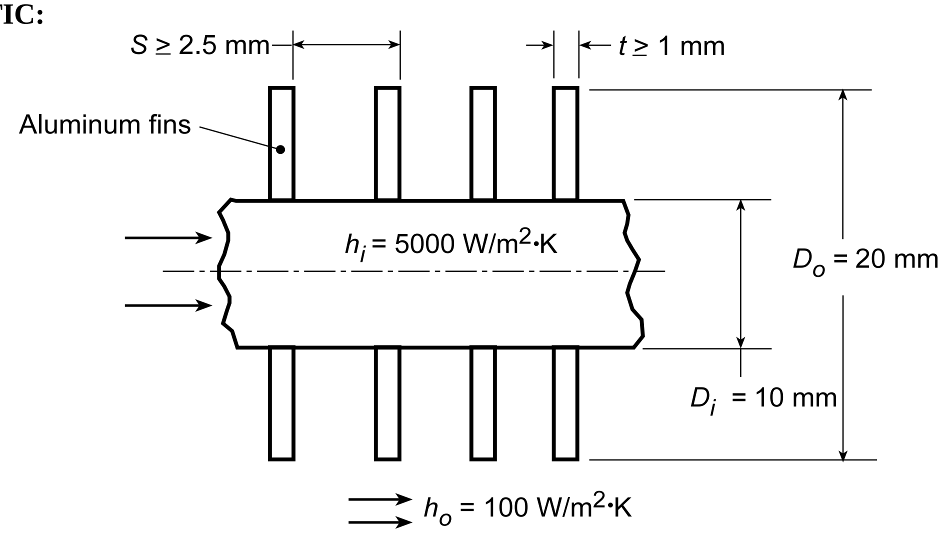
COMMENTS: (1) The largest contribution to the thermal resistance is due to convection at the interior of the tube. To increase U_o, hi could be increased by increasing $\dot{m}_1$, either by increasing $\dot{m}_c$ or decreasing N. (2) Note that $T_{m,o}$ = 302.4 K < T_{sat} = 310 K, as must be the case.

PROBLEM 11.8

KNOWN: Diameter and inner and outer convection coefficients of a condenser tube. Thickness, outer diameter, and pitch of aluminum fins.

FIND: (a) Overall heat transfer coefficient without fins, (b) Effect of fin thickness and pitch on overall heat transfer coefficient with fins.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible tube wall conduction resistance, (2) Negligible fouling and fin contact resistance, (3) One-dimensional conduction in fin.

PROPERTIES: *Table A.1*, Aluminum (T = 300 K): k = 237 W/m·K.

ANALYSIS: (a) With no fins, Eq. 11.1 yields

$$U = \left(h_i^{-1} + h_o^{-1}\right)^{-1} = \left(2\times10^{-4} + 0.01\right)^{-1} \mathrm{W/m^2 \cdot K} = 98.0\,\mathrm{W/m^2 \cdot K}$$ <

(b) With fins and a unit tube length, Eqs. 11.1 and 11.3 yield

$$\frac{1}{U_i \pi D_i} = \frac{1}{h_i \pi D_i} + \frac{1}{\eta_o h_o A'_o}$$

and $\eta_o = 1 - (A'_f / A'_o)(1 - \eta_f)$. The total fin surface area per unit length is $A'_f = N' 2\pi\left(r_{oc}^2 - r_i^2\right)$, where the number of fins per unit length is $N' = 1\mathrm{m}/S(\mathrm{m})$. The total outside surface area per unit length is $A'_o = A'_f + (1 - N't)\pi D_i$, and the fin efficiency is given by Eq. 3.91 or Fig. 3.19.

For t = 0.0015 m and S = 0.0035 m, $r_{oc} = (D_o/2) + (t/2) = 0.01075$ m, $N' \approx 286$, $A'_f = 0.163\ \mathrm{m^2/m}$, and $A'_o = (0.163 + 0.018)\ \mathrm{m^2/m} = 0.181\ \mathrm{m^2/m}$. With $r_{oc}/r_i = 2.15$, $L_c = 0.00575$ m, $A_p = 8.625 \times 10^{-6}\ \mathrm{m^2}$, and $L_c^{3/2}\left(h_o/kA_p\right)^{1/2} = 0.0964$, Fig. 3.19 yields $\eta_f \approx 0.99$. Hence, $\eta_o \approx 1 - (0.163/0.181)(0.01) = 0.99$ and

$$U_i = \left[(1/h_i) + (\pi D_i / \eta_o h_o A'_o)\right]^{-1}$$

$$U_i = \left[2\times10^{-4}\,\mathrm{m^2 \cdot K/W} + \pi\times0.01\,\mathrm{m}/0.99\times100\,\mathrm{W/m^2 \cdot K}\times0.181\,\mathrm{m^2/m}\right]^{-1} = 512\,\mathrm{W/m^2 \cdot K}$$ <

We may use the IHT *Extended Surface* Model (*Performance Calculations* for a *Circular Rectangular Fin Array*) to consider the effect of varying t and S. To maximize N', the minimum allowable value of

Continued...

S - t = 1.5 mm should be selected. It is then a matter of choosing between a large number of thin fins or a smaller number of thicker fins. Calculations were performed for the following options.

t (mm)	S (mm)	N'	U_i ($W/m^2 \cdot K$)
1	2.5	400	640
2	3.5	286	512
3	4.5	222	460
4	5.5	182	420

Since heat transfer increases with U_i, the best configuration corresponds to t = 1 mm and S = 2.5 mm, which provides the largest airside surface area.

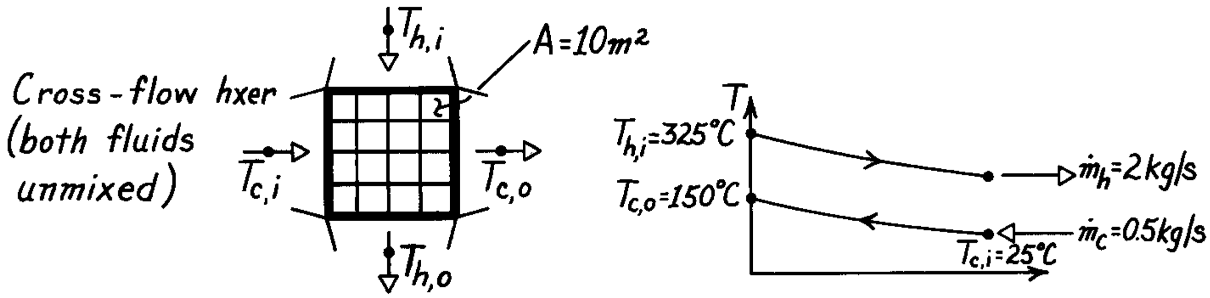
COMMENTS: The best performance is always associated with a large number of closely spaced fins, so long as the flow between adjoining fins is sufficient to maintain the convection coefficient.

PROBLEM 11.9

KNOWN: Operating conditions and surface area of a finned-tube, cross-flow exchanger.

FIND: Overall heat transfer coefficient.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible kinetic and potential energy changes, (3) Constant properties, (4) Exhaust gas properties are those of air.

PROPERTIES: *Table A-6*, Water $\left(\overline{T}_m = 87°C\right)$: $\overline{c}_p = 4203 \text{J/kg}\cdot\text{K}$; *Table A-4*, Air $\left(T_m \approx 275°C\right)$: $\overline{c}_p = 1040\,\text{J/kg}\cdot\text{K}$.

ANALYSIS: From the energy balance equations

$$q = \dot{m}_c c_{p,c}\left(T_{c,o} - T_{c,i}\right) = 0.5\text{kg/s}\times 4203\text{J/kg}\cdot\text{K}\,(150-25)\,°C = 2.63\times10^5\,\text{W}$$

$$T_{h,o} = T_{h,i} - \frac{q}{\dot{m}_h c_{p,h}} = 325°C - \frac{2.63\times10^5\,\text{W}}{2\text{kg/s}\times1040\text{J/kg}\cdot\text{K}} = 198.6°C.$$

Hence

$$U = q/A\Delta T_{\ell m} \qquad \text{where} \qquad \Delta T_{\ell m} = F\Delta T_{\ell m,CF}.$$

From Fig. 11.12, with

$$P = \frac{t_o - t_i}{T_i - t_i} = \frac{150-25}{325-25} = 0.42,\;\; R = \frac{T_i - T_o}{t_o - t_i} = \frac{325-198.6}{150-25} = 1.01,\;\; F = 0.94$$

$$\Delta T_{\ell m,CF} = \frac{(325-150)-(198.6-25)}{\ell n\dfrac{325-150}{198.6-25}} = 174.3°C.$$

Hence

$$U = \frac{q}{AF\Delta T_{\ell m,CF}} = \frac{2.63\times10^5\,\text{W}}{10\text{m}^2\times0.94\times174.3°C} = 160\ \text{W/m}^2\cdot\text{K}. \qquad <$$

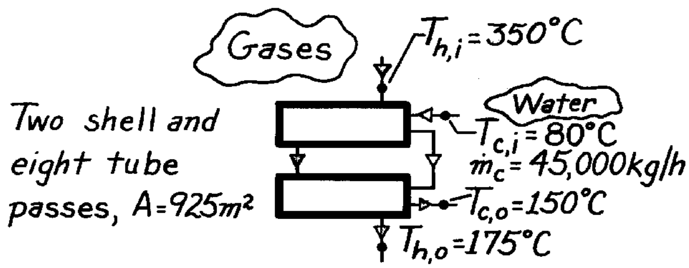
COMMENTS: From the *e* - NTU method, C_c = 2102 W/K, C_h = 2080 W/K, $(C_{min}/C_{max}) \approx 1$, $q_{max} = 6.24\times10^5$ W and *e* = 0.42. Hence, from Fig. 11.18, NTU ≈ 0.75 and U ≈ 156 W/m^2·K.

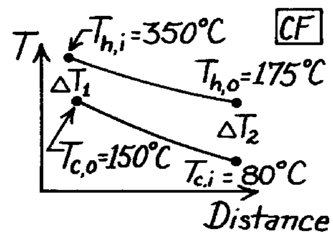
PROBLEM 11.10

KNOWN: Heat exchanger with two shell passes and eight tube passes having an area 925m^2; 45,500 kg/h water is heated from 80°C to 150°C; hot exhaust gases enter at 350°C and exit at 175°C.

FIND: Overall heat transfer coefficient.

SCHEMATIC:





ASSUMPTIONS: (1) Negligible losses to surroundings, (2) Negligible kinetic and potential energy changes, (3) Constant properties, (4) Exhaust gas properties are approximated as those of atmospheric air.

PROPERTIES: *Table A-6*, Water $\left(\overline{T}_c = (80+150)°C/2 = 388K\right)$: c = $c_{p,f}$ = 4236 J/kg·K.

ANALYSIS: The overall heat transfer coefficient follows from Eqs. 11.9 and 11.18 written in the form

$$U = q/A\,F\Delta T_{\ell m,CF}$$

where F is the correction factor for the HXer configuration, Fig. 11.11, and $\Delta T_{\ell m,CF}$ is the log mean temperature difference (CF), Eqs. 11.15 and 11.16. From Fig. 11.11, find

$$R = \frac{T_{h,i} - T_{h,o}}{T_{c,o} - T_{c,i}} = \frac{(350-175)°C}{(150-80)°C} = 2.5 \quad P = \frac{T_{c,o} - T_{c,i}}{T_{h,i} - T_{c,i}} = \frac{(150-80)°C}{(350-80)°C} = 0.26$$

find F ≈ 0.97. The log-mean temperature difference, Eqs. 11.15 and 11.17, is

$$\Delta T_{\ell m,CF} = \frac{\Delta T_1 - \Delta T_2}{\ell n(\Delta T_1/\Delta T_2)} = \frac{(350-150)°C - (175-80)°C}{\ell n\left[(350-150)/(175-80)\right]} = 141.1°C.$$

From an overall energy balance on the cold fluid (water), the heat rate is

$$q = \dot{m}_c c_c \left(T_{c,o} - T_{c,i}\right)$$

$$q = 45{,}500\,kg/h \times 1h/3600s \times 4236\,J/kg\cdot K\,(150-80)°C = 3.748\times10^6\,W.$$

Substituting values with A = 925 m^2, find

$$U = 3.748\times10^6\,W/925m^2 \times 0.97 \times 141.1K = 29.6\,W/m^2\cdot K.$$ <

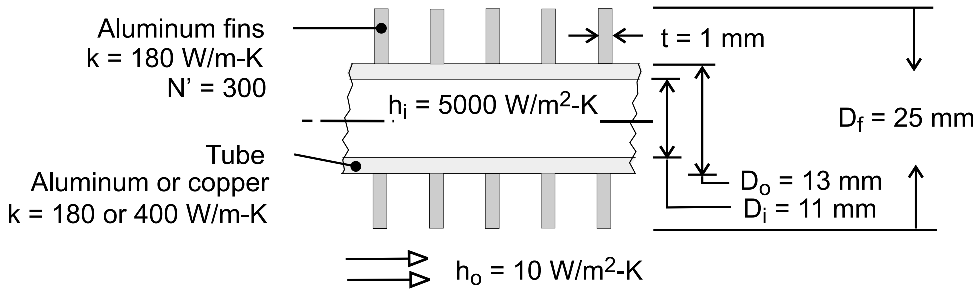
COMMENTS: Compare the above result with representative values for air-water exchangers, as given in Table 11.2. Note that in this exchanger, two shells with eight tube passes, the correction factor effect is very small, since F = 0.97.

PROBLEM 11.11

KNOWN: Dimensions and thermal conductivity of tubes with or without annular fins. Convection coefficients associated with condensation and natural convection at the inner and outer surfaces, respectively.

FIND: (a) Overall heat transfer coefficient U_i for aluminum and copper tubes without fins, (b) Value of U_i associated with adding aluminum fins.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible fouling and fin contact resistances, (2) One-dimensional conduction in fins.

ANALYSIS: (a) For unfinned, *aluminum* tubes of unit length, Eq. 11.5 yields

$$\frac{1}{U_i} = \frac{1}{h_i} + \frac{D_i \ln(Do/Di)}{2k} + \frac{1}{h_o}\left(\frac{D_i}{D_o}\right)$$

$$U_i = \left[\frac{1}{5000} + \frac{0.011 \ln(13/11)}{2\times 180} + \frac{1}{10}\left(\frac{11}{13}\right)\right]^{-1} = \left(2\times 10^{-4} + 5.1\times 10^{-6} + 846\times 10^{-4}\right)^{-1} = 11.8\,\mathrm{W/m^2\cdot K} \quad <$$

For *copper* the tube conduction resistance is reduced from 5.1×10^{-6} $m^2 \cdot K/W$ to 2.3×10^{-6}, but U_i is essentially unchanged.

$$U_i = 11.8\,\mathrm{W/m^2\cdot K} \quad <$$

(b) With fins and a unit tube length, Eqs. 11.1 and 11.3 yield

$$\frac{1}{U_i} = \frac{1}{h_i} + \frac{D_i \ln(Do/Di)}{2k} + \frac{\pi D_i}{\eta_o h_o A'_o}$$

and $\eta_o = 1 - (A'_f / A'_o)(1 - \eta_f)$. The fin surface area is $A'_f = N' 2\pi\left(r_{fc}^2 - r_o^2\right)$ and the total outer surface area is $A'_o = A'_f + (1 - N't)\pi D_o$. With t = 0.001m, $r_{fc} = r_f + t/2$ = (0.0125 + 0.0005)m = 0.0130m and $A'_f = 300\,\mathrm{m^{-1}}(2\pi)\left(0.0130^2 - 0.0065^2\right)\mathrm{m^2} = 0.239\mathrm{m}$ and $A'_o = 0.239\mathrm{m} + (1 - 0.300)\pi(0.013\mathrm{m})$ =0.268m. With $r_{2c} = r_f + t/2$ = 0.013m, $L_c = (r_f - r_o) + t/2$ = 0.0065m, r_{2c}/r_o = 2, $A_p = L_c t = 3.25 \times 10^{-6}$ m^2, and $L_c^{3/2}\left(h_o / k A_p\right)^{1/2} = 0.0685$, Fig. 3.19 yields $\eta_f \approx 0.97$. Hence, η_o = 1 – (0.239/0.268) (0.03) ≈ 0.973, and

$$U_i = \left[\frac{1}{5000} + \frac{0.011 \ln(13/11)}{360} + \frac{\pi \times 0.011}{0.973 \times 10 \times 0.244}\right]^{-1}$$

$$= \left(2\times 10^{-4} + 5.1\times 10^{-6} + 145\times 10^{-4}\right)^{-1} = 68.0\,\mathrm{W/m^2\cdot K} \quad <$$

COMMENTS: There is significant advantage to installing fins on the outer surface, which has a much smaller convection coefficient. The thermal resistance at the outer surface has been reduced from 0.0846 to 0.0145 $m^2 \cdot K/W$ and could be reduced further by increasing D_f and/or N'. However, the spacing between adjoining fins must not be so small as to restrict buoyancy driven flow in the associated air space.

PROBLEM 11.12

KNOWN: Properties and flow rates for the hot and cold fluid to a heat exchanger.

FIND: Which fluid limits the heat transfer rate of the exchanger?

ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, and (3) Negligible losses to the surroundings and kinetic and potential energy changes.

ANALYSIS: The properties and flow rates for the hot and cold fluid to the heat exchanger are tabulated below.

	Cold fluid	*Hot fluid*
Density, kg/m^3	997	1247
Specific heat, J/kg·K	4179	2564
Thermal conductivity, W/m·K	0.613	0.287
Viscosity, N·s/m^2	8.55×10^{-4}	1.68×10^{-4}
Flow rate, m^3/h	14	16

The fluid which limits the heat transfer rate of the exchanger is the minimum fluid, $C_{min} = \left(\dot{m} \cdot c\right)_{min}$. For the hot and cold fluids, find

$$C_h = \dot{m}_h\, c_h = 16\ \text{m}^3/\text{h} \times 1247\ \text{kg/m}^3 \times 2564\ \text{J/kg}\cdot\text{K} \times \left(1\text{h}/3600\text{s}\right) = 14.21\ \text{kW/K}$$

$$C_c = \dot{m}_c\, c_c = 14\ \text{m}^3/\text{h} \times 997\ \text{kg/m}^3 \times 4179\ \text{J/kg}\cdot\text{K} \times \left(1\text{h}/3600\text{s}\right) = 16.20\ \text{kW/K}$$

Hence, the hot fluid is the minimum fluid,

$$C_{min} = C_h$$ <

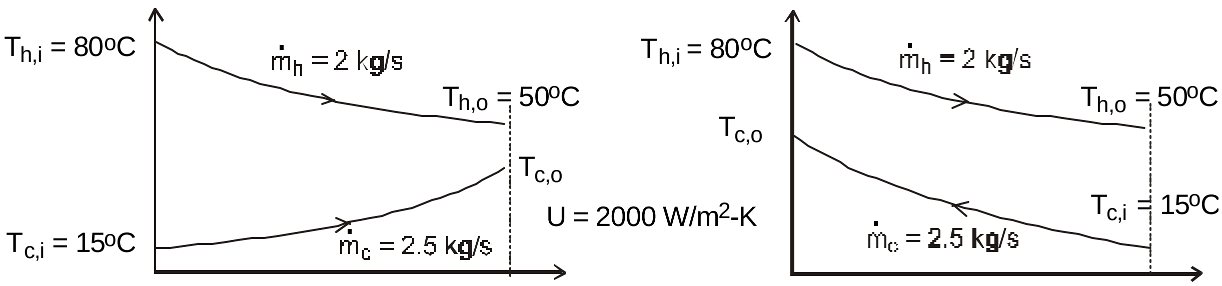
For any exchanger, the heat rate is $q = \varepsilon\, q_{max}$, where ε depends upon the exchanger type. The maximum heat rate is $q_{max} = C_{min}\,(T_{h,i} - T_{c,i})$. Hence, it is the conditions for the minimum fluid that limit the performance of the exchanger.

PROBLEM 11.13

KNOWN: Process (hot) fluid having a specific heat of 3500 J/kg·K and flowing at 2 kg/s is to be cooled from 80°C to 50°C with chilled-water (cold fluid) supplied at 2.5 ks/g and 15°C assuming an overall heat transfer coefficient of 2000 $W/m^2 \cdot K$.

FIND: The required heat transfer areas for the following heat exchanger configurations; (a) Concentric tube (CT) - parallel flow, (b) CT - counterflow, (c) Shell and tube, one-shell pass and 2 tube passes; (d) Cross flow, single pass, both fluids unmixed. Use the *IHT Tools | Heat Exchanger* models as your solution tool.

SCHEMATIC:



(a) Parallel flow hxer **(b) Counterflow hxer**

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible losses to the surroundings and kinetic and potential energy changes, (3) Overall heat transfer coefficient remains constant with different configurations, and (4) Constant properties.

ANALYSIS: The *IHT Tools | Heat Exchanger* models are based upon the effectiveness-NTU method and suited for design-type problems. The table below summarizes the results of our analysis using the IHT models including model equations, figures, and the required heat transfer area. The cold fluid outlet temperature for all configurations is $T_{c,o}$ = 35.1°C. The IHT code for the concentric tube, parallel flow heat exchanger is provided in the Comments.

Heat exchanger type	*Eqs.*	*Figs*	*A(m^2)*
(a) CT -Parallel flow	11.29b	11.14	3.09
(b) CT -Counterflow	11.30b	11.15	2.64
(c) Shell and tube (1 - sp, 2 - tp)	11.31b	11.10, 16	2.83
(d) Crossflow (1 - p, unmixed)	11.33	11.12, 18	2.84

COMMENTS: (1) Referring to the tabulated results, note that for the concentric tube exchangers, the area required for parallel flow is 17% larger than for counterflow. Under what circumstances would you choose to use the PF arrangement if the area has to be significantly larger?

(2) The shell-tube and crossflow exchangers require nearly the same heat transfer area. What are other factors that might influence your decision to select one type over the other for an application?

(3) Based upon area considerations only, the CF arrangement requires the smallest heat transfer area. What practical issues need to be considered in making a CF heat exchanger with a 2.6 m^2 area?

Continued …..

(4) The *IHT* code used for the concentric tube, parallel flow heat exchanger is shown below. Note the use of the water property function, *cp_Tx*, and the intrinsic function, *Tfluidavg*, to provide the specific heat at the mean water (cold fluid) temperature.

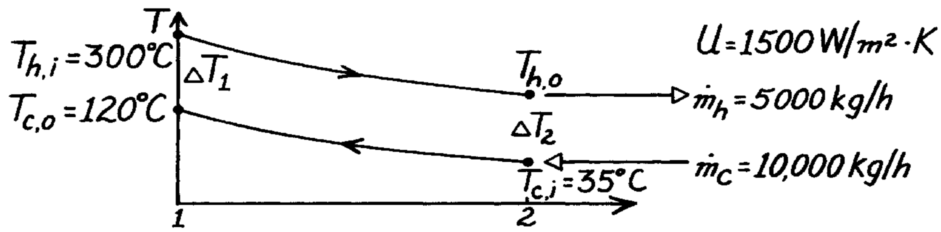
```
/" Results - energy balance only
Cc          Ch          Tco    cc     q      Tci    Thi    Tho    ch
1.045E4     7000        35.1   4180   2.1E5  15     80     50     3500*/

/" Results of sizing
A       CR      NTU     eps
3.87    0.6699  0.882   0.4615  */

// Design conditions
Thi = 80
Tho = 50
mdoth = 2
ch = 3500
mdotc = 2.5
Tci = 15
U = 2000

// For the parallel-flow, concentric-tube heat exchanger,
// For the parallel-flow, concentric-tube heat exchanger,
NTU = -ln(1 - eps * (1 + Cr))/(1 + Cr)            // Eq 11.29b
// where the heat-capacity ratio is
Cr = Cmin/Cmax
// and the number of transfer units, NTU, is
NTU = U * A/Cmin                                  // Eq 11.25
// The effectiveness is defined as
eps = q/qmax
qmax = Cmin * (Thi - Tci)                         // Eq 11.20
// See Tables 11.3 and 11.4 and Fig 11.14

// Energy balances
q = Cc * (Tco - Tci)
q = Ch * (Thi - Tho)
Cc = mdotc * cc
Ch = mdoth * ch
Cmin = Ch
Cmax = Cc

// Water property functions: T dependence, From Table A.6
// Units: T(K), p(bars):
xc = 0                                            // Quality (0=sat liquid or 1-sat vapor)
cc = cp_Tx("Water", Tcm,xc)                       // Specific heat, J/kg·K
Tcm = Tfluid_avg(Tci, Tco)                        // Mean temperature; K; intrinsic function
```

PROBLEM 11.14

KNOWN: A shell and tube Hxer (two shells, four tube passes) heats 10,000 kg/h of pressurized water from 35°C to 120°C with 5,000 kg/h water entering at 300°C.

FIND: Required heat transfer area, A_s.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible kinetic and potential energy changes, (3) Constant properties.

PROPERTIES: *Table A-6*, Water $(\bar{T}_c = 350K)$: c_p = 4195 J/kg·K; *Table A-6*, Water (Assume $T_{h,o} \approx 150°C$, $\bar{T}_h \approx 500$ K): c_p = 4660 J/kg·K.

ANALYSIS: The rate equation, Eq. 11.14, can be written in the form

$$A_s = q / U\Delta T_{\ell m} \tag{1}$$

and from Eq. 11.18,

$$\Delta T_{\ell m} = F\Delta T_{\ell m,CF} \qquad \text{where} \qquad \Delta T_{\ell m,CF} = \frac{\Delta T_1 - \Delta T_2}{\ell n(\Delta T_1/\Delta T_2)}. \tag{2,3}$$

From an energy balance on the cold fluid, the heat rate is

$$q = \dot{m}_c\, c_{p,c}\left(T_{c,o} - T_{c,i}\right) = \frac{10{,}000\ kg/h}{3600\,s/h} \times 4195\frac{J}{kg\cdot K}(120-35)K = 9.905\times10^5\ W.$$

From an energy balance on the hot fluid, the outlet temperature is

$$T_{h,o} = T_{h,i} - q/\dot{m}_h\, c_{p,h} = 300°C - 9.905\times10^5\ W \Big/ \frac{5000}{3600}\frac{kg}{s}\times 4660\frac{J}{kg\cdot K} = 147°C.$$

From Fig. 11.11, determine F from values of P and R, where P = (120 – 35)°C/(300 – 35)°C = 0.32, R = (300 – 147)°C/(120-35)°C = 1.8, and F ≈ 0.97. The log-mean temperature difference based upon a CF arrangement follows from Eq. (3); find

$$\Delta T_{\ell m} = \left[(300-120)-(147-35)\right]K \Big/ \ell n\frac{(300-120)}{(147-35)} = 143.3K. \qquad <$$

$$A_s = 9.905\times10^5 W/1500\,W/m^2\cdot K\times0.97\times143.3K = 4.75m^2 \qquad <$$

COMMENTS: (1) Check $\bar{T}_h \approx 500$ K used in property determination; $\bar{T}_h$ = (300 + 147)°C/2 = 497 K.

(2) Using the NTU-*e* method, determine first the capacity rate ratio, C_{min}/C_{max} = 0.56. Then

$$e \equiv \frac{q}{q_{max}} = \frac{C_{max}\left(T_{c,o}-T_{c,i}\right)}{C_{min}\left(T_{h,i}-T_{c,i}\right)} = \frac{1}{0.56}\times\frac{(120-35)°C}{(300-35)°C} = 0.57.$$

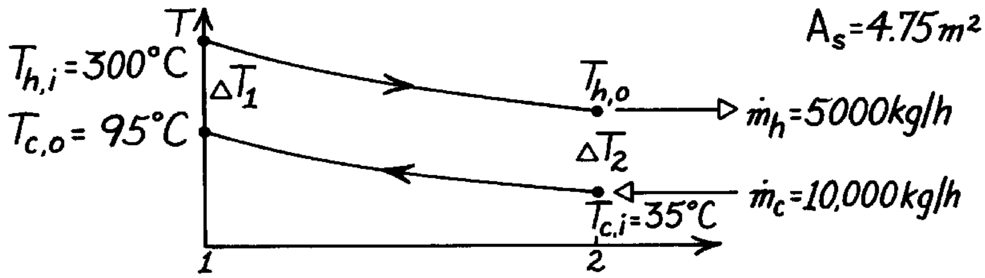
From Fig. 11.17, find that NTU = $AU/C_{min} \approx 1.1$ giving $A_s = 4.7\ m^2$.

PROBLEM 11.15

KNOWN: The shell and tube Hxer (two shells, four tube passes) of Problem 11.14, known to have an area $4.75 m^2$, provides 95°C water at the cold outlet (rather than 120°C) after several years of operation. Flow rates and inlet temperatures of the fluids remain the same.

FIND: The fouling factor, R_f.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible kinetic and potential energy changes, (3) Constant properties, (4) Thermal resistance for the clean condition is $R''_t = (1500 \text{ W/m}^2\cdot\text{K})^{-1}$.

PROPERTIES: *Table A-6*, Water ($\bar{T}_c \approx 338$ K): $c_p = 4187$ J/kg·K; *Table A-6*, Water (Assume $T_{h,o} \approx 190°C$, $\bar{T}_h \approx 520$ K): $c_p = 4840$ J/kg·K.

ANALYSIS: The overall heat transfer coefficient can be expresses as

$$U = 1/\left(R''_t + R''_f\right) \qquad \text{or} \qquad R''_f = 1/U - R''_t \tag{1}$$

where R''_t is the thermal resistance for the clean condition and R''_f, the fouling factor, represents the additional resistance due to fouling of the surface. The rate equation, Eq. 11.14 with Eq. 11.18, has the form,

$$U = q / A_s F \Delta T_{\ell m,CF} \qquad \Delta T_{\ell m,CF} = \left(\Delta T_1 - \Delta T_2\right)/\ell n\left(\Delta T_1 / \Delta T_2\right). \tag{2}$$

From energy balances on the cold and hot fluids, find

$$q = \dot{m}_c\, c_{p,c}\left(T_{c,o} - T_{c,i}\right) = (10{,}000/3600 \text{ kg/s})4187 \text{ J/kg}\cdot\text{K}(95-35)\text{K} = 6.978\times10^5 \text{ W}$$

$$T_{h,o} = T_{h,i} - q/\dot{m}_h\, c_{p,h} = 300°\text{C} - 6.978\times10^5 \text{W}/\left(5000/3600 \text{kg/s}\times 4840 \text{J/kg}\cdot\text{K}\right) = 196.2°\text{C}.$$

The factor, F, follows from values of P and R as given by Fig. 11.11 with

$$P = (95-35)/(300-35) = 0.23 \qquad R = (300-196)/(120-35) = 1.22$$

giving F ≈ 1. Based upon CF arrangement,

$$\Delta T_{\ell m,CF} = \left[(300-95)-(196-35)\right]°\text{C}/\ell n\left[(300-95)/(196.2-35)\right] = 182\text{K}.$$

Using Eq. (2), find now the overall heat transfer coefficient as

$$U = 6.978\times10^5 \text{W}/4.75\text{m}^2\times1\times182\text{K} = 806 \text{ W/m}^2\cdot\text{K}.$$

From Eq. (1), the fouling factor is

$$R''_f = \frac{1}{806 \text{ W/m}^2\cdot\text{K}} - \frac{1}{1500 \text{ W/m}^2\cdot\text{K}} = 5.74\times10^{-4} \text{m}^2\cdot\text{K/W}.$$

<

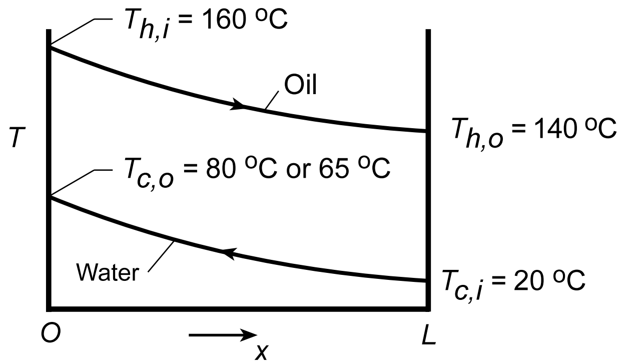
COMMENTS: Note that the effect of fouling is to nearly double ($U_{clean}/U_{fouled} = 1500/806 \approx 1.9$) the resistance to heat transfer. Note also the assumption for Th,o used for property evaluation is satisfactory.

PROBLEM 11.16

KNOWN: Inner tube diameter (D = 0.02 m) and fluid inlet and outlet temperatures corresponding to design conditions for a concentric tube heat exchanger. Overall heat transfer coefficient (U = 500 $W/m^2 \cdot K$) and desired heat rate (q = 3000 W). Cold fluid outlet temperature after three years of operation.

FIND: (a) Required heat exchanger length, (b) Heat rate, hot fluid outlet temperature, overall heat transfer coefficient, and fouling factor after three years.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat loss to the surroundings and kinetic and potential energy changes, (2) Negligible tube wall conduction resistance, (3) Constant properties.

ANALYSIS: (a) The tube length needed to achieve the prescribed conditions may be obtained from Eqs. 11.14 and 11.15 where $\Delta T_1 = T_{h,i} - T_{c,o} = 80°C$ and $\Delta T_2 = T_{h,o} - T_{c,i} = 120°C$. Hence, $\Delta T_{1m} = (120 - 80)°C/\ln(120/80) = 98.7°C$ and

$$L = \frac{q}{(\pi D) U \Delta T_{lm}} = \frac{3000\,W}{(\pi \times 0.02\,m) 500\,W/m^2 \cdot K \times 98.7°C} = 0.968\,m$$ <

(b) With $q = C_c(T_{c,o} - T_{c,i})$, the following ratio may be formed in terms of the design and 3 year conditions.

$$\frac{q}{q_3} = \frac{C_c\left(T_{c,o} - T_{c,i}\right)}{C_c\left(T_{c,o} - T_{c,i}\right)_3} = \frac{60°C}{45°C} = 1.333$$

Hence,

$$q_3 = q/1.33 = 3000\,W/1.333 = 2250\,W$$ <

Having determined the ratio of heat rates, it follows that

$$\frac{q}{q_3} = \frac{C_h\left(T_{h,i} - T_{h,o}\right)}{C_h\left(T_{h,i} - T_{h,o}\right)_3} = \frac{20°C}{160°C - T_{h,o(3)}} = 1.333$$

Hence,

$$T_{h,o(3)} = 160°C - 20°C/1.333 = 145°C$$ <

With $\Delta T_{lm,3} = (125 - 95)/\ln(125/95) = 109.3°C$,

$$U_3 = \frac{q_3}{(\pi DL)\Delta T_{lm,3}} = \frac{2250\,W}{\pi(0.02\,m)0.968\,m\left(109.3°C\right)} = 338\,W/m^2 \cdot K$$ <

Continued...

With $U = \left[\left(1/h_i \right) + \left(1/h_o \right) \right]^{-1}$ and $U_3 = \left[\left(1/h_i \right) + \left(1/h_o \right) + R''_{f,c} \right]^{-1}$,

$$R''_{f,c} = \frac{1}{U_3} - \frac{1}{U} = \left(\frac{1}{338} - \frac{1}{500} \right) m^2 \cdot K/W = 9.59 \times 10^{-4} \, m^2 \cdot K/W \qquad <$$

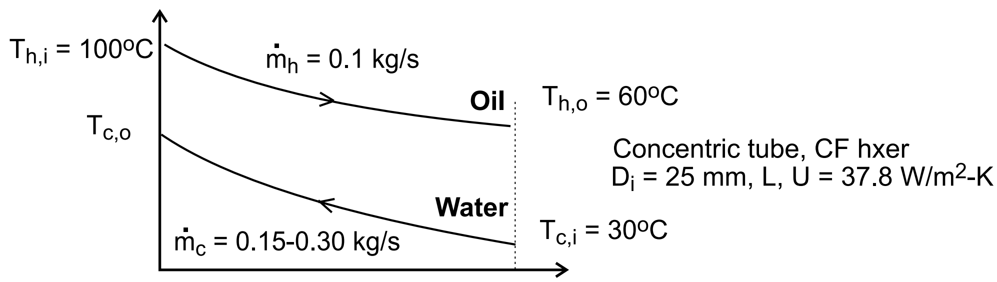
COMMENTS: Over time fouling will always contribute to a degradation of heat exchanger performance. In practice it is desirable to remove fluid contaminants and to implement a regular maintenance (cleaning) procedure.

PROBLEM 11.17

KNOWN: Counterflow, concentric tube heat exchanger of Example 11.1; maintaining the outlet oil temperature of 60°C, but with variable rate of cooling water, all other conditions remaining the same.

FIND: (a) Calculate and plot the required exchanger tube length L and water outlet temperature $T_{c,o}$ for the cooling water flow rate in the range 0.15 to 0.3 kg/s, and (b) Calculate U as a function of the water flow rate assuming the water properties are independent of temperature; justify using a constant value of U for the part (a) calculations.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible losses to the surroundings and kinetic and potential energy changes, (3) Overall heat transfer coefficient independent of water flow rate for this range, and (4) Constant properties.

PROPERTIES: *Table A-6*, Water $\left(\overline{T}_c = 35^\circ C = 308\ K\right)$: $c_p = 4178\ J/kg\cdot K$, $\mu = 725\times10^{-6}$ $N\cdot s/m^2$, $k = 0.625\ W/m\cdot K$, $Pr = 4.85$, *Table A-4*, Unused engine oil $\left(\overline{T}_h = 353\ K\right)$: $c_p = 2131\ J/kg\cdot K$.

ANALYSIS: (a) The NTU-ε method will be used to calculate the tube length L and water outlet temperature $T_{c,o}$ using this system of equations in the *IHT* workspace:

NTU relation, CF hxer, Eq. 11.30b

$$NTU = \frac{1}{C_r - 1}\ell n\frac{(\varepsilon - 1)}{(\varepsilon C_r - 1)} \qquad C_r = C_{max}/C_{min} \qquad (1, 2)$$

$$NTU = U\cdot A/C_{min} \qquad (3)$$

$$A = \pi D_i \cdot L \qquad (4)$$

Capacity rates, find minimum fluid

$$C_h = \dot{m}_h c_h = 0.1\ kg/s \times 2131\ J/kg\cdot K = 213.1\ W/K$$

$$C_c = \dot{m}_c c_c = (0.15\text{ to }0.30) kg/s \times 4178\ J/kg\cdot K = 626.7 - 1253\ W/K \qquad (5)$$

$$C_{min} = C_h \qquad (6)$$

Effectiveness and maximum heat rate, Eqs. 11.19 and 11.20

$$\varepsilon = q/q_{max} \qquad (7)$$

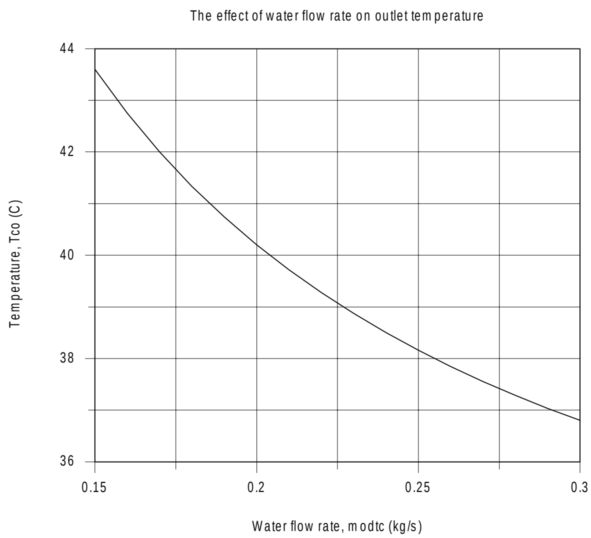
$$q_{max} = C_{min}\left(T_{h,i} - T_{c,i}\right) = C_c\left(T_{h,i} - T_{c,i}\right) \qquad (8)$$

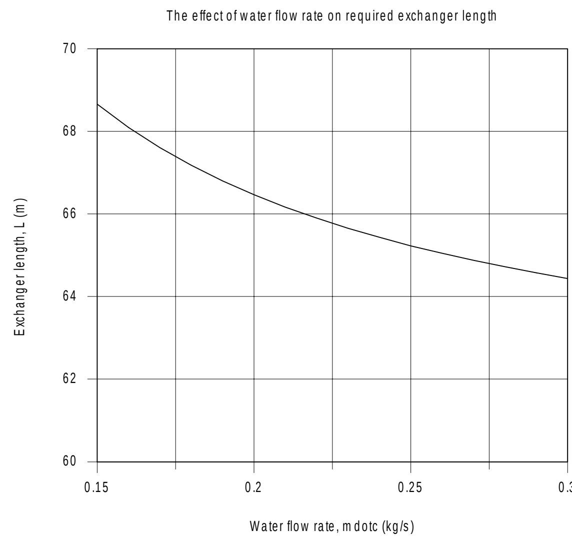
Continued …..

PROBLEM 11.17 (Cont.)

$$q = C_h\left(T_{h,i} - T_{h,o}\right) \tag{9}$$

With the foregoing equations and the parameters specified in the schematic, the results are plotted in the graphs below.





(b) The overall coefficient can be written in terms of the inner (cold) and outer (hot) side convection coefficients,

$$U = 1/\left(1/h_i + 1/h_o\right) \tag{10}$$

From Example 11.1, $h_o = 38.4\ W/m^2\cdot K$, and h_i will vary with the flow rate from Eq. 8.60 as

$$h_i = h_{i,b}\left(\dot{m}_i / \dot{m}_{i,b}\right)^{0.8} \tag{11}$$

where the subscript *b* denotes the base case when $\dot{m}_i = 0.2\ kg/s$. From these equations, the results are tabulated.

$\dot{m}_c (kg/s)$	$h_i\ (W/m^2\cdot K)$	$h_o\ (W/m^2\cdot K)$	$U\ (W/m^2\cdot K)$
015	1787	38.4	37.6
0.20	2250	38.4	37.8
0.25	2690	38.4	37.9
0.30	3112	38.4	37.9

Note that while h_i varies nearly 50%, there is a negligible effect on the value of U.

COMMENTS: Note from the graphical results, that by doubling the flow rate (from 0.15 to 0.30 kg/s), the required length of the exchanger can be decreased by approximately 6%. Increasing the flow rate is not a good strategy for reducing the length of the exchanger. However, any increase in the hot-side (oil) convection coefficient would provide a proportional decrease in the length.

PROBLEM 11.18

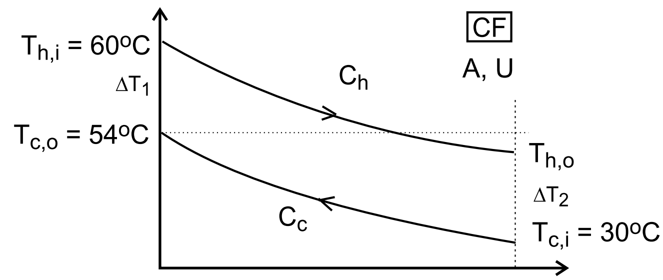
KNOWN: Concentric tube heat exchanger with area of 50 m^2 with operating conditions as shown on the schematic.

FIND: (a) Outlet temperature of the hot fluid; (b) Whether the exchanger is operating in counterflow or parallel flow; or can't tell from information provided; (c) Overall heat transfer coefficient; (d) Effectiveness of the exchanger; and (e) Effectiveness of the exchanger if its length is made very long

SCHEMATIC:

Operating conditions
Concentric tube HXer, A = 50 m^2

	Hot fluid (h)	Cold fluid (c)
Capacity rate, kW/K	6	3
Inlet temperature, ^{o}C	60	30
Outlet temperature, ^{o}C	--	54



ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible kinetic and potential energy changes, and (3) Constant properties.

ANALYSIS: From overall energy balances on the hot and cold fluids, find the hot fluid outlet temperature

$$q = C_c\left(T_{c,o} - T_{c,i}\right) = C_h\left(T_{h,i} - T_{h,o}\right) \qquad (1)$$

$$3000\ \mathrm{W/K}\ (54-30)\mathrm{K} = 6000\left(60 - T_{h,o}\right) \qquad T_{h,o} = 48°\mathrm{C} \qquad <$$

(b) HXer must be operating in counterflow (CF) since $T_{h,o} < T_{c,o}$. See schematic for temperature distribution.

(c) From the rate equation with A = 50 m^2, with Eq. (1) for q,

$$q = C_c\left(T_{c,o} - T_{c,i}\right) = UA\Delta T_{\ell m} \qquad (2)$$

$$\Delta T_{\ell m} = \frac{\Delta T_1 - \Delta T_2}{\ell m\left(\Delta T_1/\Delta T_2\right)} = \frac{(60-54)\mathrm{K} - (48-30)\mathrm{K}}{\ell n\left(6/18\right)} = 10.9°\mathrm{C} \qquad (3)$$

$$3000\ \mathrm{W/K}(54-30)\mathrm{K} = U \times 50\ \mathrm{m}^2 \times 10.9\,\mathrm{K}$$

$$U = 132\ \mathrm{W/m^2 \cdot K} \qquad <$$

(d) The effectiveness, from Eq. 11.20, with the cold fluid as the minimum fluid, $C_c = C_{min}$,

$$\varepsilon = \frac{q}{q_{max}} = \frac{C_c\left(T_{c,o} - T_{c,i}\right)}{C_{min}\left(T_{h,i} - T_{c,i}\right)} = \frac{(54-30)\mathrm{K}}{(60-30)\mathrm{K}} = 0.8 \qquad <$$

(e) For a very long CF HXer, the outlet of the minimum fluid, $C_{min} = C_c$, will approach $T_{h,i}$. That is,

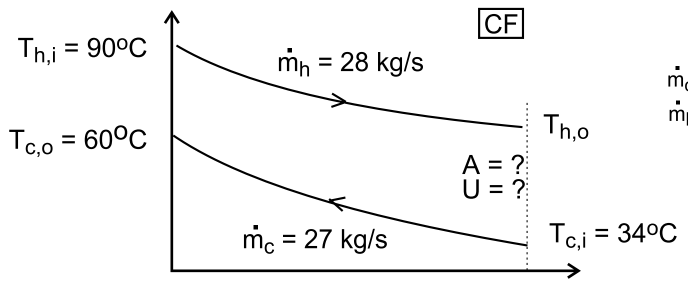
$$q \to C_{min}\left(T_{c,o} - T_{c,i}\right) \to q_{max} \qquad \varepsilon = 1 \qquad <$$

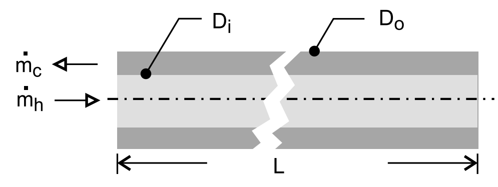
PROBLEM 11.19

KNOWN: Specifications for a water-to-water heat exchanger as shown in the schematic including the flow rate, and inlet and outlet temperatures.

FIND: (a) Design a heat exchanger to meet the specifications; that is, size the heat exchanger, and (b) Evaluate your design by identifying what features and configurations could be explored with your customer in order to develop more complete, detailed specifications.

SCHEMATIC:





Concentric N-tube, CF HXer

ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible kinetic and potential energy changes, (3) Tube walls have negligible thermal resistance, (4) Flow is fully developed, and (5) Constant properties.

ANALYSIS: (a) Referring to the schematic above and using the rate equation, we can determine the value of the UA product required to satisfy the design requirements. Sizing the heat exchanger involves determining the heat transfer area, A (tube diameter, length and number), and the associated overall convection coefficient, U, such that U × A satisfies the required UA product. Our approach has five steps: (1) *Calculate the UA product:* Select a configuration and calculate the required UA product; (2) *Estimate the area, A:* Assume a range for the overall coefficient, calculate the area and consider suitable tube diameter(s); (3) *Estimate the overall coefficient, U:* For selected tube diameter(s), use correlations to estimate hot- and cold-side convection coefficients and the overall coefficient; (4) *Evaluate first-pass design:* Check whether the A and U values (U × A) from Steps 2 and 3 satisfy the required UA product; if not, then (5) *Repeat the analysis:* Iterate on different values for area parameters until a satisfactory match is made, (U × A) = UA.

To perform the analysis, *IHT* models and tools will be used for the effectiveness-NTU method relations, internal flow convection correlations, and thermophysical properties. See the Comments section for details.

Step 1 Calculate the required UA. For the initial design, select a concentric tube, counterflow heat exchanger. Calculate UA using the following set of equations, Eqs. 11.30a,

$$\varepsilon = \frac{1-\exp\left[-\mathrm{NTU}\left(1-C_r\right)\right]}{1-C_r\exp\left[-\mathrm{NTU}\left(1-C_r\right)\right]} \qquad (1)$$

$$\mathrm{NTU} = UA/C_{min} \qquad C_r = C_{min}/C_{max} \qquad (2,3)$$

$$\varepsilon = q/q_{max} \qquad q_{max} = C_{min}\left(T_{h,i}-T_{c,i}\right) \qquad (4,5)$$

where $C = \dot{m}c_p$, and c_p is evaluated at the average mean temperature of the fluid, $\overline{T}_m = (T_{m,i} + T_{m,o})/2$. Substituting numerical values, find

$$\varepsilon = 0.464 \qquad \mathrm{NTU} = 0.8523 \quad q = 2.934\times10^{6}\,\mathrm{W} \qquad T_{h,o} = 65.0°\mathrm{C}$$

Continued …..

PROBLEM 11.19 (Cont.)

$$UA = 9.62\times10^4\,W/K \quad <$$

Step 2 Estimate the area, A. From Table 11.2, the typical range of U for water-to-water exchangers is 850 – 1700 W/m^2·K. With UA = 9.619 × 10^4 W/K, the range for A is 57 – 113 m^2, where

$$A = \pi D_i LN \qquad (6)$$

where L and N are the length and number of tubes, respectively. Consider these values of D_i with L = 10 m to describe the exchanger:

Case	D_i (mm)	L (m)	N	A (m^2)
1	25	10	73-146	57-113
2	50	10	36-72	57-113
3	75	10	24-48	57-113

<

Step 3 Estimate the overall coefficient, U. With the inner (hot) and outer (cold) fluids in the concentric tube arrangement, the overall coefficient is

$$1/U = 1/\bar{h}_i + 1/\bar{h}_o \qquad (7)$$

and the $\bar{h}$ are estimated using the Dittus-Boelter correlation assuming fully developed turbulent flow.

Coefficient, hot side, $\bar{h}_i$. For flow in the inner tube,

$$Re_{Di} = \frac{4\,\dot{m}_{h,i}}{\pi D_i \mu_h} \qquad \dot{m}_h = \dot{m}_{hi}\cdot N \qquad (8,9)$$

and the correlation, Eq. 8.60 with n = 0.3, is

$$\overline{Nu}_D = \frac{\bar{h}_i\,D_i}{k} = 0.037\,Re_{Di}^{4/5}\,Pr^{0.3} \qquad (10)$$

where properties are evaluated at the average mean temperature, $\bar{T}_h = \left(T_{hi} + T_{ho}\right)/2$.

Coefficient, cold side, $\bar{h}_o$. For flow in the annular space, $D_o - D_i$, the above relations apply where the characteristic dimension is the hydraulic diameter,

$$D_{h,o} = 4A_{c,o}/P_o \qquad A_{c,o} = \pi\left(D_o^2 - D_i^2\right)/4 \qquad P_o = \pi\left(D_o + D_i\right) \qquad (11\text{-}13)$$

To determine the outer diameter D_o, require that the inner and outer fluid flow areas are the same, that is,

$$A_{c,i} = A_{c,o} \qquad \pi D_i^2/4 = \pi\left(D_o^2 - D_i^2\right)/4 \qquad (14,15)$$

Summary of the convection coefficient calculations. The results of the analysis with L = 10 m are summarized below.

Continued …..

Case	D_i (mm)	N	A (m^2)	$\bar{h}_i$ ($W/m^2 \cdot K$)	$\bar{h}_o$ ($W/m^2 \cdot K$)	U ($W/m^2 \cdot K$)	$U \times A$ W/K
1a	25	73	57	4795	4877	2418	1.39×10^5
2a	50	36	57	2424	2465	1222	$6.91 \text{ x } 10^4$
3a	75	24	57	1616	1644	814	4.61×10^4

For all these cases, the Reynolds numbers are above 10,000 and turbulent flow occurs.

Step 4 Evaluate first-pass design. The required UA product value determined in step 1 is UA = 9.62 $\times 10^4$ W/K. By comparison with the results in the above table, note that the $U \times A$ values for cases 1a and 2a are, respectively, larger and smaller than that required. In this first-pass design trial we have identified the range of D_i and N (with L = 10 m) that could satisfy the exchanger specifications. A strategy can now be developed in *Step 5* to iterate the analysis on values for D_i and N, as well as with different L, to identify a combination that will meet specifications.

(b) What information could have been provided by the customer to simplify the analysis for design of the exchanger? Looking back at the analysis, recognize that we had to assume the exchanger configuration (type) and overall length. Will knowledge of the customer's installation provide any insight? While no consideration was given in our analysis to pumping power limitations, that would affect the flow velocities, and hence selection of tube diameter.

COMMENTS: The *IHT* workspace with the relations for step 3 analysis is shown below, including summary of key correlation parameters. The set of equations is quite stiff so that good initial guesses are required to make the initial solve.

```
/* Results, Step 3 - Di = 25 mm, N = 73, L = 10 m
A        Do        U         UA        Di       L        N
57.33    0.03536   2418      1.386E5   0.025    10       73
ReDi               ReDo      hDi       hDo
5.384E4            1.352E4   4795      4877      */

/* Results, Step 3 - Di = 50 mm, N = 36, L = 10 m
A        Do        U         UA        Di       L        N
56.55    0.07071   1222      6.912E4   0.05     10       36
ReDi               ReDo      hDi       hDo
5.459E4            1.371E4   2424      2465      */

/* Results, Step 3 - Di = 75 mm, N = 24, L = 10 m
A        Do        U         UA        Di       L        N
56.55    0.1061    814.8     4.608E4   0.075    10       24
ReDi               ReDo      hDi       hDo
5.459E4            1.371E4   1616      1644      */

// Input variables
//Di = 0.050
Di = 0.025
//Di = 0.075
//N = 36
N = 73
//N = 24
L = 10
mdoth = 28
Thi_C = 90
Tho_C = 65.0            // From Step 1
mdotc = 27
Tci_C = 34
Tco_C = 60
```

Continued …..

PROBLEM 11.19 (Cont.)

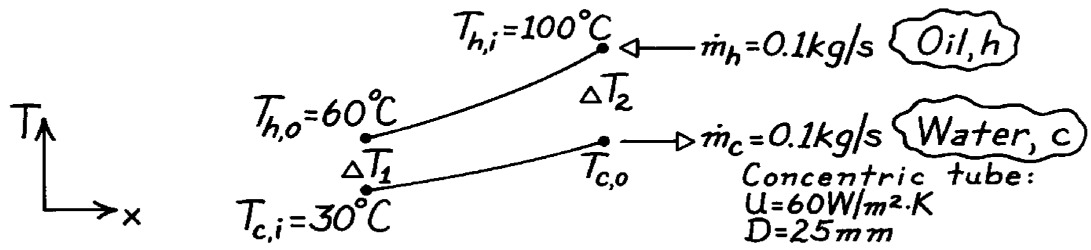
```
// Flow rate and number of tubes, inside parameters (hot)
mdoth = N * umi * rhoi * Aci
Aci = pi * Di^2 /4
1 / U = 1 / hDi + 1/ hDo
UA = U * A
A = pi * Di * L * N

// Flow rate, outside parameters (cold)
mdotc = rhoo * Aco * umo * N
Aco = Aci                          // Make cross-sectional areas of equal size
Aco = pi * (Do^2 - Di^2) / 4
Dho = 4 * Aco / P                  // hydraulic diameter
P = pi * ( Di + Do)                // wetted perimeter of the annular space

// Inside coefficient, hot fluid
NuDi = NuD_bar_IF_T_FD(ReDi,Pri,n)     // Eq 8.60
n = 0.3   // n = 0.4 or 0.3 for Tsi>Tmi or Tsi<Tmi
NuDi = hDi * Di / ki
ReDi = umi * Di / nui
/* Evaluate properties at the fluid average mean temperature, Tmi. */
Tmi = Tfluid_avg(Thi,Tho)
//Tmi = 310

// Outside coefficient, cold fluid
NuDo = NuD_bar_IF_T_FD(ReDo,Pro,nn)     // Eq 8.60
nn = 0.4   // n = 0.4 or 0.3 for Tsi>Tmi or Tsi<Tmi
NuDo= hDo * Dho / ko
ReDo= umo * Dho/ nuo
/* Evaluate properties at the fluid average mean temperature, Tmo. */
Tmo = Tfluid_avg(Tci,Tco)
//Tmo = 310

// Water property functions :T dependence, From Table A.6
// Units: T(K), p(bars);
x = 0                              // Quality (0=sat liquid or 1=sat vapor)
rhoi = rho_Tx("Water",Tmi,x)       // Density, kg/m^3
nui = nu_Tx("Water",Tmi,x)         // Kinematic viscosity, m^2/s
ki = k_Tx("Water",Tmi,x)           // Thermal conductivity, W/m·K
Pri = Pr_Tx("Water",Tmi,x)         // Prandtl number
rhoo = rho_Tx("Water",Tmo,x)       // Density, kg/m^3
nuo = nu_Tx("Water",Tmo,x)         // Kinematic viscosity, m^2/s
ko = k_Tx("Water",Tmo,x)           // Thermal conductivity, W/m·K
Pro = Pr_Tx("Water",Tmo,x)                  //Prandtl number

// Conversions
Thi_C = Thi - 273
Tho_C = Tho - 273
Tci_C = Tci - 273
Tco_C = Tco - 273
```

PROBLEM 11.20

KNOWN: Counterflow concentric tube heat exchanger.

FIND: (a) Total heat transfer rate and outlet temperature of the water and (b) Required length.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible kinetic and potential energy changes, (3) Negligible thermal resistance due to tube wall thickness.

PROPERTIES: (given):

	ρ (kg/m^3)	c_p (J/kg·K)	ν (m^2/s)	k (W/m·K)	Pr
Water	1000	4200	7×10^{-7}	0.64	4.7
Oil	800	1900	1×10^{-5}	0.134	140

ANALYSIS: (a) With the outlet temperature, $T_{c,o}$ = 60°C, from an overall energy balance on the hot (oil) fluid, find

$$q = \dot{m}_h c_h \left(T_{h,i} - T_{h,o}\right) = 0.1\,kg/s \times 1900\,J/kg \cdot K\left(100 - 60\right)°C = 7600\ W. \qquad <$$

From an energy balance on the cold (water) fluid, find

$$T_{c,o} = T_{c,i} + q/\dot{m}_c c_c = 30°C + 7600\,W/0.1\,kg/s \times 4200\,J/kg \cdot K = 48.1°C. \qquad <$$

(b) Using the LMTD method, the length of the CF heat exchanger follows from

$$q = UA\Delta T_{lm,CF} = U\left(p\,DL\right)\Delta T_{lm,CF} \qquad L = q/U\left(p\,D\right)\Delta T_{lm,CF}$$

where

$$\Delta T_{lm,CF} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\Delta T_1/\Delta T_2\right)} = \frac{\left(60-30\right)°C - \left(100-48.1\right)°C}{\ln\left(30/51.9\right)} = 40.0°C$$

$$L = 7600\,W/60\,W/m^2 \cdot K\left(p \times 0.025m\right) \times 40.0°C = 40.3m. \qquad <$$

COMMENTS: Using the ε-NTU method, find $C_{min} = C_h$ = 190 W/K and $C_{max} = C_c$ = 420 W/K. Hence

$$q_{max} = C_{min}\left(T_{h,i} - T_{c,i}\right) = 190\ W/K\left(100-30\right)K = 13{,}300\ W$$

and $\varepsilon = q/q_{max}$ = 0.571. With $C_r = C_{min}/C_{max}$ = 0.452 and using Eq. 11.30b,

$$NTU = \frac{UA}{C_{min}} = \frac{1}{C_r - 1}\ln\left(\frac{e-1}{eC_r - 1}\right) = \frac{1}{0.452-1}\ln\left(\frac{0.571-1}{0.571 \times 0.452 - 1}\right) = 1.00$$

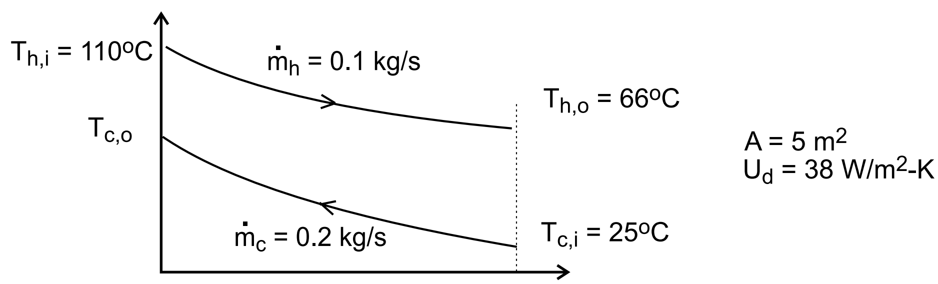
so that with $A = \pi DL$, find L = 40.3 m.

PROBLEM 11.21

KNOWN: Counterflow, concentric tube heat exchanger undergoing test after service for an extended period of time; surface area of 5 m^2 and design value for the overall heat transfer coefficient of $U_d = 38\ W/m^2 \cdot K$.

FIND: Fouling factor, if any, based upon the test results of engine oil flowing at 0.1 kg/s cooled from 110°C to 66°C by water supplied at 25°C and a flow rate of 0.2 kg/s.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible losses to the surroundings and kinetic and potential energy changes, (3) Constant properties.

PROPERTIES: *Table A-5*, Engine oil ($\overline{T}_h$ = 361 K): c = 2166 J/kg·K; *Table A-6*, Water $\left(\overline{T}_c = 304\ K, \text{ assuming } T_{c,o} = 36^\circ C\right)$: c = 4178 J/kg·K.

ANALYSIS: For the CF conditions shown in the Schematic, find the heat rate, q, from an energy balance on the hot fluid (oil); the cold fluid outlet temperature, $T_{c,o}$, from an energy balance on the cold fluid (water); the overall coefficient U from the rate equation; and a fouling factor, R, by comparison with the design value, U_d.

Energy balance on hot fluid

$$q = \dot{m}_h c_h \left(T_{h,i} - T_{h,o}\right) = 0.1\ kg/s \times 2166\ J/kg \cdot K\ (110 - 66)K = 9530\ W$$

Energy balance on the cold fluid

$$q = \dot{m}_c c_c \left(T_{c,o} - T_{c,i}\right), \qquad \text{find} \quad T_{c,o} = 36.4^\circ C$$

Rate equation

$$q = UA\Delta T_{\ell n,CF}$$

$$\Delta T_{\ell n,CF} = \frac{\left(T_{h,i} - T_{c,o}\right) - \left(T_{h,o} - T_{c,i}\right)}{\ell n\left[\left(T_{h,i} - T_{c,o}\right)/\left(T_{h,o} - T_{c,i}\right)\right]} = \frac{(110 - 36.4)^\circ C - (66 - 25)^\circ C}{\ell n[73.6/41.0]} = 55.7^\circ C$$

$$9530\ W = U \times 5\ m^2 \times 55.7^\circ C$$

$$U = 34.2\ W/m^2 \cdot K$$

Overall resistance including fouling factor

$$U = 1/\left[1/U_d + R''_f\right]$$

$$34.2\ W/m^2 \cdot K = 1/\left[1/38\ W/m^2 \cdot K + R''_f\right]$$

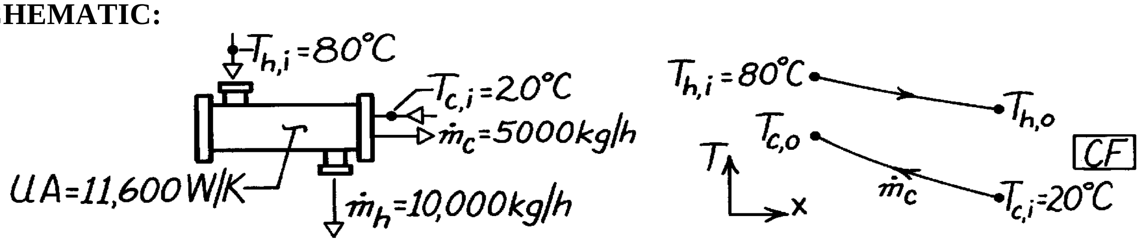
$$R''_f = 0.0029\ m^2 \cdot K/W$$ <

PROBLEM 11.22

KNOWN: Prescribed flow rates and inlet temperatures for hot and cold water; UA value for a shell-and-tube heat exchanger (one shell and two tube passes).

FIND: Outlet temperature of the hot water.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Constant properties, (3) Negligible kinetic and potential energy changes.

PROPERTIES: *Table A-6*, Water ($\overline{T}_c$ = (20 + 60)/2 = 40°C ≈ 310 K): $c_c = c_{p,f}$ = 4178 J/kg·K; Water ($\overline{T}_h$ = (80 +60)/2=70°C≈340 K): $c_h = c_{p,f}$ = 4188 J/kg·K.

ANALYSIS: From an energy balance on the hot fluid, the outlet temperature is

$$T_{h,o} = T_{h,i} - q/\dot{m}_h c_h. \tag{1}$$

The heat rate can be written in terms of the effectiveness and q_{max} as

$$q = \varepsilon q_{max} = \varepsilon C_{min}\left(T_{h,i} - T_{c,i}\right) \tag{2}$$

where for this HXer, the cold fluid is the minimum fluid giving

$$q_{max} = \left(\dot{m}c\right)_c \left(T_{h,i} - T_{c,i}\right)$$

$$q_{max} = \left(5000/3600\right) kg/s \times 4178\, J/kg\cdot K\left(80-20\right)°C = 348.2\ kW.$$

The effectiveness can be determined from Figure 11.16 with

$$NTU = \frac{UA}{C_{min}} = \frac{11{,}600\ W/K}{\left(5000/3600\right) kg/s \times 4178 J/kg\cdot K} = 2.0$$

giving, ε = 0.7 for $C_r = C_{min}/C_{max}$ = (5,000 × 4178)/(10,000 × 4188) = 0.499. Combining Eqs. (1) and (2), find

$$T_{h,o} = 80°C - \left(0.7 \times 348.2 \times 10^3 W\right) / \left(10{,}000/3600\right) kg/s \times 4188\, J/kg\cdot K$$

$$T_{h,o} = \left(80 - 21.0\right)°C = 59°C. \quad <$$

COMMENTS: (1) From an energy balance on the cold fluid, $q = (\dot{m}c)_c (T_{c,o} - T_{c,i})$, find that $T_{c,o}$ = 62°C. For evaluating properties at average mean temperatures, we should use $\overline{T}_h$ = (59 + 80)/2 = 70°C = 343 K and $\overline{T}_c$ = (20 + 62)/2 = 41°C = 314 K. Note from above that we have indeed assumed reasonable temperatures at which to obtain specific heats.

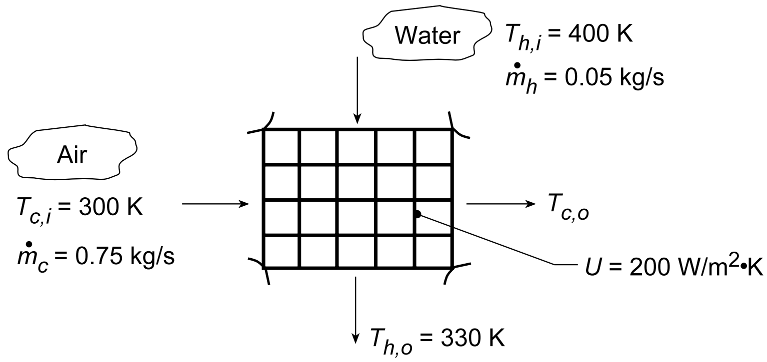
(2) We could have also used Eq. 11.31a to evaluate ε using C_r = 0.5 and NTU = 2 to obtain ε = 0.693.

PROBLEM 11.23

KNOWN: Flow rates and inlet temperatures for automobile radiator configured as a cross-flow heat exchanger with both fluids unmixed. Overall heat transfer coefficient.

FIND: (a) Area required to achieve hot fluid (water) outlet temperature, $T_{m,o}$ = 330 K, and (b) Outlet temperatures, $T_{h,o}$ and $T_{c,o}$, as a function of the overall coefficient for the range, $200 \le U \le 400$ W/m^2·K with the surface area A found in part (a) with all other heat transfer conditions remaining the same as for part (a).

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat loss to surroundings and kinetic and potential energy changes, (2) Constant properties.

PROPERTIES: *Table A.6*, Water ($\overline{T}_h$ = 365 K): $c_{p,h}$ = 4209 J/kg·K; *Table A.4*, Air $\left(\overline{T}_c \approx 310\,K\right)$: $c_{p,c}$ = 1007 J/kg·K.

ANALYSIS: (a) The required heat transfer rate is

$$q = \dot{m}_h c_{p,h}\left(T_{h,i} - T_{h,o}\right) = 0.05\,kg/s\left(4209\,J/kg\cdot K\right)70\,K = 14,732\,W\,.$$

Using the ε -NTU method,

$$C_{min} = C_h = 210.45\,W/K \qquad C_{max} = C_c = 755.25\,W/K\,.$$

Hence, C_{min}/C_{max} = 0.279 and

$$q_{max} = C_{min}\left(T_{h,i} - T_{c,i}\right) = 210.45\ W/K\,(100\ K) = 21,045\,W$$

$$\varepsilon = q/q_{max} = 14,732\,W/21,045\,W = 0.700\,.$$

Figure 11.18 yields NTU ≈ 1.5, hence,

$$A = NTU\left(C_{min}/U\right) = 1.5\times 210.45\,W/K\Big/\left(200\,W/m^2\cdot K\right) = 1.58\,m^2. \qquad <$$

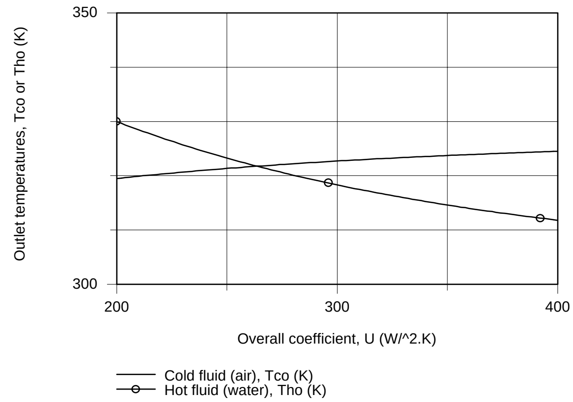
(b) Using the *IHT Heat Exchanger Tool* for *Cross-flow with both fluids unmixed* arrangement and the *Properties Tool* for *Air* and *Water*, a model was generated to solve part (a) evaluating the efficiency using Eq. 11.33. The following results were obtained:

$$A = 1.516\,m^2 \qquad NTU = 1.441 \qquad T_{c,o} = 319.5\,K$$

Using the model but assigning A = 1.516 m^2, the outlet temperature $T_{h,o}$ and $T_{c,o}$ were calculated as a function of U and the results plotted below.

Continued...

With a higher U, the outlet temperature of the hot fluid (water) decreases. A benefit is enhanced heat removal from the engine block and a cooler operating temperature. If it is desired to cool the engine with water at 330 K, the heat exchanger surface area and, hence its volume in the engine component could be reduced.



COMMENTS: (1) For the results of part (a), the air outlet temperature is

$$T_{c,o} = T_{c,i} + q/C_c \;= 300\,\text{K} + (14,732\,\text{W}/755.25\,\text{W/K}) = 319.5\,\text{K}\,.$$

(2) For the conditions of part (a), using the LMTD approach, ΔT_{lm} = 51.2 K, R = 0.279 and P = 0.7. Hence, Fig. 11.12 yields F ≈ 0.95 and

$$A = q/FU\Delta T_{lm} = (14,732\text{W})\Big/\Big[0.95\big(200\,\text{W/m}^2\cdot\text{K}\big)51.2\text{K}\Big] = 1.51\text{m}^2\,.$$

(3) The IHT workspace with the model to generate the above plot is shown below. Note that it is necessary to enter the overall energy balances on the fluids from the keyboard.

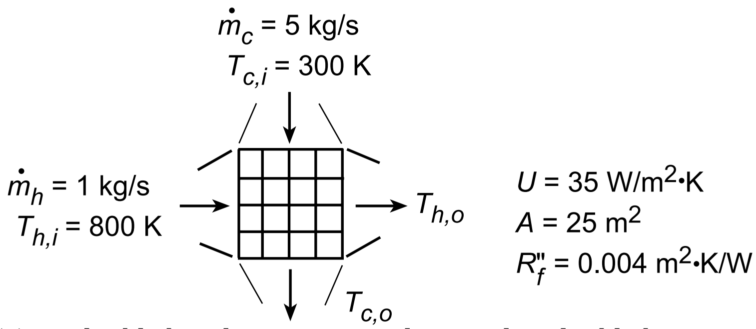
```
// Heat Exchanger Tool - Cross-flow with both fluids unmixed:
// For the cross-flow, single-pass heat exchanger with both fluids unmixed,
eps = 1 - exp((1 / Cr) * (NTU^0.22) * (exp(-Cr * NTU^0.78) - 1))      // Eq 11.33
// where the heat-capacity ratio is
Cr = Cmin / Cmax
// and the number of transfer units, NTU, is
NTU = U * A / Cmin                        // Eq 11.25
// The effectiveness is defined as
eps = q / qmax
qmax = Cmin * (Thi - Tci)                 // Eq 11.20
// See Tables 11.3 and 11.4 and Fig 11.18
// Overall Energy Balances on Fluids:
q = mdoth * cph * (Thi - Tho)
q = mdotc * cpc * (Tco - Tci)
// Assigned Variables:
Cmin = Ch                   // Capacity rate, minimum fluid, W/K
Ch = mdoth * cph            // Capacity rate, hot fluid, W/K
mdoth = 0.05                // Flow rate, hot fluid, kg/s
Thi = 400                   // Inlet temperature, hot fluid, K
Tho = 330                   // Outlet temperature, hot fluid, K; specified for part (a)
Cmax = Cc                   // Capacity rate, maximum fluid, W/K
Cc = mdotc * cpc            // Capacity rate, cold fluid, W/K
mdotc = 0.75                // Flow rate, cold fluid, kg/s
Tci = 300                   // Inlet temperature, cold fluid, K
U = 200                     // Overall coefficient, W/m^2.K
// Properties Tool - Water (h)
// Water property functions :T dependence, From Table A.6
// Units: T(K), p(bars);
xh =  0                                   // Quality (0=sat liquid or 1=sat vapor)
rhoh = rho_Tx("Water",Tmh,xh)             // Density, kg/m^3
cph = cp_Tx("Water",Tmh,xh)               // Specific heat, J/kg·K
Tmh =  Tfluid_avg(Thi,Tho )
// Properties Tool - Air(c)
// Air property functions : From Table A.4
// Units: T(K); 1 atm pressure
rhoc = rho_T("Air",Tmc)                   // Density, kg/m^3
cpc = cp_T("Air",Tmc)                     // Specific heat, J/kg·K
Tmc = Tfluid_avg(Tci,Tco)
```

PROBLEM 11.24

KNOWN: Flowrates and inlet temperatures of a cross-flow heat exchanger with both fluids unmixed. Total surface area and overall heat transfer coefficient for clean surfaces. Fouling resistance associated with extended operation.

FIND: (a) Fluid outlet temperatures, (b) Effect of fouling, (c) Effect of UA on air outlet temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat loss to surroundings and negligible kinetic and potential energy changes, (2) Constant properties, (3) Negligible tube wall resistance.

PROPERTIES: Air and gas (given): c_p = 1040 J/kg·K.

ANALYSIS: (a) With $C_{min} = C_h$ = 1 kg/s × 1040 J/kg·K = 1040 W/K and $C_{max} = C_c$ = 5 kg/s × 1040 J/kg·K = 5200 W/K, C_{min}/C_{max} = 0.2. Hence, NTU = UA/C_{min} = 35 W/m^2·K(25 m^2)/1040 W/K = 0.841 and Fig. 11.18 yields ε ≈ 0.57. With $C_{min}(T_{h,i} - T_{c,i})$ = 1040 W/K(500 K) = 520,000 W = q_{max}, Eqs. (11.21) and (11.22) yield

$$T_{h,o} = T_{h,i} - \varepsilon q_{max}/C_h = 800\,\text{K} - 0.56(520,000\,\text{W})/1040\,\text{W/K} = 520\,\text{K}$$ <

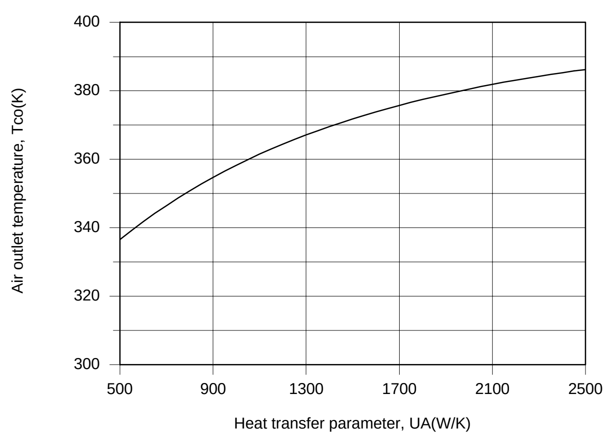
$$T_{c,o} = T_{c,i} + \varepsilon q_{max}/C_c = 300\,\text{K} + 0.56(520,000\,\text{W})/5200\,\text{W/K} = 356\,\text{K}$$ <

(b) With fouling, the overall heat transfer coefficient is reduced to

$$U_f = \left(U^{-1} + R''_f\right)^{-1} = \left[(0.029 + 0.004)\,\text{m}^2 \cdot \text{K/W}\right]^{-1} = 30.3\,\text{W/m}^2 \cdot \text{K}$$

This 13.4% reduction in performance is large enough to justify cleaning of the tubes. <

(c) Using the *Heat Exchangers* option from the IHT Toolpad to explore the effect of UA, we obtain the following result.



The heat rate, and hence the air outlet temperature, increases with increasing UA, with $T_{c,o}$ approaching a maximum outlet temperature of 400 K as UA → ∞ and ε → 1.

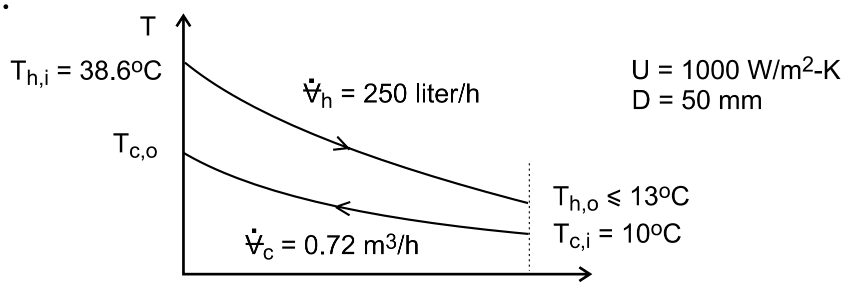
COMMENTS: Note that, for conditions of part (a), Eq. 11.33 yields a value of ε = 0.538, which reveals the level of approximation associated with reading ε from Fig. 11.18.

PROBLEM 11.25

KNOWN: Cooling milk from a dairy operation to a safe-to-store temperature, $T_{h,o} \le 13°C$, using ground water in a counterflow concentric tube heat exchanger with a 50-mm diameter inner pipe and overall heat transfer coefficient of 1000 $W/m^2 \cdot K$.

FIND: (a) The UA product required for the chilling process and the length L of the exchanger, (b) The outlet temperature of the ground water, and (c) the milk outlet temperatures for the cases when the water flow rate is halved and doubled, using the UA product found in part (a)

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible heat loss to surroundings and kinetic and potential energy changes, and (3) Constant properties.

PROPERTIES: *Table A-6*, Water $\left(\overline{T}_c = 287 \text{ K, assume } T_{c,o} = 18°C\right)$: $\rho = 1000 \text{ kg/m}^3$,

$c_p = 4187 \text{ J/kg}\cdot\text{K}$; Milk (given): $\rho = 1030 \text{ kg/m}^3$, $c_p = 3860 \text{ J/kg}\cdot\text{K}$.

ANALYSIS: (a) Using the effectiveness-NTU method, determine the capacity rates and the minimum fluid.

Hot fluid, milk:

$$\dot{m}_h = \rho_h \dot{\forall}_h = 1030 \text{ kg/m}^3 \times 250 \text{ liter/h} \times 10^{-3} \text{m}^3/\text{liter} \times 1 \text{ h}/3600 \text{ s} = 0.0715 \text{ kg/s}$$

$$C_h = \dot{m}_h c_h = 0.0715 \text{ kg/s} \times 3860 \text{ J/kg}\cdot\text{K} = 276 \text{ W/K}$$

Cold fluid, water:

$$C_c = \dot{m}_c c_c = 1000 \text{ kg/m}^3 \times \left(0.72/3600 \text{ m}^3/\text{s}\right) \times 4187 \text{ J/kg}\cdot\text{K} = 837 \text{ W/K}$$

It follows that $C_{min} = C_h$. The effectiveness of the exchanger from Eqs. 11.19 and 11.21 is

$$\varepsilon = \frac{q}{q_{max}} = \frac{C_h\left(T_{h,i} - T_{h,o}\right)}{C_{min}\left(T_{h,i} - T_{c,i}\right)} = \frac{(38.6 - 13)\text{K}}{(38.6 - 10)\text{K}} = 0.895 \quad (1)$$

The NTU can be calculated from Eq. 10.30b, where $C_r = C_{min}/C_{max} = 0.330$,

$$\text{NTU} = \frac{1}{C_r - 1} \ell n\left(\frac{\varepsilon - 1}{\varepsilon C_r - 1}\right) \quad (2)$$

$$\text{NTU} = \frac{1}{0.330 - 1} \ell n\left(\frac{0.895 - 1}{0.895 \times 0.330 - 1}\right) = 2.842$$

Continued …..

From Eq. 11.25, find UA

$$[UA] = NTU \cdot C_{min} = 2.842 \times 276\ W/K = 785\ W/K$$ <

and the exchanger tube length with A = π DL is

$$L = [UA]/\pi DU\ =\ 785\ W/K/\pi\, 0.050\ m \times 1000\ W/m^2 \cdot K = 5.0\ m$$ <

(b) The water outlet temperature, $T_{c,o}$, can be calculated from the heat rates,

$$C_h \left(T_{h,i} - T_{h,o}\right) = C_c \left(T_{c,o} - T_{c,i}\right) \tag{3}$$

$$276\ W/K\,(38.6 - 13)K = 837\ W/K\,\left(T_{c,o} - 10\right)K$$

$$T_{c,o} = 18.4^\circ C$$ <

(c) Using the foregoing Eqs. (1 - 3) in the *IHT* workspace, the hot fluid (milk) outlet temperatures are evaluated with UA = 785 W/K for different water flow rates. The results, including the hot fluid outlet temperatures, are compared to the base case, part (a).

Case	*C_c (W/K)*	*$T_{c,o}$ (°C)*	*$T_{h,o}$ (°C)*
1, halved flow rate	419	14.9	25.6
Base, part (a)	837	13	18.4
2, doubled flow rate	1675	12.3	14.3

COMMENTS: (1) From the results table in part (c), note that if the water flow rate is halved, the milk will not be properly chilled, since $T_{c,o}$ = 14.9°C > 13°C. Doubling the water flow rate reduces the outlet milk temperature by less than 1°C.

(2) From the results table, note that the water outlet temperature changes are substantially larger than those of the milk with changes in the water flow rate. Why is this so? What operational advantage is achieved using the heat exchanger under the present conditions?

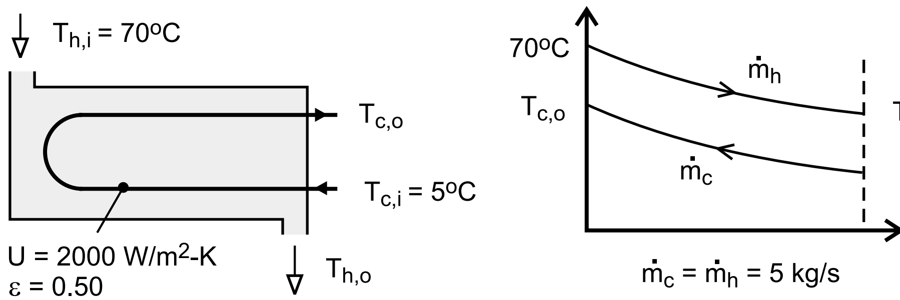
(3) The water thermophysical properties were evaluated at the average cold fluid temperature, $\overline{T}_c = \left(T_{c,i} + T_{c,o}\right)/2$. We assumed an outlet temperature of 18°C, which as the results show, was a good choice. Because the water properties are not highly temperature dependent, it was acceptable to use the same values for the calculations of part (c). You could, of course, use the properties function in *IHT* that will automatically use the appropriate values.

PROBLEM 11.26

KNOWN: Flow rate, inlet temperatures and overall heat transfer coefficient for a regenerator. Desired regenerator effectiveness. Cost of natural gas.

FIND: (a) Heat transfer area required for regenerator and corresponding heat recovery rate and outlet temperatures, (b) Annual energy and fuel cost savings.

SCHEMATIC:



ASSUMPTIONS: (a) Negligible heat loss to surroundings, (b) Constant properties.

PROPERTIES: *Table A-6*, water $\left(\overline{T}_m \approx 310\text{K}\right)$: $c_p = 4178\,\text{J/kg}\cdot\text{K}$.

ANALYSIS: (a) With $C_r = 1$ and $\varepsilon = 0.50$ for one shell and two tube passes, Eq. 11.31c yields E = 1.414. With C_{min} = 5 kg/s × 4178 J/kg·K = 20,890 W/K, Eq. 11.31b then yields

$$A = -\frac{C_{min}}{U}\frac{\ln\left[(E-1)/(E+1)\right]}{\left(1+C_r^2\right)^{1/2}} = -\frac{20{,}890\,\text{W/K}}{2000\,\text{W/m}^2\cdot\text{K}}\frac{\ln(0.171)}{1.414} = 13.05\,\text{m}^2 \qquad <$$

With ε = 0.50, the heat recovery rate is then

$$q = \varepsilon C_{min}\left(T_{h,i} - T_{c,i}\right) = 679{,}000\,\text{W} \qquad <$$

and the outlet temperatures are

$$T_{c,o} = T_{c,i} + \frac{q}{C_c} = 5°\text{C} + \frac{679{,}000\,\text{W}}{20{,}890\,\text{W/K}} = 37.5°\text{C} \qquad <$$

$$T_{h,o} = T_{h,i} - \frac{q}{C_h} = 70°\text{C} - \frac{679{,}000\,\text{W}}{20{,}890\,\text{W/K}} = 37.5°\text{C} \qquad <$$

(b) The amount of energy recovered for continuous operation over 365 days is

$$\Delta E = 679{,}000\,\text{W} \times 365\,\text{d/yr} \times 24\,\text{h/d} \times 3600\,\text{s/h} = 2.14\times10^{13}\,\text{J/yr}$$

The annual fuel savings S_A is then

$$S_A = \frac{\Delta E \times C_{ng}}{\eta} = \frac{2.14\times10^{7}\,\text{MJ/yr}\times\$0.0075/\text{MJ}}{0.9} = \$178{,}000/\text{yr} \qquad <$$

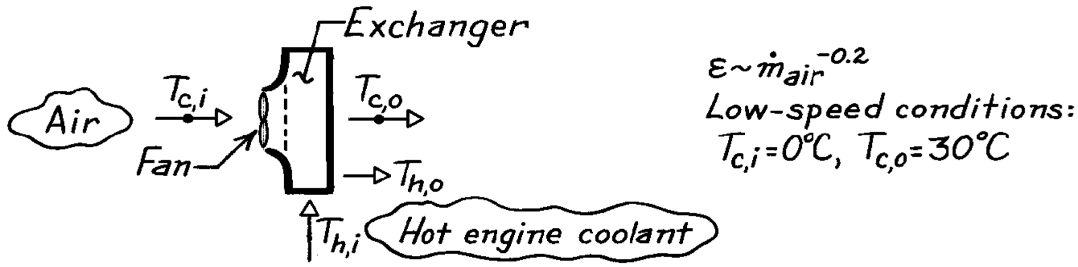
COMMENTS: (1) With $C_c = C_h$, the temperature changes are the same for the two fluids, (2) A larger effectiveness and hence a smaller value of A can be achieved with a counterflow exchanger (compare Figs. 11.15 and 11.16 for $C_r = 1$), (c) The savings are significant and well worth the cost of the heat exchanger. An additional benefit is that, with $T_{h,o}$ reduced from 70 to 37.5°C, less energy is consumed by the refrigeration system used to restore it to 5°C.

PROBLEM 11.27

KNOWN: Heat exchanger in car operating between warm radiator fluid and cooler outside air. Effectiveness of heater is $e \sim \dot{m}_{air}^{-0.2}$ since water flow rate is large compared to that of the air. For low-speed fan condition, heat warms outdoor air from 0°C to 30°C.

FIND: (a) Increase in heat added to car for high-speed fan condition causing $\dot{m}_{air}$ to be doubled while inlet temperatures remain the same, and (b) Air outlet temperature for medium-speed fan condition where air flow rate increases 50% and heat transfer increases 20%.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat losses from heat exchanger to surroundings, (2) $T_{h,i}$ and $T_{c,i}$ remain fixed for all fan-speed conditions, (3) Water flow rate is much larger than that of air.

ANALYSIS: (a) Assuming the flow rate of the water is much larger than that of air,

$$C_{min} = C_c = \dot{m}_{air}\, c_{p,c}.$$

Hence, the heat rate can be written as

$$q = e q_{max} = e C_{min}\left(T_{h,i} - T_{c,i}\right) = e \cdot \dot{m}_{air} c_{p,air}\left(T_{h,i} - T_{c,i}\right).$$

Taking the ratio of the heat rates for the high and low speed fan conditions, find

$$\frac{q_{hi}}{q_{lo}} = \frac{\left(e\dot{m}_{air}\right)_{hi}}{\left(e\dot{m}_{air}\right)_{lo}} = \frac{\left(\dot{m}_{air}^{0.8}\right)_{hi}}{\left(\dot{m}_{air}^{0.8}\right)_{lo}} = 2^{0.8} = 1.74 \qquad <$$

where we have used $e \sim \dot{m}_{air}^{-0.2}$ and recognized that for the high speed fan condition, the air flow rate is doubled. Hence the heat rate is increased by 74%.

(b) Considering the medium and low speed conditions, it was observed that,

$$\frac{q_{med}}{q_{lo}} = 1.2 \qquad\qquad \frac{\left(\dot{m}_{air}\right)_{med}}{\left(\dot{m}_{air}\right)_{lo}} = 1.5.$$

To find the outlet air temperature for the medium speed condition,

$$\frac{q_{med}}{q_{lo}} = \frac{\left[\dot{m}_{air} c_{p,c}\left(T_{c,o} - T_{c,i}\right)\right]_{med}}{\left[\dot{m}_{air} c_{p,c}\left(T_{c,o} - T_{c,i}\right)\right]_{lo}}$$

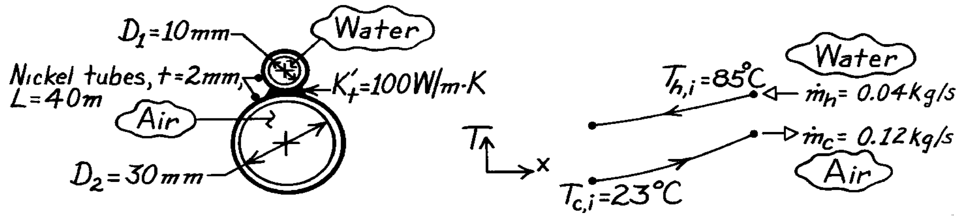
$$1.2 = \frac{1.5\ \dot{m}_{air} c_{p,c}\left(T_{c,o} - 0°C\right)}{\dot{m}_{air} c_{p,c}\left(30 - 0°C\right)} \qquad\qquad T_{c,o} = 24°C. \qquad <$$

PROBLEM 11.28

KNOWN: Counterflow heat exchanger formed by two brazed tubes with prescribed hot and cold fluid inlet temperatures and flow rates.

FIND: Outlet temperature of the air.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible loss/gain from tubes to surroundings, (2) Negligible changes in kinetic and potential energy, (3) Flow in tubes is fully developed since L/D_h = 40 m/0.030m = 1333.

PROPERTIES: *Table A-6*, Water ($\overline{T}_h$ = 335 K): $c_h = c_{p,h}$ = 4186 J/kg·K, $\mu = 453 \times 10^{-6}$ N·s/m^2, k = 0.656 W/m·K, Pr = 2.88; *Table A-4*, Air (300 K): $c_c = c_{p,c}$ = 1007 J/kg·K, $\mu = 184.6 \times 10^{-7}$ N·s/m^2, k = 0.0263 W/m·K, Pr = 0.707; *Table A-1*, Nickel ($\overline{T}$ = (23 + 85)°C/2 = 327 K): k = 88 W/m·K.

ANALYSIS: Using the NTU - ε method, from Eq. 11.30a,

$$e = \frac{1-\exp\left[-\mathrm{NTU}\left(1-C_r\right)\right]}{1-C_r\exp\left[-\mathrm{NTU}\left(1-C_r\right)\right]} \qquad \mathrm{NTU} = UA/C_{min} \qquad C_r = C_{min}/C_{max}. \qquad (1,2,3)$$

Estimate UA from a model of the tubes and flows, and determine the outlet temperature from the expression

$$e = C_c\left(T_{c,o} - T_{c,i}\right)/C_{min}\left(T_{h,i} - T_{c,i}\right). \qquad (4)$$

Water-side: $$\mathrm{Re}_D = \frac{4\dot{m}_h}{p\,Dm} = \frac{4\times 0.04\ \mathrm{kg/s}}{p\times 0.010\mathrm{m}\times 453\times 10^{-6}\,\mathrm{N\cdot s/m^2}} = 11{,}243.$$

The flow is turbulent and since fully developed, use the Dittus-Boelter correlation,

$$\overline{\mathrm{Nu}}_h = \overline{h}_h D/k = 0.023\mathrm{Re}_D^{0.8}\,\mathrm{Pr}^{0.3} = 0.023(11{,}243)^{0.8}(2.88)^{0.3} = 54.99$$

$$\overline{h}_h = 54.99\times 0.656\ \mathrm{W/m{\cdot}K}\,/\,0.01\mathrm{m} = 3{,}607\ \mathrm{W/m^2\cdot K}.$$

Air-side: $$\mathrm{Re}_D = \frac{4\dot{m}_c}{p\,Dm} = \frac{4\times 0.120\ \mathrm{kg/s}}{p\times 0.030\mathrm{m}\times 184.6\times 10^{-7}\ \mathrm{N\cdot s/m^2}} = 275{,}890.$$

The flow is turbulent and since fully developed, again use the correlation

$$\overline{\mathrm{Nu}}_c = \overline{h}_c D/K = 0.023\mathrm{Re}_D^{0.8}\,\mathrm{Pr}^{0.4} = 0.023(275{,}890)^{0.8}(0.707)^{0.4} = 450.9$$

$$\overline{h}_c = 450.9\times 0.0263\ \mathrm{W/m\cdot K}/0.030\mathrm{m} = 395.3\ \mathrm{W/m^2\cdot K}.$$

Overall coefficient: From Eq. 11.1, considering the temperature effectiveness of the tube walls and the thermal conductance across the brazed region,

Continued …..

PROBLEM 11.28 (Cont.)

$$\frac{1}{UA}=\frac{1}{(h_o hA)_h}+\frac{1}{K_t' L}+\frac{1}{(h_o hA)_c} \qquad (5)$$

where η_o needs to be evaluated for each of the tubes.

Water-side temperature effectiveness: $A_h = p D_h L = p\,(0.010m)\,40m = 1.257m^2$

$$h_{o,h} = h_{f,h} = \tanh(mL_h)/mL_h \qquad m = \left(\bar{h}_h P/kA\right)^{1/2} = (h_h/kt)^{1/2}$$

$$m = \left(3607\ W/m^2 \cdot K/88\ W/m\cdot K \times 0.002m\right)^{1/2} = 143.2m^{-1}$$

and with $L_h = 0.5\,\pi D_h$, $\eta_{o,h} = \tanh(143.2\ m^{-1} \times 0.5\,\pi \times 0.010m)/143.2\ m^{-1} \times 0.5\,\pi \times 0.010\ m = 0.435$.

Air-side temperature effectiveness: $A_c = \pi D_c L = \pi(0.030m)40m = 3.770\ m^2$

$$h_{o,c} = h_{f,c} = \tanh(mL_c)/mL_c \quad m = \left(395.3\ W/m^2\cdot K/88\ W/m\cdot K \times 0.002m\right)^{1/2} = 47.39m^{-1}$$

and with $L_c = 0.5\pi D_c$, $\eta_{o,c} = \tanh(47.39\ m^{-1} \times 0.5\,\pi \times 0.030m)/47.39\ m^{-1} \times 0.5\,\pi \times 0.030m = 0.438$.

Hence, the *overall heat transfer coefficient* using Eq. (5) is

$$\frac{1}{UA}=\frac{1}{0.435\times 3607\ W/m^2\cdot K\times 1.257m^2}+\frac{1}{100\ W/m\ K(40m)}+\frac{1}{0.438\times 395.3\ W/m^2\cdot K\times 3.770m^2}$$

$$UA = \left[5.070\times10^{-4} + 2.50\times10^{-4} + 1.533\times10^{-3}\right]^{-1} W/K = 437\ W/K.$$

Evaluating now the *heat exchanger effectiveness* from Eq. (1) with

$$\left.\begin{aligned} C_h &= \dot{m}_h c_h = 0.040kg/s \times 4186\ J/kg\cdot K = 167.4\ W/K \leftarrow C_{max} \\ C_c &= \dot{m}_c c_c = 0.120kg/s \times 1007\ J/kg\cdot K = 120.8\ W/K \leftarrow C_{min} \end{aligned}\right\} C_r = C_{min}/C_{max} = 0.722$$

$$NTU = \frac{UA}{C_{min}} = \frac{437\ W/K}{120.8\ W/K} = 3.62 \quad e = \frac{1-\exp\left[-3.62(1-0.722)\right]}{1-0.722\ \exp\left[-3.62(1-0.722)\right]} = 0.862$$

and finally from Eq. (4) with $C_{min} = C_c$,

$$0.862 = \frac{C_c\left(T_{c,o} - 23°C\right)}{C_c(85-23)°C} \qquad T_{c,o} = 76.4°C \qquad <$$

COMMENTS: (1) Using overall energy balances, the water outlet temperature is

$$T_{h,o} = T_{h,i} + (C_c/C_h)\left(T_{c,o} - T_{c,i}\right) = 85°C - 0.722(76.4-23)°C = 46.4°C.$$

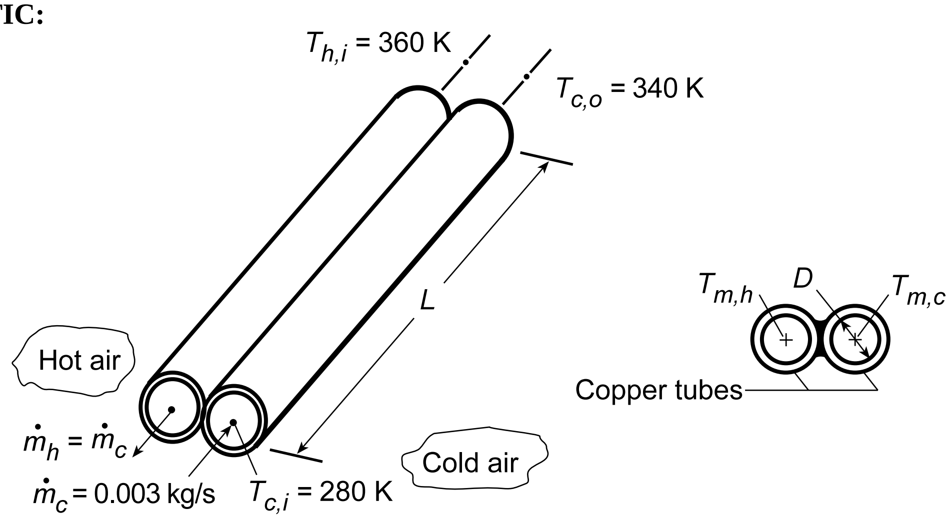
(2) To initially evaluate the properties, we assumed that $\bar{T}_h \approx 335$ K and $\bar{T}_c \approx 300$ K. From the calculated values of $T_{h,o}$ and $T_{c,o}$, more appropriate estimates of $\bar{T}_h$ and $\bar{T}_c$ are 338 K and 322 K, respectively. We conclude that proper thermophysical properties were used for water but that the estimates could be improved for air.

PROBLEM 11.29

KNOWN: Twin-tube counterflow heat exchanger with balanced flow rates, $\dot{m}$ = 0.003 kg/s. Cold airstream enters at 280 K and must be heated to 340 K. Maximum allowable pressure drop of cold airstream is 10 kPa.

FIND: (a) Tube diameter D and length L which satisfies the heat transfer and pressure drop requirements, and (b) Compute and plot the cold stream outlet temperature $T_{c,o}$, the heat rate q, and pressure drop Δp as a function of the balanced flow rate from 0.002 to 0.004 kg/s.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible heat loss to surroundings, (3) Average pressure of the airstreams is 1 atm, (4) Tube walls act as fins with 100% efficiency, (4) Fully developed flow.

PROPERTIES: *Table A.4*, Air ($\overline{T}_m$ = 310 K, 1 atm) : ρ = 1.128 kg/m^3, c_p = 1007 J/kg·K, $\mu = 18.93 \times 10^{-6}$ m^2 / s, k = 0.0270 W/m·K, Pr = 0.7056.

ANALYSIS: (a) The heat exchanger diameter D and length L can be specified through two analyses: (1) heat transfer based upon the effectiveness-NTU method to meet the cold air heating requirement and (2) pressure drop calculation to meet the requirement of 10 kPa. The *heat transfer analysis* begins by determining the effectiveness from Eq. 11.22, since $C_{min} = C_{max}$ and $C_r = 1$,

$$\varepsilon = \frac{q}{q_{max}} = \frac{C\left(T_{c,o} - T_{c,i}\right)}{C\left(T_{h,i} - T_{c,i}\right)} = \frac{(340-280)\text{K}}{(360-280)\text{K}} = 0.750 \quad (1)$$

From Table 11.4, Eq. 11.29b for $C_r = 1$,

$$\text{NTU} = \frac{\varepsilon}{1-\varepsilon} = \frac{0.750}{1-0.750} = 3 \quad (2)$$

where NTU, following its definition, Eq. 11.25, is

$$\text{NTU} = \frac{\overline{U}A}{C_{min}} \quad (3)$$

with

$$C_{min} = \dot{m}c_p = 0.003\,\text{kg/s} \times 1007\,\text{J/kg}\cdot\text{K} = 3.021\,\text{K/W} \quad (4)$$

Continued...

and $1/\overline{U}A$ represents the thermal resistance between the two fluids at $T_{m,h}$ and $T_{m,c}$ as illustrated in the above-right schematic. Since the tube walls are isothermal, it follows that

$$1/UA = 1/\overline{h}_c A + 1/\overline{h}_h A \tag{5}$$

and since the flow conditions are nearly identical $\overline{h}_c = \overline{h}_h$ so that

$$U = 0.5\overline{h} \tag{6}$$

where the heat transfer area is

$$A = \pi DL \tag{7}$$

Hence, Eq. (3) can now be expressed as

$$3 = \frac{0.5\overline{h}\left(\pi DL\right)}{3.021\,K/W}$$

$$\overline{h}DL = 5.7697 \tag{8}$$

Assuming an average mean temperature $\overline{T}_{m,c} = 310\,K$, characterize the flow with

$$Re_D = \frac{4\dot{m}}{\pi D\mu} = \frac{4\times 0.003\,kg/s}{\pi\times D\times 18.93\times 10^{-6}\,m^2/s} = \frac{201.78}{D} \tag{9}$$

and assuming the flow is both turbulent and fully developed using the Dittus-Boelter correlation, Eq. 8.57, with n = 0.4,

$$\overline{Nu}_D = \frac{\overline{h}D}{k} = 0.023\,Re_D^{0.8}\,Pr^{0.4}$$

$$\overline{h}D = 0.023\times 0.0270\,W/m\cdot K\left(201.78/D\right)^{0.8}\left(0.7056\right)^{0.4}$$

$$\overline{h}D^{1.8} = 0.0377 \tag{10}$$

The *pressure drop* for fully developed flow, Eq. 8.22a, is

$$\Delta p = f\,\frac{\rho u_m^2}{2D}\,L \tag{11}$$

where the mean velocity is $u_m = \dot{m}/(\rho\pi D^2/4)$ so that

$$\Delta p = f\,\frac{4\rho\left(\dot{m}/\rho\pi D^2\right)L}{2D} = \frac{2}{\pi^2}\,f\,\frac{m^2\rho^2 L}{D^5}$$

$$\Delta p = \frac{2}{\pi^2}\,f\,\frac{\left(0.003\,kg/s\right)^2\left(1.128\,kg/m^3\right)L}{D^5} = 2.3206\times 10^{-6}\,f\,L\,D^{-5} \tag{12}$$

Recall that the pressure drop requirement is $\Delta p = 10\ kPa = 10^4\ N/m^2$, so that Eq. (12) can be rewritten as

$$fLD^{-5} = 4.3092\times 10^{10} \tag{13}$$

Continued...

PROBLEM 11.29 (Cont.)

For the Reynolds number range, $3000 \le Re_D \le 5 \times 10^6$, Eq. 8.21 provides an estimate for the friction factor,

$$f = \left[\left(0.790\ell n\left(Re_D\right) - 1.64\right)\right]^{-2}$$

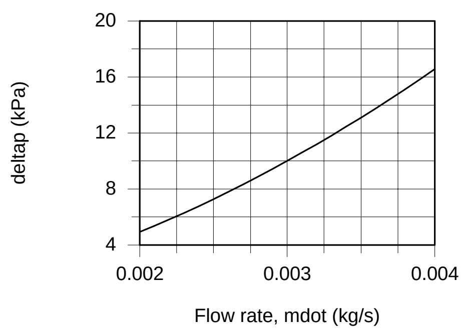
$$f = \left[\left(0.790\ell n\left(201.78/D\right) - 1.46\right)\right]^{-2} \tag{14}$$

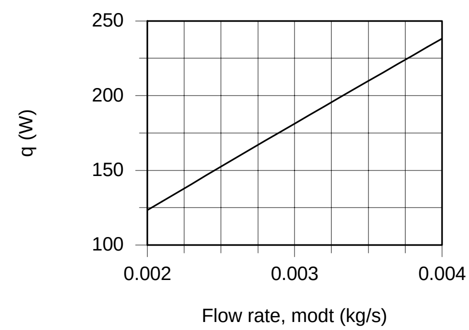
In the foregoing analysis, there are 4 unknowns (D, L, f, $\bar{h}$) and 4 equations (8, 10, 13, 14). Using the IHT workspace, find

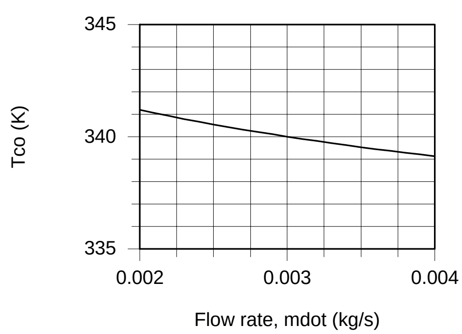
$$D = 8.96\,\text{mm} \quad L = 3.52\,\text{m} \quad f = 0.02538 \quad \bar{h} = 182.9\,\text{W/m}^2 \cdot \text{K}$$

For this configuration, Re_D = 22,520 so the flow is turbulent and since L/D = 3.52/0.00896 = 390 >> 10, the fully developed assumption is reasonable.

(b) The foregoing analysis entered into the IHT workspace was used to determine $T_{c,o}$, q and Δp as a function of the balanced flow rate, $\dot{m}$.







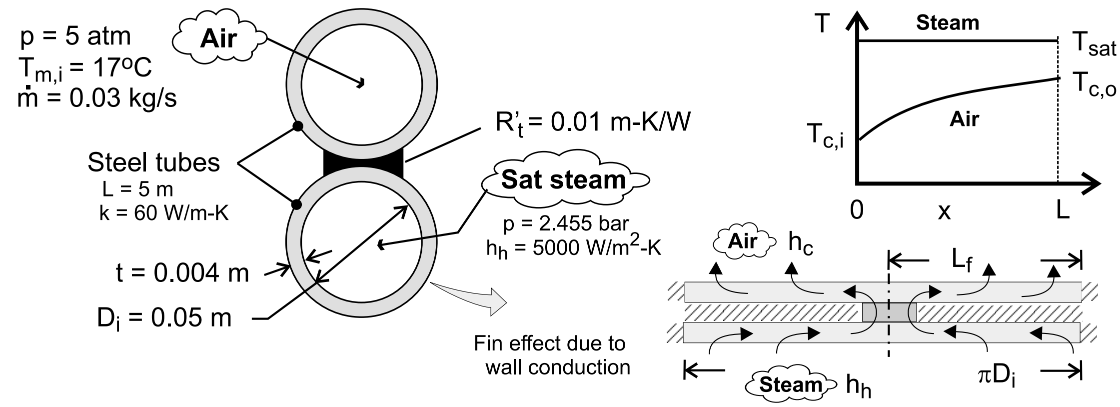
The outlet temperature of the cold air, $T_{c,o}$, is nearly insensitive to the flow rate. It follows that the heat rate, q, must be nearly proportional to the flow rate as can be seen in the q vs. $\dot{m}$ plot above. The pressure drop varies with the mean velocity squared.

PROBLEM 11.30

KNOWN: Dimensions and thermal conductivity of twin-tube, counterflow heat exchanger. Contact resistance between tubes. Air inlet conditions for one tube and pressure of saturated steam in other tube.

FIND: Air outlet temperature and condensation rate.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat exchange with surroundings, (2) Negligible kinetic and potential energy and flow work changes, (3) Fully developed air flow, (4) Negligible fouling, (5) Constant properties.

PROPERTIES: *Table A-4*, air $\left(\overline{T}_c \approx 325\,K,\ p = 5\,atm\right)$: c_p = 1008 J/kg·K, $\mu = 196.4 \times 10^{-7}$ N·s/m^2, k = 0.0281 W/m·K, Pr = 0.703. *Table A-6*, sat. steam (p = 2.455 bar): $T_{h,i} = T_{h,o}$ = 400 K, h_{fg} = 2183 kJ/kg.

ANALYSIS: With $C_{max} \to \infty$, $C_r = 0$ and Eqs. 11.22 and 11.36a yield

$$\varepsilon = \frac{T_{c,o} - T_{c,i}}{T_{h,i} - T_{c,i}} = 1 - \exp(-NTU) \tag{1}$$

From Eq. 11.1,

$$\frac{1}{UA} = \frac{1}{(\eta_o hA)_c} + \frac{R'_t}{L} + \frac{1}{(\eta_o hA)_h} \tag{2}$$

With $Re_D = 4\dot{m}/\pi D_i \mu = 0.12\ kg/s/\pi(0.05m)196.4\times10^{-7}\,N\cdot s/m^2 = 38{,}900$, the air flow is turbulent and the Dittus-Boelter correlation yields

$$h_c \approx h_{fD} = \left(\frac{k}{D_i}\right)0.023\ Re_D^{4/5}\ Pr^{0.4} = \left(\frac{0.0281\ W/m\cdot K}{0.05m}\right)0.023(38{,}900)^{4/5}(0.703)^{0.4} = 52.7\ W/m^2\cdot K$$

As shown on the inset, each tube wall may be modelled as two fins, each of length $L_f \approx \pi D_i/2$ = 0.0785m. The total surface area for heat transfer is $A_t = \pi D_i L = 0.785\ m^2 = A_c$, which is equivalent to the surface area of the fins. With $NA_f = A_t$ from Eq. 3.102, $\eta_o = \eta_f$. Because the outer surface of the tube is insulated, a wall thickness of 2t must be used in evaluating η_f. With $m = (2h/k \times 2t)^{1/2} = (h/kt)^{1/2} = [52.7\ W/m^2\cdot K/(60\ W/m\cdot K \times 0.004m)]^{1/2} = 14.8\ m^{-1}$, $L_c = L_f$ for an adiabatic tip, and mL_f = 1.163, Eq. 3.89 yields

$$\eta_f = \frac{\tanh\ mL_f}{mL_f} = \frac{0.821}{1.163} = 0.706 = \eta_{o,c}$$

Continued …..

Similarly, for the steam tube, $m = (h/kt)^{1/2} = [5{,}000\ W/m^2 \cdot K/(60\ W/m \cdot K \times 0.004 m)]^{1/2} = 144.3\ m^{-1}$ and $mL_f = 11.33$. Hence,

$$\eta_f = \frac{\tanh\ mL_f}{mL_f} = \frac{1.00}{11.33} = 0.088 = \eta_{o,h}$$

Substituting into Eq. (2),

$$UA = \left[\frac{1}{0.706 \times 52.7 \times 0.785} + \frac{0.01}{5} + \frac{1}{0.088 \times 5000 \times 0.785}\right]^{-1} \frac{W}{K} = 25.6 \frac{W}{K}$$

Hence, with $C_{min} = \left(\dot{m}\, c_p\right)_c = 0.03\ kg/s \times 1008\ J/kg \cdot K = 30.2\ W/K$, $NTU = UA/C_{min} = 0.847$ and $\varepsilon = 1 - \exp(-NTU) = 0.571$. From Eq. (1), the air outlet temperature is then

$$T_{c,o} = T_{c,i} + \varepsilon\left(T_{h,i} - T_{c,i}\right) = 17°C + 0.571(127 - 17)°C = 79.8°C \qquad <$$

The rate of heat transfer to the air is

$$q = \dot{m}\, c_p \left(T_{c,o} - T_{c,i}\right) = 0.03\, kg/s \times 1008\, J/kg \cdot K \times 62.8°C = 1900\, W$$

and the rate of condensation is

$$\dot{m}_{cond} = \frac{q}{h_{fg}} = \frac{1900\, W}{2.183 \times 10^6\, J/kg} = 8.70 \times 10^{-4}\, kg/s \qquad <$$

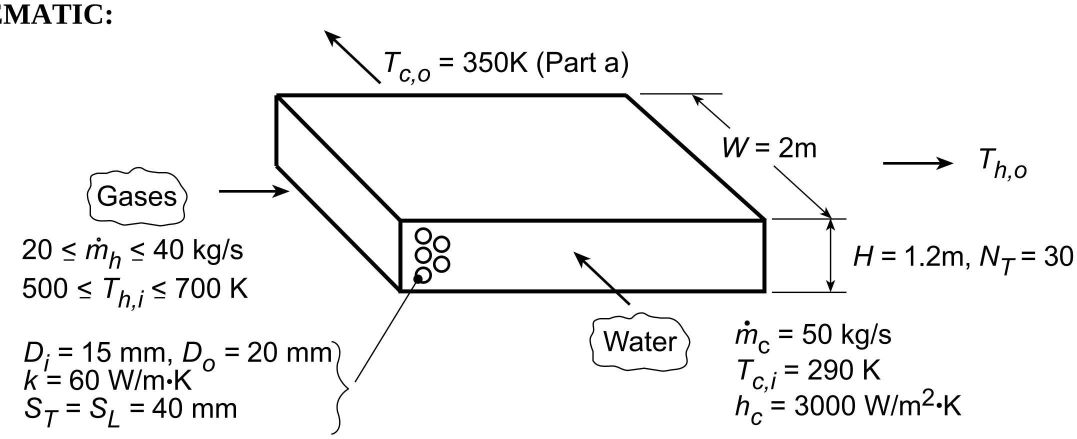
COMMENTS: (1) With $\overline{T}_c = 321.4\, K$, the initial estimate of 325K is reasonable and iteration on the property values is not necessary, (2) The major contribution to the total thermal resistance is due to air-side convection, (3) The foregoing results are independent of air pressure.

PROBLEM 11.31

KNOWN: Tube inner and outer diameters and longitudinal and transverse pitches for a cross-flow heat exchanger. Number of tubes in transverse plane. Water and gas flow rates and inlet temperatures. Water outlet temperature.

FIND: (a) Gas outlet temperature and number of longitudinal tube rows, (b) Effect of gas flowrate and inlet temperature on fluid outlet temperatures.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible kinetic and potential energy changes, (3) Constant properties, (4) Negligible fouling.

PROPERTIES: *Table A.6*, Water ($\overline{T}_c$ = 320 K: c_p = 4180 J/kg·K, $\mu = 577 \times 10^{-6}$ N·s/m^2, k_f = 0.640 W/m·K, Pr = 3.77; *Table A.4*, Air ($\overline{T}_h \approx 550$ K): c_p = 1040 J/kg·K, $\mu = 288.4 \times 10^{-7}$ N·s/m^2, k = 0.0439 W/m·K, Pr = 0.683, ρ = 0.633 kg/m^3.

ANALYSIS: (a) The required heat transfer rate is

$$q = \dot{m}_c c_{p,c}\left(T_{c,o} - T_{c,i}\right) = 50\,\text{kg/s}\left(4180\,\text{J/kg}\cdot\text{K}\right)60\,\text{K} = 1.254\times10^{7}\,\text{W}.$$

Hence, with $T_{h,o} = T_{h,i} - q/\dot{m}_h c_{p,h}$,

$$T_{h,o} = 700\,\text{K} - 1.254\times10^{7}\,\text{W}/\left(40\,\text{kg/s}\times1040\,\text{J/kg}\cdot\text{K}\right) = 398.6\,\text{K}$$ <

Use the ε - NTU method to compute the hot side HX surface area, A_H. To calculate U_h, we must find h_h. For the tube bank, S_D = 44.7 mm > $(S_T + D)/2$ = 30 mm. Hence, with $\rho V_{max} = \left[S_T/\left(S_T - D_o\right)\right]\rho V = \left[S_T/\left(S_T - D_o\right)\right]\left(\dot{m}_h/WH\right)$,

$$\rho V_{max} = (40/20)\left[40\,\text{kg/s}/(2\times1.2)\,\text{m}^2\right] = 33.3\,\text{kg/s}\cdot\text{m}^2$$

$$Re_{D,max} = \left(\rho V_{max} D_o\right)/\mu = \left[33.3\,\text{kg/s}\cdot\text{m}^2\,(0.02\,\text{m})\right]\Big/288.4\times10^{-7}\,\text{N}\cdot\text{s/m}^2 = 23{,}116.$$

From the Zhukauskas correlation, with $(Pr/Pr_s) \approx 1$, and Table 7.7,

$$\overline{Nu}_D = 0.35\,Re_D^{0.6}\,Pr^{0.36} = 0.35(23{,}116)^{0.6}(0.683)^{0.36} = 127$$

where it is assumed that $N_L > 20$. Hence,

$$h_h = \overline{Nu}_D\left(k/D_o\right) = 127\left(0.0439\,\text{W/m}\cdot\text{K}/0.02\,\text{m}\right) = 279\,\text{W/m}^2\cdot\text{K}.$$

From Eq. 11.1,

Continued...

PROBLEM 11.31 (Cont.)

$$\frac{1}{U_h} = \frac{1}{h_c}\frac{D_o}{D_i} + \frac{D_o \ln\left(D_o/D_i\right)}{2k} + \frac{1}{h_h} = \frac{1}{3000\,\mathrm{W/m^2\cdot K}}\frac{20}{15} + \frac{0.02\,\mathrm{m}\ln\left(20/15\right)}{60\,\mathrm{W/m\cdot K}} + \frac{1}{279\,\mathrm{W/m^2\cdot K}}$$

$$\frac{1}{U_h} = \left(4.44\times10^{-4} + 9.59\times10^{-5} + 3.58\times10^{-3}\right)\mathrm{m^2\cdot K/W} = 4.12\times10^{-3}\,\mathrm{m^2\cdot K/W}$$

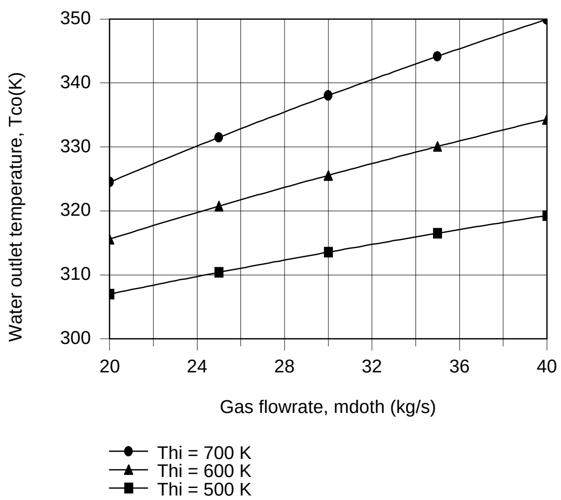
$$U_h = 243\,\mathrm{W/m^2\cdot K}.$$

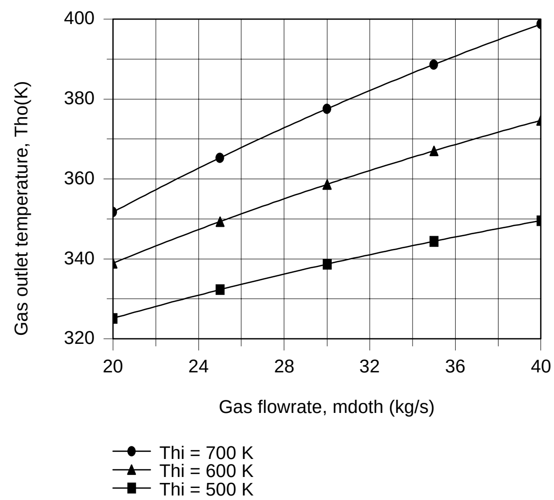
With $C_h = C_{min} = 4.160 \times 10^4$ W/K and $C_c = C_{max} = 2.09 \times 10^5$ W/K, $C_{min}/C_{max} = 0.199$ and $q_{max} = C_{min}(T_{h,i} - T_{c,i}) = 4.16 \times 10^4$ W/K(410 K) = 1.71×10^7 W. Hence, $\varepsilon = (q/q_{max}) = (1.254 \times 10^7\ \mathrm{W}/1.71 \times 10^7\ \mathrm{W}) = 0.735$. With C_{min} mixed and C_{max} unmixed, Eq. 11.35b gives NTU = 1.54 and

$$A_h = NTU\left(C_{min}/U_h\right) = 1.54\left(4.160\times10^4\ \mathrm{W/K}/243\,\mathrm{W/m^2\cdot K}\right) = 264\,\mathrm{m^2}.$$

Hence, $$N_L = \frac{A_h}{\left(\pi D_o W\right)N_T} = \frac{264\,\mathrm{m^2}}{\pi\left(0.02\right)2\left(30\right)\mathrm{m^2}} = 70$$ $<$

(b) Using the IHT *Correlations*, *Heat Exchangers* and *Properties* Toolpads to perform the parametric calculations, we obtain the following results for $N_L = 90$.





Since h_h, and hence U_h, increases with $\dot{m}_h$, q, and hence, $T_{c,o}$, increases with increasing $\dot{m}_h$, as well as with increasing $T_{h,i}$. Although q increases with $\dot{m}_h$, the proportionality is not linear ($q \propto \dot{m}_h^a$, where a < 1) and $(T_{h,i} - T_{h,o})$ must decrease with increasing $\dot{m}_h$, in which case $T_{h,o}$ must increase. From the above results, it is clear that operation is restricted to $\dot{m}_h \geq 40$ kg/s and $T_{h,i} \geq 700$ K, if corrosion of the heat exchanger surfaces is to be avoided.

COMMENTS: To check the presumed value of $h_c = 3000\ \mathrm{W/m^2\cdot K}$, compute

$$Re_D = \frac{4\left(\dot{m}_c/N\right)}{\pi D_i \mu} = \frac{4\left(50\,\mathrm{kg/s}\right)/70\times30}{\pi\left(0.015\,\mathrm{m}\right)577\times10^{-6}\ \mathrm{N\cdot s/m^2}} = 3500.$$

Hence, $$Nu_D = 0.023 Re_D^{4/5} Pr^{0.4} = 0.023\left(3500\right)^{4/5}\left(3.77\right)^{0.4} = 26.8$$

$$h_c = \left(k/D\right)Nu_D = \left(0.640\,\mathrm{W/m\cdot K}/0.015\,\mathrm{m}\right)26.8 = 1142\,\mathrm{W/m^2\cdot K}.$$

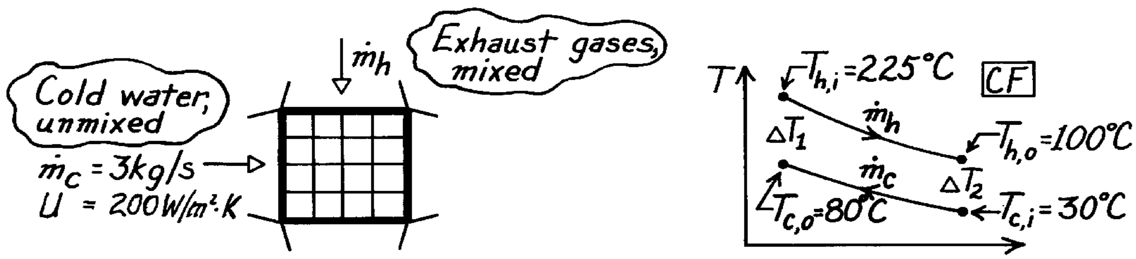
Hence, the cold side convection coefficient has been overestimated and the calculations should be repeated using the smaller value of h_c.

PROBLEM 11.32

KNOWN: Single pass, cross-flow heat exchanger with hot exhaust gases (mixed) to heat water (unmixed)

FIND: Required surface area.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible kinetic and potential energy changes, (3) Exhaust gas properties assumed to be those of air.

PROPERTIES: *Table A-6*, Water ($\overline{T}_c$ = (80 + 30)°C/2 = 328 K): c_p = 4184 J/kg·K; *Table A-4*, Air (1 atm, $\overline{T}_h$ = (100 + 225)°C/2 = 436 K): c_p = 1019 J/kg·K.

ANALYSIS: The rate equation for the heat exchanger follows from Eqs. 11.14 and 11.18. The area is given as

$$A = q / U\Delta T_{\ell m} = q / U F \Delta T_{\ell m,CF} \tag{1}$$

where F is determined from Fig. 11.13 using

$$P = \frac{80-30}{225-30} = 0.26 \text{ and } R = \frac{225-100}{80-30} = 2.50 \text{ giving } F \approx 0.92. \tag{2}$$

From an energy balance on the cold fluid, find

$$q = \dot{m}_c c_c \left(T_{c,o} - T_{c,i}\right) = 3\frac{kg}{s} \times 4184 \frac{J}{kg \cdot K}(80-30)K = 627{,}600 \text{ W}. \tag{3}$$

From Eq. 11.15, the LMTD for counter-flow conditions is

$$\Delta T_{\ell m,CF} = \frac{\Delta T_1 - \Delta T_2}{\ell n\left(\Delta T_1 / \Delta T_2\right)} = \frac{(225-80)-(100-30)}{\ell n(145/70)} °C = 103.0°C. \tag{4}$$

Substituting numerical values resulting from Eqs. (2-4) into Eq. (1), find the required surface area to be

$$A = 627{,}600\,W / 200\,W/m^2 \cdot K \times 0.92 \times 103.0K = 33.1m^2. \quad <$$

COMMENTS: Note that the properties of the exhaust gases were not needed in this method of analysis. If the ε-NTU method were used, find first $C_h/C_c = 0.40$ with $C_{min} = C_h = 5021$ W/K. From Eqs. 11.19 and 11.20, with $C_h = C_{min}$, $\varepsilon = q/q_{max} = (T_{h,i} - T_{h,o})/(T_{h,i} - T_{c,i})$ = (225 – 100)/(225 – 30) = 0.64. Using Fig. 11.19 with $C_{min}/C_{max} = 0.4$ and ε = 0.64, find NTU = $UA/C_{min} \approx 1.4$. Hence,

$$A = NTU \cdot C_{min} / U \approx 1.4 \times 5021\,W/K / 200\,W/m^2 \cdot K = 35.2m^2.$$

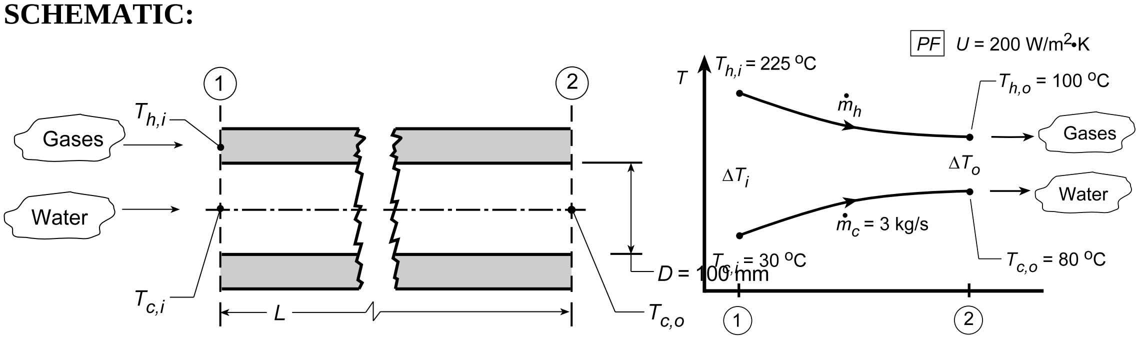
Note agreement with above result.

PROBLEM 11.33

KNOWN: Concentric tube heat exchanger operating in parallel flow (PF) conditions with a thin-walled separator tube of 100-mm diameter; fluid conditions as specified.

FIND: (a) Required length for the exchanger; (b) Convection coefficient for water flow, assumed to be fully developed; (c) Compute and plot the heat transfer rate, q, and fluid inlet temperatures, $T_{h,o}$ and $T_{c,o}$, as a function of the tube length for $60 \le L \le 400$ m with the PF arrangement and overall coefficient $\left(U = 200\,W/m^2 \cdot K\right)$, inlet temperatures ($T_{h,i}$ = 225°C and $T_{c,i}$ = 30°C), and fluid flow rates from Problem 11.23; (d) Reduction in required length relative to the value found in part (a) if the exchanger were operated in the counterflow (CF) arrangement; and (e) Compute and plot the effectiveness and fluid outlet temperatures as a function of tube length for $60 \le L \le 400$ m for the CF arrangement of part (c).

SCHEMATIC:



ASSUMPTIONS: (1) No losses to surroundings, (2) Negligible kinetic and potential energy changes, (3) Separation tube has negligible thermal resistance, (4) Water flow is fully developed, (5) Constant properties, (6) Exhaust gas properties are those of atmospheric air.

PROPERTIES: *Table A-4*, Hot fluid, Air (1 atm, $\overline{T}$ = (225 +100)°C /2 = 436 K): c_p = 1019 J/kg·K; *Table A-6*, Cold fluid, Water $\overline{T}$ = (30 + 80)°C /2 ≈ 328 K): $\rho = 1/v_f$ = 985.4 kg/m^3, c_p = 4183 J/kg·K, k = 0.648 W/m·K, $\mu = 505 \times 10^{-6}$ N·s/m^2, Pr = 3.58.

ANALYSIS: (a) From the rate equation, Eq. 11.14, with A = πDL, the length of the exchanger is

$$L = q/U \cdot \pi D \cdot \Delta T_{\ell n,PF} \,. \qquad (1)$$

The heat rate follows from an energy balance on the cold fluid, using Eq. 11.7, find

$$q = \dot{m}_c c_c \left(T_{c,o} - T_{c,i}\right) = 3\,kg/s \times 4183\,J/kg \cdot K\left(80-30\right)K = 627.5\times10^3\,W \,.$$

Using an energy balance on the hot fluid, find $\dot{m}_h$ for later use.

$$\dot{m}_h = q/c_h\left(T_{h,i} - T_{h,o}\right) = 627.5\times10^3\,W/1019\,J/kg\cdot K\left(225-100\right)K = 4.93\,kg/s \qquad (2)$$

For parallel flow, Eqs. 11.15 and 11.16,

$$\Delta T_{\ell m,PF} = \frac{\Delta T_1 - \Delta T_2}{\ell n \Delta T_1/\Delta T_2} = \frac{\left(225-30\right)^\circ C - \left(100-80\right)^\circ C}{\ell n\left(225-30\right)/\left(100-80\right)} = 76.8^\circ C \,.$$

Substituting numerical values into Eq. (1), find

$$L = 627.5\times10^3\,W/200\,W/m^2\cdot K\left(\pi\times0.1m\right)76.8K = 130m \,. \qquad <$$

Continued...

PROBLEM 11.33 (Cont.)

(b) Considering the water flow within the separator tube, from Eq. 8.6,

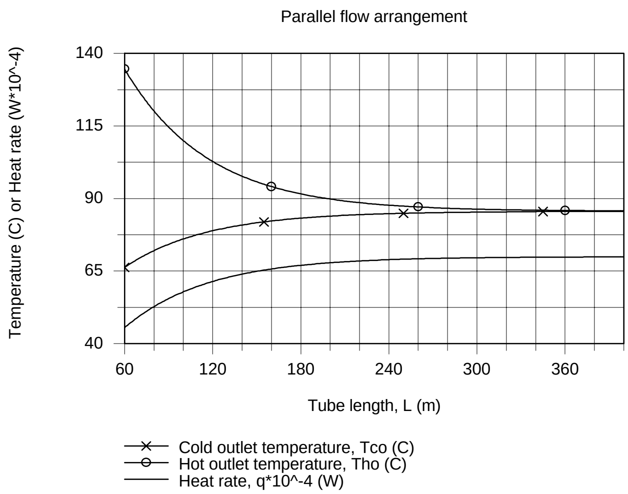
$$\mathrm{Re_D} = 4\dot{m}/\pi D\mu = 4\times 3\,\mathrm{kg/s}\Big/\left(\pi\times 0.1\mathrm{m}\times 505\times 10^{-6}\ \mathrm{N/s\cdot m^2}\right) = 75,638\,.$$

Since R_{eD} > 2300, the flow is turbulent and since flow is assumed to be fully developed, use the Dittus-Boelter correlation with n = 0.4 for heating,

$$\mathrm{Nu_D} = 0.023\,\mathrm{Re_D^{0.8}\,Pr^{0.4}} = 0.023(75,638)^{0.8}(3.58)^{0.4} = 306.4$$

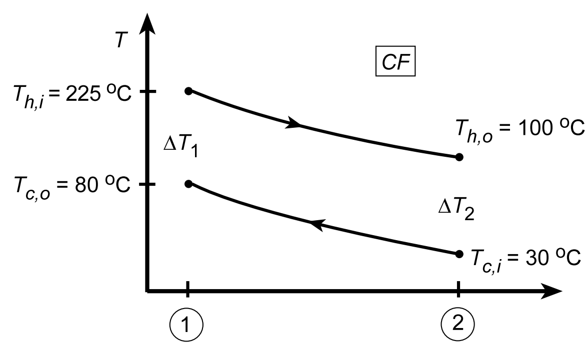
$$h = \mathrm{Nu_D}\frac{k}{D} = 306.4\times 0.648\,\mathrm{W/m\cdot K}\big/(0.1\mathrm{m}) = 1985\,\mathrm{W}\big/\mathrm{m^2\cdot K}\,.$$ <

(c) Using the *IHT Heat Exchanger Tool*, *Concentric Tube*, *Parallel Flow*, *Effectiveness relation*, and the *Properties Tool* for *Water* and *Air*, a model was developed for the PF arrangement. With U = 200 $W/m^2 \cdot K$ and prescribed inlet temperatures, $T_{h,i}$ = 225°C and $T_{c,i}$ = 30°C, the outlet temperatures, $T_{h,o}$ and $T_{c,o}$ and heat rate, q, were computed as a function of tube length L.



As the tube length increases, the outlet temperatures approach one another and eventually reach $T_{h,o}$ = $T_{c,o}$ = 85.6°C.

(d) If the exchanger as for part (a) is operated in counterflow (rather than parallel flow), the log mean temperature difference is



$$\Delta T_{\ell m,CF} = \frac{\Delta T_i - \Delta T_2}{\ell n \Delta T_1/\Delta T_2}$$

$$\Delta T_{\ell m,CF} = \frac{(225-80)-(100-30)}{\ell n\,(225-80)/100-30} = 103.0^\circ \mathrm{C}\,.$$

Using Eq. (1), the required length is

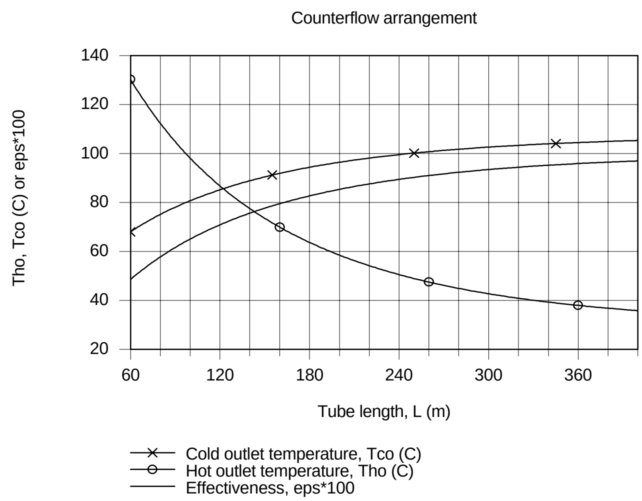
$$L = 627.5\times 10^3\ \mathrm{W}/200\,\mathrm{W}\big/\mathrm{m^2\cdot K}\times\pi\times 0.1\,\mathrm{m}\times 103.0\,\mathrm{K} = 97\,\mathrm{m}\,.$$

The reduction in required length of CF relative to PF operation is

Continued...

$$\Delta L = (L_{PF} - L_{CF})/L_{PF} = (103 - 97)/103 = 5.8\% \qquad <$$

(e) Using the *IHT Heat Exchanger Tool*, *Concentric Tube*, *Counterflow*, *Effectiveness relation*, and the *Properties Tool* for *Water* and *Air*, a model was developed for the CF arrangement. For the same conditions as part (c), but CF rather than PF, the effectiveness and fluid outlet temperatures were computed as a function of tube length L.



Note that as the length increases, the effectiveness tends toward unity, and the hot fluid outlet temperature tends toward $T_{c,i}$ = 30°C. Remember the heat rate for an infinitely long CF heat exchanger is q_{max} and the minimum fluid (hot in our case) experiences the temperature change, $T_{h,i}$ - $T_{c,i}$.

COMMENTS: (1) As anticipated, the required length for CF operations was less than for PF operation.

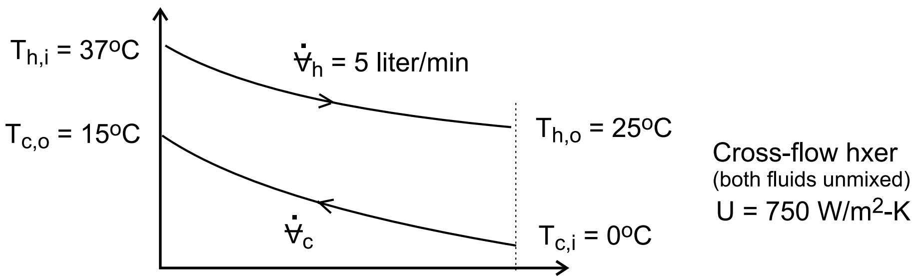
(2) Note that U is substantially less than h_i implying that the gas-side coefficient must be the controlling thermal resistance.

PROBLEM 11.34

KNOWN: Cross-flow heat exchanger (both fluids unmixed) cools blood to induce body hypothermia using ice-water as the coolant.

FIND: (a) Heat transfer rate from the blood, (b) Water flow rate, $\dot{\forall}_c$ (liter/min), (c) Surface area of the exchanger, and (d) Calculate and plot the blood and water outlet temperatures as a function of the water flow rate for the range, $2 \le \dot{\forall} \le 4$ liter/min, assuming all other parameters remain unchanged.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible losses to the surroundings and kinetic and potential energy changes, (3) Overall heat transfer coefficient remains constant with water flow rate changes, and (4) Constant properties.

PROPERTIES: *Table A-6*, Water $\left(\overline{T}_c = 280\text{K}\right)$, $\rho = 1000\ \text{kg/m}^3$, $c = 4198\ \text{J/kg}\cdot\text{K}$. Blood (given): $\rho = 1050\ \text{kg/m}^3$, $c = 3740\ \text{J/kg}\cdot\text{K}$.

ANALYSIS: (a) The heat transfer rate from the blood is calculated from an energy balance on the hot fluid,

$$\dot{m}_h = \rho_h \dot{\forall}_h = 1050\ \text{kg/m}^3 \times (5\ \text{liter/min} \times 1\ \text{min}/60\ \text{s}) \times 10^{-3}\text{m}^3/\text{liter} = 0.0875\ \text{kg/s}$$

$$q = \dot{m}_h c_h \left(T_{h,i} - T_{h,o}\right) = 0.0875\ \text{kg/s} \times 3740\ \text{J/kg}\cdot\text{K}(37 - 25)\text{K} = 3927\ \text{W} \qquad < \quad (1)$$

(b) From an energy balance on the cold fluid, find the coolant water flow rate,

$$q = \dot{m}_c\, c_c \left(T_{c,o} - T_{c,i}\right) \qquad (2)$$

$$3927\ \text{W} = \dot{m}_c \times 4198\ \text{J/kg}\cdot\text{K}\,(15 - 0)\,\text{K} \qquad \dot{m}_c = 0.0624\ \text{kg/s}$$

$$\dot{\forall}_c = \dot{m}_c / \rho_c = 0.0624\ \text{kg/s}/1000\ \text{kg/m}^3 \times 10^3\text{liter/m}^3 \times 60\ \text{s/min} = 3.74\ \text{liter/min} \qquad <$$

(c) The surface area can be determined using the effectiveness-NTU method. The capacity rates for the exchanger are

$$C_h = \dot{m}_h c_h = 327\ \text{W/K} \qquad C_c = \dot{m}_c\, c_c = 262\ \text{W/K} \qquad C_{min} = C_c \qquad (3, 4, 5)$$

From Eq. 11.19 and 11.20, the maximum heat rate and effectiveness are

Continued …..

PROBLEM 11.34 (Cont.)

$$q_{max} = C_{min}\left(T_{h,i} - T_{c,i}\right) = 262\ W/K\ (37-0)K = 9694\ W \tag{6}$$

$$\varepsilon = q / q_{max} = 3927/9694 = 0.405 \tag{7}$$

For the cross flow exchanger, with both fluids unmixed, substitute numerical values into Eq. 11.33 to find the number of transfer units, NTU, where $C_r = C_{min}/C_{max}$.

$$\varepsilon = 1 - \exp\left[(1/C_r)NTU^{0.22}\left\{\exp\left[-C_r NTU^{0.78}\right] - 1\right\}\right] \tag{8}$$

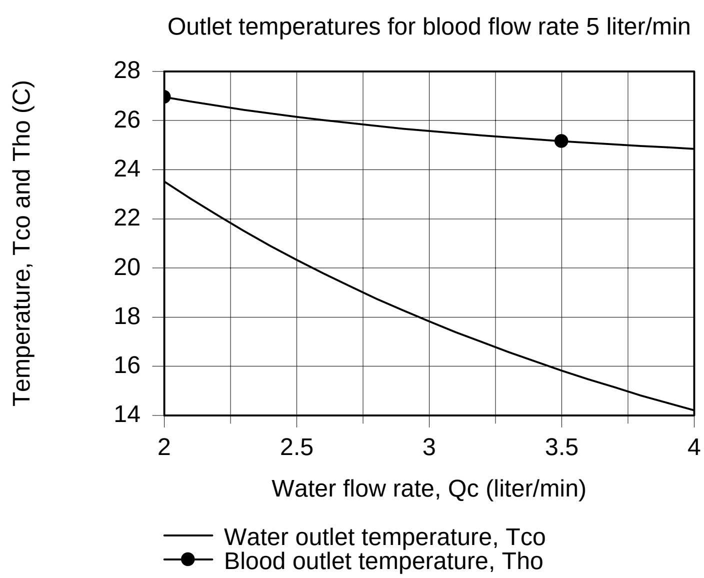
$$NTU = 0.691$$

From Eq. 11.25, find the surface area, A.

$$NTU = UA/C_{min}$$

$$A = 0.691 \times 262\ W/K / 750\ W/m^2\cdot K = 0.241\ m^2$$ <

(d) Using the foregoing equations in the *IHT* workspace, the blood and water outlet temperatures, $T_{h,o}$ and $T_{c,o}$, respectively, are calculated and plotted as a function of the water flow rate, all other parameters remaining unchanged.



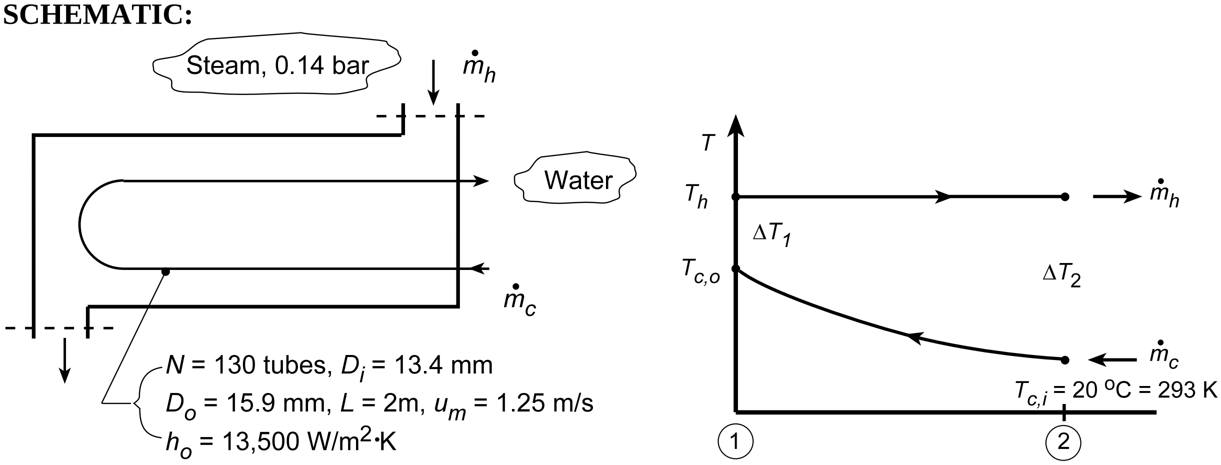
From the graph, note that with increasing water flow rate, both the blood and water outlet temperatures decrease. However, the effect of the water flow rate is greater on the water outlet temperature. This is an advantage for this application, since it is desirable to have the blood outlet temperature relatively insensitive to changes in the water flow rate. That is, if there are pressure changes on the water supply line or a slight miss-setting of the water flow rate controller, the outlet blood temperature will not change markedly.

PROBLEM 11.35

KNOWN: Steam at 0.14 bar condensing in a shell and tube HXer (one shell, two tube passes consisting of 130 brass tubes off length 2 m, D_i = 13.4 mm, D_o = 15.9 mm). Cooling water enters at 20°C with a mean velocity 1.25 m/s. Heat transfer convection coefficient for condensation on outer tube surface is h_o = 13,500 W/m^2 ·K.

FIND: (a) Overall heat transfer coefficient, U, for the HXer, outlet temperature of cooling water, $T_{c,o}$, and condensation rate of the steam $\dot{m}_h$; and (b) Compute and plot $T_{c,o}$ and $\dot{m}_h$ as a function of the water flow rate $10 \le \dot{m}_c \le 30$ kg/s with all other conditions remaining the same, but accounting for changes in U.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible kinetic and potential energy changes, (3) Constant properties, (4) Fully developed water flow in tubes.

PROPERTIES: *Table A-6,* Steam (0.14 bar): $T_{sat} = T_h$ = 327 K, h_{fg} = 2373 kJ/kg, c_p = 1898 J/kg·K; *Table A-6,* Water (Assume $T_{c,o} \approx 44°C$ or $\overline{T}_c \approx 305$ K): $v_f = 1.005 \times 10^{-3}$ m^3/kg , c_p = 4178 J/kg·K, $\mu_f = 769 \times 10^{-6}$ N·s/m^2 , k_f = 0.620 W/m·K, Pr_f = 5.2; *Table A-1,* Brass - 70/30 (Evaluate at $\overline{T} = (T_h + \overline{T}_c)/2 = 316$ K): k = 114 W/m·K.

ANALYSIS: (a) The overall heat transfer coefficient based upon the outside tube area follows from Eq. 11.5,

$$U_o = \left[\frac{1}{h_o} + \frac{r_o}{k}\ell n\frac{r_o}{r_i} + \left(\frac{r_o}{r_i}\right)\frac{1}{h_i}\right]^{-1}. \qquad (1)$$

The value for h_i can be estimated from an appropriate internal flow correlation. First determine the nature of the flow within the tubes. From Eq. 8.1,

$$Re_{D_i} = \rho u_m \frac{D_i}{\mu} = \frac{\left(1.005\times10^{-3}\,m^3/kg\right)^{-1}\times1.25\,m/s\times13.4\times10^{-3}\,m}{769\times10^{-6}\,N\cdot s/m^2} = 21,673\,.$$

The water flow is turbulent and fully developed (L/D_i = 2 m /13.4 × 10^{-3} m = 150 > 10). The Dittus-Boelter correlation with n = 0.4 is appropriate,

$$Nu_D = h_i\,D_i/k_f = 0.023\,Re_D^{0.8}\,Pr_f^{0.4} = 0.023\times(21,673)^{0.8}\,(5.2)^{0.4} = 130.9$$

Continued...

$$h_i = \frac{k_f}{D_i} Nu_D = \frac{0.620\,W/m\cdot K}{13.4\times10^{-3}m}\times130.9 = 6057\,W/m^2\cdot K\,.$$

Substituting numerical values into Eq. (1), the overall heat transfer coefficient is

$$U_o = \left[\frac{1}{13{,}500\,W/m^2\cdot K} + \frac{\left(15.9\times10^{-3}m\right)/2}{115\,W/m\cdot K}\ell n\frac{15.9}{13.4} + \frac{15.9}{13.4}\times\frac{1}{6057\,W/m^2\cdot K}\right]^{-1}$$

$$U_o = \left[7.407\times10^{-5} + 1.183\times10^{-5} + 19.590\times10^{-5}\right]^{-1} W/m^2\cdot K = 3549\,W/m^2\cdot K\,. \qquad <$$

To find the outlet temperature of the water, we'll employ the ε – NTU method. From an energy balance on the cold fluid,

$$T_{c,o} = T_{c,i} + q/C_c \qquad (3)$$

where the heat rate can be expressed as

$$q = \varepsilon q_{max} \qquad q_{max} = C_{min}\left(T_{h,i} - T_{h,o}\right). \qquad (4,5)$$

The minimum capacity rate is that of the cold water since $C_h \rightarrow \infty$. Evaluating, find

$$C_{min} = C_c = \left(\dot{m}c_p\right)_c = 22.8\,kg/s\times4178\,J/kg\cdot K = 95{,}270\,W/K\,.$$

where

$$\dot{m}_c = \left(\rho A u_m\right)N = 995.0\,kg/m^3\times\pi/4\left(0.0134m\right)^2\times1.25\,m/s\times130 = 22.8\,kg/s$$

To determine ε, use Fig. 11.16 (one shell and any multiple of tube passes) with

$$NTU = \frac{U_o A_o}{C_{min}} = \frac{3549\,W/m^2\cdot K\left(\pi 0.0159m\times2m\times130\times2\right)}{95{,}270\,W/K} = 0.968$$

where 130 and 2 represent the number of tubes and passes, respectively, to find ε ≈ 0.62. Combining Eqs. (4) and (5) into Eq. (3), find

$$T_{c,o} = T_{c,i} + \varepsilon C_{min}\left(T_{h,i} - T_{c,i}\right)/C_c = 20^\circ C + 0.62\left(327 - 293\right)K = 41.1^\circ C\,. \qquad <$$

The condensation rate of the steam is given by

$$\dot{m}_h = q/h_{fg} \qquad (6)$$

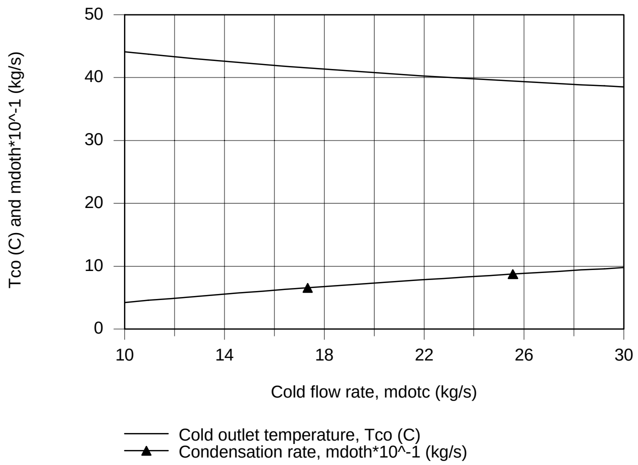
where the heat rate can be determined from Eq. (3) with $T_{c,o}$,

$$\dot{m}_h = C_c\left(T_{c,o} - T_{c,i}\right)/h_{fg} = 95{,}270\,W/K\left(41.1 - 20.0\right)K/2373\times10^3\,J/kg\cdot K = 0.85\,kg/s\,. \qquad <$$

(b) Using the *IHT Heat Exchanger Tool, All Exchangers, $C_r = 0$,* and the *Properties Tool* for *Water*, a model was developed and the cold outlet temperature and condensation rate were computed and plotted.

Continued...

PROBLEM 11.35 (Cont.)



With increasing cold flow rate, the cold outlet temperature decreases as expected. The condensation rate increases with increasing cold flow rate. Note that $T_{c,o}$ and $\dot{m}_h$ are nearly linear with the cold flow rate.

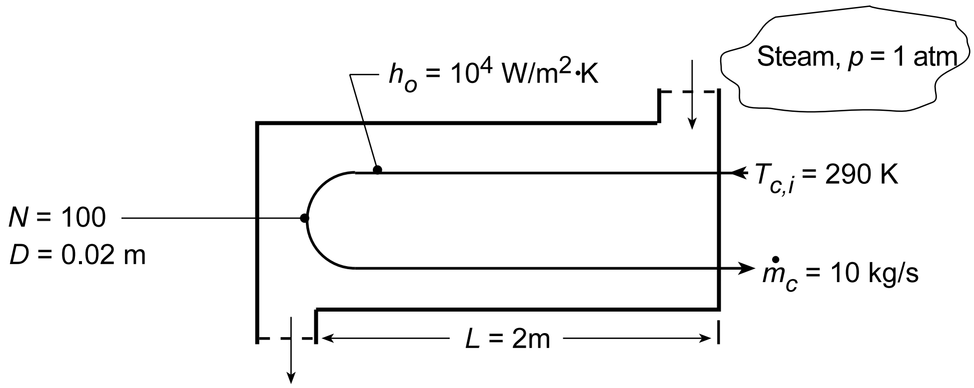
COMMENTS: For part (a) analysis, note that the assumption $T_{c,o} \approx 44°C$ used for evaluation of the cold fluid properties was reasonable. Using the IHT model of part (b), we found $T_{c,o} = 40.2°C$ and $\dot{m}_h$ = 0.812 kg /s.

PROBLEM 11.36

KNOWN: Shell-and-tube (one shell, two tube passes) heat exchanger design. Water flow rate and inlet temperature. Steam pressure and convection coefficient.

FIND: (a) Water outlet temperature, $T_{c,o}$; (b) $T_{c,o}$ as a function of flow rate, $\dot{m}_c$, for the range, $5 \le \dot{m}_c \le 20$ kg/s, with all other conditions remaining the same, but accounting for changes in the overall coefficient, U; and (c) Plot $T_{c,o}$ on the same graph considering fouling factors of $R''_f = 0.0002$ and 0.0005 $m^2 \cdot K/W$

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat loss to surroundings and kinetic and potential energy changes, (2) Negligible wall conduction and fouling resistances, (3) Constant properties.

PROPERTIES: *Table A-6*, Sat. water (p = 1.0133 bar): $T_{sat} = T = 373.1$ K; ($\bar{T}_c \approx 320$ K): $c_p = 4180$ J/kg·K, $\mu = 577 \times 10^{-6}$ N·s/m^2 , k = 0.640 W/m·K, Pr = 3.77.

ANALYSIS: Using the NTU-effectiveness method, calculate for U by finding h_i . With

$$Re_D = 4\dot{m}/\pi D\mu = \left[4\left(10\,kg/s\right)/100\right]\Big/\left[\pi\left(0.02m\right)\left(577\times10^{-6}\,N\cdot s/m^2\right)\right] = 11,033 \qquad (1)$$

and using the Dittus-Boelter correlation,

$$Nu_D = 0.023 Re_D^{4/5} Pr^{0.4} = 0.023\left(11,033\right)^{4/5}\left(3.77\right)^{0.4} = 67.05 \qquad (2)$$

$$h_i = \left(k/D\right)Nu_D = \left(0.640\,W/m\cdot K/0.02m\right)67.05 = 2146\,W\big/m^2\cdot K\,.$$

From Eq. 11.5

$$1/U = 1/h_i + 1/h_o = \left[\left(1/10,000\right)+\left(1/2146\right)\right]m^2\cdot K\big/W = 5.66\times10^{-4}\,m^2\cdot K/W \qquad (3)$$

$$U = 1766\,W\big/m^2\cdot K\,.$$

The heat transfer surface area, capacity rates and NTU are

$$A = N\left(\pi D\right)2L = 100\left(\pi 0.02m\right)2\times2m = 25.1m^2$$

$$C_{min} = C_c = 10\,kg/s\left(4180\,J/kg\cdot K\right) = 41,800\,W/K$$

$$NTU = UA\big/C_{min} = 1766\,W/m^2\cdot K\times25.1\,m^2\big/41,800\,W/K = 1.06$$

From Eq. 11.36a

Continued...

$$\varepsilon = 1 - \exp(-NTU) = 1 - \exp(-1.06) = 0.654 . \qquad (4)$$

With

$$q_{max} = C_{min}\left(T_{h,i} - T_{c,i}\right) = 41,800\,W/K\,(373.15 - 290)\,K = 3.48 \times 10^6\,W \qquad (5)$$

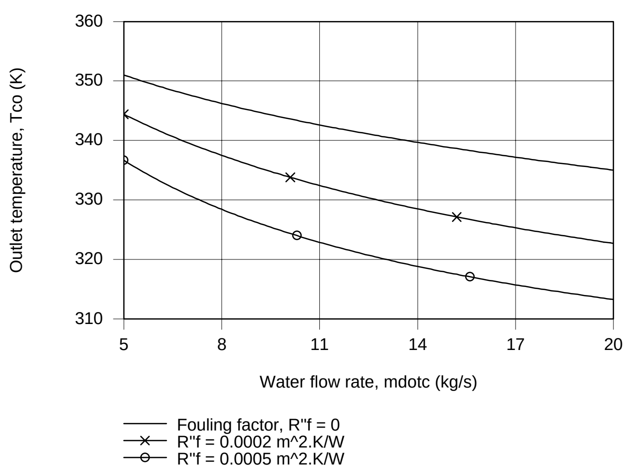
$$q = \varepsilon q_{max} = 0.654\left(3.48 \times 10^6\,W\right) = 2.27 \times 10^6\,W$$

find

$$T_{c,o} = T_{c,i} + \left(q/C_c\right) = 290\,K + \left(2.27 \times 10^6\,W/41,800\,W/K\right) = 344.4\,K . \qquad (6) <$$

(b,c) Using the *IHT Heat Exchanger Tool, All Exchangers, $C_r = 0$,* the *Properties Tool* for *Water* and the *Correlation Tool, Forced Convection, Internal Flow,* for *Turbulent, fully developed conditions,* a model was developed following the foregoing analysis to compute and plot the outlet temperature $T_{c,o}$ as a function of the cold fluid flow rate, $\dot{m}_c$. The expression for the overall coefficient, Eq.(1), was modified to include the fouling factor,

$$1/U = 1/h_i + R''_f + 1/h_o .$$



The effect of increasing the cold flow rate is to decrease the outlet temperature. The effect of the fouling resistance is to decrease the outlet temperature as well.

COMMENTS: (1) For the part (a) analysis, $\overline{T}_c$ = 317 K and the initial guess of 320 K was reasonably good.

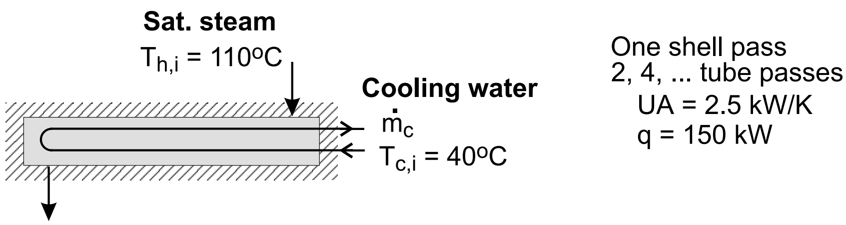
(2) In the anlysis of parts (b,c), $Re_{D,c}$ is as low as 4880, below the turbulent range (10,000) and above the laminar range (2300). We chose to treat the flow as turbulent.

PROBLEM 11.37

KNOWN: Saturated steam at 110°C condensing in a shell and tube heat exchanger (one shell pass, 2, 4, tube passes) with a UA value of 2.5 kW/K; cooling water enters at 40°C.

FIND: Cooling water flow rate required to maintain a heat rate of 150 kW; and (b) Calculate and plot the water flow rate required to provide heat rates over the range 130 to 160 kW, assuming that UA is independent of flow rate. Comment on the validity of the assumption.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible kinetic and potential energy changes, (3) UA independent of flow rate, and (4) Constant properties.

PROPERTIES: *Table A-6*, Water ($T_{m,c} = (T_{c,i} + T_{c,o})/2$ = 49.5°C = 322.5 K): $c_{p,c}$ = 4181 J/kg·K.

ANALYSIS: (a) For the shell-tube heat exchanger with any multiple of two-tube passes, from Eq. 11.36a with $C_r = 0$, using Eqs. 11.20 and 11.23,

$$\varepsilon = 1 - \exp(-NTU) \qquad NTU = UA/C_{min} \tag{1,2}$$

$$\varepsilon = q/q_{max} \qquad q_{max} = C_c\left(T_{h,i} - T_{c,i}\right) \tag{3,4}$$

By combining the equations with $C_{min} = C_c = \dot{m}_c\, c_{p,c}$,

$$\frac{q}{\dot{m}_c\, c_{p,c}\left(T_{h,i} - T_{c,i}\right)} = 1 - \exp\left(-\frac{UA}{\dot{m}_c\, c_{p,c}}\right) \tag{5}$$

Substituting numerical values, and solving using *IHT* find

$$\dot{m}_c = 1.89\ \text{kg/s}$$ <

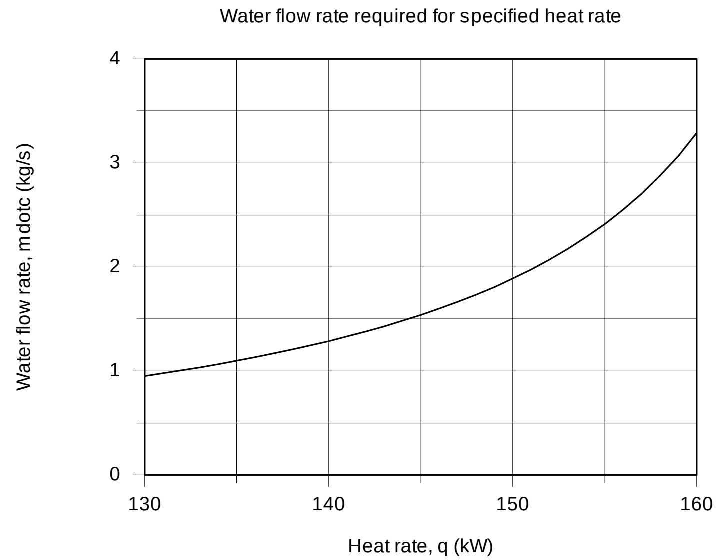
The specific heat of the cold fluid, $c_{p,c}$, is evaluated at the average of the mean inlet and outlet temperatures, $T_{m,c} = (T_{c,i} + T_{c,o})/2$, with $T_{c,o}$ determined from the energy balance equation,

$$q = \dot{m}\, c_{p,c}\left(T_{c,o} - T_{c,i}\right). \tag{6}$$

(b) Solving the above system of equations in the *IHT* workspace, the graph below illustrates the water flow rate required to provide a range of heat rates.

Continued …..

PROBLEM 11.37 (Cont.)



COMMENTS: (1) The assumption that UA is constant with flow rate is a poor one. Because the heat transfer coefficient for condensation is so high, the overall coefficient is controlled by the water-side coefficient. Presuming the flow is turbulent, from the Dittus-Boelter correlation, we'd expect $U \propto m_c^{0.8}$. Over the range of the graph above, U will vary by approximately a factor of $(3.5/1)^{0.8}$ = 2.7.

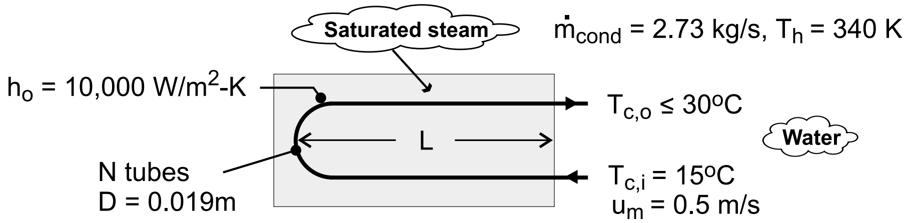
(2) If we considered UA to vary with the cold water flow rate as just described, make a sketch of $\dot{m}_c$ vs. q and compare it to the graph above.

PROBLEM 11.38

KNOWN: Temperature, convection coefficient and condensation rate of saturated steam. Tube diameter for shell-and-tube heat exchanger with one shell pass and two tube passes. Velocity and inlet and maximum allowable exit temperatures of cooling water.

FIND: (a) Minimum number of tubes and tube length per pass, (b) Effect of tube-side heat transfer enhancement on tube length.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat exchange with surroundings, (2) Negligible flow work and kinetic and potential energy changes, (3) Negligible tube wall conduction and fouling resistance, (4) Constant properties, (5) Fully developed internal flow throughout.

PROPERTIES: *Table A-6*, Sat. water (340 K): $h_{fg} = 2.342 \times 10^6$ J/kg; Sat. water $\left(\overline{T}_c = 22.5°C \approx 295\ K\right)$: ρ=998 kg/m^3, c_p = 4181 J/kg·K, $\mu = 959 \times 10^{-6}$ N·s/m^2, k = 0.606 W/m·K, Pr = 6.62.

ANALYSIS: (a) The required heat rate and the maximum allowable temperature rise of the water determine the minimum allowable flow rate. That is, with

$$q = q_{cond} = \dot{m}_{cond} h_{fg} = 2.73\ kg/s \times 2.342 \times 10^6\ J/kg = 6.39 \times 10^6\ W$$

$$\dot{m}_{c,min} = \frac{q}{c_{p,c}\left(T_{c,o} - T_{c,i}\right)} = \frac{6.39 \times 10^6\ W}{4181\ J/kg \cdot K\left(15°C\right)} = 101.9\ kg/s$$

With a specified flow rate per tube of $\dot{m}_{c,1} = \rho u_m \pi D^2/4 = 998\ kg/m^3 \times 0.5\ m/s \times \pi\ (0.019m)^2/4$ = 0.141 kg/s, the minimum number of tubes is

$$N_{min} = \frac{\dot{m}_{c,min}}{\dot{m}_{c,1}} = \frac{101.9\ kg/s}{0.141\ kg/s} = 720 \qquad <$$

To determine the corresponding tube length, we must first find the required heat transfer surface area. With $Re_D = \rho u_m D/\mu = 998\ kg/m^3\ (0.5\ m/s)\ 0.019m/959 \times 10^{-6}\ N \cdot s/m^2$ = 9,886, the Dittus-Boelter equation yields

$$\overline{h}_i = (k/D)0.023\ Re_D^{4/5}\ Pr^{0.4} = \left(0.606\ W/m \cdot K/0.019m\right)0.023\left(9886\right)^{4/5}\left(6.62\right)^{0.4} = 2454\ W/m^2 \cdot K$$

Continued …..

Hence, $U = \left[\bar{h}_i^{-1} + h_o^{-1}\right]^{-1} = 1970\ \text{W/m}^2 \cdot \text{K}$

With $C_r = 0$, $C_{min} = \dot{m}_c\ c_{p,c}$ = 101.9 kg/s × 4181 J/kg·K = 4.26 × 10^5 W/K, $q_{max} = C_{min}\,(T_{h,i} - T_{c,i})$ = 4.26 × 10^5 W/K (340 – 288) K = 2.215 × 10^7 W and $\varepsilon = q/q_{max}$ = 0.289, Eq. 11.36b yields NTU = – ln (1 – ε) = – ln (1 – 0.289) = 0.341. Hence the tube length per pass is

$$L = \frac{A}{2N\pi D} = \frac{NTU \times C_{min}}{2N\pi DU} = \frac{0.341 \times 4.26 \times 10^5\ \text{W/K}}{2 \times 720 \times \pi\,(0.019\text{m})\,1970\ \text{W/m}^2 \cdot \text{K}} = 0.858\text{m} \qquad <$$

(b) If the tube-side convection coefficient is doubled, $\bar{h}_i = 4908\ \text{W/m}^2 \cdot \text{K}$ and U = 3292 W/m^2·K. Since q, C_r, C_{min}, q_{max} and hence ε are unchanged, the number of transfer units is still NTU = 0.341. Hence, the tube length per pass is now

$$L = \frac{NTU \times C_{min}}{2\,N\pi DU} = \frac{0.341 \times 4.26 \times 10^5\ \text{W/K}}{2 \times 720 \times \pi\,(0.019\text{m})\,3292\ \text{W/m}^2 \cdot \text{K}} = 0.513\text{m} \qquad <$$

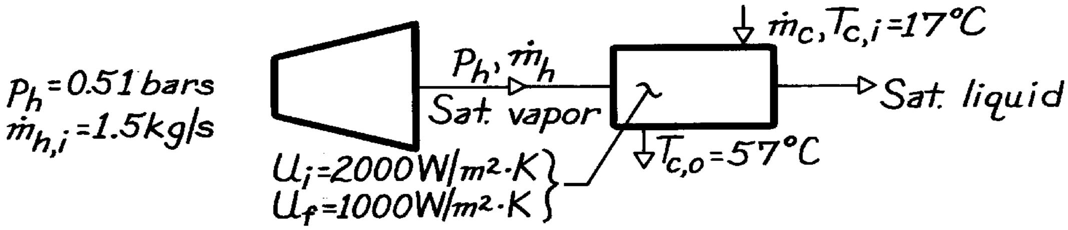
COMMENTS: Heat transfer enhancement for the flow with the smallest convection coefficient significantly reduces the size of the heat exchanger.

PROBLEM 11.39

KNOWN: Pressure and initial flow rate of water vapor. Water inlet and outlet temperatures. Initial and final overall heat transfer coefficients.

FIND: (a) Surface area for initial U and water flow rate, (b) Vapor flow rate for final U.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible wall conduction resistance.

PROPERTIES: *Table A-6*, Sat. water ($\overline{T}_c$ = 310 K): $c_{p,c}$ = 4178 J/kg·K; (p = 0.51 bars): T_{sat} = 355 K, h_{fg} = 2304 kJ/kg.

ANALYSIS: (a) The required heat transfer rate is

$$q = \dot{m}_h h_{fg} = 1.5 \text{ kg/s}\left(2.304\times10^6 \text{J/kg}\right) = 3.46\times10^6 \text{W}$$

and the corresponding heat capacity rate for the water is

$$C_c = C_{min} = q/\left(T_{c,o} - T_{c,i}\right) = 3.46\times10^6 \text{ W}/40 \text{ K} = 86{,}400 \text{ W/K}.$$

Hence, $$e = q/\left(C_{min}\left[T_{h,i} - T_{c,i}\right]\right) = 3.46\times10^6 \text{ W}/86{,}400 \text{ W/K}\left(65 \text{ K}\right) = 0.62.$$

Since $C_{min}/C_{max} = 0$, Eq. 11.36b yields

$$NTU = -\ln\left(1-e\right) = -\ln\left(1-0.62\right) = 0.97$$

and $$A = NTU\left(C_{min}/U\right) = 0.97\left(86{,}400 \text{ W/K}/2000 \text{ W/m}^2\cdot\text{K}\right) = 41.9\text{m}^2 \quad <$$

$$\dot{m}_c = C_c/c_{p,c} = 86{,}400 \text{ W/K}/4178 \text{ J/kg}\cdot\text{K} = 20.7 \text{ kg/s}. \quad <$$

(b) Using the final overall heat transfer coefficient, find

$$NTU = UA/C_{min} = 1000 \text{ W/m}^2\cdot\text{K}\left(41.9\text{m}^2\right)/86{,}400 \text{ W/K} = 0.485.$$

Since $C_{min}/C_{max} = 0$, Eq. 11.36a yields

$$e = 1 - \exp\left(-NTU\right) = 1 - \exp\left(-0.485\right) = 0.384.$$

Hence, $$q = eC_{min}\left(T_{h,i} - T_{c,i}\right) = 0.384\left(86{,}400 \text{ W/K}\right)65 \text{ K} = 2.16\times10^6 \text{ W}$$

$$\dot{m}_h = q/h_{fg} = 2.16\times10^6 \text{ W}/2.304\times10^6 \text{ J/kg} = 0.936 \text{ kg/s}. \quad <$$

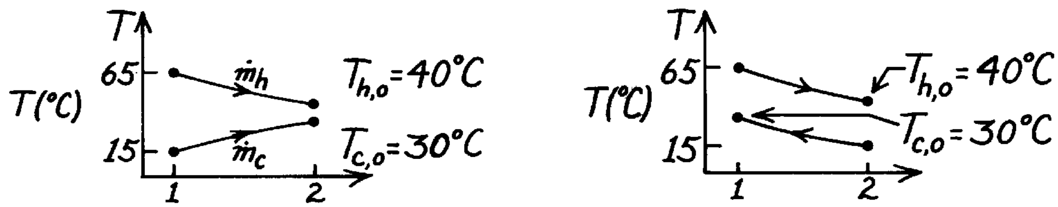
COMMENTS: The significant reduction (38%) in $\dot{m}_h$ represents a significant loss in turbine power. Periodic cleaning of condenser surfaces should be employed to minimize the adverse effects of fouling.

PROBLEM 11.40

KNOWN: Two-fluid heat exchanger with prescribed inlet and outlet temperatures of the two fluids.

FIND: (a) Whether exchanger is operating in parallel or counter flow, (b) Effectiveness of the exchanger when $C_c = C_{min}$.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat loss to the surroundings, (2) Negligible kinetic and potential energy changes, (3) Cold fluid is minimum fluid.

ANALYSIS: (a) To determine whether operation is PF or CF, consider the temperature distributions. From the distributions we note that PF or CF operation is possible.

(b) The effectiveness of the exchanger follows from Eq. 11.20,

$$\varepsilon = q / q_{max} \tag{1}$$

where from Eq. 11.19,

$$q_{max} = C_{min}\left(T_{h,i} - T_{c,i}\right). \tag{2}$$

Since $C_{min} = C_c$ and performing an energy balance on the cold fluid, Eq. (1) with Eq. (2) becomes

$$\varepsilon = C_c\left(T_{c,o} - T_{c,i}\right) / C_{min}\left(T_{h,i} - T_{c,i}\right) = \left(T_{c,o} - T_{c,i}\right) / \left(T_{h,i} - T_{c,i}\right)$$

$$\varepsilon = (30 - 15)°C / (65 - 15)°C = 0.30.$$ <

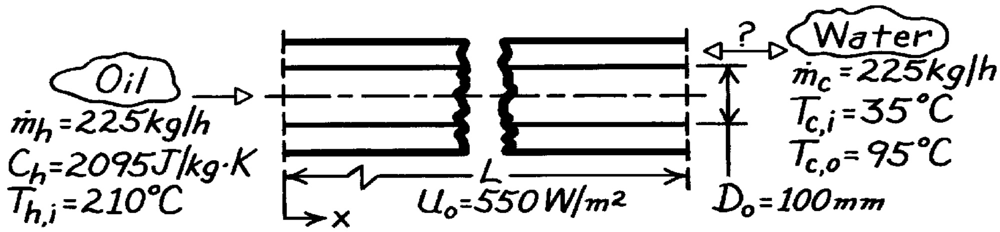
COMMENTS: If $T_{c,o}$ were greater than $T_{h,o}$, parallel-flow operation would not be possible.

PROBLEM 11.41

KNOWN: Concentric tube heat exchanger.

FIND: Length of the exchanger.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible kinetic and potential energy changes, (3) Constant properties.

PROPERTIES: *Table A-6*, Water ($\bar{T}_c$ = (35 + 95)°C/2 = 338 K): $c_{p,c}$ = 4188 J/kg·K

ANALYSIS: From the rate equation, Eq. 11.14, with $A_o = \pi D_o L$,

$$L = q / U_o \pi D_o \Delta T_{\ell m}.$$

The heat rate, q, can be evaluated from an energy balance on the cold fluid,

$$q = \dot{m}_c c_c \left(T_{c,o} - T_{c,i}\right) = \frac{225 \text{ kg/h}}{3600 \text{ s/h}} \times 4188 \text{ J/kg} \cdot \text{K}\left(95 - 35\right)\text{K} = 15{,}705 \text{ W}.$$

In order to evaluate $\Delta T_{\ell m}$, we need to know whether the exchanger is operating in CF or PF. From an energy balance on the hot fluid, find

$$T_{h,o} = T_{h,i} - q / \dot{m}_h c_h = 210°\text{C} - 15{,}705 \text{ W} / \frac{225 \text{ kg/h}}{3600 \text{s/h}} \times 2095 \frac{\text{J}}{\text{kg} \cdot \text{K}} = 90.1°\text{C}.$$

Since $T_{h,o} < T_{c,o}$ it follows that Hxer operation must be CF. From Eq. 11.15,

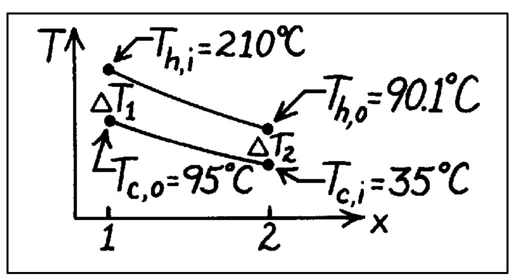
$$\Delta T_{\ell m,CF} = \frac{\Delta T_1 - \Delta T_2}{\ell n\left(\Delta T_1 / \Delta T_2\right)} = \frac{\left(210 - 95\right) - \left(90.1 - 35\right)}{\ell n\left(115/55.1\right)} °\text{C} = 81.4°\text{C}.$$

Substituting numerical values, the HXer length is

$$L = 15{,}705 \text{ W} / 550 \text{ W/m}^2 \cdot \text{K} \pi \left(0.10\text{m}\right) \times 81.4\text{K} = 1.12\text{m}.$$ <

COMMENTS: The ε-NTU method could also be used. It would be necessary to perform the hot fluid energy balance to determine if CF operation existed. The capacity rate ratio is C_{min}/C_{max} = 0.50. From Eqs. 11.19 and 11.20 with q evaluated from an energy balance on the hot fluid,

$$\varepsilon = \frac{T_{h,i} - T_{h,o}}{T_{h,i} - T_{c,i}} = \frac{210 - 90.1}{210 - 35} = 0.69.$$



From Fig. 11.15, find NTU ≈ 1.5 giving

$$L = \text{NTU} \cdot C_{min} / U_o \pi D_o \approx 1.5 \times 130.94 \frac{\text{W}}{\text{K}} / 550 \frac{\text{W}}{\text{m}^2 \cdot \text{K}} \cdot \pi \left(0.10\text{m}\right) \approx 1.14\text{m}.$$

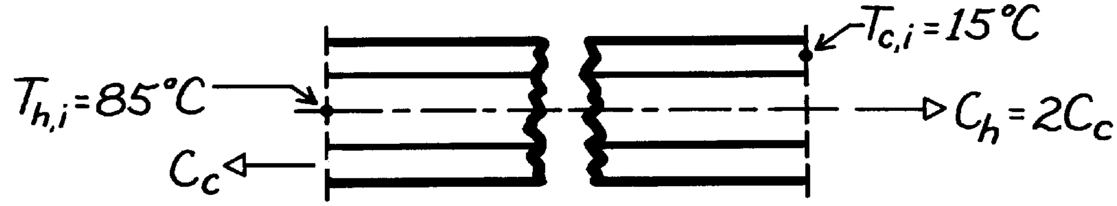
Note the good agreement in both methods.

PROBLEM 11.42

KNOWN: A *very long*, concentric tube heat exchanger having hot and cold water inlet temperatures, 85°C and 15°C, respectively; flow rate of hot water is twice that of the cold water.

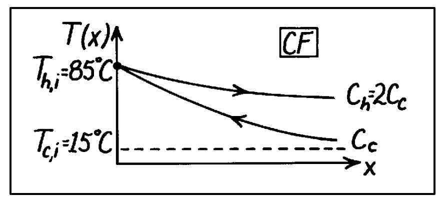
FIND: Outlet temperatures for counterflow and parallel flow operation.

SCHEMATIC:



ASSUMPTIONS: (1) Equivalent hot and cold water specific heats, (2) Negligible kinetic and potential energy changes, (3) No heat loss to surroundings.

ANALYSIS: The heat rate for a concentric tube heat exchanger with very large surface area operating in the *counterflow* mode is



$$q = q_{max} = C_{min}\left(T_{h,i} - T_{c,i}\right)$$

where $C_{min} = C_c$. From an energy balance on the hot fluid,

$$q = C_h\left(T_{h,i} - T_{h,o}\right).$$

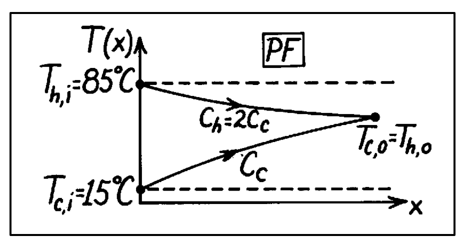
Combining the above relations and rearranging, find

$$T_{h,o} = -\frac{C_{min}}{C_h}\left(T_{h,i} - T_{c,i}\right) + T_{h,i} = -\frac{C_c}{C_h}\left(T_{h,i} - T_{c,i}\right) + T_{h,i}.$$

Substituting numerical values,

$$T_{h,o} = -\frac{1}{2}(85 - 15)°C + 85°C = 50°C. \qquad <$$

For *parallel flow* operation, the hot and cold outlet temperatures will be equal; that is, $T_{c,o} = T_{h,o}$. Hence,



$$C_c\left(T_{c,o} - T_{c,i}\right) = C_h\left(T_{h,i} - T_{h,o}\right).$$

Setting $T_{c,o} = T_{h,o}$ and rearranging,

$$T_{h,o} = \left(T_{h,i} + \frac{C_c}{C_h}T_{c,i}\right) \Big/ \left(1 + \frac{C_c}{C_h}\right)$$

$$T_{h,o} = \left(85 + \frac{1}{2}\times 15\right)°C \Big/ \left(1 + \frac{1}{2}\right) = 61.7°C. \qquad <$$

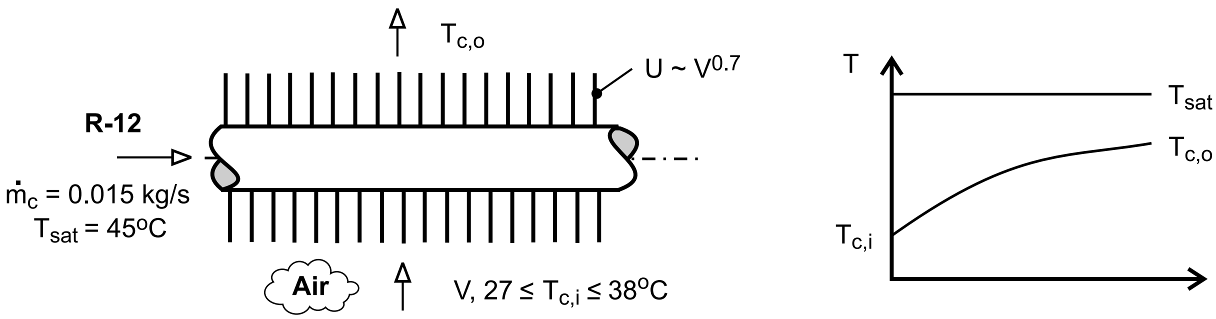
COMMENTS: Note that while $\varepsilon = 1$ for CF operation, for PF operation find $\varepsilon = q/q_{max} = 0.67$.

PROBLEM 11.43

KNOWN: Saturation temperature and condensation rate of refrigerant. Frontal area of condenser and dependence of overall coefficient on inlet velocity. Operational range of the air inlet temperature.

FIND: (a) Required heat exchanger area and air outlet temperature for prescribed air inlet velocity and temperature, (b) Variation in air velocity needed to achieve prescribed condensation rate.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Constant properties.

PROPERTIES: Given (R-12): $h_{fg} = 1.35 \times 10^5$ J/kg. *Table A-4*, air ($T_{c,i}$ = 303 K): $\rho_c = 1.17$ kg/m^3, $c_{p,c}$ = 1007 J/kg·K.

ANALYSIS: (a) With $\dot{m}_c = \rho_c V A_{fr} = 1.17\,\text{kg/m}^3 \times 2\,\text{m/s} \times 0.25\,\text{m}^2 = 0.585\,\text{kg/s}$,

$C_{min} = \dot{m}_c c_{p,c} = 589\,\text{W/K}$. Hence, from Eq. (11.19), with $T_{h,i} = T_{sat}$,

$$q_{max} = C_{min}\left(T_{h,i} - T_{c,i}\right) = 589\,\text{W/K}\left(45-30\right)\text{K} = 8,836\,\text{W}$$

and with $q = \dot{m}_h h_{fg} = 0.015\,\text{kg/s} \times 1.35 \times 10^5\,\text{J/kg} = 2025\,\text{W}$

$$\varepsilon = \frac{q}{q_{max}} = \frac{2025}{8836} = 0.229$$

From Eq. 11.36b we then obtain (for $C_r = 0$),

$$A = \frac{C_{min}}{U}NTU = -\frac{C_{min}}{U}\ln\left(1-\varepsilon\right) = -\frac{589\,\text{W/K}}{50\,\text{W/m}^2\cdot\text{K}}\ln\left(0.771\right) = 3.067\,\text{m}^2 \qquad <$$

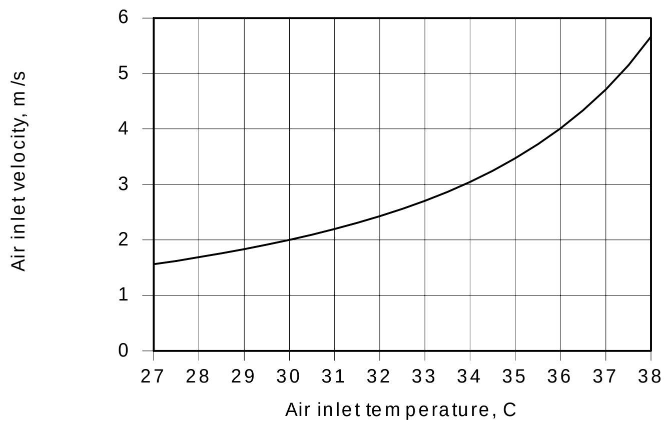
With $q = C_{min}\,(T_{c,o} - T_{c,i})$, the outlet temperature is

$$T_{c,o} = T_{c,i} + \frac{q}{C_{min}} = 30°\text{C} + \frac{2025\,\text{W}}{589\,\text{W/K}} = 33.4°\text{C} \qquad <$$

(b) With q = 2025 W, A = 3.06 m^2 and U = 50 W/m^2·K $(V/2)^{0.7}$, the foregoing equations may be solved to obtain V as a function of $T_{c,i}$.

Continued…..

PROBLEM 11.43 (Cont.)



With increasing $T_{c,i}$, the driving potential for heat transfer, $T_{h,i} - T_{c,i}$, decreases and a larger value of U, and hence V, is needed to maintain the required heat rate. For $27 \le T_{c,i} \le 38°C$, $1.56 \le V \le 5.66$ m/s and $42.1 \le U \le 103.6$ W/m^2·K.

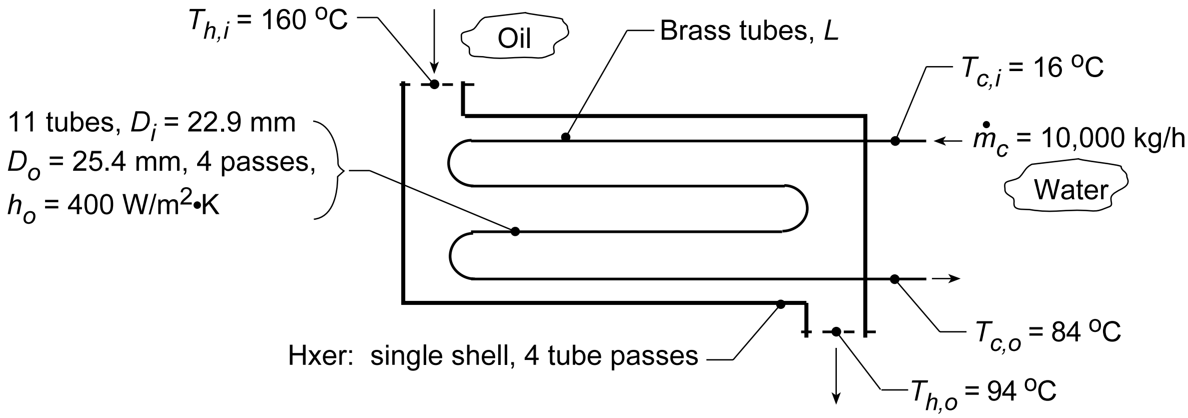
COMMENTS: The variation of V with $T_{c,i}$ is nonlinear, and, in principle, $V \to \infty$ as $T_{c,i} \to T_{sat}$.

PROBLEM 11.44

KNOWN: Conditions of oil and water for heat exchanger, one shell with 4 tube passes.

FIND: Length of exchanger tubes per pass, L; and (b) Compute and plot the effectiveness, ε, fluid outlet temperatures, $T_{h,o}$ and $T_{c,o}$, and water-side convection coefficient, h_c, as a function of the water flow rate for $5000 \le \dot{m}_c \le 15{,}000$ kg / h for the tube length found in part (a) with all other conditions remaining the same.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible kinetic and potential energy changes, (3) Constant properties, (4) Fully-developed flow in tubes.

PROPERTIES: *Table A-1*, Brass (400 K): k = 137 W/m·K; *Table A-5*, Water (323 K): ρ = 998.1 kg/m^3, k = 0.643 W/m·K, c_p = 4182 J/kg·K, μ = 548 × 10^{-6} N·s/m^2, Pr = 3.56.

ANALYSIS: (a) From an energy balance on the water, the heat rate required is

$$q = \dot{m}_c c_c \left(T_{c,o} - T_{c,i}\right) = 10{,}000/3600\,\text{kg/s} \times 4182\,\text{J/kg}\cdot\text{K}\,(84-16)^\circ\text{C} = 789{,}933\,\text{W}. \qquad (1)$$

The required tube length may be obtained from Eqs. 11.14 and 11.15,

$$q = U_o A_o F \Delta T_{\ell m,CF} \qquad (2)$$

$$\Delta T_{\ell m,CF} = \left[(160-84)^\circ\text{C} - (94-16)^\circ\text{C}\right] / \ell n\,(160-84/94-16) = 77.0^\circ\text{C}.$$

From Fig. 11.10, F = 0.86 using P = (84 - 16)/(160 - 16) = 0.47 and R = (160 - 94)/(84 - 16) = 0.97. From Eq. 11.5,

$$U_o = \left[\frac{1}{h_o} + \frac{r_o}{k}\ell n\frac{r_o}{r_i} + \frac{r_o}{r_i}\frac{1}{h_i}\right]^{-1}$$

where h_i must be estimated from the appropriate correlation. With N = 11, the number of tubes,

$$Re_D = \frac{4\dot{m}/N}{\pi D \mu} = \frac{4\times(10{,}000/3600)\,\text{kg/s}/(11)}{\pi \times 22.9\times10^{-3}\,\text{m}\times 548\times10^{-6}\,\text{N}\cdot\text{s/m}^2} = 25{,}621.$$

For fully developed turbulent flow, the Dittus-Boelter correlation with n = 0.4 yields

$$Nu_D = h_i D/k = 0.023\,Re_D^{0.8}\,Pr^{0.4} = 0.023(25{,}621)^{0.8}(3.56)^{0.4} = 128.6$$

$$h_i = Nu_D\,(k/D) = 128.6\times 0.643\,\text{W/m}\cdot\text{K}/\left(22.9\times10^{-3}\,\text{m}\right) = 3610\,\text{W/m}^2\cdot\text{K}.$$

Continued...

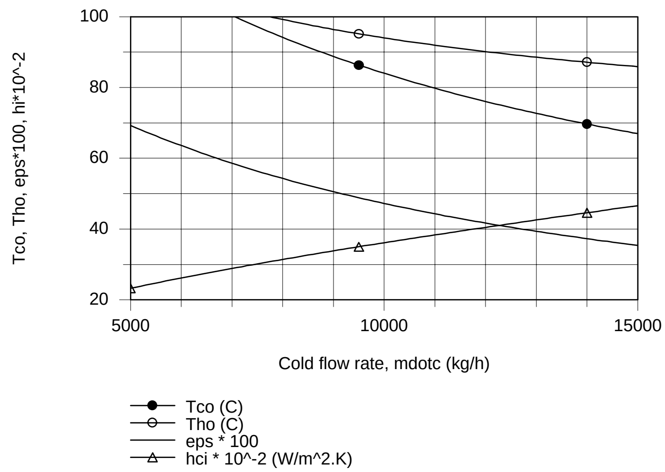
$$U_o = \left[\frac{1}{400\,\mathrm{W/m^2 \cdot K}} + \frac{25.4\times10^{-3}\,\mathrm{m}}{2\times137\,\mathrm{W/m\cdot K}} \ln\frac{25.4}{22.9} + \frac{25.4}{22.9}\times\frac{1}{3610\,\mathrm{W/m^2\cdot K}} \right]^{-1} = 355\,\mathrm{W/m^2\cdot K}\,.$$

Returning now to Eq. (2), find A_o, then the length,

$$A_o = \pi D_o L \times \text{No. of Passes} \times \text{No. of Tubes} = \pi\times25.4\times10^{-3}\,\mathrm{m}\times4\times11\,L = 3.511\,L$$

$$L = 789{,}933\,\mathrm{W} \big/ 3.511\,\mathrm{m}\times355\,\mathrm{W/m^2\cdot K}\times0.86\times77.0^\circ\,\mathrm{C} = 9.6\,\mathrm{m}$$ <

(b) Using the *IHT Heat Exchanger Tool, Shell and Tube, One-shell pass* and *N tube passes,* the *Correlation Tool, Forced Convection, Internal Flow* for *Turbulent, fully developed condition,* and the *Properties Tool* for *Water,* a model was developed using the effectiveness - NTU method to compute and plot $T_{c,o}$, $T_{h,o}$, ε, and h_i as a function of $\dot{m}_c$.



In order to avoid a boiling condition in the cold fluid, the cold flow rate should not be less than 8000 kg/h. As expected, $T_{c,o}$ and $T_{h,o}$ decrease and the internal convection coefficient increases nearly linearly with increasing flow rate. The effectiveness increases with increasing flow rate since the overall convection coefficient is increasing.

COMMENTS: (1) The thermal resistance of the brass tubes is negligible. Since $L/D_i = 400$, fully-developed conditions are reasonable.

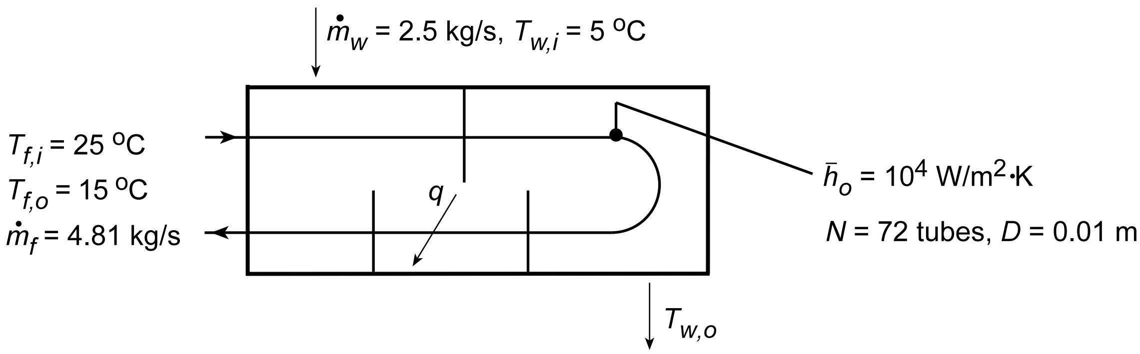
(2) In the analysis of part (b), you have to specify the capacity rate for the hot fluid in order to solve the model. From the analysis of part (a) using the model, we found L = 9.56 m and C_h = 11,974 W/K.

PROBLEM 11.45

KNOWN: Properties and flow rate of computer coolant. Diameter and number of heat exchanger tubes. Heat exchanger transfer rate and inlet temperature of computer coolant. Flow rate, specific heat, inlet temperature, and average convection coefficient of water.

FIND: (a) Tube flow convection coefficient, $\bar{h}_i$, (b) Tube length/pass required to achieve prescribed fluid outlet temperature, (c) Compute and plot the dielectric fluid outlet temperature, $T_{f,o}$, as a function of its flow rate $\dot{m}_f$ for the range $4 \le \dot{m}_f \le 6$ kg/s based upon the length/pass found in part (c), (d) the effect of ±10% change in the water flow rate, $\dot{m}_w$, on $T_{f,o}$ and (e) the effect of ±3°C change in inlet water temperature, $T_{w,i}$, on $T_{f,o}$. For parts (c, d, e), account for any changes in the overall convection coefficient, while all other conditions remain the same.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat loss to surroundings, kinetic and potential energy changes, fouling and tube wall resistance; (2) Constant properties; (3) Fully developed flow, (4) Convection coefficient on shell side, $\bar{h}_o$, remains constant for all operating conditions.

PROPERTIES: Coolant (given): c_p = 1040 J/kg·K, $\mu = 7.65 \times 10^{-4}$ kg/s·m, k = 0.058 W/m·K, Pr = 14; Water (given): c_p = 4200 J/kg·K.

ANALYSIS: (a) For flow through a single tube,

$$\mathrm{Re_D} = \frac{4\dot{m}_{f,t}}{\pi D \mu} = \frac{4(4.81\,\mathrm{kg/s})/72}{\pi(0.01\mathrm{m})7.65\times10^{-4}\,\mathrm{kg/s\cdot m}} = 11{,}120$$

and using the Dittus-Boelter correlation, find

$$h_i = (k/D)0.023\,\mathrm{Re}_D^{4/5}\,\mathrm{Pr}^{0.3} = 0.023\frac{0.058\,\mathrm{W/m\cdot K}}{0.01\mathrm{m}}(11{,}120)^{4/5}(14)^{0.3} = 508\,\mathrm{W/m^2\cdot K}\,. \quad <$$

(b) Find the capacity ratio

$$C_f = \dot{m}_f c_{p,f} = 4.81\,\mathrm{kg/s}(1040\,\mathrm{J/kg\cdot K}) = 5002\,\mathrm{W/K} = C_{min}$$

$$C_w = \dot{m}_w c_{p,w} = 2.5\,\mathrm{kg/s}(4200\,\mathrm{J/kg\cdot K}) = 10{,}500\,\mathrm{W/K} = C_{max}$$

hence, $C_r = C_{min}/C_{max} = 0.476$ and

$$\varepsilon = \frac{q}{q_{max}} = \frac{C_f\left(T_{f,i} - T_{f,o}\right)}{C_f\left(T_{f,i} - T_{w,i}\right)} = \frac{(25-15)^\circ\mathrm{C}}{(25-5)^\circ\mathrm{C}} = 0.500\,.$$

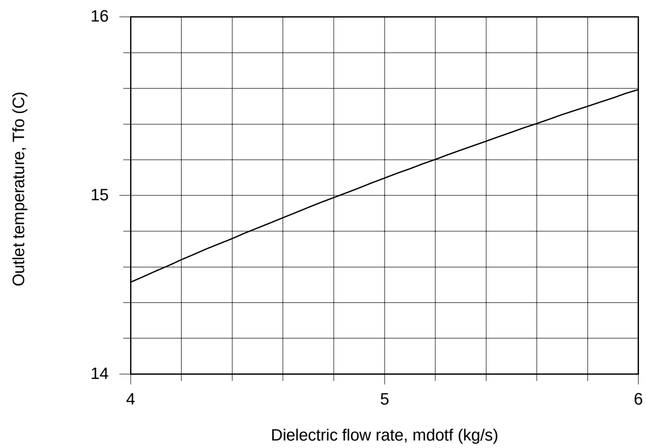
Using Fig. 11.16 with NTU = $(UA/C_{min}) = (UN\pi D2L/C_{min}) \approx 0.85$,

Continued...

$$U = \left(h_i^{-1} + h_o^{-1}\right)^{-1} = \left[(508)^{-1} + \left(10^4\right)^{-1}\right]^{-1} \text{W/m}^2 \cdot \text{K} = 483\,\text{W/m}^2 \cdot \text{K}$$

$$L = 0.85(5002\ \text{W/K}) \Big/ 144\pi\left(483\,\text{W/m}^2 \cdot \text{K}\right)0.01\text{m} = 1.95\text{m}\,.$$ <

(c) Using the *IHT Heat Exchanger Tool*, *Shell and Tube*, *One-shell pass and N-tube passes*, and the *Correlation Tool*, *Forced Convection*, *Internal Flow* for *Turbulent*, *fully developed conditions*, a model was developed using the effectiveness-NTU method employed above to compute and plot $T_{f,o}$ as a function of $\dot{m}_f$.



A change in the dielectric fluid flow rate of ±1 kg/s causes approximately ±0.5°C change in its outlet temperature.

(d) Using the above IHT model with the base conditions for part (c), the effect of a ±10% change in the water flow rate from its design value, $\dot{m}_w$ = 2.5 kg/s (2.25 ≤ $\dot{m}_w$ ≤ 2.75 kg/s) causes the dielectric fluid outlet temperature to change as

$$T_{f,o} = 15 \pm 0.14^\circ\text{C}$$ <

(e) Using the IHT model of part (c) with the base case conditions for part (c), the effect of a ±3°C in the water inlet temperature from its design value, $T_{c,i}$ = 5°C (2 ≤ $T_{c,i}$ ≤ 8°C) cause the dielectric fluid outlet temperature to change as

$$T_{f,o} = 15 \pm 1.5^\circ\text{C}$$ <

COMMENTS: (1) For the analyses of part (a), Eq. 11.31 yields NTU = 0.85 and q = 50 kW and $T_{w,o}$ = 9.76°C.

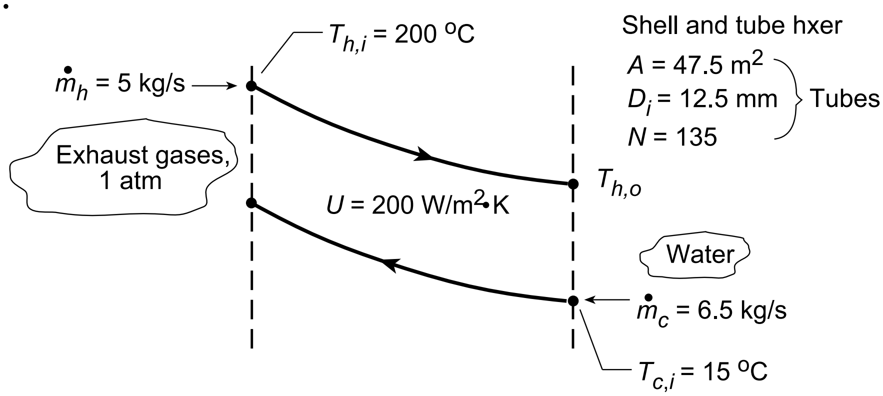
(2) The results of the analyses provide operating performance information on the effect of changes due to dielectric fluid flow rate (±1 kg/s of $\dot{m}_f$), water fluid flow rate (≤ 10% of $\dot{m}_w$) and water inlet temperature (±3°C of $T_{w,i}$) on the dielectric fluid outlet temperature, $T_{f,o}$, supplied to the computer. The greatest effect on $T_{f,o}$, is that by the input water temperature.

PROBLEM 11.46

KNOWN: Shell and tube heat exchanger with 135 tubes (one shell, double pass) of inner diameter 12.5 mm and surface area 47.5 m^2.

FIND: (a) Exchanger gas and water outlet temperatures, (b) Tube heat transfer coefficient, $\bar{h}_i$, assuming fully developed flow, (c) Compute and plot the effectiveness and fluid outlet temperatures, $T_{c,o}$ and $T_{h,o}$ for the water flow rate range $6 \le \dot{m}_c \le 12$ kg/s with all other conditions remaining the same, and (d) Hot gas inlet temperature, $T_{h,i}$, required to supply 10 kg/s of hot water with an outlet temperature of 42°C with all other conditions the same; determine also the effectiveness.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat lost to surroundings, (2) Negligible kinetic and potential energy changes, (3) Fully-developed conditions for internal flow of water in tubes, (4) Exhaust gas properties are those of air, and (5) The overall coefficient remains unchanged for the operating conditions examined.

PROPERTIES: *Table A-6*, Water ($\bar{T}_c \approx 300$ K): ρ = 997 kg/m^3, c = 4179 J/kg·K, k = 0.613 W/m·K, $\mu = 855 \times 10^{-6}$ N·s/m^2, Pr = 5.83; *Table A-4*, Air (1 atm, $\bar{T}_h \approx 400$ K): ρ = 0.8711 kg/m^3, c = 1014 J/kg·K.

ANALYSIS: (a) Using the ε-NTU method, first find the capacity rates, $C = \dot{m}c$,

$$C_c = 6.5\,\text{kg/s} \times 4179\,\text{J/kg}\cdot\text{K} = 27{,}164\,\text{W/K} \qquad C_h = 5.0\,\text{kg/s} \times 1014\,\text{J/kg}\cdot\text{K} = 5{,}070\,\text{W/K}\,.$$

Recognize that $C_h = C_{min}$ and determine

$$\frac{C_{min}}{C_{max}} = \frac{C_h}{C_c} = \frac{5{,}070}{27{,}164} = 0.19 \qquad NTU = \frac{AU}{C_{min}} = \frac{47.5\text{m}^2 \times 200\,\text{W/m}^2\cdot\text{K}}{5{,}070\,\text{W/K}} = 1.87\,.$$

From Fig. 11.16 for the shell and tube exchanger, find with NTU = 1.87 and $C_{min}/C_{max} = 0.19$ that $\varepsilon \approx 0.78$. From the definition of effectiveness,

$$\varepsilon = \frac{q}{q_{max}} = \frac{C_h\left(T_{h,i} - T_{h,o}\right)}{C_{min}\left(T_{h,i} - T_{c,i}\right)} = \frac{200 - T_{h,o}}{200 - 15} = 0.78 \quad \text{or} \quad T_{h,o} = 55.7^\circ\text{C}\,.$$ <

From energy balances on the two fluids, $C_h (T_{h,i} - T_{h,o}) = C_c (T_{c,o} - T_{c,i})$, find

$$T_{c,o} = T_{c,i} + \left(C_h/C_c\right)\left(T_{h,i} - T_{h,o}\right) = 15^\circ\text{C} + 0.19\left(200 - 55.7\right)^\circ\,\text{C} = 42.4^\circ\text{C}\,.$$ <

(b) To estimate $\bar{h}_i$ for the water, find first the Reynolds number. From Eq. 8.6,

Continued...

PROBLEM 11.46 (Cont.)

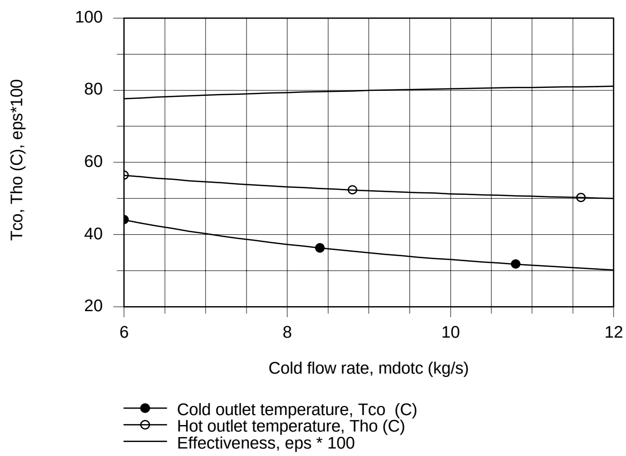
$$\mathrm{Re}_{D_i} = \frac{4\dot{m}}{\pi D_i \mu} = \frac{4\dot{m}_c/N}{\pi D_i \mu} = \frac{4 \times 6.5\,\mathrm{kg/s}/135}{\pi\, 12.5 \times 10^{-3}\,\mathrm{m} \times 855 \times 10^{-6}\,\mathrm{N/s \cdot m^2}} = 5736$$

While the flow is fully developed and turbulent, $\mathrm{Re_D}$ = 10,000 such that Dittus-Boelter correlation is not strictly applicable. However, its use allows a first estimate.

$$\overline{\mathrm{Nu}}_{D_i} = \bar{h}\, D_i / k = 0.023\, \mathrm{Re}_D^{4/5}\, \mathrm{Pr}^{0.4} = 0.023(5736)^{4/5}(5.83)^{0.4} = 47.3$$

$$\bar{h}_i = \overline{\mathrm{Nu}}_D\, k/D_i = 47.3 \times 0.613\,\mathrm{W/m^2 \cdot K}/12.5 \times 10^{-3}\,\mathrm{m} = 2320\,\mathrm{W/m^2 \cdot K}\,. \qquad <$$

(c) Using the *IHT Heat Exchanger Tool*, *Shell and Tube*, *One-shell pass and N-tube passes*, and the prescribed properties, a model was developed following the analysis of part (a) to compute and plot ε, $T_{c,o}$, and $T_{h,o}$ for a function of $\dot{m}_c$.



The outlet temperatures decrease nearly linearly with increasing cold fluid flow rate; the decrease in the cold outlet temperature is nearly twice that of the hot fluid. The change in the effectiveness with increasing flow rate is only slightly increased.

(d) Using the above IHT model, the hot inlet temperature $T_{h,i}$, required to provide $\dot{m}_c$ = 10 kg/s with $T_{c,o}$ = 42°C and the effectiveness for this operating condition are

$$T_{h,i} = 74.4^\circ\mathrm{C} \qquad \varepsilon = 0.55 \qquad <$$

COMMENTS: (1) Check that assumptions for $\overline{T}_h$ and $\overline{T}_c$ used in part (a) for evaluation of the fluid properties are satisfactory as $\overline{T}_h = 400.7$ K and $\overline{T}_c = 301.5$ K.

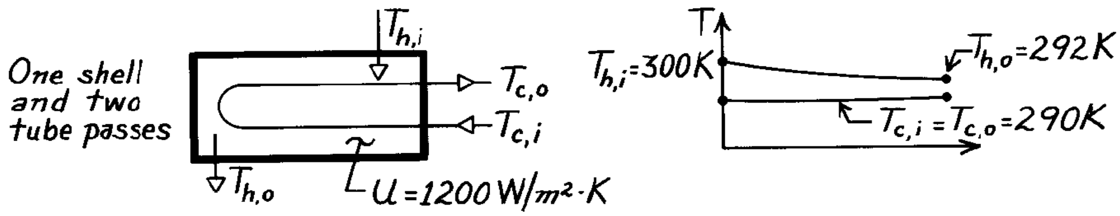
(2) From part (b), with $\bar{h}_i$ = 2320 W/m^2·K and U = 200 W/m^2·K, the shell-side convection coefficient is $\bar{h}_o$ = 219 W/m^2·K. As such, U is controlled by shell-side conditions. Assuming U as a constant in part (c) with changes in $\dot{m}_c$ is therefore reasonable. However, for part (d) with $\dot{m}_h$ doubling, we should expect U to increase.

PROBLEM 11.47

KNOWN: Power output and efficiency of an ocean energy conversion system. Temperatures and overall heat transfer coefficient of shell-and-tube evaporator.

FIND: (a) Evaporator area, (b) Water flow rate.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible kinetic and potential energy changes, (3) Constant properties.

PROPERTIES: *Table A-6*, Water ($\overline{T}_m$ = 296 K): c_p = 4181 J/kg·K.

ANALYSIS: (a) The efficiency is

$$h = \frac{\dot{W}}{q} = \frac{2\text{MW}}{q} = 0.03.$$

Hence the required heat transfer rate is

$$q = \frac{2\text{MW}}{0.03} = 66.7\text{MW}.$$

Also

$$\Delta T_{\ell m,CF} = \frac{(300-290)-(292-290)°C}{\ell n \dfrac{300-290}{292-290}} = 5°C$$

and, with P = 0 and R = ∞, from Fig. 11.10 it follows that F = 1. Hence

$$A = \frac{q}{U F \Delta T_{\ell m,CF}} = \frac{6.67\times10^7\,\text{W}}{1200\ \text{W/m}^2\cdot\text{K}\times1\times5°C}$$

$$A = 11{,}100\text{m}^2. \qquad <$$

(b) The water flow rate through the evaporator is

$$\dot{m}_h = \frac{q}{c_{p,h}\left(T_{h,i}-T_{h,o}\right)} = \frac{6.67\times10^7\,\text{W}}{4181\,\text{J/kg}\cdot\text{K}\,(300-292)}$$

$$\dot{m}_h = 1994\ \text{kg/s}. \qquad <$$

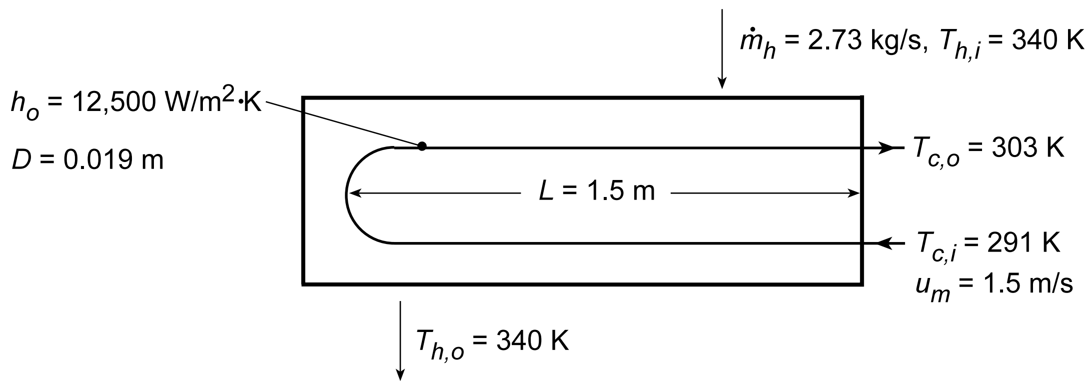
COMMENTS: (1) From the ε-NTU method, $(C_{min}/C_{max}) = 0$, $q_{max} = 8.34 \times 10^7$ W, ε = 0.80 and from Fig. 11.16, NTU ≈ 1.65, giving A = 11,500 m^2. (2) The required heat exchanger size is enormous due to the small temperature differences involved.

PROBLEM 11.48

KNOWN: Length and tube diameter for a shell-and-tube (one shell pass, multiple tube passes) heat exchanger. Flow rate and temperature of saturated steam. Condensation convection coefficient. Velocity and inlet and outlet temperatures of cooling water.

FIND: (a) Required number of tubes and if the heat exchanger length is not to exceed 1.5 m, the number of tube passes; (b) Compute and plot water outlet temperature $T_{c,o}$, and condensation rate, $\dot{m}_h$ as a function of the mean velocity for the range $0.5 \le u_m \le 3$ m/s, for the heat transfer area found from part (a), accounting for changes in the overall coefficient, but all other conditions remaining the same; and (c) Repeat the analysis of part (b) for tube diameters of 15 and 25 mm.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat loss to surroundings and changes in kinetic and potential energy, (2) Negligible tube wall conduction and fouling resistances, (3) Constant properties, and (4) The shell-side coefficient $\bar{h}_o$ remains unchanged for the operating conditions examined.

PROPERTIES: *Table A.6*, Sat. water (340 K): $h_{fg} = 2.342 \times 10^6$ J/kg; Sat. water ($\bar{T}_c$ = 297 K): $\rho = 998$ kg/m^3, $c_p = 4180$ J/kg·K, $\mu = 917 \times 10^{-6}$ kg/s·m, k = 0.609 W/m·K, Pr = 6.3.

ANALYSIS: (a) The required heat rate is

$$q = \dot{m}_h h_{fg} = 2.73\,\text{kg/s}\left(2.342\times10^6\ \text{J/kg}\right) = 6.39\times10^6\,\text{W}\,.$$

Hence, from conservation of energy,

$$\dot{m}_c = q\Big/\left[c_{p,c}\left(T_{c,o} - T_{c,i}\right)\right] = 6.39\times10^6\ \text{W}/4180\,\text{J/kg}\cdot\text{K}\left(12\text{K}\right) = 127.5\,\text{kg/s}\,.$$

Hence the number of tubes is

$$N = \dot{m}_c/\dot{m}_{c,t} = \dot{m}_c\Big/\left(\pi D^2/4\right)\rho u_m = 4\times127.5\,\text{kg/s}/\pi\left(0.019\text{m}\right)^2 998\,\text{kg/m}^3\left(1.5\,\text{m/s}\right) = 300\,. \qquad <$$

To determine the heat transfer surface area A, use the ε - NTU method. Find first

$$Re_D = \rho u_m D/\mu = 998\,\text{kg/m}^3\left(1.5\,\text{m/s}\right)\left(0.019\text{m}\right)\Big/917\times10^{-6}\ \text{kg/s}\cdot\text{m} = 31,017$$

and using the Dittus-Boelter equation,

$$h_i = \left(k/D\right)0.023\,Re_D^{4/5}\,Pr^{0.4} = \left(0.609\,\text{W/m}\cdot\text{K}/0.019\text{m}\right)0.023\left(31,017\right)^{4/5}\left(6.3\right)^{0.4} = 6034\,\text{W/m}^2\cdot\text{K}$$

$$U = \left[1/h_i + 1/h_o\right]^{-1} = \left[\left(1/6034\right) + \left(1/12,500\right)\right]^{-1}\ \text{W/m}^2\cdot\text{K} = 4070\,\text{W/m}^2\cdot\text{K}\,.$$

Continued...

PROBLEM 11.48 (Cont.)

With $C_{min} = \dot{m}_c c_{p,c}$ = 127.5 kg/s(5180 J/kg·K) = 5.33×10^5 W/K

$$\varepsilon = q/q_{max} = q/C_{min}\left(T_{h,i} - T_{c,i}\right) = 6.39\times10^6\ \mathrm{W}\Big/5.33\times10^5\ \mathrm{W/K}\left(49\mathrm{K}\right) = 0.245\,.$$

Hence, with C_r = 0, Eq. 11.36b yields NTU = -ln(1 - ε) = -ln(1 - 0.245) = 0.281,

$$A = NTU\left(C_{min}/U\right) = 0.281\left(5.33\times10^5\ \mathrm{W/K}\Big/4070\ \mathrm{W/m^2\cdot K}\right) = 36.8\mathrm{m}^2$$

The tube length, L, in terms of the number of tubes, N, and passes, P, is

$$L = A/N\cdot P\pi D$$

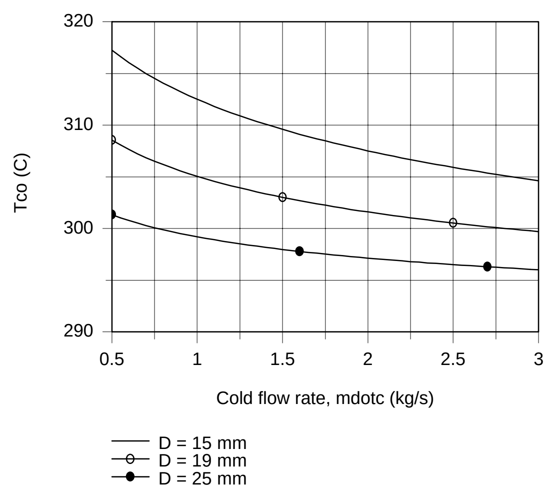
and if P = 2,

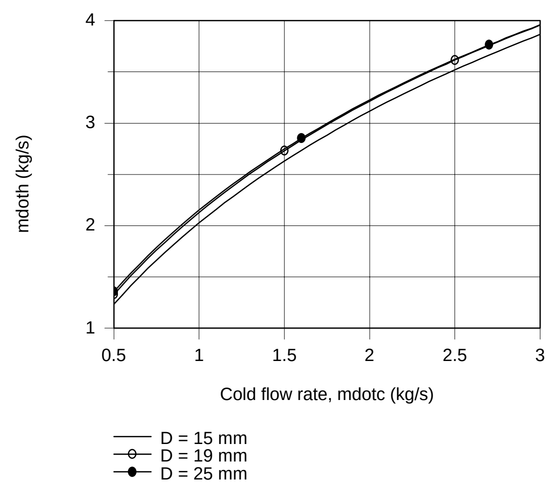
$$L = 36.8\mathrm{m}^2\Big/300\times2\times\pi\times0.019\mathrm{m} = 1.03\,\mathrm{m} \qquad <$$

which is less than the maximum length 1.5 m.

(b) Using the *IHT Heat Exchanger Tool*, *All Exchangers*, *C_r = 0*, the *Properties Tool* for *Water*, and the *Correlations Tool*, *Forced Convection*, *Internal Flow*, for *Turbulent fully developed conditions*, a model was developed following the foregoing analysis to compute $T_{c,o}$ and $\dot{m}_h$ as a function of u_m with A = 36.8 m^2 as determined from part (a). The plot is shown below with the results for part (c).

(c) The IHT model was used to compute and plot $T_{c,o}$ and $\dot{m}_h$ as a function of u_m for tube diameters of 15, 19, and 25 mm.





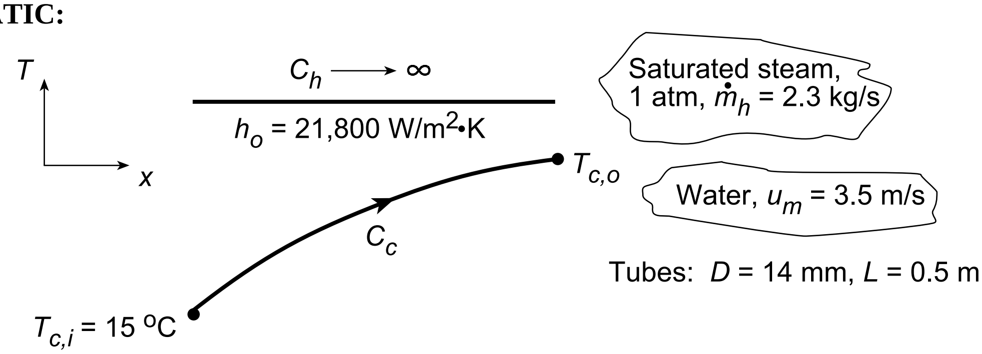
The effect of tube diameter on $T_{c,o}$ as a function of the water flow rate is significant. As D increases at any flow rate, the outlet temperatures decreases. The effect of tube diameter on the condensation rate is slight. However, the condensation rate increases markedly as the water flow rate increases.

PROBLEM 11.49

KNOWN: Shell(1)-and-tube (two passes, p = 2) heat exchanger for condensing saturated steam at 1 atm. Inlet cooling water temperature and mean velocity. Thin-walled tube diameter and length prescribed, as well as, convective heat transfer coefficient on outer tube surface, h_o.

FIND: (a) Number of tubes/pass, N, required to condense 2.3 kg/s of steam, (b) Outlet water temperature, $T_{c,o}$, (c) Maximum condensation rate possible for same water flowrate and inlet temperature, and (d) Compute and plot $T_{c,o}$ and the condensation rate, $\dot{m}_h$, for water mean velocity, u_m, in the range $1 \le u_m \le 5$ ms/, using the heat transfer surface area found in part (a) assuming the shell-side convection coefficient remains unchanged.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible kinetic and potential energy changes, (3) Negligible thermal resistance due to the tube walls.

PROPERTIES: *Table A.6*, Saturated steam (1 atm): $T_{sat} = 100°C$, $h_{fg} = 2257$ kJ/kg; Water (assume $T_{c,o} \approx 25°C$, $\overline{T}_m = (T_h + T_c)/2 \approx 295$ K): $\rho = 1/v_f = 998$ kg/m³, $c_c = c_{p,h} = 4181$ J/kg·K, $\mu = \mu_f = 959 \times 10^{-6}$ N·s/m², $k = k_f = 0.606$ W/m·K, $Pr = Pr_\ell = 6.62$.

ANALYSIS: (a) The heat transfer rate for the heat exchanger is

$$q = \dot{m}_h h_{fg} = 2.3\,\text{kg/s} \times 2257 \times 10^3\,\text{J/kg} = 5.191 \times 10^6\,\text{W} \tag{1}$$

Using the ε-NTU method, evaluate the following parameters:

Water-side heat transfer coefficient:

$$Re_D = \frac{u_m D}{\mu/\rho} = \frac{3.5\,\text{m/s} \times 0.014\,\text{m}}{959 \times 10^{-6}\,\text{N}\cdot\text{s/m}^2 / 998\,\text{kg/m}^3} = 50,993 \tag{2}$$

$$h_i = \frac{k}{D} Nu_D = \frac{k}{D} 0.023 Re_D^{0.8} Pr^{1/3} = \frac{0.606\,\text{W/m}\cdot\text{K}}{0.014\,\text{m}} \times 0.023 (50,993)^{0.8} (6.62)^{1/3} = 10,906\,\text{W/m}^2\cdot\text{K} \tag{3}$$

using the Colburn equation for fully developed turbulent conditions.

Overall coefficient:

$$\overline{U} = (1/h_i + 1/h_o)^{-1} = (1/10,906 + 1/21,800)^{-1} = 7269\,\text{W/m}^2\cdot\text{K} \tag{4}$$

Effectiveness relations: With $C_{min} = C_c$ and $\dot{m}_c = \rho(\pi D^2/4)u_m N$,

$$q = \varepsilon q_{max} = \varepsilon C_{min}\left(T_{h,i} - T_{c,i}\right) \tag{5}$$

$$C_{min} = \dot{m}_c c_c = 998\,\text{kg/m}^3 \left(\pi \times 0.014^2\,\text{m}^2/4\right) \times 3.5\,\text{m/s} \times N \times 4181\,\text{J/kg}\cdot\text{K} = 2248\,N \tag{6}$$

Continued...

$$5.191\times10^6\,\text{W} = \varepsilon\times2248\,\text{N}\left(100-15\right)\text{K}$$

$$\varepsilon N = 27.17 \quad (7)$$

From Eq. 11.36a with $C_r = 0$, the effectiveness is

$$\varepsilon = 1-\exp\left(-\text{NTU}\right) = 1-\exp\left(-0.142\right) = 0.132 \quad (8)$$

where, using $A_s = \pi DLNP$, NTU is evaluated as,

$$\text{NTU} = \frac{\overline{U}A_s}{C_{min}} = \frac{7269\,\text{W/m}^2\cdot\text{K}\left(\pi\times0.014\,\text{m}\times0.5\,\text{m}\right)\text{N}\times2}{2248\,\text{N}} = 0.142$$

Hence, using Eq. (7), the required number of tubes is

$$N = 27.17/\varepsilon = 205.8 \approx 206$$ <

and the total surface area is

$$A_s = \pi DLNP = \pi\times0.014\,\text{m}\times0.5\,\text{m}\times206\times2 = 9.06\,\text{m}^2 .$$

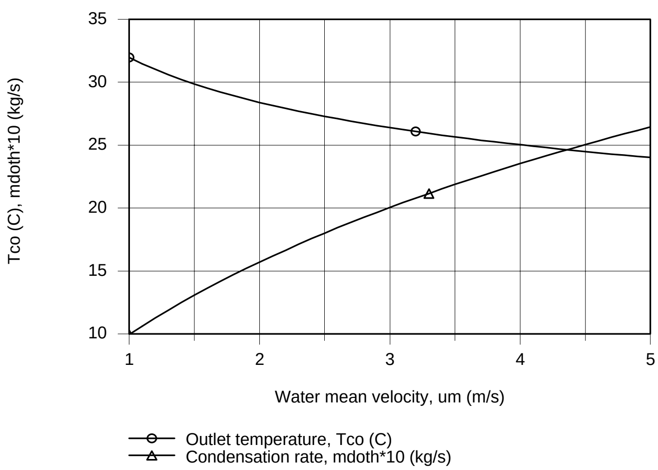
(b) The water outlet temperature with C_{min} = 2248 N = 463,090 W/K,

$$T_{c,o} = T_{c,i} + q/C_{min} = 15^\circ\text{C} + 5.191\times10^6\,\text{W}/463,090\,\text{W/K} = 26.1^\circ\text{C}$$ <

(c) The maximum condensation rate will occur when $q = q_{max}$. Hence

$$\dot{m}_{h,max} = \frac{q_{max}}{h_{fg}} = \frac{C_{min}\left(T_{h,i}-T_{c,i}\right)}{h_{fg}} = \frac{463,090\,\text{W/K}\left(100-15\right)\text{K}}{2257\times10^3\,\text{J/kg}} = 17.44\,\text{kg/s} .$$ <

(d) Using the *IHT Heat Exchanger Tool*, *All Exchangers*, $C_r = 0$, along with the *Properties Tool* for *Water*, the foregoing analysis was performed to obtain $T_{h,o}$ and $\dot{m}_h$ using the heat transfer surface area A_s = 9.06 m^2 (part a) as a function of u_m.



Note that the condensation rate increases nearly linearly with the water mean velocity. The cold water outlet temperature decreases nearly linearly with u_m. We should expect this behavior from energy balance considerations. Since h_h is nearly two times greater than h_c, $\overline{U}$ is controlled by the water side coefficient. Hence $\overline{U}$ will increase with increasing u_m.

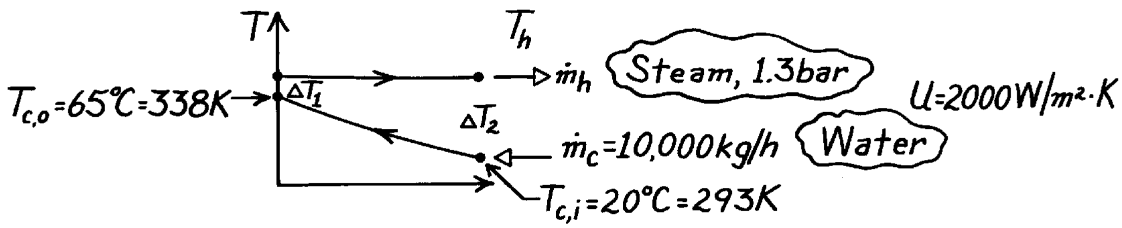
COMMENTS: Note that the assumed value for $\overline{T}_m$ to evaluate water properties in part (a) was a good choice.

PROBLEM 11.50

KNOWN: Feed water heater (single shell, two tube passes) with inlet temperature 20°C supplies 10,000 kg/h of water at 65°C by condensing steam at 1.30 bar. Overall heat transfer coefficient is 2000 W/m^2·K.

FIND: (a) Required area using LMTD and NTU approaches, (b) Steam condensation rate.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible kinetic and potential energy changes, (3) Constant properties.

PROPERTIES: *Table A-6*, Steam (1.3 bar, saturated): T_h = 380.3 K, $h_{fg} = 2238 \times 10^3$ J/kg·K; *Table A-6*, Water ($\overline{T}_c$ = 316 K): c_p = 4179 J/kg·K.

ANALYSIS: (a) Using the *LMTD approach*, from Eqs. 11.14 and 11.18,

$$A = q / U F \Delta T_{\ell m,CF} \qquad \Delta T_{\ell m,CF} = [\Delta T_1 - \Delta T_2] / \ell n (\Delta T_1 / \Delta T_2) \tag{1,2}$$

$$\Delta T_{\ell m,CF} = [(380.3-338)-(380.3-293)]\text{K} / \ell n \frac{(380.3-338)}{(380.3-293)} = 62.1\text{K}.$$

Since T_h is uniform throughout the HXer, F = 1. From an energy balance on the cold fluid,

$$q = \dot{m}_c c_{p,c} (T_{c,o} - T_{c,i}) = \frac{10{,}000}{3600}\frac{\text{kg}}{\text{s}} \times 4179 \frac{\text{J}}{\text{kg}\cdot\text{K}}(338-293)\text{K} = 5.224\times 10^5 \text{W}.$$

Substituting numerical values into Eq. (1) find that

$$A = 5.224\times 10^5 \text{W}/2000\text{W/m}^2\cdot\text{K}\times 1\times 62.1\text{K} = 4.21\text{m}^2. \qquad <$$

Using the *NTU approach*, recognize that $C_{min} = C_c$ and $C_{max} = C_h \rightarrow \infty$ so that $C_{min}/C_{max} = 0$. The effectiveness, defined by Eq. 11.20, is

$$e \equiv \frac{q}{q_{max}} = \frac{C_c (T_{c,o} - T_{c,i})}{C_{min}(T_{h,i} - T_{c,i})} = \frac{(338-293)\text{K}}{(380.3-293)\text{K}} = 0.515.$$

From Fig. 11.16, with ε = 0.52 and $C_{min}/C_{max} = 0$, find NTU ≈ 0.70. Hence,

$$A = C_{min} NTU/U = \frac{(10{,}000)/(3600)\text{kg/s}\times 4179\text{J/kg}\cdot\text{K}\times 0.70}{2000\text{ W/m}^2\cdot\text{K}} = 4.1\text{m}^2. \qquad <$$

(b) The condensation rate of steam is

$$\dot{m}_h = q/h_{fg} = 5.224\times 10^5 \text{W}/\left(2238\times 10^3 \text{J/kg}\right) = 0.233 \text{ kg/s} = 840 \text{ kg/h}. \qquad <$$

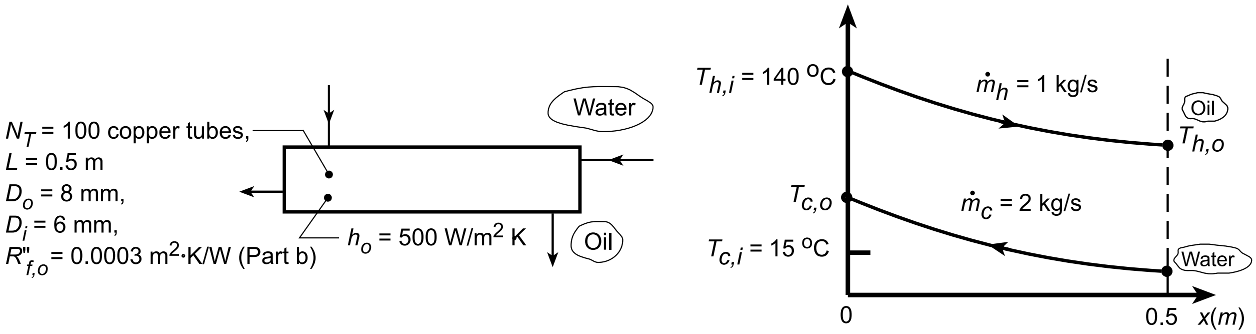
COMMENTS: Note both methods of solution given the same result. Eq. 11.31 could have been used to obtain a more precise NTU value.

PROBLEM 11.51

KNOWN: Shell-and-tube HXer with one shell and one tube pass.

FIND: (a) Oil outlet temperature for prescribed conditions, (b) Effect of fouling and water flowrate on oil outlet temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties, (2) Negligible changes in kinetic and potential energies, (3) Negligible fouling and losses to surroundings, (4) Uniform tube outer surface temperature.

PROPERTIES: *Table A.5*, Engine oil ($\overline{T}_h \approx 350$ K): $\rho_h = 854$ kg/m^3, $c_{p,h} = 2118$ J/kg·K, $\mu_h = 0.0356$ N·s/m^2, $k_h = 0.318$ W/m·K, $Pr_h = 546$; ($\overline{T}_s \approx 330$ K): $\mu_s = 0.0836$ N·s/m^2; *Table A.6*, Water ($\overline{T}_c \approx 320$ K): $c_{p,c} = 4180$ J/kg·K; *Table A.1*, Copper ($\overline{T} \approx 320$ K): k = 399 W/m·K.

ANALYSIS: (a) To determine the outlet temperature of the oil, we will need to know the overall heat transfer coefficient. From Eq. 11.5,

$$\frac{1}{UA} = \frac{1}{h_i A_i} + \frac{\ln(D_o/D_i)}{2\pi k L_t} + \frac{R''_{f,o}}{A_o} + \frac{1}{h_o A_o} \qquad (1)$$

where $h_o = 500$ W/m^2·K (water-side) and h_i (oil-side) must be estimated from an appropriate correlation. Using properties evaluated at an estimated average mean temperature $\overline{T}_h \approx 350$ K, find

$$Re_{D,h} = \frac{4\dot{m}_{h,1}}{\pi D_i \mu_h} = \frac{4\times(1\,\text{kg/s}/100)}{\pi(0.006\,\text{m})\times 0.0356\,\text{N}\cdot\text{s/m}^2} = 59.6\,. \qquad (2)$$

Since $Re_D < 2300$, the flow is laminar. To assess flow conditions, evaluate

$$Gz^{-1} = \frac{L/D_i}{Re_{D,h}\,Pr_h} = \frac{0.5\,\text{m}/0.006\,\text{m}}{59.6\times 546} = 0.00256 \qquad (3)$$

Since $Gz^{-1} < 0.05$, the flow is characterized by combined entry length conditions (Fig. 8.9), and

$$\overline{Nu}_D = 1.86\left[\left(\frac{Re_D\,Pr}{L/D}\right)^{1/3}\left(\frac{\mu}{\mu_s}\right)^{0.14}\right] \qquad (4)$$

where [ ] ≥ 2. To evaluate μ_s, assume $\overline{T}_s \approx 330$ K. Hence,

$$\overline{Nu}_D = 1.86\left[(0.00256)^{-1/3}\left(\frac{0.0356}{0.0836}\right)^{0.14}\right] = 1.86\times 6.48 = 12.1$$

Note that [ ] = 6.48 > 2 as required. Hence,

Continued...

$$\bar{h}_i = \overline{Nu}_D \frac{k}{D} = 12.1 \times 0.138\,W/m\cdot K/(0.006\,m) = 277\,W/m^2\cdot K\,.$$

With $R''_{f,o}$ = 0 and $L_t = N_t L$, Eq. 1 yields

$$\frac{1}{UA} = \frac{1}{\pi N_t L}\left(\frac{1}{h_i D_i} + \frac{\ln(D_o/D_i)}{2k} + \frac{1}{h_o D_o}\right) \qquad (5)$$

$$\frac{1}{UA} = \frac{1}{\pi \times 100 \times 0.5\,m}\left[1/\left(500\,W/m^2\cdot K \times 0.008\,m\right) + \ln(8/6)/(2 \times 399\,W/m\cdot K) + 1/\left(277\,W/m^2\cdot K \times 0.006\,m\right)\right]$$

$$\frac{1}{UA} = 6.366 \times 10^{-3}\left[0.2500 + 0.0003 + 0.6017\right] = 5.424 \times 10^{-3}\,K/W$$

$$UA = 184\ W/K$$

With knowledge of UA, we can now use the ε - NTU method to obtain the oil outlet temperature, $T_{h,o}$. Find the capacity rates, C = $\dot{m}c_p$,

$$C_c = \dot{m}_c c_{p,c} = 2\,kg/s \times 4180\,J/kg\cdot K = 8360\,W/K = C_{max}$$

$$C_h = \dot{m}_h c_{p,h} = 1\,kg/s \times 2118\,J/kg\cdot K = 2118\,W/K = C_{min}$$

$$C_r = C_{min}/C_{max} = 2118/8360 = 0.253$$

From Eq. 11.25, find

$$NTU = UA/C_{min} = 184\,W/K/(2118\,W/K) = 0.0869\,. \qquad (6)$$

For this exchanger - one shell and one pass - there are no figures (11.14-19) or relations (Table 11.3) that can be directly used to evaluate ε. However, the HXer approximates a CF concentric tube HXer; hence, use Eq. 11.30a.

$$\varepsilon = \frac{1 - \exp\left[-NTU(1 - C_r)\right]}{1 - C_r \exp\left[-NTU(1 - C_r)\right]} = \frac{1 - \exp\left[-0.0869(1 - 0.253)\right]}{1 - 0.253\exp\left[-0.0869(1 - 0.253)\right]} = 0.0824 \qquad (7)$$

From the definition of effectiveness,

$$\varepsilon = \frac{q}{q_{max}} = \frac{C_h\left(T_{h,i} - T_{h,o}\right)}{C_{min}\left(T_{h,i} - T_{c,i}\right)}$$

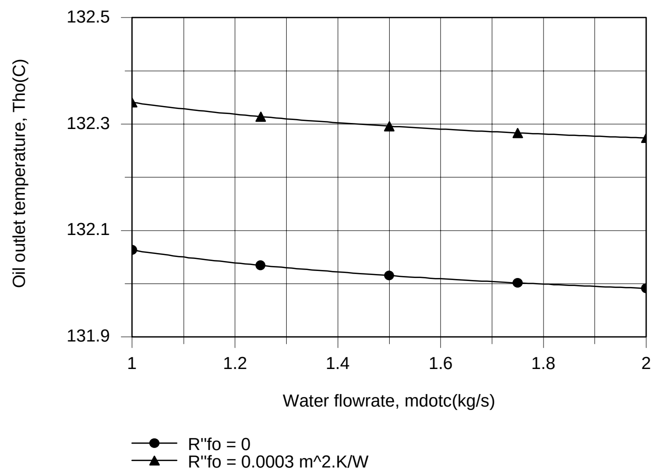
$$T_{h,o} = T_{h,i} - \varepsilon\left(T_{h,i} - T_{c,i}\right) = 140^\circ C - 0.0824(140 - 15)^\circ C = 129.7^\circ C \qquad <$$

The foregoing result indicates that $\bar{T}_h$ ≈ 408 K, which is much larger than the assumed value of 350 K. Since the properties of oil depend strongly on temperature, they should be re-evaluated and the foregoing calculations repeated until convergence is achieved. Using the *Correlations*, *Properties* and *Heat Exchangers* Toolpads of IHT, we obtain h_i = 226 W/m^2·K, UA = 159 W/K, ε = 0.064, and $T_{h,o}$ = 132°C.

Continued …..

PROBLEM 11.51 (Cont.)

(b) If the foregoing calculations are repeated with $R''_{f,o}$ = 0.0003 $m^2 \cdot K/W$, there is only a slight increase in the oil outlet temperature to $T_{h,o}$ = 132.3°C. The effect is small because the fouling resistance is approximately an order of magnitude smaller than the convection resistances. As shown below,



the effect of the water flowrate is also small, because, even for $\dot{m}_c$ = 1 kg/s, $T_{c,o}$ is only approximately 4.5°C larger than $T_{c,i}$. Although the effect of $\dot{m}_c$ on h_o has not been considered, it would also be small since the water-side convection resistance is substantially larger than the oil side resistance.

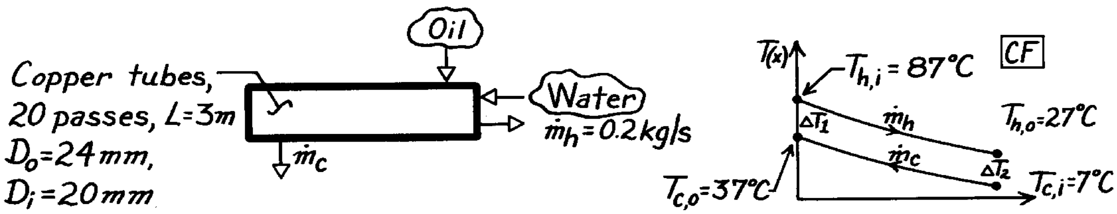
COMMENTS: In Part (a), note that the Nusselt number for the oil entrance region flow is 12.1/3.66 ≈ 3.3 times that for fully developed flow.

PROBLEM 11.52

KNOWN: Shell-and-tube heat exchanger with one shell pass and 20 tube passes.

FIND: Average convection coefficient for the outer tube surface.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible changes in kinetic and potential energies, (3) Constant properties, (4) Type of oil not specified, (5) Thermal resistance of tubes negligible; no fouling.

PROPERTIES: *Table A-6*, Water, liquid ($\overline{T}_h$ = 330 K): c_p = 4184 J/kg·K, k = 0.650 W/m·K, $\mu = 489 \times 10^{-6}$ N·s/m^2, Pr = 3.15.

ANALYSIS: To find the average coefficient for the outer tube surface, h_o, we need to evaluate h_i for the internal tube flow and U, the overall coefficient. From Eq. 11.5,

$$\frac{1}{UA} = \frac{1}{h_i A_i} + \frac{1}{h_o A_o} = \frac{1}{N_t \pi L}\left[\frac{1}{h_i D_i} + \frac{1}{h_o D_o}\right]$$

where N_t is the total number of tubes. Solving for h_o,

$$h_o = D_o^{-1}\left[(UA)^{-1} N_t \pi L - 1/h_i D_i\right]^{-1}. \qquad (1)$$

Evaluate h_i from an appropriate correlation; begin by calculating the Reynolds number.

$$Re_{D,i} = \frac{4\,\dot{m}_h}{\pi D_i \mu} = \frac{4 \times 0.2 \text{ kg/s}}{\pi\,(0.020\text{m})\,489 \times 10^{-6}\,\text{N}\cdot\text{s/m}^2} = 26{,}038.$$

Hence, flow is turbulent and since L >> D_i, the flow is likely to be fully developed. Use the Dittus-Boelter correlation with n = 0.3 since $T_s < T_m$, $Nu_D = 0.023\ Re_D^{4/5}\ Pr^{0.3}$

$$h_i = \frac{k}{D} Nu_D = \frac{0.650 \text{ W/m}\cdot\text{K}}{0.020\text{m}} \times 0.023(26{,}038)^{4/5}(3.15)^{0.3} = 3594 \text{ W/m}^2\cdot\text{K}. \qquad (2)$$

To evaluate UA, we need to employ the rate equation, written as

$$UA = q/F\Delta T_{\ell n,CF} \qquad (3)$$

where $q = \dot{m}_h\ c_{p,h}\ (T_{h,i} - T_{h,o})$ = 0.2 kg/s × 4184 J/kg·K) (87-27)°C = 50,208 W and $\Delta T_{\ell n,CF}$ = $[\Delta T_1 - \Delta T_2]/\ell n\ (\Delta T_1/\Delta T_2)$ = [(87 – 37) – (27 – 7)]°C/ ℓn (87 – 37/27 – 7) = 32.7°C. Find F ≈ 0.5 using Fig. 11.10 with P = (27 – 87)/(7 – 87) = 0.75 and R = (7 – 37)/(27 – 87) = 0.50. Substituting numerical values in Eqs. (3) and (1), find

$$UA = 50{,}208\,\text{W}/0.5 \times 32.7°\text{C} = 3071\,\text{W/K} \qquad (4)$$

$$h_o = (0.024\text{m})^{-1}\left[(3071 \text{ W/K})^{-1} \times 20 \times \pi \times 3\text{m} - 1/3594 \text{ W/m}^2\cdot\text{K} \times 0.020\text{m}\right]^{-1} = 878 \text{ W/m}^2\cdot\text{K}. \quad <$$

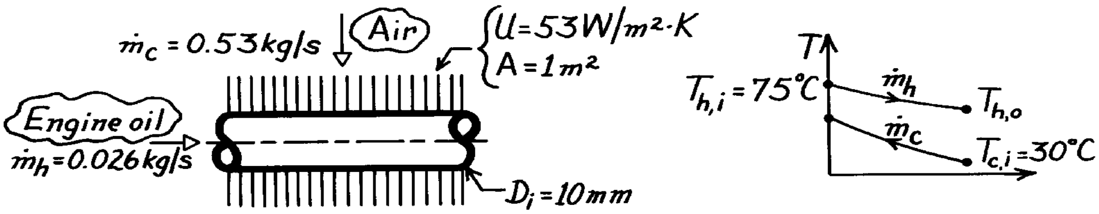
COMMENTS: Using the ε-NTU method: find C_h and C_c to obtain C_r = 0.5 and ε = 0.75. From Eq. 11.31b,c find NTU = 3.59 and UA = 3003 W/K.

PROBLEM 11.53

KNOWN: Engine oil cooled by air in a cross-flow heat exchanger with both fluids unmixed.

FIND: (a) Heat transfer coefficient on oil side of exchanger assuming fully-developed conditions and constant wall heat flux, (b) Effectiveness, and (c) Outlet temperature of the oil.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible KE and PE changes, (3) Constant properties, (4) Oil flow and thermal conditions are fully developed, (5) Oil cooling process approximates constant wall flux conditions.

PROPERTIES: *Table A-5*, Engine oil (assume $T_{h,o} \approx 45°C$, $\overline{T}_h = (45 + 75)°C/2 = 333$ K): $c_h = 2047$ J/kg·K, $\mu = 7.45 \times 10^{-2}$ N·s/m^2, k = 0.140 W/m·K; *Table A-4*, Air (assume $T_{c,o} \approx 40°C$, $\overline{T}_c = (30 + 40)°C/2 = 308$ K, 1 atm): $c_c = 1007$ J/kg·K.

ANALYSIS: (a) For the oil side, using Eq. 8.6, find,

$$\mathrm{Re}_D = 4\,\dot{m}/p\,D\,m = 4(0.026\ \mathrm{kg/s})/\left(p\,(0.01\mathrm{m})7.45\times10^{-2}\mathrm{N\cdot s/m^2}\right) = 44.4$$

Since $\mathrm{Re}_D < 2000$ the flow is laminar. For the fully-developed conditions with constant wall flux,

$$\mathrm{Nu}_D = \frac{h_i D}{k} = 4.36, \qquad h_i = 4.36\frac{k}{D} = 4.36\frac{0.140\ \mathrm{W/m\cdot K}}{0.01\mathrm{m}} = 61.0\ \mathrm{W/m^2\cdot K}. \qquad <$$

(b) The effectiveness can be determined by the ε-NTU method.

$$C_h = \dot{m}_h c_h = 0.026\ \mathrm{kg/s} \times 2047\ \mathrm{J/kg\cdot K} = 53.22\ \mathrm{W/K} \qquad C_{min} = C_h$$

$$C_c = \dot{m}_c c_c = 0.53\ \mathrm{kg/s} \times 1007\ \mathrm{J/kg\cdot K} = 533.7\ \mathrm{W/K} \qquad C_{min}/C_{max} = 0.10$$

$$\mathrm{NTU} = UA/C_{min} = 53\ \mathrm{W/m^2\cdot K} \times 1\mathrm{m^2}/53.22\ \mathrm{W/K} = 1.00.$$

Using Fig. 11.18, with $C_{min}/C_{max} = 0.1$ and NTU = 1, find $\varepsilon \approx 0.64$. <

(c) From Eqs. 11.20 and 11.19,

$$e = \frac{q}{q_{max}} = \frac{C_h\left(T_{h,i} - T_{h,o}\right)}{C_{min}\left(T_{h,i} - T_{c,i}\right)} = \frac{T_{h,i} - T_{h,o}}{T_{h,i} - T_{c,i}}.$$

Solving for $T_{h,o}$ and substituting numerical values, find

$$T_{h,o} = T_{h,i} - e\left(T_{h,i} - T_{c,i}\right) = 75°C - 0.64(75 - 30)°C = 46.2°C. \qquad <$$

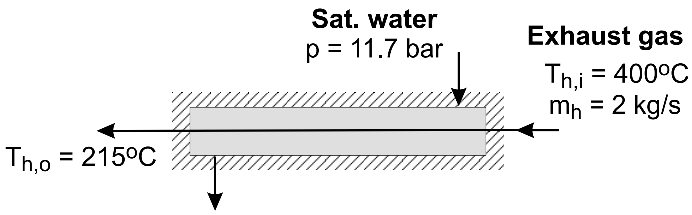
COMMENTS: Note that the $\overline{T}_h$ value at which the oil properties were evaluated is reasonable.

PROBLEM 11.54

KNOWN: Shell-tube heat exchanger with one shell and single tube pass; Tube side: exhaust gas with specified flow rate and temperature change; Shell side: supply of saturated water at 11.7 bar; Tube dimensions and thermal conductivity, and fouling resistance on gas side, $R''_{f,h}$, specified.

FIND: Number of tubes and their length if the gas velocity is not to exceed $u_{m,i}$ = 25 m/s.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible losses to the surroundings and kinetic and potential energy changes, (3) Negligible water-side thermal resistance, (4) Exhaust gas properties are those of atmospheric air, (5) Gas-side flow is fully developed, and (6) Constant properties.

PROPERTIES: *Table A-4*, Air $\left(\overline{T}_h = 581\ \text{K}\right)$: $\rho = 0.600\ \text{kg/m}^3$, $c = 1047\ \text{J/kg}\cdot\text{K}$, $\nu = 4.991\times10^{-5}\ \text{m}^2/\text{s}$, $k = 0.0457\ \text{W/m}\cdot\text{K}$, $\text{Pr} = 0.684$. *Table A-6*, Water (11.7 bar, saturated): $T_{c,i} = 460\ \text{K} = 187^\circ\text{C}$.

ANALYSIS: We'll employ the NTU-ε method to design the exchanger. Since $C_r = 0$, use Eq. 11.36b.

$$\text{NTU} = -\ell n(1-\varepsilon)$$

where the effectiveness can be evaluated from Eqs. 11.19 and 11.20.

$$C_{min} = C_h = \dot{m}_h c_h = 2\ \text{kg/s}\times1047\ \text{J/kg}\cdot\text{K} = 2094\ \text{W/K}$$

$$\varepsilon = \frac{C_h\left(T_{h,i}-T_{h,o}\right)}{C_{min}\left(T_{h,i}-T_{c,i}\right)} = \frac{(400-215)^\circ\text{C}}{(400-187)^\circ\text{C}} = 0.868$$

$$\text{NTU} = -\ell n(1-0.868) = 2.029$$

From Eq. 11.25,

$$UA = C_{min}\cdot\text{NTU} = 2094\ \text{W/K}\times2.029 = 4249\ \text{W/K} \qquad (1)$$

Considering the gas-side flow rate and velocity criteria, find the number of tubes required as

$$\dot{m}_h = N\cdot\rho_h\cdot A_c\cdot u_{m,i} = N\cdot\rho_h\left(\pi D_i^2/4\right)u_{m,i}$$

Continued …..

$$2\ \mathrm{kg/s} = N \times 0.6009\ \mathrm{kg/m^3} \times \pi(0.050\ \mathrm{m})^2/4 \times 25\ \mathrm{m/s}$$

$$N = 67.8\ \text{tubes, specify } 68 \qquad <$$

The overall coefficient, considering the convection process, fouling resistance and the tube thermal resistance, is evaluated as

$$U_i = 1/\left[R''_{f,i} + R''_{cv,i} + R''_{cd,t}\right] = 56.4\ \mathrm{W/m^2\cdot K}$$

$$R''_{f,i} = 0.0015\ \mathrm{m^2\cdot K/W}$$

$$R''_{cv,i} = 1/h_i = 1/62\ \mathrm{W/m^2\cdot K} = 0.0161\ \mathrm{m^2\cdot K/W}$$

$$R''_{cd,t} = \frac{D_i\ \ell n\left(D_o/D_i\right)}{2\ k} = \frac{0.050\ \mathrm{m}\ \ell n(58/50)}{2\times 40\ \mathrm{W/m\cdot K}} = 9.28\times 10^{-5}\ \mathrm{m^2\cdot K/W}$$

where the gas-side convection coefficient estimate is explained in the Comments section. Substituting numerical values, determine the required tube length

$$[UA] = U_i\cdot A_i = U_i\ \left(N\,\pi\,D_i\ L\right)$$

$$4249\ \mathrm{W/K} = 56.4\ \mathrm{W/m^2\cdot K} \times 68 \times \pi \times 0.050\ \mathrm{m} \times L$$

$$L = 7.1\ \mathrm{m} \qquad <$$

COMMENTS: (1) Is the assumption of negligible water-side thermal resistance reasonable? Explain why.

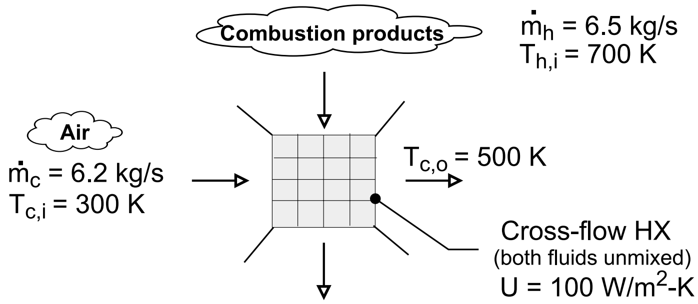
(2) Knowing the tube gas-side velocity, the usual convection correlation calculation methodology is followed. The flow is turbulent, $Re_{Di} = 2.5\times 10^4$, and assuming fully developed flow, use the Dittius-Boelter correlation, Eq. 8.60, to find $Nu_{Di} = 67.8$ and $h_i = 62.0\ \mathrm{W/m^2\cdot K}$.

PROBLEM 11.55

KNOWN: Hot and cold gas flow rates and inlet temperatures of a recuperator. Overall heat transfer coefficient. Desired cold gas outlet temperature.

FIND: (a) Required surface area, (b) Effect of surface area on cold-gas outlet temperature.

SCHEMATIC:



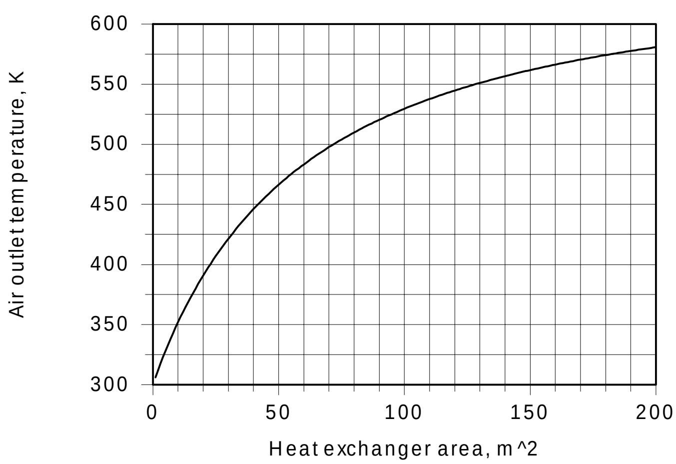
ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible kinetic and potential energy and flow work changes, (3) Constant properties.

PROPERTIES: Given: $c_{p,c} = c_{p,h} = 1040$ W/m·K.

ANALYSIS: (a) With $C_{min} = C_c = 6.2$ kg/s × 1040 J/kg·K = 6,448 W/K, $C_{max} = C_h = 6.5$ kg/s × 1040 J/kg·K = 6,760 W/K, $C_r = C_{min}/C_{max} = 0.954$, $q = C_c (T_{c,o} - T_{c,i})$ = 6,448 W/K (200 K) = 1.29×10^6 W, $q_{max} = C_{min} (T_{h,i} - T_{c,i})$ = 6,448 W/K (400 K) = 2.58×10^6 W, and $\varepsilon = q/q_{max} = 0.50$, Fig. 11.18 yields NTU ≈ 1.10. Hence

$$A = \frac{NTU \times C_{min}}{U} = \frac{1.10 \times 6,448 \text{ W/K}}{100 \text{ W/m}^2 \cdot \text{K}} = 70.9 \text{ m}^2$$ <

(b) Using the Heat Exchanger option of *IHT*, the following result was obtained



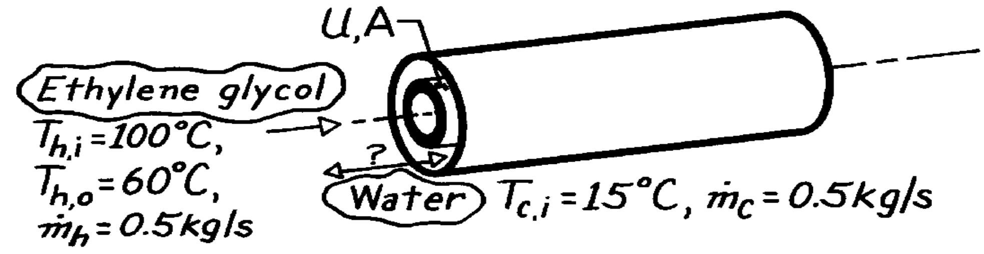
The air outlet temperature increases, of course, with increasing heat exchanger area, but the approach to the maximum possible outlet temperature, $T_{h,i}$, is slow and the heat exchanger size needed to achieve a large outlet temperature may be prohibitively expensive.

PROBLEM 11.56

KNOWN: Inlet temperature and flow rates for a concentric tube heat exchanger. Hot fluid outlet temperature.

FIND: (a) Maximum possible heat transfer rate and effectiveness, (b) Preferred mode of operation.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state operation, (2) Negligible KE and PE changes, (3) Negligible heat loss to surroundings, (4) Fixed overall heat transfer coefficient.

PROPERTIES: *Table A-5*, Ethylene glycol ($\overline{T}_m$ = 80°C): c_p = 2650 J/kg·K; *Table A-6*, Water ($\overline{T}_m$ ≈ 30°C): c_p = 4178 J/kg·K.

ANALYSIS: (a) Using the ε-NTU method, find

$$C_{min} = C_h = \dot{m}_h c_{p,h} = (0.5 kg/s)(2650 J/kg \cdot K) = 1325 W/K.$$

Hence from Eqs. 11.19 and 11.6,

$$q_{max} = C_{min}(T_{h,i} - T_{c,i}) = (1325 W/K)(100 - 15)°C = 1.13 \times 10^5 W.$$

$$q = \dot{m}_h c_{p,h}(T_{h,i} - T_{h,o}) = 0.5 kg/s(2650 J/kg \cdot K)(100 - 60)°C = 0.53 \times 10^5 W. \quad <$$

Hence from Eq. 11.20,

$$e = q / q_{max} = 0.53 \times 10^5 / 1.13 \times 10^5 = 0.47. \quad <$$

(b) From Eq. 11.7,

$$T_{c,o} = T_{c,i} + \frac{q}{\dot{m}_c c_{p,c}} = 15°C + \frac{0.53 \times 10^5}{0.5 kg/s \times 4178 J/kg \cdot K} = 40.4°C.$$

Since $T_{c,o} < T_{h,o}$, a *parallel flow* mode of operation is possible. However, with $(C_{min}/C_{max}) = (\dot{m}_h c_{p,h}/\dot{m}_c c_{p,c}) = 0.63$,

Fig. 11.14 → $(NTU)_{PF} \approx 0.95$ Fig. 11.15 → $(NTU)_{CF} \approx 0.75$.

Hence from Eq. 11.15

$$(A_{CF}/A_{PF}) = (NTU)_{CF}/(NTU)_{PF} \approx (0.75/0.95) = 0.79.$$

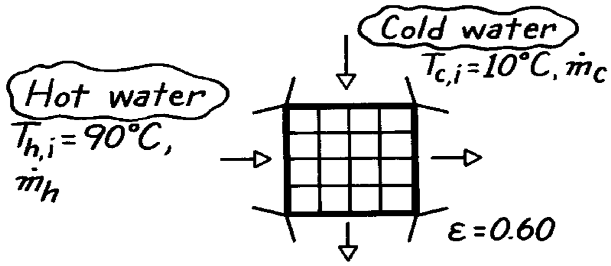
Because of the reduced size requirement, and hence capital investment, the *counterflow* mode of operation is preferred.

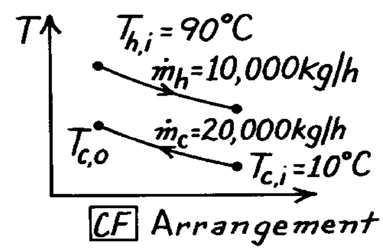
PROBLEM 11.57

KNOWN: Single-pass, cross-flow heat exchanger with both fluids (water) unmixed; hot water enters at 90°C and at 10,000 kg/h while cold water enters at 10°C and at 20,000 kg/h; effectiveness is 60%.

FIND: Cold water exit temperature, $T_{c,o}$.

SCHEMATIC:





ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible kinetic and potential energy changes, (3) Constant properties.

PROPERTIES: *Table A-6*, Water ($\overline{T}_c \approx$ (10 + 40)°C/2 ≈ 300 K): c_c = 4179 J/kg·K; *Table A-6*, Water ($\overline{T}_h \approx$ (90 + 60)°C/2 ≈ 350 K): c_h = 4195 J/kg·K.

ANALYSIS: From an energy balance on the cold fluid, Eq. 11.7, the outlet temperature can be expressed as

$$T_{c,o} = T_{c,i} + q / \dot{m}_c C_c.$$

The heat rate can be written in terms of the effectiveness and q_{max}. Using Eqs. 11.20 and 11.19,

$$q = e\, q_{max} = e\, C_{min}\left(T_{h,i} - T_{c,i}\right).$$

By inspection, it can be noted that the hot fluid is the minimum capacity fluid. Substituting numerical values,

$$q = e\left(\dot{m}_h c_h\right)\left(T_{h,i} - T_{c,i}\right)$$

$$q = 0.60\left(10,000\,\text{kg/h}/3600\,\text{s/h}\right)4195\,\text{J/kg}\cdot\text{K}\left(90-10\right)°\text{C} = 559.3\times10^3\ \text{W}.$$

The exit temperature of the cold water is then

$$T_{c,o} = 10°\text{C} + 559.3\times10^3\ \text{W} / \frac{20,000}{3600}\text{kg/s}\times 4179\,\text{J/kg}\cdot\text{K} = 34.1°\text{C}. \qquad <$$

COMMENTS: (1) The properties of the cold fluid should be evaluated at $\overline{T}$ = ($T_{c,o}$ + $T_{c,i}$)/2 = (34.1 + 10)°C/2 = 295 K. Note the analysis assumed $\overline{T}_c \approx$ 300 K, hence little error is incurred. For best precision, one should check $\overline{T}_h$ and C_h.

(2) From Fig. 11.18, the value of NTU could be determined. First evaluate the term

$$C_{min}/C_{max} = \dot{m}_h C_h / \dot{m}_c C_c = \frac{10,000\times4195}{20,000\times4179} = 0.50$$

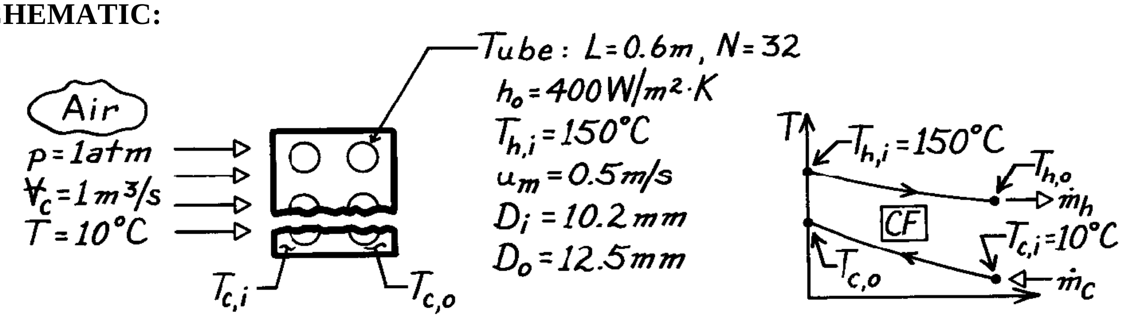
and with ε = 0.60, find NTU ≈ 1.2.

PROBLEM 11.58

KNOWN: Hxer consisting of 32 tubes in 0.6m square duct. Hot water enters tubes at 150°C with mean velocity 0.5 m/s. Atmospheric air at 10°C enters exchanger with volumetric flow rate of 1 m^3/s. Heat transfer coefficient on tube outer surfaces is 400 $W/m^2 \cdot K$.

FIND: Outlet temperatures of the fluids, $T_{c,o}$ and $T_{h,o}$.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible potential and kinetic energy changes, (3) Constant properties, (4) Hxer is a single-pass, cross-flow type with one fluid mixed (air) and the other unmixed (water), (5) Tube water flow is fully developed, (6) Negligible thermal resistance due to tube wall.

PROPERTIES: *Table A-4*, Air ($T_{c,i}$ = 10°C = 283 K, 1 atm): r = 1.2407 kg/m^3; *Table A-4*, Air (assume $T_{c,o} \approx 40°C$, $\overline{T}_c$ = (10 + 40)°C/2 = 298 K, 1 atm): c_p = 1007 J/kg·K; *Table A-6*, Water (assume $T_{h,o} \approx 140°C$, $\overline{T}_h$ = (140 + 150)°C/2 = 418 K): $r = 1/v_f = 1/1.0850 \times 10^{-3}$ m^3/kg, c_p = 4297 J/kg·K, $m_f = 188 \times 10^{-6}$ $N \cdot s/m^2$, k_f = 0.688 W/m·K, Pr_f = 1.18.

ANALYSIS: Using the ε-NTU method, first find the capacity rates.

$$C_h = \dot{m}_h c_{p,h} = (r A_c u_m)_h N \cdot c_{p,h}$$

$$C_h = \frac{1}{1.0850 \times 10^{-3} m^3/kg} \times \frac{p}{4}\left(10.2 \times 10^{-3} m\right)^2 \times 0.5 \frac{m}{s} \times 32 \times 4297 \frac{J}{kg \cdot K} = 5178 \frac{W}{K}$$

$$C_c = \dot{m}_c c_{p,c} = (r V)_c c_{p,c} = 1.2407 \frac{kg}{m^3} \times 1 m^3/s \times 1007 J/kg \cdot K = 1249 \frac{W}{K}. \qquad (1,2)$$

Note that the cold fluid is the minimum fluid, $C_c = C_{min}$. The overall heat transfer coefficient follows from Eq. 11.5,

$$U_o A_o = \left[\frac{1}{h_i A_i} + \frac{1}{h_o A_o}\right]^{-1} \qquad (3)$$

where h_i must be estimated from an appropriate internal flow correlation. The Reynolds number for water flow is

$$Re_D = \frac{r u_m D_i}{m} = \frac{\left(1/1.0850 \times 10^{-3} m^3 kg\right) \times 0.5 m/s \times \left(10.2 \times 10^{-3} m\right)}{188 \times 10^{-6} N \cdot s/m^2} = 25{,}002. \qquad (4)$$

Continued …..

The flow is turbulent and since $L/D_i = 0.6\text{m}/10.2 \times 10^{-3}\text{m} = 59$, fully developed conditions may be assumed. The Dittus-Boelter correlation with n = 0.3 is appropriate.

$$Nu_D = \frac{h_i D_i}{k} = 0.023 Re_D^{0.8} Pr^{0.3} = 0.023(25{,}002)^{0.8}(1.18)^{0.3} = 79.7$$

$$h_i = \frac{k}{D_i} Nu_D = \frac{0.688\,\text{W/m}\cdot\text{K}}{10.2\times10^{-3}\,\text{m}} \times 79.7 = 5376\,\text{W/m}^2\cdot\text{K}.$$

Substituting numerical values into Eq. (3), find

$$U_o = \left[\left(\frac{12.5\text{mm}}{10.2\text{mm}}\right)\frac{1}{5376\,\text{W/m}^2\cdot\text{K}} + \frac{1}{400\,\text{W/m}^2\cdot\text{K}}\right]^{-1} = 366.6\,\text{W/m}^2\cdot\text{K}.$$

It follows from Eq. 11.25, with $A_o = N(\pi D_o L)$, that

$$NTU = \frac{U_o A_o}{C_{min}} = 366.6\frac{\text{W}}{\text{m}^2\cdot\text{K}} \times \left(32 \times p \times 12.5\times10^{-3}\text{m}\times 0.6\text{m}\right)/1249\frac{\text{W}}{\text{K}} = 0.22.$$

From Fig. 11.19, noting that $C_{min} = C_c$ is the mixed fluid (solid curves),

$$\frac{C_{mixed}}{C_{unmixed}} = \frac{C_{min}}{C_{max}} = \frac{C_c}{C_h} = \frac{1249\,\text{W/K}}{5178\,\text{W/K}} = 0.24$$

and with NTU = 0.22 find $\varepsilon \approx 0.19$. From the definition of effectiveness, Eq. 11.20,

$$e = \frac{q}{q_{max}} = \frac{C_c\left(T_{c,o} - T_{c,i}\right)}{C_{min}\left(T_{h,i} - T_{c,i}\right)}$$

$$T_{c,o} = T_{c,i} + e\left(T_{h,i} - T_{c,i}\right) = 10°\text{C} + 0.19(150 - 10)°\text{C} = 36.6°\text{C}.$$ <

Equating the energy balances on both fluids,

$$C_c\left(T_{c,o} - T_{c,i}\right) = C_h\left(T_{h,i} - T_{h,o}\right)$$

or

$$T_{h,o} = T_{h,i} - \frac{C_c}{C_h}\left(T_{c,o} - T_{c,i}\right)$$

$$T_{h,o} = 150°\text{C} - \frac{1249\,\text{W/K}}{5178\,\text{W/K}}(36.6 - 10)°\text{C} = 143.5°\text{C}.$$ <

COMMENTS: (1) Note that the assumptions of $T_{h,o}$ and $T_{c,o}$ used in evaluating properties are reasonable.

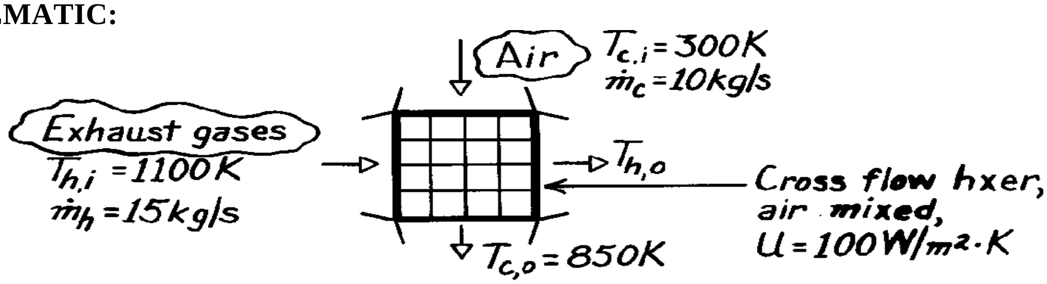
(2) Note that to calculate $\dot{m}_c$ from V, the density at 10°C is more appropriate than at $\overline{T}_c$.

PROBLEM 11.59

KNOWN: Flow rates and inlet temperatures of exhaust gases and combustion air used in a cross-flow (one fluid mixed) heat exchanger. Overall heat transfer coefficient. Desired air outlet temperature.

FIND: Required heat exchanger surface area.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible heat loss to surroundings, (3) Negligible kinetic and potential energy changes, (4) Constant properties, (5) Gas properties are those of air.

PROPERTIES: *Table A-4*, Air ($\overline{T}_m \approx 700$ K, 1 atm): $c_p = 1075$ J/kg·K.

ANALYSIS: From Eqs. 11.6 and 11.7,

$$T_{h,o} = T_{h,i} - \frac{\dot{m}_c c_{p,c}}{\dot{m}_h c_{p,h}}\left(T_{c,o} - T_{c,i}\right) = 1100K - \frac{10 \text{ kg/s}}{15 \text{ kg/s}}(850 - 300)K = 733K.$$

From Eqs. 11.15, 11.17 and 11.18,

$$\Delta T_{\ell m} = F\frac{\left(T_{h,i} - T_{c,o}\right) - \left(T_{h,o} - T_{c,i}\right)}{\ell n\left[\left(T_{h,i} - T_{c,o}\right)/\left(T_{h,o} - T_{c,i}\right)\right]} = F\frac{250 - 433}{\ell n\left(250/433\right)} = F \times 333K.$$

From Fig. 11.13, with R = (300 – 850)/(733 – 1100) = 1.50 and P = (733 – 1100)/(300 – 1100) = 0.46, $F \approx 0.73$. With

$$q = \dot{m}_h c_{p,h}\left(T_{h,i} - T_{h,o}\right) = 15 kg/s \times 1075 J/kg \cdot K\left(367K\right) = 5.92 \times 10^6 \text{ W}$$

it follows from Eq. 11.14 that

$$A = \frac{5.92 \times 10^6 \text{ W}}{100 W/m^2 \cdot K \times 0.73\left(333K\right)} = 243m^2.$$ <

COMMENTS: Using the effectiveness-NTU method, from Eq. 11.22

$$e = \frac{T_{c,o} - T_{c,i}}{T_{h,i} - T_{c,i}} = \frac{(850 - 300)K}{(1100 - 300)K} = 0.688.$$

Hence, with $C_{mixed}/C_{unmixed} = C_c/C_h = 0.67$, Fig. 11.19 gives NTU ≈ 2.3. From Eq. 11.25,

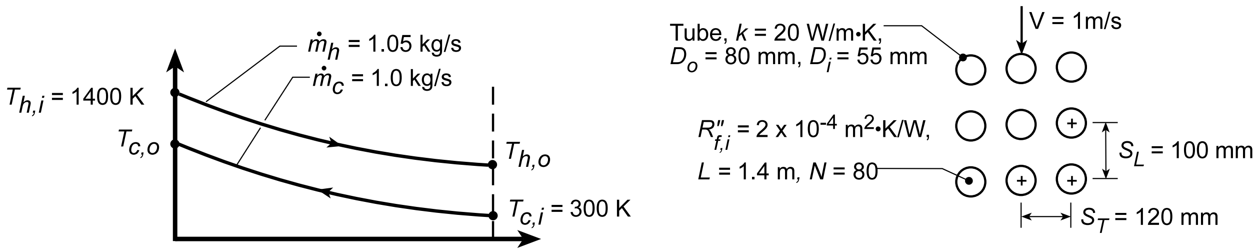
$$A = NTU\frac{C_{min}}{U} \approx 2.3\frac{10 \text{ kg/s} \times 1075 \text{ J/kg} \cdot \text{K}}{100 \text{ W/m}^2 \cdot \text{K}} \approx 247m^2.$$

PROBLEM 11.60

KNOWN: Dimensions, configuration and material of a single-pass, cross-flow heat exchanger. Inlet conditions of inner and outer flow. Fouling factor of inner surface.

FIND: (a) Percent fuel savings for prescribed conditions, (b) Effect of UA on air outlet temperature and fuel savings.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat loss to surroundings and potential and kinetic energy changes, (2) Air properties are those of atmospheric air at 300 K, (3) Gas properties are those of atmospheric air at 1400 K, (4) Tube wall temperature may be approximated as 800 K for treating variable property effects.

PROPERTIES: *Table A.4*, Air (1 atm, T = 300 K): $\nu = 15.89 \times 10^{-6}$ m^2/s, c_p = 1007 J/kg·K, k = 0.0263 W/m·K, Pr = 0.707; (T = 1400 K): $\mu = 530 \times 10^{-7}$ kg/s·m, c_p = 1207 J/kg·K, k = 0.091 W/m·K, Pr = 0.703; (T = 800 K): $\mu = 370 \times 10^{-7}$ kg/s·m, Pr = 0.709.

ANALYSIS: (a) With capacity rates of $C_c = \dot{m}_c c_{p,c}$ = 1 kg/s × 1007 J/kg·K = 1007 W/K = C_{min} and $C_h = \dot{m}_h c_{p,h}$ = 1.05 kg/s × 1207 J/kg·K = 1267 W/K = C_{max}, C_{min}/C_{max} = 0.795. The overall coefficient is

$$\frac{1}{UA} = \frac{1}{h_i A_i} + \frac{R''_{f,i}}{A_i} + \frac{\ln(D_o/D_i)}{(2\pi k L)N} + \frac{1}{h_o A_o}.$$

For flow through a single tube,

$$Re_D = \frac{4\dot{m}_h}{N\pi D_i \mu} = \frac{4 \times 1.05\,\text{kg/s}}{80\pi(0.055\,\text{m})530 \times 10^{-7}\,\text{kg/s}\cdot\text{m}} = 5733.$$

Assuming fully developed turbulent flow throughout and using the Sieder-Tate correlation,

$$Nu_D = 0.027\,Re_D^{4/5}\,Pr^{1/3}(\mu/\mu_s)^{0.14} = 0.027(5733)^{4/5}(0.703)^{1/3}(530/370)^{0.14} = 25.6$$

$$h_i = Nu_D k/D_i = 25.6(0.091\,\text{W/m}\cdot\text{K})/0.055\,\text{m} = 42.4\,\text{W/m}^2\cdot\text{K}.$$

For flow over the tube bank,

$$V_{max} = [S_T/(S_T - D_o)]V = [0.12\,\text{m}/(0.12 - 0.08)\,\text{m}]1\,\text{m/s} = 3\,\text{m/s}$$

$$Re_{D,max} = \frac{V_{max} D_o}{\nu} = \frac{3\,\text{m/s}(0.08\,\text{m})}{15.89 \times 10^{-6}\,\text{m}^2/\text{s}} = 15{,}100$$

From the Zhukauskas correlation for a tube bank,

$$\overline{Nu}_D = 0.27(15{,}100)^{0.63}(0.707)^{0.36}(0.707/0.709)^{1/4} = 102.3$$

$$\bar{h}_o = \overline{Nu}_D(k/D_o) = 102.3(0.0263\,\text{W/m}\cdot\text{K})/0.08\,\text{m} = 33.6\,\text{W/m}^2\cdot\text{K}.$$

Hence, based on the inner surface, the *overall coefficient* is

Continued...

$$\frac{1}{U_i} = \frac{1}{h_i} + R''_{f,i} + \frac{D_i \ln(D_o/D_i)}{2k} + \frac{D_i}{D_o h_o}$$

$$\frac{1}{U_i} = \left(0.0236 + 0.0002 + \frac{0.055\ln(0.08/0.055)}{40} + \frac{0.055}{0.08\times 33.6}\right) m^2 \cdot K/W$$

$$U_i = \left[(0.0236 + 0.0002 + 0.0005 + 0.0246)\, m^2 \cdot K/W\right]^{-1} = 22.3\, W/m^2 \cdot K\,.$$

Hence, $(UA)_i = U_i N\pi D_i L = 22.3\, W/m^2 \cdot K \times 80\pi\,(0.055\,m)1.4\,m = 432\, W/K$. The number of transfer units is then NTU = UA/C_{min} = 432 W/K/1007 W/K = 0.429, and with $C_{mixed}/C_{unmixed} = C_c/C_h = C_{min}/C_{max}$ = 0.795, Fig. 11.19 yields $\varepsilon \approx 0.3$ or, from Eq. 11.35 a,

$$\varepsilon = 1 - \exp\left(-C_r^{-1}\{1 - \exp[-C_r \cdot NTU]\}\right) = 0.305\,.$$

Hence, with

$$q_{max} = C_{min}(T_{h,i} - T_{c,i}) = 1007\, W/K\,(1100\,K) = 1.11\times 10^6\, W$$

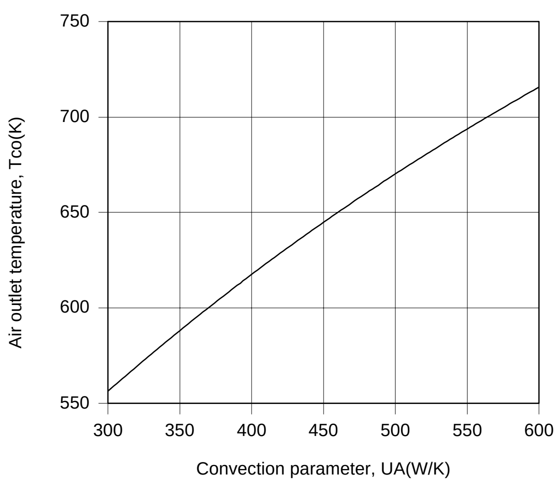
$$q = \varepsilon q_{max} = 0.305 \times 1.11\times 10^6\, W = 337,800\, W$$

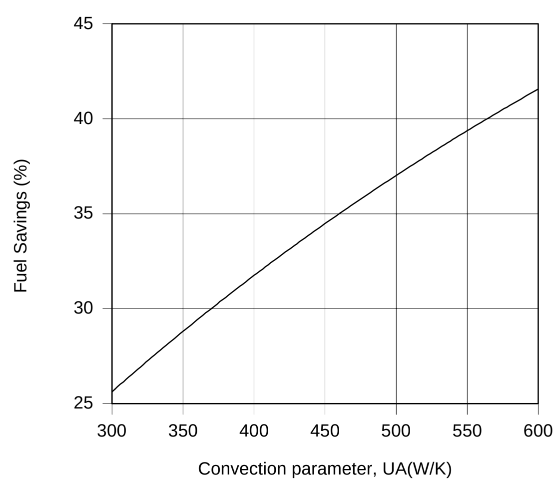
$$T_{c,o} = T_{c,i} + q/C_{min} = 300\,K + (337,800\,W/1007\,W/K) = 635\,K\,.$$

Hence,

$$\%\text{ fuel savings} \equiv FS = (\Delta T_c/10\,K)\times 1\% = (335\,K/10\,K)\times 1\% = 33.5\%$$ <

(b) Using the Heat Exchangers Toolpad of IHT to perform the parametric calculations, the following results are obtained.





Significant benefits are derived by increasing UA, with values of $T_{c,o}$ = 716 K and FS = 41.6% obtained for UA = 600 W/K. The major contributions to the total resistance are made by the inner and outer convection resistances. These contributions could be reduced by using extended surfaces on both the inner and outer surfaces.

COMMENTS: For part (a), properties of the flue gas should be evaluated at $(T_{h,i} + T_{h,o})/2$ and the calculations repeated. The Colburn equation yields

$$Nu_D = 0.023\, Re_D^{4/5}\, Pr^{1/3} = 20.8$$

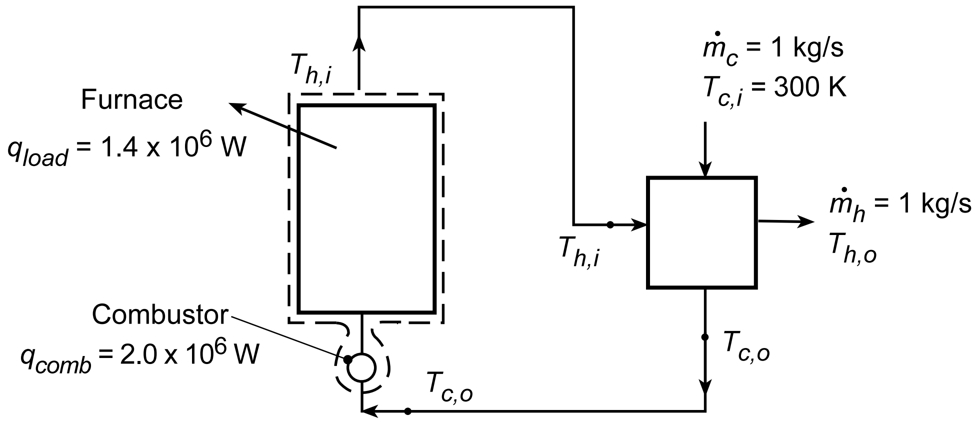
which is 19% less than the result of the Sieder-Tate correlation.

PROBLEM 11.61

KNOWN: Rate of thermal energy production in combustor and transfer to load in furnace. Cold air and flue gas flowrates and specific heats in recuperator. Recuperator cold air inlet temperature.

FIND: Recuperator hot gas inlet and outlet temperatures and air outlet temperature for a recuperator effectiveness of ε = 0.3. Value of ε needed to achieve a recuperator outlet temperature of 800 K.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible flowwork and potential and kinetic energy changes, (2) Constant properties, (3) Negligible effect of fuel addition on flowrate.

PROPERTIES: Air and gas: $c_{p,c} = c_{p,h} = 1200$ J/kg·K.

ANALYSIS: With $C_c = C_h = C_{min}$, the effectiveness of the recuperator, $\varepsilon = q/q_{max}$, may be expressed as

$$\varepsilon = \frac{C_c\left(T_{c,o} - T_{c,i}\right)}{C_{min}\left(T_{h,i} - T_{c,i}\right)} = \frac{T_{c,o} - 300\,\mathrm{K}}{T_{h,i} - 300\,\mathrm{K}} = 0.3$$

The unknown temperatures, $T_{c,o}$ and $T_{h,i}$, are also related through an energy balance performed on the air entering the combustor and leaving the furnace. Specifically,

$$C\left(T_{h,i} - T_{c,o}\right) = q_{comb} - q_{load} = 0.6 \times 10^6\,\mathrm{W}$$

where C = 1 kg/s × 1200 J/kg·K = 1200 W/K. Solving the foregoing equations, we obtain

$T_{h,i} = 1014\,\mathrm{K}$ $\qquad$ $T_{c,o} = 514$ K <

Expressing the effectiveness as

$$\varepsilon = \frac{C_h\left(T_{h,i} - T_{h,o}\right)}{C_{min}\left(T_{h,i} - T_{c,i}\right)} = \frac{1014\,\mathrm{K} - T_{h,o}}{714\,\mathrm{K}}$$

we also obtain $\quad T_{h,o} = 800$ K. <

For a combustor air inlet temperature of $T_{c,o} = 800$ K and $T_{h,i} = 1014$ K, the required effectiveness is

$$\varepsilon = \frac{T_{c,o} - T_{c,i}}{T_{h,i} - T_{c,i}} = \frac{(800 - 300)\,\mathrm{K}}{(1014 - 300)\,\mathrm{K}} = 0.70$$ <

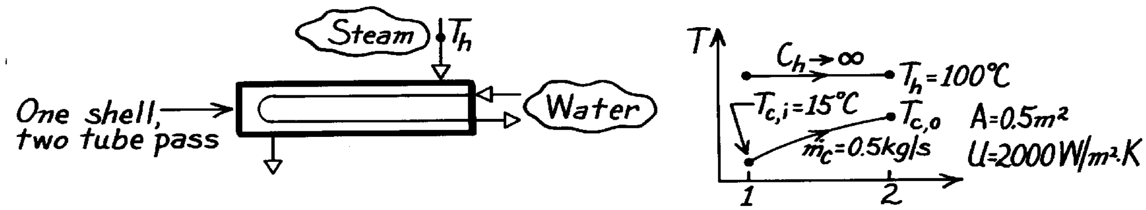
COMMENTS: The effectiveness of the recuperator may be increased by increasing NTU and hence UA, as, for example, by increasing the number of tubes.

PROBLEM 11.62

KNOWN: Single-shell, two-tube pass heat exchanger with surface area 0.5 m^2 and overall heat transfer coefficient of 2000 $W/m^2 \cdot K$; saturated steam at 100°C condenses on one side while water at a flow rate of 0.5 kg/s enters at 15°C.

FIND: (a) Outlet temperature of the water, $T_{c,o}$, (b) Rate of condensation of steam, $\dot{m}_h$.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible, kinetic and potential energy changes, (3) Constant properties.

PROPERTIES: *Table A-6*, Steam (100°C, 1 atm): h_{fg} = 2257 kJ/kg; *Table A-6*, Water ($\overline{T}_c \approx (15 + 35)°C/2 \approx 300$ K): c_c = 4179 J/kg·K.

ANALYSIS: (a) Using the ε-NTU method of analysis, recognize that the minimum capacity fluid is the cold fluid since for the hot fluid, $C_h \to \infty$. See Fig. 11.9a. That is,

$$C_{min} = \dot{m}_c c_c = 0.5 \text{ kg/s} \times 4179 \text{ J/kg} \cdot \text{K} = 2090 \text{ W/K}.$$

It follows also that,

$$NTU = AU/C_{min} = 0.5\text{m}^2 \times 2000 \text{ W/m}^2 \cdot \text{K}/2090 \text{W/K} = 0.48.$$

Using NTU = 0.48 and $C_{min}/C_{max} = 0$, find from Fig. 11.16 that ε = 0.39. Since $\dot{m}_c$ is the minimum fluid, from Eq. 11.22

$$e = \left(T_{c,o} - T_{c,i}\right) / \left(T_{h,i} - T_{c,i}\right)$$

$$T_{c,o} = T_{c,i} + e\left(T_{h,i} - T_{c,i}\right) = 15°C + 0.39(100 - 15)°C = 48.2°C. \qquad <$$

(b) The rate of steam condensation can be expressed as

$$\dot{m}_h = q / h_{fg}.$$

From Eqs. 11.19 and 11.20

$$q = e q_{max} = e C_{min}\left(T_{h,i} - T_{c,i}\right)$$

$$q = 0.39 \times 2090 \text{W/K}(100 - 15)\text{K} = 69,284 \text{W}.$$

Hence, the condensation rate is

$$\dot{m}_h = 69,284 \text{W}/2257 \times 10^3 \text{ J/kg} = 0.031 \text{kg/s}. \qquad <$$

COMMENTS: (1) Be sure to recognize why $C_h \to \infty$. Note also that $\dot{m}_c >> \dot{m}_h$.

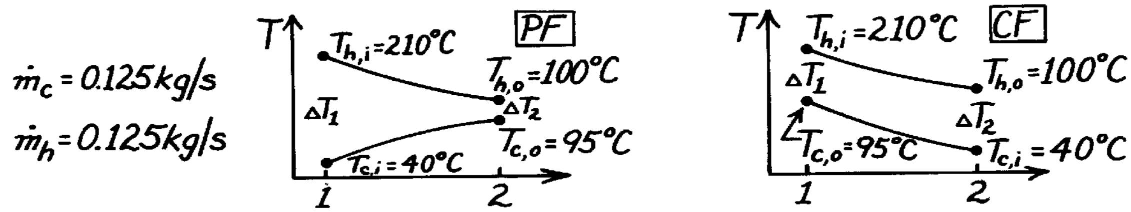
(2) Note that $\overline{T}_c = (T_{c,i} + T_{c,o})/2 = (15 + 48.2)°C/2 \approx 305$ K. This compares favorably with the value of 300 K at which properties of the cold fluid were evaluated.

PROBLEM 11.63

KNOWN: Concentric tube heat exchanger with prescribed conditions.

FIND: (a) Maximum possible heat transfer, (b) Effectiveness, (c) Whether heat exchanger should be run in PF or CF to minimize size or weight; determine ratio of required areas for the two flow conditions.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible kinetic and potential energy changes, (3) Constant properties, (4) Overall heat transfer coefficient remains unchanged for PF or CF conditions.

PROPERTIES: Hot fluid (given): c = 2100 J/kg·K; Cold fluid (given): c = 4200 J/kg·K.

ANALYSIS: (a) The maximum possible heat transfer rate is given by Eq. 11.19.

$$q_{max} = C_{min}\left(T_{h,i} - T_{c,o}\right).$$

The minimum capacity fluid is the hot fluid with $C_{min} = \dot{m}_h\ c_h$, giving

$$q_{max} = \dot{m}_h\, c_h\left(T_{h,i} - T_{c,o}\right) = 0.125\frac{kg}{s}\times 2100\frac{J}{kg\cdot K}(210-40)K = 44{,}625W. \qquad <$$

(b) The effectiveness is defined by Eq. 11.20 and the heat rate, q, can be determined from an energy balance on the cold fluid.

$$e = q/q_{max} = \dot{m}_c\, c_c\left(T_{c,o} - T_{c,i}\right)/q_{max}$$

$$e = 0.125\ kg/s\times 4200\,J/kg\cdot K\,(95-40)K/44{,}625W = 0.65. \qquad <$$

(c) Operating the heat exchanger under CF conditions will require a smaller heat transfer area than for PF conditions. The ratio of the areas is

$$\frac{A_{CF}}{A_{PF}} = \frac{q/U\,\Delta T_{\ell m,CF}}{q/U\,\Delta T_{\ell m,PF}} = \frac{\Delta T_{\ell m,PF}}{\Delta T_{\ell m,CF}}.$$

To calculate the LMTD, first find $T_{h,o}$ from overall energy balances on the two fluids.

$$T_{h,o} = T_{h,i} - \frac{\dot{m}_c\, c_c}{\dot{m}_h\, c_h}\left(T_{c,o} - T_{c,i}\right) = 210°C - \frac{0.125\times 4200}{0.125\times 2100}(95-40)°C = 100°C.$$

Using Eq. 11.15 with ΔT_1 and ΔT_2 as shown below, find $\Delta T_{\ell m} = (\Delta T_1 - \Delta T_2)/\ell n(\Delta T_1/\Delta T_2)$. Substituting values, find

$$\frac{A_{CF}}{A_{PF}} = \frac{\left[(210-40)-(100-95)\right]/\ell n\,(170/5)}{\left[(210-95)-(100-40)\right]\ell n(115/60)} = \frac{46.8°C}{84.5°C} = 0.55. \qquad <$$

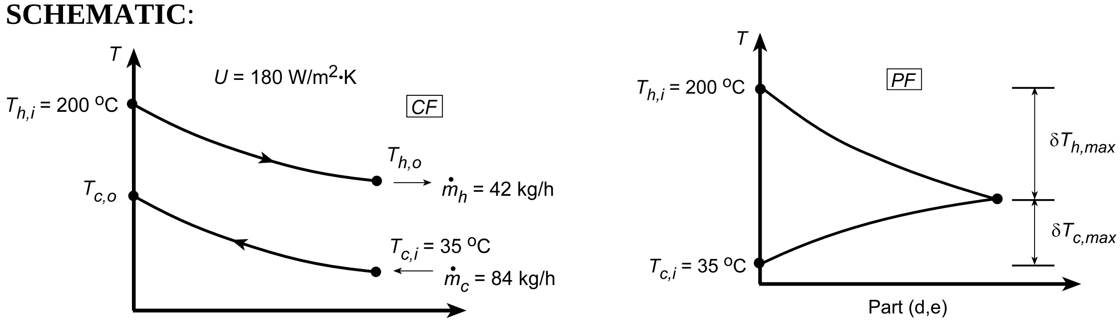
COMMENTS: In solving part (c), it is also possible to use Figs. 11.15 and 11.16 to evaluate NTU values for corresponding ε and C_{min}/C_{max} values. With knowledge of NTU it is then possible to find A_{CF}/A_{PF}.

PROBLEM 11.64

KNOWN: Concentric tube HXer with prescribed inlet fluid temperatures, fluid flow rates and overall coefficient.

FIND: (a) Maximum heat transfer rate, q_{max}; (b) Outlet fluid temperatures when area is 0.33 m^2 with CF operation; (c) Compute and plot the effectiveness, ε, and fluid outlet temperatures, $T_{c,o}$ and $T_{h,o}$, as a function of UA for the range 50 ≤ UA ≤ 1000 W/K for CF operation with all other conditions remaining the same; as UA becomes very large, find asymptotic value for $T_{h,o}$; (d) Largest heat transfer rate which could be achieved if HXer is very long with PF operation; effectiveness for this arrangement; and (e) Compute and plot ε, $T_{c,o}$ and $T_{h,o}$ as a function of UA for the range 50 ≤ UA ≤ 1000 W/K for PF operation with all other conditions remaining the same; as UA becomes very large, find asymptotic value for $T_{c,o}$ and $T_{h,o}$.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible kinetic and potential energy changes, (3) Constant properties.

PROPERTIES: *Table A-6,* Water (Assume $T_{c,o} \approx 85°C$, $\overline{T}_c \approx$ 335 K): c_c = 4186 J/kg·K, (Assume $T_{h,o} \approx$ 100°C, $\overline{T}_h \approx$ 100°C, $\overline{T}_h \approx$ 420 K): c_h = 4302 J/kg·K.

ANALYSIS: (a) With $C_{min} = C_h$, the maximum heat transfer rate from Eq. 11.19 is

$$q_{max} = C_{min}\left(T_{h,i} - T_{c,i}\right) = C_h\left(T_{h,i} - T_{c,i}\right) = \frac{42}{3600}\frac{\text{kg}}{\text{s}} \times 4302 \times \frac{\text{J}}{\text{kg}\cdot\text{K}}\left(200 - 35\right)\text{K} = 8281\,\text{W}. \quad <$$

(b) Using the ε - NTU method, find ε from values of C_{min}, C_{min}/C_{max}, and NTU.

$$C_{min} = 42/3600\,\text{kg/s} \times 4302\,\text{J/kg}\cdot\text{K} = 50.19\,\text{W/K}, \quad C_{min}/C_{max} = \frac{42\,\text{kg/h} \times 4302\,\text{J/kg}\cdot\text{K}}{84\,\text{kg/h} \times 4186\,\text{J/kg}\cdot\text{K}} = 0.514$$

$$\text{NTU} = \text{UA}/C_{min} = 180\,\text{W/m}^2\cdot\text{K} \times 0.33\text{m}^2/50.19\,\text{W/K} = 1.184.$$

Using Eq. 11.30 for counter flow operation, with $C_r = C_{min}/C_{max}$, find that

$$\varepsilon = \frac{1 - \exp\left[-\text{NTU}\left(1 - C_r\right)\right]}{1 - C_r\exp\left[-\text{NTU}\left(1 - C_r\right)\right]} = \frac{1 - \exp\left[-1.18\left(1 - 0.514\right)\right]}{1 - 0.514\exp\left[-1.18\left(1 - 0.514\right)\right]} = 0.616.$$

From the definition of effectiveness, $\varepsilon = C_h\,(T_{h,i} - T_{h,o})/C_{min}\,(T_{h,i} - T_{c,i})$, it follows that

$$T_{h,o} = T_{h,i} - \varepsilon\left(T_{h,i} - T_{c,i}\right) = 200^\circ\text{C} - 0.62\left(200 - 35\right)^\circ\text{C} = 98.4^\circ\text{C}. \quad <$$

Equating the energy balances on both fluids, $C_h\,(T_{h,i} - T_{h,o}) = C_c\,(T_{c,o} - T_{c,i})$, find

$$T_{c,o} = \left(C_h/C_e\right)\left(T_{h,i} - T_{h,o}\right) + T_{c,i} = 0.514\left(200 - 98.4\right)^\circ\text{C} + 35^\circ\text{C} = 87.2^\circ\text{C}. \quad <$$

Continued...

PROBLEM 11.64 (Cont.)

(c) Using the *IHT Heat Exchanger Tool*, *Concentric Tube, counter flow operation* and the *Properties Tool* for *Water*, a model was developed using the effectiveness NTU method employed in the previous analysis to compute ε, $T_{c,o}$ and $T_{h,o}$ as a function of UA for CF operation. The results are plotted and discussed below.

(d) For PF with same prescribed inlet conditions, the temperature distributions appear as shown above when $A \rightarrow \infty$. At the outlet, $T_{c,o} = T_{h,o}$, and from the sketch $\delta T_{h,max} + \delta T_{c,max} = (200 - 35)°C = 165°C$. From the energy balance, find

$$C_h \delta T_{h,max} = C_c \delta T_{c,max}$$

and solving simultaneously, find

$$\delta T_{h,max} = 109.0^\circ C \qquad T_{h,o} = T_{h,i} - \delta T_{h,max} = 200 - 109.0 = 91.0^\circ C.$$

The heat rate and effectiveness are

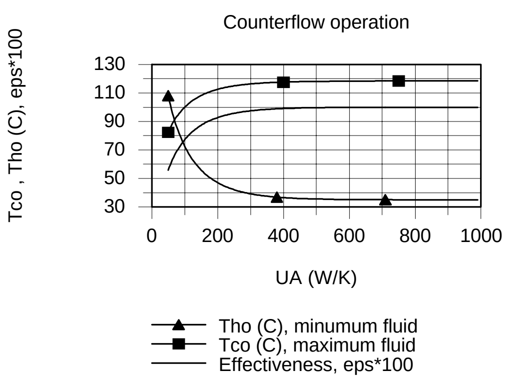
$$q = C_h \cdot \delta T_{h,max} = 50.19\, W/K \times 109.0 K = 5471 W$$

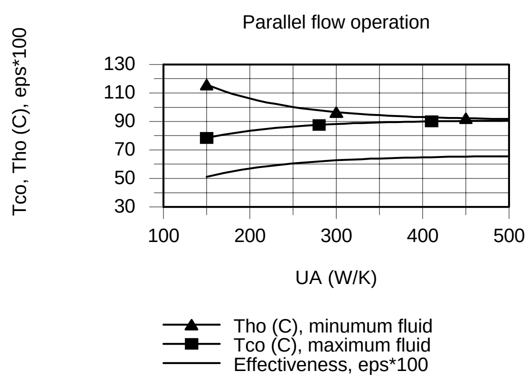
<

$$\varepsilon = q/q_{max} = 5471\, W/8,281 W = 0.661.$$

<

(e) Using the IHT model from part (c), but for PF operation, the effectiveness, $T_{c,o}$ and $T_{h,o}$ were computed and plotted as a function of UA.





COMMENTS: (1) From the plot for CF operation as UA increases, the minimum (hot) fluid outlet temperature, $T_{h,o}$, decreases to the cold fluid temperature, $T_{c,i}$. That is when $UA \rightarrow \infty$, $T_{h,o} \rightarrow T_{c,i}$. As $UA \rightarrow \infty$, the effectiveness approaches unity as expected since a very large CF heat exchanger has a heat rate q_{max} and $\varepsilon = 1$.

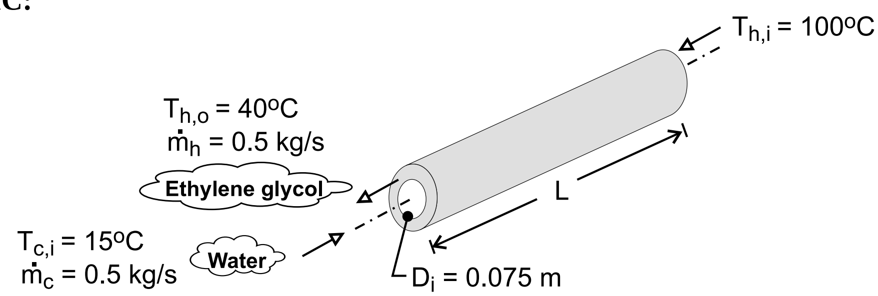
(2) From the plot for PF operation, as UA increases, $T_{h,o}$ and $T_{c,o}$ approach an asymptotic value, 91.0°C. Also, as $UA \rightarrow \infty$, the effectiveness increases, approaching 0.661, rather than unity as would be the case for CF operation.

PROBLEM 11.65

KNOWN: Flow rates and inlet temperatures of water and glycol in counterflow heat exchanger. Desired glycol outlet temperature. Heat exchanger diameter and overall heat transfer coefficient without and with spherical inserts.

FIND: (a) Required length without spheres, (b) Required length with spheres, (c) Explanation for reduction in fouling and pump power associated with using spheres.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible kinetic energy, potential energy and flow work changes, (2) Negligible heat loss to surroundings, (3) Constant properties, (4) Negligible tube wall thickness.

PROPERTIES: *Table A-5*, Ethylene glycol $\left(\bar{T}_h = 70°C\right)$: $c_{p,h}$ = 2606 J/kg·K; *Table A-6*, Water $\left(\bar{T}_c \approx 35°C\right)$: $c_{p,c}$ = 4178 J/kg·K.

ANALYSIS: (a) With $C_h = C_{min}$ = 1303 W/K and $C_c = C_{max}$ = 2089 W/K, C_r = 0.624. With actual and maximum possible heat rates of

$$q = C_h\left(T_{h,i} - T_{h,o}\right) = 1303\ W/K\left(100-40\right)°C = 78{,}180\ W$$

$$q_{max} = C_{min}\left(T_{h,i} - T_{c,i}\right) = 1303\ W/K\left(100-15\right)°C = 110{,}755\ W$$

the effectiveness is $\varepsilon = q/q_{max}$ = 0.706. From Eq. 11.30b,

$$NTU = \frac{1}{C_r - 1}\ln\left(\frac{\varepsilon - 1}{\varepsilon C_r - 1}\right) = -2.66\ \ln\left(\frac{0.294}{0.559}\right) = 1.71$$

Hence, with $A = \pi DL$ and $NTU = UA/C_{min}$,

$$L = \frac{C_{min}\ NTU}{\pi D_i U} = \frac{1303\ W/K \times 1.71}{\pi\left(0.075m\right)1000\ W/m^2\cdot K} = 9.46m \qquad <$$

(b) Since $\dot{m}_c, \dot{m}_h, T_{h,i}, T_{h,o}$ and $T_{c,i}$ are unchanged, C_r, ε and NTU are unchanged. Hence, with U = 2000 W/m²·K,

$$L = 4.73m \qquad <$$

(c) Because the spheres induce mixing of the flows, the potential for contaminant build-up on the surfaces, and hence fouling, is reduced. Although the obstruction to flow imposed by the spheres acts to increase the pressure drop, the reduction in the heat exchanger length reduces the pressure drop. The second effect may exceed that of the first, thereby reducing pump power requirements.

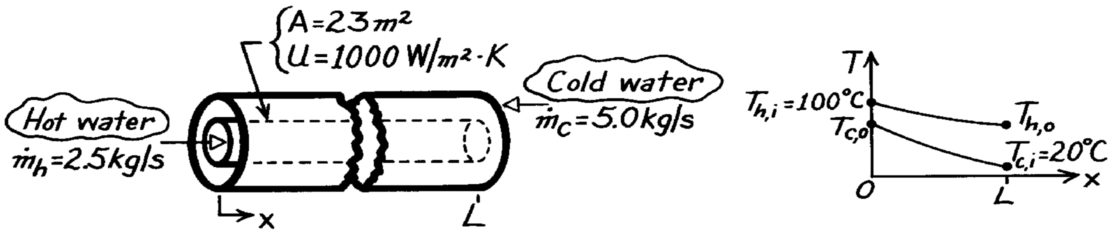
COMMENTS: The water outlet temperature is Tc,o = $T_{c,i} + q/C_c$ = 15°C + 78,180 W/2089 W/K = 52.4°C. The mean temperature $\left(\bar{T}_c = 33.7°C\right)$ is close to that used to evaluate the specific heat of water.

PROBLEM 11.66

KNOWN: Concentric tube, counter-flow heat exchanger.

FIND: Total heat transfer rate and outlet temperatures of both fluids.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible kinetic and potential energy changes, (3) Constant properties.

PROPERTIES: *Table A-6*, Water ($\overline{T}_h \approx 68°C \approx 340$ K): c_h = 4188 J/kg·K; *Table A-6*, Water ($\overline{T}_c \approx 37°C = 310$ K): c_c = 4178 J/kg·K.

ANALYSIS: Using the ε-NTU method, begin by evaluating the capacity rates.

$$C_h = \dot{m}_h c_h = 2.5\text{kg/s} \times 4188\,\text{J/kg}\cdot\text{K} = 10{,}470\,\text{W/K}$$

$$C_c = \dot{m}_c c_c = 5.0\text{kg/s} \times 4178\,\text{J/kg}\cdot\text{K} = 20{,}890\,\text{W/K}$$

Hence, $C_{min} = C_h$ and $C_{min}/C_{max} = 0.50$

From the definition, Eq. 11.25,

$$NTU = UA/C_{min} = 1000\text{W/m}^2\cdot\text{K} \times 23\text{m}^2/\left(10{,}470\text{W/K}\right) = 2.20.$$

Using values of NTU and C_{min}/C_{max}, find from Fig. 11.15, that

$$e \approx 0.80.$$

From the definition of ε, Eq. 11.20, it follows that

$$q = e\, q_{max} = e\, C_{min}\left(T_{h,i} - T_{c,i}\right) = 0.80 \times 10{,}470\text{W/K}\left(100-20\right)\text{K} = 670\text{kW}. \qquad <$$

Performing energy balances on both fluids, find

$$T_{c,o} = T_{c,i} + q/C_c = 20°C + 670\text{kW}/20{,}890\text{W/K} = 52.1°C \qquad <$$

$$T_{h,o} = T_{h,i} - q/C_h = 100°C - 670\text{kW}/10{,}470\text{W/K} = 36.0°C. \qquad <$$

COMMENTS: (1) Note that $\overline{T}_c$ = (20 + 52.1)°C/2 ≈ 310 K and $\overline{T}_h$ = (100 + 36)°C/2 = 341 K and that these values agree well with those used to evaluate the properties.

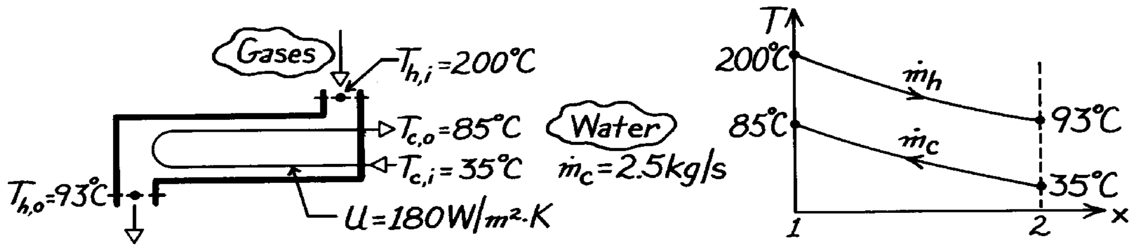
(2) Eq. 11.30 could be used to evaluate ε; the result gives ε = 0.800.

PROBLEM 11.67

KNOWN: Shell and tube heat exchanger for cooling exhaust gases with water.

FIND: Required surface area using ε-NTU method.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible changes in kinetic and potential energies, (3) Constant properties, (4) Gases have properties of air.

PROPERTIES: *Table A-6*, Water, liquid ($\overline{T}_c$ = (85 + 35)°C/2 = 333 K): c_p = 4185 J/kg·K.

ANALYSIS: Using the ε-NTU method, the area can be expressed as

$$A = NTU \cdot C_{min} / U \qquad (1)$$

where NTU must be found from knowledge of ε and $C_{min}/C_{max} = C_r$. The capacity rates are:

$$C_c = \dot{m}_c c_{p,c} = 2.5 kg/s \times 4185 J/kg \cdot K = 10,463 W/K$$

Equating the energy balance relation for each fluid,

$$C_h = C_c\left(T_{c,o} - T_{c,i}\right)/\left(T_{h,i} - T_{h,o}\right) = 10,463 W/K(85-35)/(200-93) = 4889 W/K.$$

Hence,

$$C_r = C_{min}/C_{max} = C_h/C_c = 4889/10,463 = 0.467.$$

The effectiveness of the exchanger, with $q_{max} = C_{min}(T_{h,i} - T_{c,i})$ and $C_{min} = C_h$, is

$$e = q/q_{max} = C_h\left(T_{h,i} - T_{h,o}\right)/C_h\left(T_{h,i} - T_{c,i}\right) = (200-93)/(200-35) = 0.648.$$

Considering the HXer to be a single shell with 2,4….tube passes, Eqs. 11.31b,c are appropriate to evaluate NTU.

$$NTU = -\left(1 + C_r^2\right)^{-1/2} \ell n \frac{E-1}{E+1} \qquad E = \frac{2/e_1 - (1+C_r)}{\left(1+C_r^2\right)^{1/2}}.$$

Substituting numerical values,

$$E = \frac{2/0.648 - (1+0.467)}{\left(1+0.467^2\right)^{1/2}} = 1.467 \quad NTU = -\left(1+(0.467)^2\right)^{-1/2} \ell n \frac{1.467-1}{1.467+1} = 1.51.$$

Using the appropriate numerical values in Eq. (1), the required area is

$$A = 1.51 \times 4889 W/K / 180 W/m^2 \cdot K = 40.9 m^2.$$ <

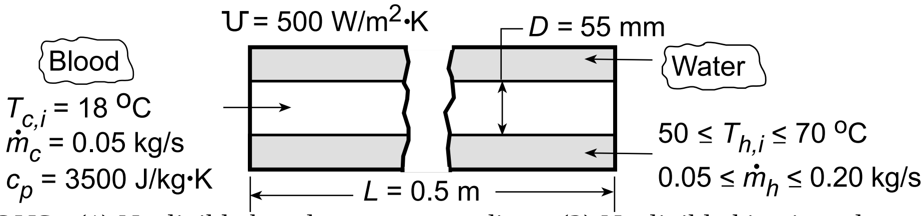
COMMENTS: Figure 11.16 could also have been used with C_r and ε to find NTU.

PROBLEM 11.68

KNOWN: Dimensions, fluid flow rates, and fluid temperatures for a counterflow heat exchanger used to heat blood.

FIND: (a) Outlet temperature of the blood, (b) Effect of water flowrate and inlet temperature on heat rate and blood outlet temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible kinetic and potential energy changes, (3) Constant properties.

PROPERTIES: *Table A.6*, Water ($\overline{T}_m \approx 55°C$): c_p = 4183 J/kg·K.

ANALYSIS: (a) Using the ε - NTU method, we first obtain $C_h = (\dot{m}_h c_{p,h}) = (0.10\ \text{kg/s} \times 4183\ \text{J/kg·K})$ = 418.3 W/K and $C_c = (\dot{m}_c c_{p,c}) = (0.05\ \text{kg/s} \times 3500\ \text{J/kg·K})$ = 175 W/K = C_{min}. Hence, (C_{min}/C_{max}) = 0.418 and

$$\text{NTU} = \frac{UA}{C_{\min}} = \frac{\left(500\,\text{W}/\text{m}^2\cdot\text{K}\right)\pi\left(0.055\,\text{m}\right)\left(0.5\,\text{m}\right)}{175\,\text{W}/\text{K}} = 0.247\,.$$

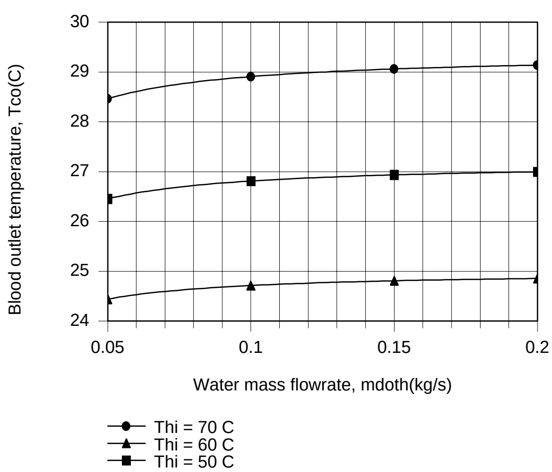
From Eq. 11.30, ε = 0.21. Hence, from Eq. 11.23

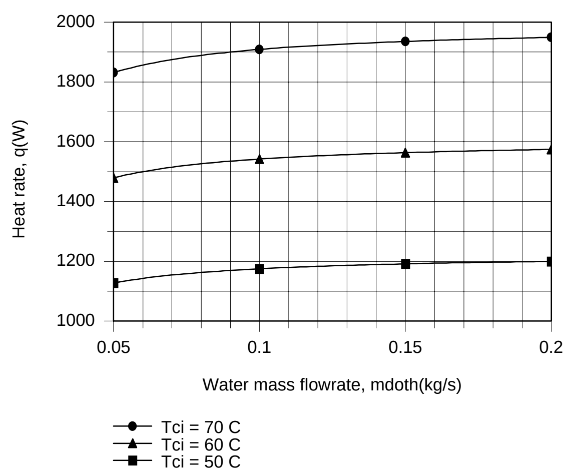
$$q = \varepsilon C_{\min}\left(T_{h,i} - T_{c,i}\right) = 0.21\left(175\,\text{W}/\text{K}\right)\left(60 - 18\right)^\circ\text{C} = 1544\,\text{W}\,.$$

From Eq. 11.7,

$$T_{c,o} = T_{c,i} + \frac{q}{C_c} = 18^\circ\text{C} + \frac{1544\,\text{W}}{175\,\text{W}/\text{K}} = 26.8^\circ\text{C}$$ <

(b) Because the variation of C_{min}/C_{max} with $\dot{m}_h$ does not have a significant effect on ε for the prescribed NTU, $T_{c,o}$ and q increase only slightly with increasing $\dot{m}_h$.





However, the water inlet temperature does have a significant effect, and accelerated heating is achieved with $T_{h,i}$ = 70°C.

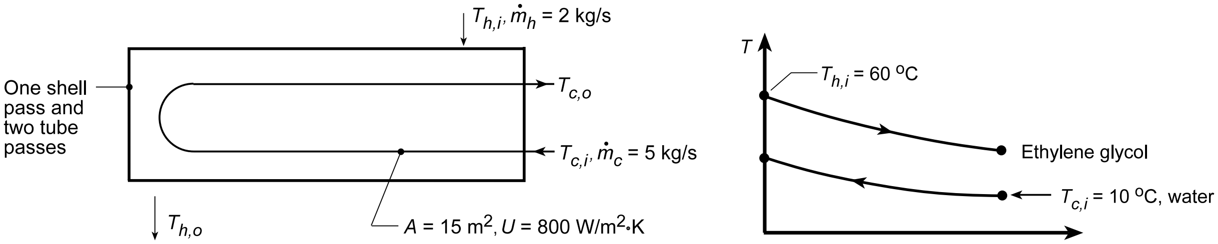
COMMENTS: With $\dot{m}_h$ = 0.2 kg/s and $T_{h,i}$ = 70°C, the outlet temperature of the blood is still below the desired level of $T_{c,o} \approx 37°C$. This value of $T_{c,o}$ could be increased by increasing L or $T_{h,i}$.

PROBLEM 11.69

KNOWN: Inlet temperatures and flow rates of water (c) and ethylene glycol (h) in a shell-and-tube heat exchanger (one shell pass and two tube passes) of prescribed area and overall heat transfer coefficient.

FIND: (a) Heat transfer rate and fluid outlet temperatures and (b) Compute and plot the effectiveness, ε, and fluid outlet temperatures, $T_{c,o}$ and $T_{h,o}$ as a function of the flow rate of ethylene glycol, $\dot{m}_h$, for the range $0.5 \le \dot{m}_h \le 5$ kg/s.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible kinetic and potential energy changes, (3) Constant properties, and (4) Overall coefficient remains unchanged.

PROPERTIES: *Table A-5*, Ethylene glycol ($\overline{T}_m \approx 40°C$): c_p = 2474 J/kg·K; *Table A-6*, Water ($\overline{T}_m \approx 15°C$): c_p = 4186 J/kg·K.

ANALYSIS: (a) Using the ε-NTU method we first obtain

$$C_h = \left(\dot{m}_h c_{p,h}\right) = \left(2\,\text{kg/s} \times 2474\,\text{J/kg}\cdot\text{K}\right) = 4948\,\text{W/K}$$

$$C_c = \left(\dot{m}_c c_{p,c}\right) = \left(5\,\text{kg/s} \times 4186\,\text{J/kg}\cdot\text{K}\right) = 20,930\,\text{W/K}\,.$$

Hence with $C_{min} = C_h$ = 4948 W/K and $C_r = C_{min}/C_{max}$ = 0.236,

$$\text{NTU} = \frac{UA}{C_{min}} = \frac{\left(800\,\text{W/m}^2\cdot\text{K}\right)15\,\text{m}^2}{4948\,\text{W/K}} = 2.43\,.$$

From Fig. 11.16, ε = 0.81 and from Eq. 11.23

$$q = \varepsilon C_{min}\left(T_{h,i} - T_{c,i}\right) = 0.81\left(4948\,\text{W/K}\right)\left(60-10\right)\text{K} = 2\times10^5\,\text{W}\,.$$ <

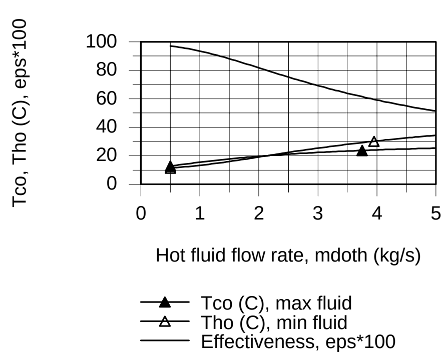
From Eqs. 11.6 and 11.7, energy balances on the fluids,

$$T_{h,o} = T_{h,i} - \frac{q}{C_h} = 60^\circ\text{C} - \frac{2\times10^5\,\text{W}}{4948\,\text{W/K}} = 19.6^\circ\text{C}$$ <

$$T_{c,o} = T_{c,i} + \frac{q}{C_c} = 10^\circ\text{C} + \frac{2\times10^5\,\text{W}}{20,930\,\text{W/K}} = 19.6^\circ\text{C}\,.$$ <

(b) Using the *IHT Heat Exchanger Tool*, *Shell and Tube*, and the *Properties Tool* for *Water* and *Ethylene Glycol*, $T_{c,o}$, $T_{h,o}$, and ε as a function of $\dot{m}_h$ were computed and plotted.

At very low C_{min}, (low $\dot{m}_h$) note that ε → 1 while $T_{h,o} \to T_{c,i}$. As $\dot{m}_h$ increases, both fluid outlet temperatures increase and the effectiveness decreases.

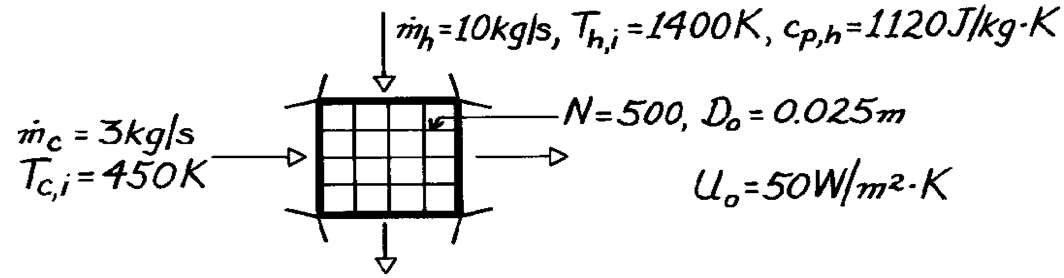


PROBLEM 11.70

KNOWN: Flow rate, specific heat and inlet temperature of gas in cross-flow heat exchanger. Flow rate and temperature of water which enters as saturated liquid and leaves as saturated vapor. Number of tubes, tube diameter and overall heat transfer coefficient.

FIND: Required tube length.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible kinetic and potential energy changes, (2) Negligible heat loss to surroundings, (3) Constant gas specific heat.

PROPERTIES: *Table A-6*, Saturated Water, (T = 450 K): $h_{fg} = 2.024 \times 10^6$ J/kg.

ANALYSIS: Use effectiveness-NTU method

$$\varepsilon = \frac{q}{q_{max}} = \frac{q}{C_{min}\left(T_{h,i} - T_{c,i}\right)} = \frac{q}{\dot{m}_h c_{p,h}\left(T_{h,i} - T_{c,i}\right)}$$

$$q = \dot{m}_c h_{fg} = 3\,kg/s \times 2.024 \times 10^6\,J/kg = 6.072 \times 10^6 W$$

$$\varepsilon = \frac{6.072 \times 10^6\,W}{10\ kg/s \times 1120 J/kg\cdot K\left(1400 - 450\right)K} = 0.571 \qquad C_{min}/C_{max} = 0.$$

From Fig. 11.19, find

$$NTU \approx 0.8 \approx U_o N \pi D_o L / C_{min}$$

$$L \approx \frac{0.8 \times 10\ kg/s \times 1120 J/kg\cdot K}{50\ W/m^2\cdot K \times 500\pi \times 0.025m} = 4.56m.$$ <

COMMENTS: (1) The gas outlet temperature is

$$T_{h,o} = T_{h,i} - q/\dot{m}_h c_{p,h} = 1400K - 6.072 \times 10^6 W/10 kg/s \times 1120 J/kg\cdot K = 857.9 K.$$

(2) Using the LMTD method,

$$\Delta T_{\ell m,CF} = \left[\left(1400 - 450\right) - \left(858 - 450\right)\right] / \ell n\left[\left(1400 - 450\right)/\left(858 - 450\right)\right] = 641K.$$

From Fig. 11.13, find F = 1, so the area and length are

$$A_o = q/U_o F\,\Delta T_{\ell m,CF} = 6.072 \times 10^6 W/\left(50\ W/m^2\cdot K \times 1 \times 641K\right) = 189m^2$$

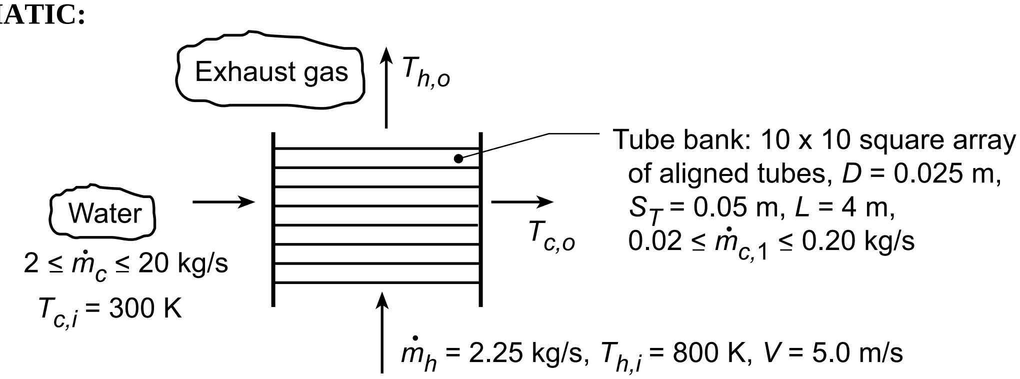
$$L = A/N\pi D_o = 189m^2/500\pi\left(0.025m\right) = 4.82m.$$

PROBLEM 11.71

KNOWN: Gas flow conditions upstream of a tube bank of prescribed geometry. Flow rate and inlet temperature of water passing through the tubes.

FIND: (a) Overall heat transfer coefficient, (b) Water and gas outlet temperatures, (c) Effect of water flow rate on heat recovery and outlet temperatures.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Negligible heat loss to the surroundings and kinetic and potential energy changes, (4) Negligible tube fouling and wall thermal resistance, (5) Fully developed water flow, (6) Gas properties are those of air.

PROPERTIES: *Table A.6*, Water (Assume $\overline{T}_m \approx 340$ K): $c_p = 4188$ J/kg·K, $\mu = 420 \times 10^{-6}$ N·s/m², k = 0.660 W/m·K, Pr = 2.66; *Table A.4*, Air (Assume $\overline{T}_m \approx 600$ K): $c_p = 1051$ J/kg·K, $\nu = 52.7 \times 10^{-6}$ m²/s, k = 0.047 W/m·K, Pr = 0.69.

ANALYSIS: (a) For the prescribed conditions, $U = (1/h_i + 1/h_o)^{-1}$. For the *internal* flow, with $\dot{m}_{c,1}$ = 0.025 kg/s,

$$Re_D = \frac{4\dot{m}_{c,1}}{\pi D \mu} = \frac{4 \times 0.025\,\text{kg/s}}{\pi(0.025\,\text{m})420 \times 10^{-6}\,\text{N}\cdot\text{s/m}^2} = 3032\,.$$

Hence, assuming turbulent flow,

$$Nu_D = 0.023\,Re_D^{4/5}\,Pr^{0.4} = 0.023(3032)^{4/5}(2.66)^{0.4} = 20.8$$

$$h_i = \frac{k}{D} Nu_D = \frac{0.660\,\text{W/m}\cdot\text{K}}{0.025\,\text{m}} 20.8 = 548\,.$$

For the external flow, $V_{max} = \frac{0.05\,\text{m}}{(0.05 - 0.025)\,\text{m}} 5.0\,\text{m/s} = 10.0\,\text{m/s}$. Hence

$$Re_{D,max} = \frac{V_{max} D}{\nu} = \frac{10\,\text{m/s} \times 0.025}{52.7 \times 10^{-6}\,\text{m}^2/\text{s}} = 4744$$

From the Zhukauskas correlation and Tables 7.7 and 7.8, $\overline{Nu}_D = (0.97)0.27\,Re_{D,max}^{0.63}\,Pr^{0.36}\left(Pr/Pr_s\right)^{1/4}$. Neglecting the Prandtl number ratio,

$$\overline{Nu}_D = (0.97)0.27(4744)^{0.63}(0.69)^{0.36} = 47.4$$

$$\overline{h}_o = \frac{k}{D}\overline{Nu}_D = \frac{0.047\,\text{W/m}\cdot\text{K}}{0.025\,\text{m}} 47.4 = 89.1\,\text{W/m}^2\cdot\text{K}\,.$$

Continued...

PROBLEM 11.71 (Cont.)

Hence, $U = (1/548 + 1/89.1)^{-1} = 76.7\ \mathrm{W/m^2 \cdot K}$. <

(b) The fluid outlet temperatures may be determined from the ε-NTU method. With $\dot{m}_c = 2.5$ kg/s, $C_c = \dot{m}_c c_{p,c} = 2.5\ \mathrm{kg/s} \times 4188\ \mathrm{J/kg \cdot K} = 10{,}470\ \mathrm{W/K}$. With $C_h = \dot{m}_h c_{p,h} = 2.25\ \mathrm{kg/s} \times 1051\ \mathrm{J/kg \cdot K} = 2365$ W/K, $C_{min}/C_{max} = C_{mixed}/C_{unmixed} = 2365/10{,}470 = 0.23$. Hence, with $A = N\pi DL = 100\pi \times 0.025\ \mathrm{m} \times 4\ \mathrm{m} = 31.4\ \mathrm{m^2}$,

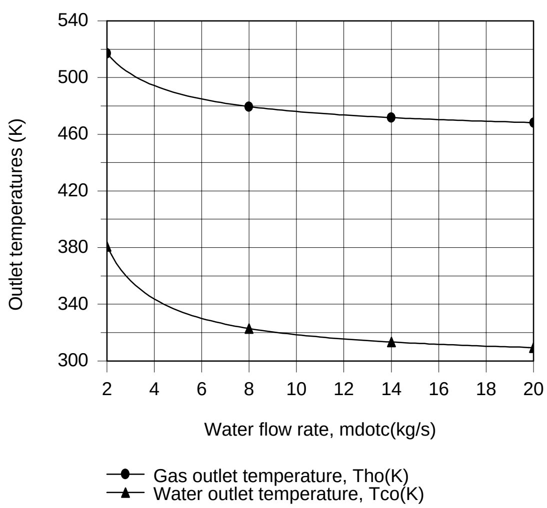
$$NTU = \frac{UA}{C_{min}} = \frac{76.7\ \mathrm{W/m^2 \cdot K}\left(31.4\ \mathrm{m^2}\right)}{2365\ \mathrm{W/K}} = 0.95$$

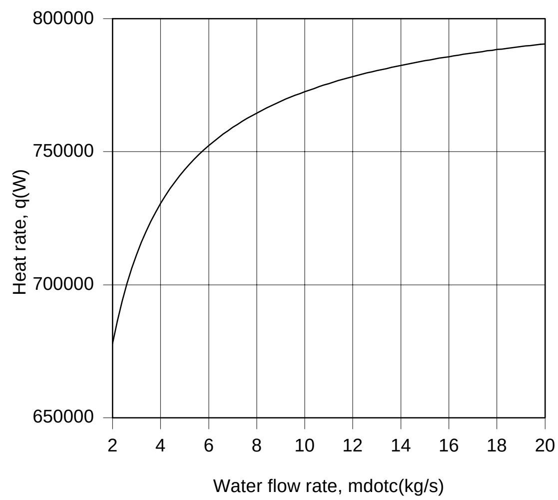
From Fig. 11.19, $\varepsilon \approx 0.61$. From Eq. 11.19, $q_{max} = C_{min}(T_{h,i} - T_{c,i}) = 2365\ \mathrm{W/K}(800 - 300)\mathrm{K} = 1.18 \times 10^6$ W. Hence, $q = \varepsilon q_{max} = 0.72 \times 10^6$ W. From Eq. 11.6b,

$$\left(T_{h,i} - T_{h,o}\right) = \frac{q}{C_h} = \frac{0.72 \times 10^6\ \mathrm{W}}{2365\ \mathrm{W/K}} = 304\ \mathrm{K} \qquad T_{h,o} = 496\ \mathrm{K}$$ <

From Eq. 11.7b, $$\left(T_{c,o} - T_{c,i}\right) = \frac{q}{C_c} = \frac{0.72 \times 10^6\ \mathrm{W}}{10{,}470\ \mathrm{W/K}} = 69\ \mathrm{K} \qquad T_{c,o} = 369\ \mathrm{K}$$ <

(c) Using the appropriate *Heat Exchangers, Correlations* and *Properties* Toolpads of IHT, the following results were obtained.





With increasing $\dot{m}_c$ (and $\dot{m}_{c,1}$), h_i increases, thereby increasing U and q. However, because the total resistance is dominated by the gas-side condition, $\dot{m}_c = 20$ kg/s only yields $U = 83.9\ \mathrm{W/m^2 \cdot K}$, despite the fact that $h_i = 2180\ \mathrm{W/m^2 \cdot K}$. Because the extent to which q increases with increasing $\dot{m}_c$ is much smaller than the increase in $\dot{m}_c$ itself, $T_{c,o}$ decreases with increasing $\dot{m}_c$. Hence, there is a trade-off between the amount of hot water and the temperature at which it is delivered. If, for example, the temperature must exceed 50°C ($T_{c,o} > 323$ K), $\dot{m}_c$ cannot exceed 8 kg/s. To maintain an acceptable value of $T_{c,o}$, while increasing $\dot{m}_c$, $\dot{m}_h$ (and V) should be increased, thereby increasing h_o, and hence U and q.

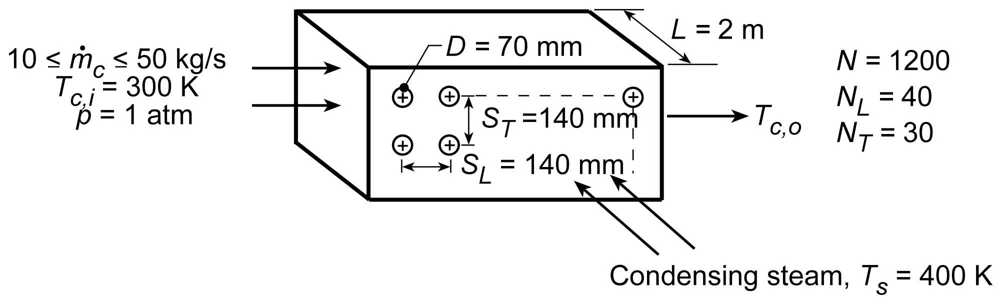
COMMENTS: If the air and water property functions of IHT are used to evaluate properties at appropriate mean values of the inlet and outlet fluid temperatures, the following, more accurate, results would be obtained for Parts (a) and (b): $\varepsilon = 0.582$, $q = 0.697 \times 10^6$ W, $T_{c,o} = 366.6$ K, $T_{h,o} = 508.8$ K, $h_i = 523\ \mathrm{W/m^2 \cdot K}$, $h_o = 86.5\ \mathrm{W/m^2 \cdot K}$ and $U = 74.2\ \mathrm{W/m^2 \cdot K}$.

PROBLEM 11.72

KNOWN: Tube arrangement in steam-to-air, cross-flow heat exchanger. Flow rate $\dot{m}_c$ and inlet temperature of air. Condensing temperature of steam.

FIND: (a) Air outlet temperature for $\dot{m}_c$ = 12 kg/s, (b) Effect of $\dot{m}_c$ on air outlet temperature, heat rate and condensation rate.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible kinetic and potential energy changes, (3) Negligible steam side convection and tube wall conduction resistance, (4) Mean air temperature is 350 K.

PROPERTIES: *Table A.4*, Air (Assume $\bar{T}_c \equiv \left(T_{c,i} + T_{c,o}\right)/2 \approx 350$ K, 1 atm): $\rho = 0.995$ kg/m^3, c_p = 1009 J/kg·K, $\nu = 20.92 \times 10^{-6}$ m^2/s, k = 0.030 W/m·K, Pr = 0.700; T_s = 400 K: Pr = 0.690.

ANALYSIS: (a) For a single-pass, cross-flow heat exchanger with one fluid mixed and the other unmixed, Fig. 11.19 can be used to obtain ε, where $C_{min}/C_{max} = C_{mixed}/C_{unmixed} = 0$ and $NTU = UA/C_{min} = U(\pi DL)N/\dot{m}_c c_p$. From Eq. 11.5, $U = \bar{h}_o$, and the Zhukauskas correlation may be used to estimate $\bar{h}_o$. The upstream velocity may be obtained from $\dot{m}_c = \rho VA \approx \rho V N_T L S_T$. Hence,

$$V = \frac{\dot{m}_c}{\rho N_T L S_T} = \frac{12\,\text{kg/s}}{0.995\,\text{kg/m}^3 \times 30 \times 2\,\text{m} \times 0.14\,\text{m}} = 1.44\,\text{m/s}.$$

For aligned tubes,

$$V_{max} = \frac{S_T}{S_T - D} V = \frac{0.14\,\text{m}}{(0.14 - 0.07)\,\text{m}} 1.44\,\text{m/s} = 2.88\,\text{m/s}$$

$$Re_{D,max} = \frac{V_{max} D}{\nu} = \frac{2.88\,\text{m/s} \times 0.07\,\text{m}}{20.92 \times 10^{-6}\,\text{m}^2/\text{s}} = 9637.$$

From Table 7.7, select values of C = 0.27 and m = 0.63. Hence,

$$\overline{Nu}_D = 0.27\, Re_{D,max}^{0.63}\, Pr^{0.36} \left(Pr/Pr_s\right)^{0.25}$$

$$\overline{Nu}_D = 0.27(9637)^{0.63}(0.70)^{0.36}(0.70/0.69)^{0.25} = 77.1$$

$$\bar{h}_o = \overline{Nu}_D \frac{k}{D} = 77.1 \frac{0.030\,\text{W/m·K}}{0.07\,\text{m}} = 33.0\,\text{W/m}^2\text{·K}.$$

Hence,

$$NTU = \frac{\bar{h}_o \pi D L N}{\dot{m}_c c_p} = \frac{33.0\,\text{W/m}^2\text{·K} \times \pi (0.07\,\text{m})\, 2\,\text{m}\,(1200)}{12\,\text{kg/s} \times 1009\,\text{J/kg·K}} = 1.44.$$

From Fig. 11.19, find ε ≈ 0.77 and then determine

Continued...

PROBLEM 11.72 (Cont.)

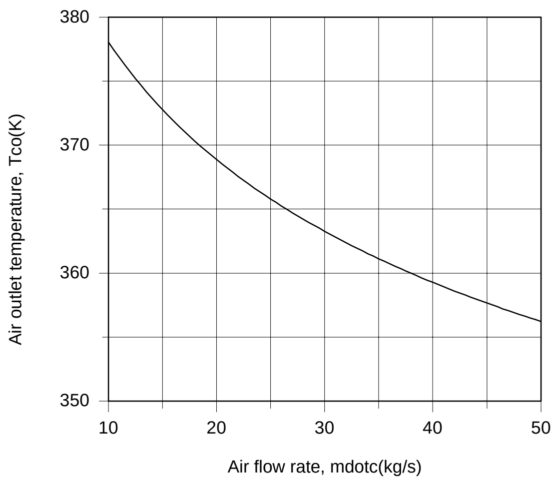
$$\varepsilon = \frac{q}{q_{max}} = \frac{\dot{m}_c c_p \left(T_{c,o} - T_{c,i}\right)}{\dot{m}_c c_p \left(T_s - T_{c,i}\right)} = \frac{T_{c,o} - T_{c,i}}{T_s - T_{c,i}}$$

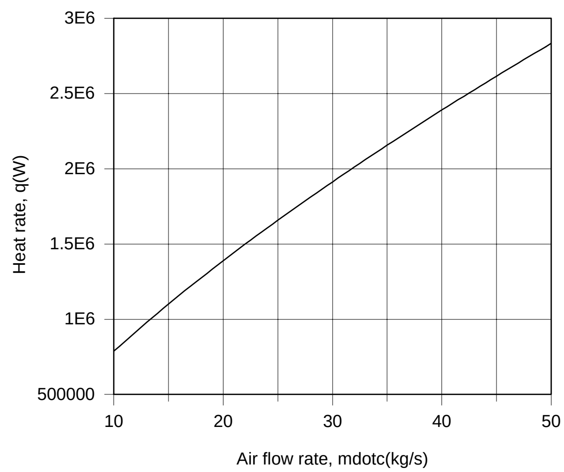
$$T_{c,o} = T_{c,i} + \varepsilon\left(T_s - T_{c,i}\right) = 300\,\text{K} + 0.77(400 - 300)\,\text{K} = 377\,\text{K} = 104^\circ\text{C}$$ <

(b) With $q = \varepsilon q_{max} = \varepsilon C_c(T_s - T_{c,i})$ and the condensation rate given by Eqs. 10.33 and 10.26,

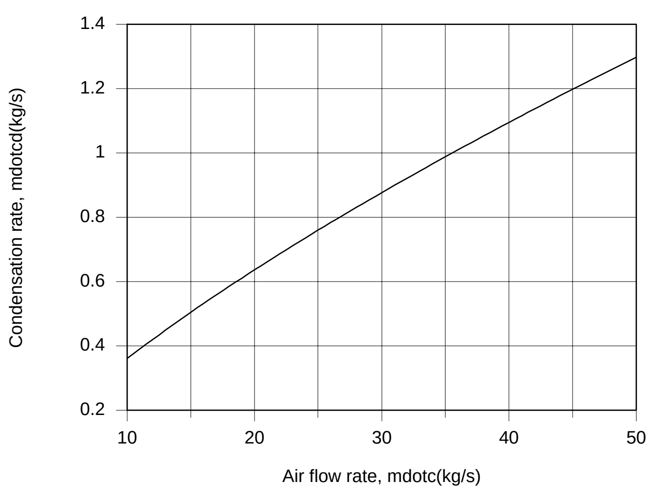
$$\dot{m}_{cd} = \frac{q}{h'_{fg}} \approx \frac{q}{h_{fg}}$$

the foregoing model may be used with the Heat Exchangers, Correlations and Properties Toolpads of IHT to determine the effect of $\dot{m}_c$ on $T_{c,o}$, q and $\dot{m}_{cd}$.





Since $\bar{h}_o$ increases with increasing $\dot{m}_c$, q must also increase. However, since the increase in q is proportionally less than the increase in $\dot{m}_c$, $T_{c,o}$ decreases with increasing $\dot{m}_c$.



The condensation rate increases proportionally with the increase in q, and if the objective is to maximize the condensation rate, the largest value of $\dot{m}_c$ should be maintained.

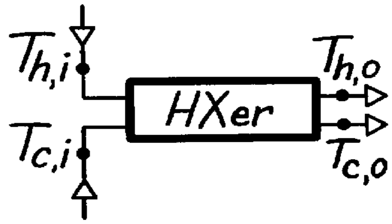
COMMENTS: If the objective is to heat the air, there is obviously a trade-off between maintaining elevated values of the flowrate and outlet temperature.

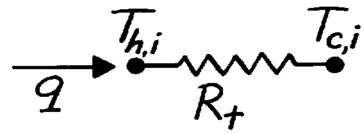
PROBLEM 11.73

KNOWN: Heat exchanger operating in parallel-flow configuration.

FIND: Expression for R_{lm}/R_t which doesn't involve temperatures. Plot result.

SCHEMATIC:





ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible change in kinetic and potential energy.

ANALYSIS: (a) For the exchanger, the rate equation is

$$q = UA\Delta T_{lm}$$

and we can define thermal resistances as

$$R_t = \left(T_{h,i} - T_{c,i}\right)/q \qquad \text{or} \qquad R_{lm} = \left(\Delta T_{lm}\right)/q = 1/UA.$$

Using the rate equation and the definition of effectiveness, find the thermal resistance based upon the inlet temperatures of the hot and cold fluids as

$$R_t = C_{min}\left(T_{h,i} - T_{c,i}\right)/C_{min} \cdot q = 1/e C_{min}.$$

The ratio of these resistances is

$$\frac{R_{lm}}{R_t} = \frac{1/UA}{1/eC_{min}} = \frac{e}{UA/C_{min}} = \frac{e}{NTU}$$

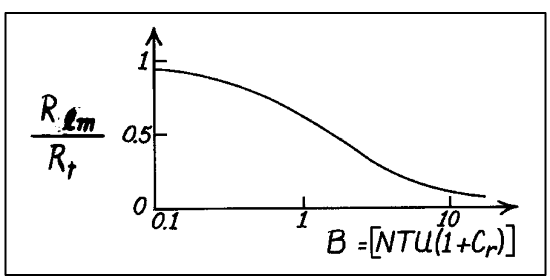
and for the parallel flow, concentric tube configuration using Eq. 11.29a,

$$\frac{R_{lm}}{R_t} = \frac{1 - \exp\left[-NTU\left(1 + C_r\right)\right]}{NTU\left(1 + C_r\right)} = \frac{1 - \exp(-B)}{B}$$ <

where $B = NTU(1 + C_r)$. Evaluating the ratio for various values of B, find

<

B	R_{lm}/R_t
0.1	0.95
0.5	0.79
1.0	0.63
3.0	0.32
5.0	0.20
10.0	0.10



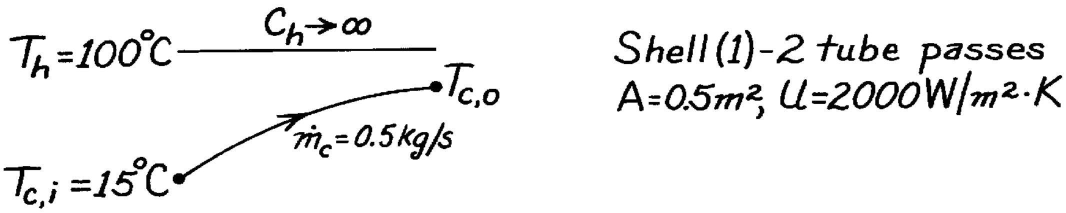
COMMENTS: (1) For $C_{max} \to \infty$, $C_r \to 0$; hence $B \to NTU$. (2) For $C_{max} \approx C_{min}$, $B \to 2NTU$ or $B \sim C_{min}^{-1}$. (3) For $B << 1$, $R_{lm}/R_t \to 1$. (4) For $B >> 1$, $R_{lm}/R_t \to B^{-1}$. (5) We conclude that care must be taken in representing heat exchangers with a thermal resistance, recognizing that the resistance will depend on flow rates for wide ranges of conditions.

PROBLEM 11.74

KNOWN: Heat exchanger condensing steam at 100°C with cooling water supplied at 15°C.

FIND: (a) Thermal resistance of the exchanger, (b) Change in thermal resistance if fouling is 0.0002 $m^2 \cdot K/W$ on each of the inner and outer tube surfaces, and (c) Plot the thermal resistance as a function of tube water inlet rate assuming all other conditions remain unchanged; comment on whether UA will remain constant if the flow rate changes.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible kinetic and potential energy changes.

PROPERTIES: *Table A-6*, Water ($\overline{T}_m$ = 42°C=315 K): c_p = 4179 J/kg·K.

ANALYSIS: (a) For an exchanger, using the rate equation,

$$q = UA\Delta T_{lm} = \left(T_{h,i} - T_{c,i}\right) / R_t,$$

the thermal resistance of the exchanger can be expressed as

$$R_t = \frac{T_{h,i} - T_{c,i}}{UA\Delta T_{lm}} = \frac{C_{min}\left(T_{h,i} - T_{c,i}\right)}{C_{min} \cdot q} = \frac{1}{C_{min}} \cdot \frac{q_{max}}{q} = \frac{1}{\varepsilon C_{min}}.$$

For the present exchanger with $C_r = 0$, use Eq. 11.36a with

$$C_{min} = \dot{m}_c c_{p,c} = 0.5\,kg/s \times 4179\,J/kg \cdot K = 2090\,W/K$$

$$NTU = UA/C_{min} = 2000\,W/m^2 \cdot K \times 0.5m^2 / 2090\,W/K = 0.478$$

$$\varepsilon = 1 - \exp(-NTU) = 0.380.$$

Hence, the thermal resistance is

$$R_t = 1/0.380 \times 2090\,W/K = 1.258 \times 10^{-3}\,K/W.$$ <

(b) With fouling present, the overall heat transfer coefficient will decrease.

No fouling: $$\frac{1}{U_o A} = \frac{1}{h_h A_h} + \frac{1}{h_c A_c}$$

With fouling: $$\frac{1}{U_f A} = \frac{1}{U_o A} + \frac{R''_{f,c}}{A_c} + \frac{R''_{f,h}}{A_h} = \frac{1}{U_o A} + \frac{2R''_f}{A}$$

Continued …..

$$\frac{1}{U_f A} = \frac{1}{2000\ \text{W/m}^2\cdot\text{K} \times 0.5\text{m}^2} + \frac{2\times 0.0002\ \text{m}^2\cdot\text{K/W}}{0.5\text{m}^2}$$

$$U_f A = 555.6\ \text{W/K}.$$

It follows that NTU = U_fA/C_{min} = 0.266 and ε_f = 0.233 giving

$$R_{t,f} = 1/e_f C_{min} = 2.050\times 10^{-3}\ \text{K/W}$$

and hence the increase in thermal resistance due to fouling is

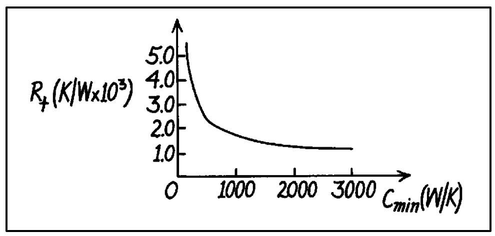
$$\left(R_{t,f} - R_t\right)/R_t = \left(2.050 - 1.258\right)/1.258 = 63\%.$$ <

(c) With no fouling, the thermal resistance, when all other conditions (U_oA = 1000 W/K) remain unchanged, depends on C_{min} only as NTU = U_oA/C_{min},

$$R_t = \frac{1}{eC_{min}} = \frac{1}{C_{min}}\left[1-\exp\left(-\frac{UA}{C_{min}}\right)\right]^{-1} = \frac{1}{C_{min}}\left[1-\exp\left(-\frac{1000\ \text{W/K}}{C_{min}}\right)\right]^{-1}$$

C_{min} (W/K)	200	400	600	1000	1500	2000	3000
R_t (K/W × 10^3)	4.967	2.723	2.055	1.582	1.370	1.271	1.176

From the plot note that R_t is a weak function of C_{min} above C_{min} > 1000 W/K, from which we conclude that using a constant R_t would be reasonable.



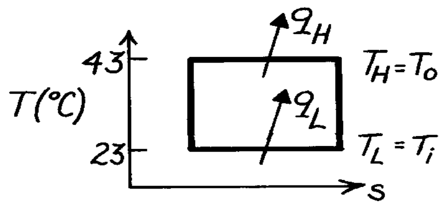
Concerning the variability of UA with changing C_{min}: if most of the resistance is on the water side and the flow is turbulent, $h_c \approx Re_D^{0.8} \approx u_m^{0.8} \approx \dot{m}_c^{0.8}$. It follows that h_c will depend significantly on changes in C_{min}. However, if h_c and h_h are of similar magnitude, the effect of C_{min} on U may not be significant.

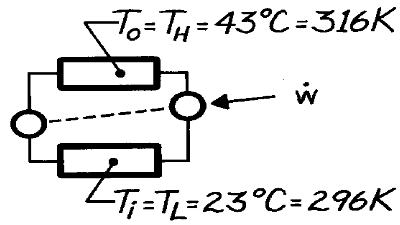
PROBLEM 11.75

KNOWN: Air conditioner modeled as a reversed Carnot heat engine, with refrigerant as the working fluid, operating between indoor and outdoor temperatures of 23 and 43°C, respectively, removing 5 kW from a building. Compressor and fan motor efficiency of 80%.

FIND: (a) Required motor power assuming negligible thermal resistances *between* the refrigerant in the condenser and the outside air and *between* the refrigerant in the evaporator and the inside air, and (b) Required power if thermal resistances are each 3×10^{-3} K/W.

SCHEMATIC:





ASSUMPTIONS: (1) Ideal heat exchanger with no losses, (2) Air conditioner behaves as reversed Carnot engine.

ANALYSIS: (a) With negligible thermal resistances, the Carnot cycle and reversed heat engine can be represented as shown above. Hence,

$$\dot{w}_{ideal} = q_H - q_L = q_L\left[(T_H / T_L) - 1\right] = 5\text{kW}\left[(316\text{ K}/296\text{ K}) - 1\right] = 0.3378 \text{ kW}.$$

Considering the fan power requirement and the efficiency of the motor,

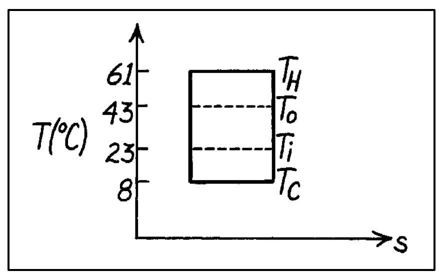
$$\dot{w}_{act} = \left(\dot{w}_{ideal} + \dot{w}_{fan}\right)/h_c = (0.3378 + 0.200)\text{kW}/0.8 = 0.672 \text{ kW}. \qquad <$$

(b) Consider now thermal resistances of $R_t = 3 \times 10^{-3}$ K/W on the high temperature (condenser) and low temperature (evaporator) sides.

Low side: in order to remove heat from the room, $T_C < T_i$. That is

$$T_i - T_C = qR_t = 5 \text{ kW}\left(3\times10^{-3}\text{ K/W}\right) = 15 \text{ K}$$

$$T_C = T_i - 15\text{ K} = 23°\text{C} - 15 \text{ K} = 8°\text{C}.$$



High side: in order to reject heat from the condenser to the outside air, $T_H > T_o$,

$$T_H - T_o = q_H R_t = q_c (T_H / T_c) R_t$$

$$T_H - (43 + 273)\text{ K} = 5 \text{ kW}\left[T_H / (8 + 273)\right] 3\times10^{-3}\text{ K/W} \qquad T_H = 333.9 \text{ K} = 61°\text{C}.$$

The work required for this cycle is

$$\dot{w}_{ideal} = q_H - q_L = q_L\left[(T_H / T_L) - 1\right] = 5 \text{ kW}\left[(61 + 273)\text{K} /(8 + 273)\text{ K} - 1\right] = 0.943\text{kW}$$

$$\dot{w}_{act} = \left(\dot{w}_{ideal} + \dot{w}_{fan}\right)/h_c = (0.943 + 0.2)\text{kW}/0.8 = 1.43 \text{ kW}. \qquad <$$

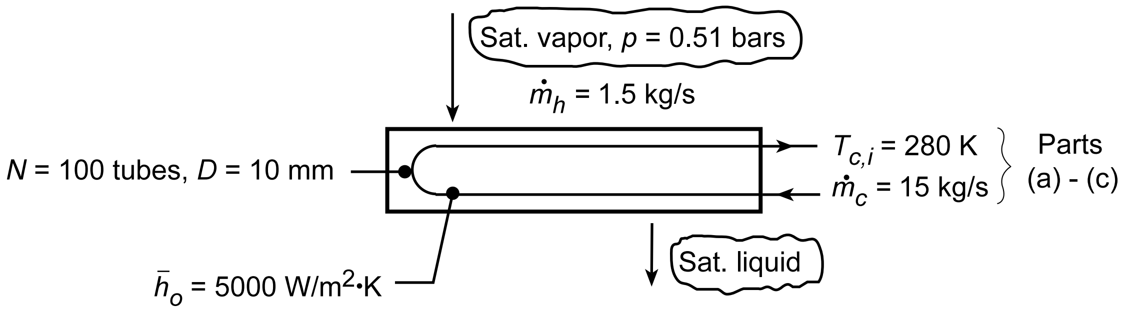
The effect of finite thermal resistances in the evaporator and condenser is to increase the power by a factor of two.

PROBLEM 11.76

KNOWN: Flow rate and pressure of saturated vapor entering a condenser. Number and diameter of condenser tubes. Water flow rate and inlet temperature. Tube outside convection coefficient.

FIND: (a) Water outlet temperature, (b) Total tube length, (c) Effect of fouling on mass condensation, (d) Effect of water flow rate and inlet temperature on condenser performance.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat loss to surroundings and potential and kinetic energy changes, (2) Constant properties, (3) Negligible wall conduction resistance and fouling (initially).

PROPERTIES: Water (given): $c_p = 4178$ J/kg·K, $\mu = 700 \times 10^{-6}$ kg/s·m, k = 0.628 W/m·K, Pr = 4.6; *Table A.6,* Sat. steam (355 K): $h_{fg} = 2.304 \times 10^6$ J/kg; With fouling: $R''_f = 0.0003\ \text{m}^2\cdot\text{K/W}$.

ANALYSIS: (a) From an energy balance, $q_h = \dot{m}_h\left(i_{h,i} - i_{h,o}\right) = \dot{m}_h h_{fg} = q_c = \dot{m}_c c_{p,c}\left(T_{c,o} - T_{c,i}\right)$, or

$$T_{c,o} = T_{c,i} + \frac{\dot{m}_h c_{c,p}}{\dot{m}_c c_{p,c}} = 280\,\text{K} + \frac{1.5\,\text{kg/s} \times 2.304 \times 10^6\ \text{J/kg}}{15\,\text{kg/s} \times 4178\,\text{J/kg}\cdot\text{K}} = 335.1\,\text{K}. \qquad <$$

(b) Since $C_r = 0$, NTU = -ln(1 - ε), where

$$\varepsilon = \frac{q}{q_{max}} = \frac{\dot{m}_c c_{p,c}\left(T_{c,o} - T_{c,i}\right)}{\dot{m}_c c_{p,c}\left(T_{h,i} - T_{c,i}\right)} = \frac{(335.1 - 280)\,\text{K}}{(355 - 280)\,\text{K}} = 0.735$$

Hence, NTU = -ln(1 - 0.735) = 1.327 = UA/C_{min}. The overall heat transfer coefficient is given by $1/U = 1/\bar{h}_i + 1/\bar{h}_o$. For the internal tube flow,

$$Re_D = \frac{4\dot{m}_{c,1}}{\pi D \mu} = \frac{4 \times 15\,\text{kg/s}/100}{\pi(0.01\,\text{m})\,700 \times 10^{-6}\ \text{kg/s}\cdot\text{m}} = 27,284$$

Hence, assuming fully developed flow with the Dittus-Boelter correlation,

$$Nu_D = 0.023\,Re_D^{4/5}\,Pr^n = 0.023(27,284)^{4/5}(4.6)^{0.4} = 149.8$$

$$\bar{h}_i = (k/D)\,Nu_D = \frac{0.628\,\text{W/m}\cdot\text{K}}{0.01\,\text{m}}149.8 = 9408\,\text{W}\big/\text{m}^2\cdot\text{K}$$

and $U = \left[(1/9408) + (1/5000)\right]^{-1}\ \text{W}\big/\text{m}^2\cdot\text{K} = 3265\,\text{W}\big/\text{m}^2\cdot\text{K}$. Hence, the heat transfer area is

$$A = \dot{m}_c c_{p,c}\,(\text{NTU}/U) = 15\,\text{kg/s}\,(4178\,\text{J/kg}\cdot\text{K})\left(1.327/3265\,\text{W}\big/\text{m}^2\cdot\text{K}\right) = 25.5\,\text{m}^2$$

and the tube length is L = A/NπD = 25.5 m²/100π(0.01 m) = 8.11 m. <

(c) With fouling, the overall heat transfer coefficient is $1/U_w = 1/U_{wo} + R''_f$. Hence,

Continued...

PROBLEM 11.76 (Cont.)

$$1/U_w = \left(3.063\times10^{-4} + 3\times10^{-4}\right) m^2\cdot K/W$$

$U_w = 1649\ W/m^2\cdot K$.

$$NTU = UA/C_{min} = \left(1649\ W/m^2\cdot K\times25.5\ m^2\right)/\left(15\ kg/s\times4178\ J/kg\cdot K\right) = 0.671$$

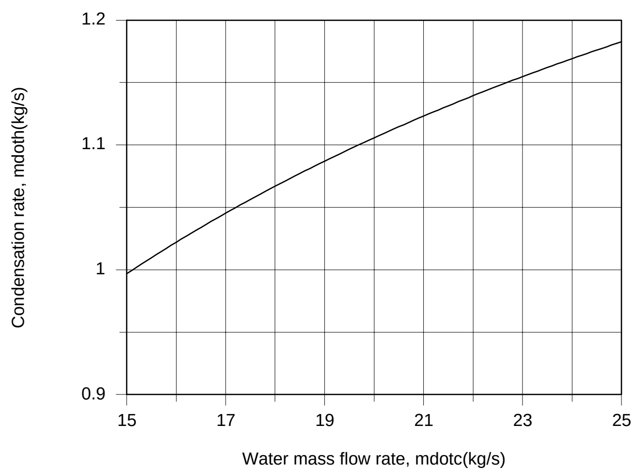
From Eq. 11.36a, $\varepsilon = 1 - \exp(-NTU) = 1 - \exp(-0.671) = 0.489$. Hence, $q = \varepsilon q_{max} = 0.489 \times 15\ kg/s \times 4178\ J/kg\cdot K(355 - 280)K = 2.30 \times 10^6\ W$. Without fouling the heat rate was

$$q = \dot{m}_h h_{fg} = 1.5\ kg/s\times2.304\times10^6\ J/kg = 3.46\times10^6\ W.$$

Hence, $\dot{m}_{h,w}/\dot{m}_{h,wo} = 2.30\times10^6/3.46\times10^6 = 0.666$. <

The condensation rate with fouling is then $\dot{m}_{h,w} = 0.666\times1.5\ kg/s = 0.998\ kg/s$.

(d) The prescribed water inlet temperature of $T_{c,i} = 280$ K is already at the lower limit of available sources, and it would not be feasible to consider smaller values. In addition, with $\bar{h}_i$ already quite large, an increase in $\dot{m}_c$ is not likely to provide a significant improvement in performance. Using the *Heat Exchanger* and *Correlations* Tools from IHT, the following results were obtained for $15 \le \dot{m}_c \le 25$ kg/s.



Over the specified range of $\dot{m}_c$, there is approximately an 18% increase in the heat rate, and hence in the condensation rate. This increase is, in part, due to the increase in $\bar{h}_i$ from 9408 to 14,160 $W/m^2\cdot K$, which increases U from 1649 to 1752 $W/m^2\cdot K$, as well as to a reduction in $T_{c,o}$ from 316.6 to 306.0 K, which increases the mean driving potential for heat transfer.

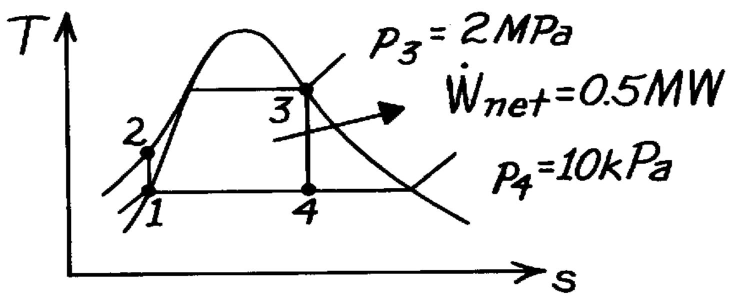
COMMENTS: There is a significant reduction in performance due to fouling, which can not be restored by increasing $\dot{m}_c$. The desired performance could be achieved by oversizing the condenser, that is, by increasing the number of tubes and/or the tube length.

PROBLEM 11.77

KNOWN: Rankine cycle with saturated steam leaving the boiler at 2 MPa and a condenser pressure of 10 kPa. Net reversible work of 0.5 MW.

FIND: (a) Thermal efficiency of ideal Rankine cycle, (b) Required cooling water flow rate to condenser at 15°C with allowable temperature rise of 10°C, and (c) Design of a shell and tube heat exchanger (one shell and multiple tube passes) to satisfy condenser flow rate and temperature rise.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible loss from condenser to surroundings, (2) Negligible kinetic and potential energy changes in heat exchanger, (3) Ideal Rankine cycle, and (4) Negligible thermal resistance on condensate side of exchanger tubes.

PROPERTIES: *Steam Tables*, (Wark, 4th Edition): (1) $p_1 = p_4$ = 10 kPa = 0.10 bar, T_{sat} = 45.8°C = 319 K, $v_f = 1.0102 \times 10^{-3}$ m^3/kg, h_f = 191.83 kJ/kg; (3) $p_2 = p_3$ = 2 Mpa = 20 bar, h_g = 2799.5 kJ/kg, s_g = 6.3409 kJ/kg·K; (4) $s_4 = s_3$ = 6.3409 kJ/kg·K, p_4 = 0.10 bar, s_f = 0.6493 kJ/kg·K, s_g = 8.1502 kJ/kg·K, h_f = 191.83 kJ/kg·K, h_{fg} = 2392.8 kJ/kg; *Table A-6*, Water (T_{sat} = 293 K): $c_{p,c}$ = 4182 J/kg·K, $\mu = 1007 \times 10^{-6}$ N·s/m^2, k = 0.603 W/m·K, Pr = 7.0. Note: 1 bar = 10^5 N/m^2 = 10^5 Pa.

ANALYSIS: (a) Referring to your thermodynamics text, find that

$$h = \frac{w_{net}}{Q_H} = \frac{w_t - w_p}{Q_H} = \frac{(h_3 - h_4) - v_1(p_2 - p_1)}{h_3 - h_2}$$

where the net work is the turbine minus the pump work. Assuming the liquid in the pump is incompressible,

$$w_p = v_1(p_2 - p_1) = 1.0102 \times 10^{-3}\,\mathrm{m^3/kg}\left(2 \times 10^6 - 10 \times 10^3\right)\mathrm{N/m^2} = 2.01\,\mathrm{kJ/kg}.$$

To find the enthalpies at states 2, 3, and 4, consider the individual processes. For the *pump*,

$$h_2 = h_1 + w_p = (191.83 + 2.01)\,\mathrm{kJ/kg} = 193.84\ \mathrm{kJ/kg}.$$

Since the exit state of the boiler is saturated at p_3 = 2 Mpa,

$$h_3 = h_g = 2799.5\ \mathrm{kJ/kg}.$$

$$Q_H = h_3 - h_2 = (2799.5 - 193.84)\,\mathrm{kJ/kg} = 2605.7\ \mathrm{kJ/kg}.$$

Since the process from 3 to 4 is isentropic, $s_4 - s_3$, hence

$$x_4 = (s_4 - s_f)/(s_g - s_f) = (6.3409 - 0.6493)/(8.1502 - 0.6493) = 0.759$$

$$h_4 = h_f + x h_{fg} = [191.83 + 0.759(2392.8)]\,\mathrm{kJ/kg} = 2007.5\,\mathrm{kJ/kg}.$$

Continued …..

PROBLEM 11.77 (Cont.)

$$w_t = h_3 - h_4 = (2799.5 - 2007.5)\,\text{kJ/kg} = 792.0\ \text{kJ/kg}.$$

Substituting appropriate values, the thermal efficiency is

$$h = \frac{(792.0 - 2.01)\,\text{kJ/kg}}{2605.7\ \text{kJ/kg}} = 0.303 = 30.3\%. \qquad <$$

(b) From an overall balance on the cycle, the heat rejected to the condenser is

$$Q_c = Q_H - w_{net} = \left[2605.7 - (792.0 - 2.01)\right]\text{kJ/kg} = 1815.7\ \text{kJ/kg}.$$

Since the net reversible power is 0.5 MW, the required steam rate (h) is

$$\dot{m}_h = \dot{W}_{net} / w_{net} = 0.5 \times 10^6\,\text{W} / (792.0 - 2.01)\,\text{kJ/kg} = 0.6329\ \text{kg/s}.$$

Hence, the heat rate to be removed by the cold water passing through the condenser is

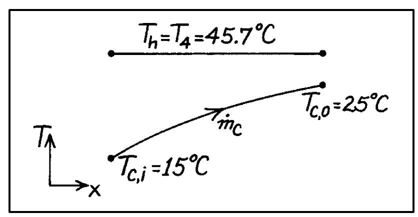
$$q_c = Q_c \cdot \dot{m}_h = \dot{m}_c\, c_{p,c}\left(T_{c,out} - T_{c,in}\right)$$

$$1815.7\ \text{kJ/kg} \times 0.6329\ \text{kg/s} = 1.149 \times 10^6\ \text{W} = \dot{m}_c \times 4182\,\text{J/kg}\cdot\text{K}(25 - 15)\,\text{K}$$

$$\dot{m}_c = 27.47\,\text{kg/s} \qquad <$$

where $c_{p,c} = c_{p,f}$ is evaluated at T_2, $T_{c,in} = 15°C$ and $T_{c,out} - T_{c,in} = 10°C$, the specified allowable rise.

(c) To design the heat exchanger we need to evaluate UA. Considering the shell-tube configuration and since $C_r = C_{min}/C_{max} = 0$,



$$e = 1 - \exp(-\text{NTU}) = 1 - \exp\left[-\left(\text{UA}/C_{min}\right)\right]$$

$$e = \frac{q}{q_{max}} = \frac{q_c}{\dot{m}_c\, c_{p,c}\left(T_h - T_{c,i}\right)}$$

$$e = \frac{1.149 \times 10^6\ \text{W}}{27.47\,\text{kg/s} \times 4182\,\text{J/kg}\cdot\text{K}(45.7 - 15)\,\text{K}} = 0.326$$

$$0.326 = 1 - \exp\left(-\frac{\text{UA}}{27.47\ \text{kg/s} \times 4182\,\text{J/kg}\cdot\text{K}}\right)$$

$$\text{UA}_s = 45{,}372\ \text{W/K}$$

where $C_{min} = \dot{m}_c\, c_{p,c}$. Our design process will involve the following steps: select tube diameter, D = 15 mm; set u_m = 2 m/s in each tube and find number of tubes; perform internal flow calculation to estimate $\overline{h}_c$ and then determine the length.

$$\dot{m}_c = r A_c \text{Nu}_m = \left(1.010 \times 10^{-3}\,\text{m}^3/\text{kg}\right)^{-1}\left(p\,(0.015\text{m})^2/4\right)2\,\text{m/s} \times \text{N} = 27.47\ \text{kg/s}$$

$$\text{N} = 78.5 \approx 79.$$

Continued …..

PROBLEM 11.77 (Cont.)

For flow in a single tube,

$$\mathrm{Re}_D = \frac{4\dot{m}_t}{\pi D \mu} = \frac{4(27.47\ \mathrm{kg/s}/79)}{\pi(0.015\mathrm{m})1007\times10^{-6}\ \mathrm{N\cdot s/m^2}} = 29{,}310.$$

Assuming the flow is fully developed and using the Dittus-Boelter correlation,

$$\mathrm{Nu} = \frac{hD}{k} = 0.023\mathrm{Re}_D^{0.8}\,\mathrm{Pr}^{0.4} = 0.023(29{,}310)^{0.8}(7.00)^{0.4} = 187.7$$

$$h = 0.603\,\mathrm{W/m\cdot K}\times187.7/0.015\mathrm{m} = 7544\ \mathrm{W/m^2\cdot K}.$$

Hence, the tube length is

$$UA_s = h(\pi DL)\,N = 45{,}372\ \mathrm{W/K}$$

$$L = 45{,}372\ \mathrm{W/K}/7544\ \mathrm{W/m^2\cdot K}\times\pi(0.015\mathrm{m})79 = 1.6\mathrm{m}$$

and our design has the following parameters:

$N = 79$ tubes $\quad\quad L = 1.6\mathrm{m} \quad\quad D = 15$ mm. <

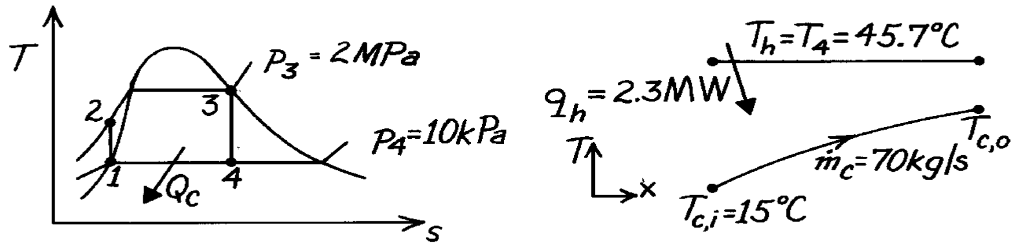
COMMENTS: (1) The selection of the tube diameter and water velocity values (15 mm, 2 m/s) was based upon prior experience; they seemed reasonable. We could, however, establish other requirements which would influence these choices such as allowable pressure drop and standard tube sizes.

PROBLEM 11.78

KNOWN: Rankine cycle with saturated steam leaving the boiler at 2 Mpa and a condenser pressure of 10 kPa. Heat rejected to the condenser of 2.3 MW. Condenser supplied with cooling water at rate of 70 kg/s at 15°C.

FIND: (a) Size of the condenser as determined by the parameter, UA, and (b) Reduction in thermal efficiency of the cycle if U decreases by 10% due to fouling assuming water flow rate and inlet temperature and the condenser steam pressure remain fixed.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible loss from condenser to surroundings, (2) Negligible kinetic and potential energy changes in exchanger, (3) Ideal Rankine cycle, (4) For fouled operating condition, $\dot{m}_c$, $T_{c,i}$ and p_4 remain the same.

PROPERTIES: *Steam Tables* (Wark, 4th Edition): See previous problem for calculations to obtain cycle enthalpies; h_1 = 191.83 kJ/kg, h_4 = 2007.5 kJ/kg.

ANALYSIS: (a) For the condenser, recognize that $C_{min} = C_c$, and $C_r = C_{min}/C_{max} = 0$,

$$e = \frac{q}{q_{max}} = 1 - \exp(-NTU) = 1 - \exp(-UA/C_{min})$$

$$C_{min} = \dot{m}_c c_{p,c} = 70\ kg/s \times 4182\ J/kg \cdot K = 292{,}740\ W/K$$

$$q_{max} = C_{min}(T_h - T_{c,i}) = 292{,}740\ W/K\,(45.7 - 15)\,K = 8.987 \times 10^6\ W.$$

$$q = q_h = 2.30 \times 10^6\ W$$

$$\frac{2.30 \times 10^6\ W}{8.987 \times 10^6\ W} = 0.256 = 1 - \exp\left(-\frac{UA}{292{,}740\ W/K}\right)$$

$$UA = 86{,}538\ W/K.$$ <

(b) In the fouled condition, U is reduced 10%, hence

$$U_f A = 0.9UA = 77{,}884\ W/K$$

and

$$NTU_f = \frac{U_f A}{C_{min}} = \frac{77{,}884\ W/K}{292{,}740\ W/K} = 0.266$$

$$e_f = 1 - \exp(-NTU_f) = 1 - \exp(-0.266) = 0.234.$$

Continued …..

If we operate the cycle at the same back pressure p_4 = 10 kPa so that T_h = 45.7°C, the heat removal rate must decrease,

$$q_h = e q_{max} = 0.234 \times 8.987 \times 10^6 \text{ W} = 2.103 \times 10^6 \text{ W}$$

since $q_{max} = C_{min}$ (T_h – $T_{c,i}$) remains the same. From the previous problem, we found the heat rejected as

$$h_4 - h_1 = (2007.5 - 191.83) \text{kJ/kg} = 1815.7 \text{ kJ/kg}$$

and hence the cycle steam rate through the *fouled* condenser is

$$\dot{m}_{h,f} = q_h / (h_4 - h_1) = 2.103 \times 10^6 \text{ W}/1815.7 \text{ kJ/kg} = 1.158 \text{ kg/s}.$$

For the *unfouled* condenser of part (a), the steam rate was

$$\dot{m}_h = 2.3 \text{MW}/1815.7 \text{ kJ/kg} = 1.267 \text{ kg/s}.$$

Hence, we see that fouling reduces the steam rate by 8.5% when U is decreased 10%. Since p_4 remains the same, the thermal efficiency remains unchanged,

$$h = 30.3\% \qquad <$$

as calculated in the previous problem. However, the net work of the cycle will decrease 8.5%.

COMMENTS: Fouling of the condenser heat exchanger has no effect on the thermal efficiency of the cycle since the back pressure at the condenser is maintained constant. The effect is, however, to reduce the heat rejection rate while maintaining exchanger flow rate and inlet temperature fixed. Comparing the conditions:

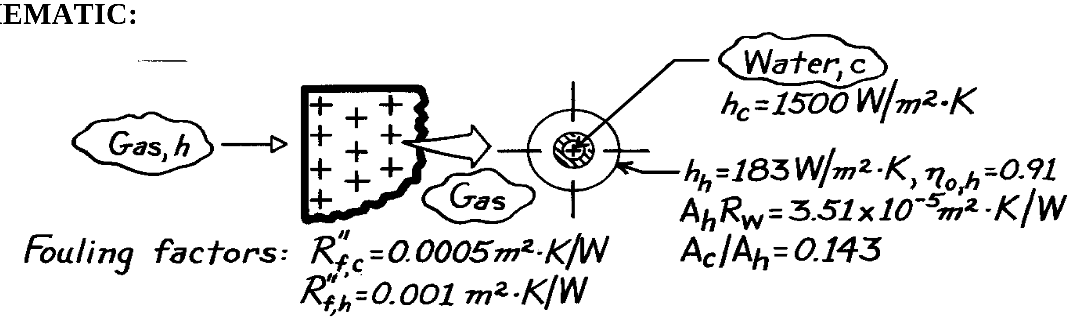
Parameter	Clean	Fouled	Change (%)
UA, W/K	86,538	77,884	-10.0
ε	0.256	0.234	-8.6
q_h, MW	2.300	2.103	-8.6
$\dot{w}_{net}$	--	--	-8.6

PROBLEM 11.79

KNOWN: Compact heat exchanger (see Example 11.6) after extended use has prescribed fouling factors on water and gas sides.

FIND: Gas-side overall heat transfer coefficient.

SCHEMATIC:



ASSUMPTIONS: (1) Heat transfer coefficients on the inside and outside (cold- and hot-sides) are the same as for the unfouled condition, (2) Temperature effectiveness of the finned hot side surface is the same as for the unfouled condition.

ANALYSIS: The overall heat transfer coefficient follows from Eq. 11.1 as

$$\frac{1}{U_h A_h} = \frac{1}{(h_o h A)_c} + \frac{R''_{f,c}}{(h_o A)_c} + R_w + \frac{R''_{f,h}}{(h_o A)_h} + \frac{1}{(h_o h A)_h}$$

where R_w and R''_f are the wall resistance and fouling factors, respectively. Multiply both sides by A_h and recognizing that $\eta_{o,c} = 1$, obtain

$$\frac{1}{U_h} = \frac{1}{h_c (A_c / A_h)} + \frac{R''_{f,c}}{(A_c / A_h)} + A_h R_w + \frac{R''_{f,h}}{h_{o,h}} + \frac{1}{h_o h_h}.$$

Substitute numerical values from Example 11.6 results (h_h, $\eta_{o,h}$, $A_h R_w$, A_c/A_h) and those from the problem statement $\left(R''_{f,h}, R''_{f,c}, h_c\right)$ to find,

$$\frac{1}{U_h} = \frac{1}{1500\ \mathrm{W/m^2 \cdot K}(0.143)}$$

$$+ \frac{0.0005\mathrm{m^2 \cdot K/W}}{(0.143)} + 3.51 \times 10^{-5}\ \mathrm{m^2 \cdot K/W} + \frac{0.001\mathrm{m^2 \cdot K/W}}{0.91} + \frac{1}{0.91 \times 183\ \mathrm{W/m^2 \cdot K}}$$

$$\frac{1}{U_h} = \left(4.662 \times 10^{-3} + 6.993 \times 10^{-3} + 3.51 \times 10^{-5} + 5.495 \times 10^{-4} + 6.005 \times 10^{-3}\right)\mathrm{m^2 \cdot K/W}$$

$$U_h = 65.4\ \mathrm{W/m^2 \cdot K}.$$ <

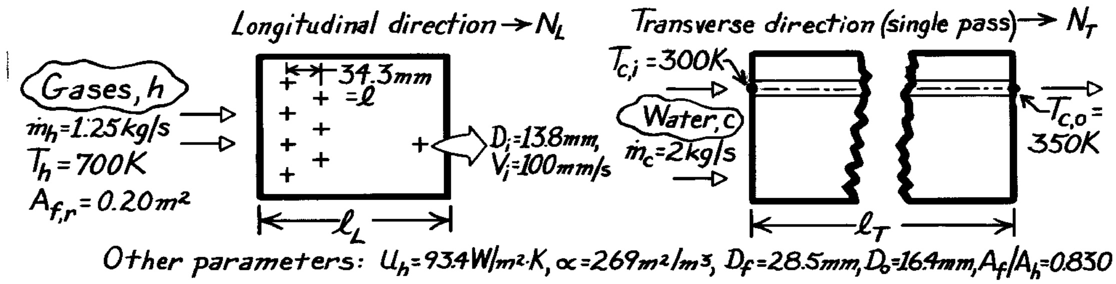
COMMENTS: For the unfouled condition, we found $U_h = 93.4$ W/m²·K from Example 11.6. Note that the thermal resistance of the tube-fin material is negligible and that fouling has a significant effect, reducing U_h by 41%.

PROBLEM 11.80

KNOWN: Compact heat exchanger with prescribed core geometry and operating parameters.

FIND: Required heat exchanger volume; number of tubes in the longitudinal and transverse directions, N_L and N_T; required tube length.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible KE and PE changes, (3) Single pass operation, (4) Gas properties are those of air.

PROPERTIES: *Table A-6*, Water ($\overline{T}_c$ = 325 K): ρ = 987.2 kg/m^3, c_p = 4182 J/kg·K; *Table A-4*, Air (Assume $T_{h,o} \approx 400$ K, $\overline{T}_h \approx 550$ K, 1 atm): c_p = 1040 J/kg·K.

ANALYSIS: To find the Hxer volume, first find A_h using the ε-NTU method. By definition,

$$V = A_h / a \qquad \text{and} \qquad A_h = NTU \cdot C_{min} / U_h. \tag{1,2}$$

Find the capacity rates, q, q_{max} and ε:

$$C_c = \dot{m}_c c_{p,c} = 2\ kg/s \times 4182\ J/kg \cdot K = 8364\ W/K$$

$$C_h = \dot{m}_h c_{p,h} = 1.25\ kg/s \times 1040\ J/kg \cdot K = 1300\ W/K \leftarrow C_{min}$$

Hence,

$$C_r = \frac{C_{min}}{C_{max}} = 0.155.$$

It follows that

$$e = \frac{q}{q_{max}} = \frac{C_c\left(T_{c,o} - T_{c,i}\right)}{C_{min}\left(T_{h,i} - T_{c,i}\right)} = \frac{8364\ W/K\left(350-300\right)K}{1300\ W/K\left(700-300\right)K} = 0.804.$$

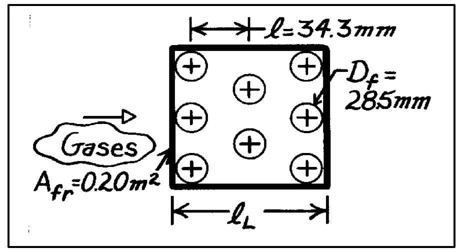
With ε = 0.804 and C_r = 0.155, find NTU ≈ 1.7 from Fig. 11.18 for a single-pass, cross flow Hxer with both fluids unmixed. Using Eqs. (2) and (1), find

$$A_h = 1.7 \times 1300\ W/K / 93.4\ W/m^2 \cdot K = 23.7 m^2$$

$$V = 23.7 m^2 / 269 m^2/m^3 = 0.0880 m^3.$$

Continued …..

To determine the number of tubes in the longitudinal direction, consider the tubular arrangement in the sketch. The Hxer volume can be written as



$$V = A_{fr} \times \ell_L \qquad (3)$$

where

$$\ell_L = (N_L - 1)\ell + D_f \qquad (4)$$

and N_L is the number of tubes in the longitudinal direction. Combining Eqs. (3) and (4) and substituting numerical values, find

$$N_L = (V / A_{fr} - D_f)/\ell + 1 \qquad (5)$$

where D_f is the overall diameter of the finned tube, and

$$N_L = \left(0.0880\text{m}^3 / 0.20\ \text{m}^2 - 0.0285\text{m}\right)/0.0343 + 1 = 13.0 \approx 13.$$ <

To determine the number of tubes in the transverse direction, compare the overall water flow rate $\dot{m}_c$ with that for a single tube, $\dot{m}_t$. That is,

$$\dot{m}_t = r_c A_t V_i \qquad (6)$$

where A_t is the tube inner cross-sectional area $\left(p D_i^2 / 4\right)$ and V_i the internal velocity. Hence,

$$N = \dot{m}_c / \dot{m}_t = (2\ \text{kg/s})/987.2\ \text{kg/m}^3 \times \frac{p}{4}(0.0138\text{m})^2 \times 0.100\ \text{m/s} = 135.4 \approx 135.$$

The total number of tubes required, N, is 135; the number in the transverse direction is

$$N_T = N / N_L = 135/13 = 10.4 \approx 11.$$ <

To determine the water tube length, recognize that the total area (A_h), less that of the finned surfaces (A_f), will be that of the water tube surface area. That is,

$$A_h - A_f = p D_o \ell_T \cdot N.$$

From specification of the core geometry, we know A_f/A_h = 0.830; solve for ℓ_T to obtain

$$\ell_T = A_h (1 - A_f / A_h)/p D_o \cdot N \qquad (7)$$

$$\ell_T = 23.7\text{m}^2 (1 - 0.830)/p\ (0.0164\text{m}) \times 135 = 0.58\text{m}.$$ <

COMMENTS: In summary we find that

Total number of tubes, N ($N_T \times N_L$)	143
Tubes in longitudinal direction, N_L	13
Tubes in transverse direction, N_T	11

with a total surface area of 27.3 m^2. The length of the exchanger is

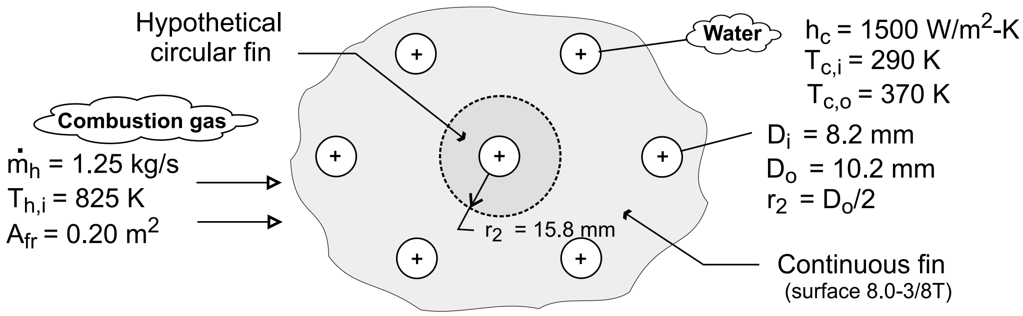
Length in longitudinal direction, ℓ_L	0.44 m
Length in transverse direction, ℓ_T	0.58 m.

PROBLEM 11.81

KNOWN: Compact heat exchanger geometry, gas-side flow rate and inlet temperature, water-side convection coefficient, water flow rate, and water inlet and outlet temperatures.

FIND: Gas-side overall heat transfer coefficient. Required heat exchanger volume.

SCHEMATIC:



ASSUMPTIONS: (1) Gas has properties of atmospheric air at an assumed mean temperature of 700 K, (2) Negligible fouling, (3) Negligible heat exchange with the surroundings and negligible kinetic and potential energy and flow work changes.

PROPERTIES: *Table A-1*, aluminum (T ≈ 300 K): k = 237 W/m·K. *Table A-4*, air (p = 1 atm, $\bar{T}$ = 700 K): c_p = 1075 J/kg·K, $\mu = 338.8 \times 10^{-7}$ N·s/m^2, Pr = 0.695. *Table A-6*, water ($\bar{T}$ = 330 K): c_p = 4184 J/kg·K.

ANALYSIS: For the prescribed heat exchanger core,

$$\frac{1}{U_h} = \frac{1}{h_c\left(A_c / A_h\right)} + A_h R_w + \frac{1}{\eta_{o,h} h_h}$$

where

$$\frac{A_c}{A_h} \approx \frac{D_i}{D_o}\left(1 - \frac{A_{f,h}}{A_h}\right) = \frac{8.2}{10.2}(1 - 0.913) = 0.070$$

The product of A_h and the wall conduction resistance is

$$A_h R_w = \frac{\ln\left(D_o / D_i\right)}{2\pi k L / A_h} = \frac{D_i \ln\left(D_o / D_i\right)}{2k\left(A_c / A_h\right)} = \frac{0.0082\text{m} \times \ln(10.2/8.2)}{2 \times 237\ \text{W/m}\cdot\text{K}\,(0.070)} = 5.39 \times 10^{-4}\ \text{m}^2\cdot\text{K/W}$$

With a gas-side mass velocity of G = $\dot{m}_h / \sigma A_{fr}$ = 1.25 kg/s/0.534 × 0.20 m^2 = 11.7 kg/s·m^2,

$$Re = \frac{G D_h}{\mu} = \frac{11.7\ \text{kg/s}\cdot\text{m}^2 \times 0.00363\text{m}}{338.8 \times 10^{-7}\ \text{N}\cdot\text{s/m}^2} = 1254$$

and Fig. 11.21 yields $j_H \approx 0.0096$. Hence,

$$h_h \approx \frac{0.0096\ G c_p}{Pr^{2/3}} = \frac{0.0096\left(11.7\ \text{kg/s}\cdot\text{m}^2\right)\left(1075\ \text{J/kg}\cdot\text{K}\right)}{(0.695)^{2/3}} = 154\ \text{W/m}^2\cdot\text{K}$$

Continued …..

With $r_{2c} = r_2 + t/2 = 15.8$ mm + 0.330 mm/2 = 15.97 mm, $r_{2c}/r_1 = 15.97/5.1 = 3.13$, $L = r_2 - r_1 = 10.7$ mm, $L_c = L + t/2 = 10.87$ mm = 0.0109m, $A_p = L_c t = 3.59 \times 10^{-6}$ m^2, and $L_c^{3/2}$ $(h_h/kA_p)^{1/2} = 0.484$, Fig. 3.19 yields $\eta_f \approx 0.77$. Hence,

$$\eta_{o,h} = 1 - \frac{A_f}{A}\left(1 - \eta_f\right) = 1 - 0.913\left(1 - 0.77\right) = 0.790$$

$$U_h^{-1} = \left(1500\ \mathrm{W/m^2 \cdot K} \times 0.07\right)^{-1} + 5.39 \times 10^{-4}\,\mathrm{m^2 \cdot K/W} + \left(0.79 \times 154\ \mathrm{W/m^2 \cdot K}\right)^{-1} = 0.0183\,\mathrm{m^2 \cdot K/W}$$

$$U_h = 54.7\ \mathrm{W/m^2 \cdot K} \qquad <$$

With $q = C_c\,(T_{c,o} - T_{c,i}) = 4184$ W/K × 80 K = 3.35×10^5 W, $q_{max} = C_{min}\,(T_{h,i} - T_{c,i}) = 1344$ W/K × 535 K = 7.19×10^5 W, $\varepsilon = 0.466$ and $C_r = 0.321$. From Figure 11.18, we then obtain NTU ≈ 0.65. The required gas-side surface area is then

$$A_h = \frac{NTU \times C_{min}}{U_h} = \frac{0.65 \times 1344\ \mathrm{W/K}}{54.7\ \mathrm{W/m^2 \cdot K}} = 16.0\ \mathrm{m^2}$$

With $\alpha = 587$ m^2/m^3, the required volume is

$$V = \frac{A_h}{\alpha} = \frac{16\,\mathrm{m^2}}{587\,\mathrm{m^2/m^3}} = 0.0273\,\mathrm{m^3} \qquad <$$

COMMENTS: (1) Although U_h is small and A_h larger for the continuous fins than for the circular fins of Example 11.6, the much larger value of α, renders the volume requirement smaller.

(2) The heat exchanger length is $L = V/A_{fr} = 0.137$ m, and the number of tube rows is

$$N_L \approx \frac{L}{S_L} + 1 = 7.23 \approx 7.$$

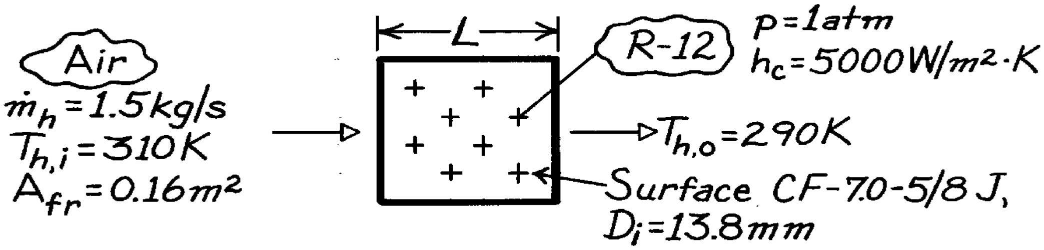
(3) The hypothetical fin radius ($r_2 = 15.8$ mm) was estimated to be the arithmetic mean of one-half the center-to-center spacing between one tube and its six neighbors.

PROBLEM 11.82

KNOWN: Cooling coil geometry. Air flow rate and inlet and outlet temperatures. Freon pressure and convection coefficient.

FIND: Required number of tube rows.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible fouling, (2) Constant properties, (3) Negligible heat loss to surroundings.

PROPERTIES: *Table A-4*, Air ($\overline{T}_h$ = 300 K, 1 atm): c_p = 1007 J/kg·K, $\mu = 184.6 \times 10^{-7}$ N·s/m^2, k = 0.0263 W/m·K, Pr = 0.707; *Table A-5*, Sat. R-12 (1 atm): $T_{sat} = T_c$ = 243 K, h_{fg} = 165 kJ/kg.

ANALYSIS: The required number of tube rows is

$$N_L = \left(L - D_f\right)/S_L + 1$$

where

$$L = V/A_{fr} \qquad V = A_h/a \qquad A_h = NTU\left(C_{min}/U_h\right)$$

$$1/U_h = 1/h_c\left(A_c/A_h\right) + A_h R_w + 1/h_{o,h} h_h.$$

From Ex. 11.6, $(A_c/A_h) = 0.143$ and $A_h R_w = 3.51 \times 10^{-5}$ m^2·K/W. With

$$G = \frac{\dot{m}_h}{s A_{fr}} = \frac{1.50 \text{ kg/s}}{0.449 \times 0.16 m^2} = 20.9 \text{ kg/s} \cdot m^2$$

$$Re = \frac{G D_h}{m} = \frac{20.9 \text{ kg/s} \cdot m^2 \times 6.68 \times 10^{-3} m}{184.6 \times 10^{-7} \text{ N} \cdot s/m^2} = 7563$$

and Fig. 11.20 gives $j_H \approx 0.0068$. Hence,

$$h_h = j_h \frac{G c_p}{Pr^{2/3}} = 0.0068 \frac{20.9 \text{ kg/s} \cdot m^2 \times 1007 \text{ J/kg} \cdot K}{(0.707)^{2/3}} = 180 \text{ W/m}^2 \cdot K.$$

With L_c = 6.18 mm and $A_p = 1.57 \times 10^{-6}$ m^2 from Ex. 11.6, $L_c^{3/2}\left(h_h/kA_p\right)^{1/2} = 0.338$ and, from Fig. 3.19, $\eta_f \approx 0.89$ for $r_{2c}/r_1 = 1.75$. Hence, as in Ex. 11.6, $\eta_{o,h} = 0.91$ and

$$1/U_h = 1/\left(5000 \text{ W/m}^2 \cdot K\right) 0.143 + 3.51 \times 10^{-5} m^2 \cdot K/W + 1/\left(0.91 \times 180 \text{ W/m}^2 \cdot K\right)$$

$$U_h = 133 \text{ W/m}^2 \cdot K.$$

Continued …..

With $C_{min}/C_{max} = 0$ and $C_{min} = \dot{m}_h c_{p,h} = 1511$ W/K,

$$\varepsilon = \frac{q}{q_{max}} = \frac{C_h\left(T_{h,i} - T_{h,o}\right)}{C_h\left(T_{h,i} - T_{c,i}\right)} = \frac{20\ \mathrm{K}}{67\ \mathrm{K}} = 0.30$$

$$\mathrm{NTU} = -\ln\left(1 - \varepsilon\right) = 0.355$$

and

$$A_h = \mathrm{NTU}\frac{C_{min}}{U_h} = 0.355\frac{1511\,\mathrm{W/K}}{133\ \mathrm{W/m^2\cdot K}} = 4.03\mathrm{m}^2.$$

Hence,

$$L = \frac{A_h}{\alpha A_{fr}} = \frac{4.03\mathrm{m}^2}{\left(269\ \mathrm{m^2/m^3}\right)0.16\mathrm{m}^2} = 0.0937\mathrm{m}$$

and

$$N_L = \frac{L - D_f}{S_L} + 1 = \frac{0.0652}{0.0343\mathrm{m}} + 1 = 2.9.$$

Hence, three or more rows must be used. <

COMMENTS: For the prescribed operating conditions, the heat rate would be

$$q = C_h\left(T_{h,i} - T_{h,o}\right) = 1511\,\mathrm{W/K}\left(20\ \mathrm{K}\right) = 30{,}220\ \mathrm{W}.$$

If R-12 enters the tubes as saturated liquid, a flow rate of at least

$$\dot{m}_c = \frac{q}{h_{fg}} = \frac{30{,}220\ \mathrm{W}}{165{,}000\,\mathrm{J/kg}} = 0.183\ \mathrm{kg/s}$$

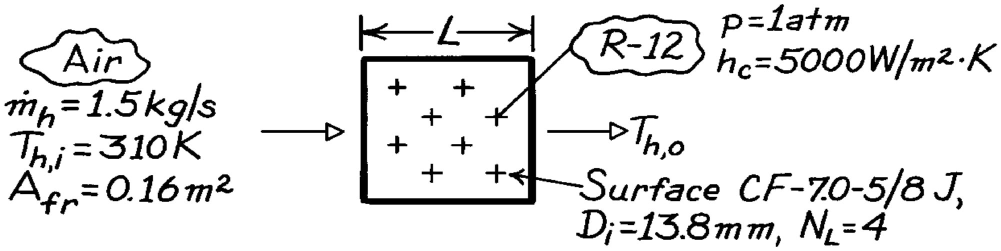
would be needed to maintain saturated conditions in the tubes.

PROBLEM 11.83

KNOWN: Cooling coil geometry. Air flow rate and inlet temperature. Freon pressure and convection coefficient.

FIND: Air outlet temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible fouling, (2) Constant properties, (3) Negligible heat loss to surroundings.

PROPERTIES: *Table A-4*, Air ($\overline{T}_h \approx 300$ K, 1 atm): $c_p = 1007$ J/kg·K, $\mu = 184.6 \times 10^{-7}$ N·s/m^2, k = 0.0263 W/m·K, Pr = 0.707; *Table A-5*, Sat. R-12 (1 atm): $T_{sat} = T_c = 243$ K, $h_{fg} = 165$ kJ/kg.

ANALYSIS: To obtain the air outlet temperature, we must first obtain the heat rate from the ε-NTU method. To find A_h, first find the heat exchanger length,

$$L \approx (N_L - 1)S_L + D_f = 3(0.0343\text{m}) + 0.0285\text{m} = 0.131\text{m}.$$

Hence,

$$V = A_{fr}L = 0.16\text{m}^2(0.131\text{m}) = 0.021\text{m}^3$$

$$A_h = aV = \left(269\text{m}^2/\text{m}^3\right)0.021\text{m}^3 = 5.65\text{m}^2.$$

The overall coefficient is

$$\frac{1}{U_h} = \frac{1}{h_c(A_c/A_h)} + A_hR_w + \frac{1}{h_{o,h}h_h}$$

where Ex. 11.6 yields $(A_c/A_h) = 0.143$ and $A_hR_w = 3.51 \times 10^{-5}$ m^2·K/W. With

$$G = \frac{\dot{m}_h}{s\,A_{fr}} = \frac{1.50\text{ kg/s}}{0.449 \times 0.16\text{ m}^2} = 20.9\text{ kg/s}\cdot\text{m}^2$$

$$Re = \frac{GD_h}{m} = \frac{20.9\text{ kg/s}\cdot\text{m}^2 \times 6.68 \times 10^{-3}\text{m}}{184.6 \times 10^{-7}\text{ N}\cdot\text{s/m}^2} = 7563.$$

Fig. 11.20 gives $j_H \approx 0.0068$. Hence,

$$h_h = j_h\frac{Gc_p}{Pr^{2/3}} = 0.0068\frac{20.9\text{ kg/s}\cdot\text{m}^2 \times 1007\text{ J/kg}\cdot\text{K}}{(0.707)^{2/3}}$$

$$h_h = 180\text{ W/m}^2\cdot\text{K}.$$

Continued …..

With L_c = 6.18 mm and $A_p = 1.57 \times 10^{-6}$ m^2 from Ex. 11.6, $L_c^{3/2}\left(h_h/kA_p\right)^{1/2}$ = 0.338 and, from Fig. 3.19, $\eta_f \approx 0.89$ for $r_{2c}/r_1 = 1.75$. Hence, as in Ex. 11.6, $\eta_{o,h} = 0.91$ and

$$\frac{1}{U_h} = \frac{1}{\left(5000\ \mathrm{W/m^2\cdot K}\right)0.143} + 3.51\times10^{-5}\,\mathrm{m^2\cdot K/W} + \frac{1}{0.91\left(180\ \mathrm{W/m^2\cdot K}\right)}$$

$$U_h = 133\ \mathrm{W/m^2\cdot K}.$$

With

$$C_{min} = C_h = \dot{m}_h\, c_{p,h} = 1.5\ \mathrm{kg/s}\left(1007\ \mathrm{J/kg\cdot K}\right) = 1511\,\mathrm{W/K}$$

$$NTU = \frac{U_h A_h}{C_{min}} = \frac{133\,\mathrm{W/m^2\cdot K} \times 5.65\mathrm{m^2}}{1511\,\mathrm{W/K}} = 0.497.$$

With $C_{min}/C_{max} = 0$, Eq. 11.36a yields

$$\varepsilon = 1 - \exp(-NTU) = 1 - \exp(-0.497) = 0.392.$$

Hence,

$$q = \varepsilon q_{max} = \varepsilon C_{min}\left(T_{h,i} - T_{c,i}\right) = 0.392\left(1511\,\mathrm{W/K}\right)67\ \mathrm{K}$$

$$q = 39{,}685\ \mathrm{W}.$$

The air outlet temperature is

$$T_{h,o} = T_{h,i} - \frac{q}{C_h} = 310\ \mathrm{K} - \frac{39{,}685\ \mathrm{W}}{1511\,\mathrm{W/K}} = 283.7\ \mathrm{K}.$$ <

COMMENTS: If R-12 enters the tubes as saturated liquid, a flow rate of at least

$$\dot{m}_c = \frac{q}{h_{fg}} = \frac{39{,}685\ \mathrm{W}}{165{,}000\,\mathrm{J/kg}} = 0.241\,\mathrm{kg/s}$$

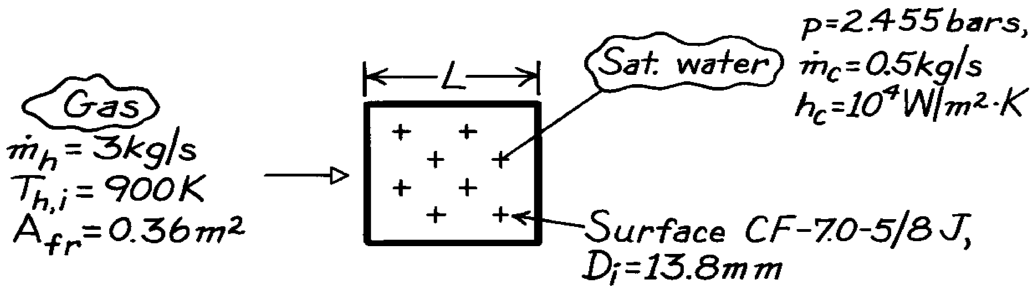
would be needed to maintain saturated conditions in the tubes.

PROBLEM 11.84

KNOWN: Cooling coil geometry. Gas flow rate and inlet temperature. Water pressure, flow rate and convection coefficient.

FIND: Required number of tube rows.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible fouling, (2) Constant properties, (3) Negligible heat loss to surroundings.

PROPERTIES: *Table A-4*, Air ($\bar{T}_h \approx 725$ K, 1 atm): $c_p = 1081$ J/kg·K, $\mu = 346.7 \times 10^{-7}$ N·s/m^2, k = 0.0536 W/m·K, Pr = 0.698; *Table A-6*, Sat. water (2.455 bar): $T_{sat} = T_c = 400$ K, h_{fg} = 2183 kJ/kg.

ANALYSIS: The required number of tube rows is

$$N_L = \frac{L - D_f}{S_L} + 1$$

where

$$L = \frac{V}{A_{fr}} \qquad V = \frac{A_h}{a} \qquad A_h = NTU\frac{C_{min}}{U_h}$$

$$\frac{1}{U_h} = \frac{1}{h_c\left(A_c / A_h\right)} + A_h R_w + \frac{1}{h_{o,h} h_h}.$$

From Ex. 11.6, $(A_c/A_h) \approx 0.143$ and

$$A_h R_w = \frac{D_i \ln\left(D_o / D_i\right)}{2k\left(A_c / A_h\right)} = \frac{(0.0138\text{m})\ln(16.4/13.8)}{2(15\text{ W/m}\cdot\text{K})(0.143)} = 5.55\times10^{-4}\text{m}^2\cdot\text{K/W}.$$

With

$$G = \frac{\dot{m}_h}{s\, A_{fr}} = \frac{3.0\text{ kg/s}}{0.449\times0.36\text{m}^2} = 18.6\text{ kg/s}\cdot\text{m}^2$$

$$Re = \frac{GD_h}{m} = \frac{18.6\text{ kg/s}\cdot\text{m}^2 \times 6.68\times10^{-3}\text{ m}}{346.7\times10^{-7}\text{ N}\cdot\text{s/m}^2} = 3576$$

and Fig. 11.20 gives $j_h \approx 0.009$. Hence,

$$h_h = j_h \frac{Gc_p}{Pr^{2/3}} = 0.009\frac{18.6\text{ kg/s}\cdot\text{m}^2 \times 1081\text{J/kg}\cdot\text{K}}{(0.698)^{2/3}} = 230\text{ W/m}^2\cdot\text{K}.$$

Continued …..

With $r_{2c}/r_1 = 1.75$, $L_c = 6.18$ mm and $A_p = 1.57 \times 10^{-6}$ m^2 from Ex. 11.6, $L_c^{3/2}\left(h_h / kA_p\right)^{1/2} = 1.52$ and Fig. 3.19 gives $\eta_f \approx 0.40$. Hence,

$$h_{o,h} = 1 - \frac{A_f}{A}\left(1 - h_f\right) = 1 - 0.83\left(1 - 0.4\right) = 0.50.$$

Hence,

$$\frac{1}{U_h} = \frac{1}{\left(10^4\ \text{W/m}^2\cdot\text{K}\right)0.143} + 5.55\times10^{-4}\ \text{m}^2\cdot\text{K/W} + \frac{1}{0.50\left(230\ \text{W/m}^2\cdot\text{K}\right)}$$

$$U_h = 100.5\ \text{W/m}^2\cdot\text{K}.$$

With

$$q = \dot{m}_c h_{fg} = 0.5\ \text{kg/s}\left(2.183\times10^6\ \text{J/kg}\right) = 1.092\times10^6\ \text{W}$$

$$C_{min} = C_h = 3.0\ \text{kg/s}\left(1081\ \text{J/kg}\cdot\text{K}\right) = 3243\ \text{W/K}$$

$$q_{max} = C_{min}\left(T_{h,i} - T_{c,i}\right) = 3243\ \text{W/K}\left(500\ \text{K}\right) = 1.622\times10^6\ \text{W}$$

find

$$e = \frac{q}{q_{max}} = \frac{1.092\times10^6\ \text{W}}{1.622\times10^6\ \text{W}} = 0.674.$$

From Eq. 11.36b

$$\text{NTU} = -\ln\left(1 - e\right) = -\ln\left(1 - 0.674\right) = 1.121.$$

Hence,

$$A_h = \text{NTU}\frac{C_{min}}{U_h} = 1.121\frac{3243\ \text{W/K}}{100.5\ \text{W/m}^2\cdot\text{K}} = 36.17\text{m}^2$$

$$L = \frac{A_h}{A_{fr}a} = \frac{36.17\text{m}^2}{0.36\text{m}^2\left(269\text{m}^2/\text{m}^3\right)} = 0.373\text{m}$$

$$N_L = \frac{L - D_f}{S_L} + 1 = \frac{373 - 28.5}{34.3} + 1 = 11.06 \approx 11. \qquad <$$

COMMENTS: The gas outlet temperature is

$$T_{h,o} = T_{h,i} - \frac{q}{C_{min}} = 900\ \text{K} - \frac{1.092\times10^6\ \text{W}}{3243\ \text{W/K}} = 564\ \text{K}.$$

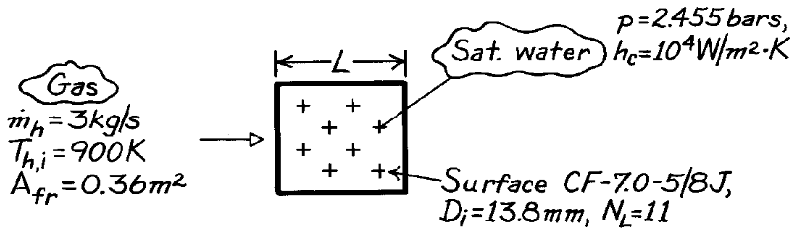
Hence $\overline{T}_h$ = (900 K + 564 K)/2 = 732 K is in good agreement with the assumed value.

PROBLEM 11.85

KNOWN: Cooling coil geometry. Gas flow rate and inlet temperature. Water pressure and convection coefficient.

FIND: Gas outlet temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible fouling, (2) Constant properties, (3) Negligible heat loss to surroundings.

PROPERTIES: *Table A-4*, Air ($\overline{T}_h \approx 725$ K, 1 atm): c_p = 1081 J/kg·K, $\mu = 346.7 \times 10^{-7}$ N·s/m^2, k = 0.0536 W/m·K, Pr = 0.698; *Table A-6*, Sat. water (2.455 bar): $T_{sat} = T_c$ = 400 K, h_{fg} = 2183 kJ/kg.

ANALYSIS: To obtain $T_{h,o}$, first obtain q from the ε-NTU method. To determine NTU, Ah must be found from knowledge of L.

$$L \approx (N_L - 1)S_L + D_f = 10(0.0343\text{m}) + 0.0285\text{m} = 0.372\text{m}.$$

Hence,

$$V = A_{fr}L = 0.36\text{m}^2(0.372\text{m}) = 0.134\text{m}^3$$

$$A_h = aV = \left(269\text{m}^2/\text{m}^3\right)0.134\text{m}^3 = 36.05\text{m}^2.$$

The overall coefficient is

$$\frac{1}{U_h} = \frac{1}{h_c(A_c/A_h)} + A_h R_w + \frac{1}{h_{o,h} h_h}.$$

From Ex. 11.6, $(A_c/A_h) \approx 0.143$ and

$$A_h R_w = \frac{D_i \ln(D_o/D_i)}{2k(A_c/A_h)} = \frac{(0.0138\text{m})\ln(16.4/13.8)}{2(15\text{ W/m}\cdot\text{K})(0.143)} = 5.55\times10^{-4}\text{m}^2\cdot\text{K/W}.$$

With

$$G = \frac{\dot{m}_h}{s\,A_{fr}} = \frac{3.0\text{ kg/s}}{0.449\times0.36\text{m}^2} = 18.6\text{ kg/s}\cdot\text{m}^2$$

$$\text{Re} = \frac{GD_h}{m} = \frac{18.6\text{ kg/s}\cdot\text{m}^2\times6.68\times10^{-3}\text{ m}}{346.7\times10^{-7}\text{ N}\cdot\text{s/m}^2} = 3576$$

and Fig. 11.20 gives $j_H \approx 0.009$. Hence,

Continued …..

PROBLEM 11.85 (Cont.)

$$h_h = j_h \frac{Gc_p}{Pr^{2/3}} = 0.009 \frac{18.6\ \text{kg/s}\cdot\text{m}^2 \times 1081\,\text{J/kg}\cdot\text{K}}{(0.698)^{2/3}}$$

$$h_h = 230\ \text{W/m}^2\cdot\text{K}.$$

With $r_{2c}/r_1 = 1.75$, $L_c = 6.18$ mm and $A_p = 1.57 \times 10^{-6}\ \text{m}^2$ from Ex. 11.6, $L_c^{3/2}\left(h_h/kA_p\right)^{1/2} = 1.52$ and Fig. 3.19 gives $\eta_f \approx 0.40$. Hence,

$$h_{o,h} = 1 - \frac{A_f}{A}(1 - h_f) = 1 - 0.83(1 - 0.4) = 0.50.$$

Hence,

$$\frac{1}{U_h} = \frac{1}{\left(10^4\ \text{W/m}^2\cdot\text{K}\right)0.143} + 5.55\times10^{-4}\,\text{m}^2\cdot\text{K/W} + \frac{1}{0.50\left(230\ \text{W/m}^2\cdot\text{K}\right)}$$

$$U_h = 100.5\ \text{W/m}^2\cdot\text{K}.$$

With

$$C_{min} = C_h = 3\,\text{kg/s}\left(1081\,\text{J/kg}\cdot\text{K}\right) = 3243\ \text{W/K}$$

$$NTU = \frac{U_h A_h}{C_{min}} = \frac{100.5\ \text{W/m}^2\cdot\text{K}\left(36.05\text{m}^2\right)}{3243\ \text{W/K}} = 1.117.$$

Since $C_{min}/C_{max} = 0$, Eq. 11.36a gives

$$e = 1 - \exp(-NTU) = 1 - \exp(-1.117) = 0.673.$$

Hence,

$$q = eC_{min}\left(T_{h,i} - T_{c,i}\right) = 0.673(3243\ \text{W/K})(500\ \text{K}) = 1.091\times10^6\ \text{W}$$

and

$$T_{h,o} = T_{h,i} - \frac{q}{C_{min}} = 900\ \text{K} - \frac{1.091\times10^6\ \text{W}}{3243\ \text{W/K}} = 564\ \text{K}.$$ <

COMMENTS: (1) The assumption of $\overline{T}_h = 725$ K is good.

(2) If water enters the tubes as saturated liquid, a flow rate of at least

$$\dot{m}_c = \frac{q}{h_{fg}} = \frac{1.091\times10^6\ \text{W}}{2.183\times10^6\ \text{J/kg}} = 0.50\ \text{kg/s}$$

would be need to maintain saturated conditions in the tubes.