

Q1:

$$\begin{aligned}
 &AN + B \sum X + C \sum X^2 = \sum Y \\
 &A \sum X + B \sum X^2 + C \sum X^3 = \sum XY \\
 &A \sum X^2 + B \sum X^3 + C \sum X^4 = \sum X^2 Y
 \end{aligned}$$

$$\begin{bmatrix} N & \sum X & \sum X^2 \\ \sum X & \sum X^2 & \sum X^3 \\ \sum X^2 & \sum X^3 & \sum X^4 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} \sum Y \\ \sum XY \\ \sum X^2 Y \end{bmatrix}$$

$$\begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} N & \sum X & \sum X^2 \\ \sum X & \sum X^2 & \sum X^3 \\ \sum X^2 & \sum X^3 & \sum X^4 \end{bmatrix}^{-1} \begin{bmatrix} \sum Y \\ \sum XY \\ \sum X^2 Y \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 & 20 \\ 0 & 20 & 0 \\ 20 & 0 & 164 \end{bmatrix}^{-1} \begin{bmatrix} 5.4 \\ -4 \\ 25.4 \end{bmatrix}$$

$$= \begin{bmatrix} 0.5296 \\ -0.008 \\ 4.2736 \end{bmatrix}$$

Differs from MATLAB

Figure 1: Hand Calculation for Quadratic Coefficients Using Least Square Method

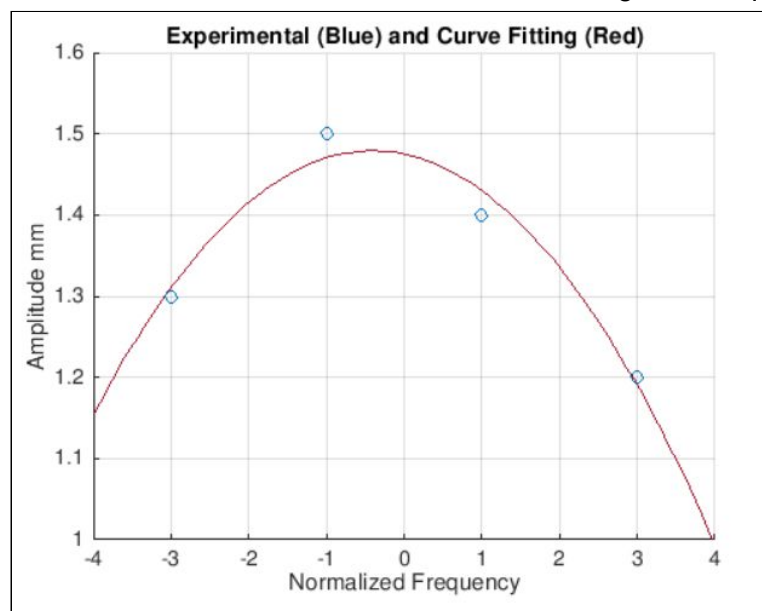


Figure 2: Normalized Frequency vs Amplitude

Matlab outputs $[A \ B \ C] = [1.4750 \ -0.0200 \ -0.0250]$, different from $[0.5296 \ -0.0080 \ 4.2736]$. This is likely because MATLAB uses a method slightly different from least square method. Natural frequency is at the peak of the curve, where the value is $[-0.3636 \ 1.4750]$. By reverse-normalizing -0.3636 , we can get 33.5Hz as natural frequency.

Q2:

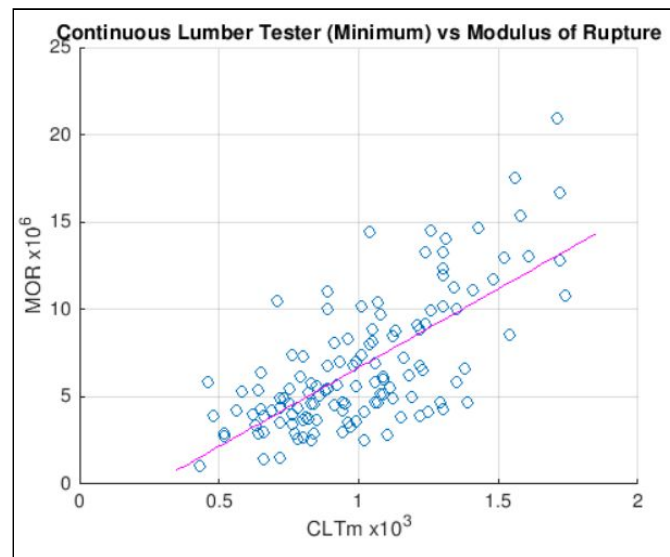


Figure 3: CLTm vs MOR

Fit Equation: $MOR = -2.3545 + 9.0208 \cdot CLTm$

$r^2: 0.496 \rightarrow r: 0.704$

Sigma: 2.6595

According to Appendix C of JRT, where $N = 131$ and $r_0 = 0.704$, there is less than 0.05% probability for r to exceed the value we have calculated from MATLAB. Therefore, the plotted correlation is highly likely to exist.

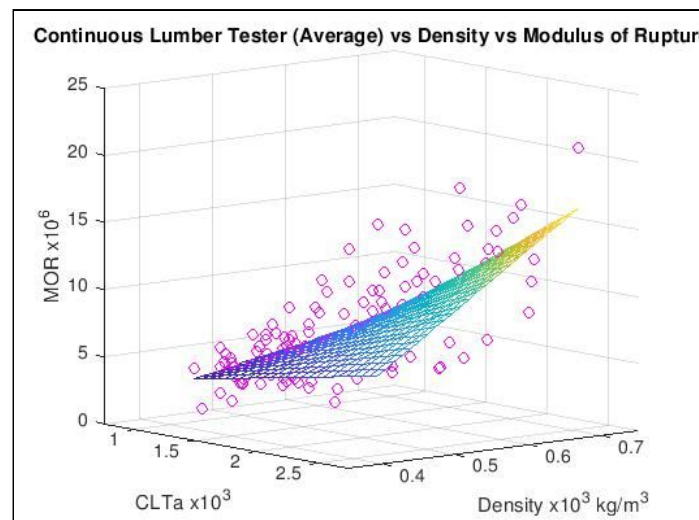


Figure 4: CLTa vs Density vs MOR

Fit Equation: $MOR = 4.6792 - 6.1082 \cdot CLTa - 7.7193 \cdot (Density) + 18.3582 \cdot CLTa \cdot (Density)$

$r^2: 0.548 \rightarrow r: 0.740$

Sigma: 2.5383

Since MOE_j is not an industry standard as the instruction has mentioned, CLTa vs Density vs MOR was plotted instead. Appendix C only over cases with two sets of data, we can't use it for Figure 4. However, we can visually make generalizations.

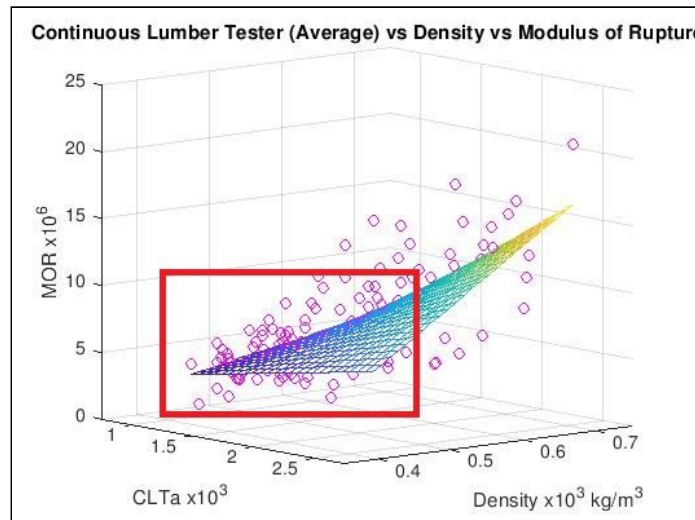


Figure 5: Visual Generalization

While there is a cluster of points around the red-boxed area, the fit is loose everywhere else. Therefore we can conclude that there is only a medium to low correlation between the datasets.