ELEC 343
Electromechanics

Spring 2019
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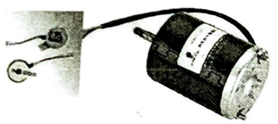
Class Webpage: http://courses.ece.ubc.ca/elec343

Module 3 (Read Chap. 3): Lab. 2
Part 1: Brushed DC Motors Fundamentals and Steady-State Analysis

Learning Objectives & Important Topics and Concepts
- Construction of DC machines, fundamentals
- Induced voltage and torque
- Equivalent circuit
- Basic types of dc machines, their characteristics
- Starting

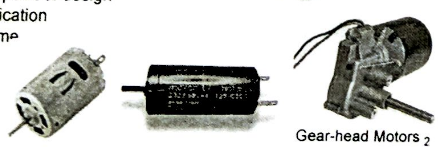
DC Machines

Range of Sizes
- Miniature dc motors – mW range
- Fractional HP (1HP = 746W)
- Medium Size – 1 ... 500 HP
- Large DC Machines – 500 HP and up



Applications
- Very popular and easy to use in various applications
- Tools, robotics, toys, medical, automotive & other industries

General Properties
- Very easy to control !
- Good torque-speed performance
- Brushes is a weak point of design
  - Limits the application
  - Limits the lifetime
  - Maintenance

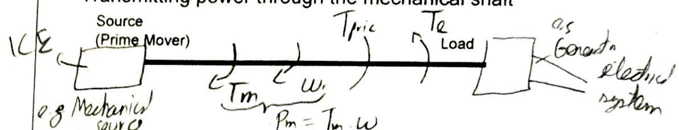


Small Motors

Gear-head Motors

Motor (Machine) Speed

Transmitting power through the mechanical shaft



Commonly used units of speed

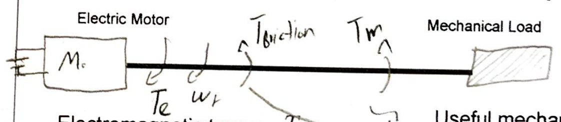
ω - $[\text{rad/sec}]$

$$m = \frac{\omega}{2\pi}\ [\text{rev/sec}]$$

$$n = 60\frac{\omega}{2\pi} = \frac{30}{\pi}\omega\ [\text{rev/min}]$$

RPM = >Most commonly used

Mechanical Speed and Torque



Electromagnetic torque T_e

Useful mechanical torque on the shaft T_m

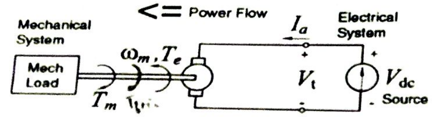
Torque balance $\quad T_e = J\dfrac{d\omega_r}{dt} + T_m + T_{mech\_loss}$

Total moment of inertia (motor and load combined) $\quad J = J_{dc\_machine} + J_{mech\_load},\quad [kg \cdot m^2]$

Mechanical loss (friction) $\quad T_{mech\_loss} = T_{fric} = D_m \omega_r$

In steady state $\quad T_e = T_m + T_{friction}$

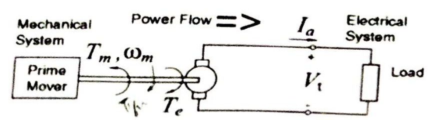
DC Machine Motoring and Generating



Mechanical load torque is applied in the direction opposite to rotation

$P_{out} = \omega_m T_m = P_m$ $\quad V_t I_a = P_{in}$

$T_e = T_m + T_{fric}$



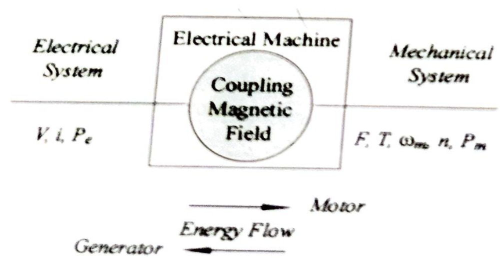
Mechanical torque is applied in the direction of rotation

$P_{in} = \omega_m T_m = P_m$ $\quad V_t I_a = P_{out}$

$T_m = T_e + T_{fric}$

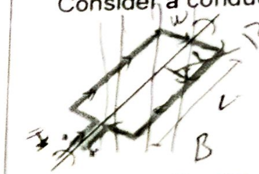
Electromechanical Interaction

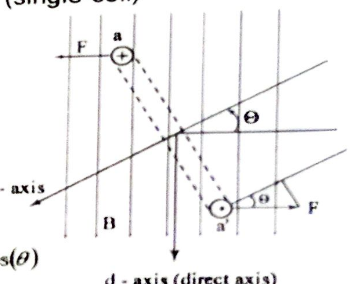
- Current-carrying conductor in magnetic field
  => mechanical force
- Conductor moves in magnetic field
  => voltage induced, emf



Force & Torque

Consider a conductor frame (single-coil)





Lorentz Force $F = lBi_a$

Torque $T_e = 2RlBi_a \cos(\theta)$

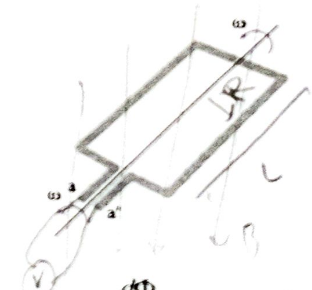
$= ABi_a \cos(\theta) = \Phi_p i_a \cos(\theta)$

NOTE: Maximum torque is when $a - axis \perp B$

and $T_e \sim \Phi_p i_a$

Induced Voltage

Consider a conductor frame (single-coil)



Faraday's Law $e = \dfrac{d\Phi}{dt}$

$\Phi = BA\sin(\theta) = B2Rl\sin(\theta)$

Let us rotate the frame with speed ω

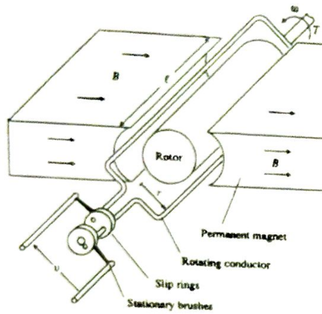
$\theta = \omega t$

$\Phi = BA\sin(\omega t) = \Phi_p \sin(\omega t)$

$e = \dfrac{d\Phi}{dt} = \omega\Phi_p \cos(\omega t)$ => $e \sim \omega\Phi_p$

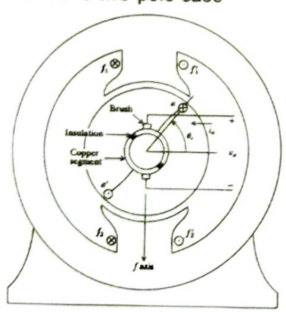
Conductor Frame (1-turn coil)

Consider a conductor frame placed on a rotor between magnetic poles



Elementary DC Machine

Consider a two-pole case



Voltage Equations

$$v_f = r_f i_f + \frac{d\lambda_f}{dt}$$

$$v_a = r_a i_a + \frac{d\lambda_a}{dt}$$

Flux linkages

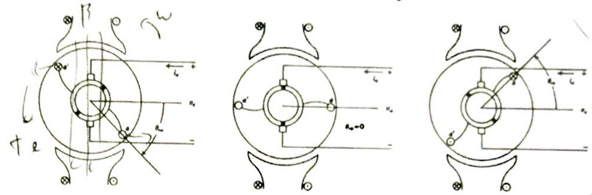
$$\lambda_f = L_{ff} i_f + L_{fa} i_a$$

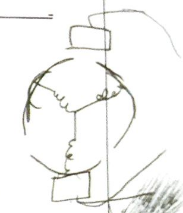
$$\lambda_a = L_a i_a + L_{af} i_f$$

Approximate the mutual inductance $L_{af} = L_{fa} = -L\cos(\theta_r)$

$e = \frac{d\lambda}{dt} = N\frac{d\Phi}{dt}$

Commutation of Elementary DC Machine





Induced voltage

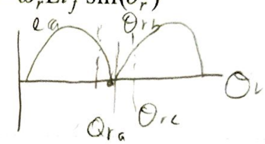
$$v_a = r_a i_a + \frac{d\lambda_a}{dt}$$

Assume

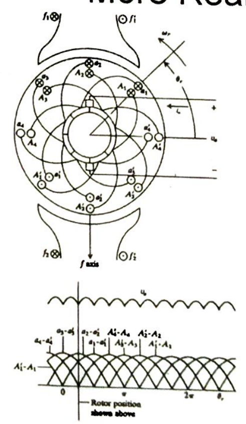
$i_a = 0$ and $\theta_r = \omega_r t$

$\lambda_a = L_a i_a + L_{af} i_f$

$e_a = \omega_r L i_f \sin(\theta_r)$

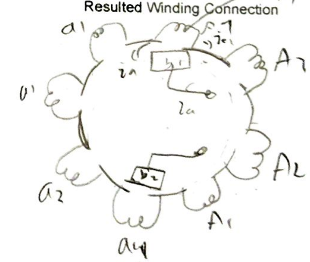


More Realistic DC Machine

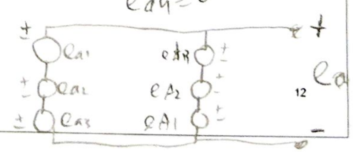


Resulted Winding Connection

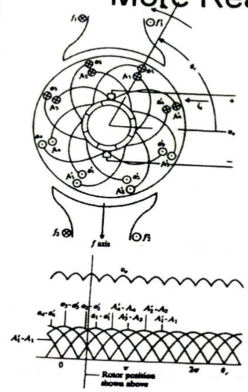
$e_{A_4} = 0$



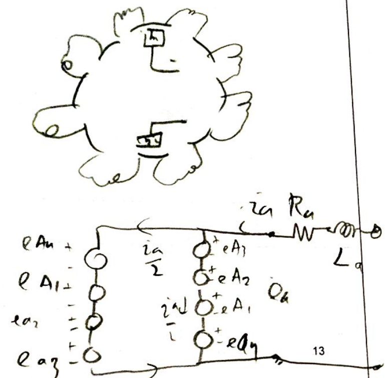
$e_{a_4} = 0$



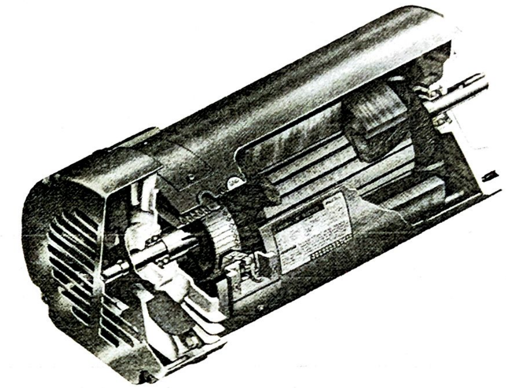
More Realistic DC Machine



Resulted Winding Connection

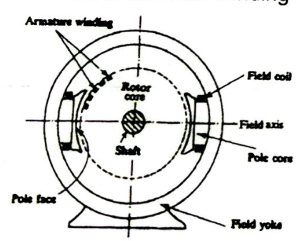


Cutaway View of a Two-Pole DC Machine

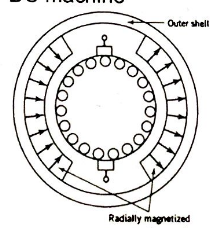


DC Machine Construction

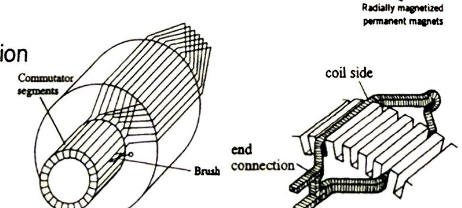
DC machine with field winding



Permanent Magnet (PM) DC machine

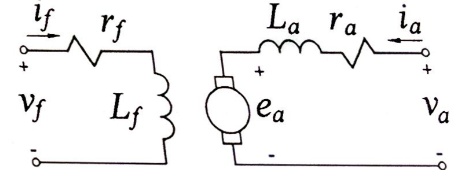


Rotor construction



Equivalent Circuit

$r_f \cdot i_f^2$ power loss in field winding



$r_a \cdot i_a^2$ power loss in armature

v_f - is the field voltage

i_f - is the field current

r_f - field winding resistance

L_f - field winding inductance

$\underbrace{P_e = e_a i_a}_{}$ Conversion power go to T_e !

v_a - applied terminal voltage

i_a - is the armature current

L_a - armature winding inductance

r_a - armature winding + brush resistance

e_a - Induced back emf (voltage)

Dynamic Equivalent Circuit

Field Winding Equations

$$v_f = r_f i_f + \frac{d\lambda_f}{dt}$$

$$\lambda_f = N_f \Phi_p = L_f i_f$$

Armature Equations

$$v_a = r_a i_a + L_a \frac{di_a}{dt} + e_a$$

$$e_a = k_1 \omega_r \Phi_p$$

Electromagnetic Torque $T_e = k_2 \Phi_p i_a$

k_1 and k_2 are machine design constants

Power balance $T_e \omega_r = k_2 \Phi_p i_a \omega_r = e_a i_a = k_1 \omega_r \Phi_p i_a$

$P_e = T_e \omega = e_a \cdot i_a$ = conversion power

$k_1 = k_2 = k$ machine Design constant

Equivalent Circuit

Recall flux $\Phi_p = \frac{\lambda_f}{N_f} = \frac{L_f}{N_f} I_f$

Induced voltage

$$e_a = k\Phi_p \omega_r = k \frac{L_f}{N_f} i_f \omega_r$$

Torque

$$T_e = k\Phi_p i_a = k \frac{L_f}{N_f} i_f i_a$$

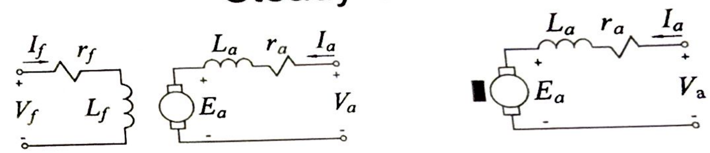
Define $L_{af} = k \frac{L_f}{N_f}$ - is the mutual inductance between field and rotating armature winding $L_{af} = \frac{N_a N_f}{\Re}$

Re-define expression for back emf and torque

$$e_a = \underbrace{L_{af} i_f}_{k_v} \omega_r = k_v \omega_r \qquad T_e = L_{af} i_f i_a = k_t i_a$$

Machine voltage/torque constant for PM machine $k_v = k_t = L_{af} i_f$

Steady-State



Field Winding Equations

$$V_f = R_f I_f$$

$$\lambda_f = N_f \Phi_p = L_f I_f$$

Armature Equations

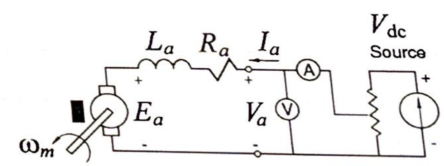
$$V_a = r_a I_a + E_a$$

$$E_a = \omega_r k_v = \omega_r L_{af} i_f$$

Electromagnetic torque $T_e = L_{af} I_f I_a = k_t I_a$

Torque balance $T_e = T_m + T_{mech\_loss}$

Example 1: No Load Test PM Motor



Assume you know R_a

Measure I_a, V_a, ω_r

$$V_a = R_a I_a + E_a$$

$$P_e = I_a E_a = \omega_r T_e$$

$$T_e = T_{friction} = \omega_r D_m$$

$$T_e = k_v I_a$$

$$E_a = k_v \omega_r$$

$k_t = k_v$ Torque / voltage machine constant

Example 2: PM DC Motor

Consider a small PM DC Motor with the following parameters:
$V_{t\_rated} = 6V$, $I_{no\_load} = 0.15A$, $R_a = 7\Omega$, and torque constant $K_t = 0.014$ [Vs/rad].
Find no-load speed n_{nl} [rpm], back emf E_a, and friction torque T_{fric} at $V_t = 6V$

$V_t = R_a I_a + E_a = R_a I_a + K_t \cdot \omega$

$\omega_{nl} = \frac{V_t - R_a I_a}{K_t} = \frac{6 - 7 \cdot 0.15}{0.014} = 353.57$ rad/sec

$n_{nl} = \frac{30}{\pi} \omega_{nl} = 3,376.4$ rpm

Back emf $E_a = K_t \cdot \omega = 0.014 \cdot 353.57 = 4.95V$

Electromagnetic torque

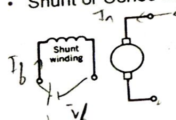
$T_e = K_t \cdot I_a = 0.014 \cdot 0.15 = 0.0021$ N·m

$T_e = \frac{P_e}{\omega} = \frac{I_a \cdot E_a}{\omega} = \frac{0.15 \cdot 4.95}{353.57} = 0.0021$ N·m

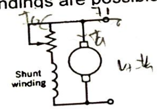
$T_{fric} = T_e = 0.0021$ N·m

Basic Types of DC Machines

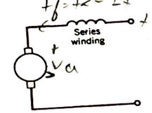
Basic DC Machines
- Shunt or Series field windings are possible



Separately Excited DC Machine



Shunt DC Machine



Series DC Machine

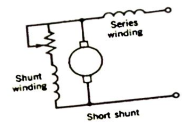
$I_f = I_a - I_L$

- Field winding is designed for up to rated armature voltage
- Field winding has large number of turns
- Field current is small compared to armature current

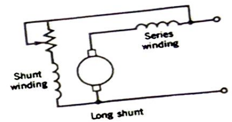
- Field winding is designed for up to rated armature current
- Field winding has small number of turns
- Field current is the same as the armature current

Basic Types of DC Machines

Compound DC Machines
- Both Shunt and Series field windings are present



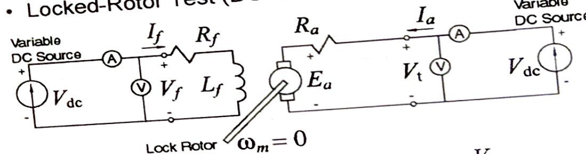
Short-Shunt Compound DC Machine



Long-Shunt Compound DC Machine

Parameters of DC Machine

- Locked-Rotor Test (DC Measurements)



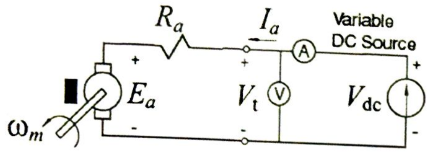
$$R_f = \frac{V_f}{I_f}$$

$$R_a = \frac{V_t}{I_a}$$

(armature + brush combined resistance at no speed!)

Parameters of DC Machine

- No Load Test for PM Motors (Friction vs. Speed)



Assume you know R_a

Measure I_a, V_t, ω_m

$$V_t = R_a I_a + E_a$$

$$T_e = K_a \Phi_p I_a = K_t I_a$$

$$P_e = I_a E_a = \omega_m T_e$$

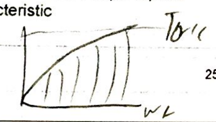
$$E_a = K_a \Phi_p \omega_m = K_v \omega_m$$

$$T_e = T_{fric}(\omega_m) = \omega_m D_m$$

$K_t = K_v$ Torque / voltage machine constant

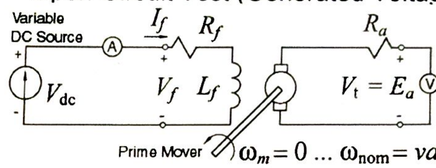
Measure Friction Torque-Speed Characteristic

$$K_t = K_v = \frac{V_a - R_a I_a}{w_r}$$



Parameters of DC Machine

- Open-Circuit Test (Generated Voltage vs. Speed)

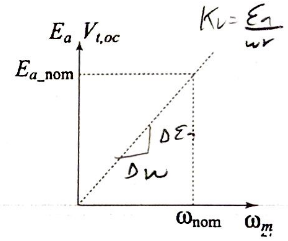


$$E_a = K_a \Phi_p \omega_m = K_a \frac{L_f}{N_f} I_f \omega_m$$

$$= \underbrace{L_{af} I_f}_{K_v} \omega_m$$

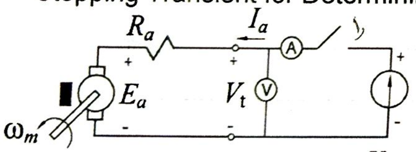
$K_v \cdot W_r$

$$K_v = \frac{E_a}{w_r}$$



Parameters of DC Machine

- Stopping Transient for Determining Inertia



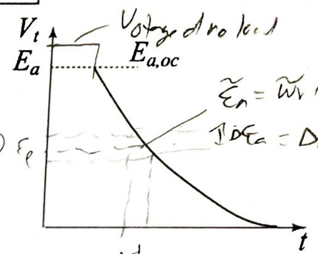
Assume you know $T_{fric}(\omega_m)$

Measure & Record I_a, V_t, ω_m

$$\cancel{T_e}^{0} = \cancel{T_m}^{0} + T_{fric}(\omega_m) + J\frac{d\omega_m}{dt}$$

$$E_a = K_a \Phi_p \omega_m = K_v \omega_m$$

$$T_{fric}(\omega_m) = -J\frac{\Delta\omega_m}{\Delta t} = -J\frac{\Delta E_a}{K_v \Delta t}$$



$E_a = w_r \cdot K_v$

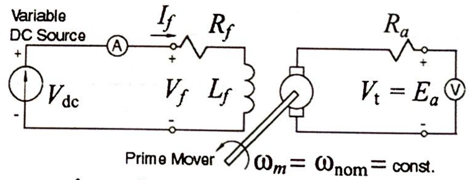
$\Delta E_a = \Delta w \cdot K_v$

$$J = \frac{T_{fric}(w_r) \cdot K_v \Delta t}{\Delta E_a}$$

make sure it's small

Parameters of DC Machine

- Open-Circuit Test (Generated Voltage vs. Field Currents)

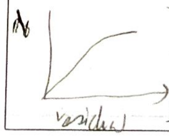


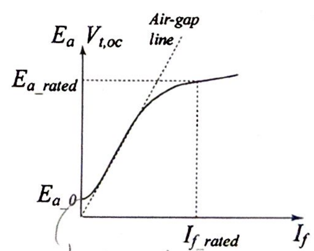
$$L_{af} = \frac{E_a}{\Delta I_f \omega_m}$$

$$\Phi_p = \frac{\lambda_f}{N_f} = \frac{L_f}{N_f} I_f$$

$$E_a = K_a \Phi_p \omega_m = K_a \frac{L_f}{N_f} I_f \omega_m$$

$$= \underbrace{L_{af} I_f}_{K_f} \omega_m$$

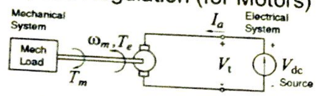


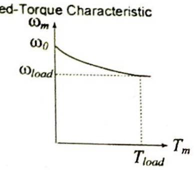


voltage due to residual flux

DC Machines Characteristics:

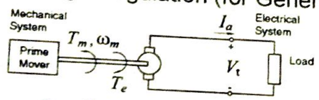
Speed Regulation (for Motors)

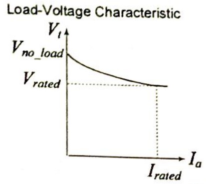




$$SR = \frac{\omega_{m,no\_load} - \omega_{m,load}}{\omega_{m,load}} 100\% = \frac{n_{no\_load} - n_{load}}{n_{load}} 100\%$$

Voltage Regulation (for Generators)

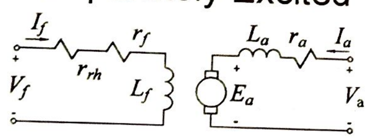




$$VR = \frac{V_{t,nc} - V_{t,load}}{V_{t,load}} 100\% = \frac{V_{no\_load} - V_{rated}}{V_{rated}} 100\%$$

Separately Excited

Field Winding Equations



Armature Equations

$$R_f = r_f + r_{rh}$$

$$V_f = R_f I_f$$

$$\lambda_f = N_f \Phi_p = L_f I_f$$

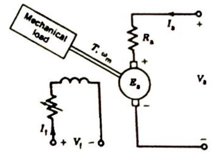
$$V_a = r_a I_a + E_a$$

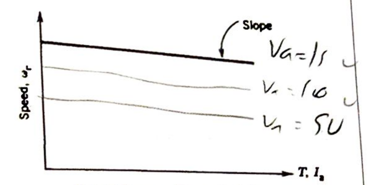
$$E_a = \omega_r L_{af} i_f$$

Electromagnetic torque $T_e = L_{af} I_f I_a$

Torque balance $T_e = T_m + T_{mech\_loss}$

Separately-Excited DC Motor





Speed-Torque Characteristic

$$V_a = R_a I_a + E_a$$

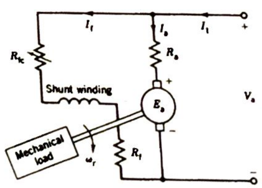
$$E_a = L_{af} I_f \omega_r$$

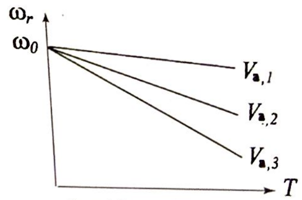
$$T = L_{af} I_f I_a$$

$$\omega_r = \frac{E_a}{L_{af} I_f} = \frac{V_a - I_a R_a}{L_{af} I_f}$$

$$\omega_r = \underbrace{\left(\frac{V_a}{L_{af} I_f}\right)}_{\text{no load speed}} - \left(\frac{R_a}{(L_{af} I_f)^2}\right) T$$

Shunt DC Motor





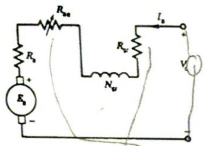
Speed-Torque Characteristic

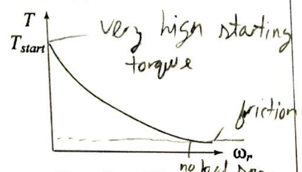
$$\omega_r = \frac{V_a}{L_{af} I_f} - \frac{R_a}{(L_{af} I_f)^2} T$$

$$L_{af} I_f = \frac{L_{af} V_a}{R_f + R_{fc}}$$

$$\omega_r = \underbrace{\frac{R_{f,total}}{L_{af}}}_{\text{No load speed}} - \underbrace{\frac{R_a R_{f,total}^2}{(L_{af} V_a)^2}}_{\text{slope}} T$$

Series DC Motor





Torque-Speed Characteristic

$$V_t = I_a(R_a + R_{sr} + R_{ae}) + L_{af} I_a \omega_m$$

$$I_a = \frac{V_a}{R_a + R_{sr} + R_{ae} + L_{af}\omega_m}$$

$$T = \frac{L_{af}V_a^2}{\underbrace{(R_a + R_{sr} + R_{ae} + L_{af}\omega_m)^2}_{\text{small}}}$$

$$T_e = L_{af} I_a^2 = \underbrace{L_{af} I_f}_{\lambda_f} \cdot I_a$$

Starting Considerations

Consider a 1.2kW, 120V, Shunt DC Motor with Ra = 2Ω, and Ia_max = 20A

Rated current $I_{a,rated} = \frac{1200}{120} = 10\text{A}$

Starting current $I_{a,start} = \frac{V_a - E_a}{R_a} = \frac{120 - 0}{2} = 60\text{A}$ Too high ! => Burn the motor !

1. Use additional starting resistor in series with armature
2. Use reduced voltage
3. Use Power Electronic Drive & Controller

Part 2: Brushed DC Motor Drives and Dynamics

Important Topics & Concepts

- Basic types of dc motor drives
- DC to DC choppers (Chap. 3.8)
- AC to DC controlled rectifiers
- Dynamic (State) equations of the separately-excited motor
- Possible model & implementation/solution
- Dynamic (State) equations of the PM motor
- Possible model & implementation/solution

Speed/Torque Control of DC Motors

Recall the Separately Excited DC Motor

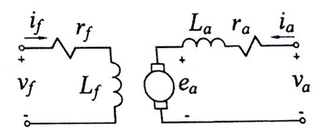
$$v_a = r_a i_a + k_v \omega_r$$

$$k_v = L_{af} i_f$$

$$\omega_r = \frac{e_a}{L_{af} i_f} = \frac{v_a - i_a R_a}{L_{af} i_f}$$

$$\omega_r = \frac{v_a - i_a r_a}{k_v}$$

$$T_e = k_v i_a$$

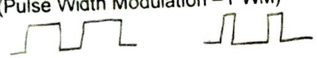


Control methods:

1. Varying the armature resistance
2. Varying the field
3. Varying the terminal voltage
4. Varying the armature current

Solid-State Converters for DC Motors

DC-DC Converters (Choppers) – used with small & large motors
- Voltage Source (VS) (Pulse Width Modulation – PWM)
  - One-quadrant
  - Two-quadrant
  - Four-quadrant

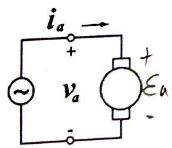


- Current Source (CS) (Hysteresis & Delta Modulation)
  - One-quadrant
  - Two-quadrant
  - Four-quadrant

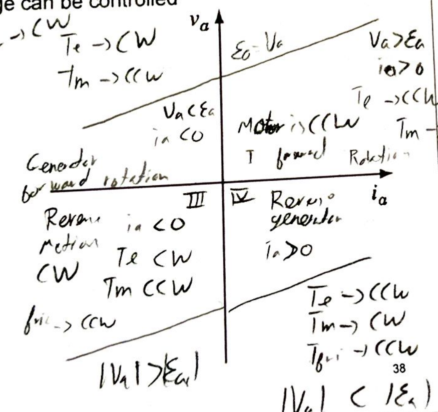
AC-DC Controlled Rectifiers – used with large motors
- Single-Phase
  - Half-wave
  - Full bridge (full wave)
- Three-Phase
  - Half-wave
  - Full bridge (full wave)

DC-DC Converters (Choppers)

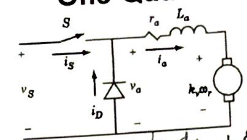
Assume a dc voltage source wherein the averaged output voltage can be controlled



$$\bar{V}_a = \frac{1}{T}\int_0^T V_a(t)dt$$

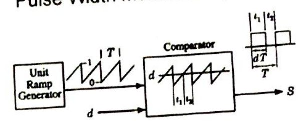


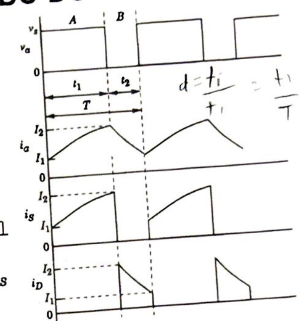
One-Quadrant VS DC-DC Converter



$V_a = r_a \cdot i_a + L_a \frac{di}{dt} + \mathcal{E}_a$

Pulse Width Modulation (PWM)





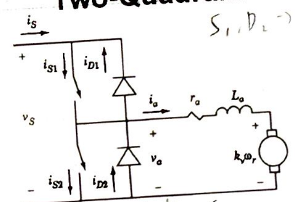
$d = \frac{t_1}{t_1 + t_1} = \frac{t_1}{T}$ duty cycle

Switch can be realized using:
- Bipolar Junction Transistor (**BJT**)
- Insulated Gate Bipolar Transistor (**IGBT**)
- Metal Oxide Semiconductor Field Effect Transistor (**MOSFET**)

$$\bar{V}_a = \frac{1}{T}\int_0^T v_a(t)dt = \underbrace{\frac{t_1}{t_1 + t_2}}_{d}V_s$$

Two-Quadrant VS DC-DC Converter

$S_1, D_2 \rightarrow$ 1st quadrant

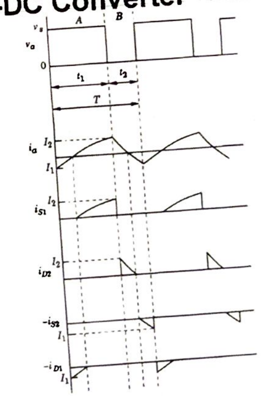


$V_a = r_a i_a + L \frac{di}{dt} + \mathcal{E}_a$

Pulse Width Modulation (PWM)

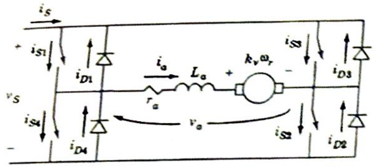
$V_a = d \cdot V_s$

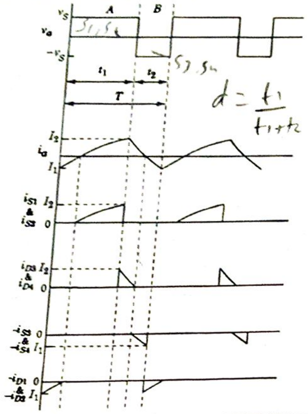
$d = \frac{t_1}{t_1 + t_2} = \frac{t_1}{T}$



H-bridge converter

Four-Quadrant VS DC-DC Converter





$d = \frac{t_1}{t_1+t_2} = \frac{t_1}{T}$

Pulse Width Modulation (**PWM**)

$$\bar{v}_a = \frac{1}{T}\int_0^T v_a(t)dt = dV_s \quad = 2\left(d-\frac{1}{2}\right)V_s$$

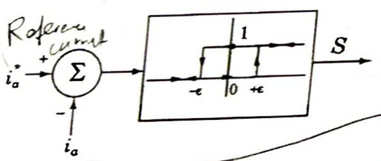
$0 < d \leq 1$

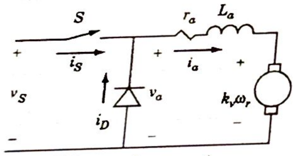
$-V_s < \bar{V}_a < +V_s$

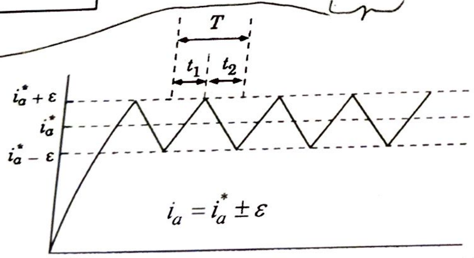
Current Source DC-DC Converter

Hysteresis Modulation (**HM**)

Reference current







$$i_a = i_a^* \pm \varepsilon$$

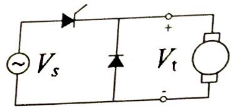
The same modulation can be applied to two- and four-quadrant choppers

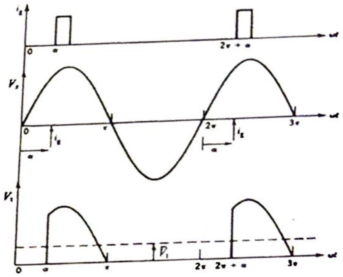
$T_e^{\Delta} = K_t \cdot i_a^{\Delta}$

Thyristor Controlled Rectifiers

Control the Averaged Output Voltage

a) Single-phase, half-wave





$$V_s = \sqrt{2}V_{rms}\sin(\omega t)$$

$$\bar{V}_t = \frac{1}{T}\int_0^T V_t(t)dt = \frac{V_{rms}}{\sqrt{2}\pi}(1+\cos(\alpha))$$

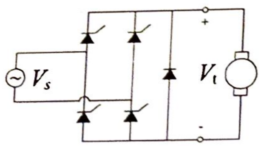
$0 \leq \alpha \leq \pi$

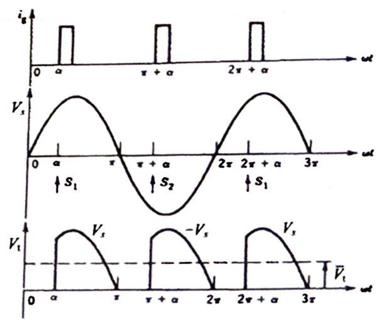
$0 \leq \bar{V}_t \leq \frac{\sqrt{2}}{\pi}V_{rms}$

Thyristor Controlled Rectifiers

Control the Averaged Output Voltage

b) Single-phase, full-wave





$$V_s = \sqrt{2}V_{rms}\sin(\omega t)$$

$$\bar{V}_t = \frac{1}{T}\int_0^T V_t(t)dt = \frac{\sqrt{2}V_{rms}}{\pi}(1+\cos(\alpha))$$

$0 \leq \alpha \leq \pi$

$0 \leq \bar{V}_t \leq \frac{2\sqrt{2}}{\pi}V_{rms}$

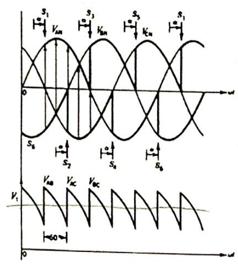
Thyristor Controlled Rectifiers

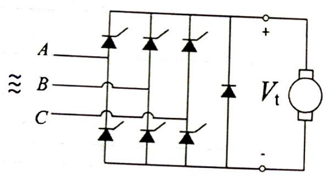
Control the Averaged Output Voltage
d) Three-phase, full-wave

$V_A = \sqrt{2}V_{rms}\sin(\omega t)$

$V_B = \sqrt{2}V_{rms}\sin(\omega t - 120^\circ)$

$V_C = \sqrt{2}V_{rms}\sin(\omega t + 120^\circ)$

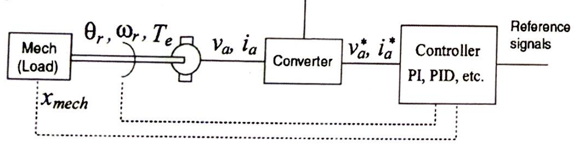




$$\overline{V}_t = \frac{1}{T}\int_0^T V_t(t)dt \qquad 0 \le \alpha \le \pi$$

$$0 \le \overline{V}_t \le \frac{3\sqrt{6}}{\pi}V_{rms}$$

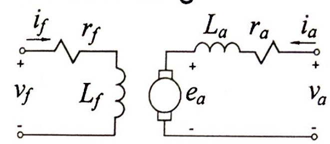
Closed-Loop Electric Drive System



Dynamic models are needed for analyzing and designing complex electro-mechanical systems!

Dynamic Modeling

Circuit is valid for transient analysis



Voltage Equations

$$v_a = r_a i_a + \frac{d\lambda_a}{dt} + e_a = r_a i_a + e_a + L_a\frac{di_a}{dt}$$

$$v_f = r_f i_f + \frac{d\lambda_f}{dt} = r_f i_f + L_f\frac{di_f}{dt}$$

Speed Equation

$$\underbrace{T_e = J\frac{d\omega_r}{dt} + T_m + D_m\omega_r}_{\text{mechanical subsystem}}$$

state variable
ω_r, I_a, I_f

Coupling Terms

$e_a = \omega_r L_{af} i_f$

$T_e = L_{af} i_f i_a$

Re-Write the Equations to Express Derivatives

State Equations 1st order system of ODE

$$\begin{cases} \dfrac{di_a}{dt} = -\dfrac{r_a}{L_a}i_a - \dfrac{1}{L_a}e_a + \dfrac{1}{L_a}v_a \\ \dfrac{di_f}{dt} = -\dfrac{r_f}{L_f}i_f + \dfrac{1}{L_f}v_f \\ \dfrac{d\omega_r}{dt} = -\dfrac{D_m}{J}\omega_r + \dfrac{1}{J}(T_e - T_m) \end{cases}$$

Coupling Terms

$e_a = \omega_r L_{af} i_f$

$T_e = L_{af} i_f i_a$

product is non-linear

Are these equations coupled or decoupled ?

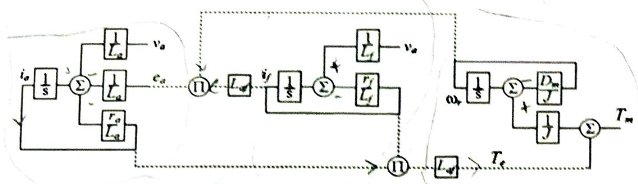
Are these equations linear or non-linear ?

How do we solve these equations ?

MatLab / simulation

Implementation of State Equations

$$\frac{di_a}{dt} = -\frac{r_a}{L_a} i_a - \frac{1}{L_a} e_a + \frac{1}{L_a} v_a \qquad \frac{d\omega_r}{dt} = -\frac{D_m}{J}\omega_r + \frac{1}{J}(T_e - T_m)$$

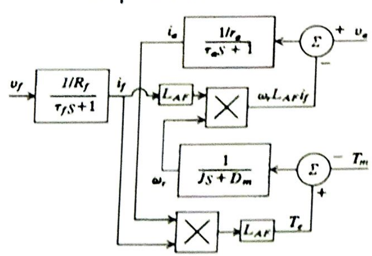


$$e_a = \omega_r L_{af} i_f \qquad \frac{di_f}{dt} = -\frac{r_f}{L_f} i_f + \frac{1}{L_f} v_f \qquad T_e = L_{af} i_f i_a$$

Try to implement it in Simulink ?

Is this a unique implementation ?

Implementation of State Equations



Re-write the equations

$$v_f = r_f(1+\tau_f s)i_f$$
$$v_a = r_a(1+\tau_a s)i_a + e_a$$
$$e_a = \omega_r L_{af} i_f$$
$$T_e - T_m = (D_m + Js)\omega_r$$
$$T_e = L_{af} i_f i_a$$

$$\omega_r = \frac{1}{(D_m + Js)}$$

Define time constants

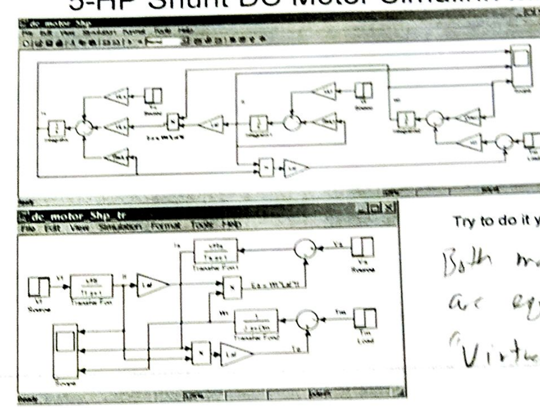
$$\tau_a = \frac{L_a}{r_a} \qquad \tau_f = \frac{L_f}{r_f}$$

$$i_a = \frac{1}{r_a(1+\tau_a s)}(V_a - e_a) \qquad i_f = \frac{1}{r_f(1+\tau_f s)} v_f$$

50

S = Laplace variable

5-HP Shunt DC Motor Simulink Model



Try to do it yourself !

Both model are equivalent

"Virtual DC machine"

Dynamic Response of a 5-HP DC Motor

Motor parameters:
Va = Vf = 240V, Rf = 240Ω, Lf = 120H, Ra = 0.6Ω, La = 0.012H, Laf = 1.8H,
J = 1kg*m^2, Dm = 1e-4 N*m*s, Tm = 29.2 N*m

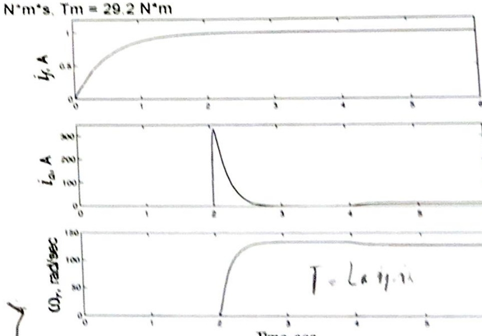
In this computer study:

Zero initial conditions assumed

Step-1: field winding is energized

Step-2: armature is energized

Step-3: load torque is applied



PM DC Motor Dynamics

State Equations

$$\frac{di_a}{dt} = -\frac{r_a}{L_a} i_a - \frac{1}{L_a} e_a + \frac{1}{L_a} v_a$$

$$\frac{d\omega_r}{dt} = -\frac{D_m}{J}\omega_r + \frac{1}{J}(T_e - T_m)$$

Coupling Terms

$$e_a = k_v \omega_r \qquad T_e = k_v i_a$$

Are these equations coupled or decoupled ?
Are these equations linear or non-linear ?
How do we solve these equations ?

Standard State-Space Form

State vector

input vector

$$\frac{d}{dt}\begin{bmatrix} i_a \\ \omega_r \end{bmatrix} = \begin{bmatrix} -\frac{r_a}{L_a} & -\frac{k_v}{L_a} \\ \frac{k_v}{J} & -\frac{D_m}{J} \end{bmatrix}\begin{bmatrix} i_a \\ \omega_r \end{bmatrix} + \begin{bmatrix} \frac{1}{L_a} & \\ & -\frac{1}{J} \end{bmatrix}\begin{bmatrix} v_a \\ T_m \end{bmatrix}$$

Linear Time invariant

(LTI system)

$$\frac{d\mathbf{x}}{dt} = \mathbf{Ax} + \mathbf{Bu}$$

Electro mechanical model

PM DC Motor Implementations

State Equations

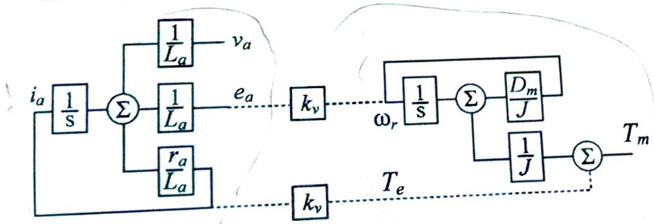
$$\frac{di_a}{dt} = -\frac{r_a}{L_a} i_a - \frac{1}{L_a} e_a + \frac{1}{L_a} v_a$$

$$\frac{d\omega_r}{dt} = -\frac{D_m}{J}\omega_r + \frac{1}{J}(T_e - T_m)$$

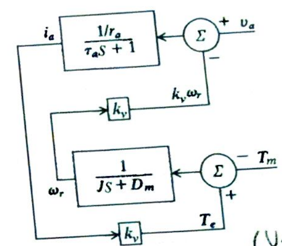
Coupling Terms

$$e_a = k_v \omega_r \qquad T_e = k_v i_a$$

Are these equations coupled or decoupled ?
Are these equations linear or non-linear ?
How do we solve these equations ?



Implementation of State Equations



Re-write the equations

$$v_a = r_a(1 + \tau_a s)i_a + e_a$$

$$e_a = k_v \omega_r$$

$$T_e - T_m = (D_m + Js)\omega_r$$

$$T_e = k_v i_a$$

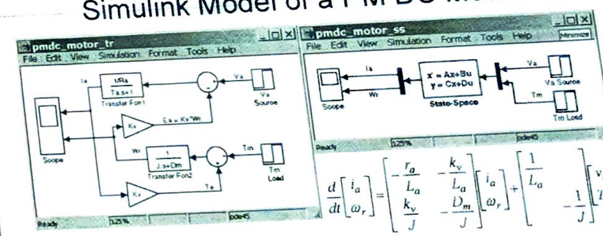
Define time constants

$$\tau_a = \frac{L_a}{r_a} \qquad \tau_f = \frac{L_f}{r_f}$$

$$i_a = \frac{1}{r_a(1+\tau_a \times s)}(v_a - e_a)$$

$$\omega_r = \frac{1}{(Js + D_m)}(T_e - T_m)$$

Simulink Model of a PM DC Motor



$$\frac{d}{dt}\begin{bmatrix} i_a \\ \omega_r \end{bmatrix} = \begin{bmatrix} -\frac{r_a}{L_a} & -\frac{k_v}{L_a} \\ \frac{k_v}{J} & -\frac{D_m}{J} \end{bmatrix}\begin{bmatrix} i_a \\ \omega_r \end{bmatrix} + \begin{bmatrix} \frac{1}{L_a} & \\ & -\frac{1}{J} \end{bmatrix}\begin{bmatrix} v_a \\ T_m \end{bmatrix}$$

$$i_a = \frac{1}{r_a(1+\tau_a s)}(v_a - e_a)$$

$$e_a = k_v \omega_r$$

$$\omega_r = \frac{1}{D_m + Js}(T_e - T_m)$$

$$T_e = k_v i_a$$

$$\begin{bmatrix} i_a \\ \omega_r \end{bmatrix} = \begin{bmatrix} 1 & \\ & 1 \end{bmatrix}\begin{bmatrix} i_a \\ \omega_r \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} v_a \\ T_m \end{bmatrix}$$

$$[T_e] \; \underbrace{(k_t \;\; 0)}_{C} \quad \underbrace{(0 \;\; 0)}_{D}$$

Dynamic Response of a 6V PM DC Motor

Motor parameters:
Va = 6V, Ra = 7Ω, La = 120mH, Kv = 0.0141 N*A/m, J = 1.06e-6 kg*m^2,
Dm = 6.01e-6 N*m*s, Tm = 3.53e-3 N*m

In this computer study:

Zero initial conditions assumed

Step-1: voltage is applied

Step-2: load torque is applied