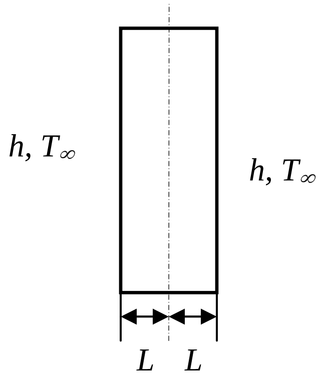
Solutions - Problem Set # 6

Problem 1:



Given: ρ_{solid} = 7817 kg/m^3; c_{solid} = 460 J/kg-°C; T_i = 500°C; T_∞= 40°C; h = 150 W/m^2-°C; k = 19 W/m-°C; $2L$ = 0.03 m

Assumptions: Radiation negligible; 1-D, unsteady cooling with constant properties.

$$Bi = \frac{hL_c}{k}; \quad L_c = \frac{2LA}{2A} = L;$$

$$Bi = \frac{150 \times 0.015}{19} = 0.118 > 0.1 \Rightarrow LPA \text{ not valid}$$

$$t^* = \alpha t / L^2$$

At this stage we do not know t to obtain t^*

Thus, let assume $t^* > 0.2$ and will check this assumption later

$\Rightarrow$ Heisler Charts are adequate

(1-term approximation of infinite series solution is adequate)

In this problem, I will use Heisler Charts:

Suggestion: Do this using 1-term approximation of infinite series solution and compare you results.

a) When $T_{(x=0,t)} = 100°\text{C}, \quad t = ?$

$$\left.\begin{aligned} \frac{T_{(x=0,t)} - T_\infty}{T_i - T_\infty} &= \frac{100-40}{500-40} = 0.13 \\ \frac{1}{Bi_M} &= \frac{k}{hL} = \frac{1}{0.118} \simeq 8.47 \end{aligned}\right\} \xrightarrow[\text{using the Heisler Chart for plane Wall}]{} F_o = \alpha t / L^2 = t^* = 18.5 > 0.2$$

$$\alpha = k / (\rho c_p) = 19/(7817 \times 460) = 5.284 \times 10^{-6} \quad \text{m}^2\text{/s}$$

Thus,

$$t = \frac{18.5 \times (0.015)^2}{5.284 \times 10^{-6}} = 787.76 \quad \text{s}$$

b) When $T_{(x=L,t)} = 100°\text{C}, \quad t = ?$

$$\left.\begin{aligned} & x/L = 1 \\ & \frac{1}{Bi_M} = \frac{k}{hL} = \frac{1}{0.118} \simeq 8.47 \end{aligned}\right\} \xrightarrow[\substack{\text{using} \\ \text{the Heisler} \\ \text{Chart for plane} \\ \text{Wall}}]{} \frac{T_{(x=L,t)} - T_\infty}{T_{(x=0,t)} - T_\infty} \simeq 0.945$$

$$T_{(x=0,t)} - T_\infty = \frac{T_{(x=L,t)} - T_\infty}{0.945} = \frac{100-40}{0.945} = 63.492°\text{C}$$

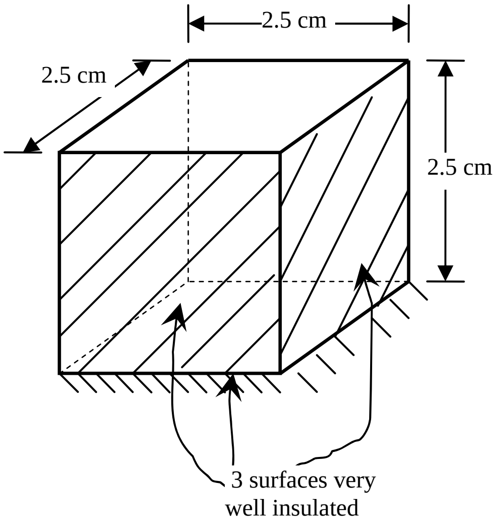
$$\Rightarrow \frac{T_{(x=0,t)} - T_\infty}{T_i - T_\infty} = \frac{63.492}{500\text{-}40} \simeq 0.138$$

$$\left.\begin{aligned} & \frac{T_{(x=0,t)} - T_\infty}{T_i - T_\infty} = 0.138 \\ & \frac{1}{Bi_M} = \frac{k}{hL} = \frac{1}{0.118} \simeq 8.47 \end{aligned}\right\} \xrightarrow[\substack{\text{using} \\ \text{the Heisler} \\ \text{Chart for plane} \\ \text{Wall}}]{} F_o = \alpha t / L^2 = t^* \simeq 18.0 > 0.2$$

Thus,

$$t = \frac{18.0 \times (0.015)^2}{5.284 \times 10^{-6}} = 766.46 \quad \text{s}$$

Problem 2:



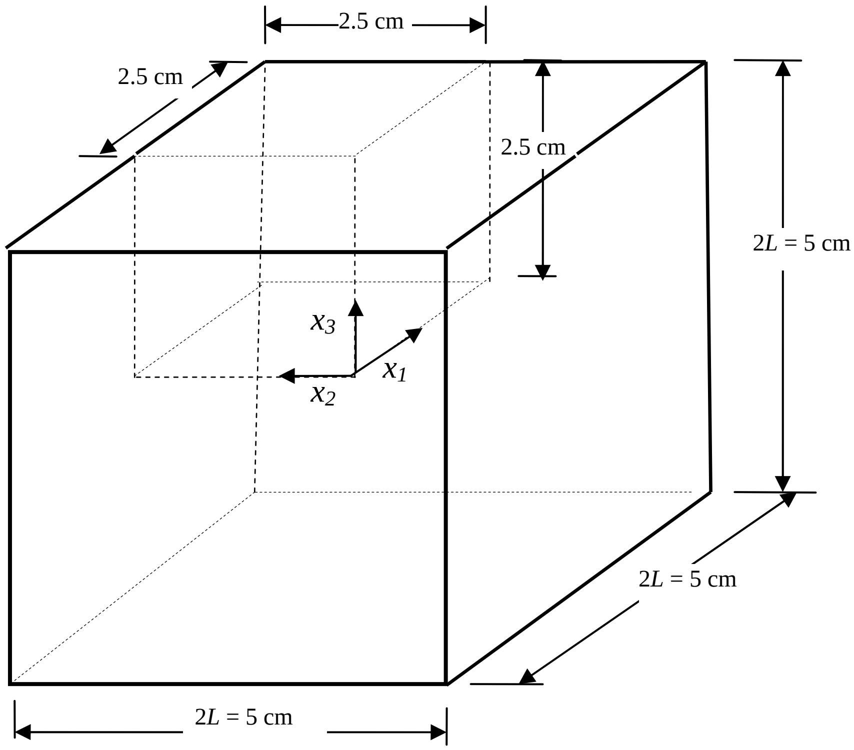
Given: k = 25 W/m-°C; ρ = 8000 kg/m^3; c = 1000 J/kg-°C; T_i = 520°C;

T_∞ = 20°C; h = 1000 W/m^2-°C;

$\alpha = k/(\rho\, c)$=3.125×10^{-6} m^2/s

Assumptions: Unsteady 3-D heat conduction; constant properties

The essentially adiabatic surfaces may be considered as symmetry surfaces. That would allow analysis of a symmetrically cooled cube of dimensions 0.05 m × 0.05 m × 0.05 m.



$$Bi = \frac{hL_c}{k};\quad L_c = \frac{\text{volume}}{\begin{array}{l}\text{Surf. total}\\ \text{exposed conv.}\end{array}} = \frac{(0.05)^3}{6\times(0.05)^2} \text{ or } \frac{(0.025)^3}{3\times(0.025)^2} = 0.00833 \quad \text{m}$$

$$Bi = \frac{hL_c}{k} = \frac{1000(0.00833)}{25} = 0.333 > 0.1 \Rightarrow LPA \text{ not valid}$$

$$t^* = \alpha t / L^2 = 3.125\times10^{-6}\times 200/\left(0.025\right)^2 = 1 > 0.2$$

Thus, $t^* > 0.2 \;\Rightarrow\;$ 1-term approximation of infinite series solution is adequate

(a) At t = 200 s the maximum temperature occurs at the center of the big cube, or the bottom front corner of the given slab. Using the product solution:

$$\frac{T_{(x_1=0,x_2=0,x_3=0,t=200\text{s})}-T_\infty}{T_i-T_\infty} = \frac{T_{(x_1=0,t=200\text{s})}-T_\infty}{T_i-T_\infty}\times\frac{T_{(x_2=0,t=200\text{s})}-T_\infty}{T_i-T_\infty}\times\frac{T_{(x3=0,t=200\text{s})}-T_\infty}{T_i-T_\infty}$$

However, because of symmetry: $\dfrac{T_{(x_1=0,t=200\text{s})}-T_\infty}{T_i-T_\infty} = \dfrac{T_{(x_2=0,t=200\text{s})}-T_\infty}{T_i-T_\infty} = \dfrac{T_{(x3=0,t=200\text{s})}-T_\infty}{T_i-T_\infty}$

Thus, $\dfrac{T_{(x_1=0,x_2=0,x_3=0,t=200\text{s})}-T_\infty}{T_i-T_\infty} = \left(\dfrac{T_{(x_1=0,t=200\text{s})}-T_\infty}{T_i-T_\infty}\right)^3$

$$\frac{T_{(x_1=0,t=200\text{s})}-T_\infty}{T_i-T_\infty} = C_B \exp[-A_B^2 t^*]$$

$$Bi_M = \frac{hL}{k} = \frac{1000\left(0.025\right)}{25} = 1$$

From Table 5.2 of Handout #5 or Table C-2, Appendix C of the Textbook (Holman, 2002):
$A_B \simeq 0.8603; C_B \simeq 1.1191$

$$\frac{T_{(x_1=0,t=200\text{s})}-T_\infty}{T_i-T_\infty} = 1.1191\exp[-\left(0.8603\right)^2\times 1] = 0.5339$$

$$\frac{T_{(x_1=0,x_2=0,x_3=0,t=200\text{s})}-T_\infty}{T_i-T_\infty} = \left(\frac{T_{(x_1=0,t=200\text{s})}-T_\infty}{T_i-T_\infty}\right)^3 = \left(0.5339\right)^3$$

$$T_{MAX} = T_{(x_1=0,x_2=0,x_3=0,t=200\text{s})} = 20+\left(520-20\right)\left(0.5339\right)^3 = 96.08°\text{C}$$

At t = 200 s the minimum temperature occurs at the outer corner of the slab or the corners of the big cube.

$x_1 = x_2 = x_3 = L \Rightarrow x_1^* = x_2^* = x_3^* = 1$

$$\frac{T_{(x_1=L,t=200\text{s})}-T_\infty}{T_i-T_\infty} = C_B \exp[-A_B^2 t^*]\cos\left(A_B\; x_1^*\right)$$

$$\frac{T_{(x_1=L,t=200\text{s})}-T_\infty}{T_i-T_\infty} = 0.5339\times\cos\left(0.8603\right) \simeq 0.3482$$

$$\frac{T_{(x_1=L,x_2=L,x_3=L,t=200\text{s})}-T_\infty}{T_i-T_\infty} = \left(\frac{T_{(x_1=L,t=200\text{s})}-T_\infty}{T_i-T_\infty}\right)^3 = \left(0.3482\right)^3$$

$$T_{MIN} = T_{(x_1=L,x_2=L,x_3=L,t=200\text{s})} = 20+\left(520-20\right)\left(0.3482\right)^3 = 41.11°\text{C}$$

(b) After 200 s, the cube is wrapped up completely. Thus, there is no heat loss after t = 200 s. The final equilibrium temperature, T_{final}, will be uniform inside the cube. An energy balance gives that the total heat loss for the period of 0 to 200s is equal to the amount of energy required for the object initially at uniform temperature of T_i cool down to a uniform temperature of T_{final}.

Thus,

$$\underset{0\le t\le 200s}{Q_{loss}} = \underset{cube}{m_{full}}\, c_p\left(T_i - T_{final}\right) \quad (1)$$

We first calculate $\underset{0\le t\le 200s}{Q_{loss}}$

Using the relation given in the handout #5:

$$\left(\frac{Q}{Q_o}\right)_{total} = \left(\frac{Q}{Q_o}\right)_1 + \left(\frac{Q}{Q_o}\right)_2\left[1-\left(\frac{Q}{Q_o}\right)_1\right] + \left(\frac{Q}{Q_o}\right)_3\left[1-\left(\frac{Q}{Q_o}\right)_2\right]\left[1-\left(\frac{Q}{Q_o}\right)_1\right]$$

Because of symmetry: $\left(\frac{Q}{Q_o}\right)_1 = \left(\frac{Q}{Q_o}\right)_2 = \left(\frac{Q}{Q_o}\right)_3$

Thus, $\left(\frac{Q}{Q_o}\right)_{total} = \left(\frac{Q}{Q_o}\right)_1 \times \left[3-\left(\frac{Q}{Q_o}\right)_1 \times \left[3-\left(\frac{Q}{Q_o}\right)_1\right]\right]$

$$\left(\frac{Q}{Q_o}\right)_1 = 1-\theta^*_{(x^*=0,t^*)}\sin\left(A_B\right)/A_B$$

$$\left(\frac{Q}{Q_o}\right)_1 = 1-\frac{T_{(x_1=0,t=200s)}-T_\infty}{T_i-T_\infty}\sin\left(A_B\right)/A_B$$

$$\left(\frac{Q}{Q_o}\right)_1 = 1-0.5339\sin\left(0.8603\right)/0.8603 = 0.5296$$

$$\left(\frac{Q}{Q_o}\right)_{total} = 0.5296\times\left[3-0.5296\times\left[3-0.5296\right]\right] = 0.8959$$

$$\underset{0\le t\le 200s}{Q_{loss}} = Q_o\times 0.8959 = \underset{cube}{m_{full}}\, c_p\left(T_i-T_\infty\right)\times 0.8959 \quad (2)$$

Thus, from Equations 1 and 2 $\Rightarrow$ $\underset{cube}{m_{full}}\, c_p\left(T_i-T_\infty\right)\times 0.8959 = \underset{cube}{m_{full}}\, c_p\left(T_i-T_{final}\right)$

$$T_{final} = T_i - \left(T_i - T_\infty\right)\times 0.8959$$

$$T_{final} = 520-\left(520-20\right)\times 0.8959 = 72.05°\text{C}$$