QUADRILATERAL

Part-I

Agenda → Quadrilateral 1

* We are left with 2 session on Triangles ie Triangle Part 3 & 4 which will be covered in this week

BYJU'S EXAM PREP

Fri 7th January → Quadrilateral 1

Sat 8th January → Quadrilateral 2

Sun 9th → [We will cover up

Mon 10th Triangles Part 3 Δ

Triangles Part 4]

Tue 11th → Quadrilateral 3

Wed 12th → Quadrilateral 4

Agenda → Quadrilateral

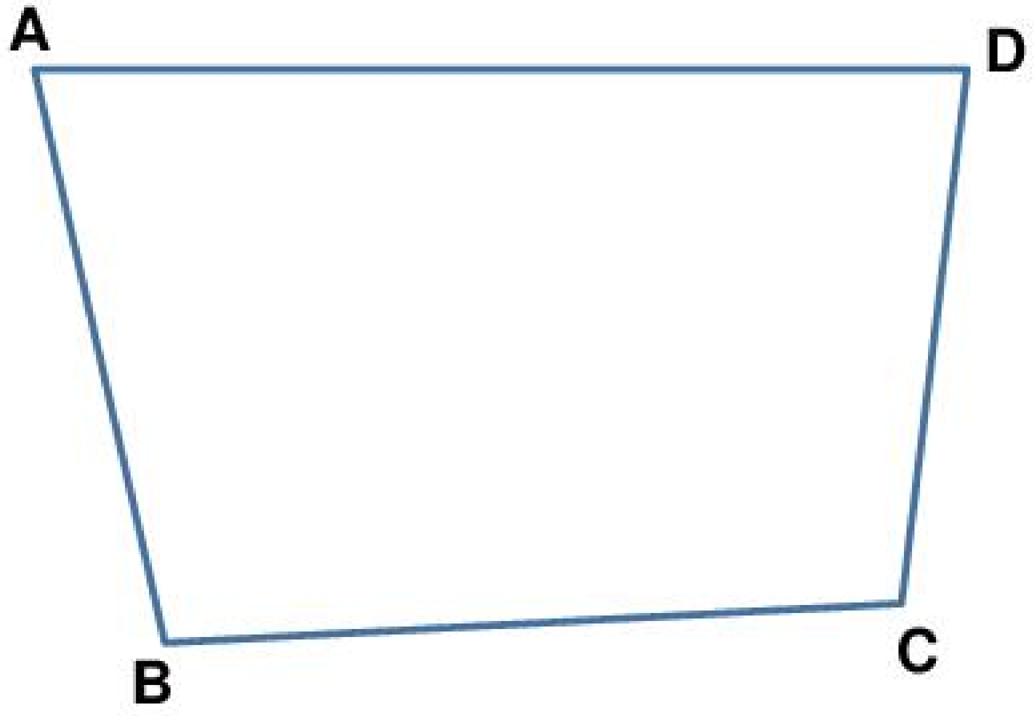
* Quadrilateral → (38-40) min

* Parallelogram → (52-54) min

QUADRILATERAL

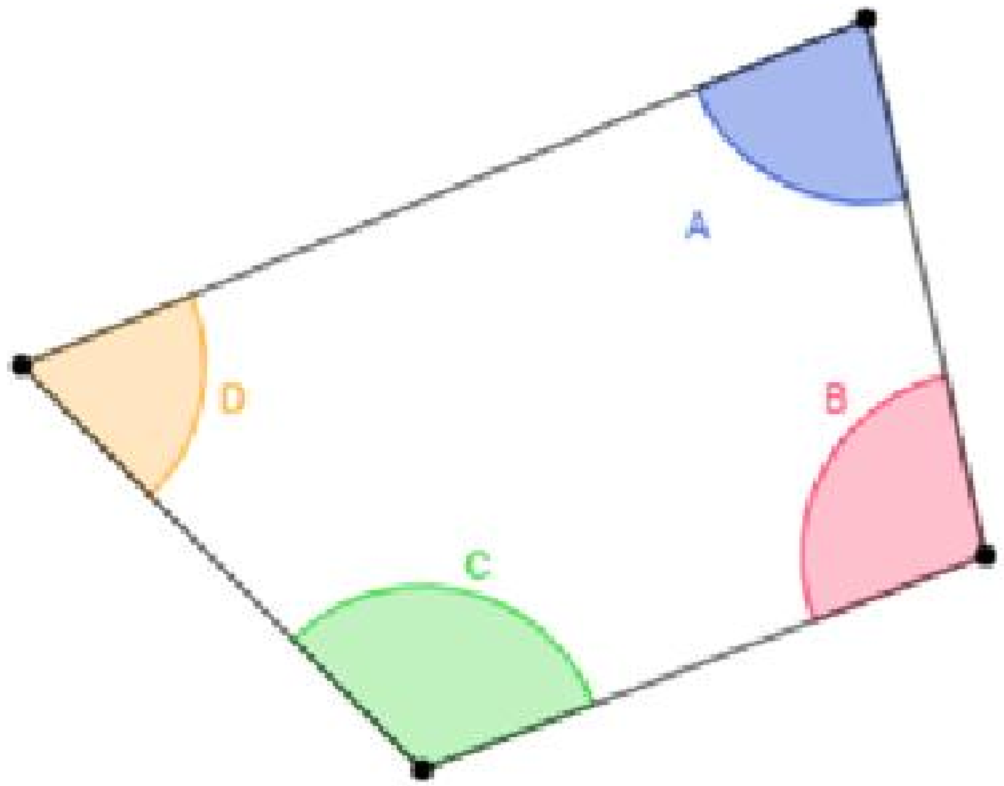
$$\underbrace{\text{QUAD}}_{4}\text{RI}\underbrace{\text{LATERAL}}_{\text{side}}$$

Def: Any four sided closed figure is called as Quadrilateral.



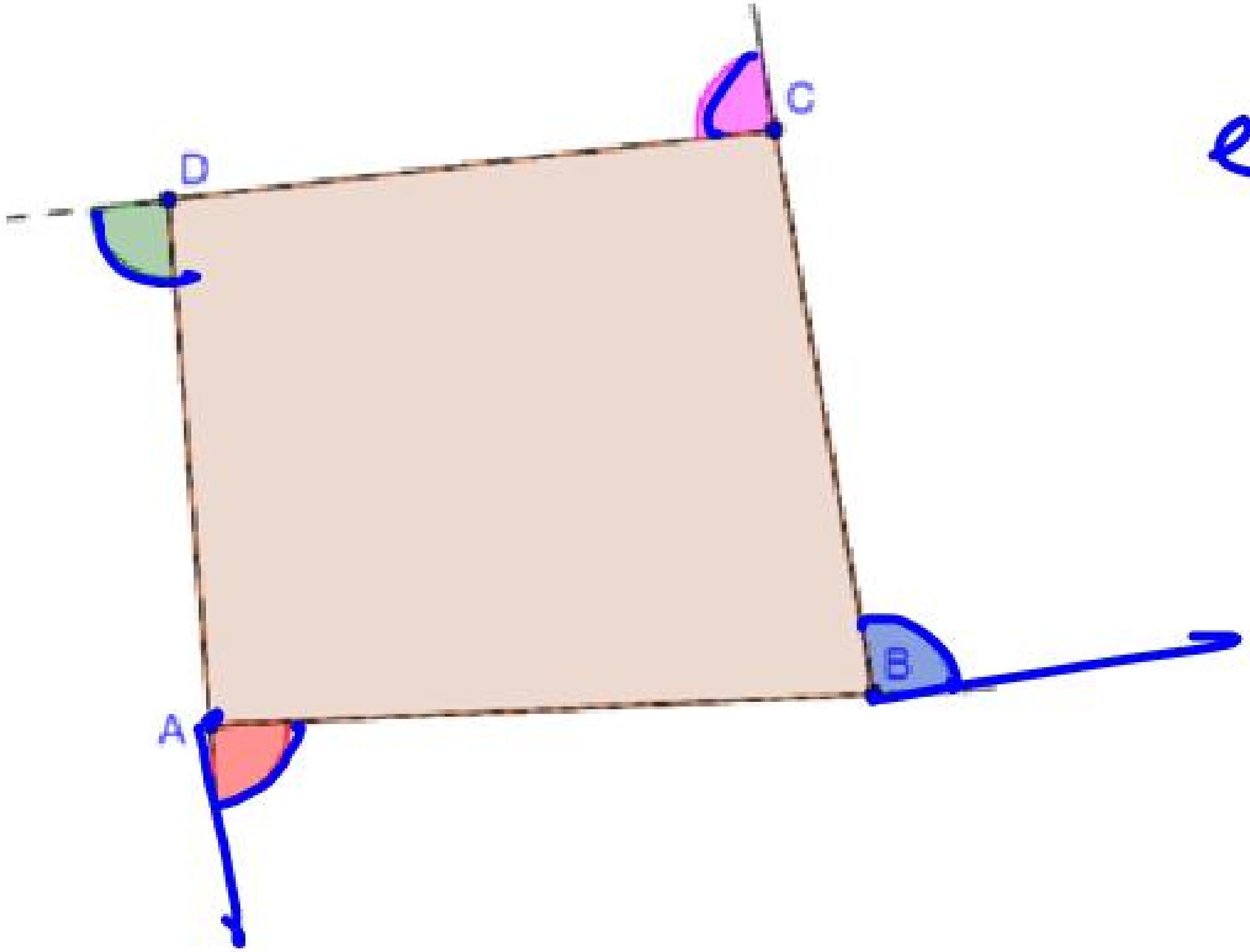
PROPERTIES

1. Sum of all interior angles of a quadrilateral = 360°



$\angle A + \angle B + \angle C + \angle D = 360^\circ$

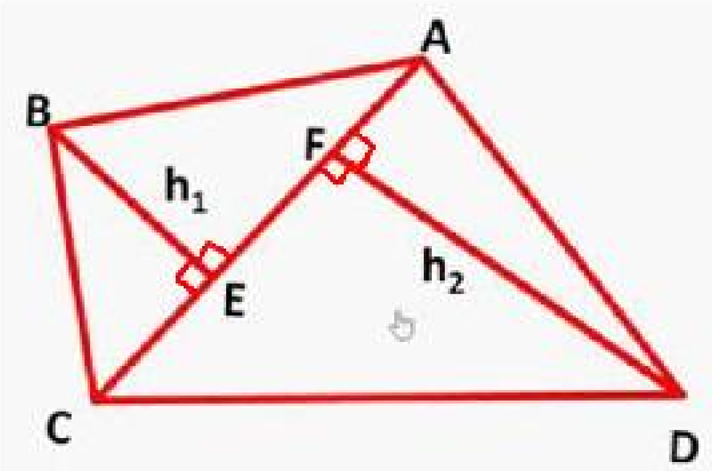
2. Sum of all exterior angles of a quadrilateral = 360°



ext∠A + ext∠B + ext∠C + ext∠D

= 360°

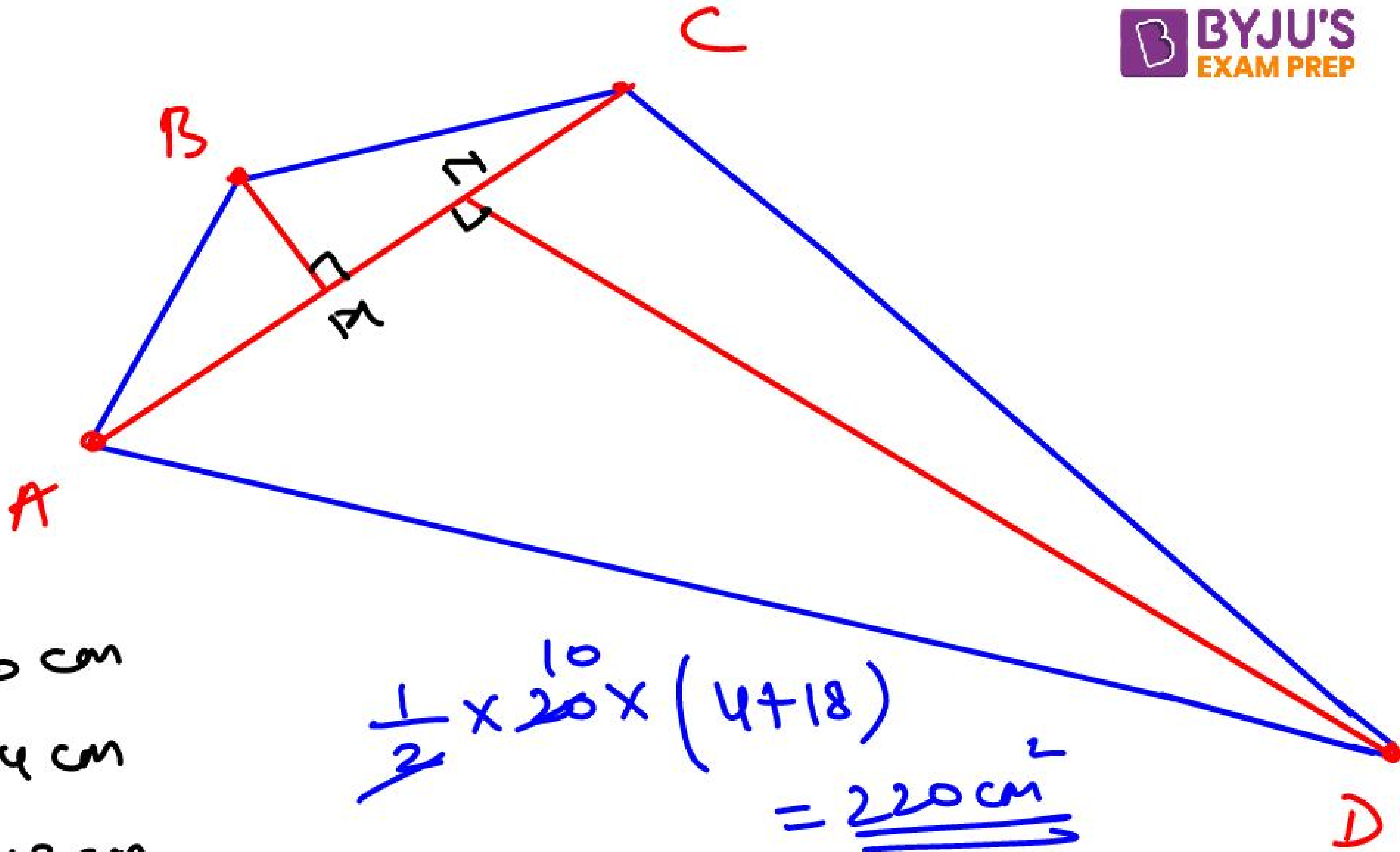
3. Area of quadrilateral ABCD :

(i) 

$$= \frac{1}{2} \times One\ of\ the\ diagonals \times Sum\ of\ \perp\ dropped\ on\ it$$

$$= \frac{1}{2} AC\left(BE + DF\right)$$

eg



AC = 20 cm

BM = 4 cm

DN = 18 cm

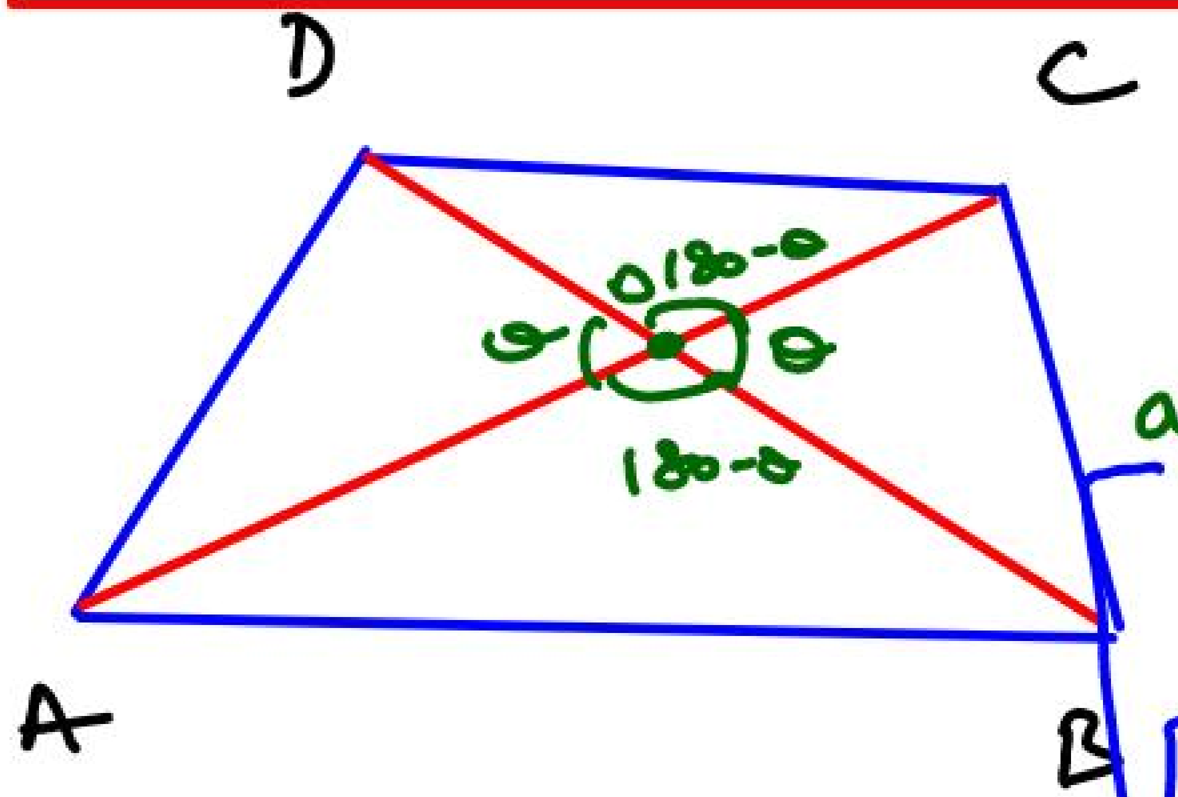
Area of quad ABCD = ??

$$\frac{1}{\cancel{2}} \times \overset{10}{\cancel{20}} \times (4+18)$$

$$= \underline{\underline{220\,cm^2}}$$

(ii) Area of quadrilateral $= \frac{1}{2} D_1 D_2 \sin\theta$

where, D_1, D_2 are diagonals of quadrilateral and θ is the angle between the diagonal.



Area of Quad ABCD

area $\triangle AOB = \boxed{\frac{1}{2}(AO)(OB)\sin(180-\theta)}$

area $\triangle BOC = \frac{1}{2}(BO)(CO)\sin\theta$

area of $\triangle COD = \boxed{\frac{1}{2}(CO)(DO)\sin(180-\theta)}$

area of $\triangle AOD = \frac{1}{2}(AO)(DO)\sin\theta$

BYJU'S EXAM PREP

area of Quad ABCD

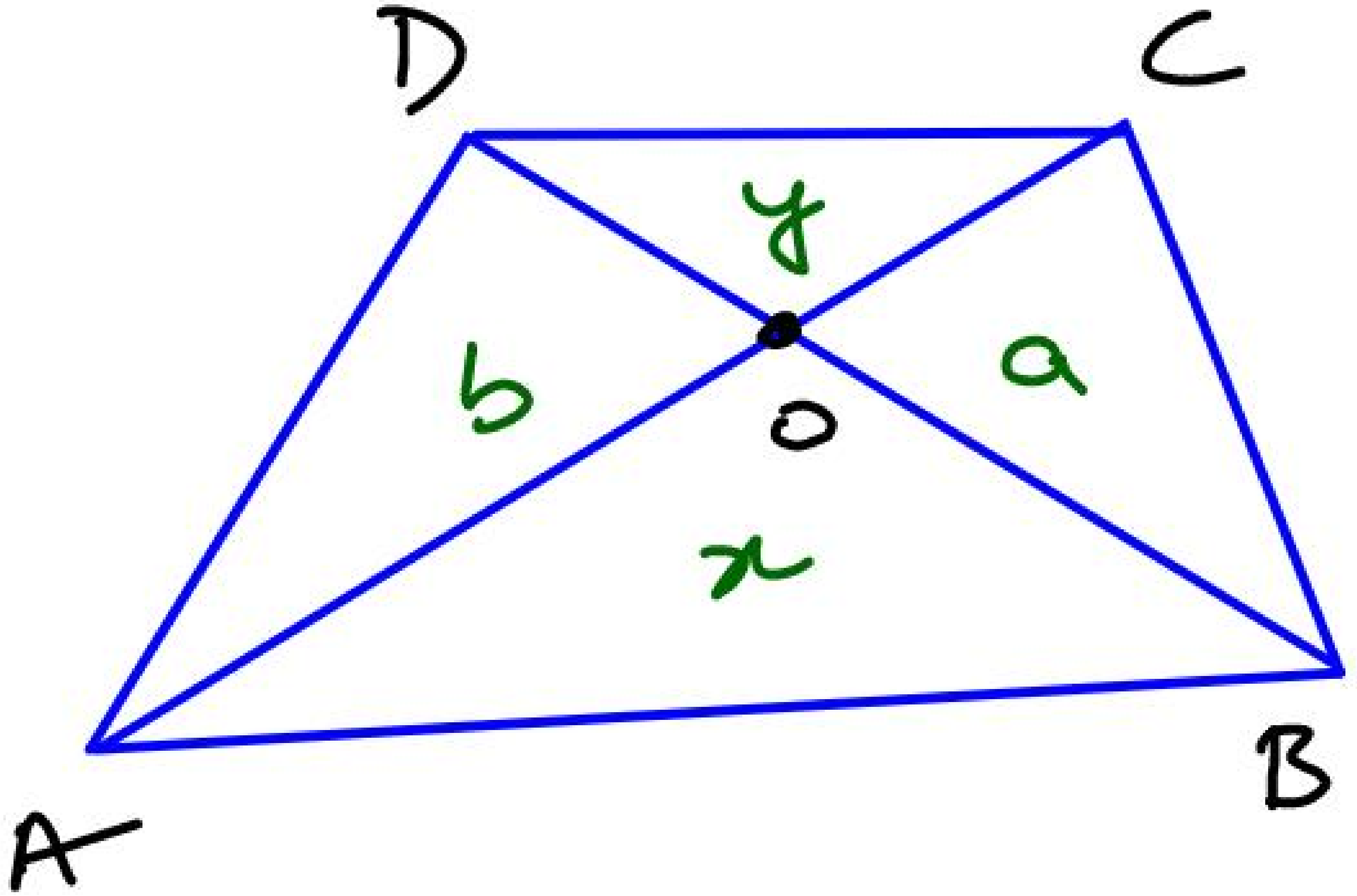
$\Rightarrow \frac{1}{2} AO \sin\theta (BO + DO) + \frac{1}{2} CO \sin\theta (BO + DO)$

$\Rightarrow \frac{1}{2} (\sin\theta) (AO + CO) (BO + DO)$

$\Rightarrow \frac{1}{2} (\sin\theta) (AC) (BD)$

4. In a quadrilateral, if AC and BD are the diagonals and they intersect at O, then

(area of ΔAOB) · (area of ΔCOD) = (area of ΔBOC) · (area of ΔAOD)



$x \cdot y = a \cdot b$

Eg1. In a quadrilateral diagonals AC and BD intersect each other at O.

If area of : $\Delta AOB = 20\ cm^2$, $\Delta BOC = 30 cm^2$ and $\Delta COD = 15\ cm^2$

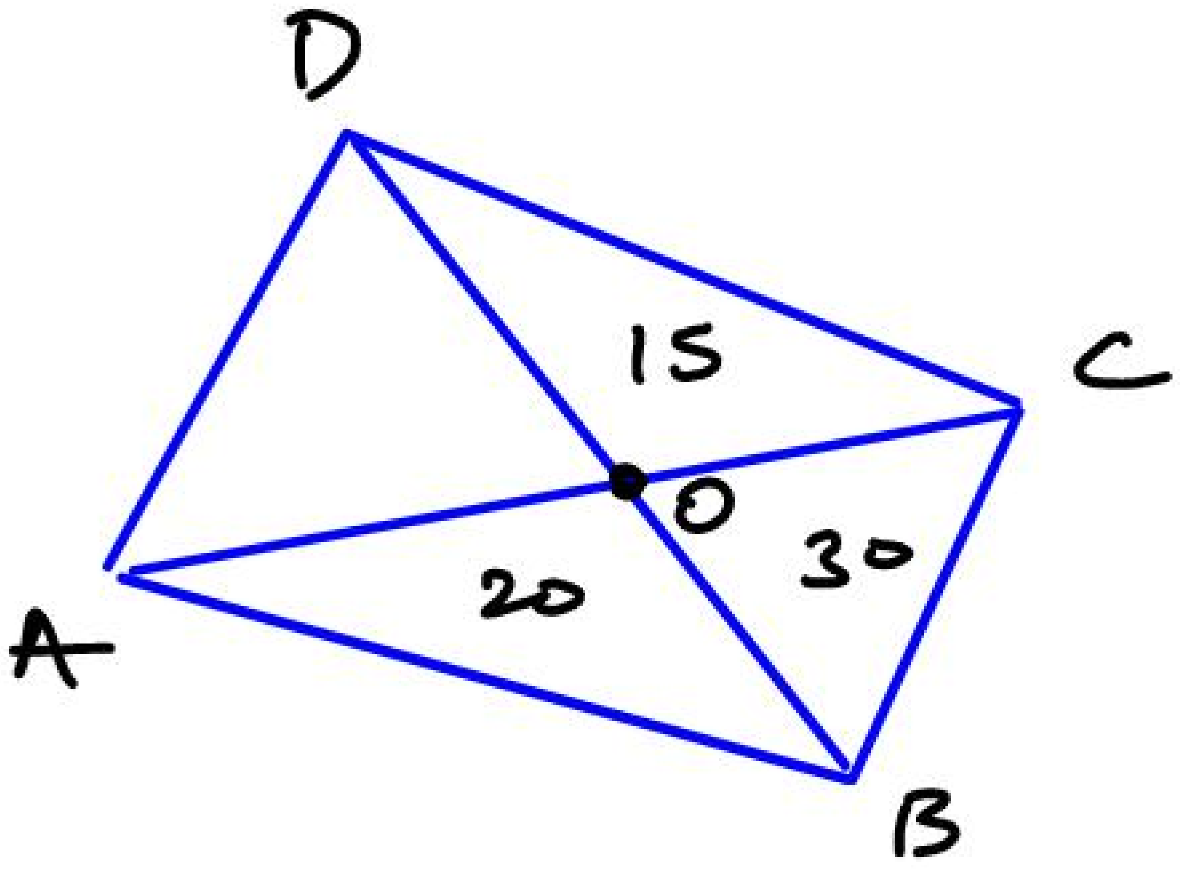
Find area of quadrilateral ABCD.

$20 \cdot 15 = 30 \cdot$ area ΔAOD

area of $\Delta AOD = 10$

$20 + 30 + 15 + 10$

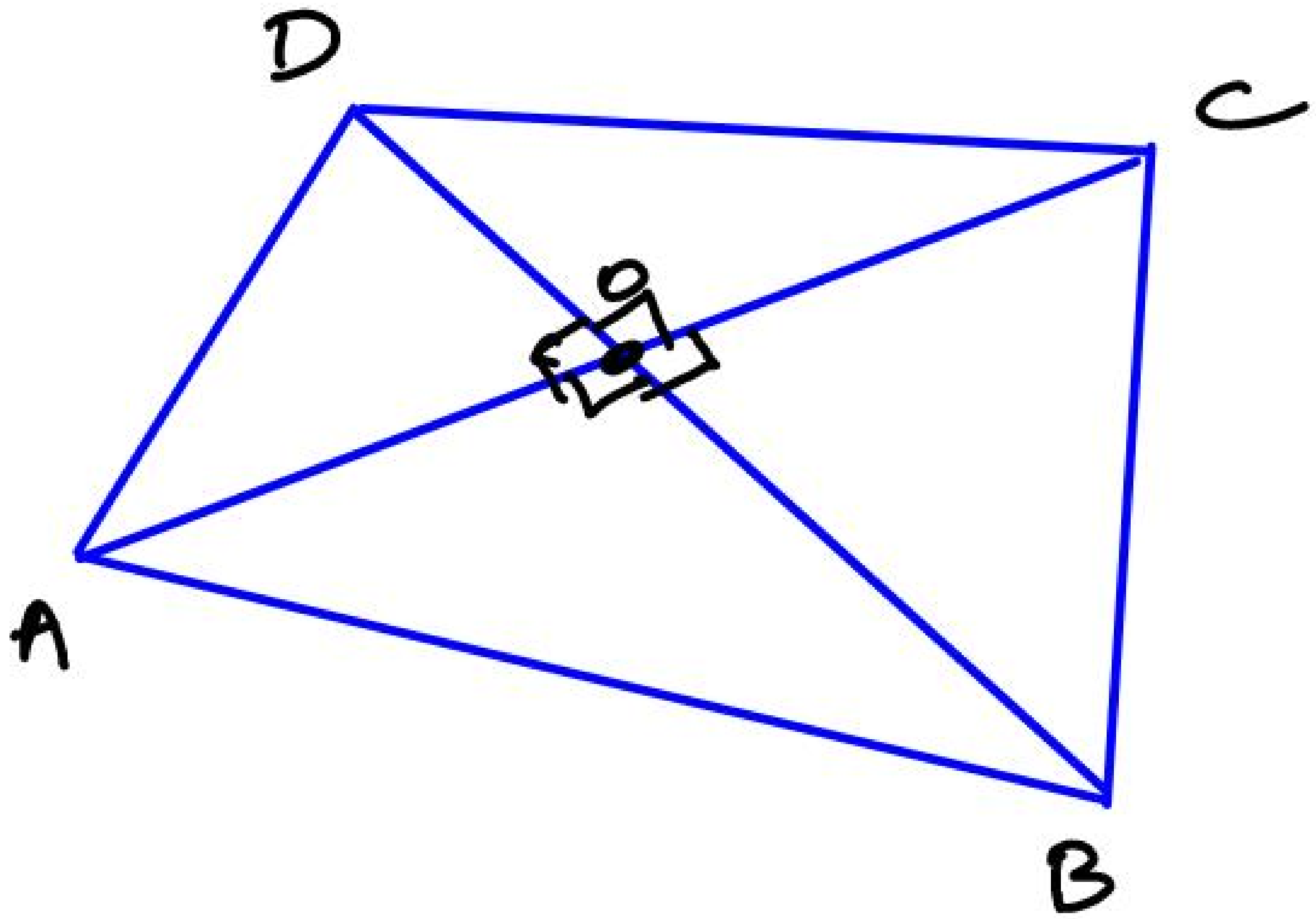
area of ABCD $= \underline{75 cm^2}$



Ans. 75 cm^2

5. If diagonals of a quadrilateral intersect each other at 90°, then :

$$\boxed{AB^2 + CD^2 = BC^2 + AD^2}$$



$AB^2 = AO^2 + BO^2$

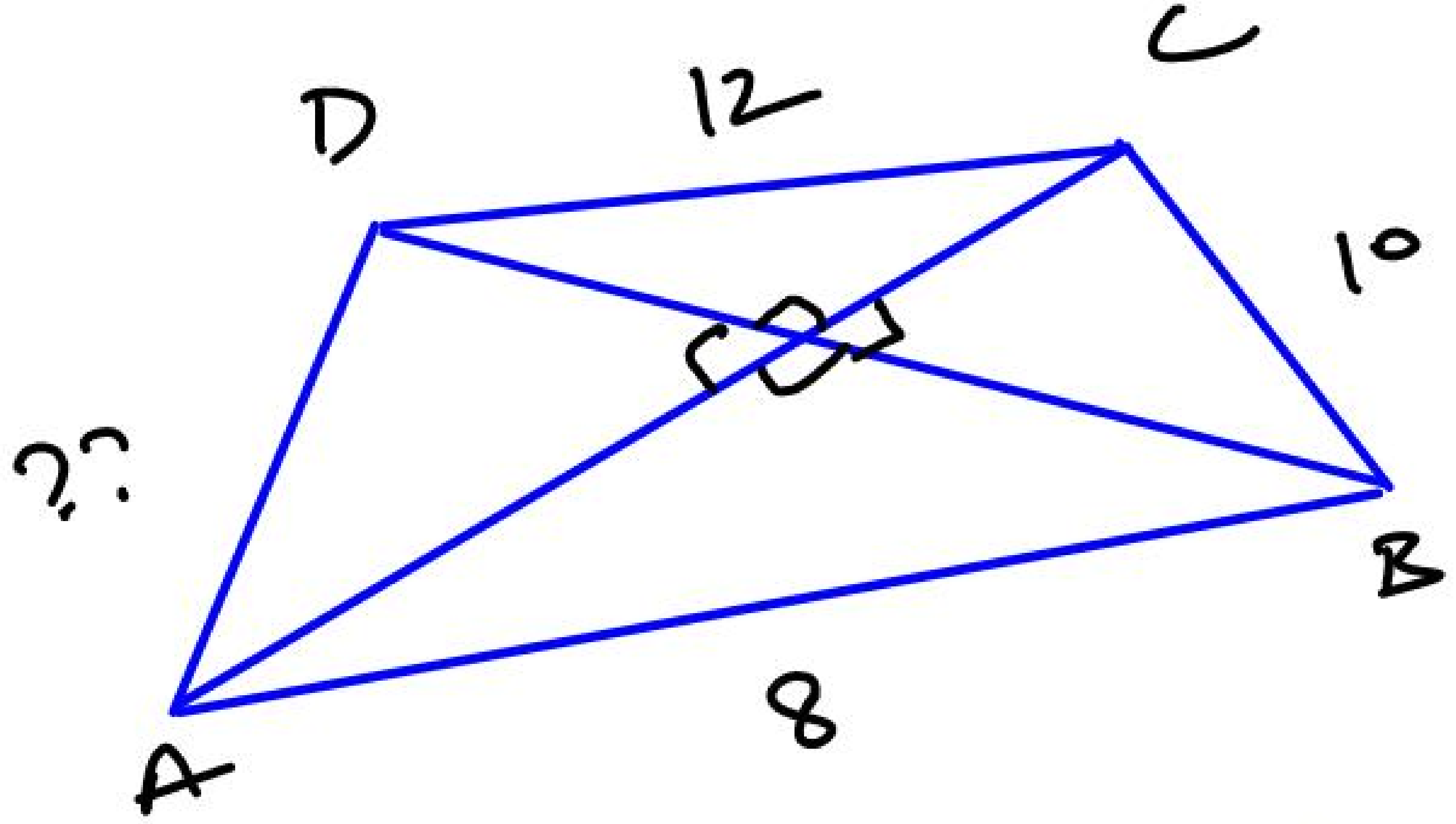
$BC^2 = BO^2 + CO^2$

$CD^2 = CO^2 + DO^2$

$AD^2 = AO^2 + DO^2$

$$\boxed{AB^2 + CD^2 = BC^2 + AD^2}$$

eg

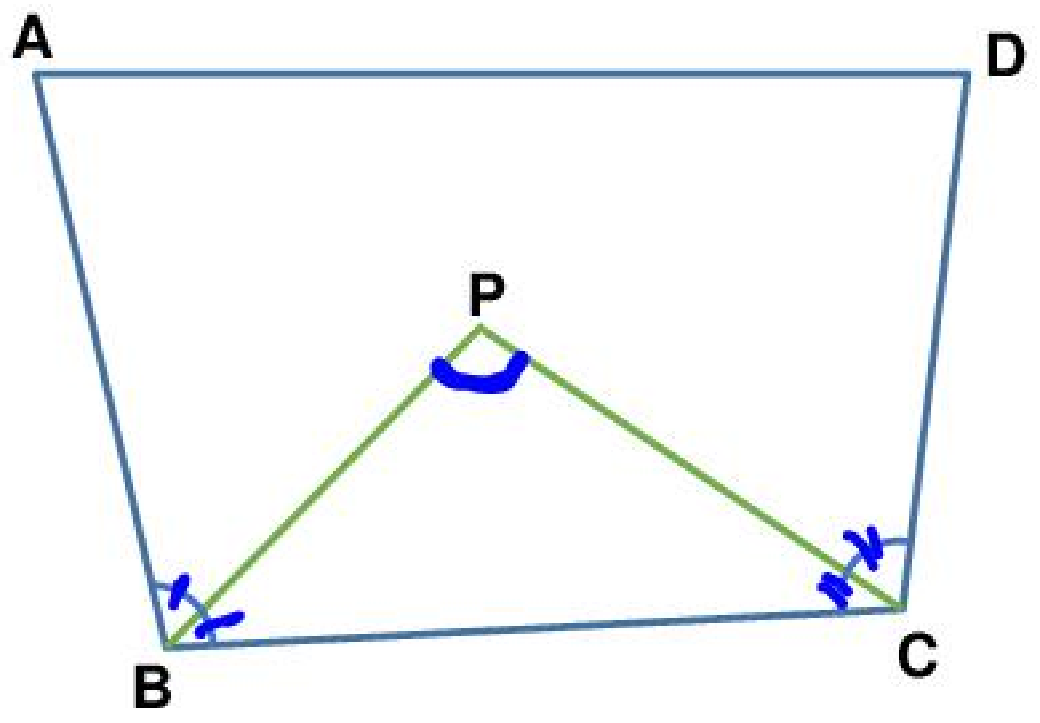


$$8^2 + 12^2 = 10^2 + (AD)^2$$

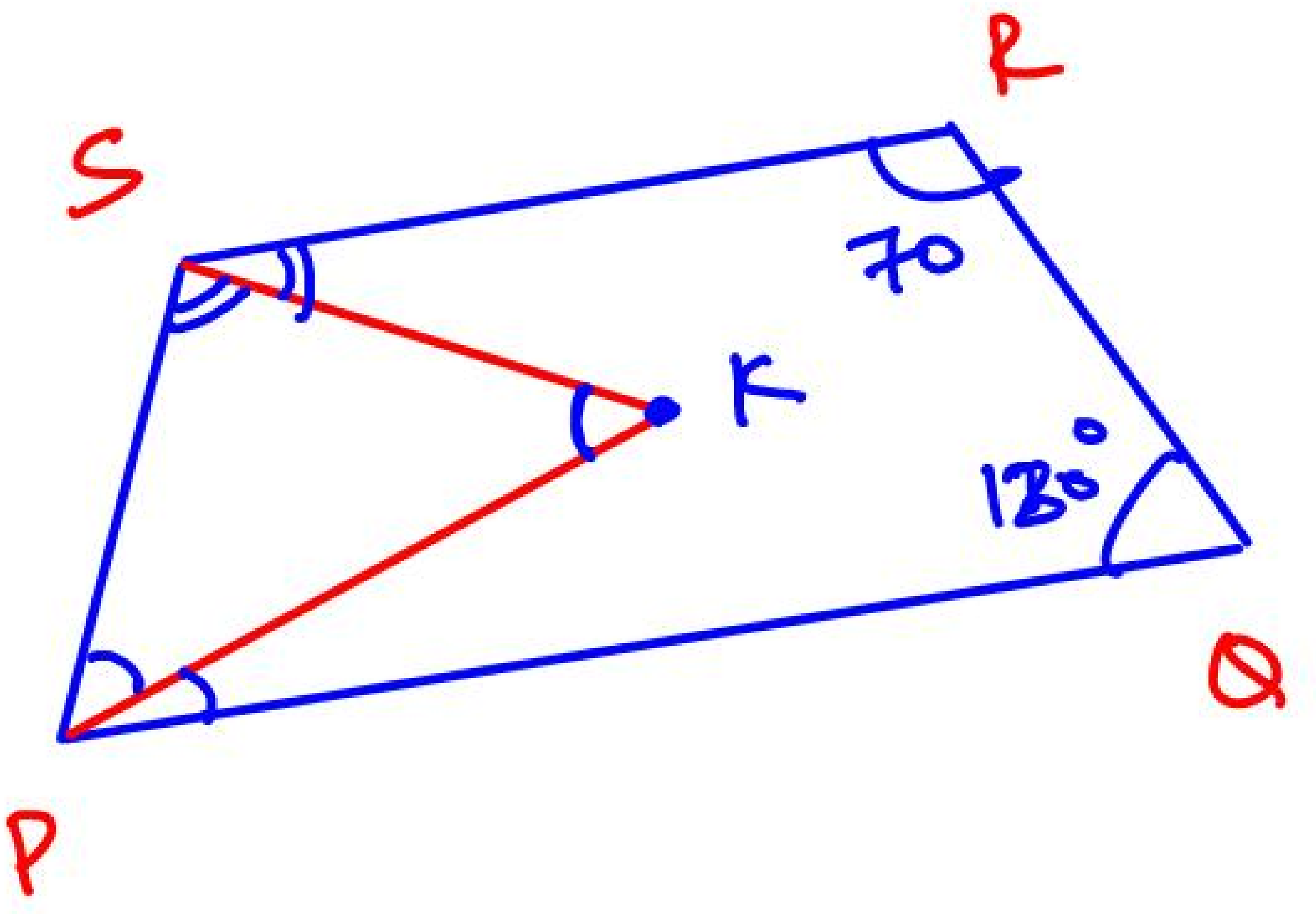
$$208 - 100 = (AD)^2$$

$$AD = \underline{\underline{6\sqrt{3}}}$$

6. If bisectors of $\angle B$ and $\angle C$ of a quadrilateral intersect each other at P, then $\boxed{\angle BPC = \frac{1}{2}(\angle A + \angle D)}$



Eg2. In a quadrilateral PQRS, bisectors of $\angle S$ and $\angle P$ meet at K. If $\angle R = 70^\circ$ & $\angle Q = 130^\circ$. Find $\angle SKP$.



$\angle P + \angle S + \angle R + \angle Q = 360$

$\angle P + \angle S = 160^\circ$

$\frac{1}{2}\angle P + \frac{1}{2}\angle S = 80$

$\triangle PSK$

$\frac{1}{2}\angle P + \frac{1}{2}\angle S + \angle SKP = 180$

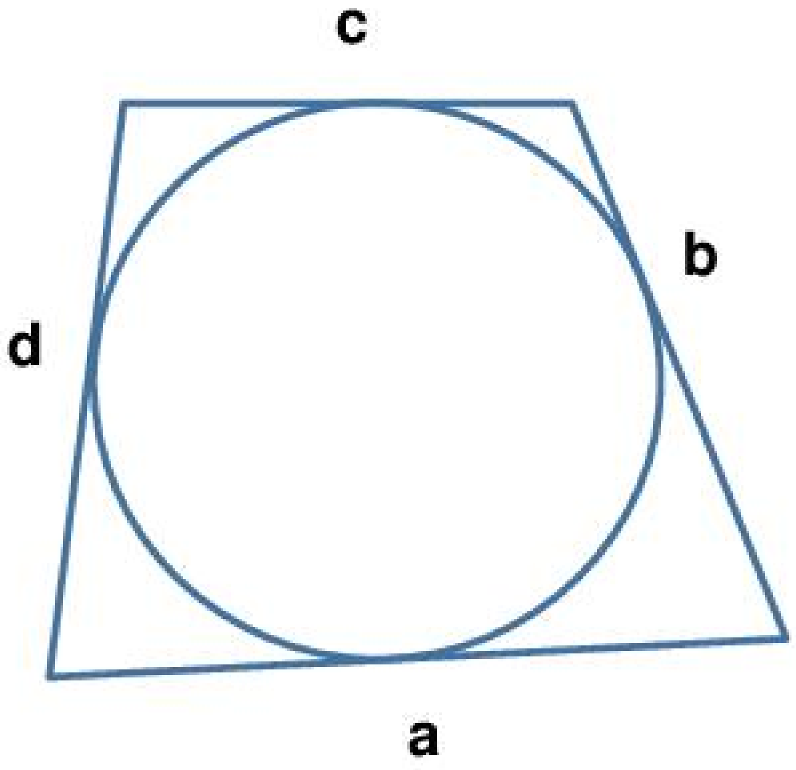
$80 + \angle SKP = 180$

$\angle SKP = 100$

7. If a circle is inscribed in quadrilateral :

v. imp

$$a + c = b + d$$



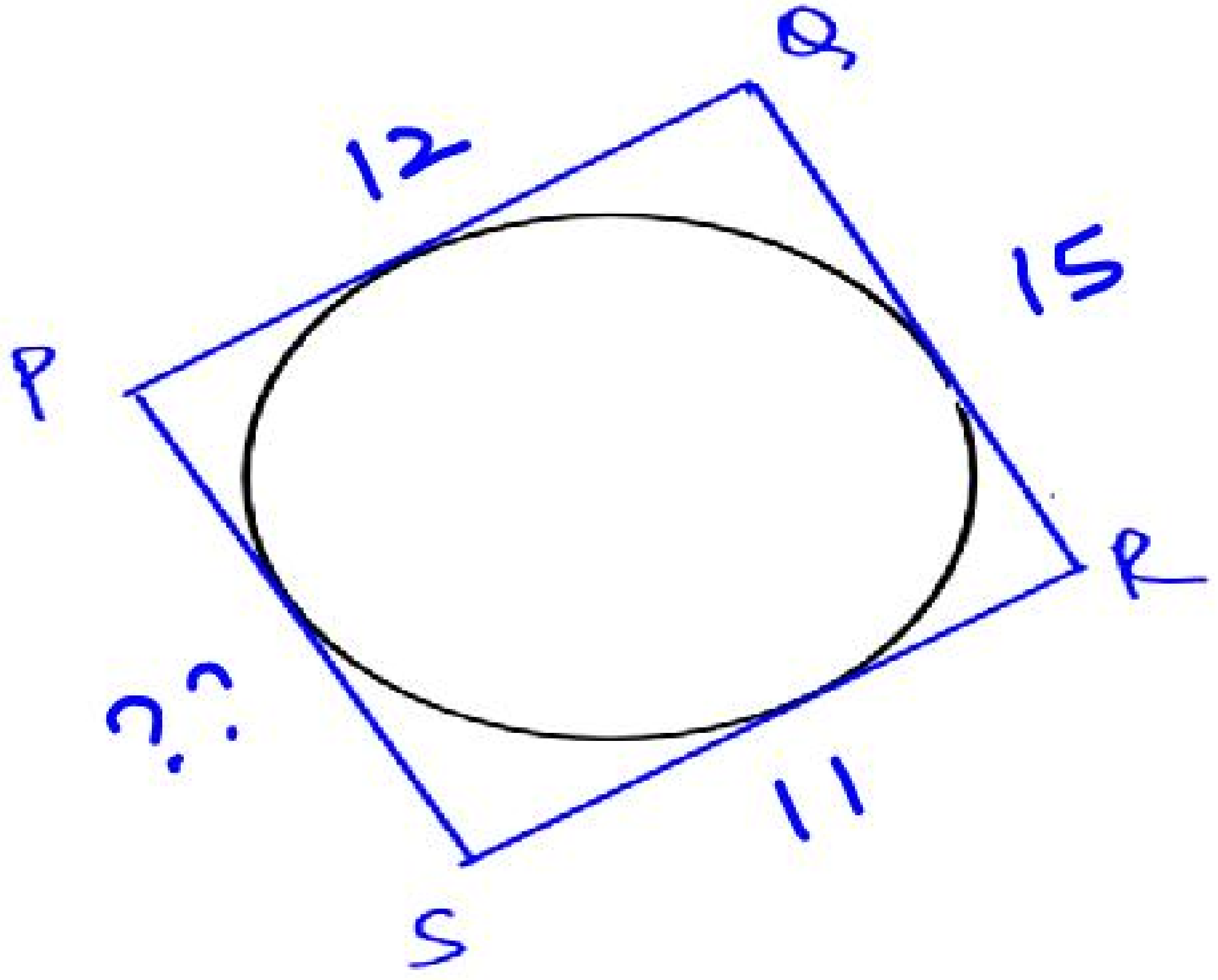
Reason :- will be discussed in Circles

Eg3. If PQ = 12 cm

QR = 15 cm

RS = 11 cm

Find PS = ??



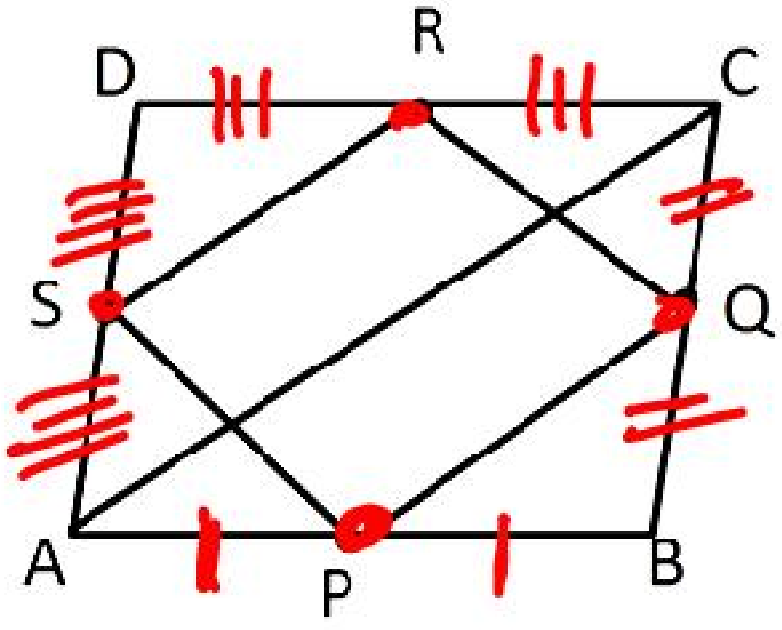
12+11 = 15 + PS

$\boxed{PS = 8}$

BYJU'S EXAM PREP

V.imp

8. **Figure formed by joining the mid-points of all sides of a quadrilateral is a parallelogram and its area is half of the quadrilateral.**



Given P, Q, R & S are mid pts of AB, BC, CD & DA respectively

$\boxed{\begin{array}{l} PQRS \rightarrow \text{Parallelogram} \\ \text{area of } PQRS \rightarrow \frac{1}{2} \text{ area } ABCD \end{array}}$

SUFFICIENT CONDITIONS FOR A QUADRILATERAL TO BE A PARALLELOGRAM

1. **If opposite sides of a quadrilateral are equal, then that is a parallelogram.**

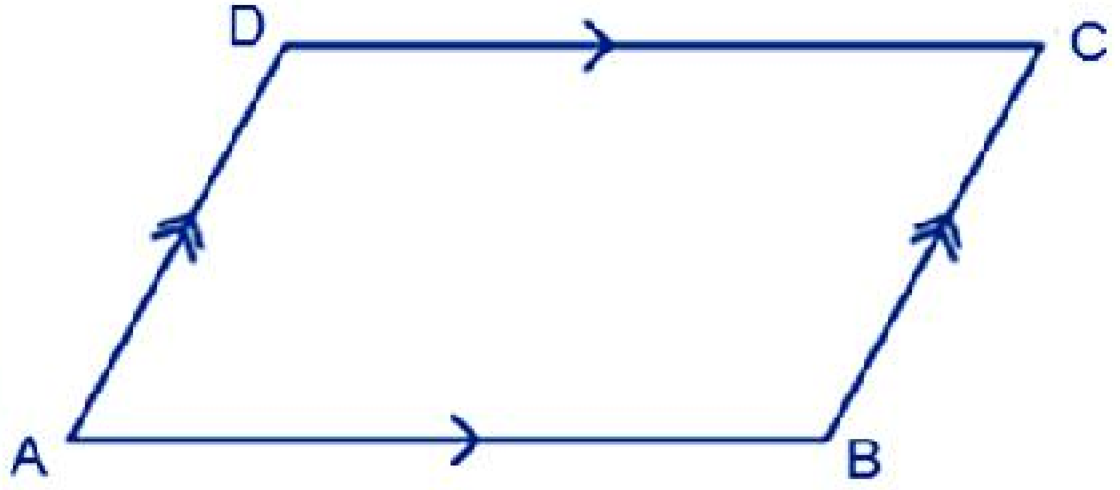
2. **If opposite angles of a quadrilateral are equal, then that is a parallelogram.**

3. **If diagonals of a quadrilateral bisect each other, then that is a parallelogram.**

4. If one pair of sides of a quadrilateral is equal and parallel, then that is a parallelogram.

PARALLELOGRAM

Def: A quadrilateral in which opposite sides are parallel.



Def

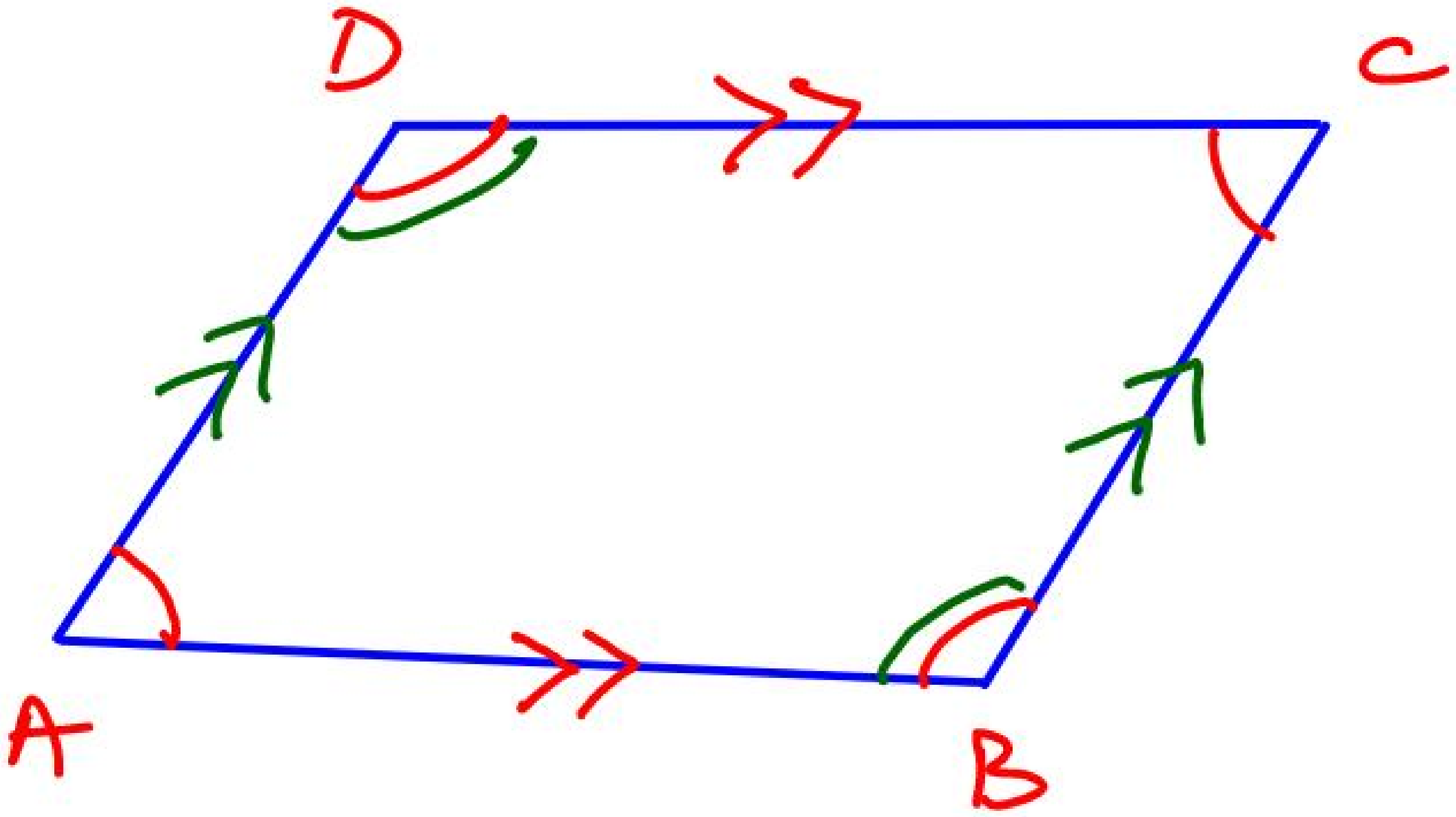
ABCD Quad

+

AB || CD & BC || AD

⇓

Parallelogram



$$\boxed{\angle A + \angle D = 180^\circ}$$

$$\angle B + \angle C = 180^\circ$$

$$\boxed{\angle A + \angle B = 180^\circ}$$

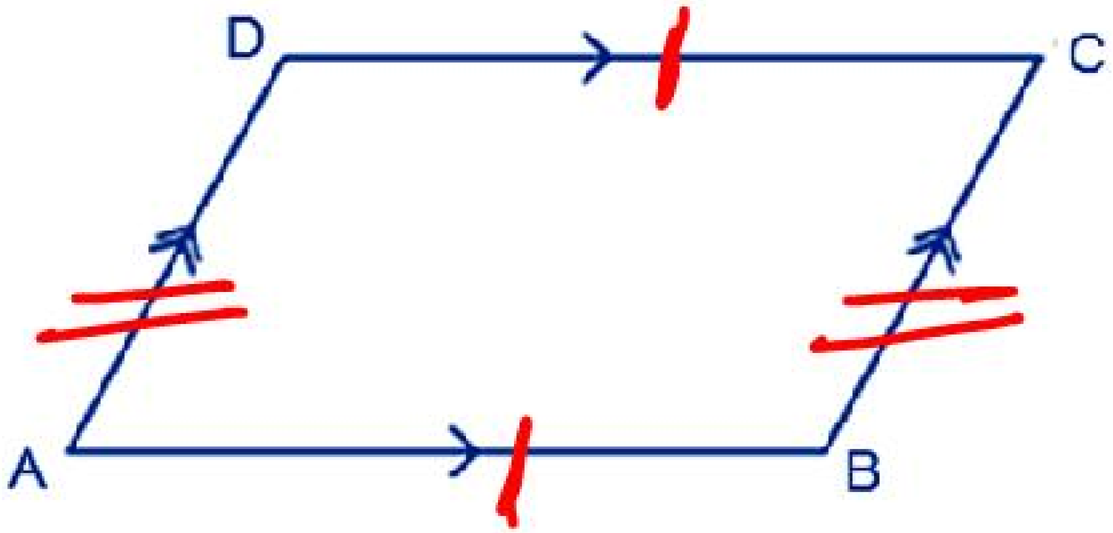
$$\angle C + \angle D = 180^\circ$$

→ opp angles are equal

→ sum of adjacent angles is 180°

PROPERTIES OF PARALLELOGRAM

1. **Opposite sides and opposite angles of parallelogram are equal.**

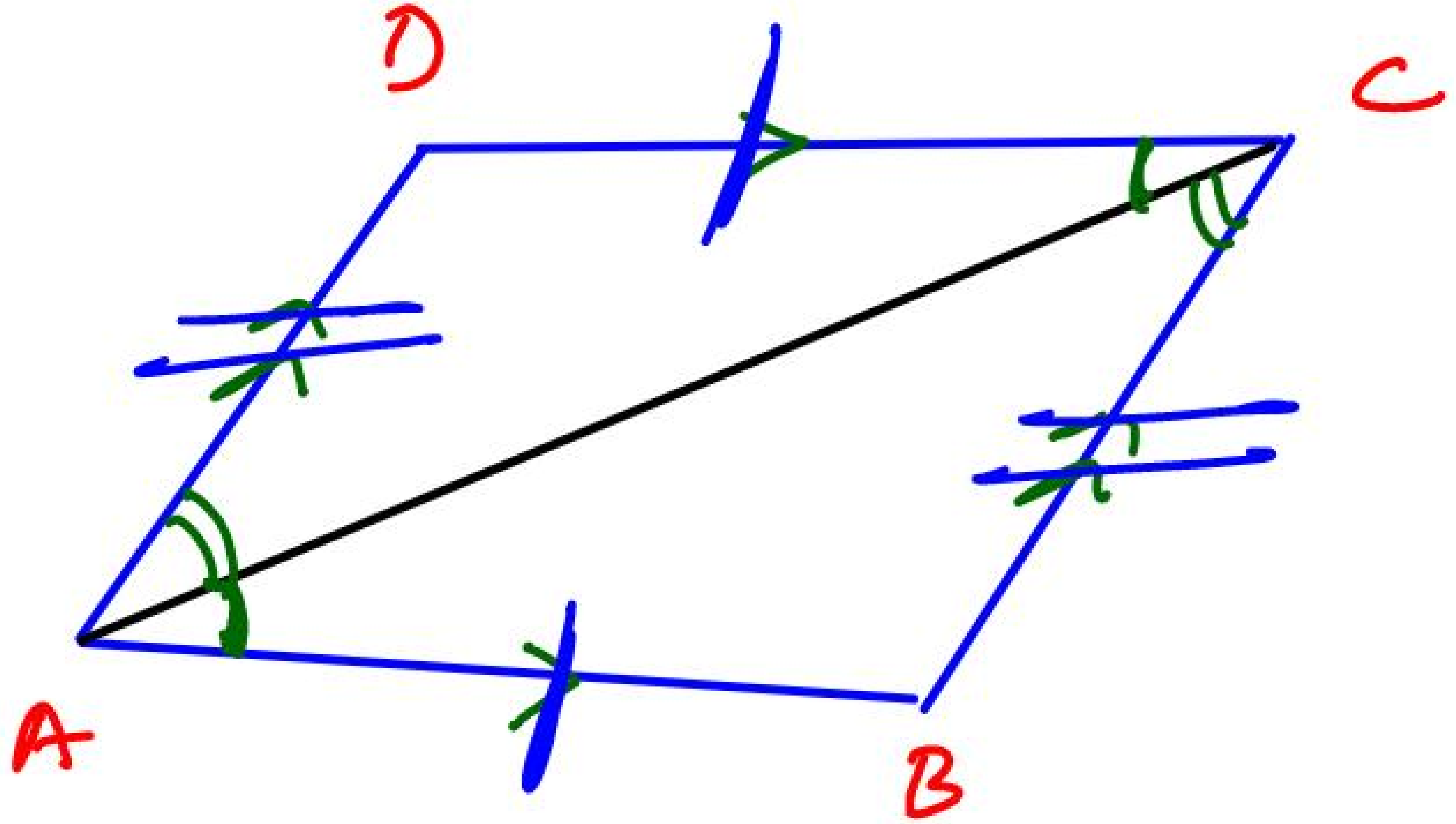


(i) AB = CD

BC = AD

(ii) $\angle A = \angle C$

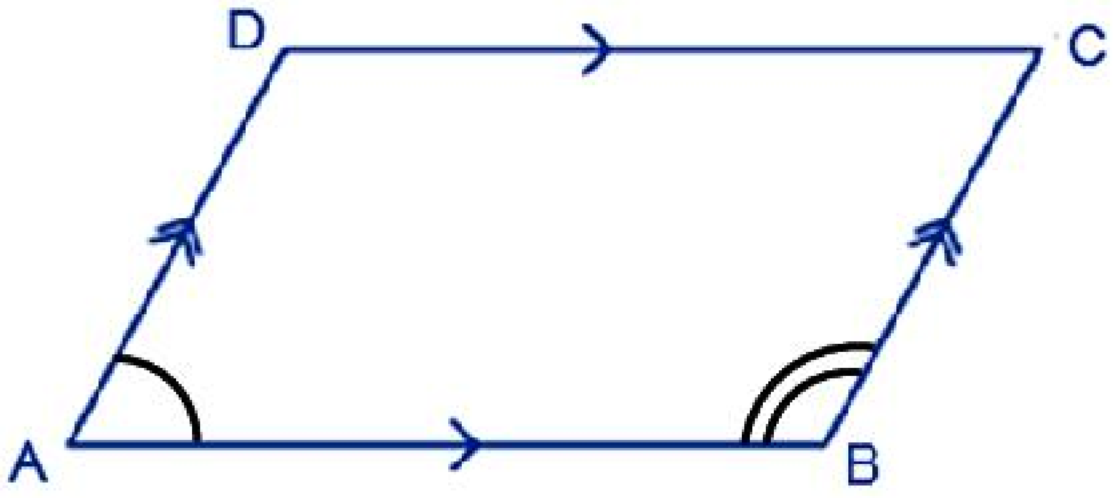
$\angle B = \angle D$



$\Delta\, C\overline{A}D \cong \Delta\, A\overline{C}B$

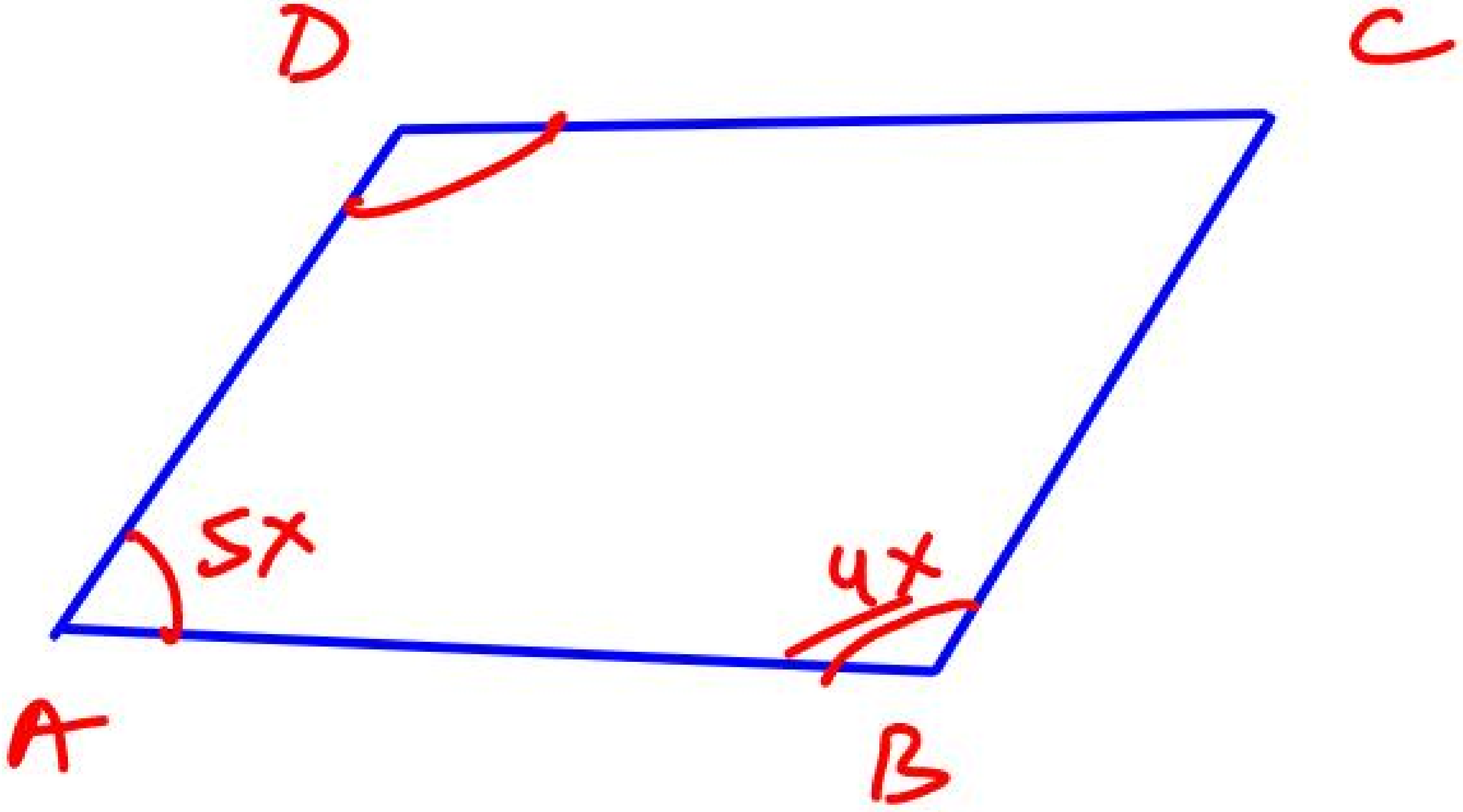
(By ASA)

2. Sum of adjacent angles of a parallelogram is 180°.



$\angle A + \angle B = 180°$

Eg4. In a parallelogram ABCD, $\angle A : \angle B = 5 : 4$
Find the value of $\angle D$.



$5x + 4x = 180$

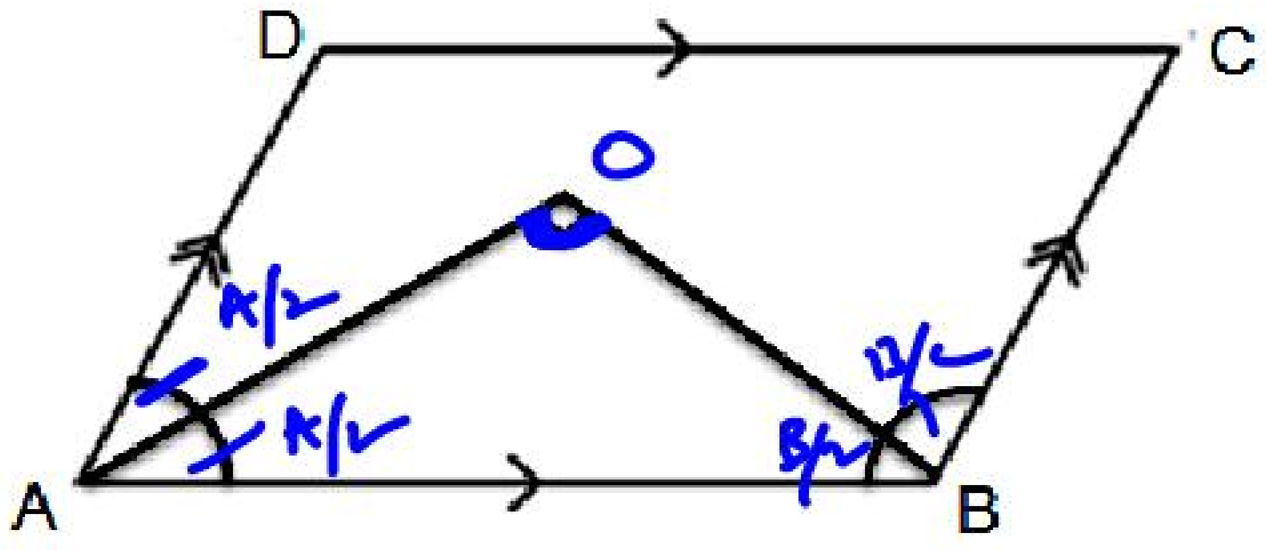
$x = 20$

$4x = 80$

$\angle D = 80^\circ$

BYJU'S EXAM PREP

3. (i) Angle bisectors of adjacent angles of a parallelogram intersect each other at 90°.



$\angle AOB = 90^\circ$

Reason

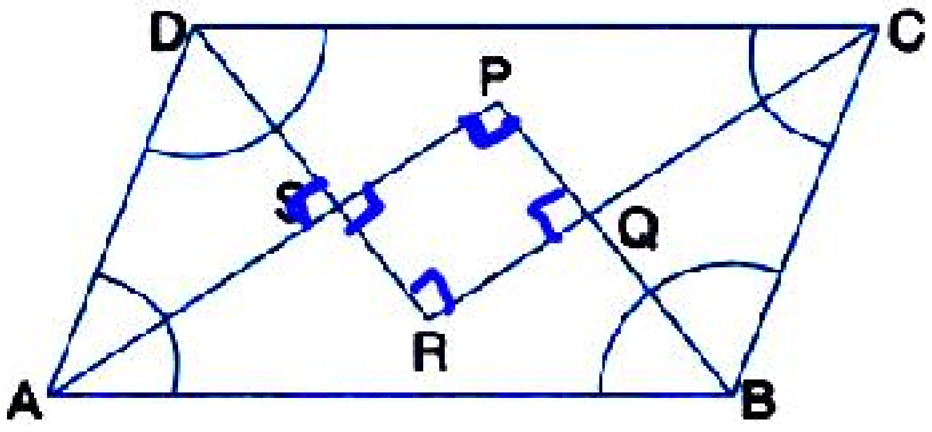
ΔAOB

$\frac{A}{2} + \frac{B}{2} + \angle AOB = 180^\circ$

$90 + \angle AOB = 180^\circ$

$\boxed{\angle AOB = 90^\circ}$

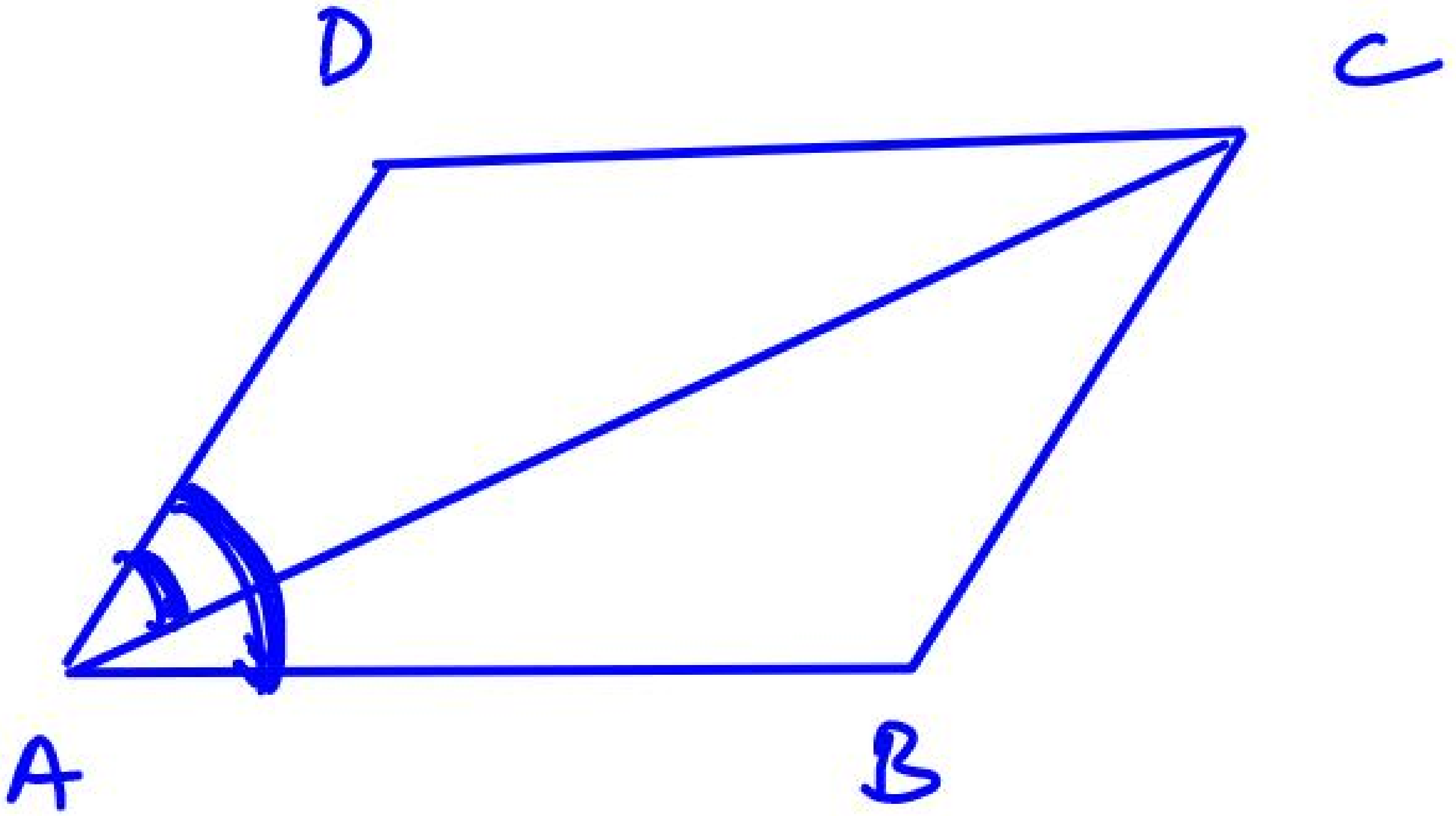
3. (ii) Angle bisector of a parallelogram forms a rectangle.



ABCD is a parallelogram.

AP, BP, CR and DR are bisectors of $\angle A$, $\angle B$, $\angle C$ & $\angle D$.

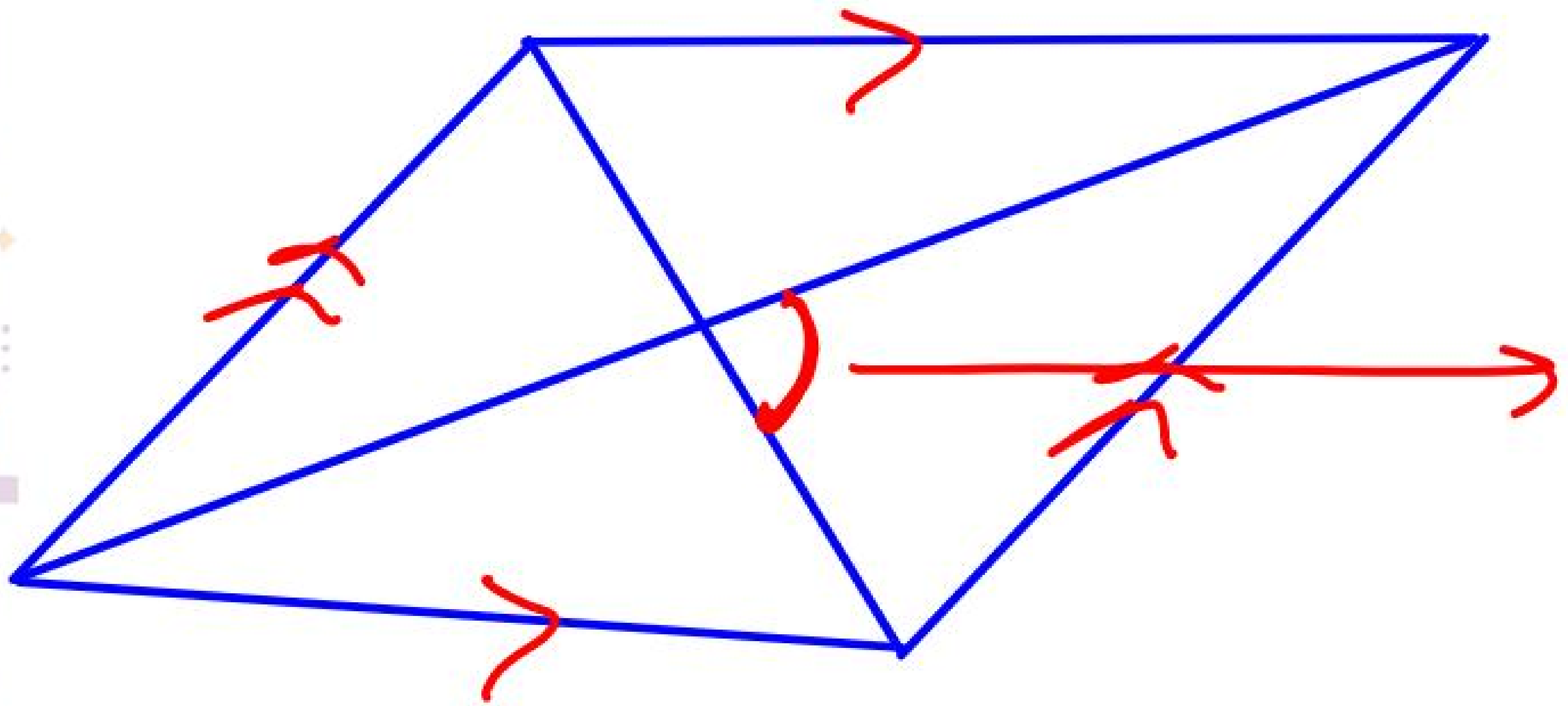
Then, PQRS is a rectangle.



$\angle A = 80^\circ$

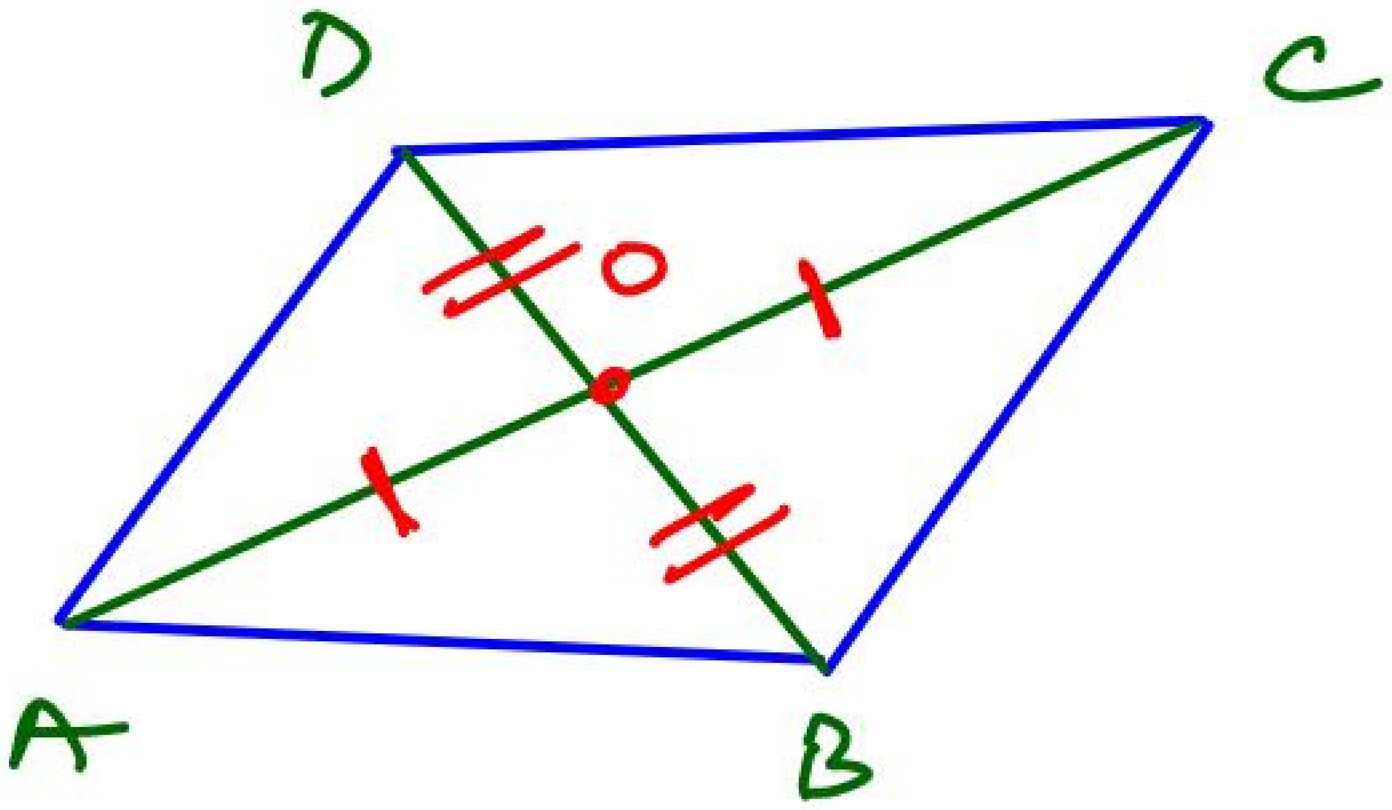
$\angle DAC$

$\longrightarrow$ Can't be determined



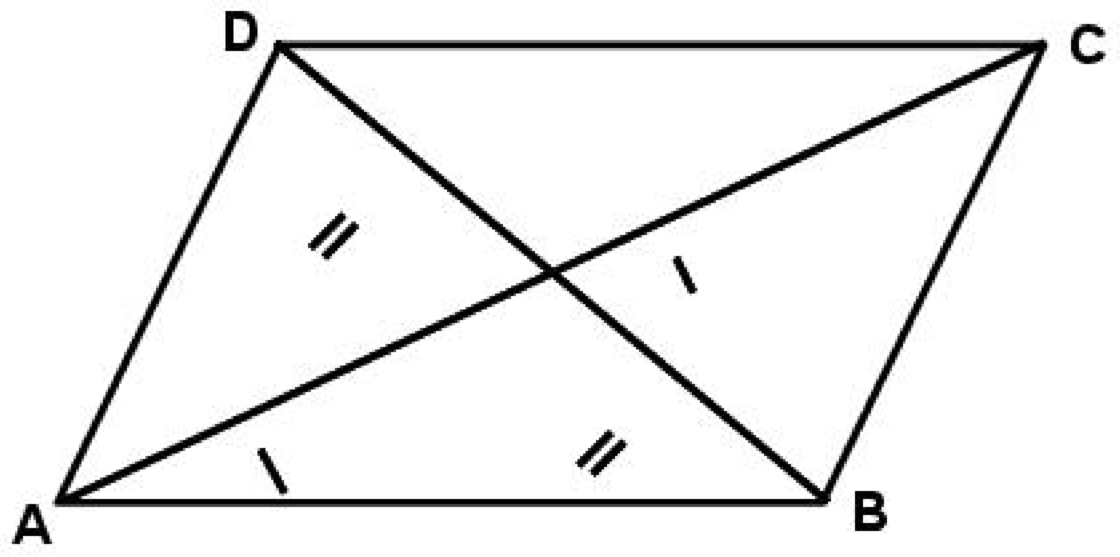
Can't say

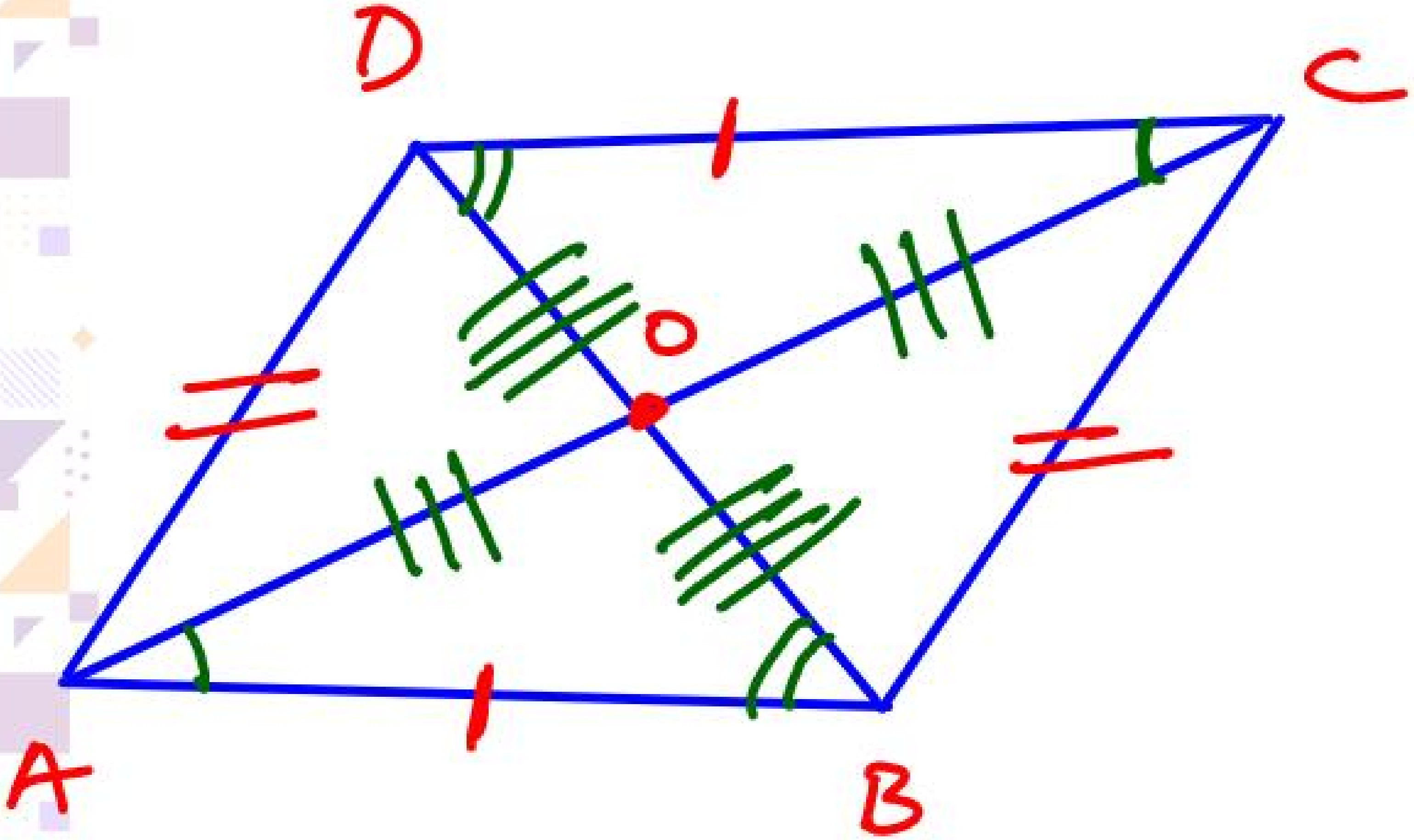
Diagonals of a parallelogram Bisect each other



$$\boxed{\begin{aligned} AO &= OC \\ BO &= OD \end{aligned}}$$

4. (i) Diagonals of a parallelogram bisect each other, but not necessarily at 90°.





To prove

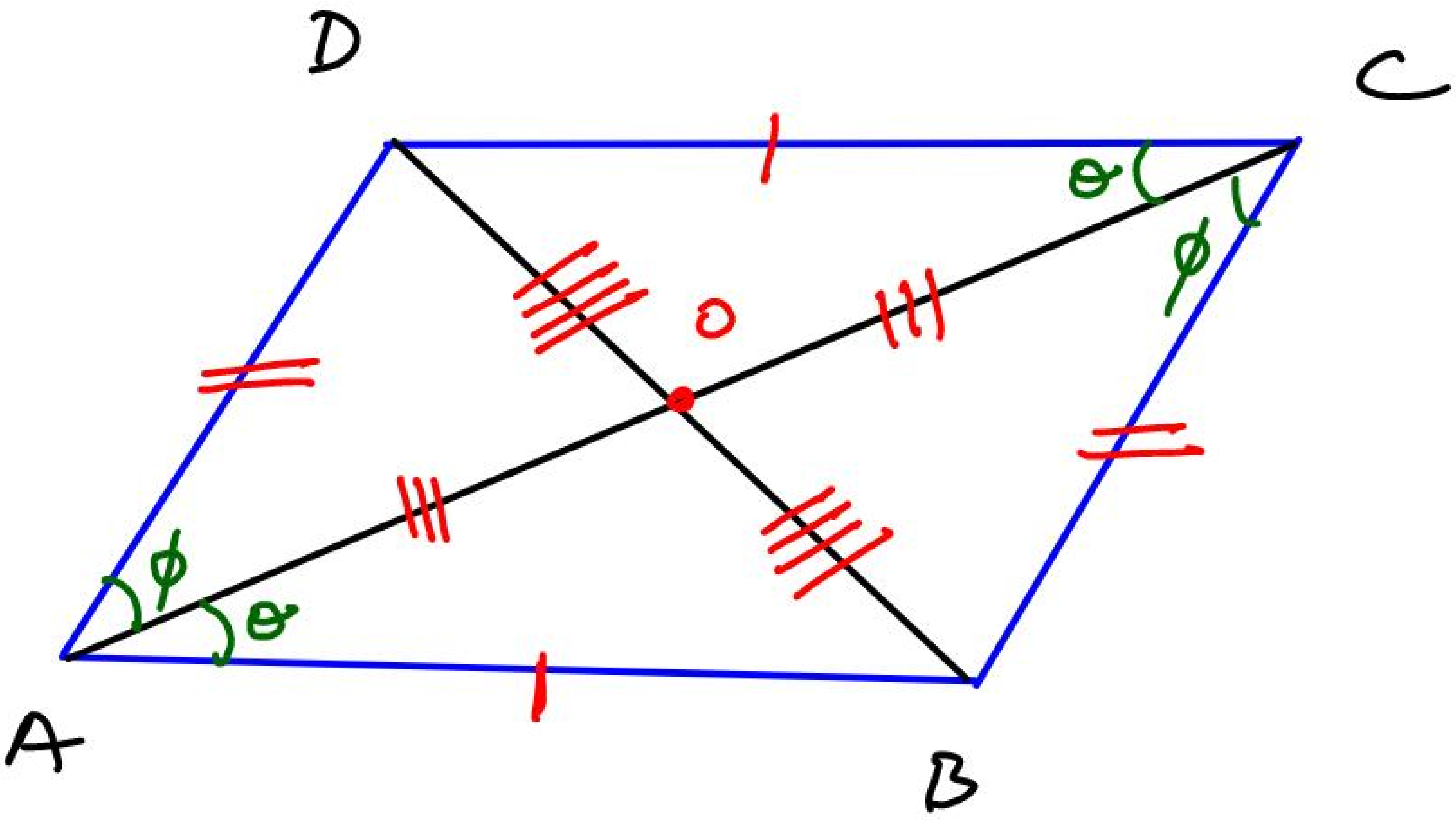
AO = OC

BO = OD

Proof

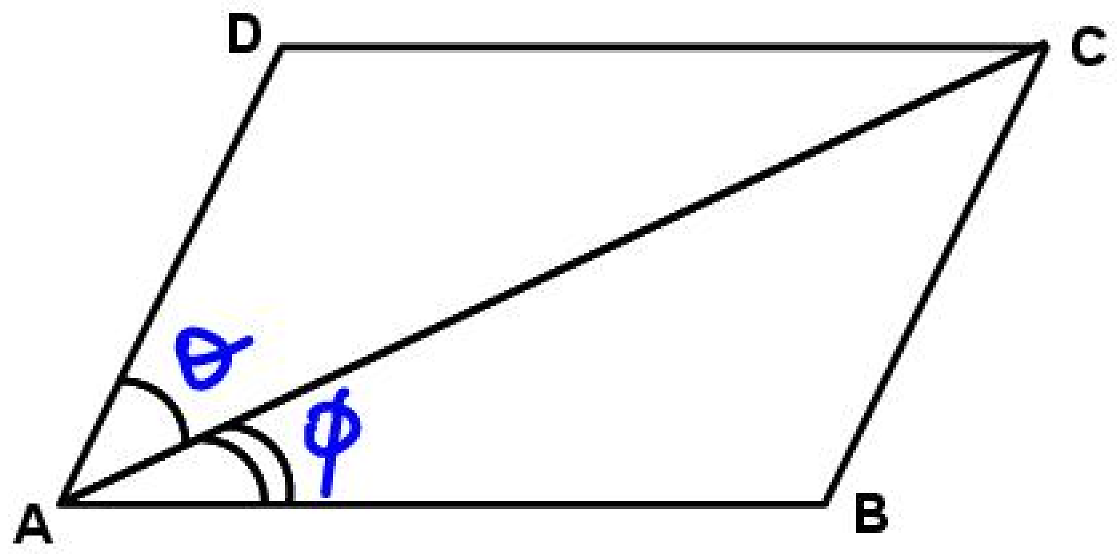
ΔAOB & ΔCOD

$\Delta \overline{A}OB \cong \Delta \overline{CO}D$ (ASA)

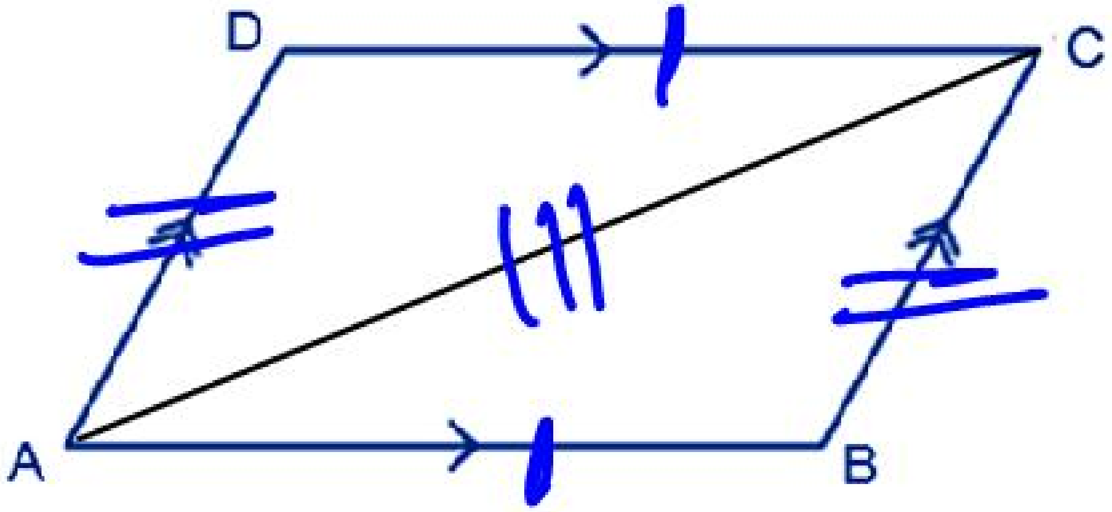

D
C
O
A
B
θ
φ
φ
θ

BYJU'S EXAM PREP

4. (ii) Diagonals of a parallelogram need not be angle bisector.



5. Diagonal of a parallelogram divides it into 2 congruent triangles.

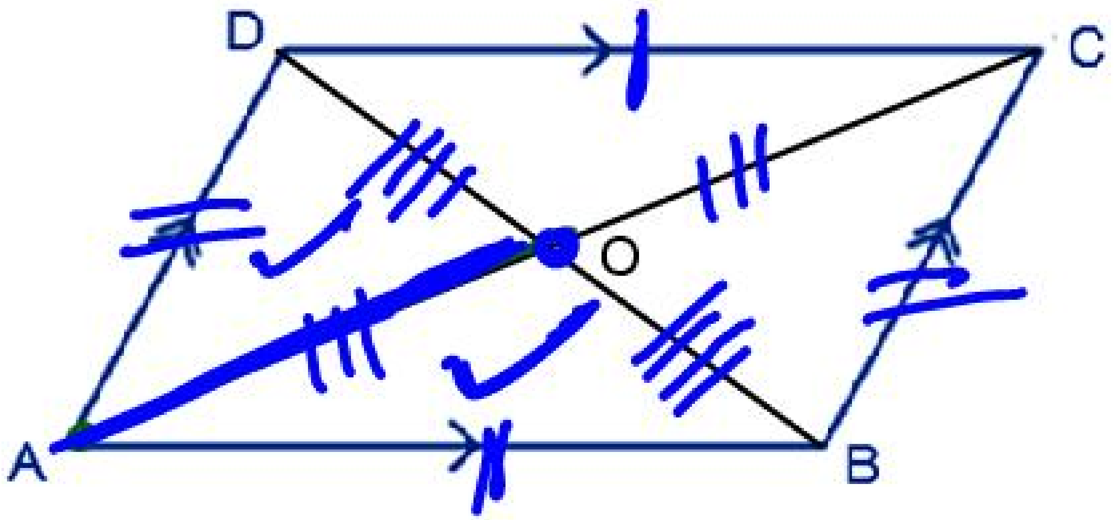


$\Delta ABC \cong \Delta CDA$

Reason $\Delta ABC \cong \Delta CDA$ (SSS)

6. If diagonals AC and BD of a parallelogram intersect each other at O.

Area of ($\Delta AOB = \Delta BOC = \Delta COD = \Delta DOA$)

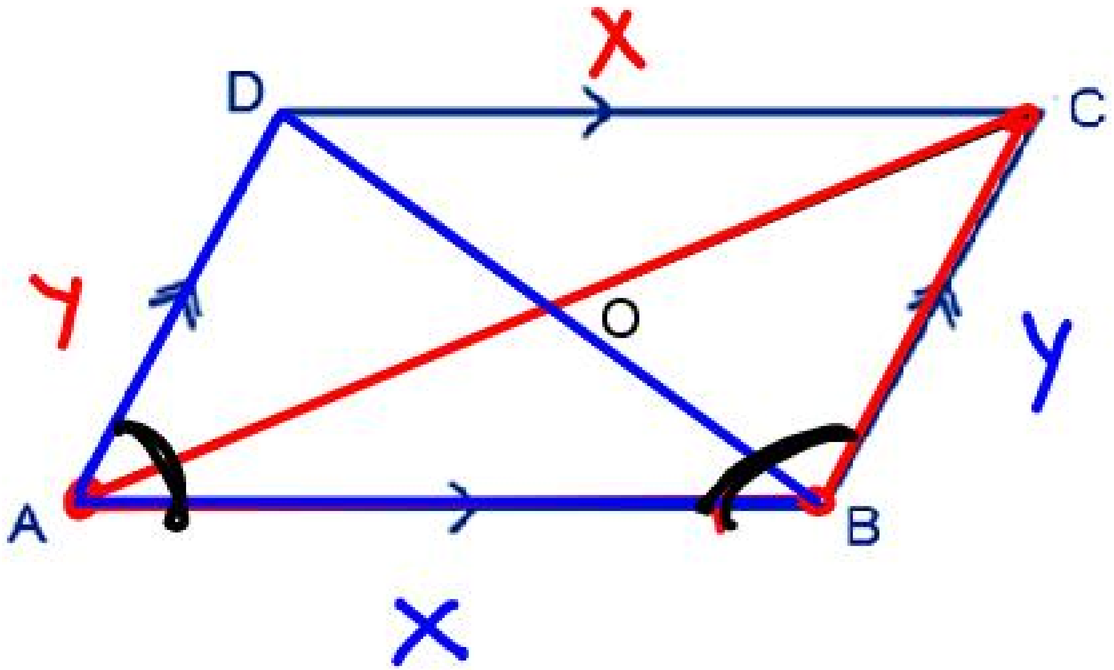


$\Delta AOB \cong \Delta COD$

$\Delta BOC \cong \Delta DOA$

ΔABD

7. $AC^2 + BD^2 = 2(AB^2 + BC^2)$



$$D_1^2 + D_2^2 = 2(x^2 + y^2)$$

$$\cos B = \frac{x^2 + y^2 - AC^2}{2xy}$$

$$\frac{x^2+y^2-AC^2}{2xy} + \frac{x^2+y^2-BD^2}{2xy} = 0$$

$$\cos A = \frac{x^2 + y^2 - BD^2}{2xy}$$

$$\boxed{AC^2 + BD^2 = 2(x^2+y^2)}$$

BYJU'S EXAM PREP

Eg2. If the 2 sides of a parallelogram are 12 cm and 15 cm and one of its diagonal is of length 17 cm. Find length of 2nd diagonal.

$$17^2 + D_2^2 = 2(12^2 + 15^2)$$

$$289 + D_2^2 = 2(144 + 225)$$

$$289 + D_2^2 = 738$$

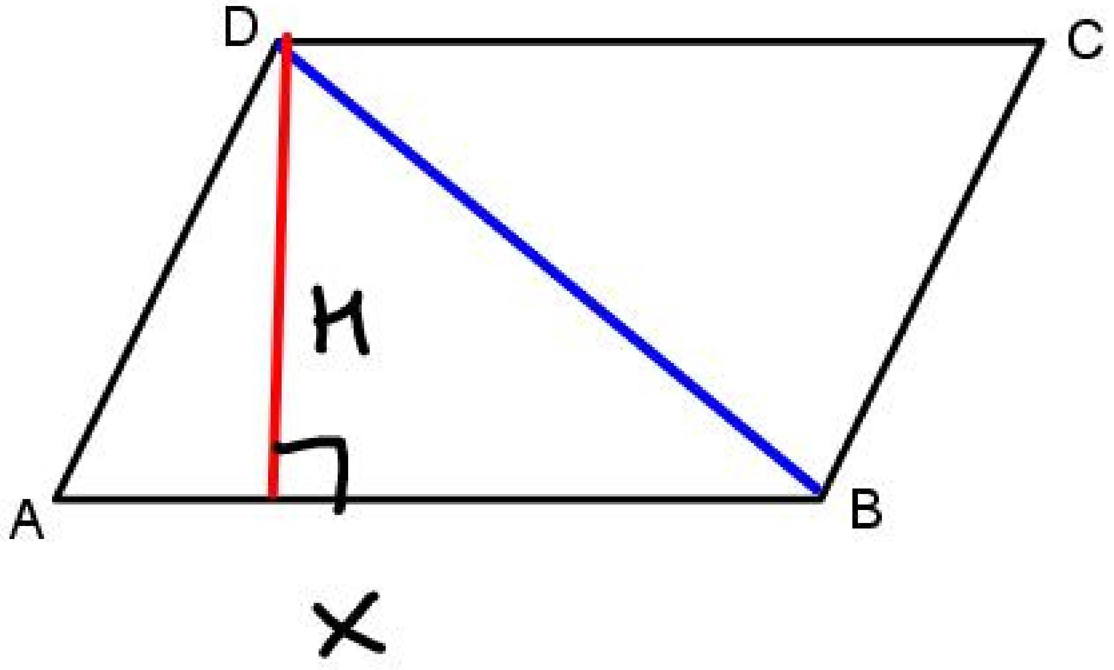
$$D_2^2 = 449$$

$$D_2 = \sqrt{449}$$

Ans. $x = \sqrt{449}$

8. Area of parallelogram :

$\boxed{\text{(i) Base} \times \text{Height}}$



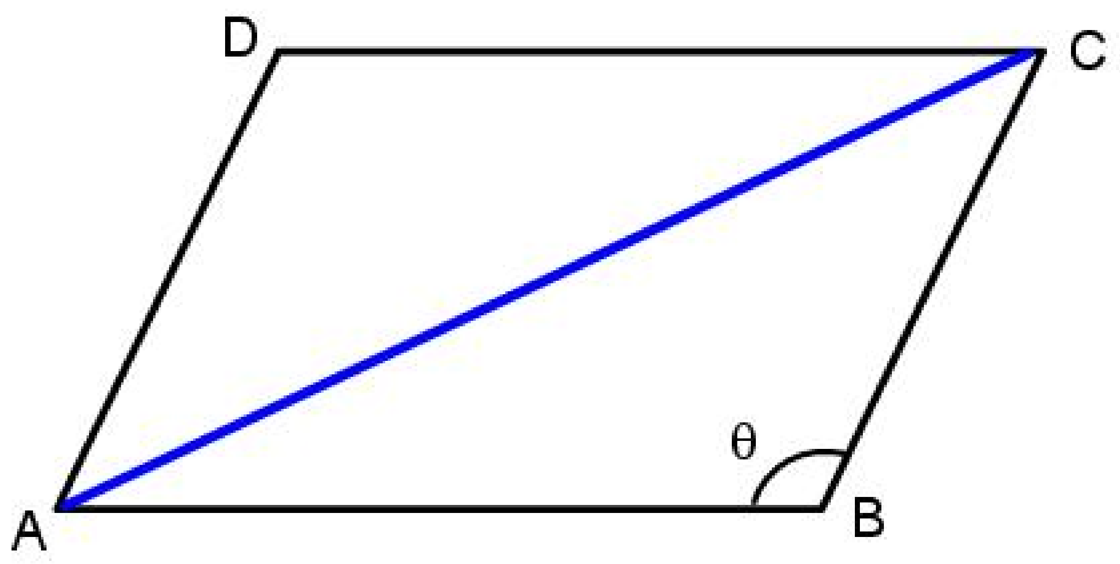
Area of ∥gm

$\rightarrow$ 2 area of ΔABD

$\rightarrow 2\left[\frac{1}{2} X \cdot H\right]$

$X \cdot H$

(ii) Area of parallelogram = $AB \cdot BC \cdot \sin\theta$

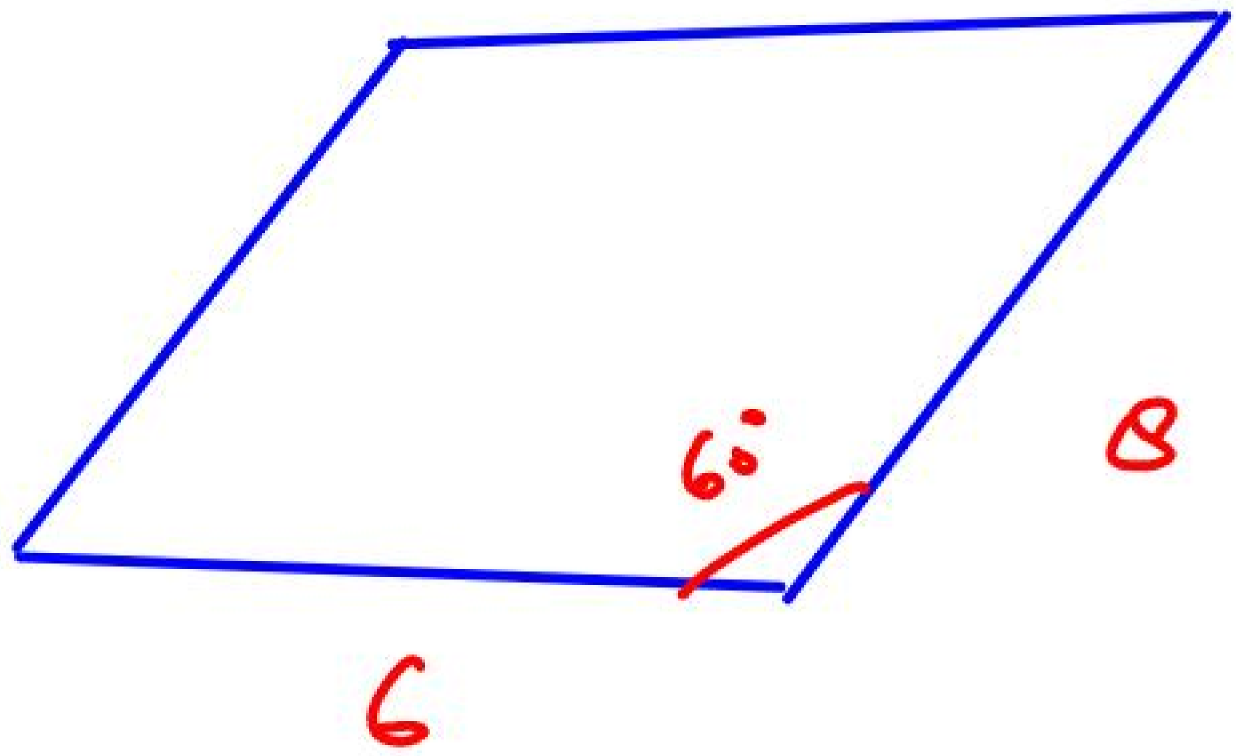


where, AB and BC are adjacent sides of a || gm and θ is the angle between them.

area of $\Delta ABC = \frac{1}{2}(AB)(BC)\sin\theta$

area ||gm ABCD $= (AB)(BC)\sin\theta$

Eg6. If 2 sides of a parallelogram are 6 cm and 8 cm and angle between them is 60°. Find area of parallelogram.

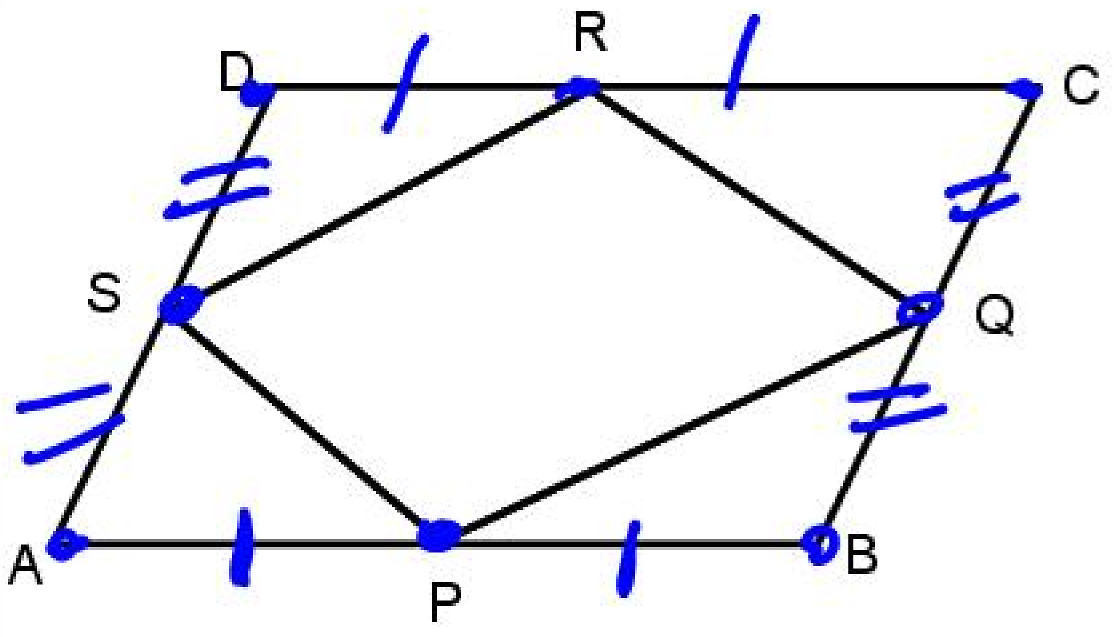


$6 \cdot 8 \cdot \sin 60$

$6 \cdot \cancel{8}^{\,4} \cdot \dfrac{\sqrt{3}}{\cancel{2}}$

$\underline{\underline{24\sqrt{3}\ cm^2}}$

9. Figure formed by joining the mid-point of all sides of a parallelogram, is a PARALLELOGRAM and its area is half of the parallelogram.



If ABCD is a ||gm

P, Q, R & S are mid pts

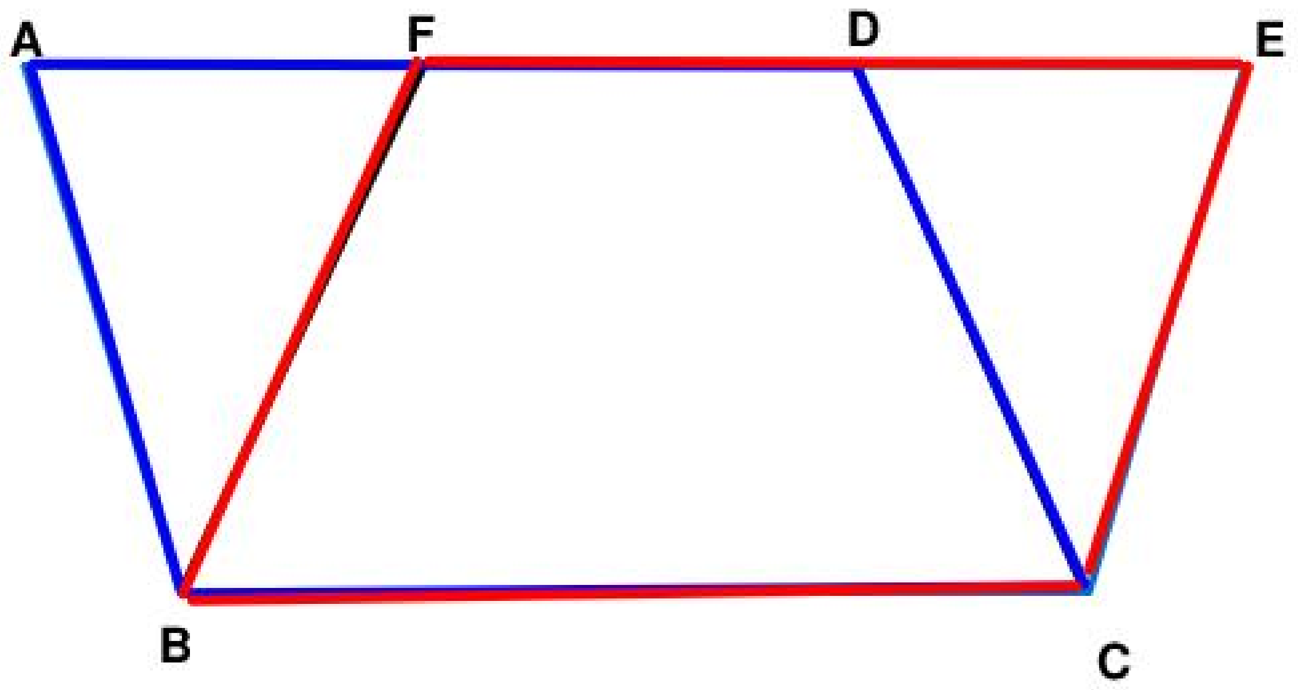
of AB, BC, CD & DA

then PQRS is a ||gm

area of PQRS = $\frac{1}{2}$ area of ABCD

10. **Parallelogram drawn on the same base and between same parallels have equal areas.**

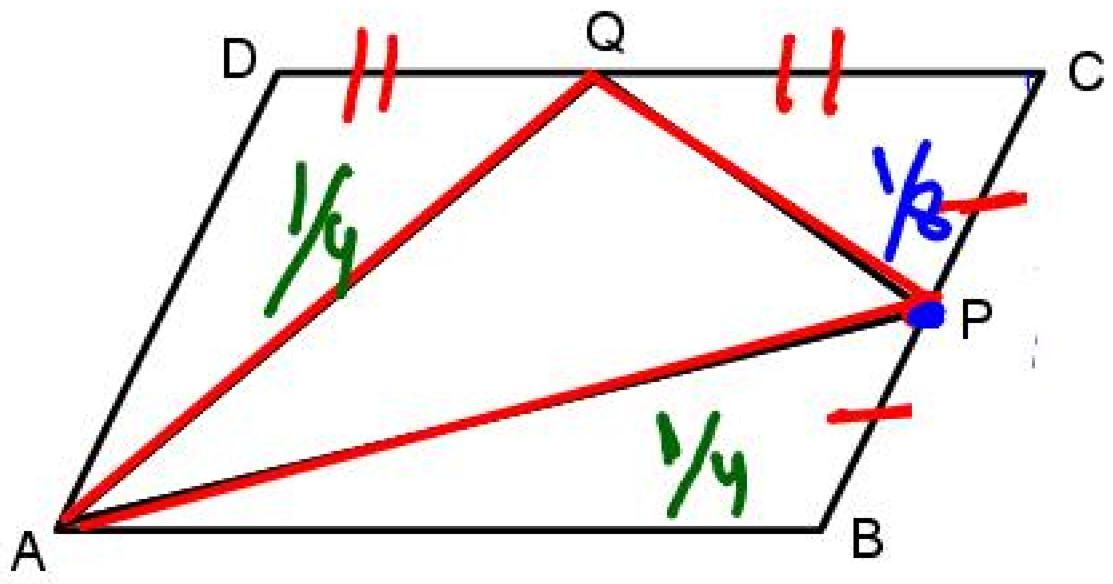
Area of | | gm ABCD = Area of | | gm BCEF



B/C Base & Height are same

BYJU'S EXAM PREP

ans 11. In a parallelogram ABCD, P, Q are mid points of BC and CD respectively.



$$\boxed{\textit{Area of } \Delta APQ = \frac{3}{8} \textit{Area of } ABCD}$$

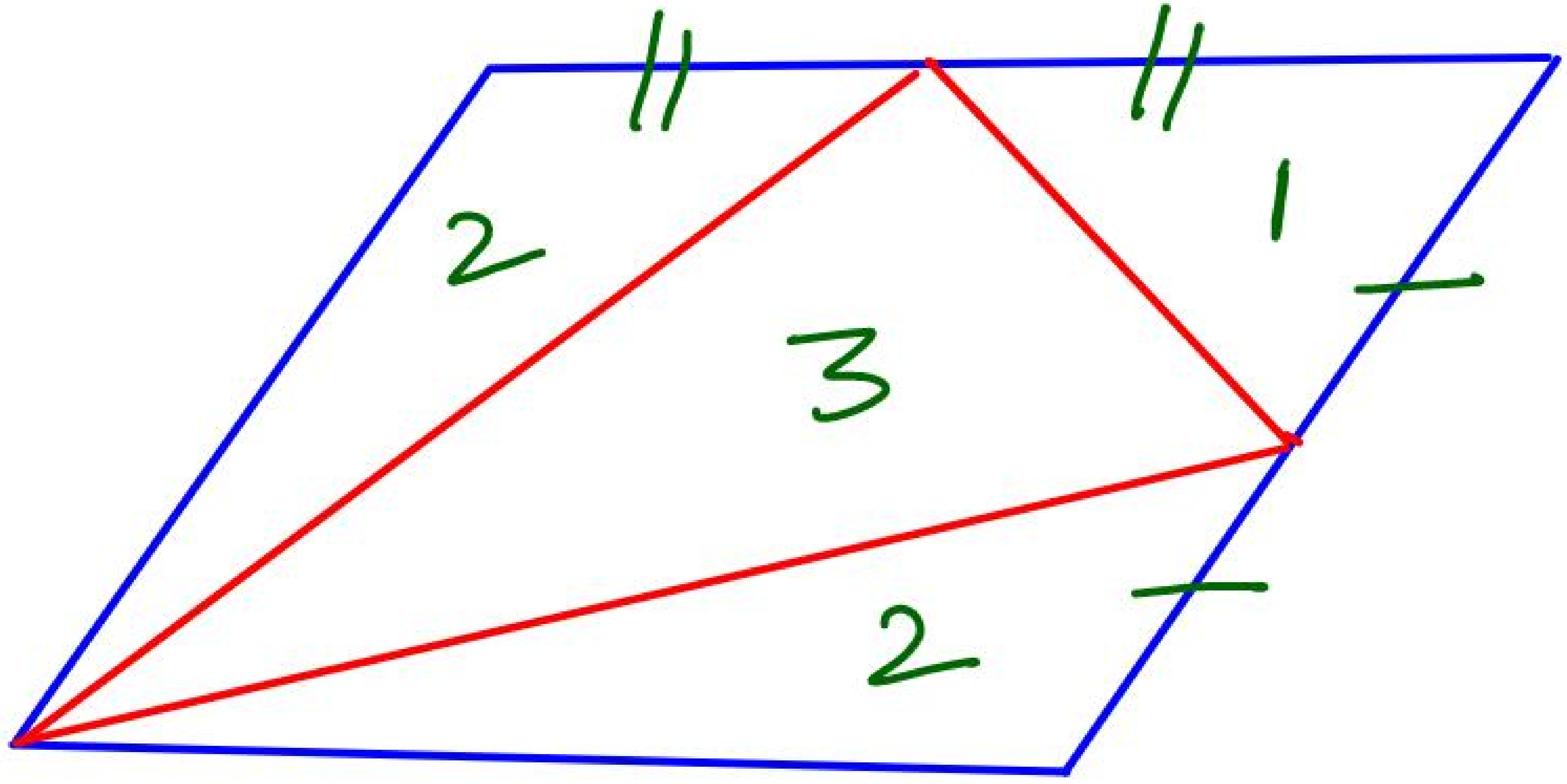
$$\frac{\text{area of } \Delta ABP}{\text{area of } ABCD} = \frac{\frac{1}{2}\cancel{AB}\cdot \cancel{H_1}}{\cancel{AB}\cdot \cancel{H_2}} = \frac{1}{4}$$

$$\frac{\text{area of } \Delta CPQ}{\text{area of } ABCD} = \frac{\frac{1}{2}\cdot \cancel{CP}\cdot \cancel{h_1}}{\cancel{BC}\cdot \cancel{h_2}} = \frac{1}{8}$$

area of ΔAPQ

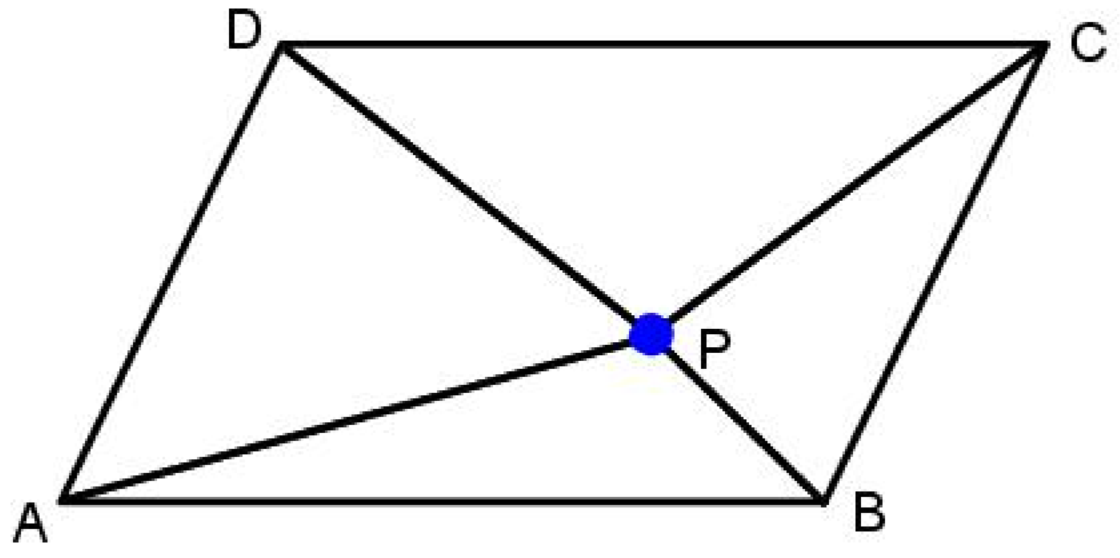
$$= 1 - \frac{1}{4} - \frac{1}{4} - \frac{1}{8}$$

$$= 3/8$$

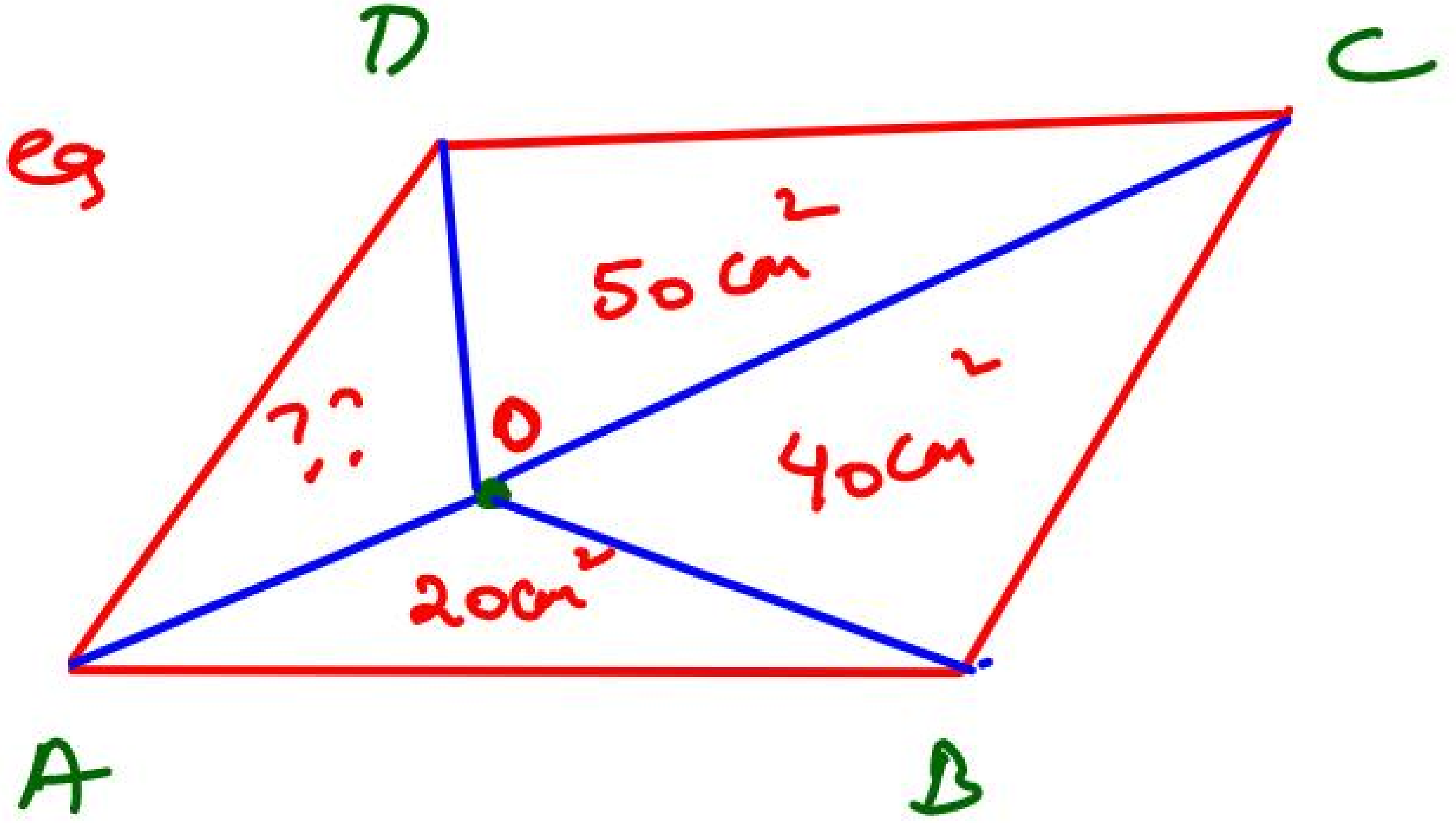

2
3
1
2

12. If P is any point in the interior of || gm ABCD, then

Area of (ΔAPB + ΔCPD) = Area of (ΔBPC + ΔAPD) = ½ || gm ABCD



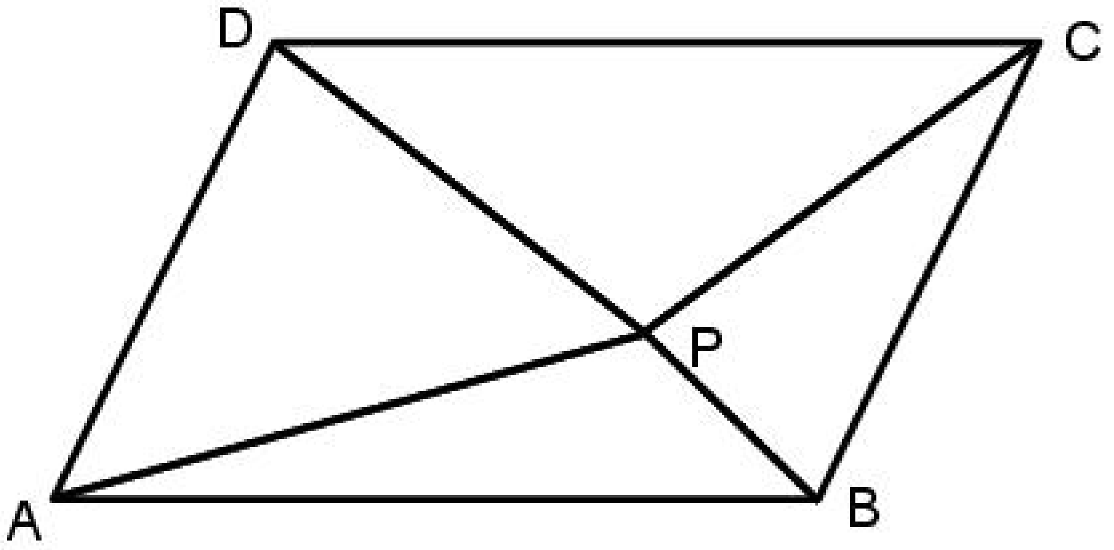
eg



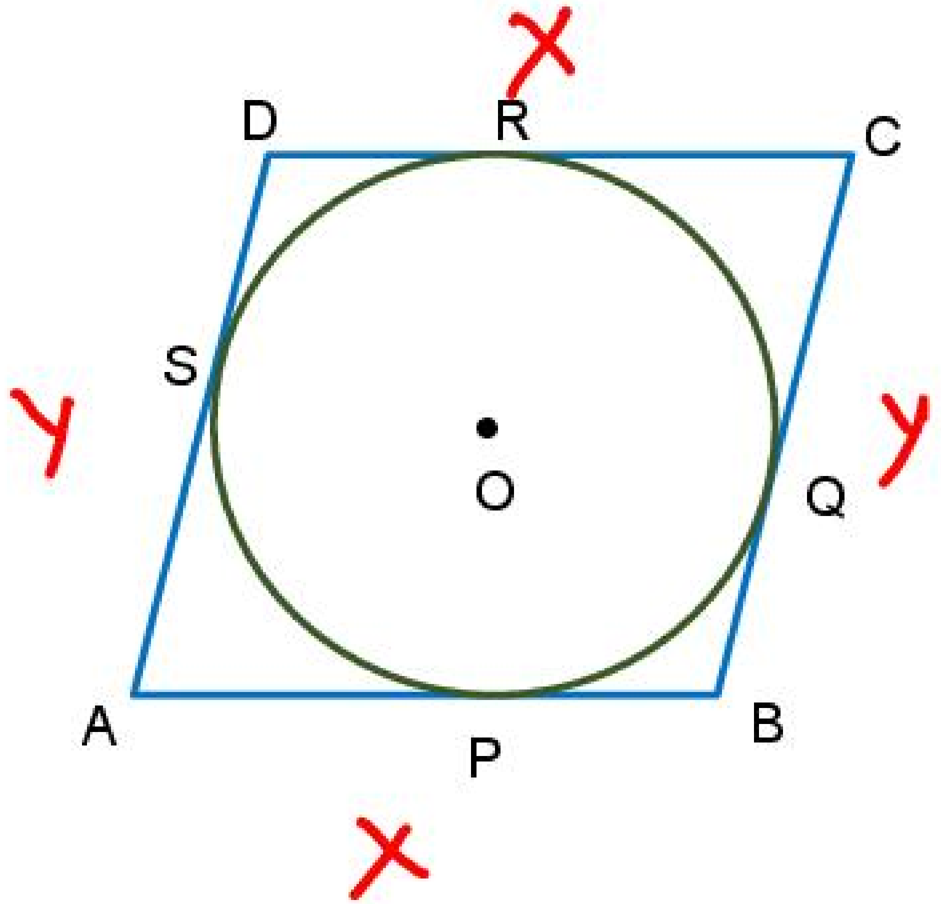
$20 + 50 = 40 + \Delta AOD$

$\Delta AOD = 30\text{cm}^2$

Reason:



PARALLELOGRAM CIRCUM SCRIBING A CIRCLE IS A RHOM BUS



$$AB = BC = CD = DA$$

Reason

$$x + x = y + y$$

$$2x = 2y$$

$$\boxed{x = y}$$

Given: ABCD be a parallelogram circumscribing a circle with centre O.

To prove: ABCD is a rhombus.

We know that the tangents drawn to a circle from an exterior point are equal in length.

Therefore, AP = AS, BP = BQ, CR = CQ and DR = DS.

Adding the above equations,

AP + BP + CR + DR = AS + BQ + CQ + DS

(AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)

AB + CD = AD + BC

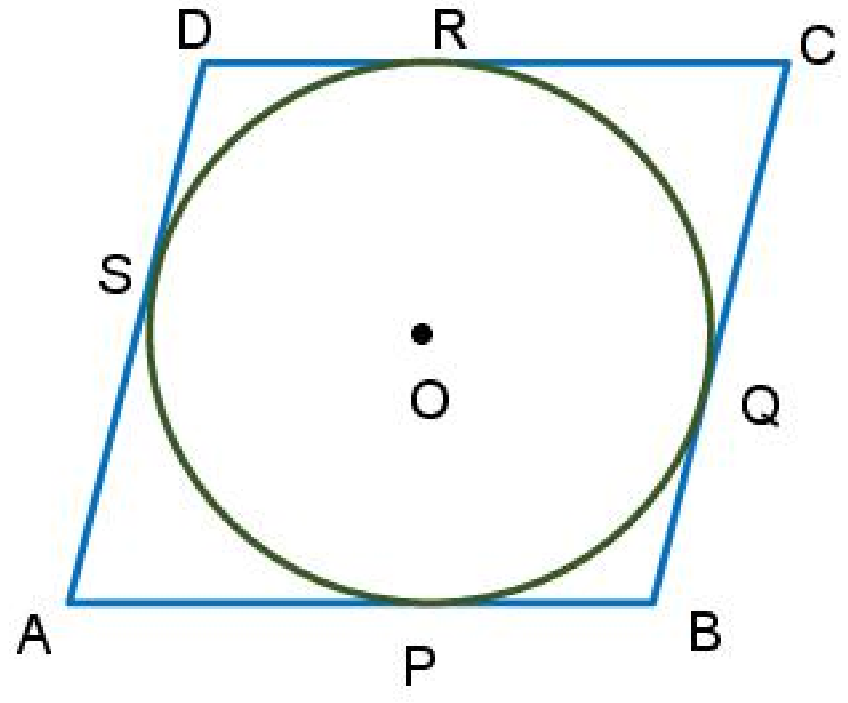
2AB = 2BC

(Since, ABCD is a parallelogram so AB = DC and AD = BC)

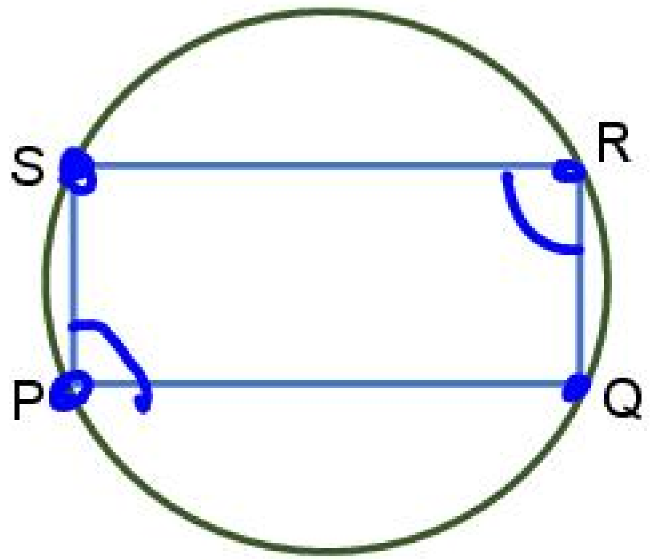
AB = BC

Therefore, AB = BC = DC = AD.

Hence, ABCD is a rhombus.



Parallelogram inscribe in a circle is rectangle.



If PQRS is || gm, then PQRS is rectangle.

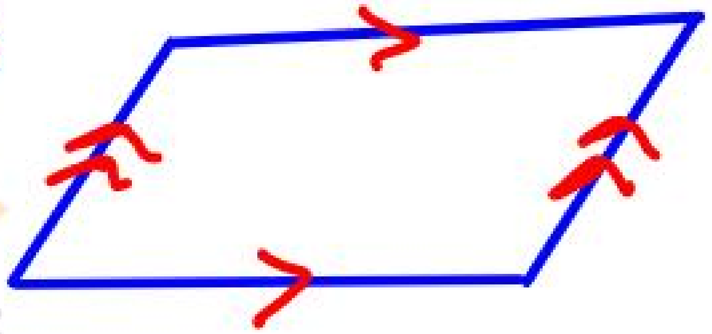
$\angle P + \angle R = 180$

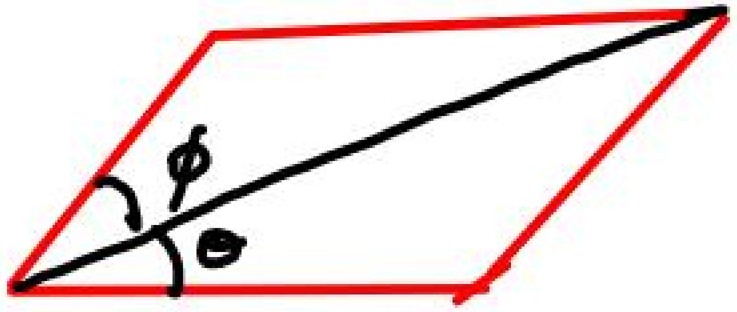
$2\angle P = 180$

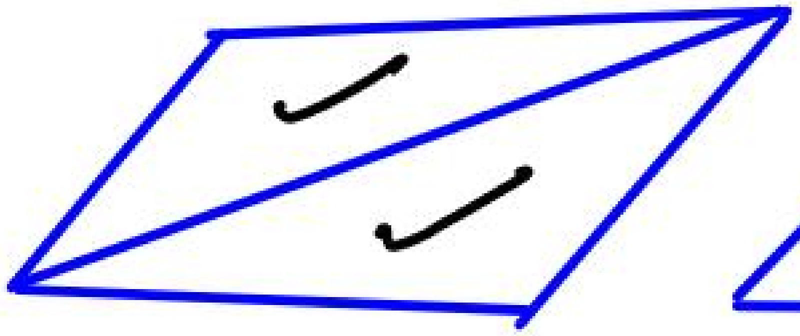
$\angle P = 90^\circ$

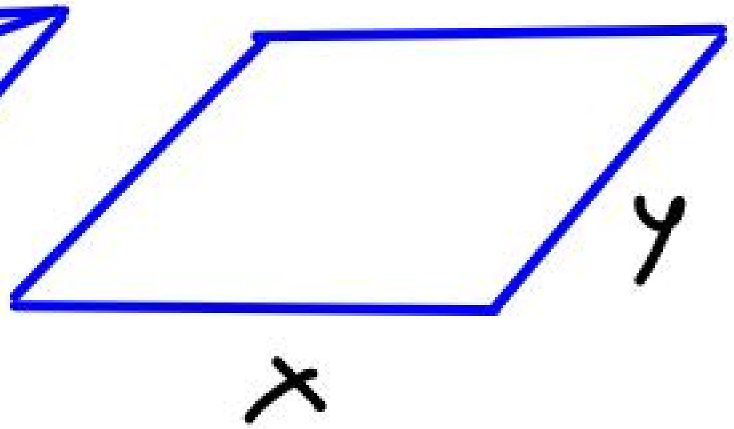
Parallelogram

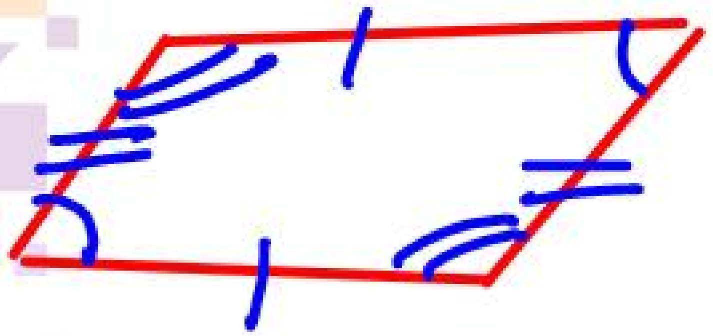
Def

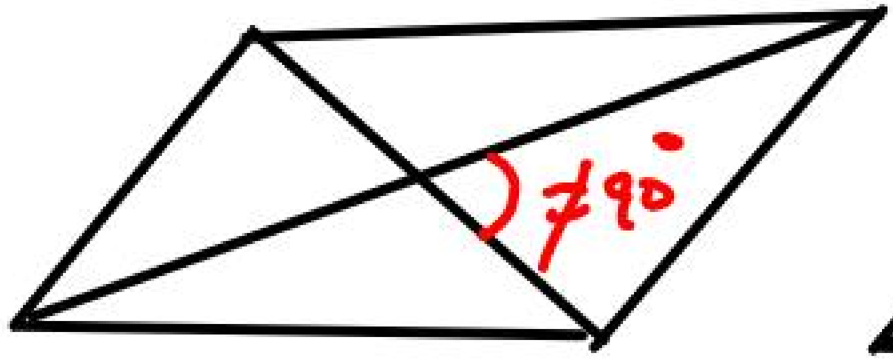


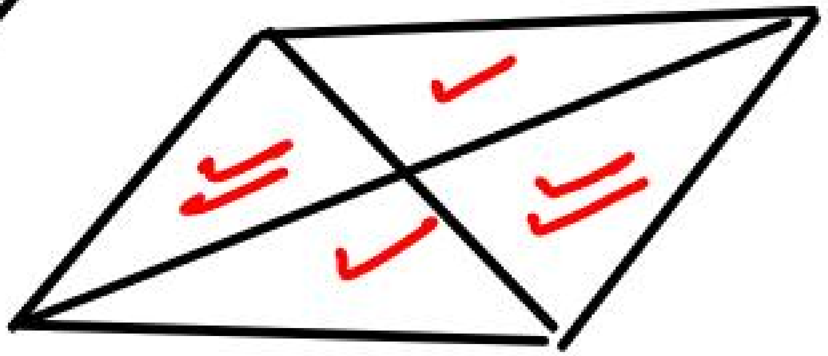






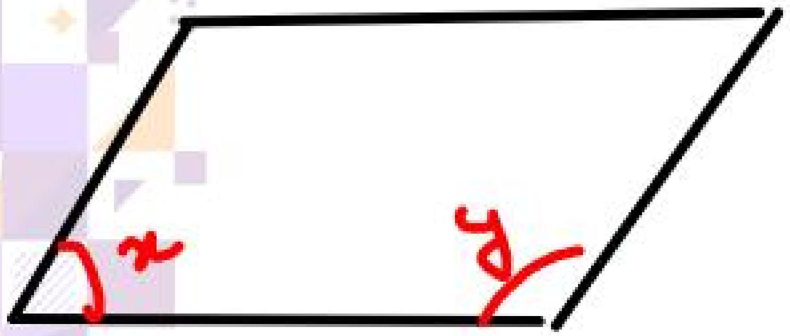




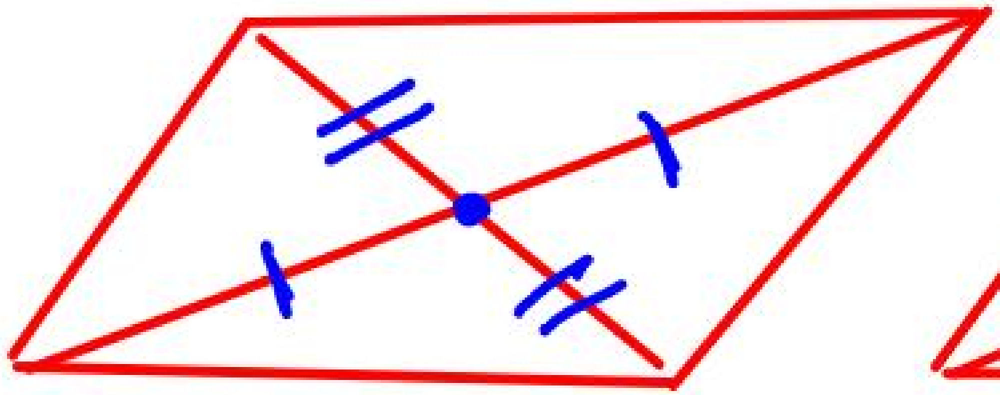


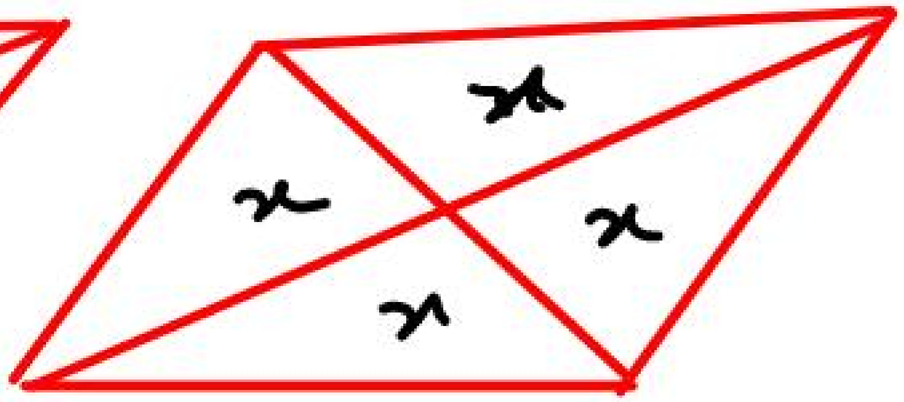
$D_1^2 + D_2^2$

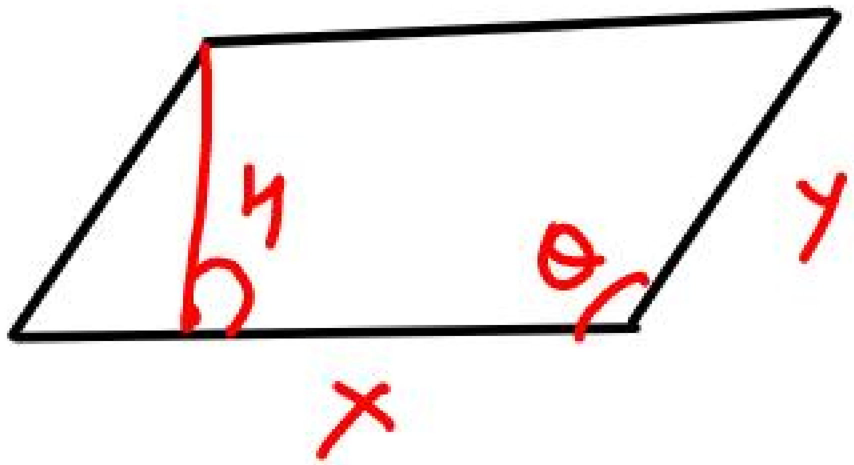
$= 2(x^2 + y^2)$

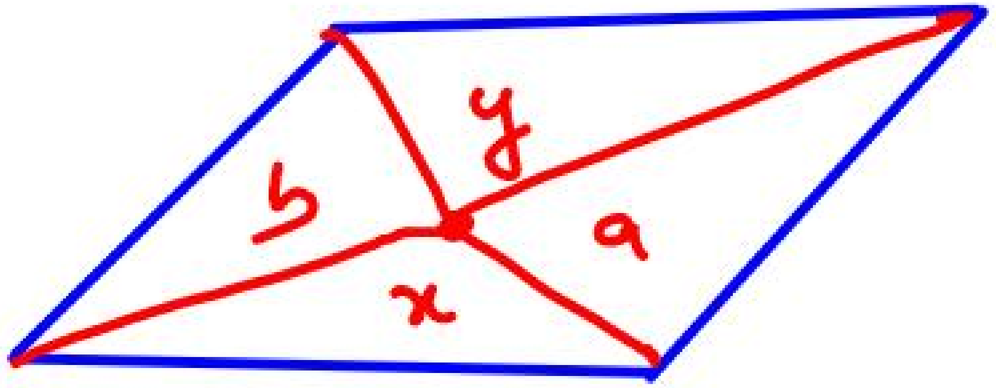


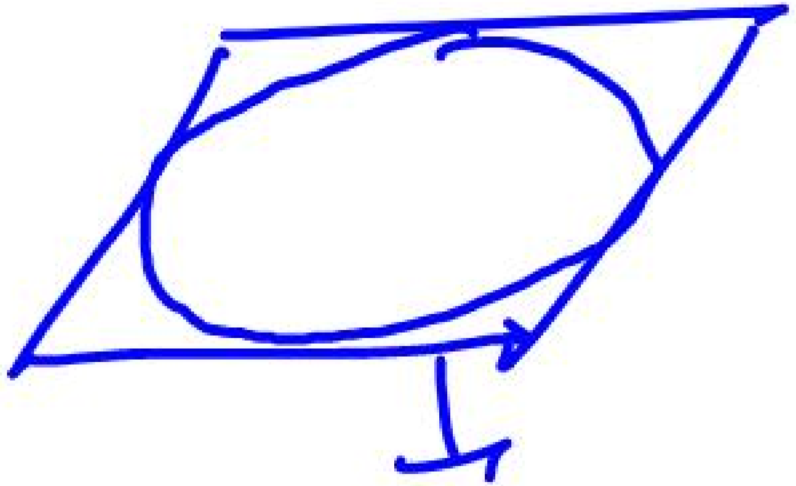
$x + y = 180^\circ$









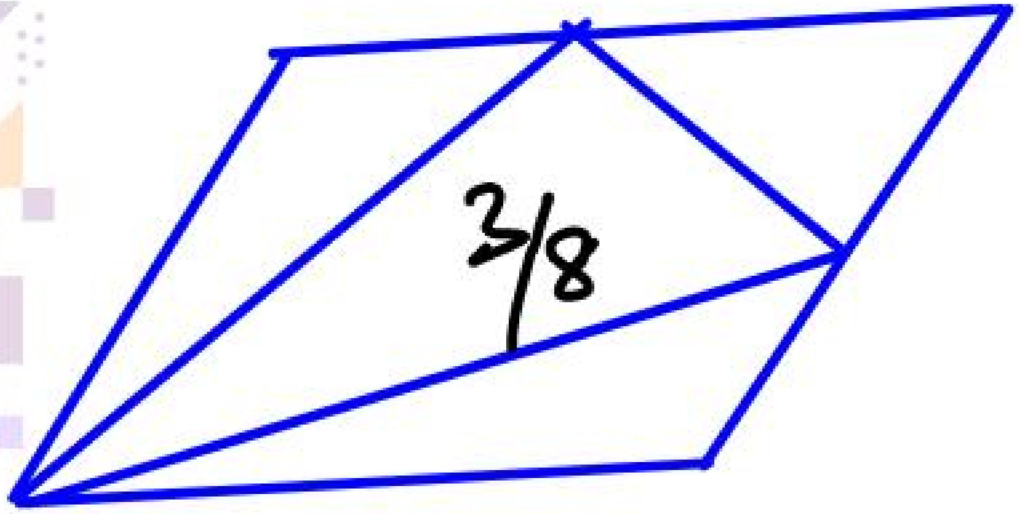


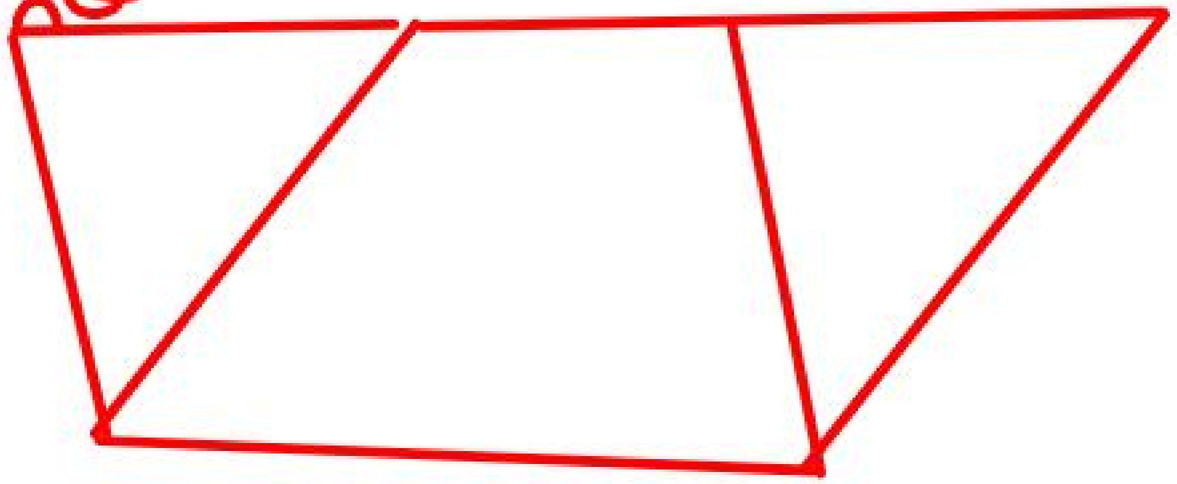
$x + y = a + b$

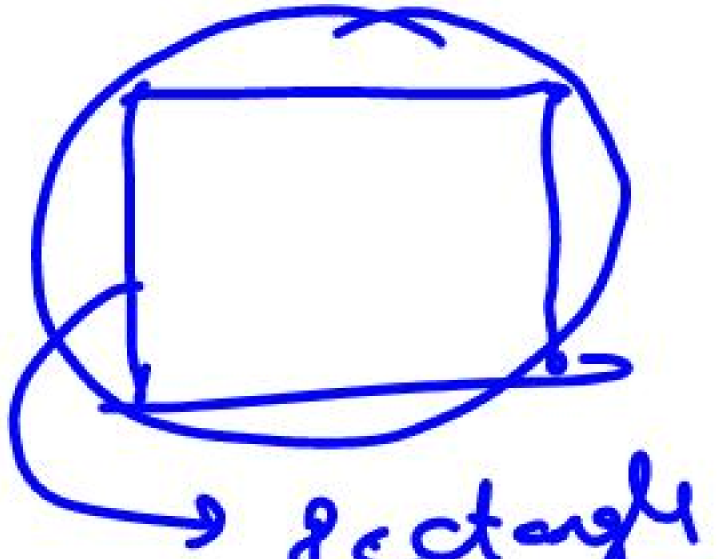
Rhombus

Area $= X \cdot H$

$= X \cdot Y \cdot \sin\theta$





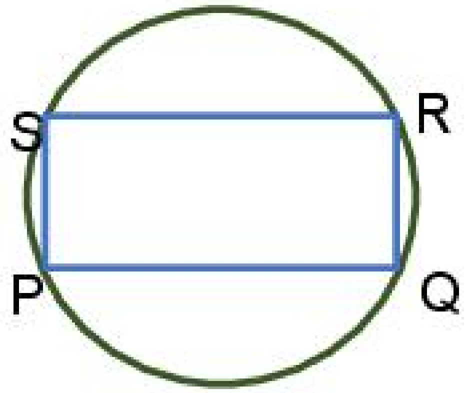


Same Base & Same ||

Rectangle

Given: PQRS is a parallelogram inscribed in a circle.

To prove: PQRS is a rectangle.



Proof: Since PQRS is a cyclic quadrilateral.

$\therefore \angle P + \angle R = 180^\circ$

$\therefore$ (Sum of opposite angles in a cyclic quadrilateral is 180°) ...(i)

But $\angle 9 = \angle R$ (In a II gm opposite angles are equal) ...(ii)

From Eqs. (i) and (ii), we get

$\angle P = \angle R = 90^\circ$

Similarly, $\angle Q = \angle S = 90$

$\therefore$ Each angle of PQRS is 90°.

Hence, PQRS is a rectangle.