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Tutorial 2

1.

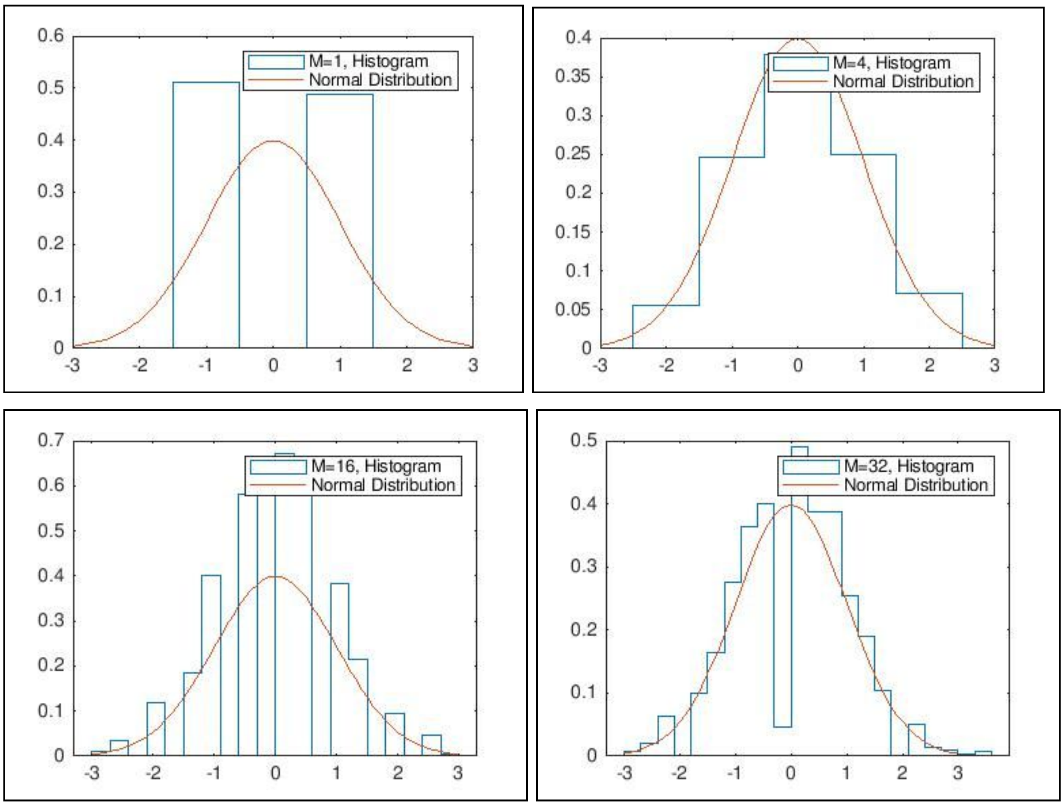


*Fig 1-4: Error distribution with varying M (M = 1,4,16,32). The larger the "error", the more normally distributed they'll be.*

*2.*

$$\sigma E = \sqrt{\left(\frac{\partial E}{\partial F}\right)^2 \sigma F^2 + \left(\frac{\partial E}{\partial L}\right)^2 \sigma L^2 + \left(\frac{\partial E}{\partial \delta}\right)^2 \sigma \delta^2 + \left(\frac{\partial E}{\partial b}\right)^2 \sigma L^2 + \left(\frac{\partial E}{\partial h}\right)^2 \sigma h^2}$$

$\frac{\partial E}{\partial F} = \frac{L^3}{3\delta\left(\frac{bh^3}{12}\right)} = \frac{.7^3}{3(.008)\left(\frac{.04(.03^3)}{12}\right)} = 1.6\times10^8$ (From lab manual) $\sigma F = .8$ $F = 80$

$\frac{\partial E}{\partial L} = \frac{L^2 F}{\delta\left(\frac{bh^3}{12}\right)} = \frac{.07^2\,80}{.008\left(\frac{.04(.03^3)}{12}\right)} = 5.4\times10^{10}$ $\sigma L = .014$ $L = .7$

$\frac{\partial E}{\partial \delta} = \frac{-FL^3}{3\delta^2\left(\frac{bh^3}{12}\right)} = \frac{-80(.7^3)}{3(.008^2)\left(\frac{.04(.03)^3}{12}\right)} = -7.6\times10^{12}$ $\sigma\delta = .00008$ $\delta = .008$

$\frac{\partial E}{\partial b} = \frac{-FL^3}{3\delta\left(\frac{b^2h^3}{12}\right)} = \frac{-80(.7^3)}{3(.008)\left(\frac{.04^2 .03^3}{12}\right)} = 3.2\times10^{11}$ $\sigma b = .0004$ $b = .04$

$\frac{\partial E}{\partial h} = \frac{-FL^3}{\delta\left(\frac{bh^4}{12}\right)} = \frac{-80(.7^3)}{.008\left(\frac{.04\,.03^4}{12}\right)} = 1.3\times10^{12}$ $\sigma h = .003$ $h = .03$

$\sigma E = 3.9\times10^9$ Pa

*a.*

*b.*

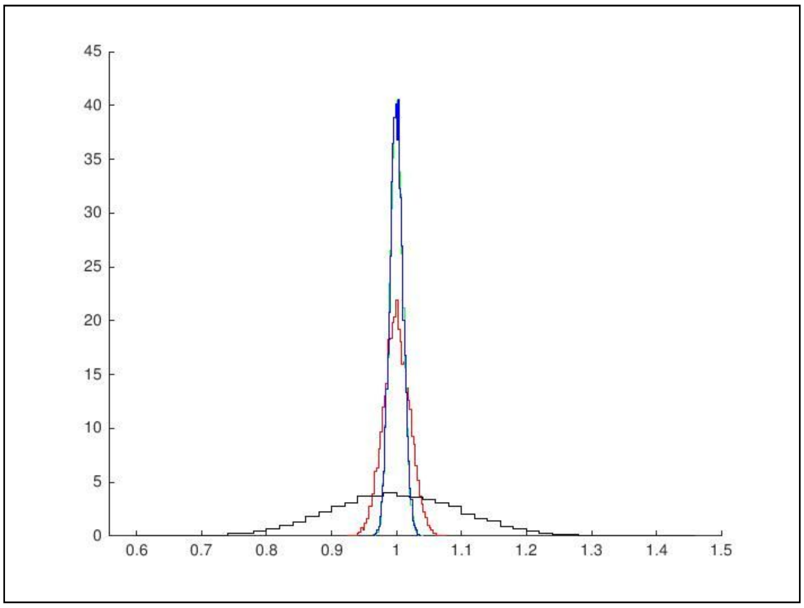


*Fig 5: Normalized values of inputs where: L/L0 is red, h/h0 is black, b/b0 is green, F/F0 is blue, delta/delta0 is magenta. All are randomly generated and normally distributed.*

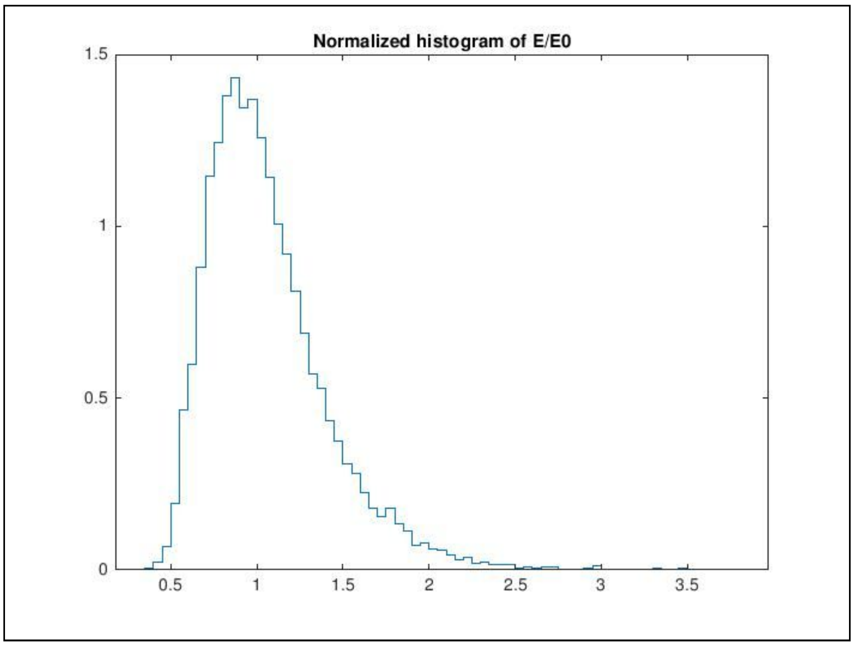


*Fig 6: Normalized value of E. Note that it is not normally due to skews discussed in Q2C. Standard Deviation E is 0.3500 *(10^10) Pa. Mean E is 1.0626*(10^10) Pa.*

c. In the uncertainty propagation equation, (dE/dx)^2 (x stands for a variable that E is function of) can be treated as the coefficient of (dx)^2. In this case, because of the different values of dE/dx, some uncertainties may affect the final propagation more than others, so normal distribution of uncertainties would not necessarily result in normally distributed value, reflected in Q2B, Fig 6.

Additionally, the larger the coefficient, the more effect it is going to have on the calculated uncertainty. From Q2A, we can see that delta has the largest dE/dx, so it is the measurement that if improved, would yield the largest benefit.

Appendixes:

**Appendix A: Code from Q1**

```
clc; clear; close all;
plotter(1, 1);
plotter(2, 4);
plotter(3, 16);
plotter(4, 32);
% Loop end here

% factor out calculation and plotting so multiple plots can be generated in 1 script
function plotter(a, M)

s = 1;
N = 1000;
M_values= [1 4 16 32];
M_size = size(M_values,2); % = 4
epsilon = zeros(M_size,N); % initialize epsilon matrix to store data

% generate the pdf for normal distribution to compare with our histograms:
x = -3:0.1:3;
norm = normpdf(x,0,1);

% A loop here to loop over M_values
   for n = 1:N
        % Generate M random plus / minus signs using rand(1,N):
      randNum = rand(1, M) > .5;
      sign = randNum*2 -1;
      delta = sign * s/sqrt(M);
      epsilon(1, n) = sum(delta);
   end
   figure(a)
   histogram(epsilon(1, :),'Normalization','pdf','DisplayStyle','stairs')
   hold on
   plot(x,norm)
   legend(['M=' num2str(M) ', Histogram'], 'Normal Distribution')
end
```

**Appendix B: Code from Q2**

```
% Tutorial 2 simulation of beam bending experiment
clear all
%% Fine Best Estimate of E
% Here are the baseline measurements, in SI
L0=0.7 ;% beam length in m
h0=0.03;% beam depth in m
b0=0.04;% beam width in m
F0=80;% applied force at end of cantilever beam, N
delta0=0.008 ;% displacement at end
% put these into the beam bending equation to get baseline estimate of E
E0=F0*(L0^3)/(3*delta0*(b0*(h0^3)/12));
disp('E0 = ' + E0);

%% Find the Standard Deviation of Each Measurands
% to construct random variables out of these, need standard deviations as
% as PERCENT OF MEAN
sLp=2;%
shp=10;
sbp=1;
sFp=1;
sdelp=1;
% express these now as standard deviations in the physical units (not
% relative values
sL=sLp*L0/100;% same units as L
sh=shp*h0/100;
sb=sbp*b0/100;
sF=sFp*F0/100;
sdel=sdelp*delta0/100;
%% Simulation: Generate Normally Distributed Random Measurements
N = 10000   ;% number of trials

% Construct normally distributed variables for all parameters:
L = L0+sL*randn(1,N);
h = h0+sh*randn(1,N);
b = b0+sb*randn(1,N);
F = F0+sF*randn(1,N);
del = delta0+sdel*rand(1,N);

E = (F.*L.*L.*L)./(3*del.*(b.*h.*h.*h/12));  %this is now a vector of values for Young?s
modulus

%% Report the Simulation Result: E = E_best +/- sigma_E

% Normalize the results relative to E0:
```

```
sErel=std(E)/E0  % st. d. of E
meanE=mean(E)/E0 % Best estimation of E

%% All variables in one histogram
%{
figure(1)
clf;
hold on
l1=histogram(L/L0,'Normalization','pdf','DisplayStyle','stairs', 'EdgeColor', 'r');
l2=histogram(h/h0,'Normalization','pdf','DisplayStyle','stairs', 'EdgeColor', 'k');
l3=histogram(b/b0,'Normalization','pdf','DisplayStyle','stairs', 'EdgeColor', 'g');
l4=histogram(F/F0,'Normalization','pdf','DisplayStyle','stairs', 'EdgeColor', 'b');
l5=histogram(del/del0,'Normalization','pdf','DisplayStyle','stairs', 'EdgeColor', 'm');
%l6=histogram(E/E0,'Normalization','pdf','DisplayStyle','stairs');
legend([l1,l2,l3,l4,l5],{'L/L0', 'h/h0', 'b/b0', 'F/F0', 'del/del0'});
legend('show');
title('All normalized values');
hold on
%}


%% Histogram of E/E0
hold on;
figure(2)
histogram(E/E0,'Normalization','pdf','DisplayStyle','stairs');
title('Normalized histogram of E/E0');
```