Jacobi's Method Sol. Hints

~~Eqn~~ Eq 1: $10x_1 - x_2 + 2x_3 = 6$

$$\Rightarrow x_1 = \frac{x_2 - 2x_3 + 6}{10}$$

$$\Rightarrow \boxed{x_1 = \frac{1}{10}x_2 - \frac{1}{5}x_3 + \frac{3}{5}}$$

~~Eqn~~ Eq 2: $-x_1 + 11x_2 - x_3 + 3x_4 = 25$

$$\Rightarrow x_2 = \frac{1}{11}(x_1 + x_3 - 3x_4 + 25)$$

$$\Rightarrow \boxed{x_2 = \frac{1}{11}x_1 + \frac{1}{11}x_3 - \frac{3}{11}x_4 + \frac{25}{11}}$$

Similarly from Eq: 3 we ~~get~~ have

$$x_3 = \frac{1}{10}(-2x_1 + x_2 + x_4 - 11)$$

$$\boxed{x_3 = -\frac{1}{5}x_1 + \frac{1}{10}x_2 + \frac{1}{10}x_4 - \frac{11}{10}}$$

and Finally

$$x_4 = \frac{1}{8}(-3x_2 + x_3 + 15)$$

$$\Rightarrow \boxed{x_4 = -\frac{3}{8}x_2 + \frac{1}{8}x_3 + \frac{15}{8}}$$

$$x_1^{(k)} = \frac{1}{10}x_2^{(k-1)} - \frac{1}{5}x_3^{(k-1)} + \frac{3}{5}$$

$$x_2^{(k)} = \frac{1}{11}x_1^{(k-1)} + \frac{1}{11}x_3 - \frac{3}{11}x_4^{(k-1)} + \frac{25}{11} \qquad k \geq 1$$

$$x_3^{(k)} = -\frac{1}{5}x_1^{(k-1)} + \frac{1}{10}x_2^{(k-1)} - \frac{11}{10}$$

$$x_4^{(k)} = -\frac{3}{8}x_2^{(k-1)} + \frac{1}{8}x_3^{(k-1)} + \frac{15}{8}$$

Scheme

Given Initial Approximation

$\vec{x}^{(0)} = (0,0,0,0)^t$ compare with $\vec{x}^{(0)} = (x_1^{(0)}, x_2^{(0)}, x_3^{(0)}, x_4^{(0)})^t$

we get $\boxed{x_1^{(0)} = 0, x_2^{(0)}, x_3^{(0)} = 0, x_4^{(0)} = 0}$ (This will be used in calculation)

For $k=1$ ★ $\Rightarrow$

$$x_1^{(1)} = \frac{1}{10}x_2^{(0)} - \frac{1}{5}x_3^{(0)} + \frac{3}{5}$$

$$x_2^{(1)} = \frac{1}{11}x_1^{(0)} + \frac{1}{11}x_3^{(0)} - \frac{3}{11}x_4^{(0)} + \frac{25}{11}$$

$$x_3^{(1)} = -\frac{1}{5}x_1^{(0)} + \frac{1}{10}x_2^{(0)} - \frac{11}{10}$$

$$x_4^{(1)} = -\frac{3}{8}x_2^{(0)} + \frac{1}{8}x_3^{(0)} + \frac{15}{8}$$

Substituting values we get

$x_1^{(1)} = 3/5$, $x_2^{(1)} = 25/11$, $x_3^{(1)} = -11/10$, $x_4^{(1)} = 15/8$

So $\vec{x}^{(1)} = (x_1^{(1)}, x_2^{(1)}, x_3^{(1)}, x_4^{(1)}) = (3/5, 25/11, -11/10, 15/8)$

OR

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$$\boxed{\vec{x}^{(1)} = (0.6, 2.2727, -1.1, 1.875)^t}$$ 1st Approx.

So $x_1^{(1)} = 0.6$, $x_2^{(1)} = 2.2727$, $x_3^{(1)} = -1.1$ and $x_4^{(1)} = 1.875$

Given stopping Criteria is:

$$\frac{\|\vec{x}^{(K)} - \vec{x}^{(K-1)}\|_\infty}{\|\vec{x}^{(K)}\|_\infty} < 10^{-3} = 0.001$$ ~~{ If True then stop otherwise continue }~~

For $K=1$

$$\frac{\|\vec{x}^{(1)} - \vec{x}^{(0)}\|_\infty}{\|\vec{x}^{(1)}\|_\infty} < 10^{-3}$$

(If True then stop otherwise calculate $\vec{x}^{(2)}$ and then check....)

Now

$$\|\vec{x}^{(1)} - \vec{x}^{(0)}\|_\infty$$

$$= \max\left\{|x_1^{(1)} - x_1^{(0)}|, |x_2^{(1)} - x_2^{(0)}|, |x_3^{(1)} - x_3^{(0)}|, |x_4^{(1)} - x_4^{(0)}|\right\}$$

$$= \max\left\{|0.6 - 0|, |2.2727 - 0|, |-1.1 - 0|, |1.875 - 0|\right\}$$

$$= \max\left\{0.6, 2.2727, +1.1, 1.875\right\}$$

$$= 2.2727$$

and $\|\vec{x}^{(1)}\|_\infty = \max\left\{|0.6|, |2.2727|, |-1.1|, |1.875|\right\}$

$$= 2.2727$$

Now since

$$\frac{\|\vec{x}^{(1)} - \vec{x}^{(0)}\|_\infty}{\|\vec{x}^{(1)}\|_\infty} = \frac{2.2727}{2.2727} = 1 < 0.001 \text{ (Not True)}$$

So we will calculate $\vec{x}^{(2)}$.

Now to find $\vec{x}^{(2)}$ we will put $K=2$ in ~~eq~~ $(\star)$ we get

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$$x_1^{(2)} = \frac{1}{10}x_2^{(1)} - \frac{1}{5}x_3^{(1)} + \frac{3}{5}$$

$$x_2^{(2)} = \frac{1}{11}x_1^{(1)} + \frac{1}{11}x_3^{(1)} - \frac{3}{11}x_4^{(1)} + \frac{25}{11}$$

$$x_3^{(2)} = -\frac{1}{5}x_1^{(1)} + \frac{1}{10}x_2^{(1)} - \frac{11}{10}$$

$$x_4^{(2)} = -\frac{3}{8}x_2^{(1)} + \frac{1}{8}x_3^{(1)} + \frac{15}{8}$$

Substituting values we get,

$$x_1^{(2)} = \frac{1}{10}(2.2727) - \frac{1}{5}(-1.1) + \frac{3}{5} = 1.04727$$

$$x_2^{(2)} = \frac{1}{11}(0.6) + \frac{1}{11}(-1.1) - \frac{3}{11}(1.875) + \frac{25}{11} = 1.7159$$

$$x_3^{(2)} = -0.8052$$

$$x_4^{(2)} = 0.8852$$

So $x_2^{(2)} = (1.04727, 1.7159, -0.8052, 0.8852)^t$

Now check $\frac{\|\vec{x}^{(2)} - \vec{x}^{(1)}\|_\infty}{\|\vec{x}^{(2)}\|_\infty} < 10^{-3}$ (If True then stop otherwise continue and calculate $\vec{x}^{(3)}$)