Assignment # 2

Question # 1

$$E_1: \quad \pi x_1 + \sqrt{2}\, x_2 - x_3 + x_4 = 0$$

$$E_2: \quad e x_1 - x_2 + x_3 + 2x_4 = 1$$

$$E_3: \quad x_1 + x_2 - \sqrt{3}\, x_3 + x_4 = 2$$

$$E_4: \quad -x_1 - x_2 + x_3 - \sqrt{5}\, x_4 = 3$$

$$\left[\begin{array}{cccc:c} 3.14 & 1.41 & -1 & 1 & 0 \\ 2.71 & -1 & 1 & 2 & 1 \\ 1 & 1 & -1.73 & 1 & 2 \\ -1 & -1 & 1 & -2.23 & 3 \end{array}\right]$$

$(E_2 - 0.863E_1) \longrightarrow E_2$, $(E_3 - 0.318E_1) \longrightarrow E_3$

$(E_4 + 0.318E_1) \longrightarrow E_4$

$$\left[\begin{array}{cccc:c} 3.14 & 1.41 & -1 & 1 & 0 \\ 0 & -2.22 & 1.86 & 1.14 & 1 \\ 0 & 0.551 & -1.41 & 0.682 & 2 \\ 0 & -0.551 & -0.682 & -1.91 & 3 \end{array}\right]$$

$(E_3 + 0.248E_2) \longrightarrow E_3$, $(E_4 - 0.248E_2) \longrightarrow E_4$

$$\left[\begin{array}{cccc|c} 3.14 & 1.41 & -1 & 1 & 0 \\ 0 & -2.22 & 1.86 & 1.13 & 1 \\ 0 & 0 & -0.953 & 0.962 & 2.24 \\ 0 & 0 & 0.221 & -2.19 & 2.57 \end{array}\right]$$

$$(E_4 + 0.228 E_3) \longrightarrow E_4$$

$$\left[\begin{array}{cccc|c} 3.14 & 1.41 & -1 & 1 & 0 \\ 0 & -2.22 & 1.86 & 1.13 & 1 \\ 0 & 0 & -0.953 & 0.862 & 2.24 \\ 0 & 0 & 0 & -1.97 & 3.27 \end{array}\right]$$

Using E_4

$$-1.97 x_4 = 3.27$$

$$x_4 = -1.65$$

Using E_3

$$-0.953 x_3 + 0.862(-1.65) = 2.47$$

$$x_3 = -4.02$$

Using E_2

$$-2.22 x_2 + 1.86(-4.02) + 1.13(-1.65)$$

$$x_2 = -4.67$$

Using E_4

$$3.14 x_1 + 1.41(-4.67) - (-40.02) + (-1.65) = 0$$

$$x_1 = 1.34$$

$$[1.34, -4.67, -4.02, -1.65]$$

Question 2

$$\left[\begin{array}{cccc:c} 3.14 & 1.41 & -1 & 1 & 0 \\ 2.71 & -1 & 1 & 2 & 1 \\ 1 & 1 & -1.73 & 1 & 2 \\ -1 & -1 & 1 & -2.23 & 3 \end{array}\right]$$

$a_{11} = 3.14 \quad a_{21} = 2.71 \quad a_{31} = 1 \quad a_{41} = -1$

$\max\{|a_{11}|, |a_{21}|, |a_{31}|, |a_{41}|\}$

$\max(a_{11}) = 3.14$

$m_{21} = \frac{2.71}{3.14} = 0.863 \qquad m_{31} = \frac{1}{3.14} = 0.318$

$m_{41} = \frac{1}{3.14} = 0.318$

$(E_2 - 0.863E_1) \rightarrow E_2, \quad (E_3 - 0.318E_1) \rightarrow E_3$

$(E_4 - 0.318E_1) \rightarrow E_4$

$$\left[\begin{array}{cccc:c} 3.14 & 1.41 & -1 & 1 & 0 \\ 0 & -2.22 & 1.87 & 1.14 & 1 \\ 0 & 0.552 & -1.41 & 0.682 & 2 \\ 0 & -0.552 & 0.682 & -1.91 & 3 \end{array}\right]$$

$\max(|a_{22}|, |a_{32}|, |a_{42}|)$

$\max\{a_{22}\} = 2.22$

$m_{32} = \frac{0.552}{2.22} = 0.249 \qquad m_{42} = \frac{0.552}{2.22} = 0.249$

$(E_3 + 0.249E_2) \rightarrow E_3 \qquad (E_4 - 0.249E_2) \rightarrow E_4$

$$\left[\begin{array}{cccc|c} 3.14 & 1.41 & -1 & 1 & 0 \\ 0 & -2.22 & 1.87 & 1.14 & 1 \\ 0 & 0 & -0.944 & 0.966 & 2.25 \\ 0 & 0 & 0.216 & -2.19 & 2.75 \end{array}\right]$$

$$\max\left[|a_{33}|, |a_{43}|\right] \Rightarrow \max(a_{33}) = 0.944$$

$$m_{43} = \frac{0.216}{0.944} = 0.229$$

$$0.229E_3 + E_4 \longrightarrow E_4$$

$$\left[\begin{array}{cccc|c} 3.14 & 1.41 & -1 & 1 & 0 \\ 0 & -2.22 & 1.87 & 1.14 & 1 \\ 0 & 0 & -0.944 & 0.966 & 2.25 \\ 0 & 0 & 0 & -1.97 & 3.26 \end{array}\right]$$

Using E_4

$$-1.97\, x_4 = 3.26$$

$$x_4 = -1.65$$

Using E_3

$$-0.944\, x_3 + 0.966(-1.65) = 2.25$$

$$x_3 = -4.07$$

Using E_2

$$-2.22\, x_2 + 1.87(-4.07) + 1.14(-1.65) = 1$$

$$x_2 = -4.72$$

Using E_1

$3.14x_1 + 1.41(-4.72) - (-4.07) + (-1.65) = 0$

$x_1 = 1.34$

$$[1.34, -4.72, -4.07, -1.65]$$

Question 3

$$\left[\begin{array}{cccc|c} 3.14 & 1.41 & -1 & 1 & 0 \\ 2.71 & -1 & 1 & 2 & 2 \\ 1 & 1 & -1.73 & 1 & 3 \\ -1 & -1 & 1 & -2.23 & 3 \end{array}\right]$$

$S_1 = 3.14 \quad S_2 = 2.71 \quad S_3 = 1.73 \quad S_4 = 2.23$

$\max\left(\frac{3.14}{3.14}, \frac{2.71}{2.71}, \frac{1}{1.73}, \frac{1}{2.23}\right)$

$\max(a_{ij}) = 3.14$

$(E_2 - 0.863E_1) \rightarrow E_2 \quad (E_3 - 0.318E_1) \rightarrow E_3$

$(E_4 + 0.318E_1) \rightarrow E_4$

$$\left[\begin{array}{cccc|c} 3.14 & 1.41 & -1 & 1 & 0 \\ 0 & -2.22 & 1.87 & 1.14 & 1 \\ 0 & 0.552 & -1.41 & 0.682 & 2 \\ 0 & -0.552 & 0.682 & -1.91 & 3 \end{array}\right]$$

$S_1 = 2.22 \qquad S_2 = 1.41 \qquad S_3 = 1.91$

$$\max\left(\frac{2.22}{2.22}, \frac{0.552}{1.41}, \frac{0.552}{1.91}\right)$$

$$\max(a_{22}) = 2.22$$

$$(E_3 + 0.249\,E_2) \rightarrow E_3 \qquad (E_4 - 0.249\,E_2) \rightarrow E_4$$

$$\left[\begin{array}{cccc|c} 3.14 & 1.41 & -1 & 1 & 0 \\ 0 & -2.22 & 1.87 & 1.14 & 1 \\ 0 & 0 & -0.944 & 0.966 & 2.25 \\ 0 & 0 & 0.216 & -2.19 & 2.75 \end{array}\right]$$

$S_1 = 0.966 \qquad S_2 = 2.19$

$$\max\left(\frac{0.944}{0.966}, \frac{0.216}{2.19}\right)$$

$$\max(a_{33}) = 0.944$$

$$(E_4 + 0.229\,E_3) \longrightarrow E_4$$

$$\left[\begin{array}{cccc|c} 3.14 & 1.41 & -1 & 1 & 0 \\ 0 & -2.22 & 1.87 & 1.14 & 1 \\ 0 & 0 & -0.944 & 0.966 & 2.25 \\ 0 & 0 & 0 & -1.97 & 3.26 \end{array}\right]$$

Using E_4

$-1.97 x_4 = 3.26$

$x_4 = -1.65$

Using E_3

$-0.944 x_3 + 0.966(-1.65) = 2.25$

$x_3 = -4.07$

Using E_2

$-2.22 x_2 + 1.87(-4.07) + 1.14(-1.65) = 1$

$x_2 = -4.72$

Using E_1

$3.14 x_1 + 1.41(-4.72) - (-4.07) + (-1.65) =$

$x_1 = 1.34$

$$[1.34, -4.72, -4.07, -1.65]$$