Mech 305-306

Tutorial 2

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Feedbacks from Tutorial 1 Report

- How many significant figures to keep? Check Section 2.2 in text.
- Don't just copy what MATLAB shows you!

- Uncertainties: almost always round to one significant figure
  - u_L = Resolution / 2 = 1mm/2 = 0.5 mm
  - Edge cases like u = 0.35, you may use 0.35 instead of 0.4
- Best Estimation: should be in the same decimal position as the uncertainty
  - Eg. Matlab gives you L = 12.3456mm, report L_{best} = 12.3mm
- Final report: $x = x_{best} + u_x$
  - $L = (12.3 \pm 0.5)$ mm

Knowledge Checklist

- Random Number Generator (Q1 & Q2)
  - How it works
  - To use it in MATLAB
- Central Limit Theorem (Q1)
  - What it says
  - Prove it using MATLAB simulation
- Error Propagation Formula (Q2)
  - Use the formula to analyze “total” error analytically
  - Simulate error propagation using RNG

Random Number Generator in MATLAB

Z = rand(1, N)

returns a 1-by-N matrix containing pseudorandom values drawn from the **standard uniform distribution on the open interval(0,1)**.

Z = randn(1, N)

returns a 1-by-N matrix containing pseudorandom values drawn from the **standard normal distribution**.

Z = poissrnd(lambda,1,N);   % Poisson Distribution RNG

Z = binornd(N,p,1,NN); % Binomial Distribution RNG

Central Limit Theorem

Random variable:

$z = x1 + x2 + x3 + \ldots xM$

Where x_i are random variables, not necessarily normally distributed

If M is larrrrrge,

→ Then z is normally distributed

Question 1

Trying to prove CLT: when M is large, epsilon becomes normally distributed!

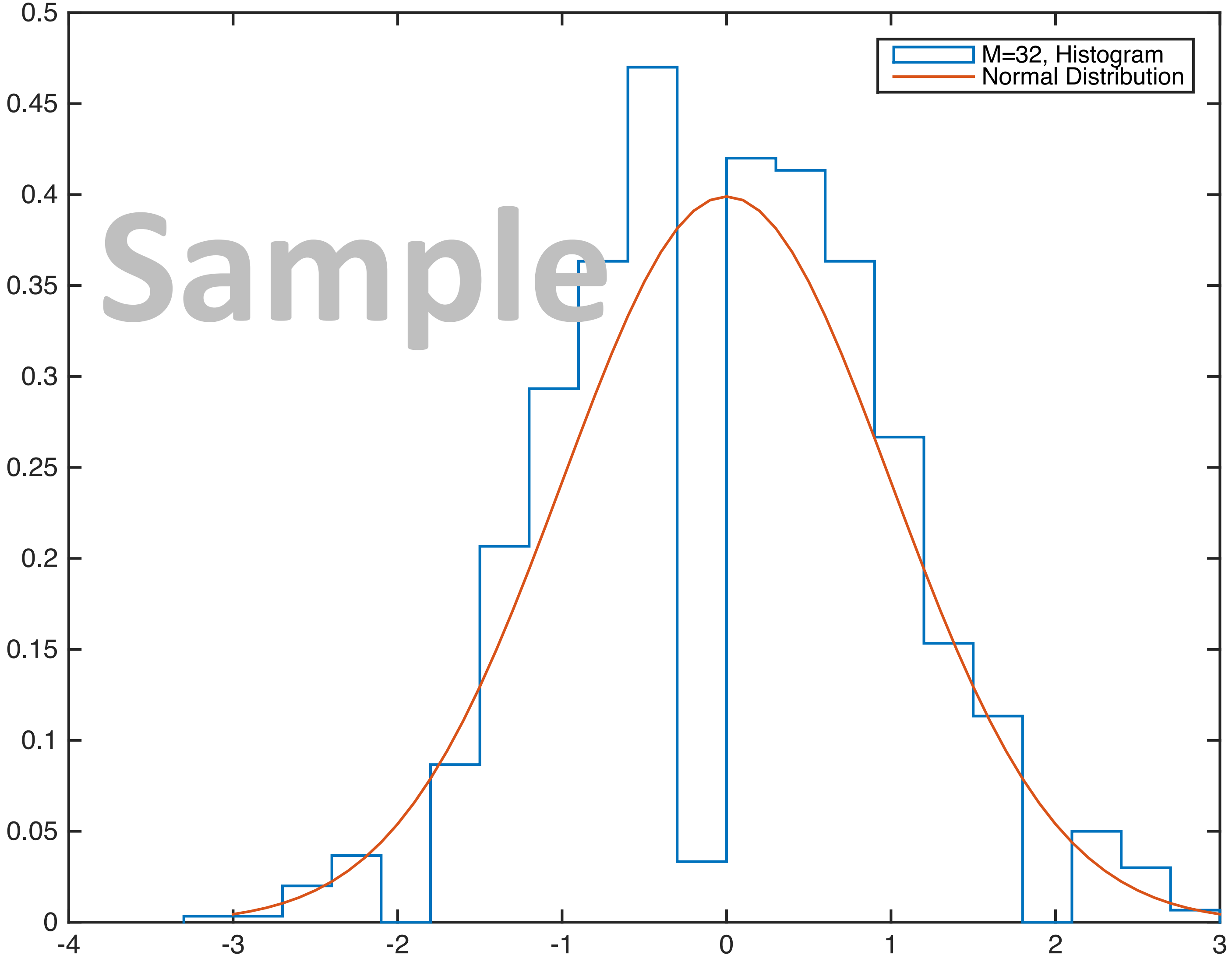
Write a Matlab script to generate random “errors” according to

$$\varepsilon_i = \sum_{j=1}^{M} \delta_j \quad ; \quad \delta_j = \pm \frac{s}{\sqrt{M}}$$

Take s=1 and try M=1,4,16,32. Plot the histograms using “pdf” normalization such that the area under them is 1. For each value of *M*, use 1000 values of e_i to obtain the histogram.

Compare with a normal distribution with mean zero and standard deviation 1.

Question 1: Sample plot



Provide 4 plots like this, with different M values.

Comment on your findings. (Did you prove CLT by these plots?)

Don’t worry if your plots look different from the samples, because these are based on *randomly* generated numbers!

Question 2

Suppose we wish to determine the Young's modulus E of wood by measuring the stiffness of a cantilever beam. We need to measure the beam length L, depth h, width b and deflection d for load F at the end. You know the relation:

$$\delta = \frac{FL^3}{3EI}$$

$$I = \frac{bh^3}{12}$$

Consider specific measurements given in the table below (next slide).

Question 2

		Error in X (sigma)	
Parameter X	Measurement	dX/X*100	dX
L (m)	0.7	2	0.014
h (m)	0.03	10	0.003
b (m)	0.04	1	0.0004
F (N)	80	1	0.8
delta (m)	0.008	1	0.00008

Question 2

Each of our measurements contain uncertainty, given below as a percent of the measured value, and as the absolute value (take dX in the table as σ).

a. Use the **error propagation formulas** in the Background to **estimate the uncertainty (σ_E) of E** derived from the measurements. (Usage of MATLAB *optional* in this step.)

b. Use the **randn** MATLAB function to **generate 10,000 sets of simulated parameters L, h, b, δ, F that go into computing E**. For each of these sets compute E using the beam formula and plot the histogram of E at the end. Compare this with the result from a.

c. Suppose you wanted to improve your measurement of E. **What measurements could be improved to give the biggest benefit? (Use your analysis from a, and test with simulations in b)**

Question 2 (a): Error Propagation Formula

Suppose that y is related to n independent measured variables $\{X_1, X_2, \ldots, X_n\}$ by a functional representation:

$$y = f(X_1, X_2, \cdots, X_n)$$

Given the uncertainties of X's around some operating points:

$$\{\overline{x}_1 \pm \Delta x_1, \overline{x}_2 \pm \Delta x_2, \cdots, \overline{x}_n \pm \Delta x_n\}$$

The expected value of $\overline{y}$ and its uncertainty Δy are:

$$\overline{y} = f(\overline{x}_1, \overline{x}_1, \cdots, \overline{x}_n)$$

$$\Delta y = \sqrt{\left(\frac{\partial f}{\partial X_1}\Delta x_1\right)^2 + \left(\frac{\partial f}{\partial X_2}\Delta x_2\right)^2 + \cdots + \left(\frac{\partial f}{\partial X_n}\Delta x_n\right)^2}\,\Bigg|_{(\overline{x}_1, \overline{x}_1, \cdots, \overline{x}_n)}$$

- Let's do this on the white board!
- You can attach a scanned/photo of the hand-written solution to this sub question.

Question 2 (b)

```
%We start by estimating E based on the given parameters
L0=0.7 ;% beam length in m
h0=0.03;% beam depth in m
b0=0.04;% beam width in m
F0=80;% applied force at end of cantilever beam, N
delta0=0.008 ;% displacement at end
```

→ `E0=F0*L0^3/3/delta0/(b0*h0^3/12)`

```
E0 =
    1.2704e+10
```

Question 2 (b) Cont'd

To simulate a new series of parameters, we need to consider the physically meaningful units (these are given to you in the table as dX:

```
sL=sLp*L0/100% same units as L (here sLp is the percentage error)
% or simpaly let sL = 0.014; this gives sigma(L)
sh=shp*h0/100
sb=sbp*b0/100
sF=sFp*F0/100
sdel=sdelp*delta0/100
```

As an example, this is how you can generate 10000 normally distributed samples of L based on the desired parameters:

```
N=10000;% number of trials
L=L0+sL*randn(1,N);
```

Question 2 Cont'd

To Now use the equation for E to calculate the new set (10000):

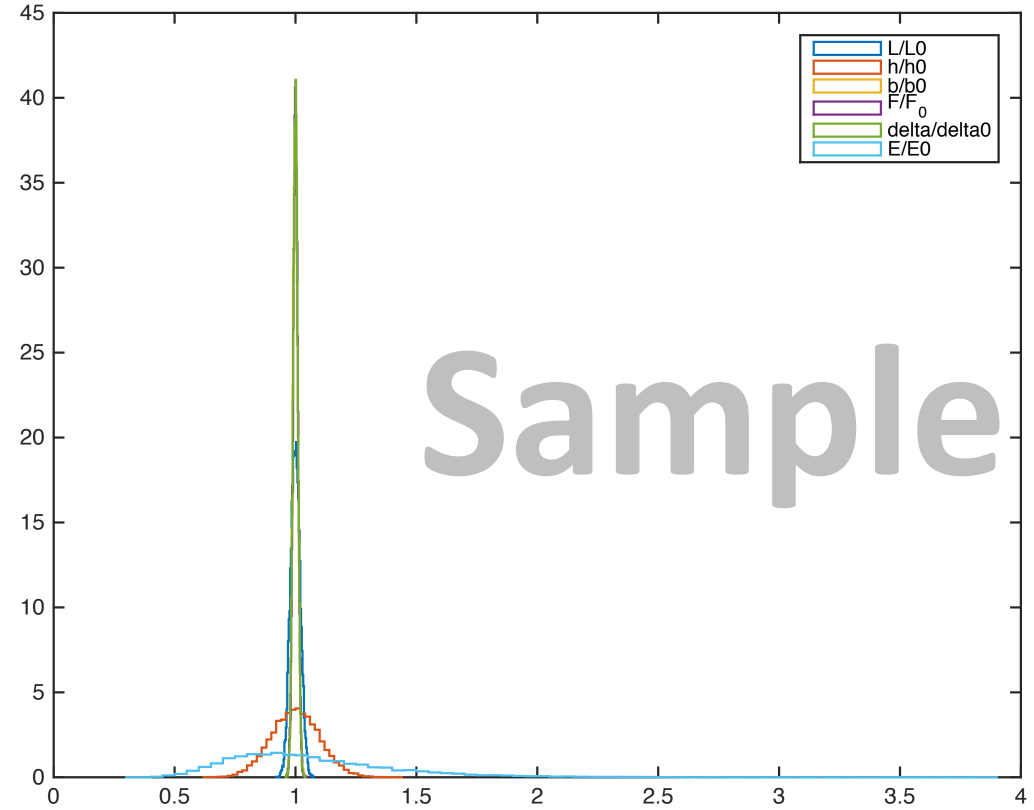
```
E=F.*L.^3./3./del./(b.*h.^3/12);
sErel=std(E)/E0
meanE=mean(E)/E0
```

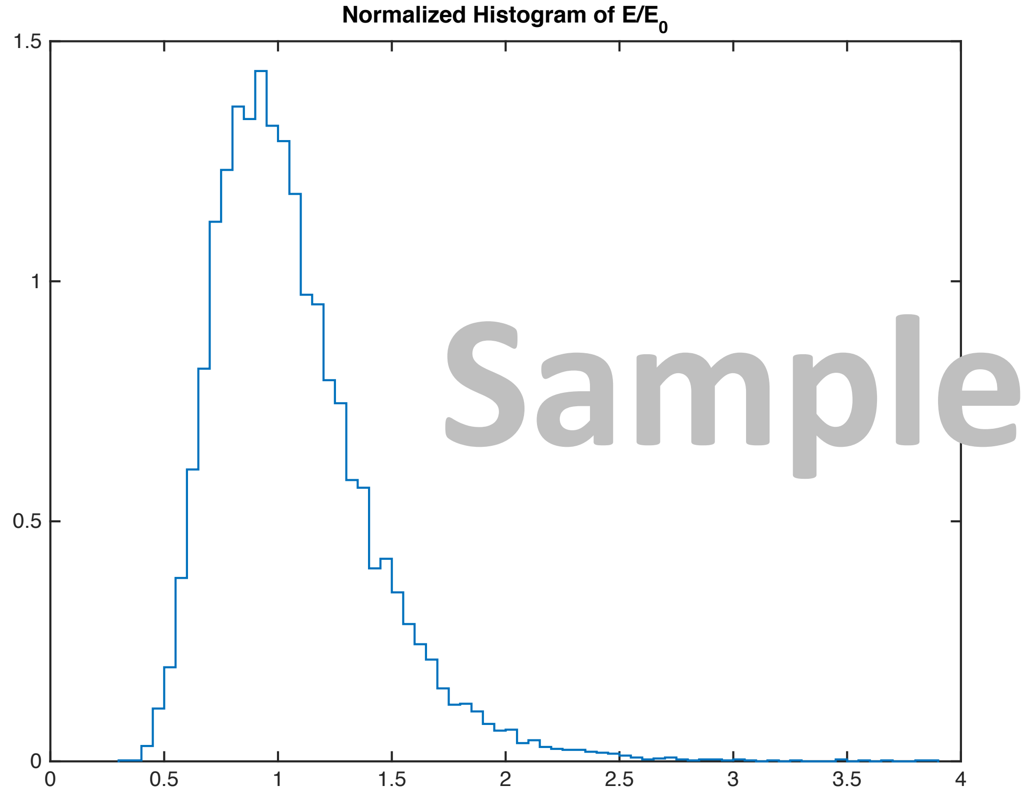
```
sErel =
    0.3549

meanE =
    1.0644
```

Question 2 Histograms

```
histogram(E/E0,'Normalization','pdf','DisplayStyle','stairs');
```





Question 2 (c)

Suppose you wanted to improve your measurement of E. **What measurements could be improved to give the biggest benefit?**

Hints:

Use your analysis from a, and test with simulations in b)