P.S. # 2

Solution for problems from textbook:

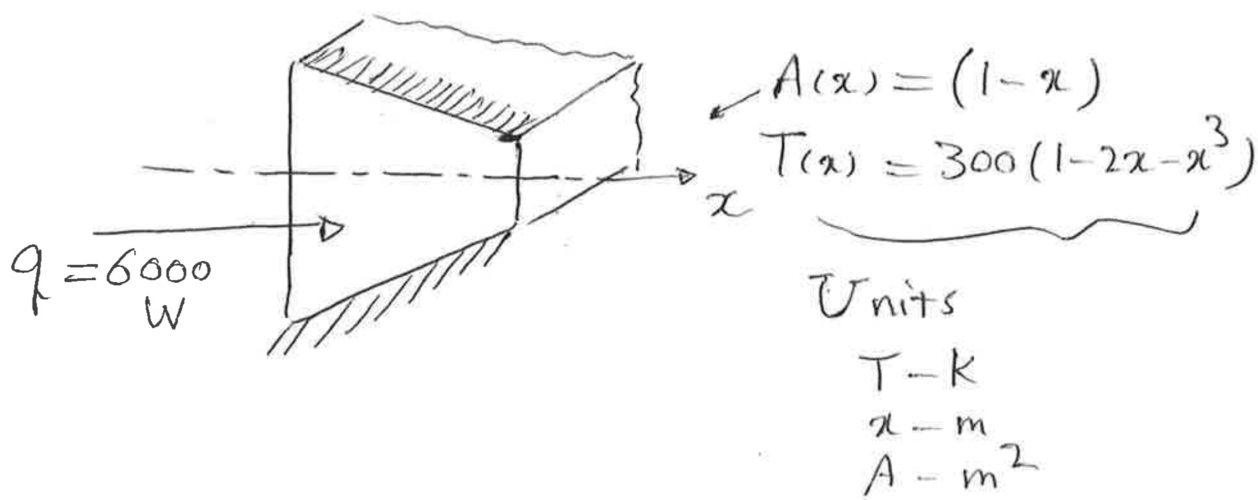
P. 2.4

Solution:

Known: Symmetric shape with prescribed variation in cross-sectional area, temperature distribution and heat rate.

Find: Expression for the thermal conductivity, K.

Schematics:




Assumption: (1) Steady-State conditions (2) One-dimensional conduction in $x$-direction, (3) No internal heat generation.

Analysis: Applying the energy balance, Eq. 1.11c, to the system, it follows that, since $\dot{E}_{in} = \dot{E}_{out}$,

$$q_x = \text{Const.} \neq f(x).$$

(#1)

Using Fourier's law, Eq. 2.1, with appropriate expressions for $A_x$ and $T$, yields

$$q_x = -kA_x \frac{dT}{dx}$$

$$6000\,W = -k \cdot (1-x)m^2 \cdot \frac{d}{dx}\left[300(1-2x-x^3)\right]\frac{K}{m}.$$

Solving for $k$ and recognizing its units are W/m.K,

$$k = \frac{-6000}{(1-x)\left[300(-2-3x^2)\right]} = \frac{20}{(1-x)(2+3x^2)}$$

Comments: (1) At $x=0$, $k = 10$ W/m.k and $k \to \infty$ as $x \to 1$

(2) Recognize that 1-D assumption is an approximation which becomes more inappropriate as the area change with $x$, and hence two-dimensional effects become more pronounced.

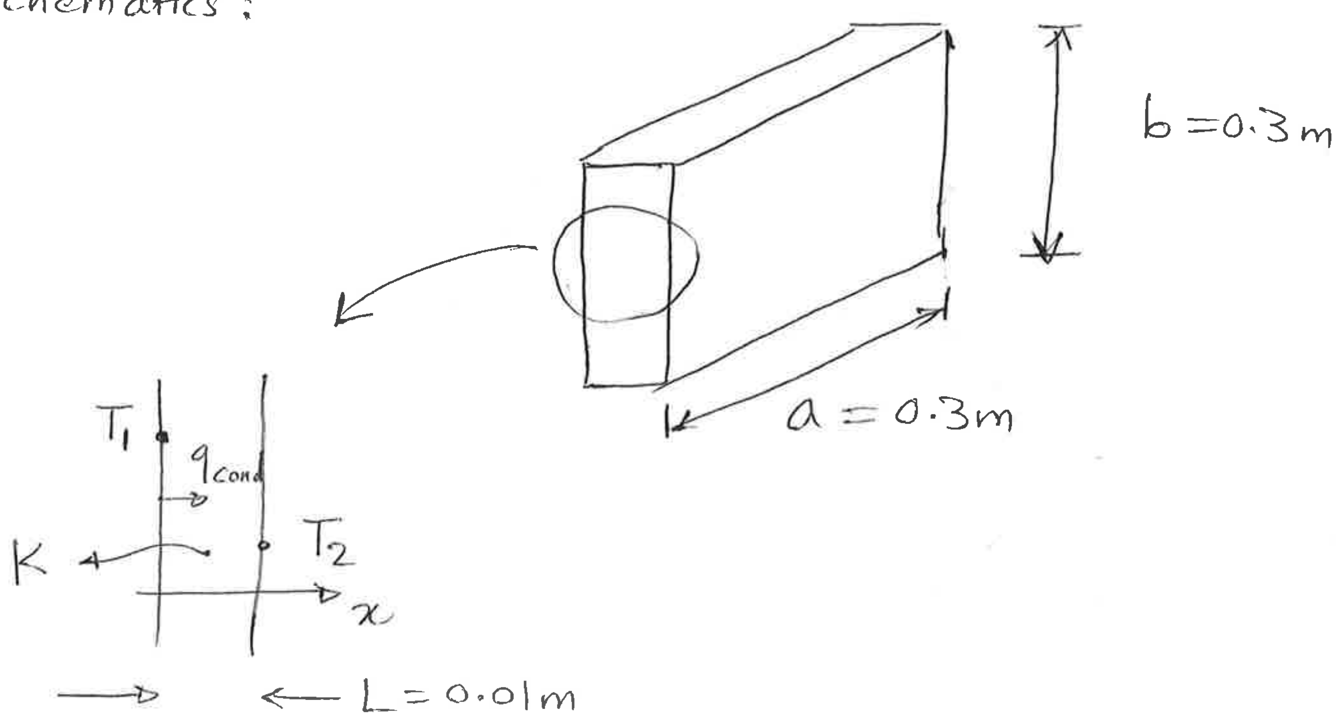
P. 2.14

Solution :

Known : Dimensions of and temperature difference across an aircraft window. Window materials and cost of energy.

Find : Heat loss through one window and cost of heating for 180 windows on 8-hour trip.

Schematics :




Assumptions : (1) Steady-state conditions, (2) One-dimensional condition in the x-direction, (3) Constant properties

Properties : Table A.3, Sodalime glass (300K) : $K_{gl} = 1.4\ W/m.K$

Analysis : From Eq. 2.1,

$$q_x = -KA\frac{dT}{dx} = Kab\frac{(T_1 - T_2)}{L}$$

For glass,

$$q_{x,g} = 1.4\left(\frac{W}{m.k}\right) \times 0.3\ (m) \times 0.3\ (m) \times \left[\frac{80^\circ C}{0.01\ m}\right] = 1010\ W$$

(#3)

The cost associated with heat loss through $N$ windows at a rate of $R = 1\,\$/\text{kW}\cdot\text{h}$ over a $t = 8\,\text{h}$ flight time is

$$C_g = N q_{x,g} R t = 130 \times 1010\,\text{W} \times 1 \frac{\$}{\text{kW}\cdot\text{h}} \times 8\,\text{h} \times 1 \frac{\text{kW}}{1000\,\text{W}}$$

$$= 1050\,\$$$

Repeating the calculation for the polycarbonate yields

$$q_{x,p} = 151\,\text{W}, \quad C_p = 157\,\$$$

while for aerogel

$$q_{x,a} = 10.1\,\text{W}, \quad C_a = 10\,\$$$

Comment : Polycabonate provides significant savings relative to glass. It is also lighter ($\rho_p = 1200\,\text{kg/m}^3$) relative to glass ($\rho_g = 2500\,\text{Kg/m}^3$). The aerogel offers the best thermal performance and is very light ($\rho_a = 2\,\text{Kg/m}^3$) but would be relatively expensive.

(#4)

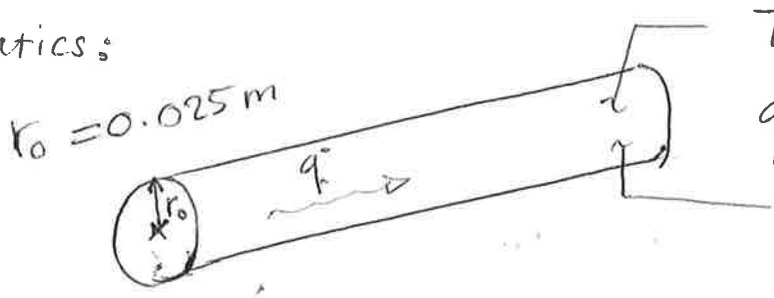
P. 2.22

Solution :

Known : Steady-state temperature distribution in a cylindrical rod having uniform heat generation of $\dot{q}_1 = 5\times10^7\ W/m^3$.

Find : (a) Steady-state centerline and surface heat transfer rates per unit length, $q'_r$. (b) Initial time rate of change of centerline and surface temperatures in response to a change in the generation rate time from $\dot{q}_1$ to $\dot{q}_2 = 10^8\ W/m^3$

Schematics:




$T(r) = 800 - 4.167\times10^5 r^2$

$\dot{q} = \dot{q}_1 = 5\times10^7\ W/m^3$

$k = 30\ W/m.k$

$\rho = 1100\ kg/m^3$

$C_p = 800\ J/kg.K$

Assumptions: (1) One-dimensional conduction in the r direction (2) Uniform generation, and (3) Steady-state for $\dot{q}_1 = 5\times10^7\ W/m^3$

Analysis : (a) From the rate equations for cylindrical coordinates,

$$q''_r = -k\frac{\partial T}{\partial r} \qquad q = -kA_r\frac{\partial T}{\partial r}$$

Hence,

$$q_r = -k(2\pi rL)\frac{\partial T}{\partial r}; \text{or} \quad \boxed{q'_r = -2\pi k r\frac{\partial T}{\partial r}} \quad (1)$$

(#5)

Where $\frac{\partial T}{\partial r}$ may be evaluated from the prescribed temperatures distribution, $T(r)$.

At $r=0$, the gradient is $(\frac{\partial T}{\partial r})=0$. Hence, from Equation (1) the heat rate is $q'_r(0)=0$

At $r=r_o$ the temperature gradient is

$$\left.\frac{\partial T}{\partial r}\right|_{r=r_o} = -2\left[4.167\times10^5 \frac{K}{m^2}\right](r_o) = -2(4.167\times10^5)(0.025m)$$

$$= -0.208\times10^5 \ K/m$$

Hence, the heat rate at the outer surface $(r=r_o)$ per unit length is

$$q'_r(r_o) = -2\pi\left[30\ W/m.K\right](0.025m)\left[-0.208\times10^5\ K/m\right]$$

$$q'_r(r_o) = 0.980\times10^5\ W/m.$$

(b) Transient (time-dependent) conditions will exist when the generation is changed, and for the prescribed assumptions, the temperature is detemined by the following form of the heat equation, Equation 2.24.

$$\frac{1}{r}\frac{\partial}{\partial r}\left[Kr\frac{\partial T}{\partial r}\right]+\dot{q}_2 = \rho c_p\frac{\partial T}{\partial t}$$

Hence

$$\frac{\partial T}{\partial t} = \frac{1}{\rho c_p}\left[\frac{1}{r}\frac{\partial}{\partial r}\left[Kr\frac{\partial T}{\partial r}\right]+\dot{q}_2\right]$$

(#6)

However, initially (at $t=0$), the temperature distribution is given by the prescribed form, $T(r) = 800 - 4.167 \times 10^5 r^2$, and

$$\frac{1}{r}\frac{\partial}{\partial r}\left[kr\frac{\partial T}{\partial r}\right] = \frac{k}{r}\frac{\partial}{\partial r}\left[r(-8.334 \times 10^5 \cdot r)\right]$$

$$= \frac{k}{r}(-16.668 \times 10^5 \cdot r)$$

$$= 30\ W/m.K\left[-16.668 \times 10^5\ K/m^2\right]$$

$$= -5 \times 10^7\ W/m^3 \text{ (the original } \dot{q} = \dot{q}_1\text{)}.$$

Hence, everywhere in the wall,

$$\frac{\partial T}{\partial t} = \frac{1}{1100\ kg/m^3 \times 800\ J/kg.K}\left[-5 \times 10^7 + 10^8\right] W/m^3$$

or $$\frac{\partial T}{\partial t} = 56.82\ K/s.$$

Comments : (1) The value of $\left(\frac{\partial T}{\partial t}\right)$ will decrease with increasing time, until a new steady-state condition is reached and once again $\left(\frac{\partial T}{\partial t}\right) = 0$.

(2.) By applying the energy conservation requirement, Equation 1.11c, to a unit length of the rod for the steady-state condition, $\dot{E}'_{in} - \dot{E}'_{out} + \dot{E}'_{gen} = 0$. Hence $q'_r(0) - q'_r(r_0) = -\dot{q}_1(\pi r_0^2)$

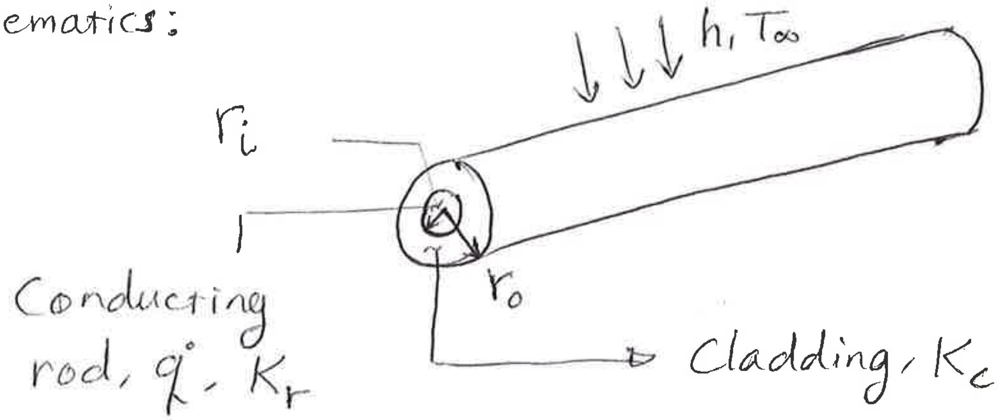
P. 2. 39

Solution:

Known: Radii and thermal conductivity of conducting rod and cladding material. Volumetric rate of thermal energy generation in the rod. Convection conditions at outer surface.

Find: Heat equation and boundary conditions for rod and cladding.

Schematics:




Assumptions: (1) Steady-state conditions, (2) One-dimensional Conduction in r, (3) Constant properties.

Analysis: From Equation 2.24, the appropriate forms of the heat equation are

Conducting Rod:

$$\frac{K_r}{r}\frac{d}{dr}\left(r\frac{dT_r}{dt}\right)+\dot{q}=0$$

Cladding:

$$\frac{d}{dr}\left(r\frac{dT_c}{dr}\right)=0$$

Appropriate boundary conditions are:

(a) $\left.\frac{dT_r}{dr}\right|_{r=0} = 0$

(b) $T_r(r_i) = T_c(r_i)$

(c) $K_r \left.\frac{dT_r}{dr}\right|_{r_i} = K_c \left.\frac{dT_c}{dr}\right|_{r_i}$

(d) $-K_c \left.\frac{dT_c}{dr}\right|_{r_o} = h\left[T_c(r_o) - T_\infty\right]$

Comments: Condition (a) corresponds to symmetry at the centerline, while the interface conditions at $r=r_i$ (b,c) correspond to requirements of thermal equilibrium and conservation of energy. Condition (d) results from conservation of energy at the outer surface. Note that contact resistance at the interface between the rod and cladding has been neglected.