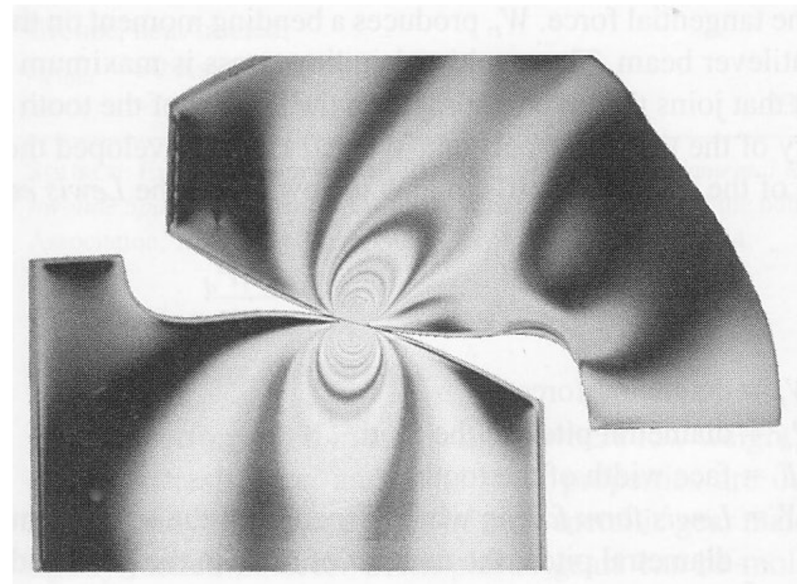
MECH 325
Gear Stress

Objectives

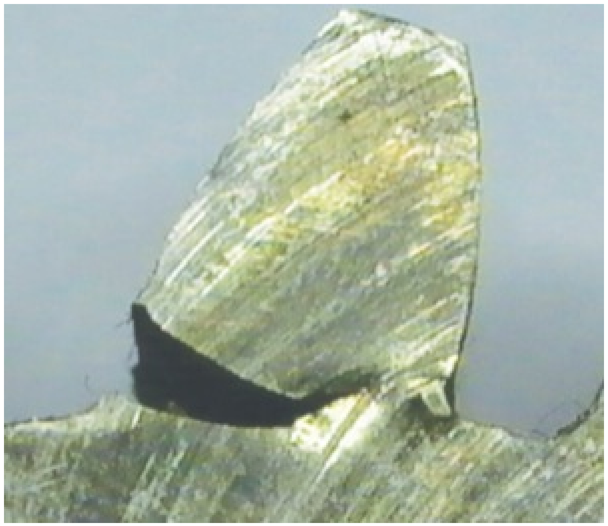
By the end of this section, you should be able to:

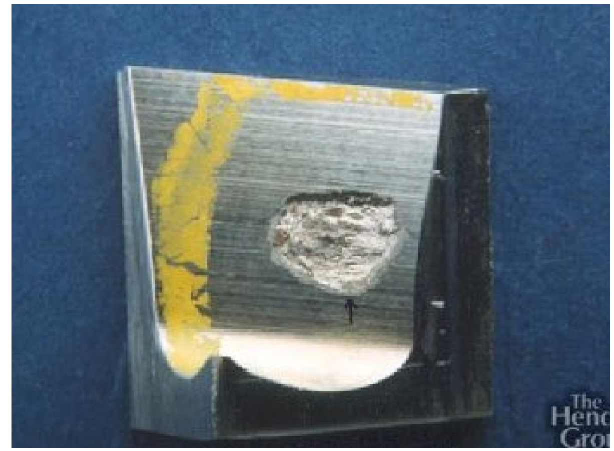
- Analyze bending and contact stress failure of spur gears using AGMA standards
- Describe the operation of power screws and cite typical applications
- Describe thread geometry for common types of power screws

Stress in Gears



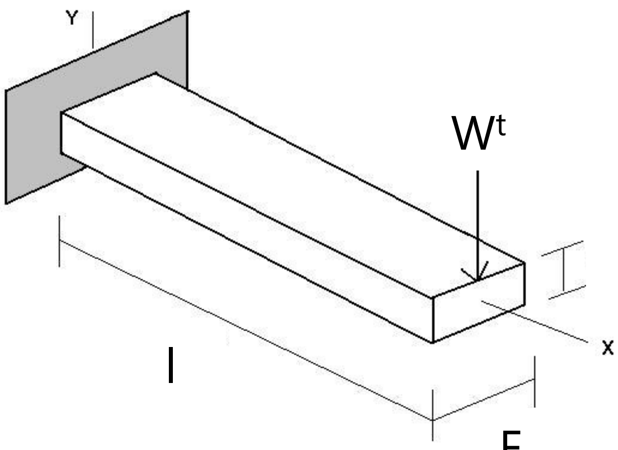
Stress Failures in Gears



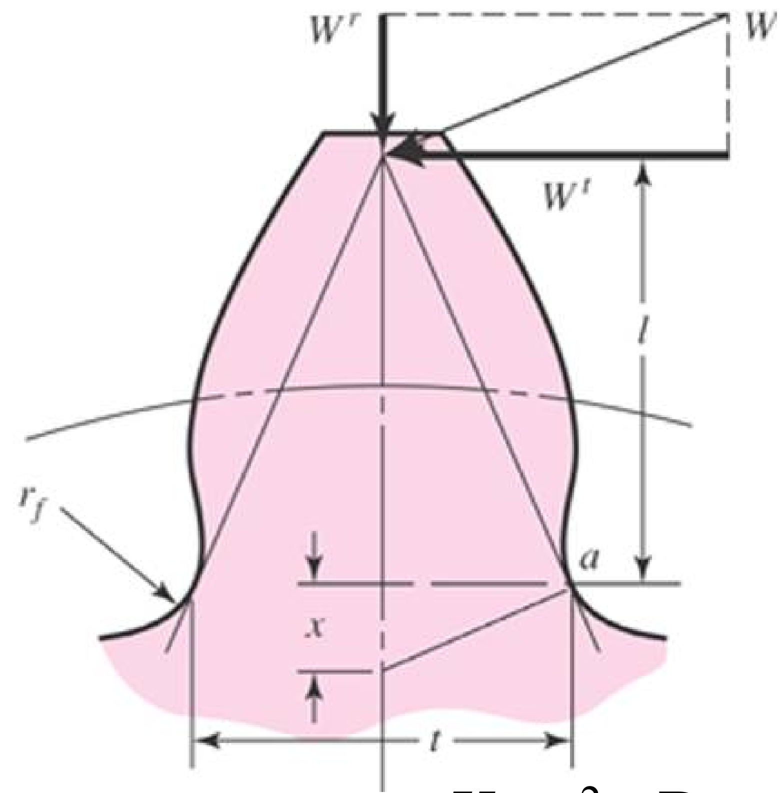


Lewis Bending Equation

Cantilever Beam
Model



$$\sigma = \frac{6W^t l}{Ft^2}$$



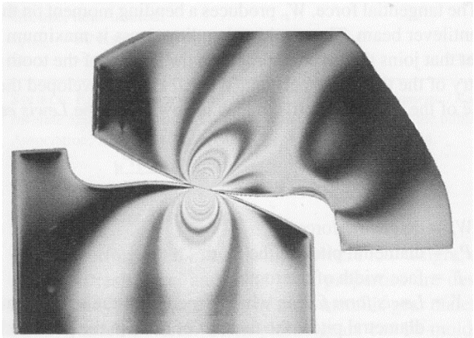
$$\sigma = \frac{W^t P}{FY}$$

$Y = \frac{2}{3}$xP

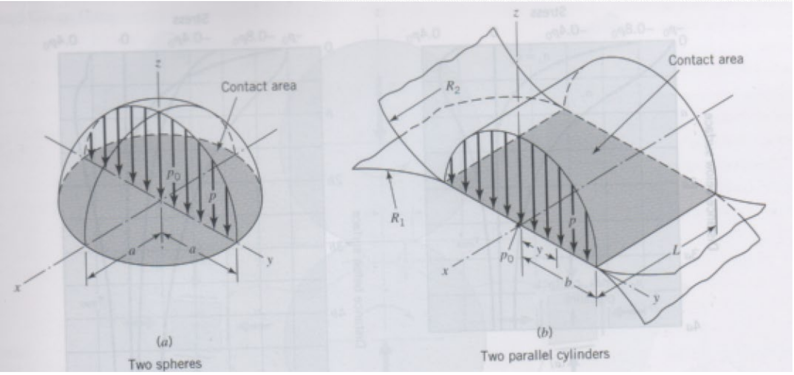
(tabulated)

Table 14-1

Hertz Contact Stress



- When curved *elastic* bodies are in contact, *finite* contact areas are developed due to deflections in the materials.
- Compressive stresses are developed as a result of these deflections.
- The stress equations are credited to Heinrich Hertz (1881)



- The following stress equations are derived (F=Force, L= Face):

$$p_0 = 0.578 \sqrt[3]{\frac{F(1/R_1 + 1/R_2)^2}{\Delta^2}}$$

$$a = 0.908 \sqrt[3]{\frac{F\Delta}{1/R_1 + 1/R_2}}$$

$$p_0 = 0.564 \sqrt{\frac{F(1/R_1 + 1/R_2)}{L\Delta}}$$

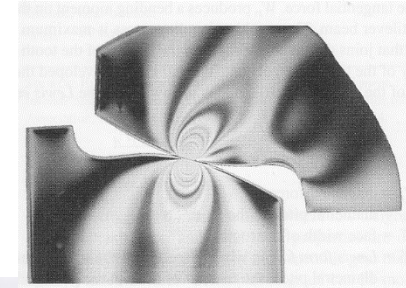
$$b = 1.13 \sqrt{\frac{F\Delta}{L(1/R_1 + 1/R_2)}}$$

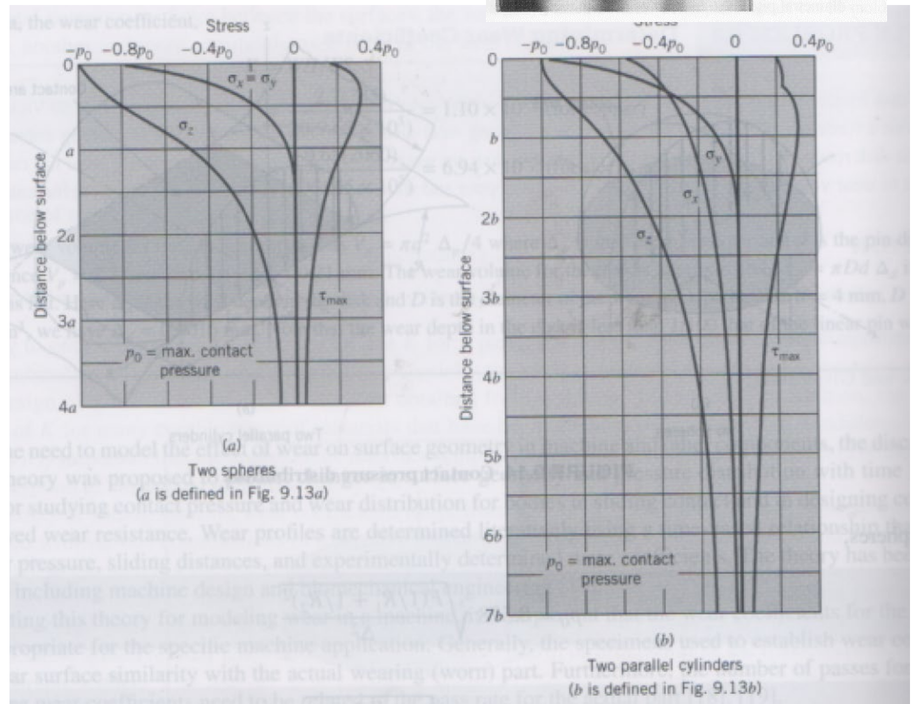
- The contact modulus, Δ, defined as;

$$\Delta = \frac{1 - \upsilon_1^2}{E_1} + \frac{1 - \upsilon_2^2}{E_2}$$

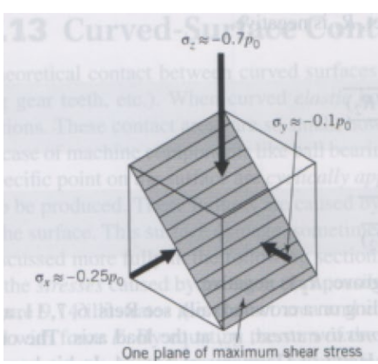
Hertz Contact Stress

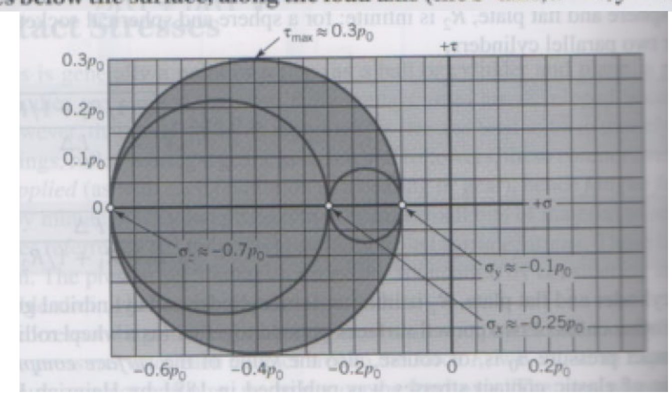
- Vertical contact stresses, σ_z, generate transverse compressive stresses, σ_x and σ_y due to Poissons ratio, υ.
- The three compressive stresses are the principal stresses for a stress element and can be plotted as a 3D Mohr Circle.
- Maximum shear stresses occur below the contact surface at a depth of approx. 0.5 the contact width, b. (Tresca Criteria).





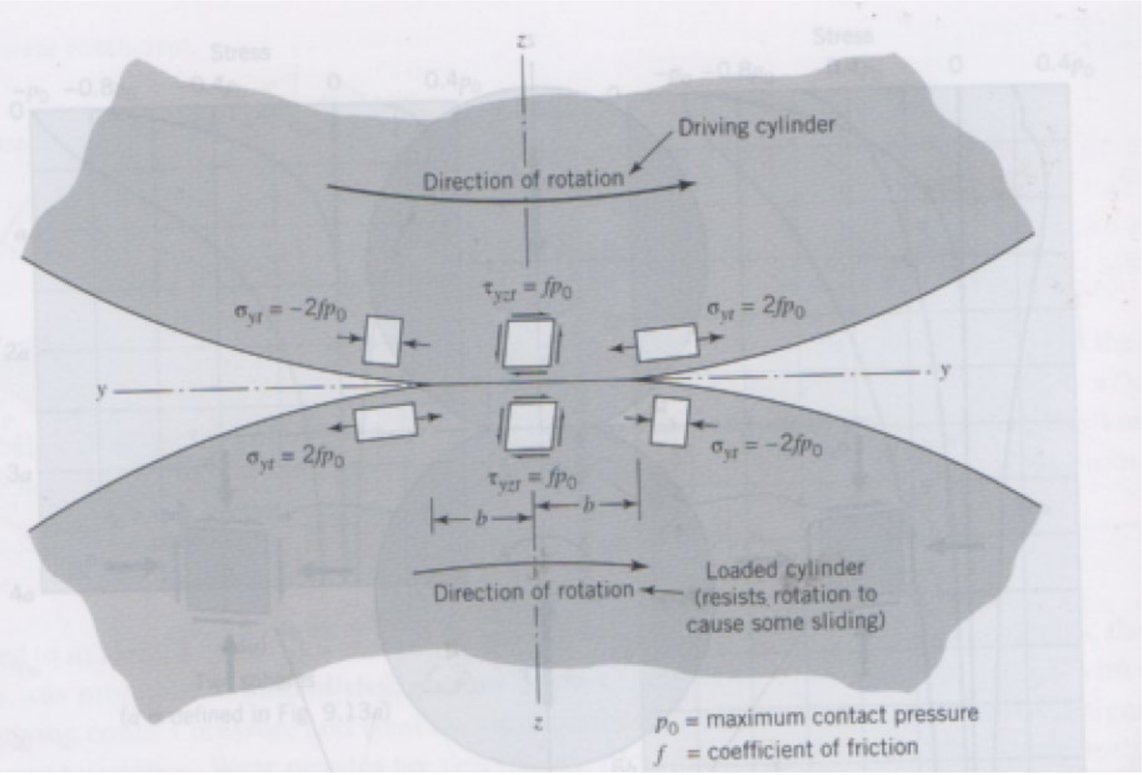
Elastic stresses below the surface, along the load axis (the z-axis; $x = 0, y = 0$; for $\nu = 0.3$).





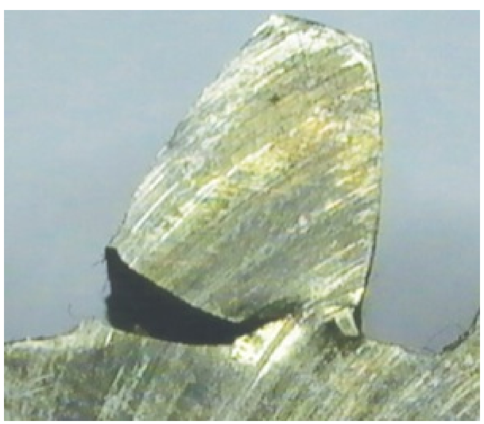
Rolling Friction Stress

- The Hertz contact stresses are for the static load condtion. With the addition of motion along with friction and sliding, the Hertz contact stresses are not the only stress that must be modelled.
- The stresses are cyclic in nature leading to *fatigue failure*.



AGMA Bending Stress Equations

$$\sigma = W^t K_o K_v K_s \frac{P_d}{F} \frac{K_m K_b}{J} \qquad \sigma_{all} = \frac{S_t}{S_F} \frac{Y_N}{K_T K_R}$$

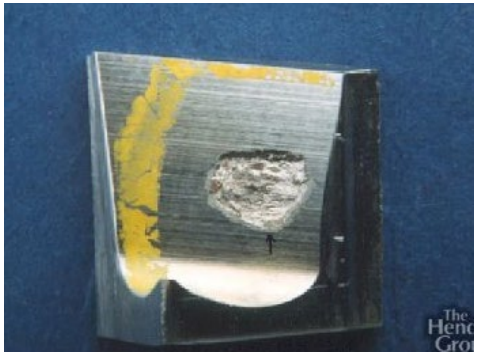


Bending stress safety factor

$$S_F = \frac{S_t}{\sigma} \frac{Y_N}{K_T K_R}$$

AGMA Contact Stress Equations

$$\sigma_c = C_P \sqrt{W^t K_o K_v K_s \frac{K_m}{d_p F} \frac{C_f}{I}} \qquad \sigma_{c,all} = \frac{S_C}{S_H} \frac{Z_N C_H}{K_T K_R}$$



Contact stress safety factor

$$S_H = \frac{S_C}{\sigma_C} \frac{Z_N C_H}{K_T K_R}$$

AGMA Empirical Factors

- Not all factors used in every case
- Refer to text for complete details

- C_f = surface condition factor
- C_H = hardness factor
- C_p = elastic coefficient
- I = geometry factor
- J = geometry factor
- K_B = rim thickness factor
- K_o = overload factor
- K_m = load distribution factor
- K_R = reliability factor
- K_s = size factor
- K_T = temperature factor
- K_v = dynamic factor
- Y_N = stress cycle factor

SPUR GEAR BENDING
Based on ANSI/AGMA 2001-D04 (U.S. customary units)

$$d_P = \frac{N_P}{P_d}$$

$$V = \frac{\pi d n}{12}$$

$$W^t = \frac{33\,000\ H}{V}$$

Gear bending stress equation Eq. (14–15)

1 [or Eq. (a), Sec. 14–10]; p. 751

Eq. (14–30); p. 751

Eq. (14–40); p. 756

$$\sigma = W^t K_o K_v K_s \frac{P_d}{F} \frac{K_m K_B}{J}$$

Fig. 14–6; p. 745

Eq. (14–27); p. 748

Table below

$_{0.99}(S_t)_{10^7}$ Tables 14–3, 14–4; pp. 740, 741

Gear bending endurance strength equation Eq. (14–17)

Fig. 14–14; p. 755

$$\sigma_{\text{all}} = \frac{S_t}{S_F} \frac{Y_N}{K_T K_R}$$

Table 14–10, Eq. (14–38); pp. 756, 755

1 if $T < 250°$F

Bending factor of safety Eq. (14–41)

$$S_F = \frac{S_t Y_N/(K_T K_R)}{\sigma}$$

Remember to compare S_F with S_H^2 when deciding whether bending or wear is the threat to function. For crowned gears compare S_F with S_H^3.

Table of Overload Factors, K_o

	Driven Machine		
Power source	Uniform	Moderate shock	Heavy shock
Uniform	1.00	1.25	1.75
Light shock	1.25	1.50	2.00
Medium shock	1.50	1.75	2.25

Figure 14–17

SPUR GEAR WEAR

Based on ANSI/AGMA 2001-D04 (U.S. customary units)

$$d_P = \frac{N_P}{P_d}$$

$$V = \frac{\pi d n}{12}$$

$$W^t = \frac{33\,000\ \mathrm{H}}{V}$$

1 [or Eq. (a), Sec. 14–10]; p. 751

Eq. (14–30); p. 751

1

Gear contact stress equation Eq. (14–16)

$$\sigma_c = C_p \left(W^t K_o K_v K_s \frac{K_m}{d_P F} \frac{C_f}{I} \right)^{1/2}$$

Eq. (14–23); p. 747

Eq. (14–27); p. 748

Eq. (14–13), Table 14–8; pp. 736, 749

Table below

$_{0.99}(S_c)_{10^7}$ Tables 14–6, 14–7; pp. 743, 744

Fig. 14–15; p. 755

Section 14–12, gear only; pp. 753, 754

Gear contact endurance strength Eq. (14–18)

$$\sigma_{c,\mathrm{all}} = \frac{S_c Z_N C_H}{S_H K_T K_R}$$

Table 14–10, Eq. (14–38); pp. 756, 755

1 if $T < 250°\mathrm{F}$

Gear only

Wear factor of safety Eq. (14–42)

$$S_H = \frac{S_c Z_N C_H / (K_T K_R)}{\sigma_c}$$

Remember to compare S_F with S_H^2 when deciding whether bending or wear is the threat to function. For crowned gears compare S_F with S_H^3.

Table of Overload Factors, K_o

	Driven Machine		
Power source	Uniform	Moderate shock	Heavy shock
Uniform	1.00	1.25	1.75
Light shock	1.25	1.50	2.00
Medium shock	1.50	1.75	2.25

Figure 14–18

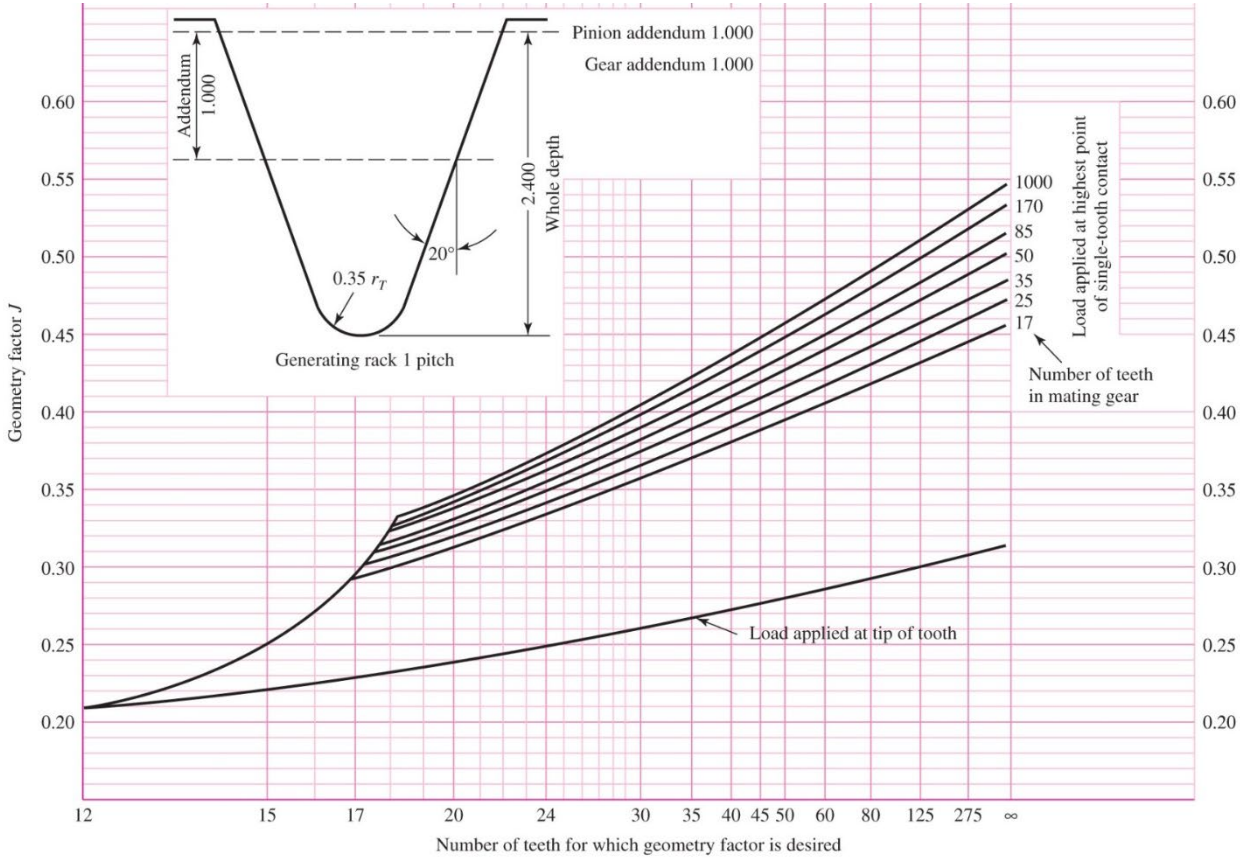
Roadmap of gear wear equations based on AGMA standards. *(ANSI/AGMA 2001-D04.)*

Geometry Factor *J*

- Accounts for shape of tooth in bending stress equation. It is related to the Lewis Equation. A *nominal* bending stress is developed.
- The value of J increases with number of teeth as the load is shared.

$$\sigma = \frac{W^t P}{FY}$$

J replaces Y



Overload (Service) Factor K_o

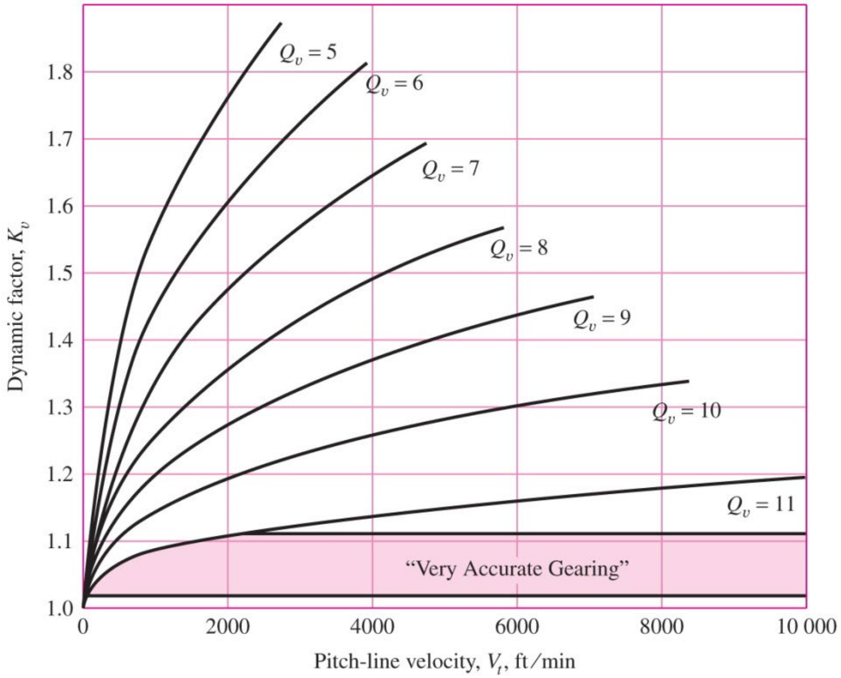
- To account for likelihood of increase in nominal tangential load due to particular application.
- Recommended values,

Table of Overload Factors, K_o

	Driven Machine		
Power source	Uniform	Moderate shock	Heavy shock
Uniform	1.00	1.25	1.75
Light shock	1.25	1.50	2.00
Medium shock	1.50	1.75	2.25

Dynamic & Size Factor K_v & K_s

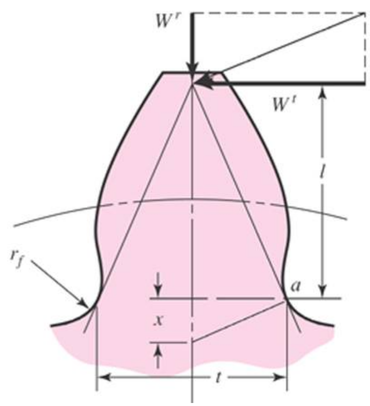
- Dynamic factor K_v is a function of tangential speed, ωr, and gear quality Q.



- Size factor K_s
- Typically $K_s = 1$

- Or use

$$K_s = \frac{1}{k_b} = 1.192\left(\frac{F\sqrt{Y}}{P}\right)^{0.0535}$$



$$Y = \frac{2}{3} \text{xP}$$

(tabulated)

Load-Distribution Factor K_m

- Accounts for non-uniform distribution of load across the line of contact.
- Depends on mounting and face width.

$$K_m = C_{mf} = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e) \qquad (14\text{–}30)$$

$$C_{mc} = \begin{cases} 1 & \text{for uncrowned teeth} \\ 0.8 & \text{for crowned teeth} \end{cases} \qquad (14\text{–}31)$$

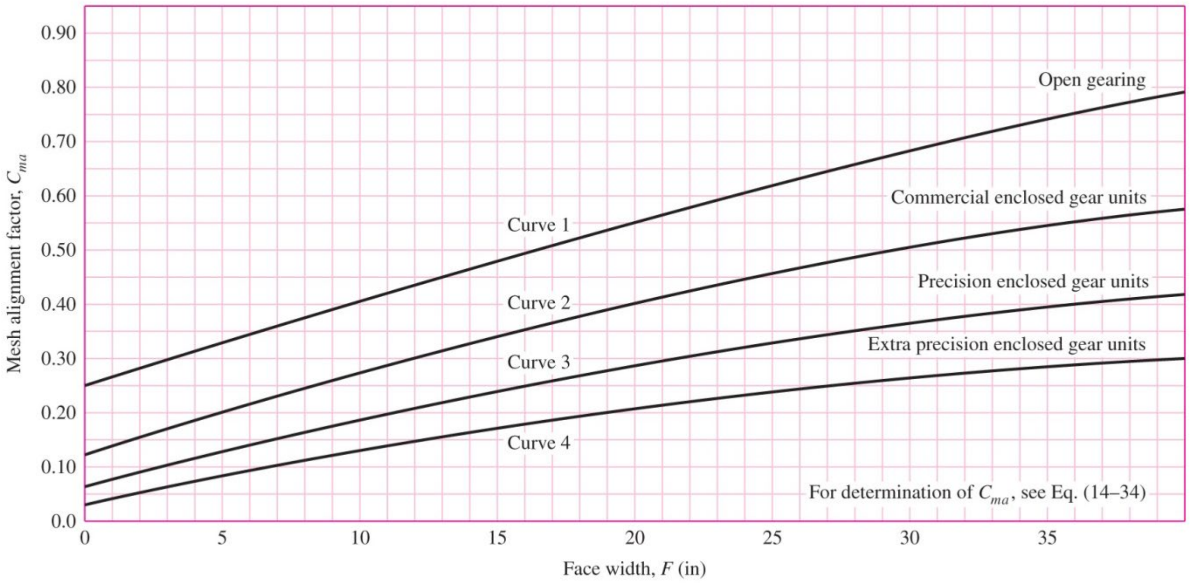
$$C_{pf} = \begin{cases} \dfrac{F}{10d_P} - 0.025 & F \le 1 \text{ in} \\ \dfrac{F}{10d_P} - 0.0375 + 0.0125F & 1 < F \le 17 \text{ in} \\ \dfrac{F}{10d_P} - 0.1109 + 0.0207F - 0.000\,228F^2 & 17 < F \le 40 \text{ in} \end{cases} \qquad (14\text{–}32)$$

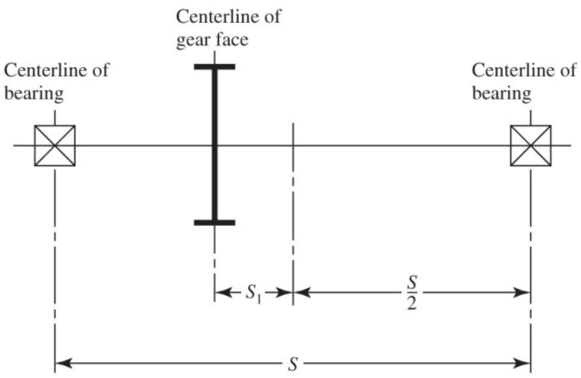
$$C_e = \begin{cases} 0.8 & \text{for gearing adjusted at assembly, or compatibility is improved by lapping, or both} \\ 1 & \text{for all other conditions} \end{cases} \qquad (14\text{–}35)$$

Load-Distribution Factor K_m

$$K_m = C_{mf} = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e) \qquad (14\text{–}30)$$

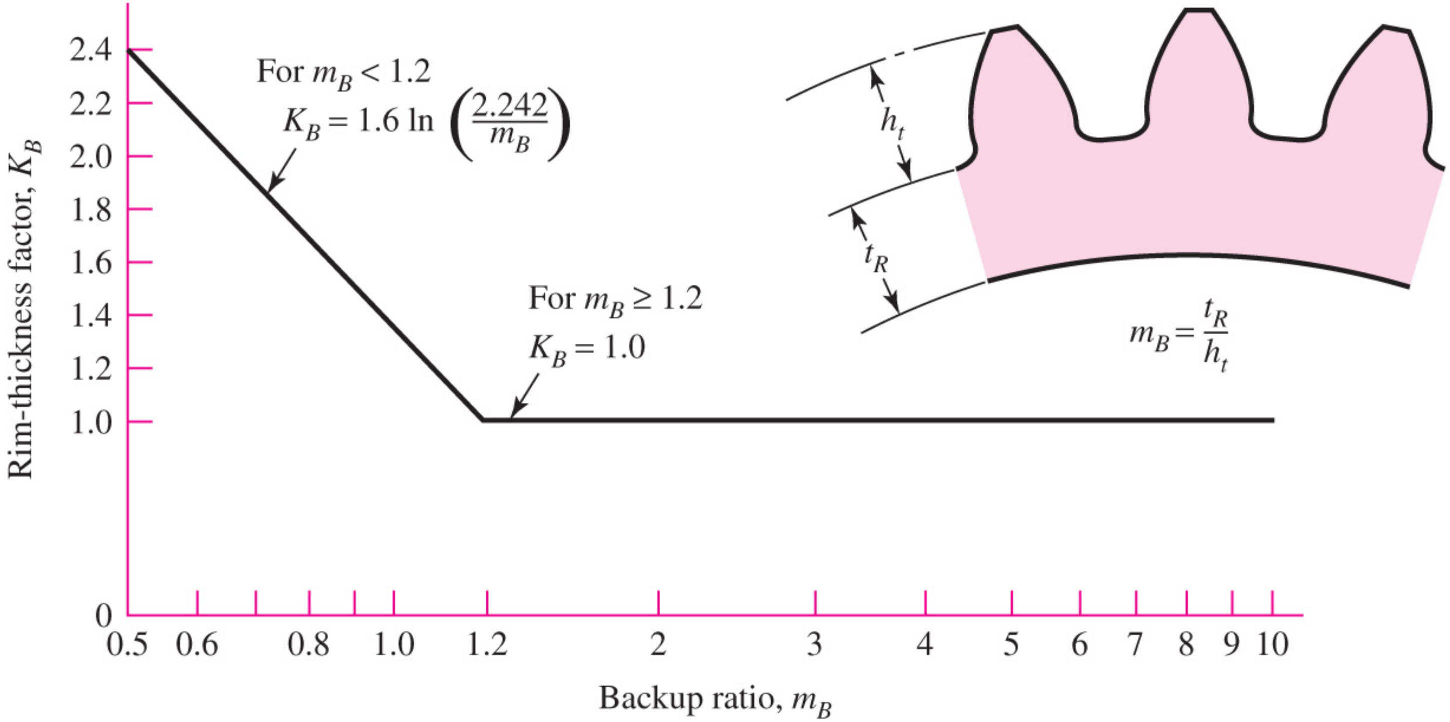
$$C_{pm} = \begin{cases} 1 & \text{for straddle-mounted pinion with } S_1/S < 0.175 \\ 1.1 & \text{for straddle-mounted pinion with } S_1/S \geq 0.175 \end{cases} \qquad (14\text{–}33)$$





Rim-Thickness Factor K_B

- Accounts for bending of rim on a gear that is not solid



AGMA Allowable Strengths, S_t

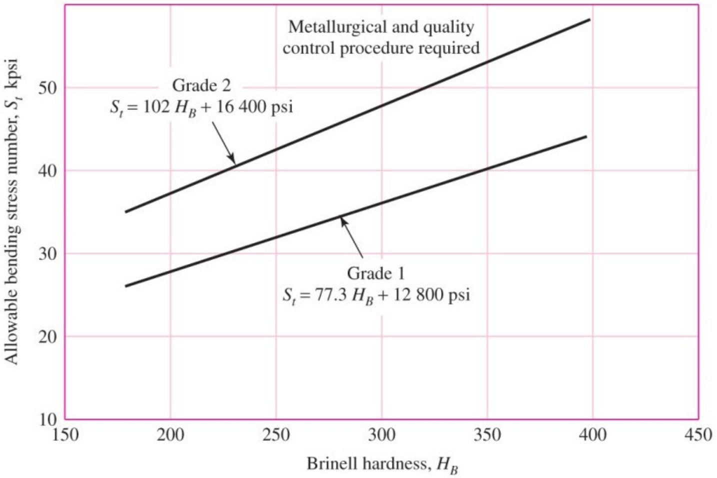
- AGMA uses *allowable stress numbers* rather than *strengths*.
- The gear strength values are only for use with the AGMA stress values, and should not be compared with other true material strengths.

Table 14–3

Repeatedly Applied Bending Strength S_t at 10^7 Cycles and 0.99 Reliability for Steel Gears

Source: ANSI/AGMA 2001-D04.

Material Designation	Heat Treatment	Minimum Surface Hardness[1]	Allowable Bending Stress N psi Grade 1	Grade 2
Steel[3]	Through-hardened	See Fig. 14–2	See Fig. 14–2	See Fig. 14–2
	Flame[4] or induction hardened[4] with type A pattern[5]	See Table 8*	45 000	55 000
	Flame[4] or induction hardened[4] with type B pattern[5]	See Table 8*	22 000	22 000
	Carburized and hardened	See Table 9*	55 000	65 000 or 70 000[6]
	Nitrided[4,7] (through-hardened steels)	83.5 HR15N	See Fig. 14–3	See Fig. 14–3
Nitralloy 135M, Nitralloy N, and 2.5% chrome (no aluminum)	Nitrided[4,7]	87.5 HR15N	See Fig. 14–4	See Fig. 14–4



AGMA Allowable Strengths, S_t

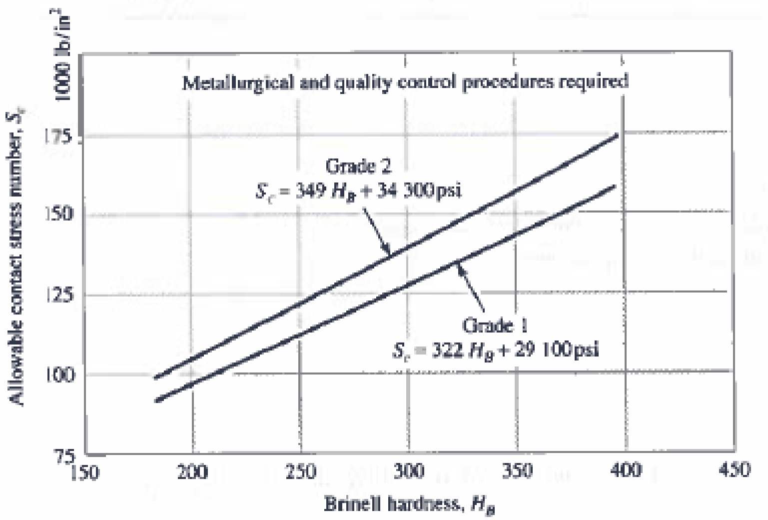
- The AGMA *allowable stress numbers* are different between bending stress and contact stress.

Table 14–6

Repeatedly Applied Contact Strength S_c at 10^7 Cycles and 0.99 Reliability for Steel Gears

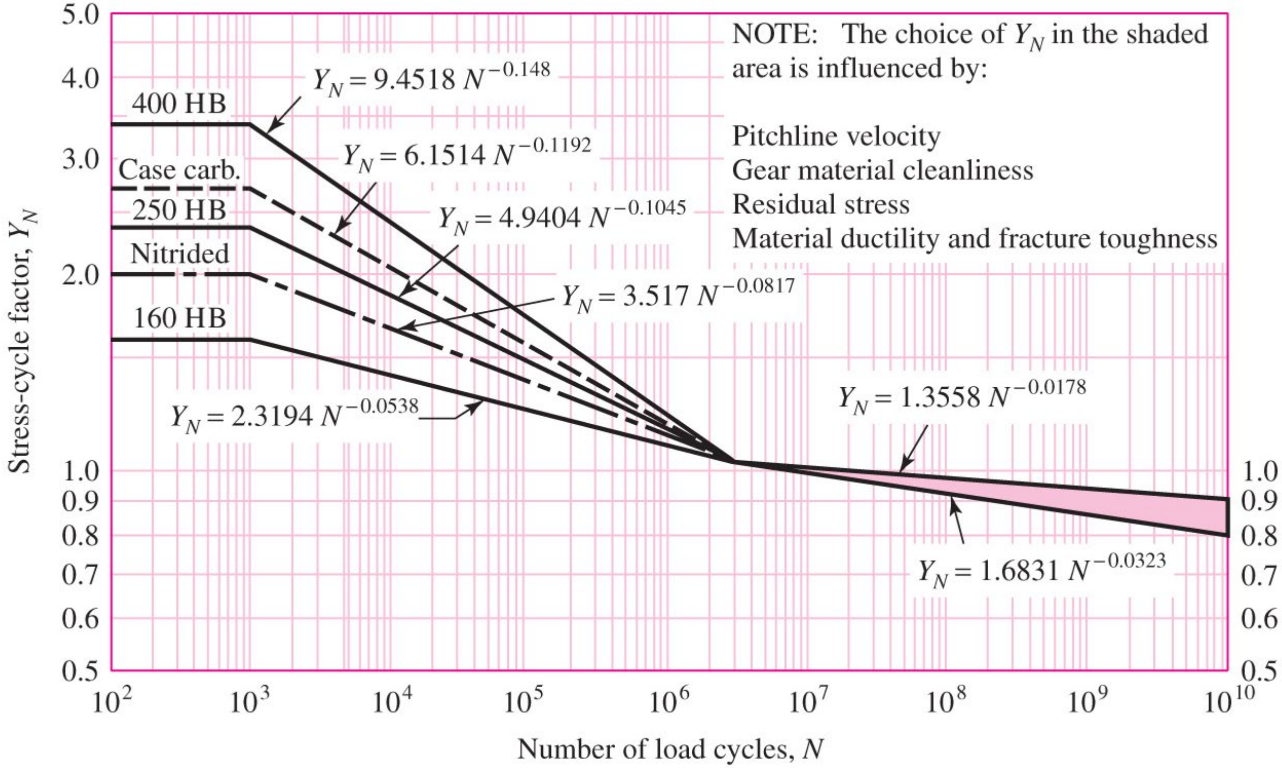
Source: ANSI/AGMA 2001-D04.

Material Designation	Heat Treatment	Minimum Surface Hardness[1]	Allowable Contact Stress Number,[2] S_c, psi Grade 1	Grade 2	Grade 3
Steel[3]	Through hardened[4]	See Fig. 14–5	See Fig. 14–5	See Fig. 14–5	—
	Flame[5] or induction hardened[5]	50 HRC	170 000	190 000	—
		54 HRC	175 000	195 000	—
	Carburized and hardened[5]	See Table 9*	180 000	225 000	275 000
	Nitrided[5] (through hardened steels)	83.5 HR15N	150 000	163 000	175 000
		84.5 HR15N	155 000	168 000	180 000
2.5% chrome (no aluminum)	Nitrided[5]	87.5 HR15N	155 000	172 000	189 000
Nitralloy 135M	Nitrided[5]	90.0 HR15N	170 000	183 000	195 000
Nitralloy N	Nitrided[5]	90.0 HR15N	172 000	188 000	205 000
2.5% chrome (no aluminum)	Nitrided[5]	90.0 HR15N	176 000	196 000	216 000



Stress-Cycle Factors Y_N and Z_N

- AGMA strengths are for 10^7 cycles
- Stress-cycle factors account for endurance of the gear train
- Fig. 14–14 gives Y_N for bending
- Fig. 14–15 gives Z_N for contact stress



Reliability Factor K_R (Y_Z)

- Accounts for statistical distributions of material fatigue failures
- Does not account for load variation
- Use Table 14–10 (Normally 99% is used K_R = 1.00)

Reliability	K_R (Y_Z)
0.9999	1.50
0.999	1.25
0.99	1.00
0.90	0.85
0.50	0.70

Temperature Factor K_T (Y_θ)

- AGMA has not established values for this factor.
- For temperatures up to 250°F (120°C), $K_T = 1$ is acceptable.

Safety Factors S_F and S_H

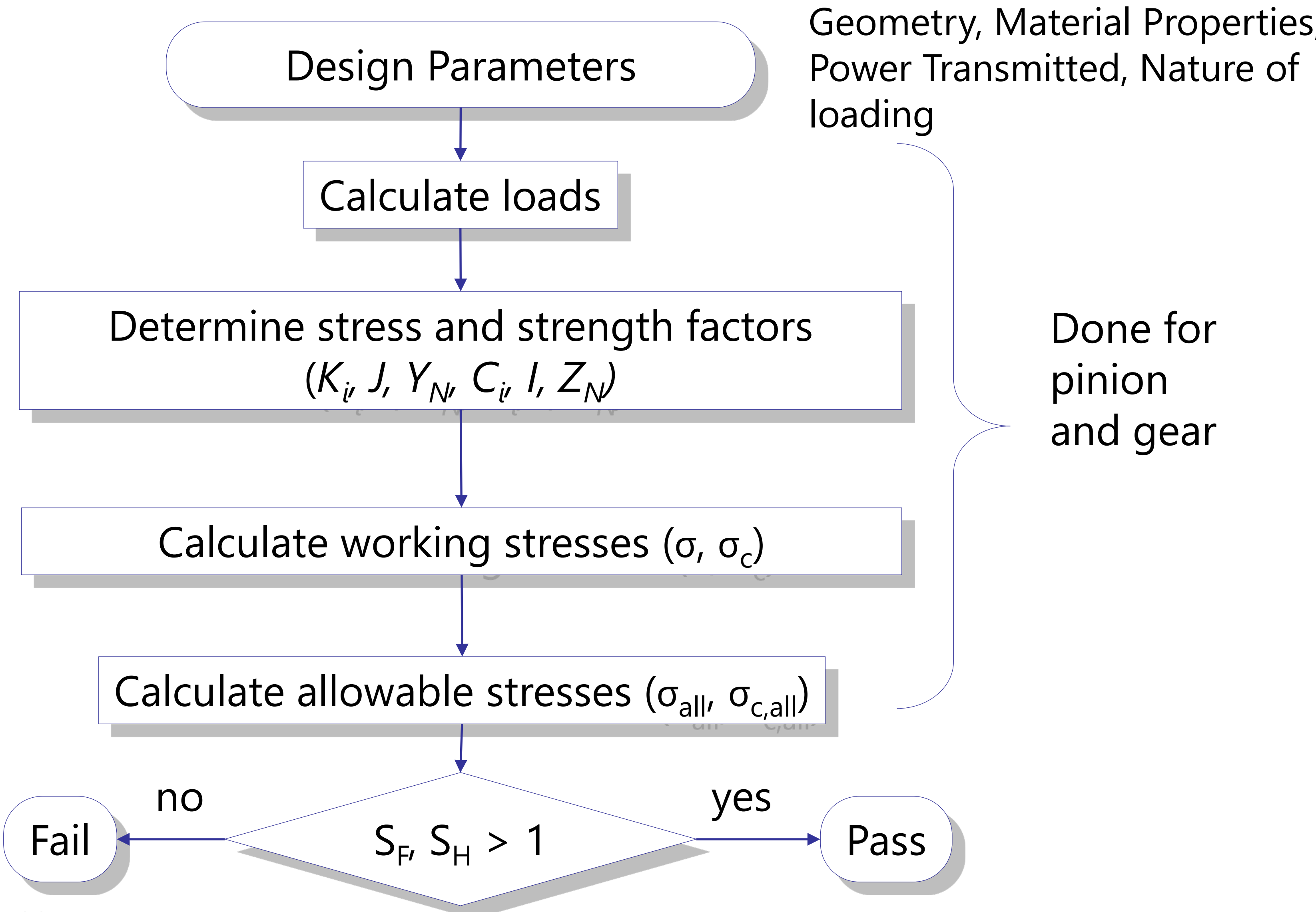
- Included as design factors in the strength equations
- Can be solved for and used as factor of safety

$$S_F = \frac{S_t Y_N/(K_T K_R)}{\sigma} = \frac{\text{fully corrected bending strength}}{\text{bending stress}} \qquad (14\text{–}41)$$

$$S_H = \frac{S_c Z_N C_H/(K_T K_R)}{\sigma_c} = \frac{\text{fully corrected contact strength}}{\text{contact stress}} \qquad (14\text{–}42)$$

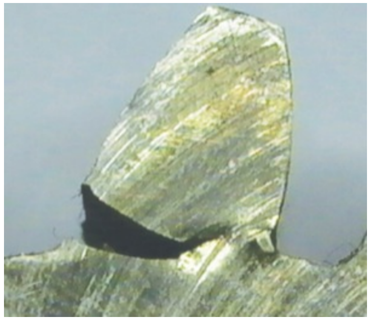
- Or, can set equal to unity, and solve for traditional factor of safety as $n = \sigma_{all}/\sigma$

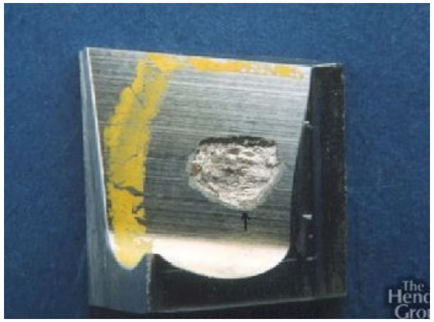
Gear Stress Analysis Procedure



Comparing S_F and S_H

- Bending stress is linear with transmitted load.
- Contact stress is not linear with transmitted load
- To compare the factors of safety between the different failure modes, to determine which is critical,
  - Compare S_F with S_H^2 for linear or helical contact
  - Compare S_F with S_H^3 for spherical contact





Design of a Gear Mesh

1. A priori decisions
   (established in advance)

2. Choose a diametral pitch, Pd.
   Examine implications on face width,
   pitch diameters, and material properties.

3. Choose pinion material.
   examine hardness requirements

4. Choose gear material.
   examine hardness requirements

Done

Design of a Gear Mesh - Details

1. A priori decisions (established in advanced)
   - Function: load, speed, reliability, life, overload factor
   - Overall design safety factor (n_d)
   - Tooth system: φ, ψ, addendum, dedendum, root fillet radius
   - Gear ratio ($m_G = N_G/N_P$)
   - Quality number (Q_v)
2. Choose a diametral pitch
   - Initially select median face width $4\pi/P$
   - Find range of necessary ultimate strengths

Design of a Gear Mesh - Details

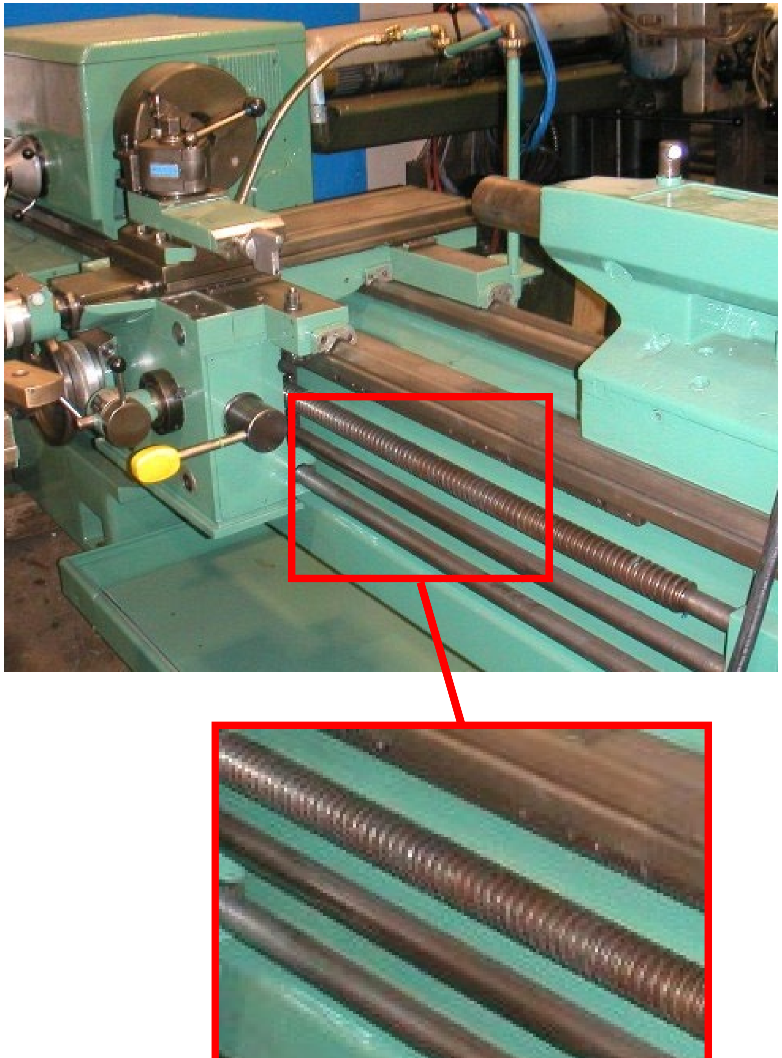
3. Choose pinion material and core hardness
   – Find face width to meet safety factor in bending
   – Choose face width
   – Check safety factor in bending
   – Find necessary contact fatigue strength, S_c
   – Choose a case hardness
   – Check safety factor in wear
4. Choose a gear material and core hardness
   – Check safety factor in bending
   – Choose a case hardness
   – Check safety factor in wear

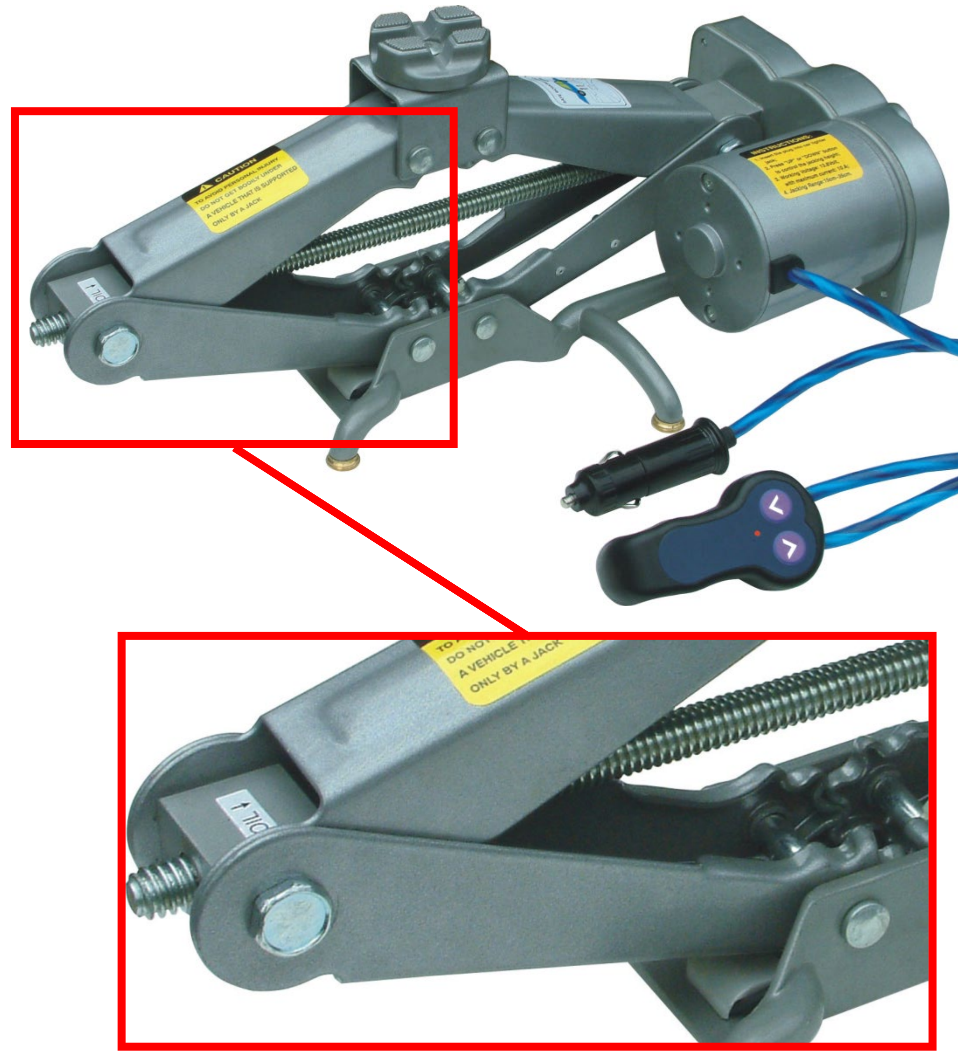
Design of a Gear Mesh - Notes

- Iterate at each stage until no decisions are changed
- If you cannot find a reasonable solution at one stage, you need to move back to the previous stage
- This is not a rigid process; you can work in any sequence
- The process is the similar for all gear types

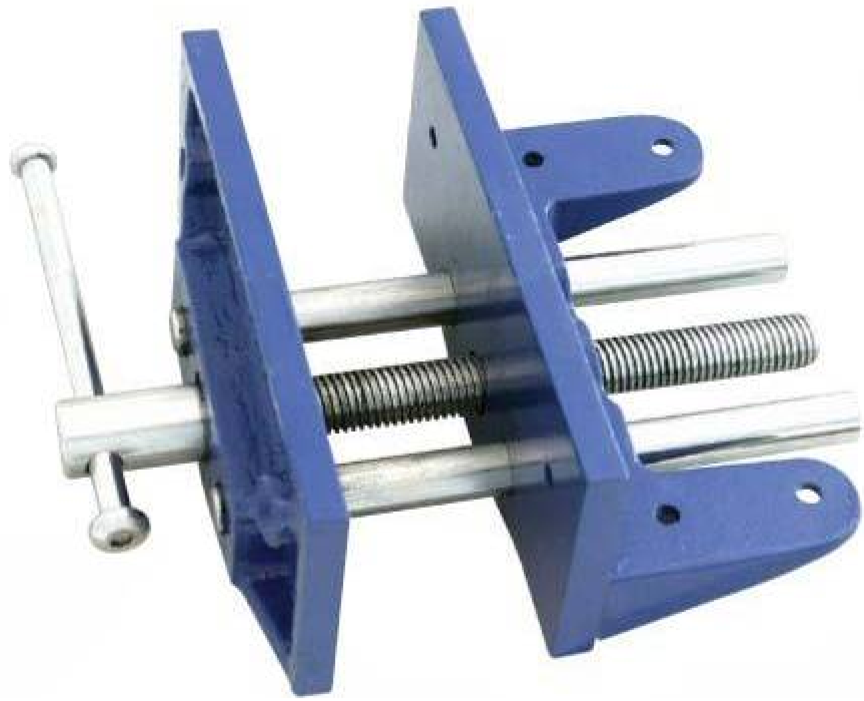
MECH 325
Power Screw Introduction

Power Screw Examples

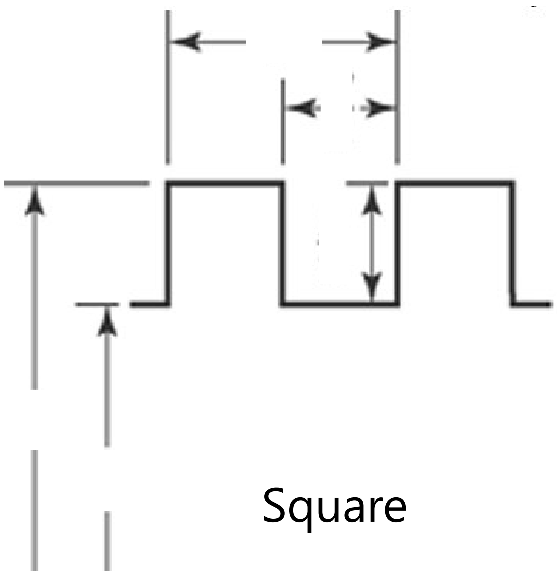




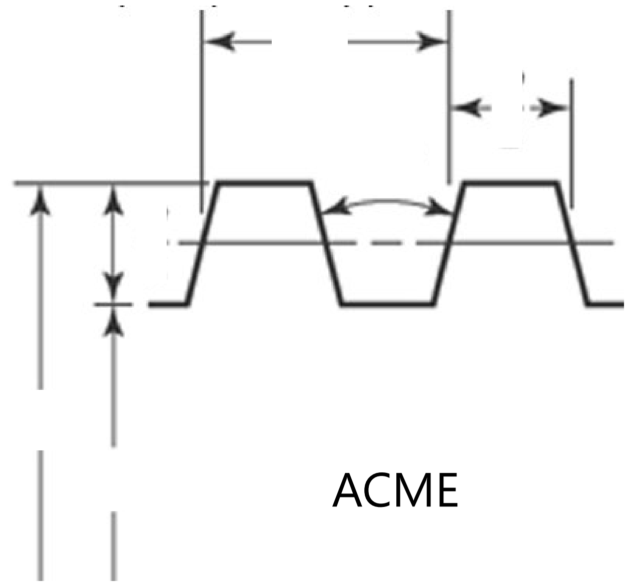
Power Screw Examples



Thread Types



Square



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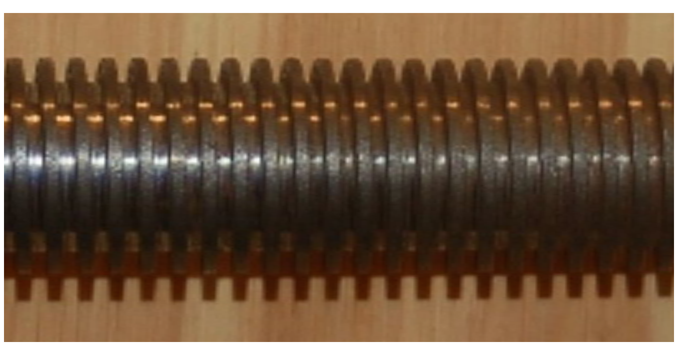
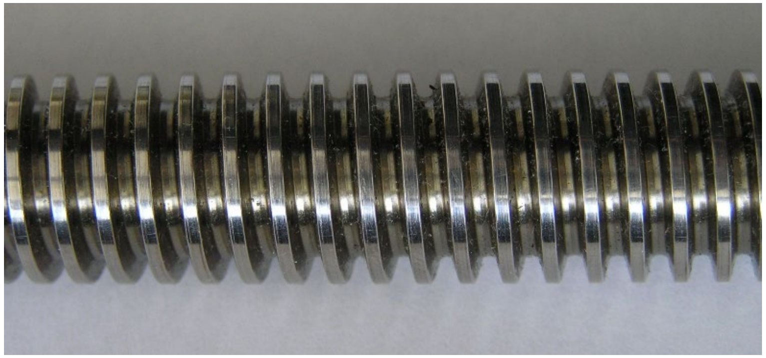




Multiple Start Threads

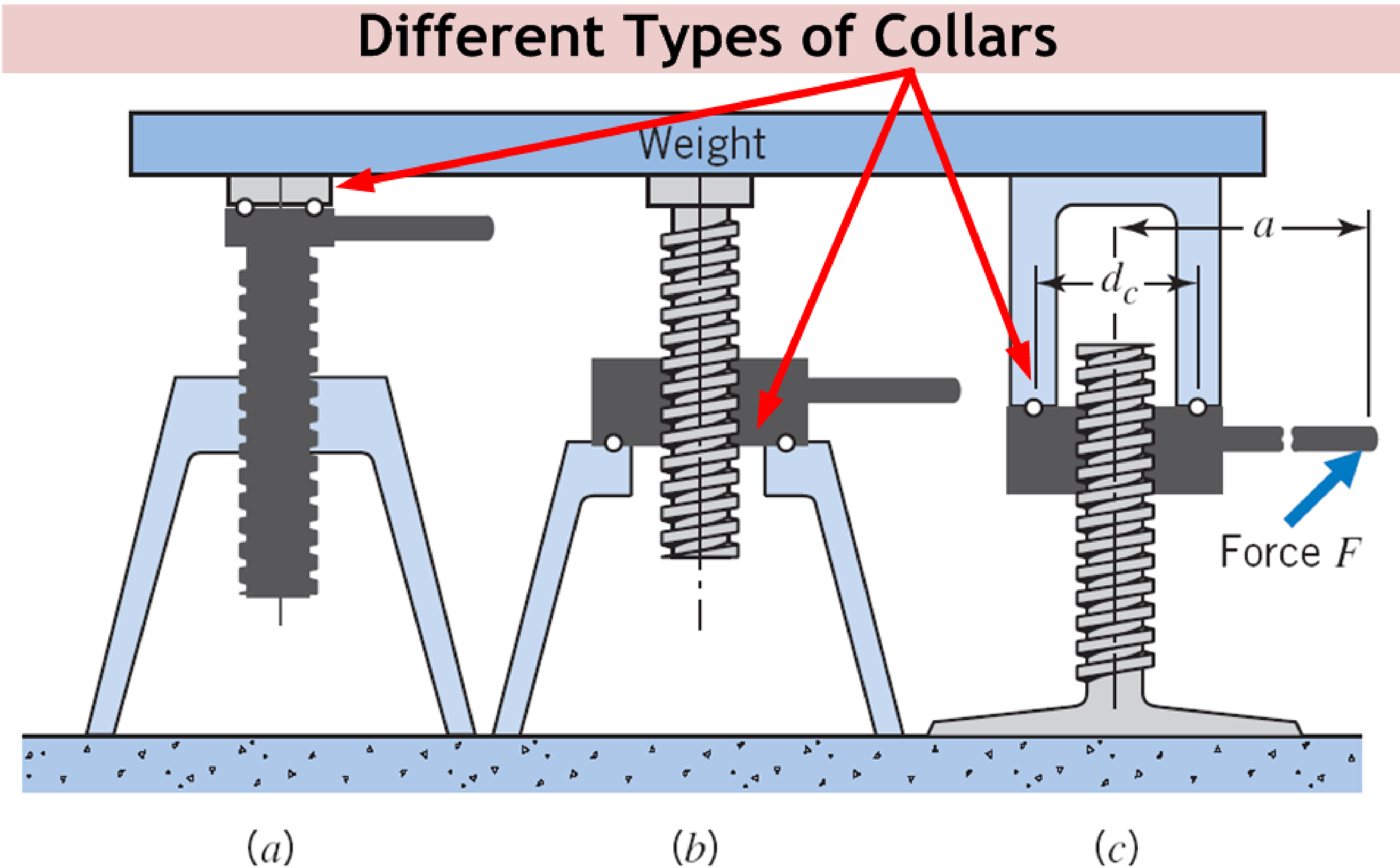
Pitch Lead

1 Start

2 Starts

4 Starts

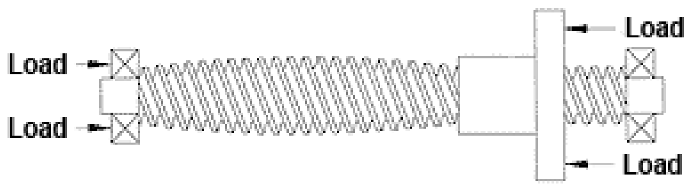
Power Screw Configurations



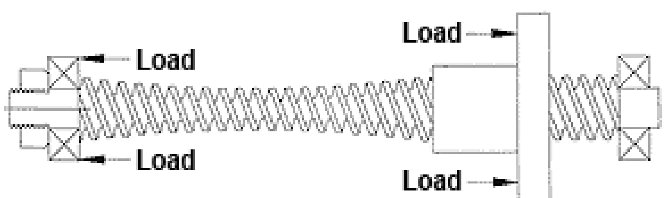
http://ocw.kfupm.edu.sa/user071/ME3070103/LEC%2034.ppt#355,24,Slide 24

Power Screw Configurations

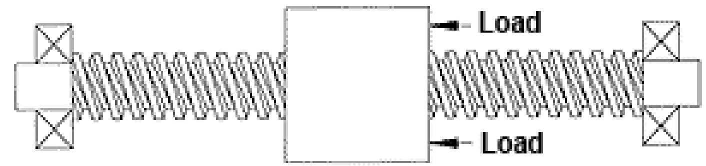
- Identify preferred configurations



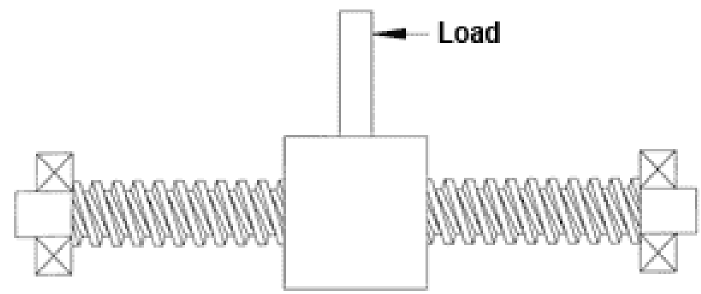
Compression Loading



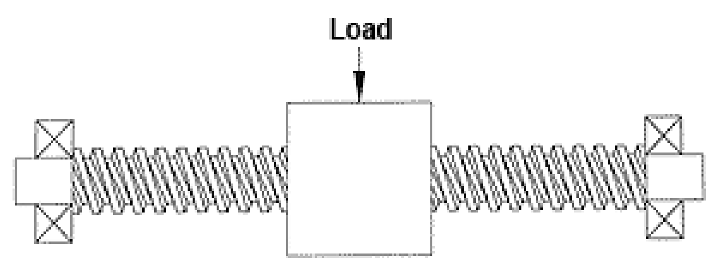
Tension Loading



Axial Center Loading



Off Center Moment Loading



Radial Loading