

**Project 2 (Pattern Formation)**  
**Complex Systems**  
**SS 2025**

---

Deadline: **June XX, 10:00** (before the lecture)

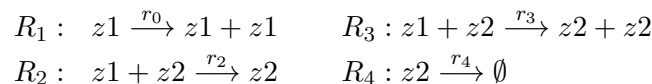
---

*The Project should be worked out in groups and presented on **June XX, 10:00**. The presentation should include the description of the task (1-2 sentences), and must address the questions outlined below. You may optionally describe problems and enlightenments, if you encountered them along the way.*

*Along this research task you may check your programming codes as outlined below. These intermediate tasks serve as check-points for you to verify that your codes are correct*

**Homework 1 (Modelling, Simulation, Programming; Analysis)**

**Modelling & Simulation:** Let us consider the following model (predator-prey):



where  $z1$  and  $z2$  denote the prey and the predator respectively. Reaction rates  $r_1 \dots r_4$  are given as:

$$\begin{array}{ll} r_1 = k_1 \cdot z1 & , \quad r_3 = k_3 \cdot z1 \cdot z2 \\ r_2 = k_2 \cdot z1 \cdot z2 & , \quad r_4 = k_4 \cdot z2 \end{array}$$

with parameters  $k_1 = 0.3$ ,  $k_2 = 0.01$ ,  $k_3 = 0.01$  and  $k_4 = 0.3$ .

a) **(1D-deterministic simulation; to be uploaded via Whiteboard)**. Implement the model above and solve the associated ODEs for initial state  $z1(t_0) = 28$  and  $z2(t_0) = 10$  for 100 time units. Plot the dynamical behaviour and discuss it in the report and supported by figures.

b) **2D-stochastic simulation; to be uploaded via Whiteboard** (Analysis to be documented in the report, discussed and supported by figures). Perform 2D stochastic simulations of the predator-prey model above on a  $100 \times 100$  units 2D continuous space. For the 2D-model, set  $k_4 = 0.1$ ,  $k_1 = 0.1 \cot(1 - |x_1(t)|/10000)$ , where  $|x_1(t)|$  denotes the number of prey at the current time  $t$ .

**Initialize.** Initialise the population of 100 prey ( $z1$ ) and 20 predators ( $z2$ ) by assigning uniformly distributed initial positions in 2D space. Tipp: Save the (x,y) position of each currently existing prey and predator in a list, e.g.  $z1(i)_{x,y}$ !

**Update.**

a) Update the positions of each predator and prey with a zero-centered Gaussian, whereby the prey and predator have  $\sigma_{z1} = 10 = \sigma_{z2}$ . You can use clipped boundaries. I.e., if the updates 'x' position was smaller 0, set to 0 and if greater 100, set to 100.

b) Check whether predator and prey are within a 'collision distance' of 20 square units, i.e., check for each predator  $z2$  whether  $(z2(i)_{x,*} - z1(j)_{x,*})^2 + (z2(i)_{*,y} - z1(j)_{*,y})^2 < 20$ . Remove the prey within collision distance (akin to reaction rate  $r_2 + r_3$ ) and with probability 0.5 produce a predator offspring.

c) Draw the positions of the predators and prey.

**Simulate and visualize.**

Simulate the 2D model above for 100 time steps and make a movie out of it.

d) Change some parameters. I.e. set  $\sigma_{z1} = 2$ ,  $\sigma_{z2} = 5$ ,  $k_4 = 0.2$ . Set the 'collision distance' to 10.

**Discuss in the report & support your statements with figures:**

- What fixed points exist for the ODE model and which do you observe in the ODE simulations (why)?
- Which dynamical behaviour did you observe for the stochastic (2D-) model and why?
- Are you satisfied with your results? (Why/Why not?)
- What could be the reasons?
- How does changing the parameters of the model affect the results?

Good luck!