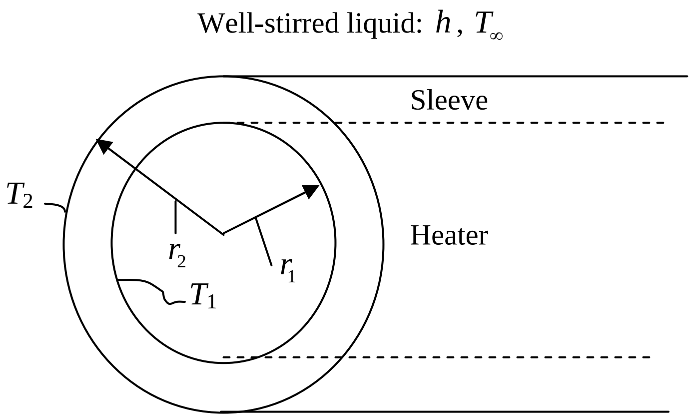
Solutions - Problem Set # 3

Problem 1:



Steady-state operation data:

$r_1 = 0.03$ m; $r_2 = 0.035$ m

$L = 1.0$ m; $T_\infty = 235.17\ ^oC$

$T_1 = 252\ ^oC$; $T_2 = 250\ ^oC$

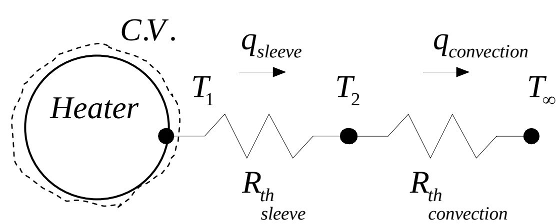
Total power input of the heater is $q = 4891.20$ W

Assumptions: 1-D Steady-state radial heat conduction; Losses from end surfaces are negligible

a) $\mathbf{k_{sleeve}}$=?

S.S. E-Bal. on the heater:

$$\left.\begin{matrix}power\ input\\ to\ heater\end{matrix}\right\} = q_{\substack{generation\\heater}} = q_{sleeve} = q_{convection}$$



$q_{sleeve} = 4891.2\text{W};$

$$R_{\substack{th\\sleeve}} = \frac{\ln(r_2/r_1)}{2\pi k_{sleeve} L} = \left(\frac{\ln(r_2/r_1)}{2\pi L}\right)\frac{1}{k_{sleeve}} = \frac{2.453\times10^{-2}}{k_{sleeve}}$$

$$q_{sleeve} = \frac{T_1 - T_2}{R_{\substack{th\\sleeve}}} \rightarrow 4891.2 = \frac{252-250}{2.453\times10^{-2}} k_{sleeve} \Rightarrow k_{sleeve} = 60 \text{ W/m-°C}$$

b) h=?

$q_{convection} = 4891.2\text{W};$

$$R_{\substack{th\\convection}} = \frac{1}{2\pi r_2 L h} = \frac{4.547}{h}$$

$$q_{convection} = \frac{T_2 - T_\infty}{R_{\substack{th\\convection}}} \rightarrow 4891.2 = \frac{250-235.17}{4.547} h \Rightarrow h = 1499.78 \text{ W/m}^2\text{-°C}$$

c) r₂=? To give $\left(T_1 - T_\infty\right)_{\min}$

$$q_{\underset{heater-liquid}{loss}} = \frac{T_1 - T_\infty}{R_{\underset{sleeve}{th}} + R_{\underset{convection}{th}}} = 4891.2\text{W=constant;}$$

$\left(T_1 - T_\infty\right)$ is minimum when $\left[R_{\underset{sleeve}{th}} + R_{\underset{convection}{th}}\right]$ is minimum

This can only happen when $r_2 = r_{crit} = \dfrac{k_{sleeve}}{h} = 60/1495.98 = 0.04$ m

For this radius the minimum temperature difference is obtained:

$$\frac{\left(T_1 - T_\infty\right)_{\min}}{\left[R_{\underset{sleeve}{th}} + R_{\underset{convection}{th}}\right]_{r_2 = r_{crit}}} = 4891.2\text{W}$$

$$\rightarrow \left(T_1 - T_\infty\right)_{\min} = 4891.2\left[\frac{\ln\left(0.04/0.03\right)}{2\pi 60} + \frac{1}{2\pi 0.04\times 1495.98}\right] = 16.71°\text{C}$$

Problem 2:

Assumptions: 1) classical fin theory applies1-D; 2) Long fin, case 1 solution applies; 3) excellent thermal contact Wall-Base, $R_{\underset{contact}{th}} = 0$ (i.e., $T_{wall} = T_{Base}$)

Given: D = 0.005 m; T_{wall} = 100°C; h = 50 W/m^2-K; T_∞ = 20°C; k_{rod}=180 W/m-K

(a)

Case 1 Solution applies}→ $\dfrac{\theta}{\theta_{Base}} = \dfrac{T - T_\infty}{T_{Base} - T_\infty} = e^{-mx}$ where $m = (hP_{c.s.}/k_{rod}A_{c.s.})^{1/2}$

$$m = \left[50\pi D/(180\pi D^2/4)\right]^{1/2} = 14.908\ \text{m}^{-1}$$

$$\Rightarrow T = 80\exp(-14.908x) + 20$$

(b)

Case 1 Solution applies}→

$$q_{\underset{Fin\to Fluid}{total\ loss}} = \sqrt{k_{rod}A_{c.s.}hP_{c.s.}}\left(T_{Base} - T_\infty\right)$$

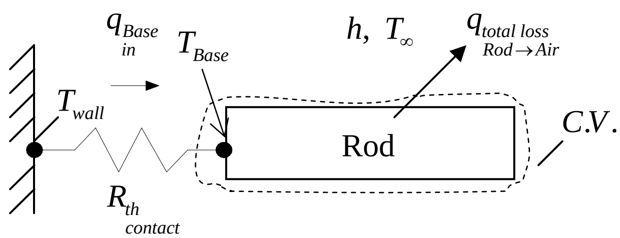
$$q_{\underset{Fin\to Fluid}{total\ loss}} = \left[180\times 1.9635\times 10^{-5}\times 50\times 0.01571\right]^{0.5}\left(80\right) = 4.215\text{W}$$

(c)

After several years of operation $q_{\underset{Fin\to Fluid}{total\ loss}}$ drops by 20 % and $R_{\underset{contact}{th}} \neq 0$ (i.e., $T_{wall} \neq T_{Base}$)

Thus the new heat loss is

$$q_{\underset{Fin\to Fluid}{total\ loss}} = q_{\underset{Rod\to Air}{total\ loss}} = 0.8\times 4.215 = 3.372\text{ W}$$



$$q_{\underset{Fin\to Fluid}{total\ loss}} = \sqrt{k_{rod}A_{c.s.}hP_{c.s.}}\left(T_{Base}-T_\infty\right)$$

$$3.372 = \left[180\times 1.9635\times 10^{-5}\times 50\times 0.01571\right]^{0.5}\left(T_{Base}-20\right)$$

$$T_{Base} = 84°\text{C}$$

S.S. E-Bal.:

$$q_{\underset{Fin\to Fluid}{total\ loss}} = q_{\underset{Rod\to Air}{total\ loss}} = q_{\underset{in}{Base}}$$

$$q_{\underset{in}{Base}} = \frac{T_{wall}-T_{base}}{R_{\underset{contact}{th}}} \Rightarrow R_{\underset{contact}{th}} = \frac{100-84}{3.372} = 4.745\ °\text{C/W}$$

$$R_{\underset{contact}{th}} = \frac{1}{\left(\pi D^2/4\right)h_{contact}} \Rightarrow h_{contact} = 10733.32\text{ W/m}^2\text{-°C}$$