Name: **Undergrad/Grad**

EMCH 585 (Spring 2018): Homework # 5

Due date: March 29, 2018 (Thursday) before the start of the class

APOGEE students may email HW solution (in pdf format) to professor

Problem #1 (For ALL students): 10 Points

Consider a $[\pm 30/0]_S$ **laminate is subjected to a combined loading of** $M_x = 25\,Nm/m, \quad and \quad \Delta T = -150\ ^0C$

Material properties: $E_1 = 155GPa \quad E_2 = 12.1GPa \quad \nu_{12} = 0.248 \quad G_{12} = 4.40GPa\ h = 0.15mm$

$X_t = 1500MPa \quad X_c = 1250MPa \quad Y_t = 50MPa \quad Y_c = 200MPa \quad S = 100MPa$

$\alpha_1 = -0.018\times10^{-6}/^0C \quad \alpha_2 = 24.3\times10^{-6}/^0C \quad \beta_1 = 146\times10^{-6}/\%M \quad \beta_2 = 4770\times10^{-6}/\%M$

$\hat{N}_x^T = 97.2\ N/m/^0C \quad \hat{N}_y^T = 232\ N/m/^0C \quad \hat{N}_x^M = 36600\ N/m/\% \quad \hat{N}_y^M = 49400\ N/m/\%$

Find the lamina failure status (safe or failed) along with failure mode of the laminate using maximum stress criteria. You may use the following template for summarizing your final answers or create your presentation style.

$X_t = 1500MPa \quad X_c = 1250MPa \quad Y_t = 50MPa \quad Y_c = 200MPa \quad S = 100MPa$						
Lamina number	**Orientation**	**Z-location**	**σ_1 MPa**	**σ_2 MPa**	**τ_{12} MPa**	**Failure status (safe/failed)**
1	**30**					
2	**-30**					
3	**0**					
4	**0**					
5	**-30**					
6	**30**					
	Failure Mode and Location:					

Problem #2 (For All students): 10 Points

Consider a $[\pm 30/0]_S$ **laminate is subjected to a combined loading of** $M_x = \pm m_x\ Nm/m, and \quad \Delta T = -150\ ^0C$

Material properties: $E_1 = 155GPa \quad E_2 = 12.1GPa \quad \nu_{12} = 0.248 \quad G_{12} = 4.40GPa\ h = 0.15mm$

$X_t = 1500MPa \quad X_c = 1250MPa \quad Y_t = 50MPa \quad Y_c = 200MPa \quad S = 100MPa$

$\alpha_1 = -0.018\times10^{-6}/^0C \quad \alpha_2 = 24.3\times10^{-6}/^0C \quad \beta_1 = 146\times10^{-6}/\%M \quad \beta_2 = 4770\times10^{-6}/\%M$

$\hat{N}_x^T = 97.2\ N/m/^0C \quad \hat{N}_y^T = 232\ N/m/^0C \quad \hat{N}_x^M = 36600\ N/m/\% \quad \hat{N}_y^M = 49400\ N/m/\%$

Find the moment Mx required to cause first ply failure (FPF) in the laminate using Tsai-Wu criteria (with Von Mises approximation).

Hint on Problem 2: The part of the problem is already solved for you as a gift ! (You should be able to calculate stresses like this).

$$\text{for each z, express stresses as}: \begin{Bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{Bmatrix}_{for\ M_x=1} m_x + \Delta T \begin{Bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{Bmatrix}_{for\ \Delta T=1}$$

In this 2-variable problem, Tsai-Wu can solve for either mx or deltaT. This problem asks for finding Mx given known deltaT. Hence substitute deltaT=-150 in the stress equation above and only one unknown left.

Stresses are already calculated for you and given below:

-Z locations

$$\text{for 30deg}(z=-0.450\text{mm})$$
$$\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{Bmatrix} = -\begin{Bmatrix}5.79\\ -0.85\\ -1.415\end{Bmatrix}\times 10^6 m_x + \begin{Bmatrix}0.368\\ -0.141\\ 0.0722\end{Bmatrix}\times 10^6 \Delta T$$

$$\text{for} -30\text{deg}\ (z=-0.30\text{mm})$$
$$\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{Bmatrix} = -\begin{Bmatrix}9.02\\ -0.875\\ 0.772\end{Bmatrix}\times 10^6 m_x + \begin{Bmatrix}0.368\\ -0.141\\ -0.0722\end{Bmatrix}\times 10^6 \Delta T$$

$$\text{for 0deg}(z=-0.150\text{mm})$$
$$\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{Bmatrix} = -\begin{Bmatrix}7.52\\ -0.618\\ -0.0858\end{Bmatrix}\times 10^6 m_x + \begin{Bmatrix}-0.356\\ -0.0977\\ 0\end{Bmatrix}\times 10^6 \Delta T$$

+Z locations

$$\text{for 30deg}(z=0.450\text{mm})$$
$$\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{Bmatrix} = \begin{Bmatrix}5.79\\ -0.85\\ -1.415\end{Bmatrix}\times 10^6 m_x + \begin{Bmatrix}0.368\\ -0.141\\ 0.0722\end{Bmatrix}\times 10^6 \Delta T$$

$$\text{for} -30\text{deg}\ (z=0.30\text{mm})$$
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$$\text{for 0deg}(z=0.150\text{mm})$$
$$\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{Bmatrix} = \begin{Bmatrix}7.52\\ -0.618\\ -0.0858\end{Bmatrix}\times 10^6 m_x + \begin{Bmatrix}-0.356\\ -0.0977\\ 0\end{Bmatrix}\times 10^6 \Delta T$$

substitute deltaT=-150 in the stress equation above.

Now simplify and use these stresses in failure criteria. Solve for multiplier mx. Choose the lowest value(s).

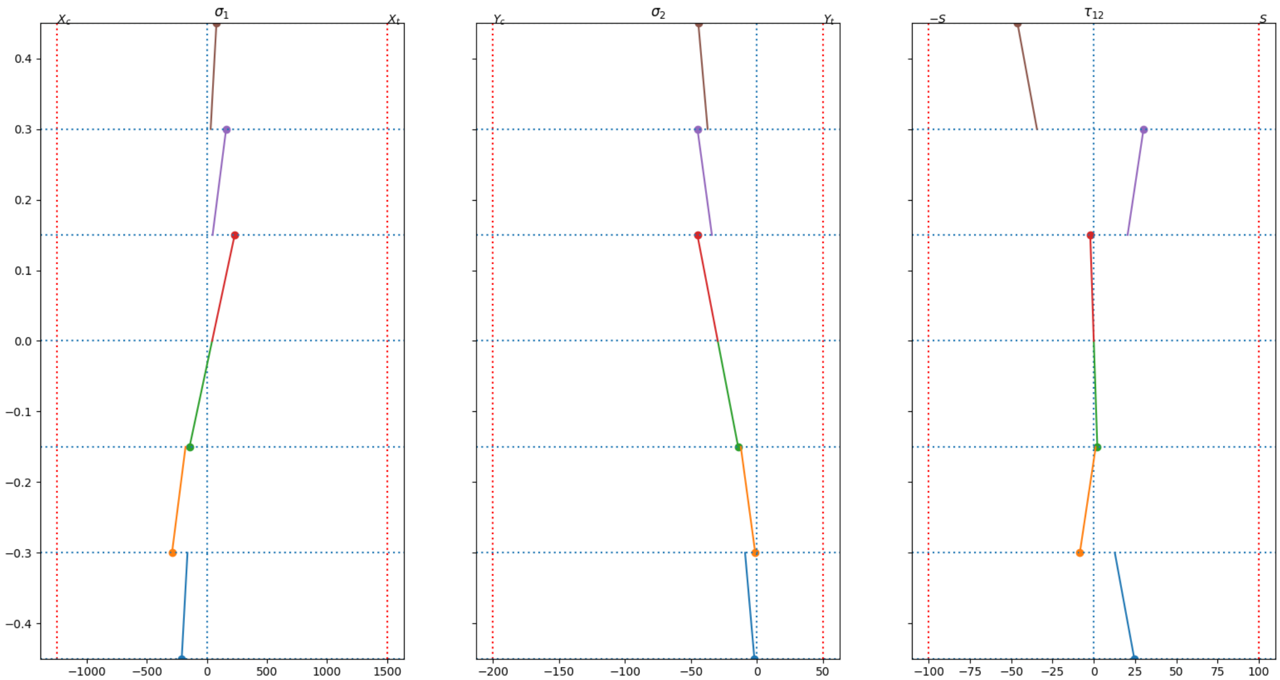
$$F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{66}\tau_{12}^2 + F_1\sigma_1 + F_2\sigma_2 + 2F_{12}\sigma_1\sigma_2 = 1$$

#1

[[0]	[[97.16]
[0]	[232.116]
[0]	[0.]
[1]	[0.]
[0]	[0.]
[0]]	[0.]]
[[[-5.789]	[[[4.383e-01]
[0.85]	[1.544e-01]
[1.415]]	[7.225e-02]]
[[-9.015]	[[4.383e-01]
[0.875]	[1.544e-01]
[-0.772]]	[-7.225e-02]]
[[-7.516]	[[-2.857e-01]
[0.618]	[1.978e-01]
[0.086]]	[1.452e-18]]
[[7.516]	[[-2.857e-01]
[-0.618]	[1.978e-01]
[-0.086]]	[-3.774e-18]]
[[9.015]	[[4.383e-01]
[-0.875]	[1.544e-01]
[0.772]]	[-7.225e-02]]
[[5.789]	[[4.383e-01]
[-0.85]	[1.544e-01]
[-1.415]]]	[7.225e-02]]]

```
[[[-210.462]
  [  -1.922]
  [  24.543]]

 [[-291.117]
  [  -1.287]
  [  -8.462]]

 [[-145.046]
  [ -14.224]
  [   2.144]]

 [[ 230.751]
  [ -45.107]
  [  -2.144]]

 [[ 159.621]
  [ -45.042]
  [  30.138]]

 [[  78.966]
  [ -44.407]
  [ -46.219]]]
```

```
 [-0.45, -0.3 , -0.15,  0.15,  0.3 ,  0.45])
```



$X_t = 1500MPa \quad X_c = 1250MPa \quad Y_t = 50MPa \quad Y_c = 200MPa \quad S = 100MPa$						
Lamina number	**Orientation**	**Z-location**	**σ_1 MPa**	**σ_2 MPa**	**τ_{12} MPa**	**Failure status (safe/failed)**
1	**30**					
2	**-30**					
3	**0**					
4	**0**					
5	**-30**					
6	**30**					
	Failure Mode and Location:					

#2

Mx= -73.34 / 25.44 (Nm/m) in (30) ply at Z=-0.450mm

Mx= -103.37 / 24.80 (Nm/m) in (-30) ply at Z=-0.300mm

Mx= -164.14 / 45.53 (Nm/m) in (0) ply at Z=-0.150mm

Mx= -45.53 / 164.14 (Nm/m) in (0) ply at Z=0.150mm

Mx= -24.80 / 103.37 (Nm/m) in (-30) ply at Z=0.300mm

Mx= -25.44 / 73.34 (Nm/m) in (30) ply at Z=0.450mm

FPF from -Mx=-24.80 (Nm/m) in (-30) ply at Z=0.300mm

FPF from Mx=24.80 (Nm/m) in (-30) ply at Z=-0.300mm

```
import numpy as np
import matplotlib.pyplot as plt

np.set_printoptions(precision=3)

def Transform(theta):
    m = np.cos( np.deg2rad(theta) )
    n = np.sin( np.deg2rad(theta) )
    return np.array([
        [m**2, n**2, 2*m*n],
        [n**2, m**2, -2*m*n],
        [-m*n, m*n,  m**2 - n**2]], np.float64)


theta = np.array([+30,-30,0,0,-30,+30])

N = theta.size
h = .15*10**-3
H = N*h

Z = np.arange(N+1)*h - .5*H

E1 = 155 * 10**9
E2 = 12.1 * 10**9
v12 = .248
G12 = 4.4 * 10**9

S = np.array([
    [1/E1, -v12/E1, 0],
    [-v12/E1, 1/E2, 0],
    [0,  0       , 1/G12]], np.float64)

T = Transform(theta)
T_ = np.rollaxis(T, 2)

Sbar = np.einsum('...jk,kl,...lm->...jm', T.T, S, T_)
Qbar = np.linalg.inv(Sbar)

A = np.sum( np.diff(Z)[:, None, None] * Qbar, axis=0)
B = (1/2)*np.sum( np.diff(Z**2)[:, None, None] * Qbar, axis=0)
D = (1/3)*np.sum( np.diff(Z**3)[:, None, None] * Qbar, axis=0)

ABD = np.block([[A,B],[B,D]])

ABD[np.abs(ABD) < 10**-8] = 0

abd = np.linalg.inv(ABD)
```

```
Xt = 1500 * 10**6
Xc = 1250 * 10**6

Yt =   50 * 10**6
Yc =  200 * 10**6
S  =  100 * 10**6


alpha_1 = -0.018 * 10**-6
alpha_2 = 24.3   * 10**-6

beta_1 =  146 * 10**-6
beta_2 = 4770 * 10**-6


alpha = np.array([alpha_1, alpha_2, 0])
beta  = np.array([beta_1, beta_2, 0])

alpha_bar = np.matmul(T.T, alpha[:, None]).reshape(N, 1, 3)
beta_bar = np.matmul(T.T, beta[:, None]).reshape(N, 1, 3)

NM = np.array([[0,0,0,1,0,0]]).T

N_t = np.sum( np.diff(Z)[:, None] *  np.sum( Qbar * alpha_bar, axis=2 ), axis=0)
M_t = (1/2)*np.sum( np.diff(Z**2)[:, None] *  np.sum( Qbar * alpha_bar, axis=2 ),
axis=0)
NM_t = np.concatenate([N_t, M_t]).reshape(6, 1)

N_m = np.sum( np.diff(Z)[:, None] *  np.sum( Qbar * beta_bar, axis=2 ), axis=0)
M_m = (1/2)*np.sum( np.diff(Z**2)[:, None] *  np.sum( Qbar * beta_bar, axis=2 ),
axis=0)

NM_m = np.concatenate([N_m, M_m]).reshape(6, 1)

dT = -150

NM_r = 25*NM + dT*NM_t

Z_ = np.hstack((Z[:N//2], Z[-N//2:]))

ref_surf = np.dot(abd, NM_r)
surf_strain = ref_surf[:3]
surf_curve = ref_surf[3:]

Strains = surf_strain + np.dot(surf_curve, Z_[None,:])
Sigma_xyz = np.matmul(Qbar, Strains.T[:,:,None])
Sigma_123 = np.matmul(T_, Sigma_xyz)
```

```
import numpy as np
import sympy as sp
from sympy import Symbol

theta = np.array([+30,-30,0,0,-30,+30])

N = theta.size
h = .15*10**-3
H = N*h

Z = np.arange(N+1)*h - .5*H
Z_ = np.hstack((Z[:N//2], Z[-N//2:]))


Xt = 1500 * 10**6
Xc = 1250 * 10**6

Yt =   50 * 10**6
Yc =  200 * 10**6

S  =  100 * 10**6

F1 = (1/Xt) - (1/Xc)
F2 = (1/Yt) - (1/Yc)

F11 = 1/(Xt*Xc)
F22 = 1/(Yt*Yc)

F66 = 1/S**2

F12 = -np.sqrt(F11*F22)

NM = np.array([
    [-5.79, .85, 1.415],
    [-9.02, .876, -.772],
    [-7.52, .618, .0858],
    [7.52, -.618, -.0858],
    [9.02, -.876, .772],
    [5.79, -.85, -1.415]]) * 10**6

NM_t = np.array([
    [.368, -.141, .0722],
    [.368, -.141, -.0722],
    [-.356, -.0977, 0],

    [-.356, -.0977, 0],
    [.368, -.141, -.0722],
    [.368, -.141, .0722]]) * 10**6
```

```
dT = -150

compression, tension = (np.inf, np.inf)

c_ply = 0
t_ply = 0

mx_var = Symbol('mx')

for i in range(N):
    s1 = NM[i, 0] * mx_var + NM_t[i, 0] * dT
    s2 = NM[i, 1] * mx_var + NM_t[i, 1] * dT
    t12 = NM[i, 2] * mx_var + NM_t[i, 2] * dT

    equ = F11*s1**2 + F22*s2**2 + F66*t12**2 + F1*s1 + F2*s2 + F12*s1*s2

    c, t = sp.solve(equ-1, mx_var)

    print('Mx= %.2f / %.2f (Nm/m) in (%d) ply at Z=%.3fmm' %(c, t, theta[i],
Z_[i]*10**3))

    compression = min(np.abs(c), compression)
    tension = min(np.abs(t), tension)

    if np.abs(c) == compression: c_ply = i
    if np.abs(t) == tension: t_ply = i

print('--------------------')
print('FPF from -Mx=%.2f (Nm/m) in (%d) ply at Z=%.3fmm' %(-compression,
theta[c_ply], Z_[c_ply]*10**3))
print('FPF from Mx=%.2f (Nm/m) in (%d) ply at Z=%.3fmm' %(tension, theta[t_ply],
Z_[t_ply]*10**3))
```