University of British Columbia
Department of Mechanical Engineering

MECH366 Modeling of Mechatronic Systems
Midterm exam

Examiner: Dr. Ryozo Nagamune
October 13 (Friday), 2017, 3pm-3:50pm

Last name, First name	
Name:	Student #:
Signature:	

Exam policies

- Allowed: One-page letter-size hand-written cheat-sheet (both sides).
- Not-allowed: PC, calculators.
- Write all your answers on this booklet. No extra sheet will be provided.
- Motivate your answers properly. (No chance to defend your answers orally.)
- 20 points in total.

Before you start ...

- Use washroom before the exam.
- Turn off your mobile phone.
- No eating.
- Questions are NOT allowed.

If you finish early ...

- Please stay at your seat until the end of exam, i.e., 3:50pm. (You are not allowed to leave the room before the end of exam, except going to washroom.)

To be filled in by the instructor/marker

Problem #	Mark	Full mark
1		5
2		5
3		10
Total		20

Extra page. Write the problem number before writing your answer.

1. Answer the following questions **concisely, within one or two lines (or even by one-word or two-words if appropriate)**.

   (a) For what purpose can a mathematical model of a physical system be used? (Giving **only one** such purpose is enough.) (1pt)

   Write your answer here.

   (b) For **thermal** systems, what is the **through** variable? (1pt)

   Write your answer here.

   (c) For **fluid** systems, what is the **across** variable? (1pt)

   Write your answer here.

   (d) In **thermal** systems, write the constitutive equation for the **T-type** element. (1pt)

   Write your answer here.

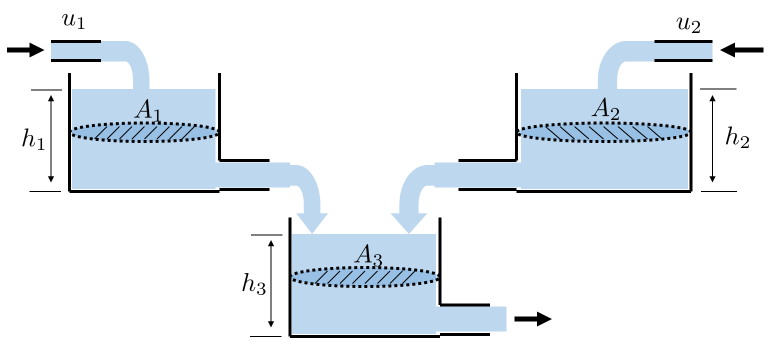
   (e) In **electrical** systems, write the constitutive equation for the **A-type** element. (1pt)

   Write your answer here.

2. Consider a three-water-tank system in the figure below. Here, A_i, $i = 1, 2, 3$, are tank section areas, h_i, $i = 1, 2, 3$, are the water heights of the tanks, and u_i, $i = 1, 2$, are input mass flow rates. The nonlinear state equation of this system is assumed to be expressed as

$$\begin{aligned}
\dot{h}_1(t) &= \frac{1}{\rho A_1}\left(-K\sqrt{h_1(t)} + u_1(t)\right), \\
\dot{h}_2(t) &= \frac{1}{\rho A_2}\left(-K\sqrt{h_2(t)} + u_2(t)\right), \\
\dot{h}_3(t) &= \frac{1}{\rho A_3}\left(-K\sqrt{h_3(t)} + K\sqrt{h_1(t)} + K\sqrt{h_2(t)}\right),
\end{aligned}$$

where ρ and K are given positive constants.



We linearize the nonlinear state equation around the situation when we maintain the water heights at $h_1(t) = h_{10}$ and $h_2(t) = h_{20}$, where h_{10} and h_{20} are given positive constant heights.

(a) Obtain the corresponding constant input flow rates $u_1(t) = u_{10}$ and $u_2(t) = u_{20}$ in terms of given constants h_{10} and h_{20}. (1pt)

(b) Obtain the corresponding constant water height $h_3(t) = h_{30}$ in terms of given constants h_{10} and h_{20}. (1pt)

Write your answer here.

(c) Derive a linearized state equation $\delta \dot{h}(t) = A\delta h(t) + B\delta u(t)$ around the equilibrium point $(h_1, h_2, h_3) = (h_{10}, h_{20}, h_{30})$ and $(u_1, u_2) = (u_{10}, u_{20})$. To answer this question, you do **not** need to use solutions obtained in (a) and (b); just use the notations (h_{10}, h_{20}, h_{30}) and (u_{10}, u_{20}). (2pt)

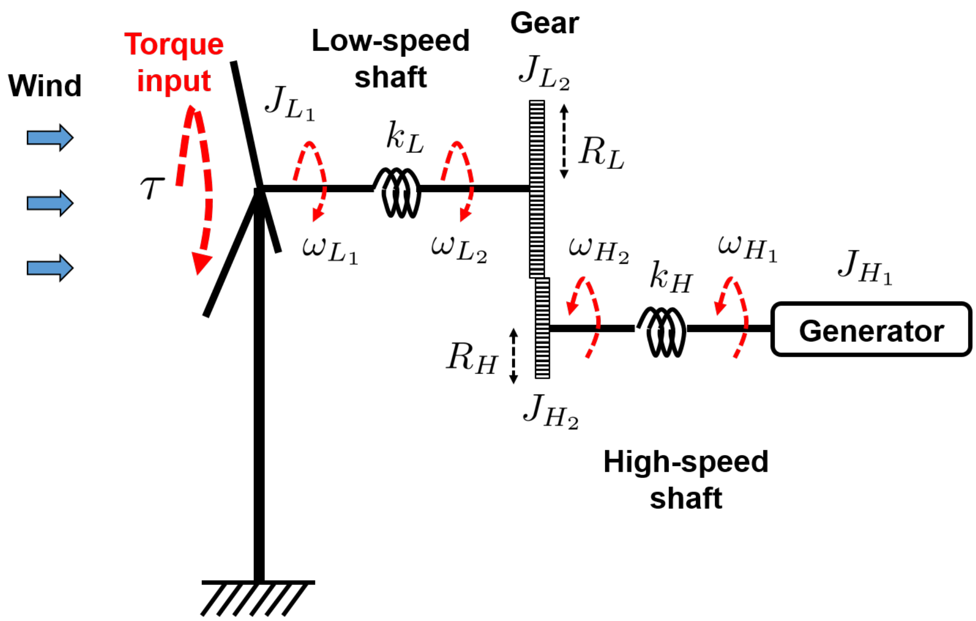
(d) Define the state vector δh and the input vector δu in the linearized model in (c). (1pt)

Write your answer here.

3. Consider a lumped model of a wind turbine system in the figure below. The notations are indicated in the figure, and given as follows. (We ignore friction and damping in this model.)

Notation	Meaning
τ	Aerodynamic torque (input)
J_{L_1} & ω_{L_1}	Moment of inertia & angular velocity of the rotor (blades)
J_{L_2} & ω_{L_2}	Moment of inertia & angular velocity of the low-speed gear
J_{H_1} & ω_{H_1}	Moment of inertia & angular velocity of the generator
J_{H_2} & ω_{H_2}	Moment of inertia & angular velocity of the high-speed gear
R_L & R_H	Radius of the low-speed gear & of the high-speed gear
k_L & k_H	Spring constant of the low-speed shaft & of the high-speed shaft

(**Note:** In this question, you do <u>not need to derive the state equation</u>.)



(a) The two rotational velocities ω_{L_2} and ω_{H_2} are related as $\omega_{H_2} = r\omega_{L_2}$ due to the gear. Obtain the positive constant r. (1pt)

Write your answer here.

Below, you can **use the notation** r, instead of using R_L and R_H.

(b) Draw a linear graph, by introducing notations appropriately. (4pt)

(c) Select the state variables. (It is fine even if you include redundant state variables.) (1pt)

(d) Write the constitutive equations for the passive elements and the gear (transformer) in the linear graph. (2pt)

(e) Write loop equations and node equations for the linear graph. (2pt)

——— (End of Midterm Exam) ———

Write your answer here.

Extra page. Write the problem number before writing your answer.

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