NUMBER THEORY PROJECT

TOPIC: PYTHAGOREAN TRIPLES

SUBMITTED TO: DOCTOR MUNNAZA NAZ

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SUBMITTED BY: GROUP 1

GROUP MEMBERS:

- HIRA 011
- HALEEMA MUNIR 015
- MAMOONA BATOOL 025
- NAGINA REHMAN 031
- SHAFIA MARYAM 044
- TOOBA KHALID 046
- UMM-E-HABIBA 048

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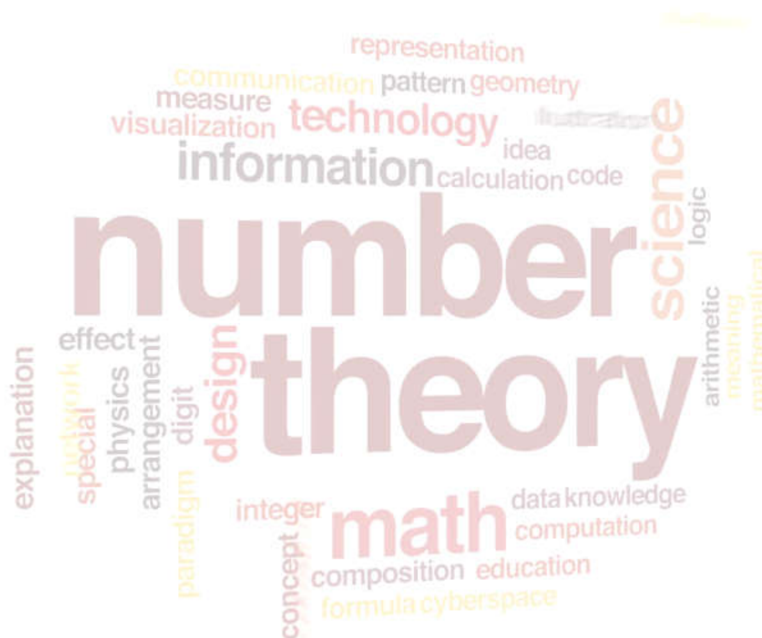
ACKNOWLEDGEMENT

We would like to express our special thanks of gratitude to our instructor DOCTOR MUNAZZA NAZ who gave us the golden opportunity to do this wonderful and informative project on the topic "PYTHAGOREAN TRIPLES", which also helped us in doing a lot of Research in "Number theory" and we came to know about so many new things. We are really thankful to them.
Regards,

GROUP 1



INTRODUCTION

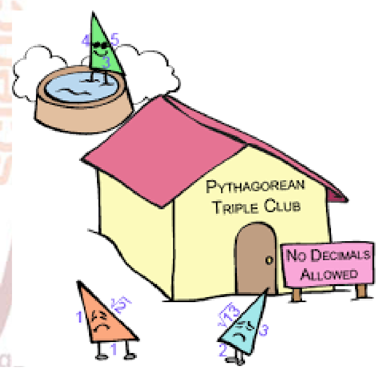
Pirates in the desert found a treasure case under a triangular wooden log. But the problem is that the case is locked by a number series which states these numbers

$$\mathbf{8, 15, -}$$

They have to fill the blank space to open the case. What would be the numbers?

In this project we will be interested in those solutions of Pythagoras equation which are interesting from number theoretic perspective. This means that we will work with the solutions of a, b, c of

$$\boldsymbol{a^2 + b^2 = c^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$



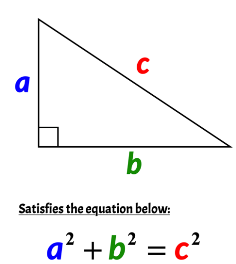
which are elements of particular subsets of real numbers e.g. natural numbers, integer, and rational number. Almost everyone knows that the credit to the result (1) goes to the school of Pythagoras. **Pythagoras theorem** states that *"In a right-angled triangle, the square of the hypotenuse side is equal to the sum of squares of the other two sides"*.

Is there any triangle all of whose sides are natural numbers? YES, there are so many of them. The most famous has sides (3,4,5).

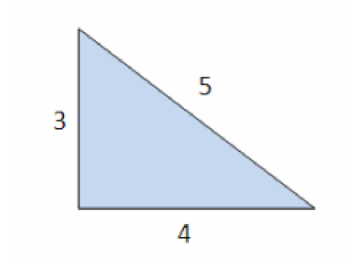
$$\mathbf{3^2 + 4^2 = 5^2} \ , \qquad \mathbf{5^2 + 12^2 = 13^2}$$

We will discuss the topic ***"PYTHAGOREAN TRIPLES".*** **Triples of positive integers satisfying the equation no (1) are called Pythagorean triples** after the ancient Greek mathematician Pythagoras. The study of Pythagorean triples as well as the general theorem of Pythagoras leads to many unexpected byways in mathematics. This triple was known to the Babylonians (who lived in the area of present Iran and Iraq) even as long as 5000 years ago! Perhaps they used it to make a new right angle triangle when constructing buildings.

Furthermore, we will learn primitive and non-primitive Pythagorean triples. In any primitive Pythagorean triple, one of the three entries must be even and it is easy to show that 2 cannot be the side of a Pythagorean triples.



In this project we will identify a Pythagorean triples and generate this formula for generating the Pythagorean triple theorem.



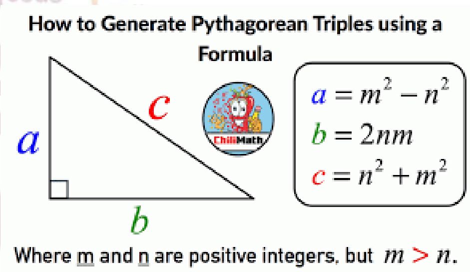
$$\boldsymbol{a = m^2 - n^2} \;,\quad \boldsymbol{b = 2nm} \;,\quad \boldsymbol{c = n^2 + m^2}$$

where m and n are positive integers, but m>n.

Certain characteristic properties that are very interesting are:

- Either a or b is divisible by 3.
- Either a or b is divisible by 4.
- Either a, b or c is divisible by 5.
- The product of a, b and c is divisible by 60.
- One of the qualities a, b, a+b, a-b are divisible by 7.
- The Pythagorean triple can have all even or one odd number.

An interesting perspective about this topic is that we deal with the numbers without fractions. There are infinite many numbers of Pythagorean triples (which will be proved further in corollary). If we multiply any constant number with the triplet, it results in a new Pythagorean triplet.



For example: *(3,4,5) is the basic Pythagorean triplet. If we multiply it by number, let's say 2.*

$2(3,4,5) = (6,8,10)$ *which satisfies the equation no (1).*

Furthermore this project will cover some interesting theorems, corollary and lemmas with proofs. After going through the whole topic, one will easily be able to answer following questions:

- How many even numbers can be there in a primitive Pythagorean triple?
- Can an even number be the largest number in Pythagorean triple?
- Are there infinite numbers of triples? How?

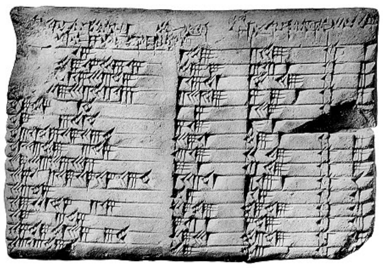
Let's go through the history of Pythagoras theorem and triples.

HISTORY

Pythagorean triples have been known since ancient times. A clay tablet named Plimpton 322 was discovered by Edgar James Banks shortly after 1900, and sold to George Arthur Plimpton in 1922, for $10. The study of Pythagorean triples began before the time of Pythagoras. We can find many evidences of its existence by exploring history. Some interesting history is described below.

Plimpton(322):

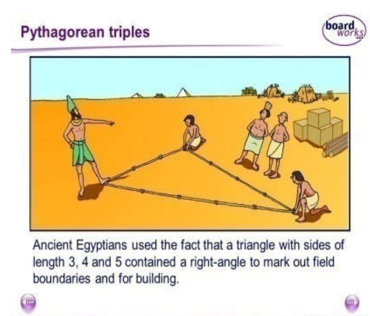
Plimpton 322 is a Babylonian clay tablet, notable as containing an example of Babylonian mathematics. This tablet, believed to have been written about 1800BC, has a table of four columns and 15 rows of number in the cuneiform script of the period. This table lists three numbers that are now called Pythagorean triples a, b, c satisfying $c^2=a^2+b^2$.



Chinese, Egyptians and Babylonians:

Around 4000 years ago, the Babylonians and the Chinese used the concept of the Pythagorean triple (3, 4, 5) to construct a right angle by dividing a long string into twelve equal parts, such that one side of triangle is three, the second side is four and third side is five section long.

The Egyptian used special right triangles to survey land by measuring 3-4-5 right angles. The Egyptians mostly understand right triangles in terms of ratios or what would now refer as Pythagorean triples.



Budhyana salba sutra:

In India (8th-2nd century BC), the Budhyana salba sutra contained a list of Pythagorean triple, a statement of the theorem and geometrical proof of the theorem of isosceles right triangle.

Pythagoras(569-475BC)

He used algebraic method to construct Pythagorean triples. He was not universally credited with this for 500 years. The basic name of this triple was derived from Pythagorean Theorem;stating that every right triangle has side lengths satisfying the formula $a^2 + b^2 = c^2$.

Pythagorean triples are useful for applications because they are whole numbers that make the **Pythagorean** Theorem true. If we are looking for the length of a side of a right triangle, and we know the lengths of two sides, check first to see if we have a right triangle whose sides are a Pythagorean triple. Pythagorean triples are used for navigation, surveying and in construction. So, people still take interest in these triples.

BACKGROUND

PYTHAGOREAN TRIPLES

1. Definition:

A Pythagorean triple is a triple of positive integers a, b, c such that a right triangle exists with legs **a, b** and hypotenuse **c**. By the Pythagorean Theorem, this is equivalent to finding positive integers a, b, c satisfying

$$a^2+b^2=c^2$$

The smallest and best known Pythagorean triples are **(3, 4, 5)**and **(5, 12, 13).** Pythagorean triples are among the oldest known solution of a **non-linear Diophantine equation**.

2. Categories:

There are two main categories of Pythagorean triples that are described below

a) *Primitive Pythagorean triple*

A Pythagorean triple is considered as primitive if it has following properties

- If all of its three numbers are relatively prime i.e. they don't have any common divisor
- They will always have one even number

- Value of c will always odd integer

For example, **(5, 12, 13)** is a primitive Pythagorean triple.

b) <u>Non- Primitive Pythagorean triple</u>

A set of numbers is considered as non-primitive if all numbers in a triple have a common divisor. By multiplying an integer k with primitive Pythagorean triple, we can get non primitive Pythagorean triple.

For example, **(10, 24, 26)** is a non-primitive Pythagorean triple with common divisor 2.

3. <u>Pythagorean triple generator:</u>

Formula for generating Pythagorean triples is more than 2000 year's problem in number theory. Euclid, Pythagoras, Plato, Fibonacci's Dickson and all other mathematician introduced formulae for generating Pythagorean triples.

- ***<u>Euclid's formula:</u>***

Euclid's formula (300BC) will generate Pythagorean triple with arbitrary pair of positive integers m and n with m>n>0. A primitive Pythagorean triples additionally require **m and n** to be of **opposite parity** and **coprime**. Then, for each side of triangle

$$\mathbf{a=m^2-n^2,\ b=2mn,\ c=m^2+n^2}$$

- ***<u>Derivation of Euclid's formula</u>***:

From $\mathbf{a^2+b^2=c^2}$ we obtain, $b^2=c^2-a^2$ and hence $(c-a)(c+a)=b^2$. Then,

$$\frac{c-a}{b}=\frac{b}{c+a} \text{.........} \ (1)$$

Since, $\frac{c+a}{b}$ is rational, we set it equal to $\frac{m}{n}$ in lowest terms. Thus, $\frac{c-a}{b}=\frac{n}{m}$, being the reciprocal of $\frac{c+a}{b}$.

Then solving

$$\frac{c}{b}+\frac{a}{b}=\frac{m.m}{n.m} \qquad \text{(Multiply}\frac{m}{m}\text{) .... (a)}$$

$$\frac{c}{b}-\frac{a}{b}=\frac{n.n}{n.m} \qquad \text{(Multiply}\frac{n}{n}\text{) .... (b)}$$

By adding (a) and (b) we get,

$$\frac{c}{b}=\frac{m^2-n^2}{2mn} \text{........} \ (s)$$

By subtracting (a) and (b) we get,

$$\frac{a}{b} = \frac{m^2+n^2}{2mn} \;\ldots\ldots\ldots \text{(t)}$$

From (s) and (t) we can easily get value of a, b, c

$$\mathbf{a=m^2-n^2\ ,\ b=2mn,\ c=m^2+n^2}$$

We can generate triple from this Pythagorean generator by taking m=2, n=1

$$\mathbf{a=(2)^2-(1)^2=3}$$

$$\mathbf{b=2\,(2)\,(1)=4}$$

$$\mathbf{c=(2)^2+(1)^2=5}$$

So, a primitive Pythagorean triple (3, 4, 5) is generated.

4. Examples:

❖ **Find all Pythagorean triples containing the integer 12?**

In order to find out all Pythagorean triple containing the integer 12, we must find all primitive triples containing a divisor of 12. These are 2, 3, 4, 6, and 12.

Such triple must have $a=m^2-n^2$, $b=2mn$, $c=m^2+n^2$

i. **For b=2mn**
   - If b=2 then m=n=1 which results in a=0 which is not allowed.
   - If b=4 then m=2, n=1 and hence a=3 and c=5 **(2, 3, 5)**
   - If b=6 then, m=3, n=1 which are **not of opposite parity**.
   - If b=12 then either m=6 , n=1 hence a=35, c=37
     Or m=3, n=2 then a=5, c=13

ii. **For $a=m^2-n^2$**
   - If a=3 then m=2, n=1 which results in b=4, c=5 **(3,4,5)**
   - **Other cases are not possible.**

iii. **For $c=m^2+n^2$**
   - **c≠3**because 9 is not a sum of two squares.
   - Other cases are not possible because c is always odd.

So, multiple of triples containing 12 are **(9, 12, 15)(35, 12, 37)(5, 12, 13)(12, 16, 20)**

❖ **Find the length of sides of triangle, where the sides have integer length and area equal perimeter?**

Let a, b, c be sides of right angle triangle then (a, b, c) is a Pythagorean triple and there must be integer and there must be (m, n) such that $a=m^2-n^2$, $b=2mn$, $c=m^2+n^2$

Because triangle is right angled so its area will be $\frac{ab}{2}$.

If area is equal to perimeter then we have $a+b+c = \frac{ab}{2}$ ……… (v)

Substituting values of a, b, c in (v) give us:

$$(m^2-n^2)+ 2mn+ (m^2+n^2) =\frac{(m^2-n^2)(2mn)}{2}$$

Simplification gives us, $2=n(m-n)$ ………(j)

(j) Tells us n=1 and n=2

If n=1 then m=3 we get (**8, 6, 10**)

If n=2 then m=3 we get (**5, 12, 13**)

So, the length of sides of triangle will be (**8, 6, 10**) or (**5, 12, 13**).

MAIN RESULTS

Lemma #01

If (a, b, c) is a primitive Pythagorean triple, then (a, b) = (b, c) = (a, c) =1.

Proof:

Suppose that (x, y, z) is a primitive Pythagorean triple and (x, y)>1. Then, there is a prime p such that $p \mid a$ & $p \mid b$.

Since, $a^2+b^2=c^2$

And $p \mid a$ & $p \mid b$

then, $p \mid a^2+b^2$

Thus, $p \mid c^2 \Rightarrow p \mid c$.

This is a contradiction, because (a, b, c) =1.

Therefore, (a, b) =1. In a similar manner, (b, c) = (a, c) =1

Next we can establish a lemma about the parity of integers of a primitive Pythagorean triple.

Lemma #02

If (a, b, c) is a primitive Pythagorean triple, then a is even and b is odd or b is even and a is odd.

Proof:

Let (a, b, c) be a primitive Pythagorean triple. By lemma#01 we know that (a, b) =1

so that, a& b cannot both be even. Also a & b cannot both be odd. If a & b were both odd, then we would have

$$a^2 \equiv b^2 \equiv 1 \pmod 4$$

so that; $$c^2 = a^2 + b^2 \equiv 2 \pmod 4.$$

This is impossible.
Therefore a is even & b is odd or vice versa.

Theorem# 01.

The triple (a, b, c) of positive integers is a primitive Pythagorean triple, with b even, if and only if there are relatively prime positive integers m and n, m > n, with m odd and n even or m even and n odd, such that

$$a = m^2 - n^2,$$
$$b = 2mn,$$
$$c = m^2 + n^2.$$

Proof:

Let (a, b, c) be a primitive Pythagorean triple. From lemma#02 either a is even and b is odd or b is even and a is odd. However, given that b is even and hence x and z is odd, such that

$$r = \frac{c+a}{2} \text{ and } s = \frac{c-a}{2}$$

Since, $$a^2 + b^2 = c^2$$

$$\Longrightarrow b^2 = c^2 - a^2$$

$\Longrightarrow$ $b^2=(c+a)(c-a)$

$\Longrightarrow$ $b^2=2r.2s$

$\Longrightarrow$ $b^2 = 4rs$

Let r and s are both squares then, $r=m^2$, $s=n^2$

Now, writing r and s in terms of m and n we get,

$$a = r - s = m^2 - n^2$$
$$b = 4rs = 4\,m^2n^2$$

$$c=r+s= m^2+n^2$$

Since, $gcd\ (a, b, c) =1$ therefore, $gcd\ (m, n) =1$, because common divisor of m and n also divides $gcd\ (a, b, c)$. Further m and n cannot be both even, because difference of two even squares is also even. However, m and n cannot be both odd, because product of two integers is odd, where b is even.

Hence either m is odd and n is even or m is even and n is odd.

Let (x, y, z) forms primitive Pythagorean triple with m > n and (m, n) =1. So,

$$a =m^2-n^2,\ b=2mn,$$

$$c = m^2+n^2$$

Then,

$$a^2+b^2= (m^2-n^2)^2+ (2mn)^2$$

$$= m^4+n^4-2m^2n^2+4m^2n^2$$

$$= (m^2+n^2)^2$$

$$= z^2$$

⇒ (a, b, c) forms Pythagorean triple.

Now, let's prove $(a, b, c) =1$.Suppose, $(a, b, c) >1$

Then there is a prime p such that p|(a, b, c) but p≠2 because a and c is odd. Since, p|a and p|c then

p ∤b because b is even.

Hence, $(a, b, c) =1$ and is a primitive Pythagorean triple.

Theorem#02

Every Pythagorean triple is a multiple of primitive Pythagorean triple.

Proof:

Let (a, b, c) be an arbitrary Pythagorean triple, with $gcd\ (a, b, c) = d$.

Then, $a = da_1,\ \ b = db_1,\ c = dc_1$

For some $a_1, b_1, c_1 \in \mathbb{Z}$ with $gcd\ (a_1, b_1, c_1) = 1$

Since, $a_1^2 + b_1^2 = c_1^2$

Hence (a_1, b_1, c_1) is a primitive Pythagorean triple.

$$\Longrightarrow \qquad (a, b, c) = d\ (a_1, b_1, c_1)$$

Theorum#03

Let (a, b, c) be a primitive Pythagorean triple. Then, (ab, (a+b) c, c^2+ab) is also a primitive Pythagorean tipple.

Proof:

First show that (a, b, c) is a Pythagorean triple, then $(ab, (a+b)c, c^2+ab)$ is a Pythagorean triple. We know that,

$$a^2+b^2=c^2 \quad \longrightarrow \qquad (2.1)$$

$$\begin{aligned} so, \quad (ab)^2+((a+b)c)^2 &= a^2b^2+(a^2+b^2+2ab)c^2 \\ &= a^2b^2+(c^2+2ab)\,c^2 \\ &= (ab)^2+2(ab)c^2+(c^2)^2 \\ &= (ab+c^2)^2 \end{aligned}$$

Hence, proved that (ab, (a+b) c, c^2+ab) is a Pythagorean triple.

Now, Let p be a prime number such that p divides ab and $c^2 + ab$. Then p must divide their

difference which is c^2. Since p is a prime, p must divide c. Similarly, if p divides ab, then p must divide a or b. There are two cases.

Case-I. If p divides both a and c, then it follows, from the equation (2.1), that p must divide b.
This is a contradiction, because (a, b, c) is a primitive triple.

Case-II. If p divides both b and c, then p must divide a by the equation (2.1).
Again a contradiction, because (a, b, c) is a primitive triple.

We conclude that the greatest common divisor of numbers ab and c^2+ab must be equal to 1.
Therefore, we proved that (ab, (a+b)c, c2 +ab) is a primitive Pythagorean triple.

This completes the proof of the Theorem

Theorem#04

> ***Corresponding to every odd positive integer n, (2n, n^2-1, n^2+1) is a Pythagorean triple***

Proof:

Suppose, $x=2n,\ y=n^2-1,\ z=n^2+1$

Then,

$$x^2+y^2=4n^2+n^4-2n^2+1$$
$$=n^4+2n^2+1$$
$$=(n^2+1)^2$$
$$=z^2$$

Hence, proved that *(2n, n^2-1, n^2+1)* is a Pythagorean triple

APPLICATIONS

COROLLARY:

FROM THEORM# 2

Corollary statement: *There are infinite many primitive Pythagorean triples.*

Proof:

Since primitive Pythagorean triples can be written as:

$X = m^2 - n^2$

$Y = 2mn$

$Z = m^2 + n^2$

Where (m,n) =1 and m, n≠0

Since there are infinite values that can be assigned to m and n. Therefore, there must be an infinite number of primitive Pythagorean triples.

FROM THEORM# 2

Corollary statement: *Let (a, b, c) be a primitive Pythagorean triple then,*

$$(ab, |a-b|c, c^2 - ab)$$

Is a Pythagorean triple too?

Proof:

Let (a, b, c) is a Pythagorean triples, then $(ab, |a-b|c, c^2 - ab)$is also a Pythagorean triple.

$$a^2+b^2=c^2 \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

so, $(ab)^2 + [(|a-b|)c]^2 = a^2b^2 + (a^2 - 2ab + b^2)c^2$

$= a^2b^2 + (c^2 - 2ab)c^2$

$= (ab)^2 - 2(ab)c^2 + (c^2)^2$

$= (ab - c^2)^2 \qquad = (c^2 - ab)^2$

Hence, proved that $(ab, |a-b|c, c^2 - ab)$ is a Pythagorean triple.

Now, let p be a prime number such that p divides ab and $c^2 - ab$. Then p must divide their summation which is c^2. Since p is a prime, p must divide c. Similarly, if p divides ab, then p must divide a or b. There are two cases.

Case- 1. If p divides both a and c, then it follows, from the equation (1), that p must divide b. This is a contradiction, because (a, b, c) is a primitive triple.

Case 2. If p divides both b and c, then p must divide a by the equation (1). Again, a contradiction, because (a, b, c) is a primitive triple.

We conclude that the greatest common divisor of numbers ab and $c^2 - ab$ must be equal to 1. Therefore, we provedthat $(ab, |a-b|c, c^2-ab)$ is a primitive Pythagorean triple.

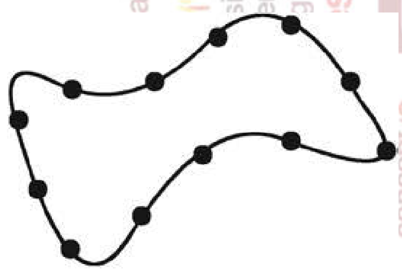
APPLICATIONS:

❒ BABYLONIAN TABLETS:

The study of Pythagorean triples began long before the time of Pythagoras. There are Babylonian tablets that contain lists of such triples, including quite large ones, indicating that the Babylonians probably had a systematic method for producing them.

❒ CONSTRUCTION OF RIGHT ANGLE:

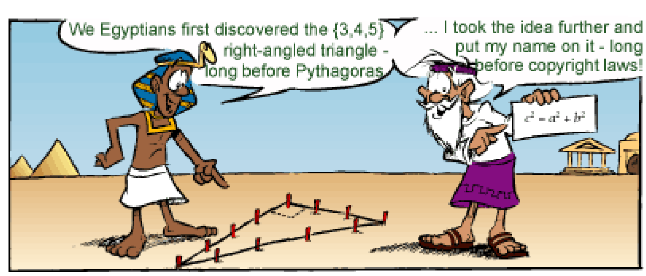
Pythagorean triples were also used in ancient Egypt. For example, a rough-and-ready way to produce a right angle is to take a piece of string mark it into 12 equal segments, tie it into the loop, and hold it taut in the form of a 3-4-5 triangle, as illustrated in figure. This provides an inexpensive right-angle tool for use on small construction project.



String with 12 knots

string pulled taut

❒ DATA ENCRYPTION AND DECRYPTION (CRYPTOGRAPHY):



We can encrypt and decrypt a text by using the New Pythagorean Triple Algorithm formulas for creating the key.

If we want to encrypt a message, we will use the formula:

$$c = m + k(mod26)$$

If we want to decrypt a message, we use:

$$m = c - k(mod26)$$

Now for creating a key, the numbers p and q are put within the New Pythagorean Triple Algorithm formulas given below to create the key.

$$x1 = 2p^2 + 2pq \quad x2 = 2p^2 - 2pq \quad x3 = 2pq$$

$$y1 = q^2 + 2pq \quad y2 = q^2 - 2pq \quad y3 = p^2 - q^2$$

$$z1 = 2p^2 + q^2 + 2pq \quad z2 = 2p^2 + q^2 - 2pq \quad z3 = p^2 + q^2$$

After we have found the values:

$$(x1, y1, z1), (x2, y2, z2), (x3, y3, z3)(mod26)$$

We can freely create the encryption key in the form:

$$x1, y1, z1, x2, y2, z2, x3, y3, z3$$

EXERCISES:

- From the book of Kenneth H. Rosen, Elementary Number Theory (Sixth Edition) we have 13th chapter **(Some nonlinear Diophantine equations)** related to Pythagorean triples. Only exercise 13.1 is related to the topics that we had cover. Here is a list of the questions related to the topics:
  - *Primitive Pythagorean triples:*
    - ✓ From question 1 to 7
    - ✓ From question 11 to 14 and also question 17, 18
  - *Nonlinear Diophantine equations and Pythagorean triples:*
    - ✓ From question 8 to 10
    - ✓ From question 15 to 16
  - *Unit circle and Pythagorean triples:*
    - ✓ From question 19 to 27

- From the book of Joseph H. Silverman, a Friendly Introduction to Number Theory (Fourth Edition) we have chapter no. 2**(Pythagorean Triples)** related to Pythagorean triples. Here is the list of the questions:
  - *Pythagorean triples:*
    - ✓ From question 1 to 9
- From the book of David M. Burton, Elementary Number Theory (Sixth Edition) we have chapter no. 12 **(Certain nonlinear Diophantine equations)** related to Pythagorean triples. Only exercise 12.1 is related to our topic. Here is the list of the question:
  - *Pythagorean triples:*
    - ✓ From question 1 to 13

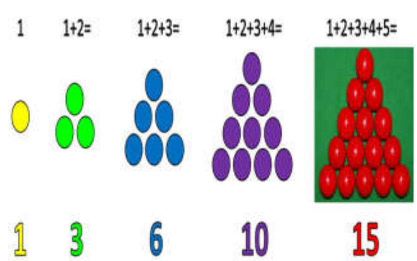
CONCLUSION:

REASONABLE NEXT STEP:

- When problems related to Pythagorean triples theorems comes, and then we are able to understand and solve them.
- It has applications in cryptography, when we read about cryptography Pythagorean Triple will be helpful.

WHAT WE LEARNED:

We learned that Pythagorean triples are the integers that fit the formula for the Pythagorean Theorem .these are whole numbers that can't be decimals because all Pythagorean triples solve the formula for the Pythagorean Theorem, we can take any Pythagorean triple and make a right triangle out of it.



Moreover, there are infinitely many Pythagorean triples and the proof was base on the fact that the difference of the squares of any two consecutive (one after the other) numbers is always an odd number.

$$2^2 - 1^2 = 4 - 1 = 3 \text{ (an odd number)}$$

$3^2 - 2^2 = 9 - 4 = 5$ (an odd number)

$4^2 - 3^2 = 16 - 9 = 7$ 9an odd number)

And there are an infinite numbers of odd numbers. Since the perfect squares from a subset of the odd numbers, and a fraction of infinity is also infinity, it follows that there must also be an infinite number of odd squares. So there are an infinite number of Pythagorean Triples.

We also construct sets of Pythagorean triples. When m & n are any two integers (m > n):

- $a = m^2 - n^2$
- $b = 2mn$
- $C = m^2 + n$

Then a, b & c form a Pythagorean triples.

Example: m = 2, n = 1

$a = 2^2 - 1^2 = 4 - 1 = 3$

$b = 2.2.1 = 4$

$c = 2^2 + 1^2 = 4+1 = 5$

Thus we obtain the first Pythagorean triple.

We also study about primitive and non-primitive Pythagorean triples, Pythagorean triple generator and solution of this theorem. By using lemma we prove that these are primitive Pythagorean triplet. By using lemma we are able to show new ways of constructing primitive Pythagorean triplet from pre determined Pythagorean triplet.

We also come to know about another interesting fact that Pythagorean also consists of

- All even numbers, or
- Two odd numbers and even numbers.

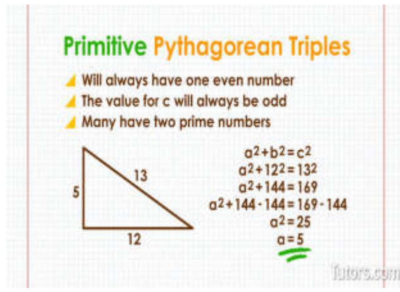
A Pythagorean triple can never be made up of all odd numbers and one odd number. This is true because

- The square of an odd number is an odd number and the square of an even number is an even number.
- The sum of two even number is an even number and the sum of an odd number and an even number is an odd number,

So when both 'a' and 'b' is even then 'c' is even too.

Similarly, when of ‘a’ and ‘b’ is odd number and other is even, ‘c’ has to be odd!

We also are able to know that primitive Pythagorean triple will always have one even number. A set of numbers is considered as a non- primitive Pythagorean triple if all the three numbers in the triples have a common divisor.



WHAT DO YOU WISH YOU HAD LEARNED BETTER:

First we study only about Pythagoras theorem, its derivation and how to solve question related to Pythagoras theorem.

Now we’ll be able to use Pythagorean triple in lemma’s and corollary and in theorems.