Numerical Simulation Methods in Geophysics, Exercise 13: I open at the close

1. MGPY+MGIN

thomas.guenther@geophysik.tu-freiberg.de

TU BERGAKADEMIE FREIBERG
TUBAF
Die Ressourcenuniversität.
Seit 1765.

Plane waves in the Earth

We consider an MT case with a horizontally polarized magnetic field up in the air.

Task: Compute electromagnetic fields in the Earth

For theory, see Theory EM (Börner)

Helmholtz equations for E and H

The magnetic field is governed by the Helmholtz equations (no displacement currents, no sources)

$$\boldsymbol{\nabla} \times \mu^{-1} \boldsymbol{\nabla} \times \mathbf{E} + \imath\omega\sigma\mathbf{E} = 0 \tag{1}$$

$$\boldsymbol{\nabla} \times \sigma^{-1} \boldsymbol{\nabla} \times \mathbf{H} + \imath\omega\mu\mathbf{H} = 0 \tag{2}$$

2D E/H polarization: $\Rightarrow \boldsymbol{\nabla} \times a \boldsymbol{\nabla} \times$ by $\boldsymbol{\nabla} \cdot a \boldsymbol{\nabla}$

$$\boldsymbol{\nabla} \cdot \boldsymbol{\nabla} E_x + \imath\omega\sigma\mu E_x = 0 \tag{3}$$

Equation to be solved

For the source field

$$\mathbf{H}(z,\omega) = \begin{bmatrix} H_x \\ 0 \\ 0 \end{bmatrix} e^{i\omega t}, \qquad z < 0$$

we end up in the equation for H_x

$$\frac{\partial^2 H_x}{\partial y^2} + \frac{\partial^2 H_x}{\partial z^2} + k^2 H_x = 0 \quad \text{with} \quad k^2 = -i\omega\mu\sigma$$

Analytical solution

For homogeneous $\sigma = \sigma_0$, the solution is

$$H_x(z) = H_x^0 e^{-ikz} = H_x^0 e^{-i\sqrt{-i\omega\mu\sigma_0}z} \tag{4}$$

Secondary field for EM

$$-\nabla^2(\mathbf{E}_0 + \mathbf{E}_s) + \imath\omega\mu\sigma(\mathbf{E}_0 + \mathbf{E}_s) = 0$$

$$-\nabla^2\mathbf{E}_0 + \imath\omega\mu\sigma_0\mathbf{E}_0 = 0$$

$$-\nabla^2\mathbf{E}_a + \imath\omega\mu\sigma\mathbf{E}_a = -\imath\omega\mu\delta\sigma\mathbf{E}_0$$

Note

Source terms only arise at anomalous conductivities and increase with primary field

Secondary field for EM

$$-\boldsymbol{\nabla}^2\mathbf{E}_a + \imath\omega\mu\sigma\mathbf{E}_a = -\imath\omega\mu\delta\sigma\mathbf{E}_0$$

leads to the discretized form ($\mathbf{A}$-stiffness, $\mathbf{M}$-mass)

$$\mathbf{A}\mathbf{E}_a + \imath\omega\mathbf{M}_\sigma\mathbf{E}_a = -\imath\omega\mathbf{M}_{\delta\sigma}\mathbf{E}_0$$

Secondary field for H

$$-\boldsymbol{\nabla} \cdot \rho \boldsymbol{\nabla} (\mathbf{H}_0 + \mathbf{H}_s) + \imath \omega \mu (\mathbf{H}_0 + \mathbf{H}_s) = 0$$

$$-\boldsymbol{\nabla} \cdot \rho_0 \boldsymbol{\nabla} \mathbf{H}_0 + \imath \omega \mu \mathbf{H}_0 = 0$$

$$-\boldsymbol{\nabla} \cdot \rho \boldsymbol{\nabla} \mathbf{H}_a + \imath \omega \mu \mathbf{H}_a = \boldsymbol{\nabla} \cdot \delta\rho \boldsymbol{\nabla} \mathbf{H}_0$$

and approximated by stiffness and mass matrix

$$\mathbf{A}_\rho \mathbf{H}_a + \imath \omega \mathbf{M} \mathbf{H}_a = \mathbf{A}_{\delta\rho} \mathbf{H}_0$$

Report II - 2D plane-wave EM

1. Design a subsurface 2D model with a good conductor in a halfspace
2. Compute the primary magnetic field analytically and numerically by solving the Helmholtz equation for homogeneous conductivity, use a constant field on the upper boundary and zero on the others
3. Visualize and compare with the analytical solution
4. Compute the secondary field for the anomalous conductivity
5. Visualize the secondary and total fields
6. Replace the good conductor with a bad conductor and repeat
7. Rerun the computations with a higher and lower (x10) period

Secondary field for H

$$-\boldsymbol{\nabla} \cdot \sigma_0^{-1} \boldsymbol{\nabla} \mathbf{H}_0 + \imath\omega\mu \mathbf{H}_0 = 0$$

$$-\boldsymbol{\nabla} \cdot \sigma^{-1} \boldsymbol{\nabla} (\mathbf{H}_0 + \mathbf{H}_s) + \imath\omega\mu (\mathbf{H}_0 + \mathbf{H}_s) = 0$$

$$-\boldsymbol{\nabla} \cdot \sigma^{-1} \boldsymbol{\nabla} \mathbf{H}_a + \imath\omega\mu \mathbf{H}_a = \boldsymbol{\nabla} \cdot \delta\sigma^{-1} \boldsymbol{\nabla} \mathbf{H}_0$$