Assignment # 1

Question 1 part (a)

$$\frac{dT_o}{dt} \propto T_s - T_o$$

$$\frac{dT_o}{dt} = K(T_s - T_o)$$

T_o = object Temperature
T_s = surrounding Temperature
K = constant

Part (b)

$T_s = 25^\circ c$

$$\frac{dT_o}{T_s - T_o} = K\,dt$$

$$\int \frac{dT_o}{25^\circ c - T_o} = \int K\,dt$$

$$-\ln(25 - T_0) = kt + c$$

$$\ln(25 - T_0) = -kt - c$$

$$e^{\ln(25 - T_0)} = e^{-kt-c}$$

$$25 - T_0 = e^{-kt-c}$$

For Cooling $25 - T_0$ is negative

$$25 - T_0 = -(25 - T_0)$$

$$-(25 - T_0) = e^{-kt-c}$$

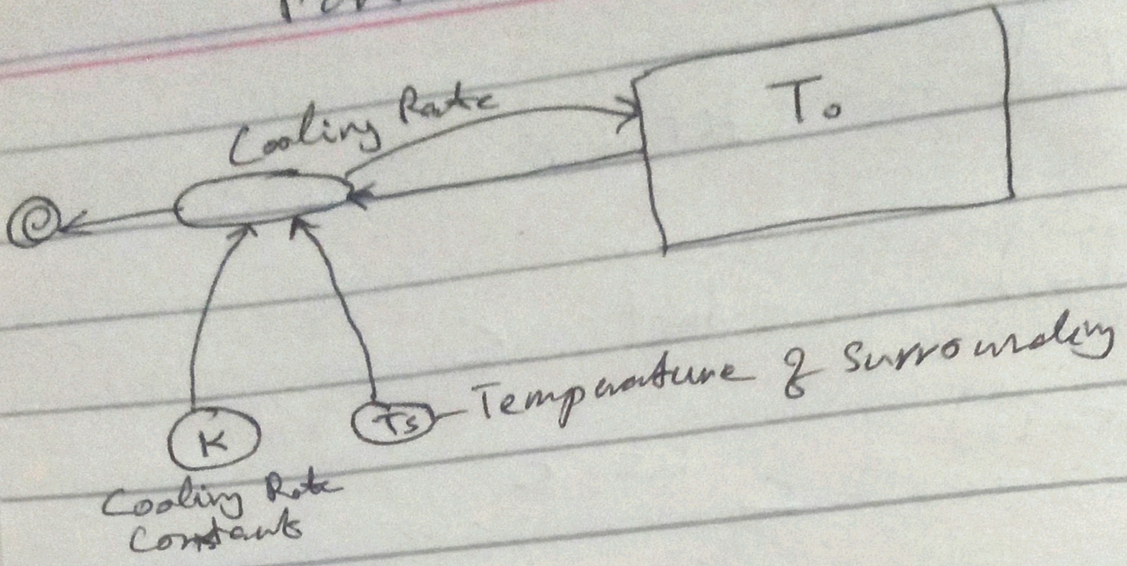
$$T_0 - 25 = e^{-kt-c}$$

$$T_0 = e^{-kt-c} + 25$$

$$T_0 = e^{-c} e^{-kt} + 25$$

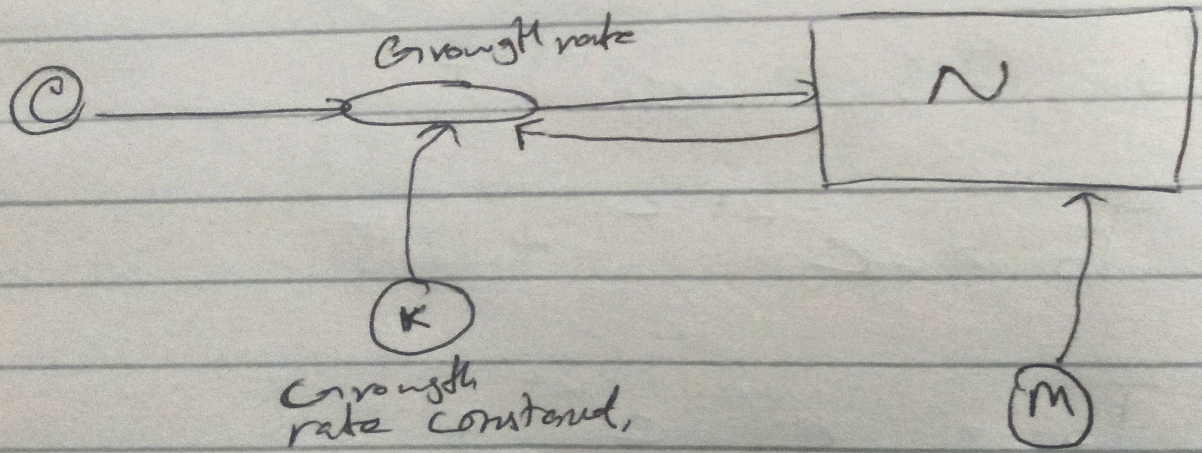
$A = e^{-c}$ $\qquad T_0 = Ae^{-kt} + 25$

Part (C)



Other answer is in the ipython notebook.

Question 2 Part (a)



Part (c)

$$\frac{dN}{dt} = kN \ln\left(\frac{M}{N}\right)$$

$u = \ln\left(\frac{M}{N}\right)$ $\quad\quad$ $u = \ln(M) - \ln(N)$

$\text{Anti}\ln(u) = M/N$

$N = \frac{M}{\text{Anti}\ln(u)}$

$$\frac{dN}{dt} = kN \ln\left(\frac{M}{N}\right)$$

$$\frac{dN}{dt} = kNu$$

$$\frac{d\,\text{Anti}\ln(u)}{dt} = k\,\text{Anti}\ln(u)\;u$$

$$\frac{d\,\text{Anti}\ln(u)}{\text{Anti}\ln(u)} = ku\,dt$$

$$\int \frac{d\,\text{Anti}\ln(u)}{\text{Anti}\ln(u)} = \int ku\,dt$$

$\ln(\text{Anti}\ln(u)) = kut + C$

$u = kut + C$

Question 3

$\frac{Q}{Q_0} = 20\%$

$\frac{Q}{Q_0} = 0.20$

We also know that half life = 573

$t_{1/2} = 5730$

$Q = Q_0 e^{-rt}$

$r = \frac{0.693}{t_{1/2}} = \frac{0.693}{5730}$

$r = 0.693/5730$

$$\frac{Q}{Q_0} = e^{-rt}$$

$$0.20 = e^{-rt}$$

$$\log(0.20) = \log(e^{-rt})$$

$$\cancel{0.20} =$$

$$-1.6094 = -rt$$

$$rt = 1.6094$$

$$t = \frac{1.6094}{r}$$

$$t = \frac{1.6094}{\frac{0.693}{5730}}$$

$$t = 13307.16$$

Question 4

$$\frac{dP}{dt} = r P(t)$$

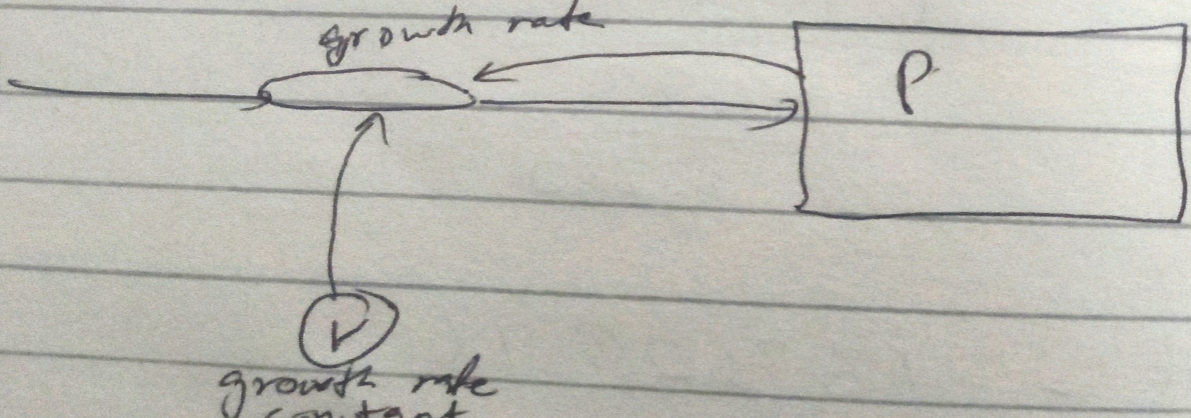
$$P(t) = P(t - \Delta t) + \Delta P$$

$$\frac{\Delta P}{\Delta t} = r P(t)$$

$$\Delta P = r P \Delta t$$

$$P(t) = P(t - \Delta t) + \Delta t \; r \; P(t+\Delta t)$$

(c)



$$\frac{dP}{dt} = \frac{dB}{dt} - \frac{dD}{dt}$$

$\frac{dD}{dt} = 0$ because no one is going to dead

$$\frac{dP}{dt} = \frac{dB}{dt}$$

$$rP = \frac{dB}{dt}$$

$$rP = 3.6\%$$

$$r = \frac{3.6\%}{10.00000} = \frac{36\cancel{00}\%}{10000\cancel{00}}$$

$$r = 0.0036$$

$$P(1) = 1 \times 10^6 + \Delta t\,(0.0036)(1 \times 10^6$$