Assignment # 3

Question # 1

$x' = -y + t \quad ; \quad x(0) = -3$

$y' = x - t \quad ; \quad y(0) = 5$

$h = 0.1$ approximate $x(0.2)$ & $y(0.2)$

formula:

$$x_{n+1} = x_n + h x'_n + \frac{h^2 x''_n}{2!} + \frac{h^3 x'''_n}{3!}$$

$$y_{n+1} = y_n + h y'_n + \frac{h^2 y''_n}{2!} + \frac{h^3 y'''_n}{3!}$$

when $n=0$

$$x_1 = x_0 + h x'_0 + \frac{h^2 x''_0}{2!} + \frac{h^3 x'''_n}{3!} \quad \text{—— } ①$$

$$y_1 = y_0 + h y'_0 + \frac{h^2 y''_0}{2!} + \frac{h^3 y'''_n}{3!} \quad \text{—— } ②$$

$x_0 = -3 \,,\; y_0 = 5 \quad t_0 = 0$

$x'_0 = -y_0 + t_0 = -5$

$y'_0 = x_0 - t_0 = -3$

$x''_0 = -y'_0 + 1 = 4$

$y''_0 = x'_0 - 1 = -6$

$y''' = x''_0 = 4$

$x'''_0 = -y''_0 = 6$

putting x_0 , x_0' , x_0'' , x_0''' & h in eq (1)

$$x_1 = -3 + 0.1(-5) + \frac{(0.1)^2(4)}{2!} + \frac{(0.1)^3 6}{3!}$$

$$x_1 = -3.479$$

putting y_0 , y_0' , y_0'' , y_0''' & h in eq (2)

$$y_1 = 5 + 0.1(-3) + \frac{(0.1)^2(-6)}{2!} + \frac{(0.1)^3 4}{3!}$$

$$y_1 = 4.6706$$

for $n = 1$

$$x_2 = x_1 + h x_1' + \frac{h^2 x_1''}{2!} + \frac{h^3 x_1'''}{3!} \quad \text{--- (3)}$$

$$y_2 = y_1 + h y_1' + \frac{h^2 y_1''}{2!} + \frac{h^3 y_1'''}{3!} \quad \text{--- (4)}$$

$x_1 = -3.479$ $\quad y_1 = 4.6706$ $\quad t_1 = 0.1$

$x_1' = -y_1 + t_1 = -4.5706$ $\quad y_1' = x_1 + t_1 = -3.579$

$x_1'' = -y_1' + 1 = 4.579$ $\quad y_1'' = x_1' + 1 = -5.5706$

$x_1''' = -y_1'' = 5.5706$ $\quad y_1''' = x_1'' = 4.579$

putting x_1, x_1', x_1'', x_1''' & h in eq (3)

$$x_2 = -3.479 + 0.1(-4.5706) + \frac{(0.1)^2}{2!}(4.579) + \frac{(0.1)^3}{3!}(5.5706)$$

$$x_2 = -3.9122$$

putting y_1, y_1', y_1'', y_1''' & h in eq (4)

$$y_2 = 4.6706 + 0.1(-3.579) + \frac{0.1}{2!}(-5.5706) + \frac{(0.1)^3}{3!}(4.579)$$

$$y_2 = 4.2894$$

$$\boxed{x(0.2) = -3.9122 \quad y(0.2) = 4.2894}$$

is the required answers.

Question # 2

$y' = 2x - 3y + 1$; $y(1) = 5$; $h = 0.1$

find $y(1.5) = ?$

For $n=0$ $x_0 = 1$ $y_0 = 5$

$$y_1 = y_0 + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4) \quad \text{— (1)}$$

$$k_1 = f(x_0, y_0) = 2x_0 - 3y_0 + 1 = -12$$

$$k_2 = f\left(x_0 + \frac{h}{2}, y_0 + \frac{hk_1}{2}\right) = f(1.05, 4.4) = -10.4$$

$$k_3 = f\left(x_0 + \frac{h}{2} + y_0 + \frac{hk_2}{2}\right) = f(1.05, 4.48) = -10.34$$

$K_4 = f(x_0 + h, y_0 + hk_3) = f(1.1, 3.966) = -8.698$

putting k_1, k_2, k_3, k_4 & y_0 in eq ①

$y_1 = 5 + \frac{0.1}{6}[-12 + 2(-10.4) + 2(-10.34) - 8.698]$

$y_1 = 3.9637$

for n = 1 $x_1 = 1.1$ $y_1 = 3.9637$

$y_2 = y_1 + \frac{h}{6}[k_1 + 2k_2 + 2k_3 + k_4]$ — ②

$k_1 = f(x_1, y_1) = -8.6911$

$k_2 = f(x_1 + \frac{h}{2}, y_1 + \frac{h}{2}k_1) = f(1.15, 3.529) = -7.287$

$k_3 = f(x_1 + \frac{h}{2}, y_1 + \frac{h}{2}k_2) = f(1.15, 3.5993) = -7.4980$

$k_4 = f(x_1 + h, y_1 + hk_3) = f(1.1, 3.2138) = -6.2416$

putting k_1 k_2 k_3 k_4 & h in eq ②

$y_2 = 3.9637 + \frac{0.1}{6}[-8.6911 + 2(-7.287) + 2(-7.4980) - 6.2416]$

$y_2 = 3.222$

For $n=2$ $x_2 = 1.2$ $y_2 = 3.222$

$$y_3 = y_2 + \frac{h}{6}\left[k_1 + 2k_2 + 2k_3 + k_4\right] \quad \text{—— (3)}$$

$k_1 = f(x_2, y_2) = f(1.2, 3.222) = -6.2661$.

$k_2 = f(x_2 + \frac{h}{2}, y_2 + \frac{hk_1}{2}) = f(1.25, 2.9087) = -5.2261$

$k_3 = f(x_2 + \frac{h}{2}, y_2 + \frac{hk_2}{2}) = f(1.25, 2.9606) = -5.3820$

$k_4 = f(x_2 + h, y_2 + hk_3) = f(1.3, 2.6837) = -4.4513$

putting k_1, k_2 k_3, k_4 & h in eq (3)

$$y_3 = 3.222 + \frac{0.1}{6}\left[-6.2661 + 2(-5.2261) + 2(-5.3820) - 4.4513\right]$$

$y_3 = 2.6897$

for $n=3$ $x_3 = 1.3$ $y_3 = 2.6897$

$$y_4 = y_3 + \frac{h}{6}\left[k_1 + 2k_2 + 2k_3 + k_4\right] \quad \text{—— (4)}$$

$k_1 = f(x_3, y_3) = f(1.3, 2.6897) = -4.4691$

$k_2 = f(x_3 + \frac{h}{2}, y_3 + \frac{hk_1}{2}) = f(1.35, 2.466) = -3.698$

$k_3 = f(x_3 + \frac{h}{2}, y_3 + \frac{hk_2}{2}) = f(1.35, 2.5048) = -3.8144$

$k_4 = f(x_3 + h, y_3 + hk_3) = f(1.4, 2.3082) = -3.1247$

putting k_1 k_2 k_3 k_4 & h in eq (4)

$$y_4 = 2.6897 + \frac{0.1}{6}\left[-4.4691 + 2(-3.698) + 2(-3.8144) - 3.1247\right]$$

$$y_4 = 2.3127$$

For $n=4$ $\quad x_4 = 1.4 \quad y_4 = 2.3127$

$$k_1 = f(x_4, y_4) = f(1.4, 2.3127) = -3.1381$$

$$k_2 = f(x_4 + \frac{h}{2}, y_4 + \frac{hk}{2}) = f(1.45, 2.1557) = -2.5673$$

$$k_3 = f(x_4 + \frac{h}{2}, y_4 + \frac{hk}{2}) = f(1.45, 2.1843) = -2.6529$$

$$k_4 = f(x_4 + h, y_4 + hk) = f(1.5, 2.0474) = -2.1422$$

putting k_1, k_2, k_3, k_4 & h in eq

$$y_5 = y_4 + \frac{h}{6}\left[k_1 + 2k_2 + 2k_3 + k_4\right]$$

$$= 2.3127 + \frac{0.1}{6}\left[-3.1381 + 2(-2.5673) + 2(-2.6529) - 2.1422\right]$$

$$y_5 = 2.050$$

$\boxed{y(1.5) = 2.050}$ is the required answers

Question # 3

$$\int_0^6 f(x)\, dx$$

Formula:

$$\int_{x_0}^{x_n} y\, dx = \frac{h}{2}\left[y_0 + 2(y_1 + y_2 \cdots + y_{n-1}) + y_n\right]$$

$x_0 = 0$ $x_1 = 1$ $x_2 = 2$ $x_3 = 3$ $x_4 = 5$

$x_6 = 6$

$y_0 = 1.56$ $y_1 = 3.64$ $y_2 = 4.62$ $y_3 = 5.12$

$y_4 = 7.05$ $y_5 = 9.22$ $y_6 = 10.44$

$$\int_0^6 y\, dx = \frac{h}{2}\left[y_0 + 2(y_1 + y_2 + y_3 + y_4 + y_5) + y_6\right]$$

$h = 1$

$$= \frac{1}{2}\left[1.56 + 2(3.64 + 4.62 + 5.12 + 7.05 + 9.22) + 10.44\right]$$

$$= \frac{1}{2}(59.3 + 12)$$

$$\boxed{\int_0^6 f(x)\, dx = 35.65}$$ Answer

Question # 4

$$f(\phi) = \int_0^{\phi} \frac{dt}{1-\frac{1}{2}\sin^2 t} \qquad y = 0.3887 = f(\phi)$$

find $\phi = ?$

Formula:

lagrange inverse interpolation

$$x = \frac{(y-y_1)(y-y_2)}{(y_0-y_1)(y_0-y_2)} x_0 + \frac{(y-y_0)(y-y_2)}{(y_1-y_0)(y_1-y_2} x_1$$

$$+ \frac{(y-y_0)(y-y_1)}{(y_2-y_0)(y_2-y_0)} x_2$$

$x_0 = 21 \qquad x_1 = 23 \qquad x_2 = 25$

$y_0 = 0.3706 \qquad y_1 = 0.4068 \qquad y_2 = 0.433$

$y = 0.3887$

$$x = \frac{(0.3887-0.4068)(0.3887-0.4433)}{(0.3706-0.4068)(0.3706-0.4433)} 21$$

$$+ \frac{(0.3887-0.3706)(0.3887-0.4433)}{(0.4068-0.3706)(0.4068-0.4433)} 23$$

$$+ \frac{(0.3887-0.3706)(0.3887-0.4068)}{(0.4433-0.3706)(0.4433-0.4068)}$$

$$= \frac{0.02075}{0.00263} + \frac{0.02221}{0.00132} - \frac{8.19025}{0.002053}$$

$$= 7.8897 + 16.8318 - 3.0871$$

$$x = 21.6344$$

$$\boxed{\phi = 21.6344}$$

Question # 05

$x = 1.3$ find $\frac{dy}{dx}$

Using Newton forward difference formula and by applying chain rule on formula

$$\frac{dy}{dx} = \frac{dy}{dp} \frac{dp}{dx}$$

$$\frac{dy}{dp} = \Delta y_0 + \frac{2p-1}{2} \Delta^2 y_0 + \frac{3p^2 - 6p + 2}{6} \Delta^3 y_0$$

$$\frac{dp}{dx} = \frac{1}{h}$$

Formula:

$$\frac{dy}{dx} = \frac{1}{h}\left[\Delta y_0 + \frac{2p-1}{2}\Delta^2 y_0 + \frac{(3p^2-6p+2)}{6}\Delta^3 y_0\right]$$

x	y	Δy_0	$\Delta^2 y$	$\Delta^3 y$
1	1			
		8		
3	9		8	
				0
		16		
5	25		8	
		24		
7	49			

$h = 2$ $\qquad p = \frac{x - x_0}{h} = \frac{1.3 - 1}{2} = 0.15$

$$\frac{dy}{dx} = \frac{1}{2}\left[8 + \frac{2(0.15-1)}{2}8 + \left(\frac{3(0.15)^2 - 6(0.15) + 2}{6}\right)0\right]$$

$$= \frac{1}{2}\left[8 - 2.8\right]$$

$$= \frac{1}{2}\left[5.2\right]$$

$$\boxed{\frac{dy}{dx} = 2.6}$$