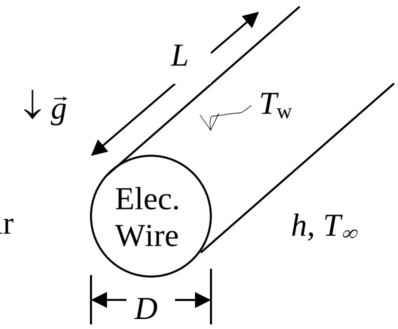
Solutions - Problem Set # 9

Problem 1:



Given: T_∞= 27°C; Cable heat dissipation rate:
q_{loss}/L =30 W/m; D = 25 mm
Assumptions: Steady-state free convection heat transfer; Radiation negligible

NOTE:
At this stage we do not have information about the wall temperature to calculate the film temperature. Thus, we guess some value for T_w for example be 50°C warmer than T_∞ (i.e., guess value T_w =77°C) to evaluate the thermophysical properties and the Ra_D number. We will check this later and improve the results if necessary once we get the actual wall temperature by recalculating the properties at new film temperature.

E-balance on the electrical cable:

$$q_{loss} = hA(T_w - T_\infty)$$

$$Gr_D = \frac{g\beta[T_w - T_\infty]D^3}{\nu^2}; \quad Ra_D = \Pr Gr_D$$

$$T_{film} = (T_w + T_\infty)/2$$

Thus, at $T_{film} = (T_w + T_\infty)/2$ =(27+77)/2 = 52°C=325.15K:

T_{film}= 325.15 K: ρ_f = 1.087 kg/m^3; $c_{p,f}$= 1007 J/kg-°C; μ_f = 19.6×10^{-6} kg/m-s; k_f = 0.028 W/m-°C

$$\beta = 1/T = 1/325.15 = 3.0755\times10^{-3}\ \text{K}^{-1}$$

$$Gr_D = \frac{g\beta[T_w - T_\infty]D^3}{\nu^2} = \frac{9.81\times3.0755\times10^{-3}\times50\times(25\times10^{-3})^3}{(19.6\times10^{-6}/1.087)^2} = 72497.3$$

$$\Pr_f = \frac{\mu_f c_{p,f}}{k_f} = \frac{19.6\times10^{-6}\times1007}{0.028} = 0.7049$$

$$Ra_D = \Pr Gr_D = 72497.3\times0.7049 = 51103.347$$

You can use either the Morgan correlation or the Churchill-Chu correlation [Property data at $T_{film} = (T_w + T_\infty)/2$]

I have used the Churchill-Chu correlation

$$Nu_{av} \triangleq \left(\frac{h_{av}D}{k_{fluid}}\right) = \left\{0.60 + \frac{0.387 Ra_D^{1/6}}{\left[1 + (0.559/\Pr)^{9/16}\right]^{8/27}}\right\}^2 \quad 10^{-5} < Ra_D < 10^{12}$$

$$Nu_{av} \triangleq \left(\frac{h_{av} D}{k_{fluid}} \right) = \left\{ 0.60 + \frac{0.387 \times (51103.347)^{1/6}}{\left[1 + (0.559/0.7049)^{9/16} \right]^{8/27}} \right\}^2 = 6.533$$

$$\frac{h_{av} D}{k_{fluid}} = 7.21695 \Rightarrow h_{av} = \frac{0.028}{25 \times 10^{-3}} \times 6.533 = 7.317 \text{ W/m}^2 - {}^\circ\text{C}$$

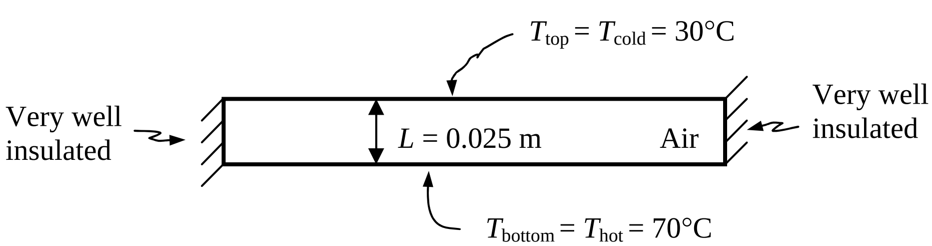
As a result:

$$q_{loss} = hA(T_w - T_\infty) \Rightarrow T_w = T_\infty + \frac{q_{loss}}{hA} = T_\infty + \frac{q_{loss}/L}{h\pi D}$$

$$T_w = 27 + \frac{30}{7.317 \times \pi \times 25 \times 10^{-3}} = 27 + 52.2 = 79.2^\circ\text{C}$$

Thus, in this case, my guess was very close; so, it is not needed to recalculate everything, and we can accept this result. However, if the temperature obtained for the wall was much different than that initially guessed, a new film temperature and Ra_D number must be calculated. Then a new *Nu* and *h* value, and from the energy balance equation a new wall temperature should be get. This process may be continued until getting a reasonably acceptable results. Generally speaking, one or two iterations provide enough precision for engineering practices.

Problem 2:



Assumptions: Steady-state free convection heat transfer; Air behaves as perfect gas; μ, k, c_p, not changing appreciably with pressure (for the range of pressure studied here); neglect radiation

Air properties at 1 atm, and $T_{mean} = 50^\circ$C:

$\rho_f = 1.087$ kg/m^3; $c_{p,f} = 1007$ J/kg-°C;
$\mu_f = 19.6 \times 10^{-6}$ kg/m-s; $k_f = 0.028$ W/m-°C

$\beta = 1/T = 1/(273.15+50) = 3.095 \times 10^{-3}$ K^{-1}

a)

$$Gr_L = \frac{g\beta[T_{hot} - T_{cold}]L^3}{\nu^2} = \frac{9.81 \times 3.095 \times 10^{-3} \times 40 \times (25 \times 10^{-3})^3}{(19.6 \times 10^{-6}/1.087)^2} = 58365.55$$

$$\Pr_f = \frac{\mu_f c_{p,f}}{k_f} = \frac{19.6 \times 10^{-6} \times 1007}{0.028} = 0.7049$$

$$Ra_L = \Pr Gr_L = 58365.55 \times 0.7049 = 41141.88 > 1708$$

Using the correlation provided in handout #9: $Nu_{av} \triangleq \left(\frac{h_{av} L}{k_{fluid}} \right) = 0.212 (Ra_L)^{1/4} = 3.019$

$$h_{av} = \frac{0.028}{25\times10^{-3}}\times 3.019 = 3.38 \ \ \text{W/m}^2\text{-°C}$$

$$q_{av} = h_{av}\left[T_h - T_c\right] = 3.38\times\left[70-30\right] = 135.2 \ \ \text{W/m}^2$$

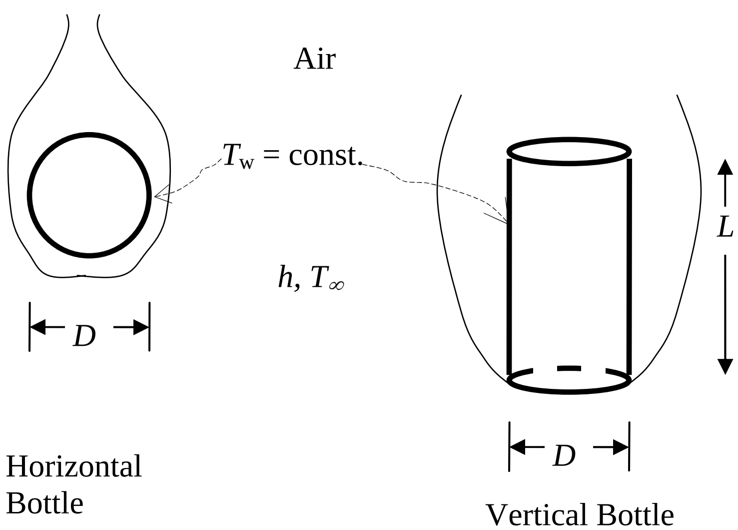
b) In order to eliminate the natural convection (have only heat conduction) in the air gap, $Ra_L = \Pr Gr_L \le 1708$, The only property of the air that change with pressure is the mass density, thus,

$$Ra_L = \Pr Gr_L = \Pr \frac{g\beta\left[T_{hot} - T_{cold}\right]L^3}{\left(\mu/\rho\right)^2} = 0.7049\times\frac{9.81\times3.095\times10^{-3}\times40\times\left(25\times10^{-3}\right)^3}{\left(19.6\times10^{-6}\right)^2}\rho^2$$

Thus, $Ra_L = 34819.6\times\rho^2 = 1708;\ \rho = \left(\frac{1708}{34819.6}\right)^{0.5} = 0.221\ \text{kg/m}^3$. And from perfect gas relation at constant temperature for any two states, 1, and 2: $P_2/P_1 = \rho_2/\rho_1$

$$P_2 = P_1\times\rho_2/\rho_1 = 1_{atm}\times\frac{0.221}{1.087} = 0.203\ \text{atm}$$

Problem 3:



The general form of Nusselt number for Natural convection is: $Nu_{av} \triangleq \left(\frac{h_{av}L_c}{k_{fluid}}\right) = C\left(Ra_{L_c}\right)^n$

Where C is a constant and n is generally a number less than unity. And, L_c is the characteristic length, depending on geometry and orientation of the object with respect to the gravitational field.

Neglecting the heat transfer from the circular ends and estimating the h values for both positions considering isothermal vertical walls, we can roughly evaluate the heat transfer from the bottle in both horizontal and vertical positions. For the horizontal bottle, using the symmetry we approximate the h value with vertical wall of height $\pi D/2$ ($L_c = \pi D/2$), and for the vertical position we use $L_c = L$. Thus,

$$Nu_{av} \triangleq \left(\frac{h_{av}L_c}{k_{fluid}}\right) = C\left(Ra_{L_c}\right)^n ; Gr_{L_c} = \frac{g\beta\left[T_w - T_\infty\right]L_c^{\ 3}}{\nu^2}; \quad Ra_{L_c} = \Pr Gr_{L_c}$$

$$Nu_{av} \triangleq \left(\frac{h_{av}L_c}{k_{fluid}}\right) = C\left(\frac{g\beta\left[T_w - T_\infty\right]L_c^{\ 3}}{\nu^2}\frac{\nu}{\alpha}\right)^n$$

$$Nu_{av} \triangleq \left(\frac{h_{av}L_c}{k_{fluid}}\right) = C\left(\frac{g\beta\left[T_w - T_\infty\right]}{\nu\alpha}\right)^n L_c^{\ 3n} \Rightarrow h_{av} = \underbrace{Ck_{fluid}\left(\frac{g\beta\left[T_w - T_\infty\right]}{\nu\alpha}\right)^n}_{\text{constant here } (\xi)} L_c^{\ 3n-1}$$

$$h_{av} = \xi L_c^{\ 3n-1}$$

Horizontal:

$$h_{av} = \xi L_c^{\ 3n-1}$$

$$h_{av,\,horizontal} = \xi\left(\pi D/2\right)^{3n-1} ; h_{av,\,vertical} = \xi\left(L\right)^{3n-1}$$

$$\frac{h_{av,\,horizontal}}{h_{av,\,vertical}} = \frac{\xi\left(\pi D/2\right)^{3n-1}}{\xi\left(L\right)^{3n-1}} = \left(\frac{\pi D/2}{L}\right)^{3n-1}$$

$$Also, L = 4D \Rightarrow \frac{h_{av,\,horizontal}}{h_{av,\,vertical}} = \left(\frac{\pi}{8}\right)^{3n-1}$$

$n = 1/4$

$$\frac{h_{av,\,horizontal}}{h_{av,\,vertical}} \simeq \left(\frac{\pi}{8}\right)^{3/4-1} = 1.263$$

NOTE:

For air as fluid and assuming bottle wall temperature of T_w =22°C and air temperature of T_∞ = 4°C, we can estimate the Ra number.

$$Ra_L = \left(\frac{g\beta\left[T_w - T_\infty\right]}{\nu\alpha}L^3\right)$$

. With the bottle height of L = 20 cm, and

T_f =13°C, $\rightarrow \nu \simeq 15.6\times10^{-6}$ m²/s; $\alpha \simeq 22.16\times10^{-6}$ m²/s

$$Ra_L = \left(\frac{g\beta\left[T_w - T_\infty\right]}{\nu\alpha}L^3\right)$$

$$Ra_L = \frac{9.81\frac{1}{(286)}\left[22-4\right]}{15.6\times10^{-6}\times22.16\times10^{-6}}\left(0.2\right)^3 \simeq 1.488\times10^7$$

For vertical wall as given in the handout #9:

$$Nu_{av} \triangleq \left(\frac{h_{av}L}{k_{fluid}}\right) = 0.59\left(Ra_L\right)^{1/4} \qquad 10^4 \le Ra_L \le 10^9$$

Thus, horizontal position enhances 26.3% the heat transfer. Please note that for short cylinders, e.g., beer can (L/D = 2) the enhancement may be insignificant.