ELEC 343
Electromechanics
Module 6:
Synchronous Motors (Chap. 7)

Spring 2019
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Class Webpage: http://courses.ece.ubc.ca/elec343

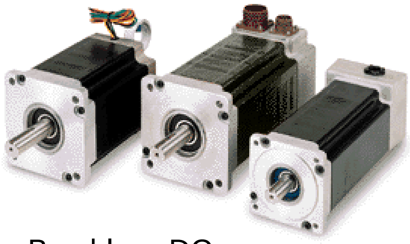
Learning Objective & Important Concepts
- Types and construction of Synchronous Motors
- Principle of torque production
- 2-phase Synchronous Motor model
- Model in *qd*-Rotor Reference Frame (RRF)
- Steady-state analysis equivalent circuit and torque-angle characteristics

Synchronous Motors (Machines)

- Portable Diesel Generators 0.5 . . . 10 kW

- Car Alternators

Synchronous



Brushless DC and Servo Motors

Rotating Magnetic Field

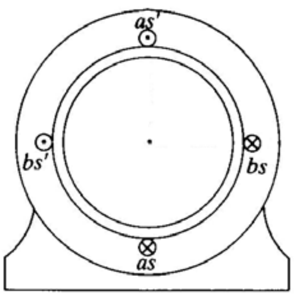
1. Given a set of ac currents shifted in time
2. Apply these currents to shifted in space stator windings

Produce MMF vector $\mathbf{F}_s$ that

$$\theta_e = \omega_e t$$

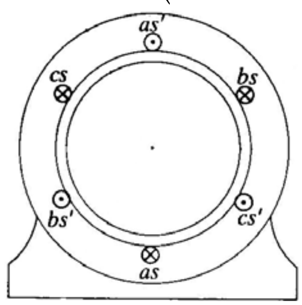
- Has constant magnitude
- Rotates in space

$$i_{as} = I_m \cos(\omega_e t)$$
$$i_{bs} = I_m \cos(\omega_e t - 90^\circ)$$



$$\mathbf{F}_s = F_m \angle \theta_e$$

$$i_{as} = I_m \cos(\omega_e t)$$
$$i_{bs} = I_m \cos(\omega_e t - 120^\circ)$$
$$i_{cs} = I_m \cos(\omega_e t + 120^\circ)$$



$$\mathbf{F}_s = (3/2) F_m \angle \theta_e$$

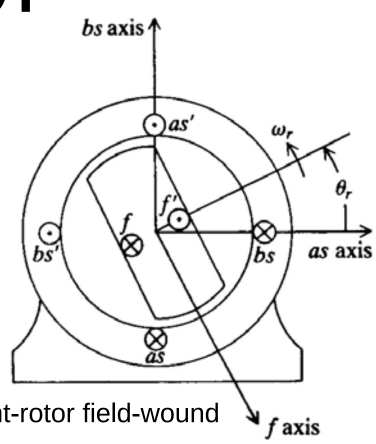
How many phases can you have ?

2, 3, 5, 6 - -

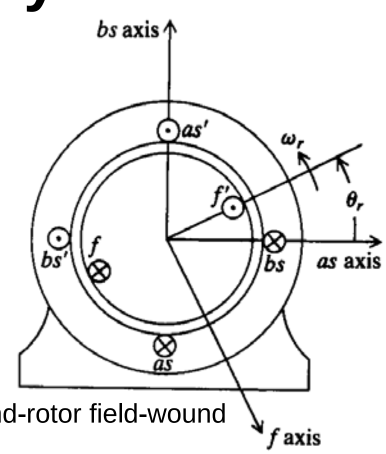
How do you change direction of rotation ?

change sequence

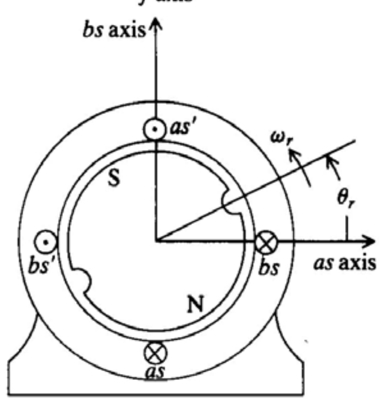
Type of Common Synchronous Motors



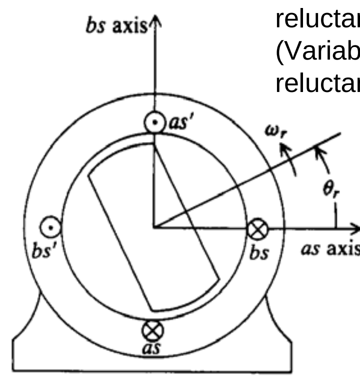
Salient-rotor field-wound



Round-rotor field-wound



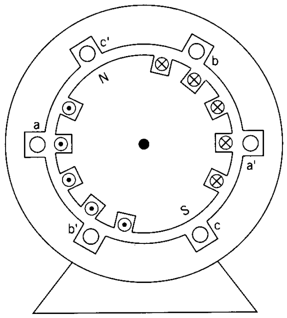
Round-rotor PM



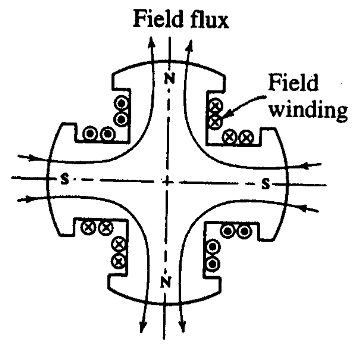
Salient-rotor reluctance (Variable reluctance)

Rotor Types

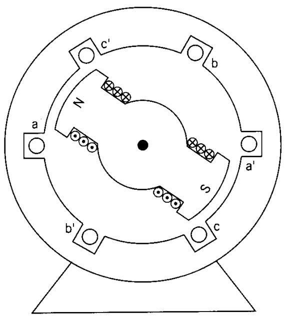
2-Pole Round Rotor

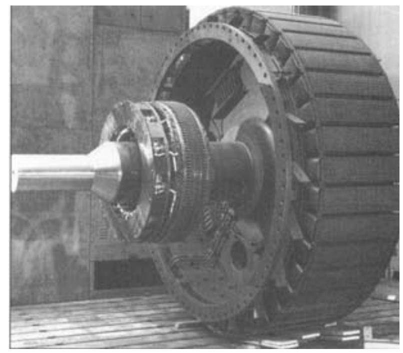


4-Pole Salient Rotor



2-Pole Salient Rotor



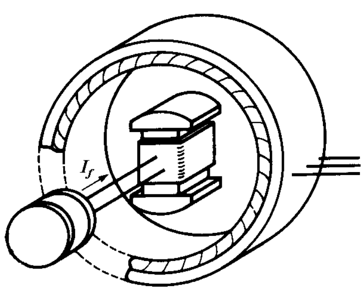


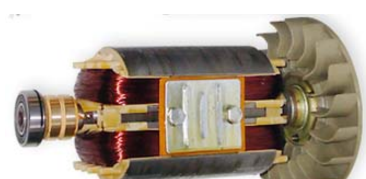
Number of Rotor Poles = Number of Stator Poles = *P*

Excitation – Establishing Rotor Field

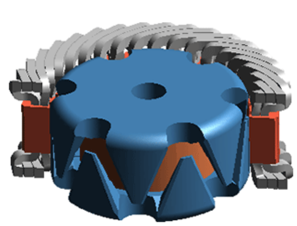
In order to establish the rotor field we need to pass DC current through the field winding

Use **Slip-Rings & Brushes** to energize the field winding from external DC source





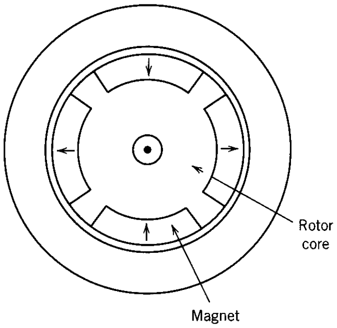
Claw-Pole Rotor - typically used in automotive alternators



Permanent Magnets (PM) Rotor Types

Use Permanent Magnets (PM) to establish the rotor field

Surface-Mounted PM Rotors

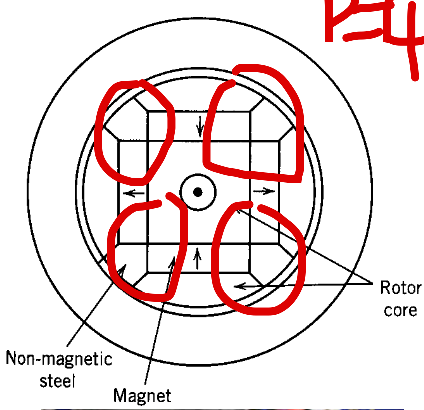


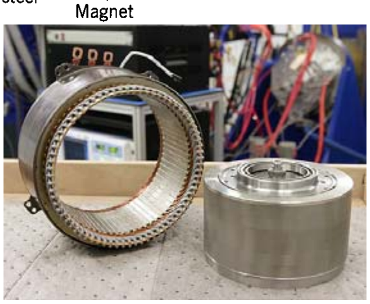
$P=2$ $P=4$ $P=8$

Permanent Magnets (PM) Rotor Types

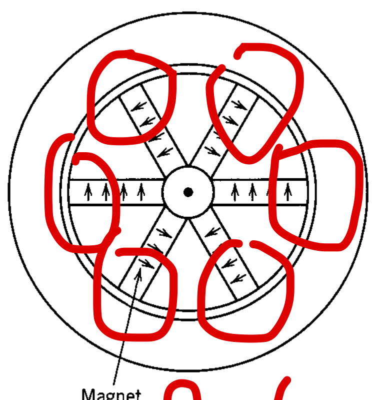
Use Permanent Magnets (PM) to establish the rotor field

Radial Buried Magnets $P=4$





Circumferential Buried Magnets



$P=6$

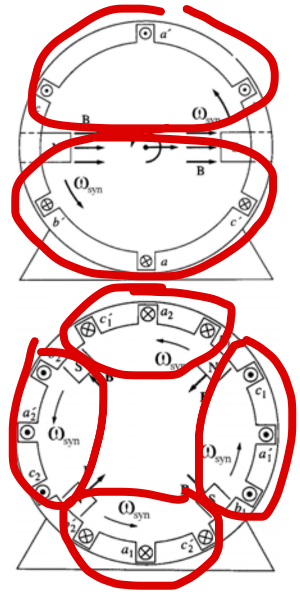
P-Pole Rotating Magnetic Field

$\theta_e = \omega_e t$ Electrical displacement

$\theta_e = 0 \rightarrow 2\pi$ One cycle of currents = 2/P revolution of magnetic poles

$\omega_{syn} = \frac{2}{P}\omega_e$ Synchronous speed is the speed of rotation of magnetic poles. This speed is 2/P of the electrical speed

$$n_{syn} = \frac{30}{\pi}\omega_{syn} = \frac{120}{P}\cdot f_e \quad [\text{rpm}]$$



What are the rotor mechanical speed & position ?

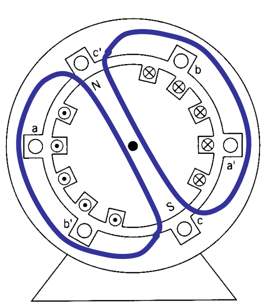
$\omega_{rm} = \frac{2}{P}\omega_e = \frac{2}{P}\omega_r$

$\theta_{rm} = \frac{2}{P}\theta_r$

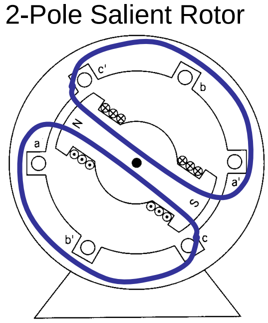
$\left.\begin{matrix}\omega_r \\ \theta_r\end{matrix}\right\}$ are referred to an equivalent 2-pole machine

Poles & Speed in Syn. Machines

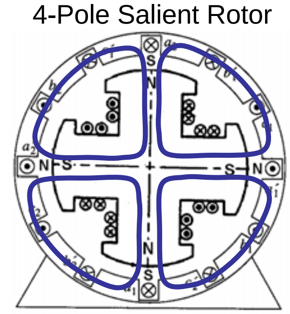
2-Pole Round Rotor



2-Pole Salient Rotor



4-Pole Salient Rotor



For North America
f_e = 60 Hz
$\omega_e = 2\pi \cdot f_e$ [rad/s]

For balanced three phase system

$$i_{as}(t) = \sqrt{2}\cdot I_{rms}\cos(\omega_e t)$$
$$i_{bs}(t) = \sqrt{2}\cdot I_{rms}\cos(\omega_e t - 120^\circ)$$
$$i_{cs}(t) = \sqrt{2}\cdot I_{rms}\cos(\omega_e t + 120^\circ)$$

Synchronous Speed in rev. per min. $n_{syn} = \frac{30}{\pi}\omega_{syn} = 60\frac{2}{P}\cdot f_e \quad [\text{rpm}]$

Electromagnetic torque $T_e \approx \frac{P}{2}\frac{\partial}{\partial\theta_r}\left(\frac{1}{2}\mathbf{i}^T\mathbf{L}(\theta_r)\mathbf{i}\right)$

$\frac{P}{2}$ - is similar to gearbox ratio

P pole	n_{syn}	ω_{syn}
2	3600	120π
4	1800	60π
6	1200	40π
8	900	30π
~~15~~	??	??
64	112.5	3.75π

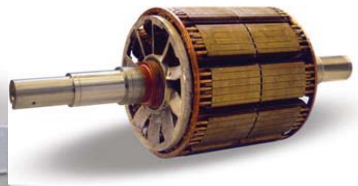
How many magnetic poles can we have?
2, 4, ~~15~~, 64…

Poles & Speed in Syn. Machines

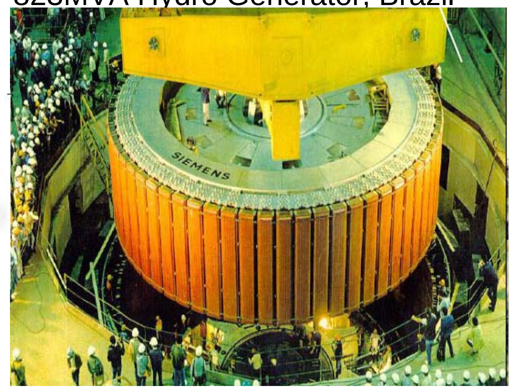
Speed $n_{syn} \propto \frac{2}{P}$ Torque $T_e \propto \frac{P}{2}$

7500 HP, 16kV Syn. Motor

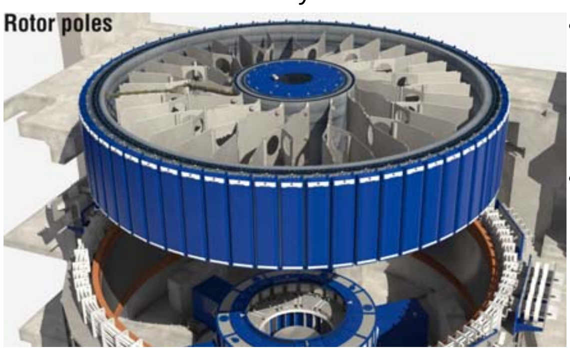
8-Pole Salient Rotor



823MVA Hydro Generator, Brazil



64-Pole Salient Rotor Hydro Generator



Generation & Utilization of Electricity

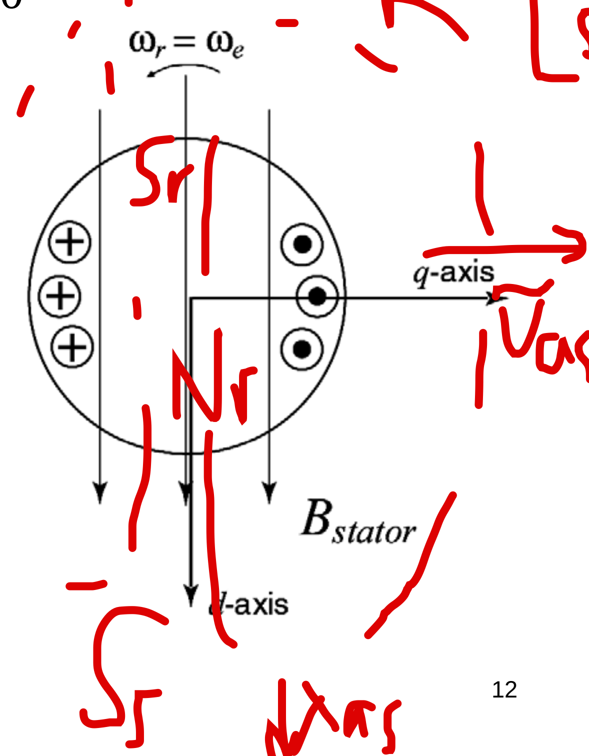
- Use low number of poles (e.g. 2, 4, 8, etc) for high-speed (e.g. 3600 – 900 rpm) low torque applications
  - Steam & thermal turbine generators, high speed motors, etc.
- Use high number of poles (e.g. 64..) for low-speed (e.g. 112.5 rpm) high torque applications
  - Hydro generators, direct-drive wind generators, low speed motors, etc.

Principle of Operation – Idle Mode

a) Assume no external torque $T_m = 0$

- Rotor poles are synchronized with the stator field
- Rotor poles are aligned with stator poles
- No torque produced $T_e = 0$

NS $\omega_r = \omega_e$ [SS]



$V_{as} = \frac{d\lambda}{dt} = \sqrt{2} V_s \cos(\omega_e t)$

$\lambda_{as} = \frac{\sqrt{2} V_s}{\omega_e} \sin(\omega_e t)$

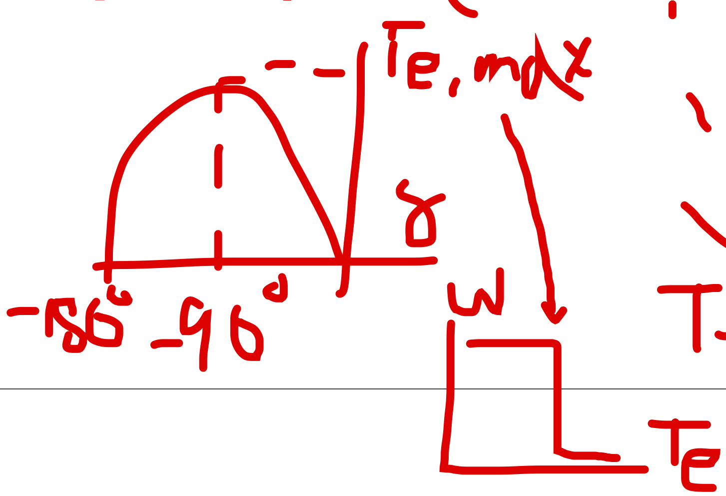
lag V_{as}

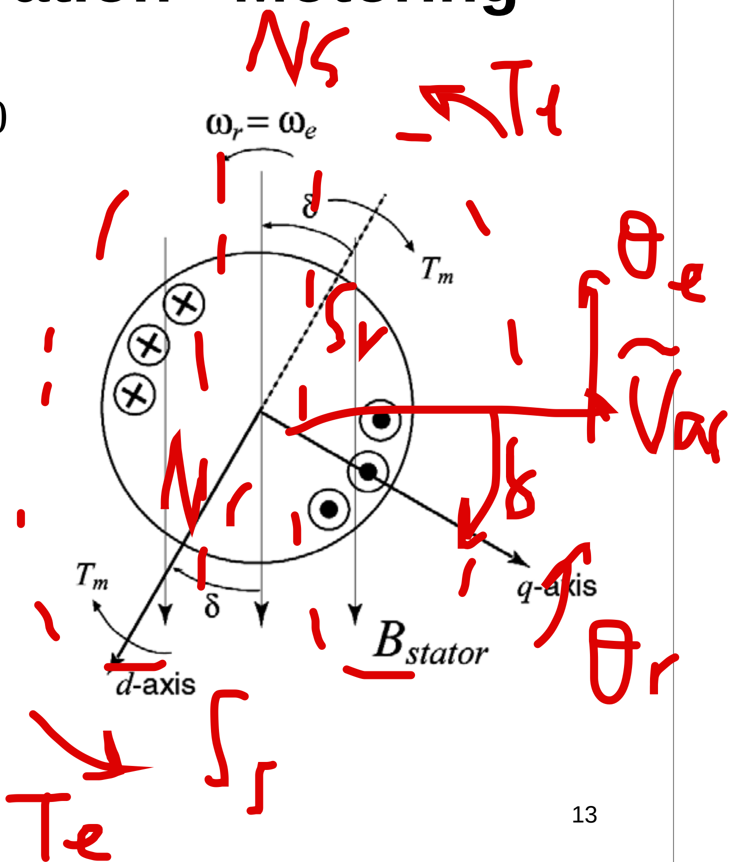
Principle of Operation - Motoring

c) Apply external torque $T_m < 0$
in the direction opposite to rotation

- Rotor poles are synchronized with the stator field
- Rotor poles **lag** stator poles
- Torque is produced T_e

$\delta = \theta_r - \theta_e$



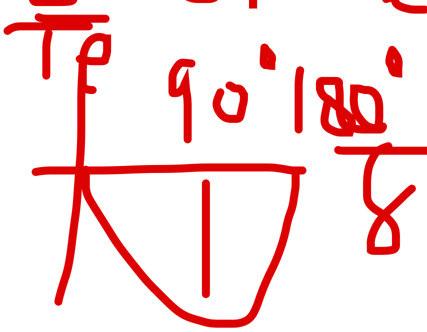


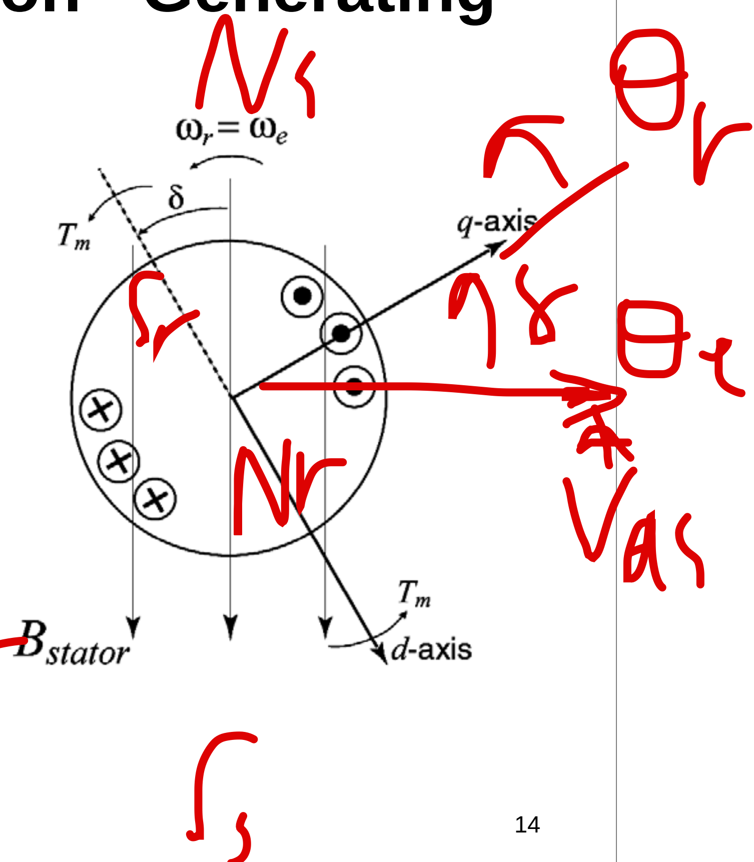
Principle of Operation - Generating

b) Apply external torque $T_m > 0$
in the direction of rotation

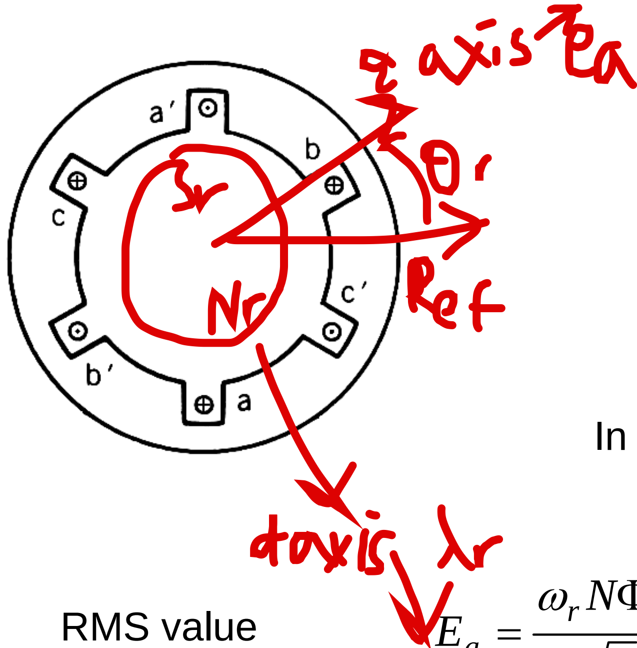
- Rotor poles are synchronized with the stator field
- Rotor poles **lead** stator poles
- Torque is produced T_e

$\delta = \theta_r - \theta_e > 0$





Induced Voltage (Open-Circuit Voltage)



$$e_a = \frac{d\lambda_a}{dt} = N\frac{d\Phi_a}{dt}$$

$$\lambda_a = \lambda_f \sin(\theta_r)$$

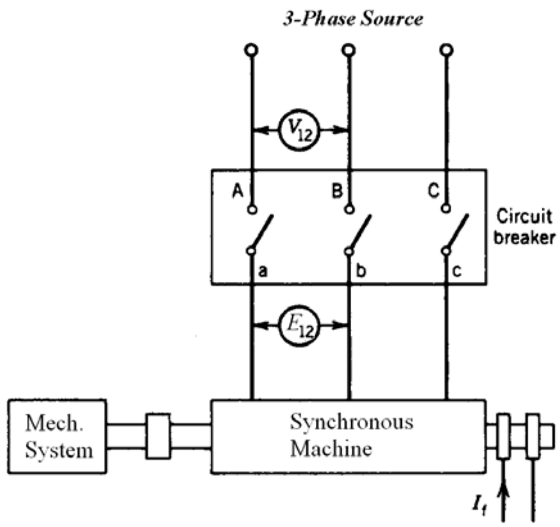
Let $\theta_r = \omega_r t$

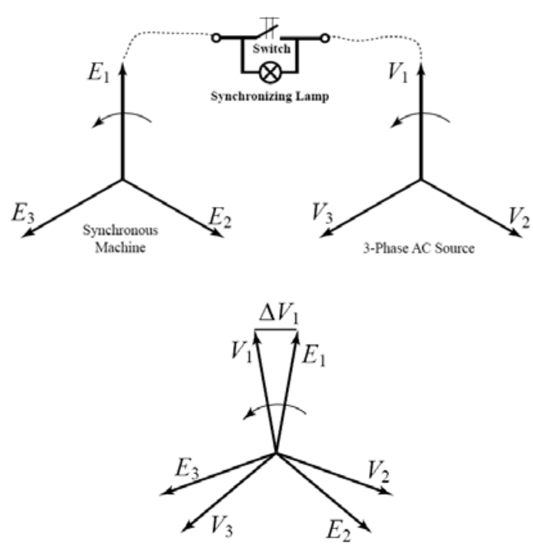
$$e_a = \frac{d\lambda_a}{dt} = \omega_r \lambda_f \cos(\theta_r)$$

In Steady-State $\omega_r = \omega_e = 2\pi f_e$

RMS value $E_a = \frac{\omega_r N \Phi_f}{\sqrt{2}} = \frac{2\pi}{\sqrt{2}} f_e N \Phi_f = 4.44 \cdot f_e \lambda_f$

Connecting Synchronous Motor/Generator to Fixed Frequency Source (Grid)

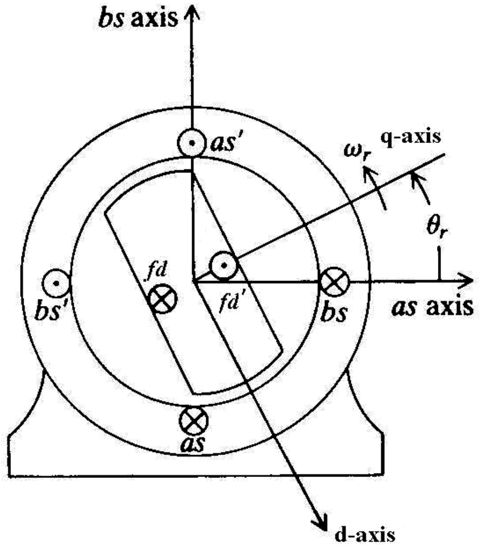


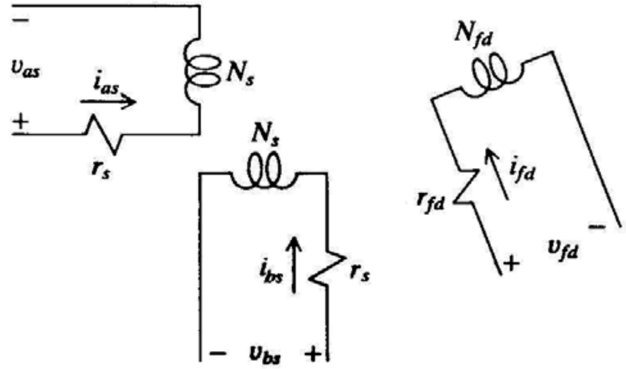


$E_a \sim X_{md} I_f \sim K_v \omega_r$

1. Make sure that phase sequence is the same (ABC & abc)
2. Bring the Syn. Machine speed corresponding to the desired frequency (60Hz) => $f_{sm} \sim f_s$
3. Adjust the magnitudes (either by field current or by source voltage) such that => $E_{sm} \sim V_s$
4. Adjust/make sure the phase difference is zero
5. Close the connecting switch

2-Phase Synchronous Motor Model





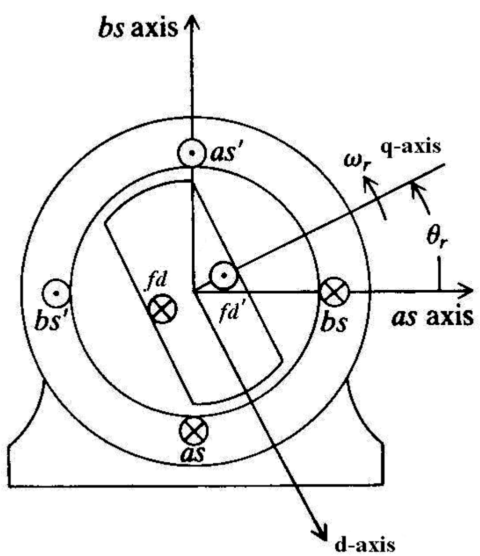
Voltage equations

AC
$$v_{as} = r_s i_{as} + \frac{d\lambda_{as}}{dt}$$

$$v_{bs} = r_s i_{bs} + \frac{d\lambda_{bs}}{dt}$$

DC
$$v_{fd} = r_{fd} i_{fd} + \frac{d\lambda_{fd}}{dt}$$

Flux Linkages & Inductances



Flux linkage equations

$$\lambda_{as} = L_{asas} i_{as} + L_{asbs} i_{bs} + L_{asfd} i_{fd}$$

$$\lambda_{bs} = L_{bsas} i_{as} + L_{bsbs} i_{bs} + L_{bsfd} i_{fd}$$

$$\lambda_{fd} = L_{fdas} i_{as} + L_{fdbs} i_{bs} + L_{fdfd} i_{fd}$$

Stator self inductances

$$L_{asas} = L_{ls} + L_{ams}(\theta_r)$$

$$L_{bsbs} = L_{ls} + L_{bms}(\theta_r)$$

$$L_{ams}(0) = \frac{N_s^2}{\Re_{mq}} = L_{mq}$$ q-axis magnetizing inductance

$$L_{ams}(90^\circ) = \frac{N_s^2}{\Re_{md}} = L_{md}$$ d-axis magnetizing inductance

q- and d-axis inductances

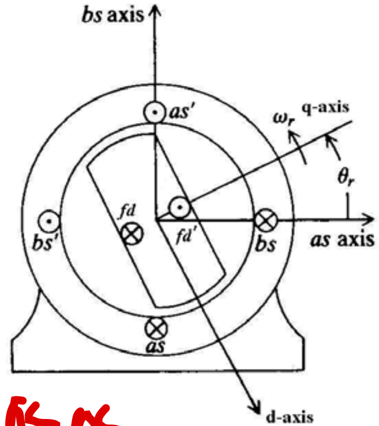
$$L_q = L_{ls} + L_{mq}$$

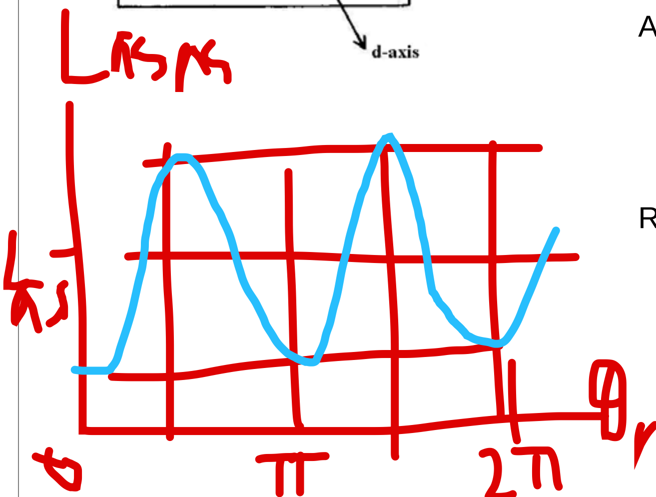
$$L_d = L_{ls} + L_{md}$$

$$L_{mq} < L_{md}$$

smaller airgap

Flux Linkages & Inductances





Stator self inductances

$$L_A = \frac{1}{2}\left(L_{mq} + L_{md}\right) = \frac{N_s^2}{2}\left(\frac{1}{\Re_{mq}} + \frac{1}{\Re_{md}}\right)$$

$$L_B = \frac{1}{2}\left(L_{md} - L_{mq}\right) = \frac{N_s^2}{2}\left(\frac{1}{\Re_{md}} - \frac{1}{\Re_{mq}}\right)$$

$$L_{mq} = L_A - L_B \qquad L_{md} = L_A + L_B$$

Assume sinusoidal variations

$$L_{asas}(\theta_r) = L_{ls} + L_A - L_B\cos(2\theta_r)$$
$$L_{bsbs}(\theta_r) = L_{ls} + L_A + L_B\cos(2\theta_r)$$

Rotor self inductance $L_{fdfd} = L_{lfd} + L_{mfd}$

field winding magnetizing inductance $L_{mfd} = \dfrac{N_{fd}^2}{\Re_{md}}$

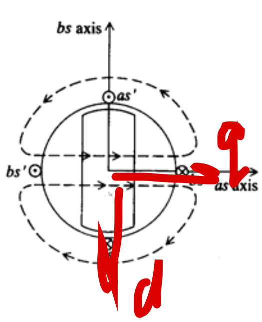
Flux Linkages & Inductances

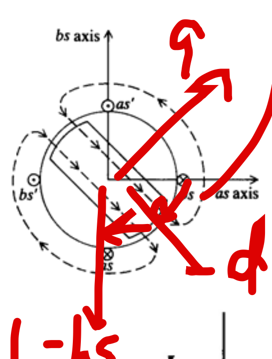
Stator mutual inductances ?

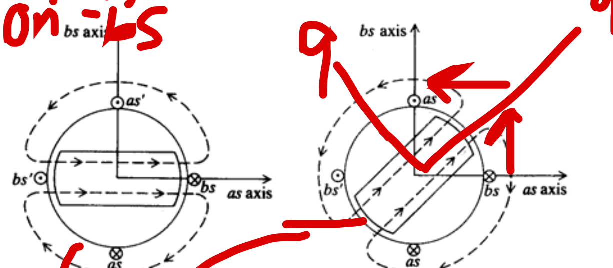
$$\lambda_{as} = L_{asas} i_{as} + L_{asbs} i_{bs} + L_{asfd} i_{fd}$$
$$\lambda_{bs} = L_{bsas} i_{as} + L_{bsbs} i_{bs} + L_{bsfd} i_{fd}$$

We should have $L_{asbs} = L_{bsas}$ as proj on d

d proj on -bs

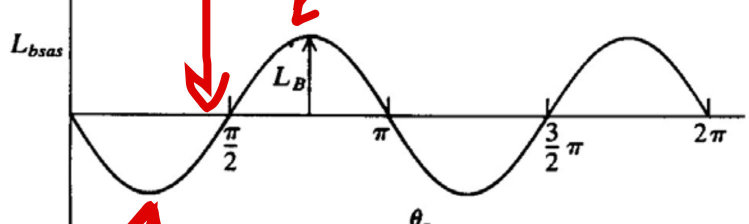






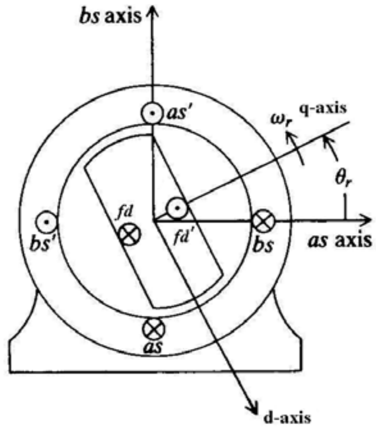
Final expression

$$L_{asbs} = L_{bsas} = -L_B\sin(2\theta_r)$$



Flux Linkages & Inductances

Summarize Inductances



Stator

$$L_{asas} = L_{ls} + L_A - L_B \cos(2\theta_r)$$
$$L_{bsbs} = L_{ls} + L_A + L_B \cos(2\theta_r)$$
self

$$L_{asbs} = L_{bsas} = -L_B \sin(2\theta_r)$$
MU

Rotor

$$L_{fdfd} = L_{lfd} + L_{mfd}$$
– Self

$$L_{asfd} = L_{sfd} \sin(\theta_r)$$
$$L_{bsfd} = -L_{sfd} \cos(\theta_r)$$
MU

Flux linkage equations

Field-to-stator mutual inductance

$$L_{sfd} = \frac{N_s N_{fd}}{\Re_{md}}$$

$$\begin{bmatrix} \lambda_{as} \\ \lambda_{bs} \\ \lambda_{fd} \end{bmatrix} = \begin{bmatrix} L_{asas} & L_{asbs} & L_{asfd} \\ L_{bsas} & L_{bsbs} & L_{bsfd} \\ L_{fdas} & L_{fdbs} & L_{fdfd} \end{bmatrix} \cdot \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{fd} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\lambda}_{abs} \\ \lambda_{fd} \end{bmatrix} = \begin{bmatrix} \mathbf{L}_s & \mathbf{L}_{sr} \\ \mathbf{L}_{sr}^T & L_{fdfd} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{i}_{abs} \\ i_{fd} \end{bmatrix}$$

Flux Linkages & Inductances

Recall

$$L_{md} = \frac{N_s^2}{\Re_{md}}$$

$$L_{mfd} = \frac{N_{fd}^2}{\Re_{md}} = \left(\frac{N_{fd}}{N_s}\right)^2 L_{md}$$

$$L_{sfd} = \frac{N_s N_{fd}}{\Re_{md}} = \frac{N_{fd}}{N_s} L_{md}$$

Use $\left(\frac{N_s}{N_{fd}}\right)^2$ and $\frac{N_s}{N_{fd}}$ to refer rotor variables to the stator side

Result

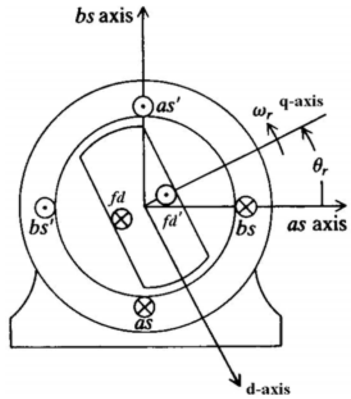
$$L'_{mfd} = L_{mfd}\left(\frac{N_s}{N_{fd}}\right)^2 = L_{md} \qquad L'_{sfd} = L_{sfd}\frac{N_s}{N_{fd}} = L_{md} \qquad L'_{lfd} = L_{lfd}\left(\frac{N_s}{N_{fd}}\right)^2$$

$$L'_{fd} = L'_{lfd} + L_{md}$$

$$\begin{bmatrix} L'_{asfd} \\ L'_{bsfd} \end{bmatrix} = \begin{bmatrix} L_{sfd}\sin(\theta_r) \\ -L_{sfd}\cos(\theta_r) \end{bmatrix} \frac{N_s}{N_{fd}} = \begin{bmatrix} L_{md}\sin(\theta_r) \\ -L_{md}\cos(\theta_r) \end{bmatrix} = \mathbf{L}'_{sr}$$

$$r'_{fd} = r_{fd}\left(\frac{N_s}{N_{fd}}\right)^2 \qquad i'_{fd} = i_{fd}\frac{N_{fd}}{N_s} \qquad v'_{fd} = v_{fd}\frac{N_s}{N_{fd}} \qquad \lambda'_{fd} = \lambda_{fd}\frac{N_s}{N_{fd}}$$

Machine Model Equations



Stator inductance matrix

$$\mathbf{L}_s = \begin{bmatrix} L_{ls} + L_A - L_B\cos(2\theta_r) & -L_B\cos(2\theta_r) \\ -L_B\cos(2\theta_r) & L_{ls} + L_A + L_B\cos(2\theta_r) \end{bmatrix}$$

Rotor-to-stator inductance matrix

$$\mathbf{L}'_{sr} = \begin{bmatrix} L_{md}\sin(\theta_r) \\ -L_{md}\cos(\theta_r) \end{bmatrix}$$

Flux linkage equation in matrix form

$$\begin{bmatrix} \boldsymbol{\lambda}_{abs} \\ \lambda'_{fd} \end{bmatrix} = \begin{bmatrix} \mathbf{L}_s & \mathbf{L}'_{sr} \\ \mathbf{L}'^{T}_{sr} & L'_{fd} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{i}_{abs} \\ i'_{fd} \end{bmatrix}$$

Voltage equation in matrix form

$$\mathbf{v}_{abs} = \mathbf{r}_s \mathbf{i}_{abs} + \frac{d\boldsymbol{\lambda}_{abs}}{dt}$$

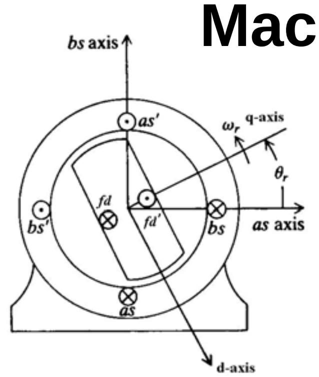
$$v'_{fd} = r'_{fd} i'_{fd} + \frac{d\lambda'_{fd}}{dt}$$

Rotor self inductance

$$L'_{fd} = L'_{lfd} + L_{md}$$

AC JS

Machine Model Equations



Electromagnetic Torque

$$T_e = \frac{P}{2} \cdot \frac{\partial W_c(\mathbf{i}, \theta_r)}{\partial \theta_r}$$

Assume magnetically linear system

$$W_c = W_f$$

$$W_f = \frac{1}{2}\mathbf{i}^T_{abs}(\mathbf{L}_s - L_{ls}\mathbf{I})\mathbf{i}_{abs} + \mathbf{i}^T_{abs}\mathbf{L}'_{sr} i'_{fd} + \frac{1}{2} i'^2_{fd} L_{md}$$

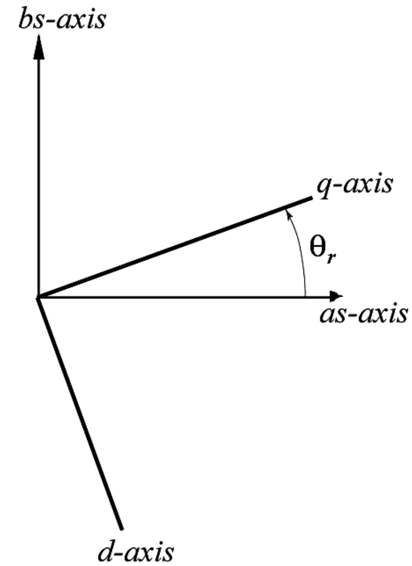
General equation for torque in Syn. Machine

$$T_e = \frac{P}{2}\left\{ \frac{L_{md} - L_{mq}}{2}\left[\left(i^2_{as} - i^2_{bs}\right)\sin(2\theta_r) - 2 i_{as} i_{bs}\cos(2\theta_r) \right] + L_{md} i'_{fd}\left[i_{as}\cos(\theta_r) + i_{bs}\sin(\theta_r) \right] \right\}$$

For Round-Rotor PM Syn. Machine

Rotor Frame Reference (RRF)

qd-frame is fixed on the rotor



Phases have variables $f = i, v, \lambda, \dots$

Define a vector $\mathbf{f}_{abs} = \begin{bmatrix} f_{as} & f_{bs} \end{bmatrix}^T$

Define change of variables to *qd*-coordinate

$$\mathbf{f}_{qds} = \begin{bmatrix} f_{qs} & f_{ds} \end{bmatrix}^T$$

$$\begin{bmatrix} f_{qs} \\ f_{ds} \end{bmatrix} = \begin{bmatrix} \cos(\theta_r) & \sin(\theta_r) \\ \sin(\theta_r) & -\cos(\theta_r) \end{bmatrix} \cdot \begin{bmatrix} f_{as} \\ f_{bs} \end{bmatrix}$$

$$\mathbf{f}_{qds} = \mathbf{K}_s^r \mathbf{f}_{abs}$$

$$\mathbf{f}_{abs} = \left(\mathbf{K}_s^r\right)^{-1} \mathbf{f}_{qds}$$

$$\left(\mathbf{K}_s^r\right)^{-1} = \mathbf{K}_s^r$$

Rotor position $\theta_r = \theta_r(0) + \int \omega_r dt$

Express all equations in transformed variables !

Voltage Equations in RRF

Original voltage equation

$$\mathbf{v}_{abs} = \mathbf{r}_s \mathbf{i}_{abs} + \frac{d\boldsymbol{\lambda}_{abs}}{dt}$$

$$v'_{fd} = r'_{fd} i'_{fd} + \frac{d\lambda'_{fd}}{dt}$$

Substitute the *qd*-variables

$$\left(\mathbf{K}_s^r\right)^{-1} \mathbf{v}_{qds} = \mathbf{r}_s \left(\mathbf{K}_s^r\right)^{-1} \mathbf{i}_{qds} + \frac{d}{dt}\left[\left(\mathbf{K}_s^r\right)^{-1} \boldsymbol{\lambda}_{qds}\right]$$

$$\mathbf{v}_{qds} = \mathbf{K}_s^r \mathbf{r}_s \left(\mathbf{K}_s^r\right)^{-1} \mathbf{i}_{qds} + \mathbf{K}_s^r \frac{d}{dt}\left[\left(\mathbf{K}_s^r\right)^{-1} \boldsymbol{\lambda}_{qds}\right]$$

$$\mathbf{K}_s^r \frac{d}{dt}\left[\left(\mathbf{K}_s^r\right)^{-1} \boldsymbol{\lambda}_{qds}\right] = \mathbf{K}_s^r \left[\frac{d}{dt}\left(\mathbf{K}_s^r\right)^{-1}\right] \boldsymbol{\lambda}_{qds} + \mathbf{K}_s^r \left(\mathbf{K}_s^r\right)^{-1} \frac{d\boldsymbol{\lambda}_{qds}}{dt}$$

$$\begin{bmatrix} \cos(\theta_r) & \sin(\theta_r) \\ \sin(\theta_r) & -\cos(\theta_r) \end{bmatrix} \cdot \omega_r \begin{bmatrix} -\sin(\theta_r) & \cos(\theta_r) \\ \cos(\theta_r) & \sin(\theta_r) \end{bmatrix} = \omega_r \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$\omega_r \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \boldsymbol{\lambda}_{qds} = \omega_r \begin{bmatrix} \boldsymbol{\lambda}_{ds} \\ -\boldsymbol{\lambda}_{qs} \end{bmatrix} = \omega_r \boldsymbol{\lambda}_{dqs}$$

Resulted voltage equation

$$\mathbf{v}_{qds} = \mathbf{r}_s \mathbf{i}_{qds} + \omega_r \boldsymbol{\lambda}_{dqs} + \frac{d\boldsymbol{\lambda}_{qds}}{dt}$$

$$v'_{fd} = r'_{fd} i'_{fd} + \frac{d\lambda'_{fd}}{dt}$$

DC SS

Flux Linkage Equations in RRF

Original equation

$$\begin{bmatrix} \boldsymbol{\lambda}_{abs} \\ \lambda'_{fd} \end{bmatrix} = \begin{bmatrix} \mathbf{L}_s & \mathbf{L}'_{sr} \\ \mathbf{L}'^{T}_{sr} & L'_{fd} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{i}_{abs} \\ i'_{fd} \end{bmatrix}$$

Substitute the *qd*-variables

$$\left(\mathbf{K}_s^r\right)^{-1} \boldsymbol{\lambda}_{qds} = \mathbf{L}_s \left(\mathbf{K}_s^r\right)^{-1} \mathbf{i}_{qds} + \mathbf{L}'_{sr} i_{fd}$$

$$\boldsymbol{\lambda}_{qds} = \mathbf{K}_s^r \mathbf{L}_s \left(\mathbf{K}_s^r\right)^{-1} \mathbf{i}_{qds} + \mathbf{K}_s^r \mathbf{L}'_{sr} i_{fd}$$

$$\lambda'_{fd} = \mathbf{L}'^{T}_{sr} \left(\mathbf{K}_s^r\right)^{-1} \mathbf{i}_{qds} + L'_{fd} i_{fd}$$

One of the major advantages of *qd*-reference frame that we get =>

$$\mathbf{K}_s^r \mathbf{L}_s \left(\mathbf{K}_s^r\right)^{-1} = \begin{bmatrix} L_{ls} + L_{mq} & 0 \\ 0 & L_{ls} + L_{md} \end{bmatrix}$$

$$\mathbf{L}'^{T}_{sr} \left(\mathbf{K}_s^r\right)^{-1} = \begin{bmatrix} 0 & L_{md} \end{bmatrix}$$

$$\mathbf{K}_s^r \mathbf{L}'_{sr} = \begin{bmatrix} 0 \\ L_{md} \end{bmatrix}$$

Resulted flux linkage equation

$$\begin{bmatrix} \lambda_{qs} \\ \lambda_{ds} \\ \lambda'_{fd} \end{bmatrix} = \begin{bmatrix} L_{ls} + L_{mq} & 0 & 0 \\ 0 & L_{ls} + L_{md} & L_{md} \\ 0 & L_{md} & L'_{lfd} + L_{md} \end{bmatrix} \cdot \begin{bmatrix} i_{qs} \\ i_{ds} \\ i'_{fd} \end{bmatrix}$$

Note: The inductance matrix is constant, and
q and *d* axes became magnetically decoupled !

Equivalent Circuit in RRF

Expanded Voltage Equations

$$v_{qs} = r_s i_{qs} + \omega_r \lambda_{ds} + \frac{d\lambda_{qs}}{dt}$$

$$v_{ds} = r_s i_{ds} - \omega_r \lambda_{qs} + \frac{d\lambda_{ds}}{dt}$$

$$v'_{fd} = r'_{fd} i'_{fd} + \frac{d\lambda'_{fd}}{dt}$$

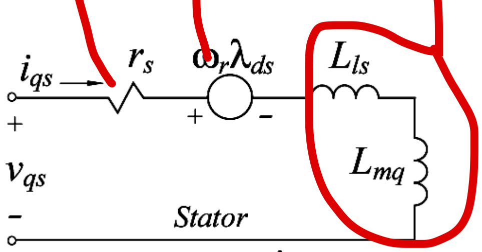
Expanded Flux Linkage Equations

$$\lambda_{qs} = L_{ls} i_{qs} + L_{mq} i_{qs} = L_q i_{qs}$$

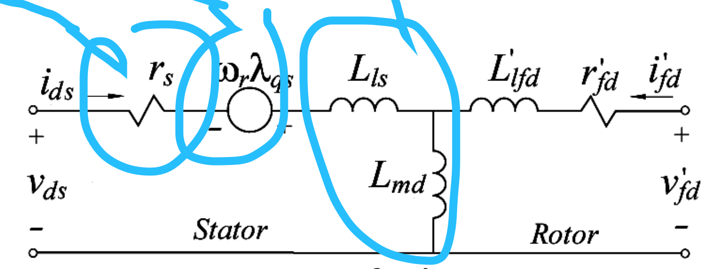
$$\lambda_{ds} = L_{ls} i_{ds} + L_{md} i_{ds} + L_{md} i'_{fd} = L_d i_{ds} + L_{md} i'_{fd}$$

$$\lambda'_{fd} = L'_{lfd} i'_{fd} + L_{md} i'_{fd} + L_{md} i_{dc} = L'_{fd} i'_{fd} + L_{md} i_{ds}$$

These equations can be realized using the following equivalent circuits



q-axis



d-axis

Note: The circuits are coupled only through the “speed-voltage” courses

This Equivalent Circuit is our Dynamic Model of the Synchronous Machine!

Electromagnetic Torque in RRF

Electromagnetic Torque $T_e = \frac{P}{2} \cdot \frac{\partial W_c(\mathbf{i}, \theta_r)}{\partial \theta_r}$

Assume magnetically linear system $W_c = W_f$

$$W_f = \frac{1}{2}\mathbf{i}_{abs}^T(\mathbf{L}_s - L_{ls}\mathbf{I})\mathbf{i}_{abs} + \mathbf{i}_{abs}^T \mathbf{L}'_{sr} i'_{fd} + \frac{1}{2} i'^2_{fd} L_{md}$$

$$T_e = \frac{P}{2} \cdot \left\{ \frac{1}{2}\mathbf{i}_{abs}^T \frac{\partial \mathbf{L}_s}{\partial \theta_r}\mathbf{i}_{abs} + \mathbf{i}_{abs}^T \frac{\partial \mathbf{L}'_{sr}}{\partial \theta_r} i'_{fd} \right\}$$

$$= \frac{P}{2}\mathbf{i}_{qds}^T \left(K_s^r\right)^{-1} \left\{ \frac{1}{2}\frac{\partial \mathbf{L}_s}{\partial \theta_r}\left(K_s^r\right)^{-1}\mathbf{i}_{qds} + \frac{\partial \mathbf{L}'_{sr}}{\partial \theta_r} i'_{fd} \right\}$$

Lmd i'fd = λm (PM)

Torque in transformed variables

$$T_e = \frac{P}{2}\left(\lambda_{ds} i_{qs} - \lambda_{qs} i_{ds}\right) = \frac{P}{2}\left[L_{md}\left(i_{ds} + i'_{fd}\right)i_{qs} - L_{mq} i_{qs} i_{ds}\right] =$$

$$= \frac{P}{2}\left[\underbrace{L_{md} i'_{fd} i_{qs}}_{\text{rotor field torque}} + \underbrace{\left(L_{md} - L_{mq}\right)}_{\text{saliency}} i_{qs} i_{ds}\right]$$

reluc torque

Steady-State Operation

Derive SS equations in order to obtain SS equivalent circuit!

In steady-state $\omega_r = \omega_e$

Xls + Xmd = Xd

Xls + Xmq = Xq

Expanded voltage equations

Balance of Torques $T_e = T_{fric} + T_m$

$$v_{qs} = r_s i_{qs} + \omega_r \lambda_{ds} + \cancel{\frac{d\lambda_{qs}}{dt}}$$

$$v_{ds} = r_s i_{ds} - \omega_r \lambda_{qs} + \cancel{\frac{d\lambda_{ds}}{dt}}$$

$$v'_{fd} = r'_{fd} i'_{fd} + \cancel{\frac{d\lambda'_{fd}}{dt}}$$

$$V_{qs} = r_s I_{qs} + \omega_e \lambda_{ds}$$

$$= r_s I_{qs} + X_d I_{ds} + X_{md} I'_{fd}$$

$$V_{ds} = r_s I_{ds} - \omega_e \lambda_{qs}$$

$$= r_s I_{ds} - X_q I_{qs}$$

$$V'_{fd} = r'_{fd} I'_{fd}$$

λqs = Lq iqs

λds = Ld ids + Lmd i'fd

Electromagnetic torque

$$T_e = \frac{P}{2}\left[L_{md}\left(i_{ds} + i'_{fd}\right)i_{qs} - L_{mq} i_{qs} i_{ds}\right] = \frac{P}{2}\left[L_{md} i'_{fd} i_{qs} + \left(L_{md} - L_{mq}\right)i_{qs} i_{ds}\right]$$

$$= \frac{P}{2} \cdot \frac{1}{\omega_e}\left[X_{md} i'_{fd} i_{qs} + \left(X_{md} - X_{mq}\right)i_{qs} i_{ds}\right]$$

Rotor Angle δ

Assume applied voltages

$$V_{as} = \sqrt{2}V_s \cos(\omega_e t)$$

$$V_{bs} = \sqrt{2}V_s \sin(\omega_e t)$$

$$\tilde{V}_{as} = V_s \angle 0^\circ$$

$$\tilde{V}_{bs} = V_s \angle -90^\circ$$

Rotor position

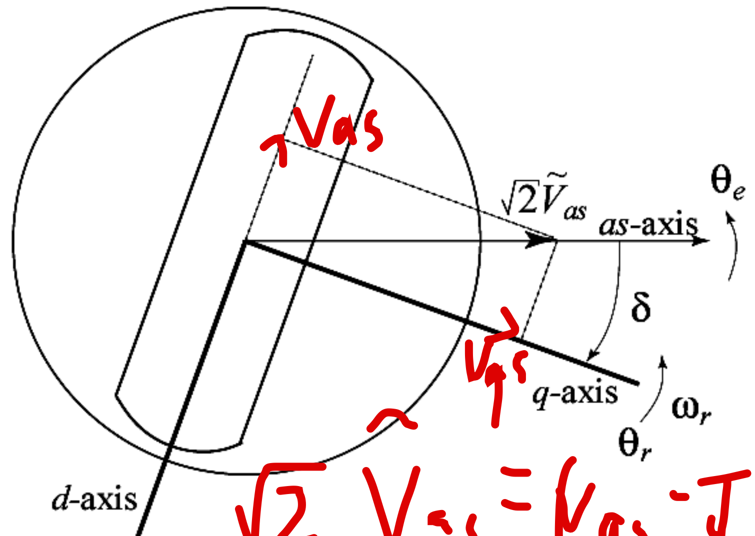
$$\theta_r = \theta_r(0) + \int \omega_r dt$$

Electrical displacement

$$\theta_e = \theta_e(0) + \int \omega_e dt$$

$$\delta = \theta_r - \theta_e$$

$$= \theta_r(0) - \theta_e(0) + \int (\omega_r - \omega_e) dt$$



$\sqrt{2}\,V_{as} = (V_{qs} - jV_{ds})\,e^{j\delta}$

How we can relate V_{qds} and V_{as}

Steady-State Operation

Combine $\sqrt{2}\tilde{V}_{as}e^{-j\delta} = V_{qs} - jV_{ds}$

$$V_{qs} = r_s I_{qs} + X_d I_{ds} + X_{md} I'_{fd}$$

$$V_{ds} = r_s I_{ds} - X_q I_{qs}$$

We get

$$\sqrt{2}\tilde{V}_{as}e^{-j\delta} = r_s I_{qs} + X_d I_{ds} + X_{md} I'_{fd} - jr_s I_{ds} + jX_q I_{qs} + X_q I_{ds} - X_q I_{ds}$$

$$= (r_s + jX_q)(I_{qs} - jI_{ds}) + (X_d - X_q) I_{ds} + X_{md} I'_{fd}$$

But $\sqrt{2}\tilde{I}_{as}e^{-j\delta} = I_{qs} - jI_{ds}$

Then $\tilde{V}_{as} = (r_s + jX_q)\tilde{I}_{as} + \frac{1}{\sqrt{2}}\left[(X_d - X_q)I_{ds} + X_{md}I'_{fd}\right]e^{j\delta}$

Excitation voltage $\tilde{E}_a = \frac{1}{\sqrt{2}}\left[(X_d - X_q)I_{ds} + X_{md}I'_{fd}\right]e^{j\delta}$

Final equation $\tilde{V}_{as} = (r_s + jX_q)\tilde{I}_{as} + \tilde{E}_a$

back EMF
as phasor

diff = δ

Steady-State Operation

Excitation voltage
Back EMF

$$\tilde{E}_a = \frac{1}{\sqrt{2}}\left[\left(X_d - X_q\right)I_{ds} + X_{md}I'_{fd}\right]e^{j\delta}$$

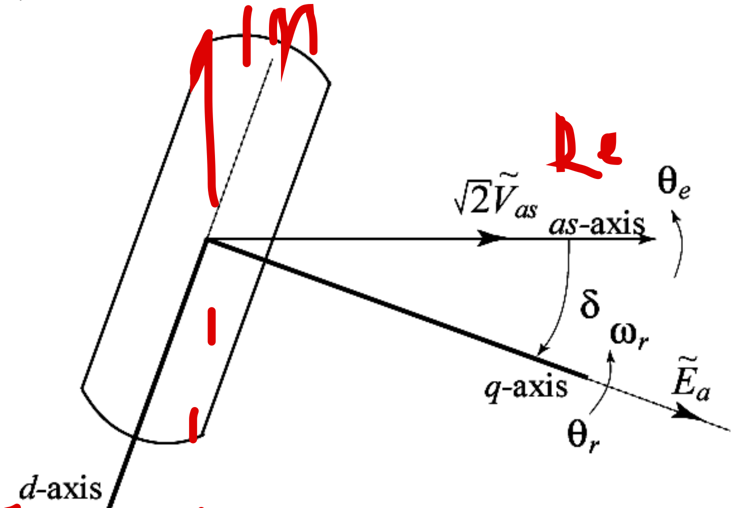
due to flux

$$= \frac{1}{\sqrt{2}}\left[\left(X_d - X_q\right)I_{ds} + E'_{xfd}\right]e^{j\delta}$$

due to saliency or reluctance

$$E'_{xfd} = X_{md}I'_{fd}$$

This would be the open-circuit voltage !

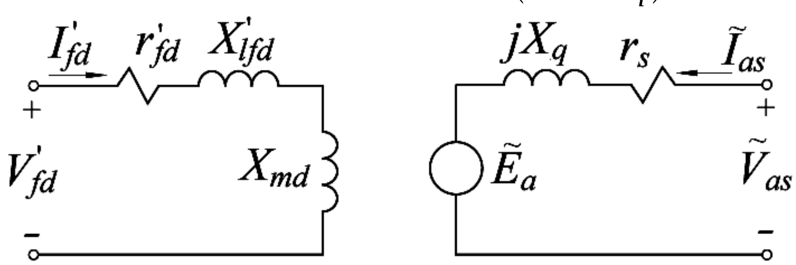


rotor flux

δ

Per-Phase Steady-State Equivalent Circuit

Stator voltage equation $\tilde{V}_{as} = \left(r_s + jX_q\right)\tilde{I}_{as} + \tilde{E}_a$



Rotor field winding *Per-phase stator circuit*

Looks like Separately Excited DC Machine, except with Phasors!

Back EMF

$$\tilde{E}_a = \frac{1}{\sqrt{2}}\left[\left(X_d - X_q\right)I_{ds} + E'_{xfd}\right]e^{j\delta} \qquad E'_{xfd} = X_{md}I'_{fd}$$

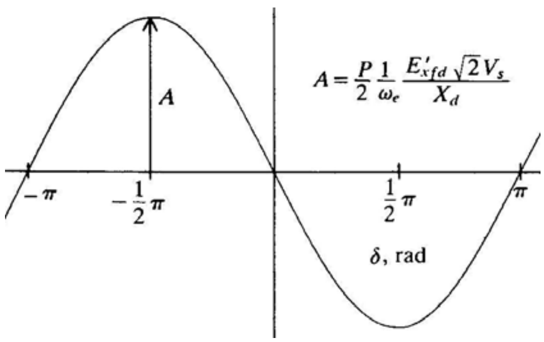
Round rotor: $\tilde{E}_a = \frac{1}{\sqrt{2}}\omega_r L_{md} I'_{fd} e^{j\delta}$ $= L\omega I$ $= \lambda\omega$

PM machine: $= \frac{1}{\sqrt{2}}\omega_r \lambda_m e^{j\delta}$

Torque-Angle Characteristic

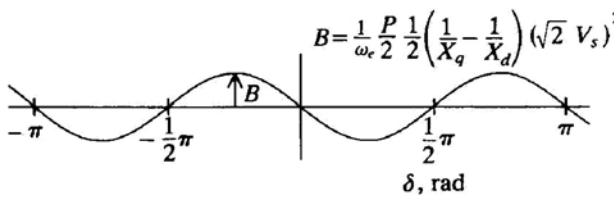
Electromagnetic torque $$T_e = \frac{P}{2} \cdot \frac{1}{\omega_e} \left[X_{md} I'_{fd} I_{qs} + \left(X_{md} - X_{mq} \right) I_{qs} I_{ds} \right]$$

Substitute for currents & neglect r_s, we can get $$T_e = -\frac{P}{2} \frac{1}{\omega_e} \left[\underbrace{\frac{E'_{xfd} \sqrt{2} V_s}{X_d} \sin(\delta)}_{\text{rotor flux}} + \underbrace{\frac{1}{2} \left(\frac{1}{X_q} - \frac{1}{X_d} \right) \left(\sqrt{2} V_s \right)^2 \sin(2\delta)}_{\text{reluctance}} \right]$$



Torque due to the rotor field

(This would be the torque of the round-rotor machine)



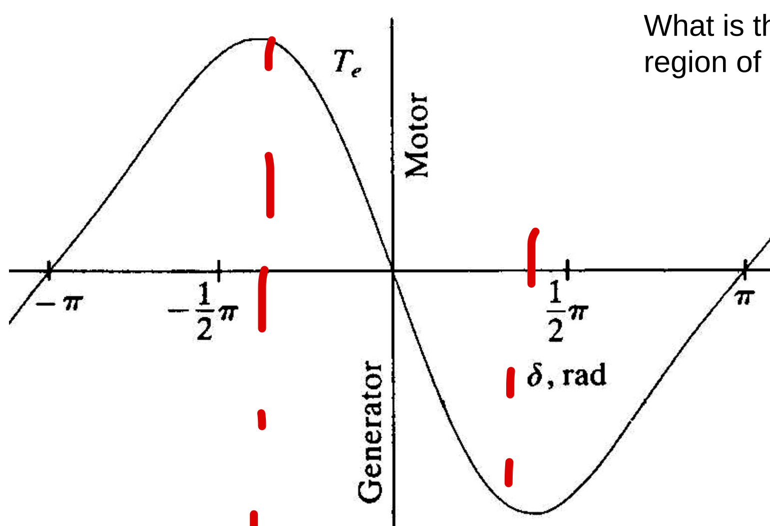
Torque due to rotor saliency

(This would be the torque of the salient-rotor reluctance synchronous motor)

Combined Torque-Angle Characteristic

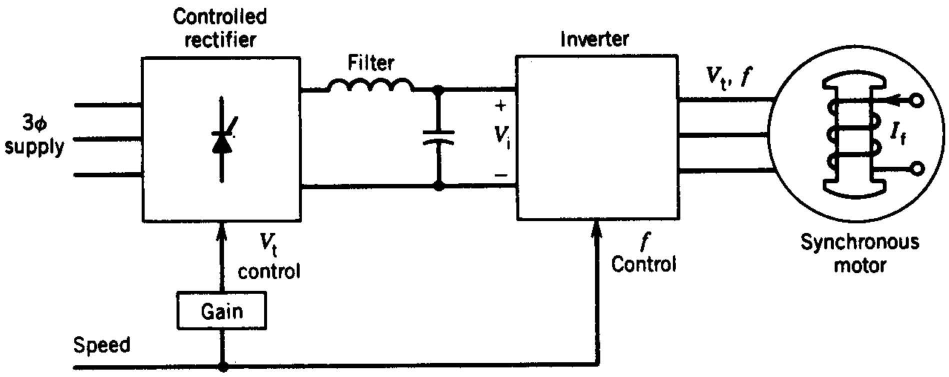
$$T_e = -\frac{P}{2} \frac{1}{\omega_e} \left[\frac{E'_{xfd} \sqrt{2} V_s}{X_d} \sin(\delta) + \frac{1}{2} \left(\frac{1}{X_q} - \frac{1}{X_d} \right) \left(\sqrt{2} V_s \right)^2 \sin(2\delta) \right]$$

What is the stable region of operation ?

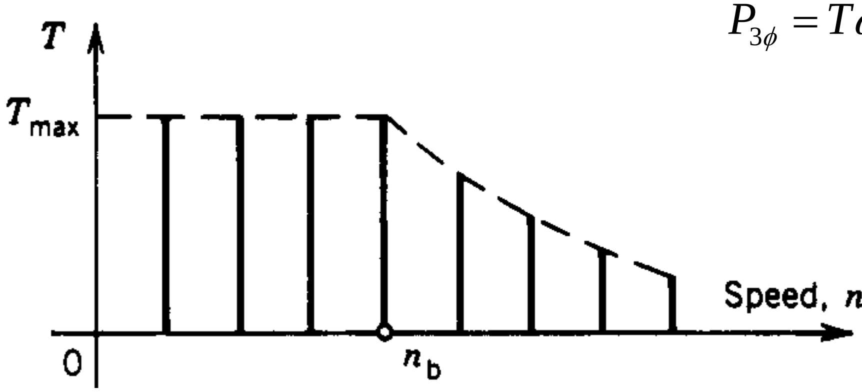


stable op ≤ 180°

Syn. Motor Speed Control



$$P_{3\phi} = T\omega_{syn} = \frac{3V_t E_f}{X_d}\sin(\delta)$$



Brushless DC Motor (Drive)

- Use PM Synchronous Machine & Variable Frequency Drive
- Use Hall-Effect or Electro-Optical sensors of the rotor position