MECH366 Modeling of Mechatronic Systems
Exercise solutions for Laplace transform

1. (a) $F(s) = \dfrac{e^{-2.5s}}{s}$

   (b) $F(s) = \dfrac{1}{s+4}$

   (c) $F(s) = \dfrac{1}{s^2}$

   (d) $F(s) = -\dfrac{3}{(s+1)^2}$

   (e) $F(s) = -\dfrac{5s}{s^2+1}$

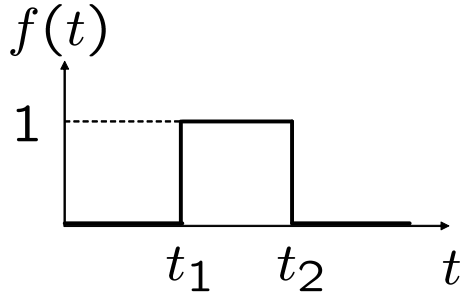
   (f) $F(s) = \mathcal{L}\left\{t\dfrac{1}{2j}\left(e^{3jt} - e^{-3jt}\right)\right\} = \dfrac{1}{2j}\left\{\dfrac{1}{(s-3j)^2} - \dfrac{1}{(s+3j)^2}\right\} = \dfrac{6s}{(s^2+9)^2}$

   (g) $F(s) = \dfrac{7(s+0.5)}{(s+0.5)^2+9}$

   (h) $F(s) = 5\mathcal{L}\left\{\cos 4t\cos(\pi/6) - \sin 4t\sin(\pi/6)\right\} = \dfrac{5}{2}\cdot\dfrac{\sqrt{3}s-4}{s^2+16}$

   (i) $F(s) = 6\mathcal{L}\left\{e^{-2t}(\sin t\cos(\pi/4) - \cos t\sin(\pi/4))\right\} = \dfrac{6}{\sqrt{2}}\cdot\dfrac{1-(s+2)}{(s+2)^2+1} = -\dfrac{6}{\sqrt{2}}\cdot\dfrac{s+1}{(s+2)^2+1}$

2. (B.5.(a)) $f(t) = u(t-t_1) - u(t-t_2)$



3. (B.5.(b)) $f(t) = \dfrac{1}{t_2-t_1}(t-t_1)u(t-t_1) - u(t-t_2) - \dfrac{1}{t_2-t_1}(t-t_2)u(t-t_2)$

$$F(s) = \frac{e^{-t_1 s} - e^{-t_2 s}}{(t_2-t_1)s^2} - \frac{e^{-t_2 s}}{s}$$

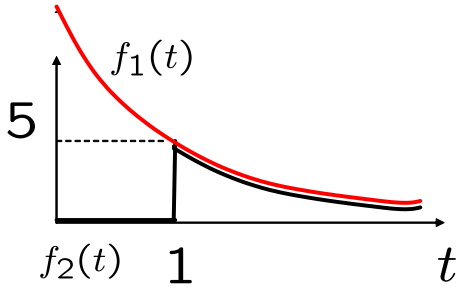
4. (B.6) For $f(t) = 4e^{-2(t-3)}u(t-3)$:

(a) $f'(t) = 4\left\{-2e^{-2(t-3)}u(t-3) + e^{-2(t-3)}\delta(t-3)\right\}$

$$\mathcal{L}\{f'(t)\} = 4\left\{-\frac{2e^{-3s}}{s+2} + e^{-3s}\right\} = 4e^{-3s}\frac{s}{s+2}$$

(b) $\mathcal{L}\{f'(t)\} = sF(s) - f(0) = s\dfrac{4e^{-3s}}{s+2} - 0 = s\dfrac{4e^{-3s}}{s+2}$

5. (a) See below.



(b) $F_1(s) = 5e^2\dfrac{1}{s+2}$, $F_2(s) = 5e^{-s}\dfrac{1}{s+2}$.