Chapter 3

MEASURES OF CENTRAL TENDENCY OR AVERAGES

3.13 (i) The decision is wrong, as an average does not reveal the whole picture.

(ii) The conclusion is wrong, as there can be several brilliant students in the class.

(iii) The conclusion is wrong, as the mean is highly affected by abnormally large or small values.

(iv) The conclusion is absurd as few people walk in the middle of the road.

3.15 (b) Now $\sum(x_i-A)^2 = \sum[x_i-M+M-A]^2$, (adding and subtracting M)

$$= \sum[(x_i-M) + (M-A)]^2$$

$$= \sum[(x_i-M)^2+(M-A)^2+2(M-A)(x_i-M)]$$

$$= \sum[(x_i-M)^2+n(M-A)^2+2(M-A)\sum(x_i-M)]$$

But $\sum(x_i-M) = 0$, as the sum of deviations taken from mean is always equal to zero. Therefore the cross product term vanishes.

Hence $\sum(x_i-A)^2 = \sum(x_i-M)^2 + n(M-A)^2$

(c) Let $\bar{x}$ denote the mean of the combined distribution. Then

$$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2 + n_3\bar{x}_3}{n_1 + n_2 + n_3}$$

where $\bar{x}_1$, $\bar{x}_2$, $\bar{x}_3$ are the means and n_1, n_2, n_3 are the frequencies of the 3 components respectively.

Substitution gives

$$\bar{x} = \frac{3(2) + 4(5.5) + 5(10)}{3 + 4 + 5} = \frac{78}{12} = 6.5$$

3.16. (c) Let $\bar{x}$ denote the mean of the combined distribution. Then

$$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2 + n_3\bar{x}_3}{n_1 + n_2 + n_3}$$

where $\bar{x}_1, \bar{x}_2, \bar{x}_3$ are the means and n_1, n_2, n_3 are the frequencies of the 3 components respectively.

Hence
$$\bar{x} = \frac{45(2) + 40(2.5) + 65(2)}{45 + 40 + 65}$$

$$= \frac{90 + 100 + 130}{150} = \frac{320}{150} = 2.13.$$

3.17. Calculation of the mean, the median and the geometric mean of the values which have been arranged in ascending order.

No.	Values (x)	log x
1	9	0.9542
2	12	1.0792
3	15	1.1761
4	15	1.1761
5	16	1.2041
6	18	1.2553
7	20	1.3010
8	20	1.3010
9	25	1.3979
10	30	1.4771
Σ	180	12.3220

Hence (i) Mean, *i.e.* $\bar{x} = \frac{\sum x}{n} = \frac{180}{10} = 18$

(ii) Median = Size of $\frac{1}{2}\left[\left(\frac{n}{2}\right)+\left(\frac{n}{2}+1\right)\right]th$ observation

($\because \frac{n}{2}$ is an integer)

$= \text{Size of } \frac{1}{2}[5\text{th} + 6\text{th}]$ observation

$= \frac{16 + 18}{2} = 17$

(iii) $\text{G.M.} = \text{anti-log}\left[\frac{\sum \log x}{n}\right]$

$= \text{anti-log}\left[\frac{12.3220}{10}\right] = \text{anti-log of } 1.2322 = 17.07$

3.18 (a) Given two positive numbers a and b.

The A.M., G.M. are H.M. are then defined as

$\text{A.M.} = \frac{a+b}{2}$, $\text{G.M.} = \sqrt{ab}$ and $\text{H.M.} = \frac{2}{1/a + 1/b} = \frac{2ab}{a+b}$

Now A.M. $\geq$ G.M. of $\frac{a+b}{2} \geq \sqrt{ab}$

or $a + b \geq 2\sqrt{ab}$ or $(a+b)^2 \geq 4ab$

or $(a+b)^2 - 4ab \geq 0$

or $(a-b)^2 \geq 0$, which is true.

Again, G.M. $\geq$ H.M., if

$\sqrt{ab} \geq \frac{2ab}{a+b}$, or $ab \geq \frac{4a^2b^2}{(a+b)^2}$

or $(a+b)^2 \geq \frac{4a^2b^2}{ab}$, or $(a+b)^2 \geq 4ab$

or $(a+b)^2 - 4ab \geq 0$ or $(a-b)^2 \geq 0$, which is true.

Hence, for two positive numbers, A.M. $\geq$ G.M. $\geq$ H.M.

(b) Calculation of the arithmetic average, the geometric mean and the harmonic mean.

	Income (x)	$\log x$	$\frac{1}{x}$
	85	1.9294	0.0118
	70	1.8451	0.0143
	10	1.0000	0.1000
	75	1.8751	0.0133
	500	2.6990	0.0020
	8	0.9031	0.1250
	42	1.6232	0.0238
	250	2.3979	0.0040
	40	1.6022	0.0250
	36	1.5563	0.0278
Σ	1116	17.4313	0.3470

$$\therefore \text{ Mean} = \frac{\sum x}{n} = \text{Rs. } \frac{1116}{10} = \text{Rs. } 111.60$$

$$\text{G.M.} = \text{anti-log } \left[\frac{\sum \log x}{n}\right]$$

$$= \text{anti-log } \left[\frac{17.4313}{10}\right] = \text{anti-log } (1.74313) = \text{Rs. } 55.35$$

$$\text{H.M.} = \frac{n}{\sum\left(\frac{1}{x}\right)} = \frac{10}{0.3470} = \text{Rs. } 28.82$$

Here G.M. is the best average.

3.19. Calculation of the arithmetic mean, the geometric mean and the harmonic mean.

	x	$\log x$	$\frac{1}{x}$
	60	1.7782	0.01667
	80	1.9031	0.01250
	90	1.9542	0.01111
	96	1.9823	0.01042
	120	2.0792	0.00833
	150	2.1761	0.00667
	200	2.3010	0.00500
	360	2.5563	0.00278
	480	2.6812	0.00208
	520	2.7160	0.00192
	1060	3.0253	0.00094
	1200	3.0792	0.00083
	1450	3.1614	0.00069
	2500	3.3979	0.00040
	7200	3.8573	0.00014
Σ	15566	38.6487	0.08048

$$\bar{x} = \frac{\sum x}{n} = \frac{15566}{15} = 1037.73$$

$$\text{G.M.} = \text{Anti-log}\left[\frac{\sum \log x}{n}\right] = \text{Anti-log}\left[\frac{38.6487}{15}\right]$$

$$= \text{Anti-log}\ (2.5766) = 377.2$$

$$\text{H.M.} = \frac{n}{\sum\left(\frac{1}{x}\right)} = \frac{15}{0.08048} = 186.7$$

3.20 (a) (i) Mean earnings $= \dfrac{60(3)+20(2)}{80} = \dfrac{220}{80} =$ Rs.2.75 per hour

(b) Calculation of the weighted mean.

Subject	Marks % (x)	Weight (w)	xw
English	73	4	292
French	82	3	246
Maths	57	3	171
Science	62	1	62
History	60	1	60
Total	---	12	831

Hence weighted mean $= \dfrac{\sum xw}{\sum w} = \dfrac{831}{12} = 69.25\%$ marks.

3.21 Calculation of the simple and weighted averages

(1) Piece goods	(2) Price per metre (Rs.) (x)	(3) Quantity (millions metres) (w)	xw
Unbleached	8.37	286	2393.82
Bleached	9.50	255	2422.50
Printed flags	9.16	64	586.24
Other sorts	9.84	172	1692.48
Dyed in piece	13.65	165	2252.25
Of dyed yarn	11.95	80	956.00
Total	62.47	1022	10303.29

(i) $\bar{x} = \dfrac{\sum x}{n} = \dfrac{62.47}{6} =$ Rs. 10.41 per metre

(ii) Weighted average $= \dfrac{\sum xw}{\sum w} = \dfrac{10303.29}{1022} =$ Rs.10.08 per metre

The weighted average price is more nearly the real average price, because the price of each and every piece goods has been multiplied by the corresponding quantity, *i.e.* properly weighted.

3.22. **We first construct the frequency table and then calculate the average bonus paid per employee, which would be the weighted mean.**

Monthly salary in rupees	Tally	Frequency (w)	Bonus (x)	xw
Exceeding 60 but not exceeding 75	|||	3	10	30
Exceeding 75 but not exceeding 90	||||	4	15	60
Exceeding 90 but not exceeding 105	~~||||~~	5	20	100
Exceeding 105 but not exceeding 120	~~||||~~	5	25	125
Exceeding 120 but not exceeding 135	~~||||~~ ||	7	30	210
Exceeding 135 but not exceeding 150	~~||||~~ |	6	35	210
Total	--	30	--	735

$$\text{Hence weighted mean} = \frac{\sum xw}{\sum w} = \frac{735}{30} = \text{Rs. } 24.50.$$

3.23. Calculation of the average age of the horses.

Age (years)	f (also w_i)	x	D	fD	mean age, $\bar{x}_i$	$\bar{x}_i w_i$
1–4	12	2.5	−9.5	−114	2.7	32.4
5–9	223	7.0	−5	−1115	7.6	1694.8
10–14	435	12.0	0	−1229	12.0	5220.0
15–19	272	17.0	5	1360	16.3	4433.6
20–24	161	22.0	10	1610	20.8	3348.8
25–29	34	27.0	15	510	25.8	877.2
30&over	6	32.0	20	120	31.8	190.8
Total	1143	--	--	+3600 2371	--	15797.6

(a) $$\text{Average age}_{\text{(simple)}} = a + \frac{\sum fD}{n}$$

$$= 12.0 + \frac{2371}{1143} = 12.0 + 2.07 = 14.07 \text{ years}$$

(b) Average age (weighted) $= \frac{\sum \bar{x}_i w_i}{\sum w_i} = \frac{15797.6}{1143} = 13.82$ years

The weighted average age is more nearly the real average age, because the mean age of each and every age-group has been multiplied by the corresponding frequency, *i.e.* properly weighted.

3.24. Calculation of the arithmetic mean, the geometric mean and the harmonic mean of the $(n+1)$ values 1, 2 4, 8, 16, ..., 2^n which are in geometric progression.

The sum of the values in G.P. is obtained by the formula

$$S = \frac{a\,(r^n - 1)}{r - 1}, \quad \text{where } r > 1$$

$$= \frac{a\,(1 - r^n)}{1 - r}, \quad \text{where } r < 1.$$

$\therefore$ Sum of the values $= \frac{1\,(2^{n+1} - 1)}{2 - 1} \quad (\because \; a = 1, r = 2)$

$$= 2^{n+1} - 1.$$

Product of the values $= 1 \times 2 \times 4 \times 8 \times 16 \times \ldots \times 2^n$

$$= 2^0 \times 2^1 \times 2^2 \times 2^3 \times 2^4 \times \ldots \times 2^n$$

$$= 2^{0+1+2+3+4+\ldots+n}$$

$$= 2^{n+1(0+n)/2} = 2^{n(n+1)/2}$$

Sum of reciprocals $= \frac{1}{1} + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots + \frac{1}{2^n}$

$$= \frac{1 - \frac{1}{2^{n+1}}}{1 - \frac{1}{2}} \quad (\because \; a = 1, r = \tfrac{1}{2})$$

$$= 2\left(1 - \frac{1}{2^{n+1}}\right)$$

Hence $\bar{x} = \dfrac{\text{Sum of the values}}{\text{No. of values}} = \dfrac{2^{n+1}-1}{n+1}$;

G.M. $= (\text{Product of the values})^{1/n+1}$

$= [2^{n(n+1)/2}]^{1/n+1} = 2^{n/2}$; and

H.M. $= \dfrac{\text{Number of values}}{\text{Sum of their reciprocals}} = \dfrac{n+1}{2\left(1-\dfrac{1}{2^{n+1}}\right)}$

3.25. Calculation of the arithmetic mean, the geometric mean and the harmonic mean of the $(n+1)$ values 1, 3, 9, 27, ..., 3^n which are in geometric progression.

The sum of the values in G.P. is obtained by the formula

$$\text{Sum} = \frac{a(r^n-1)}{r-1}, \quad \text{where } r > 1$$

$$= \frac{a(1-r^n)}{1-r}, \quad \text{where } r < 1$$

Here $a = 1$ and r (common ratio) $= 3$

$\therefore$ Sum of the values $= \dfrac{1(3^{n+1}-1)}{3-1} = \dfrac{1}{2}(3^{n+1}-1)$

Product of the values $= 1 \times 3 \times 9 \times 27 \times 81 \times \ldots \times 3^n$

$= 3^{0+1+2+3+\ldots+n}$

$= 3^{(0+n)(n+1)/2} = 3^{n(n+1)/2}$

Sum of reciprocals $= \dfrac{1}{1} + \dfrac{1}{3} + \dfrac{1}{9} + \dfrac{1}{27} + \dfrac{1}{81} + \ldots + \dfrac{1}{3^n}$

$$= \frac{1\left(1-\dfrac{1}{3^{n+1}}\right)}{1-1/3} = \frac{3}{2}\left(1-\frac{1}{3^{n+1}}\right)$$

$(\because\ a=1,\ r=\frac{1}{3})$

Hence $\bar{x} = \dfrac{\text{Sum of all values}}{\text{No. of values}} = \dfrac{\frac{1}{2}(3^{n+1} - 1)}{n + 1}$;

$$\text{G.M.} = [\text{Product of the values}]^{1/(n+1)}$$

$$= [3^{n(n+1)/2}]^{1/(n+1)} = 3^{n/2}; \text{ and}$$

$$\text{H.M.} = \frac{\text{Number of values}}{\text{Sum of their reciprocals}} = \frac{n + 1}{\frac{3}{2}\left(1 - \frac{1}{3^{n+1}}\right)}$$

3.27. The rates of his salary after getting rises come to 1.10, 1.20 and 1.25 respectively. The appropriate average of these rates is the geometric mean. Thus

$$\text{G.M.} = \sqrt[3]{1.10 \times 1.20 \times 1.25}$$

Taking logs, we have

$$\log G = \frac{1}{3}[\log 1.10 + \log 1.20 + \log 1.25]$$

$$= \frac{1}{3}[0.0414 + 0.0792 + 0.0969] = \frac{0.2175}{3} = 0.0725$$

$\therefore$ G.M. = Anti-log of 0.0725 = 1.181.

Hence the required annual percentage increase = 18.1%

3.28. (b) Calculation of the Harmonic mean which is the correct average speed of the person in this question.

$$\text{H.M.} = \frac{n}{\Sigma\left(\frac{1}{x}\right)} = \frac{2}{\frac{1}{30} + \frac{1}{60}} = \frac{2}{0.05} = 40 \text{ miles per hour}$$

(c) Calculation of the Harmonic mean which is the correct average speed of the person in the question.

$$\text{H.M} = \frac{n}{\Sigma\left(\frac{1}{x}\right)} = \frac{3}{\frac{1}{8} + \frac{1}{7.5} + \frac{1}{5.5}}$$

$$= \frac{3}{0.44015} = 6.8 \text{ miles per hour}$$

3.29. (a) As the distances are equal (and hence constant) and the times vary, therefore the correct average is the harmonic mean, which is obtained below:

Speed (x)	$1/x$
10	0.1000
15	0.0667
20	0.0500
25	0.0400
20	0.0500
30	0.0333
40	0.0250
50	0.0200
30	0.0333
40	0.0250
Σ	0.4433

Thus $\quad \text{H.M.} = \dfrac{n}{\Sigma\left(\dfrac{1}{x}\right)} = \dfrac{10}{0.4433} = 22.56$ kilometres p.h.

(b) (i) **Calculation of the harmonic mean, the correct average rate.**

$$\text{H.M.} = \frac{3}{\frac{1}{10} + \frac{1}{8} + \frac{1}{6}} = \frac{3}{0.1000 + 0.1250 + 0.1667}$$

$$= \frac{3}{0.3917} = 7.66 \text{ m.p.h.}$$

(ii) **The rates of increase in population come out to be 1.20, 1.25 and 1.44. The average rate of increase would be their G.M.**

$\therefore \quad \text{G.M.} = \sqrt[3]{1.20 \times 1.25 \times 1.44}$

Taking logs, we have

$$\log G = \frac{1}{3}[\text{og } 1.20 + \log 1.25 + \log 1.44]$$

$$= \frac{1}{3}[0.0792 + 0.0969 + 0.1584]$$

$$= \frac{0.3345}{3} = 0.1115$$

$\therefore$ G.M. = Anti-log (0.1115) = 1.293

Hence the required percentage increase = 29.3%.

3.30. Calculation of the Geometric mean and the Harmonic mean

Weekly income (Rs.)	No. of workers (f)	x	$\log x$	$f \log x$	$f(1/x)$
35–39	15	37	1.5682	23.5230	0.4054
40–44	13	42	1.6232	21.1016	0.3095
45–49	17	47	1.6721	28.4257	0.3617
50–54	29	52	1.7160	49.7640	0.5577
55–59	11	57	1.7559	19.3149	0.1930
60–64	10	62	1.7924	17.9240	0.1613
65–69	5	67	1.8261	9.1305	0.0746
Total	100	--	--	169.1837	2.0632

Now $\quad$ G.M. = Anti-log of $\left(\frac{\sum f \log x}{\sum f}\right)$ = Anti-log of $\left(\frac{169.1837}{100}\right)$

= Anti-log of (1.6918) = Rs. 49.18, and

$$\text{H.M.} = \frac{n}{\sum f\left(\frac{1}{x}\right)} = \frac{100}{2.0632} = \text{Rs. } 48.47$$

3.31. Calculation of Geometric and Harmonic means.

Variable	f	x	$\log x$	$f \log x$	$\frac{1}{x}$	$f\left(\frac{1}{x}\right)$
0 – 5	2	2.5	0.3979	0.7958	0.4000	0.8000
5 – 10	5	7.5	0.8751	4.3755	0.1333	0.6665
10 – 15	7	12.5	1.0969	7.6783	0.0800	0.5600
15 – 20	13	17.5	1.2430	16.1590	0.0571	0.7423
20 – 25	21	22.5	1.3522	28.3962	0.0444	0.9324
25 – 30	16	27.5	1.4393	23.0288	0.0364	0.5824
30 – 35	8	32.5	1.5119	12.0952	0.0308	0.2464
35 – 40	3	37.5	1.5740	4.7220	0.0267	0.0801
Total	75	--	--	97.2508	---	4.6101

Hence G.M. $= \text{Anti-log}\left[\frac{\sum f \log x}{n}\right] = \text{Anti-log}\left[\frac{97.2508}{75}\right]$

$= \text{Anti-log}(1.2966) = 19.80$; and

$$\text{H.M.} = \frac{n}{\sum f\left(\frac{1}{x}\right)} = \frac{75}{4.6101} = 16.27.$$

3.32. (i) **The salaries varied greatly, so median is more suitable average.** Arranging the data in ascending order, we get Rs.100, Rs. 950, Rs. 1500, Rs. 2100, Rs. 10000.

Thus Median = salary of $\left(\frac{n+1}{2}\right)$th person ($\because \frac{n}{2}$ is not an integer)

= salary of $(5+1)/2$ th, *i.e.* 3rd person = Rs. 1500.

(ii) **The given heights do not differ greatly, so arithmetic mean is more suitable average.**

Therefore $\bar{x} = \frac{64+65+65+66+66+67}{6} = \frac{393}{6} = 65.5''$

(ii) Here again median is more suitable average. Since $\frac{n}{2}$ is an integer, therefore median is the average value of $\left(\frac{n}{2}\right)$th and $\left(\frac{n}{2}+1\right)$th, *i.e.* 2nd and 3rd observations.

Hence Median $= \frac{18+18}{2} = 18.$

3.33. Calculation of the median, quartiles, etc.

Size of shoes	No. of pairs (f)	$c.f.$
5	2	2
$5\frac{1}{2}$	5	7
6	15	22
$6\frac{1}{2}$	30	52
7	60	112
$7\frac{1}{2}$	40	152
8	23	175
$8\frac{1}{2}$	11	186
9	4	190
$9\frac{1}{2}$	1	191
Total	191	--

Here $\frac{n}{2}$, i.e. $\frac{191}{2}$ is not an integer, therefore

$$\text{Median} = \text{Size of } \left(\left[\frac{191}{2}\right] + 1\right)th \text{ pair}$$

$$= \text{Size of } (95 + 1)th, \text{ i.e. } 96th \text{ pair} = 7$$

Again $\frac{n}{4}$, i.e. $\frac{191}{4}$ is not an integer. Therefore

$$Q_1 = \text{Size of } \left(\left[\frac{191}{4}\right] + 1\right)th \text{ pair,}$$

$$= \text{Size of } (47 + 1)th, \text{ i.e. } 48th \text{ pair} = 6\tfrac{1}{2}$$

$$Q_3 = \text{Size of } \left(\left[\frac{3 \times 191}{4}\right] + 1\right)th \text{ pair as } \frac{3n}{4} \text{ is also not an integer}$$

$$= \text{Size of } (143 + 1)th, \text{ i.e. } 144th \text{ pair} = 7\tfrac{1}{2}$$

Now $\frac{7n}{10}$, i.e. $\frac{7 \times 191}{10}$ is not an integer, therefore

$$D_7 = \text{Size of } \left(\left[\frac{7 \times 191}{10}\right] + 1\right)th \text{ pair}$$

$$= \text{Size of } (133 + 1)th, \text{ i.e. } 134th \text{ pair} = 7\tfrac{1}{2}$$

$$P_{64} = \text{Size of } \left(\left[\frac{64 \times 191}{100}\right] + 1\right)th \text{ pair as } \frac{64n}{100} \text{ is not an integer}$$

$$= \text{Size of } (122 + 1)th, \text{ i.e. } 123th \text{ pair} = 7\tfrac{1}{2}$$

3.34. Calculation of the mean, the median and the modal numbers of persons per house.

No. of persons (x)	No. of houses (f)	fx	Cumulative frequency
1	26	26	26
2	113	226	139
3	120	360	259
4	95	380	354
5	60	300	414
6	42	252	456
7	21	147	477
8	14	112	491
9	5	45	496
10	4	40	500
Total	500	1888	---

Hence $\bar{x} = \frac{\sum fx}{n} = \frac{1888}{500} = 3.78,$

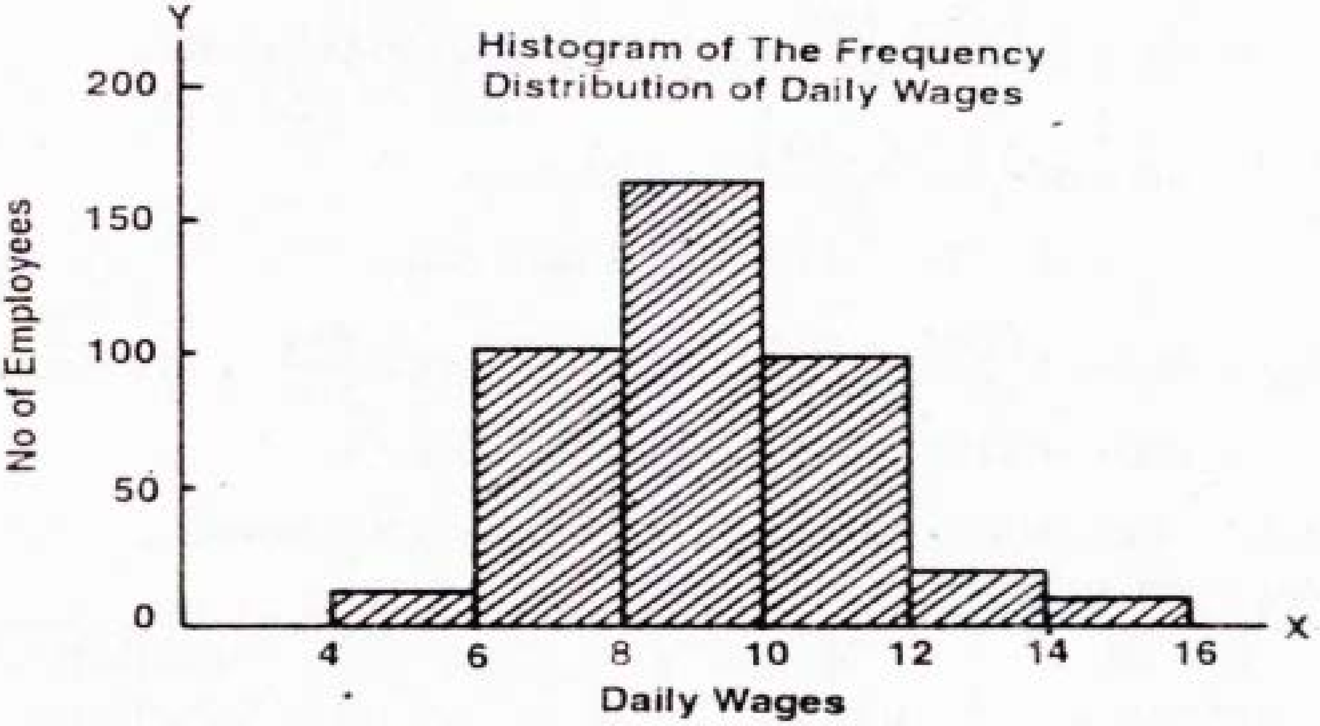
Median $=$ Average of two values with ordinal numbers $\left(\frac{n}{2}\right)$ and $\left(\frac{n}{2}\right) + 1$ as $\frac{n}{2}$, *i.e.* $\frac{500}{2}$ is an integer.

$=$ Average no. of persons of $250th$ and $251st$ houses

$= 3$, and

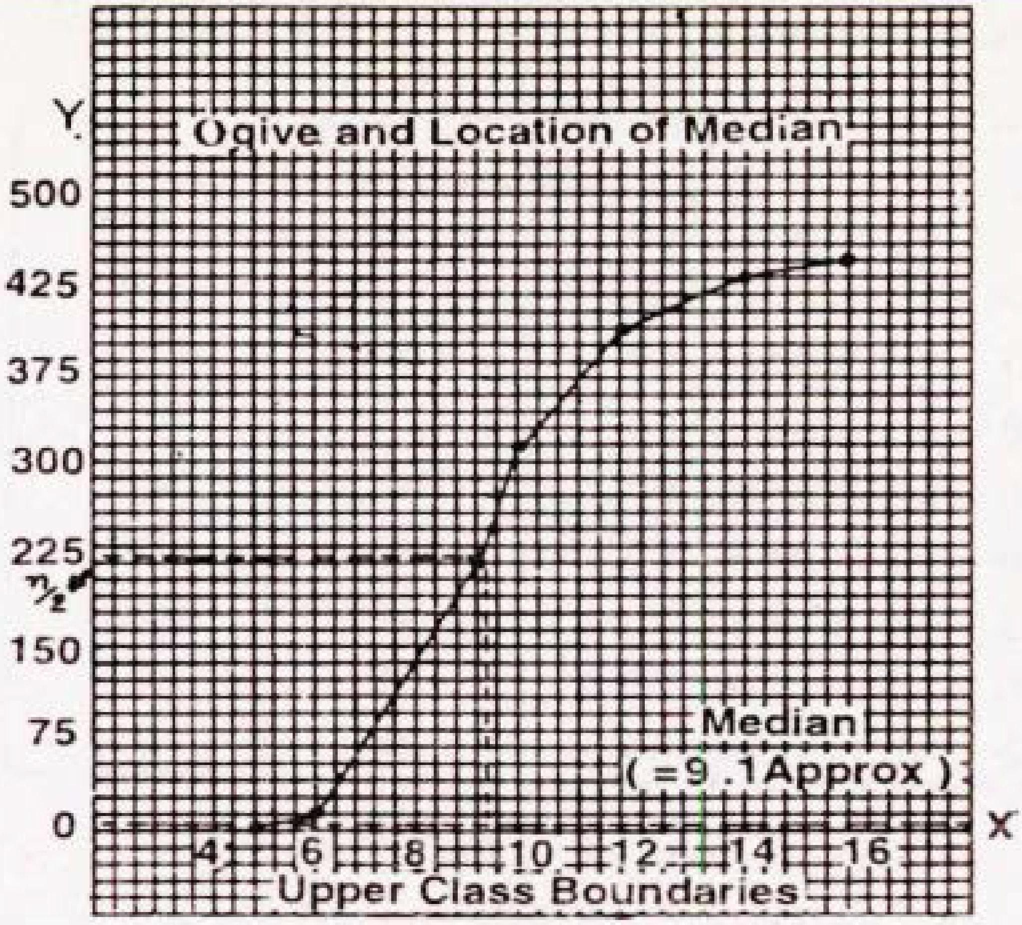
Mode $=$ No. of persons corresponding to the maximum frequency $= 3$

3.35. (i) Histogram of the daily wages.



(ii) Cumulative Frequency Distribution and its graph.

Daily wages	F
less than or equal to 4	0
less than or equal to 6	13
less than or equal to 8	124
less than or equal to 10	306
less than or equal to 12	411
less than or equal to 14	430
less than or equal to 16	437



Checking the answers by calculations.

Daily wages	f	F
4–6	13	13
6–8	111	124
8–10	182	306
10–12	105	411
12–14	19	430
14–16	7	437
Total	437	--

Median $=$ Daily wage of $\left(\frac{n}{2}\right)$*th* employee

$=$ Daily wage of $\frac{437}{2}$ *th*, *i.e.*, 218.5th employee

which lies in the group 8–10. Therefore

$$\text{Median} = l + \frac{h}{f}\left(\frac{n}{2} - C\right)$$

$$= 8 + \frac{2}{182}(218.5 - 124) = 8 + 1.04 = \text{Rs. } 9.04$$

3.36 Calculations of the median and quartile ages by formula

Age of head	f	$C.F.$
under 25	44	44
25 and under 30	79	123
30 and under 40	152	275
40 and under 50	122	397
50 and under 60	141	538
60 and under 65	100	638
65 and under 70	58	696
70 and under 75	32	728
75 and under 85	28	756

Median = Age of head of $\left(\frac{n}{2}\right)th$ household

= Age of head of $\left(\frac{756}{2}\right)th$, *i.e.* 378*th* household.

which lies in the age-group 40 and under 50. Therefore

$$\text{Median} = l + \frac{h}{f}\left(\frac{n}{2} - C\right) = 40 + \frac{10}{122}(378 - 275)$$

$$= 40 + 8.44 = 48.44 \text{ years}$$

Q_1 = Age of head of $\left(\frac{n}{4}\right)th$ household

= Age of head of $\left(\frac{756}{4}\right)th$, *i.e.* 189*th* household.

which lies in the age group 30 and under 40. Thus

$$Q_1 = l + \frac{h}{f}\left(\frac{n}{4} - C\right) = 30 + \frac{10}{152}(189 - 123)$$

$$= 30 + 4.34 = 34.34 \text{ years; and}$$

Q_3 = Age of head of $\left(\frac{3n}{4}\right)th$ household

$= \text{Age of head of } \left(\frac{3 \times 756}{4}\right) th, \text{ i.e. } 567th$ household

which lies in the age group 60 and under 65. Thus

$$Q_3 = l + \frac{h}{f}\left(\frac{3n}{4} - C\right)$$

$$= 60 + \frac{5}{100}(567 - 538) = 60 + 1.45 = 61.45 \text{ years.}$$

Graphical location is left as an exercise.

3.37. Calculation of the median and the quartiles.

Height (inches)	Number (f)	Cumulative frequency
57.5 – 60.0	6	6
60.0 – 62.5	26	32
62.5 – 65.0	190	222
65.0 – 67.5	281	503
67.5 – 70.0	412	915
70.0 – 72.5	127	1042
72.5 – 75.0	38	1080
Total	1080	---

Median $= \text{Height of } \left(\frac{n}{2}\right) th$ person

$= \text{Height of } \frac{1080}{2} th, \text{ i.e. } 540th$ person,

which lies in the group 67.5 – 70.0

$$\therefore \quad \text{Median} = l + \frac{h}{f}\left(\frac{n}{2} - C\right)$$

$$= 67.5 + \frac{2.5}{412}(540-503) = 67.5 + 0.22 = 67.72 \text{ inches.}$$

Lower quartile $=$ Height of $\left(\frac{n}{4}\right)th$ person

$=$ Height of $\frac{1080}{4}$ th, *i.e.* $270th$ person,

which lies in the group 65.0 – 67.5. Thus

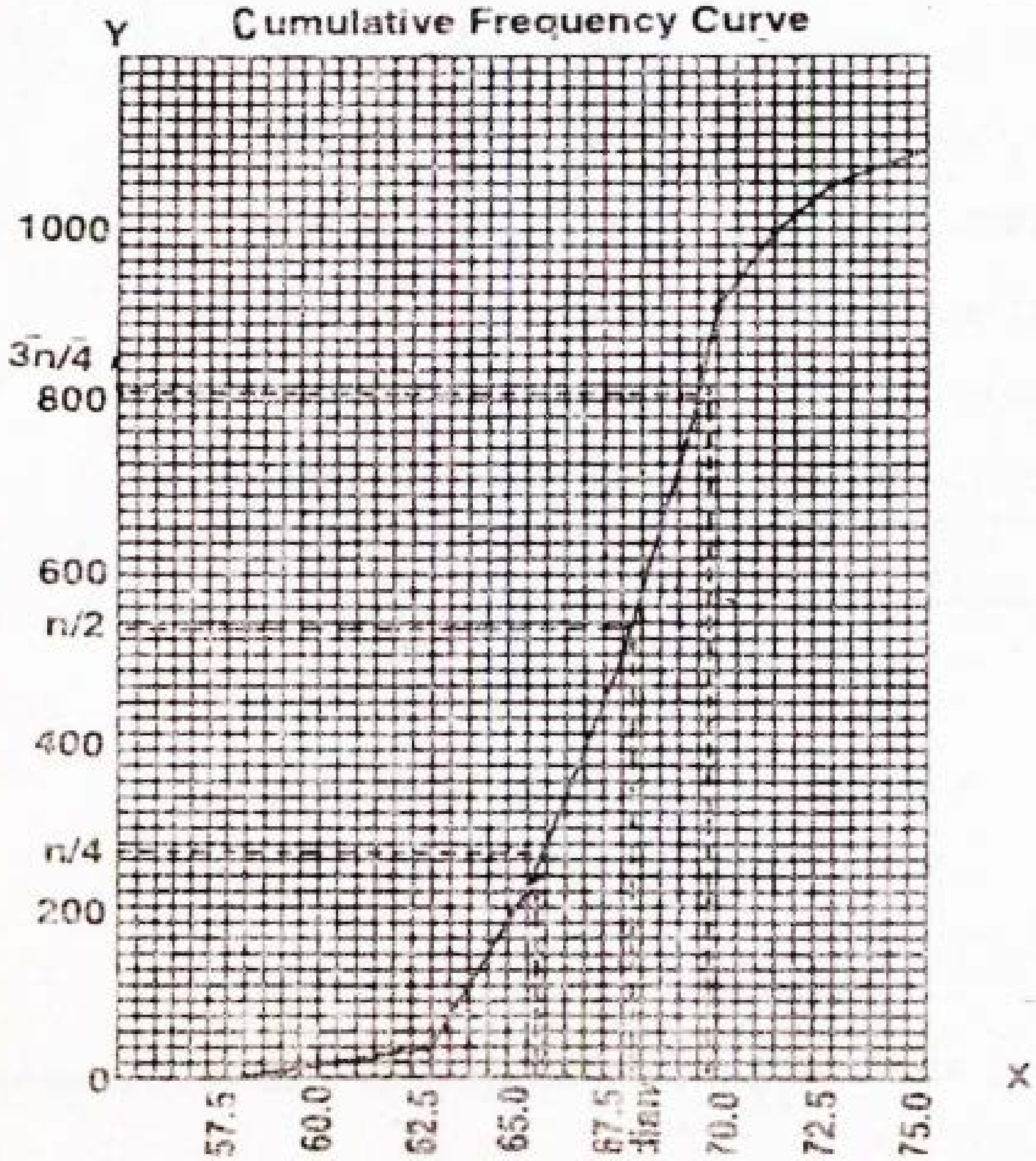
$$Q_1 = l + \frac{h}{f}\left(\frac{n}{4} - C\right)$$

$$= 65.0 + \frac{2.5}{281}(270-222) = 65.0+0.43=65.43 \text{ inches.}$$

Similarly $Q_3 = l + \frac{h}{f}\left(\frac{3n}{4} - C\right)$

$$= 67.5 + \frac{2.5}{412}(810-503) = 67.5+1.86=69.36 \text{ inches.}$$

These results are checked on the following graph



3.38. Calculation of the Median.

Classes	Class boundaries	Number (f)	F
100–104	99.5–104.5	4	4
105–109	104.5–109.5	14	18
110–114	109.5–114.5	60	78
115–119	114.5–119.5	138	216
120–124	119.5–124.5	236	452
125–129	124.5–129.5	298	750
130–134	129.5–134.5	380	1130
135–139	134.5–139.5	450	1580
140–144	139.5–144.5	500	2080
145–149	144.5–149.5	430	2510
150–154	149.5–154.5	260	2770
155–159	154.5–159.5	128	2898
160–164	159.5–164.5	66	2964
165–169	164.5–169.5	28	2992
170–174	169.5–174.5	12	3004
Total	---	3004	--

Median $= \text{Size of } \left(\frac{n}{2}\right) th$ number

$= \text{Size of } \frac{3004}{2} th$, *i.e.* 1502nd number, which lies in the group 134.5 – 139.5. Thus

$$\text{Median} = l + \frac{h}{f}\left(\frac{n}{2} - C\right)$$

$$= 134.5 + \frac{5}{450}(1502-1130) = 134.5+4.13=138.63$$

3.39. Calculation of the mean and median ages.

Age in yrs	No. of persons	F	x	fx
< 1	5	5	0.5	2.5
1–4	10	15	2.5	25.0
5–9	11	26	7.0	77.0
10–19	12	38	14.5	174.0
20–29	22	60	24.5	539.0
30–39	18	78	34.5	621.0
40–59	8	86	49.5	396.0
60–79	7	93	69.5	486.5
Total	93	---	---	2321.0

Mean age $= \dfrac{\sum fx}{n} = \dfrac{2321.0}{93} = 24.96$ years.

Median age $=$ age of $\left(\dfrac{n}{2}\right)th$ person

$=$ age of $\left(\dfrac{93}{2}\right)th$, *i.e.* $46.5th$ person which lies in the group 19.5 – 29.5

$\therefore$ Median age $= l + \dfrac{h}{f}\left(\dfrac{n}{2} - C\right)$

$= 19.5 + \dfrac{10}{22}(46.5-38) = 19.5 + \dfrac{85}{22} = 23.36$ years

3.40. (b) Calculation of the median, the lower and the upper q' artiles.

Class Interval	Class boundaries	f	F
Under 25	upto 24.5	222	222
25–29	24.5–29.5	405	627
30–34	29.5–34.5	508	1135
35–39	34.5–39.5	520	1655
40–44	39.5–44.5	525	2180
45–49	44.5–49.5	490	2670
50–54	49.5–54.5	457	3127
55–59	54.5–59.5	416	3543
60 & over	59.5 + over	166	3709
Total	---	3709	---

Median $= \text{Size of } \left(\frac{n}{2}\right) th$ observation

$= \text{Size of } \frac{3709}{2} th$, *i.e.*. 1854.5*th* observation, which lies in the group 39.5 – 44.5. Thus

$$\text{Median} = l + \frac{h}{f}\left(\frac{n}{2} - C\right)$$

$$= 39.5 + \frac{5}{525}(1854.5 - 1655) = 39.5 + 1.9 = 41.4$$

Similarly, $Q_1 = l + \frac{h}{f}\left(\frac{n}{4} - C\right)$

$$= 29.5 + \frac{5}{508}(927.25 - 627) = 29.5 + 2.96 = 32.46; \text{ and}$$

$$Q_3 = l + \frac{h}{f}\left(\frac{3n}{4} - C\right)$$

$$= 49.5 + \frac{5}{457}(2781.75 - 2670) = 49.5 + 1.22 = 50.72$$

3.41. Estimation of the mean, the median and the quartiles.

Consumption in kilowatt hours	No. of consumers (f)	x	fx	F	Class Boundries
5–24	4	14.5	58.0	4	4.5–24.5
25–44	6	34.5	207.0	10	24.5–44.5
45–64	14	54.5	763.0	24	44.5–64.5
65–84	22	74.5	1639.0	46	64.5–84.5
85–104	14	94.5	1323.0	60	84.5–104.5
105–124	5	114.5	572.5	65	104.5–124.5
125–144	7	134.5	941.5	72	124.5–144.5
145–164	3	154.5	463.5	75	144.5–164.5
Total	75	--	5967.5	--	--

$$\text{Mean} = \frac{\sum fx}{n} = \frac{5967.5}{75} = 79.57 \text{ kilowatt hours}$$

$$\text{Median} = \text{Kilowatt hours of } \left(\frac{n}{2}\right)th \text{ consumer}$$

$$= \text{Kilowatt hours of } \left(\frac{75}{2}\right)th, \text{ i.e. } 37.5th \text{ consumer}$$

which lies in the group 64.5–84.5. Thus

$$\text{Median} = l + \frac{h}{f}\left(\frac{n}{2} - C\right) = 64.5 + \frac{20}{22}(37.5 - 24)$$

$$= 64.5 + \frac{270}{22} = 76.77 \text{ kilowatt hours}$$

$$Q_1 = \text{kilowatt hours of } \left(\frac{n}{4}\right)th \text{ consumer}$$

$$= \text{kilowatt hours of } \left(\frac{75}{4}\right)th, \text{ i.e. } 18.75th \text{ consumer,}$$

which lies in the group 44.5–64.5. Therefore

$$Q_1 = l + \frac{h}{f}\left(\frac{n}{4} - C\right) = 44.5 + \frac{20}{14}(18.75 - 10)$$

$$= 44.5 + \frac{175}{14} = 57.0 \text{ kilowatt hours}$$

Similarly, $Q_3 = l + \frac{h}{f}\left(\frac{3n}{4} - C\right) = 84.5 + \frac{20}{14}(56.25 - 46)$

$$= 84.5 + \frac{205}{14} = 99.14 \text{ kilowatt hours.}$$

3.42. Calculation of the mean, the median and the quartiles.

Yield (lb)	x	f	$u\left(=\frac{x-4.0}{0.2}\right)$	fu	F
2.7–2.9	2.8	4	−6	−24	4
2.9–3.1	3.0	15	−5	−75	19
3.1–3.3	3.2	20	−4	−80	39
3.3–3.5	3.4	47	−3	−141	86
3.5–3.7	3.6	63	−2	−126	149
3.7–3.9	3.8	78	−1	−78	227
3.9–4.1	4.0	88	0	−524	315
4.1–4.3	4.2	69	1	69	384
4.3–4.5	4.4	59	2	118	443
4.5–4.7	4.6	35	3	105	478
4.7–4.9	4.8	10	4	40	488
4.9–5.1	5.0	8	5	40	496
5.1–5.3	5.2	4	6	24	500
Total	--	500	--	+396 / −128	---

Now $\bar{x} = a + \frac{\sum fu}{n} \times h$

$$= 4.0 + \frac{(-128)}{500} \times 0.2 = 4.0 - 0.05 = 3.95 \text{ lb.}$$

Median $=$ yield of $\left(\frac{n}{2}\right)th$ plot

$=$ yield of $\frac{500}{2}th$, *i.e.* 250*th* plot, which lies in the group 3.9–4.1. Thus

$$\text{Median} = l + \frac{h}{f}\left(\frac{n}{2} - C\right) = 3.9 + \frac{0.2}{88}(250 - 227)$$

$$= 3.9 + 0.05 = 3.95 \text{ lb.}$$

First quartile $= \text{yield of } \left(\frac{n}{4}\right) th$ plot

$= \text{yield of } \frac{500}{4} th$, *i.e.* $125th$ plot, which lies in the group 3.5 – 3.7

$$\therefore \quad Q_1 = l + \frac{h}{f}\left(\frac{n}{4} - C\right) = 3.5 + \frac{0.2}{63}(125-86)$$

$$= 3.5 + 0.12 = 3.62 \text{ lb, and}$$

Third quartile $= \text{yield of } \left(\frac{3n}{4}\right) th$ plot

$= \text{yield of } \frac{3(500)}{4} th$, *i.e.* $375th$ plot, which lies in the group 4.1 – 4.3. Thus

$$Q_3 = l + \frac{h}{f}\left(\frac{3n}{4} - C\right) = 4.1 + \frac{0.2}{69}(375-315)$$

$$= 4.1 + 0.17 = 4.27 \text{ lb}$$

3.43. Calculation of the average weight, the median weight, the mode, etc.

Class Boundries	No. of Seeds (f)	x	fx	F
9.95–24.95	16	17.45	279.20	16
24.95–39.95	68	32.45	2206.60	84
39.95–54.95	204	47.45	9679.80	288
54.95–69.95	233	62.45	14550.85	521
69.95–84.95	240	77.45	18588.00	761
84.95–99.95	655	92.45	60554.75	1416
99.95–114.95	803	107.45	86282.35	2219
114.95–129.95	294	122.45	36000.30	2513
129.95–144.95	21	137.45	2886.45	2534
144.95–159.95	4	152.45	609.80	2538
Total	2538	---	231638.10	--

(a) Average weight $= \frac{\sum fx}{n} = \frac{231638.10}{2538} = 91.27$ milligrams

Median = weight of $\left(\frac{n}{2}\right)th$ *seed*

$= $ weight of $\left(\frac{2538}{2}\right)th$, *i.e.* 1269*th* seed which lies in the group 84.95–99.95. Therefore

$$\text{Median} = l + \frac{h}{f}\left(\frac{n}{2} - C\right) = 84.95 + \frac{15}{655}(1269-761)$$

$$= 84.95 + 11.63 = 96.58 \text{ milligrams}$$

$$\text{Mode} = l + \frac{(f_m - f_1)}{(f_m - f_1) + (f_m - f_2)} \times h$$

$$= 99.95 + \frac{(803 - 655)}{(803 - 655) + (803 - 294)} \times 15$$

$$= 99.95 + \frac{148}{148 + 509} \times 15 = 99.95 + \frac{148}{657} \times 15$$

$$= 99.95 + 3.38 = 103.33 \text{ milligrams.}$$

(b) Q_1 = weight of $\left(\frac{n}{4}\right)th$ seed

$= $ weight of $\left(\frac{2538}{4}\right)th$, *i.e.* 634.5*th* seed which lies in the group 69.95–84.95. Thus

$$Q_1 = l + \frac{h}{f}\left(\frac{n}{4} - C\right) = 69.95 + \frac{15}{240}(634.5-521)$$

$$= 69.95 + 7.09 = 77.04 \text{ milligrams}$$

Similarly, we estimate

$$Q_3 = l + \frac{h}{f}\left(\frac{3n}{4} - C\right) = 99.95 + \frac{15}{803}(1903.5-1416)$$

$$= 99.95 + 9.11 = 109.06 \text{ milligrams;}$$

$$D_3 = l + \frac{h}{f}\left(\frac{3n}{10} - C\right) = 84.95 + \frac{15}{655}(761.4-761)$$

$$= 84.95 + 0.01 = 84.96 \text{ milligrams; and}$$

$$P_{45} = l + \frac{h}{f}\left(\frac{45n}{100} - C\right) = 84.95 + \frac{15}{655}(1142.10 - 761)$$

$$= 84.95 + 8.73 = 93.68 \text{ milligrams.}$$

3.44. Assuming a range of Rs. 55.00 to Rs. 105.00, the frequency distribution would be as below:

Group	$f(\%)$	f	x	fx
Rs. 55.00 and under Rs. 60.00	4	20	57.50	1150.00
Rs. 60.00 and under Rs. 62.50	11	55	61.25	3368.75
Rs. 62.50 and under Rs. 72.75	10	50	67.625	3381.25
Rs. 72.75 and under Rs. 78.75	15	75	75.75	5681.25
Rs. 78.75 and under Rs. 82.25	10	50	80.50	4025.00
Rs. 82.25 and under Rs. 85.25	10	50	83.75	4187.50
Rs. 85.25 and under Rs. 90.50	15	75	87.875	6590.62
Rs. 90.50 and under Rs. 95.00	10	50	92.75	4637.50
Rs. 95.00 and under Rs. 100.00	10	50	97.50	4875.00
Rs.100.00 and under Rs.105.00	5	25	102.50	2562.50
Total	100	500	--	40459.37

Now Mean wage $= \frac{\sum fx}{n}$

$$= \text{Rs. } \frac{40459.37}{500} = \text{Rs. } 80.92 \text{ approximately.}$$

3.45. (b) Calculation of the mean, the median and the mode.

Weight	f	x	$u \left(= \frac{x-149}{9}\right)$	fu	F
118–126	3	122	−3	−9	3
127–135	5	131	−2	−10	8
136–144	9	140	−1	−9	17
145–153	12	149	0	−28	29
154–162	5	158	1	5	34
163–171	4	167	2	8	38
172–180	2	176	3	6	40
Total	40	---	---	+19 −9	--

$$\text{Mean} = a + \frac{\sum fu}{n} \times h$$

$$= 149 + \frac{(-9)(9)}{40} = 149 - 2.025 = 146.975$$

$$\text{Median} = \text{Weight of } \left(\frac{n}{2}\right)th \text{ student}$$

$$= \text{Weight of } \left(\frac{40}{2}\right)th, \text{ i.e. } 20th \text{ student, which lies}$$

in the group 144.5–153.5 (class boundaries)

$$\therefore \quad \text{Median} = l + \frac{h}{f}\left(\frac{n}{2} - C\right) = 144.5 + \frac{9}{12}(20-17)$$

$$= 144.5 + 2.25 = 146.75.$$

$$\text{Mode} = l + \frac{f_m - f_1}{(f_m - f_1) + (f_m - f_2)} \times h$$

$$= 144.5 + \frac{12 - 9}{(12 - 9) + (12 - 5)} \times 9$$

$$= 144.5 + \frac{27}{10} = 147.2.$$

3.46. The calculations needed to find the mean, median and mode are given in the following table:

Max. loads (short tons)	Class Boundries	No. of cables (f)	x	fx	F
9.8 – 10.2	9.75 – 10.25	7	10.0	70.0	7
10.3 – 10.7	10.25 – 10.75	12	10.5	126.0	19
10.8 – 11.2	10.75 – 11.25	17	11.0	187.0	36
11.3 – 11.7	11.25 – 11.75	14	11.5	161.0	50
11.8 – 12.2	11.75 – 12.25	6	12.0	72.0	56
12.3 – 12.7	12.25 – 12.75	4	12.5	50.0	60
Total	---	60	--	666.0	--

Now $\bar{x} = \frac{\sum fx}{n} = \frac{666.0}{60} = 11.10$ short tons

Median = Max. load of $\left(\frac{n}{2}\right)th$ cable

= Max. load of $\left(\frac{60}{2}\right)th$ *i.e.* $30th$ cable, which lies in the group 10.75 – 11.25.

$\therefore$ Median $= l + \frac{h}{f}\left(\frac{n}{2} - C\right) = 10.75 + \frac{0.5}{17}(30 - 19)$

$= 10.75 + 0.32 = 11.07$ short tons

Mode $= l + \frac{f_m - f_1}{(f_m - f_1) + (f_m - f_2)} \times h$

$= 10.75 + \frac{(17 - 12)}{(17 - 12) + (17 - 14)} \times 0.5$

$= 10.75 + 0.31 = 11.06$ short tons.

3.47. Calculations needed to find the *modal* and the *median* wages are given in the table below:

Daily Wages (Rs.)	Class Boundries	No. of Employees (f)	F
22	21 – 23	3	3
24	23 – 25	13	16
26	25 – 27	43	59
28	27 – 29	102	161
30	29 – 31	175	336
32	31 – 33	220	556
34	33 – 35	204	760
36	35 – 37	139	899
38	37 – 39	69	968
40	39 – 41	25	993
42	41 – 43	6	999
44	43 – 45	1	1000
Total	---	1000	--

$$\text{Mode wage} = l + \frac{f_m - f_1}{(f_m - f_1) + (f_m - f_2)} \times h$$

$$= 31 + \frac{(220 - 175)}{(220 - 175) + (220 - 204)} \times 2$$

$$= 31 + \frac{45}{45 + 16} \times 2 = 31 + 1.48 = \text{Rs. } 32.48$$

$$\text{Median wage} = \text{Daily wage of } \left(\frac{n}{2}\right)th \text{ employee}$$

$$= \text{Daily wage of } \left(\frac{1000}{2}\right)th, \text{ i.e } 500th$$

employee, which lies in the group 31–33. Therefore

$$\text{Median wage} = l + \frac{h}{f}\left(\frac{n}{2} - C\right) = 31 + \frac{2}{220}(500 - 336)$$

$$= 31 + \frac{328}{220} = 31 + 1.49 = \text{Rs. } 32.49$$

3.48. (c) Calculation of mode by the empirical relation.

$$\text{Mode} = 3\,(\text{Median}) - 2\,(\text{Mean})$$

$$= 3\,(36) - 2(40.5) = 108 - 81 = 27.$$

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