PROBLEM 10.1

KNOWN: Water at 1 atm with $T_s - T_{sat} = 10°C$.

FIND: Show that the Jakob number is much less than unity; what is the physical significance of the result; does result apply to other fluids?

ASSUMPTIONS: (1) Boiling situation, $T_s > T_{sat}$.

PROPERTIES: *Table A-5* and *Table A-6*, (1 atm):

	h_{fg} (kJ/kg)	$c_{p,v}$ (J/kg·K)	T_{sat}(K)
Water	2257	2029	373
Ethylene glycol	812	2742*	470
Mercury	301	135.5*	630
R-12	165	1015*	243

* Estimated based upon value at highest temperature cited in Table A-5.

ANALYSIS: The Jakob number is the ratio of the maximum sensible energy absorbed by the vapor to the latent energy absorbed by the vapor during boiling. That is,

$$Ja = \left(c_p \Delta T\right)_v / h_{fg} = c_{p,v} \Delta T_e / h_{fg}$$

For *water* with an excess temperature $\Delta T_s = T_e - T_\infty = 10°C$, find

$$Ja = \left(2029\, J/kg \cdot K \times 10K\right) / 2257 \times 10^3 J/kg$$

$$Ja = 0.0090.$$

Since Ja << 1, the implication is that the sensible energy absorbed by the vapor is much less than the latent energy absorbed during the boiling phase change. Using the appropriate thermophysical properties for three other fluids, the Jakob numbers are:

Ethylene glycol: $Ja = \left(2742 J/kg \cdot K \times 10K\right)/812 \times 10^3 J/kg = 0.0338$ <

Mercury: $Ja = \left(135.5 J/kg \cdot K \times 10K\right)/301 \times 10^3 J/kg = 0.0045$ <

Refrigerant, R-12: $Ja = \left(1015 J/kg \cdot K \times 10K\right)/165 \times 10^3 J/kg = 0.0615$ <

For ethylene glycol and R-12, the Jakob number is larger than the value for water, but still much less than unity. Based upon these example fluids, we conclude that generally we'd expect Ja to be much less than unity.

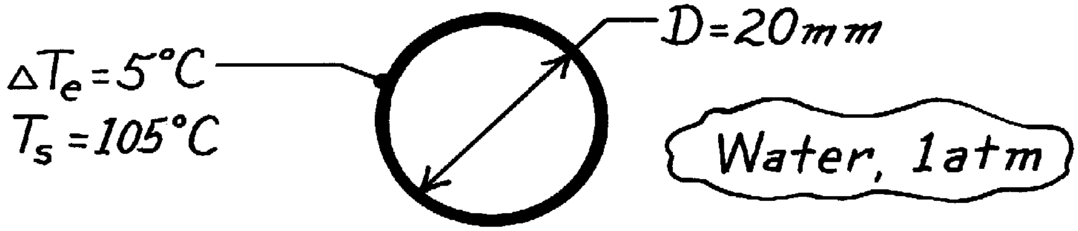
COMMENTS: We would expect the same low value of Ja for the condensation process since $c_{p,g}$ and $c_{p,f}$ are of the same order of magnitude.

PROBLEM 10.2

KNOWN: Horizontal 20 mm diameter cylinder with $\Delta T_e = T_s - T_{sat} = 5°C$ in saturated water, 1 atm.

FIND: Heat flux based upon free convection correlation; compare with boiling curve. Estimate maximum value of the heat transfer coefficient from the boiling curve.

SCHEMATIC:



ASSUMPTIONS: (1) Horizontal cylinder, (2) Free convection, no bubble information.

PROPERTIES: *Table A-6*, Water (Saturated liquid, $T_f = (T_{sat} + T_s)/2 = 102.5\ °C \approx 375K$): $r_\ell = 956.9\ kg/m^3$, $c_{p,\ell} = 4220\ J/kg\cdot K$, $m_\ell = 274 \times 10^{-6}\ N\cdot s/m^2$, $k_\ell = 0.681\ W/m\cdot K$, Pr = 1.70, $\beta = 761 \times 10^{-6}\ K^{-1}$.

ANALYSIS: To estimate the free convection heat transfer coefficient, use the Churchill-Chu correlation,

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = \left\{0.60 + \frac{0.387 Ra_D^{1/6}}{\left[1+\left(0.559/Pr\right)^{9/16}\right]^{8/27}}\right\}^2 .$$

Substituting numerical values, with $\Delta T = \Delta T_e = 5°C$, find

$$Ra_D = \frac{g\,b\,\Delta T\,D^3}{n a} = \frac{9.8\,m/s^2 \times 761\times10^{-6} K^{-1} \times 5°C\left(0.020m\right)^3}{\left[274\times10^{-6} N\cdot s/m^2\ /956.9\ kg/m^3\right]\times1.686\times10^{-7} m^2/s} = 6.178\times10^6$$

where $\alpha = k/\rho\ c_p = (0.681\ W/m\cdot K/956.9\ kg/m^3 \times 4220\ J/kg\cdot K) = 1.686 \times 10^{-7}\ m^2/s$. Note that Ra_D is within the prescribed limits of the correlation. Hence,

$$\overline{Nu}_D = \left\{0.60 + \frac{0.387\left(6.178\times10^6\right)^{1/6}}{\left[1+\left(0.559/1.70\right)^{9/16}\right]^{8/27}}\right\}^2 = 27.22$$

$$\bar{h}_{fc} = Nu_D \frac{k}{D} = \frac{27.22\times0.681 W/m\cdot K}{0.020m} = 928 W/m^2\cdot K. \qquad <$$

Hence, $q_s'' = h_{fc}\Delta T_e = 4640\ W/m^2$

From the typical boiling curve for water at 1 atm, Fig. 10.4, find at $\Delta T_e = 5°C$ that

$$q_s'' \approx 8.5\times10^3 W/m^2 \qquad <$$

The free convection correlation underpredicts (by 1.8) the boiling curve. The maximum value of h_{bc} can be estimated as

$$h_{max} \approx q''_{max}/\Delta T_e = 1.2\times10^6 MW/m^2/30°C = 40{,}000 W/m^2\cdot K. \qquad <$$

COMMENTS: (1) Note the large increase in h with a slight change in ΔT_e.

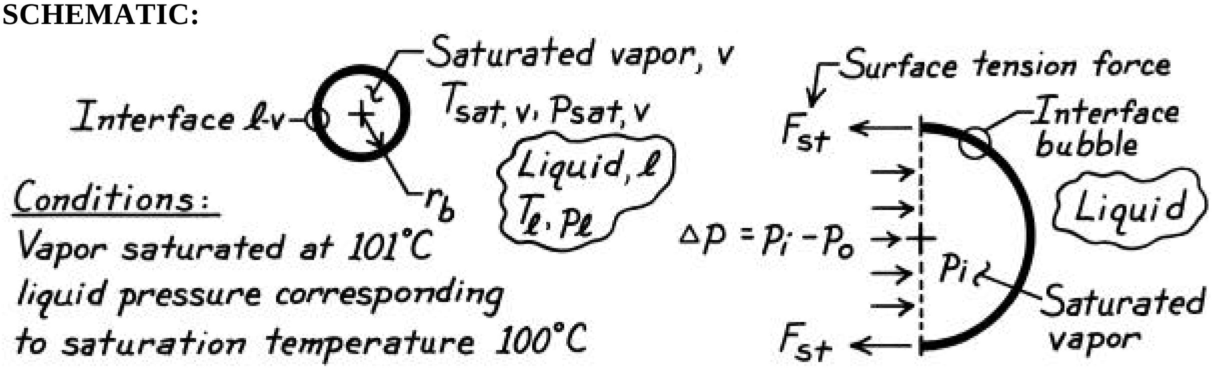
(2) The maximum value of h occurs at point P on the boiling curve.

PROBLEM 10.3

KNOWN: Spherical bubble of pure saturated vapor in mechanical and thermal equilibrium with its liquid.

FIND: (a) Expression for the bubble radius, (b) Bubble vapor and liquid states on a p-v diagram; how changes in these conditions cause bubble to collapse or grow, and (c) Bubble size for specified conditions.

SCHEMATIC:



ASSUMPTIONS: (1) Liquid-vapor medium, (2) Thermal and mechanical equilibrium.

PROPERTIES: *Table A-6*, Water (T_{sat} = 101°C = 374.15K): p_{sat} = 1.0502 bar; (T_{sat} = 100°C = 373.15K): p_{sat} = 1.0133 bar, $\sigma = 58.9 \times 10^{-3}$ N/m.

ANALYSIS: (a) For mechanical equilibrium, the difference in pressure between the vapor inside the bubble and the liquid outside the bubble will be offset by the surface tension of the liquid-vapor interface. The force balance follows from the free-body diagram shown above (right),

$$F_{st} = \left(\pi r_b^2\right)\Delta p = \left(p_i - p_o\right)\left(\pi r_b^2\right)$$

$$\left(2\pi r_b\right)\sigma = \left(\pi r_b^2\right)\left(p_i - p_o\right)$$

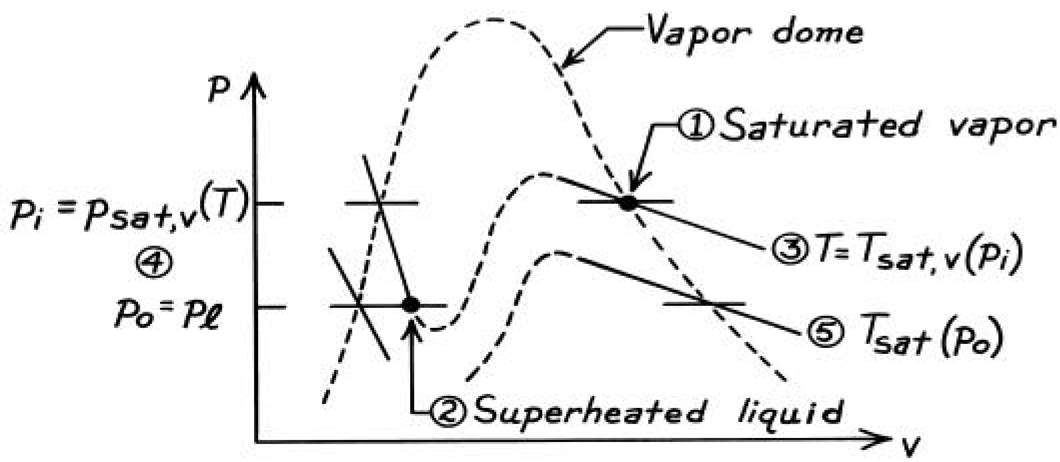
$$r_b = 2\sigma / \left(p_i - p_o\right) \qquad (1)$$

Thermal equilibrium requires that the temperatures of the vapor and liquid be equal. Since the vapor inside the bubble is saturated, $p_i = p_{sat,v}$ (T). Since $p_o < p_i$, it follows that the liquid outside the bubble must be superheated; hence, $p_o = p_\ell$ (T), the pressure of superheated liquid at T. Hence, we can write,

$$r_b = 2\sigma / \left(p_{sat,v} - p_\ell\right) \qquad (2) \quad <$$

(b) The vapor [1] and liquid [2] states are represented on the following p-v diagram. Thermal equilibrium requires both the vapor and liquid to be at the same temperature [3]. But mechanical equilibrium requires that the outside liquid pressure be less than the inside vapor pressure [4]. Hence the liquid must be in a superheated state. That is, its saturation temperature, $T_{sat}(p_o)$ [5] is less than $T_{sat}(p_i)$; $T_\ell = T_{sat}(p_o)$ and $p_o = p_\ell$.

Continued …..



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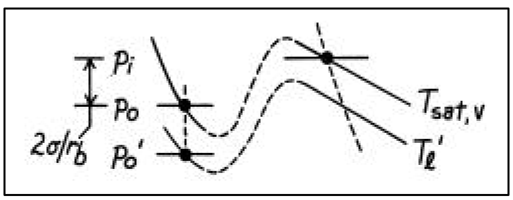
The equilibrium condition for the bubble is unstable. Consider situations for which the pressure of the surrounding liquid is greater or less than the equilibrium value. These situations are presented on portions of the p-v diagram



When $p_o' < p_o$, $T_\ell' < T_{sat,v}$ and heat must be transferred out of the bubble and vapor condenses. Hence, the bubble collapses.

A similar argument for the condition $p_o' > p_o$ leads to $T_\ell' > T_{sat,v}$ and heat is transferred into the bubble causing evaporation with the formation of vapor. Hence, the bubble begins to grow.

(c) Consider the specific conditions

$$T_{sat,v} = 101°C \qquad \text{and} \qquad T_\ell = T_{sat}(p_o) = 100°C$$

and calculate the radius of the bubble using the appropriate properties in Eq. (2).

$$r_b = 2 \times 58.9 \times 10^{-3} \frac{N}{m} / (1.0502 - 1.0133)\,\text{bar} \times \left(10^5 \frac{N}{m^2}/\text{bar}\right)$$

$$r_b = 0.032\text{mm}.$$

<

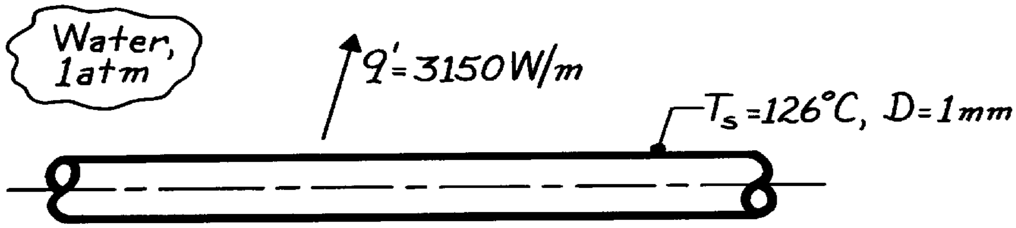
Note the small bubble size. This implies that nucleation sites of the same magnitude formed by pits and crevices are important in promoting the boiling process.

PROBLEM 10.4

KNOWN: Long wire, 1 mm diameter, reaches a surface temperature of 126°C in water at 1 atm while dissipating 3150 W/m.

FIND: (a) Boiling heat transfer coefficient and (b) Correlation coefficient, $C_{s,f}$, if nucleate boiling occurs.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Nucleate boiling.

PROPERTIES: *Table A-6*, Water (saturated, 1 atm): T_s = 100°C, $\rho_\ell = 1/v_f$ = 957.9 kg/m^3, ρ_f = $1/v_g$ = 0.5955 kg/m^3, $c_{p,\ell}$ = 4217 J/kg·K, $\mu_\ell = 279 \times 10^{-6}$ N·s/m^2, Pr_ℓ = 1.76, h_{fg} = 2257 kJ/kg, σ = 58.9×10^{-3} N/m.

ANALYSIS: (a) For the boiling process, the rate equation can be rewritten as

$$\bar{h} = q_s''/\left(T_s - T_{sat}\right) = \frac{q_s'}{\pi D}/\left(T_s - T_{sat}\right)$$

$$\bar{h} = \frac{3150\,\text{W/m}}{\pi \times 0.001\text{m}}/\left(126-100\right)°\text{C} = 1.00\times10^6\,\frac{\text{W}}{\text{m}^2}/26°\text{C} = 38{,}600\,\text{W/m}^2\cdot\text{K}. \qquad <$$

Note the heat flux is very close to q''_{max}, and nucleate boiling does exist.

(b) For nucleate boiling, the Rohsenow correlation may be solved for $C_{s,f}$ to give

$$C_{s,f} = \left\{\frac{\mu_\ell h_{fg}}{q_s''}\right\}^{1/3}\left[\frac{g\left(\rho_\ell - \rho_v\right)}{\sigma}\right]^{1/6}\left(\frac{c_{p,\ell}\Delta T_e}{h_{fg}\,Pr_\ell^n}\right).$$

Assuming the liquid-surface combination is such that n = 1 and substituting numerical values with $\Delta T_e = T_s - T_{sat}$, find

$$C_{s,f} = \left\{\frac{279\times10^{-6}\,\text{N}\cdot\text{s/m}^2\times2257\times10^3\,\text{J/kg}}{1.00\times10^6\,\text{W/m}^2}\right\}^{1/3}\left[\frac{9.8\frac{\text{m}}{\text{s}^2}\left(957.9-0.5955\right)\frac{\text{kg}}{\text{m}^3}}{58.9\times10^{-3}\,\text{N/m}}\right]^{1/6}\times$$

$$\left(\frac{4217\,\text{J/kg}\cdot\text{K}\times26\text{K}}{2257\times10^3\,\text{J/kg}\times1.76}\right)$$

$$C_{s,f} = 0.017. \qquad <$$

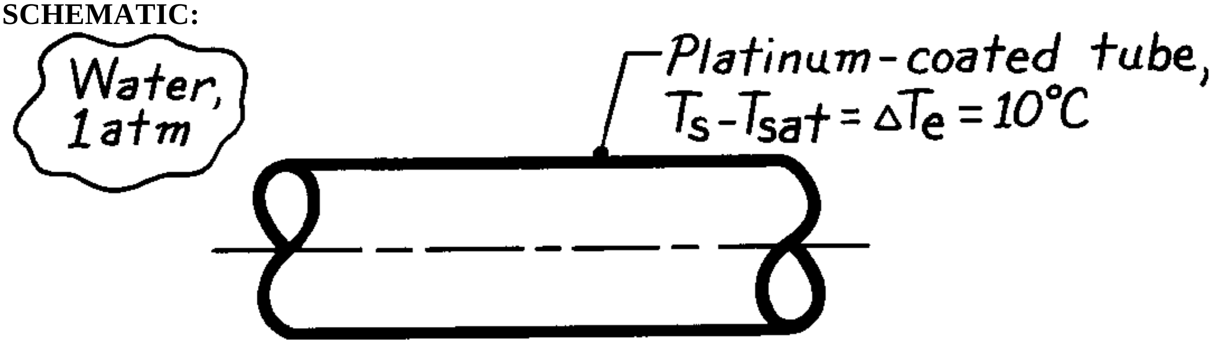
COMMENTS: By comparison with the values of $C_{s,f}$ for other water-surface combinations of Table 10.1, the $C_{s,f}$ value for the wire is large, suggesting that its surface must be highly polished. Note that the value of the boiling heat transfer coefficient is much larger than values common to single-phase convection.

PROBLEM 10.5

KNOWN: Nucleate pool boiling on a 10 mm-diameter tube maintained at ΔT_e = 10°C in water at 1 atm; tube is platinum-plated.

FIND: Heat transfer coefficient.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Nucleate pool boiling.

PROPERTIES: *Table A-6*, Water (saturated, 1 atm): T_s = 100°C, r_ℓ = $1/v_f$ = 957.9 kg/m^3, ρ_v = $1/v_g$ = 0.5955 kg/m^3, $c_{p,\ell}$ = 4217 J/kg·K, m_ℓ = 279 × 10^{-6} N·s/m^2, Pr_ℓ = 1.76, h_{fg} = 2257 kJ/kg, σ = 58.9 × 10^{-3} N/m.

ANALYSIS: The heat transfer coefficient can be estimated using the Rohsenow nucleate-boiling correlation and the rate equation

$$h = \frac{q_s''}{\Delta T_e} = \frac{m_\ell h_{fg}}{\Delta T_e}\left[\frac{g(r_\ell - r_v)}{s}\right]^{1/2}\left(\frac{c_{p,\ell}\,\Delta T_e}{C_{s,f}\,h_{fg}\,Pr_\ell^n}\right)^3.$$

From Table 10.1, find $C_{s,f}$ = 0.013 and n = 1 for the water-platinum surface combination. Substituting numerical values,

$$h = \frac{279\times10^{-6}\,\mathrm{N\cdot s/m^2}\times2257\times10^3\,\mathrm{J/kg}}{10\mathrm{K}}\left[\frac{9.8\,\mathrm{m/s^2}\,(957.9-0.5955)\,\mathrm{kg/m^3}}{58.9\times10^{-3}\,\mathrm{N/m}}\right]^{1/2}\times$$

$$\left(\frac{4217\,\mathrm{J/kg\cdot K}\times10\mathrm{K}}{0.013\times2257\times10^3\,\mathrm{J/kg}\times1.76}\right)^3$$

$$h = 13{,}690\ \mathrm{W/m^2\cdot K}.$$ <

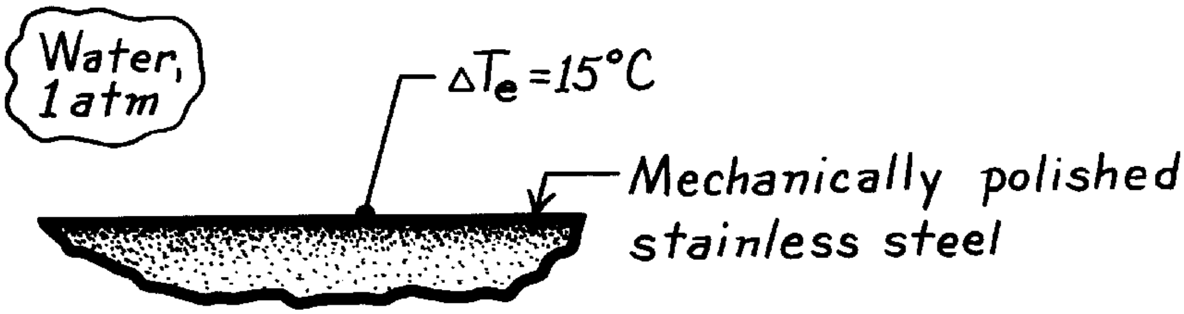
COMMENTS: For this liquid-surface combination, $q_s'' = 0.137\,\mathrm{MW/m^2}$, which is in general agreement with the *typical* boiling curve of Fig. 10.4. To a first approximation, the effect of the tube diameter is negligible.

PROBLEM 10.6

KNOWN: Water boiling on a mechanically polished stainless steel surface maintained at an excess temperature of 15°C; water is at 1 atm.

FIND: Boiling heat transfer coefficient.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Nucleate pool boiling occurs.

PROPERTIES: *Table A-6*, Saturated water (1 atm): T_{sat} = 100°C, r_ℓ = 957.9 kg/m^3, ρ_v = 0.596 kg/m^3, $c_{p,\ell}$ = 4217 J/kg·K, m_ℓ = 279 × 10^{-6} N·s/m^2, Pr_ℓ = 1.76, σ = 58.9 × 10^{-3} N/m, h_{fg} = 2257 kJ/kg.

ANALYSIS: The heat transfer coefficient can be expressed as

$$h = q_s'' / \Delta T_e$$

where the nucleate pool boiling heat flux can be estimated using the Rohsenow correlation.

$$q_s'' = m_\ell h_{fg}\left[\frac{g\left(r_\ell - r_v\right)}{s}\right]^{1/2}\left(\frac{c_{p,\ell}\Delta T_e}{C_{s,f}\, h_{fg}\, Pr_\ell^n}\right)^3 .$$

From Table 10.1, find for this liquid-surface combination, $C_{s,f}$ = 0.013 and n = 1, and substituting numerical values,

$$q_s'' = 279\times10^{-6}\,N\cdot s/m^2\times2257\times10^3\,J/kg\left[\frac{9.8\,m/s^2\left(957.9-0.596\right)kg/m^3}{58.9\times10^{-3}\,N/m}\right]^{1/2}\times$$

$$\left(\frac{4217\,J/kg\cdot K\times15°C}{0.013\times2257\,kJ/kg\times1.76}\right)^3$$

$$q_s'' = 461.9\,kW/m^2.$$

Hence, the heat transfer coefficient is

$$h = 461.9\times10^3\,W/m^2/15°C = 30{,}790\,W/m^2\cdot K.$$ <

COMMENTS: Note that this value of q_s'' for ΔT_e = 15°C is consistent with the typical boiling curve, Fig. 10.4.

PROBLEM 10.7

KNOWN: Simple expression to account for the effect of pressure on the nucleate boiling convection coefficient in water.

FIND: Compare predictions of this expression with the Rohsenow correlation for specified ΔT_e and pressures (2 and 5 bar) applied to a horizontal plate.

ASSUMPTIONS: (1) Steady-state conditions, (2) Nucleate pool boiling, (3) $C_{s,f} = 0.013$, n = 1.

PROPERTIES: *Table A-6*, Saturated water (2 bar): $r_\ell = 942.7\ \text{kg/m}^3$, $c_{p,\ell} = 4244.3\ \text{J/kg·K}$, $m_\ell = 230.7 \times 10^{-6}\ \text{N·s/m}^2$, $\Pr_\ell = 1.43$, $h_{fg} = 2203\ \text{kJ/kg}$, $\sigma = 54.97 \times 10^{-3}\ \text{N/m}$, $\rho_v = 1.1082\ \text{kg/m}^3$; Saturated water (5 bar): $r_\ell = 914.7\ \text{kg/m}^3$, $c_{p,\ell} = 4316\ \text{J/kg·K}$, $m_\ell = 179 \times 10^{-6}\ \text{N·s/m}^2$, $\Pr_\ell = 1.13$, $h_{fg} = 2107.8\ \text{kJ/kg}$, $\sigma = 48.4 \times 10^{-3}\ \text{N/m}$, $\rho_v = 2.629\ \text{kg/m}^3$.

ANALYSIS: The simple expression by Jakob [51] accounting for pressure effects is

$$h = C(\Delta T_e)^n (p/p_a)^{0.4} \qquad (1)$$

where p and p_a are the system and standard atmospheric pressures. For a horizontal plate, C = 5.56 and n = 3 for the range $15 < q_s'' < 235\ \text{kW/m}^2$. For $\Delta T_e = 10°\text{C}$,

p = 2 bar $\quad h = 5.56\,(10)^3\,(2\text{bar}/1.0133\text{bar})^{0.4} = 7{,}298\ \text{W/m}^2\cdot\text{K}, \quad q_s'' = 73\ \text{kW/m}^2$ <

p = 5 bar $\quad h = 5.56\,(10)^3\,(5\text{bar}/1.0133\text{bar})^{0.4} = 10{,}529\ \text{W/m}^2\cdot\text{K}, \quad q_s'' = 105\ \text{kW/m}^2$ <

where $q_s'' = h\Delta T_e$. The Rohsenow correlation, Eq. 10.5, with $C_{s,f} = 0.013$ and n = 1, is of the form

$$q_s'' = m_\ell h_{fg} \left[\frac{g(r_\ell - r_v)}{s}\right]^{1/2} \left[\frac{c_{p,\ell}\Delta T_e}{C_{s,f}\, h_{fg} \Pr_\ell^n}\right]^3. \qquad (2)$$

p = 2 bar: $$q_s' = 230.7 \times 10^{-6}\ \frac{\text{N}\cdot\text{s}}{\text{m}^2} \times 2203 \times 10^3\ \frac{\text{J}}{\text{kg}} \left[\frac{9.8\,\frac{\text{m}}{\text{s}^2}(942.7 - 1.1082)\,\frac{\text{kg}}{\text{m}^3}}{54.97 \times 10^{-3}\ \text{N/m}}\right]^{1/2} \times \left[\frac{4244.3\text{J/kg}\cdot\text{K} \times 10\text{K}}{0.013 \times 2203 \times 10^3\ \frac{\text{J}}{\text{kg}} \times 1.43^1}\right]^3$$

$$q_s'' = 232\ \text{kW/m}^2$$ <

p = 4 bar: $\quad q_s'' = 439\ \text{kW/m}^2.$ <

COMMENTS: For ease of comparison, the results with $p_a = 1.0133$ bar are:

	$q_s''\ (\text{kW/m}^2)$		
Correlation/p (bar)	1	2	4
Simple	56	73	105
Rohsenow	135	232	439

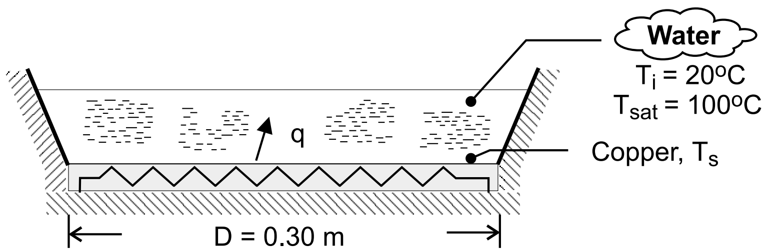
Note that the range of q_s'' is within the limits of the Simple correlation. The comparison is poor and therefore the correlation is not to be recommended. By manipulation of the Rohsenow results, find that the $(p/p_o)^m$ dependence provides $m \approx 0.75$, compared to the exponent of 0.4 in the Simple correlation.

PROBLEM 10.8

KNOWN: Diameter of copper pan. Initial temperature of water and saturation temperature of boiling water. Range of heat rates ($1 \le q \le 100$ kW).

FIND: (a) Variation of pan temperature with heat rate for boiling water, (b) Pan temperature shortly after start of heating with q = 8 kW.

SCHEMATIC:



ASSUMPTIONS: (1) Conditions of part (a) correspond to steady nucleate boiling, (2) Surface of pan corresponds to polished copper, (3) Conditions of part (b) correspond to natural convection from a heated plate to an infinite quiescent medium, (4) Negligible heat loss to surroundings.

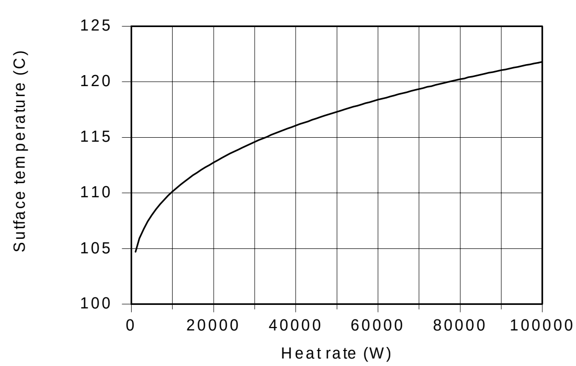
PROPERTIES: *Table A-6*, saturated water (T_{sat} = 100°C): $\rho_\ell = 957.9\ \text{kg/m}^3$, $\rho_v = 0.60\,\text{kg/m}^3$,

$c_{p,\ell} = 4217\,\text{J/kg}\cdot\text{K}$, $\mu_\ell = 279\times10^{-6}\ \text{N}\cdot\text{s/m}^2$, $Pr_\ell = 1.76$, $h_{fg} = 2.257\times10^6\ \text{J/kg}$, $\sigma = 0.0589\ \text{N/M}$.
Table A-6, saturated water (assume T_s = 100°C, T_f = 60°C = 333 K): ρ = 983 kg/m^3, $\mu = 467 \times 10^{-6}$ N·s/m^2, k = 0.654 W/m·K, Pr = 2.99, $\beta = 523 \times 10^{-6}$ K^{-1}. Hence, $\nu = 0.475 \times 10^{-6}$ m^2/s, $\alpha = 0.159 \times 10^{-6}$ m^2/s.

ANALYSIS: (a) From Eq. (10.5),

$$\Delta T_e = T_s - T_{sat} = \frac{C_{s,f}\ h_{fg}\ Pr_\ell^n}{c_{p,\ell}} \times \left\{ \frac{q_s / \mu_\ell\, h_{fg}\, A_s}{\left[g\left(\rho_\ell - \rho_v\right)/\sigma \right]^{1/2}} \right\}^{1/3}$$

For n = 1.0, $C_{s,f}$ = 0.013 and $A_s = \pi D^2/4 = 0.0707$ m^2, the following variation of T_s with q_s is obtained.



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As indicated by the correlation, the surface temperature increases as the cube root of the heat rate, permitting large increases in q for modest changes in T_s. For q = 1 kW, Ts = 104.7°C, which is barely sufficient to sustain boiling.

(b) Assuming $10^7 < Ra_L < 10^{11}$, the convection coefficient may be obtained from Eq. (9.31). Hence, with L = A_s/P = D/4 = 0.075m,

Continued …..

$$\bar{h} = \left(\frac{k}{L}\right)0.15\,Ra_L^{1/3} = \left(\frac{0.654\,W/m\cdot K}{0.075m}\right)0.15\left[\frac{9.8\,m/s^2 \times 523\times10^{-6}\,K^{-1}\left(T_s - T_i\right)\left(0.075m\right)^3}{0.475\times0.159\times10^{-12}\,m^4/s^2}\right]^{1/3}$$

$$= 1.308\left(2.86\times10^7\right)^{1/3}\left(T_s - T_i\right)^{1/3} = 400\left(T_s - T_i\right)^{1/3}$$

With As = $\pi D^2/4 = 0.0707\ m^2$, the heat rate is then

$$q = \bar{h}A_s\left(T_s - T_i\right) = \left(400\,W/m^2\cdot K^{4/3}\right)0.0707\,m^2\left(T_s - T_i\right)^{4/3}$$

With q = 8000 W,

$$T_s = T_i + 69°C = 89°C \qquad <$$

COMMENTS: (1) With $(T_s – T_i)$ = 69°C, $Ra_L = 1.97 \times 10^9$, which is within the assumed Rayleigh number range. (2) The surface temperature increases as the temperature of the water increases, and bubbles may nucleate when it exceeds 100°C. However, while the water temperature remains below the saturation temperature, the bubbles will collapse in the subcooled liquid.

PROBLEM 10.9

KNOWN: Fluids at 1 atm: mercury, ethanol, R-12.

FIND: Critical heat flux; compare with value for water also at 1 atm.

ASSUMPTIONS: (1) Steady-state conditions, (2) Nucleate pool boiling.

PROPERTIES: *Table A-5* and *Table A-6* at 1 atm,

	h_{fg} (kJ/kg)	ρ_v	r_ℓ (kg/m^3)	$\sigma \times 10^3$ (N/m)	T_{sat} (K)
Mercury	301	3.90	12,740	417	630
Ethanol	846	1.44	757	17.7	351
R-12	165	6.32	1,488	15.8	243
Water	2257	0.596	957.9	58.9	373

ANALYSIS: The critical heat flux can be estimated by the modified Zuber-Kutateladze correlation, Eq. 10.7,

$$q''_{max} = 0.149\, h_{fg}\, r_v \left[\frac{s\, g(r_\ell - r_v)}{r_v^2}\right]^{1/4}.$$

To illustrate the calculation procedure, consider numerical values for *mercury*.

$$q''_{max} = 0.149 \times 301 \times 10^3 \,\mathrm{J/kg} \times 3.90\,\mathrm{kg/m^3} \times \left[\frac{417 \times 10^{-3}\,\mathrm{N/m} \times 9.8\,\mathrm{m/s^2}\,(12{,}740 - 3.90)\,\mathrm{kg/m^3}}{\left(3.90\,\mathrm{kg/m^3}\right)^2}\right]^{1/4}$$

$$q''_{max} = 1.34\ \mathrm{MW/m^2}.$$

For the other fluids, the results are tabulated along with the ratio of the critical heat fluxes to that for water.

Flluid	$q''_{max}\left(\mathrm{MW/m^2}\right)$	$q''_{max} / q''_{max,water}$
Mercury	1.34	1.06
Ethanol	0.512	0.41
R-12	0.241	0.19
Water	1.26	1.00

<

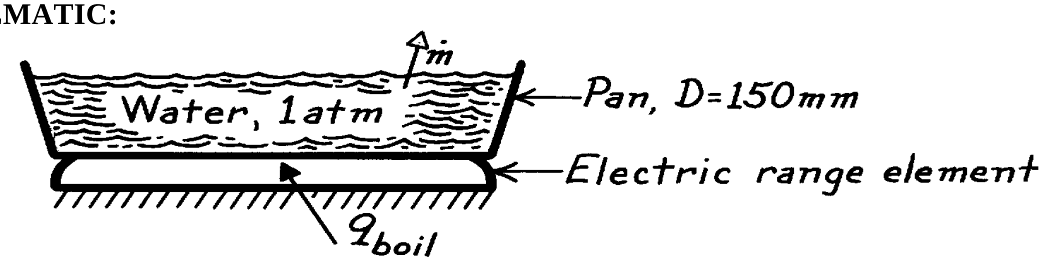
COMMENTS: Note that, despite the large difference between mercury and water properties, their critical heat fluxes are similar.

PROBLEM 10.10

KNOWN: Copper pan, 150 mm diameter and filled with water at 1 atm, is maintained at 115°C.

FIND: Power required to boil water and the evaporation rate; ratio of heat flux to critical heat flux; pan temperature required to achieve critical heat flux.

SCHEMATIC:



ASSUMPTIONS: (1) Nucleate pool boiling, (2) Copper pan is polished surface.

PROPERTIES: *Table A-6*, Water (1 atm): T_{sat} = 100°C, r_ℓ = 957.9 kg/m^3, r_v = 0.5955 kg/m^3, $c_{p,\ell}$ = 4217 J/kg·K, m_ℓ = 279 × 10^{-6} N·s/m^2, Pr_ℓ = 1.76, h_{fg} = 2257 kJ/kg, σ = 58.9 × 10^{-3} N/m.

ANALYSIS: The power requirement for boiling and the evaporation rate can be expressed as follows,

$$q_{boil} = q_s'' \cdot A_s \qquad \dot{m} = q_{boil} / h_{fg}.$$

The heat flux for nucleate pool boiling can be estimated using the Rohsenow correlation.

$$q_s'' = m_\ell h_{fg} \left[\frac{g(r_\ell - r_v)}{s} \right]^{1/2} \left(\frac{c_{p,\ell} \Delta T_e}{C_{s,f}\, h_{fg}\, Pr_\ell^n} \right)^3 .$$

Selecting $C_{s,f}$ = 0.013 and n = 1 from Table 10.1 for the polished copper finish, find

$$q_s'' = 279 \times 10^{-6} \frac{N \cdot s}{m^2} \times 2257 \times 10^3 \frac{J}{kg} \left[\frac{9.8 \frac{m}{s^2} (957.9 - 0.5955) \frac{kg}{m^3}}{589 \times 10^{-3}\, N/m} \right]^{1/2} \left(\frac{4217 \frac{J}{kg \cdot K} \times 15°C}{0.013 \times 2257 \times 10^3 \frac{J}{kg} \times 1.76} \right)^3$$

$$q_s'' = 4.619 \times 10^5\, W/m^2.$$

The power and evaporation rate are

$$q_{boil} = 4.619 \times 10^5\, W/m^2 \times \frac{p}{4} (0.150m)^2 = 8.16kW$$ <

$$\dot{m}_{boil} = 8.16kW / 2257 \times 10^3 J/kg = 3.62 \times 10^{-3} kg/s = 13 kg/h.$$ <

The maximum or critical heat flux was found in Example 10.1 as

$$q_{max}'' = 1.26 MW/m^2.$$

Hence, the ratio of the operating to maximum heat flux is

$$\frac{q_s''}{q_{max}''} = 4.619 \times 10^5\, W/m^2 / 1.26 MW/m^2 = 0.367.$$ <

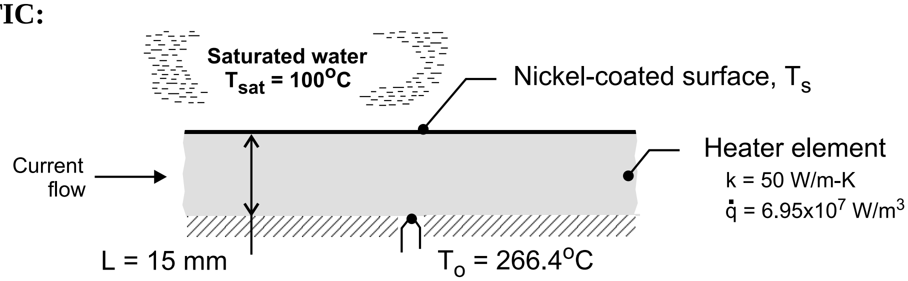
From the boiling curve, Fig. 10.4, $\Delta T_e \approx$ 30°C will provide the maximum heat flux. <

PROBLEM 10.11

KNOWN: Nickel-coated heater element exposed to saturated water at atmospheric pressure; thermocouple attached to the insulated, backside surface indicates a temperature $T_o = 266.4°C$ when the electrical power dissipation in the heater element is $6.950 \times 10^7\ W/m^3$.

FIND: (a) From the foregoing data, calculate the surface temperature, T_s, and the heat flux at the exposed surface, and (b) Using an appropriate boiling correlation, estimate the surface temperature based upon the surface heat flux determined in part (a).

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Water exposed to standard atmospheric pressure and uniform temperature, T_{sat}, and (3) Nucleate pool boiling occurs on exposed surface, (4) Uniform volumetric generation in element, and (5) Backside of heater is perfectly insulated.

PROPERTIES: *Table A-6*, Saturated water, liquid (100°C): $\rho_\ell = 1/v_f = 957.9\ kg/m^3$, $c_{p,\ell} = c_{p,f} = 4.217\ kJ/kg \cdot K$, $\mu_\ell = \mu_f = 279 \times 10^{-6} N \cdot s/m^2$, $Pr_\ell = Pr_f = 1.76$, $h_{fg} = 2257$ kJ/kg, $\sigma = 58.9 \times 10^{-3}$ N/m; Saturated water, vapor (100°C): $\rho_v = 1/v_g = 0.5955\ kg/m^3$.

ANALYSIS: (a) From Eq. 3.43, the temperature at the exposed surface, T_s, is

$$T_s = T_o - \frac{\dot{q}L^2}{2k} = 266.4°C - \frac{6.95 \times 10^7\ W/m^3 (0.015\ m)^2}{2 \times 50\ W/m \cdot K}$$

$$T_s = 110.0°C \qquad <$$

The heat flux at the exposed surface is

$$q_s'' = \dot{q}/L = 6.95 \times 10^7\ W/m^3 / 0.015\ m = 4.63 \times 10^9\ W/m^2 \qquad <$$

(b) Since $\Delta T_e = T_s - T_{sat} = (110 - 100)°C = 10°C$, nucleate pool boiling occurs and the Rohsenow correlation, Eq. 10.5, with q_s'' from part (a) can be used to estimate the surface temperature, $T_{s,c}$,

$$q_s'' = \mu_\ell h_{fg} \left[\frac{g(\rho_\ell - \rho_v)}{\sigma} \right]^{1/2} \left(\frac{c_{p,\ell} \Delta T_{e,c}}{C_{s,f}\ h_{fg}\ Pr_\ell^n} \right)^3$$

From Table 10.1, for the water-nickel surface-fluid combination, $C_{s,f} = 0.006$ and n = 1.0. Substituting numerical values, find $\Delta T_{e,c}$ and $T_{s,c}$.

Continued …..

$$4.63\times10^{9}\ \mathrm{W/m^2} = 279\times10^{-6}\ \mathrm{N\cdot s/m^2}\times2257\times10^{3}\ \mathrm{J/kg}$$

$$\times\left[\frac{9.8\ \mathrm{m/s^2}\,(957.9-0.5955)\,\mathrm{kg/m^3}}{58.9\times10^{-3}\ \mathrm{N/m}}\right]^{1/2}$$

$$\times\left(\frac{4.217\times10^{3}\ \mathrm{J/kg\cdot K}\times\Delta T_{e,c}}{0.006\times2257\times10^{3}\ \mathrm{J/kg}\times1.76}\right)^{3}$$

$$\Delta T_{e,c} = T_{s,c} - T_{sat} = 9.1^\circ\mathrm{C} \qquad\qquad T_{s,c} = 109.1^\circ\mathrm{C}$$ <

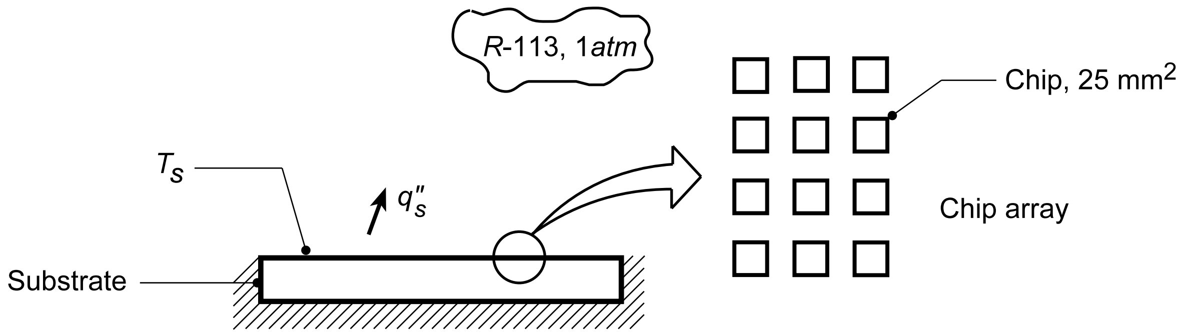
COMMENTS: From the experimental data, part (a), the surface temperature is determined from the conduction analyses as T_s = 110.0°C. Using the traditional nucleate boiling correlation with the experiential value for the heat flux, the surface temperature is estimated as $T_{s,c}$ = 109.1°C. The two approaches provide excess temperatures that are 10.0 vs. 9.1°C, which amounts to nearly a 10% difference.

PROBLEM 10.12

KNOWN: Chips on a ceramic substrate operating at power levels corresponding to 50% of the critical heat flux.

FIND: (a) Chip power level and temperature rise of the chip surface, and (b) Compute and plot the chip temperature T_s as a function of heat flux for the range $0.25 \le q''_s / q''_{max} \le 0.90$.

SCHEMATIC:



ASSUMPTIONS: (1) Nucleate boiling, (2) Fluid-surface with $C_{s,f} = 0.004$, n = 1.7 for Rohsenow correlation, (3) Backside of substrate insulated.

PROPERTIES: *Table A-5*, Refrigerant R-113 (1 atm): T_{sat} = 321 K = 48°C, ρ_ℓ = 1511 kg/m^3, ρ_v = 7.38 kg/m^3, h_{fg} = 147 kJ/kg, $\sigma = 15.9 \times 10^{-3}$ N/m; R-113, sat. liquid (given, 321 K): $c_{p,\ell}$ = 983.8 J/kg·K, $\mu_\ell = 5.147 \times 10^{-4}$ N·s/m^2, Pr_ℓ = 7.183.

ANALYSIS: (a) The operating power level (flux) is $0.50\,q''_{max}$, where the critical heat flux is estimated from Eq. l0.7 for nucleate pool boiling,

$$q''_{max} = 0.149 h_{fg} \rho_v \left[\sigma g \left(\rho_\ell - \rho_v \right) \big/ \rho_v^2 \right]^{1/4}$$

$$q''_{max} = 0.149 \times 147 \times 10^3 \frac{\text{J}}{\text{kg}} \times 7.38 \frac{\text{kg}}{\text{m}^3} \left[15.9 \times 10^{-3} \frac{\text{N}}{\text{m}} \times 9.8 \frac{\text{m}}{\text{s}^2} (1511 - 7.38) \frac{\text{kg}}{\text{m}^3} / \left(7.38 \frac{\text{kg}}{\text{m}^3} \right)^2 \right]^{1/4}$$

$$q''_{max} = 233\,\text{kW} \big/ \text{m}^2 .$$

Hence, the heat flux on a chip is 0.5×233 kW/m^2 = 116 kW/m^2 and the power level is

$$q_{chip} = q''_s \times A_s = 116 \times 10^3\,\text{W} \big/ \text{m}^2 \times 25\,\text{mm}^2 \left(10^{-3}\,\text{m/mm} \right)^2 = 2.9\,\text{W}.$$ <

To determine the chip surface temperature for this condition, use the Rohsenow equation to find $\Delta T_e = T_s - T_{sat}$ with $q''_s = 116 \times 10^3$ W/m^2. The correlation, Eq. 10.5, solved for ΔT_e is

$$\Delta T_e = \frac{C_{s,f} h_{fg} Pr_\ell^n}{c_{p,\ell}} \left(\frac{q''_s}{\mu_\ell h_{fg}} \right)^{1/3} \left[\frac{\sigma}{g \left(\rho_\ell - \rho_v \right)} \right]^{1/6} = \frac{0.004 \times 147 \times 10^3\,\text{J} \big/ \text{kg} \left(7.18 \right)^{1.7}}{983.8\,\text{J} \big/ \text{kg} \cdot \text{K}} \times$$

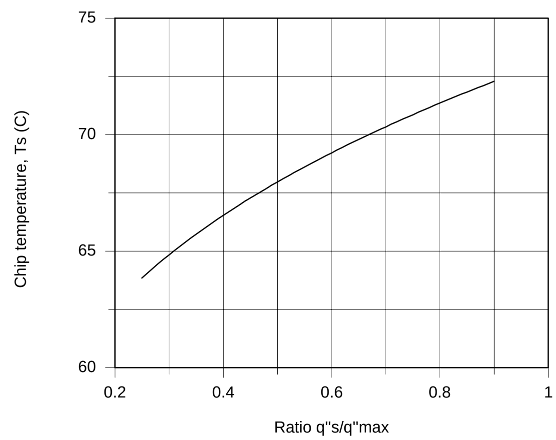
$$\left(\frac{116 \times 10^3\,\text{W} \big/ \text{m}^2 \cdot}{5.147 \times 10^{-4} \frac{\text{N} \cdot \text{s}}{\text{m}^2} \times 147 \times 10^3 \frac{\text{J}}{\text{kg}}} \right)^{1/3} \left[\frac{15.9 \times 10^{-3}\,\text{N/m}}{9.8 \frac{\text{m}}{\text{s}^2} (1511 - 7.38) \frac{\text{kg}}{\text{m}^3}} \right]^{1/6} = 19.9^\circ\text{C}.$$

Hence, the chip surface temperature is

Continued...

$$T_s = T_{sat} + \Delta T_e = 48^\circ C + 19.9^\circ C \approx 68^\circ C.$$ <

(b) Using the *IHT Correlations Tools, Boiling, Nucleate Pool Boiling -- Heat flux* and *Maximum heat flux,* the chip surface temperature, T_s, was calculated as a function of the ratio q''_s / q''_{max}. The required thermophysical properties as provided in the problem statement were entered directly into the IHT workspace. The results are plotted below.



COMMENTS: (1) Refrigerant R-113 is attractive for electronic cooling since its boiling point is slightly above room temperature. The reliability of electronic devices is highly dependent upon operating temperature.

(2) A copy of the *IHT Workspace* model used to generate the above plot is shown below.

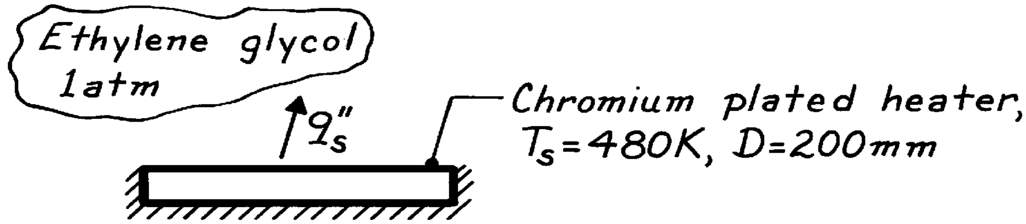
```
// Correlations Tool - Boiling, Nucleate pool boiling, Critical heat flux
q"max = qmax_dprime_NPB(rhol,rhov,hfg,sigma,g)          // Eq 10.7
g = 9.8                         // Gravitational constant, m/s^2
/*  Correlation description: Critical (maximum) heat flux for nucleate pool boiling (NPB). Eq 10.7, Table
10.1 . See boiling curve, Fig 10.4 . */
// Correlations Tool - Boiling, Nucleate pool boiling, Heat flux
qs" = qs_dprime_NPB(Csf,n,rhol,rhov,hfg,cpl,mul,Prl,sigma,deltaTe,g)    // Eq 10.5
//g = 9.8                       // Gravitational constant, m/s^2
deltaTe = Ts - Tsat             // Excess temperature, K
Ts =  Ts_C + 273                // Surface temperature, K
Ts_C = 68                       // Surface temperature, C
//Tsat =                        // Saturation temperature, K
/* Evaluate liquid(l) and vapor(v) properties at Tsat. From Table 10.1.  */
// Fluid-surface combination:
Csf =   0.004                   // Given
n =   1.7                       // Given
/*  Correlation description: Heat flux for nucleate pool boiling (NPB), water-surface combination (Cf,n), Eq
10.5, Table 10.1 . See boiling curve, Fig 10.4 . */
// Heat rates:
qsqm = qs" / q"max              // Ratio, heat flux over critical heat flux
qsqm = 0.5
// Thermophysical Properties (Given):
Tsat = 321                      // Saturation temperature, K
Tsat_C = Tsat - 273             // Saturation temperature, C
rhol = 1511                     // Density, liquid, kg/m^3
rhov = 7.38                     // Density, vapor, kg/m^3
hfg = 147000                    // Heat of vaporization, J/kg
sigma = 15.9e-3                 // Surface tension/ N/m
cpl = 983.3                     // Specific heat, saturated liquid, J/kg.K
mul  = 5.147e-4                 // Viscosity, saturated liquid, N.s/m^2
Prl = 7.183                     // Prandtl number, saturated liquid
```

PROBLEM 10.13

KNOWN: Saturated ethylene glycol at 1 atm heated by a chromium-plated heater of 200 mm diameter and maintained at 480K.

FIND: Heater power, rate of evaporation, and ratio of required power to maximum power for critical heat flux.

SCHEMATIC:



ASSUMPTIONS: (1) Nucleate pool boiling, (2) Fluid-surface, $C_{s,f}$ = 0.010 and n = 1.

PROPERTIES: *Table A-5*, Saturated ethylene glycol (1atm): T_{sat} = 470K, h_{fg} = 812 kJ/kg, ρ_f = 1111 kg/m^3, $\sigma = 32.7 \times 10^{-3}$ N/m; Saturated ethylene glycol (given, 470K): ρ_v = 1.66kg/m^3, $m_\ell = 0.38 \times 10^{-3}$ N·s/m^2, $c_{p,\ell}$ = 3280 J/kg·K, Pr_ℓ = 8.7, k_ℓ = 0.15 W/m·K.

ANALYSIS: The power requirement for boiling and the evaporation rate are $q_{boil} = q_s'' \cdot A_s$ and $\dot{m} = q_{boil}/h_{fg}$. Using the Rohsenow correlation,

$$q_s'' = m_\ell\, h_{fg}\left[\frac{g(r_\ell - r_v)}{s}\right]^{1/2}\left(\frac{c_{p,\ell}\,\Delta T_e}{C_{s,f}\, h_{fg}\, Pr_\ell^n}\right)^3$$

$$q_s'' = 0.38\times10^{-3}\,\frac{N\cdot s}{m^2}\times812\times10^3\,\frac{J}{kg}\left[\frac{9.8\,m/s^2\,(1111-1.66)\,kg/m^3}{32.7\times10^{-3}\,N/m}\right]^{1/2}\left(\frac{3280\,J/kg\cdot K\,(480-470)\,K}{0.01\times812\times10^3\,\frac{J}{kg}\,(8.7)^1}\right)^3$$

$$q_s'' = 1.78\times10^4\,W/m^2 \qquad q_{boil} = 1.78\times10^4\,W/m^2\times p/4\,(0.200m)^2 = 559\ W \qquad <$$

$$\dot{m} = 559\,W/812\times10^3\ J/kg = 6.89\times10^{-4}\,kg/s. \qquad <$$

For this fluid, the critical heat flux is estimated from Eq. 10.7,

$$q_{max}'' = 0.149\ h_{fg}\ r_v\left[s\ g(r_\ell - r_v)/r_v^2\right]^{1/4}$$

$$q_{max}'' = 0.149\times812\times10^3\,\frac{J}{kg}\times1.66\,\frac{kg}{m^3}\left[\frac{32.7\times10^{-3}\,N/m\times9.8\,m/s^2\,(1111-1.66)\,kg/m^3}{(1.66\,kg/m^3)^2}\right]^{1/4}$$

$$q_{max}'' = 6.77\times10^5\,W/m^2.$$

Hence, the ratio of the operating heat flux to the critical heat flux is,

$$\frac{q_s''}{q_{max}''} = \frac{1.78\times10^4\,W/m^2}{6.77\times10^5\,W/m^2} \approx 0.026 \qquad \text{or} \qquad 2.6\%. \qquad <$$

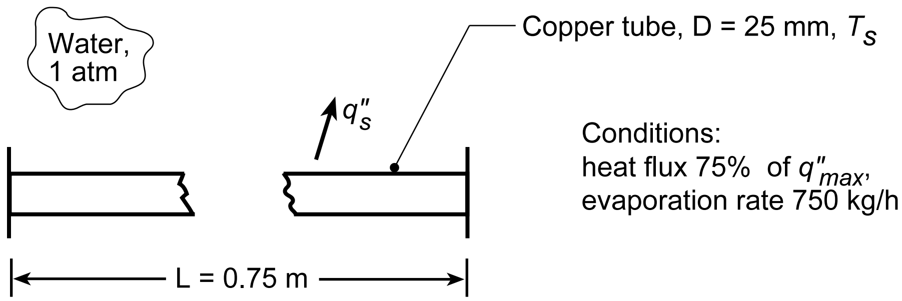
COMMENTS: Recognize that the results are crude approximations since the values for $C_{s,f}$ and n are just estimates. This fluid is not normally used for boiling processes since it decomposes at higher temperatures.

PROBLEM 10.14

KNOWN: Copper tubes, 25 mm diameter × 0.75 m long, used to boil saturated water at 1 atm operating at 75% of the critical heat flux.

FIND: (a) Number of tubes, N, required to evaporate at a rate of 750 kg/h; tube surface temperature, T_s, for these conditions, and (b) Compute and plot T_s and N required to provide the prescribed vapor production as a function of the heat flux ratio, $0.25 \le q''_s/q''_{max} \le 0.90$.

SCHEMATIC:



ASSUMPTIONS: (1) Nucleate pool boiling, (2) Polished copper tube surfaces.

PROPERTIES: *Table A-6*, Saturated water (100°C): ρ_ℓ = 957.9 kg/m^3, $c_{p,\ell}$ = 4217 J/kg·K, μ_ℓ = 279×10^{-6} N·s/m^2, Pr_ℓ = 1.76, h_{fg} = 2257 kJ/kg, $\sigma = 58.9 \times 10^{-3}$ N/m, ρ_v = 0.5955 kg/m^3.

ANALYSIS: (a) The total number of tubes, N, can be evaluated from the rate equations

$$q = q''_s A_s = q''_s N\pi DL \qquad q = \dot{m}h_{fg} \qquad N = \dot{m}h_{fg}/q''_s \pi DL. \qquad (1,2,3)$$

The tubes are operated at 75% of the critical flux (1.26 MW/m^2, see Example 10.1). Hence, the heat flux is

$$q''_s = 0.75\, q''_{max} = 0.75 \times 1.26\ \mathrm{MW/m^2} = 9.45 \times 10^5\ \mathrm{W/m^2}.$$

Substituting numerical values into Eq. (3), find

$$N = 750\,\mathrm{kg/h}\,(1\mathrm{h}/3600\mathrm{s}) \times 2257 \times 10^3\,\mathrm{J/kg} \Big/ \left(9.45 \times 10^5\,\mathrm{W/m^2} \times \pi \times 0.025\,\mathrm{m} \times 0.75\,\mathrm{m}\right) = 8.5 \approx 9. \quad <$$

To determine the tube surface temperature, use the Rohsenow correlation,

$$\Delta T_e = \frac{C_{s,f}\, h_{fg}\, Pr_\ell^n}{c_{p,\ell}} \left(\frac{q''_s}{\mu_\ell h_{fg}}\right)^{1/3} \left[\frac{\sigma}{g(\rho_\ell - \rho_v)}\right]^{1/6}.$$

From Table 10.1 for the polished copper-water combination, $C_{s,f}$ = 0.013 and n = 1.0.

$$\Delta T_e = \frac{0.013 \times 2257 \times 10^3\,\mathrm{J/kg}\,(1.76)^1}{4217\,\mathrm{J/kg \cdot K}} \left(\frac{9.45 \times 10^5\,\mathrm{W/m^2}}{279 \times 10^{-6}\,\mathrm{N \cdot s/m^2} \times 2257 \times 10^3\,\mathrm{J/kg}}\right)^{1/3} \times$$

$$\left[\frac{58.9 \times 10^{-3}\,\mathrm{N/m}}{9.8\,\mathrm{m/s^2}\,(957.9 - 0.5955)\,\mathrm{kg/m^3}}\right]^{1/6} = 19.0^\circ\mathrm{C}.$$

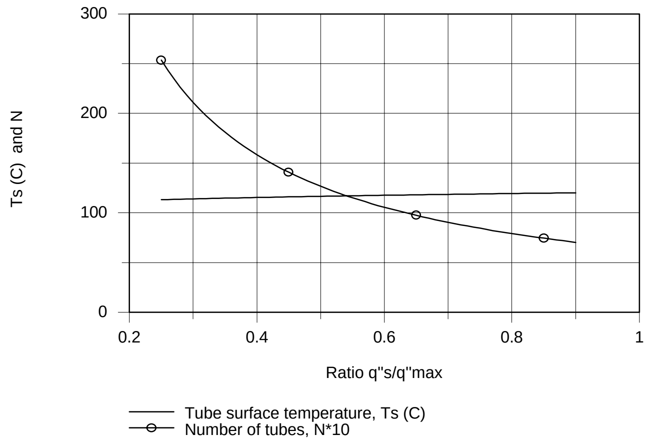
Hence,

$$T_s = T_{sat} + \Delta T_e = (100 + 19)^\circ\mathrm{C} = 119^\circ\mathrm{C}. \quad <$$

Continued...

PROBLEM 10.14 (Cont.)

(b) Using the *IHT Correlations Tool*, *Boiling*, *Nucleate Pool Boiling*, *Heat flux* and the *Properties Tool* for *Water*, combined with Eqs. (1,2,3) above, the surface temperature T_s and N can be computed as a function of q''_s/q''_{max}. The results are plotted below.



Note that the tube surface temperature increases only slightly (113 to 120°C) as the ratio q''_s/q''_{max} increases. The number of tubes required to provide the prescribed evaporation rate decreases markedly as q''_s/q''_{max} increases.

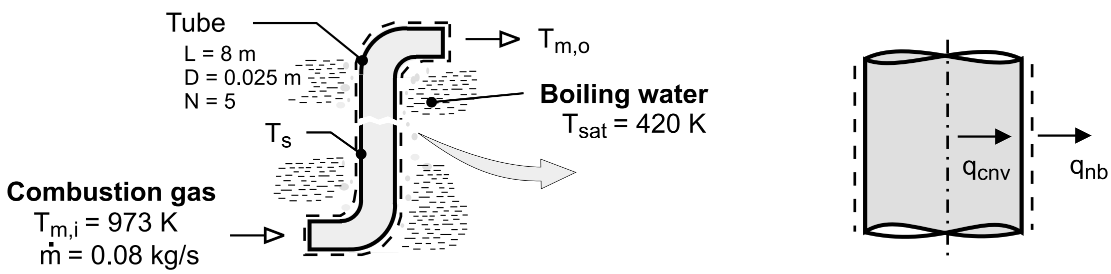
COMMENTS: (1) The critical heat flux, q''_{max} =1.26 MW/m^2, for saturated water at 1 atm is calculated in Example 10.1 using the Zuber-Kutateladze relation, Eq. 10.7. The *IHT Correlation Tool, Boiling*, *Nucleate pool boiling*, *Maximum heat flux*, with the *Properties Tool* for *Water* could also be used to determine q''_{max}.

PROBLEM 10.15

KNOWN: Diameter and length of tube submerged in pressurized water. Flowrate and inlet temperature of gas flow through the tube.

FIND: Tube wall and gas outlet temperatures.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) Uniform tube wall temperature, (3) Nucleate boiling at outer surface of tube, (4) Fully developed flow in tube, (5) Negligible flow work and potential and kinetic energy changes for tube flow, (7) Constant properties.

PROPERTIES: *Table A-6*, saturated water (p_{sat} = 4.37 bars): T_{sat} = 420 K, $h_{fg} = 2.123 \times 10^6$ J/kg, $\rho_\ell = 919\,\text{kg/m}^3$, $\rho_v = 2.4\,\text{kg/m}^3$, $\mu_\ell = 185 \times 10^{-6}\,\text{N}\cdot\text{s/m}^2$, $c_{p,\ell} = 4302\,\text{J/kg}\cdot\text{K}$, $\text{Pr}_\ell = 2.123 \times 10^6\,\text{J/kg}\cdot\text{K}$, $\sigma = 0.0494$ N/m. *Table A-4*, air $\left(p = 1\,\text{atm}, \bar{T}_m \approx 700\,\text{K}\right)$: c_p = 1075 J/kg·K, $\mu = 339 \times 10^{-7}$ N·s/m^2, k = 0.0524 W/m·K, Pr = 0.695.

ANALYSIS: From an energy balance performed for a control surface that bounds the tube, we know that the rate of heat transfer by convection from the gas to the inner surface must equal the rate of heat transfer due to boiling at the outer surface. Hence, from Eqs. (8.44), (8.45) and (10.5), the energy balance for a single tube is of the form

$$\bar{h}A_s\left[\frac{\Delta T_o - \Delta T_i}{\ln\left(\Delta T_o / \Delta T_i\right)}\right] = A_s \mu_\ell h_{fg}\left[\frac{g\left(\rho_\ell - \rho_v\right)}{\sigma}\right]^{1/2}\left(\frac{c_{p,\ell}\Delta T_e}{C_{s,f}\, h_{fg}\, \text{Pr}_\ell^n}\right)^3 \qquad (1)$$

where $\bar{U} = \bar{h}$ and $C_{s,f} = 0.013$ and n = 1.0 from Table 10.1. The corresponding unknowns are the wall temperature T_s and gas outlet temperature, $T_{m,o}$, which are also related through Eq. (8.43).

$$\frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(-\frac{\pi DL}{\dot{m}\, c_p}\bar{h}\right) \qquad (2)$$

For $\text{Re}_D = 4\dot{m}/\pi D\mu = 119,600$, the flow is turbulent, and with n = 0.3, Eq. (8.60) yields,

$$\bar{h} = h_{fd} = \left(\frac{k}{D}\right)0.023\,\text{Re}_D^{4/5}\,\text{Pr}^{0.3} = \left(\frac{0.0524\,\text{W/m}\cdot\text{K}}{0.025\text{m}}\right)0.023\left(119,600\right)^{4/5}\left(0.695\right)^{0.3} = 502\,\text{W/m}^2\cdot\text{K}$$

Solving Eqs. (1) and (2), we obtain

$$T_s = 152.6°\text{C}, \qquad T_{m,o} = 166.7°\text{C}$$ <

COMMENTS: (1) The heat rate per tube is $q = \dot{m}\, c_p$ $(T_{m,i} - T_{m,o})$ = 45,930 W, and the total heat rate is Nq = 229,600 W, in which case the rate of steam production is $\dot{m}_{steam} = q/h_{fg} = 0.108\,\text{kg/s}$.

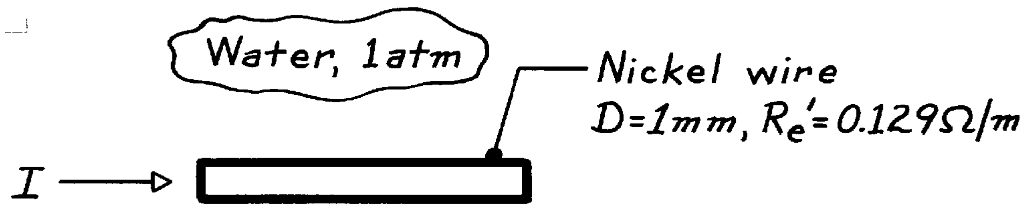
(2) It would not be possible to maintain isothermal tube walls without a large wall thickness, and T_s, as well as the intensity of boiling, would decrease with increasing distance from the tube entrance. However the foregoing analysis suffices as a first approximation.

PROBLEM 10.16

KNOWN: Nickel wire passing current while submerged in water at atmospheric pressure.

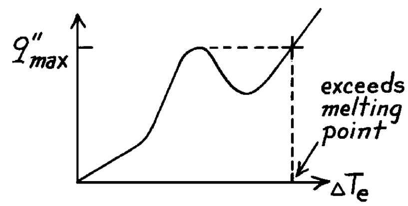
FIND: Current at which wire burns out.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Pool boiling.

ANALYSIS: The burnout condition will occur when electrical power dissipation creates a surface heat flux exceeding the critical heat flux, q''_{max}. This burn out condition is illustrated on the boiling curve to the right and in Figure 10.6.



The criterion for burnout can be Expressed as

$$q''_{max} \cdot p D = q'_{elec} \qquad\qquad q'_{elec} = I^2 R'_e. \qquad (1,2)$$

That is,

$$I = [q''_{max} p D / R'_e]^{1/2}. \qquad (3)$$

For pool boiling of water at 1 atm, we found in Example 10.1 that

$$q''_{max} = 1.26 \mathrm{MW/m^2}.$$

Substituting numerical values into Eq. (3), find

$$I = \left[1.26\times10^6 \mathrm{W/m^2} (p \times 0.001\mathrm{m})/0.129\Omega/\mathrm{m}\right]^{1/2} = 175\mathrm{A}. \qquad <$$

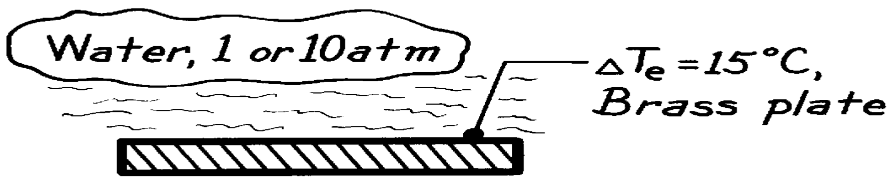
COMMENTS: The magnitude of the current required to burn out the 1 mm diameter wire is very large. What current would burn out the wire in air?

PROBLEM 10.17

KNOWN: Saturated water boiling on a brass plate maintained at 115°C.

FIND: Power required (W/m^2) for pressures of 1 and 10 atm; fraction of critical heat flux at which plate is operating.

SCHEMATIC:



ASSUMPTIONS: (1) Nucleate pool boiling, (2) ΔT_e = 15°C for both pressure levels.

PROPERTIES: *Table A-6*, Saturated water, liquid (1 atm, T_{sat} = 100°C): r_ℓ = 957.9 kg/m^3, $c_{p,\ell}$ = 4217 J/kg·K, m_ℓ $= 279 \times 10^{-6}$ N·s/m^2, Pr = 1.76, h_{fg} = 2257 kJ/kg, $\sigma = 58.9 \times 10^{-3}$ N/m; *Table A-6*, Saturated water, vapor (1 atm): ρ_v = 0.596 kg/m^3; *Table A-6*, Saturated water, liquid (10 atm = 10.133 bar, T_{sat} = 453.4 K = 180.4°C): r_ℓ = 886.7 kg/m^3, $c_{p,\ell}$ = 4410 J/kg·K, $m_\ell = 149 \times 10^{-6}$ N·s/m^2, Pr_ℓ = 0.98, h_{fg} = 2012 kJ/kg, $\sigma = 42.2 \times 10^{-3}$ N/m; *Table A-6*, Water, vapor (10.133 bar): ρ_v = 5.155 kg/m^3.

ANALYSIS: With ΔT_e = 15°C, we expect nucleate pool boiling. The Rohsenow correlation with $C_{s,f}$ = 0.006 and n = 1.0 for the brass-water combination gives

$$q''_s = m_\ell h_{fg}\left[\frac{g\left(r_\ell - r_v\right)}{s}\right]^{1/2}\left(\frac{c_{p,\ell}\Delta T_e}{C_{s,f}\, h_{fg}\, Pr_\ell^n}\right)^3$$

1 atm: $$q''_s = 279\times10^{-6}\,\mathrm{N\cdot s/m^2}\times2257\times10^3\,\mathrm{J/kg}\left[\frac{9.8\,\mathrm{m/s^2}\,(957.9-0.596)\,\mathrm{kg/m^3}}{58.9\times10^{-3}\,\mathrm{N/m}}\right]^{1/2}\times$$

$$\left(\frac{4217\,\mathrm{J/kg\cdot K}\times15\,\mathrm{K}}{0.006\times2257\times10^3\,\mathrm{J/kg}\times1.76^1}\right)^3 = 4.70\,\mathrm{MW/m^2}$$

10 atm: $$q''_s = 23.8\,\mathrm{MW/m^2}$$

From Example 10.1, $q''_{max}(1\mathrm{atm}) = 1.26\,\mathrm{MW/m^2}$. To find the critical heat flux at 10 atm, use the correlation of Eq. 10.7,

$$q''_{max} = 0.149\, h_{fg}\, r_v\left[s\, g\left(r_\ell - r_v\right)/\, r_v^2\right]^{1/4}.$$

$$q''_{max}(10\,\mathrm{atm}) = 0.149\times2012\times10^3\,\mathrm{J/kg}\times5.155\,\mathrm{kg/m^3}\times$$

$$\left[\frac{42.2\times10^{-3}\,\mathrm{N/m}\times9.8\,\mathrm{m/s^2}\,(886.7-5.16)\,\mathrm{kg/m^3}}{\left(5.155\,\mathrm{kg/m^3}\right)^2}\right]^{1/4} = 2.97\,\mathrm{MW/m^2}.$$

For both conditions, the Rohsenow correlation predicts a heat flux that exceeds the maximum heat flux, q''_{max}. We conclude that the boiling condition with ΔT_e = 15°C for the brass-water combination is beyond the inflection point (P, see Fig. 10.4) where the boiling heat flux is no longer proportional to ΔT_e^3.

$$q''_s \approx q''_{max}(1\,\mathrm{atm}) \le 1.26\,\mathrm{MW/m^2} \qquad q''_s \approx q''_{max}(10\,\mathrm{atm}) \le 2.97\,\mathrm{MW/m^2}. \quad <$$

PROBLEM 10.18

KNOWN: Zuber-Kutateladze correlation for critical heat flux, q''_{max}.

FIND: Pressure dependence of q''_{max} for water; demonstrate maximum value occurs at approximately 1/3 p_{crit}; suggest coordinates for a universal curve to represent other fluids.

ASSUMPTIONS: Nucleate pool boiling conditions.

PROPERTIES: *Table A-6*, Water, saturated at various pressures; see below.

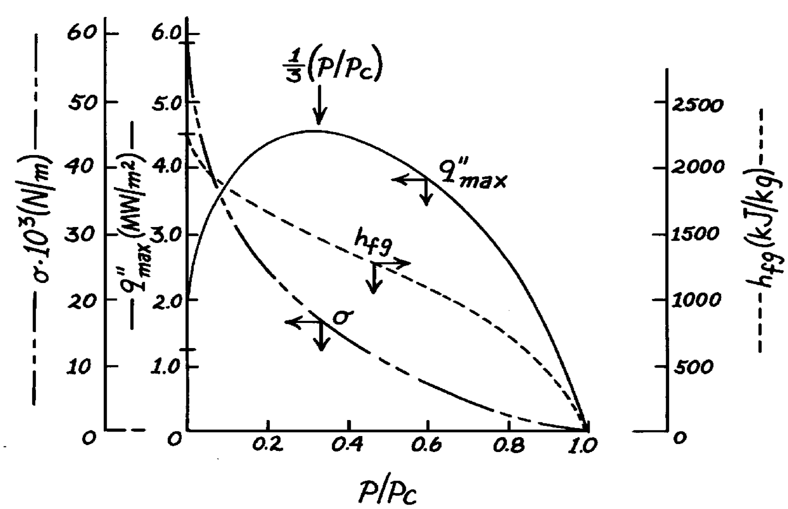
ANALYSIS: The Z-K correlation for estimating the critical heat flux, has the form

$$q''_{max} = 0.149\ r_v h_{fg} \left[\frac{g\, s\, (r_\ell - r_v)}{r_v^2} \right]^{1/4}$$

where the properties for saturation conditions are a function of pressure. The properties (Table A-6) and the values for q''_{max} are as follows:

p (bar)	p/p_c	r_ℓ (kg/m^3)	r_v (kg/m^3)	h (kJ/kg)	$\sigma \times 10^3$ (N/m)	q''_{max} (MW/m^2)
1.01	0.0045	957.9	0.5955	2257	58.9	1.258
11.71	0.053	879.5	5.988	1989	40.7	3.138
26.40	0.120	831.3	13.05	1825	31.6	3.935
44.58	0.202	788.1	22.47	1679	24.5	4.398
61.19	0.277	755.9	31.55	1564	19.7	4.549
82.16	0.372	718.4	43.86	1429	15.0	4.520
123.5	0.557	648.9	72.99	1176	8.4	4.047
169.1	0.765	562.4	117.6	858	3.5	2.905
221.2	1.000	315.5	315.5	0	0	0

The q''_{max} values are plotted as a function of p/p_c, where p_c is the critical pressure. Note the rapid decrease of hfg and σ with increasing pressure. The universal curve coordinates would be $q''_{max} / q''_{max}\,(1/3\, p_{crit})$ vs. p / p_c.



PROBLEM 10.19

KNOWN: Kutateladze's dimensional analysis and the bubble diameter parameter.

FIND: (a) Verify the dimensional consistency of the critical heat flux expression, and (b) Estimate heater size with water at 1 atm required such that the Bond number will exceed 3, i.e., Bo ≥ 3.

ASSUMPTIONS: Nucleate pool boiling.

ANALYSIS: (a) Kutateladze postulated that the critical flux was dependent upon four parameters,

$$q''_{max} = q''_{max}\left(h_{fg}, r_v, s\ , D_b\right)$$

where D_b is the bubble diameter parameter having the form

$$D_b = \left[s\ / g\left(r_\ell - r_v\right)\right]^{1/2}. \qquad (1)$$

The form of the critical heat flux expression was presumed to be

$$q''_{max} = C\ h_{fg}\ r_v^{1/2}\ D_b^{-1/2}\ s^{\,1/2} \qquad (2)$$

where C is a constant. It is not possible to derive this equation from a dimensional (Pi) analysis. We can only determine that the equation is dimensionally consistent. Using SI units, check Eq. (1) for D_b,

$$D_b => \left[\left(Nm^{-1}\right)\left(m^{-1}s^2\right)\left(kg^{-1}m^3\right)\right]^{1/2} => \left[N\left(\frac{s^2}{kg\cdot m^2}\right)m^3\right]^{1/2} => [m]$$

and in Eq. (2) for q''_{max},

$$q''_{max} => \left[\left(Jkg^{-1}\right)\left(kg^{1/2}m^{-3/2}\right)\left(m^{-1/2}\right)\left(N^{1/2}m^{-1/2}\right)\right] => \left[\frac{J}{s}\cdot\left(\frac{N\cdot s^2}{kg\cdot m}\right)^{1/2} m^{-2}\right] => \left[\frac{W}{m^2}\right].$$

Hence, the equations are dimensionally consistent.

(b) The Bond number, Bo, is defined as the ratio of the characteristic length L (width or diameter) of the heater surface to the bubble diameter parameter, D_b. That is, Bo ≡ L/D_B. The number squared is also indicative of the ratio of the buoyant to capillary forces. For water at 1 atm (see Example 10.1 for properties listing), Eq. (1) yields

$$D_b = \left[58.9\times10^{-3}\frac{N}{m}/9.8\frac{m}{s^2}(957.9 - 0.5955)\frac{kg}{m^3}\right]^{1/2} = 0.0025m = 2.5mm.$$

Eq. 10.7 for the critical heat flux is appropriate for an "infinite" heater (Bo ≥ 3). To meet this requirement, the heater dimension must be

$$L \geq Bo\cdot D_b = 3\times2.5mm = 7.5mm.$$ <

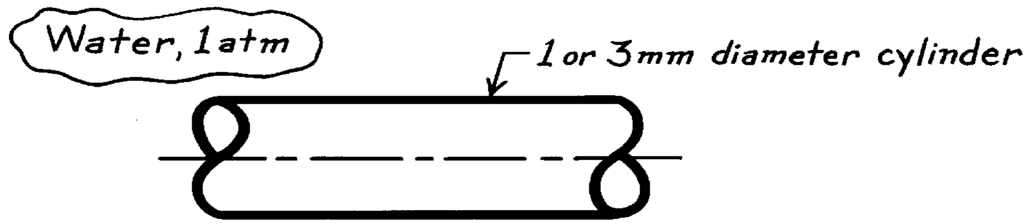
COMMENTS: As the heater size decreases (Bo decreasing), the boiling curve no longer exhibits the characteristic q''_{max} and q''_{min} features. The very small heater, such as a wire, is enveloped with vapor at small ΔT_e and film boiling occurs.

PROBLEM 10.20

KNOWN: Lienhard-Dhir critical heat flux correlation for small horizontal cylinders.

FIND: Critical heat flux for 1 mm and 3 mm diameter horizontal cylinders in water at 1 atm.

SCHEMATIC:



ASSUMPTIONS: Nucleate pool boiling.

PROPERTIES: *Table A-6*, Water (1 atm): $\rho_\ell = 957.9\ \text{kg/m}^3$, $\rho_v = 0.5955\ \text{kg/m}^3$, $\sigma = 58.9 \times 10^{-3}$ N/m.

ANALYSIS: The Lienhard-Dhir correlation for small horizontal cylinders is

$$q''_{max} = q''_{max,Z}\left[0.94\,(Bo)^{-1/4}\right] \qquad 0.15 \le Bo \le 1.2 \tag{1}$$

where $q''_{max,Z}$ is the critical heat flux predicted by the Zuber-Kutateladze correlation for the infinite heater (Eq. 10.6) and the Bond number is

$$Bo = \frac{r}{D_b} = r/\left[\sigma\ /g\left(\rho_\ell - \rho_v\right)\right]^{1/2}. \tag{2}$$

Note the characteristic length is the cylinder radius. From Example 10.1, using Eq. 10.6,

$$q''_{max,Z} = 1.11\,\text{MW/m}^2$$

and substituting property values for water at 1 atm into Eq. (2),

$$D_b = \left[58.9\times10^{-3}\,\frac{\text{N}}{\text{m}}/9.8\frac{\text{m}}{\text{s}^2}(957.9-0.5955)\frac{\text{kg}}{\text{m}^3}\right]^{1/2} = 2.51\text{mm}.$$

Substituting appropriate values into Eqs. (1) and (2),

1 mm dia cylinder Bo = 0.5 mm/2.51 mm = 0.20

$$q''_{max} = 1.11\,\text{MW/m}^2\left[0.94(0.20)^{-1/4}\right] = 1.56\,\text{MW/m}^2. \qquad <$$

3 mm dia cylinder Bo = 1.5 mm/2.51 mm = 0.60

$$q''_{max} = 1.11\,\text{MW/m}^2\left[0.94(0.60)^{-1/4}\right] = 1.19\ \text{MW/m}^2. \qquad <$$

Note that for the 3 mm diameter cylinder, the critical heat flux is 1.19/1.11 = 1.07 times larger than the value for a very large horizontal cylinder.

COMMENTS: For practical purposes a horizontal cylinder of diameter greater than 3 mm can be considered as a very large one. The critical heat flux for a 1 mm diameter cylinder is 40% larger than that for the large cylinder.

PROBLEM 10.21

KNOWN: Boiling water at 1 atm pressure on moon where the gravitational field is 1/6 that of the earth.

FIND: Critical heat flux.

ASSUMPTIONS: Nucleate pool boiling conditions.

PROPERTIES: *Table A-6*, Water (1 atm): T_{sat} = 100°C, r_ℓ = 957.9 kg/m^3, ρ_v = 0.5955 kg/m^3, h_{fg} = 2257 kJ/kg, $\sigma = 58.9 \times 10^{-3}$ N/m.

ANALYSIS: The modified Zuber-Kutateladze correlation for the critical heat flux is Eq. 10.7.

$$q''_{max} = 0.149\, r_v^{1/2} h_{fg} \left[s\, g\left(r_\ell - r_v \right) \right]^{1/4}.$$

The relation predicts the critical flux dependency on the gravitational acceleration as

$$q''_{max} \sim g^{1/4}.$$

It follows that if $g_{moon} = (1/6)\, g_{earth}$ and recognizing $q''_{max,e}$ = 1.26 MW/m^2 for earth acceleration (see Example 10.1),

$$q''_{max,moon} = q''_{max,earth} \left(g_{moon} / g_{earth} \right)^{1/4}$$

$$q''_{max,moon} = 1.26 \frac{MW}{m^2} \left(\frac{1}{6} \right)^{1/4} = 0.81\, MW/m^2.$$ <

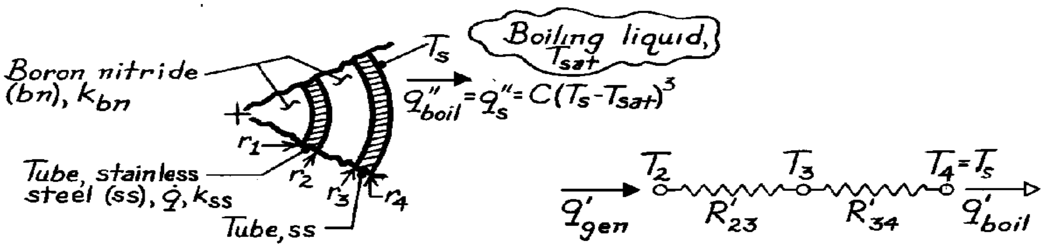
COMMENTS: Note from the discussion in Section 10.4.5 that the g1/4 dependence on the critical heat flux has been experimentally confirmed. In the nucleate pool boiling regime, the heat flux is nearly independent of the gravitational field.

PROBLEM 10.22

KNOWN: Concentric stainless steel tubes packed with dense boron nitride powder. Inner tube has heat generation while outer tube surface is exposed to boiling heat flux, $q''_s = C\left(T_s - T_{sat}\right)^3$. Saturation temperature of boiling liquid and temperature of the outer tube surface.

FIND: Expressions for the maximum temperature in the stainless steel (ss) tubes and in the boron nitride (bn).

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional (cylindrical) steady-state heat transfer in tubes and boron nitride.

ANALYSIS: Construct the thermal circuit shown above where R'_{23} and R'_{34} represent the resistances due to the boron nitride between r_2 and r_3 and to the outer stainless steel tube, respectively. From an overall energy balance,

$$q'_{gen} = q'_{boil},$$

$$\dot{q}p\left(r_2^2 - r_1^2\right) = \left(2p\, r_4\right)C\left(T_s - T_{sat}\right)^3.$$

With prescribed values for T_{sat}, T_s and C, the required volumetric heating of the inner stainless steel tube is

$$\dot{q} = \frac{2r_4}{\left(r_2^2 - r_1^2\right)}C\left(T_s - T_{sat}\right)^3.$$

Using the thermal circuit, we can write expressions for the *maximum* temperature of the stainless steel (ss) and boron nitride (bn).

Stainless steel: $T_{ss,max}$ occurs at r_1. Using the results of Section 3.4.2, the temperature distribution in a radial tube of inner and outer radii r_1 and r_2 is

$$T(r) = -\frac{\dot{q}}{2k_{ss}}r^2 + C_1 \ln r + C_2$$

for which the boundary conditions are

BC#1: $\quad r = r_1 \qquad \frac{dT}{dr} = 0 \qquad 0 = -\frac{\dot{q}}{2k_{ss}}2r_1 + \frac{C_1}{r_1} + 0 \rightarrow C_1 = +\frac{\dot{q}r_1^2}{k_{ss}}$

Continued …..

PROBLEM 10.22 (Cont.)

BC#2: $\quad r = r_2 \quad T(r_2) = T_2$

$$T_2 = -\frac{\dot{q}}{2k_{ss}} r_2^2 + \frac{\dot{q} r_1^2}{k_{ss}} \ln r_2 + C_2$$

$$C_2 = T_2 + \frac{\dot{q}}{2k_{ss}} r_2^2 - \frac{\dot{q} r_1^2}{k_{ss}} \ln r_2$$

Hence,

$$T(r) = -\frac{\dot{q}}{2k_{ss}}\left(r^2 - r_2^2\right) + \frac{\dot{q} r_1^2}{k_{ss}} \ln\left(r / r_2\right) + T_2.$$

Using the thermal circuit, find T_2 in terms of known parameters T_s, T_{sat} and C.

$$\frac{T_2 - T_s}{R'_{23} + R'_{34}} = \left(2\pi r_4\right) C \left(T_s - T_{sat}\right)^3 .$$

Hence, the maximum temperature in the inner stainless steel tube ($r = r_1$) is

$$T_{ss,max} = T(r_1) = -\frac{\dot{q}}{2k_{ss}}\left(r_1^2 - r_2^2\right) + \frac{\dot{q} r_1^2}{k_{ss}} \ln\left(r_1 / r_2\right) + T_s$$

$$+\left(R'_{23} + R'_{34}\right)\left(2\pi r_4\right) C \left(T_s - T_{sat}\right)^3$$ <

where from Eq. 3.27

$$R'_{23} = \frac{\ln\left(r_3 / r_2\right)}{2\pi k_{bn}} \qquad R'_{34} = \frac{\ln\left(r_4 / r_3\right)}{2\pi k_{ss}}.$$

Boron nitride: $T_{bn,max}$ occurs at r_1. Hence

$$T_{bn,max} = T\left(r_1\right)$$ <

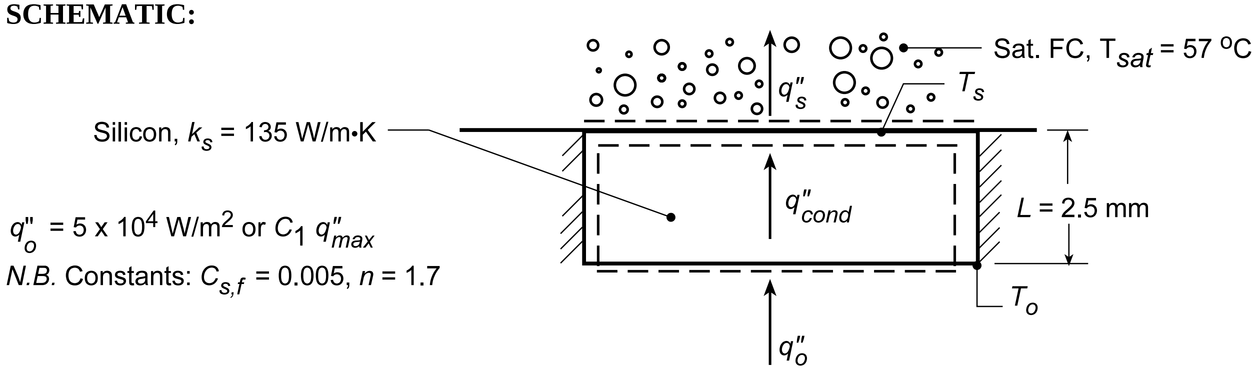
as derived above for the inner stainless steel tube.

PROBLEM 10.23

KNOWN: Thickness and thermal conductivity of a silicon chip. Properties of saturated fluorocarbon liquid.

FIND: (a) Temperature at bottom surface of chip for a prescribed heat flux and 90% of CHF, (b) Effect of heat flux on chip surface temperatures; maximum allowable heat flux.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform heat flux and adiabatic sides, hence one-dimensional conduction in chip, (3) Constant properties, (4) Nucleate boiling in liquid.

PROPERTIES: Saturated fluorocarbon (given): $c_{p,\ell}$ = 1100 J/kg·K, h_{fg} = 84,400 J/kg, ρ_ℓ = 1619.2 kg/m^3, ρ_v = 13.4 kg/m^3, $\sigma = 8.1 \times 10^{-3}$ kg/s^2, $\mu_\ell = 440 \times 10^{-6}$ kg/m·s, Pr_ℓ = 9.01.

ANALYSIS: (a) Energy balances at the top and bottom surfaces yield $q''_o = q''_{cond} = k_s(T_o - T_s)/L$ = q''_s; where T_s and q''_s are related by the Rohsenow correlation,

$$T_s - T_{sat} = \frac{C_{s,f}\, h_{fg}\, Pr_\ell^n}{c_{p,\ell}} \left(\frac{q''_s}{\mu_\ell h_{fg}} \right)^{1/3} \left[\frac{\sigma}{g(\rho_\ell - \rho_v)} \right]^{1/6}$$

Hence, for $q''_s = 5 \times 10^4$ W/m^2,

$$T_s - T_{sat} = \frac{0.005(84,400\,\text{J/kg})9.01^{1.7}}{1100\,\text{J/kg}\cdot\text{K}} \left(\frac{5\times10^4\,\text{W/m}^2}{440\times10^{-6}\,\text{kg/m}\cdot\text{s}\times84,400\,\text{J/kg}} \right)^{1/3}$$

$$\times\left[\frac{8.1\times10^{-3}\,\text{kg/s}^2}{9.807\,\text{m/s}^2\,(1619.2-13.4)\,\text{kg/m}^3} \right]^{1/6} = 15.9^\circ\text{C}$$

$$T_s = (15.9+57)^\circ\text{C} = 72.9^\circ\text{C}.$$

From the rate equation,

$$T_o = T_s + \frac{q''_o L}{k_s} = 72.9^\circ\text{C} + \frac{5\times10^4\,\text{W/m}^2\times0.0025\,\text{m}}{135\,\text{W/m}\cdot\text{K}} = 73.8^\circ\text{C} \qquad <$$

For a heat flux which is 90% of the critical heat flux (C_1 = 0.9), it follows that

$$q''_o = 0.9q''_{max} = 0.9\times0.149 h_{fg}\rho_v \left[\frac{\sigma g(\rho_\ell - \rho_v)}{\rho_v^2} \right]^{1/4} = 0.9\times0.149\times84,400\,\text{J/kg}\times13.4\,\text{kg/m}^3$$

Continued...

PROBLEM 10.23 (Cont.)

$$\times\left[\frac{8.1\times10^{-3}\,\mathrm{kg/s^2}\times9.807\,\mathrm{m/s^2}\,(1619.2-13.4)\,\mathrm{kg/m^3}}{\left(13.4\,\mathrm{kg/m^3}\right)^2}\right]^{1/4}$$

$$q''_o = 0.9\times15.5\times10^4\ \mathrm{W/m^2} = 13.9\times10^4\ \mathrm{W/m^2}$$

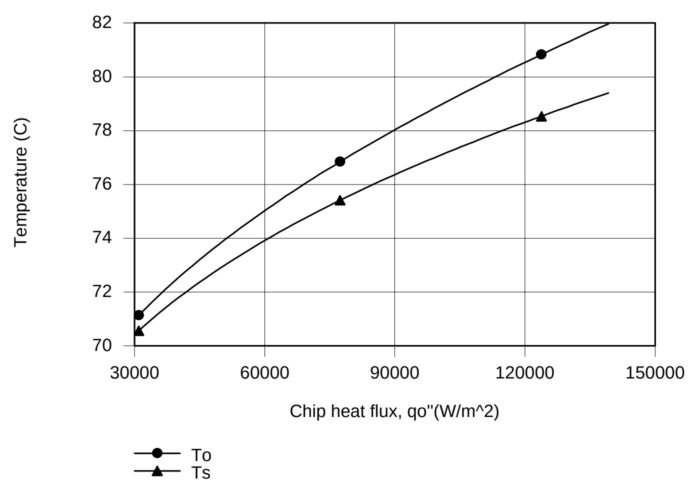
From the results of the previous calculation and the Rohsenow correlation, it follows that

$$\Delta T_e = 15.9^\circ\mathrm{C}\left(q''_o/5\times10^4\ \mathrm{W/m^2}\right)^{1/3} = 15.9^\circ\mathrm{C}\,(13.9/5)^{1/3} = 22.4^\circ\mathrm{C}$$

Hence, T_s = 79.4°C and

$$T_o = 79.4^\circ\mathrm{C} + \frac{13.9\times10^4\ \mathrm{W/m^2}\times0.0025\,\mathrm{m}}{135\,\mathrm{W/m\cdot K}} = 82^\circ\mathrm{C} \qquad <$$

(b) Using the energy balance equations with the *Correlations* Toolpad of IHT to perform the parametric calculations for $0.2 \le C_1 \le 0.9$, the following results are obtained.



The chip surface temperatures, as well as the difference between temperatures, increase with increasing heat flux. The maximum chip temperature is associated with the bottom surface, and T_o = 80°C corresponds to

$$q''_{o,max} = 11.3\times10^4\ \mathrm{W/m^2} \qquad <$$

which is 73% of CHF (q''_{max} = 15.5 × 10^4 W/m^2).

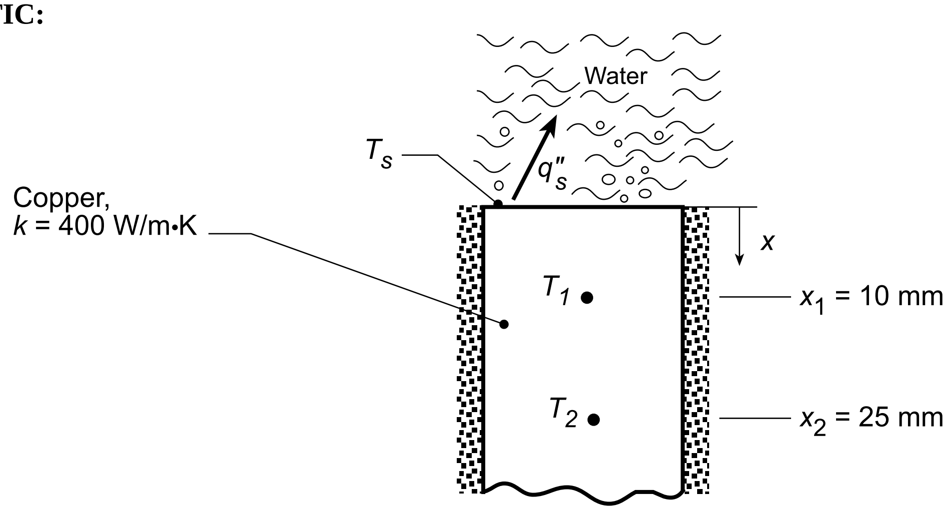
COMMENTS: Many of today's VLSI chip designs involve heat fluxes well in excess of 15 W/cm^2, in which case pool boiling in a fluorocarbon would not be an appropriate means of heat dissipation.

PROBLEM 10.24

KNOWN: Operating conditions of apparatus used to determine surface boiling characteristics.

FIND: (a) Nucleate boiling coefficient for special coating, (b) Surface temperature as a function of heat flux; apparatus temperatures for a prescribed heat flux; applicability of nucleate boiling correlation for a specified heat flux.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conduction in the bar, (2) Water is saturated at 1 atm, (3) Applicability of Rohsenow correlation with n = 1.

PROPERTIES: *Table A.6*, saturated water (100°C): ρ_ℓ = 957.9 kg/m^3, $c_{p,\ell}$ = 4217 J/kg·K, μ_ℓ = 279 × 10^{-6} N·s/m^2, Pr_ℓ = 1.76, h_{fg} = 2.257 × 10^6 J/kg, σ = 0.0589 N/m, ρ_v = 0.5955 kg/m^3.

ANALYSIS: (a) The coefficient $C_{s,f}$ associated with Eq. 10.5 may be determined if q''_s and T_s are known. Applying Fourier's law between x_1 and x_2,

$$q''_s = q''_{cond} = k\frac{T_2 - T_1}{x_2 - x_1} = 400\,\text{W/m}\cdot\text{K} \times \frac{(158.6 - 133.7)^\circ\text{C}}{0.015\,\text{m}} = 6.64 \times 10^5\,\text{W/m}^2$$

Since the temperature distribution in the bar is linear, $T_s = T_1 - (dT/dx)x_1 = T_1 - [(T_2 - T_1)/(x_2 - x_1)]x_1$. Hence,

$$T_s = 133.7^\circ\text{C} - \left[24.9^\circ\text{C}/0.015\,\text{m}\right]0.01\,\text{m} = 117.1^\circ\text{C}$$

From Eq. 10.5, with n = 1,

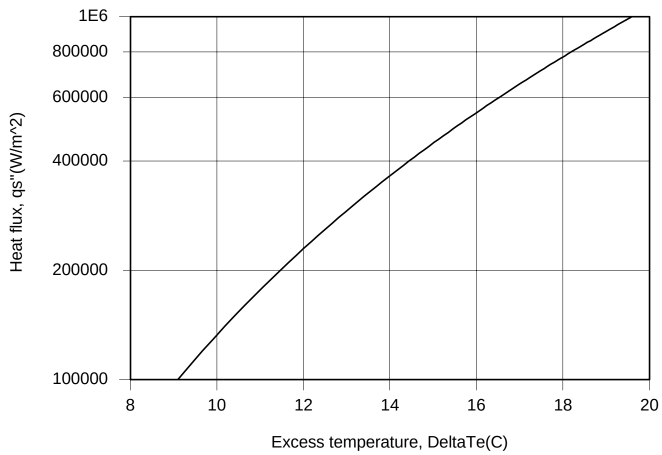
$$C_{s,f} = \frac{c_{p,\ell}\Delta T_e}{h_{fg}\,Pr_\ell}\left(\frac{\mu_\ell h_{fg}}{q''_s}\right)^{1/3}\left[\frac{g(\rho_\ell - \rho_v)}{\sigma}\right]^{1/6}$$

$$C_{s,f} = \frac{4217\,\text{J/kg}\cdot\text{K}\left(17.1^\circ\text{C}\right)}{2.257\times10^6\,\text{J/kg}\,(1.76)}\left(\frac{279\times10^{-6}\,\text{kg/s}\cdot\text{m}\times2.257\times10^6\,\text{J/kg}}{6.64\times10^5\,\text{W/m}^2}\right)^{1/3}\left[\frac{9.8\,\text{m/s}^2\times957.3\,\text{kg/m}^3}{0.0589\,\text{kg/s}^2}\right]^{1/6}$$

$C_{s,f}$ = 0.0131 <

(b) Using the appropriate IHT *Correlations* and *Properties* Toolpads, the following portion of the nucleate boiling regime was computed.

Continued...



For $q''_s = 10^6$ W/m^2 = q''_{cond}, $T_s = 119.6°C$ and

$T_1 = 144.6°C$ and $T_2 = 182.1°C$ <

With the critical heat flux given by Eq. 10.7,

$$q''_{max} = 0.149 h_{fg} \rho_v \left[\frac{\sigma g (\rho_\ell - \rho_v)}{\rho_v^2} \right]^{1/4}$$

$$q''_{max} = 0.149 \left(2.257 \times 10^6 \text{ J/kg} \right) 0.5955 \text{ kg/m}^3 \left[\frac{0.0589 \text{ kg/s}^2 \times 9.8 \text{ m/s}^2 \times 957.3 \text{ kg/m}^3}{\left(0.5955 \text{ kg/m}^3 \right)^2} \right]^{1/4}$$

$$q''_{max} = 1.25 \times 10^6 \text{ W/m}^2$$

Since $q''_s = 1.5 \times 10^6$ W/m^2 > q''_{max}, the heat flux exceeds that associated with nucleate boiling and the foregoing results can not be used.

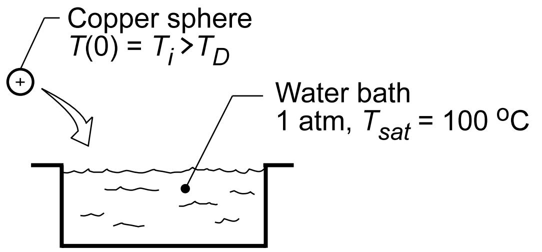
COMMENTS: For $q''_s > q''_{max}$, conditions correspond to film boiling, for which T_s may exceed acceptable operating conditions.

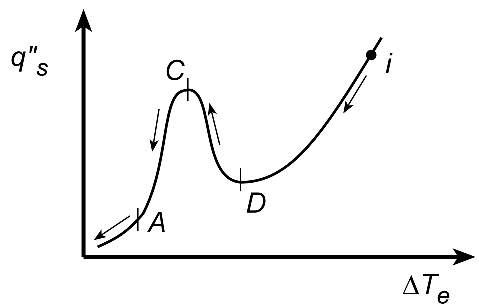
PROBLEM 10.25

KNOWN: Small copper sphere, initially at a uniform temperature, T_i, greater than that corresponding to the Leidenfrost point, T_D, suddenly immersed in a large fluid bath maintained at T_{sat}.

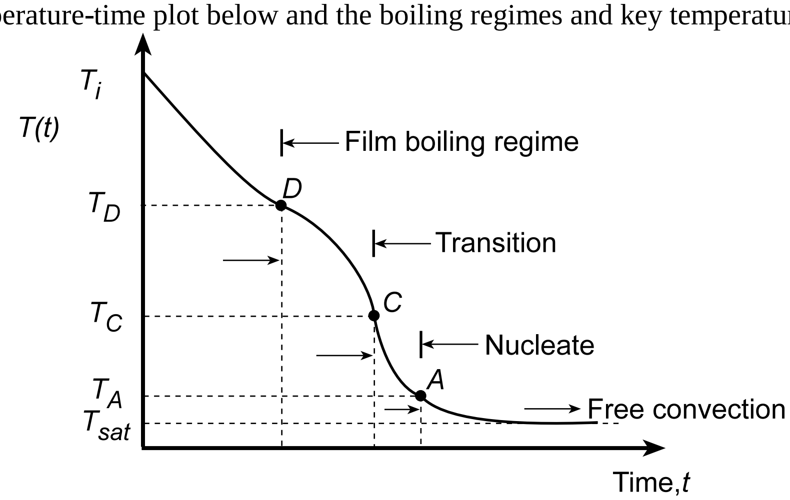
FIND: (a) Sketch the temperature-time history, T(t), during the quenching process; indicate temperature corresponding to T_i , T_D, and T_{sat} , identify regimes of film, transition and nucleate boiling and the single-phase convection regime; identify key features; and (b) Identify times(s) in this quenching process when you expect the surface temperature of the sphere to deviate most from its center temperature.

SCHEMATIC:





ANALYSIS: (a) In the right-hand schematic above, the quench process is shown on the "boiling curve" similar to Figure 10.4. Beginning at an initial temperature, $T_i > T_D$, the process proceeds as indicated by the arrows: film regime from i to D, transition regime from D to C, nucleate regime from C to A, and single-phase (free convection) from A to the condition when $\Delta T_e = T_s - T_{sat} = 0$. The quench process is shown on the temperature-time plot below and the boiling regimes and key temperatures are labeled..



The highest temperature-time change should occur in the nucleate pool boiling regime, especially near the critical flux condition, T_c . The lowest temperature-time change will occur in the single-phase, free convection regime.

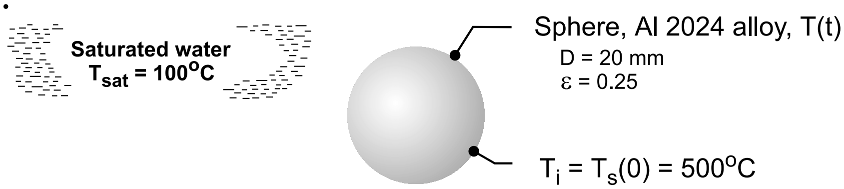
(b) The difference between the center and surface temperature will occur when $Bi = hr_o/3k \geq 0.1$. This could occur in regimes with the highest convection coefficients. For example, $h = 10{,}000\ W/m^2 \cdot K$ which might be the case for water in the nucleate boiling regime, C-A, $Bi \approx 10{,}000\ W/m^2\ (0.010m)/3\times 400\ W/m\cdot K = 0.08$. For a sphere of larger dimension, in the nucleate and film pool boiling regimes, we could expect temperature differences between the center and surface temperatures since Bi might be greater than 0.1.

PROBLEM 10.26

KNOWN: A sphere (aluminum alloy 2024) with a uniform temperature of 500°C and emissivity of 0.25 is suddenly immersed in a saturated water bath maintained at atmospheric pressure.

FIND: (a) The total heat transfer coefficient for the initial condition; fraction of the total coefficient contributed by radiation; and (b) Estimate the temperature of the sphere 30 s after it has been immersed in the bath.

SCHEMATIC:



ASSUMPTIONS: (1) Water exposed to standard atmospheric pressure and uniform temperature, T_{sat}, and (2) Lumped capacitance method is valid.

PROPERTIES: See Comment 2; properties obtained with *IHT* code.

ANALYSIS: (a) For the initial condition with T_s = 500°C, *film boiling* will occur and the coefficients due to convection and radiation are estimated using Eqs. 10.9 and 10.11, respectively,

$$\overline{Nu}_D = \frac{\bar{h}_{conv} D}{k_v} = C\left[\frac{g(\rho_\ell - \rho_v) h'_{fg} D^3}{\eta_v k_v (T_s - T_{sat})}\right]^{1/4} \tag{1}$$

$$\bar{h}_{rad} = \frac{\varepsilon\sigma\left(T_s^4 - T_{sat}^4\right)}{T_s - T_{sat}} \tag{2}$$

where C = 0.67 for spheres and $\sigma = 5.67 \times 10^{-8}$ W/m$^2\cdot$K^4. The corrected latent heat is

$$h'_{fg} = h_{fg} + 0.8\, c_{p,v}(T_s - T_{sat}) \tag{3}$$

The total heat transfer coefficient is given by Eq. 10.10a as

$$\bar{h}^{4/3} = \bar{h}_{conv}^{4/3} + \bar{h}_{rad} \cdot \bar{h}^{1/3} \tag{4}$$

The vapor properties are evaluated at the film temperature,

$$T_f = (T_s + T_{sat})/2 \tag{5}$$

while the liquid properties are evaluated at the saturation temperature. Using the foregoing relations in *IHT* (see Comments), the following results are obtained.

$\overline{Nu}_D$	$\bar{h}_{cnv}\left(W/m^2\cdot K\right)$	$\bar{h}_{rad}\left(W/m^2\cdot K\right)$	$\bar{h}\left(W/m^2\cdot K\right)$	
226	867	12.0	876	<

The radiation process contribution is 1.4% that of the total heat rate.

(b) For the lumped-capacitance method, from Section 5.3, the energy balance is

$$-\bar{h}A_s(T_s - T_{sat}) = \rho_s V c_s \frac{dT_s}{dt} \tag{6}$$

where ρ_s and c_s are properties of the sphere. To determine $T_s(t)$, it is necessary to evaluate $\bar{h}$ as a function of T_s. Using the foregoing relations in *IHT* (see Comments), the sphere temperature after 30s is

$$T_s(30s) = 333°C. \qquad <$$

Continued …..

PROBLEM 10.26 (Cont.)

COMMENTS: (1) The Biot number associated with the aluminum alloy sphere cooling process for the initial condition is Bi = 0.09. Hence, the lumped-capacitance method is valid.

(2) The *IHT* code to solve this application uses the film-boiling correlation function, the water properties function, and the lumped capacitance energy balance, Eq. (6). The results for part (a), including the properties required of the correlation, are shown at the outset of the code.

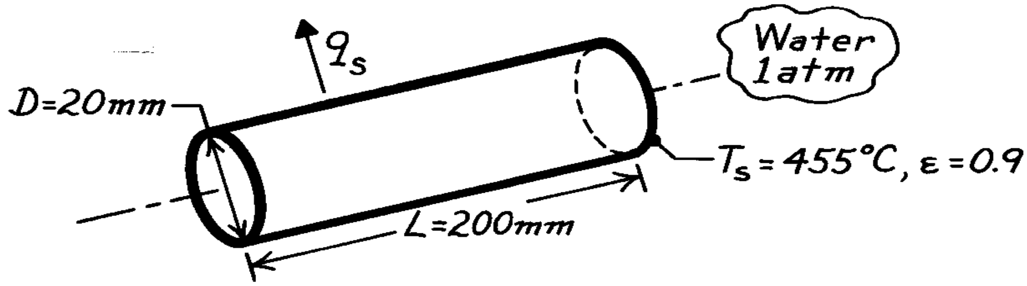
```
/* Results, Part (a): Initial condition, Ts = 500C
NuDbar          hbar     hcvbar    hradbar     F
226             875.5    866.5     11.97     0.01367    */

/* Properties: Initial condition, Ts = 500 C, Tf = 573 K
Pr      cpv     h'fg      hfg       kv       nuv       rhol     rhov
1.617   5889    3.291E6   1.406E6   0.0767   4.33E-7   712.1    45.98 */

/* Results: with initial condition, Ts = 500 C; after 30 s
Bi              F         Ts_C      hbar     t
0.09414         0.01367   500       875.5    0
0.04767         0.01587   333.2     443.3    30

// LCM analysis, energy balance
- hbar * As * (Ts - Tsat) = rhos * Vol * cps * der(Ts,t)
As = pi * D^2 / 4
Vol = pi * D^3 / 6

/*  Correlation description: coefficients for film pool boiling (FPB).  Eqs. 10.9, 10.10 and 10.11 .
See boiling curve, Fig 10.4 . */
NuDbar = NuD_bar_FPB(C,rhol,rhov,h'fg,nuv,kv,deltaTe,D,g)          // Eq 10.9
NuDbar = hcvbar * D / kv
g = 9.8                              // gravitational constant, m/s^2
deltaTe = Ts - Tsat                  // excess temperature, K
// Ts_C = 500                        // surface temperature, K
Tsat = 373                                      // saturation temperature, K
// The vapor properties are evaluated at the film temperature,Tf,
Tf = Tfluid_avg(Ts,Tsat)
// The correlation constant is 0.62 or 0.67 for cylinders or spheres,
C = 0.67
// The corrected latent heat is
h'fg = hfg + 0.80*cpv*(Ts - Tsat)
// The radiation coefficient is
hradbar = eps * sigma * (Ts^4 - Tsat^4) / (Ts - Tsat)     // Eq 10.11
sigma = 5.67E-8                      // Stefan-Boltzmann constant, W/m^2·K^4
eps =    0.25                        // surface emissivity
// The total heat transfer coefficient is
hbar^(4/3) = hcvbar^(4/3) + hradbar * hbar^(1/3)     // Eq 10.10a
F = hradbar / hbar                   // fraction contribution of radiation

// Input variables
D = 0.020
rhos = 2702                          // Sphere properties, aluminum alloy 2024
cps = 875
ks = 186
Bi = hbar * D / ks                   // Biot number

// Water property functions :T dependence, From Table A.6
// Units: T(K), p(bars);
x = 1                                // Quality (0=sat liquid or 1=sat vapor)
xx = 0
rhov = rho_Tx("Water",Tf,x)          // Density, kg/m^3
rhol = rho_Tx("Water",Tf,xx)                    // Density, kg/m^3
hfg = hfg_T("Water",Tf)              // Heat of vaporization, J/kg
cpv = cp_Tx("Water",Tf,x)            // Specific heat, J/kg·K
nuv = nu_Tx("Water",Tf,x)            // Kinematic viscosity, m^2/s
kv = k_Tx("Water",Tf,x)              // Thermal conductivity, W/m·K
Pr = Pr_Tx("Water",Tf,x)             // Prandtl number

// Conversions
Ts_C = Ts - 273
```

PROBLEM 10.27

KNOWN: Steel bar upon removal from a furnace immersed in water bath.

FIND: Initial heat transfer rate from bar.

SCHEMATIC:



ASSUMPTIONS: (1) Uniform bar surface temperature, (2) Film pool boiling conditions.

PROPERTIES: *Table A-6*, Water, liquid (1 atm, T_{sat} = 100°C): r_ℓ = 957.9 kg/m^3, h_{fg} = 2257 kJ/kg; *Table A-6*, Water, vapor ($T_f = (T_s + T_{sat})/2$ = 550K): ρ_v = 31.55 kg/m^3, $c_{p,v}$ = 4640 J/kg·K, μ_v = 18.6 × 10^{-6} N·s/m^2, k_v = 0.0583 W/m·K.

ANALYSIS: The total heat transfer rate from the bar at the instant of time it is removed from the furnace and immersed in the water is

$$q_s = \bar{h} A_s \left(T_s - T_{sat} \right) = \bar{h} A_s \Delta T_e \qquad (1)$$

where ΔT_e = 455 – 100 = 355K. According to the boiling curve of Figure 10.4, with such a high ΔT_e, film pool boiling will occur. From Eq. 10.10,

$$\bar{h}^{4/3} = \bar{h}_{conv}^{4/3} + \bar{h}_{rad} \cdot \bar{h}^{1/3} \qquad \text{or} \qquad \bar{h} = \bar{h}_{conv} + \frac{3}{4}\bar{h}_{rad} \;\left(\text{if } h_{conv} > h_{rad}\right). \qquad (2)$$

To estimate the convection coefficient, use Eq. 10.9,

$$\overline{Nu}_D = \frac{\bar{h}_{conv} D}{k_v} = C\left[\frac{g\left(r_\ell - r_v\right) h'_{fg} D^3}{n_v k_v \Delta T_e}\right]^{1/4} \qquad (3)$$

where C = 0.62 for the horizontal cylinder and $h'_{fg} = h_{fg} + 0.8\, c_{p,v}\left(T_s - T_{sat}\right)$. Find

$$\bar{h}_{conv} = \frac{0.0583\,\text{W/m}\cdot\text{K}}{0.020\ \text{m}} 0.62 \left[\frac{9.8\,\text{m/s}^2\left(957.9 - 31.55\right)\text{kg/m}^3\left[2257\times10^3 + 0.8\times4640\times355\right]\text{J/kg}\left(0.020\text{m}\right)^3}{\left(18.6\times10^{-6}/31.55\right)\text{m}^2/\text{s}\times0.0583\,\text{W/m}\cdot\text{K}\times355\text{K}}\right]^{1/4}$$

$$\bar{h}_{conv} = 690\ \text{W/m}^2\cdot\text{K}.$$

To estimate the radiation coefficient, use Eq. 10.11,

$$\bar{h}_{rad} = \frac{e\,s\left(T_s^4 - T_{sat}^4\right)}{T_s - T_{sat}} = \frac{0.9\times5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left(728^4 - 373^4\right)\text{K}^4}{355\text{K}} = 37.6\,\text{W/m}^2\cdot\text{K}.$$

Substituting numerical values into the simpler form of Eq. (2), find

$$\bar{h} = \left(690 + \left(3/4\right)37.6\right)\text{W/m}^2\cdot\text{K} = 718\,\text{W/m}^2\cdot\text{K}.$$

Using Eq. (1), the heat rate, with $A_s = \pi$ D L, is

$$q_s = 718\ \text{W/m}^2\cdot\text{K}\left(p\times0.020\text{m}\times0.200\text{m}\right)\times355\text{K} = 3.20\text{kW}. \qquad <$$

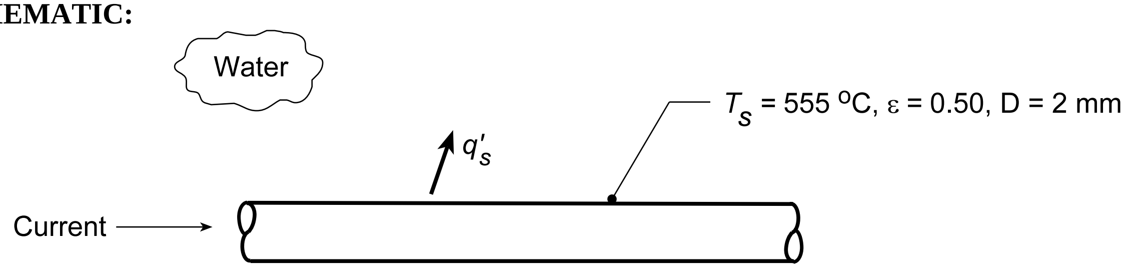
COMMENTS: For these conditions, the convection process dominates.

PROBLEM 10.28

KNOWN: Electrical conductor with prescribed surface temperature immersed in water.

FIND: (a) Power dissipation per unit length, q'_s and (b) Compute and plot q'_s as a function of surface temperature $250 \le T_s \le 650°C$ for conductor diameters of 1.5, 2.0, and 2.5 mm; separately plot the percentage contribution of radiation as a function of T_s.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Water saturated at l atm, (3) Film pool boiling.

PROPERTIES: *Table A-6*, Water, liquid (1 atm, T_{sat} = 100°C): ρ_ℓ = 957.9 kg/m^3, h_{fg} = 2257 kJ/kg; *Table A-6*, Water, vapor ($T_f = (T_s + T_{sat})/2$ = 600 K): ρ_v = 72.99 kg/m^3, $c_{p,v}$ = 8750 J/kg·K, μ_v = 22.7 × 10^{-6} N·s/m^2, k_v = 0.0929 W/m·K.

ANALYSIS: (a) The heat rate per unit length due to electrical power dissipation is

$$q'_s = \frac{q_s}{\ell} = \bar{h}\frac{A_s}{\ell}(T_s - T_{sat}) = \bar{h}\pi D\Delta T_e$$

where ΔT_e = (555 - 100)°C = 455°C. According to the boiling curve of Figure 10.4, with such a high ΔT_e, film pool boiling will occur. From Eq 10.10,

$$\bar{h}^{4/3} = \bar{h}_{conv}^{4/3} + \bar{h}_{rad} \cdot \bar{h}^{1/3} \quad \text{or} \quad \bar{h} = \bar{h}_{conv} + \frac{3}{4}\bar{h}_{rad} \quad \left(\text{if } \bar{h}_{conv} > \bar{h}_{rad}\right).$$

To estimate the convection coefficient, use Eq. 10.9,

$$\overline{Nu}_D = \frac{\bar{h}_{conv}D}{k_v} = C\left[\frac{g(\rho_\ell - \rho_v)h'_{fg}D^3}{\nu_v k_v \Delta T_e}\right]^{1/4}$$

where C = 0.62 for the horizontal cylinder and $h'_{fg} = h_{fg} + 0.8c_{p,v}(T_s - T_{sat})$. Find

$$\bar{h}_{conv} = \frac{0.0929\,\text{W/m}\cdot\text{K}}{0.002\,\text{m}} \times 0.62\left[\frac{9.8\,\text{m/s}^2\,(957.9 - 72.99)\,\text{kg/m}^3\left[2257\times10^3 + 0.8\times8750\times455\right]\text{J/kg}\,(0.002\text{m})^3}{\left(22.7\times10^{-6}/72.99\right)\text{m}^2/\text{s}\times0.0929\,\text{W/m}\cdot\text{K}\times455\,\text{K}}\right]^{1/4}$$

$$\bar{h}_{conv} = 2108\,\text{W/m}^2\cdot\text{K}.$$

To estimate the radiation coefficient, use Eq. 10.11.

$$\bar{h}_{rad} = \frac{\varepsilon\sigma\left(T_s^4 - T_{sat}^4\right)}{T_s - T_{sat}} = \frac{0.5\times5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left(828^4 - 373^4\right)\text{K}^4}{455\,\text{K}} = 28\,\text{W/m}^2\cdot\text{K}.$$

Since $h_{conv} > h_{rad}$, the simpler form of Eq. 10.10b is appropriate. Find,

$$\bar{h} = \left(2108 + (3/4)\times28\right)\text{W/m}^2\cdot\text{K} = 2129\,\text{W/m}^2\cdot\text{K}$$

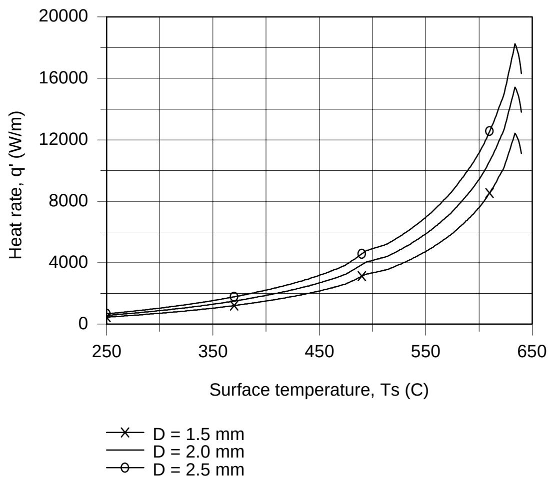
Continued...

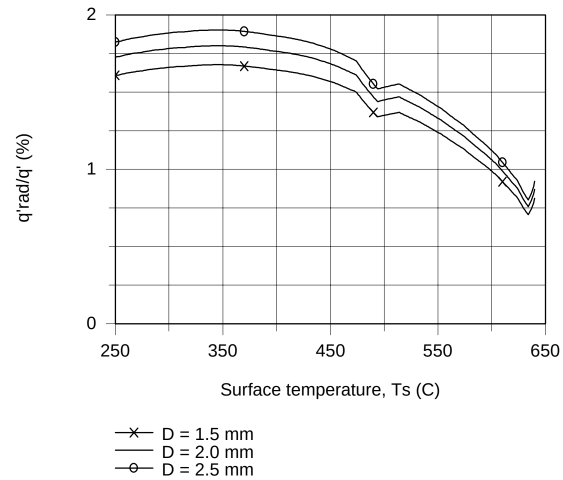
PROBLEM 10.28 (Cont.)

The heat rate is

$$q' = 2129\,\mathrm{W/m^2 \cdot K} \times \pi\,(0.002\mathrm{m}) \times 455\,\mathrm{K} = 6.09\,\mathrm{kW/m}. \qquad <$$

(b) Using the *IHT Correlations Tool, Boiling, Film Pool Boiling*, combined with the *Properties Tool* for *Water*, the heat rate, q', was calculated as a function of the surface temperature, T_s, for conductor diameters of 1.5, 2.0 and 2.5 mm. Also, plotted below is the ratio (%) of q'_{rad}/q' as a function of surface temperature.





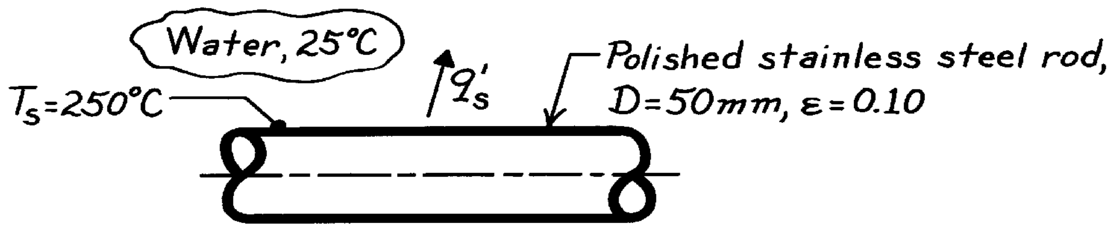
From the q' vs. T_s plot, note that the heat rate increases markedly with increasing surface temperatures, and, as expected the heat rate increases with increasing diameter. The discontinuity near T_s = 650°C is caused by the significant changes in the thermophysical properties as the film temperature, T_f, approaches the critical temperature, 647.3 K. From the q'_{rad}/q' vs. T_s plot, the maximum contribution by radiation is 2% and surprisingly doesn't occur at the maximum surface temperature. By examining a plot of q'_{rad} vs. T_s, we'd see that indeed q'_{rad} increases markedly with increasing T_s; but q'_{conv} increases even more markedly so the relative contribution of the radiation mode actually decreases with increasing temperature for T_s > 350°C.

PROBLEM 10.29

KNOWN: Horizontal, stainless steel bar submerged in water at 25°C.

FIND: Heat rate per unit length of the bar.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Film pool boiling, (3) Water at 1 atm.

PROPERTIES: *Table A-6*, Water, liquid (1 atm, Tsat = 100°C): r_ℓ = 957.9 kg/m3, hfg = 2257 kJ/kg; *Table A-6*, Water, vapor, ($T_f = (T_s + T_{sat})/2 \approx 450$K): $\rho_v = 4.81$ kg/m^3, $c_{p,v} = 2560$ J/kg·K, $\mu_v = 14.85 \times 10^{-6}$ N·s/m^2, $k_v = 0.0331$ W/m·K.

ANALYSIS: The heat rate per unit length is

$$q'_s = q_s/\ell = q''p\,D = \bar{h}p D\left(T_s - T_{sat}\right) = \bar{h}p\,D\ \Delta T_e$$

where ΔT_e = (250-100)°C = 150°C. Note from the boiling curve of Figure 10.4, that film boiling will occur. From Eq. 10.10,

$$\bar{h}^{4/3} = \bar{h}_{conv}^{4/3} + \bar{h}_{rad}\bar{h}^{1/3} \qquad \text{or} \qquad \bar{h} = \bar{h}_{conv} + \frac{3}{4}\bar{h}_{rad} \qquad \left(\text{if } \bar{h}_{conv} > \bar{h}_{rad}\right).$$

To estimate the convection coefficient, use Eq. 10.9,

$$\overline{Nu}_D = \frac{\bar{h}_{conv}D}{k_v} = C\left[\frac{g\left(r_\ell - r_v\right)h'_{fg}D^3}{n_v\,k_v\,\Delta T_e}\right]^{1/4}$$

where C = 0.62 for the horizontal cylinder and $h'_{fg} = h_{fg} + 0.8c_{p,v}\left(T_s - T_{sat}\right)$. Find

$$\bar{h}_{conv} = \frac{0.0331\text{W/m}\cdot\text{K}}{0.050\text{m}}0.62\left[\frac{9.8\text{m/s}^2\left(957.9-4.81\right)\text{kg/m}^3\left[2257\times10^3 + 0.8\times2560\text{J/kg}\cdot\text{K}\times150\text{K}\right]\left(0.050\text{m}\right)^3}{\left(14.85\times10^{-6}/4.81\right)\text{m}^2/\text{s}\times0.0331\text{ W/m}\cdot\text{K}\times150\text{K}}\right]^{1/4}$$

$$\bar{h}_{conv} = 273\text{ W/m}^2\cdot\text{K}.$$

To estimate the radiation coefficient, use Eq. 10.11,

$$\bar{h}_{rad} = \frac{e\,s\left(T_s^4 - T_{sat}^4\right)}{T_s - T_{sat}} = \frac{0.50\times5.67\times10^{-8}\text{W/m}^2\cdot\text{K}^4\left(523^4 - 373^4\right)\text{K}^4}{150\text{K}} = 11\text{ W/m}^2\cdot\text{K}.$$

Since $h_{conv} > h_{rad}$, the simpler form of Eq. 10.10 is appropriate. Find,

$$\bar{h} = \left[273 + \left(3/4\right)\times11\right]\ \text{W/m}^2\cdot\text{K} = 281\text{ W/m}^2\cdot\text{K}.$$

Using the rate equation, find

$$q'_s = 281\text{W/m}^2\cdot\text{K}\times p\times\left(0.050\text{m}\right)\times150\text{K} = 6.62\text{kW/m}.$$ <

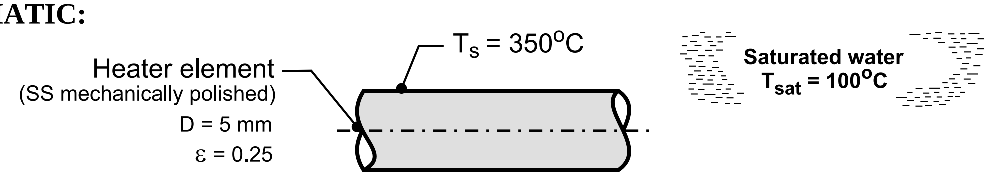
COMMENTS: The effect of the water being subcooled (T = 25°C < T_{sat}) is considered to be negligible.

PROBLEM 10.30

KNOWN: Heater element of 5-mm diameter maintained at a surface temperature of 350°C when immersed in water under atmospheric pressure; element sheath is stainless steel with a mechanically polished finish having an emissivity of 0.25.

FIND: (a) The electrical power dissipation and the rate of evaporation per unit length; (b) If the heater element were operated at the same power dissipation rate in the nucleate boiling regime, what temperature would the surface achieve? Calculate the rate of evaporation per unit length for this operating condition; and (c) Make a sketch of the boiling curve and represent the two operating conditions of parts (a) and (b). Compare the results of your analysis. If the heater element is operated in the power-controlled mode, explain how you would achieve these two operating conditions beginning with a cold element.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, and (2) Water exposed to standard atmospheric pressure and uniform temperature, T_{sat}.

PROPERTIES: *Table A-6*, Saturated water, liquid (100°C): $\rho_\ell = 957.9\ \text{kg/m}^3$, $c_{p,\ell} = 4217$ J/kg·K, $\mu_\ell = 279 \times 10^{-6}\ \text{N·s/m}^2$, $\text{Pr}_\ell = 1.76$, h_{fg} = 2257 kJ/kg, $h'_{fg} = h_{fg} + 0.80\ c_{p,v}(T_s - T_{sat})$ = 2905 kJ/kg, $\sigma = 58.9 \times 10^{-3}$ N/m; Saturated water, vapor (100°C): $\rho_v = 0.5955\ \text{kg/m}^3$; Water vapor ($T_f$ = 498 K): $\rho_v = 1/v_v = 12.54\ \text{kg/m}^3$, $c_{p,v}$ = 3236 J/kg·K, k_v = 0.04186 W/m·K, $\eta_v = 1.317 \times 10^{-6}\ \text{m}^2/\text{s}$.

ANALYSIS: (a) Since $\Delta T_e > 120°C$, the element is operating in the *film-boiling* (FB) regime. The electrical power dissipation per unit length is

$$q'_s = \bar{h}(\pi D)(T_s - T_{sat}) \qquad (1)$$

where the total heat transfer coefficient is

$$\bar{h}^{4/3} = \bar{h}_{conv}^{4/3} + \bar{h}_{rad}\,\bar{h}^{1/3} \qquad (2)$$

The convection coefficient is given by the correlation, Eq. 10.9, with C = 0.62,

$$\frac{\bar{h}_{conv} D}{k_v} = C\left[\frac{g(\rho_\ell - \rho_v) h'_{fg} D^3}{\eta_v k_v (T_s - T_{sat})}\right]^{1/4} \qquad (3)$$

$$\bar{h}_{conv} = 0.62\left[\frac{9.8\ \text{m/s}^2 (833.9 - 12.54)\ \text{kg/m}^3 \times 2.905 \times 10^6\ \text{J/kg·K}\,(0.005\ \text{m})^3}{1.31 \times 10^{-6}\ \text{m}^2/\text{s} \times 0.04186\ \text{W/m·K}\,(350 - 100)\text{K}}\right]^{1/4}$$

$$\bar{h}_{conv} = 626\ \text{W/m}^2 \cdot \text{K}$$

The radiation coefficient, Eq. (10.11), with $\sigma = 5.67 \times 10^{-8}\ \text{W/m}^2\text{·K}^4$, is

Continued …..

$$\bar{h}_{rad} = \frac{\varepsilon\sigma\left(T_s^4 - T_{sat}^4\right)}{\left(T_s - T_{sat}\right)}$$

$$\bar{h}_{rad} = \frac{0.25\sigma\left(623^4 - 373^4\right)K^4}{(350-100)K} = 4.5\ W/m^2\cdot K$$

Substituting numerical values into Eq. (2) for $\bar{h}$, and into Eq. (1) for q'_s, find

$$\bar{h} = 630\ W/m^2\cdot K$$

$$q'_s = 630\ W/m^2\cdot K\,(\pi\times 0.005\ m)(350-100)K = 2473\ W/m \quad <$$

$$q''_s = q'_s/\pi D = 0.157\ MW/m^2$$

The evaporation rate per unit length is

$$\dot{m}'_b = q'_s/h_{fg} = 3.94\ kg/h\cdot m \quad <$$

(b) For the same heat flux, $q''_s = 0.157\ MW/m^2$, using the Rohsenow correlation for the *nucleate boiling* (NB) regime, find ΔT_e, and hence T_s.

$$q''_s = \mu_\ell h_{fg}\left[\frac{g(\rho_\ell - \rho_v)}{\sigma}\right]^{1/2}\left(\frac{c_{p,\ell}\,\Delta T_e}{C_{s,f}\,h_{fg}\,Pr_\ell^n}\right)^3$$

where, from Table 10.1, for stainless steel mechanically polished finish with water, $C_{s,f}$ = 0.013 and n = 1.0.

$$0.157\times10^6 W/m^2 = 279\times10^{-6} N\cdot s/m^2 \times 2.257\times10^6 J/kg$$

$$\times\left[\frac{9.8\ m/s^2\,(957.9-0.5955)kg/m^3}{58.9\times10^{-3} N/m}\right]^{1/2}$$

$$\times\left(\frac{4217\ J/kg\cdot K\times\Delta T_e}{0.013\times2.257\times10^6 J/kg\times1.76}\right)^3$$

$$\Delta T_e = T_s - T_{sat} = 10.5\ K \qquad T_s = 110.5°C \quad <$$

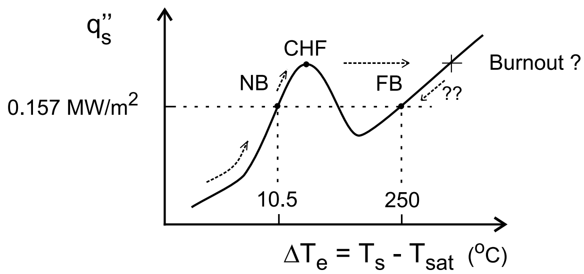
The evaporation rate per unit length is

$$\dot{m}'_b = q''_s(\pi D)h_{fg} = 3.94\ kg/h\cdot m \quad <$$

Continued …..

PROBLEM 10.30 (Cont.)

(c) The two operating conditions are shown on the boiling curve, which is fashioned after Figure 10.4. For FB the surface temperature is $T_s = 350°C$ ($\Delta T_e = 250°C$). The element can be operated at NB with the same heat flux, $q_s'' = 0.157$ MW/m^2, with a surface temperature of $T_s = 110°C$ ($\Delta T_e = 10°C$). Since the heat fluxes are the same for both conditions, the evaporation rates are the same.



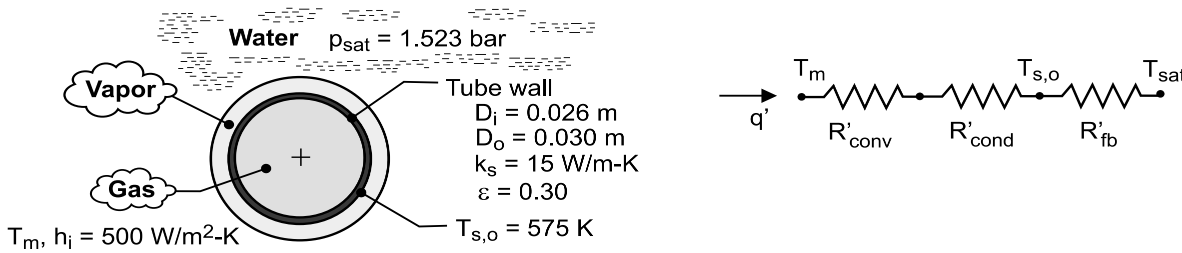
If the element is cold, and operated in a power-controlled mode, the element would be brought to the NB condition following the arrow shown next to the boiling curve near $\Delta T_e = 0$. If the power is increased beyond that for the NB point, the element will approach the critical heat flux (CHF) condition. If q_s'' is increased beyond q''_{max}, the temperature of the element will increase abruptly, and the burnout condition will likely occur. If burnout does not occur, reducing the heat flux would allow the element to reach the FB point.

PROBLEM 10.31

KNOWN: Inner and outer diameters, outer surface temperature and thermal conductivity of a tube. Saturation pressure of surrounding water and convection coefficient associated with gas flow through the tube.

FIND: (a) Heat rate per unit tube length, (b) Mean temperature of gas flow through tube.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) Uniform surface temperature, (3) Water is at saturation temperature, (4) Tube is horizontal.

PROPERTIES: *Table A-6*, saturated water, liquid (p = 1.523 bars): T_{sat} = 385 K, $\rho_\ell = 950\,\text{kg/m}^3$, h_{fg} = 2225 kJ/kg. *Table A-6*, saturated water, vapor (T_f = 480 K): $\rho_v = 9.01\,\text{kg/m}^3$, $c_{p,v}$ = 2940 J/kg·K. $\mu_v = 15.9 \times 10^{-6}\,\text{N·s/m}^2$, k_v = 0.0381 W/m·K, $\nu_v = 1.77\times10^{-6}\,\text{m}^2/\text{s}$.

ANALYSIS: (a) The heat rate per unit length is $q' = h_o \pi D_o \left(T_{s,o} - T_{sat}\right)$, where h_o includes contributions due to convection and radiation in film boiling. With C = 0.62 and $h'_{fg} = h_{fg} + 0.80\, c_{p,v}\left(T_{s,o} - T_{sat}\right) = 2.67\times10^6\,\text{J/kg}$, Eq. 10.9 yields

$$\bar{h}_{conv,o} = \left(\frac{0.0381\,\text{W/m}\cdot\text{K}}{0.030\text{m}}\right)0.62\left[\frac{9.8\,\text{m/s}^2\,(950-9)\,\text{kg/m}^3 \times 2.67\times10^6\,\text{J/kg}\,(0.03\text{m})^3}{1.77\times10^{-6}\,\text{m}^2/\text{s}\times0.0381\,\text{W/m}\cdot\text{K}\times190\,\text{K}}\right]^{1/4} = 376\,\text{W/m}^2\cdot\text{K}$$

From Eq. 10.11, the radiation coefficient is

$$\bar{h}_{rad,o} = \frac{0.30\times5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left(575^4 - 385^4\right)\text{K}^4}{(575-385)\,\text{K}} = 7.8\,\text{W/m}^2\cdot\text{K}$$

From Eq. 10.10b, it follows that

$$\bar{h}_o = \bar{h}_{conv,o} + 0.75\,\bar{h}_{rad,o} = 382\,\text{W/m}^2\cdot\text{K}$$

and the heat rate is

$$q' = \bar{h}_o \pi D_o \left(T_{s,o} - T_{sat}\right) = 382\,\text{W/m}^2\cdot\text{K}\,(\pi\times0.03\text{m})\,190\,\text{K} = 6840\,\text{W}$$ <

(b) From the thermal circuit, with $R'_{conv,i} = \left(h_i \pi D_i\right)^{-1} = \left(500\,\text{W/m}^2\cdot\text{K}\times\pi\times0.026\text{m}\right)^{-1} = 0.0245\,\text{m}\cdot\text{K/W}$

and $R'_{cond} = \ell n\left(Do/Di\right)/2\pi k_s = \ell n\left(0.030/0.026\right)/2\pi\left(15\,\text{W/m}\cdot\text{K}\right) = 0.00152\,\text{m}\cdot\text{K/W}$,

$$q' = \frac{T_m - T_{s,o}}{R_{conv,i} + R'_{cond}} = \frac{T_m - 575\,\text{K}}{(0.0245 + 0.00152)\,\text{m}\cdot\text{K/W}}$$

$$T_m = 575\,\text{K} + 6840\,\text{W/m}\left(0.0260\,\text{m}\cdot\text{K/W}\right) = 753\,\text{K}$$ <

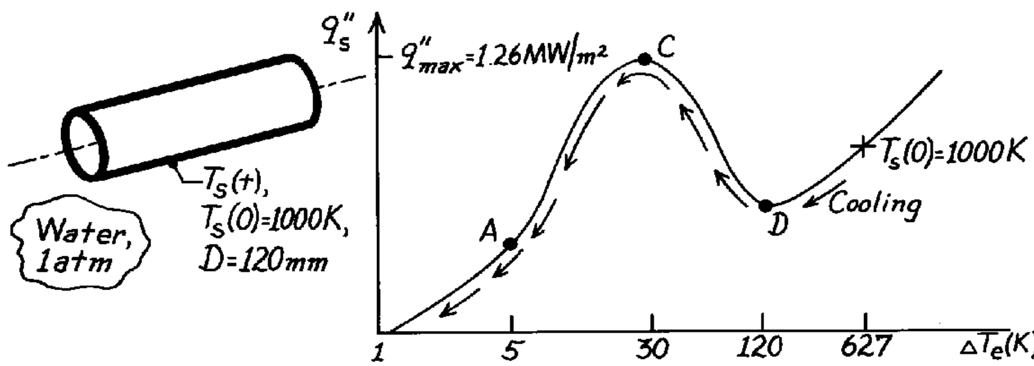
COMMENTS: Despite the large temperature of the gas, the rate of heat transfer is limited by the large thermal resistances associated with convection from the gas and film boiling. The resistance due to film boiling is $R'_{fb} = \left(\pi D_o \bar{h}_o\right)^{-1} = 0.0278\,\text{m}\cdot\text{K/W}$.

PROBLEM 10.32

KNOWN: Cylinder of 120 mm diameter at 1000K quenched in saturated water at 1 atm

FIND: Describe the quenching process and estimate the maximum heat removal rate per unit length during cooling.

SCHEMATIC:



ASSUMPTIONS: Water exposed to 1 atm pressure, T_{sat} = 100°C.

ANALYSIS: At the start of the quenching process, the surface temperature is $T_s(0)$ = 1000K. Hence, $\Delta T_e = T_s - T_{sat}$ = 1000K – 373K = 627K, and from the typical boiling curve of Figure 10.4, film boiling occurs.

As the cylinder temperature decreases, ΔT_e decreases, and the cooling process follows the boiling curve sketched above. The cylinder boiling process passes through the Leidenfrost point D, into the transition or unstable boiling regime (D → C).

At point C, the boiling heat flux has reached a maximum, q''_{max} = 1.26 MW/m^2 (see Example 10.1). Hence, the heat rate per unit length of the cylinder is

$$q'_s = q'_{max} = q''_{max}(\pi D) = 1.26\,\mathrm{MW/m^2}\left[\pi\,(0.120\mathrm{m})\right] = 0.475\,\mathrm{MW/m}. \qquad <$$

As the cylinder cools further, nucleate boiling occurs (C → A) and the heat rate drops rapidly. Finally, at point A, boiling no longer is present and the cylinder is cooled by free convection.

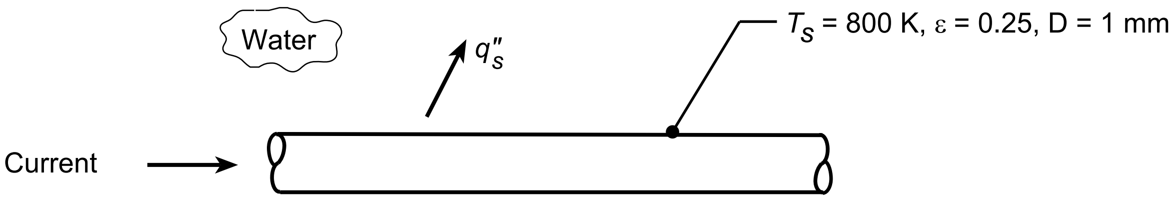
COMMENTS: Why doesn't the quenching process follow the cooling curve of Figure 10.3?

PROBLEM 10.33

KNOWN: Horizontal platinum wire of diameter of 1 mm, emissivity of 0.25, and surface temperature of 800 K in saturated water at 1 atm pressure.

FIND: (a) Surface heat flux, q_s'', when the surface temperature is T_s = 800 K and (b) Compute and plot on log-log coordinates the heat flux as a function of the excess temperature, $\Delta T_e = T_s - T_{sat}$, for the range $150 \le \Delta T_e \le 550$ K for emissivities of 0.1, 0.25, and 0.95; separately plot the percentage contribution of radiation as a function of ΔT_e.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Film pool boiling.

PROPERTIES: *Table A.6*, Saturated water, liquid (T_{sat} = 100°C, 1 atm): ρ_ℓ = 957.9 kg/m^3, h_{fg} = 2257 kJ/kg; *Table A.6*, Water, vapor ($T_f = (T_s + T_{sat})/2$ = (800 + 373)K/2 = 587 K): ρ_v = 58.14 kg/m^3, $c_{p,v}$ = 7065 J/kg·K, $\mu_v = 21.1 \times 10^{-6}$ N·s/m^2, $k_v = 81.9 \times 10^{-3}$ W/m·K.

ANALYSIS: (a) The heat flux is

$$q_s'' = \bar{h}\left(T_s - T_{sat}\right) = \bar{h}\Delta T_e$$

where ΔT_e = (800 - 373)K = 427 indicative of film boiling. From 10.10,

$$\bar{h}^{4/3} = \bar{h}_{conv}^{4/3} + \bar{h}_{rad}\bar{h}^{-1/3} \qquad \text{or} \qquad \bar{h} = \bar{h}_{conv} + (3/4)\bar{h}_{rad}$$

if $h_{rad} < h_{conv}$. Use Eq. 10.9 with C = 0.62 for a horizontal cylinder,

$$\overline{Nu}_D = \frac{\bar{h}_{conv}D}{k_v} = C\left[\frac{g\left(\rho_\ell - \rho_v\right)h_{fg}'D^3}{\nu_v k_v\left(T_s - T_{sat}\right)}\right]^{1/4}$$

$$\frac{\bar{h}_{conv}\times 0.001\,\text{m}}{81.9\times 10^{-3}\,\text{W/m}\cdot\text{K}} = 0.62\left[\frac{9.8\,\text{m/s}^2\,(957.9 - 58.14)\,\text{kg/m}^3 \times 4670\,\text{kJ/kg}\,(0.001\,\text{m})^3}{\left(21.1\times 10^{-6}\,\text{N}\cdot\text{s/m}^2/58.14\,\text{kg/m}^3\right)\times 0.0819\,\text{W/m}\cdot\text{K}\,(800-373)\,\text{K}}\right]^{1/4}$$

$$\bar{h}_{conv} = 2155\ \text{W/m}^2\cdot\text{K}$$

where $h_{fg}' = h_{fg} + 0.8c_{p,v}\left(T_s - T_{sat}\right) = 2257\,\text{kJ/kg} + 0.8\times 7065\,\text{J/kg}\cdot\text{K}\,(800-373)\,\text{K} = 4670\,\text{kJ/kg}$. To estimate the radiation coefficient, use Eq. 10.11,

$$\bar{h}_{rad} = \frac{\varepsilon\sigma\left(T_s^4 - T_{sat}^4\right)}{T_s - T_{sat}} = \frac{0.25\sigma\left(800^4 - 373^4\right)\text{K}^4}{(800-373)\,\text{K}} = 13.0\,\text{W/m}^2\cdot\text{K}.$$

Since $\bar{h}_{rad} < \bar{h}_{conv}$, use the simpler expression,

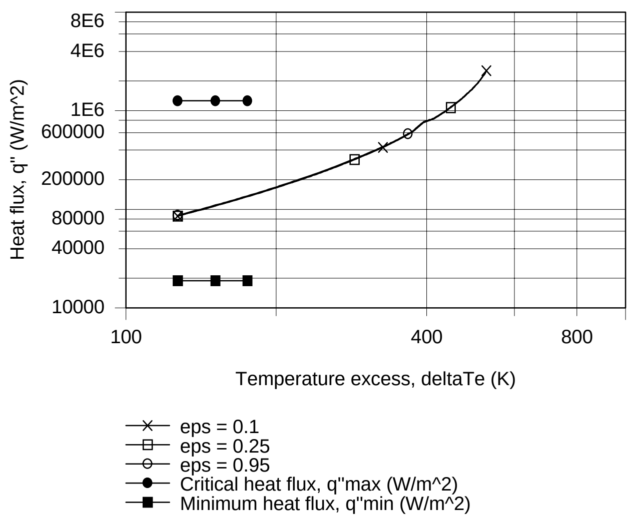
$$\bar{h} = 2155\,\text{W/m}^2\cdot\text{K} + (3/4)13.0\,\text{W/m}^2\cdot\text{K} = 2165\,\text{W/m}^2\cdot\text{K}.$$

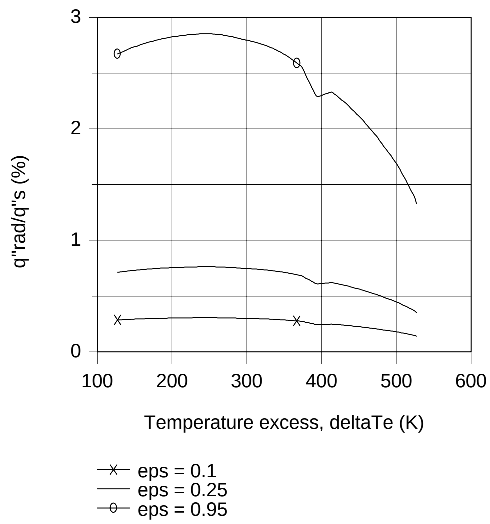
Using the rate equation, find

Continued...

$$q''_s = 2165\,\mathrm{W/m^2 \cdot K}\,(800-373)\,\mathrm{K} = 0.924\,\mathrm{MW/m^2}. \qquad <$$

(b) Using the *IHT Correlations Tool, Boiling, Film Pool Boiling*, combined with the *Properties Tool* for *Water*, the heat flux, q''_s, was calculated as a function of the excess temperature, ΔT_e for emissivities of 0.1, 0.25 and 0.95. Also plotted is the ratio (%) of q''_{rad}/q'' as a function of ΔT_e.





From the q''_s vs. ΔT_e plot, note that the heat rate increases markedly with increasing excess temperature. On the plot scale, the curves for the three emissivity values, 0.1, 0.25 and 0.95, overlap indicating that the overall effect of emissivity change on the total heat flux is slight. Also shown on the plot are the critical heat flux, q''_{max} = 1.26 MW/m^2, and the minimum heat flux, q''_{min} = 18.9kW/m^2, at the Leidenfrost point. These values are computed in Example 10.1. Note that only for the extreme value of ΔT_e is the heat flux in film pool boiling in excess of the critical heat flux. The relative contribution of the radiation mode is evident from the q''_{rad}/q''_s vs. ΔT_e plot. The maximum contribution by radiation is less than 3% and surprisingly doesn't occur at the maximum excess temperature. By examining a plot of q''_{rad} vs. ΔT_e, we'd see that indeed q''_{rad} increases markedly with increasing ΔT_e; however, q''_{conv} increases even more markedly so that the relative contribution of the radiation mode actually decreases with increasing temperature for ΔT_e > 250 K. Note that, as expected, the radiation heat flux, q''_{rad}, is proportional to the emissivity.

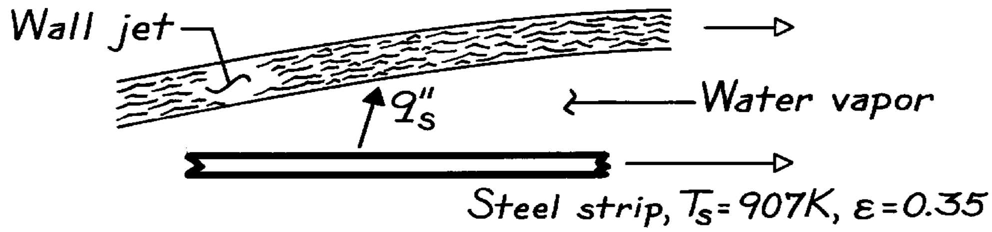
COMMENTS: Since $q''_s < q''_{max} = 1.26\,\mathrm{MW/m^2}$, the prescribed condition can only be achieved in power-controlled heating by first exceeding q''_{max} and then decreasing the flux to 0.924 MW/m^2.

PROBLEM 10.34

KNOWN: Surface temperature and emissivity of strip steel.

FIND: Heat flux across vapor blanket.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Vapor/jet interface is at T_{sat} for p = 1 atm, (3) Negligible effect of jet and strip motion.

PROPERTIES: *Table A-6*, Saturated water (100°C): r_ℓ = 957.9 kg/m^3, h_{fg} = 2257 kJ/kg; Saturated water vapor (T_f = 640K): ρ_v = 175.4 kg/m^3, $c_{p,v}$ = 42 kJ/kg·K, $\mu_v = 32 \times 10^{-6}$ N·s/m^2, k = 0.155 W/m·K, $\nu_v = 0.182 \times 10^{-6}$ m^2/s.

ANALYSIS: The heat flux is

$$q''_s = \bar{h}\Delta T_e$$

where $$\Delta T_e = 907\ \text{K} - 373\ \text{K} = 534\ \text{K}$$

and $$\bar{h}^{4/3} = \bar{h}_{conv}^{4/3} + \bar{h}_{rad}\bar{h}^{1/3} \qquad \text{or} \qquad \bar{h} = \bar{h}_{conv} + (3/4)\bar{h}_{rad}.$$

With $$h'_{fg} = h_{fg} + 0.80c_{p,v}\left(T_s - T_{sat}\right) = 2.02\times10^7\ \text{J/kg}$$

Equation 10.9 yields

$$\overline{Nu}_D = 0.62\left[\frac{9.8\,\text{m/s}^2(957.9-175.4)\text{kg/m}^3\left(2.02\times10^7\ \text{J/kg}\right)(1\ \text{m})^3}{0.182\times10^{-6}\,\text{m}^2/\text{s}(0.155\ \text{W/m}\cdot\text{K})(907-373)\text{K}}\right]^{1/4} = 6243.$$

Hence,

$$\bar{h}_{conv} = \overline{Nu}_D k_v / D = 6243\,\text{W/m}^2\cdot\text{K}(0.155\ \text{W/m}\cdot\text{K}/1\ \text{m}) = 968\ \text{W/m}^2\cdot\text{K}$$

$$\bar{h}_{rad} = \frac{e\,s\left(T_s^4 - T_{sat}^4\right)}{T_s - T_{sat}} = \frac{0.35\times5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\left(907^4 - 373^4\right)\text{K}^4}{(907-373)\text{K}}$$

$$\bar{h}_{rad} = 24\ \text{W/m}^2\cdot\text{K}$$

Hence, $$\bar{h} = 968\ \text{W/m}^2\cdot\text{K} + (3/4)\left(24\ \text{W/m}^2\cdot\text{K}\right) = 986\ \text{W/m}^2\cdot\text{K}$$

And $$q''_s = 986\ \text{W/m}^2\cdot\text{K}(907-373)\text{K} = 5.265\times10^5\ \text{W/m}^2.$$ <

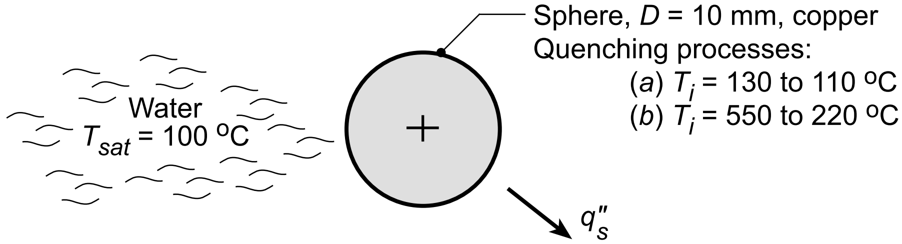
COMMENTS: The foregoing analysis is a very rough approximation to a complex problem. A more rigorous treatment is provided by Zumbrunnen et al. In ASME Paper 87-WA/HT-5.

PROBLEM 10.35

KNOWN: Copper sphere, 10 mm diameter, initially at a prescribed elevated temperature is quenched in a saturated (1 atm) water bath.

FIND: The time for the sphere to cool (a) from T_i = 130 to 110°C and (b) from T_i = 550°C to 220°C .

SCHEMATIC:



ASSUMPTIONS: (1) Sphere approximates lumped capacitance, (2) Water saturated at 1 atm.

PROPERTIES: *Table A-1*, Copper: ρ = 8933 kg/m^3; *Table A.11*, Copper (polished) : ε = 0.04, typical value; *Table A.4, Water*: as required for the pool boiling correlations.

ANALYSIS: Treating the sphere as a lumped capacitance and performing an energy balance, see Eq. 5.14,

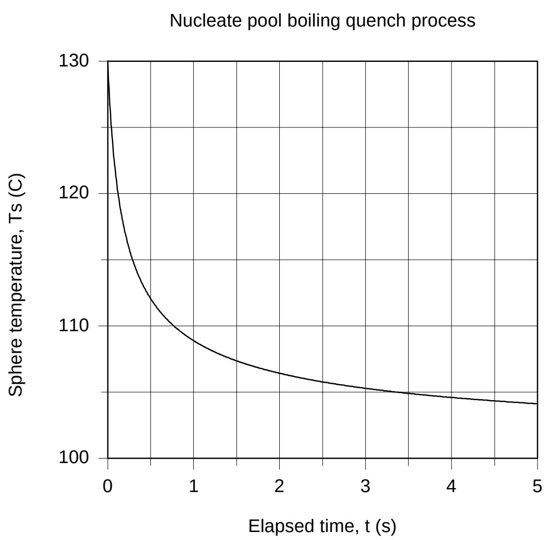
$$\dot{E}_{in} - \dot{E}_{out} = \dot{E}_{st} \qquad\qquad -q''_s \cdot A_s = \rho c \forall \frac{dT}{dt} \qquad (1,2)$$

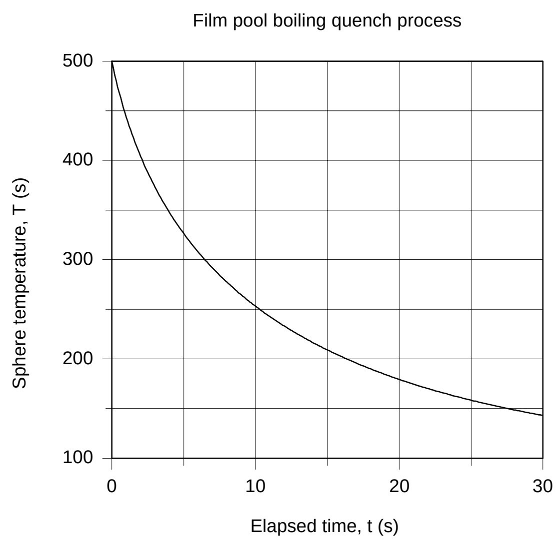
For the sphere, $V = \pi D^3 / 6$ and $A_s = \pi D^2$. Using the *IHT Lumped Capacitance Model* to solve this differential equation, we need to specify (1) the specific heat of the copper sphere as a function sphere temperature; use *IHT Properties Tool, Copper;* and (2) the heat flux, q''_s, associated with the pool boiling processes; use *IHT Correlations Tool, Boiling:*

(a) *Cooling from T_i =130° to 110°: Nucleate pool boiling*, Rohsenhow correlation, Eq. 10.5,

(b) *Cooling from T_i = 550 to 220°C : Film Pool Boiling*, Eq. 10.9 with C = 0.67 (sphere).

The thermophysical properties for water required of the correlations are provided by the *IHT Tool, Properties-Water*. The specific heat of copper as a function of sphere temperature is provided by the *IHT Tool, Properties-Copper.* The temperature-time histories for each of the cooling processes are plotted below.





Continued...

PROBLEM 10.35 (Cont.)

Using the *Explore* feature in the *IHT Plot Window*, the elapsed times for the quench process were found as:

Quench process	T_i - T_f (°C)	Δt(s)
Nucleate pool boiling	130-110	0.76
Film pool boiling	550-220	13.5

COMMENTS: (1) Comparing the elapsed times for the two processes, the nucleate pool boiling process cools 20°C in 0.76s (26.3°C/s) vs. 330°C in 13.5s (24.4°C/s) for the film pool boiling process.

(2) The IHT Workspace used to generate the temperature-time history for the nucleate pool boiling process is shown below.

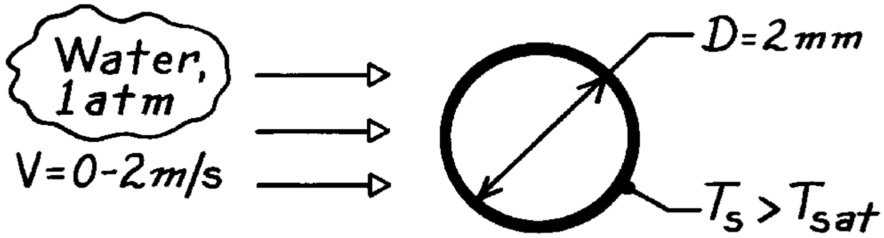
```
// Correlations Tool - Boiling, Nucleate Pool Boiling, Heat flux
qs'' = qs_dprime_NPB(Csf,n,rhol,rhov,hfg,cpl,mul,Prl,sigma,deltaTe,g)    // Eq 10.5
g = 9.8                          // Gravitational constant, m/s^2
deltaTe = Ts - Tsat              // Excess temperature, K
Ts =  Ts_C + 273                 // Surface temperature, K
//Ts_C = 130
Tsat =   100 + 273               // Saturation temperature, K
/* Evaluate liquid(l) and vapor(v) properties at Tsat. From Table 10.1 (Fill in as required), */
// fluid-surface combination:
Csf =   0.013                    // Polished copper-water combination, Table 10.1
n =   1.0
/*  Correlation description: Heat flux for nucleate pool boiling (NPB), water-surface combination (Cf,n), Eq 10.5,
Table 10.1 . See boiling curve, Fig 10.4 . */

// Properties Tool- Water:
// Water property functions :T dependence, From Table A.6
// Units: T(K), p(bars);
xv =  1                                 // Quality (0=sat liquid or 1=sat vapor)
rhov = rho_Tx("Water",Tsat,xv)          // Density, kg/m^3
hfg = hfg_T("Water",Tsat)               // Heat of vaporization, J/kg
sigma = sigma_T("Water",Tsat)           // Surface tension, N/m (liquid-vapor)
// Water property functions :T dependence, From Table A.6
// Units: T(K), p(bars);
xl = 0                                  // Quality (0=sat liquid or 1=sat vapor)
rhol = rho_Tx("Water",Tsat,xl)          // Density, kg/m^3
cpl = cp_Tx("Water",Tsat,xl)            // Specific heat, J/kg·K
mul = mu_Tx("Water",Tsat,xl)            // Viscosity, N·s/m^2
Prl = Pr_Tx("Water",Tsat,xl)            // Prandtl number

// Lumped Capacitance Model:
/* Conservation of energy requirement on the control volume, CV.  */
Edotin - Edotout = Edotst
Edotin = 0
Edotout = As * ( + qs'' )
Edotst = rho * vol * cp * Der(Ts,t)
/* The independent variables for this system and their assigned numerical values are */
As =  pi * D^2 / 4      // surface area, m^2
vol =  pi * D^3 / 6      // volume, m^3
D = 0.01
rho =  8933             // density, kg/m^3

// Properties Tool - Copper
// Copper (pure) property functions : From Table A.1
// Units: T(K)
cp = cp_T("Copper",Ts)          // Specific heat, J/kg·K
```

PROBLEM 10.36

KNOWN: Saturated water at 1 atm is heated in cross flow with velocities 0 – 2 m/s over a 2 mm-diameter tube.

FIND: Plot the critical heat flux as a function of water velocity; identify the pool boiling and transition regions between the low and high velocity ranges.

SCHEMATIC:



ASSUMPTIONS: Nucleate boiling in the presence of external forced convection.

PROPERTIES: *Table A-6*, Water (1 atm): T_{sat} = 100°C, r_ℓ = 957.9 kg/m^3, ρ_v = 0.5955 kg/m^3, h_{fg} = 2257 kJ/kg, σ = 58.9 × 10^{-3} N/m.

ANALYSIS: The Lienhard-Eichhorn correlations for forced convection boiling with cross flow over a cylinder are appropriate for estimating q''_{max}, Eqs. 10.12 and 10.13.

Low Velocity

$$q''_{max} = \frac{r_v h_{fg}}{p}\left[1+\left(\frac{4s}{r_v V^2 D}\right)^{1/3}\right]V$$

$$q''_{max} = \frac{1}{p}0.5955\frac{kg}{m^3}\times 2257\times 10^3\frac{J}{kg}\left[1+\left(\frac{4\times 58.9\times 10^{-3}\,N/m}{0.5955 kg/m^3 V^2 0.002m}\right)^{1/3}\right]V$$

$$q''_{max} = 4.2782\times 10^5 V + 2.921\times 10^6 V^{1/3}.$$

High Velocity

$$q''_{max} = \frac{r_v h_{fg}}{p}\left[\frac{1}{169}\left(\frac{r_\ell}{r_v}\right)^{3/4} + \frac{1}{19.2}\left(\frac{r_\ell}{r_v}\right)^{1/2}\left(\frac{s}{r_v V^2 D}\right)^{1/3}\right]V$$

$$q''_{max} = \frac{1}{p}0.5955\frac{kg}{m^3}\times 2257\times 10^3\frac{J}{kg}\left[\frac{1}{169}\left(\frac{957.9}{0.5955}\right)^{3/4} + \right.$$

$$\left.\frac{1}{19.2}\left(\frac{957.9}{0.5955}\right)^{1/2}\left(\frac{58.9\times 10^{-3}\,N/m}{0.5955 kg/m^3 V^2\, 0.002m}\right)^{1/3}\right]V$$

$$q''_{max} = 6.4299\times 10^5 V + 3.280\times 10^6 V^{1/3}$$

Continued …..

PROBLEM 10.36 (Cont.)

The *transition* between the low and high velocity regions occurs when

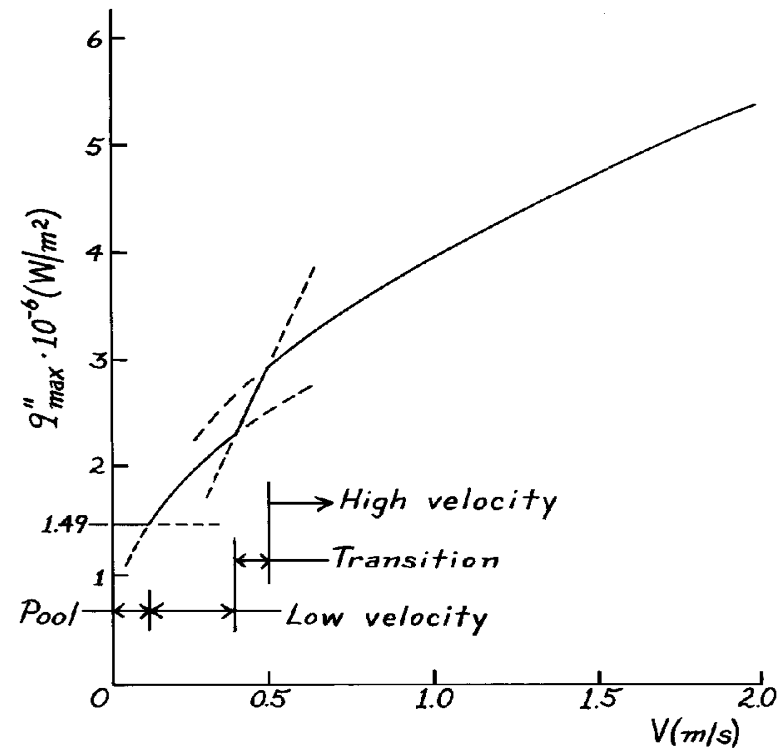
$$q''_{max} = r_v h_{fg}\ V\left[\frac{0.275}{p}\left(\frac{r_\ell}{r_v}\right)^{1/2} + 1\right]$$

$$q''_{max} = 0.5955\frac{kg}{m^3}\times 2257\times 10^3\frac{J}{kg}V\left[\frac{0.275}{p}\left(\frac{957.9}{0.5955}\right)^{1/2} + 1\right] = 6.0627\times 10^6 V. \qquad (3)$$

For pool boiling conditions when the velocity is zero, the critical heat flux must be estimated according to the correlation for the small horizontal cylinder as introduced in Problem 10.22. If the cylinder were "large," the critical heat flux would be 1.26 MW/m^2 as given by the Zuber-Kutateladze correlation, Eq. 10.7. Following the analysis of Problem 10.22, find Bo = 0.40 and the critical heat flux for the "small" 2 mm cylinder is

$$\left.q''_{max}\right)_{pool} = 1.18\times 1.26\ MW/m^2 = 1.49\ W/m^2.$$

The graph below identifies four regions: pool boiling where $q''_{max} = 1.49\ MW/m^2$ from V = 0 to 0.15 m/s and the low velocity, transition and high velocity regimes.

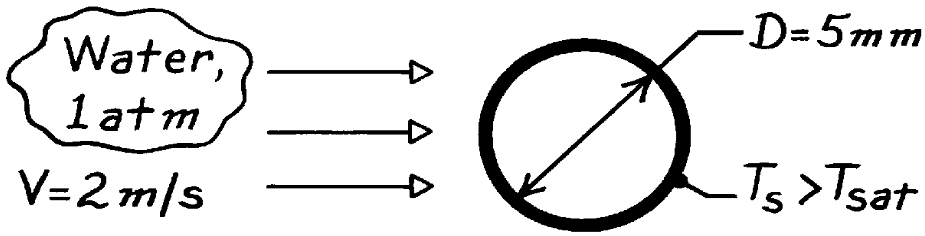


PROBLEM 10.37

KNOWN: Saturated water at 1 atm and velocity 2 m/s in cross flow over a heater element of 5 mm diameter.

FIND: Maximum heating rate, $q'[W/m]$.

SCHEMATIC:



ASSUMPTIONS: Nucleate boiling in the presence of external forced convection.

PROPERTIES: *Table A-6*, Water (1 atm): T_{sat} = 100°C, r_{ℓ} = 957.9 kg/m^3, ρ_v = 0.5955 kg/m^3, h_{fg} = 2257 kJ/kg, $\sigma = 58.9 \times 10^{-3}$ N/m.

ANALYSIS: The Lienhard-Eichhorn correlation for forced convection with cross flow over a cylinder is appropriate for estimating q''_{max}. Assuming high-velocity region flow, Eq. 10.13 with Eq. 10.14 can be written as

$$q''_{max} = \frac{r_v h_{fg} V}{p}\left[\frac{1}{169}\left(\frac{r_\ell}{r_v}\right)^{3/4} + \frac{1}{19.2}\left(\frac{r_\ell}{r_v}\right)^{1/2}\left(\frac{s}{r_v V^2 D}\right)^{1/3}\right].$$

Substituting numerical values, find

$$q''_{max} = \frac{1}{p} 0.5955 \text{kg/m}^3 \times 2257 \times 10^3 \text{J/kg} \times 2\text{m/s}\left[\frac{1}{169}\left(\frac{957.9}{0.5955}\right)^{3/4} + \right.$$

$$\left.\frac{1}{19.2}\left(\frac{957.9}{0.5955}\right)^{1/2}\left(\frac{58.9 \times 10^{-3} \text{N/m}}{0.5955 \text{kg/m}^3 (2\text{m/s})^2 \, 0.005\text{m}}\right)^{1/3}\right]$$

$$q''_{max} = 4.331 \text{MW/m}^2.$$

The high-velocity region assumption is satisfied if

$$\frac{q''_{max}}{r_v h_{fg} V} \overset{?}{<} \frac{0.275}{p}\left(\frac{r_\ell}{r_v}\right)^{1/2} + 1$$

$$\frac{4.331 \times 10^6 \text{W/m}^2}{0.5955 \text{kg/m}^3 \times 2257 \times 10^3 \text{J/kg} \times 2\text{m/s}} = 1.61 \overset{?}{<} \frac{0.275}{p}\left(\frac{957.9}{0.5955}\right)^{1/2} + 1 = 4.51.$$

The inequality is satisfied. Using the q''_{max} estimate, the maximum heating rate is

$$q'_{max} = q''_{max} \cdot p D = 4.331 \text{MW/m}^2 \times p \,(0.005\text{m}) = 68.0 \text{kW/m}. \qquad <$$

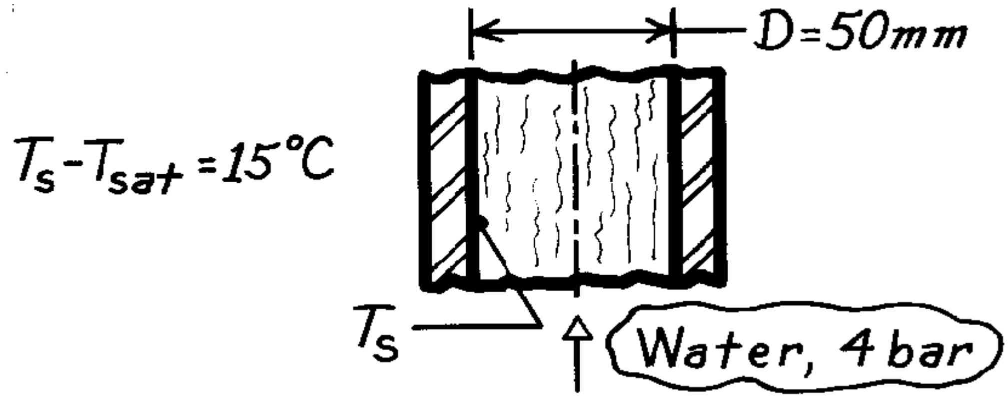
COMMENTS: Note that the effect of the forced convection is to increase the critical heat flux by 4.33/1.26 = 3.4 over the pool boiling case.

PROBLEM 10.38

KNOWN: Correlation for forced-convection local boiling inside a vertical tube.

FIND: Boiling heat transfer rate per unit length of the tube.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Local boiling occurs when tube wall is 15°C above the saturation temperature.

ANALYSIS: From experimental results, the heat transfer coefficient can be estimated by the correlation

$$h = 2.54\left(\Delta T_e\right)^3 \exp\left(\frac{p}{15.3}\right) \qquad \left[W/m^2 \cdot K\right]$$

where ΔT_e is the excess temperature, $T_s - T_{sat}$ $[K]$, and p is the pressure [bar]. The heat transfer rate per unit length is

$$q' = p\, D\, h\, \Delta T_e.$$

Evaluating the heat transfer coefficient, find

$$h = 2.54(15K)^3 \exp(4\ \text{bar}/15.3) = 11{,}134\ W/m^2 \cdot K.$$

The heat rate is then.

$$q' = p\,(0.050m) \times 11{,}134 W/m^2 \cdot K \times 15K = 26.2\ kW/m. \qquad <$$

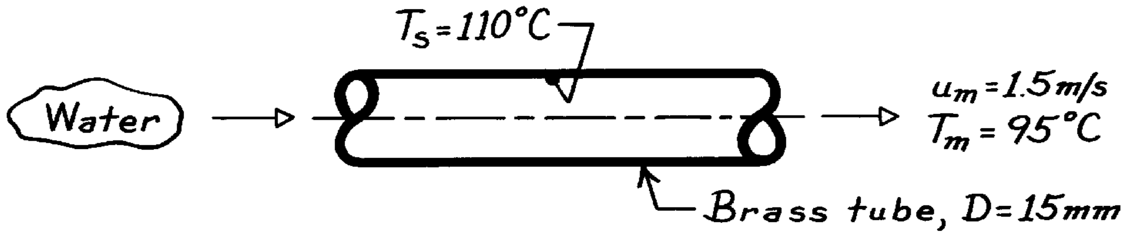
COMMENTS: The saturation temperature at 4 bar is T_{sat} = 406.5K according to Table A-6.

PROBLEM 10.39

KNOWN: Forced convection and boiling processes occur in a smooth tube with prescribed water velocity and surface temperature.

FIND: Heat transfer rate per unit length of the tube.

SCHEMATIC:



ASSUMPTIONS: (1) Fully-developed flow, (2) Nucleate boiling conditions occur on inner wall of tube, (3) Forced convection and boiling effects can be separately estimated.

PROPERTIES: *Table A-6*, Water (T_m = 95°C = 368K): $r_\ell = 1/v_f = 962$ kg/m^3, $\rho_v = 1/v_g = 0.500$ kg/m^3, $h_{fg} = 2270$ kJ/kg, $c_{p,\ell} = 4212$ J/kg·K, $m_\ell = 296 \times 10^{-6}$ N·s/m^2, $k_\ell = 0.678$ W/m·K, $Pr_\ell = 1.86$, $\sigma = 60 \times 10^{-3}$ N/m, $n_\ell = 3.08 \times 10^{-7}$ m^2/s.

ANALYSIS: Experimentation has indicated that the heat transfer rate can be estimated as the sum of the separate effects due to forced convection and boiling. On a per unit length basis,

$$q' = q'_{fc} + q'_{boil}.$$

For *forced convection*, $Re_D = u_m D / n_\ell = 1.5\text{m/s} \times 0.015\text{m} / 3.08 \times 10^{-7}\,\text{m}^2/\text{s} = 73{,}052$. Since Re > 2300, flow is turbulent and since fully developed, use the Dittus-Boelter correlation but with the 0.023 coefficient replaced by 0.019 and n = 0.4,

$$Nu_D = h D / k = 0.019 Re_D^{4/5} Pr^n$$

$$h = \frac{k}{D} Nu_D = \frac{0.678\,\text{W/m}\cdot\text{K}}{0.015\text{m}} \times 0.019 (73{,}052)^{4/5} (1.86)^{0.4} = 8563\,\text{W/m}^2\cdot\text{K}.$$

$$q'_{fc} = h p D (T_s - T_m) = 8{,}563\,\text{W/m}^2\cdot\text{K} \cdot p\,(0.015\text{m})(110-95)°\text{C} = 6052\,\text{W/m}.$$

For *boiling*, ΔT_e = (110 – 100)°C = 10°C and hence nucleate boiling occurs. From the Rohsenow equation, with C_{sf} = 0.006 and n = 1.0,

$$q''_{boil} = m_\ell h_{fg} \left[\frac{g(r_\ell - r_v)}{s} \right]^{1/2} \left[\frac{c_{p,\ell} \Delta T_e}{C_{sf}\, h_{fg}\, Pr_\ell^n} \right]^3$$

$$q'_{boil} = 296 \times 10^{-6} \frac{\text{N}\cdot\text{s}}{\text{m}^2} \times 2270 \times 10^3 \frac{\text{J}}{\text{kg}} \left[\frac{9.8\,\text{m/s}^2 (962 - 0.5)\,\text{kg/m}^3}{60 \times 10^{-3}\,\text{N/m}} \right]^{1/2} \left[\frac{4212\,\text{J/kg}\cdot\text{K} \times 10\text{K}}{0.006 \times 2270 \times 10^3 \frac{\text{J}}{\text{kg}} \times (1.86)^{1.0}} \right]^3$$

$$q''_{boil} = 1.22 \times 10^6\,\text{W/m}^2 \qquad q'_{boil} = q''_{boil} (p D) = 1.22 \times 10^6\,\text{W/m}^2\,(p \times 0.015\text{m}) = 57{,}670\,\text{W/m}.$$

The total heat rate for both processes is

$$q' = (6052 + 57{,}670)\,\text{W/m} = 6.37 \times 10^4\,\text{W/m}. \qquad <$$

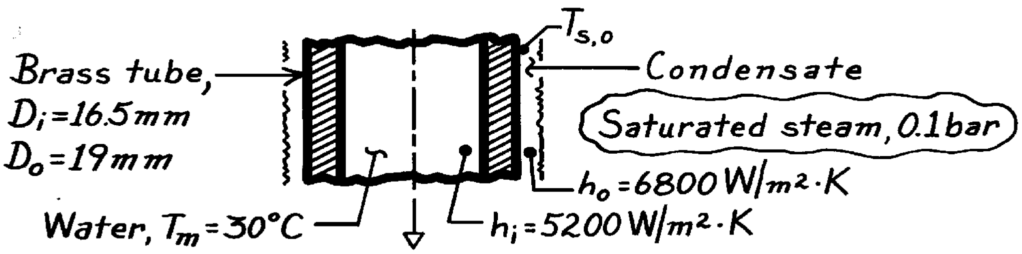
COMMENTS: Recognize that this method provides only an estimate since the processes are surely coupled.

PROBLEM 10.40

KNOWN: Saturated steam condensing on the outside of a brass tube and water flowing on the inside of the tube; convection coefficients are prescribed.

FIND: Steam condensation rate per unit length of the tube.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions.

PROPERTIES: *Table A-6*, Water, vapor (0.1 bar): $T_{sat} \approx 320K$, $h_{fg} = 2390 \times 10^3$ J/kg; *Table A-1*, Brass $\left(\overline{T} = (T_m + T_{sat})/2 \approx 300K\right)$: $k = 110\ W/m\cdot K$

ANALYSIS: The condensation rate per unit length follows from Eq. 10.33 written as

$$\dot{m}' = q'/h'_{fg} \tag{1}$$

where the heat rate follows from Eq. 10.32 using an overall heat transfer coefficient

$$q' = U_o \cdot p D_o \left(T_{sat} - T_m\right) \tag{2}$$

and from Eq. 3.31,

$$U_o = \left[\frac{1}{h_o} + \frac{D_o/2}{k}\ell n\frac{D_o}{D_i} + \frac{D_o}{D_i}\frac{1}{h_i}\right]^{-1} \tag{3}$$

$$U_o = \left[\frac{1}{6800 W/m^2\cdot K} + \frac{0.0095m}{110 W/m\cdot K}\ell n\frac{19}{16.5} + \frac{19}{16.5}\frac{1}{5200 W/m^2\cdot K}\right]^{-1}$$

$$U_o = \left[147.1\times10^{-6} + 12.18\times10^{-6} + 192.3\times10^{-6}\right]^{-1} W/m^2\cdot K = 2627 W/m^2\cdot K.$$

Combining Eqs. (1) and (2) and substituting numerical values (see below for h'_{fg}), find

$$\dot{m}' = U_o p D_o \left(T_{sat} - T_m\right)/h'_{fg}$$

$$\dot{m}' = 2627 W/m^2\cdot K p\,(0.019m)(320-303)K/2410\times10^3 J/kg = 1.11\times10^{-3} kg/s. \quad <$$

COMMENTS: (1) Note from evaluation of Eq. (3) that the thermal resistance of the brass tube is not negligible. (2) From Eq. 10.26, with Ja = $c_{p,\ell}\left(T_{sat} - T_s\right)/h_{fg}$, $h'_{fg} = h_{fg}\left[1 + 0.68 Ja\right]$. Note from expression for U_o, that the internal resistance is the largest. Hence, estimate $T_{s,o} \approx T_o - (R_o/\Sigma R)(T_o - T_m) \approx 313K$. Hence,

$$h'_{fg} \approx 2390\times10^3 J/kg\left[1 + 0.68\times4179 J/kg\cdot K\,(320-313)K / 2390\times10^3 J/kg\right]$$

$$h'_{fg} = 2410 kJ/kg$$

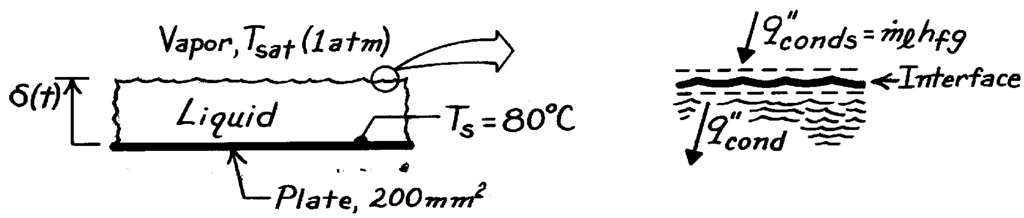
where $c_{p,\ell}$ for water (liquid) is evaluated at $T_f = (T_{s,o} + T_o)/2 \approx 317K$.

PROBLEM 10.41

KNOWN: Insulated container having cold bottom surface and exposed to saturated vapor.

FIND: Expression for growth rate of liquid layer, δ(t); thickness formed for prescribed conditions; compare with vertical plate condensate for same conditions.

SCHEMATIC:



ASSUMPTIONS: (1) Side wall effects are negligible and, (2) Vapor-liquid interface is at T_{sat}, (3) Temperature distribution in liquid is linear, (4) Constant properties.

PROPERTIES: *Table A-6*, Saturated vapor (p = 1.0133 bar): $T_{sat} = 100°C$, $\rho_v = 0.596\ kg/m^3$, $h_{fg} = 2257\ kJ/kg$; *Table A-6*, Saturated liquid ($T_f = 90°C = 363K$): $\rho_\ell = 1000\ kg/m^3$, $\mu_\ell = 313 \times 10^{-6}\ N\cdot s/m^2$, $k_\ell = 0.676\ W/m\cdot K$, $c_{p,\ell} = 4207\ J/kg\cdot K$.

ANALYSIS: Perform a surface energy balance on the interface (see above) recognizing that $\dot{m}_\ell / A = \rho_\ell\, d\delta/dt$ from an overall mass rate balance on the liquid to obtain

$$\dot{E}''_{in} - \dot{E}''_{out} = q''_{conds} - q''_{cond} = \frac{\dot{m}}{A} h_{fg} - k_\ell \frac{T_{sat} - T_s}{\delta} = \rho_\ell \frac{d\delta}{dt} h_{fg} - k_\ell \frac{T_{sat} - T_s}{\delta} = 0 \quad (1)$$

where q''_{conds} is the condensation heat flux and q''_{cond} is the conduction heat flux into the liquid layer of thickness δ with linear temperature distribution. Eq. (1) can be rewritten as

$$\rho_\ell h_{fg} \frac{d\delta}{dt} = k_\ell \frac{T_{sat} - T_s}{\delta}.$$

Separate variables and integrate with limits shown to obtain the liquid layer growth rate,

$$\int_0^\delta \delta\, d\delta = \int_0^t \frac{k_\ell \left(T_{sat} - T_s\right)}{\rho_\ell h_{fg}} dt \qquad \text{or} \qquad \delta = \left[\frac{2k_\ell \left(T_{sat} - T_s\right)}{\rho_\ell h_{fg}} t\right]^{1/2}. \qquad (2) \quad <$$

For the prescribed conditions, the liquid layer thickness and condensate formed in one hour are

$$\delta\left(1hr\right) = \left[2 \times 0.676 \frac{W}{m\cdot K}\left(100 - 80\right)°C \times 3600s / 1000\frac{kg}{m^3} \times 2257 \times 10^3 \frac{J}{kg}\right]^{1/2} = 6.57mm \quad <$$

$$M\left(1hr\right) = \rho_\ell A\delta = 1000 kg/m^3 \times 200 \times 10^{-6} m^2 \times 6.57 \times 10^{-3} m = 1.314 \times 10^{-3} kg. \quad <$$

Continued …..

PROBLEM 10.41 (Cont.)

The condensate formed on a vertical plate with the same conditions follows from Eq. 10.33,

$$M_{vp} = \dot{m}\cdot t = \bar{h}_L A\left(T_{sat} - T_s\right)\cdot t / h'_{fg}$$

where h'_{fg} and $\bar{h}_L$ follow from Eqs. 10.26 and 10.30, respectively.

$$h'_{fg} = h_{fg}\left(1 + 0.68\mathrm{Ja}\right) = h_{fg}\left(1 + 0.68 c_{p,\ell}\,\Delta T / h_{fg}\right)$$

$$h'_{fg} = 2257\times10^3\,\mathrm{J/kg}\left(1 + 0.68\times4207\frac{\mathrm{J}}{\mathrm{kg\cdot K}}(100-80)^\circ\mathrm{C}/2257\times10^3\,\mathrm{J/kg}\right) = 2314\,\mathrm{kJ/kg}$$

$$\bar{h}_L = 0.943\left[g\,r_\ell\left(r_\ell - r_v\right)k_\ell^3\,h'_{fg} / m_\ell\left(T_{sat} - T_s\right)L\right]^{1/4}$$

$$\bar{h}_L = 0.943\Big[9.8\,\mathrm{m/s^2}\times1000\,\mathrm{kg/m^3}\,(1000 - 0.596)\,\mathrm{kg/m^3}\,\left(0.676\,\mathrm{W/m\cdot K}\right)^3$$

$$\times2314\times10^3\,\mathrm{J/kg}/313\times10^{-6}\,\mathrm{N\cdot s/m^2}\,(100-80)^\circ\mathrm{C}\times0.2\mathrm{m}\Big]^{1/4}$$

$$\bar{h}_L = 8155\,\mathrm{W/m^2\cdot K}.$$

Hence,

$$M_{vp} = 8155\,\mathrm{W/m^2\cdot K}\times200\times10^{-6}\,\mathrm{m^2}\,(100-80)^\circ\mathrm{C}\times3600\mathrm{s}/2314\times10^3\,\mathrm{J/kg}$$

$$M_{vp} = 5.08\times10^{-2}\,\mathrm{kg}.$$ <

COMMENTS: (1) Note that the condensate formed by the vertical plate is an order of magnitude larger. For the vertical plate the rate of condensate formation is constant. For the container bottom surface, the rate decreases with increasing time since the conduction resistance increases as the liquid layer thickness increases.

(2) For the vertical plate, assumed to be square 14.1 × 14.1 mm, the Reynolds number, Eq. 10.35 and 10.33, is

$$\mathrm{Re}_d = \frac{4\dot{m}}{m_\ell b} = \frac{4}{m_\ell b}\frac{\bar{h}_L A\left(T_{sat} - T_s\right)}{h'_{fg}}$$

$$\mathrm{Re}_d = \frac{4}{313\times10^{-6}\,\mathrm{N\cdot s/m^2}\times14.1\times10^{-3}\,\mathrm{m}}\frac{8155\,\mathrm{W/m^2\cdot K}\left(200\times10^{-6}\,\mathrm{m^2}\right)(100-80)^\circ\mathrm{C}}{2314\,\mathrm{kJ/kg}}$$

$$\mathrm{Re}_d = 12.8.$$

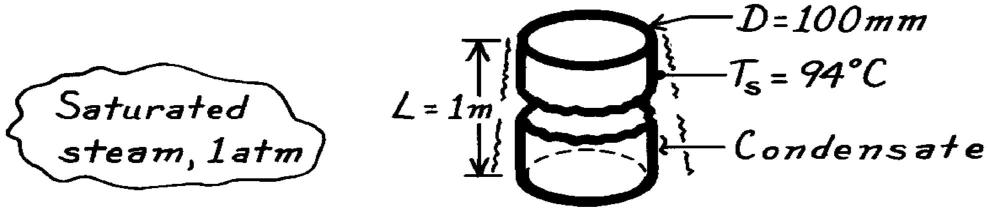
Hence, using Eq. 10.30 to estimate $\bar{h}_L$ is correct since, in fact, the film is laminar.

PROBLEM 10.42

KNOWN: Vertical tube experiencing condensation of steam on its outer surface.

FIND: Heat transfer and condensation rates.

SCHEMATIC:



ASSUMPTIONS: (1) Film condensation, (2) Negligible non-condensibles, (3) D/2 >> δ, vertical plate behavior.

PROPERTIES: *Table A-6*, Water, vapor (1.0133 bar): T_{sat} = 100°C, ρ_v = 0.596 kg/m^3, h_{fg} = 2257 kJ/kg; *Table A-6*, Water, liquid (T_f = 97°C): r_ℓ = 960.6 kg/m^3, m_ℓ = 289 × 10^{-6} N·s/m^2, $c_{p,\ell}$ = 4214 J/kg·K, k_ℓ = 0.679 W/m·K.

ANALYSIS: The heat transfer and condensation rates are

$$q = \overline{h}_L (p\, D L)(T_{sat} - T_s) \qquad\qquad \dot{m} = q / h'_{fg}$$

where $h'_{fg} = h_{fg}(1 + 0.68\text{Ja})$ and $\text{Ja} = c_{p,\ell}(T_{sat} - T_s)/h_{fg}$. Hence Ja = 4214 J/kg·K (100 - 94)K/2257 × 10^3 J/kg = 0.0112 and h'_{fg} = 2274 kJ/kg. Assume laminar film condensation and use Eq. 10.31 to estimate $\overline{h}_L$,

$$\overline{Nu}_L = \frac{\overline{h}_L L}{k_\ell} = 0.943\left[\frac{r_\ell\, g\,(r_\ell - r_v)\, h'_{fg} L^3}{m_\ell\, k_\ell\,(T_{sat} - T_s)}\right]^{1/4}$$

$$\overline{h}_L = \frac{0.679\,\text{W/m}\cdot\text{K}}{1.0\text{m}} \times 0.943\left[\frac{960.6\,\text{kg/m}^3 \times 9.8\,\text{m/s}^2\,(960.6 - 0.596)\,\text{kg/m}^3 \times 2274 \times 10^3\,\text{J/kg} \times (1\text{m})^3}{289 \times 10^{-6}\,\text{N}\cdot\text{s/m}^2 \times 0.679\,\text{W/m}\cdot\text{K}\,(100 - 94)\,\text{K}}\right]^{1/4} = 7360\,\text{W/m}^2\cdot\text{K}.$$

Hence,
$$q = 7360\,\text{W/m}^2\cdot\text{K}\,(p \times 0.100\text{m} \times 1\text{m})(100 - 94)\text{K} = 13.87\text{kW}.$$

$$\dot{m} = 13.9 \times 10^3\,\text{W}/2274 \times 10^3\,\text{J/kg} = 0.00610\text{kg/s}.$$

Check the laminar film assumption: $\text{Re}_d = 4\dot{m}/m_\ell b = 4 \times 0.00610$ kg/s/289 × 10^{-6} N·s/m^2 × (π × 0.100m) = 269. Since 30 < Re_δ < 1800, the flow is wavy, not laminar. By combining Eqs. 10.33 and 10.35 with 10.38 (see Example 10.3), find Re_δ,

$$\frac{\text{Re}_d\, m_\ell\, b\, h'_{fg}}{4 A_s (T_{sat} - T_s)} = \frac{\text{Re}_d}{1.08\text{Re}_d^{1.22} - 5.2}\cdot\frac{k_\ell}{\left(n_\ell^2/g\right)^{1/3}}$$

Continued …..

PROBLEM 10.42 (Cont.)

$$\frac{289\times10^{-6}\,\text{N}\cdot\text{s/m}^2\left(\pi\,0.10\text{m}\right)\times2274\times10^3\,\text{J/kg}}{4\left(\pi\times0.10\text{m}\times1\text{m}\right)\left(100-94\right)\text{K}}=\frac{1}{1.08\text{Re}_\delta^{1.22}-5.2}\cdot\frac{0.679\,\text{W/m}\cdot\text{K}}{\left[\left(289\times10^{-6}/960.6\right)^2\text{m}^4/\text{s}^2/9.8\,\text{m/s}^2\right]^{1/3}}$$

Solving, we obtain $\text{Re}_\delta = 311$. Using Eq. 10.38, find

$$\frac{\overline{h}_L\left(\nu_\ell^2/g\right)^{1/3}}{k_\ell}=\frac{\text{Re}_\delta}{1.08\text{Re}_\delta^{1.22}-5.2}\qquad \overline{h}_L = 8507\,\text{W/m}^2\cdot\text{K}$$

$$q = 8507\,\text{W/m}^2\cdot\text{K}\left(\pi\times0.10\text{m}\times1\text{m}\right)\left(100-94\right)\text{K} = 16.0\,\text{kW}$$ <

$$\dot{m} = 16.0\times10^3\,\text{W}/2274\times10^3\,\text{J/kg} = 7.05\times10^{-3}\,\text{kg/s}.$$ <

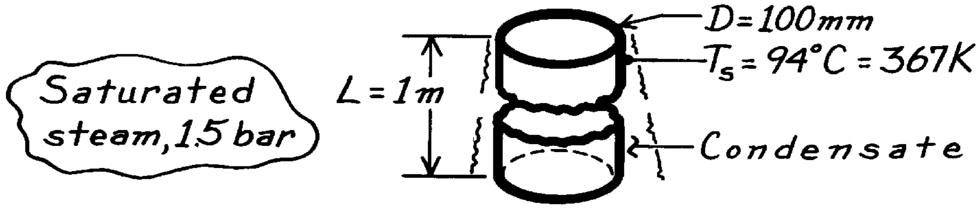
COMMENTS: To determine whether the assumption D/2 >> δ is satisfied, use Eq. 10.25 to estimate δ(L) ≈ 0.12mm. Despite the laminar film assumption, clearly the assumption is justified and the vertical plate correlation is applicable.

PROBLEM 10.43

KNOWN: Vertical tube experiencing condensation of steam on its outer surface.

FIND: Heat transfer and condensation rates.

SCHEMATIC:



ASSUMPTIONS: (1) Film condensation, (2) Negligible condensibles in steam, (3) D/2 >> δ, vertical plate behavior.

PROPERTIES: *Table A-6*, Water, vapor (1.5 bar): $T_{sat} \approx 385K$, $\rho_v = 0.876\ kg/m^3$, $h_{fg} = 2225$ kJ/kg; *Table A-6*, Water, (liquid $T_f = 376K$): $r_\ell = 956.2\ kg/m^3$, $c_{p,\ell} = 4220$ J/kg·K, $m_\ell = 271 \times 10^{-6}$ N·s/m^2, $k_\ell = 0.681$ W/m·K.

ANALYSIS: The heat transfer and condensation rates are

$$q = \overline{h}_L (p\, D L)(T_{sat} - T_s) \qquad\qquad \dot{m} = q / h'_{fg}$$

where $h'_{fg} = h_{fg}(1 + 0.68 Ja)$ and $Ja = c_{p,\ell}(T_{sat} - T_s)/h_{fg}$. Hence, Ja = 4220 J/kg·K (385 – 367)K/2225 × 10^3 J/kg = 0.0171 and h'_{fg} = 2277 kJ/kg. Assume the flow is wavy. Combine Eqs. 10.33 and 10.35 with 10.38, find Re_δ.

$$\frac{Re_d\, m_\ell\, b h'_{fg}}{4 A_s (T_{sat} - T_s)} = \frac{Re_d}{1.08 Re_d^{1.22} - 5.2} \cdot \frac{k_\ell}{\left(n^2/g\right)^{1/3}}$$

$$\frac{271 \times 10^{-6} N \cdot s/m^2 (p \times 0.10m) \times 2277 \times 10^3 J/kg}{4 \times (p \times 0.10m \times 1m)(385 - 367)K}$$

$$= \frac{1}{1.08 Re_d^{1.22} - 5.2} \cdot \frac{0.681 W/m \cdot K}{\left[\left(271 \times 10^{-6}/956.2\right)^2 m^4/s^2 / 9.8 m/s^2\right]^{1/3}}$$

$$Re_d = 832.$$

Using Eq. 10.38, find $\dfrac{\overline{h}_L \left(n_\ell^2/g\right)^{1/3}}{k_\ell} = \dfrac{Re_d}{1.08 Re_d^{1.22} - 5.2}$ $\qquad \overline{h}_L = 7{,}127 W/m^2 \cdot K.$

$$q = 7127 W/m^2 \cdot K (p \times 0.1m \times 1m)(385 - 367)K = 40.3kW \qquad <$$

$$\dot{m} = 40.3 \times 10^3 W / 2277 \times 10^3 J/kg = 0.0177 kg/s. \qquad <$$

Continued …..

COMMENTS: Since $30 < Re_{\delta} < 1800$, the wavy flow film assumption is justified. By comparing these results with those of Problem 10.42, the effect of increased pressure on condensation can be seen.

p (bar)	T_{sat}(K)	T_{sat}-T_s(K)	$\bar{h}_L \left(W/m^2 \cdot K\right)$	q (kW)	$\dot{m} \cdot 10^3 \left(kg/s\right)$
1.01	373	6	8507	16.0	7.05
1.5	385	18	7127	40.3	17.7

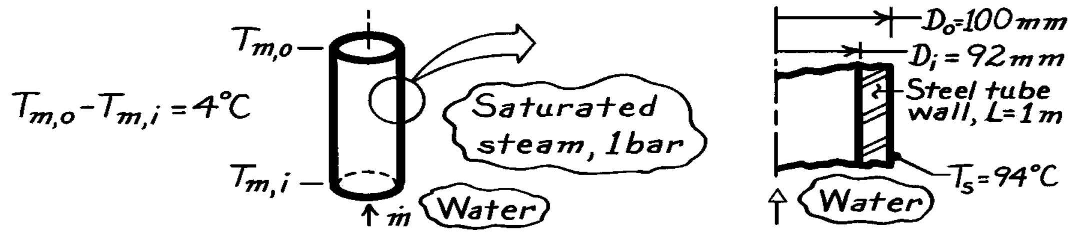
The effect of increasing the pressure from 1.01 to 1.5 bar is to increase the excess temperature three-fold, to decrease $\bar{h}_L$ by 16%, and to increase the rates by a factor of 2.5.

PROBLEM 10.44

KNOWN: Saturated steam at one atmosphere condenses on the outer surface of a vertical tube; water flow within tube experiences 4°C temperature rise.

FIND: Required flow rate to maintain tube wall at 94°C.

SCHEMATIC:



ASSUMPTIONS: (1) Laminar wavy film condensation on a vertical surface, (2) Negligible concentration of non-condensible gases in the stream, (3) Thermal resistance of tube wall is negligible, (4) Water flow is fully developed, (5) Tube wall surface is at uniform temperature T_s.

PROPERTIES: *Table A-6,* Water (Assume $\overline{T}_m \approx 300K$): $c_p = 4179$ J/kg·K, $\mu = 855 \times 10^{-6}$ N·s/m^2, k = 0.613 W/m·K, Pr = 5.83.

ANALYSIS: From the results of Problem 10.42, the heat rate for laminar wavy condensation on the outside surface of the tube was found to be q = 16.0 kW. From an energy balance on the water flowing within the tube, the flow rate is

$$\dot{m} = q / c_p \left(T_{m,o} - T_{m,i}\right) = 16.0\times10^3 W/4179 J/kg\cdot K \times 4K = 0.957 kg/s. \qquad (1)$$ <

To determine the inlet temperature of the water, the rate equation is required.

$$q = U A \Delta T_{lm} \qquad U = \frac{1}{1/h_i + 1/h_o} \qquad \Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\Delta T_1 / \Delta T_2\right)}. \qquad (2,3,4)$$

From Problem 10.42, $h_o = 8507$ W/m^2·K. Evaluate Re for the water flow using Eq. 8.6

$$Re = 4\dot{m}/p\, Dm = 4\times0.957 kg/s/p \times 0.092m \times 855\times10^{-6} N\cdot s/m^2 = 15{,}493.$$

The flow is turbulent and since fully-developed, Eq. 8.60 is an appropriate correlation.

$$Nu = h_i D_i / k = 0.023 Re_D^{4/5} Pr^{0.4} = 0.023(15{,}493)^{4/5}(5.83)^{0.4} = 104.7$$

$$h_i = Nu\cdot k / D_i = 104.7\times0.613 W/m\cdot K/0.092m = 698 W/m^2\cdot K.$$

Hence, U = 1/[1/698 + 1/8507] = 645 W/m^2·K. Substituting numerical values into the rate equation, Eq. (2), with $A = \pi D_i L$, find

$$\Delta T_{lm} = q/UA = 16.0\times10^3 W/645 W/m^2\cdot K \times (p\, 0.092m \times 1m) = 85.6K.$$

Recalling now Eq. (4), note that $\Delta T_1 - \Delta T_2 = 4K$ and that $T_{m,o} - T_{m,i} = 4K$, hence,

$$85.6K = 4K/\ln\frac{94 - T_{m,i}}{94 - \left(T_{m,i} + 4\right)} \qquad \text{giving} \qquad T_{m,i} = 6.3°C.$$

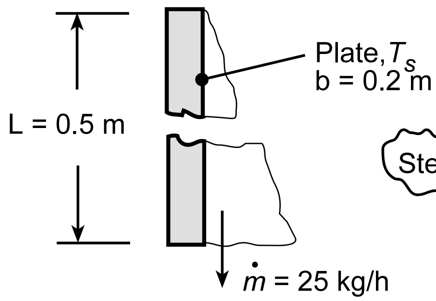
COMMENTS: Note that the $\overline{T}_m = 300K$ assumption is not reasonable and an iteration should be made. Also, it is likely that the thermal resistance of the tube wall is not negligible.

PROBLEM 10.45

KNOWN: Cooled vertical plate 500-mm high and 200-mm wide condensing saturated steam at 1 atm.

FIND: (a) Surface temperature, T_s, required to achieve a condensation rate of $\dot{m}$ = 25 kg/h, (b) Compute and plot T_s as a function of the condensation rate for the range $15 \le \dot{m} \le 50\ \text{kg/h}$, and (c) Compute and plot T_s for the same range of $\dot{m}$, but if the plate is 200 mm high and 500 mm wide (vs. 500 mm high and 200 mm wide for parts (a) and (b)).

SCHEMATIC:



ASSUMPTIONS: (1) Film condensation, (2) Negligible non-condensables in steam.

PROPERTIES: *Table A-6*, Water, vapor (1.0133 bar): T_{sat} = 100°C, $\rho_v = 0.5963\ \text{kg/m}^3$, h_{fg} = 2257 kJ/kg; *Table A-6*, Water, liquid ($T_f \approx (74 + 100)°\text{C}/2 \approx 360$ K): ρ_ℓ = 967.1 kg/m^3, $c_{p,\ell}$ = 4203 J/kg·K, $\mu_\ell = 324 \times 10^{-6}$ N·s/m^2, k_ℓ = 0.674 W/m·K.

ANALYSIS: (a) The surface temperature can be determined from the rate equation, Eq. 10.32, written as

$$T_s = T_{sat} - q/\bar{h}_L A_s = T_{sat} - \dot{m}h'_{fg}/\bar{h}_L A_s$$

where $h'_{fg} = h_{fg}(1 + 0.68\ \text{Ja})$ and $\text{Ja} = c_{p,\ell}(T_{sat} - T_s)/h_{fg}$. To evaluate T_s, we need values of h_L and h'_{fg}, both of which require knowledge of T_s. Hence, we need to assume a value of T_s and iterate the solution until good agreement with calculated T_s value is achieved. *Assume* T_s = 74°C and evaluate h'_{fg} and Re_δ.

$$h'_{fg} = 2257\ \text{kJ/kg}\left(1 + 0.68\left[4203\ \text{J/kg}\cdot\text{K}\,(100-74)\text{K}/2257\times10^3\ \text{J/kg}\right]\right) = 2331\ \text{kJ/kg}$$

$$\text{Re}_\delta = 4\dot{m}/\mu_\ell b = 4\times(25/3600)\ \text{kg/s}/324\times10^{-6}\ \text{N}\cdot\text{s/m}^2\times0.2\text{m} = 429.$$

Since $30 < \text{Re}_\delta < 1800$, the flow is wavy-laminar and Eq. 10.38 is appropriate,

$$\bar{h}_L = \frac{\text{Re}_\delta}{1.08\,\text{Re}_\delta^{1.22} - 5.2}\cdot\frac{k_\ell}{\left(\nu_\ell^2/g\right)^{1/3}}$$

$$\bar{h}_L = \frac{429}{108(429)^{1.22} - 5.2}\cdot\frac{0.674\ \text{W/m}\cdot\text{K}}{\left[\left(324\times10^{-6}/967.1\right)^2\ \text{m}^4/\text{s}^2/9.8\ \text{m/s}^2\right]^{1/3}} = 7312\ \text{W/m}^2\cdot\text{K}$$

Hence, $T_s = 100°\text{C} - (25/3600)\dfrac{\text{kg}}{\text{s}}\times2331\times10^3\dfrac{\text{J}}{\text{kg}}\Big/\left[7312\dfrac{\text{W}}{\text{m}^2\cdot\text{K}}\times(0.2\times0.5)\text{m}^2\right] = 78°\text{C}.$ <

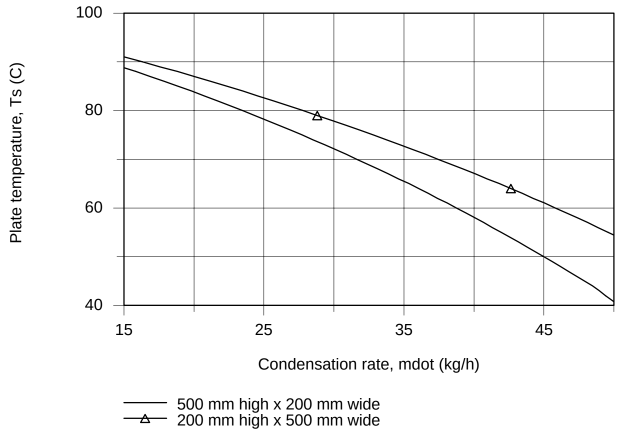
This value is to be compared to the assumed value of 74°C. See comment 1.

(b,c) Using the *IHT Correlations Tool*, *Film Condensation*, *Vertical Plate* for *laminar*, *wavy-laminar* and *turbulent regions*, combined with the *Properties Tool* for *Water*, the surface temperature T_s was

Continued...

PROBLEM 10.45 (Cont.)

calculated as a function of the condensation rate, $\dot{m}$, considering the two plate configurations as indicated in the plot below.



As expected the condensation rate increases with decreasing surface temperature. The plate with the shorter height (L = 200 mm vs 500 mm) will have the thinner boundary layer and, hence, the higher average convection coefficient. Since both plate configurations have the same total surface area, the 200-mm height plate will have the larger heat transfer and condensation rates. For the range of conditions examined, the condensate flow is in the wavy-laminar region.

COMMENTS: (1) With the IHT model developed for parts (b) and (c), the result for the part (a) conditions with $\dot{m}$ = 25 kg/h is T_s = 78.2°C (Re_δ = 439 and $\overline{h}_L = 7403\ W/m^2 \cdot K$). Hence, the assumed value (Ts = 74°C) required to initiate the analysis was a good one.

(2) A copy of the IHT Workspace model used to generate the above plot is shown below.

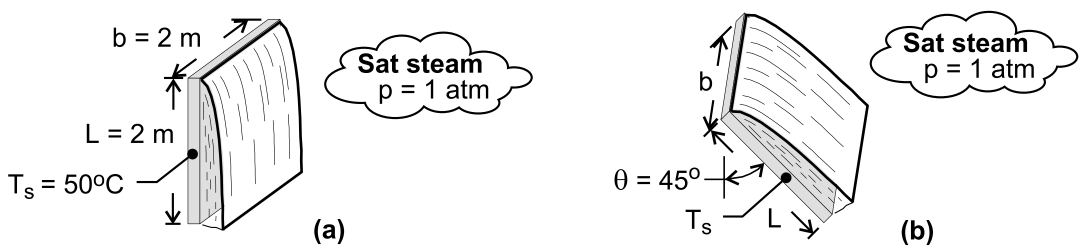
```
/* Correlations Tool
- Film Condensation, Vertical Plate, Laminar, wavy-laminar and turbulent regions: */
NuLbar = NuL_bar_FCO_VP(Redelta,Prl)     // Eq 10.37, 38, 39
NuLbar = hLbar * (nul^2 / g)^(1/3) / kl
g = 9.8                                  // Gravitational constant, m/s^2
Ts =  Ts_C + 273                         // Surface temperature, K
Ts_C = 78                                // Initial guess value used to solve the model
Tsat =  100 + 273                        // Saturation temperature, K
// The liquid properties are evaluated at the film temperature, Tf,
Tf = Tfluid_avg(Ts,Tsat)
// The condensation and heat rates are
q = hLbar * As * (Tsat - Ts)      // Eq 10.32
As = L * b                               // Surface Area, m^2
mdot = q / h'fg                          // Eq 10.33
h'fg = hfg + 0.68 * cpl * (Tsat - Ts)    // Eq 10.26
// The Reynolds number based upon film thickness is
Redelta = 4 * mdot / (mul * b)           // Eq 10.35
// Assigned Variables:
L = 0.5                      // Vertical height, m
b = 0.2                      // Width, m
mdot_h = mdot * 3600         // Condensation rate, kg/h
//mdot_h = 25                // Design value, part (a)
// Properties Tool - Water:
// Water property functions :T dependence, From Table A.6
// Units: T(K), p(bars);
xl =  0                      // Quality (0=sat liquid or 1=sat vapor)
rhol = rho_Tx("Water",Tf,xl)    // Density, kg/m^3
hfg = hfg_T("Water",Tsat)       // Heat of vaporization, J/kg
cpl = cp_Tx("Water",Tf,xl)      // Specific heat, J/kg·K
mul = mu_Tx("Water",Tf,xl)      // Viscosity, N·s/m^2
nul = nu_Tx("Water",Tf,xl)      // Kinematic viscosity, m^2/s
kl = k_Tx("Water",Tf,xl)        // Thermal conductivity, W/m·K
Prl = Pr_Tx("Water",Tf,xl)      // Prandtl number
```

PROBLEM 10.46

KNOWN: Plate dimensions, temperature and inclination. Pressure of saturated steam.

FIND: (a) Heat transfer and condensation rates for vertical plate, (b) Heat transfer and condensation rates for inclined plate.

SCHEMATIC:



ASSUMPTIONS: (1) Conditions correspond to the turbulent film region, (2) Constant properties.

PROPERTIES: *Table A-6*, saturated vapor (p=1.0133 bars): T_{sat} = 100°C, ρ_v = 0.596 kg/m^3, h_{fg} = 2257 kJ/kg. *Table A-6*, saturated liquid (T_f = 75°C): $\rho_\ell = 975\,\text{kg/m}^3$, $\mu_\ell = 375\times10^{-6}\,\text{N}\cdot\text{s/m}^2$, $k_\ell = 0.668\,\text{W/m}\cdot\text{K}$, $c_{p,\ell} = 4193\,\text{J/kg}\cdot\text{K}$.

ANALYSIS: (a) Expressing $\overline{h}_L$ in terms of Re_δ by combining Eqs. (10.33) and (10.35) and substituting into Eq. (10.38), it follows that

$$\frac{\text{Re}_\delta\,\mu_\ell h'_{fg}}{4L\left(T_{sat}-T_s\right)} = \frac{\text{Re}_\delta}{1.08\,\text{Re}_\delta^{1.22}-5.2}\cdot\frac{k_\ell}{\left(\nu_\ell^2/g\right)^{1/3}} \tag{1}$$

where, with Ja = $c_{p,\ell}\left(T_{sat}-T_s\right)/h_{fg} = 0.0929$, $h'_{fg} = h_{fg}\left(1+0.68\,\text{Ja}\right) = 2400\,\text{kJ/kg}$. From an iterative solution to Eq. (1), we obtain Re_δ = 2370, and the assumption of a turbulent film is justified. From Eqs. (10.35) and (10.33) the condensation and heat rates are then

$$\dot{m} = \frac{\mu_\ell b\,\text{Re}_\delta}{4} = 0.444\,\text{kg/s} \qquad <$$

$$q = \dot{m}\,h'_{fg} = 0.444\,\text{kg/s}\times2.4\times10^6\,\text{J/kg} = 1.065\times10^6\,\text{W} \qquad <$$

From Eq. (10.32), we also obtain $\overline{h}_L = q/\left[\left(bL\right)\left(T_{sat}-T_s\right)\right] = 5325\,\text{W/m}^2\cdot\text{K}$.

(b) With $\overline{h}_{L(incl)} \approx \left(\cos\theta\right)^{1/4}\overline{h}_L$, we obtain $\overline{h}_{L(incl)} \approx 0.917\times5325\,\text{W/m}^2\cdot\text{K} = 4880\,\text{W/m}^2\cdot\text{K}$. If the inclination reduces $\overline{h}_L$ by 8.73%, the heat and condensation rates are reduced by equivalent amounts. Hence,

$$\dot{m} = 0.407\,\text{kg/s}, \qquad q = 0.977\times10^6\ \text{W} \qquad <$$

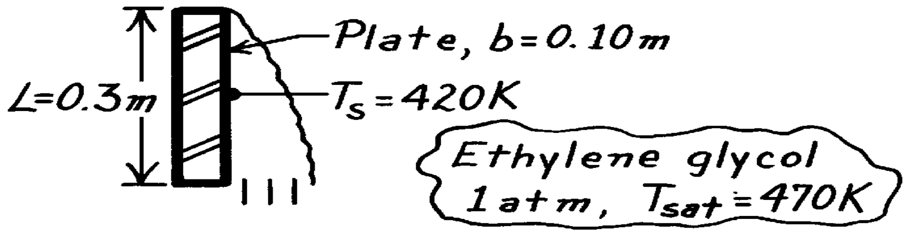
COMMENTS: The initial guess of a turbulent film region was motivated by the value of L = 2m, which was believed to be large enough for transition to turbulence. Note that the solution could also have been obtained by accessing the Film Condensation correlations of IHT, implementation of which does not require an assumption of flow conditions.

PROBLEM 10.47

KNOWN: Saturated ethylene glycol (1 atm) condensing on a vertical plate at 420K.

FIND: Heat transfer rate to the plate and condensation rate.

SCHEMATIC:



ASSUMPTIONS: (1) Film condensation, (2) Negligible non-condensible gases in vapor.

PROPERTIES: *Table A-5*, Ethylene glycol vapor (1 atm): $T_{sat} = 470K$, $\rho_v \approx 0\ kg/m^3$, $h_{fg} = 812\ kJ/kg$; *Table A-5*, Ethylene glycol, liquid ($T_f = (T_s + T_{sat})/2 \approx 445K$; use properties at upper limit of table 373K): $r_\ell = 1058.5\ kg/m^3$, $c_{p,\ell} = 2742\ J/kg \cdot K$, $m_\ell = 0.215 \times 10^{-2}\ N \cdot s/m^2$, $k_\ell = 0.263,\ W/m \cdot K$.

ANALYSIS: The heat transfer and condensation rates are given by Eqs. 10.32 and 10.33.

$$q = \overline{h}_L A_s \left(T_{sat} - T_s\right) \qquad \dot{m} = q / h'_{fg},$$

where $h'_{fg} = h_{fg}\ (1 + 0.68\ Ja)$ and $Ja = c_{p,\ell}\left(T_{sat} - T_s\right)/h_{fg}$. Substituting property values at $T_f = (T_s + T_{sat})/2$, find $h'_{fg} = 812\ kJ/kg\ (1 + 0.68\ [2742\ J/kg \cdot K\ (470 - 420)K/812 \times 10^3\ J/kg]) = 905\ kJ/kg$.

Assuming the flow is laminar, use Eq. 10.30 to evaluate $\overline{h}_L$.

$$\overline{h}_L = 0.943\left[\frac{g\, r_\ell \left(r_\ell - r_v\right) k_\ell^3 h'_{fg}}{m_\ell \left(T_{sat} - T_s\right) L}\right]^{1/4} \left[\frac{9.8 m/s^2 \times 1058.5 kg/m^3 \left(1058.5 - 0\right) kg/m^3 \left(0.263 W/m \cdot K\right)^3 \times 905 \times 10^3 J/kg}{0.215 \times 10^{-2} N \cdot s/m^2 \left(470 - 420\right) K \times 0.3m}\right]^{1/4}$$

find $\overline{h}_L = 1451\ W/m^2 \cdot K$. Using the rate equations, find

$$q = 1451\ W/m^2 \cdot K \left(0.3 \times 0.1\right) m^2 \left(470 - 420\right) K = 2.18 kW$$

$$\dot{m} = 2.18 \times 10^3 W / 905 \times 10^3 J/kg = 0.002405 kg/s = 8.66 kg/h.$$

Determine whether the flow is indeed laminar: $Re_d = 4\dot{m}/m_\ell b = 4 \times 0.002405\ kg/s / 0.215 \times 10^{-2}\ N \cdot s/m^2 \times 0.1m = 44.7$. Since $30 < Re_\delta < 1800$, the flow is in the wavy-laminar region. Hence, the correlation of Eq. 10.38 is more appropriate. Combining Eq. 10.33 and 10.35 with 10.38 (see Example 10.3),

$$\frac{Re_d\, m_\ell\, b\, h'_{fg}}{4 A_s \left(T_{sat} - T_s\right)} = \frac{Re_d}{1.08 Re_d^{1.22} - 5.2} \cdot \frac{k_\ell}{\left(n_\ell^2 / g\right)^{1/3}}$$

$$\frac{0.215 \times 10^{-2} N \cdot s/m^2 \times 0.1m \times 905 \times 10^3 J/kg}{4 \times \left(0.3 \times 0.1\right) m^2 \left(470 - 420\right) K} = \frac{1}{1.08 Re_d^{1.22} - 5.2} \frac{0.263 W/m \cdot K}{\left[\left(0.215 \times 10^{-2} / 1058.5\right) m^4/s^2 / 9.8 m/s^2\right]^{1/3}}$$

find $Re_\delta = 45$ and using Eq. 10.38, find

$$\overline{h}_L = \frac{Re_d}{1.08 Re_d^{1.22} - 5.2} \cdot \frac{k_\ell}{\left(n_\ell^2 / g\right)^{1/3}} = 1470 W/m^2 \cdot K.$$

Hence, $\qquad q = 2.21 kW \qquad \dot{m} = 2.44 \times 10^{-3} kg/s.$ <

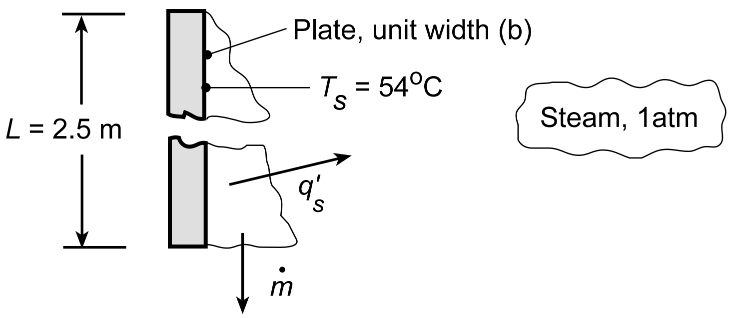
COMMENTS: Note the wavy-laminar value of Re_δ is within 1.3% of the laminar value.

PROBLEM 10.48

KNOWN: Vertical plate 2.5 m high at a surface temperature T_s = 54°C exposed to steam at atmospheric pressure.

FIND: (a) Condensation and heat transfer rates, (b) Whether turbulent flow would still exist if the height were halved, and (c) Compute and plot the condensation rates for the two plate heights (2.5 m and 1.25 m) as a function of surface temperature for the range, $54 \le T_s \le 90°C$.

SCHEMATIC:



ASSUMPTIONS: (1) Film condensation, (2) Negligible non-condensables in steam.

PROPERTIES: *Table A-6*, Water, vapor (1 atm): T_{sat} = 100°C, ρ_v = 0.596 kg/m³, h_{fg} = 2257 kJ/kg; *Table A-6*, Water, liquid (T_f = (100 + 54)°C/2 = 350 K): ρ_ℓ = 973.7 kg/m³, k_ℓ = 0.668 W/m·K, $\mu_\ell = 365 \times 10^{-6}$ N·s/m², $c_{p,\ell}$ = 4195 J/kg·K, Pr_ℓ = 2.29.

ANALYSIS: (a) The heat transfer and condensation rates are given by Eqs. 10.32 and 10.33,

$$q' = \bar{h}_L L\left(T_{sat} - T_s\right) \qquad \dot{m}' = q'/h'_{fg} \tag{1,2}$$

where, from Eq. 10.26, with Ja = $c_{p,\ell}(T_{sat} - T_s)/h_{fg}$,

$$h'_{fg} = h_{fg}\left(1 + 0.68\left[c_{p,\ell}\left(T_{sat} - T_s\right)/h_{fg}\right]\right)$$

$$h'_{fg} = 2257\,\frac{kJ}{kg}\left(1 + 0.68\left[\frac{4195\,J/kg\cdot K\left(100-54\right)K}{2257\times10^3\,J/kg}\right]\right) = 2388\,kJ/kg\,.$$

Assuming turbulent flow conditions, Eq. 10.39 is the appropriate correlation,

$$\frac{\bar{h}L\left(\nu_\ell^2/g\right)^{1/3}}{k_\ell} = \frac{Re_\delta}{8750 + 58\,Pr^{-0.5}\left(Re_\delta^{0.75} - 253\right)} \qquad Re_\delta > 1800 \tag{3}$$

Not knowing Re_δ or $\bar{h}_L$, another relation is required. Combine Eq. 10.33 and 10.35,

$$\bar{h}_L = \frac{\dot{m}h'_{fg}}{A\left(T_{sat} - T\right)} = \left(\frac{Re_\delta\,\mu_\ell b}{4}\right)\frac{h'_{fg}}{A\left(T_{sat} - T\right)}\,. \tag{4}$$

Substitute Eq. (4) for $\bar{h}_L$ into Eq. (3), with A = bL,

$$\frac{Re_\delta\,\mu_\ell b h'_{fg}}{4\left(bL\right)\left(T_{sat} - T\right)} = \frac{Re_\delta}{8750 + 58\,Pr_\ell^{-0.5}\left(Re_\delta^{0.75} - 253\right)}\cdot\frac{k_\ell}{\left(\nu_\ell^2/g\right)^{1/3}}\,. \tag{5}$$

Using appropriate properties with L = 2.5 m, find

Continued...

$$\frac{365\times10^{-6}\,\mathrm{N\cdot s/m^2}\times2388\times10^{3}\,\mathrm{J/kg}}{4\times2.5\,\mathrm{m}\,(100-54)\,\mathrm{K}} \tag{6}$$

$$=\frac{1}{8750+58(2.29)^{-0.5}\left(\mathrm{Re}_{\delta}^{0.75}-253\right)}\cdot\frac{0.668\,\mathrm{W/m\cdot K}}{\left[\left(365\times10^{-6}/973.7\right)^{2}\,\mathrm{m^4/s^2}/9.8\,\mathrm{m/s^2}\right]^{1/3}}$$

$$\mathrm{Re}_{\delta}=2979\,.$$

Note that $\mathrm{Re}_\delta > 1800$, so indeed the flow is turbulent, and using Eq. (4) or (3), find

$$\bar{h}_L=5645\,\mathrm{W/m^2\cdot K}\,.$$

From the rate equations (1) and (2), the heat transfer and condensation rates are

$$q'=5645\,\mathrm{W/m^2\cdot K}\times2.5\,\mathrm{m}\,(100-54)\,\mathrm{K}=649\,\mathrm{kW/m}$$ <

$$\dot{m}'=649\times10^{3}\,\mathrm{W/m}/2388\times10^{3}\,\mathrm{J/kg}=0.272\,\mathrm{kg/s\cdot m}\,.$$ <

(b) If the height of the plate were halved, L = 1.25 m, Eq. (6) would only need to be modified for this new value. Using the calculated values for the LHS and the last term on the RHS, Eq. (6) becomes,

$$3.78960=\frac{1}{8750+58(2.29)^{-0.5}\left(\mathrm{Re}_{\delta}^{0.75}-253\right)}\times27,493 \tag{7}$$

and after some manipulation , find

$$\mathrm{Re}_{\delta}=1280\,.$$

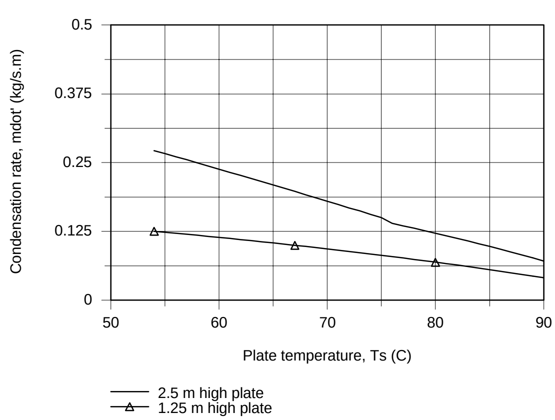
Since $1800 > \mathrm{Re}_\delta$, the flow is not turbulent, but wavy-laminar. Now the procedure follows that of Example 10.3. For L = 1.25 m with wavy-laminar flow, Eq. 10.38 is the appropriate correlation. The calculations yield these results:

$$\mathrm{Re}_{\delta}=1372 \qquad \bar{h}_L=5199\,\mathrm{W/m^2\cdot K}$$

$$q'=299\,\mathrm{kW/m} \qquad \dot{m}'=0.125\,\mathrm{kg/s\cdot m}\,.$$ <

Note that the height was decreased by a factor of 2 while the rates decreased by a factor of 2.2! Would you have expected this result?

(c) Using the *IHT Correlation Tool*, *Film Condensation*, *Vertical Plate* for *laminar, wavy-laminar*, and *turbulent regions*, combined with the *Properties Tool* for *Water*, the condensation rates were calculated as a function of the surface temperature considering the two plate heights indicated.



Continued...

PROBLEM 10.48 (Cont.)

The condensation rate decreases nearly linearly with increasing surface temperature. The inflection in the upper curve (L = 2.5 m) corresponds to the flow transition at $Re_\delta = 1800$ between wavy-laminar and turbulent. For surface temperature lower than 76°C, the flow is turbulent over the 2.5 m plate. The flow over the 1.25 m plate is always in the wavy-laminar region. The fact that the 2.5 m plate experiences turbulent flow explains the height-rate relationship mentioned in the closing sentences of part (b).

COMMENTS: A copy of the IHT model used to generate the above plot is shown below.

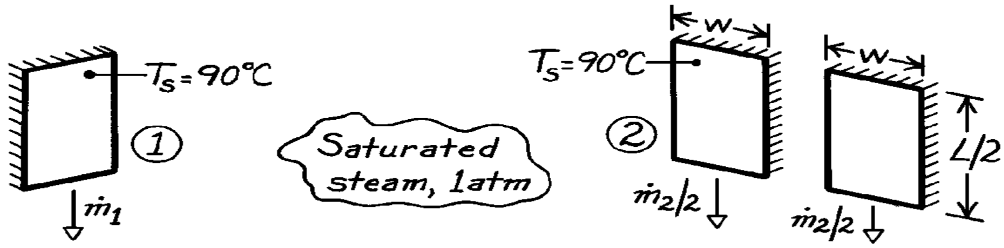
```
/* Correlations Tool
- Film Condensation, Vertical Plate, Laminar, wavy-laminar and turbulent regions: */
NuLbar = NuL_bar_FCO_VP(Redelta,Prl)     // Eq 10.37, 38, 39
NuLbar = hLbar * (nul^2 / g)^(1/3) / kl
g = 9.8                                  // Gravitational constant, m/s^2
Ts =  Ts_C + 273                         // Surface temperature, K
Ts_C = 54                                // Part (a) design condition
Tsat =  100 + 273                        // Saturation temperature, K
// The liquid properties are evaluated at the film temperature, Tf,
Tf = Tfluid_avg(Ts,Tsat)
// The condensation and heat rates are
q = hLbar * As * (Tsat - Ts)             // Eq 10.32
As = L * b                               // Surface Area, m^2
mdot = q / h'fg                          // Eq 10.33
h'fg = hfg + 0.68 * cpl * (Tsat - Ts)    // Eq 10.26
// The Reynolds number based upon film thickness is
Redelta = 4 * mdot / (mul * b)           // Eq 10.35

// Assigned Variables:
L = 1.25              // Height, m
b = 1                 // Width, m

// Properties Tool - Water:
// Water property functions :T dependence, From Table A.6
// Units: T(K), p(bars);
xl =  0                          // Quality (0=sat liquid or 1=sat vapor)
rhol = rho_Tx("Water",Tf,xl)     // Density, kg/m^3
hfg = hfg_T("Water",Tsat)        // Heat of vaporization, J/kg
cpl = cp_Tx("Water",Tf,xl)       // Specific heat, J/kg·K
mul = mu_Tx("Water",Tf,xl)       // Viscosity, N·s/m^2
nul = nu_Tx("Water",Tf,xl)       // Kinematic viscosity, m^2/s
kl = k_Tx("Water",Tf,xl)         // Thermal conductivity, W/m·K
Prl = Pr_Tx("Water",Tf,xl)       // Prandtl number
```

PROBLEM 10.49

KNOWN: Two vertical plate configurations maintained at 90°C for condensing saturated steam at 1 atm: single plate L × w and two plates each L/2 × w where L and w are the vertical and horizontal dimensions, respectively.

FIND: Which case will provide the larger heat transfer or condensation rate.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible concentration of non-condensible gases in the steam.

PROPERTIES: *Table A-6*, Saturated water vapor (1 atm): $T_{sat} = 100°C$, $\rho_v = (1/v_g) = 0.596\ kg/m^3$, $h_{fg} = 2257\ kJ/kg$; Saturated water ($T_f = (T_s + T_{sat})/2 = (90 + 100)°C/2 = 95°C = 368K$): $r_\ell = (1/v_f) = 962\ kg/m^3$, $m_\ell = 296 \times 10^{-6}\ N\cdot s/m^2$, $k_\ell = 0.678\ W/m\cdot K$, $c_{p,\ell} = 4212\ J/kg\cdot K$.

ANALYSIS: The heat transfer and condensation rates are

$$q = \bar{h}_L A_s \left(T_{sat} - T_s\right) \qquad \dot{m} = q / h'_{fg}$$

where, for the two cases,

$$\bar{h}_{L,1} A_{s,1} = \bar{h}_{L,1}(L)[L \times w] \qquad \bar{h}_{L,2} A_{s,2} = \bar{h}_{L,2}(L/2)\left[2(L/2 \times w)\right]$$

and the average convection coefficients are evaluated at L and L/2, respectively. Hence,

$$\frac{q_1}{q_2} = \frac{\dot{m}_1}{\dot{m}_2} = \frac{\bar{h}_{L,1}(L)[L \times w]}{\bar{h}_{L,2}(L/2)\left[2(L/2 \times w)\right]} = \frac{\bar{h}_{L,1}(L)}{\bar{h}_{L,2}(L/2)}.$$

For *laminar film* condensation on both plates, using the correlation of Eq. 10.30, with $\bar{h}_L \propto L^{-1/4}$,

$$q_1 / q_2 = \left(L / [L/2]\right)^{-1/4} = 0.84.$$

Hence, case 2 is preferred and provides 16% more heat transfer. <

When $Re_\delta = 30$ for case 1 with the given conditions, find from Eq. 10.37

$$\frac{\bar{h}_L \left(n_\ell^2 / g\right)^{1/3}}{k_\ell} = \frac{\bar{h}_L \left[\left(296 \times 10^{-6}\ N\cdot s/m^2 / 962\ kg/m^3\right)^2 / 9.8\ m/s^2\right]^{1/3}}{0.678\ W/m\cdot K}$$

Continued …..

PROBLEM 10.49 (Cont.)

$$\frac{\overline{h}_L \left(n_\ell^2 / g\right)^{1/3}}{k_\ell} = 1.47 \mathrm{Re}_d^{-1/3} = 1.47(30)^{-1/3}$$

$$\overline{h}_L = 15{,}061\,\mathrm{W/m^2 \cdot K}$$

and then from Eq. 10.30,

$$\overline{h}_L = 0.943\left[\frac{g\, r_\ell (r_\ell - r_v) k_\ell^3 h'_{fg}}{m_\ell (T_{sat} - T_s) L}\right]^{1/4}$$

where

$$h'_{fg} = h_{fg} + 0.68 c_{p,\ell}\left(T_{sat} - T_s\right)$$

$$h'_{fg} = 2257\,\mathrm{kJ/kg} + 0.68 \times 4212\,\mathrm{J/kg \cdot K}\,(100 - 90)\,\mathrm{K} = 2286\,\mathrm{kJ/kg},$$

$$15{,}061\,\mathrm{W/m^2 \cdot K} =$$

$$0.943\left[\frac{9.8\,\mathrm{m/s^2} \times 962\,\mathrm{kg/m^3}(962 - 0.596)\,\mathrm{kg/m^3}\left(0.678\,\mathrm{W/m \cdot K}\right)^3}{296 \times 10^{-6}\,\mathrm{N \cdot s/m^2}\,(100 - 90)\,\mathrm{K}\,L} 2286\,\mathrm{kJ/kg}\right]^{1/4}$$

$$L = 34\ \mathrm{mm}.$$

We can anticipate for other, larger values of L that the comparison of $\overline{h}_L$ values cannot be so easily made. However, according to Figure 10.15, we expect the same behavior of $\overline{h}_L$ in the *wavy* region and anticipate that indeed case 2 will provide the greater condensation rate. Note that in the turbulent region with the increase in $\overline{h}_L$ with Re_δ, we cannot conclude with certainty which case is preferred.

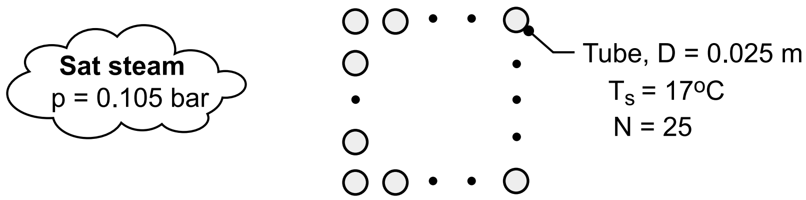
COMMENTS: In dealing with single-phase, forced or free convection, we associate thin thermal boundary layers with higher heat transfer rates. For vertical plates, we would expect the shorter plate to have the higher convection heat transfer coefficient. The results of this problem suggest the same is true for condensation on the vertical plate.

PROBLEM 10.50

KNOWN: Number, diameter and wall temperature of condenser tubes in a square array. Pressure of saturated steam around tubes.

FIND: Rates of heat transfer and condensation per unit length of the array.

SCHEMATIC:



ASSUMPTIONS: (1) Laminar film condensation on tubes, (2) Negligible concentration of noncondensable gases in steam.

PROPERTIES: *Table A-6*, saturated vapor (p_{sat} = 0.105 bar): T_{sat} = 320 K = 47°C, ρ_v = 0.0715 kg/m^3, h_{fg} = 2390 kJ/kg. *Table A-6*, saturated liquid (T_f = 32°C = 305 K): $\rho_\ell = 995\,kg/m^3$, $\mu_\ell = 769\times10^{-6}\,N\cdot s/m^2$, $k_\ell = 0.620\,W/m\cdot K$, $c_{p,\ell} = 4178\,J/kg\cdot K$.

ANALYSIS: The average heat rate per unit length for a single tube is $q_1' = \overline{h}_{D,N}(\pi D)(T_{sat} - T_s)$, where $\overline{h}_{D,N}$ is obtained from Eq. 10.41. With $Ja = c_{p,\ell}(T_{sat} - T_s)/h_{fg} = 0.052$ and $h_{fg}' = h_{fg}$ (l + 0.68 Ja) = 1.04 (2.390 × 10^6 J/kg) = 2.48 × 10^6 J/kg,

$$\overline{h}_{D,N} = 0.729\left[\frac{g\,\rho_\ell(\rho_\ell - \rho_v)k_\ell^3 h_{fg}'}{N\,\mu_\ell(T_{sat} - T_s)D}\right]^{1/4}$$

$$\overline{h}_{D,N} = 0.729\left[\frac{9.8\,m/s^2 \times 995\,kg/m^3\,(995 - 0.0715)\,kg/m^3\,(0.62\,W/m\cdot K)^3\,2.48\times10^6\,J/kg}{25\times769\times10^{-6}\,N\cdot s/m^2\,(30°C)\,0.025m}\right]^{1/4} = 3260\,W/m^2\cdot K$$

The heat rate per unit length of the array is $q' = N^2 q_1'$. Hence,

$$q' = N^2\overline{h}_{D,N}(\pi D)(T_{sat} - T_s) = 625\times3260\,W/m^2\cdot K\,(\pi\times0.025m)\,30°C = 4.79\times10^6\,W/m$$ <

The corresponding condensation rate is

$$\dot{m}' = \frac{q'}{h_{fg}'} = \frac{4.79\times10^6\,W/m}{2.48\times10^6\,J/kg} = 1.93\,kg/s\cdot m$$ <

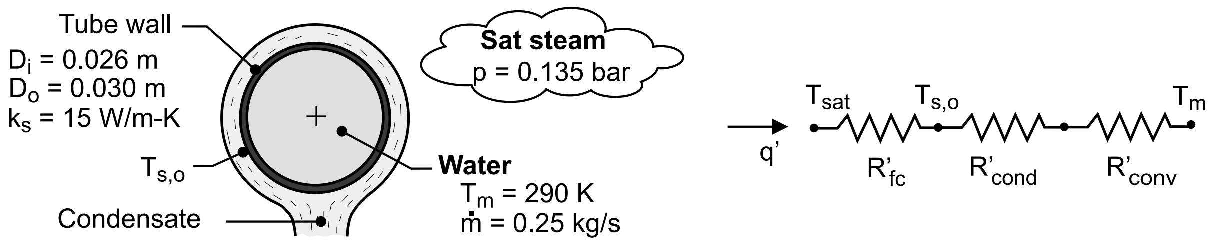
COMMENTS: Because of turbulence generation due to *splashing* from one tube to another in a vertical column, the foregoing value of $\overline{h}_{D,N}$ is expected to underestimate the actual value of $\overline{h}_{D,N}$ and hence to underpredict the heat and condensation rates.

PROBLEM 10.51

KNOWN: Tube wall diameters and thermal conductivity. Mean temperature and flow rate of water flow through tube. Pressure of saturated steam around tube.

FIND: (a) Rates of heat transfer and condensation per unit length, (b) Effect of flow rate on heat transfer.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible concentration of noncondensible gases in the steam, (2) Uniform tube surface temperatures, (3) Laminar film condensation, (4) Fully-developed internal flow, (5) Constant properties.

PROPERTIES: *Table A-6*, water (T_m = 290 K): $\mu = 0.00108\ \mathrm{N\cdot s/m^2}$, k = 0.598 W/m·K, Pr = 7.56. *Table A-6*, saturated vapor (p = 0.135 bar): T_{sat} = 325 K = 52°C, $\rho_v = 0.0904\ \mathrm{kg/m^3}$, h_{fg} = 2378 kJ/kg. *Table A-6*, saturated liquid ($T_f \approx T_{sat}$): $\rho_\ell = 987\ \mathrm{kg/m^3}$, $c_{p,\ell} = 4182\ \mathrm{J/kg\cdot K}$,

$\mu_\ell = 528\times10^{-6}\ \mathrm{N\cdot s/m^2}$, $k_\ell = 0.645\ \mathrm{W/m\cdot K}$.

ANALYSIS: (a) From the thermal circuit, the heat rate may be expressed as

$$q' = \frac{T_{sat} - T_m}{R'_{fc} + R'_{cond} + R'_{conv}} \tag{1}$$

where, $$R'_{cond} = \ell n\left(D_o / D_i\right)/2\pi k_s = 0.00152\ \mathrm{m\cdot K/W}$$

The convection resistance is $R'_{conv} = \left(\pi D_i h_i\right)^{-1}$. With $Re_D = 4\dot{m}/\pi D_i \mu = 11,336$, the flow is turbulent and the Dittus-Boelter correlation yields

$$h_i = \left(\frac{k}{D_i}\right)0.023\,Re_D^{4/5}\,Pr^{0.4} = \left(\frac{0.598\ \mathrm{W/m\cdot K}}{0.026\mathrm{m}}\right)0.023(11,336)^{4/5}(7.56)^{0.4} = 2082\ \mathrm{W/m^2\cdot K}$$

The convection resistance is then

$$R'_{conv} = \left(\pi D_i h_i\right)^{-1} = \left(\pi\times0.026\mathrm{m}\times2082\ \mathrm{W/m^2\cdot K}\right)^{-1} = 0.00588\ \mathrm{m\cdot K/W}$$

The resistance associated with the condensate film is $R'_{fc} = \left(\pi D_o \bar{h}_o\right)$, where $\bar{h}_o$ is given by Eq. 10.40. With C = 0.729,

$$\bar{h}_o = C\left[\frac{g\rho_\ell\left(\rho_\ell - \rho_v\right)k_\ell^3 h'_{fg}}{\mu_\ell\left(T_{sat} - T_{s,o}\right)D_o}\right]^{1/4} = 0.729\left[\frac{9.8\ \mathrm{m/s^2}\times987\left(987-0.09\right)\mathrm{kg^2/m^6}\left(0.645\ \mathrm{W/m\cdot K}\right)^3 h'_{fg}}{528\times10^{-6}\ \mathrm{N\cdot s/m^2}\left(325 - T_{s,o}\right)\times0.030\mathrm{m}}\right]^{1/4}$$

$$\bar{h}_o = 462\left(\frac{\mathrm{W^3\cdot kg}}{\mathrm{m^8\cdot K^3\cdot s}}\right)^{1/4}\left(\frac{h'_{fg}}{325 - T_{s,o}}\right)^{1/4}$$

where $$h'_{fg} = h_{fg} + 0.68c_{p,\ell}\left(T_{sat} - T_{s,o}\right) = 2.38\times10^6\ \mathrm{J/kg} + 2844\ \mathrm{J/kg\cdot K}\left(325 - T_{s,o}\right)$$

The unknown surface temperature may be determined from an additional rate equation, such as

Continued …..

$$q' = \frac{T_{s,o} - T_m}{R'_{cond} + R'_{conv}} \tag{2}$$

Substituting the thermal resistances into Eqs. (1) and (2), an iterative solution yields

$$T_{s,o} = 321.6\,K = 48.6°C \qquad q' = 4270\,W/m$$ <

The condensation rate is then

$$\dot{m}'_{cond} = \frac{q'}{h'_{fg}} = \frac{4270\,W/m}{2.39 \times 10^6\,J/kg} = 0.00179\,kg/s \cdot m$$ <

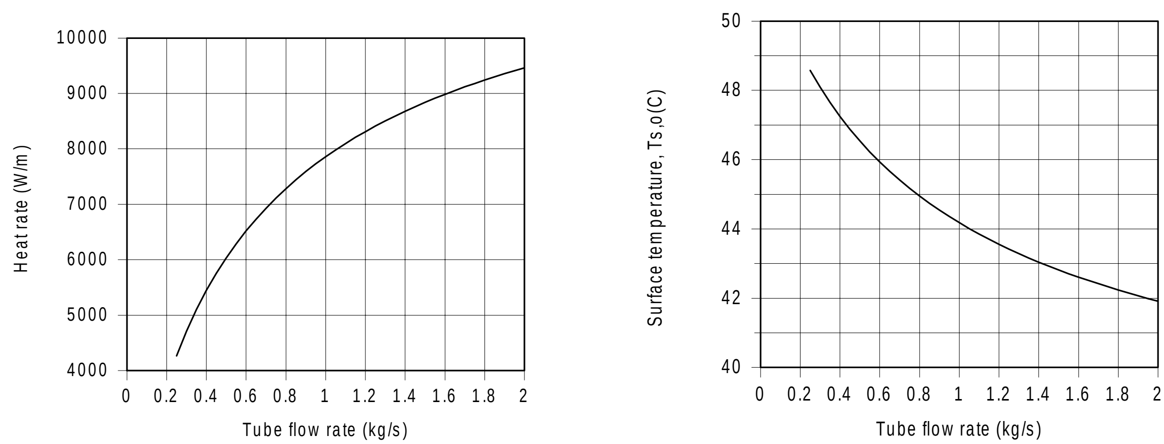
The corresponding values of the condensate convection coefficient and resistance are

$$\bar{h}_o = 13{,}380\,W/m^2 \cdot K$$

and $$R'_{fc} = 0.000793\,m \cdot K/W$$

Because R'_{conv} is much larger than R'_{cond} and R'_{fc}, attention should be paid to reducing the convection resistance in order to increase the heat rate. The resistance to heat flow by convection is the *limiting factor*.

(b) The effects of varying the flow rate are shown below



The effect of increasing $\dot{m}$ on q' is significant and is accompanied by a reduction in $T_{s,o}$.

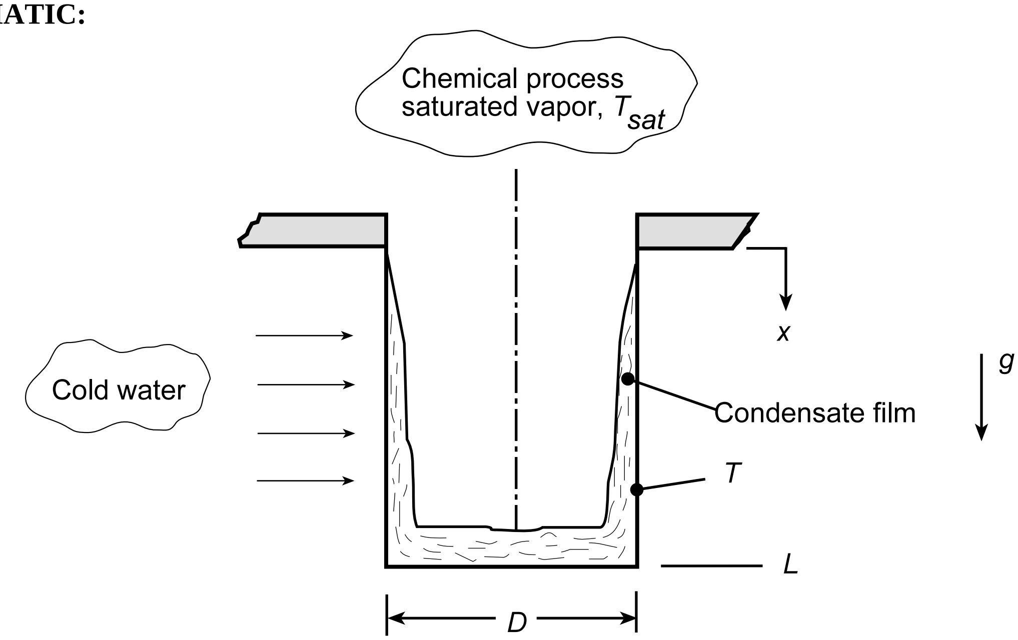
COMMENTS: (1) Use of the IHT convection and condensation correlations, as well as its temperature-dependent properties of water facilitated the numerical solution. (2) Evaluation of the film properties at T_{sat} is reasonable for part (a), since $T_f = (T_{s,o} + T_{sat})/2 = 50.3°C \approx T_{sat}$. However, with increasing $\dot{m}$ and hence decreasing $T_{s,o}$, the approximation would become inappropriate.

PROBLEM 10.52

KNOWN: Inner surface of a vertical thin-walled container of length L and diameter D experiences condensation of a saturated vapor. Container wall maintained at a uniform surface temperature by flowing cold water across its outer surface.

FIND: Expression for the time, t_f, required to fill the container with condensate assuming the condensate film is laminar. Express your result in terms of D, L, $(T_{sat} - T_s)$, g and appropriate fluid properties.

SCHEMATIC:



ASSUMPTIONS: (1) Laminar film condensation on a vertical surface, (2) Uniform temperature container wall surface, and (3) Mass of liquid condensate in the laminar film negligible compared to liquid mass on bottom of container.

ANALYSIS: From an instantaneous mass balance on the container,

$$\dot{m}(t) = \frac{dM}{dt} \tag{1}$$

Where $\dot{m}(t)$ is the condensate rate and the liquid mass in the container, M, is

$$M = \rho_\ell \left(\pi D^2/4\right)(L - x) \tag{2}$$

The condensate rate from Eq. 10.33 can be expressed as

$$\dot{m}(t) = \frac{q}{h'_{fg}} = \frac{\bar{h}_s A_s (T_{sat} - T_s)}{h'_{fg}} \tag{3}$$

where the average film coefficient over the height 0 to x from Eq. 10.30 is,

$$\bar{h}_s = 0.943\left[\frac{g\rho_\ell(\rho_\ell - \rho_v)k_\ell^3 h'_{fg}}{\mu_\ell (T_{sat} - T_s)x}\right]^{1/4} \tag{4}$$

and the surface area over which condensation occurs is

$$A_s = \pi D x \tag{5}$$

Continued...

PROBLEM 10.52 (Cont.)

Substituting Eqs (2-5) into Eq. (1),

$$0.943\left[\frac{g\rho_\ell\left(\rho_\ell-\rho_v\right)k_\ell^3 h'_{fg}}{\mu_\ell\left(T_{sat}-T_s\right)L}\right]^{1/4}\frac{L^{1/4}}{x^{1/4}}(\pi D x)\Big/h'_{fg}=-\rho_\ell\left(\pi D^2/4\right)\frac{dx}{dt} \qquad (6)$$

Separate variables and identify the limits of integration,

$$\left\{0.943\left[\frac{g\rho_\ell\left(\rho_\ell-\rho_v\right)k_\ell^3 h'_{fg}}{\mu_\ell\left(T_{sat}-T_s\right)L}\right]^{1/4}L^{1/4}(\pi D)\Big/\left[h'_{fg}\rho_\ell\left(\pi D^2/4\right)\right]\right\}\int_0^{t_f}dt=-\int_{x=L}^{0}x^{-3/4}dx \qquad (7)$$

The RHS integrates to

$$-\left[x^{1/4}\Big/(1/4)\right]_L^0=4L^{1/4} \qquad (8)$$

and solving for t_f,

$$t_f=4\left[\frac{\rho_\ell\left(\pi D^2/4\right)L h'_{fg}}{0.943\left[\frac{g\rho_\ell\left(\rho_\ell-\rho_v\right)k_\ell^3 h'_{fg}}{\mu_\ell\left(T_{sat}-T_s\right)L}\right]^{1/4}(\pi D L)\left(T_{sat}-T_s\right)}\right] \qquad <$$

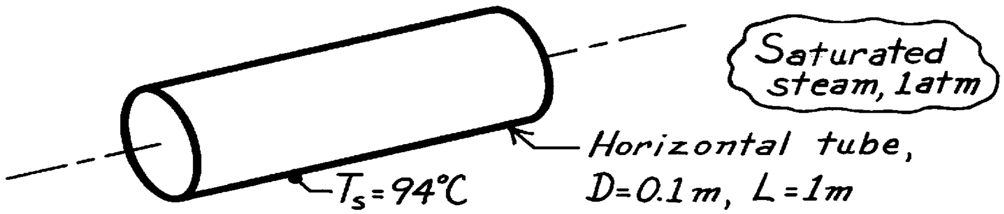
COMMENTS: The numerator and denominator in the bracketed expression are of special significance. The numerator is product of the mass in the filled container and the latent heat of vaporization; that is, the total energy removed by the cold water. What is physical significance of the denominator? Can you interpret the time-to-fill, t_f , expression in light of these terms?

PROBLEM 10.53

KNOWN: Tube of Problem 10.42 in horizontal position experiences condensation on its outer surface.

FIND: Heat transfer and condensation rates.

SCHEMATIC:



ASSUMPTIONS: (1) Laminar film condensation, (2) End effects negligible, (3) Negligible concentration of non-condensible gases in steam.

PROPERTIES: *Table A-6*, Water, vapor (1 atm): $T_{sat} = 100°C$, $\rho_v = 0.596\ kg/m^3$, $h_{fg} = 2257\ kJ/kg$; *Table A-6*, Water, liquid ($T_f = (T_s + T_{sat})/2 = 370K$): $\rho_\ell = 960.6\ kg/m^3$, $c_{p,\ell} = 4214\ J/kg\cdot K$, $\mu_\ell = 289 \times 10^{-6}\ N\cdot s/m^2$, $k_\ell = 0.679\ W/m\cdot K$.

ANALYSIS: From Eq. 10.32 with A = π D L and Eq. 10.33, the heat transfer and condensation rates are

$$q = \bar{h}_L (\pi DL)(T_{sat} - T_s) \qquad\qquad \dot{m} = q / h'_{fg}$$

where from Eq. 10.26 with $Ja = c_{p,\ell}(T_{sat} - T_s)/h_{fg}$, find

$$h'_{fg} = h_{fg}[1 + 0.68Ja] = 2257 kJ/kg\left[1 + 0.68\left[4214 J/kg\cdot K(100 - 94)K/2257 \times 10^3\ J/kg\right]\right] = 2274\frac{kJ}{kg}.$$

For laminar film condensation, Eq. 10.40 is the appropriate correlation for a cylinder with C = 0.729,

$$\bar{h}_D = 0.729\left[\frac{g\rho_\ell(\rho_\ell - \rho_v)k_\ell^3 h'_{fg}}{\mu_\ell(T_{sat} - T_s)D}\right]^{1/4}.$$

$$\bar{h}_D = 0.729\left[\frac{9.8 m/s^2 \times 960.6 kg/m^3 (960.6 - 0.596) kg/m^3 (0.679 W/m\cdot K)^3 \times 2274 \times 10^3 J/kg}{289 \times 10^{-6}\ N\cdot s/m^2 (100 - 94) K \times 0.1m}\right]^{1/4}$$

$$\bar{h}_D = 10{,}120\ W/m^2\cdot K.$$

Hence, the heat transfer and condensation rates are

$$q = 10{,}120\ W/m^2\cdot K(\pi \times 0.1m \times 1m)(100 - 94)K = 19.1kW \qquad <$$

$$\dot{m} = 19.1 \times 10^3 W/2274 \times 10^3\ J/kg = 8.39 \times 10^{-3} kg/s. \qquad <$$

COMMENTS: A comparison of the above results for the horizontal tube with those for a vertical tube (Problem 10.42) follows:

Position	$\bar{h}(W/m^2\cdot K)$	q(kW)	$\dot{m}\cdot 10^3 (kg/s)$
Vertical	8,507	16.0	7.05
Horizontal	10,120	19.1	8.39

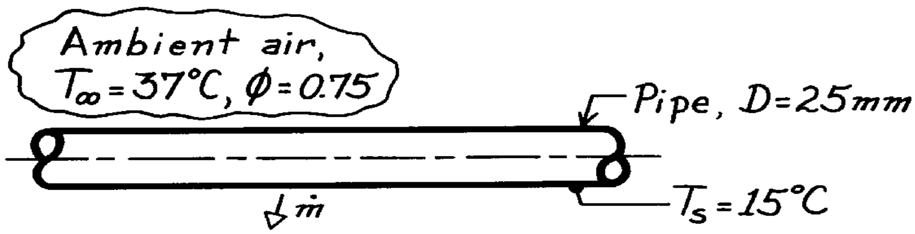
The rates are higher for the horizontal case. Why?

PROBLEM 10.54

KNOWN: Horizontal pipe passing through an air space with prescribed temperature and relative humidity.

FIND: Water condensation rate per unit length of the pipe.

SCHEMATIC:



ASSUMPTIONS: (1) Film condensation occurs on horizontal tube.

PROPERTIES: *Table A-6*, Water, vapor (T_∞ = 37°C = 310K): $p_{A,sat}$ = 0.06221 bar; *Table A-6*, Water, vapor ($p_A = \phi \cdot p_{A,sat}$ = 0.04666 bar): $T_{A,sat} \approx$ 305K, ρ_v = 0.04 kg/m^3, h_{fg} = 2426 kJ/kg; *Table A-6*, Water, liquid ($T_f = (T_s + T_{A,sat})/2$ = 297K): ρ_ℓ = 997.2 kg/m^3, $c_{p,\ell}$ = 4180 J/kg·K, $\mu_\ell = 917 \times 10^{-6}$ N·s/m^2, k_ℓ = 0.609 W/m·K.

ANALYSIS: From Eq. 10.33, the condensate rate per unit length is

$$\dot{m}' = \frac{q'}{h'_{fg}} = \frac{\bar{h}_L (\pi D)(T_{sat} - T_s)}{h'_{fg}}$$

where, from Eq. 10.26, with $Ja = c_{p,\ell}(T_{sat} - T_s)/h_{fg}$,

$$h'_{fg} = h_{fg}\left[1 + 0.68 c_{p,\ell}(T_{sat} - T_s)/h_{fg}\right] = 2426\frac{kJ}{kg}\left[1 + 0.68 \times 4180\frac{J}{kg \cdot K}(305 - 288)K/2426 \times 10^3\frac{J}{kg}\right]$$

$$h'_{fg} = 2474 \text{ kJ/kg}.$$

Note that $T_{sat} = T_{A,sat}$ is the saturation temperature of the water vapor in air at 37°C having a relative humidity ϕ = 0.75. That is, T_{sat} = 305K while T_s = 15°C = 288K. Assuming laminar film condensation on the horizontal pipe, it follows from Eq. 10.40 that,

$$\bar{h}_D = 0.729\left[\frac{g \rho_\ell (\rho_\ell - \rho_v) k_\ell^3 h'_{fg}}{\mu_\ell (T_{sat} - T_s) D}\right]^{1/4}$$

$$\bar{h}_D = 0.729\left[\frac{9.8\,m/s^2 \times 997.2\,kg/m^3 (997.2 - 0.04)\,kg/m^3 (0.609\,W/m \cdot K)^3 \times 2474 \times 10^3\,J/kg}{917 \times 10^{-6}\,N \cdot s/m^2 (305 - 288)\,K \times 0.025\,m}\right]^{1/4}$$

$$\bar{h}_D = 7925\ W/m^2 \cdot K.$$

Hence, the condensate rate is,

$$\dot{m}' = 7925\ W/m^2 \cdot K(\pi \times 0.025m)(305 - 288)K/2474 \times 10^3 J/kg$$

$$\dot{m}' = 4.28 \times 10^{-3} kg/s \cdot m. \qquad <$$

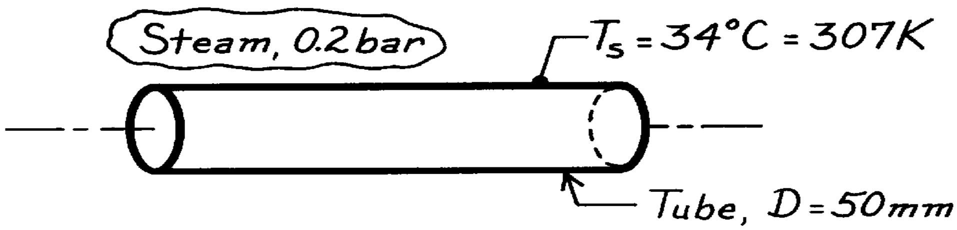
COMMENTS: The actual dropwise condensation rate exceeds the foregoing estimate.

PROBLEM 10.55

KNOWN: Horizontal tube, 50mm diameter, with surface temperature of 34°C is exposed to steam at 0.2 bar.

FIND: Estimate the heat transfer and condensation rates per unit length of the tube.

SCHEMATIC:



ASSUMPTIONS: (1) Laminar film condensation, (2) Negligible non-condensibles in steam.

PROPERTIES: *Table A-6*, Saturated steam (0.2 bar): T_{sat} = 333K, ρ_v = 0.129 kg/m^3, h_{fg} = 2358 kJ/kg; *Table A-6*, Water, liquid ($T_f = (T_s + T_{sat})/2$ = 320K): r_ℓ = 989.1 kg/m^3, $c_{p,\ell}$ = 4180 J/kg·K, m_ℓ = 577 × 10^{-6} N·s/m^2, k_ℓ = 0.640 W/m·K.

ANALYSIS: From Eqs. 10.32 and 10.33, the heat transfer and condensate rates per unit length of the tube are

$$q' = \bar{h}_D (p D)(T_{sat} - T_s) \qquad\qquad \dot{m}' = q' / h'_{fg}$$

where from Eq. 10.26 with $Ja = c_{p,\ell}(T_{sat} - T_s)/h_{fg}$,

$$h'_{fg} = h_{fg}[1 + 0.68\ Ja] = 2358\frac{kJ}{kg}\left[1 + 0.68 \times 4180 J/kg \cdot K(333 - 307)K / 2358 \times 10^3 J/kg\right]$$

$$h'_{fg} = 2432\ kJ/kg.$$

For laminar film condensation, Eq. 10.40 is appropriate for estimating $\bar{h}_D$ with C = 0.729,

$$\bar{h}_D = 0.729\left[\frac{g\, r_\ell (r_\ell - r_v) k_\ell^3 h'_{fg}}{m_\ell (T_{sat} - T_s) D}\right]^{1/4}$$

$$\bar{h}_D = 0.729\left[\frac{9.8 m/s^2 \times 989.1 kg/m^3 (989.1 - 0.129) kg/m^3 (0.640 W/m \cdot K)^3 \times 2432 \times 10^3 J/kg}{577 \times 10^{-6} N \cdot s/m^2 (333 - 307) K \times 0.050 m}\right]$$

$$\bar{h}_D = 6926\ W/m^2 \cdot K.$$

Hence, the heat transfer and condensation rates are

$$q' = 6926\ W/m^2 \cdot K(p \times 0.050m)(333 - 307)K = 28.3 kW/m \qquad <$$

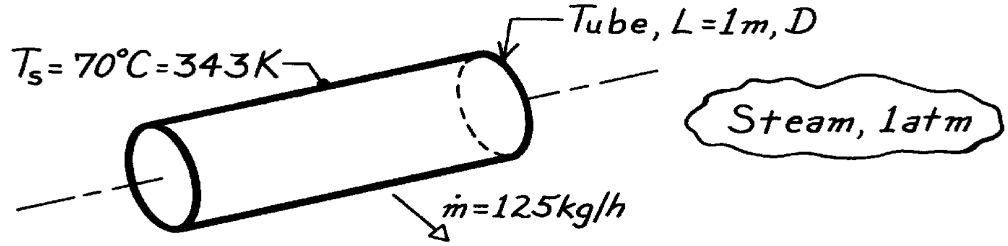
$$\dot{m}' = 28.3 \times 10^3 W/m / 2432 \times 10^3 J/kg = 1.16 \times 10^{-2} kg/s \cdot m. \qquad <$$

PROBLEM 10.56

KNOWN: Horizontal tube 1m long with surface temperature of 70°C used to condense steam at 1 bar.

FIND: Diameter required for condensation rate of 125 kg/h.

SCHEMATIC:



ASSUMPTIONS: (1) Laminar film condensation, (2) Negligible non-condensibles in steam.

PROPERTIES: *Table A-6*, Water, vapor (1 atm): T_{sat} = 100°C, ρ_v = 0.596 kg/m^3, h_{fg} = 2257 kJ/kg; *Table A-6*, Water, liquid ($T_f = (T_s + T_{sat})/2$ = 358K): r_ℓ = 968.6 kg/m^3, $c_{p,\ell}$ = 4201 J/kg·K, m_ℓ = 332×10^{-6} N·s/m^2, k_ℓ = 0.673 W/m·K.

ANALYSIS: From the rate equation, Eq. 10.33, with A = π D L, the required diameter is

$$D = \dot{m}\, h'_{fg} / p\, L\, \bar{h}_D \left(T_{sat} - T_s\right) \qquad (1)$$

where from Eq. 10.26 with $Ja = c_{p,\ell}\left(T_{sat} - T_s\right)/h_{fg}$,

$$h'_{fg} = h_{fg}\left(1 + 0.68 Ja\right) = 2257\frac{kJ}{kg}\left(1 + 0.68\frac{4201 J/kg\cdot K \times (100-70) K}{2257 \times 10^3 J/kg}\right) = 2343 kJ/kg. \qquad (2)$$

Substituting numerical values, Eq. (1) becomes

$$D = \frac{125}{3600}\frac{kg}{s} \times 2343 \times 10^3 \frac{J}{kg} / p \times 1m \times \bar{h}_D \left(100-70\right) K = 863.2 \bar{h}_D^{-1}. \qquad (3)$$

The appropriate correlation for $\bar{h}_D$ is Eq. 10.40 with C = 0.729,

$$\bar{h}_D = 0.729\left[\frac{g\, r_\ell\left(r_\ell - r_v\right) k_\ell^3\, h'_{fg}}{m_\ell\left(T_{sat} - T_s\right) D}\right]^{1/4}. \qquad (4)$$

Substitute Eq. (4) for $\bar{h}_D$ into Eq. (3) and use numerical values,

$$863.2\ D^{-1} = 0.729 \times$$

$$\left[\frac{9.8 m/s^2 \times 968.6 kg/m^3 \left(968.6 - 0.596\right) kg/m^3 \left(0.673 W/m\cdot K\right)^3 \times 2343 \times 10^3 J/kg}{332 \times 10^{-6} N\cdot s/m^2 \left(100-70\right) K \times D}\right]^{1/4}$$

$$863.2\ D^{-1} = 3693.4\ D^{-1/4}$$

$$D = 0.144 m = 144 mm. \qquad <$$

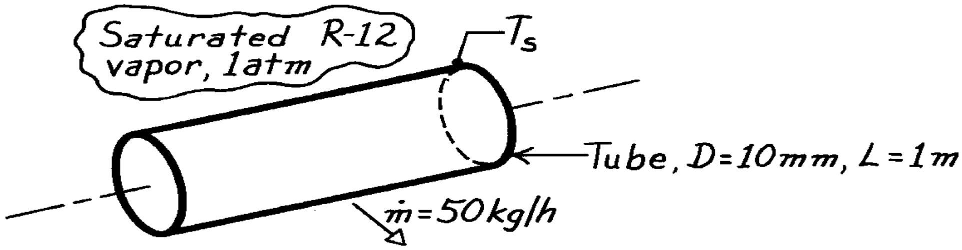
COMMENTS: Note for this situation Ja = 0.06.

PROBLEM 10.57

KNOWN: Saturated R-12 vapor at 1 atm condensing on the outside of a horizontal tube.

FIND: Tube surface temperature required for condensation rate of 50 kg/h.

SCHEMATIC:



ASSUMPTIONS: (1) Laminar film condensation, (2) Negligible non-condensibles in vapor.

PROPERTIES: *Table A-5*, R-12 Saturated vapor (1 atm): T_{sat} = 243K, ρ_v = 6.32 kg/m^3, h_{fg} = 165 kJ/kg; *Table A-5*, R-12 Saturated liquid ($T_f \approx$ 240K): ρ_ℓ = 1498 kg/m^3, $c_{p,\ell}$ = 892.3 J/kg·K, μ_ℓ = 0.0385×10^{-2} N·s/m^2, k_ℓ = 0.069 W/m·K.

ANALYSIS: The surface temperature or temperature difference can be written as follows from Eq. 10.33,

$$\Delta T = T_{sat} - T_s = \dot{m}\, h'_{fg} / \bar{h}_D\, \pi\, D\, L \qquad (1)$$

where A = π D L. To evaluate h'_{fg} and $\bar{h}_D$, we require knowledge of T_s or ΔT. Assume a ΔT = 10°C, then T_s = 233K and $T_f = (T_s + T_{sat})/2$ = 240K. From Eq. 10.26 with $Ja = c_{p,\ell}\,\Delta T / h_{fg}$, find

$$h'_{fg} = h_{fg}\left(1+0.68 Ja\right) = 165\frac{kJ}{kg}\left[1+0.68\times 892.3\frac{J}{kg\cdot K}\times 10K/165\times 10^3\frac{J}{kg}\right] = 171 kJ/kg. \qquad (2)$$

The appropriate correlation for $\bar{h}_D$ is Eq. 10.40 with C = 0.729; substitute properties and find $\bar{h}_D$ in terms of ΔT.

$$\bar{h}_D = 0.729\left[\frac{g\,\rho_\ell\left(\rho_\ell - \rho_v\right)k_\ell^3 h'_{fg}}{\mu_\ell\left(T_{sat}-T_s\right)D}\right]^{1/4}$$

$$\bar{h}_D = 0.729\left[\frac{9.8 m/s^2 \times 1498 kg/m^3\left(1498-6.32\right)kg/m^3\left(0.069 W/m\cdot K\right)^3 \times 171\times 10^3 J/kg}{0.0385\times 10^{-2} N\cdot s/m^2 \times \Delta T \times 0.010 m}\right]^{1/4}$$

$$\bar{h}_D = 3082\,\Delta T^{1/4}. \qquad (3)$$

Substitute Eq. (3) into Eq. (1) for $\bar{h}_D$, and solve for ΔT,

$$\Delta T = \frac{50}{3600} kg/s \times 171\times 10^3 J/kg / \left(3082\,\Delta T^{1/4}\right)\pi\left(0.010 m\right)\times 1m$$

$$\Delta T = 12.9K \qquad \text{or} \qquad T_s = 230K. \qquad <$$

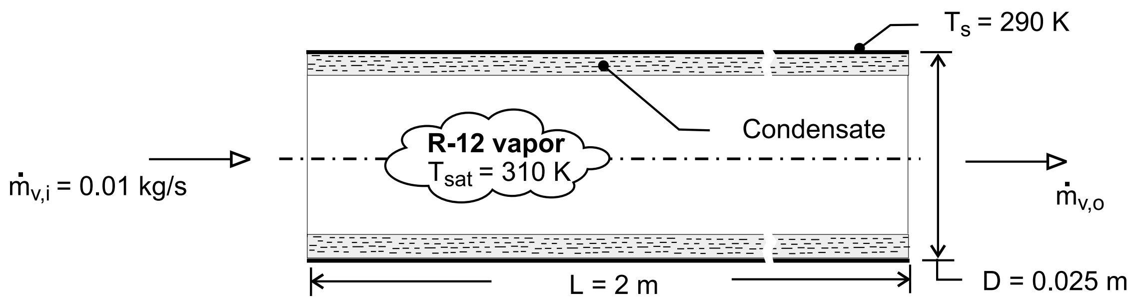
COMMENTS: We used the assumed value of T_s or ΔT only to evaluate properties. Our estimate for T_f = 240K is to be compared to the calculated value of $T_f \approx$ 236K. An iteration is probably not necessary.

PROBLEM 10.58

KNOWN: Saturation temperature and inlet flow rate of R-12. Diameter, length and temperature of tube.

FIND: Rate of condensation and outlet flow rate.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible concentration of noncondensables in vapor.

PROPERTIES: Given, R-12, saturated vapor: $\rho_v = 6$ kg/m^3, $h_{fg} = 160$ kJ/kg, $\mu_v = 150 \times 10^{-7}$ N·s/m^2. *Table A-5*, R-12, saturated liquid (T_f = 300 K): $\rho_\ell = 1306\,\text{kg/m}^3$, $c_{p,\ell} = 978\,\text{J/kg}\cdot\text{K}$, $\mu_\ell = 0.0254\,\text{N}\cdot\text{s/m}^2$, $k_\ell = 0.072\,\text{W/m}\cdot\text{K}$.

ANALYSIS: The Reynolds number associated with the inlet vapor flow is $\text{Re}_{v,i} = 4\dot{m}_{v,i}/\pi D\mu_v$ $= 0.04\,\text{kg/s}/\pi \times 0.025\text{m} \times 150 \times 10^{-7}\,\text{N}\cdot\text{s/m}^2 = 33{,}950 < 35{,}000.$ Hence, the average convection coefficient may be obtained from Eq. 10.42, where $h'_{fg} = h_{fg} + 0.375\,c_{p,\ell}\left(T_{sat} - T_s\right) = (1.6 \times 10^5 + 0.375 \times 978 \times 20)$ J/kg $= 1.67 \times 10^5$ J/kg.

$$\bar{h}_D = 0.555\left[\frac{g\rho_\ell\left(\rho_\ell - \rho_v\right)k_\ell^3\,h'_{fg}}{\mu_\ell\left(T_{sat} - T_s\right)D}\right]^{1/4} \approx 0.555\left[\frac{9.8\,\text{m/s}^2\left(1306\,\text{kg/m}^3\right)^2\left(0.072\,\text{W/m}\cdot\text{K}\right)^3 1.67\times10^5\,\text{J/kg}}{0.0254\,\text{N}\cdot\text{s/m}^2 \times 20\,\text{K} \times 0.025\text{m}}\right]^{1/4}$$

$$\bar{h}_D = 297\,\text{W/m}^2\cdot\text{K}$$

The heat rate is then

$$q = \pi D L\,\bar{h}_D\left(T_{sat} - T_s\right) = \pi \times 0.025\text{m} \times 2\text{m} \times 297\,\text{W/m}^2\cdot\text{K} \times 20\,\text{K} = 933\,\text{W}$$

and the condensation rate is

$$\dot{m}_{cond} = \frac{q}{h'_{fg}} = \frac{933\,\text{W}}{1.67\times10^5} = 0.0056\,\text{kg/s} \qquad <$$

The flow rate of vapor leaving the tube is then

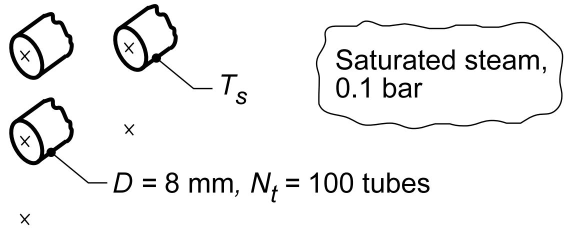
$$\dot{m}_{v,o} = \dot{m}_{v,i} - \dot{m}_{cond} = \left(0.0100 - 0.0056\right)\text{kg/s} = 0.0044\,\text{kg/s} \qquad <$$

PROBLEM 10.59

KNOWN: Array of condenser tubes exposed to saturated steam at 0.1 bar.

FIND: (a) Condensation rate per unit length of square array, (b) Options for increasing the condensation rate.

SCHEMATIC:



ASSUMPTIONS: (1) Film condensation on tubes, (2) Negligible non-condensable gases in steam.

PROPERTIES: *Table A.6*, Saturated water vapor (0.1 bar): $T_{sat} \approx 320$ K, $\rho_v = 0.072$ kg/m^3, $h_{fg} = 2390$ kJ/kg; *Table A.6*, Water, liquid ($T_f = (T_s + T_{sat})/2 = 310$ K): $\rho_\ell = 993.1$ kg/m^3, $c_{p,\ell} = 4178$ J/kg·K, $\mu_\ell = 695 \times 10^{-6}$ N·s/m^2, $k_\ell = 0.628$ W/m·K.

ANALYSIS: (a) From Eq. 10.33, the condensation rate for a N ×N square array is

$$\dot{m}' = \dot{m}/L = \bar{h}_{D,N} \cdot N_t (\pi D)(T_{sat} - T_s)/h'_{fg}$$

where $\bar{h}_{D,N}$ is the average coefficient for the tubes in a vertical array of N tubes. With Ja = $c_{p,\ell}\Delta T/h_{fg}$ = 4178 J/kg·K × (320 - 300)K/2390 × 10^3 J/kg = 0.035, Eq. 10.26 yields $h'_{fg} = h_{fg}(1 + 0.68\,\text{Ja}) = 2390$ kJ/kg(1 + 0.68 × 0.035) = 2447 kJ/kg.

For a vertical tier of N = 10 horizontal tubes, the average coefficient is given by Eq. 10.41,

$$\bar{h}_{D,N} = 0.729\left[\frac{g\rho_\ell(\rho_\ell - \rho_v)k_\ell^3 h'_{fg}}{N\mu_\ell(T_{sat} - T_s)D}\right]^{1/4}$$

$$\bar{h}_{D,N} = 0.729\left[\frac{9.8\,\text{m/s}^2 \times 993.1\,\text{kg/m}^3\,(993.1 - 0.072)\,\text{kg/m}^3\,(0.628\,\text{W/m·K})^3 \times 2447 \times 10^3\,\text{J/kg}}{10 \times 695 \times 10^{-6}\,\text{N·s/m}^2\,(320 - 300)\,\text{K} \times 0.008\,\text{m}}\right]^{1/4}$$

$$\bar{h}_{D,N} = 6210\,\text{W/m}^2\cdot\text{K}\,.$$

Hence, the condensation rate for the entire array per unit tube length is

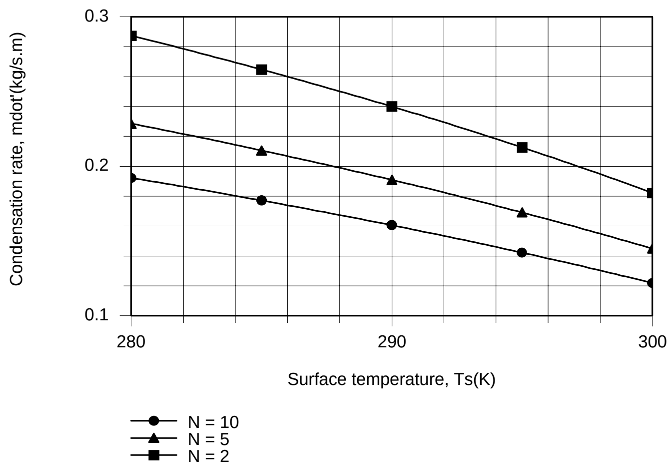
$$\dot{m}' = 6210\,\text{W/m}^2\cdot\text{K}\,(100)\pi \times 0.008\,\text{m}\,(320 - 300)\,\text{K}/2447 \times 10^3\,\text{J/kg}$$

$$\dot{m}' = 0.128\,\text{kg/s·m} = 459\,\text{kg/h·m}\,. \qquad <$$

(b) Options for increasing the condensation rate include reducing the surface temperature and/or the number of tubes in a vertical tier. By varying the temperature of cold water flowing through the tubes, it is feasible to maintain surface temperatures in the range $280 \le T_s \le 300$ K. Using the *Correlations* and *Properties* Toolpads of IHT, the following results were obtained for N = 10, 5 and 2, with $N_t = 100$ in each case. The results are based on properties evaluated at p = 0.1 bar, for which the Properties Toolpad yielded $T_{sat} = 318.9$ K.

Continued...

PROBLEM 10.59 (Cont.)



Clearly, there are significant benefits associated with reducing both T_s and N.

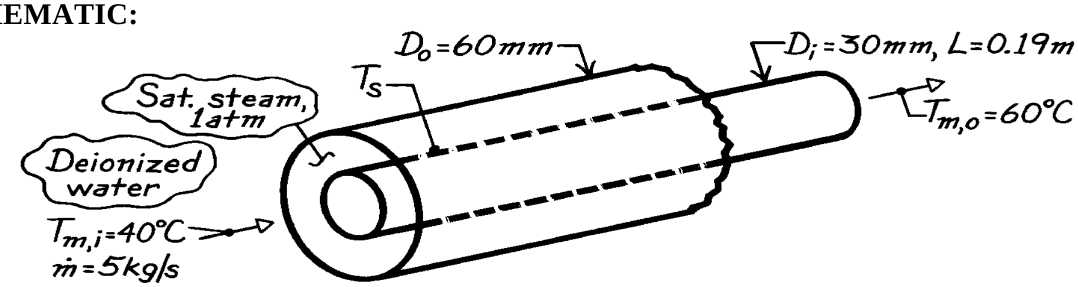
COMMENTS: Note that, since $\bar{h}_{D,N} \propto N^{-1/4}$, the average coefficient decreases with increasing N due to a corresponding increase in the condensate film thickness. From the result of part (a), the coefficient for the topmost tube is $\bar{h}_D = 6210\ W/m^2 \cdot K(10)^{1/4} = 11{,}043\ W/m^2 \cdot K$.

PROBLEM 10.60

KNOWN: Thin-walled concentric tube arrangement for heating deionized water by condensation of steam.

FIND: Estimates for convection coefficients on both sides of the inner tube. Inner tube wall outlet temperature. Whether condensation provides fairly uniform inner tube wall temperature approximately equal to the steam saturation temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible thermal resistance of inner tube wall, (2) Internal flow is fully developed.

PROPERTIES: Deionized water (given): $\rho = 982.3$ kg/m^3, $c_p = 4181$ J/kg·K, k = 0.643 W/m·K, $\mu = 548 \times 10^{-6}$ N·s/m^2, Pr = 3.56; *Table A-6*, Saturated vapor (1 atm): $T_{sat} = 100°C$, $\rho_v = (1/v_g) = 0.596$ kg/m^3, $h_{fg} = 2265$ kJ/kg; *Table A-6*, Saturated water (assume $T_s \approx 75°C$, $T_f = (75 + 100)°C/2 = 360K$): $r_\ell = (1/v_f) = 967$ kg/m^3, $m_\ell = 324 \times 10^{-6}$ N·s/m^2, $k_\ell = 0.674$ W/m·K, $c_{p,\ell} = 4203$ J/kg·K.

ANALYSIS: From an energy balance on the inner tube assuming a constant wall temperature,

$$\overline{h}_c\left(T_{sat} - T_{s,o}\right) = h_i\left(T_{s,o} - T_{m,o}\right)$$

where $\overline{h}_c$ and h_i are, respectively, the heat transfer coefficients for condensation (c) on a horizontal cylinder and internal (i) flow in a tube.

Condensation. From Eq. 10.40, for the horizontal tube,

$$\overline{h}_c = 0.729\left[\frac{g\, r_\ell\left(r_\ell - r_v\right)k_\ell^3 h'_{fg}}{m_\ell\left(T_{sat} - T_s\right)D}\right]^{1/4}$$

where $\quad h'_{fg} = h_{fg}\left\{1 + 0.68c_{p,\ell}\left(T_{sat} - T_s\right)/h_{fg}\right\}$

$$h'_{fg} = 2265\ \text{kJ/kg}\left\{1 + 0.68 \times 4203\,\text{J/kg}\cdot\text{K}\left(100 - T_s\right)/2265 \times 10^3\ \text{J/kg}\right\}$$

$$h'_{fg} = 2265\ \text{kJ/kg}\left\{1 + 1.262 \times 10^{-3}\left(100 - T_s\right)\right\}$$

$$\overline{h}_c = 0.729\Big[9.8\,\text{m/s}^2 \times 967\,\text{kg/m}^3\left(067 - 0.596\right)\text{kg/m}^3\left(0.674\,\text{W/m}\cdot\text{K}\right)^3 \times$$

$$2265\left\{1 + 1.262 \times 10^{-3}\left(100 - T_s\right)\right\}\text{kJ/kg}/324 \times 10^{-6}\,\text{N}\cdot\text{s/m}^2\left(100 - T_s\right)0.030\,\text{m}\Big]^{1/4}$$

Continued …..

$$\bar{h}_c = 2.843\times10^4\left[\frac{1+1.262\times10^{-3}(100-T_s)}{100-T_s}\right]^{1/4}.$$

Internal flow. From Eq. 8.6, evaluating properties at $\bar{T}_m$, find

$$Re_D = \frac{4\dot{m}}{p m D} = \frac{4\times5\ kg/s}{p\times548\times10^{-6}N\cdot s/m^2\times0.030\ m} = 3.872\times10^5$$

and for turbulent flow use the Colburn equation,

$$Nu_D = \frac{h_i D}{k} = 0.023 Re_D^{0.8} Pr^{1/3}$$

$$h_i = \frac{0.023\times0.643\ W/m\cdot K}{0.03\ m}\left(3.872\times10^5\right)^{0.8}(3.56)^{1/3} = 2.22\times10^4\ W/m^2\cdot K.$$ <

Substituting numerical values into the energy balance relation,

$$2.843\times10^4\left[\frac{1+1.262\times10^{-3}\left(100-T_{s,o}\right)}{100-T_{s,o}}\right]^{1/4}\left(100-T_{s,o}\right)K$$

$$= 2.22\times10^4\ W/m^2\cdot K\left(T_{s,o}-60\right)K$$

and by trial-and-error, find

$$T_{s,o}\approx75°C.$$

With this value of T_s, find that

$$\bar{h}_c = 1.29\times10^4\ W/m^2\cdot K$$ <

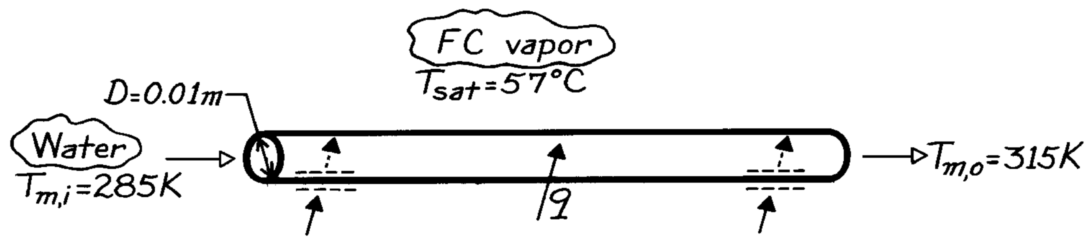
which is approximately half that for the internal flow. Hence, the tube wall cannot be at a uniform temperature. This could only be achieved if $\bar{h}_c$ □ h_i.

PROBLEM 10.61

KNOWN: Heat dissipation from multichip module to saturated liquid of prescribed temperature and properties. Diameter and inlet and outlet water temperatures for a condenser coil.

FIND: (a) Condensation and water flow rates. (b) Tube surface inlet and outlet temperatures. (c) Coil length.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions since rate of heat transfer from the module is balanced by rate of heat transfer to coil, (2) Fully developed flow in tube, (3) Negligible changes in potential and kinetic energy for tube flow.

PROPERTIES: Saturated fluorocarbon (T_{sat} = 57°C, given): k_ℓ = 0.0537 W/m·K, $c_{p,\ell}$ = 1100 J/kg·K, $h'_{fg} \approx h_{fg}$ = 84,400 J/kg. r_ℓ = 1619.2 kg/m^3, ρ_v = 13.4 kg/m^3, $\sigma = 8.1 \times 10^{-3}$ kg/s^2, $m_\ell = 440 \times 10^{-6}$ kg/m·s, Pr_ℓ = 9; *Table A-6*, Water, sat. liquid $(\overline{T}_m = 300\text{K})$: ρ = 997 kg/m^3, c_p = 4179 J/kg·K, $\mu = 855 \times 10^{-6}$ N·s/m^2, k = 0.613 W/m·K, Pr = 5.83.

ANALYSIS: (a) With

$$q = (q'' \times A)_{module} = 10^5\ \text{W/m}^2 (0.1\,\text{m})^2 = 10^3\ \text{W}$$

the condensation rate is

$$\dot{m}_{con} = \frac{q}{h'_{fg}} = \frac{10^3\ \text{W}}{84{,}400\,\text{J/kg}} = 0.0118\ \text{kg/s}$$ <

and the required water flow rate is

$$\dot{m} = \frac{q}{c_p\left(T_{m,o} - T_{m,i}\right)} = \frac{1000\ \text{W}}{4179\,\text{J/kg}\cdot\text{K}(30\text{K})} = 7.98 \times 10^{-3}\text{kg/s}.$$ <

(b) The Reynolds number for flow through the tube is

$$Re_D = \frac{4\,\dot{m}}{p\,D m} = \frac{4 \times 7.98 \times 10^{-3}\,\text{kg/s}}{p\,(0.01\text{m})855 \times 10^{-6}\,\text{N}\cdot\text{s/m}^2} = 1188.$$

Hence, the flow is laminar. Assuming a uniform wall temperature,

$$h_i = Nu_D\, k/D = 3.66\left(0.613\ \text{W/m}\cdot\text{K}/0.01\text{m}\right) = 224\ \text{W/m}^2\cdot\text{K}.$$

Continued …..

For film condensation on the outer surface, Eq. 10.40 yields

$$h_o = 0.729\left[\frac{9.8\,m/s^2\left(1619.2\ kg/m^3\right)\left(1605.8 kg/m^3\right)\left(0.0537\ W/m\cdot K\right)^3 84{,}400 J/kg}{440\times 10^{-6}\ kg/m\cdot s\times 0.01m\left(T_{sat}-T_s\right)}\right]^{1/4}$$

$$h_o = 2150\left(57-T_s\right)^{-1/4}.$$

From an energy balance on a portion of the tube surface,

$$h_o\left(T_{sat}-T_s\right) = h_i\left(T_s-T_m\right)$$

or

$$2150\left(57-T_s\right)^{3/4} = 224\left(T_s-T_m\right)$$

At the entrance where $\left(T_{m,i} = 285K\right)$, trial-and-error yields:

$$T_{s,i} = 50.6°C$$ <

and at the exit where $\left(T_{m,o} = 315K\right)$,

$$T_{s,o} = 55.4°C$$ <

(c) From Eqs. 8.44 and 8.45,

$$L = \frac{q}{h_i \pi D \Delta T_{\ell m}}$$

where

$$\Delta T_{\ell m} = \frac{\left(T_s-T_{m,i}\right)-\left(T_s-T_{m,o}\right)}{\ln\left[\left(T_s-T_{m,i}\right)/\left(T_s-T_{m,o}\right)\right]} = \frac{41-11}{\ln\left(41/11\right)} = 22.8°C$$

$$L = \frac{1000\ W}{\left(224\ W/m^2\cdot K\right)\pi\left(0.01m\right)22.8°C} = 6.23m.$$ <

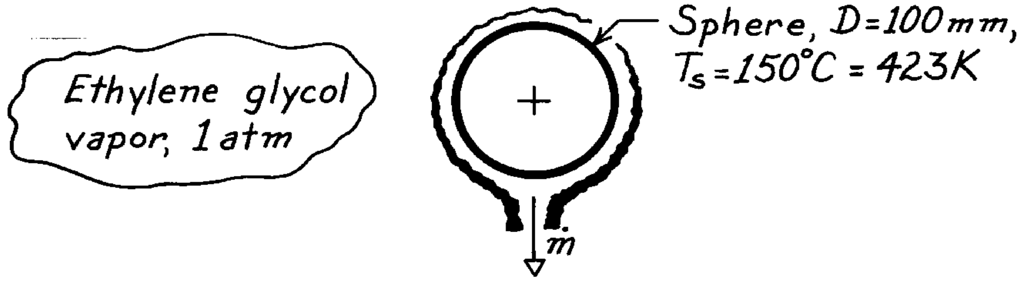
COMMENTS: Some control over system performance may be exercised by adjusting the water flow rate. By increasing $\dot{m}$, $\left(T_{m,o}-T_{m,i}\right)$ is reduced for a prescribed q. The value of h_i is increased substantially if the internal flow is turbulent.

PROBLEM 10.62

KNOWN: Saturated ethylene glycol vapor at 1 atm condensing on a sphere of 100 mm diameter having surface temperature of 150°C.

FIND: Condensation rate.

SCHEMATIC:



ASSUMPTIONS: (1) Laminar film condensation, (2) Negligible non-condensibles in vapor.

PROPERTIES: *Table A-5*, Saturated ethylene glycol, vapor (1 atm): T_{sat} = 470K, $\rho_v \approx 0$ kg/m^3, h_{fg} = 812 kJ/kg; *Table A-5*, Ethylene glycol, liquid (T_f = 423K, but use values at 373K, limit of data in table): ρ_ℓ = 1058.5 kg/m^3, $c_{p,\ell}$ = 2742 J/kg·K, $\mu_\ell = 0.215 \times 10^{-2}$ N·s/m^2, k_ℓ = 0.263 W/m·K.

ANALYSIS: The condensation rate is given by Eq. 10.33 as

$$\dot{m} = \frac{q}{h'_{fg}} = \frac{\bar{h}_L\left(\pi D^2\right)\left(T_{sat} - T_s\right)}{h'_{fg}}$$

where A = π D^2 for the sphere and h'_{fg}, with $Ja = c_{p,\ell}\,\Delta T / h_{fg}$, is given by Eq. 10.26 as

$$h'_{fg} = h_{fg}\left(1 + 0.68Ja\right) = 812\frac{kJ}{kg}\left(1 + 0.68 \times 2742\frac{J}{kg \cdot K}\left(470 - 423\right)K/812 \times 10^3\ J/kg\right) = 900\,kJ/kg.$$

The average heat transfer coefficient for the sphere follows from Eq. 10.40with C = 0.815,

$$\bar{h}_D = 0.815\left[\frac{g\,\rho_\ell\left(\rho_\ell - \rho_v\right)k_\ell^3 h'_{fg}}{\mu_\ell\left(T_{sat} - T_s\right)D}\right]^{1/4}$$

$$\bar{h}_D = 0.815\left[\frac{9.8\,m/s^2 \times 1058.5\,kg/m^3\left(1058.5 - 0\right)kg/m^3\left(0.263\,W/m \cdot K\right)^3 \times 900 \times 10^3\,J/kg}{0.215 \times 10^{-2}\,N \cdot s/m^2\left(470 - 423\right)K \times 0.100m}\right]^{1/4}$$

$$\bar{h}_D = 1674\ W/m^2 \cdot K.$$

Hence, the condensation rate is

$$\dot{m} = 1674\,W/m^2 \cdot K \times \pi\left(0.100m\right)^2\left(470 - 423\right)K/900 \times 10^3\,J/kg$$

$$\dot{m} = 2.75 \times 10^{-3}\,kg/s.$$ <

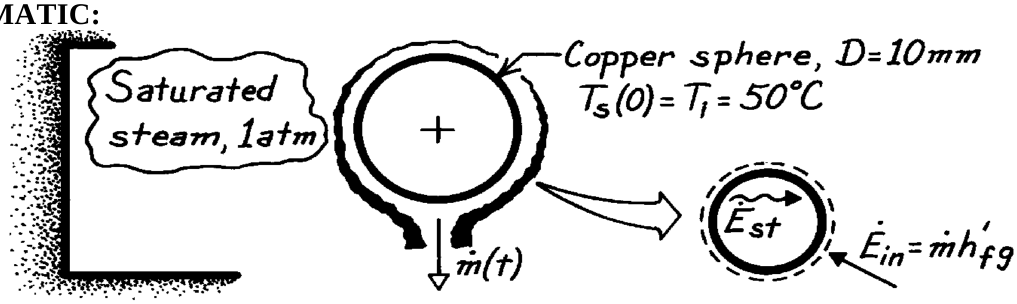
COMMENTS: Recognize this estimate is likely to be a poor one since properties were not evaluated at the proper T_f which was beyond the limit of the table.

PROBLEM 10.63

KNOWN: Copper sphere of 10 mm diameter, initially at 50°C, is placed in a large container filled with saturated steam at 1 atm.

FIND: Time required for sphere to reach equilibrium and the condensate formed during this period.

SCHEMATIC:



ASSUMPTIONS: (1) Laminar film condensation, (2) Negligible non-condensibles in vapor, (3) Sphere is spacewise isothermal, (4) Sphere experiences heat gain by condensation only.

PROPERTIES: *Table A-6*, Saturated water vapor (1 atm): T_{sat} = 100°C, ρ_v = 0.596 kg/m^3, h_{fg} = 2257 kJ/kg; *Table A-6*, Water, liquid ($T_f \approx$ (75 + 100)°C/2 = 360K): r_ℓ = 967.1 kg/m^3, $c_{p,\ell}$ = 4203 J/kg·K, m_ℓ = 324 × 10^{-6} N·s/m^2, k_ℓ = 0.674 W/m·K; *Table A-1*, Copper, pure $(\overline{T} = 75°\text{ C})$: ρ_{sp} = 8933 kg/m^3, $c_{p,sp}$ = 389 J/kg·K.

ANALYSIS: Using the lumped capacitance approach, an energy balance on the sphere provides,

$$\dot{E}_{in} - \dot{E}_{out} = \dot{E}_{st}$$

$$\dot{m}\,h'_{fg} = \overline{h}_D\,A_s\left(T_{sat} - T_s\right) = r_{sp}\,c_{p,sp}\,V_s\,\frac{dT_s}{dt}. \qquad (1)$$

Properties of the sphere, ρ_{sp} and $c_{p,sp}$,. Will be evaluated at $\overline{T}_s = (50 + 100)°C/2 = 75°C$, while water (liquid) properties will be evaluated at $\overline{T}_f = (\overline{T}_s + T_{sat})/2 = 87.5°C \approx 360K$. From Eq. 10.26 with Ja = $c_{p,\ell}\,\Delta T / h_{fg}$ where ΔT = Tsat - $\overline{T}_s$, find

$$h'_{fg} = h_{fg}\left(1 + 0.68Ja\right) = 2257\frac{kJ}{kg}\left(1 + 0.68\left[4203\frac{J}{kg\cdot K}\times(100-75)K/2257\times10^3 J/kg\right]\right) = 2328\frac{kJ}{kg}. \qquad (2)$$

To estimate the time required to reach equilibrium, we need to integrate Eq. (1) with appropriate limits. However, to perform the integration, an appropriate relation for the temperature dependence of $\overline{h}_D$ needs to be found. Using Eq. 10.40 with C = 0.815,

$$\overline{h}_D = 0.815\left[\frac{g\,r_\ell\left(r_\ell - r_v\right)k_\ell^3\,h'_{fg}}{m_\ell\left(T_{sat} - T_s\right)D}\right]^{1/4}.$$

Substitute numerical values and find,

$$\overline{h}_D = 0.815\left[\frac{9.8\,m/s^2 \times 967.1\,kg/m^3\,(967.1 - 0.596)\,kg/m^3\,(0.674\,W/m\cdot K)^3 \times 2328\times10^3 J/kg}{324\times10^{-6}\,N\cdot s/m^2\,(T_{sat} - T_s)\times 0.010m}\right]^{1/4}$$

$$\overline{h}_D = B\left(T_{sat} - T_s\right)^{-1/4} \qquad \text{where} \qquad B = 30{,}707\,W/m^2\cdot(K)^{3/4}. \qquad (3)$$

Continued …..

PROBLEM 10.63 (Cont.)

Substitute Eq. (3) into Eq. (1) for $\overline{h}_D$ and recognize $V_s / A_s = \frac{1}{6} p\, D^3 / p\, D^2 = D/6$,

$$B\left(T_{sat} - T_s\right)^{-1/4}\left(T_{sat} - T_s\right) = r_{sp}\, c_{p,sp}\left(D/6\right)\frac{dT_s}{dt}. \qquad (4)$$

Note that $d(T_s) = -\, d(T_{sat} - T_s)$; letting $\Delta T \equiv T_{sat} - T_s$ and separating variables, the energy balance relation has the form

$$\int_0^t dt = -\frac{r_{sp}\, c_{p,sp}\left(D/6\right)}{B}\int_{\Delta T_o}^{\Delta T}\frac{d\left(\Delta T\right)}{\Delta T^{3/4}} \qquad (5)$$

where the limits of integration have been identified, with $\Delta T_o = T_{sat} - T_i$ and $T_i = T_s(0)$. Performing the integration, find

$$t = -\frac{r_{sp}\, c_{p,sp}\left(D/6\right)}{B}\cdot\frac{1}{1-3/4}\left[\Delta T^{1/4} - \Delta T_o^{1/4}\right].$$

Substituting numerical values with the limits, $\Delta T = 0$ and $\Delta T_o = 100\text{-}50 = 50°C$,

$$t = -\frac{8933\,\text{kg/m}^3 \times 389\,\text{J/kg}\cdot\text{K}\left(0.010\,\text{m}/6\right)}{30{,}707\ \text{W/m}^2\cdot\text{K}^{3/4}} \times 4\left[0^{1/4} - 50^{1/4}\right]\text{K}^{1/4}$$

$$t = 2.0\,\text{s}.$$ <

To determine the total amount of condensate formed during this period, perform an energy balance on a time interval basis,

$$E_{in} - E_{out} = \Delta E = E_{final} - E_{initial}$$

$$E_{in} = r_{sp}\, c_{p,sp}\, V\left(T_{final} - T_{initial}\right] \qquad (6)$$

where $T_{final} = T_{sat}$ and $T_{initial} = T_i = T_s(0)$. Recognize that

$$E_{in} = M\, h'_{fg} \qquad (7)$$

where M is the total mass of vapor that condenses. Combining Eqs. (6) and (7),

$$M = \frac{r_{sp}\, c_{p,sp}\, V}{h'_{fg}}\left[T_{sat} - T_i\right]$$

$$M = \frac{8933\,\text{kg/m}^3 \times 389\,\text{J/kg}\cdot\text{K}\left(p/6\right)\left(0.010\,\text{m}\right)^3}{2328\times 10^3\ \text{J/kg}}\left[100 - 50\right]\text{K}$$

$$M = 3.91\times 10^{-5}\,\text{kg}.$$ <

COMMENTS: The total amount of condensate could have been evaluated from the integral,

$$M = \int_0^t \dot{m}\, dt = \int_0^t \frac{q}{h'_{fg}}\, dt = \int_0^t \frac{\overline{h}_D\, A_s\left(T_{sat} - T_s\right)dt}{h'_{fg}}$$

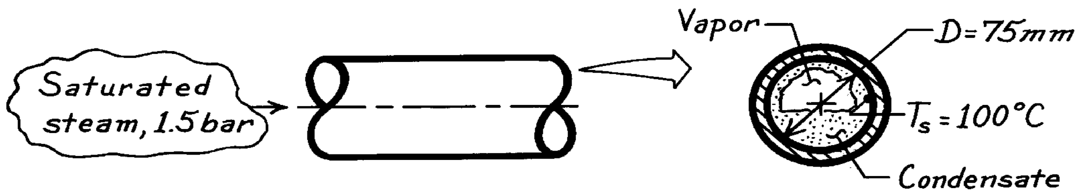
giving the same result, but with more effort.

PROBLEM 10.64

KNOWN: Saturated steam condensing on the inside of a horizontal pipe.

FIND: Heat transfer coefficient and the condensation rate per unit length of the pipe.

SCHEMATIC:



ASSUMPTIONS: (1) Film condensation with low vapor velocities.

PROPERTIES: *Table A-6*, Saturated water vapor (1.5 bar): $T_{sat} \approx 385K$, $\rho_v = 0.88\ kg/m^3$, $h_{fg} = 2225\ kJ/kg$; *Table A-6*, Saturated water ($T_f = (T_{sat} + T_s)/2 \approx 380K$): $r_\ell = 953.3\ kg/m^3$, $c_{p,\ell} = 4226\ J/kg\cdot K$, $m_\ell = 260 \times 10^{-6}\ N\cdot s/m^2$, $k_\ell = 0.683\ W/m\cdot K$.

ANALYSIS: The condensation rate per unit length follows from Eq. 10.33 with A = π D L and has the form

$$\dot{m}' = \frac{\dot{m}}{L} = \bar{h}_D (p D)(T_{sat} - T_s)/h'_{fg}$$

where $\bar{h}_D$ is estimated from the correlation of Eq. 10.42 with Eq. 10.43,

$$\bar{h}_D = 0.555\left[\frac{g\, r_\ell (r_\ell - r_v) k_\ell^3 h'_{fg}}{m_\ell (T_{sat} - T_s) D}\right]^{1/4}$$

where

$$h'_{fg} = h_{fg} + \frac{3}{8} c_{p,\ell}(T_{sat} - T_s) = 2225\times10^3 \frac{J}{kg} + \frac{3}{8}\times 4226 \frac{J}{kg\cdot K}(385 - 373)K$$

$$h'_{fg} = 2244 kJ/kg.$$

Hence,

$$\bar{h}_D = 0.555\left[\frac{9.8 m/s^2 \times 953.3 \frac{kg}{m^3}(953.3 - 0.88)\frac{kg}{m^3}(0.683 W/m\cdot K)^3\, 2244\times10^3 J/kg}{260\times10^{-6} N\cdot s/m^2 (385 - 373) K \times 0.075 m}\right]^{1/4}$$

$$\bar{h}_D = 7127 W/m^2\cdot K.$$

It follows that the condensate rate per unit length of the tube is

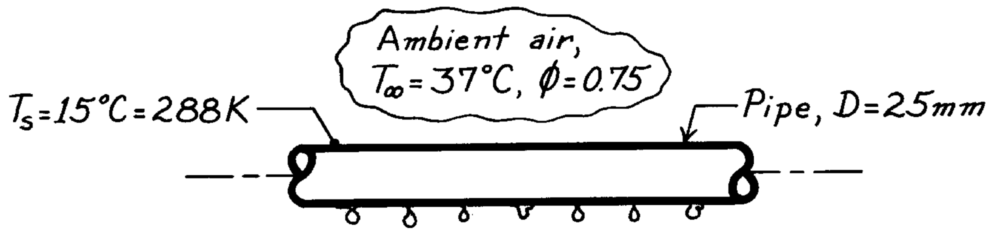
$$\dot{m}' = 7127 W/m^2\cdot K\,(p \times 0.075m)(385 - 373)K \,/\, 2225\times10^3 J/kg = 9.06\times10^{-3} kg/s\cdot m.$$ <

PROBLEM 10.65

KNOWN: Horizontal pipe passing through an air space with prescribed temperature and relative humidity.

FIND: Water condensation rate per unit length of pipe.

SCHEMATIC:



ASSUMPTIONS: (1) Drop-wise condensation, (2) Copper tube approximates well promoted surface.

PROPERTIES: *Table A-6*, Water vapor (T_∞ = 37°C = 310K): $p_{A,sat}$ = 0.06221 bar; *Table A-6*, Water vapor ($p_A = \phi \cdot p_{a,sat}$ = 0.04666 bar): T_{sat} = 305K = 32°C, h_{fg} = 2426 kJ/kg; *Table A-6*, Water, liquid ($T_f = (T_s + T_{sat})/2$ = 297K): $c_{p,\ell} = 4180 \text{ J/kg} \cdot \text{K}$.

ANALYSIS: From Eq. 10.33, the condensate rate per unit length is

$$\dot{m}' = \frac{q'}{h'_{fg}} = \frac{h_L (p\, D)(T_{sat} - T_s)}{h'_{fg}}$$

where from Eq. 10.26, with $Ja = c_{p,\ell}(T_{sat} - T_s)/h_{fg}$,

$$h'_{fg} = h_{fg}[1 + 0.68 Ja] = 2426 \frac{kJ}{kg}[1 + 0.68 \times 4180 J/kg \cdot K (305 - 288) K/2426 kJ/kg]$$

$$h'_{fg} = 2474 kJ/kg.$$

Note that T_{sat} is the saturation temperature of the water vapor in air at 37°C having a relative humidity, ϕ = 0.75. That is, T_{sat} = 305K and T_s = 15°C + 288K. For *drop-wise condensation*, the correlation of Eq. 10.44 yields

$$\bar{h}_{dc} = 51{,}104 + 2044 T_{sat} \qquad\qquad 22°C < T_{sat} < 100°C$$

where the units of $\bar{h}_{dc}$ and T_{sat} are $W/m^2 \cdot K$ and °C.

$$\bar{h}_{dc} = 51{,}104 + 2044(32°C) = 116{,}510 \text{ W/m}^2 \cdot \text{K}.$$

Hence, the condensation rate is

$$\dot{m}' = 116{,}510 \text{ W/m}^2 \cdot \text{K} (p \times 0.025 m)(305 - 288) K/2474 \times 10^3 J/kg$$

$$\dot{m}' = 6.288 \times 10^{-2} \text{ kg/s} \cdot \text{m} \qquad <$$

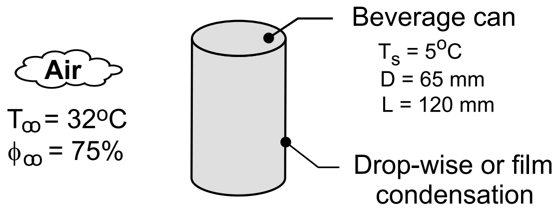
COMMENTS: From the result of Problem 10.54 assuming laminar film condensation, the condensation rate was $\dot{m}'_{film} = 4.28 \times 10^{-3} \text{ kg/s} \cdot \text{m}$ which is an order of magnitude less than for the rate assuming drop-wise condensation.

PROBLEM 10.66

KNOWN: Beverage can at 5°C is placed in a room with ambient air temperature of 32°C and relative humidity of 75%.

FIND: The condensate rate for (a) drop-wise and (b) film condensation.

SCHEMATIC:



ASSUMPTIONS: (1) Condensation on top and bottom surface of can neglected, (2) Negligible non-condensibles in water vapor-air, and (b) For film condensation, film thickness is small compared to diameter of can.

PROPERTIES: *Table A-6*, Water vapor (T_∞ = 32°C = 305 K): $p_{A,sat}$ = 0.04712 bar; Water vapor ($p_A = \phi \cdot p_{A,sat}$ = 0.03534 bar): $T_{sat} \approx$ 300 K = 27°C, h_{fg} = 2438 kJ/kg; Water, liquid ($T_f = (T_s + T_{sat})/2$ = 289 K): $c_{p,\ell}$ = 4185 J/kg·K.

ANALYSIS: From Eq. 10.33, the condensate rate is

$$\dot{m} = \frac{q}{h'_{fg}} = \frac{\bar{h}(\pi DL)(T_{sat} - T_s)}{h'_{fg}}$$

where from Eq. 10.26, with Ja = $c_{p,\ell}(T_{sat} - T_s)/h_{fg}$,

$$h'_{fg} = h_{fg}[1 + 0.68 \text{ Ja}]$$

$$h'_{fg} = 2438 \text{ kJ/kg}\left[1 + 0.68 \times 4185 \text{ J/kg}\cdot\text{K}(300 - 278)\text{K}/2438 \text{ kJ/kg}\right]$$

$$h'_{fg} = 2501 \text{ kJ/kg}$$

Note that T_{sat} is the saturation temperature of the water vapor in air at 32°C having a relative humidity of ϕ_∞ = 0.75.

(a) For drop-wise condensation, the correlation of Eq. 10.44 with T_{sat} = 300 K = 27°C yields

$$\bar{h} = \bar{h}_{dc} = 51{,}104 + 2044\, T_{sat} \qquad 22°C < T_{sat} \le 100°C$$

where the units of $\bar{h}_{dc}$ are W/m^2·K and T_{sat} are °C,

$$\bar{h}_{dc} = 51{,}104 + 2044 \times 27 = 106{,}292 \text{ W/m}^2\cdot\text{K}$$

Hence, the condensation rate is

$$\dot{m} = 1.063 \times 10^5 \text{ W/m}^2\cdot\text{K}(\pi \times 0.065 \text{ m} \times 0.125 \text{ m})(27 - 5)\text{K}/2501 \text{ kJ/kg}$$

$$\dot{m} = 0.0229 \text{ kg/s}$$ <

Continued …..

PROBLEM 10.66 (Cont.)

(b) For film condensation, we used the *IHT* tool *Correlations, Film Condensation*, which is based upon Eqs. 10.37, 10.38 or 10.39 depending upon the flow regime. The code is shown in the Comments section, and the results are

$$\mathrm{Re}_\delta = 24,\ \text{flow is laminar} \qquad\qquad \dot{m} = 0.00136\ \mathrm{kg/s}$$ <

Note that the film condensation rate estimate is nearly 20 times less than for drop-wise condensation.

COMMENTS: The *IHT* code identified in part (b) follows:

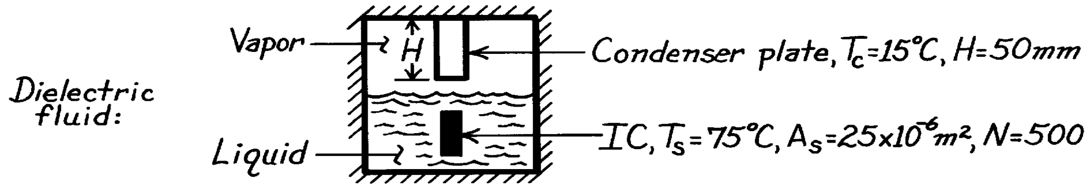
```
/* Results,          Part (b)  - input variables   and rate parameters
NuLbar               Redelta   hLbar     mdot       D       L       Ts      Tsat
0.5093               24.05     6063      0.001362   0.065   0.125   278     300    */

/* Thermophysical properties evaluated at Tf; hfg at Tsat
Prl     Tf           cpl       h'fg      hfg        kl      mul       nul
7.81    289          4185      2.501E6   2.438E6    0.5964  0.001109  1.11E-6*/

// Other input variables required in the correlation
L = 0.125
b = pi * D
D = 0.065

/*  Correlation description: Film condensation (FCO) on a vertical plate (VP). If Redelta<29,
laminar region,  Eq 10.37 . If 31<Redelta<1750, wavy-laminar region,  Eq 10.38 . If Redelta>=1850,
turbulent region,  Eq 10.26, 10.32, 10.33, 10.35, 10.39 . In laminar-wavy and wavy-turbulent transition
regimes, function interpolates between laminar and wavy, and wavy and turbulent correlations.  See
Fig 10.15 .  */
NuLbar = NuL_bar_FCO_VP(Redelta,Prl)          // Eq 10.37, 38, 39
NuLbar = hLbar * (nul^2 / g)^(1/3) / kl
g = 9.8                                 // gravitational constant, m/s^2
Ts =  5 + 273                           // surface temperature, K
Tsat = 300                              // saturation temperature, K
// The liquid properties are evaluated at the film temperature, Tf,
Tf= (Ts + Tsat) / 2
// The condensation and heat rates are
q = hLbar * As * (Tsat - Ts)            // Eq 10.32
As = L * b                              // surface Area, m^2
mdot = q / h'fg                                    // Eq 10.33
h'fg = hfg + 0.68 * cpl * (Tsat - Ts)   // Eq 10.26; hfg evaluated at Tsat
// The Reynolds number based upon film thickness is
Redelta = 4 * mdot / (mul * b)          // Eq 10.35

// Water property functions :T dependence, From Table A.6
// Units: T(K), p(bars);
x = 0                                   // Quality (0=sat liquid or 1=sat vapor)
hfg = hfg_T("Water",Tsat)               // Heat of vaporization, J/kg; evaluated at Tsat
cpl = cp_Tx("Water",Tf,x)               // Specific heat, J/kg·K
mul = mu_Tx("Water",Tf,x)               // Viscosity, N·s/m^2
nul = nu_Tx("Water",Tf,x)               // Kinematic viscosity, m^2/s
kl = k_Tx("Water",Tf,x)                 // Thermal conductivity, W/m·K
Prl = Pr_Tx("Water",Tf,x)               // Prandtl number
```

PROBLEM 10.67

KNOWN: Surface temperature and area of integrated circuits submerged in a dielectric fluid of prescribed properties. Height and temperature of condenser plates.

FIND: (a) Heat dissipation by an integrated circuit, (b) Condenser surface area needed to balance heat load.

SCHEMATIC:



ASSUMPTIONS: (1) Nucleate pool boiling in liquid, (2) Laminar film condensation of vapor, (3) Negligible heat loss to surroundings.

PROPERTIES: Dielectric fluid (given, T_{sat} = 50°C): r_ℓ = 1700kg/m^3, $c_{p,\ell}$ = 1005J/kg·K, m_ℓ = 6.80 × 10^{-4} kg/s·m, k_ℓ = 0.062W/m·K, Pr_ℓ = 11, σ = 0.013 kg/s^2, h_{fg} = 1.05 × 10^5 J/kg, $C_{s,f}$ = 0.004, n = 1.7.

ANALYSIS: (a) For nucleate pool boiling,

$$q_s'' = m_\ell h_{fg}\left[\frac{g\left(r_\ell - r_v\right)}{s}\right]^{1/2}\left(\frac{c_{p,\ell}\Delta T_e}{C_{s,f}h_{fg}\,Pr_\ell^n}\right)^3 \approx 6.8\times10^{-4}\,\mathrm{kg/s\cdot m}\left(1.05\times10^5\,\mathrm{J/kg}\right)$$

$$\times\left[\frac{9.8\,\mathrm{m/s^2}\times1700\,\mathrm{kg/m^3}}{0.013\,\mathrm{kg/s^2}}\right]^{1/2}\left(\frac{1005\,\mathrm{J/kg\cdot K}\times25\mathrm{K}}{0.004\times1.05\times10^5\ \mathrm{J/kg}\times11^{1.7}}\right)^3 = 84{,}530\,\mathrm{W/m^2}$$

$$q_s = A_s q_s'' = 84{,}530\,\mathrm{W/m^2}\times25\times10^{-6}\ \mathrm{m^2} = 2.11\mathrm{W}. \qquad <$$

(b) For laminar film condensation on a vertical surface, $\overline{Nu}_L = 0.943\left[\dfrac{g r_\ell\left(r_\ell - r_v\right)h_{fg}'L^3}{m_\ell k_\ell\left(T_{sat}-T_s\right)}\right]^{1/4}$

$$h_{fg}' = h_{fg}\left(1+0.68\frac{c_{p,\ell}\Delta T}{h_{fg}}\right) = 1.05\times10^5 + 0.68\left(1005\,\mathrm{J/kg\cdot K}\times35\mathrm{K}\right) = 1.29\times10^5\ \mathrm{J/kg}$$

$$\overline{Nu}_L \approx 0.943\left[\frac{9.8\,\mathrm{m/s^2}\left(1700\,\mathrm{kg/m^3}\right)^2 1.29\times10^5\ \mathrm{J/kg}\left(0.05\ \mathrm{m}\right)^3}{6.8\times10^{-4}\ \mathrm{kg/s\cdot m}\left(0.062\ \mathrm{W/m\cdot K}\right)\left(35\mathrm{K}\right)}\right]^{1/4} = 703$$

$$\overline{h}_L = \left(k_\ell/L\right)\overline{Nu}_L = \left(0.062\ \mathrm{W/m\cdot K}/0.05\mathrm{m}\right)703 = 872\ \mathrm{W/m^2\cdot K}$$

$$q_c = \overline{h}_L A_c\left(T_{sat}-T_c\right) = 872\ \mathrm{W/m^2\cdot K}\left(35\mathrm{K}\right)A_c = 30{,}500 A_c\left(\mathrm{m^2}\right)$$

To balance the heat load, $q_c = Nq_s$. Hence

$$A_c = \frac{500\times2.11\ \mathrm{W}}{30{,}500\ \mathrm{W/m^2}} = 0.0346\ \mathrm{m^2} \qquad <$$

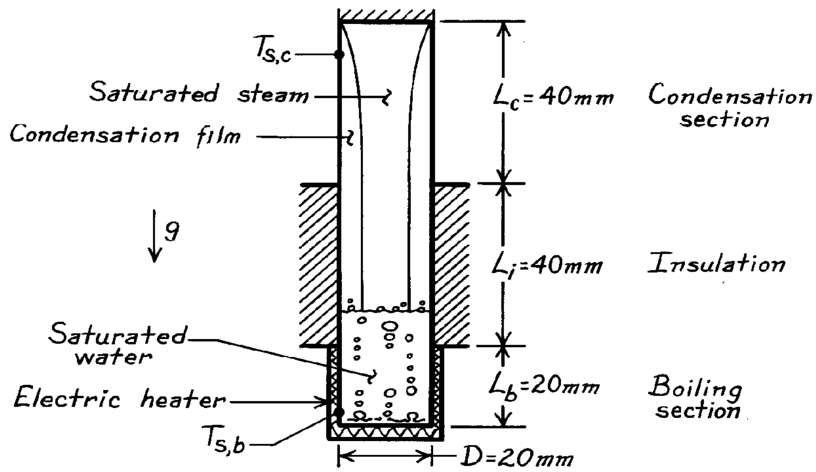
COMMENTS: (1) With A_c = 0.0346m^2 and H = 0.05m, the total condenser width is W = A_c/H = 692mm. (2) With $\dot{m}_c/b = \Gamma = q_c/h_{fg}'W = 1055\mathrm{W}/1.29\times10^5\mathrm{J/kg}\times0.692\mathrm{m} = 0.0118\,\mathrm{kg/s\cdot m}$, $Re_d = 4\Gamma/m_\ell$ = 4(0.0118kg/s·m)/6.8 × 10^{-4} kg/s·m = 69.4. Hence condensate film is in the laminar-wavy regime, and a more accurate estimate of A_c would require iteration.

PROBLEM 10.68

KNOWN: Thin-walled thermosyphon. Absorbs heat by boiling saturated water at atmospheric pressure on boiling section L_b. Rejects heat by condensing vapor into a thick film which falls length of condensation section L_c back into boiling section.

FIND: (a) Mean surface temperature, Ts,b, of the boiling surface if nucleate boiling flux is 30% critical flux, (b) Mean surface temperature, Ts,c of condensation section, and (c) Total condensation flow rate, $\dot{m}$, in thermosyphon. Explain how to determine whether film is laminar, wavy-laminar or turbulent.

SCHEMATIC:



ASSUMPTIONS: (1) Laminar film condensation occurs in condensation section which approximates a vertical plate, (2) Boiling and condensing section are separated by insulated length L_i, (3) Top surface of condensation section is insulated, (4) For condensation, liquid properties evaluated at T_f = 90°C.

PROPERTIES: *Table A-6*, Saturated water (100°C): $r_\ell = 1/v_f = 957.9\ \text{kg/m}^3$, $c_{p,\ell}$ = 4217 J/kg·K, $m_\ell = 279\times10^{-6}\ \text{N}\cdot\text{s/m}^2$, $\text{Pr}_\ell = 1.76$, h_{fg} = 2257 kJ/kg, $\sigma = 58.9\times10^{-3}$ N/m; Saturated vapor (100°C): $\rho_v = 1/v_g = 0.5955\ \text{kg/m}^3$; Saturated water (90°C): $r_\ell = 1/v_f = 964.9\text{kg/m}^3$, $c_{p,\ell}$ = 4207 J/kg·K, $m_\ell = 313\times10^{-6}$ N·s/m^2, $k_\ell = 0.676$ W/m·K.

ANALYSIS: (a) The heat flux for the boiling section is 30% the critical heat flux which at atmospheric pressure is

$$q''_{s,b} = 0.30 q''_{max} = 0.30\times1.26\times10^6\ \text{W/m}^2 = 3.78\times10^5\ \text{W/m}^2.$$

Using the Rohsenow correlation for nucleate boiling with T_{sat} = 100°C and typical values for the surface of $C_{s,f}$ = 0.0130 and n = 1.0, find

$$q''_{s,b} = m_\ell h_{fg}\left[\frac{g(r_\ell - r_v)}{s}\right]^{1/2}\left(\frac{c_{p,\ell}(T_{s,b}-T_{sat})}{C_{s,f}h_{fg}\text{Pr}_\ell^n}\right)^3$$

$$3.78\times10^5\ \text{W/m}^2 = 279\times10^{-6}\ \text{N}\cdot\text{s/m}^2 \times 2257\times10^3\ \text{J/kg}\times$$

$$\left[\frac{9.8\text{m/s}^2(957.9-0.5955)\text{kg/m}^3}{58.9\times10^{-3}\ \text{N/m}}\right]^{1/2}\left(\frac{4217\ \text{J/kg}\cdot\text{K}(T_{s,b}-100)}{0.013\times2257\times10^3\ \text{J/kg}1.76^{1.0}}\right)^3$$

Continued …..

$$T_{s,b} = 114.0^\circ C.$$ <

(b) The heat transferred into the boiling section must be rejected by film condensation,

$$q_c = q_b = q''_{s,b}\left[\pi D^2/4 + \pi D L_b\right]$$

$$q_c = 3.78\times 10^5\ \mathrm{W/m^2}\left[\pi\,(0.020\mathrm{m})^2/4 + \pi\,(0.020\mathrm{m})\times 0.020\mathrm{m}\right] = 592\ \mathrm{W}.$$

The mean surface temperature can be determined from the rate equation

$$q_c = \bar{h}_{Lc}\left(\pi D L_c\right)\left(T_{sat} - T_{s,c}\right)$$

where the convection coefficient is determined from Eq. 10.30,

$$\bar{h}_{Lc} = 0.943\left[\frac{g\rho_\ell\left(\rho_\ell - \rho_v\right)k_\ell^3 h'_{fg}}{\mu_\ell\left(T_{sat} - T_{s,c}\right)L_c}\right]^{1/4}$$

$$\bar{h}_{Lc} = 0.943\left[\frac{9.8\,\mathrm{m/s^2}\times 964.9\,\mathrm{kg/m^3}\left(964.9 - 0.5955\right)\mathrm{kg/m^3}\left(0.676\,\mathrm{W/m\cdot K}\right)^3 2257\times 10^3\ \mathrm{J/kg}}{313\times 10^{-6}\ \mathrm{N\cdot s/m^2}\left(100 - T_{s,c}\right)0.040\ \mathrm{m}}\right]^{1/4}$$

where $h'_{fg} = h_{fg}\left\{1 + 0.68 c_{p,\ell}\left(T_{sat} - T_{s,c}\right)/h_{fg}\right\}$

$$h'_{fg} = 2257\times 10^3\ \mathrm{J/kg}\left\{1 + 0.68\times 4207\ \mathrm{J/kg\cdot K}\left(100 - T_{s,c}\right)/2257\times 10^3\ \mathrm{J/kg}\right\} \approx 2257\times 10^3\ \mathrm{J/kg}.$$

Hence, $\bar{h}_{Lc} = 2.517\times 10^4\left(100 - T_{s,c}\right)^{-1/4}$.

Using the rate equation, now find $T_{s,c}$ by trial-and-error,

$$592\ \mathrm{W} = 2.517\times 10^4\left(100 - T_{s,c}\right)^{-1/4}\left(\pi\times 0.020\mathrm{m}\times 0.040\mathrm{m}\right)\left(100 - T_{s,c}\right)\mathrm{K}$$

$$9.358 = \left(100 - T_{s,c}\right)^{0.75}$$

$$T_{s,c} = 80.3^\circ C.$$ <

(c) The condensation rate in the condenser section is

$$\dot{m} = q_c/h'_{fg} = 592\mathrm{W}/\left(2257\times 10^3\ \mathrm{J/kg}\right) = 2.623\times 10^{-4}\ \mathrm{kg/s}$$ <

and from Eq. 10.35,

$$Re_d = \frac{4\dot{m}}{\mu_\ell b} = \frac{4\dot{m}}{\mu_\ell\left(\pi D\right)} = \frac{4\times 2.623\times 10^{-4}\ \mathrm{kg/s}}{313\times 10^{-6}\ \mathrm{N\cdot s/m^2}\left(\pi\times 0.020\mathrm{m}\right)} = 53.3.$$

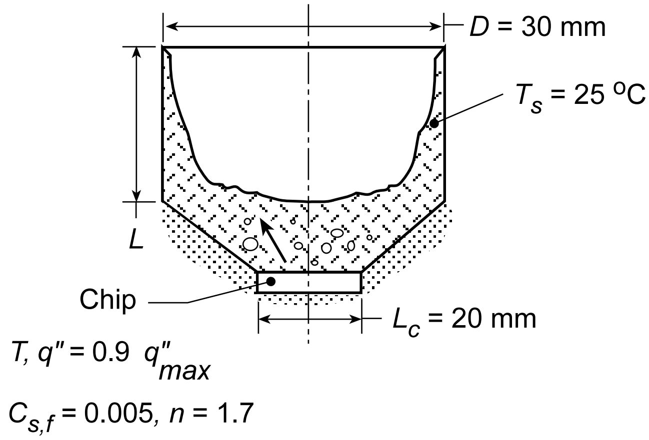
Since $30 < Re_\delta < 1800$, we conclude the film is laminar-wavy.

PROBLEM 10.69

KNOWN: Thermosyphon configuration for cooling a computer chip of prescribed size.

FIND: (a) Chip temperature and total power dissipation when chip operates at 90% of critical heat flux, (b) Required condenser length.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) Saturated liquid/vapor conditions, (3) Negligible heat transfer from bottom of chip.

PROPERTIES: Fluorocarbon (prescribed): T_{sat} = 57°C, $c_{p,\ell}$ = 1100 J/kg·K, h_{fg} = 84,400 J/kg, ρ_ℓ = 1619.2 kg/m^3, ρ_v = 13.4 kg/m^3, σ = 8.1 × 10^{-3} kg/s^2, μ_ℓ = 440 × 10^{-6} kg/m·s, Pr_ℓ = 9.01, k_ℓ = 0.054 W/m·K, $\nu_\ell = \mu_\ell/\rho_\ell$ = 0.272 × 10^{-6} m^2/s.

ANALYSIS: (a) With $q'' = 0.9\ q''_{max}$ and the critical heat flux given by Eq. 10.7, the chip power dissipation is

$$q = 0.9 L_c^2 \times 0.149 h_{fg} \rho_v \left[\frac{\sigma g\left(\rho_\ell - \rho_v\right)}{\rho_v^2}\right]^{1/4}$$

$$q = 0.9\left(0.02\,\text{m}\right)^2 \times 0.149\left(84,400\,\text{J/kg}\right)13.4\,\text{kg/m}^3\left[\frac{0.0081\,\text{kg/s}^2\left(9.8\,\text{m/s}^2\right)\left(1605.8\,\text{kg/m}^3\right)}{\left(13.4\,\text{kg/m}^3\right)^2}\right]^{1/4}$$

$$q_c = 0.9\left(4\times10^{-4}\,\text{m}^2\right)1.55\times10^5\,\text{W/m}^2 = 55.7\,\text{W}$$ <

With operation at q'' = 1.40 × 10^5 W/m^2 in the nucleate boiling region, Eq. 10.5 yields

$$T = T_{sat} + \frac{C_{s,f} h_{fg} Pr_\ell^n}{c_{p,\ell}}\left(\frac{q''}{\mu_\ell h_{fg}}\right)^{1/3}\left[\frac{\sigma}{g\left(\rho_\ell - \rho_v\right)}\right]^{1/6}$$

$$T = 57^\circ\text{C} + \frac{0.005\left(84,400\,\text{J/kg}\right)\left(9.01\right)^{1.7}}{1100\,\text{J/kg}\cdot\text{K}}\left(\frac{1.40\times10^5\,\text{W/m}^2}{4.4\times10^{-4}\,\text{kg/m}\cdot\text{s}\times84,400\,\text{J/kg}}\right)^{1/3}\left[\frac{0.0081\,\text{kg/s}^2}{9.8\,\text{m/s}^2\left(1605.8\,\text{kg/m}^3\right)}\right]^{1/6}$$

Continued...

$$T = 57^\circ C + 22.4^\circ C = 79.4^\circ C$$ <

(b) The power dissipated by the chip must be balanced by the rate of heat transfer from the condensing section. Hence, with $A = \pi DL$, Eq. 10.32 yields the requirement that

$$\bar{h}_L L = \frac{q}{\pi D\left(T_{sat} - T_s\right)} = \frac{55.7\,W}{\pi(0.03\,m)\left(32^\circ C\right)} = 18.5\,W/m\cdot K$$

To determine $\bar{h}_L$, we combine Eqs. 10.33 and 10.35 to obtain $Re_\delta = 4q/\mu_\ell b h'_{fg}$, where $b = \pi D = 0.0942$ m and $h'_{fg} = h_{fg} + 0.68 c_{p,1}\left(T_{sat} - T_s\right) = 84{,}400\,J/kg + 0.68(1100\,J/kg\cdot K)32^\circ C$ = 108,300 J/kg. Hence, Re_δ = 4(55.7 W)/4.4 $\times 10^{-4}$ kg/m·s(0.0942 m)108,300 J/kg = 49.6 and the condensate film is in the laminar-wavy region. Hence, from Eq. 10.38

$$\bar{h}_L = \frac{k_\ell}{\left(\nu_\ell^2/g\right)^{1/3}} \frac{Re_\delta}{1.08 Re_\delta^{1.22} - 5.2} = \frac{0.054\,W/m\cdot K \times 0.409}{\left[\left(0.272\times10^{-6}\,m^2/s\right)^2 \big/ 9.8\,m/s^2\right]^{1/3}} = 1130\,W/m^2\cdot K$$

in which case,

$$L = \frac{18.5\,W/m\cdot K}{1130\,W/m^2\cdot K} = 0.0164\,m = 16.4\,mm$$ <

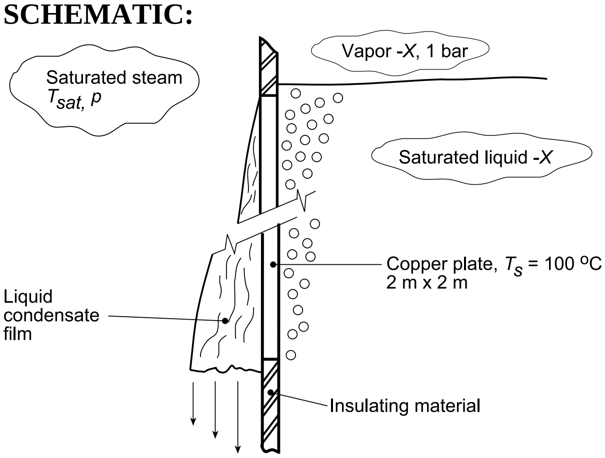
COMMENTS: The chip operating temperature (T = 79.4°C) is not excessive, and the proposed scheme provides a compact means of cooling high performance chips.

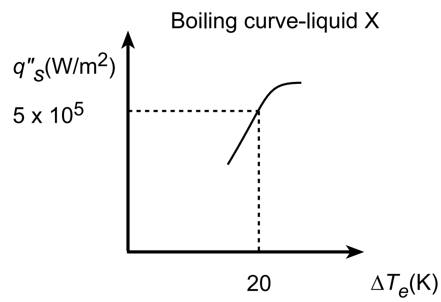
PROBLEM 10.70

KNOWN: Copper plate, 2m × 2m, in a condenser-boiler section maintained at $T_s = 100°C$ separates condensing saturated steam and nucleate-pool boiling of saturated liquid X.

FIND: (a) Rates of evaporation and condensation (kg/s) for the two fluids and (b) Saturation temperature T_{sat} and pressure p for the steam, assuming that film condensation occurs.

SCHEMATIC:





ASSUMPTIONS: (1) Steady-state conditions, (2) Isothermal copper plate.

PROPERTIES: *Fluid-X* (Given, 1 atm): $T_{sat} = 80°C$, $h_{fg} = 700$ kJ/kg, portion of boiling curve shown above for operating condition, $\Delta T_e = T_s - T_{sat} = (100 - 80)°C = 20°C$, $q_s'' = 5 \times 10^4\ W/m^2$; *Table A.4*, Water (saturated, 1 atm): $T_{sat} = 100°C$, $h_{fg} = 2.25 \times 10^6$ J/kg; Water (saturated, T_{sat}): as required in part (b); Water (saturated, $T_f = (T_{sat} + T_s)/2$): as required in part (b).

ANALYSIS: (a) For fluid-X, with $\Delta T_e = T_s - T_{sat} = (100 - 80)°C = 20$ K, the heat flux from the boiling curve is

$$q_s'' = 50,000\,W/m^2$$

and the heat rate from the copper plate section into liquid-X is

$$q_s = q_s'' \times A_s = 50,000\,W/m^2 \times (2\times 2)m^2 = 200,000\,W$$

From an energy balance around liquid-X, the evaporation rate for fluid-X is

$$\dot{m}_X = q_s/h_{fg,X} = 200,000\,W/700,000\,J/kg = 0.286\,kg/s$$ <

The heat rate into the copper plate section from the steam is $q_s = 200,000$ W, and from an energy balance around the condensate film, the condensation rate for steam (w)

$$\dot{m}_w = q_s/h'_{fg,w} = 200,000\,W/2.25\times 10^6\,J/kg = 0.0889\,kg/s$$

where we are assuming that $T_{sat,w}$ is only a few degrees above T_s so that $h'_{fg'} \approx h_{fg}$.

(b) The condensation heat rate, Eq. 10.32 can be expressed as

$$q_s = \bar{h}_L A_s \left(T_{sat} - T_s\right)$$

and assuming laminar film condensation, Eq. 10.30,

$$\bar{h}_L = 0.943\left[\frac{g\rho_\ell\left(\rho_\ell - \rho_v\right)k_\ell^3 h'_{fg}}{\mu_\ell k_\ell\left(T_{sat} - T_s\right)}\right]^{1/4}$$

Continued...

Recognize that with q_s , A_s and T_s known, this relation can be used to determine T_{sat} , and from the steam table, the corresponding p_{sat} can be found. The vapor properties (v) are evaluated at T_{sat} while the liquid properties (ℓ) are evaluated at the film temperature $T_f = (T_{sat} + T_s)/2$. An iterative solution is required, beginning by assuming a value for T_{sat} , evaluate properties and check whether the rate equation returns the assumed value for T_{sat} . Using the *IHT Correlations Tool, Film Condensation, Vertical Plate* for the *laminar* region, the results are

$$T_{sat} = 381.7\,\text{K} \qquad p_{sat} = 1.367\,\text{bar}$$

for which $Re_\delta = 661$, so that the flow is wavy-laminar, not laminar. Repeating the analysis but with the *IHT Tool* for the *laminar*, *wavy-laminar*, *turbulent* regions, the results with $Re_\delta = 652$ are

$$T_{sat} = 379.6\,\text{K} \qquad P_{sat} = 1.27\,\text{bar}$$ <

COMMENTS: A copy of the IHT model for determining T_{sat} and p_{sat} for part (b) is shown below.

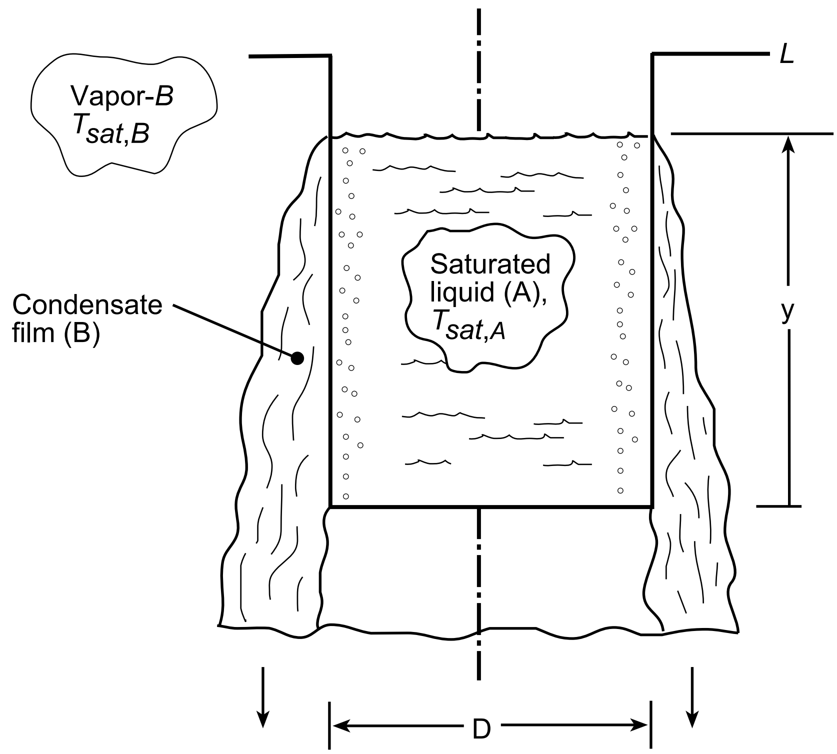
```
// Correlations Tool -
//Film Condensation, Vertical Plate, laminar, wavy-laminar, turbulent regions
NuLbar = NuL_bar_FCO_VP(Redelta,Prl)         // Eq 10.37, 38, 39
NuLbar = hLbar * (nul^2 / g)^(1/3) / kl
g = 9.8                                     // Gravitational constant, m/s^2
Ts = 100 + 273                              // Surface temperature, K
Tsat =     380                              // Saturation temperature, K; explore over range to match q
// The liquid properties are evaluated at the film temperature, Tf,
Tf = Tfluid_avg(Ts,Tsat)
// The condensation and heat rates are
q = hLbar * As * (Tsat - Ts)                // Eq 10.32
As = L * b     // Surface Area, m^2
mdot = q / h'fg                             // Eq 10.33
h'fg = hfg + 0.68 * cpl * (Tsat - Ts)       // Eq 10.26
// The Reynolds number based upon film thickness is
Redelta = 4 * mdot / (mul * b)              // Eq 10.35
/*  Correlation description: Film condensation (FCO) on a vertical plate (VP). If Redelta<29, laminar
region,  Eq 10.37 . If 31<Redelta<1750, wavy-laminar region,  Eq 10.38 . If Redelta>=1850, turbulent
region,  Eq 10.22, 10.32, 10.33, 10.35, 10.39 . In laminar-wavy and wavy-turbulent transition regimes,
function interpolates between laminar and wavy, and wavy and turbulent correlations.  See Fig 10.15 .  */
// Assigned Variables:
L = 2                                       // Plate height, m
b = 2                                       // Plate width, m
//q = 200000                                // Heat rate, W; required heat rate for suitable Tsat
// Properties Tool - Water:
// Water property functions :T dependence, From Table A.6
// Units: T(K), p(bars);
xl =  0                                     // Quality (0=sat liquid or 1=sat vapor)
pl = psat_T("Water", Tf)                    // Saturation pressure, bar
vl = v_Tx("Water",Tf,xl)                    // Specific volume, m^3/kg
rhol = rho_Tx("Water",Tf,xl)                // Density, kg/m^3
cpl = cp_Tx("Water",Tf,xl)                  // Specific heat, J/kg·K
mul = mu_Tx("Water",Tf,xl)                  // Viscosity, N·s/m^2
nul = nu_Tx("Water",Tf,xl)                  // Kinematic viscosity, m^2/s
kl = k_Tx("Water",Tf,xl)                    // Thermal conductivity, W/m·K
Prl = Pr_Tx("Water",Tf,xl)                  // Prandtl number
// Water property functions :T dependence, From Table A.6
// Units: T(K), p(bars);
xv =  1                                     // Quality (0=sat liquid or 1=sat vapor)
pv = psat_T("Water", Tsat)                  // Saturation pressure, bar
vv = v_Tx("Water",Tsat,xv)                  // Specific volume, m^3/kg
rhov = rho_Tx("Water",Tsat,xv)              // Density, kg/m^3
hfg = hfg_T("Water",Tsat)                   // Heat of vaporization, J/kg
cpv = cp_Tx("Water",Tsat,xv)                // Specific heat, J/kg·K
muv = mu_Tx("Water",Tsat,xv)                // Viscosity, N·s/m^2
nuv = nu_Tx("Water",Tsat,xv)                // Kinematic viscosity, m^2/s
kv = k_Tx("Water",Tsat,xv)                  // Thermal conductivity, W/m·K
Prv = Pr_Tx("Water",Tsat,xv)                // Prandtl number
```

PROBLEM 10.71

KNOWN: Thin-walled container filled with a low boiling point liquid (A) at $T_{sat,A}$. Outer surface of container experiences laminar-film condensation with the vapor of a high-boiling point fluid (B). Laminar film extends from the location of the liquid-A free surface. The heat flux for nucleate pool boiling in liquid-A along the container wall is given as $q''_{npb} = C(T_s - T_{sat})^3$, where C is a known empirical constant.

FIND: (a) Expression for the average temperature of the container wall, T_s; assume that the properties of fluids A and B are known; (b) Heat rate supplied to liquid-A, and (c) Time required to evaporate all the liquid-A in the container, assuming that initially the container is filled, y = L.

SCHEMATIC:



ASSUMPTIONS: (1) Nucleate pool boiling occurs on the inner surface of the container with liquid-A, (2) Laminar film condensation occurs on the outer surface of the container with fluid-B over the liquid-A free surface, y, and (3) Negligible wall thermal resistance.

ANALYSIS: (a) Perform an energy balance on the control surface about the container wall along locations experiencing boiling (A) and condensation (B) as shown in the schematic above.

$$\dot{E}''_{in} - \dot{E}''_{out} = 0 \qquad (1)$$

$$q''_{cond} - q''_{npb} = 0 \qquad (2)$$

$$\bar{h}_y (\pi D y)\left(T_{sat,B} - T_s\right) - (\pi D y) C\left(T_s^3 - T_{sat,A}\right) = 0$$

$$\bar{h}_y \left(T_{sat,B} - T_s\right) = C\left(T_s - T_{sat,A}\right)^3 \qquad (3) \; <$$

where $\bar{h}_y$ is the average convection coefficient for laminar film condensation over the surface length 0 to y. From Eq. 10.30 and 10.26,

Continued...

PROBLEM 10.71 (Cont.)

$$\bar{h}_y = 0.943\left[\frac{g\rho_\ell(\rho_\ell - \rho_v)k_\ell^3 h'_{fg}}{\mu_\ell(T_{sat} - T_s)y}\right]_B^{1/4} \tag{3}$$

$$h'_{fg} = h_{fg,B} + 0.68c_{p,B}\left(T_{sat,B} - T_s\right) \tag{4}$$

where the properties are for fluid-B.

(b) The heat flux supplied to liquid-A is, from Eq. (2), $q''_{cond} = q''_{npb}$. Since $\bar{h}_y$ is a function of y, T_s and, hence, the heat fluxes will be functions of y, the height of liquid A in the container.

(c) To determine the dry-out time, t_f, begin with an energy balance on the inside of the container (fluid-A). The heat transfer supplied to liquid-A results in an evaporation rate of liquid-A,

$$q''_{npb}(\pi Dy) - \frac{dM}{dt}h_{fg} = 0 \tag{4}$$

where M is the mass of liquid-A in the container,

$$M = \rho_{\ell,A}\left(\pi D^2/4\right)y \tag{5}$$

Substituting Eq. (5) into (4), separating variables and identifying integration limits, find

$$C\left(T_s - T_{sat,A}\right)^3(\pi Dy) = \frac{d}{dt}\left[\rho_{\ell,A}\left(\pi D^2/4\right)y\right]h_{fg}$$

$$\int_0^{t_f} dt = t_f = \frac{\rho_{\ell,A}\left(\pi D^2/4\right)h_{fg}}{C\pi D}\int_L^0 \frac{dy}{\left(T_s - T_{sat,A}\right)^3 y} \tag{6}$$

The definite integral could be numerically evaluated using values for $T_s(y)$ obtained by solving Eq. (3).