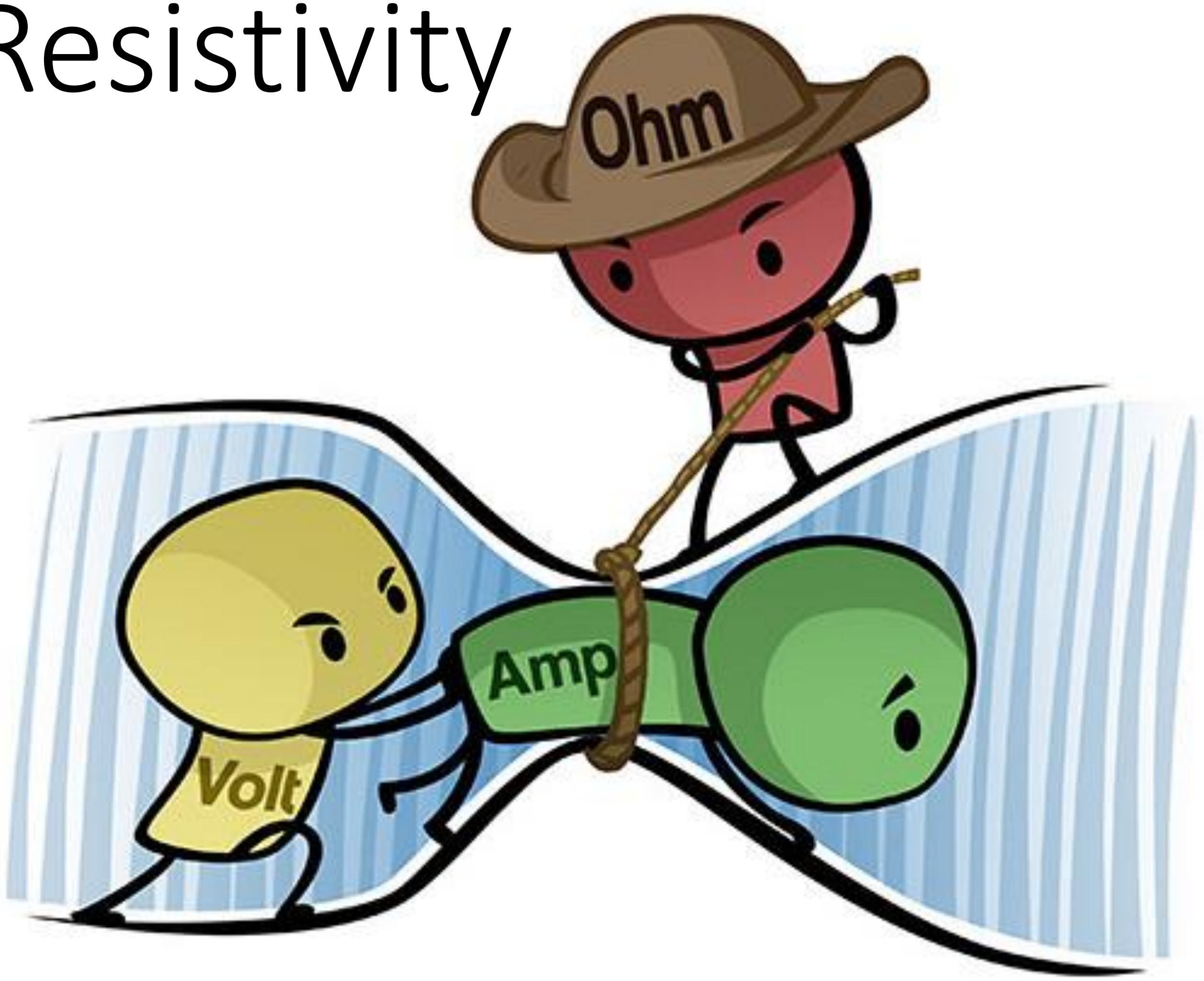
Near Surface Geophysical methods for site characterization
3. Electrical Resistivity



1. Playing with magnetics
2. Introduction into ERT
3. Electrical resistivity
4. Potential & current
5. Electrode arrays
6. Sounding & Mapping
7. A case study from Borkum

Gravity and magnetics: wrap-up

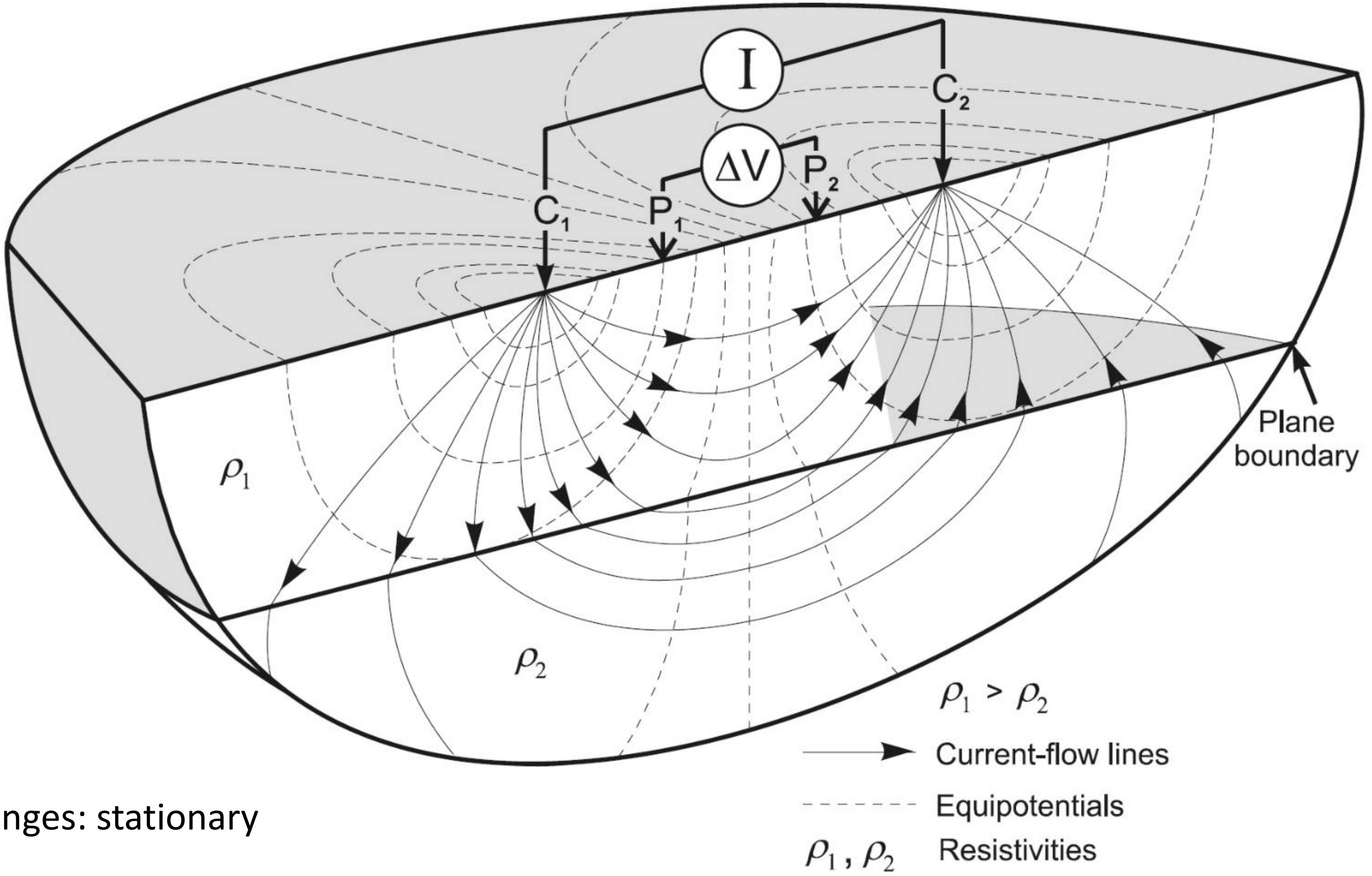
- Size of anomaly depends on size and density contrast
- Width of anomaly indicates depth of body
- Difference between 2D (cylinder) and 3D (sphere)
- Real data need modelling
- Large ambiguity of subsurface models
- Data are overlain by much stronger regional field
- Separation techniques necessary to extract local field
- Shape of magnetic (total field) anomaly depending on location

Electrical resistivity tomography

Maxwells equations

$$
\begin{aligned}
\nabla \times \mathbf{H} &= \mathbf{j} + \frac{\partial}{\partial t}\mathbf{D} \\
\nabla \times \mathbf{E} &= -\frac{\partial}{\partial t}\mathbf{B} \\
\nabla \cdot \mathbf{B} &= 0 \\
\nabla \cdot \mathbf{D} &= q
\end{aligned}
$$

Ohm‘s law $\mathbf{j} = \sigma \mathbf{E}$



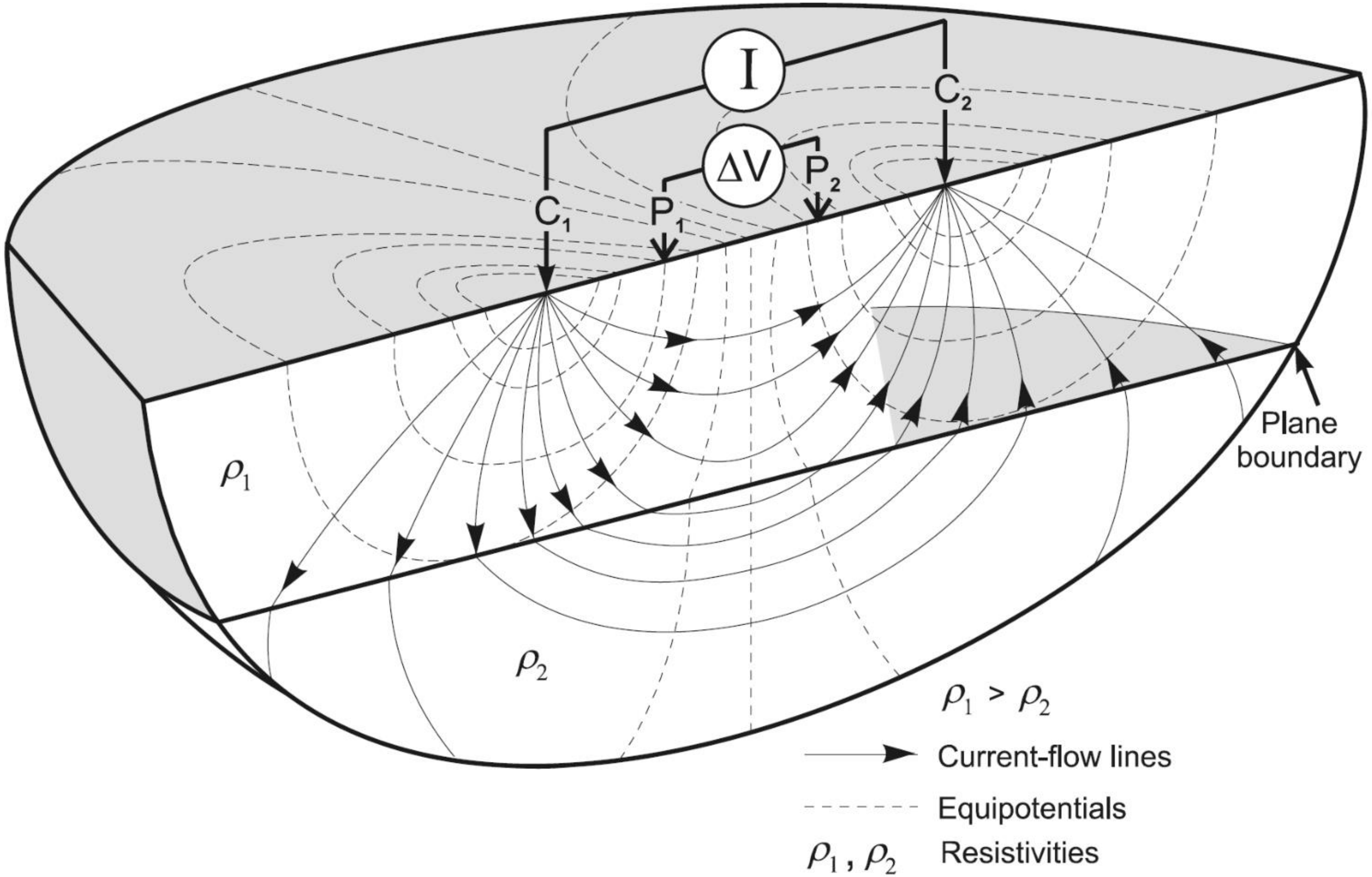
No temporal changes: stationary

Electrical resistivity methods (direct current)

Maxwells equations

$$\begin{aligned} \nabla \times \mathbf{H} &= \mathbf{j} \\ \nabla \times \mathbf{E} &= 0 \\ \nabla \cdot \mathbf{B} &= 0 \\ \nabla \cdot \mathbf{D} &= q \end{aligned}$$

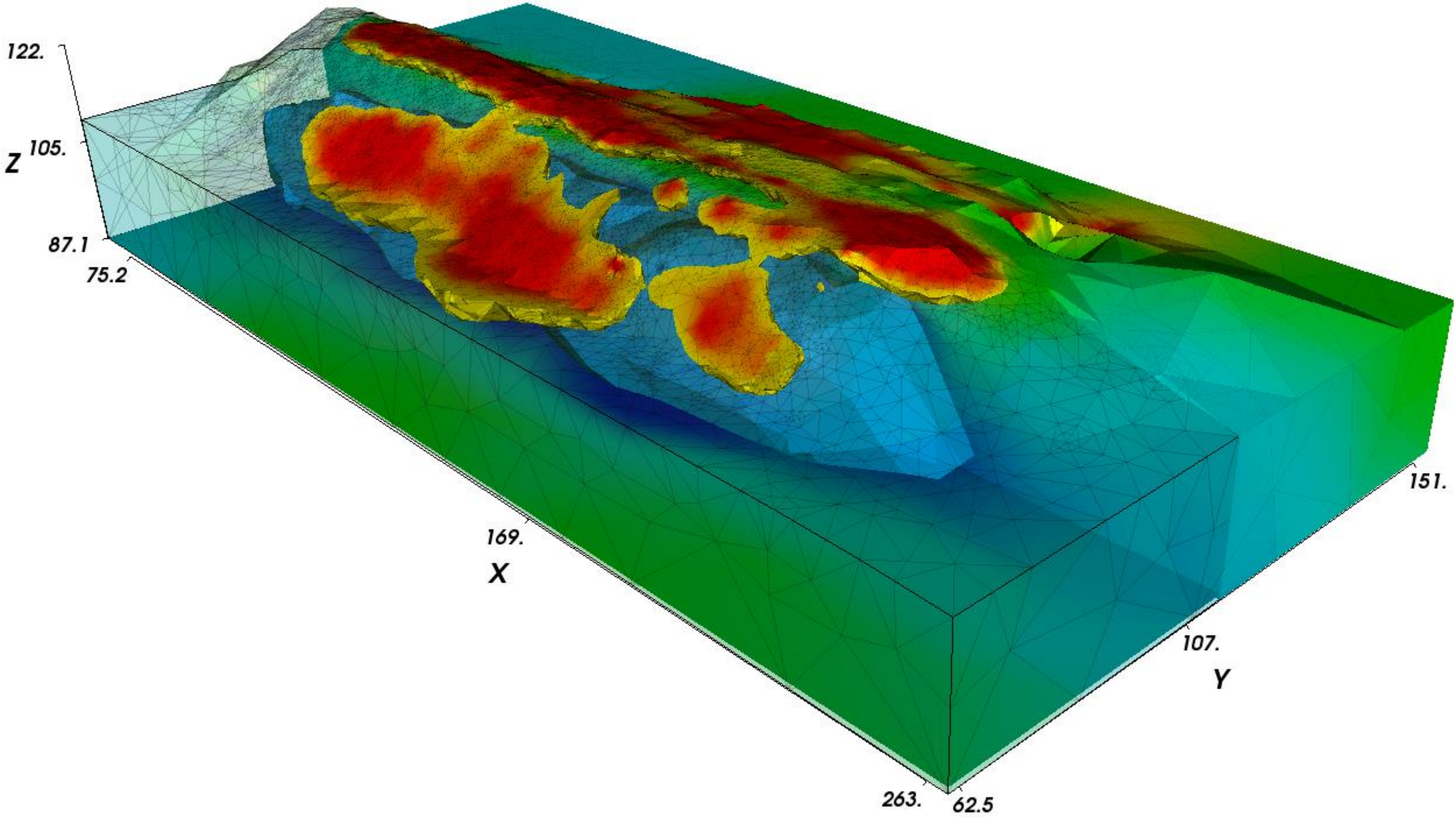
Ohm‘s law $\mathbf{j} = \sigma \mathbf{E}$



Fields of application

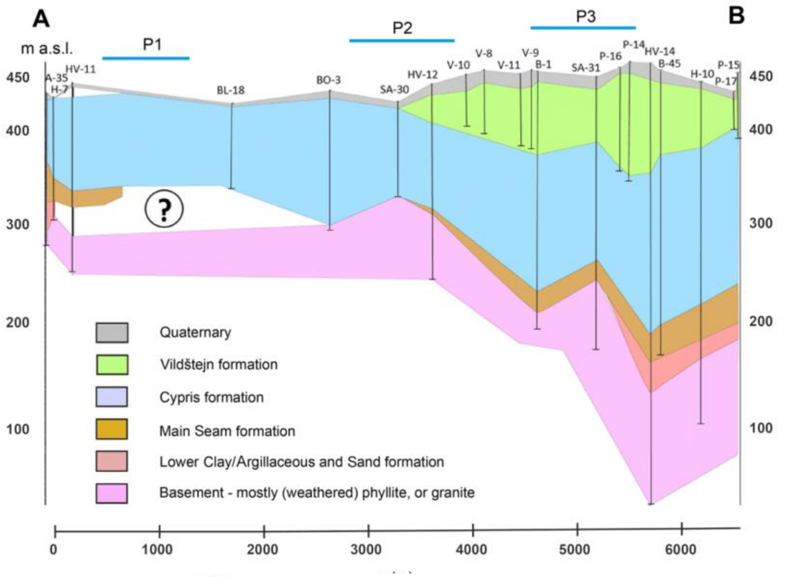
- Engineering geology (bedrock detection, void search, tree tomography)
- Hydrogeology (aquifer system architecture, contaminants)
- Saltwater intrusion into freshwater
- Geological investigation (faults)
- Archaeology (foundations, slag heaps)
- Agriculture and precision farming (water and clay content)
- Geohazards (landslides)
- Process monitoring (water and solute transport,

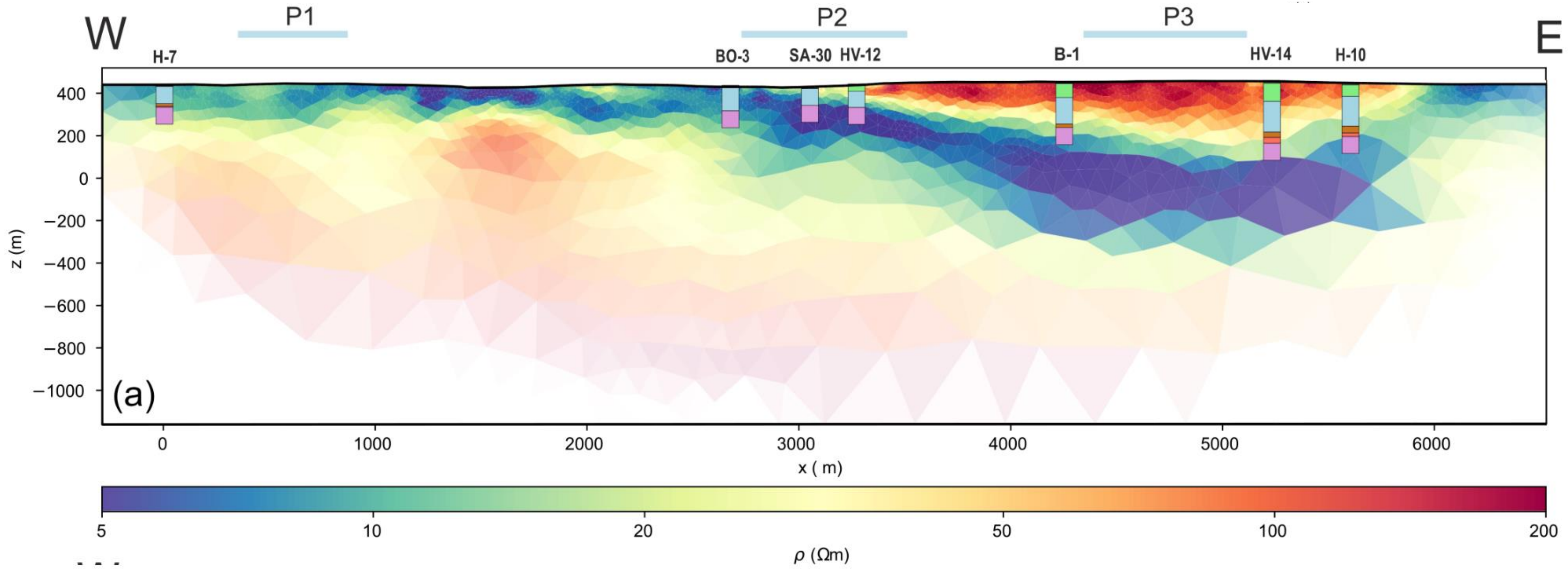
3D Investigation of a mining heap



Large-scale geology

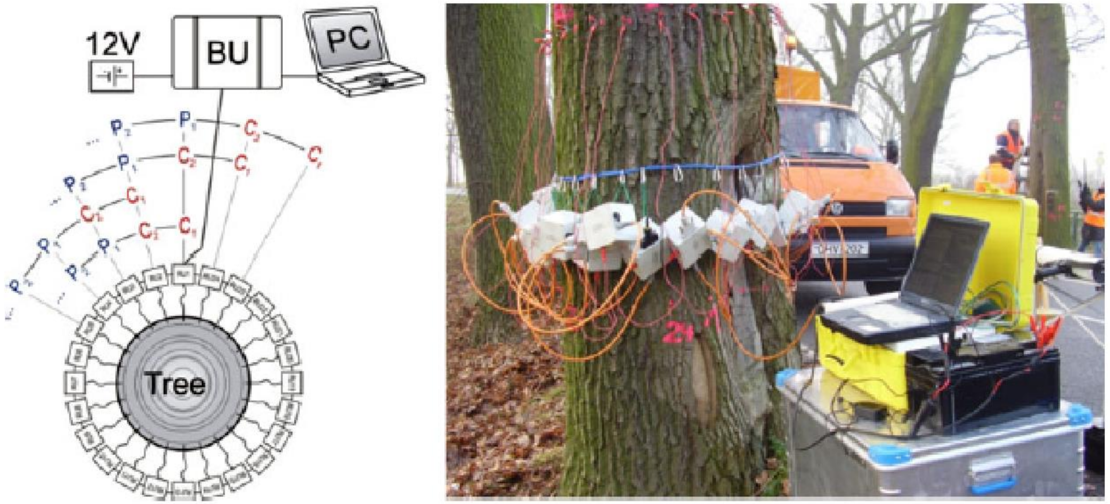
Nickschick et al. (2019)

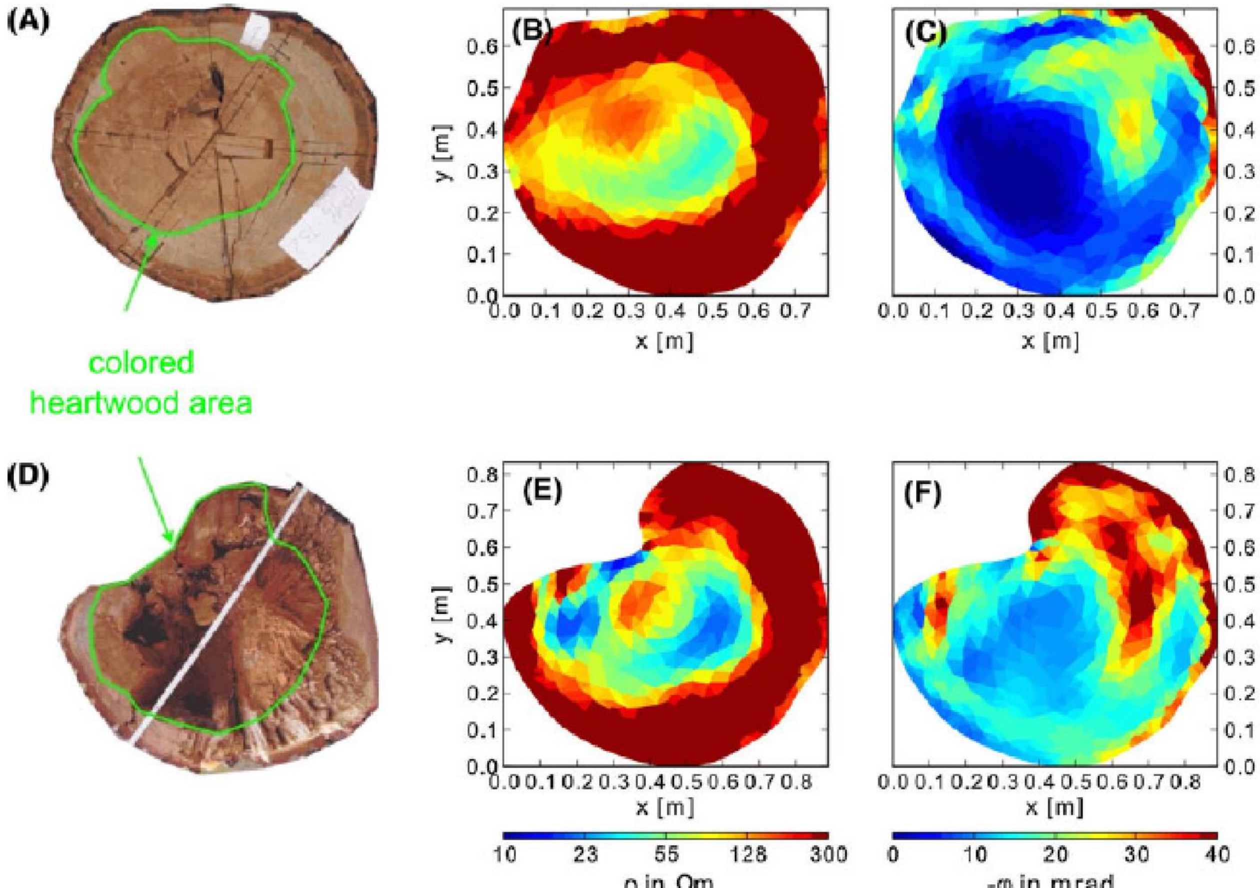


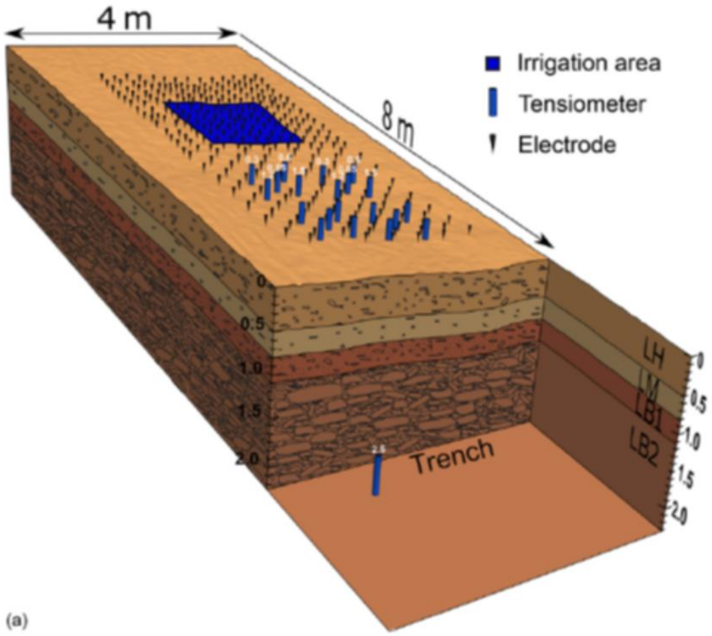


Tree tomography

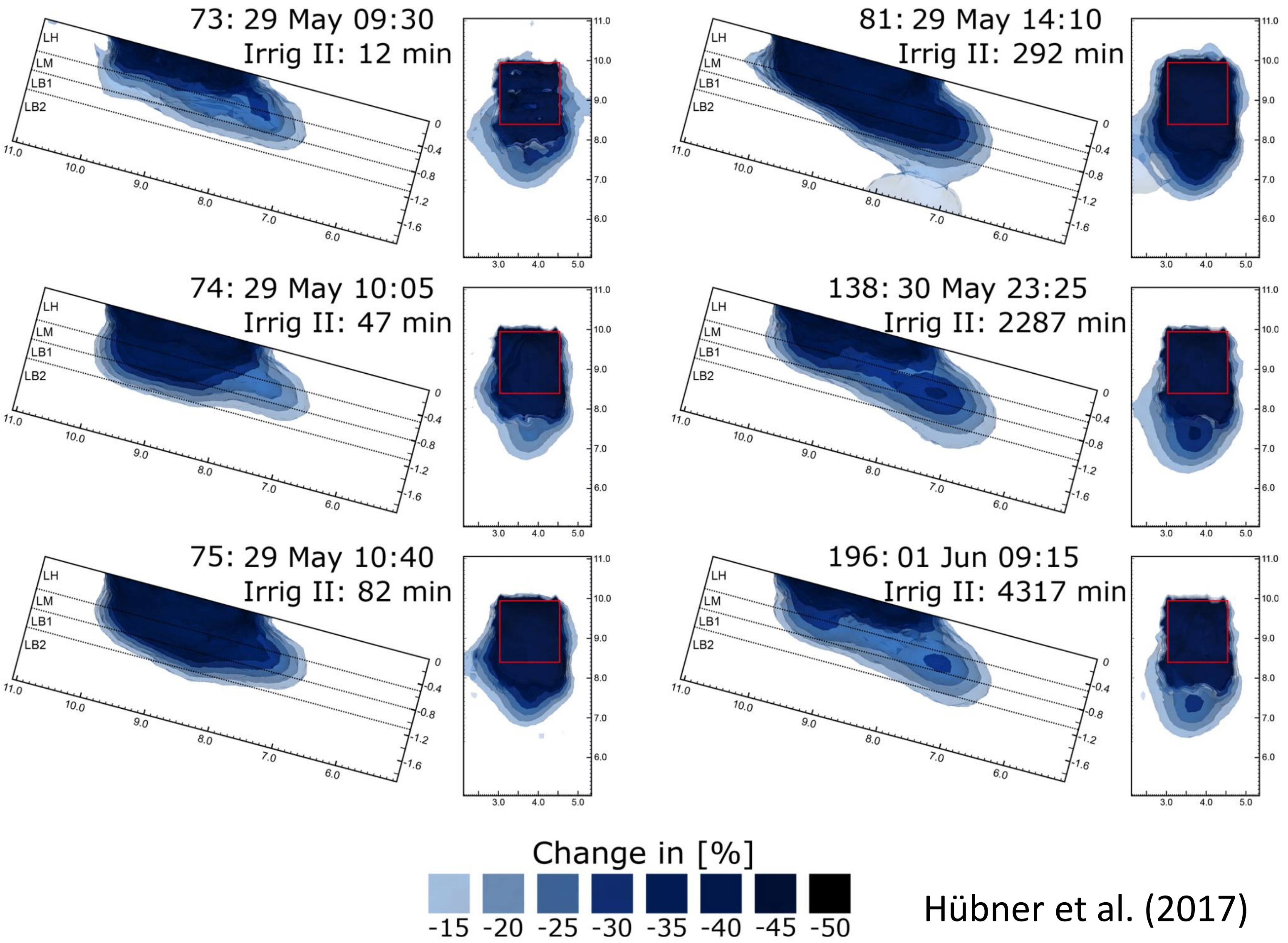
Fig. 2 *Left*: sketch of the field measurement set-up. BU—base unit of the system, C_1, C_2—shifting current electrodes, P_1, P_2—shifting potential electrodes for a dipol-dipol-ring configuration. *Right*: field test





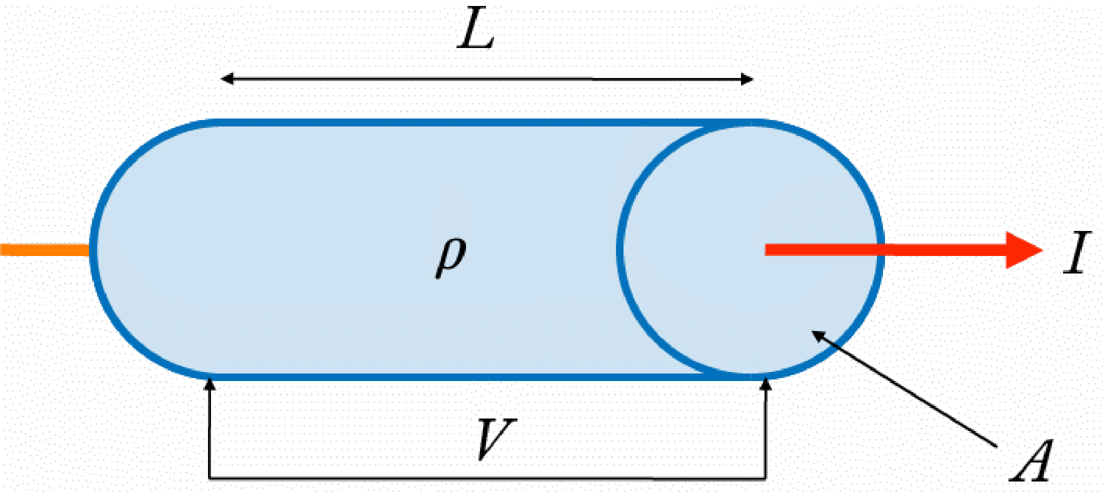


Infiltration monitoring



Hübner et al. (2017)

Resistance and resistivity



$$R = \frac{V}{I} = \rho\left(\frac{L}{A}\right)\ [\Omega]$$

$$\rho = \frac{V}{I}\left(\frac{A}{L}\right)\ [\Omega \cdot \mathrm{cm}]$$

V : voltage (V)
ρ : resistivity (Ω-cm)
I : current (A)
A : cross-sectional area (cm^2)
R : Resistance (Ω)
L : length (cm)

- Constant electric field $E = V/L$
- Current I distributes on area A
  $J = I / A$
- **Ohms law**: current density proportional to electric field
  $E = \rho J$ or $J = \sigma E$ *with* $\sigma = 1/\rho$

➔ $\rho = E / J = (V/L) / (I/A)$

➔ ρ = V/I * A/L = R * G

Resistivity is a material property

Unit: Ωm=Vm/A .

G .. Geometric factor (m)

Conductivity

σ = 1 / ρ in S/m=A/Vm

Resistivity

- From diamond (1e18 Ωm) to metal (1e-7 Ωm)
- Ionic conduction: water content ion content
- surface conduction (clay minerals)

Material	**Resistivity (in Ωm)** minimum	maximum
gravel	50 (water saturated)	>10^4 (dry)
sand	50 (water saturated)	>10^4 (dry)
silt	20	50
loam	30	100
clay (wet)	5	30
clay (dry)		>1000
peat, humus, sludge	15	25
sandstone	<50 (wet, jointed)	>10^5 (compact)
limestone	100 (wet, jointed)	>10^5 (compact)
schist	50 (wet, jointed)	>10^5 (compact)
igneous and metamorphic rock	<100 (weathered, wet)	>10^6 (compact)
rock salt	30 (wet)	>10^6 (compact)
domestic and industrial waste	<1	>1000 (plastic)
natural water	10	300
sea water (35‰ NaCl)	0.25	
saline water (brine)	<0.15	

Knödel et al. 2007

Resistivity

- Ionische Leitfähigkeit: Wassergehalt Ionengehalt
- Grenzflächen-Leitfähigkeit (Tonminerale)

Material	Minimum	Maximum in Ωm
Sand	50 (wassergesättigt)	$> 10^4$ (trocken)
Schluff	20	50
Geschiebemergel	30	70
Lößlehm	30	100
Ton (erdfeucht)	3	30
Ton (trocken)		> 1000
Torf, Humus, Schlick	15	25
Moorböden	10	150
Braunkohle	10	150
Erdöl	10^9	10^{12}
Sandstein	< 50 (klüftig, feucht)	$> 10^5$ (kompakt)
Kalkstein	100 (klüftig, feucht)	$> 10^5$ (kompakt)
Tonschiefer	50 (klüftig, feucht)	$> 10^5$ (kompakt)
Magmatite, Metamorphite	150 (verwittert, feucht)	$> 10^6$ (kompakt)
Schwarzschiefer	< 1	50
Steinsalz	30 (feucht)	$> 10^6$ (trocken)
Destilliertes Wasser		$> 10^3$
Schneefirn		$> 10^5$

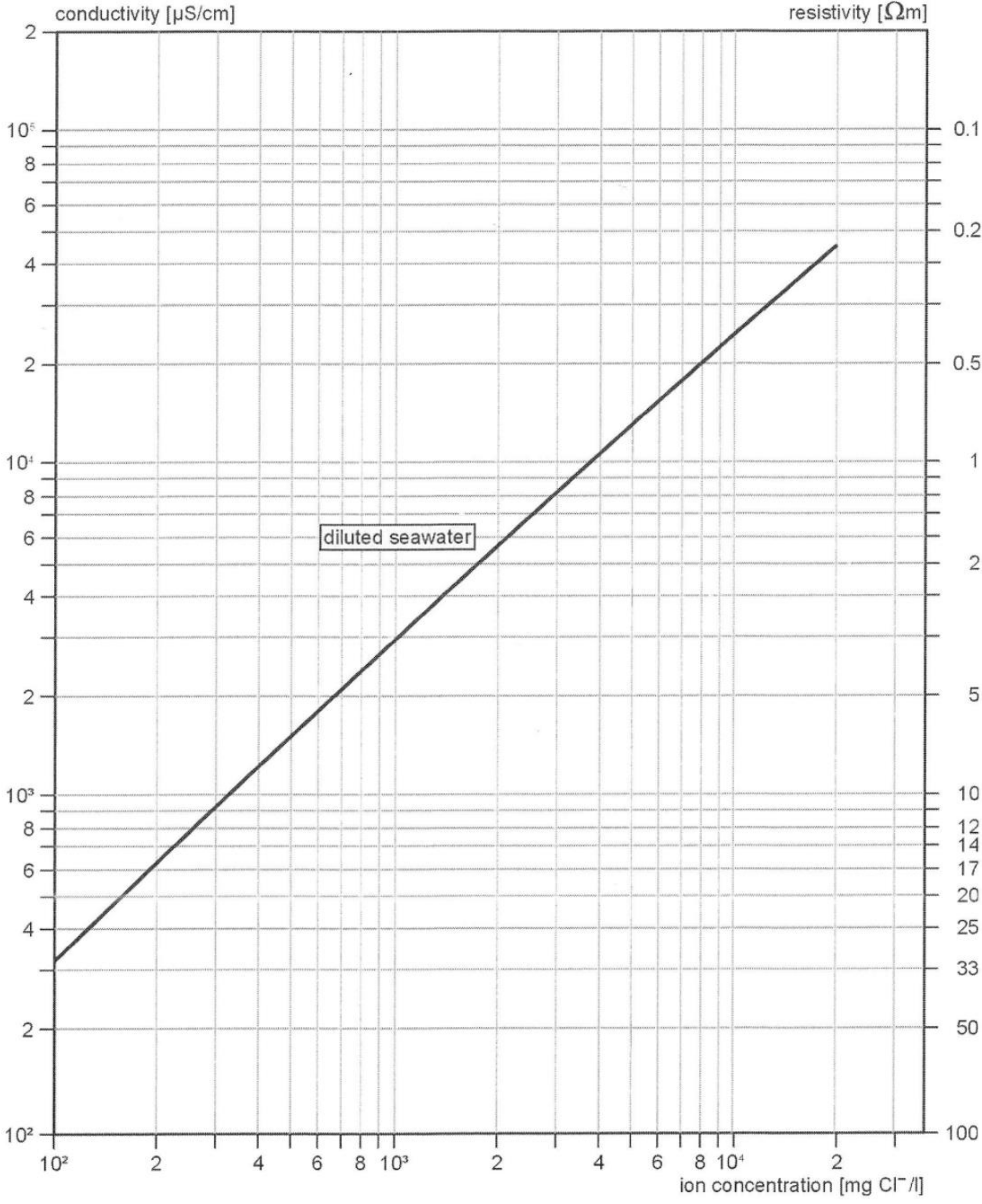
Knödel et al. 1995

Conductivity in porous media

Archie Gleichung

$$\sigma = \sigma_w \phi^m$$

σ: elektrische Leitfähigkeit des Gesteins
σ_w: elektrische Leitfähigkeit des Porenfluids
ϕ: Porosität
m: Zementationsfaktor



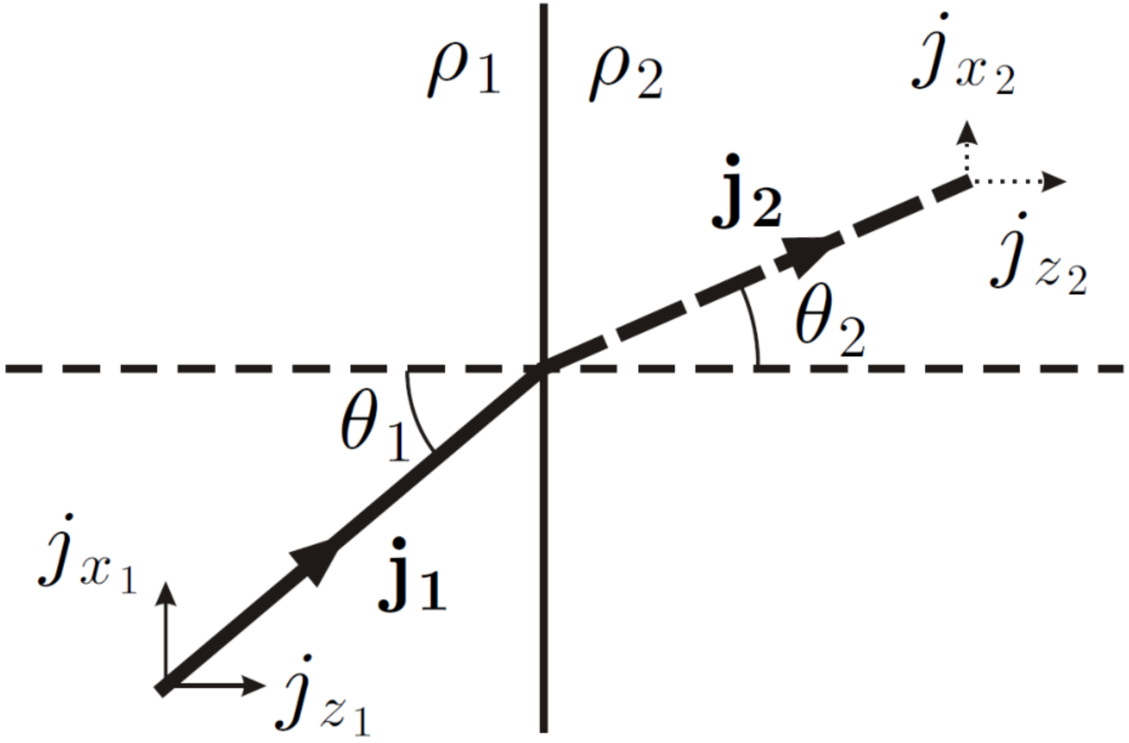
Continuity equation

$$\nabla \times \mathbf{H} = \mathbf{j}$$

$$\nabla \cdot (\nabla \times \mathbf{H}) = \nabla \cdot \mathbf{j} = 0$$

$$\nabla \cdot (\sigma \mathbf{E}) = 0$$
$$\nabla \cdot (-\sigma \nabla V) = 0$$

I (in) = I (out)



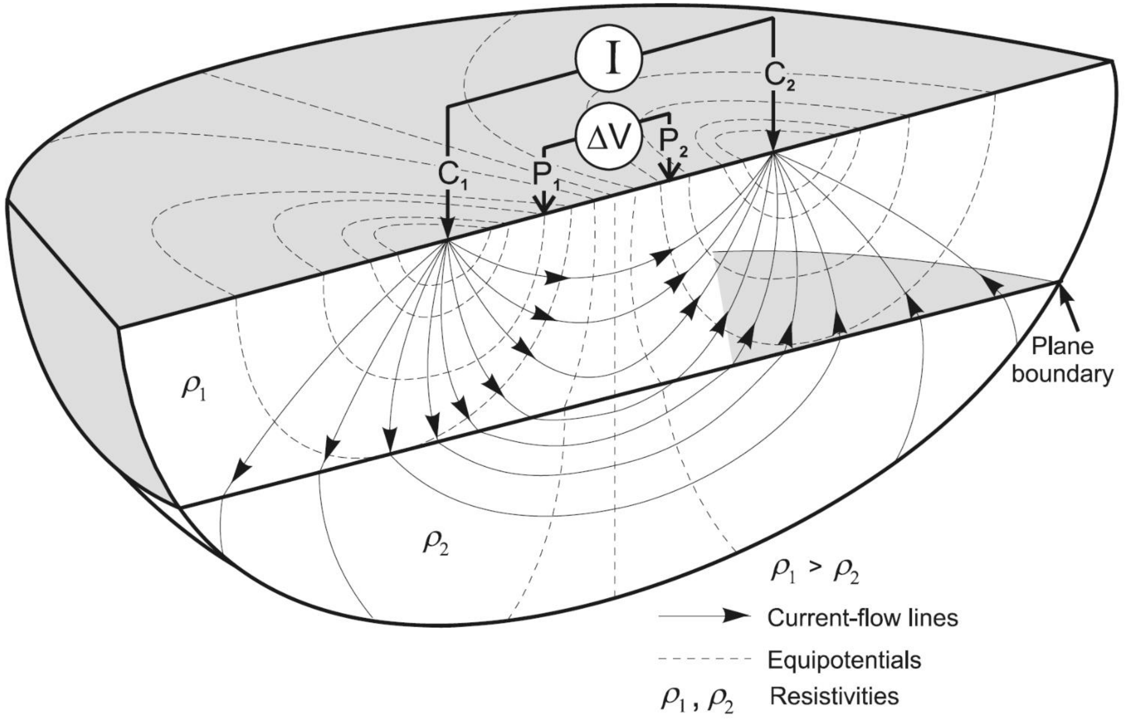
- Current density at boundary
  $E_1 = E_2$ ➔ $\rho_1 j_{1x} = \rho_2 j_{2x}$
  $j_{z1} = j_{z2}$

Boerner (2016)

Point source potential and geometric factor

$$\nabla \cdot (\sigma \nabla \varphi) = -I\delta(\vec{r} - \vec{r}_s)$$

Integration over surface
Current spreads over the surface of a sphere with radius *r*
j = *I* / *A* = *I* / (4π r^2)
U = ρ *I* / (4π *r)*
➔ ρ = U / *I* * 4π *r* = *R* * *k* ➔ *k*=4π*r*
➔ *At the surface k*=2π*r*



$$\begin{aligned} V_{MN} &= V_M - V_N = V_{AM} - V_{BM} - (V_{AN} - V_{BN}) \\ &= \frac{\rho I}{2 \cdot \pi} \left(\frac{1}{r_{AM}} - \frac{1}{r_{BM}} - \frac{1}{r_{AN}} + \frac{1}{r_{BN}} \right) \end{aligned}$$

$$k = \frac{2\pi}{\left(\frac{1}{r_{AP}} - \frac{1}{r_{BP}} - \frac{1}{r_{AQ}} + \frac{1}{r_{BQ}} \right)}$$