THE UNIVERSITY OF BRITISH COLUMBIA

Department of Electrical and Computer Engineering

EECE 373: Electromechanical Energy Conversion and Transmission

Final Exam
April 21, 2004

Magnetic Circuits, Transformers, DC Machines, Induction Machines, Synchronous Machines, Stepper Motors

Surname:_____

First Name:_____

Student ID:_____

Signature:_____

- **Close notes and books.**

- You are allowed to have only a **calculator** and a **pen/pencil**.

- Show you work including **derivations**, **comments**, **assumptions**, and **units** wherever appropriate.

- Use back side of each page or ask for additional pages if you need extra space to write your answers.

- Exams suspected of cheating and/or turned in late will not be marked – **failed exam**.

- You have about 90 minutes to answer the following questions:

Problem	Points	Max.
1		16
2		17
3		17
4		17
5		17
6		16
Total		100

Problem 1 (16pts)

Consider the magnetic system shown below. The length of the air-gap is such that the air-gap reluctance is $\Re_g = 8.75 \times 10^5 \, At/Wb$. The effective length and material of the core are such that the effective reluctance of the magnetic path inside the core is $\Re_c = 1.25 \times 10^5 \, At/Wb$. The cross-sectional area of the core and air-gap is $A_c = A_g = 10 cm^2$. The coil has 100 turns and dc resistance of 3Ω. Neglect flux leakage, fringing, and core losses. Assume that an external voltage source is connected to the coil.
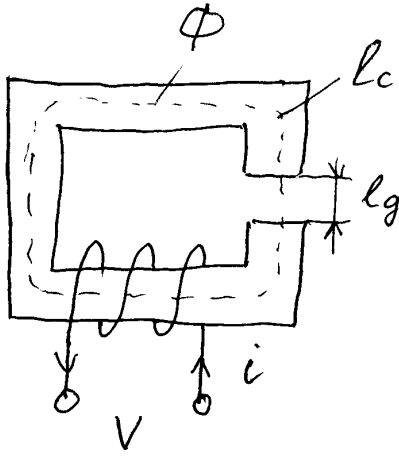
(a) (8pts) Draw an equivalent **magnetic** circuit, show the direction of mmf F and the flux Φ

(b) (8pts) Calculate the rms value of ac voltage required to produce the flux density in the air-gap $B_g = 1T$ rms
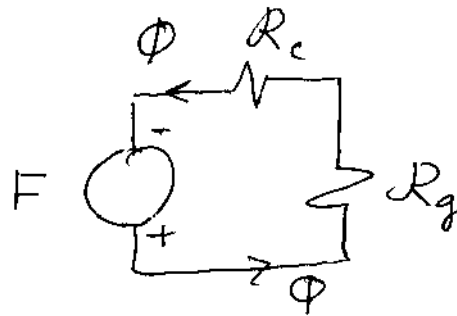
$$F = N \cdot I = (R_g + R_c)\Phi$$

$$\Phi = B \cdot A$$

$$N \cdot I = (R_g + R_c) \cdot B_g \cdot A_c$$

$$I = \frac{(R_g + R_c) \cdot B_g \cdot A_c}{N} = 10 A$$

$$L = \frac{N^2}{R_{total}} \quad ; \quad X = \frac{\omega \cdot N^2}{(R_g + R_c)} = 3.77 \, \Omega$$

$$2 = \sqrt{R^2 + X^2} = 4.818 \, \Omega$$

$$V = I \cdot 2 = \underline{48.18 \, V \ rms}$$

Problem 2 (17pts)

Consider a single-phase 60-Hz transformer with the following parameters: $R_1 = 1\Omega$, $X_1 = 3\Omega$, $X_m = 30\Omega$, $R_2 = 1.2\Omega$, and $X_2 = 5\Omega$, respectively. Assume that both windings have the same number of turns and that the primary side is connected to a 240-V ac source.

(a) (8pts) Based on the given information, sketch the T-equivalent circuit and label all its elements

(b) (9pts) Assume a resistor load $R_{load} = 40\ \Omega$ is connected to the secondary side. Calculate the voltage regulation VR expressed in percent.

Recall: $VR = \dfrac{V_{no\_load} - V_{load}}{V_{load}} \cdot 100\%$
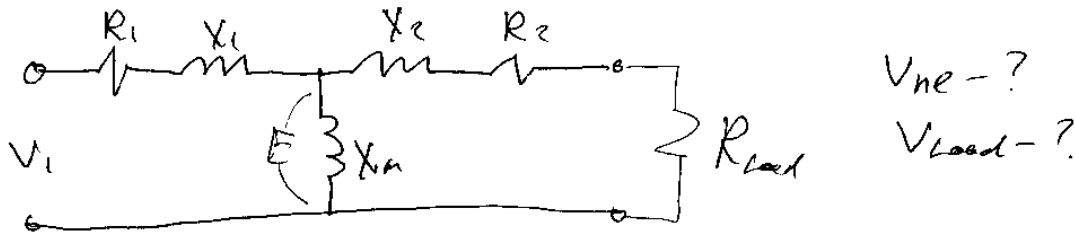
V_{ne} $V_{ne} = V_{oc}$

$$I_{1,oc} = \frac{V_1}{R_1 + j(X_1 + X_m)} = 0.2202 - j7.266$$

$$V_{ne} = I_{1,oc} \cdot jX_m = 217.98 + j6.6\ 055 = 218.08\ V_{(rms)}$$

V_{load} $Z_{in} = R_1 + jX_1 + \dfrac{jX_m \cdot (R_1 + R_{load} + jX_2)}{j(X_m + X_2) + R_1 + R_{load}} \simeq 13.69 + j22.2$

$$I_1 = \frac{V_1}{Z_{in}} = 4.823 - j7.83$$

$$E = V_1 - I_1 \cdot (R_1 + jX_1) = 211.69 - 6.69$$

$$I_2 = \frac{E}{R_2 + R_{load} + jX_2} = 5.044 - j0.773$$

$$V_2 = I_2 \cdot R_{load} = 201.77 - j30.932 = 204.126\ V_{rms}$$

$$VR = \frac{218.08 - 204.126}{204.126} = 0.0684$$
$$= 6.84\ \%$$

Problem 3 (17pts)

Consider a **Permanent-Magnet DC Machine** with the armature resistance $R_a = 1\,\Omega$. When the machine is connected to a 12-V battery it draws the armature current $I_a = 0.2$-A and the shaft speed is $n = 1800\,\text{rpm}$ CCW at no load. The battery voltage and friction torque can be assumed constant.

(a) (8pts) Calculate the torque constant K_t and the friction torque T_{fric}

(b) (9pts) Assume that you need to use this DC Machine as a generator to charge the battery. Calculate the external mechanical torque T_m required to produce the charging current of 1A. In what direction, CW or CCW, should this torque be applied? What is the resulting speed n?

a) $E_a = 12 - 1 \cdot 0.2 = 11.8\,V$

$$\omega_{ne} = 188.4956\,^r/s$$

$E_a = K_t \cdot \omega$

$K_t = \dfrac{E_a}{\omega} = \dfrac{11.8}{188.4956} = 0.0626$

$T_{fric} = I_{ne} \cdot K_t = 0.0125\,N \cdot m$

b) $I_a = 1\,A$

$E_a = 12 + 1 \cdot 1 = 13\,V!$ $\omega = \dfrac{E_a}{K_t}$; $n = \dfrac{30 \cdot E_a}{\pi K_t} = 1983.1\,rpm$

$T_e = K_t \cdot I_a = 0.0626$

$T_m = T_e + T_{fric} = 0.751\,N \cdot m$ __CCW__
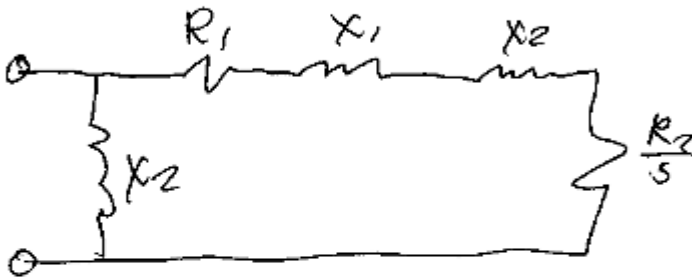
Problem 4 (17pts)
Consider a 60Hz, 208V (line-to-line), Y-connected, NEMA Clsass B (Squirrel-Cage) Induction Motor with the following per-phase parameters: $R_1 = 1\Omega$, $R_2 = 1.5\Omega$, $X_1 = X_2 = 3\Omega$, and $X_m = 40\Omega$ (all referred to the stator). The motor is supplied with the nominal (rated) voltage and is driving a mechanical load. The speed of the motor shaft is $n = 840$ rpm. You can neglect core and friction losses and use an approximate equivalent circuit. Determine the following:

(a) (4pts) Number of poles p
(b) (4pts) Slip s
(c) (5pts) Stator current I_1 and power factor PF
(d) (4pts) Total (3-phase) input real power

$$3600 - 2$$
$$1800 - 4$$
$$1200 - 6$$
$$900 - 8$$

$$\underline{p = 8}$$

$$S = \frac{900 - 840}{900} = 0.0667 = 6.67\%$$

$$Z = \frac{jX_m\left(R_1 + \frac{R_2}{S} + jX_1 + jX_2\right)}{R_1 + \frac{R_2}{S} + j(X_1 + X_2 + X_m)} = 14.09 + j12.4$$

$$I_1 = \frac{V_1}{Z} = \frac{208}{\sqrt{3}\,(14.09 + j12.4)} = 4.79 - 4.22j$$

$$PF = \cos\varphi = \cos(angle(I_1)) = 0.75$$

$$P_{3\varphi} = 3 I_1 V_1 \cdot \cos\varphi = 1.728 \text{ kW}$$

Problem 5 (17pts)

Consider a Salient-Pole Synchronous Generator with the following parameters: $X_q = 0.5\,pu$, $X_d = 0.9\,pu$ (the stator winding resistance is negligible). The excitation is adjusted so that the open-circuit voltage is $E_f = 1.2969\,pu$. The generator is supplying a resistive load that draws rated (nominal) current $I_a = 1\,pu$ and the terminal voltage is $V_t = 1\,pu$.

(a) (8pts) Sketch a phasor diagram and label all relevant axes, angles, and phasors
(b) (9pts) Calculate the rotor angle δ in degrees

$$E_s = V_t + jX_d I_d + jX_q I_q$$
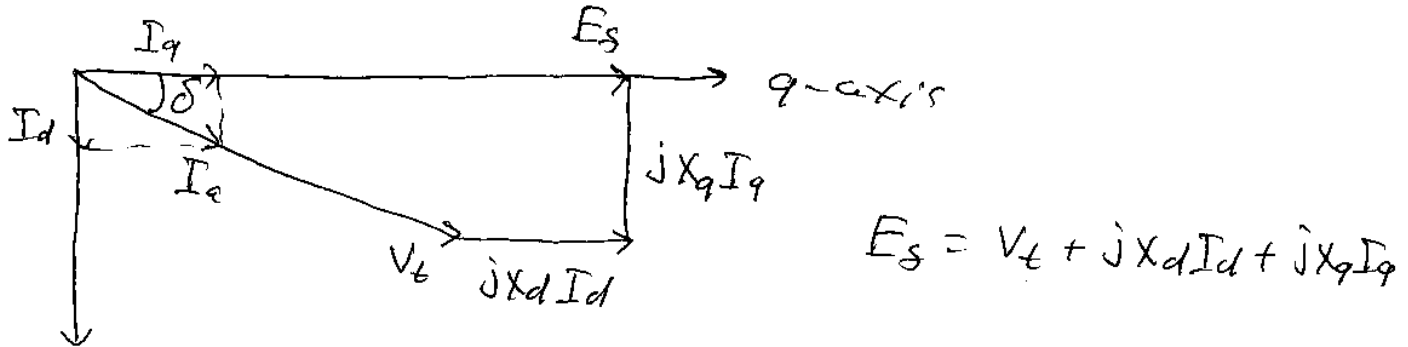
$$V_t \cdot \sin\delta = X_q \cdot I_q = X_q \cdot I_a \cdot \cos\delta$$

$$\tan\delta = \frac{X_q I_a}{V_t} = X_q = 0.5; \quad \delta = a\tan(X_q) = 26.6°$$
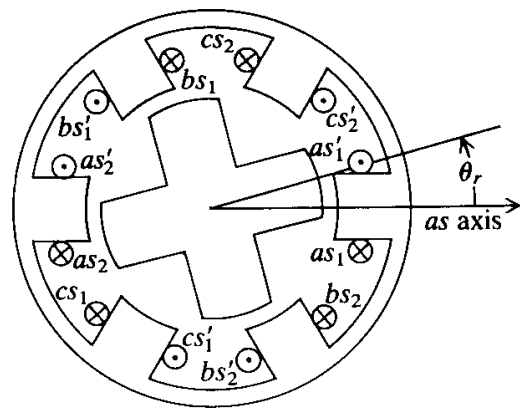
$$E_f = V_t \cdot \cos\delta + X_d \cdot I_a \cdot \sin\delta$$

$$\sin\delta = \frac{X_q I_a \cdot \cos\delta}{V_t}$$

$$E_f = V_t \cos\delta + X_d I_a \cdot \frac{X_q I_a \cdot \cos\delta}{V_t} = \left[V_t + \frac{X_d X_q I_a^2}{V_t}\right]\cos\delta$$

$$\delta = a\cos\left[\frac{E_f}{V_t + \frac{X_d X_q I_a^2}{V_t}}\right] = 26.6°$$

Problem 6 (16pts)

Consider a single-stack stepper motor shown below. Complete the following table:

2pts	Stator tooth pitch (STP)	60 degrees
2pts	Rotor tooth pitch (RTP)	90 degrees
3pts	Step length $\Delta\Theta$	-30 degrees
3pts	Number of steps per revolution (Resolution)	12
3pts	Assume sequence of pulses as B – C – A, determine the direction of rotation (CW or CCW)	CCW
3pts	Assume you have a 3-phase pulse generator to supply this motor. The generator produces 6 pulses per second. What is the motor speed in rpm?	30 rpm