a place of mind

MECH366 : Modeling of Mechatronic Systems

L9 : Modeling of electrical systems

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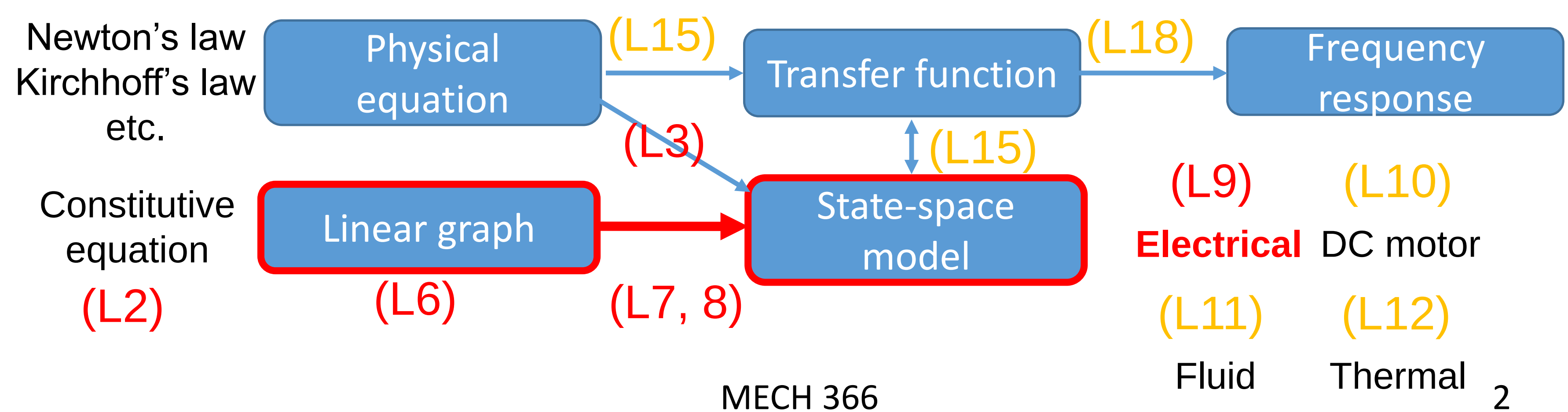
Review and today's topic

- Up to now, we have studied for mechanical systems
  - How to draw linear graphs
  - How to derive state-space models from linear graphs
- Today, we will study modeling of electrical systems.
- Various models and their relations



$\underline{\quad}$: State variable

	Constitutive relation for		
	Energy storage element		Energy dissipating element
System type	A-Type	T-Type	D-Type
Mechanical (translational)	Mass	Spring	Viscous Damper
v: velocity across var.	$m\underline{\dot{v}} = f$	$\underline{\dot{f}} = kv$	$f = bv$
f: force through var.	m: mass	k: stiffness	b: damping const.
Electrical	Capacitor	Inductor	Resistor
v: voltage across var.	$C\underline{\dot{v}} = i$	$L\underline{\dot{i}} = v$	$v = Ri$
i: current through	C: capacitance	L: inductance	R: resistance
Thermal	Thermal capacitor	None	Thermal resistor
T: temperature	$C_t\dot{T} = Q$		$T = R_tQ$
Q: heat transfer rate	C: thermal capacitance		R_t: thermal resistance
Fluid	Fluid capacitor	Fluid inertor	Fluid resistor
P: pressure difference	$C_f\dot{P} = Q$	$I_f\dot{Q} = P$	$P = R_fQ$
Q: volume flow rate	C_f: fluid capacitance	I_f: fluid inertance	R_f: fluid resistance

power

$$\mathcal{P} = fv$$

$$\mathcal{P} = iv$$

Energy expressions based on across and through variables

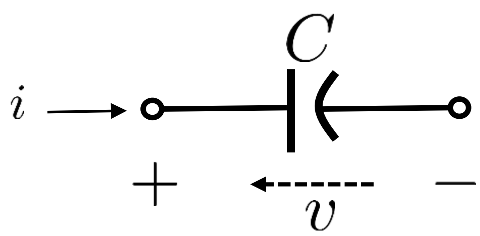
	A-type element	T-type element
Mechanical v : Across variable f : Through variable	Kinetic energy $\frac{1}{2}mv^2$	Potential energy $\left(\frac{1}{2}kx^2 =\right)\frac{1}{2}\frac{f^2}{k}$
Electrical v : Across variable i : Through variable	Electrostatic energy $\frac{1}{2}Cv^2$	Electromagnetic energy $\frac{1}{2}Li^2$
Thermal T : Across variable Q : Through variable	Thermal energy $\int Q = C_t T$	N/A
Fluid P : Across variable Q : Through variable	Potential energy $\frac{1}{2}C_f P^2$	Kinetic energy $\frac{1}{2}I_f Q^2$

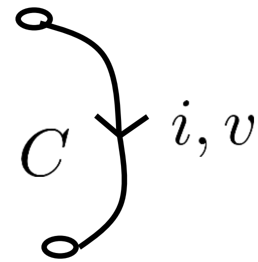
Linear graph representation

- Single-port elements
  - Energy storage elements
  - Energy dissipation elements
  - Energy sources
- Two-port elements (Energy transfer elements)
  - Transformer
  - (Gyrator)

Linear graph representation
Electrical single-port elements

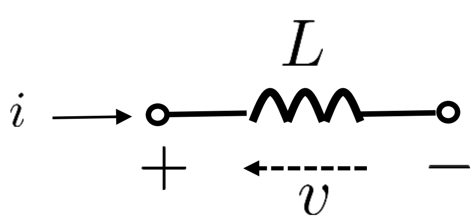
Energy storage element

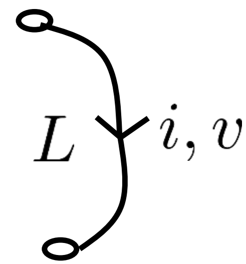




$$\dot{v} = \frac{1}{C} i$$

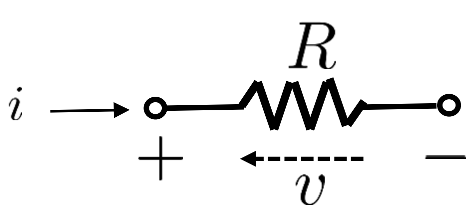
Energy storage element

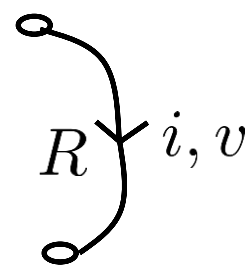




$$\dot{i} = \frac{1}{L} v$$

Energy dissipation element





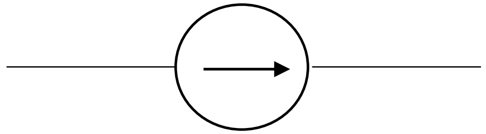
$$v = Ri$$

Linear graph representation

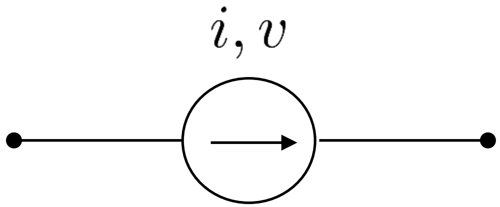
- Single-port elements
  - Energy storage elements
  - Energy dissipation elements
  - Energy sources
- Two-port elements (Energy transfer elements)
  - Transformer
  - (Gyrator)

Linear graph representation Electrical energy sources

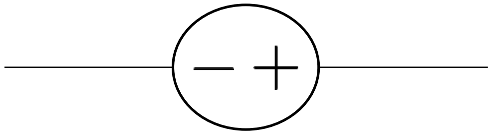
- Current source



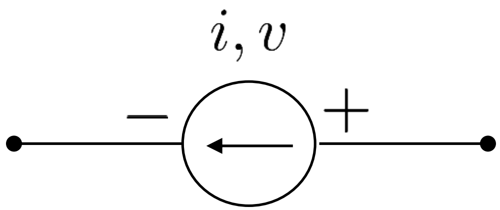
Linear graph



- Voltage source



Linear graph



Linear graph representation

- Single-port elements
  - Energy storage elements
  - Energy dissipation elements
  - Energy sources
- Two-port elements (Energy transfer elements)
  - Transformer
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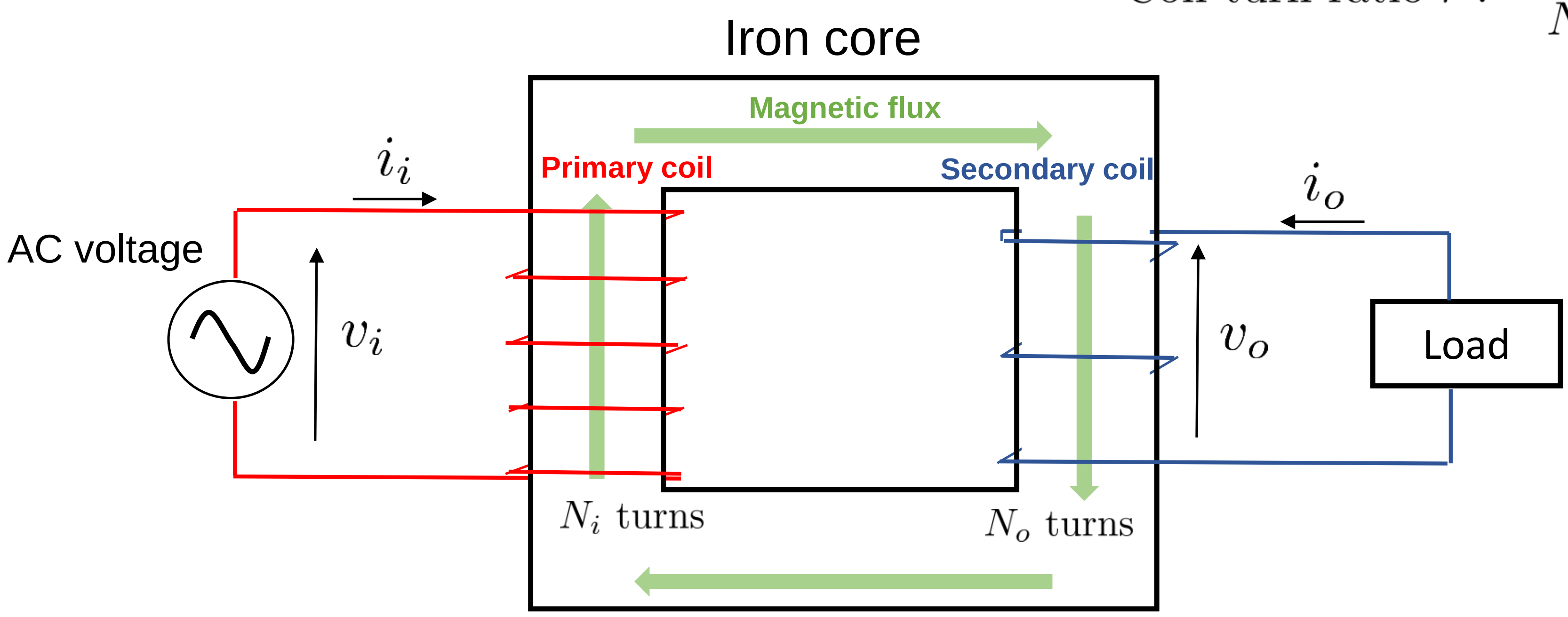
Electrical transformer

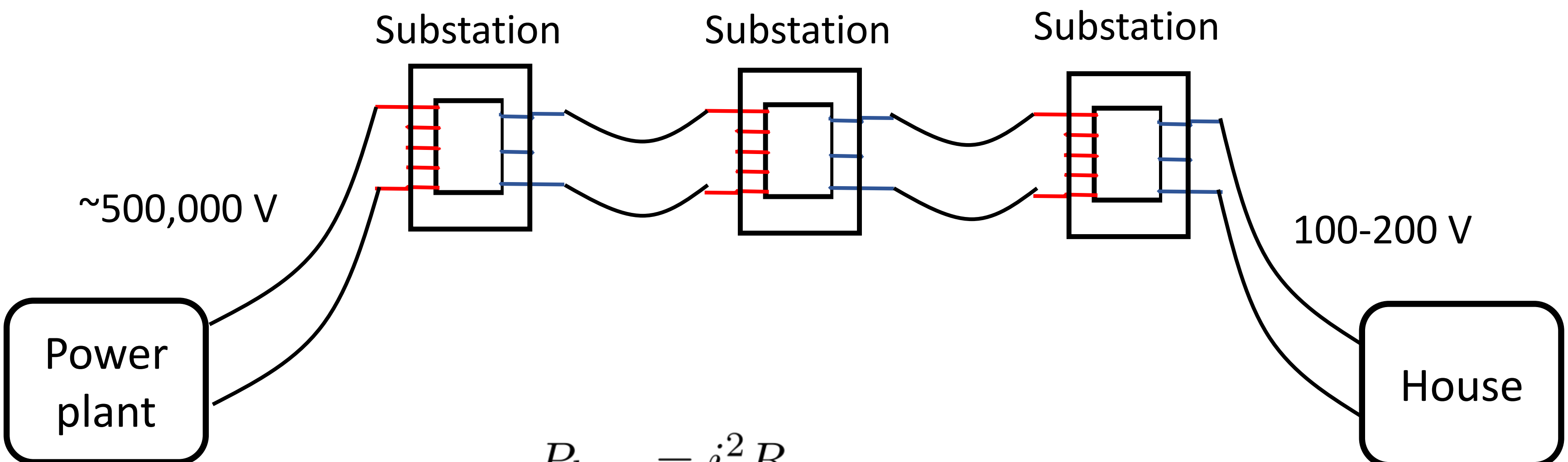
- Passive device to step-up or step-down AC voltage.
  $(r > 1)$ $(r < 1)$

$$\text{Coil turn ratio } r := \frac{N_o}{N_i}$$



Usage of electric transformer

Substations have electric transformers, to transform AC voltage from high to low, or low to high



$$P_{loss} = i^2 R$$

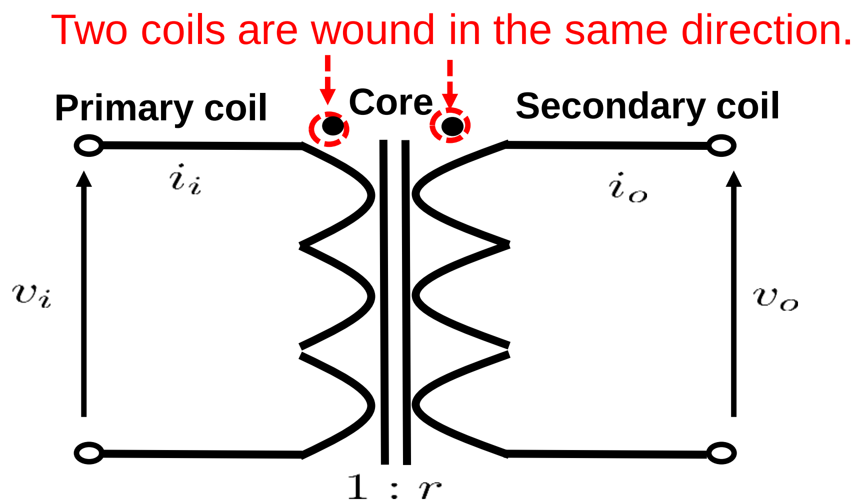
To reduce power heat losses during the transmission, power is sent with high voltage (small current).

Linear graph representation
Two-port element: Transformer

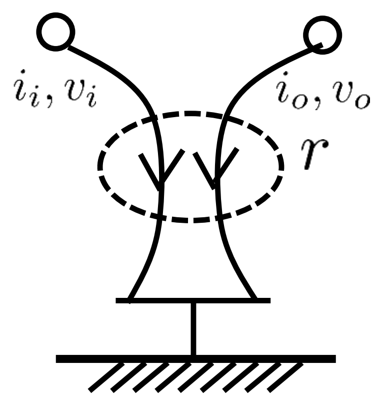
- Voltage ratio $v_o = r v_i$
- Conservation of power

$$i_i v_i + i_o v_o = 0$$

$$\Rightarrow \quad i_o = -\frac{v_i}{v_o} i_i = -\frac{1}{r} i_i$$



Linear graph

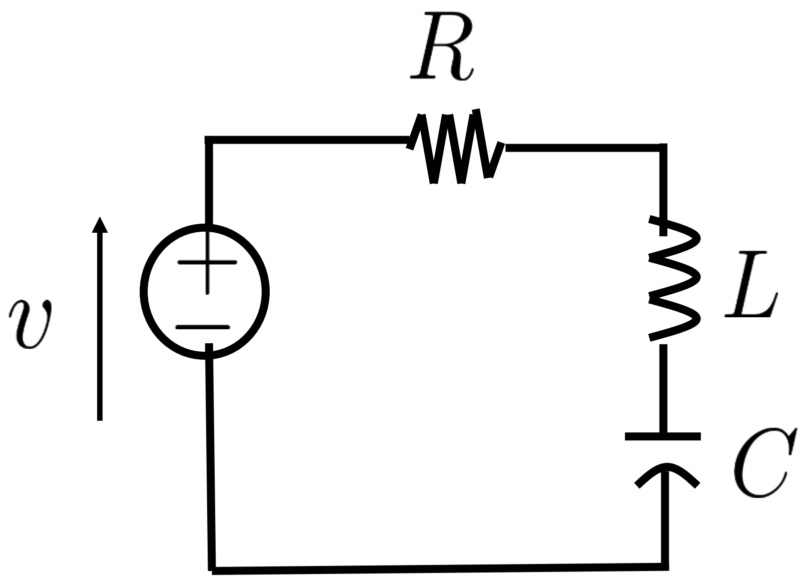


$$v_o = r v_i$$

$$i_o = -\frac{1}{r} i_i$$

Example: RLC circuit (series)

- Circuit diagram



Example: RLC circuit (series) (cont'd)

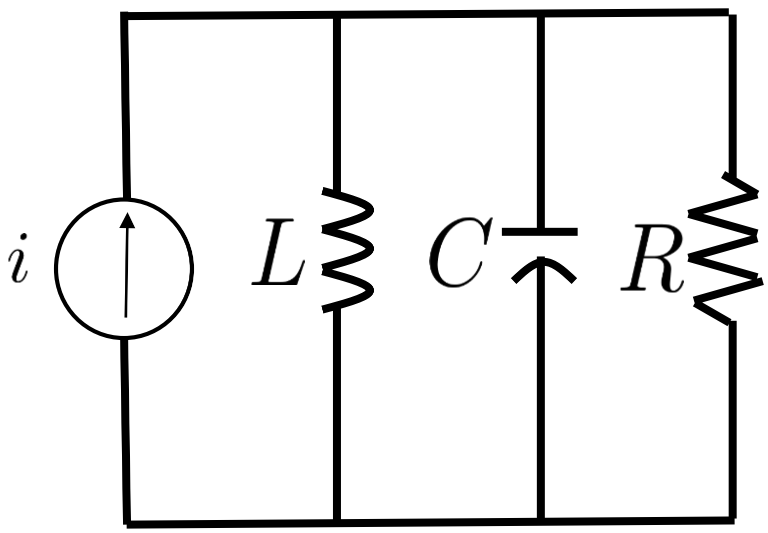
- Linear graph drawing
- State variable selection
- Loop & node equations
- State equation

Take a node for each different voltage point.

Direct all arrows on passive elements away from sources and toward the reference node.

Example: RLC circuit (parallel)

- Circuit diagram



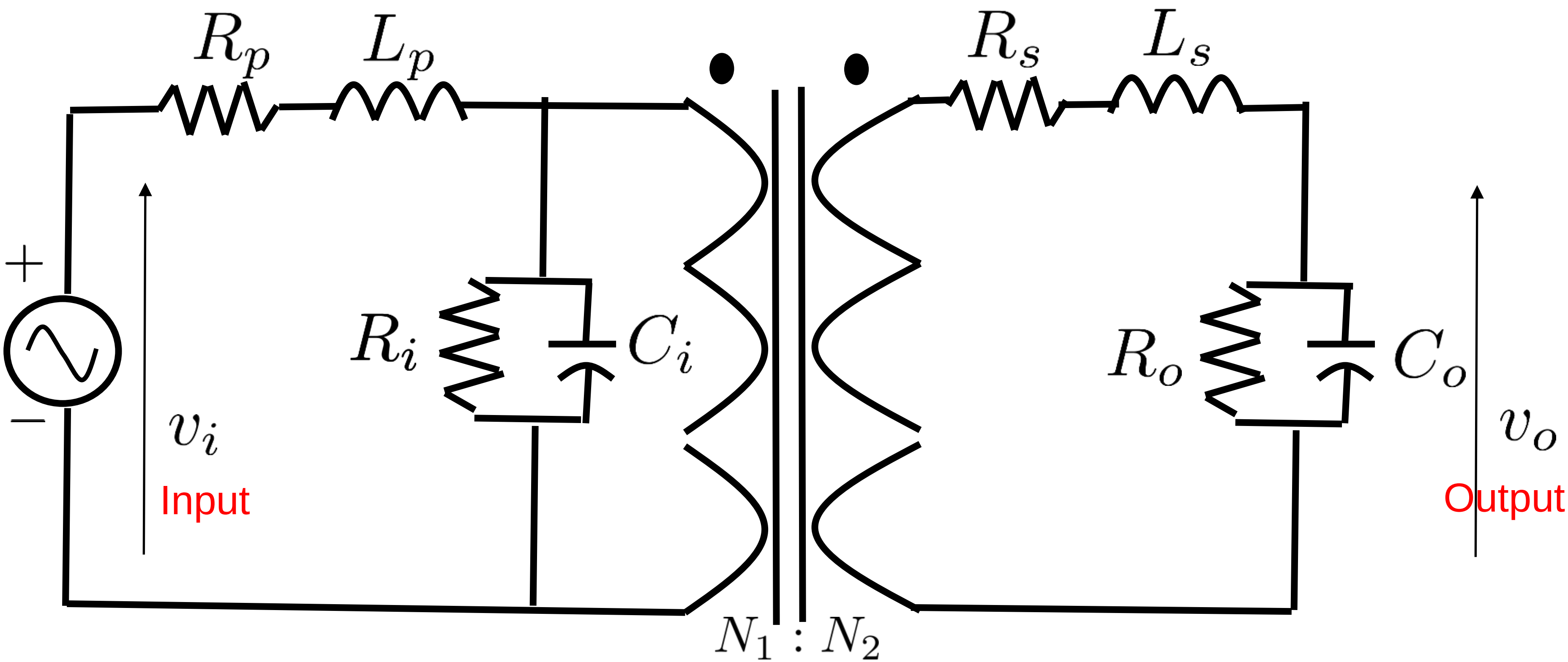
Example: RLC circuit (parallel) (cont'd)

- Linear graph drawing
- State variable selection
- Loop & node equations
- State equation

Take a node for each different voltage point.

Direct all arrows on passive elements away from sources and toward the reference node.

An exercise



Summary

- Linear graph for electrical systems
  - Single-port elements
    - Energy storage elements
    - Energy dissipation elements
    - Energy sources
  - Two-port elements (Energy transfer elements)
    - Transformer
    - (Gyrator)
- Derivation of state-space models from linear graphs
- Homework 3: Due Oct 7 (Monday), 3pm