UBC - Department of Mechanical Engineering
MECH 375 – Heat Transfer <u>Section: 101</u>
3 Credits / [3,0,1]

INSTRUCTOR: Dr. N. Atabaki, P.Eng.,

Office:
Room 104, Rusty Hut
2275 East Mall, UBC, Vancouver
Phone#: 604 827 4065
E-Mail: natabaki@mail.ubc.ca

OFFICE HOURS: Mondays 10:00 to 11:00 RH 104
or by appointment (e-mail)

T.AS.:
1- Alireza Jalali, alirezaa.jalaali@gmail.com
Office: ICICS X227

2- Amir Maleki Zamenjani,
amir.maleki.zamenjani@gmail.com
Office: Pulp and Paper Center

LECTURES: MECH 375 101 Mon/Wed/Fri 9:00 to 10:00 McLeod 228

TUTORIALS: MECH 375 T1A Wednesdays 10:00 to 11:00 McLeod 202

COURSE WEBSITE: **http://www.vista.ubc.ca/**

PREREQUISITES: Either (a) MECH 222 or (b) one of CHBE 241, PHYS 257 and one of CHBE 251, CIVL 215, MECH 280.

COURSE DESCRIPTION
This course is designed to introduce students to the basic concepts of heat transfer. Topics:

Conduction heat transfer:
- Steady and unsteady heat conduction.
- Fin Theory.

Convection heat transfer:
- Governing equations;
- Dimensionless parameters;
- Analogy between momentum and heat transfer;
- Design correlations for convection heat transfer.

Radiation heat transfer:
- Black- and gray-body radiation;
- Shape factors;
- Enclosure theory

Heat exchangers

COURSE LEARNING OBJECTIVES (CLO)

a. Develop a comprehensive and integrated skill to describe the physical mechanisms responsible for heat transfer

b. Think critically about different aspects that influence heat transfer

c. State/explain the significance of the basic "laws" of heat transfer

d. Build mathematical models for heat transfer problems (simple but still practical problems)

e. Critically examine a heat transfer system

Specific Objectives

At the end of this course, it is expected that students will able to:

- Identify the principal components of a heat transfer system
- Build simple mathematical models for heat transfer problems
- Solve steady-state and transient heat conduction problems
- Calculate thermal resistances
- Identify the importance of insulation in minimizing heat losses
- Calculate the contact thermal resistances between two surfaces
- Apply the fin theory and identify the importance of extended surfaces (fins) in enhancing heat transfer
- Calculate drag coefficient in external flow
- Calculate the rate of convection heat transfer from a surface to a stagnant or moving fluid
- Calculate pressure drop in duct flow
- Calculate the rate of convection heat transfer in duct flow
- Calculate the rate of radiation heat transfer between surfaces
- Design simple heat exchangers

TEXTBOOK

Fundamentals of Heat and Mass Transfer
F.P., Incropera, and D.P., DeWitt, T.L. Bergman, A.S. Lavine, 7th Edition, Wiley, 2011.
(Available in UBC bookstore)

REFERENCE HEAT TRANSFER TEXTBOOKS

1. **Principles of Heat Transfer**
   F., Kreith, and M.S., Bohn, 6th Edition, Brooks/Cole, Pacific Grove, California, 2001.

2. **Heat Transfer**
   J.P. Holman, 10th Edition, McGraw-Hill, 2010

OTHER USEFUL REFERENCES

1. **Principles of Engineering Thermodynamics**
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GRADING POLICY

Suggested Homeworks (0%)
11 Problem sets; these are **recommended**; **NOT** to be turned in for grading.

Quizzes (10%)
Two quizzes are designed (5% each) to be held ***<u>during the tutorial hours</u>***:
1- Quiz 1 **Wednesday, October 3,** 2012.
2- Quiz 2 **Wednesday, November 14,** 2012.

Midterm (30%)
A 50 min midterm exam is designed to be held on **Monday, October 22,** 2012.

Final Exam (60%)
Date and location of the final exam will be determined later.

LIST OF TOPICS	Corresponding Chapters in Textbook	Estimated Length
1) Introduction	**1, 2**	1 week
2) One-Dimensional Steady-State Heat Conduction	**3**	2 weeks
3) Multi-Dimensional Steady-State Heat Conduction	**4**	1 week
4) Transient (Unsteady) Heat Conduction	**5**	2 weeks
5) Convection Heat Transfer	**6 , 7, 8, 9**	3 weeks
6) Radiation Heat Transfer	**12, 13**	2 weeks
7) Heat Exchangers	**11**	2 weeks

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DO THEM ALWAYS

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MECH 375 HEAT TRANSFER – HANDOUT #1

INTRODUCTION TO HEAT TRANSFER

TOPICS

- **What is heat transfer?**
- **How does heat transfer compare with thermodynamics?**
- **Why study heat transfer?**
- **Objectives of the course**
- **Modes of heat transfer**

HEAT TRANSFER: A subject that deals with energy transfer between material bodies solely as a result of temperature differences between them. In particular, it deals with the physical mechanisms responsible for this energy transfer, and the formulation and application of mathematical models for the prediction of the total amount and the rate of this energy transfer.

THERMODYNAMICS (classical)

Concerned with:

- Systems in equilibrium or quasi-equilibrium
- Overall (total) energy transfer from or to a system
- End states of processes
- Conservation of energy
- Direction of energy transfer

Not concerned with:

- Physical mechanisms
- Temperature distributions

HEAT TRANSFER

- Builds on: Mathematics, Thermodynamics, and Fluid Mechanics
- Deals with systems in equilibrium *and* nonequilibrium
- Mechanisms and rate of heat transfer
- Temperature distributions within systems

Why Study heat Transfer?

Power and Process Industries

Transportation

Materials Engineering

Electronics Industry

Environment

Our Existence Depends on it

OBJECTIVES OF THE COURSE

1. Describe the physical mechanisms responsible for heat transfer
2. State / Explain the significance of the basic “laws” of heat transfer
3. Mathematical models of heat transfer
4. Empirical correlations
5. Applications (to simple yet practical problems)

MODES OF HEAT TRANSFER

1. Conduction

• Heat transfer due to intermolecular interactions, or at atomic or subatomic levels

2. Convection [= conduction + advection]

• Advection: overall transport due to motion of a medium [typically a fluid]

3. Radiation

• Energy transport due to electromagnetic waves. [No need for medium]

4. Mixed Mode Heat Transfer

Any combination of the 3

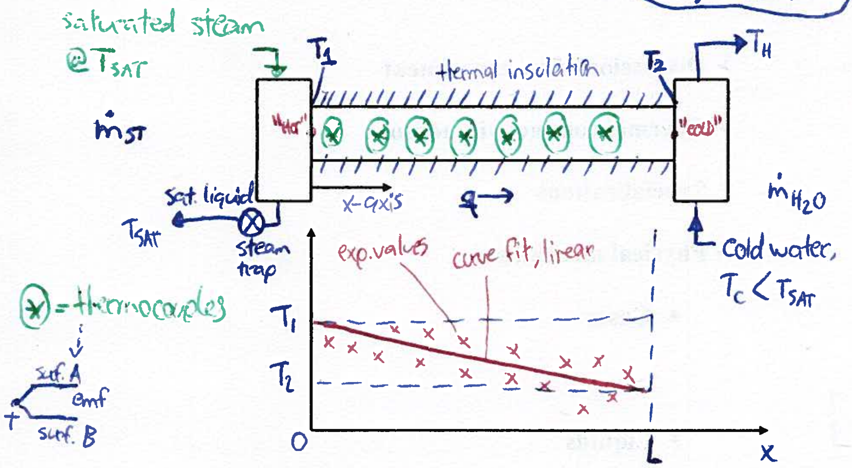
INTRODUCTION TO CONDUCTION HEAT TRANSFER

TOPICS

- Discussion of an experiment
- Thermal conductivity tensor
- Specializations
- Physical mechanisms
  - Gases
  - Liquids
  - Solids

INTRODUCTION TO CONDUCTION HEAT TRANSFER

- **Consider the following experiment:** *Steady state, no change over time



L : Length of the bar [m]

A_{CS} : Area of cross-section of the bar [m^2]

$T_1, T_2, T_C, T_H, T_{SAT}$: Temperatures (see figure) [°C]

$\dot{m}_{H_2O}$: Mass flow rate of the cooling water [kg/s]

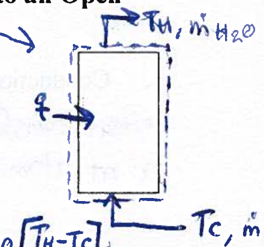
c_{p,H_2O} : Specific heat at constant pressure of the cooling water [J/kg-°C]

q : Total rate of heat transfer through the bar (left to right) [W] → 2nd Law of thermo

*Heat moves from high to low, but it can be reversed at a cost (electricity)

*This problem is about 2nd Law of thermo: Heat moves from high to low.

- **First Law of Thermodynamics Applied to an Open System: Cooling Water Chamber**



Assumptions:

*Heat loss negligible
*Steady state
*ΔKE and ΔPE negligible

Observations:

rate of heat transfer ← $q = \dot{m}_{H_2O}(h_H - h_C) = \dot{m}_{H_2O} c_{p,H_2O} [T_H - T_C] = \dot{m}_{ST} h_{fg}$; h_{fg} is enth. of evap.

1. Heat is transferred through the bar from the hot side to the cold side

2. Temperature distribution inside the bar is essentially one-dimensional and linear in the x direction

"$T = ax + b$"

3. (i) L, A_{CS} (cross section of bar) constant; vary $(T_1 - T_2)$: Then, $q \sim (T_1 - T_2)$ *higher ... means ... q

   (ii) A_{CS}, $(T_1 - T_2)$ constant; vary L: Then, $q \sim \frac{1}{L}$ *more length means more resistance

   (iii) L, $(T_1 - T_2)$ constant; vary A_{CS}: Then, $q \sim A_{CS}$ more cross ar... to move hea...

   (iv) Vary L, $(T_1 - T_2)$, A_{CS}: Then, $q \sim \frac{(T_1 - T_2) A_{CS}}{L}$ combine

Aside (P.K.): $\frac{dT}{dx} = \lim_{\Delta x \to 0} \left[\frac{T_{x+\Delta x} - T_x}{\Delta x} \right]$

In this problem:

(i) $(T_1 - T_2)/L = -\frac{dT}{dx}$ } Note: T vs. x b.c. $\frac{dT}{dx} = \frac{T_2 - T_1}{L}$, essentially

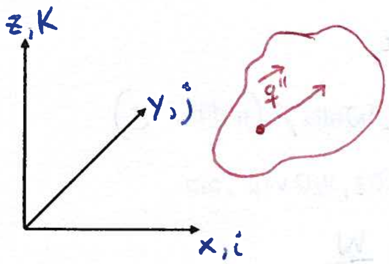
(ii) $q \sim \frac{A_{cs}(T_1 - T_2)}{L}$; or $q / A_{CS} = q''$ $[W/m^2]$

units q is Watts, so q'' is Heat flux in positive x direction

Define a constant of proportionality:

Let $q''_x \triangleq -K\frac{dT}{dx}$; here, k: coefficient of thermal conductivity $\left[\frac{W}{m - °C}\right]$

- **IN GENERAL**



(i) $\bar{q}''$: Heat Flux Vector
It has both direction and magnitude

(ii) $k = f[x, y, z, T, P,$ direction, Material, Phase$]$

(iii) $\vec{\nabla} T = \frac{\partial T}{\partial x}\hat{i} + \frac{\partial T}{\partial y}\hat{j} + \frac{\partial T}{\partial z}\hat{k}$

Thermal Conductivity Tensor:

red mass is:
- 3D
- AnisoTropic (different structure of materials in different directions)
- Heterogeneous

$$\vec{q}'' = -[K][\vec{\nabla} T]$$

$$[K] = \begin{bmatrix} K_{xx} & K_{xy} & K_{xz} \\ K_{yx} & K_{yy} & K_{yz} \\ K_{zx} & K_{zy} & K_{zz} \end{bmatrix}$$

Specializations:

(1) Orthotropic Materials:
- different [K] in different directions of a material
- ex: wood, laminated composite materials

$$\rightarrow [K] = \begin{bmatrix} K_{xx} & 0 & 0 \\ 0 & K_{yy} & 0 \\ 0 & 0 & K_{zz} \end{bmatrix}$$

(2) Isotropic Materials: ~~[illegible]~~
K: independent of direction $\Rightarrow [K] = \begin{bmatrix} K & 0 & 0 \\ 0 & K & 0 \\ 0 & 0 & K \end{bmatrix}$

Thus for a given material and phase: $k = f[x, y, z, T, P]$

In the Cartesian coordinate system:

$$\vec{q} = -[K][\vec{\nabla} T]$$

In general, for isotropic materials:

$$\boxed{\vec{q} = -K\vec{\nabla} T = -K\left[\frac{\partial T}{\partial x}\hat{i} + \frac{\partial T}{\partial y}\hat{j} + \frac{\partial T}{\partial z}\hat{k}\right]}$$

Fourier's Law of Heat Conduction

(3) Homogeneous and isotropic materials:
K is independent of direction and location
Thus, for a given material and phase:

$$K = f[T, P]$$

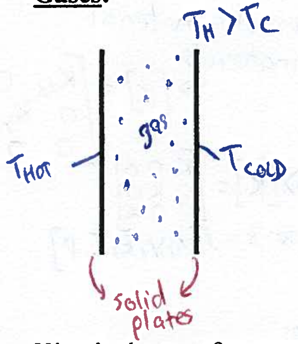
(4) Constant-property material:

$$K = \text{constant}$$

* for most problems we study, K is constant
* If $K = f[T, P]$, look at tables for right value
* In the T & P ranges of interest, K is essentially constant around the mean value.

- **Physical Mechanisms Responsible for Heat Conduction**

Gases:



- Molecules in Random Motion
- Momentum and energy transfer caused by: Molecular Collisions

Kinetic theory of monatomic ideal gases gives:

*Atoms are point mass
*All collisions are elastic $\Rightarrow$ $\boxed{\frac{1}{2} m \overline{v^2} = \frac{3}{2} K_B T_{ABS}}$

where

κ_B : Boltzmann Constant [=1.3803 × 10^{-23} J/K]

m : mass of the atom [Kg]

$\overline{v^2}$: mean square of speed of atoms [(m^2/s^2)]

T_{ABS} : Absolute Temperature [K]

Thus, $\sqrt{\overline{v^2}} = v_{rms} = \sqrt{\dfrac{3\kappa_B T_{ABS}}{m}} \propto \sqrt{\dfrac{T_{ABS}}{m}}$

root mean square velocity of atom

The rate of energy transfer by atomic collision is proportional to the frequency of collisions.

For a monatomic ideal gas:

$$k \propto v_{rms} \propto \sqrt{\frac{T_{ABS}}{m}} \quad ; \quad or \quad k \propto \sqrt{T_{abs}} \quad and \quad k \propto \sqrt{\frac{1}{m}}$$

Rules of Thumb: At room temperature

Light Gases: For example, H_2

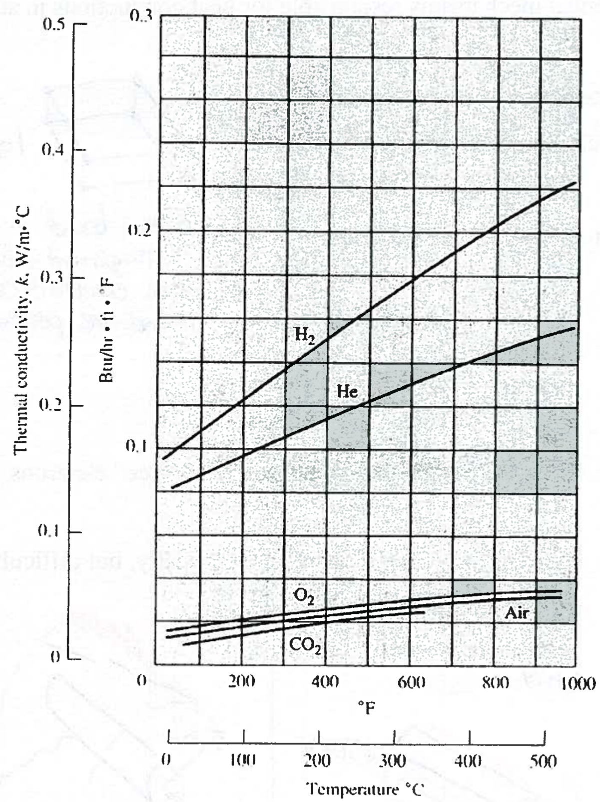
$K \sim 0.1$ Watts / (meter – °C)

Heavy Gases: For example, CO_2, H_2O vapor, air

$K \sim 0.01 \frac{W}{m\text{-}°C}$

For gases, k is essentially invariant with pressure in the range of 0.01 atm to 100 atm.

Figure 1-4 | Thermal conductivities of some typical gases [1 W/m · °C = 0.5779 Btu/h · ft · °F].



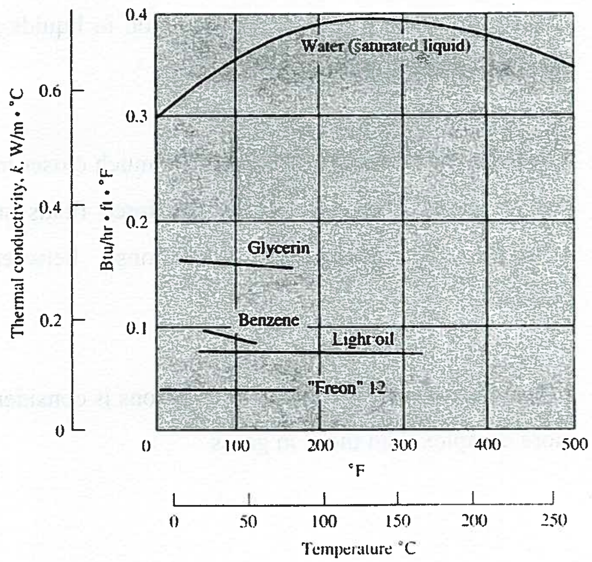
(Figure from: J.P., Holman, 2002)

K_{air} at room temp is about 0.027 W/m-°C, good insulator

Liquids:

- Physical mechanisms of heat conduction in liquids are the same as those in gases
- However, the molecules in liquids are much closer together than in gases; thus, inter-molecular force fields strongly influence the interactions (collisions) between the molecules
- Actual phenomena of molecular collisions is considerably more complex than those in gases

Figure 1-5 | Thermal conductivities of some typical liquids.



(Figure from: J.P., Holman, 2002)

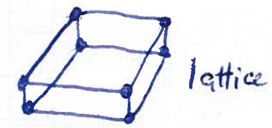
water @ Room Temp, $K_{water} \sim \boxed{0.6\ W/m\text{-}°C}$

Solids:

Physical mechanisms responsible for heat conductions in solids:

- Interaction of molecules/atoms which vibrate about mean positions
  "vibration of lattice"
- Interaction of free electrons

$K = K_{electrons} + K_{lattice}$

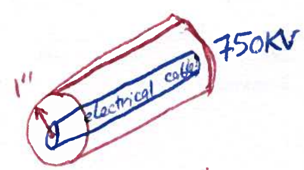


- atoms bound in a periodic arrangement → lattice
- free electrons can move around and are present

Notes:

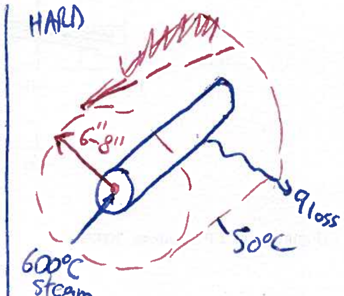
- Electrical conduction is due solely to "free" electrons
- It is relatively easy to insulate electrically, but difficult to insulate thermally.

EASY



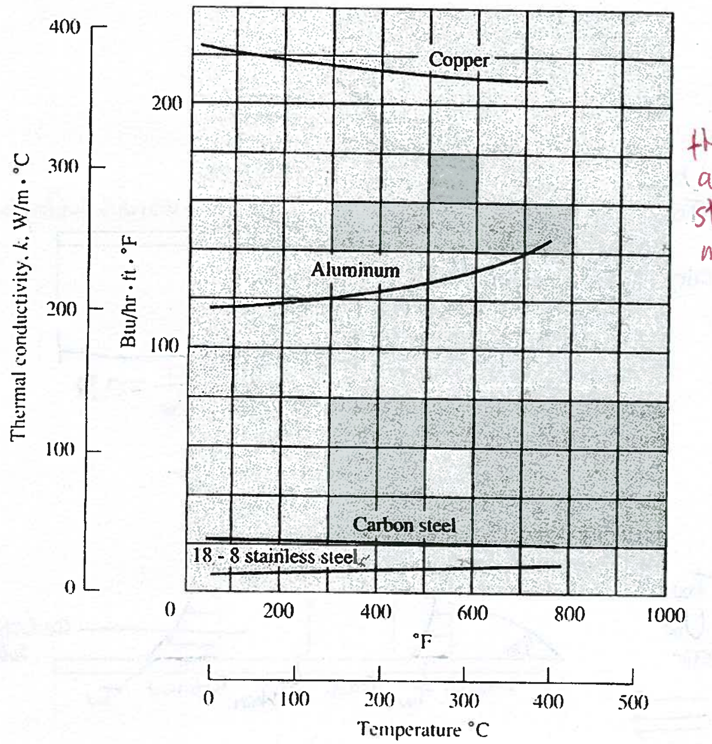
1"-thick electrical insulation makes it okay

HARD



6-8"-thick thermal insulation is still warm (50°C)

Figure 1-6 | Thermal conductivities of some typical solids.



(Figure from: J.P., Holman, 2002)

these are standard materials

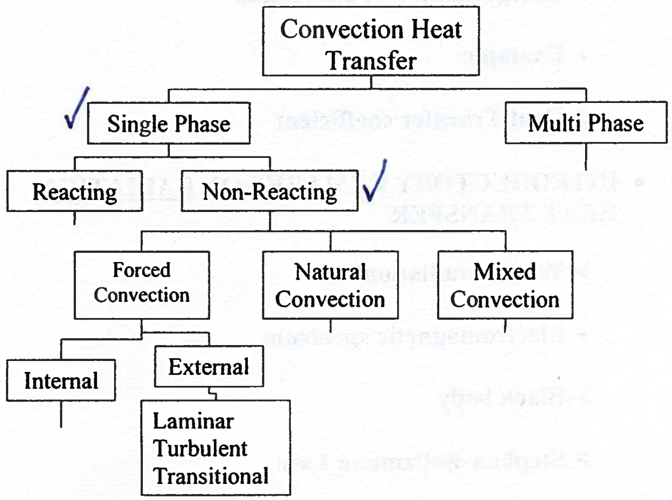
A copper has high thermal conductivity (~400 W/m-°C)

TOPICS

- **INTRODUCTION TO <u>CONVECTION</u>**
  - ➢ **What is convection?**
  - ➢ **Categorization of convection**
  - ➢ **Example**
  - ➢ **Heat Transfer coefficient**
- **INTRODUCTORY REMARKS ON <u>RADIATION</u> HEAT TRANSFER**
  - ➢ **What is radiation?**
  - ➢ **Electromagnetic spectrum**
  - ➢ **Black body**
  - ➢ **Stephan-Boltzmann Law**
  - ➢ **Emissivity / Real bodies**
  - ➢ **Other radiation Properties**
  - ➢ **Gray bodies**
  - ➢ **Radiation exchange between two gray bodies**

CONVECTION HEAT TRANSFER

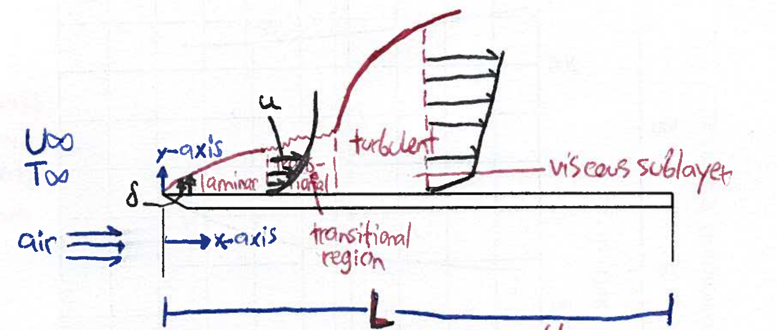
- **Heat transfer in the presence of overall material motion (typically, fluid flow)**
- **Categorization**



* flow is imposed externally by means of ~~fan~~ fan, pump, wind

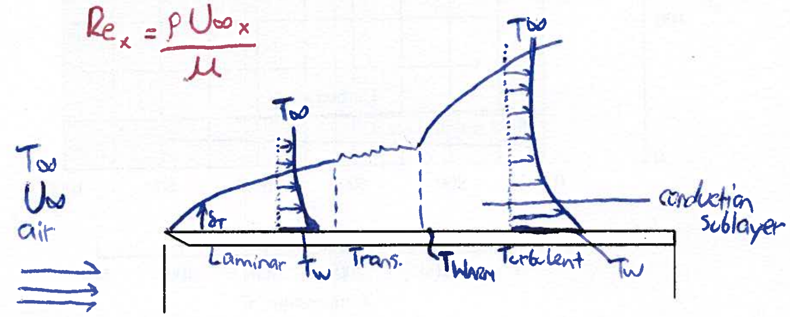
* flow is due to buoancy forces within the fluid

- **Example**: External forced convection over a flat plate



δ: Boundary Layer Thickness $y = \delta \rightarrow \frac{u}{U_\infty} \simeq 0.99$

$Re_x = \frac{\rho U_\infty x}{\mu}$



δ_T: Thermal Boundary Layer Thickness $y = \delta_T$ $\frac{T_w - T}{T_w - T_\infty} \simeq 0.99$

$\boxed{q''_w = ?}$

$$\begin{bmatrix}\text{At any given x, the heat} \\ \text{flux from the top surface} \\ \text{of the plate to the fluid}\end{bmatrix} = q''_{w_x} = -K_{air} \left.\frac{dT}{dy}\right|_{y=0} \left[\frac{W}{m^2}\right]$$

Notes:

(1) At the plate-fluid interface, the heat transfer is by conduction (because $\vec{v}_w = 0$). Convection heat transfer comes into the picture because: $\left.\frac{dT}{dy}\right|_{y=0}$ is influenced by the flow

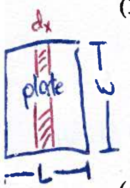
(2) Convection heat transfer coefficient:

$$h = \frac{q''_{w_x}}{(T_w - T_\infty)} = \frac{-K_{FLUID} \left.\frac{dT}{dy}\right|_{y=0}}{(T_w - T_\infty)} \quad \left[\frac{W}{m^2 - {}^oC}\right]$$

Newton's Cooling Law

not enthalpy

(3) Total rate of heat transfer from the top surface of the plate to the fluid:



$$q_{Total\ Top} = \int_0^L q''_w W dx = \int_0^L q''_w \frac{(T_w - T_\infty)}{(T_w - T_\infty)} W dx = \int_0^L h_x W (T_w - T_\infty) dx \ [W]$$

(4) $h_{av} = \left(\frac{q_{Total\ Top} / Area_{Total\ Top}}{(T_w - T_\infty)_{mean}}\right) = \left(\frac{(q'')_{av}}{(T_w - T_\infty)_{mean}}\right)$ Here, in this example T_w is constant

$(T_w - T_\infty) =$ constant $\Rightarrow h_{avg} = \frac{1}{L}\int_0^L h_x dx$

(5) $h_x = fnc[x, \rho, \mu, U_\infty, C_p, K, \text{geometry}...]$

Table 1-3 I Approximate values of convection heat-transfer coefficients.

Mode	h W/m²·°C	h Btu/h·ft²·°F
Free convection, $\Delta T = 30$ C		
Vertical plate 0.3 m [1 ft] high in air	4.5	0.79
Horizontal cylinder, 5-cm diameter, in air	6.5	1.14
Horizontal cylinder, 2-cm diameter, in water	890	157
Heat transfer across 1.5-cm vertical air gap with $\Delta T = 60$ C	2.64	0.46
Fine wire in air, $d = 0.02$ mm, $\Delta T = 55$ C	490	86
Forced convection		
Airflow at 2 m/s over 0.2-m square plate	12	2.1
Airflow at 35 m/s over 0.75-m square plate	75	13.2
Airflow at Mach number = 3, p = 1/20 atm, $T_\infty = -40$ C, across 0.2-m square plate	56	9.9
Air at 2 atm flowing in 2.5-cm-diameter tube at 10 m/s	65	11.4
Water at 0.5 kg/s flowing in 2.5-cm-diameter tube	3500	616
Airflow *across* 5-cm-diameter cylinder with velocity of 50 m/s	180	32
Liquid bismuth at 4.5 kg/s and 420 C in 5.0-cm-diameter tube	3410	600
Airflow at 50 m/s across fine wire, $d = 0.04$ mm	3850	678
Boiling water		
In a pool or container	2500–35,000	440–6200
Flowing in a tube	5000–100,000	880–17,600
Condensation of water vapor, 1 atm		
Vertical surfaces	4000–11,300	700–2000
Outside horizontal tubes	9500–25,000	1700–4400

(Table from: J.P., Holman, 2002)

$$\boxed{q = h_{av} A (T_w - T_\infty)}$$

this is how you calculate convection heat transfer

INTRODUCTORY REMARKS ON RADIATION HEAT TRANSFER

- In contrast to conduction and convection heat transfer, a material medium is not necessary for heat transfer by radiation
- Thermal radiation: Heat transfer by electromagnetic waves
- λ: wavelength [m, μm]

  c: speed of light [m/s]

  ς: freq. of radiation [Hz, s^{-1}]

  $\lambda = \frac{c}{\varsigma}$
- c_o, Speed of light in vacuum: 3×10^8 m/s
- λ is commonly expressed in microns (micrometer):

  $1\ \mu m = 10^{-6} m$

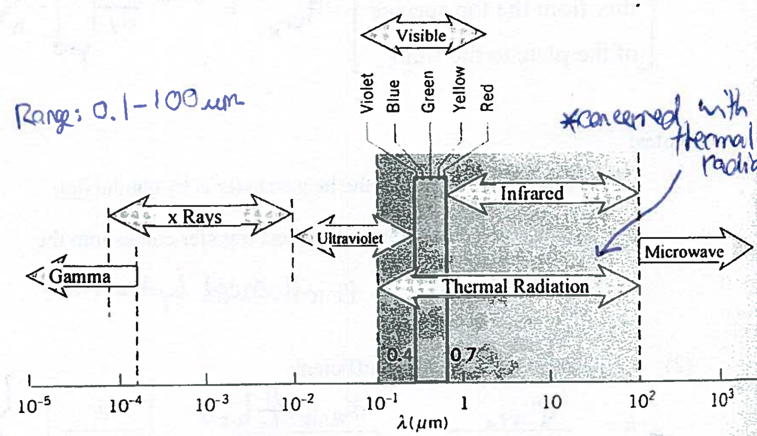
  $\boxed{1\ \mathring{A} = 10^{-10} m}$

  Angstrom

- **Electromagnetic Spectrum:**

Range: 0.1-100 μm

*concerned with thermal radiation

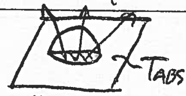


(Figure from "Fundamental of Heat and Mass transfer," by F.P. Incropera and D.P. DeWitt, 1994)

Black Body: It is an ideal surface with the following properties

- **Ideal Emitter:** For a given Temperature and wavelength, no surface can emit more energy than a black body
- **Ideal Absorber:** It absorbs all incident radiation regardless of wavelength and direction
- **Diffuse Emitter:** The radiation emitted by a black body is independent of direction

hemispherical



- A black body emits radiation according to the following relation:

$e_b = \sigma T_{Abs}^4$ } Stefan-Boltzmann Law $[W/m^2]$

e_b : emissive power $[W/m^2]$; σ : Stefan-Boltzmann Constant $= 5.669 \times 10^{-8}\ W/m^2\text{-}K^4$

T_{Abs} : Absolute Temperature $[K]$

Real Bodies:

The rate at which radiation is emitted by real bodies is always less than that emitted by a black body at the same temperature

e : Emissive power of a real body $\left[\frac{W}{m^2}\right]$

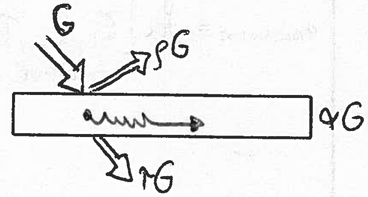
e_b : Emissive power of a black body $= \sigma T_{ABS}^4$ $\left[\frac{W}{m^2}\right]$

Total hemispherical emissivity: $\varepsilon = \frac{e}{e_b}$, $0 < \varepsilon \leq 1.0$

For black body, $\varepsilon = 1.0$

Other Radiation Properties:

irradiation $[W/m^2]$



α : Absorbtivity $0 \leq \alpha \leq 1$

ρ : Reflectivity of surface $0 \leq \rho \leq 1$

τ : Transmissivity $0 \leq \tau \leq 1$

$G = (\rho + \alpha + \tau) G \Rightarrow \boxed{1 = \rho + \alpha + \tau}$

Gray Bodies:

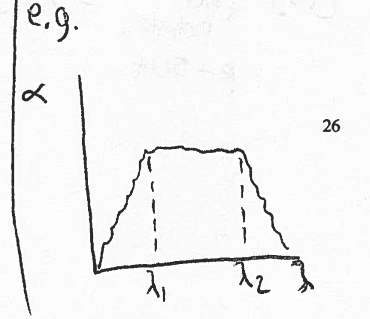
ε_λ and α_λ are independent of λ over our concerned range of λ (0.1-100 μm)

Kirchoff's Law for Diffuse and Gray surfaces:

$\alpha_\lambda = \varepsilon_\lambda$

Gray surface $\boxed{\varepsilon = \alpha}$

e.g.

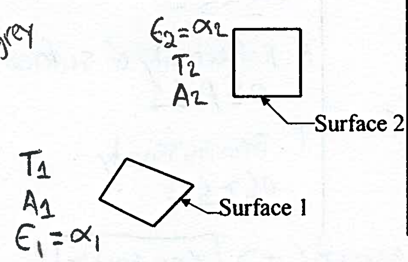


we assume $\varepsilon = \alpha$ despite it not entirely being true

Radiation Exchange between Two Gray Surfaces:

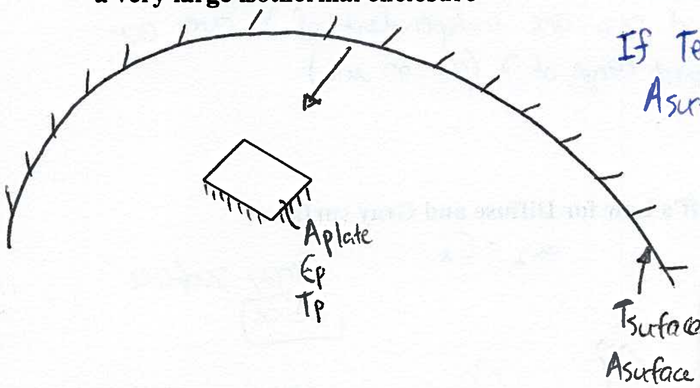
Assumptions: i) Surfaces are isothermal; ii) Irradiation on each surface is uniform

both surfaces are grey



$q_{Rad,Net,1\to2} = fnc[\epsilon_1, \epsilon_2, T_1, T_2, A_1, A_2,$ relative angle$]$

Radiation exchange from a surface to the "surroundings" or a very large isothermal enclosure



If Temp. are uniform

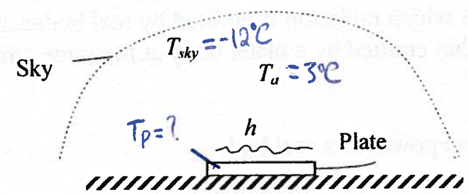
$A_{surf} \gg A_{plate}$

$$[W] \quad q_{\substack{Net\\radiation\\p-surf}} = A_p \epsilon_p \sigma [T_p^4 - T_{surf}^4]$$

Example:

A thin plate of 1 m^2 is facing the sky. The bottom of the plate is perfectly insulated. Assume there is no solar radiation.

The ambient air temperature, T_a, is 3°C. The convection heat transfer coefficient h is 10 W/m^2-°C, and the surface emissivity, ε = 0.9. The average sky temperature is estimated as T_{sky}= -12°C. Assume there is radiant heat exchange with the sky. Find the equilibrium temperature of the plate.

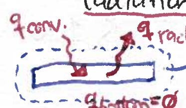


Ans.: -1.2°C

$A_{plate} = 1\,m^2$

Assumptions:
1. steady-state
2. all temperatures are uniform and constant
3. no solar radiation
4. it's a thin plate: heat transfer from edges are negligible
5. No heat transfer from bottom surface of the plate
6. plate and sky behave as grey surface

* heat transfer for plate:
  - convection with Tair
  - radiation with Tsky



I assume $T_{sky} < T_{plate} < T_{air}$

$q_{conv} = h A_p (T_a - T_p)$

$q_{rad} = \epsilon_p A_p (T_p^4 - T_{sky}^4)$, $A_p \ll A_{sky}$

$q_{bott} = \emptyset$

$$\boxed{q_{net\,c.v.} = q_{conv} - q_{rad} = 0}$$

Steady-state energy balance

$$q_{\substack{net\,to\\sys}} - \cancel{W_{sys}} = \cancel{\frac{\delta E_{sys}}{\partial t}}$$

$$h \cancel{A_p}(T_a - T_p) = \epsilon_p \cancel{A_p}(T_p^4 - T_{sky}^4)$$

$$\boxed{① \; T_{P\,ABS} = T_{a\,ABS} - \frac{\sigma\epsilon_p}{\cdot}(T_{p\,ABS}^4 - T_{sur}^4)}$$

iterative solution (see back)

iterative Solution from example

* Successive Substitution Method
  - guess T_p, put in (1)
    - Find another $T_{p\,new}$
    - compare $|T_{p\,new} - T_{p\,guess}| \leq 10^{-2}$
    - re-do until within range

Final answer: $T_p = -1.176°C \simeq \boxed{-1.2°C}$

MECH 375 HEAT TRANSFER - HANDOUT # 2

HEAT CONDUCTION IN ISOTROPIC MATERIALS

TOPICS

- **Three-dimensional unsteady heat conduction: Governing equation**
- **Steady-State One-Dimensional Heat Conduction in Isotropic Materials**
  - **Plane Wall**
    - **Governing Equation**
    - **No-source, k=constant**
    - **Resistance Analogy**
    - **Constant source, k = const., sym. BCs**
  - **Long Hollow Cylinder**
    - **Governing Equation**
    - **No-source, k=constant**
    - **Resistance Analogy**
    - **Constant source, k = const., solid cylinder**
  - **Hollow Sphere**
    - **Governing Equation**
    - **No-source, k=constant**
    - **Resistance Analogy**
    - **Constant source, k = const., solid sphere**

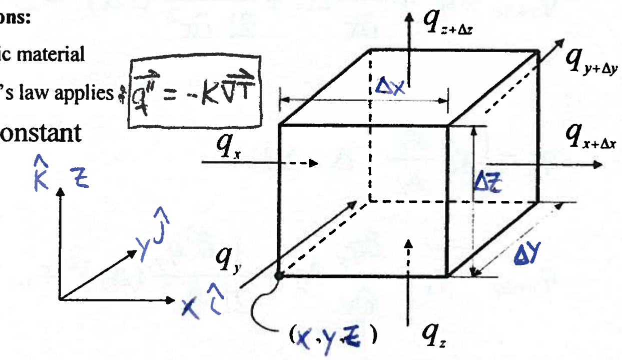
- **Convection Heat Transfer from a Surface**
- **Multilayer Heat Conduction / Overall Heat Transfer Coefficient**
- **Insulation / Coating of Curved Surfaces**
- **Thermal Contact Resistance**

Unsteady Three-Dimensional Heat Conduction: Governing Equation

Heat flux vector [W/m^2]: $\vec{q''} = q''_x\,\vec{i} + q''_y\,\vec{j} + q''_z\,\vec{k}$

Assumptions:

- Isotropic material
- Fourier's law applies $\boxed{\vec{q''} = -k\vec{\nabla T}}$
- $\rho = \text{constant}$



Energy balance on a control volume (closed system here):

$$q_{net-to-sys} - \cancelto{0}{\dot{W}_{by-sys}} = \frac{\partial E_{sys}}{\partial t}$$

$$\underbrace{\left\{q_x - q_{x+\Delta x} + q_y - q_{y+\Delta y} + q_z - q_{z+\Delta z}\right\}}_{\text{Net Rate of Heat added by conduction}} + \underbrace{q_{Gen-sys}}_{\text{Rate of heat generated inside the control volume}} = \underbrace{\frac{\partial E_{sys}}{\partial t}}_{\text{Rate of energy accumulated inside control volume}}$$

$q_x = q''_x \cdot \Delta y \Delta z$, and so on

$$q_x = \left(-K\frac{\partial T}{\partial x}\right)\Delta y\,\Delta z$$

$$q_{x+\Delta x} = q_x + \frac{\partial q_x}{\partial x}\Delta x + \frac{1}{2!}\frac{\partial^2 q_x}{\partial x^2}(\Delta x)^2 + \ldots$$

$$q_y = \left(-K\frac{\partial T}{\partial y}\right)\Delta x\,\Delta z$$

$$q_{y+\Delta y} = q_y + \frac{\partial q_y}{\partial y}\Delta y + \frac{1}{2!}\frac{\partial^2 q_y}{\partial y^2}(\Delta y)^2 + \ldots$$

$$q_z = \left(-K\frac{\partial T}{\partial z}\right)\Delta x\,\Delta y$$

$$q_{z+\Delta z} = q_z + \frac{\partial q_z}{\partial z}\Delta z + \frac{1}{2!}\frac{\partial^2 q_z}{\partial z^2}(\Delta z)^2 + \ldots$$

$$q_{Gen.} = S\,\Delta x\Delta y\Delta z \quad ; \quad \frac{\partial E_{sys}}{\partial t} = \frac{\partial(m_{sys}\, e)}{\partial t}$$

volumetric rate of heat generated $[W/m^2]$

$$= \rho\Delta x\Delta y\Delta z\,\frac{\partial e}{\partial t}$$

$$= \rho\Delta x\Delta y\Delta z\, C_v\frac{\partial T}{\partial t}$$

Thus, energy balance (E-Bal) can be expressed as follows:

$$-\frac{\partial}{\partial x}\left[\left(-k\frac{\partial T}{\partial x}\right)_x \Delta y\Delta z\right]\Delta x - \frac{1}{2!}\frac{\partial^2}{\partial x^2}\left[\left(-k\frac{\partial T}{\partial x}\right)_x \Delta y\Delta z\right](\Delta x)^2 - \ldots$$

$$-\frac{\partial}{\partial y}\left[\left(-k\frac{\partial T}{\partial y}\right)_y \Delta x\Delta z\right]\Delta y - \frac{1}{2!}\frac{\partial^2}{\partial y^2}\left[\left(-k\frac{\partial T}{\partial y}\right)_y \Delta x\Delta z\right](\Delta y)^2 - \ldots$$

$$-\frac{\partial}{\partial z}\left[\left(-k\frac{\partial T}{\partial z}\right)_z \Delta x\Delta y\right]\Delta z - \frac{1}{2!}\frac{\partial^2}{\partial z^2}\left[\left(-k\frac{\partial T}{\partial z}\right)_z \Delta x\Delta y\right](\Delta z)^2 - \ldots$$

$$+S\ \Delta x\Delta y\Delta z = \rho\ \Delta x\Delta y\Delta z\ c_v\frac{\partial T}{\partial t}$$

Note: For a substance with ρ = constant:

$$c_v = c_p = c$$ *please show this

* Divide E-Bal Eq. by $(\mathrm{Vol})_{c.v.} = \Delta x\Delta y\Delta z$ and take limit $\Delta x \to 0$; $\Delta y \to 0$; $\Delta z \to 0$

Result:

$$\underbrace{\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)}_{\text{Net Rate of Heat Added by Conduction per unit Volume}} + \underbrace{S}_{\text{Rate of heat generation per unit volume}} = \underbrace{\rho c\frac{\partial T}{\partial t}}_{\text{Rate energy accumulation per unit volume}}$$

In general, for compressible isotropic substances:

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+S=\rho c_p\frac{\partial T}{\partial t}$$

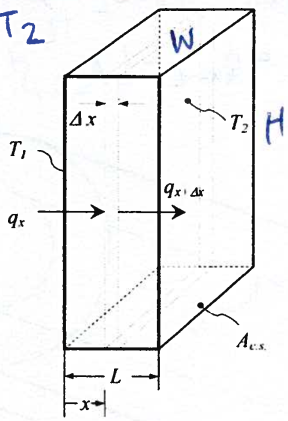
or, $$\left.\vec{\nabla}.\left(k\vec{\nabla}T\right)+S=\rho c_p\frac{\partial T}{\partial t}\right\}$$ 3D-unsteady heat conduction equation, isotropic materials

- Three-dimensional, unsteady heat conduction equation

Steady-State One-Dimensional Heat Conduction [Isotropic Materials]

1. Plane Wall

$T_1 > T_2$



$W \gg L$

$H \gg L$

S.S. E-Balance on CV

$q_x - q_{\Delta x + x} + q_{gen} = 0$

S.S. 1-D Heat Conduction Equation

$$q_{x+\Delta x} = q_x + \frac{dq_x}{dx}\Delta x + \frac{1}{2!}\frac{d^2q}{dx^2}(\Delta x)^2 + \ldots$$

$q_x = -k\dfrac{dT}{dx}A_{c.s.}$; substitute; divide by $A_{c.s.}\Delta x$; take limit $\Delta x \to 0$

$$\frac{d}{dx}\left(k\frac{dT}{dx}\right)+S=0$$

- needs 2 Boundary Conditions to make this problem solvable

(2^{nd} order O.D.E.)

1(a): S.S., 1-D, No-Source ($S = 0$), k = Constant

Gov. Eq.: $\dfrac{d}{dx}\left(k\dfrac{dT}{dx}\right)+\cancel{S}^{0}=0$, $\dfrac{d^2T}{dx^2}=0$

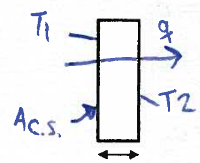
B.Cs.: (i) at $x = 0$, $T = T_1$

(ii) at $x = L$, $T = T_2$

Solution: $T = C_1 x + C_2$ integrate with respect to x

Boundary Conditions (i) $C_2 = T_1$ (ii) $T_2 = C_1 L + T_1$ $\Rightarrow \dfrac{T-T_1}{T_2-T_1} = \dfrac{x}{L}$

Net rate of heat transfer through the *plane* wall



$$q = \left(-k\frac{dT}{dx}\right)_{A_{cs}} = -k\frac{T_2-T_1}{L}A_{c.s.}$$

or $q = \dfrac{T_1-T_2}{[L/kA_{c.s.}]}$ [W]

Notes:
- $S = \varnothing$
- k = constant
- 1-D, SS, Cartesian

Resistance Analogy [Electric Circuits]

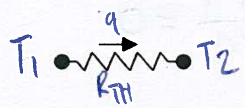
$$q = \frac{T_1-T_2}{L/kA_{c.s.}} = \frac{T_1-T_2}{R_{Thermal}}$$

□ $\dfrac{L}{kA_{c.s.}} = R_{TH}\left[\dfrac{°C}{W}\right]$

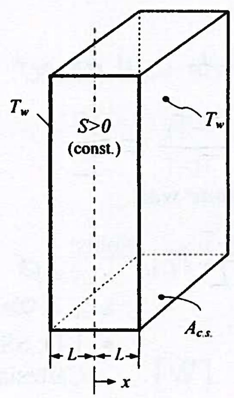
$\boxed{S=0}$

Thus, $R_{th,wall} = \dfrac{L}{kA_{c.s.}}$ [°C/W]

Restrictions:
$\boxed{S=0}$; k = constant;
1-D, SS-Cart.



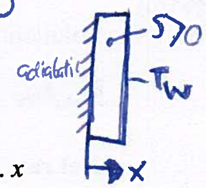
1(b): S.S., 1-D, S = Constant, k = Constant, Symmetric B.Cs.



Gov. Eq.: $\dfrac{d^2T}{dx^2} + S/k = 0$

B.Cs.: (i) at $x = 0$, $\dfrac{dT}{dx} = 0$ adiabatic

(ii) at $x = L$ $T = T_w$



Solution: Integrate twice w.r.t. x

$$dT/dx + \int (S/k)\,dx = c_1$$

$$\Rightarrow dT/dx = -(S/k)x + c_1$$

$$T = -\frac{1}{2}\frac{S}{k}x^2 + c_1 x + c_2$$

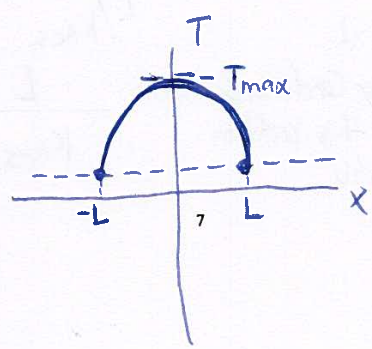
$$\Rightarrow dT/dx = -(S/k)x + c_1$$

Applying B.Cs.:

$$(i) \rightarrow c_1 = 0; \quad (ii) \rightarrow T_w = -\frac{1}{2}(S/k)L^2 + c_2$$

Thus, $$T - T_w = \frac{1}{2}\frac{S}{k}L^2\left[1-\left(\frac{x}{L}\right)^2\right]$$

Note: $T_{max} = T_{x=0} = T_w + \frac{1}{2}\frac{S}{k}L^2$

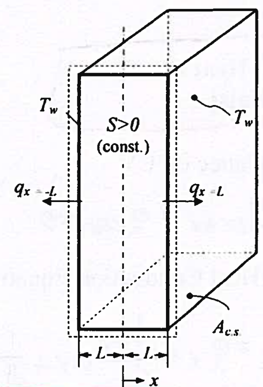


For this case: $T - T_w = \frac{1}{2}\frac{S}{K}L^2\left[1-\left(\frac{x}{L}\right)^2\right]$

Rate of heat transfer out of wall at $x = L$ } $q_{x=L} = -K\dfrac{dT}{dx}\Big|_{x=L} A_{c.s.} = [L A_{c.s.} S]$

Rate of heat transfer out of wall at $x = -L$ } $q_{x=-L} = -K\dfrac{dT}{dx}\Big|_{x=-L} A_{c.s.} = [L A_{c.s.} S]$

Consider SS overall E-Bal. on the wall

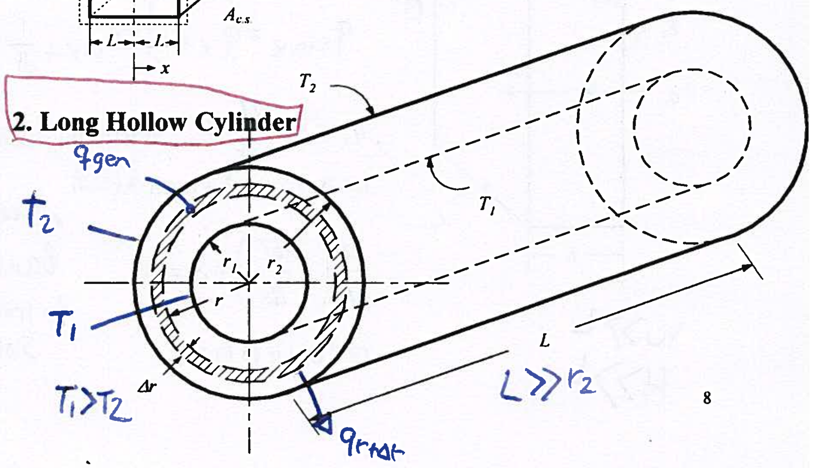


$$q_{Net\ to\ wall} + q_{Gen} = 0$$

$$-q_{x=L} - q_{x=-L} + \int_{-L}^{L} S A_{c.s.}\,dx$$

$$q_{x=L} = q_{x=-L} = L A_{c.s.} S$$

2. Long Hollow Cylinder



S.S. E-Balance on CV

$q_r = q_{r+\Delta r} - q_{gen}$

- Long $L >> 2r_2$
- SS, 1-D radial heat conduction $T = T(r)$

S.S. 1-D Heat Conduction Equation

$q_{r+\Delta r} = q_r + \frac{dq_r}{dr}\Delta r + \frac{1}{2!}\frac{d^2 q_r}{dr^2}(\Delta r)^2 + \ldots$

$q_{gen} = S'(2\pi r \Delta \ell)$

$$q_r = -k\frac{dT}{dr}A_{\perp to\, r} = -k\frac{dT}{dr}(2\pi r L)$$

Substitute; divide by the volume$(2\pi r\, \Delta r\, L)$; take limit $\Delta r \to 0$

$$\frac{1}{r}\frac{d}{dr}\left(rk\frac{dT}{dr}\right) + S = 0$$:2nd order O.D.E.

- 2 Boundary conditions needed to solve
- variables K and $\cancel{g}$ needed

S

2(a): S.S., 1-D Radial, No-Source ($S = 0$), k = Constant

Gov. Eq.: $\cancel{\frac{1}{r}}\frac{d}{dr}\left[r\frac{dT}{dr}\right] = 0$

B.Cs.: (i) at $r = r_1$, $T = T_1$

(ii) at $r = r_2$, $T = T_2$, where $T_1 > T_2$

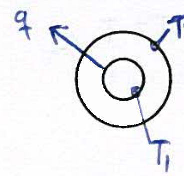
Solution: Integrate twice w.r.t.

$$rdT/dr = c_1 \Rightarrow dT/dr = c_1/r$$

$$T = c_1 \ln r + c_2$$

Applying B.Cs.: $\frac{T - T_2}{T_1 - T_2} = \frac{\ln(r_2/r)}{\ln(r_2/r_1)}$ please show this in problems

Net rate of heat transfer through the ***hollow*** cylinder



$$q_r = \left(-k\frac{dT}{dr}\right)\overbrace{2\pi r\ell}^{\text{Area}} = \frac{(T_1 - T_2)2\pi k\ell}{\ln(r_2/r_1)}$$

or $q_r = \frac{T_1 - T_2}{\ln(r_2/r_1)/(2\pi k\ell)}$ [W]

Notes:
- $S = \emptyset$
- k = constant
- 1-D, SS, Radial

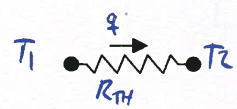
Resistance Analogy [Electric Circuits]

$$q_r = \quad = \frac{T_1 - T_2}{R_{TH\, hollow\, cylinder}}$$

$$R_{TH} \equiv \frac{\ln(r_2/r_1)}{2\pi k\ell}$$

Thus, $R_{th,long\ hollow\ cylinder} = \frac{\ln(r_2/r_1)}{2\pi kL}$ [°C/W]

Restrictions:
$S = 0$; k = const.;
1-D radial; SS

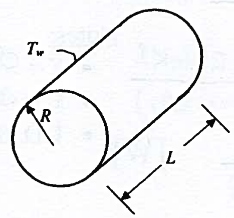


$S = 0$ for this analogy

2(b): S.S., 1-D Radial, S = Constant, k = Constant

Long Solid Cylinder

Gov. Eq.: $\frac{1}{r}\frac{d}{dr}\left[rK\frac{dT}{dr}\right]+S=\emptyset$



B.Cs.: (i) at $r = 0$, T is finite

(ii) at $r = R$, $T = T_{WALL}$

Solution: Integrate twice w.r.t. r

$$\frac{dT}{dr} = -\frac{1}{2}\frac{S}{k}r + \frac{c_1}{r}$$

$$T = -\frac{1}{4}\frac{S}{k}r^2 + c_1 \ln r + c_2$$

Applying B.Cs.:

$(i) \rightarrow c_1 = 0;\quad (ii) \rightarrow T_w = -\frac{1}{4}(S/k)R^2 + c_2$

Thus, $T - T_w = \frac{1}{4}\frac{S}{k}R^2\left[1-\left(\frac{r}{R}\right)^2\right]$

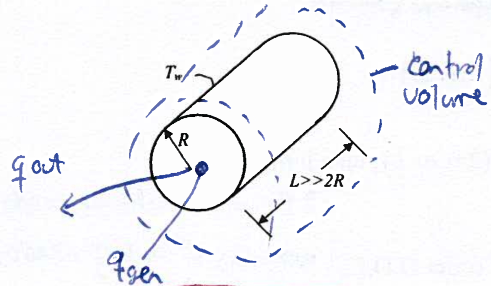
Note: $T_{max} = \boxed{T_{r=0} = T_w + \frac{1}{4}\frac{S}{k}R^2}$

For this case: $T - T_w = \frac{1}{4}\frac{S}{k}R^2\left[1-\left(\frac{r}{R}\right)^2\right]$

Rate of heat transfer out of the cylinder at $r = R$ $\Big\}$ $q_{r=R} = \left(-K\frac{dT}{dr}\Big|_{r=R}\right)\underbrace{2\pi R L}_{\text{surface area}}$

$q_{r=R} = (\pi r^2 L)S$ please show this!

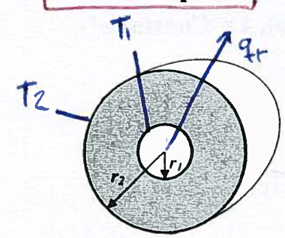
Consider SS overall E-Bal. on the solid cylinder

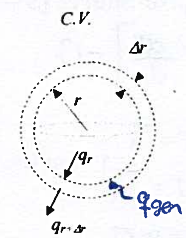


$q_{out} = q_{Gen} = \int_{vol} s\, dV = S\int dV$, S is constant

$\Rightarrow q_{out} = S\pi R^2 L$

3. Hollow Sphere





$T_1 > T_2$

- SS, 1-D Radial Heat Conduction $T = T(r)$

$q_r + q_{gen} = q_{r+\Delta r}$

$q_r = \left(-K\frac{dT}{dr}\right)4\pi r^2$

$q_{gen} = S\,4\pi r^2\Delta r$

$q_{r+\Delta r} = q_r + \frac{dq_r}{dr}\Delta r + \frac{1}{2!}\frac{d^2q_r}{dr^2}(\Delta r^2) + \ldots$

S.S. E-Bal. on CV

$q_r + q_{gen} = q_{r+\Delta r}$

S.S. 1-D Radial Heat Conduction Equation

$$q_r = -k\frac{dT}{dr}A_{\perp \text{ to } r} = -k\frac{dT}{dr}\left(4\pi r^2\right)$$

Substitute; divide by CV volume$\left(4\pi r^2\ \Delta r\right)$; take limit $\Delta r \to 0$

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 k\frac{dT}{dr}\right)+S=0$$

· 2 Boundary Conditions needed

· K and S are needed too

2nd order O.D.E.

3(a): S.S., 1-D Radial, No-Source ($S = 0$), k = Constant

Gov. Eq.: $\frac{d}{dr}\left[r^2\frac{dT}{dr}\right] = 0$ *integrate twice w.r.t. r and apply boundary conditions

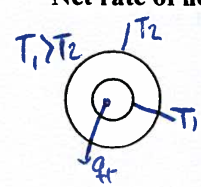
B.Cs.: (i) at $r = r_1$, $T = T_1$
(ii) at $r = r_2$, $T = T_2$, where $T_1 > T_2$

Solution:

$$\frac{T-T_2}{T_1-T_2} = \frac{(1/r - 1/r_2)}{(1/r_1 - 1/r_2)}$$

please show this

Net rate of heat transfer through the *hollow* sphere



$q_r = \left(-k\frac{dT}{dr}\right)4\pi r^2$

$q_r = \dfrac{T_1 - T_2}{\frac{1}{4\pi k}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]}$ [W]

Notes:
- $S = 0$
- k = constant
- 1-D, SS, Radial

Resistance Analogy [Electric Circuits]

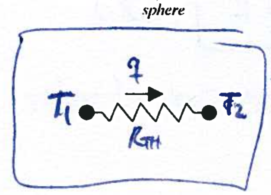
$q_r = \qquad = \dfrac{T_1 - T_2}{R_{TH\,spherical}}$

$$R_{TH} = \frac{\frac{1}{r_1} - \frac{1}{r_2}}{4\pi k}$$

Thus, $R_{th,\,hollow\ sphere} = \dfrac{[1/r_1 - 1/r_2]}{4\pi k}$ [°C/W]

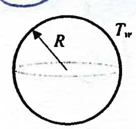
Restrictions:
$S = 0$; k = const.;
1-D, Rad.; SS.

requires



3(b): S.S., 1-D Radial, S = Constant, k = Constant

Solid Sphere



Gov. Eq.: $\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right)+\frac{S}{k}=0$

B.Cs.: (i) at $r = 0$, T is finite

(ii) at $r = R$, $T=T_w$

Solution: Integrate twice w.r.t. r

$$\frac{dT}{dr}=-\frac{1}{3}\frac{S}{k}r+\frac{c_1}{r^2}$$

$$T=-\frac{1}{6}\frac{S}{k}r^2-\frac{c_1}{r}+c_2$$

Applying B.Cs.:

$$(i)\rightarrow c_1=0;\quad (ii)\rightarrow T_w=-\frac{1}{6}(S/k)R^2+c_2$$

Thus, $$T-T_w=\frac{1}{6}\frac{S}{k}R^2\left[1-\left(\frac{r}{R}\right)^2\right]$$

Note: T_{max} = $\boxed{T_{r=0} = T_w + \frac{1}{6}\frac{S}{K}R^2}$, $S>0$

Farthest point from the cooling boundary

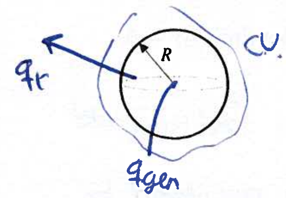
For this case: $T-T_w=\frac{1}{6}\frac{S}{K}R^2\left[1-\left(\frac{r}{R}\right)^2\right]$

Rate of heat transfer out of the sphere at $r = R$ $\Big\}$ $q_{r=R}=-K\left.\frac{dT}{dr}\right|_{r=R}4\pi R^2$

$$=\underbrace{\left(\frac{4}{3}\pi R^3\right)}_{\text{volume of sphere}}S$$

please show this

Consider SS overall E-Bal. on the solid sphere



$$q_{out}=q_{Gen}=\int_{vol}S\,dV$$

S is constant

$$\Rightarrow \boxed{q_{out}=\frac{4}{3}\pi R^3 S} = q_{gen}$$

governing steady-state equation:

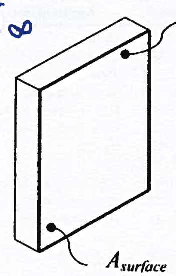
$$\boxed{\vec{\nabla}\cdot(K\vec{\nabla}T)+S=\rho C_p\frac{\partial T}{\partial t}}$$

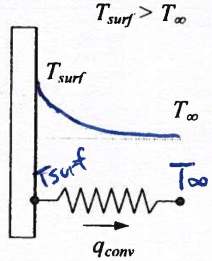
3D unsteady heat conduction equation for isotropic materials

CONVECTION HEAT TRANSFER FROM A SURFACE

Resistance Analogy:

$T_{surf} > T_\infty$





$q_{conv} = A_{surf}\, h\, [T_{surf} - T_\infty]$

$q_{conv} = \dfrac{T_{SURF} - T_\infty}{(1/A_{surf} h)}$

$q_{conv} = \dfrac{T_{SURF} - T_\infty}{R_{TH\ convection}}$

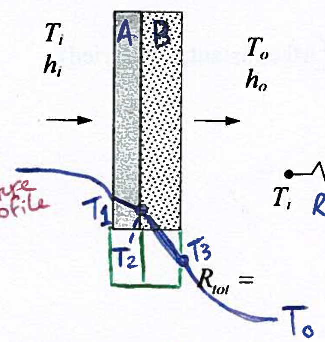
Thus, $R_{th,conv} = \dfrac{1}{h A_{surf}}$

MULTILAYER HEAT CONDUCTION / OVERALL HEAT TRANSFER COEFFICIENT

A) Multilayer Plane Wall

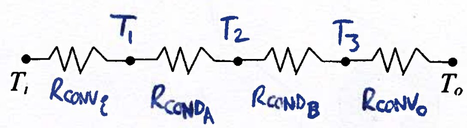
*Assume no heat generation

Restrictions:
$S = 0$



$T_i > T_o$

$q \rightarrow$



$R_{tot} =$

$\boxed{R_{TOT} = R_{CONV_i} + R_{COND_A} + R_{COND_B} + R_{CONV_o}}$

$K_A > K_B$

Materials A and B have no contact resistances

*high thermal conductivity means ~~high~~ low resistance, means low ΔT.

$q = \dfrac{T_i - T_o}{R_{TOT}}$; $R_{CONV_i} = \dfrac{1}{h_i A}$ $\quad R_{COND_A} = \dfrac{L_A}{K_A A}$ $\quad R_{COND_B} = \dfrac{L_B}{K_B A}$ $\quad R_{CONV_o} = \dfrac{1}{h_o A}$

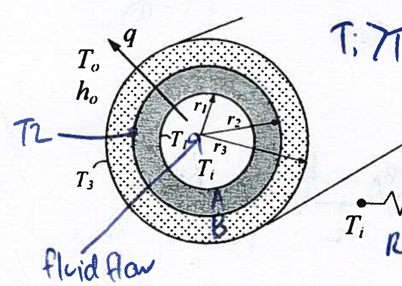
$$q = \frac{T_i - T_o}{\dfrac{1}{h_i A} + \dfrac{L_A}{k_A A} + \dfrac{L_B}{k_B A} + \dfrac{1}{h_o A}} = UA\,\Delta T_{overall}$$

$$U = \frac{1}{AR_{tot}} = \frac{1}{\dfrac{1}{h_i} + \dfrac{L_A}{k_A} + \dfrac{L_B}{k_B} + \dfrac{1}{h_o}} \quad \left[\frac{W}{m^2\text{-}°C}\right]$$

overall heat transfer coefficient, U factor

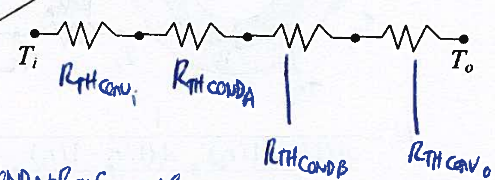
B) Multilayer Cylindrical Systems

Restrictions:
$S = 0$
Steady-state



$T_i > T_o$

$q \rightarrow$



$R_{tot} = R_{TH\,CONV_i} + R_{TH\,COND_A} + R_{TH\,COND_B} + R_{TH\,CONV_o}$

$q = \dfrac{T_i - T_o}{R_{TOT}} = (UA)(T_i - T_o)$

$(UA) = U_i A_i = U_o A_o$

$R_{TH\,COND_A} = \dfrac{\ln(r_2/r_1)}{2\pi K_A L}$ $\quad R_{TH\,COND_B} = \dfrac{\ln(r_3/r_2)}{2\pi K_B L}$

$R_{TH\,conv_i} = \dfrac{1}{h_i A_i}$ $\quad R_{TH\,CONV_o} = \dfrac{1}{h_o A_o}$

$$q=\frac{T_i - T_o}{\frac{1}{h_iA_i}+\frac{\ln(r_2/r_1)}{2\pi k_A L}+\frac{\ln(r_3/r_2)}{2\pi k_B L}+\frac{1}{h_oA_o}}=U_iA_i\,\Delta T_{overall}=U_oA_o\,\Delta T_{overall}$$

$$U_i=\frac{1}{\frac{1}{h_i}+\frac{A_i\ln(r_2/r_1)}{2\pi k_A L}+\frac{A_i\ln(r_3/r_2)}{2\pi k_B L}+\frac{A_i}{h_oA_o}} \rightarrow \text{Overall, heat transfer Coefficient based on inner surface}$$

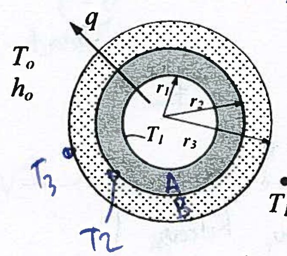
$$U_o=\frac{1}{\frac{A_o}{h_iA_i}+\frac{A_o\ln(r_2/r_1)}{2\pi k_A L}+\frac{A_o\ln(r_3/r_2)}{2\pi k_B L}+\frac{1}{h_o}} \rightarrow \text{" " outer surface}$$

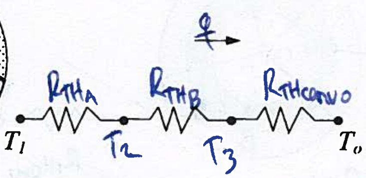
C) Multilayer Spherical Systems

Restrictions:

$S=0$

$T_i > T_o$





$$U_i=\frac{1}{\frac{A_i(1/r_1-1/r_2)}{4\pi k_A}+\frac{A_i(1/r_2-1/r_3)}{4\pi k_B}+\frac{A_i}{h_oA_o}}$$

$$q=\frac{T_i-T_o}{R_{Tot}}$$

$$=(UA)(T_i-T_o)$$

$$U_o=\frac{1}{\frac{A_o(1/r_1-1/r_2)}{4\pi k_A}+\frac{A_o(1/r_2-1/r_3)}{4\pi k_B}+\frac{1}{h_o}}$$

$$(UA)=U_iA_i=U_oA_o$$

$$R_{TH}=\left(\frac{1}{r_1}-\frac{1}{r_2}\right)\Big/(4\pi k_A)$$

$$R_{TH\,COND}=\frac{L}{KA}$$

INSULATION / COATING OF CURVED SURFACES

Material
Porous

T_1
air pocket
T_2

$$q=\frac{\Delta T}{R}$$

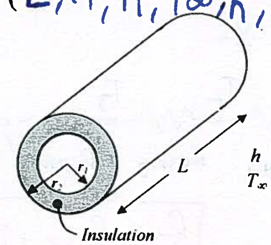
Table 2-1 I Insulation types and applications

	Type	Temperature range, °C	Thermal conductivity, mW/m · °C	Density, kg/m³	Application
1	Linde evacuated superinsulation	−240–1100	0.0015–0.72	Variable	Many
2	Urethane foam	−180–150	16–20	25–48	Hot and cold pipes
3	Urethane foam	−170–110	16–20	32	Tanks
4	Cellular glass blocks	−200–200	29–108	110–150	Tanks and pipes
5	Fiberglass blanket for wrapping	−80–290	22–78	10–50	Pipe and pipe fittings
6	Fiberglass blankets	−170–230	25–86	10–50	Tanks and equipment
7	Fiberglass preformed shapes	−50–230	32–55	10–50	Piping
8	Elastomeric sheets	−40–100	36–39	70–100	Tanks
9	Fiberglass mats	60–370	30–55	10–50	Pipe and pipe fittings
10	Elastomeric preformed shapes	−40–100	36–39	70–100	Pipe and fittings
11	Fiberglass with vapor barrier blanket	−5–70	29–45	10–32	Refrigeration lines
12	Fiberglass without vapor barrier jacket	to 250	29–45	24–48	Hot piping
13	Fiberglass boards	20–450	33–52	25–100	Boilers, tanks, heat exchangers
14	Cellular glass blocks and boards	20–500	29–108	110–150	Hot piping
15	Urethane foam blocks and boards	100–150	16–20	25–65	Piping
16	Mineral fiber preformed shapes	to 650	35–91	125–160	Hot piping
17	Mineral fiber blankets	to 750	37–81	125	Hot piping
18	Mineral wool blocks	450–1000	52–130	175–290	Hot piping
19	Calcium silicate blocks, boards	230–1000	32–65	100–160	Hot piping, boilers, chimney linings
20	Mineral fiber blocks	to 1100	52–130	210	Boilers and tanks

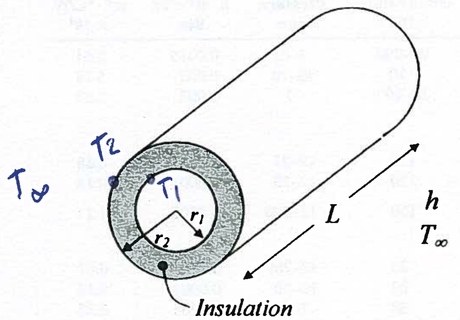
Critical Radius: Conduction-Convection Systems

1- Cylindrical Geometry

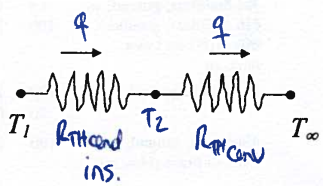
($L, r_1, T_1, T_\infty, h, K_{ins}$) (All constant; r_2 varied)



$S=0$; steady-state



Resistance Analogy:



Thus, $q = \dfrac{(T_1 - T_\infty)}{\underbrace{\dfrac{\ln(r_2/r_1)}{2\pi k_{ins} L}}_{R_{TH\ COND}} + \underbrace{\dfrac{1}{(2\pi r_2 L)h}}_{R_{TH\ CONV}}}$ $[W]$

Notes:

1- As r_2 ↑, with r_1, and k_{ins} fixed, $R_{TH\ COND}$ ↑

2- As r_2 ↑, with h = constant, $R_{TH\ CONV}$ ↓

3- Thus, as r_2 ↑, $[R_{th,ins} + R_{th,conv}]$ could ↑ or ↓; Therefore, for fixed $(T_1 - T_\infty)$, q could ↑ or ↓ as r_2 ↑

Question: is a $q_{max} > q_{bare}$ possible?

$$\left.\frac{\partial q}{\partial r_2}\right|_{r_2=r_{crit}} = 0 \Rightarrow \frac{0-(T_1-T_\infty)\left[\dfrac{1}{2\pi k_{ins} L}\dfrac{1}{r_2} - \left(\dfrac{1}{r_2^2}\right)\left[\dfrac{1}{2\pi L h}\right]\right]}{\left[R_{TH\ COND} + R_{TH\ CONV}\right]^2}$$

When $r_2 = r_{crit}$,

$= 0 \rightarrow r_{crit} = \dfrac{k_{ins}}{h}$

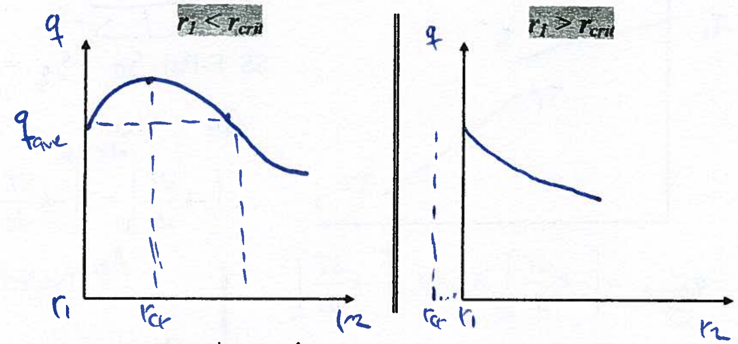
$$\left[\frac{1}{2\pi k_{ins} L r_2} - \frac{1}{(2\pi L) h r_2^2}\right] = 0$$

or $r_{crit\ cylinder} = \dfrac{k_{ins}}{h}$ ex: fiberglass, air → $K_{in} = 0.04$, $h = 10\ W/m^2\,°C$

Restrictions

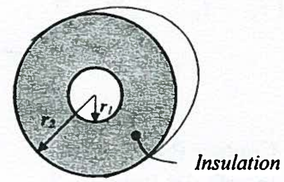
- h constant
- k_{ins} constant
- T_∞, T_1, L, r_1 constant

Additional Comments



Note: T_2 **always** ↓ as r_2 ↑

2- Spherical Geometry



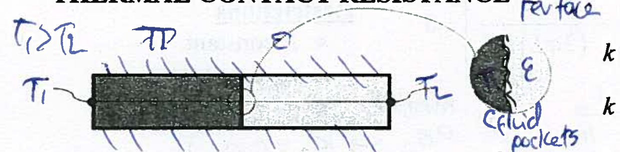
$r_{crit\ sphere} = \dfrac{2k_{ins}}{h}$

Restrictions

- h, k_{ins} constant
- T_∞, T_1, r_1 constant

please show this!!

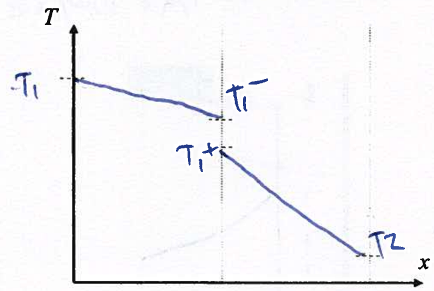
THERMAL CONTACT RESISTANCE



$k_{ins} \ll k_{\pi}$

$k_{ins} \ll k_{\varepsilon}$

Suppose: $k_{\pi} > k_{\varepsilon}$



SS. E-Bal ($\dot{S}_{\pi} = \dot{S}_{\varepsilon} = 0$):

$q_{\pi} = q_{\text{across interface}} = q_{\varepsilon}$

$$\left[-k\frac{dT}{dx}\right]_{\pi} = \left[-k\frac{dT}{dx}\right]_{\varepsilon}$$

$A_{\pi} = A_{interface} = A_{\varepsilon}$

$$q_{\substack{through\\composite\\rod}} = \left[-k\frac{dT}{dx}\right]_{\Gamma} A_{c.s.} = A_{c.s.}\left[-k\frac{dT}{dx}\right]_{\Sigma}$$

Or

$$q_{\substack{through\\composite\\rod}} \,\square\, A_{c.s.}h_{contact}\left[T_I - T_{I'}\right] \,\square\, \frac{\left[T_I - T_{I'}\right]}{R_{th.\,contact}}$$

$$R_{th.\,contact} = \frac{1}{h_{contact}A_{c.s.}} \left[\frac{°C}{W}\right];\quad h_{contact} = \frac{q/A_{c.s.}}{\left[T_I - T_{I'}\right]} \left[\frac{W}{m^2 - °C}\right]$$

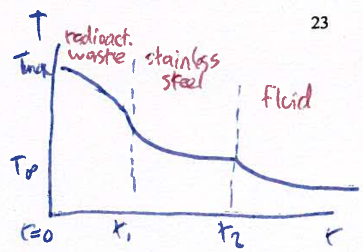
Note:

$R_{th.\,contact}$ **depends on:**

- Contacting materials
- Surface Roughness
- Contact Pressure
- Interfacial fluid

(i) Assumptions

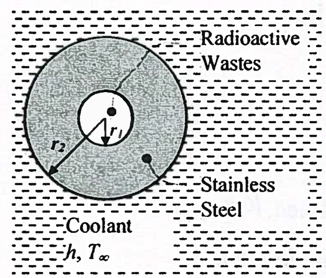
1- S.S. Heat Conduction Problem
2- 1-D Radial Heat Conduction
3- No thermal contact resistance
4- Constant properties
5- Radiation Heat Trans negligible



(J.P. Holman, 2002)

Surface type	Roughness μin	Roughness μm	Temperature, °C	Pressure, atm	$1/h_c$ h · ft² · °F/Btu	$1/h_c$ m² · °C/W × 10⁴
416 Stainless, ground, air	100	2.54	90–200	3–25	0.0015	2.64
304 Stainless, ground, air	45	1.14	20	40–70	0.003	5.28
416 Stainless, ground, with 0.001-in brass shim, air	100	2.54	30–200	7	0.002	3.52
Aluminum, ground, air	100	2.54	150	12–25	0.0005	0.88
	10	0.25	150	12–25	0.0001	0.18
Aluminum, ground, with 0.001-in brass shim, air	100	2.54	150	12–200	0.0007	1.23
Copper, ground, air	50	1.27	20	12–200	0.00004	0.07
Copper, milled, air	150	3.81	20	10–50	0.0001	0.18
Copper, milled, vacuum	10	0.25	30	7–70	0.0005	0.88

Example

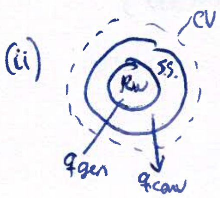


Radioactive wastes (k_{rw} = 20 W/m-K) are stored in a spherical stainless steel (k_{ss} = 15 W/m-K) container (r_1 = 0.5m; r_2 =0.6 m). Heat is generated within the wastes at a uniform rate of 10^5 W/m³. The outer surface of the container is convectively cooled: h = 1000 W/m²-K; T_∞=25°C. The thermal contact resistance at the inner surface of the container is negligible (RW are in the form of slurry).

(i) Draw a qualitatively accurate T vs. r profile (steady-state).
(ii) Evaluate the steady-state outer surface temperature, T_2.
(iii) What is the steady-state container inner surface temperature?
(iv) Obtain an expression for T vs. r in the radioactive wastes.
(v) What is the maximum temperature inside the radioactive wastes?

Ans. (ii) T_2 = 36.6°C, (iii) T_1 = 129.4°C; (v) T_{max} = 337.7°C.

(ii)



Energy balance, steady-state:

$q_{net\ to\ sys} - \dot{W}_{sys} = \frac{d}{dt}E_{sys}$ ($\dot{W}_{sys} \to 0$, $\frac{d}{dt}E_{sys} \to 0$)

$q_{net\ rw} + \cancel{q_{gen/sys}} = q_{conv}$

$\int_0^{r_1} \dot{S}_{rw} 4\pi r^2 dr = h 4\pi r_2^2 (T_2 - T_\infty)$

$\dot{S}_{rw} \frac{4}{3}\pi r_1^3 = h 4\pi r_2^2 (T_2 - T_\infty)$ $\boxed{T_2 = 36.6°C}$

MECH-375 HEAT TRANSFER - HANDOUT # 3

FIN THEORY / EXTENDED SURFACE HEAT TRANSFER

Section 3.6, Chapter 3, Incropera et al., 2007.

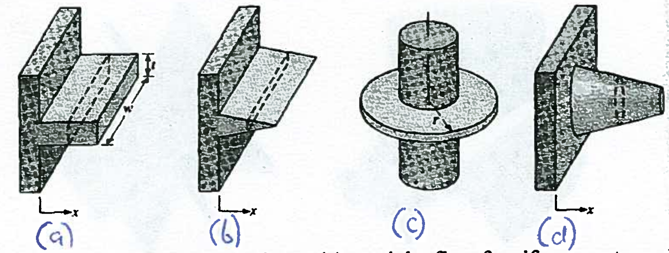
TOPICS

- Examples
- Quasi one-dimensional heat conduction
- Derivation of the fin equation
- Note on solutions of the fin equation
- Classical fin theory
  - ✓ General solution
  - ✓ Four particular solutions (Cases A, B, C, and D)
- Rate of heat loss from the fin
- Fin efficiency
- Corrected length approach
- Fin efficiency (design) charts
- Fin effectiveness
- Resistance analogy for fins

• most fins made of copper or aluminum

Rinfret

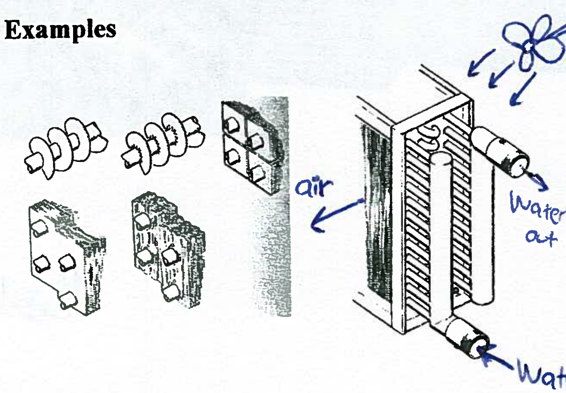
Fin Theory / Extended Surface Heat Transfer



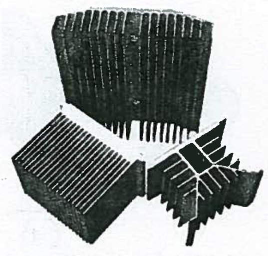
Some typical fin configurations: (a) straight fin of uniform rectangular cross-section; (b) straight fin of uniform triangular cross-section; (c) annular fin; and (d) pin fin of truncated conical shape (figure taken from "Fundamentals of Heat and Mass transfer," by F.P. Incropera and D.P. DeWitt, 4th Edition, 1996)

- Conduction-convection systems Here: ex: electronics cooling
- Conduction-radiation system ex: space application
- Conduction-convection-radiation systems

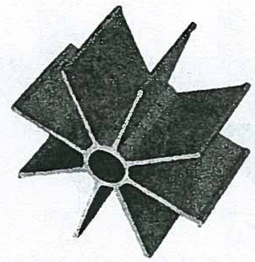
Examples



Water Coil
(Source: Rosaler, R.C., HVAC Handbook, McGrawhill, 2004)

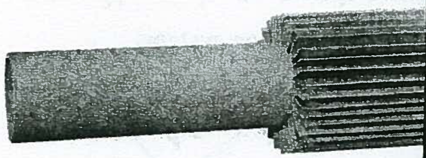


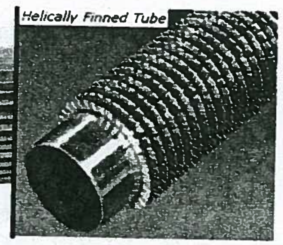
Heat sinks

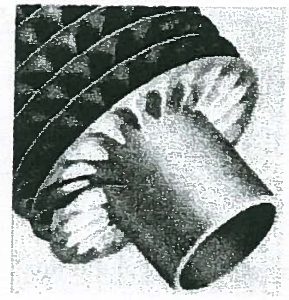


Pipe with external fins

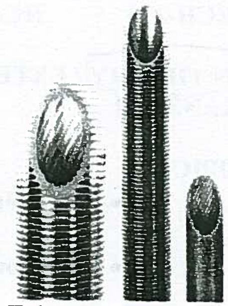
Longitudinally Finned Tube



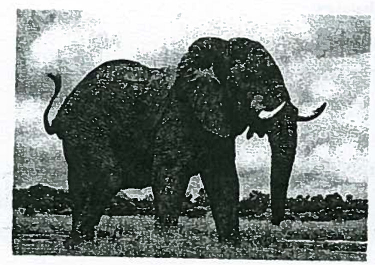




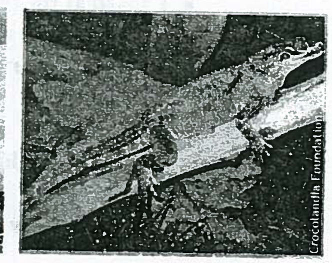
Spirally finned tubes



Tubes with annular fins



African elephant



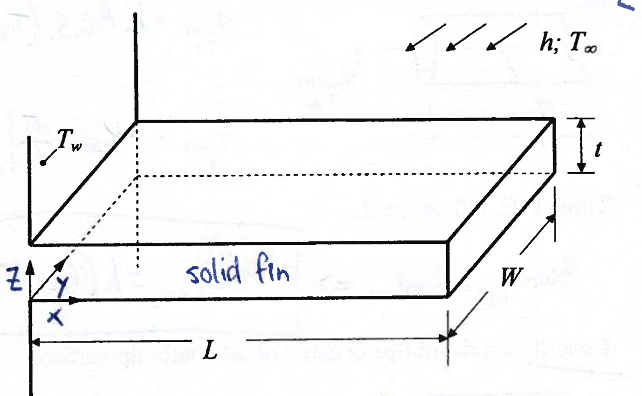
Philippine sail-fin lizard

Real-Life Examples

Quasi one-dimensional heat conduction

(Steady-state; conduction-convection systems)

Assume $W \gg T$, $L \gg t$



Strictly 3D; $T_{Fin} = T(x,y,z)$

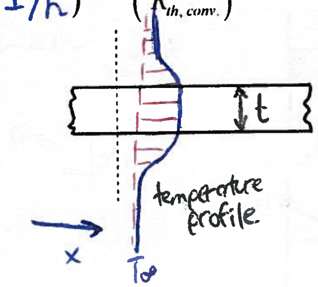
However: If $L \gg t$ (in practice, $L \gtrsim 6t$)

And $\left(\frac{ht}{k_{solid}}\right) \leq 0.2$ $\Rightarrow T_{Fin} = T(x)$

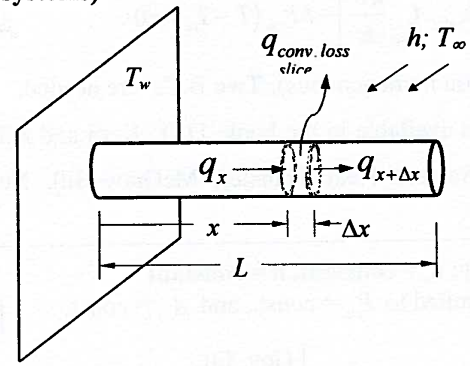
$$\frac{(t/K)}{(1/h)} = \frac{(R_{th,\,cond.\,in\,c.s.})}{(R_{th,\,conv.})}$$

Quasi 1-D

if it's ~~smaller~~ Larger than ~~[illegible]~~ 0.2 Convection is smaller than thermal resistance so not much is loss to environment



Derivation of the fin equation (S.S.; conduction-convection systems)

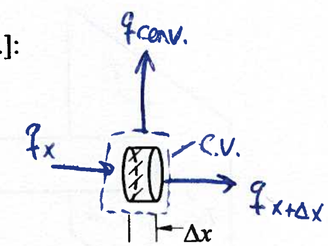


Assume:

- $S = 0$
- Steady state
- Quasi 1-D
- $T_w > T_\infty$

Energy balance on a slice of the fin [c.v.]:

$q_{net_{sys}} = 0 \Rightarrow q_x - q_{x+\Delta x} - q_{conv} = 0$



$$q_x = -k_{solid} \frac{dT}{dx} A_{\substack{c.s.\\Fin}}$$

$$q_{x+\Delta x} = q_x + \frac{dq_x}{dx}\Delta x + \frac{1}{2!}\frac{d^2 q_x}{dx^2}(\Delta x)^2 + \ldots$$

$$q_{\substack{conv\,loss\\surf\,slice}} = hA_{\substack{surf\\slice}}\left(T_{surf} - T_\infty\right) \approx h(P_{c.s.}\Delta x)(T - T_\infty)$$ Assume $T_{surf_x} \cong T_x$

* Substitute in E-Bal.; divide by Δx; take limit $\Delta x \to 0$:

$$\frac{d}{dx}\left[K_{solid} A_{c.s.} \frac{dT}{dx}\right] - hP_{c.s.}(T - T_\infty) = 0$$

2nd-order O.D.E. (none-homogeneous); two bound. conditions

$P_{c.s.}$ = Perimeter

Note on solutions of the fin equation

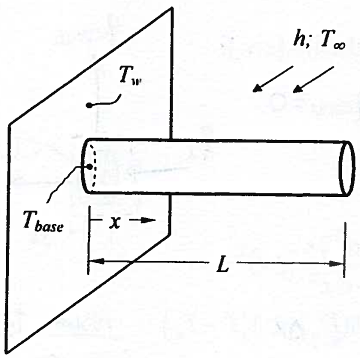
$$\frac{d}{dx}\left[k_{solid}A_{\substack{c.s.\\Fin}}\frac{dT}{dx}\right]-hP_{c.s.}(T-T_\infty)=0$$

2^{nd}-order O.D.E. (non homogeneous); **Two** B.Cs. are needed.

Numerous solutions available in the book: D.Q., Kern and A.D. Kraus, Extended Surface Heat Transfer, McGraw-Hill, New York, 1972.

Classical fin theory: k_s = constant, h = constant

Here, attention is limited to $P_{c.s.}$ = const., and $A_{c.s.}$ = const. $P_{c.s.}$ = perimeter of cross section



Gov. Eq.

$$\frac{d^2T}{dx^2}-\left(\frac{h\,P_{c.s.}}{K_s\,A_{c.s.}}\right)(T-T_\infty)=0$$

Note: $(hP_{c.s.}/k_sA_{c.s.})$ has dimensions of $[L^{-2}]$

Let: $m^2=\dfrac{h\,P_{c.s.}}{K_s\,A_{c.s.}}>0$

Then, $\dfrac{d^2T}{dx^2}-m^2\,(T-T_\infty)=0$

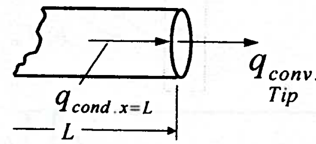
B.C. (i): at $x=0$, $T=T_{base}$

Note: If thermal contact resistance at the fin base $(x=0)$ is negligible: $T_{x=0}=T_{Base}=T_{WALL}$

B.C (ii) at $x=L$??

B.C. (ii): Typically, the following four cases are considered:

Case A: Convection from the tip surface



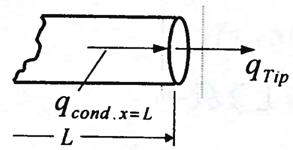
$q_{\substack{conv.\\Tip}}=h\,A_{c.s.}(T_{x=L}-T_\infty)$

$q_{\substack{cond.\\x=L}}=-K_{solid}\left.\frac{dT}{dx}\right|_{x=L}$

Thus, B.C. (*ii*): at $x=L$,

$q_{conv\,tip}=q_{cond\,x=L}\Rightarrow\boxed{-K_s\left.\frac{dT}{dx}\right|_{x=L}=h(T_{x=L}-T_\infty)}$

Case B: Insulated tip surface / or adiabatic tip surface

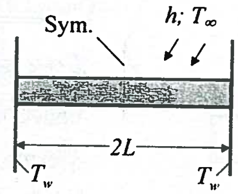


$q_{Tip}=0$

(1) $q_{\substack{cond.\\x=L}}=-k_s\left.\frac{dT}{dx}\right|_{x=L}A_{c.s.}=0$ $\left.\frac{dT}{dx}\right|_{x=L}=0$

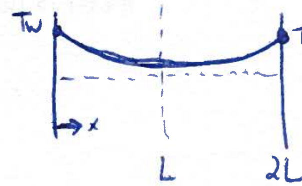
Thus, B.C. (*ii*): at $x=L$, $\therefore\left.\frac{dT}{dx}\right|_{x=L}=0$, because K_s & A_{cs} have val

Symmetry conditions at $x=L$:



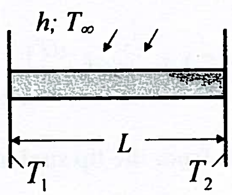
(2) $q_{\substack{across\\sym.surf}}=0=-k_s\left.\frac{dT}{dx}\right|_{x=L}A_{c.s.}$

$\Rightarrow\left.\frac{dT}{dx}\right|_{x=L}=0$ (∴ Case B)



Case C: Prescribed temperature at tip ($x = L$)

B.C. (*ii*): at $x = L$, $T = T_{x=L}$ is fixed or given

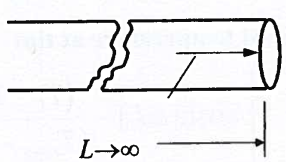


$T_1 \neq T_2$

Case D: Long fin ($L \to \infty$); in practice $mL \geq 5$ $\rightarrow m^2 = \dfrac{h P_{c.s.}}{K_s A_{c.s.}}$

B.C. (*ii*): at $x = L$, $T_{x=L} = T_\infty$

if $mL \geq 5$, defined as "long fin"



$T_{Tip} \approx T_\infty$

General solution $\dfrac{d^2T}{dx^2} - m^2(T - T_\infty) = 0$

Let $\theta = (T - T_\infty)$. Then the fin equation can be expressed as follows (Note: Here, $A_{c.s.}$ = constant and $P_{c.s.}$ = constant)

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0 \text{ and the general solution:}$$

$$\theta = C_1 e^{-mx} + C_2 e^{+mx}$$

or $\theta = A \sinh(mx) + B \cosh(mx)$

$\sinh(\lambda) = \dfrac{e^{\lambda} - e^{-\lambda}}{2}$

$\cosh(\lambda) = \dfrac{e^{\lambda} + e^{-\lambda}}{2}$

$\tanh(\lambda) = \dfrac{\sinh(\lambda)}{\cosh(\lambda)}$

Solution to Case A (convection from the tip surface)

B.C. (*i*): at $x = 0$, $T = T_{Base} \Rightarrow \theta = (T_{Base} - T_\infty) = \theta_{Base}$

Thus, at x = 0: $\theta_{Base} = A \times 0 + B \times 1 \Rightarrow B = \theta_{Base}$

$\dfrac{d\theta}{dx} = Am\cosh(mx) + Bm\sinh(mx)$

B.C. (ii): at $x = L$ $\quad -k_s \left.\dfrac{dT}{dx}\right|_{x=L} = h(T_{x=L} - T_\infty)$

$$-k_s \left.\frac{d\theta}{dx}\right|_{x=L} = h\theta_L$$

Thus, at x = L:

$$-k_s[Am\cosh(mL) + Bm\sinh(mL)] = h[A\sinh(mL) + B\cosh(mL)]$$

A=? (please work this out). Thus,

$$\frac{\theta}{\theta_{Base}} = \frac{T - T_\infty}{T_{Base} - T_\infty} = \frac{\cosh[m(L-x)] + (h/mk_s)\sinh[m(L-x)]}{\cosh[mL] + (h/mk_s)\sinh[mL]}$$

Solution to Case B (insulated tip) $x=L$ $\left.\frac{dT}{dx}\right|_{x=L}=0$

$$\frac{\theta}{\theta_{Base}}=\frac{T-T_\infty}{T_{Base}-T_\infty}=\frac{\cosh[m(L-x)]}{\cosh[mL]}$$; please derive this

Solution to Case C (prescribed temperature at tip) $x=L$ $T=T_{x=L}$ Fixed

$\theta=\theta_L$

$=(T_{x=L}-T_\infty)$

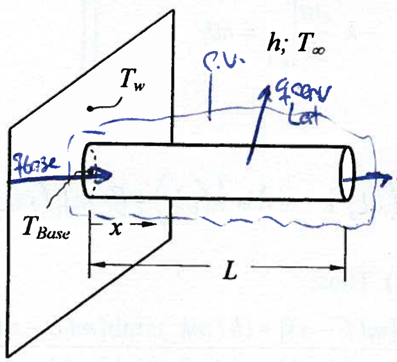
$$\frac{\theta}{\theta_{Base}}=\frac{T-T_\infty}{T_{Base}-T_\infty}=\frac{(\theta_L/\theta_{Base})\sinh[mx]+\sinh[m(L-x)]}{\sinh[mL]}$$

Please derive this solution

Solution to Case D (long fin; $L\to\infty$)

$$\frac{\theta}{\theta_{Base}}=\frac{T-T_\infty}{T_{Base}-T_\infty}=e^{-mx}$$; please derive this

Rate of heat loss from the fin to the surrounding fluid



S.S. E-Bal on the fin:

$q_{Net\ to\ sys.}=0$

or $q_{Base}=q_{conv\ Lat}+q_{conv\ Tip}$

Thus,

$q_{\substack{total\ loss\\ Fin\to Fluid}} = q_{base}$

$= q_{conv_{lat}} + q_{conv_{tip}}$

$q_{Base} = -K_s A_{cs}\left.\frac{dT}{dx}\right|_{x=0}$

$q_{\substack{conv.\\ lateral}} = \int_0^L h(T-T_\infty)P_{c.s.}\,dx$

$q_{\substack{conv.\\ tip}} = h(T_{x=L}-T_\infty)A_{c.s.}$

Thus,

$$q_{\substack{total\ loss\\ Fin\to Fluid}}=\int_0^L h(T-T_\infty)P_{c.s.}dx+h(T_{x=L}-T_\infty)A_{\substack{c.s.\\ tip}}=-k_s\left.\frac{dT}{dx}\right|_{x=0}A_{\substack{c.s.\\ Base}}$$

Rate of heat loss for **Case A (convection from the tip surface)**

$q_{\substack{total\ loss\\ Fin\to Fluid}}=q_{Base}= -KA_{c.s.}\left.\frac{dT}{dx}\right|_{x=0}$

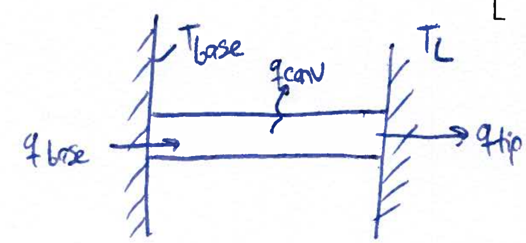
$$q_{\substack{total\ loss\\ Fin\to Fluid}}=\sqrt{k_sA_{c.s.}hP_{c.s.}}\,(T_{Base}-T_\infty)\left[\frac{\sinh[mL]+(h/mk_s)\cosh[mL]}{\cosh[mL]+(h/mk_s)\sinh[mL]}\right]$$

Rate of heat loss for **Case B (insulated tip)**

$$q_{\substack{total\ loss\\ Fin\to Fluid}}=\sqrt{k_sA_{c.s.}hP_{c.s.}}\,(T_{Base}-T_\infty)\tanh[mL] = -K\left.\frac{dT}{dx}\right|_{x=0}A_{cs}$$

Rate of heat loss for **Case C (prescribed temperature at tip)**

$$q_{base}\ q_{\substack{total\ loss\\ from\ Base}}=\sqrt{k_sA_{c.s.}hP_{c.s.}}\,(T_{Base}-T_\infty)\left[\frac{\cosh[mL]-\dfrac{(T_L-T_\infty)}{(T_{Base}-T_\infty)}}{\sinh[mL]}\right]$$



$q_{base}=-K\left.\frac{dT}{dx}\right|_{x=0}A_{c.s.}$

$q_{tip}=-K\left.\frac{dT}{dx}\right|_{x=L}A_{c.s.}$

$q_{conv}=q_{base}-q_{tip}$

Rate of Heat Loss for **Case D (long fin; $L\rightarrow\infty$)**

$$q_{\substack{total\ loss\\ Fin\rightarrow Fluid}} = \sqrt{k_s A_{c.s.} h P_{c.s.}}\,(T_{Base} - T_\infty) = -k_s \frac{dT}{dx}\Big|_{x=0} A_{c.s.}$$

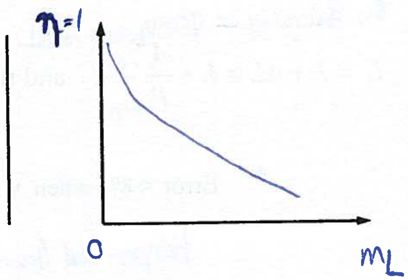
Fin efficiency

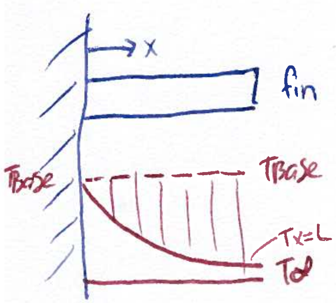
$$\eta_{Fin} = \frac{q_{actual\ fin \rightarrow fluid}}{q_{fin\rightarrow fluid \text{ if entire fin were at } T_{Base}}}$$

For example, consider **Case B (insulated tip)**

$$\eta_{Fin,Case\,B} = \frac{\sqrt{k_s A_{c.s.} h P_{c.s.}}\,\cancel{(T_{Base} - T_\infty)}\tanh[mL]}{h P_{c.s.} L\,\cancel{(T_{Base}-T_\infty)}} = \frac{\tanh(mL)}{mL}$$

Show: $\eta_{\substack{Fin\\ Case\,B}} = 1$ when $mL \rightarrow 0$





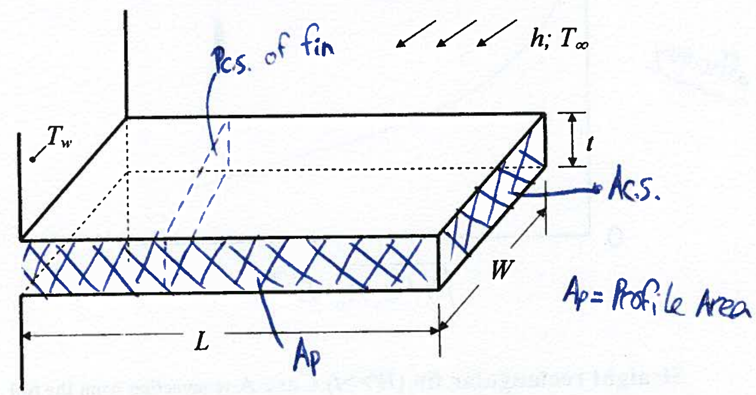
Fin efficiency (continued)

$$\eta_{Fin} = \frac{[\text{Actual rate of heat loss from the fin}]}{[\text{Rate of heat loss if the entire fin were at the base temperature}]_{\substack{\text{Same}\\ \text{tip condition}}}}$$

For **Case B (insulated tip)**

$$\eta_{Fin,Case\,B} = \frac{\tanh[mL]}{mL}\ ; \text{ where } m = \sqrt{hP_{c.s.}/k_s A_{c.s.}}$$

Consider a straight fin of rectangular cross-section: • Assume w>>t



$$mL = \sqrt{hP_{c.s.}/k_s A_{c.s.}}\ L = \sqrt{h(2W+2t)/k_s W t}\ L \approx \sqrt{2h/k_s t}\ L$$

$\boxed{mL \approx \sqrt{2h/k_s L t}\ L^{3/2}}$; **define profile area:** $A_p = Lt$; then

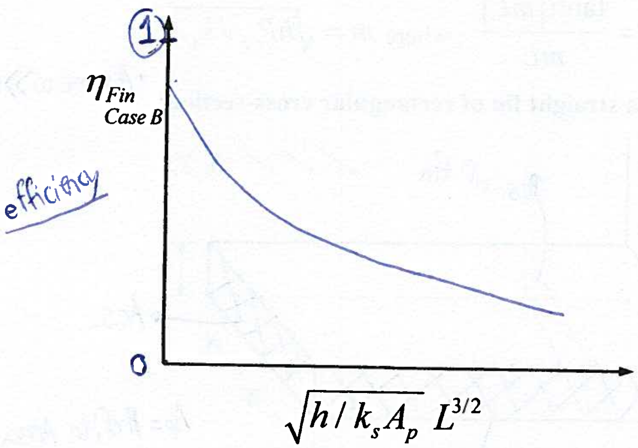
$$\boxed{mL \simeq \sqrt{2}\sqrt{h/k_s A_p}\cdot L^{3/2}}$$

$$\eta_{Fin} = fnc(mL) = fnc(\sqrt{h/k_s A_p}\, L^{3/2})$$

Straight rectangular fin (*W*>>*t*) Case B (insulated tip)

$$\eta_{Fin,Case\,B} = \frac{\tanh[mL]}{mL}$$

Thus, $\eta_{Fin,Case\,B} = fnc(mL) = fnc(\sqrt{h/k_s A_p}\, L^{3/2})$



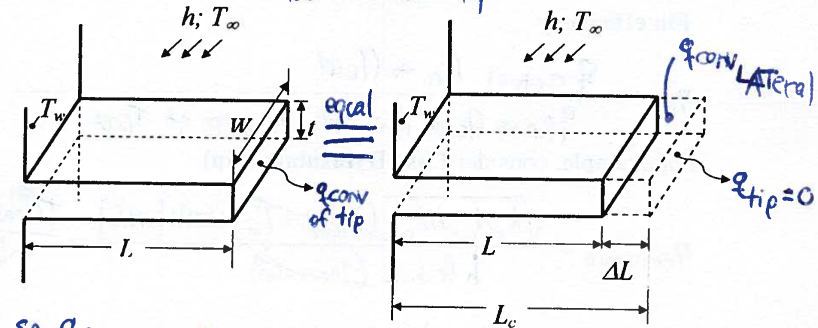
Straight rectangular fin (*W*>>*t*) Case A (convection from the tip)

$\eta_{Fin\,Case\,A}$ = (Please work this out)

Difficult, so we introduce the corrected length approach

Corrected length: L_c may be used to approximate the total fin rate of heat loss in Case A using the results for Case B

add on insulated tip

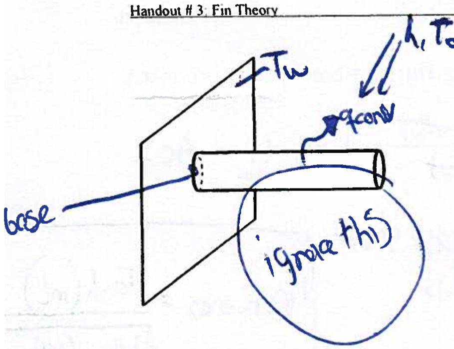


So, $q_{conv\,Tip} \simeq q_{conv\,lateral}$ at ΔL & $A_{c.s} = \text{Perimeter}_{c.s.}\,\Delta L = \boxed{P_{cs}\,\Delta L}$

$$L_c = L + \Delta L \cong L + \frac{A_{c.s.\,tip}}{P_{c.s.\,tip}};\quad \text{and if } W >> t, \text{ then } L_c = L + \frac{t}{2}$$

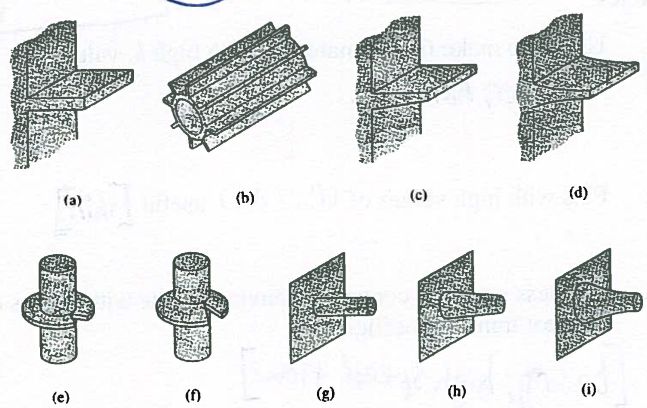
$$\text{Error} < 8\% \text{ when } \sqrt{ht/k_s} \le \frac{1}{2}$$

Harper and Brown (1700's)



$$L_c = L + \Delta L \cong L + \frac{A_{c.s.\,tip}}{P_{c.s.\,tip}}$$

$$L_c = L +$$



Schematic diagrams of different types of fins: (a) longitudinal fin of rectangular profile; (b) cylindrical tube with fins of rectangular profile; (c) longitudinal fin of trapezoidal profile; (d) longitudinal fin of parabolic profile; (e) cylindrical tube with radial fin of rectangular profile; (f) cylindrical tube with radial fin of truncated conical profile; (g) cylindrical pin fin; (h) truncated conical spine; (i) parabolic spine.

(Figure taken from "Principles of Heat Transfer," by F. Kreith and M.S. Bohn, 6th Edition, 2001)

$$\frac{d}{dx}\left[k A_{c.s.} \frac{dT}{dx}\right] - h P_{c.s.} (T - T_\infty) = 0$$

$A_{c.s.}$ and $P_{c.s.}$ are functions of x

Fin efficiency (design) charts [based on insulated tip, Case B]

$$\eta_{Fin} = fnc(\sqrt{h / k_s A_p}\; L_c^{3/2}, \text{geometric parameters})$$

- Calculate A_p with L_c
- Use appropriate L_c and A_p for the fin geometries of interest

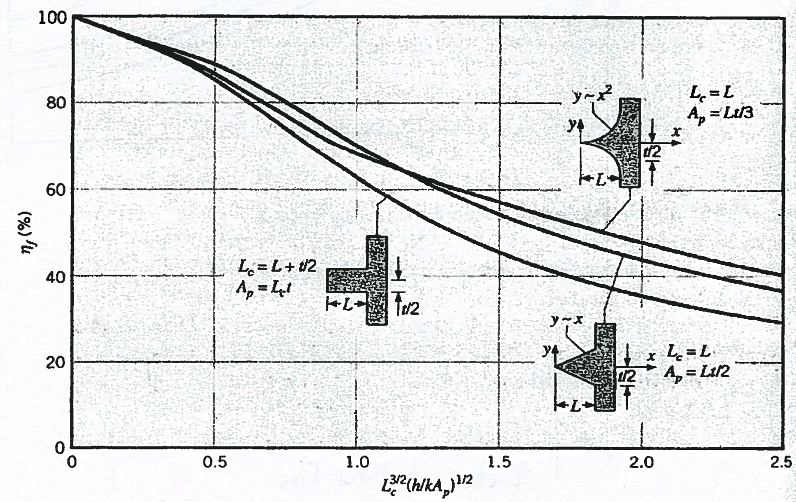


FIGURE 3.18 Efficiency of straight fins (rectangular, triangular, and parabolic profiles).

Figure taken from "Fundamentals of Heat and Mass Transfer" by Incropera et al. 2007

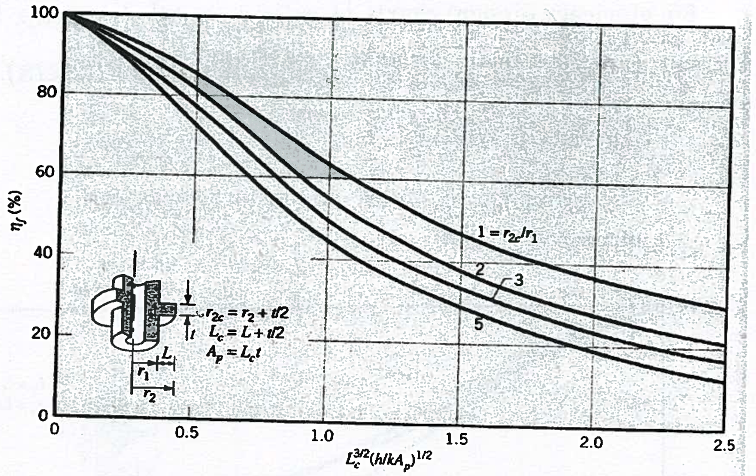


FIGURE 3.19 Efficiency of annular fins of rectangular profile.

Figure taken from "Fundamentals of Heat and Mass Transfer" by Incropera et al. 2007

Fin effectiveness

$$\varepsilon_{Fin} \equiv \frac{[\ q_{actual}\ \text{with Fin}\ ]}{[\ q_{actual}\ \text{without Fin}\ ]}$$

$\varepsilon_{Fin} > 1$ desirable. In practice, $\varepsilon_{fin} \geq 5$

To make fins thermally attractive solution

$L \to \infty$

Example: Consider Case D (long fin; k_s = const.; $A_{c.s.}$ = const.)

$$\varepsilon_{\substack{Fin\\Case\ D}} = \frac{\sqrt{k_s A_{c.s.} h P_{c.s.}}\ \cancel{(T_{Base} - T_\infty)}}{A_{\substack{c.s.\\Base}}\ h\cancel{(T_{Base} - T_\infty)}}\ ;\quad \text{Here, } A_{\substack{c.s.\\Base}} = A_{c.s.}$$

Thus, $\boxed{\varepsilon_{\substack{Fin\\Case\ D}} = \sqrt{\frac{k_s P_{c.s.}}{A_{c.s.} h}}}$ Please show this

$$\boxed{\text{Effectiveness} = \frac{\tanh(mL)}{\sqrt{hA_{c.s.}/kP_{c.s.}}}}$$

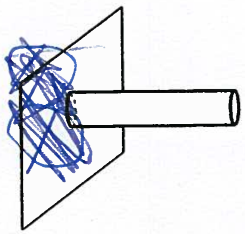
Notes:

1. Useful to make fins of materials with high k_s values
   Copper; Aluminum

2. Fins with high values of $(P_{c.s.} / A_{c.s.})$ useful [thin]

3. Fins less useful in convective environments with values of the heat transfer coefficient, h
   [boiling; high speed flow]
   h, very high

Resistance analogy for fins



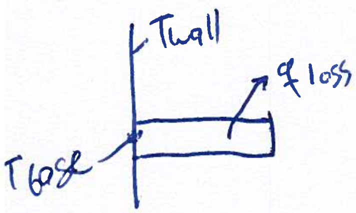
$$\eta_{Fin} \equiv \frac{q_{actual\ with\ fin}}{\underbrace{q_{\ Entire\ fin\ at\ the\ base\ temperature}}_{A_{\substack{surface\\ total}}\, h(T_{Base}-T_\infty)}}$$

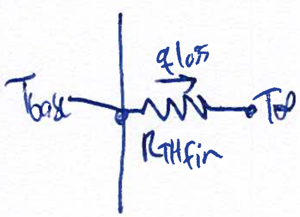
$$q_{actual\ with\ fin} = \eta_{Fin} A_{\substack{surf\\ total}} h(T_{Base} - T_\infty)$$

$$q_{actual\ with\ fin} = \frac{T_{Base}-T_\infty}{1/\eta_{fin} A_{\substack{surf\\ total}}\, h} = \frac{\Delta T}{R_{TH\ fin}}$$

$$\Rightarrow R_{\substack{th\\ Fin}} = \frac{1}{\eta_{fin} A_{\substack{surf\\ total}} h}$$

← Total surface of fin exposed to convection





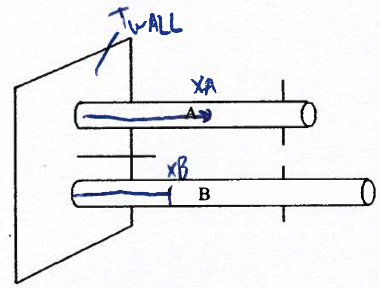
$$\frac{T^A_{XA}-T_\infty}{T_{Base}-T_\infty} = \frac{T^B_{XB}-T_\infty}{T_{Base}-T_\infty}$$

$$\exp(-m_A x_A) = \exp(-m_B x_B)$$

$$\boxed{m_a x_a = m_B x_B}$$

Example

Consider two long, straight, slender rods, A and B, of the same diameter, but made of different materials. One end of each of these rods is attached to a wall: the thermal contact is excellent in both cases. The wall is maintained at a temperature of T_{wall} =100 °C. The outer surfaces of the rods are exposed to ambient air at 20 °C. Measurements indicate that the temperatures of the rods are equal at the positions x_A = 0.15 m and x_B = 0.075 m, where x is measured from the wall surface, along the length of the rods. The thermal conductivity of rod A, k_A, is known to be 70 W/m-°C. Determine the value of the thermal conductivity of rod B; k_B = ?



Ans. k_B = 17.5 W/m-°C

$T_{WALL} = T_{BASE} = 100°C$

$T^A_{x=x_A} = T^B_{x=x_B}$

*steady state quasi one D heat transfer

*classical fin theory applies

*Long fins → case D solution applies

*No thermal contact resistance @base $T_{WALL} = T_{BASE}$

Case D

$$\frac{T-T_\infty}{T_{Base}-T_\infty} = e^{-mx}$$

$$\boxed{m = \sqrt{\frac{hP_{c.s.}}{KA_{c.s.}}}}$$

So, $m = \sqrt{\frac{hP_{cs}}{KA_{c.s.}}} = y\frac{1}{\sqrt{K}} \Rightarrow \frac{M_A}{M_B} = \sqrt{\frac{K_B}{K_A}}$ $\therefore K_B = K_A\left(\frac{X_B}{X_A}\right)^2 = 70\left(\frac{0.075}{0.15}\right)^2 = \boxed{17.5}$

MECH-375 HEAT TRANSFER - HANDOUT # 4

Topics

- **Multidimensional Steady-State Heat Conduction in Isotropic Materials**
  - **Note on mathematical model and related issues**
  - **Note on methods of solution**
  - **Brief discussion of analytical methods**
  - **Graphical method**
    - ✓ **Basis**
    - ✓ **Synopsis**
    - ✓ **Limitations**
    - ✓ **Advantages**
  - **Heat conduction shape factors**
  - **Table of heat conduction shape factors**

$\boxed{1.01}$

0.4 A

Multidimensional Steady-State Heat Conduction in Isotropic Materials

Note on mathematical model and related issues

B.C.'s ; k; S;

Governing equation:

$$\bar{\nabla}.(k\bar{\nabla}T)+S=0$$

Required problem-specific information: k and S; details of calculation domain; and boundary conditions } If well posed,

Solutions are generally possible

Note on methods of solution

The mathematical model may be solved using:

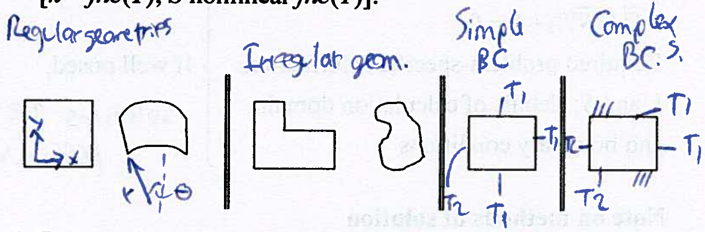
1. Analytical Methods
2. Graphical Methods
3. Heat Conduction Shape Factors
4. Numerical Methods (MECH-462, FEA; Suggested reading: Chap. 4, Sec. 4.4 and 4.5 in the textbook, Incropera et al., 2007)

Finite diff. Method (FDM) & Finite Element Method (FEM)

Brief discussion of analytical methods

1. Usually involve product-solution technique; principle of superposition; separation of variables; and infinites series:
   - Cartesian geometries – Fourier series
   - Cylindrical geometries – Bessel function series
   - Spherical geometries – Legend & polynomial series

2. Work well in linear problems with: regular geometries; constant k and S; and simple boundary conditions.
3. Do not work well in problems with one or more of the following features: irregular geometries; inhomogeneous materials; complex boundary conditions; or nonlinearities [$k = fnc(T)$; S nonlinear $fnc(T)$].

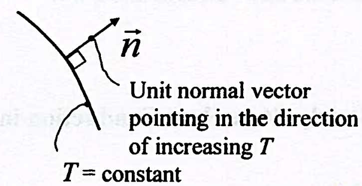


4. Even when they work, analytical methods produce solutions that typically involve infinite series: Thus, if specific solutions are needed for designs, computers are required.
5. Main use today: produce design charts; develop heat conduction shape factors; serve as checks on numerical solutions.
6. Suggested reading: Chapter 4, Section 4.2, in the textbook, Incropera et al., 2007.

Graphical method

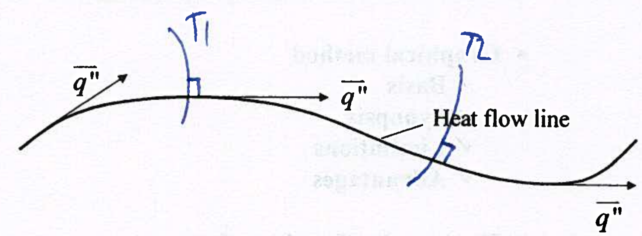
Basis: $\overline{q}'' = -k\bar{\nabla}T$ } Fourier's law of heat conduction

$\bar{\nabla}T$: Represents both in magnitude and direction, the maximum spatial rate of *increase* of temperature.



$$\bar{\nabla}T \equiv \left(\frac{\partial T}{\partial n}\right)\vec{n}$$

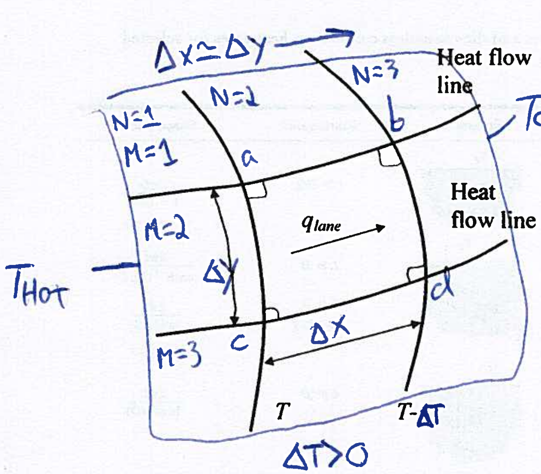
- Thus, $\overline{q}''$ is normal ($\perp$ 90°) to isotherms
- Heat flow line [analogous to streamline in fluid dynamics]: a curve that is tangent to $\overline{q}''$ at all points along it.
  - ➢ Schematic



∴ [Heat Flow Lines] [Isotherms]

Synopsis: This technique requires the following: (i) a drawing (by the user) of a network of curvilinear squares, using isotherms and heat flow lines; and (ii) some approximate analytical considerations

$$\frac{ab+cd}{2} \simeq \frac{ac+bd}{2}$$



$$q_{lane} = K \frac{(\Delta T)_{drop}}{\Delta x}(\Delta y L) \quad \boxed{\Delta x = \Delta y}$$

$$q_{lane} \approx K (\Delta T)_{drop} L$$

$$q_{total} = M \left[\frac{(\Delta T)_{overall-drop}}{N}\right] kL$$

$$q_{total} = \frac{M}{N} L k (T_{hot} - T_{cold})$$

L: Length normal to the page

N: # of temperature increments

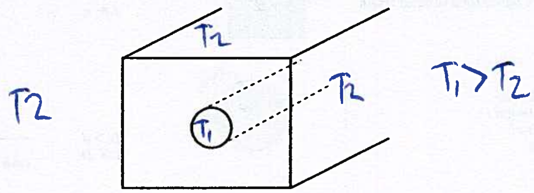
M: # of heat flow lanes

- Suggested reading: Chapter 4, Section **4S.1**, in the textbook (Supplemental and Website Material)

Limitations (above-mentioned graphical method):

a) 2-D, S.S., k = constant, $S = 0$

b) Applicable to problems in which there are only two (different) uniform boundary temperatures (T_{hot} and T_{cold})



c) Accuracy depends on the skill and patience of the user

Advantages (above-mentioned graphical method):

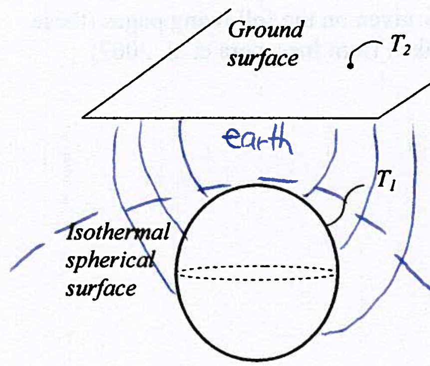
a) Quick way of determining heat transfer rates and isotherms in problems with the following features: 2-D, S.S., k = constant, $S = 0$, only two uniform boundary temperatures, and irregular domains

b) Provides physical insights

Heat conduction shapes factors [Section 4.3, Textbook]

Applicable to problems that allow the following approximations and restrictions: S.S., k = constant, $S = 0$, and only two uniform boundary temperatures

For example:



$$q_{total,\, T_1 \to T_2} \approx \frac{(T_1 - T_2)}{R_{th}}$$

$$R_{th} = \frac{1}{S k}$$

S: Heat transfer conduction shape factor [m]

Limitations (heat conduction shape factors):

a) S.S., k = constant, $\dot{S} = 0$, and only two uniform boundary temperatures

b) Only total rate of heat transfer can be calculated

c) Temperature distribution and related quantities cannot be calculated

Advantages (heat conduction shape factors):

a) Quick and effective way of doing design calculations

b) Can be used to develop the thermal resistance for heat conduction problems that have complex geometries, but respect the above-mentioned limitations

Table of heat conduction shape factors (S)

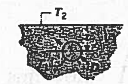
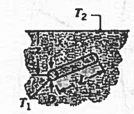
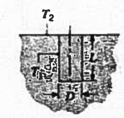
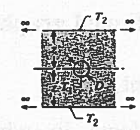
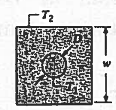
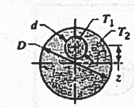
➤ Expressions for S given on the following pages (these expressions are taken from Incropera et al. 2007)

➤ $q_{total \atop T_1 \to T_2} = S\,k\,(T_1 - T_2)$

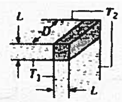
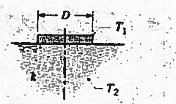
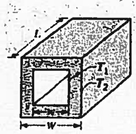
$$R_{th} = \frac{1}{S\,k}$$

TABLE 4.1 Conduction shape factors and dimensionless conduction heat rates for selected systems.

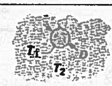
(a) Shape factors [$q = Sk(T_1 - T_2)$]

System	Schematic	Restrictions	Shape Factor
Case 1 Isothermal sphere buried in a semi-infinite medium		$z > D/2$	$\dfrac{2\pi D}{1 - D/4z}$
Case 2 Horizontal isothermal cylinder of length L buried in a semi-infinite medium		$L \gg D$ $L \gg D$, $z > 3D/2$	$\dfrac{2\pi L}{\cosh^{-1}(2z/D)}$ $\dfrac{2\pi L}{\ln(4z/D)}$
Case 3 Vertical cylinder in a semi-infinite medium		$L \gg D$	$\dfrac{2\pi L}{\ln(4L/D)}$
Case 4 Conduction between two cylinders of length L in infinite medium		$L \gg D_1, D_2$ $L \gg w$	$\dfrac{2\pi L}{\cosh^{-1}\left(\dfrac{4w^2 - D_1^2 - D_2^2}{2D_1D_2}\right)}$
Case 5 Horizontal circular cylinder of length L midway between parallel planes of equal length and infinite width		$z \gg D/2$ $L \gg z$	$\dfrac{2\pi L}{\ln(8z/\pi D)}$
Case 6 Circular cylinder of length L centered in a square solid of equal length		$w > D$ $L \gg w$	$\dfrac{2\pi L}{\ln(1.08\, w/D)}$
Case 7 Eccentric circular cylinder of length L in a cylinder of equal length		$D > d$ $L \gg D$	$\dfrac{2\pi L}{\cosh^{-1}\left(\dfrac{D^2 + d^2 - 4z^2}{2Dd}\right)}$

(Table extracted from Incropera et al. 2007)

System	Schematic	Restrictions	Shape Factor
Case 8 Conduction through the edge of adjoining walls		$D > 5L$	$0.54D$
Case 9 Conduction through corner of three walls with a temperature difference ΔT_{1-2} across the walls		$L \ll$ length and width of wall	$0.15L$
Case 10 Disk of diameter D and temperature T_1 on a semi-infinite medium of thermal conductivity k and temperature T_2		None	$2D$
Case 11 Square channel of length L		$\frac{W}{w} < 1.4$	$\frac{2\pi L}{0.785 \ln (W/w)}$
		$\frac{W}{w} > 1.4$	$\frac{2\pi L}{0.930 \ln (W/w) - 0.050}$
		$L \gg W$	

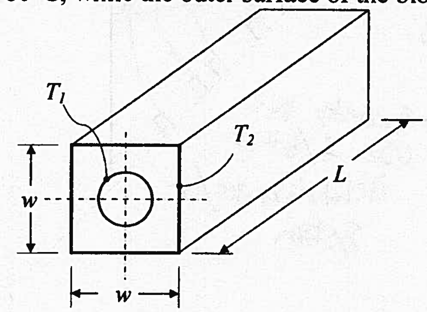
(b) Dimensionless conduction heat rates [$q = q_{ss}^* k A_s (T_1 - T_2)/L_c$; $L_c = (A_s/4\pi)^{1/2}$]

System	Schematic	Active Area, A_s	q_{ss}^*
Case 12 Isothermal sphere of diameter D and temperature T_1 in an infinite medium of temperature T_2		πD^2	1
Case 13 Infinitely thin, isothermal disk of diameter D and temperature T_1 in an infinite medium of temperature T_2		$\frac{\pi D^2}{2}$	$\frac{2\sqrt{2}}{\pi} = 0.900$
Case 14 Infinitely thin rectangle of length L, width w, and temperature T_1 in an infinite medium of temperature T_2		$2wL$	0.932
Case 15 Cuboid shape of height d with a square footprint of width D and temperature T_1 in an infinite medium of temperature T_2		$2D^2 + 4Dd$	d/D — q_{ss}^*: 0.1 — 0.943; 1.0 — 0.956; 2.0 — 0.961; 10 — 1.111

(Table extracted from Incropera et al. 2007)

Example:

A hole of diameter $D = 1.0$ m is drilled through the center of a solid block of square cross section with $w = 2$ m on a side. The hole is drilled along the length, $L = 6$ m of the block. The block thermal conductivity is $k = 200$ W/m-°C. A warm fluid passing through the hole maintains an inner surface temperature of $T_1 = 80$°C, while the outer surface of the block is kept at $T_2 = 20$°C.

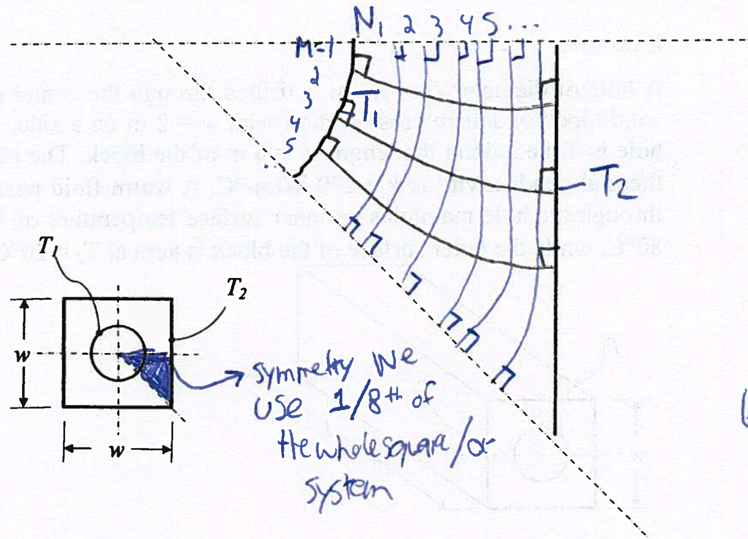


a- Using the flux plot method (graphical method) determine the rate of heat transfer through the block.

b- Using results obtained in part (a), what is the shape factor for this system?

c- Determine the accuracy of your result by referring to Table 3.1 of the textbook (shape factors for prescribed systems).

Ans.: a) q □ 576 kW; b) S □ 48 m; c) $S_{Table-3.1} = 48.95$ m



a) $q_{tot\,1/8} = \frac{M}{N} K (T_1 - T_2) \times L = \frac{5}{2}(200)(80-26)6$

$= 72000\ W$

$q_{tot} = 8 \times q_{tot\,1/8} = 576000\ W$

b) $q = SK(T_1 - T_2) = \frac{ML}{N} K (T_1 - T_2)\, 8$

$S = \cancel{4/8}\, \frac{ML}{N} 8 = 48m$

c) Tables: $S = \frac{2\pi L}{Ln(1.08w/D)} = \frac{2\pi 6}{Ln\left(\frac{1.08 \times 2}{1}\right)} = \boxed{48.95m}$

MECH-375 HEAT TRANSFER - HANDOUT # 5

Topics

➢ **Multidimensional Unsteady Heat Conduction in Isotropic Materials**

- **Note on mathematical model and related issues**
- **Note on methods of solution**
- **Lumped parameter analysis (LPA)**
- **Use of LPA in an experimental technique for the determination of h_{av}**
- **Semi-infinite solids**
- **Conduction-convection systems with Bi > 0.1**
  - ✓ **One-dimensional systems: one-term solutions; design charts**
  - ✓ **Multidimensional systems: product solution and other techniques**

Multidimensional Unsteady Heat Conduction in Isotropic Materials

Note on mathematical model and related issues

Governing equation:

$$\vec{\nabla}.(k\vec{\nabla}T) + S = \rho c_p \frac{\partial T}{\partial t}$$

$$\rho c_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + S$$

Required problem-specific information:
$k, S, \rho, and\, c_p$; details of calculation domain; boundary conditions; and initial condition

Mathematical model If well posed, Solutions are possible

Note on methods of solution

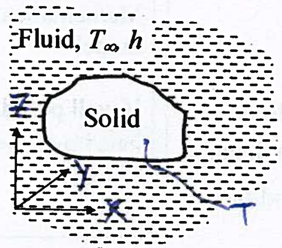
The mathematical model may be solved using:

1. Analytical methods involving product-solution technique, principle of superposition, separation of variables, and infinite series (see Lecture Handout # 4 for notes on limitations and usefulness of such methods)
2. Graphical Methods (Bender-Schmidt Plots)
3. Lumped parameter analysis (LPA)
4. Laplace transform
5. Design charts Heisler; Gröber
6. Numerical Methods (MECH- 462, FEA; Suggested reading: Chapter 5, Section 5.10, in the Textbook, Incopera et al. 2007)

- FDM
- FVM
- FEM

Lumped parameter analysis (LPA)
[Will be developed and demonstrated in this course in the context of conduction-convection systems]

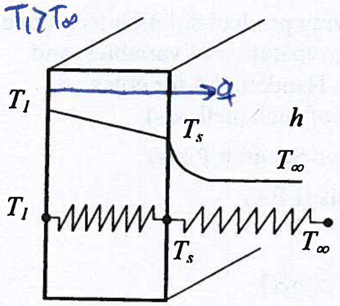
Key idea:



$T = T(x, y, z, t) \Rightarrow T(t)$

Aside: Consider S.S., $S = 0$, k = constant heat conduction in a plane wall:

$T_1 > T_\infty$



$$q = \frac{T_1 - T_\infty}{R_{th\,wall} + R_{th\,conv}} = \frac{T_s - T_\infty}{R_{th\,conv}}$$

$$\text{or } T_1 - T_\infty = [1 + \frac{R_{th\,wall}}{R_{th\,conv}}](T_s - T_\infty)$$

$\boxed{\text{Biot \#:}}$ → dimensionless

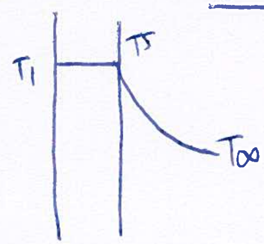
$$\text{Bi} = \frac{R_{th,wall}}{R_{th,conv}} = \frac{L/(k_{solid}A_{c.s.})}{1/(hA_{c.s.})} = \frac{hL}{k_{solid}}$$

$$\rightarrow T_1 - T_\infty = [1 + \text{Bi}](T_s - T_\infty)$$

If Bi << 1, $(T_1 - T_\infty) \simeq (T_s - T_\infty)\} \Rightarrow \boxed{T_1 = T_s}$

Conclusion: If Bi << 1, LPA is VALID

If B<<1

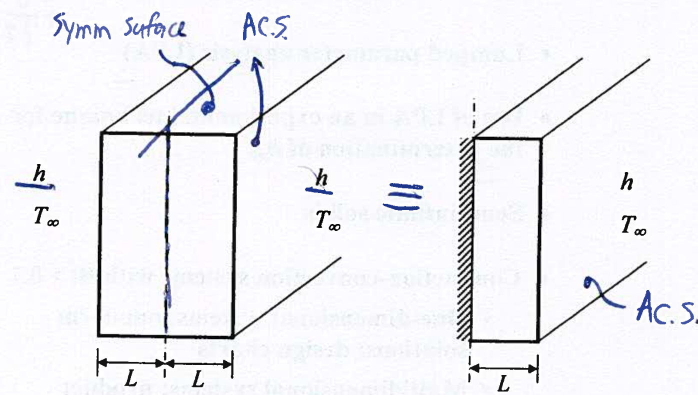


Notes:

1) In engineering practice, if Bi ≤ 0.1, then even in unsteady problems with/without source term, LPA is considered valid
2) In general, Bi $\equiv hL_c/k_{solid}$. Here, L_c is the characteristic length:

$$L_c = \frac{\text{Volume of the Solid}}{\text{Surface Area Exposed to Convection System}}$$

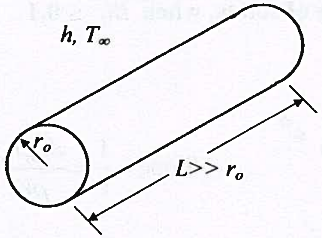
Example 1: Convectively cooled plane wall



$$L_c = \frac{(2L)A_{cs}}{2A_{cs}} = L \qquad L_c = \frac{LA_{cs}}{A_{cs}} = L$$

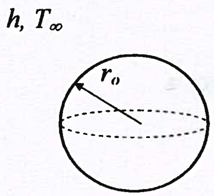
$$Bi = \frac{hL}{k_{solid}}$$

Example 2: Convectively cooled long solid cylinder



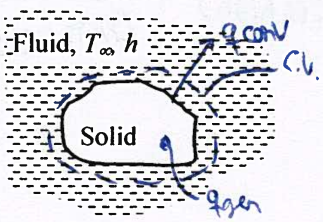
$$L_c = \frac{\pi r_o^2 L}{2\pi r_o L} = \frac{r_o}{2}$$

Example 3: Convectively cooled solid sphere



$$L_c = \frac{4/3\,\pi r_o^3}{4\pi r_o^2} = \frac{r_o}{3}$$

Governing equation [LPA]:



$$\int_V S\,dV - A_{\substack{surf\\conv}} h(T - T_\infty) = \rho V c \frac{dT}{dt}$$

Energy balance on solid:

$$q_{gen} - q_{conv.} - \cancel{\dot{W}_{by\,sys}}^{\,0} = \frac{dE_{sys}}{dt}$$

$$q_{gen} = \int_{vol} S\, dV_{ol}$$

$$q_{conv.} = h A_s (T - T_\infty)$$

$$\frac{dE_{sys}}{dt} = \rho V C_v \frac{dT}{dt}$$

$$\dot{W}_{by\,sys} = \emptyset$$

if ρ_{solid} = constant

$C_v = C_p = C$

LPA solution: ρ = constant, $\boxed{S = 0}$, and (h, T_∞, c) all constant

I.C. : at $t = 0$ $T = T_{ini}$ = constant

Governing Equation: $-A_{\substack{surf\\conv}} h(T - T_\infty) = \rho V c \frac{dT}{dt}$

Let $\theta = (T - T_\infty)$, then: $\rightarrow A_s h \theta = \rho V c \frac{d\theta}{dt}$

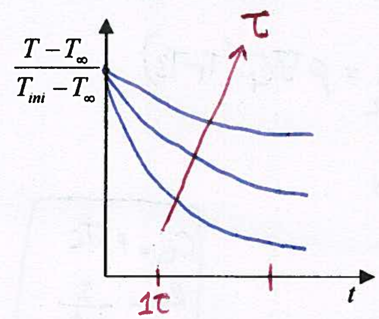
or $\frac{d\theta}{\theta} = (-A_s h / \rho V c)\,dt$

Integrating with respect to t:

$$\ln\theta = \frac{-A_s * h}{\rho V c} t + C_1$$

At $t = 0$, $\theta = (T_{ini} - T_\infty) = \theta_{ini}$

Thus, $\ln\left(\frac{\theta}{\theta_{ini}}\right) = -\frac{A_{surf} h}{\rho V c} t$ *or* $\frac{T - T_\infty}{T_{ini} - T_\infty} = \exp\left[-\frac{A_{surf} h}{\rho V c} t\right] = \exp^{-t/\tau}$



$$\tau = \underbrace{\left[\frac{\rho V c}{A_{surf} h}\right]}_{\text{time constant}} \rightarrow [\,s\,]$$

Notes:

1) when $t = one\,time\,constant, \tau \Rightarrow \dfrac{T-T_\infty}{T_{ini}-T_\infty} = e^{-1} \cong 0.368$

2) $\dfrac{A_{surf}h}{\rho Vc}t = h\underbrace{\dfrac{1}{(V/A_{surf})}}_{L_c}\dfrac{1}{\rho c}t \times \dfrac{k_{solid}}{k_{solid}} \times \dfrac{L_c}{L_c} = \left(\dfrac{hL_c}{K_{solid}}\right)\left(\dfrac{K_{solid}}{\rho C}\right)\left(\dfrac{t}{L_c^2}\right)$

Let $\alpha \equiv \dfrac{K}{\rho C_p}$ } $[m^2/s]$ Thermal diffusivity

and $Fo \equiv \dfrac{\alpha t}{L_c^2}$ Fourier Number [dimensionless time]

$Fo \propto \dfrac{\text{Rate of H cond}}{\text{Rate of therm. energy storage}}$

then, $\dfrac{\theta}{\theta_{ini}} = \exp[-BiFo]$ or $\ln\left(\dfrac{\theta}{\theta_{ini}}\right) = -BiFo$

3) <u>Total</u> heat loss during $t_1 \le t \le t_2$:

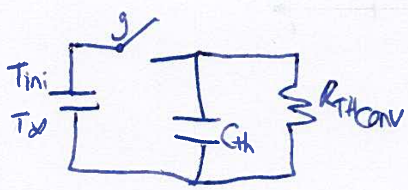
$@t=t_1 \rightarrow T=T_1$

$@t=t_2 \rightarrow T=T_2$

$Q = \left[m_{sys}\, C\,(T_{ini}-T)\right]_{1-2}$

$[J]\quad Q_{t_1-t_2} = \rho V C_p (T_1 - T_2)$

4) Electrical network analogy

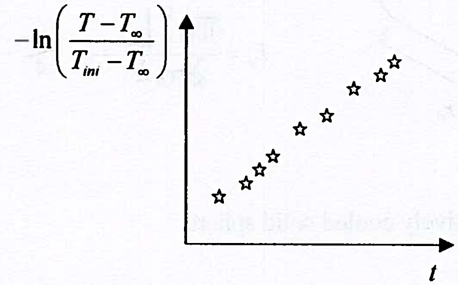


$C_{th} = \rho V C$

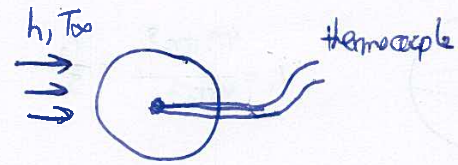
$R_{th} = \dfrac{1}{hA_s}$

$\dfrac{1}{\tau} = \dfrac{1}{R_{th}C_{th}}$

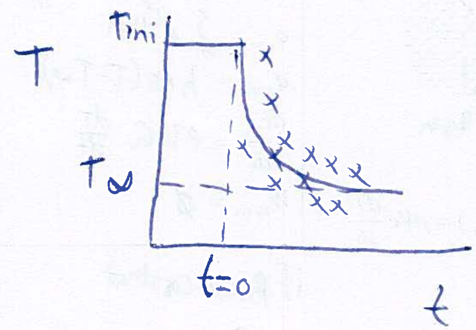
Use of LPA in an experimental technique for the determination of h_{av} on surfaces of solids, when $Bi_{L_c} \le 0.1$



$\text{Slope} = \dfrac{1}{\tau} = \dfrac{A_{surf}h_{av}}{\rho Vc}$



• Small sphere, copper
• radius is small, K is high } Assume $Bi \ll 0.1$
LPA is valid



Example ☆ Get from Emre

A long wire of 1 mm diameter is submerged in an oil bath of temperature $T_\infty = 25$°C. The wire has an electrical resistance per unit length of 0.01 Ω/m. If a current of $I = 100$ A flows through the wire and the convection coefficient is $h = 500$ W/m^2-K:

a) What is the steady state temperature of the wire?

b) From the time the current is applied, how long does it take for the wire to reach a temperature that is within 1°C of the steady-state value? Initially, with no current, it may be assumed that the wire is uniformly at the same temperature of the oil, T_∞.

$\rho_{\text{wire}} = 8000$ kg/m^3

$c_{\text{wire}} = 500$ J/kg-K

$k_{\text{wire}} = 20$ W/m-K

Calculations were done for this copy wire, It will too much to copy'

Ans.: a) 88.7°C; b) 8.31 s

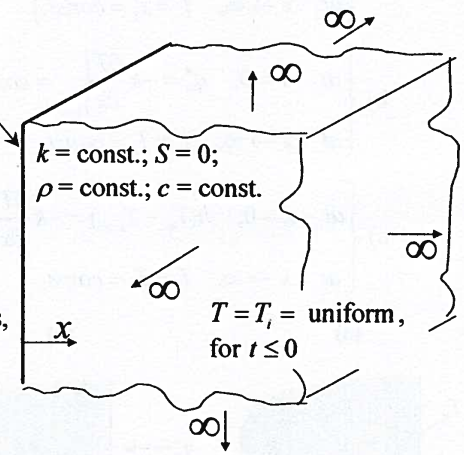
Transient heat conduction in semi-infinite solids

Common B.C.s (imposed) on the surface: for $t > 0$

a) Constant temperature, $T_o \neq T_i$

b) Constant heat flux, q''_0

c) Convection boundary conditions, h, T_∞



Mathematical model:

Governing Equation: $\dfrac{\partial}{\partial x}\left(k\dfrac{\partial T}{\partial x}\right) = \rho c\dfrac{\partial T}{\partial t}$

k is constant and thermal diffusivity is defined as $\alpha \equiv \dfrac{k}{\rho c}$, thus:

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

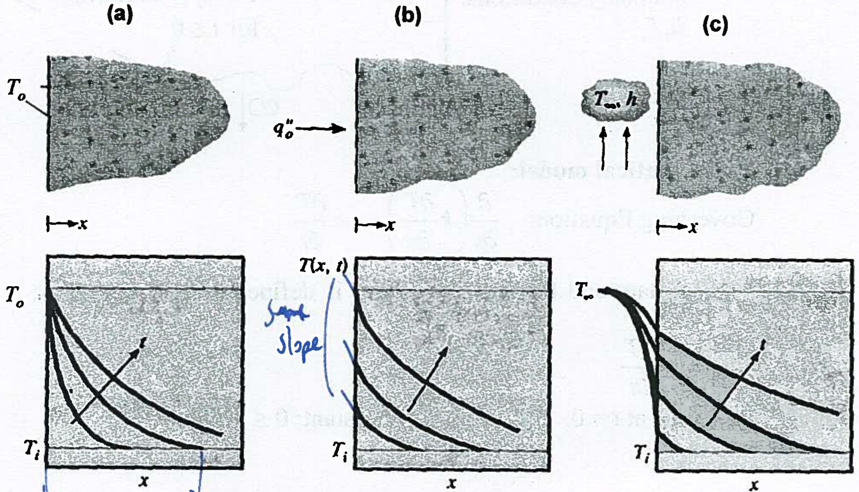
<u>I.C.:</u> at $t = 0$, $T = T_i$ = constant; $0 \le x \le \infty$

<u>B.C.s</u>

a) $\left\{\begin{array}{lll} at & x=0, & T=T_o=const. \\ at & x\to\infty, & T=T_i=const. \end{array}\right\}$ *for all* $t>0$

b) $\left\{\begin{array}{lll} at & x=0, & q_o''=-k\left.\dfrac{\partial T}{\partial x}\right|_{x=0}=const. \\ at & x\to\infty, & T=T_i=const. \end{array}\right\}$ *for all* $t>0$

c) $\left\{\begin{array}{lll} at & x=0, & h(T_\infty-T_{x=0})=-k\left.\dfrac{\partial T}{\partial x}\right|_{x=0} \\ at & x\to\infty, & T=T_i=const. \end{array}\right\}$ *for all* $t>0$



δ = penetration depth

Figures are from Incropera and DeWitt, 1994.

$\delta=\delta(t)$

Solutions:

a) $\left\{ \dfrac{T(x,t)-T_o}{T_i-T_o}=\text{erf}\left(\dfrac{x}{2\sqrt{\alpha t}}\right)\right.$

Penetration depth: $\delta(t)$ Case A
When $x=\delta(t)$,
$$\frac{T(\delta,t)-T_0}{T_i-T_o}=0.99=erf\left(\frac{\delta}{2\sqrt{\alpha t}}\right)$$

Notes:

1) Error function:

$\text{erf}(\eta)\equiv\dfrac{2}{\sqrt{\pi}}\int_0^\eta e^{-\eta^2}d\eta$; Values of $\text{erf}(\eta)$ are tabulated in Table 5-1 (page 14) and also in Table A-1, page 593 of Holman, 2002.

2) Complementary error function: $\text{erfc}(\eta)\equiv 1-\text{erf}(\eta)$

b) $\left\{ T(x,t)-T_i=\dfrac{2q_o''\sqrt{\alpha t/\pi}}{k}\exp\left(\dfrac{-x^2}{4\alpha t}\right)-\dfrac{q_o''x}{k}\text{erfc}\left(\dfrac{x}{2\sqrt{\alpha t}}\right)\right.$

c) $\left\{ \dfrac{T(x,t)-T_i}{T_\infty-T_i}=\text{erfc}\left(\dfrac{x}{2\sqrt{\alpha t}}\right)-\exp\left[\dfrac{hx}{k}+\dfrac{h^2\alpha t}{k^2}\right]\text{erfc}\left[\dfrac{x}{2\sqrt{\alpha t}}+\dfrac{h\sqrt{\alpha t}}{k}\right]\right.$

Note: Solution to case c) is presented graphically in the figure on the next page (extracted from your textbook, Holman).

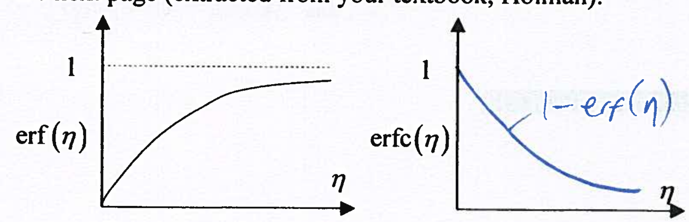
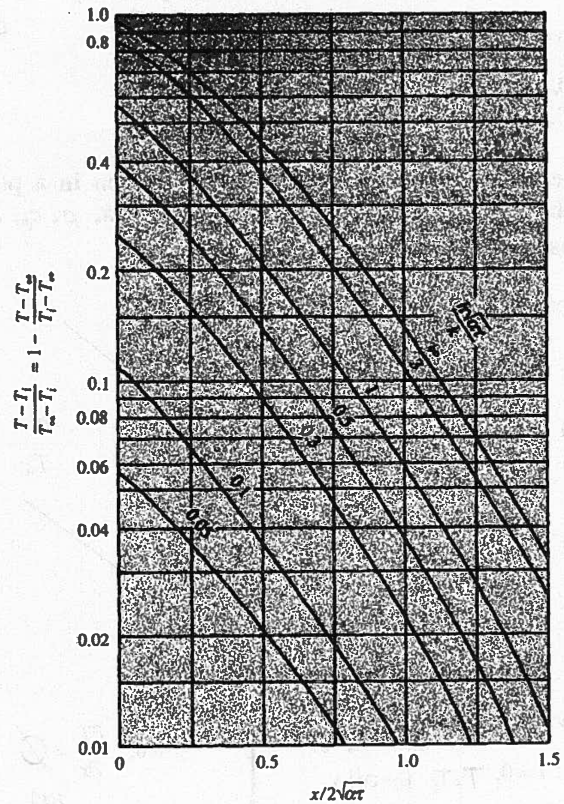


Figure 4-5 | Temperature distribution in the semi-infinite solid with convection boundary condition.



(Figure extracted from Heat Transfer by J.P. Holman, 9th Edition, 2002)

Table 5.1: Values of error function

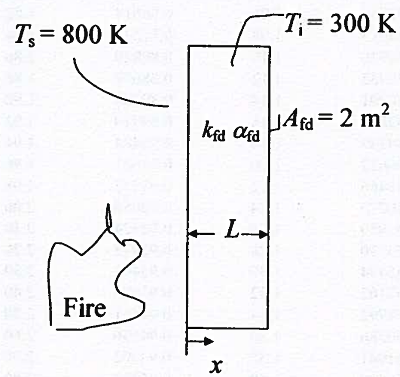
x	$erf(x)$	x	$erf(x)$	x	$erf(x)$
0.00	0.00000	0.76	0.71754	1.52	0.96841
0.02	0.02256	0.78	0.73001	1.54	0.97059
0.04	0.04511	0.80	0.74210	1.56	0.97263
0.06	0.06762	0.82	0.75381	1.58	0.97455
0.08	0.09008	0.84	0.76514	1.60	0.97635
0.10	0.11246	0.86	0.77610	1.62	0.97804
0.12	0.13476	0.88	0.78669	1.64	0.97962
0.14	0.15695	0.90	0.79691	1.66	0.98110
0.16	0.17901	0.92	0.80677	1.68	0.98249
0.18	0.20094	0.94	0.81627	1.70	0.98379
0.20	0.22270	0.96	0.82542	1.72	0.98500
0.22	0.24430	0.98	0.83423	1.74	0.98613
0.24	0.26570	1.00	0.84270	1.76	0.98719
0.26	0.28690	1.02	0.85084	1.78	0.98817
0.28	0.30788	1.04	0.85865	1.80	0.98909
0.30	0.32863	1.06	0.86614	1.82	0.98994
0.32	0.34913	1.08	0.87333	1.84	0.99074
0.34	0.36936	1.10	0.88020	1.86	0.99147
0.36	0.38933	1.12	0.88679	1.88	0.99216
0.38	0.40901	1.14	0.89308	1.90	0.99279
0.40	0.42839	1.16	0.89910	1.92	0.99338
0.42	0.44749	1.18	0.90484	1.94	0.99392
0.44	0.46622	1.20	0.91031	1.96	0.99443
0.46	0.48466	1.22	0.91553	1.98	0.99489
0.48	0.50275	1.24	0.92050	2.00	0.99532
0.50	0.52050	1.26	0.92524	2.10	0.997020
0.52	0.53790	1.28	0.92973	2.20	0.998137
0.54	0.55494	1.30	0.93401	2.30	0.998857
0.56	0.57162	1.32	0.93806	2.40	0.999311
0.58	0.58792	1.34	0.94191	2.50	0.999593
0.60	0.60386	1.36	0.94556	2.60	0.999764
0.62	0.61941	1.38	0.94902	2.70	0.999866
0.64	0.63459	1.40	0.95228	2.80	0.999925
0.66	0.64938	1.42	0.95538	2.90	0.999959
0.68	0.66378	1.44	0.95830	3.00	0.999978
0.70	0.67780	1.46	0.96105	3.20	0.999994
0.72	0.69143	1.48	0.96365	3.40	0.999998
0.74	0.70468	1.50	0.96610	3.60	1.000000

Example

It is desired to determine the **minimum allowable** wall thickness for a fire door (2 m^2). The door is designed according to the criterion that once exposed to fire in one side (assume surface temperature reaches 800 K) its midpoint temperature will not exceed 400 K after 1 h. The door is initially at a uniform temperature of 300 K.

a) Find the door thickness; b) Find the heat flux at the surface of the door on the fire side and after 1 h; c) What is the total amount of energy (J) that will be transferred from the fire over the period of 1 h?

k_{fd} = 5 W/m-K; $\alpha_{fd} = 7 \times 10^{-7}$ m^2/s



Ans.: a) 18.12 cm; b) 28097.3 W/m^2, c) 404.6 MJ

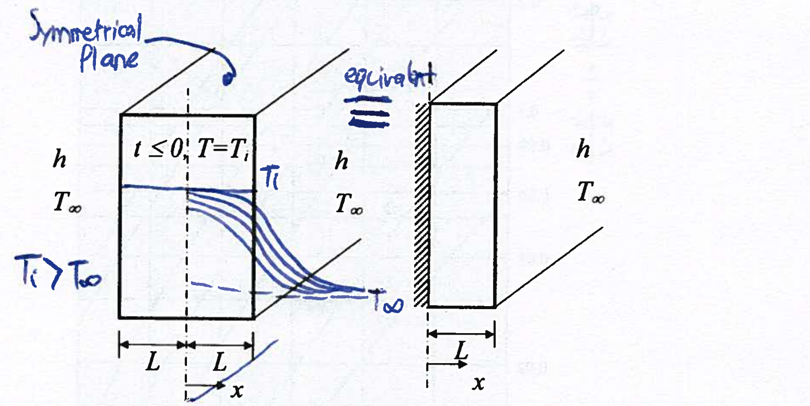
Transient Heat Conduction with Convection B.C.: Bi > 0.1 [spatial variations within solid are important]

LPA not Valid!!

Governing equation: $\vec{\nabla}.(k\vec{\nabla}T) + S = \rho c_p \dfrac{\partial T}{\partial t}$

k, ρ, c_p, and S } All specified
B.C.s and I.C. } Problem specific (specified)

One-dimensional unsteady heat conduction in a plane wall with symmetric convection B.C.s [S =0; k, ρ, c_p, h, T_∞ all constant; and $Bi_{L_c} > 0.1$]



Gov. equation: $\dfrac{\partial^2 T}{\partial x^2} = \dfrac{1}{\alpha}\dfrac{\partial T}{\partial t}$

I.C.: $t = 0$, $T = T_i$ for all x

$\therefore\ T = fnc[x, t, \rho, c_p, k, h, T_\infty, T_i]$

B.C.s:

$x = 0,\quad \dfrac{\partial T}{\partial x} = \emptyset$

$x = L,\quad -k\left.\dfrac{\partial T}{\partial x}\right|_{x=L} = h(T_{x=L} - T_\infty)$

- This problem may be solved analytically or numerically

Example for Handout 5, Slide #15

OCT.15,201

a) Case A: $\frac{T_{*}-T_s}{T_i-T_s} = erf\left(\frac{x}{2\sqrt{\alpha t}}\right)$

$$\frac{400-800}{300-800} = erf\left(\frac{L/2}{2\sqrt{\alpha t}}\right) = 0.8$$

Using table 5.1: "value of error function", we find using $erf(x)=0.8$

interpolate

$$\frac{L/2}{2\sqrt{\alpha t}} = 0.902 \quad \therefore L = 4\sqrt{7\times10^{-7}\times3600s}\times0.902 = 0.1812m$$

$\boxed{\text{ANS: } 18.12cm}$

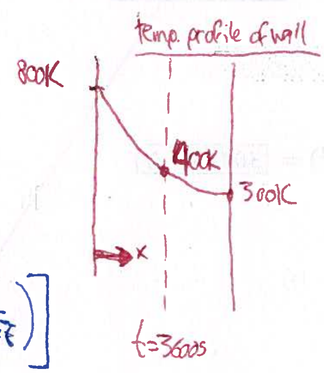
for Design Purposes, $L = 7\frac{1}{4}'' = 18.4cm$

- Penetration depth after 1 hour:

$$0.99 = \frac{T_{\delta pt}-T_s}{T_i-T_s} = erf\left[\frac{\delta}{2\sqrt{\alpha t}}\right] \Rightarrow \delta_{t=3600s} = 0.183m$$

$\delta < 7\frac{1}{4}''$ thus semi-infinite assumption is valid

b) $q_0'' = ?$



$$\rightarrow q''_{x=0} = -K\left.\frac{\partial T}{\partial x}\right|_{x=0}$$

what is: $q''_{x=0,\,t=3600s}$?

$$q''_{x=0,t} = -K\frac{\partial}{\partial x}\left[T_s + (T_i - T_s)\,erf\left(\frac{x}{2\sqrt{\alpha t}}\right)\right]$$

$$\frac{d}{dx}erf(u) = \frac{2}{\sqrt{\pi}}e^{-u^2}\left(\frac{du}{dx}\right)$$

$U = U(x)$

$$q''_{x=0,t} = -K\left[(T_i-T_s)\frac{1}{2\sqrt{\alpha t}}\frac{2}{\sqrt{\pi}}\exp\left(-\left[\frac{x}{2\sqrt{\alpha t}}\right]^2\right)\right]$$

$$\therefore \boxed{q''_{x=0,t} = \frac{K(T_s-T_i)}{\sqrt{\pi\alpha t}}}$$

$$\boxed{q''_{x=0,t=3600} = 28{,}097.3\,W/m^2}$$

c) What is total energy transferred from fire over 1 hr?

$$Q_{0\le t\le 3600s} = \int_0^{3600} q''_{x=0,t}\,A\,dt$$

$$= \int_0^{3600} \frac{k(T_s-T_i)}{\sqrt{\pi\alpha t}}\,A\,dt = \frac{k(T_s-T_i)A}{\sqrt{\pi\alpha}}\int_0^{3600}\frac{1}{\sqrt{t}}\,dt$$

$$\therefore Q = \frac{5(800-300)2}{\sqrt{\pi(7\times10^{-7})}}\left[2t^{1/2}\right]_0^{3600} = \boxed{404.6\,MJ}$$

- It is important at this stage to look at dimensionless analysis and its advantages

Dimensionless variables:

$$\theta^* = \frac{T(x,t)-T_\infty}{T_i - T_\infty};$$

$$x^* = x/L;$$

$$Fo = t^* = \frac{\alpha t}{L^2} = \text{Fourier\#}$$

Dimensionless governing equation:

$$\frac{\partial^2 \theta^*}{\partial x^{*2}} = \frac{\partial \theta^*}{\partial t^*}$$

I.C.: $t^* = 0,$ $\theta^* = 1$

B.C.s:

$$x^* = 0, \quad \left.\frac{\partial \theta^*}{\partial x^*}\right|_{x^*=0} = 0$$

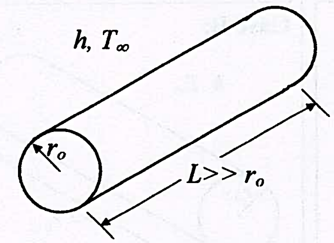
$$x^* = 1, \quad \left.\frac{\partial \theta^*}{\partial x^*}\right|_{x^*=1} = -\underbrace{\left(\frac{hL}{k}\right)}_{Bi_{Modified} = Bi_M}\theta^*_{x=1}$$

$$\therefore \ \theta^* = fnc[x^*, t^*; Bi_M] \quad where \quad Bi_M = \frac{hL}{k_{solid}}$$

Advantages of dimensionless analysis:

- θ^* is not dependent on particular values of the following: $x, t, L, \rho, c_p, k, h, T_\infty, T_i$
- For a prescribed geometry, θ^* is a function of x^*, t^*, Bi_M: only dimensionless parameter here is Bi_M
- Advantage is taken of this feature of the dimensionless formulation in the preparation of plans for experiments, general presentation of results, and preparation of design charts [Heisler (1947); Gröber (1961)]

One-dimensional unsteady heat conduction in a long solid cylinder with convection B.C. [S =0; k, ρ, c_p, h, T_∞ all constant; and $Bi_{L_c} > 0.1$]



Dimensionless variables:

$$\theta^* = \frac{T(r,t)-T_\infty}{T_i - T_\infty}; \ r^* = \frac{r}{r_o}; \ t^* = \frac{\alpha t}{r_o^2}$$

Dimensionless governing equation:

$$\frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r^*\frac{\partial \theta^*}{\partial r^*}\right) = \frac{\partial \theta^*}{\partial t^*}$$

I.C.: $t^* = 0,$ $\theta^* = 1$

B.C.s:

$$r^* = 0, \quad \left.\frac{\partial \theta^*}{\partial r^*}\right|_{r^*=0} = 0$$

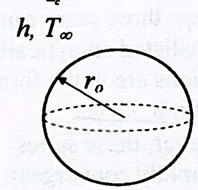
$$r^* = 1, \quad \left.\frac{\partial \theta^*}{\partial r^*}\right|_{r^*=1} = -\underbrace{\left(\frac{hr_o}{k_{solid}}\right)}_{Bi_M}\theta^*\big|_{r^*=1}$$

$$\therefore \ \theta^* = fnc[r^*, t^*; Bi_M]$$

$$where \quad Bi_M = \frac{hr_o}{k_{solid}}$$

$$Bi_{Lc} = \frac{hr_o/2}{K} \neq Bi_m$$

One-dimensional unsteady heat conduction in a solid sphere with convection B.C. [S =0; k, ρ, c_p, h, T_∞ all constant; and $Bi_{L_c} > 0.1$]



Dimensionless variables:

$$\theta^* = \frac{T(r,t)-T_\infty}{T_i - T_\infty}; \ r^* = \frac{r}{r_o}; \ t^* = \frac{\alpha t}{r_o^2}$$

Dimensionless governing equation:

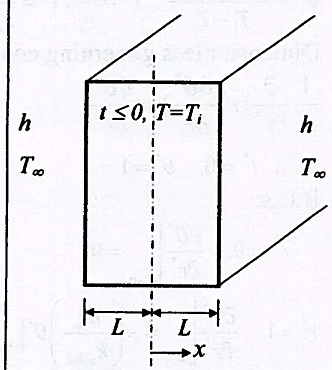
$$\frac{1}{r^{*2}}\frac{\partial}{\partial r^*}\left(r^{*2}\frac{\partial \theta^*}{\partial r^*}\right) = \frac{\partial \theta^*}{\partial t^*}; \ \text{I.C.: } t^* = 0, \quad \theta^* = 1$$

$$\text{B.C.s: } r^* = 0, \ \left.\frac{\partial \theta^*}{\partial r^*}\right|_{r^*=0} = 0; \ r^* = 1, \ \left.\frac{\partial \theta^*}{\partial r^*}\right|_{r^*=1} = -\underbrace{\left(\frac{hr_o}{k_{solid}}\right)}_{Bi_M}\theta^*\big|_{r^*=1}$$

$$\therefore \ \theta^* = fnc[r^*, t^*; Bi_M] \text{ where } Bi_M = \frac{hr_o}{k_{solid}}; \ Bi_{Lc} = \frac{hr_o/3}{K}$$

One-dimensional unsteady heat conduction in solids with convection B.C. [S =0; k, ρ, c_p, h, T_∞ all constant; and Bi_{L_c} > 0.1]

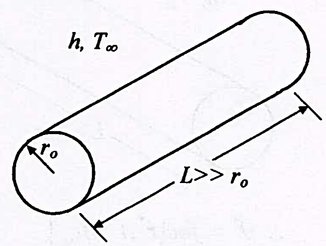
Case A:



Symmetrically cooled/heated plane wall

$$Bi_M = \frac{hL}{k_{solid}};\ x^* = \frac{x}{L};\ t^* = \frac{\alpha t}{L^2}$$

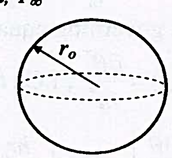
Case B:



Long solid cylinder

$$Bi_M = \frac{hr_o}{k_{solid}};\ r^* = \frac{r}{r_o};\ t^* = \frac{\alpha t}{r_o^2}$$

Case C:



Solid sphere

$$Bi_M = \frac{hr_o}{k_{solid}};\ r^* = \frac{r}{r_o};\ t^* = \frac{\alpha t}{r_o^2}$$

Notes:

- One-dimensional transient heat conduction in these three cases can be predicted analytically: solutions are in the form of infinite series;
- However, these series are rapidly convergent;
- For $t^* \ge 0.2$, one-term approximation of infinite series is excellent: [Error $\le \pm 2\%$]

For $t^* \ge 0.2$, use the one-term approximation presented below. Please read **Appendix C** of the textbook, Holman, 2002.

Case A: Symmetrically cooled/heated plane wall [Note: $\theta = (T - T_\infty)$]

$$\theta^*(x^*,t^*) = \frac{\theta}{\theta_i} = \frac{T(x,t)-T_\infty}{T_i - T_\infty} = \underbrace{C_B \exp[-A_B^2 t^*]}_{\theta^*(0,t^*)}\underbrace{\cos(A_B x^*)}_{\text{Radians}}$$

Where $x^* = x/L$; $t^* = \dfrac{\alpha t}{L^2}$

$$\theta^*(0,t^*) = \frac{\theta_0}{\theta_i} = \frac{T(x=0,t)-T_\infty}{T_i - T_\infty} = C_B \exp(-A_B^2 t^*)$$

$$\frac{Q}{Q_o} = 1 - [\theta^*(0,t^*)]\left[\frac{\sin(A_B)}{A_B}\right]$$

And $\begin{cases} Q = 2\int_0^t q_{x=L}\,dt\ [J] \\ Q_o = M_{sys} C_p (T_i - T_\infty)\ [J] \rightarrow Q_o = \text{total potential for heat loss} \end{cases}$

Notes: Radians

C_B, A_B are functions of Bi_M and the geometry

Table 5.2 (on page 22) provides values of C_B, A_B for different values of Bi_M and three different geometries

Case B: Long solid cylinder

$$\theta^*(r^*,t^*) = \frac{\theta}{\theta_i} = \frac{T(r,t)-T_\infty}{T_i - T_\infty} = \underbrace{C_B \exp[-A_B^2 t^*]}_{\theta^*(0,t^*)} J_0(A_B r^*)$$

Where $r^* = r/r_o$; $t^* = \dfrac{\alpha t}{r_o^2}$ | J_o: Bessel function of the first kind, order zero

$$\theta^*(r^*=0,t^*) = \frac{\theta_0}{\theta_i} = \frac{T(r=0,t)-T_\infty}{T_i - T_\infty} = C_B \exp[-A_B^2 t^*]$$

$$\frac{Q}{Q_o} = 1 - 2[\theta^*(r^*=0,t^*)]\left[\frac{J_1(A_B)}{A_B}\right]$$

J_1: Bessel function of the first kind, order one

and $\begin{cases} Q = \int_0^t q_{r=r_0}\,dt\ [J] \\ Q_o = M_{sys} C_p (T_i - T_\infty)\ [J] \end{cases}$

Case C: Solid sphere

$$\theta^*(r^*,t^*) = \frac{\theta}{\theta_i} = \frac{T(r,t)-T_\infty}{T_i - T_\infty} = \underbrace{C_B \exp[-A_B^2 t^*]}_{\theta^*(r^*=0,t^*)} \frac{\sin(A_B r^*)}{A_B r^*}$$

Where $r^* = r/r_o$; $t^* = \dfrac{\alpha t}{r_o^2}$

$$\theta^*(r^*=0,t^*) = \frac{\theta_0}{\theta_i} = \frac{T(r=0,t)-T_\infty}{T_i - T_\infty} = C_B \exp[-A_B^2 t^*]$$

$$\frac{Q}{Q_o} = 1 - 3[\theta^*(r^*=0,t^*)]\left[\frac{\sin(A_B) - A_B\cos(A_B)}{A_B^3}\right]$$

And $\begin{cases} Q = \int_0^t q_{r=r_o}\, dt \quad [J] \\ Q_o = M_s c_p (T_i - T_\infty) \quad [J] \end{cases}$

Notes:

- For cases A, B, and C, values of C_B, A_B as functions of Bi_M are given in Table 5.2 on the next page and also in Table 5.1, of the Textbook ($A_B \equiv \zeta_1$ & $C_B \equiv C_1$)
- Values of $J_0(\xi)$ and $J_1(\xi)$ as functions of ξ are given in Table 5.3 on the next page and also in Table B.4 Appendix B of the Textbook. Note: $dJ_0(\xi)/d\xi = -J_1(\xi)$
- Graphical representations of the above-mentioned one-term solutions for cases A, B, and C are presented in Heisler Charts. For these cases, graphical representations of Q/Q_o are given in design charts produced by Gröber et al. These charts are attached (see pages 23, 24, and 25).

Table 5.2
Values of A_B and C_B for different values of Bi_M and Cases A, B, and C

	Case A		Case B		Case C	
	Plane Wall		Long Cylinder		Sphere	
Bi_M	A_B	C_B	A_B	C_B	A_B	C_B
0.01	0.0998	1.0017	0.1412	1.0025	0.1730	1.0030
0.02	0.1410	1.0033	0.1995	1.0050	0.2445	1.0060
0.04	0.1987	1.0066	0.2814	1.0099	0.3450	1.0120
0.06	0.2425	1.0098	0.3438	1.0148	0.4217	1.0179
0.08	0.2791	1.0130	0.3960	1.0197	0.4860	1.0239
0.1	0.3111	1.0161	0.4417	1.0246	0.5423	1.0298
0.2	0.4328	1.0311	0.6170	1.0483	0.7593	1.0592
0.3	0.5218	1.0451	0.7465	1.0712	0.9208	1.0880
0.4	0.5932	1.0580	0.8516	1.0931	1.0528	1.1164
0.5	0.6533	1.0701	0.9408	1.1143	1.1656	1.1441
0.6	0.7051	1.0814	1.0185	1.1345	1.2644	1.1713
0.7	0.7506	1.0919	1.0873	1.1539	1.3525	1.1978
0.8	0.7910	1.1016	1.1490	1.1724	1.4320	1.2236
0.9	0.8274	1.1107	1.2048	1.1902	1.5044	1.2488
1.0	0.8603	1.1191	1.2558	1.2071	1.5708	1.2732
2.0	1.0769	1.1785	1.5995	1.3384	2.0288	1.4793
3.0	1.1925	1.2102	1.7887	1.4191	2.2889	1.6227
4.0	1.2646	1.2287	1.9081	1.4698	2.4556	1.7202
5.0	1.3138	1.2403	1.9898	1.5029	2.5704	1.7870
6.0	1.3496	1.2479	2.0490	1.5253	2.6537	1.8338
7.0	1.3766	1.2532	2.0937	1.5411	2.7165	1.8674
8.0	1.3978	1.2570	2.1286	1.5526	2.7654	1.8920
9.0	1.4149	1.2598	2.1566	1.5611	2.8044	1.9106
10.0	1.4289	1.2620	2.1795	1.5677	2.8363	1.9249
20.0	1.4961	1.2699	2.2881	1.5919	2.9857	1.9781
30.0	1.5202	1.2717	2.3261	1.5973	3.0372	1.9898
40.0	1.5325	1.2723	2.3455	1.5993	3.0632	1.9942
50.0	1.5400	1.2727	2.3572	1.6002	3.0788	1.9962
100.0	1.5552	1.2731	2.3809	1.6015	3.1102	1.9990

Table 5.3
Zeroth- and first-order Bessel functions of the first kind

ξ	$J_0(\xi)$	$J_1(\xi)$
0.0	1.0000	0.0000
0.1	0.9975	0.0499
0.2	0.9900	0.0995
0.3	0.9776	0.1483
0.4	0.9604	0.1960
0.5	0.9385	0.2423
0.6	0.9120	0.2867
0.7	0.8812	0.3290
0.8	0.8463	0.3688
0.9	0.8075	0.4059
1.0	0.7652	0.4400
1.1	0.7196	0.4709
1.2	0.6711	0.4983
1.3	0.6201	0.5220
1.4	0.5669	0.5419
1.5	0.5118	0.5579
1.6	0.4554	0.5699
1.7	0.3980	0.5778
1.8	0.3400	0.5815
1.9	0.2818	0.5812
2.0	0.2239	0.5767
2.1	0.1666	0.5683
2.2	0.1104	0.5560
2.3	0.0555	0.5399
2.4	0.0025	0.5202
2.6	−0.0968	−0.4708
2.8	−0.1850	−0.4097
3.0	−0.2601	−0.3391
3.2	−0.3202	−0.2613

Notes:

$$(Bi_M)_{Plane\,Wall} = \frac{hL}{k_{solid}}; \quad (Bi_M)_{Long\,Solid\,Cylinder} = \frac{hr_o}{k_{solid}}; \quad (Bi_M)_{Solid\,Sphere} = \frac{hr_o}{k_{solid}}$$

$Bi_{LC} \neq Bi_M$

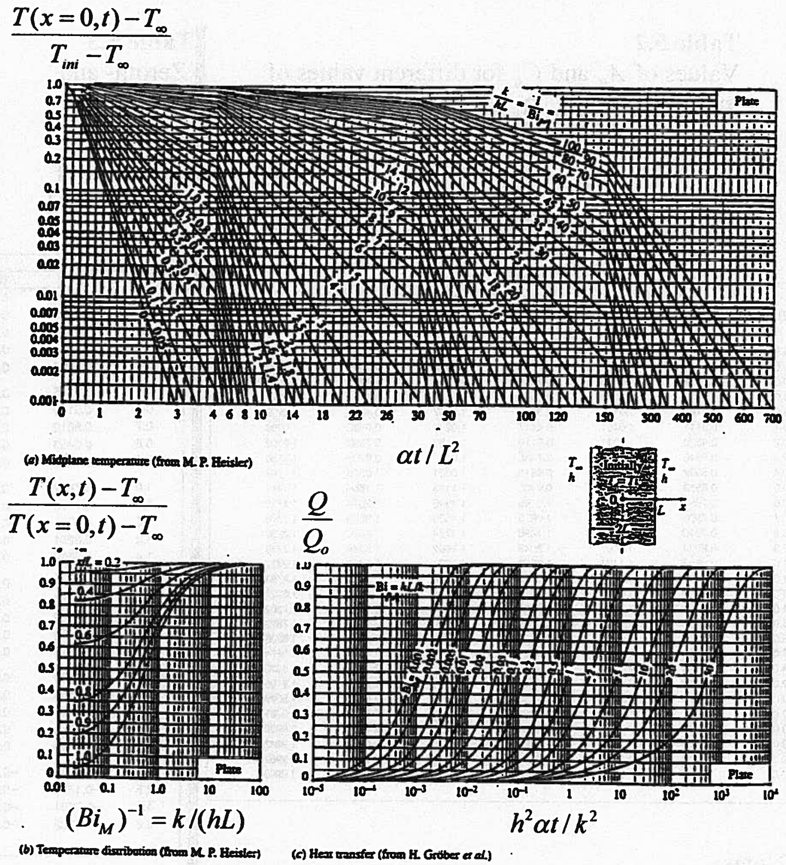


Figure 5.1: Heisler and Gröber charts for a symmetrically cooled plane wall

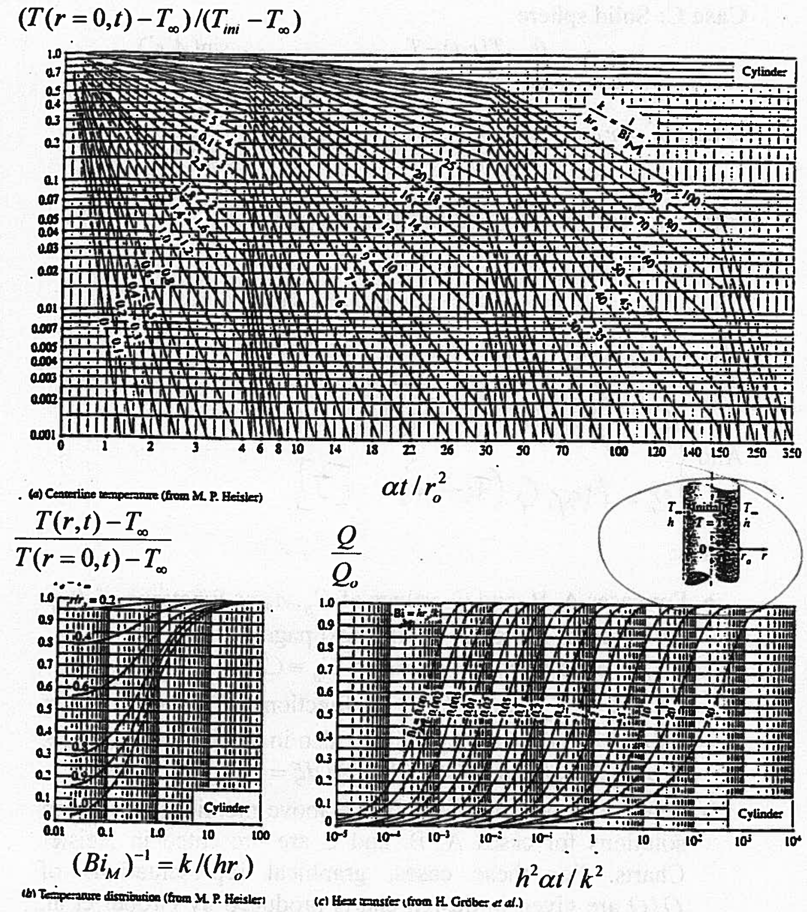


Figure 5.2: Heisler and Gröber charts for a long solid cylinder

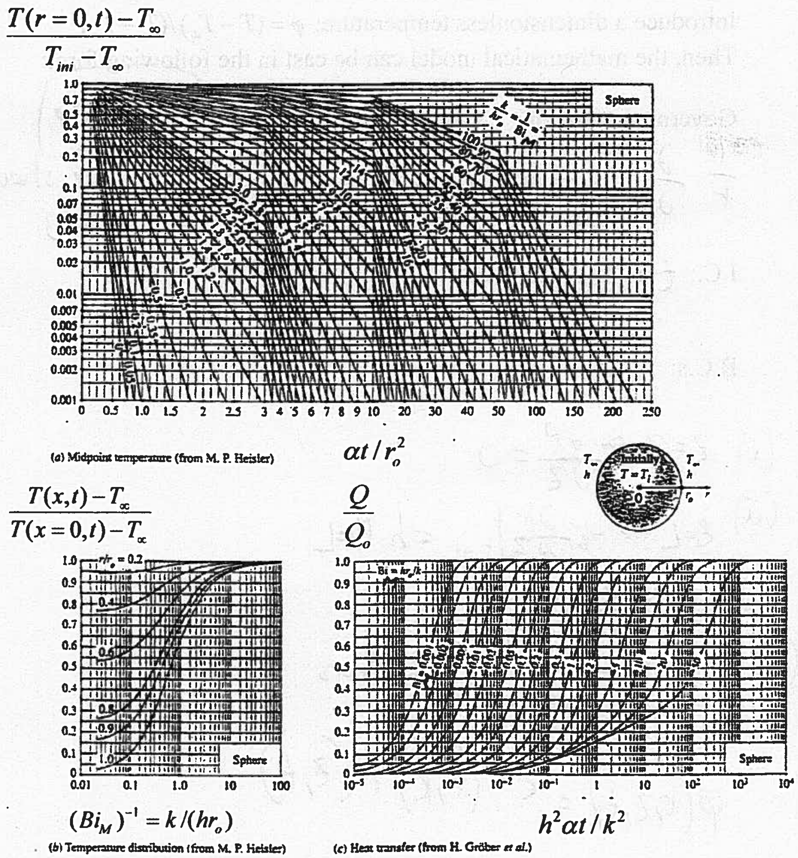


Figure 5.3: Heisler and Gröber charts for a solid sphere

Example

A one-dimensional plane wall with a thickness of 0.1 m initially at a uniform temperature of 250°C is suddenly immersed in an oil bath at 30°C. assuming the convection heat transfer coefficient for the wall in the bath is h = 500 W/m^2-K, calculate the surface temperature of the wall 9 min after immersion. What would be the amount of heat loss per unit area for the wall along this 9 min period (in MJ/m^2). The properties of the wall are:

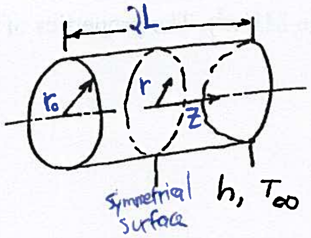
ρ_{wall} = 7835 kg/m^3
c_{wall} = 465 J/kg-K
k_{wall} = 50 W/m-K

Assume for this wall the height and width are much larger than the thickness of it.

[redacted]

Multidimensional unsteady heat conduction in solids with convection B.C. [S =0; k, ρ, c_p, h, T_∞ all constant; and $Bi_{L_c} > 0.1$]: Predictions using product solution technique

- **Example: Consider two-dimensional unsteady heat conduction in a short cylinder with convection B.C. [S =0; k, ρ, c_p, h, T_∞ all constant; and $Bi_{L_c} > 0.1$]**



Governing equation:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\partial^2 T}{\partial z^2}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$\alpha = k/(\rho c_p)$:thermal diffusivity

I.C.: $t = 0$, $T = T_i$

Four B.C.s are needed:

(i) at $z = 0$, $\left.\frac{\partial T}{\partial z}\right|_{z=0} = 0$

ii) at $z = L$, $-k\left.\frac{\partial T}{\partial z}\right|_{z=L} = h(T_{z=L} - T_\infty)$

(iii) at $r = 0$, $\left.\frac{\partial T}{\partial r}\right|_{r=0} = 0$

(iv) at $r = r_o$, $-k\left.\frac{\partial T}{\partial r}\right|_{r=r_0} = h(T_{r=r_0} - T_\infty)$

Introduce a dimensionless temperature: $\phi = (T - T_\infty)/(T_i - T_\infty)$
Then, the mathematical model can be cast in the following form:

$\alpha = k/\rho c_p$

Governing equation:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \varphi}{\partial r}\right)+\frac{\partial^2 \varphi}{\partial z^2}=\frac{1}{\alpha}\frac{\partial \varphi}{\partial t}$$

*please show this

I.C.: $t = 0 \Rightarrow \varphi = 1$

B.C.s: $t > 0$

(i) $z = 0 \rightarrow \frac{\partial \varphi}{\partial z} = 0$

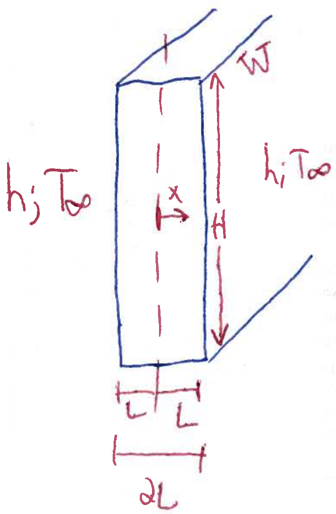
(ii) $z = L \rightarrow -k\left.\frac{\partial \varphi}{\partial z}\right|_{z=L} = h\,\varphi_{z=L}$

(iii) $r = 0 \rightarrow \frac{\partial \varphi}{\partial r} = 0$

(iv) $r = r_0 \rightarrow -k\left.\frac{\partial \varphi}{\partial r}\right|_{r=r_0} = h\,\varphi_{r=r_0}$

$$\varphi(r,z,t) = C(r,t)\,P(z,t)$$

Example for Handout 5, Slide #26



Given:

$W \gg 2L$

$H \gg 2L$

1D-Transient Heat Conduction Problem

$\rho = 7835\ kg/m^3$

$C = 465\ J/kg\cdot K$

$k = 50\ W/m\cdot K$

$T_{ini} = 250°C$

$2L = 0.1\ m$

Q: a) $T(x=L, t=9\,min) = ??$

LPA process

Check for LPA

$$Bi_{Lc} = \frac{hLc}{k_{solid}} \;;\; Lc = \frac{\text{Volume}}{\text{Area exposed to convection}} = L$$

$$Bi_{Lc} = \frac{500 \times 0.5}{50} = 0.5 \quad \therefore \text{ Since } Bi_{Lc} > 0.1,\ \underline{\text{LPA not valid}}$$

t^* process

$$\text{Fourier number} = t^* = \frac{\alpha t}{L^2} \;\&\; \alpha = \frac{k}{\rho C} = \frac{50}{7835 \times 465} = 1.37 \times 10^{-5}\ m^2/s$$

$$\therefore t^* = \frac{1.37\times10^{-5}\ (9\,min \times 60\,sec/min)}{0.05^2} = 2.964 > 0.2$$

$\therefore$ one term approximation adequate

$$\theta^*(x^*, t^*) = \frac{T - T_\infty}{T_i - T_\infty} = C_B \exp\left(-A_B^2 t^*\right) \cos\left(A_B x^*\right)$$

$x^* = \frac{x}{L}$ at $x = L \Rightarrow x^* = 1.0$

$Bi_M = \frac{hL}{K} = 0.5 \rightarrow$ use Table 5.2 $\rightarrow$ $A_B = 0.6533$, $C_B = 1.0701$

$t = 9\text{min} \rightarrow t^* = 2.964$

$$\therefore \frac{T-30}{250-30} = 1.0701 \exp\left(-(0.6533)^2 2.964\right) \times \cos\underbrace{\left(0.6533 \times 1\right)}_{\text{RADIANS}}$$

$\therefore \boxed{T = 82.76^\circ}$ for $x = 0.05$ at $t = 9\text{min}$
$= L$

2 techniq.

Use Chart Heisler

$$\left.\begin{aligned} t^* &= 2.964 \approx 3 \\ \frac{1}{Bi_M} &= 2 \end{aligned}\right\} \frac{T_0 - T_\infty}{T_i - T_\infty} \simeq 0.3 \qquad \frac{x}{L} = 1.0 \rightarrow \frac{T - T_\infty}{T_0 - T_\infty} \simeq 0.76$$

Combine: $\frac{T - T_\infty}{T_i - T_\infty} = 0.3 \times 0.76 = 0.228$

$$\frac{T-30}{250-30} = 0.228 \Rightarrow \boxed{T \simeq 80.16^\circ C}$$

b) $Q_{0\text{-}9\,min} = ?$

$$\frac{Q}{Q_0} = 1 - \left(\theta^*_{(x^*=0,t^*)}\right)\left[\frac{\sin(Ab)}{Ab}\right]$$

$$\theta^*_{(x^*=0,t^*=2.964)} = 1.0701\exp\left[-(0.6533)^2(2.964)\right] = 0.3$$

$$\begin{cases} Q_0 = m_{sys}C(T_i - T_\infty) \\ \dfrac{Q}{Q_0} = 1 - 0.3\left[\dfrac{\sin(0.6533)}{0.6533}\right] = 0.721 \end{cases}$$

$$\therefore Q = Q_0(0.721) = \rho V c_p (T_i - T_\infty)0.721$$

$$= 7835(2L \times A_{cs} \times c_p)(T_i - T_\infty)0.721$$

$$= 7835(2\times 0.05)A_{cs}(4.65)(250-30)0.721$$

$$= 57.79 \times A_{cs} \text{ MJ}$$

$$\boxed{\frac{Q}{A_{cs}} = 57.79\text{ MJ}}\,/m^2$$

---

2 techniques

Chart Gröber

$$\left.\begin{aligned} \frac{h^2\alpha t}{k^2} &= 0.741 \\ Bi_M &= 0.5 \end{aligned}\right\} \frac{Q}{Q_0} \simeq 0.7 \longrightarrow \boxed{\frac{Q}{A_{cs}} = 56.1\text{ MJ}/m^2}$$

Propose a product solution: $\phi(r,z,t) = P(z,t)C(r,t)$

Notes: (i) $\dfrac{\partial^2\phi}{\partial z^2} = C\dfrac{\partial^2 P}{\partial z^2}$ (ii) $\dfrac{\partial\phi}{\partial r} = P\dfrac{\partial C}{\partial r}$

(iii) $\dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\dfrac{\partial\phi}{\partial r}\right) = P\dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\dfrac{\partial C}{\partial r}\right)$ (iv) $\dfrac{\partial\phi}{\partial t} = C\dfrac{\partial P}{\partial t} + P\dfrac{\partial C}{\partial t}$

Thus, the governing equations can be rewritten as follows:

$$P\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial C}{\partial r}\right) + C\frac{\partial^2 P}{\partial z^2} = \frac{1}{\alpha}\left[C\frac{\partial P}{\partial t} + P\frac{\partial C}{\partial t}\right]$$

or

$$\frac{1}{P}\left[\frac{\partial^2 P}{\partial z^2} - \frac{1}{\alpha}\frac{\partial P}{\partial t}\right] + \frac{1}{C}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial C}{\partial r}\right) - \frac{1}{\alpha}\frac{\partial C}{\partial t}\right] = 0$$

This equation is satisfied if:

$$\frac{\partial^2 P}{\partial z^2} - \frac{1}{\alpha}\frac{\partial P}{\partial t} \quad \text{and} \quad \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial C}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial C}{\partial t}$$

B.C. (i) is satisfied if:

$z=0 \Rightarrow \dfrac{\partial P}{\partial z} = 0$

B.C. (ii) is satisfied if:

$z=L \rightarrow -k\dfrac{\partial P}{\partial z}\Big|_{z=L} = hP_{z=L}$

I.C.: at $t = 0$, $\phi = PC = 1$

This I.C. is satisfied if

at $t = 0$, $P =$ 1 and $C =$ 1

B.C. (iii) is satisfied if:

$r=0 \rightarrow \dfrac{\partial C}{\partial r}\Big|_{r=0} = 0$

B.C. (iv) is satisfied if:

$r=r_0 \rightarrow -k\dfrac{\partial C}{\partial r}\Big|_{r=r_0} = hC_{r=r_0}$

I.C.: at $t = 0$, $\phi = PC = 1$

1-D unsteady Plane wall heat conduction

1D unsteady long cylinder heat conduction

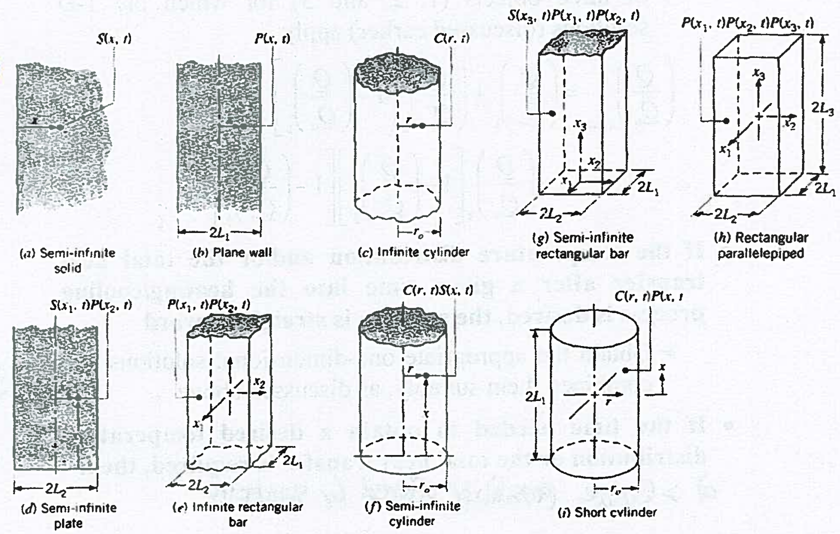
- **Product solution approach: temperature distribution for multidimensional unsteady heat conduction in solids with convection B.C. [$S=0$; k, ρ, c_p, h, T_∞ all constant; and $Bi_{L_c} > 0.1$]**

Notation for one-dimensional unsteady solutions:

Plane-wall: $P(x,t) = \dfrac{T(x,t)-T_\infty}{T_i - T_\infty}$

Long-cylinder: $C(r,t) = \dfrac{T(r,t)-T_\infty}{T_i - T_\infty}$

Semi-infinite solid: $S(x,t) = \dfrac{T(x,t)-T_\infty}{T_i - T_\infty} = 1 - \overbrace{\left(\dfrac{T(x,t)-T_i}{T_\infty - T_i}\right)}^{\text{Semi-infinite Solution for Case C}}$



In general, for three-dimensional unsteady problems:

$$\left(\frac{T-T_\infty}{T_i-T_\infty}\right)_{Full\ 3-D\ solid} = \prod_{j=1,3}\left(\frac{T-T_\infty}{T_i-T_\infty}\right)_{\substack{intersection\\ solid\ j}}$$

- **Total heat transfer for multidimensional unsteady heat conduction in solids with convection B.C. [S =0; k, ρ, c_p, h, T_∞ all constant; and $Bi_{L_c} > 0.1$] – Work of Langston**
  - ➢ For solids that can be constructed by the intersection of two objects (1 and 2) for which the 1-D solutions (discussed earlier) apply:

$$\left(\frac{Q}{Q_o}\right)_{total} = \left(\frac{Q}{Q_o}\right)_1 + \left(\frac{Q}{Q_o}\right)_2 \left[1 - \left(\frac{Q}{Q_o}\right)_1\right]$$

  - ➢ For solids that can be constructed by the intersection of three objects (1, 2, and 3) for which the 1-D solutions (discussed earlier) apply:

$$\left(\frac{Q}{Q_o}\right)_{total} = \left(\frac{Q}{Q_o}\right)_1 + \left(\frac{Q}{Q_o}\right)_2 \left[1 - \left(\frac{Q}{Q_o}\right)_1\right] + \left(\frac{Q}{Q_o}\right)_3 \left[1 - \left(\frac{Q}{Q_o}\right)_2\right]\left[1 - \left(\frac{Q}{Q_o}\right)_1\right]$$

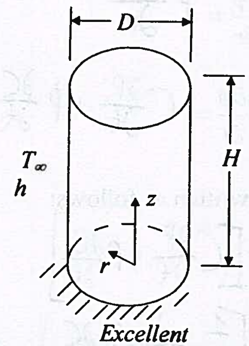
- **If the temperature distribution and/or the total heat transfer after a given time into the heating/cooling process is desired, the solution is straightforward**
  - ➢ Obtain the appropriate one-dimensional solutions and combined them suitably, as discussed above
- **If the time needed to obtain a desired temperature distribution or the total heat transfer is required, then:**

a) ➢ Explore possibility offered by symmetry

b) ➢ Explore options provided by one-term approximation to 1-D unsteady problem

→ c) if a) & b) not useful, go to iterative procedure

Example



Given Data
$H = D = 0.04$ m
$h = 2500$ W/m²-K; $T_\infty = 225$°C
$\rho_{solid} = 8000$ kg/m³
$k_{solid} = 50$ W/m-K
$c_{p,\,solid} = 800$ J/kg-K
$\alpha_{solid} = k_{solid}/(\rho_{solid}\, c_{p,\,solid}) = 7.8125\times10^{-6}$ m²/s
Initial temp. (uniform) $T_{ini} = 25$°C

Find:
(i) $T(r = 0 \text{ m}, z = 0 \text{ m}, t = 60 \text{ s}) = ?$
(ii) At t = 60 s, $T_{max} = ?$
(iii) For $0 \le t \le 60$ s, $Q_{\text{to cylinder}} = ?$

1) check LPA:

$Bi_{Lc} = \frac{hLc}{K}$; $Lc = \frac{V}{A_{convection}}$

$Lc = \frac{(\pi D^2/4)H}{\pi D^2/4 + \pi D H}$

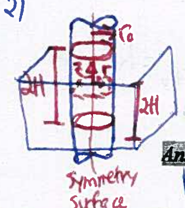
$\therefore Bi_{Lc} = 0.4 > 0.1$

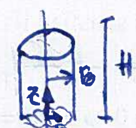
** $\boxed{\text{LPA NOT VALID}}$ **

3) $\varphi = \frac{T_{r=0,z=0,t=60} - T_\infty}{T_i - T_\infty} = P(z,t)\big|_{z=0,\,t=60s} \left[C(r,t)\big|_{r=0,\,t=60s}\right]$

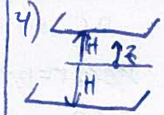
r=0
z=0
t=60s

2)





4)



$t^*_{\text{plane wall}} = \frac{\alpha t}{H^2} = \frac{7.8125\times10^{-6}(60)}{(0.04)^2} = 0.293 > 0$

$\therefore$ $\boxed{\text{one-term approx. is valid}}$

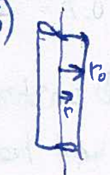
Ans.: T(0,0,60) = 193.08°C; $T_{\max\,@t=60s}$ = 215.28°C; $Q_{\text{to cylinder (0-60)}}$ = 57.85 kJ

5) $P_{(z=0,t=60)} = C_B \exp(-A_B^2 t^*)$

$Bi_m = \frac{hH}{K} = 2.0 \rightarrow$ use Table $\left[\begin{matrix} A_B = 1.0769 \\ C_B = 1.1785 \end{matrix}\right]$

$\therefore P_{(z=0,t=60)} = \boxed{0.839}$

6)



$C_{(r=0,t=60)}$

$t^*_{\text{long cylinder}} = \frac{\alpha t}{r_o^2} = 1.1719 > 0$

$\boxed{\text{one-term approx is valid}}$

$\therefore C = C_B$

HANDOUT 5 EXAMPLE, SLIDE 32

#6) $C = C_B \exp(-A_B^2 t)$

Use table: $A_B = 1.2558$
$C_B = 1.2071$

$Bi_m = \frac{hr_0}{K} = 1.0$

$\therefore C(r=0, t=60) = \boxed{0.1902}$

$\varphi(r=0, z=0, t=60) = P(z=0, t=60s)\, C(r=0, t=60s)$

$\varphi = 0.839 \times 0.1902 = 0.1596$

$T(r=0, z=0, t=60) = \varphi(T_i - T_\infty) + T_\infty = \boxed{193.08°C}$

MECH-375 HEAT TRANSFER - HANDOUT # 6

Convection Heat Transfer-I
Introduction

plates

René

Topics

- **Introduction**
  - **Example 1: Fluid flow and heat over a flat plate at zero angle of attack**
  - **Example 2: Fluid flow and heat transfer in a pipe of circular cross-section**
- **Some Background Material in Fluid Mechanics**
  - **Continuity equation**
  - **Fluid stresses**
    - ✓ **Nomenclature**
    - ✓ **Newtonian fluid**
    - ✓ **Stokes viscosity law**
  - **Momentum equations**
    - ✓ **Cauchy equations**
    - ✓ **Navier-Stokes equations**
- **Energy equation [Newtonian fluids]**
  - **Derivation [two-dimensional planar flows]**
  - **Specialization to perfect gases and simple incompressible fluids**

Introduction

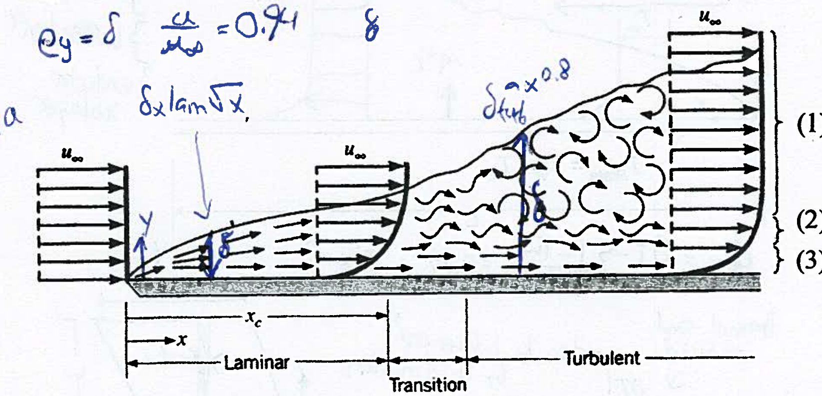
Example – 1: Fluid flow and heat transfer over a flat plate at zero angle of attack

$e_y = \delta \quad \frac{u}{u_\infty} = 0.99 \quad \delta$

Fluid=0 m/s at interface

$\delta_x \text{lam} \sqrt{x}$

$\delta_{turb} \propto x^{0.8}$



(Figure from Incropera, and DeWitt, 1994)

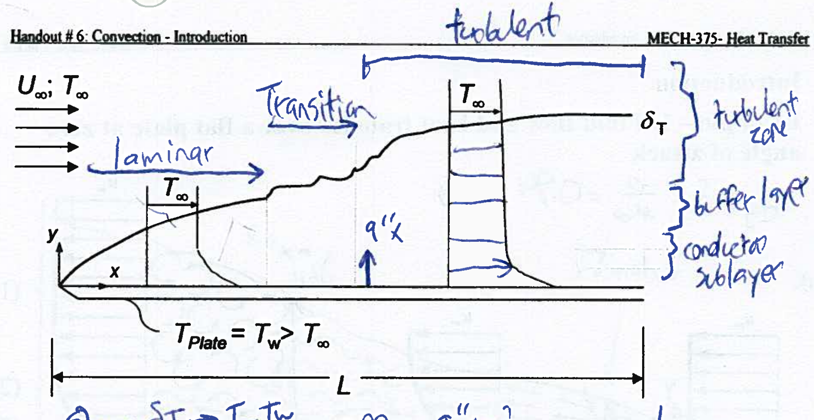
in turbulent boundary layer

(1) Turbulent zone: transport is dominated by turbulent mixing

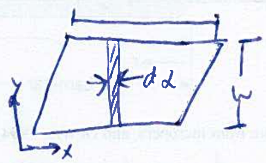
(2) Buffer layer: Diffusion and turbulent mixing are comparable

(3) Laminar Viscous Sublayer: transport is dominated by diffusion

- Local or "running" Reynolds number, $Re_x = \rho U_\infty x / \mu$
- Transition region: $1 \times 10^5 \le Re_x \le 1 \times 10^6$
- In engineering analyses: $Re_{x_{crit}} = (\rho U_\infty x_{crit} / \mu) = 5 \times 10^5$ for flow over a flat plate at zero angle of attack



$@y = \delta T \rightarrow \frac{T - T_w}{T_\infty - T_w} = 0.99 \quad q''_x = ?$



thermal cond. of fluid

heavily influenced by fluid motion

$$q''_{\substack{plate\to fluid\\ @x}} = -k\left.\frac{\partial T}{\partial y}\right|_{y=0} = h_x(T_w - T_\infty)$$

$$q_{total\ top\ surf} = \int_0^L q''_{\substack{plate\to fluid\\ @x}} W dx = \int_0^L h_x(T_w - T_\infty) W dx = h_{av} A_{total\ top\ surf}(T_w - T_\infty)_{av}$$

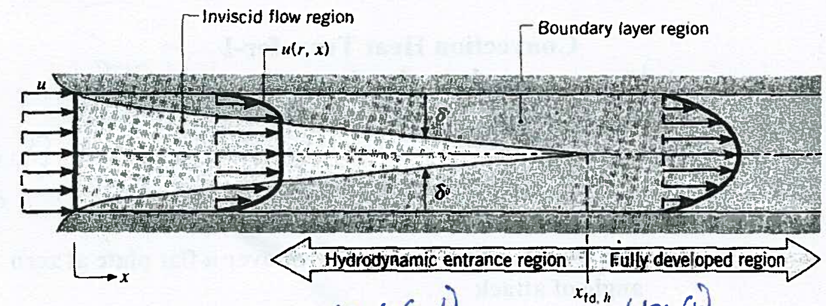
$$h_{av} = \frac{(q_{total\ top\ surf} / A_{tota\ top\ surf})}{(T_w - T_\infty)} = \frac{1}{L}\int_0^L h_x dx$$

if Twall constant

$$h_x = fnc(x, \rho, \mu, U_\infty, k, c_p, T_w, T_\infty, surf\ geom, ...)$$

Objectives:
- ✓ Physical mechanisms; governing equations, dimensionless parameters
- ✓ Predictions: i) analytical solutions; ii) empirical correlations

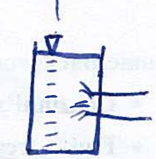
Example – 2: Fluid flow and heat transfer in a pipe of circular cross-section



(Figure from Incropera, and DeWitt, 1994)

$u = u(r,t)$ $\quad$ $u = u(r)$

$v = v(r,t)$ $\quad$ $v = 0$



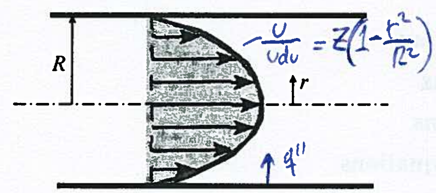
$$Re_D \approx \frac{\rho U_{av} D}{\mu}$$

use this $\left\{ \begin{array}{l} Re_D \le 2300 \\ Re_D > 2300 \end{array} \right.$

$2300 < Re < 4000$ transitional

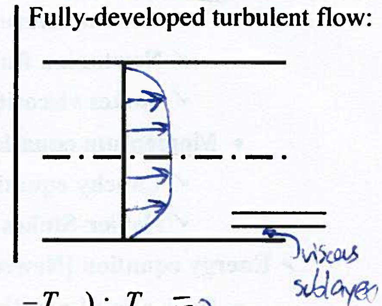
$Re > 4000$ Turbulent

Fully-developed laminar flow ((Poiseuille flow):



$$\frac{u}{u_{av}} = 2\left(1 - \frac{r^2}{R^2}\right)$$

Fully-developed turbulent flow:



$$q''_{\substack{pipe\ wall\\ \to fluid}} = k(\partial T / \partial r)|_{r = r_{pipe\ inside}} = h_{local}(T_{w,local} - T_{bulk})\ ;\ T_{bulk} = \frac{\int_{A_{cs}} \overbrace{\rho u}^{\dot m} c_p T dA}{\int_{A_{cs}} \rho u c_p dA}$$

$$h_{local} = fnc(z, \rho, \mu, U_{av}, k, c_p, T_w, T_{bulk}, geom, ...)$$

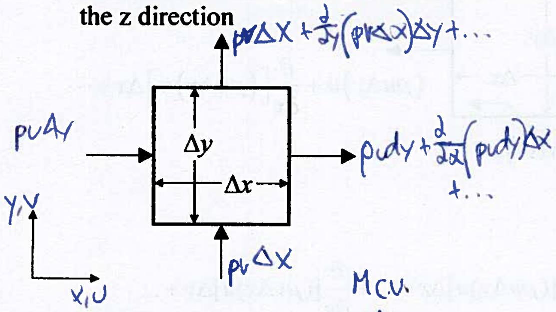
Objectives: Same as those stated for Example -1

Some Background Material in Fluid Mechanics

Continuity equation

[mathematical statement of the principle of conservation of Mass]

- Two-dimensional planar, Cartesian coordinate system; unit depth in the z direction



{[Rate of accumulation of mass inside CV]
+
[Net rate of flow of mass out of CV]}
= [Rate of mass creation]
= 0

$$\begin{bmatrix}\text{Rate of accumulation}\\ \text{of mass inside CV}\end{bmatrix} = \frac{\partial}{\partial t}(\overbrace{\rho\Delta x\Delta y}^{M_{CV}}) = \frac{\partial \rho}{\partial t}\Delta x\Delta y$$

$$\begin{bmatrix}\text{Net rate of flow of}\\ \text{mass out of CV}\end{bmatrix} = \left\{\rho u + \frac{\partial}{\partial x}(\rho u)\Delta x + \ldots\right\}\Delta y - \rho u\Delta y$$

$$+\left\{\rho v + \frac{\partial}{\partial y}(\rho v)\Delta y + \ldots\right\}\Delta x - \rho v\Delta x$$

$$= \frac{\partial}{\partial x}(\rho u)\Delta x\Delta y + \frac{\partial}{\partial y}(\rho v)\Delta x\Delta y + \ldots$$

$\therefore$, mass balance on CV gives:

$$\frac{\partial \rho}{\partial t}\Delta x\Delta y + \frac{\partial}{\partial x}(\rho u)\Delta x\Delta y + \frac{\partial}{\partial y}(\rho v)\Delta x\Delta y + \ldots = 0$$

Divide by $\Delta x\Delta y$; take limit $\Delta x \to 0; \Delta y \to 0$

$$\underbrace{\frac{\partial \rho}{\partial t}}_{\text{Rate of accumulation of mass}} + \underbrace{\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v)}_{\text{Net rate of mass flow out}} = 0 \quad \text{or} \quad \frac{\partial \rho}{\partial t} + div(\rho\bar{v}) = 0$$

If, ρ = constant, then:
$div(\bar{v}) = 0$
In this course,
$(\rho = \text{constant}) \equiv$
incompressible

*No compressible flow in this course

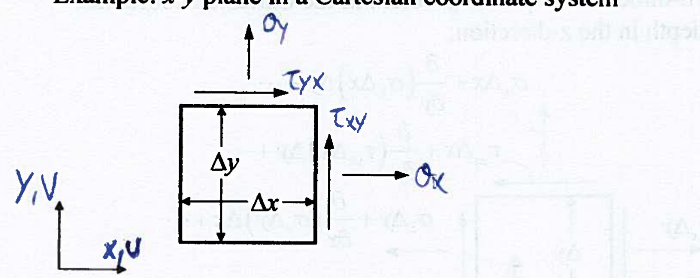
Fluid stresses [viscous fluid flow]

Nomenclature:

Normal stresses: σ ; Shear stress: τ

Subscript convention: $\underset{\substack{\text{plane on which}\\ \text{the stress acts; and}\\ \text{direction of the stress}}}{\sigma_x}$; $\underset{\substack{\text{plane on which}\\ \text{the stress acts}}}{\tau_{x,y}}$ {direction in which the stress acts

Example: x-y plane in a Cartesian coordinate system



Similar convention is adopted for the x-z and y-z planes: Please draw them

Newtonian fluid: [Fluid viscous stress] $\propto$ [Rate of strain]

Stokes viscosity law [G.G. Stokes (1845)]:

$$\sigma_x = -p + 2\mu\frac{\partial u}{\partial x} - \frac{2}{3}\mu div(\bar{v}) \qquad \tau_{xy} = \tau_{yx} = \mu\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)$$

$$\sigma_y = -p + 2\mu\frac{\partial v}{\partial y} - \frac{2}{3}\mu div(\bar{v}) \qquad \tau_{yz} = \tau_{zy} = \mu\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)$$

$$\sigma_z = -p + 2\mu\frac{\partial w}{\partial z} - \frac{2}{3}\mu div(\bar{v}) \qquad \tau_{xz} = \tau_{zx} = \mu\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right)$$

u, v, w: velocity components in the x, y, and z directions, respectively

p: static pressure

Notes: 1) Stokes hypothesis has been invoked: $\mu' = (\lambda + \frac{2}{3}\mu) = 0$;

μ': coefficient of bulk viscosity; λ: second coefficient of viscosity;

2) μ: dynamic viscosity [kg/m-s $\equiv$ Pa-s];

3) Incompressible fluid: $div(\bar{v}) = 0$

$\nu = \frac{\mu}{\rho}$

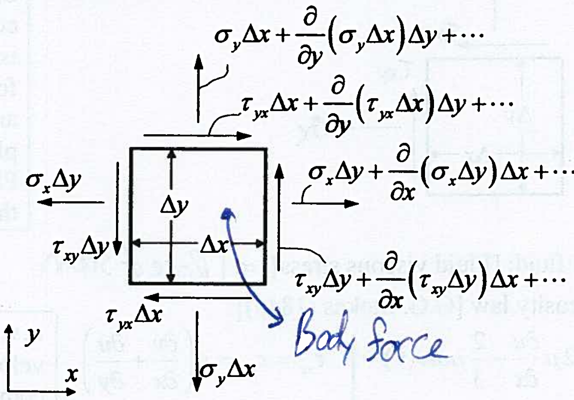
Kinematic viscosity (m²/s)

Momentum equations [mathematical statement of Newton's second law; also referred to as "conservation of momentum" when applied to a CV]

$$\begin{bmatrix}\text{Rate of accumulation}\\ \text{of momentum inside CV}\end{bmatrix}+\begin{bmatrix}\text{Net rate of transport}\\ \text{of momentum out of CV}\end{bmatrix}$$

$$=\sum \bar{F}_{\substack{surface,\,acting\\ on\,fluid\,inside\,CV}}+\sum \bar{F}_{\substack{body,\,acting\\ on\,fluid\,inside\,CV}}$$

Consider a two-dimensional planar problem in the Cartesian coordinate system; unit depth in the z-direction:



x-momentum equation:

$$\sum \bar{F}_{\substack{surface\\ in\,the\,x\,direc}}=\frac{\partial}{\partial x}(\sigma_x)\Delta x\Delta y+\frac{\partial}{\partial y}(\tau_{yx})\Delta x\Delta y+\ldots$$

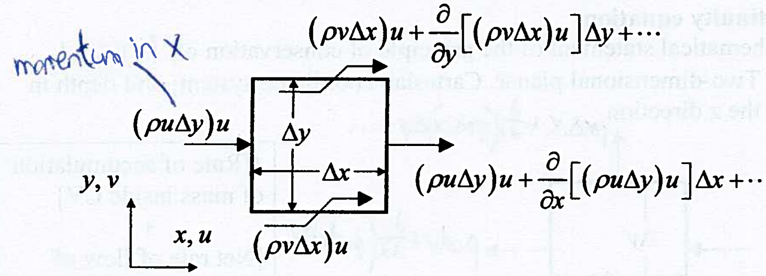
$$\sum \bar{F}_{\substack{surface\\ in\,the\,y\,direc}}=\frac{\partial}{\partial y}(\sigma_y)\Delta x\Delta y+\frac{\partial}{\partial x}(\tau_{xy})\Delta x\Delta y+\ldots$$

$$\sum \bar{F}_{\substack{body\\ in\,the\,x\,direc}}=B_x\Delta x\Delta y\ ;\ \sum \bar{F}_{\substack{body\\ in\,the\,y\,direc}}=B_y\Delta x\Delta y$$

B_x : Body force in the x direction per unit volume [N/m^3]

B_y : Body force in the y direction per unit volume [N/m^3]

Rate of advection transport of x direction momentum



$$\begin{bmatrix}\text{Net rate of flow}\\ \text{of } x\text{-direction}\\ \text{mom. out of CV}\end{bmatrix}=\frac{\partial}{\partial x}[(\rho u\Delta y)u]\Delta x+\ldots+\frac{\partial}{\partial y}[(\rho v\Delta x)u]\Delta y+\ldots$$

$$=\frac{\partial}{\partial x}[\rho uu]\Delta x\Delta y+\frac{\partial}{\partial y}[\rho vu]\Delta x\Delta y+\ldots$$

$$\begin{bmatrix}\text{Net rate of accumulation of}\\ x\text{-direction momentum inside CV}\end{bmatrix}=\frac{\partial}{\partial t}[(\rho\Delta x\Delta y)u]=\frac{\partial}{\partial t}[\rho u]\Delta x\Delta y$$

Therefore, conservation of x-direction momentum over the CV gives:

$$\frac{\partial}{\partial t}[\rho u]\Delta x\Delta y+\frac{\partial}{\partial x}[\rho uu]\Delta x\Delta y+\frac{\partial}{\partial y}[\rho vu]\Delta x\Delta y+\ldots$$

$$=\frac{\partial}{\partial x}(\sigma_x)\Delta x\Delta y+\frac{\partial}{\partial y}(\tau_{yx})\Delta x\Delta y+\ldots+B_x\Delta x\Delta y$$

Divide by $\Delta x\Delta y$; take limit $\Delta x\to 0$; $\Delta y\to 0$:

$$\boxed{\frac{\partial}{\partial t}(\rho u)+\frac{\partial}{\partial x}(\rho uu)+\frac{\partial}{\partial y}(\rho vu)=B_x+\frac{\partial}{\partial x}(\sigma_x)+\frac{\partial}{\partial y}(\tau_{yx})}\quad \boxed{\to x\text{-mom eq.}}$$

These are the Cauchy equations

Similarly:

$$\boxed{\frac{\partial}{\partial t}(\rho v)+\frac{\partial}{\partial x}(\rho uv)+\frac{\partial}{\partial y}(\rho vv)=B_y+\frac{\partial}{\partial y}(\sigma_y)+\frac{\partial}{\partial x}(\tau_{xy})}\quad \boxed{\to y\text{-mom eq.}}$$

For a Newtonian fluid (use Stokes viscosity law in the Cauchy equations) – **Result: Navier-Stokes equations (1823; 1845)**

x-momentum equation (conservative form):

$$\frac{\partial}{\partial t}(\rho u)+\frac{\partial}{\partial x}(\rho uu)+\frac{\partial}{\partial y}(\rho vu)=-\frac{\partial p}{\partial x}$$

$$+\frac{\partial}{\partial x}\left\{\underbrace{\mu\left[2\frac{\partial u}{\partial x}-\frac{2}{3}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)\right]}_{\text{normal stress}}\right\}+\frac{\partial}{\partial y}\left\{\underbrace{\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)}_{\text{shear stress}}\right\}+B_x$$

y-momentum equation (conservative form):

$$\frac{\partial}{\partial t}(\rho v)+\frac{\partial}{\partial x}(\rho uv)+\frac{\partial}{\partial y}(\rho vv)=-\frac{\partial p}{\partial y}$$

$$+\frac{\partial}{\partial y}\left\{\mu\left[2\frac{\partial v}{\partial y}-\frac{2}{3}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)\right]\right\}+\frac{\partial}{\partial x}\left\{\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right\}+B_y$$

The continuity is $\frac{\partial}{\partial t}(\rho)+\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)=0$; use this equation in the above-mentioned x- and y-momentum equations:

Navier-Stokes equations (1823; 1845): Two-dimensional, Cartesian

$$\rho\left[\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right]=-\frac{\partial p}{\partial x}$$

$$+\frac{\partial}{\partial x}\left\{\mu\left[2\frac{\partial u}{\partial x}-\frac{2}{3}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)\right]\right\}+\frac{\partial}{\partial y}\left\{\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right\}+B_x$$

$$\rho\left[\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right]=-\frac{\partial p}{\partial y}$$

$$+\frac{\partial}{\partial y}\left\{\mu\left[2\frac{\partial v}{\partial y}-\frac{2}{3}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)\right]\right\}+\frac{\partial}{\partial x}\left\{\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right\}+B_y$$

For a Newtonian fluid with constant properties (ρ , μ are constant)

Continuity equation: $\left.\frac{\partial}{\partial t}(\rho)+div(\rho\bar{v})=0\right\}$ reduces to $div(\vec{v})=0$

Navier-Stokes equations with $\bar{B}=\rho\bar{g}$ reduce to:

$$\rho\frac{D\bar{v}}{Dt}=-\bar{\nabla}p+\rho\bar{g}+\mu\nabla^2(\bar{v})$$

where $\left.\frac{D}{Dt}(\,)=\frac{\partial}{\partial t}(\,)+\bar{v}.\bar{\nabla}(\,)\right\}\Rightarrow$

"Total" derivative
"Substantial" derivative
"Particle" derivative
"Material" derivative

Summary of equations that govern two-dimensional planar flow of constant-property Newtonian fluids, with $\bar{B}=\rho\bar{g}=-\rho(g_x\bar{i}+g_y\bar{j})$:

Continuity: $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$

x-momentum: $\rho\left[\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right]=-\frac{\partial P}{\partial x}+\mu\left[\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right]$

y-momentum: $\rho\left[\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right]=-\frac{\partial P}{\partial y}+\mu\left[\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right]$

In these equations, the term P is given by the following equation:

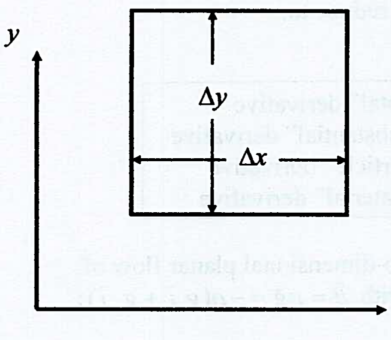
$\left.P=p+\rho g_x x+\rho g_y y\right\}\Rightarrow$

This is the so-called Reduced pressure

It accounts for the influence of the Gravitational force

Energy equation

[Two-dimensional planar flows; Cartesian coordinate system (x,y); unit depth in the z direction; Newtonian fluid]



$$\begin{bmatrix}\text{Internal + kinetic}\\ \text{per unit mass of the fluid}\end{bmatrix} = e + (V^2/2)$$

Where

$V^2 = u^2 + v^2$

Let:

$R_{adv} \Rightarrow$ rate of transport of $\{e+(V^2/2)\}$ by advection

$R_{cond} \Rightarrow$ rate of transport of e by conduction

- Fourier's law applies

$$R_{adv,x+\Delta x} - R_{adv,x} = \left[(\rho u \Delta y)(e+\frac{V^2}{2}) + \frac{\partial}{\partial x}\left\{(\rho u\Delta y)(e+\frac{V^2}{2})\right\}\Delta x + ...\right]$$

$$-(\rho u \Delta y)(e+\frac{V^2}{2}) = \left[\frac{\partial}{\partial x}\left\{(\rho u)(e+\frac{V^2}{2})\right\}\right]\Delta x\Delta y + ...$$

$$R_{cond,x+\Delta x} - R_{cond,x} = \left[(-k\frac{\partial T}{\partial x})\Delta y + \frac{\partial}{\partial x}\left[(-k\frac{\partial T}{\partial x})\Delta y\right]\Delta x + ...\right] - (-k\frac{\partial T}{\partial x})\Delta y$$

$$= \frac{\partial}{\partial x}(-k\frac{\partial T}{\partial x})\Delta y\Delta x + ...$$

$$\dot{W}_{\substack{\text{net on sys}\\ \text{by body and surface forces}\\ \text{in the x direction}}} = (B_x\Delta x\Delta y)u + \frac{\partial}{\partial x}\{(\sigma_x\Delta y)u\}\Delta x + \frac{\partial}{\partial y}\{(\tau_{yx}\Delta x)u\}\Delta y + ...$$

Similar expressions for the rates of transport in the y direction and also net rate of work done on system by forces in the y direction

$$\begin{bmatrix}\text{Rate of accumulation of}\\ \{e+(V^2/2)\}\text{ within CV}\end{bmatrix} = \frac{\partial E}{\partial t} = \frac{\partial}{\partial t}[m_{sys}\{e+\frac{V^2}{2}\}] = \Delta x\Delta y\frac{\partial}{\partial t}\left[\rho\{e+\frac{V^2}{2}\}\right]$$

Energy balance on CV:

$$\begin{bmatrix}\text{Net rate of transport}\\ \text{of }\{e+(V^2/2)\}\text{ out of}\\ \text{CV by advection}\end{bmatrix} + \begin{bmatrix}\text{Net rate of transport}\\ \text{of } e \text{ out of CV by}\\ \text{conduction}\end{bmatrix}$$

$$+\begin{bmatrix}\text{Rate of accumulation}\\ \text{of }\{e+(V^2/2)\}\text{ within CV}\end{bmatrix} = \begin{bmatrix}\text{Net rate of work}\\ \text{done on CV by body}\\ \text{and surface forces}\end{bmatrix} + \begin{bmatrix}\text{Rate of}\\ \text{heat generation}\\ \text{within CV}\end{bmatrix}$$

$$\left[\frac{\partial}{\partial x}\left\{(\rho u)(e+\frac{V^2}{2})\right\}\right]\Delta x\Delta y + \left[\frac{\partial}{\partial y}\left\{(\rho v)(e+\frac{V^2}{2})\right\}\right]\Delta x\Delta y + ...$$

$$+\left\{\frac{\partial}{\partial x}(-k\frac{\partial T}{\partial x})\right\}\Delta y\Delta x + \left\{\frac{\partial}{\partial y}(-k\frac{\partial T}{\partial y})\right\}\Delta y\Delta x + \frac{\partial}{\partial t}\left[\rho\{e+\frac{V^2}{2}\}\right]\Delta x\Delta y$$

$$= (B_x u + B_y v)\Delta x\Delta y + \frac{\partial}{\partial x}\{\sigma_x u\}\Delta x\Delta y + \frac{\partial}{\partial y}\{\tau_{yx}u\}\Delta x\Delta y + ...$$

$$+\frac{\partial}{\partial y}\{\sigma_y v\}\Delta x\Delta y + \frac{\partial}{\partial x}\{\tau_{xy}v\}\Delta x\Delta y + ... + S\Delta x\Delta y$$

Divide by $\Delta x\Delta y$; take limit $\Delta x \to 0$, $\Delta y \to 0$;
subtract $u(x-$direction Cauchy eq.) and $v(y-$direction Cauchy eq.);
use Stokes law of viscosity; expand advection terms; use continuity

$$\boxed{\rho\frac{\partial e}{\partial t} + \rho u\frac{\partial e}{\partial x} + \rho v\frac{\partial e}{\partial y} = \frac{\partial}{\partial x}(k\frac{\partial T}{\partial x}) + \frac{\partial}{\partial y}(k\frac{\partial T}{\partial y}) - p\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + \mu\Phi + S}$$

Th. E-Bal. Eq.

$$\boxed{\mu\Phi = \mu\left[\left\{\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right\}^2 + 2\left\{\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right\} - \frac{2}{3}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)^2\right]}$$

Viscous Dispersion Term

h (specific enthalpy)

Introduce specific enthalpy: $i = e + p\upsilon = e + p/\rho$ Note: $\upsilon = 1/\rho$

Then the thermal energy-balance equation can be rewritten as follows:

$$\boxed{\rho\frac{\partial i}{\partial t} + \rho u\frac{\partial i}{\partial x} + \rho v\frac{\partial i}{\partial y} = \frac{\partial}{\partial x}(k\frac{\partial T}{\partial x}) + \frac{\partial}{\partial y}(k\frac{\partial T}{\partial y}) + \left(\frac{Dp}{Dt}\right) + \mu\Phi + S}$$

For a perfect gas: $p = \rho R T_{abs}$ and $e = e(T)$ $\} \Rightarrow i = i(T)$; $di/dT = c_p$; and the thermal energy-balance equation can be rewritten as follows:

$$\boxed{\rho c_p(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}) = \frac{\partial}{\partial x}(k\frac{\partial T}{\partial x}) + \frac{\partial}{\partial y}(k\frac{\partial T}{\partial y}) + \left(\frac{Dp}{Dt}\right) + \mu\Phi + S}$$

For a simple incompressible fluid: In this course, assumed to be equivalent to $\rho = 1/v = \text{constant}$. Thus,

$$e = e(T,v)\ \} \Rightarrow de = \left(\frac{\partial e}{\partial T}\right)_v dT + \left(\frac{\partial e}{\partial v}\right)_T dv = c_v dT \text{ ; note that } c_v = \left(\frac{\partial e}{\partial T}\right)_v$$

and $di = d(e + pv) = de + pdv + vdp = c_v dT + vdp$; thus

$$\left.\left(\frac{\partial i}{\partial T}\right)_p = c_p = \frac{de}{dT} = c_v = c\ \right\}$$ For a simple incompressible substance

Also, the continuity equation gives: $div(\bar{v}) = \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0$

Thus, the thermal energy-balance equation ($\Rightarrow$energy equation)

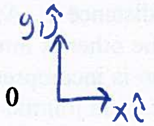
$$\rho\frac{\partial e}{\partial t} + \rho u\frac{\partial e}{\partial x} + \rho v\frac{\partial e}{\partial y} = \frac{\partial}{\partial x}(k\frac{\partial T}{\partial x}) + \frac{\partial}{\partial y}(k\frac{\partial T}{\partial y}) - p\underbrace{\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)}_{\text{incompressible}=0} + \mu\Phi + S$$

reduces to the following:

$$\boxed{\rho c_p(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}) = \frac{\partial}{\partial x}(k\frac{\partial T}{\partial x}) + \frac{\partial}{\partial y}(k\frac{\partial T}{\partial y}) + \mu\Phi + S}$$

Summary for incompressible Newtonian fluids with ρ= constant and constant properties in 2D Cartesian with no body forces:

The continuity is



$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$\nabla\cdot(\vec{V}) = 0$

$\vec{V} = u\hat{i} + v\hat{j}$

x-momentum: $\rho\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] = -\frac{\partial P}{\partial x} + \mu\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right]$

y-momentum: $\rho\left[\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right] = -\frac{\partial P}{\partial y} + \mu\left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right]$

Energy Equation:

$$\rho c_p(\frac{\partial T}{\partial t} + \overbrace{u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}}^{\text{advection}}) = k\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right] + \mu\Phi + S$$

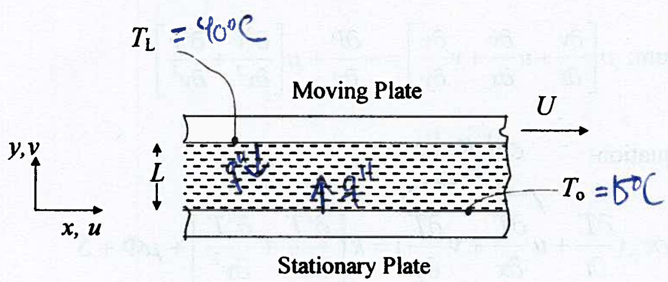
$$\mu\Phi = \mu\left[\left\{\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right\}^2 + 2\left\{\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right\}\right]$$

Example:

Consider parallel flow between two infinite plates. The plates are separated with a distance L. As shown in the figure below, one plate is stationary while the other is moving with a uniform velocity. Assume the fluid filling the gap is incompressible. This situation is referred as Couette flow and occurs, e.g., in journal bearing.

Consider $U = 15$ m/s, $L = 4$ mm, $T_o = 15°C$ and $T_L = 40°C$ and the fluid is oil ($\rho = 800$ kg/m^3; $k = 0.15$ W/m-K; $\mu = 0.8$ N-s/m^2).

Calculate a) The heat flux to each of the plates; b) What is the maximum temperature in the oil?



$U = 15$ m/s $\rho = 800$ Kg/m^3

$L = 4$ mm $K = 0.15$ W/mK

$T_o = 15°C$ $\mu = 0.8$ N-s/m^2

$T_L = 40°C$

Ans. a) $q''_{y=0} = -23.44$ kW/m^2; $q''_{y=L} = +21.56$ kW/m^2; b) $T_{max} = 177.76°C$

*steady state
*two dimensions
*incompressible / constant properties
*no heat generation
* Parallel flow

Continuity Equation

$$\cancel{\frac{\partial \rho}{\partial t}}^{0} + \frac{\partial}{\partial t}(\rho u) + \frac{\partial (\rho v)}{\partial y} = 0$$

parallel flow → $v = 0$

ρ constant $\frac{\partial u}{\partial x} = 0$

this means u is only a fluid

$$\rho\left[\cancel{\frac{\partial u}{\partial t}}^{0} + u\cancel{\frac{du}{dx}}^{0} + \cancel{v}^{0}\frac{\partial u}{\partial y}\right] + \mu\left[\cancel{\frac{\partial^2 u}{\partial x^2}}^{0} + \frac{\partial^2 u}{\partial y^2}\right] = \frac{\partial P}{\partial x}$$

$$-\cancel{\frac{\partial P}{\partial t}}^{0} + \mu\frac{d^2 u}{dy^2} = 0$$

Example [Slide 15, Handout 6]

~~continuity equation:~~ $\cancel{\frac{\partial \rho}{\partial t} + \ldots}$

Energy equation for compressible flow

$$\rho c_p\left(\cancel{\frac{\partial T}{\partial t}} + \cancel{u\frac{\partial T}{\partial x}} + \cancel{v\frac{\partial T}{\partial y}}\right) = \cancel{\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)} + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \mu\Phi + \cancel{\dot{S}}$$

0 Steady-state   0   0 $v=0$ 0   0 no generation

Simplified: $\mu\Phi$ = viscous dissipation $= \mu\left[\left[\cancel{\frac{\partial v}{\partial y}} + \cancel{\frac{\partial u}{\partial x}}\right]^2 + 2\left[\left(\cancel{\frac{\partial u}{\partial x}}\right)^2 + \left(\cancel{\frac{\partial v}{\partial y}}\right)^2\right] - \frac{2}{3}\left[\cancel{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}}\right]^2\right]$

$v=0$ 0 $v=0$ 0 continuity

$$\boxed{\therefore 0 = k\frac{dT^2}{dy^2} + \mu\left(\frac{du}{dy}\right)^2}$$

$$\boxed{k\frac{d^2T}{dy^2} = -\mu\left(\frac{V}{L}\right)^2} \quad \therefore \quad \boxed{\frac{u}{y} = \frac{V}{L}}$$

Boundary conditions: ~~T=T0, L, T0=~~

$y=0$ @ $T=T_0$

$y=L$ @ $T=T_L$

gives:

$$\boxed{T(y) = T_0 + \frac{\mu}{2k}V^2\left[\frac{y}{L} - \left(\frac{y}{L}\right)^2\right] + \frac{T_L - T_0}{L}\,y}$$

$q''_{y=0} = ?$ & $q''_{y=L} = ?$

$$q'' = -k\frac{\partial T}{\partial y} = -k\left[\frac{\mu}{2k}U^2\left[\frac{1}{L} - \frac{2y}{L^2}\right] + \frac{T_L - T_0}{L}\right]$$

$$\boxed{\begin{aligned} q''_{y=0} &= -\frac{\mu}{2}\frac{U^2}{L} - k\frac{T_L - T_0}{L} \\ q''_{y=L} &= \frac{\mu}{2}\frac{U^2}{L} - k\frac{T_L - T_0}{L}\end{aligned}} \rightarrow \text{putting in numbers} \rightarrow \boxed{\begin{aligned} q''_{y=0} &= -23437.5\ \mathrm{W/m^2} \\ q''_{y=L} &= 21562.5\ \mathrm{W/m^2}\end{aligned}}$$

$T_{max} = ?$

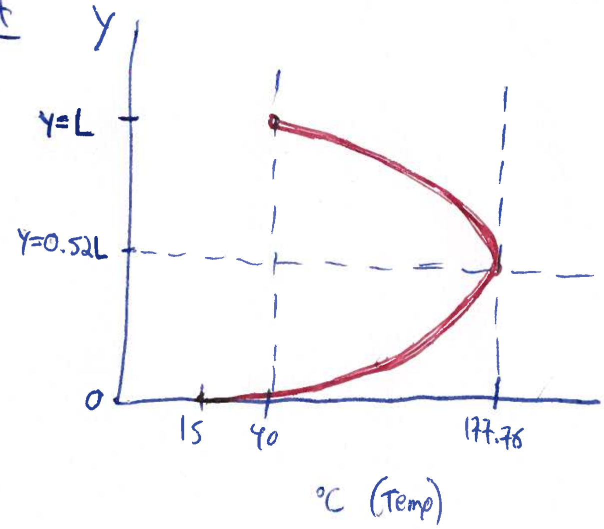
$\frac{dT}{dy} = 0 \rightarrow y = y_{critical}$

$T_{max} = T_{y = y_{critical}}$

$\therefore T_{max} = 177.76°C,\ y_{cr} = 0.52L$

$\hookrightarrow T_0 = 15°C\ ;\ T_L = 40°C$

plot



MECH-375 HEAT TRANSFER - HANDOUT # 7
Convection Heat Transfer – II

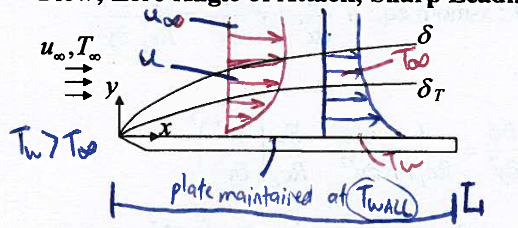
External Forced Convection [Textbook: Chapter 7]

Topics

- **Laminar Flow and Heat Transfer over a Smooth Flat Plate: Uniform Flow; Zero Angle of Attack; Sharp Leading Edge**
  - **Prandtl's boundary layer approximations**
  - **Governing equations**
  - **Dimensionless analysis**
  - **Notes**
  - **Analogy between momentum and heat transfer**
  - **Similarity solutions of Blasius and Pohlhausen**
  - **Correlation for heat transfer in liquid metal flows**
  - **Churchill-Ozoe correlation**
- **Turbulent Flow and Heat Transfer-Smooth Flat Plate: Uniform Flow; Zero Angle of Attack; Sharp Leading Edge**
  - **Correlations for local results in the turbulent region**
  - **Correlations for average results for laminar + turbulent flow over the plate**
- **Cylinder of Circular Cross-Section in Cross-Flow: Physics**
- **Empirical Correlations for External Forced Convection**
  - **Cylinder of circular cross-section in cross-flow**
  - **Cylinders of non-circular cross-section in cross-flow**
  - **Sphere in cross-flow**
  - **Tube banks in cross-flow**

René [illegible]

Laminar Flow and Heat Transfer over a Smooth Flat Plate: Uniform Flow; Zero Angle of Attack; Sharp Leading Edge



Newtonian fluid
Constant ρ, μ, c_p, k
T_w = constant $> T_\infty$
Two-dimensional
Steady-state conditions
Smooth surface

Prandtl's boundary layer approximations

1) $\delta / L << 1$; $\delta_T / L << 1$; 2) $\left.\frac{\partial}{\partial x}(\,) << \frac{\partial}{\partial y}(\,)\right\} \Rightarrow$ $\boxed{\text{Diffusion in the } x \text{ direction negligible}}$

3) $\left.\frac{v}{U_\infty} << 1 \text{ or } \frac{\partial P}{\partial y} = 0 \text{ at any } x \right\} \Rightarrow$ at any x, $P|_x$ = constant

Governing equations [Note: ρ, μ, c_p, k all constant]

Outer region (free stream): $u = U_\infty$ =constant ; $v = 0$; T_∞ =constant
P = constant or $dP/dx = 0$

Inside the boundary layer:

Continuity equation: $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$; x-momentum: $\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \mu \frac{\partial^2 u}{\partial y^2}$

y-momentum: $\left.\frac{\partial P}{\partial y} = 0 \right\} \Rightarrow P_{inside\ boundary\ layer} = P_{free\ stream} = P_\infty$ = constant, for flat plates

Energy equation: $\rho c_p \left[u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right] = k \frac{\partial^2 T}{\partial y^2} + \mu \left(\frac{\partial u}{\partial y} \right)^2$

Dimensionless analysis

Let :

$$\boxed{\begin{array}{l} x^* = x/L \,;\, y^* = y/L \,;\, P^* = (P - P_{ref})/(\rho U_\infty^2/2) \\ u^* = u/U_\infty \,;\, v^* = v/U_\infty \,;\, \phi = (T_w - T)/(T_w - T_\infty) \end{array}}$$

$\frac{1}{2}\rho U_\infty^2$ ⇒ Dynamic Pressure

Dimensionless governing equations:

Continuity eq.: $\frac{\partial u^*}{\partial x^*}+\frac{\partial v^*}{\partial y^*}=0$; x-mom eq.: $u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}=\frac{1}{Re_L}\frac{\partial^2 u^*}{\partial y^{*2}}$

y-momentum: $(\partial P^*/\partial y^*)=0$

Energy equation: $u^*\frac{\partial \phi}{\partial x^*}+v^*\frac{\partial \phi}{\partial y^*}=\frac{1}{Re_L Pr}\frac{\partial^2\phi}{\partial y^{*2}}-\frac{Ec}{Re_L}\left(\frac{\partial u^*}{\partial y^*}\right)^2$

Dimensionless parameters:

$Re_L=\frac{\rho U_\infty L}{\mu}$ } Reynolds # defined as: $\frac{\text{inertia forces}}{\text{viscous forces}}$

$Pr=\frac{\mu c_p}{k}=\frac{(\mu/\rho)}{k/(\rho c_p)}=\frac{\upsilon}{\alpha}$ } Prandtl # defined as: $\frac{\text{Rate of diffusion of momentum}}{\text{Rate of diffusion of heat}}$

$Ec=\frac{U_\infty^2}{c_p(T_w-T_\infty)}$ } Eckert # defined as: $\frac{\text{Kin. Energ. Advected}}{\text{change of enthalpy across boundary layer}}$

Notes:

1) $u^*=f_1(x^*,y^*;Re_L)$; 2) $v^*=g_1(x^*,y^*;Re_L)$

3) $\phi=\left(\frac{T_w-T}{T_w-T_\infty}\right)=\Im(x^*,y^*;Re_L,Pr,Ec)$

4) $\tau_w=\mu\left.\frac{\partial u}{\partial y}\right|_{y=0}=\left(\frac{\mu U_\infty}{L}\right)\left.\frac{\partial u^*}{\partial y^*}\right|_{y^*=0}$

Therefore, skin friction coefficient:

$$c_f=\frac{\tau_w}{\frac{1}{2}\rho U_\infty^2}=2\left(\frac{\mu}{\rho U_\infty L}\right)\left.\frac{\partial u^*}{\partial y^*}\right|_{y^*=0}=\frac{2}{Re_L}\left.\frac{\partial u^*}{\partial y^*}\right|_{y^*=0}\quad\text{or}\quad c_f=\frac{2}{Re_L}f_2(x^*;Re_L)$$

τ_w = Drag Friction Force / Asurf

5) $h_x=\frac{q''_{w,x}}{(T_w-T_\infty)}=\frac{-k_{fluid}(\partial T/\partial y)|_{y=0}}{(T_w-T_\infty)}=\frac{k_{fluid}(\partial\phi/\partial y^*|_{y^*=0}}{L}$

or $Nu_x=\left(\frac{h_x L}{k_{fluid}}\right)=\left.\frac{\partial\phi}{\partial y^*}\right|_{y^*=0}$ } $=\frac{h\,(\Delta T)}{k_{fluid}\frac{(\Delta T)}{L}}\propto\frac{\text{Heat Transf. by convection}}{\text{Heat Transf. by Pure conduction}}$

Nu: Nusselt # → dimensionless heat transfer coefficient

$Nu_x=\Im_2(x^*;Re_L,Pr,Ec)$

Analogy between momentum and heat transfer

Physical basis: Both momentum and energy are transported by advection and diffusion -- so, if source terms are similar and boundary conditions are similar, then distribution of momentum and energy would be similar

→ $u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}=\frac{1}{Re_L}\frac{\partial^2 u^*}{\partial y^{*2}}$ if $Ec \ll 1$ & $Pr \simeq 1.0$

→ $u^*\frac{\partial\phi}{\partial x^*}+v^*\frac{\partial\phi}{\partial y^*}=\frac{1}{Re_L Pr}\frac{\partial^2\phi}{\partial y^{*2}}-\frac{Ec}{Re_L}\left(\frac{\partial u^*}{\partial y^*}\right)^2$

Boundary conditions:

@ $y^*=0$, $u^*=0$; $\phi=0$

@ $y^*=\infty$, $u^*=1$; $\phi=1$

thus, if $Pr=1$ and $Ec \ll 1$ then U^* and φ distribution would be identical

Therefore, if $Ec \ll 1$, Viscous Dissipation is neglected

and, in addition, if $Pr = 1$, then the distributions of u^* and ϕ in this problem would be Identical

Thus, $\left.\frac{\partial u^*}{\partial y^*}\right|_{y^*=0} = \left.\frac{\partial \phi}{\partial y^*}\right|_{y^*=0}$ or $\left.\frac{c_f Re_L}{2} = Nu \right\}$ for … $Ec \ll 1$; $Pr = 1$

Define Stanton number as follows: $St = \frac{h}{\rho U_\infty c_p} = \frac{Nu}{Re_L Pr}$ Reynolds Analogy

Then, if $Pr = 1$, $\boxed{\frac{1}{2}c_f = St}$ → Restrictions: $Ec \ll 1$; $Pr = 1$; smooth flat plate; zero angle of attack; Newtonian fluid; laminar

Notes:

1) Stanton number

$$St \equiv \left.\frac{h}{\rho U_\infty c_p}\right\} \Rightarrow \left(\begin{matrix}\text{Rate of heat transfer}\\ \text{by convection}\end{matrix}\right) \Big/ \left(\begin{matrix}\text{Rate of transport of}\\ \text{entahlpy by advection}\end{matrix}\right)$$

$\nu = \frac{\mu}{\rho}$

Also sometimes referred to as modified Nusselt number

2) Peclet number: $Pe \equiv Pr\, Re_L$

$\boxed{Pr = \nu/\alpha}$ → Rate of transport of enthalpy by advection

$$Pe = \left(\frac{\nu}{\alpha}\right)\left(\frac{\rho U_\infty L}{\mu}\right) = \frac{U_\infty L}{\alpha} = \frac{U_\infty L}{k/(\rho c_p)} = \left.\frac{\rho c_p U_\infty}{(k/L)}\right\} \Rightarrow$$

Rate of transport of energy by conduction

3) Modified Reynolds analogy or the Chilton-Colburn analogy:

$\left.\frac{1}{2}c_{f,x} = St_x Pr^{2/3} = j_H \right\}$ for $0.6 \le Pr \le 60$; j_H : Colburn "j" factor useful in heat exchanger

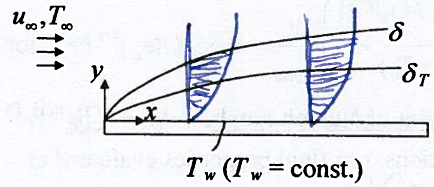
4) The Chilton-Colburn analogy is also valid for the following situations:

a) Flat plate at zero angle of attack with: surface roughness; and laminar and/or turbulent flow

b) Laminar and/or turbulent flow over streamlined objects, provided there is no flow separation from the surface of the object

Laminar Flow and Heat Transfer over a Smooth Flat Plate: Uniform Flow; Zero Angle of Attack; Sharp Leading Edge

Similarity solutions of Blasius and Pohlhausen (Isothermal surface)



Restrictions: Smooth flat plate; zero angle of attack; uniform U_∞ and T_∞; constant-property Newtonian fluid; steady, two-dimensional, laminar flow and heat transfer ($Re_L \le 5\text{x}10^5$); $Ec \ll 1$; Prandtl's boundary-layer approximations valid.

Results:

(See Section 7.2 of the Textbook)

1) $\delta = \frac{5.0}{\sqrt{\rho U_\infty /(\mu x)}}$; or $\frac{\delta}{x} = \frac{5.0}{\sqrt{Re_x}}$

where $Re_x = (\rho U_\infty x/\mu) \le 5\text{x}10^5$

Notes: a) Higher the Re_x, thinner the δ;

and (b) $\delta_{laminar\ flow} = x^{1/2}$

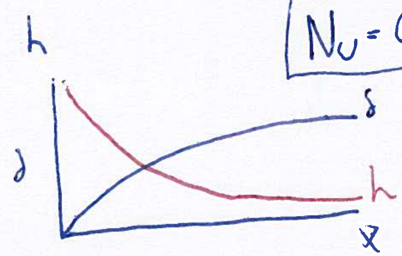
2) $\tau_{w\,@any\,x} = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} = 0.332 U_\infty \sqrt{\rho \mu U_\infty / x}$; thus, the local skin friction

coefficient is $c_{f,x} = \frac{\tau_{w,x}}{\frac{1}{2}\rho U_\infty^2} = 0.664(Re_x)^{-1/2}$. Note that $c_{f,x} = \Im(Re_x)^{\beta}$

3) $Nu_x = \frac{h_x x}{k_{fluid}} = \frac{\{q''_{w,x}/(T_w - T_\infty)\}x}{k_{fluid}} = 0.332(Re_x)^{1/2}Pr^{1/3}$ for $0.6 \le Pr \le 60$

4) $(\delta/\delta_T) = Pr^{1/3}$

$\boxed{Nu = C Re^m Pr^n}$



5) $$c_{\underset{0\le x\le x_1}{f,av}} = \frac{\underset{0\le x\le x_1}{\tau_{w,av}}}{\frac{1}{2}\rho U_\infty^2} = \frac{(\underset{0\le x\le x_1}{Drag_{Viscous}})/(x_1 W)}{\frac{1}{2}\rho U_\infty^2} = 1.328(Re_{x_1})^{-1/2} = 1.328\left(\frac{\rho U_\infty L}{\mu}\right)^{1/2}$$

6) $$Nu_{\underset{0\le x\le x_1}{av,x_1}} = \frac{(\underset{0\le x\le x_1}{h_{av}})x_1}{k_{fluid}} = \left(\frac{\{\underset{0\le x\le x_1}{q_{w\to fluid}}/(x_1 W)\}}{(T_w - T_\infty)}\right)\frac{x_1}{k_{fluid}} = 0.664(Re_{x_1})^{1/2}Pr^{1/3} \text{ for}$$

$0.6 \le Pr \le 60$. Important to note form of Nusselt number: $Nu = C Re^m Pr^n$

7) In the above-mentioned correlations, use fluid properties evaluated at the film temperature: $T_{film} = (T_w + T_\infty)/2$

8) Correlation for laminar flows of liquid metals: $Pr \ll 1$

$$Nu_x = \frac{h_x x}{k_{fluid}} = 0.565(Re_x Pr)^{1/2} \quad \text{for } Pr \le 0.05 \quad Pe_x = (Re_x Pr) \ge 100$$

9) For fluid with very low Pr numbers like liquid metals or very high Pr number fluids equation given in part (3) is not valid. For wide range of Pr numbers for laminar flow over isothermal surfaces use the Churchill-Ozoe correlation:

$$Nu_x = \frac{0.3387\,Re^{1/2}\,Pr^{1/3}}{[1+(0.0468/Pr)^{2/3}]^{1/4}} \Big\} \text{ for } Pe_x = (Re_x Pr) \ge 100$$

$\boxed{\text{Liquid Metals}}$

Constant Surface Heat Flux

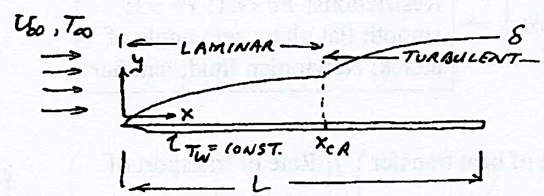
10) The above relations are valid for laminar heat transfer from a constant temperature surface. For the case where the surface heat flux is constant and the flow is laminar use:

$$Nu_x = \frac{h_x x}{k_{fluid}} = \frac{\{q''_{w,x}/(T_w - T_\infty)\}x}{k_{fluid}} = 0.453(Re_x)^{1/2}Pr^{1/3} \text{ for } 0.6 \le Pr \le 60$$

11) For wide range of Pr numbers for laminar flow over surfaces with constant heat flux the Churchill-Ozoe correlation is given:

$$Nu_x = \frac{0.4637\,Re^{1/2}\,Pr^{1/3}}{[1+(0.0207/Pr)^{2/3}]^{1/4}} \Big\} \text{ for } Pe_x = (Re_x Pr) \ge 100$$

Turbulent Flow and Heat Transfer over a Smooth Flat Plate: Uniform Flow; Zero Angle of Attack; Sharp Leading Edge



Newtonian fluid
Constant ρ, μ, c_p, k
T_w = constant $> T_\infty$
Two-dimensional
Steady-state conditions
Smooth surface

$$Re_{x_{cr}} = \frac{\rho U_\infty x_{cr}}{\mu} = 5\text{x}10^5$$

Notes: a) The transition region typically lies in the following range of the "running" Reynolds number: ;

b) In engineering practice, this transition region is ignored, and it is assumed that the laminar→turbulent transition occurs suddenly at $Re_{x_{cr}} = 5\text{x}10^5$

Local results in the turbulent flow region ($Re_x > 5\text{x}10^5$)
[Based on the works of: Schlichting; Shultz-Grunow]

1) $$\underset{\text{in the turb region}}{c_{f,x}} = \left(\frac{\tau_{w,x}}{\rho U_\infty^2/2}\right)_{\text{in the turb region}} = 0.0592\,Re_x^{-1/5} \text{ for } 5\text{x}10^5 < Re_x < 10^7$$

- Uncertainties in this correlation are $\le \pm 10\%$

- This correlation may also be used with uncertainties $\leq \pm 15\%$ for values of Re_x up to 10^8

2) $(\delta / x)_{in\, the\, turb\, region} = 0.37\, Re^{-1/5}$

Notes: $\delta_{turbulent} = x^{0.8}$; $\delta_{laminar} = x$

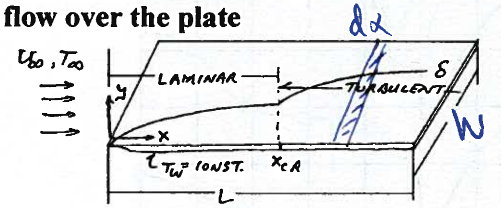
3) $(\delta / \delta_T)_{turbulent} = 1$

4) Applying the fluid-friction analogy $\frac{1}{2} c_{f,x} = St_x\, Pr^{2/3}$

$$\underset{\substack{in\ the\\ turb\ region}}{Nu_x} = (St_x\, Re_x\, Pr)_{\substack{in\ the\\ turb\ region}} = 0.0296\, Re_x^{4/5} Pr^{1/3} \text{ for } 5\text{x}10^5 < Re_x < 10^7$$

- Uncertainties in this correlation are $\leq \pm 10\%$
- This correlation may also be used with uncertainties $\leq \pm 15\%$ for values of Re_x up to 10^8

Correlations for average results for "mixed" (laminar + turbulent) flow over the plate



It is assumed here that laminar-turbulent transition takes place suddenly at $Re_{x_{cr}} = (\rho U_\infty x_{cr} / L) = 5\text{x}10^5$

Here: $Re_L > 5\text{x}10^5$

$$\underset{0\le x\le L}{h_{av}} = \left(\frac{q_{plate\to fluid} / A_{surf\ plate}}{T_w - T_\infty} \right) = \frac{1}{(T_w - T_\infty)(WL)} \int_0^L h_x (T_w - T_\infty) W dx$$

$$= \frac{1}{L} \left[\int_0^{x_{cr}} h_{lam,x} dx + \int_{x_{cr}}^{L} h_{turb,x} dx \right]$$

Note: Here, $(T_w - T_\infty)$ is constant

1) $$\underset{0\le x\le L}{Nu_{av}} = \frac{(\underset{0\le x\le L}{h_{av}})L}{k_{fluid}} = [0.664 Re_{x_{cr}}^{1/2} + 0.037 (Re_L^{4/5} - Re_{x_{cr}}^{4/5})] Pr^{1/3}$$

Valid for $0.6 \leq Pr \leq 60$ and $5\text{x}10^5 < Re_L < 10^8$

2) With $Re_{x_{cr}} = 5\text{x}10^5$,

$$\underset{0\le x\le L}{Nu_{av}} = \frac{(\underset{0\le x\le L}{h_{av}})L}{k_{fluid}} = [0.037\, Re_L^{4/5} - 871] Pr^{1/3}$$

Valid for $0.6 \leq Pr \leq 60$ and $5\text{x}10^5 < Re_L < 10^8$

~~Valid~~ Applies to the whole plate $0 \le x \le L$

3) $$\underset{0\le x\le L}{c_{f,av}} = \frac{\underset{0\le x\le L}{\tau_{w,av}}}{\rho U_\infty^2 / 2} = \left[\frac{0.074}{Re_L^{1/5}} - \frac{1742}{Re_L} \right]$$

Valid for $5\text{x}10^5 < Re_L < 10^8$ and with $Re_{x_{cr}} = 5\text{x}10^5$. Note that $\underset{0\le x\le L}{\tau_{w,av}} \equiv$ [Viscous Drag$_{0\le x\le L}$]/[Surface Area$_{0\le x\le L}$]

4) If $L >> x_{cr}$, then $Re_L >> Re_{x_{cr}}$ and

$$\underset{0\le x\le L}{Nu_{av}} = \frac{(\underset{0\le x\le L}{h_{av}})L}{k_{fluid}} = 0.037\, Re_L^{4/5}\, Pr^{1/3}$$

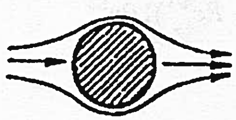
$$\underset{0\le x\le L}{c_{f,av}} = \frac{\underset{0\le x\le L}{\tau_{w,av}}}{\rho U_\infty^2 / 2} = \frac{0.074}{Re_L^{1/5}}$$

Valid for $5\text{x}10^5 < Re_L < 10^8$ and $0.6 \leq Pr \leq 60$

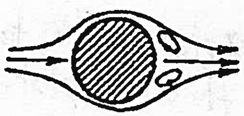
5) The correlations given above in point 4) also apply if the boundary layer is tripped at the leading edge ($x = 0$) and the flow is turbulent over the whole plate ($0 \leq x \leq L$)

6) In turbulent heat transfer, generally speaking the correlations obtained for isothermal surfaces can be applied to the surface constant flux case.

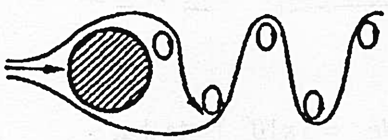
ylinder of Circular Cross-Section in Cross-Flow: Physics



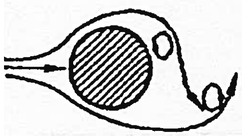
$Re_D = \dfrac{\rho U_\infty D}{\mu} \le 5$ } Regime of unseparated flow



$5 \le Re_D \le 40$ } A fixed pair of Föppl vortices in the wake of the cylinder

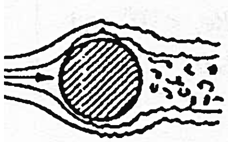


Periodic the von Karman vortex street: $40 \le Re_D \le 150$

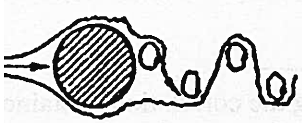


$150 \le Re_D \le 300$ } Transition to turbulence in vortex street

$300 \le Re_D \le 1.4\text{x}10^5$ } Vortex street is fully turbulent; flow is increasingly three-dimensional



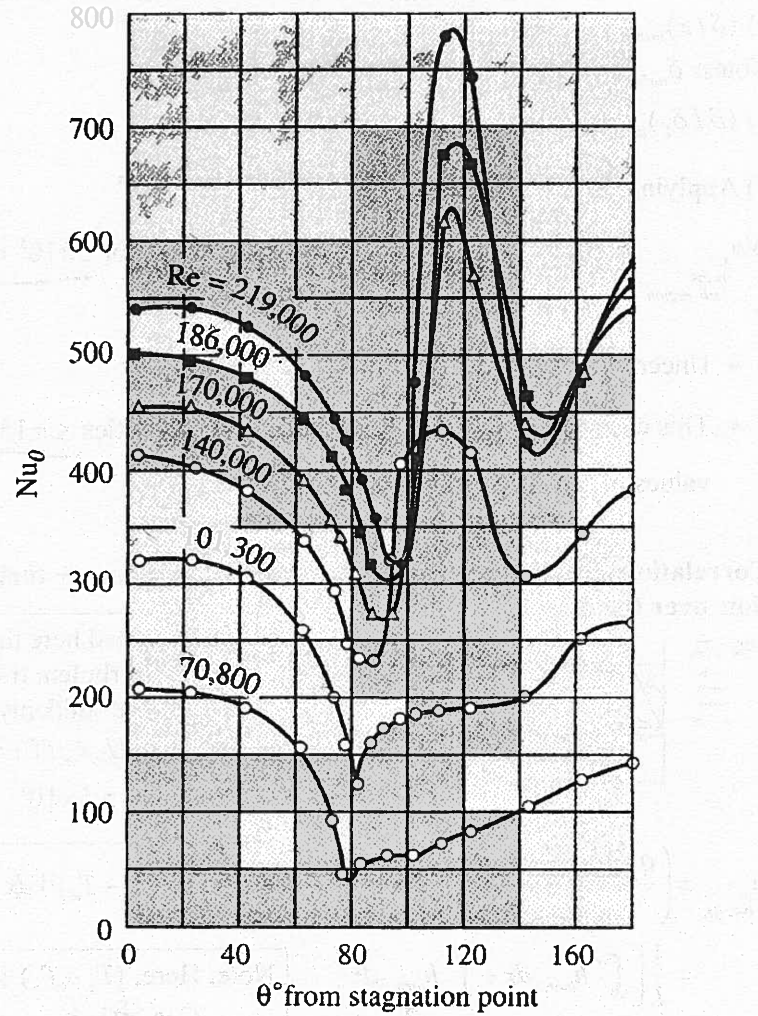
$1.4\text{x}10^5 \le Re_D \le 3.5\text{x}10^6$ } Laminar boundary layer undergoes transition to turbulence on the surface of the cylinder; the wake is narrower and disorganized; no vortex street is apparent



$3.5\text{x}10^6 \le Re_D < ??$ } Reestablishment of the turbulent vortex street; the boundary layer is turbulent and the wake thinner

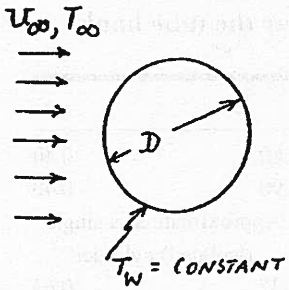
ketches/Descriptions adapted from the work of John H. Lienhard

Variation of local Nusselt number over a cylinder of circular cross-sectior in uniform cross-flow [W.H. Giedt and E.R.G. Eckert]



Empirical Correlations for External Forced Convection

A: Cylinder of circular cross-section in uniform cross-flow



(i) Knudsen and Katz correlation

$$Nu_{av} = \frac{h_{av}D}{k_{fluid}} = C\,Re_D^n Pr^{1/3}$$

- All properties are evaluated at T_{film}

- The values of C and n should be obtained from the following table:

Re_{d}	C	n
0.4–4	0.989	0.330
4–40	0.911	0.385
40–4000	0.683	0.466
4000–40,000	0.193	0.618
40,000–400,000	0.0266	0.805

(ii) Whitaker correlation [Note form: $Nu = (C_1 Re_D^{m_1} + C_2 Re_D^{m_2})Pr^n$]

$$Nu_{av} = \frac{h_{av}D}{k_{fluid}} = (0.4Re_D^{1/2} + 0.06Re_D^{2/3})Pr^{0.4}\left(\frac{\mu_\infty}{\mu_w}\right)^{1/4}$$

- All properties are evaluated at T_∞, except μ_w which is evaluated at T_w
- Range of validity of this correlation:
  $0.65 \le Pr \le 300$; $40 \le Re_D \le 10^5$; and $0.25 \le (\mu_\infty/\mu_w) \le 5.2$

Handout # 7: External Forced Convection MECH-375- Heat Transfe

B: Cylinders of non-circular cross-section in uniform cross-flow

Jacob correlation: $Nu_{av} = \dfrac{h_{av}L_c}{k_{fluid}} = C Re_{L_c}^m Pr^{1/3}$

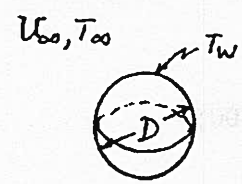
L_c = Characteristic length (see table below)

All properties evaluated at the film temperature: $T_{film} = 0.5(T_w + T_\infty)$

Geometry	Re_{L_c}	C	m
Square			
$U_\infty \rightarrow$ (diamond) L_c	5×10^3–10^5	0.246	0.588
$U_\infty \rightarrow$ (square) L_c	5×10^3–10^5	0.102	0.675
Hexagon			
$U_\infty \rightarrow$ L_c	5×10^3–1.95×10^4	0.160	0.638
	1.95×10^4–10^5	0.0385	0.782
$U_\infty \rightarrow$ L_c	5×10^3–10^5	0.153	0.638
Vertical plate			
$U_\infty \rightarrow$ L_c	4×10^3–1.5×10^4	0.228	0.731

C: Sphere in cross-flow

Whitaker correlation



$$Nu_{av} = \frac{h_{av}D}{k_{fluid}} = 2 + [0.4Re_D^{1/2} + 0.06Re_D^{2/3}]Pr^{0.4}\left(\frac{\mu}{\mu}\right.$$

- All properties are evaluated at T_∞, except μ_w which is evaluated at T_w
- Range of validity of this correlation:
  $0.7 \le Pr \le 380$; $3.5 \le Re_D \le 7.6\text{x}10^4$

Heat Transfer across Tube Banks

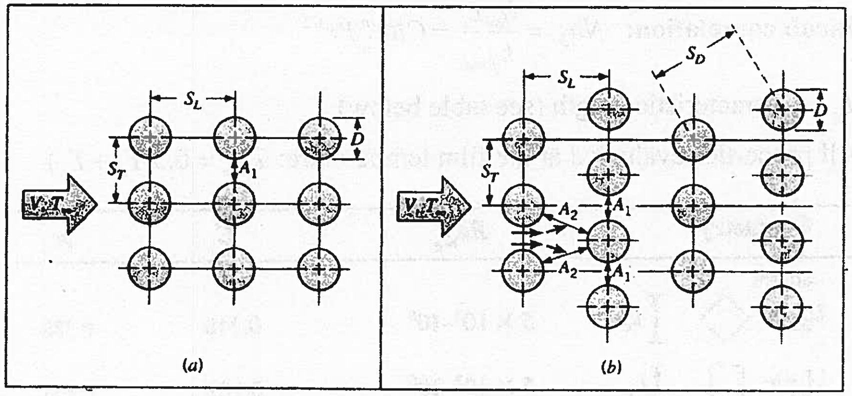


FIGURE 7.11 Tube arrangements in a bank. (a) Aligned. (b) Staggered.

Zukauskas correlation:

$$Nu_{av} = \frac{h_{av} d}{k_{fluid}} = C Re^{n}_{D_{Max}} Pr^{0.36}\left(\frac{Pr}{Pr_w}\right)^{1/4}$$

- $Re_{D_{Max}} = \dfrac{\rho V_{\max} D}{\mu}$, where:

  Aligned arrangement

  $V_{\max} = V\left[S_T/(S_T - D)\right]$

  Staggered arrangement

  $V_{\max} = V\left[S_T/(S_T - D)\right]$ *if* $S_D > (S_T + D)/2$

  $V_{\max} = V(S_T/2)/\left[(S_D - D)\right]$ *if* $S_D < (S_T + D)/2$
- Range of validity of this correlation: $0.7 \le Pr \le 500$; $10 \le Re_{D_{Max}} \le 10^6$

- All properties except Pr_w are evaluated at T_∞, and the values of the constants are given below.

TABLE 7.7 Constants of Equation 7.64 for the tube bank in cross flow [15]

Configuration	$Re_{D,\max}$	C	m
Aligned	10–10^2	0.80	0.40
Staggered	10–10^2	0.90	0.40
Aligned	10^2–10^3	Approximate as a single (isolated) cylinder	
Staggered	10^2–10^3		
Aligned ($S_T/S_L > 0.7$)[a]	10^3–2×10^5	0.27	0.63
Staggered ($S_T/S_L < 2$)	10^3–2×10^5	$0.35(S_T/S_L)^{1/5}$	0.60
Staggered ($S_T/S_L > 2$)	10^3–2×10^5	0.40	0.60
Aligned	2×10^5–2×10^6	0.021	0.84
Staggered	2×10^5–2×10^6	0.022	0.84

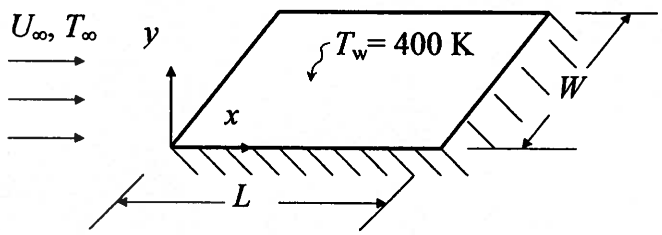
[a]For $S_T/S_L < 0.7$, heat transfer is inefficient and aligned tubes should not be used.

TABLE 7.8 Correction factor C_2 of Equation 7.65 for $N_L < 20$ ($Re_{D,\max} \gtrsim 10^3$) [15]

N_L	1	2	3	4	5	7	10	13	16
Aligned	0.70	0.80	0.86	0.90	0.92	0.95	0.97	0.98	0.99
Staggered	0.64	0.76	0.84	0.89	0.92	0.95	0.97	0.98	0.99

Example 1

As shown schematically in the figure below, nitrogen at $P_\infty = 50$ kPa flows over the top surface of a smooth, isothermal, flat plate with a sharp leading edge, at zero angle of attack. For this flow: $U_\infty = 100$ m/s and $T_\infty = 300$ K. The length of the plate in the flow direction is $L = 1.2$ m and its width is $W = 1$ m. Calculate the rate of heat transfer from the top surface of the plate to nitrogen.

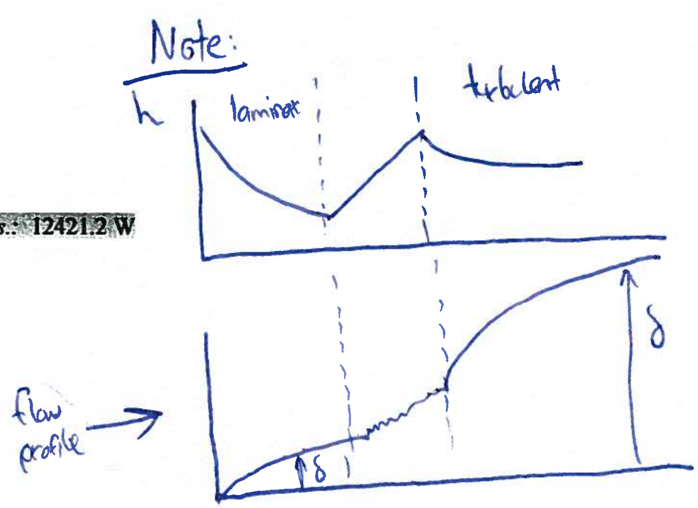


Properties of N_2 at 350 K and 1 atm (101.325 kPa):

$\rho = 0.998$ kg/m^3; $c_p = 1043.35$ J/kg-°C; $k = 0.029775$ W/m-°C; $\mu = 19.91\times 10^{-6}$ kg/m-s.

Note: Assume for the range of interest, c_p, k and μ of nitrogen not strongly pressure dependent.

Note:



Ans.: 12421.2 W

* $T_\infty \neq 350$K, & $\neq 101$ kPa ∴ we need a table to find properties of N_2 under given conditions

Assumptions

- steady-state heat transfer problem
- N_2 behaves as a perfect gas
- c_p, k, μ not strong function of pressure
- $Ec \ll 1$; viscous dissipation negligible
- Radiation H.T. negligible

$P = \rho R T$

$$\frac{P_1}{P_2} = \frac{\rho_1}{\rho_2}\Big|_{T=\text{constant}} \Rightarrow \rho_{P=50kPa} = \rho_{P=101kPa}\left(\frac{50{,}000\,Pa}{101{,}325\,Pa}\right) = \boxed{0.4925\ kg/}$$

$$T_{film} = \frac{T_w + T_\infty}{2} = 350K$$

$$Re_L = \frac{\rho U_\infty L}{\mu} = \frac{0.4925\times100\times1.2}{19.91\times10^{6}} = 2.97\times10^6$$

$Re_L > Re_{cr}$ (5×10^5)

$$\underset{0\le x\le L}{Nu_{av}} = \frac{h_{av}L}{k_{fluid}} = (0.037 Re_L^{0.8} - 871)Pr^{1/3}$$

$$Pr = \nu/\alpha = \frac{\mu c_p}{k_{fluid}} = 0.698$$

$$Nu_{av} = 4171.9 \rightarrow h_{av} = \frac{4171.9\times k_f}{L} \rightarrow h_{av} = 103.5\ W/m^2\text{-}°C$$

$$q = h_{av} A_{plate}(T_w - T_\infty) = 103.5(1.2\times1)(400-300)$$

$$\boxed{= 12{,}421.2\ W}$$

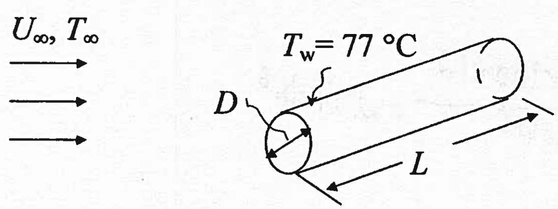
ample 2

simple constant-temperature hot-wire anemometer is to be designed to termine the velocity of the air in a uniform flow. In this design, the air locity is to be deduced from measurements of the electrical current quired to maintain a metal wire of diameter 0.5 mm at a constant nperature of $T_w = 77°C$ in a cross-flow of air at a temperature of $T_\infty =$ °C. Ignore radiation heat transfer.

a) Assuming that the Reynolds number falls in the range $40 \le \mathrm{Re}_D \le 1000$, develop a relationship between the electrical current, I, in the wire, the velocity, U_∞ of the air, and the electrical resistance of the wire per unit length, $\gamma = R_{elec} / L$

b) Use the result obtained in part (a) to establish a relationship between the fractional change in the current, $\Delta I / I$, in response to the corresponding fractional change in the air velocity, $\Delta U_\infty / U_\infty$.

c) Calculate the electrical current required when $U_\infty = 10$ m/s and the electrical resistivity of the metal, $\sigma_{elec} = 17.1 \times 10^{-5}$ Ω.m.
Note: $R_{elec} = \sigma_{elec} L_{wire} / A_{\substack{c.s. \\ wire}}$



c) I 0.206 A (Knudsen and Katz correlation)

MECH-375 HEAT TRANSFER - HANDOUT # 8

Convection Heat Transfer – III: Internal Forced Convection

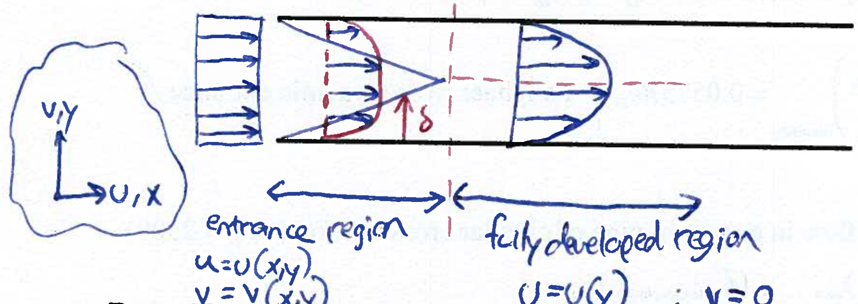
Topics

- ➢ **Some Background Material**
  - **Developing and fully-developed flows; critical Reynolds number; examples of hydraulic diameter**
  - **Hydrodynamic entrance length: laminar and turbulent**
- ➢ **Bulk Temperature**
  - **Use in differential and overall energy balances**
  - **Local and average heat transfer coefficients / LMTD**
- ➢ **Thermally Fully-Developed region**
- ➢ **Analytical and Empirical Correlations for Fully-Developed Flow and Heat Transfer in Straight Circular Pipes**
  - **Laminar flow and heat transfer: Poiseuille flow; Darcy / Fanning friction factors; Nusselt numbers for given-flux and constant-wall-temperature boundary conditions**
  - **Turbulent flow and heat transfer**
    - ✓ **Colebrook-White correlation; Moody diagram**
    - ✓ **Dittus-Boelter; Sieder-Tate correlation; Petukhov correlation; Gnielinski correlation; Skupinski and Seban-Shimazaki correlations for liquid metals**
    - ✓ **Chilton-Colburn analogy**
- ➢ **Entrance Effect on Heat Transfer in Circular Ducts**
- ➢ **Flow and Heat Transfer in Ducts of Non-Circular Cross-Section: Laminar; Turbulent**

Kené Kinfet

Some Background Material

Developing and fully-developed flows



Reynolds number based on average velocity and the hydraulic diamet

$Re_{D_h} = \frac{\rho u_{av} D_h}{\mu}$; $\underbrace{D_h}_{\text{Hydraulic diameter}} = \frac{4A_{c.s.}}{P_{\text{wetted by fluid}}}$; $u_{av} = \int \rho u \, dA_{c.s} / \rho A_{c.s.}$

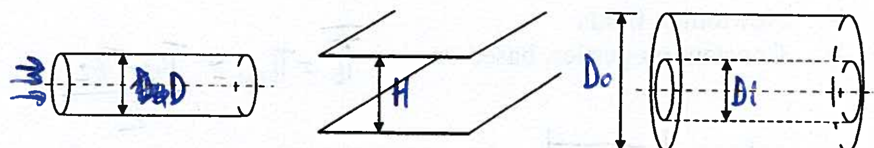
Critical Reynolds number: $(Re_{D_h})_{crit} =$ 2300 } in engineering analyses

If $Re_{D_h} \leq$ 2300 then the flow is laminar

If $Re_{D_h} >$ 4000 then the flow is turbulent ← in this course

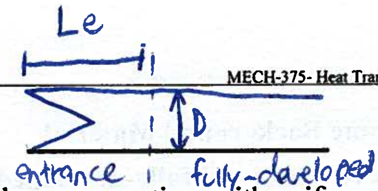
$2300 < Re_{D_H} < 4000$ transitional region

Examples of hydraulic diameter:



$D_H = D$ $D_h = 2H$ $D_H = D_o - D_i$

please show these



ydrodynamic entrance length

aminar flow in a straight pipe of circular cross-section, with uniform *let* velocity distribution ($Re_D = (\rho u_{av} D/\mu) \le 2300$):

$$\left(\frac{L_{entrance,hydro}}{D}\right)_{laminar} = 0.0575 Re_D \quad \Big\}$$ Langhaar hydrodynamic entrance ngth

urbulent flow in a straight pipe of circular cross-section ($Re_D > 2300$):

$(L_{entrance,hydro})_{turb}$ $(L_{entrance,hydro})_{laminar}$ }

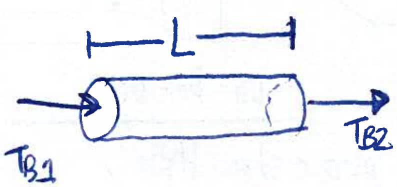
ypically, $2 \le \left(\frac{L_{entrance,hydro}}{D}\right)_{turb} \le 10$. Thus, for turbulent flows, if

$/D) \ge 10$, fully-developed conditions are commonly assumed.

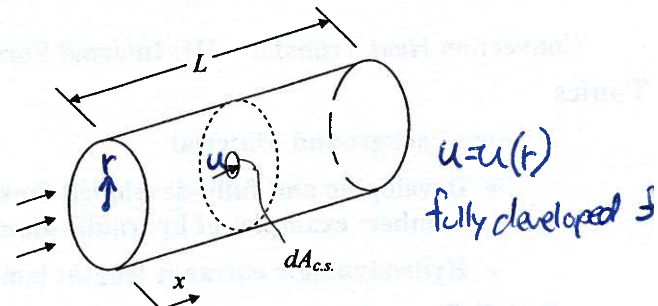
practical applications, usually, $(L_{pipe}/D) >>$: thus for turbulent ows, it is often valid to assume that fully-developed conditions prevail roughout the duct of interest.

this course, we will be concentrating mostly on:

- Fully-developed flows
- Newtonian fluids
- Constant properties, based on ….. $\overline{T_b} = T_{b\,av} = \frac{T_{b1}+T_{b2}}{2}$



Bulk Temperature

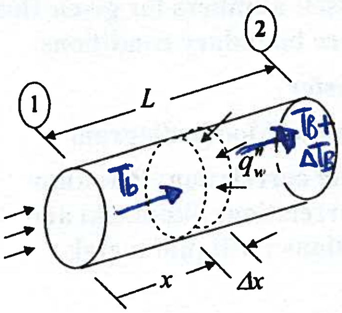


Definition: $T_b = \int_{A_{c.s.}} \rho u c_p T dA_{c.s.} \Big/ \int_{A_{c.s.}} \rho u c_p dA_{c.s.}$ Thus, T_b is a <u>capacity-rate-weighted average temperature in the c-s of the duct.</u>

If c_p = constant, then: $T_b = \int_{A_{c.s.}} \rho u c_p T dA_{c.s.} \Big/ \int_{A_{c.s.}} \rho u c_p dA_{c.s.} = \int_{A_{c.s.}} \rho u T dA_{c.s.} \Big/ \dot m$

Use of the bulk temperature in energy balances

(i) Differential energy balance on a slice of the duct



$$\bar{q}''_w Peri_{c.s.}\Delta x = \dot m c_p \Delta T_b$$

Thus,

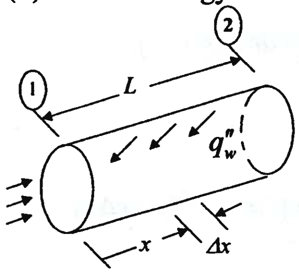
$$\frac{dT_b}{dx} = \frac{\bar{q}''_w Peri_{c.s.}}{\dot m c_p}$$

Restrictions:
Steady-state
Constant properties
$Ec << 1$ no visc dis
$Pe >> 1$ no axial cond

Here, it has also be assumed that changes in enthalpy due to changes in pressure are small compared to those due to changes in temperature.

$$\bar{q}''_w = \frac{1}{Peri_{c.s.}} \int_{Peri_{c.s.}} q''_w ds \quad \Big\}$$ perimeter averaged heat flux

(ii) Overall energy balance on the duct between sections 1 and 2:



$$\frac{dT_b}{dx} = \frac{\bar{q}_w'' Peri_{c.s.}}{\dot{m}c_p}$$

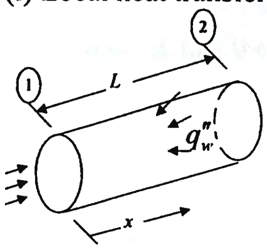
$$\int_{T_{b,1}}^{T_{b,2}} dT_b = \int_{x_1}^{x_2} \frac{\bar{q}_w'' Peri_{c.s.}}{\dot{m}c_p} dx$$

Restrictions:
Steady-state
Constant properties
$Ec \ll 1$
$Pe \gg 1$

Thus, $\dot{m}c_p(T_{b,2} - T_{b,1}) = \int_{x_1}^{x_2} \bar{q}_w'' Peri_{c.s.} dx = q_{\substack{conv,total \\ wall \to fluid}}$ }

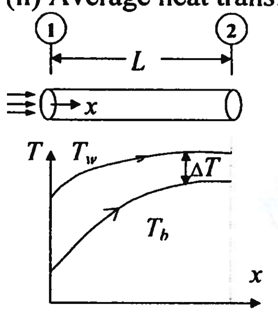
Definition of convection heat transfer coefficients

(i) Local heat transfer coefficient



$$h_{local} = \frac{(q_w'')_{local}}{(T_w - T_b)_{local}}$$

(ii) Average heat transfer coefficient



$$h_{av} = \frac{q_{w,av}''}{(T_w - T_b)_{mean}} = \frac{q_{total} / (Area_{duct\ wall\ surface})}{(T_w - T_b)_{mean}}$$

Recommended $(T_w - T_b)_{mean}$

Log-mean temperature difference (LMTD)

$$\Delta T_{LMTD} = \frac{(T_w - T_b)_1 - (T_w - T_b)_2}{\ln[(T_w - T_b)_1 / (T_w - T_b)_2]}$$

$\rightarrow \boxed{q = h_{avg} A \Delta T_{LMTD}} = \dot{m}C_p(T_{b2} - T_{b1})$

✓ Very good in correlating experimental data

✓ Appears naturally in forced convection analyses with T_w = constant

Thermally Fully-Developed Region [Requires fully-developed FLOW

In the thermally fully-developed region, the relative shape of the temperature profile no longer changes.

That is $\theta = (T_w - T)/(T_w - T_b) \neq fnc(x)$; $h = $ constant

Thus, $\frac{\partial \theta}{\partial r} = \frac{\partial}{\partial r}[(T_w - T)/(T_w - T_b)]$ is also not a function of x

At each position x, T_w and T_b are constant and not a function of r

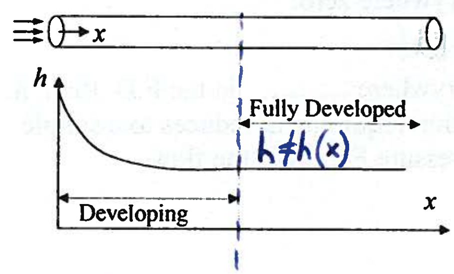
This can be used to show that at the wall ($r = r_o$):

$$\frac{\partial}{\partial r}\left[\frac{(T_w - T)}{(T_w - T_b)}\right]_{r=r_o} = \frac{-\partial T/\partial r|_{r=r_o}}{(T_w - T_b)} \neq fnc(x)$$

By definition $q_w'' = -k\left.\frac{\partial T}{\partial r}\right|_{r=r_o} = h(T_w - T_b)$

Thus, $\frac{h}{k} = \frac{-\partial T/\partial r|_{r=r_o}}{(T_w - T_b)} \neq fnc(x)$ $\boxed{h = \text{constant}}$

In the thermally fully-developed region, $h = \frac{q_w''}{T_w - T_b} = \frac{-k\frac{\partial T}{\partial r}}{T_w -}$



In the thermally fully-developed region:
✓ h is invariant with x
✓ Or, equivalently,
$\theta = (T_w - T)/(T_w - T_b) \neq fnc($

his region can be established at a sufficiently large distance downstream f the inlet plane for several commonly encountered boundary conditions. or example:

q''_w = constant : thin-walled pipe heated electrically
↳ well insulated from outside

] q'_w = constant and T_w = constant in the cross-section, but varies axially: Relatively thick-walled pipe heated electrically; well insulated

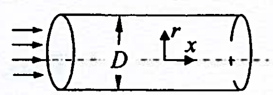
T_w = constant peripherally and axially: Tubes in boiler
Boiling/Condensation

nalytical and Empirical Correlations for Fully-Developed Flow and leat Transfer in Straight Pipes of Circular Cross-Section Newtonian fluids; constant properties; $Ec \ll 1$]

aminar flow and heat transfer

$Re_D = \dfrac{\rho u_{av} D}{\mu} = \dfrac{4\dot{m}}{\mu \pi D} \leq 2300$]

$D = 2r_o$

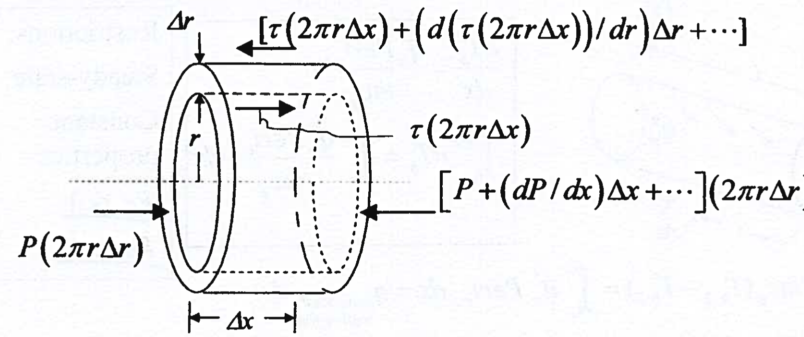


oiseuille flow:
ı fully developed region, it is important to note at both the radial velocity component, v, and the gradient of the axial elocity component $(\partial u/\partial x)$ are everywhere zero:

$v = 0$ and $(\partial u/\partial x) = 0$. $u = u(r)$

hus, the net axial momentum is everywhere ……….. in the F.D. Region. s a result, the momentum conservation requirement reduces to a simple alance between Shear Forces and Pressure Forces in the flow.

Consider an element of the fluid:



$$-[d(r\tau)/dr + \cdots]2\pi\Delta x\Delta r - [(dP/dx) + \cdots](2\pi r\Delta r\Delta x) = 0$$

Divide by volume $2\pi r\Delta r\Delta x$ and take the limit $\Delta x \to 0$ and $\Delta r \to 0$:

$$-\frac{1}{r}\frac{d}{dr}(r\tau) - \frac{dP}{dx} = 0$$

Thus, the x momentum equation is reduced to:

$$\frac{\mu}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right) = \frac{dP}{dx} \qquad P = P(x)$$

BCs: $r = 0 \rightarrow du/dr = 0$
$r = r_o \rightarrow u = 0$

Solution:

$u = u_c[1 - (r/r_o)^2]$; Here, u_c : centerline velocity

$$u_c = -\frac{1}{4\mu}\left(\frac{dP}{dx}\right)r_o^2$$

$$u_{av} = \int_0^{r_o} u 2\pi r dr \Big/ (\pi r_o^2) \Rightarrow u_{av} = u_c/2 = -\frac{1}{8\mu}\left(\frac{dP}{dx}\right)r_o^2$$

P is constant

Darcy friction factor [Note: Definition applies to both laminar and turbulent flows]: $f_{Darcy} = \dfrac{-(dP/dx)D}{\rho u_{av}^2/2}$; here, P is the Reduced Pressure

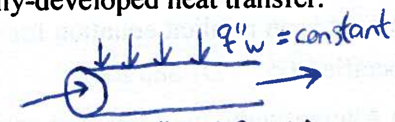
For Poiseuille flow, $f_{Darcy} = \dfrac{[(P_1 - P_2)/L]D}{\rho u_{av}^2/2} = \dfrac{64}{Re_D}$ or $f_{Darcy} Re_D = 64$

$Re_D \leq 2300$

Fanning friction factor [Note: Definition applies to both laminar and turbulent flows]: $f_{Fanning} = \dfrac{\tau_w}{\rho u_{av}^2/2}$; here, τ_w is the shear stress at wall

For Poiseuille flow, $f_{Fanning} = \dfrac{-\mu(du/dr)_{r=r_o}}{\rho u_{av}^2/2} = \dfrac{16}{Re_D}$ or $f_{Fanning} Re_D = 16$

Nusselt number values for laminar fully-developed heat transfer:



A) Uniform wall heat flux B.C.:

$q''_{w \to fluid} = k_{fluid}(dT/dr)_{r=r_o} = \text{constant}$ $\quad q''_w \neq fcn(x)$

and also h = constant. Thus, in this case,

$q''_w = h(T_w - T_b)$

$$\left.\frac{\partial T}{\partial x}\right|_{fd} = \left.\frac{dT_w}{dx}\right|_{fd} = \left.\frac{dT_b}{dx}\right|_{fd} = \text{constant} \quad q''_w = \text{constant}$$

please show this

Thus the heat transfer due to axial heat conduction is zero: $\left(\partial^2 T/\partial x^2\right) = 0$

and also neglecting the viscous dissipation terms, the governing energy equation is:

$$u\frac{\partial T}{\partial x} + \cancel{v\frac{\partial T}{\partial r}} = \frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \cancel{\alpha\frac{\partial^2 T}{\partial x^2}}$$

0, parallel flow, v=0 $\qquad$ 0

$$\therefore u\frac{\partial T}{\partial x} = \frac{\alpha}{r}\frac{\partial}{\partial r}\left[r\frac{\partial T}{\partial r}\right]$$

For fully developed region, the velocity boundary layer approximations are satisfied. That is, the radial component of velocity $v = 0$, and $(\partial u/\partial x) = 0$ everywhere. The parabolic profile developed earlier is give for the axial velocity, u. Thus, the governing equation becomes:

$$u_c[1-(r/r_o)^2]\frac{dT_b}{dx} = \frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)$$

Or, the axial heat convection is balanced with the radial heat conductior

The BCs are:

$r = 0 \rightarrow T$ is finite

$r = r_o \rightarrow T = T_w$

Solution: $T(r) = T_w - \dfrac{2u_{av}r_o^2}{\alpha}\dfrac{dT_b}{dx}\left[\dfrac{3}{16} + \dfrac{1}{16}\left(\dfrac{r}{r_o}\right)^4 - \dfrac{1}{4}\left(\dfrac{r}{r_o}\right)^2\right]$

The bulk temperature was given as,

$$T_b = \int_{A_{c.s.}} \rho u c_p T dA_{c.s.} \Big/ \int_{A_{c.s.}} \rho u c_p dA_{c.s.} = \int_{A_{c.s.}} \rho u T dA_{c.s.} \Big/ \dot{m}$$

Using the temperature and velocity profiles in this equation and workin out the integration, an expression for the bulk temperature can be obtained.

$T_b - T_w = \dfrac{11}{48}\dfrac{u_{av}r_o^2}{\alpha}\dfrac{dT_b}{dx}$ and using the fact that $\dfrac{dT_b}{dx} = \dfrac{q''_w \pi D}{\dot{m}c_p} = \dfrac{2q''_w}{\rho u_{av} r_o c_p}$

$$T_b - T_w = \frac{11}{48}\frac{2r_o}{k}q''_w \Rightarrow \boxed{h = \frac{48}{11}\frac{k}{D}} \; ; h = \frac{q''_w}{T_w - T_B}$$

Thus, $\boxed{Nu = \dfrac{hD}{k_{fluid}} = 4.364}$

$$Nu = \frac{hD}{k_{fluid}} = 4.364 \Bigg\} \rightarrow \begin{cases} \text{Uniform wall heat flux B.C.:} \\ \dot{q}_{w \rightarrow fluid} = k_{fluid}(dT/dr)_{r=r_o} = \text{constant} \end{cases}$$

or constant wall temperature:

$$Nu = \frac{hD}{k_{fluid}} = 3.658 \Bigg\} \rightarrow \begin{cases} \text{Uniform wall temperature B.C.:} \\ T_w = T_{r=r_o} = \text{constant} \end{cases}$$

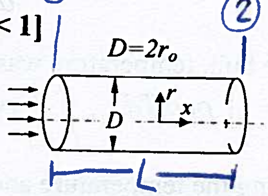
Note: These two thermal boundary conditions and the corresponding values of the Nusselt number represent extremes (bounding values) w.r.t. other thermal boundary conditions that yield the thermal fully-developed region: $h_{\substack{fully-developed \\ T_w=constant}} \le h_{\substack{fully-developed \\ other\ B.C.s}} \le h_{\substack{fully-developed \\ \dot{q}_w=constant}}$

Sparrow & Patankar 1972

Turbulent flow and heat transfer
[Newtonian fluids; constant properties; $Ec \ll 1$]

1

2



$$Re_D = \frac{\rho u_{av} D}{\mu} = \frac{4\dot{m}}{\mu \pi D} > 2300]$$

Notes:

1) Turbulent flows more frequently encountered in practice than laminar flows / Except in ...Microfluid Mechanics, Compact Heat Exchanger

2) $L_{\substack{entrance \\ laminar}} > L_{\substack{entrance \\ turbulent}}$, for the same D } because of ...Turbulent Mixing.

3) In turbulent flows, if $(L/D) \ge 10$, the flow may be assumed to be fully developed; the heat transfer too may be assumed to be fully developed, provide the thermal boundary conditions allow this [see notes on p. 7]

(4) As $L_{\substack{entrance \\ turbulent}}$ are quite small, attention in practical problems involving turbulent flow is usually focused mainly on the fully-developed regio – as is done in this course.

(5) In turbulent flows: $Nu_{\substack{fully-developed \\ \dot{q}_w=constant\ B.C.}} = Nu_{\substack{fully-developed \\ T_w=constant\ B.C.}}$ Pr ≪ 1 except for liquid m

(6) Thus, the Nu correlations given in this section can be used for all B.C. that yield the thermally fully-developed region [see notes on p. 7]

Colebrook-White (C-W) correlation for Darcy friction factor

$$\frac{1}{\sqrt{f_{Darcy}}} = -2.0 \log_{10}\left[\frac{(\varepsilon_{rms}/D)}{3.7} + \frac{2.51}{Re_D \sqrt{f_{Darcy}}}\right]$$

Notes: 1) ε_{rms} is the root mean square of roughness of the pipe wa

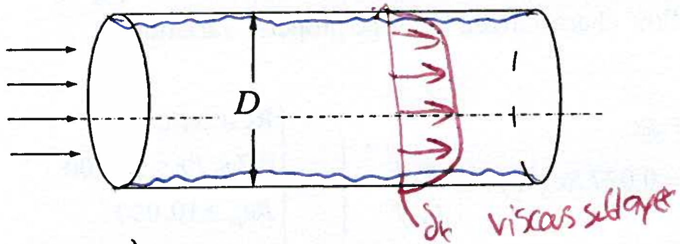
2) This is an implicit equation for f_{Darcy}: an iterative solution is needed fo specified (ε_{rms}/D) and Re_D.

3) Alternatively, the following **explicit approximation of the C-W correlation** may be used:

$$f_{Darcy} = \left\{-2.0 \log_{10}\left[\frac{(\varepsilon_{rms}/D)}{3.7} - \frac{5.02}{Re_D} \log_{10}\left(\frac{(\varepsilon_{rms}/D)}{3.7} + \frac{14.49}{Re_D}\right)\right]\right\}^{-2}$$

4) The C-W correlation is used to generate the Moody diagram (see page 14)

5) Smooth and rough pipes



$\delta_{viscous\ sublayer} \geq 3.0\varepsilon_{rms}$ } Hydraulically smooth

$\delta_{viscous\ sublayer} < 3.0\varepsilon_{rms}$ } Hydraulically rough

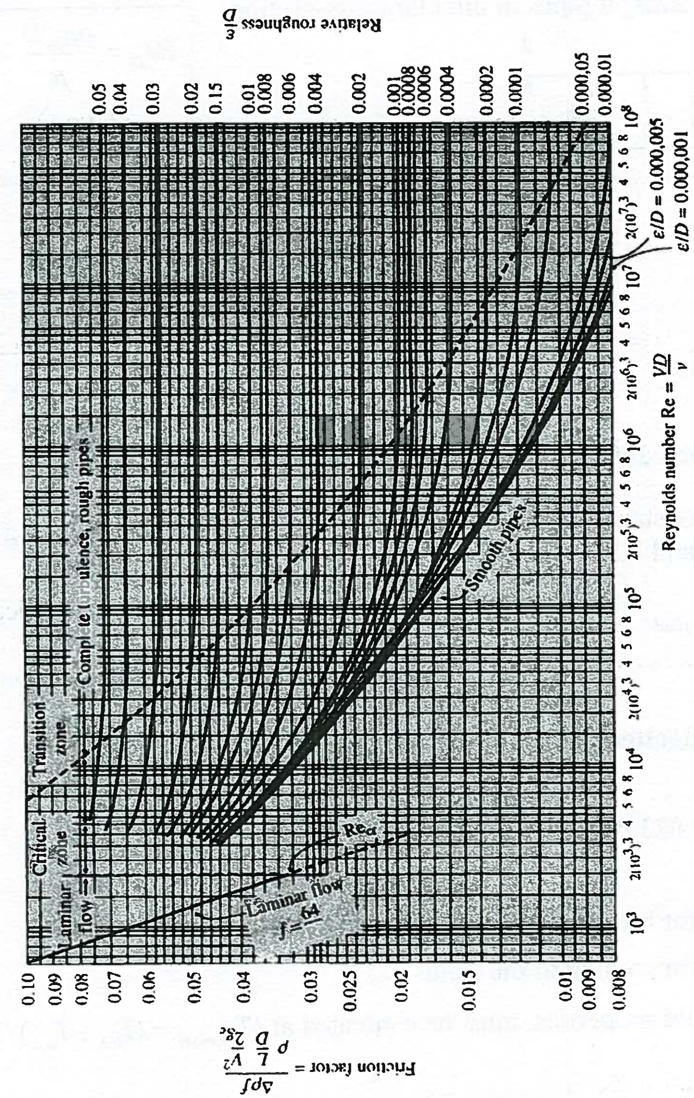
7) Holman, 2002, proposes for smooth pipe:

$$f_{\substack{Darcy\\smooth\\pipes}} = \frac{0.316}{\mathrm{Re}_d^{1/4}} \qquad \mathrm{Re}_d \leq 2\times10^5$$

8) For fully-developed turbulent flow in smooth pipes, the **Filonenko** correlation may also be used (not as accurate as the C-W correlation with $\varepsilon_{rms} = 0$, but simpler to use) to calculate the Darcy friction factor:

$$f_{darcy} = \frac{[(P_1 - P_2)/L]D}{\rho u_{av}^2/2} = \left[1.82\log_{10}(Re_D) - 1.64\right]^{-2}$$

Moody Diagram

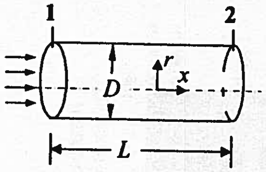


*commercial pipe: $\varepsilon \simeq 46\,\mu m$

cast iron: $\varepsilon \approx 260\,\mu m$

Concrete pipe: $\varepsilon \approx 300\text{-}3000\,\mu m$

lusselt number correlations for heat transfer *fully-developed turbulent 'ow* in straight pipes of circular cross-section



$$Re_D = \frac{\rho u_{av} D}{\mu} = \frac{4\dot{m}}{\mu \pi D}$$

$$Pr = \left(\frac{\mu c_p}{k}\right)_{fluid}$$

$$Nu_D = \frac{hD}{k_{fluid}}$$

$$q_{\substack{total \\ wall \to fluid}} = (\pi D L) h (\Delta T)_{LMTD} \quad \Big| \quad (\Delta T)_{LMTD} = \frac{(T_w - T_b)_1 - (T_w - T_b)_2}{\ln[(T_w - T_b)_1 / (T_w - T_b)_2]}$$

Holman 2002, suggests: $q_{\substack{total \\ wall \to fluid}} = (\pi D L) h (T_w - \frac{T_{b1} + T_{b2}}{2})$

For constant thermophysical properties; $Ec << 1$; $Pe >> 1$; negligible K.E. and P.E. changes; and steady-state conditions:

$q_{\substack{total \\ wall \to fluid}} = \boxed{(\pi D L) h (\Delta T)_{LMTD} = \dot{m} c_p (T_{b,2} - T_{b,1})}$ } Note: Implications of the ....

ttus-Boelter correlation [applies only to smooth pipes]

$$_D = 0.023 Re_D^{0.8} Pr^n \; \} \rightarrow \begin{cases} \text{Restrictions:} \\ 0.6 \le Pr \le 160 \\ Re_D \ge 10{,}000 \end{cases}$$

= 0.4 for heating of the fluid

= 0.3 for cooling of the fluid

All fluid properties, must be evaluated at $(T_b)_{mean} = (T_{b,1} + T_{b,2})/2$

Sieder-Tate correlation [applies only to smooth pipes]

(For flow characterized by large property variations)

$$Nu_D = 0.027 Re_D^{0.8} Pr^{1/3} \left(\frac{\mu}{\mu_w}\right)^{0.14} \; \} \rightarrow \begin{cases} \text{Restrictions:} \\ 0.7 \le Pr \le 16{,}700 \\ Re_D \ge 10{,}000 \\ (L/D) \ge 10 \end{cases}$$

- All fluid properties, except μ_w, must be evaluated at $(T_b)_{mean} = (T_{b,1} + T_{b,2})/2$
- μ_w is the dynamic viscosity of the fluid at $T = T_w$

Petukhov, correlation [applies to both smooth and rough tubes]

$$Nu_D = \left[\frac{(f_{Darcy}/8) Re_D Pr}{1.07 + 12.7 (f_{Darcy}/8)^{1/2} (Pr^{2/3} - 1)}\right] \left(\frac{\mu_b}{\mu_w}\right)^n \; \} \rightarrow \begin{cases} \text{Restrictions:} \\ 0.5 \le Pr \le 2000 \\ 10^4 \le Re_D \le 5\text{x}1 \\ 0.8 \le \frac{\mu_b}{\mu_w} \le 40 \end{cases}$$

$n = 0.11$ for heating of the fluid

$n = 0.25$ for cooling of the fluid

$n = 0$ for constant heat flux or for gases.

- Calculate f_{Darcy} values using the C-W correlation (explicit approx)
- For **smooth pipes**, f_{Darcy} may be obtained from the C-W correlation with $\varepsilon_{rms} = 0$, or the **Filonenko** correlation.

xample 1

onsider fully developed flow of air inside a pipe with circular cross ection. The pipe internal diameter is D = 0.01 m, and the pipe wall mperature is maintained at T_w = 140 °C. The air average velocity is $_{av}$ = 2 m/s. If the inlet bulk temperature for the air is T_{b1} = 20°C, what ould be the length of the pipe to get an exit air temperature of $_{b2}$ = 84°C?

ns. L = 0.406 m.

xample 2

onsider fully developed flow of a fluid inside a smooth pipe with rcular cross section. The pipe internal diameter is D = 0.08 m, and the ipe wall temperature is maintained at T_w = 300 °C. The air average elocity is u_{av} = 3 m/s and the pipe is L = 5 m long. If the inlet bulk mperature for the air is T_{b1} = 60°C, find T_{b2} = ?, and $q_{\text{total to fluid}}$?

luid properties are assumed constant and given as:

ρ = 1.5 kg/m^3; c_p = 1200 J/kg-°C;

ν = 16× 10^{-6} m^2/s; k = 0.025 W/m-°C.

ns. T_{b2} = 189.1°C, and $q_{\text{total to fluid}}$ = 3504.1 W.

$$\frac{dT_b}{T_b - T_w} = \frac{-h\pi D\,dx}{\dot m C_p} \Longrightarrow \int_{T_{b1}}^{T_{b2}} \frac{dT_b}{T_b - T_w} = \int_0^L \frac{-h\pi D\,dx}{\dot m C_p}$$

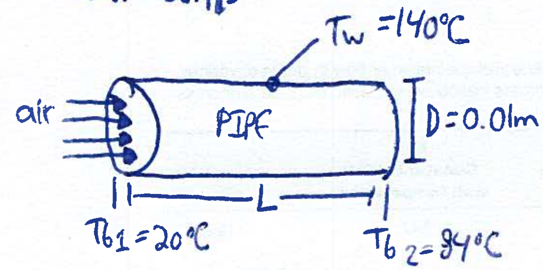
$$\boxed{\ln\left(\frac{T_{b2}-T_w}{T_{b1}-T_w}\right) = \frac{-h\pi D}{\dot m C_p} L}$$

$$\dot m c_p (T_{b2}-T_{b1}) = \frac{h(\pi D L)\,[T_{b2}-T_{b1}]}{\ln\left(T_{b2}-T_w / T_{b1}-T_w\right)}$$

$$\boxed{\dot m C_p (T_{b2}-T_{b1}) = h A_{surf}\, \Delta T_{LMTD}} \rightarrow L = 0.406\text{m}$$

Example 1 - What is L?

$u_{av} = 2$ m/s



- steady state constant property fluid flow and heat transfer
- fully developed
- $Ec \ll 1$ no viscous dissipation

$Re_D = \frac{\rho u_{av} D}{\mu}$ → go to Air Tables at end of textbook

$T_b = \frac{T_{b1} + T_{b2}}{2} = \frac{84+20}{2} = 52°C = \underline{325.15K}$ → use this parameter for air ta

$\therefore \rho = 1.0877$ Kg/m^3

$C_p = 1007.55$ J/Kg°C

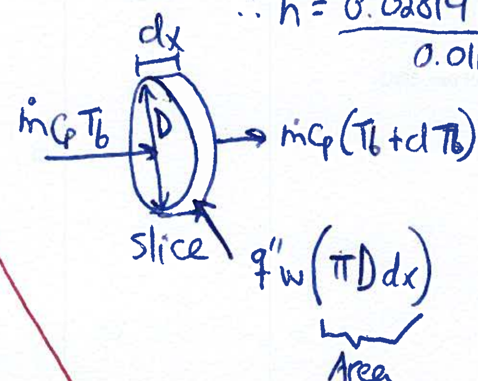
$K = 0.02814$ W/m°C

$\mu = 1.9606 \times 10^{-5}$ Kg/mS

$Re_D = 1109.56 < 2300$ ∴ Laminar Flow

T_w constant means $Nu_D = \frac{hD}{K} = 3.658$

$\therefore h = \frac{0.02814(3.658)}{0.01m} = 10.294$ W/m^2°C



$\boxed{q_w'' = h(T_w - T_b)}$

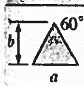
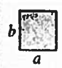
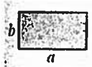
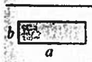
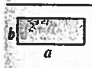
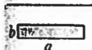
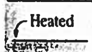
Energy Balance

$$\frac{dT_b}{dx} = \frac{q_w'' \pi D}{\dot m C_p} = \frac{h\pi D(T_w - T_b)}{\dot m C_p}$$

Flow and Heat Transfer in Ducts of Non-Circular Cross-Section

A- Laminar flow and heat transfer [$Re_{D_h} = \rho u_{av} D_h / \mu \leq 2300$]

Table 6-1 | Heat transfer and fluid friction for fully developed laminar flow in ducts of various cross sections. Average Nusselt numbers based on hydraulic diameters of cross sections.

Geometry ($L/D_h > 100$)	Nu_H Constant axial wall heat flux	Nu_T Constant axial wall temperature	$f Re_{D_h}/4$
 $\frac{b}{a} = \frac{\sqrt{3}}{2}$	3.111	2.47	13.333
 $\frac{b}{a} = 1$	3.608	2.976	14.227
	4.002	3.34	15.054
 $\frac{b}{a} = \frac{1}{2}$	4.123	3.391	15.548
	4.364	3.657	16.000
 $\frac{b}{a} = \frac{1}{4}$	5.331	4.44	18.23
 $\frac{b}{a} = \frac{1}{3}$	4.79	3.96	17.25
 $\frac{b}{a} = \frac{1}{8}$	6.490	5.597	20.585
 $\frac{b}{a} = 0$	8.235	7.541	24.000
 Heated / Insulated $\frac{b}{a} = 0$	5.385	4.861	24.000

(Data from Holman, 2002)

B- Turbulent flow and heat transfer in ducts of non-circular cross-section [$Re_{D_h} = \rho u_{av} D_h / \mu > 2300$]

- Use specific correlations, if available
- Use experimental data, if available, to propose correlations of the following forms:

$$f_{Darcy} = \frac{-(dP/dx)\,D_h}{(\rho u_{av}^2/2)} = \Im\, Re^{\beta} \quad \text{and} \quad Nu_{D_h} = \frac{h\,D_h}{k_{fluid}} = C\, Re^m\, Pr^n$$

- Use the Chilton-Colburn analogy when either pressure drop or he transfer data are given:

$$\frac{1}{8} f_{Darcy} = St\, Pr^{2/3} \quad ; \text{or} \quad \frac{1}{8}\left[\frac{\{(P_1 - P_2)/L\}\,D_h}{\rho u_{av}^2/2}\right] = \left(\frac{h}{\rho u_{av} c_p}\right) Pr^{2/3}$$

- Calculate $f_{Darcy} = [\{(P_1 - P_2)/L\}\,D_h/(\rho u_{av}^2/2)]$ and $Nu_{D_h} = (h\,D_h/$ using the abovementioned correlations for pipes of circular cross-section, but with the hydraulic diameter, $D_h = (4A_{c.s.}/Peri_{c.s.,wetted})$ <u>replacing</u> the diameter, D, and the Reynolds number based on hydraulic diameter, $Re_{D_h} = \rho u_{av} D_h / \mu$, <u>replacing</u> the Reynolds nu based on the diameter, $Re_D = \rho u_{av} D / \mu$
- All thermophysical properties of the fluid should be evaluated at $(T_b)_{mean} = (T_{b,1} + T_{b,2})/2$
- $q_{\substack{total \\ wall \to fluid}} = (A_{wall\ surface})\, h\, (\Delta T)_{LMTD}$ | $(\Delta T)_{LMTD} = \frac{(T_w - T_b)_1 - (T_w -}{\ln[(T_w - T_b)_1/(T_w -}$

- For constant thermophysical properties; $Ec \ll 1$; $Pe \gg 1$; negligit K.E. and P.E. changes; and steady-state conditions:

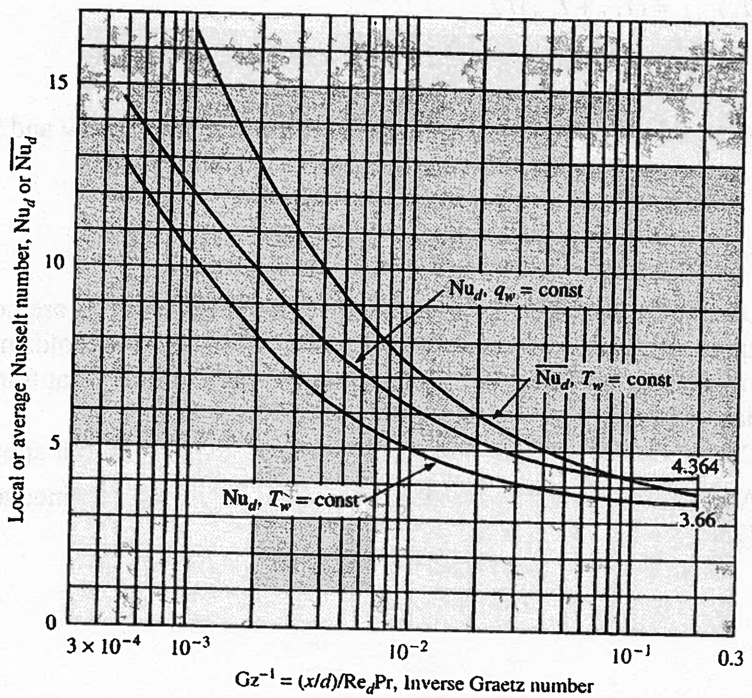
  $q_{\substack{total \\ wall \to fluid}} = (A_{wall\ surface})\, h\, (\Delta T)_{LMTD} = \dot{m} c_p (T_{b,2} - T_{b,1})$ } Note:

  Implications of the ....

trance Effect on Heat Transfer in Ducts

- Laminar flow and heat transfer [$Re_D = \rho u_{av} D/\mu \leq 2300$]

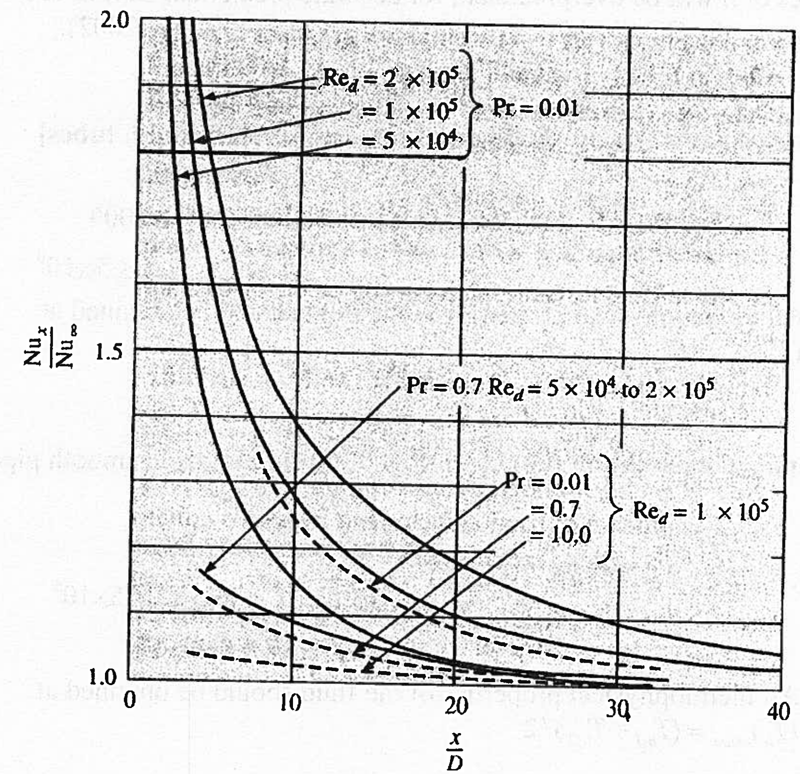
Figure 6-5 | Local and average Nusselt numbers for circular tube thermal entrance regions in fully developed laminar flow.



(Figure from Holman, 2002)

B- Turbulent flow and heat transfer in ducts of non-circular cross-section [$Re_D = \rho u_{av} D/\mu > 2300$]

Figure 6-6 | Turbulent thermal entry Nusselt numbers for circular tubes with q_w = constant.



(Figure from Holman, 2002)

- Could also obtain rough values of f_{Darcy} using the Moody diagram
- All fluid properties, except μ_w, must be evaluated at $(T_b)_{mean} = (T_{b,1} + T_{b,2})/2$

Note: The SiederTate and the Petukhov, correlations may be used as a first approximation for $2300 \le Re_D \le 10^4$ with the understanding that the values of h will be overpredicted; for accurate predictions of h in this range use the Gnielinski correlation (Not given in Holman, 2002).

Gnielinski correlation [applies to both smooth and rough tubes]

$$Nu_D = \left[\frac{(f_{Darcy}/8)(Re_D - 1000)\,Pr}{1 + 12.7(f_{Darcy}/8)^{1/2}(Pr^{2/3} - 1)} \right] \Bigg\} \rightarrow \begin{cases} \text{Restrictions:} \\ 0.5 \le Pr \le 2000 \\ 3000 \le Re_D \le 5\text{x}10^6 \end{cases}$$

- All thermophysical properties of the fluid should be obtained at $(T_b)_{mean} = (T_{b,1} + T_{b,2})/2$

Skupinshi correlation [turbulent flow of **liquid metals** in smooth pipes]

$$Nu_D = 4.82 + 0.0185(Re_D\,Pr)^{0.827} \Big\} \rightarrow \begin{cases} \text{Restrictions:} \\ q''_w = constant\ B.C. \\ 3.6\times10^3 \le Re_D \le 9.05\times10^5 \\ 10^2 \le (Re_D\,Pr) \le 10^4 \end{cases}$$

- All thermophysical properties of the fluid should be obtained at $(T_b)_{mean} = (T_{b,1} + T_{b,2})/2$

Seban-Shimazaki correlation [turbulent flow of **liquid metals** in sm pipes]

$$Nu_D = 5.0 + 0.025(Re_D\,Pr)^{0.8} \Big\} \rightarrow \begin{cases} \text{Restrictions:} \\ T_w = constant\ B.C. \\ (Re_D\,Pr) \ge 100 \end{cases}$$

- All thermophysical properties of the fluid should be obtained at $(T_b)_{mean} = (T_{b,1} + T_{b,2})/2$

Chilton-Colburn analogy for fully-developed turbulent flow and hea transfer in straight pipes:

$$\frac{1}{8} f_{Dacry} = St\,Pr^{2/3} \quad \text{or} \quad \frac{1}{8}\left[\frac{\{(P_1 - P_2)/L\}\,D}{\rho u_{av}^2/2}\right] = \left(\frac{h}{\rho u_{av} c_p}\right) Pr^{2/3}$$

- Use this analogy when the above-mentioned correlations are not applicable (when the roughness is unknown; or the Reynolds num value is outside range; …) ***and*** either pressure drop or heat transf data are given.
- Calculate f_{Darcy} values using the C-W correlation (explicit approx
- All thermophysical properties of the fluid should be obtained at $(T_b)_{mean} = (T_{b,1} + T_{b,2})/2$

MECH-375 HEAT TRANSFER - HANDOUT # 9

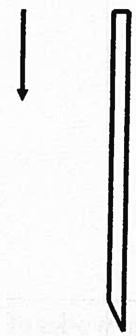
Convection Heat Transfer – IV: Buoyancy-Driven Natural and Mixed Convection in a Gravitational Force Field

Topics

- **Natural Convection**
  - **Some basic considerations**
  - **Thermal volumetric expansion coefficient**
  - **Boussinesq approximation**
  - **Governing equations**
  - **Dimensionless parameters**
  - **General form of correlations for Nusselt number**
  - **Some comments on design correlations**
  - **Recommended design correlations**
    - ✓ **External natural convection**
    - ✓ **Natural convection in enclosed spaces**
- **Mixed Convection**
  - **Some basic considerations**
  - **External mixed convection**
  - **Internal mixed convection in pipes**

Natural Convection

Some basic considerations



Suppose the density of the fluid, ρ,

Specific volume:

Thermal volumetric expansion coefficient: β □ =

For a perfect gas: $\beta = 1/T_{abs}$

Boussinesq approximation

1.

2.

Governing equations [Newtonian fluid; steady-state; $Ec << 1$]

- Continuity equation
- Momentum equations [ ]
- Energy equation
- A key conclusion

$$h_{@\text{any }x} \,\square \left[-k_{fluid} (\partial T / \partial y)_{\substack{\text{in fluid} \\ @y=0}} \right]_{@\text{ corresponding } x} \Big/ (T_w - T_\infty)$$

$$= fnc[x, \rho_{ref}, \beta, \mu, c_p, k, (T_w - T_\infty), g, geometry, ...]$$

imensionless parameters

or a given geometry: $Nu_{av} = \dfrac{h_{av} L_c}{k_{fluid}} = fnc[Gr, Pr]$ where

$$r = \left.\frac{\beta g (T_w - T_\infty) L_c^3}{(\mu/\rho)^2}\right\} \text{ Grashof \#} \propto$$

$$r = \left.\frac{\mu c_p}{k} = \frac{\nu}{\alpha}\right\} \text{ Prandtl \#} \propto$$

eneral form of correlation for Nusselt number

$$Ju_{av} = \frac{h_{av} L_c}{k_{fluid}} = fnc(Gr, Pr) =$$

$$\text{a} =$$

Based on the works of Eckert, Schmidt and Beckmann, and Ostrach

ome comments on design correlations

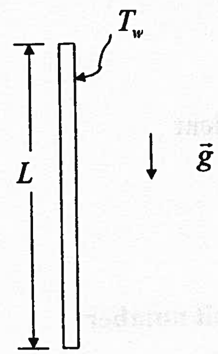
lumerous design correlations for both external and internal natural onvection are available in the published literature. For example:

1. Holman, J.P., *Heat Transfer*, 9th Edition, Chapter 7, McGraw-Hill, 2002.
2. Kreith, F. and Bohn, M.S., *Principles of Heat Transfer*, 6th Edition, Chapter 5, Table 5.2, Brooks-Cole, CA, 2001.
3. Incropera, F.P. and DeWitt, D.P., *Fundamentals of Heat and Mass Transfer*, Chpater 9, 5th Edition, John Wiley & Sons, New York, 2002.
4. Raithby, G.D. and Hollands, K.G.T., *Natural Convection Heat Transfer*, in *CRC Handbook of Mechanical Engineering*, F. Kreith, Editor, CRC Press, Boca Raton, Florida, 1998.

Recommended Design Correlations

A: External Natural Convection

1: Vertical Flat Plate: Isothermal Surface



$$Gr_L = \frac{g\beta [T_w - T_\infty] L^3}{\nu^2}; \quad \Pr = \frac{\mu c_p}{k} = \frac{\nu}{\alpha}; \quad Ra_L =$$

- Obtain fluid properties at $T_{film} = [T_w + T_\infty]/2$

McAdams (Holman, 2002) correlation:

$$Nu_{av} = \left(\frac{h_{av} L}{k_{fluid}}\right) = 0.59 (Ra_L)^{1/4} \qquad 10^4 \le Ra_L \le 10^9$$

- **Churchill-Chu** correlations:

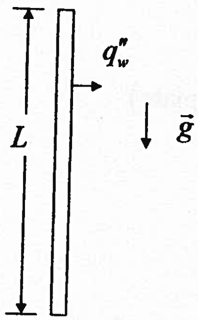
$$Nu_{av} = \left(\frac{h_{av} L}{k_{fluid}}\right) = 0.68 + \frac{0.670 Ra_L^{1/4}}{\left[1 + (0.492/\Pr)^{9/16}\right]^{4/9}} \qquad \text{for } Ra_L \le 10^9$$

$$Nu_{av} = \left(\frac{h_{av} L}{k_{fluid}}\right) = \left[0.825 + \frac{0.387 Ra_L^{1/6}}{\left[1 + (0.492/\Pr)^{9/16}\right]^{8/27}}\right]^2 \qquad \text{for } Ra_L > 10^9$$

Notes:

1. These correlations apply to the T_w =constant boundary condition
2. If the plate is highly conductive, these correlations may also be applied to q''_w =constant boundary condition.

2: Vertical Flat Plate: Wall Constant-Heat-Flux (q''_w=constant)



Local Gr_x and Ra_x are defined as:

$$Gr_x = \frac{g\beta q''_w x^4}{k\nu^2}; \quad \Pr = \frac{\mu c_p}{k} = \frac{\nu}{\alpha}; \quad Ra_x = Gr_x \Pr$$

To obtain the average Nu number:

- Guess the wall average temperature, $T_{w,av}$
- Obtain fluid properties at $T_{film} = \left[T_{w,av} + T_\infty\right]/2$
- Use the following correlations (given in Holman, 2002), and find $Nu_{x=L}$

Local Nusselt for **Laminar** regime:

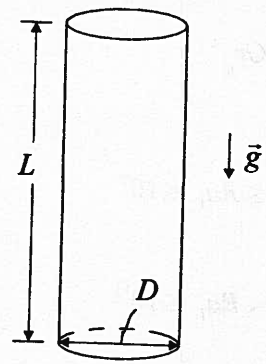
$$Nu_x = \left(\frac{h_x x}{k_{fluid}}\right) = 0.60 Ra_x^{1/5} \qquad \text{for } 10^6 \le Ra_x \le 10^{12}$$

Local Nusselt for **Turbulent** regime:

$$Nu_x = \left(\frac{h_x x}{k_{fluid}}\right) = 0.17 Ra_x^{1/4} \qquad \text{for } 10^{12} < Ra_x \le 10^{16}$$

- Laminar regime: $h_{av} = 1.25 h_{x=L}$; Turbulent regime: $h_{av} = h_{x=L}$
- Obtain the new $T_{w,av}$, and repeat the abovementioned procedure until convergence.

3: Vertical Cylinder



$$Gr_L = \frac{g\beta\left[T_w - T_\infty\right]L^3}{\nu^2}$$

For vertical cylinder:

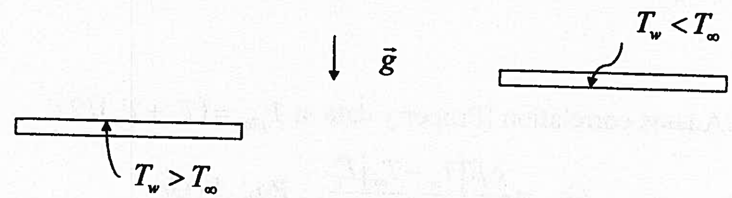
- If   <<   , use the correlations give for vertical flat plate. The condition is met when: $D/L \ge 35/Gr_L^{1/4}$
- If the $D/L < 35/Gr_L^{1/4}$, the flat plate result for the average heat transfer coefficient should be multiplied by a factor F given below (transverse curvature plays a role):

$$F = 1.3\left[(L/D)/Gr_D\right]^{0.25} + 1.0$$

Where: $Gr_D = \dfrac{g\beta\left[T_w - T_\infty\right]D^3}{\nu^2}$

4: Horizontal Flat Plate: Isothermal Surface

4-a) Hot plate facing upward; Cold plate facing downward



se Lloyd –Moran correlation [Property data at $T_{film} = (T_w + T_\infty)/2$]:

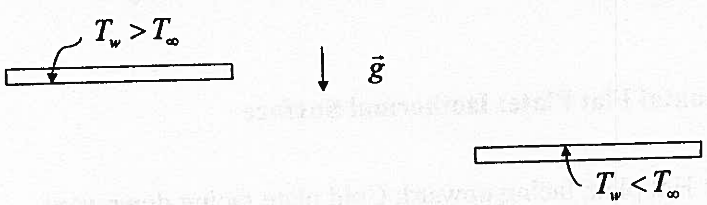
$$Gr_{L_c} = \frac{g\beta[T_w - T_\infty]L_c^3}{\nu^2}; \quad Ra_{L_c} = \Pr Gr_{L_c}$$

$$Nu_{av} = \left(\frac{h_{av}L_c}{k_{fluid}}\right) = 0.54Ra_{L_c}^{1/4} \quad \text{for } 10^4 \le Ra_{L_c} \le 10^7$$

$$Nu_{av} = \left(\frac{h_{av}L_c}{k_{fluid}}\right) = 0.15Ra_{L_c}^{1/3} \quad \text{for } 10^7 < Ra_{L_c} \le 10^{11}$$

Notes: $L_c = (\text{Surface area of the plate})/(\text{Perimeter of the plate})$

4-b) Hot plate facing downward; Cold plate facing upward



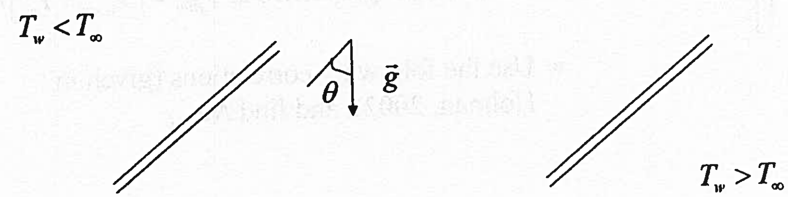
Use McAdams correlation [Property data at $T_{film} = (T_w + T_\infty)/2$]:

$$Gr_{L_c} = \frac{g\beta[T_w - T_\infty]L_c^3}{\nu^2}; \quad Ra_{L_c} = \Pr Gr_{L_c}$$

$$Nu_{av} = \left(\frac{h_{av}L_c}{k_{fluid}}\right) = 0.27Ra_{L_c}^{1/4} \quad \text{for } 10^5 \le Ra_{L_c} \le 10^{10}$$

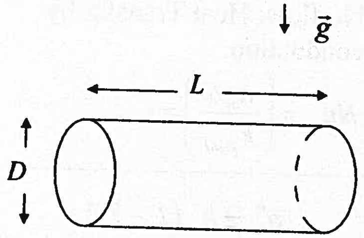
Notes: $L_c = (\text{Surface area of the plate})/(\text{Perimeter of the plate})$

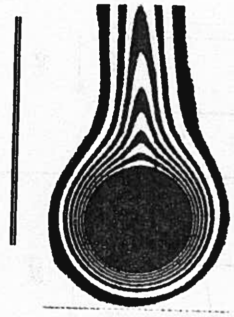
5: Inclined Flat Plate



- For the ……. surface of cooled plates ($T_w < T_\infty$) and the ……. Surface of heated plates ($T_w > T_\infty$) use the vertical flat plate correlations with g replaced by $g\cos\theta$. This approach is valid for $0 < \theta < 60$.
- For all other cases, look for specialized correlations/data.

6: Horizontal Cylinder (long: $L >> D$)





Interferometer Photograph by Prof. E.R.G. Eckert. (Kreith and Bohn, 2001).

Here $L_c = D$

$$Gr_D = \frac{g\beta[T_w - T_\infty]D^3}{\nu^2}; \quad Ra_D = \Pr Gr_D$$

Use Morgan correlation [Property data at $T_{film} = (T_w + T_\infty)/2$]

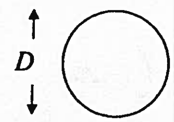
$$Nu_{av} = \left(\frac{h_{av}D}{k_{fluid}}\right) = C.Ra_D^n$$

Ra_D	C	n
10^{-10} to 10^{-2}	0.675	0.058
10^{-2} to 10^{2}	1.02	0.148
10^{2} to 10^{4}	0.850	0.188
10^{4} to 10^{7}	0.480	1/4
10^{7} to 10^{12}	0.125	1/3

Or alternatively use the Churchill-Chu correlation:

$$Nu_{av} = \left(\frac{h_{av}D}{k_{fluid}}\right) = \left\{0.60 + \frac{0.387 Ra_D^{1/6}}{\left[1 + (0.559/\Pr)^{9/16}\right]^{8/27}}\right\}^2 \qquad Ra_D \le 10^{12}$$

7: Spheres



$$Gr_D = \frac{g\beta[T_w - T_\infty]D^3}{\nu^2}; \quad Ra_D = \Pr Gr_D$$

Use Churchill correlation [Property data at $T_{film} = (T_w + T_\infty)/2$]

$$Nu_{av} = \left(\frac{h_{av}D}{k_{fluid}}\right) = 2 + \frac{0.589 Ra_D^{1/4}}{\left[1 + (0.469/\Pr)^{9/16}\right]^{4/9}} \qquad Ra_D < 10^{11} \text{ and } \Pr \ge 0.7$$

8: Irregular Solids with Isothermal Surfaces in Natural Convection

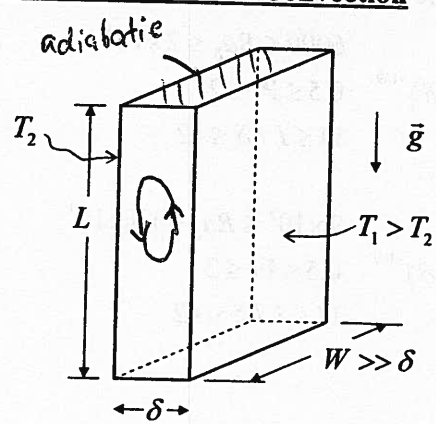
For the first estimation use Lienhard correlation:

$$Nu_{av} = \left(\frac{h_{av}L_c}{k_{fluid}}\right) = 0.52 Ra_{L_c}^{0.25} \qquad 10^4 \le Gr_{L_c}\Pr \le 10^9$$

Where the characteristic length, L_c: Is the distance fluid particle travels i boundary layer

B: Internal Natural Convection

1: Vertical Rectangular Cavity



- Average heat flux:

$$q''_{av} = \frac{q}{WL} = h_{av}(T_1 - T_2)$$

$$Gr_\delta = \frac{g\beta[T_1 - T_2]\delta^3}{\nu^2}; \; Ra_\delta = \Pr C$$

- If $Ra_\delta \le 1000$, influence of natural convection is very small.

Thus, $Nu_{av} = \left(\dfrac{h_{av}\delta}{k_{fluid}}\right) = 1$ $\quad q''_{av} = h_{av}(T_1 - T_2) = \dfrac{k_f}{\delta}(T_1 - T_2)$

- For large Ra_δ use the following correlations: $\dfrac{L}{\delta}$: aspect ratio

$$Nu_{av} = \left(\frac{h_{av}\delta}{k_{fluid}}\right) = 0.42 Ra_\delta^{1/4} \Pr^{0.012} (L/\delta)^{-0.3} \qquad \begin{array}{l} 10^4 \le Ra_\delta < 10^7 \\ 1 \le \Pr < 2\times10^4 \\ 10 \le L/\delta < 40 \end{array}$$

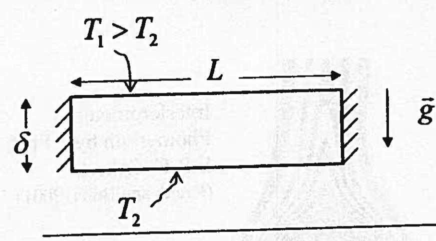
$$Nu_{av} = \left(\frac{h_{av}\delta}{k_{fluid}}\right) = 0.046 Ra_\delta^{1/3} \qquad \begin{array}{l} 10^6 \le Ra_\delta < 10^9 \\ 1 \le \Pr < 20 \\ 1 \le L/\delta < 40 \end{array}$$

- For large Ra_δ and small Pr use the following correlations:

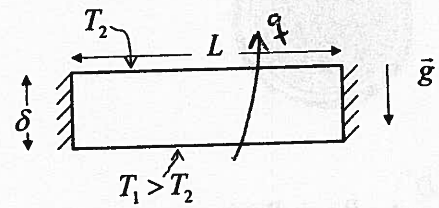
$$Nu_{av} = \left(\frac{h_{av}\delta}{k_{fluid}}\right) = 0.197 Ra_\delta^{1/4} (L/\delta)^{-1/9} \qquad \begin{array}{l} 6000 \le Ra_\delta \le 2\times10^5 \\ 0.5 \le \Pr \le 2 \\ 11 \le L/\delta \le 42 \end{array}$$

$$Nu_{av} = \left(\frac{h_{av}\delta}{k_{fluid}}\right) = 0.073 Ra_\delta^{1/3} (L/\delta)^{-1/9} \qquad \begin{array}{l} 2\times10^5 \le Ra_\delta < 1.1\times10^7 \\ 0.5 \le \Pr \le 2 \\ 11 \le L/\delta \le 42 \end{array}$$

2: Horizontal Rectangular Cavity



- This situation corresponds to a thermally stable stratification; No flow; Heat Transfer by conduction.
- $Nu_{av} = \left(\dfrac{h_{av}\delta}{k_{fluid}}\right) = 1$

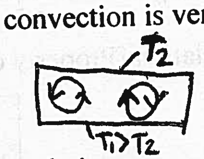


$q''_{av} = h_{av}(T_1 - T_2)$

$$Gr_\delta = \frac{g\beta[T_1 - T_2]\delta^3}{\nu^2}; \; Ra_\delta = \Pr Gr_\delta$$

- If $Ra_\delta \le 1708$, influence of natural convection is very small.

Thus, $Nu_{av} = \left(\dfrac{h_{av}\delta}{k_{fluid}}\right) = 1$

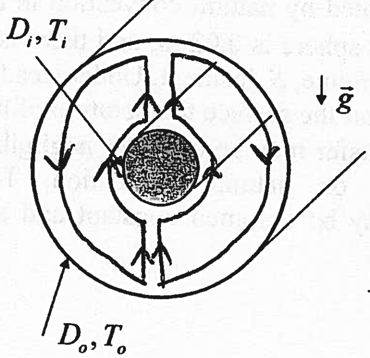


- For large Ra_δ use the following correlations:

$$Nu_{av} = \left(\frac{h_{av}\delta}{k_{fluid}}\right) = 0.212 Ra_\delta^{1/4} \qquad \begin{array}{l} 7000 \le Ra_\delta < 3\times10^5 \\ 0.5 \le \Pr < 2 \end{array}$$

$$Nu_{av} = \left(\frac{h_{av}\delta}{k_{fluid}}\right) = 0.059 Ra_\delta^{0.4} \qquad \begin{array}{l} 1708 \le Ra_\delta < 7000 \\ 0.5 \le \Pr < 2 \end{array}$$

3: Horizontal Concentric Cylinders (Isothermal)



Ti>To

- Rate of heat transfer:

$$q = \frac{(T_i - T_o)}{\ln(D_o / D_i)/2\pi k_{eff} L} \quad \text{And,}$$

$$Gr_\delta = \frac{g\beta[T_i - T_o]\delta^3}{\nu^2}; \; Ra_\delta = \Pr Gr_\delta$$

Where $\delta = r_o - r_i$

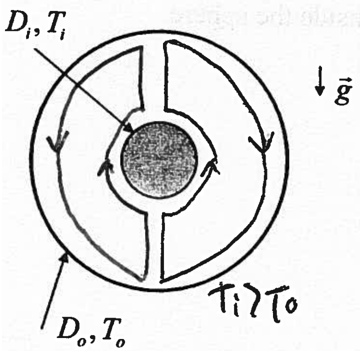
$$\frac{k_{eff}}{k_{fluid}} = 0.11(Gr_\delta \Pr)^{0.29} \qquad 6000 \le Gr_\delta \Pr \le 10^6, \quad 1 \le \Pr \le 5000$$

$$\frac{k_{eff}}{k} = 0.40(Gr_\delta \Pr)^{0.20} \qquad 10^6 \le Gr_\delta \Pr \le 10^8, \quad 1 \le \Pr \le 5000$$

f: effective thermal conductivity

$$T_{film} = \frac{T_i + T_o}{2}$$

4: Concentric Spheres (Isothermal)



Ti>To

- Rate of heat transfer:

$$q = \frac{(T_i - T_o)}{(1/r_i - 1/r_o)/4\pi k_{eff}} \quad \text{And,}$$

$$Gr_\delta = \frac{g\beta[T_i - T_o]\delta^3}{\nu^2}; \; Ra_\delta = \Pr Gr_\delta$$

Where $\delta = r_o - r_i$

$$T_{film} = \frac{T_i + T_o}{2}$$

$$\frac{k_{eff}}{k} = 0.228(Gr_\delta \Pr)^{0.226} \qquad 1.2\times10^2 \le Gr_\delta \Pr \le 1.1\times10^9, \quad 0.7 \le \Pr \le 4150, \quad 0.25 \le \delta / r_i \le 1.5$$

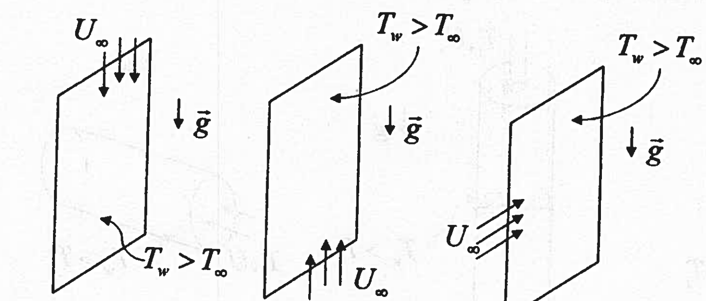
C: Mixed Convection (Combined Forced and Natural Convection)

Reynolds #, $\mathrm{Re}_{L_c} = \frac{\rho U_c L_c}{\mu}$; $\quad Gr_{L_c} = \frac{g\beta(\Delta T)_c L_c^3}{(\mu/\rho)^2}$;

Richardson → Buoyancy Forces / Inertia Forces

$$\frac{Gr_c}{\mathrm{Re}_c^2} = \begin{cases} < 0.1 & \text{Forced convection dominates} \\ 0.1 \le (\;) \le 10 & \text{Mixed convection} \\ > 10 & \text{Natural convection dominate} \end{cases}$$

1: External Mixed Convection:



Opposing Flow — Aiding Flow or assisting — Transverse Flow ommon practice [1st approximation] (Incropera and DeWitt, 1994):

- For Aiding (assisting) and Transverse Flows:

$$(Nu)^n_{mixed} = (Nu)^n_{forced} + (Nu)^n_{Natural}$$

$n = 3$ Assisting flow over vertical plates

$n = 3.5$ Transverse flow over vertical plates

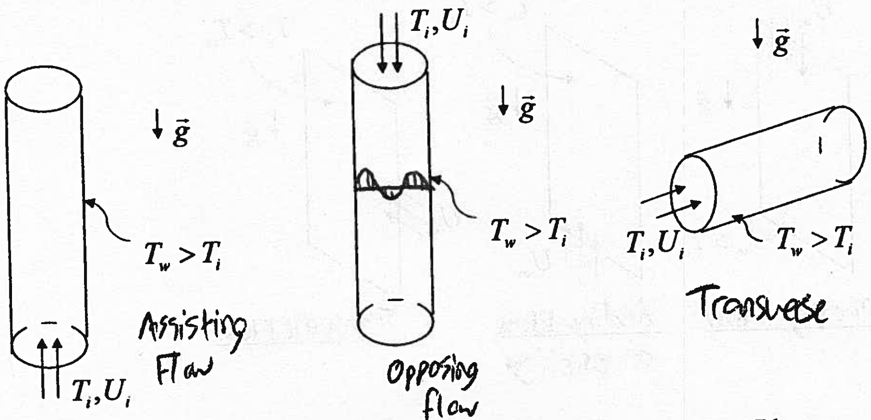
$n = 4$ Transverse flow over cylinders or spheres

- For Opposing Flows:

$$(Nu)^n_{mixed} = (Nu)^n_{forced} - (Nu)^n_{Natural}$$

$n = 3$ Opposing flow over vertical plates

: Internal Mixed Convection in Pipes:



Works of Metais and Eckert; Martinelli, Brawn and Gauvin… Please see Figures 7-13 and 7.14 in J.P. Holman 2002, for summary of available correlations.

Example

A solid sphere (k_s = 0.49 W/m-°C) is cooled by natural convection in a inert gas: T_∞ = 20°C. The diameter of the sphere is 0.02 m, and there is constant rate of heat generation per unit volume, S, inside it. Under steady state conditions, measurements indicate that the surface temperature of th sphere is T_w = 100°C. Radiation heat transfer may be consider negligibl in this problem, in order to focus on natural convection. Th thermophysical properties of the gas may be assumed constant and ar given:

k_g = 0.025 W/m-°C; $c_{p,g}$ = 1000 J/kg-°C;

$\mu_g = 2\times10^{-5}$ kg/m-s; ρ_g = 1.0 kg/m^3; β_g = 0.003 K^{-1}

a) Calculate the volumetric rate of heat generation, S, inside the sphere

b) Calculate the maximum temperature inside the sphere.

Ans.: a) S = 252240 W/m^3; T_{max} = 108.58°C

MECH-375 HEAT TRANSFER - HANDOUT # 10

Radiation Heat Transfer

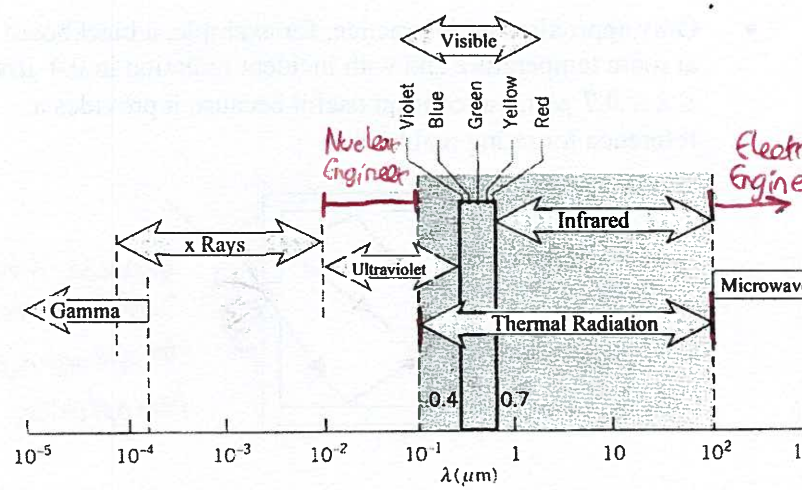
For radiation, all temperatures are in Kelvin

Topics

- ➢ **Some Characteristics of Thermal Radiation**
- ➢ **Spectrum of Electromagnetic Radiation**
- ➢ **Blackbody: Definition; Stefan-Boltzmann Law; Real Bodies; Gray Bodies**
- ➢ **Basic Radiation Properties; Kirchoff's Law**
- ➢ **Radiation Shape Factors; Summation and Reciprocity Relations**
- ➢ **Radiation Heat Transfer between Two Black Surfaces Forming an Enclosure**
- ➢ **Emissive Power of a Blackbody: Planck's Radiation Relation; Wien's Displacement Law; Total Hemispherical Blackbody Emissive Power**
- ➢ **Emissive Power of a Real Body; Emissivity; Absorption; Absorptivity**
- ➢ **Examples I, II, III, and IV**
- ➢ **Blackbody Radiation Functions**
- ➢ **Radiation Heat Transfer between Diffuse-Gray Surfaces in an Enclosure**
    - **Irradiation; radiosity; assumptions; enclosure schematic; surface and space radiation resistances; governing equations**
    - **Two- and three-surface diffuse-gray enclosures; radiation shields**

Some Characteristics of Thermal Radiation

- Matter (an intermediate medium) is not necessary for a surface to exchange thermal radiation with another surface
- Member of the large family of electromagnetic waves (spectrum shown below): other members include X-rays, visible light, radio waves, …
- Electromagnetic radiation emitted by a material solely as a result of its temperature
- Thermal Radiation: 0.1 $\mu m \le \lambda \le$ 100 μm
- Notes: visible light (0.4 $\mu m \le \lambda \le$ 0.7 μm); x-rays ($10^{-4}\ \mu m \le \lambda \le 10^{-2}\ \mu m$); microwaves ($\lambda \ge 100\ \mu m$)



(Figure from "Fundamental of Heat and Mass transfer," by F.P. Incropera and D.P. DeWitt, 1994)

Blackbody

- Diffuse emitter and absorber (independent of direction)
- At any given temperature, T, and λ, no surface can emit more energy than a black body
- Absorbs *all* incident radiation regardless of wavelength & direction
- Emits radiation according to the **Stefan-Boltzmann law**:

  $e_b = \sigma T_{abs}^4$ [W/m^2] (T in Kelvin)

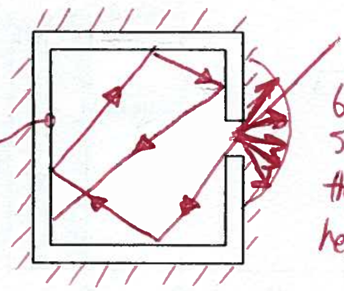
  e_b: Total Hemispherical Black Body Emissive Power

  σ: Stefan-Boltzmann constant = $\boxed{5.669 \times 10^{-8}\ \text{W/m}^2\text{-K}^4}$

  T_{abs}: Absolute temperature [K]

- Only approximated in practice, for example, a blackboard at room temperature and with incident radiation in 0.4 μm $\leq \lambda \leq$ 0.7 μm, but concept useful because it provides a reference for rating real bodies.

Constant internal temp.



because it hits every wall surface, the heat leaving the opening is diffused and hemispherical (all directions)

Non-black bodies: $e = \varepsilon e_b = \varepsilon \sigma T_{abs}^4$ [W/m^2]; here, ε is the emissivity of the surface; and $0 \leq \varepsilon \leq 1$

$\varepsilon = 1$ for Black Body

Gray bodies: The emissivity, ε, is usually a function of the wavelength, λ, of the radiation. A gray body is one for which ε is independent of λ.

grey bodies: $\boxed{\varepsilon \neq \varepsilon(\lambda)}$

Basic Radiation Properties

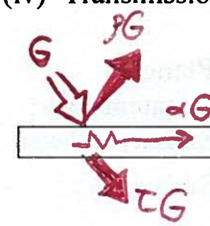
(i) Emission of radiation: emissivity, ε

(ii) Absorption of radiation: absorptivity, α

(iii) Reflection of radiation: reflectivity, ρ

(iv) Transmission of radiation: transmissivity, τ

$\varepsilon; \alpha; \rho; \tau$ all in the $\boxed{0-}$ range



G: Total hemispherical incident radiation (irradiation) [W/m^2]

ρG: Reflected radiation [W/m^2]

αG: Absorbed radiation [W/m^2]

τG; Transmitted radiation [W/m^2]

A balance on the incident radiation gives: $G =$

Thus, $\rho + \alpha + \tau = 1$

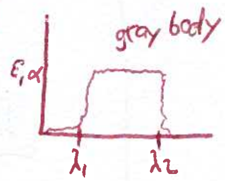
Opaque materials: $\tau = \emptyset$ $\} \Rightarrow \alpha + \rho = 1 \rightarrow \rho = 1 - \alpha$

100% transparent materials: $\tau = 1.0$ $\} \Rightarrow \alpha = \emptyset$; $\rho = \emptyset$

Blackbody: $\alpha = 1$ $\} \Rightarrow \tau = \emptyset$; $\rho = \emptyset$

In general: α, ρ, τ are functions of λ and angle of the incident radiation, and also surface properties and temperature.

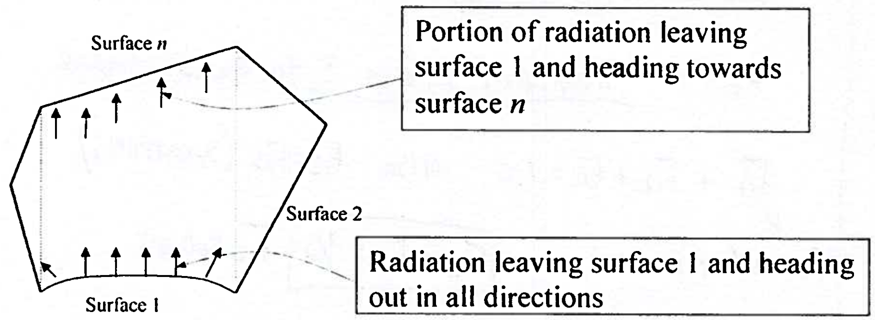
Kirchoff's law: $\boxed{\alpha_\lambda = \varepsilon_\lambda}$ $\} \Rightarrow \begin{cases} \text{For a gray body: } \varepsilon \neq \varepsilon(\lambda) \\ \alpha = \alpha_\lambda \text{ and } \varepsilon = \varepsilon_\lambda \text{; thus, } \alpha = \varepsilon \end{cases}$



Radiation Shape Factors

[Also called angle, configuration, and view factors]

Consider an enclosure of N isothermal surfaces:



- F_{m-n}: Fraction of total radiation energy leaving surface m and heading towards (and intercepted by) surface n $0 \leq F_{m-n} \leq 1$
- If each of the surfaces m and n are gray, radiate diffusely, and are isothermal, then F_{m-n} depends only on their relative orientation and their relative sizes and shapes
- For a complete enclosure of N diffuse isothermal surfaces:

$$F_{1-1} + F_{1-2} + F_{1-3} + \text{-----} + F_{1-n} + \text{-----} F_{1-N} = 1.0$$

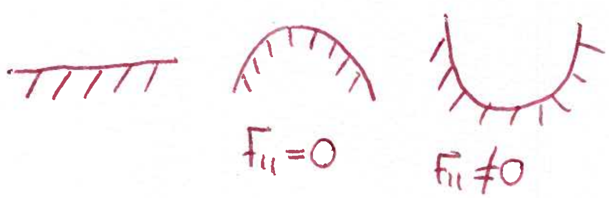
Thus, $\sum_{n=1}^{N} F_{1-n} = 1$ or $\sum_{n=1}^{N} F_{m-n} = 1.0 \} \Rightarrow$ Summation Relationship

- For any two gray, diffuse, isothermal surfaces:

$A_m F_{m-n} = A_n F_{n-m}$ $\} \Rightarrow$ Reciprocity Relationship

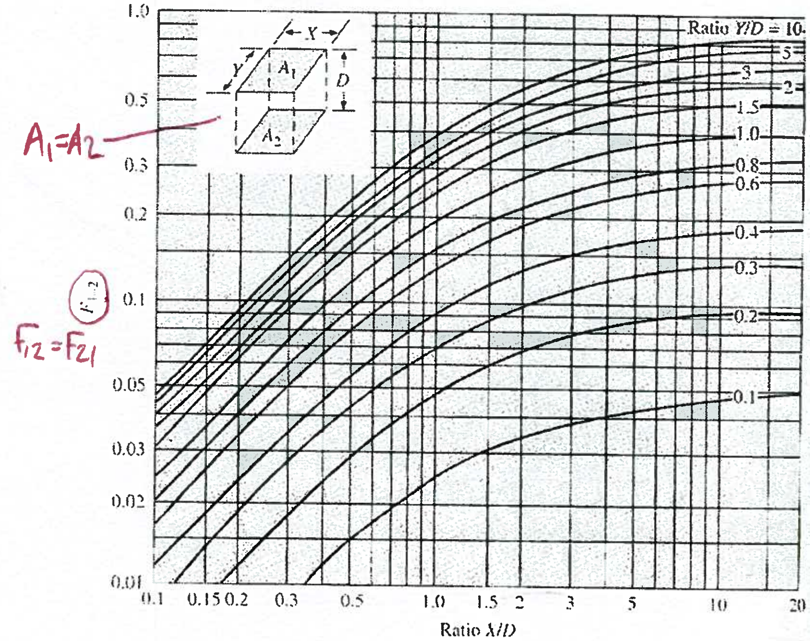
- Always look for symmetry → example →

Because of symmetry, $\boxed{F_{13} = F_{12}}$ (b/c 2=3)

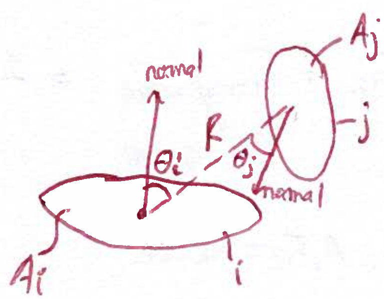


- Sample shape factors (view, configuration, angle factors)

Figure 8-12 | Radiation shape factor for radiation between parallel rectangles.



(Figure extracted from Heat Transfer by J.P. Holman, 9th Edition, 2002)

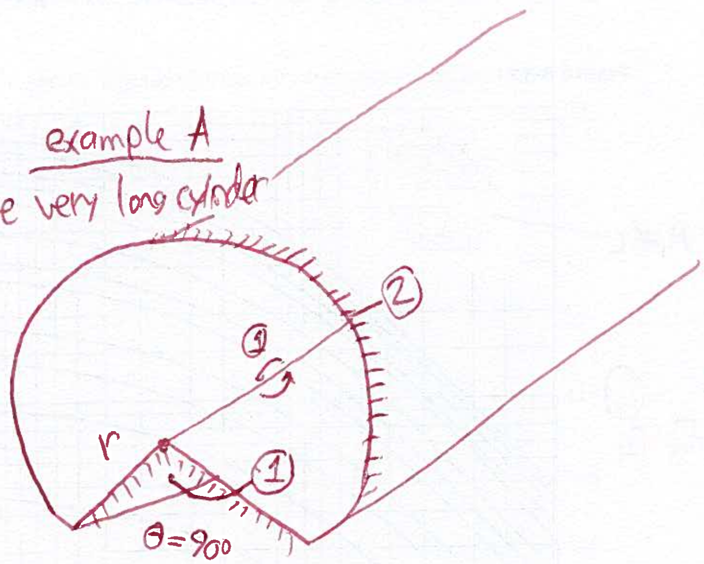


R=distance from i-j

$$A_i F_{ij} = \iint_{A_i\,A_j} \frac{\cos\theta_i \cos\theta_j}{\pi R^2}\, dA_i\, dA_j$$

example A

*Assume very long cylinder



$F_{12} = ?$

$F_{24} = ?$

Surface 2 is 3/4 of circle
surface 1 is both inside surfaces

$$\sum_{N=1}^{3} F_{1N} = 1.0$$

$$\cancel{F_{11}} + F_{12} + \cancel{F_{1\text{-end surface}}} = 1.0 \Rightarrow F_{12} = 1.0$$

Flat surface

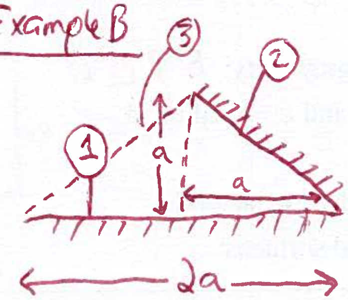
0, long cylinder

Symmetry: $A_1 F_{12} = A_2 F_{21}$

$$\therefore F_{21} = \frac{A_1}{A_2} F_{12}$$

$$= \frac{2rL}{3/4\,\pi D L}(1.0) = 0.424 \quad (\text{since } 2r = D)$$

Example B



$F_{12} = ?$

$F_{21} = ?$

*put an imaginary surface 3 to enclose figure

$\cancel{F_{11}} + F_{12} + F_{13} = 1.0$ also $F_{12} = F_{13}$ (symmetry)

flat surface, 0 $\quad \therefore \boxed{F_{12} = F_{13} = 1/2}$ F_{12} solved

$$A_2 F_{21} = F_{12} A_1 \rightarrow F_{21} = \frac{A_1}{A_2} F_{12} = \frac{2a}{\sqrt{2}\,a}\left(\frac{1}{2}\right) = \boxed{0.707}$$

two disks of exact dimensions

=A2

·F21

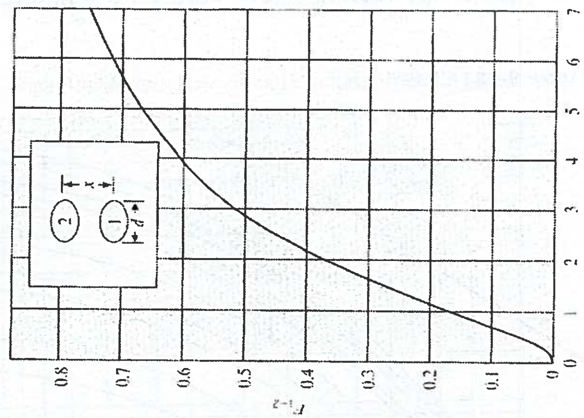
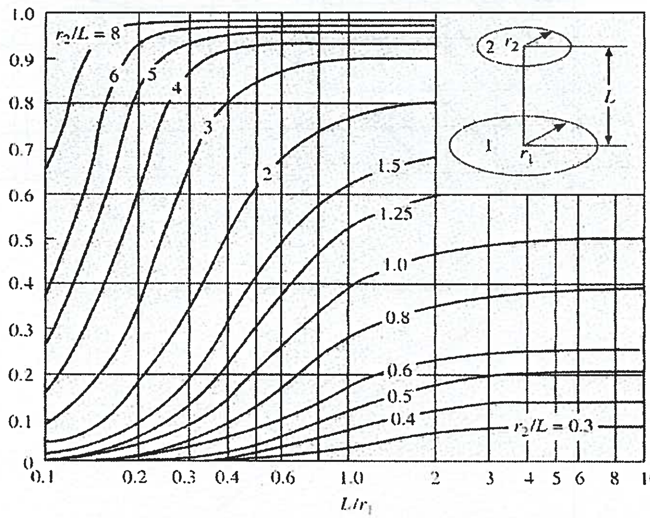


Figure 8-13 | Radiation shape factor for radiation between parallel disks.

Figure 8-16 | Radiation shape factor for radiation between two parallel coaxial disks.



Here, $A_1 \neq A_2$

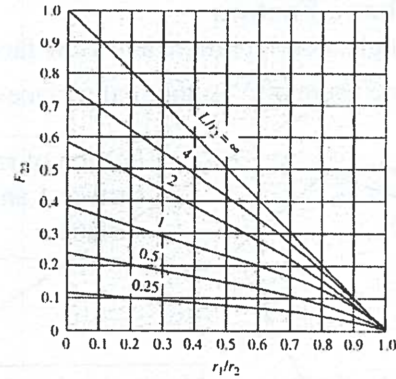
this gives F_{12} only

↓

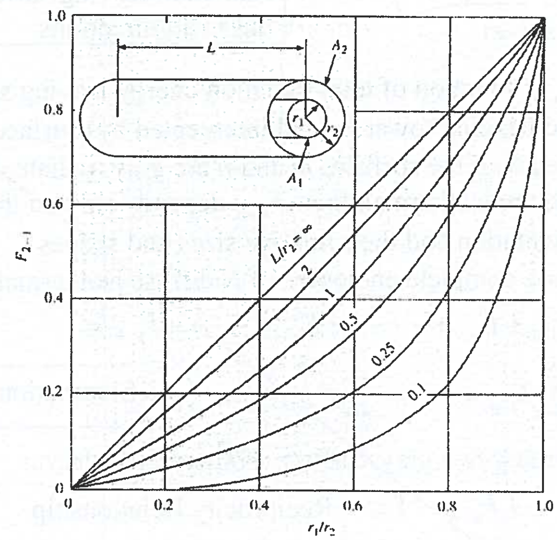
to find $\boxed{F_{21} = \frac{A_1}{A_2} F_{12}}$

(Figures extracted from Heat Transfer by J.P. Holman, 9th Edition, 2002)

Figure 8-15 | Radiation shape factors for two concentric cylinders of finite length. (a) Outer cylinder to itself; (b) outer cylinder to inner cylinder.



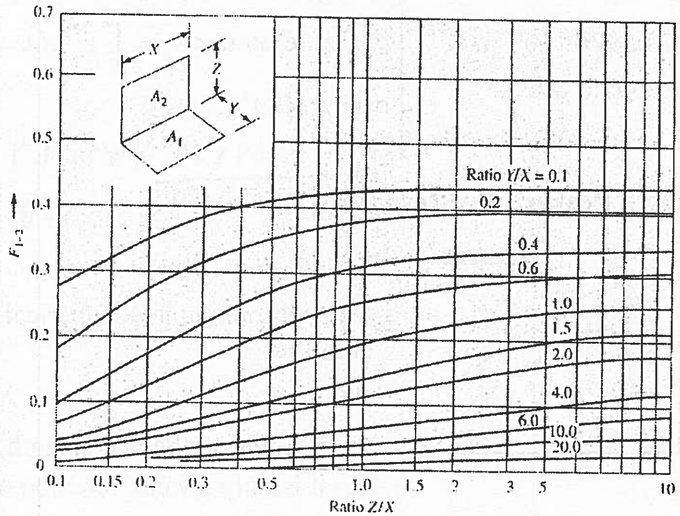
(a)

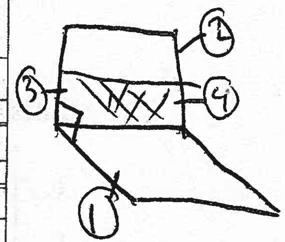


(b)

(Figures extracted from Heat Transfer by J.P. Holman, 9th Edition, 2002)

Figure 8-14 | Radiation shape factor for radiation between perpendicular rectangles with a common edge.

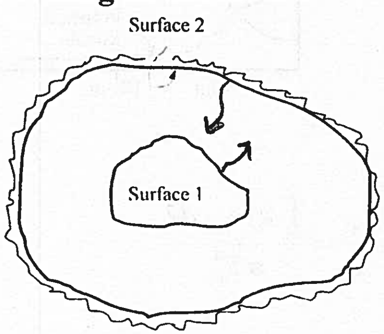




$F_{12} = F_{13} - F_{14}$

(Figures extracted from Heat Transfer by J.P. Holman, 9th Edition, 2002)

Radiation Heat Transfer between Two Black Surfaces Forming an Enclosure



$\varepsilon_1 = \varepsilon_2 = 1.0 \quad \alpha_1 = \alpha_2 = 1$

Both black surfaces are isothermal: $T_1 \neq T_2$

[Amount of radiation energy leaving surface 1 and intercepted, and absorbed, by surface 2] $= F_{1-2}(A_1 e_{b,1})$

$\boxed{e_{b_1} = \sigma T_1^4 \ \frac{W}{m^2}}$ — emissive power of a Black Body

[Amount of radiation energy emitted by surface 2] = $A_2 e_{b,2}$

$e_{b2} = \sigma T_2^4$

[Amount of radiation energy emitted by surface 2 and incident on, and absorbed by, itself] = $F_{2-2}(A_2 e_{b,2})$

[Net rate of radiation heat loss from surface 2] = $q_{2\,\text{net rad loss}}$

$$q_{2\,\text{net rad loss}} = A_2 e_{b,2} - (F_{1-2} A_1 e_{b,1} + F_{2-2} A_2 e_{b,2})$$

Using the reciprocity relation: $A_1 F_{1-2} = A_2 F_{2-1}$

The summation relation gives: $F_{2-1} + F_{2-2} = 1$ or $F_{2-2} = 1 - F_{21}$

$$\therefore q_{2\,\text{net rad loss}} = A_2 e_{b,2} - \left[A_2 F_{2-1} e_{b,1} + A_2 (1 - F_{2-1}) e_{b,2}\right]$$

$$= \cancel{A_2 e_{b,2}} - A_2 F_{2-1} e_{b,1} - \cancel{A_2 e_{b,2}} + A_2 F_{2-1} e_{b,2}$$

or $$q_{2\,\text{net rad loss}} = A_2 F_{2-1}(e_{b,2} - e_{b,1}) = A_2 F_{2-1}\sigma(T_{2,abs}^4 - T_{1,abs}^4).$$

Similarly,

$$* \quad q_{1\,\text{net rad loss}} = A_1 F_{1-2}(e_{b,1} - e_{b,2}) = A_1 F_{1-2}\sigma(T_{1,abs}^4 - T_{2,abs}^4) = -q_2$$

Emissive Power of a Blackbody

Hemispherical monochromatic emissive power of a black body at a given temperature, T } $e_{b,\lambda}$ $\left[\frac{W}{m^2 - m}\right]$

depends on λ

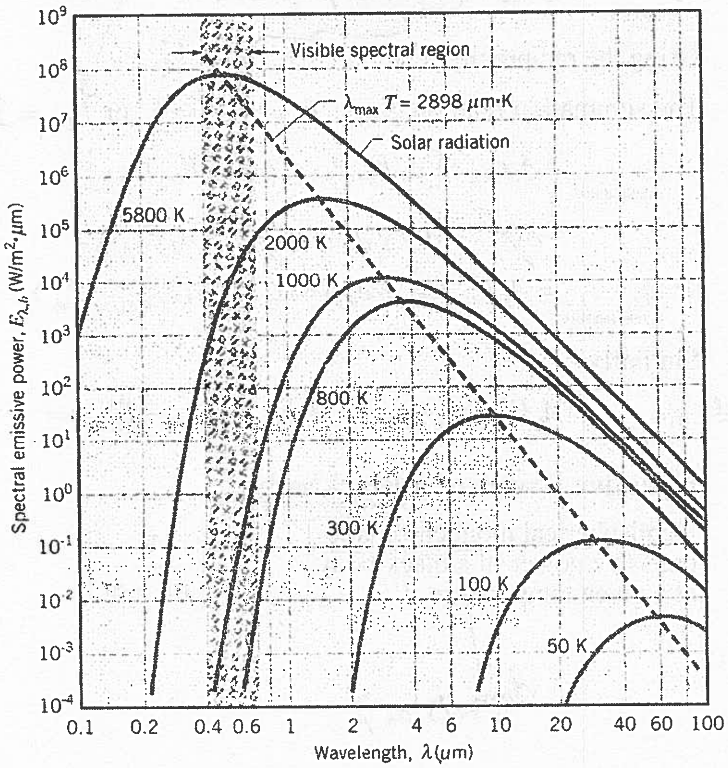
Planck's radiation relation for a blackbody

Monochromatic emissive power of a blackbody:

$$e_{b,\lambda} = \frac{C_1}{\lambda^5\left[\exp\left\{\frac{C_2}{\lambda T_{abs}}\right\}-1\right]} \quad [\mathrm{W/m^3}] \Big\} \left[\mathrm{W/m^2\text{-}\mu m}\right]$$

$C_1 = 3.7418 \times 10^{-16}$ W-m^2 : First radiation constant

$C_2 = 1.4388 \times 10^{-2}$ m-K : Second radiation constant



(Figures extracted from Incropera and DeWitt, 4th Edition, 1994)

Wien's displacement law:

$$\lambda_{max} T_{abs} = 2.8976 \times 10^{-3} \text{ m-K}$$
$$= 2897.6\ \mu\text{m-K}$$

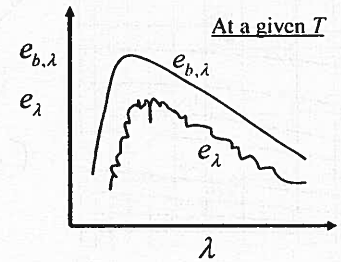
Hemispherical total emissive power of a blackbody: $e_b = \int_0^\infty e_{b,\lambda} d\lambda = \sigma T_{abs}^4$

$\sigma = (\pi / C_2)^4 (C_1 / 15)$

or $\sigma = 5.669 \times 10^{-8}$ [W/m^2-K^4]

Stefan Boltman

Emissive Power of a Real Body

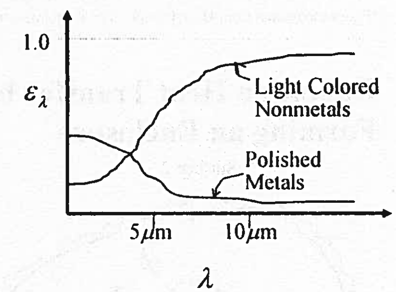


$\varepsilon_\lambda = (e_\lambda / e_{b,\lambda}) \} \Rightarrow$ Monochromatic hemispherical emissivity

$0 \le \varepsilon_\lambda \le 1$

ε_λ depends on the wavelength, λ; but it is only a weak function of T

Typical behavior:



Total emissive power:

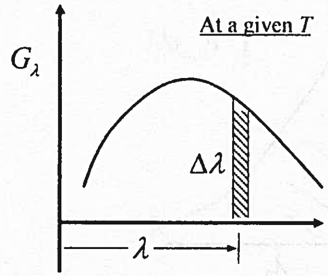
$$e = \int_0^\infty e_\lambda \, d\lambda$$

$$e = \int_0^\infty \varepsilon_\lambda e_{b\lambda} \, d\lambda$$

plats equation

Overall or total emissivity: $\varepsilon = \dfrac{e}{e_b} = \dfrac{\int_0^\infty \varepsilon_\lambda e_{b,\lambda} d\lambda}{\sigma T_{abs}^4}$

Absorption



G_λ: Spectral irradiation [W/m^2-m]

The rate at which radiation of wavelength λ is incident on a surface per unit area of the surface and per unit wavelength interval Δλ about λ.

Surface may exhibit selective absorption with respect to the wavelength of the incident radiation.

Hemispherical monochromatic absorptivity } α_λ

$\tau = \dfrac{\int_0^\infty \tau_\lambda G_\lambda d\lambda}{\int_0^\infty G_\lambda d\lambda}$

For a blackbody: α_λ = 1.0

$G = \int_0^\infty G_\lambda d\lambda$

$\rho = \dfrac{\int_0^\infty \rho_\lambda G_\lambda d\lambda}{\int_0^\infty G_\lambda d\lambda}$

Total absorptivity: $\alpha = \int_0^\infty \alpha_\lambda G_\lambda d\lambda \Big/ \int_0^\infty G_\lambda d\lambda$

$\alpha + \rho + \tau = 1.0$ also $\alpha_\lambda + \rho_\lambda + \tau_\lambda = 1.0$

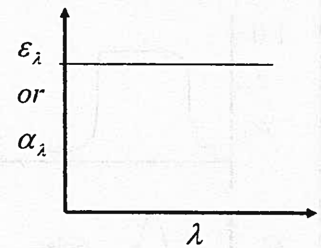
Kirchoff's Law: $\varepsilon_\lambda = \alpha_\lambda \} \rightarrow$ [For diffuse incident radiation]

Proved by Max Planck (1959).

gray $\varepsilon \neq \varepsilon(\lambda)$

thus, $\alpha \cong \varepsilon$ for gray surfaces

Gray Body: α_λ and ε_λ are independent of λ

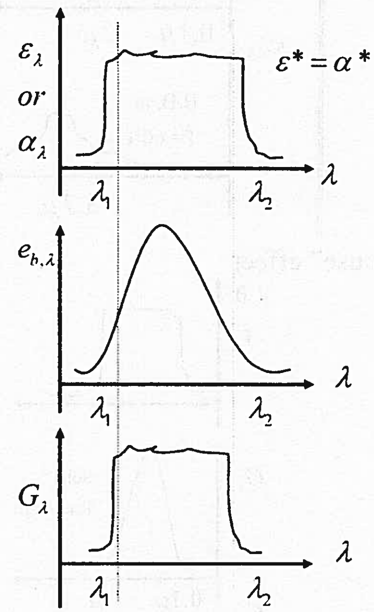


$$\varepsilon = \frac{\int_0^\infty \varepsilon_\lambda e_{b,\lambda}\, d\lambda}{\int_0^\infty e_{b,\lambda}\, d\lambda} = \varepsilon_\lambda$$

$$\alpha = \frac{\int_0^\infty \alpha_\lambda G_\lambda\, d\lambda}{\int_0^\infty G_\lambda\, d\lambda} = \alpha_\lambda$$

But, Kircho
law gives
$\alpha_\lambda = \varepsilon_\lambda$
Thus, for a g
body:
$\alpha = \varepsilon$

Example I: Quite a few real bodies behave as gray bo especially in certain wavelength bands



$$\varepsilon = \frac{\int_0^\infty \varepsilon_\lambda e_{b,\lambda}\, d\lambda}{\int_0^\infty e_{b,\lambda}\, d\lambda}$$

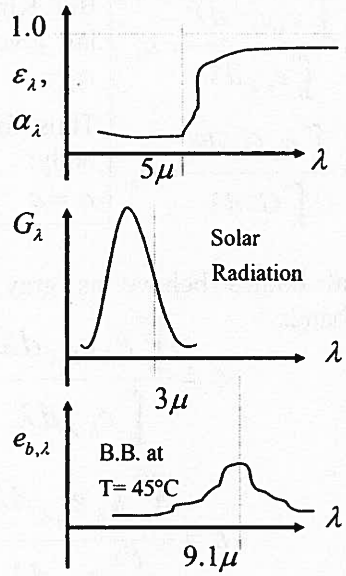
$$\varepsilon = \frac{\int_{\lambda_1}^{\lambda_2} \varepsilon_\lambda e_{b,\lambda}\, d\lambda}{\int_{\lambda_1}^{\lambda_2} e_{b,\lambda}\, d\lambda} =$$

$$\alpha = \frac{\int_0^\infty \alpha_\lambda G_\lambda\, d\lambda}{\int_0^\infty G_\lambda\, d\lambda}$$

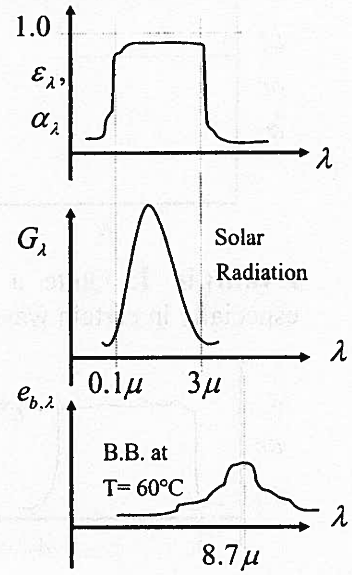
$$\alpha = \frac{\int_{\lambda_1}^{\lambda_2} \alpha_\lambda G_\lambda\, d\lambda}{\int_{\lambda_1}^{\lambda_2} G_\lambda\, d\lambda} =$$

The irradiation and the surface emission are concentrated in region for which the spectral properties of the surface approximately constant.

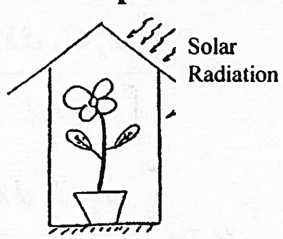
Example II: White robe problem

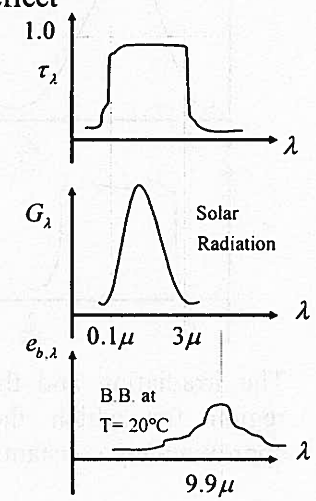


Example III: Special structure of solar collectors: $\alpha = 1$; $\varepsilon = 0$



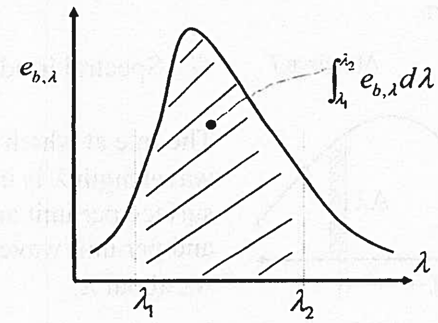
Example IV: The "greenhouse" effect





- Plate glass windows and roof
- Let in solar radiation
- Do not let out thermal radiation from surfaces inside

Blackbody Radiation Functions



$$\int_{\lambda_1}^{\lambda_2} e_{b,\lambda}\, d\lambda = \int_0^{\lambda_2} e_{b,\lambda}\, d\lambda - \int_0^{\lambda_1} e_{b,\lambda}\, d\lambda$$

Planck's radiation relation: $e_{b,\lambda} = \dfrac{C_1}{\lambda^5\left[\exp\left(\dfrac{C_2}{\lambda T_{abs}}\right) - 1\right]}$

Notes: 1) $\int_0^\lambda e_{b,\lambda}\, d\lambda = fnc(\lambda, T_{abs}) \rightarrow$ depends on both λ and T_{abs}, individually;

2) $\dfrac{\int_0^\lambda e_{b,\lambda}\, d\lambda}{\sigma T_{abs}^4} = fnc(\lambda T_{abs}) \rightarrow$ depends on the product (λT_{abs})

Blackbody radiation function: $\dfrac{E_b(0 \rightarrow \lambda T_{abs})}{\sigma T_{abs}^4} = \dfrac{\int_0^\lambda e_{b,\lambda}\, d\lambda}{\sigma T_{abs}^4}$. Then,

total surface Area

$$\left[\begin{array}{l}\text{Fraction of total blackbody} \\ \text{radiation that lies in } \lambda_1 \le \lambda \le \lambda_2\end{array}\right] = \frac{E_b(0 \rightarrow \lambda_2 T_{abs})}{\sigma T_{abs}^4} - \frac{E_b(0 \rightarrow \lambda_1 T_{abs})}{\sigma T_{abs}^4}$$

$$\left\{\frac{E_b(0 \rightarrow \lambda T_{abs})}{\sigma T_{abs}^4}\right\} \rightarrow \text{Tabulated as a function of } (\lambda T_{abs})$$

Table 8-1 | Radiation functions.

λT $\mu m \cdot K$	$E_{b\lambda}/T^5$ $\frac{W}{m^2 \cdot K^5 \cdot \mu m} \times 10^{11}$	$\frac{E_{b_0-\lambda T}}{\sigma T^4}$	λT $\mu m \cdot K$	$E_{b\lambda}/T^5$ $\frac{W}{m^2 \cdot K^5 \cdot \mu m} \times 10^{11}$	$\frac{E_{b_0-\lambda T}}{\sigma T^4}$
1000	0.02110	0.00032	6300	0.42760	0.76180
1100	0.04846	0.00091	6400	0.41128	0.76920
1200	0.09329	0.00213	6500	0.39564	0.77631
1300	0.15724	0.00432	6600	0.38066	0.78316
1400	0.23932	0.00779	6700	0.36631	0.78975
1500	0.33631	0.01285	6800	0.35256	0.79609
1600	0.44359	0.01972	6900	0.33940	0.80219
1700	0.55603	0.02853	7000	0.32679	0.80807
1800	0.66872	0.03934	7100	0.31471	0.81373
1900	0.77736	0.05210	7200	0.30315	0.81918
2000	0.87858	0.06672	7300	0.29207	0.82443
2100	0.96994	0.08305	7400	0.28146	0.82949
2200	1.04990	0.10088	7500	0.27129	0.83436
2300	1.11768	0.12002	7600	0.26155	0.83906
2400	1.17314	0.14025	7700	0.25221	0.84359
2500	1.21659	0.16135	7800	0.24326	0.84796
2600	1.24868	0.18311	7900	0.23468	0.85218
2700	1.27029	0.20535	8000	0.22646	0.85625
2800	1.28242	0.22788	8100	0.21857	0.86017
2900	1.28612	0.25055	8200	0.21101	0.86396
3000	1.28245	0.27322	8300	0.20375	0.86762
3100	1.27242	0.29576	8400	0.19679	0.87115
3200	1.25702	0.31809	8500	0.19011	0.87456
3300	1.23711	0.34009	8600	0.18370	0.87786
3400	1.21352	0.36172	8700	0.17755	0.88105
3500	1.18695	0.38290	8800	0.17164	0.88413
3600	1.15806	0.40359	8900	0.16596	0.88711
3700	1.12739	0.42375	9000	0.16051	0.88999
3800	1.09544	0.44336	9100	0.15527	0.89277
3900	1.06261	0.46240	9200	0.15024	0.89547
4000	1.02927	0.48085	9300	0.14540	0.89807
4100	0.99571	0.49872	9400	0.14075	0.90060
4200	0.96220	0.51599	9500	0.13627	0.90304
4300	0.92892	0.53267	9600	0.13197	0.90541
4400	0.89607	0.54877	9700	0.12783	0.90770
4500	0.86376	0.56429	9800	0.12384	0.90992
4600	0.83212	0.57925	9900	0.12001	0.91207
4700	0.80124	0.59366	10,000	0.11632	0.91415
4800	0.77117	0.60753	10,200	0.10934	0.91813
4900	0.74197	0.62088	10,400	0.10287	0.92188
5000	0.71366	0.63372	10,600	0.09685	0.92540
5100	0.68628	0.64606	10,800	0.09126	0.92872
5200	0.65983	0.65794	11,000	0.08606	0.93184
5300	0.63432	0.66935	11,200	0.08121	0.93479
5400	0.60974	0.68033	11,400	0.07670	0.93758
5500	0.58608	0.69087	11,600	0.07249	0.94021
5600	0.56332	0.70101	11,800	0.06856	0.94270
5700	0.54146	0.71076	12,000	0.06488	0.94505
5800	0.52046	0.72012	12,200	0.06145	0.94728
5900	0.50030	0.72913	12,400	0.05823	0.94939
6000	0.48096	0.73778	12,600	0.05522	0.95139
6100	0.46242	0.74610	12,800	0.05240	0.95329
6200	0.44464	0.75410	13,000	0.04976	0.95509

Handout # 10: Radiation MECH-375- Heat Transfer

Table 8-1 | (*Continued*).

λT $\mu m \cdot K$	$E_{b\lambda}/T^5$ $\frac{W}{m^2 \cdot K^5 \cdot \mu m} \times 10^{11}$	$\frac{E_{b_0-\lambda T}}{\sigma T^4}$	λT $\mu m \cdot K$	$E_{b\lambda}/T^5$ $\frac{W}{m^2 \cdot K^5 \cdot \mu m} \times 10^{11}$	$\frac{E_{b_0-\lambda T}}{\sigma T^4}$
13,200	0.04728	0.95680	19,800	0.01151	0.98515
13,400	0.04494	0.95843	20,000	0.01110	0.98555
13,600	0.04275	0.95998	21,000	0.00931	0.98735
13,800	0.04069	0.96145	22,000	0.00786	0.98886
14,000	0.03875	0.96285	23,000	0.00669	0.99014
14,200	0.03693	0.96418	24,000	0.00572	0.99123
14,400	0.03520	0.96546	25,000	0.00492	0.99217
14,600	0.03358	0.96667	26,000	0.00426	0.99297
14,800	0.03205	0.96783	27,000	0.00370	0.99367
15,000	0.03060	0.96893	28,000	0.00324	0.99429
15,200	0.02923	0.96999	29,000	0.00284	0.99482
15,400	0.02794	0.97100	30,000	0.00250	0.99529
15,600	0.02672	0.97196	31,000	0.00221	0.99571
15,800	0.02556	0.97288	32,000	0.00196	0.99607
16,000	0.02447	0.97377	33,000	0.00175	0.99640
16,200	0.02343	0.97461	34,000	0.00156	0.99669
16,400	0.02245	0.97542	35,000	0.00140	0.99695
16,600	0.02152	0.97620	36,000	0.00126	0.99719
16,800	0.02063	0.97694	37,000	0.00113	0.99740
17,000	0.01979	0.97765	38,000	0.00103	0.99759
17,200	0.01899	0.97834	39,000	0.00093	0.99776
17,400	0.01823	0.97899	40,000	0.00084	0.99792
17,600	0.01751	0.97962	41,000	0.00077	0.99806
17,800	0.01682	0.98023	42,000	0.00070	0.99819
18,000	0.01617	0.98081	43,000	0.00064	0.99831
18,200	0.01555	0.98137	44,000	0.00059	0.99842
18,400	0.01496	0.98191	45,000	0.00054	0.99851
18,600	0.01439	0.98243	46,000	0.00049	0.99861
18,800	0.01385	0.98293	47,000	0.00046	0.99869
19,000	0.01334	0.98340	48,000	0.00042	0.99877
19,200	0.01285	0.98387	49,000	0.00039	0.99884
19,400	0.01238	0.98431	50,000	0.00036	0.99890
19,600	0.01193	0.98474			

Notes: 1) These values are based on $C_1 = 3.7418 \times 10^{-16}$ W-m^2 and $C_2 = 1.4388 \times 10^{-2}$ m-K; 2) $\dfrac{E_b(0 \to \lambda T_{abs})}{\sigma T_{abs}^4} = \dfrac{\int_0^{\lambda} e_{b,\lambda}\, d\lambda}{\sigma T_{abs}^4}$

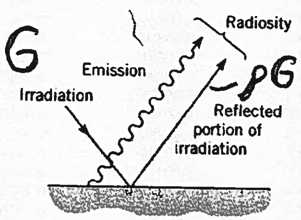
Radiation Heat Transfer between Diffuse-Gray Surfaces in an Enclosure

xx $\boxed{\alpha = \varepsilon}$

Irradiation: Total rate of incident radiation on a surface per unit time and per unit area $\rightarrow G$ [W/m^2]

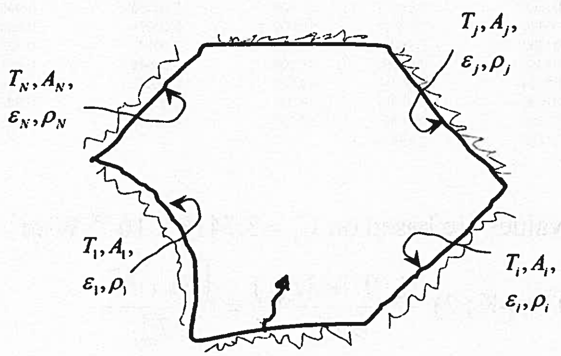
Radiosity: Total rate at which radiation leaves a surface per unit time and per unit area $\rightarrow J$ [W/m^2]

$\varepsilon = \dfrac{e}{e_b}$, $e_b = \sigma T^4$



For a given surface:
$J = \varepsilon e_b + \rho G$

Enclosure schematic:

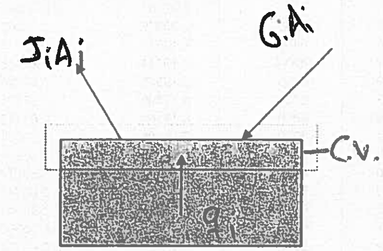


Total number of surfaces: N

Assumptions: [steady-state]

1. All surfaces are diffuse-gray $[\alpha_\lambda = \alpha ; \varepsilon_\lambda = \varepsilon ; \alpha = \varepsilon]$
2. Each surface is uniform in temperature (isothermal)
3. Reflectivity, ρ, and emissivity, ε, are constant over each surface
4. All surfaces are opaque ($\tau = 0$)
5. G and J are uniform over each surface

Consider a surface *i*:



Radiation surface energy balance:

$q_i = q_i^{Net\ Rad} = A_i J_i - A_i G_i$

i.e., Net energy leaving the surface

But $J_i = e_i + \rho_i G_i$;

or $J_i = \varepsilon_i e_{b,i} + \rho_i G_i$

For an opaque diffuse-gray surface: $\alpha + \rho + \overset{0}{\cancel{\tau}} = 1$; and $\alpha = 1-\rho$

Thus, $\rho = 1-\alpha \quad = 1-\varepsilon$ since $\varepsilon = \alpha$

Therefore, $\rho_i = 1 - \alpha_i = 1 - \varepsilon_i$

and $J_i = \varepsilon_i e_{b,i} + (1-\varepsilon_i) G_i$ or $G_i = (J_i - \varepsilon_i e_{b,i})/(1-\varepsilon_i)$

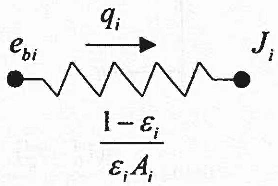
$$q_i / A_i = J_i - \frac{J_i - \varepsilon_i e_{bi}}{1-\varepsilon_i} = \frac{e_{bi} - J_i}{\frac{1-\varepsilon_i}{\varepsilon_i}}$$

Therefore,

$$q_i^{Net\,Rad} = A_i\left[J_i - \frac{(J_i - \varepsilon_i e_{b,i})}{(1-\varepsilon_i)}\right] = A_i\left[\frac{J_i - J_i\varepsilon_i - J_i + \varepsilon_i e_{b,i}}{1-\varepsilon_i}\right]$$

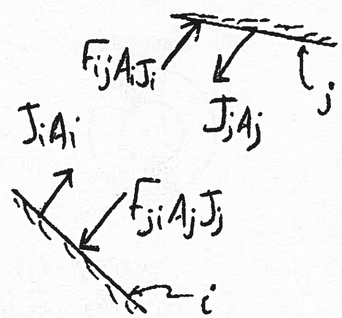
or $q_i^{Net\,Rad} = (e_{b,i} - J_i)\Big/\left(\frac{1-\varepsilon_i}{\varepsilon_i A_i}\right)$ For B.B. $\rightarrow e_{bi} = J_i$ please show this

Surface radiation resistance:



$$\left.\begin{array}{l}\text{surface}\\ \text{resistance}\end{array}\right\} = \frac{1-\varepsilon_i}{A_i\varepsilon_i}$$

Net radiation exchange between two diffuse-gray surfaces



$q_{i\to j}^{Net\,Rad} = (J_i A_i)F_{i-j} - (J_j A_j)F_{j-i}$

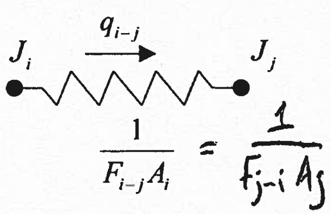
But reciprocity relation gives:

$A_i F_{i-j} = A_j F_{ji}$

Thus, $q_{i\to j}^{Net\,Rad} = (J_i - J_j)A_i F_{i-j}$

or $q_{i\to j}^{Net\,Rad} = \dfrac{(J_i - J_j)}{1/(A_i F_{i-j})} = \boxed{\dfrac{J_i - J_j}{1/A_j F_{ji}}}$

Space or geometrical radiation resistance:

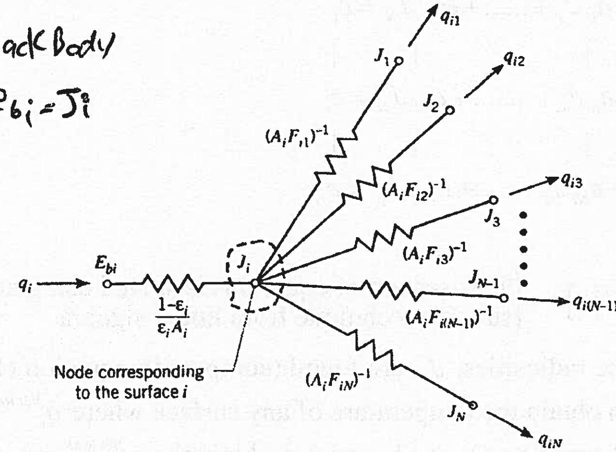


space resistance $= \boxed{\dfrac{1}{F_{ij}A_i}}$

Network representation of radiation exchange between surface and all other surfaces of the enclosure:

Black Body

$e_{bi} = J_i$



(Figures extracted from Incropera and DeWitt, 4th Edition, 1994)

Two key equations

$$q_i = \frac{e_{b,i} - J_i}{(1-\varepsilon_i)/(A_i\varepsilon_i)} = \sum_{j=1}^{N}\frac{J_i - J_j}{\{1/(A_i F_{i-j})\}}\Bigg\} \rightarrow (1)\ \text{use this equation if}$$

$\boxed{\text{Temperature } T_i \text{ is Kno}}$

$\rightarrow e_{bi} = \sigma T_i^4$

$$q_i^{Net\,Rad} = \sum_{j=1}^{N}\frac{J_i - J_j}{\{1/(A_i F_{i-j})\}}\Bigg\} \rightarrow (2)\ \text{use this equation if}$$

$\boxed{q_i \text{ is Sp}}$

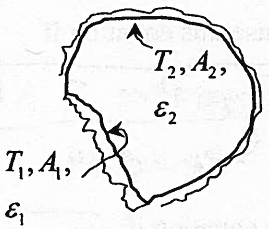
Governing equations for radiation exchange in the *N*-surface diffuse-gray enclosure

$$
\begin{array}{ccccccc}
a_{11} J_1 + \ldots\ldots + & a_{1i} J_i + \ldots\ldots + & a_{1N} J_N & = c_1 \\
| & | & | & | \\
a_{i1} J_1 + \ldots\ldots + & a_{ii} J_i + \ldots\ldots + & a_{iN} J_N & = c_i \\
| & | & | & | \\
a_{N1} J_1 + \ldots\ldots + & a_{Ni} J_i + \ldots\ldots + & a_{NN} J_N & = c_N
\end{array}
$$

or $[A][J]=[C]$ $\} \rightarrow$ {This system of equations is solved using any suitable technique from linear algebra}

Note: Once the radiosities, J_i, are found (computed), equation (1) can be used to obtain the temperature of any surface where $q_i^{Net\,Rad}$ is specified, and equation (2) can be used to obtain the $q_i^{Net\,Rad}$ at any surface where the temperature is specified.

Two-Surface Diffuse-Gray Enclosure



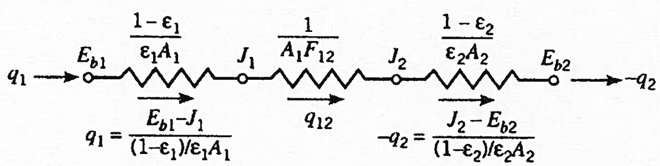
Notes:

1) $q_1^{Net\,Rad} = q_{1-2}^{Net\,Rad} = -q_{2-1}^{Net\,Rad}$

2) $e_{b,1} = \sigma T_{1,abs}^4$

3) $e_{b,2} = \sigma T_{2,abs}^4$

Equivalent circuit:

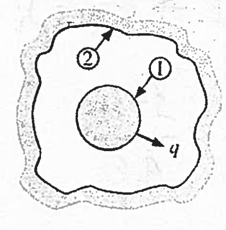


$E_{b_i} = e_{b_1} = \sigma T_i^4$

Examples: $q_{12} = \dfrac{e_{b_1} - e_{b_2}}{\frac{1-\varepsilon_1}{A_1\varepsilon_1} + \frac{1}{A_1 F_{12}} + \frac{1-\varepsilon_2}{A_2\varepsilon_2}}$

Figure 8-30 | Radiation heat transfer between simple two-body diffuse, gray surfaces. In all cases $F_{12} = 1.0$.

Small convex object in large enclosure

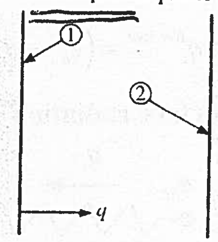


$A_2 \gg A_1$

Please show

$q = A_1 \epsilon_1 \sigma (T_1^4 - T_2^4)$
for $A_1/A_2 \to 0$

Infinite parallel planes

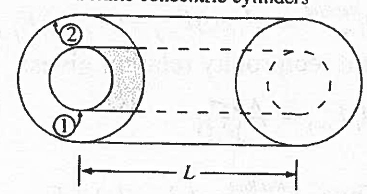


$F_{12} =$

$A_1 =$

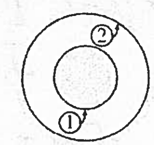
$(q/A) = \dfrac{\sigma(T_1^4 - T_2^4)}{1/\epsilon_1 + 1/\epsilon_2 - 1}$
with $A_1 = A_2$

Infinite concentric cylinders



$q = \dfrac{\sigma A_1 (T_1^4 - T_2^4)}{1/\epsilon_1 + (1/\epsilon_2 - 1)(r_1/r_2)}$
with $A_1/A_2 = r_1/r_2$; $r_1/L \to 0$

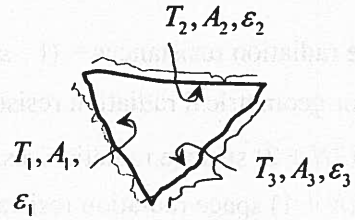
Concentric spheres

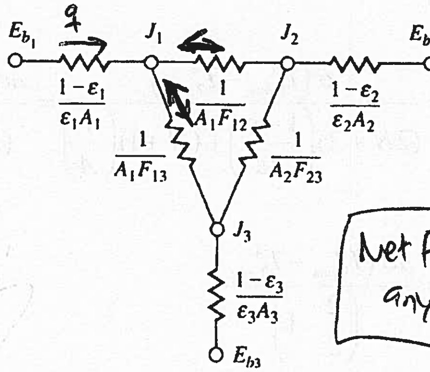


$q = \dfrac{\sigma A_1 (T_1^4 - T_2^4)}{1/\epsilon_1 + (1/\epsilon_2 - 1)(r_1/r_2)^2}$
for $A_1/A_2 = (r_1/r_2)^2$

(Table extracted from Holman, 2002)

Three-Surface Diffuse-Gray Enclosures

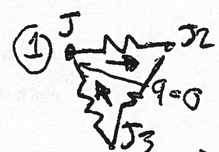




reradiating ~~radiating~~ surface

$q_{net} = 0$

$e_b = J$



different

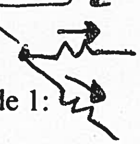
Net flow of current in any node = ∅

de 1

$\frac{\;\; - J_i}{\;\;} + \frac{J_2 - J_1}{\frac{1}{A_1 F_2}}$

$\frac{J_3 - J_1}{\frac{1}{A_1 F_3}} = 0$

blackbody $e_{bi} = J_i$



NOTE:

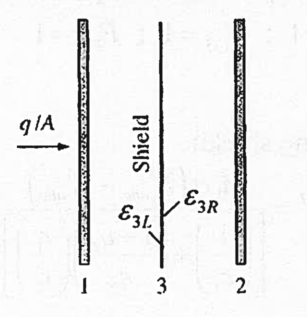
i) Net flow of current to any one node = 0. For example, for Node 1:

$$\frac{e_{b,1} - J_1}{\frac{1-\varepsilon_1}{A_1\varepsilon_1}} + \frac{J_2 - J_1}{\frac{1}{A_1 F_{12}}} + \frac{J_3 - J_1}{\frac{1}{A_1 F_{13}}} = 0;$$

ii) If one of the surfaces be perfectly insulated from one side, and convection effects on the other side be negligible, the surface is considered as *reradiating surface*. In this case, the net radiation transfer for the reradiating surface is zero, i.e., $q^{net}_{\substack{\text{Rerad.}\\ \text{surf.}}} = 0$, and $e_{b,\substack{\text{Rerad.}\\ \text{surf.}}} = J_{\substack{\text{Rerad.}\\ \text{surf.}}}$.

Radiation Shields

Example 1: Two large parallel flat plates, with a thin-plate radiation shield in between them



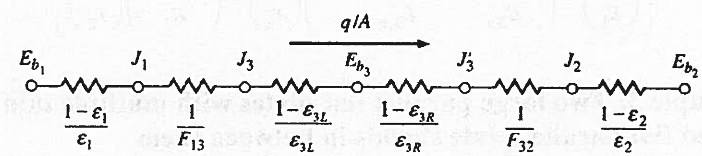
$A_1 = A_2 = A_3 = A \;;\; F_{1-3} = F_{3-2} = 1$

With no shield:

$$q_1^{Net\,Rad} = q_{1-2}^{Net\,Rad} = \frac{A\sigma\left(T_{1,abs}^4 - T_{2,}^4\right.}{[(1/\varepsilon_1)+(1/\varepsilon_2)}$$

Aluminum Polished: ε≈

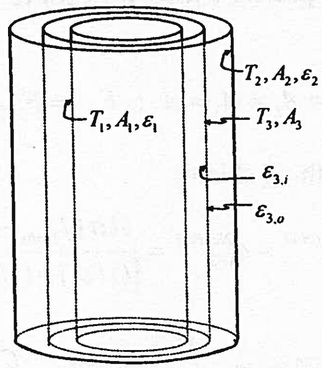
Equivalent circuit with shield:



★★

$$\boxed{q_1^{Net\,Rad} = \frac{A\sigma\left(T_{1,abs}^4 - T_{2,abs}^4\right)}{\left[\left(\frac{1}{\varepsilon_1}\right)+\left(\frac{1}{\varepsilon_2}\right)+\left(\frac{1-\varepsilon_{3,L}}{\varepsilon_{3,L}}\right)+\left(\frac{1-\varepsilon_{3,R}}{\varepsilon_{3,R}}\right)\right]}}$$

Example 2: Two long concentric cylindrical shells, with a thin-walled cylindrical radiation shield in between them



$$A_1 = 2\pi r_1 L \;;\; A_2 = 2\pi r_2 L \;;\; A_3 = 2\pi r_3 L$$

$$F_{1-2} = 1 \;;\; F_{1-3} = 1 \;;\; F_{3-2} = 1$$

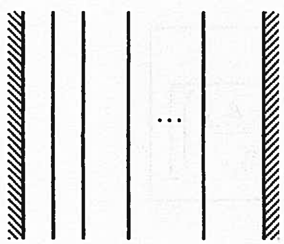
With no shield:

$$q_{1-2}^{Net\,Rad} = \frac{A_1 \sigma \left(T_{1,abs}^4 - T_{2,abs}^4\right)}{\left[\left(\frac{1}{\varepsilon_1}\right) + \left(\frac{1-\varepsilon_2}{\varepsilon_2}\right)\left(\frac{r_1}{r_2}\right)\right]}$$

With shield,

$$q_{1-2}^{Net\,Rad} = \frac{A_1 \sigma \left(T_{1,abs}^4 - T_{2,abs}^4\right)}{\left[\left(\frac{1}{\varepsilon_1}\right) + \left(\frac{1-\varepsilon_{3,i}}{\varepsilon_{3,i}} + \frac{1-\varepsilon_{3,o}}{\varepsilon_{3,o}} + 1\right)\left(\frac{r_1}{r_3}\right) + \left(\frac{1-\varepsilon_2}{\varepsilon_2}\right)\left(\frac{r_1}{r_2}\right)\right]}$$

Example 3: Two large parallel flat plates with multiple thin-walled flat parallel-plate shields in between them



Assumptions:

1) $A_1 = A_2 = \ldots\ldots = A$

2) All shape factors are equal to one ($F_{i-j} = 1$)

3) All surfaces have the same emissivity: ε

- Two end-plates, 1 and 2; N shields (thin, large, flat parallel plates)
- All surface radiation resistances = $(1-\varepsilon)/(A\varepsilon)$
- All space or geometrical radiation resistances = $1/(A_i F_{i-j}) = 1/A$
- There are $(2N+2)$ surface radiation resistances
- There are $(N+1)$ space radiation resistances

Therefore,

$$q_{\substack{1-2 \\ with\,N\,shields}}^{Net\,Rad} = \frac{\sigma\left(T_{1,abs}^4 - T_{2,abs}^4\right)}{(2N+2)\left(\frac{1-\varepsilon}{A\varepsilon}\right) + (N+1)\left(\frac{1}{A}\right)} = \frac{A\sigma\left(T_{1,abs}^4 - T_{2,abs}^4\right)}{(N+1)\left(\frac{2}{\varepsilon} - 1\right)}$$

and

$$q_{\substack{1-2 \\ with\,no\,shields}}^{Net\,Rad} = \frac{A\sigma\left(T_{1,abs}^4 - T_{2,abs}^4\right)}{\left(\frac{2}{\varepsilon} - 1\right)}$$

Thus, $\left[q_{\substack{1-2 \\ with\,N\,shields}}^{Net\,Rad} \Big/ q_{\substack{1-2 \\ with\,no\,shields}}^{Net\,Rad}\right] = \left(\frac{1}{N+1}\right)$

Example-1

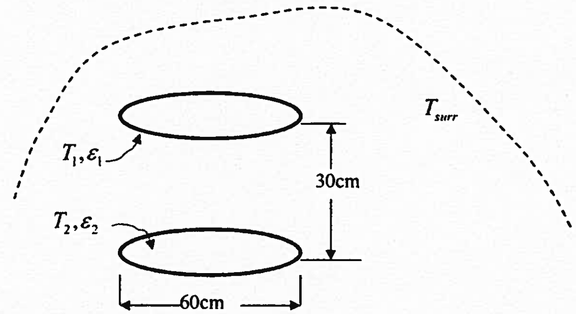
Ordinary clear glass transmits 91% of the normal incident radiation in the wavelength of 0.35 to 3.0 μm, and is opaque to longer and shorter wavelengths. Estimate the percentage of solar radiation that will be transmitted. (Assume the sun radiate as a black body at 5800 K, and, for this problem, sun rays are normal to the glass surface)

Ans. 82.57%

Handout # 10: Radiation MECH-375- Heat Transfer

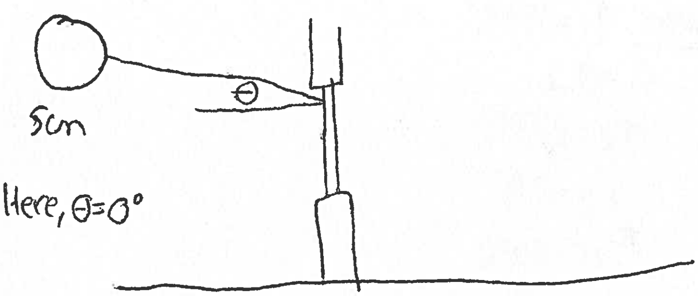
Example-2

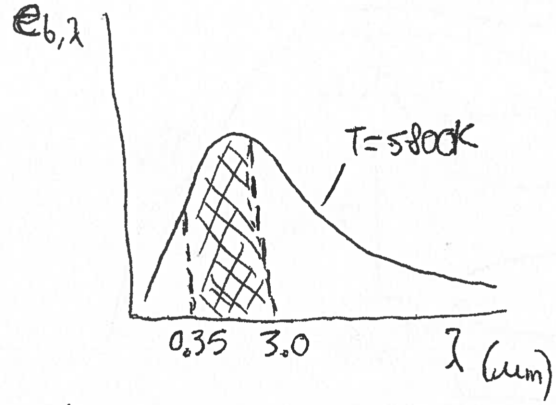
As illustrated in figure bellow, two parallel disks of 60 cm diame are exchanging heat by radiation with each other and also with surrounding. The bottom face of the upper disk (surface 1) is at 500°C, and its emissivity ε_1= 0.2, while the upper face of the lo disk (surface 2) is at T_2= 227°C, and its emissivity ε_2= 0.4. ' surrounding is at T_{surr} = 60°C and may assume behaving as a bl body. Neglecting the heat transfer by convection, what is the net of radiation exchange for surface 1 and 2?



Ans.: $q_1^{rad} = 1062.19$ W; $q_2^{rad} = 131.96$ W

Handout #10, Example 1





$\lambda_1 = 0.35\,\mu m \rightarrow \lambda_1 T = 0.35(5{,}800) = 2030\,\mu mK$

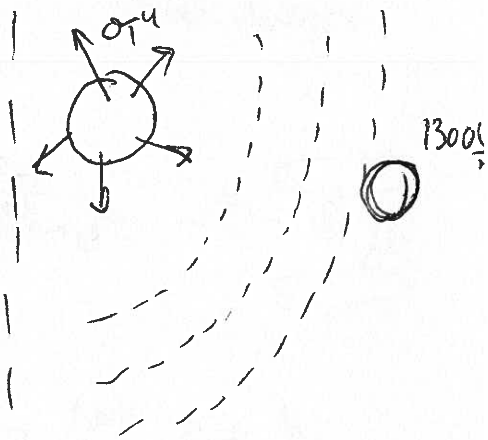
$\lambda_2 = 3\,\mu m \rightarrow \lambda_2 T = 3(5{,}800) = 17{,}400\,\mu mK$

Using Table $\boxed{8\text{-}1}$ $\rightarrow \lambda_1 T \longrightarrow \dfrac{E_b(0\rightarrow\lambda_1 T)}{\sigma T^4} = 0.07162$

$\lambda_2 T \longrightarrow \dfrac{E_b(0\rightarrow\lambda_2 T)}{\sigma T^4} = 0.97899$

$\therefore \dfrac{E_b(\lambda_1\rightarrow\lambda_2)}{\sigma T^4} = 0.97899 - 0.07162 = 0.90737$

sidenote



as distance increa[ses] intensity of heat decrea[ses] exponentially

$\therefore \tau_{window} = 0.91$ for $0.35 \le \lambda \le 3.0$ (given in example, 91%)

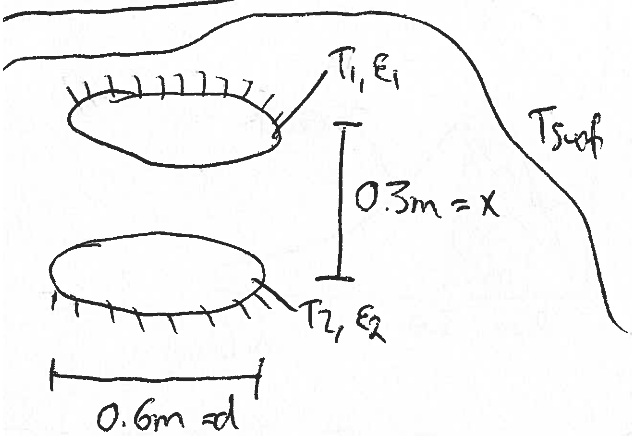
and $\tau_{window} = 0$ for $\lambda < 0.35$

$\tau_{window} = 0$ for $\lambda > 3.0$

Transmitted Solar Radiation : $= 0.91 \times 0.90737 = 0.8257$

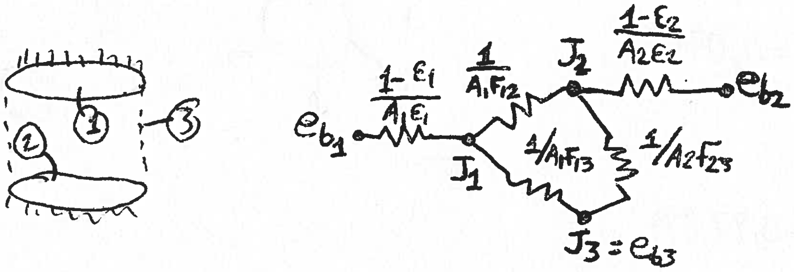
ANS: $\boxed{82.57\%}$

Handout #10, Example 2



given: $T_1 = 500°C$
$\varepsilon_1 = 0.2$
$T_2 = 227°C$
$\varepsilon_2 = 0.4$
$T_{surf} = 60°C$
$\varepsilon_{surr} = 1.0$

*neglecting constant heat transfer
*steady-state



surrounding environment is a black body, so we don't need to put it in here.
$\therefore e_{b_3} = J_3$ (black-body assumption)

$$\frac{d}{x} = 2 \xrightarrow{\text{chart}} F_{12} = F_{21} \simeq 0.37$$

$$\underbrace{\cancel{F_{11}}}_{0,\ \text{flat plate}} + F_{12} + F_{13} = 1.0 \quad \therefore F_{13} = 1 - F_{12} = 0.63$$

By symmetry, $F_{13} = F_{23} = 0.63$

$$A_1 = A_2 = \frac{\pi d^2}{4} = 0.283\,m^2$$

$$e_{b_1} = \sigma T_1^4 = [5.669\times10^{-8}][773.15K]^4 = 20{,}256.3\,W/m^2$$

$$e_{b_2} = \sigma T_2^4 = [5.669\times10^{-8}][500.15K]^4 = 3{,}547.38\,W/m^2$$

$$*J_3 = e_{b_3} = \sigma T_3^4 = [5.669\times10^{-8}][333.15]^4 = 698.34\,W/m^2$$

*Net flow of current to any one node = 0

$$\text{Node 2: } \frac{e_{b2} - J_2}{(1-\varepsilon_2)/A_2\varepsilon_2} + \frac{J_1 - J_2}{1/A_1F_{12}} + \frac{J_3 - J_2}{1/A_2F_{23}} = 0$$

* 2 equations, 2 unknowns:

$\boxed{\text{Node 1}}$: $J_1 = 4404.56 + 0.296 J_2$

$\boxed{\text{Node 2:}}$ $J_2 = 1683.04 + 0.222 J_1$

$\therefore J_1 = 5247.57 \text{ W/m}^2$

$J_2 = 2848 \text{ W/m}^2$

$$\therefore q^{rad}_{net-1} = \frac{e_{b1} - J_1}{(1-\varepsilon_1)/\varepsilon_1 A_1} = \boxed{1062.19 W}$$

$$\therefore q^{rad}_{net-2} = \frac{e_{b2} - J_2}{(1-\varepsilon_2)/\varepsilon_2 A_2} = \boxed{131.96 W}$$

MECH-375 HEAT TRANSFER - HANDOUT # 11

Heat Exchangers

Topics

- ➢ **Definition**
- ➢ **Categorization / Basic Types of Heat Exchangers**
- ➢ **Heat Exchanger Design: Overview**
- ➢ **Overall Heat Transfer Coefficient**
- ➢ **Log-Mean Temperature Difference (LMTD)**
- ➢ **Heat Exchanger Analysis**
    - **LMTD method**
        - ✓ **Parallel-flow heat exchangers**
        - ✓ **Counter-flow heat exchangers**
    - **ε-NTU method**
    - **ε-NTU design charts**
    - **LMTD method *vs.* ε-NTU method**

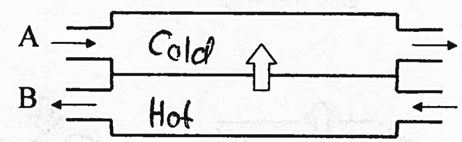
Heat Exchangers: Definition

Devices used to transfer heat from a hot fluid to a cold fluid

Categorization / Basic types of Heat Exchangers

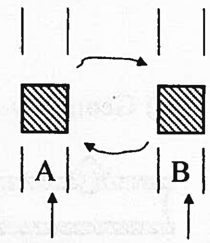
(i) Recuperator / Regenerator

Cyclic device



(a) Recuperator

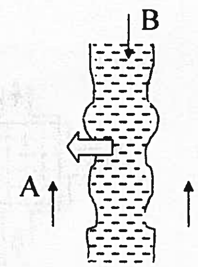
*constant transfer from hot to cold



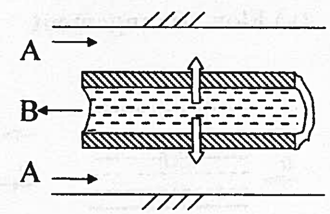
(b) Regenerator

e.g. : thermal whe
enthalpy wh

(ii) Direct contact / Transmural heat transfer



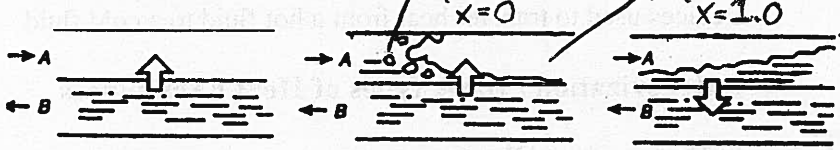
(c) Direct contact heat transfer
[Heat transfer across interface between the fluids]



(d) Transmural heat transfer
[Heat transfer through walls separating the fluids]

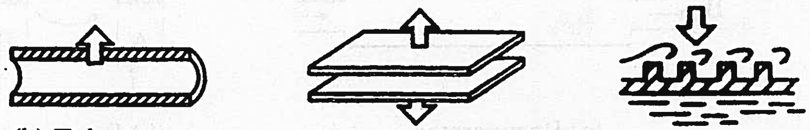
(iii) Single-phase / Two-phase

liquid phase x=0 vapor phase x=1.0



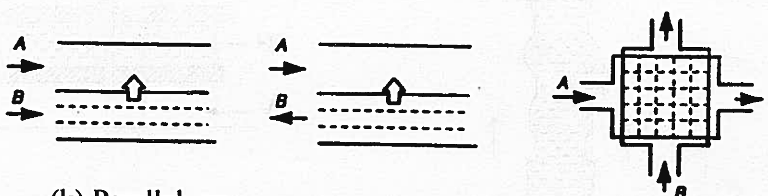
(e) Single-phase (f) Evaporation [Boilers; steam generators] (g) Condensation [Condensers]

(iv) Geometry



(h) Tube-type (i) Plate-type (j) Enhanced-surface

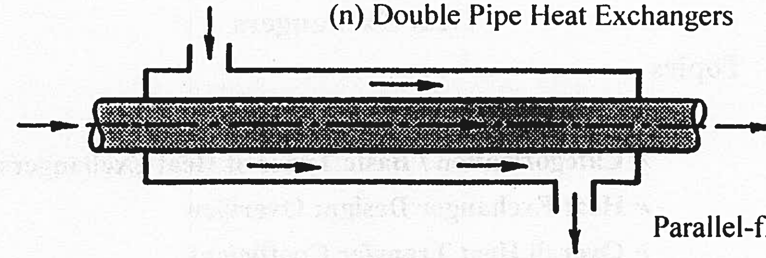
(v) Flow arrangement



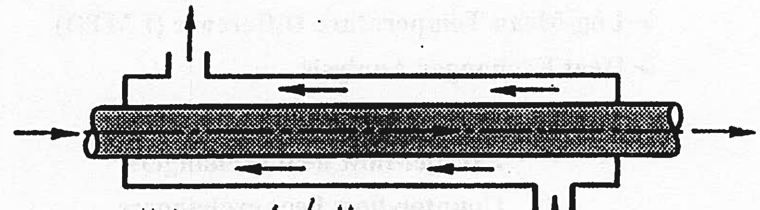
(k) Parallel- or co-current flow (l) Counter-flow (m) Cross-flow

(vi) Construction details

(n) Double Pipe Heat Exchangers

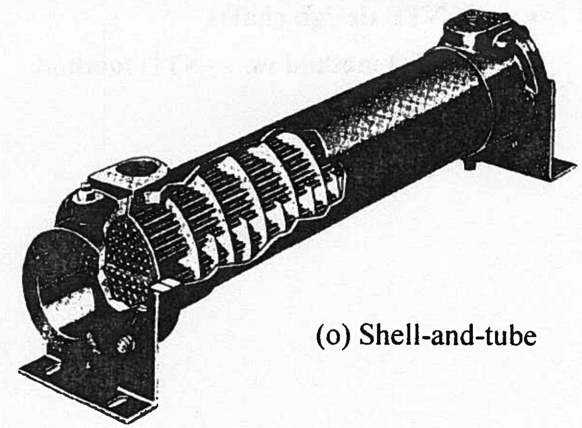


Parallel-flow

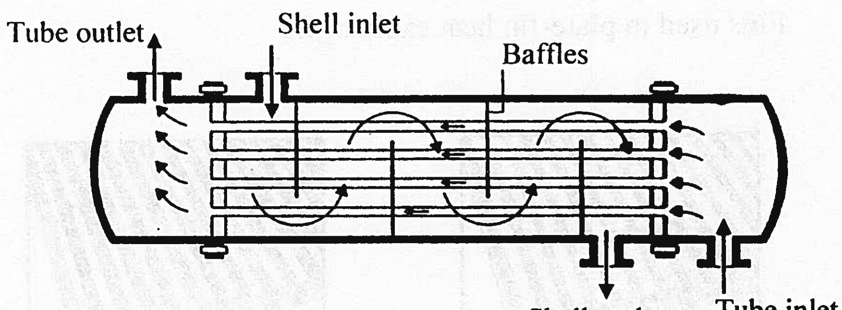


*this one is better

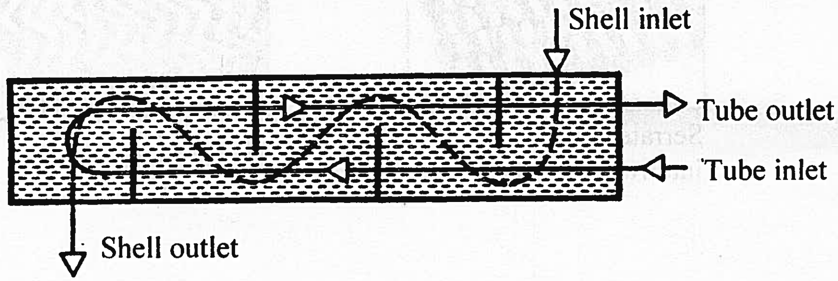
Counter-flow



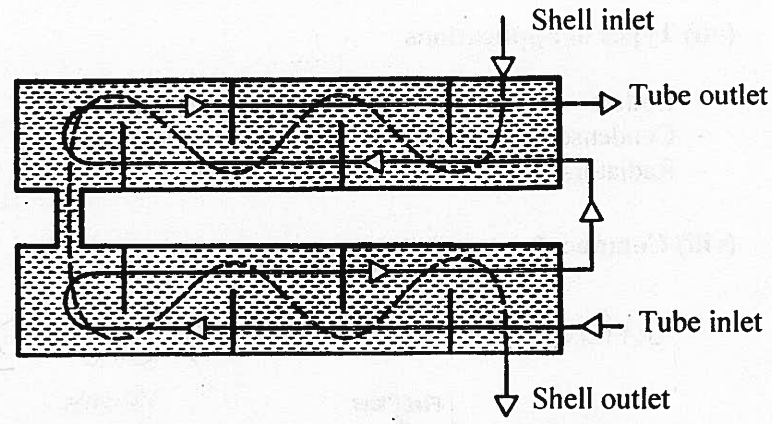
(o) Shell-and-tube



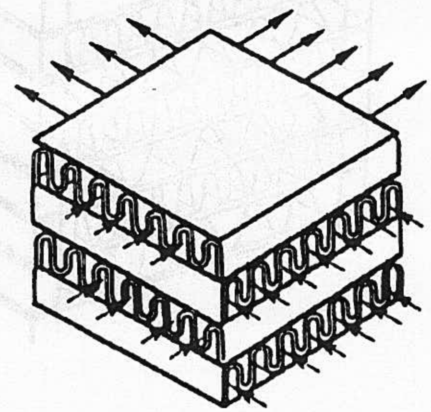
(o) One-shell-pass / One-tube-pass



(p) One-shell-pass / Two-tube-pass



(q) Two-shell-pass / Four-tube-pass



Cross-flow
Heat Exchan

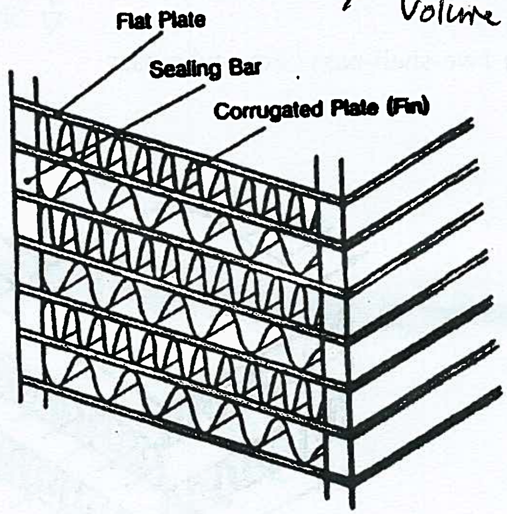
(r) Plate-type

(vii) Types of applications

- Boilers
- Condensers
- Radiators
- Cooling towers
- Air heaters
- Chillers

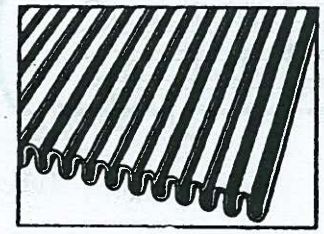
(viii) Compact heat exchangers

$$\frac{\text{Surface Area for Heat Transfer}}{\text{Core Volume}} \geq \frac{700\,m^2}{m^3}$$

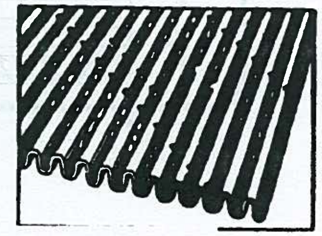


Construction details of a plate-fin compact heat exchanger

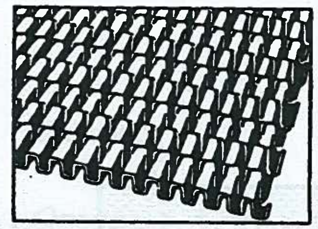
Fins used in plate-fin heat exchangers



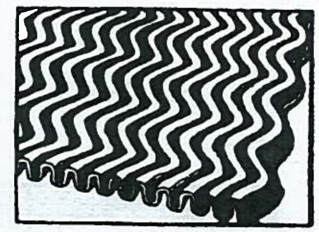
Plain



Perforated



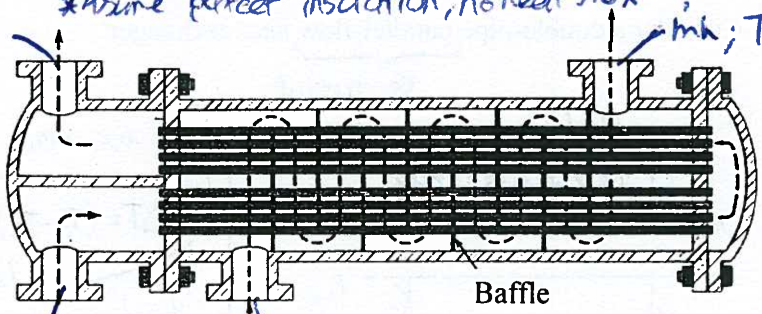
Serrated or interrupted-surface



Herringbone

Heat Exchanger Design: Overview

*Assume perfect insulation, no heat flux



$q_{loss\ ambient} \ll q_{hot\ to\ cold\ fluid}$

One-shell-pass and two-tube-pass heat exchanger

[$\dot{m}_c$, $\dot{m}_h$, T_{ci}, T_{co}, T_{hi}, T_{ho}, C_c, C_h] ← specific heat

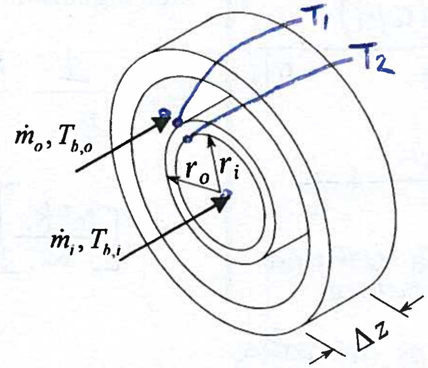
1st Law of thermodynamics can be used to fix one of these, given the rest.

- Surface area for heat transfer
- k_{tube}; corrosion characteristics; mechanical properties
- Fluid-flow direction
- Geometry
- Degree of scaling
- Overall pumping power
- Manufacturing aspects
- Transportation
- Reliability
- Costs

*Design depends on all of these parameters

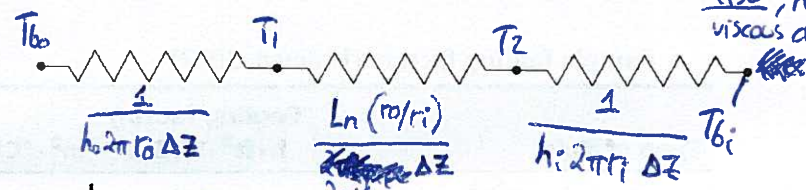
*optimization of heat exchangers is complex

Overall Heat Transfer Coefficient



- Steady State
- Suppose $T_{b,o} > T_{b,i}$
- ΔT that drives the heat transfer $\} = T_{bo}$
- Fully developed conditi[ons] prevail: thus, h_o & h_i (constant ; also, n[o] heat conduction in ... also, n[o] viscous d[issipation]

Resistance analogy:



and $q_{slice} =$ → $T_{bo} - T_{bi} \Big/ \left[\frac{1}{h_o 2\pi r_o \Delta z} + \frac{\ln(r_o/r_i)}{2\pi k_{pipe} \Delta z} + \frac{1}{h_i 2\pi r_i \ldots}\right.$

or $q_{slice} = 2\pi\Delta z\left(T_{b,o} - T_{b,i}\right) \Big/ \left[\frac{1}{h_o r_o} + \frac{\ln(r_o/r_i)}{k_{tube}} + \frac{1}{h_i r_i}\right]$

Let $q_{slice} = U_o 2\pi r_o \Delta z\left(T_{b,o} - T_{b,i}\right) = U_i 2\pi r_i \Delta z\left(T_{b,o} - T_{b,i}\right)$

$q = UA(T_{bo} - T_{bi}) = U_i A_i (T_{bo} - T_{bi}) = U_o A_o (T_{bo} - \ldots$

can be defined on inside or outside surfaces

$UA = U_i A_i = U_o A_o$

$\boxed{U = \text{overall heat transfer coefficient } [W/m^2 K]}$

then $\frac{1}{U_o} = \frac{1}{h_o} + \frac{r_o \ln(r_o/r_i)}{k_{pipe}} + \frac{r_o}{r_i h_i}$

and $\frac{1}{U_i} = \frac{r_i}{r_o h_o} + \frac{r_i \ln(r_o/r_i)}{k_{pipe}} + \frac{1}{h_i}$

U_o: Overall heat transfer coefficient based on outer surface

U_i: " " based on inner surface

Fouling factor: minus!

$$R_{Foul} = \frac{1}{U_{foul}} - \frac{1}{U_{clean}}$$

$$\left[\frac{m^2 K}{W}\right]$$

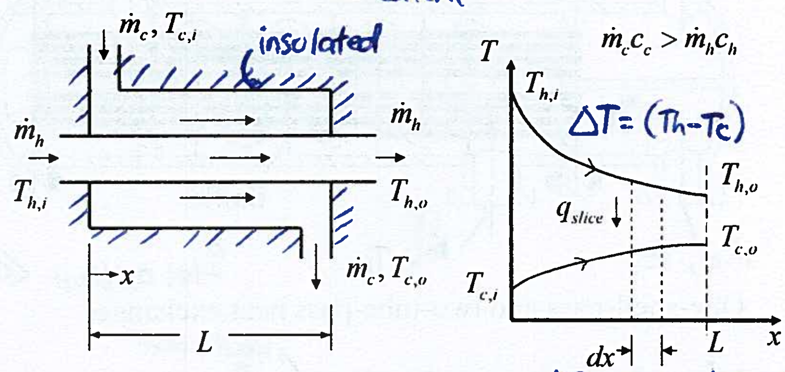
Sample fouling factors (Holman, 2002)

Type of fluid	Fouling factor, h·ft²·°F/Btu	m²·°C/W
Seawater, below 125°F	0.0005	0.00009
Above 125°F	0.001	0.002
Treated boiler feedwater above 125°F	0.001	0.0002
Fuel oil	0.005	0.0009
Quenching oil	0.004	0.0007
Alcohol vapors	0.0005	0.00009
Steam, non-oil-bearing	0.0005	0.00009
Industrial air	0.002	0.0004
Refrigerating liquid	0.001	0.0002

Log-Mean Temperature Difference (LMTD)

Consider a double-pipe parallel-flow heat exchanger

co-current



Solution — ΔT varies logarit

So you can't tal

1st Law of thermodynamics: $q_{total} = \dot m_h C_h (T_{hi} - T_{ho}) = \dot m_c C_c (T_c$

average

Heat exchanger design theory:

$$q_{total} = (UA)(\Delta T)_{mean}; \quad (\Delta T)_{mean} =$$

$q_{slice} = -\dot m_h C_h dT_h = \dot m_c C_c dT_c$ | $q_{slice} = \{U_{perimeter}\} dx (T_h - T_c)$

$$dT_h = \frac{U_{perimeter}\, dx (T_h - T_c)}{-\dot m_h C_h}; \quad dT_c = \frac{U_{peri}\, dx (T_h - T_c)}{\dot m_c C_c}$$

and $dT_h - dT_c = d(T_h - T_c) = -\underbrace{(U\,Peri)}(T_h - T_c)dx\left[\frac{1}{\dot{m}_h c_h} + \frac{1}{\dot{m}_c c_c}\right]$

or $\displaystyle\int_{inlet}^{outlet} \frac{d(T_h - T_c)}{(T_h - T_c)} = \int_{inlet}^{outlet} -(U\,Peri)\left[\frac{1}{\dot{m}_h c_h} + \frac{1}{\dot{m}_c c_c}\right]dx$

$\therefore \ln\left[\frac{T_{ho} - T_{co}}{T_{hi} - T_{ci}}\right] = -(U\,Peri\,\underbrace{L}_{x_{outlet} - x_{inlet}})\left[\frac{1}{\dot{m}_h c_h} + \frac{1}{\dot{m}_c c_c}\right]$

but $\dot{m}_h c_h = q_{total} / (T_{h,i} - T_{h,o})$; $\dot{m}_c c_c = q_{total} / (T_{c,o} - T_{c,i})$

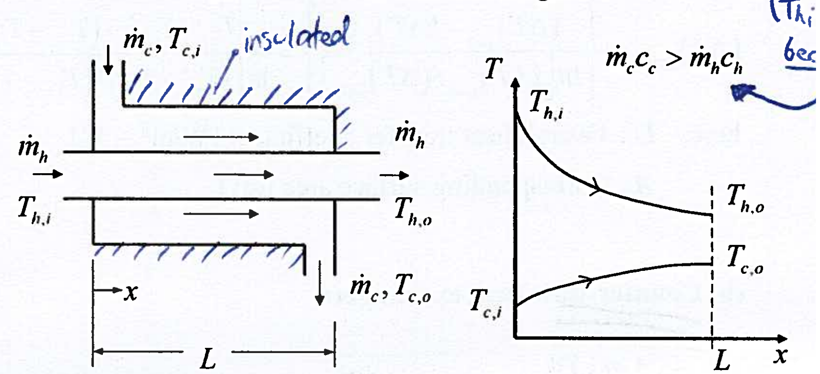
$$\therefore q_{total} = (UA_{surf})\underbrace{\left\{\frac{(T_h - T_c)_{x=0} - (T_h - T_c)_{x=L}}{\ln\left[(T_h - T_c)_{x=0} / (T_h - T_c)_{x=L}\right]}\right\}}_{\Delta T_{LMTD} \text{ for parallel flow double-pipe heat exchanger}} = (UA_{surf})(\Delta T)_{mean}$$

Heat Exchanger Analysis [steady-state]

Two main methods are used: (i) the LMTD method; and (ii) the effectiveness-number-of-transfer-units (ε-NTU) method

LMTD method

(a) Parallel-flow (or co-current) heat exchangers Here, $(T_{co} - T_{ci})$ $(T_{hi}$ bec



{Heat Capacity Rate} = C = $\dot{m}c$ [W/K]

Overall energy balance:

Assumptions:

(i) Heat losses to the ambient (surroundings) are negligible

(ii) Negligible viscous dissipation ($Ec \ll 1$) for both the hot and the cold fluid streams

(iii) K.E. and P.E. changes are negligible

(iv) $di = c_p dT$

(v) properties based on average temperature are essentially constant

Then, application of the first law of thermodynamics gives:

$$\left.\begin{array}{l} \dot{m}_h c_h\left(T_{h,i}-T_{h,o}\right)=\dot{m}_c c_c\left(T_{c,o}-T_{c,i}\right) \\ \text{or} \quad C_h\left(T_{h,i}-T_{h,o}\right)=C_c\left(T_{c,o}-T_{c,i}\right) \end{array}\right\} \rightarrow \text{(A)}$$

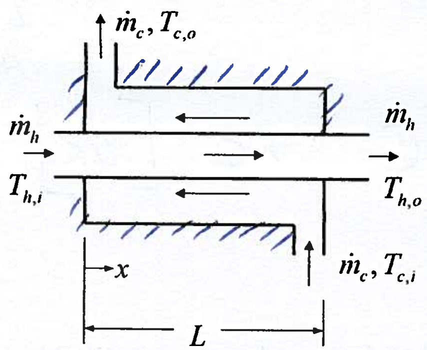
$$q_{total}=(UA)(\Delta T)_{LMTD} \quad \} \rightarrow \text{(B)}$$

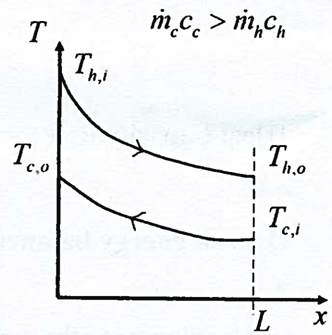
$$(\Delta T)_{LMTD}=\left\{\frac{(\Delta T)_{x=0}-(\Delta T)_{x=L}}{\ln\left[(\Delta T)_{x=0}/(\Delta T)_{x=L}\right]}\right\}=\frac{\left(T_{h,i}-T_{c,i}\right)-\left(T_{h,o}-T_{c,o}\right)}{\ln\left[\left(T_{h,i}-T_{c,i}\right)/\left(T_{h,o}-T_{c,o}\right)\right]}$$

here, U: Overall heat transfer coefficient [W/m^2 – °C]

A: Corresponding surface area [m^2]

(b) **Counter-flow heat exchangers**





Overall energy balance:

Assumptions: Same as those for parallel-flow heat exchangers

$$\left.\begin{array}{l} q=\dot{m}_h c_h\left(T_{h,i}-T_{h,o}\right)=\dot{m}_c c_c\left(T_{c,o}-T_{c,i}\right) \\ \text{or} \quad C_h\left(T_{h,i}-T_{h,o}\right)=C_c\left(T_{c,o}-T_{c,i}\right) \end{array}\right\} \rightarrow \text{(A)}$$

$$q_{total}=(UA)(\Delta T)_{LMTD} \quad \} \rightarrow \text{(B)}$$

$$(\Delta T)_{LMTD}=\left\{\frac{(\Delta T)_{x=0}-(\Delta T)_{x=L}}{\ln\left[(\Delta T)_{x=0}/(\Delta T)_{x=L}\right]}\right\}=\frac{\left(T_{h,i}-T_{c,o}\right)-\left(T_{h,o}-T_{c,i}\right)}{\ln\left[\left(T_{h,i}-T_{c,o}\right)/\left(T_{h,o}-T_{c,i}\right)\right]}$$

ε-NTU method

(First proposed by Nusselt; extensively developed by Kays and London)

$$\text{Effectiveness, } \varepsilon=\frac{\text{[Actual rate of heat transfer]}}{\text{[Max. possible rate of heat transfer from the hot fluid stream to the cold one]}} = \frac{q_a}{q_{max}}$$

$$\varepsilon=\begin{cases}\left\{\left(T_{c,o}-T_{c,i}\right)/\left(T_{h,i}-T_{c,i}\right)\right\} & \text{when } \dot{m}_c c_c<\dot{m}_h c_h \\ \left\{\left(T_{h,i}-T_{h,o}\right)/\left(T_{h,i}-T_{c,i}\right)\right\} & \text{when } \dot{m}_h c_h<\dot{m}_c c_c\end{cases}$$

$$\text{or} \quad \varepsilon=\frac{\left|\Delta T_b\right|_{\text{fluid stream with } (\dot{m}c)_{min}}}{\left|\Delta T_b\right|_{\text{Max. in heat exchanger}}}$$

$q_{actual} = \dot{m}_h C_h (T_{hi} - T_{ho}) = \dot{m}_c C_c (T_{co} - T_{ci})$

q_{max} possible $= (T_{hi} - T_{ci})(\dot{m}c_{min})$

whichever is smallest betw cold and hot fluid

Number of Transfer Units } NTU = $\left\{ \dfrac{\text{Heat transfer rate per degree of } \Delta T_{LMTD}}{\text{Heat transfer rate per degree 'rise' in temperature of the fluid stream with } (\dot{m}c)_{min}} \right\} = \dfrac{UA}{(\dot{m}c)_{min}}$ $= U_i A_i = U_o A_o$

Minimum capacity rate ratio} $R_{min} = \dfrac{(\dot{m}c)_{min}}{(\dot{m}c)_{max}} = \dfrac{C_{min}}{C_{max}}$

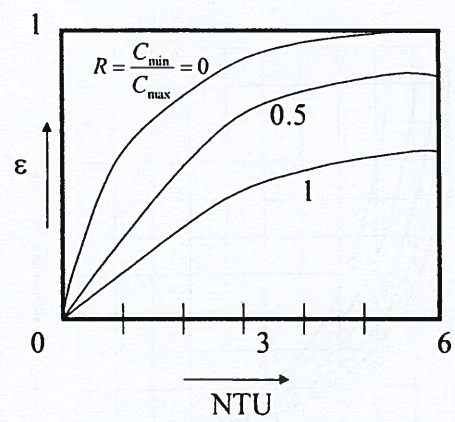
It can be shown that:

$\varepsilon = fnc(NTU, R_{min}, \text{Geometry, Flow Arrangement})$

ε-NTU design charts

Graphical representations of the above-mentioned ε-NTU functional relationship are available for many heat exchanger configurations.

A TYPICAL ε-NTU is shown below:



Some typical values of NT

- Automobile radiator: 0.5-1.0
- Regenerators used in turbine engines: 5-50
- Heat exchangers in L plants: ≥ 200

Notes:

1. For all heat exchangers (regardless of configuration / flow arrangement) when $R = 0$; Typically : Boiling or Condensation (two-phase flow)

$$\varepsilon = 1 - \exp(-NTU)$$

2. ε-NTU design charts for selected heat exchanger geometries a flow arrangements are also shown on the next four pages. The design charts have been taken from your textbook by Holman 2002.

NTU

Figure 10-12 | Effectiveness for parallel-flow exchanger performance.

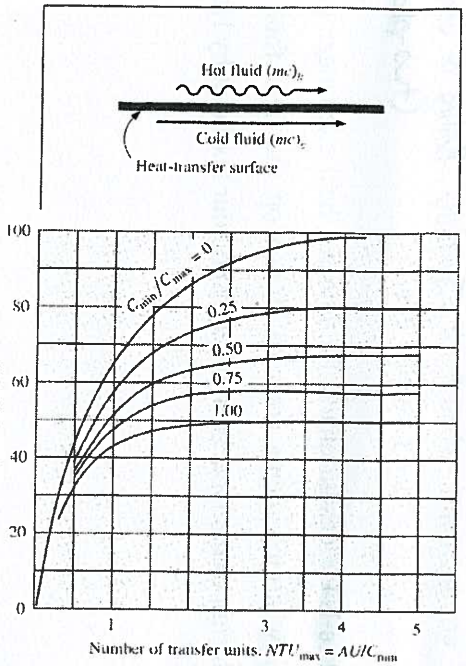
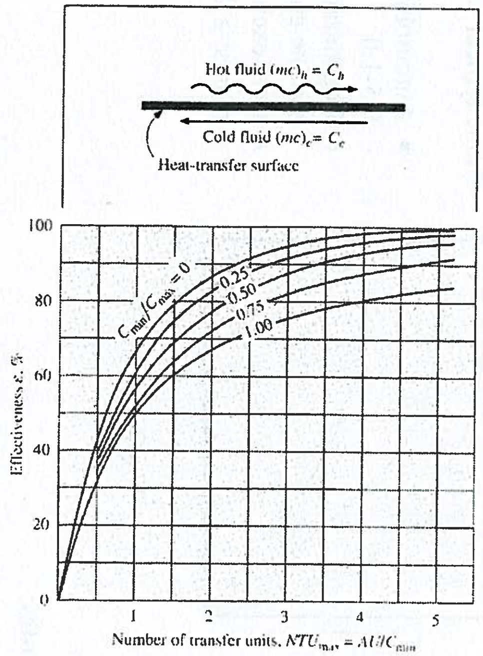


Figure 10-13 | Effectiveness for counterflow exchanger performance.



(Figure extracted from Heat Transfer by J.P. Holman, 9th Edition, 2002)

Dashed Lines | C_{max}=unmixed
C_{min}=mixed

Solid Lines | C_{min}=mixed
C_{max}=unmixed

Figure 10-14 | Effectiveness for cross-flow exchanger with one fluid mixed.

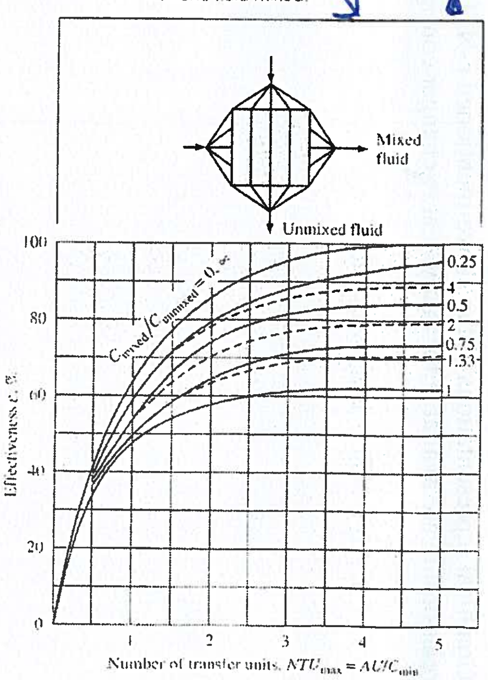
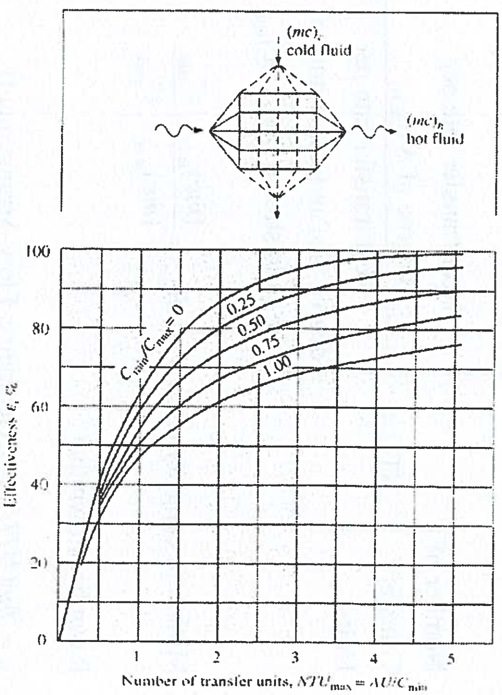
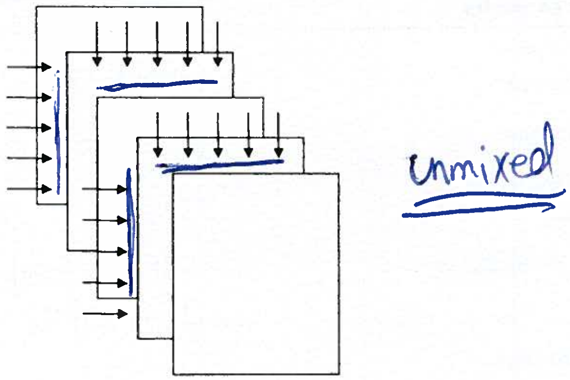


Figure 10-15 | Effectiveness for cross-flow exchanger with fluids unmixed.

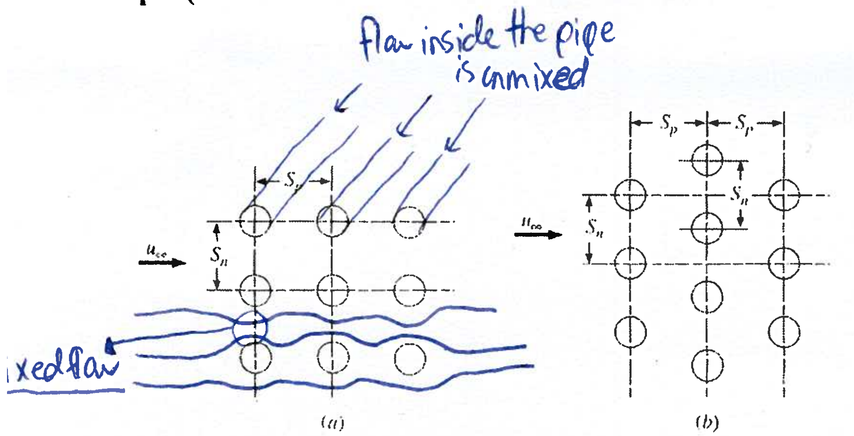


(Figure extracted from Heat Transfer by J.P. Holman, 9th Edition, 2002)

Example



Example (



(a) In-line tube rows; (b) staggered tube rows

Handout # 11: Heat Exchangers MECH-375- Heat Tran

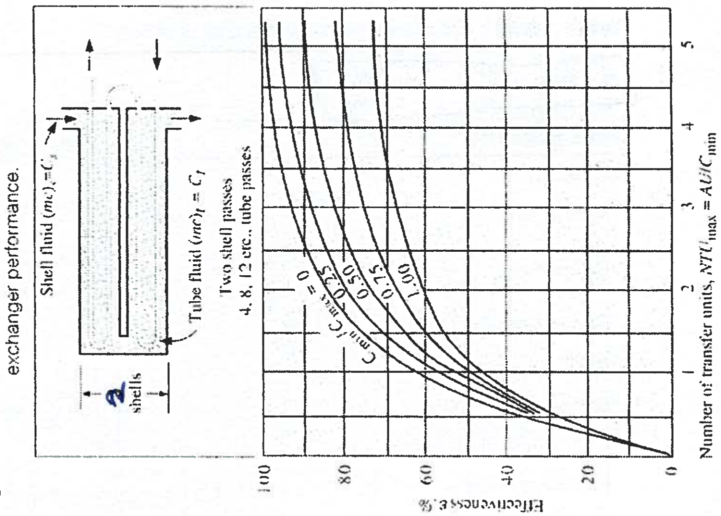


Figure 10-17 | Effectiveness for 2-4 multipass counterflow exchanger performance.

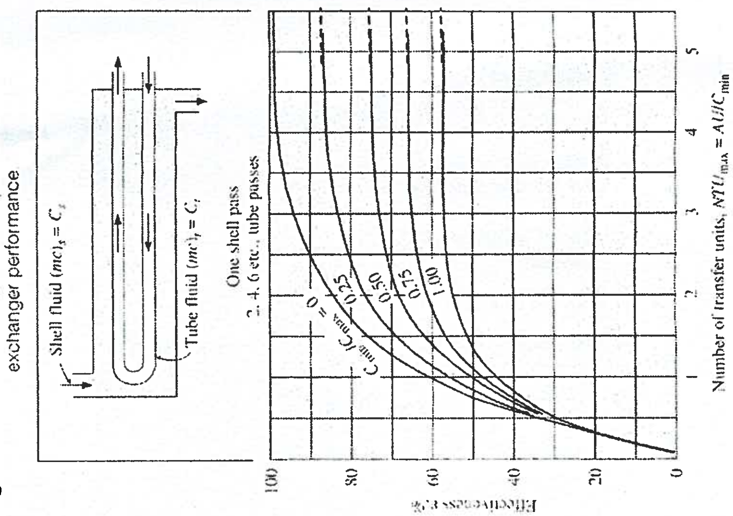


Figure 10-16 | Effectiveness for 1-2 parallel counterflow exchanger performance.

Table 10-3 | Heat-exchanger effectiveness relations.

$N = \text{NTU} = \dfrac{UA}{C_{min}}$ $\quad C = \dfrac{C_{min}}{C_{max}}$

Flow geometry	Relation
Double pipe:	
Parallel flow	$\epsilon = \dfrac{1-\exp[-N(1+C)]}{1+C}$
Counterflow	$\epsilon = \dfrac{1-\exp[-N(1-C)]}{1-C\exp[-N(1-C)]}$
Counterflow, $C = 1$	$\epsilon = \dfrac{N}{N+1}$
Cross flow:	
Both fluids unmixed	$\epsilon = 1-\exp\left[\dfrac{\exp(-NCn)-1}{Cn}\right]$ where $n = N^{-0.22}$
Both fluids mixed	$\epsilon = \left[\dfrac{1}{1-\exp(-N)} + \dfrac{C}{1-\exp(-NC)} - \dfrac{1}{N}\right]$
C_{max} mixed, C_{min} unmixed	$\epsilon = (1/C)\{1-\exp[-C(1-e^{-N})]\}$
C_{max} unmixed, C_{min} mixed	$\epsilon = 1-\exp\{-(1/C)[1-\exp(-NC)]\}$
Shell and tube:	
One shell pass, 2, 4, 6, tube passes	$\epsilon = 2\left\{1+C+(1+C^2)^{1/2} \times \dfrac{1+\exp[-N(1+C^2)^{1/2}]}{1-\exp[-N(1+C^2)^{1/2}]}\right\}^{-1}$
Multiple shell passes, $2n$, $4n$, $6n$ tube passes (ϵ_p = effectiveness of each shell pass, n = number of shell passes)	$\epsilon = \dfrac{[(1-\epsilon_p C)/(1-\epsilon_p)]^n - 1}{[(1-\epsilon_p C)/(1-\epsilon_p)]^n - C}$
Special case for $C = 1$	$\epsilon = \dfrac{n\epsilon_p}{1+(n-1)\epsilon_p}$
All exchangers with $C = 0$	$\epsilon = 1-e^{-N}$

Table 10-4 | NTU relations for heat exchangers.

$C = C_{min}/C_{max}$ $\quad \epsilon$ = effectiveness $\quad N = \text{NTU} = UA/C_{min}$

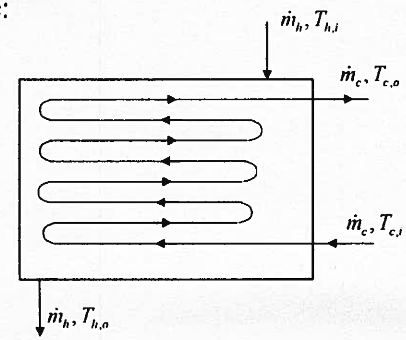
Flow geometry	Relation
Double pipe:	
Parallel flow	$N = \dfrac{-\ln[1-(1+C)\epsilon]}{1+C}$
Counterflow	$N = \dfrac{1}{C-1}\ln\left(\dfrac{\epsilon-1}{C\epsilon-1}\right)$
Counterflow, $C = 1$	$N = \dfrac{\epsilon}{1-\epsilon}$
Cross flow:	
C_{max} mixed, C_{min} unmixed	$N = -\ln\left[1+\dfrac{1}{C}\ln(1-C\epsilon)\right]$
C_{max} unmixed, C_{min} mixed	$N = \dfrac{-1}{C}\ln[1+C\ln(1-\epsilon)]$
Shell and tube:	
One shell pass, 2, 4, 6, tube passes	$N = -(1+C^2)^{-1/2} \times \ln\left[\dfrac{2/\epsilon-1-C-(1+C^2)^{1/2}}{2/\epsilon-1-C+(1+C^2)^{1/2}}\right]$
All exchangers, $C = 0$	$N = -\ln(1-\epsilon)$

Example 1

A clean shell-and-tube heat exchanger is used to heat 2.5 kg/s of water from 15°C to 85°C. The heating is accomplished by passing hot oil, available at 160°C, through the shell side of the heat exchanger. The mass flow rate of the oil is 5.19 kg/s, and this is known to provide an average heat transfer coefficient of $h_o = 400$ W/m^2-K on the outside surface of the tubes. Ten identical tubes are used to pass the water through the shell. Each tube is made of a material of very high thermal conductivity and a smooth surface, and its wall is thin, with $D_o = D_i = 0.025$ m. Each tubes makes eight passes through the shell. The following data are available:

Oil properties: $c_p = 2350$ J/kg-K

Water properties: $c_p = 4182$ J/kg-K;
$\mu = 5.48 \times 10^{-4}$ kg/m-s;
$k = 0.643$ W/m-K;
and $\rho = 1000$ kg/m^3.

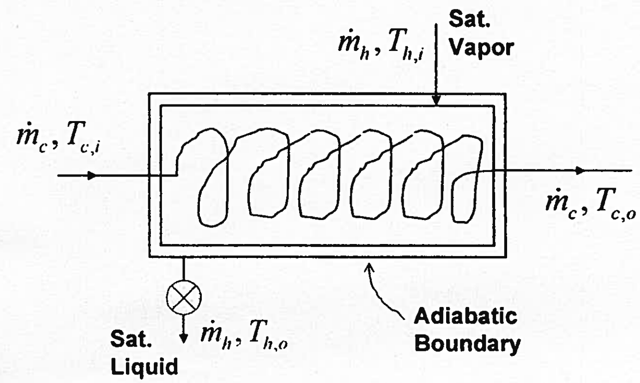


(a) What is the exit temperature of the hot oil?
(b) What is the effectiveness of the heat exchanger?
(c) What is the heat transfer coefficient inside the tubes?
(d) What is the overall heat transfer coefficient?
(e) How long must *each* tube be to accomplish the desired heating?
(f) After four years of operation, $\dot m_h$, $\dot m_c$, $T_{h,i}$, and $T_{c,i}$ are maintained at their original values, but fouling causes a 30% drop in the overall rate of heat transfer. What is the fouling factor for the dirty heat exchanger?

Ans.: a) 100°C; b) 0.483; c) 3059.7 W/m^2-°C; d) 353.75 W/m^2-°C; e) 37.6 m; f) 1.884×10^{-3} m^2-°C/W

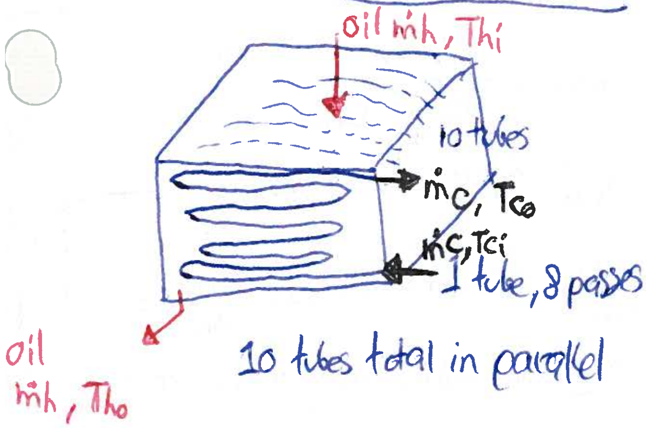
Example 2

Consider the condenser shown in the adjoining figure. The pressu drop for the condensing (hot) fluid stream is negligible compared the absolute pressure, p_{sat}. Derive a relationship between tl effective, ε, and the number-of-transfer-units, NTU, for this he exchanger.



Ans.: $\varepsilon = 1 - \exp(-NTU)$

Handout 11, Ex. 1, Slide 25



10 tubes total in parallel

Assumptions

* Constant properties; all tubes are smooth
* steady-state heat transfer between oil and water
* Heat loss to ambient is negligible
* $E_c \ll 1$ i.e. no viscous dissipation

a) $T_{ho} = ?$

$$q = \dot{m}_c C_c (T_{co} - T_{ci}) = \dot{m}_h C_h (T_{hi} - T_{ho})$$

$$\frac{\dot{m}_c C_c (T_{co} - T_{ci})}{\dot{m}_h C_h} = T_{hi} - T_{ho}$$

$$T_{ho} = T_{hi} - \frac{\dot{m}_c C_c (T_{co} - T_{ci})}{\dot{m}_h C_h} = 160 - \frac{2.5 \times 4182 (85-15)}{5.19 \times 2350} = 99.99 \approx \boxed{100°C}$$

b) $\varepsilon = ?$ $\quad \varepsilon = \frac{q_{actual}}{q_{max}}$

water: $\dot{m}_c C_c = 2.5 \times 4182 = 10455$ W/°C

oil: $\dot{m}_h C_h = 5.19 \times 2350 = 12,196$ W/°C

$\therefore (\dot{m}c)_{min} = \dot{m}_c C_c$

$$\therefore \varepsilon = \frac{\cancel{\dot{m}_c C_c} (T_{co} - T_{ci})}{\cancel{\dot{m}_c C_c} (T_{hi} - T_{ci})} = \frac{85-15}{160-15} = 0.488 \quad \boxed{\varepsilon = 48.3\%}$$

c) h_i inside tubes [water]

$$\dot m_{c\,\text{one tube}} = \frac{\dot m c}{10} = 0.25\,\text{kg/s}$$

$$Re_{D_i} = \frac{\rho u_{av} D_i}{\mu}\ ; \quad \dot m_{c\,\text{one tube}} = \frac{\pi D_i^2}{4}\rho u_{av}$$

$$u_{av} = \frac{0.25}{1000\,\pi\,\frac{(0.025)^2}{4}} = 0.5093\,\text{m/s}$$

$\therefore Re_{D_i} = 23234.3 > 2300$ $\therefore$ Turbulent flow; smooth pipe

use dittus boelter correlation:

$$Nu_{D_i} = \frac{h_i D_i}{k_{fluid}} = 0.023\, Re_{D_i}^{0.8}\, Pr^{n}$$, $n = 0.4$ because it is heating, $n = 0.3$ if cooling

$$Pr_{water} = \frac{\mu c_p}{k} = 3.564$$

$\therefore \boxed{h_i = 3059.7\,\text{W/m}^2\,{}^\circ\text{C}}$ (other method gives $h_i = 3300$, correct as well)

d) $U \Rightarrow \frac{1}{U_i} = \frac{1}{h_i} + \frac{r_i \ln\left(\frac{r_o}{r_i}\right)}{k_{tube}} + \frac{r_i}{r_o}\frac{1}{h_o}$

Here, $D_i \simeq D_o$ (thin tube) so $\ln\left(\frac{r_o}{r_i}\right) \to 0$

$$\therefore \frac{1}{U} = \frac{1}{h_i} + \frac{1}{h_o} = \frac{1}{3059.7} + \frac{1}{400} \rightarrow \boxed{U = 353.75\,\text{W/m}^2\,{}^\circ\text{C}}$$

e) $L = ?$

$\varepsilon = fnc(R_{min}, NTU)$ $\rightarrow$ $\varepsilon = 0.483$

$NTU = \frac{UA}{(\dot m c)_{min}} = ?$ $\rightarrow$ $R_{min} = \frac{(\dot m c)_{min}}{(\dot m c)_{max}} = 0.857$ $\rightarrow$ chart gives $NTU \simeq 1.0$

$\hookrightarrow A = \frac{NTU\,[(\dot m c)_{min}]}{U} = \frac{1 \times 10{,}455}{353.75} = \cancel{29.56}$

part f)

ANS: $U_{foul} = 212.34$; $R_{foul} = 1.884 \times 10^{-3}$

$$R_{fouling} = \frac{1}{U_{foul}} - \frac{1}{U_{clean}}$$

- $U_{clean} = 353.75$
- $\frac{q_{fouled}}{q_{clean}} = 0.7 \rightarrow$ find $\varepsilon_{fouled} = \frac{q_{fouled}}{q_{max}} = 0.338$

use chart ($R_{min} = 0.857$) & $\varepsilon_{fouled} \rightarrow$ find $NTU = 0.6 = \frac{UA}{(\dot m c)_{min}}$

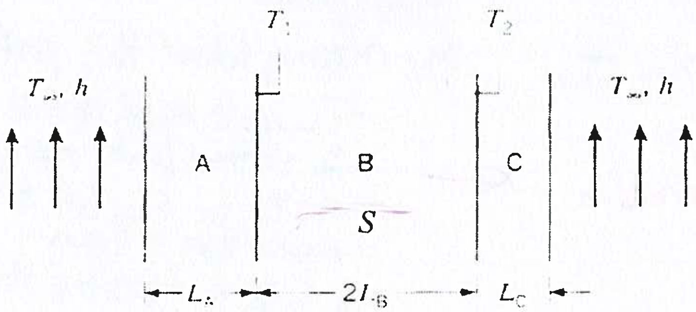
8
—
50

Department of Mechanical Engineering
MECH 375: Heat Transfer-1 [5% of the Total Marks]

Notes: This is an open Notes/Textbook exam. Quiz period: 30 min
State your assumptions concisely. Answer on the questionnaire.

Problem [50 Marks]:

Consider one-dimensional conduction in a plane composite wall. The outer surfaces are exposed to a fluid at 25°C and a convection heat transfer coefficient of 1000 W/m$^2\cdot$K. The middle wall B experiences uniform heat generation S while there is no generation in walls A and C. The temperatures at the interfaces are T_1 = 261°C and T_2 = 211°C.



h= 1000W/m²K

k_A = 25 W/m·K L_A = 30 mm
k_C = 50 W/m·K L_B = 30 mm
L_C = 20 mm

(a) Assuming negligible contact resistance at the interfaces, determine the volumetric heat generation S

(b) Assuming $k_B<k_A$, sketch a qualitatively accurate temperature distribution within the system, showing its important features

(c) Consider conditions corresponding to a loss of coolant at the exposed surface of material A (h=0). Determine T_2, and sketch a qualitatively accurate temperature distribution within the system, showing its important features

Note: Assume steady-state heat transfer, and neglect radiation heat transfer.

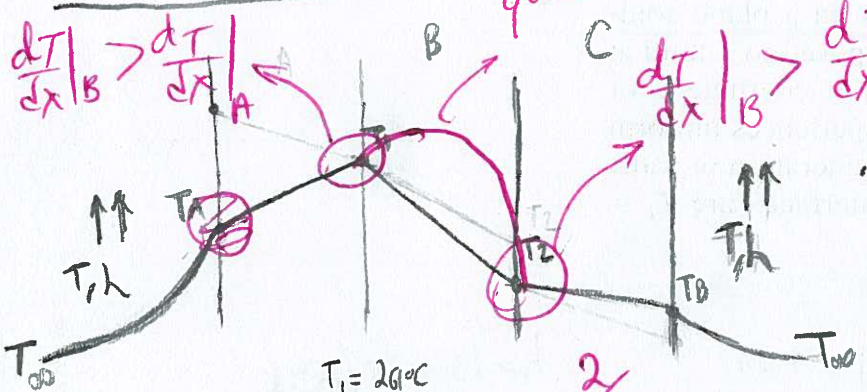
a) **(30 Marks)**; b) **(8 Marks)**; c) **(12 Marks)**

Good Luck!

OK!

(b) ~~In general~~, High thermal conductivity means a lower resistance, which means a lower ΔT

Temperature profile



$T_1 = 261°C$

$T_2 = 211°C$

since $K_B < K_C$

where: $K_B < K_A < K_C$

$\therefore \Delta T$ in $B > \Delta T$ in $A > \Delta T$ in C

as represented through gradients of temp. profile

My reasoning behind this profile is:

- T_∞ must equal on both sides, so start/end points of profile is equal
- $T_1 > T_2$, so from left to right a negative slope is shown
- T_A to T_1 is a positive slope, because it gets warmer as you approach B. Same logic for T_B to T_2
- Slopes are a property of K values

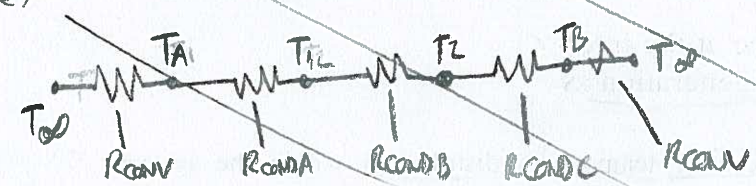
2/

1D Conduction + heat generation

↓

quadratic

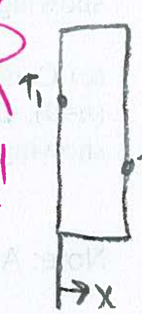
(a) Multilayer Plane Wall



$R_{TOT} = 2R_{conv} + R_{COND\,A} + R_{COND\,B} + R_{COND\,C}$

NOT symmetric $T_1 > T_2$

(a) $\frac{d^2T}{dx^2} + \frac{\dot{S}}{K} = 0$ $\int d\dot{V}$, $d\dot{V} =$ boundary conditions: ~~at $x=0$, $\frac{dT}{dx} = \emptyset$~~ at $x=L$, $T=T_2$



See the Solution!

integrat twice with respect to x:

$$\frac{dT}{dx} + \int (\dot{S}/K)\,dx = C_1$$

$$\frac{dT}{dx} = -(\dot{S}/K)x + C_1$$

$$T = -\frac{1}{2}\frac{\dot{S}}{K}x^2 + C_1 x + C_2$$

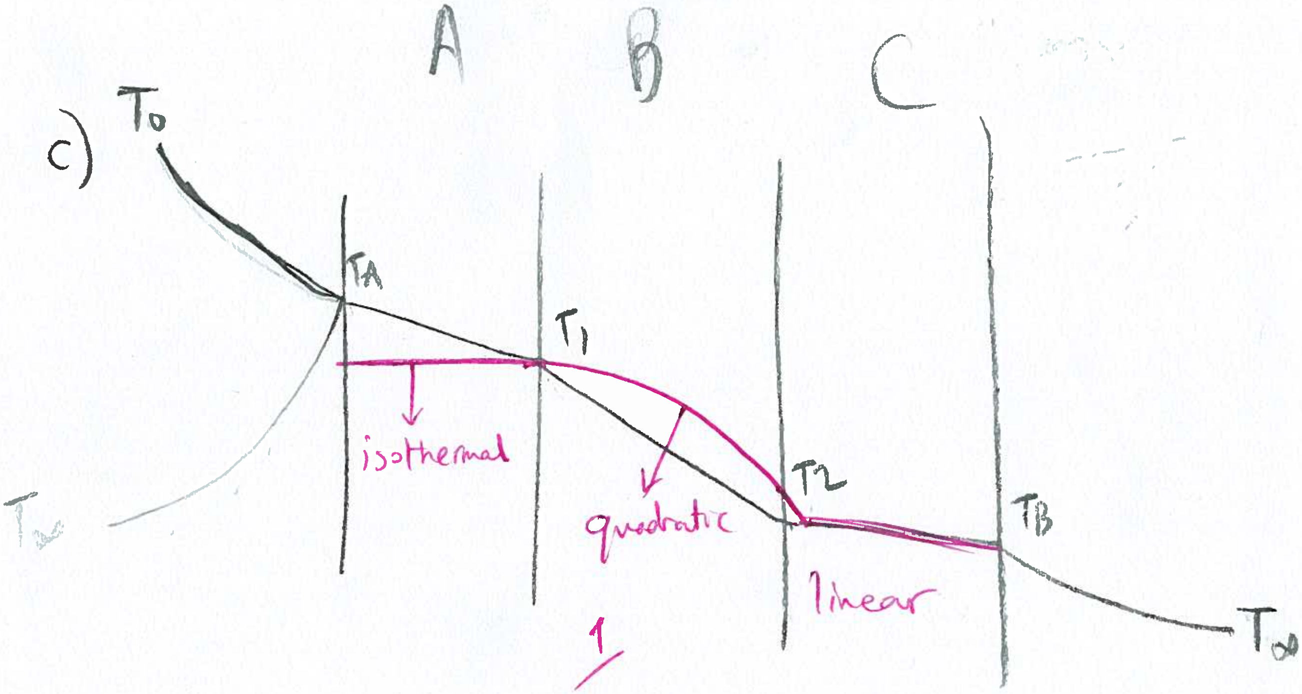
$$\rightarrow \frac{dT}{dx} = -\left(\frac{\dot{S}}{K}\right)x + C_1$$

$$T_1 - T_2 = \frac{1}{2}\frac{\dot{S}}{K}L^2\left[1 - \left(\frac{x}{L}\right)^2\right]$$

$$100 = \frac{\dot{S}}{K_B}(30\times10^{-3}\,m)\left[1 - \left(\frac{x}{30\times10^{-3}\,m}\right)^2\right]$$

5/

c)



loss of coolant at A $\Rightarrow h=0$ @A

∴ Again, slopes show K in regards to each wall A, B, C

- $T_0 > T_\infty$, since there is no more coolant on left side
- Since right side is cooler, more heat that way ↳

See the solution!

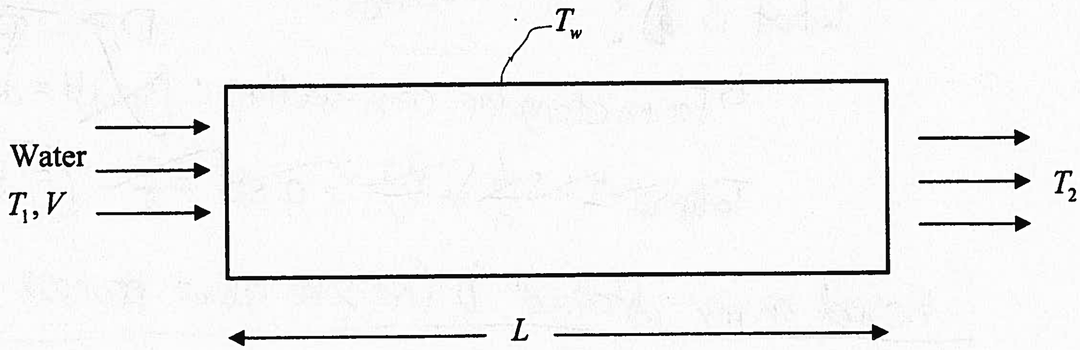
$\frac{10}{50}$

Department of Mechanical Engineering
MECH 375: Heat Transfer-1 [5% of the Total Marks]

Notes: This is an open Notes/Textbook exam. Quiz period: 30 min
State your assumptions concisely. Answer on the questionnaire.

Name: René Rinfret **Student ID#:** 34929091

Problem 1 [50 Marks]: Water is to be heated in steady-state operation from 15°C to 85°C as it flows through a rectangular (2.5 cm × 4 cm) smooth duct. The duct is equipped with an electric resistance heater that provides *uniform* heating throughout the surface of the duct. The outer surface of the heater is well insulated. The system is to provide hot water at a rate of 15 L/min.



a) Determine the electrical power needed to heat water (~~25~~ 20 **Marks)**

b) The inner surface temperature of the duct at the exit is measured as 120°C. Find the length of the tube. (~~25~~ 30 **Marks)**

Good Luck

given: $T_{b1} = 15°C$ $\rho = 1000\ kg/m^3$ $k = 0.6\ \frac{W}{mK}$
$T_{b2} = 85°C$ $C_p = 4180$
$U = V = 15\ L/min$ $\mu = 10^{-3}\ Pa \cdot s$

IGNORE!

$U = \frac{15L}{min} \times \frac{m^3}{1000L} \times \frac{1\ min}{60}$
$U = 2.5 \times 10^{-4}\ m^3/s$
$U = U/A = \frac{2.5 \times 10^{-4}}{(2.5 \times 10^{-3})(4 \times 10^{-3})}$
$\boxed{U = 25\ m/s}$

a) $q = \dot{m} C_p (T_{b2} - T_{b1})$
where $\dot{m} = \frac{\rho u_{av} D^2 \pi}{4}$

See next page

$Re_{DH} = \frac{\rho U D_h}{\mu}$

$D_H = \frac{4(2.5 \times 10^{-3})(4 \times 10^{-3})}{(2 \times 2.5 \times 10^{-3})(2 \times 4 \times 10^{-3})}$

$Re_{DH} = 1000(\ldots)(25)(\ldots)/1$

a) so, $q = \dot{m} C_p (T_{62} - T_{61})$ $\frac{10}{20}$

$$\dot{m} = \frac{\rho u_{av} D_h^2 \pi}{4}$$

$$u_{av} = \frac{15L}{min} \times \left(\frac{1m^3}{1000L}\right) \times \frac{1min}{60sec} \times \frac{1}{(2.5\times10^{-3})(4\times10^{-3})} = \boxed{25m/s}$$

water

what is D?

$D = \frac{4A}{P}$

↳ For rectangular cross-sections: $\cancel{D_h = 2H} = 2(4\times10^{-3}\,m)$

Table ~~8.1~~: $\frac{b}{a} = \frac{2.5}{4} = 0.625 = \frac{5}{8}$ $\boxed{= 8\times10^{-3}} = D$

Based on my obtained D (not sure about process...)

$$q = \left[\frac{1000(25m/s)(8\times10^{-3})^2 \pi}{4}\right] 4180 (85°C - 15°C)$$

the only variable that could be wrong here is D...

$\therefore$ $\boxed{\cancel{q = 36.77KW}}$ ANS

6) $T_w = 120°C, T_{b_1} = 15, T_{b_2} = 85$

~~For~~ T_w constant: ~~$Nu = \frac{hD}{K} = 3.658$~~

T_w is not Constant

Again, assume D is correct from part a) and is 8×10^{-3} m ~~(D = 2H for rectangular)~~ → I think ... this is the only varia[ble] that could be wrong in my entire process. Calculations and equations are correct... This also means we assume q from part a) is correct for calculations in part b). Thank you!!

So: $h = \frac{K(3.658)}{D} = \frac{3.658(0.6)}{8\times10^{-3}} = \boxed{274.35\ W/m^2°C}$

use of ΔT_{LMTD} is not correct, because T_w is not Const

$q = h A_{surf} \cancel{\Delta T_{LMTD}} = 36{,}770W$ (from part a)

$$\Delta T_{LMTD} = \frac{(T_w - T_{b_1}) - (T_w - T_{b_2})}{\ln((T_w - T_{b_1})/(T_w - T_{b_2}))} = 63.72$$

$$A_{surf} = (2.5\times10^{-3})(4\times10^{-3})L = 1\times10^{-5}(L)$$

$$\therefore L = \left(\frac{q}{h\Delta T}\right) \Big/ (1\times10^{-5}) = \boxed{\cancel{2103}\ Km}$$

ANS

wow...