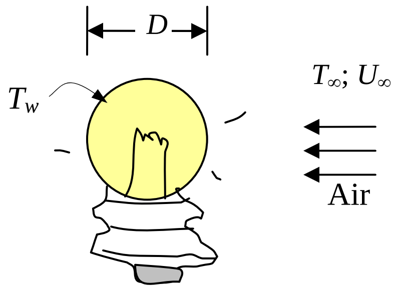
Solutions - Problem Set # 8

Problem 1:



Given: Air at T_∞= 25°C; U_∞=0.5 m/s. Incandescent bulb of 50 W; T_w= 140°C; bulb diameter, D = 50 mm

Assumptions: Steady-state fluid flow and heat transfer in cross flow over sphere with constant surface temperature.

What is the rate of heat loss by convection to the air?

The rate of heat loss by convection:

$$q_{\underset{loss}{conv.}} = h_{av} A_{bulb} \left(T_w - T_\infty\right)$$

we need h; Whitaker correlation can be used (All properties are evaluated at T_∞, except μ_w which is evaluated at T_w) :

$$Nu_{av} \triangleq \frac{h_{av} D}{k_{fluid}} = 2 + [0.4 Re_D^{1/2} + 0.06 Re_D^{2/3}] Pr^{0.4} \left(\frac{\mu_\infty}{\mu_w}\right)^{1/4}$$

at T_∞= 25°C= 298.15K: ρ = 1.161 kg/m^3; c_p = 1007 J/kg-°C; μ_∞ = 18.46×10^{-6} kg/m-s; k = 0.0263W/m-°C
at T_w= 140°C = 413.15K: μ_w = 23.45×10^{-6} kg/m-s

$$\mathrm{Re_D} = \frac{\rho_\infty U_\infty D}{\mu_\infty} = \frac{1.161 \times 0.5 \times 50 \times 10^{-3}}{18.46 \times 10^{-6}} = 1572.3$$

$$\mathrm{Pr}_\infty = \frac{\mu_\infty c_{p,\infty}}{k_\infty} = \frac{18.46 \times 10^{-6} \times 1007}{0.0263} = 0.707; \quad \frac{\mu_\infty}{\mu_w} = \frac{18.46 \times 10^{-6}}{23.45 \times 10^{-6}} = 0.7872$$

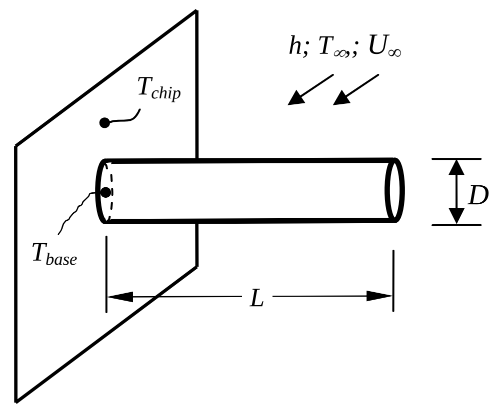
$$Nu_{av} \triangleq \frac{h_{av} \times 50 \times 10^{-3}}{0.0263} = 2 + [0.4 \times \left(1572.3\right)^{1/2} + 0.06 \times \left(1572.3\right)^{2/3}] \times \left(0.707\right)^{0.4} \times \left(0.7872\right)^{1/4} = 21.66$$

$$h_{av} = 11.39 \text{ W/m}^2 - °\text{C}$$

$$\text{Thus, } q_{\underset{loss}{conv.}} = h_{av} A_{bulb} \left(T_w - T_\infty\right) = 11.39 \times 4\pi \times \left(\frac{50 \times 10^{-3}}{2}\right)^2 \times \left(140 - 25\right) \Rightarrow q_{\underset{loss}{conv.}} = 10.29 \text{ W}$$

Please note that the rest of the light wattage (39.71 W) is dissipated by radiation both in the visible and long wave length range.

Problem 2:



Given: Air at $T_\infty = 300$ K; $U_\infty = 10$ m/s; fin dimensions: $L = 12$ mm and $D = 2$ mm. $T_{Base} = 350$ K; $k_{copper} = 380$ W/m-°C

Assumptions: Steady-state fluid flow and heat transfer; perfect thermal contact between the chip and the fin base. Cross flow over cylindrical object. Radiation is negligible. Classical fin theory applies k_s = constant, h = constant with $P_{c.s.}$ = const., and $A_{c.s.}$ = const.

a)

This is cross flow over cylinder. The correlation of Knudsen and Katz may be used to obtain the heat transfer coefficient.

$$Nu_{av} \triangleq \frac{h_{av}D}{k_f} = CRe_D^n Pr_f^{1/3}$$

The properties in this correlation are based of film temperature: $T_{film} = \left(T_\infty + T_{fin,av}\right)/2$. The average fin surface temperature is defined as $T_{fin,av} = \frac{1}{L}\int_0^L T dx$, and $T_{Base} < T_{fin,av} < T_{Tip}$. However, as we are dealing with a relatively short fin made of high thermal conductivity material cooled in air, one may assume a small temperature drop from the fin base to its tip. Thus, as a first estimation, we assume $T_{film} = \left(T_\infty + T_{Base}\right)/2$. We will check this later when we obtain the heat transfer coefficient.

Thus, $T_{film} = \left(300 + 350\right)/2 = 325$ K

at T_{film}= 325 K: ρ_f = 1.087 kg/m^3; $c_{p,f}$= 1007 J/kg-°C; μ_f = 19.6×10^{-6} kg/m-s; k_f = 0.028 W/m-°C

$$\mathrm{Re_D} = \frac{\rho_f U_\infty D}{\mu_f} = \frac{1.087\times 10\times 2\times 10^{-3}}{19.6\times 10^{-6}} = 1109.18 \underbrace{\rightarrow}_{Table 6-2} C = 0.683; n = 0.466$$

$$\mathrm{Pr}_f = \frac{\mu_f c_{p,f}}{k_f} = \frac{19.6\times 10^{-6}\times 1007}{0.028} = 0.7049$$

$$Nu_{av} \triangleq \frac{h_{av}D}{k_f} = CRe_D^n Pr_f^{1/3} = 0.683\left(1109.18\right)^{0.466}\left(0.7049\right)^{1/3} = 15.95$$

$$h_{av} = \frac{k_f}{D}\times 15.95 = \frac{0.028}{2\times 10^{-3}}\times 15.95 = 223.3 \text{ W/m}^2 - °\text{C}$$

To check the choice for $T_{film} = \left(T_\infty + T_{Base}\right)/2$, we can now obtain the tip temperature: $T_{tip} = T_{x=L}$ with Case 2 fin solutions (convection from the tip surface):

$$\frac{T - T_\infty}{T_{Base} - T_\infty} = \frac{\cosh[m(L-x)] + (h/mk_s)\sinh[m(L-x)]}{\cosh[mL] + (h/mk_s)\sinh[mL]} \Rightarrow \frac{T_{x=L} - T_\infty}{T_{Base} - T_\infty} = \frac{1}{\cosh[mL] + (h/mk_s)\sinh[mL]}$$

$$\frac{T_{x=L}-T_\infty}{T_{Base}-T_\infty}=\frac{1}{\cosh[mL]+(h/mk_s)\sinh[mL]}$$

$$m=\left(\frac{hP_{c.s.}}{k_sA_{c.s.}}\right)^{0.5}=\left(\frac{h\pi D}{k_s\pi D^2/4}\right)^{0.5}=\left[223.3\times 4/\left(380\times 2\times 10^{-3}\right)\right]^{0.5}=34.28\ \mathrm{m}^{-1}$$

and $mL=34.28\times 12\times 10^{-3}=0.411$; and $h/mk_s=223.3/(34.28\times 380)=0.0171$

$$\Rightarrow T_{x=L}=300+\frac{350-300}{\cosh[0.411]+(0.0171)\sinh[0.411]}=345.75\ \mathrm{K};$$

thus, $345.75\ \mathrm{K}<T_{fin,av}<350\ \mathrm{K}$ and in engineering practice our initial assumption is justified!

b)

This is **case 2** fin, convection from the tip surface. The rate of heat loss by convection:

$$q_{\substack{total\ loss\\ Fin\to Fluid}}=\sqrt{k_sA_{c.s.}hP_{c.s.}}\,(T_{Base}-T_\infty)\left[\frac{\sinh[mL]+(h/mk_s)\cosh[mL]}{\cosh[mL]+(h/mk_s)\sinh[mL]}\right]$$

Where $m=\left(\frac{hP_{c.s.}}{k_sA_{c.s.}}\right)^{0.5}=34.28\ \mathrm{m}^{-1}$; $mL=0.411$; $h/mk_s=0.0171$

$$\sqrt{k_sA_{c.s.}hP_{c.s.}}=\sqrt{380\times\frac{\pi^2(2\times 10^{-3})^3}{4}\times 223.3}=0.0409\ \mathrm{W/^\circ C};$$

$$q_{\substack{total\ loss\\ Fin\to Fluid}}=\sqrt{k_sA_{c.s.}hP_{c.s.}}\,(T_{Base}-T_\infty)\left[\frac{\sinh[mL]+(h/mk_s)\cosh[mL]}{\cosh[mL]+(h/mk_s)\sinh[mL]}\right]$$

$$q_{\substack{total\ loss\\ Fin\to Fluid}}=0.0409\times(350-300)\left[\frac{\sinh[0.411]+0.0171\times\cosh[0.411]}{\cosh[0.411]+0.0171\times\sinh[0.411]}\right]=0.826\ \mathrm{W}$$

c)

Total rate of heat transfer from the chip:

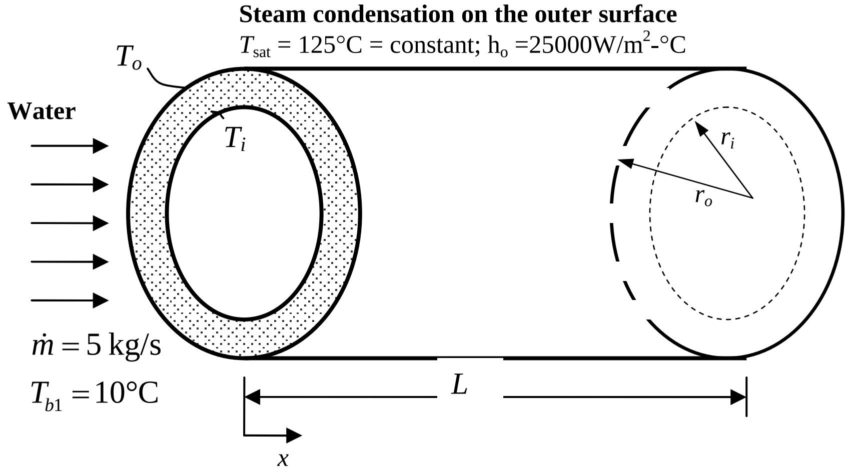
$$q_{\substack{total\\ chip}}=q_{\substack{chip\\ unfinned}}+q_{\substack{total\ loss\\ Fin\to Fluid}}$$

$$q_{\substack{chip\\ unfinned}}=h_{av}A_{unfinned}(T_{chip}-T_\infty)=h_{av}A_{unfinned}(T_{Base}-T_\infty)$$

$$q_{\substack{chip\\ unfinned}}=h_{av}\left(A_{\substack{total\\ chip}}-A_{\substack{c.s\\ fin}}\right)(T_{Base}-T_\infty)=223.3\times\left(\left(4\times 10^{-3}\right)^2-\frac{\pi\left(2\times 10^{-3}\right)^2}{4}\right)(350-300)=0.1435\ \mathrm{W}$$

$$q_{\substack{total\\ chip}}=0.1435+0.826=0.97\ \mathrm{W}$$

Problem 3:



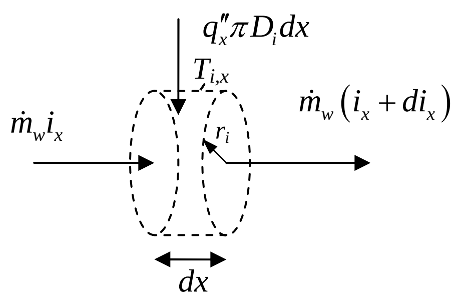
Given: k_{pipe} = 25 W/m-°C ; D_i = 0.05 m, D_o = 0.06 m;
Water flowing inside the pipe: $\dot{m} = 5$ kg/s; $T_{b1} = 10°C$, water properties: ρ = 1000 kg/m^3; c_p = 4180 J/kg-°C; μ = 1.2× 10^{-3} kg/m-s; k = 0.585 W/m-°C
Steam condenses on outer surface: T_{sat} = 125°C; h_{fg} = 2.2 × 10^6 J/kg; $h_{outside}$ = 25000 W/m^2-°C

Assumptions: Steady-state fully-developed flow inside the pipe; constant properties; Ec<<1; Pe>>1; smooth tube; axial heat conduction in tube wall is negligible in compare to radial heat conduction.

a) $\dot{m}_{condensate} = ?$

$q_{total} = \dot{m}_{condensate} h_{fg} = \dot{m}_{water} c_{p,water} \left(T_{b2} - T_{b1}\right)$; Thus, we need $T_b \big|_{x=L} = T_{b2}$.

Energy balance on an infinitesimal C.V. for water inside the pipe:



$$\dot{m}_w \left(i_x + di_x\right) - \dot{m}_w i_x = q''_x \pi D_i dx$$

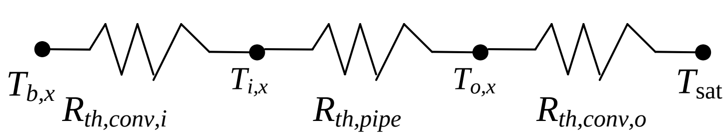
where i_x : specific enthalpy of water J/kg

$$\dot{m}_w di_x = q''_x \pi D_i dx \Rightarrow \dot{m}_w c_p dT_{b,x} = q''_x \pi D_i dx$$

$$\text{or } \dot{m}_w c_p dT_{b,x} = h_i \left(T_i - T_{b,x}\right) \pi D_i dx = h_i \pi D_i L \left(T_i - T_{b,x}\right) \frac{dx}{L}$$

$$\text{or } \dot{m}_w c_p dT_{b,x} = \frac{\left(T_i - T_{b,x}\right)}{1/\left(h_i \pi D_i L\right)} \frac{dx}{L} = \frac{\left(T_i - T_{b,x}\right)}{R_{th,conv,i}} \frac{dx}{L}$$

Using resistance analogy for steady-state one-D radial heat transfer from the condensate to the water at each position x:



$$\dot{m}_w c_p dT_{b,x} = \frac{\left(T_i - T_{b,x}\right)}{R_{th,conv,i}}\frac{dx}{L} = \frac{\left(T_{sat} - T_{b,x}\right)}{R_{th,conv,i} + R_{\substack{th,cond.\\pipe\\wall}} + R_{th,conv,o}}\frac{dx}{L}$$

$$R_{th,conv,i} = \frac{1}{h_i \pi D_i L}; R_{\substack{th,cond.\\pipe\\wall}} = \frac{\ln\left(D_o / D_i\right)}{2\pi k_{pipe} L}; R_{th,conv,o} = \frac{1}{h_o \pi D_o L}$$

$$\dot{m}_w c_p dT_{b,x} = \frac{\left(T_{sat} - T_{b,x}\right)}{\sum R_{th}}\frac{dx}{L} \Rightarrow \frac{dT_{b,x}}{\left(T_{sat} - T_{b,x}\right)} = \frac{1}{\dot{m}_w c_p L \sum R_{th}} dx$$

$\dot{m}_w, c_p, L, T_{sat}$, and $\sum R_{th}$ are all constant:

$$\int_{T_{b1}}^{T_{b2}} \frac{dT_{b,x}}{\left(T_{sat} - T_{b,x}\right)} = \frac{1}{\dot{m}_w c_p L \sum R_{th}} \int_0^L dx$$

$$\Rightarrow \ln\left[\frac{\left(T_{sat} - T_{b1}\right)}{\left(T_{sat} - T_{b2}\right)}\right] = \frac{1}{\dot{m}_w c_p \sum R_{th}} \qquad (1)$$

please note that this equation can be manipulated to get the form of ΔT_{LMTD} :

$$\dot{m}_w c_p \left(T_{b2} - T_{b1}\right) = (UA)\Delta T_{LMTD} = \frac{\Delta T_{LMTD}}{\sum R_{th}}$$

and here $\Delta T_{LMTD} = \dfrac{\left(T_{sat} - T_{b1}\right) - \left(T_{sat} - T_{b2}\right)}{\ln\left[\dfrac{\left(T_{sat} - T_{b1}\right)}{\left(T_{sat} - T_{b2}\right)}\right]} = \dfrac{T_{b2} - T_{b1}}{\ln\left[\dfrac{\left(T_{sat} - T_{b1}\right)}{\left(T_{sat} - T_{b2}\right)}\right]}$

thus, from Eq. (1) $T_{b2} = T_{sat} - \left(T_{sat} - T_{b1}\right) / \exp\left[\dfrac{1}{\dot{m}_w c_p \sum R_{th}}\right]$

Now we need to estimate h_i:

$$\mathrm{Re_D} = \frac{\rho_{water} U_{av} D_i}{\mu_{water}} = \frac{\rho_{water} U_{av}\left(\pi D_i^2 / 4\right)}{\mu_{water}}\left(\frac{4}{\pi D_i}\right) = \frac{4\dot{m}_{water}}{\pi D_i \mu_{water}} = \underbrace{1.061\times 10^5}_{Turbulent} > 2300$$

$$\mathrm{Pr}_{water} = \frac{\mu_{water} c_{p,water}}{k_{water}} = 8.574$$

Here, I will use Seider-Tate correlation (please redo using other available correlations.

$$Nu_{av} \triangleq \frac{h_{av} D_i}{k_{water}} = 0.027 Re_D^{0.8} Pr_f^{\ 1/3}\left(\frac{\mu}{\mu_{wall}}\right)^{0.14}$$

note: all properties in this prob. are constant. Thus, $\left(\dfrac{\mu}{\mu_{wall}}\right) = 1$

$$h_{av} = \frac{k_{water}}{D}\times 0.027 Re_D^{0.8} Pr_f^{\ 1/3} = \frac{0.585}{0.05}\times 0.027\times\left(1.061\times 10^5\right)^{0.8}\left(8.574\right)^{1/3} = 6779.3\ \mathrm{W/m^2 - {}^\circ C}$$

$$R_{th,conv,i} = \frac{1}{h_i \pi D_i L} = \frac{1}{6779.3 \times \pi \times 0.05 \times 10} = 9.3906 \times 10^{-5} \text{ °C/W;}$$

$$R_{\substack{th,cond.\\ pipe\\ wall}} = \frac{\ln(D_o / D_i)}{2\pi k_{pipe} L} = \frac{\ln(0.06/0.05)}{2\pi \times 25 \times 10} = 1.1607 \times 10^{-4} \text{ °C/W}$$

$$R_{th,conv,o} = \frac{1}{h_o \pi D_o L} = \frac{1}{25000 \times \pi \times 0.06 \times 10} = 2.122 \times 10^{-5} \text{ °C/W}$$

$$\sum R_{th} = 9.3906 \times 10^{-5} + 1.1607 \times 10^{-4} + 2.122 \times 10^{-5} = 2.312 \times 10^{-4}$$

$$\text{and } T_{b2} = 125 - (125 - 10)/\exp\left[\frac{1}{5 \times 4180 \times 2.312 \times 10^{-4}}\right] = 31.5\text{°C}$$

$$q_{total} = \dot{m}_{condensate} h_{fg} = \dot{m}_{water} c_{p,water} (T_{b2} - T_{b1})$$

$$\Rightarrow \dot{m}_{condensate} = \frac{5 \times 4180 \times (31.5 - 10)}{2.2 \times 10^6} = 0.20425 \text{ kg/s}$$

b) What is the total pressure drop in the pipe?

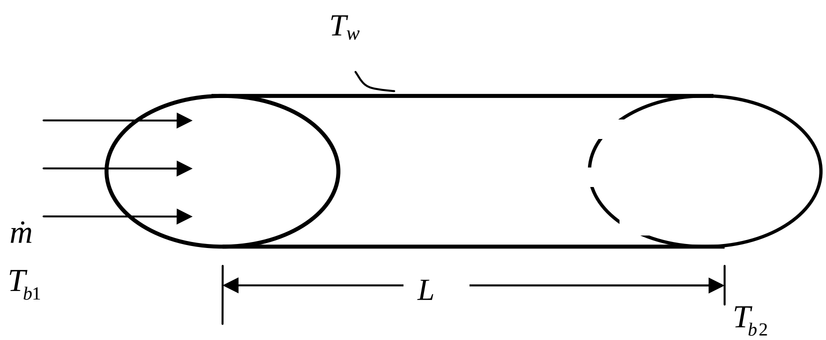
Using Filonenko correlation for smooth pipes:

$$f_{darcy} = \frac{[(P_1 - P_2)/L] D_i}{\rho_{water} u_{av}^2 / 2} = \left[1.82 \log_{10}(Re_{D_i}) - 1.64\right]^{-2} \Rightarrow f_{darcy} = 0.01775$$

$$u_{av} = \frac{\dot{m}_{water}}{\rho_{water} (\pi D_i^2 / 4)} = 2.546 \text{ m/s}$$

$$\Rightarrow (P_1 - P_2) = \frac{10}{0.05} \times 0.01775 \times \frac{1000 \times (2.546)^2}{2} = 11.5 \text{ kPa}$$

Problem 4:



Given: Duct geometry: Perimeter of cross section = 0.08 m; Cross sectional area = 5 × 10^{-4} m^2. L = 2 m. Flow and heat transfer: u_{av} = 8 m/s; T_w = 200°C; T_{b1} = 20°C; T_{b2} = 160°C
Fluid properties: ρ = 1 kg/m^3; c_p = 1000 J/kg-°C; μ = 2× 10^{-5} kg/m-s; k = 0.025 W/m-°C

Assumptions: Steady-state fully developed fluid flow and heat transfer in a duct of non-circular cross section and unknown roughness; constant properties; Ec<<1; Pe>>1.

a) Find the average heat transfer coefficient;

$$q_{\substack{total\\ to\ fluid}} = \dot{m}_w c_p \left(T_{b2} - T_{b1}\right) = \left(hA_{surf}\right)\Delta T_{LMTD} \qquad \text{(please see HD # 8)}$$

and here $\Delta T_{LMTD} = \dfrac{\left(T_w - T_{b1}\right) - \left(T_w - T_{b2}\right)}{\ln\left[\dfrac{\left(T_w - T_{b1}\right)}{\left(T_w - T_{b2}\right)}\right]} = \dfrac{T_{b2} - T_{b1}}{\ln\left[\dfrac{\left(T_w - T_{b1}\right)}{\left(T_w - T_{b2}\right)}\right]}$

$$q_{\substack{total\\ to\ fluid}} = \dot{m}_w c_p \left(T_{b2} - T_{b1}\right) = \left(\rho A_{cs} u_{av}\right) c_p \left(T_{b2} - T_{b1}\right)$$

$$q_{\substack{total\\ to\ fluid}} = \left(1 \times 5 \times 10^{-4} \times 8\right) \times 1000 \times \left(160 - 20\right) = 560 \text{ W}$$

$$q_{\substack{total\\ to\ fluid}} = \left(hA_{surf}\right)\Delta T_{LMTD}; \quad A_{surf} = P_{c.s.}L = 0.08 \times 2 = 0.16 \text{ m}^2$$

$$\Delta T_{LMTD} = \frac{T_{b2} - T_{b1}}{\ln\left[\dfrac{\left(T_w - T_{b1}\right)}{\left(T_w - T_{b2}\right)}\right]} = \frac{\left(160 - 20\right)}{\ln\left[\dfrac{\left(200 - 20\right)}{\left(200 - 160\right)}\right]} = 93.08°\text{C} \Rightarrow h = \frac{560}{0.16 \times 93.08} = 37.6 \text{ W/m}^2\text{-°C}$$

Please note that, for constant pipe wall temperature, in your textbook (Holman, 2002), Chapter 6, they have used the following temperature difference between the fluid and the pipe wall: $\Delta T = (T_w - \dfrac{T_{b1} + T_{b2}}{2})$ instead of ΔT_{LMTD} presented above. In fact, this is not precise for the constant wall temperature case, as from point 1 to 2 the bulk temperature is not varying linearly. In our problem, this approximate temperature difference is $\Delta T = (T_w - \dfrac{T_{b1} + T_{b2}}{2}) = 200 - \dfrac{160 + 20}{2} = 110$ °C instead of $\Delta T_{LMTD} = 93.08$°C. ΔT_{LMTD} is introduced in Chapter 10 (Heat Exchangers) of the textbook.

b) What is the total pressure drop in the pipe?

$$D_H = \frac{4A_{c.s}}{P_{c.s.}} = 4 \times 5 \times 10^{-4} / 0.08 = 2.5 \times 10^{-2} \text{ m}; \ \text{Re}_{D_H} = \frac{\rho u_{av} D_H}{\mu} = \frac{1 \times 8 \times 2.5 \times 10^{-2}}{2 \times 10^{-5}} = 10000 > 2300 \quad \text{Turbulent}$$

$$\Pr = \frac{\mu c_p}{k} = \frac{2 \times 10^{-5} \times 1000}{0.025} = 0.8$$

This is Turbulent fluid flow in a non-circular c.s. duct with unknown roughness; since no specific correlations are available here, we use the Chilton-Colburn Analogy.

$$St\,\Pr^{2/3} = \frac{f}{8} = \frac{1}{8}\left\{\frac{\Delta P_{drop}}{L}\frac{D_H}{0.5\rho u_{av}^{\ 2}}\right\}; \qquad St \triangleq \frac{h}{\rho u_{av} c_p} = \frac{37.6}{1 \times 8 \times 1000} = 4.7 \times 10^{-3}$$

$$\frac{1}{8}\left\{\frac{\Delta P_{drop}}{L}\frac{D_H}{0.5\rho u_{av}^{\ 2}}\right\} = 4.7 \times 10^{-3} \times \left(0.8\right)^{2/3} = 4.05 \times 10^{-3}$$

$$\Delta P_{drop} = P_1 - P_2 = 8 \times \frac{2}{2.5 \times 10^{-2}} \times \left(0.5 \times 1 \times 8^2\right) \times 4.05 \times 10^{-3} = 82.94 \text{ N/m}^2$$