a place of mind

UBC

MECH366 : Modeling of Mechatronic Systems

L19 : Bode diagram of first-order and second-order systems

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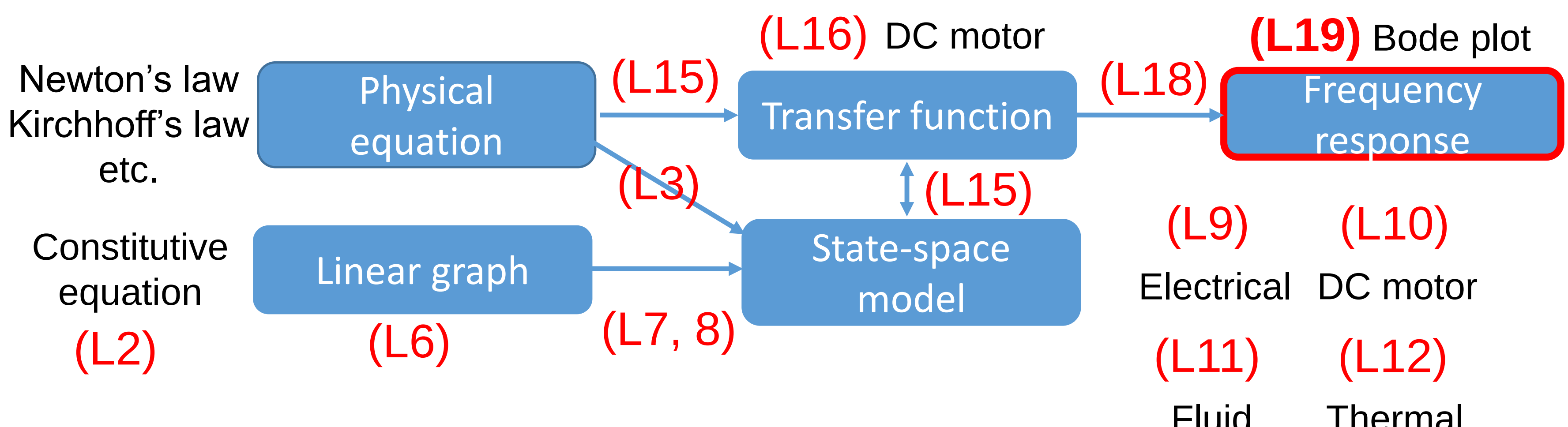
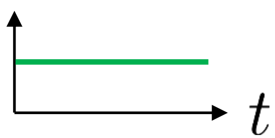
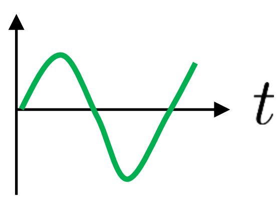
Today’s topic & class schedule

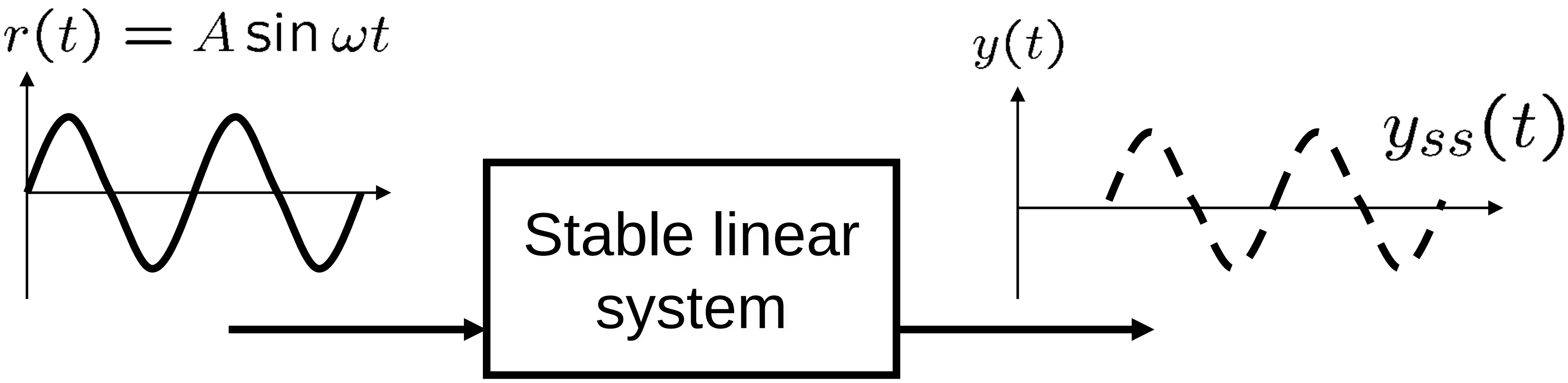
- L18: Nov 15 (Fri): Frequency response
- L19: Nov 18 (Mon): Bode diagram (Lab 4 report content, report due Nov 25, 6pm)
- L20: Nov 22 (Fri): Simulink, overdamped system
- L21: Nov 25 (Mon): Stability, course summary



Response analyses (useful for modeling and controller design)

$G(s)$	$\dfrac{K}{Ts+1}$	$\dfrac{K\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$
Step response 	(L16) 	(L17) underdamped  (L20) overdamped 
Frequency response (L18) 	(L19) **Slide 13**	(L19) **Slide 16**

Frequency response (review)

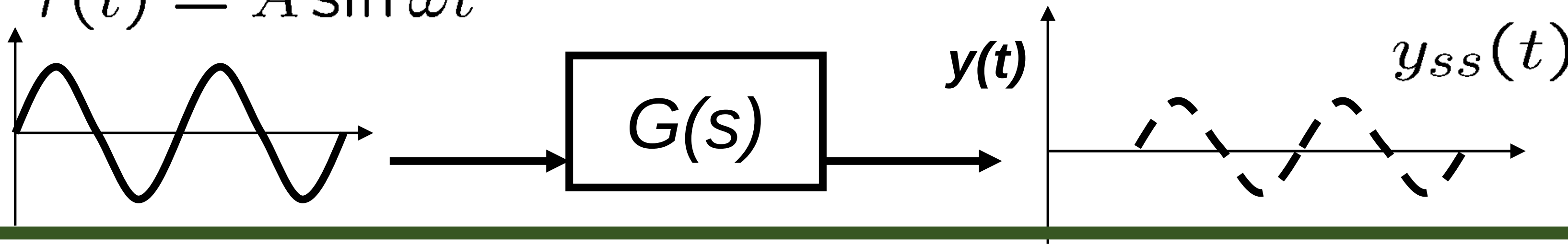


- We would like to analyze a system property by applying a *sinusoidal input* *r(t)* and observing a response *y(t)*.
- Steady state response $y_{ss}(t)$ (after transient dies out) of a system to sinusoidal inputs is called ***frequency response***.

Response to sinusoidal input (review)

- What is the steady state output of a stable linear system when the input is sinusoidal?

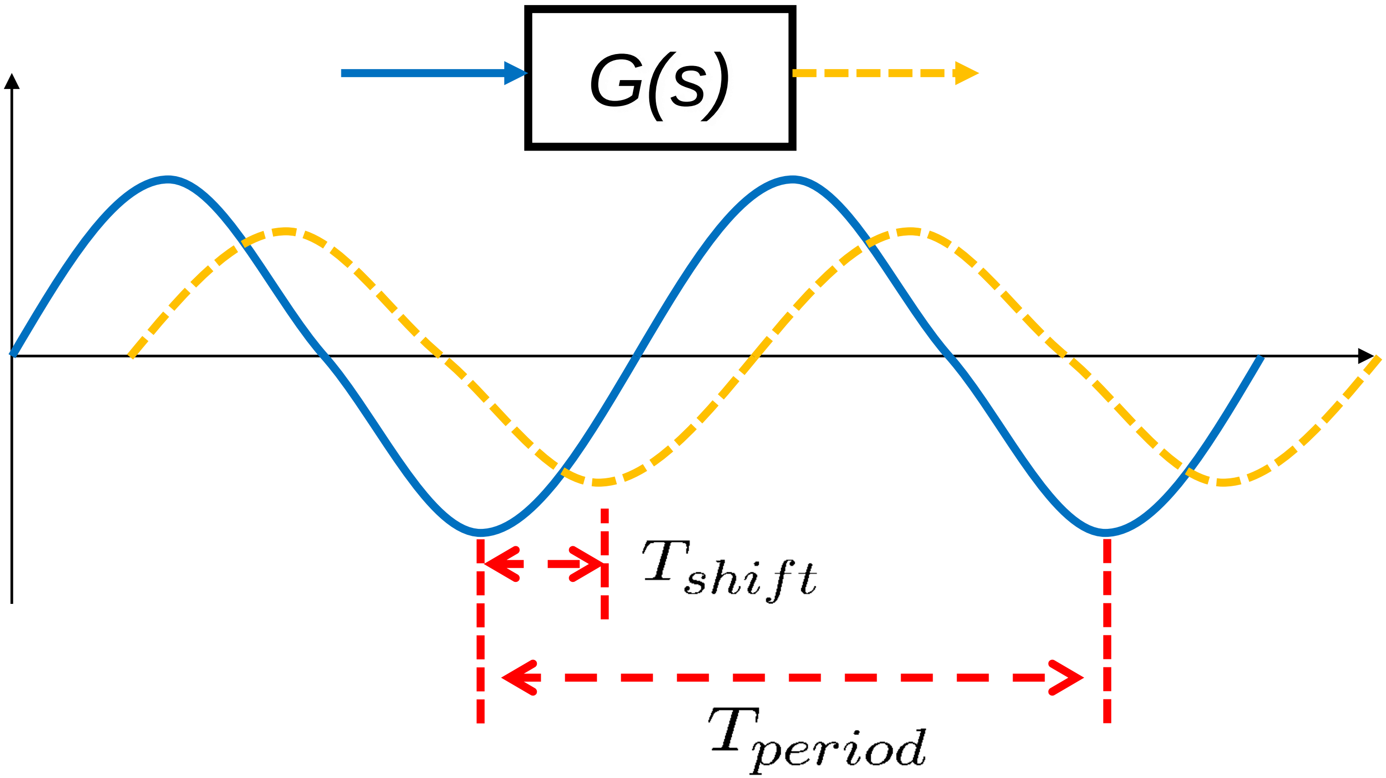
$r(t) = A \sin \omega t$



- Steady state output $y_{ss}(t) = A|G(j\omega)| \sin(\omega t + \angle G(j\omega))$
  - Frequency is same as the input frequency ω
  - Amplitude is that of input (A) multiplied by $|G(j\omega)|$
  - Phase shifts $\angle G(j\omega)$

Gain

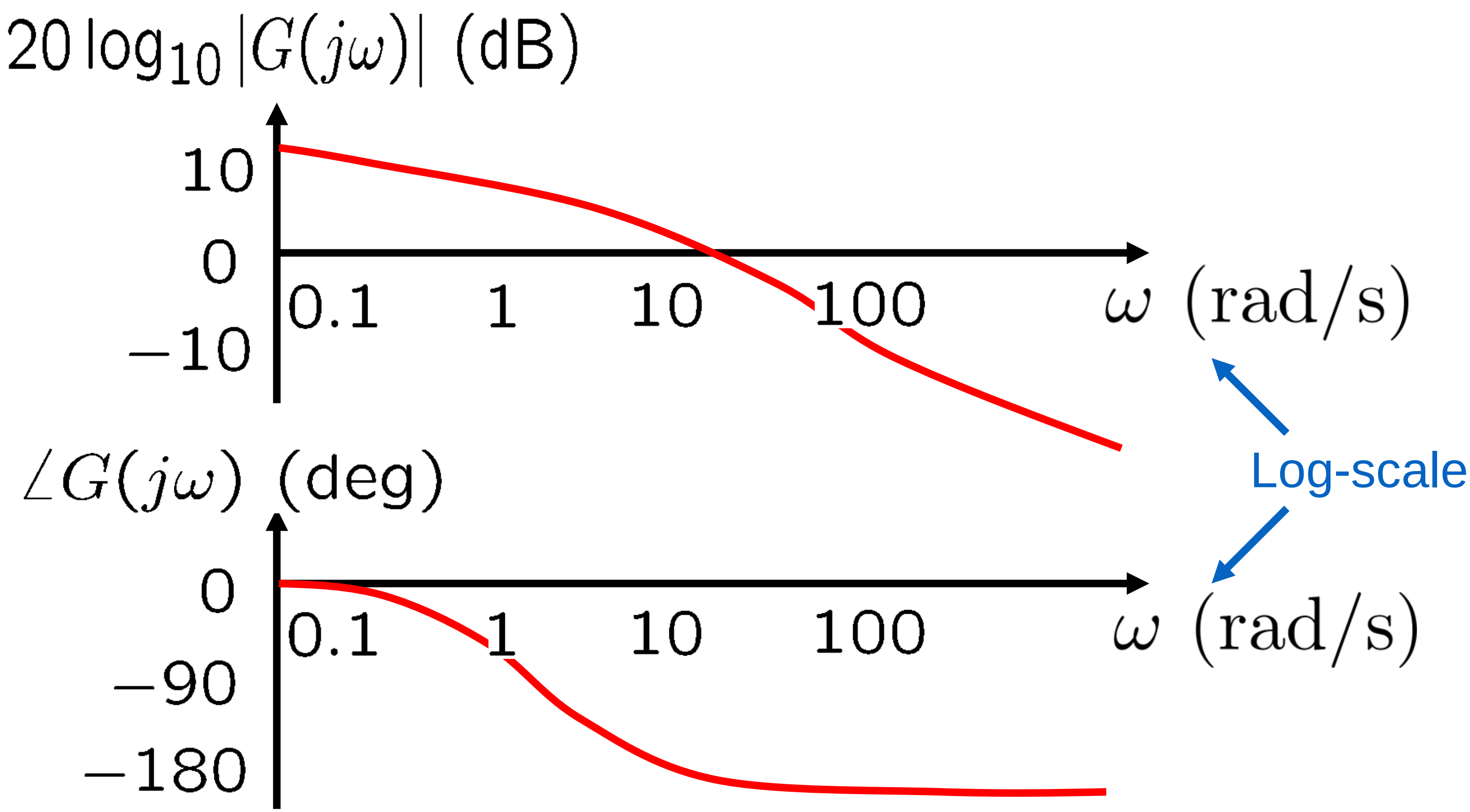
Phase shift (review)



$$\frac{T_{shift}}{T_{period}} = \frac{-\angle G(j\omega)}{360^\circ} \quad \Rightarrow \quad \angle G(j\omega) = -\frac{T_{shift}}{T_{period}} \times 360^\circ$$

Bode plot (Bode diagram) of $G(j\omega)$

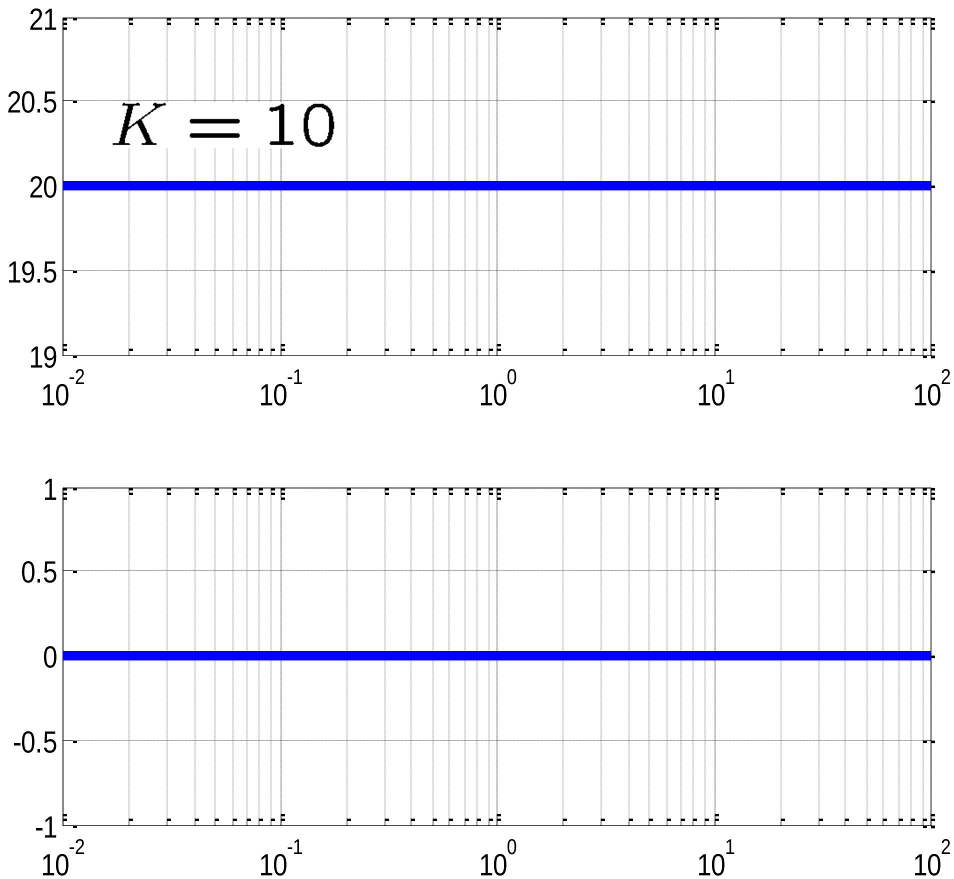
- Bode diagram consists of gain plot & phase plot



Bode plot of a constant gain

$$G(s) = K \Rightarrow |G(j\omega)| = K, \ \angle G(j\omega) = 0^{\circ}, \ \forall \omega$$

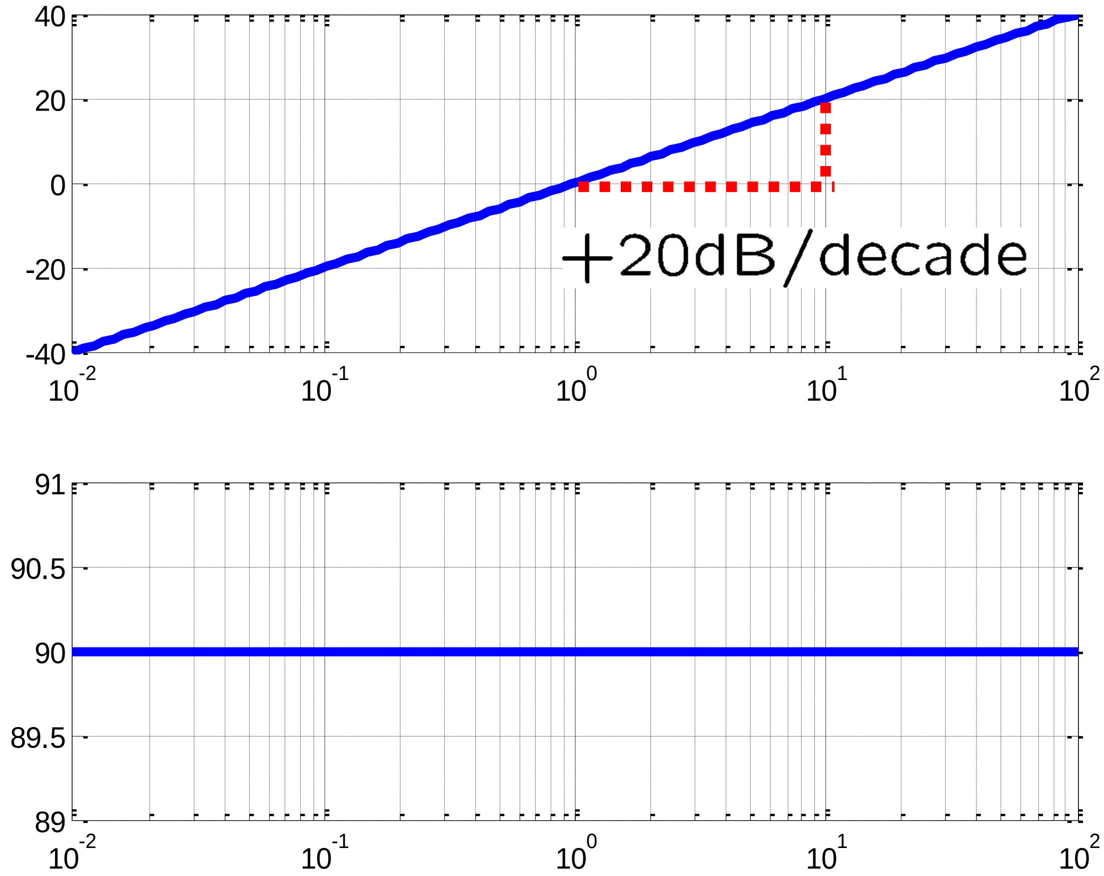
(for all)



K	$20 \log_{10} K$
100	40 dB
10	20 dB
2	≈ 6 dB
1	0 dB
0.1	-20 dB
0.01	-40 dB

Bode plot of a differentiator

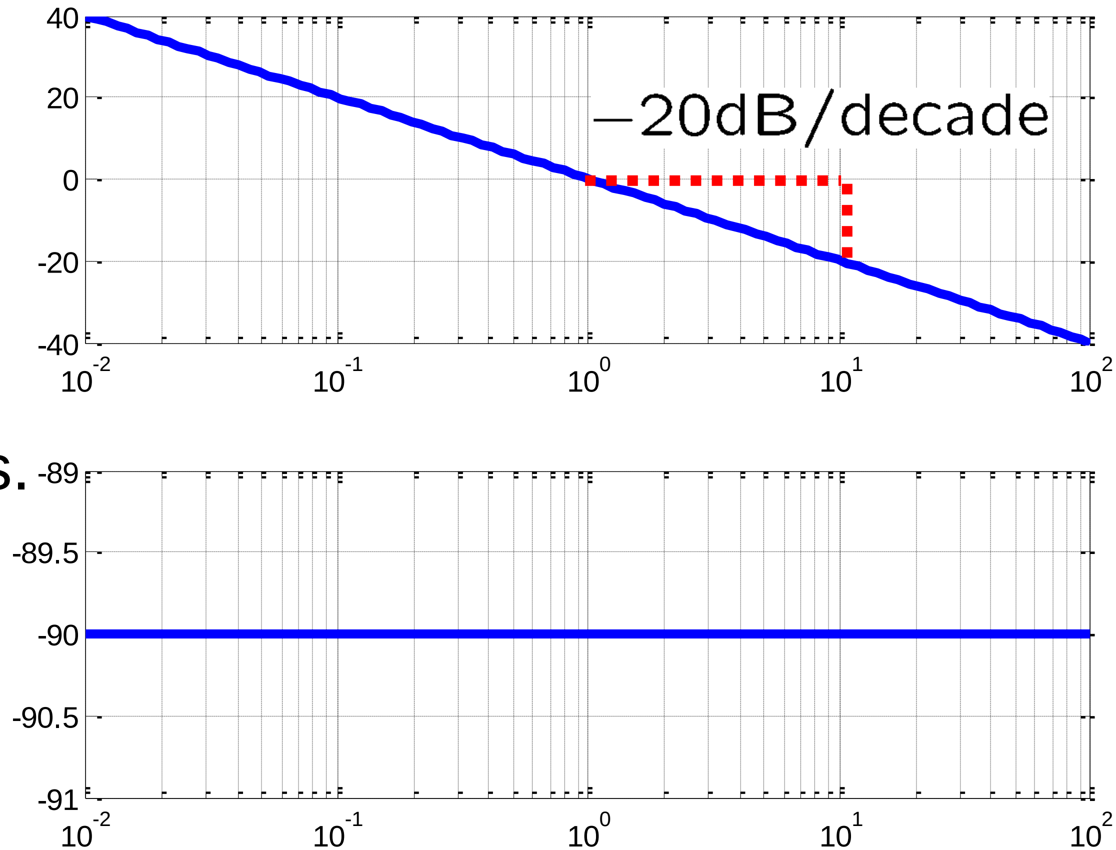
$$G(s) = s \Rightarrow |G(j\omega)| = \omega,\ \angle G(j\omega) = \angle j\omega = 90^{\circ},\ \forall \omega$$



Bode plot of an integrator

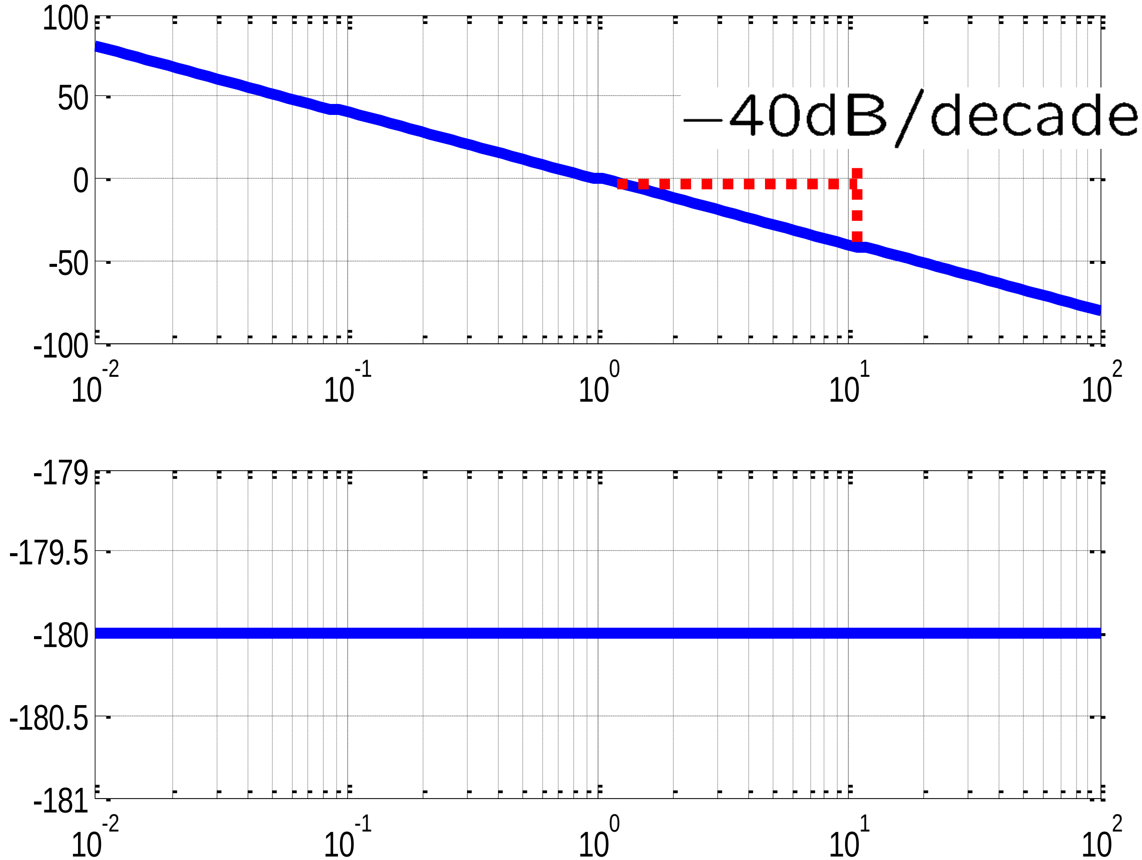
$$G(s)=\frac{1}{s}\Rightarrow |G(j\omega)|=\frac{1}{\omega},\ \angle G(j\omega)=\angle\frac{1}{j\omega}=-90^\circ,\ \forall\omega$$

Mirror image of the Bode plot of *G(s)=s* with respect to ω-axis.



Bode plot of a double integrator

$$G(s)=\frac{1}{s^2}\Rightarrow |G(j\omega)|=\frac{1}{\omega^2},\ \angle G(j\omega)=\angle\frac{1}{(j\omega)^2}=-180^\circ,\ \forall\omega$$

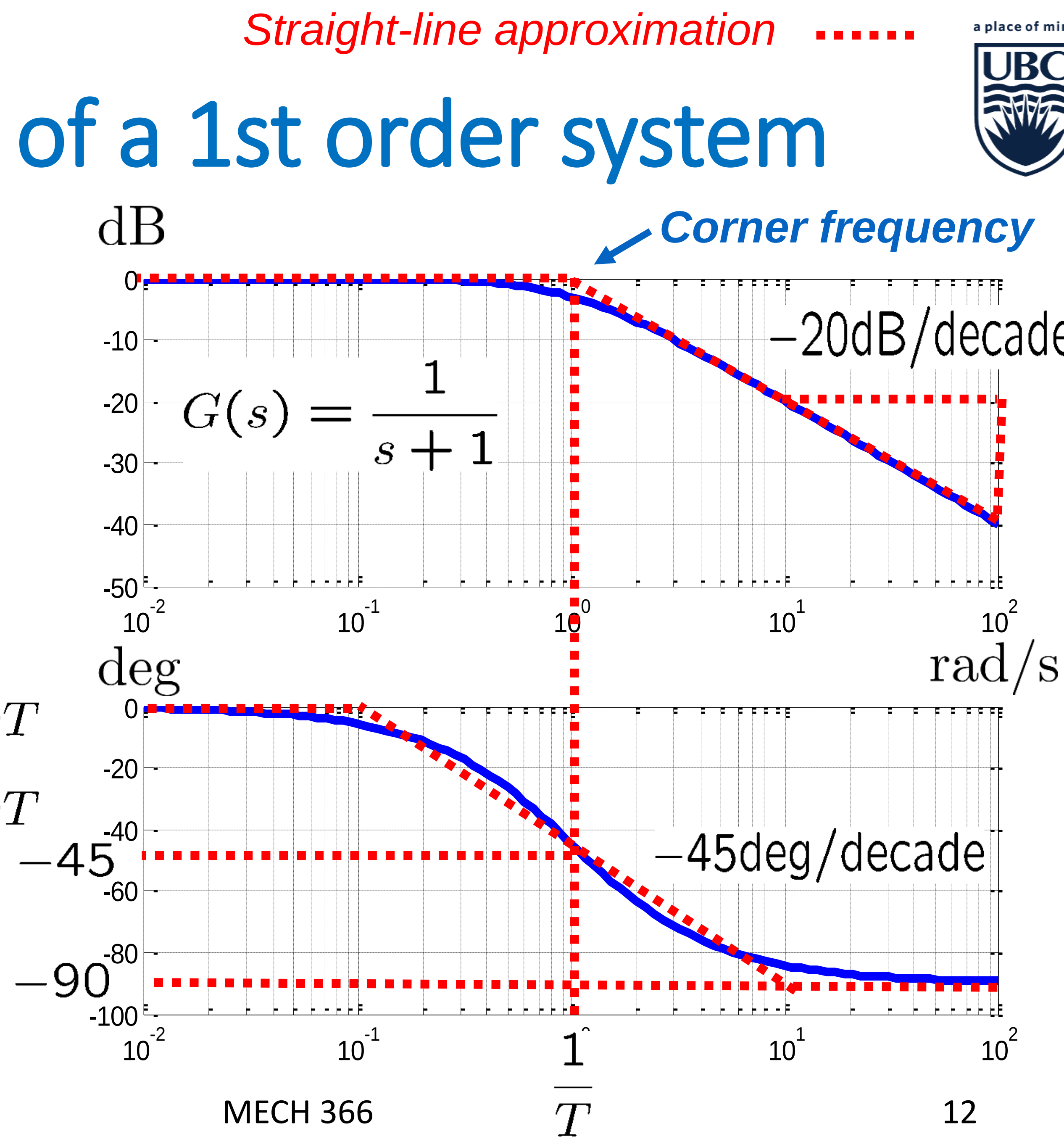


Bode plot of a 1st order system

$$G(s) = \frac{1}{Ts+1}$$

$$G(j\omega) = \frac{1}{j\omega T + 1}$$

$$\approx \begin{cases} 1 & \text{if } 1 \gg \omega T \\ \frac{1}{j\omega T} & \text{if } 1 \ll \omega T \end{cases}$$



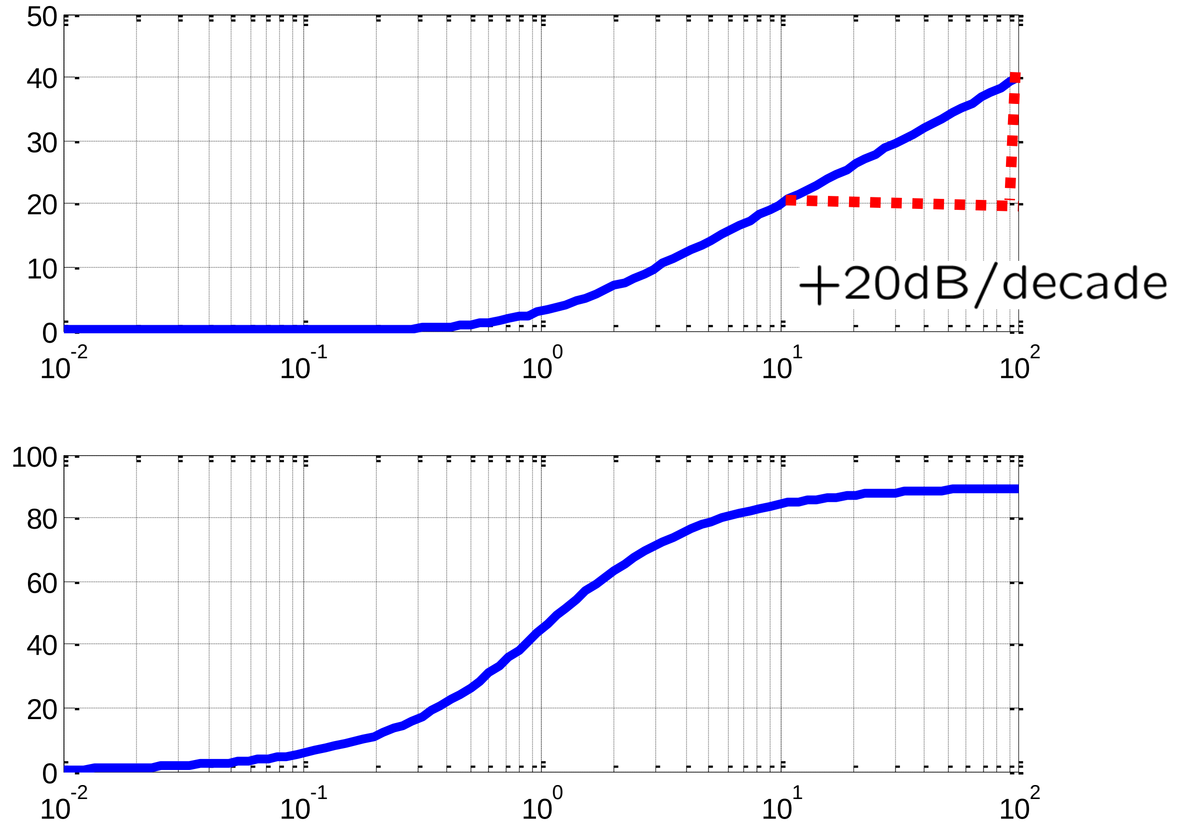
Exercises of sketching Bode plot

$$G(s) = \frac{1}{s+1} \qquad G(s) = \frac{1}{0.1s+1} \qquad G(s) = \frac{1}{10s+1}$$

Bode plot of an inverse system

$$G(s) = Ts + 1 = \left(\frac{1}{Ts+1}\right)^{-1}$$

Mirror image of the original Bode plot with respect to ω-axis.



Bode plot of a 2nd order system

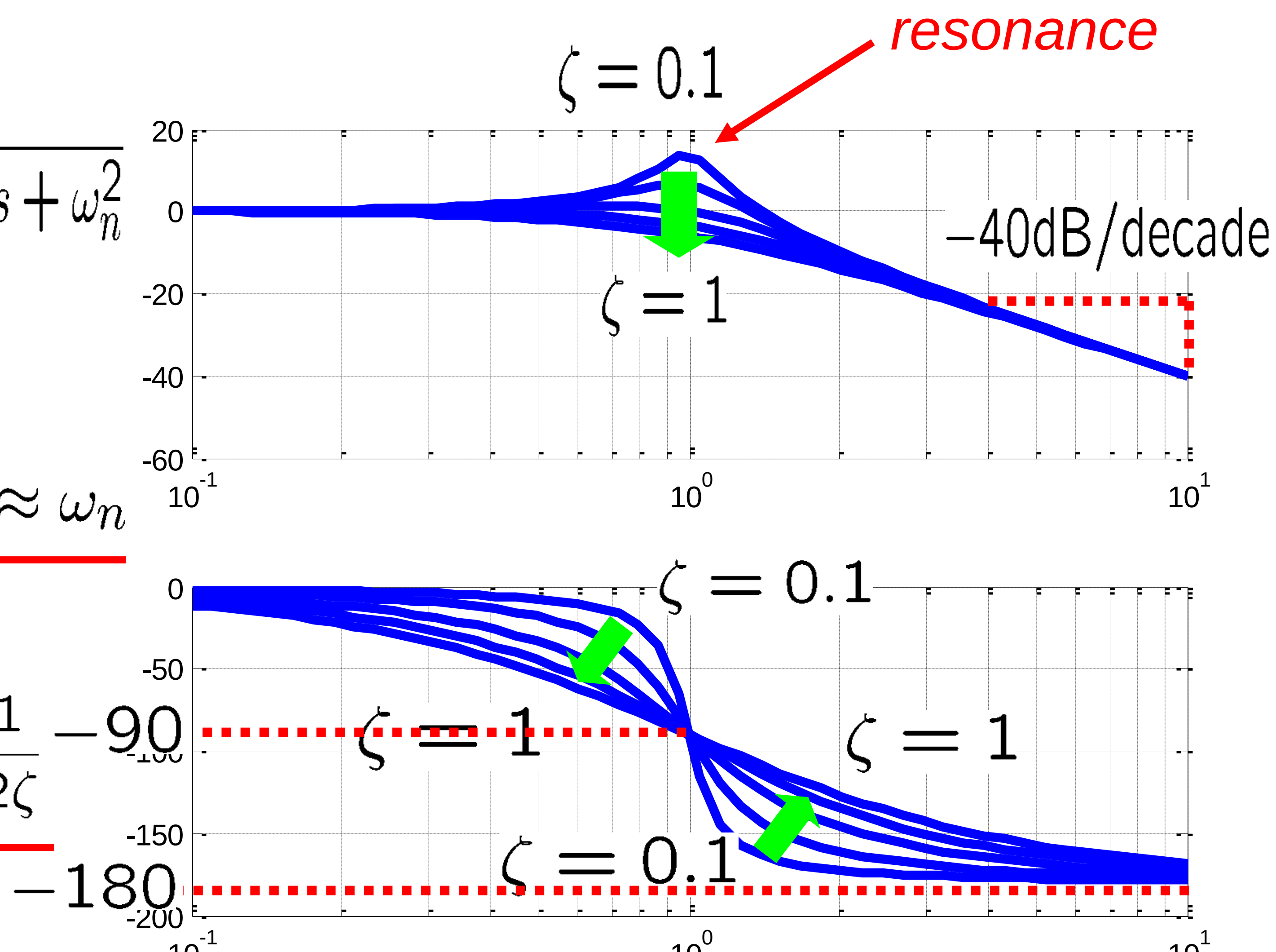
$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Resonant freq.

$$\omega_n\sqrt{1-2\zeta^2} \approx \omega_n$$

Peak gain

$$\frac{1}{2\zeta\sqrt{1-\zeta^2}} \approx \frac{1}{2\zeta}$$



An advantage of Bode plot

- Bode plot of a series connection $G_1(s)G_2(s)$ is the addition of each Bode plot of G_1 and G_2.
  - Gain

$$20\log_{10}|G_1(j\omega)G_2(j\omega)| = 20\log_{10}|G_1(j\omega)| + 20\log_{10}|G_2(j\omega)|$$

  - Phase

$$\angle G_1(j\omega)G_2(j\omega) = \angle G_1(j\omega) + \angle G_2(j\omega)$$

- Later, we use this property to design *C(s)* so that *G(s)C(s)* has a "desired" shape of Bode plot. (in Controls course MECH467)

Short proofs (optional)

- Use polar representation

$$G_1(j\omega) = |G_1(j\omega)|e^{j\angle G_1(j\omega)} \qquad G_2(j\omega) = |G_2(j\omega)|e^{j\angle G_2(j\omega)}$$

Then, $$\begin{aligned} G_1(j\omega)G_2(j\omega) &= |G_1(j\omega)||G_2(j\omega)|e^{j\angle G_1(j\omega)}e^{j\angle G_2(j\omega)} \\ &= |G_1(j\omega)||G_2(j\omega)|e^{j\{\angle G_1(j\omega)+\angle G_2(j\omega)\}} \end{aligned}$$

Therefore,

$$20\log_{10}|G_1(j\omega)G_2(j\omega)| = 20\log_{10}|G_1(j\omega)|\cdot|G_2(j\omega)| = 20\log_{10}|G_1(j\omega)| + 20\log_{10}|G_2(j\omega)|$$

$$\angle G_1(j\omega)G_2(j\omega) = \angle G_1(j\omega) + \angle G_2(j\omega)$$

Example 1

- Sketch the Bode plot of a transfer function

$$\boxed{G(s) = \frac{10}{s}}$$

1. Decompose *G(s)* into a product form:

$$G(s) = 10 \cdot \frac{1}{s}$$

2. Sketch a Bode plot for each component on the same graph.

3. Add them all on both gain and phase plots.

Example 1 (cont'd)

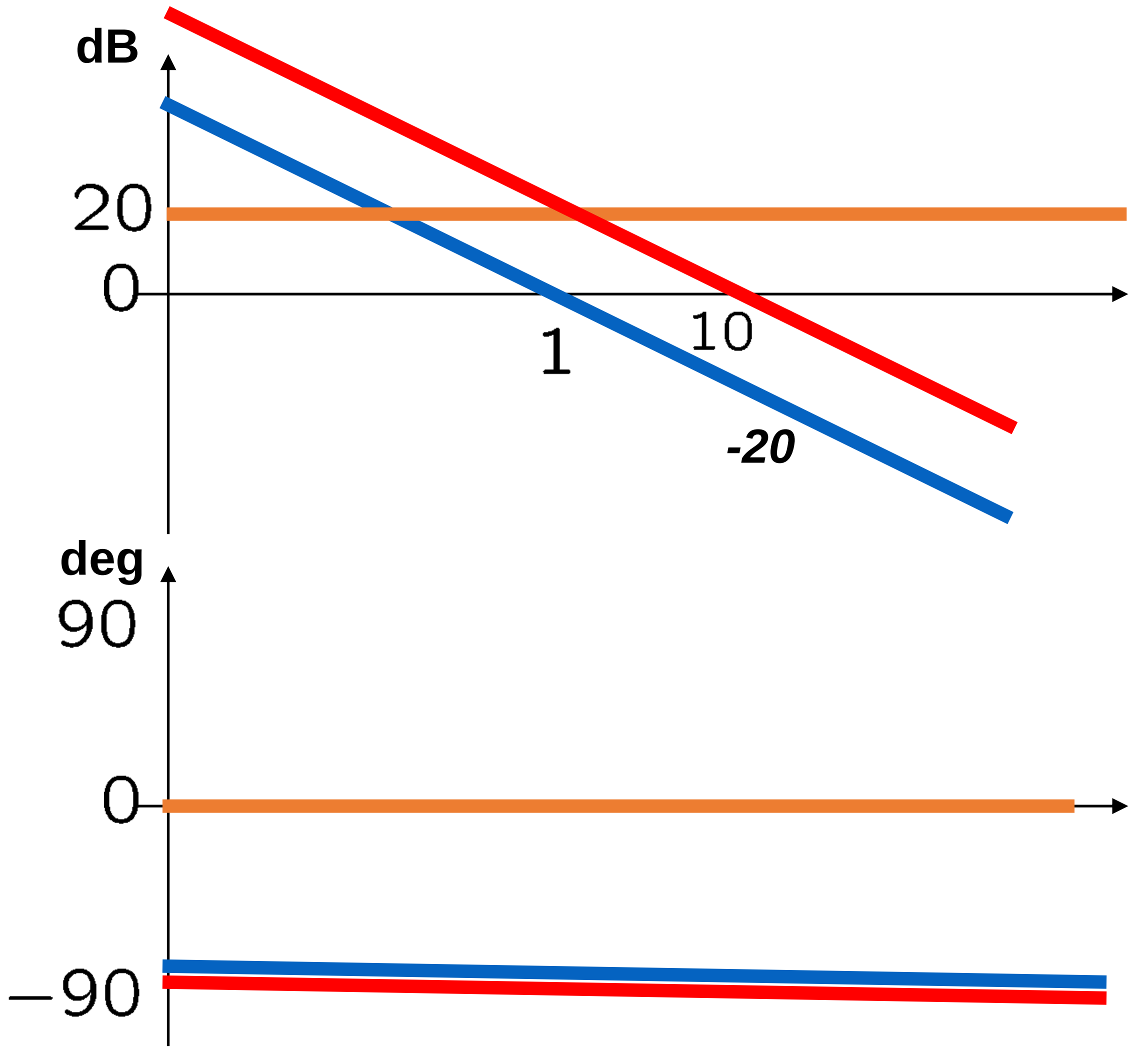
$$G(s) = 10$$

$$\times$$

$$G(s) = \frac{1}{s}$$

$$\Downarrow$$

$$G(s) = \frac{10}{s}$$



Example 2

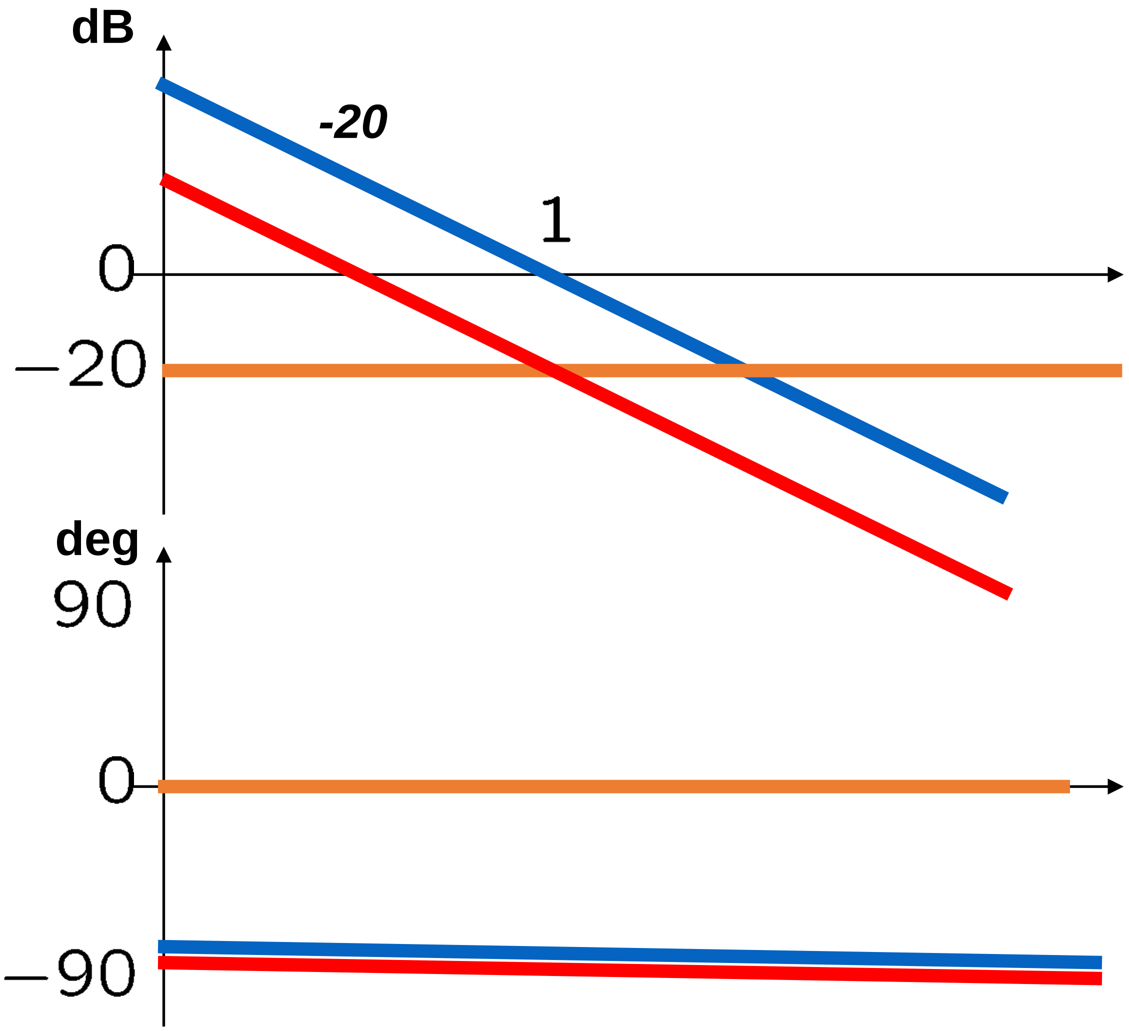
$$G(s) = 0.1$$

$$\times$$

$$G(s) = \frac{1}{s}$$

$$\Downarrow$$

$$G(s) = \frac{0.1}{s}$$



Example 3

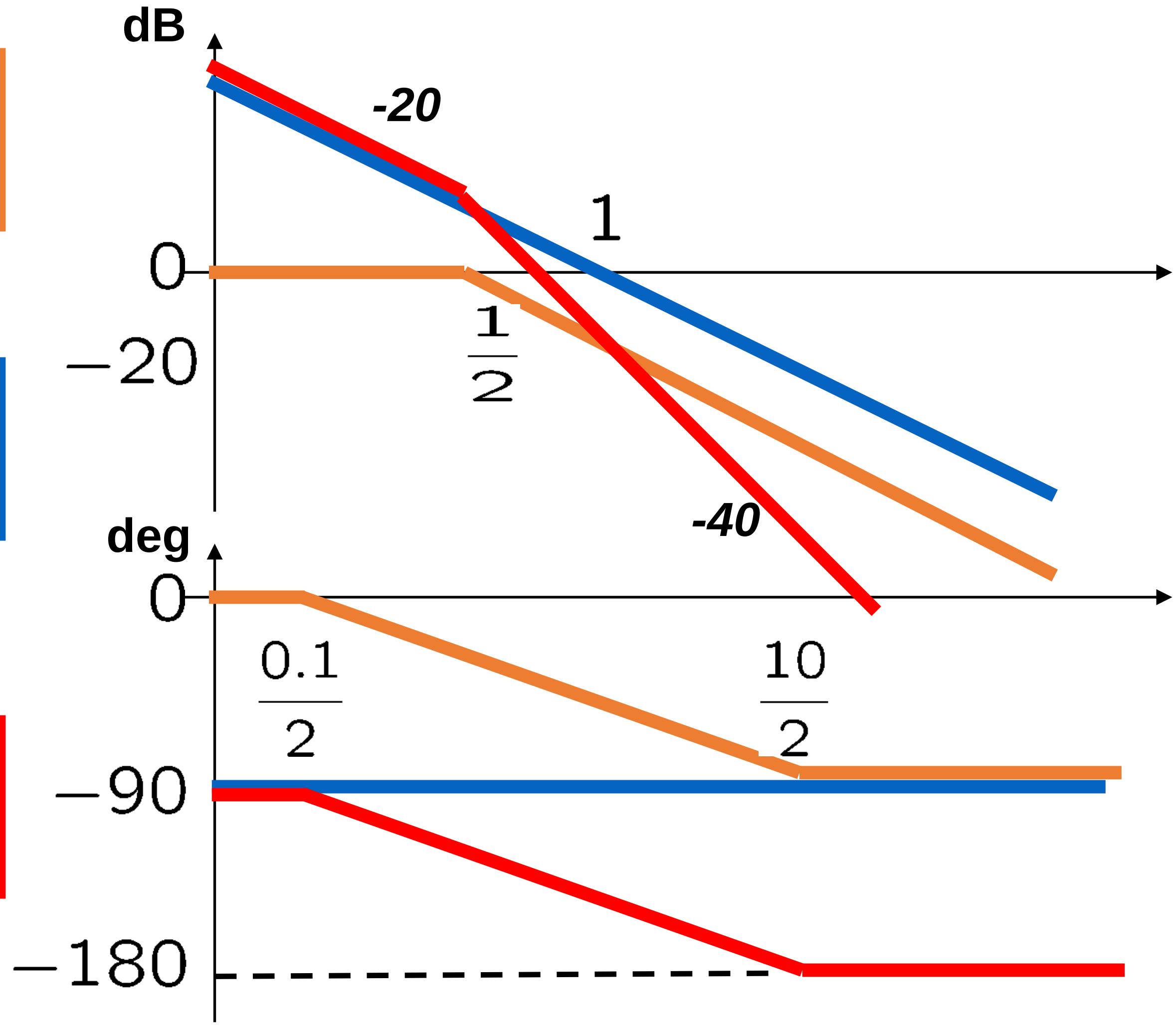
$$G(s) = \frac{1}{2s+1}$$

$$\times$$

$$G(s) = \frac{1}{s}$$

$$\Downarrow$$

$$G(s) = \frac{1}{s(2s+1)}$$



Summary

- How to sketch/draw Bode diagram (bode.m)
- Next,
  - Simulink
  - Step response of overdamped systems
- Project: Fridays Nov 22, 29 (presentation)
- Lab 4 report: Due Nov 25 (Monday), 6pm