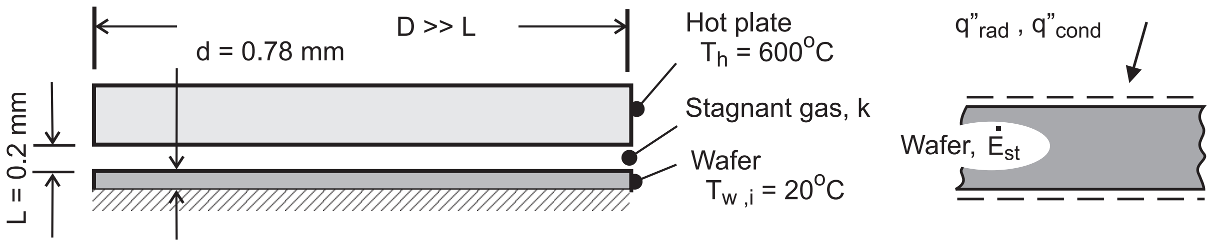
PROBLEM 1.41

KNOWN: Hot plate-type wafer thermal processing tool based upon heat transfer modes by conduction through gas within the gap and by radiation exchange across gap.

FIND: (a) Radiative and conduction heat fluxes across gap for specified hot plate and wafer temperatures and gap separation; initial time rate of change in wafer temperature for each mode, and (b) heat fluxes and initial temperature-time change for gap separations of 0.2, 0.5 and 1.0 mm for hot plate temperatures 300 < T_h < 1300°C. Comment on the relative importance of the modes and the influence of the gap distance. Under what conditions could a wafer be heated to 900°C in less than 10 seconds?

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions for flux calculations, (2) Diameter of hot plate and wafer much larger than gap spacing, approximating plane, infinite planes, (3) One-dimensional conduction through gas, (4) Hot plate and wafer are blackbodies, (5) Negligible heat losses from wafer backside, and (6) Wafer temperature is uniform at the onset of heating.

PROPERTIES: Wafer: ρ = 2700 kg/m^3, c = 875 J/kg·K; Gas in gap: k = 0.0436 W/m·K.

ANALYSIS: (a) The radiative heat flux between the hot plate and wafer for T_h = 600°C and T_w = 20° C follows from the rate equation,

$$q''_{rad} = \sigma\left(T_h^4 - T_w^4\right) = 5.67\times10^{-8}\,W/m^2\cdot K^4\left((600+273)^4 - (20+273)^4\right)K^4 = 32.5\,kW/m^2 \quad <$$

The conduction heat flux through the gas in the gap with L = 0.2 mm follows from Fourier's law,

$$q''_{cond} = k\frac{T_h - T_w}{L} = 0.0436\,W/m\cdot K\frac{(600-20)K}{0.0002\ m} = 126\ kW/m^2 \quad <$$

The initial time rate of change of the wafer can be determined from an energy balance on the wafer at the instant of time the heating process begins,

$$\dot{E}''_{in} - \dot{E}''_{out} = \dot{E}''_{st} \qquad\qquad \dot{E}''_{st} = \rho c\, d\left(\frac{dT_w}{dt}\right)_i$$

where $\dot{E}''_{out} = 0$ and $\dot{E}''_{in} = q''_{rad}$ or q''_{cond}. Substituting numerical values, find

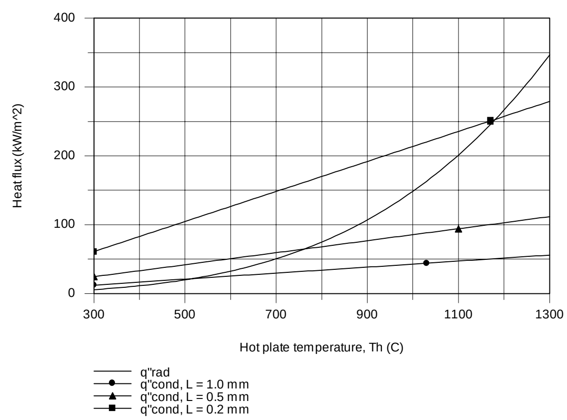
$$\left.\frac{dT_w}{dt}\right)_{i,rad} = \frac{q''_{rad}}{\rho c d} = \frac{32.5\times10^3\,W/m^2}{2700\,kg/m^3\times875\ J/kg\cdot K\times0.00078\ m} = 17.6\ K/s \quad <$$

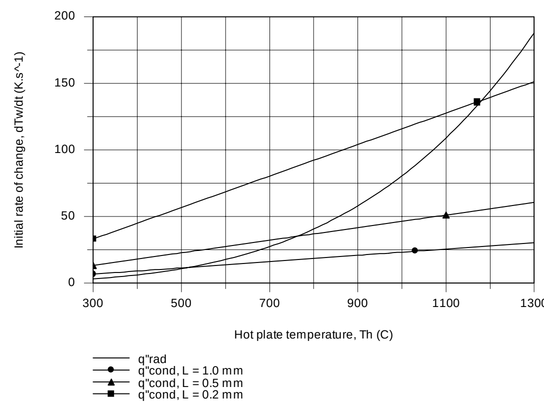
$$\left.\frac{dT_w}{dt}\right)_{i,cond} = \frac{q''_{cond}}{\rho c d} = 68.4\ K/s \quad <$$

Continued …..

PROBLEM 1.41 (Cont.)

(b) Using the foregoing equations, the heat fluxes and initial rate of temperature change for each mode can be calculated for selected gap separations L and range of hot plate temperatures T_h with T_w = 20°C.





In the left-hand graph, the conduction heat flux increases linearly with T_h and inversely with L as expected. The radiative heat flux is independent of L and highly non-linear with T_h, but does not approach that for the highest conduction heat rate until T_h approaches 1200°C.

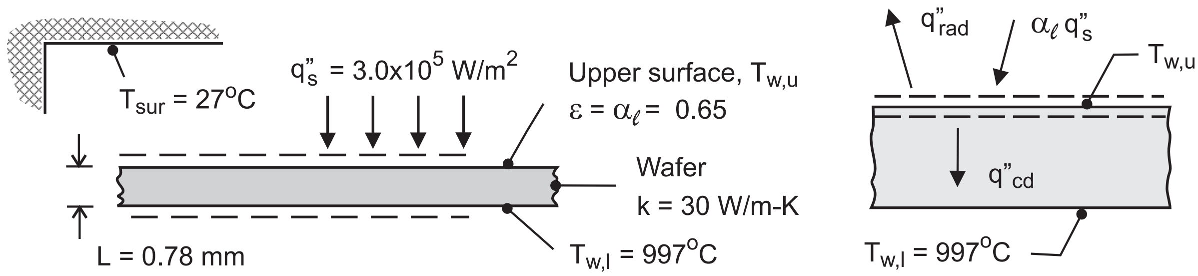
The general trends for the initial temperature-time change, $(dT_w/dt)_i$, follow those for the heat fluxes. To reach 900°C in 10 s requires an average temperature-time change rate of 90 K/s. Recognizing that (dT_w/dt) will decrease with increasing T_w, this rate could be met only with a very high T_h and the smallest L.

PROBLEM 1.42

KNOWN: Silicon wafer, radiantly heated by lamps, experiencing an annealing process with known backside temperature.

FIND: Whether temperature difference across the wafer thickness is less than 2°C in order to avoid damaging the wafer.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in wafer, (3) Radiation exchange between upper surface of wafer and surroundings is between a small object and a large enclosure, and (4) Vacuum condition in chamber, no convection.

PROPERTIES: Wafer: k = 30 W/m·K, $\varepsilon = \alpha_\ell = 0.65$.

ANALYSIS: Perform a surface energy balance on the upper surface of the wafer to determine $T_{w,u}$. The processes include the absorbed radiant flux from the lamps, radiation exchange with the chamber walls, and conduction through the wafer.

$$\dot{E}''_{in} - \dot{E}''_{out} = 0$$

$$\alpha_\ell q''_s - q''_{rad} - q''_{cd} = 0$$

$$\alpha_\ell q''_s - \varepsilon\sigma\left(T_{w,u}^4 - T_{sur}^4\right) - k\frac{T_{w,u} - T_{w,\ell}}{L} = 0$$

$$0.65 \times 3.0 \times 10^5 \mathrm{W/m^2} - 0.65 \times 5.67 \times 10^{-8} \mathrm{W/m^2 \cdot K^4}\left(T_{w,u}^4 - (27+273)^4\right)\mathrm{K^4}$$

$$-30 \mathrm{W/m \cdot K}\left[T_{w,u} - (997+273)\right]\mathrm{K}/0.00078\ \mathrm{m} = 0$$

$$T_{w,u} = 1273\,\mathrm{K} = 1000°\mathrm{C}$$ <

COMMENTS: (1) The temperature difference for this steady-state operating condition, $T_{w,u} - T_{w,l}$, is larger than 2°C. Warping of the wafer and inducing slip planes in the crystal structure could occur.

(2) The radiation exchange rate equation requires that temperature must be expressed in kelvin units. Why is it permissible to use kelvin or Celsius temperature units in the conduction rate equation?

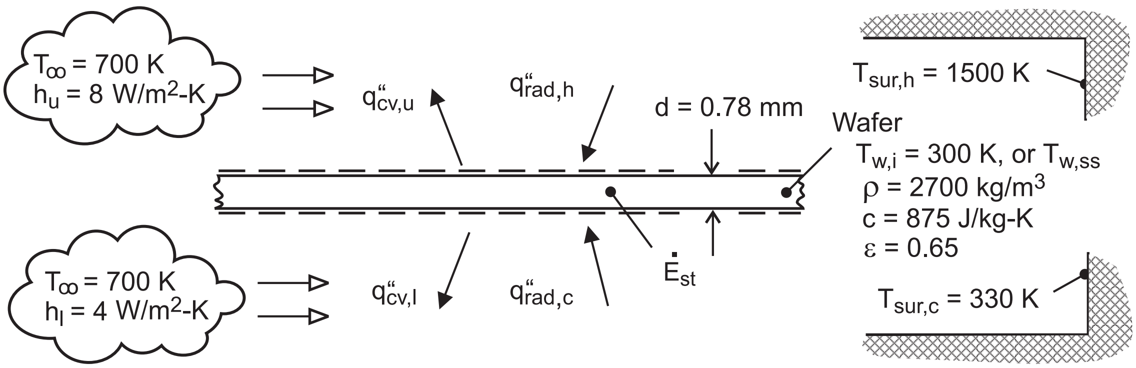
(3) Note how the surface energy balance, Eq. 1.12, is represented schematically. It is essential to show the control surfaces, and then identify the rate processes associated with the surfaces. Make sure the directions (in or out) of the process are consistent with the energy balance equation.

PROBLEM 1.43

KNOWN: Silicon wafer positioned in furnace with top and bottom surfaces exposed to hot and cool zones, respectively.

FIND: (a) Initial rate of change of the wafer temperature corresponding to the wafer temperature $T_{w,i} = 300\,K$, and (b) Steady-state temperature reached if the wafer remains in this position. How significant is convection for this situation? Sketch how you'd expect the wafer temperature to vary as a function of vertical distance.

SCHEMATIC:



ASSUMPTIONS: (1) Wafer temperature is uniform, (2) Transient conditions when wafer is initially positioned, (3) Hot and cool zones have uniform temperatures, (3) Radiation exchange is between small surface (wafer) and large enclosure (chamber, hot or cold zone), and (4) Negligible heat losses from wafer to mounting pin holder.

ANALYSIS: The energy balance on the wafer illustrated in the schematic above includes convection from the upper (u) and lower (l) surfaces with the ambient gas, radiation exchange with the hot- and cool-zone (chamber) surroundings, and the rate of energy storage term for the transient condition.

$$\dot{E}''_{in} - \dot{E}''_{out} = \dot{E}''_{st}$$

$$q''_{rad,h} + q''_{rad,c} - q''_{cv,u} - q''_{cv,l} = \rho c d \frac{dT_w}{dt}$$

$$\varepsilon\sigma\left(T^4_{sur,h} - T^4_w\right) + \varepsilon\sigma\left(T^4_{sur,c} - T^4_w\right) - h_u\left(T_w - T_\infty\right) - h_l\left(T_w - T_\infty\right) = \rho c d \frac{dT_w}{dt}$$

(a) For the initial condition, the time rate of temperature change of the wafer is determined using the energy balance above with $T_w = T_{w,i} = 300\,K$,

$$0.65 \times 5.67 \times 10^{-8}\,W/m^2 \cdot K^4 \left(1500^4 - 300^4\right)K^4 + 0.65 \times 5.67 \times 10^{-8}\,W/m^2 \cdot K^4 \left(330^4 - 300^4\right)K^4$$

$$-8\,W/m^2 \cdot K\left(300 - 700\right)K - 4\,W/m^2 \cdot K\left(300 - 700\right)K =$$

$$2700\,kg/m^3 \times 875\,J/kg \cdot K \times 0.00078\,m\left(dT_w/dt\right)_i$$

$$\left(dT_w/dt\right)_i = 104\,K/s$$ <

(b) For the steady-state condition, the energy storage term is zero, and the energy balance can be solved for the steady-state wafer temperature, $T_w = T_{w,ss}$.

Continued …..

PROBLEM 1.43 (Cont.)

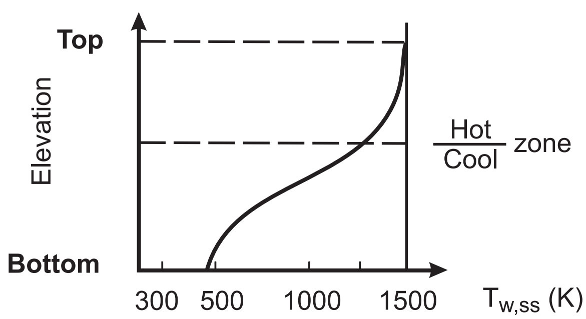
$$0.65\sigma\left(1500^4 - T_{w,ss}^4\right)K^4 + 0.65\sigma\left(330^4 - T_{w,ss}^4\right)K^4$$

$$-8\,W/m^2\cdot K\left(T_{w,ss} - 700\right)K - 4\,W/m^2\cdot K\left(T_{w,ss} - 700\right)K = 0$$

$$T_{w,ss} = 1251\ K$$ <

To determine the relative importance of the convection processes, re-solve the energy balance above ignoring those processes to find $\left(dT_w / dt\right)_i = 101\ K/s$ and $T_{w,ss} = 1262\ K$. We conclude that the radiation exchange processes control the initial time rate of temperature change and the steady-state temperature.

If the wafer were elevated above the present operating position, its temperature would increase, since the lower surface would begin to experience radiant exchange with progressively more of the hot zone chamber. Conversely, by lowering the wafer, the upper surface would experience less radiant exchange with the hot zone chamber, and its temperature would decrease. The temperature-distance trend might appear as shown in the sketch.

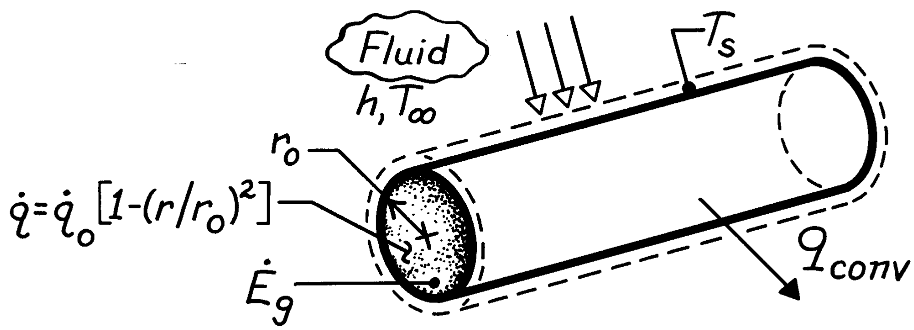


PROBLEM 1.44

KNOWN: Radial distribution of heat dissipation in a cylindrical container of radioactive wastes. Surface convection conditions.

FIND: Total energy generation rate and surface temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible temperature drop across thin container wall.

ANALYSIS: The rate of energy generation is

$$\dot{E}_g = \int \dot{q} dV = \dot{q}_o \int_0^{r_o} \left[1-(r/r_o)^2\right] 2\pi r L dr$$

$$\dot{E}_g = 2\pi L \dot{q}_o \left(r_o^2/2 - r_o^2/4\right)$$

or per unit length,

$$\dot{E}_g' = \frac{\pi \dot{q}_o r_o^2}{2}. \qquad <$$

Performing an energy balance for a control surface about the container yields, at an instant,

$$\dot{E}_g' - \dot{E}_{out}' = 0$$

and substituting for the convection heat rate per unit length,

$$\frac{\pi \dot{q}_o r_o^2}{2} = h(2\pi r_o)(T_s - T_\infty)$$

$$T_s = T_\infty + \frac{\dot{q}_o r_o}{4h}. \qquad <$$

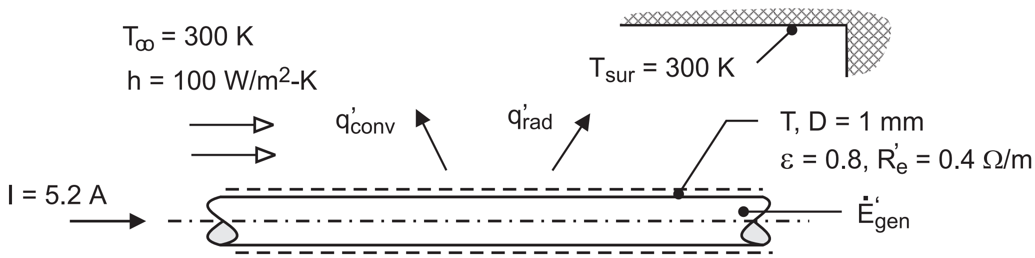
COMMENTS: The temperature within the radioactive wastes increases with decreasing r from T_s at r_o to a maximum value at the centerline.

PROBLEM 1.45

KNOWN: Rod of prescribed diameter experiencing electrical dissipation from passage of electrical current and convection under different air velocity conditions. See Example 1.3.

FIND: Rod temperature as a function of the electrical current for $0 \le I \le 10$ A with convection coefficients of 50, 100 and 250 $W/m^2 \cdot K$. Will variations in the surface emissivity have a significant effect on the rod temperature?

SCHEMATIC:



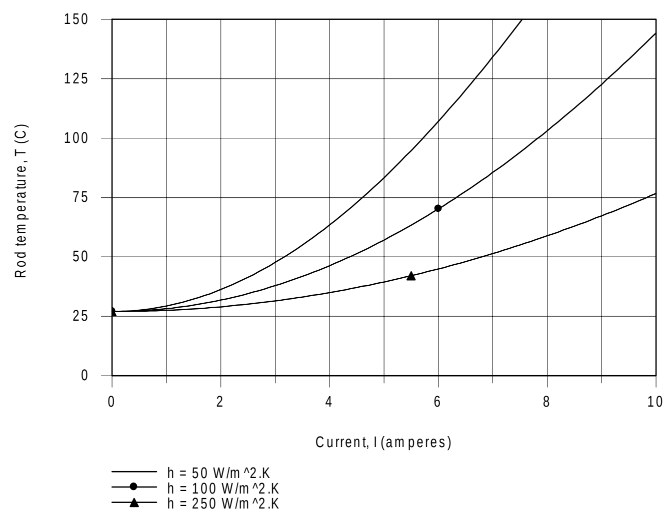
ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform rod temperature, (3) Radiation exchange between the outer surface of the rod and the surroundings is between a small surface and large enclosure.

ANALYSIS: The energy balance on the rod for steady-state conditions has the form,

$$q'_{conv} + q'_{rad} = \dot{E}'_{gen}$$

$$\pi D h \left(T - T_\infty\right) + \pi D \varepsilon \sigma \left(T^4 - T_{sur}^4\right) = I^2 R'_e$$

Using this equation in the Workspace of IHT, the rod temperature is calculated and plotted as a function of current for selected convection coefficients.



COMMENTS: (1) For forced convection over the cylinder, the convection heat transfer coefficient is dependent upon air velocity approximately as $h \sim V^{0.6}$. Hence, to achieve a 5-fold change in the convection coefficient (from 50 to 250 $W/m^2 \cdot K$), the air velocity must be changed by a factor of nearly 15.

Continued …..

PROBLEM 1.45 (Cont.)

(2) For the condition of I = 4 A with h = 50 W/m^2·K with T = 63.5°C, the convection and radiation exchange rates per unit length are, respectively, $q'_{cv} = 5.7\ W/m$ and $q'_{rad} = 0.67\ W/m$. We conclude that convection is the dominate heat transfer mode and that changes in surface emissivity could have only a minor effect. Will this also be the case if h = 100 or 250 W/m^2·K?

(3) What would happen to the rod temperature if there was a "loss of coolant" condition where the air flow would cease?

(4) The Workspace for the IHT program to calculate the heat losses and perform the parametric analysis to generate the graph is shown below. It is good practice to provide commentary with the code making your solution logic clear, and to summarize the results. It is also good practice to show plots in *customary* units, that is, the units used to prescribe the problem. As such the graph of the rod temperature is shown above with Celsius units, even though the calculations require temperatures in kelvins.

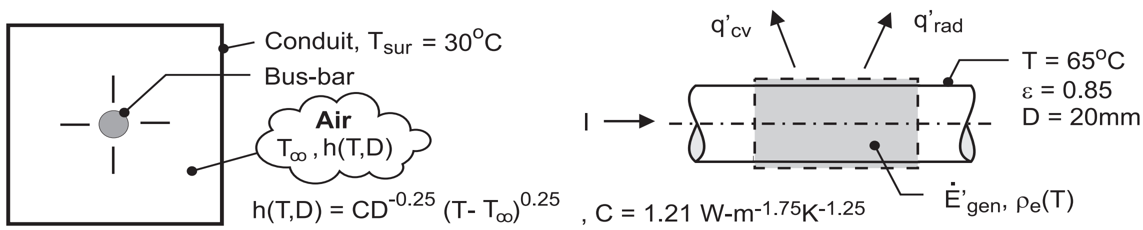
```
// Energy balance; from Ex. 1.3, Comment 1
-q'cv - q'rad + Edot'g = 0
q'cv = pi*D*h*(T - Tinf)
q'rad = pi*D*eps*sigma*(T^4 - Tsur^4)
sigma = 5.67e-8

// The generation term has the form
Edot'g = I^2*R'e
qdot = I^2*R'e / (pi*D^2/4)

// Input parameters
D = 0.001
Tsur = 300
T_C = T – 273          // Representing temperature in Celsius units using _C subscript
eps = 0.8
Tinf = 300
h = 100
//h = 50               // Values of coefficient for parameter study
//h = 250
I = 5.2                // For graph, sweep over range from 0 to 10 A
//I = 4                // For evaluation of heat rates with h = 50 W/m^2.K
R'e = 0.4

/*  Base case results: I = 5.2 A with h = 100 W/m^2.K, find T = 60 C (Comment 2 case).
Edot'g      T       T_C     q'cv    q'rad   qdot            D       I       R'e
            Tinf    Tsur    eps     h       sigma
10.82       332.6   59.55   10.23   0.5886  1.377E7         0.001   5.2     0.4
            300     300     0.8     100     5.67E-8   */

/* Results: I = 4 A with h = 50 W/m^2.K, find q'cv =  5.7 W/m and q'rad = 0.67 W/m
Edot'g      T       T_C     q'cv    q'rad   qdot    D       I       R'e
      Tinf  Tsur    eps     h       sigma
6.4         336.5   63.47   5.728   0.6721  8.149E6 0.001   4       0.4
      300   300     0.8     50      5.67E-8    */
```

PROBLEM 1.46

KNOWN: Long bus bar of prescribed diameter and ambient air and surroundings temperatures. Relations for the electrical resistivity and free convection coefficient as a function of temperature.

FIND: (a) Current carrying capacity of the bus bar if its surface temperature is not to exceed 65°C; compare relative importance of convection and radiation exchange heat rates, and (b) Show graphically the operating temperature of the bus bar as a function of current for the range $100 \le I \le 5000$ A for bus-bar diameters of 10, 20 and 40 mm. Plot the ratio of the heat transfer by convection to the total heat transfer for these conditions.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Bus bar and conduit are very long in direction normal to page, (3) Uniform bus-bar temperature, (4) Radiation exchange between the outer surface of the bus bar and the conduit is between a small surface and a large enclosure.

PROPERTIES: Bus-bar material, $\rho_e = \rho_{e,o}\left[1+\alpha\left(T-T_o\right)\right]$, $\rho_{e,o} = 0.0171\mu\Omega\cdot m$, $T_o = 25°C$, $\alpha = 0.00396\,K^{-1}$.

ANALYSIS: An energy balance on the bus-bar for a unit length as shown in the schematic above has the form

$$\dot{E}'_{in} - \dot{E}'_{out} + \dot{E}'_{gen} = 0$$

$$-q'_{rad} - q'_{conv} + I^2R'_e = 0$$

$$-\varepsilon\pi D\sigma\left(T^4 - T_{sur}^4\right) - h\pi D\left(T - T_\infty\right) + I^2\rho_e / A_c = 0$$

where $R'_e = \rho_e / A_c$ and $A_c = \pi D^2/4$. Using the relations for $\rho_e(T)$ and $h(T,D)$, and substituting numerical values with T = 65°C, find

$$q'_{rad} = 0.85\pi\left(0.020m\right)\times5.67\times10^{-8}\,W/m^2\cdot K^4\left(\left[65+273\right]^4 - \left[30+273\right]^4\right)K^4 = 223\,W/m$$ <

$$q'_{conv} = 7.83\,W/m^2\cdot K\;\pi\left(0.020m\right)\left(65-30\right)K = 17.2\,W/m$$ <

where $h = 1.21\,W\cdot m^{-1.75}\cdot K^{-1.25}\left(0.020m\right)^{-0.25}\left(65-30\right)^{0.25} = 7.83\,W/m^2\cdot K$

$$I^2R'_e = I^2\left(198.2\times10^{-6}\Omega\cdot m\right)/\pi\left(0.020\right)^2 m^2/4 = 6.31\times10^{-5}I^2\,W/m$$

where $\rho_e = 0.0171\times10^{-6}\Omega\cdot m\left[1+0.00396\,K^{-1}\left(65-25\right)K\right] = 198.2\,\mu\Omega\cdot m$

The maximum allowable current capacity and the ratio of the convection to total heat transfer rate are

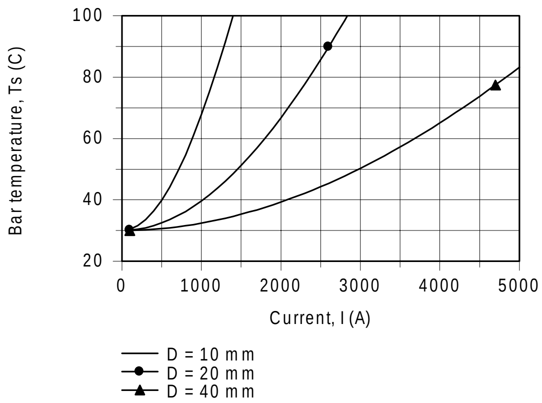
$$I = 1950\,A \qquad q'_{cv}/\left(q'_{cv}+q'_{rad}\right) = q'_{cv}/q'_{tot} = 0.072$$ <

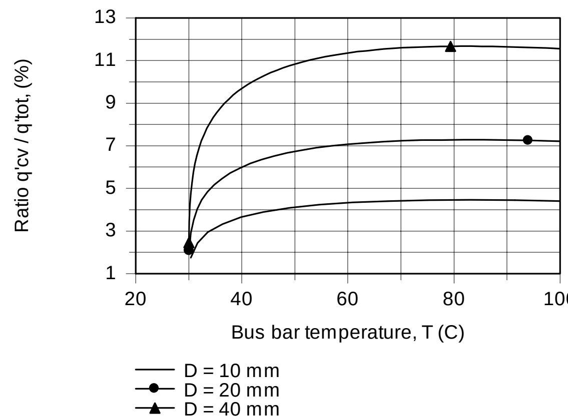
For this operating condition, convection heat transfer is only 7.2% of the total heat transfer.

(b) Using these equations in the Workspace of IHT, the bus-bar operating temperature is calculated and plotted as a function of the current for the range $100 \le I \le 5000$ A for diameters of 10, 20 and 40 mm. Also shown below is the corresponding graph of the ratio (expressed in percentage units) of the heat transfer by convection to the total heat transfer, q'_{cv}/q'_{tot}.

Continued …..

PROBLEM 1.46 (Cont.)





COMMENTS: (1) The trade-off between current-carrying capacity, operating temperature and bar diameter is shown in the first graph. If the surface temperature is not to exceed 65°C, the maximum current capacities for the 10, 20 and 40-mm diameter bus bars are 960, 1950, and 4000 A, respectively.

(2) From the second graph with q'_{cv} / q'_{tot} vs. T, note that the convection heat transfer rate is always a small fraction of the total heat transfer. That is, radiation is the dominant mode of heat transfer. Note also that the convection contribution increases with increasing diameter.

(3) The Workspace for the IHT program to perform the parametric analysis and generate the graphs is shown below. It is good practice to provide commentary with the code making your solution logic clear, and to summarize the results.

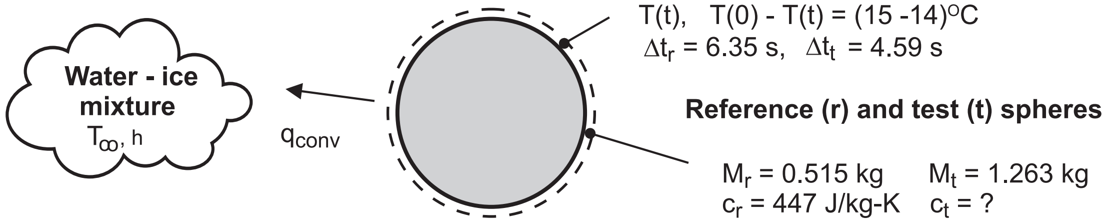
```
/* Results: base-case conditions, Part (a)
I        R'e       cvovertot hbar    q'cv    q'rad   rhoe      D      Tinf_C   Ts_C
         Tsur_C    eps
1950     6.309E-5  7.171     7.826   17.21   222.8   1.982E-8  0.02   30       65
         30        0.85 */

// Energy balance, on a per unit length basis; steady-state conditions
// Edot'in - Edot'out + Edot'gen = 0
-q'cv - q'rad + Edot'gen = 0
q'cv = hbar * P * (Ts - Tinf)
P = pi * D
q'rad = eps * sigma * (Ts^4 - Tsur^4)
sigma = 5.67e-8
Edot'gen = I^2 * R'e
R'e = rhoe / Ac
rhoe = rhoeo * (1 + alpha * (Ts - To) )
To = 25 + 273
Ac = pi * D^2 / 4

// Convection coefficient
hbar = 1.21 * (D^-0.25) * (Ts - Tinf)^0.25          // Compact convection coeff. correlation
// Convection vs. total heat rates
cvovertot = q'cv / (q'cv + q'rad) * 100

// Input parameters
D = 0.020
// D = 0.010                  // Values of diameter for parameter study
// D = 0.040
// I = 1950                   // Base case condition unknown
rhoeo = 0.01711e-6
alpha = 0.00396
Tinf_C  = 30
Tinf = Tinf_C + 273
Ts_C = 65                     // Base case condition to determine current
Ts = Ts_C + 273
Tsur_C = 30
Tsur = Tsur_C + 273
eps = 0.85
```

PROBLEM 1.47

KNOWN: Elapsed times corresponding to a temperature change from 15 to 14°C for a reference sphere and test sphere of unknown composition suddenly immersed in a stirred water-ice mixture. Mass and specific heat of reference sphere.

FIND: Specific heat of the test sphere of known mass.

SCHEMATIC:



ASSUMPTIONS: (1) Spheres are of equal diameter, (2) Spheres experience temperature change from 15 to 14°C, (3) Spheres experience same convection heat transfer rate when the time rates of surface temperature are observed, (4) At any time, the temperatures of the spheres are uniform, (5) Negligible heat loss through the thermocouple wires.

PROPERTIES: Reference-grade sphere material: $c_r = 447$ J/kg K.

ANALYSIS: Apply the conservation of energy requirement at an instant of time, Eq. 1.11a, after a sphere has been immersed in the ice-water mixture at T_∞.

$$\dot{E}_{in} - \dot{E}_{out} = \dot{E}_{st}$$

$$-q_{conv} = Mc\frac{dT}{dt}$$

where $q_{conv} = h\,A_s\left(T - T_\infty\right)$. Since the temperatures of the spheres are uniform, the change in energy storage term can be represented with the time rate of temperature change, dT/dt. The convection heat rates are equal at this instant of time, and hence the change in energy storage terms for the reference (r) and test (t) spheres must be equal.

$$M_r\ c_r\ \left.\frac{dT}{dt}\right)_r = M_t\ c_t\ \left.\frac{dT}{dt}\right)_t$$

Approximating the instantaneous differential change, dT/dt, by the difference change over a short period of time, ΔT/Δt, the specific heat of the test sphere can be calculated.

$$0.515\ \text{kg} \times 447\,\text{J/kg}\cdot\text{K}\ \frac{(15-14)\text{K}}{6.35\,\text{s}} = 1.263\,\text{kg} \times c_t \times \frac{(15-14)\text{K}}{4.59\,\text{s}}$$

$$c_t = 132\ \text{J/kg}\cdot\text{K} \qquad <$$

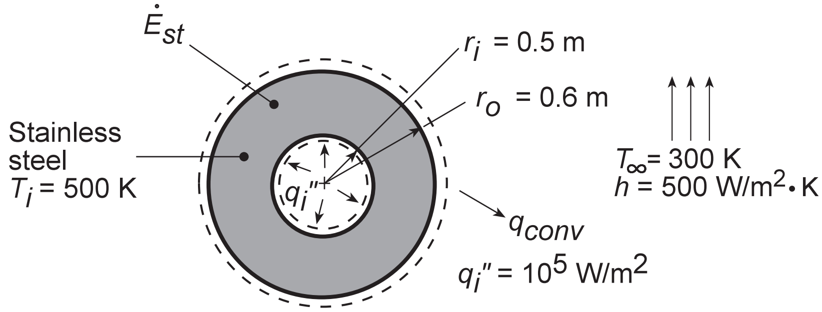
COMMENTS: Why was it important to perform the experiments with the reference and test spheres over the same temperature range (from 15 to 14°C)? Why does the analysis require that the spheres have uniform temperatures at all times?

PROBLEM 1.48

KNOWN: Inner surface heating and new environmental conditions associated with a spherical shell of prescribed dimensions and material.

FIND: (a) Governing equation for variation of wall temperature with time. Initial rate of temperature change, (b) Steady-state wall temperature, (c) Effect of convection coefficient on canister temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible temperature gradients in wall, (2) Constant properties, (3) Uniform, time-independent heat flux at inner surface.

PROPERTIES: *Table A.1*, Stainless Steel, AISI 302: ρ = 8055 kg/m^3, c_p = 510 J/kg·K.

ANALYSIS: (a) Performing an energy balance on the shell at an instant of time, $\dot{E}_{in} - \dot{E}_{out} = \dot{E}_{st}$. Identifying relevant processes and solving for dT/dt,

$$q_i''\left(4\pi r_i^2\right) - h\left(4\pi r_o^2\right)\left(T - T_\infty\right) = \rho \frac{4}{3}\pi\left(r_o^3 - r_i^3\right)c_p \frac{dT}{dt}$$

$$\frac{dT}{dt} = \frac{3}{\rho c_p\left(r_o^3 - r_i^3\right)}\left[q_i'' r_i^2 - h r_o^2\left(T - T_\infty\right)\right].$$

Substituting numerical values for the initial condition, find

$$\left.\frac{dT}{dt}\right)_i = \frac{3\left[10^5 \frac{W}{m^2}(0.5m)^2 - 500\frac{W}{m^2\cdot K}(0.6m)^2(500-300)K\right]}{8055\frac{kg}{m^3}510\frac{J}{kg\cdot K}\left[(0.6)^3 - (0.5)^3\right]m^3}$$

$$\left.\frac{dT}{dt}\right)_i = -0.089\,K/s.$$ <

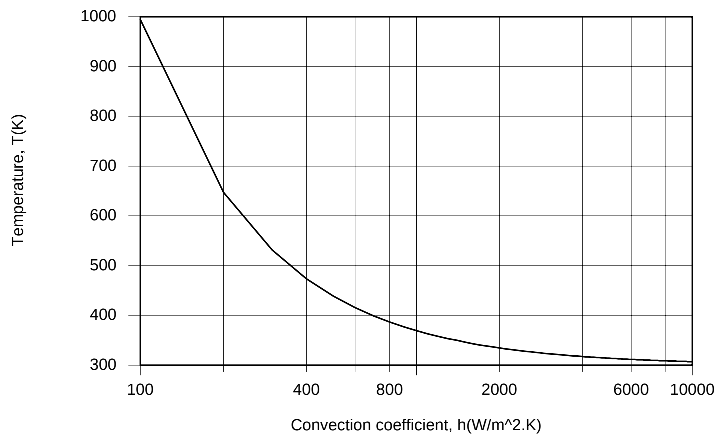
(b) Under steady-state conditions with $\dot{E}_{st}$ = 0, it follows that

$$q_i''\left(4\pi r_i^2\right) = h\left(4\pi r_o^2\right)\left(T - T_\infty\right)$$

Continued …..

$$T = T_\infty + \frac{q_i''}{h}\left(\frac{r_i}{r_o}\right)^2 = 300\text{K} + \frac{10^5\,\text{W/m}^2}{500\,\text{W/m}^2\cdot\text{K}}\left(\frac{0.5\text{m}}{0.6\text{m}}\right)^2 = 439\text{K} \qquad <$$

(c) Parametric calculations were performed using the IHT *First Law Model* for an *Isothermal Hollow Sphere*. As shown below, there is a sharp increase in temperature with decreasing values of $h < 1000$ $\text{W/m}^2\cdot\text{K}$. For $T > 380$ K, boiling will occur at the canister surface, and for $T > 410$ K a condition known as film boiling (Chapter 10) will occur. The condition corresponds to a precipitous reduction in h and increase in T.



Although the canister remains well below the melting point of stainless steel for $h = 100$ $\text{W/m}^2\cdot\text{K}$, boiling should be avoided, in which case the convection coefficient should be maintained at $h > 1000$ $\text{W/m}^2\cdot\text{K}$.

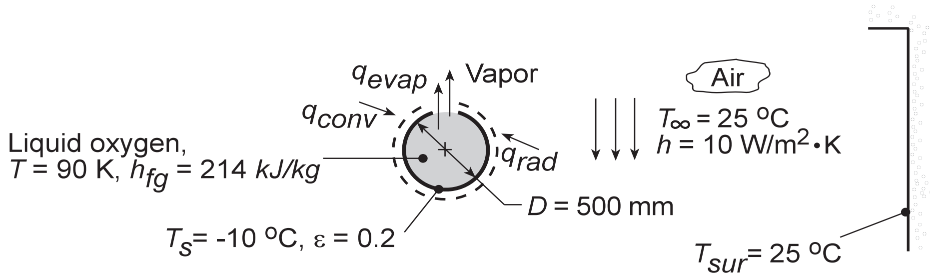
COMMENTS: The governing equation of part (a) is a first order, nonhomogenous differential equation with constant coefficients. Its solution is $\theta = (S/R)\left(1 - e^{-Rt}\right) + \theta_i e^{-Rt}$, where $\theta \equiv T - T_\infty$, $S \equiv 3q_i'' r_i^2 / \rho c_p \left(r_o^3 - r_i^3\right)$, $R = 3hr_o^2/\rho c_p \left(r_o^3 - r_i^3\right)$. Note results for $t \to \infty$ and for $S = 0$.

PROBLEM 1.49

KNOWN: Boiling point and latent heat of liquid oxygen. Diameter and emissivity of container. Free convection coefficient and temperature of surrounding air and walls.

FIND: Mass evaporation rate.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Temperature of container outer surface equals boiling point of oxygen.

ANALYSIS: (a) Applying an energy balance to a control surface about the container, it follows that, at any instant,

$$\dot{E}_{in} - \dot{E}_{out} = 0 \qquad \text{or} \qquad q_{conv} + q_{rad} - q_{evap} = 0 .$$

The evaporative heat loss is equal to the product of the mass rate of vapor production and the heat of vaporization. Hence,

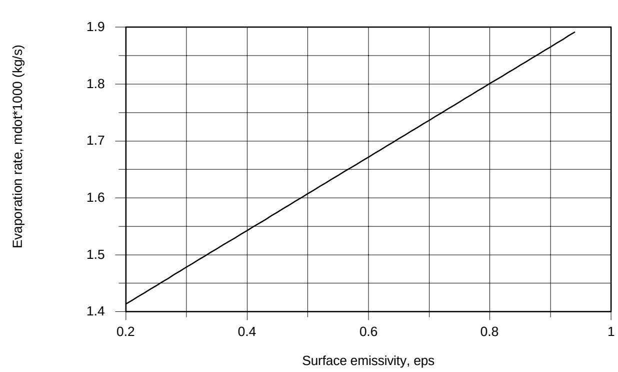
$$\left[h\left(T_\infty - T_s\right) + \varepsilon\sigma\left(T_{sur}^4 - T_s^4\right)\right]A_s - \dot{m}_{evap}h_{fg} = 0 \qquad (1)$$

$$\dot{m}_{evap} = \frac{\left[h\left(T_\infty - T_s\right) + \varepsilon\sigma\left(T_{sur}^4 - T_s^4\right)\right]\pi D^2}{h_{fg}}$$

$$\dot{m}_{evap} = \frac{\left[10\,\mathrm{W/m^2\cdot K}\left(298-263\right)\mathrm{K} + 0.2\times5.67\times10^{-8}\,\mathrm{W/m^2\cdot K^4}\left(298^4 - 263^4\right)\mathrm{K^4}\right]\pi\left(0.5\,\mathrm{m}\right)^2}{214\,\mathrm{kJ/kg}}$$

$$\dot{m}_{evap} = \frac{\left(350+35.2\right)\mathrm{W/m^2}\left(0.785\ \mathrm{m^2}\right)}{214\,\mathrm{kJ/kg}} = 1.41\times10^{-3}\ \mathrm{kg/s}. \qquad <$$

(b) Using the energy balance, Eq. (1), the mass rate of vapor production can be determined for the range of emissivity 0.2 to 0.94. The effect of increasing emissivity is to increase the heat rate into the container and, hence, increase the vapor production rate.



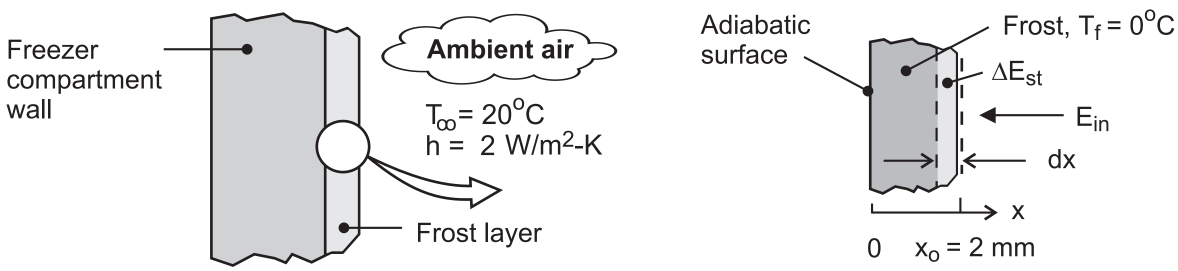
COMMENTS: To reduce the loss of oxygen due to vapor production, insulation should be applied to the outer surface of the container, in order to reduce q_{conv} and q_{rad}. Note from the calculations in part (a), that heat transfer by convection is greater than by radiation exchange.

PROBLEM 1.50

KNOWN: Frost formation of 2-mm thickness on a freezer compartment. Surface exposed to convection process with ambient air.

FIND: Time required for the frost to melt, t_m.

SCHEMATIC:



ASSUMPTIONS: (1) Frost is isothermal at the fusion temperature, T_f, (2) The water melt falls away from the exposed surface, (3) Negligible radiation exchange at the exposed surface, and (4) Backside surface of frost formation is adiabatic.

PROPERTIES: Frost, $\rho_f = 770\ kg/m^3$, $h_{sf} = 334\ kJ/kg$.

ANALYSIS: The time t_m required to melt a 2-mm thick frost layer may be determined by applying an energy balance, Eq 1.11b, over the differential time interval dt and to a differential control volume extending inward from the surface of the layer dx. From the schematic above, the energy *in* is the convection heat flux over the time period dt and the change in energy storage is the latent energy change within the control volume, $A_s \cdot dx$.

$$E_{in} - E_{out} = E_{st}$$

$$q''_{conv} A_s dt = dU_{\ell at}$$

$$h A_s \left(T_\infty - T_f\right) dt = -\rho_f A_s h_{sf} dx$$

Integrating both sides of the equation and defining appropriate limits, find

$$h\left(T_\infty - T_f\right)\int_0^{t_m} dt = -\rho_f h_{sf} \int_{x_o}^{0} dx$$

$$t_m = \frac{\rho_f h_{sf} x_o}{h\left(T_\infty - T_f\right)}$$

$$t_m = \frac{700\ kg/m^3 \times 334 \times 10^3\ J/kg \times 0.002m}{2\ W/m^2 \cdot K\left(20-0\right)K} = 11,690\ s = 3.2\ \text{hour}$$ <

COMMENTS: (1) The energy balance could be formulated intuitively by recognizing that the total heat *in* by convection during the time interval $t_m \left(q''_{cv} \cdot t_m\right)$ must be equal to the total latent energy for melting the frost layer $\left(\rho\, x_o h_{sf}\right)$. This equality is directly comparable to the derived expression above for t_m.

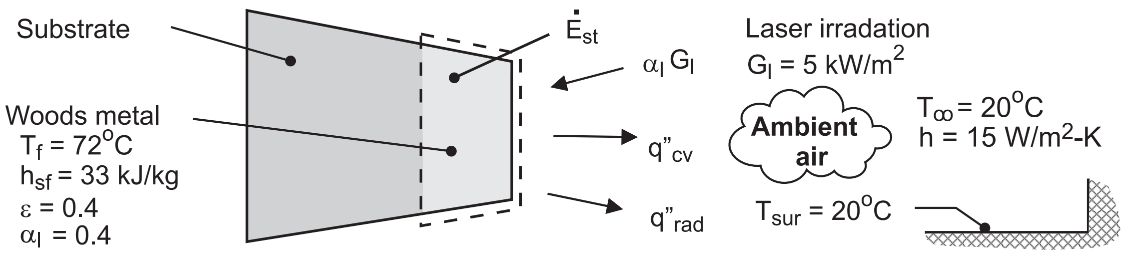
(2) Explain why the energy storage term in the analysis has a negative sign, and the limits of integration are as shown. *Hint*: Recall from the formulation of Eq. 1.11b, that the storage term represents the change between the final and initial states.

PROBLEM 1.51

KNOWN: Vertical slab of Woods metal initially at its fusion temperature, T_f, joined to a substrate. Exposed surface is irradiated with laser source, $G_\ell\left(W/m^2\right)$.

FIND: Instantaneous rate of melting per unit area, $\dot{m}''_m$ (kg/s·m^2), and the material removed in a period of 2 s, (a) Neglecting heat transfer from the irradiated surface by convection and radiation exchange, and (b) Allowing for convection and radiation exchange.

SCHEMATIC:



ASSUMPTIONS: (1) Woods metal slab is isothermal at the fusion temperature, T_f, and (2) The melt runs off the irradiated surface.

ANALYSIS: (a) The instantaneous rate of melting per unit area may be determined by applying an energy balance, Eq 1.11a, on the metal slab at an instant of time neglecting convection and radiation exchange from the irradiated surface.

$$\dot{E}''_{in} - \dot{E}''_{out} = \dot{E}''_{st} \qquad \alpha_\ell G_\ell = \frac{d}{dt}\left(-M'' h_{sf}\right) = -h_{sf}\frac{dM''}{dt}$$

where $dM''/dt = \dot{m}''_m$ is the time rate of change of mass in the control volume. Substituting values,

$$0.4 \times 5000\,W/m^2 = -33{,}000\,J/kg \times \dot{m}''_m \qquad \dot{m}''_m = -60.6\times10^{-3}\,kg/s\cdot m^2 \qquad <$$

The material removed in a 2s period per unit area is

$$M''_{2s} = \dot{m}''_m \cdot \Delta t = 121\,g/m^2 \qquad <$$

(b) The energy balance considering convection and radiation exchange with the surroundings yields

$$\alpha_\ell G_\ell - q''_{cv} - q''_{rad} = -h_{sf}\,\dot{m}''_m$$

$$q''_{cv} = h\left(T_f - T_\infty\right) = 15\,W/m^2\cdot K\left(72-20\right)K = 780\,W/m^2$$

$$q''_{rad} = \varepsilon\sigma\left(T_f^4 - T_\infty^4\right) = 0.4\times5.67\times10^{-8}\,W/m^2\cdot K\left(\left[72+273\right]^4 - \left[20+273\right]^4\right)K^4 = 154\,W/m^2$$

$$\dot{m}''_m = -32.3\times10^{-3}\,kg/s\cdot m^2 \qquad M_{2s} = 64\,g/m^2 \qquad <$$

COMMENTS: (1) The effects of heat transfer by convection and radiation reduce the estimate for the material removal rate by a factor of two. The heat transfer by convection is nearly 5 times larger than by radiation exchange.

(2) Suppose the work piece were horizontal, rather than vertical, and the melt puddled on the surface rather than ran off. How would this affect the analysis?

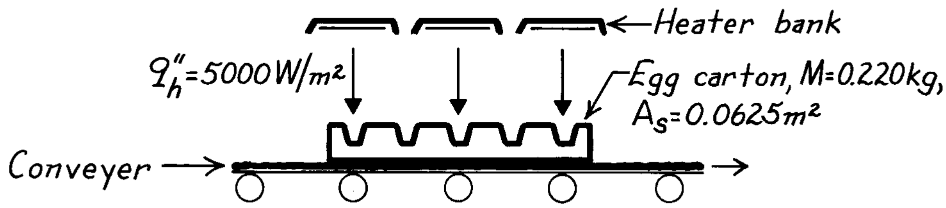
(3) Lasers are common heating sources for metals processing, including the present application of melting (heat transfer with phase change), as well as for heating work pieces during milling and turning (laser-assisted machining).

PROBLEM 1.52

KNOWN: Hot formed paper egg carton of prescribed mass, surface area and water content exposed to infrared heater providing known radiant flux.

FIND: Whether water content can be reduced from 75% to 65% by weight during the 18s period carton is on conveyor.

SCHEMATIC:

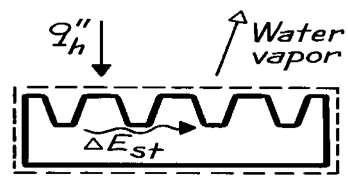


ASSUMPTIONS: (1) All the radiant flux from the heater bank is absorbed by the carton, (2) Negligible heat loss from carton by convection and radiation, (3) Negligible mass loss occurs from bottom side.

PROPERTIES: Water (given): h_{fg} = 2400 kJ/kg.

ANALYSIS: Define a control surface about the carton, and write the conservation of energy requirement for an interval of time, Δt,

$$E_{in} - E_{out} = \Delta E_{st} = 0$$



where E_{in} is due to the absorbed radiant flux, q_h'', from the heater and E_{out} is the energy leaving due to evaporation of water from the carton. Hence.

$$q_h'' \cdot A_s \cdot \Delta t = \Delta M \cdot h_{fg}.$$

For the prescribed radiant flux q_h'',

$$\Delta M = \frac{q_h'' A_s \Delta t}{h_{fg}} = \frac{5000\ W/m^2 \times 0.0625\ m^2 \times 18s}{2400\ kJ/kg} = 0.00234\ kg.$$

The chief engineer's requirement was to remove 10% of the water content, or

$$\Delta M_{req} = M \times 0.10 = 0.220\ kg \times 0.10 = 0.022\ kg$$

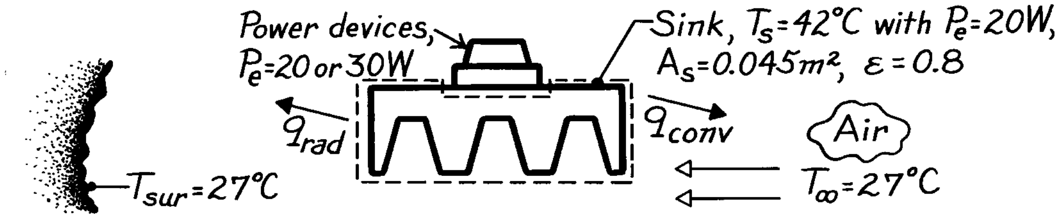
which is nearly an order of magnitude larger than the evaporative loss. Considering heat losses by convection and radiation, the actual water removal from the carton will be less than ΔM. Hence, the purchase should not be recommended, since the desired water removal cannot be achieved. $<$

PROBLEM 1.53

KNOWN: Average heat sink temperature when total dissipation is 20 W with prescribed air and surroundings temperature, sink surface area and emissivity.

FIND: Sink temperature when dissipation is 30 W.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) All dissipated power in devices is transferred to the sink, (3) Sink is isothermal, (4) Surroundings and air temperature remain the same for both power levels, (5) Convection coefficient is the same for both power levels, (6) Heat sink is a small surface within a large enclosure, the surroundings.

ANALYSIS: Define a control volume around the heat sink. Power dissipated within the devices is transferred into the sink, while the sink loses heat to the ambient air and surroundings by convection and radiation exchange, respectively.

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$P_e - hA_s\left(T_s - T_\infty\right) - A_s\varepsilon\sigma\left(T_s^4 - T_{sur}^4\right) = 0. \qquad (1)$$

Consider the situation when P_e = 20 W for which T_s = 42°C; find the value of h.

$$h = \left[P_e / A_s - \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right)\right] / \left(T_s - T_\infty\right)$$

$$h = \left[20\ \text{W}/0.045\ \text{m}^2 - 0.8 \times 5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4\left(315^4 - 300^4\right)\text{K}^4\right] / \left(315 - 300\right)\text{K}$$

$$h = 24.4\ \text{W}/\text{m}^2 \cdot \text{K}.$$

For the situation when P_e = 30 W, using this value for h with Eq. (1), obtain

$$30\ \text{W} - 24.4\ \text{W/m}^2 \cdot \text{K} \times 0.045\ \text{m}^2\left(T_s - 300\right)\text{K}$$
$$-0.045\ \text{m}^2 \times 0.8 \times 5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4\left(T_s^4 - 300^4\right)\text{K}^4 = 0$$

$$30 = 1.098\left(T_s - 300\right) + 2.041 \times 10^{-9}\left(T_s^4 - 300^4\right).$$

By trial-and-error, find

$$T_s \approx 322\ \text{K} = 49^\circ\text{C}. \qquad <$$

COMMENTS: (1) It is good practice to express all temperatures in kelvin units when using energy balances involving radiation exchange.

(2) Note that we have assumed A_s is the same for the convection and radiation processes. Since not all portions of the fins are completely exposed to the surroundings, $A_{s,rad}$ is less than $A_{s,conv}$ = A_s.

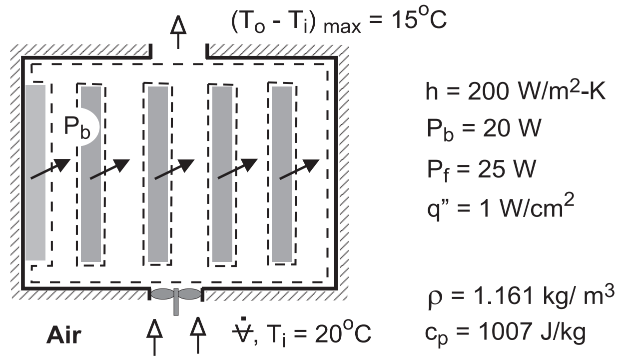
(3) Is the assumption that the heat sink is isothermal reasonable?

PROBLEM 1.54

KNOWN: Number and power dissipation of PCBs in a computer console. Convection coefficient associated with heat transfer from individual components in a board. Inlet temperature of cooling air and fan power requirement. Maximum allowable temperature rise of air. Heat flux from component most susceptible to thermal failure.

FIND: (a) Minimum allowable volumetric flow rate of air, (b) Preferred location and corresponding surface temperature of most thermally sensitive component.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) Constant air properties, (3) Negligible potential and kinetic energy changes of air flow, (4) Negligible heat transfer from console to ambient air, (5) Uniform convection coefficient for all components.

ANALYSIS: (a) For a control surface about the air space in the console, conservation of energy for an open system, Eq. (1.11e), reduces to

$$\dot{m}(u+pv)_i - \dot{m}(u+pv)_o + q - \dot{W} = 0$$

where $u + pv = i$, $q = 5P_b$, and $\dot{W} = -P_f$. Hence, with $\dot{m}(i_i - i_o) = \dot{m}c_p(T_i - T_o)$,

$$\dot{m}c_p(T_o - T_i) = 5\,P_b + P_f$$

For a maximum allowable temperature rise of 15°C, the required mass flow rate is

$$\dot{m} = \frac{5\,P_b + P_f}{c_p(T_o - T_i)} = \frac{5\times 20\ W + 25\ W}{1007\ J/kg\cdot K\left(15\ ^\circ C\right)} = 8.28\times 10^{-3}\,kg/s$$

The corresponding volumetric flow rate is

$$\forall = \frac{\dot{m}}{\rho} = \frac{8.28\times 10^{-3}\,kg/s}{1.161\ kg/m^3} = 7.13\times 10^{-3}\,m^3/s \qquad <$$

(b) The component which is most susceptible to thermal failure should be mounted at the bottom of one of the PCBs, where the air is coolest. From the corresponding form of Newton's law of cooling, $q'' = h(T_s - T_i)$, the surface temperature is

$$T_s = T_i + \frac{q''}{h} = 20^\circ C + \frac{1\times 10^4\ W/m^2}{200\ W/m^2\cdot K} = 70^\circ C \qquad <$$

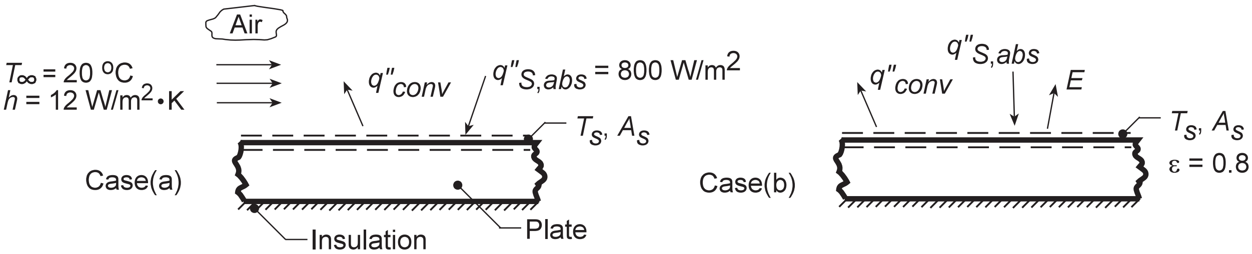
COMMENTS: (1) Although the mass flow rate is invariant, the volumetric flow rate increases as the air is heated in its passage through the console, causing a reduction in the density. However, for the prescribed temperature rise, the change in ρ, and hence the effect on $\dot{\forall}$, is small. (2) If the thermally sensitive component were located at the top of a PCB, it would be exposed to warmer air (T_o = 35°C) and the surface temperature would be T_s = 85°C.

PROBLEM 1.55

KNOWN: Top surface of car roof absorbs solar flux, $q''_{S,abs}$, and experiences for case (a): convection with air at T_∞ and for case (b): the same convection process and radiation emission from the roof.

FIND: Temperature of the plate, T_s, for the two cases. Effect of airflow on roof temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible heat transfer to auto interior, (3) Negligible radiation from atmosphere.

ANALYSIS: (a) Apply an energy balance to the control surfaces shown on the schematic. For an instant of time, $\dot{E}_{in} - \dot{E}_{out}$ = 0. Neglecting radiation emission, the relevant processes are convection between the plate and the air, q''_{conv}, and the absorbed solar flux, $q''_{S,abs}$. Considering the roof to have an area A_s,

$$q''_{S,abs} \cdot A_s - hA_s\left(T_s - T_\infty\right) = 0$$

$$T_s = T_\infty + q''_{S,abs}/h$$

$$T_s = 20^\circ C + \frac{800 W/m^2}{12 W/m^2 \cdot K} = 20^\circ C + 66.7^\circ C = 86.7^\circ C \qquad <$$

(b) With radiation emission from the surface, the energy balance has the form

$$q''_{S,abs} \cdot A_s - q_{conv} - E \cdot A_s = 0$$

$$q''_{S,abs} A_s - hA_s\left(T_s - T_\infty\right) - \varepsilon A_s \sigma T_s^4 = 0.$$

Substituting numerical values, with temperature in absolute units (K),

$$800 \frac{W}{m^2} - 12 \frac{W}{m^2 \cdot K}\left(T_s - 293 K\right) - 0.8 \times 5.67 \times 10^{-8} \frac{W}{m^2 \cdot K^4} T_s^4 = 0$$

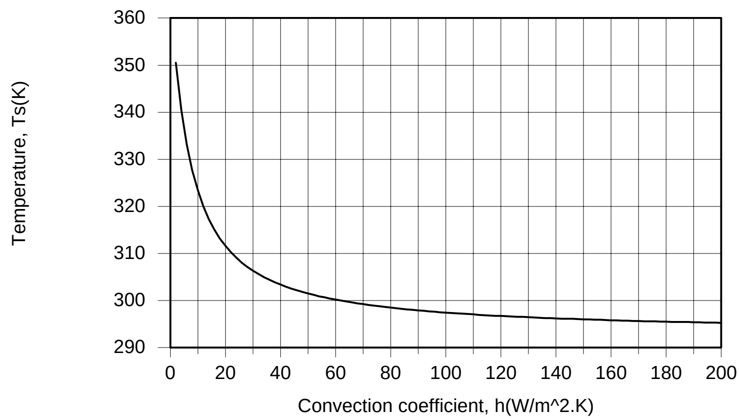
$$12 T_s + 4.536 \times 10^{-8} T_s^4 = 4316$$

It follows that T_s = 320 K = 47°C. $\qquad <$

Continued.….

PROBLEM 1.55 (Cont.)

(c) Parametric calculations were performed using the IHT *First Law Model* for an *Isothermal Plane Wall*. As shown below, the roof temperature depends strongly on the velocity of the auto relative to the ambient air. For a convection coefficient of h = 40 W/m^2·K, which would be typical for a velocity of 55 mph, the roof temperature would exceed the ambient temperature by less than 10°C.



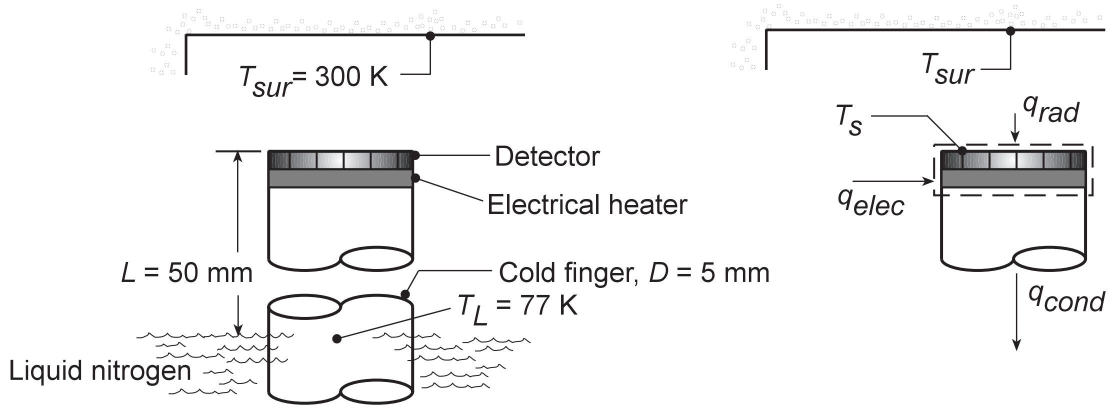
COMMENTS: By considering radiation emission, T_s decreases, as expected. Note the manner in which q''_{conv} is formulated using Newton's law of cooling; since q''_{conv} is shown leaving the control surface, the rate equation must be $h\left(T_s - T_\infty\right)$ and not $h\left(T_\infty - T_s\right)$.

PROBLEM 1.56

KNOWN: Detector and heater attached to cold finger immersed in liquid nitrogen. Detector surface of $\varepsilon = 0.9$ is exposed to large vacuum enclosure maintained at 300 K.

FIND: (a) Temperature of detector when no power is supplied to heater, (b) Heater power (W) required to maintain detector at 195 K, (c) Effect of finger thermal conductivity on heater power.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction through cold finger, (3) Detector and heater are very thin and isothermal at T_s, (4) Detector surface is small compared to enclosure surface.

PROPERTIES: Cold finger (given): k = 10 W/m·K.

ANALYSIS: Define a control volume about detector and heater and apply conservation of energy requirement on a rate basis, Eq. 1.11a,

$$\dot{E}_{in} - \dot{E}_{out} = 0 \tag{1}$$

where

$$\dot{E}_{in} = q_{rad} + q_{elec}; \qquad \dot{E}_{out} = q_{cond} \tag{2,3}$$

Combining Eqs. (2,3) with (1), and using the appropriate rate equations,

$$\varepsilon A_s \sigma \left(T_{sur}^4 - T_s^4\right) + q_{elec} = kA_s\left(T_s - T_L\right)/L. \tag{4}$$

(a) Where q_{elec} = 0, substituting numerical values

$$0.9 \times 5.67 \times 10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left(300^4 - T_s^4\right)\text{K}^4 = 10\,\text{W/m}\cdot\text{K}\left(T_s - 77\right)\text{K}/0.050\,\text{m}$$

$$5.103 \times 10^{-8}\left(300^4 - T_s^4\right) = 200\left(T_s - 77\right)$$

$$T_s = 79.1\,\text{K}$$ <

Continued…..

PROBLEM 1.56 (Cont.)

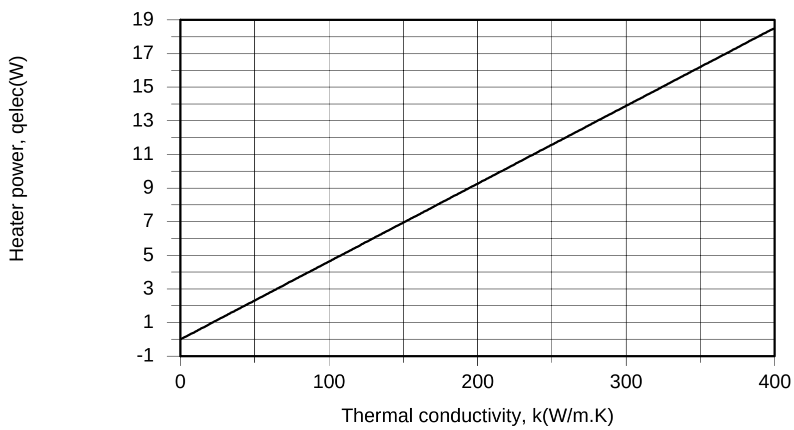
(b) When T_s = 195 K, Eq. (4) yields

$$0.9\times[\pi(0.005\,\text{m})^2/4]\times5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left(300^4-195^4\right)\text{K}^4+q_{elec}$$

$$=10\,\text{W/m}\cdot\text{K}\times[\pi(0.005\,\text{m})^2/4]\times(195-77)\,\text{K}/0.050\,\text{m}$$

$$q_{elec}=0.457\,\text{W}=457\,\text{mW}$$ <

(c) Calculations were performed using the *First Law Model* for a *Nonisothermal Plane Wall*. With net radiative transfer to the detector fixed by the prescribed values of T_s and T_{sur}, Eq. (4) indicates that q_{elec} increases linearly with increasing k.



Heat transfer by conduction through the finger material increases with its thermal conductivity. Note that, for k = 0.1 W/m·K, q_{elec} = -2 mW, where the minus sign implies the need for a heat *sink*, rather than a heat source, to maintain the detector at 195 K. In this case q_{rad} exceeds q_{cond}, and a heat sink would be needed to dispose of the difference. A conductivity of k = 0.114 W/m·K yields a precise balance between q_{rad} and q_{cond}. Hence to circumvent heaving to use a heat sink, while minimizing the heater power requirement, k should exceed, but remain as close as possible to the value of 0.114 W/m·K. Using a graphite fiber composite, with the fibers oriented normal to the direction of conduction, Table A.2 indicates a value of k ≈ 0.54 W/m·K at an average finger temperature of $\bar{T}$ = 136 K. For this value, q_{elec} = 18 mW

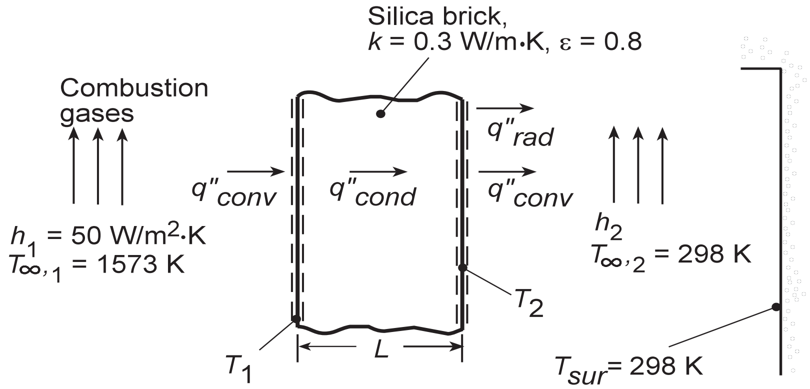
COMMENTS: The heater power requirement could be further reduced by decreasing ε.

PROBLEM 1.57

KNOWN: Conditions at opposite sides of a furnace wall of prescribed thickness, thermal conductivity and surface emissivity.

FIND: Effect of wall thickness and outer convection coefficient on surface temperatures. Recommended values of L and h_2.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Negligible radiation exchange at surface 1, (4) Surface 2 is exposed to large surroundings.

ANALYSIS: The unknown temperatures may be obtained by simultaneously solving energy balance equations for the two surface. At surface 1,

$$q''_{conv,1} = q''_{cond}$$

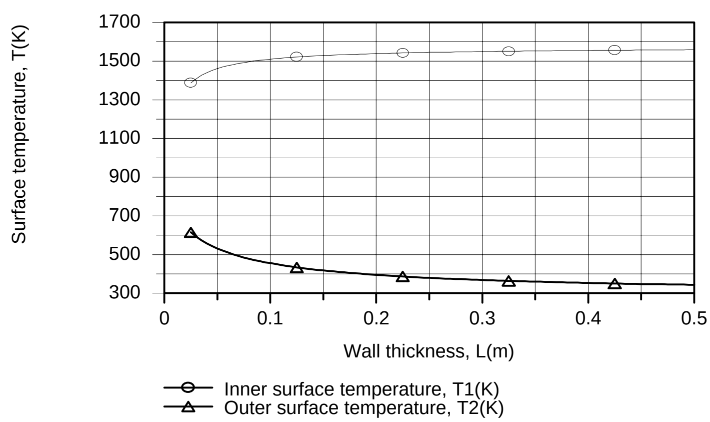
$$h_1\left(T_{\infty,1} - T_1\right) = k\left(T_1 - T_2\right)/L \quad (1)$$

At surface 2,

$$q''_{cond} = q''_{conv} + q''_{rad}$$

$$k\left(T_1 - T_2\right)/L = h_2\left(T_2 - T_{\infty,2}\right) + \varepsilon\sigma\left(T_2^4 - T_{sur}^4\right) \quad (2)$$

Using the IHT *First Law Model* for a *Nonisothermal Plane Wall,* we obtain

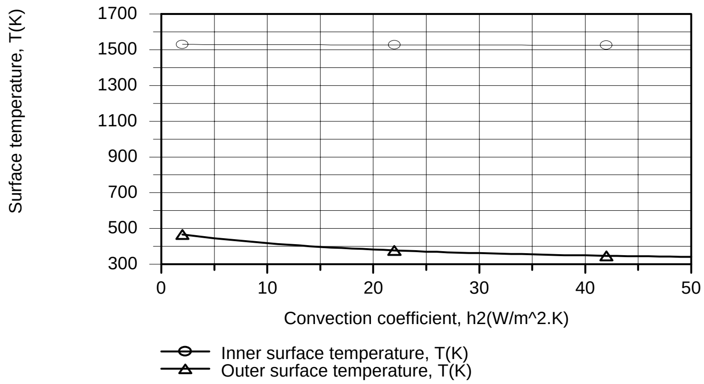


Continued …..

PROBLEM 1.57 (Cont.)

Both q''_{cond} and T_2 decrease with increasing wall thickness, and for the prescribed value of h_2 = 10 W/m^2·K, a value of L ≥ 0.275 m is needed to maintain T_2 ≤ 373 K = 100 °C. Note that inner surface temperature T_1, and hence the temperature difference, $T_1 - T_2$, increases with increasing L.

Performing the calculations for the prescribed range of h_2, we obtain



For the prescribed value of L = 0.15 m, a value of h_2 ≥ 24 W/m^2·K is needed to maintain T_2 ≤ 373 K. The variation has a negligible effect on T_1, causing it to decrease slightly with increasing h_2, but does have a strong influence on T_2.

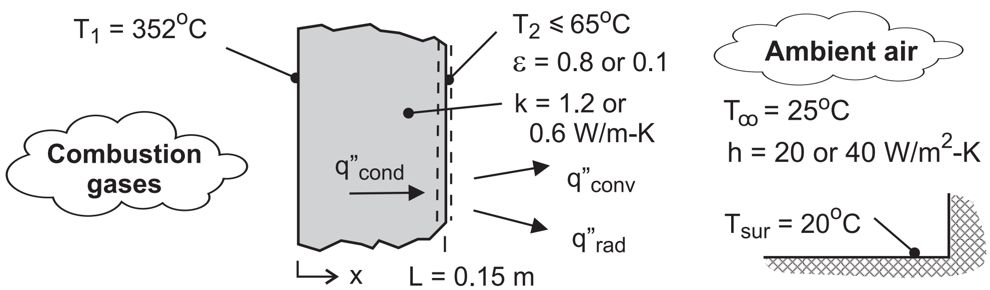
COMMENTS: If one wishes to avoid use of active (forced convection) cooling on side 2, reliance will have to be placed on free convection, for which h_2 ≈ 5 W/m^2·K. The minimum wall thickness would then be L = 0.40 m.

PROBLEM 1.58

KNOWN: Furnace wall with inner surface temperature $T_1 = 352°C$ and prescribed thermal conductivity experiencing convection and radiation exchange on outer surface. See Example 1.5.

FIND: (a) Outer surface temperature T_2 resulting from decreasing the wall thermal conductivity k or increasing the convection coefficient h by a factor of two; benefit of applying a low emissivity coating ($\varepsilon < 0.8$); comment on the effectiveness of these strategies to reduce risk of burn injury when $T_2 \le 65°C$; and (b) Calculate and plot T_2 as a function of h for the range $20 \le h \le 100$ W/m^2·K for three materials with k = 0.3, 0.6, and 1.2 W/m·K; what conditions will provide for safe outer surface temperatures.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in wall, (3) Radiation exchange is between small surface and large enclosure, (4) Inner surface temperature remains constant for all conditions.

ANALYSIS: (a) The surface (x = L) energy balance is

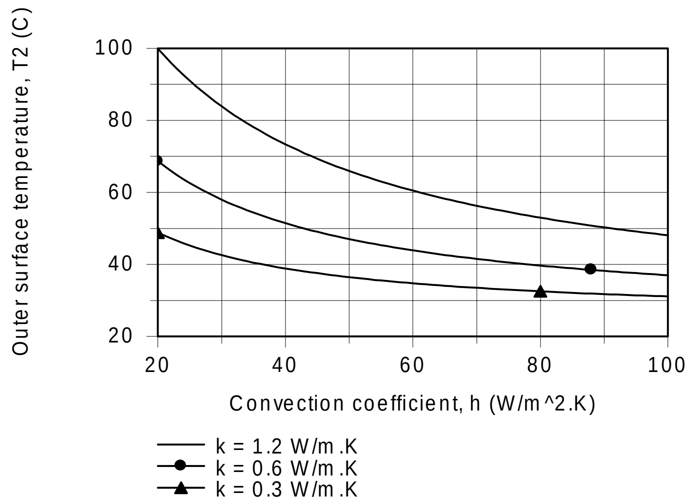
$$k\frac{T_1 - T_2}{L} = h\left(T_2 - T_\infty\right) + \varepsilon\sigma\left(T_2^4 - T_{sur}^4\right)$$

With $T_1 = 352°C$, the effects of parameters h, k and ε on the outer surface temperature are calculated and tabulated below.

Conditions	$k\left(W/m\cdot K\right)$	$h\left(W/m^2\cdot K\right)$	ε	$T_2\left(°C\right)$
Example 1.5	1.2	20	0.8	100
Decrease k by ½	0.6	20	0.8	69
Increase h by 2	1.2	40	0.8	73
Change k and h	0.6	40	0.8	51
Decrease ε	1.2	20	0.1	115

(b) Using the energy balance relation in the Workspace of IHT, the outer surface temperature can be calculated and plotted as a function of the convection coefficient for selected values of the wall thermal conductivity.

Continued …..



COMMENTS: (1) From the parameter study of part (a), note that decreasing the thermal conductivity is more effective in reducing T_2 than is increasing the convection coefficient. Only if both changes are made will T_2 be in the safe range.

(2) From part (a), note that applying a low emissivity coating is not beneficial. Did you suspect that before you did the analysis? Give a physical explanation for this result.

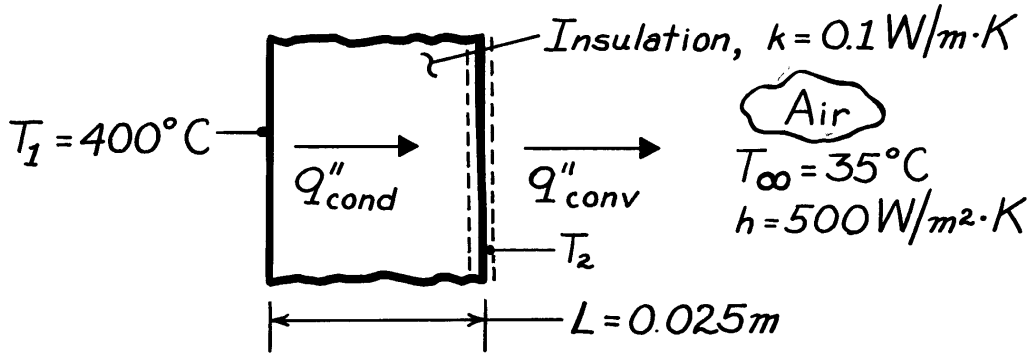
(3) From the parameter study graph we conclude that safe wall conditions ($T_2 \le 65°C$) can be maintained for these conditions: with k = 1.2 W/m·K when h > 55 $W/m^2 \cdot K$; with k = 0.6 W/m·K when h > 25 $W/m^2 \cdot K$; and with k = 0.3 W/m·K when h > 20 W/m·K.

PROBLEM 1.59

KNOWN: Inner surface temperature, thickness and thermal conductivity of insulation exposed at its outer surface to air of prescribed temperature and convection coefficient.

FIND: Outer surface temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in the insulation, (3) Negligible radiation exchange between outer surface and surroundings.

ANALYSIS: From an energy balance at the outer surface at an instant of time,

$$q''_{cond} = q''_{conv}.$$

Using the appropriate rate equations,

$$k\frac{(T_1 - T_2)}{L} = h(T_2 - T_\infty).$$

Solving for T_2, find

$$T_2 = \frac{\frac{k}{L}T_1 + h\,T_\infty}{h + \frac{k}{L}} = \frac{\frac{0.1\text{ W/m}\cdot\text{K}}{0.025\text{m}}\left(400°\text{C}\right) + 500\frac{\text{W}}{\text{m}^2\cdot\text{K}}\left(35°\text{C}\right)}{500\frac{\text{W}}{\text{m}^2\cdot\text{K}} + \frac{0.1\text{ W/m}\cdot\text{K}}{0.025\text{m}}}$$

$$T_2 = 37.9°\text{C}.$$ <

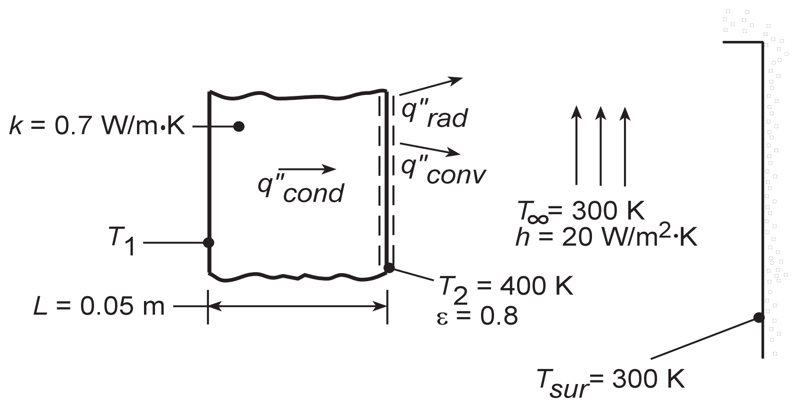
COMMENTS: If the temperature of the surroundings is approximately that of the air, radiation exchange between the outer surface and the surroundings will be negligible, since T_2 is small. In this case convection makes the dominant contribution to heat transfer from the outer surface, and assumption (3) is excellent.

PROBLEM 1.60

KNOWN: Thickness and thermal conductivity, k, of an oven wall. Temperature and emissivity, ε, of front surface. Temperature and convection coefficient, h, of air. Temperature of large surroundings.

FIND: (a) Temperature of back surface, (b) Effect of variations in k, h and ε.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional conduction, (3) Radiation exchange with large surroundings.

ANALYSIS: (a) Applying an energy balance, Eq. 1.13, at an instant of time to the front surface and substituting the appropriate rate equations, Eqs. 1.2, 1.3a and 1.7, find

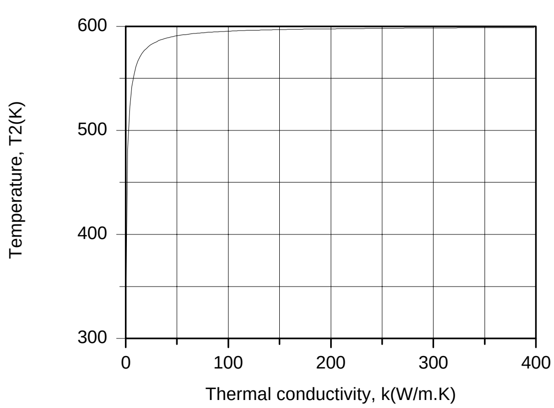
$$k\frac{T_1 - T_2}{L} = h\left(T_2 - T_\infty\right) + \varepsilon\sigma\left(T_2^4 - T_{sur}^4\right).$$

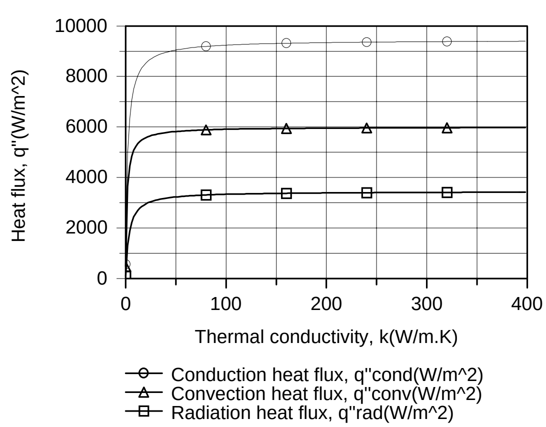
Substituting numerical values, find

$$T_1 - T_2 = \frac{0.05\,\text{m}}{0.7\,\text{W/m}\cdot\text{K}}\left[20\frac{\text{W}}{\text{m}^2\cdot\text{K}}100\,\text{K} + 0.8\times5.67\times10^{-8}\frac{\text{W}}{\text{m}^2\cdot\text{K}^4}\left[\left(400\,\text{K}\right)^4 - \left(300\,\text{K}\right)^4\right]\right] = 200\,\text{K}.$$

Since T_2 = 400 K, it follows that T_1 = 600 K. <

(b) Parametric effects may be evaluated by using the IHT *First Law* Model for a *Nonisothermal Plane Wall*. Changes in k strongly influence conditions for k < 20 W/m·K, but have a negligible effect for larger values, as T_2 approaches T_1 and the heat fluxes approach the corresponding limiting values

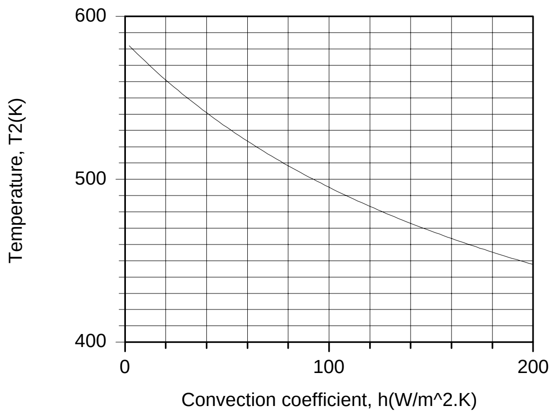


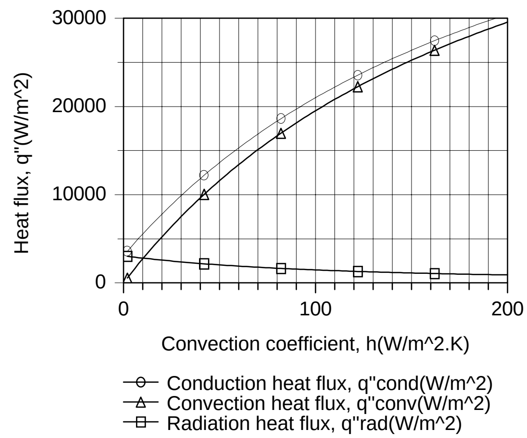


PROBLEM 1.60 (Cont.)

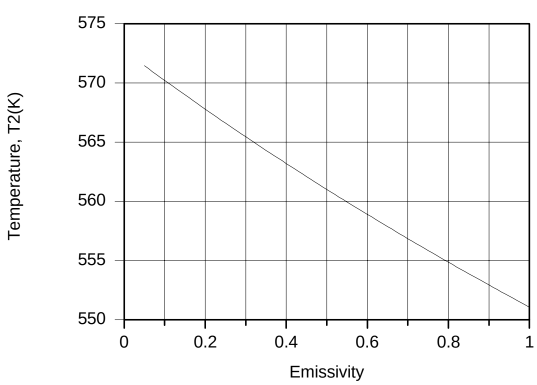
The implication is that, for k > 20 W/m·K, heat transfer by conduction in the wall is extremely efficient relative to heat transfer by convection and radiation, which become the *limiting* heat transfer processes. Larger fluxes could be obtained by increasing ε and h and/or by decreasing T_∞ and T_{sur}.

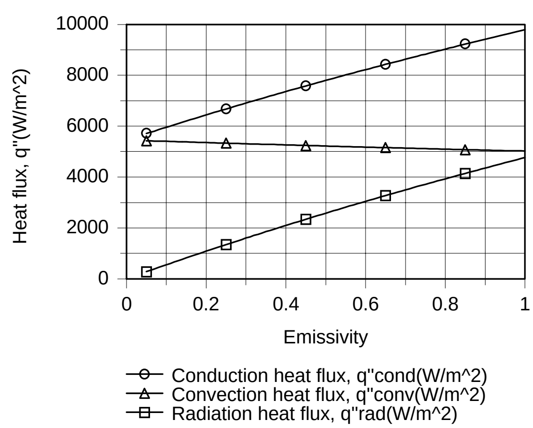
With increasing h, the front surface is cooled more effectively (T_2 decreases), and although q''_{rad} decreases, the reduction is exceeded by the increase in q''_{conv}. With a reduction in T_2 and fixed values of k and L, q''_{cond} must also increase.





The surface temperature also decreases with increasing ε, and the increase in q''_{rad} exceeds the reduction in q''_{conv}, allowing q''_{cond} to increase with ε.





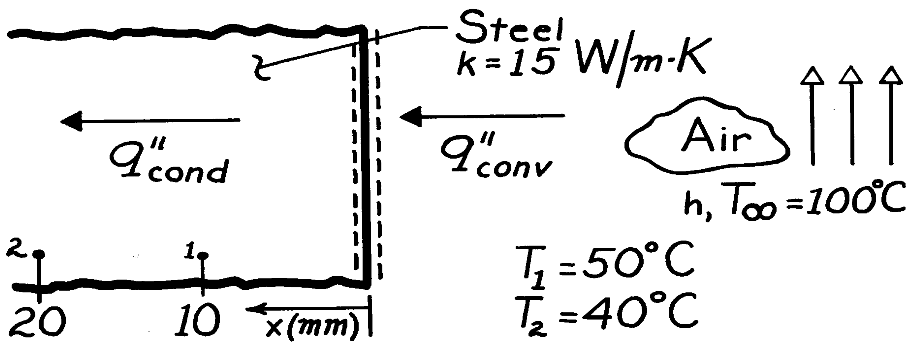
COMMENTS: Conservation of energy, of course, dictates that, irrespective of the prescribed conditions, $q''_{cond} = q''_{conv} + q''_{rad}$.

PROBLEM 1.61

KNOWN: Temperatures at 10 mm and 20 mm from the surface and in the adjoining airflow for a thick steel casting.

FIND: Surface convection coefficient, h.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional conduction in the x-direction, (3) Constant properties, (4) Negligible generation.

ANALYSIS: From a surface energy balance, it follows that

$$q''_{cond} = q''_{conv}$$

where the convection rate equation has the form

$$q''_{conv} = h\left(T_\infty - T_0\right),$$

and q''_{cond} can be evaluated from the temperatures prescribed at surfaces 1 and 2. That is, from Fourier's law,

$$q''_{cond} = k\frac{T_1 - T_2}{x_2 - x_1}$$

$$q''_{cond} = 15\frac{W}{m\cdot K}\frac{(50-40)^\circ C}{(20\text{-}10)\times 10^{-3}m} = 15,000\ W/m^2.$$

Since the temperature gradient in the solid must be linear for the prescribed conditions, it follows that

$$T_0 = 60°C.$$

Hence, the convection coefficient is

$$h = \frac{q''_{cond}}{T_\infty - T_0}$$

$$h = \frac{15,000\ W/m^2}{40^\circ C} = 375\ W/m^2\cdot K. \qquad <$$

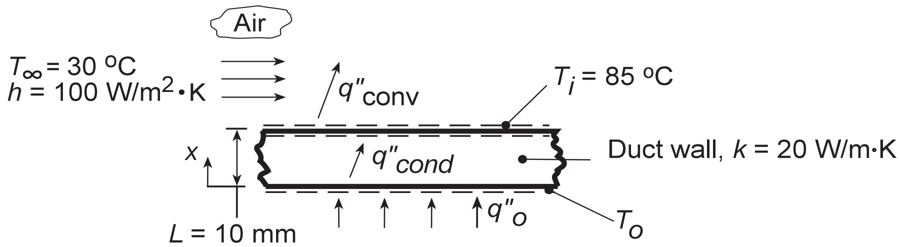
COMMENTS: The accuracy of this procedure for measuring h depends strongly on the validity of the assumed conditions.

PROBLEM 1.62

KNOWN: Duct wall of prescribed thickness and thermal conductivity experiences prescribed heat flux q''_o at outer surface and convection at inner surface with known heat transfer coefficient.

FIND: (a) Heat flux at outer surface required to maintain inner surface of duct at T_i = 85°C, (b) Temperature of outer surface, T_o, (c) Effect of h on T_o and q''_o.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in wall, (3) Constant properties, (4) Backside of heater perfectly insulated, (5) Negligible radiation.

ANALYSIS: (a) By performing an energy balance on the wall, recognize that $q''_o = q''_{cond}$. From an energy balance on the top surface, it follows that $q''_{cond} = q''_{conv} = q''_o$. Hence, using the convection rate equation,

$$q''_o = q''_{conv} = h\left(T_i - T_\infty\right) = 100\,\mathrm{W/m^2\cdot K}\left(85-30\right)^\circ\mathrm{C} = 5500\,\mathrm{W/m^2}. \qquad <$$

(b) Considering the duct wall and applying Fourier's Law,

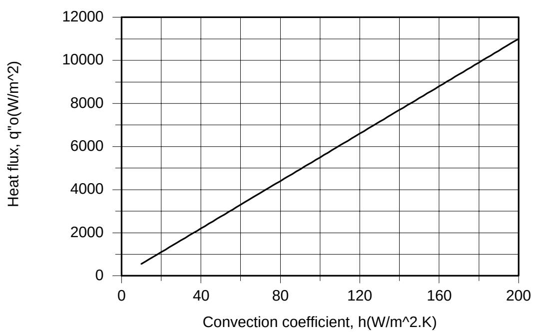
$$q''_o = k\frac{\Delta T}{\Delta X} = k\frac{T_o - T_i}{L}$$

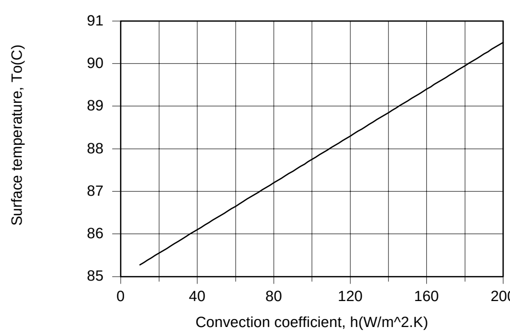
$$T_o = T_i + \frac{q''_o L}{k} = 85^\circ\mathrm{C} + \frac{5500\,\mathrm{W/m^2}\times 0.010\,\mathrm{m}}{20\,\mathrm{W/m\cdot K}} = \left(85+2.8\right)^\circ\mathrm{C} = 87.8^\circ\mathrm{C}. \qquad <$$

(c) For T_i = 85°C, the desired results may be obtained by simultaneously solving the energy balance equations

$$q''_o = k\frac{T_o - T_i}{L} \qquad \text{and} \qquad k\frac{T_o - T_i}{L} = h\left(T_i - T_\infty\right)$$

Using the IHT *First Law* Model for a *Nonisothermal Plane Wall*, the following results are obtained.





Since q''_{conv} increases linearly with increasing h, the applied heat flux q''_o must be balanced by an increase in q''_{cond}, which, with fixed k, T_i and L, necessitates an increase in T_o.

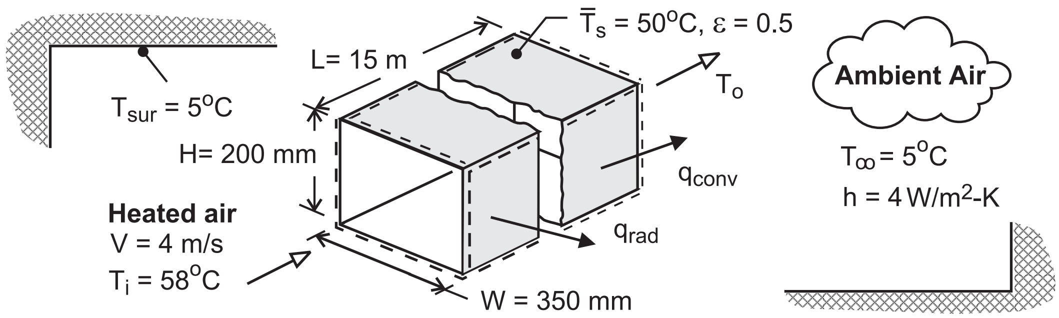
COMMENTS: The temperature difference across the wall is small, amounting to a maximum value of $\left(T_o - T_i\right)$ = 5.5°C for h = 200 W/m^2·K. If the wall were thinner (L < 10 mm) or made from a material with higher conductivity (k > 20 W/m·K), this difference would be reduced.

PROBLEM 1.63

KNOWN: Dimensions, average surface temperature and emissivity of heating duct. Duct air inlet temperature and velocity. Temperature of ambient air and surroundings. Convection coefficient.

FIND: (a) Heat loss from duct, (b) Air outlet temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) Constant air properties, (3) Negligible potential and kinetic energy changes of air flow, (4) Radiation exchange between a small surface and a large enclosure.

ANALYSIS: (a) Heat transfer from the surface of the duct to the ambient air and the surroundings is given by Eq. (1.10)

$$q = hA_s\left(T_s - T_\infty\right) + \varepsilon A_s \sigma\left(T_s^4 - T_{sur}^4\right)$$

where $A_s = L\ (2W + 2H) = 15\ m\ (0.7\ m + 0.5\ m) = 16.5\ m^2$. Hence,

$$q = 4\ W/m^2 \cdot K \times 16.5\ m^2\left(45^\circ C\right) + 0.5 \times 16.5\ m^2 \times 5.67 \times 10^{-8}\ W/m^2 \cdot K^4\left(323^4 - 278^4\right)K^4$$

$$q = q_{conv} + q_{rad} = 2970\ W + 2298\ W = 5268\ W \qquad <$$

(b) With $i = u + pv$, $\dot{W} = 0$ and the third assumption, Eq. (1.11e) yields,

$$\dot{m}\left(i_i - i_o\right) = \dot{m}c_p\left(T_i - T_o\right) = q$$

where the sign on q has been reversed to reflect the fact that heat transfer is *from* the system.
With $\dot{m} = \rho VA_c = 1.10\ kg/m^3 \times 4\ m/s\left(0.35m \times 0.20m\right) = 0.308\ kg/s,$ the outlet temperature is

$$T_o = T_i - \frac{q}{\dot{m}c_p} = 58^\circ C - \frac{5268\ W}{0.308\ kg/s \times 1008\ J/kg \cdot K} = 41^\circ C \qquad <$$

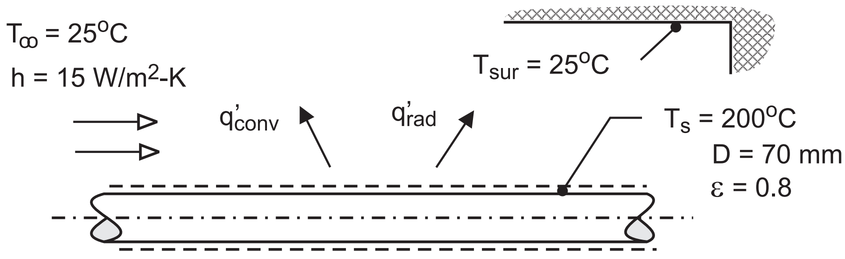
COMMENTS: The temperature drop of the air is large and unacceptable, unless the intent is to use the duct to heat the basement. If not, the duct should be insulated to insure maximum delivery of thermal energy to the intended space(s).

PROBLEM 1.64

KNOWN: Uninsulated pipe of prescribed diameter, emissivity, and surface temperature in a room with fixed wall and air temperatures. See Example 1.2.

FIND: (a) Which option to reduce heat loss to the room is more effective: reduce by a factor of two the convection coefficient (from 15 to 7.5 $W/m^2 \cdot K$) or the emissivity (from 0.8 to 0.4) and (b) Show graphically the heat loss as a function of the convection coefficient for the range $5 \le h \le 20$ $W/m^2 \cdot K$ for emissivities of 0.2, 0.4 and 0.8. Comment on the relative efficacy of reducing heat losses associated with the convection and radiation processes.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Radiation exchange between pipe and the room is between a small surface in a much larger enclosure, (3) The surface emissivity and absorptivity are equal, and (4) Restriction of the air flow does not alter the radiation exchange process between the pipe and the room.

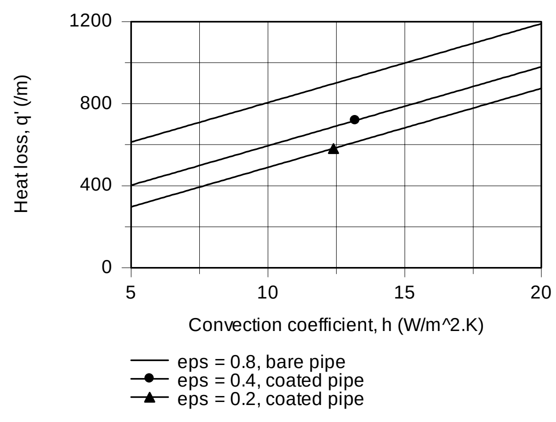
ANALYSIS: (a) The heat rate from the pipe to the room per unit length is

$$q' = q'/L = q'_{conv} + q'_{rad} = h(\pi D)(T_s - T_\infty) + \varepsilon(\pi D)\sigma\left(T_s^4 - T_{sur}^4\right)$$

Substituting numerical values for the two options, the resulting heat rates are calculated and compared with those for the conditions of Example 1.2. We conclude that both options are comparably effective.

Conditions	$h\left(W/m^2 \cdot K\right)$	ε	$q'(W/m)$
Base case, Example 1.2	15	0.8	998
Reducing h by factor of 2	7.5	0.8	788
Reducing ε by factor of 2	15	0.4	709

(b) Using IHT, the heat loss can be calculated as a function of the convection coefficient for selected values of the surface emissivity.



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COMMENTS: (1) In Example 1.2, Comment 3, we read that the heat rates by convection and radiation exchange were comparable for the base case conditions (577 vs. 421 W/m). It follows that reducing the key transport parameter (h or ε) by a factor of two yields comparable reductions in the heat loss. Coating the pipe to reduce the emissivity might to be the more practical option as it may be difficult to control air movement.

(2) For this pipe size and thermal conditions (T_s and T_∞), the minimum possible convection coefficient is approximately 7.5 $W/m^2 \cdot K$, corresponding to free convection heat transfer to quiescent ambient air. Larger values of h are a consequence of forced air flow conditions.

(3) The Workspace for the IHT program to calculate the heat loss and generate the graph for the heat loss as a function of the convection coefficient for selected emissivities is shown below. It is good practice to provide commentary with the code making your solution logic clear, and to summarize the results.

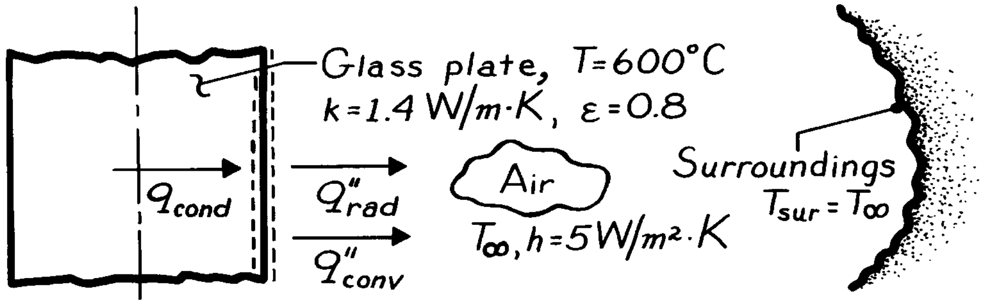
```
// Heat loss per unit pipe length; rate equation from Ex. 1.2
q' = q'cv + q'rad
q'cv = pi*D*h*(Ts - Tinf)
q'rad = pi*D*eps*sigma*(Ts^4 - Tsur^4)
sigma = 5.67e-8

// Input parameters
D = 0.07
Ts_C = 200        // Representing temperatures in Celsius units using _C subscripting
Ts = Ts_C +273
Tinf_C = 25
Tinf = Tinf_C + 273
h = 15            // For graph, sweep over range from 5 to 20
Tsur_C = 25
Tsur = Tsur_C + 273
eps = 0.8
//eps = 0.4       // Values of emissivity for parameter study
//eps = 0.2

/* Base case results
Tinf   Ts     Tsur   q'       q'cv    q'rad   D      Tinf_C  Ts_C   Tsur_C
       eps    h      sigma
298    473    298    997.9    577.3   420.6   0.07   25      200    25
       0.8    15     5.67E-8      */
```

PROBLEM 1.65

KNOWN: Conditions associated with surface cooling of plate glass which is initially at 600°C. Maximum allowable temperature gradient in the glass.

FIND: Lowest allowable air temperature, T_∞

SCHEMATIC:



ASSUMPTIONS: (1) Surface of glass exchanges radiation with large surroundings at $T_{sur} = T_\infty$, (2) One-dimensional conduction in the x-direction.

ANALYSIS: The maximum temperature gradient will exist at the surface of the glass and at the instant that cooling is initiated. From the surface energy balance, Eq. 1.12, and the rate equations, Eqs. 1.1, 1.3a and 1.7, it follows that

$$-k\frac{dT}{dx} - h\left(T_s - T_\infty\right) - \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right) = 0$$

or, with $(dT/dx)_{max} = -15°C/mm = -15{,}000°C/m$ and $T_{sur} = T_\infty$,

$$-1.4\frac{W}{m\cdot K}\left[-15{,}000\frac{°C}{m}\right] = 5\frac{W}{m^2\cdot K}\left(873 - T_\infty\right)K$$

$$+0.8\times 5.67\times 10^{-8}\frac{W}{m^2\cdot K^4}\left[873^4 - T_\infty^4\right]K^4.$$

T_∞ may be obtained from a trial-and-error solution, from which it follows that, for T_∞ = 618K,

$$21{,}000\frac{W}{m^2} \approx 1275\frac{W}{m^2} + 19{,}730\frac{W}{m^2}.$$

Hence the lowest allowable air temperature is

$$T_\infty \approx 618K = 345°C.$$ <

COMMENTS: (1) Initially, cooling is determined primarily by radiation effects.

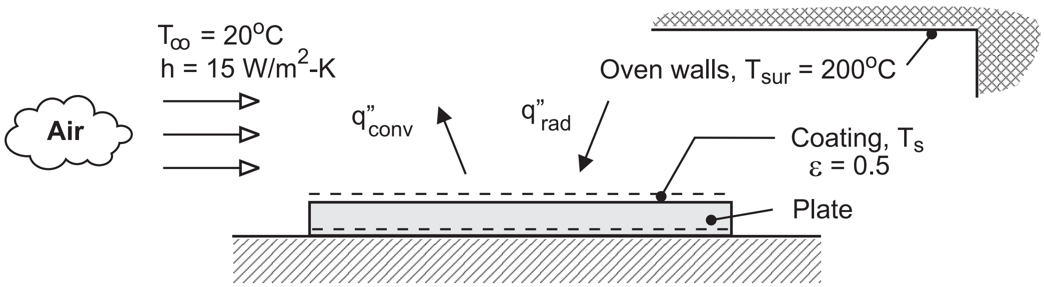
(2) For fixed T_∞, the surface *temperature gradient* would *decrease* with *increasing* time into the cooling process. Accordingly, T_∞ could be decreasing with increasing time and still keep within the maximum allowable temperature gradient.

PROBLEM 1.66

KNOWN: Hot-wall oven, in lieu of infrared lamps, with temperature T_{sur} = 200°C for heating a coated plate to the cure temperature. See Example 1.6.

FIND: (a) The plate temperature T_s for prescribed convection conditions and coating emissivity, and (b) Calculate and plot T_s as a function of T_{sur} for the range $150 \le T_{sur} \le 250$°C for ambient air temperatures of 20, 40 and 60°C; identify conditions for which acceptable curing temperatures between 100 and 110°C may be maintained.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible heat loss from back surface of plate, (3) Plate is small object in large isothermal surroundings (hot oven walls).

ANALYSIS: (a) The temperature of the plate can be determined from an energy balance on the plate, considering radiation exchange with the hot oven walls and convection with the ambient air.

$$\dot{E}''_{in} - \dot{E}''_{out} = 0 \qquad \text{or} \qquad q''_{rad} - q''_{conv} = 0$$

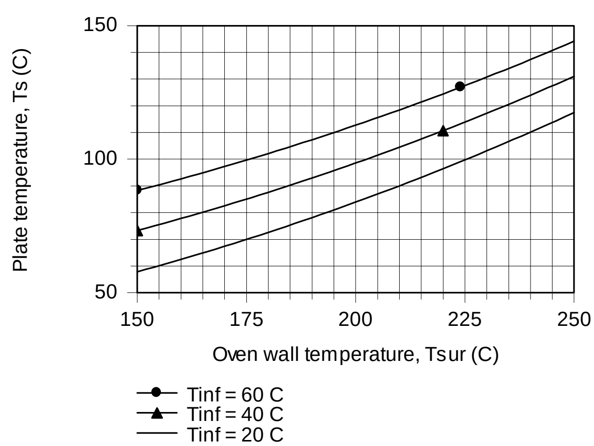
$$\varepsilon\sigma\left(T_{sur}^4 - T_s^4\right) - h\left(T_s - T_\infty\right) = 0$$

$$0.5 \times 5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4 \left(\left[200 + 273\right]^4 - T_s^4\right)\text{K}^4 - 15\ \text{W/m}^2 \cdot \text{K}\left(T_s - \left[20 + 273\right]\right)\text{K} = 0$$

$$T_s = 357\ \text{K} = 84°\text{C}$$

<

(b) Using the energy balance relation in the Workspace of IHT, the plate temperature can be calculated and plotted as a function of oven wall temperature for selected ambient air temperatures.



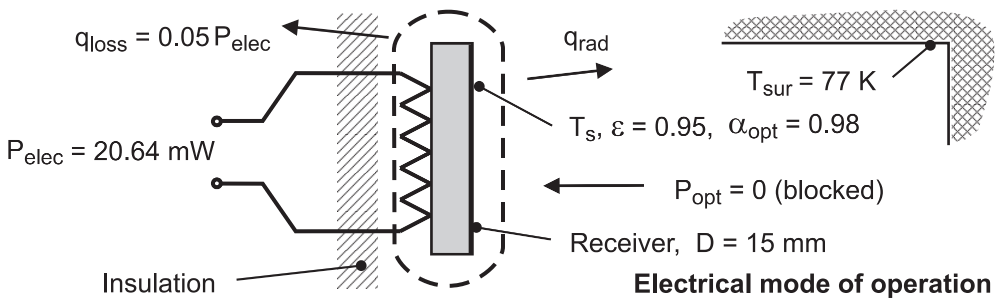
COMMENTS: From the graph, acceptable cure temperatures between 100 and 110°C can be maintained for these conditions: with T_∞ = 20°C when $225 \le T_{sur} \le 240$°C; with T_∞ = 40°C when $205 \le T_{sur} \le 220$°C; and with T_∞ = 60°C when $175 \le T_{sur} \le 195$°C.

PROBLEM 1.67

KNOWN: Operating conditions for an electrical-substitution radiometer having the same receiver temperature, T_s, in electrical and optical modes.

FIND: Optical power of a laser beam and corresponding receiver temperature when the indicated electrical power is 20.64 mW.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Conduction losses from backside of receiver negligible in optical mode, (3) Chamber walls form large isothermal surroundings; negligible effects due to aperture, (4) Radiation exchange between the receiver surface and the chamber walls is between small surface and large enclosure, (5) Negligible convection effects.

PROPERTIES: Receiver surface: ε = 0.95, $\alpha_{opt} = 0.98$.

ANALYSIS: The schematic represents the operating conditions for the *electrical mode* with the optical beam blocked. The temperature of the receiver surface can be found from an energy balance on the receiver, considering the electrical power input, conduction loss from the backside of the receiver, and the radiation exchange between the receiver and the chamber.

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$P_{elec} - q_{loss} - q_{rad} = 0$$

$$P_{elec} - 0.05\,P_{elec} - \varepsilon\, A_s \sigma \left(T_s^4 - T_{sur}^4\right) = 0$$

$$20.64\times10^{-3}\,\text{W}\,(1-0.05) - 0.95\left(\pi 0.015^2/4\right)\text{m}^2 \times 5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left(T_s^4 - 77^4\right)\text{K}^4 = 0$$

$$T_s = 213.9\ \text{K} \qquad <$$

For the *optical mode* of operation, the optical beam is incident on the receiver surface, there is no electrical power input, and the receiver temperature is the same as for the electrical mode. The optical power of the beam can be found from an energy balance on the receiver considering the absorbed beam power and radiation exchange between the receiver and the chamber.

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$\alpha_{opt}\, P_{opt} - q_{rad} = 0.98\,P_{opt} - 19.60\,\text{mW} = 0$$

$$P_{opt} = 19.99\,\text{mW} \qquad <$$

where q_{rad} follows from the previous energy balance using T_s = 213.9K.

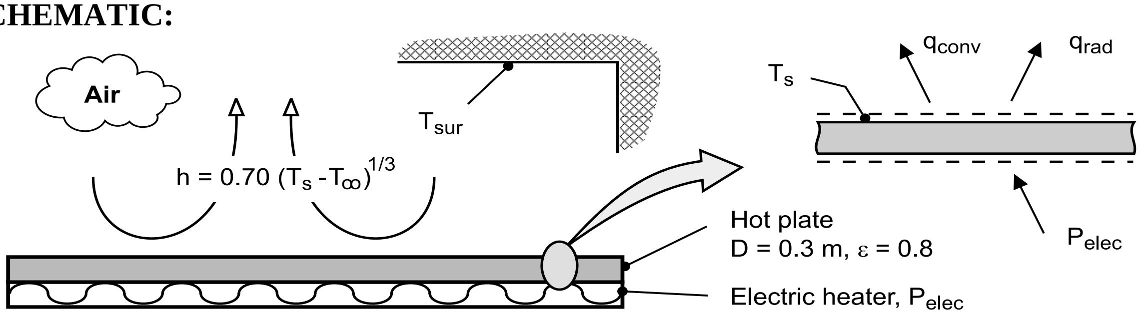
COMMENTS: Recognizing that the receiver temperature, and hence the radiation exchange, is the same for both modes, an energy balance could be directly written in terms of the absorbed optical power and equivalent electrical power, $\alpha_{opt}\, P_{opt} = P_{elec} - q_{loss}$.

PROBLEM 1.68

KNOWN: Surface temperature, diameter and emissivity of a hot plate. Temperature of surroundings and ambient air. Expression for convection coefficient.

FIND: (a) Operating power for prescribed surface temperature, (b) Effect of surface temperature on power requirement and on the relative contributions of radiation and convection to heat transfer from the surface.

SCHEMATIC:



ASSUMPTIONS: (1) Plate is of uniform surface temperature, (2) Walls of room are large relative to plate, (3) Negligible heat loss from bottom or sides of plate.

ANALYSIS: (a) From an energy balance on the hot plate, $P_{elec} = q_{conv} + q_{rad} = A_p\left(q''_{conv} + q''_{rad}\right)$.

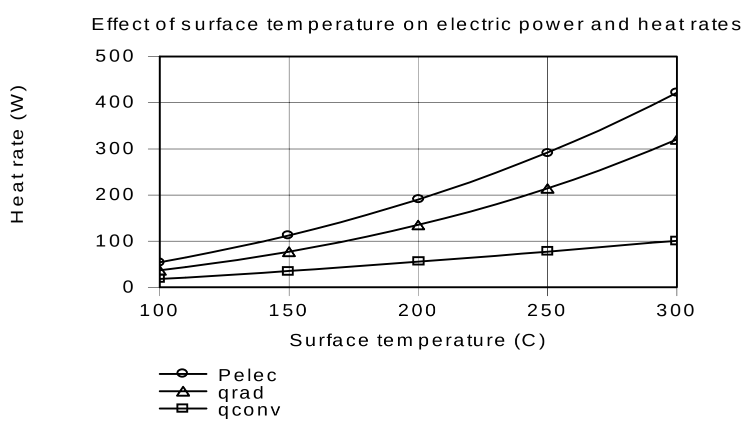
Substituting for the area of the plate and from Eqs. (1.3a) and (1.7), with $h = 0.70\,(T_s - T_\infty)^{1/3}$, it follows that

$$P_{elec} = \left(\pi D^2/4\right)\left[0.70\left(T_s - T_\infty\right)^{4/3} + \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right)\right]$$

$$P_{elec} = \pi\left(0.3\text{m}\right)^2/4\left[0.70\left(175\right)^{4/3} + 0.8\times5.67\times10^{-8}\left(473^4 - 298^4\right)\right]\ \text{W/m}^2$$

$$P_{elec} = 0.0707\ \text{m}^2\left[685\ \text{W/m}^2 + 1913\ \text{W/m}^2\right] = 48.4\ \text{W} + 135.2\ \text{W} = 190.6\ \text{W} \qquad <$$

(b) As shown graphically, both the radiation and convection heat rates, and hence the requisite electric power, increase with increasing surface temperature.



However, because of its dependence on the fourth power of the surface temperature, the increase in radiation is more pronounced. The significant relative effect of radiation is due to the small convection coefficients characteristic of natural convection, with $3.37 \le h \le 5.2\ \text{W/m}^2\cdot\text{K}$ for $100 \le T_s < 300°\text{C}$.

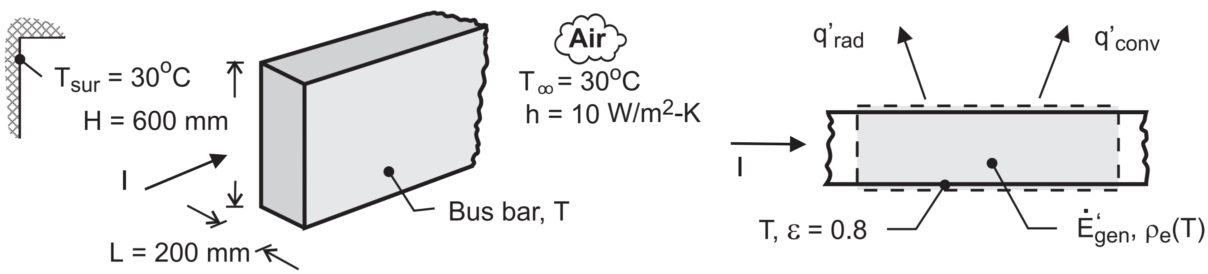
COMMENTS: Radiation losses could be reduced by applying a low emissivity coating to the surface, which would have to maintain its integrity over the range of operating temperatures.

PROBLEM 1.69

KNOWN: Long bus bar of rectangular cross-section and ambient air and surroundings temperatures. Relation for the electrical resistivity as a function of temperature.

FIND: (a) Temperature of the bar with a current of 60,000 A, and (b) Compute and plot the operating temperature of the bus bar as a function of the convection coefficient for the range $10 \le h \le 100$ W/m^2·K. Minimum convection coefficient required to maintain a safe-operating temperature below 120°C. Will increasing the emissivity significantly affect this result?

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Bus bar is long, (3) Uniform bus-bar temperature, (3) Radiation exchange between the outer surface of the bus bar and its surroundings is between a small surface and a large enclosure.

PROPERTIES: Bus-bar material, $\rho_e = \rho_{e,o}\left[1+\alpha\left(T-T_o\right)\right]$, $\rho_{e,o} = 0.0828\,\mu\Omega\cdot m$, $T_o = 25°C$, $\alpha = 0.0040\,K^{-1}$.

ANALYSIS: (a) An energy balance on the bus-bar for a unit length as shown in the schematic above has the form

$$\dot{E}'_{in} - \dot{E}'_{out} + \dot{E}'_{gen} = 0 \qquad\qquad -q'_{rad} - q'_{conv} + I^2 R'_e = 0$$

$$-\varepsilon P\sigma\left(T^4 - T_{sur}^4\right) - hP\left(T - T_\infty\right) + I^2\rho_e / A_c = 0$$

where $P = 2(H+W)$, $R'_e = \rho_e / A_c$ and $A_c = H \times W$. Substituting numerical values,

$$-0.8\times 2(0.600+0.200)\,m\times 5.67\times 10^{-8}\,W/m^2\cdot K^4\left(T^4 - [30+273]^4\right)K^4$$

$$-10\,W/m^2\cdot K\times 2(0.600+0.200)\,m\left(T-[30+273]\right)K$$

$$+(60,000\,A)^2\left\{0.0828\times 10^{-6}\,\Omega\cdot m\left[1+0.0040\,K^{-1}\left(T-[25+273]\right)K\right]\right\}/(0.600\times 0.200)\,m^2 = 0$$

Solving for the bus-bar temperature, find $\quad T = 426\ K = 153°C.$ <

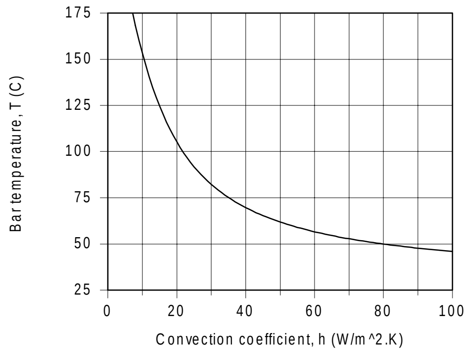
(b) Using the energy balance relation in the Workspace of IHT, the bus-bar operating temperature is calculated as a function of the convection coefficient for the range $10 \le h \le 100$ W/m^2·K. From this graph we can determine that to maintain a safe operating temperature below 120°C, the minimum convection coefficient required is

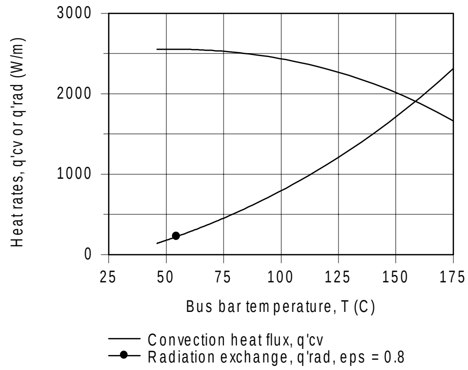
$$h_{min} = 16\ W/m^2\cdot K. \qquad <$$

Continued …..

PROBLEM 1.69 (Cont.)

Using the same equations, we can calculate and plot the heat transfer rates by convection and radiation as a function of the bus-bar temperature.





Note that convection is the dominant mode for low bus-bar temperatures; that is, for low current flow. As the bus-bar temperature increases toward the safe-operating limit (120°C), convection and radiation exchange heat transfer rates become comparable. Notice that the relative importance of the radiation exchange rate increases with increasing bus-bar temperature.

COMMENTS: (1) It follows from the second graph that increasing the surface emissivity will be only significant at higher temperatures, especially beyond the safe-operating limit.

(2) The Workspace for the IHT program to perform the parametric analysis and generate the graphs is shown below. It is good practice to provide commentary with the code making your solution logic clear, and to summarize the results.

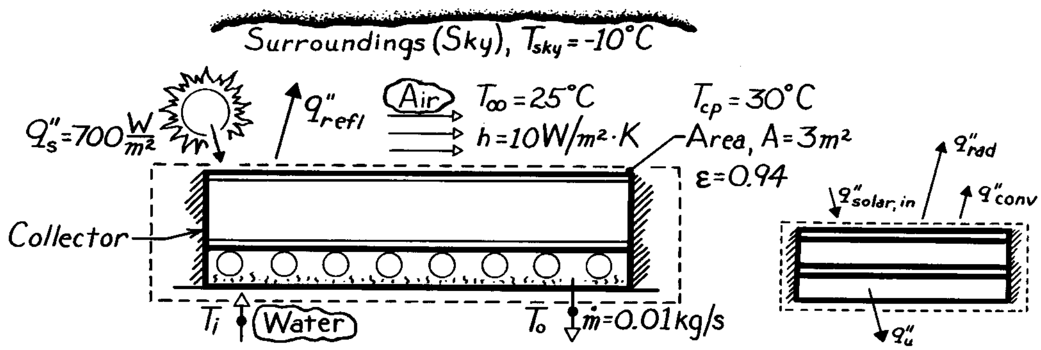
```
/* Results for base case conditions:
Ts_C   q'cv   q'rad   rhoe       H     I     Tinf_C   Tsur_C   W     alpha
       eps    h
153.3  1973   1786    1.253E-7   0.6   6E4   30       30       0.2   0.004
       0.8    10   */

// Surface energy balance on a per unit length basis
-q'cv - q'rad + Edot'gen = 0
q'cv = h * P * (Ts - Tinf)
P = 2 * (W + H)          // perimeter of the bar experiencing surface heat transfer
q'rad = eps * sigma * (Ts^4 - Tsur^4) * P
sigma = 5.67e-8
Edot'gen = I^2 * Re'
Re' = rhoe / Ac
rhoe = rhoeo * ( 1 + alpha * (Ts - Teo))
Ac = W * H

// Input parameters
I = 60000
alpha = 0.0040           // temperature coefficient, K^-1; typical value for cast aluminum
rhoeo = 0.0828e-6        // electrical resistivity at the reference temperature, Teo; microohm-m
Teo = 25 + 273           // reference temperature, K
W = 0.200
H = 0.600
Tinf_C = 30
Tinf = Tinf_C + 273
h = 10
eps = 0.8
Tsur_C = 30
Tsur = Tsur_C + 273
Ts_C = Ts - 273
```

PROBLEM 1.70

KNOWN: Solar collector designed to heat water operating under prescribed solar irradiation and loss conditions.

FIND: (a) Useful heat collected per unit area of the collector, q''_u, (b) Temperature rise of the water flow, $T_o - T_i$, and (c) Collector efficiency.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) No heat losses out sides or back of collector, (3) Collector area is small compared to sky surroundings.

PROPERTIES: *Table A.6,* Water (300K): c_p = 4179 J/kg·K.

ANALYSIS: (a) Defining the collector as the control volume and writing the conservation of energy requirement on a per unit area basis, find that

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \dot{E}_{st}.$$

Identifying processes as per above right sketch,

$$q''_{solar} - q''_{rad} - q''_{conv} - q''_u = 0$$

where $q''_{solar} = 0.9\ q''_S$; that is, 90% of the solar flux is absorbed in the collector (Eq. 1.6). Using the appropriate rate equations, the useful heat rate per unit area is

$$q''_u = 0.9\ q''_S - \varepsilon\sigma\left(T_{cp}^4 - T_{sky}^4\right) - h\left(T_s - T_\infty\right)$$

$$q''_u = 0.9\times700\frac{W}{m^2} - 0.94\times5.67\times10^{-8}\frac{W}{m^2\cdot K^4}\left(303^4 - 263^4\right)K^4 - 10\frac{W}{m^2\cdot K}(30-25)^\circ C$$

$$q''_u = 630\ W/m^2 - 194\ W/m^2 - 50\ W/m^2 = 386\ W/m^2.$$ <

(b) The total useful heat collected is $q''_u \cdot A$. Defining a control volume about the water tubing, the useful heat causes an enthalpy change of the flowing water. That is,

$$q''_u \cdot A = \dot{m}c_p\left(T_i - T_o\right) \qquad \text{or}$$

$$\left(T_i - T_o\right) = 386\ W/m^2 \times 3m^2 / 0.01 kg/s \times 4179 J/kg\cdot K = 27.7^\circ C.$$ <

(c) The efficiency is $\eta = q''_u / q''_S = \left(386\ W/m^2\right)/\left(700\ W/m^2\right) = 0.55$ or 55%. <

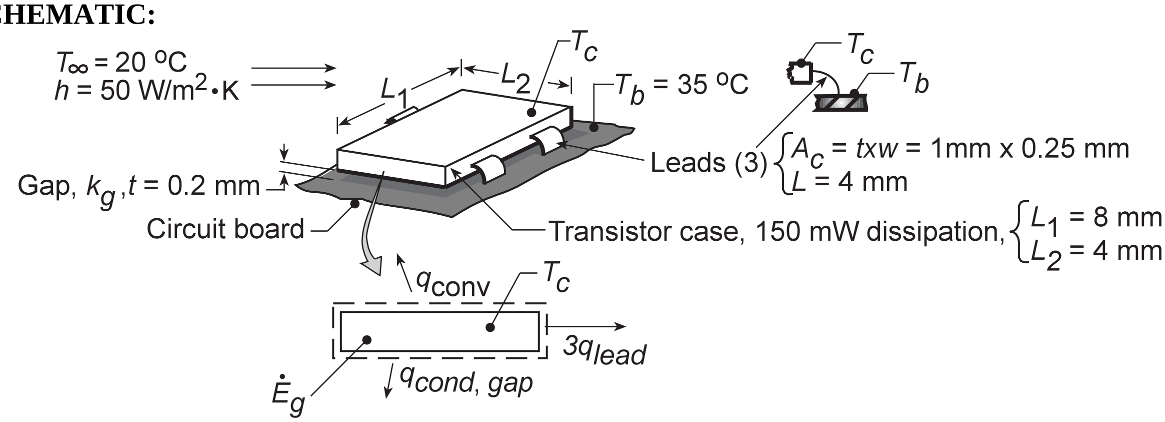
COMMENTS: Note how the sky has been treated as large surroundings at a uniform temperature T_{sky}.

PROBLEM 1.71

KNOWN: Surface-mount transistor with prescribed dissipation and convection cooling conditions.

FIND: (a) Case temperature for mounting arrangement with air-gap and conductive paste between case and circuit board, (b) Consider options for increasing $\dot{E}_g$, subject to the constraint that T_c = 40°C.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Transistor case is isothermal, (3) Upper surface experiences convection; negligible losses from edges, (4) Leads provide conduction path between case and board, (5) Negligible radiation, (6) Negligible energy generation in leads due to current flow, (7) Negligible convection from surface of leads.

PROPERTIES: (Given): Air, $k_{g,a}$ = 0.0263 W/m·K; Paste, $k_{g,p}$ = 0.12 W/m·K; Metal leads, k_ℓ = 25 W/m·K.

ANALYSIS: (a) Define the transistor as the system and identify modes of heat transfer.

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = \Delta\dot{E}_{st} = 0$$

$$-q_{conv} - q_{cond,gap} - 3q_{lead} + \dot{E}_g = 0$$

$$-hA_s(T_c - T_\infty) - k_g A_s \frac{T_c - T_b}{t} - 3k_\ell A_c \frac{T_c - T_b}{L} + \dot{E}_g = 0$$

where $A_s = L_1 \times L_2$ = 4 × 8 mm² = 32 × 10⁻⁶ m² and A_c = t × w = 0.25 × 1 mm² = 25 × 10⁻⁸ m².

Rearranging and solving for T_c,

$$T_c = \left\{hA_s T_\infty + \left[k_g A_s/t + 3(k_\ell A_c/L)\right]T_b + \dot{E}_g\right\} / \left[hA_s + k_g A_s/t + 3(k_\ell A_c/L)\right]$$

Substituting numerical values, with the *air-gap condition* ($k_{g,a}$ = 0.0263 W/m·K)

$$T_c = \left\{50\,\text{W/m}^2\cdot\text{K} \times 32\times10^{-6}\,\text{m}^2 \times 20^\circ\text{C} + \left[\left(0.0263\,\text{W/m}\cdot\text{K}\times32\times10^{-6}\,\text{m}^2/0.2\times10^{-3}\,\text{m}\right)\right.\right.$$

$$\left.\left. +3\left(25\,\text{W/m}\cdot\text{K}\times25\times10^{-8}\,\text{m}^2/4\times10^{-3}\,\text{m}\right)\right]35^\circ\text{C}\right\} / \left[1.600\times10^{-3} + 4.208\times10^{-3} + 4.688\times10^{-3}\right]\text{W/K}$$

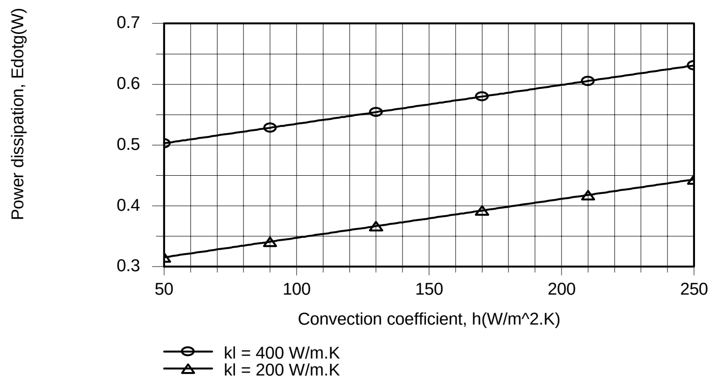
$$T_c = 47.0^\circ\text{C}.$$ <

Continued.….

PROBLEM 1.71 (Cont.)

With the *paste condition* ($k_{g,p}$ = 0.12 W/m·K), T_c = 39.9°C. As expected, the effect of the conductive paste is to improve the coupling between the circuit board and the case. Hence, T_c decreases.

(b) Using the keyboard to enter model equations into the workspace, IHT has been used to perform the desired calculations. For values of k_ℓ = 200 and 400 W/m·K and convection coefficients in the range from 50 to 250 W/m^2·K, the energy balance equation may be used to compute the power dissipation for a maximum allowable case temperature of 40°C.



As indicated by the energy balance, the power dissipation increases linearly with increasing h, as well as with increasing k_ℓ. For h = 250 W/m^2·K (enhanced air cooling) and k_ℓ = 400 W/m·K (copper leads), the transistor may dissipate up to 0.63 W.

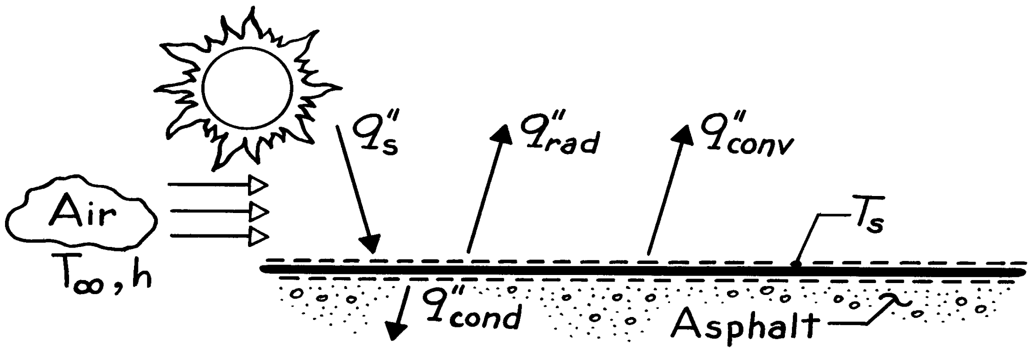
COMMENTS: Additional benefits may be derived by increasing heat transfer across the gap separating the case from the board, perhaps by inserting a highly conductive material in the gap.

PROBLEM 1.72(a)

KNOWN: Solar radiation is incident on an asphalt paving.

FIND: Relevant heat transfer processes.

SCHEMATIC:



The relevant processes shown on the schematic include:

q''_S Incident solar radiation, a large portion of which $q''_{S,abs}$, is absorbed by the asphalt surface,

q''_{rad} Radiation emitted by the surface to the air,

q''_{conv} Convection heat transfer from the surface to the air, and

q''_{cond} Conduction heat transfer from the surface into the asphalt.

Applying the surface energy balance, Eq. 1.12,

$$q''_{S,abs} - q''_{rad} - q''_{conv} = q''_{cond}.$$

COMMENTS: (1) q''_{cond} and q''_{conv} could be evaluated from Eqs. 1.1 and 1.3, respectively.

(2) It has been assumed that the pavement surface temperature is higher than that of the underlying pavement and the air, in which case heat transfer by conduction and convection are from the surface.

(3) For simplicity, radiation incident on the pavement due to atmospheric emission has been ignored (see Section 12.8 for a discussion). Eq. 1.6 may then be used for the absorbed solar irradiation and Eq. 1.5 may be used to obtain the emitted radiation q''_{rad}.

(4) With the rate equations, the energy balance becomes

$$q''_{S,abs} - \varepsilon\,\sigma\,T_s^4 - h\left(T_s - T_\infty\right) = -k\left.\frac{dT}{dx}\right]_s.$$

PROBLEM 1.72(b)

KNOWN: Physical mechanism for microwave heating.

FIND: Comparison of (a) cooking in a microwave oven with a conventional radiant or convection oven and (b) a microwave clothes dryer with a conventional dryer.

(a) Microwave cooking occurs as a result of volumetric thermal energy generation *throughout* the food, without heating of the food container or the oven wall. Conventional cooking relies on radiant heat transfer from the oven walls and/or convection heat transfer from the air space to the surface of the food and subsequent heat transfer by conduction to the core of the food. Microwave cooking is more efficient and is achieved in less time.

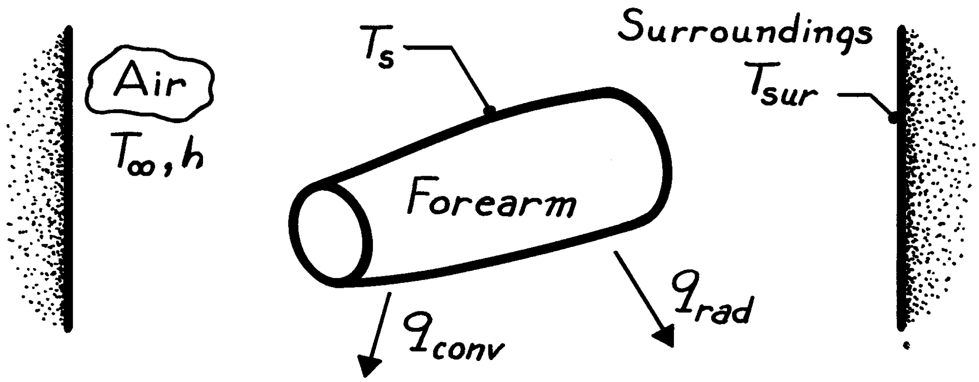
(b) In a microwave dryer, the microwave radiation would heat the water, but not the fabric, directly (the fabric would be heated indirectly by energy transfer from the water). By heating the water, energy would go directly into evaporation, unlike a conventional dryer where the walls and air are first heated electrically or by a gas heater, and thermal energy is subsequently transferred to the wet clothes. The microwave dryer would still require a rotating drum and air flow to remove the water vapor, but is able to operate more efficiently and at lower temperatures. For a more detailed description of microwave drying, see *Mechanical Engineering*, March 1993, page 120.

PROBLEM 1.72(c)

KNOWN: Surface temperature of exposed arm exceeds that of the room air and walls.

FIND: Relevant heat transfer processes.

SCHEMATIC:



Neglecting evaporation from the surface of the skin, the only relevant heat transfer processes are:

q_{conv}	Convection heat transfer from the skin to the room air, and
q_{rad}	Net radiation exchange between the surface of the skin and the surroundings (walls of the room).

You are not imagining things. Even though the room air is maintained at a fixed temperature ($T_\infty = 15°C$), the inner surface temperature of the outside walls, T_{sur}, will decrease with decreasing outside air temperature. Upon exposure to these walls, body heat loss will be larger due to increased q_{rad}.

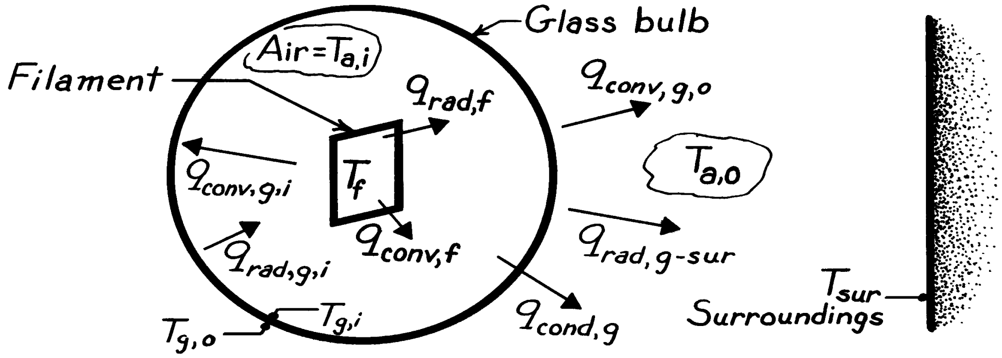
COMMENTS: The foregoing reasoning assumes that the thermostat measures the true room air temperature and is shielded from radiation exchange with the outside walls.

PROBLEM 1.72(d)

KNOWN: Tungsten filament is heated to 2900 K in an air-filled glass bulb.

FIND: Relevant heat transfer processes.

SCHEMATIC:



The relevant processes associated with the filament and bulb include:

$q_{rad,f}$	Radiation emitted by the tungsten filament, a portion of which is transmitted through the glass,
$q_{conv,f}$	Free convection from filament to air of temperature $T_{a,i} < T_f$,
$q_{rad,g,i}$	Radiation emitted by inner surface of glass, a small portion of which is intercepted by the filament,
$q_{conv,g,i}$	Free convection from air to inner glass surface of temperature $T_{g,i} < T_{a,i}$,
$q_{cond,g}$	Conduction through glass wall,
$q_{conv,g,o}$	Free convection from outer glass surface to room air of temperature $T_{a,o} < T_{g,o}$, and
$q_{rad,g-sur}$	Net radiation heat transfer between outer glass surface and surroundings, such as the walls of a room, of temperature $T_{sur} < T_{g,o}$.

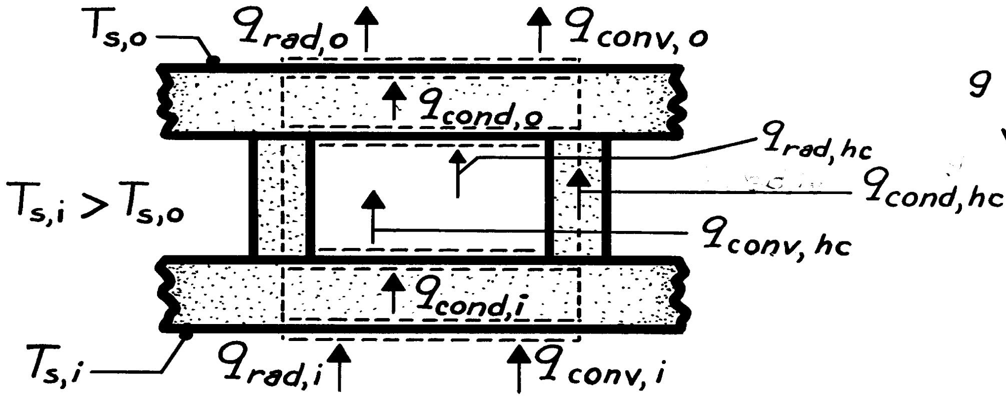
COMMENTS: If the glass bulb is evacuated, no convection is present within the bulb; that is, $q_{conv,f} = q_{conv,g,i} = 0$.

PROBLEM 1.72(e)

KNOWN: Geometry of a composite insulation consisting of a honeycomb core.

FIND: Relevant heat transfer processes.

SCHEMATIC:



The above schematic represents the cross section of a single honeycomb cell and surface slabs. Assumed direction of gravity field is downward. Assuming that the bottom (inner) surface temperature exceeds the top (outer) surface temperature $\left(T_{s,i} > T_{s,o}\right)$, heat transfer is in the direction shown.

Heat may be transferred to the inner surface by convection and radiation, whereupon it is transferred through the composite by

$q_{cond,i}$	Conduction through the inner solid slab,
$q_{conv,hc}$	Free convection through the cellular airspace,
$q_{cond,hc}$	Conduction through the honeycomb wall,
$q_{rad,hc}$	Radiation between the honeycomb surfaces, and
$q_{cond,o}$	Conduction through the outer solid slab.

Heat may then be transferred from the outer surface by convection and radiation. Note that for a single cell under steady state conditions,

$$q_{rad,i} + q_{conv,i} = q_{cond,i} = q_{conv,hc} + q_{cond,hc}$$

$$+ q_{rad,hc} = q_{cond,o} = q_{rad,o} + q_{conv,o}.$$

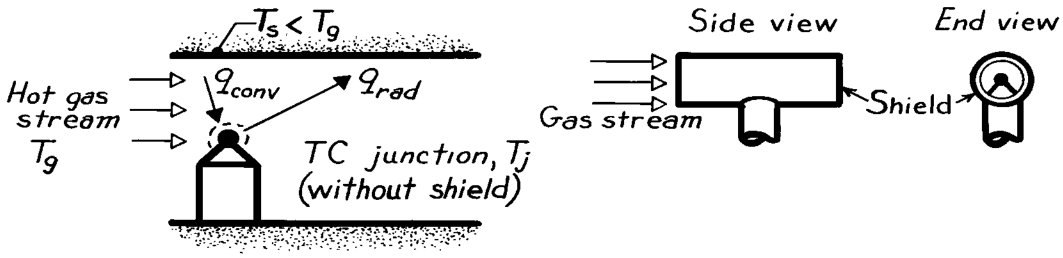
COMMENTS: Performance would be enhanced by using materials of low thermal conductivity, k, and emissivity, ε. Evacuating the airspace would enhance performance by eliminating heat transfer due to free convection.

PROBLEM 1.72(f)

KNOWN: A thermocouple junction is used, with or without a radiation shield, to measure the temperature of a gas flowing through a channel. The wall of the channel is at a temperature much less than that of the gas.

FIND: (a) Relevant heat transfer processes, (b) Temperature of junction relative to that of gas, (c) Effect of radiation shield.

SCHEMATIC:



ASSUMPTIONS: (1) Junction is small relative to channel walls, (2) Steady-state conditions, (3) Negligible heat transfer by conduction through the thermocouple leads.

ANALYSIS: (a) The relevant heat transfer processes are:

q_{rad}	Net radiation transfer from the junction to the walls, and
q_{conv}	Convection transfer from the gas to the junction.

(b) From a surface energy balance on the junction,

$$q_{conv} = q_{rad}$$

or from Eqs. 1.3a and 1.7,

$$h\,A\left(T_j - T_g\right) = \varepsilon\,A\,\sigma\left(T_j^4 - T_s^4\right).$$

To satisfy this equality, it follows that

$$T_s < T_j < T_g.$$

That is, the junction assumes a temperature between that of the channel wall and the gas, thereby sensing a temperature which is less than that of the gas.

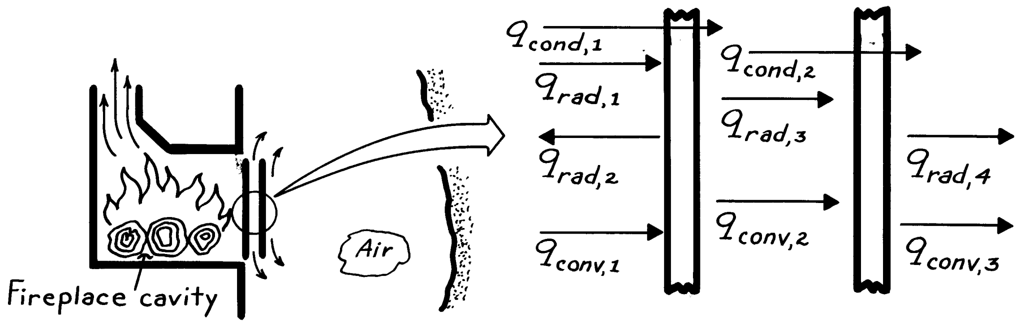
(c) The measurement error $\left(T_g - T_j\right)$ is reduced by using a radiation shield as shown in the schematic. The junction now exchanges radiation with the shield, whose temperature must exceed that of the channel wall. The radiation loss from the junction is therefore reduced, and its temperature more closely approaches that of the gas.

PROBLEM 1.72(g)

KNOWN: Fireplace cavity is separated from room air by two glass plates, open at both ends.

FIND: Relevant heat transfer processes.

SCHEMATIC:



The relevant heat transfer processes associated with the double-glazed, glass fire screen are:

$q_{rad,1}$	Radiation from flames and cavity wall, portions of which are absorbed and transmitted by the two panes,
$q_{rad,2}$	Emission from inner surface of inner pane to cavity,
$q_{rad,3}$	Net radiation exchange between outer surface of inner pane and inner surface of outer pane,
$q_{rad,4}$	Net radiation exchange between outer surface of outer pane and walls of room,
$q_{conv,1}$	Convection between cavity gases and inner pane,
q_{conv2}	Convection across air space between panes,
$q_{conv,3}$	Convection from outer surface to room air,
$q_{cond,1}$	Conduction across inner pane, and
$q_{cond,2}$	Conduction across outer pane.

COMMENTS: (1) Much of the luminous portion of the flame radiation is transmitted to the room interior.

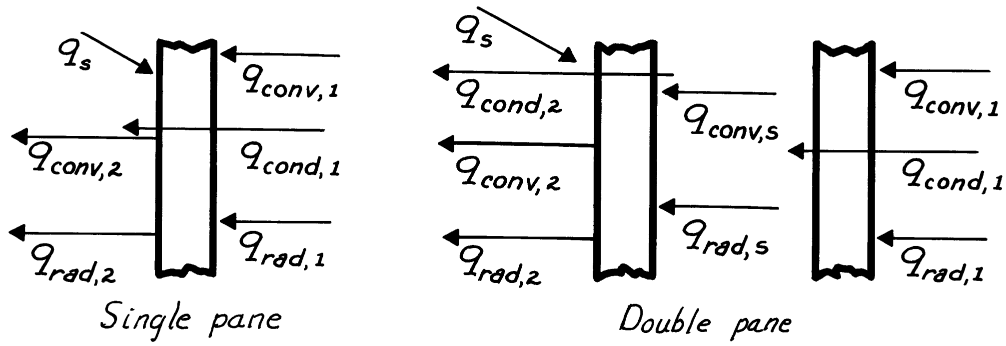
(2) All convection processes are buoyancy driven (free convection).

PROBLEM 1.73(a)

KNOWN: Room air is separated from ambient air by one or two glass panes.

FIND: Relevant heat transfer processes.

SCHEMATIC:



The relevant processes associated with single (above left schematic) and double (above right schematic) glass panes include.

$q_{conv,1}$	Convection from room air to inner surface of first pane,
$q_{rad,1}$	Net radiation exchange between room walls and inner surface of first pane,
$q_{cond,1}$	Conduction through first pane,
$q_{conv,s}$	Convection across airspace between panes,
$q_{rad,s}$	Net radiation exchange between outer surface of first pane and inner surface of second pane (across airspace),
$q_{cond,2}$	Conduction through a second pane,
$q_{conv,2}$	Convection from outer surface of single (or second) pane to ambient air,
$q_{rad,2}$	Net radiation exchange between outer surface of single (or second) pane and surroundings such as the ground, and
q_S	Incident solar radiation during day; fraction transmitted to room is smaller for double pane.

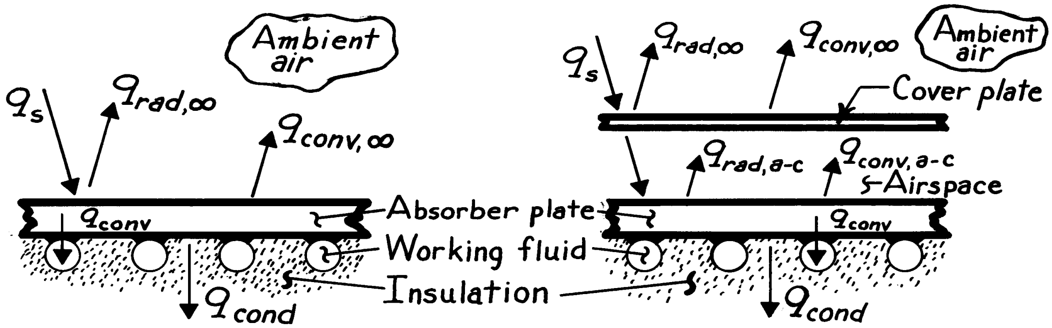
COMMENTS: Heat loss from the room is significantly reduced by the double pane construction.

PROBLEM 1.73(b)

KNOWN: Configuration of a flat plate solar collector.

FIND: Relevant heat transfer processes with and without a cover plate.

SCHEMATIC:



The relevant processes without (above left schematic) and with (above right schematic) include:

q_S	Incident solar radiation, a large portion of which is absorbed by the absorber plate. Reduced with use of cover plate (primarily due to reflection off cover plate).
$q_{rad,\infty}$	Net radiation exchange between absorber plate or cover plate and surroundings,
$q_{conv,\infty}$	Convection from absorber plate or cover plate to ambient air,
$q_{rad,a-c}$	Net radiation exchange between absorber and cover plates,
$q_{conv,a-c}$	Convection heat transfer across airspace between absorber and cover plates,
q_{cond}	Conduction through insulation, and
q_{conv}	Convection to working fluid.

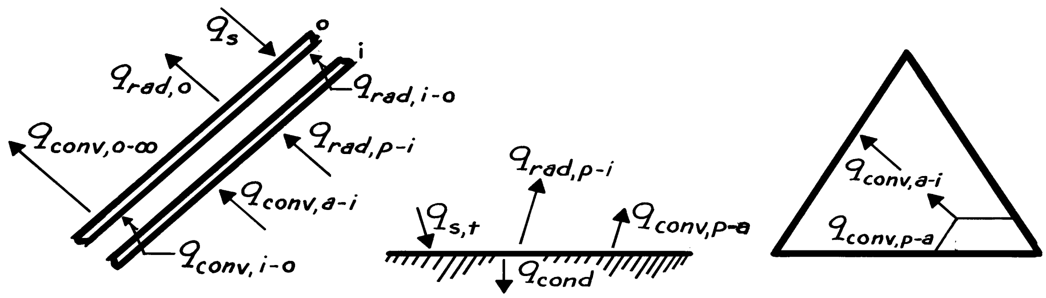
COMMENTS: The cover plate acts to significantly reduce heat losses by convection and radiation from the absorber plate to the surroundings.

PROBLEM 1.73(c)

KNOWN: Configuration of a solar collector used to heat air for agricultural applications.

FIND: Relevant heat transfer processes.

SCHEMATIC:



Assume the temperature of the absorber plates exceeds the ambient air temperature. At the *cover plates*, the relevant processes are:

$q_{conv,a-i}$	Convection from inside air to inner surface,
$q_{rad,p-i}$	Net radiation transfer from absorber plates to inner surface,
$q_{conv,i-o}$	Convection across airspace between covers,
$q_{rad,i-o}$	Net radiation transfer from inner to outer cover,
$q_{conv,o-\infty}$	Convection from outer cover to ambient air,
$q_{rad,o}$	Net radiation transfer from outer cover to surroundings, and
q_S	Incident solar radiation.

Additional processes relevant to the *absorber plates* and *airspace* are:

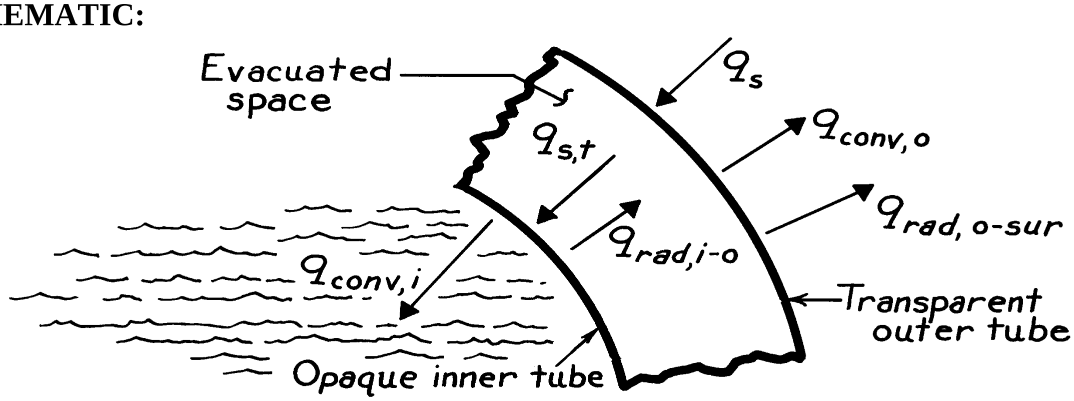
$q_{S,t}$	Solar radiation transmitted by cover plates,
$q_{conv,p-a}$	Convection from absorber plates to inside air, and
q_{cond}	Conduction through insulation.

PROBLEM 1.73(d)

KNOWN: Features of an evacuated tube solar collector.

FIND: Relevant heat transfer processes for one of the tubes.

SCHEMATIC:



The relevant heat transfer processes for one of the evacuated tube solar collectors includes:

q_S	Incident solar radiation including contribution due to reflection off panel (most is transmitted),
$q_{conv,o}$	Convection heat transfer from outer surface to ambient air,
$q_{rad,o\text{-}sur}$	Net rate of radiation heat exchange between outer surface of outer tube and the surroundings, including the panel,
$q_{S,t}$	Solar radiation transmitted through outer tube and incident on inner tube (most is absorbed),
$q_{rad,i\text{-}o}$	Net rate of radiation heat exchange between outer surface of inner tube and inner surface of outer tube, and
$q_{conv,i}$	Convection heat transfer to working fluid.

There is also conduction heat transfer through the inner and outer tube walls. If the walls are thin, the temperature drop across the walls will be small.