# University of British Columbia
## Department of Mechanical Engineering

## MECH 305/306. Test 2. March 19, 2019

**Suggested Time**: 60 min **Allowed Time:** 70 min

**Materials admitted**: Pen, pencil, eraser, straightedge, Mech 2 style calculator, one letter-size sheet of paper (both sides) for personally hand-written notes.

There are 4 questions on this test. You are asked to answer all the 4 questions on this test paper.

The purpose of this test is to evaluate your knowledge of the course material. Orderly presentation demonstrates your knowledge most clearly, while disorganized and unprofessional work creates serious doubt. Marks are assigned accordingly. A bonus of up to 3 marks will be given for exemplary presentation.

Complete the section below **during** the examination time **only.**

NAME:________________________________ Section (305/306) _________

SIGNATURE:___________________________________________

STUDENT NUMBER:____________________________________

| **Question** | **Mark Received** | **Maximum Mark** |
|---|---|---|
| 1 | | 15 |
| 2 A **or** B | | 15 |
| 3 | | 15 |
| 4 | | 15 |
| Presentation | | 3 |
| **Total** | | 60 (+3) |

Name: ________________________________________

1. A panel on an aircraft wing is attached to the frame with 100 rivets. In a prior test of 10,000 rivets, one rivet failed. The aircraft wing is designed to be tolerant of a few faulty rivets and will work safely with at least 98 good rivets out of 100.

   a) [5 marks] What is the probability that a wing is unsafe?

The wing is unsafe if there are more than 2 bad rivets. Let q be the probability that there are 0, 1, 2 bad rivets; probability(unsafe)=1-q. This is a binomial problem where the probability p of a good rivet is 1-1/10,000=0.9999

$$q = \frac{100!}{(100-98)!\,98!}p^{98}(1-p)^{2} + \frac{100!}{(100-99)!\,99!}p^{99}(1-p)^{1} + \frac{100!}{(0)!\,100!}p^{100}(1-p)^{0}$$

$$1 - q = 1.61x10^{-7}$$

2 marks for recognizing that this a binomial distribution problem and that the probability of a single rivet failure is 1/10000.

2 marks for setting up the correct terms to get the probability of an unsafe wing (and not 1 minus this chance, for example).

1 mark for executing the calculation. Note that this can be a little tricky. If you round off the terms too early you will very likely get the wrong answer because the 3 terms of the binomial distribution are quite different orders of magnitude. When you add them, you need to keep the precision of the smallest terms.

b) The aircraft company is considering the use of an ultrasonic non-destructive test on the wing. The test is pretty good at identifying a faulty wing, in which case a red sign flashes “Faulty!”. The probability of indicating “faulty” when the wing IS faulty is 0.999. The probability of indicating “faulty” when the wing is actually ok is 0.01.

i. [5 marks] Suppose we try this test on the wing and the flashing red sign comes on. What is your best estimate of the probability that the wing *really is faulty?*

Use Bayes: A=event wing is faulty ; B= event the *test* indicates faulty.

We want to know

$$P(A|B) = \frac{P(B|A)}{P(B)} P(A)$$

Part a) gives P(A)=1.61e-7 and the wording of the question gives P(B|A)=0.999. To get the “unconditional” probability of a test indicating “faulty”, we need to consider the two ways that we can get a faulty reading: a correct positive and a false positive.

$$P(B) = 0.999(1.61x10^{-7}) + 0.01(1 - 1.61x10^{-7}) \approx 0.01$$

The result is that P(A|B)=1.61e-5.

2 marks for use of Bayes with the correct interpretation of events A, B.

3 marks for correct assignment of probabilities for use in the formula.

[this probably could also be done using a probability tree; up to full marks if clear and accurate]

If P(A) is way off, the final result will be way off as well. For example, if P(A) is calculated as 0.01, the final result will be 0.5.

ii. [5 marks] Would you recommend that the aircraft company run the test? Why?

Nearly all (>99.99%) of the tests indicating “faulty” will be false positives, so this is a very poor test to catch the rare faulty wing. However, only about 1% of the tests will indicate faulty, so it is possible that this test could be used for screening, but it would need to be followed up with more accurate testing for those 1 in 100 planes that get “caught”.

Some students calculated very high values for P(A|B) above, say 0.5. In this case, for ii, they should conclude that test is worthwhile (consider the consequences of letting a faulty aircraft leave the factory).

5 marks for a sensible discussion of the result. Both answers “yes” and “no” are acceptable if properly explained.

Name: ________________________________________

2. Do **EITHER** Option A **OR** Option B

**OPTION A**

In traditional steel heat-treating, the part to be treated is brought to a high temperature, then plunged into a liquid bath. The cooling rate is quite important and this will depend on the convection heat transfer coefficient $h$ between the liquid in the bath and the steel surface. After considering the theory behind this, we aim to measure $h$ [W/m$^2$/$^o$C].

[6 marks] Suppose the steel part has a spatially uniform temperature $T$ (but varies with time $t$). How would you relate $T(t)$ to the convection coefficient?

Assuming spatially uniform T and uniform h over the surface,

$$MC\frac{dT}{dt} = hA(T_{sur} - T)$$

Mass M, area A and heat capacity C would be constants, as would the temperature of the surroundings, $T_{sur}$. If we measure T(t) we can differentiate it and solve for h in the equation above.

i. [4 marks] Outline how you would run an experiment to measure $h$. Note carefully the parameters you would need to measure and how you might measure them.

Place several thermcouples in the object (to check for uniformity), acquire T(t) after the object is heated up then plunged in the liquid. Parameters M, C, A to be determined from scale, property data tables and geometry measurements, respectively.

ii. [5 marks] Discuss the main contributors to random and systematic errors in your measurement of $h$.

Thermocouple noise would be amplified in the differentiation, giving random errors. Thermocouple offsets could give a systematic error. Non-uniformity of the object temperature is likely the largest cause of bias, but other heat losses (say, through the support of the object) could bias results as well (they are not considred in the model above).

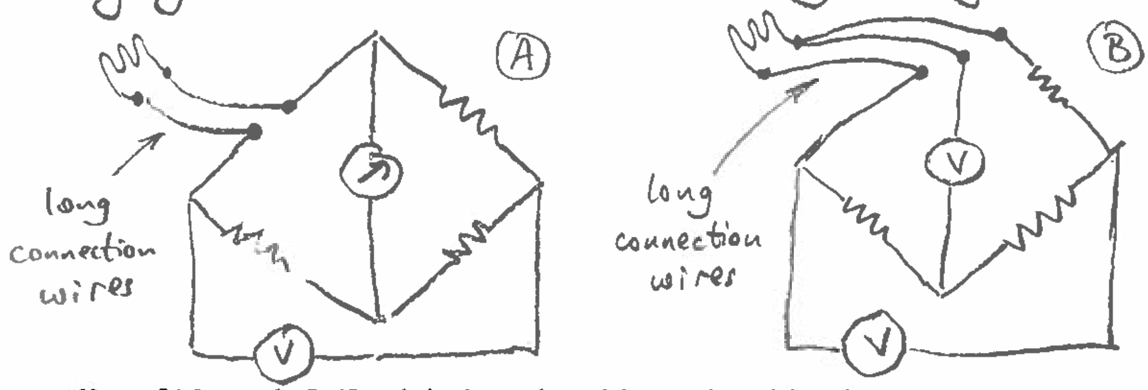
Name: ______________________________

**OPTION B**

When wiring the strain gauges in the pop-can pressure measurement lab, you used a 3-strand wire for your connections.

i. [5 marks] What is the purpose of this connection method?

The purpose of the 3-wire connection method is to eliminate the effect of resistance changes in the strain gauge connection wires caused by changes in temperature.




ii. [10 marks] Explain how it achieves its objectives.

Commonly, a strain gauge is physically some significant distance away from the rest of the bridge components. Thus, the connection wires can get quite long and have significant resistance. In circuit (A), and changes in the connection wire resistance due to temperature changes will get added to resistance change of the strain gauge, and will create false strain readings called "thermal output". In circuit (B), the outer two wires are connected in adjacent arms of the bridge, so potential effects of thermal resistance change will cancel out. The resistance change in the centre wire is not significant because the voltage indicator has very high impedance.

Name: ______________________________

3. Short answer questions (answer both).

   a) [8 marks] There are two apples, an orange and a peach in a bowl. A visitor comes and, without looking, randomly takes two of the fruits from the bowl. She then looks at the two fruits she has taken and tells that one of them is an apple. What is the probability that the second fruit is also an apple ?

There are six possible combinations for any two fruits taken from the bowl:

Apple 1 + Apple 2

Apple 1 + Orange

Orange + Apple 2

Apple 1 + Peach

Peach + Apple 2

Orange + Peach

The last possible combination is not acceptable because it does not contain an apple.

Thus, there are five acceptable possibilities, one of which is for the second apple.

→ Probability the second fruit is an apple $= \underline{\frac{1}{5}}$

Name: ________________________________________

3. Short answer questions (answer both).

a) [8 marks] There are two apples, an orange and a peach in a bowl. A visitor comes and, without looking, randomly takes two of the fruits from the bowl. She then looks at the two fruits she has taken and tells that one of them is an apple. What is the probability that the second fruit is also an apple?

We should evaluate the probability from the perspective of the audience watching the person selecting the fruit. The audience cannot see the fruit. The phrase "tells them that one of them is an apple" means "tells them that AT LEAST one of them is an apple".

Let A=event that the second fruit is an apple, B= event that the fruit picker calls out "I have an apple".

We seek P(A|B)=P(B|A) P(A)/P(B). Rather than evaluate each term as shown here, use P(B|A)P(A)=P(B&A) which is the probability that both picked fruits are apples.

P(B&A)= P(picking an apple on first pick) x P(picking an apple on $2^{nd}$ pick)

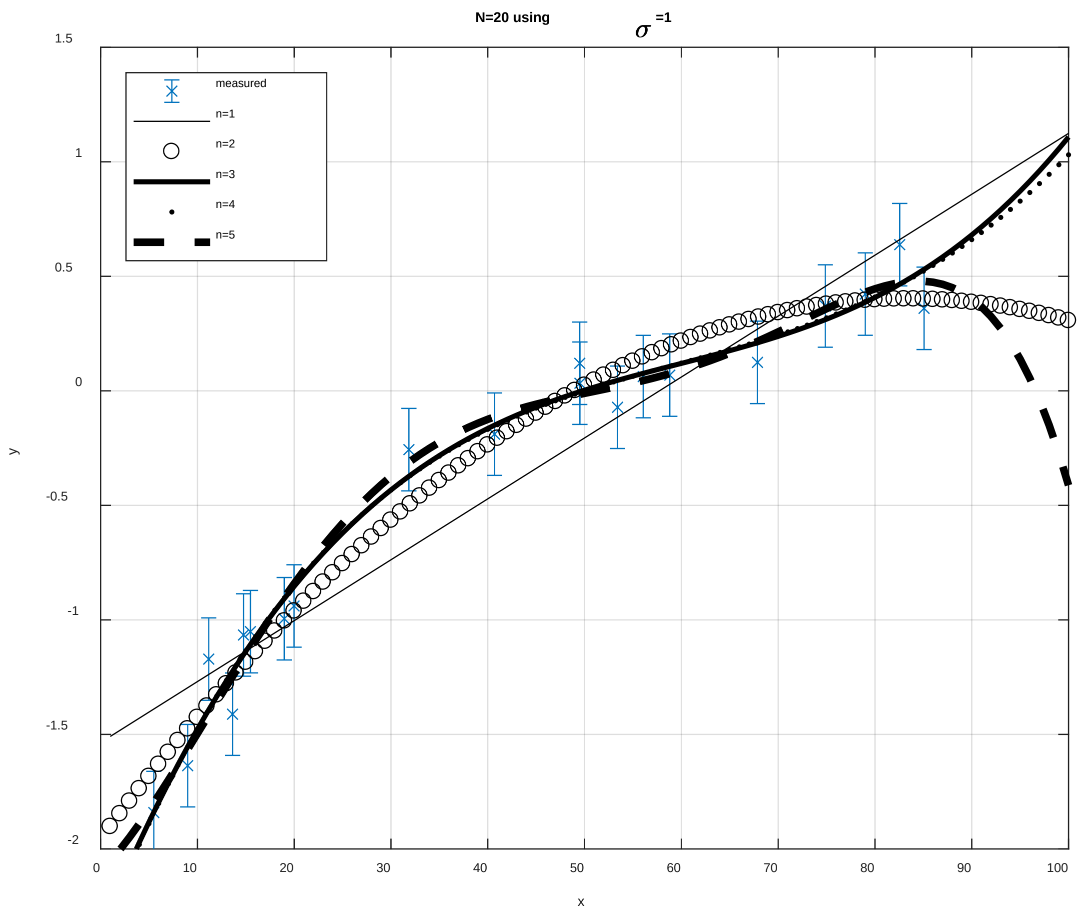
P(apple on first pick)=1/2 (we are picking from Ap Ap Or Pe, half are apples)
P(apple on $2^{nd}$ pick)= 1/3 (remaining fruits are Ap Or Pe)
➔ P(B&A)=1/6

P(B)= 1 – P(no apples)=1- P(Or & Pe)
By the logic used to find P(B&A), we get P(B)=1-1/6=5/6

**Thus, P(A|B)=P(B|A) P(A)/P(B)= (1/6)/(5/6)= 1/5**

Name: ________________________________________

b) [7 marks] Measurements (x, with 1 standard deviation error bars) are fit to different degrees of polynomials (n=1 to 5). Which degree looks most appropriate? Explain why.




As emphasized in class, adding more terms to a regression model *always* reduces the total error so it is no surprise that n=5 matches the data best. However, this is NOT the best choice for a model. **The cubic (n=3) fits the results nearly as well (within 1 standard deviation of the measurements) so this is the simplest good model and our best choice.** [7 marks]

Reasonable people could pick n=4 on this basis but it is so close to the n=3 results there is really no justification for the more complex model. [4 marks].

Up to 3 marks for picking n=1,2 if there is a clear statement that you want the simplest model, or up to 3 marks if you picked n=5 and made it clear that you were going for low residuals.

Name: ___________________________________

4. In a phase-stepping interferometer, the measured light intensity $I$ varies with phase according to relationship

$$I = A + B \sin \phi$$

where $A$ and $B$ are constants and $\phi$ is the local phase. It is desired to evaluate $A$, $B$ and $\phi$ from four measurements of $I$, where a piezo actuator has been used to advance the phase by $\pi/2$ between measurements. Under these circumstances

$$I_i = A + B \sin(\phi + (i\text{-}1)*\pi/2) \quad \text{for } i = 1, 2, 3, 4$$

You are asked to develop least-squares best-fit formulas for $A$, $B$ and $\phi$ in terms of the measured phase-stepped intensities $I_1$, $I_2$, $I_3$, $I_4$.

Hint: You have four equations and three unknowns, $A$, $B$ and $\phi$. You can linearize your least-squares equations by instead working with $A$, $B\sin\phi$ and $B\cos\phi$ as your unknowns. Also note that $\sin(\phi+\pi/2) = \cos\phi$ and $\cos(\phi+\pi/2) = -\sin\phi$.

The four measurements are

$$\begin{aligned} I_1 &= A + B\sin(\phi + 0) \\ I_2 &= A + B\sin(\phi + \tfrac{\pi}{2}) \\ I_3 &= A + B\sin(\phi + \pi) \\ I_4 &= A + B\sin(\phi + \tfrac{3}{2}\pi) \end{aligned} \quad \longrightarrow \quad \begin{aligned} I_1 &= A + B\sin\phi \\ I_2 &= A + B\cos\phi \\ I_3 &= A - B\sin\phi \\ I_4 &= A - B\cos\phi \end{aligned}$$

In matrix form:

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} A \\ B\sin\phi \\ B\cos\phi \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}$$

$\uparrow$ $\underset{\sim}{G}$

Name: ______________________________

We form the least-squares solution by premultiplying by $\underset{\sim}{G}^T$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} A \\ B\sin\phi \\ B\cos\phi \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}$$

Do the matrix multiplication:

$$\begin{bmatrix} 4 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} A \\ B\sin\phi \\ B\cos\phi \end{bmatrix} = \begin{bmatrix} I_1+I_2+I_3+I_4 \\ I_1-I_3 \\ I_2-I_4 \end{bmatrix}$$

Very conveniently, the least-squares matrix is diagonal.

$$\rightarrow \quad \underline{A = \frac{I_1+I_2+I_3+I_4}{4}}$$

$$B\sin\phi = \frac{I_1-I_3}{2} \qquad B\cos\phi = \frac{I_2-I_4}{2}$$

$$\rightarrow \quad B = \sqrt{(B\sin\phi)^2+(B\cos\phi)^2} = \underline{\frac{\sqrt{(I_1-I_3)^2+(I_2-I_4)^2}}{2}}$$

$$\underline{\tan\phi = \frac{B\sin\phi}{B\cos\phi} = \frac{I_1-I_3}{I_2-I_4}}$$