PROBLEM 9.1

KNOWN: Tabulated values of density for water and definition of the volumetric thermal expansion coefficient, β.

FIND: Value of the volumetric expansion coefficient at 300K; compare with tabulated values.

PROPERTIES: *Table A-6*, Water (300K): $\rho = 1/v_f = 1/1.003 \times 10^{-3}\ m^3/kg = 997.0\ kg/m^3$, $\beta = 276.1 \times 10^{-6}\ K^{-1}$; (295K): $\rho = 1/v_f = 1/1.002 \times 10^{-3}\ m^3/kg = 998.0\ kg/m^3$; (305K): $\rho = 1/v_f = 1/1.005 \times 10^{-3}\ m^3/kg = 995.0\ kg/m^3$.

ANALYSIS: The volumetric expansion coefficient is defined by Eq. 9.4 as

$$b = -\frac{1}{r}\left(\frac{\partial r}{\partial \mathrm{T}}\right)_p .$$

The density change with temperature at constant pressure can be estimated as

$$\left(\frac{\partial r}{\partial \mathrm{T}}\right)_p \approx \left(\frac{r_1 - r_2}{\mathrm{T}_1 - \mathrm{T}_2}\right)_p$$

where the subscripts (1,2) denote the property values just above and below, respectively, the condition for T = 300K denoted by the subscript (o). That is,

$$b_o \approx -\frac{1}{r_o}\left(\frac{r_1 - r_2}{\mathrm{T}_1 - \mathrm{T}_2}\right)_p .$$

Substituting numerical values, find

$$b_o \approx \frac{-1}{997.0\ \mathrm{kg/m^3}}\ \frac{(995.0 - 998.0)\ \mathrm{kg/m^3}}{(305 - 295)\,\mathrm{K}} = 300.9 \times 10^{-6}\ \mathrm{K}^{-1}. \qquad \<$$

Compare this value with the tabulation, $\beta = 276.1 \times 10^{-6}\ K^{-1}$, to find our estimate is 8.7% high.

COMMENTS: (1) The poor agreement between our estimate and the tabulated value is due to the poor precision with which the density change with temperature is estimated. The tabulated values of β were determined from accurate equation of state data.

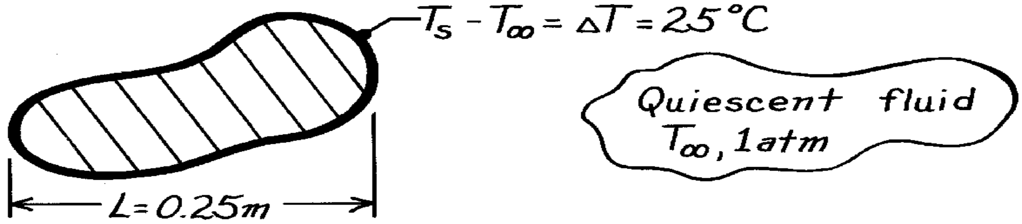
(2) Note that β is negative for T < 275K. Why? What is the implication for free convection?

PROBLEM 9.2

KNOWN: Object with specified characteristic length and temperature difference above ambient fluid.

FIND: Grashof number for air, hydrogen, water, ethylene glycol for a pressure of 1 atm.

SCHEMATIC:



ASSUMPTIONS: (1) Thermophysical properties evaluated at T_f = 350K, (2) Perfect gas behavior, ($\beta = 1/T_f$).

PROPERTIES: Evaluate at 1 atm, T_f = 350K:

Table A-4, Air: $\nu = 20.92 \times 10^{-6}\ m^2/s$; Hydrogen: $\nu = 143 \times 10^{-6}\ m^2/s$

Table A-6, Water (Sat. liquid): $\nu = \mu_f v_f = 37.5 \times 10^{-6}\ m^2/s$, $\beta_f = 0.624 \times 10^{-3}\ K^{-1}$

Table A-5, Ethylene glycol: $\nu = 3.17 \times 10^{-6}\ m^2/s$, $\beta = 0.65 \times 10^{-3}\ K^{-1}$.

ANALYSIS: The Grashof number is given by Eq. 9.12,

$$Gr_L = \frac{g b \left(T_s - T_\infty\right) L^3}{n^2}.$$

Substituting numerical values for *air* with $\beta = 1/T_f$, find

$$Gr_{L,air} = \frac{9.8 m/s^2 \times (1/350K)\ (25K)\ (0.25m)^3}{\left(20.92 \times 10^{-6}\ m^2/s\right)^2}$$

$$Gr_{L,air} = 2.50 \times 10^7. \qquad <$$

Performing similar calculations for the other fluids, find

$$Gr_{L,hyd} = 5.35 \times 10^5 \qquad <$$

$$Gr_{L,water} = 1.70 \times 10^6 \qquad <$$

$$Gr_{L,eth} = 2.48 \times 10^8. \qquad <$$

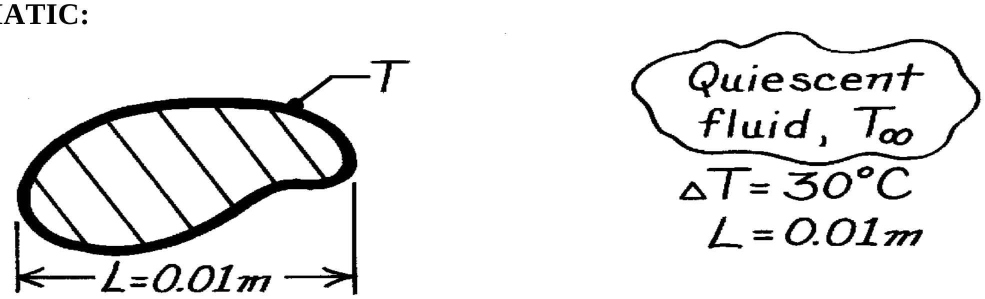
COMMENTS: Higher values of Gr_L imply increased free convection flows. However, other properties affect the value of the heat transfer coefficient.

PROBLEM 9.3

KNOWN: Relation for the Rayleigh number.

FIND: Rayleigh number for four fluids for prescribed conditions.

SCHEMATIC:



ASSUMPTIONS: (1) Perfect gas behavior for specified gases.

PROPERTIES: *Table A-4*, Air (400K, 1 atm): $\nu = 26.41 \times 10^{-6}$ m^2/s, $\alpha = 38.3 \times 10^{-6}$ m^2/s, $\beta = 1/T = 1/400K = 2.50 \times 10^{-3}$ K^{-1}; *Table A-4,* Helium (400K, 1 atm): $\nu = 199 \times 10^{-6}$ m^2/s, $\alpha = 295 \times 10^{-6}$ m^2/s, $\beta = 1/T = 2.50 \times 10^{-3}$ K^{-1}; *Table A-5*, Glycerin (12°C = 285K): $\nu = 2830 \times 10^{-6}$ m^2/s, $\alpha = 0.964 \times 10^{-7}$ m^2/s, $\beta = 0.475 \times 10^{-3}$ K^{-1}; *Table A-6*, Water (37°C = 310K, sat. liq.): $\nu = \mu_f v_f = 695 \times 10^{-6}$ $N \cdot s/m^2 \times 1.007 \times 10^{-3}$ $m^3/kg = 0.700 \times 10^{-6}$ m^2/s, $\alpha = k_f v_f/c_{p,f} = 0.628$ $W/m \cdot K \times 1.007 \times 10^{-3}$ $m^3/kg/4178$ $J/kg \cdot K = 0.151 \times 10^{-6}$ m^2/s, $\beta_f = 361.9 \times 10^{-6}$ K^{-1}.

ANALYSIS: The Rayleigh number, a dimensionless parameter used in free convection analysis, is defined as the product of the Grashof and Prandtl numbers.

$$\mathrm{Ra_L} \equiv \mathrm{Gr} \cdot \mathrm{Pr} = \frac{\mathrm{g}b\Delta \mathrm{T L}^3}{n^2} \frac{m \mathrm{c_p}}{\mathrm{k}} = \frac{\mathrm{g}b\Delta \mathrm{T L}^3}{n^2} \cdot \frac{(nr)\mathrm{c_p}}{\mathrm{k}} = \frac{\mathrm{g}b\Delta \mathrm{T L}^3}{na}$$

where $\alpha = k/\rho c_p$ and $\nu = \mu/\rho$. The numerical values for the four fluids follow:

Air (400K, 1 atm)

$$\mathrm{Ra_L} = 9.8\mathrm{m/s^2}\ (1/400\mathrm{K})\ 30\mathrm{K}(0.01\mathrm{m})^3/26.41\times10^{-6}\ \mathrm{m^2/s}\times38.3\times10^{-6}\ \mathrm{m^2/s} = 727 \quad <$$

Helium (400K, 1 atm)

$$\mathrm{Ra_L} = 9.8\mathrm{m/s^2}(1/400\mathrm{K})\ 30\mathrm{K}(0.01\mathrm{m})^3/199\times10^{-6}\mathrm{m^2/s}\times295\times10^{-6}\mathrm{m^2/s} = 12.5 \quad <$$

Glycerin (285K)

$$\mathrm{Ra_L} = 9.8\mathrm{m/s^2}\left(0.475\times10^{-3}\mathrm{K^{-1}}\right)\ 30\mathrm{K}\ (0.01\mathrm{m})^3/2830\times10^{-6}\mathrm{m^2/s}\times0.964\times10^{-7}\mathrm{m^2/s} = 512 \quad <$$

Water (310K)

$$\mathrm{Ra_L} = 9.8\mathrm{m/s^2}\left(0.362\times10^{-3}\mathrm{K^{-1}}\right)\ 30\mathrm{K}\ (0.01\mathrm{m})^3/0.700\times10^{-6}\mathrm{m^2/s}\times0.151\times10^{-6}\mathrm{m^2/s} = 9.35\times10^5. \quad <$$

COMMENTS: (1) Note the wide variation in values of Ra for the four fluids. A large value of Ra implies enhanced free convection, however, other properties affect the value of the heat transfer coefficient.

PROBLEM 9.4

KNOWN: Form of the Nusselt number correlation for natural convection and fluid properties.

FIND: Expression for figure of merit F_N and values for air, water and a dielectric liquid.

PROPERTIES: Prescribed. Air: k = 0.026 W/m·K, b = 0.0035 K^{-1}, ν = 1.5 × 10^{-5} m^2/s, Pr = 0.70. Water: k = 0.600 W/m·K, b = 2.7 × 10^{-4} K^{-1}, ν = 10^{-6} m^2/s, Pr = 5.0. Dielectric liquid: k = 0.064 W/m·K, b = 0.0014 K^{-1}, ν = 10^{-6} m^2/s, Pr = 25

ANALYSIS: With $Nu_L \sim Ra^n$, the convection coefficient may be expressed as

$$h \sim \frac{k}{L}\left(\frac{g b \Delta T L^3}{a n}\right)^n \sim \frac{\left(g \Delta T L^3\right)^n}{L}\left(\frac{k\, b^n}{a^n n^n}\right)$$

The figure of merit is therefore

$$F_N = \frac{k\, b^n}{a^n n^n}$$ <

and for the three fluids, with n = 0.33 and $a = n/\mathrm{Pr}$,

$F_N\left(W \cdot s^{2/3}/m^{7/3} \cdot K^{4/3}\right)$	Air	Water	Dielectric
	5.8	663	209

<

Water is clearly the superior heat transfer fluid, while air is the least effective.

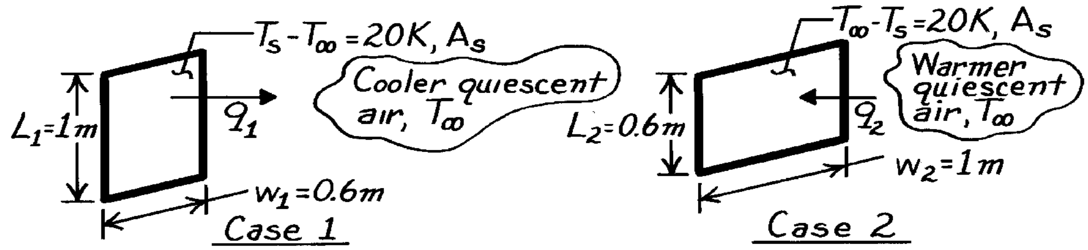
COMMENTS: The figure of merit indicates that heat transfer is enhanced by fluids of large k, large b and small values of a and ν.

PROBLEM 9.5

KNOWN: Heat transfer rate by convection from a vertical surface, 1m high by 0.6m wide, to quiescent air that is 20K cooler.

FIND: Ratio of the heat transfer rate for the above case to that for a vertical surface that is 0.6m high by 1m wide with quiescent air that is 20K warmer.

SCHEMATIC:



ASSUMPTIONS: (1) Thermophysical properties independent of temperature; evaluate at 300K; (2) Negligible radiation exchange with surroundings, (3) Quiescent ambient air.

PROPERTIES: *Table A-4*, Air (300K, 1 atm): $\nu = 15.89 \times 10^{-6}$ m^2/s, $\alpha = 22.5 \times 10^{-6}$ m^2/s.

ANALYSIS: The rate equation for convection between the plates and quiescent air is

$$q = \overline{h}_L A_s \Delta T \tag{1}$$

where ΔT is either $(T_s - T_\infty)$ or $(T_\infty - T_s)$; for both cases, $A_s = Lw$. The desired heat transfer ratio is then

$$\frac{q_1}{q_2} = \frac{\overline{h}_{L1}}{\overline{h}_{L2}}. \tag{2}$$

To determine the dependence of $\overline{h}_L$ on geometry, first calculate the Rayleigh number,

$$Ra_L = g\, b\, \Delta T L^3 / n a \tag{3}$$

and substituting property values at 300K, find,

Case 1: $Ra_{L1} = 9.8\ \text{m/s}^2\ (1/300\text{K})\ 20\text{K}\ (1\text{m})^3 / 15.89 \times 10^{-6}\ \text{m}^2/\text{s} \times 22.5 \times 10^{-6}\ \text{m}^2/\text{s} = 1.82 \times 10^9$

Case 2: $Ra_{L2} = Ra_{L1}\,(L_2/L_1)^3 = 1.82 \times 10^4\ (0.6\text{m}/1.0\text{m})^3 = 3.94 \times 10^8$.

Hence, Case 1 is turbulent and Case 2 is laminar. Using the correlation of Eq. 9.24,

$$\overline{Nu}_L = \frac{\overline{h}_L L}{k} = C\left(Ra_L\right)^n \qquad \overline{h}_L = \frac{k}{L} C\, Ra_L^n \tag{4}$$

where for *Case 1*: $C_1 = 0.10$, $n_1 = 1/3$ and for *Case 2*: $C_2 = 0.59$, $n_2 = 1/4$. Substituting Eq. (4) into the ratio of Eq. (2) with numerical values, find

$$\frac{q_1}{q_2} = \frac{(C_1/L_1)\,Ra_{L1}^{n_1}}{(C_2/L_2)\,Ra_{L2}^{n_2}} = \frac{(0.10/1\text{m})\left(1.82\times10^9\right)^{1/3}}{(0.59/0.6\text{m})\left(3.94\times10^8\right)^{1/4}} = 0.881 \qquad <$$

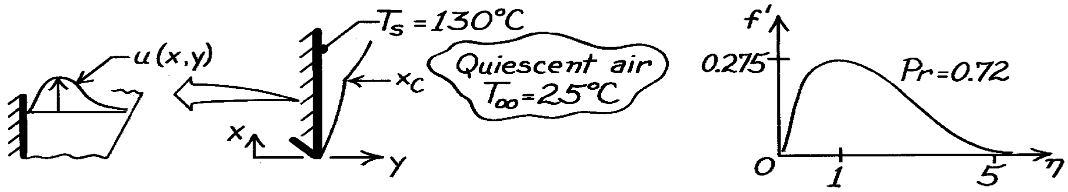
COMMENTS: Is this result to be expected? How do you explain this effect of plate orientation on the heat rates?

PROBLEM 9.6

KNOWN: Large vertical plate with uniform surface temperature of 130°C suspended in quiescent air at 25°C and atmospheric pressure.

FIND: (a) Boundary layer thickness at 0.25 m from lower edge, (b) Maximum velocity in boundary layer at this location and position of maximum, (c) Heat transfer coefficient at this location, (d) Location where boundary layer becomes turbulent.

SCHEMATIC:



ASSUMPTIONS: (1) Isothermal, vertical surface in an extensive, quiescent medium, (2) Boundary layer assumptions valid.

PROPERTIES: *Table A-4*, Air $\left(T_f = \left(T_s + T_\infty\right)/2 = 350\text{K, 1 atm}\right)$: $\nu = 20.92 \times 10^{-6}$ m^2/s, k = 0.030 W/m·K, Pr = 0.700.

ANALYSIS: (a) From the similarity solution results, Fig. 9.4 (see above right), the boundary layer thickness corresponds to a value of $\eta \approx 5$. From Eqs. 9.13 and 9.12,

$$y = h\,x\left(Gr_x / 4\right)^{-1/4} \qquad (1)$$

$$Gr_x = g b \left(T_s - T_\infty\right) x^3 / n^2 = 9.8\frac{m}{s^2} \times \frac{1}{350K}\left(130 - 25\right) K\, x^3 / \left(20.92 \times 10^{-6} m^2/s\right)^2 = 6.718 \times 10^9\, x^3 \qquad (2)$$

$$y \approx 5\left(0.25m\right)\left(6.718 \times 10^9 \left(0.25\right)^3 / 4\right)^{-1/4} = 1.746 \times 10^{-2} m = 17.5 \text{ mm.} \qquad (3) <$$

(b) From the similarity solution shown above, the maximum velocity occurs at $\eta \approx 1$ with $f'(h) = 0.275$. From Eq.9.15, find

$$u = \frac{2n}{x} Gr_x^{1/2}\ f'(h) = \frac{2 \times 20.92 \times 10^{-6} m^2/s}{0.25m}\left(6.718 \times 10^9 \left(0.25\right)^3\right)^{1/2} \times 0.275 = 0.47 \text{ m/s.} <$$

The maximum velocity occurs at a value of $\eta = 1$; using Eq. (3), it follows that this corresponds to a position in the boundary layer given as

$$y_{max} = 1/5\ \left(17.5 \text{ mm}\right) = 3.5 \text{ mm.} <$$

(c) From Eq. 9.19, the local heat transfer coefficient at x = 0.25 m is

$$Nu_x = h_x x/k = \left(Gr_x / 4\right)^{1/4}\ g\left(Pr\right) = \left(6.718 \times 10^9 \left(0.25\right)^3 / 4\right)^{1/4} 0.586 = 41.9$$

$$h_x = Nu_x k/x = 41.9 \times 0.030 \text{ W/m}\cdot\text{K}/0.25 \text{ m} = 5.0 \text{ W/m}^2\cdot\text{K.} <$$

The value for g(Pr) is determined from Eq. 9.20 with Pr = 0.700.

(d) According to Eq. 9.23, the boundary layer becomes turbulent at x_c given as

$$Ra_{x,c} = Gr_{x,c} Pr \approx 10^9 \qquad x_c \approx \left[10^9 / 6.718 \times 10^9 \left(0.700\right)\right]^{1/3} = 0.60 \text{ m.} <$$

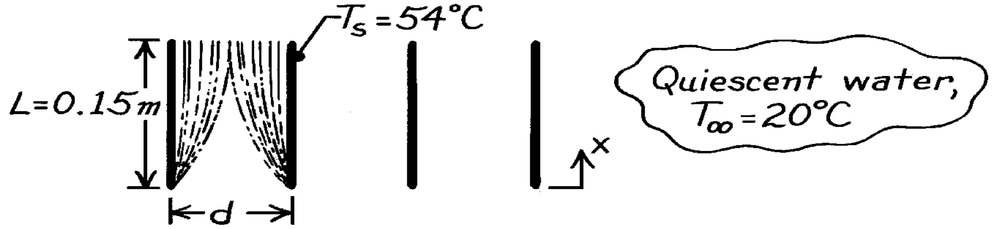
COMMENTS: Note that $\beta = 1/T_f$ is a suitable approximation for air.

PROBLEM 9.7

KNOWN: Thin, vertical plates of length 0.15m at 54°C being cooled in a water bath at 20°C.

FIND: Minimum spacing between plates such that no interference will occur between free-convection boundary layers.

SCHEMATIC:



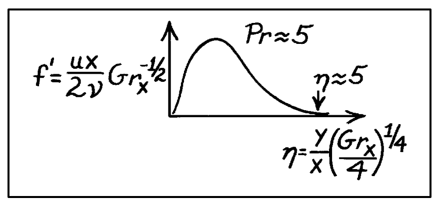
ASSUMPTIONS: (a) Water in bath is quiescent, (b) Plates are at uniform temperature.

PROPERTIES: *Table A-6*, Water ($T_f = (T_s + T_\infty)/2 = (54 + 20)°C/2 = 310K$): $\rho = 1/v_f = 993.05$ kg/m^3, $\mu = 695 \times 10^{-6}$ N·s/m^2, $\nu = \mu/\rho = 6.998 \times 10^{-7}$ m^2/s, Pr = 4.62, $\beta = 361.9 \times 10^{-6}$ K^{-1}.

ANALYSIS: The minimum separation distance will be twice the thickness of the boundary layer at the trailing edge where x = 0.15m. Assuming laminar, free convection boundary layer conditions, the similarity parameter, η, given by Eq. 9.13, is

$$h = \frac{y}{x}\left(Gr_x / 4\right)^{1/4}$$

where y is measured normal to the plate (see Fig. 9.3). According to Fig. 9.4, the boundary layer thickness occurs at a value $\eta \approx 5$.
It follows then that,



$$y_{bl} = h\ x\ \left(Gr_x / 4\right)^{-1/4}$$

where $\quad Gr_x = \dfrac{g\, b\ \left(T_s - T_\infty\right) x^3}{n^2}$

$$Gr_x = 9.8\ \text{m/s}^2 \times 361.9 \times 10^{-6}\ \text{K}^{-1}(54-20)\,\text{K} \times (0.15\text{m})^3 / \left(6.998 \times 10^{-7}\ \text{m}^2/\text{s}\right)^2 = 8.310 \times 10^8. \quad <$$

Hence, $\quad y_{bl} = 5 \times 0.15\text{m}\left(8.310 \times 10^8 / 4\right)^{-1/4} = 6.247 \times 10^{-3}\text{m} = 6.3\ \text{mm}$

and the minimum separation is

$$d = 2\ y_{bl} = 2 \times 6.3\ \text{mm} = 12.6\ \text{mm}. \quad <$$

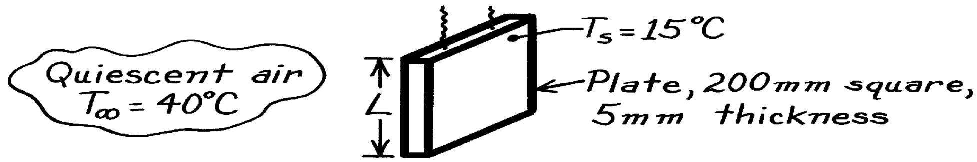
COMMENTS: According to Eq. 9.23, the critical Grashof number for the onset of turbulent conditions in the boundary layer is $Gr_{x,c}\, Pr \approx 10^9$. For the conditions above, $Gr_x\, Pr = 8.31 \times 10^8 \times 4.62 = 3.8 \times 10^9$. We conclude that the boundary layer is indeed turbulent at x = 0.15m and our calculation is only an estimate which is likely to be low. Therefore, the plate separation should be greater than 12.6 mm.

PROBLEM 9.8

KNOWN: Square aluminum plate at 15°C suspended in quiescent air at 40°C.

FIND: Average heat transfer coefficient by two methods – using results of boundary layer similarity and results from an empirical correlation.

SCHEMATIC:



ASSUMPTIONS: (1) Uniform plate surface temperature, (2) Quiescent room air, (3) Surface radiation exchange with surroundings negligible, (4) Perfect gas behavior for air, $\beta = 1/T_f$.

PROPERTIES: *Table A-4*, Air ($T_f = (T_s + T_\infty)/2 = (40 + 15)°C/2 = 300K$, 1 atm): $\nu = 15.89 \times 10^{-6}$ m^2/s, k = 0.0263 W/m·K, $\alpha = 22.5 \times 10^{-6}$ m^2/s, Pr = 0.707.

ANALYSIS: Calculate the Rayleigh number to determine the boundary layer flow conditions,

$$Ra_L = g\,b\,\Delta T\,L^3/n\,a$$

$$Ra_L = 9.8\text{ m/s}^2\,(1/300K)\,(40-15)°C\,(0.2m)^3/\left(15.89\times10^{-6}m^2/s\right)\left(22.5\times10^{-6}m^2/s\right) = 1.827\times10^7$$

where $\beta = 1/T_f$ and $\Delta T = T_\infty - T_s$. Since $Ra_L < 10^9$, the flow is laminar and the *similarity solution* of Section 9.4 is applicable. From Eqs. 9.21 and 9.20,

$$\overline{Nu}_L = \frac{\bar{h}_L L}{k} = \frac{4}{3}\left(Gr_L/4\right)^{1/4} g(Pr)$$

$$g(Pr) = \frac{0.75\,Pr^{1/2}}{\left[0.609 + 1.221\,Pr^{1/2} + 1.238\,Pr\right]^{1/4}}$$

and substituting numerical values with $Gr_L = Ra_L/Pr$, find

$$g(Pr) = 0.75(0.707)^{1/2}/\left[0.609 + 1.22(0.707)^{1/2} + 1.238\times0.707\right]^{1/4} = 0.501$$

$$\bar{h}_L = \left(\frac{0.0263\text{ W/m}\cdot K}{0.20m}\right)\times\frac{4}{3}\left(\frac{1.827\times10^7/0.707}{4}\right)^{1/4}\times0.501 = 4.42\text{ W/m}^2\cdot K. \quad <$$

The appropriate empirical correlation for estimating $\bar{h}_L$ is given by Eq. 9.27,

$$\overline{Nu}_L = \frac{\bar{h}_L L}{k} = 0.68 + \frac{0.670\,Ra_L^{1/4}}{\left[1+(0.492/Pr)^{9/16}\right]^{4/9}}$$

$$\bar{h}_L = (0.0263\text{ W/m}\cdot K/0.20m)\left[0.68 + 0.670\left(1.827\times10^7\right)^{1/4}/\left[1+(0.492/0.707)^{9/16}\right]^{4/9}\right]$$

$$\bar{h}_L = 4.42\text{ W/m}^2\cdot K. \quad <$$

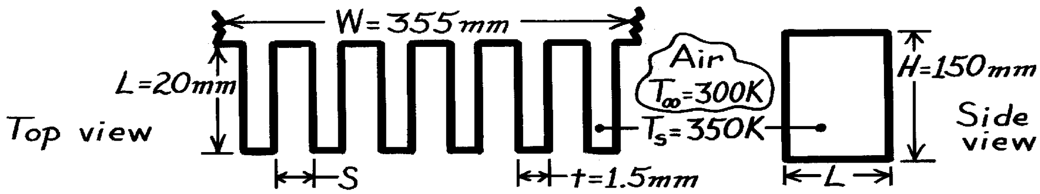
COMMENTS: The agreement of $\bar{h}_L$ calculated by these two methods is excellent. Using the Churchill-Chu correlation, Eq. 9.26, find $\bar{h}_L = 4.87$ W/m·K. This relation is not the most accurate for the laminar regime, but is suitable for both laminar and turbulent regions.

PROBLEM 9.9

KNOWN: Dimensions of vertical rectangular fins. Temperature of fins and quiescent air.

FIND: (a) Optimum fin spacing, (b) Rate of heat transfer from an array of fins at the optimal spacing.

SCHEMATIC:



ASSUMPTIONS: (1) Fins are isothermal, (2) Radiation effects are negligible, (3) Air is quiescent.

PROPERTIES: *Table A-4*, Air (T_f = 325K, 1 atm): $\nu = 18.41 \times 10^{-6}$ m^2/s, k = 0.0282 W/m·K, Pr = 0.703.

ANALYSIS: (a) If fins are too close, boundary layers on adjoining surfaces will coalesce and heat transfer will decrease. If fins are too far apart, the surface area becomes too small and heat transfer decreases. $S_{op} \approx \delta_{x=H}$. From Fig. 9.4, the edge of boundary layer corresponds to

$$h = (d/\mathrm{H})\ (\mathrm{Gr_H}/4)^{1/4} \approx 5.$$

$$\text{Hence, } \mathrm{Gr_H} = \frac{g b\,(\mathrm{T_s} - \mathrm{T_\infty})\mathrm{H}^3}{n\,\mathrm{H2}} = \frac{9.8\ \mathrm{m/s^2}(1/325\mathrm{K})\ 50\mathrm{K}\ (0.15\mathrm{m})^3}{\left(18.41\times10^{-6}\mathrm{m^2/s}\right)^2} = 1.5\times10^7$$

$$d\,(\mathrm{H}) = 5(0.15\mathrm{m})/\left(1.5\times10^7/4\right)^{1/4} = 0.017\mathrm{m} = 17\mathrm{mm} \qquad S_{op} \approx 34\mathrm{mm}. \qquad <$$

(b) The number of fins N can be found as

$$\mathrm{N} = \mathrm{W}/\left(\mathrm{S_{op}} + \mathrm{t}\right) = 355/35.5 = 10$$

and the rate is $\quad q = 2\ \mathrm{N}\ \overline{\mathrm{h}}(\mathrm{H}\cdot\mathrm{L})\ (\mathrm{T_s} - \mathrm{T_\infty}).$

For laminar flow conditions

$$\overline{\mathrm{Nu}}_\mathrm{H} = 0.68 + 0.67\ \mathrm{Ra_L^{1/4}}/\left[1+(0.492/\mathrm{Pr})^{9/16}\right]^{4/9}$$

$$\overline{\mathrm{Nu}}_\mathrm{H} = 0.68 + 0.67\left(1.5\times10^7\times0.703\right)^{1/4}/\left[1+(0.492/0.703)^{9/16}\right]^{4/9} = 30$$

$$\overline{\mathrm{h}} = \mathrm{k}\ \mathrm{Nu_H}/\mathrm{H} = 0.0282\ \mathrm{W/m\cdot K}(30)/0.15\ \mathrm{m} = 5.6\ \mathrm{W/m^2\cdot K}$$

$$q = 2(10)5.6\ \mathrm{W/m^2\cdot K}\,(0.15\mathrm{m}\times0.02\mathrm{m})\ (350-300)\mathrm{K} = 16.8\ \mathrm{W}. \qquad <$$

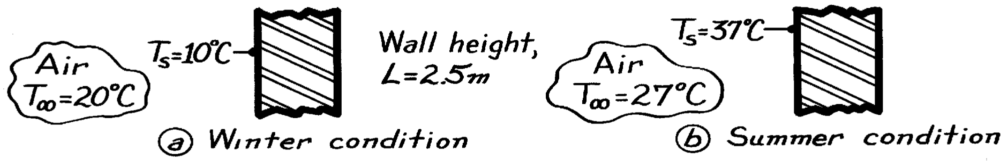
COMMENTS: Part (a) result is a conservative estimate of the optimum spacing. The increase in area resulting from a further reduction in S would more than compensate for the effect of fluid entrapment due to boundary layer merger. From a more rigorous treatment (see Section 9.7.1), $S_{op} \approx$ 10 mm is obtained for the prescribed conditions.

PROBLEM 9.10

KNOWN: Interior air and wall temperatures; wall height.

FIND: (a) Average heat transfer coefficient when $T_\infty = 20°C$ and $T_s = 10°C$, (b) Average heat transfer coefficient when $T_\infty = 27°C$ and $T_s = 37°C$.

SCHEMATIC:



ASSUMPTIONS: (a) Wall is at a uniform temperature, (b) Room air is quiescent.

PROPERTIES: *Table A-4*, Air (T_f = 298K, 1 atm): $\beta = 1/T_f = 3.472 \times 10^{-3}$ K^{-1}, $\nu = 14.82 \times 10^{-6}$ m^2/s, k = 0.0253 W/m·K, $\alpha = 20.9 \times 10^{-6}$ m^2/s, Pr = 0.710; (T_f = 305K, 1 atm): $\beta = 1/T_f = 3.279 \times 10^{-3}$ K^{-1}, $\nu = 16.39 \times 10^{-6}$ m^2/s, k = 0.0267 W/m·K, $\alpha = 23.2 \times 10^{-6}$ m^2/s, Pr = 0.706.

ANALYSIS: The appropriate correlation for the average heat transfer coefficient for free convection on a vertical wall is Eq. 9.26.

$$\overline{Nu}_L = \frac{\bar{h}L}{k} = \left\{0.825 + \frac{0.387\, Ra_L^{0.1667}}{\left[1+\left(0.492/Pr\right)^{0.563}\right]^{0.296}}\right\}^2$$

where $Ra_L = g\, \beta\, \Delta T\, L^3/\nu\alpha$, Eq. 9.25, with $\Delta T = T_s - T_\infty$ or $T_\infty - T_s$.

(a) Substituting numerical values typical of *winter* conditions gives

$$Ra_L = \frac{9.8\ \text{m/s}^2 \times 3.472\times10^{-3}\ \text{K}^{-1}\left(20-10\right)\ \text{K}\ \left(2.5\text{m}\right)^3}{14.82\times10^{-6}\text{m}^2/\text{s}\times20.96\times10^{-6}\text{m}^2/\text{s}} = 1.711\times10^{10}$$

$$\overline{Nu}_L = \left\{0.825 + \frac{0.387\left(1.711\times10^{10}\right)^{0.1667}}{\left[1+\left(0.492/0.710\right)^{0.563}\right]^{0.296}}\right\}^2 = 299.6.$$

Hence, $\bar{h} = \overline{Nu}_L\ k/L = 299.6\left(0.0253\ \text{W/m}\cdot\text{K}\right)/2.5\text{m} = 3.03\ \text{W/m}^2\cdot\text{K}.$ <

(b) Substituting numerical values typical of *summer* conditions gives

$$Ra_L = \frac{9.8\ \text{m/s}^2 \times 3.279\times10^{-3}\ \text{K}^{-1}\left(37-27\right)\ \text{K}\ \left(2.5\ \text{m}\right)^3}{23.2\times10^{-6}\text{m}^2/\text{s}\times16.39\times10^{-6}\text{m}^2/\text{s}} = 1.320\times10^{10}$$

$$\overline{Nu}_L = \left\{0.825 + \frac{0.387\left(1.320\times10^{10}\right)^{0.1667}}{\left[1+\left(0.492/0.706\right)^{0.563}\right]^{0.296}}\right\}^2 = 275.8.$$

Hence, $\bar{h} = \overline{Nu}_L\ k/L = 275.8\times0.0267\ \text{W/m}\cdot\text{K}/2.5\text{m} = 2.94\ \text{W/m}^2\cdot\text{K}.$ <

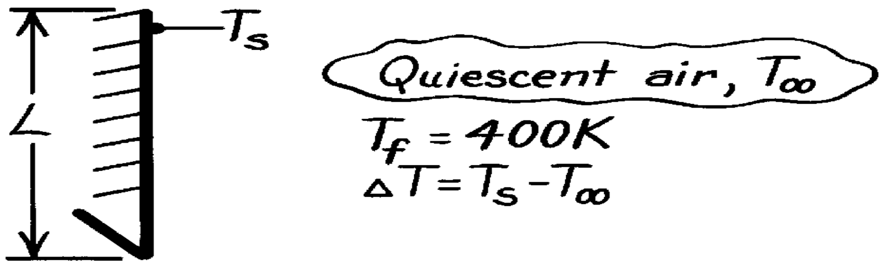
COMMENTS: There is a small influence due to T_f on $\bar{h}$ for these conditions. We should expect radiation effects to be important with such low values of $\bar{h}$.

PROBLEM 9.11

KNOWN: Vertical plate experiencing free convection with quiescent air at atmospheric pressure and film temperature 400 K.

FIND: Form of correlation for average heat transfer coefficient in terms of ΔT and characteristic length.

SCHEMATIC:



ASSUMPTIONS: (1) Air is extensive, quiescent medium, (2) Perfect gas behavior.

PROPERTIES: *Table A-6*, Air (T_f = 400K, 1 atm): $\nu = 26.41 \times 10^{-6}\ m^2/s$, k = 0.0338 W/m·K, $\alpha = 38.3 \times 10^{-6}\ m^2/s$.

ANALYSIS: Consider the correlation having the form of Eq. 9.24 with Ra_L defined by Eq. 9.25.

$$\overline{Nu}_L = \bar{h}_L L/k = CRa_L^n \tag{1}$$

where

$$Ra_L = \frac{gb\left(T_s - T_\infty\right)L^3}{na} = \frac{9.8\ m/s^2\left(1/400\ K\right)\Delta T \cdot L^3}{26.41\times10^{-6} m^2/s \times 38.3\times10^{-6} m^2/s} = 2.422\times10^7\,\Delta T \cdot L^3. \tag{2}$$

Combining Eqs. (1) and (2),

$$\bar{h}_L = \left(k/L\right) CRa_L^n = \frac{0.0338\ W/m\cdot K}{L} C\left(2.422\times10^7 \Delta T L^3\right)^n. \tag{3}$$

From Fig. 9.6, note that for laminar boundary layer conditions, $10^4 < Ra_L < 10^9$, C = 0.59 and n = 1/4. Using Eq. (3),

$$\bar{h} = 1.40\left[L^{-1}\left(\Delta T \cdot L^3\right)^{1/4}\right] = 1.40\left(\frac{\Delta T}{L}\right)^{1/4} \qquad <$$

For turbulent conditions in the range $10^9 < Ra_L < 10^{13}$, C = 0.10 and n = 1/3. Using Eq. (3),

$$\bar{h}_L = 0.98\left[L^{-1}\left(\Delta T \cdot L^3\right)^{1/3}\right] = 0.98\Delta T^{1/3}. \qquad <$$

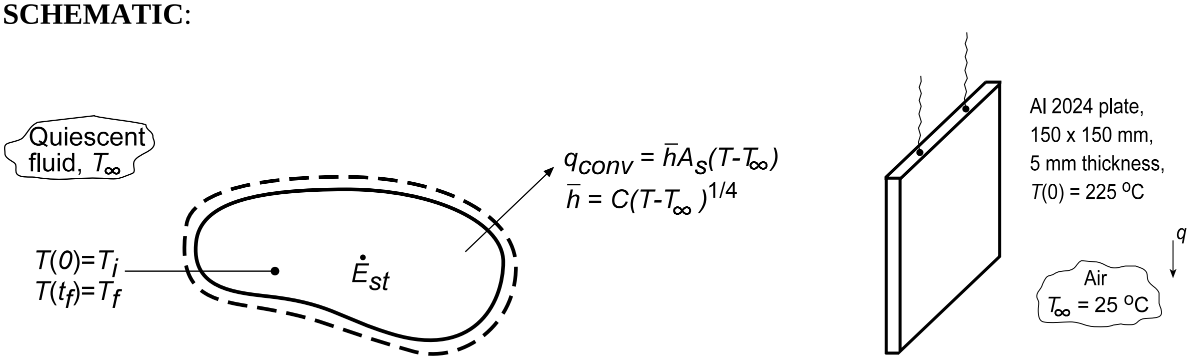
COMMENTS: Note the dependence of the average heat transfer coefficient on ΔT and L for laminar and turbulent conditions. The characteristic length L does not influence $\bar{h}_L$ for turbulent conditions.

PROBLEM 9.12

KNOWN: Temperature dependence of free convection coefficient, $\bar{h} = C\Delta T^{1/4}$, for a solid suddenly submerged in a quiescent fluid.

FIND: (a) Expression for cooling time, t_f, (b) Considering a plate of prescribed geometry and thermal conditions, the time required to reach 80°C using the appropriate correlation from Problem 9.10 and (c) Plot the temperature-time history obtained from part (b) and compare with results using a constant $\bar{h}_o$ from an appropriate correlation based upon an average surface temperature $\bar{T} = (T_i + T_f)/2$.

SCHEMATIC:



ASSUMPTIONS: (1) Lumped capacitance approximation is valid, (2) Negligible radiation, (3) Constant properties.

PROPERTIES: *Table A.1*, Aluminum alloy 2024 $\left(\bar{T} = (T_i + T_f)/2 \approx 400\,K\right)$: ρ = 2770 kg/m^3, c_p = 925 J/kg·K, k = 186 W/m·K; *Table A.4*, Air ($\bar{T}_{film}$ = 362 K): ν = 2.221 × 10^{-5} m^2/s, k = 0.03069 W/m·K, α = 3.187 × 10^{-5} m^2/s, Pr = 0.6976, β = 1/$\bar{T}_{film}$.

ANALYSIS: (a) Apply an energy balance to a control surface about the object, $-\dot{E}_{out} = \dot{E}_{st}$, and substitute the convection rate equation, with $\bar{h} = C\Delta T^{1/4}$, to find

$$-CA_s(T - T_\infty)^{5/4} = d/dt(\rho VcT). \qquad (1)$$

Separating variables and integrating, find

$$dT/dt = -(CA_s/\rho Vc)(T - T_\infty)^{5/4}$$

$$\int_{T_i}^{T_f} \frac{dT}{(T - T_\infty)^{5/4}} = -\left(\frac{CA_s}{\rho Vc}\right)\int_0^{t_f} dt \qquad -4(T - T_\infty)^{-1/4}\Big|_{T_i}^{T_f} = -\frac{CA_s}{\rho Vc}t_f$$

$$t_f = \frac{4\rho Vc}{CA_s}\left[(T_f - T_\infty)^{-1/4} - (T_i - T_\infty)^{-1/4}\right] = \frac{4\rho Vc}{CA_s(T_i - T_\infty)^{1/4}}\left[\left(\frac{T_i - T_\infty}{T_f - T_\infty}\right)^{1/4} - 1\right]. \qquad (2) \;<$$

(b) Considering the aluminum plate, initially at T(0) = 225°C, and suddenly exposed to ambient air at $T_\infty = 25°C$, from Problem 9.10 the convection coefficient has the form

$$\bar{h}_i = 1.40\left(\frac{\Delta t}{L}\right)^{1/4} \qquad \bar{h}_i = C\Delta T^{1/4}$$

where C = 1.40/L$^{1/4}$ = 1.40/(0.150)$^{1/4}$ = 2. 2496 $W/m^2 \cdot K^{3/4}$. Using Eq. (2), find

Continued...

PROBLEM 9.12 (Cont.)

$$t_f = \frac{4\times 2770\,\text{kg}/\text{m}^3\left(0.150^2\times 0.005\right)\text{m}^3\times 925\,\text{J/kg}\cdot\text{K}}{2.2496\,\text{W}/\text{m}^2\cdot\text{K}^{3/4}\times 2\times(0.150\text{m})^2\,(225-25)^{1/4}\,\text{K}^{1/4}}\left[\left(\frac{225-25}{80-25}\right)^{1/4}-1\right] = 1154\text{s}$$

(c) For the vertical plate, Eq. 9.27 is an appropriate correlation. Evaluating properties at

$$\overline{T}_{film} = \left(\overline{T}_s + T_\infty\right)/2 = \left(\left(T_i + T_f\right)/2 + T_\infty\right)/2 = 362\text{ K}$$

where $\overline{T}_s = 426\text{K}$, the average plate temperature, find

$$Ra_L = \frac{g\beta\left(\overline{T}_s - T_\infty\right)L^3}{\nu\alpha} = \frac{9.8\,\text{m/s}^2\,(1/362\text{K})(426-298)\text{K}\,(0.150\text{m})^3}{2.221\times 10^{-5}\,\text{m}^2/\text{s}\times 3.187\times 10^{-5}\,\text{m}^2/\text{s}} = 1.652\times 10^7$$

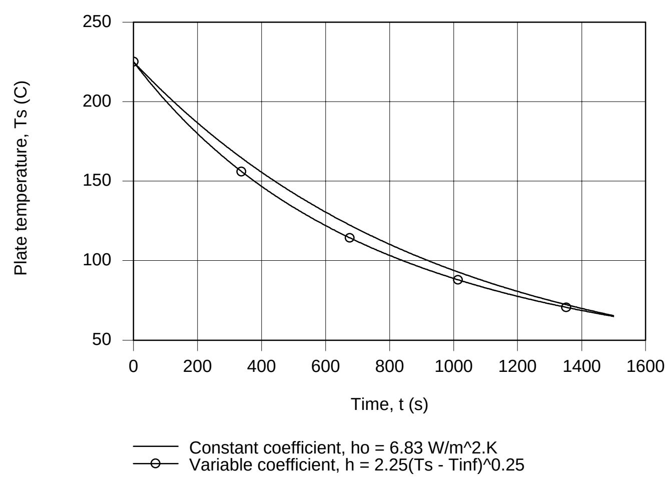
$$\overline{Nu}_L = 0.68 + \frac{0.670 Ra_L^{1/4}}{\left[1+(0.492/Pr)^{9/16}\right]^{4/9}} = 0.68 + \frac{0.670\left(1.652\times 10^7\right)^{1/4}}{\left[1+(0.492/0.6976)^{9/16}\right]^{4/9}} = 33.4$$

$$\overline{h}_o = \frac{k}{L}\overline{Nu}_L = \frac{0.03069\,\text{W/m}\cdot\text{K}}{0.150\text{m}}\times 33.4 = 6.83\,\text{W}/\text{m}^2\cdot\text{K}$$

From Eq. 5.6, the temperature-time history with a constant convection coefficient is

$$T(t) = T_\infty + \left(T_i - T_\infty\right)\exp\left[-\left(\overline{h}_o A_s/\rho V c\right)t\right] \tag{3}$$

where $A_s/V = 2L^2/(L\times L\times w) = 2/w = 400\text{m}^{-1}$. The temperature-time histories for the h = $C\Delta T^{1/4}$ and $\overline{h}_o$ analyses are shown in plot below.



COMMENTS: (1) The times to reach $T(t_o)$ = 80°C were 1154 and 1212s for the variable and constant coefficient analysis, respectively, a difference of 5%. For convenience, it is reasonable to evaluate the convection coefficient as described in part (b).

(2) Note that $Ra_L < 10^9$ so indeed the expression selected from Problem 9.10 was the appropriate one.

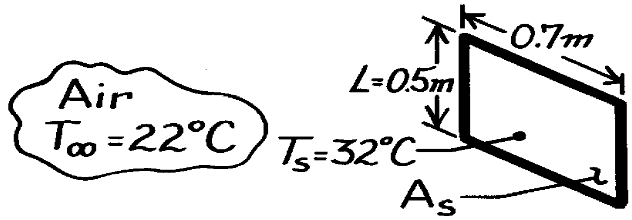
(3) Recognize that if the emissivity of the plate were unity, the average linearized radiation coefficient using Eq. (1.9) is $\overline{h}_{rad} = 11.0\,\text{W}/\text{m}^2\cdot\text{K}$ and radiative exchange becomes an important process.

PROBLEM 9.13

KNOWN: Oven door with average surface temperature of 32°C in a room with ambient air at 22°C.

FIND: Heat loss to the room. Also, find effect on heat loss if emissivity of door is unity and the surroundings are at 22°C.

SCHEMATIC:



ASSUMPTIONS: (1) Ambient air is quiescent, (2) Surface radiation effects are negligible.

PROPERTIES: *Table A-4*, Air (T_f = 300K, 1 atm): $\nu = 15.89 \times 10^{-6}$ m^2/s, k = 0.0263 W/m·K, α = 22.5×10^{-6} m^2/s, Pr = 0.707, $\beta = 1/T_f = 3.33 \times 10^{-3}$ K^{-1}.

ANALYSIS: The heat rate from the oven door surface by convection to the ambient air is

$$q = \bar{h} A_s \left(T_s - T_\infty \right) \tag{1}$$

where $\bar{h}$ can be estimated from the free-convection correlation for a vertical plate, Eq. 9.26,

$$\overline{Nu}_L = \frac{\bar{h}L}{k} = \left\{ 0.825 + \frac{0.387 Ra_L^{1/6}}{\left[1 + \left(0.492/Pr\right)^{9/16}\right]^{8/27}} \right\}^2 . \tag{2}$$

The Rayleigh number, Eq. 9.25, is

$$Ra_L = \frac{g\, b \left(T_s - T_\infty\right) L^3}{n a} = \frac{9.8\,m/s^2 \left(1/300K\right)\left(32-22\right)K \times 0.5^3 m^3}{15.89 \times 10^{-6} m^2/s \times 22.5 \times 10^{-6} m^2/s} = 1.142 \times 10^8.$$

Substituting numerical values into Eq. (2), find

$$\overline{Nu}_L = \left\{ 0.825 + \frac{0.387\left(1.142 \times 10^8\right)^{1/6}}{\left[1 + \left(0.492/0.707\right)^{9/16}\right]^{8/27}} \right\}^2 = 63.5$$

$$\bar{h}_L = \frac{k}{L}\overline{Nu}_L = \frac{0.0263\,W/m\cdot K}{0.5m} \times 63.5 = 3.34\,W/m^2\cdot K.$$

The heat rate using Eq. (1) is

$$q = 3.34\,W/m^2\cdot K \times \left(0.5 \times 0.7\right) m^2 \left(32 - 22\right) K = 11.7W.$$ <

Heat loss by radiation, assuming ε = 1, is

$$q_{rad} = e A_s s \left(T_s^4 - T_{sur}^4\right)$$

$$q_{rad} = 1\left(0.5 \times 0.7\right) m^2 \times 5.67 \times 10^{-8} W/m^2\cdot K^4 \left[\left(273+32\right)^4 - \left(273+22\right)^4\right] K^4 = 21.4W.$$ <

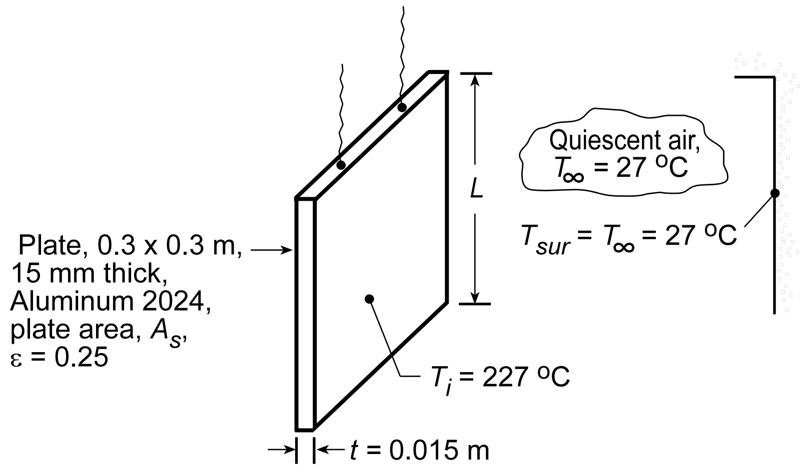
Note that heat loss by radiation is nearly double that by free convection. Using Eq. (1.9), the radiation heat transfer coefficient is h_{rad} = 6.4 W/m^2·K, which is twice the coefficient for the free convection process.

PROBLEM 9.14

KNOWN: Aluminum plate (alloy 2024) at an initial uniform temperature of 227°C is suspended in a room where the ambient air and surroundings are at 27°C.

FIND: (a) Expression for time rate of change of the plate, (b) Initial rate of cooling (K/s) when plate temperature is 227°C, (c) Validity of assuming a uniform plate temperature, (d) Decay of plate temperature and the convection and radiation rates during cooldown.

SCHEMATIC:



ASSUMPTIONS: (1) Plate temperature is uniform, (2) Ambient air is quiescent and extensive, (3) Surroundings are large compared to plate.

PROPERTIES: *Table A.1*, Aluminum alloy 2024 (T = 500 K): $\rho = 2770\ \text{kg/m}^3$, k = 186 W/m·K, c = 983 J/kg·K; *Table A.4*, Air (T_f = 400 K, 1 atm): $\nu = 26.41 \times 10^{-6}\ \text{m}^2/\text{s}$, k = 0.0388 W/m·K, $\alpha = 38.3 \times 10^{-6}\ \text{m}^2/\text{s}$, Pr = 0.690.

ANALYSIS: (a) From an energy balance on the plate with free convection and radiation exchange, $-\dot{E}_{out} = \dot{E}_{st}$, we obtain

$$-\bar{h}_L 2A_s\left(T_s - T_\infty\right) - \varepsilon 2A_s\sigma\left(T_s^4 - T_{sur}^4\right) = \rho A_s tc\frac{dT}{dt} \quad \text{or} \quad \frac{dT}{dt} = \frac{-2}{\rho tc}\left[\bar{h}_L\left(T_s - T_\infty\right) + \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right)\right] \quad <$$

where T_s, the plate temperature, is assumed to be uniform at any time.

(b) To evaluate (dT/dt), estimate $\bar{h}_L$. First, find the Rayleigh number,

$$Ra_L = g\beta\left(T_s - T_\infty\right)L^3/\nu\alpha = \frac{9.8\,\text{m/s}^2\,(1/400\,\text{K})(227-27)\text{K}\times(0.3\,\text{m})^3}{26.41\times10^{-6}\,\text{m}^2/\text{s}\times38.3\times10^{-6}\,\text{m}^2/\text{s}} = 1.308\times10^8 .$$

Eq. 9.27 is appropriate; substituting numerical values, find

$$\overline{Nu}_L = 0.68 + \frac{0.670Ra_L^{1/4}}{\left[1+\left(0.492/Pr\right)^{9/16}\right]^{4/9}} = 0.68 + \frac{0.670\left(1.308\times10^8\right)^{1/4}}{\left[1+\left(0.492/0.690\right)^{9/16}\right]^{4/9}} = 55.5$$

$$\bar{h}_L = \overline{Nu}_L k/L = 55.5\times0.0338\,\text{W/m}\cdot\text{K}/0.3\,\text{m} = 6.25\,\text{W/m}^2\cdot\text{K}$$

Continued...

PROBLEM 9.14 (Cont.)

$$\frac{dT}{dt} = \frac{-2}{2770\,\text{kg/m}^3 \times 0.015\,\text{m} \times 983\,\text{J/kg}\cdot\text{K}} \times$$

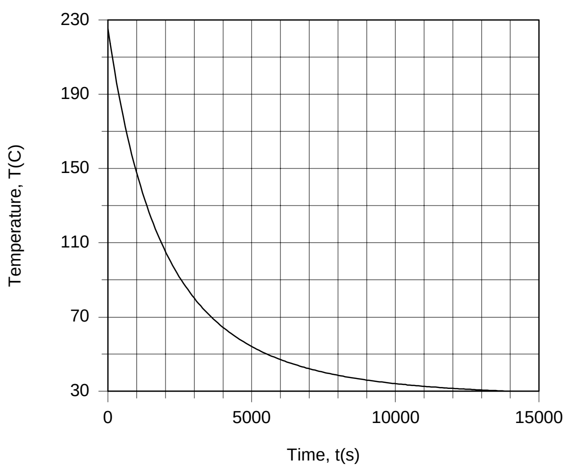
$$\left[6.25\,\text{W/m}^2\cdot\text{K}\,(227-27)\,\text{K} + 0.25\left(5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\right)\left(500^4 - 300^4\right)\text{K}^4\right] = -0.099\,\text{K/s}. \quad <$$

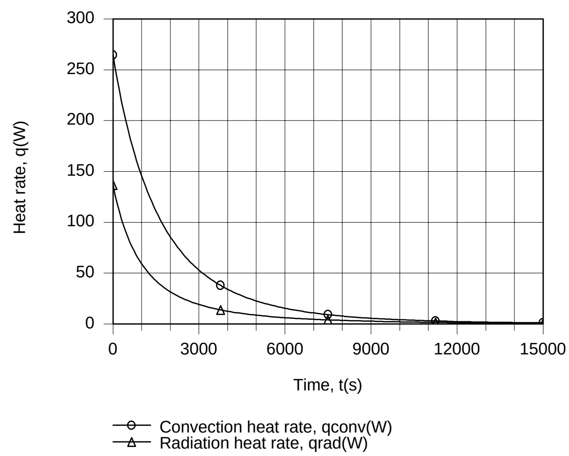
(c) The uniform temperature assumption is justified if the Biot number criterion is satisfied. With $L_c \equiv (V/2A_s) = (A_s \cdot t/2A_s) = (t/2)$ and $\overline{h}_{tot} = \overline{h}_{conv} + \overline{h}_{rad}$, Bi = $\overline{h}_{tot}(t/2)/k \le 0.1$. Using the linearized radiation coefficient relation, find

$$\overline{h}_{rad} = \varepsilon\sigma\left(T_s + T_{sur}\right)\left(T_s^2 + T_{sur}^2\right) = 0.25\left(5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\right)(500+300)\left(500^2+300^2\right)\text{K}^3 = 3.86\,\text{W/m}^2\cdot\text{K}$$

Hence, Bi = (6.25 + 3.86) $\text{W/m}^2\cdot\text{K}$(0.015 m/2)/186 W/m·K = 4.07×10^{-4}. Since Bi << 0.1, the assumption is appropriate.

(d) The temperature history of the plate was computed by combining the *Lumped Capacitance Model* of IHT with the appropriate *Correlations* and *Properties* Toolpads.





Due to the small values of $\overline{h}_L$ and $\overline{h}_{rad}$, the plate cools slowly and does not reach 30°C until t ≈ 14000s = 3.89h. The convection and radiation rates decrease rapidly with increasing t (decreasing T), thereby decelerating the cooling process.

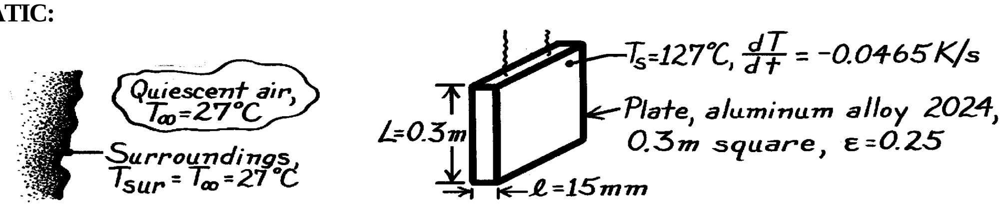
COMMENTS: The reduction in the convection rate with increasing time is due to a reduction in the thermal conductivity of air, as well as the values of $\overline{h}_L$ and T.

PROBLEM 9.15

KNOWN: Instantaneous temperature and time rate of temperature change of a vertical plate cooling in a room.

FIND: Average free convection coefficient for the prescribed conditions; compare with standard empirical correlation.

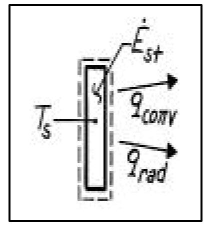
SCHEMATIC:



ASSUMPTIONS: (1) Uniform plate temperature, (2) Quiescent room air, (3) Large surroundings.

PROPERTIES: *Table A-1*, Aluminum alloy 2024 (T_s = 127°C = 400K): ρ = 2770 kg/m^3, c_p = 925 J/kg·K; *Table A-4*, Air ($T_f = (T_s + T_\infty)/2$ = 350K, 1 atm): $\nu = 20.92 \times 10^{-6}$ m^2/s, k = 0.020 W/m·K, $\alpha = 29.9 \times 10^{-6}$ m^2/s, Pr = 0.700.

ANALYSIS: From an energy balance on the plate considering free convection *and* radiation exchange,



$$\dot{E}_{in} - \dot{E}_{out} = \dot{E}_{st}$$

$$-\overline{h}_L (2A_s)(T_s - T_\infty) - \varepsilon (2A_s) \sigma \left(T_s^4 - T_{sur}^4\right) = \rho A_s \ell c_p \frac{dT}{dt}.$$

Noting that the plate area is $2A_s$, solving for $\overline{h}_L$, and substituting numerical values, find

$$\overline{h}_L = \left[-\rho \ell c_p \frac{dT}{dt} - 2\varepsilon\sigma \left(T_s^4 - T_{sur}^4\right)\right] / 2\left(T_s - T_\infty\right)$$

$$\overline{h}_L = \left[-2770\,\text{kg/m}^3 \times 0.3\text{m} \times 925\,\text{J/kg}\cdot\text{K}\,(-0.0465\,\text{K/s}) - 2 \times 0.25 \times 5.67 \times 10^{-8}\,\text{W/m}^2\cdot\text{K}^4 \left(400^4 - 300^4\right)\text{K}^4\right]$$

$$/\ 2(127 - 27)^\circ\,\text{C} = (8.936 - 2.455)\,\text{W/m}^2\cdot\text{K}$$

$$\overline{h}_L = 6.5\,\text{W/m}^2\cdot\text{K}.$$ <

To select an appropriate empirical correlation, first evaluate the Rayleigh number,

$$Ra_L = g\beta\Delta T L^3/\nu\alpha$$

$$Ra_L = 9.8\,\text{m/s}^2\,(1/350\,\text{K})(127 - 27)\,\text{K}\,(0.3\text{m})^3 / \left(20.92 \times 10^{-6}\,\text{m}^2/\text{s}\right)\left(29.9 \times 10^{-6}\,\text{m}^2/\text{s}\right) = 1.21 \times 10^8.$$

Since $Ra_L < 10^9$, the flow is laminar and Eq. 9.27 is applicable,

$$\overline{Nu}_L = \frac{\overline{h}_L L}{k} = 0.68 + \frac{0.670 Ra_L^{1/4}}{\left[1 + (0.492/Pr)^{9/16}\right]^{4/9}}$$

$$\overline{h}_L = \left(\frac{0.030\,\text{W/m}\cdot\text{K}}{0.3\text{m}}\right)\left\{0.68 + 0.670\left(1.21 \times 10^8\right)^{1/4} / \left[1 + (0.492/0.700)^{9/16}\right]^{4/9}\right\} = 5.5\,\text{W/m}^2\cdot\text{K}.$$ <

COMMENTS: (1) The correlation estimate is 15% lower than the experimental result. (2) This transient method, useful for obtaining an average free convection coefficient for spacewise isothermal objects, requires $Bi \le 0.1$.

PROBLEM 9.16

KNOWN: Person, approximated as a cylinder, experiencing heat loss in water or air at 10°C.

FIND: Whether heat loss from body in water is 30 times that in air.

ASSUMPTIONS: (1) Person can be approximated as a vertical cylinder of diameter D = 0.3 m and length L = 1.8 m, at 25°C, (2) Loss is only from the lateral surface.

PROPERTIES: *Table A-4*, Air $\left(\overline{T} = (25+10)^\circ C/2 = 290K, 1atm\right)$: k = 0.0293 W/m·K, ν = 19.91×10^{-6} m^2/s, $\alpha = 28.4 \times 10^{-6}$ m^2/s; *Table A-6*, Water (290K): k = 0.598 W/m·K, $\nu = \mu v_f$ = 1.081×10^{-6} m^2/s, $\alpha = k v_f / c_p = 1.431 \times 10^{-7}$ m^2/s, $\beta_f = 174 \times 10^{-6}$ K^{-1}.

ANALYSIS: In both water (wa) and air (a), the heat loss from the lateral surface of the cylinder approximating the body is

$$q = \overline{h} p DL (T_s - T_\infty)$$

where T_s and T_∞ are the same for both situations. Hence,

$$\frac{q_{wa}}{q_a} = \frac{\overline{h}_{wa}}{\overline{h}_a}$$

Vertical cylinder in air:

$$Ra_L = \frac{g b \Delta T L^3}{n a} = \frac{9.8 m/s^2 \times (1/290K)(25-10)K(1.8m)^3}{19.91\times10^{-6} m^2/s \times 28.4\times10^{-6} m^2/s} = 5.228\times10^9$$

Using Eq. 9.24 with C = 0.1 and n = 1/3,

$$\overline{Nu}_L = \frac{\overline{h}_L L}{k} = C Ra_L^n = 0.1\left(5.228\times10^9\right)^{1/3} = 173.4 \qquad \overline{h}_L = 2.82 \text{ W/m}^2\cdot\text{K}.$$

Vertical cylinder in water:

$$Ra_L = \frac{9.8 m/s^2 \times 174\times10^{-6} K^{-1}(25-10)K(1.8m)^3}{1.081\times10^{-6} m^2/s \times 1.431\times10^{-7} m^2/s} = 9.643\times10^{11}$$

Using Eq. 9.24 with C = 0.1 and n = 1/3,

$$\overline{Nu}_L = \frac{\overline{h} L}{k} = C Ra_L^n = 0.1\left(9.643\times10^{11}\right)^{1/3} = 978.9 \qquad \overline{h}_L = 328 \text{ W/m}^2\cdot\text{K}.$$

Hence, from this analysis we find

$$\frac{q_{wa}}{q_a} = \frac{328 \text{ W/m}^2\cdot\text{K}}{2.8 \text{ W/m}^2\cdot\text{K}} = 117$$

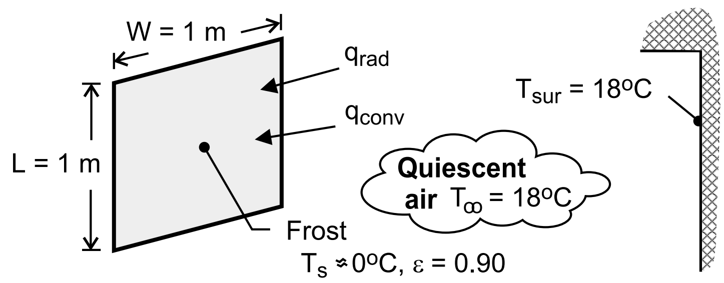
which compares poorly with the claim of 30.

PROBLEM 9.17

KNOWN: Dimensions of window pane with frost formation on inner surface. Temperature of room air and walls.

FIND: Heat loss through window.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) Surface of frost is isothermal with $T_s \approx 0°C$, (3) Radiation exchange is between a small surface (window) and a large enclosure (walls of room), (4) Room air is quiescent.

PROPERTIES: *Table A-4*, air (T_f = 9°C = 282 K): k = 0.0249 W/m·K, $\nu = 14.3 \times 10^{-6}$ m²/s, $\alpha = 20.1 \times 10^{-6}$ m²/s, Pr = 0.712, $\beta = 3.55 \times 10^{-3}$ K^{-1}.

ANALYSIS: Under steady-state conditions, the heat loss through the window corresponds to the rate of heat transfer to the frost by convection and radiaiton.

$$q = q_{conv} + q_{rad} = W \times L\left[\bar{h}\left(T_\infty - T_s\right) + \varepsilon\sigma\left(T_{sur}^4 - T_s^4\right)\right]$$

With $Ra_L = g\beta\left(T_\infty - T_s\right)L^3/\alpha\nu = 9.8\,m/s^2 \times 0.00355\,K^{-1} \times 18\,K\left(1m\right)^3/\left(14.3 \times 20.1 \times 10^{-12}\,m^4/s^2\right)$

$= 2.18 \times 10^9$, Eq. (9.26) yields

$$\overline{Nu}_L = \left\{0.825 + \frac{0.387\,Ra_L^{1/6}}{\left[1 + \left(0.492/Pr\right)^{9/16}\right]^{8/27}}\right\}^2 = 156.5$$

$$\bar{h} = Nu_L\frac{k}{L} = 156.5\left(\frac{0.0249\,W/m\cdot K}{1m}\right) = 3.9\,W/m^2\cdot K$$

$$q = 1m^2\left[3.9\,W/m^2\cdot K\left(18K\right) + 0.90 \times 5.67 \times 10^{-8}\,W/m^2\cdot K^4\left(291^4 - 273^4\right)\right]$$

$$= 70.2\,W + 82.5\,W = 152.7\,W$$ <

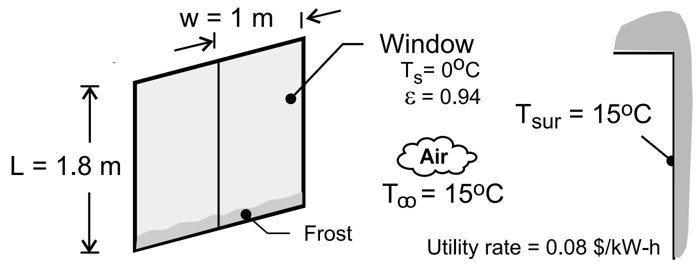
COMMENTS: (1) The thickness of the frost layer does not affect the heat loss, since the inner surface of the layer remains at $T_s \approx 0°C$. However, the temperature of the glass/frost interface decreases with increasing thickness, from a value of 0°C for negligible thickness. (2) Since the thermal boundary layer thickness is zero at the top of the window and has its maximum value at the bottom, the temperature of the glass will actually be largest and smallest at the top and bottom, respectively. Hence, frost will first begin to form at the bottom.

PROBLEM 9.18

KNOWN: During a winter day, the window of a patio door with a height of 1.8 m and width of 1.0 m shows a frost line near its base.

FIND: (a) Explain why the window would show a frost layer at the base of the window, rather than at the top, and (b) Estimate the heat loss through the window due to free convection and radiation. If the room has electric baseboard heating, estimate the daily cost of the window heat loss for this condition based upon the utility rate of 0.08 $/kW·h.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Window has a uniform temperature, (3) Ambient air is quiescent, and (4) Room walls are isothermal and large compared to the window.

PROPERTIES: *Table A-4*, Air ($T_f = (T_s + T_\infty)/2 = 280$ K, 1 atm): $\nu = 14.11 \times 10^{-6}$ m^2/s, k = 0.0247 W/m·K, $\alpha = 1.986 \times 10^{-5}$ m^2/s.

ANALYSIS: (a) For these winter conditions, a frost line could appear and it would be at the bottom of the window. The boundary layer is thinnest at the top of the window, and hence the heat flux from the warmer room is greater than compared to that at the bottom portion of the window where the boundary layer is thicker. Also, the air in the room may be stratified and cooler near the floor compared to near the ceiling.

(b) The heat loss from the room to the window having a uniform temperature $T_s = 0°C$ by convection and radiation is

$$q_{loss} = q_{cv} + q_{rad} \tag{1}$$

$$q_{loss} = A_s\left[\bar{h}_L\left(T_\infty - T_s\right) + \varepsilon\sigma\left(T_{sur}^4 - T_s^4\right)\right] \tag{2}$$

The average convection coefficient is estimated from the Churchill-Chu correlation, Eq. 9.26, using properties evaluated at $T_f = (T_s + T_\infty)/2$.

$$\overline{Nu}_L = \frac{\bar{h}_L L}{k} = \left\{0.825 + \frac{0.387\,Ra_L^{1/6}}{\left[1+\left(0.492/Pr\right)^{9/16}\right]^{8/27}}\right\}^2 \tag{3}$$

$$Ra_L = g\beta T\left(T_\infty - T_s\right)L^3/\nu\alpha \tag{4}$$

Substituting numerical values in the correlation expressions, find

$$Ra_L = 1.084\times10^{10} \qquad \overline{Nu}_L = 258.9 \qquad \bar{h}_L = 3.6\ W/m^2\cdot K$$

Continued …..

Using Eq. (2), the heat loss with $\sigma = 5.67 \times 10^{-8}\ \mathrm{W/m^2 \cdot K^4}$ is

$$q_{loss} = (1 \times 1.8)\,\mathrm{m}^2 \left[3.6\ \mathrm{W/m^2 \cdot K}\,(15\ \mathrm{K}) + 0.940\,\sigma\left(288^4 - 273^4\right)\mathrm{K}^4\right]$$

$$q_{loss} = (96.1 + 127.1)\,\mathrm{W} = 223\ \mathrm{W}$$

The daily cost of the window heat loss for the given utility rate is

$$\text{cost} = q_{loss} \times (\text{utility rate}) \times 24\ \text{hours}$$

$$\text{cost} = 223\ \mathrm{W} \times (10^{-3}\ \mathrm{kW/W}) \times 0.08\ \$/\mathrm{kW-h} \times 24\ \mathrm{h}$$

$$\text{cost} = 0.43\ \$/\text{day}$$ $<$

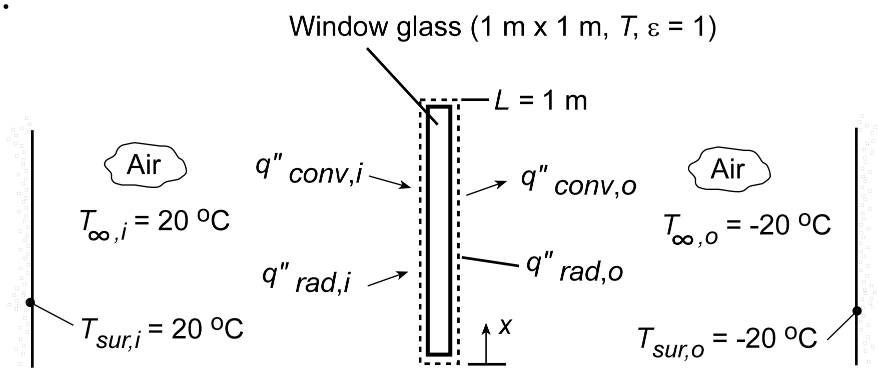
COMMENTS: Note that the heat loss by radiation is 30% larger than by free convection.

PROBLEM 9.19

KNOWN: Room and ambient air conditions for window glass.

FIND: Temperature of the glass and rate of heat loss.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible temperature gradients in the glass, (3) Inner and outer surfaces exposed to large surroundings.

PROPERTIES: *Table A.4*, air ($T_{f,i}$ and $T_{f,o}$): Obtained from the IHT *Properties* Tool Pad.

ANALYSIS: Performing an energy balance on the window pane, it follows that $\dot{E}_{in} = \dot{E}_{out}$, or

$$\varepsilon\sigma\left(T_{sur,i}^4 - T^4\right) + \overline{h}_i\left(T_{\infty,i} - T\right) = \varepsilon\sigma\left(T^4 - T_{sur}^4\right) + \overline{h}_o\left(T - T_{\infty,o}\right)$$

where $\overline{h}_i$ and $\overline{h}_o$ may be evaluated from Eq. 9.26.

$$\overline{Nu}_L = \left\{0.825 + \frac{0.387 Ra_L^{1/6}}{\left[1 + (0.492/Pr)^{9/16}\right]^{8/27}}\right\}^2$$

Using the *First Law* Model for an *Isothermal Plane Wall* and the *Correlations* and *Properties* Tool Pads of IHT, the energy balance equation was formulated and solved to obtain

$T = 273.8$ K <

The heat rate is then $q_i = q_o$, or

$$q_i = L^2\left[\varepsilon\sigma\left(T_{sur,i}^4 - T^4\right) + \overline{h}_i\left(T_\infty - T\right)\right] = 174.8\,W$$ <

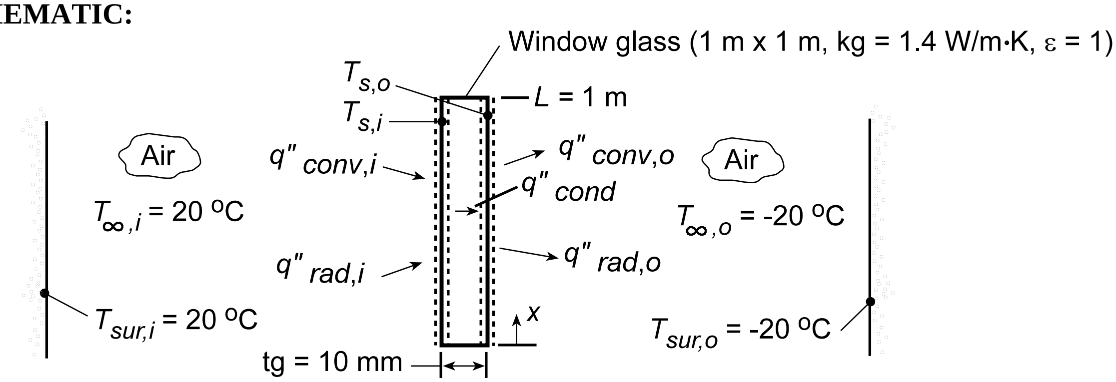
COMMENTS: The radiative and convective contributions to heat transfer at the inner and outer surfaces are $q_{rad,i} = 99.04$ W, $q_{conv,i} = 75.73$ W, $q_{rad,o} = 86.54$ W, and $q_{conv,o} = 88.23$ W, with corresponding convection coefficients of $\overline{h}_i = 3.95\ W/m^2\cdot K$ and $\overline{h}_o = 4.23\ W/m^2\cdot K$. The heat loss could be reduced significantly by installing a double pane window.

PROBLEM 9.20

KNOWN: Room and ambient air conditions for window glass. Thickness and thermal conductivity of glass.

FIND: Inner and outer surface temperatures and heat loss.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in the glass, (3) Inner and outer surfaces exposed to large surroundings.

PROPERTIES: *Table A.4*, air ($T_{f,i}$ and $T_{f,o}$): Obtained from the IHT *Properties* Tool Pad.

ANALYSIS: Performing energy balances at the inner and outer surfaces, we obtain, respectively,

$$\varepsilon\sigma\left(T_{sur,i}^4 - T_{s,i}^4\right) + \bar{h}_i\left(T_{\infty,i} - T_{s,i}\right) = (kg/tg)\left(T_{s,i} - T_{s,o}\right) \quad (1)$$

$$(kg/tg)\left(T_{s,i} - T_{s,o}\right) = \varepsilon\sigma\left(T_{s,o}^4 - T_{sur,o}^4\right) + \bar{h}_o\left(T_{s,o} - T_{\infty,o}\right) \quad (2)$$

where Eq. 9.26 may be used to evaluate $\bar{h}_i$ and $\bar{h}_o$

$$\overline{Nu}_L = \left\{0.825 + \frac{0.387 Ra_L^{1/6}}{\left[1 + (0.492/Pr)^{9/16}\right]^{8/27}}\right\}^2$$

Using the *First Law* Model for *One-dimensional Conduction* in a *Plane Wall* and the *Correlations* and *Properties* Tool Pads of IHT, the energy balance equations were formulated and solved to obtain

$T_{s,i} = 274.4$ K $\qquad$ $T_{s,o} = 273.2$ K $\qquad$ <

from which the heat loss is

$$q = \frac{k_g L^2}{t_g}\left(T_{s,i} - T_{s,o}\right) = 168.8\,\text{W}$$ <

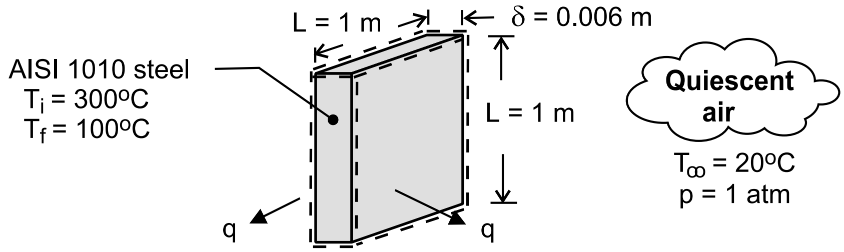
COMMENTS: By accounting for the thermal resistance of the glass, the heat loss is smaller (168.8 W) than that determined in the preceding problem (174.8 W) by assuming an isothermal pane.

PROBLEM 9.21

KNOWN: Plate dimensions, initial temperature, and final temperature. Air temperature.

FIND: (a) Initial cooling rate, (b) Time to reach prescribed final temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Plate is spacewise isothermal as it cools (lumped capacitance approximation), (2) Negligible heat transfer from minor sides of plate, (3) Thermal boundary layer development corresponds to that for an isolated plate (negligible interference between adjoining boundary layers). (4) Negligible radiation. (5) Constant properties.

PROPERTIES: *Table A-1*, AISI 1010 steel $\left(\bar{T} = 473\,K\right)$: ρ = 7854 kg/m^3, c = 513 J/kg·K. *Table A-4*, air ($T_{f,i}$ = 433 K): $\nu = 30.4 \times 10^{-6}$ m^2/s, k = 0.0361 W/m·K, $\alpha = 44.2 \times 10^{-6}$ m^2/s, Pr = 0.687, β = 0.0023 K^{-1}.

ANALYSIS: (a) The initial rate of heat transfer is $q_i = \bar{h}\,A_s\left(T_i - T_\infty\right)$, where $A_s \approx 2\,L^2 = 2\,m^2$. With $Ra_{L,i} = g\beta\,(T_i - T_\infty)L^3/\alpha\nu$ = 9.8 m/s^2 × 0.0021 (280)1m^3/44.2 × 10^{-6} m^2/s × 30.4 × 10^{-6} m^2/s = 4.72 × 10^9, Eq. 9.26 yields

$$\bar{h} = \frac{0.0361\,W/m\cdot K}{1m}\left\{0.825 + \frac{0.387\left(4.72\times10^9\right)^{1/6}}{\left[1+\left(0.492/0.687\right)^{9/16}\right]^{8/27}}\right\}^2 = 7.16\,W/m^2\cdot K$$

Hence, $$q_i = 7.16\,W/m^2\cdot K \times 2m^2 \times 280°C = 4010\,W \quad <$$

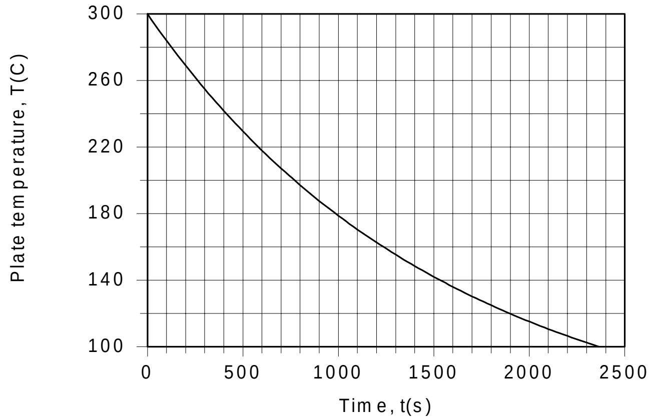
(b) From an energy balance at an instant of time for a control surface about the plate, $-q = \dot{E}_{st} = \rho\,L^2\,\delta\,c\,dT/dt$, the rate of change of the plate temperature is

$$\frac{dT}{dt} = -\frac{\bar{h}\,2L^2\left(T - T_\infty\right)}{\rho\,L^2\,\delta c} = -\frac{2\bar{h}}{\rho\,\delta c}\left(T - T_\infty\right)$$

where the Rayleigh number, and hence $\bar{h}$, changes with time due to the change in the temperature of the plate. Integrating the foregoing equation with the DER function of IHT, the following results are obtained for the temperature history of the plate.

Continued …..

PROBLEM 9.21 (Cont.)



The time for the plate to cool to 100°C is

$$t \approx 2365\,\text{s}$$ <

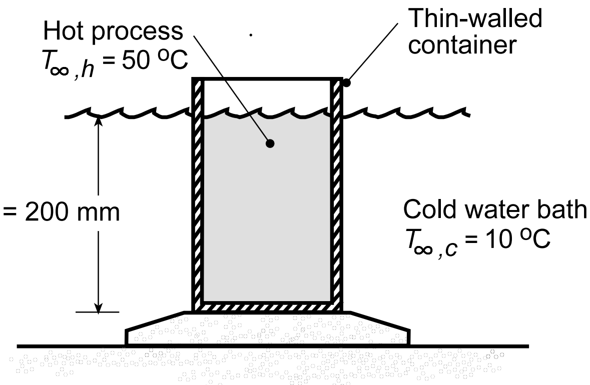
COMMENTS: (1) Although the plate temperature is comparatively large and radiation emission is significant relative to convection, much of the radiation leaving one plate is intercepted by the adjoining plate if the spacing between plates is small relative to their width. The net effect of radiation on the plate temperature would then be small. (2) Because of the increase in β and reductions in ν and α with increasing t, the Rayleigh number decreases only slightly as the plate cools from 300°C to 100°C (from 4.72×10^9 to 4.48×10^9), despite the significant reduction in $(T - T_\infty)$. The reduction in $\overline{h}$ from 7.2 to 5.6 $W/m^2\cdot K$ is principally due to a reduction in the thermal conductivity.

PROBLEM 9.22

KNOWN: Thin-walled container with hot process fluid at 50°C placed in a quiescent, cold water bath at 10°C.

FIND: (a) Overall heat transfer coefficient, U, between the hot and cold fluids, and (b) Compute and plot U as a function of the hot process fluid temperature for the range $20 \le T_{\infty,h} \le 50$°C.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Heat transfer at the surfaces approximated by free convection from a vertical plate, (3) Fluids are extensive and quiescent, (4) Hot process fluid thermophysical properties approximated as those of water, and (5) Negligible container wall thermal resistance.

PROPERTIES: *Table A.6*, Water (assume $T_{f,h}$ = 310 K): $\rho_h = 1/1.007 \times 10^{-3} = 993$ kg/m^3, $c_{p,h} = 4178$ J/kg·K, $\nu_h = \mu_h/\rho_h = 695 \times 10^{-6}$ N·s/m^2/993 kg/m^3 = 6.999×10^{-7} m^2/s, $k_h = 0.628$ W/m·K, $Pr_h = 4.62$, $\alpha_h = k_h/\rho_h c_{p,h} = 1.514 \times 10^{-7}$ m^2/s, $\beta_h = 361.9 \times 10^{-6}$ K^{-1} ; *Table A.6*, Water (assume $T_{f,c}$ = 295 K): $\rho_c = 1/1.002 \times 10^{-3} = 998$ kg/m^3, $c_{p,c} = 4181$ J/kg·K, $\nu_c = \mu_c/\rho_c = 959 \times 10^{-6}$ N·s/m^2/998 kg/m^3 = 9.609×10^{-7} m^2/s, k_c = 0.606 W/m·K, $Pr_c = 6.62$, $\alpha_c = k_c/\rho_c c_{p,c} = 1.452\times 10^{-7}$ m^2/s, $\beta_c = 227.5 \times 10^{-6}$ K^{-1}.

ANALYSIS: (a) The overall heat transfer coefficient between the hot process fluid, $T_{\infty,h}$, and the cold water bath fluid, $T_{\infty,c}$, is

$$U = \left(1/\bar{h}_h + 1/\bar{h}_c\right)^{-1} \quad (1)$$

where the average free convection coeffieicnts can be estimated from the vertical plate correlation Eq. 9.26, with the Rayleigh number, Eq. 9.25,

$$\overline{Nu}_L = \left\{0.825 + \frac{0.387 Ra_L^{1/6}}{\left[1+\left(0.492/Pr\right)^{9/16}\right]^{8/27}}\right\}^2 \qquad Ra_L = \frac{g\beta\Delta T L^3}{\nu\alpha} \quad (2,3)$$

To affect a solution, assume $T_s = \left(T_{\infty,h} - T_{\infty,i}\right)/2 = 30^\circ C = 303\,K$, so that the hot and cold fluid film temperatures are $T_{f,h}$ = 313 K ≈ 310 K and $T_{f,c}$ = 293 K ≈ 295 K. From an energy balance across the container walls,

$$\bar{h}_h\left(T_{\infty,h} - T_s\right) = \bar{h}_c\left(T_s - T_{\infty,c}\right) \quad (4)$$

the surface temperature T_s can be determined. Evaluating the correlation parameters, find:

Hot process fluid:

$$Ra_{L,h} = \frac{9.8\,m/s^2 \times 361.9\times10^{-6}\,K^{-1}\,(50-30)\,K\,(0.200\,m)^3}{6.999\times10^{-7}\,m^2/s \times 1.514\times10^{-7}\,m^2/s} = 5.357\times10^9$$

Continued...

$$\overline{Nu}_{L,h} = \left\{0.825 + \frac{0.387\left(5.357\times10^{9}\right)^{1/6}}{\left[1+\left(0.492/4.62\right)^{9/16}\right]^{8/27}}\right\}^2 = 251.5$$

$$\bar{h}_h = \overline{Nu}_{L,h}\,\frac{h_h}{L} = 251.5\times0.628\,\mathrm{W/m^2\cdot K}/0.200\mathrm{m} = 790\,\mathrm{W/m^2\cdot K}$$

Cold water bath:

$$Ra_{L,c} = \frac{9.8\dot{m}/s^2\times227.5\times10^{-6}\,K^{-1}\,(30-10)\,K\,(0.200m)^3}{9.609\times10^{-7}\,m^2/s\times1.452\times10^{-7}\,m^2/s} = 2.557\times10^{9}$$

$$\overline{Nu}_{L,c} = \left\{0.825 + \frac{0.387\left(2.557\times10^{9}\right)^{1/6}}{\left[1+\left(0.492/6.62\right)^{9/16}\right]^{8/27}}\right\}^2 = 203.9$$

$$\bar{h}_c = 203.9\times0.606\,\mathrm{W/m\cdot K}/0.200\,\mathrm{m} = 618\,\mathrm{W/m^2\cdot K}$$

From Eq. (1) find

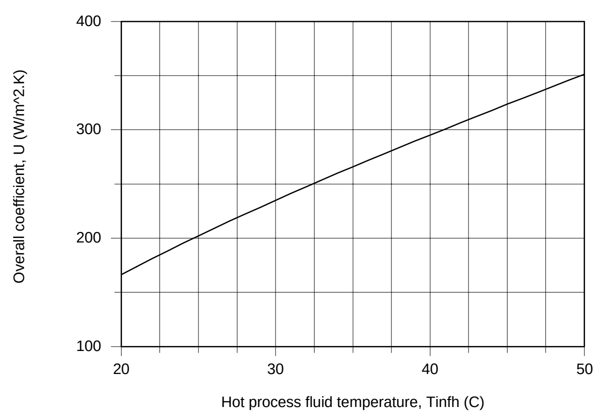
$$U = \left(1/790 + 1/618\right)^{-1}\,\mathrm{W/m^2\cdot K} = 347\,\mathrm{W/m^2\cdot K}$$ <

Using Eq.(4), find the resulting surface temperature

$$790\,\mathrm{W/m^2\cdot K}\,(50-T_s)\,\mathrm{K} = 618\,\mathrm{W/m^2\cdot K}\,(T_s-30)\,\mathrm{K} \qquad T_s = 32.4^\circ\mathrm{C}$$

Which compares favorably with our assumed value of 30°C.

(b) Using the *IHT Correlations Tool, Free Convection, Vertical Plate* and following the foregoing approach, the overall coefficient was computed as a function of the hot fluid temperature and is plotted below. Note that U increases almost linearly with $T_{\infty,h}$.



COMMENTS: For the conditions of part (a), using the IHT model of part (b) with thermophysical properties evaluated at the proper film temperatures, find U = 352 W/m·K with T_s = 32.4°C. Our approximate solution was a good one.

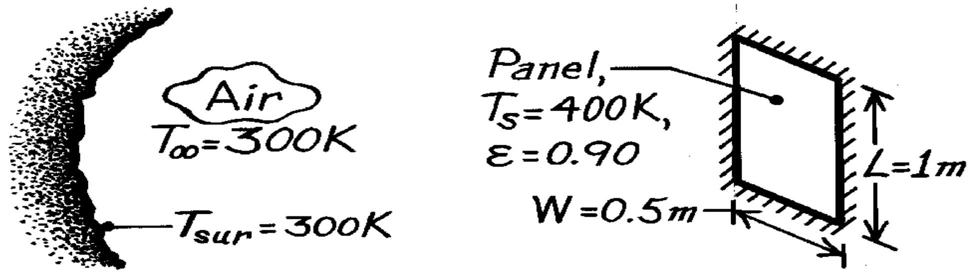
(2) Because the set of equations for part (b) is quite stiff, when using the IHT model you should follow the suggestions in the IHT Example 9.2 including use of the intrinsic function Tfluid_avg (T1,T2).

PROBLEM 9.23

KNOWN: Height, width, emissivity and temperature of heating panel. Room air and wall temperature.

FIND: Net rate of heat transfer from panel to room.

SCHEMATIC:



ASSUMPTIONS: (1) Quiescent air, (2) Walls of room form a large enclosure, (3) Negligible heat loss from back of panel.

PROPERTIES: *Table A-4*, Air (T_f = 350K, 1 atm): $\nu = 20.9 \times 10^{-6}\ m^2/s$, k = 0.03 W/m·K, $\alpha = 29.9 \times 10^{-6}\ m^2/s$, Pr = 0.700.

ANALYSIS: The heat loss from the panel by convection and radiation exchange is

$$q = \bar{h}A\left(T_s - T_\infty\right) + es\,A\left(T_s^4 - T_{sur}^4\right).$$

With

$$Ra_L = \frac{gb\left(T_s - T_\infty\right)L^3}{an} = \frac{9.8\,m/s^2\left(1/350K\right)\left(100K\right)\left(1m\right)^3}{\left(20.9\right)\left(29.9\right)\times 10^{-12}\,m^4/s^2} = 4.48 \times 10^9$$

and using the Churchill-Chu correlation for free convection from a vertical plate,

$$\overline{Nu}_L = \frac{\bar{h}L}{k} = \left\{0.825 + \frac{0.387 Ra_L^{1/6}}{\left[1 + \left(0.492/Pr\right)^{9/16}\right]^{8/27}}\right\}^2 = 196$$

$$\bar{h} = 196k/L = 196 \times 0.03\,W/m\cdot K/1m = 5.87\,W/m^2\cdot K.$$

Hence,

$$q = 5.86\,W/m^2\cdot K\left(0.5m^2\right)100K$$
$$+0.9 \times 5.67 \times 10^{-8}\,W/m^2\cdot K^4\left(0.5m^2\right)\left[\left(400\right)^4 - \left(300\right)^4\right]K$$

$$q = 293W + 447\ W = 740W.$$ <

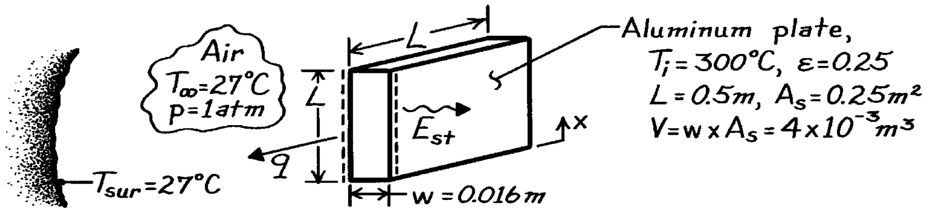
COMMENTS: As is typical of free convection in gases, heat transfer by surface radiation is comparable to, if not larger than, the convection rate. The *relative* contribution of free convection would increase with decreasing L and T_s.

PROBLEM 9.24

KNOWN: Initial temperature and dimensions of an aluminum plate. Condition of the plate surroundings. Plate emissivity.

FIND: (a) Initial cooling rate, (b) Validity of assuming negligible temperature gradients in the plate during the cooling process.

SCHEMATIC:



ASSUMPTIONS: (1) Plate temperature is uniform, (2) Chamber air is quiescent, (3) Chamber surface is much larger than that of plate, (4) Negligible heat transfer from edges.

PROPERTIES: *Table A-1*, Aluminum (573K): k = 232 W/m·K, c_p = 1022 J/kg·K, ρ = 2702 kg/m^3; *Table A-4*, Air (T_f = 436K, 1 atm): ν = 30.72 × 10^{-6} m^2/s, α = 44.7 × 10^{-6} m^2/s, k = 0.0363 W/m·K, Pr = 0.687, β = 0.00229 K^{-1}.

ANALYSIS: (a) Performing an energy balance on the plate,

$$-q = -2A_s\left[\bar{h}\left(T-T_\infty\right)+es\left(T^4-T_{sur}^4\right)\right]=\dot{E}_{st}=rVc_p\left[dT/dt\right]$$

$$dT/dt=-2\left[\bar{h}\left(T-T_\infty\right)+es\left(T^4-T_{sur}^4\right)\right]/\,rwc_p$$

Using the correlation of Eq. 9.27, with

$$Ra_L=\frac{gb\left(T_i-T_\infty\right)L^3}{na}=\frac{9.8\text{m/s}^2\times0.00229\text{K}^{-1}\left(300-27\right)\text{K}\left(0.5\text{m}\right)^3}{30.72\times10^{-6}\text{m}^2/\text{s}\times44.7\times10^{-6}\text{m}^2/\text{s}}=5.58\times10^8$$

$$\bar{h}=\frac{k}{L}\left\{0.68+\frac{0.670Ra_L^{1/4}}{\left[1+\left(0.492/\text{Pr}\right)^{9/16}\right]^{4/9}}\right\}=\frac{0.0363}{0.5}\left\{0.68+\frac{0.670\left(5.58\times10^8\right)^{1/4}}{\left[1+\left(0.492/0.687\right)^{9/16}\right]^{4/9}}\right\}$$

$$\bar{h}=5.8\text{W/m}^2\cdot\text{K}. \qquad <$$

Hence the initial cooling rate is

$$\frac{dT}{dt}=-\frac{2\left(5.8\text{W/m}^2\cdot\text{K}\left(300-27\right)^\circ\text{C}+0.25\times5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4\left[\left(573\text{K}\right)^4-\left(300\text{K}\right)^4\right]\right)}{2702\text{kg/m}^3\left(0.016\text{m}\right)1022\text{J/kg}\cdot\text{K}}$$

$$\frac{dT}{dt}=-0.136\text{K/s}. \qquad <$$

(b) To check the validity of neglecting temperature gradients across the plate thickness, calculate Bi = h_{eff} (w/2)/k where h_{eff} = q''_{tot} /(T_i - T_∞) = (1583 + 1413) W/m^2/273 K = 11.0 W/m^2·K. Hence

$$Bi=\left(11\text{W/m}^2\cdot\text{K}\right)\left(0.008\text{m}\right)/232\text{W/m}\cdot\text{K}=3.8\times10^{-4} \qquad <$$

and the assumption is excellent.

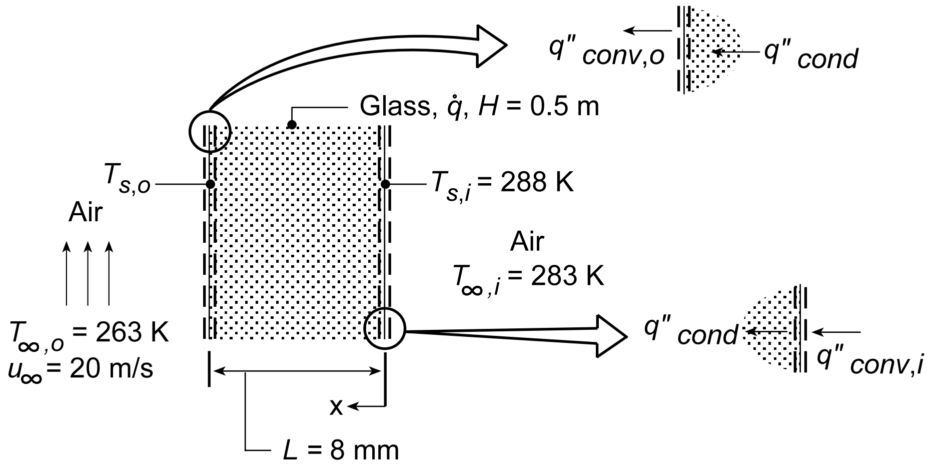
COMMENTS: (1) Longitudinal (x) temperature gradients are likely to be more severe than those associated with the plate thickness due to the variation of h with x. (2) Initially $q''_{conv} \approx q''_{rad}$.

PROBLEM 9.25

KNOWN: Boundary conditions associated with a rear window experiencing uniform volumetric heating.

FIND: (a) Volumetric heating rate $\dot{q}$ needed to maintain inner surface temperature at $T_{s,i}$ = 15°C, (b) Effects of $T_{\infty,o}$, u_∞, and $T_{\infty,i}$ on $\dot{q}$ and $T_{s,o}$.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, one-dimensional conditions, (2) Constant properties, (3) Uniform volumetric heating in window, (4) Convection heat transfer from interior surface of window to interior air may be approximated as free convection from a vertical plate, (5) Heat transfer from outer surface is due to forced convection over a flat plate in parallel flow.

PROPERTIES: *Table A.3*, Glass (300 K): k = 1.4 W/m·K: *Table A.4*, Air ($T_{f,i}$ = 12.5°C, 1 atm): ν = 14.6×10^{-6} m^2/s, k = 0.0251 W/m·K, α = 20.59×10^{-6} m^2/s, β = (1/285.5) = 3.503×10^{-3} K^{-1}, Pr = 0.711; ($T_{f,o} \approx 0$°C): ν = 13.49×10^{-6} m^2/s, k = 0.0241 W/m·K, Pr = 0.714.

ANALYSIS: (a) The temperature distribution in the glass is governed by the appropriate form of the heat equation, Eq. 3.39, whose general solution is given by Eq. 3.40.

$$T(x) = -(\dot{q}/2k)x^2 + C_1 x + C_2 .$$

The constants of integration may be evaluated by applying appropriate boundary conditions at x = 0. In particular, with T(0) = $T_{s,i}$, $C_2 = T_{s,i}$. Applying an energy balance to the inner surface, $q''_{cond} = q''_{conv,i}$

$$-k\left.\frac{dT}{dx}\right|_{x=0} = \bar{h}_i\left(T_{\infty,i} - T_{s,i}\right) \qquad -k\left.\left(-\frac{\dot{q}}{k}x + C_1\right)\right|_{x=0} = \bar{h}_i\left(T_{\infty,i} - T_{s,i}\right)$$

$$C_1 = -\left(\bar{h}_i/k\right)\left(T_{\infty,i} - T_{s,i}\right)$$

$$T(x) = -(\dot{q}/2k)x^2 - \frac{\bar{h}_i\left(T_{\infty,i} - T_{s,i}\right)}{k}x + T_{s,i} \qquad (1)$$

The required generation may then be obtained by formulating an energy balance at the outer surface, where $q''_{cond} = q''_{conv,o}$. Using Eq. (1),

$$-k\left.\frac{dT}{dx}\right|_{x=L} = \bar{h}_o\left(T_{s,o} - T_{\infty,o}\right) \qquad (2)$$

Continued...

$$-k\left.\frac{dT}{dx}\right|_{x=L} = -k\left(-\frac{\dot{q}L}{k}\right) + \bar{h}_i\left(T_{\infty,i} - T_{s,i}\right) = \dot{q}L + \bar{h}_i\left(T_{\infty,i} - T_{s,i}\right) \qquad (3)$$

Substituting Eq. (3) into Eq. (2), the energy balance becomes

$$\dot{q}L = \bar{h}_o\left(T_{s,o} - T_{\infty,o}\right) + \bar{h}_i\left(T_{s,i} - T_{\infty,i}\right) \qquad (4)$$

where $T_{s,o}$ may be evaluated by applying Eq. (1) at x = L.

$$T_{s,o} = -\frac{\dot{q}L^2}{2k} - \frac{\bar{h}_i\left(T_{\infty,i} - T_{s,i}\right)}{k}L + T_{s,i}\,. \qquad (5)$$

The *inside* convection coefficient may be obtained from Eq. 9.26. With

$$Ra_H = \frac{g\beta\left(T_{s,i} - T_{\infty,i}\right)H^3}{\nu\alpha} = \frac{9.8\,\mathrm{m/s^2}\left(3.503\times10^{-3}\,\mathrm{K^{-1}}\right)(15-10)\,\mathrm{K}\,(0.5\,\mathrm{m})^3}{14.60\times10^{-6}\,\mathrm{m^2/s}\times20.59\times10^{-6}\,\mathrm{m^2/s}} = 7.137\times10^7\,,$$

$$\overline{Nu}_H = \left[0.825 + \frac{0.387Ra_H^{1/6}}{\left[1+(0.492/Pr)^{9/16}\right]^{8/27}}\right]^2 = \left[0.825 + \frac{0.387\left(7.137\times10^7\right)^{1/6}}{\left[1+(0.492/0.711)^{9/16}\right]^{8/27}}\right]^2 = 56$$

$$\bar{h}_i = \overline{Nu}_H\,\frac{k}{H} = \frac{56\times0.0251\,\mathrm{W/m\cdot K}}{0.5\,\mathrm{m}} = 2.81\,\mathrm{W/m^2\cdot K}$$

The *outside* convection coefficient may be obtained by first evaluating the Reynolds number. With

$$Re_H = \frac{u_\infty H}{\nu} = \frac{20\,\mathrm{m/s}\times0.5\,\mathrm{m}}{13.49\times10^{-6}\,\mathrm{m^2/s}} = 7.413\times10^5$$

and with $Re_{x,c} = 5\times10^5$, mixed boundary layer conditions exist. Hence,

$$\overline{Nu}_H = \left(0.037\,Re_H^{4/5} - 871\right)Pr^{1/3} = \left[0.037\left(7.413\times10^5\right)^{4/5} - 871\right](0.714)^{1/3} = 864$$

$$\bar{h}_o = \overline{Nu}_H\,(k/H) = \left(864\times0.0241\,\mathrm{W/m\cdot K}\right)/0.5\,\mathrm{m} = 41.6\,\mathrm{W/m^2\cdot K}\,.$$

Eq. (5) may now be expressed as

$$T_{s,o} = -\frac{\dot{q}\,(0.008\,\mathrm{m})^2}{2\left(1.4\,\mathrm{W/m\cdot K}\right)} - \frac{2.81\,\mathrm{W/m^2\cdot K}\,(10-15)\,\mathrm{K}}{1.4\,\mathrm{W/m\cdot K}}\times0.008\,\mathrm{m} + 288\,\mathrm{K} = -2.286\times10^{-5}\dot{q} + 288.1\,\mathrm{K}$$

or, solving for $\dot{q}$,

$$\dot{q} = -43{,}745\left(T_{s,o} - 288.1\right) \qquad (6)$$

and substituting into Eq. (4),

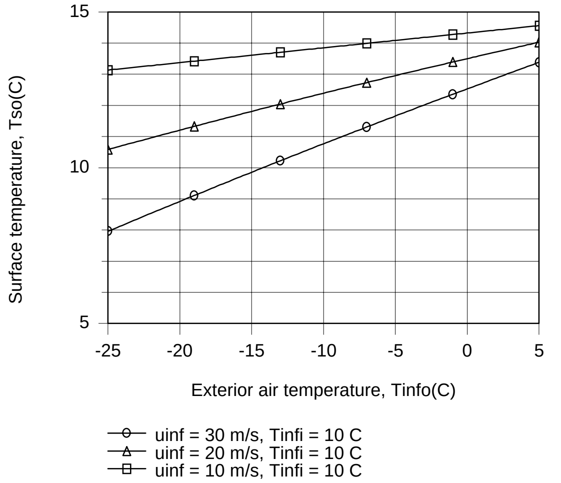
$$-43{,}745\left(T_{s,o} - 288.1\right)(0.008\,\mathrm{m}) = 41.6\,\mathrm{W/m^2\cdot K}\left(T_{s,o} - 263\,\mathrm{K}\right) + 2.81\,\mathrm{W/m^2\cdot K}\,(288\,\mathrm{K} - 283\,\mathrm{K})\,.$$

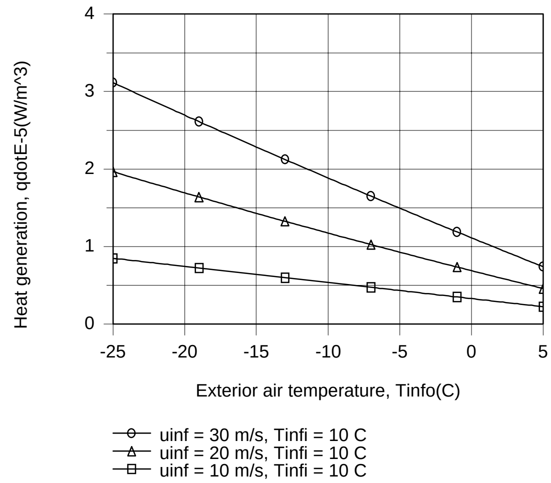
It follows that $T_{s,o}$ = 285.4 K in which case, from Eq. (6)

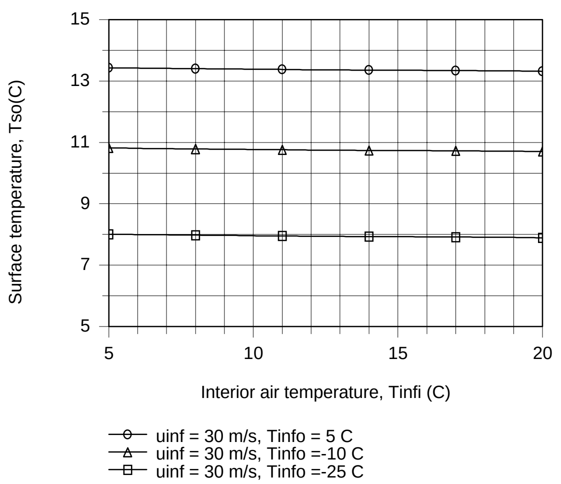
$$\dot{q} = 118\,\mathrm{kW/m^3}\,. \qquad <$$

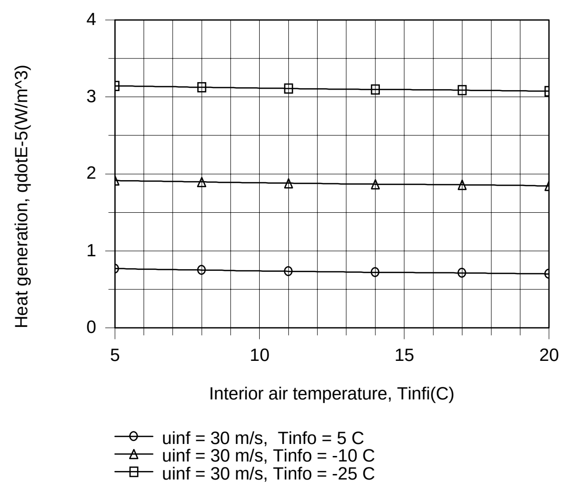
(b) The parametric calculations were performed using the *One-Dimensional, Steady-state Conduction* Model of IHT with the appropriate *Correlations* and *Properties* Tool Pads, and the results are as follows.

Continued...









For fixed $T_{s,i}$ and $T_{\infty,i}$, $T_{s,o}$ and $\dot{q}$ are strongly influenced by $T_{\infty,o}$ and u_∞, increasing and decreasing, respectively, with increasing $T_{\infty,o}$ and decreasing and increasing, respectively with increasing u_∞. For fixed $T_{s,i}$ and u_∞, $T_{s,o}$ and $\dot{q}$ are independent of $T_{\infty,i}$, but increase and decrease, respectively, with increasing $T_{\infty,o}$.

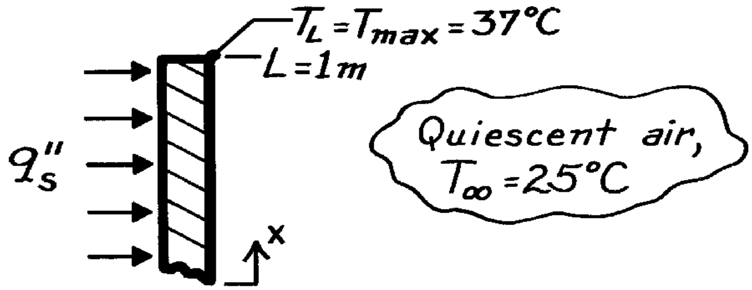
COMMENTS: In lieu of performing a surface energy balance at x = L, Eq. (4) may also be obtained by applying an energy balance to a control volume about the entire window.

PROBLEM 9.26

KNOWN: Vertical panel with uniform heat flux exposed to ambient air.

FIND: Allowable heat flux if maximum temperature is not to exceed a specified value, T_{max}.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties, (2) Radiative exchange with surroundings negligible.

PROPERTIES: *Table A-4*, Air ($T_f = (T_{L/2} + T_\infty)/2 = (35.4 + 25)°C/2 = 30.2°C = 303K$, 1 atm): $\nu = 16.19 \times 10^{-6}\ m^2/s$, $k = 26.5 \times 10^{-3}$ W/m·K, $\alpha = 22.9 \times 10^{-6}\ m^2/s$, Pr = 0.707.

ANALYSIS: Following the treatment of Section 9.6.1 for a vertical plate with uniform heat flux (constant q''_s), the heat flux can be evaluated as

$$q''_s = \bar{h}\Delta T_{L/2} \qquad \text{where} \qquad \Delta T_{L/2} = T_s(L/2) - T_\infty \tag{1,2}$$

and $\bar{h}$ is evaluated using an appropriate correlation for a constant temperature vertical plate. From Eq. 9.28,

$$\Delta T_x \equiv T_x - T_\infty = 1.15(x/L)^{1/5}\Delta T_{L/2} \tag{3}$$

and recognizing that the maximum temperature will occur at the top edge, x = L, use Eq. (3) to find

$$\Delta T_{L/2} = (37-25)^\circ C/1.15(1/1)^{1/5} = 10.4^\circ C \qquad \text{or} \qquad T_{L/2} = 35.4^\circ C.$$

Calculate now the Rayleigh number based upon $\Delta T_{L/2}$, with $T_f = (T_{L/2} + T_\infty)/2 = 303K$,

$$Ra_L = \frac{g b \Delta T L^3}{n a} \qquad \text{where} \qquad \Delta T = \Delta T_{L/2} \tag{4}$$

$$Ra_L = 9.8 m/s^2 (1/303K) \times 10.4K (1m)^3 / 16.19 \times 10^{-6} m^2/s \times 22.9 \times 10^{-6} m^2/s = 9.07 \times 10^8.$$

Since $Ra_L < 10^9$, the boundary layer flow is laminar; hence the correlation of Eq. 9.27 is appropriate,

$$\overline{Nu}_L = \frac{\bar{h}L}{k} = 0.68 + \frac{0.670 Ra_L^{1/4}}{\left[1 + (0.492/Pr)^{9/16}\right]^{4/9}} \tag{5}$$

$$\bar{h} = \left[\frac{0.0265 W/m \cdot K}{1m}\right]\left\{0.68 + 0.670\left(9.07 \times 10^8\right)^{1/4} / \left[1 + (0.492/0.707)^{9/16}\right]^{4/9}\right\} = 2.38 W/m^2 \cdot K.$$

From Eqs. (1) and (2) with numerical values for $\bar{h}$ and $\Delta T_{L/2}$, find

$$q''_s = 2.38 W/m^2 \cdot K \times 10.4^\circ C = 24.8 W/m^2.$$ <

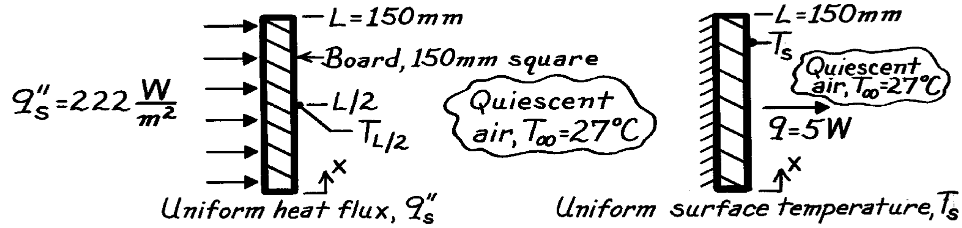
COMMENTS: Recognize that radiation exchange with the environment will be significant. Assuming $\bar{T}_s = T_{L/2}$, $T_{sur} = T_\infty$ and $\varepsilon = 1$, find $q''_{rad} = s\left(\bar{T}_s^{-4} - T_{sur}^4\right) = 66 W/m^2$.

PROBLEM 9.27

KNOWN: Vertical circuit board dissipating 5W to ambient air.

FIND: (a) Maximum temperature of the board assuming uniform surface heat flux and (b) Temperature of the board for an isothermal surface condition.

SCHEMATIC:



ASSUMPTIONS: (1) Either uniform q_s'' or T_s on the board, (2) Quiescent room air.

PROPERTIES: *Table A-4*, Air (T_f =($T_{L/2}$ + T_∞)/2 or (T_s + T_∞)/2, 1 atm), values used in iterations:

Iteration	T_f(K)	$\nu\cdot 10^6(m^2/s)$	$k\cdot 10^3$(W/m·K)	$\alpha\cdot 10^6(m^2/s)$	Pr
1	312	17.10	27.2	24.3	0.705
2	323	18.20	28.0	25.9	0.704
3	318	17.70	27.6	25.2	0.704
4	320	17.90	27.8	25.4	0.704

ANALYSIS: (a) For the uniform heat flux case (see Section 9.6.1), the heat flux is

$$q_s'' = \bar{h}\Delta T_{L/2} \qquad \text{where} \qquad \Delta T_{L/2} = T_{L/2} - T_\infty \qquad (1,2)$$

and $$q_s'' = q/A_s = 5\,W/(0.150m)^2 = 222\,W/m^2.$$

The maximum temperature on the board will occur at x = L and from Eq. 9.28 is

$$\Delta T_x = 1.15(x/L)^{1/5}\,\Delta T_{L/2} \qquad (3)$$

$$T_L = T_{max} = T_\infty + 1.15\,\Delta T_{L/2}.$$

The average heat transfer coefficient $\bar{h}$ is estimated from a vertical (uniform T_s) plate correlation based upon the temperature difference $\Delta T_{L/2}$. Recognize that an iterative procedure is required: (i) assume a value of $T_{L/2}$, use Eq. (2) to find $\Delta T_{L/2}$; (ii) evaluate the Rayleigh number

$$Ra_L = g\,b\Delta T_{L/2}L^3/na \qquad (4)$$

and select the appropriate correlation (either Eq. 9.26 or 9.27) to estimate $\bar{h}$; (iii) use Eq. (1) with values of $\bar{h}$ and $\Delta T_{L/2}$ to find the calculated value of q_s''; and (iv) repeat this procedure until the calculated value for q_s'' is close to q_s'' = 222 W/m^2, the required heat flux.

Continued …..

PROBLEM 9.27 (Cont.)

To evaluate properties for the correlation, use the film temperature,

$$T_f = \left(T_{L/2} + T_\infty\right)/2. \tag{5}$$

Iteration #1: Assume $T_{L/2}$ = 50°C and from Eqs. (2) and (5) find

$$\Delta T_{L/2} = (50-27)^\circ \mathrm{C} = 23^\circ \mathrm{C} \qquad\qquad T_f = (50+27)^\circ \mathrm{C}/2 = 312\mathrm{K}.$$

From Eq. (4), with $\beta = 1/T_f$, the Rayleigh number is

$$\mathrm{Ra}_L = 9.8\mathrm{m/s}^2 (1/312\mathrm{K}) \times 23^\circ\mathrm{C}(0.150\mathrm{m})^3 / \left(17.10\times10^{-6}\mathrm{m}^2/\mathrm{s}\right)\times\left(24.3\times10^{-6}\mathrm{m}^2/\mathrm{s}\right) = 5.868\times10^6.$$

Since $\mathrm{Ra}_L < 10^9$, the flow is laminar and Eq. 9.27 is appropriate

$$\overline{\mathrm{Nu}}_L = \frac{\bar{h}L}{k} = 0.68 + \frac{0.670\mathrm{Ra}_L^{1/4}}{\left[1+(0.492/\mathrm{Pr})^{9/16}\right]^{4/9}}$$

$$\bar{h}_L = \frac{0.0272\mathrm{W/m\cdot K}}{0.150\mathrm{m}}\left\{0.68 + 0.670\left(5.868\times10^6\right)^{1/4} / \left[1+(0.492/0.705)^{9/16}\right]^{4/9}\right\} = 4.71\mathrm{W/m^2\cdot K}.$$

Using Eq. (1), the calculated heat flux is

$$q_s'' = 4.71\mathrm{W/m^2\cdot K}\times 23^\circ\mathrm{C} = 108\mathrm{W/m^2}.$$

Since q_s'' < 222 W/m^2, the required value, another iteration with an increased estimate for $T_{L/2}$ is warranted. Further iteration results are tabulated.

Iteration	$T_{L/2}$(°C)	$\Delta T_{L/2}$(°C)	T_f(K)	Ra_L	$\bar{h}\left(\mathrm{W/m^2\cdot K}\right)$	$q_s''\left(\mathrm{W/m^2}\right)$
2	75	48	323	1.044×10^7	5.58	267
3	65	38	318	8.861×10^6	5.28	200
4	68	41	320	9.321×10^6	5.39	221

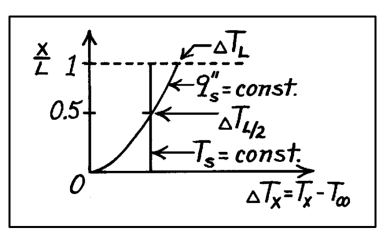
After Iteration 4, close agreement between the calculated and required q_s'' is achieved with $T_{L/2}$ = 68°C. From Eq. (3), the maximum board temperature is

$$T_L = T_{max} = 27^\circ\mathrm{C} + 1.15(41)^\circ\mathrm{C} = 74^\circ\mathrm{C}.$$ <

(b) For the uniform temperature case, the procedure for estimation of the average heat transfer coefficient is the same. Hence,

$$T_s = T_{L/2}\Big|_{q_s''} = 68^\circ\mathrm{C}.$$ <

COMMENTS: In both cases, q = 5W and $\bar{h} = 5.38\mathrm{W/m^2}$. However, the temperature distributions for the two cases are quite different as shown on the sketch. For q_s'' = constant, $\Delta T_x \sim x^{1/5}$ according to Eq. 9.28.

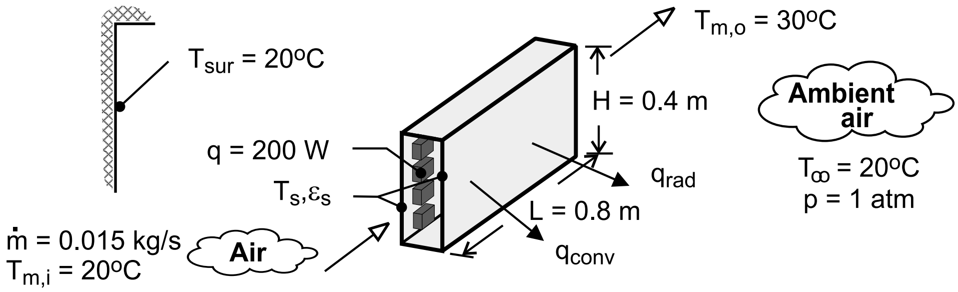


PROBLEM 9.28

KNOWN: Coolant flow rate and inlet and outlet temperatures. Dimensions and emissivity of channel side walls. Temperature of surroundings. Power dissipation.

FIND: (a) Temperature of sidewalls for $\varepsilon_s = 0.15$, (b) Temperature of sidewalls for $\varepsilon_s = 0.90$, (c) Sidewall temperatures with loss of coolant for $\varepsilon_s = 0.15$ and $\varepsilon_s = 0.90$.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) Negligible heat transfer from top and bottom surfaces of duct, (3) Isothermal side walls, (4) Large surroundings, (5) Negligible changes in flow work and potential and kinetic energies of coolant, (6) Constant properties.

PROPERTIES: *Table A-4*, air $\left(\bar{T}_m = 298\,\text{K}\right)$: c_p = 1007 J/kg·K. Air properties required for the free convection calculations depend on T_s and were evaluated as part of the iterative solution obtained using the IHT software.

ANALYSIS: (a) The heat dissipated by the components is transferred by forced convection to the coolant (q_c), as well as by natural convection (q_{conv}) and radiation (q_{rad}) to the ambient air and the surroundings. Hence,

$$q = q_c + q_{conv} + q_{rad} = 200\,\text{W} \tag{1}$$

$$q_c = \dot{m}c_p\left(T_{m,o} - T_{m,i}\right) = 0.015\,\text{kg/s} \times 1007\,\text{J/kg}\cdot\text{K} \times 10°\text{C} = 151\,\text{W} \tag{2}$$

$$q_{conv} = 2\bar{h}A_s\left(T_s - T_\infty\right) \tag{3}$$

where $A_s = H \times L = 0.32\,\text{m}^2$ and $\bar{h}$ is obtained from Eq. 9.26, with $Ra_H = g\beta\left(T_s - T_\infty\right)H^3/\alpha\nu$.

$$\bar{h} = \frac{k}{H}\left\{0.825 + \frac{0.387\,Ra_H^{1/6}}{\left[1 + \left(0.492/Pr\right)^{9/16}\right]^{8/27}}\right\}^2 \tag{3a}$$

$$q_{rad} = 2A_s\varepsilon_s\sigma\left(T_s^4 - T_{sur}^4\right) \tag{4}$$

Substituting Eqs. (2) – (4) into (1) and solving using the IHT software with $\varepsilon_s = 0.15$, we obtain

$$T_s = 308.8\,\text{K} = 35.8°\text{C}$$ <

The corresponding heat rates are q_{conv} = 39.6 W and q_{rad} = 9.4 W.

(b) For $\varepsilon_s = 0.90$ and q_c = 151 W, the solution to Eqs. (1) – (4) yields

Continued …..

$$T_s = 301.8\,\text{K} = 28.8^\circ\text{C}$$ <

with q_{conv} = 18.7 W and q_{rad} = 30.3 W. Hence, enhanced emission from the surface yields a lower operating temperature and heat transfer by radiation now exceeds that due to conduction.

(c) With loss of coolant flow, we can expect all of the heat to be dissipated from the sidewalls (q_c = 0). Solving Eqs. (1), (3) and (4), we obtain

$$\varepsilon_s = 0.15: \qquad T_s = 341.8\,\text{K} = 68.8^\circ\text{C}$$ <

$$q_{conv} = 165.9\,\text{W}, \qquad q_{rad} = 34.1\,\text{W}$$

$$\varepsilon_s = 0.90: \qquad T_s = 322.5\,\text{K} = 49.5^\circ\text{C}$$ <

$$q_{conv} = 87.6\,\text{W}, \qquad q_{rad} = 112.4\,\text{W}$$

Since the temperature of the electronic components exceeds that of the sidewalls, the value of T_s = 68.8°C corresponding to ε_s = 0.15 may be unacceptable, in which case the high emissivity coating should be applied to the walls.

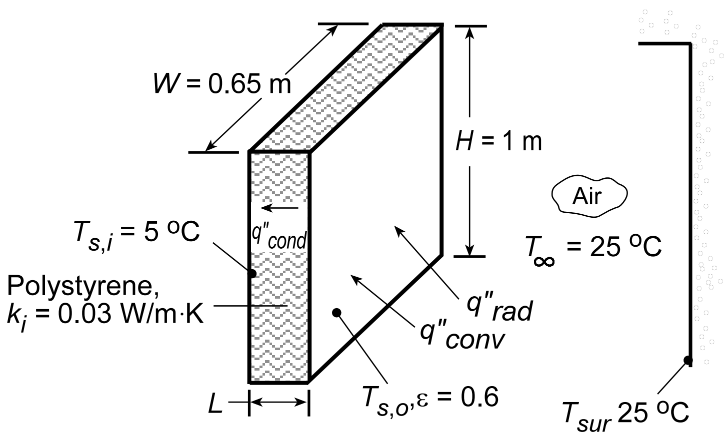
COMMENTS: For the foregoing cases the convection coefficient is in the range $3.31 \le \overline{h} \le 5.31$ W/m^2·K, with the smallest value corresponding to (q_c = 151 W, ε_s = 0.90) and the largest value to (q_c = 0, ε_s = 0.15). The radiation coefficient is in the range $0.93 \le h_{rad} \le 5.96$ W/m^2·K, with the smallest value corresponding to (q_c = 151 W, ε_s = 0.15) and the largest value to (q_c = 0, ε_s = 0.90).

PROBLEM 9.29

KNOWN: Dimensions, interior surface temperature, and exterior surface emissivity of a refrigerator door. Temperature of ambient air and surroundings.

FIND: (a) Heat gain with no insulation, (b) Heat gain as a function of thickness for polystyrene insulation.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible thermal resistance of steel and polypropylene sheets, (3) Negligible contact resistance between sheets and insulation, (4) One-dimensional conduction in insulation, (5) Quiescent air.

PROPERTIES: *Table A.4*, air (T_f = 288 K): $\nu = 14.82 \times 10^{-6}$ m²/s, $\alpha = 20.92 \times 10^{-6}$ m²/s, k = 0.0253 W/m·K, Pr = 0.71, $\beta = 0.00347$ K^{-1}.

ANALYSIS: (a) Without insulation, $T_{s,o} = T_{s,i}$ = 278 K and the heat gain is

$$q_{wo} = \bar{h}A_s\left(T_\infty - T_{s,i}\right) + \varepsilon\sigma A_s\left(T_{sur}^4 - T_{s,i}^4\right)$$

where A_s = HW = 0.65 m². With a Rayleigh number of $Ra_H = g\beta\left(T_\infty - T_{s,i}\right)H^3/\alpha\nu$ = 9.8 m/s²(0.00347 K^{-1})(20 K)(1)³/(20.92 × 10^{-6} m²/s)(14.82 × 10^{-6} m²/s) = 2.19 × 10⁹, Eq. 9.26 yields

$$\overline{Nu}_H = \left\{0.825 + \frac{0.387\left(2.19\times10^9\right)^{1/6}}{\left[1+\left(0.492/0.71\right)^{9/16}\right]^{8/27}}\right\}^2 = 156.6$$

$$\bar{h} = \overline{Nu}_H\left(k/H\right) = 156.6\left(0.0253\,W/m\cdot K/1m\right) = 4.0\,W/m^2\cdot K$$

$$q_{wo} = 4.0\,W/m^2\cdot K\left(0.65\,m^2\right)\left(20\,K\right) + 0.6\left(5.67\times10^{-8}\,W/m^2\cdot K^4\right)\left(0.65\,m^2\right)\left(298^4 - 278^4\right)K^4$$

$$q_{wo} = \left(52.00 + 42.3\right)W = 94.3\,W$$ <

(b) With the insulation, $T_{s,o}$ may be determined by performing an energy balance at the outer surface, where $q''_{conv} + q''_{rad} = q''_{cond}$, or

$$\bar{h}\left(T_\infty - T_{s,o}\right) + \varepsilon\sigma\left(T_{sur}^4 - T_{s,o}^4\right) = \frac{k_i}{L}\left(T_{s,o} - T_{s,i}\right)$$

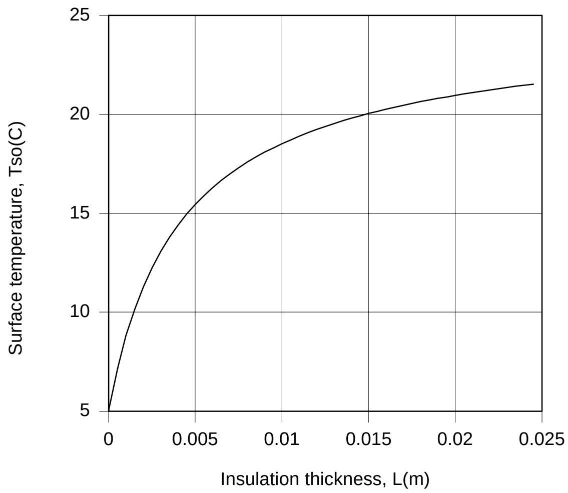
Using the *IHT First Law* Model for a *Nonisothermal Plane Wall* with the appropriate *Correlations* and *Properties* Tool Pads and evaluating the heat gain from

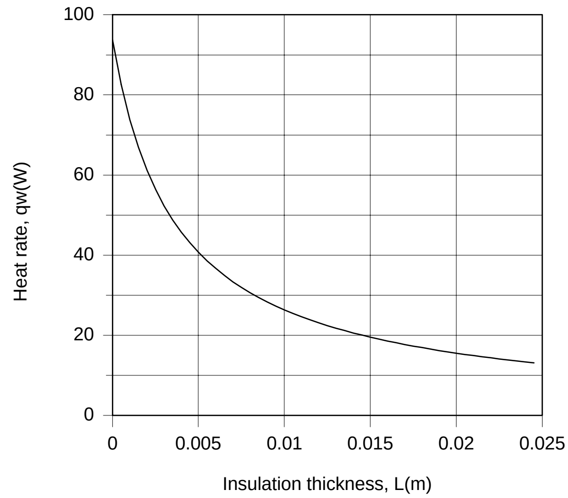
Continued...

PROBLEM 9.29 (Cont.)

$$q_w = \frac{k_i A_s}{L}\left(T_{s,o} - T_{s,i}\right)$$

the following results are obtained for the effect of L on $T_{s,o}$ and q_w.





The outer surface temperature increases with increasing L, causing a reduction in the rate of heat transfer to the refrigerator compartment. For L = 0.025 m, $\overline{h}$ = 2.29 W/m^2·K, h_{rad} = 3.54 W/m^2·K, q_{conv} = 5.16 W, q_{rad} = 7.99 W, q_w = 13.15 W, and $T_{s,o}$ = 21.5°C.

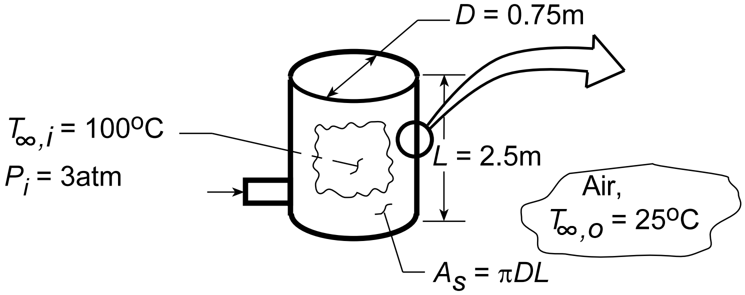
COMMENTS: The insulation is extremely effective in reducing the heat load, and there would be little value to increasing L beyond 25 mm.

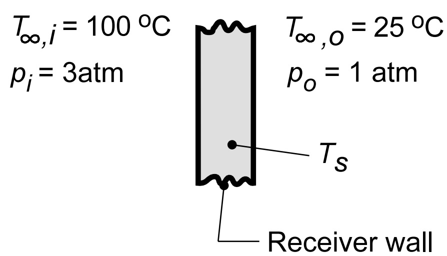
PROBLEM 9.30

KNOWN: Air receiving tank of height 2.5 m and diameter 0.75 m; inside air is at 3 atm and 100°C while outside ambient air is 25°C.

FIND: (a) Receiver wall temperature and heat transfer to the ambient air; assume receiver wall is T_s = 60°C to facilitate use of the free convection correlations; (b) Whether film temperatures $T_{f,i}$ and $T_{f,o}$ were reasonable; if not, use an iteration procedure to find consistent values; and (c) Receiver wall temperatures, $T_{s,i}$ and $T_{s,o,}$ considering radiation exchange from the exterior surface ($\varepsilon_{s,o}$ = 0.85) and thermal resistance of the wall (20 mm thick, k = 0.25W/m·K); represent the system by a thermal circuit.

SCHEMATIC:

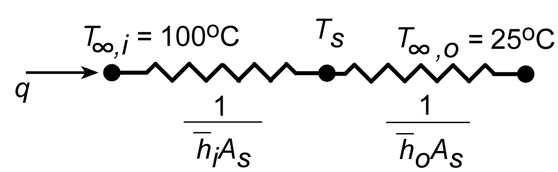




ASSUMPTIONS: (1) Surface radiation effects are negligible, parts (a,b), (2) Losses from top and bottom of receiver are negligible, (3) Thermal resistance of receiver wall is negligible compared to free convection resistance, parts (a,b), (4) Interior and exterior air is quiescent and extensive.

PROPERTIES: *Table A-4,* Air (assume $T_{f,o}$ = 315 K, 1 atm): $\nu = 1.74 \times 10^{-5}$ m²/s, k = 0.02741 W/m·K, $\alpha = 2.472 \times 10^{-5}$ m²/s, Pr =0.7049; *Table A-4,* Air (assume $T_{f,i}$ = 350 K, 3 atm): $\nu = 2.092 \times 10^{-5}$ m²/s/3= 6.973×10^{-6} m²/s, k = 0.030 W/m·K, $\alpha = 2.990 \times 10^{-5}$ m²/s/3 = 9.967×10^{-6} m²/s, Pr = 0.700. Note that the pressure effect is present for ν and α since ρ(1 atm) = 1/3ρ(3 atm); other properties (c_p, k, μ) are assumed independent of pressure.

ANALYSIS: The heat transfer rate from the receiver follows from the thermal circuit,



$$q = \frac{\Delta T}{R_t} = \frac{T_{\infty,i} - T_{\infty,o}}{1/\bar{h}_o A_s + 1/\bar{h}_i A_s} = \frac{A_s\left(T_{\infty,i} - T_{\infty,o}\right)}{1/\bar{h}_o + 1/\bar{h}_i} \text{s} \quad (1)$$

where $\bar{h}_o$ and $\bar{h}_i$ must be estimated from free convection correlations. We must assume a value of T_s in order to obtain first estimates for $\Delta T_o = T_s - T_{\infty,o}$ and $\Delta T_i = T_{\infty,o} - T_s$ as well as $T_{f,o}$ and $T_{f,i}$. Assume that T_s = 60°C, then ΔT_o = 60 - 25 = 35°C, $T_{f,o}$ = 315 K and ΔT_i = 100 - 60 = 40°C, and $T_{f,i}$ = 350 K.

$$Ra_{L,o} = \frac{g\beta\Delta T L^3}{\nu\alpha} = \frac{9.8\,\text{m/s}^2\,(1/315\,\text{K}) \times 35\,\text{K}\,(2.5\,\text{m})^3}{1.74\times10^{-5}\,\text{m}^2/\text{s} \times 2.472\times10^{-5}\,\text{m}^2/\text{s}} = 3.952\times10^{10}$$

$$Ra_{L,i} = \frac{9.8\,\text{m/s}^2\,(1/350\,\text{K}) \times 40\,\text{K}\,(2.5\,\text{m})^3}{6.973\times10^{-6}\,\text{m}^2/\text{s} \times 9.967\times10^{-6}\,\text{m}^2/\text{s}} = 2.518\times10^{11}$$

Approximating the receiver wall as a vertical plate, Eq. 9.26 yields

Continued...

$$\overline{Nu}_{L,o} = \left[0.825 + \frac{0.387 Ra_{L,o}^{1/6}}{\left[1+\left(0.492/Pr\right)^{9/16}\right]^{8/27}}\right]^2 = \left[0.825 + \frac{0.387\left(3.952\times10^{10}\right)^{1/6}}{\left[1+\left(0.492/0.7049\right)^{9/16}\right]^{8/27}}\right]^2 = 390.0$$

$$\overline{Nu}_{L,i} = \frac{\overline{h}_{L,i}L}{k} = \left[0.825 + \frac{0.387\left(2.518\times10^{11}\right)^{1/6}}{\left[1+\left(0.492/0.700\right)^{9/16}\right]^{8/27}}\right]^2 = 706.4$$

$$\overline{h}_{L,o} = \frac{0.02741\,W/m\cdot K}{2.5m}\times 390.0 = 4.27\,W/m^2\cdot K \qquad \overline{h}_{L,i} = \frac{0.030\,W/m\cdot K}{2.5m}\times 706.4 = 8.48\,W/m^2\cdot K$$

From Eq. (1),

$$q = \pi\times0.75m\times2.5m\left(100-25\right)K\Big/\left[\frac{1}{4.27}+\frac{1}{8.48}\right]m^2/K\cdot W = 1225W$$ <

Also,

$$T_s = T_{\infty,i} - q/\overline{h}_i\,A_s = 100^\circ C - 1255\,W\Big/\left(8.48\,W/m^2\cdot K\times\pi\times0.75\,m\times2.5\,m\right) = 74.9^\circ C$$ <

(b) From the above result for T_s, the computed film temperatures are

$$T_{f,o} = 323\,K \qquad T_{f,i} = 360\,K$$

as compared to assumed values of 315 and 350 K, respectively. Using *IHT Correlation Tools* for the *Free Convection*, *Vertical Plate*, and the thermal circuit representing Eq. (1) to find T_s, rather than using as assumed value,

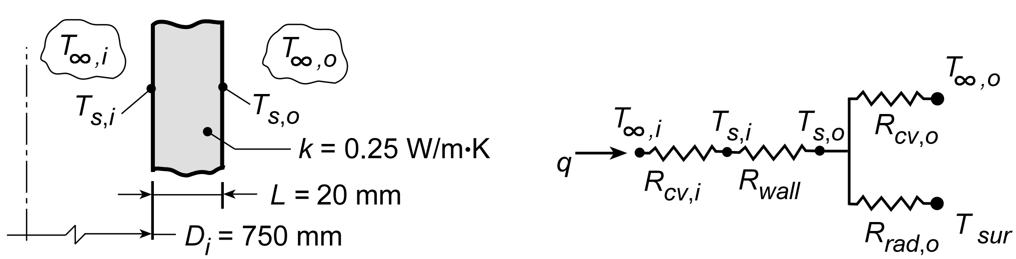
$$\frac{T_{\infty,o}-T_s}{1/\overline{h}_o} = \frac{T_s - T_{\infty,o}}{1/\overline{h}_o}$$

we found

q = 1262 W $\qquad\qquad$ T_s = 71.4°C <

with $T_{f,o}$ = 321K and 359 K. The iteration only influenced the heat rate slightly.

(c) Considering effects due to thermal resistance of the tank wall and radiation exchange, the thermal resistance network representing the system is shown below.



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Using the *IHT Model*, *Thermal Network*, with the *Correlation Tool for Free Convection*, *Vertical Plate*, and *Properties Tool* for *Air*, a model was developed which incorporates all the foregoing equations of parts (a,b), but includes the thermal resistance of the wall, Table 3.3,

$$R_{wall} = \frac{\ell n\left(D_i / D_o\right)}{2\pi Lk} \qquad\qquad D_o = D_i + 2\times t$$

Continued...

The results of the analyses are tabulated below showing for comparison those from parts (a) and (b):

Part	$R_{cv,i}$ (K/W)	R_w (K/W)	$R_{cv,o}$ (K/W)	R_{rad} (K/W)	$T_{s,i}$ (°C)	$T_{s,o}$ (°C)	q W
(a)	0.0200	0	0.0398	∞	74.9*	74.9*	1255
(b)	0.0227	0	0.0367	∞	71.4	71.4	1262
(c)	0.0219	0.0132	0.0419	0.0280	68.4	49.3	1445

*Recall we assumed T_s = 60°C in order to simplify the correlation calculation with fixed values of ΔT_i, ΔT_o as well as $T_{f,o}$, $T_{f,i}$.

COMMENTS: (1) In the table note the slight difference between results using assumed values for T_f and ΔT in the correlations (part (a)) and the exact solution (part (b)).

(2) In the part (c) results, considering thermal resistance of the wall and the radiation exchange process, the net effect was to reduce the overall thermal resistance of the system and, hence, the heat rate increased.

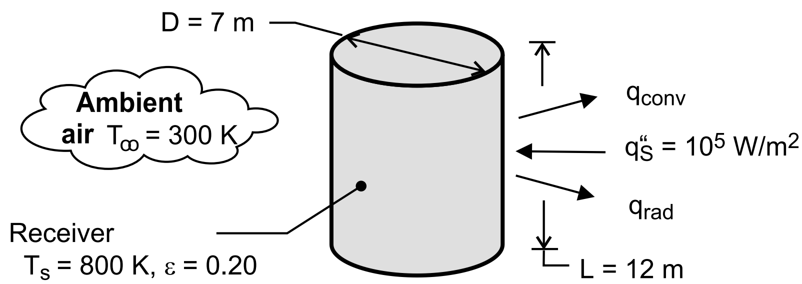
(3) In the part (c) analysis, the *IHT Thermal Resistance Network* model was used to create the thermal circuit and generate the required energy balances. The convection resistances were determined from appropriate *Convection Correlation Tools*. The code was developed in two steps: (1) Solve the energy balance relations from the *Network* with assigned values for h_i and h_o to demonstrate that the energy relations were correct and then (2) Call in the *Convection Correlations* and solve with variable coefficients. Because this equation set is very stiff, we used the intrinsic heat transfer function *Tfluid_avg* and followed these steps in the solution: Step (1): Assign constant values to the film temperatures, T_{fi} and T_{fo}, and to the temperature differences in the convection correlations, ΔT_i and ΔT_o; and in the *Initial Guesses* table, restrain all thermal resistances to be positive (minimum value = 1e-20); *Solve*; Step (2): Allow the film temperatures to be unknowns but keep assigned variables for the temperature differences; use the *Load* option and *Solve*. Step (3): Repeat the previous step but allowing the temperature differences to be unknowns. Even though you get a "successful solve" message, repeat the *Load-Solve* sequence until you see no changes in key variables so that you are assured that the Solver has fully converged on the solution.

PROBLEM 9.31

KNOWN: Dimensions and emissivity of cylindrical solar receiver. Incident solar flux. Temperature of ambient air.

FIND: (a) Heat loss and collection efficiency for a prescribed receiver temperature, (b) Effect of receiver temperature on heat losses and collector efficiency.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) Ambient air is quiescent, (3) Incident solar flux is uniformly distributed over receiver surface, (4) All of the incident solar flux is absorbed by the receiver, (5) Negligible irradiation from the surroundings, (6) Uniform receiver surface temperature, (7) Curvature of cylinder has a negligible effect on boundary layer development, (8) Constant properties.

PROPERTIES: *Table A-4*, air (T_f = 550 K): k = 0.0439 W/m·K, $\nu = 45.6 \times 10^{-6}$ m^2/s, $\alpha = 66.7 \times 10^{-6}$ m^2/s, Pr = 0.683, $\beta = 1.82 \times 10^{-3}$ K^{-1}.

ANALYSIS: (a) The total heat loss is

$$q = q_{rad} + q_{conv} = A_s \varepsilon \sigma T_s^4 + \bar{h} A_s \left(T_s - T_\infty\right)$$

With $Ra_L = g\beta(T_s - T_\infty)L^3/\nu\alpha = 9.8\ \text{m/s}^2\ (1.82 \times 10^{-3}\ \text{K}^{-1})\ 500\text{K}\ (12\text{m})^3/(45.6 \times 66.7 \times 10^{-12}\ \text{m}^4/\text{s}^2) = 5.07 \times 10^{12}$, Eq. 9.26 yields

$$\bar{h} = \frac{k}{L}\left\{0.825 + \frac{0.387\, Ra_L^{1/6}}{\left[1+\left(0.492/Pr\right)^{9/16}\right]^{8/27}}\right\}^2 = \frac{0.0439\,\text{W/m}\cdot\text{K}}{12\text{m}}\left\{0.825+42.4\right\}^2 = 6.83\,\text{W/m}^2\cdot\text{K}$$

Hence, with $A_s = \pi DL = 264\ \text{m}^2$

$$q = 264\,\text{m}^2 \times 0.2 \times 5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left(800\,\text{K}\right)^4 + 264\,\text{m}^2 \times 6.83\,\text{W/m}^2\cdot\text{K}\left(500\,\text{K}\right)$$

$$q = q_{rad} + q_{conv} = 1.23\times10^6\,\text{W} + 9.01\times10^5\,\text{W} = 2.13\times10^6\,\text{W} \qquad <$$

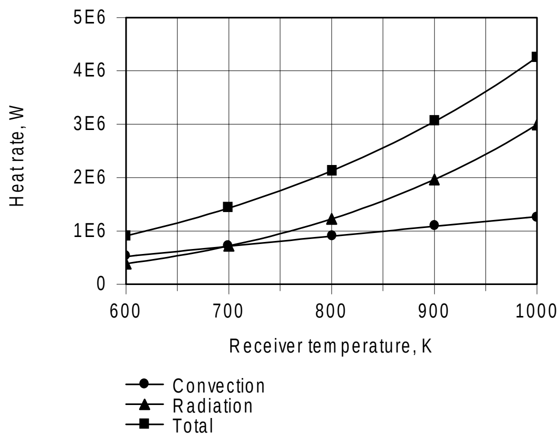
With $A_s q_s'' = 2.64\times10^7$ W, the collector efficiency is

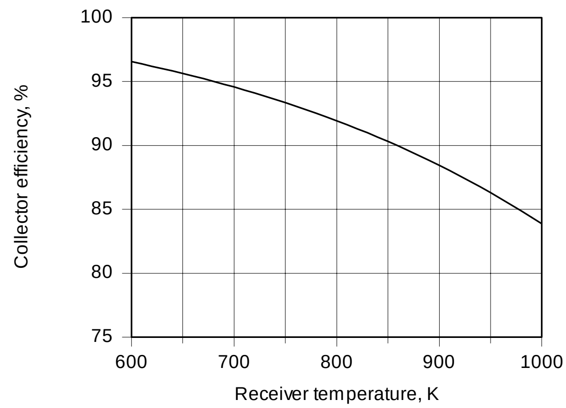
$$\eta = \left(\frac{A_s q_s'' - q}{A_s q_s''}\right)100 = \frac{\left(2.64\times10^7 - 2.13\times10^6\right)\text{W}}{2.64\times10^7\ \text{W}}(100) = 91.9\% \qquad <$$

Continued …..

PROBLEM 9.31 (Cont.)

(b) As shown below, because of its dependence on temperature to the fourth power, q_{rad} increases more significantly with increasing T_s than does q_{conv}, and the effect on the efficiency is pronounced.





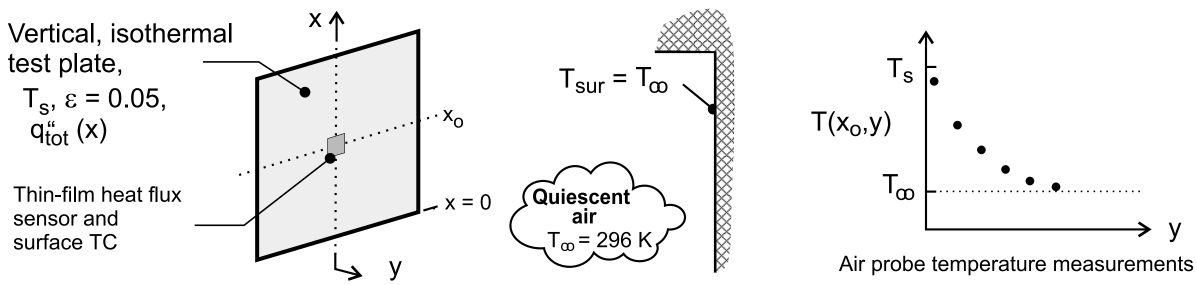
COMMENTS: The collector efficiency is also reduced by the inability to have a perfectly absorbing receiver. Partial reflection of the incident solar flux will reduce the efficiency by at least several percent.

PROBLEM 9.32

KNOWN: An experimental apparatus for measuring the local convection coefficient and the boundary layer temperature distribution for a heated vertical plate immersed in an extensive, quiescent fluid.

FIND: (a) An expression for estimating the radiation heat flux from the sensor as a function of the surface emissivity, surroundings temperature, and the quantity (T_s - T_∞); (b) Using this expression, apply the correction to the measured total heat flux, q''_{tot}, (see Table 1 below for data) to obtain the onvection heat flux, q''_{cv}, and calculate the convection coefficient; (c) Calculate and plot the local convection coefficient, $h_x(x)$, as a function the x-coordinate using the similarity solution, Eqs. 9.19 and 9.20; on the same graph, plot the experimental points; comment on the comparison between the experimental and analytical results; and (d) Compare the experimental boundary-layer air temperature measurements (see Table 2 below for data) with results from the similarity solution, Fig. 9.4(b). Summarize the results of your analysis using the similarity parameter, η, and the dimensionless temperature, T^*. Comment on the comparison between the experimental and analytical results.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Test plate at a uniform temperature, (3) Ambient air is quiescent, (4) Room walls are isothermal and at the same temperature as the plate.

PROPERTIES: *Table A-4*, Air (T_f =(T_s + T_∞)/2 = 303 K, 1 atm): $\nu = 16.19 \times 10^{-6}$ m^2/s, Pr = 0.707, $\beta = 1/T_f$.

ANALYSIS: (a) The radiation heat flux from the sensor as a function of the surface emissivity, surroundings temperature, and the quantity (T_s - T_∞) follows from Eqs. (1.8) and (1.9)

$$q''_{rad} = \bar{h}_{rad}\left(T_s - T_\infty\right) \qquad \bar{h}_{rad} = \varepsilon\sigma\left(T_s + T_\infty\right)\left(T_s^2 + T_\infty^2\right) \tag{1,2}$$

where $T_{sur} = T_\infty$. Since $T_s \approx T_\infty$, $\bar{h}_{rad} \approx 4\varepsilon\sigma\bar{T}^3$ where $\bar{T} = \left(T_s + T_\infty\right)/2$.

(b) Using the above expression, the radiation heat flux, q''_{rad}, is calculated. This correction is applied to the measured total heat flux, q''_{tot}, to obtain the convection heat flux, q''_{cv}, from which the local convection coefficient, $h_{x,exp}$ is calculated.

$$q''_{cv} = q''_{tot} - q''_{rad} \tag{3}$$

$$h''_{x,exp} = q''_{cv} / \left(T_s - T_\infty\right) \tag{4}$$

Continued …..

The heat flux sensor data are given in the first row of the table below, and the subsequent rows labeled (b) are calculated using Eqs. (1, 3, 4).

Table 1
Heat flux sensor data and convection coefficient calculation results

		$T_s - T_\infty = 7.7$ K					
	x (mm)	25	75	175	275	375	475
Data	q''_{tot} (W/m^2)	41.4	27.2	22.0	20.1	18.3	17.2
(b)	q''_{rad} (W/m^2)	2.28	2.28	2.28	2.28	2.28	2.28
(b)	q''_{cv} (W/m^2)	39.12	24.92	19.72	17.82	16.02	14.92
(b)	$h_{x,exp}$ (W/m^2·K)	5.08	3.24	2.56	2.31	2.08	1.94
(c)	$h_{x,ss}$ (W/m^2·K)	4.44	3.37	2.73	2.44	2.26	2.13

(c) The similarity solution for the vertical surface, Section 9.4, provides the expression for the local Nusselt number in terms of the dimensionless parameters T^* and η. Using Eqs. (9.19) and (9.20),

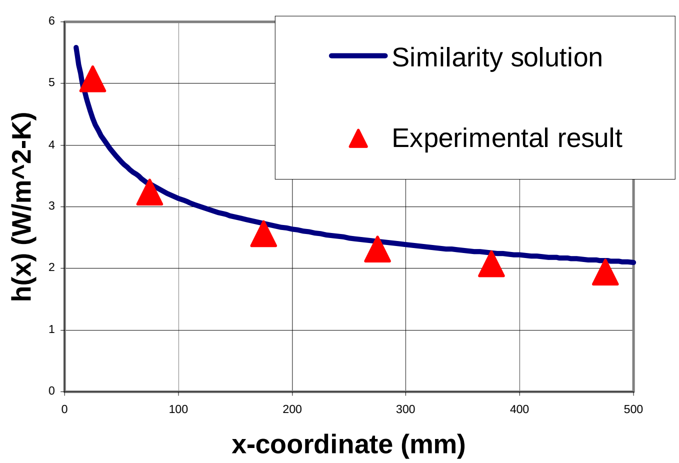
$$Nu_x = \frac{h_{x,ss}x}{k} = \left(Gr_x/4\right)^{1/4} g\left(Pr\right) \qquad (5)$$

$$g\left(Pr\right) = \frac{0.75\ Pr^{1/2}}{\left(0.609 + 1.221\ Pr^{1/2} + 1.238\ Pr\right)^{1/4}} \qquad (6)$$

where the local Grashof number is

$$Gr_x = g\beta\left(T_s - T_\infty\right)x^3/\nu^2 \qquad (7)$$

and the thermophysical properties are evaluated at the film temperature, $T_f = (T_s + T_\infty)/2$. Using the above relations in the *IHT* workspace along with the properties library for air, the convection coefficient $h_{x,ss}$ is calculated for selected values of x. The results are shown in Table 1 above and the graph below compared to the experimental results.



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PROBLEM 9.32 (Cont.)

The experimental results and the calculated similarity solution coefficients are in good agreement. Except near the leading edge, the experimental results are systematically lower than those from the similarity solution.

(d) The experimental boundary-layer air temperature measurements for three discrete y-locations at two x-locations are shown in the first two rows of the table below. From Eq. 9.13, the similarity parameter is

$$\eta = \frac{y}{x}\left(\frac{Gr_x}{4}\right)^{1/4}$$

and the dimensionless temperature for the experimental data are

$$T^*_{exp} = \frac{T - T_\infty}{T_s - T_\infty}$$

Figure 9.4(b) is used to obtain the dimensionless temperature from the similarity solution, T^*_{ss}, for the required values of η and are tabulated below.

Table 2

Boundary-layer air temperature data and similarity solution results

	$T_s - T_\infty = 7.3$ K					
	x = 200 mm, $Gr_x = 8.9\times10^6$			x = 400 mm, $Gr_x = 7.2\times10^7$		
y (mm)	2.5	5.0	10.0	2.5	5.0	10.0
$T(x,y) - T_\infty$ (K)	5.5	3.8	1.6	5.9	4.5	2.0
T^*_{exp}	0.753	0.521	0.219	0.808	0.616	0.274
η	0.48	0.97	1.93	0.41	0.81	1.63
T^*_{ss}	0.77	0.55	0.22	0.79	0.62	0.28

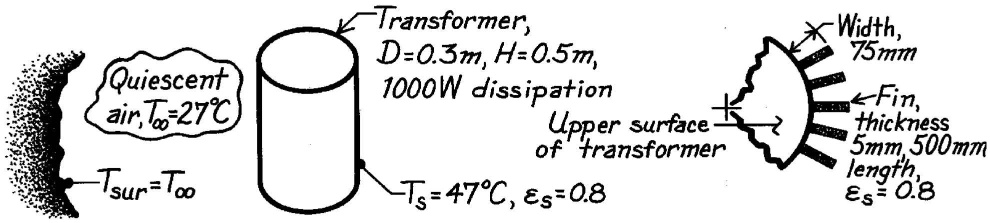
The experimentally determined dimensionless temperatures, T^*_{exp}, are systematically lower than those from the similarity solution T^*_{ss}. The agreement is excellent at the x = 400 mm location, ranging from less than 1% near the wall to 2% far from the wall. For the x = 200 mm location, nearer to the leading edge, where the boundary layer is thinner and the boundary layer temperature gradient is higher, the agreement is good, but near the wall the differences are larger. Note that for both x-locations far from the wall, T^*_{exp} and T^*_{ss} are in excellent agreement. Would you have expected that behavior?

PROBLEM 9.33

KNOWN: Transformer which dissipates 1000 W whose surface is to be maintained at 47°C in quiescent air and surroundings at 27°C.

FIND: Power removal (a) by free convection and radiation from lateral and upper horizontal surfaces and (b) with 30 vertical fins attached to lateral surface.

SCHEMATIC:



ASSUMPTIONS: (1) Fins are isothermal at lateral surface temperature, T_s, (2) Vertical fins and lateral surface behave as vertical plate, (3) Transformer has isothermal surfaces and loses heat only on top and side.

PROPERTIES: *Table A-4*, Air (T_f = (27+47)°C/2=310K, 1 atm): $\nu = 16.90 \times 10^{-6}$ m^2/s, k = 27.0 × 10^{-3} W/m·K, $\alpha = 23.98 \times 10^{-6}$ m^2/s, Pr = 0.706, $\beta = 1/T_f$.

ANALYSIS: (a) For the vertical lateral (lat) and top horizontal (top) surfaces, the heat loss by radiation and convection is

$$q = q_{lat} + q_{top} = \left(\bar{h}_{lat} + h_r\right) p\, DL \left(T_s - T_\infty\right) + \left(\bar{h}_{top} + h_r\right)\left(p\,^2 D/4\right)\left(T_s - T_\infty\right)$$

where, from Eq. 1.9, the linearized radiation coefficient is

$$h_r = es\,\left(T_s + T_\infty\right)\left(T_s^2 + T_\infty^2\right)$$

$$h_r = 0.8 \times 5.67 \times 10^{-8} \mathrm{W/m^2 \cdot K^4}\left(320 + 300\right)\mathrm{K}\left(320^2 + 300^2\right)\mathrm{K}^2 = 5.41 \mathrm{W/m^2 \cdot K}.$$

The free convection coefficient for the lateral and top surfaces is:

Lateral-vertical plate: Using Eq. 9.26 with

$$Ra_L = \frac{gb\left(T_s - T_\infty\right)H^3}{na} = \frac{9.8\mathrm{m/s^2}\left(1/310\mathrm{K}\right)\left(47-27\right)\mathrm{K}\left(0.5\mathrm{m}\right)^3}{16.90 \times 10^{-6}\mathrm{m^2/s} \times 23.98 \times 10^{-6}\mathrm{m^2/s}} = 1.950 \times 10^8$$

$$\overline{Nu}_L = \left\{0.825 + \frac{0.387 Ra_L^{1/6}}{\left[1 + \left(0.492/Pr\right)^{9/16}\right]^{8/27}}\right\}^2$$

$$\overline{Nu}_L = \left\{0.825 + \frac{0.387\left(1.950 \times 10^8\right)^{1/6}}{\left[1 + \left(0.492/0.706\right)^{9/16}\right]^{8/27}}\right\}^2 = 74.5$$

$$\bar{h}_{lat} = \overline{Nu}_L \cdot k/H = 74.5 \times 0.027 \mathrm{W/m \cdot K}/0.5\mathrm{m} = 4.02 \mathrm{W/m^2 \cdot K}.$$

Continued …..

Top-horizontal plate: Using Eq. 9.30 with

$$L_c = A_s / P = \frac{\pi D^2/4}{\pi D} = D/4 = 0.075\,\text{m}$$

$$Ra_L = \frac{g\beta\left(T_s - T_\infty\right)L_c^3}{\nu\alpha} = \frac{9.8\,\text{m/s}^2\left(1/310\,\text{K}\right)\left(47-27\right)\text{K}\left(0.075\,\text{m}\right)^3}{16.90\times10^{-6}\,\text{m}^2/\text{s}\times 23.98\times10^{-6}\,\text{m}^2/\text{s}} = 6.598\times10^5$$

$$\overline{Nu}_L = 0.54 Ra_L^{1/4} = 0.54\left(6.598\times10^5\right)^{1/4} = 15.39$$

$$\bar{h}_{top} = \overline{Nu}_L \cdot k/L_c = 15.39\times0.027\,\text{W/m}\cdot\text{K}/0.075\,\text{m} = 5.54\,\text{W/m}^2\cdot\text{K}.$$

Hence, the heat loss by convection and radiation is

$$q = \left(4.02+5.41\right)\text{W/m}^2\cdot\text{K}\left(\pi\times0.30\,\text{m}\times0.50\,\text{m}\right)\left(47-20\right)\text{K}$$
$$+\left(5.54+5.41\right)\text{W/m}^2\cdot\text{K}\left(\pi\times0.30^2\,\text{m}^2/4\right)\left(47-20\right)\text{K}$$

$$q = \left(88.9+15.5\right)\text{W} = 104\,\text{W}. \qquad <$$

(b) The effect of adding the vertical fins is to increase the area of the lateral surface to

$$A_{wf} = \left[\pi DH - 30\left(t\cdot H\right)\right] + 30\times2\left(w\cdot H\right)$$

$$A_{wf} = \left[\pi\, 0.30\,\text{m}\times0.50\,\text{m} - 30\left(0.005\times0.500\right)\text{m}^2\right] + 30\times2\left(0.075\times0.500\right)\text{m}^2$$

$$A_{wf} = \left[0.471-0.075\right]\text{m}^2 + 2.25\,\text{m}^2 = 2.646\,\text{m}^2.$$

where t and w are the thickness and width of the fins, respectively. Hence, the heat loss is now

$$q = q_{lat} + q_{top} = \left(\bar{h}_{lat} + h_r\right)A_{wf}\left(T_s - T_\infty\right) + q_{top}$$

$$q = \left(4.02+5.41\right)\text{W/m}^2\times2.646\,\text{m}^2\times20\,\text{K} + 15.5\,\text{W} = 515\,\text{W}. \qquad <$$

Adding the fins to the lateral surface increases the heat loss by a factor of five.

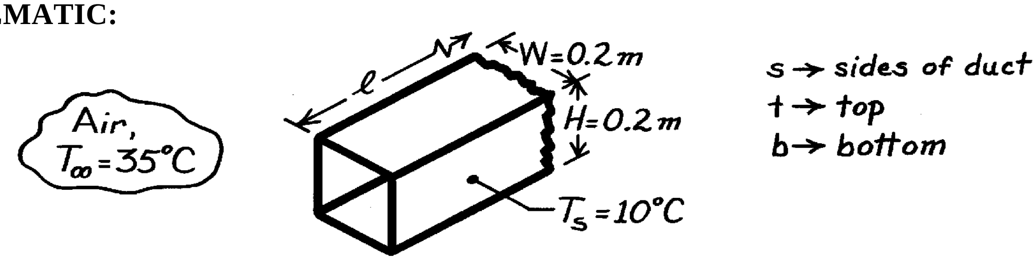
COMMENTS: Since the fins are not likely to have 100% efficiency, our estimate is optimistic. Further, since the fins see one another, as well as the lateral surface, the radiative heat loss is over predicted.

PROBLEM 9.34

KNOWN: Surface temperature of a long duct and ambient air temperature.

FIND: Heat gain to the duct per unit length of the duct.

SCHEMATIC:



ASSUMPTIONS: (1) Surface radiation effects are negligible, (2) Ambient air is quiescent.

PROPERTIES: *Table A-4*, Air ($T_f = (T_\infty + T_s)/2 \approx 300K$, 1 atm): $\nu = 15.89 \times 10^{-6}\ m^2/s$, k = 0.0263 W/m·K, $\alpha = 22.5 \times 10^{-6}\ m^2/s$, Pr = 0.707, $\beta = 1/T_f$.

ANALYSIS: The heat gain to the duct can be expressed as

$$q' = 2q'_s + q'_t + q'_b = \left(2\bar{h}_s \cdot H + \bar{h}_t \cdot W + \bar{h}_b \cdot W\right)\left(T_\infty - T_s\right). \tag{1}$$

Consider now correlations to estimate $\bar{h}_s$, $\bar{h}_t$, and $\bar{h}_b$. From Eq. 9.25, for the sides with L ≡ H,

$$Ra_L = \frac{g b \left(T_\infty - T_s\right) L^3}{n a} = \frac{9.8\,m/s^2\,(1/300K)(35-10)K \times (0.2m)^3}{15.89\times10^{-6}m^2/s \times 22.5\times10^{-6}m^2/s} = 1.827\times10^7. \tag{2}$$

Eq. 9.27 is appropriate to estimate $\bar{h}_s$,

$$\overline{Nu}_L = 0.68 + \frac{0.670 Ra_L^{1/4}}{\left[1+\left(0.492/Pr\right)^{9/16}\right]^{4/9}} = 0.68 + \frac{0.670\left(1.827\times10^7\right)^{1/4}}{\left[1+\left(0.492/0.707\right)^{9/16}\right]^{4/9}} = 34.29$$

$$\bar{h}_s = \overline{Nu}_L \cdot k/L = 34.29 \times 0.0263\,W/m\cdot K/0.2m = 4.51\,W/m^2\cdot K.$$

(3)

For the top and bottom portions of the duct, $L \equiv A_s/P \approx W/2$, (see Eq. 9.29), find the Rayleigh number from Eq. (2) with L = 0.1 m, $Ra_L = 2.284 \times 10^6$. From the correlations, Eqs. 9.31 and 9.32 for the top and bottom surfaces, respectively, find

$$\bar{h}_t = \frac{k}{(W/2)} \times 0.15 Ra_L^{1/3} = \frac{0.0263\,W/m\cdot K}{0.1m} \times 0.15\left(2.284\times10^6\right)^{1/3} = 5.17\,W/m^2\cdot K. \tag{4}$$

$$\bar{h}_b = \frac{k}{(W/2)} \times \frac{0.0263\,W/m\cdot K}{0.1m} \times 0.27\left(2.284\times10^6\right)^{1/4} = 2.76\,W/m^2\cdot K. \tag{5}$$

The heat rate, Eq. (1), can now be evaluated using the heat transfer coefficients estimated from Eqs. (3), (4), and (5).

$$q' = \left(2\times4.51\,W/m^2\cdot K\times0.2m + 5.17\,W/m^2\cdot K\times0.2m + 2.76\,W/m^2\cdot K\times0.2m\right)(35-10)\,K$$

$$q' = 84.8\,W/m.$$ <

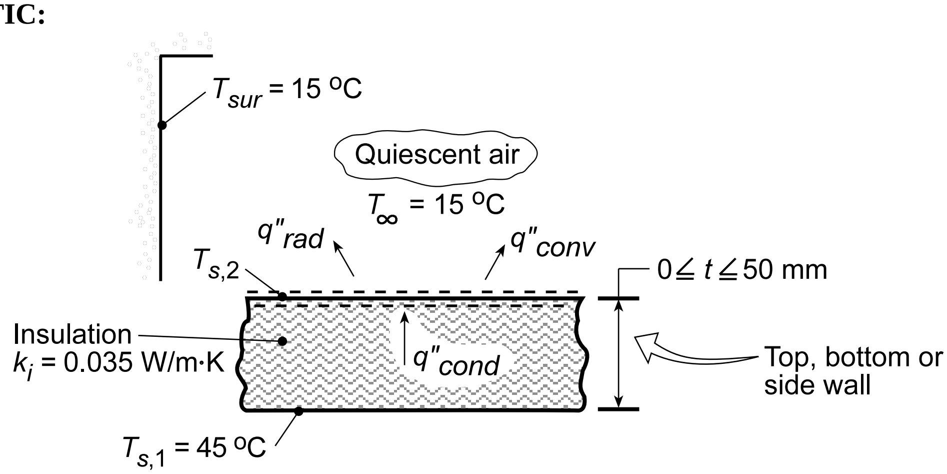
COMMENTS: Radiation surface effects will be significant in this situation. With knowledge of the duct emissivity and surroundings temperature, the radiation heat exchange could be estimated.

PROBLEM 9.35

KNOWN: Inner surface temperature and dimensions of rectangular duct. Thermal conductivity, thickness and emissivity of insulation.

FIND: (a) Outer surface temperatures and heat losses from the walls, (b) Effect of insulation thickness on outer surface temperatures and heat losses.

SCHEMATIC:



ASSUMPTIONS: (1) Ambient air is quiescent, (2) One-dimensional conduction, (3) Steady-state.

PROPERTIES: *Table A.4*, air (obtained from *Properties* Tool Pad of IHT).

ANALYSIS: (a) The analysis follows that of Example 9.3, except the surface energy balance must now include the effect of radiation. Hence, $q''_{cond} = q''_{conv} + q''_{rad}$, in which case

$$(k_i/t)(T_{s,1} - T_{s,2}) = \bar{h}(T_{s,2} - T_\infty) + h_r(T_{s,2} - T_{sur})$$

where $h_r = \varepsilon\sigma(T_{s,2} + T_{sur})(T_{s,2}^2 + T_{sur}^2)$. Applying this expression to each of the top, bottom and side walls, with the appropriate correlation obtained from the *Correlations* Tool Pad of IHT, the following results are determined for t = 25 mm.

Sides: $T_{s,2}$ = 19.3°C, $\bar{h}$ = 2.82 W/m²·K, h_{rad} = 5.54 W/m²·K

Top: $T_{s,2}$ = 19.3°C, $\bar{h}$ = 2.94 W/m²·K, h_{rad} = 5.54 W/m²·K $<$

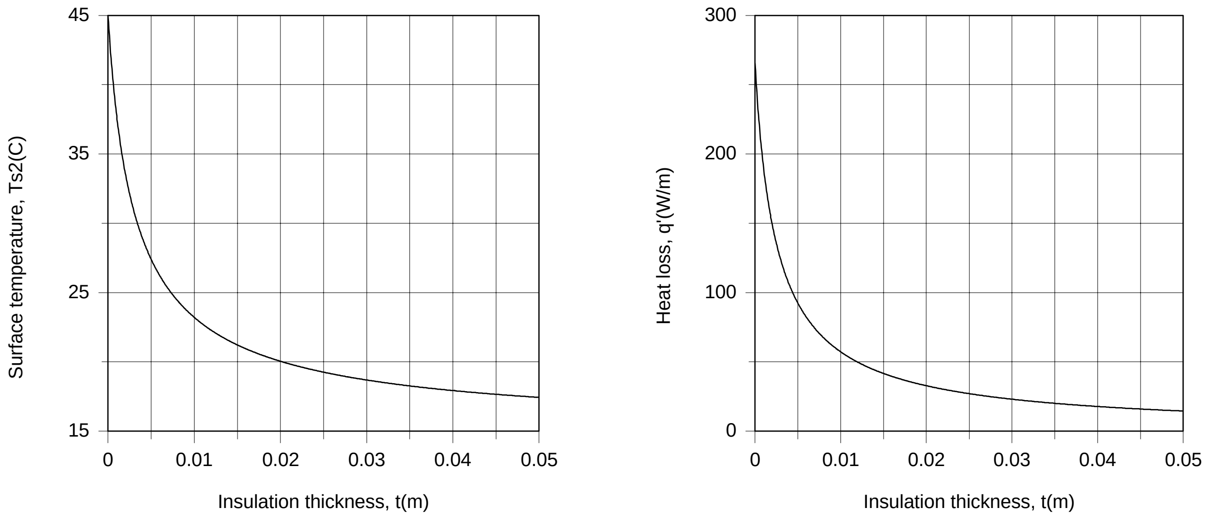
Bottom: $T_{s,2}$ = 20.1°C, $\bar{h}$ = 1.34 W/m²·K, h_{rad} = 5.56 W/m²·K

With $q'' = q''_{cond}$, the surface heat losses may also be evaluated, and we obtain

Sides: $q' = 2Hq''$ = 21.6 W/m; *Top:* $q' = wq''$ = 27.0 W/m; *Bottom:* $q' = wq''$ = 26.2 W/m $<$

(b) For the top surface, the following results are obtained from the parametric calculations

Continued...



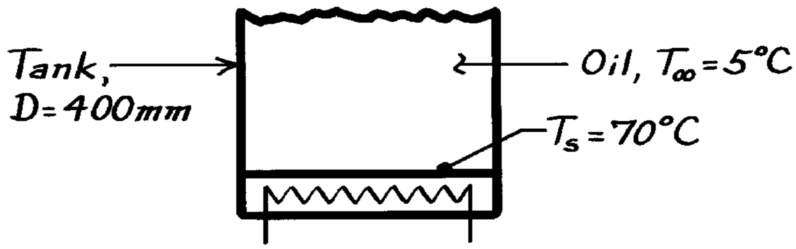
COMMENTS: Contrasting the heat rates of part (a) with those predicted in Comment 1 of Example 9.3, it is evident that radiation is significant and increases the total heat loss from 57.6 W/m to 74.8 W/m. As shown in part (b), reductions in $T_{s,o}$ and q' may be effected by increasing the insulation thickness above 0.025 W/m·K, although attendant benefits diminish with increasing t.

PROBLEM 9.36

KNOWN: Electric heater at bottom of tank of 400mm diameter maintains surface at 70°C with engine oil at 5°C.

FIND: Power required to maintain 70°C surface temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Oil is quiescent, (2) Quasi-steady state conditions exist.

PROPERTIES: *Table A-5*, Engine Oil ($T_f = (T_\infty + T_s)/2 = 310K$): $\nu = 288 \times 10^{-6}$ m^2/s, k = 0.145 W/m·K, $\alpha = 0.847 \times 10^{-7}$ m^2/s, $\beta = 0.70 \times 10^{-3}$ K^{-1}.

ANALYSIS: The heat rate from the bottom heater surface to the oil is

$$q = \bar{h} A_s \left(T_s - T_\infty \right)$$

where $\bar{h}$ is estimated from the appropriate correlation depending upon the Rayleigh number Ra_L, from Eq. 9.25, using the characteristic length, L, from Eq. 9.29,

$$L = \frac{A_s}{P} = \frac{\pi D^2/4}{\pi D} = \frac{D}{4} = \frac{0.4\text{m}}{4} = 0.1\text{m}.$$

The Rayleigh number is

$$Ra_L = \frac{g\beta\left(T_s - T_\infty\right)L^3}{\nu\alpha}$$

$$Ra_L = \frac{9.8\text{m/s}^2 \times 0.70\times10^{-3}\text{K}^{-1}\left(70-5\right)\text{K}\times0.1^3\text{m}^3}{288\times10^{-6}\text{m}^2/\text{s}\times0.847\times10^{-7}\text{m}^2/\text{s}} = 1.828\times10^7.$$

The appropriate correlation is Eq. 9.31 giving

$$\overline{Nu}_L = \frac{\bar{h}L}{k} = 0.15Ra_L^{1/3} = 0.15\left(1.828\times10^7\right)^{1/3} = 39.5$$

$$\bar{h} = \frac{k}{L}\overline{Nu}_L = \frac{0.145\text{W/m}\cdot\text{K}}{0.1\text{m}}\times39.5 = 57.3\text{W/m}^2\cdot\text{K}.$$

The heat rate is then

$$q = 57.3\text{W/m}^2\cdot\text{K}\left(\pi/4\right)\left(0.4\text{m}\right)^2\left(70-5\right)\text{K} = 468\text{W}.$$ <

COMMENTS: Note that the characteristic length is D/4 and not D; however, A_s is based upon D.

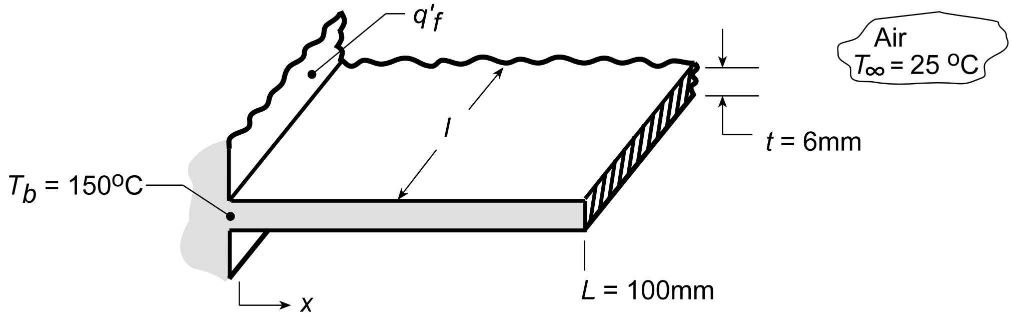
Recognize that if the oil is being continuously heated by the plate, T_∞ could change. Hence, here we have analyzed a quasi-steady state condition.

PROBLEM 9.37

KNOWN: Horizontal, straight fin fabricated from plain carbon steel with thickness 6 mm and length 100 mm; base temperature is 150°C and air temperature is 26°C.

FIND: (a) Fin heat rate per unit width, q_f', assuming an average fin surface temperature $\overline{T}_s = 125^\circ C$ for estimating free convection and linearized radiation coefficient; how sensitive is q_f' to the assumed value for $\overline{T}_s$?; (b) Compute and plot the heat rate, q_f' as a function of emissivity $0.05 \le \varepsilon \le 0.95$; show also the fraction of the total heat ratio due to radiation exchange.

SCHEMATIC:



ASSUMPTIONS: (1) Air is quiescent medium, (2) Surface radiation effects are negligible, (3) One dimensional conduction in fin, (4) Characteristic length, $L_c = A_s/P = \ell L\left(2\ell + 2L\right) \approx L/2$.

PROPERTIES: Plain carbon steel, Given $\left(\overline{T}_{fin} \approx 125^\circ C \approx 400\,K\right)$: $k = 57\,W/m\cdot K$, $\varepsilon = 0.5$; *Table A-4*, Air ($T_f = \left(\overline{T}_{fin} + T_\infty\right)/2 = \left(125 + 25\right)^\circ C/2 \approx$ 350 K, 1 atm): $\nu = 20.92 \times 10^{-6}\ m^2/s$, $\alpha = 29.9 \times 10^{-6}\ m^2/s$, k = 0.030 W/m·K, Pr = 0.70, $\beta = 1/T_f$.

ANALYSIS: (a) We estimate $\overline{h}$ as the average of the values for a heated plate facing upward and a heated plate facing downward. See Table 9.2, Case 3(a) and (b). Begin by evaluating the Rayleigh number, using Eq. 9.29 for L_c.

$$Ra_L = \frac{g\beta\left(\overline{T}_{fin} - T_\infty\right)L_c^3}{\nu\alpha} = \frac{9.8\,m/s^2\left(1/350K\right)\left(125-25\right)K \times \left(0.1\,m/2\right)^3}{20.92\times10^{-6}\,m^2/s \times 29.9\times10^{-6}\,m^2/s} = 5.595\times10^5$$

An average fin temperature of $\overline{T}_{fin} \approx 125^\circ C$ has been assumed in evaluating properties and Ra_L. According to Table 9.2, Eqs. 9.30 and 9.32 are appropriate. For the *upper* fin surface, Eq. 9.30,

$$\overline{Nu}_L = \overline{h}\,L_c/k = 0.54 Ra_L^{1/4} = 0.54\left(5.595\times10^5\right)^{1/4} = 14.77$$

$$\overline{h}_{upper} = \overline{Nu}_L\,k/L_c = 14.77\times0.030\,W/m\cdot K/0.05\,m = 8.86\,W/m^2\cdot K.$$

For the *lower* fin surface, Eq. 9.32,

$$\overline{Nu}_L = \overline{h}\,L/k = 0.27 Ra_L^{1/4} = 0.27\left(5.595\times10^5\right)^{1/4} = 7.384$$

$$\overline{h}_{lower} = \overline{Nu}_L\,k/L = 7.384\times0.030\,W/m\cdot K/0.05\,m = 4.43\,W/m^2\cdot K.$$

The linearized radiation coefficient follows from Eq. 1.9

$$\overline{h}_r = \varepsilon\sigma\left(\overline{T}_{fin} + T_{sur}\right)\left(\overline{T}_{fin}^2 + T_{sur}^2\right)$$

$$\overline{h}_r = 0.5\times5.67\times10^{-8}\,W/m^2\cdot K^4\left(398 + 298\right)\left(398^2 + 298^2\right)K^3 = 4.88\,W/m^2\cdot K$$

Continued …..

PROBLEM 9.37 (Cont.)

Hence, the average heat transfer coefficient for the fin is

$$\bar{h} = \left(\bar{h}_{upper} + \bar{h}_{lower}\right)/2 + \bar{h}_r = \left[(8.86 + 4.43)/2 + 4.88\right] W/m^2\cdot K = 11.53\, W/m^2\cdot K$$

Assuming the fin experiences convection at the tip, from Eq. 3.72,

$$q_f = M \tanh(mL)$$

$$M = \left(\bar{h}PkA_c\right)^{1/2}\theta_b = \left(11.53\, W/m^2\cdot K \times 2\ell \times 57\, W/m\cdot K\left(6\times10^{-3} m\times\ell\right)\right)^{1/2}(150-25)K = 352.1\, W$$

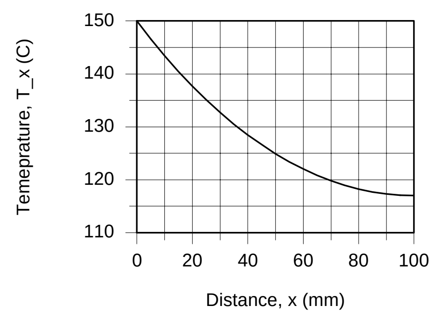
$$m = \left(\bar{h}\, P/k\, A_c\right)^{1/2} = \left(11.53\, W/m^2\cdot K \times 2\ell/57\, W/m\cdot K\left(6\times10^{-3} m\times\ell\right)\right)^{1/2} = 8.236\, m^{-1}$$

$$mL = 8.236\, m^{-1} \times 0.1\, m = 0.824$$

$$q_f' = q_f/\ell = 352.1\ W/m \times \tanh(0.824) = 238\ W/m\ .$$ <

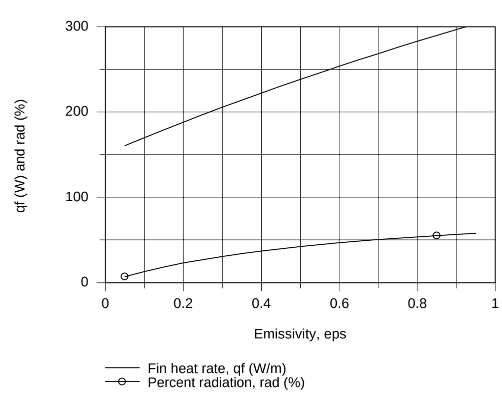
To determine how sensitive the estimate for $\bar{h}$ is to the choice of the average fin surface temperature, the foregoing calculations were repeated using the *IHT Correlations Tool and Extended Surface Model* and the results are tabulated below; coefficients have units $W/m^2\cdot K$,

$\bar{T}_{fin}\left(^\circ C\right)$	125	135	145
$\bar{h}_{upper}$	4.43	4.54	4.64
$\bar{h}_{lower}$	8.86	9.08	9.28
$\bar{h}_r$	4.88	5.11	5.35
$\bar{h}$	11.5	11.9	12.3
$q'(W/m)$	238	245	252



The temperature distribution for the $\bar{T}_{fin} = 125^\circ C$ case is shown above. With $\bar{T}_{fin} = 145^\circ C$, the tip temperature is about $2^\circ C$ higher. It appears that $\bar{T}_{fin} = 125^\circ C$ was a reasonable choice. Note $\bar{T}_{fin}$ is the value at the mid length.

(b) Using the IHT code developed for part (a), the fin heat rate, q_f, was plotted as a function of the emissivity. In this analysis, the convection and radiation coefficients were evaluated for an average fin temperature $\bar{T}_{fin}$ evaluated at L/2. On the same plot we have also shown rad (%) = $\left(\bar{h}_r/\bar{h}\right)\times 100$, which is the portion of the total heat rate due to radiation exchange.

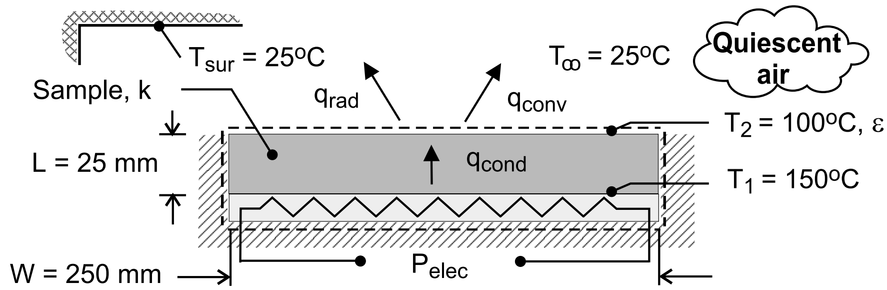


PROBLEM 9.38

KNOWN: Width and thickness of sample material. Rate of heat dissipation at bottom surface of sample and temperatures of top and bottom surfaces. Temperature of quiescent air and surroundings.

FIND: Thermal conductivity and emissivity of the sample.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional conduction in sample, (3) Quiescent air, (4) Sample is small relative to surroundings, (5) All of the heater power dissipation is transferred through the sample, (6) Constant properties.

PROPERTIES: *Table A-4*, air (T_f = 335.5K): $\nu = 19.5 \times 10^{-6}$ m²/s, k = 0.0289 W/m·K, $\alpha = 27.8 \times 10^{-6}$ m²/s, Pr = 0.703, $\beta = 0.00298$ K⁻¹.

ANALYSIS: The thermal conductivity is readily obtained by applying Fourier's law to the sample. Hence, with q = P_{elec},

$$k = \frac{P_{elec}/W^2}{(T_1 - T_2)/L} = \frac{70\,W/(0.250m)^2}{50°C/0.025m} = 0.560\,W/m\cdot K \qquad <$$

The surface emissivity may be obtained by applying an energy balance to a control surface about the sample, in which case

$$P_{elec} = q_{conv} + q_{rad} = \left[\bar{h}(T_2 - T_\infty) + \varepsilon\sigma\left(T_2^4 - T_{sur}^4\right)\right]W^2$$

$$\varepsilon = \frac{\left(P_{elec}/W^2\right) - \bar{h}(T_2 - T_\infty)}{\sigma\left(T_2^4 - T_{sur}^4\right)}$$

With L = $A_s/P = W^2/4W = W/4$ = 0.0625m, $Ra_L = g\beta(T_2 - T_\infty)L^3/\nu\alpha = 9.86 \times 10^5$ and Eq. 9.30 yields

$$\bar{h} = \frac{\overline{Nu}_L\,k}{L} = \frac{k}{L}0.54\,Ra_L^{1/4} = \frac{0.0289\,W/m\cdot K}{0.0625m}0.54\left(9.86\times10^5\right)^{1/4} = 7.87\,W/m^2\cdot K \qquad <$$

Hence,

$$\varepsilon = \frac{70\,W/(0.250m)^2 - 7.87\,W/m^2\cdot K\,(75°C)}{5.67\times10^{-8}\,W/m^2\cdot K^4\left(373^4 - 298^4\right)K^4} = 0.815 \qquad <$$

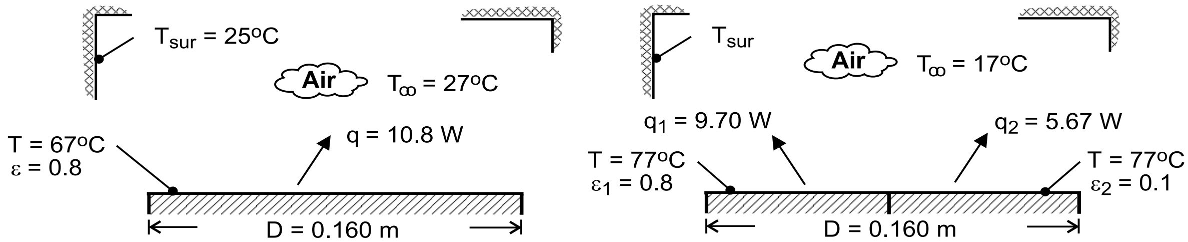
COMMENTS: The uncertainty in the determination of ε is strongly influenced by uncertainties associated with using Eq. 9.30. If, for example, $\bar{h}$ is overestimated by 10%, the actual value of ε would be 0.905.

PROBLEM 9.39

KNOWN: Diameter, power dissipation, emissivity and temperature of gage(s). Air temperature (Cases A and B) and temperature of surroundings (Case A).

FIND: (a) Convection heat transfer coefficient (Case A), (b) Convection coefficient and temperature of surroundings (Case B).

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) Quiescent air, (3) Net radiation exchange from surface of gage approximates that of a small surface in large surroundings, (4) All of the electrical power is dissipated by convection and radiation heat transfer from the surface(s) of the gage, (5) Negligible thickness of strip separating semi-circular disks of Part B, (6) Constant properties.

PROPERTIES: *Table A-4*, air (T_f = 320K): $\nu = 17.9 \times 10^{-6}$ m^2/s, $\alpha = 25.5 \times 10^{-6}$ m^2/s, k = 0.0278 W/m·K, Pr = 0.704, $\beta = 0.00313$ K^{-1}.

ANALYSIS: (a) With $q = q_{conv} + q_{rad} = P_{elec}$ and $A_s = \pi D^2/4 = 0.0201$ m^2,

$$\bar{h}_{meas} = \frac{P_{elec} - \varepsilon\sigma A_s \left(T^4 - T_{sur}^4\right)}{A_s \left(T - T_\infty\right)} = \frac{10.8\,\text{W} - 0.8 \times 5.67 \times 10^{-8}\,\text{W/m}^2 \cdot \text{K}^4 \times 0.0201\,\text{m}^2 \left(340^4 - 300^4\right)\text{K}^4}{0.0201\,\text{m}^2\,(40\,\text{K})} = 7.46\,\text{W/m}^2 \cdot \text{K} \quad <$$

With $L = A_s/P = D/4 = 0.04$ m and $Ra_L = g\beta\,(T - T_\infty)L^3/\nu\alpha = 1.72 \times 10^5$, Eq. 9.30 yields

$$\bar{h} = \frac{k}{L} 0.54\, Ra_L^{1/4} = \frac{0.0278\,\text{W/m} \cdot \text{K} \times 0.54 \left(1.72 \times 10^5\right)^{1/4}}{0.04\text{m}} = 7.64\,\text{W/m}^2 \cdot \text{K} \quad <$$

Agreement between the two values of $\bar{h}$ is well within the uncertainty of the measurements.

(b) Since the semi-circular disks have the same temperature, each is characterized by the same convection coefficient and $q_{conv,1} = q_{conv,2}$. Hence, with

$$P_{elec,1} = q_{conv,1} + \varepsilon_1 \sigma \left(A_s/2\right)\left(T^4 - T_{sur}^4\right) \qquad (1)$$

$$P_{elec,2} = q_{conv,2} + \varepsilon_2 \sigma \left(A_s/2\right)\left(T^4 - T_{sur}^4\right) \qquad (2)$$

$$T_{sur} = \left[T^4 - \frac{P_{elec,1} - P_{elec,2}}{(\varepsilon_1 - \varepsilon_2)\sigma\left(A_s/2\right)}\right]^{1/4} = \left[(350)^4 - \frac{4.03\,\text{W}}{0.7 \times 5.67 \times 10^{-8}\,\text{W/m}^2 \cdot \text{K}^4 \times 0.01\,\text{m}^2}\right]^{1/4}$$

$$T_{sur} = 264\,\text{K} \quad <$$

From Eq. (1), the convection coefficient is then

$$\bar{h}_{meas} = \frac{P_{elec,1} - \varepsilon_1 \sigma \left(A_s/2\right)\left(T^4 - T_{sur}^4\right)}{\left(A_s/2\right)\left(T - T_\infty\right)} = \frac{9.70\,\text{W} - 4.60\,\text{W}}{(0.01 \times 60)\,\text{m}^2 \cdot \text{K}} = 8.49\,\text{W/m}^2 \cdot \text{K} \quad <$$

With $Ra_L = 2.58 \times 10^5$, Eq. 9.30 yields

$$\bar{h} = \frac{k}{L} 0.054\, Ra_L^{1/4} = \frac{0.0278\,\text{W/m} \cdot \text{K}}{0.04\text{m}} 0.54 \left(2.58 \times 10^5\right)^{1/4} = 8.46\,\text{W/m}^2 \cdot \text{K} \quad <$$

Again, agreement between the two values of $\bar{h}$ is well within the experimental uncertainty of the measurements.

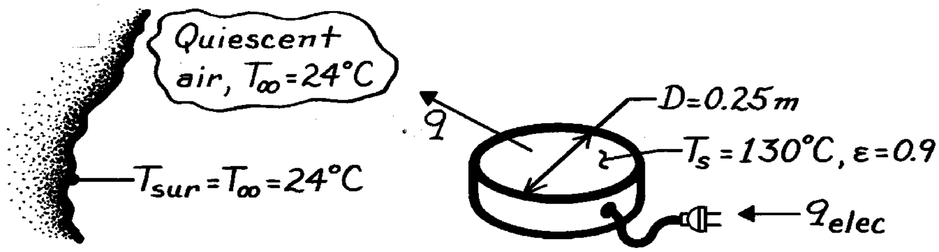
COMMENTS: Because the semi-circular disks are at the same temperature, the characteristic length corresponds to that of the circular disk, L = D/4.

PROBLEM 9.40

KNOWN: Horizontal, circular grill of 0.2m diameter with emissivity 0.9 is maintained at a uniform surface temperature of 130°C when ambient air and surroundings are at 24°C.

FIND: Electrical power required to maintain grill at prescribed surface temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Room air is quiescent, (2) Surroundings are large compared to grill surface.

PROPERTIES: *Table A-4*, Air ($T_f = (T_\infty + T_s)/2 = (24 + 130)°C/2 = 350K$, 1 atm): $\nu = 20.92 \times 10^{-6}\ m^2/s$, $k = 0.030\ W/m\cdot K$, $\alpha = 29.9 \times 10^{-6}\ m^2/s$, $\beta = 1/T_f$.

ANALYSIS: The heat loss from the grill is due to free convection with the ambient air and to radiation exchange with the surroundings.

$$q = A_s\left[\bar{h}\left(T_s - T_\infty\right) + es\left(T_s^4 - T_{sur}^4\right)\right]. \qquad (1)$$

Calculate Ra_L from Eq. 9.25,

$$Ra_L = gb\left(T_s - T_\infty\right)L_c^3/na$$

where for a horizontal disc from Eq. 9.29, $L_c = A_s/P = (\pi D^2/4)/\pi D = D/4$. Substituting numerical values, find

$$Ra_L = \frac{9.8 m/s^2 \left(1/350K\right)\left(130-24\right)K\left(0.25m/4\right)^3}{20.92\times10^{-6} m^2/s \times 29.9\times10^{-6} m^2/s} = 1.158\times10^6.$$

Since the grill is an *upper surface heated*, Eq. 9.30 is the appropriate correlation,

$$\overline{Nu}_L = \bar{h}_L L_c/k = 0.54 Ra_L^{1/4} = 0.54\left(1.158\times10^6\right)^{1/4} = 17.72$$

$$\bar{h}_L = \overline{Nu}_L\, k/L_c = 17.72\times0.030 W/m\cdot K/\left(0.25m/4\right) = 8.50 W/m^2\cdot K. \qquad (2)$$

Substituting from Eq. (2) for $\bar{h}$ into Eq. (1), the heat loss or required electrical power, q_{elec}, is

$$q = \frac{p}{4}\left(0.25m\right)^2\left[8.50\frac{W}{m^2\cdot K}\left(130-24\right)K + 0.9\times5.67\times10^{-8}\frac{W}{m^2\cdot K^4}\left(\left(130+273\right)^4 - \left(24+273\right)^4\right)K^4\right]$$

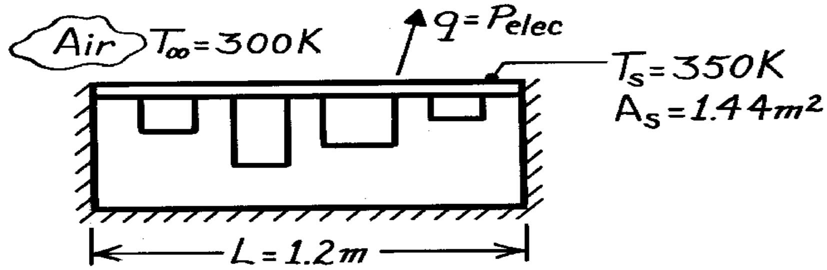
$$q = 44.2W + 46.0W = 90.2W.$$ <

COMMENTS: Note that for this situation, free convection and radiation modes are of equal importance. If the grill were highly polished such that $\varepsilon \approx 0.1$, the required power would be reduced by nearly 50%.

PROBLEM 9.41

KNOWN: Plate dimensions and maximum allowable temperature. Freestream temperature.
FIND: Maximum allowable power dissipation.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Negligible heat loss from sides and bottom, (4) Isothermal plate.

PROPERTIES: *Table A-4*, Air (T_f = 325K, 1 atm): $\nu = 18.4 \times 10^{-6}$ m^2/s, k = 0.028 W/m·K, $\alpha = 26.2 \times 10^{-6}$ m^2/s.

ANALYSIS: The power dissipated by convection is

$$P_{elec} = q = \bar{h} A_s \left(T_s - T_\infty \right).$$

With $\quad L = A_s / P = (1.2\text{m})^2 / 4(1.2\text{m}) = 0.3\text{m}$

$$Ra_L = \frac{g b \left(T_s - T_\infty\right) L^3}{na} = \frac{9.8\text{m/s}^2 (325\text{K})^{-1} (50\text{K}) (0.3\text{m})^3}{\left(18.4 \times 10^{-6} \text{m}^2/\text{s}\right)\left(26.2 \times 10^{-6} \text{m}^2/\text{s}\right)}$$

$$Ra_L = 8.44 \times 10^7.$$

With the upper surface heated, Eq. 9.31 yields

$$\overline{Nu}_L = \frac{\bar{h} L}{k} = 0.15 Ra_L^{1/3} = 65.8$$

$$\bar{h} = 65.8 \frac{0.028\text{W/m} \cdot \text{K}}{0.3\text{m}} = 6.14\text{W/m}^2 \cdot \text{K}$$

and the power dissipated is

$$q = 6.14\text{W/m}^2 \cdot \text{K} (1.2\text{m})^2 (50\text{K})$$

$$P_{elec} = q = 442\text{W}. \qquad <$$

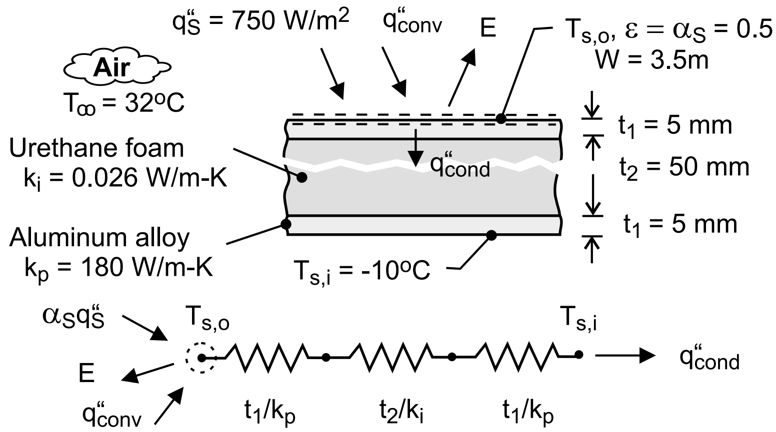
COMMENTS: This result corresponds to an average surface heat flux of 442 W/1.44 m^2 = 307 W/m^2 = 0.03 W/cm^2, which is extremely small. Heat dissipation by free convection in this manner is a poor option compared to the heat flux with forced convection (u_∞ = 15 m/s) of 0.15 W/cm^2.

PROBLEM 9.42

KNOWN: Material properties, inner surface temperature and dimensions of roof of refrigerated truck compartment. Solar irradiation and ambient temperature.

FIND: Outer surface temperature of roof and rate of heat transfer to compartment.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible irradiation from the sky, (2) $T_{s,o} > T_\infty$ (hot surface facing upward) and $Ra_L > 10^7$, (3) Constant properties.

PROPERTIES: *Table A-4*, air (p = 1 atm, $T_f \approx 310K$): $\nu = 16.9 \times 10^{-6}$ m^2/s, k = 0.0270 W/m·K, Pr = 0.706, $\alpha = \nu/Pr = 23.9 \times 10^{-6}$ m^2/s, $\beta = 0.00323$ K^{-1}.

ANALYSIS: From an energy balance for the outer surface,

$$\alpha_S G_S - q''_{conv} - E = q''_{cond} = \frac{T_{s,o} - T_{s,i}}{R''_{tot}}$$

$$\alpha_S G_S - \overline{h}\left(T_{s,o} - T_\infty\right) - \varepsilon\sigma T_{s,o}^4 = \frac{T_{s,o} - T_{s,i}}{2R''_p + R''_i}$$

where $R''_p = \left(t_1 / k_p\right) = 2.78 \times 10^{-5} \text{m}^2 \cdot \text{K/W}$ and $R''_i = \left(t_2 / k_i\right) = 1.923\,\text{m}^2 \cdot \text{K/W}$. For a hot surface facing upward and $Ra_L = g\beta\left(T_{s,o} - T_\infty\right)L^3 / \alpha\nu > 10^7$, $\overline{h}$ is obtained from Eq. 9.31. Hence, with cancellation of L,

$$\overline{h} = \frac{k}{L} 0.15\,Ra_L^{1/3} = 0.15 \times 0.0270\,\text{W/m}\cdot\text{K}\left(\frac{9.8\,\text{m/s}^2 \times 0.00323\,\text{K}^{-1}}{16.9 \times 23.9 \times 10^{-12}\,\text{m}^4/\text{s}^2}\right)^{1/3}\left(T_{s,o} - T_\infty\right)^{1/3}$$

$$= 1.73\,\text{W/m}^2 \cdot \text{K}^{4/3}\left(T_{s,o} - 305\,\text{K}\right)^{4/3}$$

Hence,

$$0.5\left(750\,\text{W/m}^2 \cdot \text{K}\right) - 1.73\,\text{W/m}^2 \cdot \text{K}^{4/3}\left(T_{s,o} - 305\right)^{4/3} - 0.5 \times 5.67 \times 10^{-8}\,\text{W/m}^2 \cdot \text{K}^4\,T_{s,o}^4 = \frac{T_{s,o} - 263\text{K}}{\left(5.56 \times 10^{-5} + 1.923\right)\text{m}^2 \cdot \text{K/W}}$$

Solving, we obtain $\quad T_{s,o} = 318.3\text{K} = 45.3°\text{C}$ $\quad$ <

Hence, the heat load is $\quad q = \left(W \cdot L_t\right)q''_{cond} = \left(3.5\text{m} \times 10\text{m}\right)\dfrac{(45.3 + 10)°\text{C}}{1.923\,\text{m}^2 \cdot \text{K/W}} = 1007\,\text{W}$ $\quad$ <

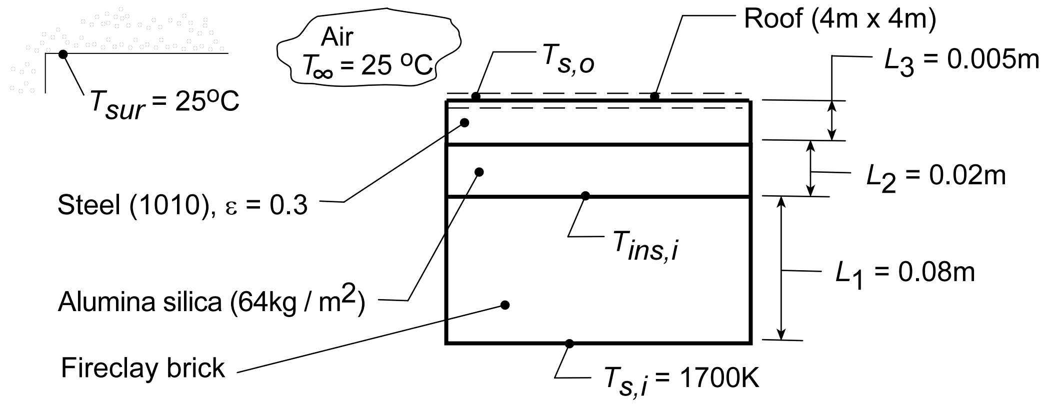
COMMENTS: (1) The thermal resistance of the aluminum panels is negligible compared to that of the insulation. (2) The value of the convection coefficient is $\overline{h} = 1.73\left(T_{s,o} - T_\infty\right)^{1/3} = 4.10\,\text{W/m}^2 \cdot \text{K}$.

PROBLEM 9.43

KNOWN: Inner surface temperature and composition of a furnace roof. Emissivity of outer surface and temperature of surroundings.

FIND: (a) Heat loss through roof with no insulation, (b) Heat loss with insulation and inner surface temperature of insulation, and (c) Thickness of fire clay brick which would reduce the insulation temperature, $T_{ins,i}$, to 1350 K.

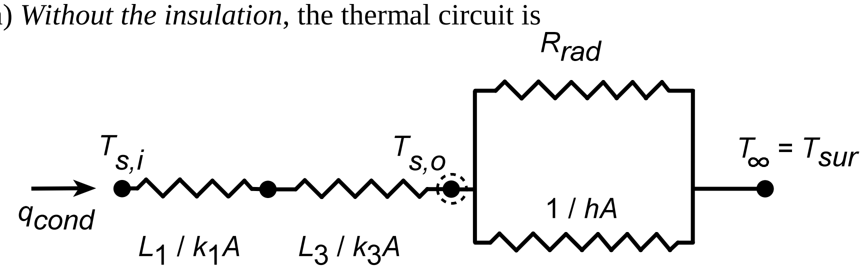
SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction through the composite wall, (3) Negligible contact resistance, (4) Constant properties.

PROPERTIES: *Table A-4*, Air ($T_f \approx 400$ K, 1 atm): k = 0.0338 W/m·K, $\nu = 26.4 \times 10^{-6}$ m²/s, $\alpha = 38.3 \times 10^{-6}$ m²/s, Pr = 0.69, $\beta = (400\text{ K})^{-1} = 0.0025\text{ K}^{-1}$; *Table A-1*, Steel 1010 (600 K): k = 48.8 W/m·K; *Table A-3* Alumina-Silica blanket (64 kg/m³, 750 K): k = 0.125 W/m·K; *Table A-3*, Fire clay brick (1478 K): k = 1.8 W/m·K.

ANALYSIS: (a) *Without the insulation*, the thermal circuit is



Performing an energy balance at the outer surface, it follows that

$$q_{cond} = q_{conv} + q_{rad} \qquad \frac{T_{s,i} - T_{s,o}}{L_1/k_1A + L_3/k_3A} = hA\left(T_{s,o} - T_\infty\right) + \varepsilon\sigma A\left(T_{s,o}^4 - T_{sur}^4\right) \quad (1,2)$$

where the radiation term is evaluated from Eq. 1.7. The characteristic length associated with free convection from the roof is, from Eq. 9.29 $L = A_s/P = 16\text{m}^2/16\text{ m} = 1\text{ m}$. From Eq. 9.25, with an assumed value for the film temperature, T_f = 400 K,

$$Ra_L = \frac{g\beta\left(T_{s,o} - T_\infty\right)L^3}{\nu\alpha} = \frac{9.8\text{ m/s}^2\left(0.0025\text{ K}^{-1}\right)\left(T_{s,o} - T_\infty\right)(1\text{ m})^3}{26.4\times10^{-6}\text{ m}^2/\text{s}\times38.3\times10^{-6}\text{ m}^2/\text{s}} = 2.42\times10^7\left(T_{s,o} - T_\infty\right)$$

Hence, from Eq. 9.31

$$h = \frac{k}{L}0.15Ra_L^{1/3} = \frac{0.0338\text{ W/m}\cdot\text{K}}{1\text{ m}}0.15\left(2.42\times10^7\right)^{1/3}\left(T_{s,o} - T_\infty\right)^{1/3} = 1.47\left(T_{s,o} - T_\infty\right)^{1/3}\text{ W/m}^2\cdot\text{K}.(3)$$

Continued...

PROBLEM 9.43 (Cont.)

The energy balance can now be written

$$\frac{(1700 - T_{s,o})\,\text{K}}{(0.08\,\text{m}/1.8\,\text{W/m}\cdot\text{K} + 0.005\,\text{m}/48.8\,\text{W/m}\cdot\text{K})} = 1.47\left(T_{s,o} - 298\,\text{K}\right)^{4/3}$$

$$+0.3\times 5.67\times 10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left[T_{s,o}^4 - (298\,\text{K})^4\right]$$

and from iteration, find $T_{s,o} \approx 895$ K. Hence,

$$q = 16\,\text{m}^2\left\{1.47(895-298)^{4/3}\,\text{W/m}^2 + 0.3\times 5.67\times 10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left[(895\,\text{K})^4 - (298\,\text{K})^4\right]\right\}$$

$$q = 16\,\text{m}^2\{7,389 + 10,780\}\,\text{W/m}^2 = 2.91\times 10^5\,\text{W}. \qquad <$$

(b) *With the insulation*, an additional conduction resistance is provided and the energy balance at the outer surface becomes

$$\frac{T_{s,i} - T_{s,o}}{L_1/k_1A + L_2/k_2A + L_3/k_3A} = hA\left(T_{s,o} - T_\infty\right) + \varepsilon\sigma A\left(T_{s,o}^4 - T_{sur}^4\right) \qquad (4)$$

$$\frac{(1700 - T_{s,o})\,\text{K}}{(0.08\,\text{m}/1.8 + 0.02/0.125 + 0.005/48.8)\,\text{m}^2\cdot\text{K/W}} = 1.47\left(T_{s,o} - 298\,\text{K}\right)^{4/3}$$

$$+0.3\times 5.67\times 10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left[T_{s,o}^4 - (298\,\text{K})^4\right].$$

From an iterative solution, it follows that $T_{s,o} \approx 610$ K. Hence,

$$q = 16\,\text{m}^2\left\{1.47(610-298)^{4/3}\,\text{W/m}^2 + 0.3\times 5.67\times 10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left[(610\,\text{K})^4 - (298\,\text{K})^4\right]\right\}$$

$$q = 16\,\text{m}^2\{3111 + 2221\}\,\text{W/m}^2 = 8.53\times 10^4\,\text{W}. \qquad <$$

The insulation inner surface temperature is given by

$$q = \frac{T_{s,i} - T_{ins,i}}{L_1/k_1A}.$$

Hence

$$T_{ins,i} = -q\frac{L_1}{k_1A} + T_{s,i} = -8.53\times 10^4\,\text{W}\,\frac{0.08\,\text{m}}{1.8\,\text{W/m}\cdot\text{K}\times 16\,\text{m}^2} + 1700\,\text{K} = 1463\,\text{K}. \qquad <$$

(c) To determine the required thickness L_1 of the fire clay brick to reduce $T_{ins,i}$ = 1350 K, we keyboarded Eq. (4) into the IHT Workspace and found

L_1 = 0.13 m. <

COMMENTS: (1) The accuracy of the calculations could be improved by re-evaluating thermophysical properties at more appropriate temperatures.

(2) Convection and radiation heat losses from the roof are comparable. The relative contribution of radiation increases with increasing $T_{s,o}$, and hence decreases with the addition of insulation.

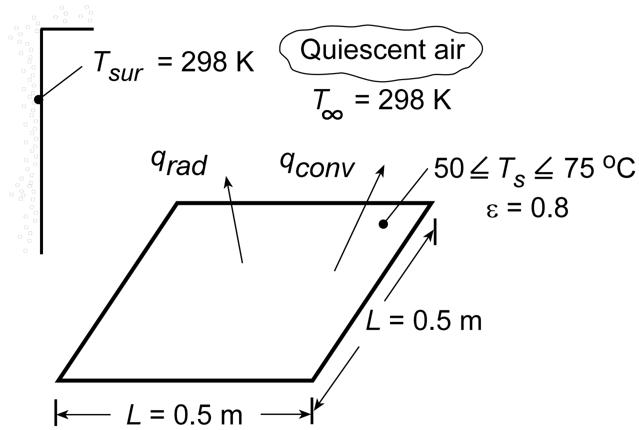
(3) Note that with the insulation, $T_{ins,i}$ = 1463 K exceeds the melting point of aluminum (933 K). Hence, molten aluminum, which can seep through the refractory, would penetrate, and thereby degrade the insulation, under the specified conditions.

PROBLEM 9.44

KNOWN: Dimensions and emissivity of top surface of amplifier. Temperature of ambient air and large surroundings.

FIND: Effect of surface temperature on convection, radiation and total heat transfer from the surface.

SCHEMATIC:



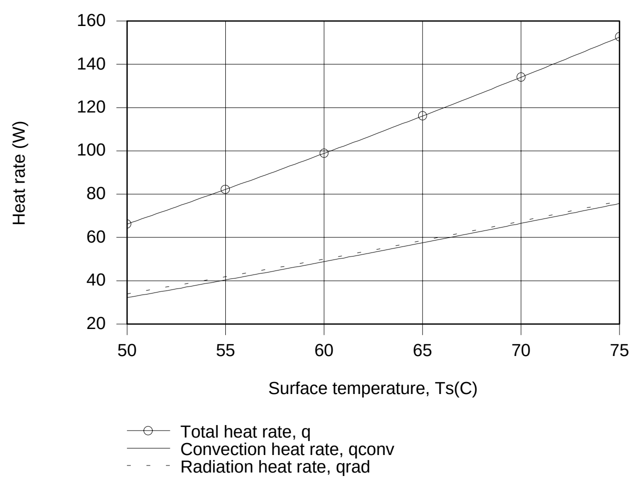
ASSUMPTIONS: (1) Steady-state, (2) Quiescent air.

PROPERTIES: *Table A.4,* air (Obtained from *Properties* Tool Pad of IHT).

ANALYSIS: The total heat rate from the surface is $q = q_{conv} + q_{rad}$. Hence,

$$q = \overline{h} A_s (T_s - T_\infty) + \varepsilon \sigma A_s \left(T_s^4 - T_{sur}^4\right)$$

where $A_s = L^2 = 0.25\ m^2$. Using the *Correlations* and *Properties* Tool Pads of IHT to evaluate the average convection coefficient for the upper surface of a heated, horizontal plate, the following results are obtained.



Over the prescribed temperature range, the radiation and convection heat rates are virtually identical and the heat rate increases from approximately 66 to 153 W.

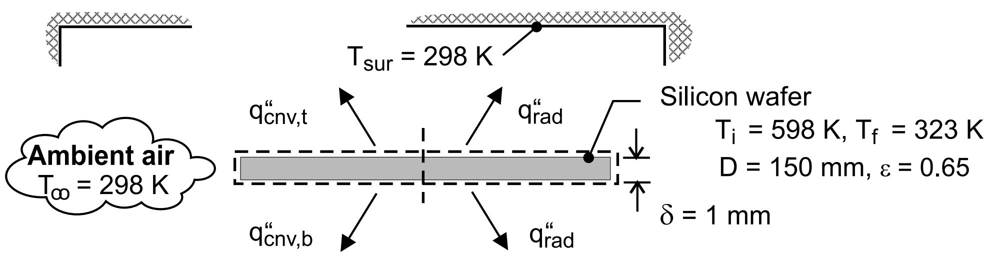
COMMENTS: A surface temperature above 50°C would be excessive and would accelerate electronic failure mechanisms. If operation involves large power dissipation (> 100 W), the receiver should be vented.

PROBLEM 9.45

KNOWN: Diameter, thickness, emissivity and initial temperature of silicon wafer. Temperature of air and surrounding.

FIND: (a) Initial cooling rate, (b) Time required to achieve prescribed final temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat transfer from side of wafer, (2) Large surroundings, (3) Wafer may be treated as a lumped capacitance, (4) Constant properties, (5) Quiescent air.

PROPERTIES: *Table A-1*, Silicon ($\overline{T}$ = 187°C = 460K): ρ = 2330 kg/m^3, c_p = 813 J/kg·K, k = 87.8 W/m·K. *Table A-4*, Air ($T_{f,i}$ = 175°C = 448K): $\nu = 32.15 \times 10^{-6}$ m^2/s, k = 0.0372 W/m·K, $\alpha = 46.8 \times 10^{-6}$ m^2/s, Pr = 0.686, β = 0.00223 K^{-1}.

SOLUTION: (a) Heat transfer is by natural convection and net radiation exchange from top and bottom surfaces. Hence, with $A_s = \pi D^2/4 = 0.0177$ m^2,

$$q = A_s\left[\left(\overline{h}_t + \overline{h}_b\right)\left(T_i - T_\infty\right) + 2\varepsilon\sigma\left(T_i^4 - T_{sur}^4\right)\right]$$

where the radiation flux is obtained from Eq. 1.7, and with $L = A_s/P = 0.0375$m and $Ra_L = g\beta\left(T_i - T_\infty\right)L^3/\alpha\nu = 2.30 \times 10^5$, the convection coefficients are obtained from Eqs. 9.30 and 9.32. Hence,

$$\overline{h}_t = \frac{k}{L}\left(0.54\,Ra_L^{1/4}\right) = \frac{0.0372\,\text{W/m}\cdot\text{K} \times 11.8}{0.0375\text{m}} = 11.7\,\text{W/m}^2\cdot\text{K}$$

$$\overline{h}_b = \frac{k}{L}\left(0.27\,Ra_L^{1/4}\right) = \frac{0.0372\,\text{W/m}\cdot\text{K} \times 5.9}{0.0375\text{m}} = 5.9\,\text{W/m}^2\cdot\text{K}$$

$$q = 0.0177\,\text{m}^2\left[\left(11.7+5.9\right)\text{W/m}^2\cdot\text{K}\left(300\text{K}\right) + 2\times0.65\times5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left(598^4 - 298^4\right)\text{K}^4\right]$$

$$q = 0.0177\,\text{m}^2\left[\left(5280 + 8845\right)\text{W/m}^2\right] = 250\,\text{W}$$ <

(b) From the generalized lumped capacitance model, Eq. 5.15,

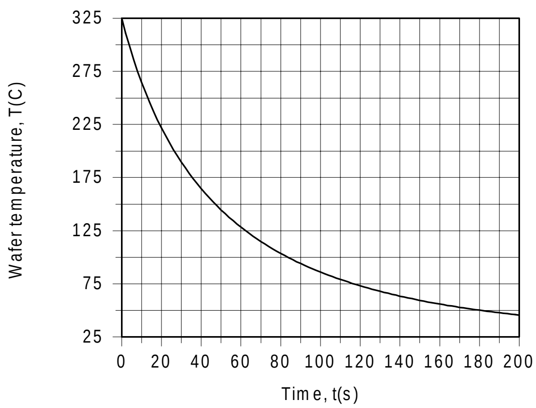
$$\rho c A_s \delta \frac{dT}{dt} = -\left[\left(\overline{h}_t + \overline{h}_b\right)\left(T - T_\infty\right) + 2\varepsilon\sigma\left(T^4 - T_{sur}^4\right)\right]A_s$$

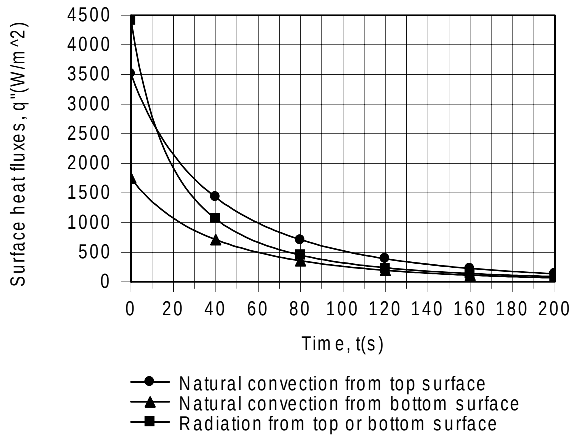
$$\int_{T_i}^{T} dT = -\int_0^t\left[\frac{\left(\overline{h}_t + \overline{h}_b\right)\left(T - T_\infty\right) + 2\varepsilon\sigma\left(T^4 - T_{sur}^4\right)}{\rho c \delta}\right]dt$$

Continued …..

Using the DER function of IHT to perform the integration, thereby accounting for variations in $\overline{h}_t$ and $\overline{h}_b$ with T, the time t_f to reach a wafer temperature of 50°C is found to be

$$t_f\left(T = 320\text{K}\right) = 181\text{s} \qquad <$$





As shown above, the rate at which the wafer temperature decays with increasing time decreases due to reductions in the convection and radiation heat fluxes. Initially, the surface radiative flux (top or bottom) exceeds the heat flux due to natural convection from the top surface, which is twice the flux due to natural convection from the bottom surface. However, because q''_{rad} and q''_{cnv} decay approximately as T^4 and $T^{5/4}$, respectively, the reduction in q''_{rad} with decreasing T is more pronounced, and at t = 181s, q''_{rad} is well below $q''_{cnv,t}$ and only slightly larger than $q''_{cnv,b}$.

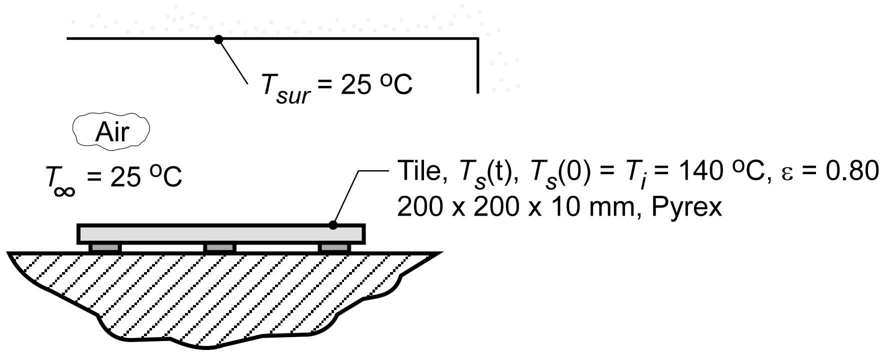
COMMENTS: With $\overline{h}_{r,i} = \varepsilon\sigma\left(T_i + T_{sur}\right)\left(T_i^2 + T_{sur}^2\right) = 14.7\,\text{W/m}^2\cdot\text{K}$, the largest cumulative coefficient of $\overline{h}_{tot} = \overline{h}_{r,i} + \overline{h}_{t,i} = 26.4\,\text{W/m}^2\cdot\text{K}$ corresponds to the top surface. If this coefficient is used to estimate a Biot number, it follows that $Bi = \overline{h}_{tot}\left(\delta/2\right)/k = 1.5\times10^{-4} \ll 1$ and the lumped capacitance approximation is excellent.

PROBLEM 9.46

KNOWN: Pyrex tile, initially at a uniform temperature T_i = 140°C, experiences cooling by convection with ambient air and radiation exchange with surroundings.

FIND: (a) Time required for tile to reach the safe-to-touch temperature of T_f = 40°C with free convection and radiation exchange; use $\overline{T} = (T_i + T_f)/2$ to estimate the average free convection and linearized radiation coefficients; comment on how sensitive result is to this estimate, and (b) Time-to-cool if ambient air is blown in parallel flow over the tile with a velocity of 10 m/s.

SCHEMATIC:



ASSUMPTIONS: (1) Tile behaves as spacewise isothermal object, (2) Backside of tile is perfectly insulated, (3) Surroundings are large compared to the tile, (4) For forced convection situation, part (b), assume flow is fully turbulent.

PROPERTIES: *Table A.3*, Pyrex (300 K): ρ = 2225 kg/m^3, c_p = 835 J/kg·K, k = 1.4 W/m·K, ε = 0.80 (given); *Table A.4*, Air $\left(T_f = (\overline{T}_s + T_\infty)/2 = 330.5\,\text{K}, 1\,\text{atm}\right)$: ν = 18.96 × 10^{-6} m^2/s, k = 0.0286 W/m·K, α = 27.01 × 10^{-6} m^2/s, Pr = 0.7027, β = 1/T_f.

ANALYSIS: (a) For the lumped capacitance system with a constant coefficient, from Eq. 5.6,

$$\frac{T_s(t) - T_\infty}{T_i - T_\infty} = \exp\left[-\left(\frac{\overline{h}A_s}{\rho V c}\right)t\right] \tag{1}$$

where $\overline{h}$ is the combined coefficient for the convection and radiation processes,

$$\overline{h} = \overline{h}_{cv} + \overline{h}_{rad} \tag{2}$$

and $\qquad A_s = L^2 \qquad\qquad V = L^2 d \qquad$ (3,4)

The linearized radiation coefficient based upon the average temperature, $\overline{T}_s$, is

$$\overline{T}_s = (T_i + T_f)/2 = (140 + 40)^\circ\,\text{C}/2 = 90^\circ\,\text{C} = 363\,\text{K} \tag{5}$$

$$\overline{h}_{rad} = \varepsilon\sigma\left(\overline{T}_s + T_{sur}\right)\left(\overline{T}_s^2 + T_{sur}^2\right) \tag{6}$$

$$\overline{h}_{rad} = 0.8 \times 5.67 \times 10^{-8}\,\text{W/m}^2\cdot\text{K}^4\,(363 + 298)\left(363^2 + 298^2\right)\text{K}^3 = 6.61\,\text{W/m}^2\cdot\text{K}$$

The free convection coefficient can be estimated from the correlation for the flat plate, Eq. 9.30, with

$$Ra_L = \frac{g\beta\Delta T L^3}{\nu\alpha} \qquad\qquad L = A_s/P = L^2/4L = 0.25L \tag{7,8}$$

Continued...

PROBLEM 9.46 (Cont.)

$$Ra_L = \frac{9.8\,m/s^2\,(1/330\,K)(363-298)\,K\,(0.25\times0.200\,m)^3}{18.96\times10^{-6}\,m^2/s\times27.01\times10^{-6}\,m^2/s} = 4.712\times10^5$$

$$\overline{Nu}_L = 0.54 Ra_L^{1/4} = 0.54\left(4.712\times10^5\right)^{1/4} = 14.18$$

$$\bar{h}_{cv} = \overline{Nu}_L\,k/L = 14.18\times0.0286\,W/m\cdot K/0.25\times0.200\,m = 8.09\,W/m^2\cdot K$$

From Eq. (2), it follows

$$\bar{h} = (6.61+8.09)\,W/m^2\cdot K = 14.7\,W/m^2\cdot K$$

From Eq. (1), with $A_s/V = 1/d$, where d is the tile thickness, the time-to-cool is found as

$$\frac{40-25}{140-25} = \exp\left[-\frac{14.7\,W/m^2\cdot K\times t_f}{2225\,kg/m^3\times0.010\,m\times835\,J/kg\cdot K}\right]$$

$$t_f = 2574\,s = 42.9\,min$$ <

Using the *IHT Lumped Capacitance Model* with the *Correlations Tool, Free Convection, Flat Plate*, we can perform the analysis where both h_{cv} and h_{rad} are evaluated as a function of the tile temperature. The time-to-cool is

$$t_f = 2860\,s = 47.7\,min$$ <

which is 10% higher than the approximate value.

(b) Considering parallel flow with a velocity, $u_\infty = 10\,m/s$ over the tile, the Reynolds number is

$$Re_L = \frac{u_\infty L}{\nu} = \frac{10\,m/s\times0.200\,m}{18.96\times10^{-6}\,m^2/s} = 1.055\times10^5$$

but, assuming the flow is turbulent at the upstream edge, use Eq. 7.41 to estimate $\bar{h}_{cv}$,

$$\overline{Nu}_L = 0.037\,Re_L^{4/5}\,Pr^{1/3} = 0.037\left(1.055\times10^5\right)^{4/5}(0.7027)^{1/3} = 343.3$$

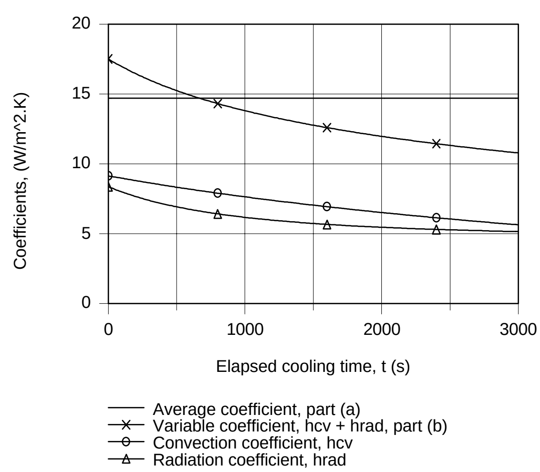
$$\bar{h}_{cv} = \overline{Nu}_L k/L = 343.3\times0.0286\,W/m\cdot K/0.200\,m = 49.1\,W/m^2\cdot K$$

Hence, using Eqs. (2) and (1), find

$$\bar{h} = 57.2\,W/m^2\cdot K \qquad t_f = 661\,s = 11.0\,min$$ <

COMMENTS: (1) For the conditions of part (a), Bi = hd/k= 14.7 W/m^2·K × 0.01m / 1.4 W/m·K = 0.105. We conclude that the lumped capacitance analysis is marginally applicable. For the condition of part (b), Bi = 0.4 and, hence, we need to consider spatial effects as explained in Section 5.4. If we considered spatial effects, would our estimates for the time-to-cool be greater or less than those from the foregoing analysis?

(2) For the conditions of part (a), the convection and radiation coefficients are shown in the plot below as a function of cooling time. Can you use this information to explain the relative magnitudes of the t_f estimates?

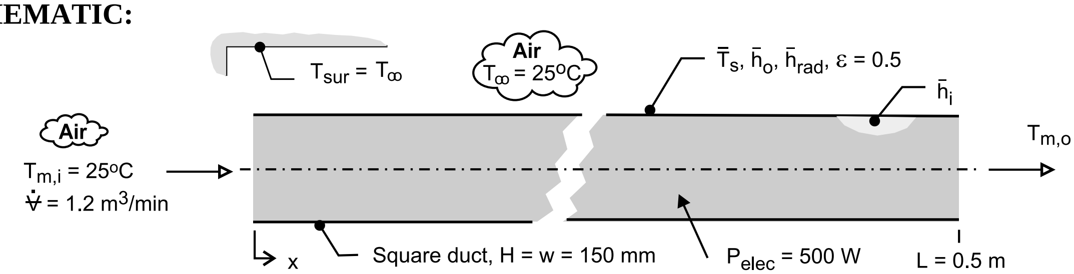


PROBLEM 9.47

KNOWN: Stacked IC boards within a duct dissipating 500 W with prescribed air flow inlet temperature, flow rate, and internal convection coefficient. Outer surface has emissivity of 0.5 and is exposed to ambient air and large surroundings at 25°C.

FIND: Develop a model to estimate outlet temperature of the air, $T_{m,o}$, and the average surface temperature of the duct, $\overline{T}_s$, following these steps: (a) Estimate the average free convection for the outer surface, $\overline{h}_o$, assuming an average surface temperature of 37°C; (b) Estimate the average (linearized) radiation coefficient for the outer surface, $\overline{h}_{rad}$, assuming an average surface temperature of 37°C; (c) Perform an overall energy balance on the duct considering (i) advection of the air flow, (ii) dissipation of electrical power in the ICs, and (iii) heat transfer from the fluid to the ambient air and surroundings. Express the last process in terms of thermal resistances between the fluid and the mean fluid temperature, $\overline{T}_m$, and the outer temperatures T_∞ and T_{sur}; (d) Substituting numerical values into the expression of part (c), calculate $T_{m,o}$ and $\overline{T}_s$; comment on your results and the assumptions required to develop your model.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible potential energy, kinetic energy and flow work changes for the internal flow, (3) Constant properties, (4) Power dissipated in IC boards nearly uniform in longitudinal direction, (5) Ambient air is quiescent, and (5) Surroundings are isothermal and large relative to the duct.

PROPERTIES: *Table A-4*, Air ($T_f = (\overline{T}_s + T_\infty)/2 = 304$ K): $\nu = 1.629 \times 10^{-5}\ m^2/s$, $\alpha = 2.309 \times 10^{-5}\ m^2/s$, k = 0.0266 W/m·K, $\beta = 0.003289\ K^{-1}$, Pr = 0.706.

ANALYSIS: (a) *Average, free-convection coefficient over the duct*. Heat loss by free convection occurs on the vertical sides and horizontal top and bottom. The methodology for estimating the average coefficient assuming the average duct surface temperature $\overline{T}_s$ = 37°C follows that of Example 9.3. For the *vertical sides*, from Eq. 9.25 with L = H, find

$$Ra_L = \frac{g\beta\left(\overline{T}_s - T_\infty\right)H^3}{\nu\alpha}$$

$$Ra_L = \frac{9.8\ m/s^2 \times 0.003289\ K^{-1}(37-25)K \times (0.150\ m)^3}{1.629\times10^{-5}\ m^2/s \times 2.309\times10^{-5} m^2/s} = 3.47\times10^6$$

The free convection is laminar, and from Eq. 9.27,

$$\overline{Nu}_L = 0.68 + \frac{0.670\ Ra_L^{1/4}}{\left[1+(0.492/Pr)^{9/16}\right]^{4/9}}$$

Continued …..

$$\overline{Nu}_L = \frac{\bar{h}_v H}{k} = 0.68 + \frac{0.670\times\left(3.47\times10^6\right)^{1/4}}{\left[1+\left(0.492/0.706\right)^{9/16}\right]^{4/9}} = 23.2$$

$$\bar{h}_v = 4.11\ \text{W/m}^2\cdot\text{K}$$

For the top and bottom surfaces, $L_c = (A_s/P) = (w \times L)/(2w + 2L) = 0.0577$ m, hence, $Ra_L = 1.974 \times 10^5$ and with Eqs. 9.30 and 9.32, respectively,

Top surface: $\overline{Nu}_L = \frac{\bar{h}_t L_c}{k} = 0.54\ Ra_L^{1/4}$; $\bar{h}_t = 5.25\ \text{W/m}^2\cdot\text{K}$

Bottom surface: $\overline{Nu}_L = \frac{\bar{h}_b L_c}{k} = 0.27\ Ra_L^{1/4}$; $\bar{h}_b = 2.62\ \text{W/m}^2\cdot\text{K}$

The average coefficient for the entire duct is

$$\bar{h}_{cv,o} = \left(2\bar{h}_v + \bar{h}_t + \bar{h}_b\right)/2 = \left(2\times4.11+5.25+2.62\right)\text{W/m}^2\cdot\text{K} = 4.02\ \text{W/m}^2\cdot\text{K} \quad <$$

(b) *Average (linearized) radiation coefficient over the duct*. Heat loss by radiation exchange between the duct outer surface and the surroundings on the vertical sides and horizontal top and bottom. With $\bar{T}_s$ = 37°C, from Eq. 1.9,

$$\bar{h}_{rad} = \varepsilon\sigma\left(\bar{T}_s + T_{sur}\right)\left(\bar{T}_s^2 + T_{sur}^2\right)$$

$$\bar{h}_{rad} = 0.5\times5.67\times10^{-8}\text{W/m}^2\cdot\text{K}^4\left(310+298\right)\left(310^2+298^2\right)\text{K}^3 = 3.2\ \text{W/m}^2\cdot\text{K} \quad <$$

(c) *Overall energy balance on the fluid in the duct.* The control volume is shown in the schematic below and the energy balance is

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = 0$$

$$-q_{adv} + P_{elec} - q_{out} = 0 \tag{1}$$

The advection term has the form, with $\dot{m} = \dot{\forall}\rho$,

$$q_{adv} = \dot{m}\,c_p\left(T_{m,o} - T_{m,i}\right) \tag{2}$$

and the heat rate q_{out} is represented by the thermal circuit shown below and has the form, with $T_{sur} = T_\infty$,

$$q_{out} = \frac{\bar{T}_m - T_\infty}{R_{cv,i} + \left(1/R_{cv,o} + 1/R_{rad}\right)^{-1}} \tag{3}$$

where $\bar{T}_m$ is the average mean temperature of the fluid, $(T_{m,i} + T_{m,o})/2$. The thermal resistances are evaluated with $A_s = 2(w + H)\,L$ as

$$R_{cv,i} = 1/\bar{h}_i A_s \tag{4}$$

$$R_{cv,o} = 1/\bar{h}_{cv,o} A_s \tag{5}$$

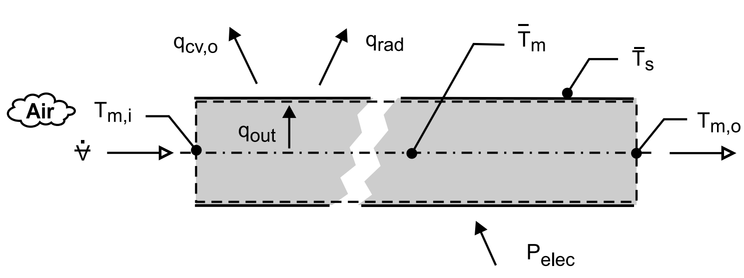
$$R_{rad} = 1/\bar{h}_{rad} A_s \tag{6}$$

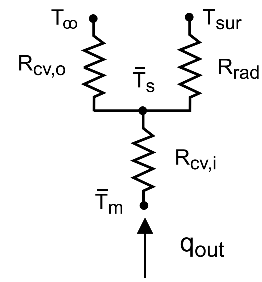
Continued …..

PROBLEM 9.47 (Cont.)

Using this energy balance, the outlet temperature of the air can be calculated. From the thermal circuit, the average surface temperature can be calculated from the relation

$$q_{out} = \left(\overline{T}_m - \overline{T}_s\right) / R_{cv,i} \tag{7}$$





(d) *Calculating $T_{m,o}$ and* $\overline{T}_s$. Substituting numerical values into the expressions of Part (c), find

$$T_{m,o} = 45.7°C \qquad \overline{T}_s = 34.0°C$$ <

The heat rates and thermal resistance results are

$$q_{adv} = 480.5 \text{ W} \qquad q_{out} = 19.5 \text{ W}$$

$$R_{cv,i} = 0.020 \text{ K/W} \qquad R_{cv,o} = 0.250 \text{ K/W} \qquad R_{rad} = 0.313 \text{ K/W}$$

COMMENTS: (1) We assumed $\overline{T}_s$ = 37°C for estimating $\overline{h}_{cv,o}$ and $\overline{h}_{rad}$, whereas from the energy balance we found the value was 34.0°C. Performing an interative solution, with different assumed $\overline{T}_s$ we would find that the results are not sensitive to the $\overline{T}_s$ value, and that the foregoing results are satisfactory.

(2) From the results of Part (d) for the heat rates, note that about 4% of the electrical power is transferred from the duct outer surface. The present arrangement does not provide a practical means to cool the IC boards.

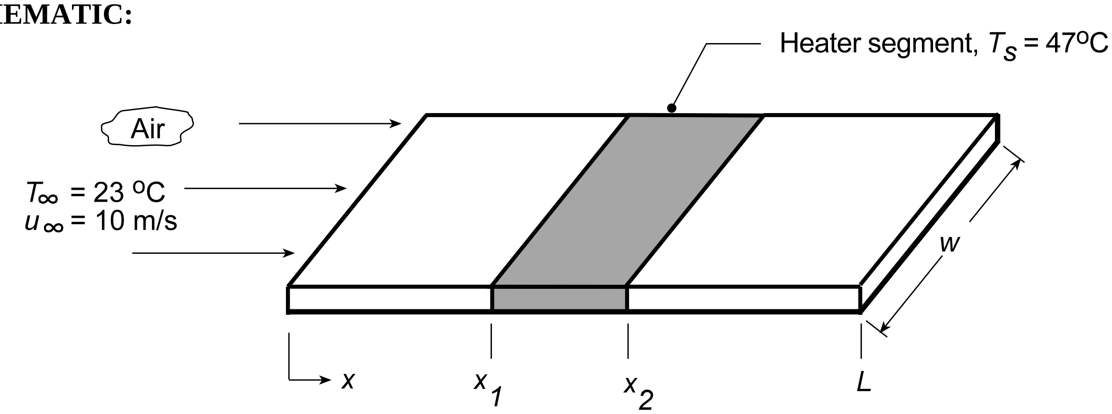
(3) Note that $T_{m,i} < T_s < T_{m,o}$. As such, we can't utilize the usual log-mean temperature (LMTD) expression, Eq. 8.45, in the rate equation for the internal flow analysis. It is for this reason we used the overall coefficient approach representing the heat transfer by the thermal circuit. The average surface temperature of the duct, $\overline{T}_s$, is only used for the purposes of estimating $\overline{h}_{cv,o}$ and $\overline{h}_{rad}$. We represented the effective temperature difference between the fluid and the ambient/surroundings as $\overline{T}_m - T_\infty$. Because the fluid temperature rise is not very large, this assumption is a reasonable one.

PROBLEM 9.48

KNOWN: Parallel flow of air over a highly polished aluminum plate flat plate maintained at a uniform temperature T_s = 47°C by a series of segmented heaters.

FIND: (a) Electrical power required to maintain the heater segment covering the section between x_1 = 0.2 m and x_2 = 0.3m and (b) Temperature that the surface would reach if the air blower malfunctions and heat transfer occurs by free, rather than forced, convection.

SCHEMATIC:



ASSUMPTIONS : (l) Steady-state conditions, (2) Backside of plate is perfectly insulated, (3) Flow is turbulent over the entire length of plate, part (a), (4) Ambient air is extensive, quiescent at 23°C for part (b).

PROPERTIES: *Table A.4*, Air ($T_f = (T_s + T_\infty)/2$ = 308K): $\upsilon = 16.69 \times 10^{-6}$ m²/s, k = 0.02689 W/m·K, $\alpha = 23.68 \times 10^{-6}$ m²/s, Pr = 0.7059, $\beta = 1/T_f$; *Table A.12*, Aluminum, highly polished: ε = 0.03.

ANALYSIS: (a) The power required to maintain the segmented heater (x_1 - x_2) is

$$P_e = \bar{h}_{x1-x2}(x_2 - x_1)\,w\,(T_s - T_\infty) \qquad (1)$$

where $\bar{h}_{x1-x2}$ the average coefficient for the section between x_1 and x_2 can be evaluated as the average of the local values at x_1 and x_2,

$$\bar{h}_{x1-x2} = \left(h(x_1) + h(x_2)\right)/2 \qquad (2)$$

Using Eq. 7.37 appropriate for fully turbulent flow, with $Re_x = u_\infty x / k$,

$$Nu_{x1} = 0.0296\, Re_x^{4/5}\, Pr^{1/3}$$

$$Nu_{X1} = 0.0296\left(\frac{10\text{m/s} \times 0.2\text{m}}{16.69 \times 10^{-6}\ \text{m}^2/\text{s}}\right)^{4/5}(0.7059)^{1/3} = 304.6$$

$$h_{x1} = Nu_{x1}\,k/x_1 = 304.6 \times 0.02689\ \text{W/m}\cdot\text{K}/0.2\text{m} = 40.9\ \text{W/m}^2\cdot\text{K}$$

$$Nu_{x2} = 421.3 \qquad h_{x2} = 37.8\ \text{W/m}^2\cdot\text{K}$$

Hence, from Eq (2) to obtain $\bar{h}_{x1-x2}$ and Eq. (1) to obtain P_e,

$$\bar{h}_{x1-x2} = (40.9 + 37.8)\ \text{W/m}^2\cdot\text{K}/2 = 39.4\ \text{W/m}^2\cdot\text{K}$$

$$P_e = 39.4\ \text{W/m}^2\cdot\text{K}\,(0.3 - 0.2)\text{m} \times 0.2\text{m}\,(47 - 23)^\circ\text{C} = 18.9\ \text{W} \qquad <$$

Continued...

(b) Without the airstream flow, the heater segment experiences free convection and radiation exchange with the surroundings,

$$P_e = \left[\bar{h}_{cv}\left(T_s - T_\infty\right) + \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right)\right]\left(x_2 - x_1\right)w \qquad (3)$$

We will assume that the free convection coefficient, $\bar{h}_{cv}$, for the segment is the same as that for the entire plate. Using the correlation for a flat plate, Eq. 9.30, with

$$Ra_L = \frac{g\beta\Delta T L_c^3}{\nu\alpha} \qquad L_c = \frac{A_s}{P} = \frac{0.2 \times 0.5\,\text{m}^2}{2(0.2 + 0.5)\,\text{m}} = 0.0714\,\text{m}$$

and evaluating properties at T_f = 308 K,

$$Ra_L = \frac{9.8\,\text{m/s}^2\,(1/308\,\text{K})(47 - 23)(0.0714\,\text{m})^3}{16.69 \times 10^{-6}\,\text{m/s}^2 \times 23.68 \times 10^{-6}\,\text{m}^2/\text{s}} = 7.033 \times 10^5$$

$$\overline{Nu}_L = 0.54 Ra_L^{1/4} = 0.54\left(7.033 \times 10^5\right)^{1/4} = 15.64$$

$$\bar{h}_{cv} = \overline{Nu}_L\, k/L_c = 15.64 \times 0.02689\,\text{W/m}\cdot\text{K}/0.0714\,\text{m} = 5.89\,\text{W/m}^2\cdot\text{K}$$

Substituting numerical values into Eq. (3),

$$18.9\,\text{W} = \left[5.89\,\text{W/m}^2\cdot\text{K}\left(T_s - 296\right) + 0.03 \times 5.67 \times 10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left(T_s^4 - 296^4\right)\right]\left(0.3 - 0.2\right)\text{m} \times 0.2\,\text{m}$$

$$T_s = 447\,\text{K} = 174^\circ\text{C} \qquad <$$

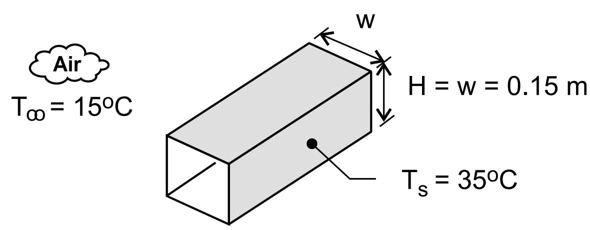
COMMENTS: Recognize that in part (b), the assumed value for T_f = 308 K is a poor approximation. Using the above relations in the IHT work space with the *Properties Tool*, find that T_s = 406 K = 133 °C using the properly evaluated film temperature (T_f) and temperature difference (ΔT) in the correlation. From this analysis, $\bar{h}_{cv} = 8.29\,\text{W/m}^2\cdot\text{K}$ and h_{rad} = 0.3 W/m^2·K. Because of the low emissivity of the plate, the radiation exchange process is not significant.

PROBLEM 9.49

KNOWN: Correlation for estimating the average free convection coefficient for the exterior surface of a long horizontal rectangular cylinder (duct) exposed to a quiescent fluid. Consider a horizontal 0.15 m-square duct with a surface temperature of 35°C passing through ambient air at 15°C.

FIND: (a) Calculate the average convection coefficient and the heat rate per unit length using the H-D correlation, (b) Calculate the average convection coefficient and the heat rate per unit length considering the duct as formed by vertical plates (sides) and horizontal plates (top and bottom), and (c) Using an appropriate correlation, calculate the average convection coefficient and the heat rate per unit length for a duct of circular cross-section having a diameter equal to the wetted perimeter of the rectangular duct of part (a). Do you expect the estimates for parts (b) and (c) to be lower or higher than those obtained with the H-D correlation? Explain the differences, if any.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Ambient air is quiescent, (3) Duct surface has uniform temperature.

PROPERTIES: *Table A-4*, air ($T_f = (T_s + T_\infty)/2 = 298$ K, 1 atm): $\nu = 1.571 \times 10^{-5}$ m^2/s, k = 0.0261 W/m·K, $\alpha = 2.22 \times 10^{-5}$ m^2/s, Pr = 0.708.

ANALYSIS: (a) The Hahn-Didion (H-D) correlation [ASHRAE Proceedings, Part 1, pp 262-67, 1972] has the form

$$\overline{Nu}_p = 0.55\,Ra_p^{1/4}\left(\frac{H}{p}\right)^{1/8} \qquad\qquad Ra_p \le 10^7$$

where the characteristic length is the half-perimeter, p = (w + H), and w and H are the horizontal width and vertical height, respectively, of the duct. The thermophysical properties are evaluated at the film temperature. Using *IHT*, with the correlation and thermophysical properties, the following results were obtained.

Ra_p	$\overline{Nu}_D$	$\bar{h}_p\ (W/m^2\cdot K)$	$q'_p\ (W/m)$	
5.08×10^7	42.6	3.71	44.5	<

where the heat rate per unit length of the duct is

$$q'_p = \bar{h}_p\, 2(H + w)(T_s - T_\infty).$$

(b) Treating the duct as a combination of horizontal (*top*: hot-side up and *bottom*: hot-side down) and two vertical plates (v) as considered in Example 9.3, the following results were obtained

$\bar{h}_t$ (W/m^2·K)	$\bar{h}_b$ (W/m^2·K)	$\bar{h}_v$ (W/m^2·K)	$\bar{h}_{hv}$ (W/m^2·K)	q'_{hv} (W/m)	
5.62	2.81	4.78	4.50	54.0	<

Continued

where the average coefficient and heat rate per unit length for the horizontal-vertical plate duct are

$$\bar{h}_{hv} = \left(\bar{h}_t + \bar{h}_b + 2\bar{h}_v\right)/4$$

$$q'_{hv} = \bar{h}_{hv} 2(H + w)(T_s - T_\infty).$$

(c) Consider a circular duct having a wetted perimeter equal to that of the rectangular duct, for which the diameter is

$$\pi D = 2(H + w) \qquad\qquad D = 0.191\text{ m}$$

Using the Churchill-Chu correlation, Eq. 9.34, the following results are obtained.

Ra_D	$\overline{Nu}_D$	$\bar{h}_D\left(W/m^2 \cdot K\right)$	$q'_D\,(W/m)$	
1.31×10^7	30.6	4.19	50.3	<

where the heat rate per unit length for the circular duct is

$$q'_D = \pi D\,\bar{h}_D\,(T_s - T_\infty).$$

COMMENTS: (1) The H-D correlation, based upon experimental measurements, provided the lowest estimate for $\bar{h}$ and q'. The circular duct analysis results are in closer agreement than are those for the horizontal-vertical plate duct.

(2) An explanation for the relative difference in $\bar{h}$ and q' values can be drawn from consideration of the boundary layers and induced flows around the surfaces. Viewing the cross-section of the square duct, recognize that flow induced by the bottom surface flows around the vertical sides, joining the vertical plume formed on the top surface. The flow over the vertical sides is quite different than would occur if the vertical surface were modeled as an *isolated* vertical surface. Also, flow from the top surface is likewise modified by flow rising from the sides, and doesn’t behave as an *isolated* horizontal surface. It follows that treating the duct as a combination of horizontal-vertical plates (hv results), each considered as *isolated*, would over estimate the average coefficient and heat rate.

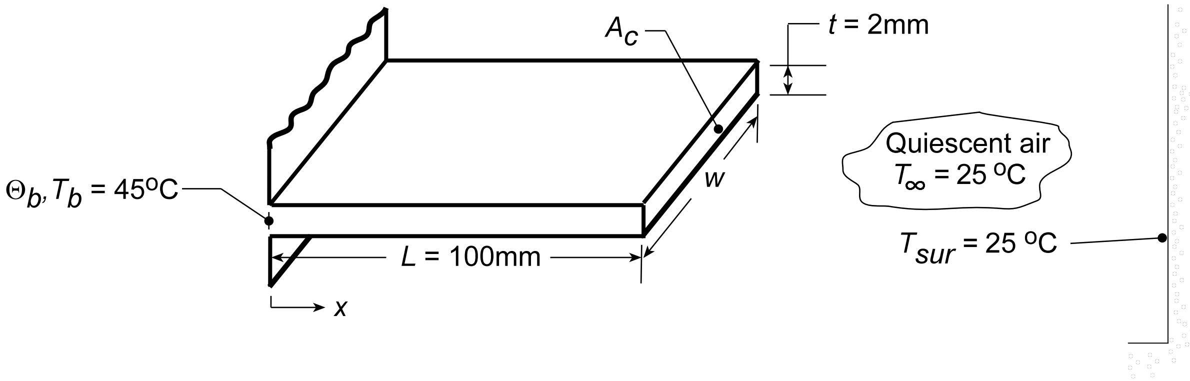
(3) It follows that flow over the horizontal cylinder more closely approximates the situation of the square duct. However, the flow is more streamlined; thinnest along the bottom, and of increasing thickness as the flow rises and eventually breaks away from the upper surface. The edges of the duct disrupt the rising flow, lowering the convection coefficient. As such, we expect the horizontal cylinder results to be systematically higher than for the H-D correlation that accounts for the edges.

PROBLEM 9.50

KNOWN: Straight, rectangular cross-sectioned fin with prescribed geometry, base temperature, and environmental conditions.

FIND: (a) Effectiveness considering only free convection with average coefficient, (b) Effectiveness considering also radiative exchange, (c) Finite-difference equations suitable for considering local, rather than average, values.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) One-dimensional conduction in fin, (4) Width of fin much larger than length, w >> L, (5) Uniform heat transfer coefficient over length for Parts (a) and (b).

PROPERTIES: *Table A-1*, Aluminum alloy 2024-T6 ($T \approx (45 + 25) / 2 = 35\ °C \approx 300$ K), k = 177 W/m·K; *Table A-11*, Aluminum alloy 2024-T6 (Given), $\varepsilon = 0.82$; *Table A-4*. Air ($T_f \approx 300$ K), $\nu = 15.89 \times 10^{-6}$ m²/s, k = 26.3×10^{-3} W/m·K, $\alpha = 22.5 \times 10^{-6}$ m²/s, $\beta = 1/T_f = 33.3 \times 10^{-3}$ K⁻¹.

ANALYSIS: (a) The effectiveness of a fin is determined from Eq. 3.81

$$\varepsilon = q_f \big/ \bar{h} A_{c,b} \theta_b \tag{1}$$

where $\bar{h}$ is the average heat transfer coefficient. The fin heat transfer follows from Eq. 3.72

$$q_f = M \frac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL} \tag{2}$$

where

$$M = \left(hPkA_c\right)^{1/2} \theta_b \qquad \text{and} \qquad m = \left(hP/kA_c\right)^{1/2}. \tag{3,4}$$

Horizontal, flat plate correlations assuming $T_f = (T_b + T_\infty) / 2 \approx 300$ K may be used to estimate $\bar{h}$, Eqs. 9.30 to 9.32. Calculate first the Rayleigh number

$$Ra_{L_c} = \frac{g\beta\left(\bar{T}_s - T_\infty\right)L_c^3}{\nu\alpha} \tag{5}$$

where $\bar{T}_s$ is the average temperature of the fin surface and L_c is the characteristic length from Eq. 9.29,

$$L_c \equiv \frac{A_s}{P} = \frac{L \times w}{2L + 2w} \approx \frac{L}{2}. \tag{6}$$

Substituting numerical values,

$$Ra_{L_c} = \frac{9.8\,m/s^2 \times 1/300K \times (310 - 298)K\left(100 \times 10^{-3}/2\right)^3 m^3}{22.5 \times 10^{-6}\,m^2/s \times 15.89 \times 10^{-6}\,m^2/s} = 1.37 \times 10^5 \tag{7}$$

Continued...

PROBLEM 9.50 (Cont.)

where $\overline{T}_s \approx (T_b + T_f)/2 = 310K$. Recognize the importance of this assumption which must be justified for a precise result. Using Eq. 9.30 and 9.32 for the upper and lower surfaces, respectively,

$$Nu_{L_c} = 0.54\left(1.37\times10^5\right)^{1/4} = 10.4, \qquad \overline{h}_u = Nu_{L_c} \times \frac{k}{L_c} = \frac{0.0263\,W/m\cdot K}{\left(100\times10^{-8}/2\right)m} = 5.47\,W/m^2\cdot K$$

$$Nu_{L_c} = 0.27\left(1.37\times10^5\right)^{1/4} = 5.20, \qquad \overline{h}_\ell = 2.73\,W/m^2\cdot K$$

The average value is estimated as $\overline{h}_c = \left(\overline{h}_u + \overline{h}_\ell\right)/2 = 4.10\,W/m^2\cdot K$. Using this value in Eqs. (3) and (4), find

$$M = \left[4.10\,W/m^2\cdot K\,(2w)\,m \times 177\,W/m\cdot K\left(w\times2\times10^{-3}\right)m^2\right]^{1/2}(45-25)^\circ C = 34.1w\ W$$

$$m = \left(\overline{h}_c P/kA_c\right)^{1/2} = \left[4.1\,W/m^2\cdot K\,(2w)\,m\Big/177\,W/m\cdot K\left(w\times2\times10^{-3}\right)m^2\right]^{1/2} = 4.81\,m^{-1}.$$

Substituting these values into Eq. (2), with mL = 0.481 and $q_f/w = q_f'$.

$$q'_f = 34.1\,W/m \times \frac{\sinh 0.481 + \left(4.1\,W/m^2\cdot K/4.81\,m^{-1}\times177\,W/m\cdot K\right)\cosh 0.481}{\cosh 0.481 + \left(4.86\times10^{-3}\right)\sinh 0.481} = 15.2\,W/m$$

and then from Eq. (1), the effectiveness is

$$\varepsilon = 15.2\,W/m\times w\Big/4.1\,W/m^2\cdot K\left(w\times2\times10^{-3}m\right)(45-25)^\circ C = 92.7. \qquad <$$

(b) If radiation exchange with the surroundings is considered, use Eq. 1.9 to determine

$$\overline{h}_r = \varepsilon\sigma\left(\overline{T}_s + T_{sur}\right)\left(\overline{T}_s^2 + T_{sur}^2\right) = 0.82\times5.67\times10^{-8}\,W/m^2\cdot K^4\,(310+298)(310^2+298^2)K^3 = 5.23\,W/m^2\cdot K.$$

This assumes the fin surface is gray-diffuse and small compared to the surroundings. Using $\overline{h} = \overline{h}_c + \overline{h}_r$ where $\overline{h}_c$ is the convection parameter from part (a), find $\overline{h} = (4.10+5.23)\,W/m^2\cdot K = 9.33\,W/m^2\cdot K$,

$M = 51.4w\ W$, $m = 7.26\,m^{-1}$, $q_f' = 31.8\,W/m$ giving

$$\varepsilon = 85.2 \qquad <$$

(c) To perform the numerical method, we used the *IHT Finite Difference Equation Tool* for *1-D, SS*, extended surfaces. The convection coefficient for each node was expressed as

$$h_{tot,m} = \overline{h}_u(T_m) + \overline{h}_\ell(T_m)/2 + \overline{h}_r(T_m)$$

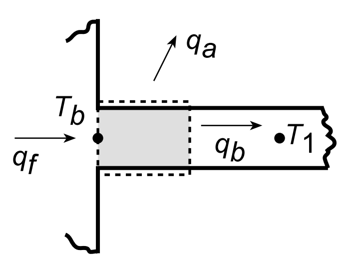
The effectiveness was calculated from Eq. (1) where the fin heat rate is determined from an energy balance on the base node.

$$q_f = q_{cond} + q_{cv} + q_{rad}$$

$$q_b = q_{cond} = kA_c\,(T_b - T_1)/\Delta x$$

$$q_a = q_{cv} + q_{rad} = \overline{h}_{tot,b}\,(P\cdot\Delta w/2)(T_b - T_{inf})$$

$$\overline{h}_{tot,b} = \left(\overline{h}_u(T_b) + \overline{h}_\ell(T_b)\right)/2 + \overline{h}_r(T_b)$$



Continued...

PROBLEM 9.50 (Cont.)

The results of the analysis (15 nodes, $\Delta x = L/15$)

$$q_f = 33.6\,W/m \qquad \varepsilon = 83.2$$ <

COMMENTS: (1) From the analytical treatments, parts (a) and (b), considering radiation exchange nearly doubles the fin heat rate (31.8 vs. 15.2 W/m) and reduces the effectiveness from 92.7 to 85.2. The numerical method, part (c) considering local variations for h_c and h_{rad}, provides results for q_f' and ε which are in close agreement with the analytical method, part (b).

(2) The *IHT Finite Difference Equation Tool* provides a powerful approach to solving a problem as tedious as this one. Portions of the work space are copied below to illustrate the general logic of the analysis.

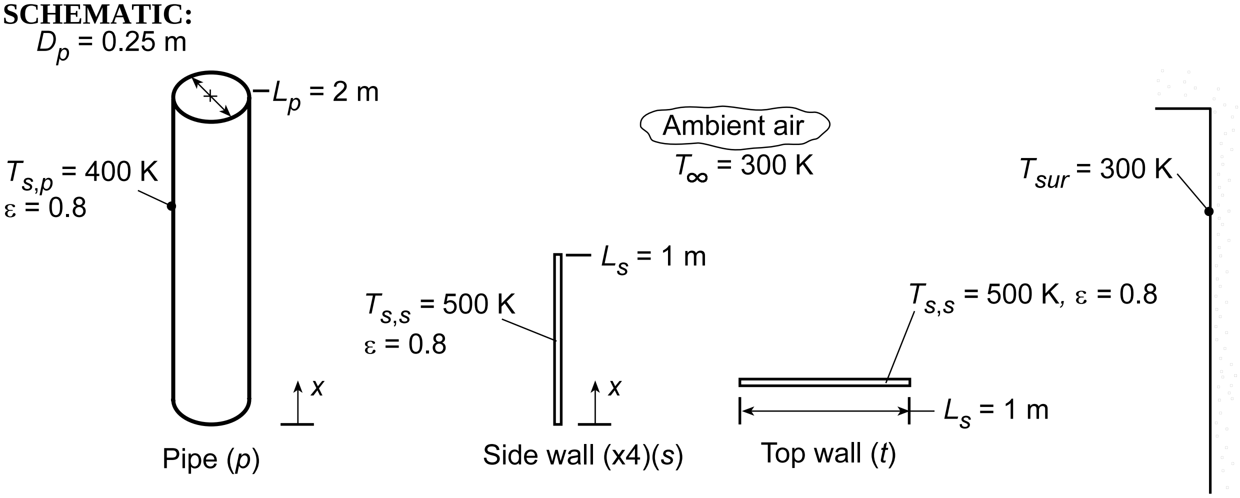
```
// Method of Solution:  The Finite-Difference Equation tool for One-Dimensional, Steady-State
Conditions for an extended surface was used to write 15 nodal equations.  The convection and linearized
radiation  coefficient for each node was separately calculated by a User-Defined Function.  */

// User-Defined Function - Upper surface convection coefficients:
/*  FUNCTION h_up ( Ts )
h_up = 0.0263 / 0.05 * NuLcu
NuLcu = 0.54 * (11,421 * (Ts - 298) )^0.25
RETURN h_up
END   */

// User-Defined Function - Linearized radiation coefficients:
/* FUNCTION h_rad ( Ts )
h_rad = 0.82 * sigma * (Ts + 298 ) * (Ts^2 + 298^2 )
sigma = 5.67e-8
RETURN h_rad
END */

/* Node 1: extended surface interior node;e and w labeled 2 and b. */
0.0 = fd_1d_xsur_i(T1,T2,Tb,k,qdot,Ac,P,deltax,Tinf,htot1,q''a)
q''a = 0        // Applied heat flux, W/m^2; zero flux shown
qdot = 0
htot1 = ( h_up(T1) + h_do(T1) ) / 2   +  h_rad(T1)

/* Node 2: extended surface interior node;e and w labeled 2 and b. */
0.0 = fd_1d_xsur_i(T2,T3,T1,k,qdot,Ac,P,deltax,Tinf,htot2,q''a)
htot2 = ( h_up(T2) + h_do(T2) ) / 2   +  h_rad(T2)

/* Node 15: extended surface end node, e-orientation; w labeled inf. */
0.0 = fd_1d_xend_e(T15,T14,k,qdot,Ac,P,deltax,Tinf,htot15,q''a,Tinf,htot15,q''a)
htot15 = ( h_up(T15) + h_do(T15) ) / 2   +  h_rad(T15)

// Assigned Variables:
Tb = 45 + 273           // Base temperature, K
Tinf = 25 + 273         // Ambient temperature, K
Tsur = 25 + 273         // Surroundings temperature, K
L  = 0.1                // Length of fin, m
deltax = L / 15         // Space increment, m
k = 177                 // Thermal conductivity, W/m.K; fin material
Ac = t * w              // Cross-sectional area, m^2
t = 0.002               // Fin thickness, m
w = 1                   // Fin width, m; unity value selected
P = 2 * w               // Perimeter, m
Lc = L / 2              // Characteristic length, convection correlation, m

// Fin heat rate and effectiveness
qf = qcond + qcvrad                                  // Heat rate from the fin base, W
qcond = k * Ac * (Tb - T1 ) / deltax                 // Heat rate, conduction, W
qcvrad =  htotb * P * deltax / 2 * ( Tb - Tinf )     // Heat rate, combined radiation convection, W
htotb = ( h_up(Tb) + h_do(Tb) ) / 2   +  h_rad(Tb)   // Total heat transfer coefficient, W/m^2.K
eff = qf / ( htotb * Ac * (Tb - Tinf) )              // Effectivenss
```

PROBLEM 9.51

KNOWN: Dimensions, emissivity and operating temperatures of a wood burning stove. Temperature of ambient air and surroundings.

FIND: Rate of heat transfer.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) Quiescent air, (3) Negligible heat transfer from pipe elbow, (4) Free convection from pipe corresponds to that from a vertical plate.

PROPERTIES: *Table A.4*, air (T_f = 400 K): $\nu = 26.41 \times 10^{-6}$ m²/s, k = 0.0338 W/m·K, $\alpha = 38.3 \times 10^{-6}$ m²/s, $\beta = 0.0025$ K^{-1}, Pr = 0.69. *Table A.4*, air (T_f = 350 K): $\nu = 20.92 \times 10^{-6}$ m²/s, k = 0.030 W/m·K, $\alpha = 29.9 \times 10^{-6}$ m²/s, Pr = 0.70, $\beta = 0.00286$ K^{-1}.

ANALYSIS: Three distinct contributions to the heat rate are made by the 4 side walls, the top surface, and the pipe surface. Hence $q_t = 4q_s + q_t + q_p$, where each contribution includes transport due to convection and radiation.

$$q_s = \bar{h}_s L_s^2 \left(T_{s,s} - T_\infty\right) + h_{rad,s} L_s^2 \left(T_{s,s} - T_{sur}\right)$$

$$q_t = \bar{h}_t L_s^2 \left(T_{s,s} - T_\infty\right) + h_{rad,s} L_s^2 \left(T_{s,s} - T_{sur}\right)$$

$$q_p = \bar{h}_p \left(\pi D_p L_p\right)\left(T_{s,p} - T_\infty\right) + h_{rad,p} \left(\pi D_p L_p\right)\left(T_{s,p} - T_{sur}\right)$$

The radiation coefficients are

$$h_{rad,s} = \varepsilon\sigma\left(T_{s,s} + T_{sur}\right)\left(T_{s,s}^2 + T_{sur}^2\right) = 12.3\,\mathrm{W/m^2 \cdot K}$$

$$h_{rad,p} = \varepsilon\sigma\left(T_{s,p} + T_{sur}\right)\left(T_{s,p}^2 + T_{sur}^2\right) = 7.9\,\mathrm{W/m^2 \cdot K}$$

For the stove side walls, $Ra_{L,s} = g\beta\left(T_{s,s} - T_\infty\right)L_s^3 / \alpha\nu = 4.84 \times 10^9$. Similarly, with $(A_s/P) = L_s^2/4L_s = 0.25$ m, $Ra_{L,t} = 7.57 \times 10^7$ for the top surface, and with L_p = 2 m, $Ra_{L,p} = 3.59 \times 10^{10}$ for the stove pipe.

For the side walls and the pipe, the average convection coefficient may be determined from Eq. 9.26,

$$\overline{Nu}_L = \left\{0.825 + \frac{0.387 Ra_L^{1/6}}{\left[1 + \left(0.492/Pr\right)^{9/16}\right]^{8/27}}\right\}^2$$

Continued...

which yields $\overline{Nu}_{L,s}$ = 199.9 and $\overline{Nu}_{L,p}$ = 377.6. For the top surface, the average coefficient may be obtained from Eq. 9.31,

$$\overline{Nu}_L = 0.15 Ra_L^{1/3}$$

which yields $\overline{Nu}_{L,t}$ = 63.5. With $\bar{h} = \overline{Nu}(k/L)$, the convection coefficients are

$$\bar{h}_s = 6.8\,W/m^2\cdot K, \quad \bar{h}_t = 8.6\,W/m^2\cdot K, \quad \bar{h}_p = 5.7\,W/m^2\cdot K$$

Hence,

$$q_s = \left(\bar{h}_s + h_{rad,s}\right)L_s^2\left(T_{s,s} - 300\,K\right) = 19.1\,W/m^2\cdot K\left(1\,m^2\right)\left(200\,K\right) = 3820\,W$$

$$q_t = \left(\bar{h}_t + h_{rad,s}\right)L_s^2\left(T_{s,s} - 300\,K\right) = 20.9\,W/m^2\cdot K\left(1\,m^2\right)\left(200\,K\right) = 4180\,W$$

$$q_p = \left(\bar{h}_p + h_{rad,p}\right)\left(\pi D_p L_p\right)\left(T_{s,p} - 300\,K\right) = 13.6\,W/m^2\cdot K\left(\pi \times 0.25\,m \times 2\,m\right)\left(100\,K\right) = 2140\,W$$

and the total heat rate is

$$q_{tot} = 4q_s + q_t + q_p = 21{,}600\,W \qquad <$$

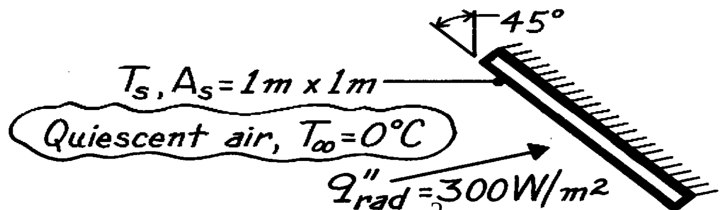
COMMENTS: The amount of heat transfer is significant, and the stove would be capable of maintaining comfortable conditions in a large, living space under harsh (cold) environmental conditions.

PROBLEM 9.52

KNOWN: Plate, 1m × 1m, inclined at 45° from the vertical is exposed to a net radiation heat flux of 300 W/m^2; backside of plate is insulated and ambient air is at 0°C.

FIND: Temperature plate reaches for the prescribed conditions.

SCHEMATIC:



ASSUMPTIONS: (1) Net radiation heat flux (300 W/m^2) includes exchange with surroundings, (2) Ambient air is quiescent, (3) No heat losses from backside of plate, (4) Steady-state conditions.

PROPERTIES: *Table A-4*, Air (assuming T_s = 84°C, $T_f = (T_s + T_\infty)/2$ = (84 + 0)°C/2 = 315K, 1 atm): ν = 17.40 × 10^{-6} m^2/s, k = 0.0274 W/m·K, α = 24.7 × 10^{-6} m^2/s, Pr = 0.705, β = 1/T_f.

ANALYSIS: From an energy balance on the plate, it follows that $q''_{rad} = q''_{conv}$. That is, the net radiation heat flux into the plate is equal to the free convection heat flux to the ambient air. The temperature of the surface can be expressed as

$$T_s = T_\infty + q''_{rad}/\overline{h}_L \tag{1}$$

where $\overline{h}_L$ must be evaluated from an appropriate correlation. Since this is the *bottom surface of a heated inclined plate*, "*g*" may be replaced by "*g cos q*"; hence using Eq. 9.25, find

$$Ra_L = \frac{g\cos q b\,(T_s - T_\infty)L^3}{na} = \frac{9.8\,\text{m/s}^2 \times \cos 45°(1/315\text{K})(84-0)\,\text{K}\,(1\text{m})^3}{17.40\times10^{-6}\,\text{m}^2/\text{s}\times24.7\times10^{-6}\,\text{m}^2/\text{s}} = 4.30\times10^9.$$

Since $Ra_L > 10^9$, conditions are turbulent and Eq. 9.26 is appropriate for estimating $\overline{Nu}_L$

$$\overline{Nu}_L = \left\{0.825 + \frac{0.387 Ra_L^{1/6}}{\left[1+(0.492/\text{Pr})^{9/16}\right]^{8/27}}\right\}^2 \tag{2}$$

$$\overline{Nu}_L = \left\{0.825 + \frac{0.387\left(4.30\times10^9\right)^{1/6}}{\left[1+(0.492/0.705)^{9/16}\right]^{8/27}}\right\}^2 = 193.2$$

$$\overline{h}_L = \overline{Nu}_L\, k/L = 193.2\times0.0274\,\text{W/m}\cdot\text{K}/1\text{m} = 5.29\,\text{W/m}^2\cdot\text{K}. \tag{3}$$

Substituting $\overline{h}_L$ from Eq. (3) into Eq. (1), the plate temperature is

$$T_s = 0°\text{C} + 300\,\text{W/m}^2/5.29\,\text{W/m}^2\cdot\text{K} = 57°\text{C}.$$ <

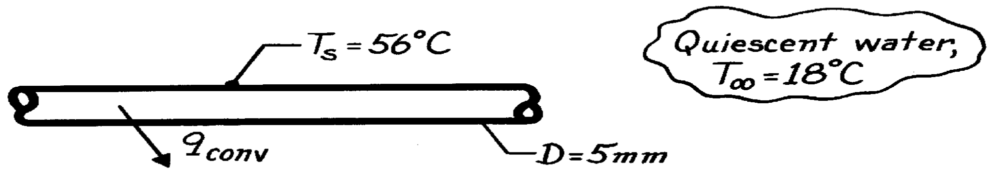
COMMENTS: Note that the resulting value of T_s ≈ 57°C is substantially lower than the assumed value of 84°C. The calculation should be repeated with a new estimate of T_s, say, 60°C. An alternate approach is to write Eq. (2) in terms of T_s, the unknown surface temperature and then combine with Eq. (1) to obtain an expression which can be solved, by trial-and-error, for T_s.

PROBLEM 9.53

KNOWN: Horizontal rod immersed in water maintained at a prescribed temperature.

FIND: Free convection heat transfer rate per unit length of the rod, q'_{conv}

SCHEMATIC:



ASSUMPTIONS: (1) Water is extensive, quiescent medium.

PROPERTIES: *Table A-6*, Water ($T_f = (T_s + T_\infty)/2 = 310K$): $\rho = 1/v_f = 993.0\ kg/m^3$, $\nu = \mu/\rho = 695 \times 10^{-6}\ N{\cdot}s/m^2/993.0\ kg/m^3 = 6.999 \times 10^{-7}\ m^2/s$, $\alpha = k/\rho c = 0.628\ W/m{\cdot}K/993.0\ kg/m^3 \times 4178\ J/kg{\cdot}K = 1.514 \times 10^{-7}\ m^2/s$, Pr = 4.62, $\beta = 361.9 \times 10^{-6}\ K^{-1}$.

ANALYSIS: The heat rate per unit length by free convection is given as

$$q'_{conv} = \overline{h}_D \cdot p D \left(T_s - T_\infty\right). \tag{1}$$

To estimate $\overline{h}_D$, first find the Rayleigh number, Eq. 9.25,

$$Ra_D = \frac{g\, b \left(T_s - T_\infty\right) D^3}{na} = \frac{9.8 m/s^2 \left(361.9 \times 10^{-6} K^{-1}\right)(56-18)\,K\left(0.005m\right)^3}{6.999 \times 10^{-7} m^2/s \times 1.514 \times 10^{-7} m^2/s} = 1.587 \times 10^5$$

and use Eq. 9.34 for a horizontal cylinder,

$$\overline{Nu}_D = \left\{0.60 + \frac{0.387 Ra_D^{1/6}}{\left[1 + \left(0.599/Pr\right)^{9/16}\right]^{8/27}}\right\}^2$$

$$\overline{Nu}_D = \left\{0.60 + \frac{0.387\left(1.587 \times 10^5\right)^{1/6}}{\left[1 + \left(0.599/4.62\right)^{9/16}\right]^{8/27}}\right\}^2 = 10.40$$

$$\overline{h}_D = \overline{Nu}_D\, k/D = 10.40 \times 0.628 W/m \cdot K/0.005m = 1306 W/m^2 \cdot K. \tag{2}$$

Substituting for $\overline{h}_D$ from Eq. (2) into Eq. (1),

$$q'_{conv} = 1306 W/m^2 \cdot K \times p\left(0.005m\right)\left(56-18\right)K = 780 W/m. \quad <$$

COMMENTS: (1) Note the relatively large value of $\overline{h}_D$; if the rod were immersed in air, the heat transfer coefficient would be close to 5 $W/m^2{\cdot}K$.

(2) Eq. 9.33 with appropriate values of C and n from Table 9.1 could also be used to estimate $\overline{h}_D$. Find

$$\overline{Nu}_D = C Ra_D^n = 0.48\left(1.587 \times 10^5\right)^{0.25} = 9.58$$

$$\overline{h}_D = \overline{Nu}_D\, k/D = 9.58 \times 0.628 W/m \cdot K/0.005m = 1203 W/m^2 \cdot K.$$

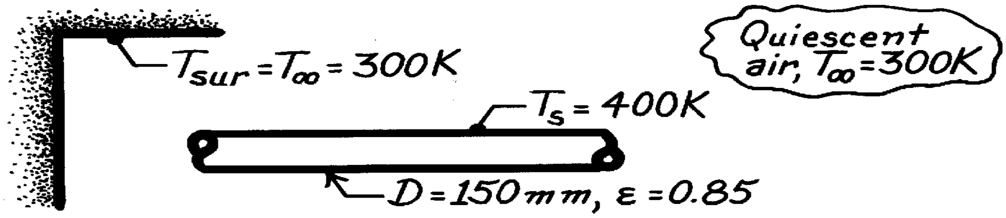
By comparison with the result of Eq. (2), the disparity of the estimates is ~8%.

PROBLEM 9.54

KNOWN: Horizontal, uninsulated steam pipe passing through a room.

FIND: Heat loss per unit length from the pipe.

SCHEMATIC:



ASSUMPTIONS: (1) Pipe surface is at uniform temperature, (2) Air is quiescent medium, (3) Surroundings are large compared to pipe.

PROPERTIES: *Table A-4*, Air ($T_f = (T_s + T_\infty)/2 = 350K$, 1 atm): $\nu = 20.92 \times 10^{-6}\ m^2/s$, $k = 0.030\ W/m\cdot K$, $\alpha = 29.9 \times 10^{-6}\ m^2/s$, $Pr = 0.700$, $\beta = 1/T_f = 2.857 \times 10^{-3}\ K^{-1}$.

ANALYSIS: Recognizing that the heat loss from the pipe will be by free convection to the air and by radiation exchange with the surroundings, we can write

$$q' = q'_{conv} + q'_{rad} = \pi D\left[\bar{h}_D\left(T_s - T_\infty\right) + \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right)\right]. \qquad (1)$$

To estimate $\bar{h}_D$, first find Ra_L, Eq. 9.25, and then use the correlation for a horizontal cylinder, Eq. 9.34,

$$Ra_L = \frac{g\beta\left(T_s - T_\infty\right)D^3}{\nu\alpha} = \frac{9.8m/s^2\left(1/350K\right)\left(400-300\right)K\left(0.150m\right)^3}{20.92\times10^{-6}m^2/s\times29.9\times10^{-6}m^2/s} = 1.511\times10^7$$

$$\overline{Nu}_D = \left\{0.60 + \frac{0.387Ra_L^{1/6}}{\left[1+\left(0.559/Pr\right)^{9/16}\right]^{8/27}}\right\}^2$$

$$\overline{Nu}_D = \left\{0.60 + \frac{0.387\left(1.511\times10^7\right)^{1/6}}{\left[1+\left(0.559/0.700\right)^{9/16}\right]^{8/27}}\right\}^2 = 31.88$$

$$\bar{h}_D = \overline{Nu}_D \cdot k/D = 31.88\times0.030W/m\cdot K/0.15m = 6.38W/m^2\cdot K. \qquad (2)$$

Substituting for $\bar{h}_D$ from Eq. (2) into Eq. (1), find

$$q' = \pi\left(0.150m\right)\left[6.38W/m^2\cdot K\left(400-300\right)K + 0.85\times5.67\times10^{-8}W/m^2\cdot K^4\left(400^4 - 300^4\right)K^4\right]$$

$$q' = 301W/m + 397W/m = 698W/m.$$ <

COMMENTS: (1) Note that for this situation, heat transfer by radiation and free convection are of equal importance.

(2) Using Eq. 9.33 with constants C,n from Table 9.1, the estimate for $\bar{h}_D$ is

$$\overline{Nu}_D = CRa_L^n = 0.125\left(1.511\times10^7\right)^{0.333} = 30.73$$

$$\bar{h}_D = \overline{Nu}_D k/D = 30.73\times0.030W/m\cdot K/0.150m = 6.15W/m^2\cdot K.$$

The agreement is within 4% of the Eq. 9.34 result.

PROBLEM 9.55

KNOWN: Diameter and outer surface temperature of steam pipe. Diameter, thermal conductivity, and emissivity of insulation. Temperature of ambient air and surroundings.

FIND: Effect of insulation thickness and emissivity on outer surface temperature of insulation and heat loss.

SCHEMATIC: See Example 9.4, Comment 2.

ASSUMPTIONS: (1) Pipe surface is small compared to surroundings, (2) Room air is quiescent.

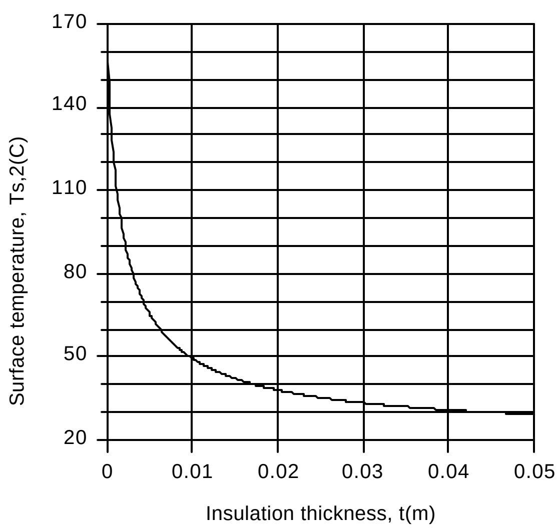
PROPERTIES: *Table A.4*, air (evaluated using *Properties* Tool Pad of IHT).

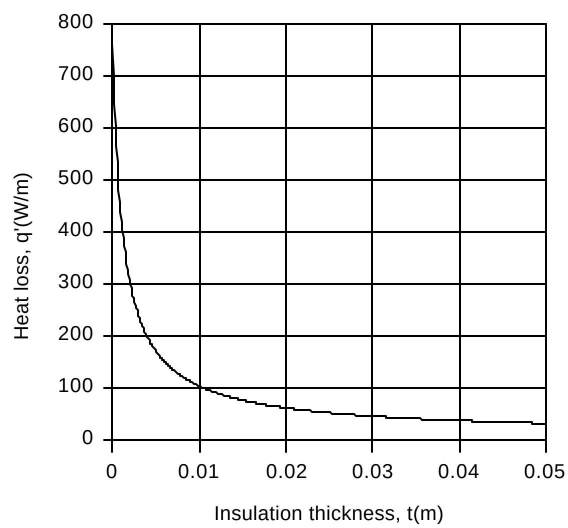
ANALYSIS: The appropriate model is provided in Comment 2 of Example 9.4 and includes use of the following energy balance to evaluate $T_{s,2}$,

$$q'_{cond} = q'_{conv} + q'_{rad} \equiv q'$$

$$\frac{2\pi k_i \left(T_{s,1} - T_{s,2}\right)}{\ln\left(r_2/r_1\right)} = \bar{h} 2\pi r_2 \left(T_{s,2} - T_\infty\right) + \varepsilon 2\pi r_2 \sigma \left(T_{s,2}^4 - T_{sur}^4\right)$$

from which the total heat rate q' can then be determined. Using the IHT *Correlations* and *Properties* Tool Pads, the following results are obtained for the effect of the insulation thickness, with $\varepsilon = 0.85$.

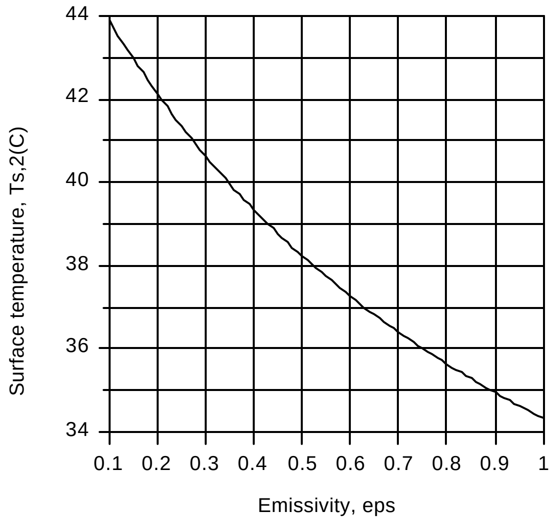


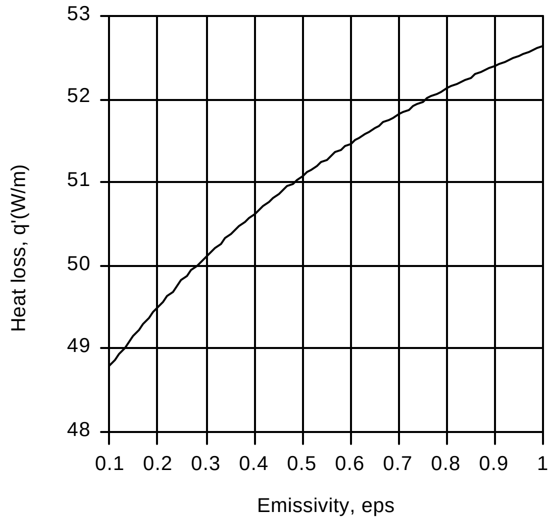


The insulation significantly reduces $T_{s,2}$ and q', and little additional benefits are derived by increasing t above 25 mm. For t = 25 mm, the effect of the emissivity is as follows.

Continued...

PROBLEM 9.55 (Cont.)





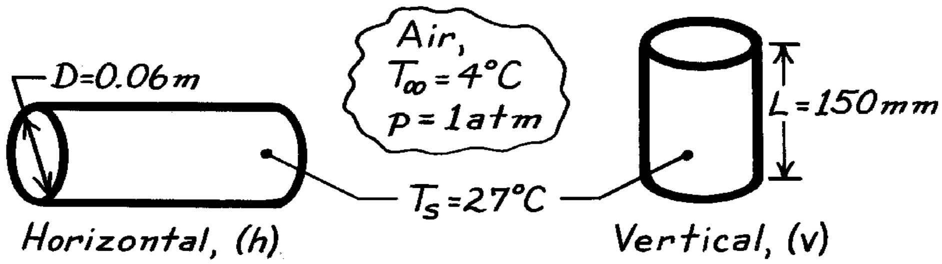
Although the surface temperature decreases with increasing emissivity, the heat loss increases due to an increase in net radiation to the surroundings.

PROBLEM 9.56

KNOWN: Dimensions and temperature of beer can in refrigerator compartment.

FIND: Orientation which maximizes cooling rate.

SCHEMATIC:



ASSUMPTIONS: (1) End effects are negligible, (2) Compartment air is quiescent, (3) Constant properties.

PROPERTIES: *Table A-4*, Air (T_f = 288.5K, 1 atm): $\nu = 14.87 \times 10^{-6}$ m^2/s, k =0.0254 W/m·K, α = 21.0×10^{-6} m^2/s, Pr = 0.71, $\beta = 1/T_f = 3.47 \times 10^{-3}$ K^{-1}.

ANALYSIS: The ratio of cooling rates may be expressed as

$$\frac{q_v}{q_h} = \frac{\overline{h}_v}{\overline{h}_h}\frac{p\,DL}{p\,DL}\frac{(T_s - T_\infty)}{(T_s - T_\infty)} = \frac{\overline{h}_v}{\overline{h}_h}.$$

For the *vertical* surface, find

$$Ra_L = \frac{gb\,(T_s - T_\infty)}{na}L^3 = \frac{9.8\,\mathrm{m/s^2} \times 3.47 \times 10^{-3}\,\mathrm{K^{-1}}\,(23°\mathrm{C})}{\left(14.87 \times 10^{-6}\,\mathrm{m^2/s}\right)\left(21 \times 10^{-6}\,\mathrm{m^2/s}\right)}L^3 = 2.5 \times 10^9 L^3$$

$$Ra_L = 2.5 \times 10^9 (0.15)^3 = 8.44 \times 10^6,$$

and using the correlation of Eq. 9.26, $$\overline{Nu}_L = \left\{0.825 + \frac{0.387\left(8.44 \times 10^6\right)^{1/6}}{\left[1 + (0.492/0.71)^{9/16}\right]^{8/27}}\right\}^2 = 29.7.$$

Hence $$\overline{h}_L = \overline{h}_v = \overline{Nu}_L\frac{k}{L} = 29.7\frac{0.0254\,\mathrm{W/m \cdot K}}{0.15\,\mathrm{m}} = 5.03\,\mathrm{W/m^2 \cdot K}.$$

For the *horizontal* surface, find $Ra_D = \frac{gb\,(T_s - T_\infty)}{na}D^3 = 2.5 \times 10^9 (0.06)^3 = 5.4 \times 10^5$

and using the correlation of Eq. 9.34, $$\overline{Nu}_D = \left\{0.60 + \frac{0.387\left(5.4 \times 10^5\right)^{1/6}}{\left[1 + (0.559/0.71)^{9/16}\right]^{8/27}}\right\}^2 = 12.24$$

$$\overline{h}_D = \overline{h}_h = \overline{Nu}_D\frac{k}{D} = 12.24\frac{0.0254\,\mathrm{W/m \cdot K}}{0.06\,\mathrm{m}} = 5.18\,\mathrm{W/m^2 \cdot K}.$$

Hence $$\frac{q_v}{q_h} = \frac{5.03}{5.18} = 0.97.$$ <

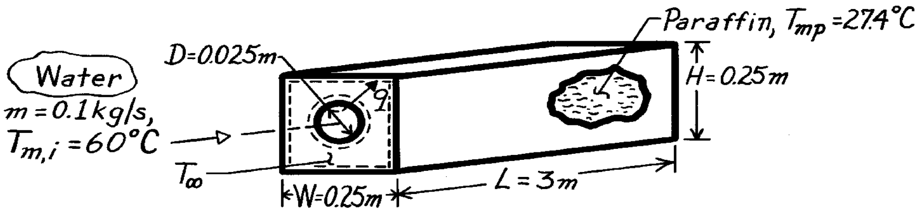
COMMENTS: In view of the uncertainties associated with Eqs. 9.26 and 9.34 and the neglect of end effects, the above result is inconclusive. The cooling rates are approximately the same.

PROBLEM 9.57

KNOWN: Length and diameter of tube submerged in paraffin of prescribed dimensions. Properties of paraffin. Inlet temperature, flow rate and properties of water in the tube.

FIND: (a) Water outlet temperature, (b) Heat rate, (c) Time for complete melting.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible k.e. and p.e. changes for water, (2) Constant properties for water and paraffin, (3) Negligible tube wall conduction resistance, (4) Free convection at outer surface associated with horizontal cylinder in an infinite quiescent medium, (5) Negligible heat loss to surroundings, (6) Fully developed flow in tube.

PROPERTIES: Water (given): c_p = 4185 J/kg·K, k = 0.653 W/m·K, $\mu = 467 \times 10^{-6}$ kg/s·m, Pr = 2.99; Paraffin (given): T_{mp} = 27.4°C, h_{sf} = 244 kJ/kg, k = 0.15 W/m·K, $\beta = 8 \times 10^{-4}\ K^{-1}$, ρ = 770 kg/m^3, $\nu = 5 \times 10^{-6}\ m^2/s$, $\alpha = 8.85 \times 10^{-8}\ m^2/s$.

ANALYSIS: (a) The overall heat transfer coefficient is

$$\frac{1}{U} = \frac{1}{h_i} + \frac{1}{h_o}.$$

To estimate h_i, find $\quad Re_D = \frac{4\dot{m}}{\pi D \mu} = \frac{4 \times 0.1\,kg/s}{\pi \times 0.025m \times 467 \times 10^{-6}\,kg/s\cdot m} = 10{,}906$

and noting the flow is turbulent, use the Dittus-Boelter correlation

$$Nu_D = 0.023 Re_D^{4/5} Pr^{0.3} = 0.023(10{,}906)^{4/5}(2.99)^{0.3} = 54.3$$

$$h_i = \frac{Nu_D\, k}{D} = \frac{54.3 \times 0.653\,W/m\cdot K}{0.025m} = 1418\,W/m^2\cdot K.$$

To estimate h_o, find

$$Ra_D = \frac{g\beta\left(T_s - T_\infty\right)D^3}{\nu\alpha} = \frac{\left(9.8\,m/s^2\right)8 \times 10^{-4}\,K^{-1}(55 - 27.4)K\,(0.025m)^3}{5 \times 10^{-6}\,m^2/s \times 8.85 \times 10^{-8}\,m^2/s}$$

$$Ra_D = 7.64 \times 10^6$$

and using the correlation of Eq. 9.34, $\quad \overline{Nu}_D = \left\{0.60 + \frac{0.387 Ra_D^{1/6}}{\left[1 + \left(0.559/Pr\right)^{9/16}\right]^{8/27}}\right\}^2 = 35.0$

$$h_o = \overline{Nu}_D \frac{k}{D} = 35.0 \frac{0.15\,W/m\cdot K}{0.025m} = 210\,W/m^2\cdot K.$$

Alternatively, using the correlation of Eq. 9.33,

Continued …..

$$Nu_D = CRa_D^n \text{ with } C = 0.48,\ n = 0.25 \qquad Nu_D = 25.2$$

$$h_o = 25.2\frac{0.15\,W/m\cdot K}{0.025m} = 151\,W/m^2\cdot K.$$

The significant difference in h_o values for the two correlations may be due to difficulties associated with high Pr applications of one or both correlations. Continuing with the result from Eq. 9.34,

$$\frac{1}{\overline{U}} = \frac{1}{h_i} + \frac{1}{\overline{h}_o} = \frac{1}{1418} + \frac{1}{210} = 5.467\times10^{-3} m^2\cdot K/W$$

$$\overline{U} = 183\,W/m^2\cdot K.$$

Using Eq. 8.46, find

$$\frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \exp\left(-\frac{p\,DL}{\dot{m}c_p}\overline{U}\right) = \exp\left(-\frac{p\times0.025m\times3m}{0.1kg/s\times4185J/kg\cdot K}183\frac{W}{m^2\cdot K}\right)$$

$$T_{m,o} = T_\infty - \left(T_\infty - T_{m,i}\right)0.902 = \left[27.4 - (27.4-60)0.902\right]°C$$

$$T_{m,o} = 56.8°C. \qquad <$$

(b) From an energy balance, the heat rate is

$$q = \dot{m}c_p\left(T_{m,i} - T_{m,o}\right) = 0.1kg/s\times4185J/kg\cdot K\,(60-56.8)K = 1335W \qquad <$$

or using the rate equation,

$$q = \overline{U}A\,\Delta T_{\ell m} = 183W/m^2\cdot K\,p\,(0.025m)3m\,\frac{(60-27.4)K - (56.8-27.4)K}{\ell n\dfrac{60-27.4}{56.8-27.4}}$$

$$q = 1335W.$$

(c) Applying an energy balance to a control volume about the paraffin,

$$E_{in} = \Delta E_{st}$$

$$q\cdot t = r V h_{sf} = rL\left[WH - pD^2/4\right]h_{sf}$$

$$t = \frac{770kg/m^3\times3m}{1335W}\left[(0.25m)^2 - \frac{p}{4}(0.025m)^2\right]2.44\times10^5 J/kg$$

$$t = 2.618\times10^4 s = 7.27h. \qquad <$$

COMMENTS: (1) The value of $\overline{h}_o$ is overestimated by assuming an infinite quiescent medium. The fact that the paraffin is enclosed will increase the resistance due to free convection and hence decrease q and increase t.

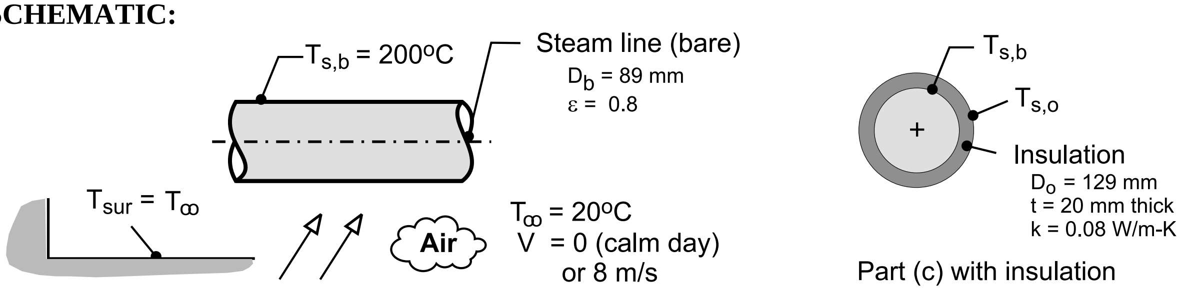
(2) Using $\overline{h}_o = 151\,W/m^2\cdot K$ results in $\overline{U} = 136\,W/m^2\cdot K, T_{m,o} = 57.6°C,$ q = 1009 W and t = 9.62 h.

PROBLEM 9.58

KNOWN: A long uninsulated steam line with a diameter of 89 mm and surface emissivity of 0.8 transports steam at 200°C and is exposed to atmospheric air and large surroundings at an equivalent temperature of 20°C.

FIND: (a) The heat loss per unit length for a calm day when the ambient air temperature is 20°C; (b) The heat loss on a breezy day when the wind speed is 8 m/s; and (c) For the conditions of part (a), calculate the heat loss with 20-mm thickness of insulation (k = 0.08 W/m·K). Would the heat loss change significantly with an appreciable wind speed?

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Calm day corresponds to quiescent ambient conditions, (3) Breeze is in crossflow over the steam line, (4) Atmospheric air and large surroundings are at the same temperature; and (5) Emissivity of the insulation surface is 0.8.

PROPERTIES: *Table A-4*, Air ($T_f = (T_s + T_\infty)/2$ = 383 K, 1 atm): $\nu = 2.454 \times 10^{-5}$ m^2/s, k = 0.03251 W/m·K, $\alpha = 3.544 \times 10^{-5}$ m^2/s, Pr = 0.693.

ANALYSIS: (a) The heat loss per unit length from the pipe by convection and radiation exchange with the surroundings is

$$q'_b = q'_{cv} + q'_{rad}$$

$$q'_b = \bar{h}_D P_b \left(T_{s,b} - T_\infty\right) + \varepsilon P_b \sigma \left(T_{s,b}^4 - T_\infty^4\right) \qquad P_b = \pi D_b \tag{1,2}$$

where D_b is the diameter of the bare pipe. Using the Churchill-Chu correlation, Eq. 9.34, for *free convection* from a horizontal cylinder, estimate $\bar{h}_D$

$$\overline{Nu}_D = \frac{\bar{h} D_b}{k} = \left\{0.60 + \frac{0.387\ Ra_D^{1/6}}{\left[1+\left(0.559/Pr\right)^{9/16}\right]^{8/27}}\right\}^2 \tag{3}$$

where properties are evaluated at the film temperature, $T_f = (T_s + T_\infty)/2$ and

$$Ra_D = \frac{g\beta\left(T_s - T_\infty\right)D_b^3}{\nu\alpha} \tag{4}$$

Substituting numerical values, find for the *bare* steam line

Ra_D	$\overline{Nu}_D$	$\bar{h}_D$ (W/m^2·K)	q'_{cv} (W/m)	q'_{rad} (W/m)	q'_b (W/m)
3.73×10^6	21.1	7.71	388	541	929

<

Continued …..

(b) For forced convection conditions with V = 8 m/s, use the Churchill-Bernstein correlation, Eq. 7.56,

$$\overline{Nu}_D = \frac{\bar{h}_D D_b}{k} = 0.3 + \frac{0.62\ Re_D^{1/2}\ Pr^{1/3}}{\left[1+\left(0.4/Pr\right)^{2/3}\right]^{1/4}}\left[1+\left(\frac{Re_D}{282,000}\right)^{5/8}\right]^{4/5}$$

where $Re_D = VD/\nu$. Substituting numerical values, find

Re_D	$\overline{Nu}_D$	$\bar{h}_{D,b}$ (W/m^2·K)	q'_{cv} (W/m)	q'_{rad} (W/m)	q'_b (W/m)
2.17×10^4	82.5	30.1	1517	541	2058

<

(c) With 20-mm thickness insulation, and for the calm-day condition, the heat loss per unit length is

$$q'_{ins} = \left(T_{s,o} - T_\infty\right)/R'_{tot} \qquad (1)$$

$$R'_t = R'_{ins} + \left[1/R'_{cv} + 1/R'_{rad}\right]^{-1} \qquad (2)$$

where the thermal resistance of the insulation from Eq. 3.28 is

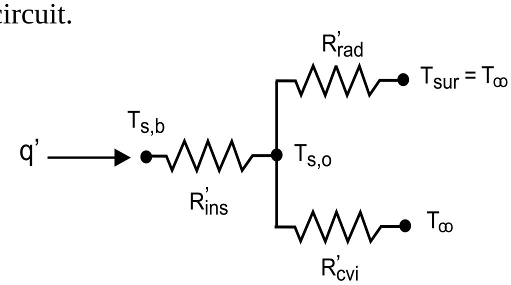
$$R'_{ins} = \ell n\left(D_o/D_b\right)/\left[2\pi k\right] \qquad (3)$$

and the convection and radiation thermal resistances are

$$R'_{cv} = 1/\left(\bar{h}_{D,o}\pi D_o\right) \qquad (4)$$

$$R'_{rad} = 1/\left(\bar{h}_{rad}\ \pi D_o\right) \qquad \bar{h}_{rad,o} = \varepsilon\sigma\left(T_{s,o}+T_\infty\right)\left(T_{s,o}^2 + T_\infty^2\right) \qquad (5,6)$$

The outer surface temperature on the insulation, $T_{s,o}$, can be determined by an energy balance on the *surface node of* the thermal circuit.



$$\frac{T_{s,b} - T_{s,o}}{R'_{ins}} = \frac{T_{s,o} - T_\infty}{\left[1/R'_{cv} + 1/R'_{rad}\right]^{-1}}$$

Substituting numerical values with $D_{b,o}$ = 129 mm, find the following results.

$R'_{ins} = 0.7384\ m\cdot K/W$ $\qquad$ $\bar{h}_{D,o} = 5.30\ W/m^2\cdot K$

$R'_{cv} = 0.4655\ K/W$ $\qquad$ $\bar{h}_{rad} = 5.65\ W/m^2\cdot K$

$R'_{rad} = 0.4371\ K/W$ $\qquad$ $q'_{ins} = 187\ W/m$ <

$T_{s,o} = 62.1°C$

Continued …..

COMMENTS: (1) For the calm-day conditions, the heat loss by radiation exchange is 58% of the total loss. Using a reflective shield (say, $\varepsilon = 0.1$) on the outer surface could reduce the heat loss by 50%.

(2) The effect of a 8-m/s breeze over the steam line is to increase the heat loss by more than a factor of two above that for a calm day. The heat loss by radiation exchange is approximately 25% of the total loss.

(3) The effect of the 20-mm thickness insulation is to reduce the heat loss to 20% the rate by free convection or to 9% the rate on the breezy day. From the results of part (c), note that the insulation resistance is nearly 3 times that due to the combination of the convection and radiation process thermal resistances. The effect of increased wind speed is to reduce R'_{cv}, but since R'_{ins} is the dominant resistance, the effect will not be very significant.

(4) Comparing the free convection coefficients for part (a), $D_b = 89$ mm with $T_{s,b} = 200°C$, and part (b), $D_{b,o} = 129$ mm with $T_{s,o} = 62.1°C$, it follows that $\bar{h}_{D,o}$ is less than $\bar{h}_{D,b}$ since, for the former, the steam line diameter is larger and the diameter smaller.

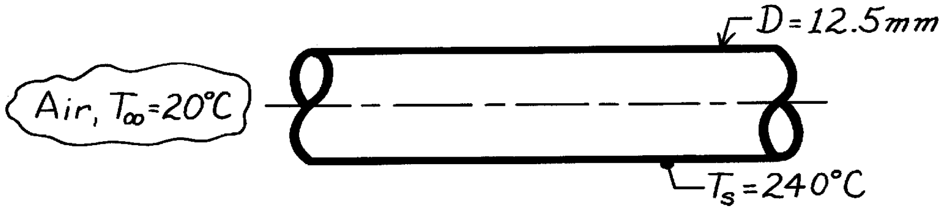
(5) The convection correlation models in *IHT* are especially useful for applications such as the present one to eliminate the tediousness of evaluating properties and performing the calculations. However, it is essential that you have experiences in hand calculations with the correlations before using the software.

PROBLEM 9.59

KNOWN: Horizontal tube, 12.5mm diameter, with surface temperature 240°C located in room with an air temperature 20°C.

FIND: Heat transfer rate per unit length of tube due to convection.

SCHEMATIC:



ASSUMPTIONS: (1) Ambient air is quiescent, (2) Surface radiation effects are not considered.

PROPERTIES: *Table A-4*, Air (T_f = 400K, 1 atm): $\nu = 26.41 \times 10^{-6}$ m^2/s, k= 0.0338 W/m·K, α = 38.3×10^{-6} m^2/s, Pr = 0.690, $\beta = 1/T_f = 2.5 \times 10^{-3}$ K^{-1}.

ANALYSIS: The heat rate from the tube, per unit length of the tube, is

$$q' = \bar{h}\, p\, D\left(T_s - T_\infty\right)$$

where $\bar{h}$ can be estimated from the correlation, Eq. 9.34,

$$\overline{Nu}_D = \left\{0.60 + \frac{0.387 Ra_D^{1/6}}{\left[1 + (0.559/Pr)^{9/16}\right]^{8/27}}\right\}^2 .$$

From Eq. 9.25,

$$Ra_D = \frac{g b\left(T_s - T_\infty\right)D^3}{n a} = \frac{9.8\,m/s^2 \times 2.5 \times 10^{-3} K^{-1}(240-20)K \times \left(12.5 \times 10^{-3} m\right)^3}{26.41 \times 10^{-6} m^2/s \times 38.3 \times 10^{-6} m^2/s} = 10{,}410.$$

Hence,

$$\overline{Nu}_D = \left\{0.60 + \frac{0.387(10{,}410)^{1/6}}{\left[1 + (0.559/0.690)^{9/16}\right]^{8/27}}\right\}^2 = 4.40$$

$$\bar{h} = \frac{k}{D}\overline{Nu}_D = \frac{0.0338\,W/m \cdot K}{12.5 \times 10^{-3} m} \times 4.40 = 11.9\,W/m^2 \cdot K.$$

The heat rate is

$$q' = 11.9\,W/m^2 \cdot K \times p\left(12.5 \times 10^{-3} m\right)(240-20)K = 103\,W/m.$$ <

COMMENTS: Heat loss rate by radiation, assuming an emissivity of 1.0 for the surface, is

$$q'_{rad} = e P s\left(T_s^4 - T_\infty^4\right) = 1 \times p\left(12.5 \times 10^{-3} m\right) \times 5.67 \times 10^{-8} \frac{W}{m^2 \cdot K^4}\left[(240+273)^4 - (20+273)^4\right]K^4$$

$$q'_{rad} = 138\,W/m.$$

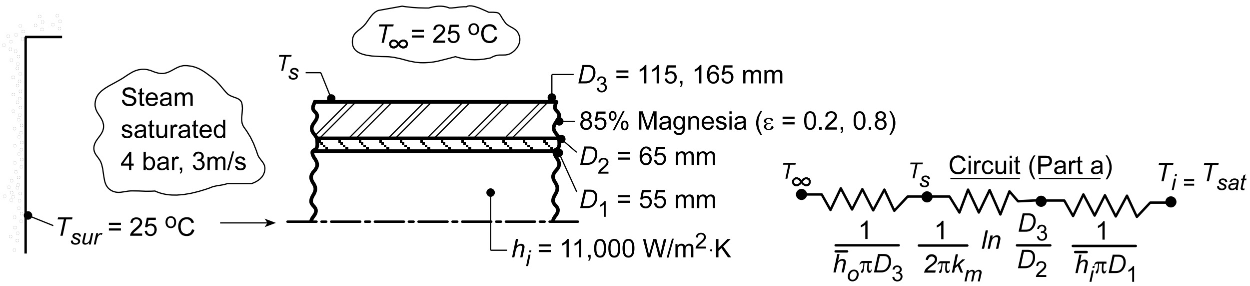
Note that P = π D. Note also this estimate assumes the surroundings are at ambient air temperature. In this instance, $q'_{rad} > q'_{conv}$.

PROBLEM 9.60

KNOWN: Insulated steam tube exposed to atmospheric air and surroundings at 25°C.

FIND: (a) Heat transfer rate by free convection to the room, per unit length of the tube; effect on quality, x, at outlet of 30 m length of tube; (b) Effect of radiation on heat transfer and quality of outlet flow; (c) Effect of emissivity and insulation thickness on heat rate.

SCHEMATIC:



ASSUMPTIONS: (1) Ambient air is quiescent, (2) Negligible surface radiation (part a), (3) Tube wall resistance negligible.

PROPERTIES: Steam tables, steam (sat., 4 bar): h_f = 566 kJ/kg, T_{sat} = 416 K, h_g = 2727 kJ/kg, h_{fg} = 2160 kJ/kg, $v_g = 0.476 \times 10^3$ m³/kg; *Table A.3*, magnesia, 85% (310 K): k_m = 0.051 W/m·K; *Table A.4*, air (assume T_s = 60°C, T_f = (60 + 25)°C/2 = 315 K, 1 atm): $\nu = 17.4 \times 10^{-6}$ m²/s, k = 0.0274 W/m·K, $\alpha = 24.7 \times 10^{-6}$ m²/s, Pr = 0.705, $T_f = 1/315$ K $= 3.17 \times 10^{-3}$ K^{-1}.

ANALYSIS: (a) The heat rate per unit length of the tube (see sketch) is given as,

$$q' = \frac{T_i - T_\infty}{R'_t} \qquad \text{where} \qquad \frac{1}{R'_t} = \left[\frac{1}{\bar{h}_o \pi D_3} + \frac{1}{2\pi k_m}\ln\frac{D_3}{D_2} + \frac{1}{\bar{h}_i \pi D_1}\right]^{-1} \qquad (1,2)$$

To estimate $\bar{h}_o$, we have assumed $T_s \approx 60°C$ in order to calculate Ra_L from Eq. 9.25,

$$Ra_D = \frac{g\beta(T_s - T_\infty)D_3^3}{\nu\alpha} = \frac{9.8\,\text{m/s}^2 \times 3.17\times10^{-3}\,\text{K}^{-1}(60-25)\,\text{K}(0.115\,\text{m})^3}{17.4\times10^{-6}\,\text{m}^2/\text{s}\times 24.7\times10^{-6}\,\text{m}^2/\text{s}} = 3.85\times10^6.$$

The appropriate correlation is Eq. 9.34; find

$$\overline{Nu}_D = \left\{0.60 + \frac{0.387(Ra_D)^{1/6}}{\left[1+(0.559/Pr)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.60 + \frac{0.387\left(3.85\times10^6\right)^{1/6}}{\left[1+(0.559/0.705)^{9/16}\right]^{8/27}}\right\}^2 = 21.4$$

$$\bar{h}_o = \frac{k}{D_3}\overline{Nu}_D = \frac{0.0274\,\text{W/m}\cdot\text{K}}{0.115\,\text{m}}\times 21.4 = 5.09\,\text{W/m}^2\cdot\text{K}.$$

Substituting numerical values into Eq. (2), find

$$\frac{1}{R'_t} = \left[\frac{1}{5.09\,\text{W/m}^2\cdot\text{K}\times\pi 0.115\,\text{m}} + \frac{1}{2\pi\times0.051\,\text{W/m}\cdot\text{K}}\ln\frac{115}{65} + \frac{1}{11{,}000\,\text{W/m}^2\cdot\text{K}\pi\times0.055\,\text{m}}\right]^{-1} = 0.430\,\text{W/m}\cdot\text{K}$$

and from Eq. (1), $\qquad q' = 0.430\,\text{W/m}\cdot\text{K}(416-298)\,\text{K} = 50.8\,\text{W/m}$ <

Continued...

We need to verify that the assumption of T_s = 60°C is reasonable. From the thermal circuit,

$$T_s = T_\infty + q'/\bar{h}_o \pi D_3 = 25^\circ C + 50.8\,W/m \Big/ \left(5.09\,W/m^2 \cdot K \times \pi \times 0.115\,m\right) = 53^\circ C.$$

Another calculation using T_s = 53°C would be appropriate for a more precise result.

Assuming q' is constant, the enthalpy of the steam at the outlet (L = 30 m), h_2, is

$$h_2 = h_1 - q' \cdot L/\dot{m} = 2727\,kJ/kg - 50.8\,W/m \times 30\,m/14.97\,kg/s = 2625\,kJ/kg$$

where $\dot{m} = \rho_g A_c u_m$ with $\rho_g = 1/v_g$ and A_c = $\pi D_1^2/4$. For negligible pressure drop,

$$x = \left(h_2 - h_f\right)/h_{fg} = (2625 - 566)\,kJ/kg / (2160\,kJ/kg) = 0.953. \qquad <$$

(b) With radiation, we first determine T_s by performing an energy balance at the outer surface, where

$$q'_i = q'_{conv,o} + q'_{rad}$$

$$\frac{T_i - T_s}{R'_i} = \bar{h}_o \pi D_3 \left(T_s - T_\infty\right) + \pi D_3 \varepsilon \sigma \left(T_s^4 - T_{sur}^4\right)$$

and

$$R'_i = \frac{1}{\bar{h}_i \pi D_1} + \frac{1}{2\pi k_m} \ln \frac{D_3}{D_2}$$

From knowledge of T_s, $q'_i = \left(T_i - T_s\right)/R'_i$ may then be determined. Using the *Correlations* and *Properties* Tool Pads of IHT to determine $\bar{h}_o$ and the properties of air evaluated at $T_f = (T_s + T_\infty)/2$, the following results are obtained.

Condition	T_s (°C)	q'_i (W/m)
ε = 0.8, D_3 = 115 mm	41.8	56.9
ε = 0.8, D_3 = 165 mm	33.7	37.6
ε = 0.2, D_3 = 115 mm	49.4	52.6
ε = 0.2, D_3 = 165 mm	38.7	35.9

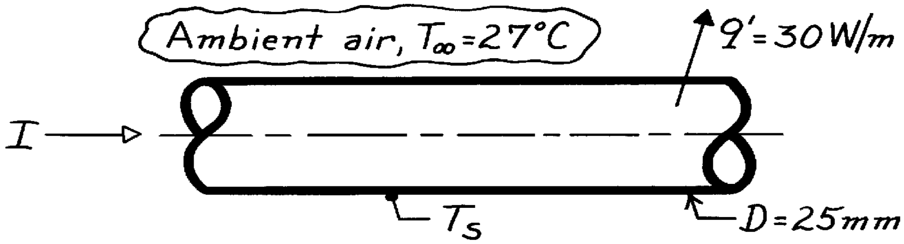
COMMENTS: Clearly, a significant reduction in heat loss may be realized by increasing the insulation thickness. Although T_s, and hence $q'_{conv,o}$, increases with decreasing ε, the reduction in q'_{rad} is more than sufficient to reduce the heat loss.

PROBLEM 9.61

KNOWN: Dissipation rate of an electrical cable suspended in air.

FIND: Surface temperature of the cable, T_s.

SCHEMATIC:



ASSUMPTIONS: (1) Quiescent air, (2) Cable in horizontal position, (3) Negligible radiation exchange.

PROPERTIES: *Table A-4*, Air ($T_f = (T_s + T_\infty)/2 = 325$K, based upon initial estimate for T_s, 1 atm): $\nu = 18.41 \times 10^{-6}$ m^2/s, k = 0.0282 W/m·K, $\alpha = 26.2 \times 10^{-6}$ m^2/s, Pr = 0.704.

ANALYSIS: From the rate equation on a unit length basis, the surface temperature is

$$T_s = T_\infty + q'/\pi D\bar{h}$$

where $\bar{h}$ is estimated by an appropriate correlation. Since such a calculation requires knowledge of T_s, an iteration procedure is required. Begin by assuming $T_s = 77°C$ and calculated Ra_D,

$$Ra_D = g\beta\Delta T D^3/\nu\alpha \quad \text{where} \quad \Delta T = T_s - T_\infty \quad \text{and} \quad T_f = \left(T_s + T_\infty\right)/2 \qquad (1,2,3)$$

For air, $\beta = 1/T_f$, and substituting numerical values,

$$Ra_D = 9.8\frac{m}{s^2}(1/325K)(77-27)K(0.025m)^3/18.41\times10^{-6}\frac{m^2}{s}\times26.2\times10^{-6}\frac{m^2}{s} = 4.884\times10^4.$$

Using the Churchill-Chu relation, find $\bar{h}$.

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = \left\{0.60 + \frac{0.387Ra_D^{1/6}}{\left[1+\left(0.559/Pr\right)^{9/16}\right]^{8/27}}\right\}^2 \qquad (4)$$

$$\bar{h} = \frac{0.0282W/m\cdot K}{0.025m}\left\{0.60 + \frac{0.387\left(4.884\times10^4\right)^{1/6}}{\left[1+\left(0.559/0.704\right)^{9/16}\right]^{8/27}}\right\}^2 = 7.28W/m^2\cdot K.$$

Substituting numerical values into Eq. (1), the calculated value for T_s is

$$T_s = 27°C + \left(30W/m\right)/\pi\times0.025m\times7.28W/m^2\cdot K = 79.5°C.$$

This value is very close to the assumed value (77°C), but an iteration with a new value of 79°C is warranted. Using the same property values, find for this iteration:

$$Ra_D = 5.08\times10^4 \qquad \bar{h} = 7.35W/m^2\cdot K \qquad T_s = 79°C. \qquad <$$

We conclude that $T_s = 79°C$ is a good estimate for the surface temperature.

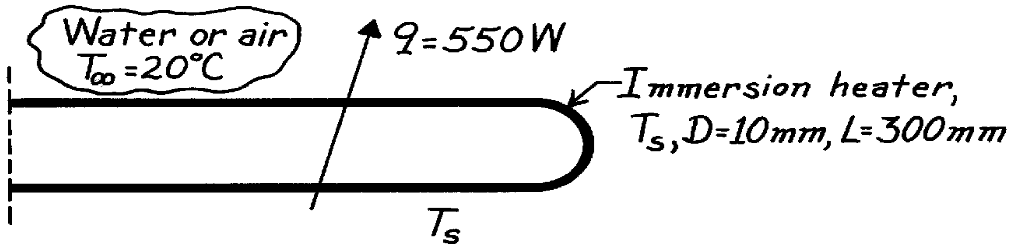
COMMENTS: Recognize that radiative exchange is likely to be significant and would have the effect of reducing the estimate for T_s.

PROBLEM 9.62

KNOWN: Dissipation rate of an immersion heater in a large tank of water.

FIND: Surface temperature in water and, if accidentally operated, in air.

SCHEMATIC:



ASSUMPTIONS: (1) Quiescent ambient fluid, (2) Negligible radiative exchange.

PROPERTIES: *Table A-6*, Water and *Table A-4*, Air:

	T(K)	$k \cdot 10^3$(W/m·K)	$\nu \cdot 10^7$(μ/ρ,m^2/s)	$\alpha \cdot 10^7$($k/\rho c_p$,m^2/s)	Pr	$\beta \cdot 10^6$(K^{-1})
Water	315	634	6.25	1.531	4.16	400.4
Air	1500	100	2400	3500	0685	666.7

ANALYSIS: From the rate equation, the surface temperature, T_s, is

$$T_s = T_\infty + q/\left(p D L \bar{h}\right) \tag{1}$$

where $\bar{h}$ is estimated by an appropriate correlation. Since such a calculation requires knowledge of T_s, an iteration procedure is required. Begin by assuming for *water* that T_s = 64°C such that T_f = 315K. Calculate the Rayleigh number,

$$Ra_D = \frac{g b \Delta T D^3}{n a} = \frac{9.8 m/s^2 \times 400.4 \times 10^{-6} K^{-1} (64-20) K (0.010m)^3}{6.25 \times 10^{-7} m^2/s \times 1.531 \times 10^{-7} m^2/s} = 1.804 \times 10^6. \tag{2}$$

Using the Churchill-Chu relation, find

$$\overline{Nu}_D = \frac{\bar{h} D}{k} = \left\{ 0.60 + \frac{0.387 Ra_D^{1/6}}{\left[1 + (0.559/Pr)^{9/16}\right]^{8/27}} \right\}^2 \tag{3}$$

$$\bar{h} = \frac{0.634 W/m \cdot K}{0.01m} \left\{ 0.60 + \frac{0.387 \left(1.804 \times 10^6\right)^{1/6}}{\left[1 + (0.559/4.16)^{9/16}\right]^{8/27}} \right\}^2 = 1301 W/m^2 \cdot K.$$

Substituting numerical values into Eq. (1), the calculated value for T_s in *water* is

$$T_s = 20°C + 550W/p \times 0.010m \times 0.30m \times 1301 W/m^2 \cdot K = 64.8°C.$$ <

Continued …..

PROBLEM 9.62 (Cont.)

Our initial assumption of $T_s = 64°C$ is in excellent agreement with the calculated value.

With accidental operation in *air*, the heat transfer coefficient will be nearly a factor of 100 less. Suppose $\overline{h} \approx 25\,W/m^2\cdot K$, then from Eq. (1), $T_s \approx 2360°C$. Very likely the heater will burn out. Using air properties at $T_f \approx 1500K$ and Eq. (2), find $Ra_D = 1.815 \times 10^2$. Using Eq. 9.33, $Nu_D = CRa_D^n$ with C= 0.85 and n = 0.188 from Table 9.1, find $\overline{h} = 22.6\,W/m^2\cdot K$. Hence, our first estimate for the surface temperature in *air* was reasonable,

$$T_s \approx 2300°C. \quad <$$

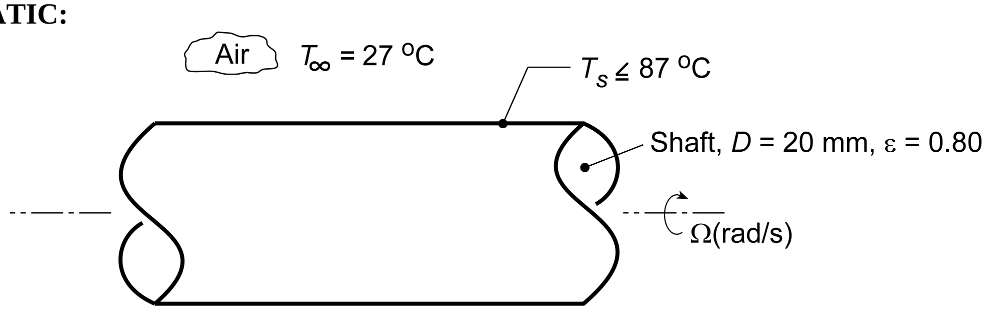
However, radiation exchange will be the dominant mode, and would reduce the estimate for T_s. Generally such heaters could not withstand operating temperatures above 1000°C and safe operation in air is not possible.

PROBLEM 9.63

KNOWN: Motor shaft of 20-mm diameter operating in ambient air at T_∞ = 27°C with surface temperature $T_s \le 87$°C.

FIND: Convection coefficients and/or heat removal rates for different heat transfer processes: (a) For a rotating horizontal cylinder as a function of rotational speed 5000 to 15,000 rpm using the recommended correlation, (b) For free convection from a horizontal stationary shaft; investigate whether mixed free and forced convection effects for the range of rotational speeds in part (a) are significant using the recommended criterion; (c) For radiation exchange between the shaft having an emissivity of 0.8 and the surroundings also at ambient temperature, $T_{sur} = T_\infty$; and (d) For cross flow of ambient air over the stationary shaft, required air velocities to remove the heat rates determined in part (a).

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Shaft is horizontal with isothermal surface.

PROPERTIES: *Table A.4*, Air ($T_f = (T_s + T_\infty)/2$ = 330 K, 1 atm): $\nu = 18.91 \times 10^{-6}$ m²/s , k = 0.02852 W/m·K, $\alpha = 26.94 \times 10^{-6}$ m²/s, Pr = 0.7028, $\beta = 1/T_f$.

ANALYSIS: (a) The recommended correlation for the a horizontal rotating shaft is

$$\overline{Nu}_D = 0.133\,Re_D^{2/3}\,Pr^{1/3} \qquad Re_D < 4.3\times10^5 \qquad 0.7 < Pr < 670$$

where the Reynolds number is

$$Re_D = \Omega D^2/\nu$$

and $\Omega\ (\text{rad/s})$ is the rotational velocity. Evaluating properties at $T_f = (T_s + T_\infty)/2$, find for ω = 5000 rpm,

$$Re_D = (5000\text{rpm}\times 2\pi\ \text{rad/rev}/60\text{s/min})(0.020\text{m})^2 \Big/ 18.91\times10^{-6}\ \text{m}^2/\text{s} = 11{,}076$$

$$\overline{Nu}_D = 0.133(11{,}076)^{2/3}(0.7028)^{1/3} = 58.75$$

$$\bar{h}_{D,rot} = \overline{Nu}_D\, k/D = 58.75\times0.02852\ \text{W/m}\cdot\text{K}/0.020\text{m} = 83.8\ \text{W}\big/\text{m}^2\cdot\text{K} \qquad <$$

The heat rate per unit shaft length is

$$q'_{rot} = \bar{h}_{D,rot}(\pi D)(T_s - T_\infty) = 83.8\ \text{W}\big/\text{m}^2\cdot\text{K}\,(\pi\times0.020\text{m})(87-27)^\circ\text{C} = 316\ \text{W/m} \qquad <$$

The convection coefficient and heat rate as a function of rotational speed are shown in a plot below.

(b) For the stationary shaft condition, the free convection coefficient can be estimated from the Churchill-Chu correlation, Eq. (9.34) with

Continued...

$$\mathrm{Ra_D} = \frac{g\beta\Delta T D^3}{\nu\alpha}$$

$$\mathrm{Ra_D} = \frac{9.8\,\mathrm{m/s^2}\,(1/330\mathrm{K})(87-27)\,\mathrm{K}\,(0.020\mathrm{m})^3}{18.91\times10^{-6}\,\mathrm{m^2/s}\times 26.94\times10^{-6}\,\mathrm{m^2/s}} = 27,981$$

$$\overline{\mathrm{Nu}}_\mathrm{D} = \left\{0.60 + \frac{0.387\mathrm{Ra_D^{1/6}}}{\left[1+(0.559/\mathrm{Pr})^{9/16}\right]^{8/27}}\right\}^2$$

$$\overline{\mathrm{Nu}}_\mathrm{D} = \left\{0.60 + \frac{0.387\,(27,981)^{1/6}}{\left[1+(0.559/0.7028)^{9/16}\right]^{8/27}}\right\}^2 = 5.61$$

$$\bar{h}_{\mathrm{D,fc}} = \overline{\mathrm{Nu}}_\mathrm{D}\,k/D = 5.61\times0.02852\,\mathrm{W/m\cdot K}/0.020\mathrm{m} = 8.00\,\mathrm{W/m^2\cdot K}$$

$$q'_{fc} = 8.00\,\mathrm{W/m^2\cdot K}\,(\pi\times0.020\mathrm{m})(87-27)^\circ\,\mathrm{C} = 30.2\,\mathrm{W/m} \qquad <$$

Mixed free and forced convection effects may be significant if

$$\mathrm{Re_D} < 4.7\left(\mathrm{Gr_D^3}/\mathrm{Pr}\right)^{0.137}$$

where $\mathrm{Gr_D} = \mathrm{Ra_D}/\mathrm{Pr}$, find using results from above and in part (a) for ω = 5000 rpm,

$$11,076 \;\;?<?\;\; 4.7\left[(27,981/0.7028)^3/0.7018\right]^{0.137} = 383$$

We conclude that free convection effects are not significant for rotational speeds above 5000 rpm.

(c) Considering radiation exchange between the shaft and the surroundings,

$$h_{rad} = \varepsilon\sigma\,(T_s + T_{sur})\left(T_s^2 + T_{sur}^4\right)$$

$$h_{rad} = 0.8\times5.67\times10^{-8}\,\mathrm{W/m^2\cdot K}\,(360+300)\left(360^2+300^2\right)\mathrm{K^3} = 6.57\,\mathrm{W/m^2\cdot K} \qquad <$$

and the heat rate by radiation exchange is

$$q'_{rad} = h_{rad}\,(\pi D)(T_s - T_{sur})$$

$$q'_{rad} = 6.57\,\mathrm{W/m^2\cdot K}\,(\pi\times0.020\mathrm{m})(87-27)\mathrm{K} = 24.8\,\mathrm{W/m} \qquad <$$

(d) For cross flow of ambient air at a velocity V over the shaft, the convection coefficient can be estimated using the Churchill-Bernstein correlation, Eq. 7.57, with

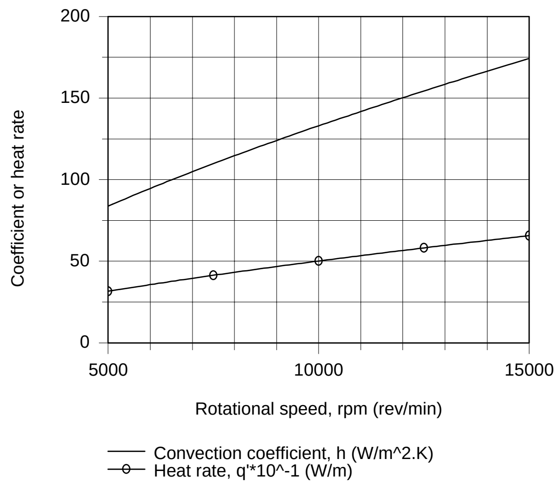
$$\mathrm{Re_{D,cf}} = \frac{VD}{\nu}$$

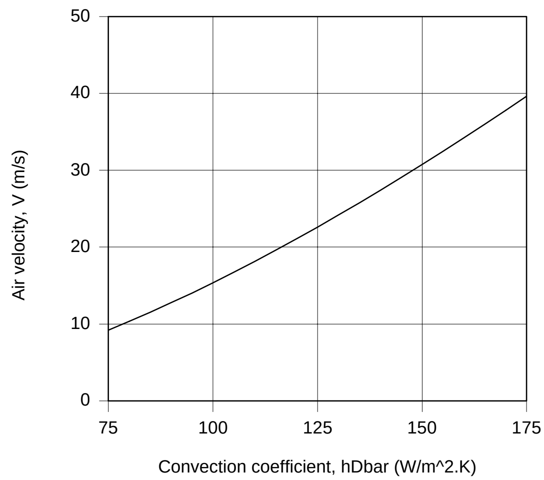
$$\overline{\mathrm{Nu}}_{\mathrm{D,cf}} = \bar{h}_{\mathrm{D,cf}}\,D/k = 0.3 + \frac{0.62\,\mathrm{Re_{D,cf}^{1/2}}\,\mathrm{Pr^{1/3}}}{\left[1+(0.4/\mathrm{Pr})^{2/3}\right]^{1/4}}\left[1+\left(\frac{\mathrm{Re_{D,cf}}}{282,000}\right)^{5/8}\right]^{4/5}$$

Continued...

PROBLEM 9.63 (Cont.)

From the plot below (left) for the rotating shaft condition of part (a), $\bar{h}_{D,rot}$ vs. rpm, note that the convection coefficient varies from approximately 75 to 175 $W/m^2 \cdot K$. Using the *IHT Correlations Tool, Forced Convection, Cylinder*, which is based upon the above relations, the range of air velocities V required to achieve $\bar{h}_{D,cf}$ in the range 75 to 175 $W/m^2 \cdot K$ was computed and is plotted below (right).





Note that the air cross-flow velocities are quite substantial in order to remove similar heat rates for the rotating shaft condition.

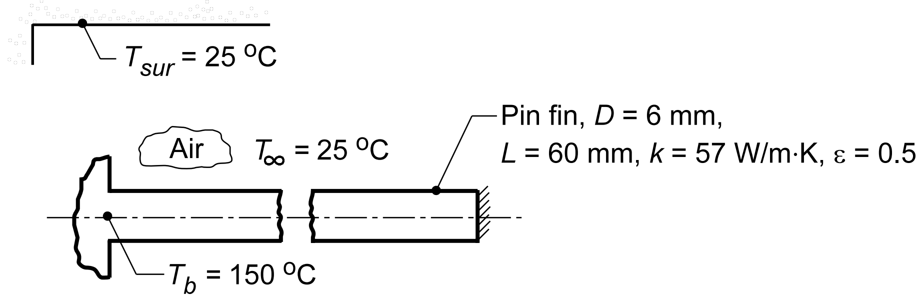
COMMENTS: We conclude for the rotational speed and surface temperature conditions, free convection effects are not significant. Further, radiation exchange, part (c) result, is less than 10% of the convection heat loss for the lowest rotational speed condition.

PROBLEM 9.64

KNOWN: Horizontal pin fin of 6-mm diameter and 60-mm length fabricated from plain carbon steel (k = 57 W/m·K, ε = 0.5). Fin base maintained at T_b = 150°C. Ambient air and surroundings at 25°C.

FIND: Fin heat rate, q_f, by two methods: (a) Analytical solution using average fin surface temperature of $\overline{T}_s = 125^\circ C$ to estimate the free convection and linearized radiation coefficients; comment on sensitivity of fin heat rate to choice of $\overline{T}_s$; and, (b) Finite-difference method when coefficients are based upon local temperatures, rather than an average fin surface temperature; compare result of the two solution methods.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in the pin fin, (3) Ambient air is quiescent and extensive, (4) Surroundings are large compared to the pin fin, and (5) Fin tip is adiabatic.

PROPERTIES: *Table A.4*, Air ($T_f = \left(\overline{T}_s + T_\infty\right)/2$ = 348 K): ν = 20.72 × 10^{-6} m^2/s, k = 0.02985 W/m·K, α = 29.60 × 10^{-6} m^2/s, Pr = 0.7003, β = $1/T_f$.

ANALYSIS: (a) The heat rate for the pin fin with an adiabatic tip condition is, Eq. 3.76,

$$q_f = M \tanh(mL) \tag{1}$$

$$M = \left(\overline{h}_{tot} P k A_c\right)^{1/2} \theta_b \qquad m = \left(hP/kA_c\right)^{1/2} \tag{2,3}$$

$$P = \pi D \qquad A_c = \pi D^2/4 \qquad \theta_b = T_b - T_\infty \tag{4-6}$$

and the average coefficient is the sum of the convection and linearized radiation processes, respectively,

$$\overline{h}_{tot} = \overline{h}_{fc} + \overline{h}_{rad} \tag{7}$$

evaluated at $\overline{T}_s = 125^\circ C$ with $\overline{T}_f = \left(\overline{T}_s + T_\infty\right)/2 = 75^\circ C = 348\,K$.

Estimating $\overline{h}_{fc}$ *:* For the horizontal cylinder, Eq. 9.34, with

$$Ra_D = \frac{g\beta\Delta T D^3}{\nu\alpha}$$

Continued …..

PROBLEM 9.64 (Cont.)

$$Ra_D = \frac{9.8\,m/s^2\,(1/348K)(125-25)(0.006m)^3}{20.72\times10^{-6}\,m^2/s\times29.60\times10^{-6}\,m^2/s} = 991.79$$

$$\overline{Nu}_D = \left\{0.60 + \frac{0.387\,Ra_D^{1/6}}{\left[1+(0.559/Pr)^{9/16}\right]^{8/27}}\right\}^2$$

$$\overline{Nu}_D = \left\{0.60 + \frac{0.387\,(991.79)^{1/6}}{\left[1+(0.559/0.7003)^{9/16}\right]^{8/27}}\right\}^2 = 2.603$$

$$\overline{h}_{fc} = \overline{Nu}_D\,k/D = 2.603\times0.02985\,W/m\cdot K\big/0.006m = 12.95\,W/m^2\cdot K$$

Calculating $\overline{h}_{rad}$ **:** The linearized radiation coefficient is

$$\overline{h}_{rad} = \varepsilon\sigma\left(\overline{T}_s + T_{sur}\right)\left(\overline{T}_s^2 + T_{sur}^2\right) \qquad (8)$$

$$\overline{h}_{rad} = 0.5\times5.67\times10^{-8}\,W/m^2\cdot K^4\,(398+298)\left(398^2+298^2\right)K^3 = 4.88\,W/m^2\cdot K$$

Substituting numerical values into Eqs. (1-7) , find

$$q_{fin} = 2.04W \qquad <$$

with $\theta_b = 125\,K$, $A_c = 2.827\times10^{-5}m^2$, $P = 0.01885\,m$, $m = 2.603\,m^{-1}$, $M = 2.909\,W$, and $\overline{h}_{tot} = 17.83\,W/m^2\cdot K$.

Using the *IHT Model*, *Extended Surfaces, Rectangular Pin Fin,* with the *Correlations Tool* for *Free Convection* and the *Properties Tool* for *Air*, the above analysis was repeated to obtain the following results.

$\overline{T}_s\,(^\circ C)$	115	120	125	130	135
$q_f\,(W)$	1.989	2.012	2.035	2.057	2.079
$\left(q_f - q_{f,o}\right)/q_{fo}$ (%)	-2.3	-1.1	0	+1.1	+2.2

The fin heat rate is not very sensitive to the choice of $\overline{T}_s$ for the range T_s = 125 ± 10 °C. For the base case condition, the fin tip temperature is T(L) = 114 °C so that $\overline{T}_s \approx (T(L) + T_b)/2$ = 132°C would be consistent assumed value.

Continued …..

PROBLEM 9.64 (Cont.)

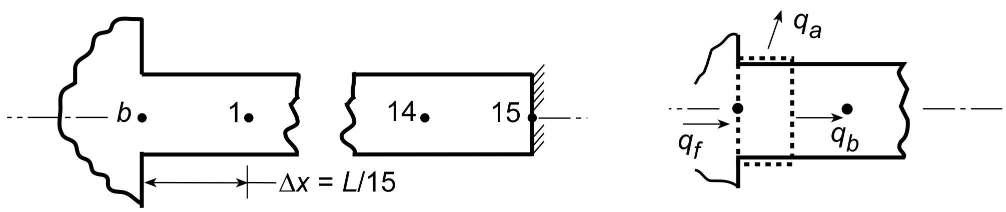
(b) Using the *IHT Tool, Finite-Difference Equation, Steady- State, Extended Surfaces*, the temperature distribution was determined for a 15-node system from which the fin heat rate was determined. The local free convection and linearized radiation coefficients $h_{tot} = h_{fc} + h_{rad,}$ were evaluated at local temperatures, T_m, using IHT with the *Correlations Tool, Free Convection, Horizontal Cylinder,* and the *Properties Tool* for *Air*, and Eq. (8). The local coefficient h_{tot} vs. T_s is nearly a linear function for the range $114 \le T_s \le 150°C$ so that it was reasonable to represent h_{tot} (T_s) as a *Lookup Table Function.* The fin heat rate follows from an energy balance on the base node, (see schematic next page)

$$q_f = q_a + q_b = (0.08949 + 1.879)\,W = 1.97\,W$$ <

$$q_a = h_b\,(P\Delta x/2)(T_b - T_\infty)$$

$$q_b = kA_c\,(T_b - T_1)/\Delta x$$

where T_b = 150°C, T_1 = 418.3 K = 145.3°C, and h_b = h_{tot} (T_b) = l8.99 $W/m^2 \cdot K$.



Considering variable coefficients, the fin heat rate is -3.3% lower than for the analytical solution with the assumed $\overline{T}_s$ = 125°C.

COMMENTS: (1) To validate the FDE model for part (b), we compared the temperature distribution and fin heat rate using a constant h_{tot} with the analytical solution ($\overline{T}_s$ = 125°C). The results were identical indicating that the 15-node mesh is sufficiently fine.

(2) The fin temperature distribution (K) for the IHT finite-difference model of part (b) is

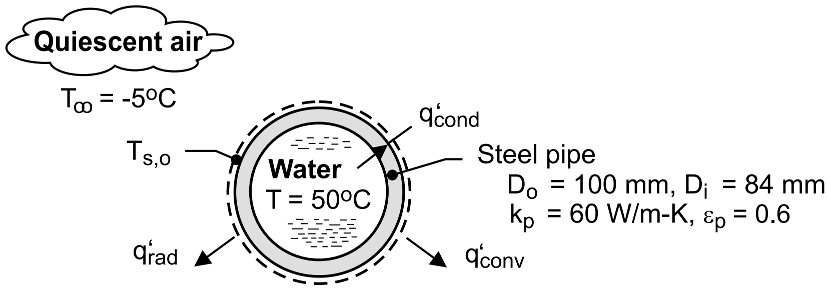
Tb	T01	T02	T03	T04	T05	T06	T07
423	418.3`	414.1	410.3	406.8	403.7	401	398.6
T08	T09	T10	T11	T12	T13	T14	T15
396.6	394.9	393.5	392.4	391.7	391.2	391	390.9

PROBLEM 9.65

KNOWN: Diameter, thickness, emissivity and thermal conductivity of steel pipe. Temperature of water flow in pipe. Cost of producing hot water.

FIND: Cost of daily heat loss from an uninsulated pipe.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) Negligible convection resistance for water flow, (3) Negligible radiation from pipe surroundings, (4) Quiescent air, (5) Constant properties.

PROPERTIES: *Table A-4*, air (p = 1 atm, $T_f \approx 295K$): $k_a = 0.0259$ W/m·K. $\nu = 15.45 \times 10^{-6}$ m^2/s, $\alpha = 21.8 \times 10^{-6}$ m^2/s, Pr = 0.708, $\beta = 3.39 \times 10^{-3}$ K^{-1}.

ANALYSIS: Performing an energy balance for a control surface about the outer surface, $q'_{cond} = q'_{conv} + q'_{rad}$, it follows that

$$\frac{T - T_{s,o}}{R'_{cond}} = \bar{h}\pi D_o \left(T_{s,o} - T_\infty\right) + \varepsilon_p \pi D_o \sigma T_{s,o}^4 \tag{1}$$

where $R'_{cond} = \ell n\left(D_o / D_i\right)/2\pi k_p = \ell n\left(100/84\right)/2\pi\left(60\,W/m\cdot K\right) = 4.62\times10^{-4}\,m\cdot K/W$. The convection coefficient may be obtained from the Churchill and Chu correlation. Hence, with Ra_D = $g\beta\,(T_{s,o} - T_\infty)\; D_o^3/\alpha\nu = 9.8\,m/s^2 \times 3.39\times10^{-3}K^{-1}\left(0.1m\right)^3\left(T_{s,o} - 268K\right)/\;\left(21.8\times15.45\times10^{-12}m^4/s^2\right)$

$= 98,637\;\left(T_{s,o} - 268\right)$,

$$\overline{Nu}_D = \left\{0.60 + \frac{0.387\,Ra_D^{1/6}}{\left[1+\left(0.559/Pr\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.60 + 2.182\left(T_{s,o} - 268\right)^{1/6}\right\}^2$$

$$\bar{h} = \frac{k_a}{D_o}\overline{Nu}_D = 0.259\,W/m^2\cdot K\left\{0.60 + 2.182\left(T_{s,o} - 268\right)^{1/6}\right\}^2$$

Substituting the foregoing expression for $\bar{h}$, as well as values of R'_{cond}, D_o, ε_p and σ into Eq. (1), an iterative solution yields

$$T_{s,o} = 322.9\,K = 49.9°C$$

It follows that $\bar{h} = 6.10\,W/m^2\cdot K$, and the heat loss per unit length of pipe is

$$q' = q'_{conv} + q'_{rad} = 6.10\,W/m^2\cdot K\left(\pi\times0.1m\right)54.9K + 0.6\left(\pi\times0.1m\right)5.67\times10^{-8}\,W/m^2\cdot K^4\left(322.9K\right)^4$$

$$= \left(105.2 + 116.2\right)W/m = 221.4\,W/m$$

The corresponding daily energy loss is $\quad Q' = 0.221\,kW/m\times24\,h/d = 5.3\,kW\cdot h/m\cdot d$

and the associated cost is $\quad C' = \left(5.3\,kW\cdot h/m\cdot d\right)\left(\$0.05/kW\cdot h\right) = \$0.265/m\cdot d$ <

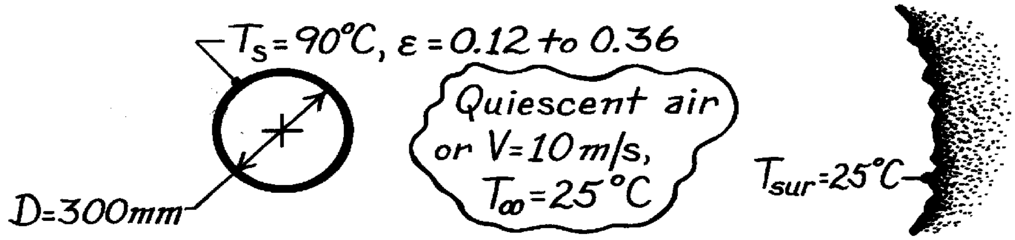
COMMENTS: (1) The heat loss is significant, and the pipe should be insulated. (2) The conduction resistance of the pipe wall is negligible relative to the combined convection and radiation resistance at the outer surface. Hence, the temperature of the outer surface is only slightly less than that of the water.

PROBLEM 9.66

KNOWN: Insulated, horizontal pipe with aluminum foil having emissivity which varies from 0.12 to 0.36 during service. Pipe diameter is 300 mm and its surface temperature is 90°C.

FIND: Effect of emissivity degradation on heat loss with ambient air at 25°C and (a) quiescent conditions and (b) cross-wind velocity of 5 m/s.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Surroundings are large compared to pipe, (3) Pipe has uniform temperature.

PROPERTIES: *Table A-4*, Air (T_f = (90 + 25)°C/2 = 330K, 1 atm): $\nu = 18.9 \times 10^{-6}$ m^2/s, k = 28.5×10^{-3} W/m·K, $\alpha = 26.9 \times 10^{-6}$ m^2/s, Pr = 0.703.

ANALYSIS: The heat loss per unit length from the pipe is

$$q' = \bar{h}P\left(T_s - T_\infty\right) + es\,P\left(T_s^4 - T_{sur}^4\right)$$

where P = πD and $\bar{h}$ needs to be evaluated for the two ambient air conditions.

(a) *Quiescent air*. Treating the pipe as a horizontal cylinder, find

$$Ra_D = \frac{gb\left(T_s - T_\infty\right)D^3}{na} = \frac{9.8\,m/s^2\,(1/330K)(90-25)K\,(0.30m)^3}{18.9\times10^{-6}m^2/s\times26.9\times10^{-6}m^2/s} = 1.025\times10^8$$

and using the Churchill-Chu correlation for $10^{-5} < Re_D < 10^{12}$.

$$\overline{Nu}_D = \left\{0.60 + \frac{0.387Ra_D^{1/6}}{\left[1+\left(0.559/Pr\right)^{9/16}\right]^{8/27}}\right\}^2$$

$$\overline{Nu}_D = \left\{0.60 + \frac{0.387\left(1.025\times10^8\right)^{1/6}}{\left[1+\left(0.559/0.703\right)^{9/16}\right]^{8/27}}\right\}^2 = 56.93$$

$$\bar{h}_D = \overline{Nu}_D\,k/D = 56.93\times0.0285\ W/m\cdot K/0.300m = 5.4\,W/m^2\cdot K.$$

Continued …..

PROBLEM 9.66 (Cont.)

Hence, the heat loss is

$$q' = 5.4\,\mathrm{W/m^2\cdot K}(p\,0.30\mathrm{m})(90-25)\mathrm{K} + e \times 5.67\times10^{-8}\mathrm{W/m^2\cdot K}(p\,0.300\mathrm{m})\left(363^4 - 298^4\right)\mathrm{K}^4$$

$$q' = 331 + 506e \quad \begin{cases} e = 0.12 \rightarrow q' = (331+61) = 392\,\mathrm{W/m} & < \\ e = 0.36 \rightarrow q' = (331+182) = 513\,\mathrm{W/m} & < \end{cases}$$

The radiation effect accounts for 16 and 35%, respectively, of the heat rate.

(b) *Cross-wind condition*. With a cross-wind, find

$$Re_D = \frac{VD}{n} = \frac{10\,\mathrm{m/s}\times0.30\mathrm{m}}{18.9\times10^{-6}\mathrm{m^2/s}} = 1.587\times10^5$$

and using the Hilpert correlation where C = 0.027 and m = 0.805 from Table 7.2,

$$\overline{Nu}_D = C\,Re_D^m\,Pr^{1/3} = 0.027\left(1.587\times10^5\right)^{0.805}(0.703)^{1/3} = 368.9$$

$$\bar{h}_D = Nu_D \cdot k/D = 368.9\times0.0285\,\mathrm{W/m\cdot K}/0.300\mathrm{m} = 35\,\mathrm{W/m^2\cdot K}.$$

Recognizing that *combined* free and forced convection conditions may exist, from Eq. 9.64 with n = 3,

$$Nu_m^3 = Nu_F^3 + Nu_N^3 \qquad \bar{h}_m = \left(5.4^3 + 35^3\right)^{1/3} = 35\,\mathrm{W/m^2\cdot K}$$

we find forced convection dominates. Hence, the heat loss is

$$q' = 35\,\mathrm{W/m^2\cdot K}(p\,0.300\mathrm{m})(90-25)\mathrm{K} + e\times5.67\times10^{-8}\mathrm{W/m^2\cdot K}(p\,0.300\mathrm{m})\left(393^4 - 298^4\right)\mathrm{K}^4$$

$$q' = 2144 + 853e \quad \begin{cases} e = 0.12 \rightarrow q' = 2144 + 102 = 2246\,\mathrm{W/m} & < \\ e = 0.36 \rightarrow q' = 2144 + 307 = 2451\,\mathrm{W/m} & < \end{cases}$$

The radiation effect accounts for 5 and 13%, respectively, of the heat rate.

COMMENTS: (1) For high velocity wind conditions, radiation losses are quite low and the degradation of the foil is not important. However, for low velocity and quiescent air conditions, radiation effects are significant and the degradation of the foil can account for a nearly 25% change in heat loss.

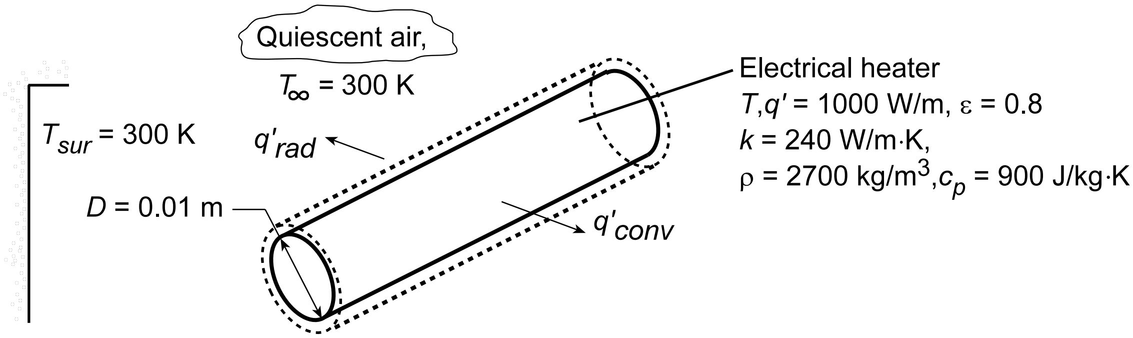
(2) The radiation coefficient is in the range 0.83 to 2.48 $\mathrm{W/m^2\cdot K}$ for ε = 0.12 and 0.36, respectively. Compare these values with those for convection.

PROBLEM 9.67

KNOWN: Diameter, emissivity, and power dissipation of cylindrical heater. Temperature of ambient air and surroundings.

FIND: Steady-state temperature of heater and time required to come within 10°C of this temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Air is quiescent, (2) Duct wall forms large surroundings about heater, (3) Heater may be approximated as a lumped capacitance.

PROPERTIES: *Table A.4*, air (Obtained from *Properties* Tool Pad of IHT).

ANALYSIS: Performing an energy balance on the heater, the final (steady-state) temperature may be obtained from the requirement that $q' = q'_{conv} + q'_{rad}$, or

$$q' = \overline{h}(\pi D)(T - T_\infty) + h_r(\pi D)(T - T_{sur})$$

where $\overline{h}$ is obtained from Eq. 9.34 and $h_r = \varepsilon\sigma(T + T_{sur})\left(T^2 + T_{sur}^2\right)$. Using the *Correlations* Tool Pad of IHT to evaluate $\overline{h}$, this expression may be solved to obtain

T = 854 K = 581°C <

Under transient conditions, the energy balance is of the form, $\dot{E}'_{st} = q' - q'_{conv} - q'_{rad}$, or

$$\rho c_p\left(\pi D^2/4\right)dT/dt = q' - \overline{h}(\pi D)(T - T_\infty) - h_r(\pi D)(T - T_{sur})$$

Using the IHT *Lumped Capacitance* model with the *Correlations* Tool Pad, the above expression is integrated from t = 0, for which T_i = 562.4 K, to the time for which T = 844 K. The integration yields

t = 183s <

The value of T_i = 562.4 K corresponds to the steady-state temperature for which the power dissipation is balanced by convection and radiation (see solution to Problem 7.44).

COMMENTS: The forced convection coefficient (Problems 7.43 and 7.44) of 105 W/m^2·K is much larger than that associated with free convection for the steady-state conditions of this problem (14.6 W/m^2·K). However, because of the correspondingly larger heater temperature, the radiation coefficient with free convection (42.9 W/m^2·K) is much larger than that associated with forced convection (15.9 W/m^2·K).

PROBLEM 9.68

KNOWN: Cylindrical sensor of 12.5 mm diameter positioned horizontally in quiescent air at 27°C.

FIND: An expression for the free convection coefficient as a function of only $\Delta T = T_s - T_\infty$ where T_s is the sensor temperature.

ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform temperature over cylindrically shaped sensor, (3) Ambient air extensive and quiescent.

PROPERTIES: *Table A-4*, Air (T_f, 1 atm): $\beta = 1/T_f$ and

T_s (°C)	T_f (K)	$\nu \times 10^6$ m^2/s	$\alpha \times 10^6$ m^2/s	$k \times 10^3$ W/m·K	Pr
30	302	16.09	22.8	26.5	0.707
55	314	17.30	24.6	27.3	0.705
80	327	18.61	26.5	28.3	0.703

ANALYSIS: For the cylindrical sensor, using Eqs. 9.25 and 9.34,

$$Ra_D = \frac{g b \Delta T D^3}{n a} \qquad \overline{Nu}_D = \frac{\bar{h}_D D}{k} = \left\{0.60 + \frac{0.387 Ra_D^{1/6}}{\left[1+\left(0.559/Pr\right)^{9/16}\right]^{8/17}}\right\}^2 \qquad (1,2)$$

where properties are evaluated at $(T_f = T_s + T_\infty)/2$. With $30 \le T_s \le 80°C$ and $T_\infty = 27°C$, $302 \le T_f \le 326$ K. Using properties evaluated at the mid-range of T_f, $\bar{T}_f = 314K$, find

$$Ra_D = \frac{9.8 m/s^2 \left(1/314K\right)\Delta T\left(0.0125m\right)^3}{17.30\times10^{-6} m^2/s \times 24.6\times10^{-6} m^2/s} = 143.2\Delta T$$

$$\bar{h}_D = \frac{0.0273 W/m\cdot K}{0.0125m}\left\{0.60 + \frac{0.387\left(143\Delta T\right)^{1/6}}{\left[1+\left(0.559/0.705\right)^{9/16}\right]^{8/27}}\right\}^2$$

$$\bar{h}_D = 2.184\left\{0.60 + 0.734\Delta T^{1/6}\right\}^2. \qquad (3) \quad <$$

COMMENTS: (1) The effect of using a fixed film temperature, $\bar{T}_f = 314K = 41°C$, for the full range $30 \le T_s \le 80°C$ can be seen by comparing results from the approximate Eq. (3) and the correlation, Eq. (2), with the proper film temperature. The results are summarized in the table.

		Correlation			Eq. (3)
T_s (°C)	$\Delta T = T_s - T_\infty$ (°C)	Ra_D	$\overline{Nu}_D$	$\bar{h}_D\left(W/m^2\cdot K\right)$	$\bar{h}_D\left(W/m^2\cdot K\right)$
30	3	518	2.281	4.83	4.80
55	28	4011	3.534	7.72	7.71

The approximate expression for $\bar{h}_D$ is in excellent agreement with the correlation.

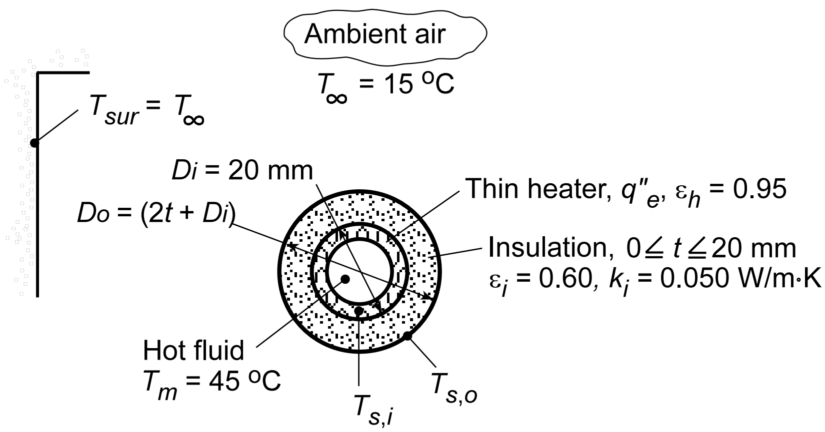
(2) In calculating heat rates it may be important to consider radiation exchange with the surroundings.

PROBLEM 9.69

KNOWN: Thin-walled tube mounted horizontally in quiescent air and wrapped with an electrical tape passing hot fluid in an experimental loop.

FIND: (a) Heat flux q''_e from the heating tape required to prevent heat loss from the hot fluid when (a) neglecting and (b) including radiation exchange with the surroundings, (c) Effect of insulation on q''_e and convection/radiation rates.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Ambient air is quiescent and extensive, (3) Surroundings are large compared to the tube.

PROPERTIES: *Table A.4*, Air ($T_f = (T_s + T_\infty)/2 = (45 + 15)°C/2 = 303$ K, 1 atm): $\nu = 16.19 \times 10^{-6}$ m^2/s, $\alpha = 22.9 \times 10^{-6}$ m^2/s, k = 26.5×10^{-3} W/m·K, Pr = 0.707, $\beta = 1/T_f$.

ANALYSIS: (a,b) To prevent heat losses from the hot fluid, the heating tape temperature must be maintained at T_m; hence $T_{s,i} = T_m$. From a surface energy balance,

$$q''_e = q''_{conv} + q''_{rad} = \left(\bar{h}_{D_i} + h_r\right)\left(T_{s,i} - T_\infty\right)$$

where the linearized radiation coefficient, Eq. 1.9, is $h_r = \varepsilon\sigma\left(T_{s,i} + T_\infty\right)\left(T_{s,i}^2 + T_\infty^2\right)$, or

$$h_r = 0.95 \times 5.67 \times 10^{-8}\,\mathrm{W/m^2 \cdot K^4}\,(318 + 288)\left(318^2 + 288^2\right)\mathrm{K}^3 = 6.01\,\mathrm{W/m^2 \cdot K}.$$

Neglecting radiation: For the horizontal cylinder, Eq. 9.34 yields

$$Ra_D = \frac{g\beta\left(T_{s,i} - T_\infty\right)D_i^3}{\nu\alpha} = \frac{9.8\,\mathrm{m/s^2}\,(1/303\,\mathrm{K})(45 - 15)\,\mathrm{K}\,(0.020\,\mathrm{m})^3}{16.19 \times 10^{-6}\,\mathrm{m^2/s} \times 22.9 \times 10^{-6}\,\mathrm{m^2/s}} = 20{,}900$$

$$\overline{Nu}_D = \frac{\bar{h}_{D_i} D_i}{k} = \left\{0.60 + \frac{0.386 Ra_D^{1/6}}{\left[1 + (0.559/Pr)^{9/16}\right]^{8/27}}\right\}^2$$

Continued ….

PROBLEM 9.69 (Cont.)

$$\bar{h}_{D_i} = \frac{0.0265\,\text{W/m}\cdot\text{K}}{0.020\,\text{m}}\left\{0.60 + \frac{0.386(20,900)^{1/6}}{\left[1+(0.559/0.707)^{9/16}\right]^{8/27}}\right\}^2 = 6.90\,\text{W/m}^2\cdot\text{K}$$

Hence, neglecting radiation, the required heat flux is

$$q''_e = 6.90\,\text{W/m}^2\cdot\text{K}(45-15)\text{K} = 207\,\text{W/m}^2\cdot\text{K}$$ <

Considering radiation: The required heat flux considering radiation is

$$q''_e = (6.90+6.01)\,\text{W/m}^2\cdot\text{K}(45-15)\text{K} = 387\,\text{W/m}^2\cdot\text{K}$$ <

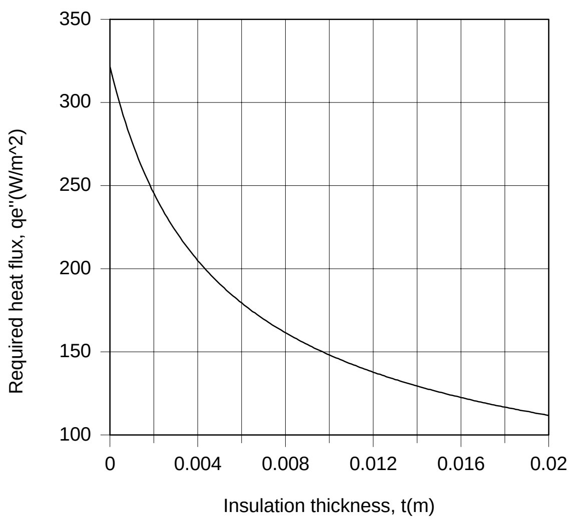
(c) With insulation, the surface energy balance must be modified to account for an increase in the outer diameter from D_i to $D_o = D_i + 2t$ and for the attendant thermal resistance associated with conduction across the insulation. From an energy balance at the inner surface of the insulation,

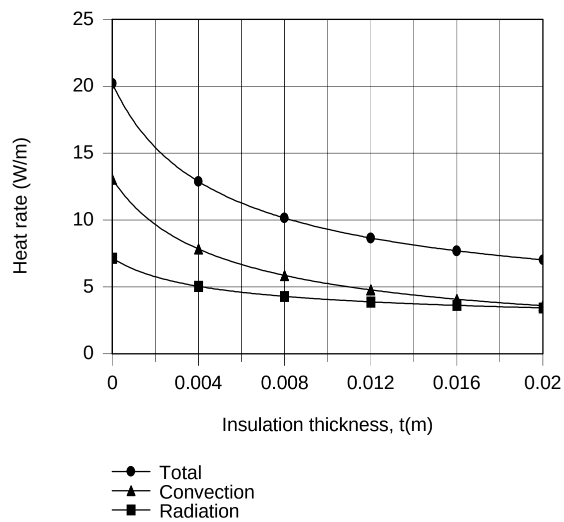
$$q''_e(\pi D_i) = q'_{cond} = \frac{2\pi k_i(T_m - T_{s,o})}{\ln(D_o/D_i)}$$

and from an energy balance at the outer surface,

$$q'_{cond} = q'_{conv} + q'_{rad} = \pi D_o\left(\bar{h}_{D_o} + h_r\right)\left(T_{s,o} - T_\infty\right)$$

The foregoing expressions may be used to determine $T_{s,o}$ and q''_e as a function of t, with the IHT *Correlations* and *Properties* Tool Pads used to evaluate $\bar{h}_{D_o}$. The desired results are plotted as follows.





By adding 20 mm of insulation, the required power dissipation is reduced by a factor of approximately 3. Convection and radiation heat rates at the outer surface are comparable.

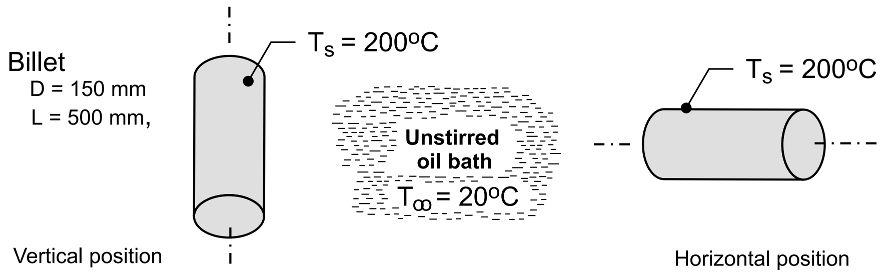
COMMENTS: Over the range of insulation thickness, $T_{s,o}$ decreases from 45°C to 20°C, while $\bar{h}_{D_o}$ and h_r decrease from 6.9 to 3.5 W/m^2·K and from 3.8 to 3.3 W/m^2·K, respectively.

PROBLEM 9.70

KNOWN: A billet of stainless steel AISI 316 with a diameter of 150 mm and length 500 mm emerges from a heat treatment process at 200°C and is placed into an unstirred oil bath maintained at 20°C.

FIND: (a) Determine whether it is advisable to position the billet in the bath with its centerline horizontal or vertical in order decrease the cooling time, and (b) Estimate the time for the billet to cool to 30°C for the better positioning arrangement.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions for part (a), (2) Oil bath approximates a quiescent fluid, (3) Consider only convection from the lateral surface of the cylindrical billet; and (4) For part (b), the billet has a uniform initial temperature.

PROPERTIES: *Table A-5*, Engine oil ($T_f = (T_s + T_\infty)/2$): see Comment 1. *Table A-1*, AISI 316 (400 K): $\rho = 8238\ \text{kg/m}^3$, $c_p = 468$ J/kg·K, k = 15 W/m·K.

ANALYSIS: (a) For the purpose of determining whether the horizontal or vertical position is preferred for faster cooling, consider only free convection from the lateral surface. The heat loss from the lateral surface follows from the rate equation

$$q = \bar{h}\,A_s\left(T_s - T_\infty\right)$$

Vertical position. The lateral surface of the cylindrical billet can be considered as a vertical surface of height L, width P = πD, and area A_s = PL. The Churchill-Chu correlation, Eq. 9.26, is appropriate to estimate $\bar{h}_L$,

$$\overline{Nu}_L = \frac{\bar{h}_L L}{k} = \left\{0.825 + \frac{0.387\ Ra_L^{1/6}}{\left[1+\left(0.492/Pr\right)^{9/16}\right]^{8/27}}\right\}^2$$

$$Ra_L = \frac{g\beta\left(T_s - T_\infty\right)L^3}{\nu\alpha}$$

with properties evaluated at $T_f = (T_s + T_\infty)/2$.

Horizontal position. In this position, the billet is considered as a long horizontal cylinder of diameter D for which the Churchill-Chu correlation of Eq. 9.34 is appropriate to estimate $\bar{h}_D$,

$$\overline{Nu}_L = \frac{\bar{h}_D D}{k} = \left\{0.60 + \frac{0.387\ Ra_D^{1/6}}{\left[1+\left(0.55/Pr\right)^{9/16}\right]^{8/27}}\right\}^2$$

Continued …..

$$Ra_D = \frac{g\beta(T_s - T_\infty)D^3}{\nu\alpha}$$

with properties evaluated at T_f. The heat transfer area is also A_s = PL.

Using the foregoing relations in *IHT* with the thermophysical properties library as shown in Comment 1, the analysis results are tabulated below.

$Ra_L = 1.36\times10^{11}$	$\overline{Nu}_L = 801$	$\bar{h}_L = 218\ W/m^2\cdot K$	(vertical)
$Ra_D = 3.67\times10^{9}$	$\overline{Nu}_D = 245$	$\bar{h}_D = 221\ W/m^2\cdot K$	(horizontal)

Recognize that the orientation has a small effect on the convection coefficient for these conditions, but we'll select the horizontal orientation as the preferred one.

(b) Evaluate first the Biot number to determine if the lumped capacitance method is valid.

$$Bi = \frac{\bar{h}_D(D_o/2)}{k} = \frac{221\ W/m^2\cdot K(0.150\ m/2)}{15\ W/m\cdot K} = 1.1$$

Since Bi >> 0.1, the spatial effects are important and we should use the one-term series approximation for the infinite cylinder, Eq. 5.49. Since $\bar{h}_D$ will decrease as the billet cools, we need to estimate an average value for the cooling process from 200°C to 30°C. Based upon the analysis summarized in Comment 1, use $\bar{h}_D = 119\ W/m^2\cdot K$. Using the transient model for the infinite cylinder in *IHT*, (see Comment 2) find for $T(r_o, t_o)$ = 30°C,

$$t_o = 3845\ s = 1.1\ h$$ <

COMMENTS: (1) The *IHT* code using the convection correlation functions to estimate the coefficients is shown below. This same code was used to calculate $\bar{h}_D$ for the range $30 \le T_s \le 200$°C and determine that an average value for the cooling period of part (b) is 119 W/m^2·K.

```
/* Results - convection coefficients, Ts = 200 C
hDbar  hLbar    D        L       Tinf_C   Ts_C
221.4  217.5    0.15     0.5     20       200      */

/* Results - correlation parameters, Ts = 200 C
NuDbar          NuLbar   Pr      RaD      RaL
244.7           801.3    219.2   3.665E9  1.357E11 */

/* Results - properties, Ts = 200 C; Tf = 383 K
Pr     alpha     beta    deltaT  k        nu        Tf
219.2  7.188E-8  0.0007  180     0.1357   1.582E-5  383

/*  Correlation description: Free convection (FC), long horizontal cylinder (HC),
10^-5<=RaD<=10^12, Churchill-Chu correlation, Eqs 9.25 and 9.34 . See Table 9.2 .  */
NuDbar = NuD_bar_FC_HC(RaD,Pr)                   // Eq 9.34
NuDbar = hDbar * D / k
RaD = g * beta * deltaT * D^3 / (nu * alpha)     //Eq 9.25
deltaT = abs(Ts - Tinf)
g = 9.8     // gravitational constant, m/s^2
// Evaluate properties at the film temperature, Tf.
Tf = Tfluid_avg(Tinf,Ts)
```

Continued …..

PROBLEM 9.70 (Cont.)

```
/*  Correlation description: Free convection (FC) for a vertical plate (VP),  Eqs 9.25 and 9.26 .
See Table 9.2 .  */
NuLbar = NuL_bar_FC_VP(RaL,Pr)              // Eq 9.26
NuLbar = hLbar * L / k
RaL = g * beta * deltaT * L^3 / (nu * alpha)       //Eq 9.25

// Input variables
D = 0.15
L = 0.5
Tinf_C = 20
Ts_C = 200

// Engine Oil property functions : From Table A.5
// Units: T(K)
nu = nu_T("Engine Oil",Tf)              // Kinematic viscosity, m^2/s
k = k_T("Engine Oil",Tf)                // Thermal conductivity, W/m·K
alpha = alpha_T("Engine Oil",Tf)        // Thermal diffusivity, m^2/s
Pr = Pr_T("Engine Oil",Tf)              // Prandtl number
beta = beta_T("Engine Oil",Tf)          // Volumetric coefficient of expansion, K^(-1)

// Conversions
Tinf_C = Tinf - 273
Ts_C = Ts - 273
```

(2) The portion of the *IHT* code used for the transient analysis is shown below. Recognize that we have not considered heat losses from the billet end surfaces, also, we should consider the billet as a three-dimensional object rather than as a long cylinder.

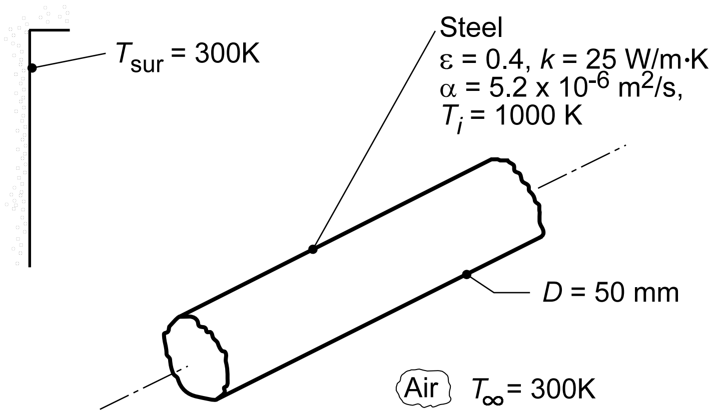
```
/* Results - time to cool to 30 C, center and surface temperatures
D      T_xt_C   Ti_C    Tinf_C   r       h      t
0.15   30.01    200     20       0.075   119    3845   */
0.15   33.19    200     20       0       119    3845

// Transient conduction model, cylinder (series solution)
// The temperature distribution T(r,t) is
T_xt = T_xt_trans("Cylinder",rstar,Fo,Bi,Ti,Tinf)    // Eq 5.47
// The dimensionless parameters are
rstar = r / ro
Bi = h * ro / k
Fo= alpha * t / ro^2
alpha = k/ (rho * cp)
```

PROBLEM 9.71

KNOWN: Diameter, initial temperature and emissivity of long steel rod. Temperature of air and surroundings.

FIND: (a) Average surface convection coefficient, (b) Effective radiation coefficient, (c,d) Maximum allowable conveyor time.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible effect of forced convection, (2) Constant properties, (3) Large surroundings, (4) Quiescent air.

PROPERTIES: Stainless steel (given): k = 25 W/m·K, $\alpha = 5.2 \times 10^{-6}$ m^2/s; *Table A.4,* Air (T_f = 650 K, 1 atm): $\nu = 6.02 \times 10^{-5}$ m^2/s, $\alpha = 8.73 \times 10^{-5}$ m^2/s, k = 0.0497 W/m·K, Pr = 0.69.

ANALYSIS: (a) For free convection from a horizontal cylinder,

$$Ra_D = \frac{g\beta\left(T_s - T_\infty\right)D^3}{\alpha\nu} = \frac{9.8\,\mathrm{m/s^2}\left(1/650\,\mathrm{K}\right)\left(0.05\,\mathrm{m}\right)^3}{6.02\times 8.73\times 10^{-10}\,\mathrm{m^4/s^2}} = 2.51\times 10^5$$

The Churchill and Chu correlation yields

$$\overline{Nu}_D = \left\{0.60 + \frac{0.387 Ra_D^{1/6}}{\left[1+\left(0.559/Pr\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.60 + \frac{0.387\left(2.51\times 10^5\right)^{1/6}}{\left[1+\left(0.559/0.69\right)^{9/16}\right]^{8/27}}\right\}^2 = 9.9$$

$$\bar{h} = \overline{Nu}_D\, k/D = 9.9\left(0.0497\,\mathrm{W/m\cdot K}\right)/0.05\,\mathrm{m} = 9.84\,\mathrm{W/m^2\cdot K} \qquad <$$

(b) The radiation heat transfer coefficient is

$$h_r = \varepsilon\sigma\left(T_s + T_{sur}\right)\left(T_s^2 + T_{sur}^2\right) = 0.4\times 5.67\times 10^{-8}\,\mathrm{W/m^2\cdot K^4}\,(1000$$

$$+300)\mathrm{K}\left[(1000)^2 + (300)^2\right]\mathrm{K}^2 = 32.1\,\mathrm{W/m^2\cdot K} \qquad <$$

(c) For the long stainless steel rod and the initial values of $\bar{h}$ and h_r,

$$Bi = \left(\bar{h} + h_r\right)\left(r_o/2\right)/k = 42.0\,\mathrm{W/m^2\cdot K}\times 0.0125\,\mathrm{m}/25\,\mathrm{W/m\cdot K} = 0.021.$$

Hence, the lumped capacitance method can be used.

$$\frac{T - T_\infty}{T_i - T_\infty} = \frac{600\,\mathrm{K}}{700\,\mathrm{K}} = \exp\left(-Bi\cdot Fo\right) = \exp\left(-0.021 Fo\right)$$

Continued...

$$\mathrm{Fo} = 7.34 = \alpha t \big/ (r_o/2)^2 = 0.0333t$$

$t = 221$ s. <

(d) Using the IHT *Lumped Capacitance* Model with the *Correlations* and *Properties* Tool Pads, a more accurate estimate of the maximum allowable transit time may be obtained by evaluating the numerical integration,

$$\int_o^t dt = -\frac{\rho c_p D}{4} \int_{1000\,K}^{900\,K} \frac{dT}{\left(\bar{h} + h_r\right)\left(T - T_\infty\right)}$$

where $\rho c_p = k/\alpha = 4.81 \times 10^6\ \mathrm{J/K \cdot m^3}$. The integration yields

$t = 245$ s <

At this time, the convection and radiation coefficients are $\bar{h}$ = 9.75 and h_r = 24.5 W/m^2·K, respectively.

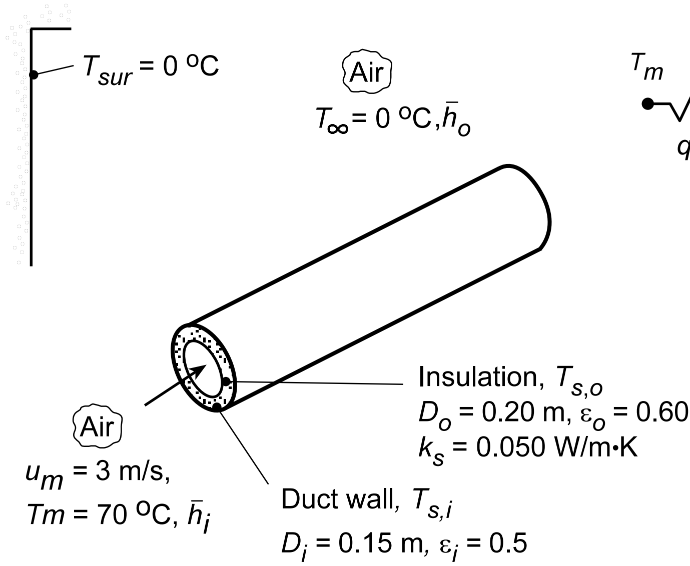
COMMENTS: Since $\bar{h}$ and h_r decrease with increasing time, the maximum allowable conveyor time is underestimated by the result of part (c).

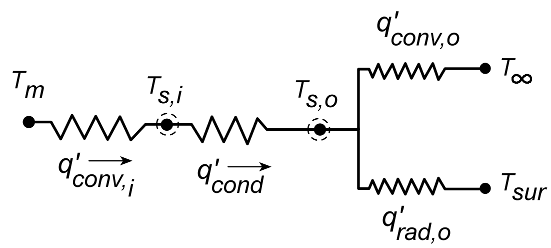
PROBLEM 9.72

KNOWN: Velocity and temperature of air flowing through a duct of prescribed diameter. Temperature of duct surroundings. Thickness, thermal conductivity and emissivity of applied insulation.

FIND: (a) Duct surface temperature and heat loss per unit length with no insulation, (b) Surface temperatures and heat loss with insulation.

SCHEMATIC:





ASSUMPTIONS: (1) Steady-state conditions, (2) Fully-developed internal flow, (3) Negligible duct wall resistance, (4) Duct outer surface is diffuse-gray, (5) Outside air is quiescent, (6) Pressure of inside and outside air is atmospheric.

PROPERTIES: *Table A.4*, Air (T_m = 70°C): $\nu = 20.22 \times 10^{-6}$ m²/s, Pr = 0.70, k = 0.0295 W/m·K; *Table A.4*, Air ($T_f \approx 27$°C): $\nu = 15.89 \times 10^{-6}$ m²/s, Pr = 0.707, k = 0.0263 W/m·K, $\alpha = 22.5 \times 10^{-6}$ m²/s, β = 0.00333 K^{-1}.

ANALYSIS: (a) Performing an energy balance on the duct wall with no insulation ($T_{s,i} = T_{s,o}$),

$$q'_{conv,i} = q'_{conv,o} + q'_{rad,o} \qquad h_i(\pi D_i)(T_m - T_{s,i}) = h_o(\pi D_i)(T_{s,i} - T_\infty) + \varepsilon_i \sigma(\pi D_i)\left(T_{s,i}^4 - T_{sur}^4\right)$$

with $Re_{D,i} = u_m D_i/\nu = 3\text{ m/s} \times 0.15\text{ m}/(20.22 \times 10^{-6}\text{ m}^2/\text{s}) = 2.23 \times 10^4$, the internal flow is turbulent, and from the Dittus-Boelter correlation,

$$h_i = \frac{k}{D_i} 0.023 Re_{D,i}^{4/5} Pr^{0.3} = \frac{0.0295\text{ W/m}\cdot\text{K}}{0.15\text{ m}} 0.023\left(2.23\times10^4\right)^{4/5}(0.7)^{0.3} = 12.2\text{ W/m}^2\cdot\text{K}.$$

For free convection, the Rayleigh number is

$$Ra_{D,i} = \frac{g\beta\left(T_{s,i} - T_\infty\right)D_i^3}{\nu\alpha} = \frac{9.8\text{ m/s}^2\,(0.0033)\left(T_{s,i} - 273\right)(0.15)^3\text{ m}^3}{15.89\times10^{-6}\text{ m}^2/\text{s}\times22.5\times10^{-6}\text{ m}^2/\text{s}} = 3.08\times10^5\left(T_{s,i} - T_\infty\right)$$

and from Eq. 9.34,

$$\bar{h}_o = \frac{k}{D_i}\left[0.60 + \frac{0.387 Ra_{D,i}^{1/6}}{\left[1 + (0.559/Pr)^{9/16}\right]^{8/27}}\right]^2 = \frac{0.0263}{0.15}\left[0.60 + \frac{0.387\left[3.08\times10^5\left(T_{s,i} - T_\infty\right)\right]^{1/6}}{\left[1 + (0.559/0.707)^{9/16}\right]^{8/27}}\right]^2$$

Continued...

$$\bar{h}_o = 0.175\left[0.60 + 2.64\left(T_{s,i} - T_\infty\right)^{1/6}\right]^2$$

Hence

$$12.2\left(343 - T_{s,i}\right) = 0.175\left\{0.60 + 2.64\left(T_{s,i} - 273\right)^{1/6}\right\}^2\left(T_{s,i} - 273\right) + 0.5 \times 5.67 \times 10^{-8}\left[T_{s,i}^4 - (273)^4\right]$$

A trial-and-error solution gives $\quad T_{s,i} \approx 314.7\,\text{K} \approx 41.7^\circ\text{C}$ $\quad <$

The heat loss per unit length is then

$$q' = q'_{conv,i} \approx 12.2(\pi \times 0.15)(70 - 42) \approx 163\,\text{W/m}. \qquad <$$

(b) Performing energy balances at the inner and outer surfaces, we obtain, respectively,

$$q'_{conv,i} = q'_{cond}$$

or,

$$\bar{h}_i\left(\pi D_i\right)\left(T_m - T_{s,i}\right) = \frac{2\pi k_s\left(T_{s,i} - T_{s,o}\right)}{\ln\left(D_o/D_i\right)}$$

and,

$$q'_{cond} = q'_{conv,o} + q'_{rad,o}$$

or,

$$\frac{2\pi k_s\left(T_{s,i} - T_{s,o}\right)}{\ln\left(D_o/D_i\right)} = \bar{h}_o\left(\pi D_o\right)\left(T_s - T_\infty\right) + \varepsilon_o \sigma\left(\pi D_o\right)\left(T_{s,o}^4 - T_{sur}^4\right)$$

Using the IHT workspace with the *Correlations* and *Properties* Tool Pads to solve the energy balances for the unknown surface temperatures, we obtain

$$T_{s,i} = 60.8^\circ\text{C} \qquad T_{s,o} = 12.5^\circ\text{C} \qquad <$$

With the heat loss per unit length again evaluated from the inside convection process, we obtain

$$q' = q'_{conv,i} = 52.8\,\text{W/m} \qquad <$$

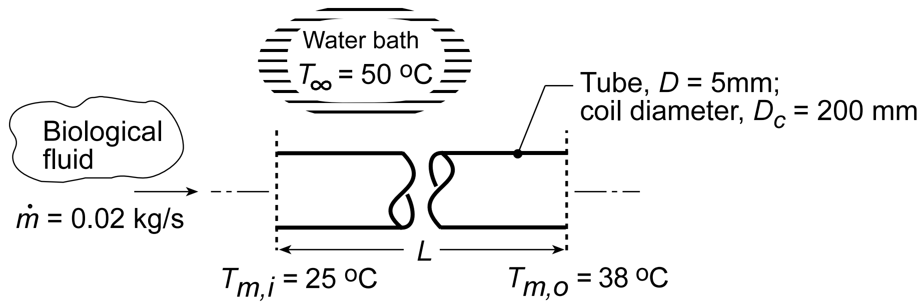
COMMENTS: For part (a), the outside convection coefficient is $\bar{h}_o$ = 5.4 W/m^2·K < h_i. The outside heat transfer rates are $q'_{conv,o}$ ≈ 106 W/m and $q'_{rad,o}$ ≈ 57 W/m. For part (b), $\bar{h}_o$ = 3.74 W/m^2·K, $q'_{conv,o}$ = 29.4 W/m, and $q'_{rad,o}$ = 23.3 W/m. Although $T_{s,i}$ increases with addition of the insulation, there is a substantial reduction in $T_{s,o}$ and hence the heat loss.

PROBLEM 9.73

KNOWN: Biological fluid with prescribed flow rate and inlet temperature flowing through a coiled, thin-walled, 5-mm diameter tube submerged in a large water bath maintained at 50°C.

FIND: (a) Length of tube and number of coils required to provide an exit temperature of $T_{m,o}$ = 38°C, and (b) Variations expected in $T_{m,o}$ for a ±10 % change in the mass flow rate for the tube length determined in part (a) .

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Coiled tube approximates a horizontal tube experiencing free convection in a quiescent, extensive medium (water bath), (3) Biological fluid has thermophysical properties of water, and (4) Negligible tube wall thermal resistance.

PROPERTIES: *Table A.4* Water - cold side ($T_{m,c} = (T_{m,i} + T_{m,o}) / 2$ = 304.5 K) : $c_{p,c}$ = 4178 J/kg·K, $\mu_c = 777.6 \times 10^{-6}$ N·s/m^2 , k_c = 0.6193 W/m·K , Pr_c = 5.263 ; *Table A.4,* Water - hot side ($\overline{T}_f = (T_s + T_\infty)/2$ = 320.1 K, see comment 1) : ρ_h = 989.1 kg/m^3 , $c_{p,h}$ = 4180 J/kg·K, $\mu_h = 575.6 \times 10^{-6}$ N·s/m^2, k_h = 0.6401 W/m·K , Pr_h = 3.76, $\nu_h = \mu_h/\rho_h = 5.827 \times 10^{-7}$ m^2/s , $\alpha_h = k_h / \rho_h c_{ph}$, = 15.48×10^{-8} m^2/s.

ANALYSIS: (a) Following the treatment of Section 8.3.3, the coil experiences internal flow of the cold biological fluid (c) and free convection with the external hot fluid (h). From Eq. 8.46a,

$$\frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \exp\left(-\frac{\overline{U}PL}{\dot{m}c_{p,c}}\right) \qquad \overline{U} = \left(1/\overline{h}_c + 1/\overline{h}_h\right)^{-1} \tag{1}$$

with P = πD and for the overall coefficient $\overline{U}$, $\overline{h}_c$ and $\overline{h}_h$ are the average convection coefficients for internal flow and external free convection, respectively. These coefficients are estimated as follows.

Internal flow, $\overline{h}_c$*:* To characterize the flow, calculate the Reynolds number,

$$Re_{D,c} = \frac{4\dot{m}}{\pi D \mu_c} = \frac{4 \times 0.02\,\text{kg/s}}{\pi \times 0.005\text{m} \times 777.6 \times 10^{-6}\,\text{N}\cdot\text{s/m}^2} = 6550 \tag{3}$$

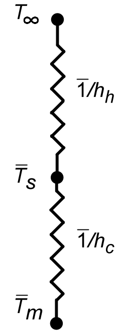
evaluating properties at $\overline{T}_m = \left(T_{m,i} + T_{m,o}\right)/2 = (25+38)^\circ\text{C}/2 = 31.5^\circ\text{C} = 304.5\text{K}$. Note that $Rc_{D,c}$ is between the laminar upper limit (2300) and the turbulent lower limit (10,000). To provide a conservative estimate, we choose to consider the flow as laminar and anticipate that the flow will be fully developed. From Eq. 8.55, $Nu_{D,c}$ = 3.66,

$$\overline{h}_c = Nu_{D,c}\, k_c / D\, 3.66 \times 0.6193\,\text{W/m}\cdot\text{K}/0.005\text{m} = 453\,\text{W/m}^2\cdot\text{K} \tag{4}$$

External free convection , $\overline{h}_h$: For the horizontal tube, Eq. 9.34, with

$$Ra_{D,h} = \frac{g\beta_h \Delta T D^3}{\nu_h \alpha_h} \qquad \Delta T = \overline{T}_s - T_\infty \tag{5,6}$$

Continued...



where $\overline{T}_s$ is the average tube wall temperature determined from the thermal circuit for which

$$\overline{h}_c\left(\overline{T}_m - \overline{T}_s\right) = \overline{h}_h\left(\overline{T}_s - T_\infty\right) \qquad (7)$$

and the average film temperature at which to evaluate properties is

$$\overline{T}_{f,c} = \left(\overline{T}_s + T_\infty\right)/2 \qquad (8)$$

We need to guess a value for $\overline{T}_s$ and iterate the solution of the system of equations until all the equations are satisfied. See Comments 1 and 2.

Results of the analysis: Using the foregoing relations in IHT (see Comment 2) the following results were obtained

$$\overline{U} = 313.4\,\mathrm{W/m^2\cdot K}, \qquad \overline{h}_c = 453\,\mathrm{W/m^2\cdot K} \qquad \overline{h}_h = 1015\,\mathrm{W/m^2\cdot K}$$

$$\overline{T}_{m,c} = 304.5\,\mathrm{K}, \qquad \overline{T}_{f,h} = 320.1\,\mathrm{K}, \qquad \overline{T}_s = 317.0\,\mathrm{K} \qquad L = 12.46\,\mathrm{m} \quad <$$

From knowledge of the tube length with the diameter of the coil D_c = 200 mm, the number of coils required is

$$N = \frac{L}{\pi D_c} = \frac{12.46\,\mathrm{m}}{\pi \times 0.200\,\mathrm{m}} = 19.8 \approx 20 \quad <$$

(b) With the length fixed at L = 12.46 m, we can backsolve the foregoing IHT workspace model to find what effect a ±10% change in the mass flow rate has on the outlet temperature, $T_{m,o}$. The results of the analysis are tabulated below.

$\dot{m}\,(\mathrm{kg/s})$	0.018	0.02	0.022
$T_{m,o}\,(^\circ\mathrm{C})$	38.95	38.00	37.17

That is, a ±10 % change in the flow rate causes a ±1°C change in the outlet temperature. While this change seems quite small, the effect on biological processes can be significant.

COMMENTS: (1) For the hot fluid, the Properties section shows the relevant thermophysical properties evaluated at the proper average (rather than a guess value for the film temperature).

(2) For the tube L/D = 12.46m/0.005m = 2492 which is substantially greater than the entrance length criterion, $0.05Re_D = 0.05\times 6550 = 328$. Hence, the assumption of fully developed internal flow is justified.

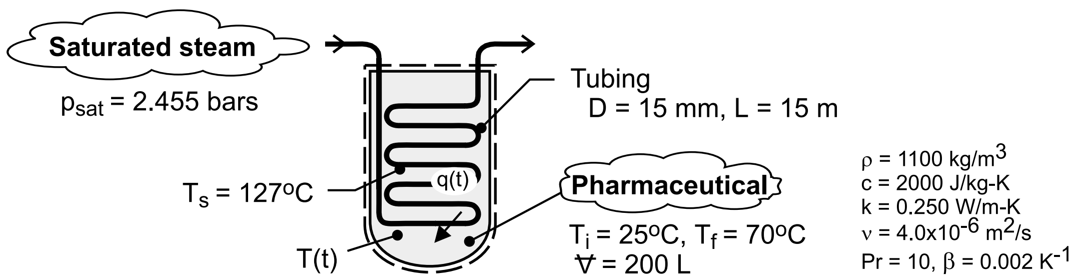
(3) The IHT model for the system can be constructed beginning with the *Rate Equation Tools, Tube Flow, Constant Surface Temperature* along with the *Correlation Tools* for *Free Convection, Horizontal Cylinder* and *Internal Flow, Laminar, Fully Developed Flow* and the *Properties Tool* for the hot and cold fluids (water). The full set of equations is extensive and very stiff. Review of the IHT Example 8.5 would be helpful in understanding how to organize the complete model.

PROBLEM 9.74

KNOWN: Volume, thermophysical properties, and initial and final temperatures of a pharmaceutical. Diameter and length of submerged tubing. Pressure of saturated steam flowing through the tubing.

FIND: (a) Initial rate of heat transfer to the pharmaceutical, (b) Time required to heat the pharmaceutical to 70°C and the amount of steam condensed during the process.

SCHEMATIC:



ASSUMPTIONS: (1) Pharmaceutical may be approximated as an infinite, quiescent fluid of uniform, but time-varying temperature, (2) Free convection heat transfer from the coil may be approximated as that from a heated, horizontal cylinder, (3) Negligible thermal resistance of condensing steam and tube wall, (4) Negligible heat transfer from tank to surroundings, (5) Constant properties.

PROPERTIES: *Table A-4*, Saturated water (2.455 bars): T_{sat} = 400K = 127°C, $h_{fg} = 2.183 \times 10^6$ J/kg. Pharmaceutical: See schematic.

ANALYSIS: (a) The initial rate of heat transfer is $q = \bar{h} A_s (T_s - T_i)$, where $A_s = \pi DL = 0.707\ m^2$ and $\bar{h}$ is obtained from Eq. 9.34. With $\alpha = \nu/Pr = 4.0 \times 10^{-7}\ m^2/s$ and $Ra_D = g\beta (T_s - T_i) D^3/\alpha\nu$ = $9.8\ m/s^2\ (0.002\ K^{-1})\ (102K)\ (0.015m)^3/16 \times 10^{-13}\ m^4/s^2 = 4.22 \times 10^6$,

$$\overline{Nu}_D = \left\{0.60 + \frac{0.387\, Ra_D^{1/6}}{\left[1+(0.559/Pr)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.60 + \frac{0.387\left(4.22\times10^6\right)^{1/6}}{\left[1+(0.559/10)^{9/16}\right]^{8/27}}\right\}^2 = 27.7$$

Hence, $$\bar{h} = Nu_D\, k/D = 27.7 \times 0.250\ W/m\cdot K/0.015m = 462\ W/m^2\cdot K$$

and $$q = \bar{h} A_s (T_s - T_i) = 462\ W/m^2\cdot K \times 0.707\ m^2\ (102°C) = 33{,}300\ W$$ <

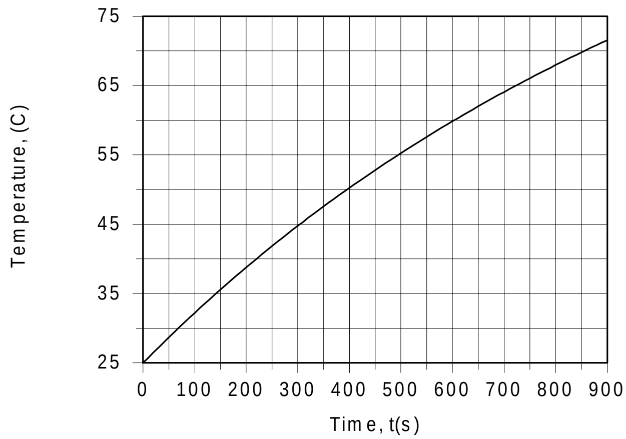
(b) Performing an energy balance at an instant of time for a control surface about the liquid,

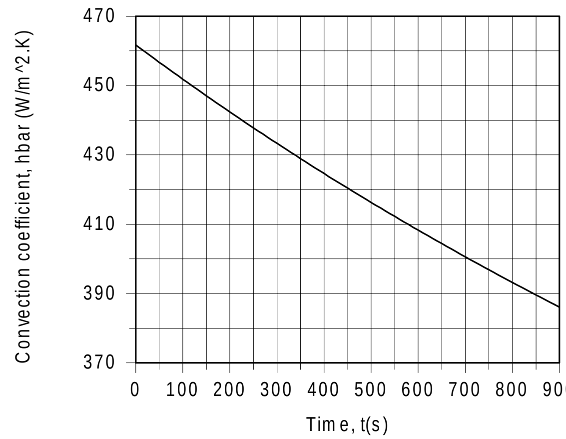
$$\frac{d(\rho \forall c T)}{dt} = q(t) = \bar{h}(t) A_s (T_s - T(t))$$

where the Rayleigh number, and hence $\bar{h}$, changes with time due to the change in the temperature of the liquid. Integrating the foregoing equation using the DER function of IHT, the following results are obtained for the variation of T and $\bar{h}$ with t.

Continued …..

PROBLEM 9.74 (Cont.)





The time at which the liquid reaches 70°C is

$$t_f \approx 855\,\text{s}$$ <

The rate at which T increases decreases with increasing time due to the corresponding reduction in ($T_s - T$), and hence reductions in Ra_D, $\overline{h}$ and q. The Rayleigh number decreases from 4.22×10^6 to 2.16×10^6, while the heat rate decreases from 33,300 to 14,000 W. The convection coefficient decreases approximately as $(T_s - T)^{1/3}$, while $q \sim (T_s - T)^{4/3}$. The latent energy released by the condensed steam corresponds to the increase in thermal energy of the pharmaceutical. Hence, $m_c h_{fg} = \rho \forall c\left(T_f - T_i\right)$, and

$$m_c = \frac{\rho \forall c\left(T_f - T_i\right)}{h_{fg}} = \frac{1100\,\text{kg/m}^3 \times 0.2\,\text{m}^3 \times 2000\,\text{J/kg}\cdot\text{K} \times 45°\text{C}}{2.183 \times 10^6\,\text{J/kg}} = 9.07\,\text{kg}$$ <

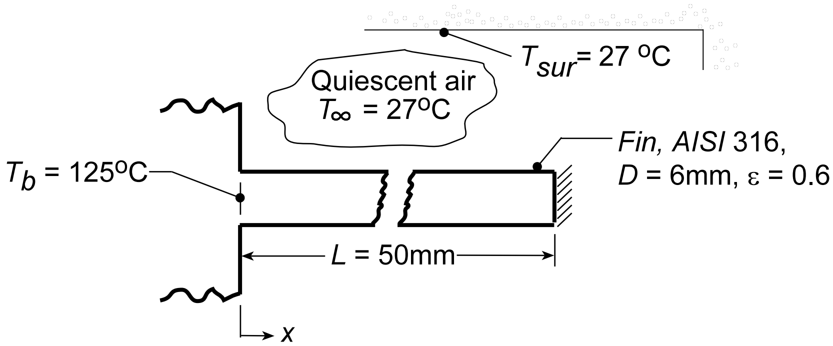
COMMENTS: (1) Over such a large temperature range, the fluid properties are likely to vary significantly, particularly ν and Pr. A more accurate solution could therefore be performed if the temperature dependence of the properties were known. (2) Condensation of the steam is a significant process expense, which is linked to the equipment (capital) and energy (operating) costs associated with steam production.

PROBLEM 9.75

KNOWN: Fin of uniform cross section subjected to prescribed conditions.

FIND: Tip temperature and fin effectiveness based upon (a) *average* values for free convection and radiation coefficients and (b) *local* values using a numerical method of solution.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Surroundings are isothermal and large compared to the fin, (3) One-dimensional conduction in fin, (4) Constant fin properties, (5) Tip of fin is insulated, (6) Fin surface is diffuse-gray.

PROPERTIES: *Table A-4,* Air (T_f = 325 K, 1 atm): $\nu = 18.41 \times 10^{-6}$ m²/s, k = 0.0282 W/m·K, $\alpha = 26.2 \times 10^{-6}$ m²/s, Pr = 0.704, $\beta = 1/T_f = 3.077 \times 10^{-3}$ K^{-1}; *Table A-1,* Steel AISI 316 $\left(\bar{T}_s = 350\,\text{K}\right)$: k = 14.3 W/m·K.

ANALYSIS: (a) *Average value* $\bar{h}_c$ *and* $\bar{h}_r$: From Table 3.4 for a fin of constant cross section with an insulated tip and constant heat transfer coefficient $\bar{h}$, the tip temperature (x = L) is given by Eq. 3.75,

$$\theta_L = \theta_b \frac{\cosh m(L-x)}{\cosh mL} = \theta_b / \cosh(mL) \qquad m = \left(\bar{h}P/kA_c\right)^{1/2} \tag{1,2}$$

where $\theta_L = T_L - T_\infty$ and $\theta_b = T_b - T_\infty$. For this situation, the average heat transfer coefficient is

$$\bar{h} = \bar{h}_c + \bar{h}_r \tag{3}$$

and is evaluated at the average temperature of the fin. The fin effectiveness ε_f follows from Eqs. 3.81 and 3.76

$$\varepsilon_f \equiv q_f / \bar{h}A_{c,b}\theta_b, \qquad q_f = M \cdot \tanh(mL), \qquad M = \left(\bar{h}PkA_c\right)^{1/2}\theta_b. \tag{4,5,6}$$

To estimate the coefficients, assume a value of $\bar{T}_s$; the lowest $\bar{T}_s$ occurs when the tip reaches T_∞. That is,

$$\bar{T}_s = \left(\bar{T}_\infty + T_b\right)/2 = (27+125)^\circ\,\text{C}/2 = 76^\circ \approx 350\,\text{K} \qquad T_f = \left(\bar{T}_s + T_\infty\right)/2 = 325\ \text{K}.$$

The free convection coefficient can be estimated from Eq. 9.33,

$$\overline{Nu}_D = \frac{\bar{h}_c D}{k} = CRa_D^n \tag{7}$$

$$Ra_D = \frac{g\beta\Delta T D^3}{\nu\alpha} = \frac{9.8\,\text{m/s}^2 \times 3.077\times10^{-3}\,\text{K}^{-1}(350-300)\,\text{K}\,(0.006\,\text{m})^3}{18.41\times10^{-6}\ \text{m}^2/\text{s}\times 26.2\times10^{-6}\ \text{m}^2/\text{s}} = 675$$

and from Table 9.1 with $10^2 < Ra_L < 10^4$, C = 0.850 and n = 0.188. Hence

Continued...

$$\bar{h}_C = \frac{0.0282\ \text{W/m}\cdot\text{K}}{0.006\text{m}} \times 0.850(675)^{0.188} = 13.6\ \text{W/m}^2\cdot\text{K}. \tag{8}$$

The radiation coefficient is estimated from Eq. 1.9,

$$\bar{h}_r = \varepsilon\sigma\left(\bar{T}_s + T_{sur}\right)\left(\bar{T}_s^2 + T_{sur}^2\right)$$

$$\bar{h}_r = 0.6\times 5.67\times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4\,(350+300)\left(350^2+300^2\right)\text{K}^3 = 4.7\ \text{W/m}^2\cdot\text{K} \tag{9}$$

Hence, the average coefficient, Eq. (3), is

$$\bar{h} = (13.6+4.7)\,\text{W/m}^2\cdot\text{K} = 18.3\,\text{W/m}^2\cdot\text{K}.$$

Evaluate the fin parameters, Eq. (2) and (6) with

$$P = \pi D = \pi\times 0.006\text{m} = 1.885\times 10^{-2}\,\text{m} \qquad A_C = \pi D^2/4 = \pi\,(0.006\text{m})^2/4 = 2.827\times 10^{-5}\,\text{m}^2$$

$$m = \left(18.3\,\text{W/m}^2\cdot\text{K}\times 1.885\times 10^{2}\ \text{m}/14.3\,\text{W/m}\cdot\text{K}\times 2.827\times 10^{-5}\right)^{1/2} = 29.21\text{m}^{-1}$$

$$M = \left(18.3\ \text{W/m}^2\cdot\text{K}\times 1.885\times 10^{-2}\,\text{m}\times 14\cdot 3\,\text{W/m}\cdot\text{K}\times 2.827\times 10^{-5}\,\text{m}^2\right)^{1/2}(125-27)\,\text{K} = 1.157\ \text{W}.$$

From Eq. (1), the tip temperature is

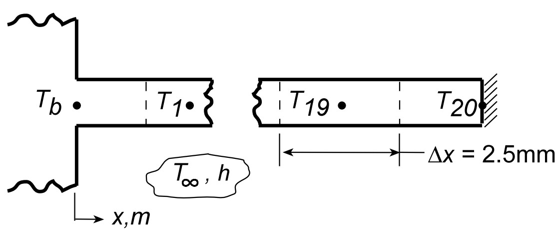
$$\theta_L = T_L - T_b = (125-27)\,\text{K}/\cosh\left(29.21\text{m}^{-1}\times 0.050\text{m}\right) = 43.2\text{K} \qquad T_L = 70.2^\circ\text{C} = 343\,\text{K}. \quad <$$

Note this value of T_L provides for $\bar{T}_s \approx 370$ K; so we underestimated $\bar{T}_s$. For best results, an iteration is warranted. The fin effectiveness, Eqs. (4) and (5), is

$$q_f = 1.157\,\text{W}\tanh\left(29.21\,\text{m}^{-1}\times 0.050\,\text{m}\right) = 1.039\,\text{W}$$

$$\varepsilon_f = 1.039\,\text{W}/18.3/\ \text{W/m}^2\cdot\text{K}\times 2.827\times 10^{-5}\,\text{m}^2\,(125-27)\,\text{K} = 20.5. \quad <$$

(b) *Local values* h_c *and* h_r: Consider the nodal arrangement for using a numerical method to find the tip temperature T_L, the heat rate q_f, and the fin effectiveness ε.



From an energy balance on a control volume about node m, the finite-difference equation is of the form

$$T_m = \left[T_{m+1} + T_{m-1} + \left(h_c + h_r\right)\left(4\Delta x^2/kD\right)T_\infty\right]\Big/\left[2 + \left(h_r + h_c\right)\left(4\Delta x^2/kD\right)\right]. \tag{10}$$

The local coefficient h_c follows from Eq. (3), with Eq. 9.33, yielding

$$h_C = \frac{k}{D}CRa_D^n$$

$$h_C = \frac{0.0282\,\text{W/m}\cdot\text{K}}{0.006\,\text{m}} \times 0.850\left(675\left[\Delta T/(350-300)\right]\right)^{0.188} = 6.517\left(T_m - 300\right)^{0.188}. \tag{11}$$

The local coefficient h_r follows from Eq. (9),

$$h_r = 0.6\times 5.67\times 10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left(T_m + 300\right)\left(T_m^2 + 300^2\right)$$

Continued...

$$h_r = 3.402\times10^{-8}(T_m + 300)\left(T_m^2 + 300^2\right). \qquad (12)$$

The 20-node system of finite-difference equations based upon Eq. (10) with the variable coefficients h_c and h_r prescribed Eqs. (11) and (12), respectively, can be solved simultaneously using IHT or another approach. The temperature distribution is

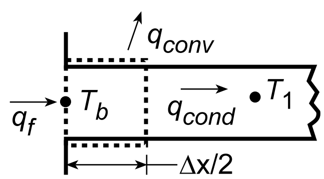
Node	T_m(K)	Node	T_m(K)	Node	T_m(K)	Node	T_m(K)
1	391.70	6	367.61	11	353.02	16	345.49
2	385.95	7	364.03	12	351.00	17	344.70
3	380.70	8	360.81	13	349.25	18	344.15
4	375.92	9	357.91	14	347.75	19	343.82
5	371.56	10	355.32	15	346.50	20	343.71

From these results the tip temperature is

$$T_L = T_{fd} = 343.7\,\text{K} = 70.7^\circ\text{C}. \qquad <$$

The fin heat rate follows from an energy balance for the control surface about node b.

$$q_f = q_{conv} + q_{cond}$$



$$q_f = h_b P \frac{\Delta x}{2}(T_b - T_\infty) + kA_c \frac{T_b - T_1}{\Delta x}$$

where h_b follows from Eqs. (11) and (12), with $T_b = 125^\circ\text{C} = 398$ K,

$$h_b = 6.517(398-300)^{0.188} + 3.402\times10^{-8}(398+300)\left(398^2+300^2\right) = 21.33\,\text{W/m}^2\cdot\text{K}$$

$$q_f = 21.33\,\text{W/m}^2\cdot\text{K}\times1.855\times10^{-2}\,\text{m}(0.0025\,\text{m}/2)(398-300)\text{K}$$

$$+14.3\,\text{W/m}\cdot\text{K}\times2.827\times10^{-5}\,\text{m}^2\,\frac{(398-391.70)\text{K}}{0.0025\text{m}} = (0.049+1.018)\text{W} = 1.067\,\text{W}.$$

The effectiveness follows from Eq. (4)

$$\varepsilon_f = 1.067\,\text{W}/21.33\,\text{W/m}^2\cdot\text{K}\times2.827\times10^{-5}\text{m}^2(125-27)\text{K} = 18.1$$

COMMENTS: (1) The results by the two methods of solution compare as follows:

Coefficients	T(L),K	q_f(W)	ε_f
average	343.1	1.039	20.5
local	343.7	1.067	18.1

The temperature predictions are in excellent agreement and the heat rates very close, within 4%.

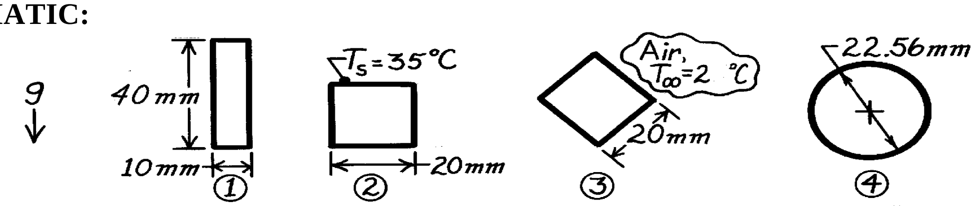
(2) To obtain the finite-different equation for node n = 20, use Eq. (10) but consider the adiabatic surface as a symmetry plane.

PROBLEM 9.76

KNOWN: Horizontal tubes of different shapes each of the same cross-sectional area transporting a hot fluid in quiescent air. Lienhard correlation for immersed bodies.

FIND: Tube shape which has the minimum heat loss to the ambient air by free convection.

SCHEMATIC:



ASSUMPTIONS: (1) Ambient air is quiescent, (2) Negligible heat loss by radiation, (3) All shapes have the same cross-sectional area and uniform surface temperature.

PROPERTIES: *Table A-4*, Air ($T_f \approx 300K$, 1 atm): $\nu = 15.89 \times 10^{-6}\ m^2/s$, $\alpha = 22.5 \times 10^{-6}\ m^2/s$, k = 0.0263 W/m·K, Pr = 0.707, $\beta = 1/T_f$.

ANALYSIS: The Lienhard correlation approximates the laminar convection coefficient for an immersed body on which the boundary layer does not separate from the surface by $\overline{Nu}_\ell = \left(\bar{h}\ell\right)/k = 0.52 Ra_\ell^{1/4}$, where the characteristic length, ℓ, is the length of travel of the fluid in the boundary layer across the shape surface. The heat loss per unit length from any shape is $q' = \bar{h}P\left(T_s - T_\infty\right)$. For the shapes,

$$Ra_\ell = \frac{g b \Delta T \ell^3}{n a} = \frac{9.8\,m/s^2\,(1/300K)(35-25)K\,\ell^3 m^3}{15.89\times10^{-6} m^2/s \times 22.5\times10^{-6} m^2/s} = 9.137\times10^8\,\ell^3$$

$$\bar{h}_\ell = \left(0.0263\,W/m\cdot K/\ell\right)0.52\left(9.137\times10^8\ell^3\right)^{1/4} = 2.378\ell^{-1/4}.$$

For the shapes, ℓ is half the total wetted perimeter P. Evaluating $\bar{h}_\ell$ and q', find

Shape	P (mm)	ℓ(mm)	$\bar{h}_\ell\ (W/m^2\cdot K)$	$q'(W/m)$
1	2 × 40 + 2 × 10 = 100	50	5.03	5.03
2	4 × 20 = 80	40	5.32	4.26
3	4 × 20 = 80	40	5.32	4.26
4	π × 22.56 = 70.9	35.4	5.48	3.89

Hence, it follows that shape 4 has the minimum heat loss. <

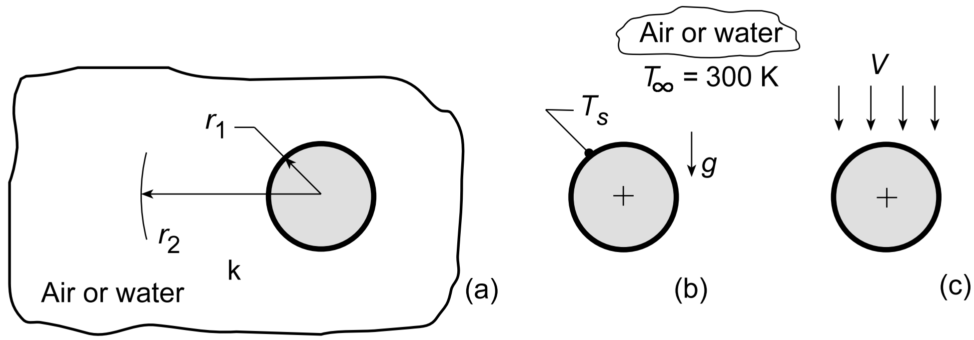
COMMENTS: Using the Lienhard correlation for a sphere of D = 22.56 mm, find $\ell = 35.4mm$, the same as for a cylinder, namely, $h_4 = 5.48\ W/m^2\cdot K$. Using the Churchill correlation, Eq. 9.35, find $\bar{h} = 7.69 W/m^2\cdot K$. Hence, the approximation for the sphere is 29% low. For a cylinder, using Eq. 9.34, find $\bar{h} = 5.15 W/m^2\cdot K$. The approximation for the cylinder is 6% high.

PROBLEM 9.77

KNOWN: Sphere of 2-mm diameter immersed in a fluid at 300 K.

FIND: (a) The conduction limit of heat transfer from the sphere to the quiescent, extensive fluid, $Nu_{D,cond} = 2$; (b) Considering free convection, surface temperature at which the Nusselt number is twice that of the conduction limit for the fluids air and water; and (c) Considering forced convection, fluid velocity at which the Nusselt number is twice that of the conduction limit for the fluids air and water.

SCHEMATIC:



ASSUMPTIONS: (1) Sphere is isothermal, (2) For part (a), fluid is stationary, and (3) For part (b), fluid is quiescent, extensive.

ANALYSIS: (a) Following the hint provided in the problem statement, the thermal resistance of a hollow sphere, Eq. 3.36 of inner and outer radii, r_1 and r_2 , respectively, and thermal conductivity k, is

$$R_{t,cond} = \frac{1}{4\pi k}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \tag{1}$$

and as $r_2 \to \infty$, that is the medium is extensive

$$R_{t,cond} = \frac{1}{4\pi k r_1} = \frac{1}{2\pi k D} \tag{2}$$

The Nusselt number can be expressed as

$$Nu = \frac{hD}{k} \tag{3}$$

and the conduction resistance in terms of a convection coefficient is

$$R_{t,cond} = \frac{1}{hA_s} = \frac{1}{h\pi D^2} \tag{4}$$

Combining Eqs. (3) and (4)

$$Nu_{D,cond} = \frac{\left(1/R_{t,cond}\pi D^2\right)D}{k} = \frac{\left[1/(1/2\pi kD)\left(\pi D^2\right)\right]D}{k} = 2 \qquad <$$

(b) For free convection, the recommended correlation, Eq. 9.35, is

$$\overline{Nu}_D = 2 + \frac{0.589 Ra_D^{1/4}}{\left[1 + (0.469/Pr)^{9/16}\right]^{4/9}}$$

Continued...

$$\mathrm{Ra_D} = \frac{g\beta\Delta T D^3}{\nu\alpha} \qquad \Delta T = T_s - T_\infty$$

where properties are evaluated at $T_f = (T_s + T_\infty) / 2$. What value of T_s is required for $\overline{\mathrm{Nu}}_D = 4$ for the fluids air and water? Using the *IHT Correlations Tool, Free Convection, Sphere* and the *Properties Tool* for *Air* and *Water*, find

Air: $\overline{\mathrm{Nu}} \le 3.1$ for all T_s <

Water: $T_s = 301.1\mathrm{K}$ <

(c) For forced convection, the recommended correlation, Eq. 7.59, is

$$\overline{\mathrm{Nu}}_D = 2 + \left(0.4\,\mathrm{Re}_D^{1/2} + 0.06\,\mathrm{Re}_D^{2/3}\right)\mathrm{Pr}^{0.4}\left(\mu/\mu_s\right)^{1/4}$$

$$\mathrm{Re_D} = VD/\nu$$

where properties are evaluated at T_∞, except for μ_s evaluated at T_s .What value of V is required for $\overline{\mathrm{Nu}}_D = 4$ if the fluids are air and water? Using the *IHT Correlations Tool, Forced Convection, Sphere* and the *Properties Tool* for *Air* and *Water,* find

Air: V = 0.17 m/s *Water*: V = 0.00185 m/s <

COMMENTS: (1) For water, $\overline{\mathrm{Nu}}_D = 2\times\overline{\mathrm{Nu}}_{D,cond}$ can be achieved by $\Delta T \approx 1$ for free convection and with very low velocity, V< 0.002 m/s, for forced convection.

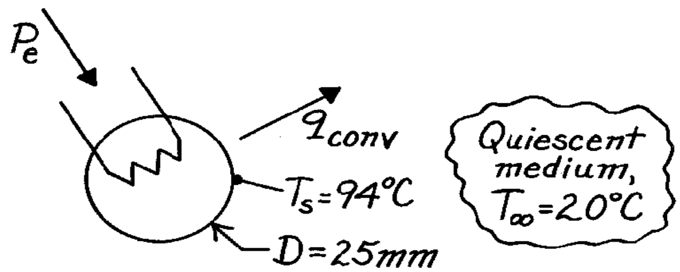
(2) For air, $\overline{\mathrm{Nu}}_D = 2\times\overline{\mathrm{Nu}}_{D,cond}$ can be achieved in forced convection with low velocities, V< 0.2 m/s. In free convection, $\overline{\mathrm{Nu}}_D$ increases with increasing T_s and reaches a maximum, $\overline{\mathrm{Nu}}_{D,max} = 3.1$, around 450 K. Why is this so? Hint: Plot $\mathrm{Ra_D}$ as a function of T_s and examine the role of β and ΔT as a function of T_s .

PROBLEM 9.78

KNOWN: Sphere with embedded electrical heater is maintained at a uniform surface temperature when suspended in various media.

FIND: Required electrical power for these media: (a) atmospheric air, (b) water, (c) ethylene glycol.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible surface radiation effects, (2) Extensive and quiescent media.

PROPERTIES: Evaluated at $T_f = (T_s + T_\infty)/2 = 330K$:

	$\nu \cdot 10^6$, m^2/s	$k \cdot 10^3$, W/m·K	$\alpha \cdot 10^6$, m^2/s	Pr	$\beta \cdot 10^3$, K^{-1}
Table A-4, Air (1 atm)	18.91	28.5	26.9	0.711	3.03
Table A-6, Water	0.497	650	0.158	3.15	0.504
Table A-5, Ethylene glycol	5.15	260	0.0936	55.0	0.65

ANALYSIS: The electrical power (P_e) required to offset convection heat transfer is

$$q_{conv} = \bar{h} A_s (T_s - T_\infty) = p \bar{h} D^2 (T_s - T_\infty). \qquad (1)$$

The free convection heat transfer coefficient for the sphere can be estimated from Eq. 9.35 using Eq. 9.25 to evaluate Ra_D.

$$\overline{Nu}_D = 2 + \frac{0.589 Ra_D^{1/4}}{\left[1+(0.469/Pr)^{9/16}\right]^{4/9}} \begin{cases} Pr \ge 0.7 \\ Ra_D \le 10^{11} \end{cases} \qquad Ra_D = \frac{g\, b\, \Delta T\, D^3}{na}. \qquad (2,3)$$

(a) For air

$$Ra_D = \frac{9.8 m/s^2 \left(3.03\times10^{-3} K^{-1}\right)(94-20) K (0.025 m)^3}{18.91\times10^{-6} m^2/s \times 26.9\times10^{-6} m^2/s} = 6.750\times10^4$$

$$\bar{h}_D = \frac{k}{D}\overline{Nu}_D = \frac{0.02285 W/m\cdot K}{0.025 m}\left\{2+\frac{0.589\left(6.750\times10^4\right)^{1/4}}{\left[1+(0.469/0.711)^{9/16}\right]^{4/9}}\right\} = 10.6 W/m^2\cdot K$$

$$q_{conv} = p \times 10.6 W/m^2\cdot K (0.025 m)^2 (94-20) K = 1.55 W.$$

Continued …..

(b,c) Summary of the calculations above and for water and ethylene glycol:

Fluid	Ra_D	$\bar{h}_D\left(W/m^2 \cdot K\right)$	q(W)	
Air	6.750×10^4	10.6	1.55	<
Water	7.273×10^7	1299	187	<
Ethylene glycol	15.82×10^6	393	57.0	<

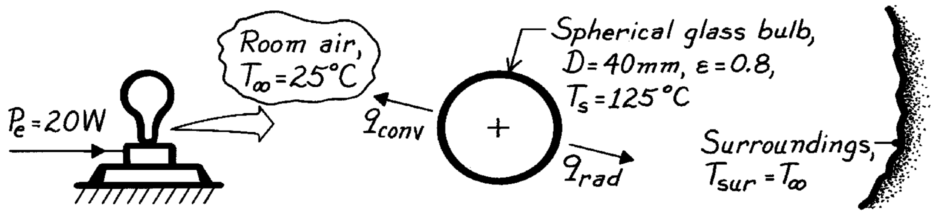
COMMENTS: Note large differences in the coefficients and heat rates for the fluids.

PROBLEM 9.79

KNOWN: Surface temperature and emissivity of a 20W light bulb (spherical) operating in room air

FIND: Heat loss from bulb surface.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Quiescent room air, (3) Surroundings much larger than bulb.

PROPERTIES: *Table A-4*, Air ($T_f = (T_s + T_\infty)/2 = 348K$, 1 atm): $\nu = 20.72 \times 10^{-6}\ m^2/s$, k = 0.0298 W/m·K, $\alpha = 29.6 \times 10^{-6}\ m^2/s$, Pr = 0.700, $\beta = 1/T_f$.

ANALYSIS: Heat loss from the surface of the bulb is by free convection and radiation. The rate equations are

$$q = q_{conv} + q_{rad} = \bar{h} A_s (T_s - T_\infty) + e\, A_s\, s \left(T_s^4 - T_{sur}^4\right)$$

where $A_s = \pi D^2$. To estimate $\bar{h}$ for free convection, first evaluate the Rayleigh number.

$$Ra_D = \frac{g\,b\,\Delta T\,D^3}{na} = \frac{9.8\,m/s^2\,(1/348K)(125-25)\,K\,(0.040m)^3}{20.72\times10^{-6}m^2/s\times29.6\times10^{-6}m^2/s} = 2.93\times10^5.$$

Since $Pr \geq 0.7$ and $Ra_D < 10^{11}$, the Churchill relation, Eq. 9.35, is appropriate.

$$\overline{Nu}_D = 2 + \frac{0.589 Ra_D^{1/4}}{\left[1+(0.469/Pr)^{9/16}\right]^{4/9}} = 2 + \frac{0.589\left(2.93\times10^5\right)^{1/4}}{\left[1+(0.469/0.700)^{9/16}\right]^{4/9}} = 12.55$$

$$\bar{h} = \overline{Nu}_D\, k/D = 12.55(0.0298\,W/m\cdot K)/0.040m = 9.36\,W/m^2\cdot K.$$

Substituting numerical values, the heat loss from the bulb is,

$$q = p\,(0.040m)^2\left[9.36\frac{W}{m^2\cdot K}(125-25)K + 0.80\times5.67\times10^{-8}\frac{W}{m^2\cdot K^4}\left[(125+273)^4 - (25+273)^4\right]K^4\right]$$

$$q = (4.70 + 3.92)W = 8.62W. \qquad <$$

COMMENTS: (1) The contributions of convection and radiation to the surface heat loss are comparable.

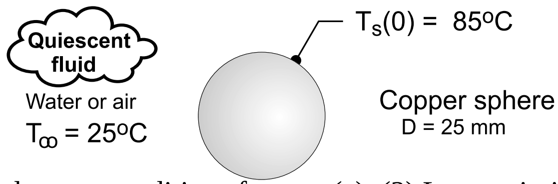
(2) The remaining heat loss (20 – 8.62 = 11.4 W) is due to the transmission of radiant energy (light) through the bulb and heat conduction through the base.

PROBLEM 9.80

KNOWN: A copper sphere with a diameter of 25 mm is removed from an oven at a uniform temperature of 85°C and allowed to cool in a quiescent fluid maintained at 25°C.

FIND: (a) Convection coefficients for the initial condition for the cases when the fluid is air and water, and (b) Time for the sphere to reach 30°C when the cooling fluid is air and water using two different approaches, average coefficient and numerically integrated energy balance.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions for part (a); (2) Low emissivity coating makes radiation exchange negligible for the in-air condition; (3) Fluids are quiescent, and (4) Constant properties.

PROPERTIES: *Table A-4*, Air (T_f = (25 + 85)°C/2 = 328 K, 1 atm): $\nu = 1.871 \times 10^{-5}$ m²/s, k = 0.0284 W/m·K, $\alpha = 2.66 \times 10^{-5}$ m²/s, Pr = 0.703, $\beta = 1/T_f$; *Table A-6*, Water (T_f = 328 K): $\nu = 5.121 \times 10^{-7}$ m²/s, k = 0.648 W/m·K, $\alpha = 1.57 \times 10^{-7}$ m²/s, Pr = 3.26, $\beta = 4.909 \times 10^{-4}$ K^{-1}; *Table A-1*, Copper, pure ($\overline{T}$ = (25 + 85)°C/2 = 328 K): ρ = 8933 kg/m³, c = 382 J/kg·K, k = 399 W/m·K.

ANALYSIS: (a) For the initial condition, the average convection coefficient can be estimated from the Churchill-Chu correlation, Eq. 9.35,

$$\overline{Nu}_D = \frac{\overline{h}_D D}{k} = 2 + \frac{0.589\ Ra_D^{1/4}}{\left[1+\left(0.469/Pr\right)^{9/16}\right]^{4/9}} \tag{1}$$

$$Ra_D = \frac{g\beta\left(T_s - T_\infty\right)D^3}{\nu\alpha} \tag{2}$$

with properties evaluated at $T_f = (T_s + T_\infty)/2$ = 328 K. Substituting numerical values find these results:

Fluid	T_s(°C)	T_f(K)	Ra_D	Nu_D	$\overline{h}_D$ (W/m²·K)	
Air	85	328	$5.62{\times}10^4$	8.99	10.2	<
Water	85	328	$5.61{\times}10^7$	46.8	1213	<

(b) To establish the validity of the lumped capacitance (LC) method, calculate the Biot number for the worst condition (air).

$$Bi = \frac{\overline{h}_D\left(D/3\right)}{k} = 10.2\ W/m^2 \cdot K\left(0.025\ m/3\right)/399\ W/m \cdot K = 2.1{\times}10^{-4}$$

Since Bi << 0.1, the sphere can be represented by this energy balance for the cooling process

$$\dot{E}_{in} - \dot{E}_{out} = \dot{E}_{st} \qquad\qquad -q_{cv} = Mc\frac{dT_s}{dt}$$

$$-\overline{h}_D\ A_s\left(T_s - T_\infty\right) = \rho Vc\frac{dT_s}{dt} \tag{3}$$

where $A_s = \pi D^2$ and $V = \pi D^3/6$. Two approaches are considered for evaluating appropriate values for $\overline{h}_D$.

Average coefficient. Evaluate the convection coefficient corresponding to the average temperature of the sphere, $\overline{T}_s$ = (30 + 85)°C/2 = 57.5°C, for which the film temperature is $T_f = (\overline{T}_s + T_\infty)/2$. Using the foregoing analyses of part (a), find these results.

Continued …..

Fluid	$\overline{T}_s$ (°C)	T_f(K)	Ra_D	Nu_D	$\overline{h}_D$ (W/m^2·K)
Air	57.5	314	3.72×10^4	8.31	9.09
Water	57.5	314	$1.99\text{x}10^7$	37.1	940

Numerical integration of the energy balance equation. The more accurate approach is to numerically integrate the energy balance equation, Eq. (3), with $\overline{h}_D$ evaluated as a function of T_s using Eqs. (1) and (2). The properties in the correlation parameters would likewise be evaluated at T_f, which varies with T_s. The integration is performed in the *IHT* workspace; see Comment 3.

Results of the lumped-capacitance analysis. The results of the LC analyses using the two approaches are tabulated below, where t_o is the time to cool from 85°C to 30°C:

	t_o (s)	
Approach	Air	Water
Average coefficient	3940	39
Numerical coefficient	4600	49

COMMENTS: (1) For these condition, the convection coefficient for the water is nearly two orders of magnitude higher than for air.

(2) Using the average-coefficient approach, the time-to-cool, t_o, values for both fluids is 15-20% faster than the more accurate numerical integration approach. Evaluating the average coefficient at $\overline{T}_s$ results in systematically over estimating the coefficient.

(3) The *IHT* code used for numerical integration of the energy balance equation and the correlations is shown below for the fluid water.

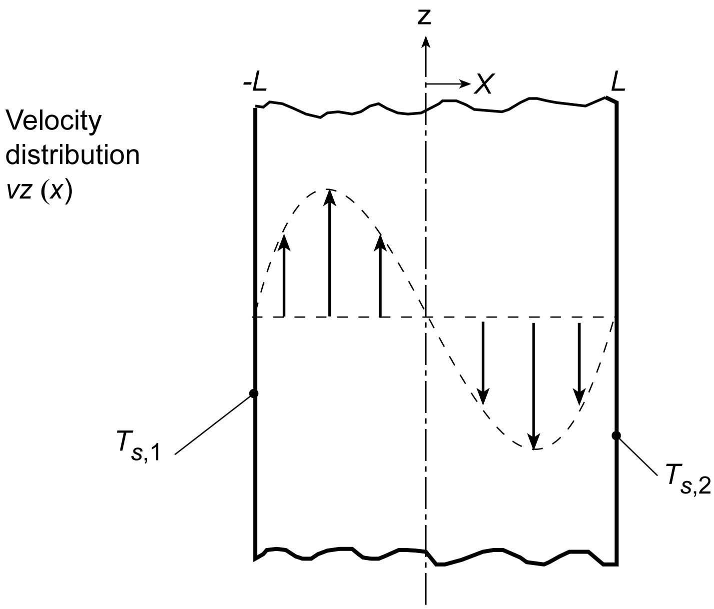
```
//  LCM energy balance
- hDbar * As * (Ts - Tinf) = M * cps * der(Ts,t)
As = pi * D^2
M = rhos * Vs
Vs = pi * D^3 / 6

// Input variables
D = 0.025
// Ts = 85 + 273                // Initial condition, Ts
Tinf_C = 25
rhos = 8933                     // Table A.1, copper, pure
cps = 382
ks =  399

/*  Correlation description: Free convection (FC), sphere (S),  RaD<=10^11, Pr >=0.7, Churchill
correlation, Eqs 9.25 and 9.35 . See Table 9.2 .  */
NuDbar = NuD_bar_FC_S(RaD,Pr)                // Eq 9.35
NuDbar = hDbar * D / k
RaD = g * beta * deltaT * D^3 / (nu * alpha)        //Eq 9.25
deltaT = abs(Ts - Tinf)
g = 9.8     // gravitational constant, m/s^2
// Evaluate properties at the film temperature, Tf.
Tf = Tfluid_avg(Tinf,Ts)

// Water property functions :T dependence, From Table A.6
// Units: T(K), p(bars);
x = 0                           // Quality (0=sat liquid or 1=sat vapor)
nu = nu_Tx("Water",Tf,x)        // Kinematic viscosity, m^2/s
k = k_Tx("Water",Tf,x)          // Thermal conductivity, W/m·K
Pr = Pr_Tx("Water",Tf,x)        // Prandtl number
beta = beta_T("Water",Tf)       // Volumetric coefficient of expansion, K^(-1) (f, liquid, x = 0)
alpha = k / (rho * cp)          // Thermal diffusivity, m^2/s

// Conversions
Ts_C = Ts - 273
Tinf_C = Tinf - 273
```

PROBLEM 9.81

KNOWN: Temperatures and spacing of vertical, isothermal plates.

FIND: (a) Shape of velocity distribution, (b) Forms of mass, momentum and energy equations for laminar flow, (c) Expression for the temperature distribution, (d) Vertical pressure gradient, (e) Expression for the velocity distribution.

SCHEMATIC:



ASSUMPTIONS: (1) Laminar, incompressible, fully-developed flow, (2) Constant properties, (3) Negligible viscous dissipation, (4) Boussinesq approximation.

ANALYSIS: (a) For the prescribed conditions, there must be buoyancy driven ascending and descending flows along the surfaces corresponding to $T_{s,1}$ and $T_{s,2}$, respectively (see schematic). However, conservation of mass dictates equivalent rates of *upflow* and *downflow* and, assuming constant properties, inverse symmetry of the velocity distribution about the midplane.

(b) For fully-developed flow, which is achieved for *long* plates, $v_x = 0$ and the continuity equation yields

$$\partial v_z / \partial z = 0 \qquad <$$

Hence, there is no net transfer of momentum or energy by advection, and the corresponding equations are, respectively,

$$0 = -(dp/dz) + \mu\left(d^2 v_z / dx^2\right) - \rho\,(g/g_c) \qquad <$$

$$0 = (dT^2/dx^2) \qquad <$$

(c) Integrating the energy equation twice, we obtain

$$T = C_1 x + C_2$$

and applying the boundary conditions, $T(-L) = T_{s,1}$ and $T(L) = T_{s,2}$, it follows that $C_1 = -(T_{s,1} - T_{s,2})/2L$ and $C_2 = (T_{s,1} + T_{s,2})/2 \equiv T_m$, in which case,

$$\frac{T - T_m}{T_{s,1} - T_{s,2}} = -\frac{x}{2L} \qquad <$$

Continued...

PROBLEM 9.81 (Cont.)

(d) From hydrostatic considerations and the assumption of a constant density ρ_m, the balance between the gravitational and net pressure forces may be expressed as $dp/dz = -\rho_m(g/g_c)$. The momentum equation is then of the form

$$0 = \mu\left(d^2v_z/dx^2\right) - (\rho - \rho_m)(g/g_c)$$

or, invoking the Boussinesq approximation, $\rho - \rho_m \approx -\beta\rho_m(T - T_m)$,

$$d^2v_z/dx^2 = -(\beta\rho_m/\mu)(g/g_c)(T - T_m)$$

or, from the known temperature distribution,

$$d^2v_z/dx^2 = (\beta\rho_m/2\mu)(g/g_c)\left(T_{s,1} - T_{s,2}\right)(x/L) \qquad <$$

(e) Integrating the foregiong expression, we obtain

$$dv_z/dx = (\beta\rho_m/4\mu)(g/g_c)\left(T_{s,1} - T_{s,2}\right)\left(x^2/L\right) + C_1$$

$$v_z = (\beta\rho_m/12\mu)(g/g_c)\left(T_{s,1} - T_{s,2}\right)\left(x^3/L\right) + C_1x + C_2$$

Applying the boundary conditions $v_z(-L) = v_z(L) = 0$, it follows that $C_1 = -(\beta\rho_m/12\mu)(g/g_c)(T_{s,1} - T_{s,2})L$ and $C_2 = 0$. Hence,

$$v_z = \left(\beta\rho_m L^2/12\mu\right)(g/g_c)\left(T_{s,1} - T_{s,2}\right)\left[\left(x^3/L^3\right) - (x/L)\right] \qquad <$$

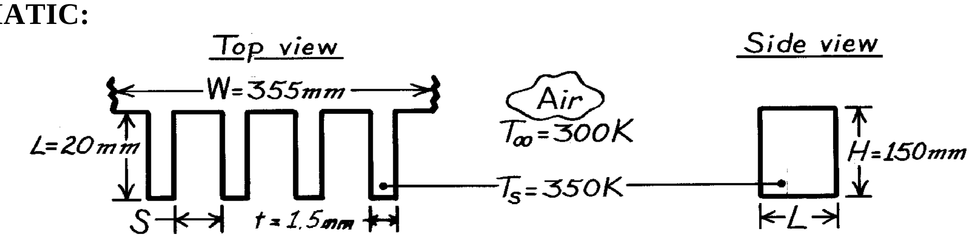
COMMENTS: The validity of assuming fully-developed conditions improves with increasing plate length and would be satisfied precisely for infinite plates.

PROBLEM 9.82

KNOWN: Dimensions of vertical rectangular fins. Temperature of fins and quiescent air.

FIND: Optimum fin spacing and corresponding fin heat transfer rate.

SCHEMATIC:



ASSUMPTIONS: (1) Isothermal fins, (2) Negligible radiation, (3) Quiescent air, (4) Negligible heat transfer from fin tips, (5) Negligible radiation.

PROPERTIES: *Table A-4*, Air (T_f = 325K, 1 atm): $\nu = 18.41 \times 10^{-6}$ m^2/s, k = 0.0282 W/m·K, $\alpha = 26.1 \times 10^{-6}$ m^2/s, Pr = 0.703.

ANALYSIS: From Table 9.3

$$S_{opt} = 2.71\left(Ra_S / S^3 H\right)^{-1/4} = 2.71\left[\frac{gb\,(T_s - T_\infty)}{an\,H}\right]^{-1/4}$$

$$S_{opt} = 2.71\left[\frac{9.8\,m/s^2\,(325K)^{-1}(50K)}{26.1\times10^{-6} m^2/s \times 18.4\times10^{-6} m^2/s \times 0.15m}\right]^{-1/4} = 7.12mm \qquad <$$

From Eq. 9.45 and Table 9.3

$$\overline{Nu}_s = \left[\frac{576}{(Ra_S S/L)^2} + \frac{2.87}{(Ra_S S/L)^{1/2}}\right]^{-1/2}$$

$$Ra_S(S/L) = \frac{gb\,(T_s - T_\infty) S^4}{an\,H} = \frac{9.8\,m/s^2\,(325K)^{-1}(50K)\left(7.12\times10^{-3} m\right)^4}{25.4\times10^{-6} m^2/s \times 18.4\times10^{-6} m^2/s \times 0.15m}$$

$$Ra_S(S/L) = 53.2$$

$$\overline{Nu}_S = \left[\frac{576}{(53.2)^2} + \frac{2.87}{(53.2)^{1/2}}\right]^{-1/2} = [0.204 + 0.393]^{-1/2} = 1.29$$

$$\bar{h} = \overline{Nu}_S\, k/S = 1.29\left(0.0282 W/m\cdot K/0.00712m\right) = 5.13 W/m^2\cdot K.$$

With N = W/(t + S) = (355 mm)/(8.62×10^{-3} m) = 41.2 ≈ 41,

$$q = 2N\bar{h}(L\times H)(T_s - T_\infty) = 82\left(5.13 W/m^2\cdot K\right)(0.02m\times0.15m)50K$$

$$q = 63.1W. \qquad <$$

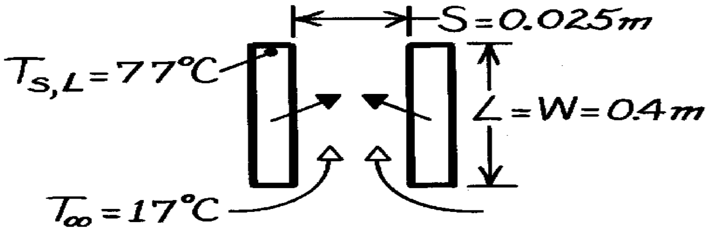
COMMENTS: S_{opt} = 7.12 mm is considerably less than the value of 34 mm predicted from previous considerations. Hence, the corresponding value of q = 63.1 W is considerably larger than that of the previous predication.

PROBLEM 9.83

KNOWN: Length, width and spacing of vertical circuit boards. Maximum allowable board temperature.

FIND: Maximum allowable power dissipation per board.

SCHEMATIC:



ASSUMPTIONS: (1) Circuit boards are flat with uniform heat flux at each surface, (2) Negligible radiation.

PROPERTIES: *Table A-4*, Air $\left(\bar{T} = 320\text{K}, 1\text{atm}\right)$: $\nu = 17.9 \times 10^{-6}$ m^2/s, k = 0.0278 W/m·K, α = 25.5×10^{-6} m^2/s.

ANALYSIS: From Eqs. 9.41 and 9.46 and Table 9.3,

$$\frac{q_s''}{T_{s,L} - T_\infty}\frac{S}{k} = \left[\frac{48}{Ra_S^* S/L} + \frac{2.51}{\left(Ra_S^* S/L\right)^{2/5}}\right]^{-1/2}$$

where $$Ra_S^* \frac{S}{L} = \frac{g\beta q_s'' S^5}{k\,\alpha\nu L} = \frac{9.8\,\text{m/s}^2 \,(320\text{K})^{-1} (0.025\text{m})^5\, q_s''}{0.0278\,\text{W/m}\cdot\text{K}\left(25.5\times10^{-6}\text{m}^2/\text{s}\right)\left(17.9\times10^{-6}\text{m}^2/\text{s}\right)0.4\text{m}}$$

$$Ra_S^* \frac{S}{L} = 58.9 q_s''$$

and $$\frac{q_s''}{T_{s,L} - T_\infty}\frac{S}{k} = \frac{0.025\text{m}\cdot q_s''}{(60\ \text{K})0.0278\,\text{W/m}\cdot\text{K}} = 0.015 q_s''.$$

Hence, $$0.015 q_s'' = \left[\frac{0.815}{q_s''} + \frac{0.492}{(q_s'')^{0.4}}\right]^{-1/2}.$$

A trial-and-error solution yields

$$q_s'' = 287\ \text{W/m}^2.$$

Hence, $$q = 2A_s q_s'' = 2(0.4\text{m})^2\left(287\,\text{W/m}^2\right) = 91.8\ \text{W}.$$ <

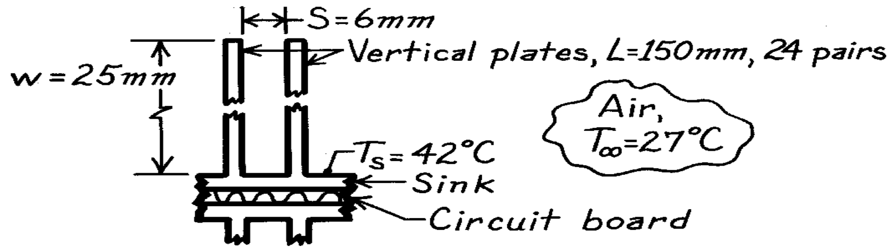
COMMENTS: Larger heat rates may be achieved by using a fan to superimpose a forced flow on the buoyancy driven flow.

PROBLEM 9.84

KNOWN: Array of isothermal vertical fins attached to heat sink at 42°C with ambient air temperature at 27°C.

FIND: (a) Heat removal rate for 24 pairs of fins and (b) Optimum fin spacing for maximizing heat removal rate if overall size of sink is to remain unchanged.

SCHEMATIC:



ASSUMPTIONS: (1) Fins form vertical, symmetrically heated, isothermal plates, (2) Negligible radiation effects, (3) Ambient air is quiescent.

PROPERTIES: *Table A-4*, Air $\left(T_f = (T_s + T_\infty)/2 = (42+27)/2°C = 308K, 1atm\right)$:

$n = 16.69\times10^{-6} m^2/s$, $a = 23.5\times10^{-6} m^2/s$, $k = 26.9\times10^{-3}$ W/m·K.

ANALYSIS: Considering the fins as vertical isothermal plates, the heat rate can be determined from Eq. 9.37 with the Elenbaas correlation, Eq. 9.36,

$$Ra_S = \frac{gb\,(T_s - T_\infty)S^3}{na} = \frac{9.8 m/s^2 (1/308\ K)(42-27)K(0.006\ m)^3}{16.69\times10^{-6} m^2/s \times 23.5\times10^{-6} m^2/s} = 262.9$$

$$\overline{Nu}_S = \left(\frac{q/A_s}{T_s - T_\infty}\right)\frac{S}{k} = \frac{1}{24}Ra_S(S/L)\left\{1 - \exp\left[-\frac{35}{Ra_S(S/L)}\right]\right\}^{3/4}$$

$$\overline{Nu}_S = \frac{1}{24}(262.9)\left(\frac{0.006m}{0.150m}\right)\left\{1 - \exp\left[-\frac{35}{262.9(0.006m/0.150m)}\right]\right\}^{3/4} = 0.4263$$

find the heat rate as

$$q_s = A_s\,(T_s - T_\infty)\frac{k}{S}\overline{Nu}_S$$

$$q_s = 2\times2\times24(0.025\times0.150)m^2(42-27)K\frac{0.0269 W/m\cdot K}{0.006\ m}0.4263$$

$$q_s = 10.4\ W.$$
<

(b) For symmetric isothermal plates, from Table 9.3, the optimum spacing to maximize the heat rate with L = 0.150 m is

$$S_{opt} = 2.71\left(Ra_S/S^3L\right)^{-1/4} = 2.71\left[\frac{gb\,(T_s - T_\infty)}{na}\frac{1}{L}\right]^{-1/4} = 9.03\ mm$$

Continued …..

and using Eq. 9.45 with values of C_1 and C_2 from Table 9.3,

$$\mathrm{Ra}_S = \frac{g\beta\left(T_s - T_\infty\right)S_{opt}^3}{\nu\alpha} = 896$$

$$\overline{\mathrm{Nu}}_S = \left[\frac{q/A_s}{T_s - T_\infty}\right]\frac{S}{k} = \left[\frac{C_1}{\left(\mathrm{Ra}_S S/L\right)^2} + \frac{C_2}{\left(\mathrm{Ra}_S S/L\right)^{1/2}}\right]^{-1/2}$$

and solving for the heat rate,

$$q = 0.240\mathrm{m}^2\left(42-27\right)\mathrm{K}\frac{0.0269\ \mathrm{W/m\cdot K}}{0.00903\ \mathrm{m}}\left[\frac{576}{\left(896\times 9/150\right)^2} + \frac{2.87}{\left(896\times 9/150\right)^{1/2}}\right]^{-1/2}$$

$$q = 14.0\ \mathrm{W}$$ <

where with spacing S_{opt} = 9.03 mm, rather than 6 mm, the available space for the fins has decreased by (9.03 – 6)/6 = 50%; hence only 16 pairs are possible and $A_s = 4 \times 16(0.025 \times 0.150)\mathrm{m}^2 = 0.240\ \mathrm{m}^2$.

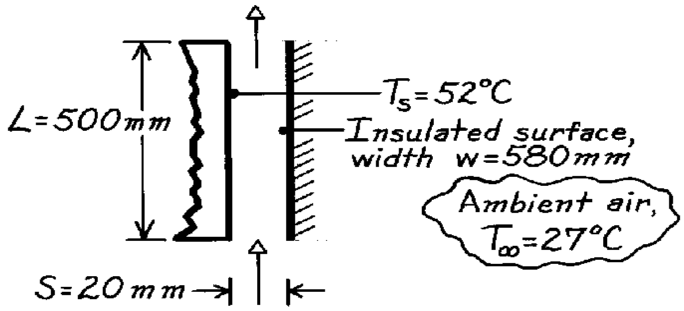
COMMENTS: Note that with S_{opt} = 9 mm, the convection coefficient is increased by (3.88 – 1.91)1.91 = 103%. However, the increased spacing reduces the number of surfaces possible within the given space constraint.

PROBLEM 9.85

KNOWN: Vertical air vent in front door of dishwasher with prescribed width and height. Spacing between isothermal and insulated surface of 20 mm.

FIND: (a) Heat loss from the tub surface and (b) Effect on heat rate of changing spacing by ± 10 mm.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Vent forms vertical parallel isothermal/adiabatic plates, (3) Ambient air is quiescent.

PROPERTIES: *Table A-4*, ($T_f = (T_s + T_\infty)/2 = 312.5K$, 1 atm): $\nu = 17.15 \times 10^{-6}$ m^2/s, $\alpha = 24.4 \times 10^{-6}$ m^2/s, k = 27.2×10^{-3} W/m·K, $\beta = 1/T_f$.

ANALYSIS: The vent arrangement forms two vertical plates, one is isothermal, T_s, and the other is adiabatic $(q'' = 0)$. The heat loss can be estimated from Eq. 9.37 with the correlation of Eq. 9.45 using C_1 = 144 and C_2 = 2.87 from Table 9.3:

$$Ra_S = \frac{gb\,(T_s - T_\infty)S^3}{na} = \frac{9.8\,\text{m/s}^2\,(1/312.5\text{ K})(52-27)\text{K}\,(0.020\text{m})^3}{17.15\times10^{-6}\text{m}^2/\text{s}\times24.4\times10^{-6}\text{m}^2/\text{s}} = 14{,}988$$

$$q = A_s\,(T_s - T_\infty)\frac{k}{S}\left[\frac{C_1}{(Ra_S S/L)^2} + \frac{C_2}{(Ra_S S/L)^{1/2}}\right]^{-1/2} = (0.500\times0.580)\text{m}^2\times$$

$$(52-27)\text{K}\,\frac{0.0272\text{W/m}\cdot\text{K}}{0.020\text{m}}\left[\frac{C_1}{(Ra_S S/L)^2} + \frac{C_2}{(Ra_S S/L)^{1/2}}\right]^{-1/2} = 28.8\text{W}. <$$

(b) To determine the effect of the spacing at S = 30 and 10 mm, we need only repeat the above calculations with these results

S (mm)	Ra_S	q (W)	
10	1874	26.1	<
30	50,585	28.8	<

Since it would be desirable to minimize heat losses from the tub, based upon these calculations, you would recommend a decrease in the spacing.

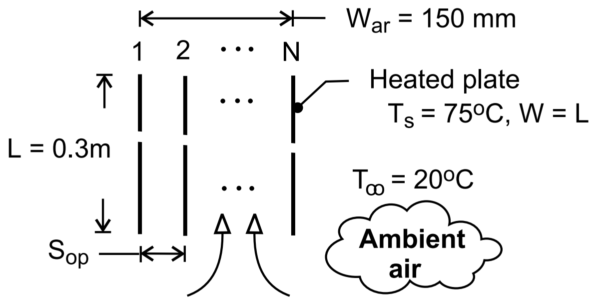
COMMENTS: For this situation, according to Table 9.3, the spacing corresponding to the maximum heat transfer rate is $S_{max} = (S_{max}/S_{opt}) \times 2.15(Ra_S/S^3L)^{-1/4}$ = 14.5 mm. Find q_{max} = 28.5 W. Note that the heat rate is not very sensitive to spacing for these conditions.

PROBLEM 9.86

KNOWN: Dimensions, spacing and temperature of plates in a vertical array. Ambient air temperature. Total width of the array.

FIND: Optimal plate spacing for maximum heat transfer from the array and corresponding number of plates and heat transfer.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) Negligible plate thickness, (3) Constant properties.

PROPERTIES: *Table A-4*, air $(p = 1\,\text{atm},\ \overline{T} = 320\text{K})$: $\nu = 17.9 \times 10^{-6}$ m^2/s, k = 0.0278 W/m·K, $\alpha = 25.5 \times 10^{-6}$ m^2/s, Pr = 0.704, $\beta = 0.00313$ K^{-1}.

ANALYSIS: With $Ra_S/S^3L = g\beta(T_s - T_\infty)/\alpha\nu L = (9.8\text{ m/s}^2 \times 0.00313\text{ K}^{-1} \times 55°\text{C})/(25.5 \times 17.9 \times 10^{-12}\text{ m}^4/\text{s}^2 \times 0.3\text{m}) = 1.232 \times 10^{10}\text{ m}^4$, from Table 9.3, the spacing which maximizes heat transfer for the array is

$$S_{opt} = \frac{2.71}{\left(Ra_S/S^3L\right)^{1/4}} = \frac{2.71}{\left(1.232\times10^{10}\,\text{m}^{-4}\right)^{1/4}} = 8.13\times10^{-3}\,\text{m} = 8.13\,\text{mm} \qquad <$$

With the requirement that $(N - 1)\, S_{opt} \le W_{ar}$, it follows that N ≤ 1 + 150 mm/8.13 mm = 19.4, in which case

$$N = 19 \qquad <$$

The corresponding heat rate is $q = N(2WL)\overline{h}(T_s - T_\infty)$, where, from Eq. 9.45 and Table 9.3,

$$\overline{h} = \frac{k}{S}\overline{Nu}_S = \frac{k}{S}\left[\frac{576}{(Ra_S S/L)^2} + \frac{2.87}{(Ra_S S/L)^{1/2}}\right]^{1/2}$$

With $Ra_S\, S/L = (Ra_S/S^3L)S^4 = 1.232 \times 10^{10}\text{ m}^{-4} \times (0.00813\text{m})^4 = 53.7$,

$$\overline{h} = \frac{0.0278\,\text{W/m}\cdot\text{K}}{0.00457\text{m}}\left[\frac{576}{(53.7)^2} + \frac{2.87}{(53.7)^{1/2}}\right] = 6.08(0.200 + 0.392) = 3.60\,\text{W/m}^2\cdot\text{K}$$

$$q = 19(2\times0.3\text{m}\times0.3\text{m})3.60\,\text{W/m}^2\cdot\text{K}\times55°\text{C} = 677\,\text{W} \qquad <$$

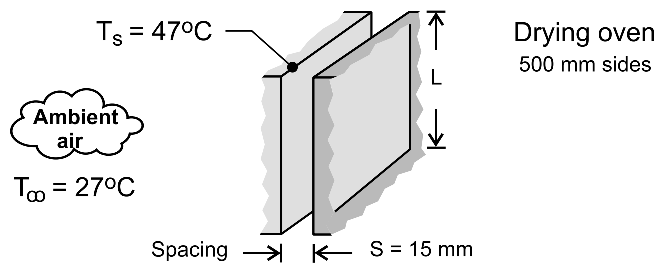
COMMENTS: It would be difficult to fabricate heater plates of thickness $\delta \ll S_{opt}$. Hence, subject to the constraint imposed on W_{ar}, N would be reduced, where $N \le 1 + W_{ar}/(S_{opt} + \delta)$.

PROBLEM 9.87

KNOWN: A bank of drying ovens is mounted on a rack in a room with an ambient temperature of 27°C; the cubical ovens are 500 mm to a side and the spacing between the ovens is 15 mm.

FIND: (a) Estimate the heat loss from the facing side of an oven when its surface temperature is 47°C, and (b) Explore the effect of the spacing dimension on the heat loss. At what spacing is the heat loss a maximum? Describe the boundary layer behavior for this condition. Can this condition be analyzed by treating the oven side-surface as an isolated vertical plate?

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Adjacent oven sides form a vertical channel with symmetrically heated plates, (3) Room air is quiescent, and channel sides are open to the room air, and (4) Constant properties.

PROPERTIES: *Table A-4*, Air ($T_f = (T_s + T_\infty)/2 = 310$ K, 1 atm): $\nu = 1.69 \times 10^{-5}$ m^2/s, k = 0.0270 W/m·K, $\alpha = 2.40 \times 10^{-5}$ m^2/s, Pr = 0.706, $\beta = 1/T_f$.

ANALYSIS: (a) For the isothermal plate channel, Eq. 9.45 with Eqs. 9.37 and 9.38, allow for calculation of the heat transfer from a plate to the ambient air.

$$\overline{Nu}_S = \left[\frac{C_1}{Ra_S S/L} + \frac{C_2}{(Ra_S S/L)^{1/2}}\right]^{-1/2} \qquad (1)$$

$$\overline{Nu}_S = \frac{q/A}{T_s - T_\infty}\,\frac{S}{k} \qquad (2)$$

$$Ra_S = \frac{g\beta(T_s - T_\infty)S^3}{\alpha\nu} \qquad (3)$$

where, from Table 9.3, for the *symmetrical isothermal plates*, $C_1 = 576$ and $C_2 = 2.87$. Properties are evaluated at the film temperature T_f. Substituting numerical values, evaluate the correlation parameters and the heat rate.

$$Ra_S = \frac{9.8\ \text{m/s}^2\,(1/310\ \text{K})(47-27)\text{K}(0.015\ \text{m})^3}{2.40\times10^{-5}\ \text{m}^2/\text{s}\times1.69\times10^{-5}\ \text{m}^2/\text{s}} = 5267$$

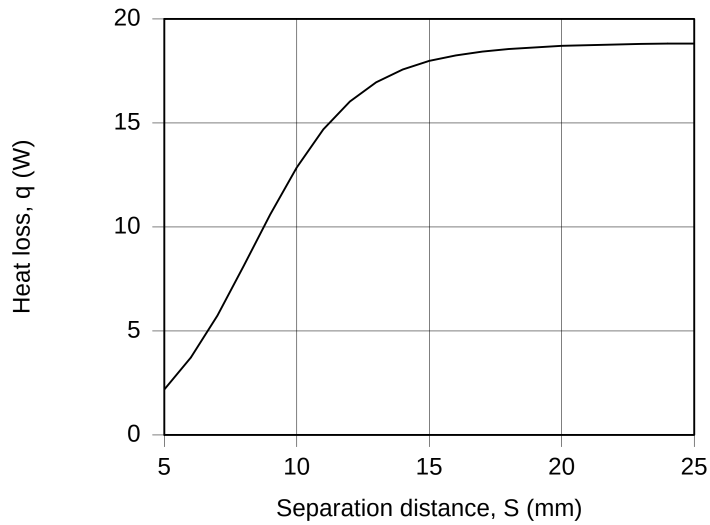
$$\overline{Nu}_S = \left[\frac{576}{5267\times0.015\ \text{m}/0.50\ \text{m}} + \frac{2.87}{(5267\times0.015\ \text{m}/0.050\ \text{m})^{1/2}}\right]^{-1/2} = 1.994$$

$$1.994 = \frac{q/(0.50\times0.50)\text{m}^2}{(47-27)\text{K}}\,\frac{0.015\ \text{m}}{0.0274\ \text{W/m}\cdot\text{K}} \qquad q = 18.0\ \text{W} \qquad <$$

Continued …..

PROBLEM 9.87 (Cont.)

(b) Using the foregoing relations in *IHT*, the heat rate is calculated for a range of spacing S.



Note that the heat rate increases with increasing spacing up to about S = 20 mm. This implies that for S > 20 mm, the side wall of the oven behaves as an *isolated vertical plate*. From the treatment of the vertical channel, Section 9.7.1, the spacing to provide maximum heat rate from a plate occurs at S_{max} which, from Table 9.3, is evaluated by

$$S_{max} = 1.71\, S_{opt} = 0.01964 \text{ m} = 19.6 \text{ mm}$$

$$S_{opt} = 2.71\left(Ra_S / S^3 L\right)^{-1/4} = 0.01147 \text{ m}$$

For the condition S = S_{max}, the spacing is sufficient that the boundary layers on the plates do not overlap.

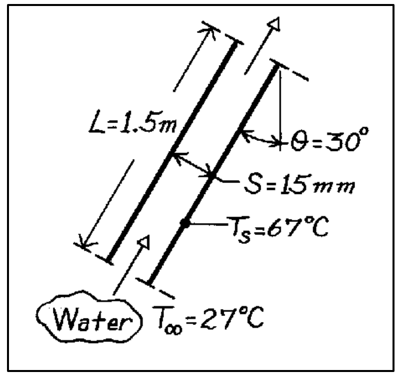
COMMENTS: Using the Churchill-Chu correlation, Eq. 9.26, for the isolated vertical plate, where the characteristic dimension is the height L, find q = 20.2 W ($Ra_L = 1.951 \times 10^8$ and $\bar{h}_L$ = 4.03 $W/m^2 \cdot K$). This value is slightly larger than that from the channel correlation when S > S_{max}, but a good approximation.

PROBLEM 9.88

KNOWN: Inclination angle of parallel plate solar collector. Plate spacing. Absorber plate and inlet temperature.

FIND: Rate of heat transfer to collector fluid.

SCHEMATIC:



ASSUMPTIONS: (1) Flow in collector corresponds to buoyancy driven flow between parallel plates with quiescent fluids at the inlet and outlet, (2) Constant properties.

PROPERTIES: *Table A-6*, Water $(\overline{T} = 320K)$: ρ = 989 kg/m^3, c_p = 4180 J/kg·K, μ = 577 × 10^{-6} kg/s·m, k = 0.640 W/m·K, β = 436.7 × 10^{-6} K^{-1}.

ANALYSIS: With

$$a = \frac{k}{r c_p} = \frac{0.640 W/m \cdot K}{989 kg/m^3 (4180 J/kg \cdot K)} = 1.55 \times 10^{-7} m^2/s$$

$$n = (m/r) = \left(577 \times 10^{-6} kg/s \cdot m\right)/989 kg/m^3 = 5.83 \times 10^{-7} m^2/s$$

find

$$Ra_S \frac{S}{L} = \frac{gb(T_s - T_\infty)S^4}{anL} = \frac{9.8 m/s^2 \left(436.7 \times 10^{-6} K^{-1}\right)(40K)(0.015m)^4}{\left(1.55 \times 10^{-7} m^2/s\right)\left(5.83 \times 10^{-7} m^2/s\right)1.5m}$$

$$Ra_S \frac{S}{L} = 6.39 \times 10^4.$$

Since $Ra_S(S/L) > 200$, Eq. 9.47 may be used,

$$\overline{Nu}_S = 0.645\left[Ra_S(S/L)\right]^{1/4} = 0.645\left(6.39 \times 10^4\right)^{1/4} = 10.3$$

$$\overline{h} = \overline{Nu}_S \frac{k}{S} = 10.3(0.64 W/m \cdot K/0.015m) = 438 W/m^2 \cdot K.$$

Hence the heat rate is

$$q = \overline{h}A(T_s - T_\infty) = 438 W/m^2 \cdot K(1.5m)(67 - 27)K = 26{,}300 W/m.$$ <

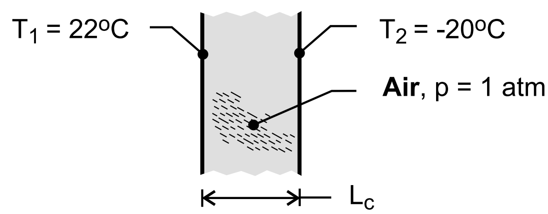
COMMENTS: Such a large heat rate would necessitate use of a concentrating solar collector for which the normal solar flux would be significantly amplified.

PROBLEM 9.89

KNOWN: Critical Rayleigh number for onset of convection in vertical cavity filled with atmospheric air. Temperatures of opposing surfaces.

FIND: Maximum allowable spacing for heat transfer by conduction across the air. Effect of surface temperature and air pressure.

SCHEMATIC:



ASSUMPTIONS: (1) Critical Rayleigh number is $Ra_{L,c} = 2000$, (2) Constant properties.

PROPERTIES: *Table A-4*, air [T = $(T_1 + T_2)/2 = 1°C = 274K$]: $\nu = 13.6 \times 10^{-6}$ m^2/s, k = 0.0242 W/m·K, $\alpha = 19.1 \times 10^{-6}$ m^2/s, $\beta = 0.00365$ K^{-1}.

ANALYSIS: With $Ra_{L,c} = g\,\beta\,(T_1 - T_2)\,L_c^3/\alpha\nu$,

$$L_c = \left[\frac{\alpha\nu\,Ra_{L,c}}{g\beta\left(T_1 - T_2\right)}\right]^{1/3} = \left[\frac{19.1\times13.6\times10^{-12}\,m^4/s^2\times2000}{9.8\,m/s^2\times0.00365\,K^{-1}\times42°C}\right]^{1/3} = 0.007m = 7\,mm \qquad <$$

The critical value of the spacing, and hence the corresponding thermal resistance of the air space, increases with a decreasing temperature difference, $T_1 - T_2$, and decreasing air pressure. With $\nu = \mu/\rho$ and $\alpha \equiv k/\rho c_p$, both quantities increase with decreasing p, since ρ decreases while μ, k and c_p are approximately unchanged.

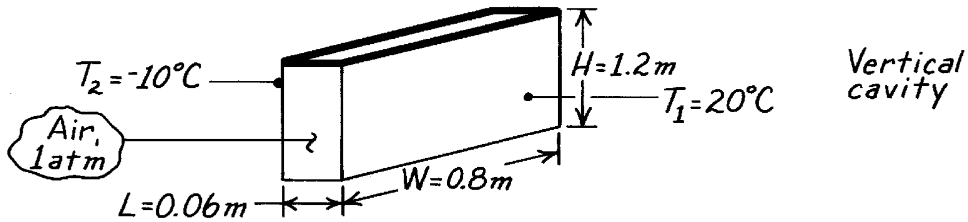
COMMENTS: (1) For the prescribed conditions and L_c = 7 mm, the conduction heat flux across the air space is $q'' = k\left(T_1 - T_2\right)/L_c = 0.0242\,W/m\cdot K\times42°C/0.007m = 145\,W/m^2$, (2) With triple pane construction, the conduction heat loss could be reduced by a factor of approximately two, (3) Heat loss is also associated with radiation exchange between the panes.

PROBLEM 9.90

KNOWN: Temperatures and dimensions of a window-storm window combination.

FIND: Rate of heat loss by free convection.

SCHEMATIC:



ASSUMPTIONS: (1) Both glass plates are of uniform temperature with insulated interconnecting walls and (2) Negligible radiation exchange.

PROPERTIES: *Table A-4*, Air (278K, 1 atm): $\nu = 13.93 \times 10^{-6}\ m^2/s$, k = 0.0245 W/m·K, $\alpha = 19.6 \times 10^{-6}\ m^2/s$, Pr = 0.71, $\beta = 0.00360\ K^{-1}$.

ANALYSIS: For the vertical cavity,

$$Ra_L = \frac{g\,b\,(T_1 - T_2)L^3}{an} = \frac{9.8\,m/s^2\left(0.00360K^{-1}\right)(30°C)(0.06m)^3}{19.6\times10^{-6}m^2/s\times13.93\times10^{-6}m^2/s}$$

$$Ra_L = 8.37\times10^5.$$

With (H/L) = 20, Eq. 9.52 may be used as a first approximation for Pr = 0.71,

$$\overline{Nu}_L = 0.42 Ra_L^{1/4}\,Pr^{0.012}\,(H/L)^{-0.3} = 0.42\left(8.37\times10^5\right)^{1/4}(0.71)^{0.012}(20)^{-0.3}$$

$$\overline{Nu}_L = 5.2$$

$$\bar{h} = \overline{Nu}_L \frac{k}{L} = 5.2\frac{0.0245W/m\cdot K}{0.06m} = 2.1W/m^2\cdot K.$$

The heat loss by free convection is then

$$q = \bar{h}A(T_1 - T_2)$$

$$q = 2.1W/m^2\cdot K(1.2m\times0.8m)(30°C) = 61W.$$ <

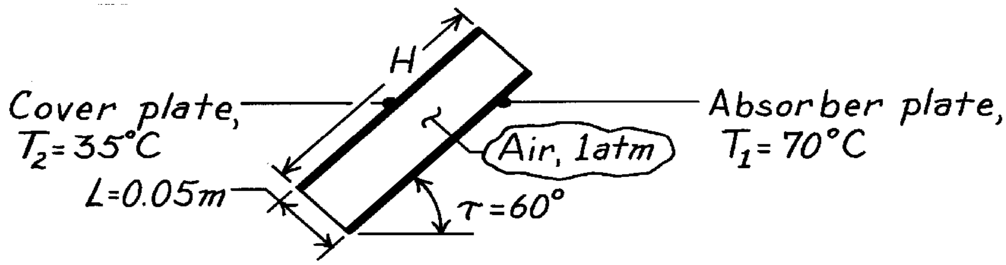
COMMENTS: In such an application, radiation losses should also be considered, and infiltration effects could render heat loss by free convection significant.

PROBLEM 9.91

KNOWN: Absorber plate and cover plate temperatures and geometry for a flat plate solar collector.

FIND: Heat flux due to free convection.

SCHEMATIC:



ASSUMPTIONS: (1) Aspect ratio, H/L, is greater than 12.

PROPERTIES: *Table A-4*, Air (325K, 1 atm): $\nu = 18.4 \times 10^{-6}$ m^2/s, k = 0.028 W/m·K, $\alpha = 26.2 \times 10^{-6}$ m^2/s, Pr = 0.703, $\beta = 3.08 \times 10^{-3}$ K^{-1}.

ANALYSIS: For the inclined enclosure,

$$Ra_L = \frac{g\beta(T_1 - T_2)L^3}{\alpha\nu} = \frac{9.8\,m/s^2\left(3.08\times10^{-3}K^{-1}\right)(70-35)°C\,(0.05m)^3}{\left(26.2\times10^{-6}m^2/s\right)\left(18.4\times10^{-6}m^2/s\right)}$$

$$Ra_L = 2.74\times10^5.$$

With $\tau < \tau^* = 70°$, Table 9.4,

$$\overline{Nu}_L = 1 + 1.44\left[1 - \frac{1708}{Ra_L \cos\tau}\right]^{\bullet}\left[1 - \frac{1708(\sin 1.8\tau)^{1.6}}{Ra_L \cos\tau}\right] + \left[\left(\frac{Ra_L \cos\tau}{5830}\right)^{1/3} - 1\right]^{\bullet}$$

$$\overline{Nu}_L = 1 + 1.44(0.99)(0.99) + 1.86 = 4.28$$

$$\bar{h} = \overline{Nu}_L \frac{k}{L} = 4.28\frac{0.028\,W/m\cdot K}{0.05m} = 2.4\,W/m^2\cdot K.$$

Hence, the heat flux is

$$q'' = h(T_1 - T_2) = 2.4\,W/m^2\cdot K\,(70-35)°C$$

$$q'' = 84\,W/m^2.$$ <

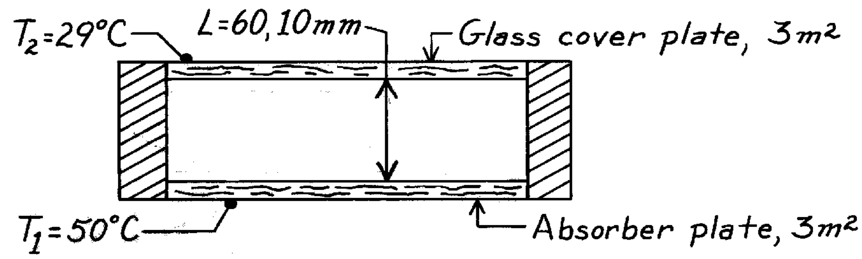
COMMENTS: Radiation exchange between the absorber and cover plates will also contribute to heat loss from the collector.

PROBLEM 9.92

KNOWN: Horizontal solar collector cover and absorber plates.

FIND: Heat loss from absorber to cover plate for spacings (a) 60mm and (b) 10mm.

SCHEMATIC:



ASSUMPTIONS: (1) Both plates are of uniform temperature with insulated interconnecting walls, (2) Surface radiation effects are negligible.

PROPERTIES: *Table A-4*, Air ($T_f = (T_1 + T_2)/2 = 312K$, 1 atm): $\nu = 17.15 \times 10^{-6}\ m^2/s$, $k = 0.0272\ W/m\cdot K$, $\alpha = 24.35 \times 10^{-6}\ m^2/s$, $Pr = 0.705$, $\beta = 1/T_f = 3.21 \times 10^{-3}\ K^{-1}$.

ANALYSIS: The heat rate between the plates is

$$q = q'' \cdot A_s = \bar{h} A_s \left(T_1 - T_2 \right) \qquad (1)$$

where h can be estimated by an appropriate correlation depending upon the magnitude of the Rayleigh number, Eq. 9.25,

$$Ra_L = g\, b \left(T_1 - T_2\right) L^3 / n a. \qquad (2)$$

(a) For separation distance L = 60mm,

$$Ra_L = \frac{9.8\,m/s^2 \times 3.21 \times 10^{-3} (50 - 29) K (0.060m)^3}{17.15 \times 10^{-6} m^2/s \times 24.35 \times 10^{-6} m^2/s} = 3.417 \times 10^5. \qquad (3)$$

As a first approximation, Eq. 9.49 is appropriate (note $Ra_L > 3 \times 10^5$),

$$\bar{h} = \overline{Nu}_L \cdot \frac{k}{L} = \frac{k}{L}\left[0.069 Ra_L^{1/3} Pr^{0.074}\right] \qquad (4)$$

$$\bar{h} = \frac{0.0272 W/m \cdot K}{0.060m}\left[0.069\left(3.417 \times 10^5\right)^{1/3} (0.705)^{0.074}\right] = 2.13 W/m^2 \cdot K$$

$$q = 2.13 W/m^2 \cdot K \times 3m^2 (50 - 29) K = 134W. \qquad <$$

(b) For separation distance L = 10mm, from Eq. (3) it follows that $Ra_L = (10/60)^3 \times 3.417 \times 10^5 = 1582$. Since $Ra_L < 1700$, heat transfer occurs by conduction only, such that

$$\overline{Nu}_L = \frac{\bar{h} L}{k} = 1 \qquad \text{or} \qquad \bar{h} = 1 \times \frac{0.0272 W/m \cdot K}{0.010m} = 2.72 W/m^2 \cdot K$$

$$q = 2.72 W/m^2 \cdot K \times 3m^2 (50 - 29) K = 171W. \qquad <$$

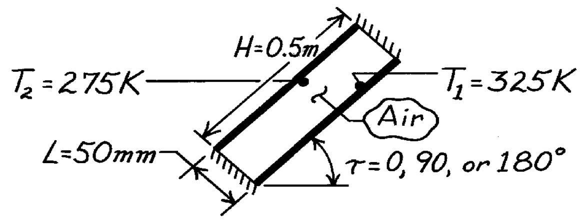
COMMENTS: Note that as L increases, from 10mm to 60mm, the heat rate decreases. However, For L ≥ 60mm, the heat rate will not change. This follows from Eq. 4 which, for $Ra > 3 \times 10^5$, $\bar{h}$ is independent of separation distance L.

PROBLEM 9.93

KNOWN: Rectangular cavity of two parallel, 0.5m square plates with insulated inter-connecting sides and with prescribed separation distance and surface temperatures.

FIND: Heat flux between surfaces for three orientations of the cavity: (a) Vertical τ = 90°C, (b) Horizontal with τ = 0°, and (c) Horizontal with τ = 180°.

SCHEMATIC:



ASSUMPTIONS: (1) Radiation exchange is negligible, (2) Air is at atmospheric pressure.

PROPERTIES: *Table A-4*, Air ($T_f = (T_1 + T_2)/2$ = 300K, 1 atm): $\nu = 15.89 \times 10^{-6}\ m^2/s$, k = 0.0263 W/m·K, $\alpha = 22.5 \times 10^{-6}\ m^2/s$, Pr = 0.707, $\beta = 1/T_f = 3.333 \times 10^{-3}\ K^{-1}$.

ANALYSIS: The convective heat flux between the two cavity plates is

$$q''_{conv} = \overline{h}\left(T_1 - T_2\right)$$

where $\overline{h}$ is estimated from the appropriate enclosure correlation which will depend upon the Rayleigh number. From Eq. 9.25, find

$$Ra_L = \frac{g\,b\,(T_1-T_2)L^3}{na} = \frac{9.8\,m/s^2 \times 3.333\times10^{-3}K^{-1}(325-275)K\,(0.05m)^3}{15.89\times10^{-6}m^2/s \times 22.5\times10^{-6}m^2/s} = 5.710\times10^5.$$

Note that H/L = 0.5/0.05 = 10, a factor which is important in selecting correlations.

(a) With τ = 90°, for a vertical cavity, Eq. 9.50, is appropriate,

$$\overline{Nu}_L = 0.22\left(\frac{Pr}{0.22+Pr}Ra_L\right)^{0.28}\left(\frac{H}{L}\right)^{-1/4} = 0.22\left(\frac{0.707}{0.22+0.707}\times5.71\times10^5\right)^{0.28}(10)^{-1/4} = 4.692$$

$$\overline{h}_L = \frac{k}{L}\overline{Nu}_L = \frac{0.0263\,W/m\cdot K}{0.05m}\times4.692 = 2.47\,W/m^2\cdot K$$

$$q''_{conv} = 2.47\,W/m^2\cdot K\,(325-275)K = 123\,W/m^2.$$ <

(b) With τ = 0* for a horizontal cavity heated from below, Eq. 9.49 is appropriate.

$$\overline{h} = \frac{k}{L}\overline{Nu}_L = 0.069\frac{k}{L}Ra_L^{1/3}Pr^{0.074} = 0.069\frac{0.0263\,W/m\cdot K}{0.05m}\left(5.710\times10^5\right)^{1/3}(0.707)^{0.074}$$

$$\overline{h} = 2.92\,W/m^2\cdot K$$

$$q''_{conv} = 2.92\,W/m^2\cdot K\,(325-275)K = 146\,W/m^2.$$ <

(c) For τ = 180° corresponding to the horizontal orientation with the heated plate on the top, heat transfer will be by conduction. That is,

$$\overline{Nu}_L = 1 \qquad \text{or} \qquad \overline{h}_L = \overline{Nu}_L\cdot\frac{k}{L} = 1\times0.0263\,W/m\cdot K/(0.05m) = 0.526\,W/m^2\cdot K.$$

$$q''_{conv} = 0.526\,W/m^2\cdot K\,(325-275)K = 26.3\,W/m^2.$$ <

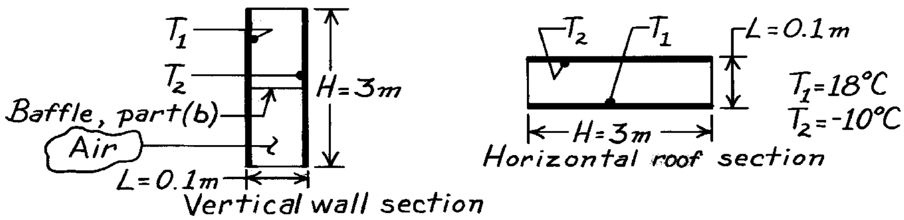
COMMENTS: Compare the heat fluxes for the various orientations and explain physically their relative magnitudes.

PROBLEM 9.94

KNOWN: Horizontal flat roof and vertical wall sections of same dimensions exposed to identical temperature differences.

FIND: (a) Ratio of convection heat rate for horizontal section to that of the vertical section and (b) Effect of inserting a baffle at the mid-height of the vertical wall section on the convection heat rate.

SCHEMATIC:



ASSUMPTIONS: (1) Ends of sections and baffle adiabatic, (2) Steady-state conditions.

PROPERTIES: *Table A-4*, Air $\left(\overline{T} = (T_1 + T_2)/2 = 277\text{K}, 1\text{atm}\right)$: $\nu = 13.84 \times 10^{-6}$ m^2/s, k = 0.0245 W/m·K, $\alpha = 19.5 \times 10^{-6}$ m^2/s, Pr = 0.713.

ANALYSIS: (a) The ratio of the convection heat rates is

$$\frac{q_{hor}}{q_{vert}} = \frac{\overline{h}_{hor} A_s \Delta T}{\overline{h}_{vert} A_s \Delta T} = \frac{\overline{h}_{hor}}{\overline{h}_{vert}}. \qquad (1)$$

To estimate coefficients, recognizing both sections have the same characteristics length, L = 0.1m, with $Ra_L = g\beta\Delta T L^3/\nu\alpha$ find

$$Ra_L = \frac{9.8\,\text{m/s}^2 \times (1/277\text{K})(18-(-10))\text{K}\,(0.1\text{m})^3}{13.84\times10^{-6}\text{m}^2/\text{s}\times19.5\times10^{-6}\text{m}^2/\text{s}} = 3.67\times10^6.$$

The appropriate correlations for the sections are Eqs. 9.49 and 9.52 (with H/L = 30),

$$\overline{Nu}_L\Big|_{hor} = 0.069 Ra_L^{1/3}\, Pr^{0.074} \qquad \overline{Nu}_L\Big|_{vert} = 0.42 Ra_L^{1/4}\, Pr^{0.012}\,(H/L)^{-0.3}. \qquad (3,4)$$

Using Eqs. (3) and (4), the ratio of Eq. (1) becomes,

$$\frac{q_{hor}}{q_{vert}} = \frac{0.069 Ra_L^{1/3}\, Pr^{0.074}}{0.42 Ra_L^{1/4}\, Pr^{0.012}\,(H/L)^{-0.3}} = \frac{0.069\left(3.67\times10^6\right)^{1/3}(0.713)^{0.074}}{0.42\left(3.67\times10^6\right)^{1/4}(0.713)^{0.012}(30)^{-0.3}} = 1.57.$$ <

(b) The effect of the baffle in the vertical wall section is to reduce H/L from 30 to 15. Using Eq. 9.52, it follows,

$$\frac{q_{baf}}{q} = \frac{\overline{h}_{baf}}{\overline{h}} = \frac{(H/L)_{baf}^{-0.3}}{(H/L)^{-0.3}} = \left(\frac{15}{30}\right)^{-0.3} = 1.23.$$ <

That is, the effect of the baffle is to increase the convection heat rate.

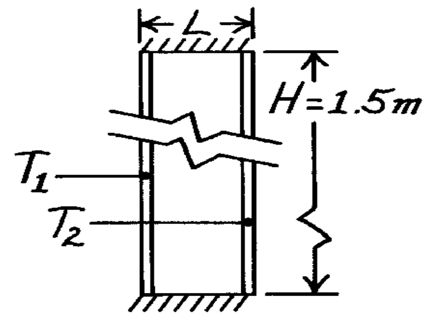
COMMENTS: (1) Note that the heat rate for the horizontal section is 57% larger than that for the vertical section for the same ($T_1 - T_2$). This indicates the importance of heat losses from the ceiling or roofs in house construction. (2) Recognize that for Eq. 9.52, the Pr > 1 requirement is not completely satisfied. (3) What is the physical explanation for the result of part (b)?

PROBLEM 9.95

KNOWN: Double-glazed window of variable spacing L between panes filled with either air or carbon dioxide.

FIND: Heat transfer across window for variable spacing when filled with either gas. Consider these conditions (outside, T_1; inside, T_2): winter (-10, 20°C) and summer (35°C, 25°C).

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Radiation exchange is negligible, (3) Gases are at atmospheric pressure, (4) Perfect gas behavior.

PROPERTIES: *Table A-4*: Winter, $\overline{T} = (-10 + 20)°C/2 = 288K$, Summer, $\overline{T} = (35 + 25)°C/2 = 303K$:

Gas (1 atm)	$\overline{T}$ (K)	α ($m^2/s \times 10^6$)	ν ($m^2/s \times 10^6$)	$k \times 10^3$ (W/m·K)
Air	288	20.5	14.82	24.9
Air	303	22.9	16.19	26.5
CO_2	288	10.2	7.78	15.74
CO_2	303	11.2	8.55	16.78

ANALYSIS: The heat flux by convection across the window is

$$q'' = h(T_1 - T_2)$$

where the convection coefficient is estimated from the correlation of Eq. 9.53 for large aspect ratios 10< H/L < 40,

$$\overline{Nu}_L = \overline{h}L/k = 0.046 Ra_L^{1/4}.$$

Substituting numerical values for winter (w) and summer (s) conditions,

$$Ra_{L,w,air} = \frac{9.8 m/s^2 (1/288K)(20-(-10))KL^3}{20.5\times10^{-6} m^2/s \times 14.82\times10^{-6} m^2/s} = 3.360\times10^9 L^3$$

$$Ra_{L,s,air} = 8.724\times10^8 L^3 \qquad Ra_{L,w,CO_2} = 1.286\times10^{10} L^3 \qquad Ra_{L,s,CO_2} = 3.378\times10^9 L^3$$

the heat transfer coefficients are

$$\overline{h}_{w,air} = (0.0249 W/m\cdot K/L)\times 0.046\left(3.360\times10^9 L^3\right)^{1/4} = 0.276 L^{-1/4}$$

$$h_{s,air} = 0.209 L^{-1/4} \quad h_{w,CO_2} = 0.244 L^{-1/4} \qquad h_{s,CO_2} = 0.186 L^{-1/4}.$$

For a separation distance such that H/L = 40, the maximum aspect ratio for the correlation, with H = 1.5 m, L = 37.5 mm find

$$q''_{w,air} = 18.8 W/m^2 \quad q''_{s,air} = 4.7 W/m^2 \quad q''_{w,CO_2} = 16.6 W/m^2 \quad q''_{s,CO_2} = 4.2 W/m^2.$$

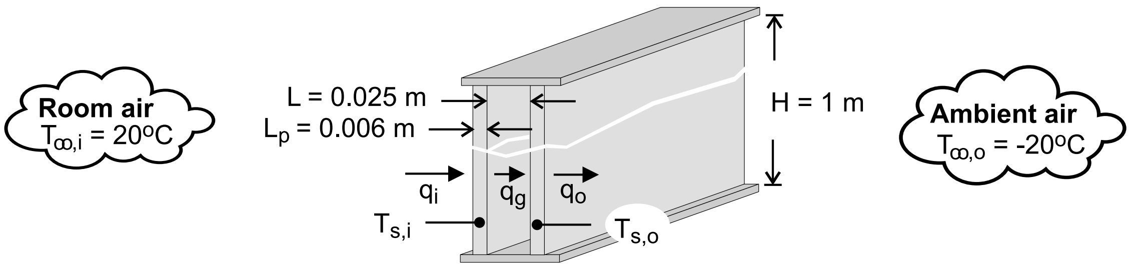
Using CO_2 rather than air reduces the heat loss/gain by approximately 12%. Note the winter heat rate for this window is nearly 4 times that for summer.

PROBLEM 9.96

KNOWN: Dimensions of double pane window. Thickness of air gap. Temperatures of room and ambient air.

FIND: (a) Temperatures of glass panes and heat rate through window, (b) Resistance of glass pane relative to smallest convection resistance.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) Negligible glass pane thermal resistance, (3) Constant properties.

PROPERTIES: *Table A-3*, Plate glass: $k_p = 1.4$ W/m·K. *Table A-4*, Air (p = 1 atm). $T_{f,i} = 287.6$K: $\nu_i = 14.8 \times 10^{-6}$ m^2/s, $k_i = 0.0253$ W/m·K, $\alpha_i = 20.9 \times 10^{-6}$ m^2/s, $Pr_i = 0.710$, $\beta_i = 0.00348$ K^{-1}. $\overline{T} = (T_{s,i} + T_{s,o})/2 = 272.8$K: $\nu = 13.49 \times 10^{-6}$ m^2/s, $k = 0.0241$ W/m·K, $\alpha = 18.9 \times 10^{-6}$ m^2/s, Pr = 0.714, $\beta = 0.00367$ K^{-1}. $T_{f,o} = 258.2$K: $\nu_o = 12.2 \times 10^{-6}$ m^2/s, $k_o = 0.0230$W/m·K, $\alpha = 17.0 \times 10^{-6}$ m^2/s, Pr = 0.718, $\beta_o = 0.00387$ K^{-1}.

ANALYSIS: (a) The heat rate may be expressed as

$$q = q_o = \overline{h}_o H^2 \left(T_{s,o} - T_{\infty,o}\right) \tag{1}$$

$$q = q_g = \overline{h}_g H^2 \left(T_{s,i} - T_{s,o}\right) \tag{2}$$

$$q = q_i = \overline{h}_i H^2 \left(T_{\infty,i} - T_{s,i}\right) \tag{3}$$

where $\overline{h}_o$ and $\overline{h}_i$ may be obtained from Eq. (9.26),

$$\overline{Nu}_H = \left\{0.825 + \frac{0.387\,Ra_H^{1/6}}{\left[1+\left(0.492/Pr\right)^{9/16}\right]^{8/27}}\right\}^2$$

with $Ra_H = g\beta_o\left(T_{s,o} - T_{\infty,o}\right)H^3/\alpha_o\nu_o$ and $Ra_H = g\beta_i\left(T_{\infty,i} - T_{s,i}\right)H^3/\alpha_i\nu_i$, respectively. Assuming $10^4 < Ra_L < 10^7$, $\overline{h}_g$ is obtained from

$$\overline{Nu}_L = 0.42\,Ra_L^{1/4}\,Pr^{0.012}\left(H/L\right)^{-0.3}$$

where $Ra_L = g\beta\left(T_{s,i} - T_{s,o}\right)L^3/\alpha\nu$. A simultaneous solution to Eqs. (1) – (3) for the three unknowns yields

Continued …..

$$T_{s,i} = 9.1°\text{C}, \quad T_{s,o} = -9.6°\text{C}, q = 35.7 \text{ W}$$ <

where $\overline{h}_i = 3.29\,\text{W/m}^2 \cdot \text{K}$, $\overline{h}_o = 3.45\,\text{W/m}^2 \cdot \text{K}$ and $\overline{h}_g = 1.90\,\text{W/m}^2 \cdot \text{K}$.

(b) The unit conduction resistance of a glass pane is $R''_{cond} = L_p / k_p = 0.00429\,\text{m}^2 \cdot \text{K/W}$, and the smallest convection resistance is $R''_{conv,o} = \left(1/\overline{h}_o\right) = 0.290\,\text{m}^2 \cdot \text{K/W}$. Hence,

$$R''_{cond} << R''_{conv,min}$$ <

and it is reasonable to neglect the thermal resistance of the glass.

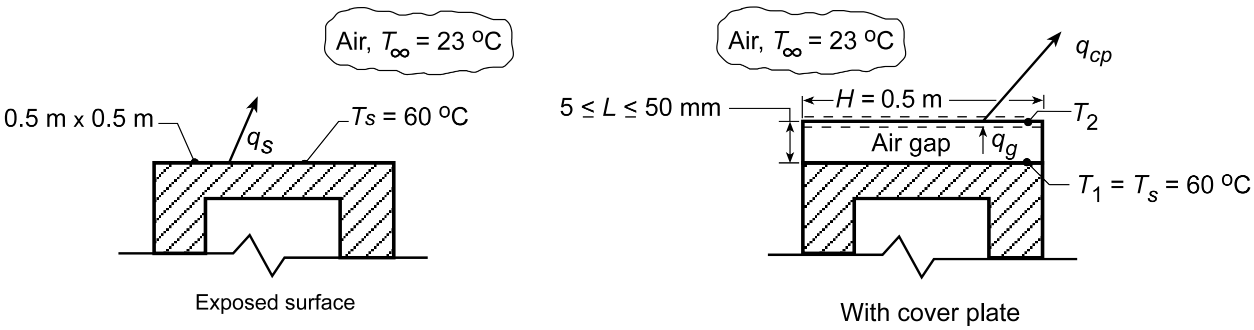
COMMENTS: (1) Assuming a heat flux of 35.7 W/m^2 through a glass pane, the corresponding temperature difference across the pane is $\Delta T = q''\left(L_p / k_p\right) = 0.15°\text{C}$. Hence, the assumption of an isothermal pane is good. (2) Equations (1) – (3) were solved using the IHT workspace and the temperature-dependent air properties provided by the software. The property values provided in the PROPERTIES section of this solution were obtained from the software.

PROBLEM 9.97

KNOWN: Top surface of an oven maintained at 60°C.

FIND: (a) Reduction in heat transfer from the surface by installation of a cover plate with specified air gap; temperature of the cover plate, (b) Effect of cover plate spacing.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Oven surface at $T_1 = T_s$ for both cases, (3) Negligible radiative exchange with surroundings and across air gap.

PROPERTIES: *Table A.4,* Air ($T_f = (T_s + T_\infty)/2 = 315$ K, 1 atm): $\nu = 17.40 \times 10^{-6}$ m²/s, k = 0.0274 W/m·K, $\alpha = 24.7 \times 10^{-6}$ m²/s; *Table A.4*, Air ($\overline{T} = (T_1 + T_2)/2$ and $T_{f2} = (T_2 + T_\infty)/2$): Properties obtained form *Correlations Toolpad* of IHT.

ANALYSIS: (a) The convective heat loss from the exposed top surface of the oven is $q_s = \overline{h} A_s(T_s - T_\infty)$. With $L = A_s/P = (0.5\text{ m})^2/(4 \times 0.5\text{ m}) = 0.125$ m,

$$Ra_L = \frac{g\beta\Delta T L^3}{\nu\alpha} = \frac{9.8\,\text{m/s}^2\,(1/315\,\text{K})(60-23)^\circ\text{C}\,(0.125\,\text{m})^3}{17.40\times10^{-6}\,\text{m/s}^2 \times 24.7\times10^{-6}\,\text{m/s}^2} = 5.231\times10^6 .$$

The appropriate correlation for a heated plate facing upwards, Eq. 9.30, is

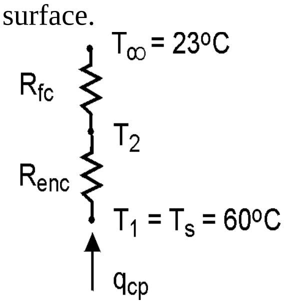
$$\overline{Nu}_L = \frac{\overline{h}L}{k} = 0.54 Ra_L^{1/4} \qquad 10^4 \le Ra_L \le 10^7$$

$$\overline{h} = \left(\frac{0.0274\,\text{W/m}\cdot\text{K}}{0.125\,\text{m}}\right) \times 0.54\left(5.231\times10^6\right)^{1/4} = 5.66\,\text{W/m}^2\cdot\text{K}$$

Hence, the heat rate for the exposed surface is

$$q_s = 5.66\,\text{W/m}^2\cdot\text{K}\,(0.5\,\text{m})^2\,(60-23)^\circ\text{C} = 52.4\,\text{W} .$$ <

With the cover plate, the surface temperature ($T_s = T_2$) is unknown and must be obtained by performing an energy balance at the top surface.



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Equating heat flow across the gap to that from the top surface, $q_g = q_{cp}$. Hence, for a unit surface area,

$$\bar{h}_g \left(T_1 - T_2\right) = \bar{h}_{cp} \left(T_2 - T_\infty\right)$$

where $\bar{h}_{cp}$ is obtained from Eq. 9.30 and $\bar{h}_g$ is evaluated from Eq. 9.49.

$$\overline{Nu}_L = \frac{\bar{h}_g L}{k} = 0.069 Ra_L^{1/3} Pr^{0.074}$$

Entering this expression from the keyboard and Eq. 9.30 from the *Correlations* Toolpad, with the *Properties* Toolpad used to evaluate air properties at $\bar{T}$ and T_{fs}, IHT was used with L = 0.05 m to obtain

$T_2 = 35.4°C$ $\qquad\qquad$ $q_{cp} = 13.5$ W $\qquad$ <

where $\bar{h}_g = 2.2\ W/m^2 \cdot K$ and $\bar{h}_{cp} = 4.4\ W/m^2 \cdot K$. Hence, the effect of installing the cover plate creating the enclosure is to reduce the heat loss by

$$\frac{q_s - q_{cp}}{q_s} \times 100 = \frac{52.4 - 13.5}{52.4} \times 100 = 74\%.$$ <

Note, however, that for L = 0.05 m, $Ra_L = 2.05 \times 10^5$ is slightly less than the lower limit of applicability for Eq. 9.49.

(b) If we use the foregoing model to evaluate T_2 and q_{cp} for $0.005 \le L \le 0.05$ m, we find that there is no effect. This seemingly unusual result is a consequence of the fact that, in Eq. 9.49, $\overline{Nu}_L \propto Ra_L^{1/3}$, in which case $\bar{h}_g$ is independent of L. However, Ra_L and Nu_L do decrease with decreasing L, eventually approaching conditions for which transport across the airspace is determined by conduction and not convection. If transport is by conduction, the heat rate must be determined from Fourier's law, for which $q_g'' = (k/L)(T_1 - T_2)$ and the equivalent, *pseudo*, Nusselt number is $\overline{Nu}_L = \bar{h}L/k = 1$. If this expression is used to determine $\bar{h}_g$ in the energy balance, q_{cp} increases with decreasing L. The results would only apply if there is negligible advection in the airspace and hence for Rayleigh numbers less than 1708, which corresponds to L ≈ 10.5 mm. For this value of L, $q_{cp} = 15.4$ W exceeds that previously determined for L = 50 mm. Hence, there is little variation in q_{cp} over the range 10.5 < L < 50 mm. However, q_{cp} increases with decreasing L below 10.5 mm, achieving a value of 24.2 W for L = 5 mm. Hence, a value of L slightly larger than 10.5 mm could be considered an optimum.

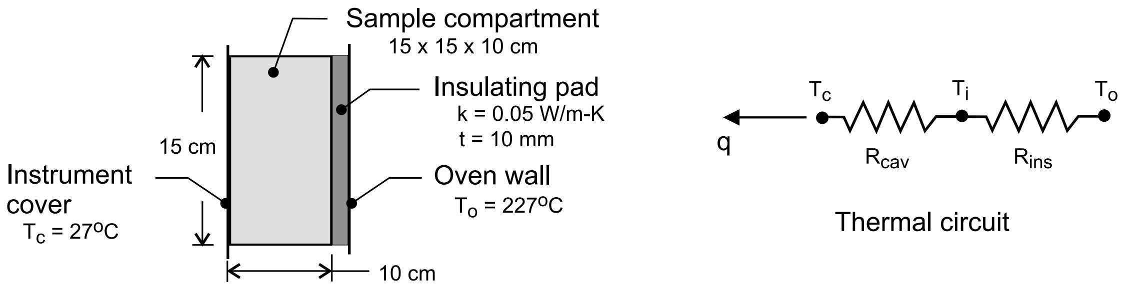
COMMENTS: Radiation exchange across the cavity and with the surroundings is likely to be significant and should be considered in a more detailed analysis.

PROBLEM 9.98

KNOWN: The sample compartment of an optical instrument in the form of a rectangular cavity; one face in contact with instrument cover maintained at 27°C; other face having 10-mm thick insulation pad in contact with oven wall maintained at 227°C.

FIND: Heat gain to the instrument, and average air temperature in the compartment.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible heat losses from the edges of the rectangular-cavity shaped compartment, and (3) Constant properties.

PROPERTIES: *Table A-4*, air ($T_f = \overline{T}_{air} = (T_i + T_c)/2$ = 354 K, 1 atm): $\nu = 2.132 \times 10^{-5}$ m^2/s, k = 0.0303 W/m·K, $\alpha = 3.051 \times 10^{-5}$ m^2/s, Pr = 0.699, $\beta = 1/T_f$.

ANALYSIS: The system comprised of the rectangular cavity and the insulating pad can be represented by the thermal circuit shown above. The heat gain to the instrument and the hot-side temperature of the cavity, T_i, can be expressed as

$$q = \frac{T_o - T_c}{R_{ins} + R_{cav}} \qquad \frac{T_o - T_i}{R_{ins}} = \frac{T_i - T_c}{R_{cav}} \tag{1,2}$$

The thermal resistance of the insulating pad is

$$R_{ins} = \frac{L_i}{k_i A_s} = \frac{0.010\ \text{m}}{0.05\ \text{W/m}\cdot\text{K}\left(0.15 \times 0.15\right)\text{m}^2} = 8.89\ \text{K/W} \tag{3}$$

The thermal resistance of the rectangular cavity is

$$R_{cav} = 1/\left(\overline{h}\,A_s\right) \tag{4}$$

where L_c is the cavity width and the average convection coefficient follows from Eq. 9.51 (since H/L_c = 15/10 = 1.5),

$$\overline{Nu}_L = \frac{\overline{h}\,L_c}{k} = 0.18\left(\frac{Pr}{0.2 + Pr}Ra_{Lc}\right)^{0.29} \tag{5}$$

$$Ra_{Lc} = g\beta\left(T_i - T_c\right)L_c^3/\alpha\nu \tag{6}$$

where the properties are evaluated at the average cavity air temperature

$$T_f = \overline{T}_{air} = \left(T_i + T_c\right)/2 \tag{7}$$

Recognize that the system of equations needs to be solved iteratively by initially guessing values of T_i or solved simultaneously using equation-solving software with a properties library. The results are:

$q = 10.4$ W $\qquad \overline{T}_{air} = 134°$C <

COMMENTS: Other parameters resulting from the analyses are:

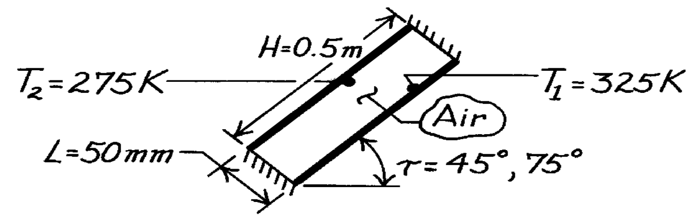
$\overline{Nu}_{Lc} = 14.3 \qquad \overline{Ra}_L = 4.57 \times 10^6 \qquad \overline{h} = 4.33\ \text{W/m}^2\cdot\text{K} \qquad R_{cav} = 10.28\ \text{K/W}$

PROBLEM 9.99

KNOWN: Rectangular cavity of two parallel, 0.5m square plates with insulated boundaries and with prescribed separation distance and surface temperatures.

FIND: Convective heat flux between surfaces for tilt angles of (a) 45° and (b) 75°.

SCHEMATIC:



ASSUMPTIONS: (1) Radiation exchange is negligible, (2) Cavity air is 1 atm.

PROPERTIES: *Table A-4*, Air ($T_f = (T_1 + T_2)/2 = 300K$, 1 atm): $k = 0.0263$ W/m·K, $\nu = 15.89 \times 10^{-6}$ m^2/s, $\alpha = 22.5 \times 10^{-6}$ m^2/s, Pr = 0.707, $\beta = 1/T_f = 3.333 \times 10^{-3}$ K^{-1}.

ANALYSIS: (a) The convective heat flux between the plates is $q'' = \overline{h}(T_1 - T_2)$ where $\overline{h}$ is estimated from the appropriate correlation with

$$Ra_L = \frac{g\,b\,(T_1 - T_2)L^3}{na} = \frac{9.8\,\text{m/s}^2 \times 3.333\times10^{-3}\text{K}^{-1}(325-275)\text{K}(0.05\text{m})^3}{22.5\times10^{-6}\text{m}^2/\text{s}\times15.89\times10^{-6}\text{m}^2/\text{s}} = 5.710\times10^5.$$

For H/L = 0.5m/0.05m = 10 and $\tau < \tau^*$ ($\tau^* \approx 64°$ from Table 9.4), Eq. 9.55 is suitable,

$$\overline{Nu}_L = \overline{Nu}_L(t = 0)\left[\frac{\overline{Nu}_L\ (t = 90)}{\overline{Nu}_L\ (t = 0)}\right]^{t/t^*}(\sin t^*)^{t/4t^*}. \qquad (1)$$

For $\overline{Nu}_L\ (t = 90°)$, Eq. 9.50 is appropriate,

$$\overline{Nu}_L\ (t = 90°) = 0.22\left(\frac{Pr}{0.2 + Pr}Ra_L\right)^{0.28}\left(\frac{H}{L}\right)^{-1/4} = 0.22\left(\frac{0.707}{0.2+0.707}5.71\times10^5\right)^{0.28}(10)^{-1/4} = 4.72.$$

For $\overline{Nu}_L\ (t = 0°)$, Eq. 9.49 is appropriate,

$$\overline{Nu}_L\ (t = 0°) = 0.069Ra_L^{1/3}Pr^{0.074} = 0.069\times\left(5.71\times10^5\right)^{1/3}(0.707)^{0.074} = 5.58.$$

Substituting numerical values into Eq. (1) with $\tau = 45°$,

$$\overline{Nu}_L = 5.58[4.72/5.58]^{45/64}(\sin 64)^{45/4\times64} = 4.86$$

$$\overline{h} = \overline{Nu}_L\ k/L = 4.86\times0.0263\,\text{W/m}\cdot\text{K}/0.05\text{m} = 2.56\,\text{W/m}^2\cdot\text{K}.$$

$$q'' = 2.56\,\text{W/m}^2\cdot\text{K}(325 - 275)\text{K} = 128\,\text{W/m}^2. \qquad <$$

(b) For $\tau = 75°$, $\tau > \tau^*$, the critical tilt angle, Eq. 9.56 is appropriate for estimating $\overline{h}$.

$$\overline{Nu}_L = \overline{Nu}_L(t = 90)\cdot(\sin t)^{1/4} = 4.72(\sin 75°)^{1/4} = 4.68$$

$$\overline{h} = \overline{Nu}_L\ k/L = 4.68\times0.0263\,\text{W/m}\cdot\text{K}/0.05\text{m} = 2.46\,\text{W/m}^2\cdot\text{K}.$$

$$q'' = 2.46\,\text{W/m}^2\cdot\text{K}(325 - 275)\text{K} = 123\,\text{W/m}^2. \qquad <$$

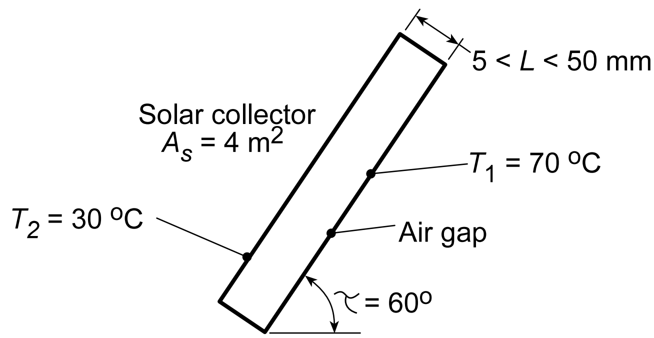
COMMENTS: Note that $\overline{Nu}_L\ (t = 0) > \overline{Nu}_L\ (t = 90°)$. For the cavity conditions there is little change in $\overline{h}$ for tilt angles, τ, from 45° to 90°.

PROBLEM 9.100

KNOWN: Dimensions and surface temperatures of a flat-plate solar collector.

FIND: (a) Heat loss across collector cavity, (b) Effect of plate spacing on the heat loss.

SCHEMATIC:



ASSUMPTIONS: Negligible radiation.

PROPERTIES: *Table A.4*, Air ($\overline{T} = (T_1 + T_2)/2 = 323$ K): $\nu = 18.2 \times 10^{-6}$ m^2/s, k = 0.028 W/m·K, $\alpha = 25.9 \times 10^{-6}$ m^2/s, $\beta = 0.0031$ K^{-1}.

ANALYSIS: (a) Since H/L = 2 m/0.03 m = 66.7 > 12, $\tau < \tau^*$ and Eq. 9.54 may be used to evaluate the convection coefficient associated with the air space. Hence, $q = \overline{h} A_s(T_1 - T_2)$, where $\overline{h} = (k/L)\overline{Nu}_L$ and

$$\overline{Nu}_L = 1 + 1.44\left[1 - \frac{1708}{Ra_L \cos\tau}\right]^{\bullet}\left[1 - \frac{1708(\sin 1.8\tau)^{1.6}}{Ra_L \cos\tau}\right] + \left[\left(\frac{Ra_L \cos\tau}{5830}\right)^{1/3} - 1\right]^{\bullet}$$

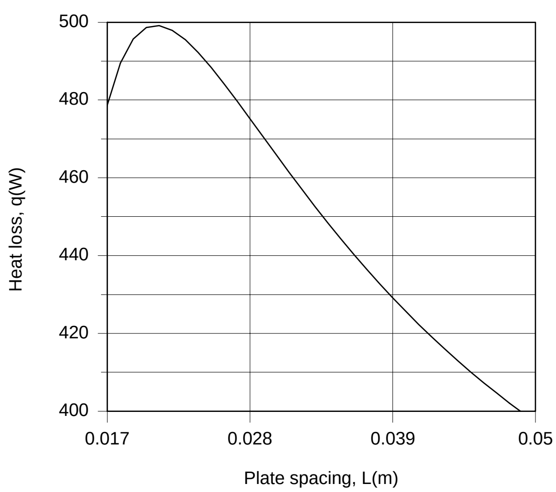
For L = 30 mm, the Rayleigh number is

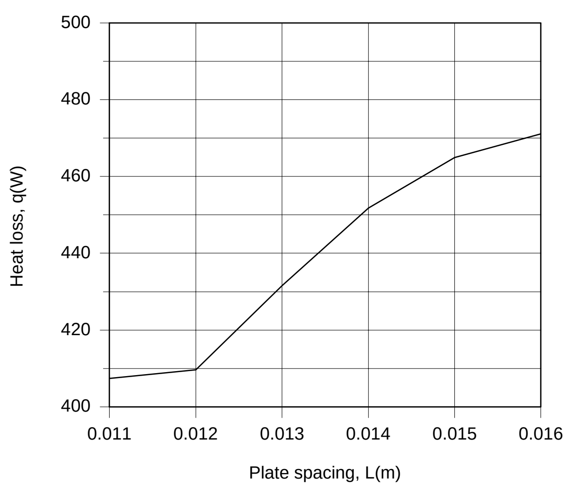
$$Ra_L = \frac{g\beta(T_1 - T_2)L^3}{\alpha\nu} = \frac{9.8\,m/s^2\left(0.0031\,K^{-1}\right)\left(40^\circ C\right)(0.03\,m)^3}{25.9\times10^{-6}\,m^2/s \times 18.2\times10^{-6}\,m^2/s} = 6.96\times10^4$$

and $Ra_L \cos\tau = 3.48 \times 10^4$. It follows that $\overline{Nu}_L = 3.12$ and $\overline{h} = (0.028$ W/m·K/0.03 m$)3.12 = 2.91$ W/m^2·K. Hence,

$$q = 2.91\,W/m^2\cdot K\left(4\,m^2\right)\left(40^\circ C\right) = 466\,W$$ <

(b) The foregoing model was entered into the workspace of IHT, and results of the calculations are plotted as follows.





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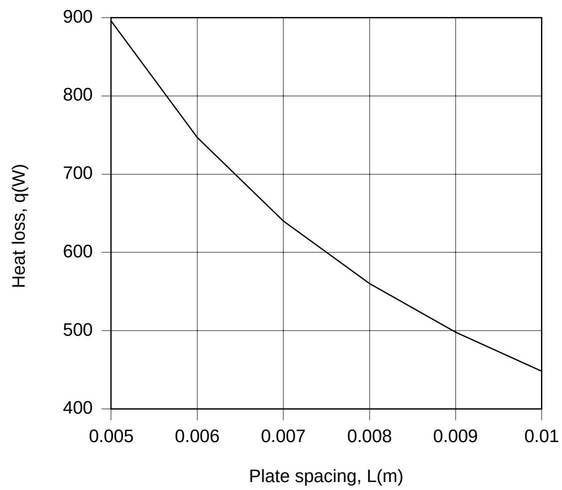
PROBLEM 9.100 (Cont.)



The plots are influenced by the fact that the third and second terms on the right-hand side of the correlation are set to zero at $L \approx 0.017$ m and $L \approx 0.011$ m, respectively. For the range of conditions, minima in the heat loss of $q \approx 410$ W and $q = 397$ W are achieved at $L \approx 0.012$ m and $L = 0.05$ m, respectively. Operation at $L \approx 0.02$ m corresponds to a maximum and is clearly undesirable, as is operation at $L < 0.011$ m, for which conditions are conduction dominated.

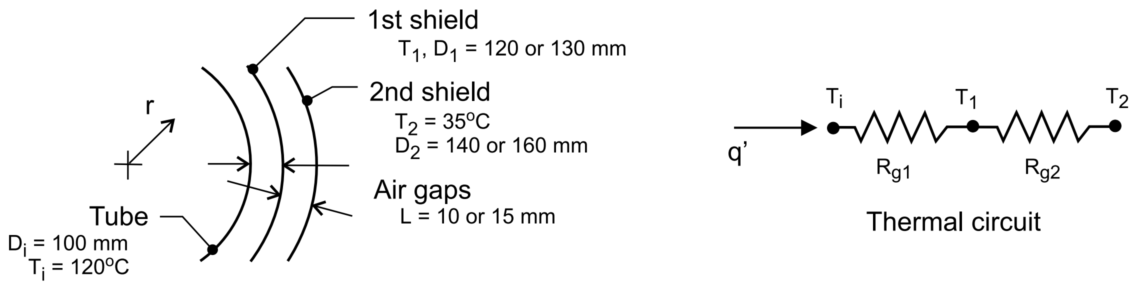
COMMENTS: Because the convection coefficient is low, radiation effects would be significant.

PROBLEM 9.101

KNOWN: Cylindrical 120-mm diameter radiation shield of Example 9.5 installed concentric with a 100-mm diameter tube carrying steam; spacing provides for an air gap of L = 10 mm.

FIND: (a) Heat loss per unit length of the tube by convection when a second shield of diameter 140 m is installed; compare the result to that for the single shield calculation of the example; and (b) The heat loss per unit length if the gap dimension is made L = 15 mm (rather than 10 mm). Do you expect the heat loss to increase or decrease?

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, and (b) Constant properties.

PROPERTIES: *Table A-4*, Air ($T_f = (T_s + T_\infty)/2 = 350$ K, 1 atm): $\nu = 20.92 \times 10^{-6}$ m^2/s, k = 0.030 W/m·K, Pr = 0.700.

ANALYSIS: (a) The thermal circuit representing the tube with two concentric cylindrical radiation shields having gap spacings L = 10 mm is shown above. The heat loss per unit length by convection is

$$q' = \frac{T_i - T_2}{R'_{g1} + R'_{g2}} \tag{1}$$

where the R'_g represents the thermal resistance of the annular gap (spacing). From Eq. 9.58, 59 and 60, find

$$R'_g = \frac{\ell n\left(D_o / D_i\right)}{2\pi k_{eff}} \tag{2}$$

$$\frac{k_{eff}}{k} = 0.386\left(\frac{Pr}{0.861 + Pr}\right)^{1/4}\left(Ra_c^*\right)^{1/4} \tag{3}$$

$$Ra_c^* = \frac{\left[\ell n\left(D_o / D_i\right)\right]^4}{L^3\left(D_i^{-3/5} + D_o^{-3/5}\right)^5} Ra_L \tag{4}$$

$$Ra_L = g\beta\left(T_o - T_i\right)L^3 / \alpha\nu \tag{5}$$

where the properties are evaluated at the average temperature of the bounding surfaces, $T_f = (T_i + T_o)/2$. Recognize that the above system of equations needs to be solved iteratively by initial guess values of T_1, or solved simultaneously using equation-solving software with a properties library. The results are tabulated below.

Continued …..

PROBLEM 9.101 (Cont.)

(b) Using the foregoing relations, the analyses can be repeated with L = 15 mm, so that D_i = 130 mm and D_2 = 160 mm. The results are tabulated below along with those from Example 9.5 for the single-shield configuration.

Shields	L(mm)	R'_{g1} (m·K/W)	R'_{g2} (m·K/W)	R'_{tot} (m·K/W)	T_1(°C)	q' (W/m)
1	10	0.7658	---	0.76	---	100
2	10	1.008	0.8855	1.89	74.8	44.9
2	15	0.9773	0.8396	1.82	74.3	46.8

COMMENTS: (1) The effect of adding the second shield is to more than double the thermal resistance of the shields to convection heat transfer.

(2) The effect of gap increase from 10 to 15 mm for the two-shield configuration is slight. Increasing L allows for greater circulation in the annular space, thereby reducing the thermal resistance.

(3) Note the difference in thermal resistances for the annular spaces R'_{g1} of the one-and two-shield configurations with L = 10 mm. Why are they so different (0.7658 vs. 1.008 m·K/W, respectively)?

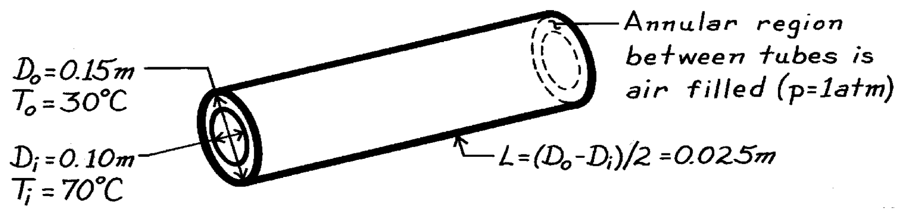
(4) See Example 9.5 for details on how to evaluate the properties for use with the correlation.

PROBLEM 9.102

KNOWN: Operating conditions of a concentric tube solar collector.

FIND: Convection heat transfer per unit length across air space between tubes.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Long tubes.

PROPERTIES: *Table A-4*, Air (T = 50°C, 1 atm): $\nu = 18.2 \times 10^{-6}$ m^2/s, k = 0.028 W/m·K, $\alpha = 25.9 \times 10^{-6}$ m^2/s, Pr = 0.71, β = 0.0031 K^{-1}.

ANALYSIS: For the annular region

$$Ra_L = \frac{g\,b\,(T_s - T_\infty)L^3}{na} = \frac{\left(9.8\,m/s^2\right)\left(0.0031K^{-1}\right)(70-30)\,°C\,(0.025m)^3}{\left(18.2\times10^{-6}m^2/s\right)\left(25.9\times10^{-6}m^2/s\right)}$$

$$Ra_L = 4.03\times10^4.$$

Hence, from Eq. 9.60,

$$Ra_c^* = \frac{\left[\ln(0.15/0.10)\right]^4}{(0.025m)^3\left[(0.10)^{-3/5}+(0.15)^{-3/5}\right]^5}\times4.03\times10^4 = 3857.$$

Accordingly, Eq. 9.59 may be used, in which case

$$k_{eff} = 0.386k\left(\frac{Pr}{0.861+Pr}\right)^{1/4}\left(Ra_c^*\right)^{1/4}$$

$$k_{eff} = 0.386(0.028W/m\cdot K)\left(\frac{0.71}{0.861+0.71}\right)^{1/4}(3857)^{1/4} = 0.07W/m\cdot K.$$

From Eq. 9.58, it then follows that

$$q' = \frac{2p\,k_{eff}}{\ln(D_o/D_i)}(T_i - T_o) = \frac{2p\,(0.07W/m\cdot K)}{\ln(0.15/0.10)}(70-30)\,°C = 43.4W/m.$$ <

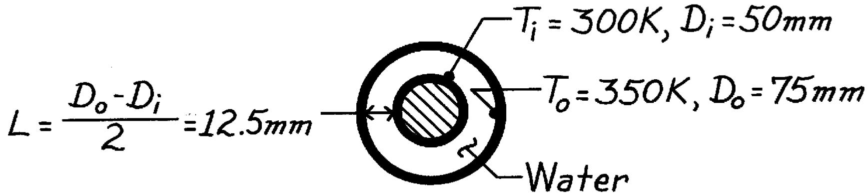
COMMENTS: An additional heat loss is related to thermal radiation exchange between the inner and outer surfaces.

PROBLEM 9.103

KNOWN: Annulus formed by two concentric, horizontal tubes with prescribed diameters and surface temperatures is filled with water.

FIND: Convective heat transfer rate per unit length of the tubes.

SCHEMATIC:



ASSUMPTIONS: Steady-state conditions.

PROPERTIES: *Table A-6*, Water (T_f = 325K): $\rho = (1/1.013 \times 10^{-3}\ m^3/kg)$, c_p = 4182 J/kg·K, $\mu = 528 \times 10^{-6}\ N{\cdot}s/m^2$, k = 0.645 W/m·K, Pr = 3.42, $\beta = 471.2 \times 10^{-6}\ K^{-1}$.

ANALYSIS: From Eqs. 9.58 and 9.59,

$$q' = \frac{2\pi\, k_{eff}}{\ell n\left(D_o/D_i\right)}\left(T_i - T_o\right) \qquad \frac{k_{eff}}{k} = 0.386\left(\frac{Pr}{0.861 + Pr}\right)^{1/4}\left(Ra_c^*\right)^{1/4}. \qquad (1,2)$$

From Eq. 9.60,

$$Ra_c^* = \left[\ell n\left(D_o/D_i\right)\right]^4 \cdot Ra_L / L^3\left(D_i^{-3/5} + D_o^{-3/5}\right)^5. \qquad (3)$$

The Rayleigh number follows from Eq. 9.25 using $\nu = \mu/\rho$ and $\alpha = k/\rho\, c_p$,

$$Ra_L = \frac{g\,\beta\left(T_o - T_i\right)L^3}{\nu\alpha} = \frac{9.8\,m/s^2 \times 471.2\times10^{-6}\,K^{-1}\left(350-300\right)K\times\left(12.5\times10^{-3}\,m\right)^3}{\left(528\times10^{-6}\,N\cdot s/m^2\times1.013\times10^{-3}\,\dfrac{m^3}{kg}\right)\times\left(\dfrac{0.645\,W/m\cdot K\times1.013\times10^{-3}\,m^3/kg}{4182\,J/kg\cdot K}\right)}$$

$$Ra_L = 5.396\times10^5.$$

Using this value in Eq. (3), find

$$Ra_c^* = \left[\ell n\left(\frac{75}{50}\right)\right]^4 \times 5.396\times10^5 / \left(12.5\times10^{-3}\,m\right)^3\left(\left[50\times10^{-3}\,m\right]^{-3/5} + \left[75\times10^{-3}\,m\right]^{-3/5}\right)^5 = 5.164\times10^4$$

and then evaluating Eq. (2), find

$$\frac{k_{eff}}{k} = 0.386\left(\frac{3.42}{0.861+3.42}\right)^{1/4}\left(5.164\times10^4\right)^{1/4} = 5.50$$

$$k_{eff} = 5.50\times0.645\,W/m\cdot K = 3.55\,W/m\cdot K.$$

The heat rate from Eq. (1) is then,

$$q' = \frac{2\pi\times3.55\,W/m\cdot K}{\ell n\left(75/50\right)}\left(350-300\right)K = 2.75\,kW/m. \qquad <$$

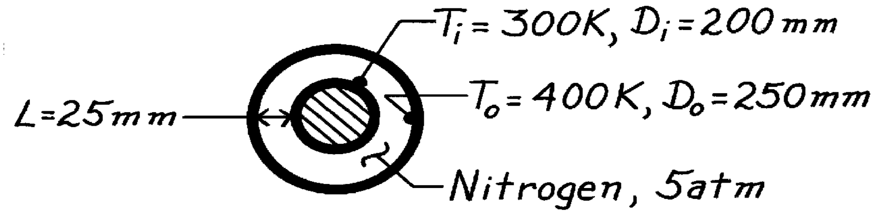
COMMENTS: Note that the Ra_c^* value is within prescribed limits for Eq. 9.50 or Eq. (3). Note also the characteristic length in Ra_L is L = (D_o – D_i)/2, the annulus gap.

PROBLEM 9.104

KNOWN: Annulus formed by two concentric, horizontal tubes with prescribed diameters and surface temperatures is filled with nitrogen at 5 atm.

FIND: Convective heat transfer rate per unit length of the tubes.

SCHEMATIC:



ASSUMPTIONS: (1) Thermophysical properties k, μ, and Pr, are independent of pressure, (2) Density is proportional to pressure, (3) Perfect gas behavior.

PROPERTIES: *Table A-4*, Nitrogen $\left(\overline{T} = \left(T_i + T_o\right)/2 = 350\text{K},\ 5\text{ atm}\right)$: k = 0.0293 W/m·K, μ = 200×10^{-7} N·s/m^2, ρ(5 atm) = 5 ρ (1 atm) = 5 × 0.9625 kg/m^3 = 4.813 kg/m^3, Pr = 0.711, ν = μ/ρ = 4.155×10^{-6} m^2/s, α = k/ρc = 0.0293 W/m·K/(4.813 kg/m^3 × 1042 J/kg·K) = 5.842×10^{-6} m^2/s.

ANALYSIS: From Eqs. 9.58 and 9.59

$$q' = \frac{2p\, k_{eff}}{\ell n\left(D_o/D_i\right)}\left(T_o - T_i\right) \qquad \frac{k_{eff}}{k} = 0.386\left(\frac{Pr}{0.861 + Pr}\right)^{1/4}\left(Ra_c^*\right)^{1/4}. \qquad (1,2)$$

From Eq. 9.60,

$$Ra_c^* = \left[\ell n\left(D_o/D_i\right)\right]^4 Ra_L / L^3\left(D_i^{-3/5} + D_o^{-3/5}\right)^5.$$

(3)

The Rayleigh number, Ra_L, follows from Eq. 9.25, and Ra_c^* from Eq. (3),

$$Ra_L = \frac{gb\left(T_o - T_i\right)L^3}{an} = \frac{9.8\text{m/s}^2\left(1/350\text{K}\right)\left(400 - 300\right)\text{K}\left(0.025\text{m}\right)^3}{5.842\times10^{-6}\text{m}^2/\text{s}\times4.155\times10^{-6}\text{m}^2/\text{s}} = 1.802\times10^6.$$

$$Ra_c^* = \left[\ell n\frac{250}{200}\right]^4 \times 1.802\times10^6/\left(0.025\text{m}\right)^3\left(0.20^{-3/5} + 0.25^{-3/5}\right)^5\text{m}^3 = 98{,}791$$

and then evaluating Eq. (2),

$$\frac{k_{eff}}{k} = 0.386\left(\frac{0.711}{0.861 + 0.711}\right)^{1/4}\left(98{,}791\right)^{1/4} = 5.61.$$

Hence, the heat rate, Eq. (1), becomes

$$q' = \frac{2p \times 5.61 \times 0.0293\text{W/m}\cdot\text{K}}{\ell n\left(250/200\right)}\left(400 - 300\right)\text{K} = 463\text{W/m}. \qquad <$$

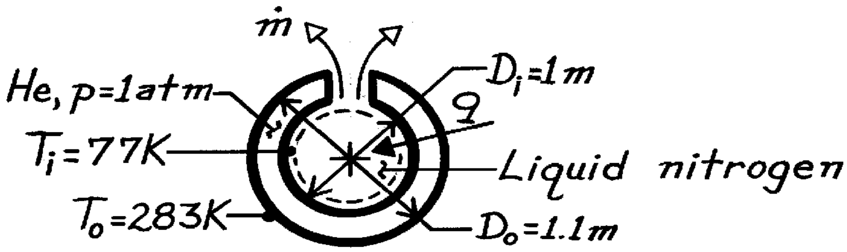
COMMENTS: Note that the heat loss by convection is nearly six times that for conduction. Radiation transfer is likely to be important for this situation. The effect of nitrogen pressure is to decrease ν which in turn increases Ra_L; that is, free convection heat transfer will increase with increase in pressure.

PROBLEM 9.105

KNOWN: Diameters and temperatures of concentric spheres.

FIND: Rate at which stored nitrogen is vented.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible radiation.

PROPERTIES: Liquid nitrogen (given): $h_{fg} = 2 \times 10^5$ J/kg; *Table A-4*, Helium ($\overline{T} = (T_i + T_o)/2$ = 180K, 1 atm): $\nu = 51.3 \times 10^{-6}$ m^2/s, k = 0.107 W/m·K, $\alpha = 76.2$ m^2/s, Pr = 0.673, $\beta = 0.00556$ K^{-1}.

ANALYSIS: Performing an energy balance for a control surface about the liquid nitrogen, it follows that

$$q = q_{conv} = \dot{m} h_{fg}.$$

From the Raithby and Hollands expressions for free convection between concentric spheres,

$$q_{conv} = k_{eff} p \left(D_i D_o / L\right)\left(T_o - T_i\right)$$

$$k_{eff} = 0.74 k \left[\Pr/\left(0.861 + \Pr\right)\right]^{1/4} \left(Ra_s^*\right)^{1/4}$$

where
$$Ra_s^* = \left[\frac{L}{\left(D_o D_i\right)^4} \frac{Ra_L}{\left(D_i^{-7/5} + D_o^{-7/5}\right)^5}\right]$$

$$Ra_L = \frac{g b \left(T_o - T_i\right) L^3}{na} = \frac{9.8\,\text{m/s}^2 \left(0.00556\,\text{K}^{-1}\right)\left(206\,\text{K}\right)\left(0.05\,\text{m}\right)^3}{\left(51.3 \times 10^{-6}\,\text{m}^2/\text{s}\right)\left(76.2 \times 10^{-6}\,\text{m}^2/\text{s}\right)} = 3.59 \times 10^5$$

$$Ra_s^* = \frac{0.05\,\text{m}}{\left(1.10\,\text{m}^2\right)^4} \frac{3.59 \times 10^5}{\left[1 + (1.1)^{-7/5}\right]^5 \text{m}^{-7}} = 529$$

$$k_{eff} = 0.74\left(0.107\,\text{W/m}\cdot\text{K}\right)\left[0.673/\left(0.861 + 0.673\right)\right]^{1/4} (529)^{1/4} = 0.309\,\text{W/m}\cdot\text{K}.$$

Hence, $$q_{conv} = \left(0.309\,\text{W/m}\cdot\text{K}\right) p \left(1.10\,\text{m}^2/0.05\,\text{m}\right) 206\ \text{K} = 4399\ \text{W}.$$

The rate at which nitrogen is lost from the system is therefore

$$\dot{m} = q_{conv}/h_{fg} = 4399\,\text{W}/2 \times 10^5\,\text{J/kg} = 0.022\ \text{kg/s}.$$ <

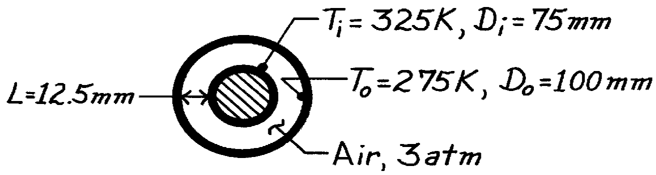
COMMENTS: The heat gain and mass loss are large. Helium should be replaced by a noncondensing gas of smaller k, or the cavity should be evacuated.

PROBLEM 9.106

KNOWN: Concentric spheres with prescribed surface temperatures.

FIND: Convection heat transfer rate.

SCHEMATIC:



ASSUMPTIONS: (1) Quiescent air in void space, (2) Density ~ pressure; other properties independent, (3) Perfect gas behavior.

PROPERTIES: *Table A-4*, Air (T_f = 300K, 3 atm): $\beta = 3.33 \times 10^{-3}$ K^{-1}, $\nu = 1/3 \times 15.89 \times 10^{-6}$ m^2/s, k = 0.263 W/m·K, $\alpha = 1/3 \times 22.5 \times 10^{-6}$ m^2/s, Pr = 0.707.

ANALYSIS: The heat transfer rate due to free convection is

$$q = k_{eff}\ p\left(D_i D_o / L\right)\left(T_i - T_o\right) \qquad (9.61)$$

where

$$\frac{k_{eff}}{k} = 0.74\left(\frac{Pr}{0.861 + Pr}\right)^{1/4}\left(Ra_s^*\right)^{1/4} \qquad (9.62)$$

$$Ra_s^* = \left[\frac{L}{\left(D_o D_i\right)^4}\ \frac{Ra_L}{\left(D_i^{-7/5} + D_o^{-7/5}\right)^5}\right] \qquad (9.63)$$

$$Ra_L = \frac{g\,b\left(T_i - T_o\right)L^3}{na}. \qquad (9.25)$$

Substituting numerical values in the above expressions, find that

$$Ra_L = \frac{9.8\,m/s^2 \times 3.333\times10^{-3}K^{-1}\left(325 - 375\right)K\left(12.5\times10^{-3}\right)^3 m^3}{(1/3)15.89\times10^{-6}m^2/s(1/3)22.5\times10^{-6}m^2/s} = 80{,}928$$

$$Ra_s^* = \left[\frac{\left(12.5\times10^{-3}\right)m}{\left(100\times10^{-3}\times75\times10^{-3}\right)^4 m^4}\cdot\frac{80{,}928}{\left(\left(75\times10^{-3}m\right)^{-7/5} + \left(100\times10^{-3}m\right)^{-7/5}\right)^5}\right] = 330.1$$

$$\frac{k_{eff}}{k} = 0.74\left(\frac{0.707}{0.861 + 0.707}\right)^{1/4}\left(330.1\right)^{1/4} = 2.58.$$

Hence, the heat rate becomes

$$q = \left(2.58\times0.263\frac{W}{m\cdot K}\right)p\left(\frac{75\times10^{-3}m\times100\times10^{-3}m}{12.5\times10^{-3}m}\right)\left(325 - 275\right)K = 64.0W. \qquad <$$

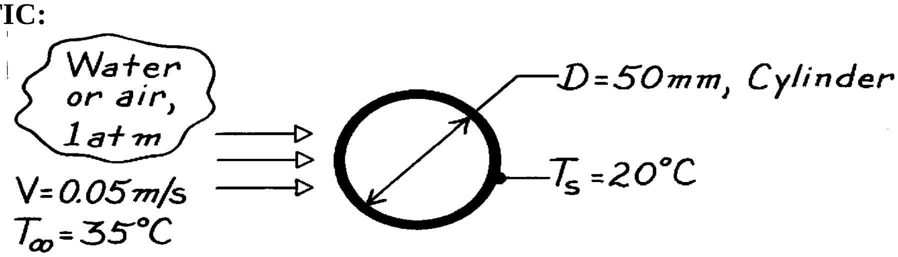
COMMENTS: Note the manner in which the thermophysical properties vary with pressure. Assuming perfect gas behavior, $\rho \sim p$. Also, k, μ and c_p are independent of pressure. Hence, Pr is independent of pressure, but $\nu = \mu/\rho \sim p^{-1}$ and $\alpha = k/\rho c \sim p^{-1}$.

PROBLEM 9.107

KNOWN: Cross flow over a cylinder with prescribed surface temperature and free stream conditions.

FIND: Whether free convection will be significant if the fluid is water or air.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties, (2) Combined free and forced heat transfer.

PROPERTIES: *Table A-6*, Water ($T_f = (T_\infty + T_s)/2 = 300K$): $\nu = \mu\, v_f = 855 \times 10^{-6}$ N·s/m^2 $\times$ 1.003 $\times 10^{-3}$ m^3/kg = 8.576×10^{-7} m^2/s, $\beta = 276.1 \times 10^{-6}$ K^{-1}; *Table A-4*, Air (300K, 1 atm): $\nu = 15.89 \times 10^{-6}$ m^2/s, $\beta = 1/T_f = 3.333 \times 10^{-3}$ K^{-1}.

ANALYSIS: Following the discussion of Section 9.9, the general criterion for delineating the relative significance of free and forced convection depends upon the value of Gr/Re^2. If free convection is significant.

$$Gr_D / Re_D^2 \geq 1 \tag{1}$$

where $$Gr_D = g\, b\, (T_\infty - T_s) D^3 / n^2 \quad \text{and} \quad Re_D = VD/n. \tag{2,3}$$

(a) When the surrounding fluid is *water*, find

$$Gr_D = 9.8\,m/s^2 \times 276.1 \times 10^{-6} K^{-1} (35-20) K (0.05m)^3 / \left(8.576 \times 10^{-7} m^2/s\right)^2 = 68,980$$

$$Re_D = 0.05\,m/s \times 0.05m / 8.576 \times 10^{-7} m^2/s = 2915$$

$$Gr_D / Re_D^2 = 68,980/2915^2 = 0.00812.$$ <

We conclude that since $Gr_D / Re_D^2 << 1$, free convection is not significant. It is apparent that forced convection dominates the heat transfer process.

(b) When the surrounding fluid is *air*, find

$$Gr_D = 9.8\,m/s^2 \times 3.333 \times 10^{-3} K^{-1} (35-20) K (0.05m)^3 / \left(15.89 \times 10^{-6} m^2/s\right)^2 = 242,558$$

$$Re_D = 0.05\,m/s \times 0.05m / 15.89 \times 10^{-6} m^2/s = 157$$

$$Gr_D / Re_D^2 = 242,558/157^2 = 9.8.$$ <

We conclude that, since $Gr_D / Re_D^2 >> 1$, free convection dominates the heat transfer process.

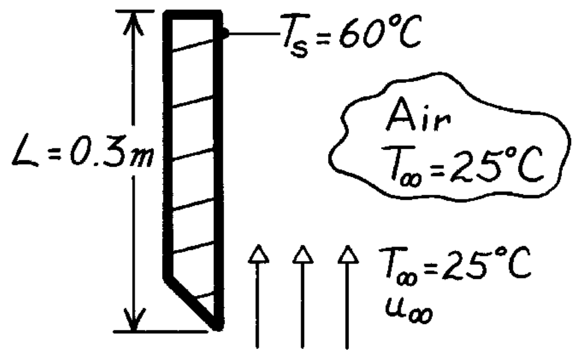
COMMENTS: Note also that for the air flow situation, surface radiation exchange is likely to be significant.

PROBLEM 9.108

KNOWN: Parallel air flow over a uniform temperature, heated vertical plate; the effect of free convection on the heat transfer coefficient will be 5% when $Gr_L / Re_L^2 = 0.08$.

FIND: Minimum vertical velocity required of air flow such that free convection effects will be less than 5% of the heat rate.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Criterion for combined free-forced convection determined from experimental results.

PROPERTIES: *Table A-4*, Air ($T_f = (T_s + T_\infty)/2 = 315K$, 1 atm): $\nu = 17.40 \times 10^{-6}$ m^2/s, $\beta = 1/T_f$.

ANALYSIS: To delineate flow regimes, according to Section 9.9, the general criterion for predominately forced convection is that

$$Gr_L / Re_L^2 << 1. \tag{1}$$

From experimental results, when $Gr_L / Re_L^2 \approx 0.08$, free convection will be equal to 5% of the total heat rate.

For the vertical plate using Eq. 9.12,

$$Gr_L = \frac{g\,b\,(T_1 - T_2)\,L^3}{n^2} = \frac{9.8\,m/s^2 \times 1/315K \times (60-25)K \times (0.3m)^3}{\left(17.40\times10^{-6} m^2/s\right)^2} = 9.711\times10^7.$$

(2)

For the vertical plate with forced convection,

$$Re_L = \frac{u_\infty L}{n} = \frac{u_\infty (0.3m)}{17.4\times10^{-6} m^2/s} = 1.724\times10^4\, u_\infty. \tag{3}$$

By combining Eqs. (2) and (3),

$$\frac{Gr_L}{Re_L^2} = \frac{9.711\times10^7}{\left[1.724\times10^4\, u_\infty\right]^2} = 0.08$$

find that

$$u_\infty = 2.02 m/s.$$ <

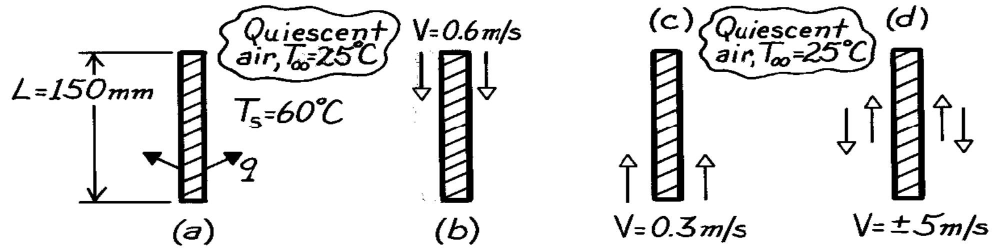
That is, when $u_\infty \geq 2.02$ m/s, free convection effects will not exceed 5% of the total heat rate.

PROBLEM 9.109

KNOWN: Vertical array of circuit boards 0.15m high with maximum allowable uniform surface temperature for prescribed ambient air temperature.

FIND: Allowable electrical power dissipation per board, $q'[W/m]$, for these cooling arrangements: (a) Free convection only, (b) Air flow downward at 0.6 m/s, (c) Air flow upward at 0.3 m/s, and (d) Air flow upward or downward at 5 m/s.

SCHEMATIC:



ASSUMPTIONS: (1) Uniform surface temperature, (2) Board horizontal spacing sufficient that boundary layers don't interfere, (3) Ambient air behaves as quiescent medium, (4) Perfect gas behavior.

PROPERTIES: *Table A-4*, Air ($T_f = (T_s + T_\infty)/2 \approx 315K$, 1 atm): $\nu = 17.40 \times 10^{-6}\ m^2/s$, $k = 0.0274\ W/m\cdot K$, $\alpha = 24.7 \times 10^{-6}\ m^2/s$, $Pr = 0.705$, $\beta = 1/T_f$.

ANALYSIS: (a) For *free convection* only, the allowable electrical power dissipation rate is

$$q' = \overline{h}_L(2L)(T_s - T_\infty) \tag{1}$$

where $\overline{h}_L$ is estimated using the appropriate correlation for free convection from a vertical plate. Find the Rayleigh number,

$$Ra_L = \frac{g\,b\,\Delta T\,L^3}{na} = \frac{9.8\,m/s^2\,(1/315K)(60-25)\,K(0.150m)^3}{17.4\times10^{-6}m^2/s \times 24.7\times10^{-6}m^2/s} = 8.551\times10^6. \tag{2}$$

Since $Ra_L < 10^9$, the flow is laminar. With Eq. 9.27 find

$$\overline{Nu}_L = \frac{\overline{h}L}{k} = 0.68 + \frac{0.670 Ra_L^{1/4}}{\left[1+(0.492/Pr)^{9/16}\right]^{4/9}} = 0.68 + \frac{\left(0.670\left[8.551\times10^6\right]^{1/4}\right)}{\left[1+(0.492/0.705)^{9/16}\right]^{4/9}} = 28.47 \tag{3}$$

$$\overline{h}_L = (0.0274\,W/m\cdot K/0.150m)\times 28.47 = 5.20\,W/m^2\cdot K.$$

Hence, the allowable electrical power dissipation rate is,

$$q' = 5.20\,W/m^2\cdot K(2\times0.150m)(60-25)°C = 54.6\,W/m.$$ <

(b) With *downward velocity* V = 0.6 m/s, the possibility of mixed forced-free convection must be considered. With $Re_L = VL/\nu$, find

$$\left(Gr_L/Re_L^2\right) = \left(\frac{Ra_L}{Pr}/Re_L^2\right) \tag{4}$$

$$\left(Gr_L/Re_L^2\right) = \left(8.551\times10^6/0.705\right)/\left(0.6\,m/s\times0.150m/17.40\times10^{-6}m^2/s\right)^2 = 0.453.$$

Continued …..

Since $\left(Gr_L / Re_L^2\right) \sim 1$, flow is mixed and the average heat transfer coefficient may be found from a correlating equation of the form

$$\overline{Nu}^n = Nu_F^n \pm Nu_N^n \tag{5}$$

where n = 3 for the vertical plate geometry and the minus sign is appropriate since the natural convection (N) flow opposes the forced convection (F) flow. For the forced convection flow, Re_L = 5172 and the flow is laminar; using Eq. 7.31,

$$\overline{Nu}_F = 0.664\ Re_L^{1/2}\ Pr^{1/3} = 0.664(5172)^{1/2}(0.705)^{1/3} = 42.50. \tag{6}$$

Using $\overline{Nu}_N = 28.47$ from Eq. (3), Eq. (5) now becomes

$$\overline{Nu}^3 = \left(\frac{\bar{h}L}{k}\right)^3 = (42.50)^3 - (28.47)^3 \qquad \overline{Nu} = 37.72$$

$$\bar{h} = \left(\frac{0.0274\,W/m\cdot K}{0.150m}\right) \times 37.72 = 6.89\,W/m^2\cdot K.$$

Substituting for $\bar{h}$ into the rate equation, Eq. (1), the allowable power dissipation with a downward velocity of 0.6 m/s is

$$q' = 6.89\,W/m^2\cdot K\,(2\times 0.150m)\,(60-25)°C = 72.3\,W/m. \qquad <$$

(c) With an *upward velocity* V = 0.3 m/s, the positive sign of Eq. (5) applies since the N-flow is assisting the F-flow. For forced convection, find

$$Re_L = VL/\nu = 0.3\,m/s \times 0.150\,m/\left(17.40\times 10^{-6}\,m^2/s\right) = 2586.$$

The flow is again laminar, hence Eq. (6) is appropriate.

$$\overline{Nu}_F = 0.664(2586)^{1/2}(0.705)^{1/3} = 30.05.$$

From Eq. (5), with the positive sign, and $\overline{Nu}_N$ from Eq. (4),

$$\overline{Nu}^3 = (30.05)^3 + (28.47)^3 \quad \text{or} \quad \overline{Nu} = 36.88 \quad \text{and} \quad \bar{h} = 6.74\,W/m^2\cdot K.$$

From Eq. (1), the allowable power dissipation with an upward velocity of 0.3 m/s is

$$q' = 6.74\,W/m^2\cdot K\,(2\times 0.150m)\,(60-25)°C = 70.7\,W/m. \qquad <$$

(d) With a *forced convection* velocity V = 5 m/s, very likely forced convection will dominate. Check by evaluating whether $\left(Gr_L\ Re_L^2\right) << 1$ where Re_L = VL/ν = 5 m/s × 0.150m/(17.40 × 10^{-6} m^2/s) = 43,103. Hence,

$$\left(Gr_L / Re_L^2\right) = \left(\frac{Ra_L}{Pr} / Re_L^2\right) = \left(8.551\times 10^6 / 0.705\right)/43{,}103^2 = 0.007.$$

The flow is not mixed, but pure forced convection. Using Eq. (6), find

$$\bar{h} = (0.0274\,W/m\cdot K/0.150m)\,0.664(43{,}103)^{1/2}(0.705)^{1/3} = 22.4\,W/m^2\cdot K$$

and the allowable dissipation rate is

$$q' = 22.4\,W/m^2\cdot K\,(2\times 0.150m)\,(60-25)°C = 235\,W/m. \qquad <$$

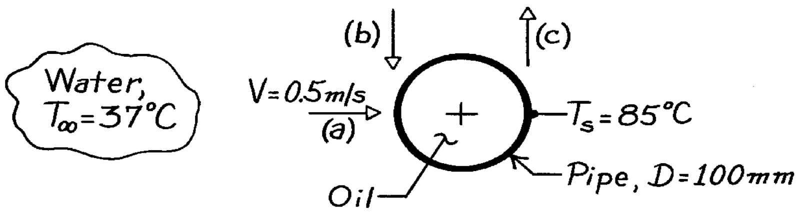
COMMENTS: Be sure to compare dissipation rates to see relative importance of mixed flow conditions.

PROBLEM 9.110

KNOWN: Horizontal pipe passing hot oil used to heat water.

FIND: Effect of water flow direction on the heat rate.

SCHEMATIC:



ASSUMPTIONS: (1) Uniform pipe surface temperature, (2) Constant properties.

PROPERTIES: *Table A-6*, Water ($T_f = (T_s + T_\infty)/2 \approx 335$K): $\nu = \mu_f\, v_f = 4.625 \times 10^{-7}$ m^2/s, k = 0.656 W/m·K, $\alpha = k\, v_f/c_p = 1.595 \times 10^{-7}$ m^2/s, Pr = 2.88, $\beta = 535.5 \times 10^{-6}$ K^{-1}; *Table A-6*, Water (T_∞ = 310K): $\nu = \mu_f\, v_f = 6.999 \times 10^{-7}$ m^2/s, k = 0.028 W/m·K, Pr = 4.62; *Table A-6*, Water (T_s = 358K): Pr = 2.07

ANALYSIS: The rate equation for the flow situations is of the form

$$q' = \bar{h}\,(\pi D)\,(T_s - T_\infty).$$

To determine whether mixed flow conditions are present, evaluate $\left(Gr_D / Re_D^2\right)$.

$$Gr_D = \frac{g\,\beta\Delta T\, D^3}{\nu^2} = \frac{9.8\,\text{m/s}^2 \times 535.5\times10^{-6}\,\text{K}^{-1}(85-37)\,\text{K}\,(0.100\text{m})^3}{\left(4.625\times10^{-7}\,\text{m}^2/\text{s}\right)^2} = 1.178\times10^9$$

$$Re_D = VD/\nu = 0.5\,\text{m/s}\times0.100\text{m}/6.999\times10^{-7}\,\text{m}^2/\text{s} = 7.144\times10^4.$$

It follows that $\left(Gr_D / Re_D^2\right) = 0.231$; since this ratio is of order unity, the flow condition is mixed. Using Eq. 9.64, $\overline{Nu}^n = \overline{Nu}_F^n \pm \overline{Nu}_N^n$ and for the three flow arrangements,

(a) Transverse flow: (b) Opposing flow: (c) Assisting flow:

$$\overline{Nu}^4 = \overline{Nu}_F^4 + \overline{Nu}_N^4 \qquad \overline{Nu}^3 = \overline{Nu}_F^3 - \overline{Nu}_N^3$$

$$\overline{Nu}^3 = \overline{Nu}_F^3 + \overline{Nu}_N^3$$

For *natural convection* from the cylinder, use Eq. 9.34 with Ra = Gr·Pr.

$$\overline{Nu}_N = \left\{0.60 + \frac{0.387 Ra_D^{1/6}}{\left[1+(0.559/Pr)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.60 + \frac{0.387\left(1.178\times10^9\times2.88\right)^{1/6}}{\left[1+(0.559/2.88)^{9/16}\right]^{8/27}}\right\}^2 = 201.2$$

For *forced convection* in cross flow over the cylinder, from Table 7-4 use

$$\overline{Nu}_F = C\, Re_D^m\, Pr^n\, (Pr/Pr_s)^{1/4}$$

$$\overline{Nu}_F = 0.26\left(7.144\times10^4\right)^{0.6}(4.62)^{0.37}(4.62/2.07)^{1/4} = 457.5$$

Continued …..

where n = 0.37 since Pr ≤ 10. The results of the calculations are tabulated.

Flow	$\overline{Nu}$	$\bar{h}\left(W/m^2\cdot K\right)$	$q'\times10^{-4}\,(W/m)$
(a) Transverse	461.7	3029	4.57
(b) Opposing	444.1	2913	4.39
(c) Assisting	470.1	3083	4.65

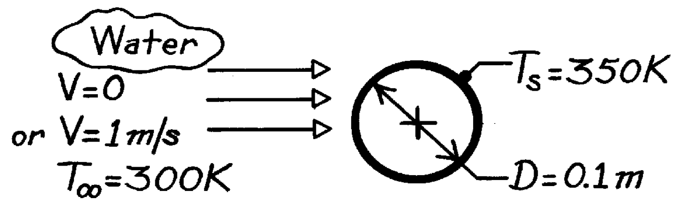
COMMENTS: Note that the flow direction has a minor effect (<6%) for these conditions.

PROBLEM 9.111

KNOWN: Diameter and surface temperature of long tube housing heat dissipating electronic components. Temperature of cooling water.

FIND: (a) Heat dissipation per unit length to quiescent water, (b) Percent enhancement for imposed cross flow.

SCHEMATIC:



ASSUMPTIONS: Constant properties evaluated at T_f.

PROPERTIES: *Table A-6*, Water (325K): ρ = 987 kg/m^3, $\mu = 528 \times 10^{-6}$ kg/s·m, $\nu = \mu/\rho = 0.535 \times 10^{-6}$ m^2/s, k = 0.645 W/m·K, Pr = 3.42, $\beta = 471 \times 10^{-6}$ K^{-1}.

ANALYSIS: (a) With

$$\mathrm{Ra_D} = \frac{b g \Delta T D^3}{n^2}\mathrm{Pr} = \frac{9.8\,\mathrm{m/s^2} \times 471\times10^{-6}\,\mathrm{K^{-1}}(50\ \mathrm{K})(0.1\mathrm{m})^3\, 3.42}{\left(0.535\times10^{-6}\mathrm{m^2/s}\right)^2} = 2.76\times10^9$$

use the Churchill and Chu correlation

$$\overline{\mathrm{Nu}}_D = \left\{0.60 + \frac{0.387\mathrm{Ra}_D^{1/6}}{\left[1+(0.559/\mathrm{Pr})^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.60 + \frac{0.387\left(2.76\times10^9\right)^{1/6}}{\left[1+(0.559/3.42)^{9/16}\right]^{8/27}}\right\}^2 = 191$$

$$\bar{h} = \overline{\mathrm{Nu}}_D\,(k/D) = 191(0.645\mathrm{W/m\cdot K}/0.1\mathrm{m}) = 1232\mathrm{W/m^2\cdot K}.$$

Hence, $$q' = p D\bar{h}\left(T_s - T_\infty\right) = p\,(0.1\mathrm{m})1232\mathrm{W/m^2\cdot K}\,(350-300)\mathrm{K} = 19.4\mathrm{kW/m}.$$ <

(b) Using the Hilpert correlation (for which properties are evaluated at T_f), it follows that, for pure forced convection,

$$\mathrm{Re}_D = \frac{VD}{n} = \frac{1\,\mathrm{m/s}\times0.1\mathrm{m}}{0.535\times10^{-6}\mathrm{m^2/s}} = 1.87\times10^5.$$

Hence, using the Hilpert correlation,

$$\overline{\mathrm{Nu}}_{D,F} = 0.027\mathrm{Re}_D^{0.805}\,\mathrm{Pr}^{1/3} = 0.027\left(1.87\times10^5\right)^{0.805}(3.42)^{1/3} = 713.$$

For mixed convection with n = 4,

$$\overline{\mathrm{Nu}}^n = \overline{\mathrm{Nu}}_F^n + \overline{\mathrm{Nu}}_N^n = (713)^4 + (191)^4 = 2.59\times10^{11} \quad \overline{\mathrm{Nu}} = 714$$

$$\bar{h} = \overline{\mathrm{Nu}}(k/D) = 714(0.645\mathrm{W/m\cdot K}/0.1\mathrm{m}) = 4605\mathrm{W/m^2\cdot K}.$$

Hence, $$q' = \bar{h} p D\left(T_s - T_\infty\right) = 4605\mathrm{W/m^2\cdot K}\times p\,(0.1\mathrm{m})(350-300)\mathrm{K} = 72.3\mathrm{kW/m}.$$ <

The cross flow enhances the heat rate by a factor of 72.3/19.4 = 3.7.

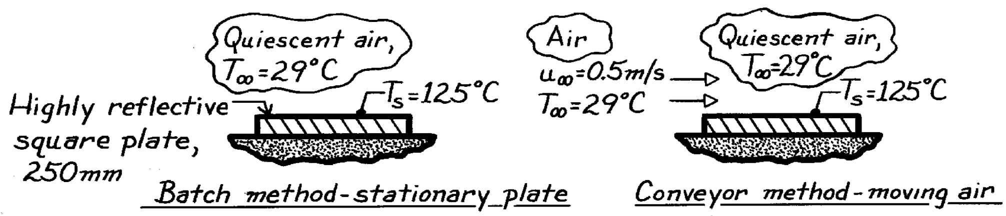
COMMENTS: (1) With V = 1 m/s, heat transfer is dominated by forced convection.

PROBLEM 9.112

KNOWN: Horizontal square panel removed from an oven and cooled in quiescent or moving air.

FIND: Initial convection heat rates for both methods of cooling.

SCHEMATIC:



ASSUMPTIONS: (1) Quasi-steady state conditions, (2) Backside of plates insulated, (3) Air flow is in the length-wise (not diagonal) direction, (4) Constant properties, (5) Radiative exchange negligible.

PROPERTIES: *Table A-4*, Air ($T_f = (T_\infty + T_s)/2 = 350$K, 1 atm): $\nu = 20.92 \times 10^{-6}$ m^2/s, k = 0.030 W/m·K, $\alpha = 29.9 \times 10^{-6}$ m^2/s, Pr = 0.700, $\beta = 1/T_f$.

ANALYSIS: The initial heat transfer rate from the plates by convection is given by the rate equation $q = \bar{h} A_s (T_s - T_\infty)$. Test for the existence of combined free-forced convection by calculation of the ratio Gr_L / Re_L^2. Use the same characteristic length in both parameters, L = 250mm, the side length.

$$Gr_L = \frac{g\,b\,\Delta T\,L^3}{n^2} = \frac{9.8\text{m/s}^2 (1/350\text{K})(125-29)\text{K}(0.250\text{m})^3}{\left(20.92\times10^{-6}\text{m/s}^2\right)^2} = 9.597\times10^7$$

$$Re_L = u_\infty L/n = 0.5\text{m/s}\times0.250\text{m}/\left(20.92\times10^{-6}\text{m}^2/\text{s}\right) = 5.975\times10^3.$$

Since $Gr_L / Re_L^2 = 2.69$ flow is mixed. For the *stationary plate*, $Ra_L = Gr_L \cdot Pr = 6.718 \times 10^7$ and Eq. 9.31 is the appropriate correlation,

$$\overline{Nu}_N = \frac{\bar{h}L}{k} = 0.15 Ra_L^{1/3} = 0.15\left(6.718\times10^7\right)^{1/3} = 60.9$$

$$\bar{h} = (0.030\text{W/m}\cdot\text{K}/0.250\text{m})\times60.9 = 7.31\text{W/m}^2\cdot\text{K}.$$

$$q = 7.31\text{W/m}^2\cdot\text{K}\times(0.250\text{m})^2(125-29)\text{K} = 43.9\text{W}.$$ <

For the *plate with moving air*, $Re_L = 5.975 \times 10^3$ and the flow is laminar.

$$\overline{Nu}_F = 0.664\,Re_L^{1/2}\,Pr^{1/3} = 0.664\left(5.975\times10^3\right)^{1/2}(0.700)^{1/3} = 45.6.$$

For combined free-forced convection, use the correlating equation with n = 7/2.

$$\overline{Nu}^{7/2} = \overline{Nu}_F^{7/2} + \overline{Nu}_N^{7/2} = (45.6)^{7/2} + (60.9)^{7/2} \qquad \overline{Nu} = 66.5.$$

$$\bar{h} = \overline{Nu}\,k/L = 66.5(0.030\text{W/m}\cdot\text{K}/0.25\text{m}) = 7.99\text{W/m}^2\cdot\text{K}$$

$$q = 7.99\text{W/m}^2\cdot\text{K}(0.250\text{m})^2(125-29)\text{K} = 47.9\text{W}.$$ <

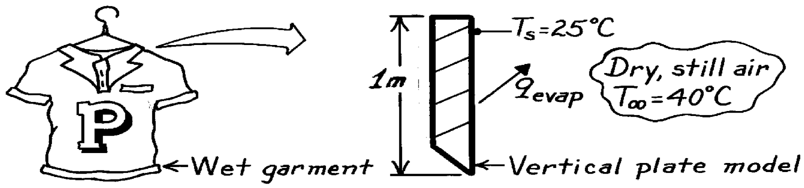
COMMENTS: (1) The conveyor method provides only slight enhancement of heat transfer.

PROBLEM 9.113

KNOWN: Wet garment at 25°C hanging in a room with still, dry air at 40°C.

FIND: Drying rate per unit width of garment.

SCHEMATIC:



ASSUMPTIONS: (1) Analogy between heat and mass transfer applies, (2) Water vapor at garment surface is saturated at T_s, (3) Perfect gas behavior of vapor and air.

PROPERTIES: *Table A-4*, Air ($T_f \approx (T_s + T_\infty)/2 = 305K$, 1 atm): $\nu = 16.39 \times 10^{-6}$ m^2/s; *Table A-6*, Water vapor (T_s = 298K, 1 atm): $p_{A,s}$ = 0.0317 bar, $\rho_{A,s} = 1/v_f$ = 0.02660 kg/m^3; *Table A-8*, Air-water vapor (305 K): $D_{AB} = 0.27 \times 10^{-4}$ m^2/s, Sc = ν/DAB = 0.607.

ANALYSIS: The drying rate per unit width of the garment is

$$\dot{m}'_A = \bar{h}_m \cdot L\left(r_{A,s} - r_{A,\infty}\right)$$

where $\bar{h}_m$ is the mass transfer coefficient associated with a vertical surface that models the garment. From the heat and mass transfer analogy, Eq. 9.24 and Fig. 9.6 yield

$$\overline{Sh}_L = 0.59\left(Gr_L\, Sc\right)^{1/4}$$

where GrL = $g\Delta\rho L^3/\rho\nu^2$ and $\Delta\rho = \rho_s - \rho_\infty$. Since the still air is dry, $\rho_\infty = \rho_{B,\infty} = p_{B,\infty}/R_B T_\infty$, where $R_B = \Re/\mathcal{M}_B = 8.314 \times 10^{-2}$ m^3·bar/kmol·K/29 kg/kmol = 0.00287 m^3·bar/kg·K. With $p_{B,\infty}$ = 1 atm = 1.0133 bar,

$$r_\infty = \frac{1.0133 \text{ bar}}{0.00287 \text{ m}^3 \cdot \text{bar/kg} \cdot \text{K} \times 313 \text{ K}} = 1.1280 \text{ kg/m}^3$$

The density of the air/vapor mixture at the surface is $\rho_s = \rho_{A,s} + \rho_{B,s}$. With $p_{B,s}$ = 1 atm – $p_{A,s}$ = 1.0133 bar – 0.0317 bar = 0.9816 bar,

$$r_{B,s} = \frac{p_{B,s}}{R_B T_s} = \frac{0.9816 \text{ bar}}{0.00287\left(\text{m}^3 \cdot \text{bar/kg} \cdot \text{K}\right) \times 298 \text{ K}} = 1.1477 \text{ kg/m}^3$$

Hence, ρ_s = (0.0266 + 1.1477) kg/m^3 = 1.1743 kg/m^3 and $\rho = (\rho_s + \rho_\infty)/2$ = 1.512 kg/m^3. The Grashof number is then

$$Gr_L = \frac{9.8 \text{ m/s}^2 \times (1.1743 - 1.1280) \text{ kg/m}^3 (1 \text{ m})^3}{1.1512 \text{ kg/m}^3 \times \left(16.39 \times 10^{-6} \text{m}^2/\text{s}\right)^2} = 1.467 \times 10^9$$

and (Gr_L Sc) = 8.905×10^8. The convection coefficient is then

$$\bar{h}_m = \frac{D_{AB}}{L}\overline{Sh}_L = \frac{0.27 \times 10^{-4} \text{m}^2/\text{s}}{1 \text{ m}} \times 0.59\left(8.905 \times 10^8\right)^{1/4} = 0.00275 \text{ m/s}$$

The drying rate is then

$$\dot{m}'_A = 2.750 \times 10^{-3} \text{m/s} \times 1.0 \text{m}\left(0.0226 - 0\right) \text{kg/m}^3 = 6.21 \times 10^{-5} \text{ kg/s} \cdot \text{m}.$$ <

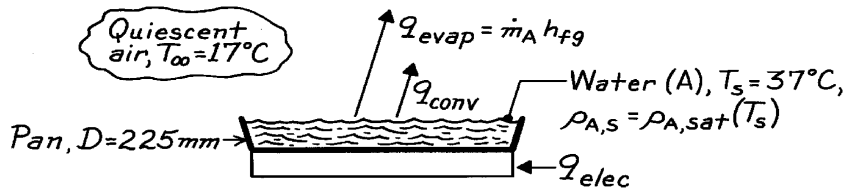
COMMENTS: Since $\rho_s > \rho_\infty$, the buoyancy driven flow *descends* along the garment.

PROBLEM 9.114

KNOWN: Circular pan of water at 37°C exposed to dry, still air at 17°C.

FIND: Evaporation rate and total heat transfer rate from the pan.

SCHEMATIC:



ASSUMPTIONS: (1) Dry room air, (2) Negligible radiation, (3) Water vapor and air behave as perfect gases.

PROPERTIES: *TableA-4*, Air ($T_f = (T_s + T_\infty)/2$=300K, 1 atm): $\nu = 15.89 \times 10^{-6}$ m^2/s, k = 0.0263 W/m·K, Pr = 0.707, $\beta = 1/T_f$; *Table A-6*, Water (T_s = 310K): $\rho_{A,s} = \rho_{A,sat} = 1/v_g = 0.04361$ kg/m^3, $p_{A,s}$ = 0.0622 bar, h_{fg} = 2414 kJ/kg; *Table A-8*, Air-water vapor (1 atm, T_f = 300K): $D_{AB} \approx 0.26 \times 10^{-4}$ m^2/s, Sc = ν/D_{AB} = 0.611

ANALYSIS: The evaporation rate and total heat transfer rate from the pan are

$$\dot{m}_A = \overline{h}_m A_s \left(r_{A,s} - r_{A,\infty} \right) = \overline{h}_m A_s \, r_{A,sat} \left(T_s \right) \qquad q = q_{conv} + q_{evap} = \overline{h} \, A_s \left(T_s - T_\infty \right) + \dot{m}_A \, h_{fg} \qquad (1,2)$$

where $A_s = \pi D^2/4$. The convection coefficients can be estimated from the free convection correlation for a horizontal, circular plate with $L = A_s/P = D/4 = 0.0563$ m.

To determine the appropriate convection correlation, we must first determine ρ_∞ and ρ_s. Since $\phi_\infty = 0$ and the gas constant for air is $R_B = \mathfrak{R}/\mathcal{M}_B$ =8.314 × 10-2 m^3·bar/kmol·K/29 kg/kmol = 0.00287 m^3·bar/kg·K,

$$r_\infty = \frac{p_{B,\infty}}{R_B T_\infty} = \frac{1.0133\text{bar}}{0.00287 \text{ m}^3 \cdot \text{bar/kg} \cdot \text{K} \times 290 \text{ K}} = 1.2175 \text{ kg/m}^3$$

At the surface, $\rho_s = \rho_{A,s} + \rho_{B,s}$. With $p_{B,s}$ = 1.0133 bar – $p_{A,s}$ = 0.9511 bar, $\rho_{B,s} = p_{B,s}/R_B T_s$ = 1.0690 kg/m3 and

$$r_s = r_{A,s} + r_{B,s} = \left(0.0436 + 1.0690\right) \text{kg/m}^3 = 1.1126 \text{ kg/m}^3$$

From Eq. 9.65, with $\rho = (\rho_s + \rho_\infty)/2 = 1.1651$ kg/m^3, the Grashof number is

$$Gr_L = \frac{g\left(r_\infty - r_s\right)L^3}{r n^2} = \frac{9.8 \text{ m/s}^2 \left(0.0524 \text{ kg/m}^3\right)\left(0.0563 \text{ m}\right)^3}{1.1651 \text{kg/m}^3 \left(15.89 \times 10^{-6} \text{m}^2/\text{s}\right)^2} = 3.12 \times 10^5$$

in which case $Ra_L = Gr_L \, Pr = 2.21 \times 10^5$ and $Gr_L \, Sc = 1.91 \times 10^5$. From Eq. 9.30,

$$h = \left(\frac{k}{L}\right) 0.54 \, Ra_L^{1/4} = \left(\frac{0.0263 \text{ W/m} \cdot \text{K}}{0.0563 \text{ m}}\right) 0.54 \left(2.21 \times 10^5\right)^{1/4} = 5.47 \text{ W/m}^2 \cdot \text{K}$$

Continued …..

PROBLEM 9.114 (Cont.)

and from its mass transfer analog,

$$h_m = \frac{D_{AB}}{L} 0.54\left(Gr_L\, Sc\right)^{1/4} = \left(\frac{0.26\times10^{-4}\,m^2/s}{0.0563\ m}\right)0.54\left(1.91\times10^5\right)^{1/4} = 0.00521\,m/s$$

Substituting numerical values into the rate equations, Eqs. (1) and (2), find

$$\dot{m}_A = 5.21\times10^{-3}\,m/s\left(p\left(0.225m\right)^2/4\right)\times0.04361\,kg/m^3 = 9.034\times10^{-6}\,kg/s = 32.5\,g/h$$ <

$$q = 5.47\ W/m^2\cdot K\left(p\left(0.225m\right)^2/4\right)\left(37-17\right)K + 9.034\times10^{-6}\,kg/s\times2414\times10^3\,J/kg$$

$$q = \left(4.4 + 21.8\right)W = 26.2\ \ W.$$ <

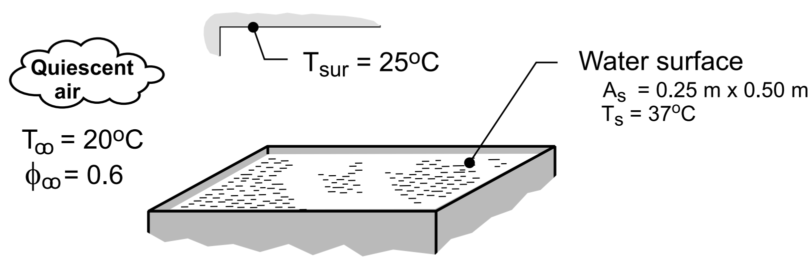
COMMENTS: As expected, the heat loss is more strongly influenced by the loss of latent energy.

PROBLEM 9.115

KNOWN: A water bath maintained at a uniform temperature of 37°C with top surface exposed to draft-free air and uniform temperature walls in a laboratory.

FIND: (a) The heat loss from the surface of the bath by radiation exchange with the surroundings; (b) Calculate the Grashof number using Eq. 9.65 with a characteristic length L that is appropriate for the exposed surface of the water bath; (c) Estimate the free convection heat transfer coefficient using the result for Gr_L obtained in part (b); (d) Invoke the heat-mass analogy and use an appropriate correlation to estimate the mass transfer coefficient using Gr_L; calculate the water evaporation rate on a daily basis and the heat loss by evaporation; and (e) Calculate the total heat loss from the surface and compare relative contributions of the sensible, latent and radiative effects. Review assumptions made in your analysis, especially those relating to the heat-mass analogy.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Laboratory air is quiescent, (3) Laboratory walls are isothermal and large compared to water bath exposed surface, (4) Emissivity of the water surface is 0.96, (5) Heat-mass analogy is applicable, and (6) Constant properties.

PROPERTIES: *Table A-6*, Water vapor (T_∞ = 293 K): $\rho_{A,\infty,sat}$ = 0.01693 kg/m^3; (T_s = 310 K): $\rho_{A,s}$ = 0.04361 kg/m^3, $h_{fg} = 2.414 \times 10^6$ J/kg; *Table A-4*, Air (T_∞ = 293 K, 1 atm): $\rho_{B,\infty}$ = 1.194 kg/m^3; (T_s = 310 K, 1 atm): $\rho_{B,s}$ = 1.128 kg/m^3; ($T_f = (T_s + T_\infty)/2$ = 302 K, 1 atm): $\nu_B = 1.604 \times 10^{-5}$ m^2/s, k = 0.0270 W/m·K, Pr = 0.706; *Table A-8*, Water vapor-air (T_f = 302 K, 1 atm): $D_{AB} = 0.24 \times 10^{-4}$ m^2/s $(302/298)^{3/2} = 2.65 \times 10^{-5}$ m^2/s.

ANALYSIS: (a) Using the linearized form of the radiation exchange rate equation, the heat rate and radiation coefficient can be estimated.

$$h_{rad} = \varepsilon\sigma\left(T_s + T_{sur}\right)\left(T_s^2 + T_{sur}^2\right) \tag{1}$$

$$h_{rad} = 0.96\sigma\left(310 + 298\right)\left(310^2 + 298^2\right)K^3 = 6.12\ W/m^2 \cdot K$$ <

$$q_{rad} = h_{rad}A_s\left(T_s - T_{sur}\right) \tag{2}$$

$$q_{rad} = 6.12\ W/m^2 \cdot K \times \left(0.25 \times 0.50\right)m^2 \times \left(37 - 25\right)K = 9.18\ W$$

(b) The general form of the Grashof number, Eq. 9.65, applied to natural convection flows driven by concentration gradients

$$Gr_L = g\left(\rho_\infty - \rho_s\right)L^3 / \rho\nu^2 \tag{3}$$

where L is the characteristic length defined in Eq. 9.29 as L = A_s/P, where A_s and P are the exposed surface area and perimeter, respectively; ρ_s and ρ_∞ are the density of the mixture at the surface and in the quiescent fluid, respectively; and, ρ is the mean boundary layer density, $(\rho_\infty + \rho_s)/2$, and ν is the kinematic viscosity of fluid B, evaluated at the film temperature $T_f = (T_s + T_\infty)/2$. Using the property values from above,

Continued …..

$$\rho_s = \rho_{A,s} + \rho_{B,s} = (0.04361 + 1.128)\,\mathrm{kg/m^3} = 1.1716\ \mathrm{kg/m^3}$$

$$\rho_\infty = \rho_{A,\infty} + \rho_{B,\infty} = \phi_\infty \rho_{A,\infty,sat} + \rho_{B,\infty}$$

$$\rho_\infty = (0.6 \times 0.01693 + 1.194)\,\mathrm{kg/m^3} = 1.2042\ \mathrm{kg/m^3}$$

$$\rho = (\rho_s + \rho_\infty)/2 = 1.4601\ \mathrm{kg/m^3}$$

Substituting numerical values in Eq. (3), find the Grashof number.

$$Gr_L = \frac{9.8\ \mathrm{m/s^2}\,(1.2042 - 1.1716)\,\mathrm{kg/m^3} \times (0.0833\ \mathrm{m})^3}{1.4601\ \mathrm{kg/m^3}\left(1.604 \times 10^{-5}\,\mathrm{m^2/s}\right)^2}$$

$$Gr_L = 4.916 \times 10^5 \qquad <$$

where the characteristic length is defined by Eq. 9.29,

$$L = A_s / P = (0.25 \times 0.5)\,\mathrm{m^2} / 2(0.25 + 0.50)\,\mathrm{m} = 0.0833\ \mathrm{m}$$

(c) The free convection heat transfer coefficient for the horizontal surface, Eq. 9.30, for *upper surface of heated plate*, is estimated as follows:

$$Ra_L = Gr_L\,Pr_L = 4.916 \times 10^5 \times 0.706 = 3.471 \times 10^5$$

$$\overline{Nu}_L = \frac{\bar{h}L}{k} = 0.54\ Ra_L^{1/4} = 13.11$$

$$\bar{h} = 13.11 \times 0.0270\ \mathrm{W/m\cdot K} / 0.0833\ \mathrm{m} = 4.25\ \mathrm{W/m^2 \cdot K} \qquad <$$

(d) Invoking the heat-mass analogy, the mass transfer coefficient is estimated as follows,

$$Ra_{L,m} = Gr_L Sc = 4.916 \times 10^5 \times 0.605 = 2.975 \times 10^5$$

where the Schmidt number is given as

$$Sc = \nu / D_{AB} = 1.604 \times 10^{-5}\,\mathrm{m^2/s} / 2.65 \times 10^{-5}\,\mathrm{m^2/s} = 0.605$$

The correlation has the form

$$\overline{Sh}_L = \frac{\bar{h}_m L}{D_{AB}} = 0.54\ Ra_{L,m}^{1/4} = 12.61$$

$$\bar{h}_m = 12.61 \times 2.65 \times 10^{-5}\,\mathrm{m^2/s} / 0.0833\ \mathrm{m} = 0.00401\ \mathrm{m/s} \qquad <$$

The water evaporation rate on a daily basis is

$$n_A = \bar{h}_m A_s \left(\rho_{A,sat} - \rho_{A,\infty}\right)$$

$$n_A = 0.00401\ \mathrm{m/s}\,(0.25 \times 0.50)\,\mathrm{m^2}\,(0.04361 - 0.6 \times 0.01693)\,\mathrm{kg/m^3}$$

Continued …..

$$nA = 1.677 \times 10^{-5} \text{kg/s} = 1.45 \text{ kg/day}$$ <

and the *heat loss by evaporation* is

$$q_{evap} = n_A \, h_{fg} = 1.677 \times 10^{-5} \text{kg/s} \times 2.414 \times 10^{6} \text{J/kg} = 40.5 \text{ W}$$ <

(e) The *convective heat loss* is that of free convection,

$$q_{cv} = \overline{h} A_s \left(T_s - T_\infty\right)$$

$$q_{cv} = 4.25 \text{ W/m}^2 \times \left(0.25 \times 0.50\right) \text{m}^2 \left(37 - 20\right) \text{K} = 9.02 \text{ W}$$ <

In summary, the *total heat loss* from the surface of the bath, which must be supplied as electrical power to the bath heaters, is

$$q_{tot} = q_{rad} + q_{cv} + q_{evap}$$

$$q_{tot} = \left(9.18 + 9.02 + 40.5\right) \text{W} = 59 \text{ W}$$ <

The *sensible heat losses* are by convection (q_{rad} + q_{cv}), which represent 31% of the total; the balance is the *latent loss* by evaporation, 69%.