Inverse Probleme in der Geophysik
Vorlesung (Vertretung K. Spitzer)
TU Bergakademie Freiberg, SS 2020

Teil 12: Abschluss der Veranstaltung

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Heutige Veranstaltung

Abschluss nichtlineare Probleme

- Auflösungsmatritzen für nichtlineare Probleme (pdf)
- Inversion mit mehreren Datentypen (1D-MT JNB und Beleg Teil 2)
- Inversion mit mehreren Modellgrößen (NMR)
- 2D-Geoelektrik-Vorführung (JNB) mit struktureller A-priori-Information
- Parameter-Informationen und Regionskonzept (BERT)
- Joint inversion
  - klassische JI: gleicher Parameter (pg: DC+EM)
  - petrophysikalische JI: Überführung auf klassisch (pg Example)
  - strukturell gekoppelte Inversion (pg 1D MRS)
  - voll gekoppelte (hydrogeophysikalische) Inversion (pg)
- Alternative Inversionsmethoden (Genetische Algorithmen)

Auflösungsmatritzen nichtlinearer Probleme

Iterativ ($\mathbf{m}^{k+1} = \mathbf{m}^k + \Delta\mathbf{m}^k$) gelöstes inverses Subproblem:

$$(\mathbf{S}^T\mathbf{S} + \lambda^2\mathbf{C}^T\mathbf{C})\Delta\mathbf{m}^k = \mathbf{S}^T(\mathbf{d} - \mathbf{f}(\mathbf{m})) - \lambda^2\mathbf{C}^T\mathbf{C}(\mathbf{m}^k - \mathbf{m}^0)$$

$$\mathbf{m}^{k+1} = \mathbf{m}^k + \mathbf{S}^\dagger(\mathbf{d} - \mathbf{f}(\mathbf{m}^k)) + \mathbf{C}^\dagger\mathbf{C}(\mathbf{m}^k - \mathbf{m}^0)$$

Mit

$$\mathbf{d} = \mathbf{f}(\mathbf{m}^{true}) + \mathbf{n} = \mathbf{f}(\mathbf{m}^k) - \mathbf{S}(\mathbf{m}^{true} - \mathbf{m}^k) + \mathbf{n}$$

ergibt sich

$$\begin{aligned}
\mathbf{m}^{est} &= \mathbf{m}^k + (\mathbf{S}^\dagger\mathbf{S} + \mathbf{C}^\dagger\mathbf{C})(\mathbf{m}^{true} - \mathbf{m}^k) &&+\mathbf{S}^\dagger\mathbf{n} && (1)\\
&= \mathbf{S}^\dagger\mathbf{S}\mathbf{m}^{true} - (\mathbf{S}^\dagger\mathbf{S} - \mathbf{C}^\dagger\mathbf{C})\mathbf{m}^k + \mathbf{C}^\dagger\mathbf{C}\mathbf{m}^0 &&+\mathbf{S}^\dagger\mathbf{n} && (2)\\
&= \mathbf{R}^M\mathbf{m}^{true} + (\mathbf{I} - \mathbf{R}^M)\mathbf{m}^0 &&+\mathbf{S}^\dagger\mathbf{n} && (3)
\end{aligned}$$

wie bei linearem Problem, hängt aber von der Sensitivität ab. $\mathbf{R}^D$ analog. -

Inversion verschiedener Datentypen

Beispiel Magnetotellurik

Modell: spezifischer Widerstand als Funktion der Tiefe (logarithmisch)

$$\mathbf{m} = [\log \rho_1, \log \rho_2, \ldots, \log \rho_M]^T$$

Daten: scheinbarer spezifischer Widerstand Amplitude (log) und Phase

$$\mathbf{d} = [\log \rho_1^a, \ldots, \log \rho_N^a, \phi_1, \ldots, \phi_N]^T$$

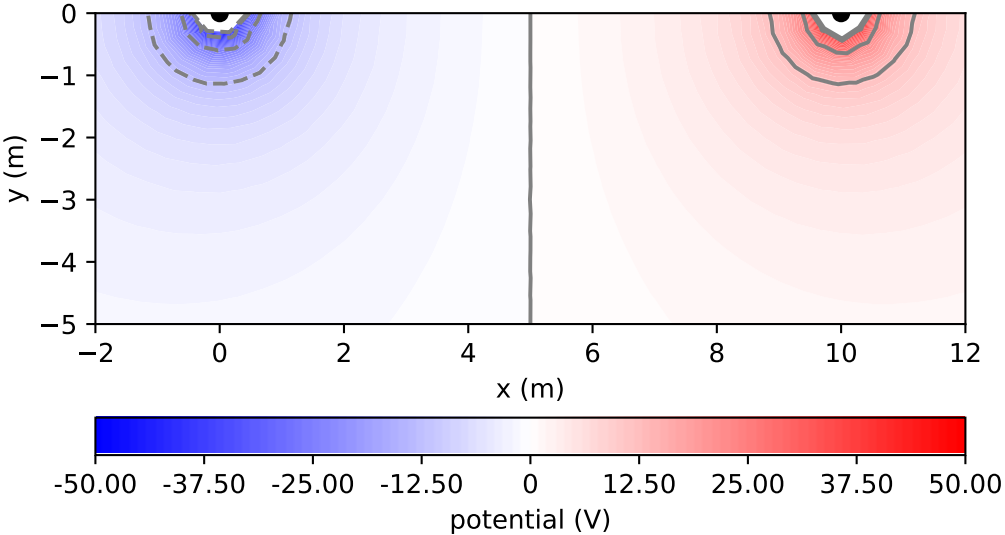
Einheitenproblem gelöst durch Fehlermodell $\delta\rho^a$ und $\delta\phi$

$$\varepsilon = [\delta\rho^a/\rho_1^a, \ldots, \delta\rho^a/\rho_N^a, \delta\phi, \ldots, \delta\phi]^T$$

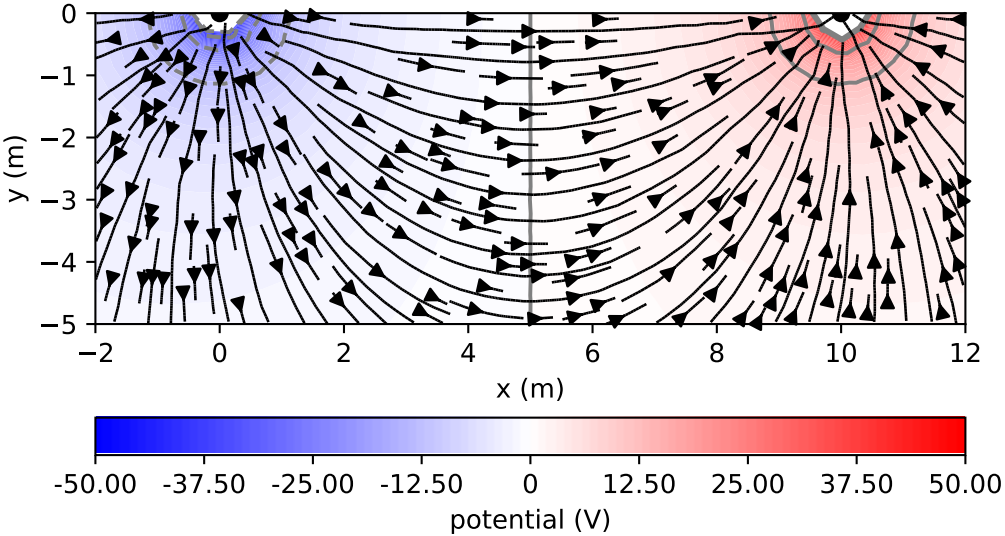
Jacobi-Matrix durch vertikales Aneinander-hängen von Matritzen

$$S_i = [\partial \log \rho_1^a/\partial\rho, \ldots, \partial \log \rho_N^a/\partial \log \rho, \partial\phi_1/\partial \log \rho, \ldots, \partial\phi_N/\partial \log \rho]^T$$

Electrical resistivity tomography - Potential and current



Electrical resistivity tomography - Potential and current



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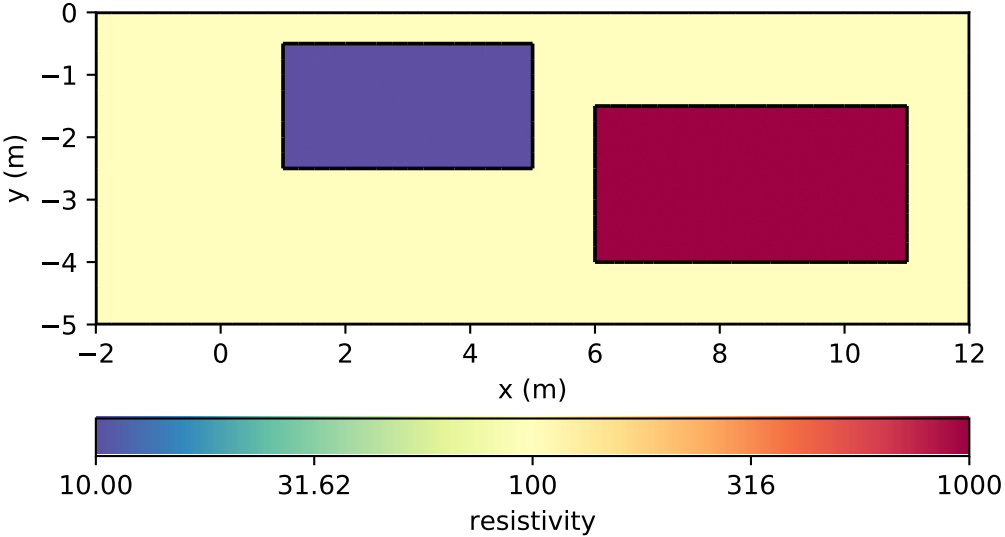

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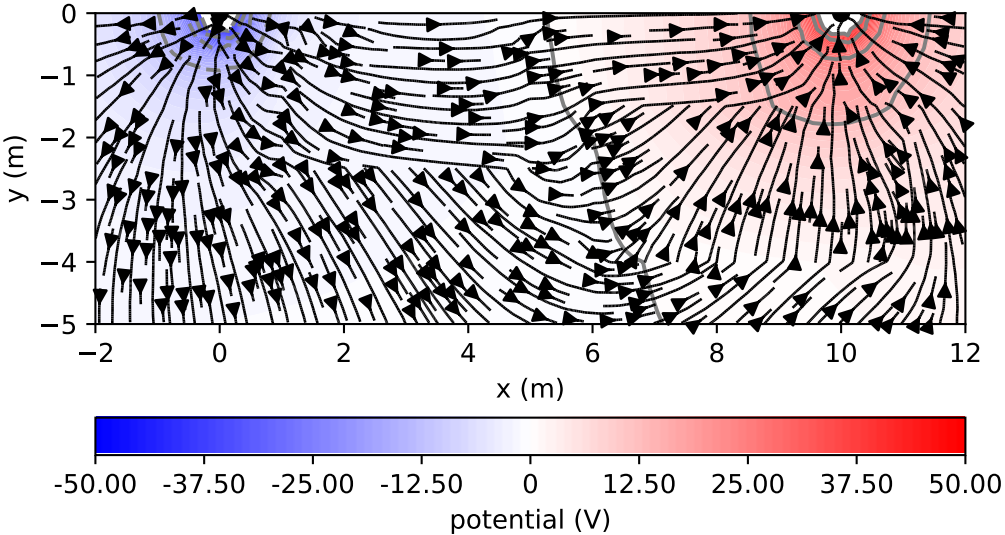
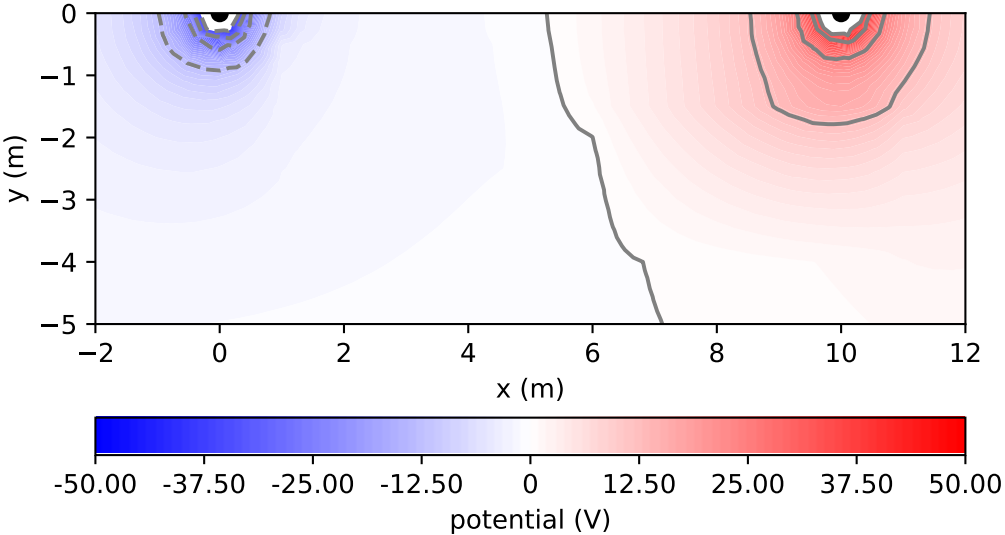


Electrical resistivity tomography - Potential and current



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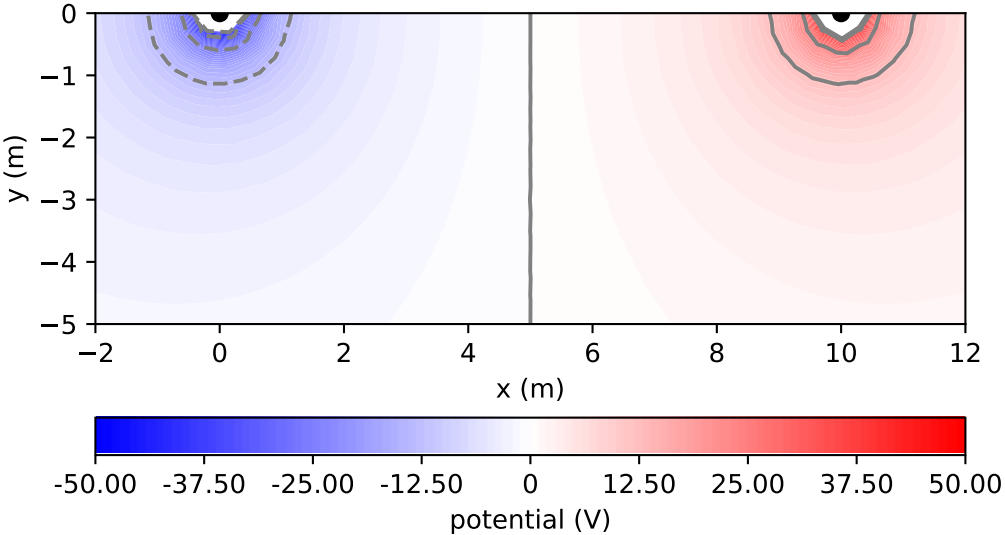
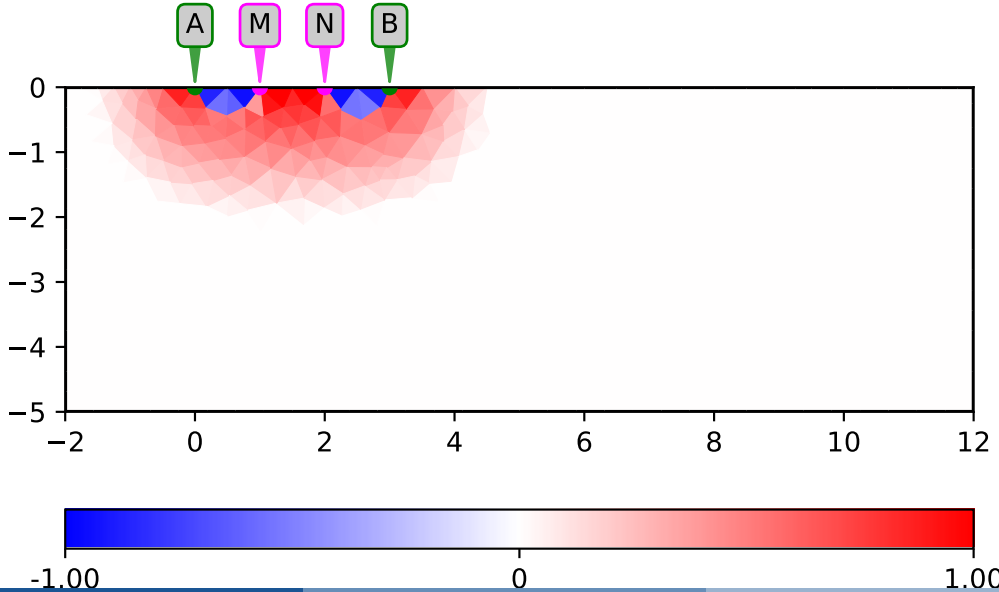
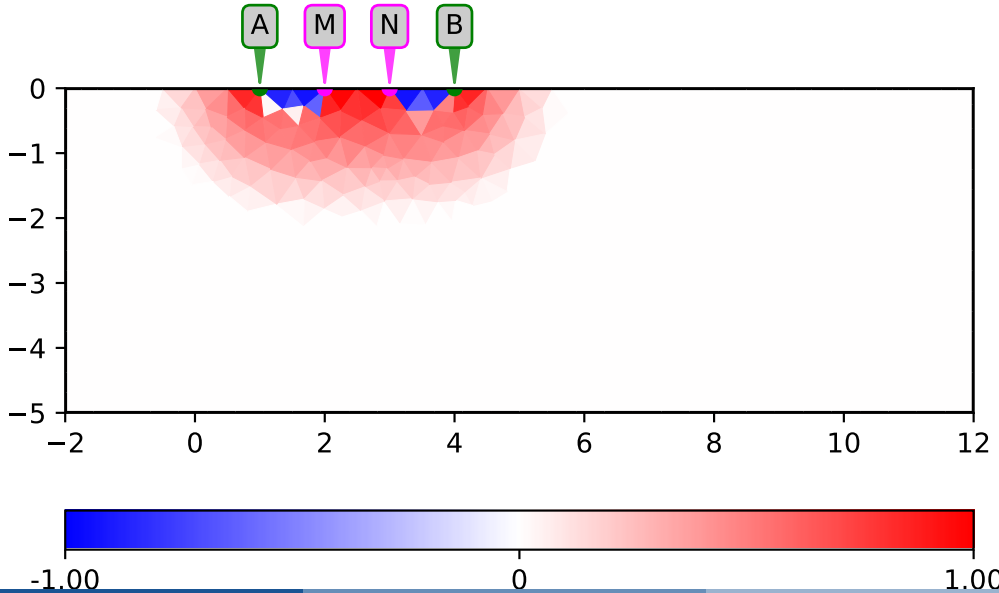


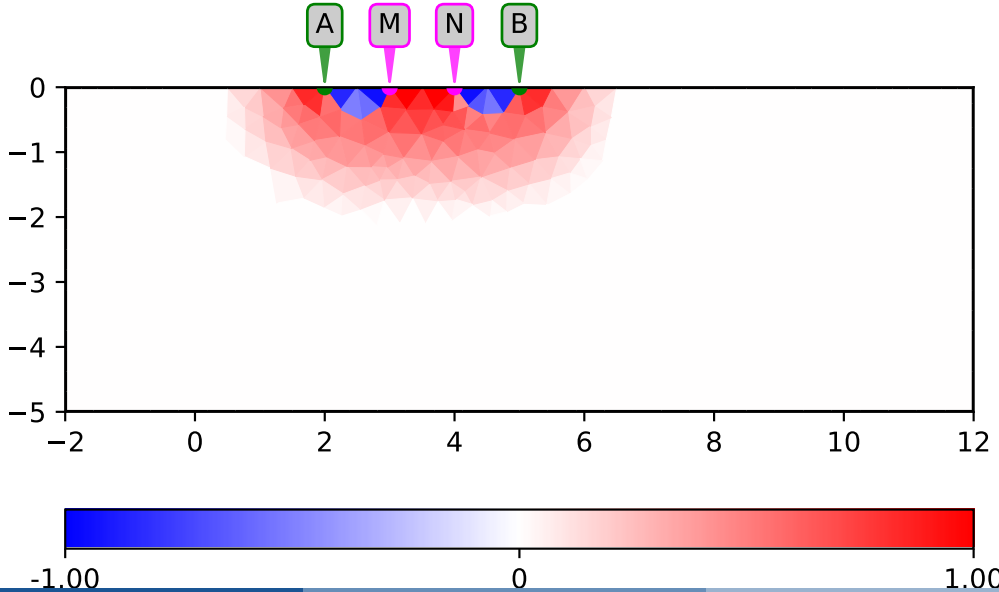
Electrical resistivity tomography - Sensitivity



Electrical resistivity tomography - Sensitivity



Electrical resistivity tomography - Sensitivity



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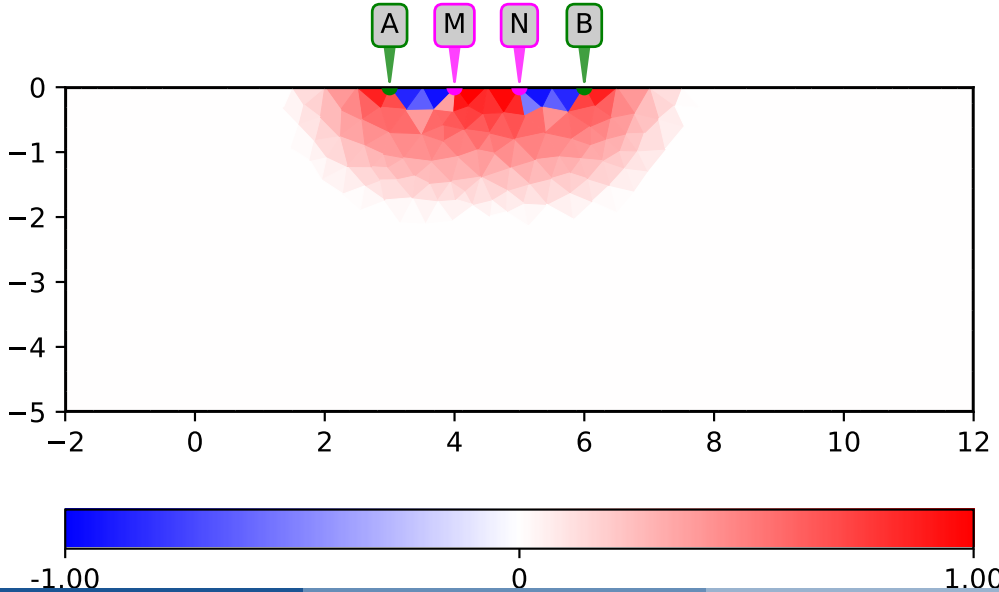

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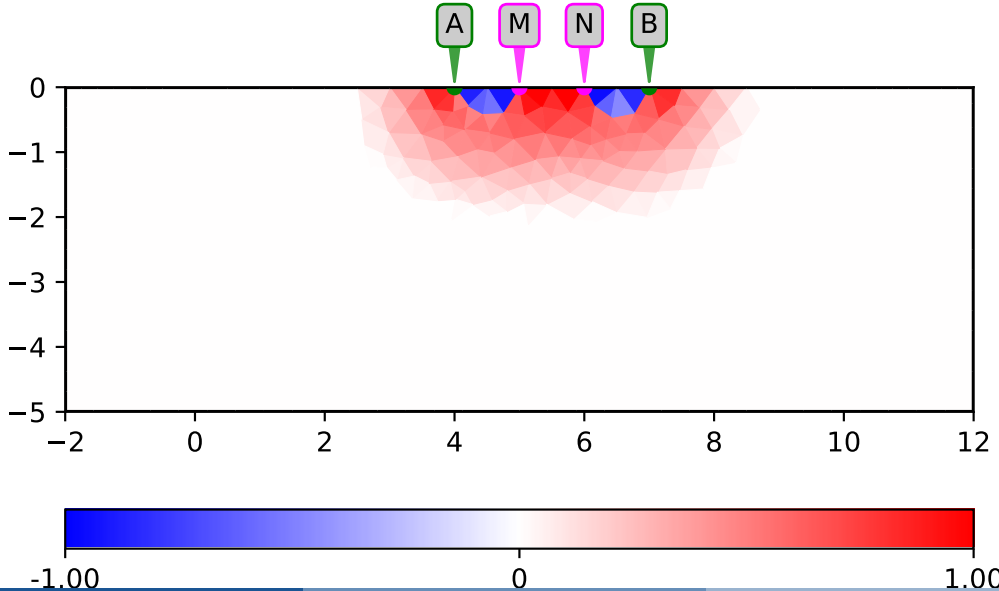

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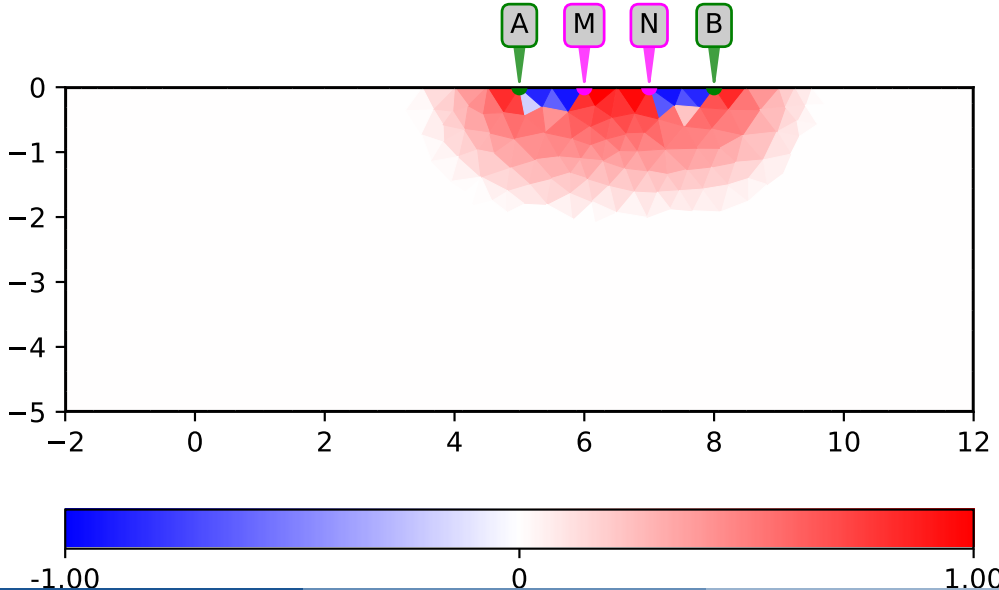

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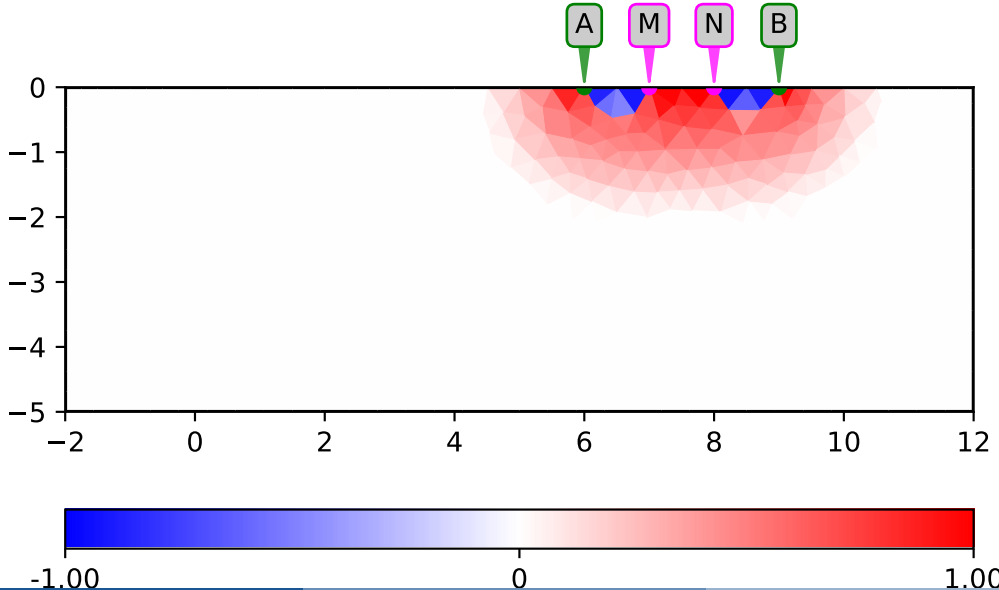

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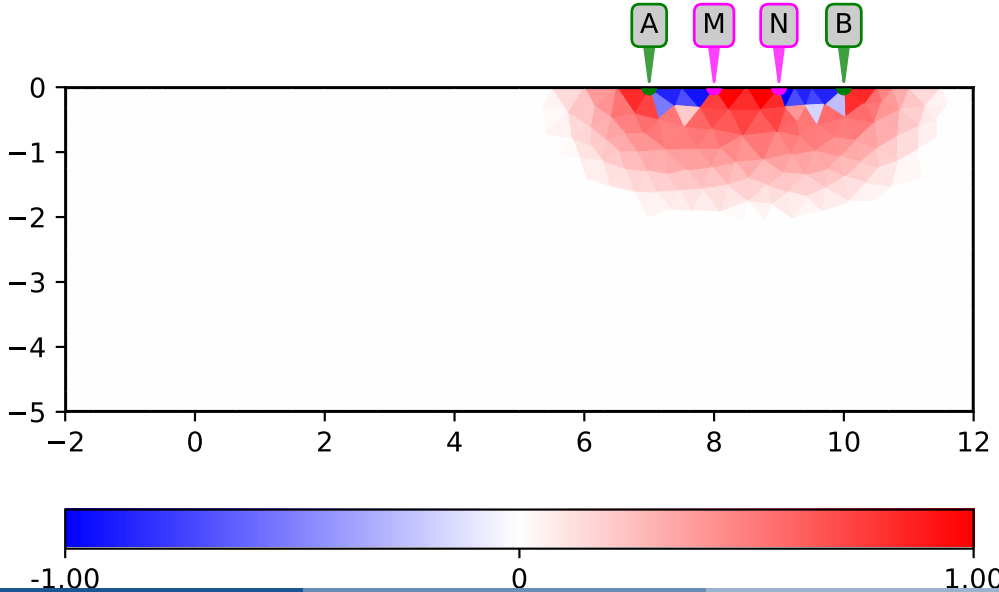

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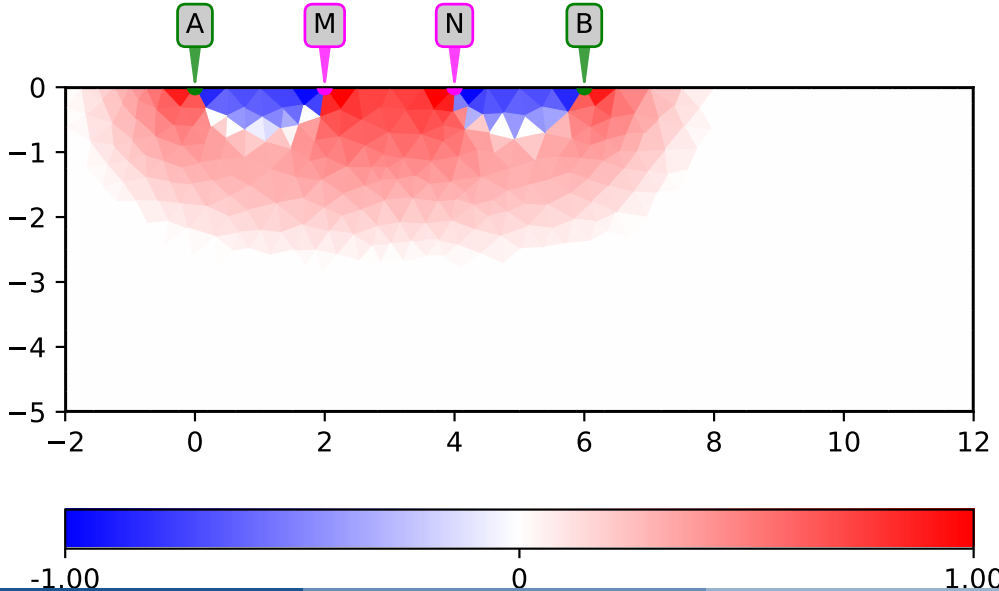

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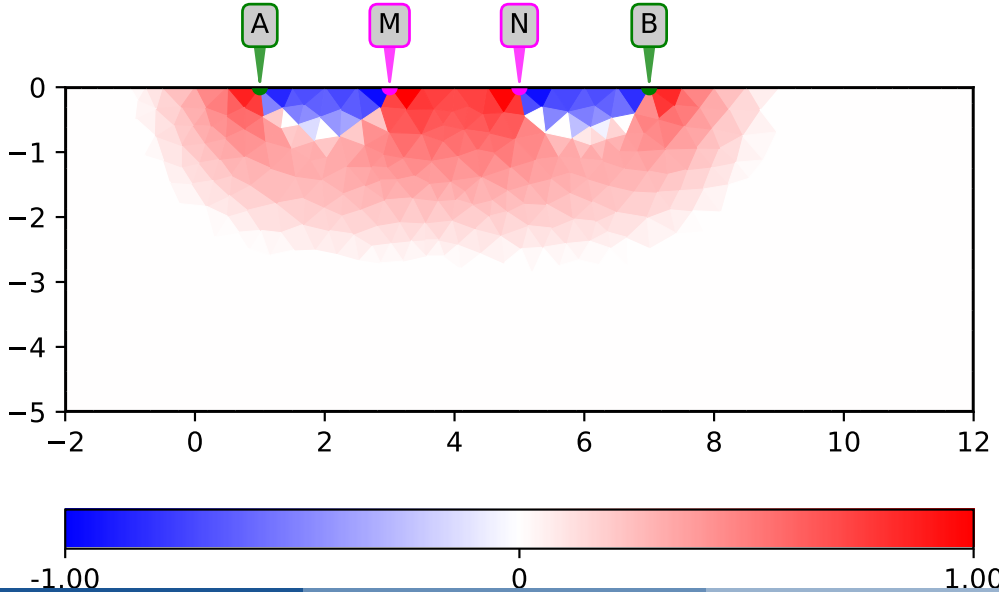

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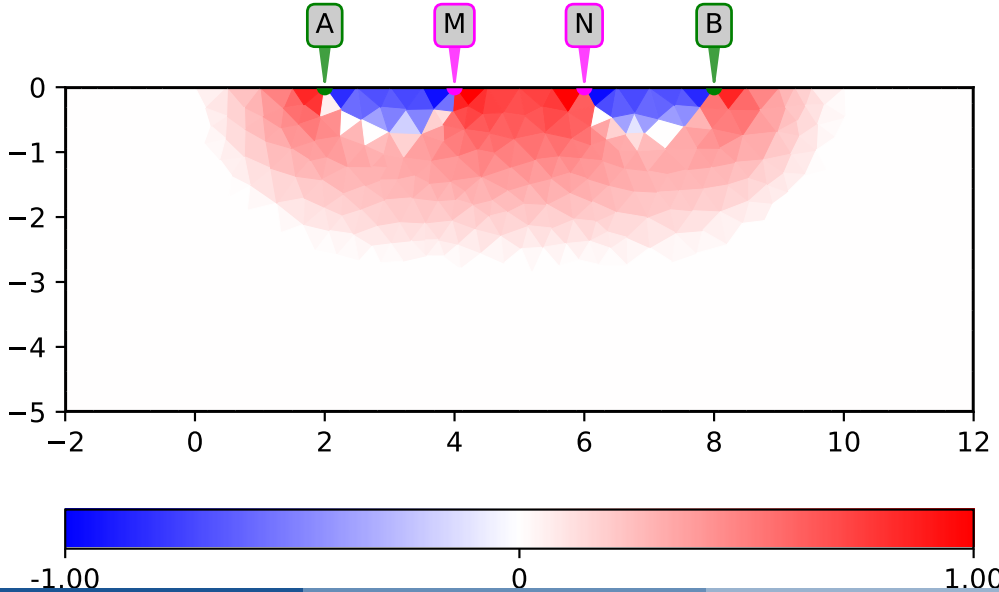

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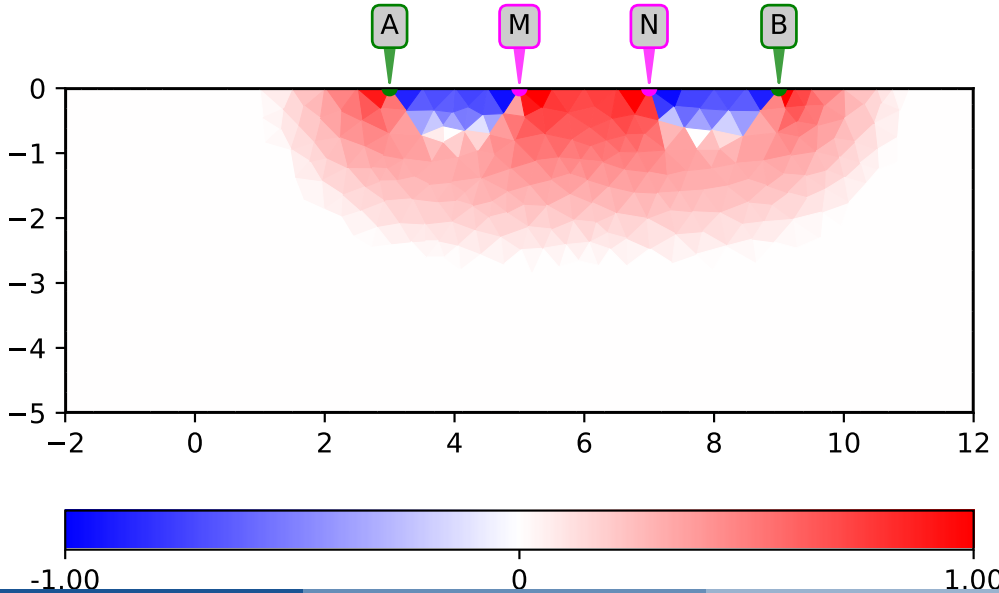

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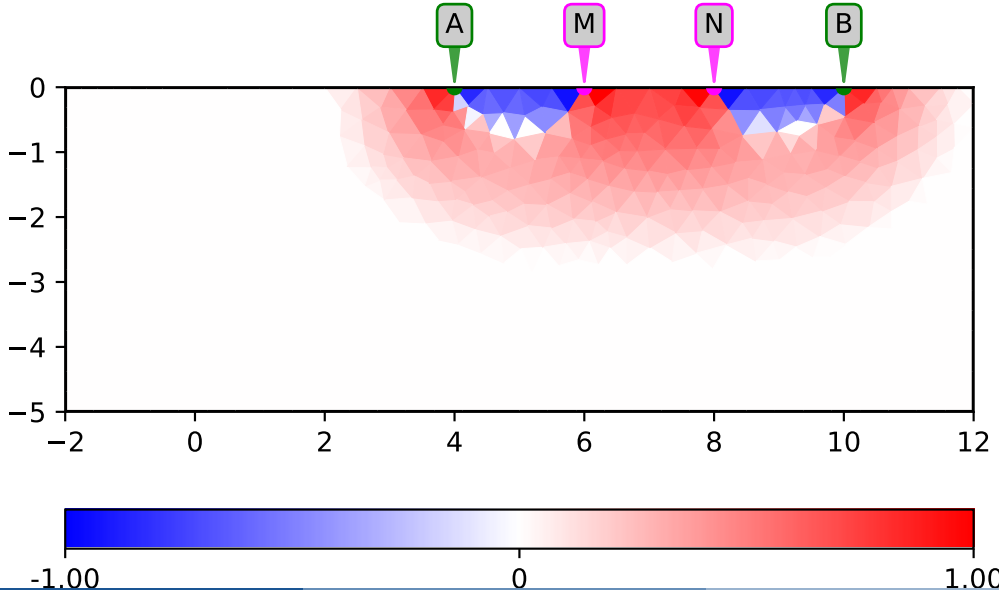

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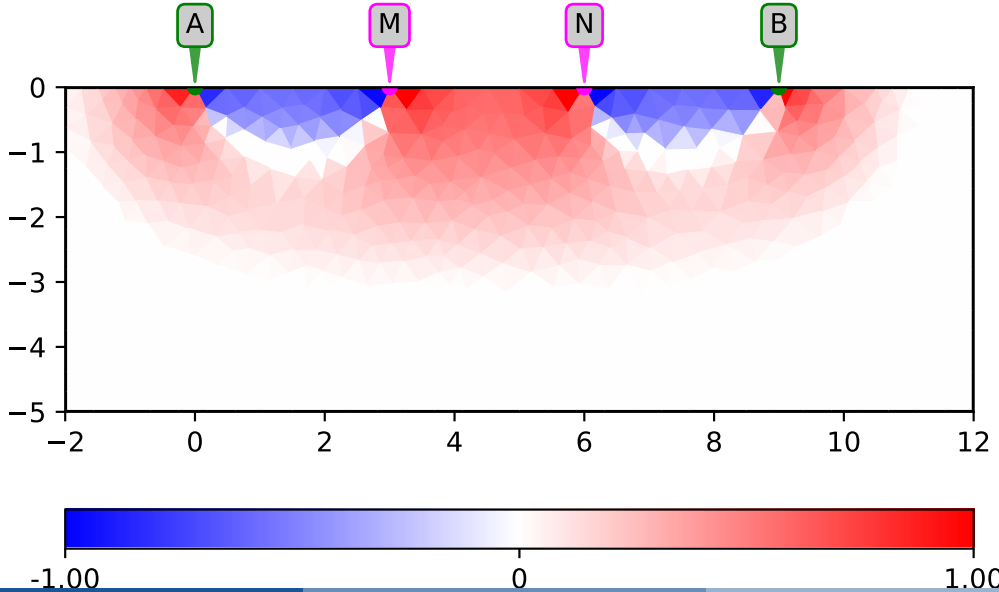

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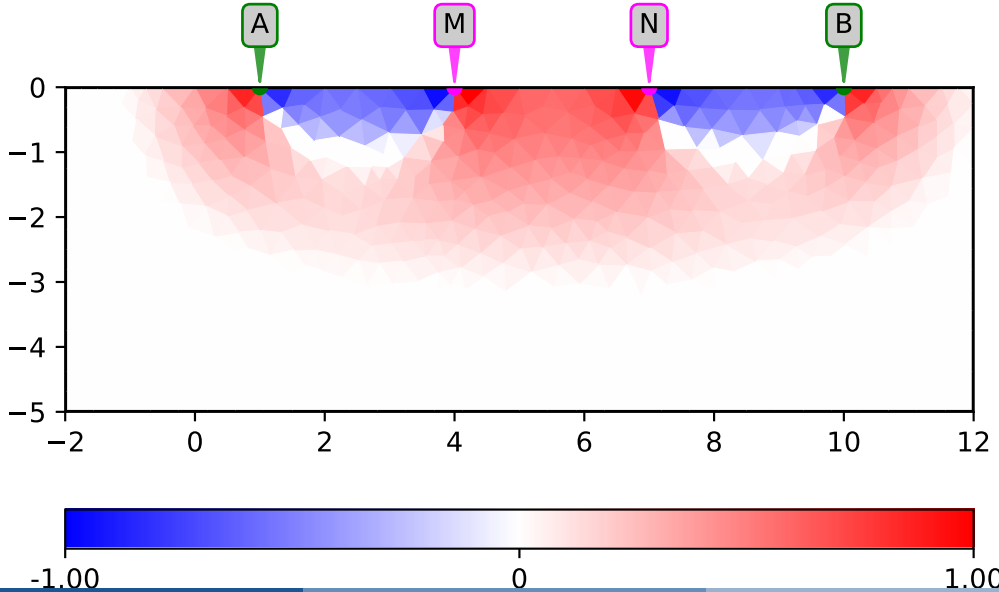


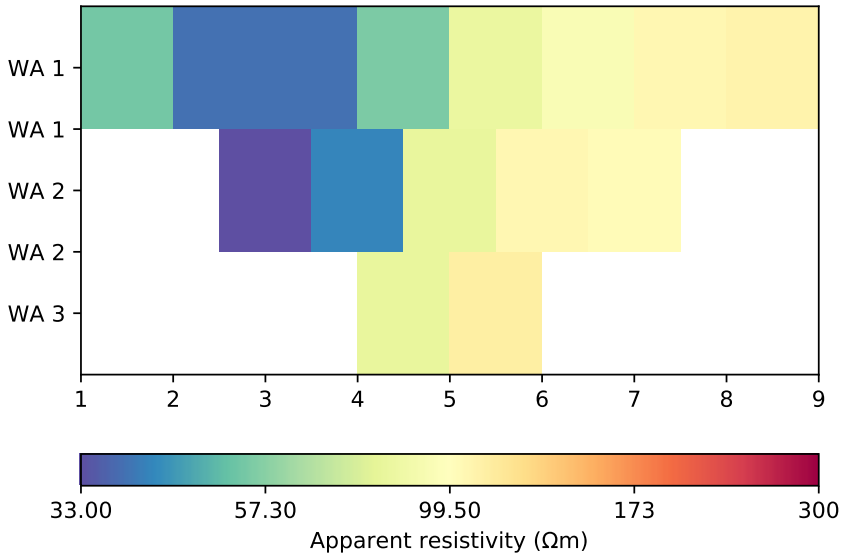
Electrical resistivity tomography - data

Pseudosection

coloured data table
overlain by data errors

Inversion

reconstruct resistivity image
ambiguous ⇒ smoothest distribution
improve by additional data



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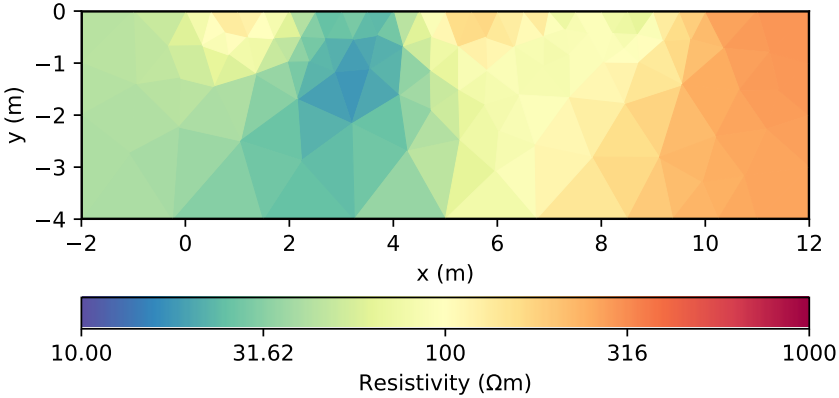

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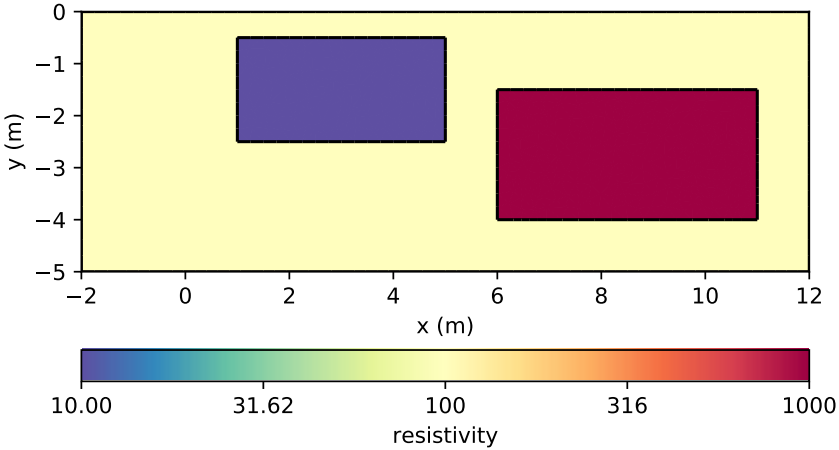
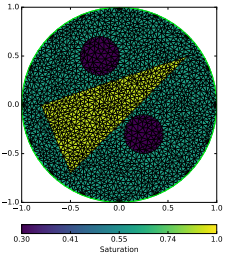
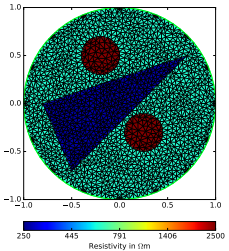
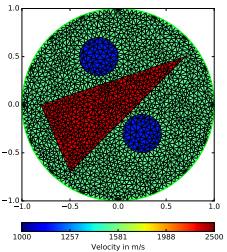


Petrophysical Joint Inversion ERT/Ultrasonic



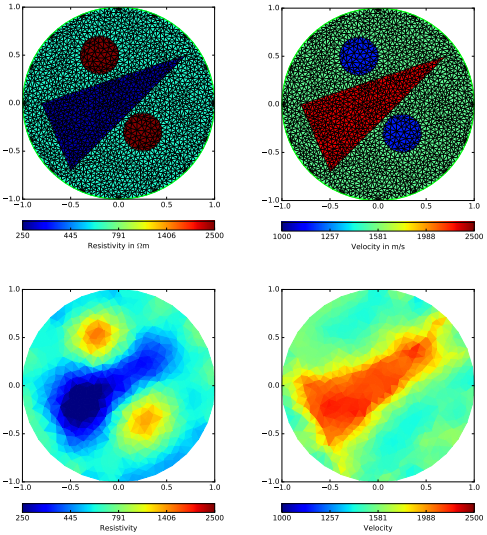




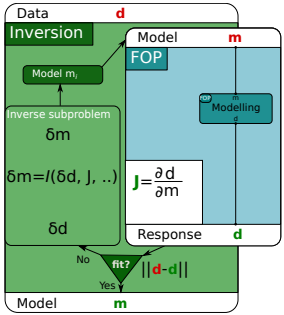
Synthetic model

- Circular heterogeneous domain
- ERT + Ultrasonic with 16 sensors
- Target: saturation S_w
- Assumption: known porosity Φ=40%
- Archie-Equation
- Wyllie-Equation
- Data ρ_a, t with noise (2%, 10 μs)

Inversion ERT/Ultrasonic



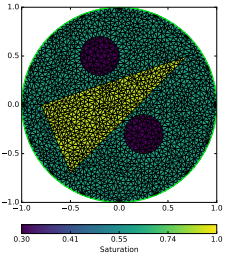
Separate inversion for ρ and v_p

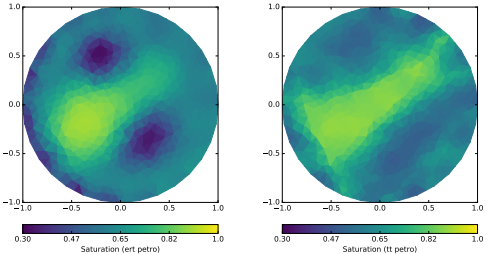


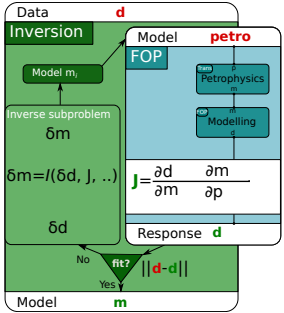

```
fERT = ERTManager.createFop()
iERT = Inversion(fERT, rhoa)
resistivity = iERT.run()

fSRT = Traveltime.createFop()
```

Petrophysical Inversion ERT/Ultrasonic

Separate petrophysical inversion for S_w

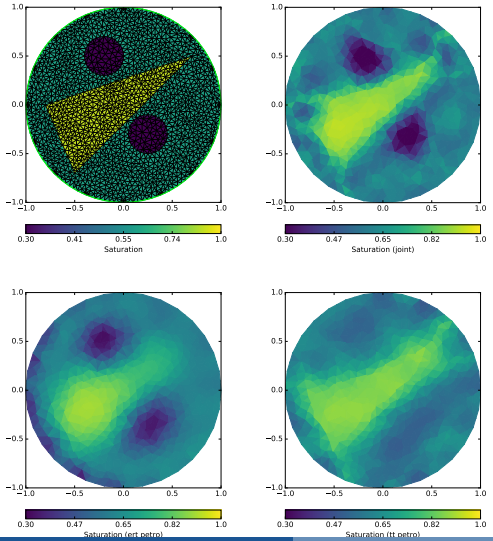




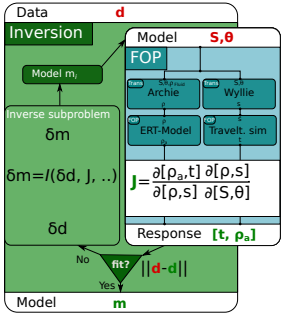



```
fERTpetro = PetroModelling(
fERT, Archie(phi=0.4, rhof=20))
fSRTpetro = PetroModelling(
fSRT, Wyllie(phi=0.4))
```

Petrophysical Joint-Inversion ERT/Ultrasonic



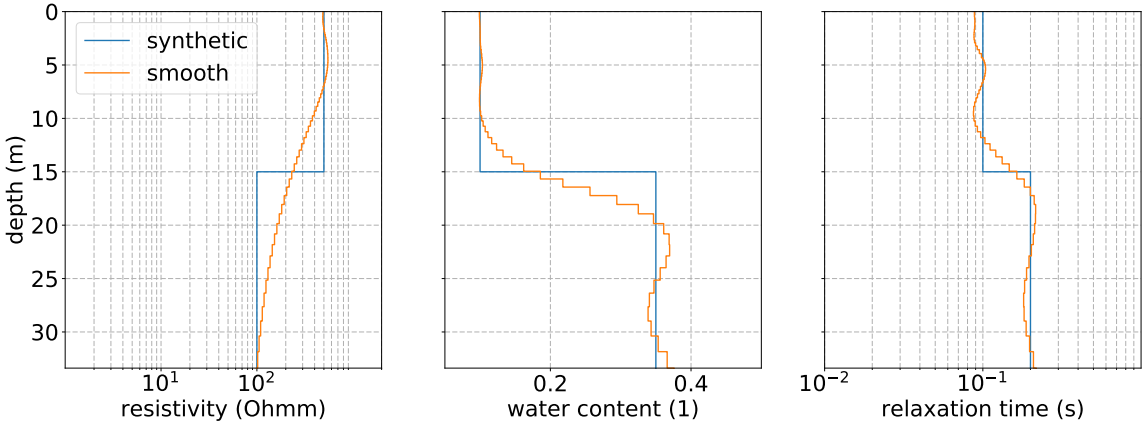
Common inversion for S_w




```
fArchie = PetroModelling(
fERT, Archie(phi=0.4, rhof=20))
fWyllie = PetroModelling(
fTT, Wyllie(phi=0.4))
fop = JointModelling([fArchie,
```

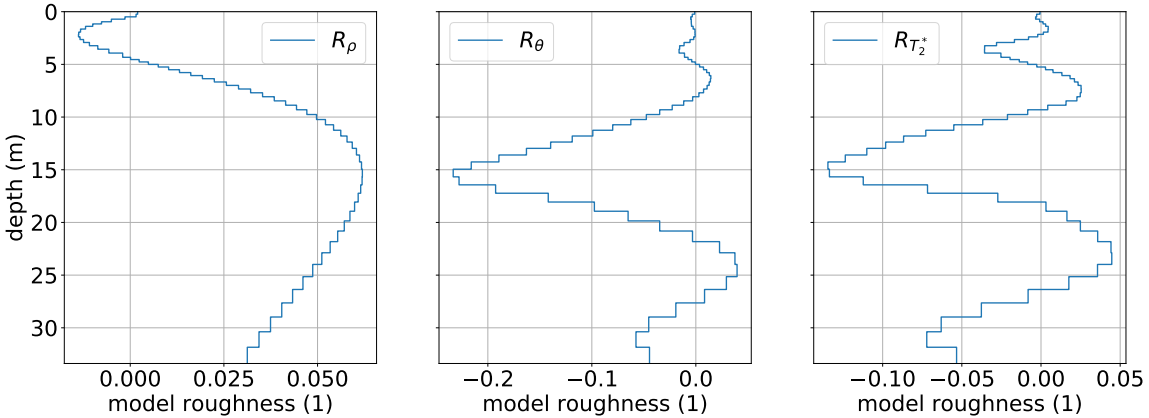
Structurally constrained inversion of VES and MRS data

Smooth inversion



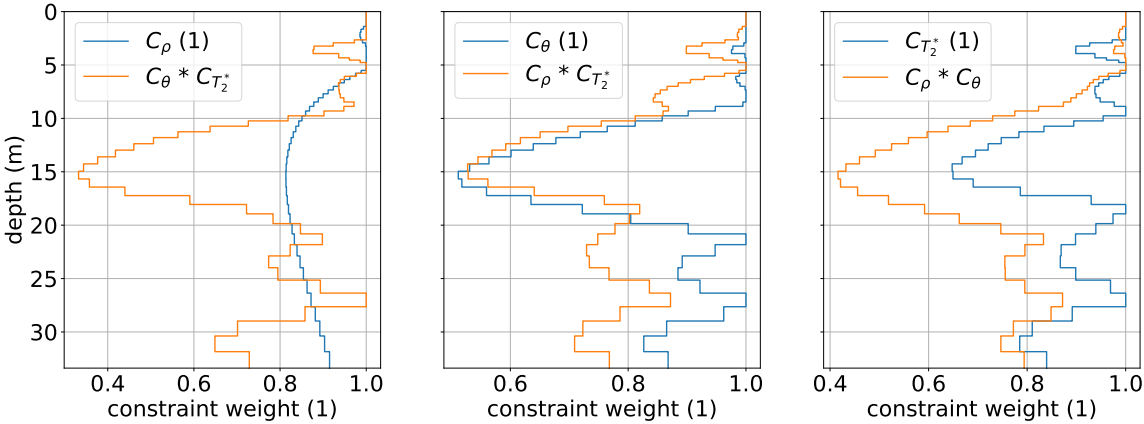
Structurally constrained inversion of VES and MRS data

Roughness



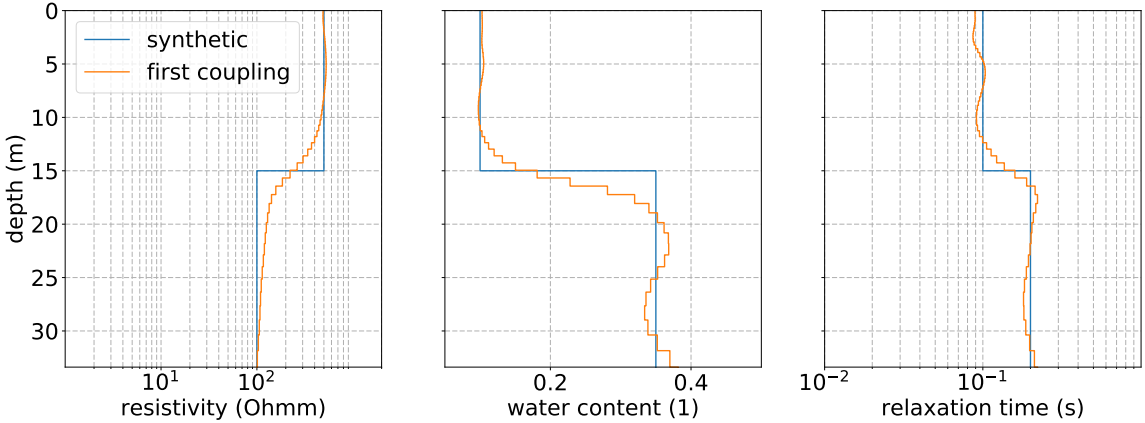
Structurally constrained inversion of VES and MRS data

Constraint weights



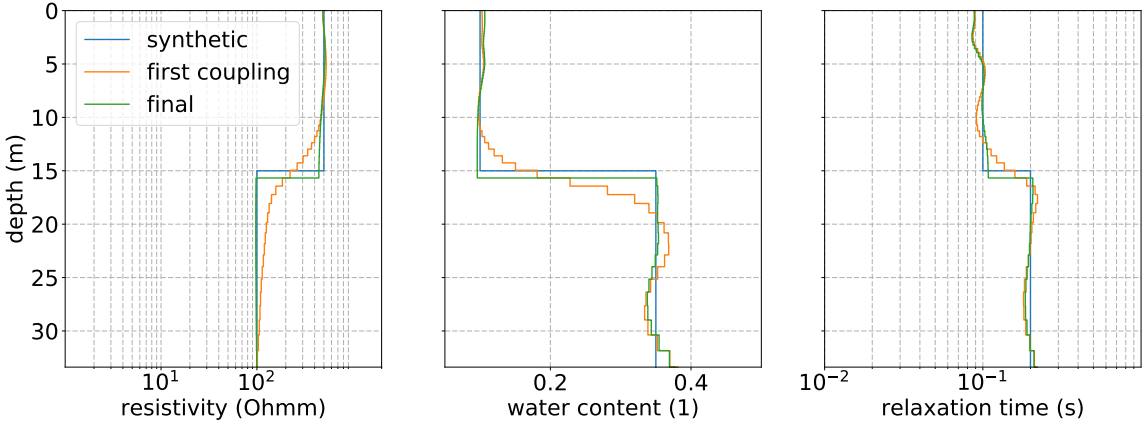
Structurally constrained inversion of VES and MRS data

First coupling



Structurally constrained inversion of VES and MRS data

Final result



Inversion verschiedener Modellparameter

Problemangepasste Regularisierung

Inhalt...

Globale Inversionsstrategien

Bio-inspirierte Inversionsalgorithmen

- Genetische Algorithmen (GA)
- Simulated Annealing (SA)
- Particle Swarm Optimization (PSO)
- Ant Colony System (ACS)
- Artificial Bee colony Algorithm