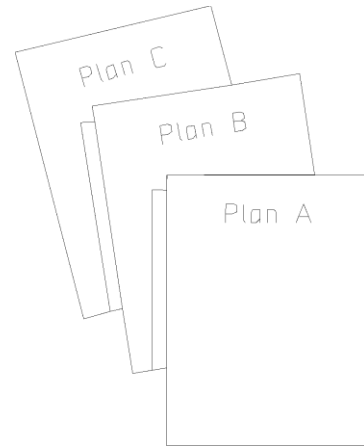


Pruning (Part I)

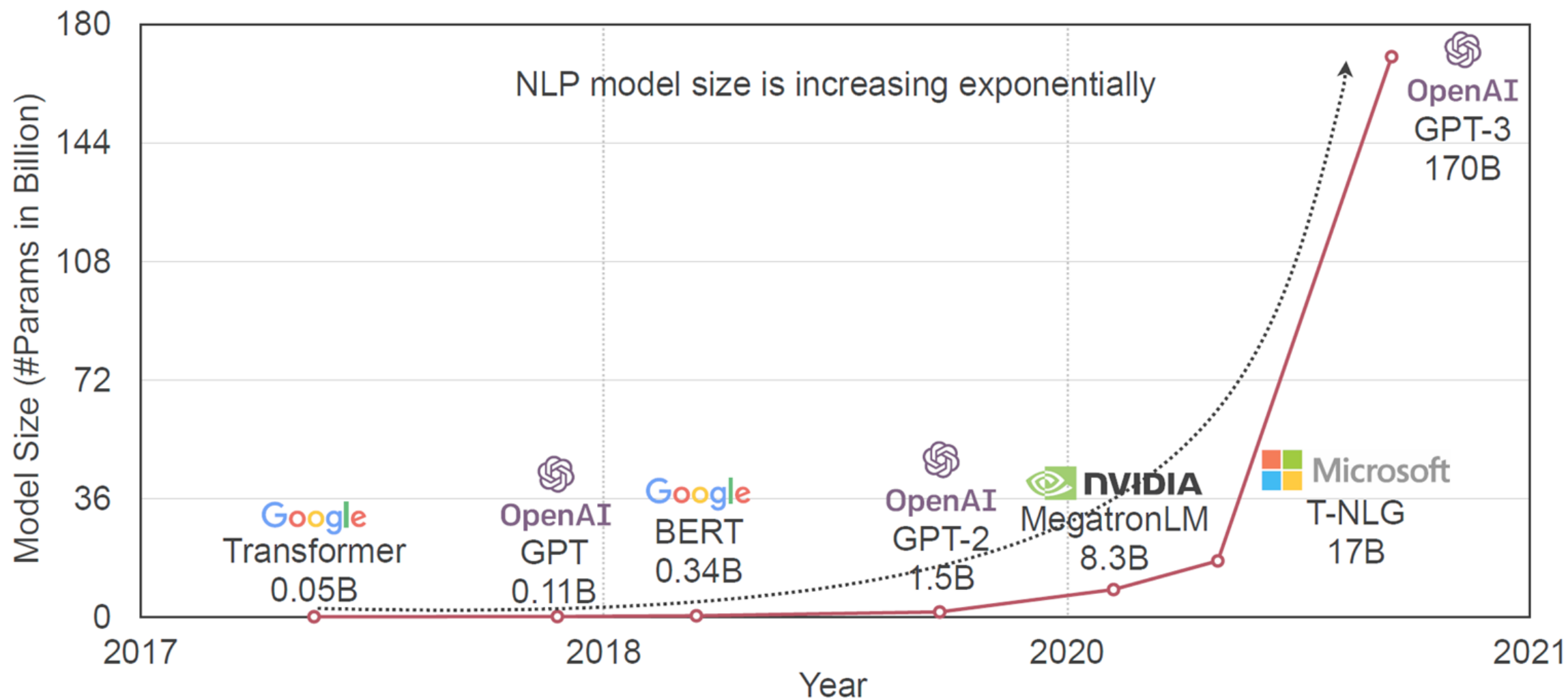
서울대학교 컴퓨터공학부 이영기

Overview

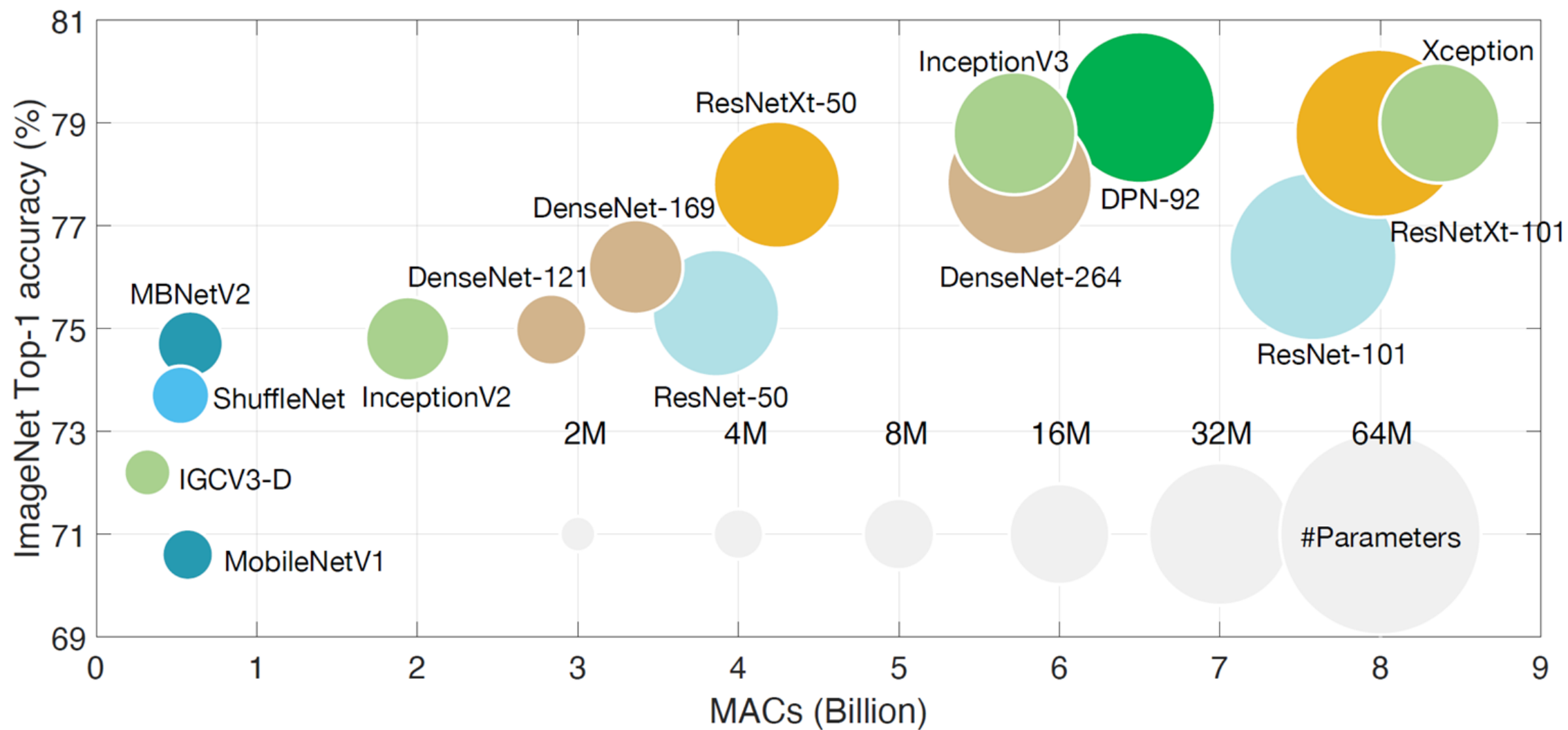
- Objective
 - Understand neural network pruning to reduce the storage and computation requirements
 - Learn different granularities and criteria of neural network pruning
- Content
 - Introduction to pruning
 - Pruning granularity: Fine-grained / Pattern-based / Channel-level pruning
 - Pruning criterion: Magnitude-based / Scaling-based pruning
- After this module, you should be able to
 - Grasp the concept of neural network pruning and its effects on deep learning models
 - Understand various types of pruning and their advantages and disadvantages



Today's AI is too BIG!



Today's AI is too BIG!



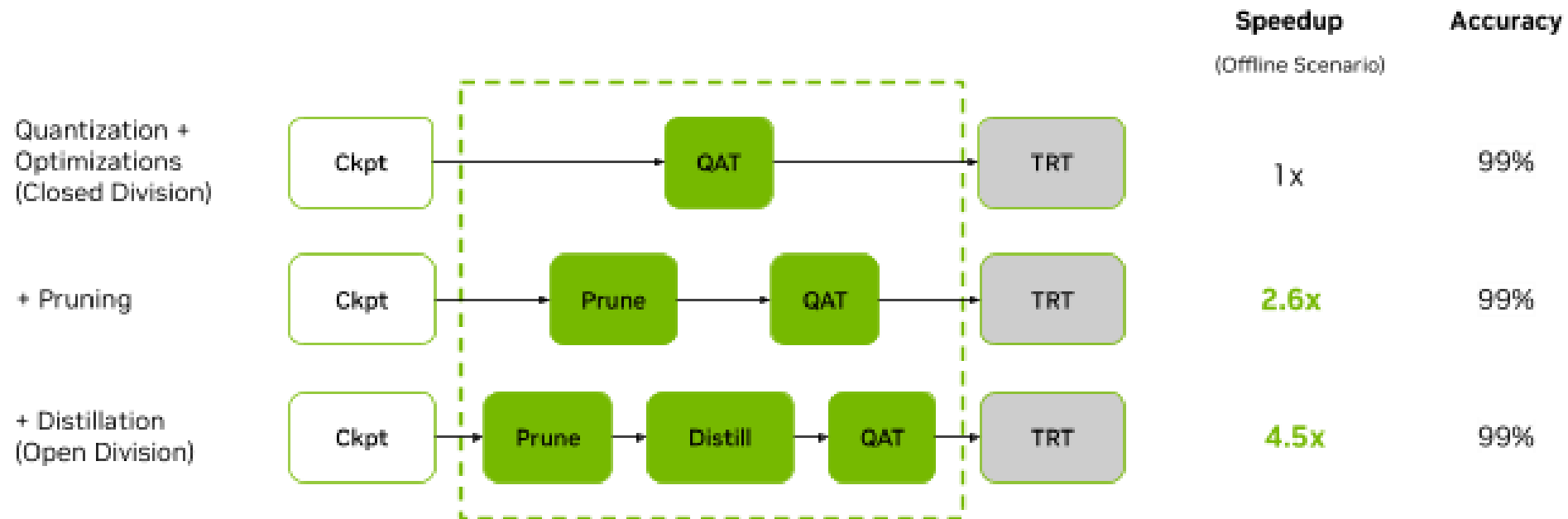
MLPerf (the Olympic Game for AI Computing)

- The open division submission on BERT
 - More than 4x while maintaining 99% accuracy

	Closed Division	Open Division	Speedup
Offline samples/sec	1029	4609	4.5x

MLPerf (the Olympic Game for AI Computing)

- The key techniques are pruning, distillation and quantization

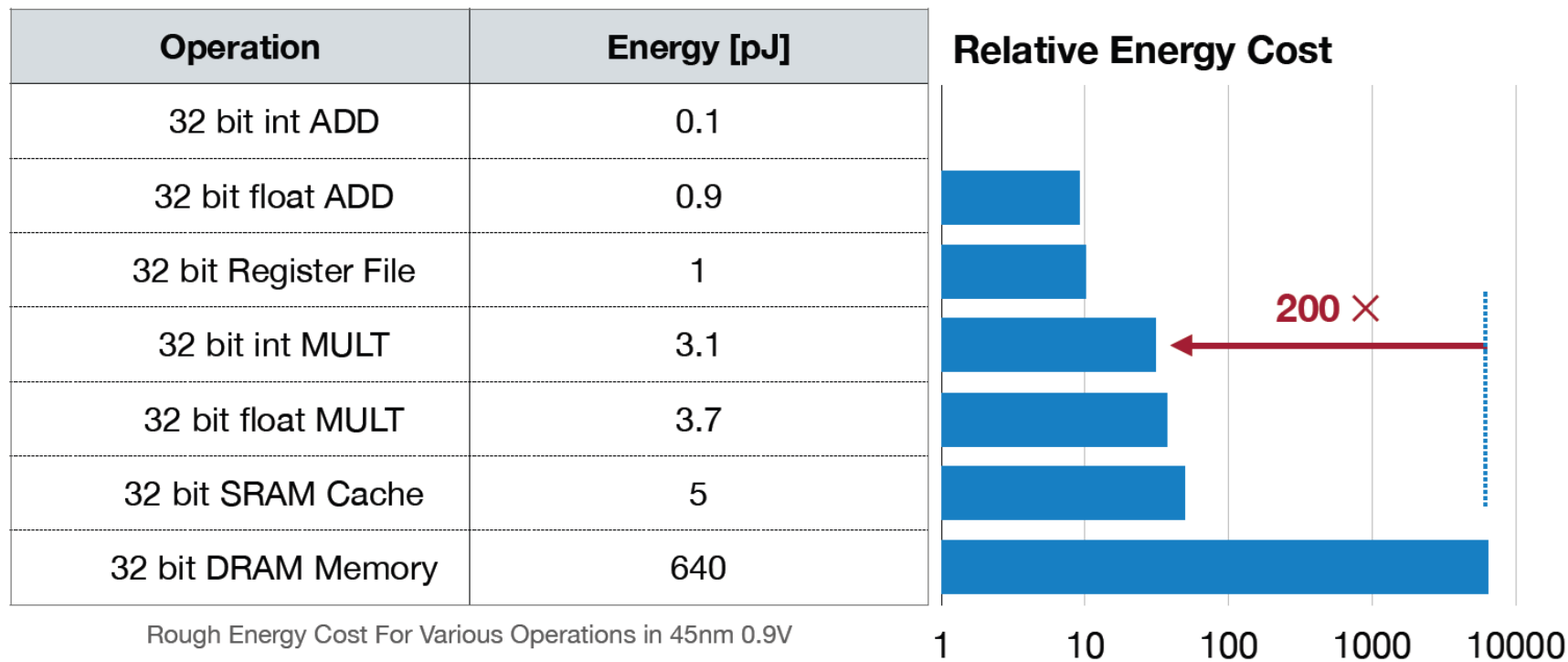


Neural Network Pruning

- A widely-used technique that can reduce the parameter counts of neural networks by more than 90%.
- It decreases the storage(memory) requirements and improves computation efficiency of neural networks.

Memory is Expensive

- Data movement ☐ More memory reference ☐ more energy

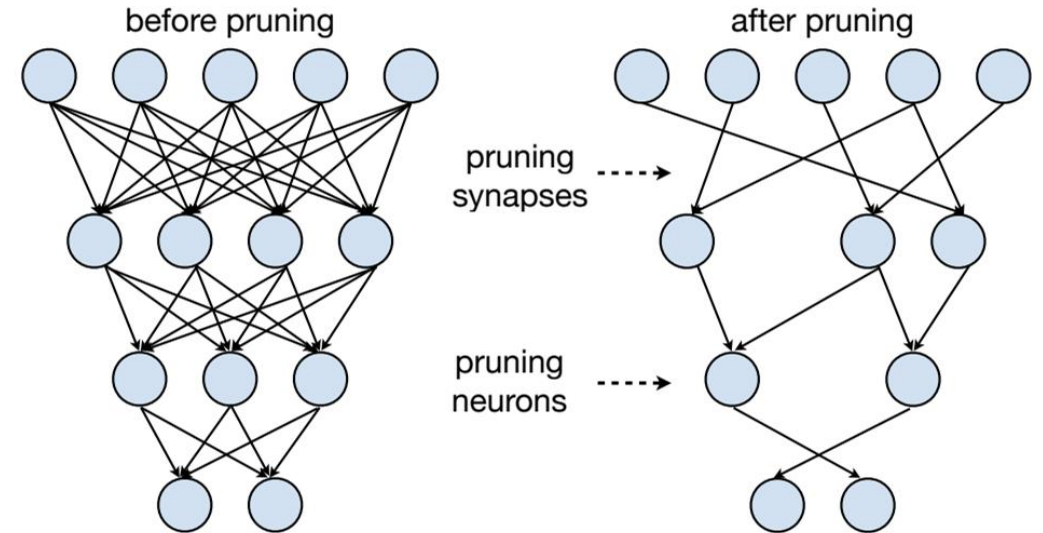


$$1 \text{ [Image of DRAM chip]} = 200 \times +$$

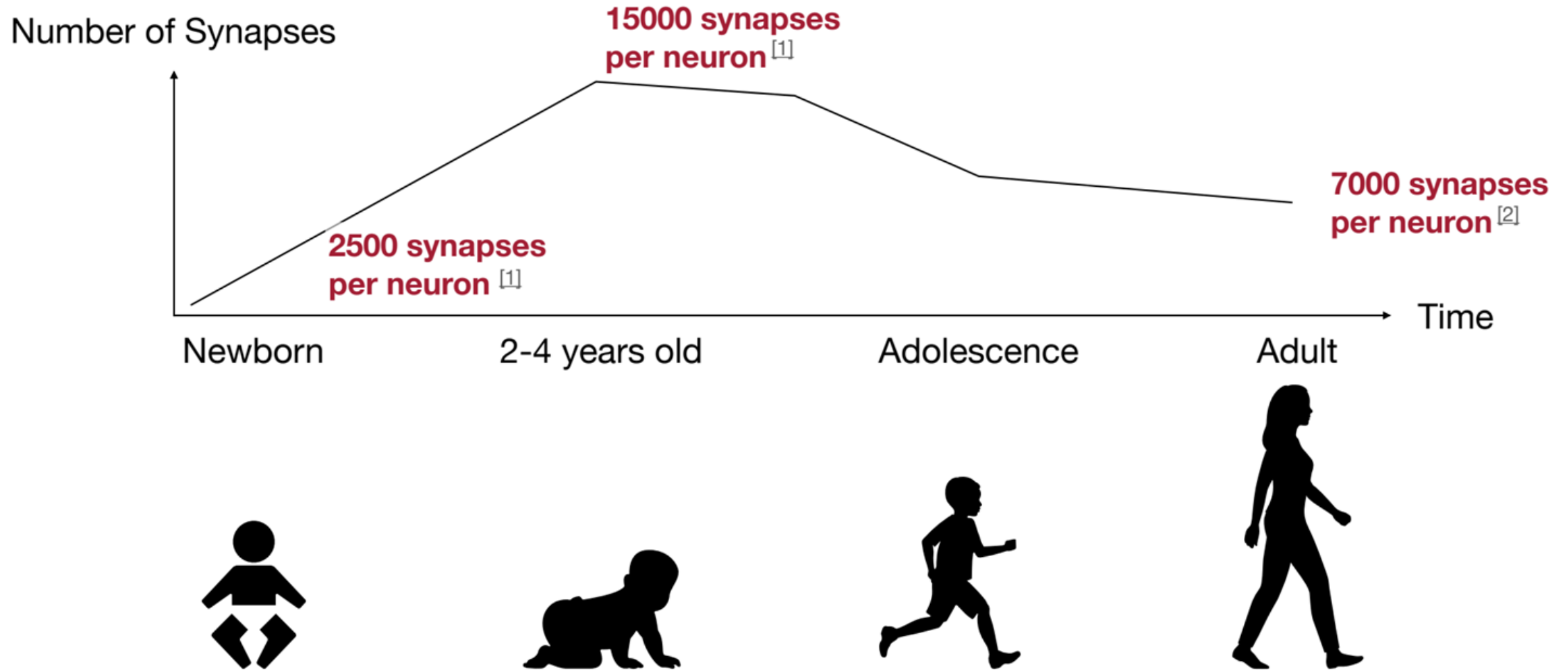
How should we make deep learning more efficient?

Neural Network Pruning

- Introduction to Pruning
 - What is pruning?
 - How should we formulate pruning?
- Determine the Pruning Granularity
 - In what pattern should we prune the neural network?
- Determine the Pruning Criterion
 - What synapses/neurons should we prune?
- Determine the Pruning Ratio
 - What should the target sparsity be for each layer?
- Fine-tune/Train Pruned Neural Network
 - How should we improve the performance of pruned models?

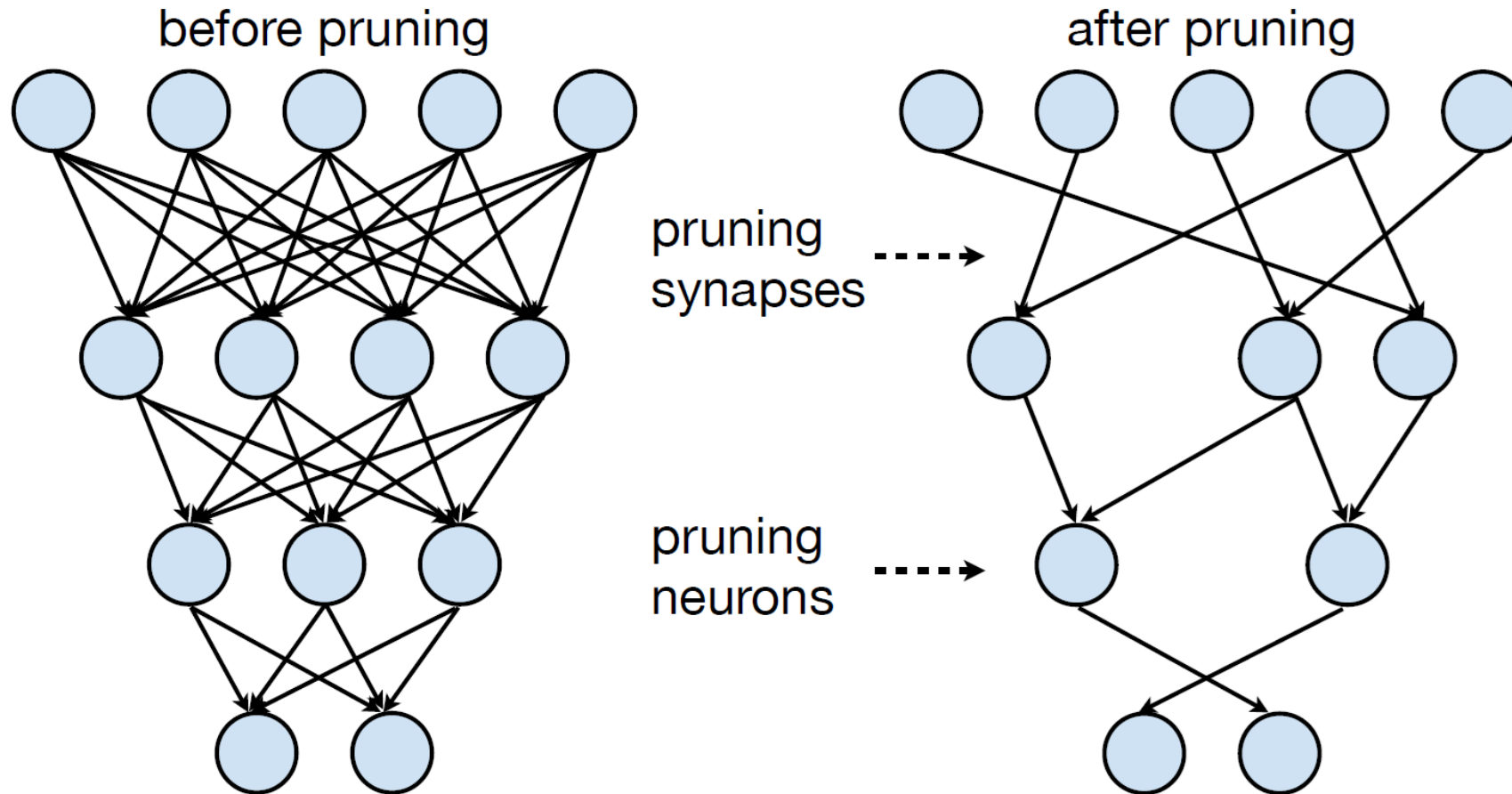


Pruning Happens in Human Brain

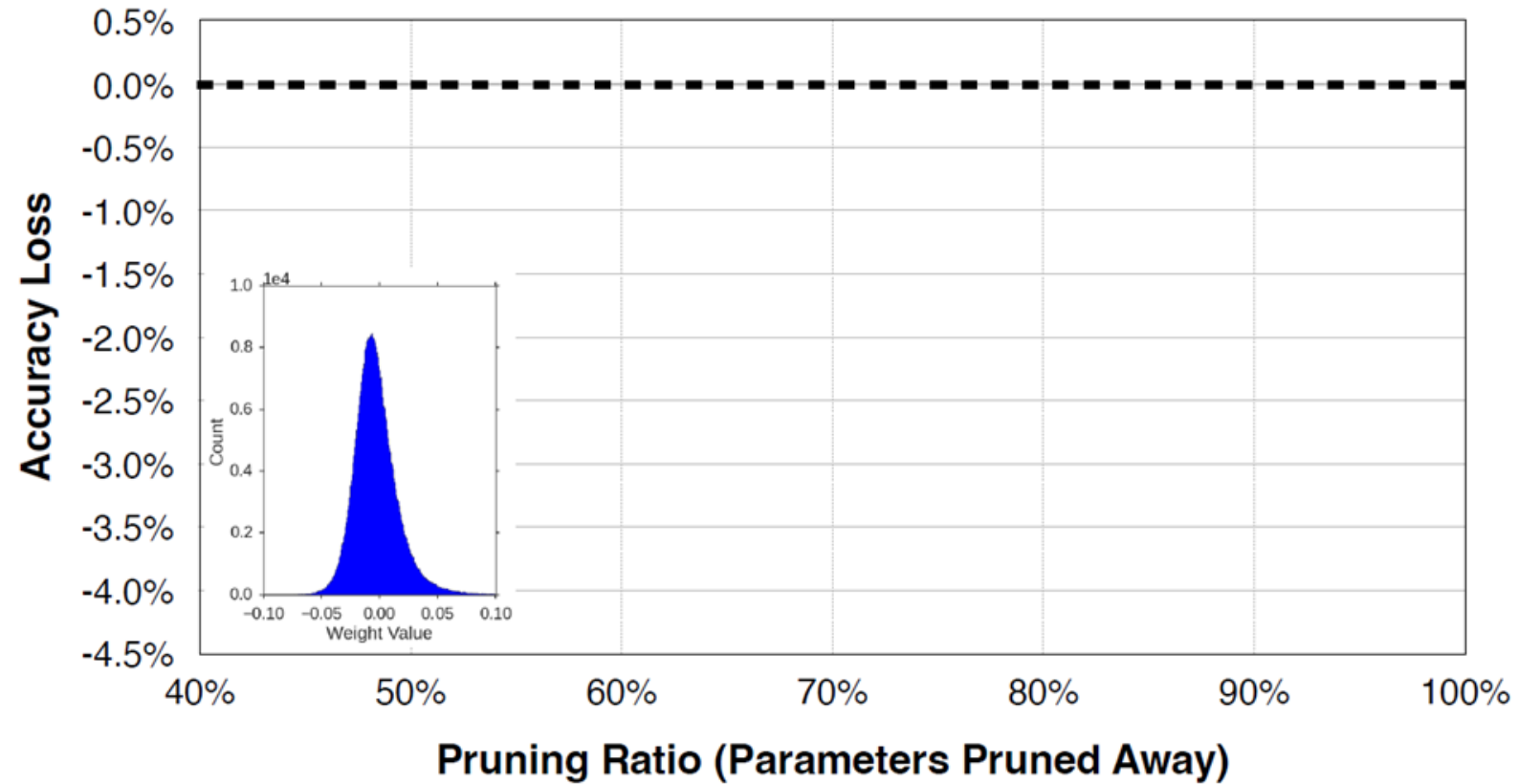
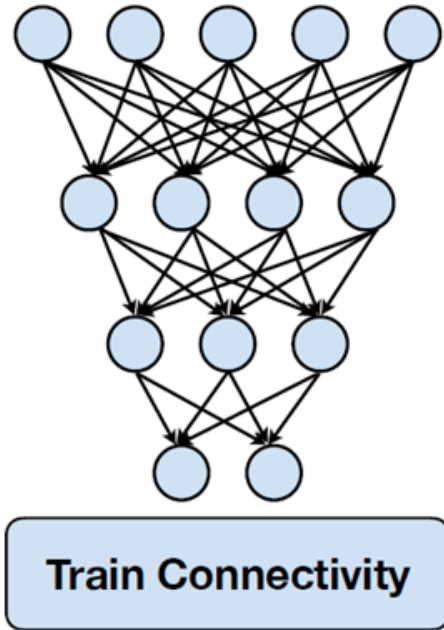


Neural Network Pruning

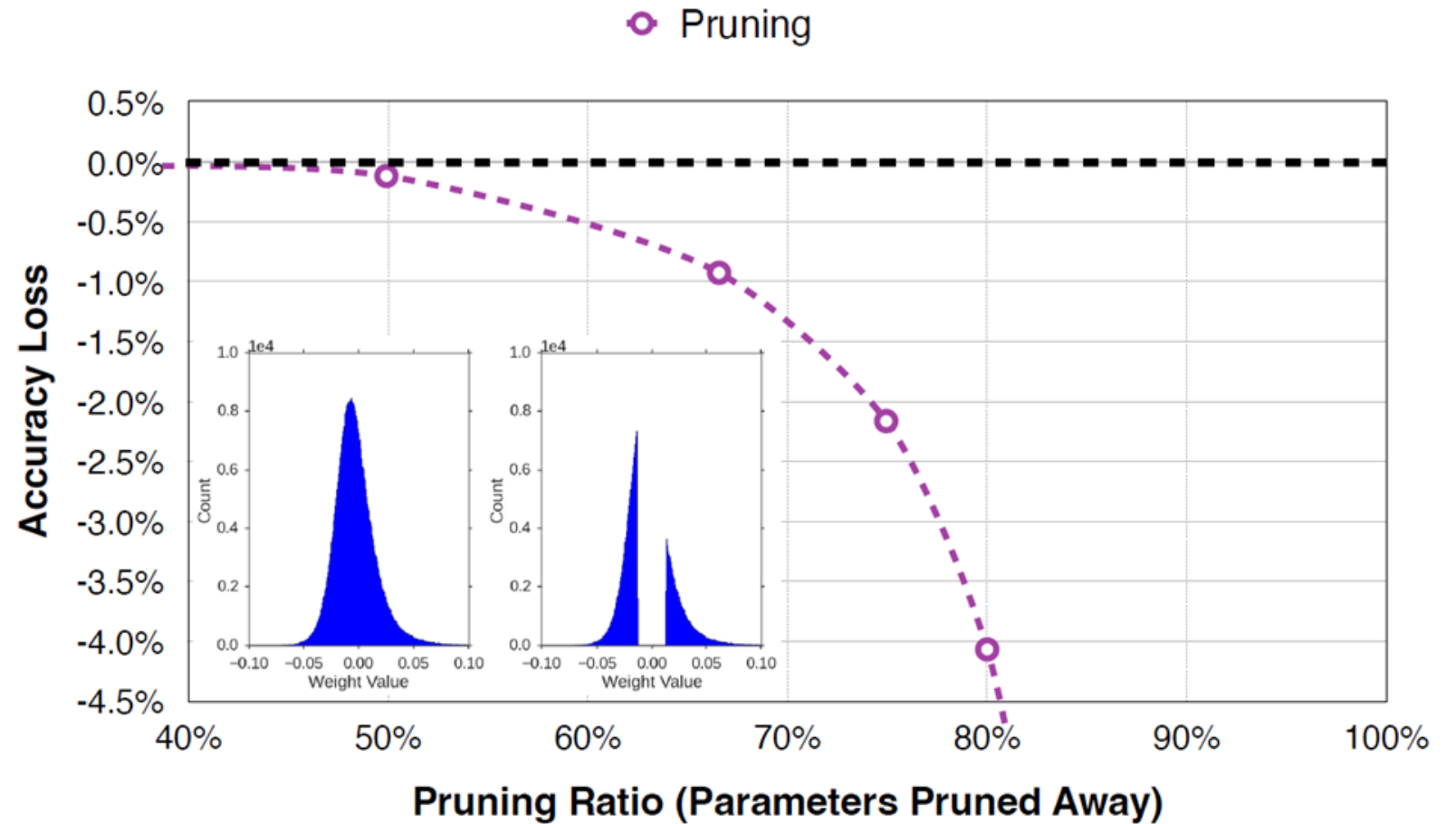
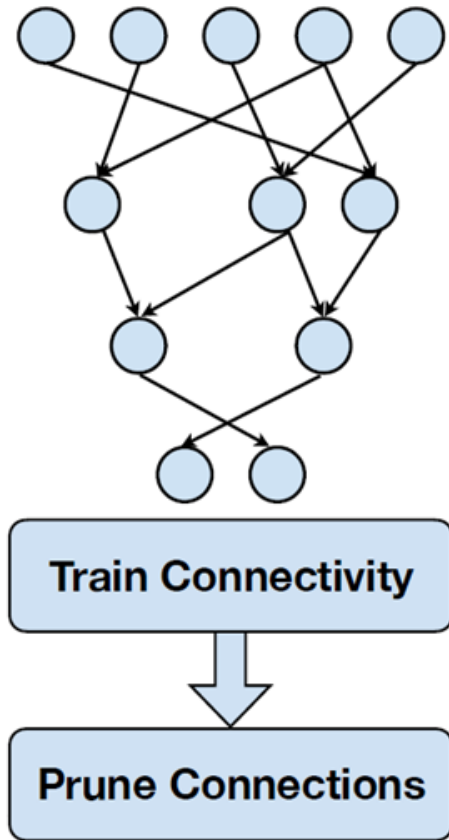
- Make neural network smaller by removing synapses and neurons



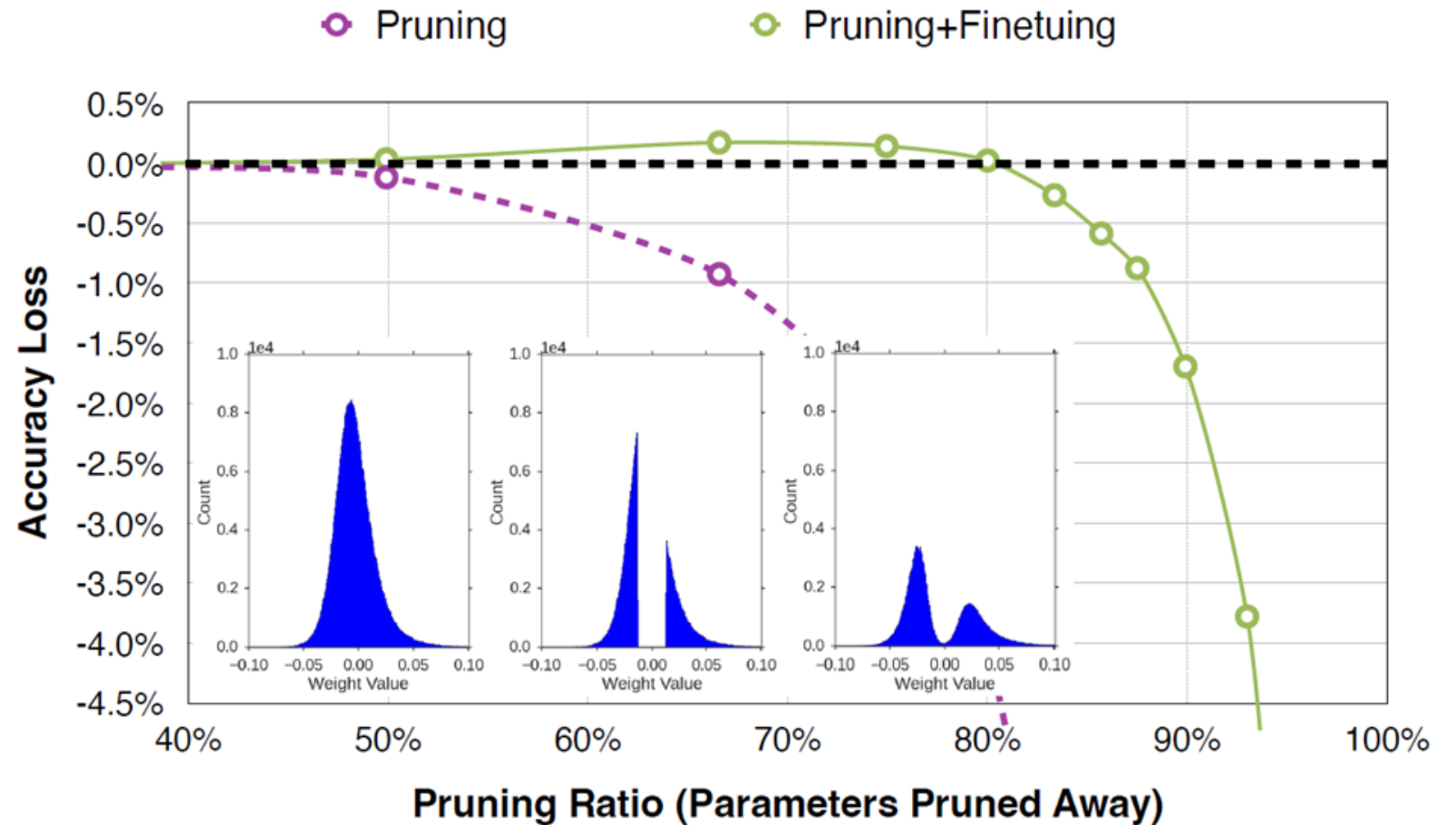
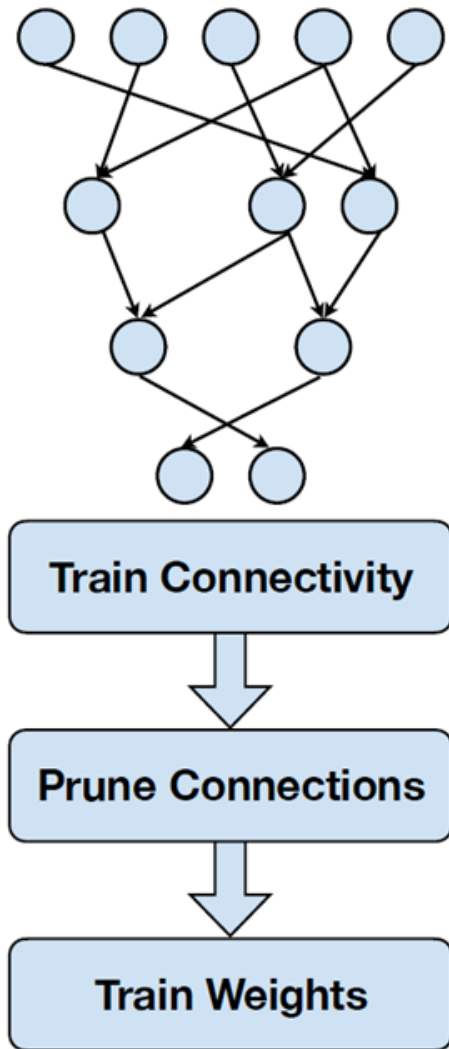
Neural Network Pruning



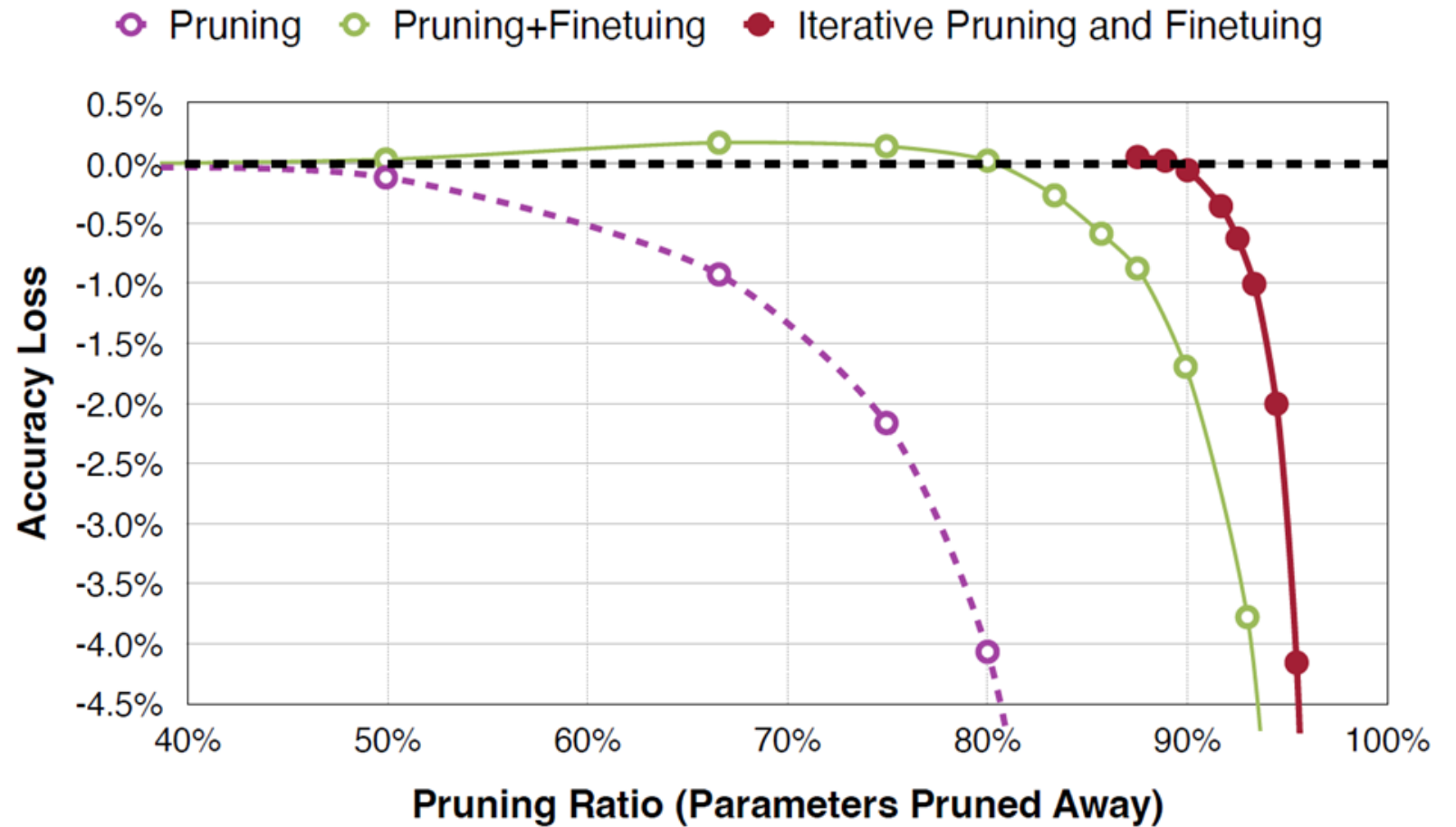
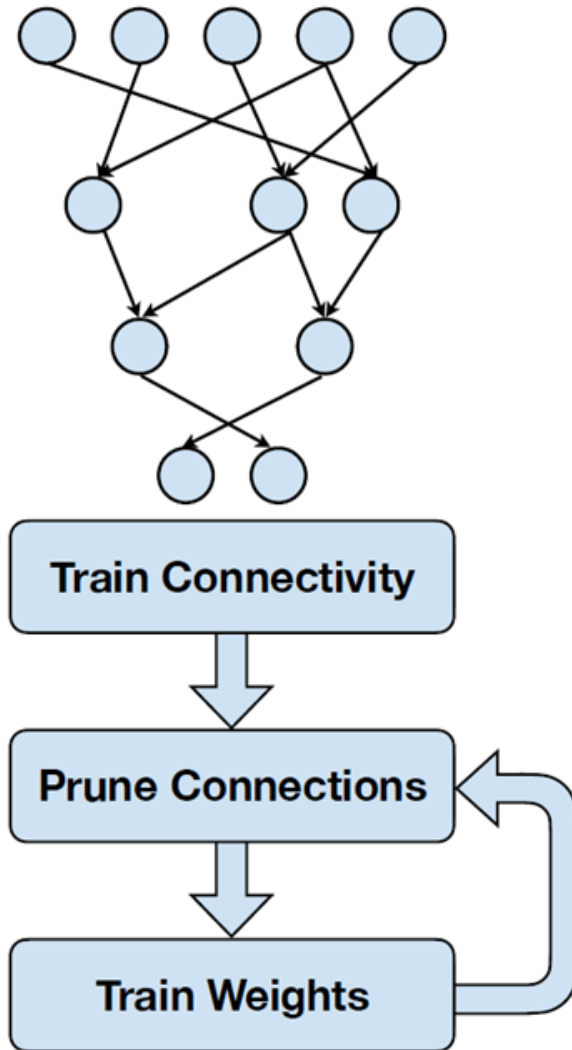
Neural Network Pruning



Neural Network Pruning



Neural Network Pruning



Neural Network Pruning

Neural Network	# Parameters			MACs
	Before Pruning	After Pruning	Reduction	Reduction
AlexNet	61 M	6.7 M	9 x	3 x
VGG-16	138 M	10.3 M	12 x	5 x
GoogleNet	7 M	2.0 M	3.5 x	5 x
ResNet50	26 M	7.47 M	3.4 x	6.3 x
SqueezeNet	1M	0.38 M	3.2 x	3.5 x

Pruning saves up to 12x parameter storage without accuracy drop

Neural Network Pruning: Example

- Pruning the NeuralTalk LSTM does not hurt image caption quality.



Baseline: a basketball player in a white uniform is playing with a ball .

Pruned 90%: a basketball player in a white uniform is playing with a basketball.



Baseline: a brown dog is running through a grassy field.

Pruned 90%: a brown dog is running through a grassy area.



Baseline: a man is riding a surfboard on a wave.

Pruned 90%: a man in a wetsuit is riding a wave on a beach.

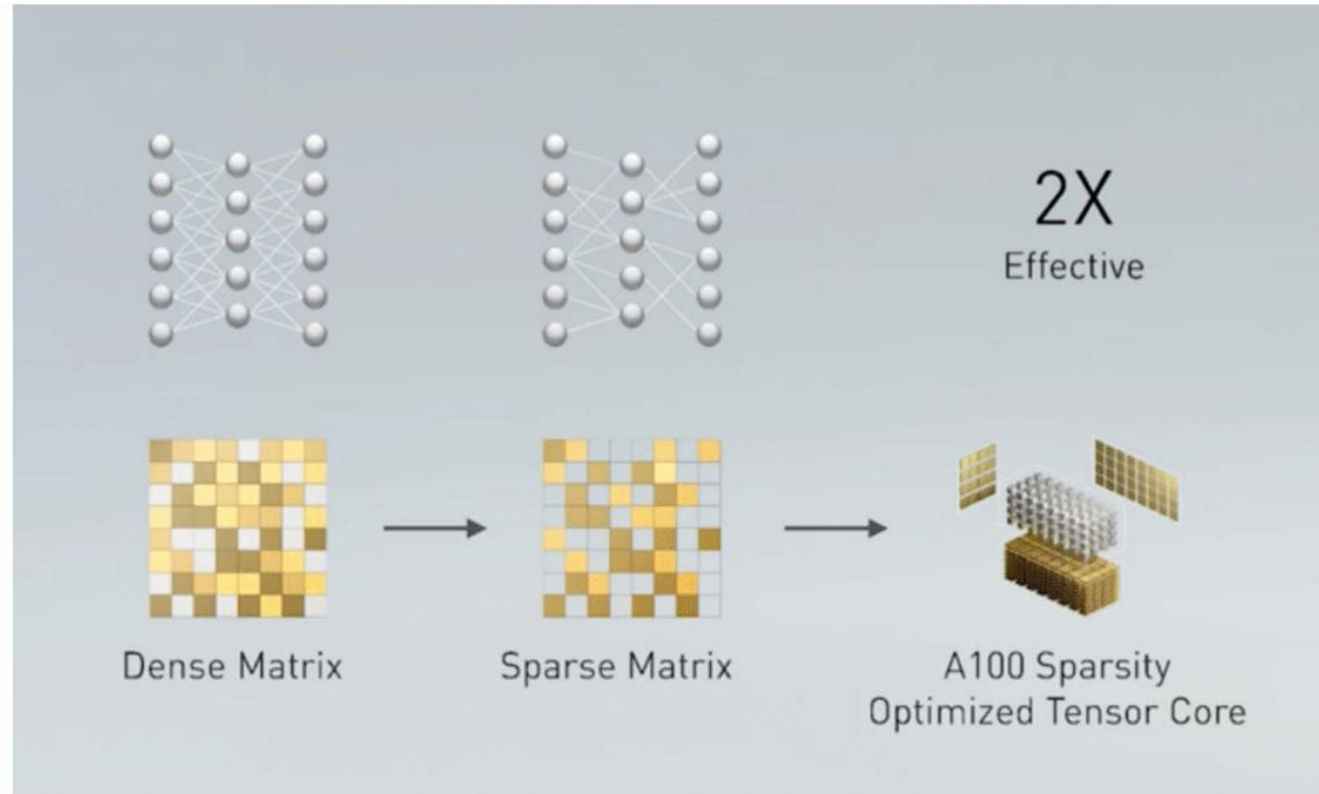


Baseline: a soccer player in red is running in the field.

Pruned 95%: a man in a red shirt and black and white black shirt is running through a field.

Pruning in the Industry

Hardware support for Sparsity for Nvidia A100 GPU



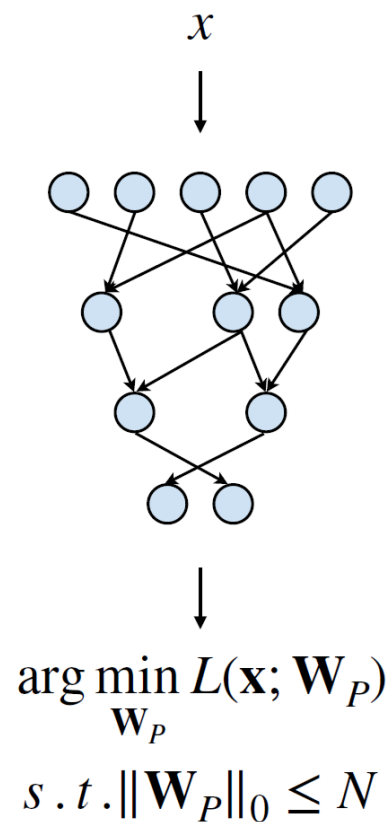
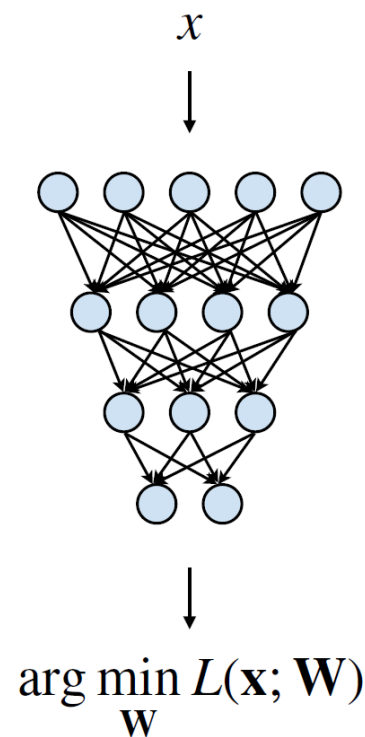
2:4 sparsity in A100 GPU
2X peak performance, 1.5X measured BERT speedup

How Should We Formulate Pruning?

- In general, we could formulate the pruning as follows:

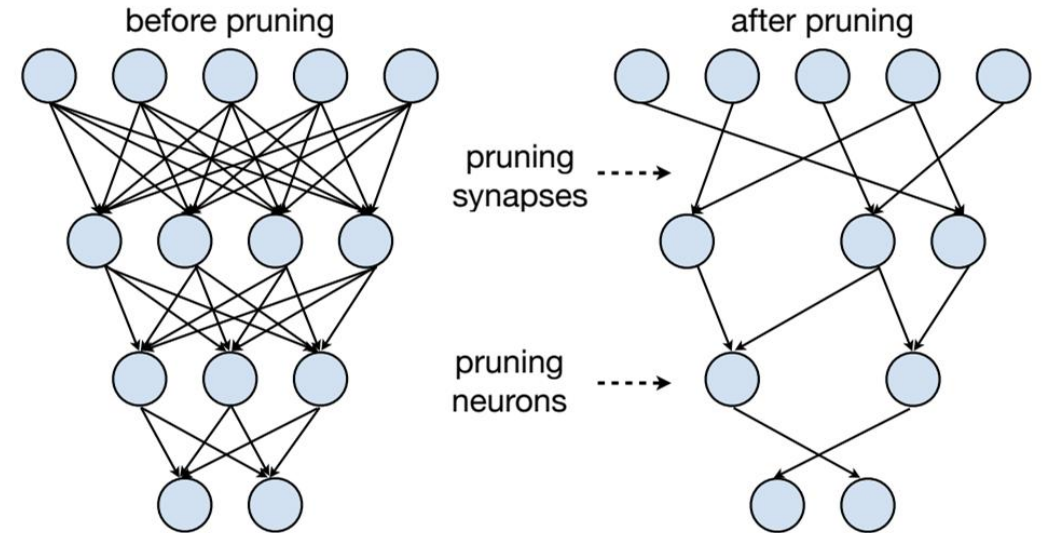
$$\arg \min_{\mathbf{W}_p} L(\mathbf{x}; \mathbf{W}_p) \text{ subject to } \|\mathbf{W}_p\|_0 < N$$

- L represents the objective function for neural network training.
- \mathbf{x} is input, \mathbf{W} is original weights, \mathbf{W}_p is pruned weights.
- $\|\mathbf{W}_p\|_0$ calculates the #nonzeros in \mathbf{W}_p , and N is the target #nonzeros .



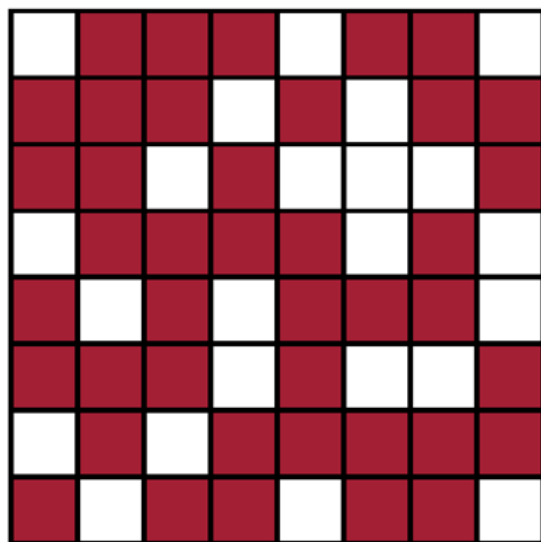
Neural Network Pruning

- ~~Introduction to Pruning~~
 - ~~What is pruning?~~
 - ~~How should we formulate pruning?~~
- **Determine the Pruning Granularity**
 - In what pattern should we prune the neural network?
- **Determine the Pruning Criterion**
 - What synapses/neurons should we prune?
- **Determine the Pruning Ratio**
 - What should the target sparsity be for each layer?
- **Fine-tune/Train Pruned Neural Network**
 - How should we improve the performance of pruned models?



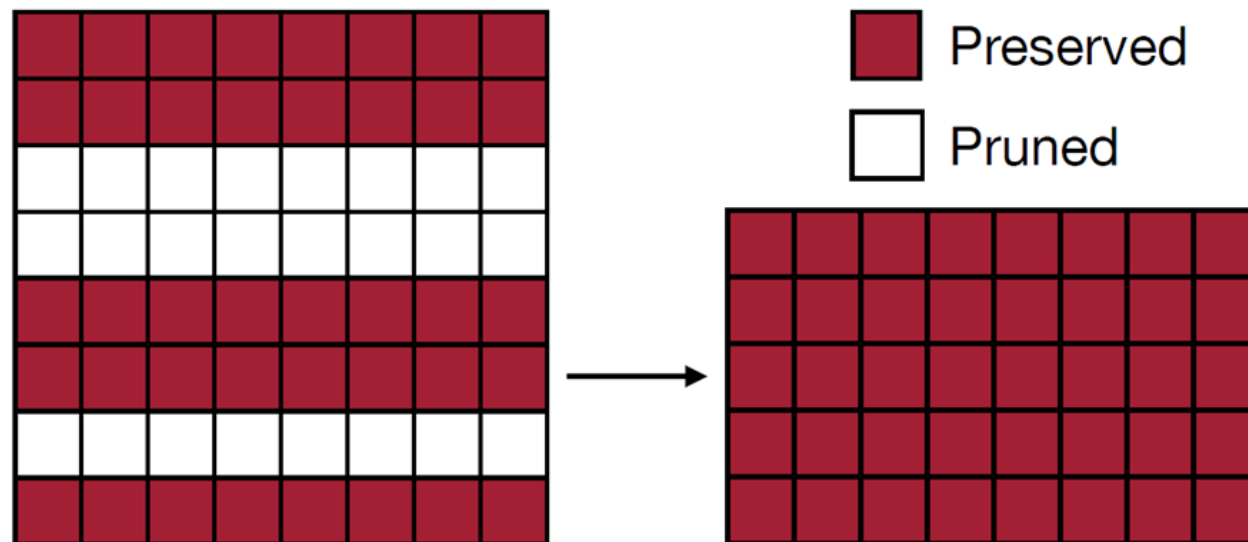
Pruning at Different Granularities

- Pruning is performed at different granularities, from structured to non-structured.



Fine-grained/Unstructured

- More flexible pruning index choice
- Hard to accelerate
(irregular data expression)



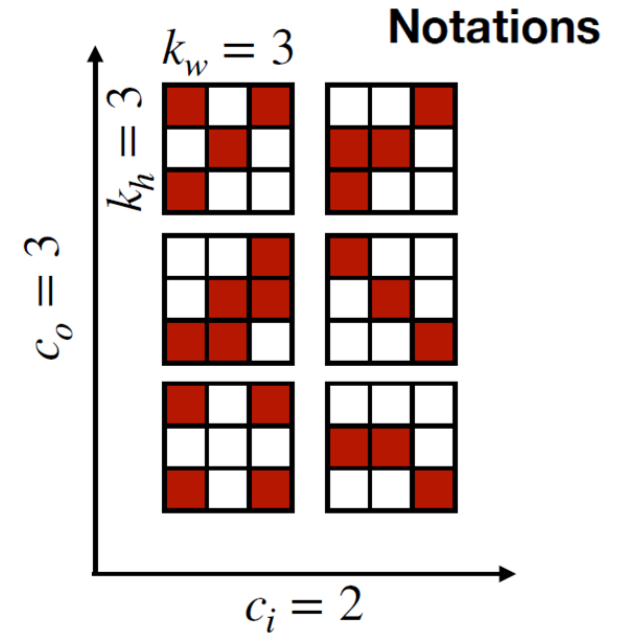
Coarse-grained/Structured

- Less flexible pruning index choice
(a subset of the fine-grained case)
- Easy to accelerate (just a smaller matrix!)

Pruning at Different Granularities

● Convolution layer pruning

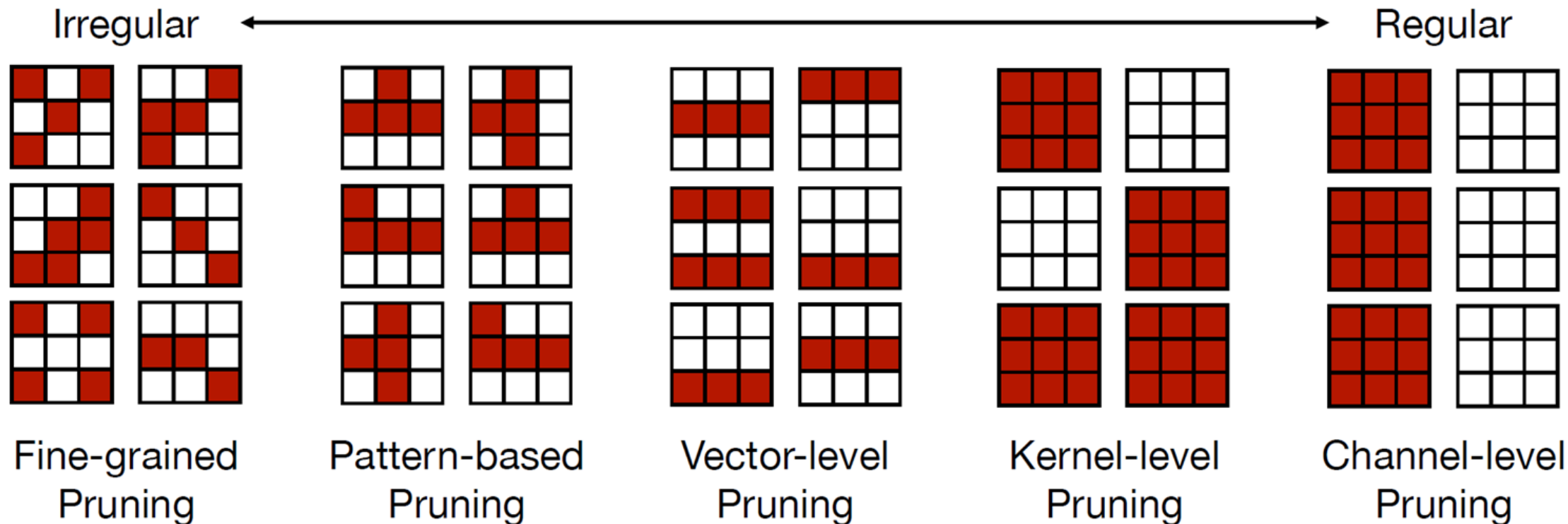
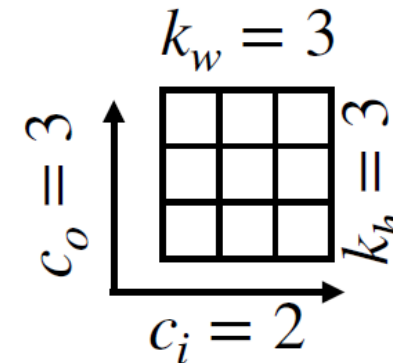
- The weights of convolutional layers have 4 dimensions:
 - c_i : input channels (or channels)
 - c_o : output channels (or filters)
 - k_h : kernel size height
 - k_w : kernel size width
- The 4 dimensions give us more choices to select pruning granularities.



Pruning at Different Granularities

- Convolution layer pruning

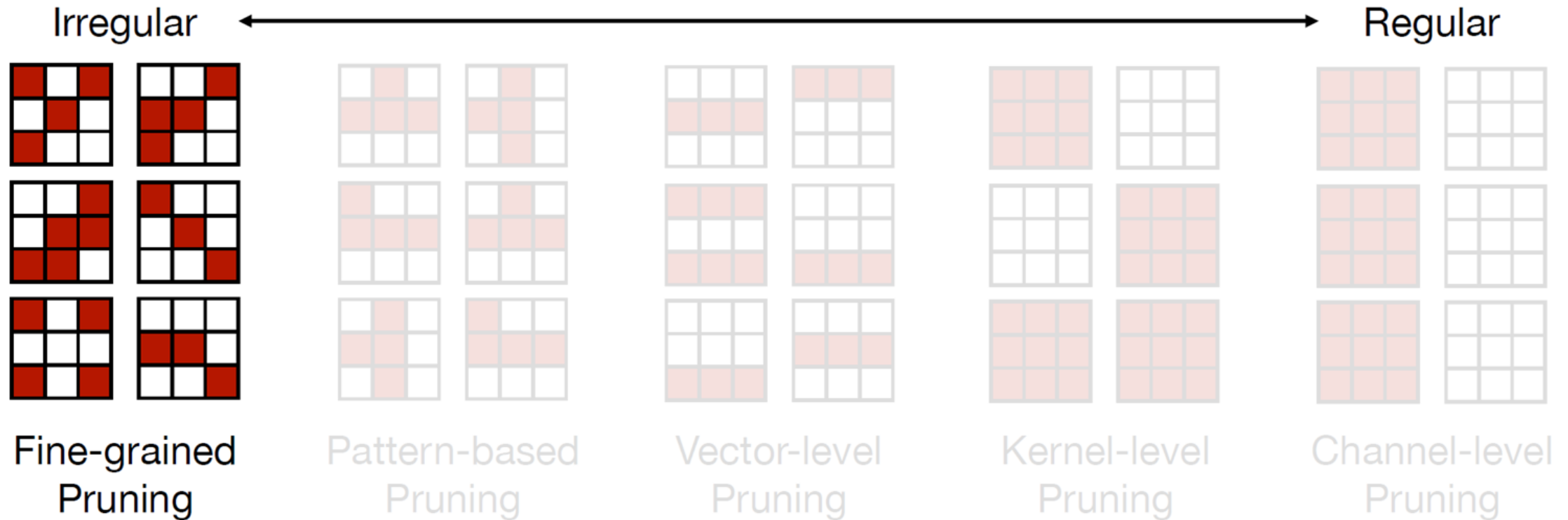
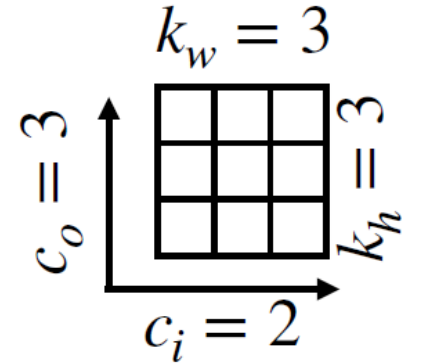
■ Preserved
□ Pruned



Pruning at Different Granularities

- Convolution layer pruning

■ Preserved
□ Pruned



Fine-grained Pruning

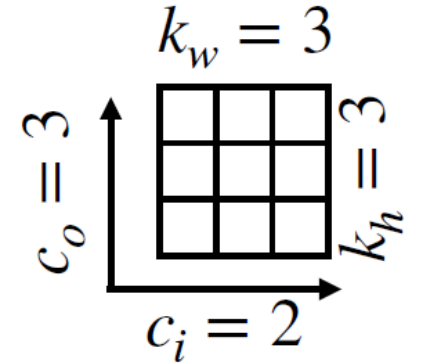
- Flexible pruning indices
- Usually large compression ratio since we can flexibly find “redundant” weights
- Can deliver speed up on some custom hardware (e.g., EIE) but not GPU

Neural Network	#Parameters		
	Before Pruning	After Pruning	Reduction
AlexNet	61 M	6.7 M	9 ×
VGG-16	138 M	10.3 M	12 ×
GoogleNet	7 M	2.0 M	3.5 ×
ResNet50	26 M	7.47 M	3.4 ×

Pruning at Different Granularities

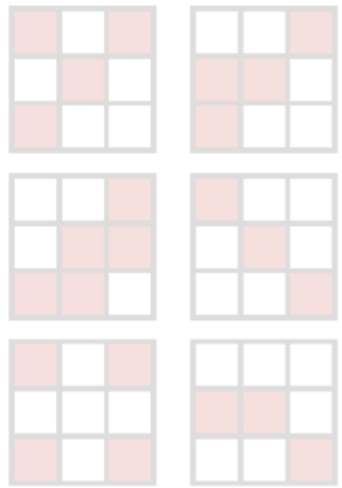
- Convolution layer pruning

■ Preserved
□ Pruned

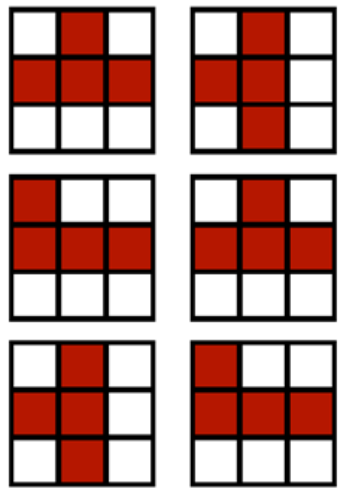


Irregular ←

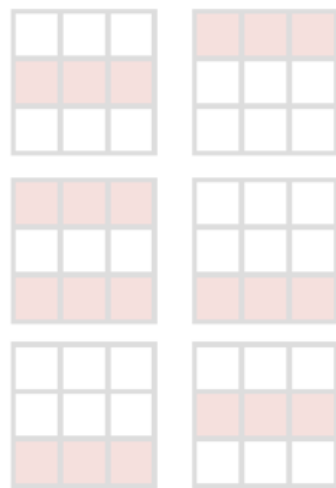
→ Regular



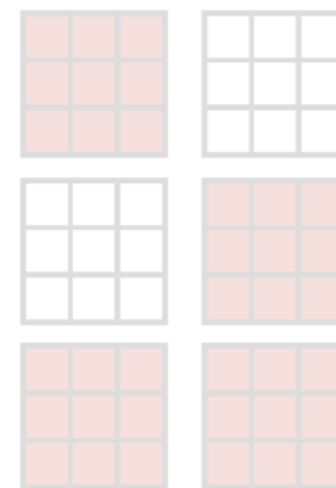
Fine-grained
Pruning



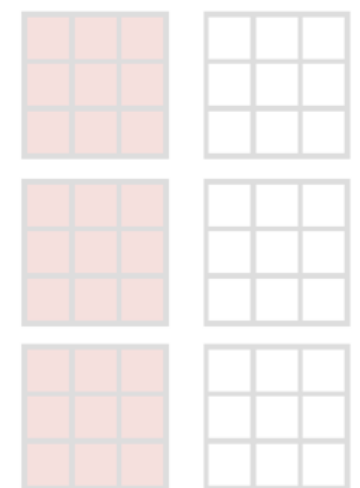
Pattern-based
Pruning



Vector-level
Pruning



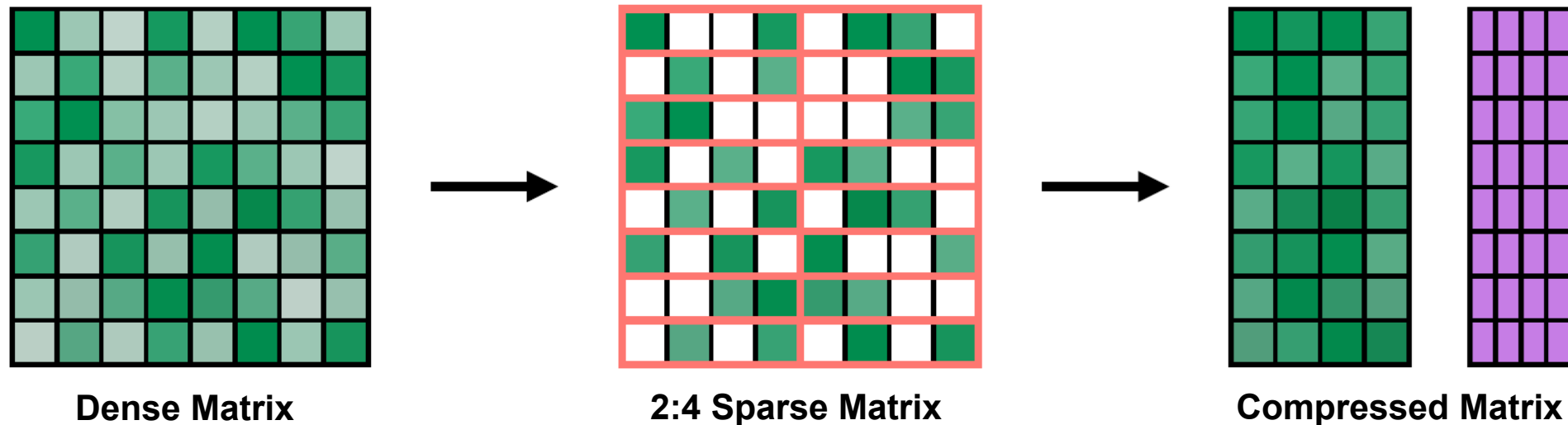
Kernel-level
Pruning



Channel-level
Pruning

Pattern-based Pruning

- N:M sparsity
 - N:M sparsity means that in each contiguous M elements, N of them are pruned.
 - A classic case in 2:4 sparsity (50% sparsity).
 - It is supported by NVIDIA's Ampere GPU Architecture, which delivers up to 2x speed up.
 - Usually maintains accuracy (tested on varieties of tasks)



Pattern-based Pruning

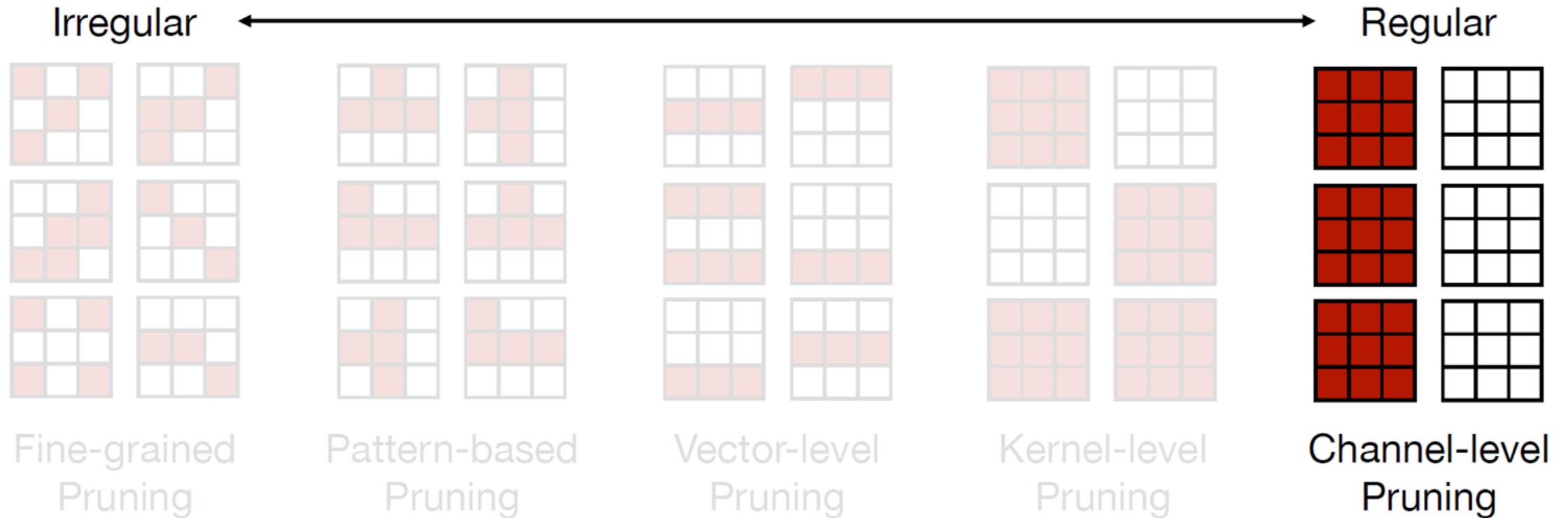
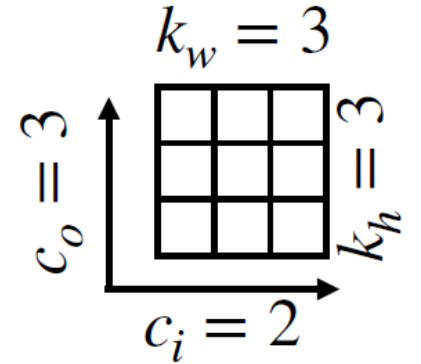
- N:M sparsity
 - N:M sparsity means that in each contiguous M elements, N of them are pruned.
 - A classic case in 2:4 sparsity (50% sparsity).
 - It is supported by NVIDIA's Ampere GPU Architecture, which delivers up to 2x speed up.
 - Usually maintains accuracy (tested on varieties of tasks)

Network	Data Set	Metric	Dense FP16	Sparse FP16
ResNet-50	ImageNet	Top-1	76.1	76.2
ResNeXt-101_32x8d	ImageNet	Top-1	79.3	79.3
Xception	ImageNet	Top-1	79.2	79.2
SSD-RN50	COCO2017	bbAP	24.8	24.8
MaskRCNN-RN50	COCO2017	bbAP	37.9	37.9
FairSeq Transformer	EN-DE WMT'14	BLEU	28.2	28.5
BERT-Large	SQuAD v1.1	F1	91.9	91.9

Pruning at Different Granularities

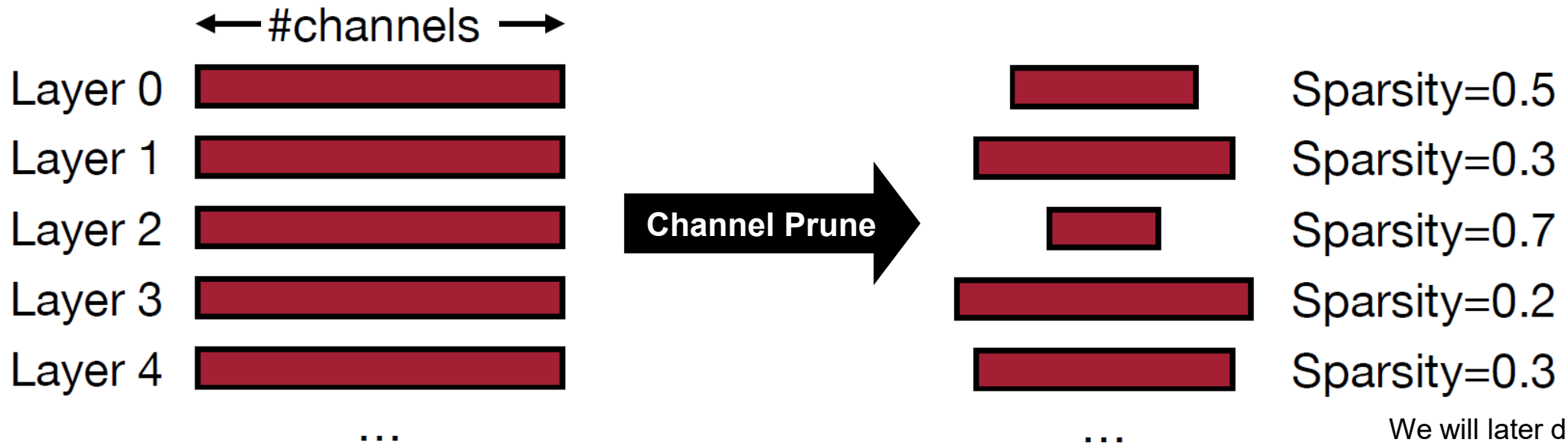
- Convolution layer pruning

■ Preserved
□ Pruned



Channel-level Pruning

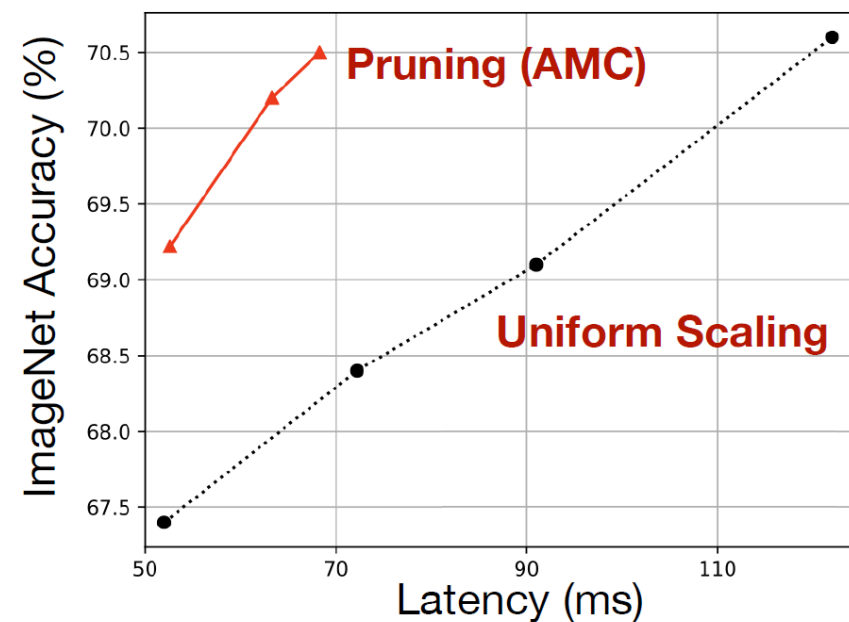
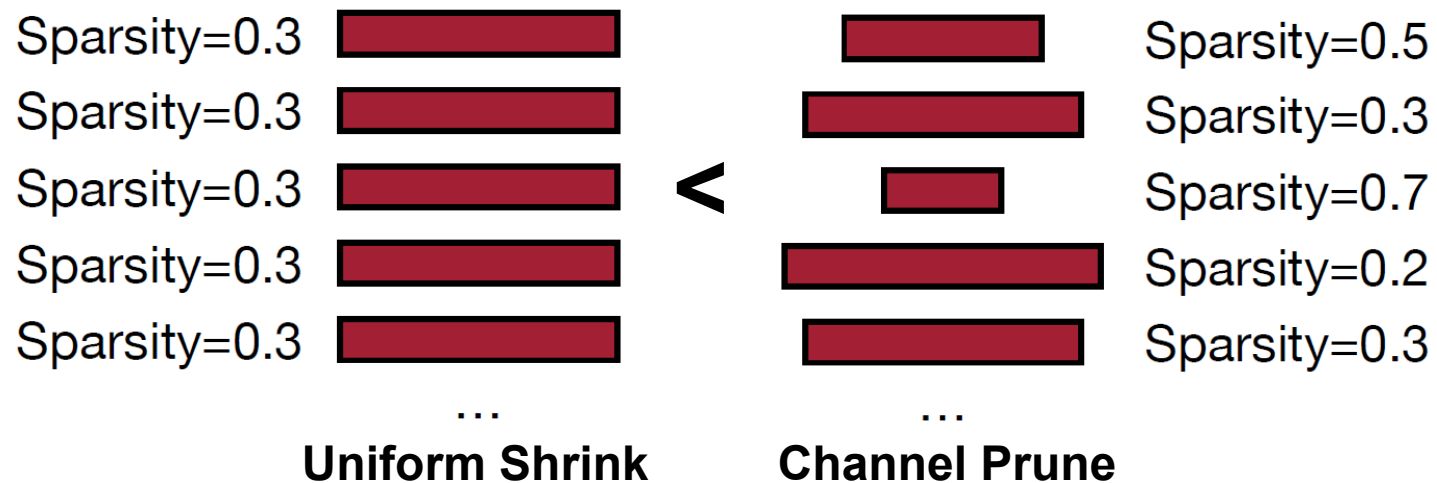
- Direct speed up due to reduced channel numbers (leading to an NN with smaller #channels)
- Smaller compression ratio than fine-grained pruning



We will later discuss
how to determine sparsity ratios.

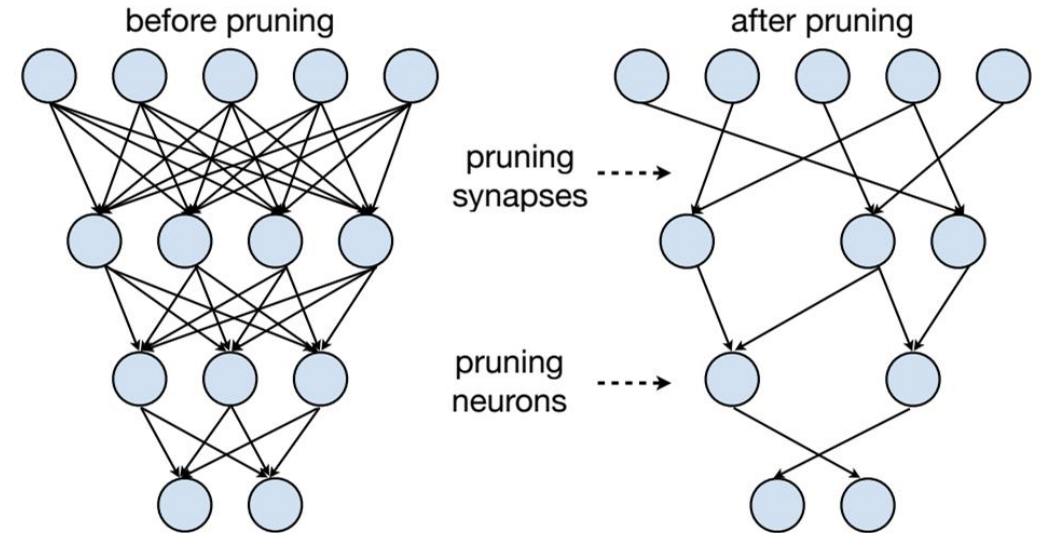
Channel-level Pruning

- Channel pruning has larger compression ratio than uniform shrink.



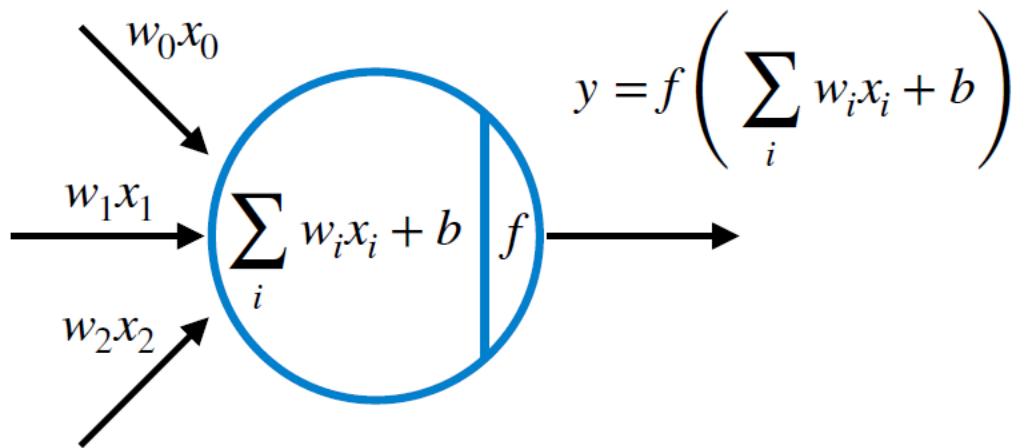
Neural Network Pruning

- ~~Introduction to Pruning~~
 - ~~What is pruning?~~
 - ~~How should we formulate pruning?~~
- ~~Determine the Pruning Granularity~~
 - ~~In what pattern should we prune the neural network?~~
- Determine the Pruning Criterion
 - What synapses/neurons should we prune?
- Determine the Pruning Ratio
 - What should the target sparsity be for each layer?
- Fine-tune/Train Pruned Neural Network
 - How should we improve the performance of pruned models?



Selection of Synapses to Prune

- When removing parameters from a neural network model,
 - The less important the parameters being removed, the better the performance of the pruned network.



Example

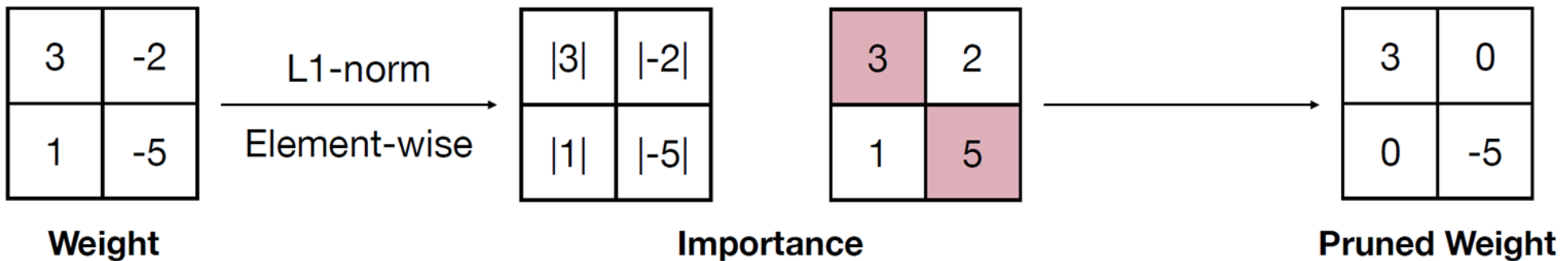
$$f(\cdot) = \text{ReLU}(\cdot), \quad W = [10, -8, 0.1]$$
$$\Rightarrow y = \text{ReLU}(10x_0 - 8x_1 + 0.1x_2)$$

If only one weight will be removed,
Which one? Why?

Magnitude-based Pruning

- Magnitude-based pruning considers weights with larger absolute values are more important than other weights.
 - For element-wise pruning,

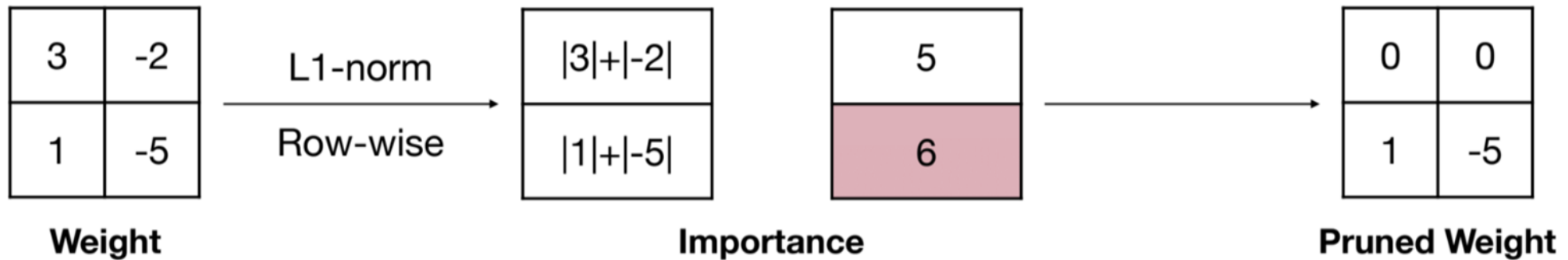
$$\text{Importance} = |W|$$



Magnitude-based Pruning

- Magnitude-based pruning considers weights with larger absolute values are more important than other weights.
 - For row-wise pruning, the L1-norm magnitude can be defined as

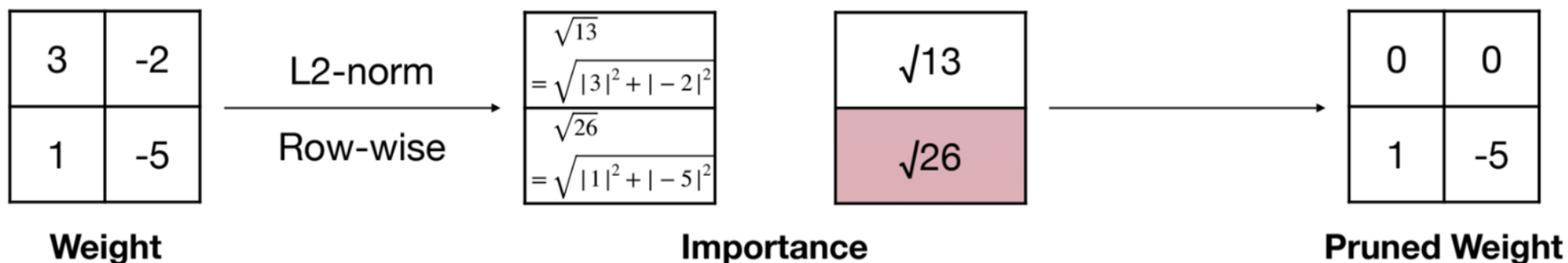
$$Importance = \sum_{i \in S} |w_i|, \text{ where } \mathbf{W}^{(S)} \text{ is the structural set } S \text{ of parameters } \mathbf{W}$$



Magnitude-based Pruning

- Magnitude-based pruning considers weights with larger absolute values are more important than other weights.
 - For row-wise pruning, the L2-norm magnitude can be defined as

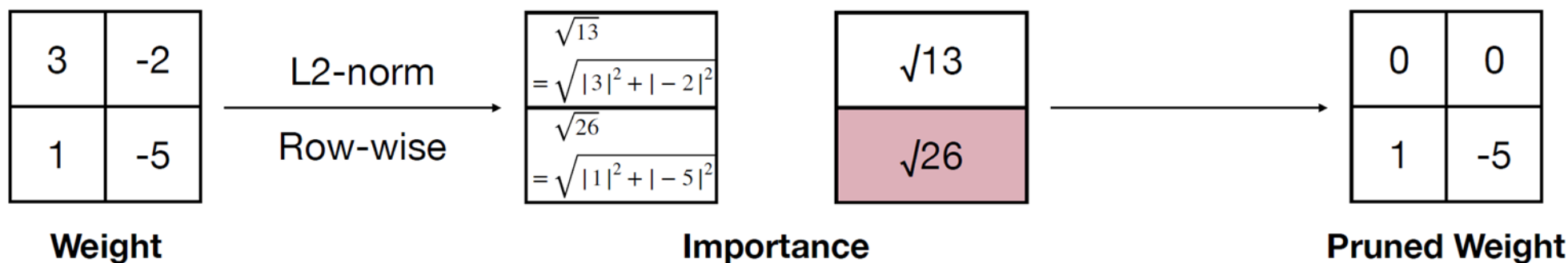
$$Importance = \sqrt{\sum_{i \in S} |w_i|^2}, \text{ where } \mathbf{W}^{(S)} \text{ is the structural set } S \text{ of parameters } \mathbf{W}$$



Magnitude-based Pruning

- Magnitude-based pruning considers weights with larger absolute values are more important than other weights.
 - For row-wise pruning, the L_p -norm magnitude can be defined as

$$Importance = \|\mathbf{W}^{(S)}\|_p = \left(\sum_{i \in S} |w_i|^p \right)^{\frac{1}{p}}, \text{ where } \mathbf{W}^{(S)} \text{ is a structural set of parameters}$$



Taylor Expansion Analysis on Pruning Error

- Evaluate pruning error induced by pruned synapses
- The task of training NN is to minimize the objective function $L(\mathbf{x}; \mathbf{W})$.
 - The importance of a parameter can be quantified by the error on loss function induced by removing it.
- The induced error can be approximated by a Taylor series.

$$\delta L = L(\mathbf{x}; \mathbf{W}) - L(\mathbf{x}; \mathbf{W}_P = \mathbf{W} - \delta \mathbf{W}) = \sum_i g_i \delta w_i + \frac{1}{2} \sum_i h_{ii} \delta w_i^2 + \frac{1}{2} \sum_{i \neq j} h_{ij} \delta w_i \delta w_j + O(\|\delta \mathbf{W}\|^3)$$

where $g_i = \frac{\partial L}{\partial w_i}, h_{i,j} = \frac{\partial^2 L}{\partial w_i \partial w_j}$

Second-Order-based Pruning

- Prunes by minimization of the second-order loss change approximation

$$\delta L = L(\mathbf{x}; \mathbf{W}) - L(\mathbf{x}; \mathbf{W}_P = \mathbf{W} - \delta \mathbf{W}) = \sum_i g_i \delta w_i + \frac{1}{2} \sum_i h_{ii} \delta w_i^2 + \frac{1}{2} \sum_{i \neq j} h_{ij} \delta w_i \delta w_j + O(\|\delta \mathbf{W}\|^3)$$

where $g_i = \frac{\partial L}{\partial w_i}$, $h_{i,j} = \frac{\partial^2 L}{\partial w_i \partial w_j}$

Second-Order-based Pruning

- Optimal Brain Damage assumes that
 - The objective function L is nearly quadratic: the last term is neglected
 - The neural network training has converged: first-order terms are neglected
 - The error caused by deleting each parameter is independent: cross terms are neglected

$$\delta L_i = L(\mathbf{x}; \mathbf{W}) - L(\mathbf{x}; \mathbf{W}_P | w_i = 0) \approx \frac{1}{2} h_{ii} w_i^2$$

- The synapses with smaller induced error $|\delta L_i|$ will be removed; that is to say,

$$importance_{w_i} = |\delta L_i| = \frac{1}{2} h_{ii} w_i^2$$

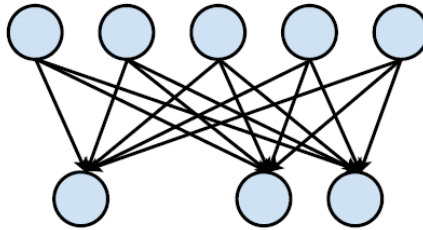
* h_{ii} is non-negative

Selection of Neurons to Prune

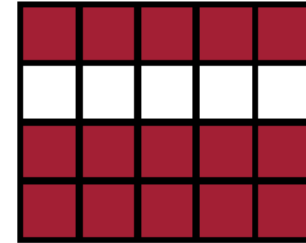
- When removing neurons from a neural network model, the less useful the neurons being removed the better the performance of the pruned network.

Neuron pruning is coarse-grained pruning.

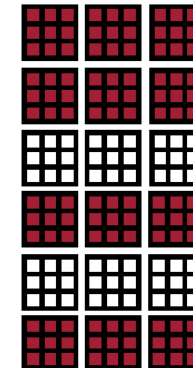
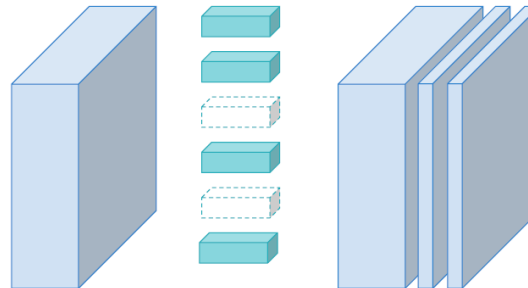
Neuron pruning
in Linear Layer



Weight Matrix

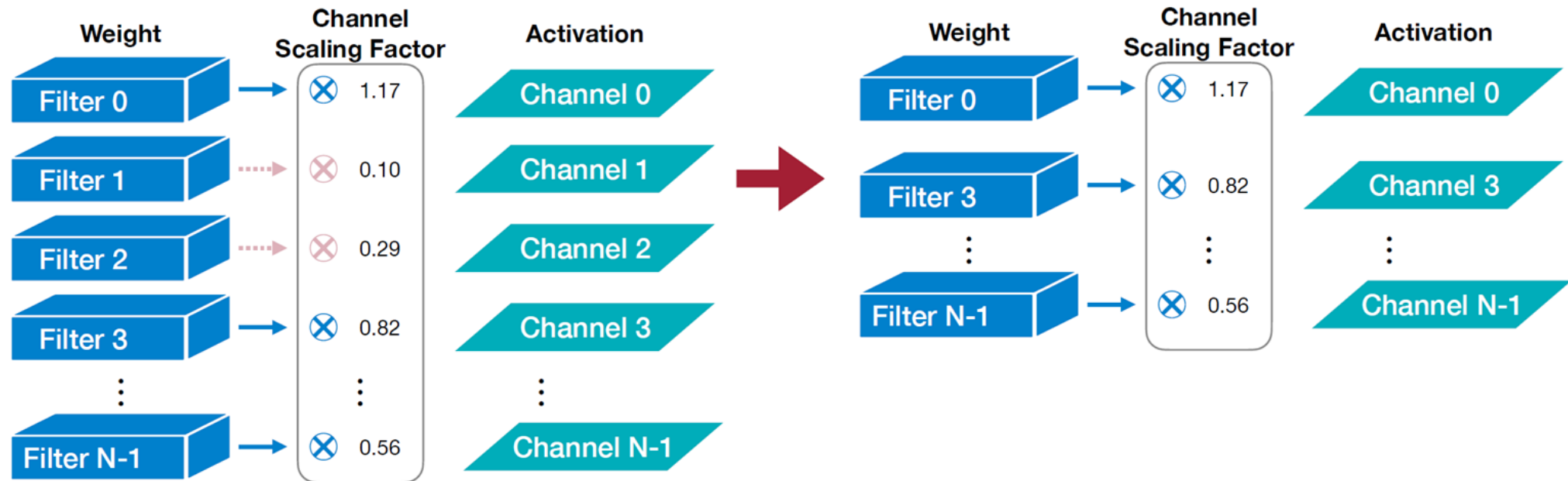


Channel pruning
in Convolution Layer



Scaling-based Pruning

- A scaling factor is associated with each filter (i.e. output channel) in conv layers.
 - The scaling factor is multiplied to the output of that channel
 - The scaling factors are trainable parameters
- The filters/output channels with small scaling factor magnitude will be pruned.



Percentage-of-Zero-Based Pruning

- ReLU activation will generate zeros in the output activation.
- Similar to magnitude of weights, the Average Percentage of Zero activations measures importance of the neurons.

Output Activations	Height = 4	Width = 4								Width = 4																																																							
		<table><tr><td>0</td><td>0.1</td><td>0.5</td><td>1</td></tr><tr><td>1.2</td><td>0.6</td><td>0.3</td><td>0.2</td></tr><tr><td>0</td><td>0.5</td><td>0</td><td>0.3</td></tr><tr><td>0.2</td><td>0</td><td>0</td><td>0.8</td></tr></table>				0	0.1	0.5	1	1.2	0.6	0.3	0.2	0	0.5	0	0.3	0.2	0	0	0.8	<table><tr><td>0.1</td><td>0.5</td><td>0</td><td>0</td></tr><tr><td>0.2</td><td>0.3</td><td>0</td><td>1</td></tr><tr><td>0.1</td><td>0</td><td>0</td><td>0.5</td></tr><tr><td>0.1</td><td>0.6</td><td>0.7</td><td>0.1</td></tr></table>				0.1	0.5	0	0	0.2	0.3	0	1	0.1	0	0	0.5	0.1	0.6	0.7	0.1	<table><tr><td>0</td><td>0</td><td>0.8</td><td>0</td></tr><tr><td>0.7</td><td>0</td><td>0.6</td><td>0.1</td></tr><tr><td>1.2</td><td>1</td><td>0</td><td>0.2</td></tr><tr><td>0.5</td><td>0</td><td>0.3</td><td>0.5</td></tr></table>				0	0	0.8	0	0.7	0	0.6	0.1	1.2	1	0	0.2	0.5	0	0.3	0.5				
		0	0.1	0.5	1																																																												
		1.2	0.6	0.3	0.2																																																												
		0	0.5	0	0.3																																																												
0.2	0	0	0.8																																																														
0.1	0.5	0	0																																																														
0.2	0.3	0	1																																																														
0.1	0	0	0.5																																																														
0.1	0.6	0.7	0.1																																																														
0	0	0.8	0																																																														
0.7	0	0.6	0.1																																																														
1.2	1	0	0.2																																																														
0.5	0	0.3	0.5																																																														
Channel = 3																																																																	
Batch = 2																																																																	
	Height = 4					Width = 4																																																											
						<table><tr><td>0.5</td><td>0</td><td>0.2</td><td>0.1</td></tr><tr><td>0</td><td>0.2</td><td>1.2</td><td>0</td></tr><tr><td>1.2</td><td>0</td><td>0.2</td><td>0.3</td></tr><tr><td>0.2</td><td>0.4</td><td>0</td><td>0</td></tr></table>				0.5	0	0.2	0.1	0	0.2	1.2	0	1.2	0	0.2	0.3	0.2	0.4	0	0	<table><tr><td>0.1</td><td>0.5</td><td>0</td><td>0</td></tr><tr><td>0</td><td>0.8</td><td>0</td><td>1</td></tr><tr><td>0.1</td><td>0</td><td>0.1</td><td>1.0</td></tr><tr><td>0.2</td><td>0</td><td>1.0</td><td>0</td></tr></table>				0.1	0.5	0	0	0	0.8	0	1	0.1	0	0.1	1.0	0.2	0	1.0	0	<table><tr><td>0</td><td>0.8</td><td>0.1</td><td>0</td></tr><tr><td>0.2</td><td>0</td><td>0</td><td>0.3</td></tr><tr><td>0</td><td>0.4</td><td>0</td><td>0.5</td></tr><tr><td>0.2</td><td>0</td><td>0.3</td><td>0</td></tr></table>				0	0.8	0.1	0	0.2	0	0	0.3	0	0.4	0	0.5	0.2	0	0.3	0
		0.5	0	0.2	0.1																																																												
		0	0.2	1.2	0																																																												
		1.2	0	0.2	0.3																																																												
0.2	0.4	0	0																																																														
0.1	0.5	0	0																																																														
0	0.8	0	1																																																														
0.1	0	0.1	1.0																																																														
0.2	0	1.0	0																																																														
0	0.8	0.1	0																																																														
0.2	0	0	0.3																																																														
0	0.4	0	0.5																																																														
0.2	0	0.3	0																																																														
Channel = 3																																																																	

Regression-based Pruning

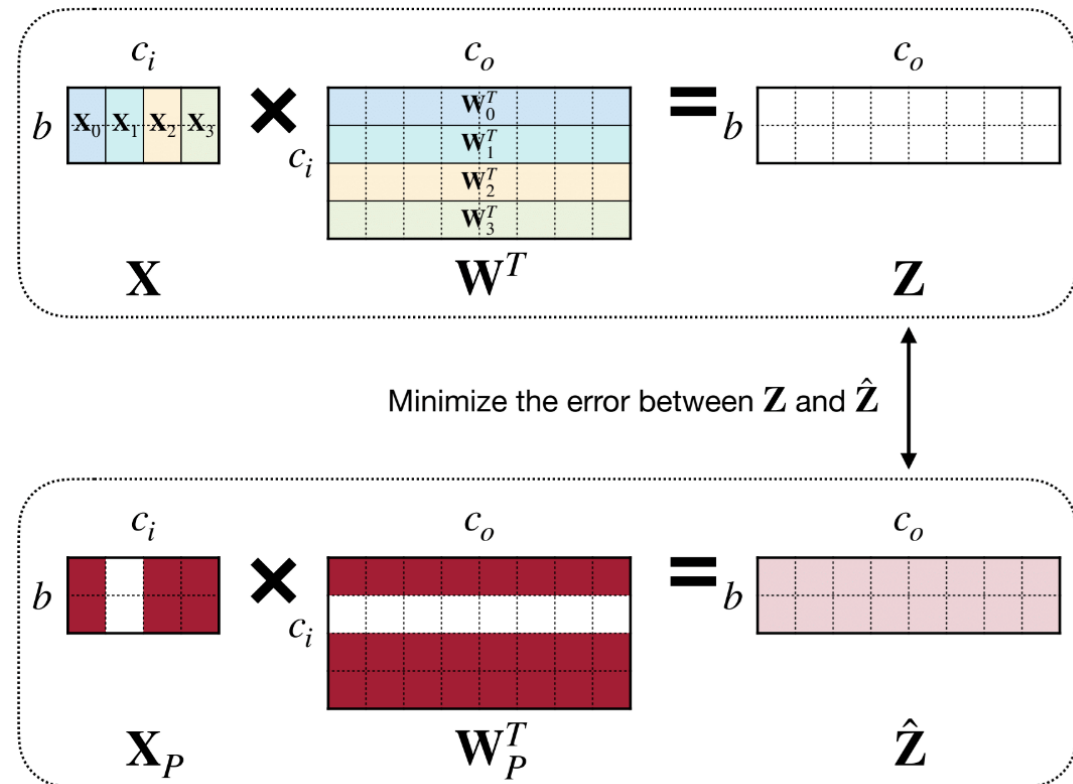
- Instead of considering the pruning error of the objective function $L(\mathbf{x}; \mathbf{W})$, regression-based pruning minimizes the reconstruction error of the corresponding layer's outputs.

$$\mathbf{Z} = \mathbf{X}\mathbf{W}^T = \sum_{c=0}^{c_i-1} \mathbf{X}_c \mathbf{W}_c^T$$

$$\arg \min_{\mathbf{W}, \beta} \|\mathbf{Z} - \hat{\mathbf{Z}}\|_F^2 = \|\mathbf{Z} - \sum_{c=0}^{c_i-1} \beta_c \mathbf{X}_c \mathbf{W}_c^T\|_F^2$$

$$\|\beta\|_0 \leq N_c$$

- β is coefficient vector for channel selection.
 $\beta_c = 0$ means channel c is pruned.
- N_c is the number of nonzero channels
- Fix \mathbf{W} , solve β for channel selection
- Fix β , solve \mathbf{W} to minimize reconstruction error



Summary

- In this lecture, we learned:
 - What is pruning
 - Granularities of pruning
 - Criteria to select weights to prune
- In the next lecture, we will cover:
 - How to find pruning ratio for each layer
 - How to train/fine-tune the pruned layer
 - Lottery ticket hypothesis