# Artificial Intelligence
# CS-401

**Chapter # 04**

# Local Search & Optimization Problems

**Dr. Hafeez Ur Rehman**

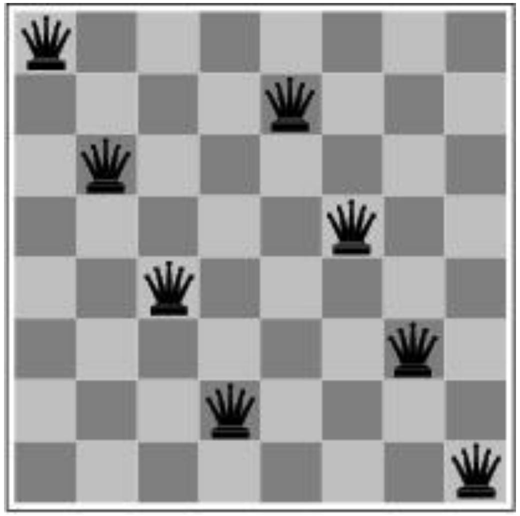
**(Email: hafeez.urrehman@nu.edu.pk)**

# Outline

- Local search techniques and optimization
  - Hill-climbing
  - Simulated annealing
  - Local Beam Search
  - Genetic algorithms

## Local search and optimization

- Previously: **systematic exploration** of search space.
  - Path to goal is solution to problem
- YET, for some problems **path is irrelevant**.
  - E.g 8-queens
  - Factory Floor Layout
  - Automatic programming
  - Integrated circuit design
  - Vehicle routing

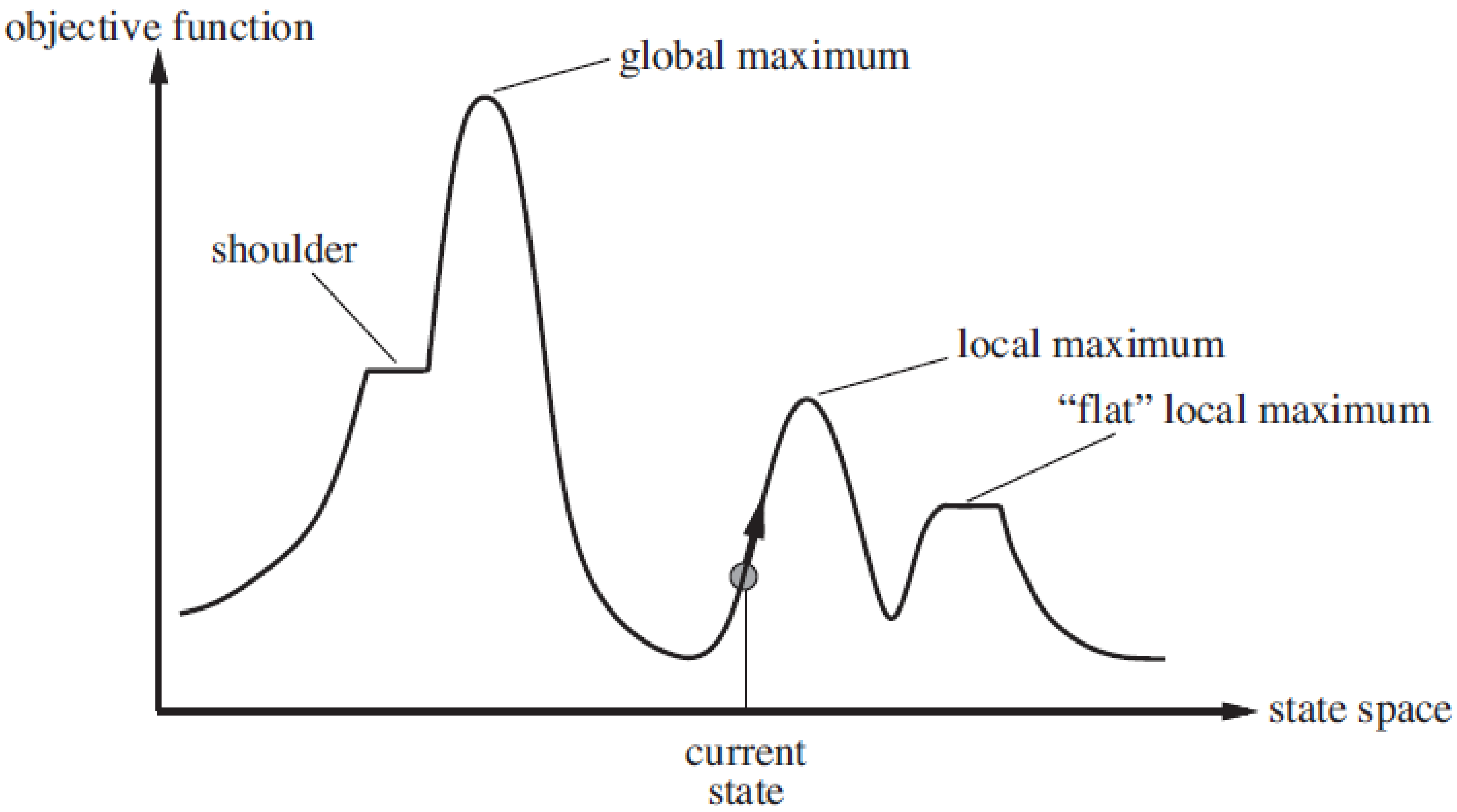


- For such problems different algorithms can be used
  - Local search

# Local search and optimization

- Local search
  - Keep track of single current state
  - Move only to neighboring states
  - Ignore paths
- Advantages:
  - Use very little memory
  - Can often find reasonable solutions in large or infinite (continuous) state spaces.
- “Pure optimization” problems
  - All states have an objective function
  - Goal is to find **state with max (or min)** objective value
  - Some problems do not quite fit into path-cost/goal-state formulation e.g. nature provides **reproductive fitness**, Darwin evolution seems to optimize it.
  - Local search can do quite well on these problems.

# “Landscape” of search



## Hill-climbing search algorithm (1)

**function** HILL-CLIMBING( *problem*) **return** a state that is a local maximum

**input:** *problem*, a problem

**local variables:** *current***, a node.**

*neighbor***, a node.**

*current* ← MAKE-NODE(INITIAL-STATE[*problem*])

**loop do**

*neighbor* ← a highest valued successor of *current*

**if** VALUE [*neighbor*] ≤ VALUE[*current*]

**then return** STATE[*current*]

*current* ← *neighbor*

# Hill-climbing search

- "a loop that continuously moves in the direction of increasing value"
  - terminates when a peak is reached
  - Aka greedy local search

- Value can be either
  - Objective function value
  - Heuristic function value (minimized)

- Hill climbing **does not look ahead** of the immediate neighbors of the current state.

- Can **randomly** choose among the set of best successors, if multiple have the best value

- Characterized as "**trying to find the top of Mount Everest while in a thick fog**"

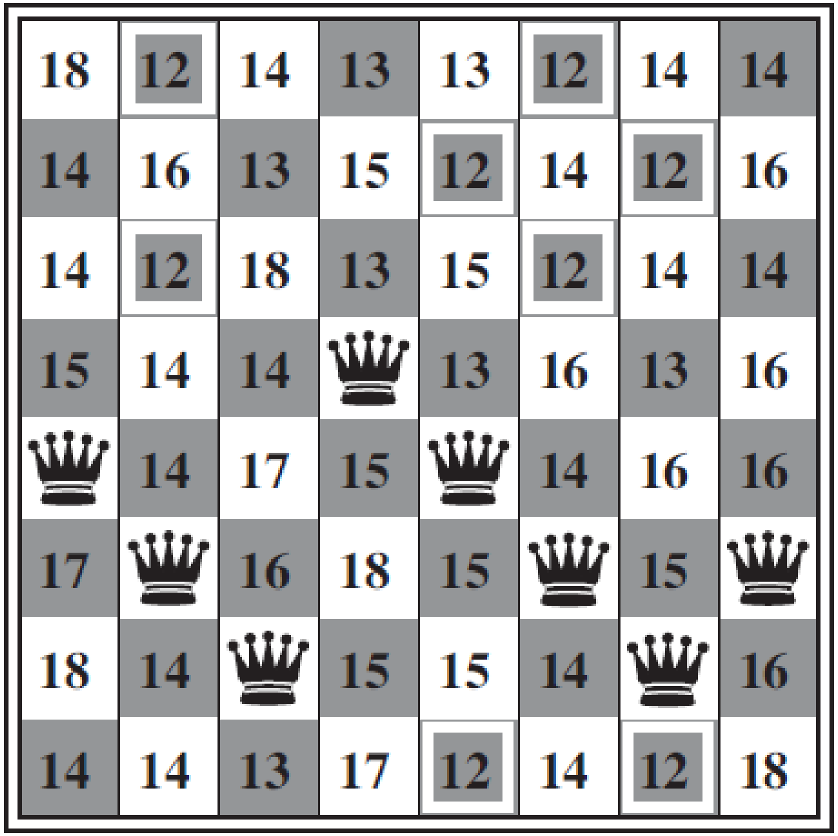
## Hill-climbing example

- 8-queens problem, complete-state formulation
  - All 8 queens on the board in some configuration

- Successor function:
  - move a single queen to another square in the same column.

- Example of a heuristic function *h(n)*:
  - the number of pairs of queens that are attacking each other (directly or indirectly)
  - (so we want to minimize this)

# Hill-climbing example

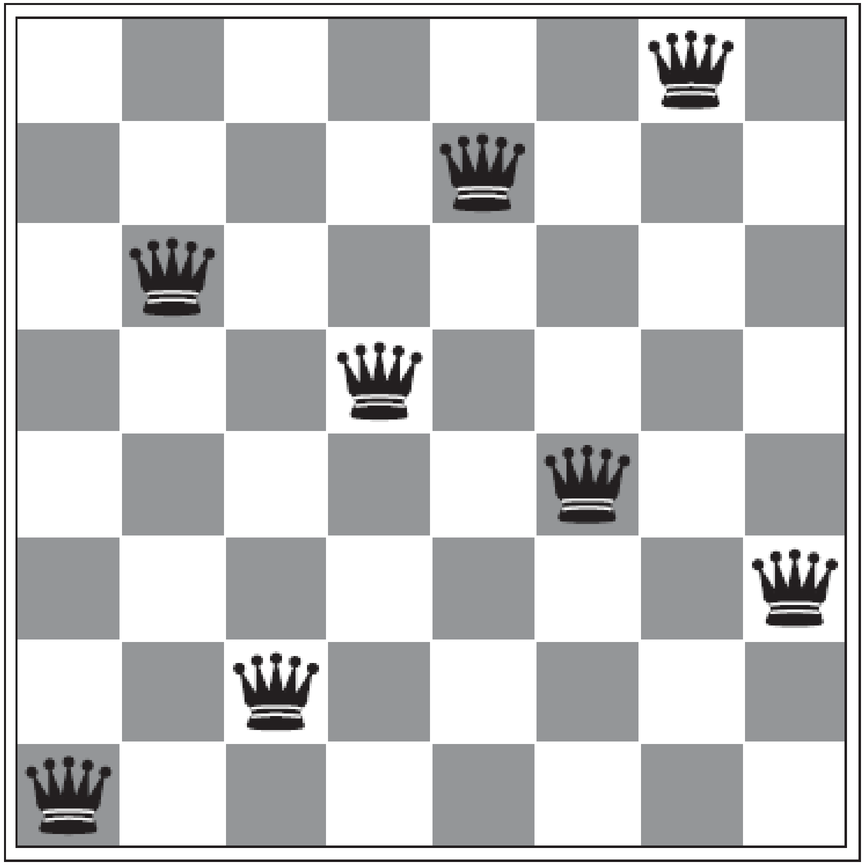



Current state: h=17

h is the number of queens attacking each other.

Shown is the h-value for each possible successor in each column. Best moves are marked.

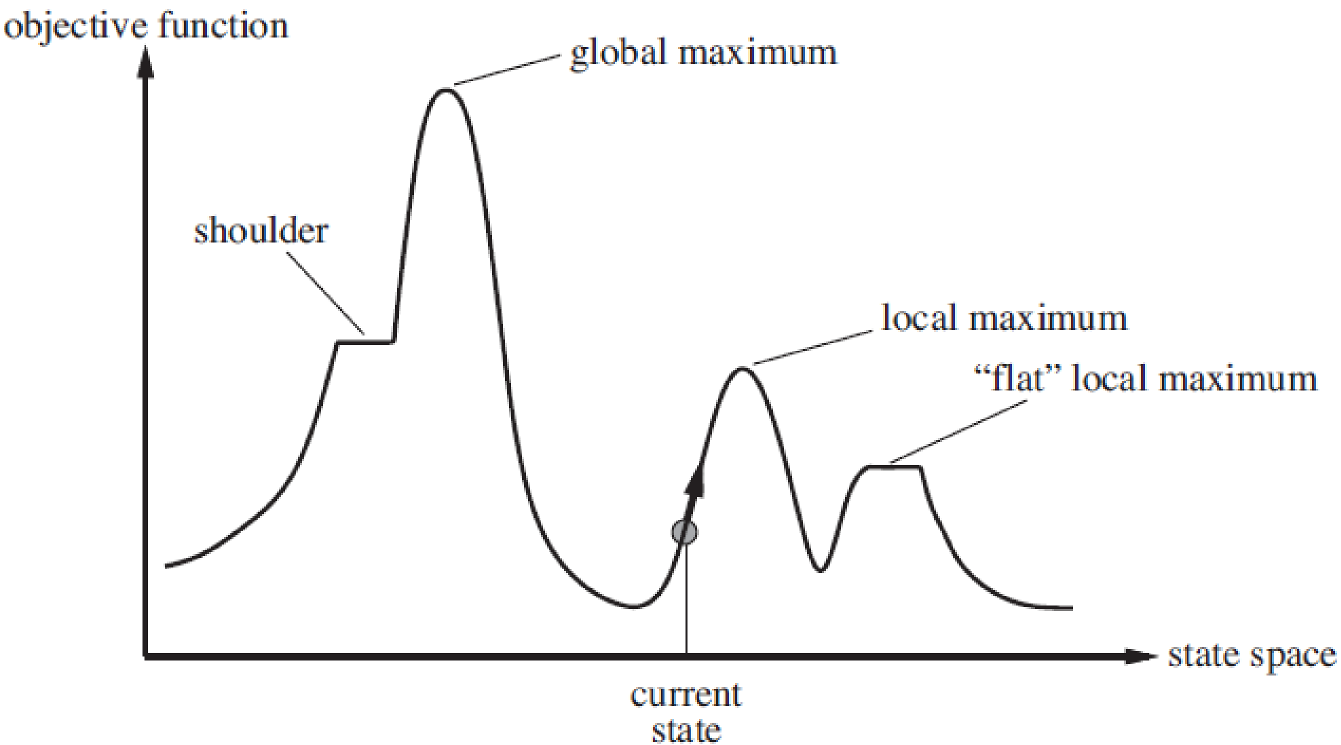
# A local minimum for 8-queens



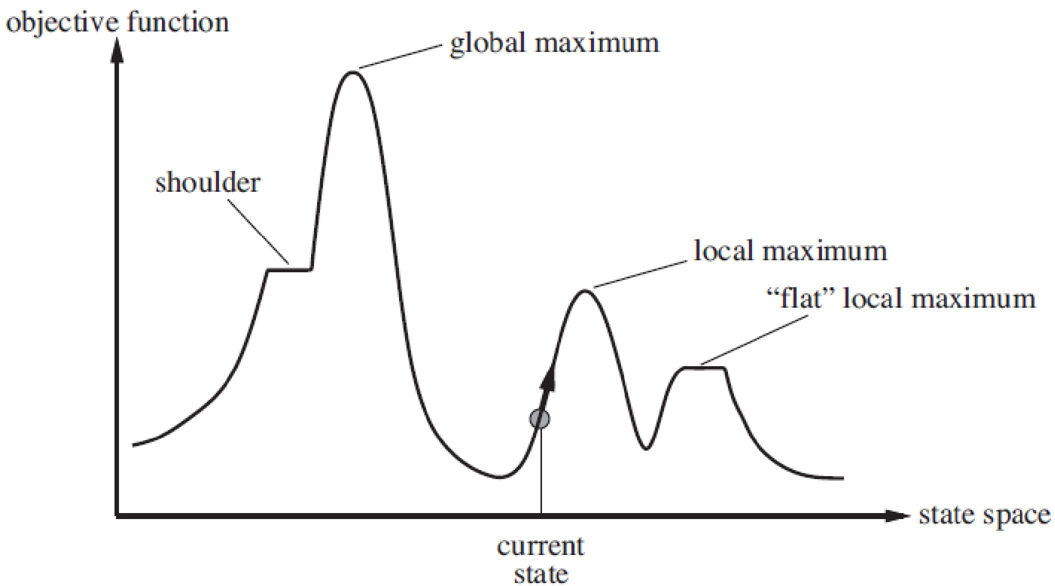
**A local minimum in the 8-queens state space (h=1)**

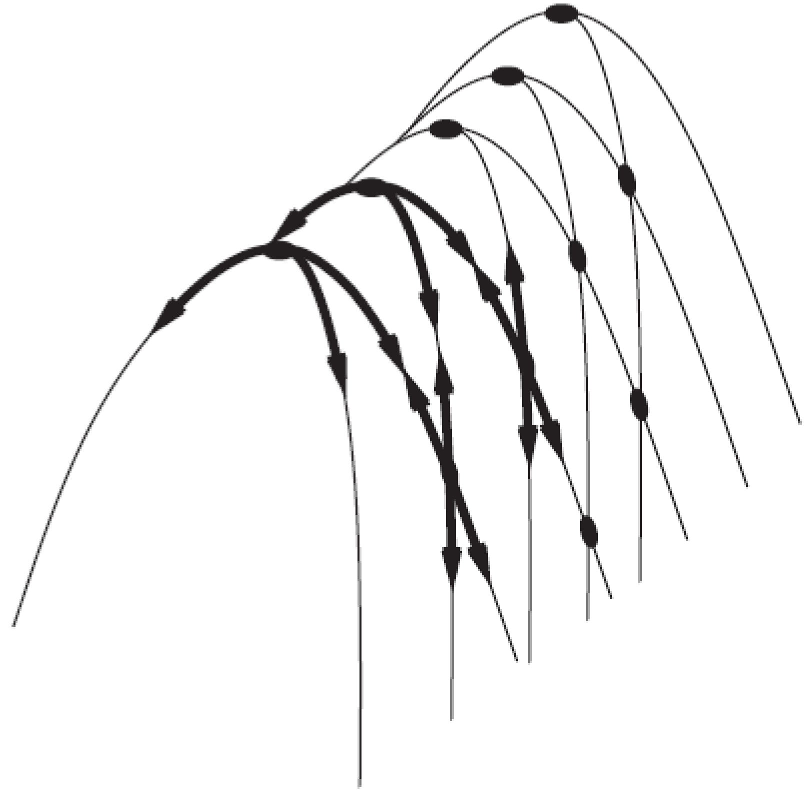
# Problems: Hill climbing and local maxima

- **Local Maxima:** When local maxima exists, hill climbing is suboptimal.
- Simple (often effective) solution
  - Multiple random restarts



# Problems: Other drawbacks



- **Ridge** = sequence of local maxima difficult for greedy algorithms to navigate (shown above right).

- **Plateau** = an area of the state space where the evaluation function is flat, shoulder region is a type of plateau.

## Performance of hill-climbing on 8-queens (Statistics)

- Randomly generated 8-queens starting states…
- 14% the time it solves the problem
- 86% of the time it get stuck at a local minimum
- However…
  - Takes only 4 steps on average when it succeeds
  - And 3 on average when it gets stuck
  - (for a state space with ~17 million states)

## Possible solution…sideways moves

- If no downhill (uphill) moves, allow sideways moves in hope that algorithm can escape
  - Need to place a limit on the possible number of sideways moves to avoid infinite loops
- For 8-queens
  - Now **allow sideways moves with a limit of 100**
  - Raises percentage of problem instances solved from 14 to 94%
  - However on average….
    - 21 steps for every successful solution
    - 64 for each failure

# Hill-climbing variations

1. **Stochastic hill-climbing**
   - **Random selection** among the uphill moves.
   - The selection probability can vary with the **steepness of the uphill move.**
   - This usually converges slowly than **steepest ascent**.
2. **First-choice hill-climbing**
   - Stochastic hill climbing by **generating successors randomly until a better one is found**
   - Useful when there are a very large number of successors
3. **Random-restart hill-climbing**
   - Hill Climbing from randomly generated initial states
   - Tries to avoid getting stuck in local maxima.
   - Complete

# Hill-climbing with random restarts

- Different variations
  - For each restart: run until termination v. Run for a fixed time
  - Run a fixed number of restarts or run indefinitely

- Analysis
  - Say each search has probability p of success then expected no of restarts required is: 1/p.
    - E.g., for 8-queens, p = 0.14 with no sideway moves

  - **Expected number of restarts?**

    **Ans:** 1/p
  - **Expected number of steps taken?**

    **Ans:** p x avg. of success + (1-p) avg. steps of failure

## Search using Simulated Annealing (2)

- Simulated Annealing = hill-climbing with non-deterministic search (i.e. randomness)

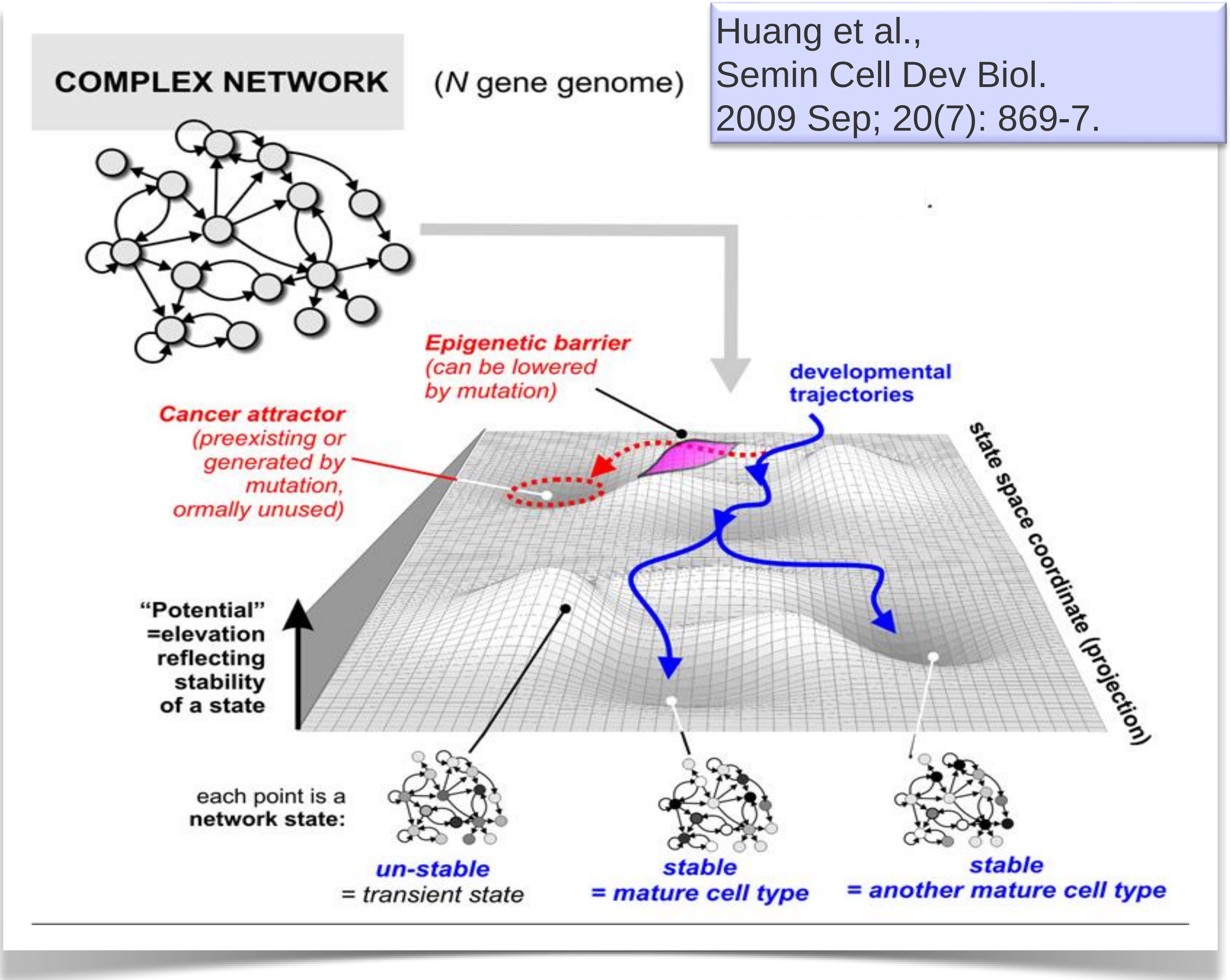
- Basic ideas:
  - like hill-climbing identify the quality of the local improvements
  - instead of picking the best move, pick one randomly
  - say the **change** in objective function is $\delta$
  - if $\delta$ is **positive**, then move to that state
  - **otherwise**:
    - move to this state with probability proportional to $\delta$
    - thus: worse moves (very large negative $\delta$) are executed less often
  - however, there is always a chance of escaping from local maxima
  - over time, make it less likely to accept locally bad moves
  - (Can also make the size of the move random as well, i.e., allow "large" steps in state space)

## Physical Interpretation of Simulated Annealing

- **A Physical Analogy:**
  - imagine letting a ball roll downhill on the function surface
    - this is like hill-climbing (for minimization)
  - now imagine **shaking the surface, while the ball rolls, gradually reducing the amount of shaking**
    - this is like simulated annealing
- **Annealing** = physical process of cooling a liquid or metal until particles achieve a certain frozen crystal state
  - simulated annealing:
    - free variables are like particles
    - seek "low energy" (high quality) configuration
    - get this by slowly reducing temperature T, which particles move around randomly

# Physical Interpretation of Simulated Annealing

Huang et al.,
Semin Cell Dev Biol.
2009 Sep; 20(7): 869-7.



# Simulated annealing

**function** SIMULATED-ANNEALING( *problem*, *schedule*) **return** a solution state

**input:** *problem*, a problem

*schedule*, a mapping from time to temperature

**local variables:** *current***,** a node.

*next***,** a node.

*T***,** a "temperature" controlling the probability of downward steps

*current* $\leftarrow$ MAKE-NODE(INITIAL-STATE[*problem*])

**for t $\leftarrow$ 1 to $\infty$ do**

$T \leftarrow schedule[t]$

**if** $T = 0$ **then return** *current*

*next* $\leftarrow$ a randomly selected successor of *current*

$\Delta E \leftarrow$ VALUE[*next*] - VALUE[*current*]

**if** $\Delta E > 0$ **then** *current* $\leftarrow$ *next*

**else** *current* $\leftarrow$ *next* only with probability $e^{\Delta E / T}$

# More Details on Simulated Annealing

- Lets say there are 3 moves available, with changes in the objective function of d1 = -0.1, d2 = 0.5,  d3 = -5. (Let T = 1).
- pick a move randomly:
  - if d2 is picked, move there.
  - if d1 or d3 are picked, probability of move = exp(d/T)
  - move 1: prob1 = exp(-0.1) = 0.9,
    - i.e., 90% of the time we will accept this move
  - move 3: prob3 = exp(-5) = 0.05
    - i.e., 5% of the time we will accept this move

- T = “temperature” parameter
  - high T  => probability of “locally bad” move is higher
  - low T   => probability of “locally bad” move is lower
  - typically, T is decreased as the algorithm runs longer
    - i.e., there is a “temperature schedule”

# Simulated Annealing in Practice

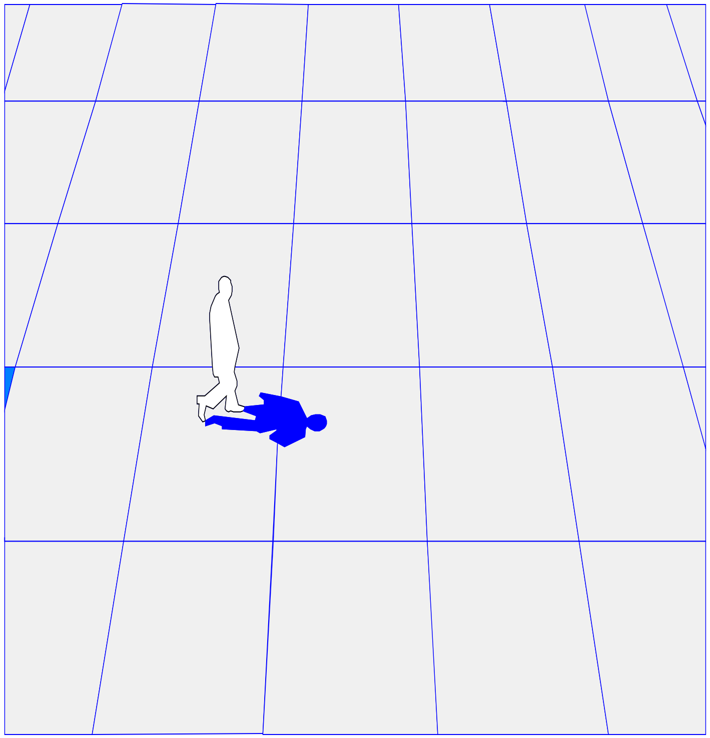
- method proposed in 1983 by IBM researchers for solving **VLSI layout** problems (Kirkpatrick et al, *Science*, 220:671-680, 1983).
  - Very-large-scale integration (**VLSI**) is the process of creating an integrated circuit (IC) by combining thousands of transistors into a single chip. **VLSI** began in the 1970s when complex semiconductor and communication technologies were being developed. The microprocessor is a **VLSI** device.
  - theoretically will always find the global optimum (the best solution)

- useful for some problems, but can be very slow
    - slowness comes about because T must be decreased very gradually to retain optimality
  - In practice how do we decide the rate at which to decrease T? (this is a practical problem with this method)

# Local beam search (3)

- Keep track of *k* states instead of one
  - Initially: *k* randomly selected states
  - Next: determine all successors of *k* states
  - If any of successors is goal → finished
  - Else **select *k* best from ALL successors** and repeat.
- Major difference with random-restart search
  - Information is shared among *k* search threads.
- Can suffer from lack of diversity.
  - Stochastic beam search
    - choose k successors at random proportional to state quality.

# Genetic Algorithms



*"Genetic Algorithms are good at taking large, potentially huge search spaces and navigating them, looking for optimal combinations of things, solutions you might not otherwise find in a lifetime."*

- Salvatore Mangano

*Computer Design*, May 1995

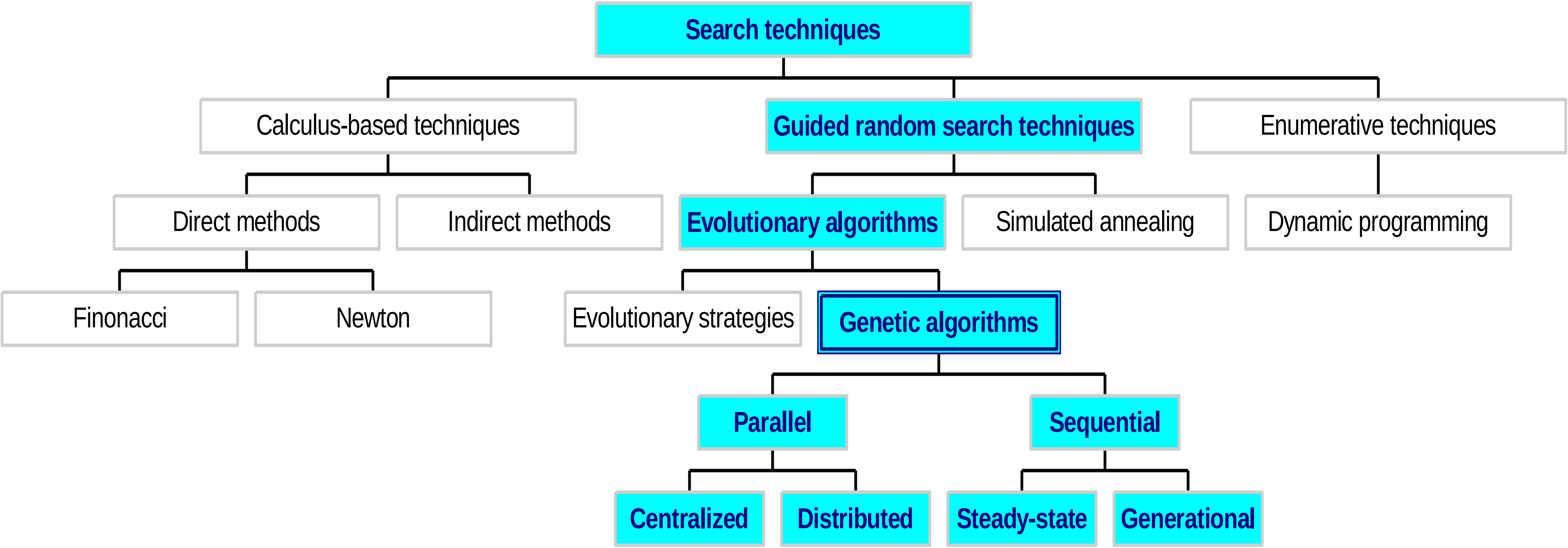
# The Genetic Algorithm

- Directed search algorithms based on the mechanics of biological evolution
- Developed by John Holland, University of Michigan (1970's)
  - To understand the adaptive processes of natural systems
  - To design artificial systems software that retains the robustness of natural systems

# The Genetic Algorithm (cont.)

- Provide efficient, effective techniques for optimization and machine learning applications
- Widely-used today in business, scientific and engineering circles

# Classes of Search Techniques



# Components of a GA

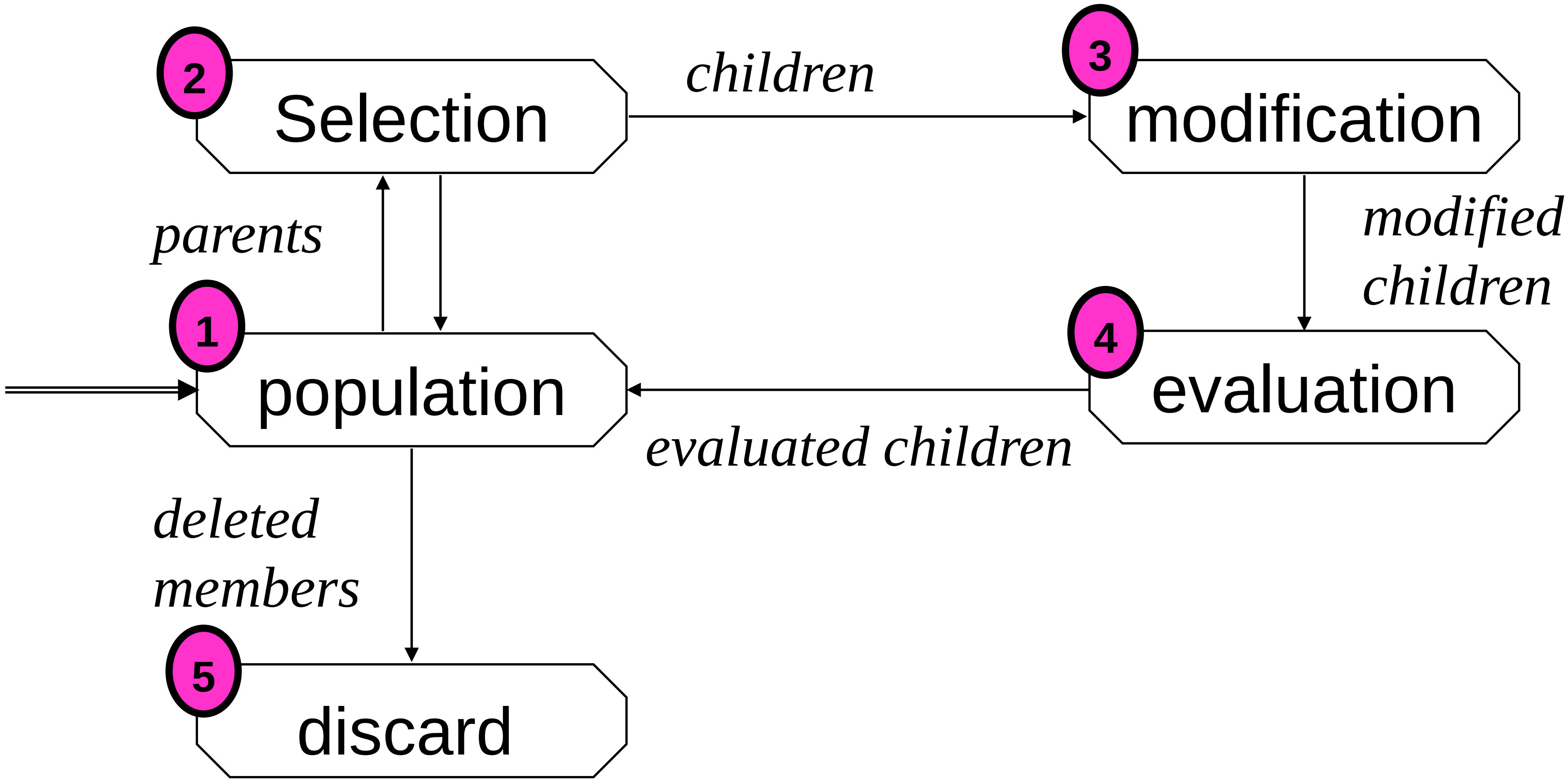
A problem to solve, and ...

- Encoding technique (*gene, chromosome*)
- Initialization procedure *(creation)*
- Evaluation function *(environment)*
- Selection of parents *(reproduction)*
- Genetic operators *(mutation, recombination)*
- Parameter settings *(practice and art)*

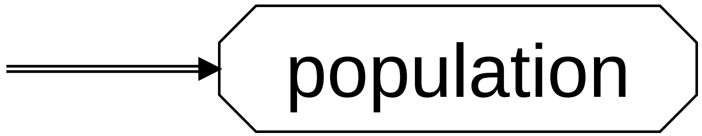
# Simple Genetic Algorithm

```
{
    initialize population;
    evaluate population;
    while TerminationCriteriaNotSatisfied
    {
        select parents for reproduction;
        perform recombination and mutation;
        evaluate population;
    }
}
```

# The GA Cycle of Reproduction



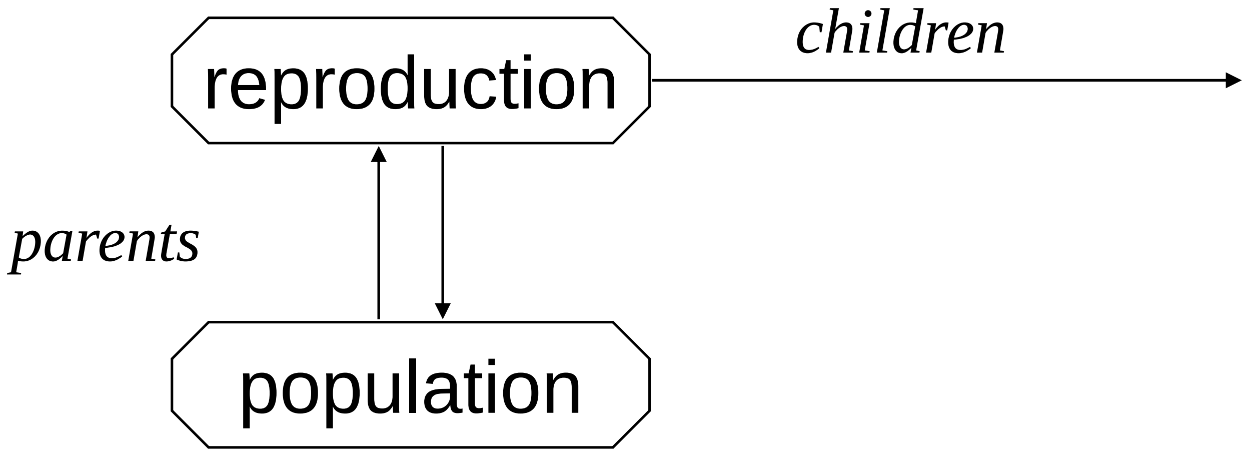
# 1- Population




Chromosomes could be:

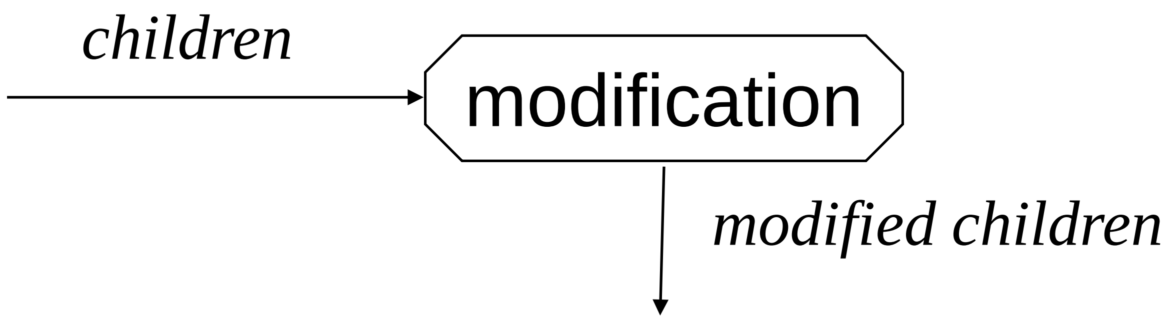
- Bit strings (0101 ... 1100)
- Real numbers (43.2 -33.1 ... 0.0 89.2)
- Permutations of element (E11 E3 E7 ... E1 E15)
- Lists of rules (R1 R2 R3 ... R22 R23)
- Program elements (genetic programming)
- ... any data structure ...

# 2- Reproduction/Selection




Parents are selected at random with selection chances biased in relation to chromosome evaluations.

# 3- Chromosome Modification




- Modifications are stochastically triggered
- Operator types are:
  - Mutation
  - Crossover (recombination)

# Crossover: Recombination

| | | | | |
|---|---|---|---|---|
| | $(0\ 1\ \overset{*}{\boxed{1}}\ 0\ 1\ 0\ 0\ 0)$ | | $(0\ 1\ \boxed{0}\ 0\ 1\ 0\ 0\ 0)$ | C1 |
| P1 | | $\longrightarrow$ | | |
| P2 | $(1\ 1\ \boxed{0}\ 1\ 1\ 0\ 1\ 0)$ | | $(1\ 1\ \boxed{1}\ 1\ 1\ 0\ 1\ 0)$ | C2 |

Crossover is a critical feature of genetic algorithms:

- It greatly accelerates search early in evolution of a population
- It leads to effective combination of schemata (subsolutions on different chromosomes)

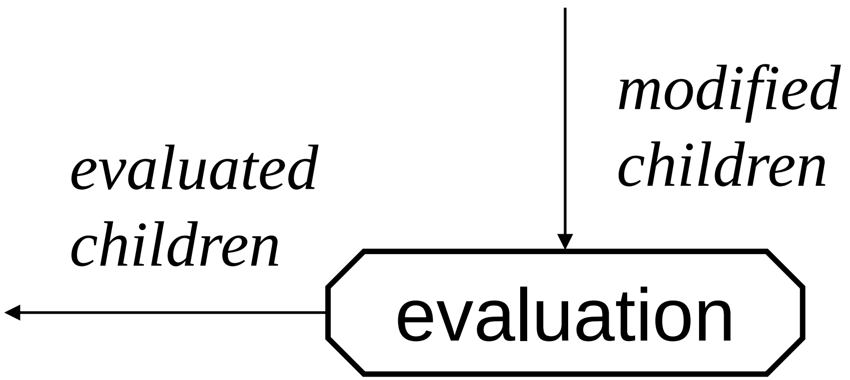
# Mutation: Local Modification

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Before: | (1 | 0 | 1 | 1 | 0 | 1 | 1 | 0) |
| After: | (0 | 1 | 1 | 0 | 0 | 1 | 1 | 0) |

| | | | | |
|---|---|---|---|---|
| Before: | (1.38 | -69.4 | 326.44 | 0.1) |
| After: | (1.38 | -67.5 | 326.44 | 0.1) |

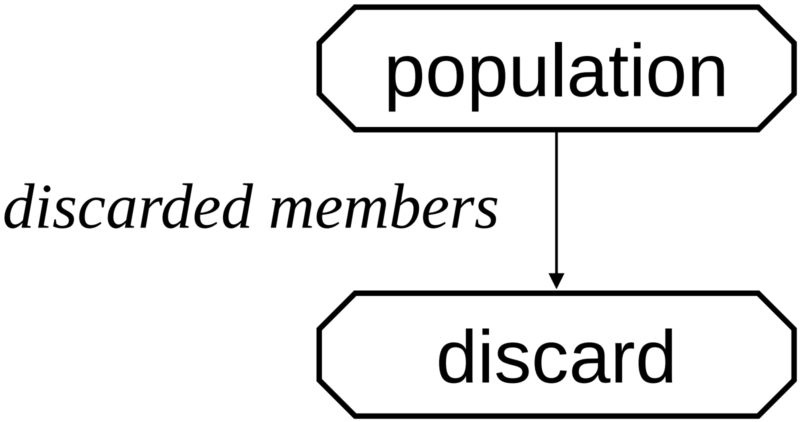
- Causes movement in the search space (local or global)
- Restores lost information to the population

# 4- Evaluation



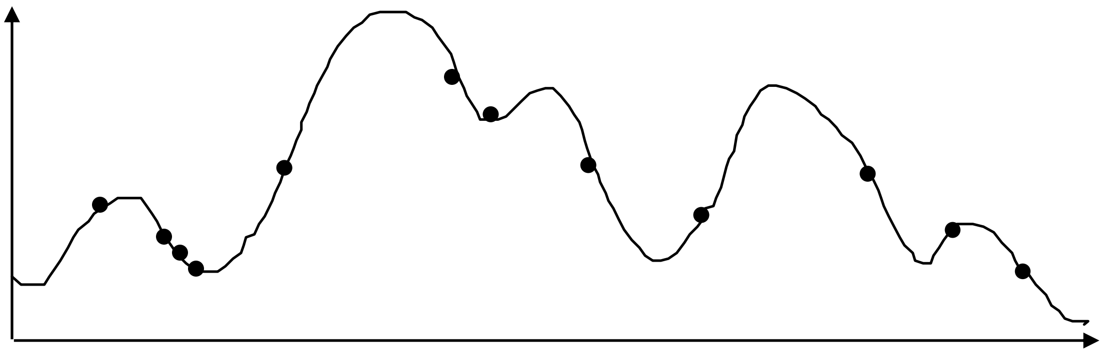

- The evaluator decodes a chromosome and assigns it a fitness measure
- The evaluator is the only link between a classical GA and the problem it is solving

# 5- Deletion

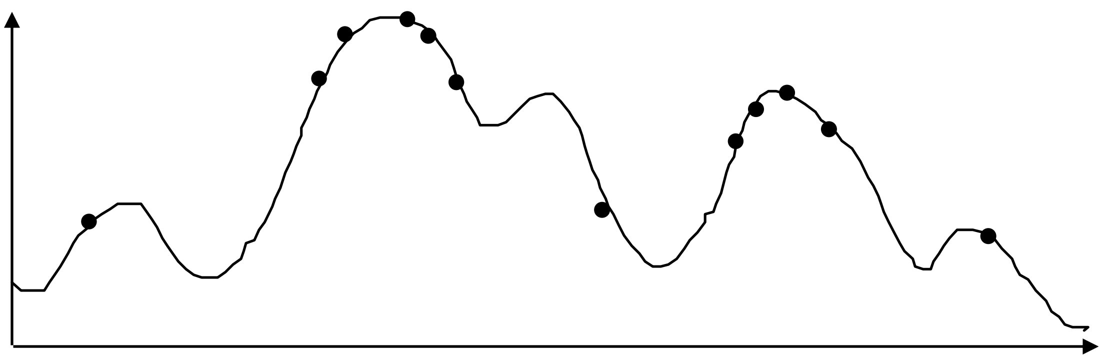



- *Generational* GA:
  entire populations replaced with each iteration
- *Steady-state* GA:
  a few members replaced each generation

# An Abstract Example

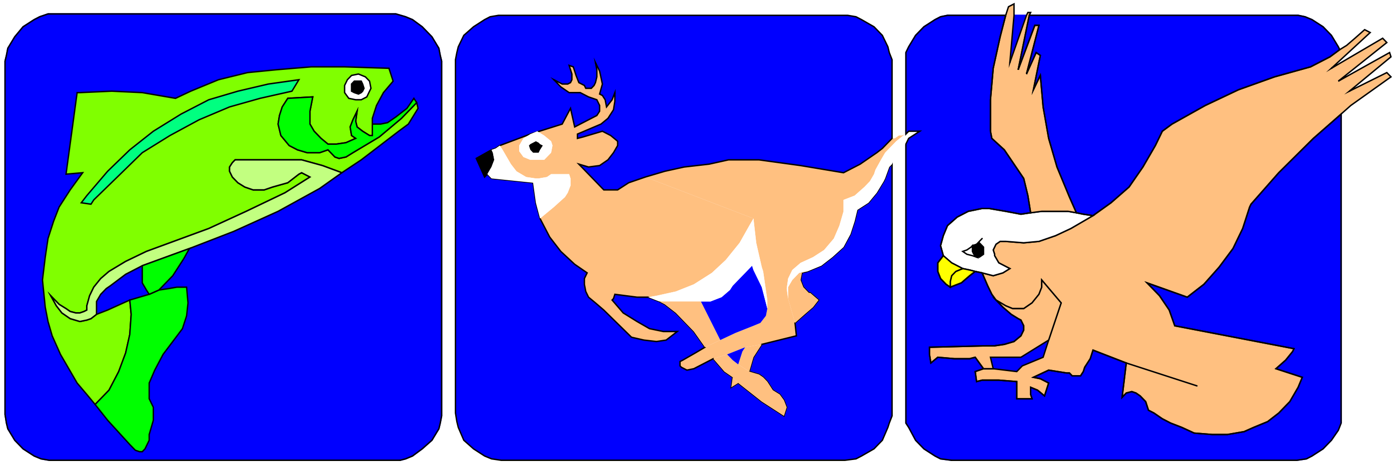


*Distribution of Individuals in Generation 0*



*Distribution of Individuals in Generation N*

# A Simple Example



*“The Gene is by far the most sophisticated program around.”*

- Bill Gates, *Business Week*, June 27, 1994

# A Simple Example

The Traveling Salesman Problem:

Find a tour of a given set of cities so that

- each city is visited only once
- the total distance traveled is minimized

# Representation & Selection

Representation is an ordered list of city numbers known as an *order-based* GA.

| | | | |
|---|---|---|---|
| 1) London | 3) Dunedin | 5) Beijing | 7) Tokyo |
| 2) Venice | 4) Singapore | 6) Phoenix | 8) Victoria |

CityList1 (3 5 7 2 1 6 4 8)

CityList2 (2 5 7 6 8 1 3 4)

# Crossover

Crossover combines inversion and recombination:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | * | | | * | | |
| Parent1 | (3 | 5 | $\boxed{7}$ | $\boxed{2}$ | $\boxed{1}$ | $\boxed{6}$ | 4 | 8) |
| Parent2 | (2 | 5 | 7 | 6 | 8 | 1 | 3 | 4) |
| Child | (5 | 8 | $\boxed{7}$ | $\boxed{2}$ | $\boxed{1}$ | $\boxed{6}$ | 3 | 4) |

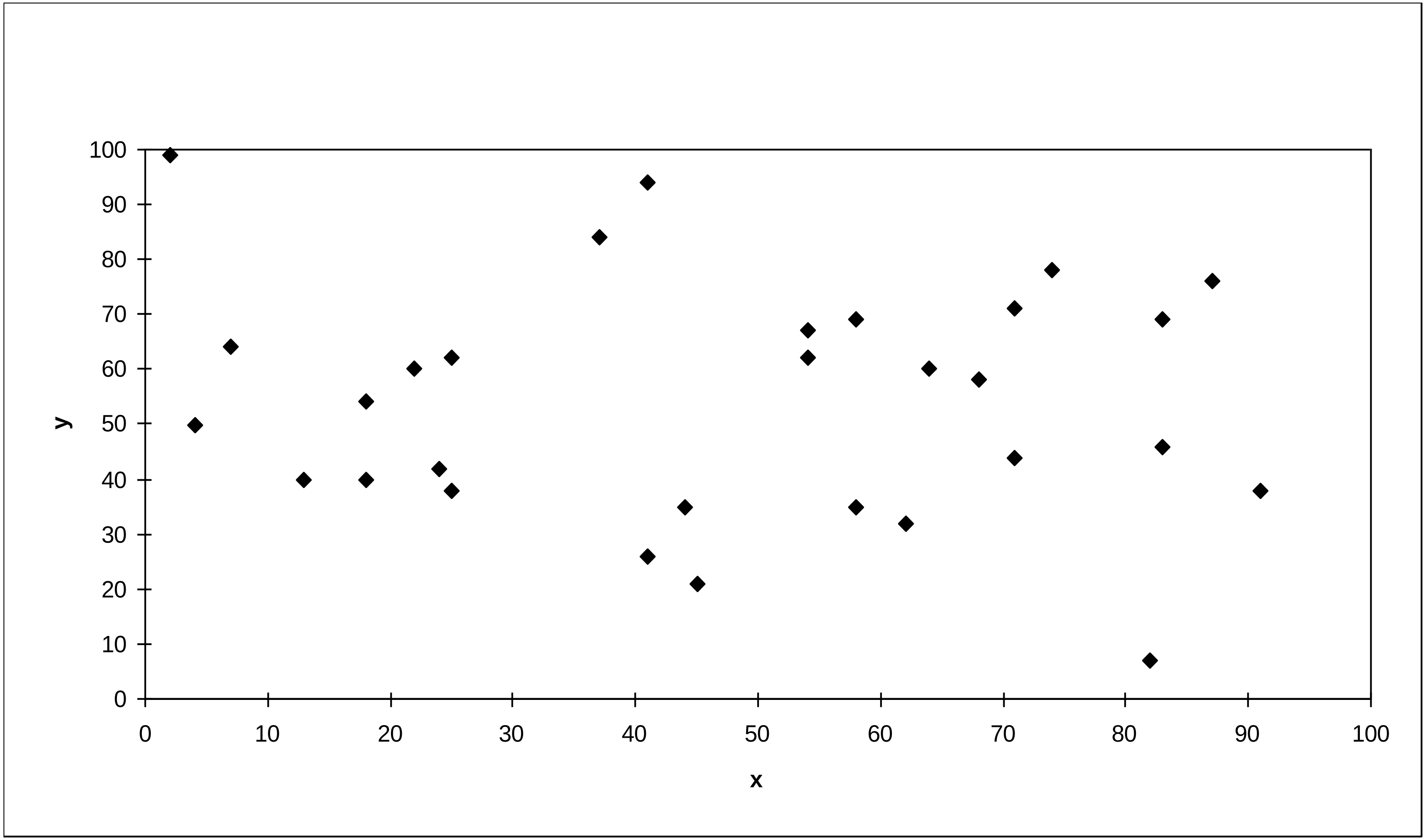
This operator is called the *Order1* crossover.

# Mutation

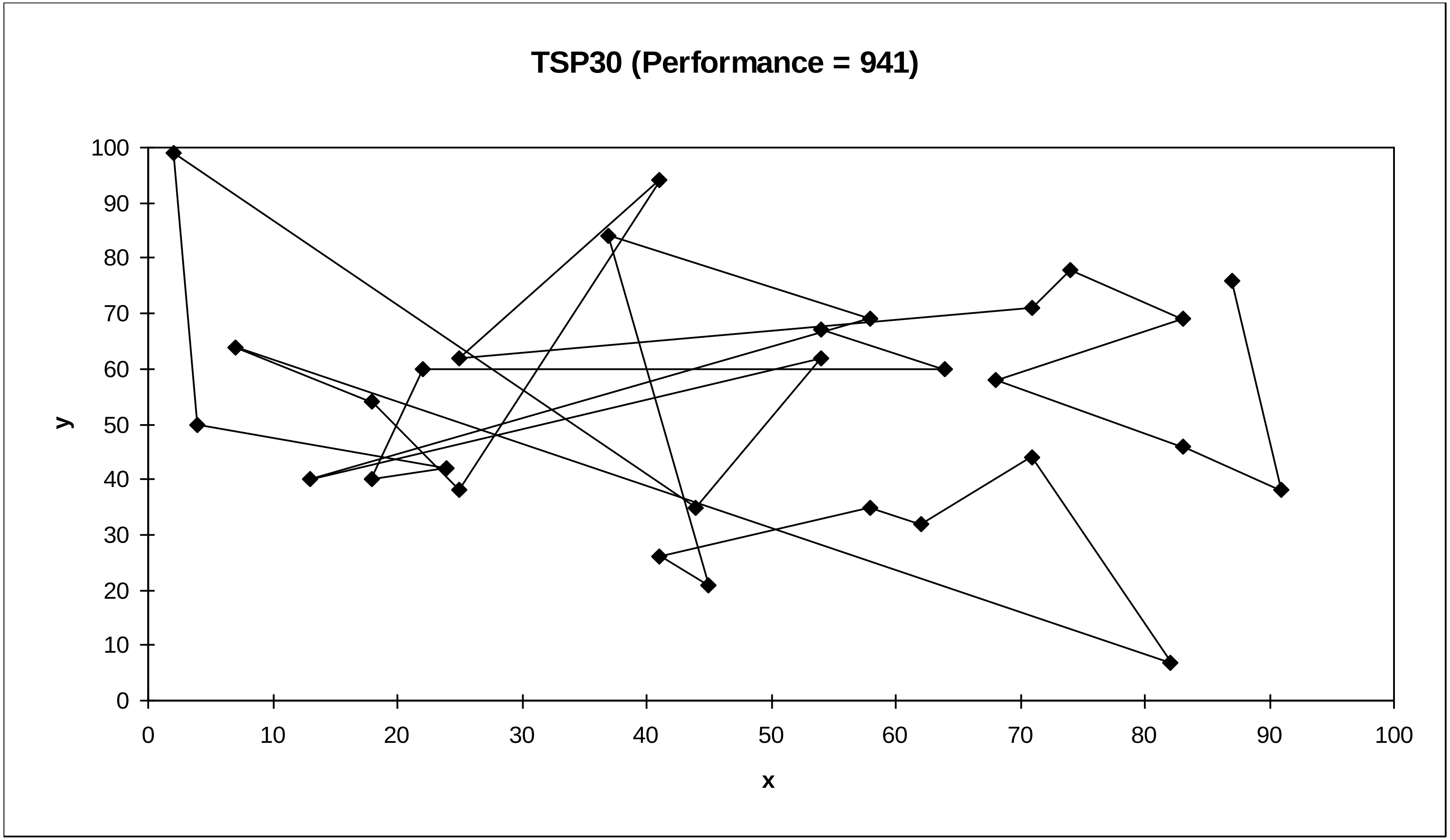
Mutation involves reordering of the list:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | * | | | * | | |
| Before: | (5 | 8 | 7 | 2 | 1 | 6 | 3 | 4) |
| After: | (5 | 8 | 6 | 2 | 1 | 7 | 3 | 4) |

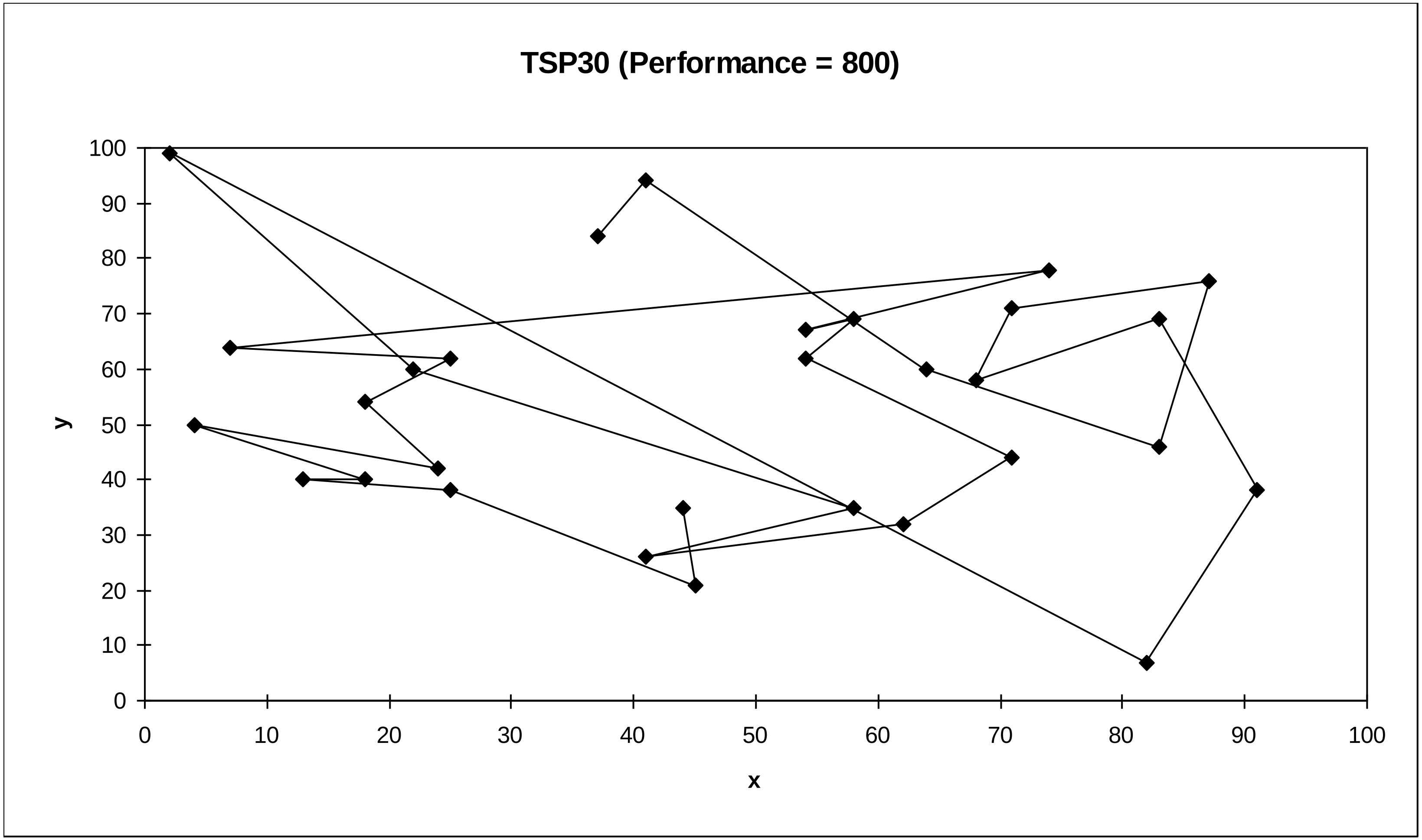
# TSP Example: 30 Cities



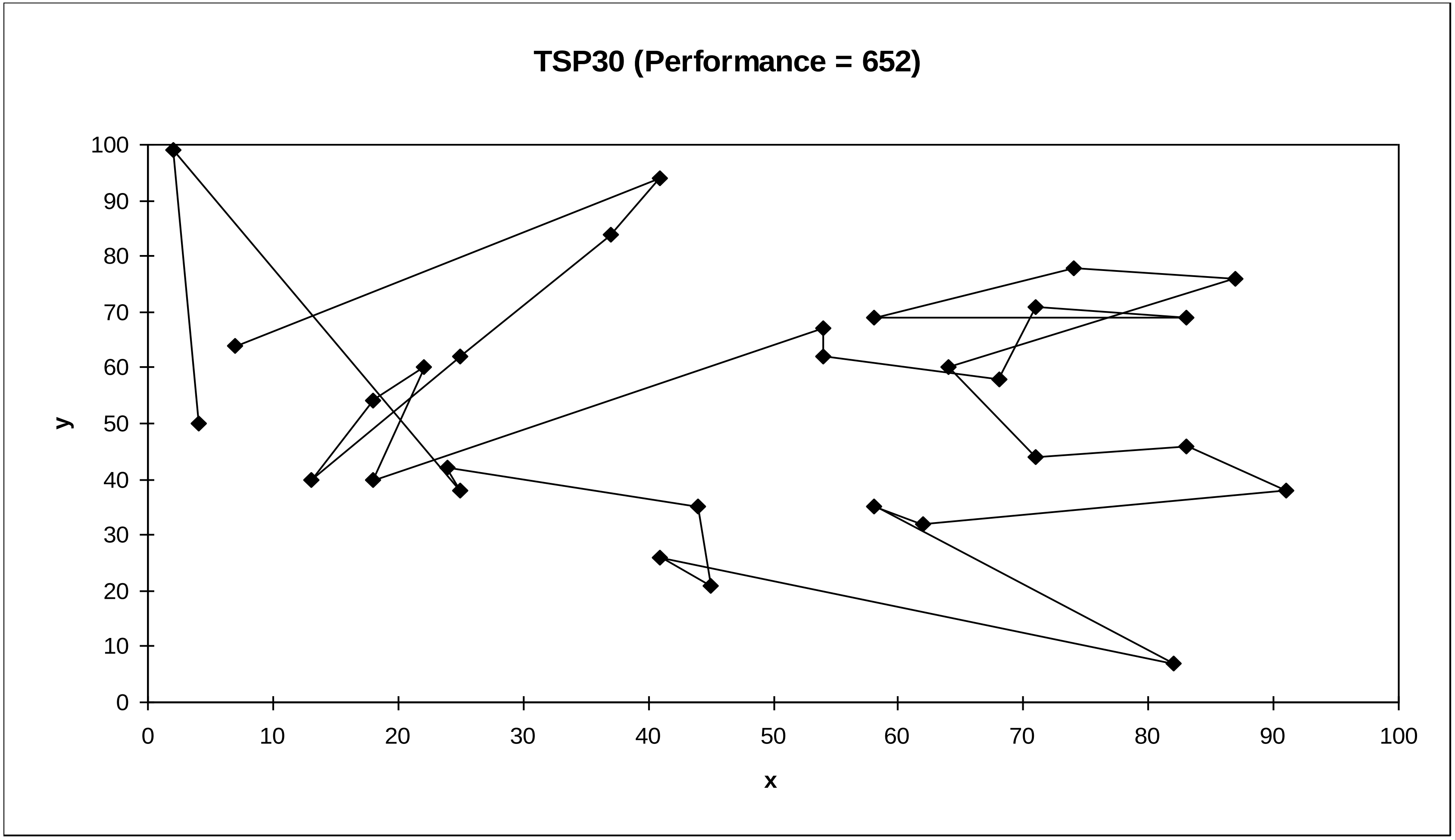
# Solution $_i$ (Distance = 941)



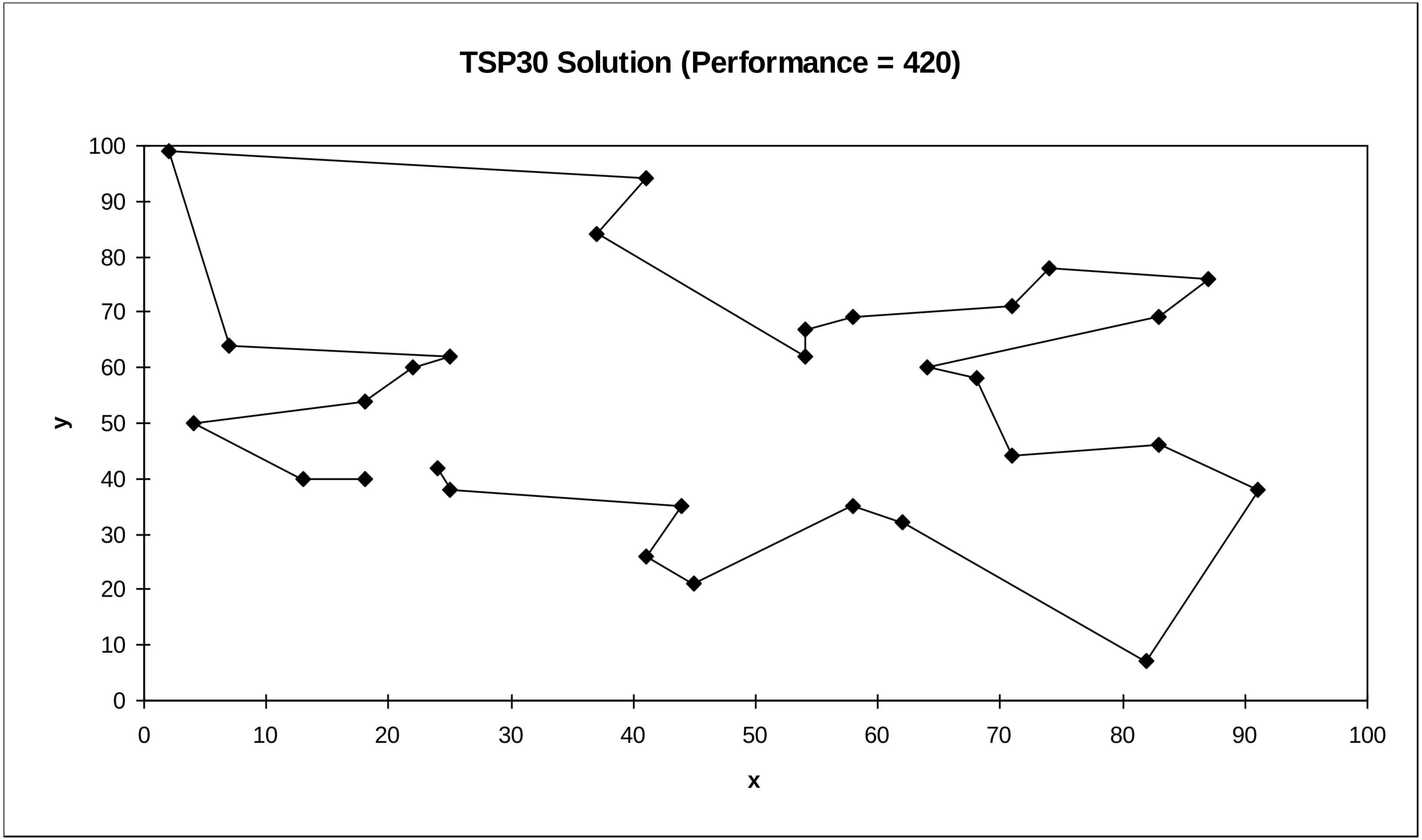
# Solution $_j$(Distance = 800)



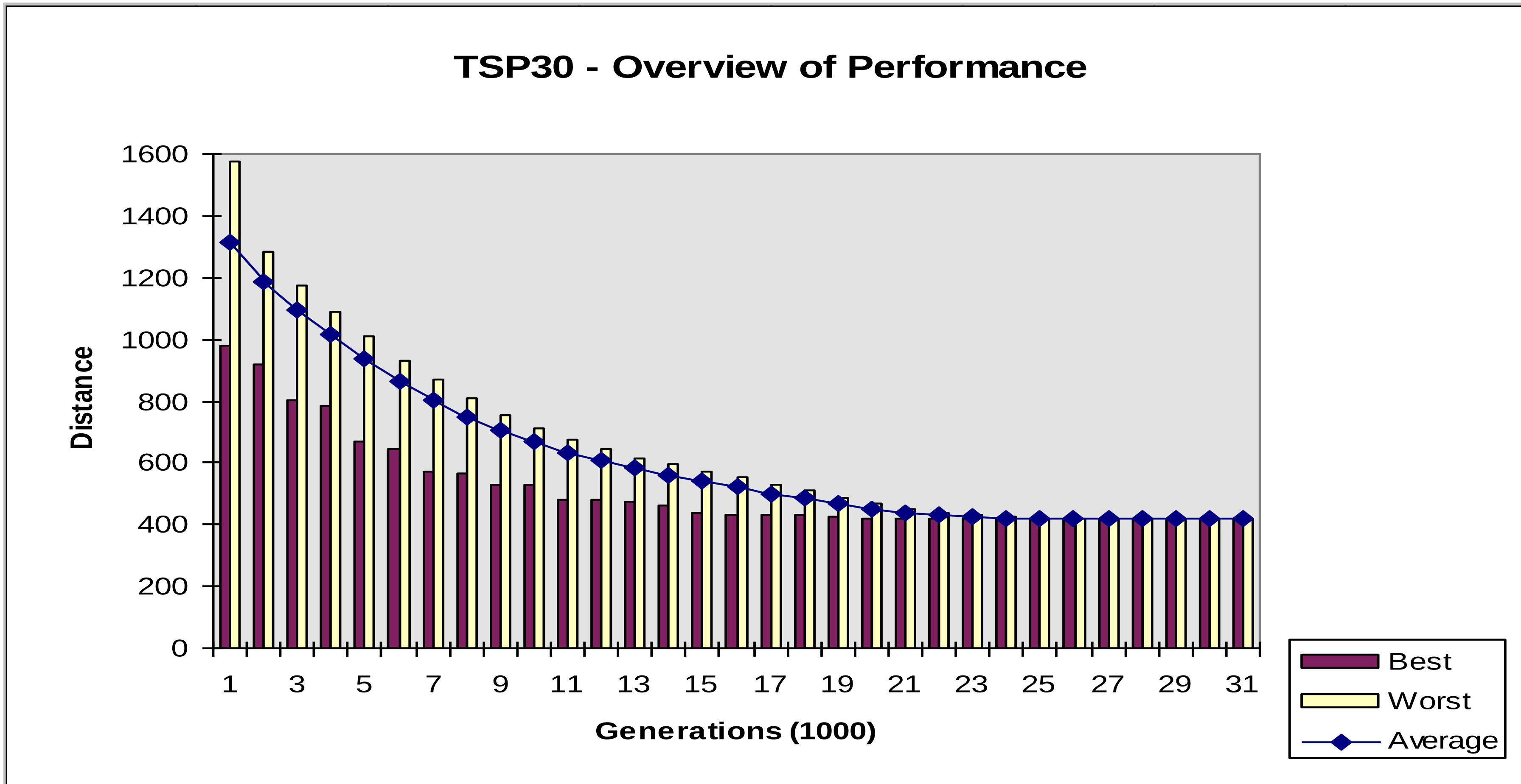
# Solution $_k$(Distance = 652)



# Best Solution (Distance = 420)



# Overview of Performance



# Genetic algorithms- 8 Queen Example

- A state is represented as a string over a finite alphabet (e.g. binary)
  - 8-queens
    - State = position of 8 queens each in a column

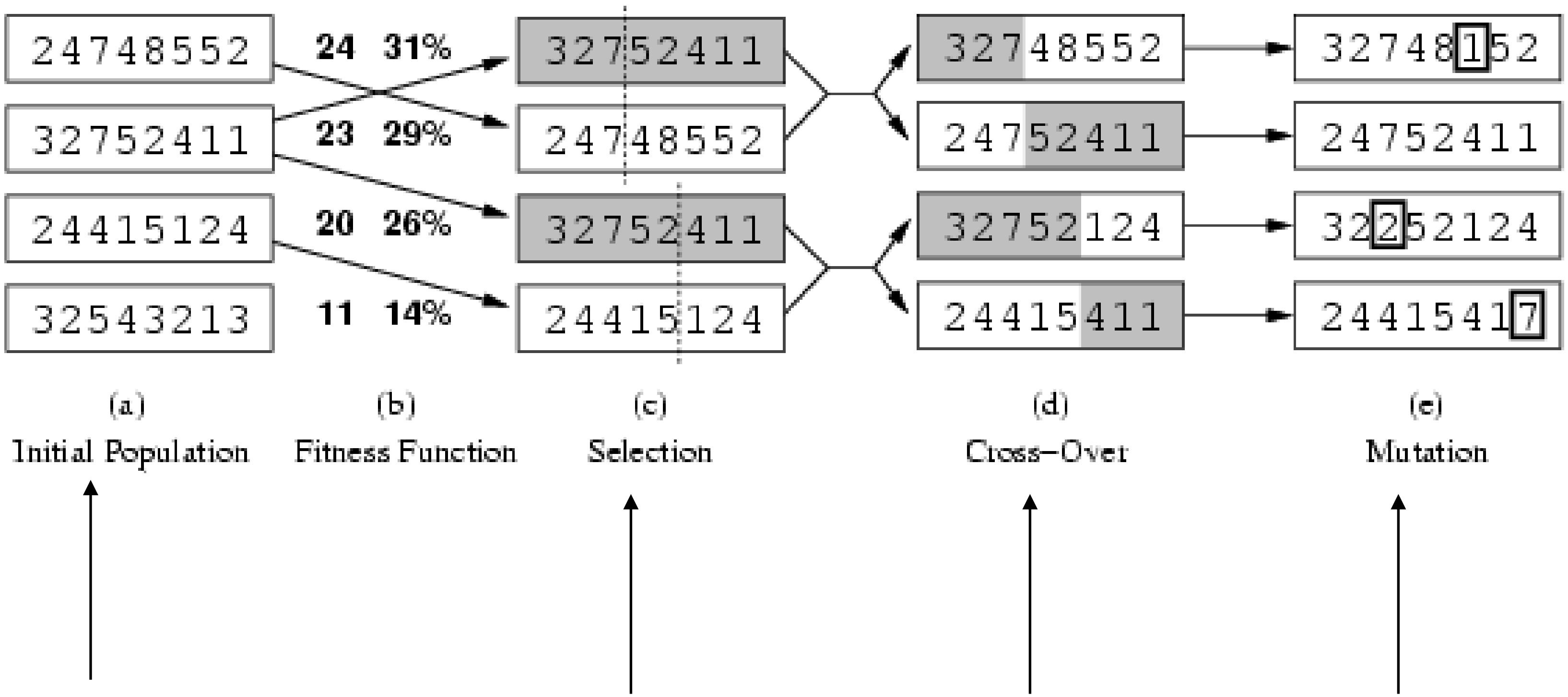
      => 8 x log(8) bits = 24 bits (for binary representation)
- **Start with *k* randomly generated states (population)**
- **Evaluation function (fitness function).**
  - Higher values for better states.
  - Opposite to heuristic function, e.g., # non-attacking pairs in 8-queens
  - Solution has a **value of 28**
- **Produce the next generation of states by "simulated evolution"**
  - **Random selection**
  - **Crossover**
  - **Random mutation**

# Genetic algorithms

**Fitness function:** number of non-attacking pairs of queens (min = 0, max = 8 × 7/2 = 28)

24/(24+23+20+11) = 31%

23/(24+23+20+11) = 29% etc




4 states for 8-queens problem

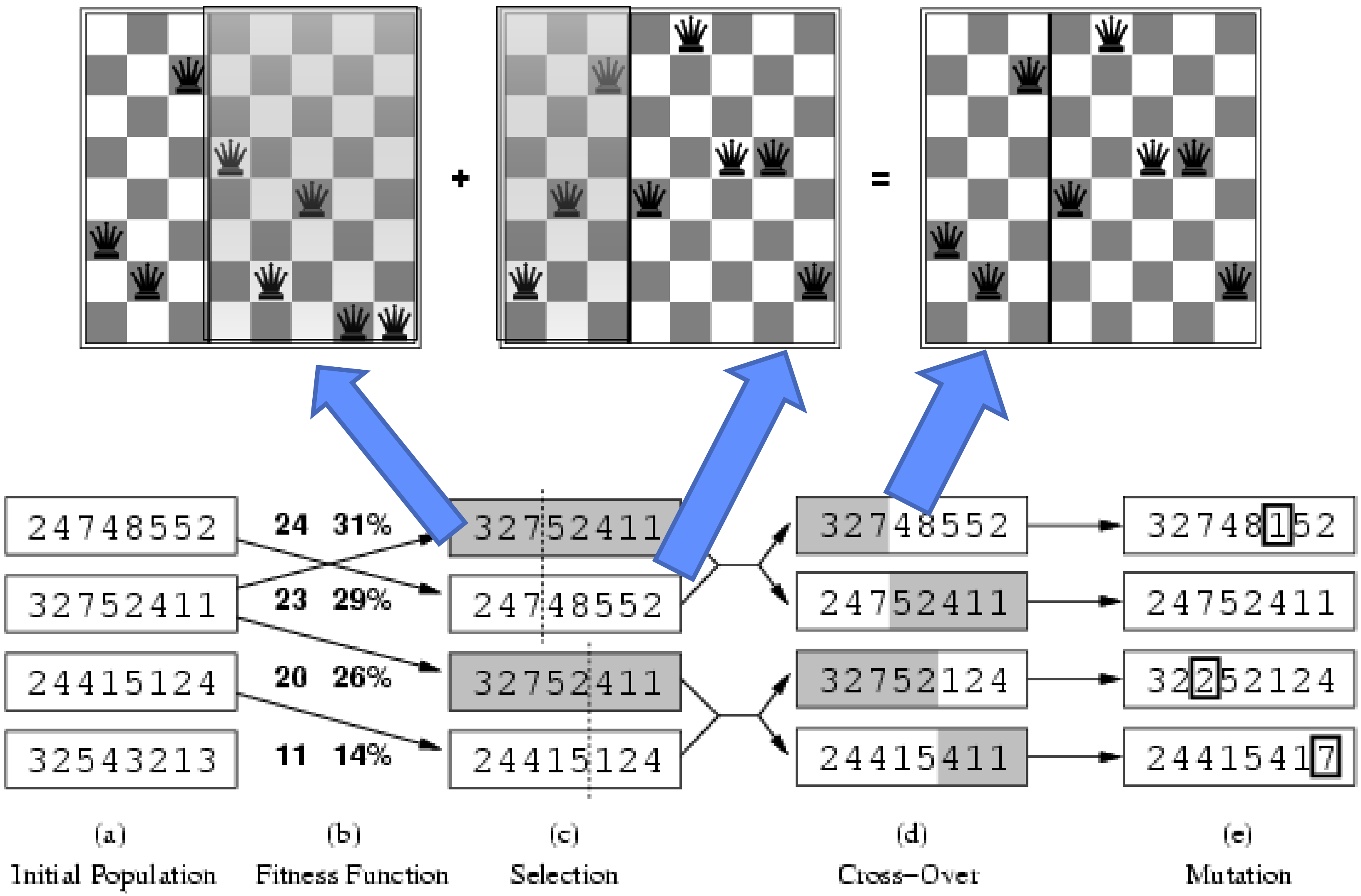
2 pairs of 2 states randomly selected based on fitness. Random crossover points selected

New states after crossover

Random mutation applied

# Genetic algorithms

Like Simulated Annealing **Cross Over** takes **large** steps at the **beginning** of the search process while **small** steps when the population (individuals) are quite similar.



# Genetic algorithm pseudocode

**function** GENETIC_ALGORITHM( *population*, FITNESS-FN) **return** an individual

**input:** *population*, a set of individuals

FITNESS-FN, a function which determines the quality of the individual

**repeat**

*new_population* ← empty set

**loop for** i **from** 1 **to** SIZE(*population*) **do**

*x* ← RANDOM_SELECTION(*population*, FITNESS_FN)

*y* ← RANDOM_SELECTION(*population*, FITNESS_FN)

*child* ← REPRODUCE(*x*,*y*)

**if** (small random probability) **then** *child* ← MUTATE(*child* )

add *child* to *new_population*

*population* ← *new_population*

**until** some individual is fit enough or enough time has elapsed

**return** the best individual

# Issues for GA Practitioners

- Choosing basic implementation issues:
  - representation
  - population size, mutation rate, ...
  - selection, deletion policies
  - crossover, mutation operators
- Termination Criteria
- Performance, scalability
- Solution is only as good as the evaluation function (often hardest part)

# Benefits of Genetic Algorithms

- Concept is easy to understand
- Modular, separate from application
- Supports multi-objective optimization
- Good for “noisy” environments
- Always an answer; answer gets better with time
- Inherently parallel; easily distributed

# Benefits of Genetic Algorithms (cont.)

- Many ways to speed up and improve a GA-based application as knowledge about  problem domain is gained
- Easy to exploit previous or alternate solutions
- Flexible building blocks for hybrid applications
- Substantial history and range of use