

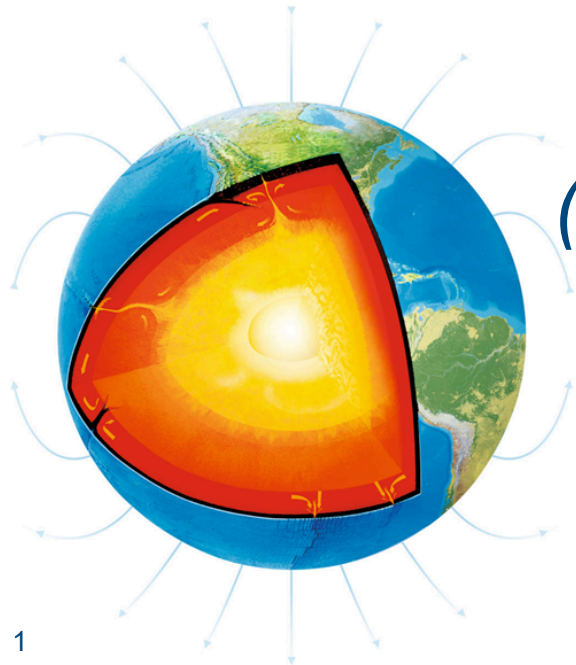
Einführung in die Geophysik

Teil 2: Gravimetrie

2. BSc Geophysik und Geoinformatik (BGIP)

Thomas Günther

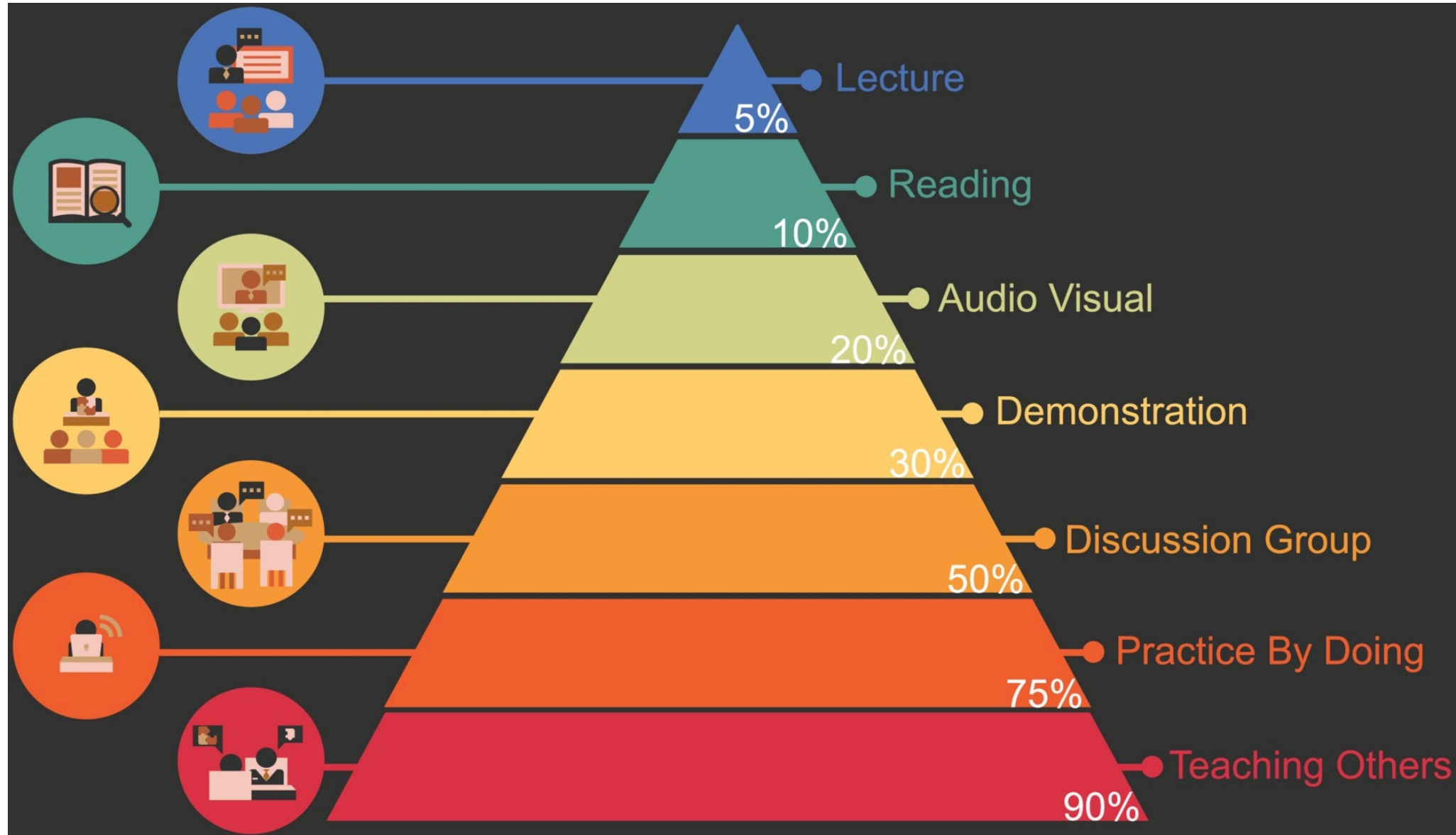
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Previously on AnGy

- Organisatorisches
- What is love? Ähm Geophysik
- Allgemeine vs. Angewandte Geophysik
- Teilgebiete (allgemein)
- Workflow Untergrunds-Erkundung
- Methoden der Angewandten Geophysik

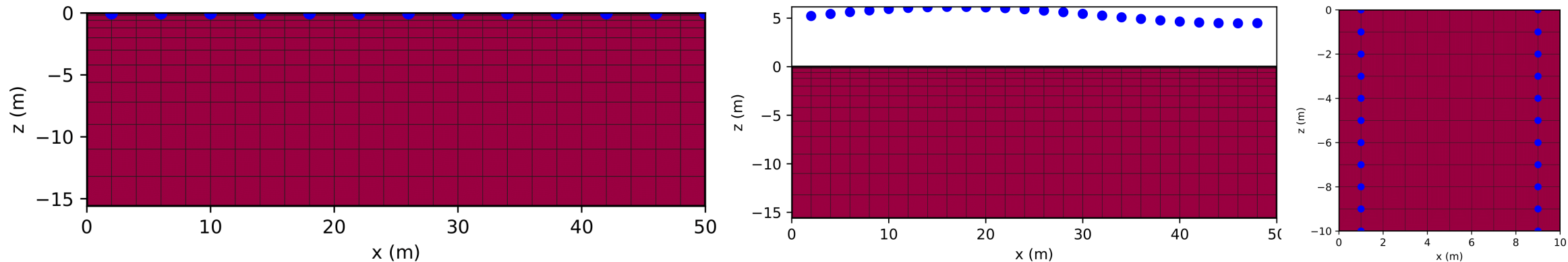
Learning pyramid



Geophysikalische Messungen

Aufprägung eines **Anregefelds** oder Nutzung eines **natürlichen Felds**

Messung eines physikalischen Felds (Erdoberfläche, Luft, Erdinneres)

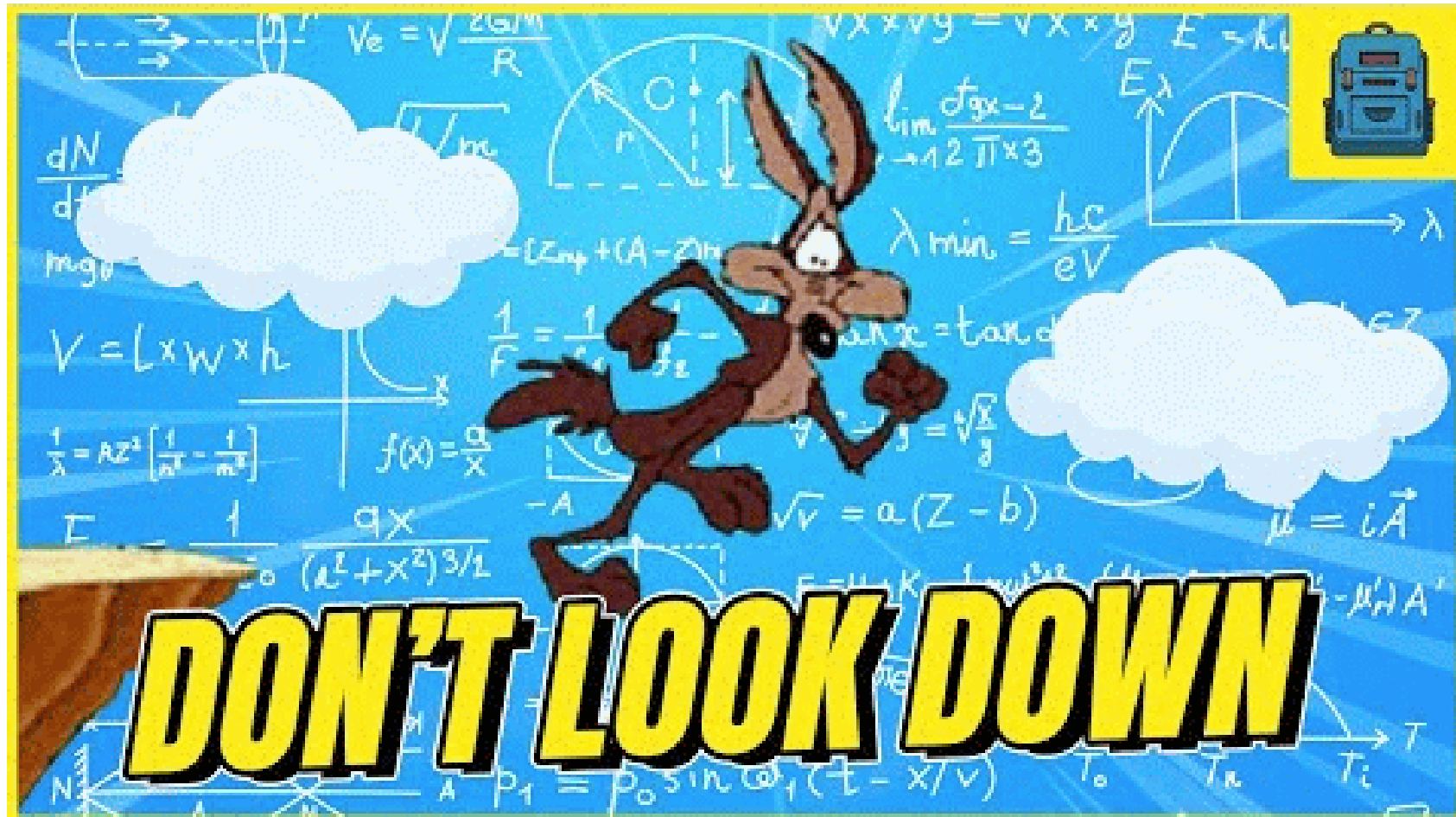


Geophysikalische Bildgebung: Rekonstruktion einer
Parameterverteilung in der Erde \Rightarrow Geologische Interpretation

Methoden der Angewandten Geophysik

Methode	Parameter	Gleichung	Anwendung
Gravimetrie	Dichte	Gravitation	Geologie
Geomagnetik	Suszeptibilität	Poisson-Eötv	UXO, Bergbau
Geoelektrik	Leitfähigkeit	Poisson-Gl.	Grundw., Umwelt
Induz. Polar.	Aufladbarkeit	Poisson-Gl.	Bergbau, Umwelt
Induktive EM	Leitfähigkeit	Maxwell-Gl.	Bergbau, Grundw.
NMR	Wassergehalt	Bloch-Gl.	Grundwasser
GPR	Permittivität	Wellengl.	Grundwasser
Seismik	Geschwindigkeit	Wellengl.	Geologie, Tektonik

Gravitation und Schwerefeld



History

Sir Isaac Newton (1643–1727): Philosophiae Naturalis Principia Mathematica (1687): Newton's laws of motion

1. inertia: an object remains at rest or continues to move at a constant velocity and direction if no force is acting on it.
2. action: The sum of forces $\vec{\mathbf{F}}$ acting on an object is equal to the mass m of the object multiplied by the acceleration $\vec{\mathbf{a}}$: $\vec{\mathbf{F}} = m\vec{\mathbf{a}}$
3. action=reaction: When an object exerts a force on another, the other exerts a force of equal magnitude but opposite direction on the first.

Newton's law of gravitation

$$\vec{\mathbf{F}} = -G \frac{m_1 m_2}{r^2} \frac{\vec{\mathbf{r}}}{r}$$

G -gravitational constant = $6.6743\text{e-}11 \text{ m}^3/\text{kg}/\text{s}^2$

$$\vec{\mathbf{g}} = -G \frac{M}{r^2} \frac{\vec{\mathbf{r}}}{r}$$

Earth's mass: $5.9722\text{e}24 \text{ kg} \Rightarrow g=9.8\text{xxxxxxxxxxx m/s}^2$.

A strange unit: Gal (after Galileo)

1Gal = 1e-2 m/s², 1mGal=1e-5 m/s², 1μGal=1e-8m/s²

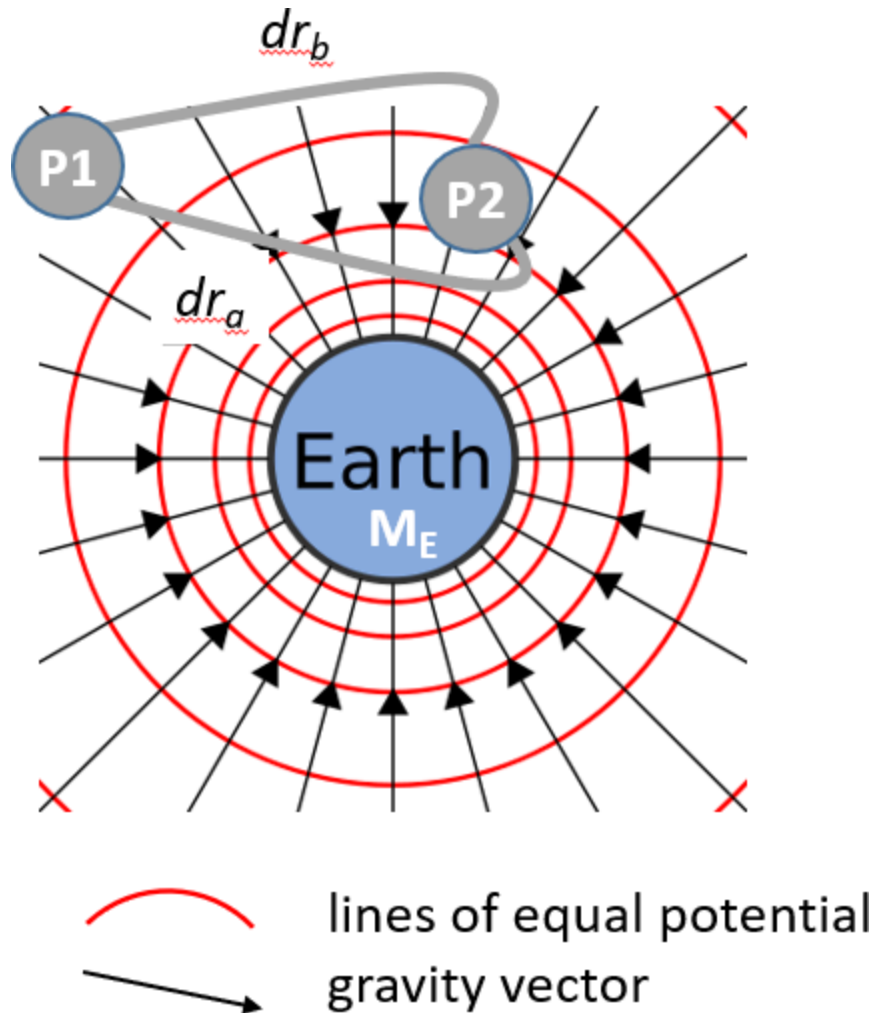
Earths acceleration: $\approx 1000 \text{ Gal} - 9.8 \times 10^{-3} \text{ m/s}^2$

typical anomalies (deviations): geological (1-10mGal), man-made 1μGal

gravity acceleration is a vector $\vec{g} = (g_x, g_y, g_z)$

in geophysical applications: $g \approx g_z$

The gravity potential



Conservative field: total work moving a mass from P1 to P2 is independent on path

If $P1=P2$ (closed loop): work is 0

$$E = - \int_{P_1}^{P_2} \vec{\mathbf{F}} d\mathbf{r} = - \int_{P_1}^{P_2} \frac{mGM}{r^2} dr$$

$$E = -m \frac{GM}{r} \Big|_{P_1}^{P_2} = mGM \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

Übergang zu Kontinuum

$$\vec{g} = -G \int_{\text{body}} \frac{dm}{r^2} \frac{\vec{r}}{r}$$

und mit Dichte $\varrho = m/V$ (z.B. Wasser 1000kg/m³ oder 1g/cm³)

$$\vec{g} = -G \int_{\text{body}} \frac{\varrho}{r^2} \frac{\vec{r}}{r} dV$$

Dichte von Sedimenten

Sedimente, Sedimentgesteine und Böden				
	Bereich	Durchschnitt (feucht)	Bereich	Durchschnitt (trocken)
		kg/m ³		kg/m ³
Alluviale Sedimente	1960 - 2000	1980	1500 - 1600	1540
Tone	1630 - 2600	1980	1300 - 2400	1700
Löß	1400 - 1930	1640	1000 - 1600	1200
Schotter	1700 - 2400	2000	1400 - 2200	1950
Sande	1700 - 2300	2000	1400 - 1800	1600
Böden	1200 - 2400	1920	1000 - 2000	1460
Sandsteine	1610 - 2760	2350	1600 - 2680	2240
Tonsteine	1770 - 3200	2400	1560 - 3200	2100
Kalksteine	1930 - 2900	2550	1740 - 2760	2110
Dolomite	2280 - 2900	2700	2040 - 2540	2300

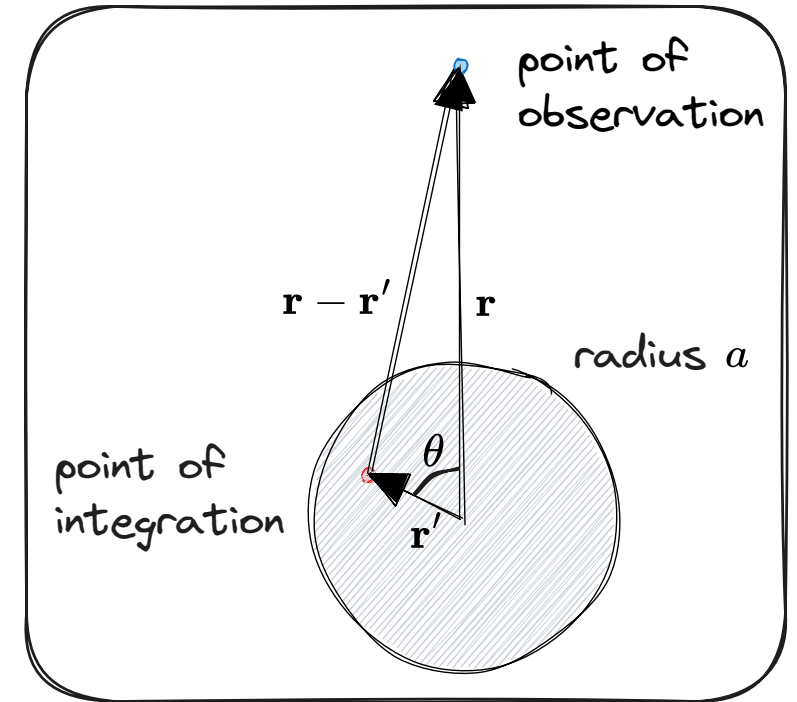
Dichte magmatischer & metamorpher Gesteine

Magmatite	Bereich	Durchschnitt
	kg/m ³	
Obsidian	2200 - 2400	2300
Dazit	2350 - 2800	2580
Andesit	2400 - 2800	2610
Granit	2500 - 2810	2640
Granodiorit	2670 - 2790	2730
Basalt	2700 - 3300	2990
Gabbro	2700 - 3500	3030
Peridotit	2780 - 3370	3150

Metamorphe Gesteine	Bereich	Durchschnitt
	kg/m ³	
Quarzite	2500 - 2700	2600
Schiefer	2390 - 2900	2640
Granulite	2520 - 2730	2650
Phyllite	2680 - 2800	2740
Marmor	2600 - 2900	2750
Serpentinite	2400 - 3100	2780
Gneise	2590 - 3000	2800
Amphibolite	2900 - 3040	2960
Eklogite	3200 - 3540	3370

Newtons shell theorem

1. A spherical symmetric body causes a gravitational force on external objects as if all of its mass were concentrated in a point in its center.
2. If the body is a spherically symmetric shell, then no net gravitational force is exerted by the shell on any object inside regardless of the objects's location within the shell.



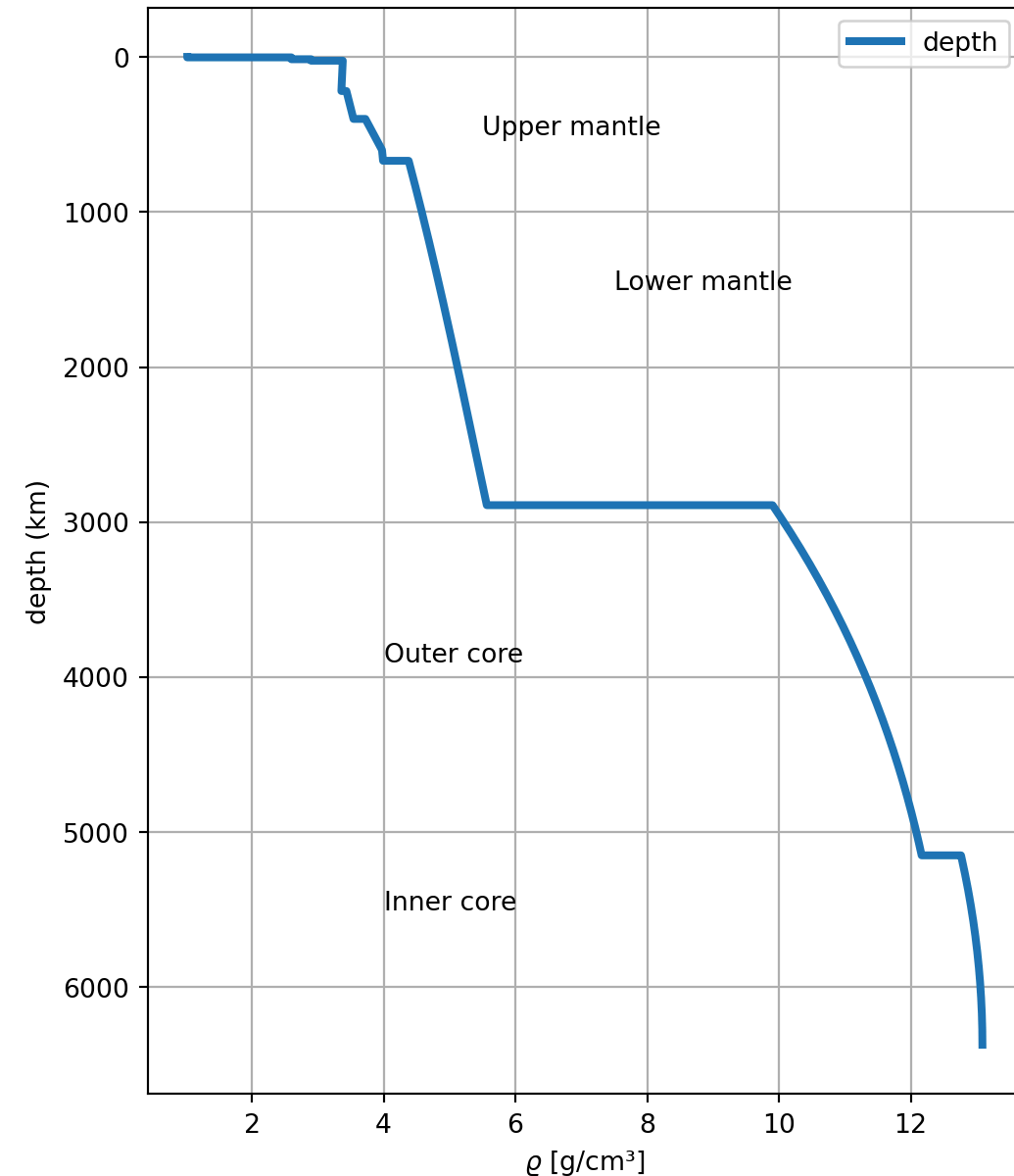
Proof: [Potential theory](#)

PREM - Density

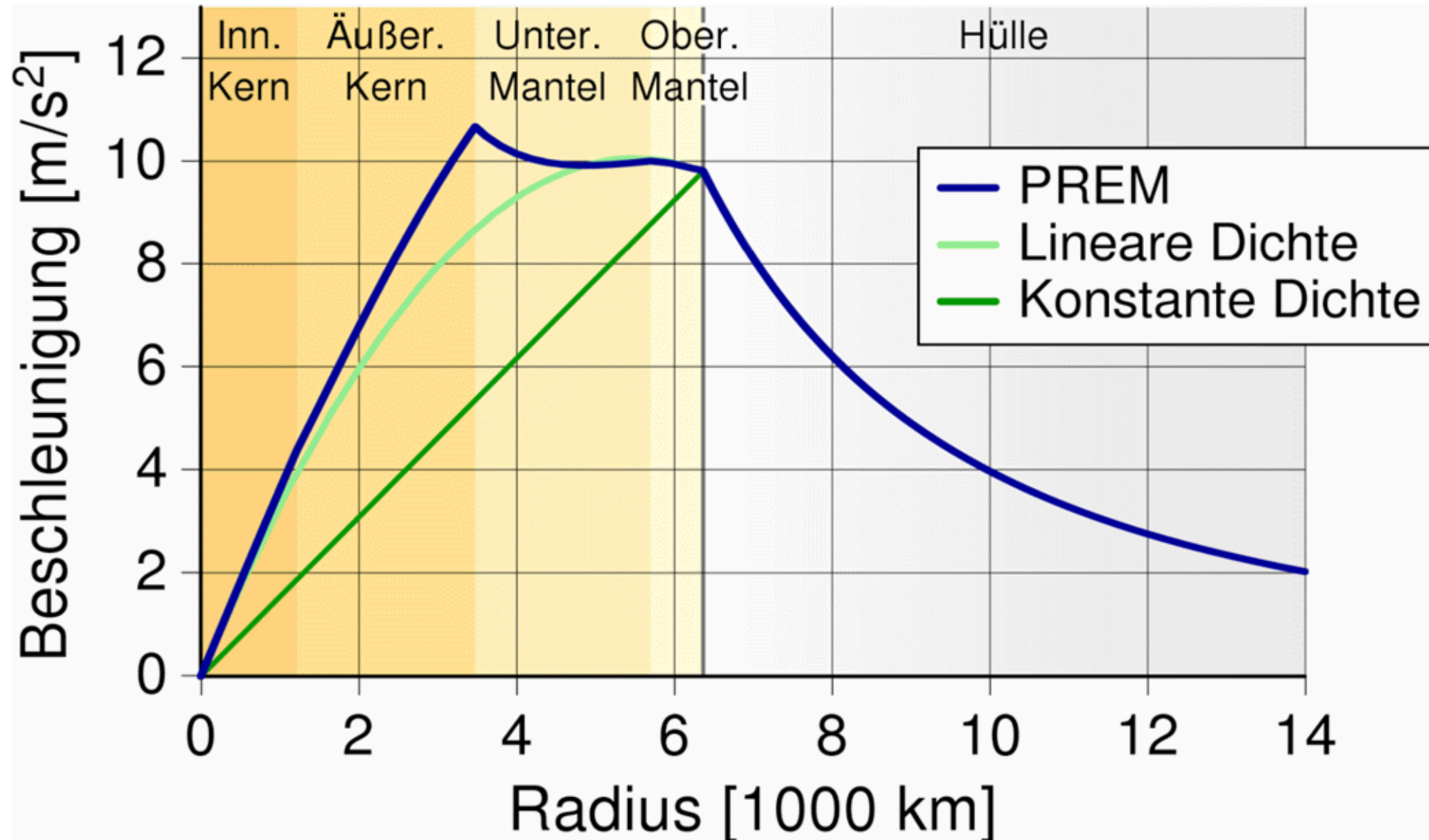
```
# !pip install rockhound
import rockhound as rh

prem = rh.fetch_prem()
fig, ax = plt.subplots(figsize=(6, 7.5))
prem.plot("density", "depth", lw=3,
          legend=False, ax=ax)

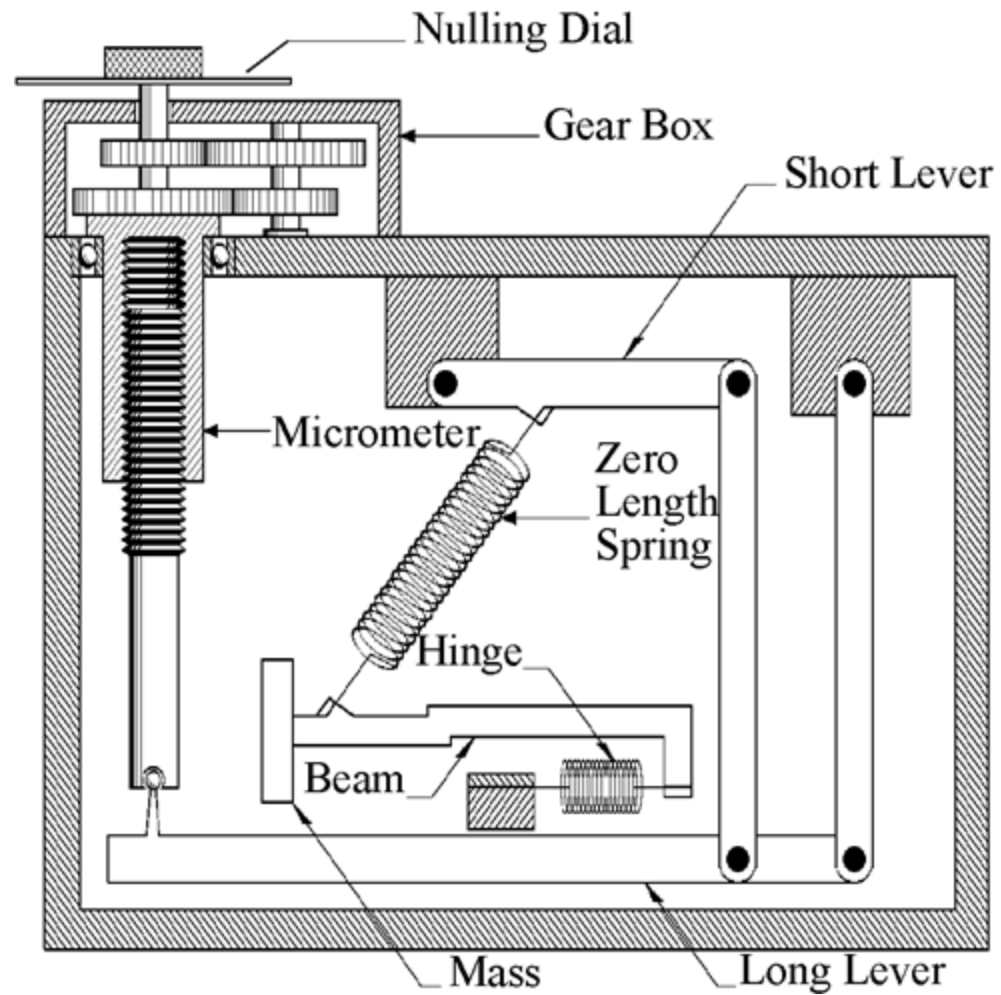
ax.invert_yaxis()
ax.grid()
ax.legend()
ax.set_xlabel(r"$\varrho$ [g/cm3]");
ax.set_ylabel("depth (km)")
ax.text(4, 5500, "Inner core")
ax.text(4, 3900, "Outer core")
ax.text(7.5, 1500, "Lower mantle")
ax.text(5.5, 500, "Upper mantle");
```



Fallbeschleunigung in der Erde



Gravimetrie



Lacoste-Romberg-Gravimeter

Freiluftkorrektur

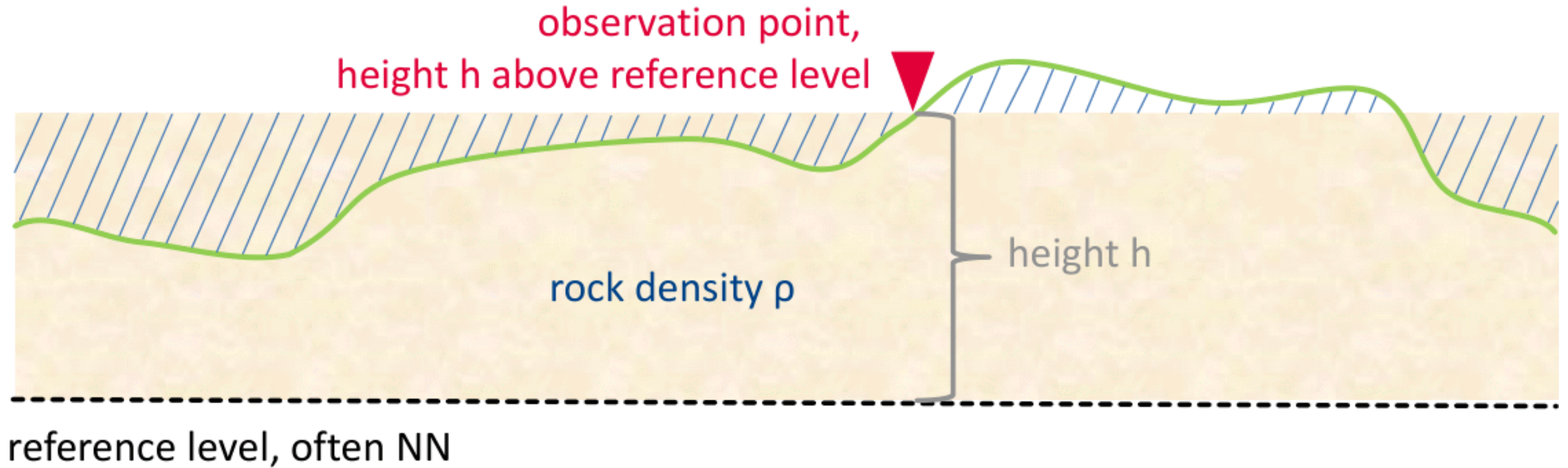
$$\vec{g} = -G \frac{M}{r^2} \frac{\vec{r}}{r}$$

Ableitung in radiale Richtung

$$\frac{\partial g}{\partial r} = 2G \frac{M}{r^3} = -2 \frac{g_z}{r}$$

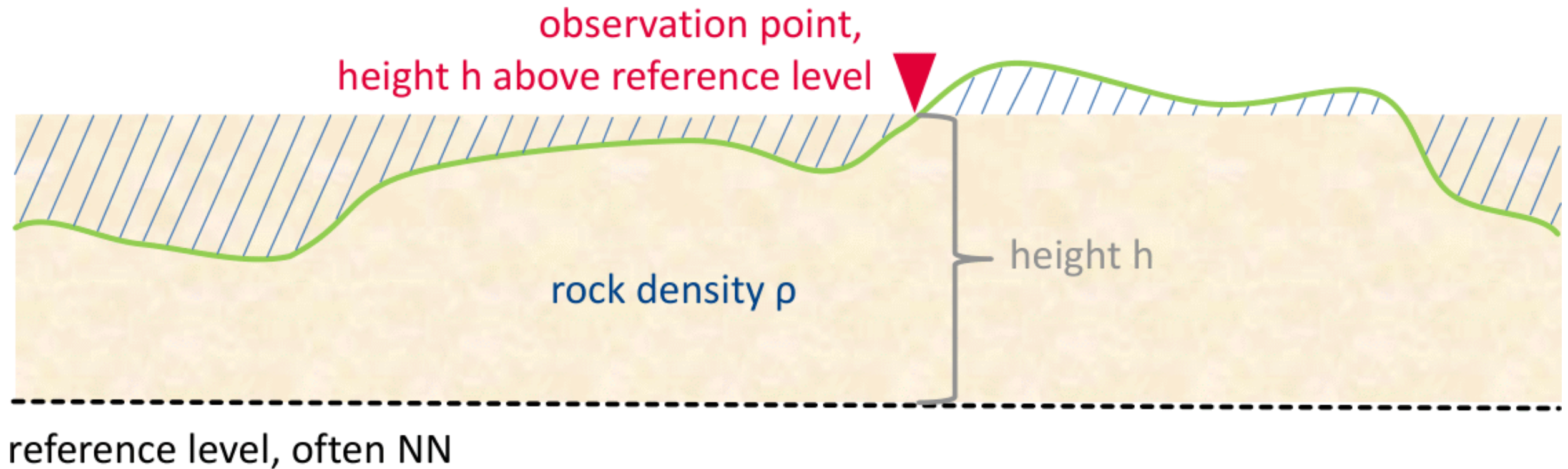
Erdoberfläche ($r = R_E = 6371 \text{ km}$): $g_{zz} = \frac{\partial g_z}{\partial r} = -0.308 \text{ mGal/m}$

Bouguer-Korrektur



Bouguer-Platte: $\delta g_B = 2\pi G \rho h$

Bouguer-Anomalie

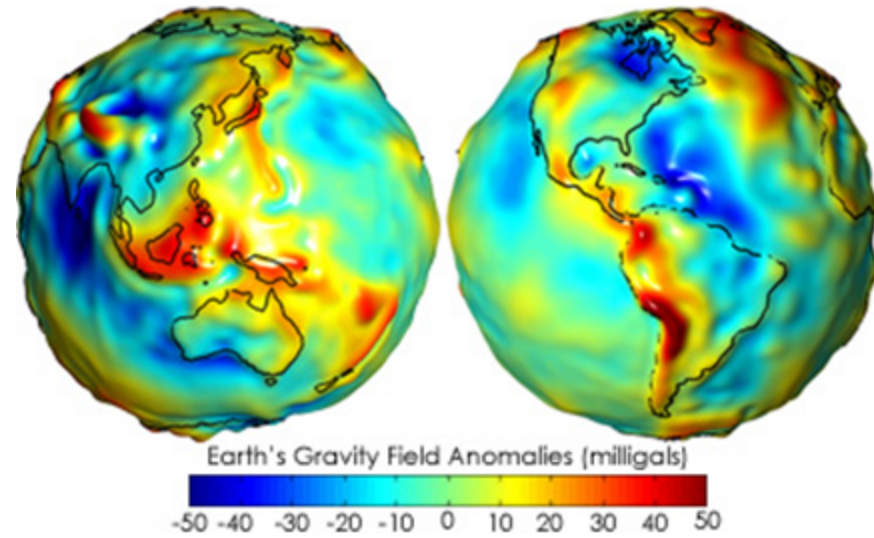


$$g'' = g_{obs} - [\gamma - 0.3086h - 2\pi G \rho h - \delta g_{Topo}]$$

Normalschwere Referenzellipsoid: $\gamma = \gamma_e(1 + \beta \sin^2 \phi - \beta_1 \sin^2 2\phi)$

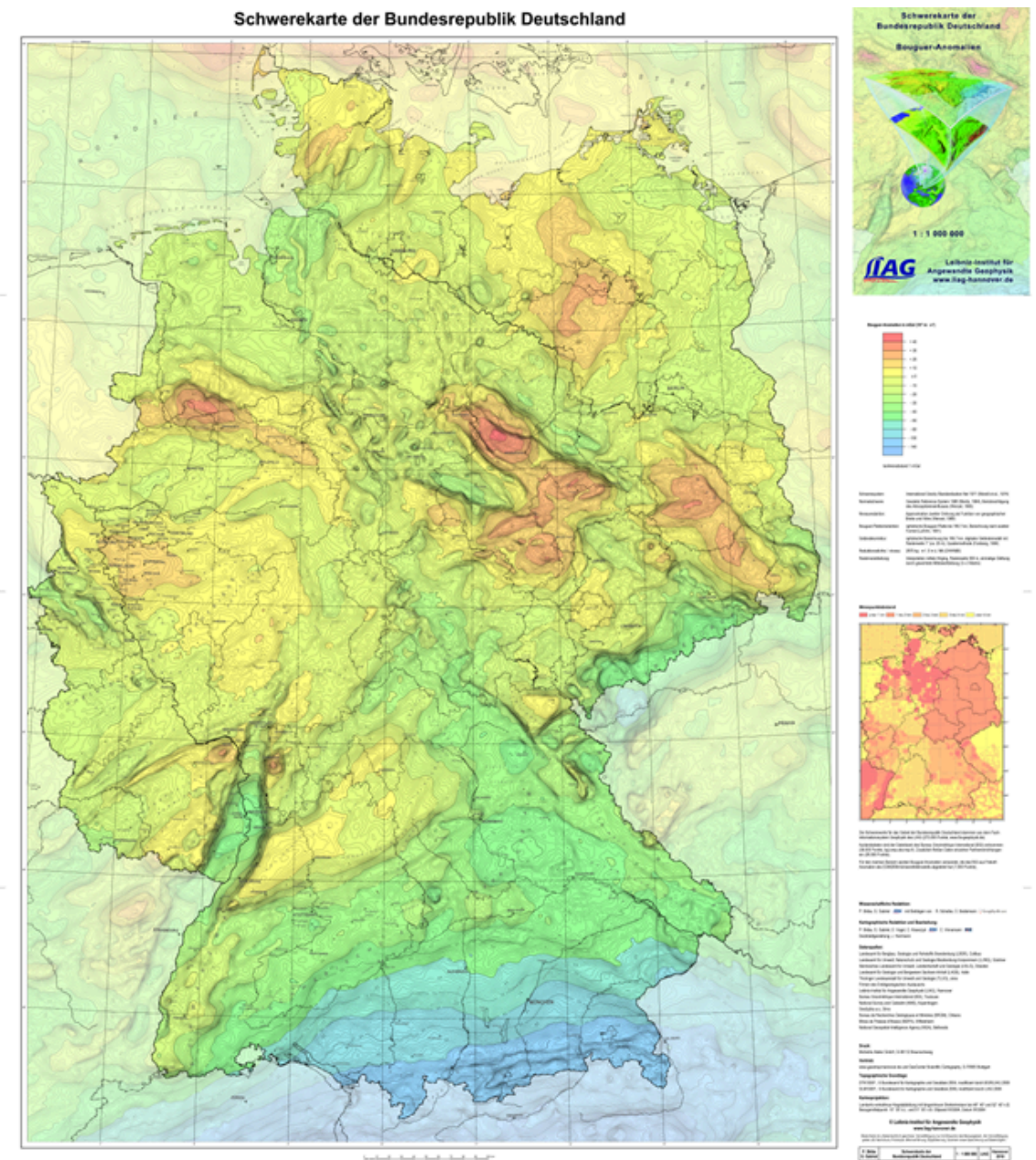
Globale Schwereanomalien

Bouguer-Schwere weltweit

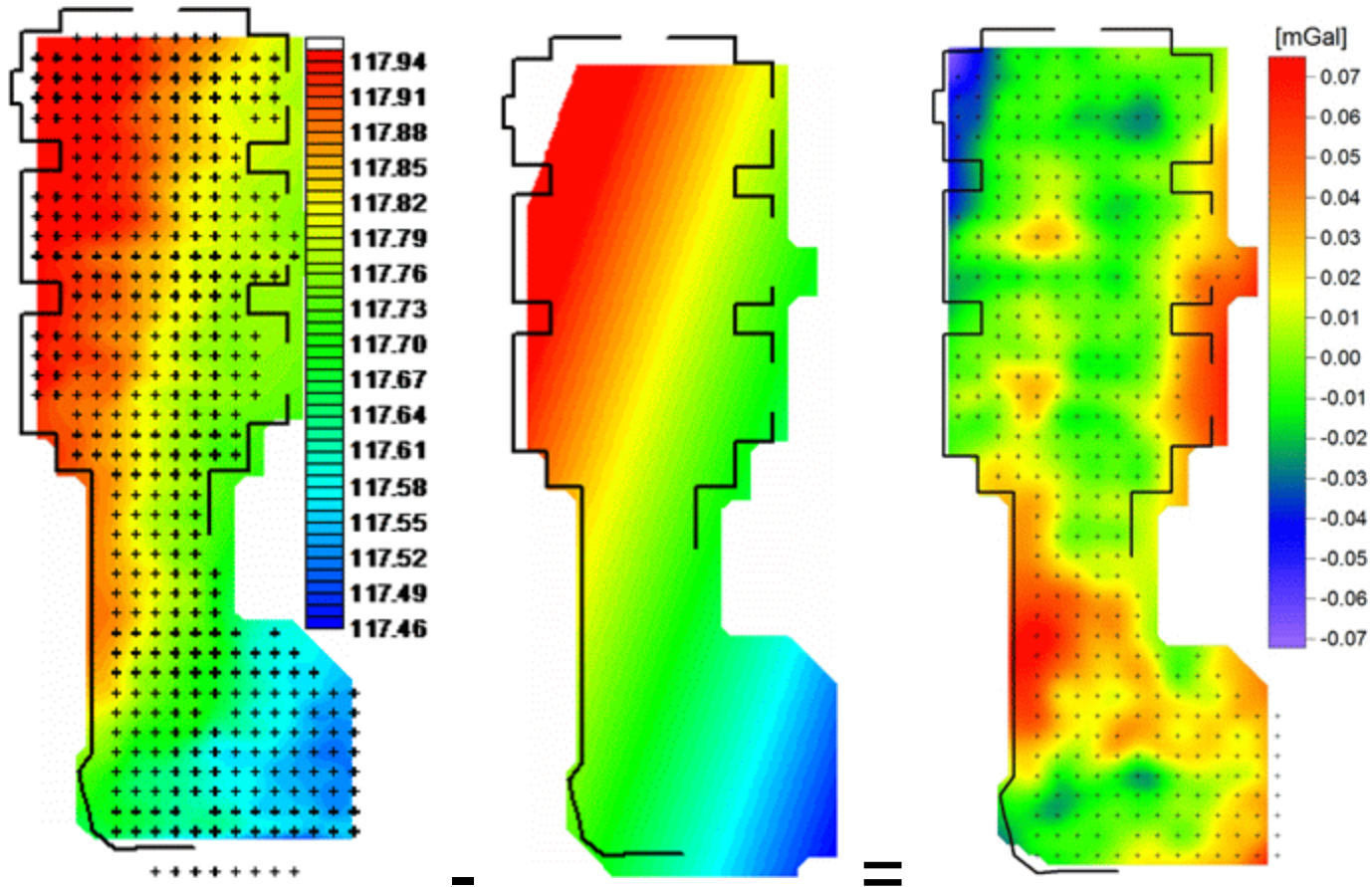


Center for space research, U Texas, Austin

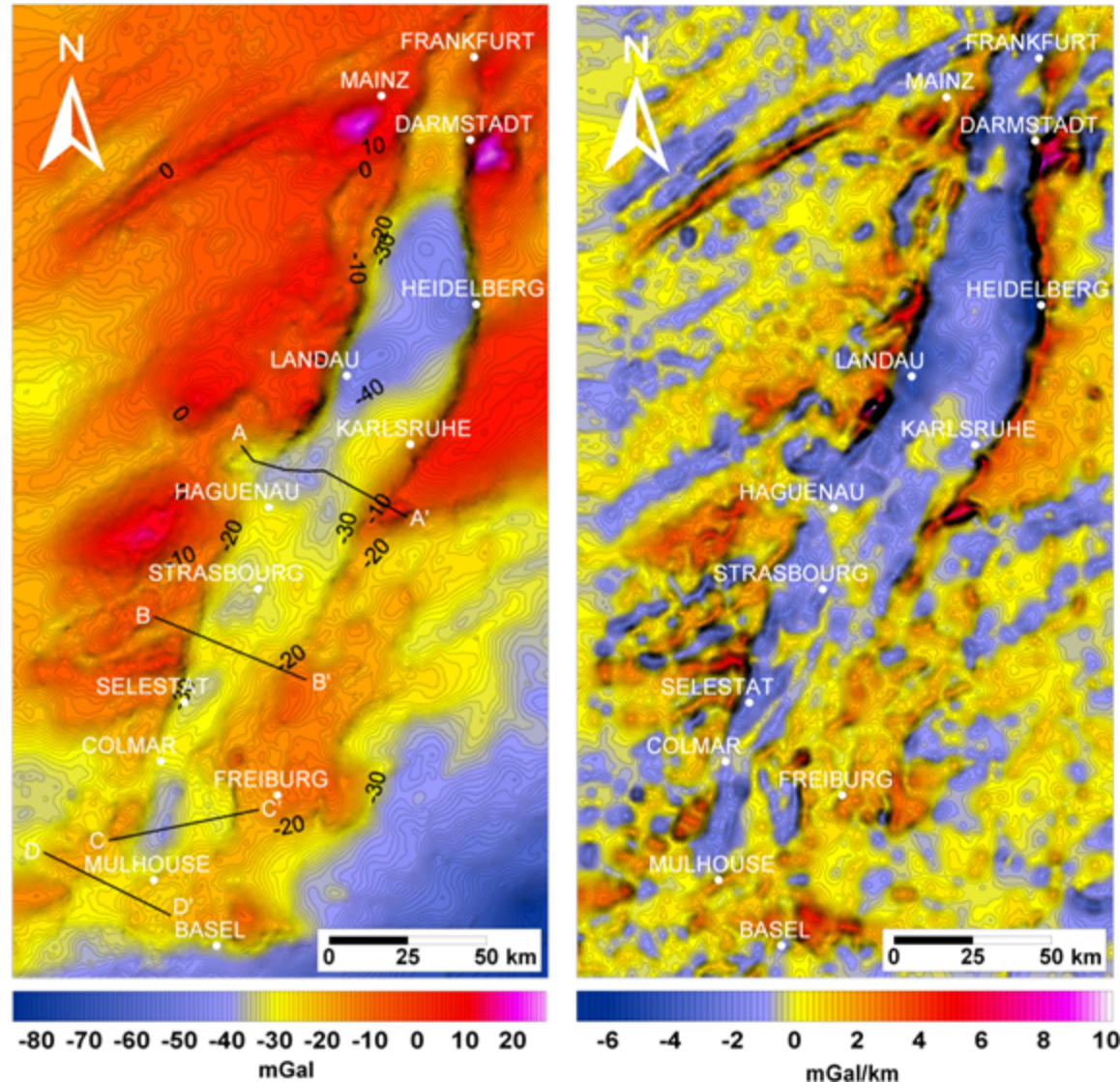
⇒ Dichteunterschiede in Kruste



Residuenbestimmung



Anomalie Schwere und Schweregradient



Kartierung des Rheingrabens
(Rotstein et al., 2006,
Tectonophysics 425, 55-70)

Vertikal-Gradient hebt
(tektonische) Strukturen hervor

Störkörper-Formeln

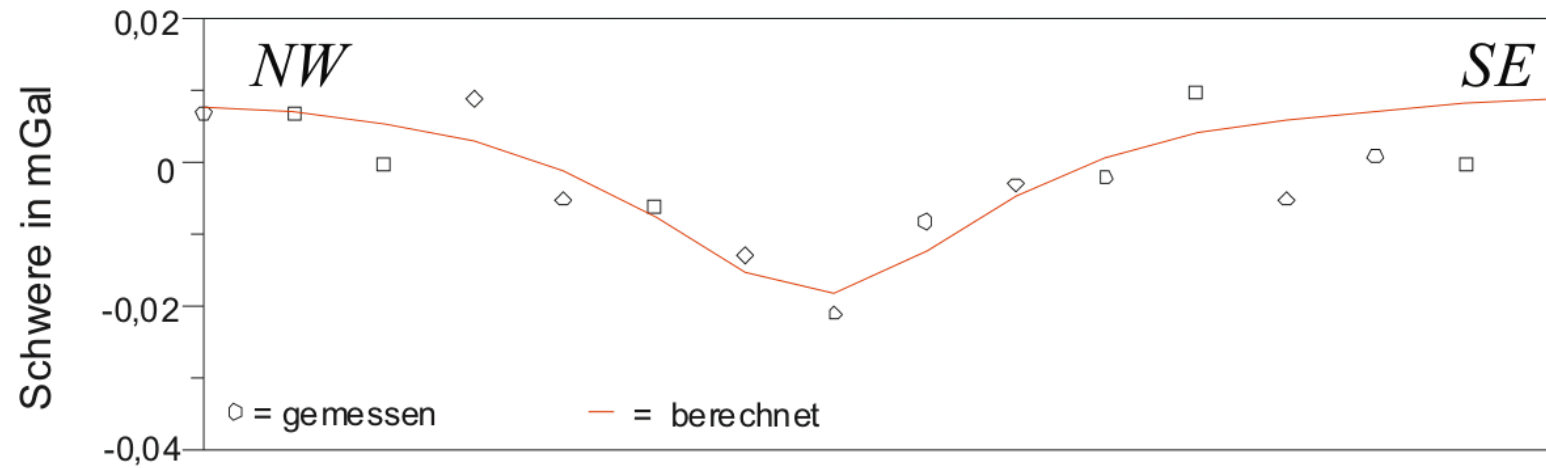
Kugel (=Punktmasse)

$$\vec{g} = Gm \frac{\vec{r}}{r^3} = \Delta \varrho G \frac{4}{3} \pi R^3 \frac{\vec{r}}{r^3}$$

Liegender Zylinder

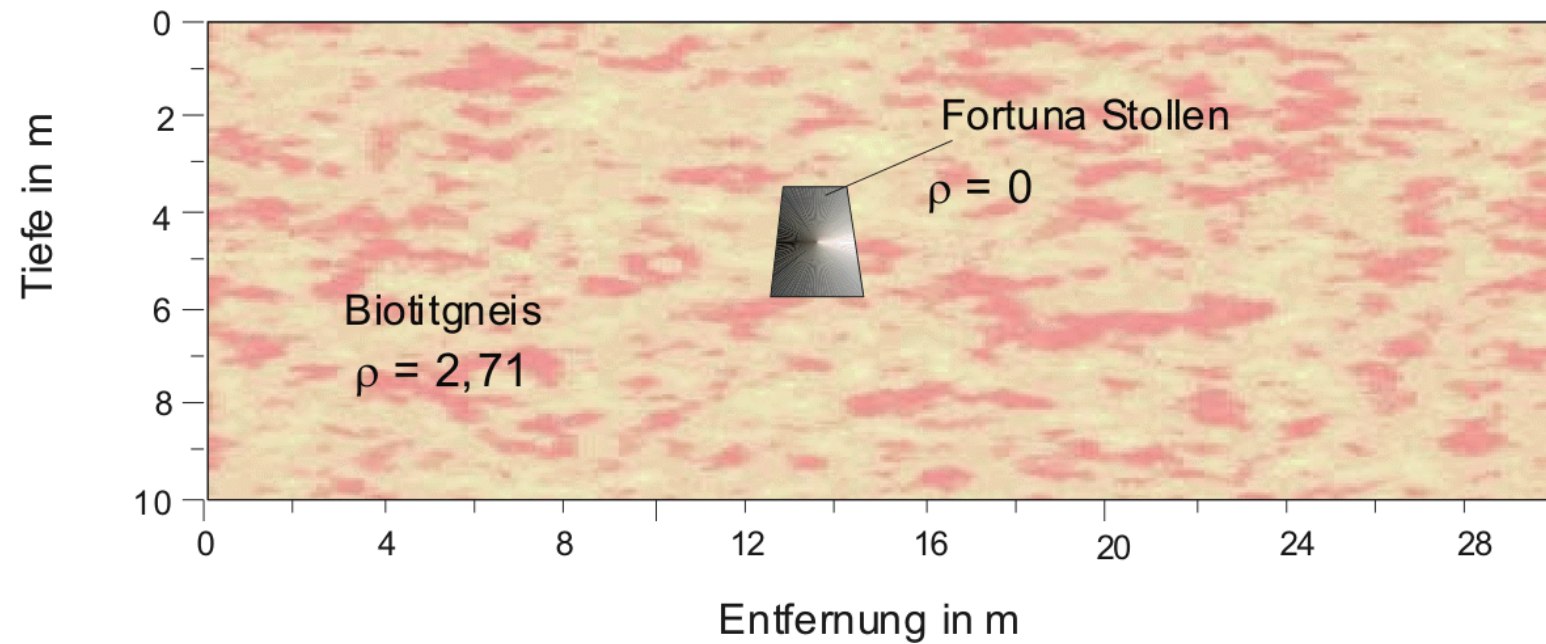
$$\vec{g} = GA \Delta \varrho \frac{\vec{r}}{r^3} = \Delta \varrho G 2\pi R^2 \frac{d}{r^2}$$

Hohlraumsuche Fortunastollen



Lindner & Casten
(Knoedel et al. 2005)

1 Geophysiker, 1
Meßgehilfe (ich)



Wrap up

1. Focus: geological exploration (large-scale), void search
2. anomalies 0.1-1-10-30 mGal ($1\text{e-}5 \text{ m/s}^2$) 1Mio times smaller than g!
3. variable (spatially & temporally) background field
4. very accurate instruments necessary ($\sim 10\mu\text{Gal}$ for near surface)
5. caused by density (kg/m^3) contrasts: air 0, water 1000, loose sediments 1500-2000, consolidated 2000-2500, hardrocks 2500-3500
6. equivalent models (ambiguity)
7. Solution by forward modelling (simple) structures \Rightarrow structural geological model (seismics helps a lot)
8. Further reading: Knoedel textbooks (DE or EN)