Inverse Problems in Geophysics
Part 1: Introduction

2. MGPY+MGIN

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Introduction

- Content and dates
- Literature and Links
- Data and model
- Simple matrix problem

Subject and Objectives

Inversion

Determine a **model** describing the subsurface that is able to **explain our data**!

Aim of the lecture

- Understand the process of imaging/inversion
- Actively control the inversion process

Content

Schedule

Lectures Wednesday, 11:30-13, MEI-0150

15 slots: 2.4., 9.4., 16.4., 23.4., 30.4., 7.5., 14.5., (21.5.), 28.5., 4.6., 11.6., 18.6., 25.6., 2.7., 9.7.

Exercises Friday, 11:30-13, CIP pool MEI1203a

14 slots: 4.4., 11.4., 25.4., (2.5.), 9.5., 16.5., 23.5., (30.5.), 6.6., 13.6., 20.6., 27.6., 4.7., 11.7.

Grade: submitting a report including codes

What should you know already?

- Higher mathematics: differential equations, algebra (1.-2. BSc)
- Experimental and theoretical physics: governing equations
- Numerics for engineers (2. BSc)
- Programming (1. BSc), Software development (3. BSc)
- Geophysics: feeling for physical fields & methods
- Electromagnetics (5. BSc), Theory EM,
- now: Scientific programming, HPC, seismic imaging

Topics to be covered

- recap on partial differential equations
- (1D) heat equation: stationary and instationary (Geothermics course)
- 2D: magnetotellurics
- 3D DC modelling (content of Spitzer videos)
- 2D ground-penetrating radar (EM) and pressure waves (seismics)
- excurse to hydrodynamic modelling
- modelling the Eikonal equation (the travelling saleman)
- exercises: code FD & FE by hand, use packages to obtain feeling

Literature

- Menke (2018): Geophysical Data Analysis: Discrete Inverse Theory, Academic
- Gubbins, Tarantola, Zhdanov
- Richter (2020): Inverse Problems: Basics, Theory and Applications in Geophysics
- Günther (2004): Inversion Methods and Resolution Analysis for the 2D/3D Reconstruction of Resistivity Structures from DC Measurements PhD thesis

Further links

- pyGIMLi: Python Geophysical Inversion and Modelling Library https://pygimli.org
- Geoscience.XYZ: https://geosci.xyz
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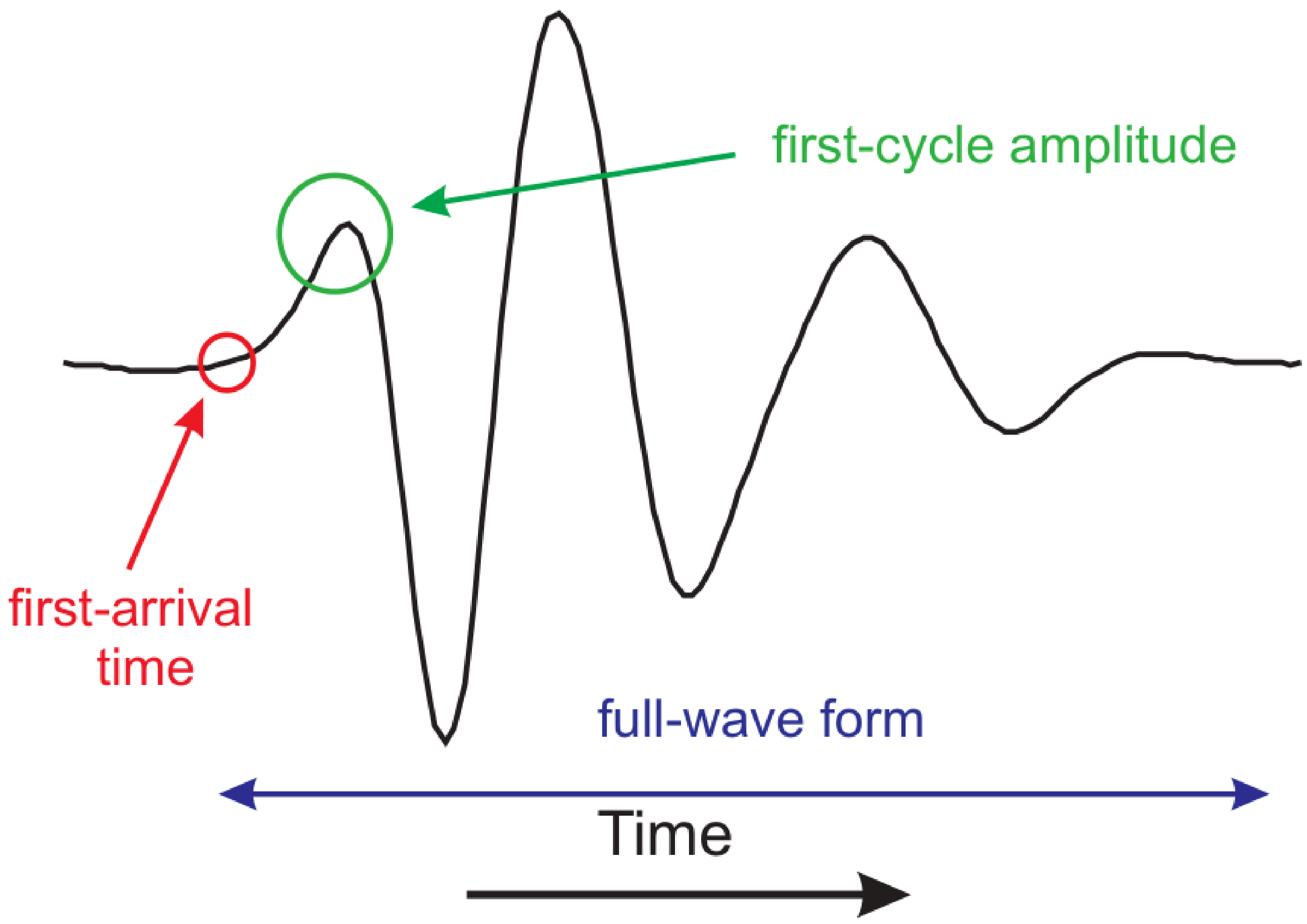
- https://github.com/halbmy/IJulia - Julia Notebooks for Inverse Problems

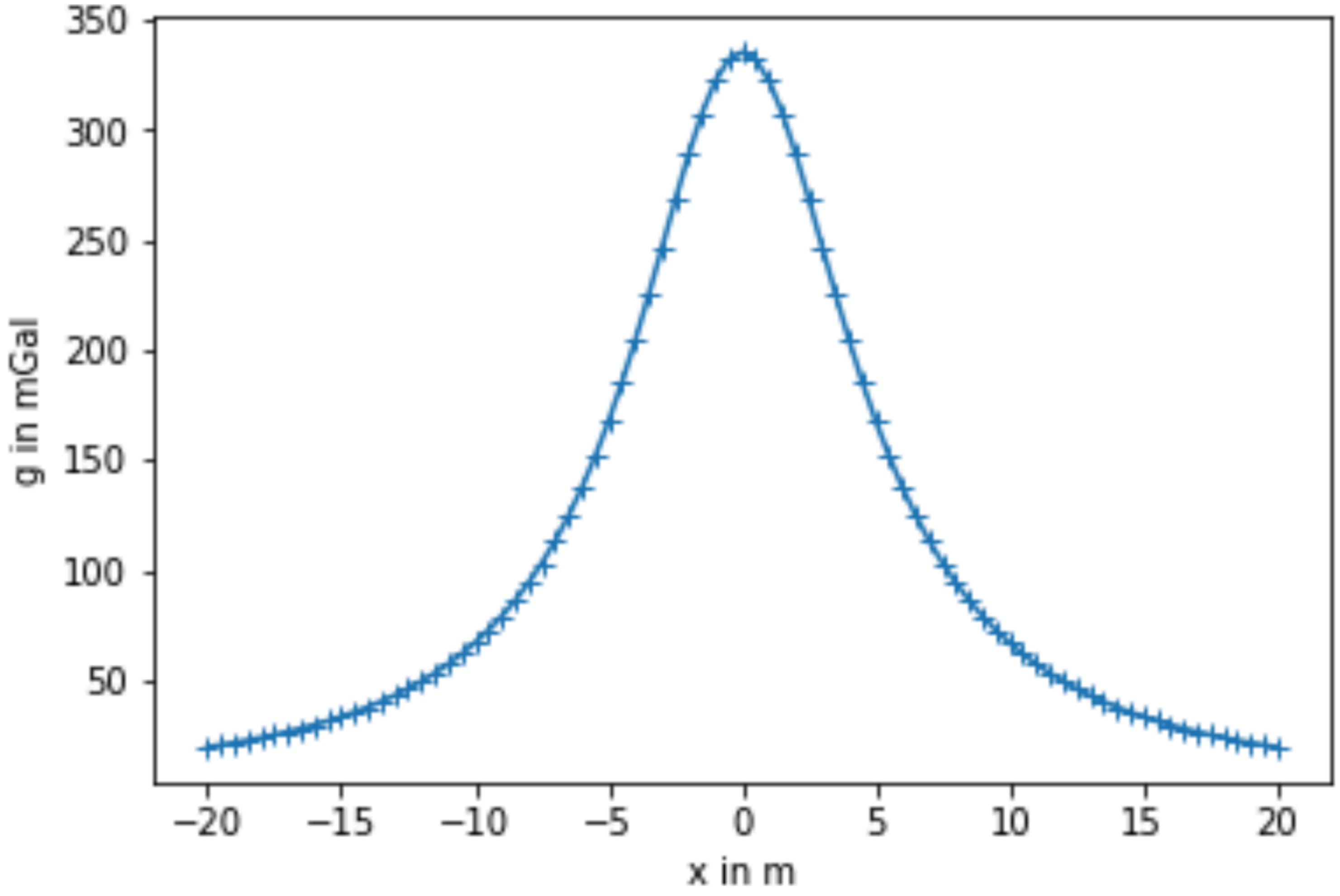
Data in seismology

time series of acceleration



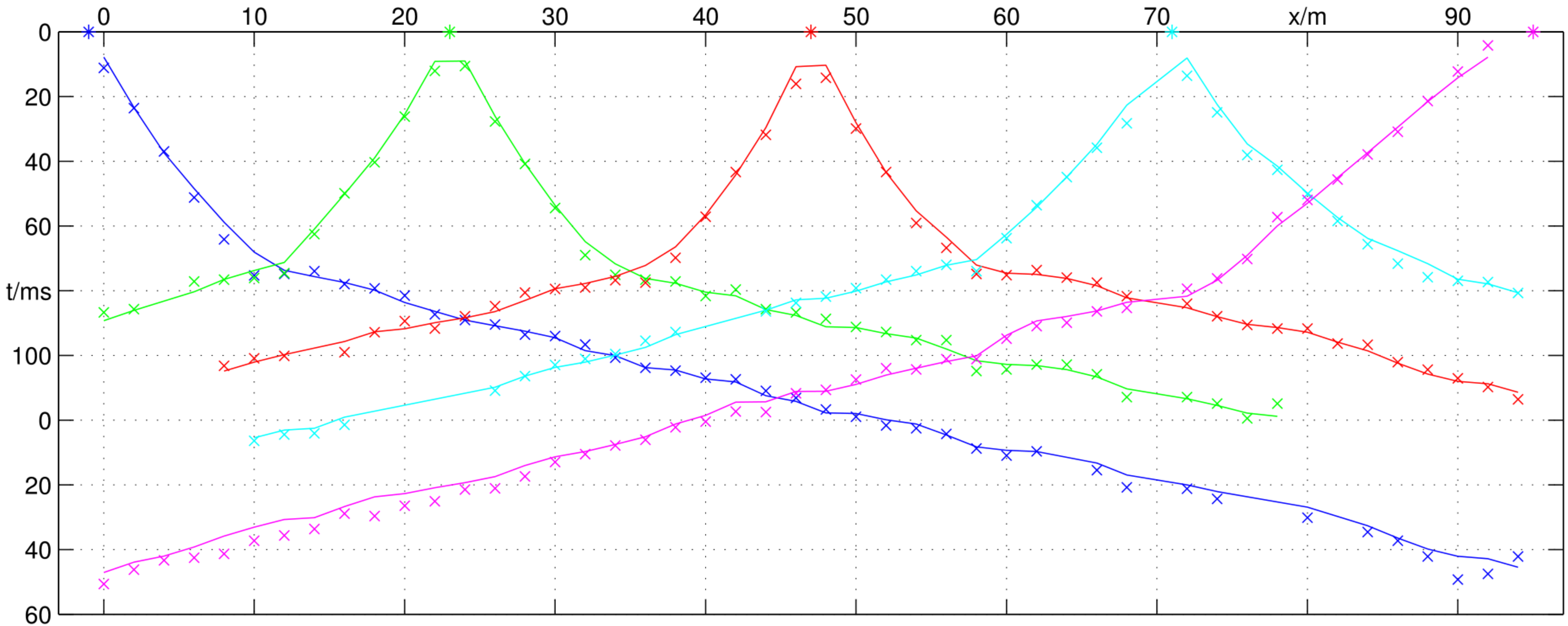
Data in gravimetry

gravity measures at discrete positions



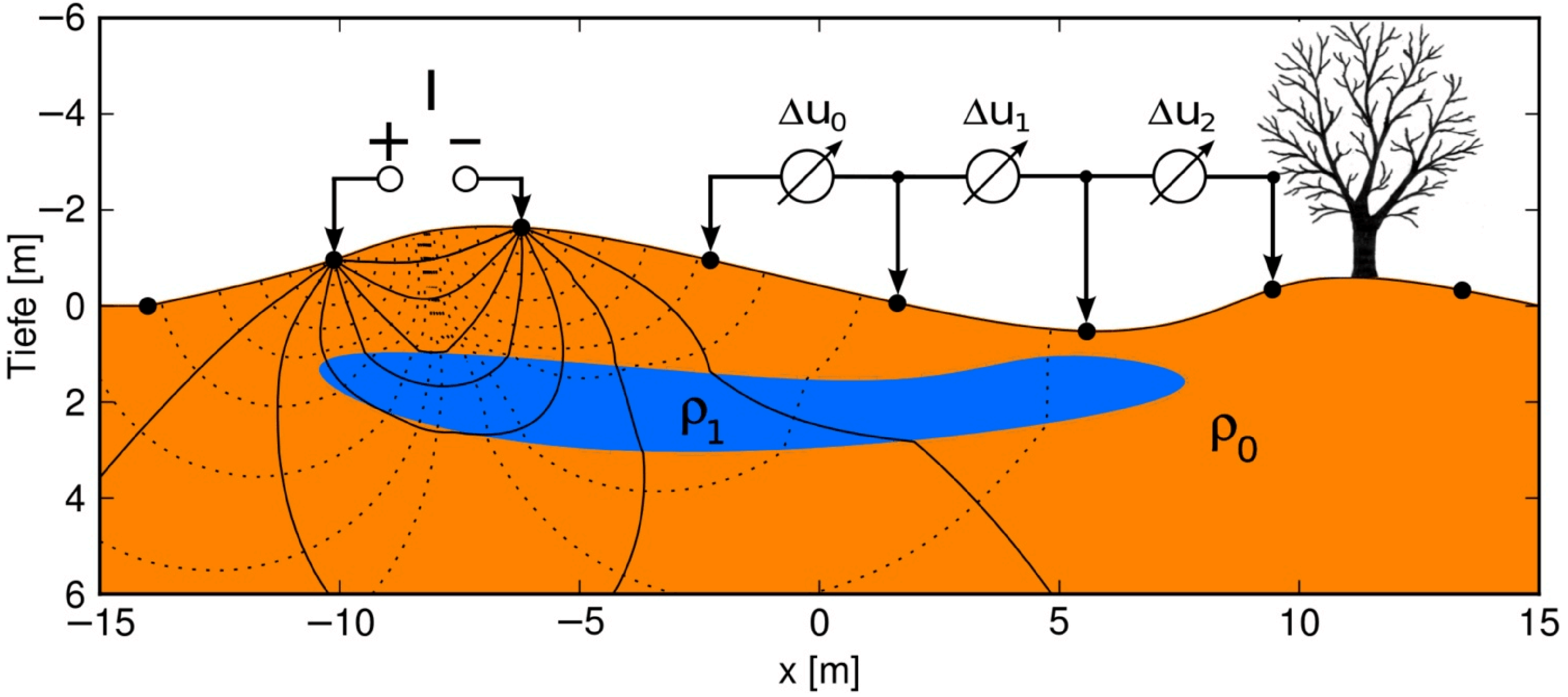
Data in travel-time tomography

traveltimes between shot and geophones



Data in electrical resistivity tomography (ERT)

current and voltage combinations



Data

- can be a discretized function of space, time or frequency, and plotted as such (curves)
- can depend on several sensor (shot/geophone, ABMN) and visualized as several curves or a coloured matrix (crossplot, pseudosection)

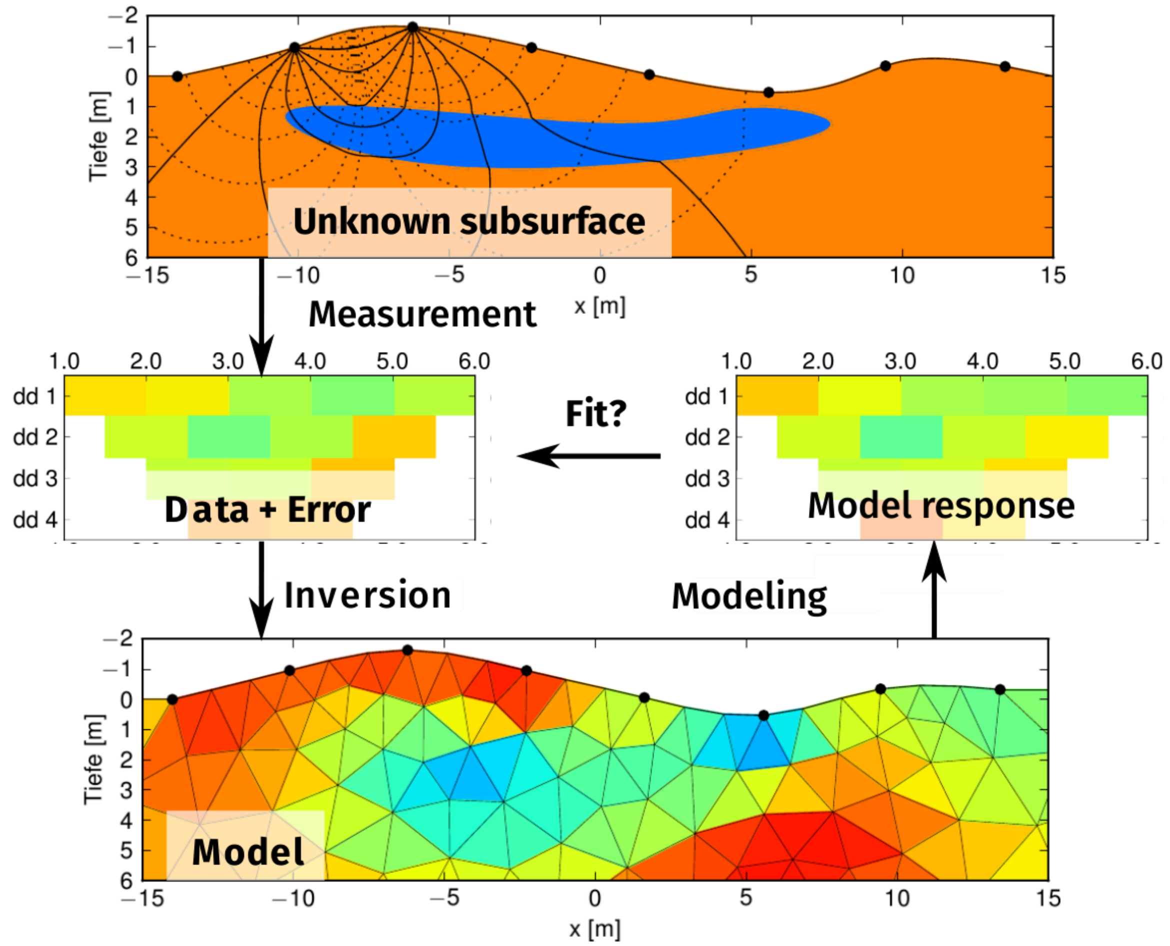
Data are subsumed in the *data vector*

$$\mathbf{d} = [d_1, d_2, \ldots, d_N]^T$$

consist of the *model response* $\mathbf{f}(\mathbf{m})$ plus noise $\mathbf{n}$: $\mathbf{d} = \mathbf{f}(\mathbf{m}) + \mathbf{n}$

Geophysical Workflow

1. Data acquisition
2. Preprocessing (QC, filtering)
3. Parameterization (mesh)
4. **Inversion**
5. Fit of data & model response
6. Postprocessing & visualization
7. Interpretation



Models in Geophysics

subsurface described by *model vector* $\mathbf{m} = [m_1, m_2, \ldots, m_M]^T$

Independent parameters:

- seismology: earthquake location, stress, principal axis
- gravity: position and depth of anomaly, size, density contrast
- spectroscopy (e.g. SIP): function parameters (e.g. Cole-Cole)

Discrete functions of space, time, frequency

- refraction: depth (and velocity) of refractor
- distribution of a parameter in space (and time)

Inversion

Inverse problem

Determine $\mathbf{m}$ so that the data are explained

$$\mathbf{d} \approx \mathbf{f}(\mathbf{m})$$

Minimization problem

$$\|\mathbf{d} - \mathbf{f}(\mathbf{m})\| \to \min$$

Take into account errors: Explain model within error ($\mathbf{e}$) bounds

$$\|\frac{\mathbf{d} - \mathbf{f}(\mathbf{m})}{\mathbf{e}}\| \to \min$$

Correctness

Well-posed problems (Hadamard)

- There is a solution,
- it is uniquely defined and
- depends steadily from input data (small variations lead to small model deviations)

Ill-posed problem

- There is no model to fit the data perfectly
- Many models can fit the data within errors
- Small data variations can lead to large model deviations

Occam’s razor - a fundamental rule

novacula Occami

Pluralitas non est ponenda sine neccesitate!

(William of Ockham, Scottish philosopher and theologian, 14th century)

Principle of Parsimony

Entities must not be multiplied beyond necessity.

Of two competing theories, the simpler explanation of an entity is to be preferred.

Linear problems

$\mathbf{f}(\mathbf{m})$ is linear with respect to $\mathbf{m} \Rightarrow$ write as matrix-vector product

$$\mathbf{d} = \mathbf{G}\mathbf{m} + \mathbf{n}$$

Examples

Gravimetry, Magnetics, Magnetic Resonance, VSP, straight-ray tomography, regression

Problem

$\mathbf{m} = \mathbf{G}^{-1}\mathbf{d}$? No, because is usually not invertible, not even quadratic.

Types of inverse problems

Important

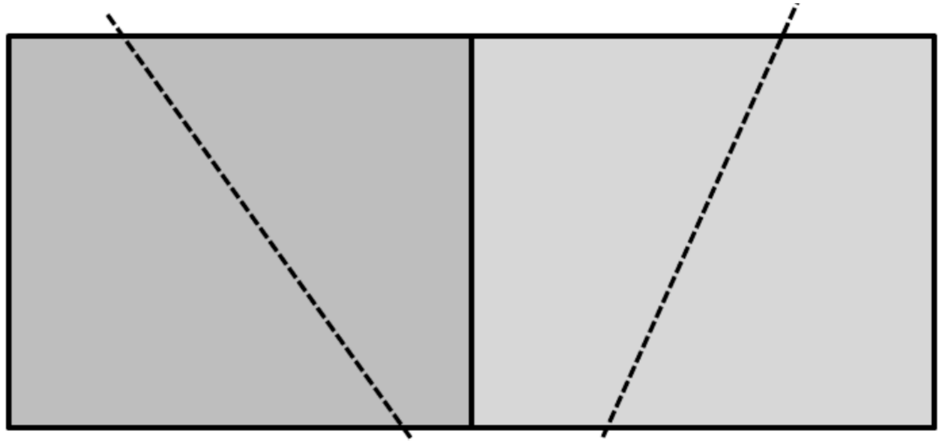
- every row stands for a measurement (data point)
- every column represents a model parameter

$$\mathbf{d} = \mathbf{G}\mathbf{m} + \mathbf{n} \Rightarrow \mathbf{G} \in \Re^{\mathbf{N}\times\mathbf{M}}$$

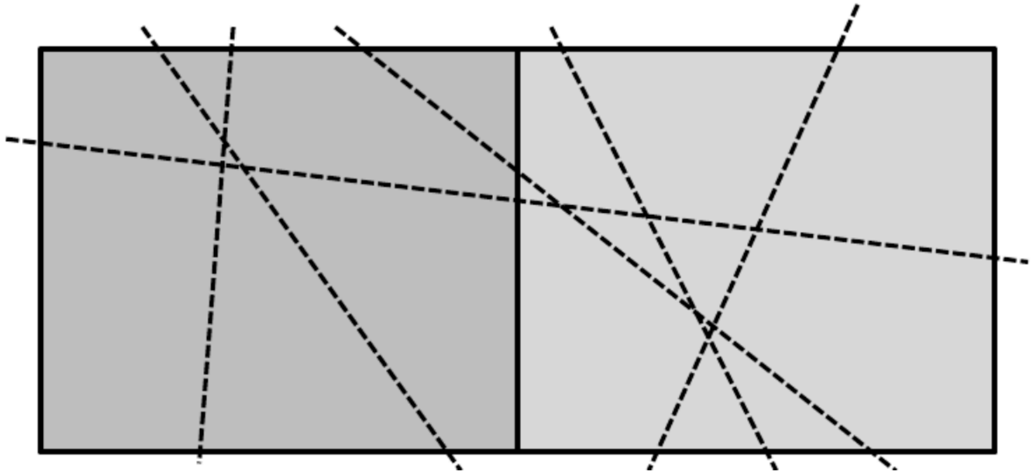
- $N = M$: even-determined
- $N > M$: over-determined problem (no existing solution)
- $N < M$: under-determined problem (no unique solution)
- mostly: (both over- and) under-determined model parts

Types of inverse problems (Menke, 2012)

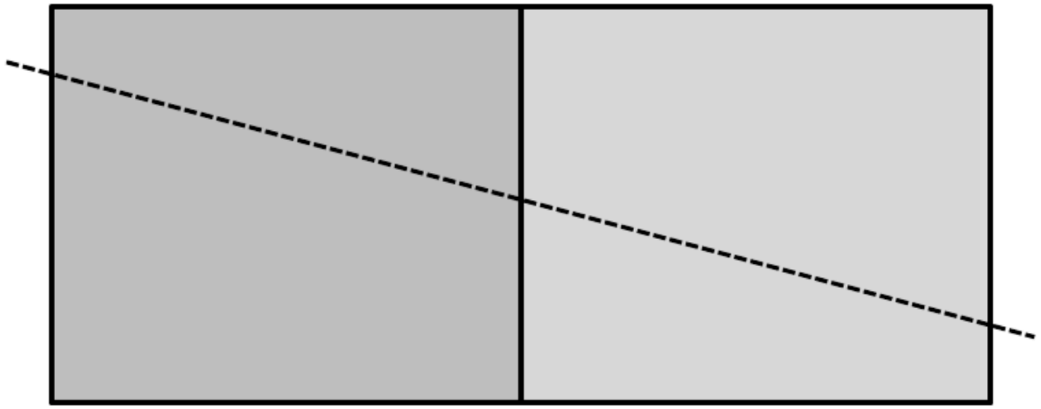
Even-determined



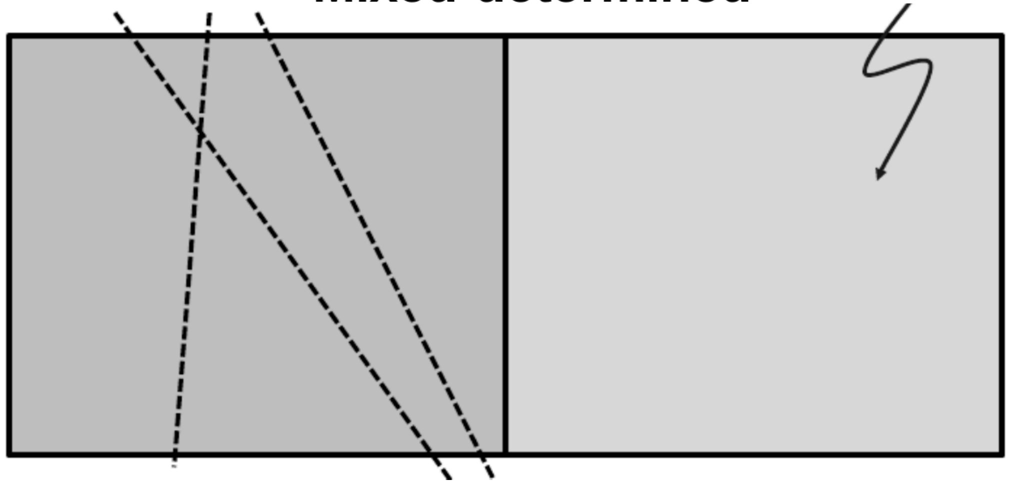
Over-determined



Under-determined



Mixed-determined



A simple matrix problem

$$m_1 - m_2 = -1$$
$$2m_1 - m_2 = 0$$
$$m_1 + m_2 = 2.5$$

$$m_2 = \frac{d_i - G_{i1} \cdot m_1}{G_{i2}}$$