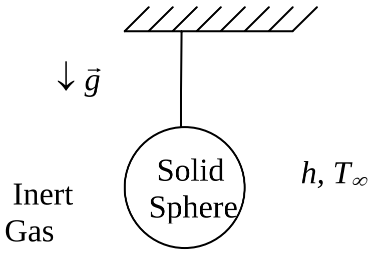
Solutions - Problem Set # 5

Problem 1:

Given: ρ_{solid} = 5000 kg/m^3; c_{solid} = 1000 J/kg-°C; T_i = 1000°C; T_∞= 200°C; $h = \xi(T-T_\infty)^{0.25}$

$T_{t=30min}$= 523.4°C; D = 0.04 m

Assumptions: LPA valid; Radiation negligible; unsteady cooling.



E-balance:

$$\frac{dE}{dt} + hA\left(T - T_\infty\right) = 0$$

$$\frac{dT}{dt} = -\frac{hA\left(T-T_\infty\right)}{\rho c V} = -\frac{\xi\left(T-T_\infty\right)^{5/4} 4\pi r^2}{\rho c \frac{4}{3}\pi r^3} = -\left(\frac{3\xi}{\rho c r}\right)\left(T-T_\infty\right)^{5/4}$$

$$\frac{dT}{\left(T-T_\infty\right)^{5/4}} = -\left(\frac{3\xi}{\rho c r}\right)dt \qquad IC: \quad t=0 \quad T=T_i$$

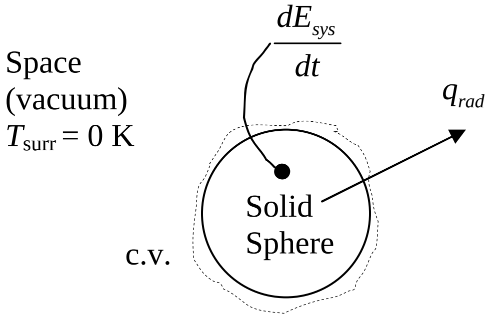
$$-4\left(T-T_\infty\right)^{-1/4} = -\left(\frac{3\xi}{\rho c r}\right)t + C_1 \Rightarrow C_1 = -4\left(T_i - T_\infty\right)^{-1/4}$$

Thus, $-4\left(T-T_\infty\right)^{-1/4} + 4\left(T_i - T_\infty\right)^{-1/4} = -\left(\frac{3\xi}{\rho c r}\right)t$

Using the data given, $-4\left(523.4-200\right)^{-1/4} + 4\left(1000-200\right)^{-1/4} = -\left(\frac{3\xi}{5000\times1000\times0.02}\right)1800$

$$\xi = 3.54 \text{ W/m}^2 - {}^\circ\text{C}^{5/4}$$

Problem 2:



Given: ρ_{solid} = 2707 kg/m^3; c_{solid} = 896 J/kg-°C; T_i = 50°C; T_{surr} = 0 K; ε_{solid} =1 (black body)

Assumptions: LPA valid; no convection heat transfer/only radiation heat transfer; unsteady cooling.

E-balance:

$$\frac{dE}{dt} + A\varepsilon\sigma\left(T^4 - T_{surr}^4\right) = 0$$

Important Note: Absolute Temperatures must be used.

$$\frac{dT}{dt} = -\frac{A\sigma T^4}{\rho c V}$$

$$\frac{dT}{T^4} = -\frac{A\sigma}{\rho c V}dt \qquad IC: \quad t = 0 \quad T = T_i$$

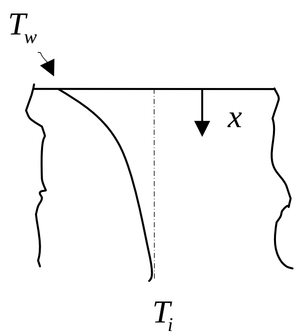
$$-\frac{1}{3}T^{-3} = -\frac{A\sigma}{\rho c V}t + C_1 \Rightarrow C_1 = -\frac{1}{3}T_i^{-3}$$

$$\text{Thus,} \quad t = \frac{\rho c D}{18\sigma}\left[\frac{1}{T^3} - \frac{1}{T_i^3}\right]$$

$T_i = 50 + 273.15 = 323.15$ K; $T_{req.} = -110 + 273.15 = 163.15$ K; $t_{req.} = ?$

$$t_{req.} = \frac{2707\times 896\times 0.05}{18\times 5.668\times 10^{-8}}\left[\frac{1}{163.15^3} - \frac{1}{323.15^3}\right] = 23849.3\,\text{s} = 6\text{h } 37\text{ min } 29\text{s}$$

Problem 3:



Given: ρ_{ss} = 7817 kg/m^3; c_{ss} = 460 J/kg-°C; k_{ss} = 17 W/m-°C; T_i = 300°C; T_w = 100°C.

Assumptions: Thick plate of stainless steel behaves as a semi-infinite solid. 1-D unsteady heat conduction; constant properties

(a) $T_{0.3\text{m},\,t}$=200°C; find t.

$$\frac{T(x,t) - T_w}{T_i - T_w} = \text{erf}\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

$$\frac{200-100}{300-100} = 0.5 = \text{erf}\left(\frac{x}{2\sqrt{\alpha t}}\right) \Rightarrow \frac{x}{2\sqrt{\alpha t}} \simeq 0.477$$

$$\alpha = \frac{k}{\rho c_p} = \frac{17}{7817\times 460} = 4.73\times 10^{-6} \quad \text{m}^2/\text{s}$$

$$\frac{0.03}{2\sqrt{4.73\times 10^{-6}t}} = 0.477 \Rightarrow t = 209.1\,\text{s}$$

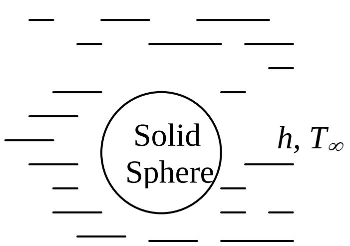
(b) $q''_{x=0,t=209.1s} = ?$

$$q''_{x=0,t} = -k\left.\frac{dT}{dx}\right|_{x=0,t} = \frac{k\left(T_w - T_i\right)}{\sqrt{\pi\alpha t}}$$

$$q''_{x=0,t} = \frac{17\left(100-300\right)}{\sqrt{\pi 4.73\times 10^{-6}\times 209.1}} = -60995.3\ \text{W/m}^2$$

thus, $q''_{x=0,t}$ <0 and is outward

Problem 4:



Given: α = 9.5×10^{-7} m^2/s; k = 1.52 W/m-°C; T_i = 25°C; T_∞ = 200°C; h = 110 W/m^2-°C; D = 0.025 m.

Assumptions: Unsteady heat conduction; constant properties

$$Bi = \frac{hL_c}{k} = \frac{h\left(r_o/3\right)}{k} = \frac{110\left(0.025/6\right)}{1.52} = 0.3 > 0.1 \Rightarrow LPA \text{ not valid}$$

$$t^* = \alpha t / r_o^{\,2} = 9.5\times10^{-7}\times240/\left(0.0125\right)^2 = 1.4592$$

Thus, $t^* > 0.2 \;\Rightarrow$ 1-term approximation of infinite series solution is adequate

(a) $T(r=0, t=240s) = ?$

$$\theta^*(r^*=0,t^*) = \frac{T(r=0,t)-T_\infty}{T_i - T_\infty} = C_B \exp[-A_B^2 t^*]$$

$$Bi_M = \frac{hr_o}{k} = \frac{110\left(0.025/2\right)}{1.52} = 0.9046$$

From Table 5.2 of Handout #5:

$A_B \simeq 1.5075; C_B \simeq 1.2499$

$$\theta^*(r^*=0,t^*) = \frac{T(r=0,t)-200}{25-200} = 1.2499\exp[-\left(1.5075\right)^2\times1.4592] = 4.5363\times10^{-2}$$

$$T(r=0, t=240s) = 192.06°\text{C}$$

(b) $T(r=0.0064m, t=240s) = ?$

(Note: For a depth of 6.1 mm from the surface of the sphere, r = 6.4 mm)

$$\theta^*(r^*,t^*) = C_B \exp[-A_B^2 t^*]\frac{\sin(A_B r^*)}{A_B r^*}$$

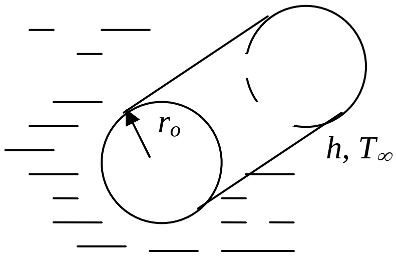
$$\theta^*(r^*,t^*) = \frac{T(r,t)-T_\infty}{T_i - T_\infty} = \theta^*(r^*=0,t^*)\times\frac{\sin(A_B r^*)}{A_B r^*}$$

$$r^* = 0.0064/0.0125 = 0.512$$

$$\frac{T(r,t)-200}{25-200} = 4.5363\times10^{-2}\times\frac{\sin(1.5075\times0.512)}{1.5075\times0.512} = 4.0991\times10^{-2}$$

$$T(r=0.0064\text{ m}, t=240\text{ s}) = 192.83°\text{C}$$

Problem 5:



Given: D = 2r_o = 8cm; ρ = 7900 kg/m^3; c = 480 J/kg-°C; k = 35 W/m-°C; T_i = 400°C; T_∞= 80°C and h = 450 W/m^2-°C.

Assumptions: Unsteady heat conduction; constant properties; long cylinder (L>>D); radiation negligible

$$Bi = \frac{hL_c}{k} = \frac{h\left(r_o/2\right)}{k} = \frac{450\left(0.04/2\right)}{35} = 0.257 > 0.1 \Rightarrow LPA \text{ not valid}$$

At this stage we do not know t to obtain $t^* = \alpha t / r_o^{\,2}$

Thus, let assume $t^* > 0.2$ and will check this assumption later

$\Rightarrow$ 1-term approximation of infinite series solution is adequate

(a) $T(r = 0.04, t) = 180;\quad \text{t} = ?$

$$\theta^*(r^*,t^*) = \frac{T(r,t) - T_\infty}{T_i - T_\infty} = C_B \exp[-A_B^2 t^*] \mathrm{J}_0(A_B r^*)$$

$$Bi_M = \frac{hr_o}{k} = \frac{450\left(0.04\right)}{35} = 0.5143 \underset{Table\text{-}5.2}{\underbrace{\rightrightarrows}} \begin{array}{l} A_B \simeq 0.9519 \\ C_B \simeq 1.1172 \end{array}$$

$$\frac{180-80}{400-80} = \underbrace{1.1172 \times \exp[-\left(0.9519\right)^2 t^*]}_{\theta^*(r^*=0,t^*)} \mathrm{J}_0(0.9519 \times 1)$$

using Table 5.3:

$\mathrm{J}_0(0.9519) \simeq 0.7855$

thus,

$$0.3125 = 1.1172 \times \exp[-\left(0.9519\right)^2 t^*] \times 0.7855$$

$$\Rightarrow \exp[-\left(0.9519\right)^2 t^*] = 0.3561 \Rightarrow t^* = 1.1395 > 0.2$$

thus, the initial assumption is ok!

$$t^* = \alpha t / r_o^{\,2}; \alpha = k / \left(\rho c_p\right) = 9.23 \times 10^{-6}$$

$$\Rightarrow t = t^* r_o^{\,2} / \alpha = 197.53 \text{ s}$$

(b) $T(r=0, t=197.53\,\text{s}) = ?$

$$\theta^*(r^*=0, t^*) = \frac{T(r=0,t)-T_\infty}{T_i - T_\infty} = C_B \times \exp[-(A_B)^2 t^*]$$

$$\theta^*(r^*=0, t^*) = 1.1172 \times \exp[-(0.9519)^2 \times 1.1395] = 0.3978$$

$$\theta^*(r^*=0, t^*=1.1395) = \frac{T(r=0, t=197.53\,\text{s}) - 80}{400-80} = 0.3978 \Rightarrow T(r=0, t=197.53\,\text{s}) = 207.3°\text{C}$$

(c) $Q_{100s \le t \le 200s} = ?$

$$\left.\begin{aligned} t &= 200\,s, \quad t^* = \alpha t / r_o^2 = 1.1538 > 0.2 \\ t &= 100\,s, \quad t^* = \alpha t / r_o^2 = 0.5769 > 0.2 \end{aligned}\right\} \text{ one term app. ok!}$$

$$\frac{Q}{Q_o} = 1 - 2C_B \exp[-A_B^2 t^*]\left[\frac{J_1(A_B)}{A_B}\right]$$

$$\boxed{\begin{array}{l}\text{using Table 5.3:} \\ J_1(0.9519) \simeq 0.4236\end{array}}$$

$$Q_{100s \le t \le 200s} = Q_{0s \le t \le 200s} - Q_{0s \le t \le 100s}$$

$$\frac{Q_{100s \le t \le 200s}}{Q_o} = \frac{Q_{0s \le t \le 200s}}{Q_o} - \frac{Q_{0s \le t \le 100s}}{Q_o}$$

$$\frac{Q_{0s \le t \le 200s}}{Q_o} = 1 - 2 \times 1.1172 \times \exp[-(0.9519)^2 \times 1.1538]\left[\frac{J_1(0.9519)}{0.9519}\right] = 0.6508$$

$$\frac{Q_{0s \le t \le 100s}}{Q_o} = 1 - 2 \times 1.1172 \times \exp[-(0.9519)^2 \times 0.5769]\left[\frac{J_1(0.9519)}{0.9519}\right] = 0.4105$$

$$\frac{Q_{100s \le t \le 200s}}{Q_o} = 0.6508 - 0.4105 = 0.2403$$

$$Q_o = mc_p(T_i - T_\infty) = \rho \pi r_o^2 L c_p (T_i - T_\infty)$$

$$\Rightarrow Q_o = 6099414.7 \times L \quad \text{J}$$

$$\Rightarrow \frac{Q_{100s \le t \le 200s}}{L} = 0.2403 \times 6099414.7 = 1465689.4 \quad \text{J/m}$$