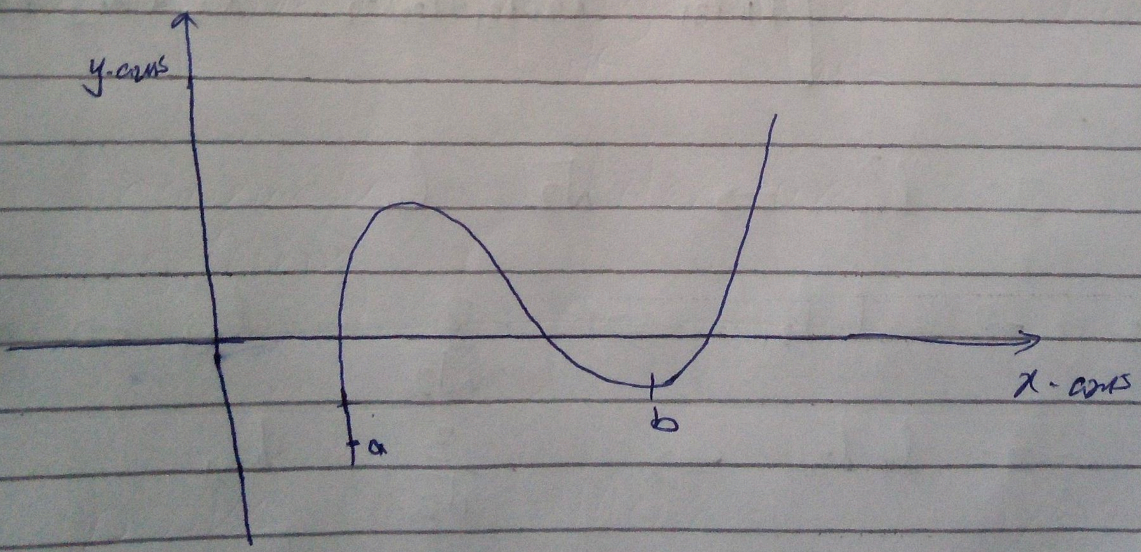
Question # 1

a)

Accordingly to intermediate theorem, if we consider a interval [a, b] if function value at 'a' is some negative real value and function value at 'b' is some positive real value then we say that in the given interval at some point there will be x-axis intercept which is root of the non-linear equation.

(b)



in interval [a, b] there equation have two roots,

Question # 3

(a)

$$A = \begin{bmatrix} 2 & -2 & -3 \\ -2 & 5 & 4 \\ -3 & 4 & 5 \end{bmatrix}$$

$$A = \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{21} & a_{22} & a_{32} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} l_{11} & l_{21} & l_{31} \\ 0 & l_{22} & l_{32} \\ 0 & 0 & l_{33} \end{bmatrix}$$

$$= \begin{bmatrix} l_{11}^2 & l_{11}l_{21} & l_{11}l_{31} \\ l_{11}l_{21} & l_{21}^2 + l_{22}^2 & l_{21}l_{31} + l_{22}l_{32} \\ l_{11}l_{31} & l_{21}l_{31} + l_{22}l_{32} & l_{31}^2 + l_{32}^2 + l_{33}^2 \end{bmatrix}$$

a_{11} : $2 = l_{11}^2 \Rightarrow \sqrt{2} = l_{11}$

a_{21} : $-2 = l_{11}l_{21} \Rightarrow \dfrac{-2}{\sqrt{2}} = l_{21} \Rightarrow -\sqrt{2} = l_{21}$

a_{31} : $-3 = l_{11}l_{31} \Rightarrow \dfrac{-3}{\sqrt{2}} = l_{31}$

a_{22} : $5 = l_{21}^2 + l_{22}^2 \Rightarrow 5 = 2 + l_{22}^2 \Rightarrow l_{22}^2 = 3$

$\Rightarrow l_{22} = \sqrt{3}$

a_{32} := $4 = l_{21}l_{31} + l_{22}l_{32} \Rightarrow 4 = -\cancel{\sqrt{2}}\left(\frac{-3}{\cancel{\sqrt{2}}}\right) + \sqrt{3}\,l_{32}$

$\Rightarrow \sqrt{3}\,l_{32} = 4 - 3 \Rightarrow l_{32} = \frac{1}{\sqrt{3}}$

a_{33} : $5 = l_{31}^2 + l_{32}^2 + l_{33}^2 \Rightarrow 5 = \left(-\frac{3}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2 + l_{33}^2$

$\Rightarrow 5 = \frac{9}{2} + \frac{1}{3} + l_{33}^2 \Rightarrow 5 = \frac{27+2}{6} + l_{33}^2$

$\Rightarrow 5 = \frac{29}{6} + l_{33}^2 \Rightarrow l_{33}^2 = 5 - \frac{29}{6}$

$\Rightarrow l_{33}^2 = \frac{30-29}{6} \Rightarrow l_{33}^2 = \frac{1}{6} \Rightarrow l_{33} = \frac{1}{\sqrt{6}}$

$$A = LL^t = \begin{bmatrix} \sqrt{2} & 0 & 0 \\ -\sqrt{2} & \sqrt{3} & 0 \\ -3/\sqrt{2} & 1/\sqrt{3} & 1/\sqrt{6} \end{bmatrix} \begin{bmatrix} \sqrt{2} & -\sqrt{2} & -3/\sqrt{2} \\ 0 & \sqrt{3} & 1/\sqrt{3} \\ 0 & 0 & 1/\sqrt{6} \end{bmatrix}$$

(b)

$Ly = b$

~~$y_1 = \sqrt{2}$~~

$\sqrt{2}\, y_1 = 1$

$\Rightarrow \quad y_1 = \frac{1}{\sqrt{2}}$

$-\sqrt{2}\, y_1 + \sqrt{3}\, y_2 = 2 \quad \Rightarrow \quad -\sqrt{2}\left(\frac{1}{\sqrt{2}}\right) + \sqrt{3}\, y_2 = 2$

$-1 + \sqrt{3}\, y_2 = 2 \quad \Rightarrow \quad y_2 = \frac{3}{\sqrt{3}}$

$y_2 = \sqrt{3}$

$-\frac{3}{\sqrt{2}}\, y_1 + \frac{1}{\sqrt{3}}\, y_2 + \frac{1}{\sqrt{6}}\, y_3 = 3$

$\Rightarrow \quad -\frac{3}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right) + \frac{1}{\sqrt{3}}\sqrt{3} + \frac{1}{\sqrt{6}}\, y_3 = 3$

$-\frac{3}{2} + 1 + \frac{1}{\sqrt{6}}\, y_3 = 3 \quad \Rightarrow \frac{1}{\sqrt{6}}\, y_3 = 2 + \frac{3}{2}$

$y_3 = 7\sqrt{6}/2$

$$y = \begin{bmatrix} 1/\sqrt{2} \\ \sqrt{3} \\ 7\sqrt{6}/2 \end{bmatrix}$$

$$L^t x = y$$

$$\sqrt{2}x_1 = \frac{1}{\sqrt{2}} \implies x_1 = \frac{1}{2} = 0.5$$

$$-\sqrt{2}\,x_1 + \sqrt{3}\,x_2 = \sqrt{3} \implies -\sqrt{2}\left(\frac{1}{2}\right) + \sqrt{3}\,x_2 = \sqrt{3}$$

$$\implies -\frac{1}{\sqrt{2}} + \sqrt{3}\,x_2 = \sqrt{3} \implies \sqrt{3}\,x_2 = \sqrt{3} + \frac{1}{\sqrt{2}}$$

$$\implies x_2 = \frac{\sqrt{3}}{\sqrt{3}} + \frac{1}{\sqrt{2}\sqrt{3}} \implies x_2 = 1 + \frac{1}{\sqrt{6}}$$

$$x_2 = \frac{\sqrt{6}+1}{\sqrt{6}} = 1.40$$

$$-\frac{3}{\sqrt{2}}x_1 + \frac{1}{\sqrt{3}}x_2 + \frac{1}{\sqrt{6}}x_3 = \frac{7\sqrt{6}}{2}$$

$$-\frac{3}{\sqrt{2}}\,\frac{1}{2} + \frac{1}{\sqrt{3}}\left(\frac{\sqrt{6}+1}{\sqrt{6}}\right) + \frac{1}{\sqrt{6}}x_3 = \frac{7\sqrt{6}}{2}$$

$$-\frac{3}{2\sqrt{2}} + \frac{\sqrt{6}+1}{3\sqrt{2}} + \frac{1}{\sqrt{6}}x_3 = \frac{7\sqrt{6}}{2}$$

$$x_3 = \sqrt{6}\left(\frac{3}{2\sqrt{2}} - \frac{\sqrt{6}+1}{3\sqrt{2}} + \frac{7\sqrt{6}}{2}\right)$$

$$= \sqrt{6}\,(1.06 - 1.53 + 8.57)$$

$$x_3 = \sqrt{6}\,(8.10)$$

$$x_3 = 19.84$$

$$x_2 \begin{bmatrix} 0.5 \\ 1.40 \\ 19.84 \end{bmatrix}$$

Question # 6

lagrange inverse interpolation

$$x = \frac{(y-y_1)(y-y_2)(y-y_3)}{(y_0-y_1)(y_0-y_2)(y_0-y_3)}x_0 + \frac{(y-y_0)(y-y_2)(y-y_3)}{(y_1-y_0)(y_1-y_2)(y_1-y_3)}x_1$$

$$+ x_2\frac{(y-y_0)(y-y_1)(y-y_3)}{(y_2-y_0)(y_2-y_1)(y_2-y_3)} + x_3\frac{(y-y_0)(y-y_1)(y-y_2)}{(y_3-y_0)(y_3-y_1)(y_3-y_2)}$$

$x_0 = 0.4846$ $x_1 = 0.4937$
$x_2 = 0.5027$ $x_3 = 0.5116$

$y_0 = 0.46$ $y_1 = 0.47$
$y_2 = 0.48$ $y_3 = 0.49$

$y = 0.50$ find x for y

$\cancel{x = \frac{(0.50-0.47)(0.50-0.48)(0.50-0.49)}{0.}}$

putting values in formula we have

$$x = \frac{(0.03)(0.02)(0.01)}{(-0.01)(-0.02)(-0.03)} 0.4846 + \frac{(0.04)(0.02)(0.01)}{(0.01)(-0.01)(0.02)}$$

$$0.4937 + \frac{(0.04)(0.03)(0.01)}{(0.02)(0.01)(-0.01)} 0.5027 + \frac{(0.04)(0.03)(0.02)}{(0.01)(0.02)(0.03)}$$

$$x = \frac{0.000006}{-0.000006} 0.4846 + \frac{0.000008}{0.000002} 0.4937$$

$$+ \frac{0.000012}{-0.000002} 0.5027 + \frac{0.000024}{0.000006} 0.5116$$

$\cancel{x = 556 - 2(0.4846) + 4(0.4937)}$

$x = -0.4846 + 1.9748 - 3.0162 + 2.0464$
$x = 0.5204$

Question # 5

Using Newton's Backward difference

x	y	∇	∇^2	∇^3	∇^4	∇^5
40	180					
50	204	24				
60	226	22	-2			
70	250	24	2	+4		
80	276	26	2	0	-4	
90	304	28	2	0	0	4
	y_5	∇y_5	$\nabla^2 y_5$	$\nabla^3 y_5$	$\nabla^4 y_5$	$\nabla^5 y_5$

$h = 10 \qquad n = 5 \qquad x_5 = x_n = 90$

$x = 84$

$$P = \frac{x - x_n}{h} = \frac{84 - 90}{10} = \frac{-6}{10} =$$

$P = -0.6$

$$y_5(x) = y_5 + P\nabla y_5 + \frac{P(P+1)}{2!}\nabla^2 y_5 + \frac{P(P+1)(P+2)}{3!}\nabla^3 y_5$$

$$+ \frac{P(P+1)(P+2)(P+3)}{4!}\nabla^4 y_5 + \frac{P(P+1)(P+2)(P+3)(P+4)}{5!}\nabla^5 y_5$$

$$= 304 + (-16.8) + \frac{(-0.24)}{2}2 + 0 + 0 + (-0.0913)$$

$$= 304 - 16.8 - 0.24 - 0.0913$$

$y_5(x) = 286.86$

Question # 4

$\frac{x}{y}$?

(a)

$$\left[\begin{array}{ccc|c} \pi & -e & \sqrt{2} & \sqrt{11} \\ \pi & e & -e^2 & 0 \\ \sqrt{5} & -\sqrt{6} & 1 & \pi \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 3.14 & -2.72 & 1.41 & 3.32 \\ 3.14 & 2.72 & -7.39 & 0 \\ 2.23 & 2.45 & 1 & 3.14 \end{array}\right]$$

$(E_2 - E_1) \rightarrow E_2$; ~~3.32~~

$$\left[\begin{array}{ccc|c} 3.14 & -2.72 & 1.41 & 3.32 \\ 0 & 5.44 & -8.80 & -3.32 \\ 2.23 & -2.45 & 1 & 3.14 \end{array}\right]$$

~~$(E_3 - 1.41 E_1) \rightarrow E_3$~~

$(E_3 - 0.71\,E_1) \rightarrow E_3$

$$\left[\begin{array}{ccc|c} 3.14 & -2.72 & 1.41 & 3.32 \\ 0 & 5.44 & -8.80 & -3.32 \\ 0 & -0.520 & 0.001 & 0.7828 \end{array}\right]$$

$(E_3 + 0.095\,E_2) \rightarrow E_3$

$$\left[\begin{array}{ccc|c} 3.14 & -2.72 & 1.41 & 3.32 \\ 0 & 5.44 & -8.80 & -3.32 \\ 0 & 0 & -0.835 & 0.468 \end{array}\right]$$

$-0.835x_3 = 0.468$

$x_3 = -\dfrac{0.468}{0.835} = -0.560$

$5.44x_2 - 8.80(x_3) = -3.32$

$5.44x_2 + 4.93 = -3.32$

$5.44x_2 = -3.32 - 4.93$

$x_2 = -\dfrac{8.25}{5.44}$

$x_2 = -1.51$

$$3.14x_1 - 2.72x_2 + 1.41x_3 = 3.32$$

$$3.14x_1 + 4.10 + (-0.790) = 3.32$$

$$3.14x_1 = 3.32 - 4.10 + 0.790$$

$$x_1 = \frac{0.01}{3.14}$$

$$x_1 = 0.003$$

$$[0.003, -1.51, -0.560]$$

(b)

We use Gauss-Seidal Method Scheme

$$x_1 = +\frac{e x_2}{\pi} - \frac{\sqrt{2}}{\pi} + \frac{\sqrt{11}}{\pi}$$

$$x_2 = -\frac{\pi^2}{e} x_1 + \frac{e^2}{e} x_3.$$

$$x_3 = -\sqrt{5}x_1 + \sqrt{6}x_2$$

this is the scheme

when we ~~foun~~ start from (0, 0, 0) then on next iteration we we previous value that is founded in the iteration.

Question 8

$x(0) = 0.5 \qquad y(0) = 0.2$

$n = 0$

$$x_1 = x_0 + h x_0' + \frac{h^2 x''}{2!} + h^3 \frac{x'''}{3!} \quad \text{--- (1)}$$

$$y_1 = y_0 + h y_0' + \frac{h^2 y_0''}{2!} + \frac{h^3 y'''}{3!} \quad \text{--- (2)}$$

$x_0 = 0.5 \qquad y_0 = 0.2 \qquad t_0 = 0$

$x_0' = 6x_0 + y_0 + 6t_0$ | $y_0' = 6.6$

$x_0' = 6(0.5) + (0.2) + 6(0)$

$x_0' = 3.2$

$x_0'' = 31.8$ $\qquad$ $y_0'' = 22.6$

$x_0''' = 213$ $\qquad$ $y_0''' = 195$

putting value of x_0, x_0', x_0'', x_0''' and h in eq ①

$$x_1 = 0.5 + 0.1(3.2) + \frac{(0.1)^2}{2!} 31.8 + \frac{(0.1)^3}{3!} 213$$

$$x_1 = 1.0145$$

$$y_1 = 0.2 + 0.1(6.6) + \frac{(0.1)^2}{2!}(22.6) + \frac{(0.1)^3}{3!} 195$$

$$y_1 = 1.006$$

For $n = 1$

$$x_2 = x_1 + h x_1' + \frac{h^2}{2!} x_1'' + \frac{h^3}{3!} x^3 \quad -③$$

$$y_2 = y_1 + h y_1' + \frac{h^2}{2!} y'' + \frac{h^3}{3!} y''' \quad -④$$

$x_1 = 1.0148$ $\qquad y_1 = 1.006$

$x_1' = 7.69$ $\qquad y_1' = 10.08$

$x_1'' = 62.22$ $\qquad y_1'' = 51.00$

$x_1''' = 424.32$ $\qquad y_1''' = 401.88$

putting the value of x, x_1', x_1'', x_1''' and h in eq (3)

$$x_2 = x_1 + h x_1' + \frac{h^2 x_1''}{2!} + \frac{h^3 x^3}{3!}$$

$$x_2 = 1.014 + 0.1(7.69) + \frac{(0.1)^2\, 62.22}{2!} + \frac{(0.1)^3\, 422}{3!}$$

$$x_2 = 2.156$$

$$y_2 = 1.006 + 0.1(10.08) + \frac{(0.1)^2\, 51}{2!} + \frac{(0.1)^3\, 401.88}{3!}$$

$$y_2 = 2.4175$$

$x(0.2) = 2.156$ $\qquad y(0.2) = 2.417$