

# ELEC 343

## Electromechanics

### Module 7: Brushless DC Motors (Chap. 8)

Spring 2019

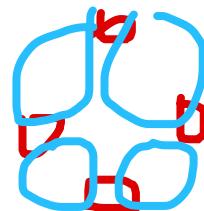
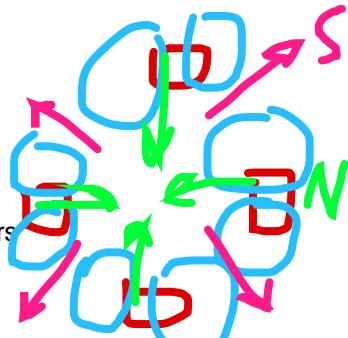
Instructor: Dr. Juri Jatskevich

Class Webpage:

<http://courses.ece.ubc.ca/elec343>

#### Important Topics & Concepts

- Types and construction of commonly Brushless DC and Servo Motors
- Principle of operation & torque production
- 2-phase BLDC Motor model
- Model in qd-Rotor Reference Frame (RRF)
- Steady-state analysis characteristics
- Similarity with conventional brushed DC Motors
- 3-phase BLDC Motor model
- 6-step 180/120 deg inverter operation
- Principle of PWM drive of Brushless DC Motors
- Simulink Model implementation



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### Typical Brushless DC Motors

Computers

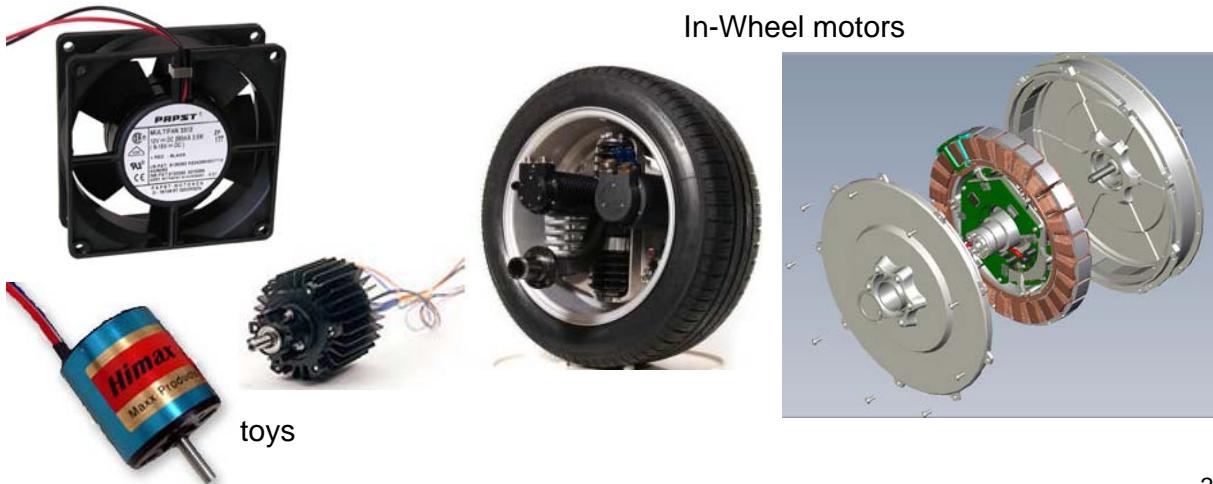


Propulsion & Transportation



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In-Wheel motors

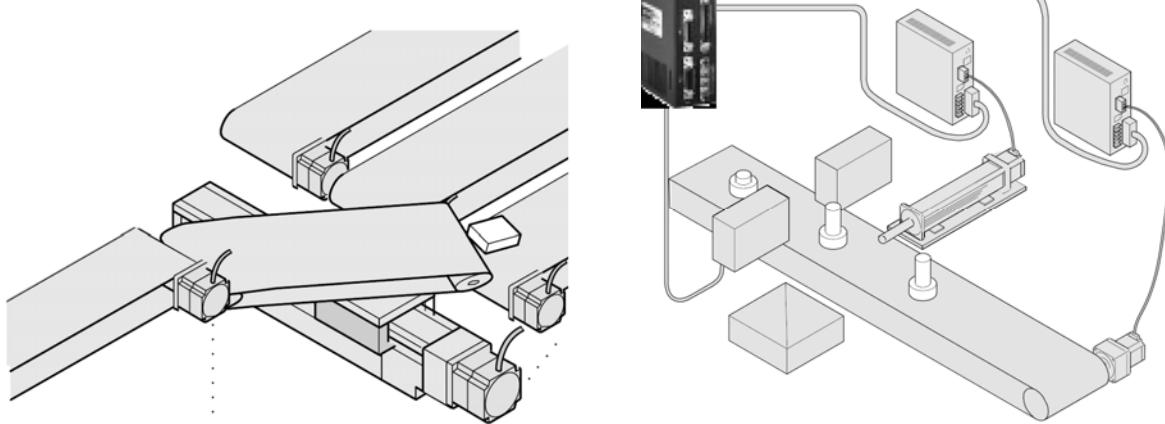


toys

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# Typical Brushless DC Motors

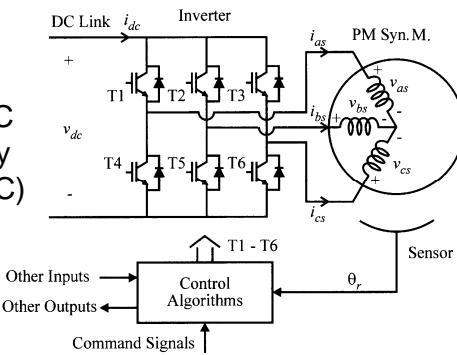
Servo motors for positioning and industrial automation



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## Typical Configurations of BLDC Motors

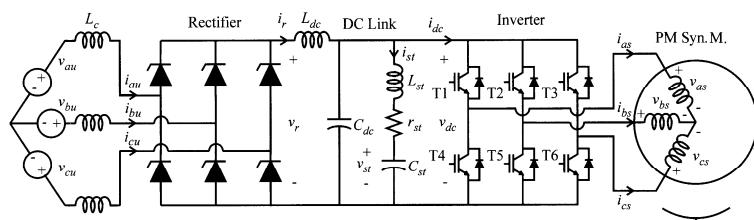
**DC input:**  
requires  
additional DC  
power supply  
(24 – 48V DC)



BLDC Motor Drivers



### Brushless AC or DC Servo Motors



**3-Phase AC input:**  
no additional power  
supply is needed  
(208/120 V AC)

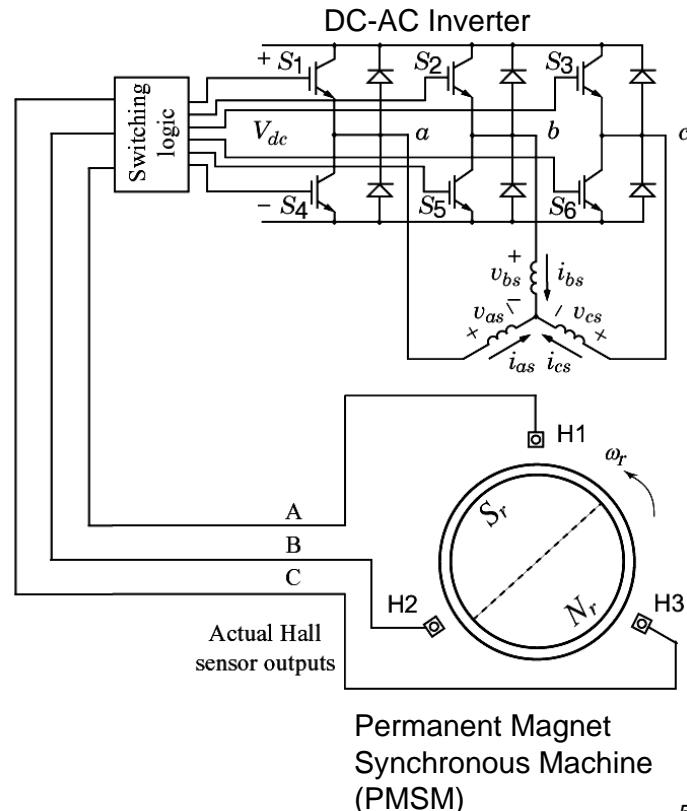


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# Hall-Sensor-Driven BLDC Machine

## Features

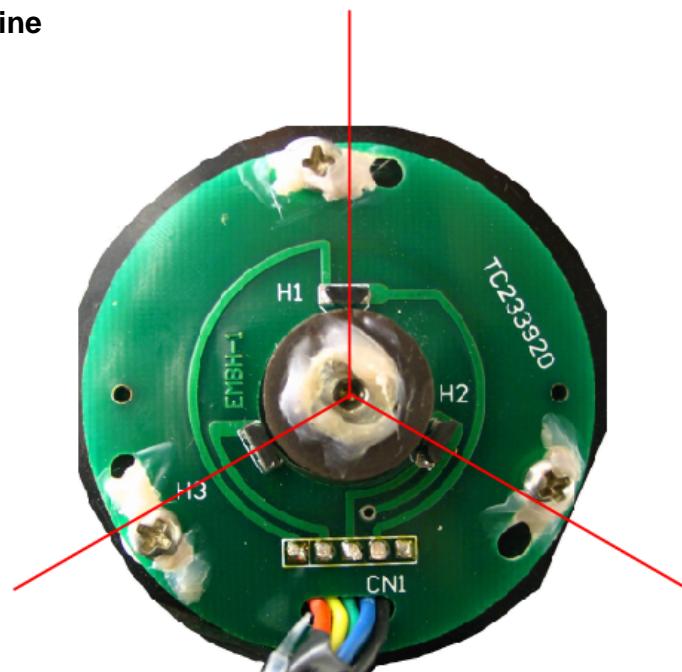
- Simple & low cost
- Work with various loads
- Drivers work with different motors
- Motors work with different drivers
- Good speed/torque characteristics
- High dynamic response
- High efficiency
- Wide speed ranges



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# Hall Sensors in BLDC Motors

Typical Industrial BLDC Machine



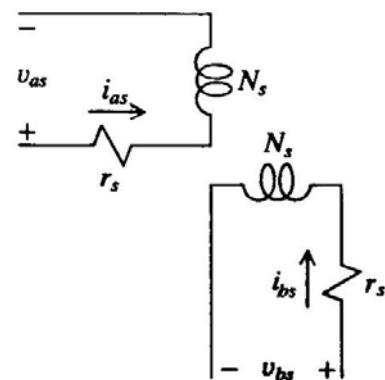
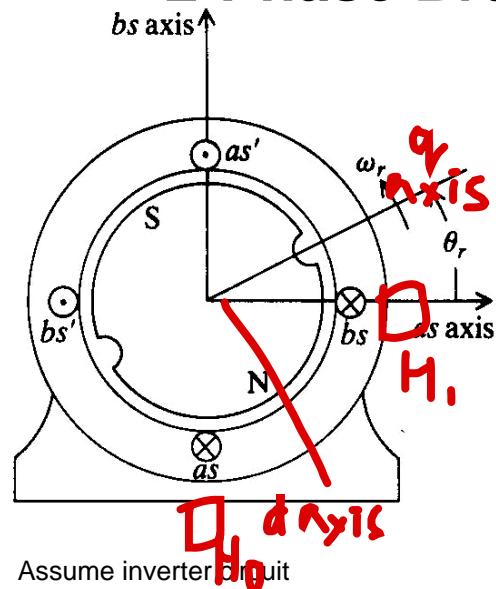
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# Brushless DC Motor

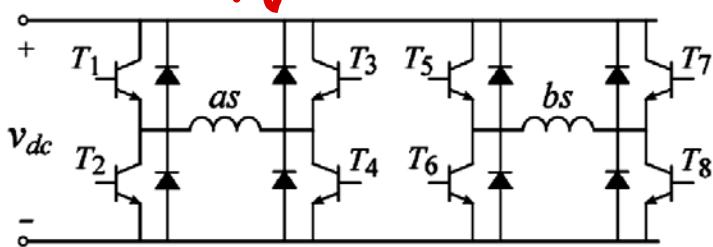


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## 2-Phase Brushless DC Motor



Hall-effect sensors control the inverter

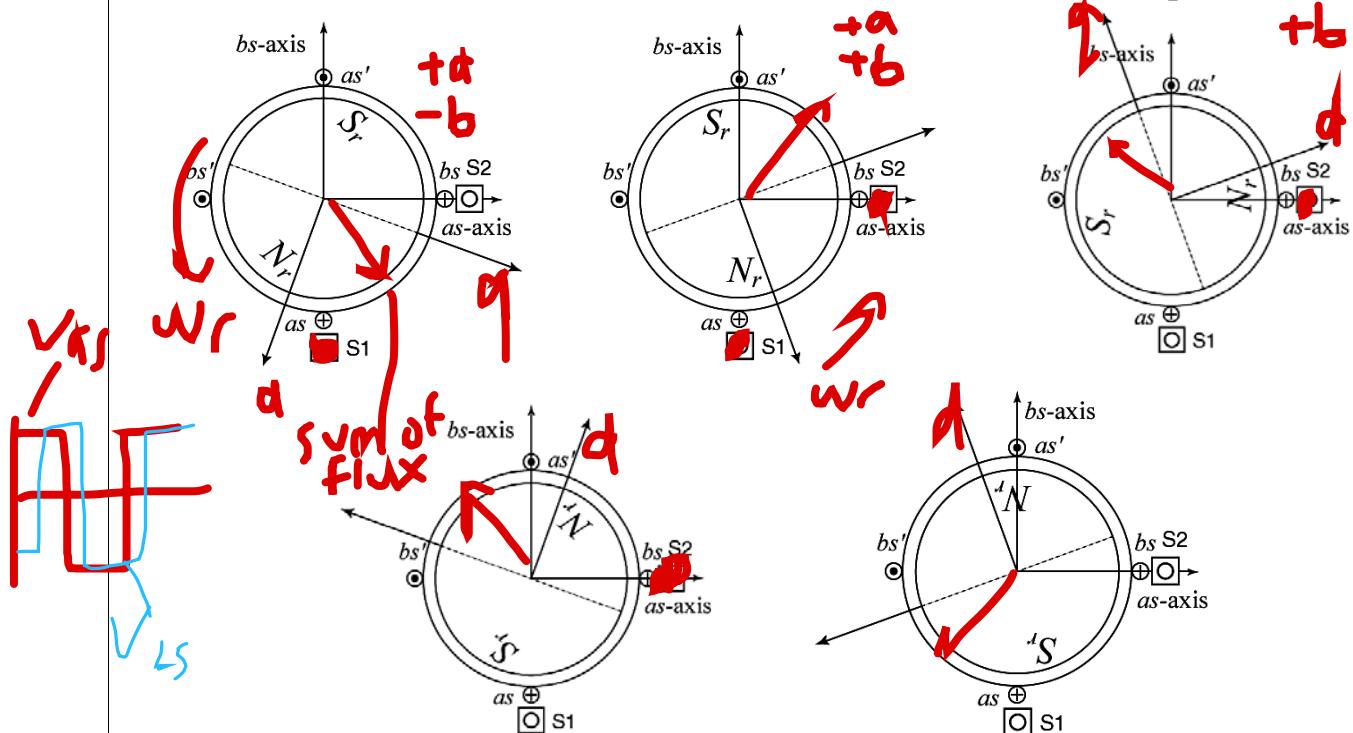


T1 & T4 => as positive  
T2 & T3 => as negative

T5 & T8 => bs positive  
T6 & T7 => bs negative

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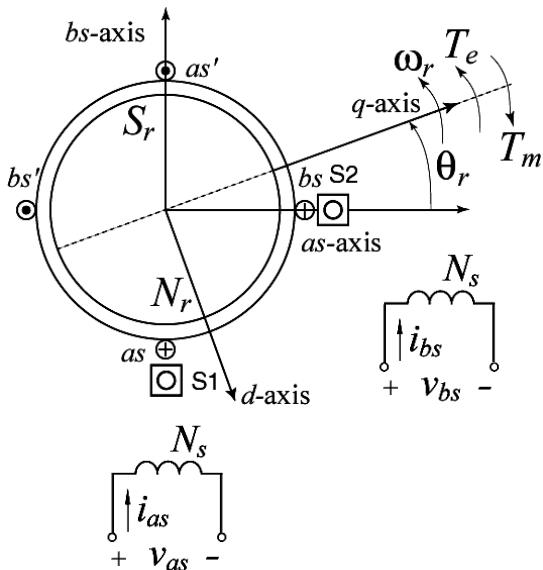
# Brushless DC Motor Principle



Hall-effect sensors: Under N – on (positive), under S – off (negative)

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## 2-Phase Brushless DC Motor Model



Assume stator self inductances

$$L_{asas} = L_{ls} + L_{ms}$$

$$L_{bsbs} = L_{ls} + L_{ms}$$

Voltage equations

$$v_{as} = r_s i_{as} + \frac{d\lambda_{as}}{dt}$$

$$v_{bs} = r_s i_{bs} + \frac{d\lambda_{bs}}{dt}$$

$$\mathbf{v}_{abs} = \mathbf{r}_s \mathbf{i}_{abs} + \frac{d\lambda_{abs}}{dt}$$

Flux linkage equations

$$\begin{bmatrix} \lambda_{as} \\ \lambda_{bs} \end{bmatrix} = \begin{bmatrix} L_{asas} & 0 \\ 0 & L_{bsbs} \end{bmatrix} \cdot \begin{bmatrix} i_{as} \\ i_{bs} \end{bmatrix} + \begin{bmatrix} \lambda_{asm} \\ \lambda_{bsm} \end{bmatrix}$$

$$\lambda_{abs} = \mathbf{L}_s \mathbf{i}_{abs} + \lambda_m$$

Flux linkage due to Permanent Magnet

$$\lambda_m = \begin{bmatrix} \lambda_{asm} \\ \lambda_{bsm} \end{bmatrix} = \lambda_m \begin{bmatrix} \sin(\theta_r) \\ -\cos(\theta_r) \end{bmatrix}$$

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## 2-Phase Brushless DC Motor Model

Electromagnetic Torque  $T_e = \frac{P}{2} \cdot \frac{\partial W_c(\mathbf{i}, \theta_r)}{\partial \theta_r}$  Assume magnetically linear system  $W_c = W_f$

$$\begin{aligned} W_f &= \frac{1}{2} \mathbf{i}_{abs}^T (\mathbf{L}_s - L_{ls} \mathbf{I}) \mathbf{i}_{abs} + \mathbf{i}_{abs}^T \boldsymbol{\lambda}_m \\ &= \frac{1}{2} L_{ms} i_{as}^2 + \frac{1}{2} L_{ms} i_{bs}^2 + i_{as} \lambda_{asm} + i_{bs} \lambda_{bsm} \\ &= \frac{1}{2} L_{ms} (i_{as}^2 + i_{bs}^2) + \lambda_m [i_{as} \sin(\theta_r) - i_{bs} \cos(\theta_r)] \end{aligned}$$

$$T_e = \frac{P}{2} \lambda_m [i_{as} \cos(\theta_r) + i_{bs} \sin(\theta_r)]$$

Mechanical System

$$J_{total} \frac{d\omega_{rm}}{dt} = T_e - T_{fric} - T_m$$

Rotor position

$$\theta_r = \theta_r(0) + \int \omega_r dt$$

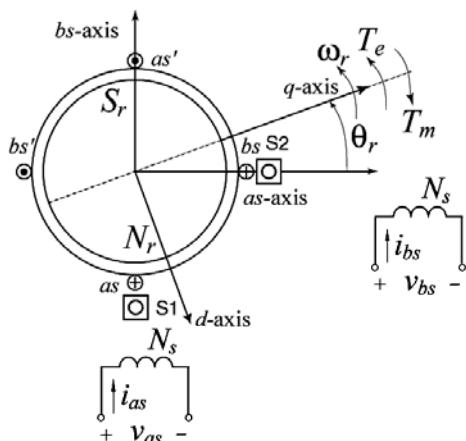
Recall for P-pole motors we can define:

$$\omega_r = \frac{P}{2} \omega_{rm}$$

$$\theta_r = \frac{P}{2} \theta_{rm}$$

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## 2-Phase Brushless DC Motor Model



Mechanical System

$$J_{total} \frac{d\omega_{rm}}{dt} = T_e - T_{fric} - T_m$$

$$\theta_r = \theta_r(0) + \int \omega_r dt$$

$$\omega_r = \frac{P}{2} \omega_{rm} \quad \theta_r = \frac{P}{2} \theta_{rm}$$

Voltage equations = State equations

$$\frac{d\lambda_{abs}}{dt} = -\mathbf{r}_s \mathbf{i}_{abs} + \mathbf{v}_{abs}$$

$$= -\mathbf{r}_s \mathbf{L}_s^{-1} (\lambda_{abs} - \lambda_m) + \mathbf{v}_{abs}$$

Flux linkage equations

$$\lambda_{abs} - \lambda_m = \mathbf{L}_s \mathbf{i}_{abs}$$

$$\mathbf{i}_{abs} = \mathbf{L}_s^{-1} (\lambda_{abs} - \lambda_m)$$

Electromagnetic Torque

$$T_e = \frac{P}{2} \lambda_m [i_{as} \cos(\theta_r) + i_{bs} \sin(\theta_r)]$$

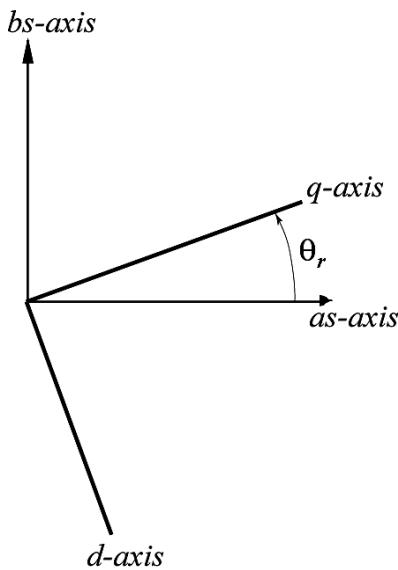
How similar is this to a conventional brushed DC motor?

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# Apply Rotor Frame Reference (RRF)

*qd*-frame is fixed on the rotor



Phases have variables  $f = i, v, \lambda, \dots$

Define a vector  $\mathbf{f}_{abs} = [f_{as} \ f_{bs}]^T$

Define change of variables to *qd*-coordinate

$$\mathbf{f}_{qds} = [f_{qs} \ f_{ds}]^T$$

$$\begin{bmatrix} f_{qs} \\ f_{ds} \end{bmatrix} = \begin{bmatrix} \cos(\theta_r) & \sin(\theta_r) \\ \sin(\theta_r) & -\cos(\theta_r) \end{bmatrix} \cdot \begin{bmatrix} f_{as} \\ f_{bs} \end{bmatrix}$$

$$\mathbf{f}_{qds} = \mathbf{K}_s^r \mathbf{f}_{abs}$$

$$\mathbf{f}_{abs} = (\mathbf{K}_s^r)^{-1} \mathbf{f}_{qds}$$

$$(\mathbf{K}_s^r)^{-1} = \mathbf{K}_s^r = (\mathbf{K}_s^r)^T$$

Rotor position  $\theta_r = \theta_r(0) + \int \omega_r dt$

Express all equations in transformed *qd* variables !

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## Voltage Equations in RRF

Original voltage equation

$$\mathbf{v}_{abs} = \mathbf{r}_s \mathbf{i}_{abs} + \frac{d\boldsymbol{\lambda}_{abs}}{dt}$$

Substitute the *qd*-variables

$$(\mathbf{K}_s^r)^{-1} \mathbf{v}_{qds} = \mathbf{r}_s (\mathbf{K}_s^r)^{-1} \mathbf{i}_{qds} + \frac{d}{dt} \left[ (\mathbf{K}_s^r)^{-1} \boldsymbol{\lambda}_{qds} \right]$$

$$\mathbf{v}_{qds} = \mathbf{K}_s^r \mathbf{r}_s (\mathbf{K}_s^r)^{-1} \mathbf{i}_{qds} + \mathbf{K}_s^r \frac{d}{dt} \left[ (\mathbf{K}_s^r)^{-1} \boldsymbol{\lambda}_{qds} \right]$$

$$\mathbf{K}_s^r \frac{d}{dt} \left[ (\mathbf{K}_s^r)^{-1} \boldsymbol{\lambda}_{qds} \right] = \mathbf{K}_s^r \left[ \frac{d}{dt} (\mathbf{K}_s^r)^{-1} \right] \boldsymbol{\lambda}_{qds} + \mathbf{K}_s^r (\mathbf{K}_s^r)^{-1} \frac{d\boldsymbol{\lambda}_{qds}}{dt}$$

$$\begin{bmatrix} \cos(\theta_r) & \sin(\theta_r) \\ \sin(\theta_r) & -\cos(\theta_r) \end{bmatrix} \cdot \omega_r \begin{bmatrix} -\sin(\theta_r) & \cos(\theta_r) \\ \cos(\theta_r) & \sin(\theta_r) \end{bmatrix} = \omega_r \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$\omega_r \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \boldsymbol{\lambda}_{qds} = \omega_r \begin{bmatrix} \boldsymbol{\lambda}_{ds} \\ -\boldsymbol{\lambda}_{qs} \end{bmatrix} = \omega_r \boldsymbol{\lambda}_{dqs}$$

Resulted voltage equation

$$\mathbf{v}_{qds} = \mathbf{r}_s \mathbf{i}_{qds} + \omega_r \boldsymbol{\lambda}_{dqs} + \frac{d\boldsymbol{\lambda}_{qds}}{dt}$$

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# Flux Linkage Equations in RRF

Original equation

$$\begin{bmatrix} \lambda_{as} \\ \lambda_{bs} \end{bmatrix} = \begin{bmatrix} L_{ls} + L_{ms} & 0 \\ 0 & L_{ls} + L_{ms} \end{bmatrix} \cdot \begin{bmatrix} i_{as} \\ i_{bs} \end{bmatrix} + \lambda_m \begin{bmatrix} \sin(\theta_r) \\ -\cos(\theta_r) \end{bmatrix}$$

$$\lambda_{abs} = \mathbf{L}_s \mathbf{i}_{abs} + \lambda_m$$

Substitute the *qd*-variables

$$(\mathbf{K}_s^r)^{-1} \lambda_{qds} = \mathbf{L}_s (\mathbf{K}_s^r)^{-1} \mathbf{i}_{qds} + \lambda_m$$

$$\lambda_{qds} = \boxed{\mathbf{K}_s^r \mathbf{L}_s (\mathbf{K}_s^r)^{-1} \mathbf{i}_{qds}} + \boxed{\mathbf{K}_s^r \lambda_m}$$

where

$$\boxed{\mathbf{K}_s^r \mathbf{L}_s (\mathbf{K}_s^r)^{-1}} = \begin{bmatrix} L_{ls} + L_{ms} & 0 \\ 0 & L_{ls} + L_{ms} \end{bmatrix}$$

$$\begin{bmatrix} \cos(\theta_r) & \sin(\theta_r) \\ \sin(\theta_r) & -\cos(\theta_r) \end{bmatrix} \cdot \lambda_m \begin{bmatrix} \sin(\theta_r) \\ -\cos(\theta_r) \end{bmatrix} = \lambda_m \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Resulted flux linkage equation

$$\begin{bmatrix} \lambda_{qs} \\ \lambda_{ds} \end{bmatrix} = \begin{bmatrix} L_{ls} + L_{ms} & 0 \\ 0 & L_{ls} + L_{ms} \end{bmatrix} \cdot \begin{bmatrix} i_{qs} \\ i_{ds} \end{bmatrix} + \lambda_m \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\lambda_{qs} = (L_{ls} + L_{ms}) i_{qs} = L_{ss} i_{qs}$$

$$\lambda_{ds} = (L_{ls} + L_{ms}) i_{ds} + \lambda_m = L_{ss} i_{ds} + \lambda_m$$

LSS

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## 2-Phase Brushless DC Motor Model in *qd* RRF

Expanded Voltage Equations

$$v_{qs} = r_s i_{qs} + \omega_r \lambda_{ds} + \frac{d\lambda_{qs}}{dt} = r_s i_{qs} + \omega_r L_{ss} i_{ds} + \omega_r \lambda_m + L_{ss} \frac{di_{qs}}{dt}$$

$$v_{ds} = r_s i_{ds} - \omega_r \lambda_{qs} + \frac{d\lambda_{ds}}{dt} = r_s i_{ds} - \omega_r L_{ss} i_{qs} + L_{ss} \frac{di_{ds}}{dt}$$

$$\frac{d}{dt} \begin{bmatrix} i_{qs} \\ i_{ds} \end{bmatrix} = \begin{bmatrix} L_{ss} & 0 \\ 0 & L_{ss} \end{bmatrix}^{-1} \left\{ \begin{bmatrix} -r_s & -\omega_r L_{ss} \\ \omega_r L_{ss} & -r_s \end{bmatrix} \begin{bmatrix} i_{qs} \\ i_{ds} \end{bmatrix} + \begin{bmatrix} -\omega_r \lambda_m \\ 0 \end{bmatrix} + \begin{bmatrix} v_{qs} \\ v_{ds} \end{bmatrix} \right\}$$

Electromagnetic Torque

$$\begin{bmatrix} i_{as} \\ i_{bs} \end{bmatrix} = \begin{bmatrix} \cos(\theta_r) & \sin(\theta_r) \\ \sin(\theta_r) & -\cos(\theta_r) \end{bmatrix} \begin{bmatrix} i_{qs} \\ i_{ds} \end{bmatrix} = \begin{bmatrix} i_{qs} \cos(\theta_r) + i_{ds} \sin(\theta_r) \\ i_{qs} \sin(\theta_r) - i_{ds} \cos(\theta_r) \end{bmatrix}$$

$$T_e = \frac{P}{2} \lambda_m [i_{as} \cos(\theta_r) + i_{bs} \sin(\theta_r)]$$

$$= \frac{P}{2} \lambda_m [i_{qs} \cos^2(\theta_r) + i_{ds} \sin(\theta_r) \cos(\theta_r) + i_{qs} \sin^2(\theta_r) - i_{ds} \sin(\theta_r) \cos(\theta_r)]$$

$$\bar{T}_e = \frac{P}{2} \lambda_m i_{qs} = K_T i_{qs} \quad \text{← PM DC Ma}$$

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# Steady-State Operation

Voltage Equations

$$V_{qs} = r_s I_{qs} + \omega_r L_{ss} I_{ds} + \omega_r \lambda_m$$

$$V_{ds} = r_s I_{ds} - \omega_r L_{ss} I_{qs}$$

$$v_{qs} = r_s i_{qs} + \omega_r L_{ss} i_{ds} + \omega_r \lambda_m + L_{ss} \frac{di_{qs}}{dt}$$

$$v_{ds} = r_s i_{ds} - \omega_r L_{ss} i_{qs} + L_{ss} \frac{di_{ds}}{dt}$$

Electromagnetic Torque

$$T_e = \frac{P}{2} \lambda_m I_{qs}$$

$$T_e = \frac{P}{2} \frac{\lambda_m r_s}{r_s^2 + \omega_r^2 L_{ss}^2} \left( V_{qs} - \frac{\omega_r L_{ss}}{r_s} V_{ds} - \omega_r \lambda_m \right)$$

Assume applied voltages

$$V_{as} = \sqrt{2} V_s \cos(\omega_e t + \phi_v)$$

$$V_{bs} = \sqrt{2} V_s \sin(\omega_e t + \phi_v)$$

$$\phi_v = \theta_e - \theta_r$$

$$= \theta_e(0) - \theta_r(0) + \int (\omega_e - \omega_r) dt$$

$$\text{Voltages in RRF } V_{qs} = \sqrt{2} V_s \cos(\phi_v) = 2\sqrt{V_s}$$

$$V_{ds} = -\sqrt{2} V_s \sin(\phi_v) = 0$$

Common Operation Mode: Set  $\phi_v = 0$

$$I_{ds} = \frac{\omega_r L_{ss}}{r_s} I_{qs}$$

$$V_{qs} = r_s I_{qs} + \omega_r L_{ss} \frac{\omega_r L_{ss}}{r_s} I_{qs} + \omega_r \lambda_m$$

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# Steady-State Operation

Brushless DC Motor in COM

- Voltage equation

$$V_{qs} = \frac{r_s^2 + \omega_r^2 L_{ss}^2}{r_s} I_{qs} + \omega_r \lambda_m$$

- Back emf

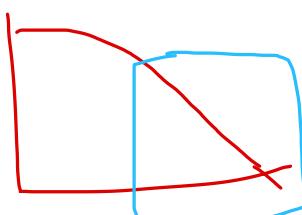
$$E_{a,peak} = \omega_r \lambda_m$$

- Electromagnetic torque

$$T_e = \frac{P}{2} \lambda_m I_{qs}$$

- Torque-speed characteristic

$$T_e = \frac{P}{2} \frac{\lambda_m r_s}{r_s^2 + \omega_r^2 L_{ss}^2} (V_{qs} - \omega_r \lambda_m)$$



PM Brushed DC Motor

- Voltage equation

$$V_a = r_a I_a + \omega_r k_v$$

- Back emf

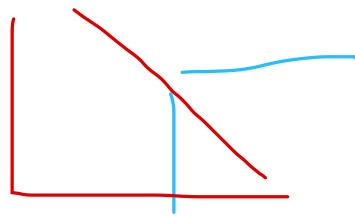
$$E_a = \omega_r k_v$$

- Electromagnetic torque

$$T_e = k_v I_a$$

- Torque-speed characteristic

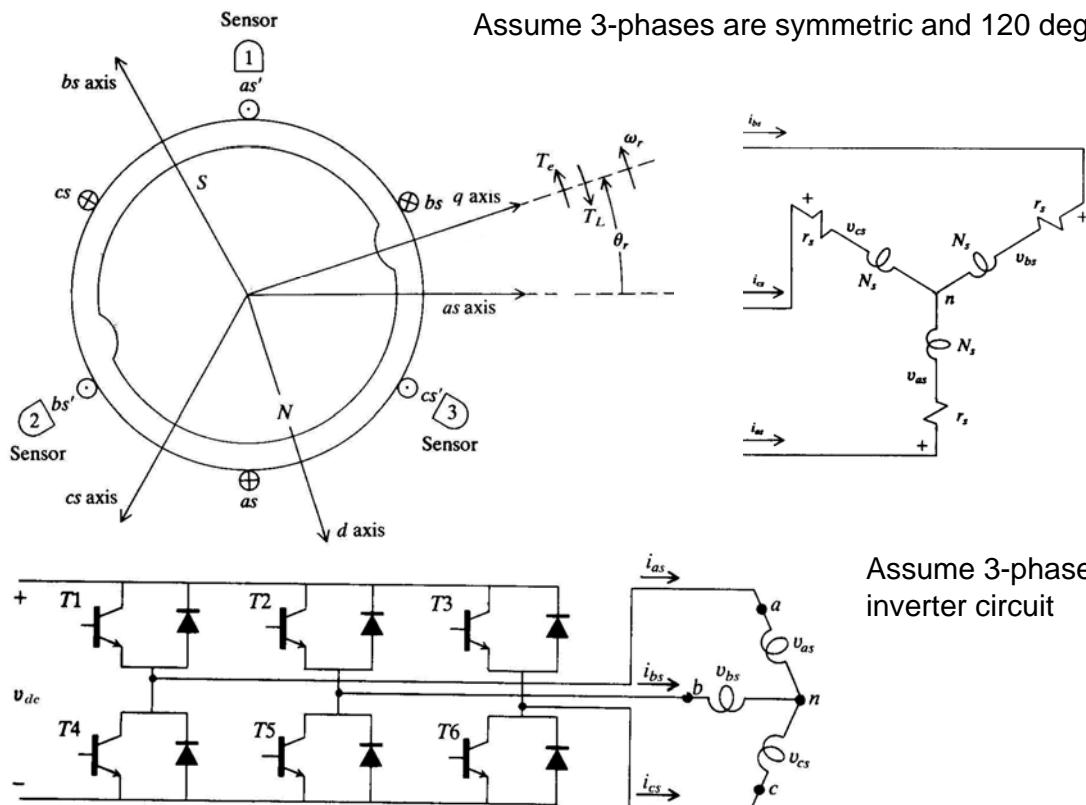
$$T_e = \frac{k_v}{r_s} (V_a - \omega_r \lambda_m)$$



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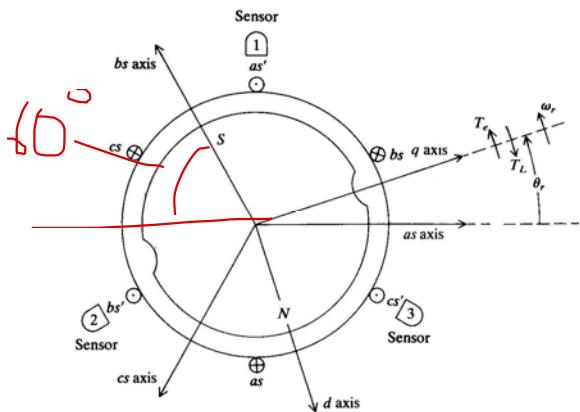
# 3-Phase Brushless DC Motor

Assume 3-phases are symmetric and 120 deg apart



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# 3-Phase Brushless DC Motor Model



Voltage equations

$$v_{as} = r_s i_{as} + \frac{d\lambda_{as}}{dt}$$

$$v_{bs} = r_s i_{bs} + \frac{d\lambda_{bs}}{dt}$$

$$v_{cs} = r_s i_{cs} + \frac{d\lambda_{cs}}{dt}$$

$$\mathbf{v}_{abcs} = \mathbf{r}_s \mathbf{i}_{abcs} + \frac{d\lambda_{abcs}}{dt}$$

Flux linkage equations

$$\begin{bmatrix} \lambda_{as} \\ \lambda_{bs} \\ \lambda_{cs} \end{bmatrix} = \begin{bmatrix} L_{ls} + L_{ms} & -0.5L_{ms} & -0.5L_{ms} \\ -0.5L_{ms} & L_{ls} + L_{ms} & -0.5L_{ms} \\ -0.5L_{ms} & -0.5L_{ms} & L_{ls} + L_{ms} \end{bmatrix} \cdot \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix} + \lambda_m \begin{bmatrix} \sin(\theta_r) \\ \sin(\theta_r - 120) \\ \sin(\theta_r + 120) \end{bmatrix}$$

$$\lambda_{abcs} = \mathbf{L}_s \mathbf{i}_{abcs} + \lambda_m$$

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# 3-Phase Brushless DC Motor Model

Electromagnetic Torque  $T_e = \frac{P}{2} \cdot \frac{\partial W_c(\mathbf{i}, \theta_r)}{\partial \theta_r}$  Assume magnetically linear system  $W_c = W_f$

$$W_f = \frac{1}{2} \mathbf{i}_{abcs}^T (\mathbf{L}_s - L_{ls} \mathbf{I}) \mathbf{i}_{abcs} + \mathbf{i}_{abcs}^T \boldsymbol{\lambda}_m$$

$$T_e = \frac{P}{2} \lambda_m \left[ \left( i_{as} - \frac{1}{2} i_{bs} - \frac{1}{2} i_{cs} \right) \cos(\theta_r) + \frac{\sqrt{3}}{2} (i_{bs} - i_{cs}) \sin(\theta_r) \right]$$

Mechanical System

$$J_{total} \frac{d\omega_{rm}}{dt} = T_e - T_{fric} - T_m$$

Rotor position

$$\theta_r = \theta_r(0) + \int \omega_r dt$$

Recall for P-pole motors:

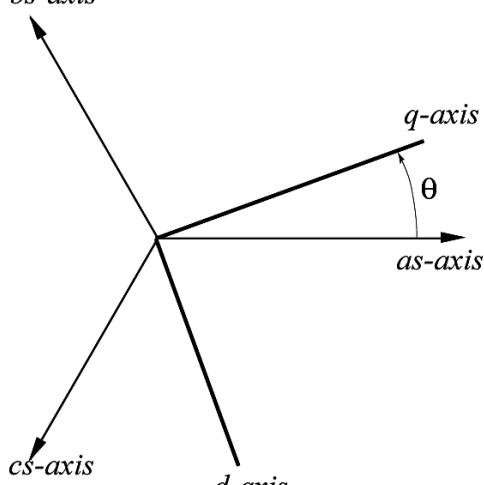
$$\omega_r = \frac{P}{2} \omega_{rm}$$

$$\theta_r = \frac{P}{2} \theta_{rm}$$

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## Apply Rotating Frame of Reference (RRF)

*qd*-frame is fixed on the rotor  
*bs*-axis



$$(\mathbf{K}_s^r)^{-1} = \begin{bmatrix} \cos(\theta_r) & \sin(\theta_r) \\ \cos(\theta_r - 120^\circ) & \sin(\theta_r - 120^\circ) \\ \cos(\theta_r + 120^\circ) & \sin(\theta_r + 120^\circ) \end{bmatrix}$$

Pseudo inverse !

Consider that each phase has variables

$$f = i, v, \lambda, \dots$$

Define a vector  $\mathbf{f}_{abcs} = [f_{as} \ f_{bs} \ f_{cs}]^T$

Define change of variables to *qd*-coordinate

$$\mathbf{f}_{qds} = [f_{qs} \ f_{ds}]^T$$

$$\begin{bmatrix} f_{qs} \\ f_{ds} \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \cos(\theta) & \cos(\theta - 120^\circ) & \cos(\theta + 120^\circ) \\ \sin(\theta) & \sin(\theta - 120^\circ) & \sin(\theta + 120^\circ) \end{bmatrix} \cdot \begin{bmatrix} f_{as} \\ f_{bs} \\ f_{cs} \end{bmatrix}$$

$$\mathbf{f}_{qds} = \mathbf{K}_s \mathbf{f}_{abcs}$$

$$\mathbf{f}_{abcs} = (\mathbf{K}_s^r)^{-1} \mathbf{f}_{qds}$$

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# Voltage Equations in RRF

Original voltage equation

$$\mathbf{v}_{abcs} = \mathbf{r}_s \mathbf{i}_{abcs} + \frac{d\lambda_{abcs}}{dt}$$

Substitute the  $qd$ -variables

$$(\mathbf{K}_s^r)^{-1} \mathbf{v}_{qds} = \mathbf{r}_s (\mathbf{K}_s^r)^{-1} \mathbf{i}_{qds} + \frac{d}{dt} \left[ (\mathbf{K}_s^r)^{-1} \lambda_{qds} \right]$$

$$\mathbf{v}_{qds} = \mathbf{K}_s^r \mathbf{r}_s (\mathbf{K}_s^r)^{-1} \mathbf{i}_{qds} + \mathbf{K}_s^r \frac{d}{dt} \left[ (\mathbf{K}_s^r)^{-1} \lambda_{qds} \right]$$

$$\mathbf{K}_s^r \frac{d}{dt} \left[ (\mathbf{K}_s^r)^{-1} \lambda_{qds} \right] = \mathbf{K}_s^r \left[ \frac{d}{dt} (\mathbf{K}_s^r)^{-1} \right] \lambda_{qds} + \mathbf{K}_s^r (\mathbf{K}_s^r)^{-1} \frac{d\lambda_{qds}}{dt}$$

Following the same procedure as for 2-phase case, the resulted voltage equation is

$$\mathbf{v}_{qds} = \mathbf{r}_s \mathbf{i}_{qds} + \omega_r \lambda_{dqs} + \frac{d\lambda_{qds}}{dt}$$

where

$$\lambda_{dqs} = \begin{bmatrix} \lambda_{ds} \\ -\lambda_{qs} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \lambda_{qds}$$

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# Flux Linkage Equations in RRF

Original equation

$$\begin{bmatrix} \lambda_{as} \\ \lambda_{bs} \\ \lambda_{cs} \end{bmatrix} = \begin{bmatrix} L_{ls} + L_{ms} & -0.5L_{ms} & -0.5L_{ms} \\ -0.5L_{ms} & L_{ls} + L_{ms} & -0.5L_{ms} \\ -0.5L_{ms} & -0.5L_{ms} & L_{ls} + L_{ms} \end{bmatrix} \cdot \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix} + \lambda_{asm} \begin{bmatrix} \sin(\theta_r) \\ \sin(\theta_r - 120) \\ \sin(\theta_r + 120) \end{bmatrix}$$

$$\lambda_{abcs} = \mathbf{L}_s \mathbf{i}_{abcs} + \lambda_m$$

Substitute the  $qd$ -variables

$$(\mathbf{K}_s^r)^{-1} \lambda_{qds} = \mathbf{L}_s (\mathbf{K}_s^r)^{-1} \mathbf{i}_{qds} + \lambda_m$$

$$\lambda_{qds} = \mathbf{K}_s^r \mathbf{L}_s (\mathbf{K}_s^r)^{-1} \mathbf{i}_{qds} + \mathbf{K}_s^r \lambda_m$$

where

$$\mathbf{K}_s^r \mathbf{L}_s (\mathbf{K}_s^r)^{-1} = \begin{bmatrix} L_{ls} + \frac{3}{2}L_{ms} & 0 \\ 0 & L_{ls} + \frac{3}{2}L_{ms} \end{bmatrix}$$

$$\mathbf{K}_s^r \lambda_m = \lambda_m \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Resulted flux linkage equation

$$\begin{bmatrix} \lambda_{qs} \\ \lambda_{ds} \end{bmatrix} = \begin{bmatrix} L_{ss} & 0 \\ 0 & L_{ss} \end{bmatrix} \cdot \begin{bmatrix} i_{qs} \\ i_{ds} \end{bmatrix} + \lambda_m \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\lambda_{qs} = L_{ss} i_{qs}$$

$$\lambda_{ds} = L_{ss} i_{ds} + \lambda_m$$

Define

$$L_{ss} = L_{ls} + \frac{3}{2}L_{ms}$$

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# 3-Phase Brushless DC Motor Model in qd RRF

Expanded Voltage Equations

$$v_{qs} = r_s i_{qs} + \omega_r \lambda_{ds} + \frac{d\lambda_{qs}}{dt} = r_s i_{qs} + \omega_r L_{ss} i_{ds} + \omega_r \lambda_m + L_{ss} \frac{di_{qs}}{dt}$$

$$v_{ds} = r_s i_{ds} - \omega_r \lambda_{qs} + \frac{d\lambda_{ds}}{dt} = r_s i_{ds} - \omega_r L_{ss} i_{qs} + L_{ss} \frac{di_{ds}}{dt}$$

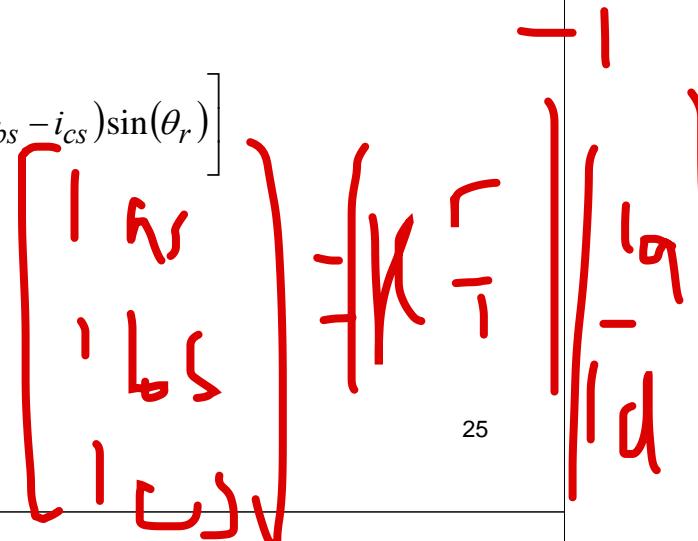
$$\frac{d}{dt} \begin{bmatrix} i_{qs} \\ i_{ds} \end{bmatrix} = \begin{bmatrix} L_{ss} & 0 \\ 0 & L_{ss} \end{bmatrix}^{-1} \left\{ \begin{bmatrix} -r_s & -\omega_r L_{ss} \\ \omega_r L_{ss} & -r_s \end{bmatrix} \begin{bmatrix} i_{qs} \\ i_{ds} \end{bmatrix} + \begin{bmatrix} -\omega_r \lambda_m \\ 0 \end{bmatrix} + \begin{bmatrix} v_{qs} \\ v_{ds} \end{bmatrix} \right\}$$

Electromagnetic Torque

$$T_e = \frac{P}{2} \lambda_m \left[ \left( i_{as} - \frac{1}{2} i_{bs} - \frac{1}{2} i_{cs} \right) \cos(\theta_r) + \frac{\sqrt{3}}{2} (i_{bs} - i_{cs}) \sin(\theta_r) \right]$$

$$= \frac{3}{2} \frac{P}{2} \lambda_m i_{qs}$$

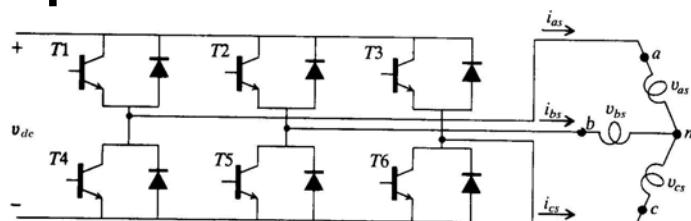
Steady-State Operation = Same as 2-phase motor !



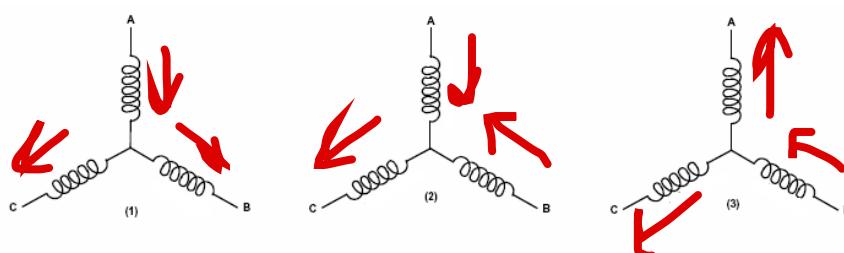
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## 180-Deg., 6-Step BLDC Motor Drive

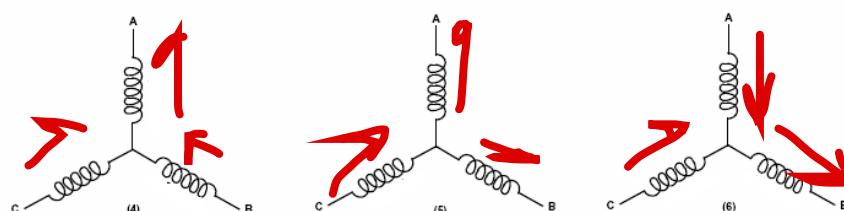
Step	Rotor Position	Active Transistors
1	-30 to 30	T1, T5, T6
2	30 to 90	T1, T2, T6
3	90 to 150	T2, T4, T6
4	150 to 210	T2, T3, T4
5	210 to 270	T3, T4, T5
6	270 to 330	T1, T3, T5



Hall-effect sensors control the inverter



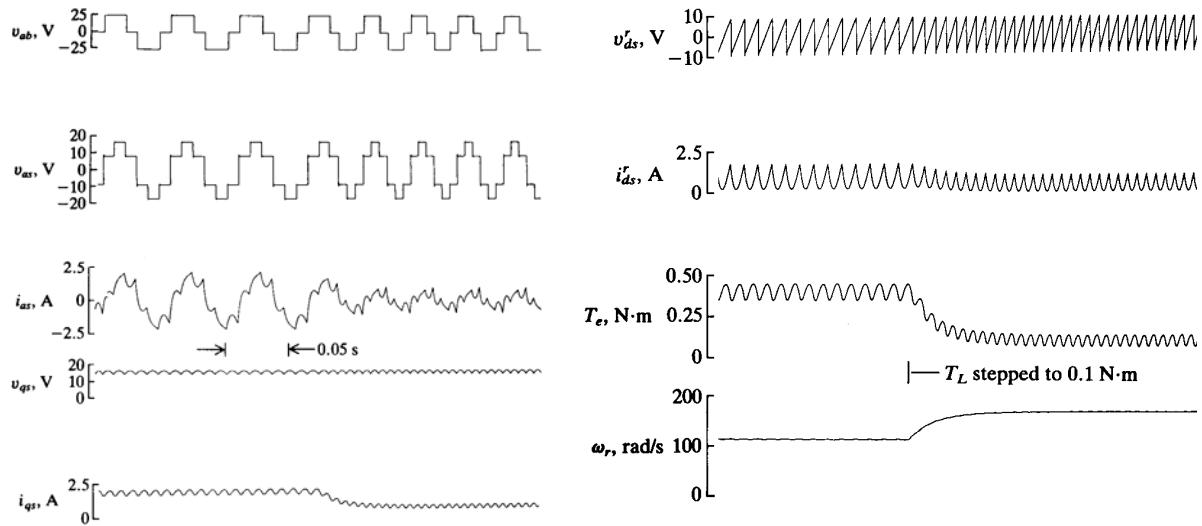
T1 => as positive  
 T4 => as negative  
 T2 => bs positive  
 T5 => bs negative  
 T3 => cs positive  
 T6 => cs negative



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# 3-Phase Brushless DC Motor Drive

6-Step, 180 deg. Operation:  
Typical waveforms

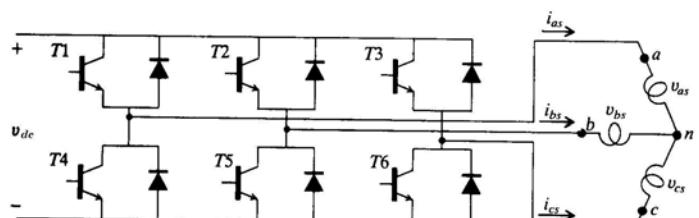


Note: current harmonics cause the torque ripple !

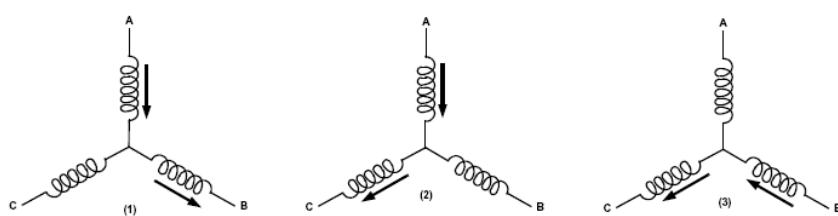
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# 120-Deg., 6-Step BLDC Motor Drive

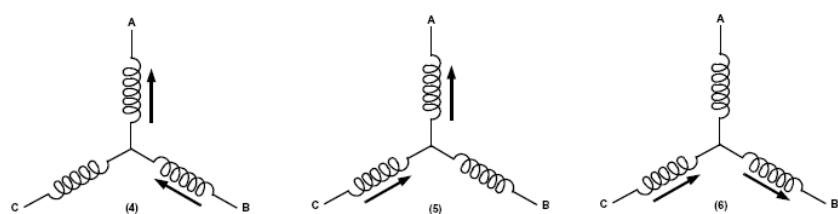
Step	Rotor Position	Active Transistors
1	-30 to 30	T1, T5
2	30 to 90	T1, T6
3	90 to 150	T2, T6
4	150 to 210	T2, T4
5	210 to 270	T3, T4
6	270 to 330	T3, T5



Hall-effect sensors control the inverter



T1 => as positive  
 T4 => as negative  
 T2 => bs positive  
 T5 => bs negative  
 T3 => cs positive  
 T6 => cs negative

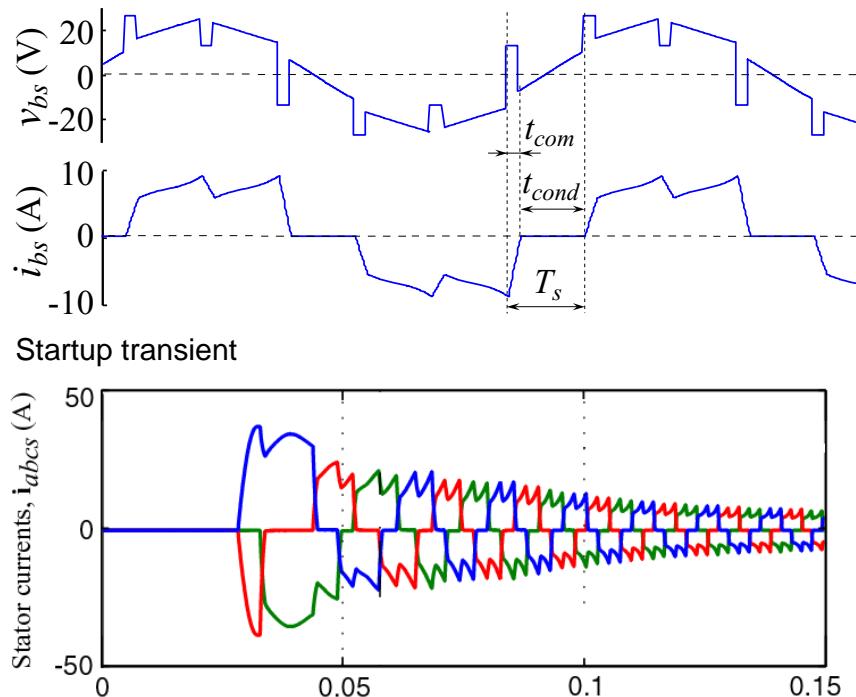


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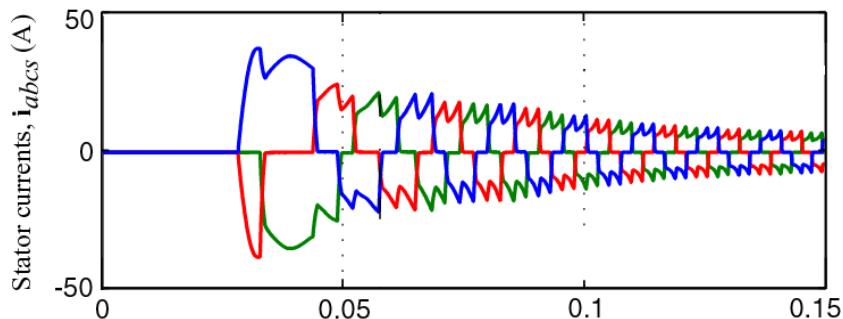
# 3-Phase Brushless DC Motor Drive

ELEC 343 S19, M7

6-Step, 120 deg. Operation: Typical steady state waveforms



Startup transient



Note: current harmonics will cause the torque ripple !

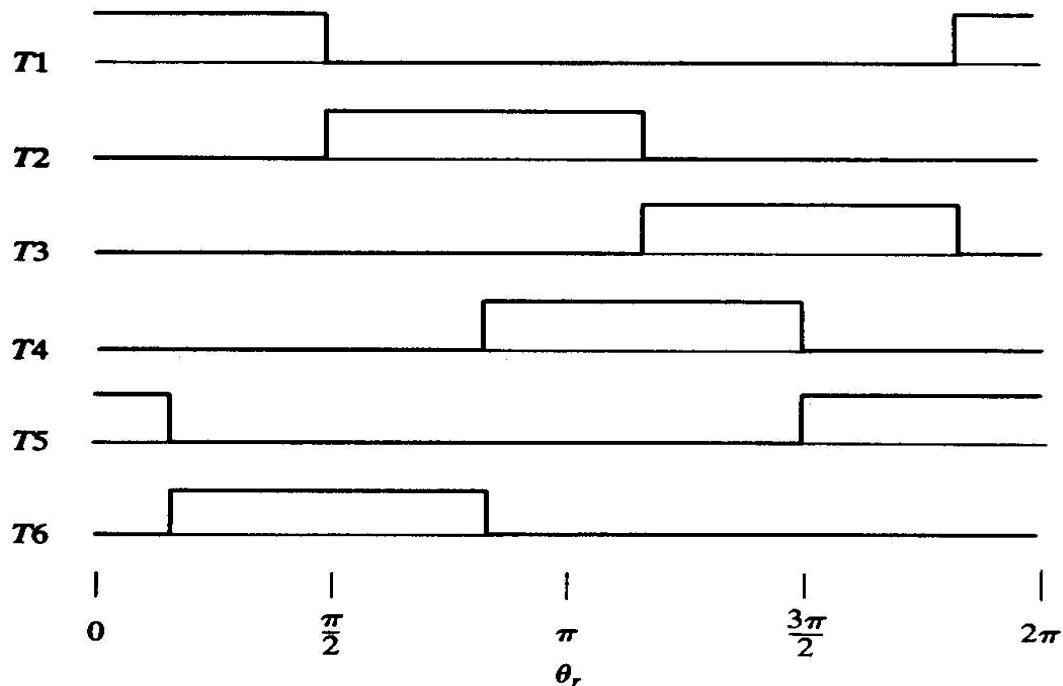
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# 3-Phase Brushless DC Motor Drive

ELEC 343 S19, M7

6-Step, 120 deg. Operation with PWM

Apply PWM to control the average voltage !



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# Possible Implementation

Voltage Equations

$$v_{qs} = r_s i_{qs} + \omega_r L_{ss} i_{ds} + \omega_r \lambda_m + L_{ss} \frac{di_{qs}}{dt}$$

$$v_{ds} = r_s i_{ds} - \omega_r L_{ss} i_{qs} + L_{ss} \frac{di_{ds}}{dt}$$

Electromagnetic Torque

$$T_e = \frac{P}{2} \lambda_m I_{qs}$$

$$v_{qs} = r_s (1 + \tau_s s) i_{qs} + \omega_r L_{ss} i_{ds} + \omega_r \lambda_m$$

$$v_{ds} = r_s (1 + \tau_s s) i_{ds} - \omega_r L_{ss} i_{qs}$$

$$i_{qs} = \frac{1/r_s}{\tau_s s + 1} (v_{qs} - \omega_r L_{ss} i_{ds} - \omega_r \lambda_m)$$

$$i_{ds} = \frac{1/r_s}{\tau_s s + 1} (v_{ds} + \omega_r L_{ss} i_{qs})$$

$$\text{Stator time constant } \tau_s = \frac{L_{ss}}{r_s}$$

Mechanical System

Rotor Position

$$\theta_r = \theta_r(0) + \int \omega_r dt$$

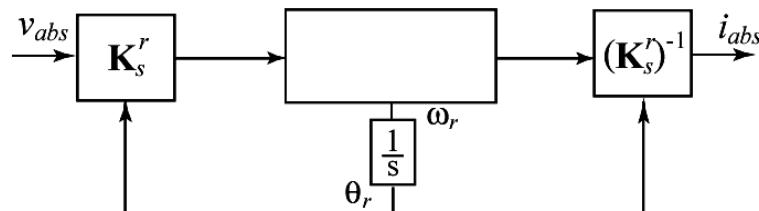
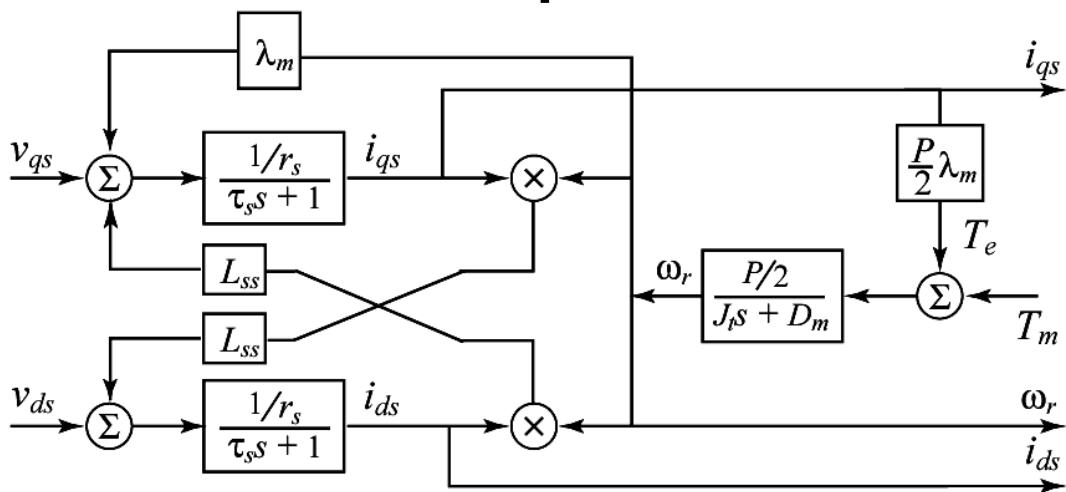
$$\theta_r = \theta_r(0) + \frac{1}{s} \omega_r$$

$$J_t \frac{2}{P} \frac{d\omega_r}{dt} = T_e - D_m \frac{2}{P} \omega_r - T_m$$

$$\omega_r = \frac{P/2}{J_t s + D_m} (T_e - T_m)$$

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# Possible Implementation



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# Possible Implementation

