We Know that

Equation of the line passing through the points (x_0, y_0) and (x_1, y_1) is :

$$\boxed{\frac{y-y_0}{y_1-y_0} = \frac{x-x_0}{x_1-x_0} \quad \text{——— (i)}}$$

We can write (i) in another nice form. Let's see.

From (i) we have

$$y - y_0 = \left(\frac{x-x_0}{x_1-x_0}\right)(y_1 - y_0)$$

$$\Rightarrow y = y_0 + \left(\frac{x-x_0}{x_1-x_0}\right)y_1 - \left(\frac{x-x_0}{x_1-x_0}\right)y_0$$

$$= y_0 + \left(\frac{x-x_0}{x_1-x_0}\right)y_1 + \left(\frac{x-x_0}{x_0-x_1}\right)y_0$$

$$\Rightarrow y = \left(1 + \frac{x-x_0}{x_0-x_1}\right)y_0 + \left(\frac{x-x_0}{x_1-x_0}\right)y_1$$

$$\Rightarrow y = \left(\frac{\cancel{x_0} - x_1 + x - \cancel{x_0}}{x_0 - x_1}\right) y_0 + \frac{x - x_0}{x_1 - x_0} y_1$$

$$\Rightarrow y = \frac{x - x_1}{x_0 - x_1} y_0 + \frac{x - x_0}{x_1 - x_0} y_1$$

$$= l_0(x) y_0 + l_1(x) y_1$$, where $l_0(x) = \frac{x - x_1}{x_0 - x_1}$

and $l_1(x) = \frac{x - x_0}{x_1 - x_0}$

$$\boxed{\begin{gathered} \Rightarrow y = \sum_{i=0}^{1} l_i(x) y_i \\ \text{where } l_0(x) = \frac{x - x_1}{x_0 - x_1} \text{ and} \\ l_1(x) = \frac{x - x_0}{x_1 - x_0} \end{gathered}}$$

<u>Note that</u> $l_0(x_0) = \frac{x_0 - x_1}{x_0 - x_1} = 1$

and $l_0(x_1) = \frac{x_1 - x_1}{x_0 - x_1} = 0$

Also

$l_1(x_0) = 0$ and $l_1(x_1) = 1$.

$$\sum_{i=0}^{1} l_i(x) = l_0(x) + l_1(x) = \frac{x-x_0}{x_0-x_1} + \frac{x-x_0}{x_1-x_0}$$

$$= \frac{x-x_0}{x_0-x_1} - \frac{(x-x_0)}{x_0-x_1}$$

$$= \frac{x-x_0-(x-x_0)}{x_0-x_1}$$

$$= \frac{\cancel{x}-x_0-\cancel{x}+x_0}{x_0-x_1}$$

$$= \frac{x_0-x_1}{x_0-x_1}$$

$$= 1$$