

# Near Surface Geophysical methods for site characterization

## 4. Geophysical Inversion (of ERT data)

Repetition ERT: Surface Arrays & Sensitivity

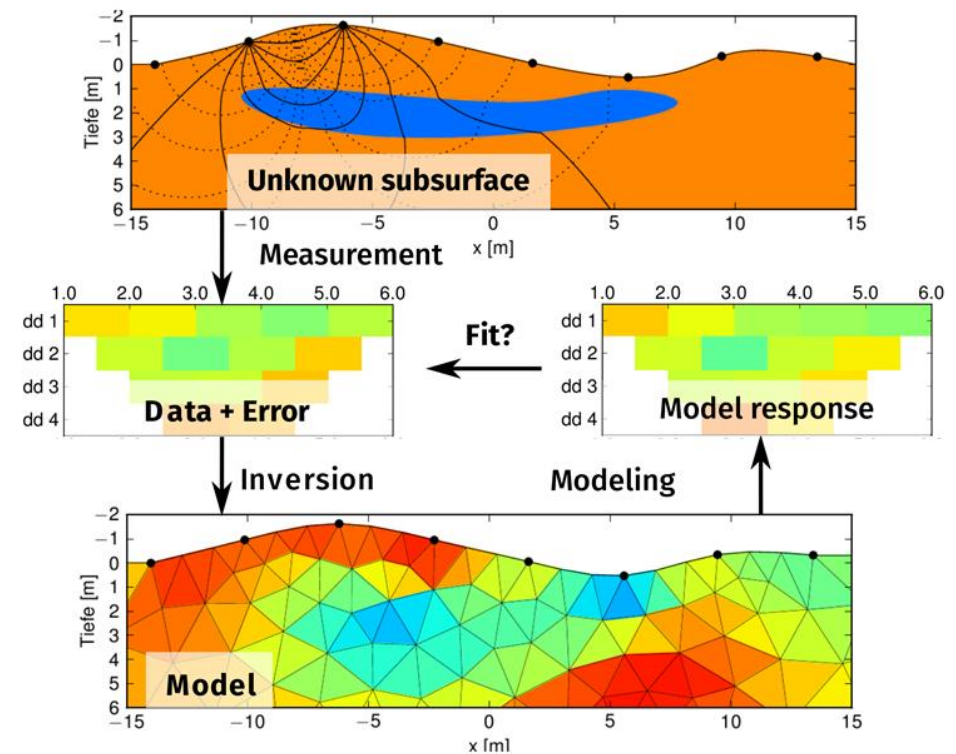
Crosshole ERT Arrays & Sensivities

Fundamentals of Geophysical Inversion

Objective and Ambiguity

Comparison of different ERT arrays

A recent case study from Borkum

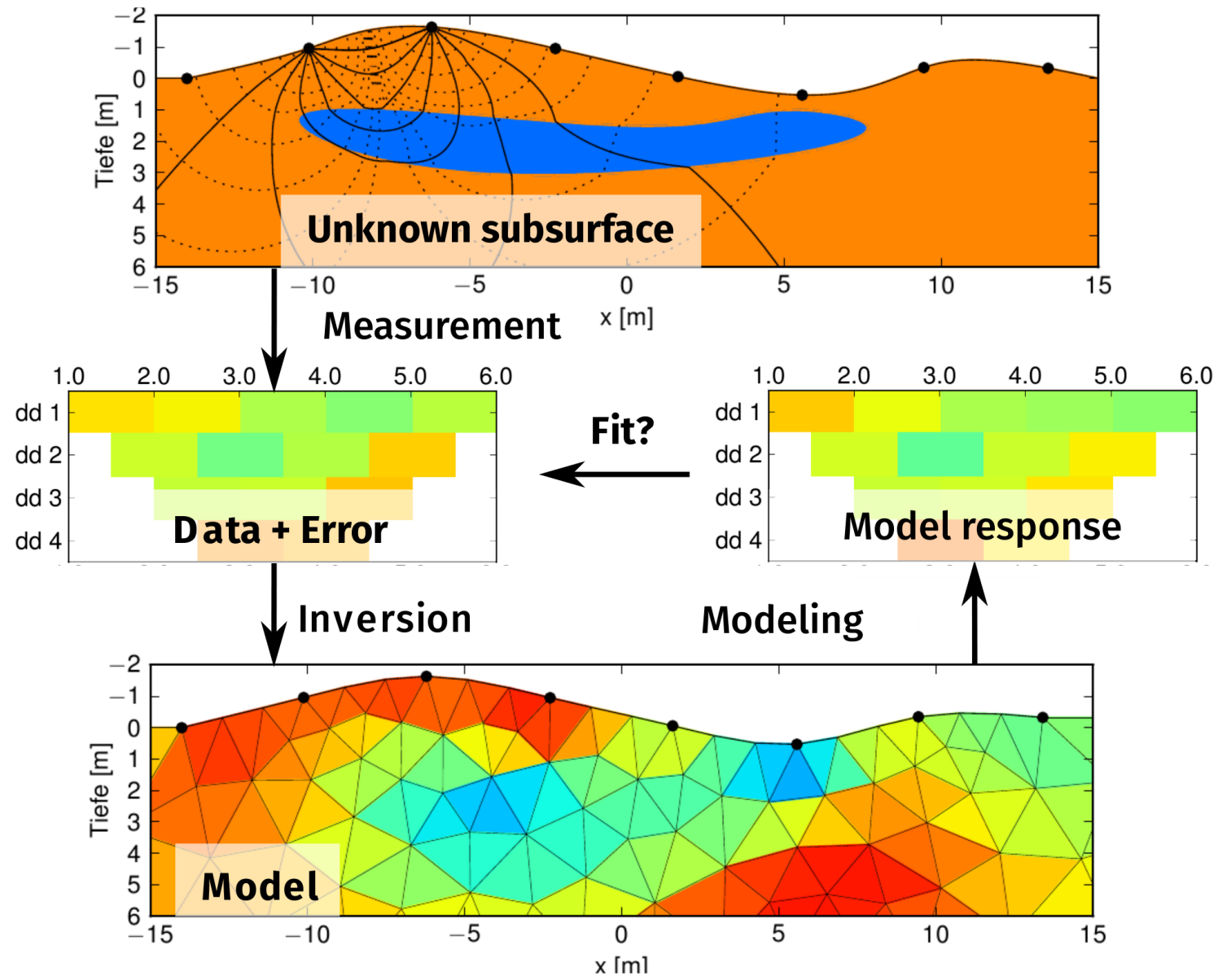


# Electrical Resistivity tomography

- Four electrodes: 2 for current (A-B), 2 for voltage (M-N)
- Different arrangement of these: geoelectrical arrays
- Mapping: moving a constant array along a profile  
typical array: Wenner array A—M—N—B ➔ Notebook
- Sounding: Increasing the (current) electrode spacing  
Typical array: Schlumber array (Vertical Electrical Sounding VES)
- Combination of sounding & mapping = 2D ERT
- Sensitivity shows area of influence

# Geophysical Inversion

- 1 Data acquisition
- 2 Preprocessing (quality check and filtering)
- 3 Parameterization (i.e., mesh generation)
- 4 Inversion
- 5 Evaluate fit between measured & simulated data
- 6 Postprocessing & visualization of final model(s)
- 7 Interpretation



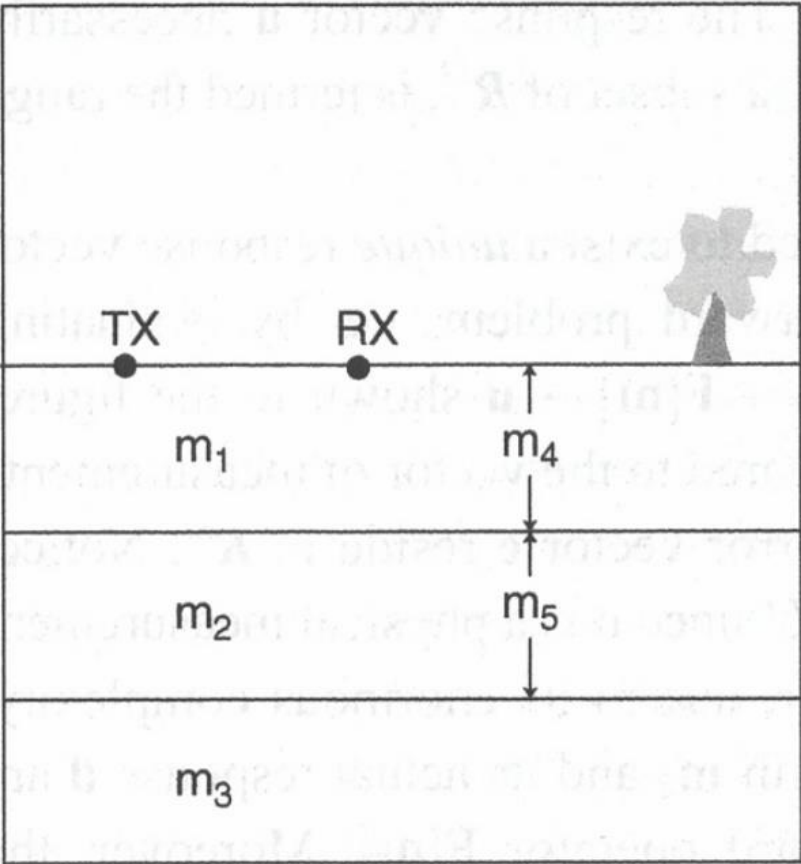
# Model types and dimensions

1D: soundings, refraction

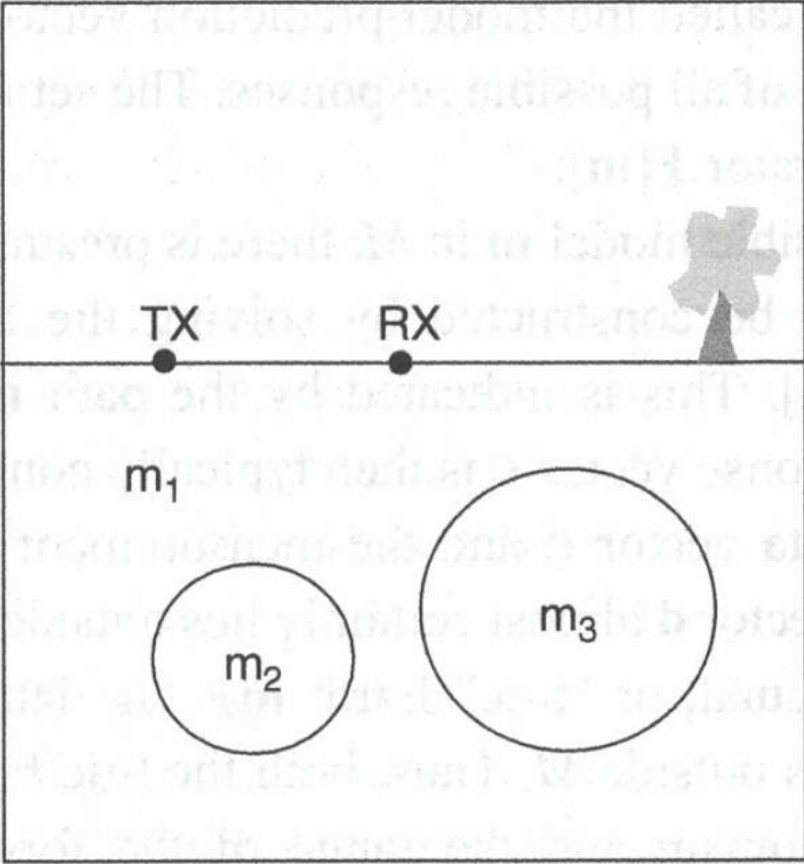
0D: gravity, magnetics

2D,3D: ERT, seismics, EM, grav/mag

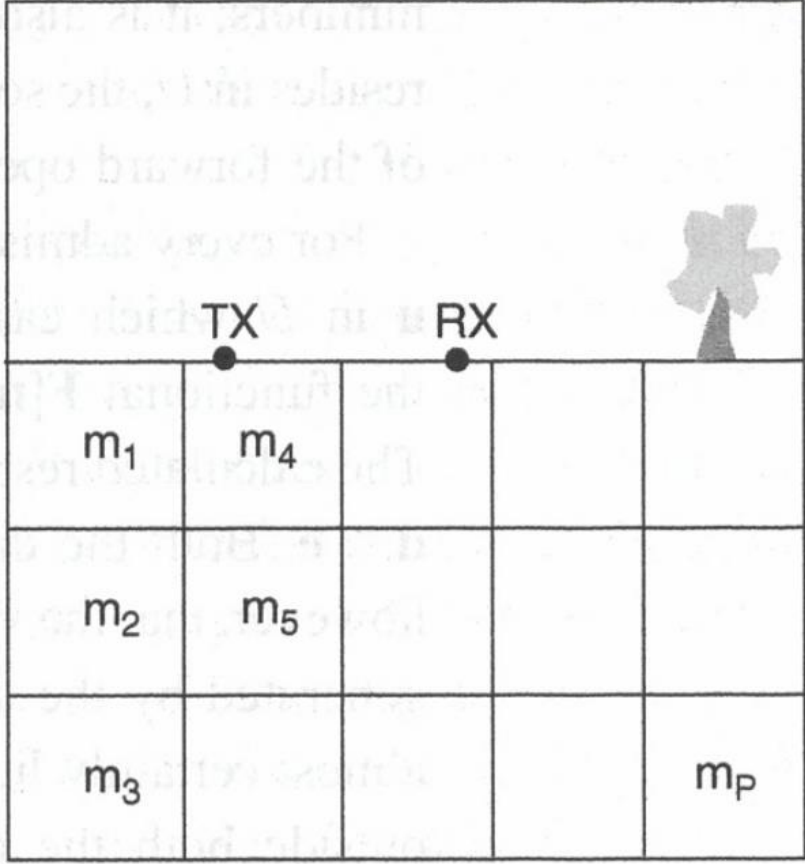
(a) layered model



(b) parametric model



(c) gridded model



# A fundamental principle: Occams razor

William of Occam, monk in scotland, 14th century

*Pluralitas non est ponenda sine neccesitate!*

*Don't make it more complicated than necessary.*

*Of all models explaining your data, choose the simplest!*

How can we define „simple“?

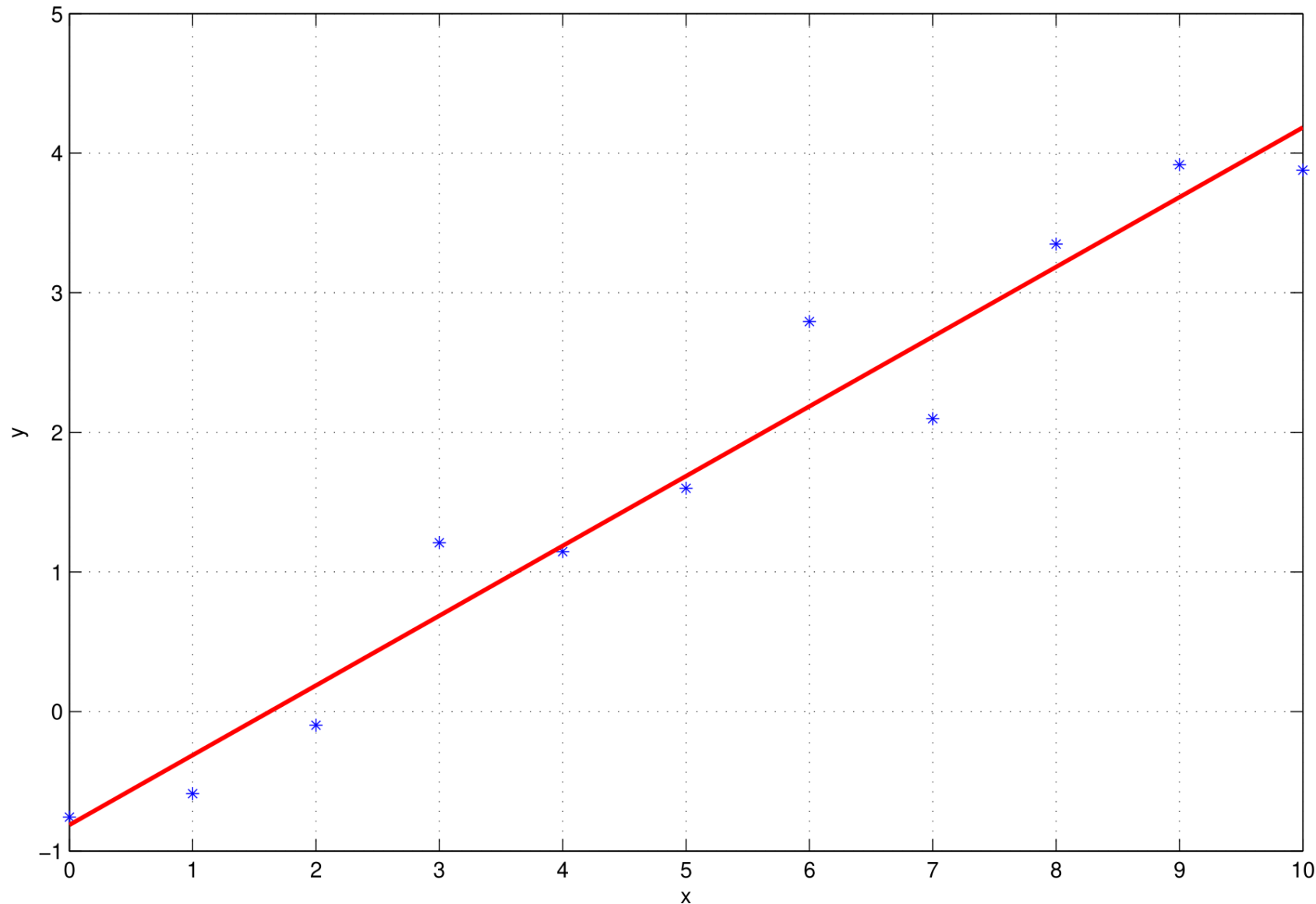
- Least number of model parameters (e.g. layers)
- Least number or strength of contrasts: smoothness
- Estimate probabilities, information content or model entropy
- Fitting the data no more than the should be fitted

# Geophysical Inversion

- Objective: find a model  $\mathbf{m}$  that explains the data reasonably
- Forward response  $f(\mathbf{m})$  should be close to data  $d$  (within error  $\varepsilon$ )
- Minimization of an objective function in a least-squares (regression) sense
- Chi-square (mean misfit)  $\chi^2 = \phi_d / N$

$$\Phi_d = \sum_{i=1}^N \left( \frac{d_i - f_i(\mathbf{m})}{\epsilon_i} \right)^2$$

# Simplest inverse problem: Linear regression



Data  $y$  given on measuring locations  $x$

$$y = a + b * x = G * x$$

$$G = [1 \ x]$$

$$M(a,b) = (G^T * G)^{-1} G^T x$$

Ready formula

# Minimization methods

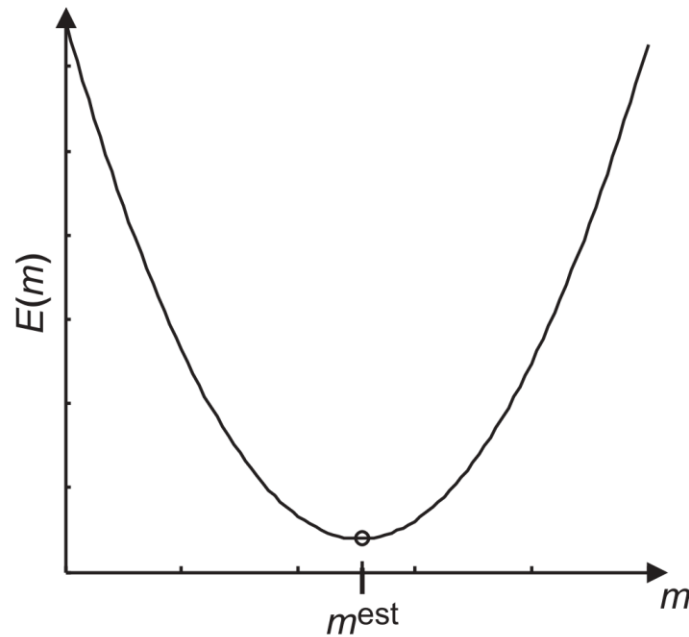
- Trial and error (hand or interactive modelling)
- Grid Search (test all possible combinations in a grid)
- Machine Learning, Neural Networks (training necessary)
- Global search: Genetic Algorithms, Monte Carlo methods
- Gradient methods: Steepest descent, conjugate gradients
- Newton methods: Use second derivative of objective function



# Objective function types

**Linear**

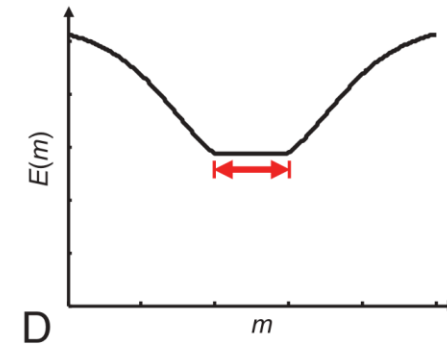
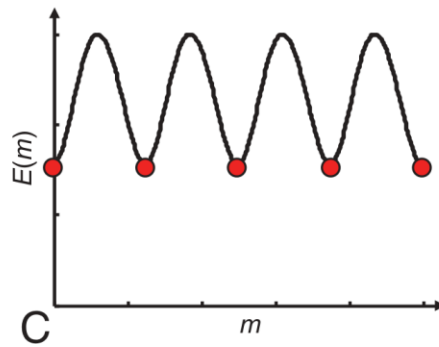
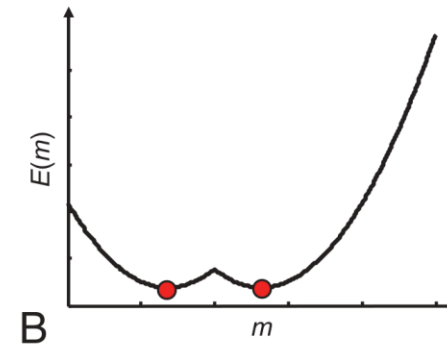
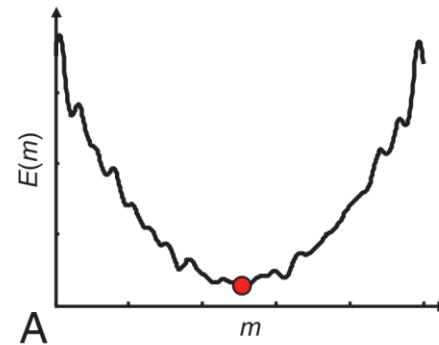
$$\mathbf{d} = \mathbf{G}\mathbf{m}$$



Clear minimum

**Non-linear**

Undulated  $\mathbf{d} = \mathbf{G}(\mathbf{m})$  Similar models

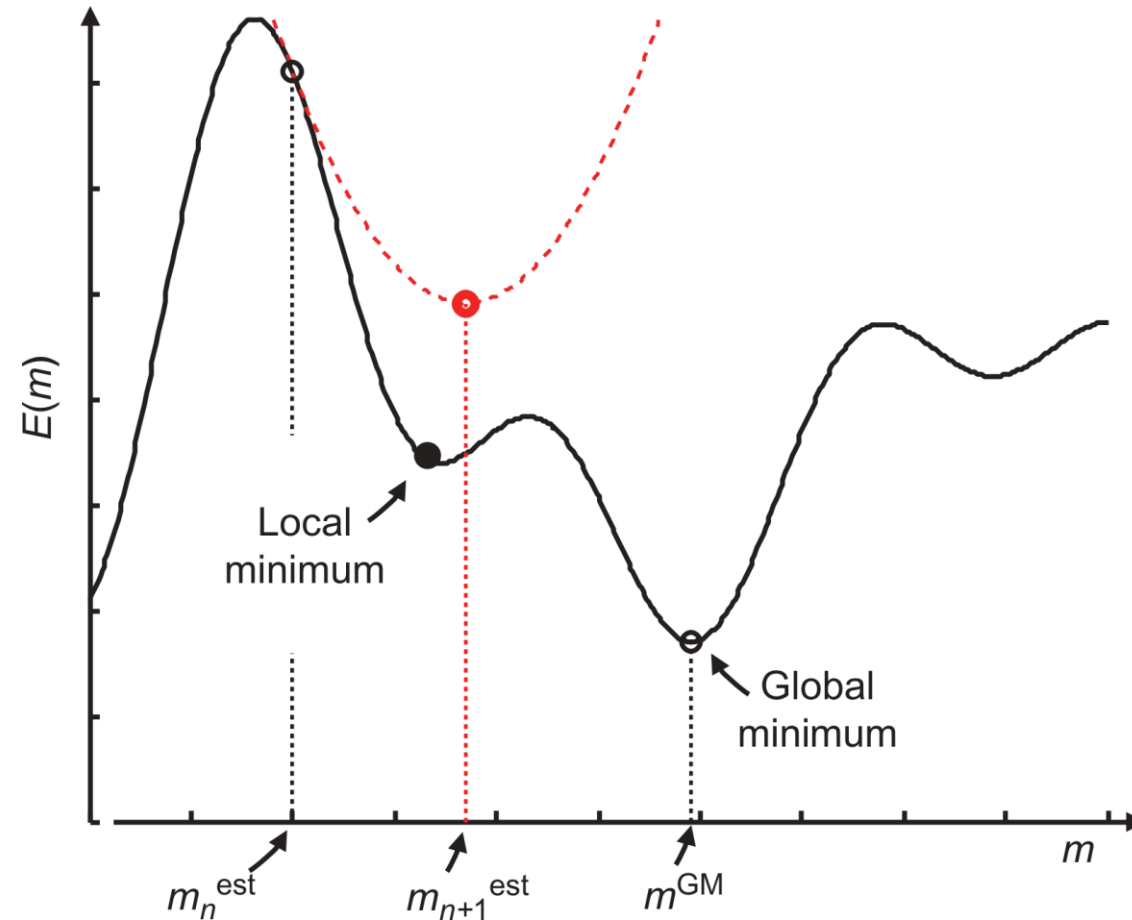


Additional constraints on  
valid model parameters

Periodic: modes

Broad minimum

# Iterative minimization by gradient methods



# Choice of simplicity

