Freie Universität Berlin
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Project 2 (Pattern Formation)
Complex Systems
SS 2025

Deadline: June 12, 10:00 (**before** the lecture)

The Project should be worked out in groups and presented on June 12, 10:00. The presentation should include the description of the task (1-2 sentences), and must address the questions outlined below. You may optionally describe problems and enlightenments, if you encountered them along the way.
Along this research task you may check your programming codes as outlined below. These intermediate tasks serve as check-points for you to verify that your codes are correct

Homework 1 (Modelling, Simulation, Programming; Analysis)
Modelling & Simulation: Let us consider the following model (predator-prey):

$$R_1 : z1 \xrightarrow{r_0} z1 + z1 \qquad R_3 : z1 + z2 \xrightarrow{r_3} z2 + z2 \tag{1}$$
$$R_2 : z1 + z2 \xrightarrow{r_2} z2 \qquad R_4 : z2 \xrightarrow{r_3} \varnothing \tag{2}$$

where $z1$ and $z2$ denote the prey and the predator respectively. Reaction rates $r_1 \dots r_4$ are given as:

$$\begin{aligned} r_1 &= \quad k_1 \cdot z1 \quad , \quad r_3 = k_3 \cdot z1 \cdot z2 \\ r_2 &= k_2 \cdot z1 \cdot z2 \quad , \quad r_4 = k_4 \cdot z2 \end{aligned}$$

with parameters $k_1 = 0.3$, $k_2 = 0.01$, $k_3 = 0.01$ and $k_4 = 0.3$.
a) (**1D-deterministic simulation; to be uploaded via Whiteboard**). Implement the model above and solve the associated ODEs for initial state $z1(t_0) = 28$ and $z2(t_0) = 10$ for 100 time units. Plot the dynamical behaviour and discuss it in the report and supported by figures.
b) **2D-stochastic simulation; to be uploaded via Whiteboard** (Analysis to be documented in the report, discussed and supported by figures). Perform 2D stochastic simulations of the predator-prey model above on a 100×100 units 2D continuous space. For the 2D-model, set $k_4 = 0.1$, $k_1 = 0.1 \cdot (1 - |x_1(t)|/10000)$, where $|x_1(t)|$ denotes the number of prey at the current time t.
Initialize. Initialise the population of 100 prey ($z1$) and 20 predators ($z2$) by assigning uniformly distributed initial positions in 2D space. Tipp: Save the (x,y) position of each currently existing prey and predator in a list, e.g. $z1(i)_{x,y}$!
Update.
a) Update the positions of each predator and prey with a zero-centered Gaussian, whereby the prey and predator have $\sigma_{z1} = 10 = \sigma_{z1}$. You can use clipped boundaries. I.e., if the updates 'x' position was smaller 0, set to 0 and if greater 100, set to 100.
b) Check whether predator and prey are within a 'collision distance' of 20 square units, i.e., check for each predator z_2 whether $(z2(i)_{x,*} - z1(j)_{x,*})^2 + ((z2(i)_{*,y} - z1(j)_{*,y})^2 < 20$. Remove the prey within collision distance (akin to reaction rate $r_2 + r_3$) and with probability 0.5 produce a predator offspring.
c) Draw the positions of the predators and prey.

Simulate and visualize.
Simulate the 2D model above for 100 time steps and make a movie out of it.
d) Change some parameters. I.e. set $\sigma_{z1} = 2$, $\sigma_{z1} = 5$, $k_4 = 0.2$. Set the 'collision distance' to 10.
Discuss in the report & support your statements with figures:

- What fixed points exist for the ODE model and which do you observe in the ODE simulations (why)?
- Which dynamical behaviour did you observe for the stochastic (2D-) model and why?
- Are you satisfied with your results? (Why/Why not?)
- What could be the reasons?
- How does changing the parameters of the model affect the results?

Good luck!