# ELEC 343
## Electromechanics

Spring 2019
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Class Webpage: http://courses.ece.ubc.ca/elec343

## Module 3 (Read Chap. 3):
### Part 1: Brushed DC Motors Fundamentals and Steady-State Analysis

Learning Objectives & Important Topics and Concepts

- Construction of DC machines, fundamentals
- Induced voltage and torque
- Equivalent circuit
- Basic types of dc machines, their characteristics
- Starting





# DC Machines


### Range of Sizes

- Miniature dc motors – mW range
- Fractional HP (1HP = 746W)
- Medium Size – 1 … 500 HP
- Large DC Machines – 500 HP and up

### Applications

- Very popular and easy to use in various applications
- Tools, robotics, toys, medical, automotive & other industries

### General Properties

- Very easy to control !
- Good torque-speed performance
- Brushes is a weak point of design
  - Limits the application
  - Limits the lifetime
  - Maintenance



Small Motors

Gear-head Motors

# Motor (Machine) Speed

Transmitting power through the mechanical shaft

Source
(Prime Mover)

Load

Commonly used units of speed

$$\omega \text{ - } \left[\mathrm{rad/sec}\right]$$

$$\mathrm{m} = \frac{\omega}{2\pi} \left[\mathrm{rev/sec}\right]$$

$$\mathrm{n} = 60\frac{\omega}{2\pi} = \frac{30}{\pi}\omega \left[\mathrm{rev/min}\right]$$

RPM = >Most commonly used





# Mechanical Speed and Torque

Electric Motor

Mechanical Load

Electromagnetic torque $T_e$

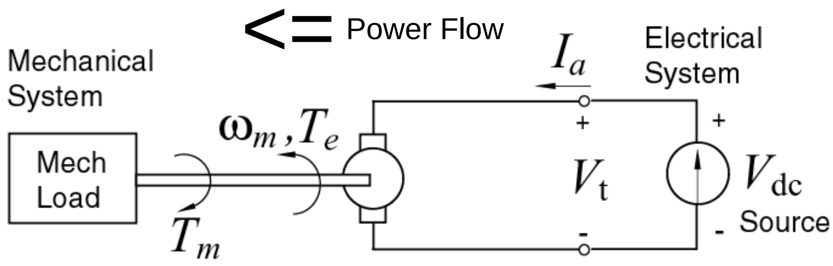
Useful mechanical torque on the shaft $T_m$

Torque balance $$T_e = J\frac{d\omega_r}{dt} + T_m + T_{mech\_loss}$$

Total moment of inertia (motor and load combined) $$J = J_{dc\_machine} + J_{mech\_load}, \quad [kg \cdot m^2]$$

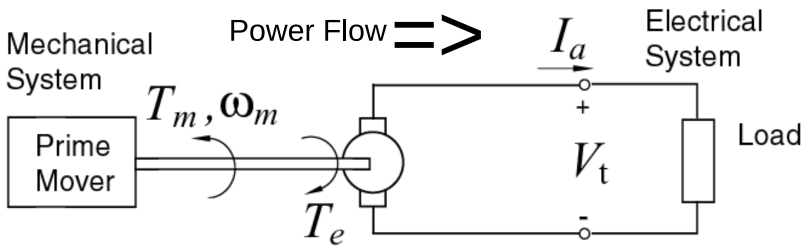
Mechanical loss (friction) $$T_{mech\_loss} = T_{fric} = D_m \omega_r$$

# DC Machine Motoring and Generating




Mechanical load torque is applied in the direction opposite to rotation

$$P_{out} = \omega_m T_m = P_m \qquad V_t I_a = P_{in}$$




Mechanical torque is applied in the direction of rotation

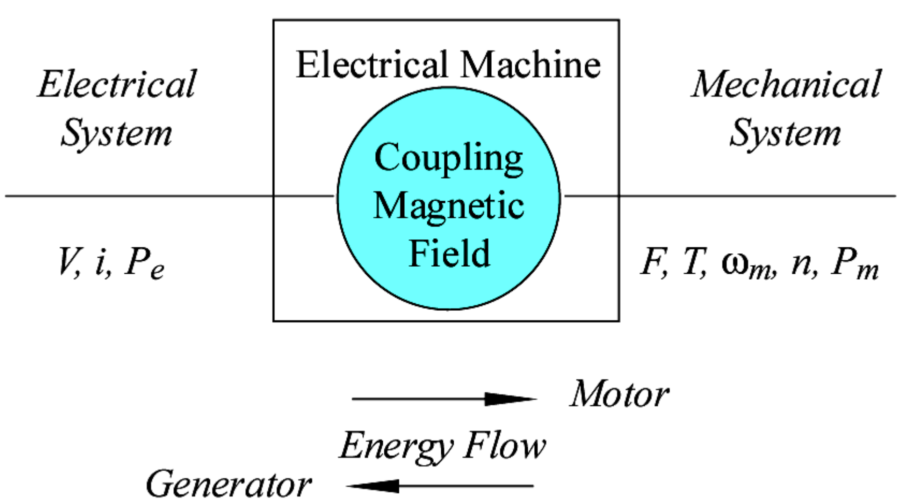
$$P_{in} = \omega_m T_m = P_m \qquad V_t I_a = P_{out}$$





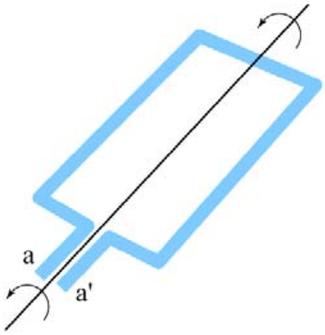
# Electromechanical Interaction

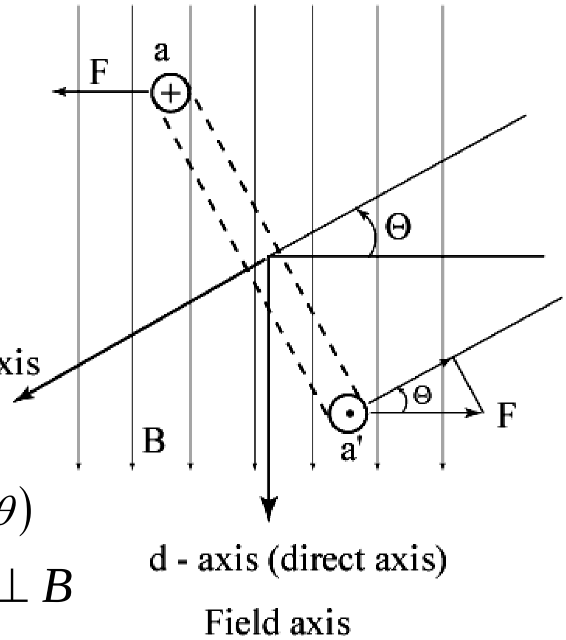
- Current-carrying conductor in magnetic field
  => mechanical force
- Conductor moves in magnetic field
  => voltage induced, emf



# Force & Torque

## Consider a conductor frame (single-coil)




Lorentz Force $F = lBi_a$

Torque $T_e = 2RlBi_a \cos(\theta)$

$$= ABi_a \cos(\theta) = \Phi_p i_a \cos(\theta)$$

NOTE: Maximum torque is when $a - axis \perp B$

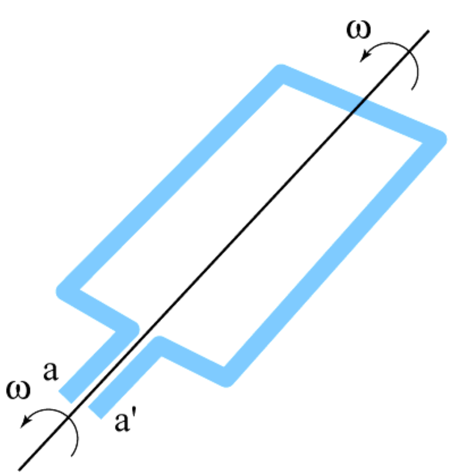
and $T_e \sim \Phi_p i_a$





# Induced Voltage

## Consider a conductor frame (single-coil)




Faraday's Law $e = \frac{d\Phi}{dt}$

$$\Phi = BA\sin(\theta) = B2Rl\sin(\theta)$$

Let us rotate the frame with speed $\omega$

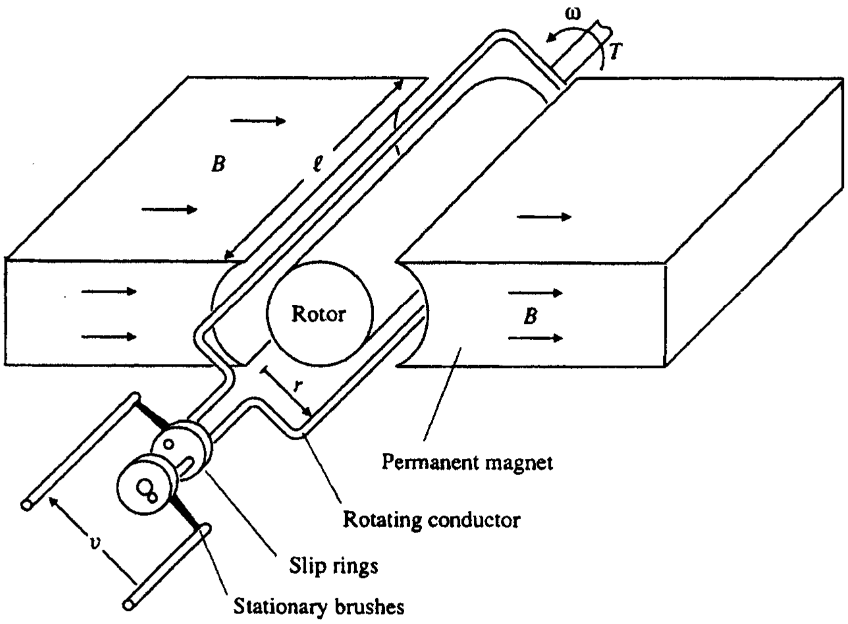
$$\theta = \omega t$$

$$\Phi = BA\sin(\omega t) = \Phi_p \sin(\omega t)$$

$$e = \frac{d\Phi}{dt} = \omega\Phi_p \cos(\omega t) \quad \Rightarrow \quad e \sim \omega\Phi_p$$

# Conductor Frame (1-turn coil)

Consider a conductor frame placed on a rotor between magnetic poles

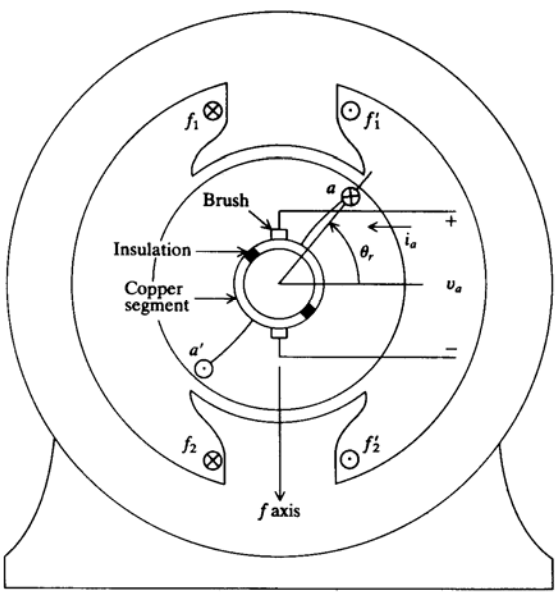







# Elementary DC Machine

Consider a two-pole case




Voltage Equations

$$v_f = r_f i_f + \frac{d\lambda_f}{dt}$$

$$v_a = r_a i_a + \frac{d\lambda_a}{dt}$$

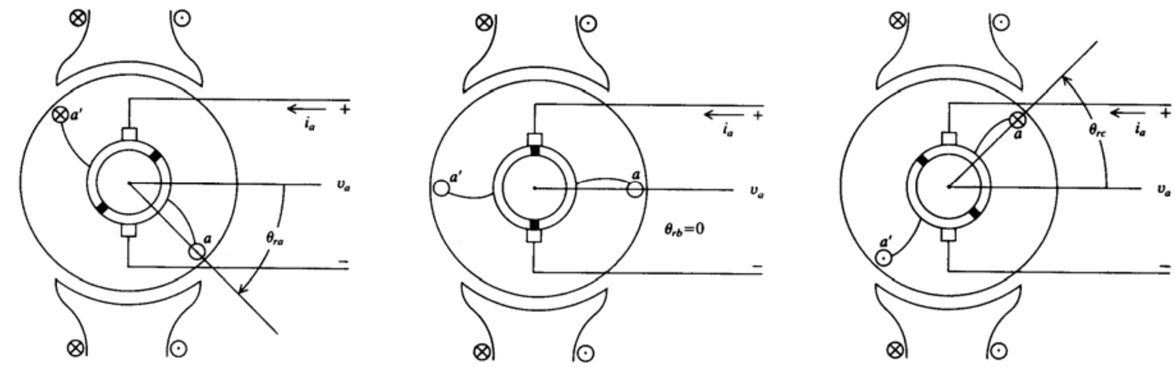
Flux linkages

$$\lambda_f = L_{ff} i_f + L_{fa} i_a$$

$$\lambda_a = L_a i_a + L_{af} i_f$$

Approximate the mutual inductance $L_{af} = L_{fa} = -L\cos(\theta_r)$

# Commutation of Elementary DC Machine



Induced voltage

$$e_a = \omega_r L i_f \sin(\theta_r)$$

$$v_a = r_a i_a + \frac{d\lambda_a}{dt}$$

Assume

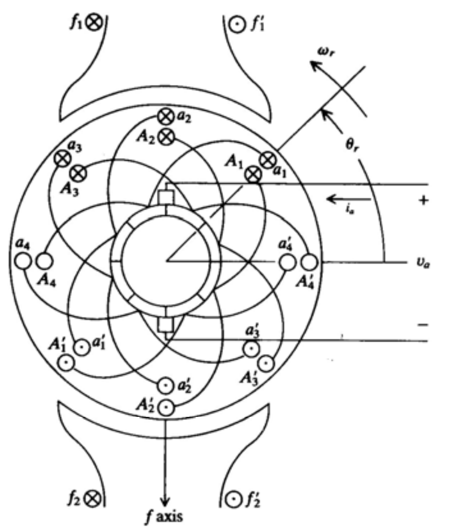
$i_a = 0$ and $\theta_r = \omega_r t$

$$\lambda_a = L_a i_a + L_{af} i_f$$

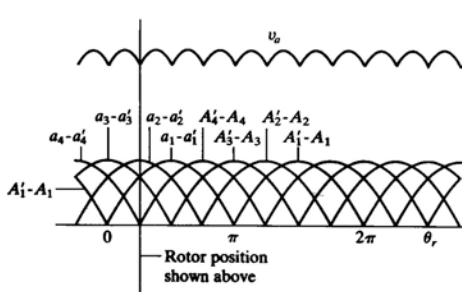




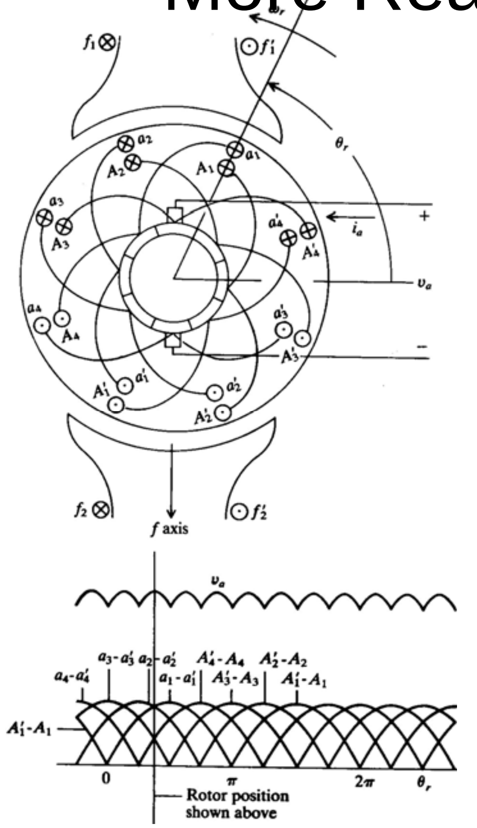
# More Realistic DC Machine




Resulted Winding Connection



# More Realistic DC Machine



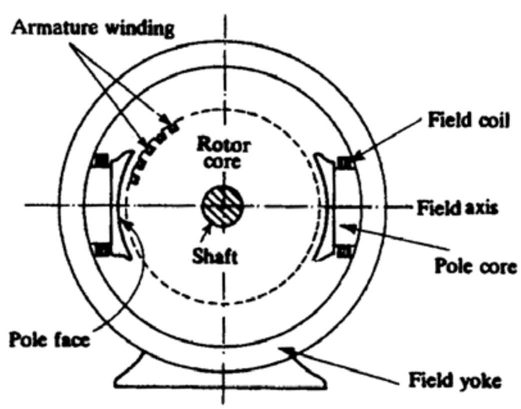

Resulted Winding Connection



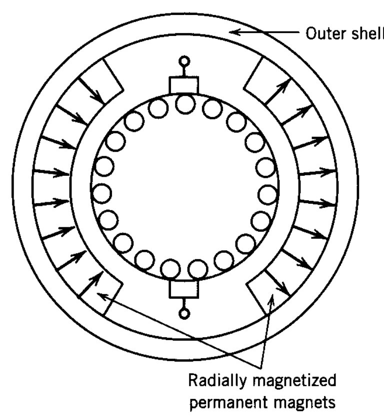


# DC Machine Construction

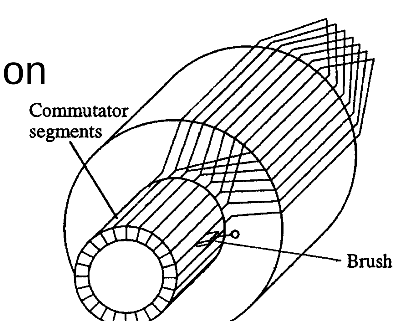
DC machine with field winding

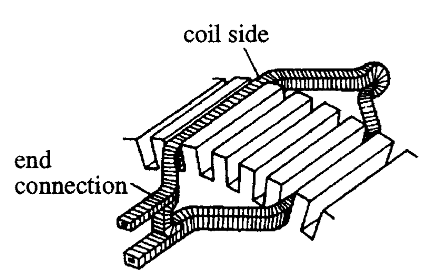



Permanent Magnet (PM) DC machine

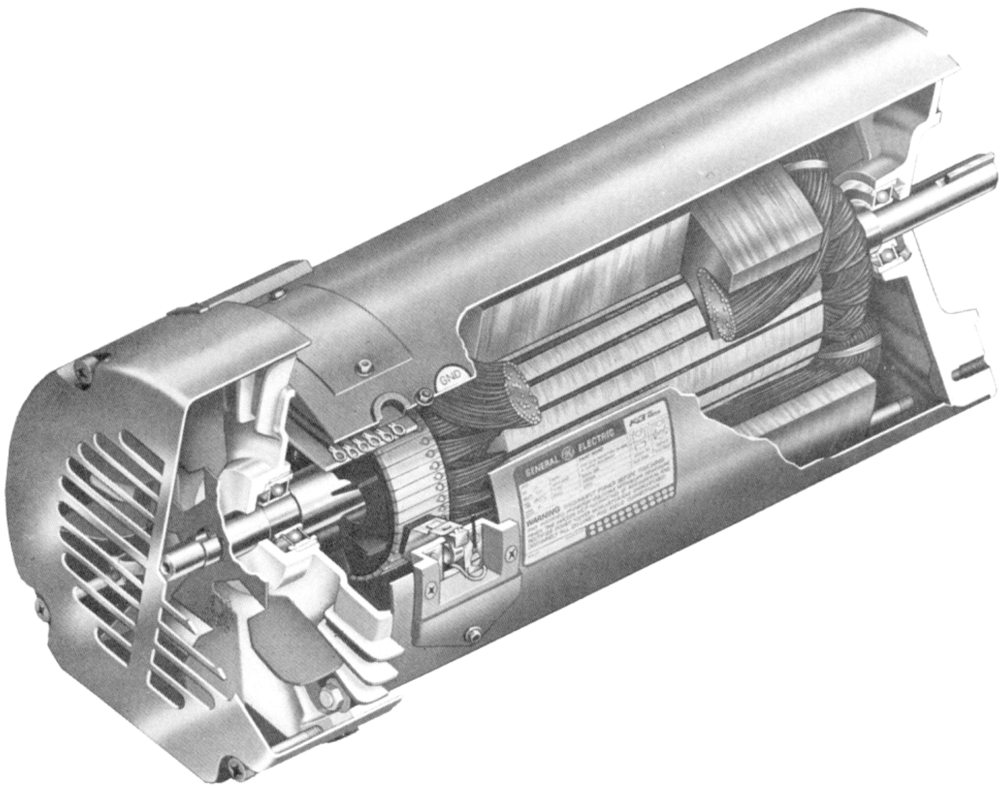



Rotor construction



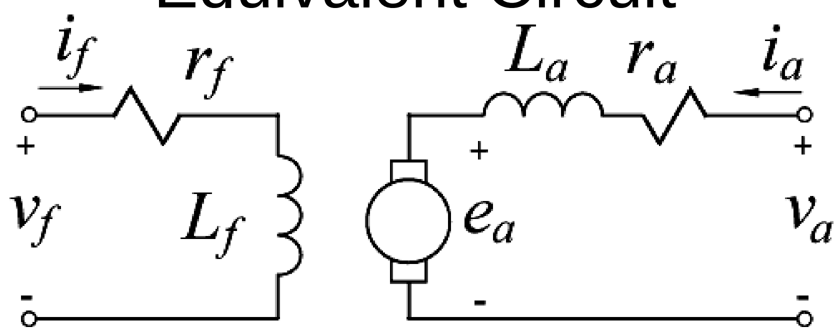
# Cutaway View of a Two-Pole DC Machine








# Equivalent Circuit




$v_f$ - is the field voltage

$i_f$ - is the field current

$r_f$ - field winding resistance

$L_f$ - field winding inductance

$v_a$ - applied terminal voltage

$i_a$ - is the armature current

$L_a$ - armature winding inductance

$r_a$ - armature winding + brush resistance

$e_a$ - Induced back emf (voltage)

# Dynamic Equivalent Circuit

Field Winding Equations

$$v_f = r_f i_f + \frac{d\lambda_f}{dt}$$

$$\lambda_f = N_f \Phi_p = L_f i_f$$

Armature Equations

$$v_a = r_a i_a + L_a \frac{di_a}{dt} + e_a$$

$$e_a = k_1 \omega_r \Phi_p$$

Electromagnetic Torque $T_e = k_2 \Phi_p i_a$

Power balance $T_e \omega_r = k_2 \Phi_p i_a \omega_r = e_a i_a = k_1 \omega_r \Phi_p i_a$





# Equivalent Circuit

Recall flux $\Phi_p = \frac{\lambda_f}{N_f} = \frac{L_f}{N_f} I_f$

Induced voltage

$$e_a = k\Phi_p \omega_r = k\frac{L_f}{N_f} i_f \omega_r$$

Torque

$$T_e = k\Phi_p i_a = k\frac{L_f}{N_f} i_f i_a$$

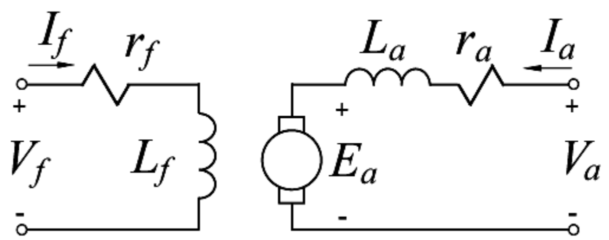
Define $L_{af} = k\frac{L_f}{N_f}$ - is the mutual inductance between field and rotating armature winding $L_{af} = \frac{N_a N_f}{\Re}$

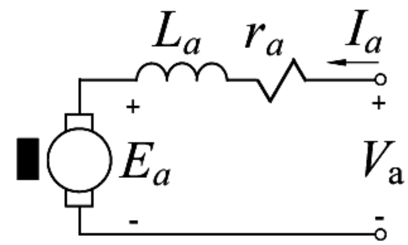
Re-define expression for back emf and torque

$$e_a = L_{af} i_f \omega_r = k_v \omega_r \qquad T_e = L_{af} i_f i_a = k_t i_a$$

Machine voltage/torque constant for PM machine $k_v = k_t = L_{af} i_f$

# Steady-State




Field Winding Equations

$$V_f = R_f I_f$$

$$\lambda_f = N_f \Phi_p = L_f I_f$$

Armature Equations

$$V_a = r_a I_a + E_a$$

$$E_a = \omega_r k_v = \omega_r L_{af} i_f$$

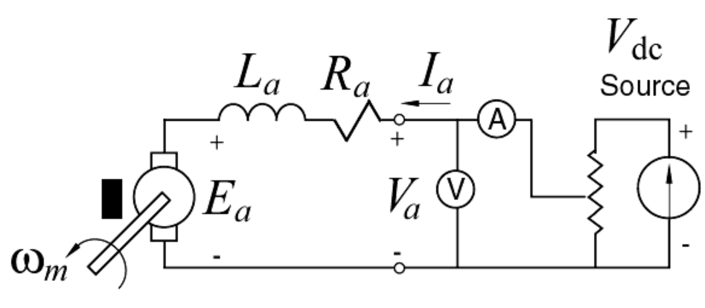
Electromagnetic torque $T_e = L_{af} I_f I_a = k_t I_a$

Torque balance $T_e = T_m + T_{mech\_loss}$





# Example 1: No Load Test PM Motor




Assume you know $R_a$

Measure $I_a, V_a, \omega_r$

$$V_a = R_a I_a + E_a$$

$$P_e = I_a E_a = \omega_r T_e$$

$$T_e = T_{friction} = \omega_r D_m$$

$$T_e = k_v I_a$$

$$E_a = k_v \omega_r$$

$$k_t = k_v$$ Torque / voltage machine constant

# Example 2: PM DC Motor

Consider a small PM DC Motor with the following parameters:
$V_{t\_rated} = 6V$, $I_{no\_load} = 0.15A$, $R_a = 7\Omega$, and torque constant $K_t = 0.014$ [Vs/rad].
Find no-load speed $n_{nl}$ [rpm], back emf $E_a$, and friction torque $T_{fric}$ at $V_t = 6V$

$$V_t = R_a \cdot I_a + E_a = R_a I_a + K_t \cdot \omega$$

$$\omega_{nl} = \frac{V_t - R_a I_a}{K_t} = \frac{6 - 7 \cdot 0.15}{0.014} = 353.57 \text{ rad/sec}$$

$$n_{nl} = \frac{30}{\pi} \omega_{nl} = 3{,}376.4 \text{ rpm}$$

Back emf $E_a = K_t \cdot \omega = 0.014 \cdot 353.57 = 4.95V$

Electromagnetic torque

$$T_e = K_t \cdot I_a = 0.014 \cdot 0.15 = 0.0021 \text{ N}\cdot\text{m}$$

$$T_e = \frac{P_e}{\omega} = \frac{I_a \cdot E_a}{\omega} = \frac{0.15 \cdot 4.95}{353.57} = 0.0021 \text{ N}\cdot\text{m}$$

$$T_{fric} = T_e = 0.0021 \text{ N}\cdot\text{m}$$

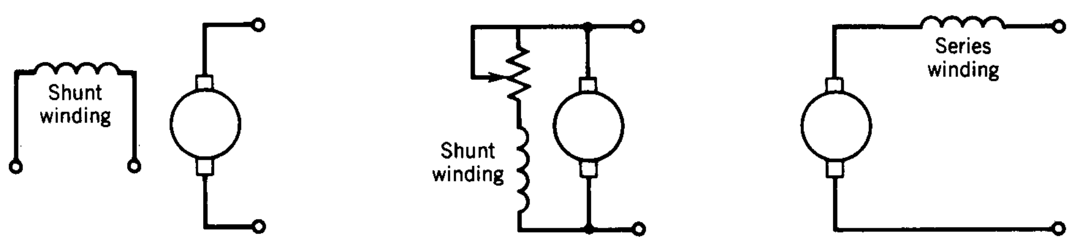




# Basic Types of DC Machines

## Basic DC Machines

- Shunt or Series field windings are possible




Separately Excited DC Machine

Shunt DC Machine

Series DC Machine

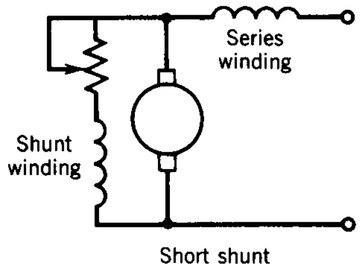
- Field winding is designed for up to rated armature voltage
- Field winding has large number of turns
- Field current is small compared to armature current

- Field winding is designed for up to rated armature current
- Field winding has small number of turns
- Field current is the same as the armature current

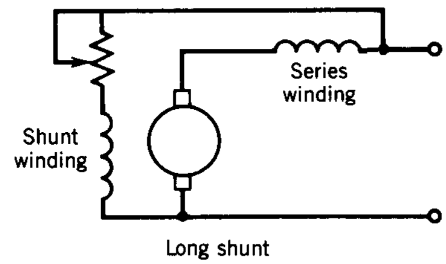
# Basic Types of DC Machines

## Compound DC Machines

- Both Shunt and Series field windings are present




Short-Shunt
Compound
DC Machine



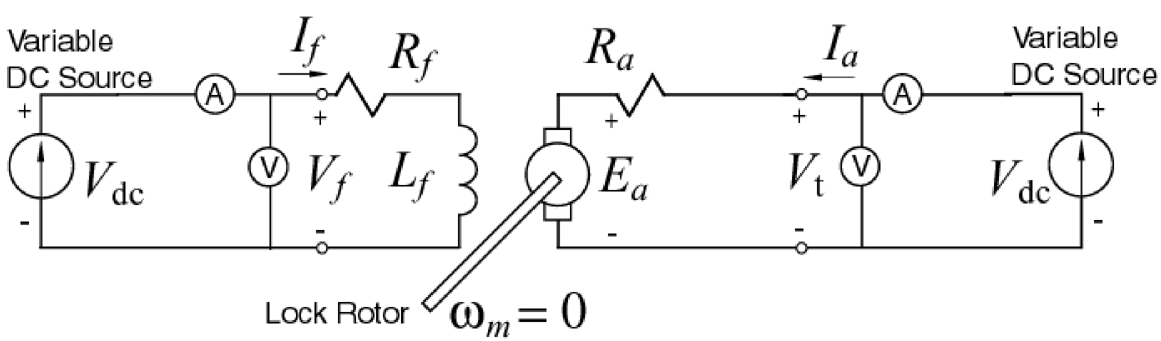

Long-Shunt
Compound
DC Machine





# Parameters of DC Machine

- Locked-Rotor Test (DC Measurements)



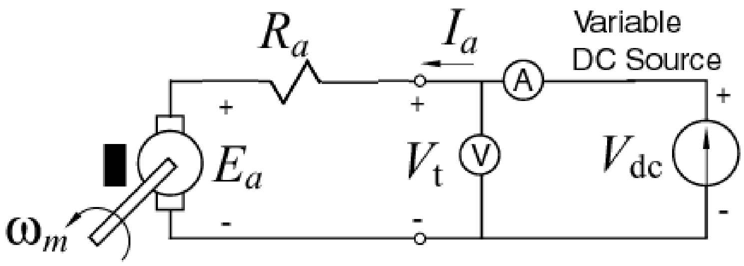

$$R_f = \frac{V_f}{I_f}$$

$$R_a = \frac{V_t}{I_a}$$

(armature + brush combined resistance at no speed!)

# Parameters of DC Machine

- No Load Test for PM Motors (Friction vs. Speed)




Assume you know $R_a$

Measure $I_a, V_t, \omega_m$

$$V_t = R_a I_a + E_a$$

$$T_e = K_a \Phi_p I_a = K_t I_a$$

$$P_e = I_a E_a = \omega_m T_e$$

$$E_a = K_a \Phi_p \omega_m = K_v \omega_m$$

$$T_e = T_{fric}(\omega_m) = \omega_m D_m$$

$K_t = K_v$ Torque / voltage machine constant

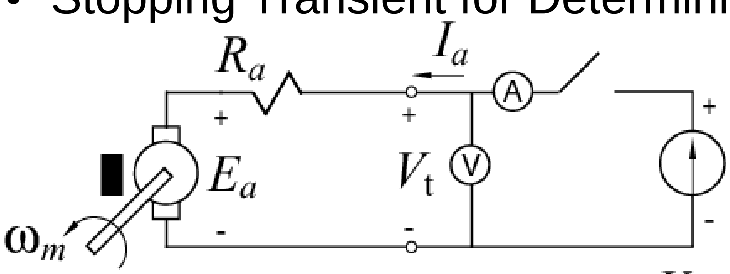
Measure Friction Torque-Speed Characteristic





# Parameters of DC Machine

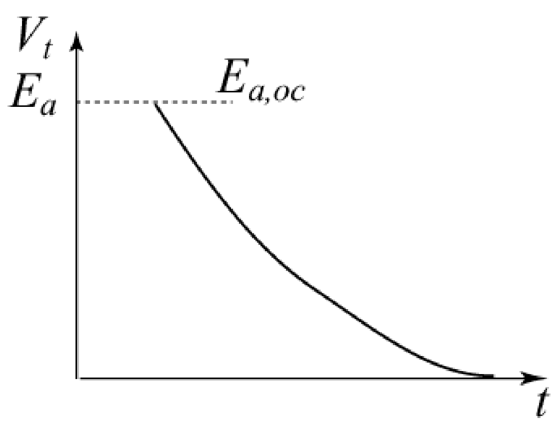
- Stopping Transient for Determining Inertia




Assume you know $T_{fric}(\omega_m)$

Measure & Record $I_a, V_t, \omega_m$

$$T_e = T_m + T_{fric}(\omega_m) + J\frac{d\omega_m}{dt}$$

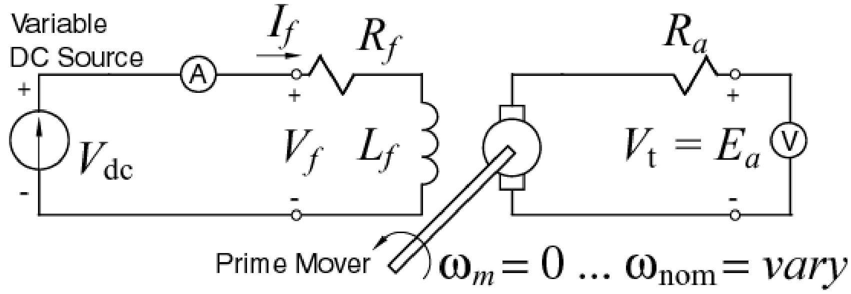



$$E_a = K_a \Phi_p \omega_m = K_v \omega_m$$

$$T_{fric}(\omega_m) = -J\frac{\Delta\omega_m}{\Delta t} = -J\frac{\Delta E_a}{K_v \Delta t}$$

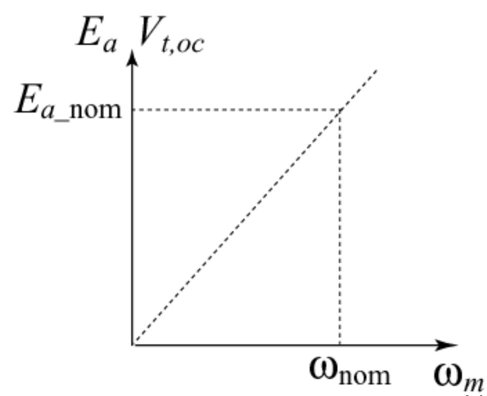
# Parameters of DC Machine

- Open-Circuit Test (Generated Voltage vs. Speed)




$$E_a = K_a \Phi_p \omega_m = K_a \frac{L_f}{N_f} I_f \omega_m$$

$$= L_{af} I_f \omega_m$$

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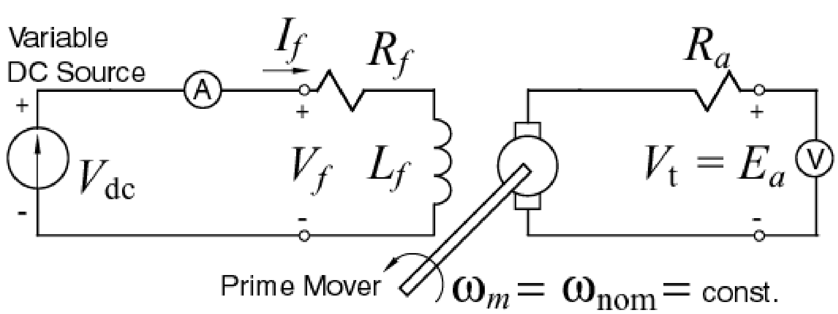
# Parameters of DC Machine

- Open-Circuit Test (Generated Voltage vs. Field Currents)

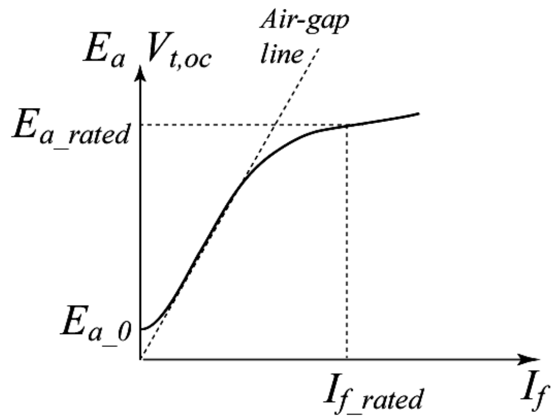



$$L_{af} = \frac{E_a}{I_f \omega_m}$$

$$\Phi_p = \frac{\lambda_f}{N_f} = \frac{L_f}{N_f} I_f$$

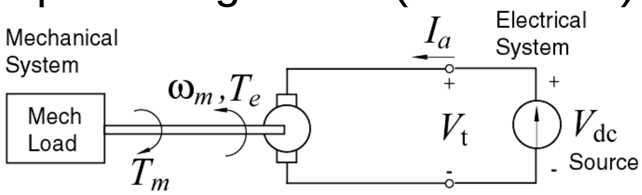
$$E_a = K_a \Phi_p \omega_m = K_a \frac{L_f}{N_f} I_f \omega_m$$

$$= L_{af} I_f \omega_m$$

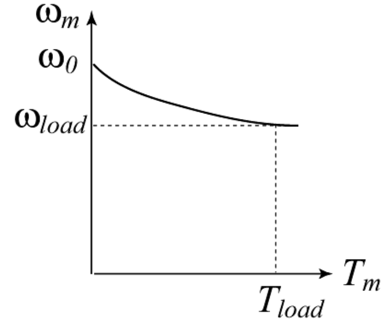


# DC Machines Characteristics:

## Speed Regulation (for Motors)

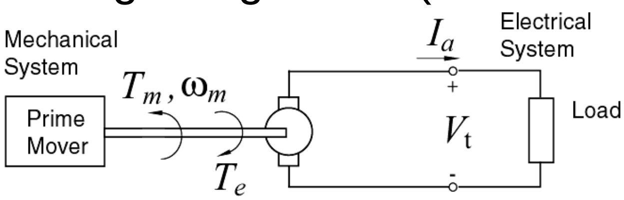



Speed-Torque Characteristic

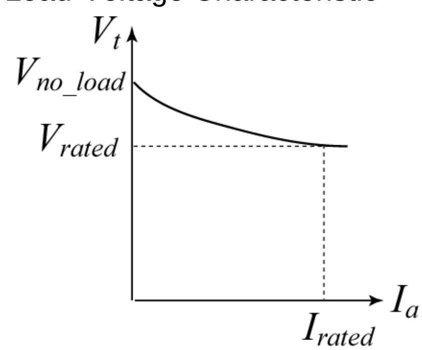



$$SR = \frac{\omega_{m,no\_load} - \omega_{m,load}}{\omega_{m,load}} 100\% = \frac{n_{no\_load} - n_{load}}{n_{load}} 100\%$$

## Voltage Regulation (for Generators)




Load-Voltage Characteristic



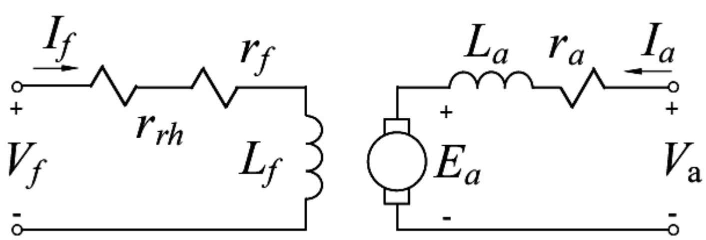

$$VR = \frac{V_{t,oc} - V_{t,load}}{V_{t,load}} 100\% = \frac{V_{no\_load} - V_{nom}}{V_{nom}} 100\%$$





# Separately Excited

Field Winding Equations




Armature Equations

$$R_f = r_f + r_{rh}$$

$$V_f = R_f I_f$$

$$\lambda_f = N_f \Phi_p = L_f I_f$$

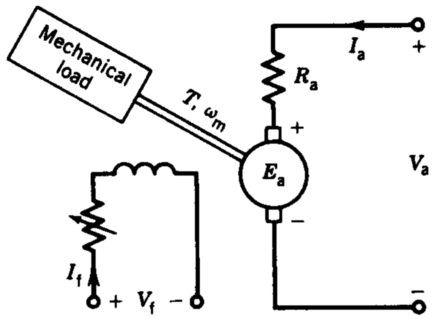
$$V_a = r_a I_a + E_a$$

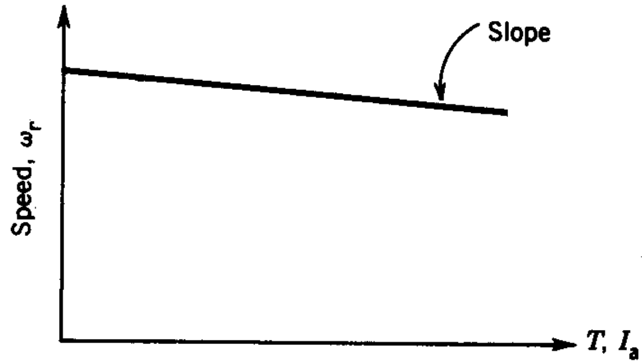
$$E_a = \omega_r L_{af} i_f$$

Electromagnetic torque $T_e = L_{af} I_f I_a$

Torque balance $T_e = T_m + T_{mech\_loss}$

# Separately-Excited DC Motor




Speed-Torque Characteristic

$$V_a = R_a I_a + E_a$$

$$E_a = L_{af} I_f \omega_r$$

$$T = L_{af} I_f I_a$$

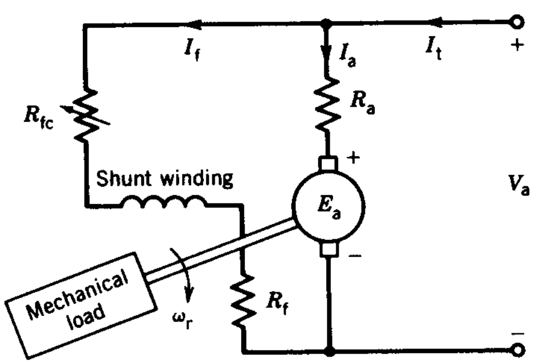
$$\omega_r = \frac{E_a}{L_{af} I_f} = \frac{V_a - I_a R_a}{L_{af} I_f}$$

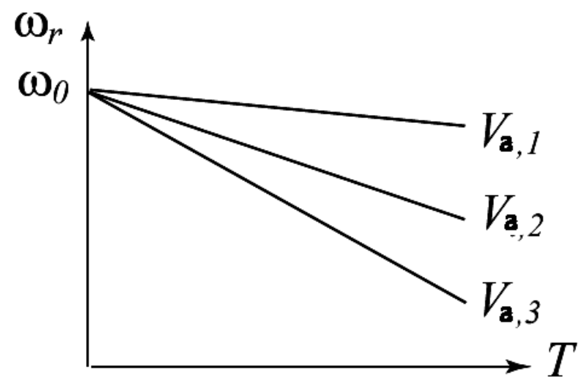
$$\omega_r = \frac{V_a}{L_{af} I_f} - \frac{R_a}{(L_{af} I_f)^2} T$$





# Shunt DC Motor



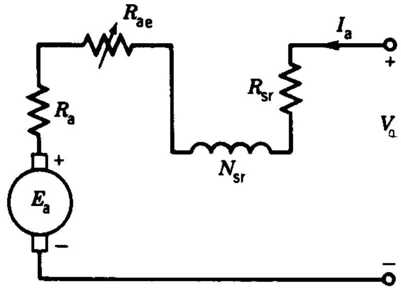

Speed-Torque Characteristic

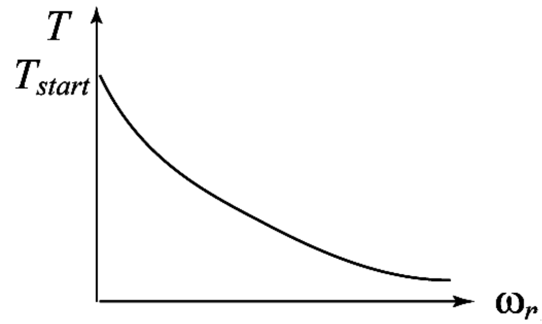
$$\omega_r = \frac{V_a}{L_{af} I_f} - \frac{R_a}{(L_{af} I_f)^2} T$$

$$L_{af} I_f = \frac{L_{af} V_a}{R_f + R_{fc}}$$

$$\omega_r = \frac{R_{f,total}}{L_{af}} - \frac{R_a R_{f,total}^2}{\left(L_{af} V_a\right)^2} T$$

# Series DC Motor




Torque-Speed Characteristic

$$V_t = I_a\left(R_a + R_{sr} + R_{ae}\right) + L_{af} I_a \omega_m$$

$$I_a = \frac{V_a}{R_a + R_{sr} + R_{ae} + L_{af}\omega_m}$$

$$T = \frac{L_{af} V_a^2}{\left(R_a + R_{sr} + R_{ae} + L_{af}\omega_m\right)^2}$$

$$T_e = L_{af} I_a^2$$





# Starting Considerations

Consider a 1.2kW, 120V, Shunt DC Motor with Ra = 2Ω, and Ia_max = 20A

Rated current $I_{a,rated} = \frac{1200}{120} = 10\text{A}$

Starting current $I_{a,start} = \frac{V_a - E_a}{R_a} = \frac{120 - 0}{2} = 60\text{A}$ Too high ! => Burn the motor !

1. Use additional starting resistor in series with armature
2. Use reduced voltage
3. Use Power Electronic Drive & Controller

# Part 2: Brushed DC Motor Drives and Dynamics

Important Topics & Concepts

- Basic types of dc motor drives
- DC to DC choppers (Chap. 3.8)
- AC to DC controlled rectifiers
- Dynamic (State) equations of the separately-excited motor
- Possible model & implementation/solution
- Dynamic (State) equations of the PM motor
- Possible model & implementation/solution





# **Speed/Torque Control of DC Motors**

Recall the Separately Excited DC Motor

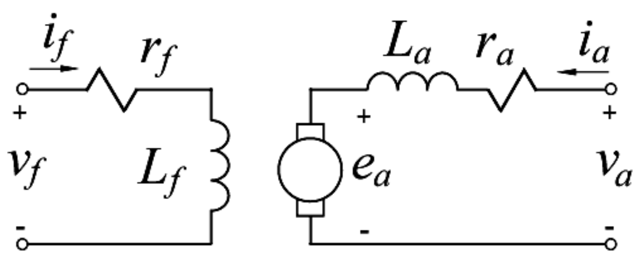
$$v_a = r_a i_a + k_v \omega_r$$

$$k_v = L_{af} i_f$$

$$\omega_r = \frac{e_a}{L_{af} i_f} = \frac{v_a - i_a R_a}{L_{af} i_f}$$

$$\omega_r = \frac{v_a - i_a r_a}{k_v}$$

$$T_e = k_v i_a$$




Control methods:

1. Varying the armature resistance
2. Varying the field
3. Varying the terminal voltage
4. Varying the armature current

# Solid-State Converters for DC Motors

DC-DC Converters (Choppers) – used with small & large motors

- Voltage Source (VS) (Pulse Width Modulation – PWM)
  - One-quadrant
  - Two-quadrant
  - Four-quadrant
- Current Source (CS) (Hysteresis & Delta Modulation)
  - One-quadrant
  - Two-quadrant
  - Four-quadrant

AC-DC Controlled Rectifiers – used with large motors

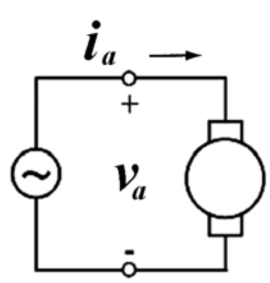
- Single-Phase
  - Half-wave
  - Full bridge (full wave)
- Three-Phase
  - Half-wave
  - Full bridge (full wave)

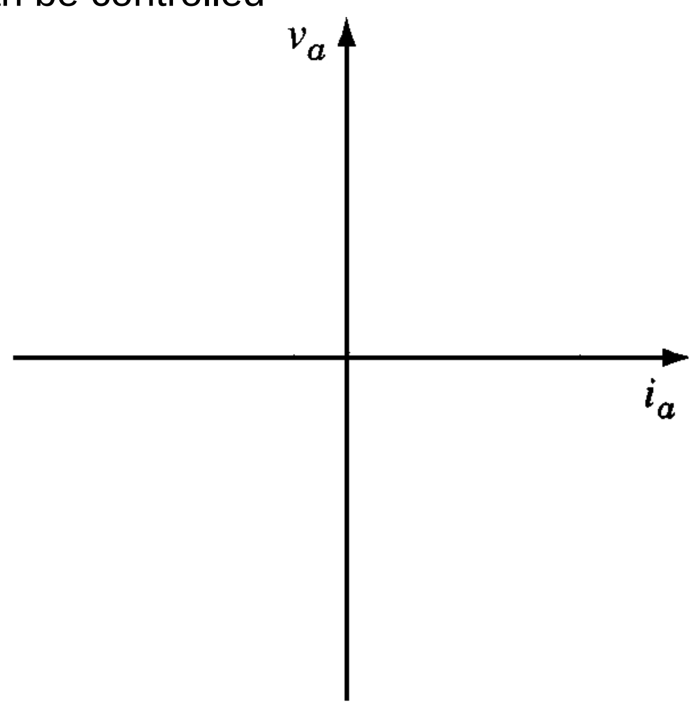




# DC-DC Converters (Choppers)

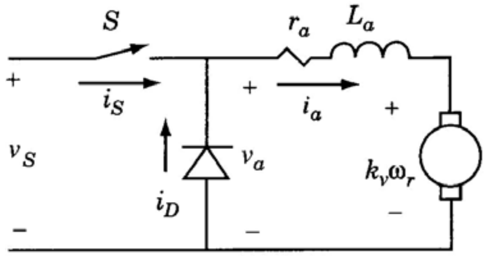
Assume a dc voltage source wherein the averaged output voltage can be controlled

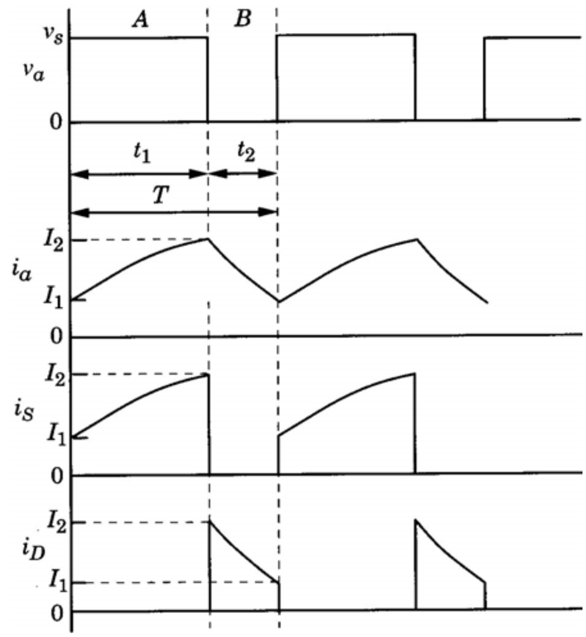


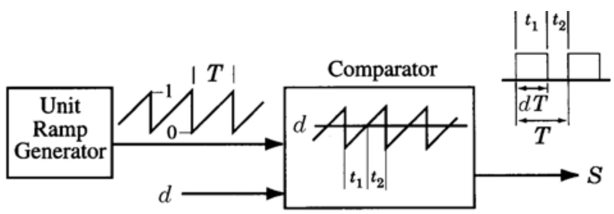

$$\overline{V_a} = \frac{1}{T}\int_0^T V_a(t)dt$$

# One-Quadrant VS DC-DC Converter




Pulse Width Modulation (**PWM**)




Switch can be realized using:

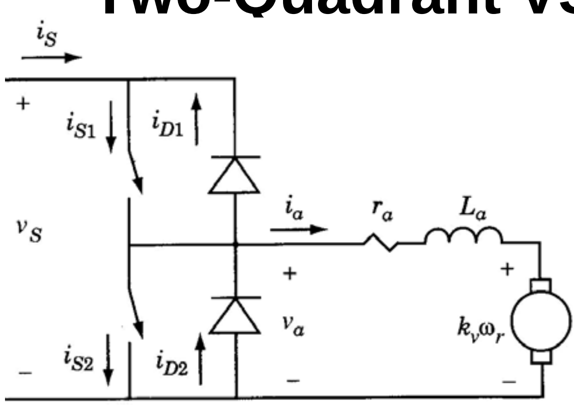
- Bipolar Junction Transistor (**BJT**)
- Insulated Gate Bipolar Transistor (**IGBT**)
- Metal Oxide Semiconductor Field Effect Transistor (**MOSFET**)

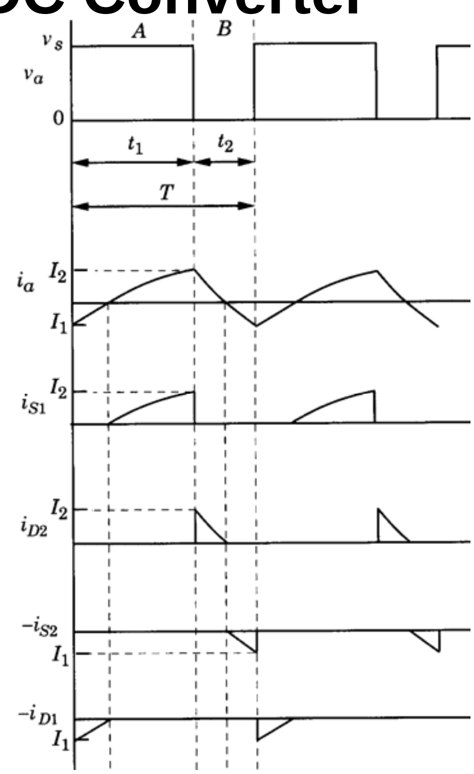
$$\bar{v}_a = \frac{1}{T}\int_0^T v_a(t)dt = \frac{t_1}{t_1 + t_2} V_s$$





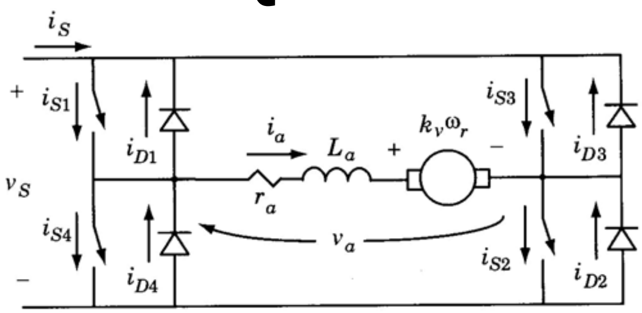
# Two-Quadrant VS DC-DC Converter

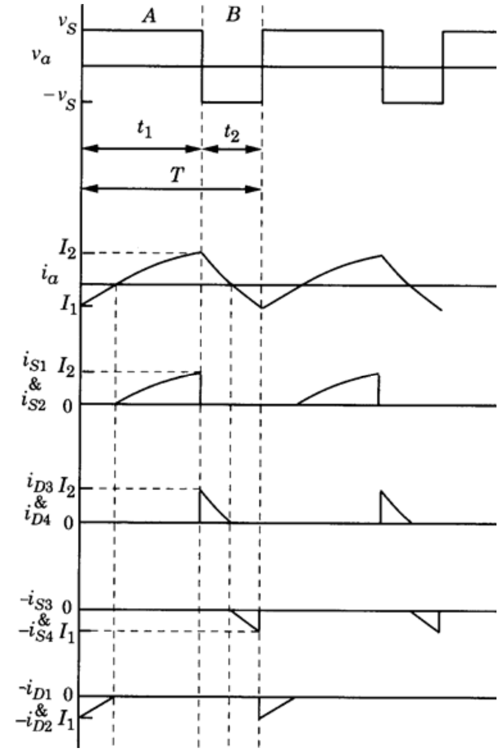



Pulse Width Modulation (**PWM**)

# Four-Quadrant VS DC-DC Converter




Pulse Width Modulation (**PWM**)

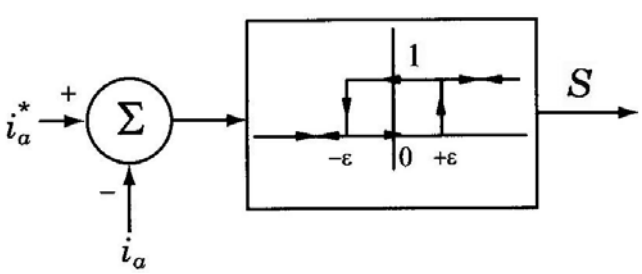
$$\bar{v}_a = \frac{1}{T}\int_0^T v_a(t)dt = dV_s$$

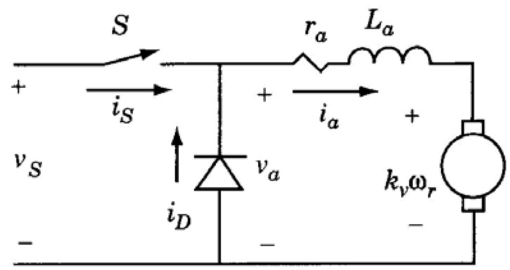


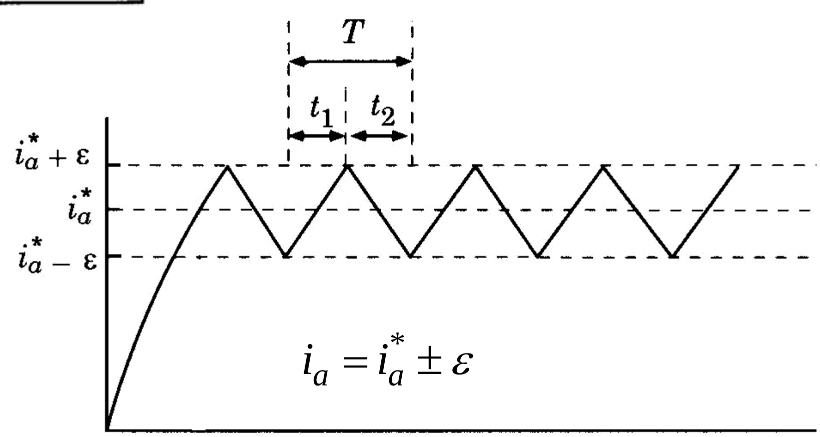


# Current Source DC-DC Converter

Hysteresis Modulation (**HM**)



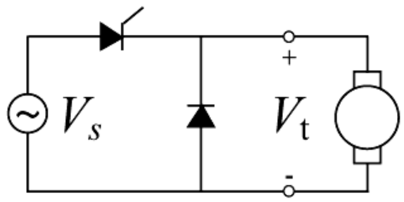

$$i_a = i_a^* \pm \varepsilon$$

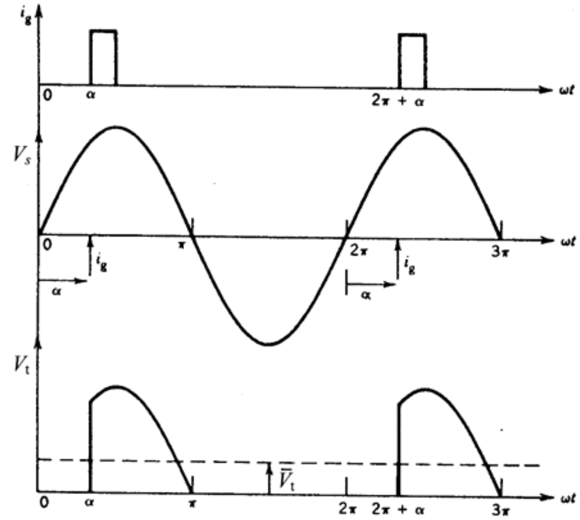
The same modulation can be applied to two- and four-quadrant choppers

# Thyristor Controlled Rectifiers

Control the Averaged Output Voltage

a) Single-phase, half-wave




$$V_s = \sqrt{2} V_{rms} \sin(\omega t)$$

$$\overline{V_t} = \frac{1}{T}\int_0^T V_t(t)dt = \frac{V_{rms}}{\sqrt{2}\pi}(1+\cos(\alpha))$$

$$0 \le \alpha \le \pi$$

$$0 \le \overline{V_t} \le \frac{\sqrt{2}}{\pi} V_{rms}$$

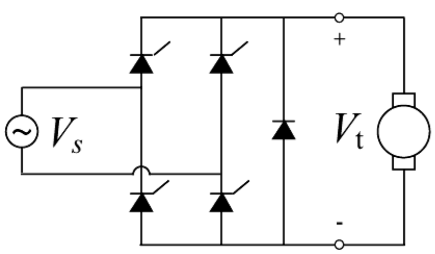


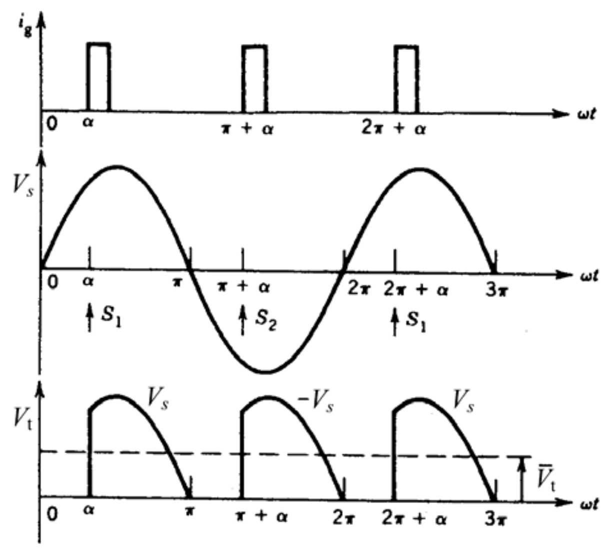


# Thyristor Controlled Rectifiers

Control the Averaged Output Voltage

b) Single-phase, full-wave




$$V_s = \sqrt{2} V_{rms} \sin(\omega t)$$

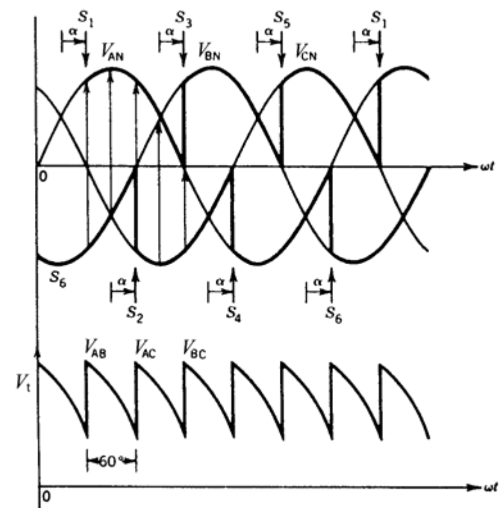
$$\overline{V_t} = \frac{1}{T}\int_0^T V_t(t)dt = \frac{\sqrt{2}V_{rms}}{\pi}(1+\cos(\alpha))$$

$$0 \le \alpha \le \pi$$

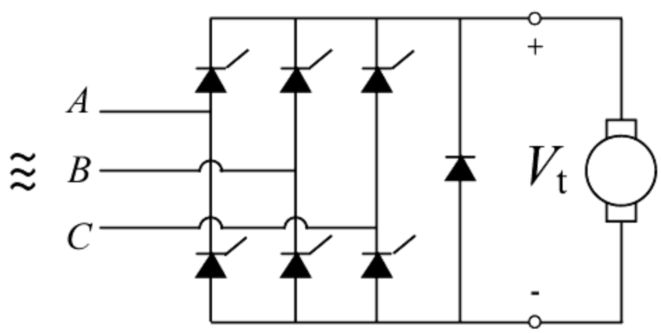
$$0 \le \overline{V_t} \le \frac{2\sqrt{2}}{\pi} V_{rms}$$

# Thyristor Controlled Rectifiers

Control the Averaged Output Voltage
d) Three-phase, full-wave




$$V_A = \sqrt{2}V_{rms}\sin(\omega t)$$
$$V_B = \sqrt{2}V_{rms}\sin(\omega t - 120^\circ)$$
$$V_C = \sqrt{2}V_{rms}\sin(\omega t + 120^\circ)$$



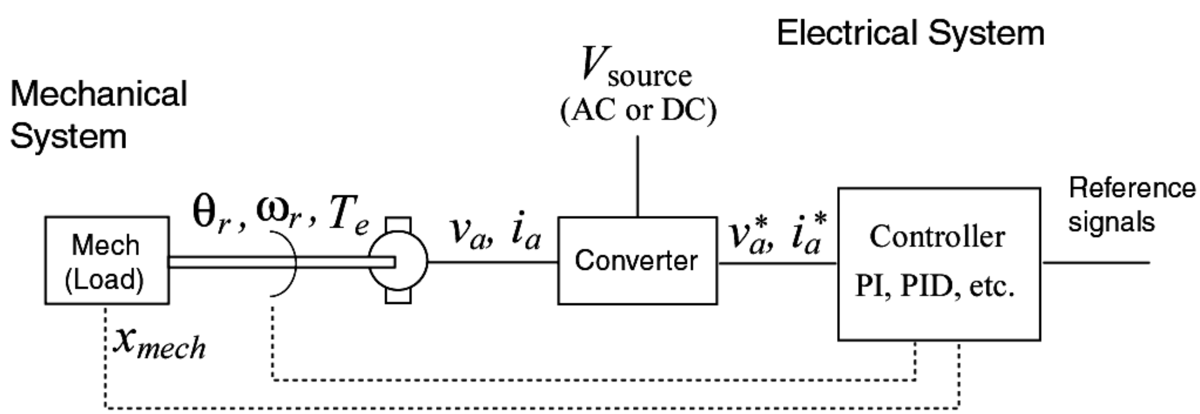

$$\overline{V_t} = \frac{1}{T}\int_0^T V_t(t)dt \qquad 0 \le \alpha \le \pi$$

$$0 \le \overline{V_t} \le \frac{3\sqrt{6}}{\pi}V_{rms}$$





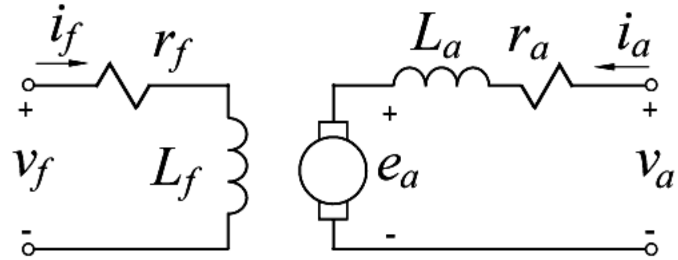
# Closed-Loop Electric Drive System




Dynamic models are needed for analyzing and designing complex electro-mechanical systems!

# Dynamic Modeling

Circuit is valid for transient analysis




Voltage Equations

$$v_a = r_a i_a + \frac{d\lambda_a}{dt} + e_a = r_a i_a + e_a + L_a \frac{di_a}{dt}$$

$$v_f = r_f i_f + \frac{d\lambda_f}{dt} = r_f i_f + L_f \frac{di_f}{dt}$$

Speed Equation

$$T_e = J\frac{d\omega_r}{dt} + T_m + D_m\omega_r$$

Coupling Terms

$$e_a = \omega_r L_{af} i_f$$

$$T_e = L_{af} i_f i_a$$





# Re-Write the Equations to Express Derivatives

State Equations

$$\frac{di_a}{dt} = -\frac{r_a}{L_a} i_a - \frac{1}{L_a} e_a + \frac{1}{L_a} v_a$$

$$\frac{di_f}{dt} = -\frac{r_f}{L_f} i_f + \frac{1}{L_f} v_f$$

$$\frac{d\omega_r}{dt} = -\frac{D_m}{J}\omega_r + \frac{1}{J}\left(T_e - T_m\right)$$

Coupling Terms

$$e_a = \omega_r L_{af} i_f$$

$$T_e = L_{af} i_f i_a$$

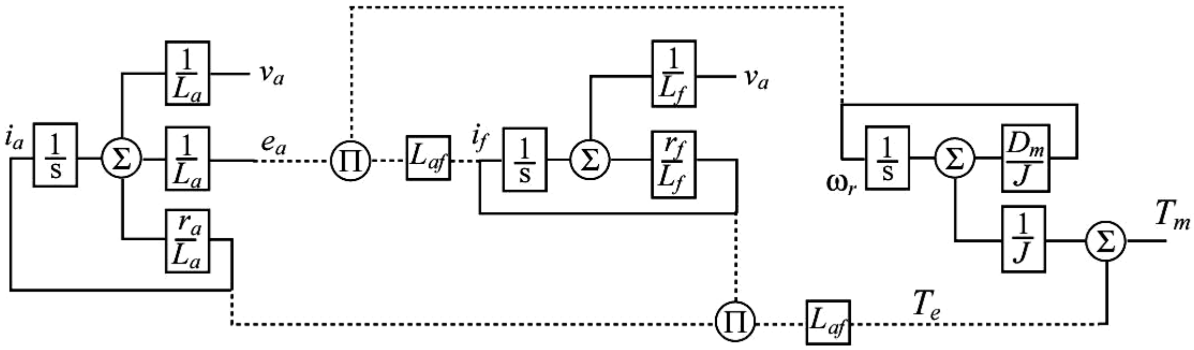
Are these equations coupled or decoupled ?

Are these equations linear or non-linear ?

How do we solve these equations ?

# Implementation of State Equations

$$\frac{di_a}{dt} = -\frac{r_a}{L_a} i_a - \frac{1}{L_a} e_a + \frac{1}{L_a} v_a \qquad \frac{d\omega_r}{dt} = -\frac{D_m}{J}\omega_r + \frac{1}{J}\left(T_e - T_m\right)$$




$$e_a = \omega_r L_{af} i_f \qquad \frac{di_f}{dt} = -\frac{r_f}{L_f} i_f + \frac{1}{L_f} v_f \qquad T_e = L_{af} i_f i_a$$

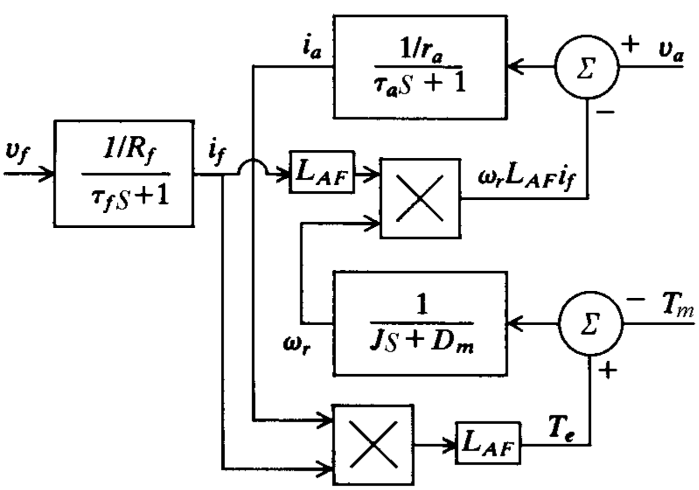
Try to implement it in Simulink ?

Is this a unique implementation ?





# Implementation of State Equations




Re-write the equations

$$v_f = r_f\left(1 + \tau_f s\right) i_f$$

$$v_a = r_a\left(1 + \tau_a s\right) i_a + e_a$$

$$e_a = \omega_r L_{af} i_f$$

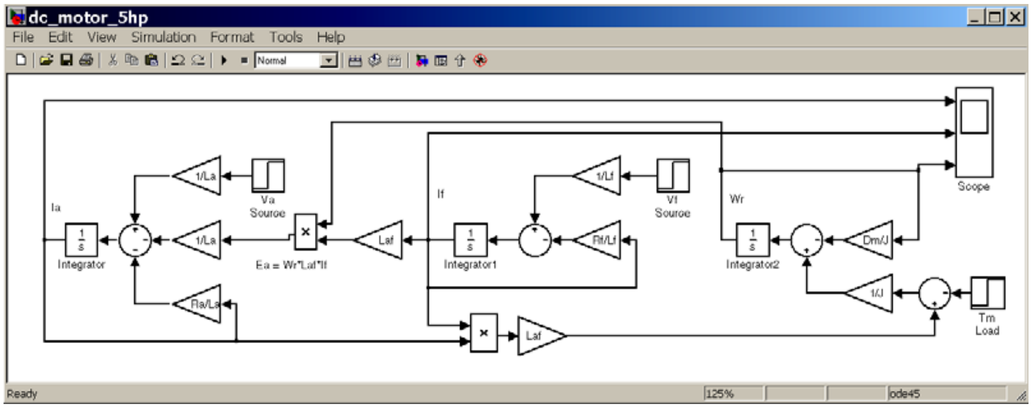
$$T_e - T_m = \left(D_m + Js\right)\omega_r$$

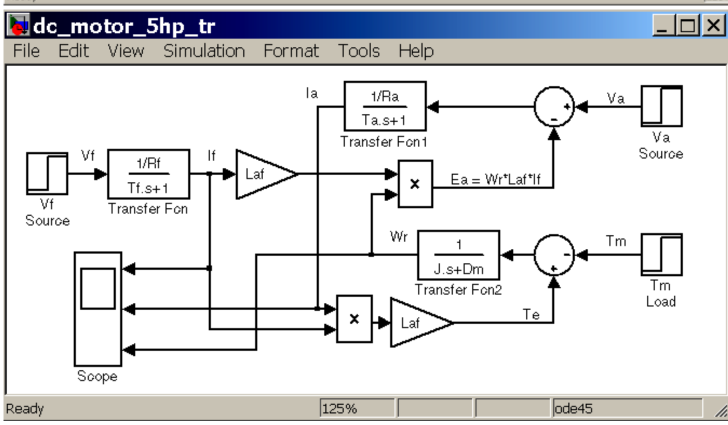
$$T_e = L_{af} i_f i_a$$

Define time constants

$$\tau_a = \frac{L_a}{r_a} \qquad \tau_f = \frac{L_f}{r_f}$$

# 5-HP Shunt DC Motor Simulink Model




Try to do it yourself !





# Dynamic Response of a 5-HP DC Motor

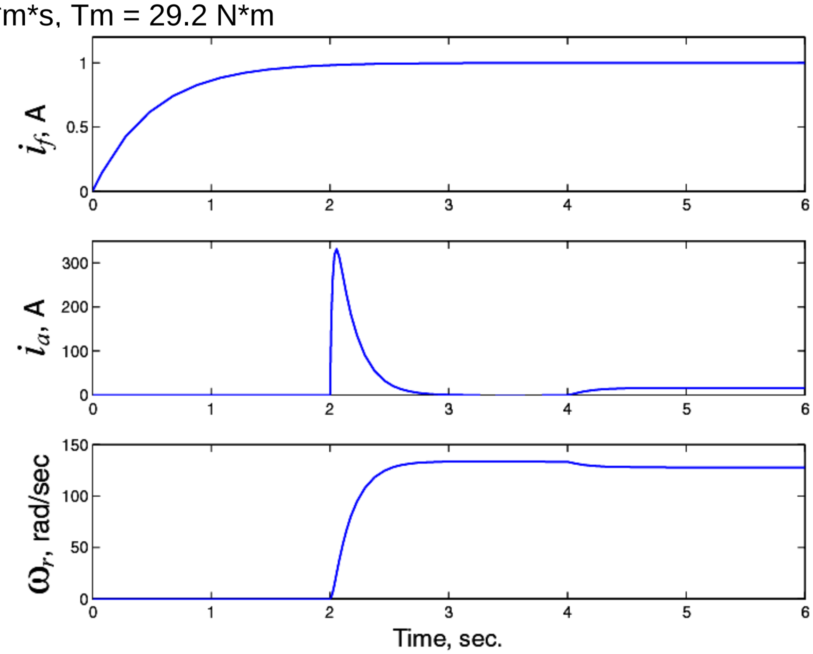
Motor parameters:

Va = Vf = 240V, Rf = 240Ω, Lf = 120H, Ra = 0.6Ω, La = 0.012H, Laf = 1.8H,
J = 1kg*m^2, Dm = 1e-4 N*m*s, Tm = 29.2 N*m

In this computer study:

Zero initial conditions assumed

Step-1: field winding is energized
Step-2: armature is energized
Step-3: load torque is applied



# PM DC Motor Dynamics



State Equations

$$\frac{di_a}{dt} = -\frac{r_a}{L_a} i_a - \frac{1}{L_a} e_a + \frac{1}{L_a} v_a$$

$$\frac{d\omega_r}{dt} = -\frac{D_m}{J} \omega_r + \frac{1}{J}\left(T_e - T_m\right)$$

Coupling Terms

$$e_a = k_v \omega_r \qquad T_e = k_v i_a$$

Are these equations coupled or decoupled ?

Are these equations linear or non-linear ?

How do we solve these equations ?

Standard State-Space Form

$$\frac{d}{dt}\begin{bmatrix} i_a \\ \omega_r \end{bmatrix} = \begin{bmatrix} -\frac{r_a}{L_a} & -\frac{k_v}{L_a} \\ \frac{k_v}{J} & -\frac{D_m}{J} \end{bmatrix} \begin{bmatrix} i_a \\ \omega_r \end{bmatrix} + \begin{bmatrix} \frac{1}{L_a} & \\ & -\frac{1}{J} \end{bmatrix} \begin{bmatrix} v_a \\ T_m \end{bmatrix}$$

$$\frac{d\mathbf{x}}{dt} = \mathbf{Ax} + \mathbf{Bu}$$



# PM DC Motor Implementations



State Equations

$$\frac{di_a}{dt} = -\frac{r_a}{L_a} i_a - \frac{1}{L_a} e_a + \frac{1}{L_a} v_a$$

$$\frac{d\omega_r}{dt} = -\frac{D_m}{J} \omega_r + \frac{1}{J}\left(T_e - T_m\right)$$

Coupling Terms

$$e_a = k_v \omega_r \qquad T_e = k_v i_a$$

Are these equations coupled or decoupled ?

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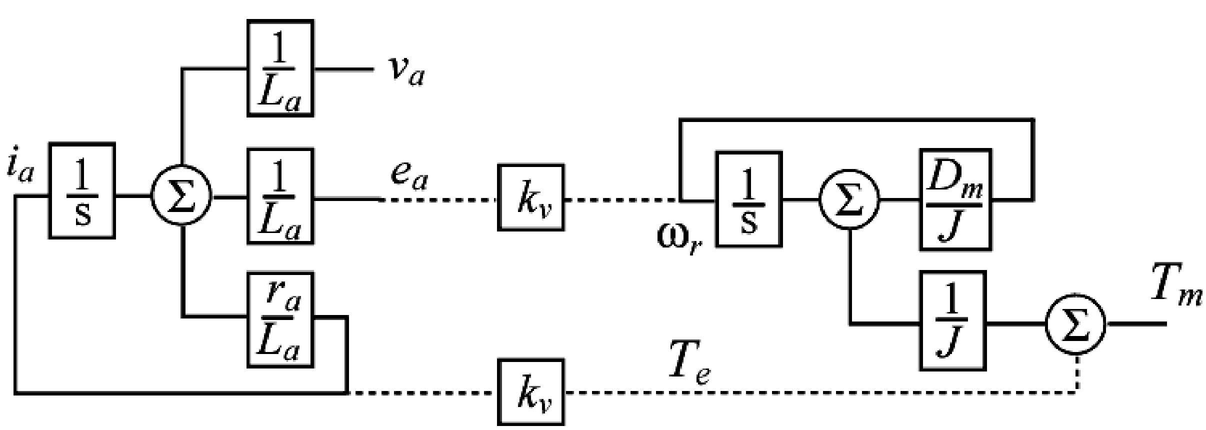
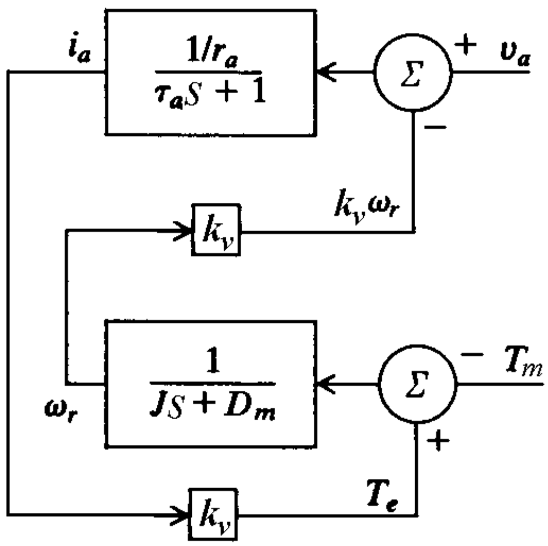
How do we solve these equations ?



# Implementation of State Equations




Re-write the equations

$$v_a = r_a(1+\tau_a s)i_a + e_a$$

$$e_a = k_v\omega_r$$

$$T_e - T_m = (D_m + Js)\omega_r$$

$$T_e = k_v i_a$$

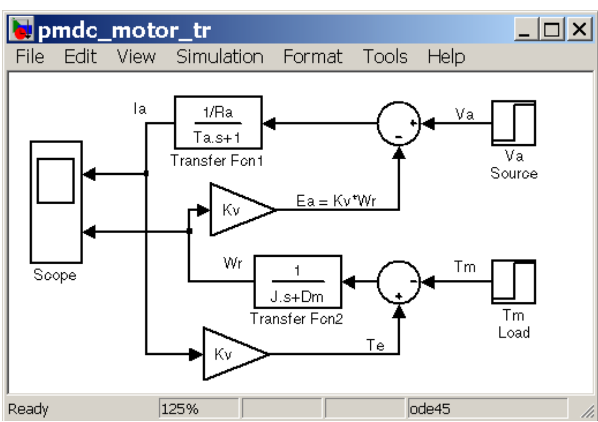
Define time constants

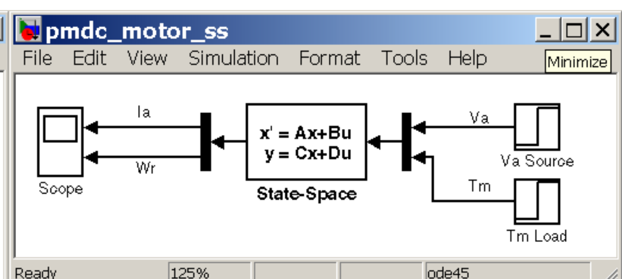
$$\tau_a = \frac{L_a}{r_a} \qquad \tau_f = \frac{L_f}{r_f}$$





# Simulink Model of a PM DC Motor




$$\frac{d}{dt}\begin{bmatrix} i_a \\ \omega_r \end{bmatrix} = \begin{bmatrix} -\frac{r_a}{L_a} & -\frac{k_v}{L_a} \\ \frac{k_v}{J} & -\frac{D_m}{J} \end{bmatrix}\begin{bmatrix} i_a \\ \omega_r \end{bmatrix} + \begin{bmatrix} \frac{1}{L_a} & \\ & -\frac{1}{J} \end{bmatrix}\begin{bmatrix} v_a \\ T_m \end{bmatrix}$$

$$i_a = \frac{1}{r_a(1+\tau_a s)}(v_a - e_a)$$

$$e_a = k_v\omega_r$$

$$\omega_r = \frac{1}{D_m + Js}(T_e - T_m)$$

$$T_e = k_v i_a$$

$$\begin{bmatrix} i_a \\ \omega_r \end{bmatrix} = \begin{bmatrix} 1 & \\ & 1 \end{bmatrix}\begin{bmatrix} i_a \\ \omega_r \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} v_a \\ T_m \end{bmatrix}$$

# Dynamic Response of a 6V PM DC Motor

Motor parameters:
Va = 6V, Ra = 7Ω, La = 120mH, Kv = 0.0141 N*A/m, J = 1.06e-6 kg*m^2,
Dm = 6.01e-6 N*m*s, Tm = 3.53e-3 N*m

In this computer study:

Zero initial conditions assumed

Step-1: voltage is applied

Step-2: load torque is applied