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MECH366 : Modeling of Mechatronic Systems

L2 : Modeling procedure
Analogies among different domains

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Review and today's topics

- Last lecture's key messages
  - Models play an important role in mechatronic systems.
  - Modeling is not an easy task.
- Today's topics
  - Dynamic models
  - Modeling procedure for dynamic models
  - Analogies among different domains (Mechanical, electrical, thermal, fluid)

    * Details of analogies will not be covered in today's class, but will be followed up in later classes.

Dynamic models

- Static model
  - Present output depends on only present input.
  - Input-output relation is represented by an algebraic equation (or look-up table).
  - Example: $x = (1/k)f$

    f : force input

    x : displacement output
- Dynamic model (This course's interest)
  - Present output depends on past and present input.
  - Input-output relation is represented by a differential equation (ODE, PDE).
  - Example: $\dot{v} = (1/m)f$

    f : force input

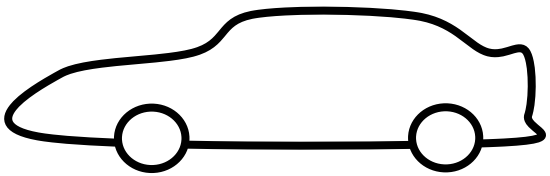
    v : velocity output

Modeling procedure (How to obtain dynamic models)

1. Identify a physical system to be modeled, and associated input and output variables.
2. Simplify the physical system with basic elements (see slide 14), based on your assumptions.
3. By applying physical laws (Newton's second law, Kirchhoff's law etc.) to the basic elements, obtain differential equations.
4. Identify (estimate) parameter values in the differential equations.
5. Validate the obtained model (differential equations with estimated parameter values) experimentally. If the model turns out to be invalid (the model is invalidated), go back to Step 2 with modifications.

1. Identify a physical system to be modeled, and I/O variables

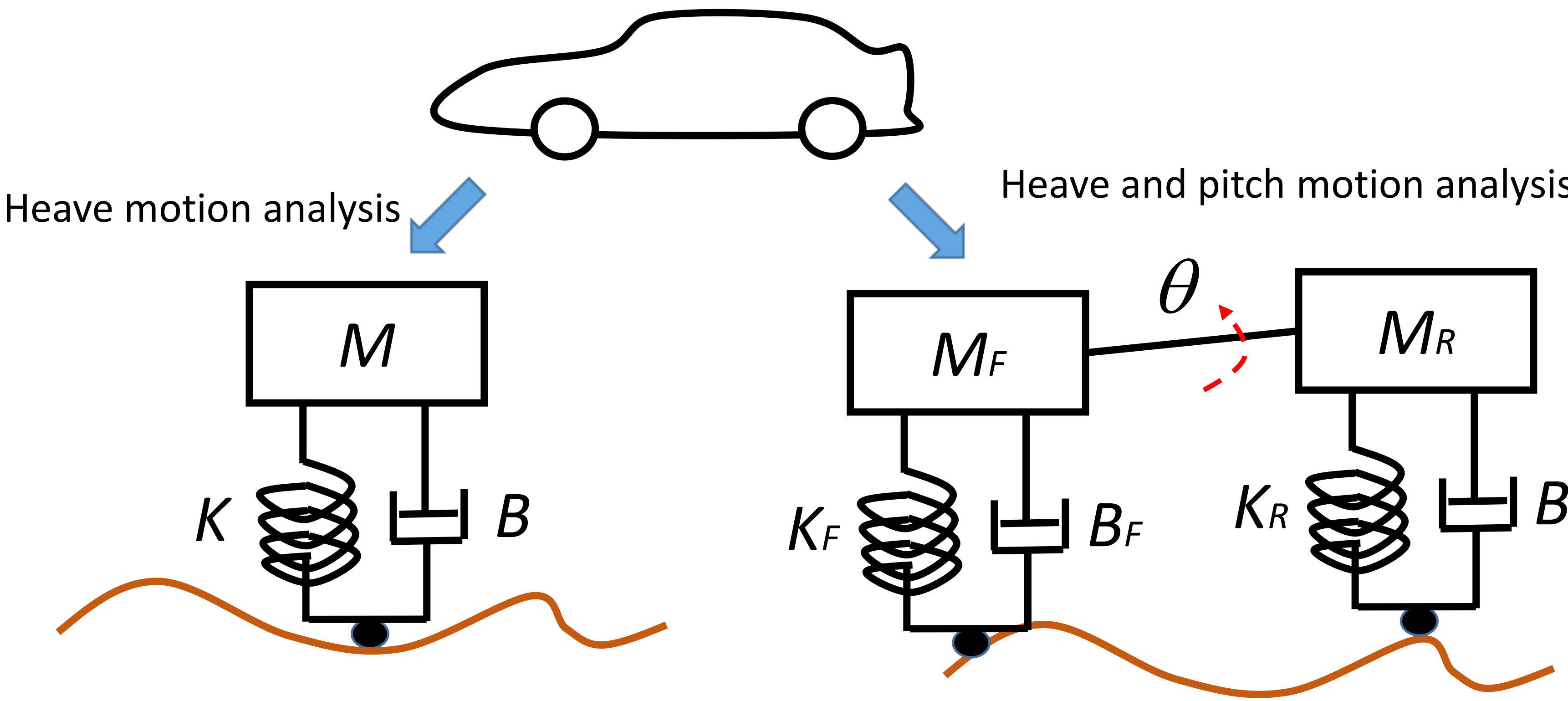
- Modeling depends on the purpose of models. (How do you want to use a model?)
- Automobile example



  - Cruise control
    (Input: throttle valve opening angle, output: car speed)
  - Direction control
    (Input: Steering wheel angle, output: direction)
  - Comfort analysis
    (Input: ground displacement, output: roll/pitch motion)

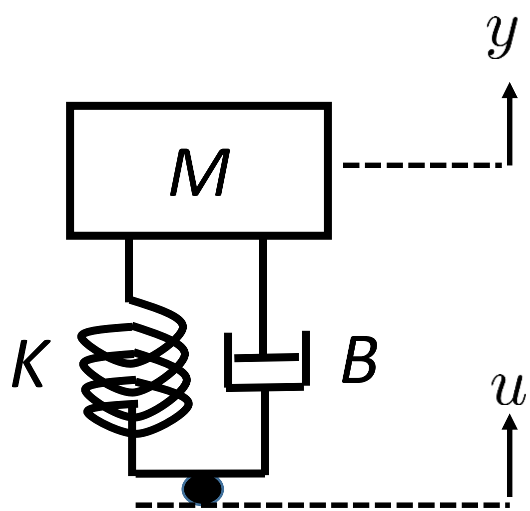
2. Simplify the physical system with basic (lumped) elements

- Automobile comfort analysis example



3. Obtain a differential equation by using physical laws

- Heave motion analysis



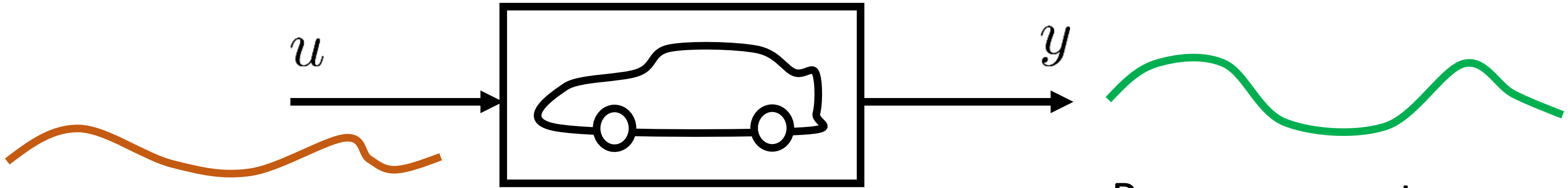
Newton's second law

Equation of motion

$$M\ddot{y} = -K(y-u) - B(\dot{y}-\dot{u})$$

4. Identify (estimate) parameter values in the differential equations

- Excite the physical system



Ground displacement input

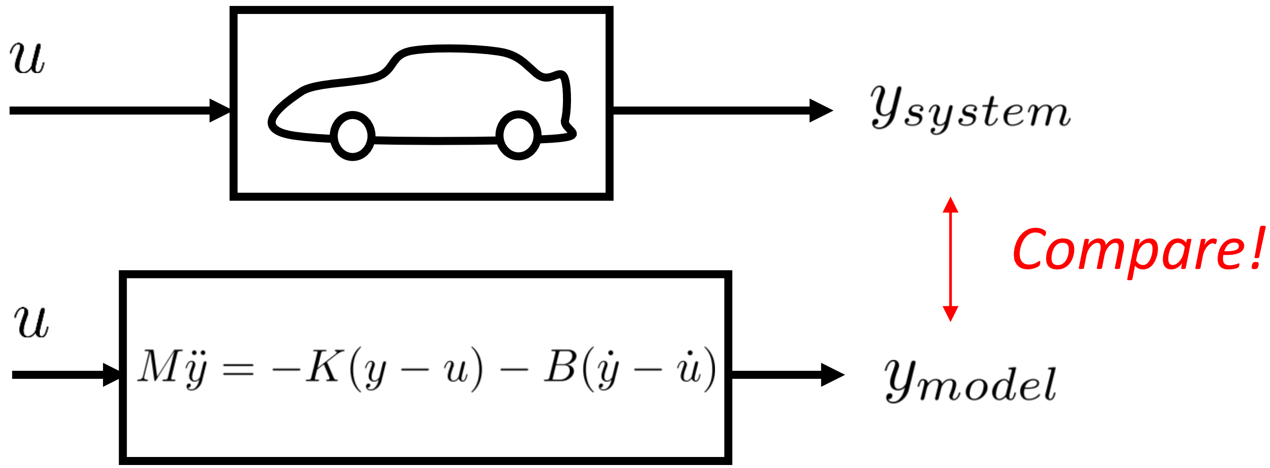
Passenger seat displacement output

Parameter estimation method (Step response technique, frequency response technique)

$$M\ddot{y} = -K(y-u) - B(\dot{y}-\dot{u})$$

5. Model validation (to increase the confidence of the model)

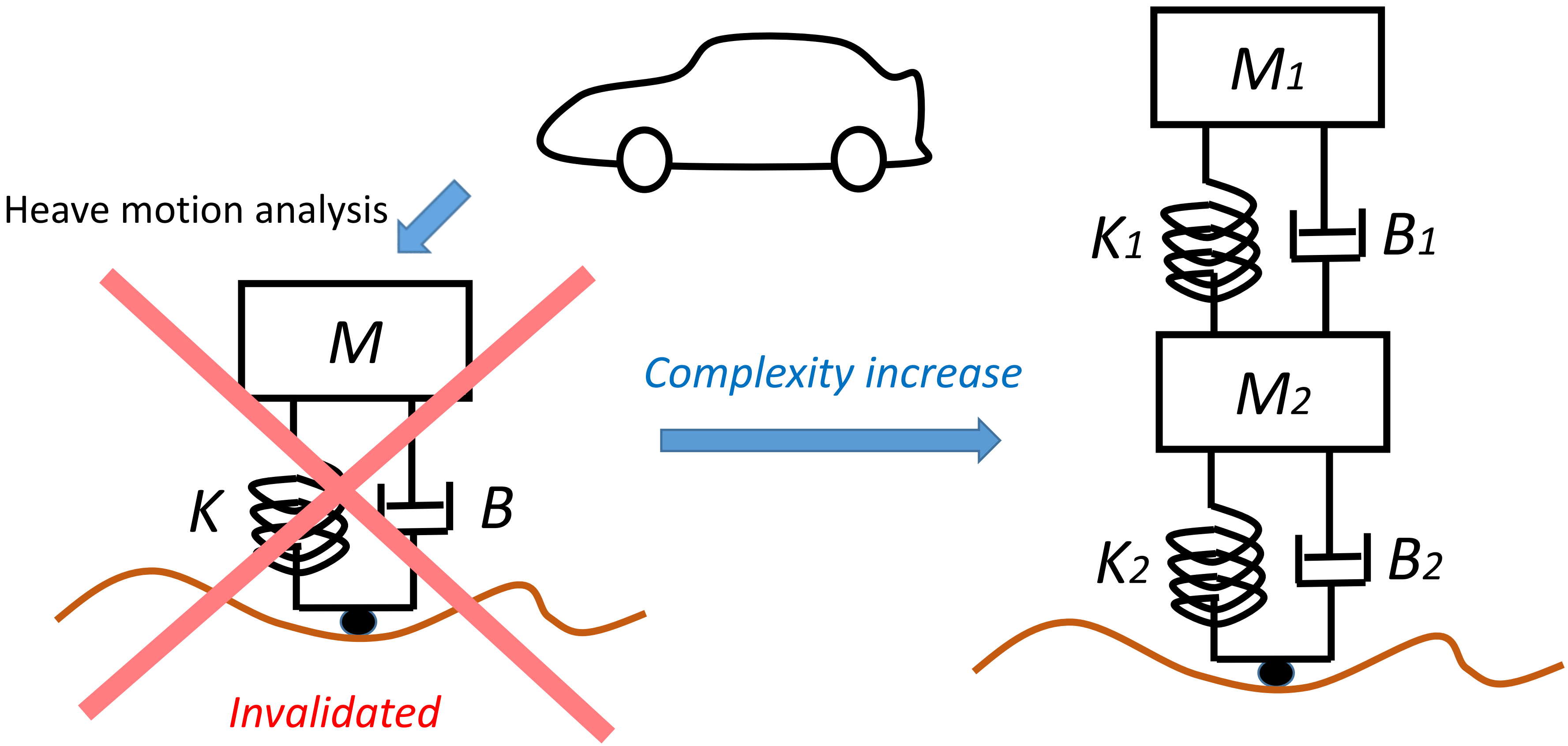
- Excite both the physical system and the model with various inputs, and compare the outputs



- If the model is invalidated, go back to Step 2 with modified assumptions. (next slide)

2. Simplify the physical system with basic (lumped) elements

- Automobile comfort analysis example



Modeling procedure Summary

Assumptions (Step 2)
(Model structure)
+
Physical laws (Step 3)
+
Experimental data (Step 4)

Modeling

Physical system → Model

Model validation

Physical system ⟷ Model

Experimental data (Step 5)

Data-driven (machine learning) modeling procedure

Assumptions (Step 2)
(Model structure)
+
~~Physical laws (Step 3)~~
+
Experimental data (Step 4)

Modeling

Physical system

Model

Model validation

Experimental data (Step 5)

Analogies (Table in the next slide)

- Analogies exist among mechanical, electrical, thermal and fluid systems.
- Understanding the analogies will facilitate the modeling, especially when using the linear graph.
- In each domain, basic elements are categorized as
  - Energy storage elements
    - A-type (Element’s energy is a function of an across variable.)
    - T-type (Element’s energy is a function of a through variable.)
  - Energy dissipating elements
- Power = (through variable)*(across variable)

	Constitutive Relation for		
	Energy Storage Elements		Energy Dissipating Elements
System Type	A-type (Across) Element	T-type (Through) Element	D-type (Dissipative) Element
Translatory-Mechanical v = velocity across variable f = force through variable $\mathcal{P} = fv$ power	*Mass* $m\frac{dv}{dt} = f$ (Newton's second law) m = mass	*Spring* $\frac{df}{dt} = kv$ (Hooke's law) k = stiffness	*Viscous Damper* $f = bv$ b = damping constant
Electrical v = voltage across variable i = current through variable $\mathcal{P} = iv$ power	*Capacitor* $C\frac{dv}{dt} = i$ C = capacitance	*Inductor* $L\frac{di}{dt} = v$ L = inductance	*Resistor* $Ri = v$ R = resistance
Thermal across variable T = temperature difference $[K]$ Q = heat transfer rate $[J/s], [W]$ through variable $\mathcal{P} = Q$ power	*Thermal Capacitor* $C_t\frac{dT}{dt} = Q$ C_t = thermal capacitance	*None*	*Thermal Resistor* $R_t Q = T$ R_t = thermal resistance
Fluid across variable P = pressure difference $[N/m^2]$ Q = volume flow rate $[m^3/s]$ through variable $\mathcal{P} = QP$ power	*Fluid Capacitor* $C_f\frac{dP}{dt} = Q$ C_f = fluid capacitance	*Fluid Inertor* $I_f\frac{dQ}{dt} = P$ I_f = inertance	*Fluid Resistor* $R_f Q = P$ R_f = fluid resistance

Energy expressions based on across and through variables

	A-type element	T-type element
Mechanical v : Across variable f : Through variable	Kinetic energy $\frac{1}{2}mv^2$	Potential energy $\left(\frac{1}{2}kx^2 =\right) \frac{1}{2}\frac{f^2}{k}$
Electrical v : Across variable i : Through variable	Electrostatic energy $\frac{1}{2}Cv^2$	Electromagnetic energy $\frac{1}{2}Li^2$
Thermal T : Across variable Q : Through variable	Thermal energy $\int Q = C_t T$	N/A
Fluid P : Across variable Q : Through variable	Potential energy $\frac{1}{2}C_f P^2$	Kinetic energy $\frac{1}{2}I_f Q^2$

Summary

Project group	Lab group
G1	A1, A2, A3
G2	A4, A5, A6
G3	B1, B2, B3
G4	B4, B5, B6

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- Today's topics
  - Modeling procedure
  - Analogies between different domains (mechanical, electrical, thermal, fluid)
- "Homework"
  - Think about physical systems for the project available for you. Discuss people around you.
  - Sep 13 (Fri): Meeting for project topic discussion at Kaiser 1160. (There is no laboratory session.)

  G1: 10am, **G2**: 11am, **G3**: 1pm, **G4**: 2pm