Solutions - Problem Set # 7

Problem 1:

Given: A_s = 0.02 m^2; T_w = 75°C; T_∞ = 25°C

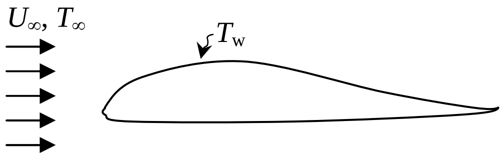
ρ = 1 kg/m^3; c_p = 1000 J/kg-°C; k = 0.025 W/m-°C

Experimental results:
Run 1: U_∞ = 5 m/s; q_{total}= 200 W; D_1 = 0.01 N
Run 2: U_∞ = 24.02 m/s; q_{total}= 600 W

Assumptions: Steady-state flow;

Ec<<1 (negligible viscous dissipation);

$Nu = C\,\mathrm{Re}^m\,\mathrm{Pr}^n$; Chilton-Colburn Analogy applies



a)

$$q_{total} = \bar{q}''_w A_s = hA\left(T_w - T_\infty\right)$$

$$h_1 = \frac{q_{total-1}/A_s}{\left(T_w - T_\infty\right)} = \frac{200/0.02}{\left(75-25\right)} = 200 \text{ W/m}^2\text{-°C}$$

$$h_2 = \frac{q_{total-2}/A_s}{\left(T_w - T_\infty\right)} = \frac{600/0.02}{\left(75-25\right)} = 600 \text{ W/m}^2\text{-°C}$$

b)

$$Nu = C\,\mathrm{Re}^m\,\mathrm{Pr}^n$$

$$\frac{hL_c}{k} = C\left(\frac{\rho U_\infty L_c}{\mu}\right)^m \left(\frac{\mu c_p}{k}\right)^n$$

In this problem:

L_c, μ, ρ, c_p, k are all constant, so

$$h = \underbrace{\Upsilon}_{\text{constant}} U_\infty^{\;m}$$

Using the data provided,
the exponent m can be calculated:

$$\frac{h_1}{h_2} = \left(\frac{U_{\infty,1}}{U_{\infty,2}}\right)^m \Rightarrow m = \frac{\ln\left(h_1/h_2\right)}{\ln\left(U_{\infty,1}/U_{\infty,2}\right)}$$

$$m = \frac{\ln\left(200/600\right)}{\ln\left(5/24.02\right)} = 0.7$$

and the constant $\Upsilon = \dfrac{h_1}{U_{\infty,1}^{\;m}} = 64.826 \quad \dfrac{\text{W/m}^2\text{-°C}}{\left(\text{m/s}\right)^{0.7}}$

Thus, when U_∞, = 13.46 m/s:

$$h = \Upsilon\, U_\infty^{\;m} = 64.826\times\left(13.46\right)^{0.7} = 400.02 \text{ W/m}^2\text{-°C}$$

$$q = hA\Delta T = 400.02\times 0.02\times\left(75-25\right) = 400.02 \text{ W}$$

c)

ρ = 1 kg/m^3; c_p = 1000 J/kg-°C;

k = 0.025 W/m-°C; and In Run 1:

Friction Drag Total = 0.01 N

$$\bar{c}_f = \frac{\bar{\tau}_w}{\frac{1}{2}\rho U_\infty^2} = \frac{Drag/A_s}{\frac{1}{2}\rho U_\infty^2} = \frac{0.01/0.02}{\frac{1}{2}\times 1\times 25} = 0.04$$

The Chilton-Colburn Analogy gives: $\dfrac{\bar{c}_f}{2} = \bar{St}\,\mathrm{Pr}^{2/3}$;

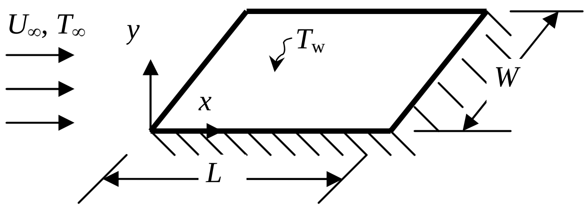
$$\bar{St} = \frac{h}{\rho U_\infty c_p} = \frac{200}{1\times 5\times 1000} = 0.04$$

$$\Rightarrow \mathrm{Pr}^{2/3} = \frac{0.5\times 0.04}{0.04} = 0.5 \Rightarrow \mathrm{Pr} = 0.3536$$

$$\mathrm{Pr} = \frac{\mu c_p}{k} = 0.3536$$

$$\Rightarrow \mu = 0.3536\times 0.025/1000 = 8.84\times 10^{-6}\,\text{kg/m-s}$$

Problem 2:



Given: $L = 1$ m, $W = 1$m. $T_w = 50$°C
Experimental data show the followings:
Run 1: Laminar flow throughout:
$U_{\infty,1} = 2$ m/s; $T_{\infty,1} = 10$°C; $\text{Drag}_1 = 8.4\times10^{-3}$ N;
and $q_{\text{total},1} = 168$ W
Run 2: $U_{\infty,2}$ =? m/s; $T_{\infty,2} = 10$°C; $\text{Drag}_1 = 0.5854$ N;
and $q_{\text{total},2} = 1170.8$ W
$\rho = 1$ kg/m^3; $c_p = 1000$ J/kg-°C

Assumptions: Steady-state flow; Ec<<1 (negligible viscous dissipation); Chilton-Colburn Analogy applies

a)

$$q_{total} = hA(T_w - T_\infty); \quad \text{Drag}=\overline{\tau}_w A; \text{ and } A = 1\,\text{m}^2$$

$$h_1 = 168/(1\times40) = 4.2\ \text{W/m}^2\text{-°C}; \quad \overline{\tau}_{w,1} = 8.4\times10^{-3}\ \text{N/m}^2$$

$$h_2 = 1170.8/(1\times40) = 29.27\ \text{W/m}^2\text{-°C}; \quad \overline{\tau}_{w,2} = 0.5854\ \text{N/m}^2$$

The Chilton-Colburn Analogy gives: $\frac{\overline{c}_f}{2} = \overline{St}\,\text{Pr}^{2/3}; \quad \overline{St} = \frac{\overline{h}}{\rho U_\infty c_p}$

$$\left.\begin{aligned} &\text{Run 1: } \frac{1}{2}\left[\frac{\overline{\tau}_{w,1}}{0.5\rho U_{\infty,1}^2}\right] = \frac{\overline{h}_1}{\rho U_{\infty,1} c_p}\text{Pr}^{2/3} \\ &\text{Run 2: } \frac{1}{2}\left[\frac{\overline{\tau}_{w,2}}{0.5\rho U_{\infty,2}^2}\right] = \frac{\overline{h}_2}{\rho U_{\infty,2} c_p}\text{Pr}^{2/3} \end{aligned}\right\} \Rightarrow \frac{U_{\infty,2}}{U_{\infty,1}} = \frac{\overline{h}_1}{\overline{h}_2}\frac{\overline{\tau}_{w,2}}{\overline{\tau}_{w,1}} \Rightarrow U_{\infty,2} = 2\times\frac{4.2}{29.27}\times\frac{0.5854}{8.4\times10^{-3}} = 20\ \text{m/s}$$

b)

Assume in Run 1, $\text{Re} < 5\times10^5$

using the exact solution (Blasius similarity sol.):

$$c_{f,x} = \frac{\tau_{w,x}}{\frac{1}{2}\rho U_\infty^2} = 0.664(Re_x)^{-1/2} \text{ and}$$

$$\underset{0\le x\le L}{c_{f,av}} = \frac{1}{L}\int_0^L c_{f,x}\,dx = 1.328(Re_L)^{-1/2}$$

$$\underset{0\le x\le L}{c_{f,av}} = \frac{\underset{0\le x\le L}{\tau_{w,av}}}{\frac{1}{2}\rho U_\infty^2}; \text{ and } Re_L = \frac{\rho U_\infty L}{\mu}$$

$$\Rightarrow \underset{0\le x\le L}{c_{f,av}} = \frac{\underset{0\le x\le L}{\tau_{w,av}}}{\frac{1}{2}\rho U_\infty^2} = 1.328\left(\frac{\rho U_\infty L}{\mu}\right)^{-1/2}$$

Using data from Run 1:

$$\mu = \left[\frac{\underset{0\le x\le L}{\tau_{w,av}}}{\frac{1.328}{2}\rho U_\infty^2}\right]^2 \rho U_\infty L = \left[\frac{8.4\times10^{-3}}{\frac{1.328}{2}\times1\times2^2}\right]^2 1\times2\times1$$

$$\mu = 2\times10^{-5}\ \text{kg/m-s}$$

Check the Re_L (Run 1)

$$\text{Re}_\text{L} = \frac{\rho U_\infty L}{\mu} = \frac{1\times2\times1}{2\times10^{-5}} = 10^5 < 5\times10^5$$

$$\underset{0\le x\le L}{Nu_{av,L}} \triangleq \frac{(\underset{0\le x\le L}{h_{av}})L}{k_{fluid}} = 0.664(Re_L)^{1/2}Pr^{1/3}$$

$$\text{Pr} = \frac{\mu c_p}{k};$$

$$\Rightarrow \frac{\underset{0\le x\le L}{h_{av}}\,L}{k_{fluid}} = 0.664(Re_L)^{1/2}\left(\frac{\mu c_p}{k}\right)^{1/3}$$

$$\Rightarrow k^{-2/3} = \frac{0.664(Re_L)^{1/2}}{\underset{0\le x\le L}{h_{av}}\,L}\left(\mu c_p\right)^{1/3}$$

$$k = \frac{0.664\times(10^5)^{1/2}}{4.2\times1}\left(2\times10^{-5}\times1000\right)^{1/3}$$

$$k = 0.02\ \text{W/m-°C}$$

c) Run 2

$$q_{0.1m\le x\le 1m} = q_{0\le x\le 1m} - q_{0\le x\le 0.1m}$$

in Run 2, $q_{0\le x\le 1m} = 1170.8$ W

$$\text{at } x = 0.1m,\ \mathrm{Re}_{x=0.1} = \frac{\rho U_{\infty,2} 0.1}{\mu} = \frac{1\times 20\times 0.1}{2\times 10^{-5}}$$

$$\mathrm{Re}_{x=0.1} = 10^5 < 5\times 10^5;\ \mathrm{Pr} = \frac{\mu c_p}{k} = 1$$

$$Nu_{\substack{av\\0\le x\le 0.1}} \triangleq \frac{(h_{\substack{av\\0\le x\le 0.1}})0.1}{k_{fluid}} = 0.664(Re_{0.1})^{1/2} Pr^{1/3}$$

$$h_{\substack{av\\0\le x\le 0.1}} = \frac{0.02}{0.1} 0.664\times(10^5)^{1/2}\times 1^{1/3} = 41.995 \text{ W/m}^2\text{-°C}$$

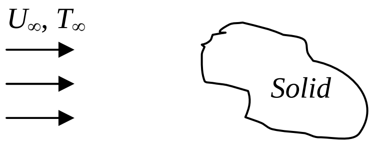
$$q_{0\le x\le 0.1m} = h_{\substack{av\\0\le x\le 0.1}}\left(A_{0\le x\le 0.1}\right)\left(T_w - T_\infty\right)$$

$$q_{0\le x\le 0.1m} = 41.995\times(0.1\times 1)(40) = 167.98 \text{ W}$$

$$q_{0.1m\le x\le 1m} = 1170.8 - 167.98$$

$$\Rightarrow q_{0.1m\le x\le 1m} = 1002.82 \text{ W}$$

Problem 3:



Given: U_∞ = (10+0.05 t) m/s; Vol.$_{solid}$ = 8 ×10^{-6} m^3; A$_{solid}$ = 2 ×10^{-3} m^2; T_i = 420°C; T_∞ = 20°C.
Fluid properties: ρ = 1 kg/m^3; c_p = 1000 J/kg-°C; μ = 2× 10^{-5} kg/m-s; k = 0.02 W/m-°C
Solid properties: ρ = 2000 kg/m^3; c_p = 500 J/kg-°C; k = 400 W/m-°C

Measurements:

(i) inside the solid: T = T(t);
(ii) at t = 0 s, T = T_i = 420°C and dT/dt = -5.4°C/s;
(iii) at t = 60 s, T = 188.9 °C and dT/dt = -2.6 °C/s.

Assumptions: Transient fluid flow and heat transfer; LPA is valid; $Nu = C\,\mathrm{Re}^m\,\mathrm{Pr}^n$; All given properties remain constant

a)

Governing diff. eq.:

$$\rho_{solid} c_{p,solid} V \frac{dT}{dt} = -hA\left(T - T_\infty\right);$$

$$t = 0s,\quad T_{t=0} = T_i = 420°\text{C}$$

$$dT/dt = -5.4°\text{C/s}$$

$$t = 60s,\quad T_{t=60s} = 188.9°\text{C}$$

$$dT/dt = -2.6°\text{C/s}$$

$$thus,\ h_{t=0} = \frac{-\rho_{solid} c_{p,solid} V}{A\left(T_{t=0} - T_\infty\right)}\left(\frac{dT}{dt}\right)_{t=0}$$

$$h_{t=0} = \frac{-2000\times 500\times 8\times 10^{-6}}{2\times 10^{-3}\left(420-20\right)}\times -5.4$$

$$h_{t=0} = 54 \text{ W/m}^2\text{-°C}$$

$$h_{t=60} = \frac{-\rho_{solid} c_{p,solid} V}{A\left(T_{t=60} - T_\infty\right)}\left(\frac{dT}{dt}\right)_{t=60}$$

$$h_{t=60} = \frac{-2000\times 500\times 8\times 10^{-6}}{2\times 10^{-3}\left(188.9-20\right)}\times -2.6$$

$$h_{t=60} = 61.575 \text{ W/m}^2\text{-°C}$$

$$Nu = C\,\mathrm{Re}^m\,\mathrm{Pr}^n$$

$$\frac{hL_c}{k_{fluid}} = C\left(\frac{\rho_{fluid} U_\infty L_c}{\mu_{fluid}}\right)^m \left(\frac{\mu_{fluid} c_{p,fluid}}{k_{fluid}}\right)^n$$

$\Rightarrow h = \psi\left(U_\infty\right)^m$ where ψ is a constant

as was given $U_\infty = (10 + 0.05\,t)$; thus,

$$t = 0s,\quad U_{\infty,0} = 10 \text{ m/s};\ h_{t=0} = 54 \text{ W/m}^2\text{-°C}$$

$$t = 60s,\quad U_{\infty,60} = 13 \text{ m/s};\ h_{t=60} = 61.575 \text{ W/m}^2\text{-°C}$$

$$thus,\ \left.\begin{array}{r} 54 = \psi\left(10\right)^m \\ 61.575 = \psi\left(13\right)^m \end{array}\right\} \Rightarrow \left.\begin{array}{r} m = 0.5 \\ \psi = 17.08 \end{array}\right\} \Rightarrow \boxed{h = 17.08\left(U_\infty\right)^{0.5}}$$

b)

Governing diff. eq.:

$$\rho_{solid} c_{p_{solid}} V \frac{dT}{dt} = -hA(T - T_\infty);$$

$h = 17.08(U_\infty)^{0.5}$; and $U_\infty = (10 + 0.05\ t)$ thus,

$h = 17.08(10 + 0.05\ t)^{0.5}$ or

$$\frac{dT}{(T - T_\infty)} = -\frac{17.08 \times A}{\rho_{solid} c_{p_{solid}} V}(10 + 0.05\ t)^{0.5}\ dt$$

$$\int_{T_i}^{T} \frac{dT}{(T - T_\infty)} = -\frac{17.08 \times 2 \times 10^{-3}}{2000 \times 500 \times 8 \times 10^{-6}} \int_0^t (10 + 0.05\ t)^{0.5}\ dt$$

$$\ln\left(\frac{(T - T_\infty)}{(T_i - T_\infty)}\right) = -4.27 \times 10^{-3} \frac{1}{0.05}\left[\frac{2}{3}(10 + 0.05\ t)^{3/2}\right]_0^t$$

$$\ln\left(\frac{T_{t=100s} - 20}{420 - 20}\right) = -56.933 \times 10^{-3} \times \left[(10 + 0.05\ t)^{3/2}\right]_0^{100s}$$

$$\ln\left(\frac{T_{t=100s} - 20}{400}\right) = -56.933 \times 10^{-3} \times \left[(15)^{3/2} - (10)^{3/2}\right] = -1.507$$

$$T_{t=100s} = 20 + 400\exp(-1.507) = 20 + 88.63$$

$$\boxed{T_{t=100s} = 108.63°\text{C}}$$