Problem Set # 7

Textbook Problems

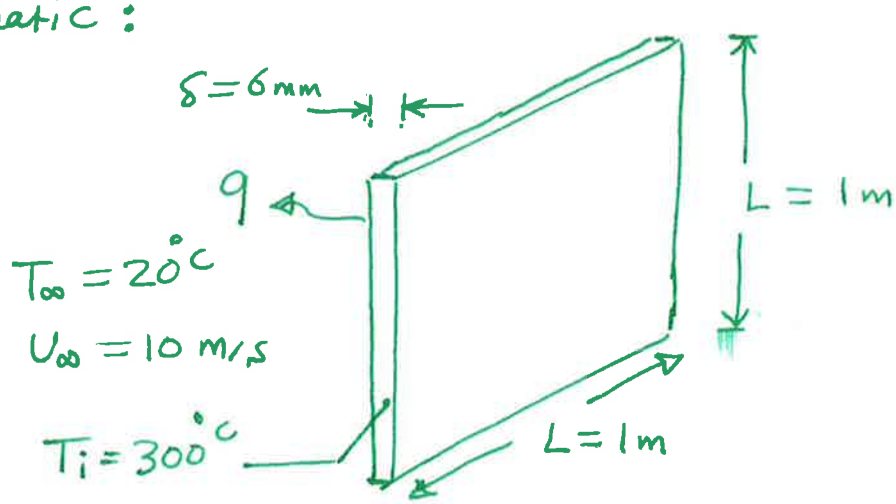
Problem 7.24

Solution:

Known: Plate dimensions and initial temperature. Velocity and temperature of air in parallel flow over plates.

Find: Initial rate of heat transfer from plate. Rate of change of plate temperature.

Schematic:



Assumptions: (1) Negligible radiation, (2) Negligible effect of conveyor velocity on boundary layer development (3) Plates are isothermal, (4) Negligible heat transfer from sides of plate, (5) $Re_{x,c} = 5 \times 10^5$ (6) Constant properties.

Properties: Table A-1, AISI 1010 Steel (573 K):
$K_p = 49.2\ W/m.K$, $C = 549\ J/Kg \cdot K$
$\rho = 7832\ Kg/m^3$. Table A-4, Air ($p = 1\,atm$, $T_f = 433\,K$): $\nu = 30.4 \times 10^{-6}\ m^2/s$, $K = 0.0361\ \frac{W}{m.K}$
$Pr = 0.688$

Analysis: The initial rate of heat transfer from a plate is:

$$q = 2\bar{h}A_s(T_i - T_\infty) = 2\bar{h}L^2(T_i - T_\infty)$$

with $Re_L = U_\infty L/\nu = 10\,\text{m/s} \times 1\,\text{m} / 30.4\times10^{-6}\,\text{m}^2/\text{s}$

$$= 3.29 \times 10^5$$

Flow is laminar over the entire surface and

$$\overline{Nu}_L = 0.664\,Re_L^{1/2}\,Pr^{1/3} = 0.664(3.29\times10^5)^{1/2}(0.688)^{1/3}$$

$$= 336$$

$$\bar{h} = (K/L)\,\overline{Nu}_L = (0.0361\,\tfrac{W}{m\cdot K}/1\,m)\,336 = 12.1\,\tfrac{W}{m^2\cdot K}$$

Hence,

$$q = 2 \times 12.1\,\frac{W}{m^2\cdot K}\,(1\,m^2)(300-20)^\circ C = 6780\,W$$

Performing an energy balance at an instant of time for a control surface about the plate: $-\dot{E}_{out} = \dot{E}_{st}$

$$\rho\delta L^2 c \left.\frac{dT}{dt}\right|_i = \bar{h}\,2L^2(T_i - T_\infty)$$

$$\left.\frac{dT}{dt}\right|_i = \frac{2(12.1\,W/m^2\cdot K)(300-20)^\circ C}{7832\,kg/m^3 \times 0.006\,m \times 549\,J/kg\cdot K} = -0.26\,^\circ C/s$$

Comments: (1) with $Bi = \bar{h}(\delta/2)/K_p = 7.4\times10^{-4}$, use of the lumped capacitance method is appropriate.

(2) Despite the large plate temperature and the small convection coefficient, if adjoining plates

are in close proximity, radiation exchange with the surroundings will be small and the assumption of negligible radiation is justifiable.

(3)

Problem 7.42

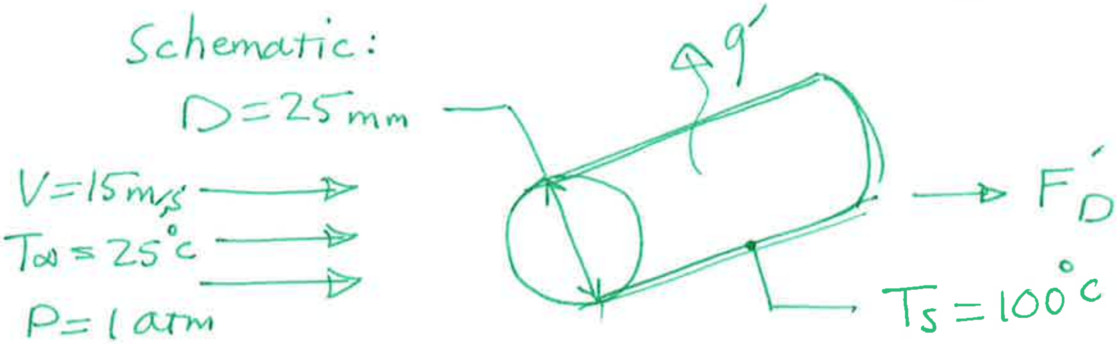
Solution:

Known: Conditions associated with air in cross flow over a pipe

Find: (a) Drag force per unit length of pipe

(b) Heat transfer per unit length of pipe.

Schematic:



Assumptions: (1) Steady-state conditions (2) Uniform cylinder surface temperature.

(3) Negligible radiation effects.

Properties: Table A-4, Air (T_f = 335 K, 1atm):

$\nu = 19.31 \times 10^{-6}\ m^2/s$, $\rho = 1.048\ kg/m^3$

$k = 0.0288\ \frac{W}{m\text{-}K}$, $Pr = 0.702$

Analysis: (a) From the difinition of the drag coefficient with $A_f = DL$ find

$$F_D = C_D A_f \frac{\rho V^2}{2}$$

$$F_D' = C_D D \frac{\rho D^2}{2}$$

with

$$Re_D = \frac{VD}{\nu} = \frac{15\ m/s \times (0.025)\ m}{19.31 \times 10^{-6}\ m^2/s} = 1.942 \times 10^4$$

From Fig 7.8, $C_D \approx 1.1$, Hence

$$F_D = 1.1(0.025\,m)\,1.048\,kg/m^3\,(15\,m/s)^2/2 = 3.24\,N/m$$

(b) Using Hilpert's ralation, with $C = 0.193$ and $m = 0.618$ from Table 7.2,

$$\bar{h} = \frac{k}{D} C Re_D^m Pr^{1/3} = \frac{0.0288\ W/m.k}{0.025} \times 0.193(1.942\times10^4)^{0.618} \times (0.702)^{1/3}$$

$$\Rightarrow \bar{h} = 88\ \frac{W}{m^2.K}$$

Hence, the heat rate per unit length is

$$q' = \bar{h}(\pi D)(T_s - T_\infty) = 88\ W/m^2.K\ (\pi \times 0.025m)(100-25)^\circ C$$
$$= 520\ W/m$$

Comments: Using the Zukauskas correlation and evaluating properties at T_∞ ($\nu = 15.71\times10^{-6}\ m^2/s$, $k = 0.0261\ \frac{W}{m.K}$, $Pr = 0.707$), but with $Pr_s = 0.695$ at T_s,

$$\bar{h} = \frac{0.0261}{0.025}\,0.26\left(\frac{15\times0.025}{15.71\times10^{-6}}\right)^{0.6}(0.707)^{0.37} \times (0.707/0.695)^{1/4} = 102\ W/m^2.K$$

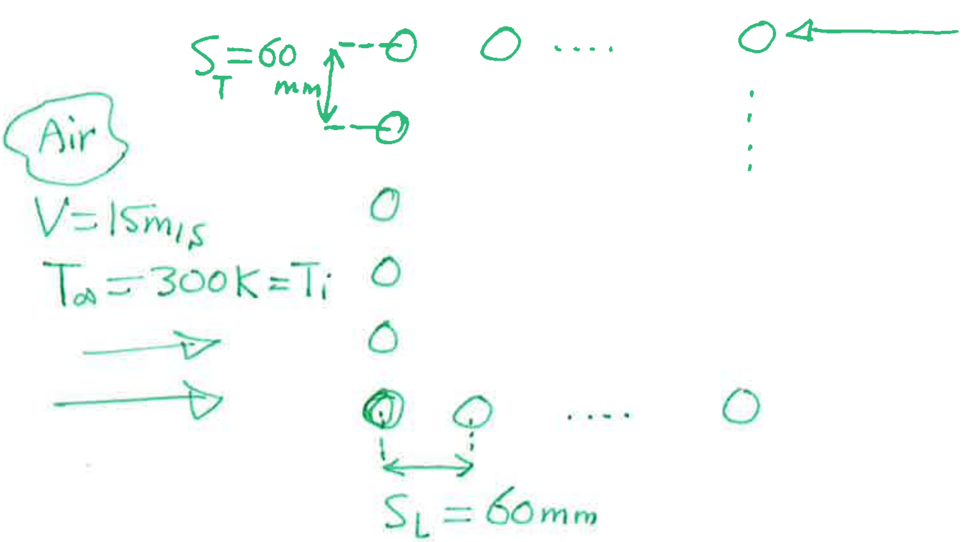
This result agrees with that obtained from Hilpert's relation to within the uncertainty normally associated with convection correlations.

Problem 7-88

Solution:

Known: Surface temperature and geometry of a tube bank. Velocity and temperature of air in cross-flow.

Find: (a) Air outlet temperature (b) Pressure drop and fan power requirements.



Tube, $D = 30$ mm

$T_s = 373K$

$N_L = 10,\ N_T = 7$

$N = 70$

Assumptions: (1) Steady-state conditions, (2) Negligible radiation, (3) Air pressure is approximately one atmosphere (4) Uniform surface temperature

Properties: Table A-4 Air (300K, 1atm): $\rho = 1.1614\ kg/m^3$

$C_p = 1007\ J/kg \cdot K$ $\nu = 15.89 \times 10^{-6}\ m^2/s$

$K = 0.0263\ W/m \cdot K$ $Pr = 0.707$: (373K): $Pr = 0.695$

Analysis: (a) The air temperature increases exponentially with

$$T_o = T_s - (T_s - T_i) \exp\left(-\frac{\pi D N \bar{h}}{\rho V N_T S_T C_p}\right).$$

with $V_{max} = \dfrac{S_T}{S_T - D} V = \dfrac{60}{30} 15 \frac{m}{s} = 30 \frac{m}{s}$;

$$Re_{D,max} = \frac{30 \text{ m/s} \times 0.03\text{m}}{15.89 \times 10^{-6} \text{ m}^2/\text{s}} = 56{,}639.$$

Tables 7.7 and 7.8 give $C = 0.27$, $m = 0.63$ and $C_2 = 0.97$

Hence from the Zukauskas correlation,

$$\overline{Nu}_D = 0.27(0.97)(56{,}639)^{0.63}(0.707)^{0.36}(0.707/0.695)^{1/4}$$

$$= 229$$

$$\bar{h} = \overline{Nu}_D \, k/D = 229 \times 0.0263 \frac{W}{m.K} / 0.03\text{m} = 201 \frac{W}{m^2.K}$$

Hence, $T_o = 373\text{K} - (373 - 300)\text{k} \exp\left(- \dfrac{\pi \times 0.03 \times 70 \times 201}{1.1614 \times 15 \times 7 \times 0.06 \times 1007}\right)$

$$T_o = 373 - 73 \times 0.835 = 312\text{K} = 39^\circ C$$

(b) with $Re_{D_{max}} = 5.66 \times 10^4$, $P_L = 2$, $(P_T - 1)/(P_L - 1) = 1$,

Fig. 7.13 yields $f \approx 0.19$ and $\chi = 1$.

Hence,

$$\Delta P = N_L \chi \left(\frac{\rho V_{max}^2}{2}\right) f = 10 \left(\frac{1.1614 \text{ kg/m}^3 \times (30 \text{ m/s})^2}{2}\right) 0.19$$

$$= 993 \text{ N/m}^2 = 0.00993 \text{ bar}.$$

The fan power requirement is

$$P = \dot{m}_a \Delta P/\rho = \rho V N_T S_T L \, \Delta P/\rho = 15 \text{ m/s} \times 7 \times 0.06\text{m} \times 1\text{m} \times 993 \text{ N/m}^2 = 6.26 \text{ KW}$$

Comments: The heat rate is

$$q = \dot{m}_a C_p (T_o - T_i) = \rho V N_T S_T L C_p (T_o - T_i)$$

$$q = 1.1614 \text{ kg/m}^3 \times 15 \text{ m/s} \times 7 \times 0.06 \text{ m} \times 1 \text{ m} \times 1007 \frac{\text{J}}{\text{kg}\cdot\text{K}} \times (312 - 300)\text{ K} = 88.4 \text{ kW}$$