SVM 보조자료

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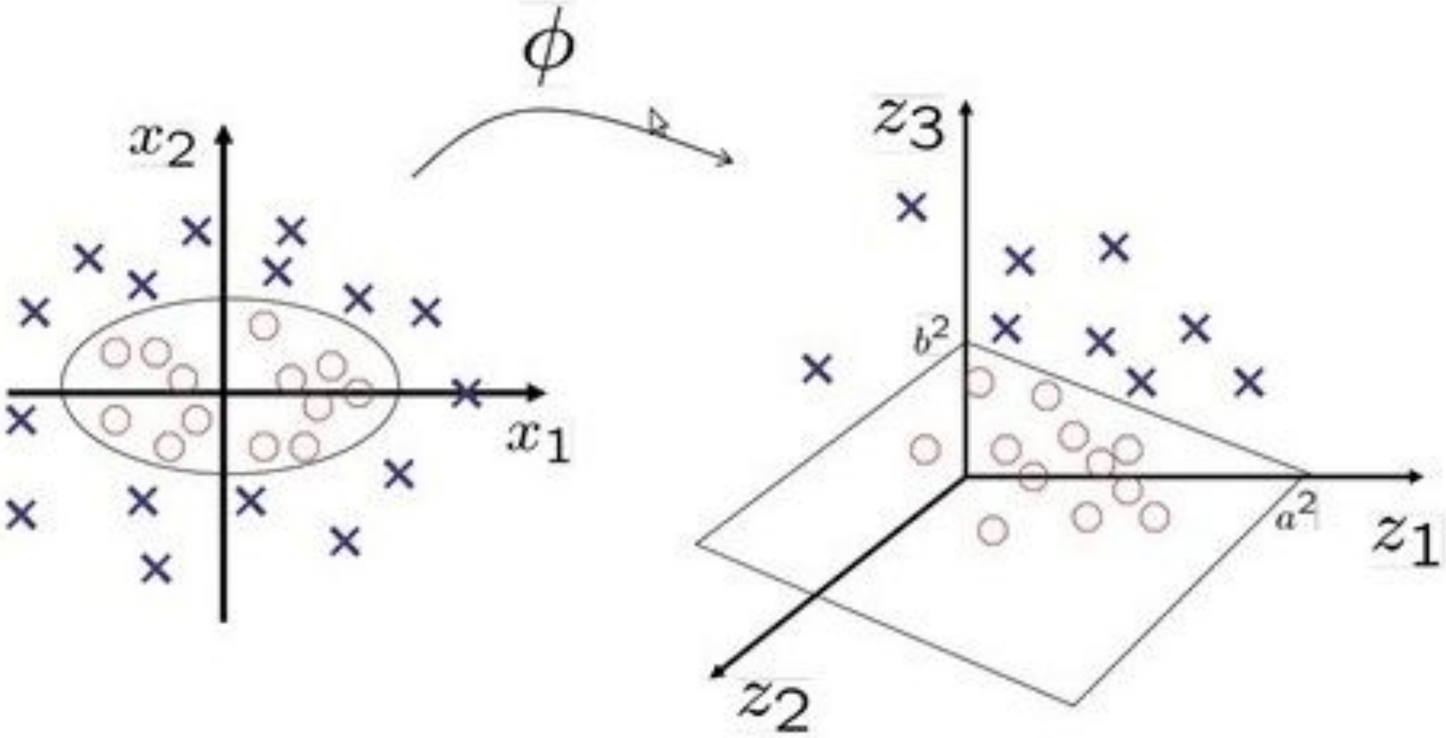

$$\phi : (x_1, x_2) \longrightarrow (x_1^2, \sqrt{2}x_1x_2, x_2^2)$$

- 차원 변환

$$\Phi: (p_1, p_2) \rightarrow (p_1^2, \sqrt{2}p_1p_2, p_2^2)$$

- 커널 표현

$$\begin{aligned}\Phi(p) \cdot \Phi(\mathrm{q}) &= \left(\mathrm{p}_1^2, \sqrt{2}p_1p_2, p_2^2\right)\left(q_1^2, \sqrt{2}q_1q_2, q_2^2\right)^T \\ &= p_1^2q_1^2 + 2p_1q_1p_2q_2 + p_2^2q_2^2 \\ &= (p_1q_1 + p_2q_2)^2 \\ &= (p \cdot q)^2\end{aligned}$$

Polynomial 커널

$$k(x_1, x_2) = (\gamma(x_1^T x_2) + \theta)^d$$

예:

$$\gamma = 1, \theta = 1, d = 3$$

$$\begin{aligned} k(x_1, x_2) &= (x_1^T x_2 + 1)^4 \\ &= x_1^4 x_2^4 + 4x_1^3 x_2^3 + 6x_1^2 x_2^2 + 4x_1 x_2 + 1 \\ &= (x_1^4, 2x_1^3, \sqrt{6}x_1, 2x_1, 1)^T (x_2^4, 2x_2^3, \sqrt{6}x_2, 2x_2, 1) \end{aligned}$$

기저 함수:

$$\begin{aligned} \phi_1(x) &= x^4 \\ \phi_2(x) &= 2x^3 \\ \phi_3(x) &= \sqrt{6}x^2 \\ \phi_4(x) &= 2x \\ \phi_5(x) &= 1 \end{aligned}$$

RBF 커널

$$K_{\mathrm{RBF}}(\mathbf{x}, \mathbf{x}') = \exp\left[-\gamma \left\|\mathbf{x} - \mathbf{x}'\right\|^2\right]$$

Gamma = spread

Example with $\gamma = \frac{1}{2}$

$$\begin{aligned}
k(x_1, x_2) &= \exp\left(-\frac{\left\|x_1 - x_2\right\|^2}{2}\right) \\
&= \exp\left(-\frac{x_1^T x_1}{2} - \frac{x_2^T x_2}{2} + \; x_1^T x_2\right) \\
&= \exp\left(-\frac{x_1^T x_1}{2}\right) \exp\left(-\frac{x_2^T x_2}{2}\right) \exp\left(x_1^T x_2\right) \\
&= C \exp\left(x_1^T x_2\right) \\
&\approx C\left(1 + \left(x_1^T x_2\right) + \frac{1}{2!}\left(x_1^T x_2\right)^2 + \frac{1}{3!}\left(x_1^T x_2\right)^3 + \cdots\right)
\end{aligned}$$