THE UNIVERSITY OF BRITISH COLUMBIA

Department of Electrical and Computer Engineering

EECE 373: Electromechanical Energy Conversion and Transmission

Final Exam
December 11, 2004

Magnetic Circuits, Transformers, DC Machines, Induction Machines, and Synchronous Machines

Surname:______________________________

First Name:______________________________

Student ID:______________________________

Signature:______________________________

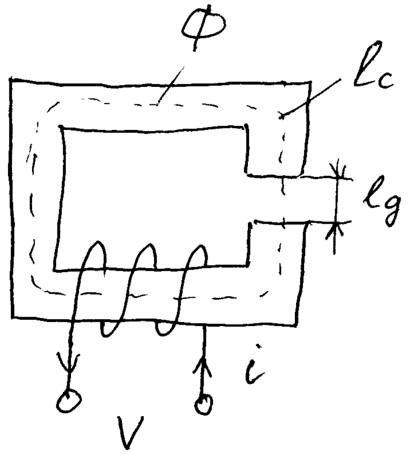
Solution

Problem	Points	Max.
1		20
2		20
3		20
4		20
5		20
Total		100

- **Close notes and books.**
- You are allowed to have only a **calculator** and a **pen/pencil**.
- Show you work including **derivations**, **comments**, **assumptions**, and **units** wherever appropriate.
- Use back side of each page or ask for additional pages if you need extra space to write your answers.
- Exams suspected of cheating and/or turned in late will not be marked – **failed exam**.
- Cheating may result in your suspension and/or withdrawal from the university
- You have **90** minutes to answer the following questions:

Problem 1 (20pts)

Consider the magnetic system shown below. The core and air-gap have reluctances are $\Re_c = 0.5\times10^6\, At/Wb$ and $\Re_g = 4.5\times10^6\, At/Wb$, respectively. The cross-sectional area of the core and air-gap is $A_c = A_g = 2.3cm^2$. Neglect flux leakage, fringing, and core losses. The dc resistance of the coil is $3\,\Omega$. The coil is connected to a 60Hz ac source which provides $V = 25V$ (rms) and the resulted current is $I = 5A$ (rms).

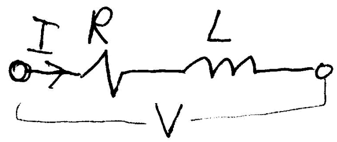


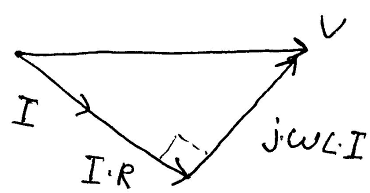
(a) (5pts) Draw an equivalent **electric** circuit and sketch the corresponding qualitative phasor diagram

(b) (5pts) Calculate the inductance L and the number of turns N (round it to the closest integer)

(c) (5pts) Calculate the flux Φ (rms value)

(d) (5pts) Calculate the flux density in the air-gap B_g (rms value)





b) $V = 25\ V$; $I = 5\ A$; $R = 3\,\Omega$

$Z = V/I = 25/5 = 5\,\Omega$

$X = \sqrt{Z^2 - R^2} = \sqrt{25-9} = 4\,\Omega$

$L = X/\omega = \frac{4}{60\cdot 2\cdot\pi} = \boxed{0.0106\ H}$

$L = \frac{N^2}{\mathcal{R}_g + \mathcal{R}_c}$

$N = \sqrt{L\cdot(\mathcal{R}_g + \mathcal{R}_c)} \approx \boxed{230}$

$N\cdot I = F = \Phi\cdot(\mathcal{R}_g + \mathcal{R}_c)$

c) $\Phi = \frac{F}{\mathcal{R}_g + \mathcal{R}_c} = \frac{N\cdot I}{\mathcal{R}_g + \mathcal{R}_c} = \frac{230\cdot 5}{5e^6} = \boxed{2.3e{-4}\ Wb}$

d) $\Phi = B\cdot A \Rightarrow B = \frac{\Phi}{A_c} = \frac{2.3e^{-4}}{2.3e^{-4}} = \boxed{1\ T}$

Problem 2 (20pts)

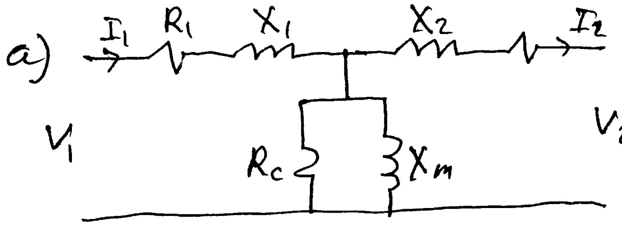
Consider a single-phase 120V, 60Hz transformer with the following parameters: $R_1 = 1\,\Omega$, $X_1 = 2\,\Omega$, $X_m = 100\,\Omega$, $R_c = 200\,\Omega$, $R_2 = 1\,\Omega$, and $X_2 = 2\,\Omega$, respectively. Assume that both windings have the same number of turns. Assume **T-equivalent circuit** in the following questions:

(a) (5pts) Sketch the equivalent circuits and label its parameters and variables (just the symbols)

(b) (8pts) Assume an Open-Circuit Test wherein the primary winding is supplied from a 120V ac source. Calculate primary current $I_{1,oc}$, power real P_{oc}, and secondary voltage $V_{2,oc}$

(c) (7pts) Assume a Short-Circuit Test. As you know, the Short-Circuit Test should be performed at reduced voltage in order to avoid very high currents. Assume that a maximum allowable primary current is $I_{1,\max} = I_{1,sc} = 5A$, calculate the required primary voltage $V_{1,sc}$, and real power P_{sc}

a)



$$Z_1 = R_1 + jX_1 = 1 + j2$$

$$Z_2 = R_2 + jX_2 = 1 + j2$$

$$Z_m = \frac{jX_m \cdot R_c}{jX_m + R_c} = 40 + j80$$

b) OCT

$$I_{1,oc} = \frac{V_1}{Z_1 + Z_m} = \frac{120}{41 + j82} = 0.5854 - j\cdot 1.17 = 1.31\ A(rms)$$

$$I_{1,oc} = 1.31 \angle -63.435^\circ \ ; \quad PF = \cos(63.435^\circ) = 0.4472$$

$$V_{2,oc} = I_{1,oc} \cdot Z_m = 117.07\ V(rms)$$

$$P_{oc} = V_1 \cdot I_{1,oc} \cdot \cos\varphi = V_1 \cdot \mathrm{real}(I_{1,oc}) = 120 \cdot 0.5854 = 70.2\,W$$

c) SCT

$$Z_{sc} = Z_1 + \frac{Z_2 \cdot Z_m}{Z_2 + Z_m} = 1.976 + j\cdot 3.951$$

$$|Z_{sc}| = 4.4176\ \Omega \ ; \quad I_{1,sc} = 5\ A(rms)$$

$$V_{1,sc} = |Z_{sc}| \cdot I_{1,sc} = 22.09\ V(rms)$$

$$I_{1,sc} = \frac{V_{1,sc}}{Z_{sc}} = 2.2361 - j\cdot 4.472 = 5\ A(rms)$$

$$P_{sc} = V_{1,sc} \cdot I_{1,sc} \cdot \cos\varphi = V_{1,sc} \cdot \mathrm{real}(I_{1,sc}) = 22.09 \cdot 2.2361 =$$
$$= 49.4\,W$$

$$I_{2,sc} = \frac{V_{1,sc} - I_{1,sc} \cdot Z_1}{Z_2} = 2.1815 - j4.363 = 4.87\ A(rms)$$

Problem 3 (20pts)

Consider a 120V **Shunt DC Motor** with the following parameters $R_a = 0.5\,\Omega$, $R_f = 200\,\Omega$. When the machine is connected to a 120V dc source it draws the armature current $I_a = 10A$ and the shaft speed is $n = 1800$ rpm CCW under a given mechanical load. The friction losses are $P_{fric} = 50W$

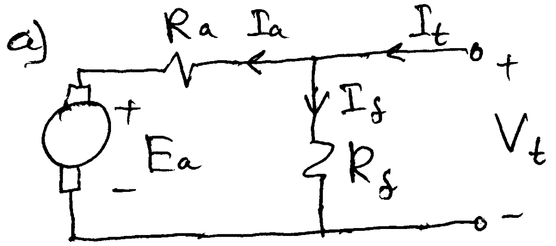
(a) (5pts) Sketch the equivalent steady-state electric circuit and label all parameters and variables

(b) (5pts) Determine the generated voltage E_a

(c) (5pts) Determine the load torque T_m and its direction (circle CW or CCW)

(d) (5pts) Determine the efficiency of the motor η

a)



b) $E_a = V_t - R_a \cdot I_a =$

$= 120 - 0.5 \cdot 10 = \boxed{115V}$

c) $\omega = n \cdot \frac{\pi}{30} = 188.496$ r/s

$P_e = E_a \cdot I_a = 1150$ W

$P_m = P_e - P_{fric} = 1150 - 50 = 1100$ W

$P_m = T_m \cdot \omega \Rightarrow T_m = \frac{P_m}{\omega} = \frac{1100}{188.496} = \boxed{5.84\ N\cdot m}$

$\boxed{CW}$

d) $\eta = \frac{P_{out}}{P_{in}}$

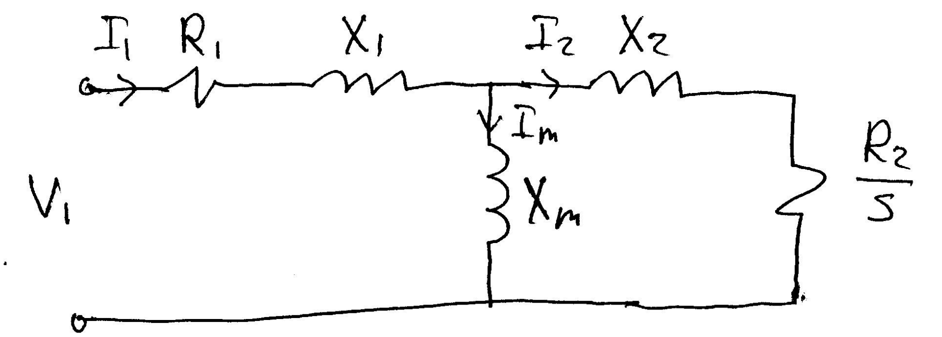
$P_{out} = P_m$; $P_{in} = V_t \cdot I_t = V_t \cdot \left(I_a + \frac{V_t}{R_f}\right) = 120 \cdot (10 + 0.6)$

$= 1272$ W

$\eta = \frac{1100}{1272} = 0.865 = \underline{86.5\%}$

Problem 4 (20pts)

(a) (10pts) Sketch a per-phase equivalent circuit of an Induction Machine (IEEE Recommended, most commonly used) and label all elements and variables.

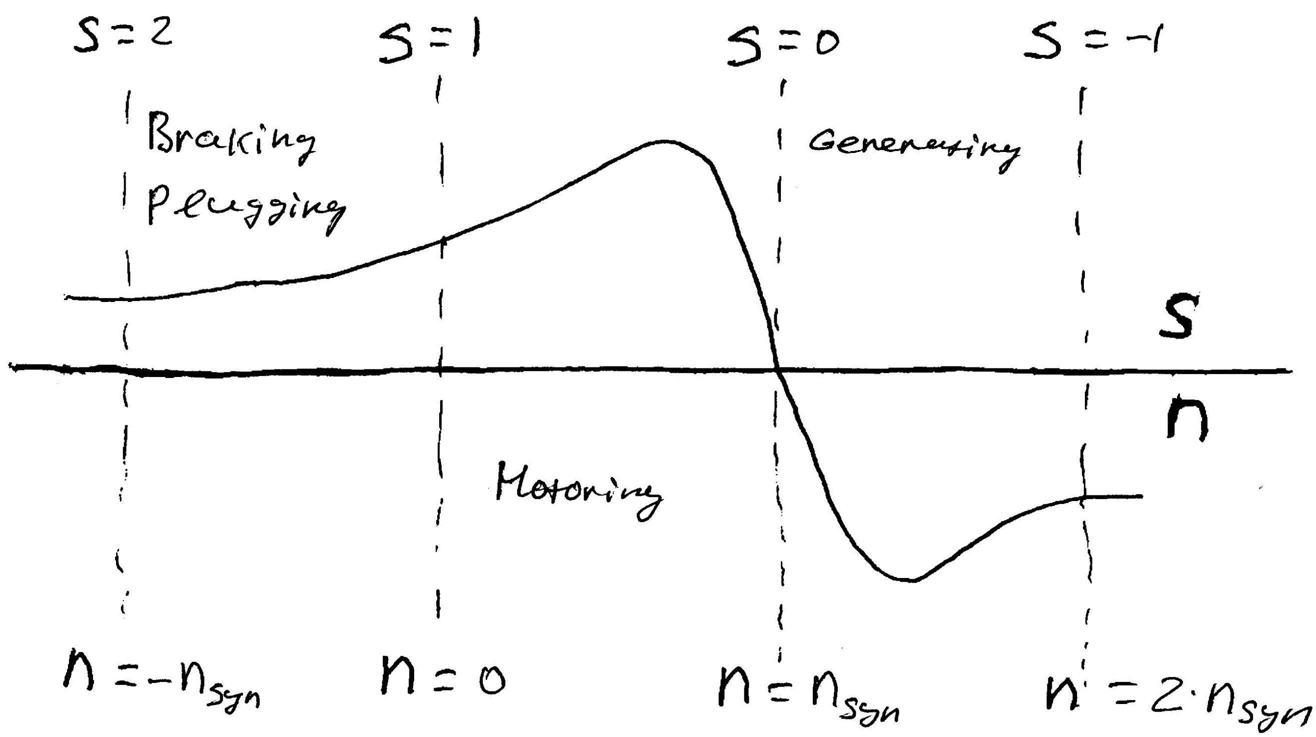


(b) (10pts) Consider a typical Torque-Speed Characteristic of an Induction machine shown below.

a. (4pts) Identify and label the regions corresponding to the three modes of operation

b. (3pts) Label the slip values (assume integers)

c. (3pts) Label the speed values ($\pm n_{syn}$,0, etc.)



Problem 5 (20pts)

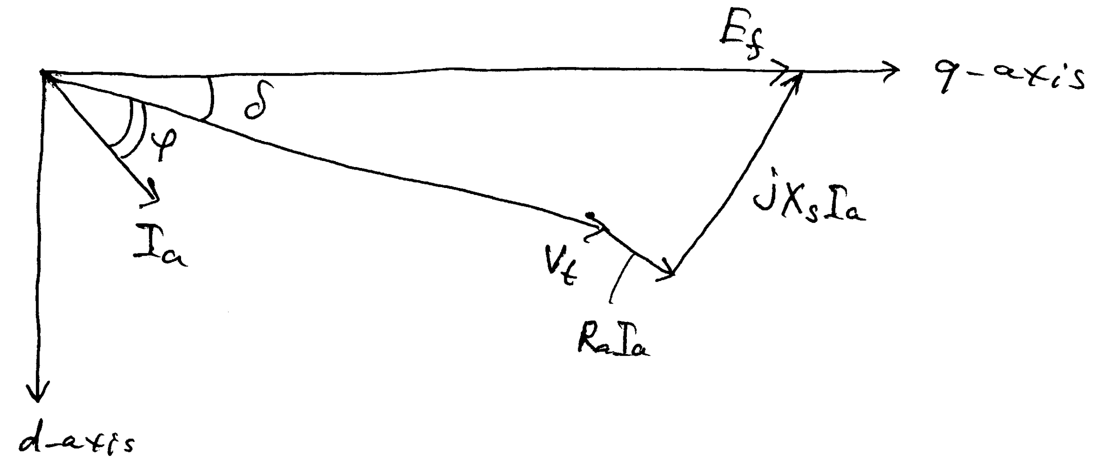
Consider a 3-phase, 60Hz, 208V (line-to-line) 2-pole **Permanent Magnet Round-Rotor Synchronous Generator** with the following parameters: Rated (nominal) output electrical power $S_{3\phi} = 1.2kVA$, per-phase stator resistance and synchronous reactance are $R_a = 1\,\Omega$ and $X_s = 20\,\Omega$, respectively. Assume that a nominal load with 0.7 lagging power factor is connected to the generator terminals.

(a) (6pts) Sketch a phasor diagram and label the q- and d- axes, all relevant angles, and phasors

(b) (7pts) Calculate the induced voltage E_f and the rotor angle δ in degrees

(c) (7pts) Assume mechanical rotational losses $P_{mech\_loss} = 50W$, calculate the efficiency η

a)



b) $E_f = V_t + (R_a + jX_s)I_a$

$$I_a = \frac{S}{\sqrt{3}\cdot V_{Line}} = \frac{1200}{\sqrt{3}\cdot 208} = 3.331\ A(rms)$$

$\varphi = a\cos(PF) = a\cos(0.7) = 45.57°$

$I_a = 3.331\angle -45.57° = 2.3316 - j\cdot 2.3787\ A$

$$E_f = \frac{208}{\sqrt{3}} + (1 + j20)\cdot(2.3316 - j\cdot 2.3787)$$

$= 168.99 + j\,44.253 = 175.66\angle +14.59$

$\delta = 14.59$ degrees

c) $P_{out} = S\cdot PF = 1200\cdot 0.7 = 840\ W$

$P_{in} = P_{out} + P_{mech\_loss} + 3\cdot R_a\cdot I_a^2 = 923.3\ W$

$\eta = \frac{P_{out}}{P_{in}} = 0.91 = 91\%$