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The Most Comprehensive
Preparation App For All Exams

CI RCLE

Part-I

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Agenda → Circles Part I

* Basic Concepts &

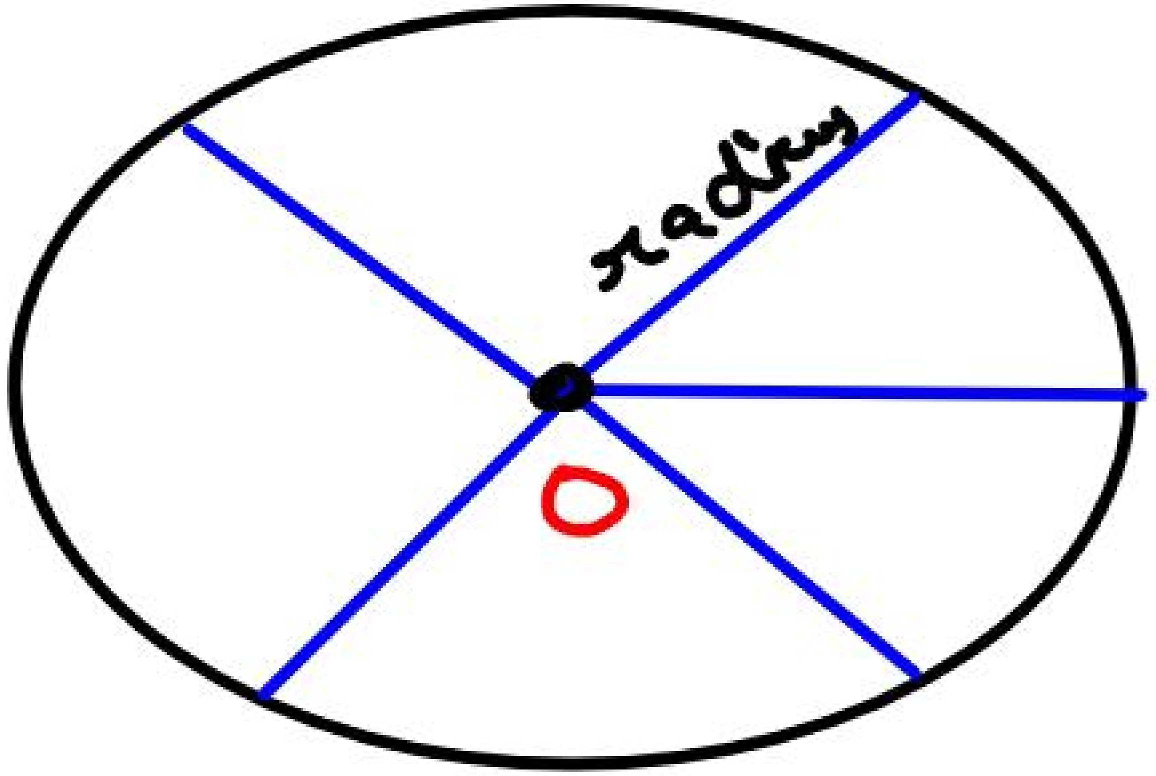
Theorems of Circles

Homework → 25 Questions

BASIC TERMINOLOGIES

Circle is a collection of all those points which are **at a fixed distance from a certain given point**.

O → centre



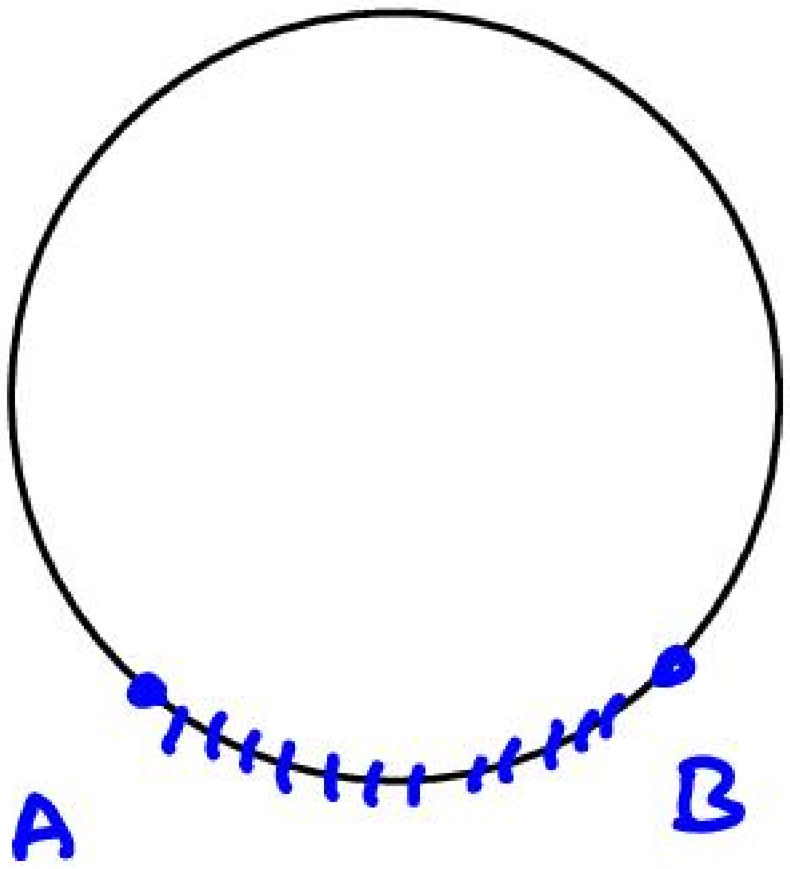
Centre

Circumference

Radius

Arc

$\overset{\frown}{AB}$



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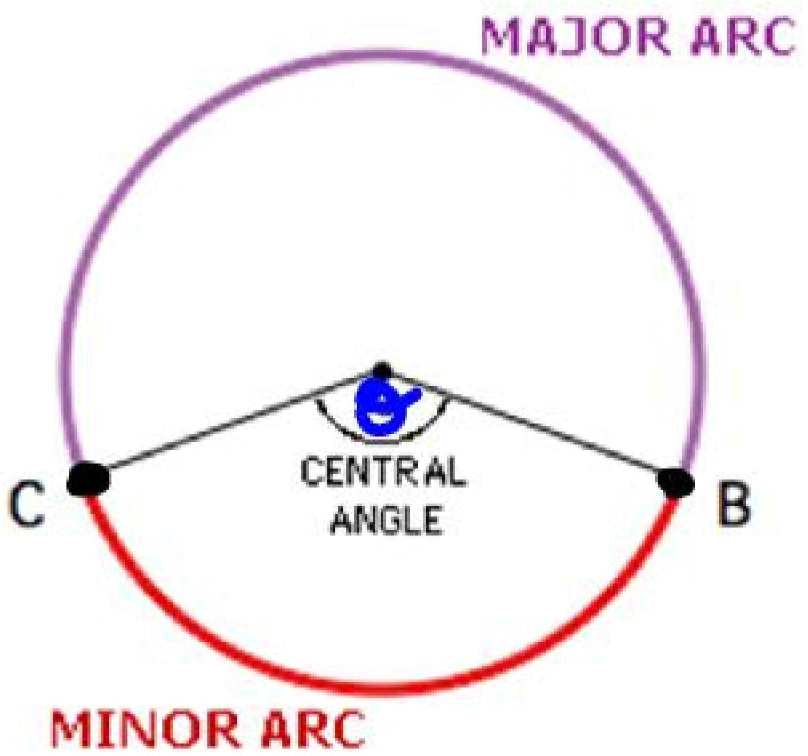
Minor Arc & Major Arc



$$\text{Length of Arc} = \frac{2\pi r\theta}{360}$$

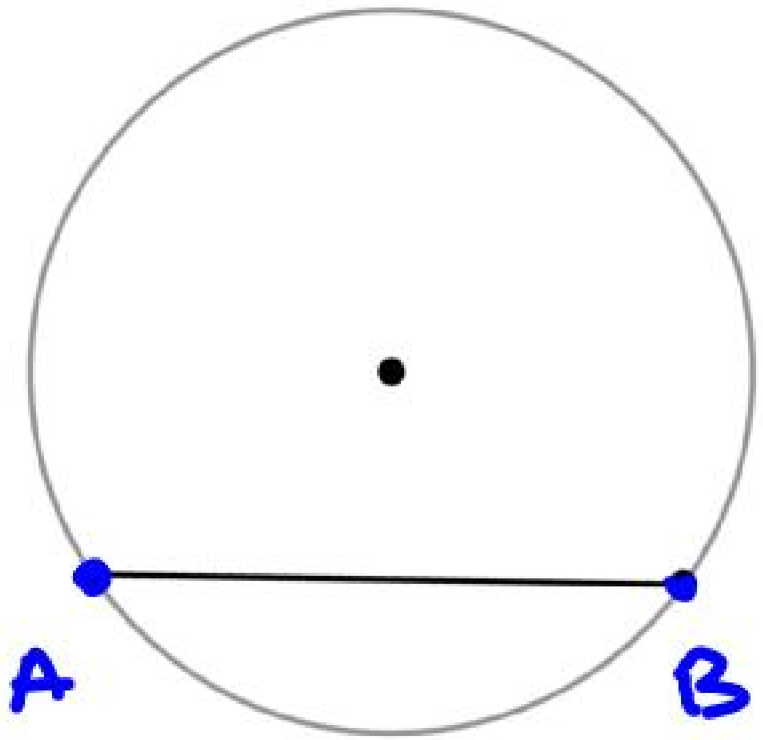
If nothing is given in the question, it is always considered as minor arc.

$360^\circ \rightarrow 2\pi r$

$1^\circ \rightarrow \dfrac{2\pi r}{360}$

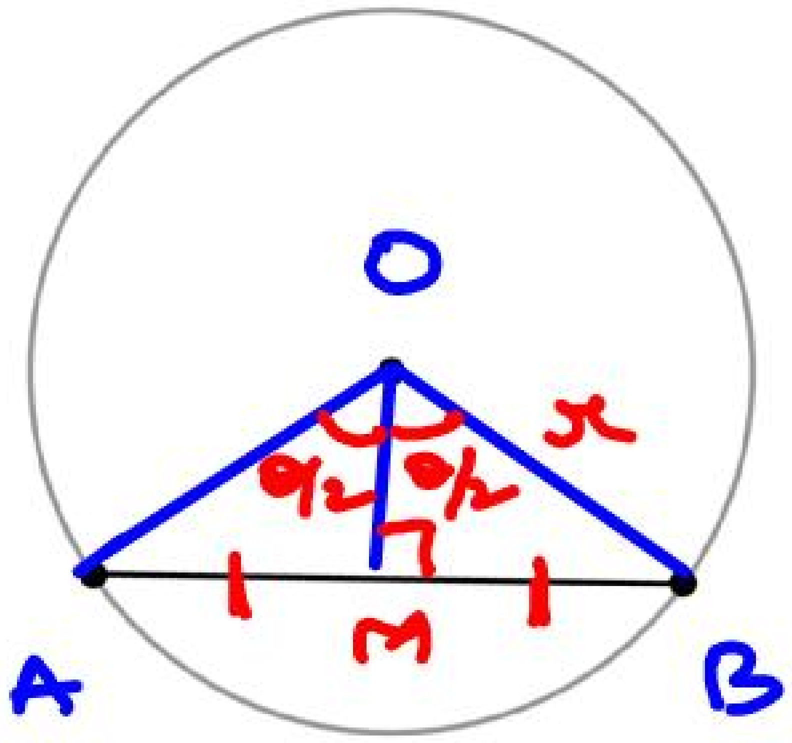
$\theta \rightarrow \dfrac{2\pi r\theta}{360}$

Chord



$\overline{AB}$

Length of a chord of a circle

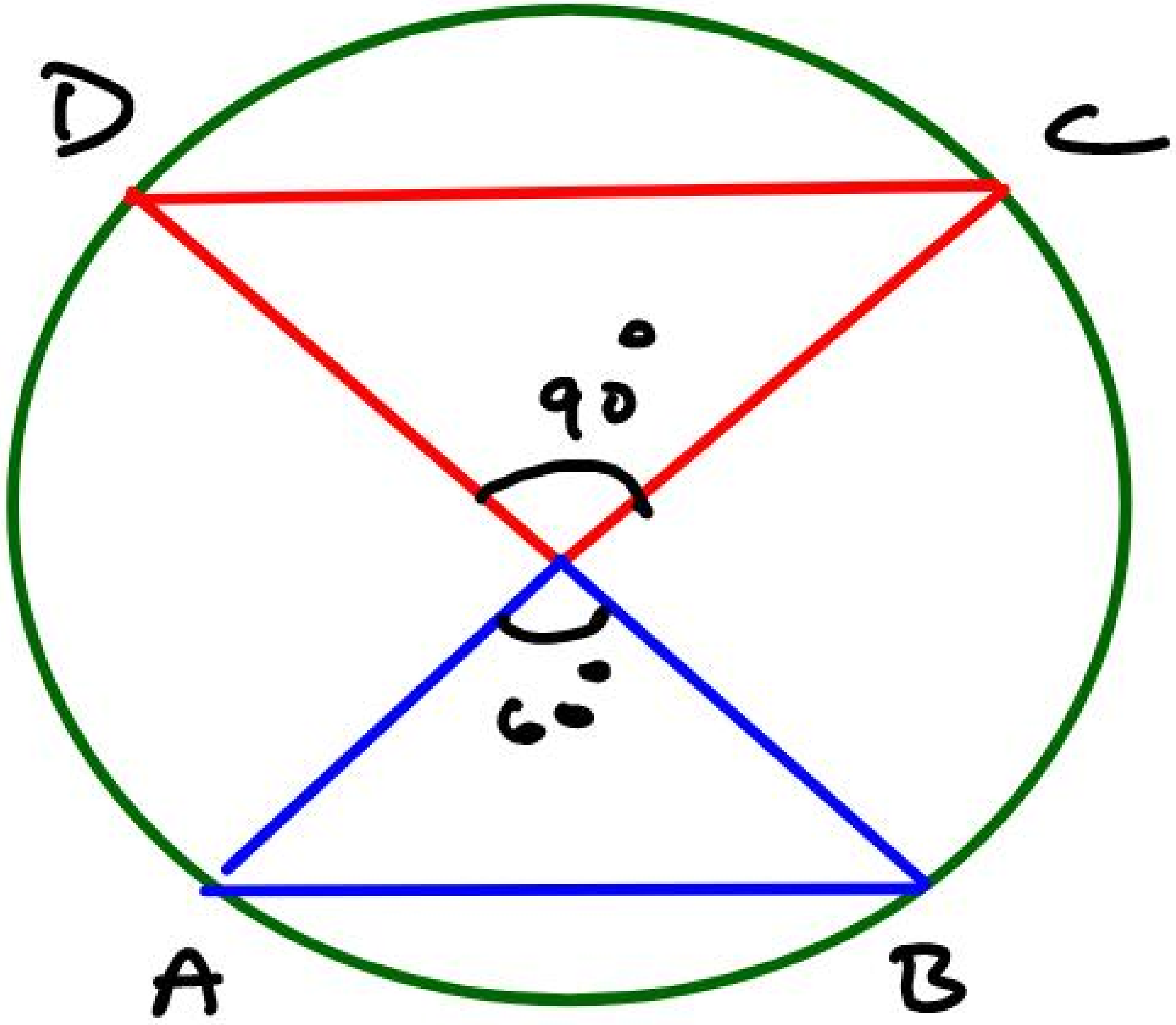


$\triangle OMB$

$$\sin\theta/2 = \frac{MB}{OB}$$

$$MB = r\sin\theta/2$$

$$\boxed{AB = 2r\sin\theta/2}$$

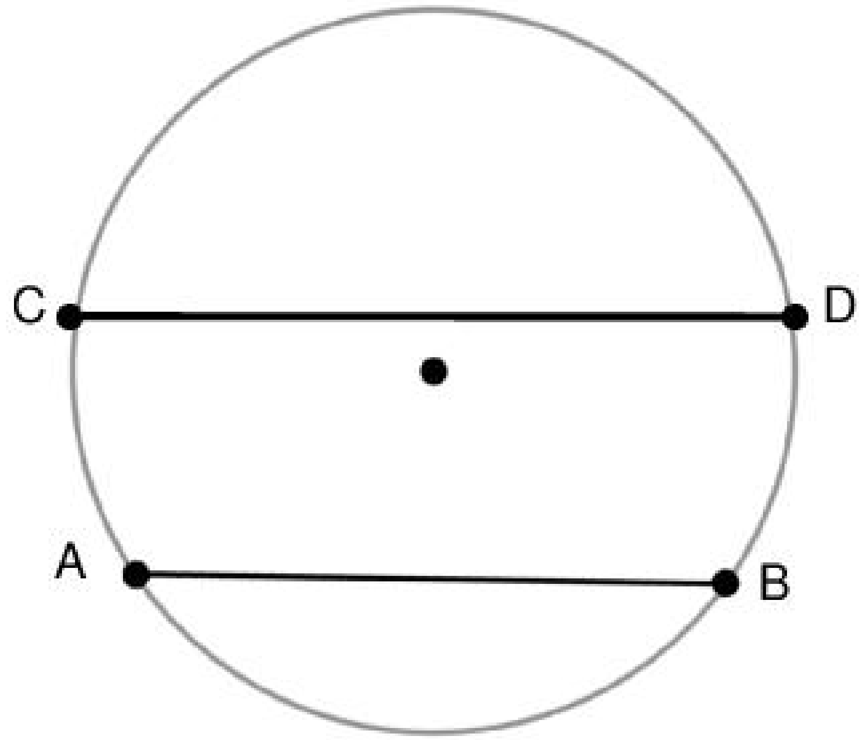


Find $\frac{CD}{AB}$ = ??

$$\frac{\cancel{2r}\sin 45}{\cancel{2r}\sin 30}$$

$$\frac{1}{\sqrt{2}} \cdot 2 = \frac{\sqrt{2}}{1}$$

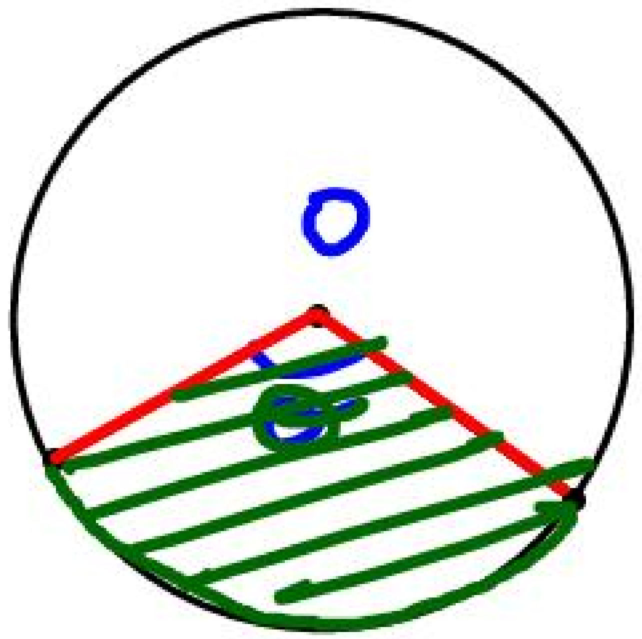
Chord, which is more closer to the centre is larger.



CD > AB

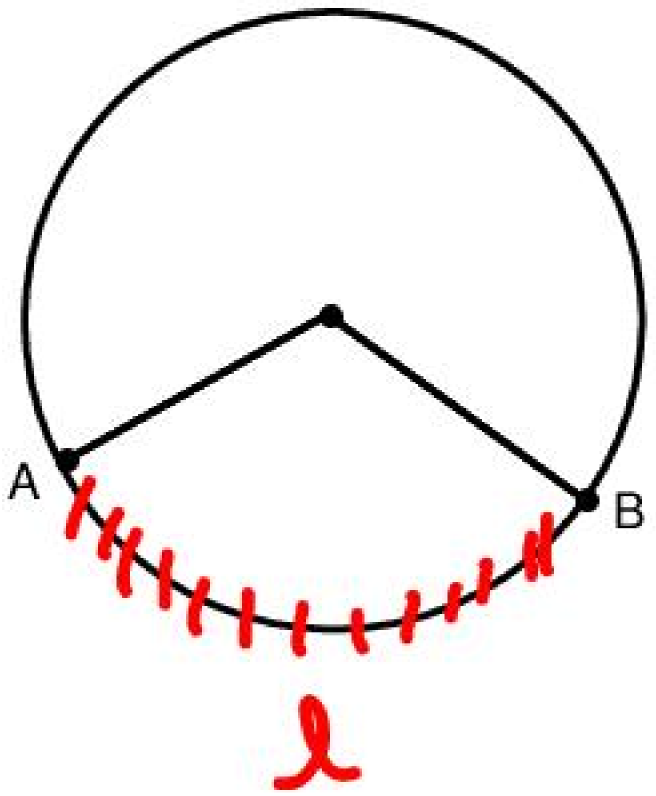
* Diameter is the larger chord

SECTOR OF A CIRCLE



AREA OF SECTOR = $\dfrac{\pi r^2 \theta}{360^\circ}$

Length of the Arc AB (l) = $\dfrac{2\pi r\theta}{360^\circ}$



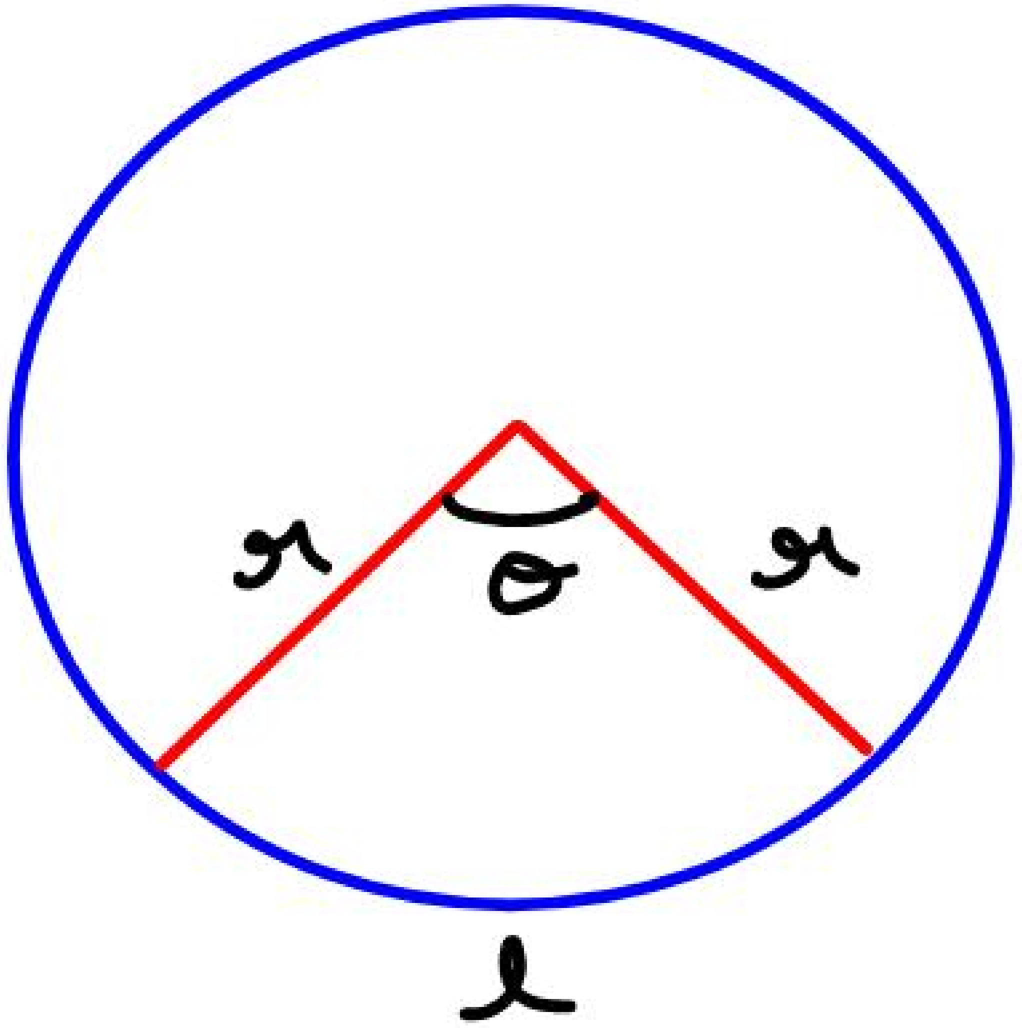
Area of sector = $\dfrac{1}{2}lr$

$$A = \frac{\pi r^2 \theta}{360}$$

$$l = \frac{2\pi r \theta}{360}$$

$$\boxed{A = \frac{1}{2} l r}$$

$$\frac{A}{l} = \frac{r}{2}$$



If $l = 6cm$

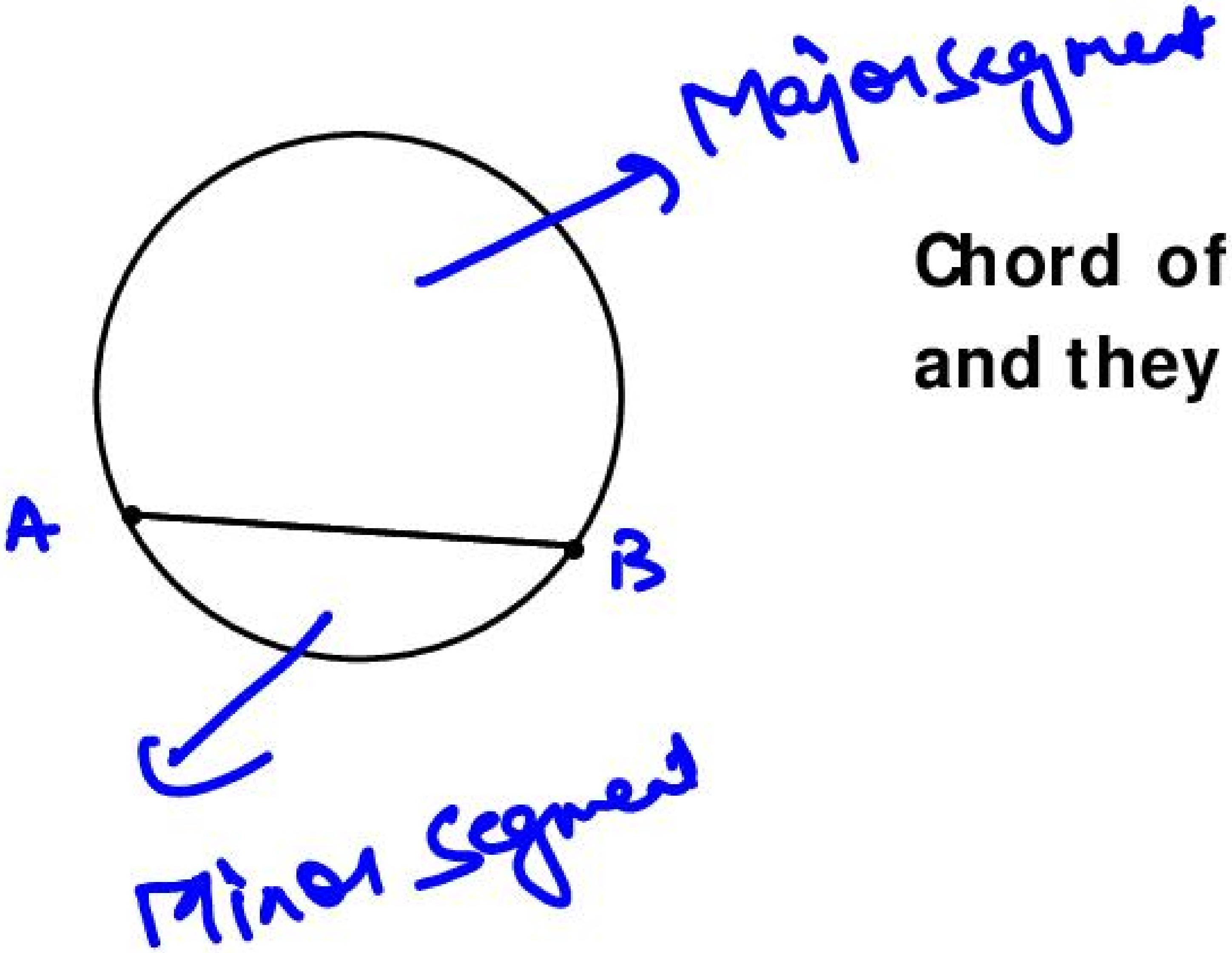
$r = 5cm$

Area of sector ??

$\frac{1}{2} \cdot 6 \cdot 5$

$= \underline{15cm^2}$

SEGMENT OF A CIRCLE



Chord of a circle divides a circle in 2 segments and they are alternate segments to each other.

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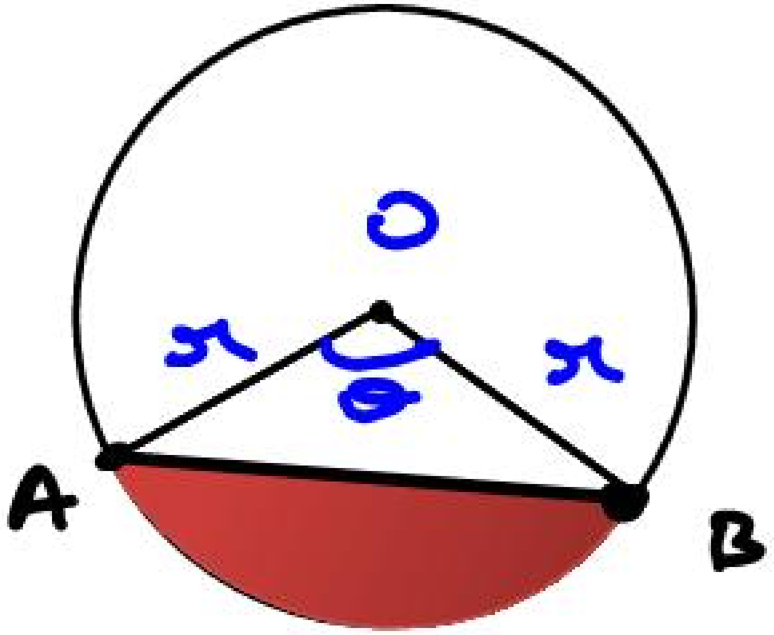
AREA OF SEGMENT = Area of sector – Area of ΔAOB

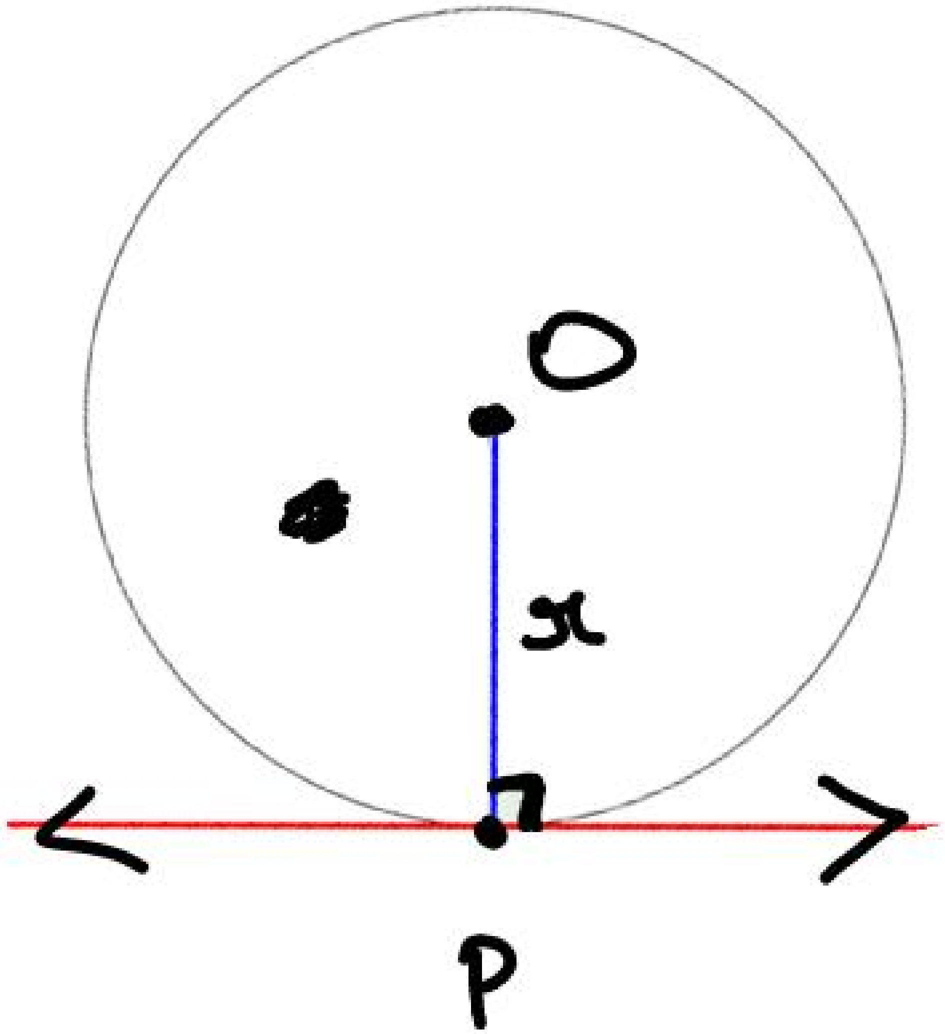


ans

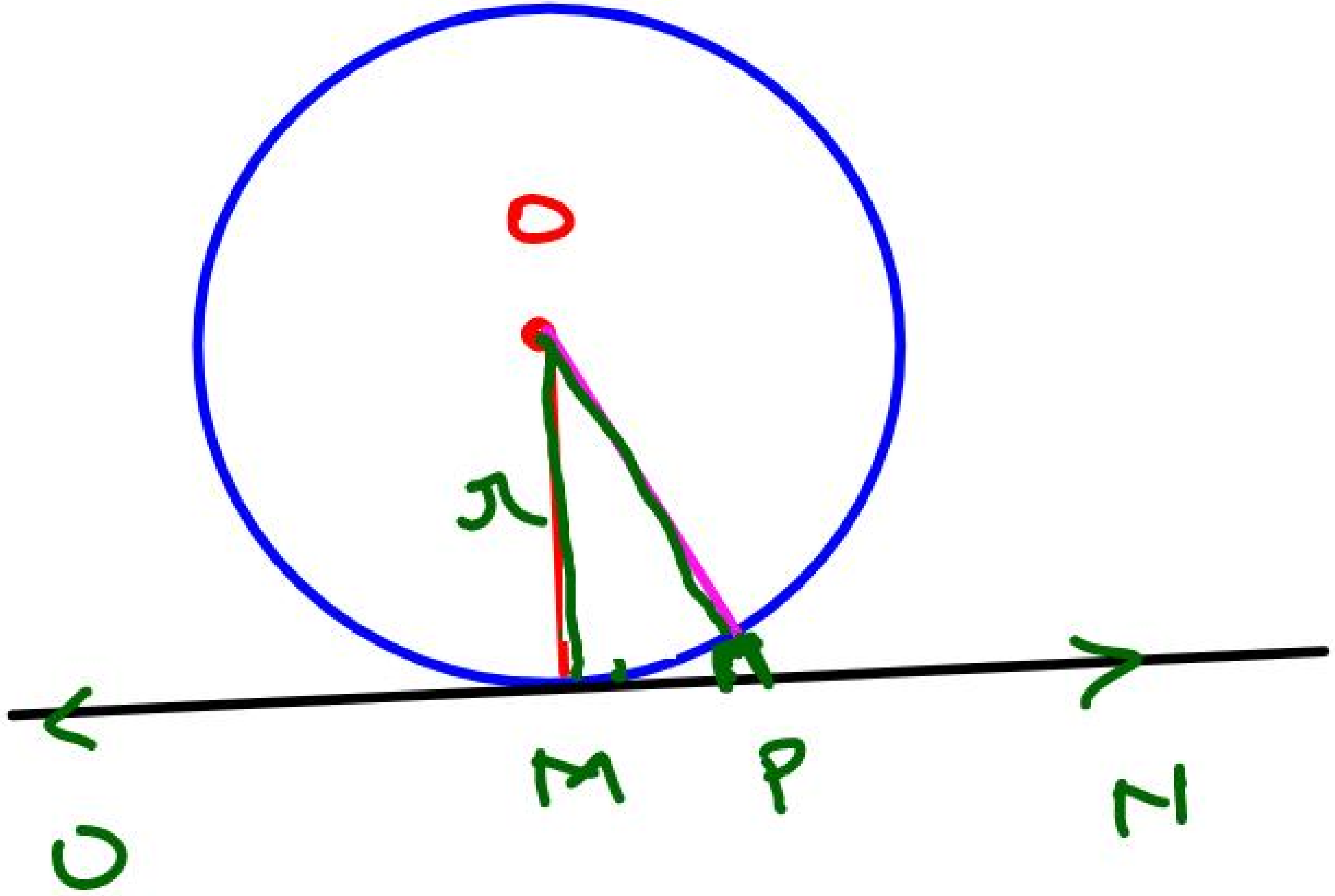
$$\frac{\pi r^2\theta}{360^\circ} - \frac{1}{2}r^2\sin\theta$$

TANGENT

A tangent is a line that touches the circle at exactly one point.



P → Point of contact



To prove

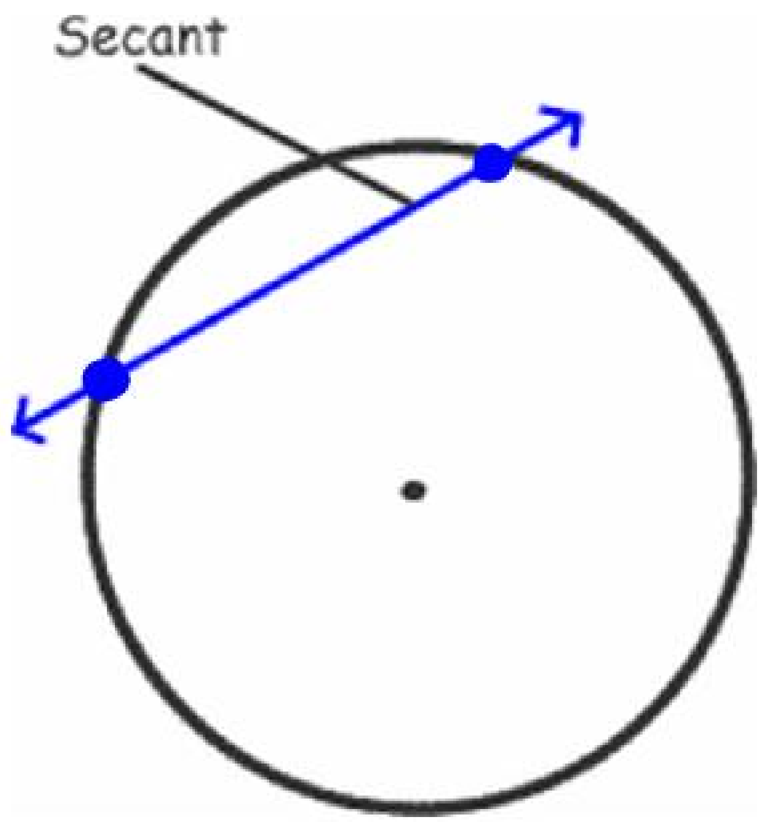
$\angle OMN = 90^\circ$

let $\angle OMN \neq 90^\circ$

$OP \perp OM$

ΔOMP

SECANT



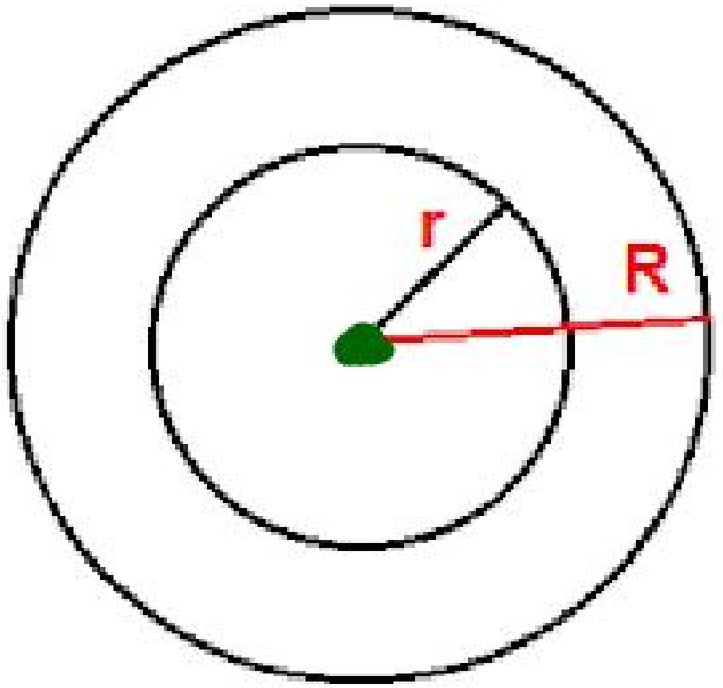
A secant is a line which intersect the circle at 2 distinct points.

DIFFERENCE BETWEEN A CHORD AND A SECANT

chord → " line segment"

secant → " line"

CONCENTRIC CIRCLES



Circles with the same centre.

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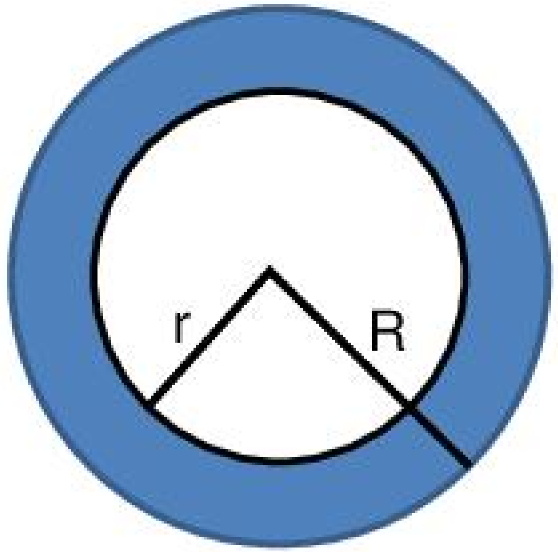
AREA ENCLOSED BY TWO CONCENTRIC CIRCLES



If R and r are radii of two concentric circles, then

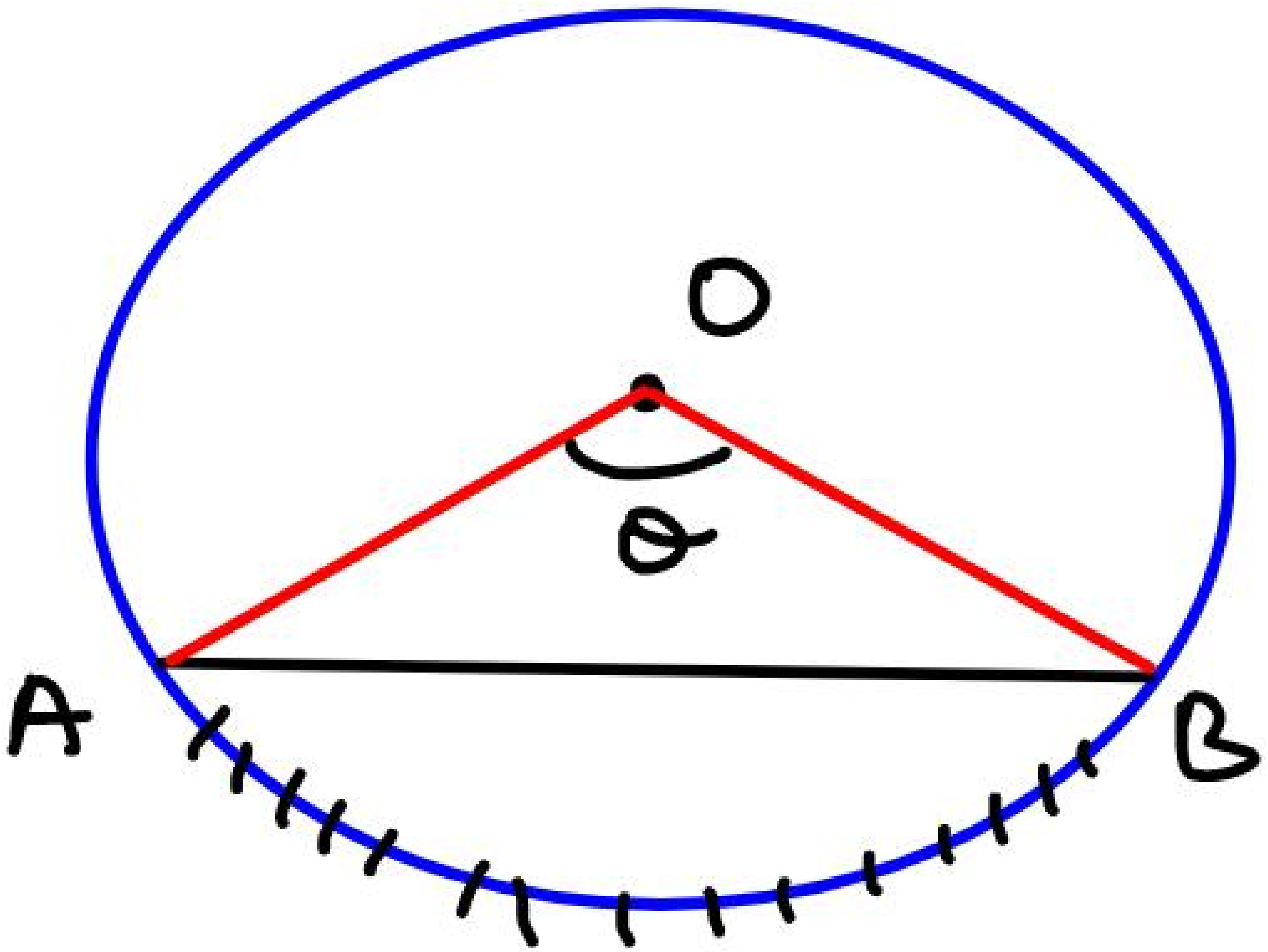
Area enclosed by the two circles $= \pi \boldsymbol{R}^2 - \pi \boldsymbol{r}^2$

$$= \pi\left(\boldsymbol{R}^2 - \boldsymbol{r}^2\right)$$

$$= \pi\left(\boldsymbol{R} + \boldsymbol{r}\right)\left(\boldsymbol{R} - \boldsymbol{r}\right)$$

Congruent Circle : Circle having same radius.

BASIC THEOREMS RELATED TO CIRCLE



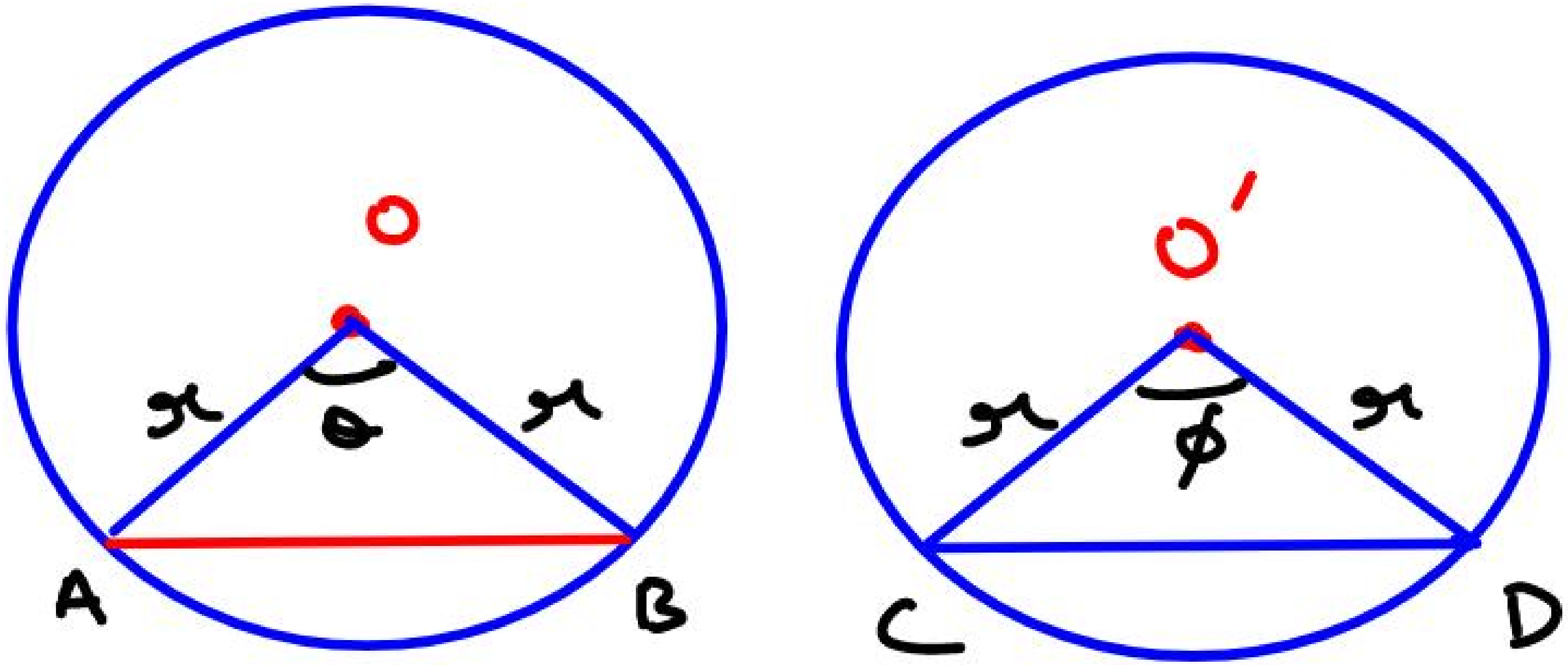
$AB \rightarrow$ chord

$\overset{\frown}{AB} \rightarrow$ Arc

θ

1. In same circle / Congruent circle

(i) Equal chords of a circle subtends equal angle at the centre.



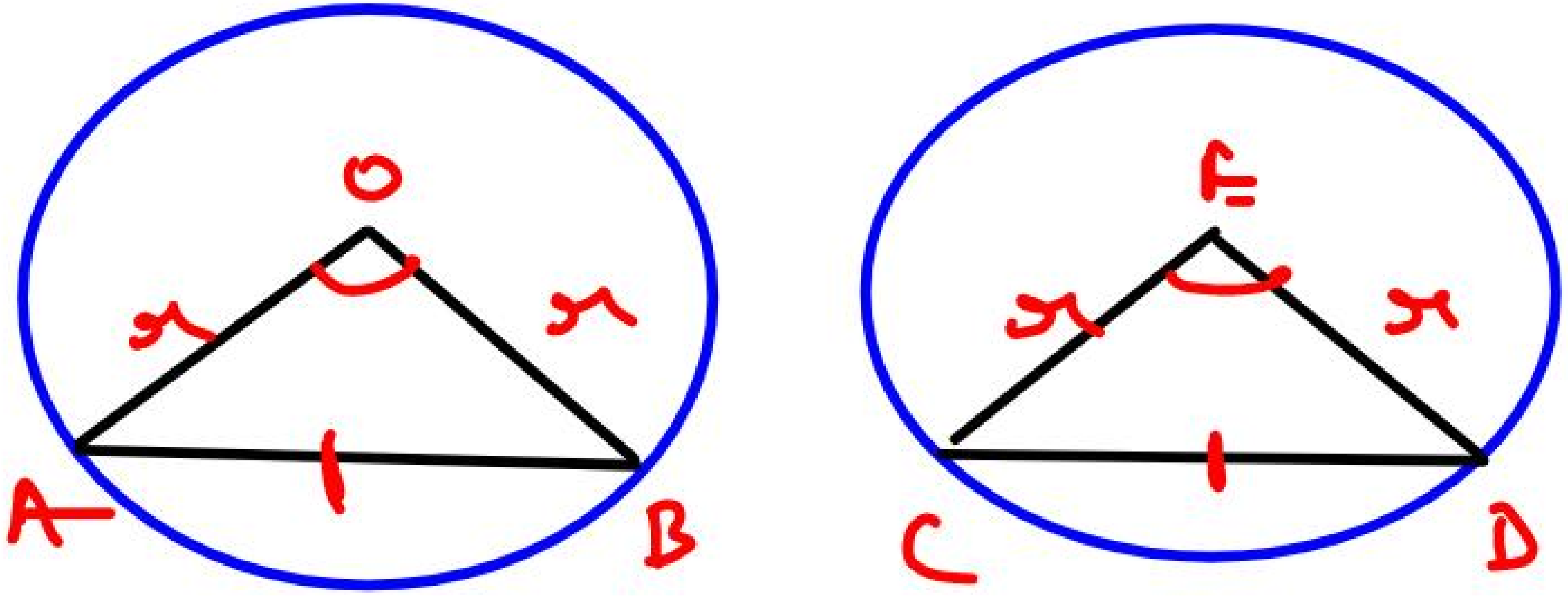
$AB = CD$

$\overset{\frown}{AB} = \overset{\frown}{CD}$

$\theta = \phi$

* If out of three component any one is equal than remaining two are equal.

Equal chord subtends equal angles at centre

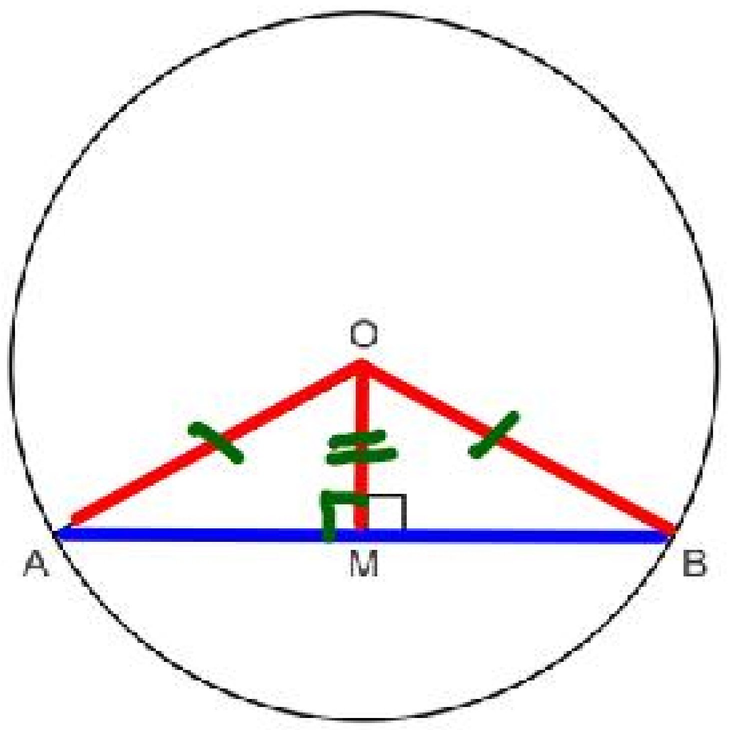


$AB = CD$

$\triangle AOB \cong \triangle CED$

$\angle AOB = \angle CED$

2. **Perpendicular dropped from the centre of a circle bisect the chord.**



Given, OM ⊥ AB

AM = MB

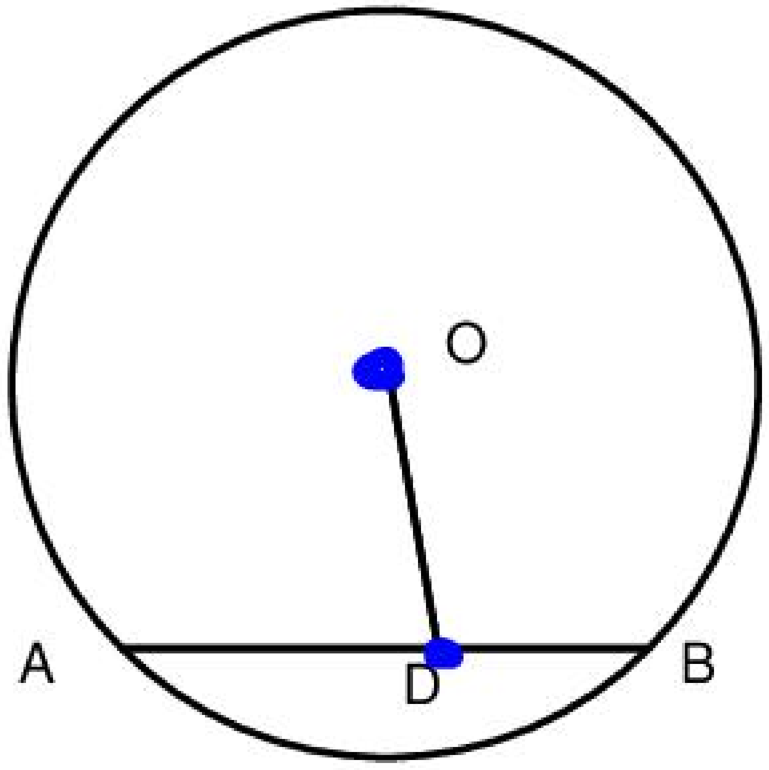
Given
→ O is centre
OM ⊥ AB
To prove AM = MB

$\Delta OMA \cong \Delta OMB$
[By RHS]

$\boxed{MA = MB}$

Converse

The line joining the centre of a circle to the mid-point of a chord is perpendicular to the chord.

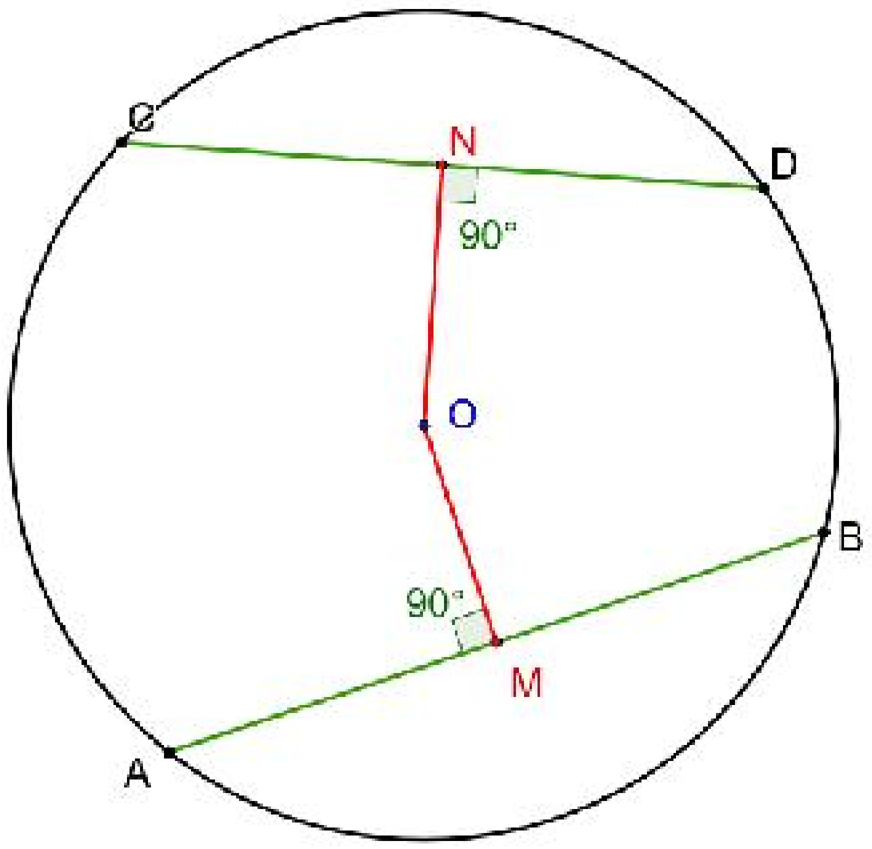


Given → O is centre

$AD = DB$

$\angle ODA = \angle ODB = \underline{\underline{90^\circ}}$

3. Chords equidistant from the centre of the circle are equal.



Given, ON = OM

AB = CD

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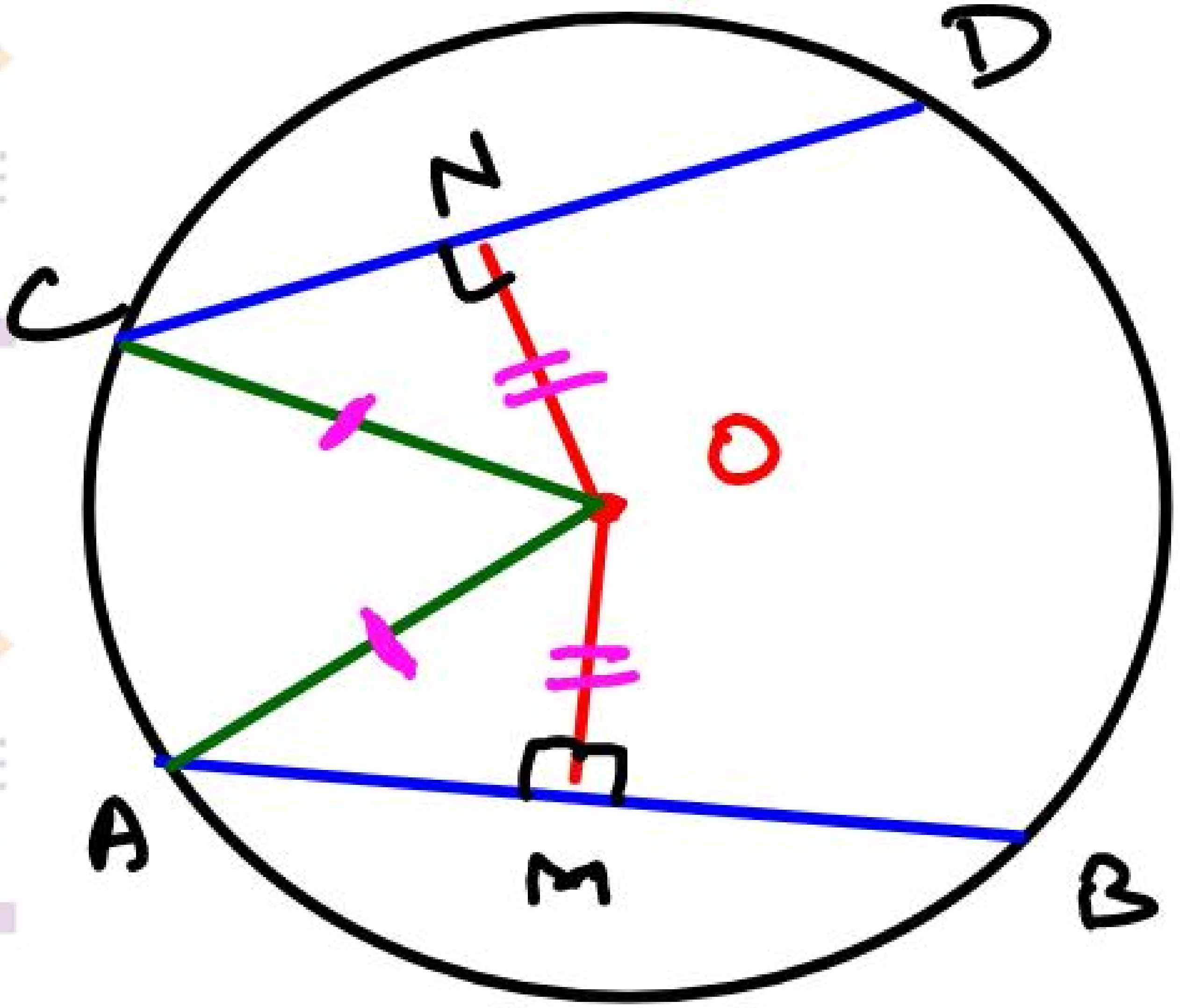
Chords equidistant from the centre are equal



Given OM = ON

To prove AB = CD

Proof

△OMA ≅ △ONC

(By RHS)

AM = CN

$\boxed{AB = CD}$

Converse :

If 2 chords are equal then their distance from centre is also equal.

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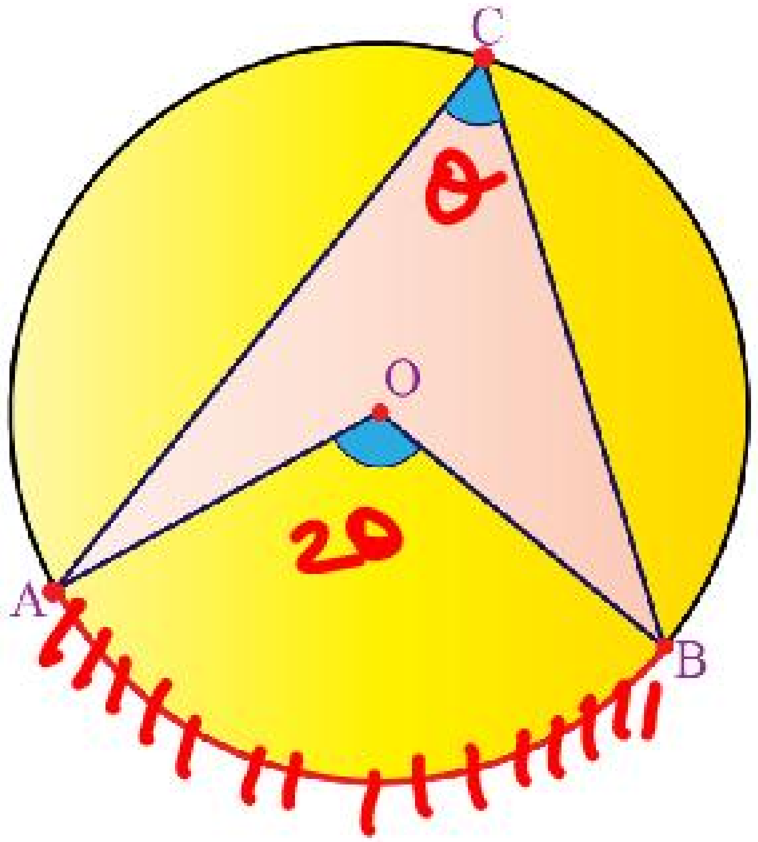
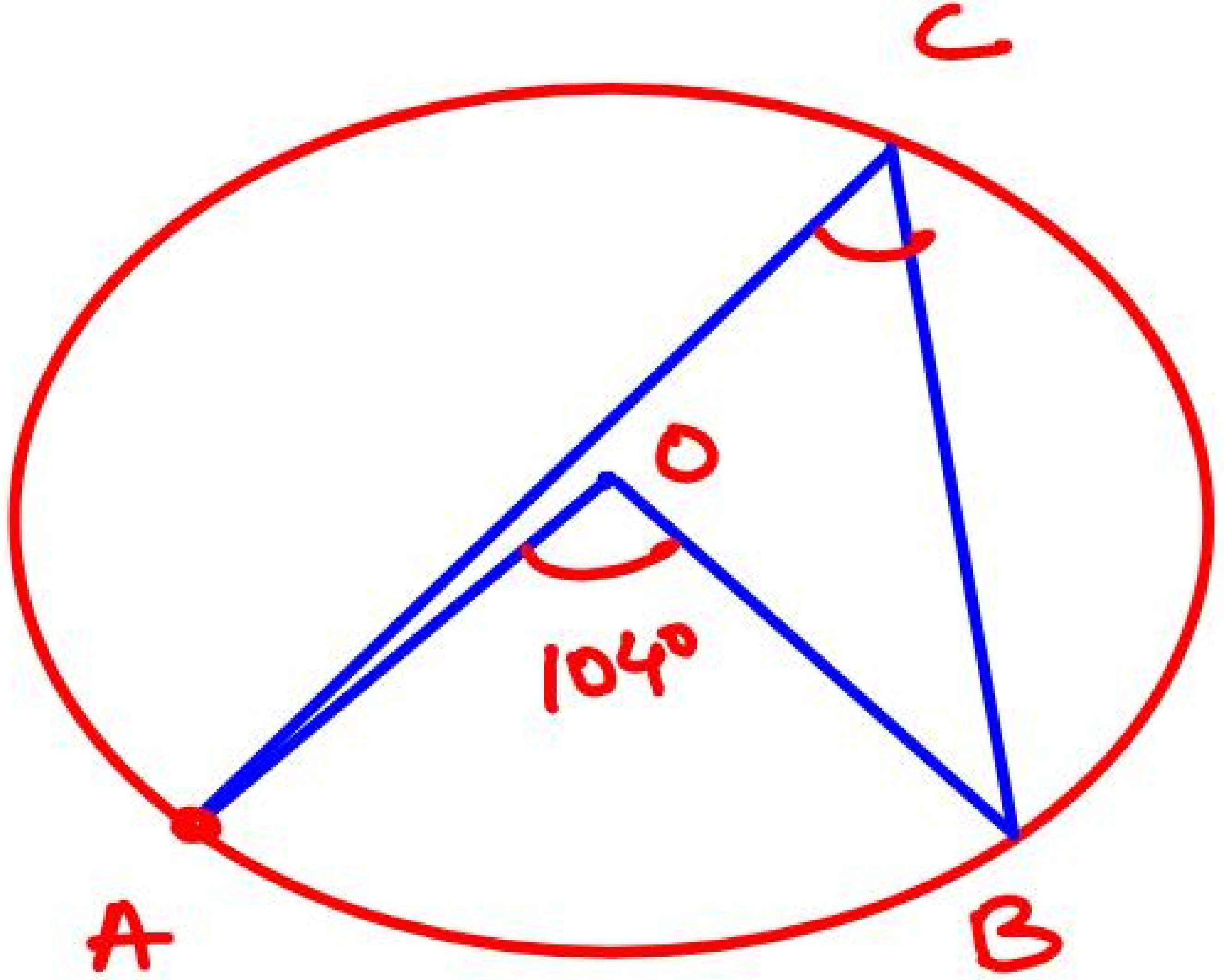
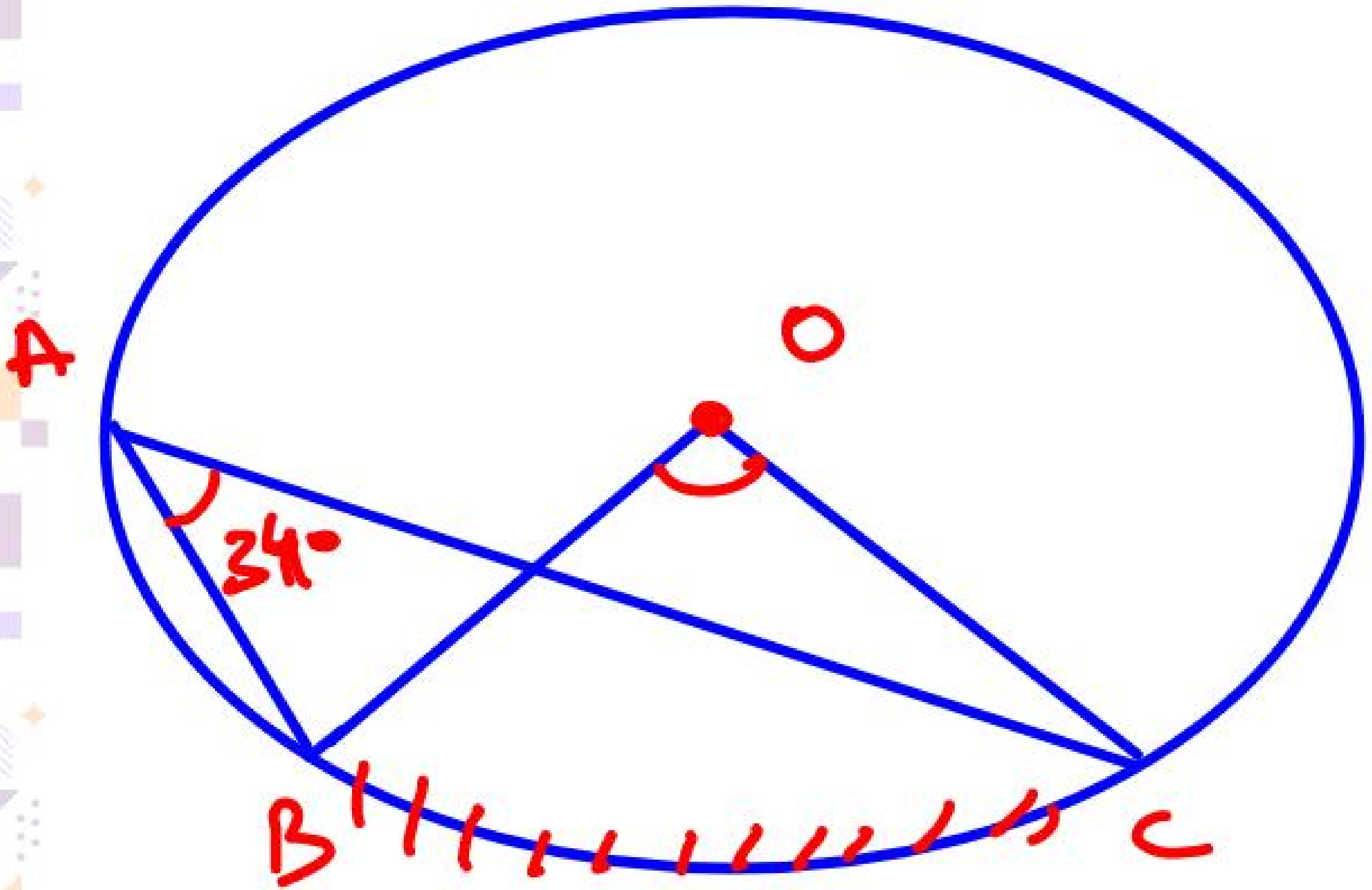
4 (i) . Angle made by an arc at the centre of circle is twice of the angle made by the same arc on the circumference of the circle (except the arc).



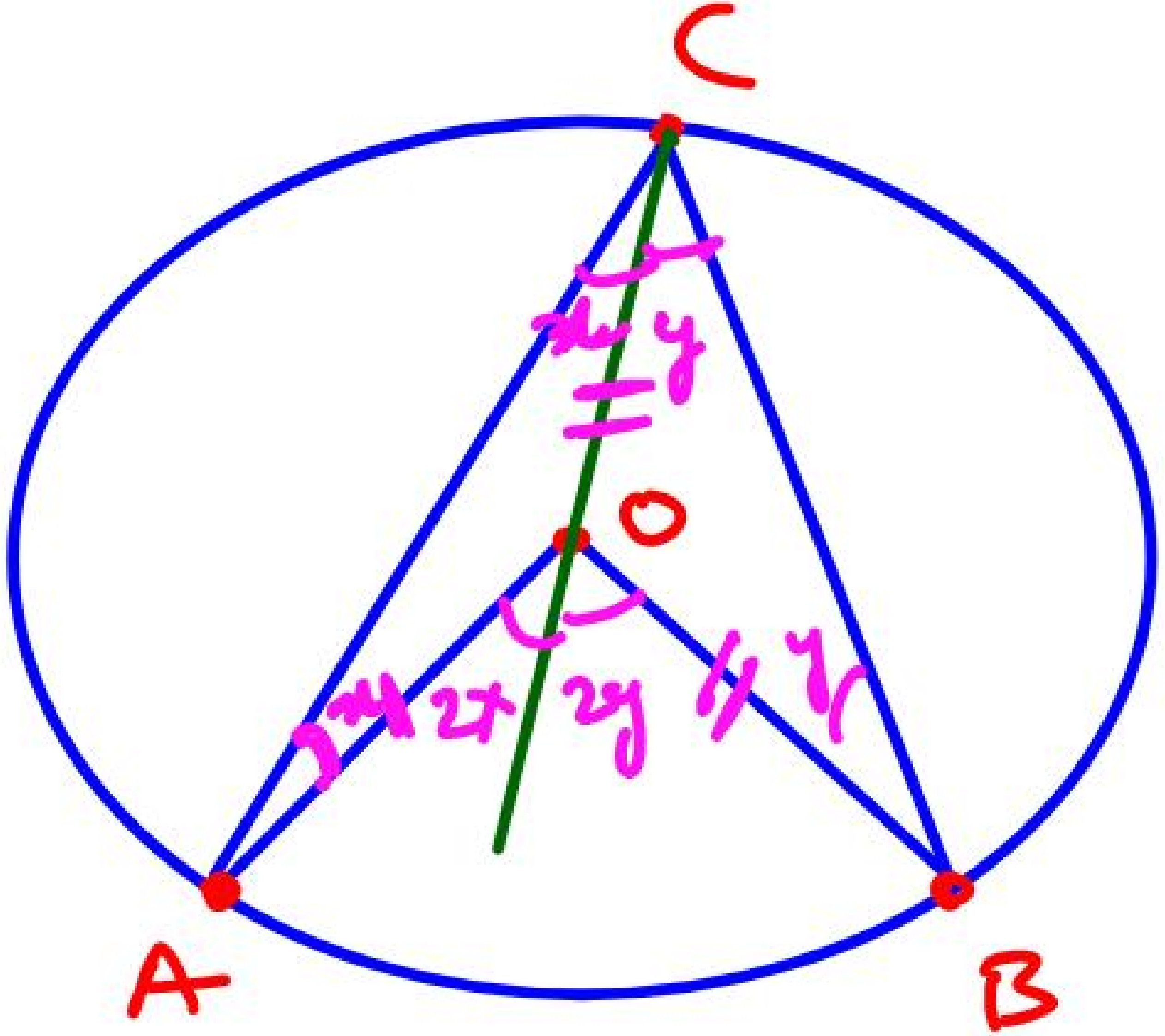


If $\angle AOB = 104^\circ$

$\angle ACB = 52^\circ$



$\angle BOC = 68°$



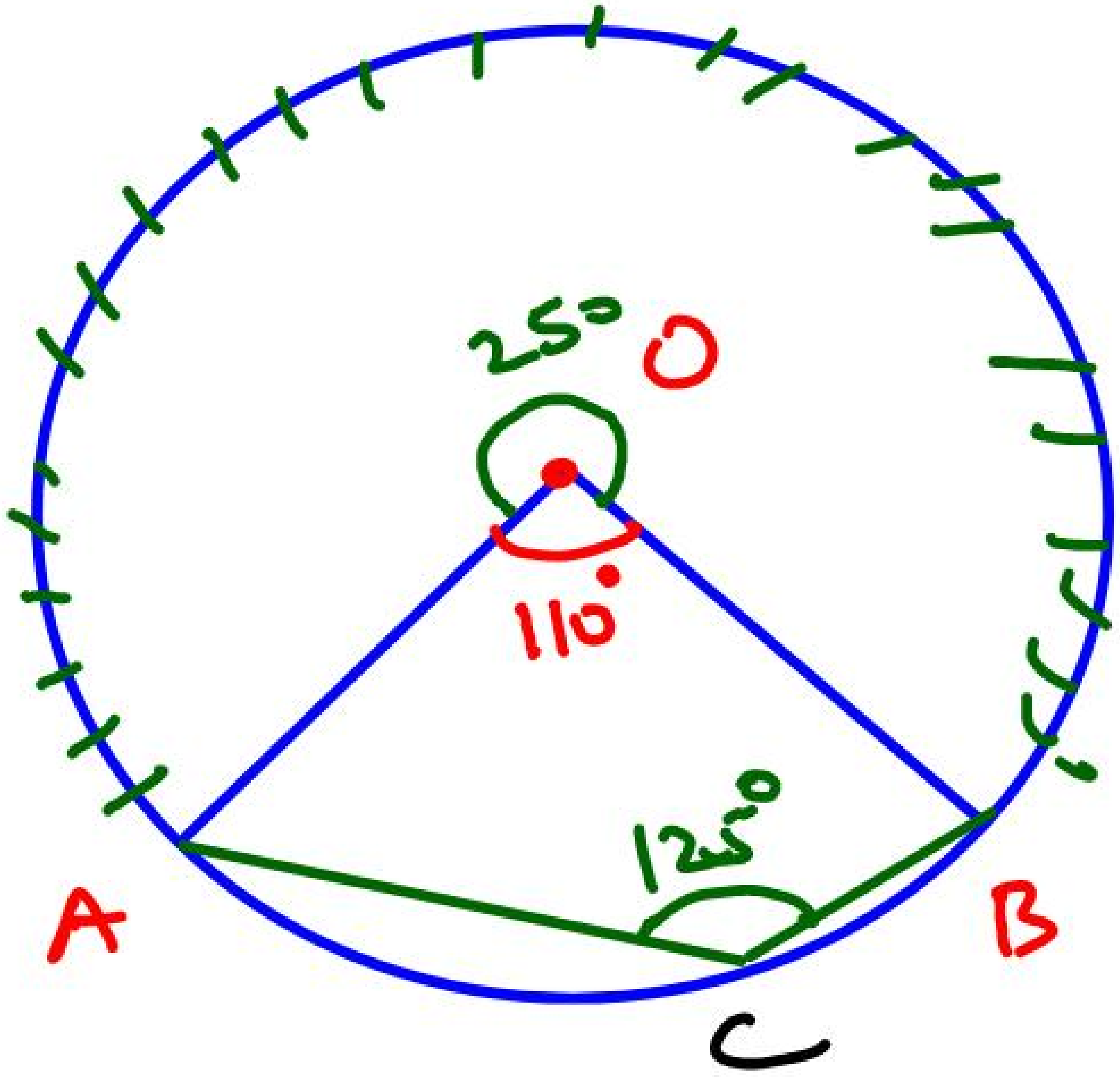
Given

→ O is the centre of circle

To prove $\angle AOB = 2\angle ACB$

$\angle AOB = 2(x+y)$

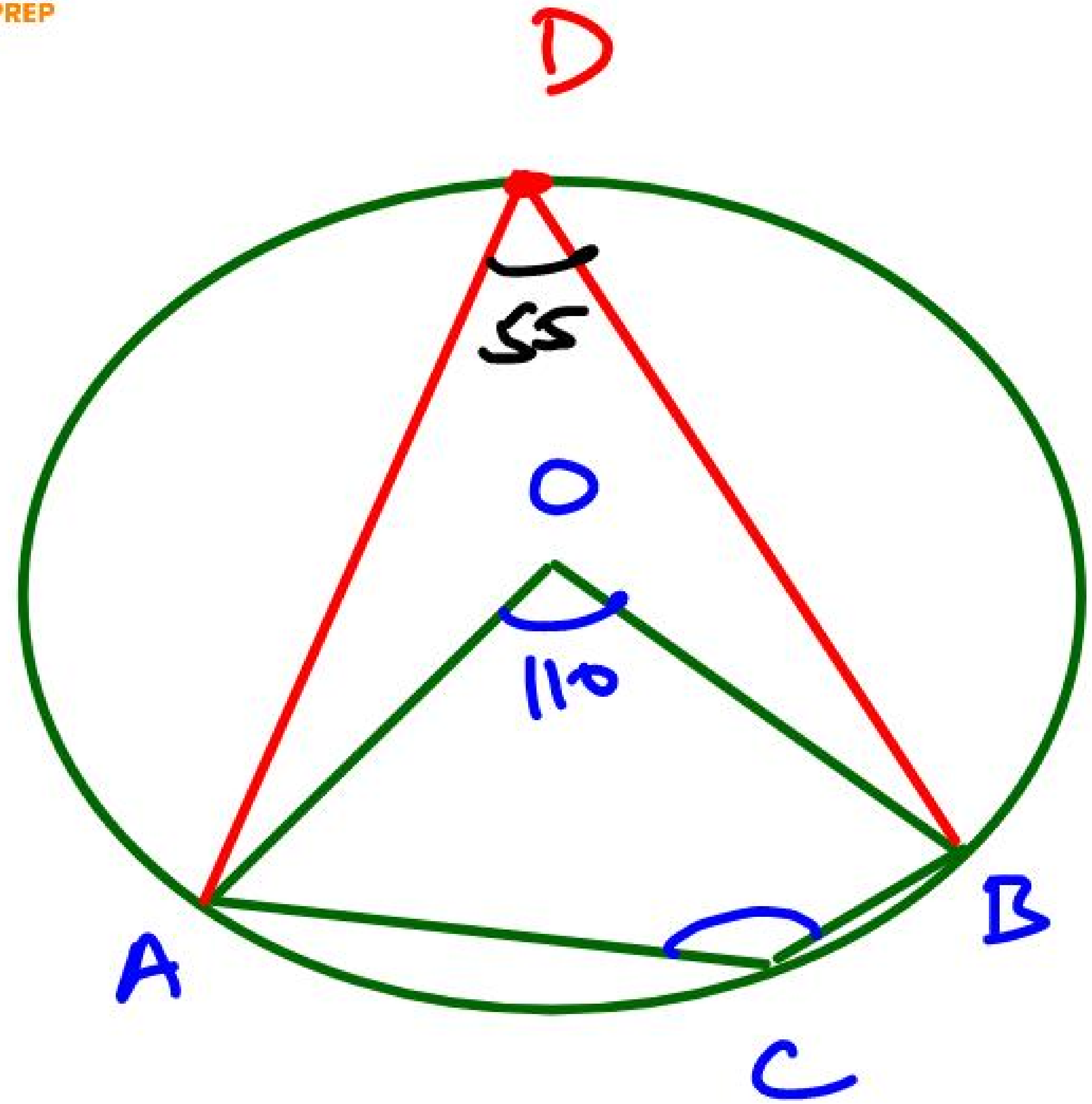
$$\boxed{\angle AOB = 2\angle ACB}$$



Find $\angle ACB = ??$

I^{st}

$\angle ACB \rightarrow 125^\circ$



A CBD

$\longrightarrow$ Cyclic Quad

$55 + \angle ACB = 180$

$\angle ACB = 125^\circ$

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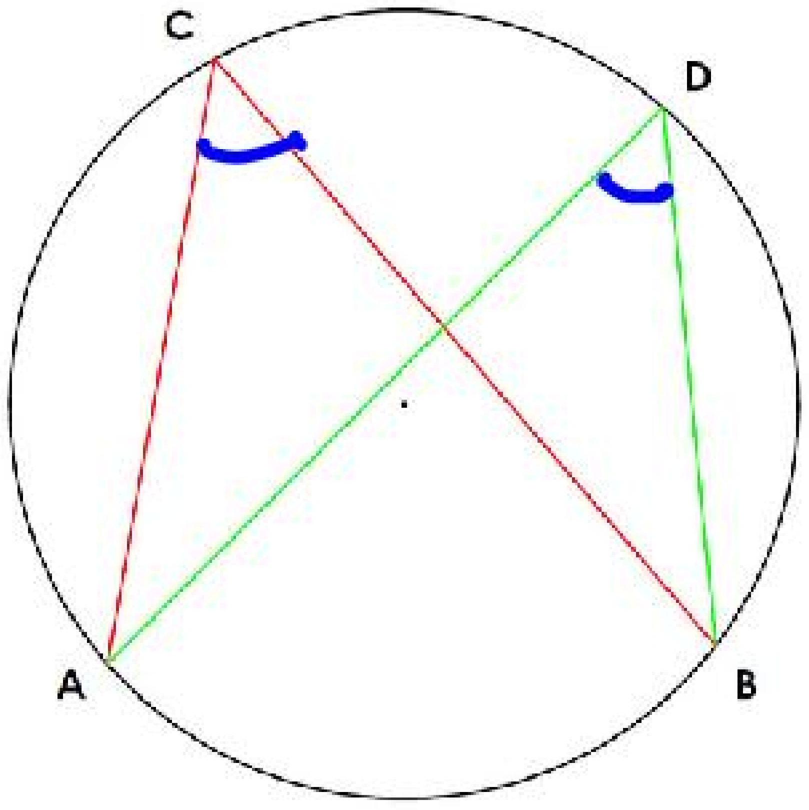
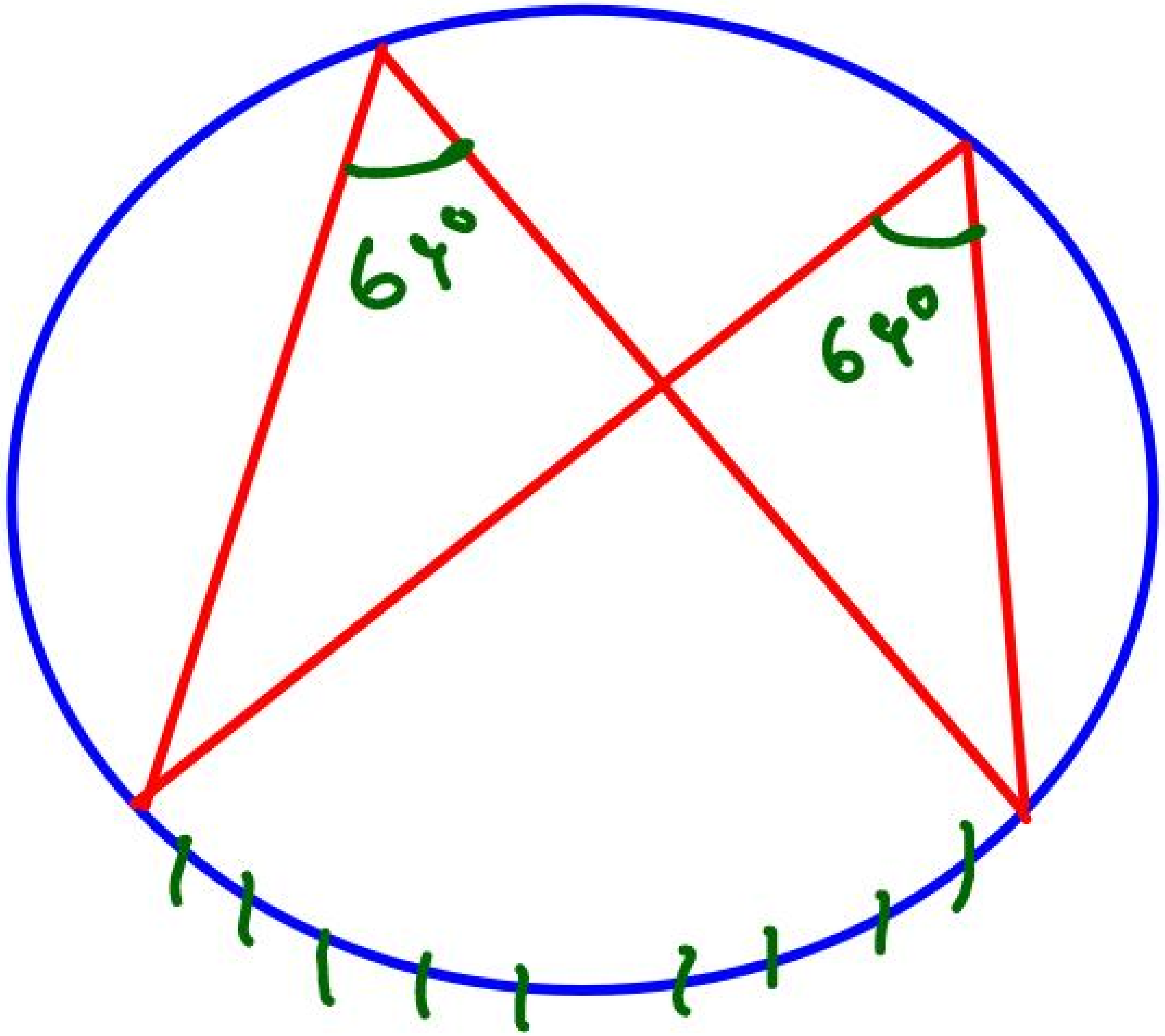
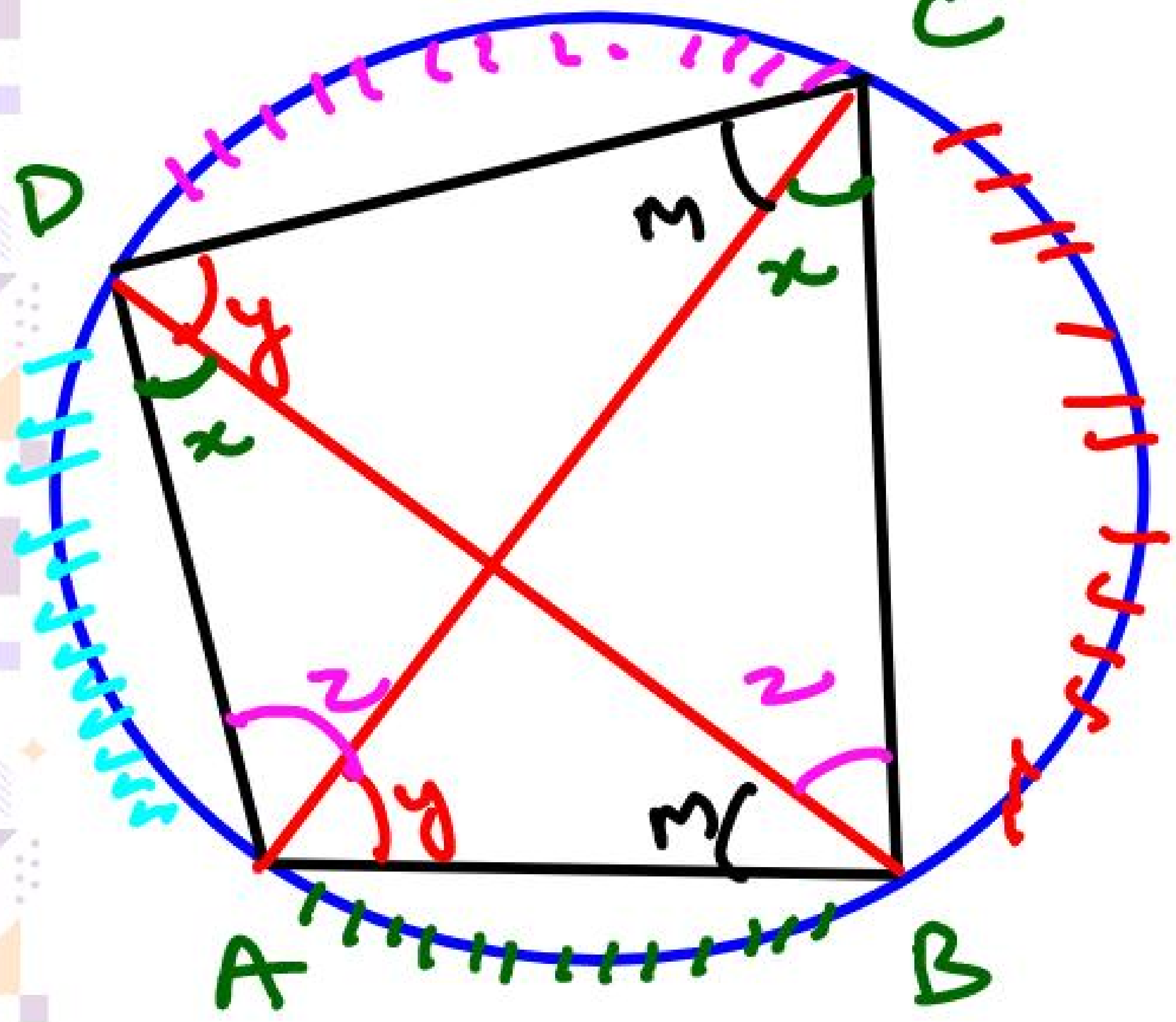
4 (ii) . Angles in the same segment of a circle are equal.



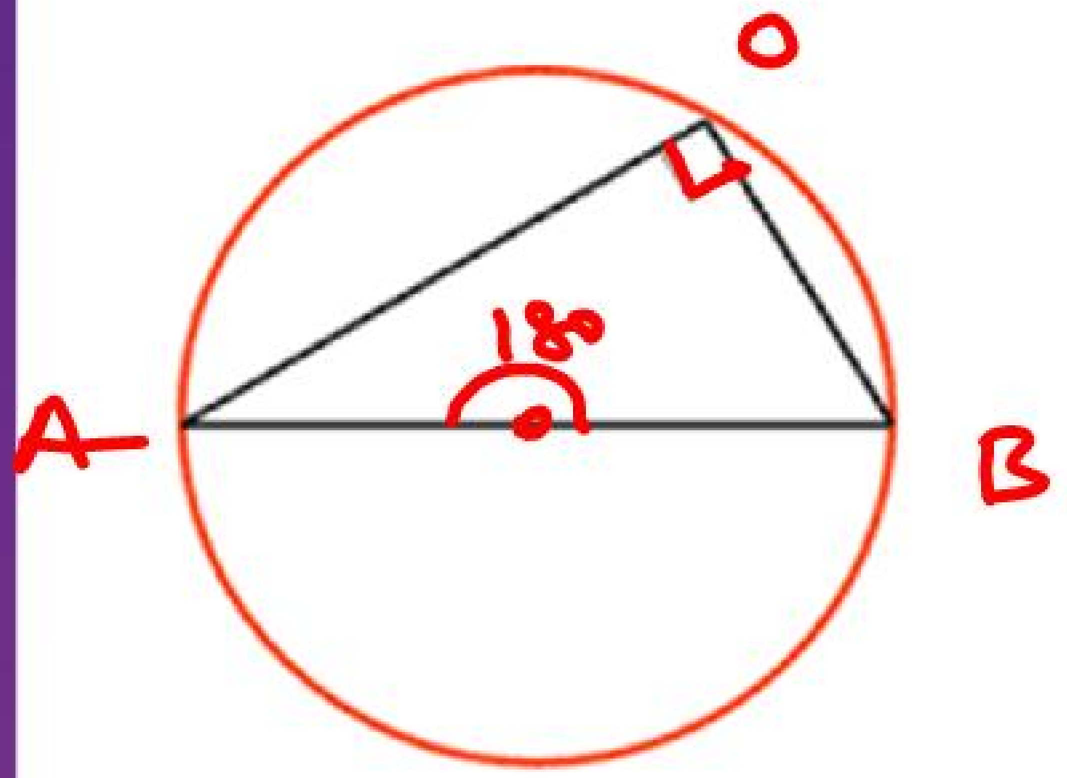
$\angle ACB = \angle ADB$

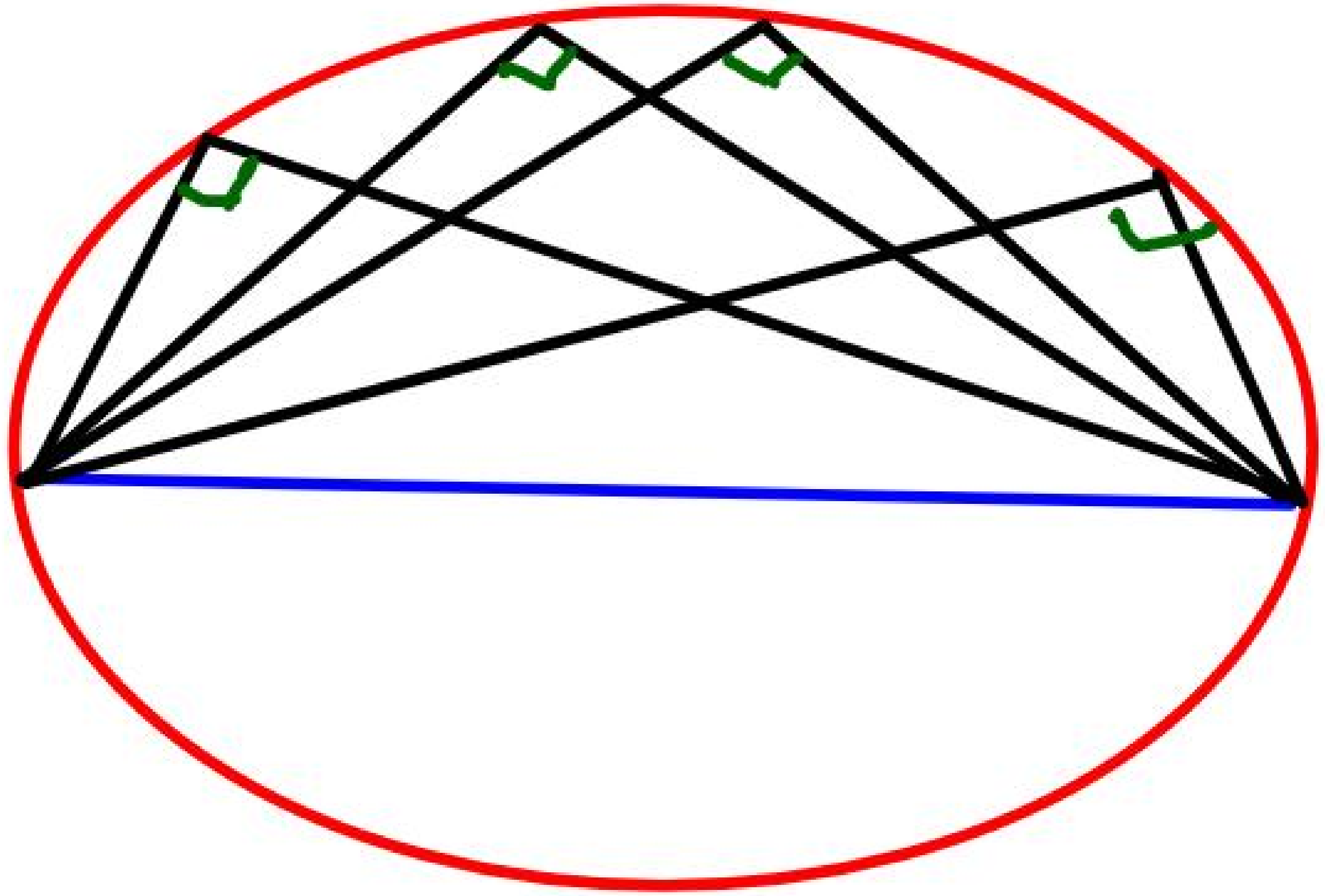

64°
64°



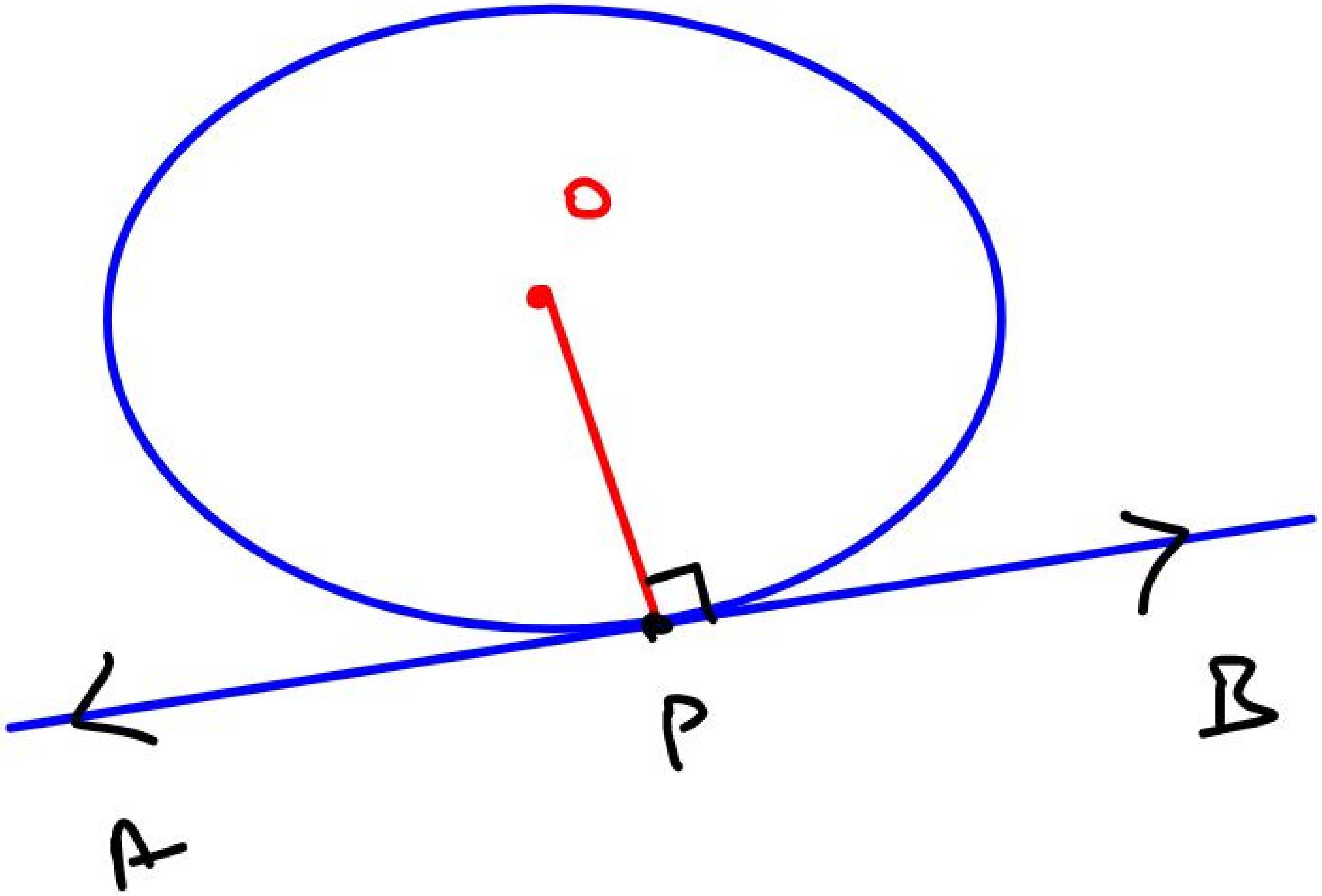
Sum of Opp angles of Cyclic Quad is 180°

4 (iii). Angles in a semi-circle is always a right angle.

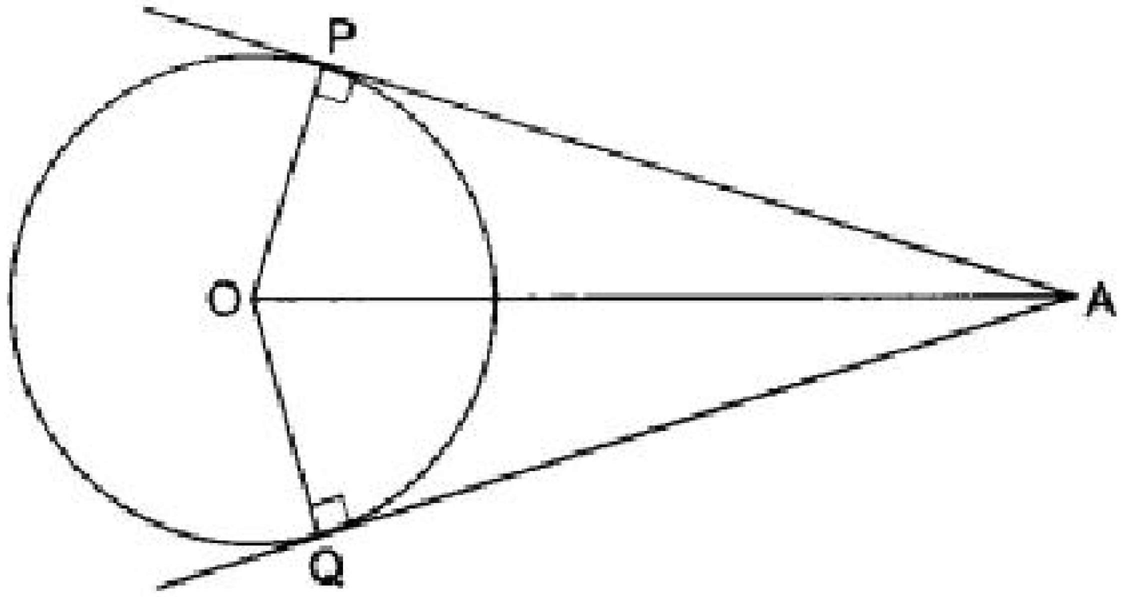




5. Tangent



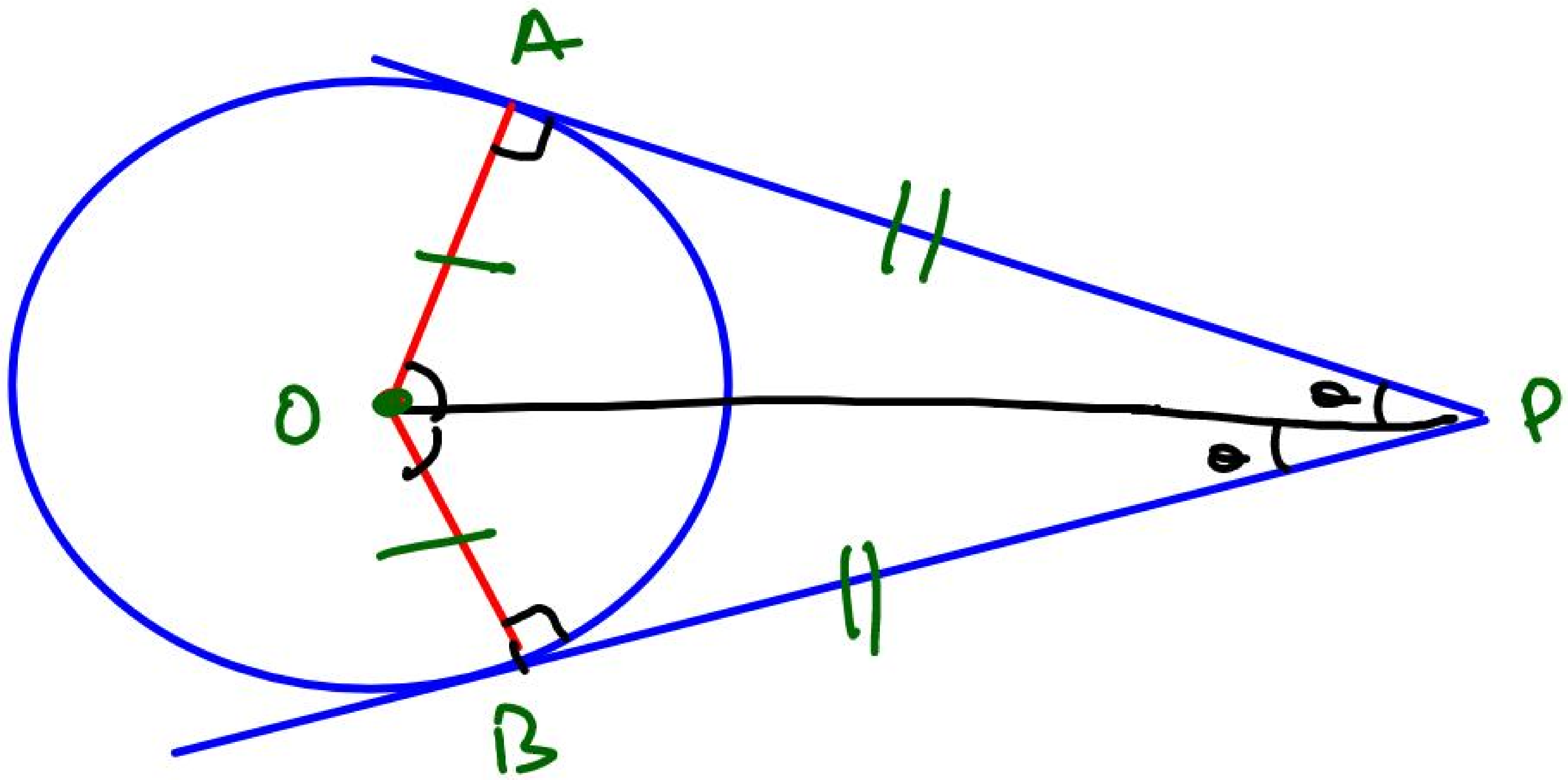
Tangents drawn from an external point are always equal.

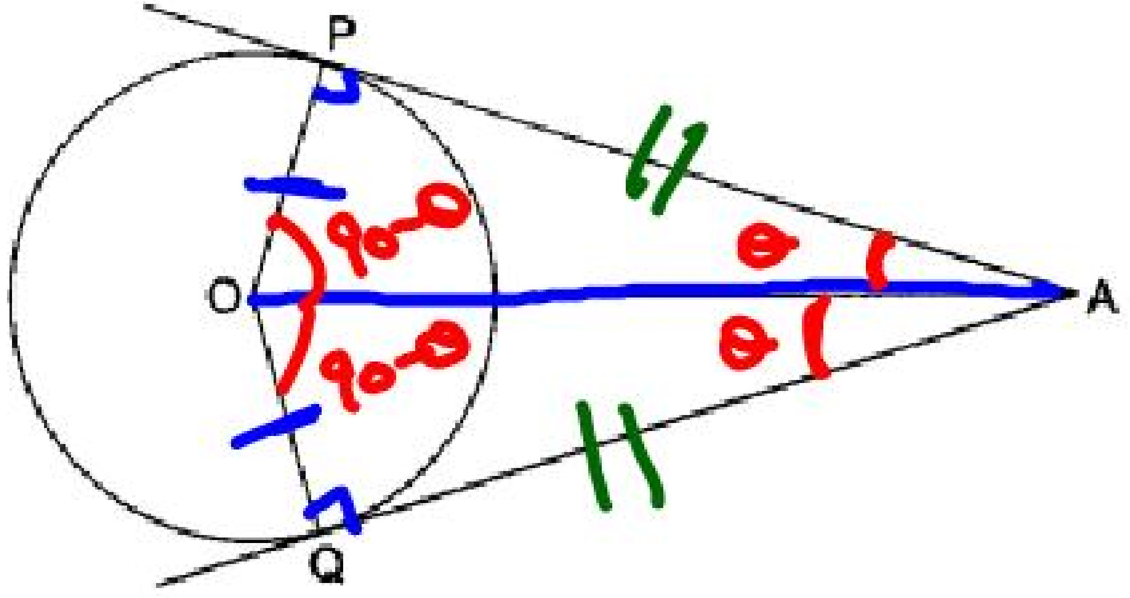


* $AP = AQ$

* $\angle OAP = \angle OAQ$

* $\angle AOP = \angle AOQ$


A
O
B
P
θ
θ



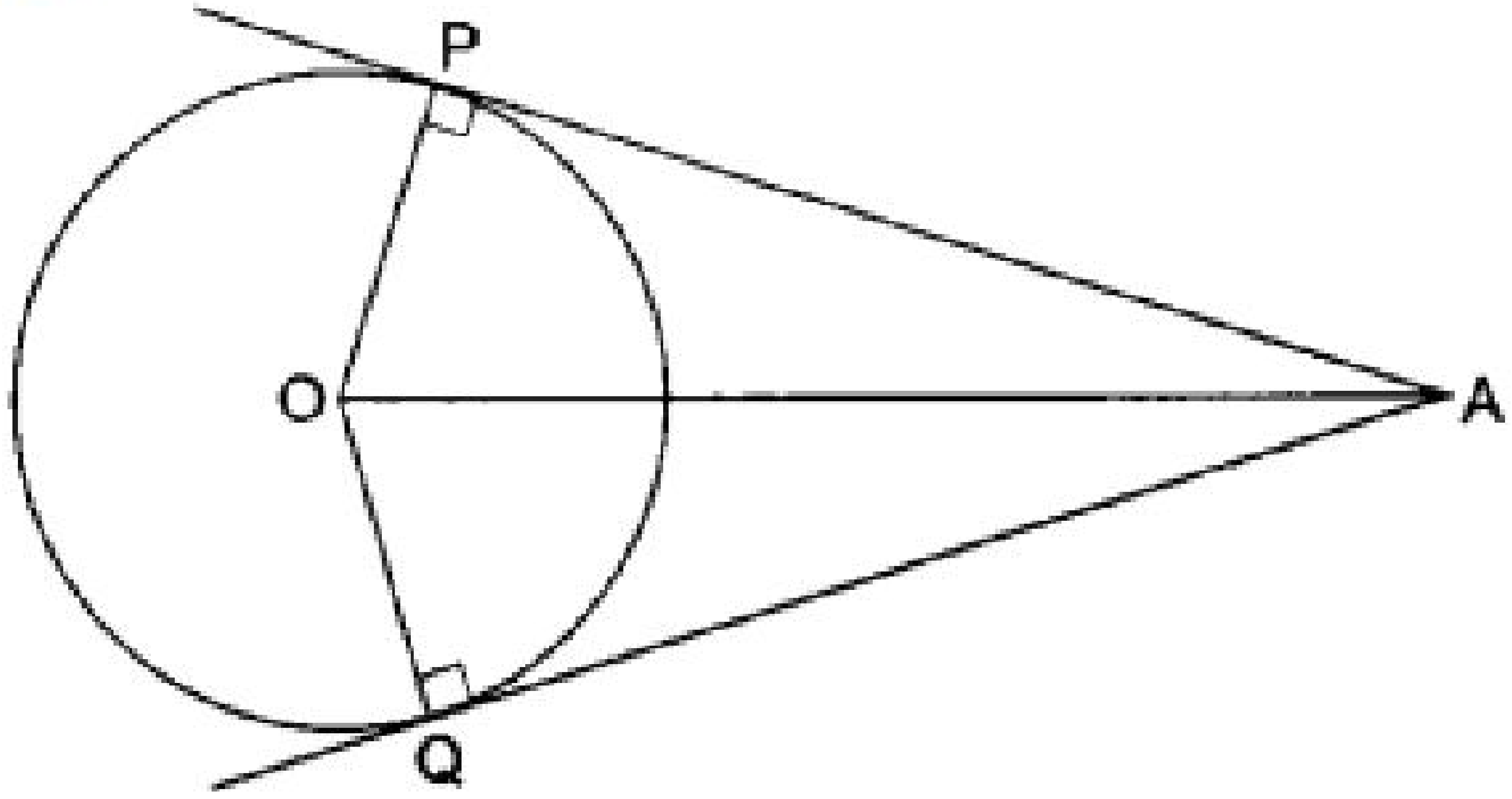
Given → O is centre of circle

To prove AP = AQ

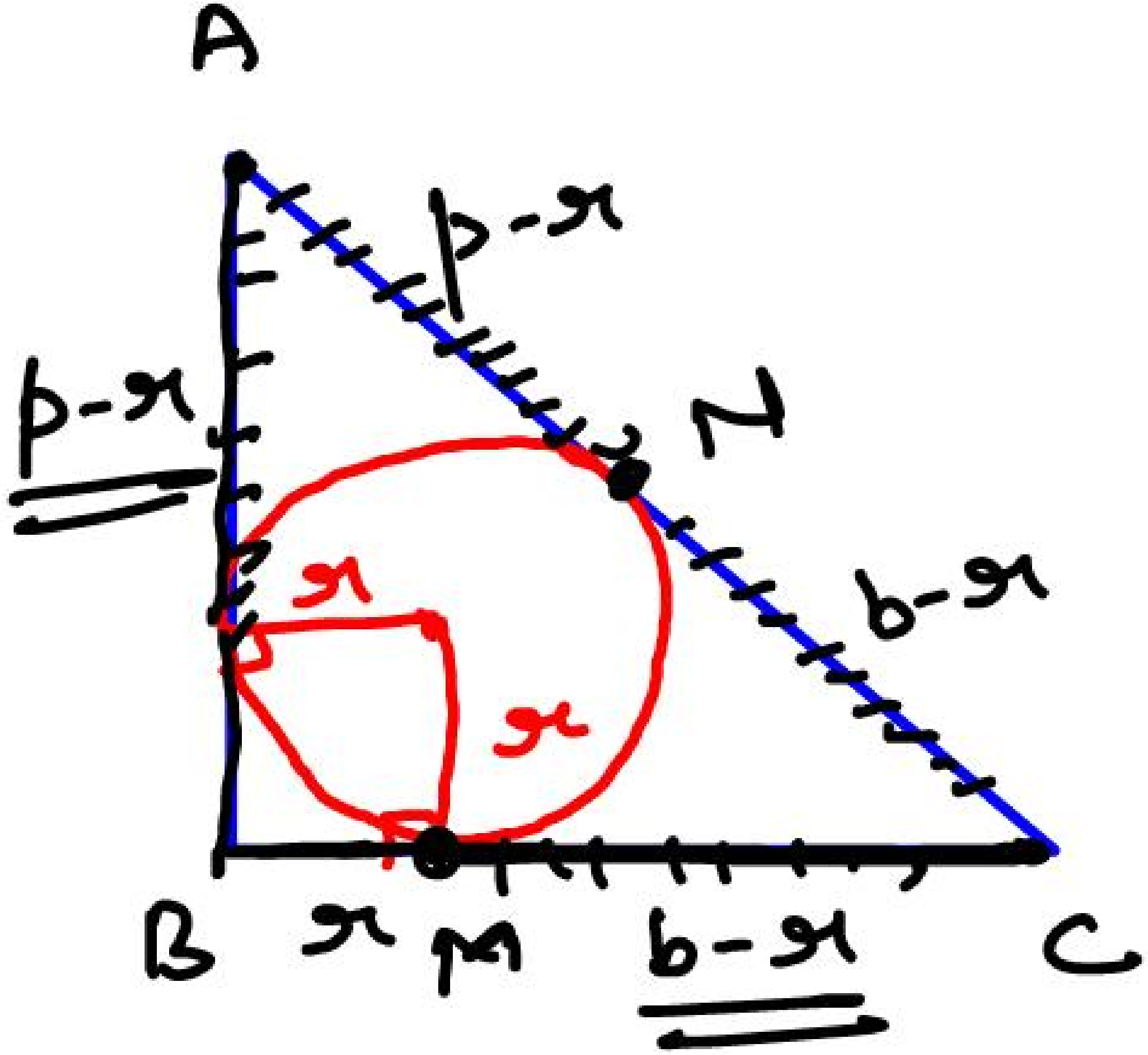
Proof

$\triangle OAP \cong \triangle OAQ$

AP = AQ


P
O
A
Q

*

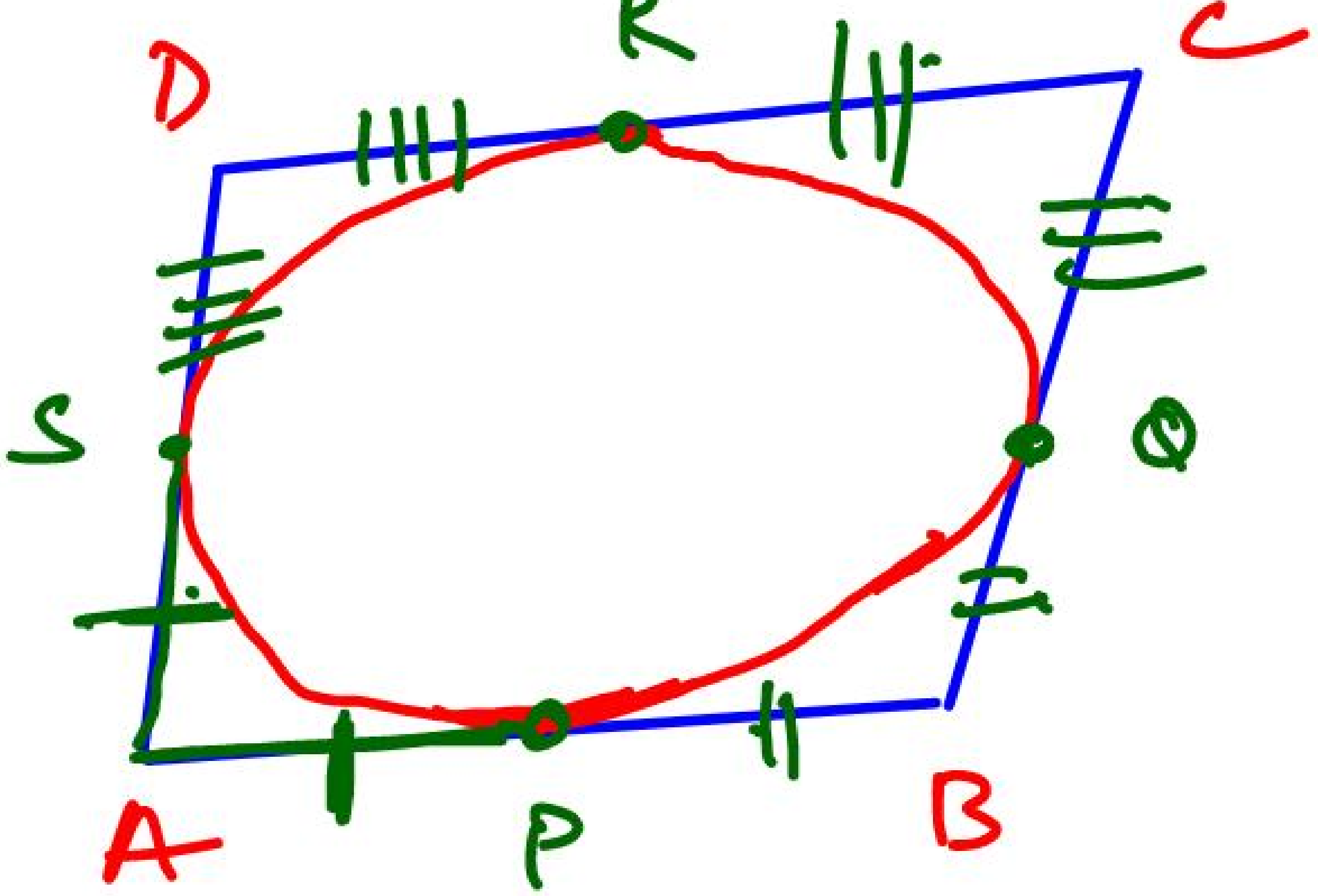


$$r = \frac{b+p-h}{2}$$

$$(p-r)+(b-r) = h$$

$$2r = p+b-h$$

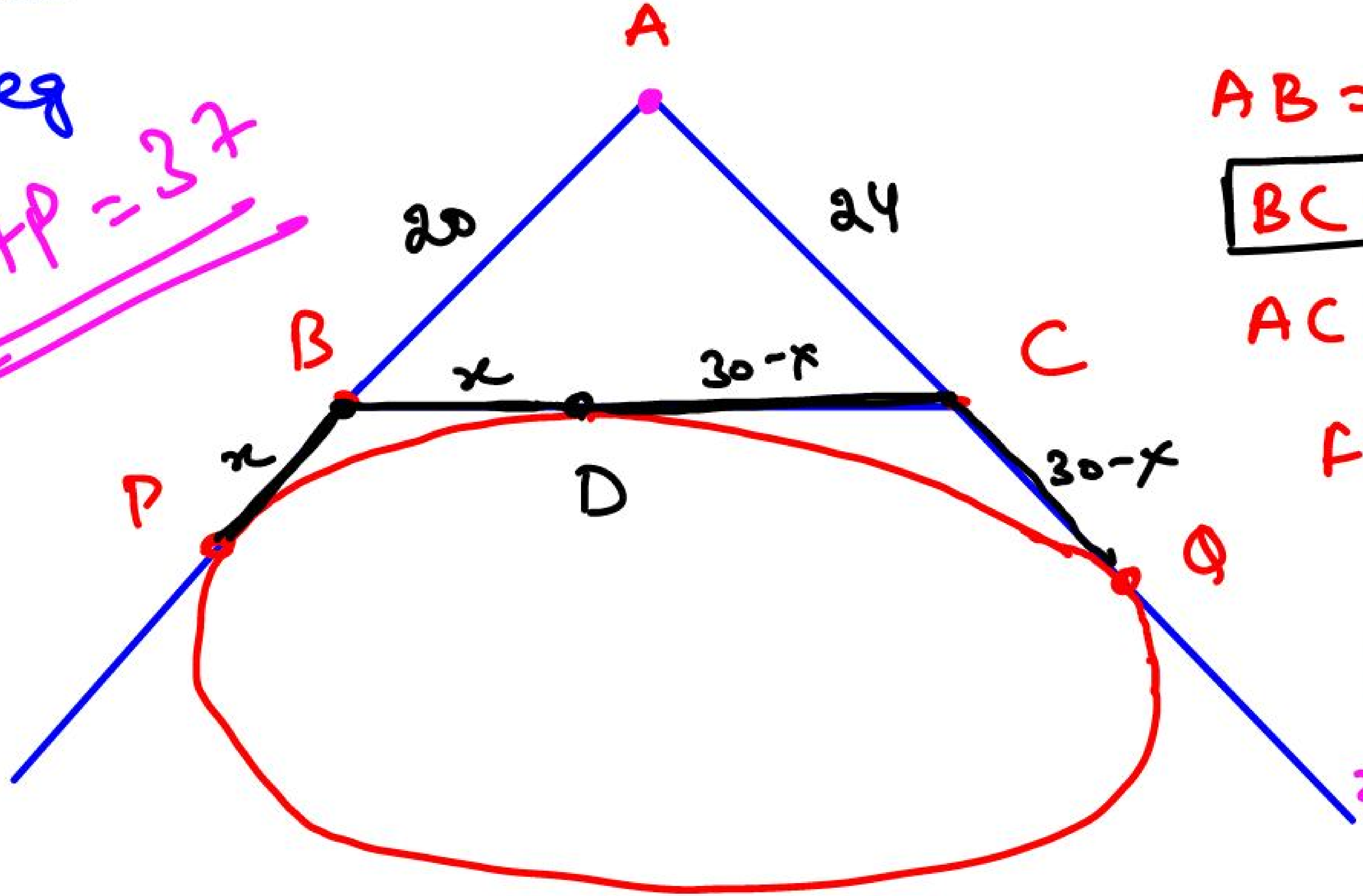
$$\boxed{r = \frac{p+b-h}{2}}$$



$\boxed{AB + CD = BC + AD}$

eg

$AP = 37$



$AB = 20\text{ cm}$

$\boxed{BC = 30\text{ cm}}$

$AC = 24\text{ cm}$

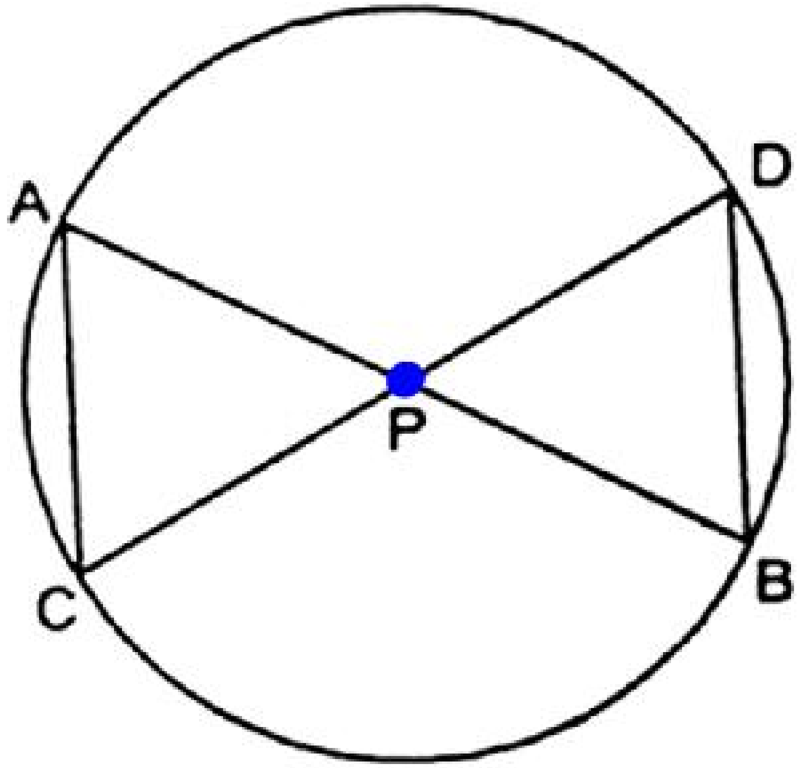
Find $AP = ??$

$AP = AQ$

$20 + x = 24 + 30 - x$

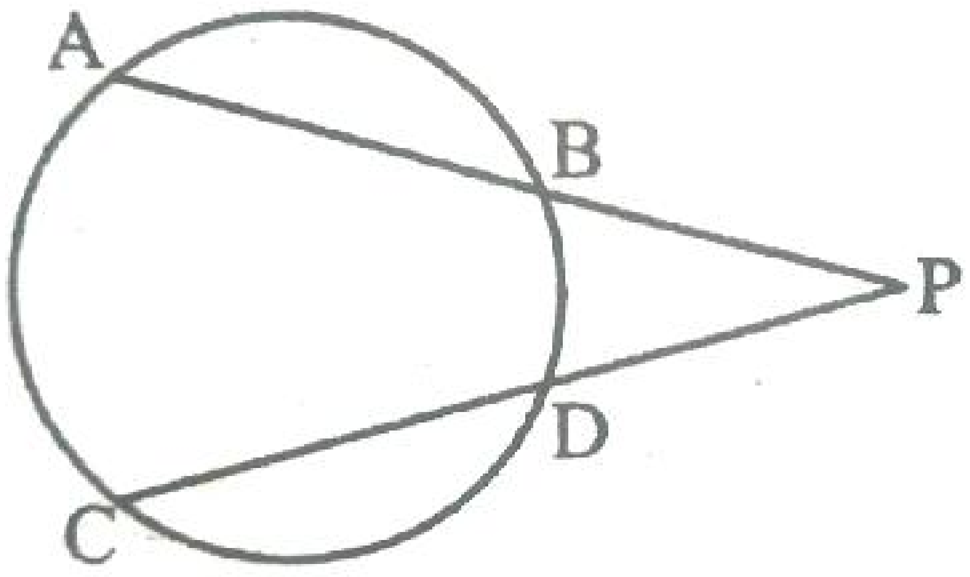
$\boxed{x = 17}$

6 (i). If 2 chords AB and CD intersect each other at P.



$$AP \cdot BP = CP \cdot DP$$

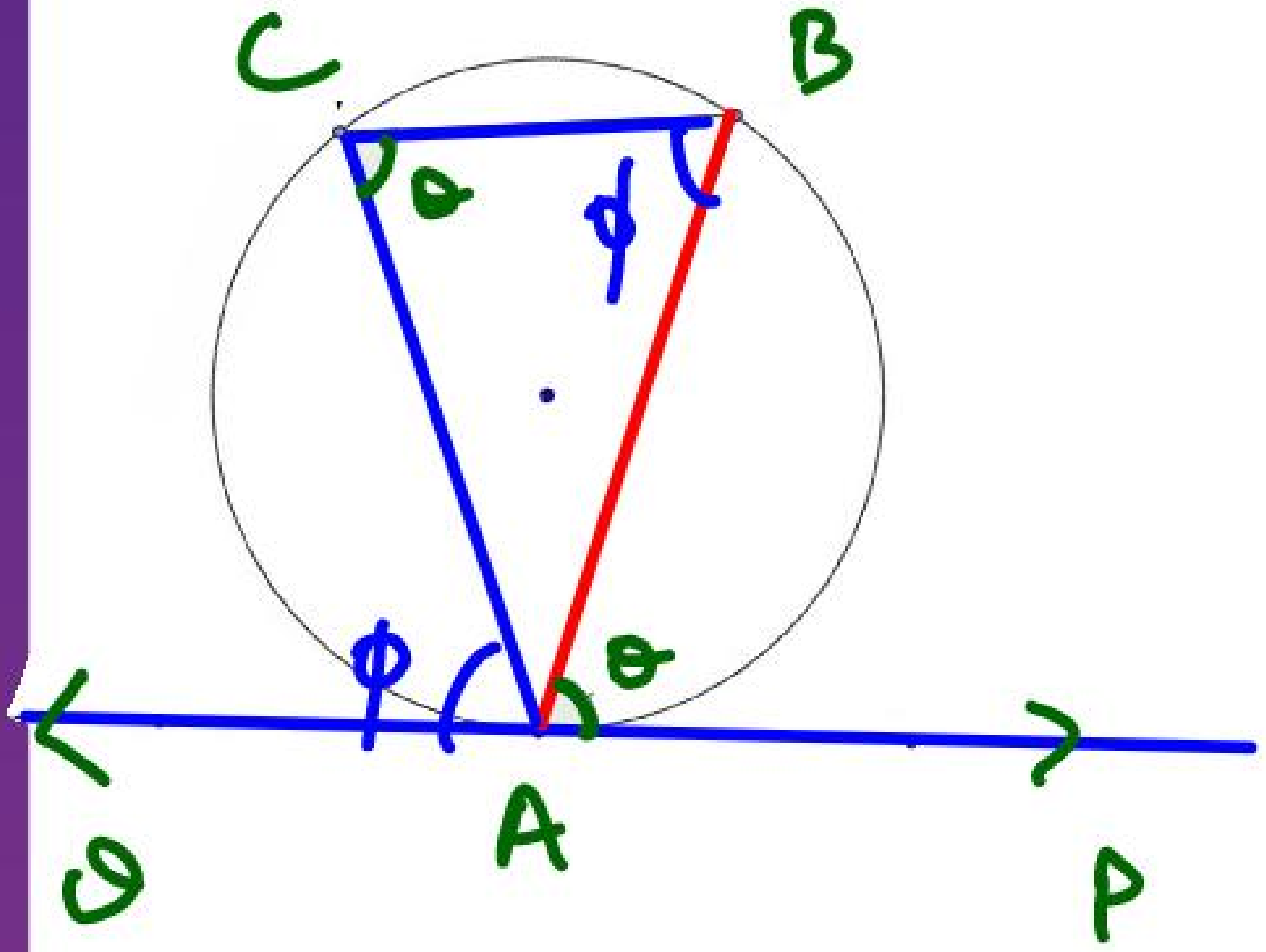
6 (ii). If 2 chords AB and CD intersect each other externally at P.



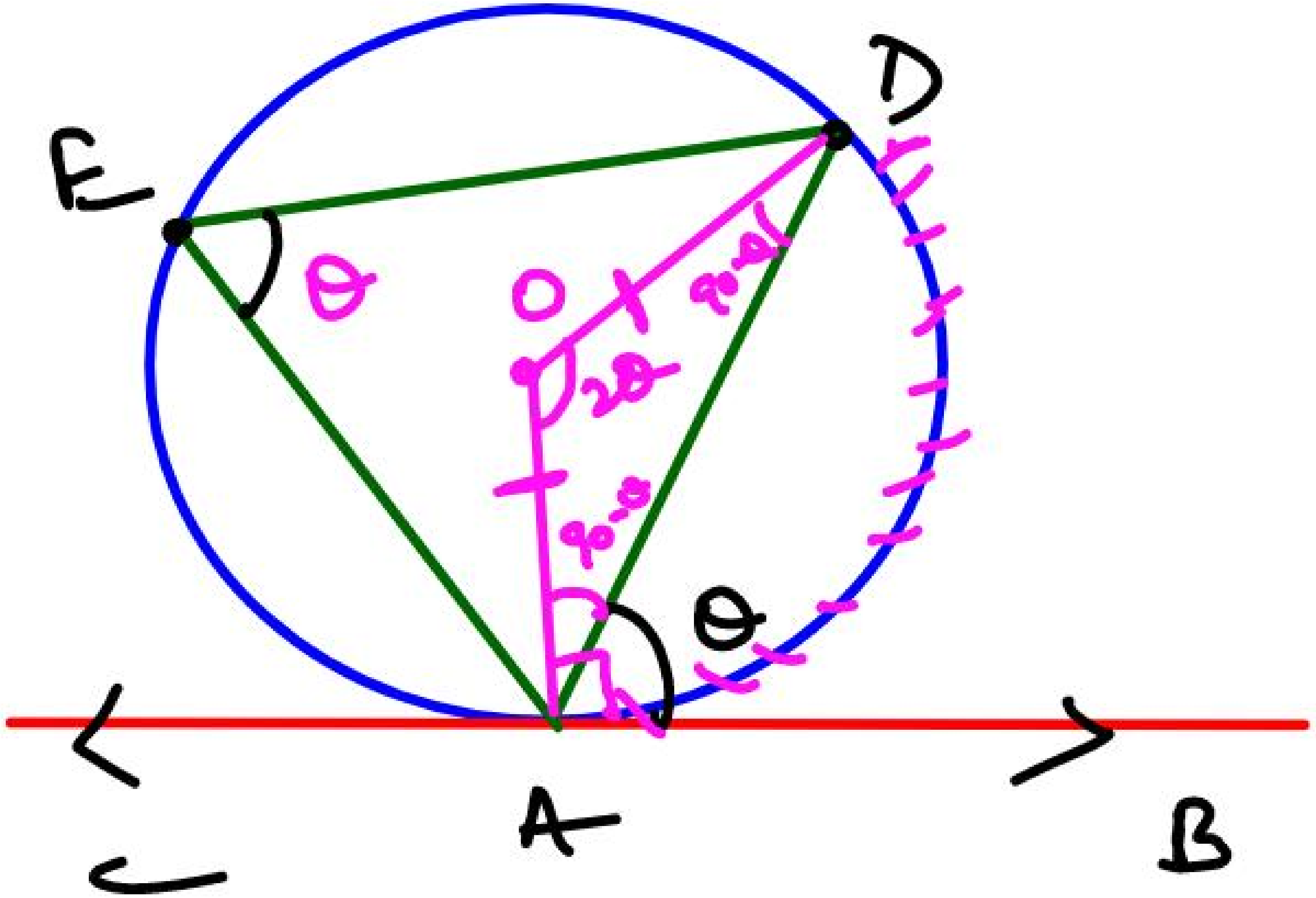
ans

$$\boxed{AP \cdot BP = CP \cdot DP}$$

7. Alternate segment theorem

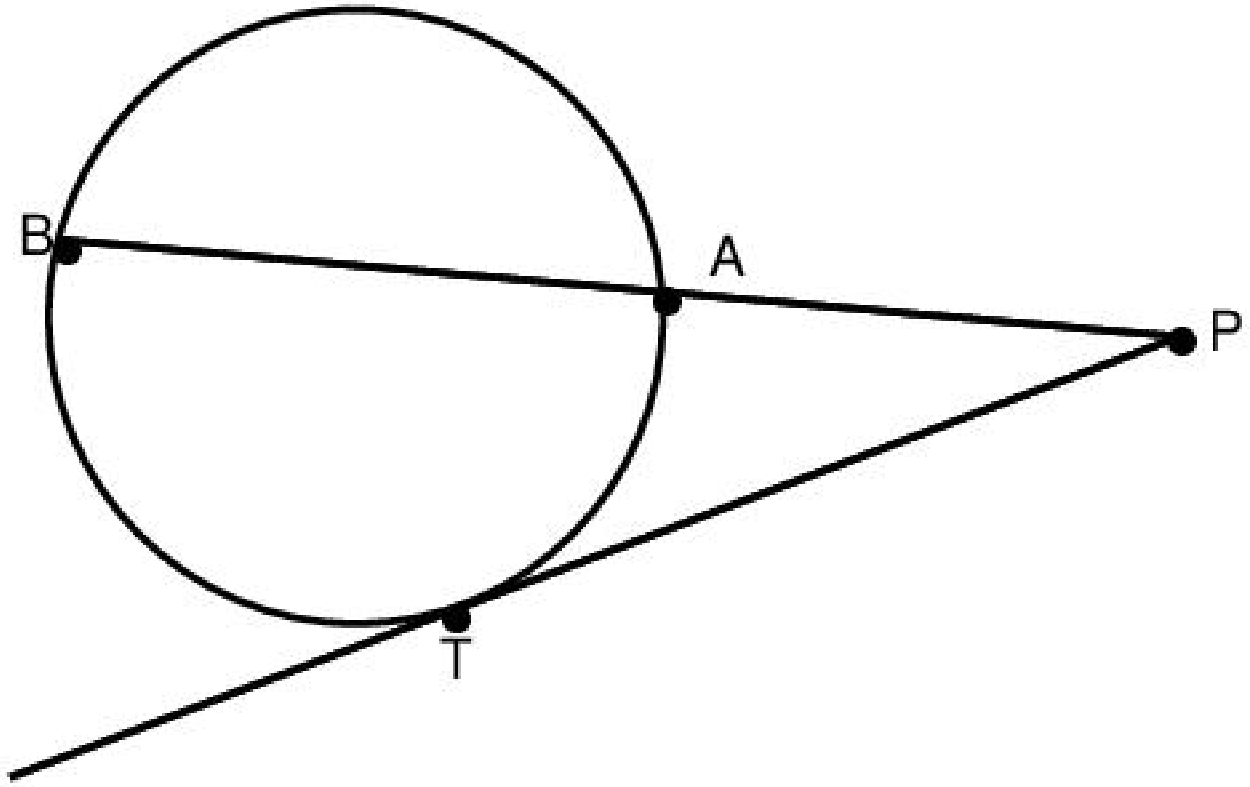


Angle made by a chord with the tangent of a circle is always equal the angle made by the same chord in alternate segment.

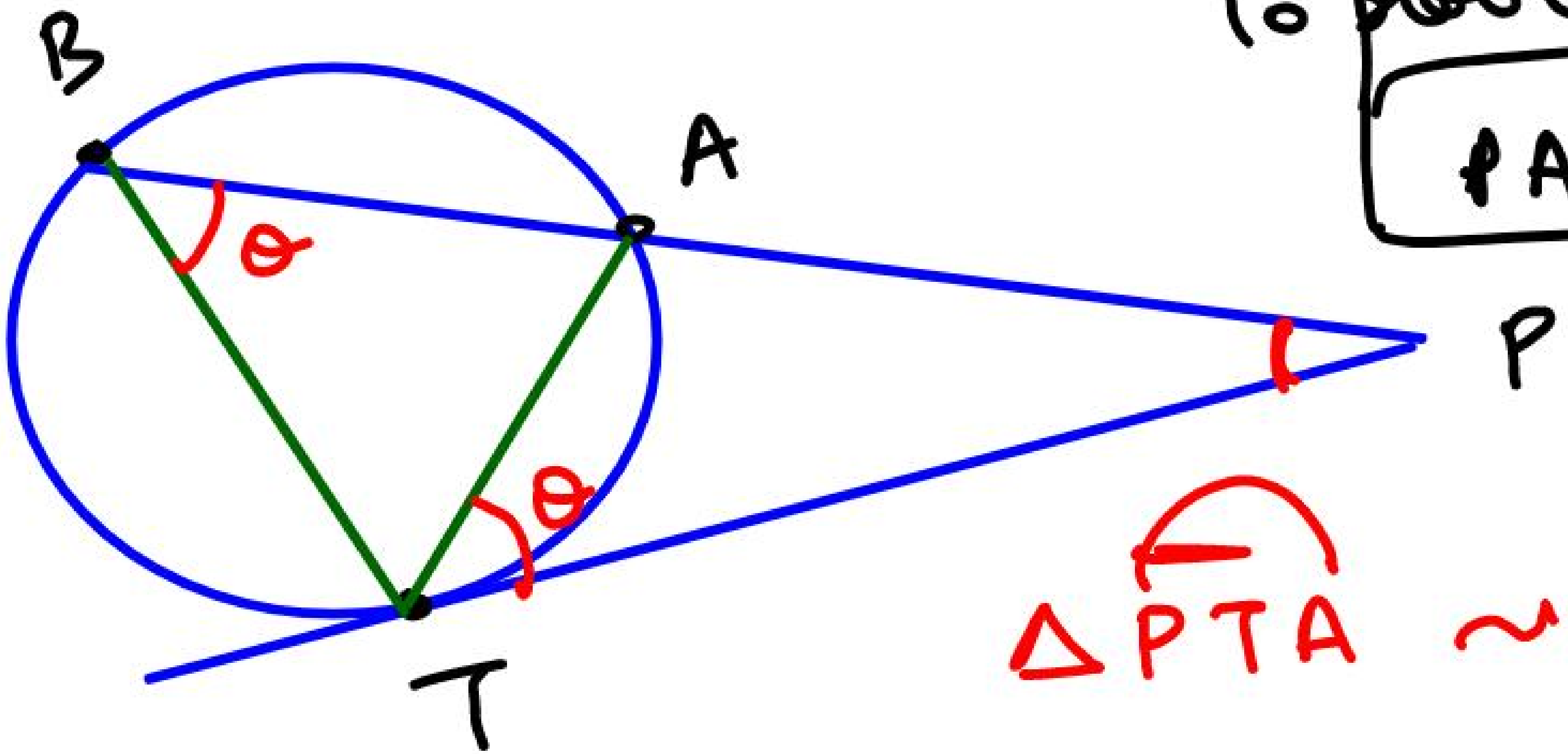


To prove $\angle AED = \theta$

8. Tangent – Secant Theorem



$$\boxed{PT^2 = PA \times PB}$$



To prove

$$\boxed{PA \times PB = (PT)^2}$$

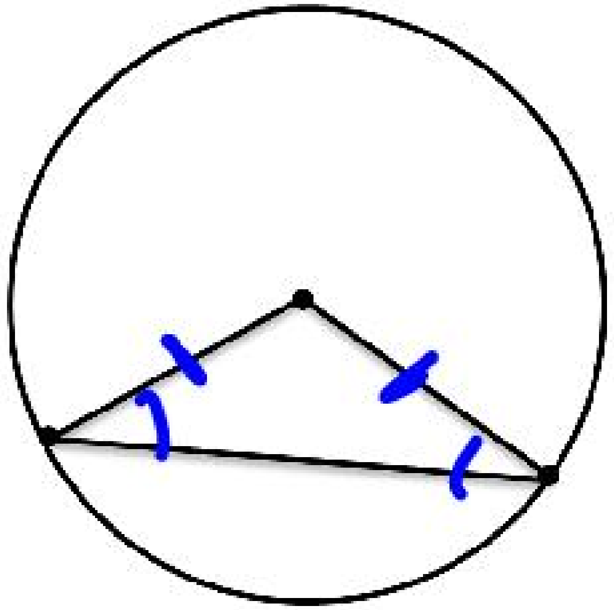
$$\Delta PTA \sim \Delta PBT$$

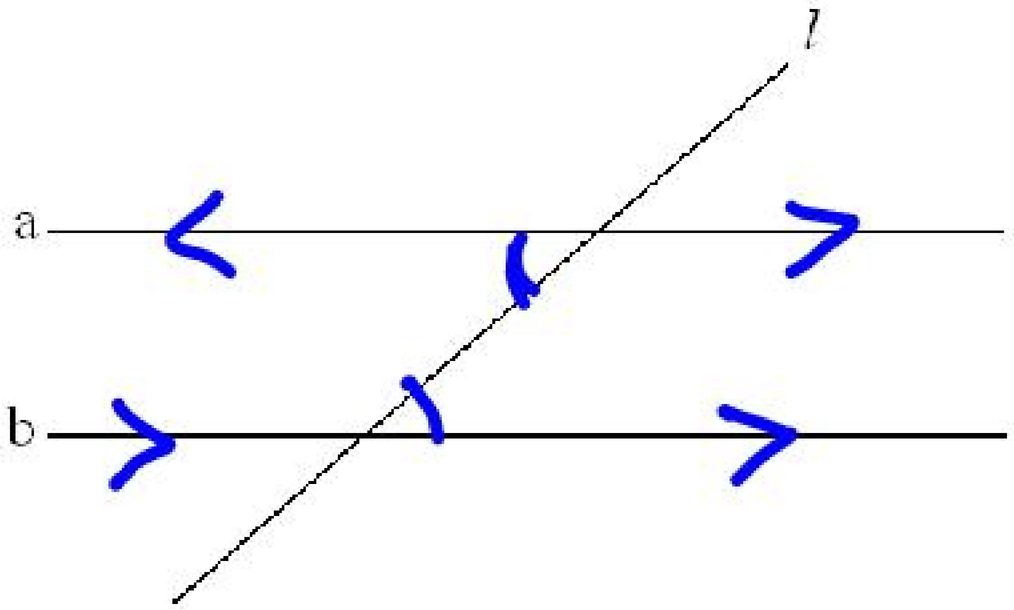
$$\frac{PT}{PB} = \frac{PA}{PT}$$

Whenever you do questions on circles, focus on :

(i) Central angle $\rightleftarrows$ Circumference

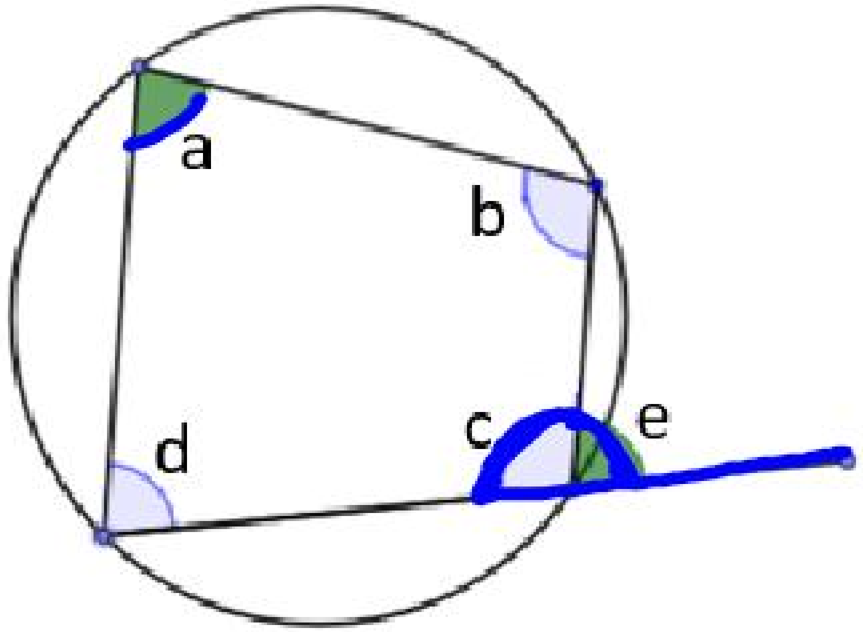
(ii) Diameter $\longrightarrow$ Angle in a semi-circle

(iii) 

(iv) 

Cyclic Quadrilateral

A cyclic quadrilateral has all its vertices on the circumference of the circle.



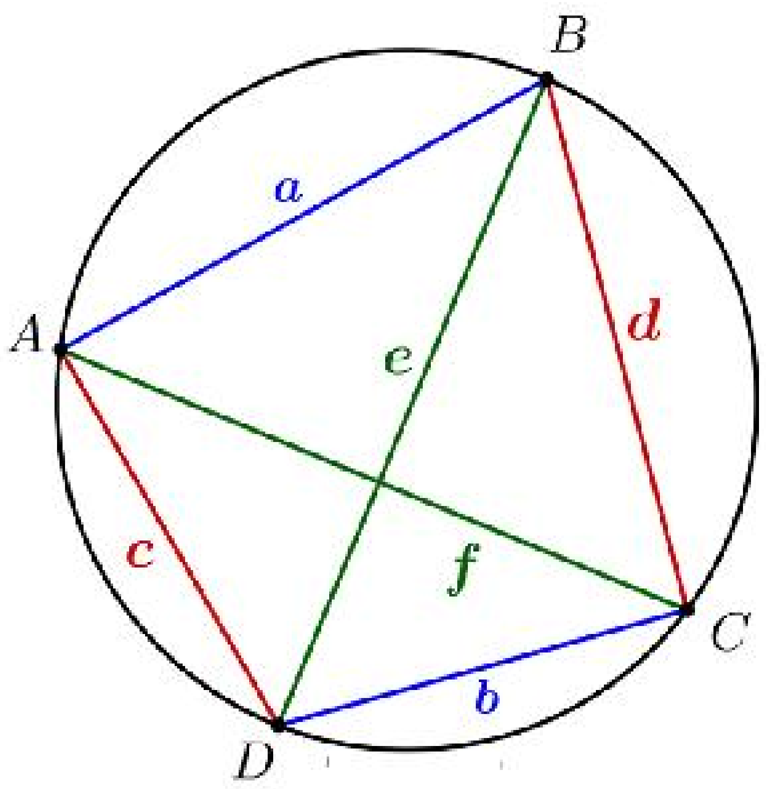
Opposite angles add up to 180°

$$\angle a + \angle c = 180°$$
$$\angle b + \angle d = 180°$$

Exterior angle is equal to the interior opposite angle

$$\angle a = \angle e$$

Ptolemy's Theorem



imp

$$AC \times BD = AB \times CD + BC \times AD$$

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Eg. In a cyclic quadrilateral ABCD, AB = 8 cm, BC = 9 cm, CD = 6 cm and DA = 4 cm. If the value BD is 7 cm, the value of AC is:

(a) 10 ~~(b) 12~~

(c) 14 (d) 16

$AC \times BD = 8 \cdot 6 + 9 \cdot 4$

$AC \times 7 = 84$

$AC = 12$

Ans. (b)

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If the sides of a cyclic quadrilateral is a, b, c and d.

Area of cyclic quadrilateral = $\sqrt{(s-a)(s-b)(s-c)(s-d)}$

Where, s is semi-perimeter of cyclic quadrilateral.

$$s = \frac{a+b+c+d}{2}$$

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Eg. Find the area of a cyclic quadrilateral whose sides are 5 cm, 2 cm, 5 cm and 8 cm.

(a) 10 cm^2
(b) 20 cm^2
(c) 40 cm^2
(d) 25 cm^2

$$s = \frac{5+2+5+8}{2}$$

$$= 10$$

$$\text{Area} = \sqrt{(10-5)(10-2)(10-5)(10-8)}$$

$$= \sqrt{5 \cdot 8 \cdot 5 \cdot 2}$$

$$= \underline{\underline{20\,cm^2}}$$

Ans. (b)

Practice Questions

Q1. Find the distance between 2 parallel chords of length 24 cm and 32 cm. If both the chords lie on opposite side of centre and radius of circle is 20 cm.

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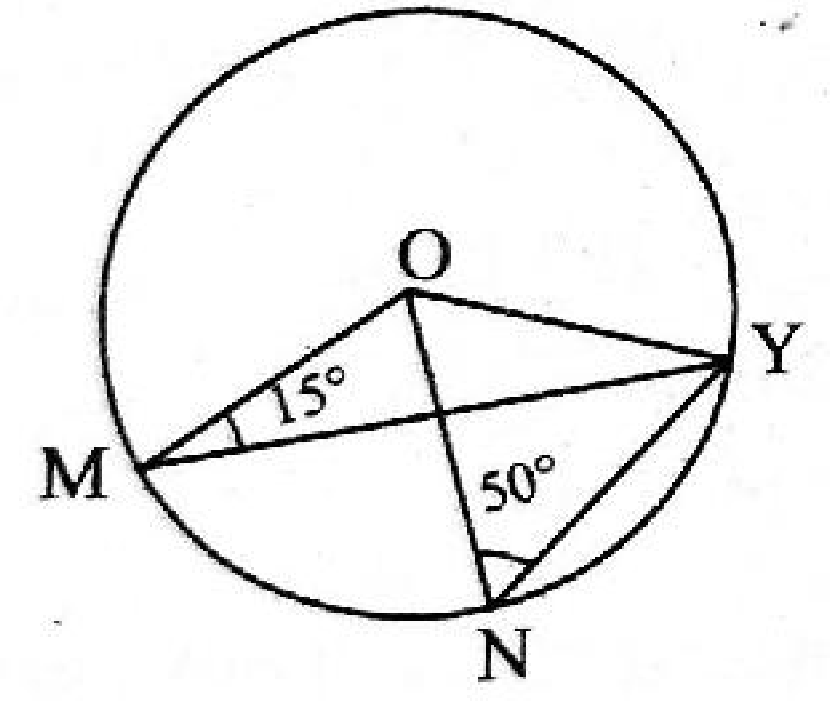
Ans. 28 cm

Q2. In the given figure, $\angle ONY = 50°$ and $\angle OMY = 15°$, then the value of the $\angle MON$ is:

(a) 30° (b) 40°

(c) 20° (d) 70°



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Ans. (d)

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Q3. P and Q are the mid-points of two chords (not diameters) AB and AC, respectively of the circle with centre at a point O. The line OP and OQ are produced to meet the circle, respectively, at the points R and S. T is any point on the major arc between the points R and S of the circle. If $\angle BAC = 32°$, $\angle RTS = ?$

(a) 32°

(b) 64°

(c) 74°

(d) 106°

Ans. (c)

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Q4. P and Q are two points on a circle with centre at O. R is a point on the minor arc at the circle between the points P and Q. The tangents to the circle at the points P and Q meet each other at the point S. If $\angle PSQ = 20^\circ$, then $\angle PRQ = ?$

(a) 80° (b) 200°

(c) 160° (d) 100°

Ans. (d)

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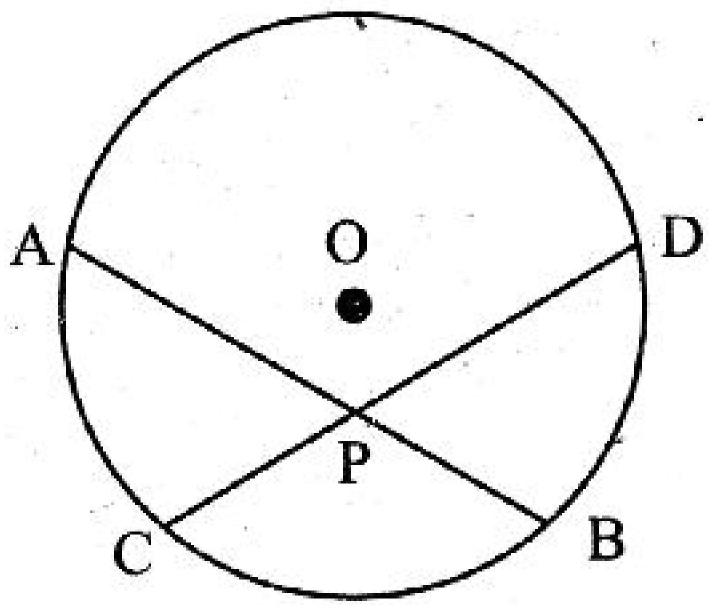
Q5. In the given figure, two chords AB and CD intersects at point P and O is the centre of the circle. If AP = 3 PB, AB = 24 cm and CP = 9 cm, then CD is

(a) 10 cm (b) 12 cm

(c) 15 cm (d) 21 cm



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Ans. (d)

Q6. The tangents at two points A and B on the circle with the centre O intersects at P. If in quadrilateral PAOB, $\angle AOB : \angle APB = 5 : 1$, the measure of $\angle APB$ is:

(a) $30°$ (b) $15°$

(c) $45°$ (d) $60°$

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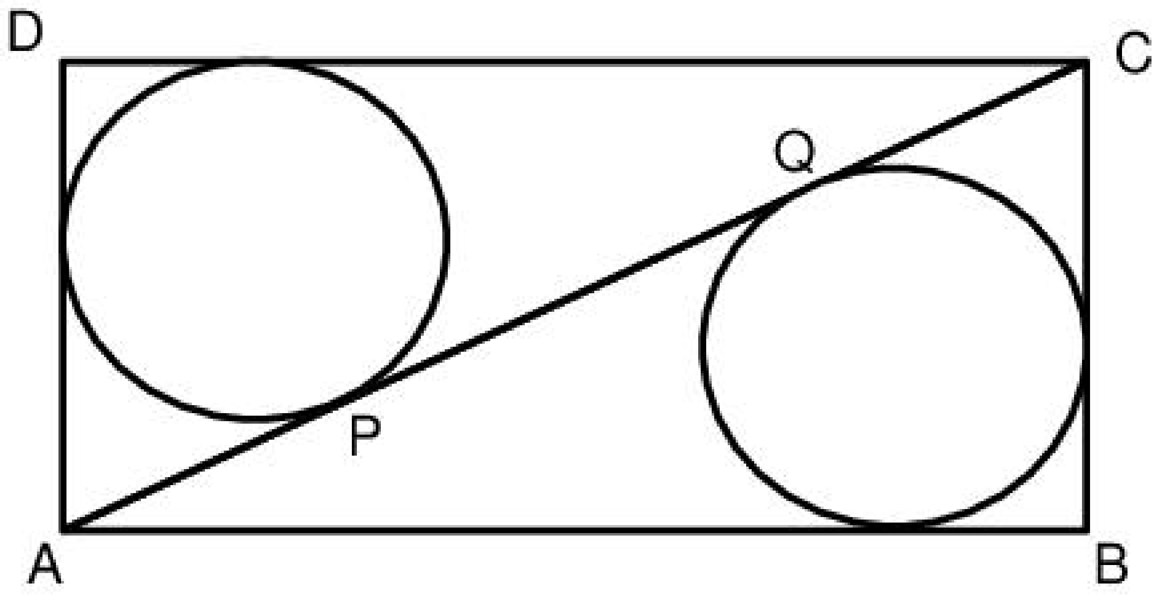
Ans. (a)

Q7. ABCD is a rectangle.

BC = 15 cm, AB = 20 cm

Find PQ.



Ans. 5 cm

Shortcut: $PQ = l - b$

where, l = length of rectangle

b = breadth of rectangle

Q8. ABCD is an isosceles trapezium with parallel sides AB = 25 cm and CD = 9 cm. A circle is inscribed in ABCD. Find diameter of inscribed circle.

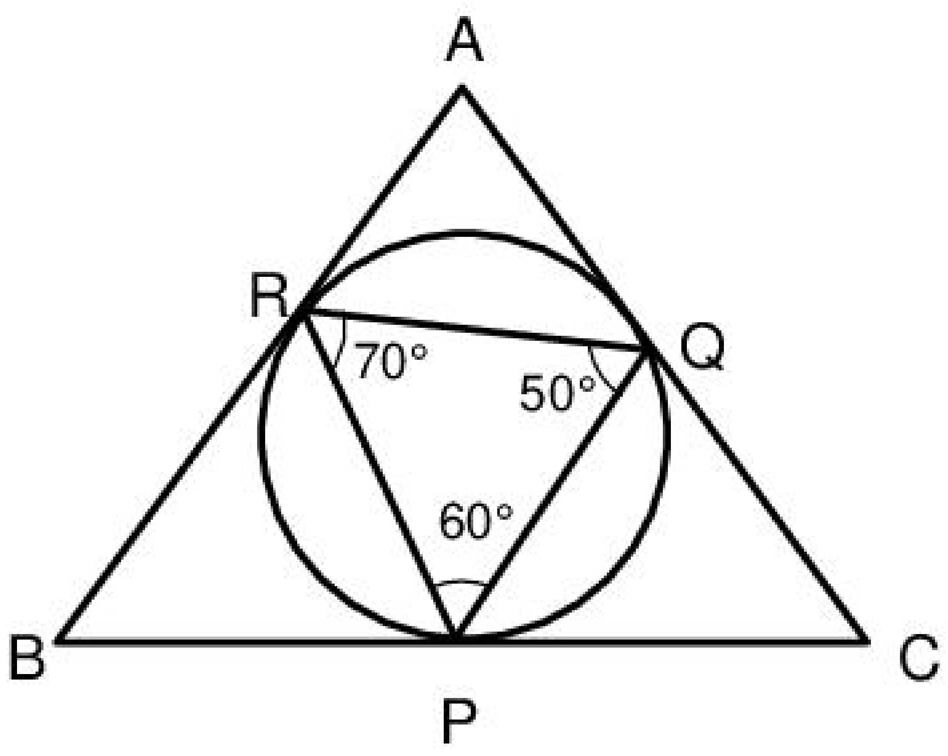
Ans. 15 cm

Shortcut:

Diameter = $\sqrt{ab}$

where, a and b are parallel sides of isosceles trapezium.

Q9. Find $\angle A$, $\angle B$ and $\angle C$.



Ans. $\angle A = 60°$

$\angle B = 80°$

$\angle C = 40°$

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Q10. P and Q are two points on a circle with centre at O. R is a point on the minor arc of the circle, between the points P and Q. The tangents to the circle at the points. P and Q meet each other at the point S. If $\angle PSQ = 20°$, then $\angle PRQ = ?$

(a) 80°
(b) 200°
(c) 160°
(d) 100°

Ans. (d)

Q11. The tangents are drawn at the extremities of diameter AB of a circle with centre P. If a tangent to the circle at the point C intersects the other two tangents at Q and R then the measure of the $\angle QPR$ is:

(a) 45°
(b) 60°
(c) 90°
(d) 180°

Ans. (c)

Q12. A, B and C are three points on a circle with centre O. The tangent at C meets BA produced to T. If $\angle ATC = 30°$ and $\angle ACT = 48°$, then what is the value of $\angle AOB$?

(a) 78°
(b) 96°
(c) 102°
(d) 108°

Ans. (d)

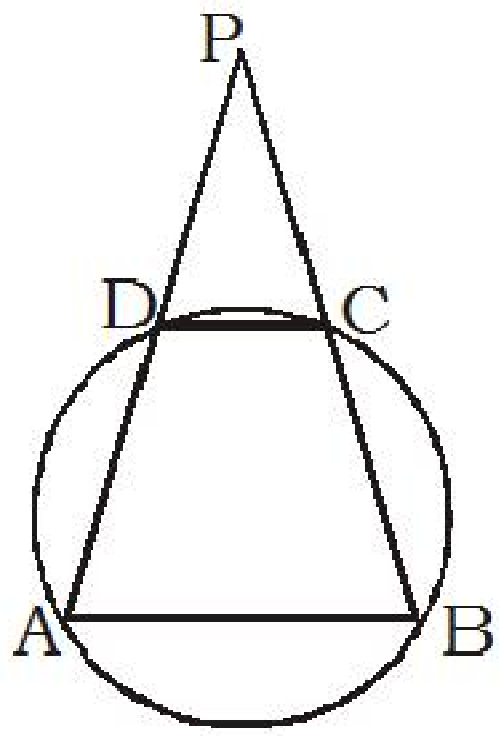
Q13. ABCD is a cyclic quadrilateral AB and DC are produced to meet at P.

If $\angle ADC = 70°$ and $\angle DAB = 60°$, then the $\angle PBC + \angle PCB$ is:

(a) 130°
(b) 150°
(c) 155°
(d) 180°

Ans. (a)

Q14. In the figure given above, if $\angle BAD = 60°$, $\angle ADC = 105°$, then what is $\angle DPC$ equal to?



(a) 40°
(b) 45°
(d) 60°
(c) 50°

Ans. (b)

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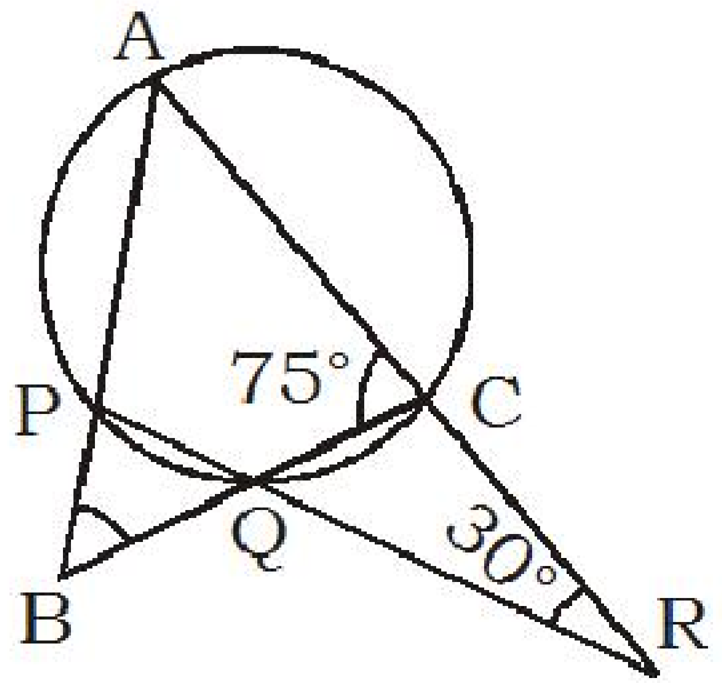
Q15. In the given figure, what is $\angle CBA$?



(a) 30°   (b) 45°

(d) 50°   (c) 60°

Ans. (d)

Q16. ABCD is a cyclic quadrilateral. The tangents at A and C intersect each other at P. If $\angle ABC = 100°$, then what is $\angle APC$ equal to?

(a) 10°

(b) 20°

(d) 30°

(c) 40°

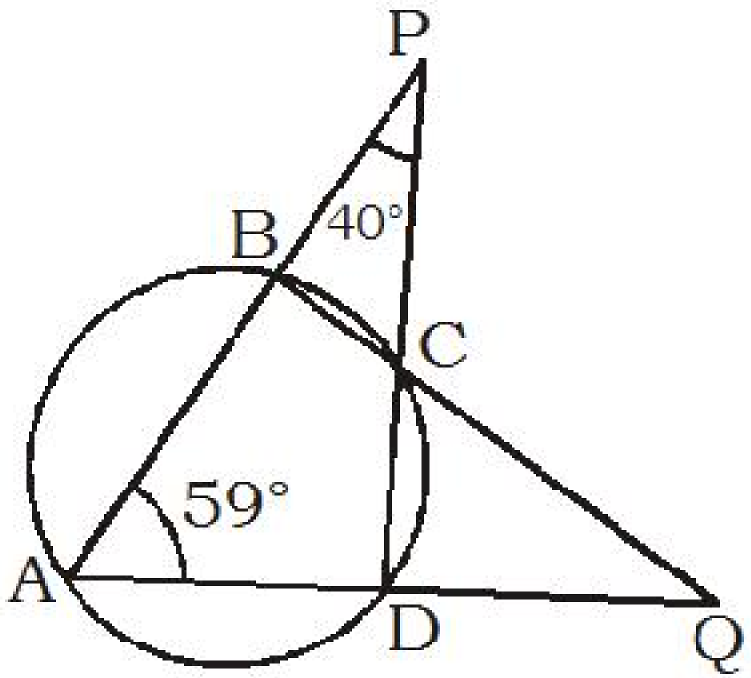
Ans. (b)

Q17. Two circles C_1 and C_2 touch each other internally at P. Two lines PCA and PDB meet the circles C_1, in C, D and C_2 in A, B respectively. If $\angle BDC = 120°$, the value of $\angle ABD$ is equal to:

(a) 60°
(b) 80°
(c) 100°
(d) 120°

Ans. (a)

Q18. In the given figure, if $\angle PAQ = 59°$, $\angle APD = 40°$, then what is $\angle AQB$?

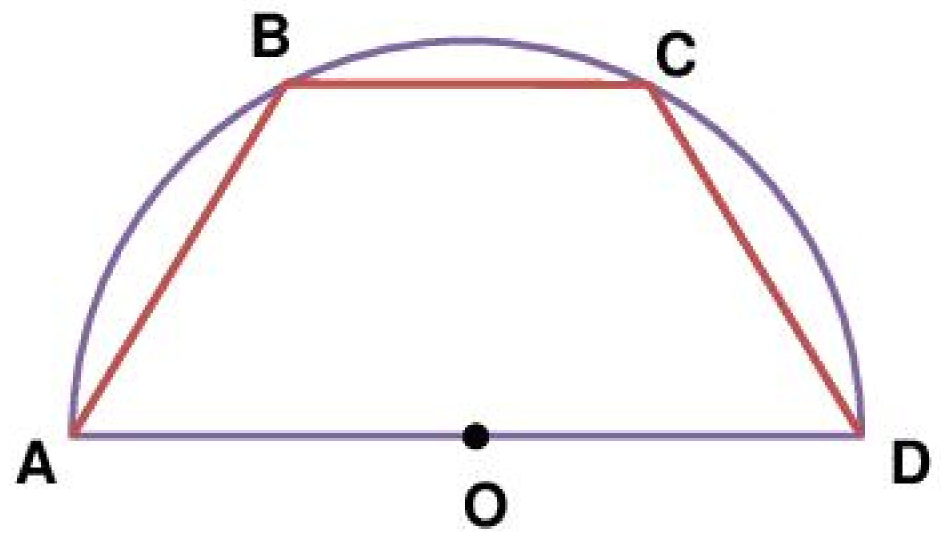


(a) 19°
(b) 20°
(c) 22°
(d) 27°

Ans. (c)

Q19. If AD || BC, AB = CD = 1 cm and AD = 4 cm

Find BC. (O is centre of semicircle)

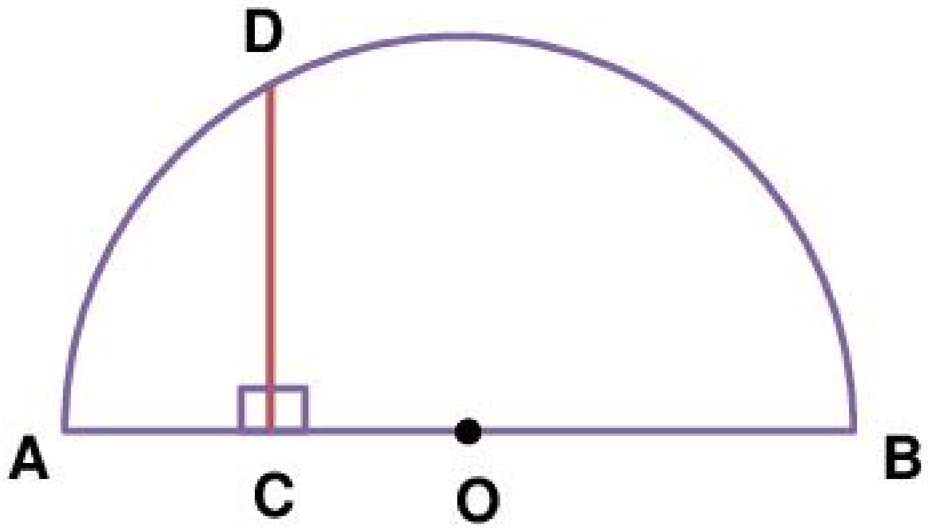


Ans. 3.5 cm

Q20. If AC = 3 cm, CD = 9 cm

(O is centre of semicircle)

Find area of semi-circle.



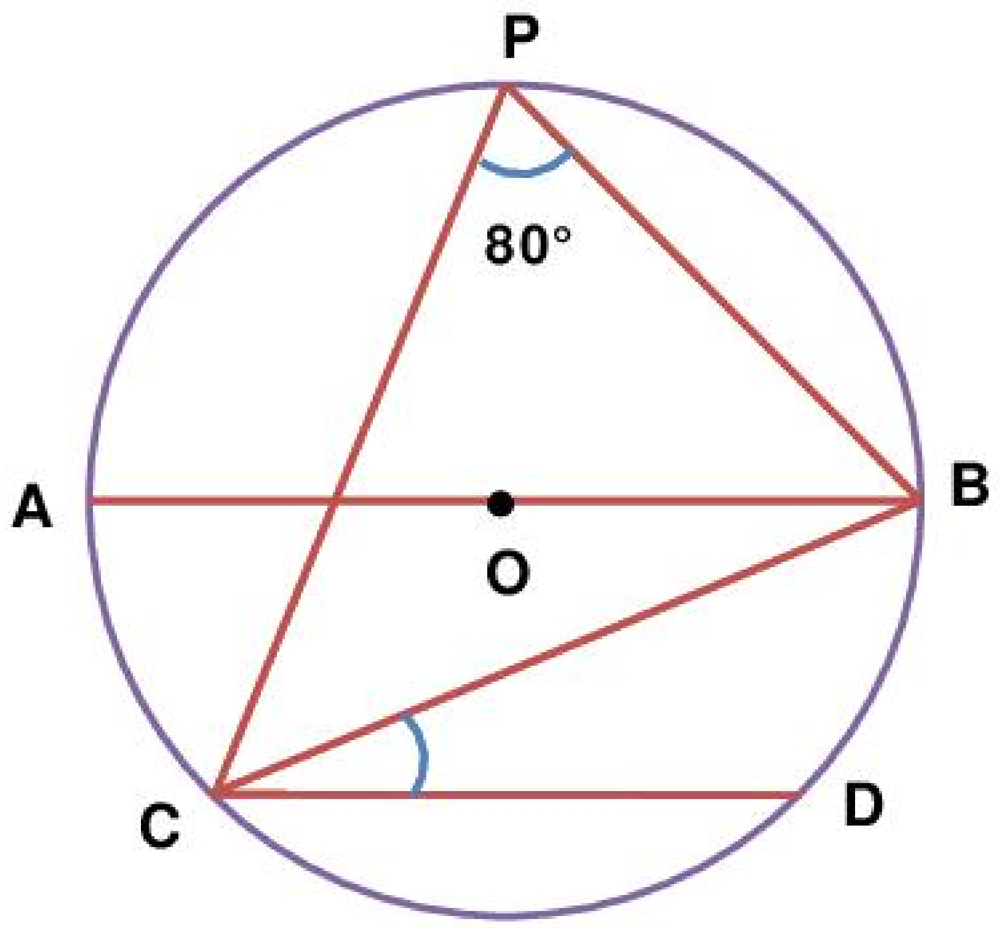
Ans. $$\frac{225\pi}{2}$$

Q21. If AB || CD

$\angle CPB = 80°$

(O is centre of circle)

Find $\angle BCD$.



Ans. 10°

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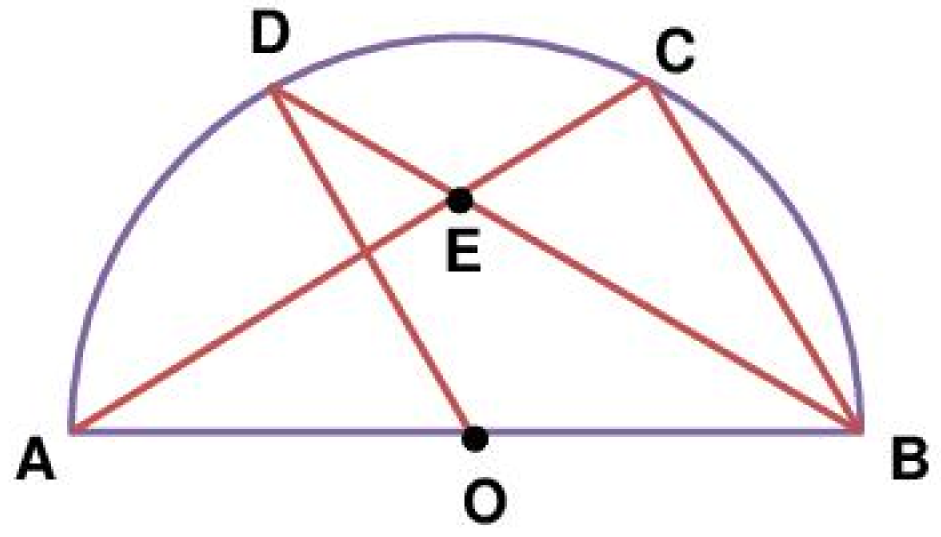
Q22. AB is diameter

$\angle DOC = 80°$ (O is centre of circle)

Find $\angle DEC$.



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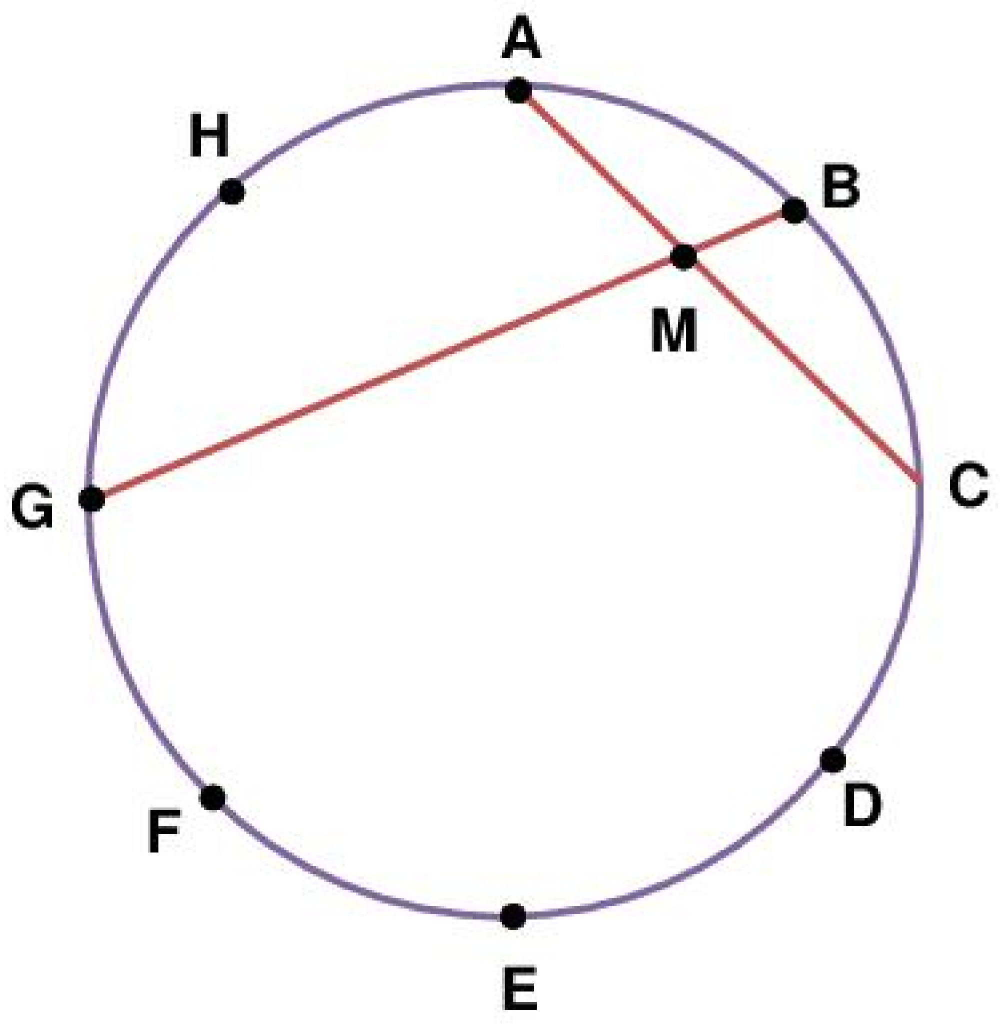
Ans. 130°

Q23. If $\overset{\frown}{AB} = \overset{\frown}{BC} = \overset{\frown}{CD} = \overset{\frown}{DE} = \overset{\frown}{EF} = \overset{\frown}{FG} = \overset{\frown}{GH} = \overset{\frown}{HA}$

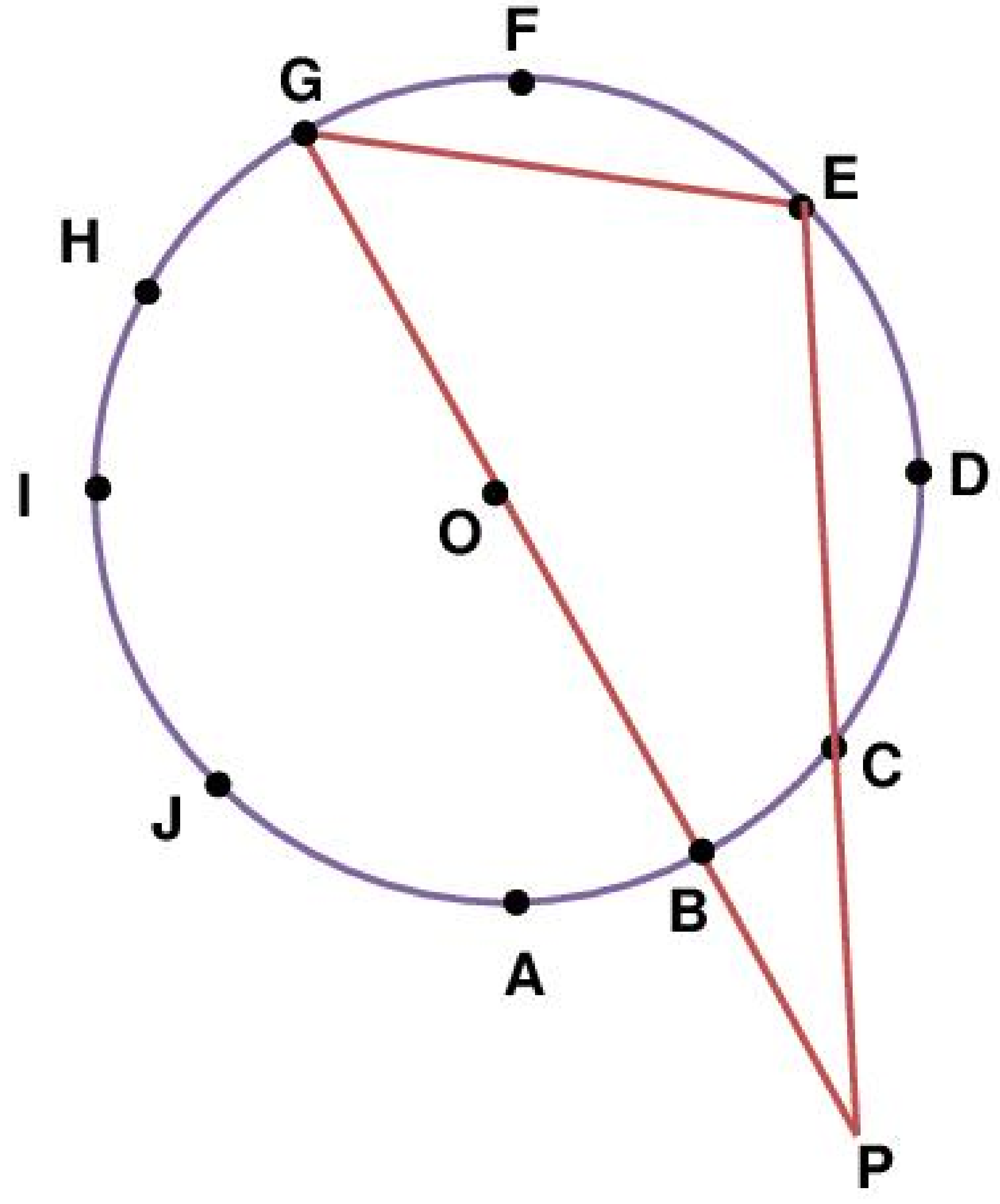
Find $\angle AMB$.



Ans. 112.5°

Q24. *If* $\overset{\frown}{AB} = \overset{\frown}{BC} = \overset{\frown}{CD} = \overset{\frown}{DE} = \overset{\frown}{EF} = \overset{\frown}{FG} = \overset{\frown}{GH} = \overset{\frown}{HI} = \overset{\frown}{IJ} = \overset{\frown}{JA}$

Find $\angle EPG$.



Ans. 18°