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The Most Comprehensive
Preparation App For All Exams

TRI ANGLE

Part-III

* 2 sessions were left

Triangles Part 3

& Triangles Part 4

11th Jan

9 – 10:30 am	Triangles Part 3
11:00 – 12:30	Triangles Part 4

12th Jan

11:00 – 12:30 pm	Quad Part 3
1:00 – 2:30 pm	Quad Part 4

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Agenda : Triangles Part 3

32 min

* Mid point Theorem & its converse → (34 - 36) min
* Congruency →

28 min

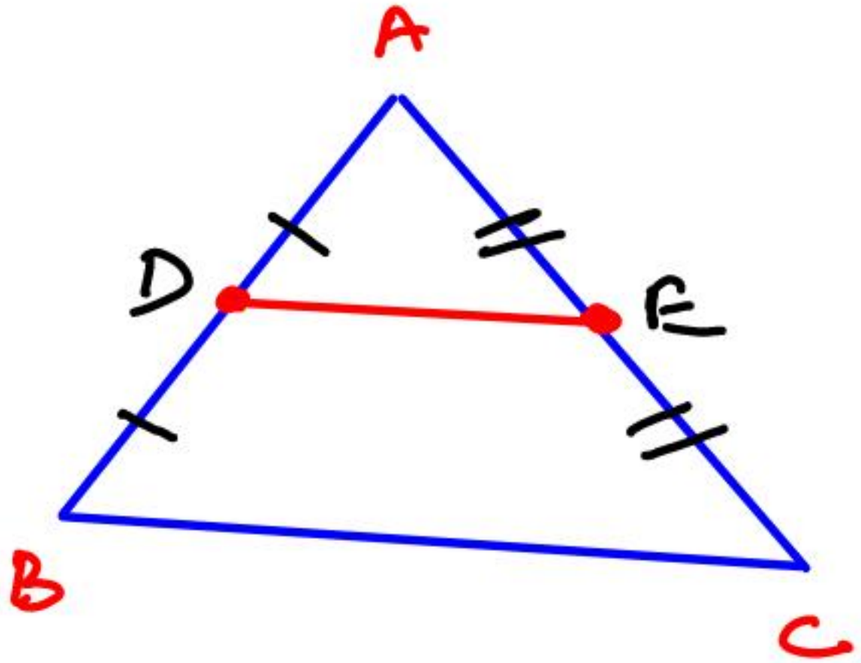
* Orthocentre → (28 - 30) min
* Circumcentre →

5 Practice Question → (22-24) min

Homework →

MID-POINT THEOREM

If we join mid-points of any 2 sides of a Δ by a line segment then that line segment will be parallel to the third side and half of it.



Given : D, E are mid pts of AB & AC

$\Rightarrow$ DE ∥ BC

Δ DE $= \frac{1}{2}$ BC

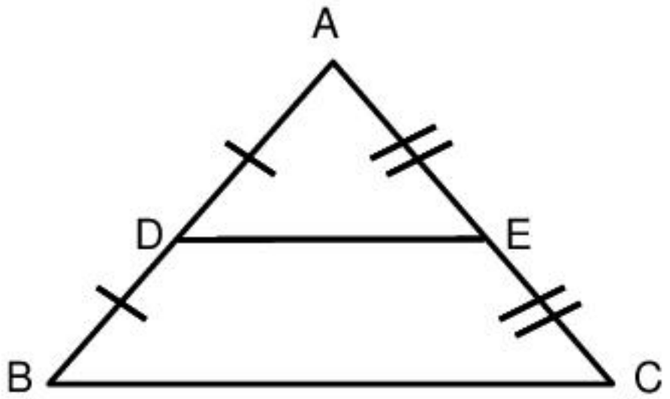
2. Given,

D is mid-point of AB.

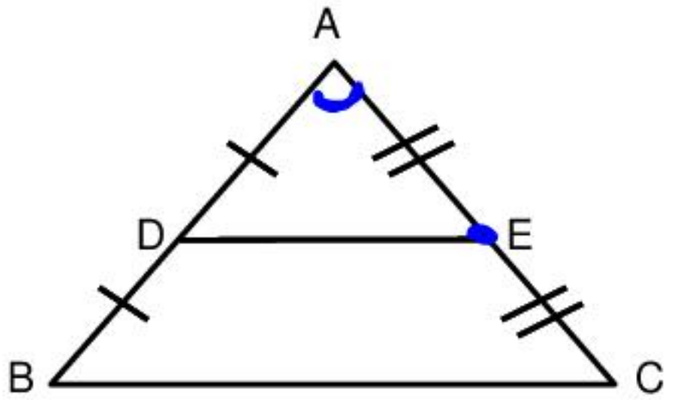
E is mid-point of AC.

$$DE \parallel BC$$

$$DE = \frac{1}{2}BC$$



Proof of Mid-point theorem:



Given, D, E are mid-point of AB & AC.

To prove: (i) $DE \,||\, BC$

(ii) $DE = \frac{1}{2}BC$

Proof: AD : AB = 1 : 2

AE : AC = 1 : 2 $\angle A = \angle A$

$\Delta ADE \sim \Delta ABC$ (SAS similarity)

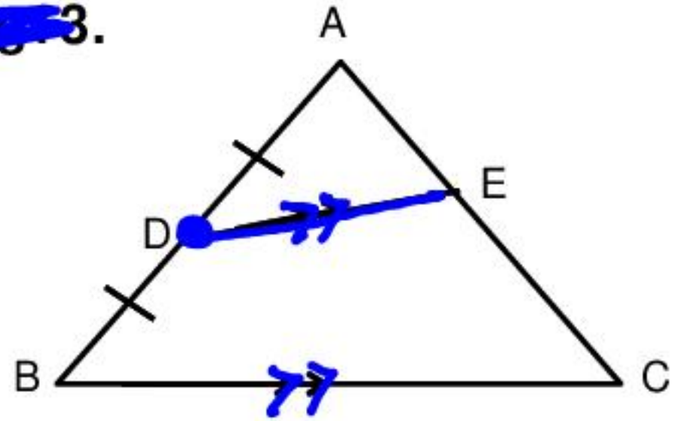
$\angle ADE = \angle ABC$ (Corresponding angles)

$DE \,||\, BC$

$DE = \frac{1}{2}BC$

CONVERSE OF MID-POINT THEOREM

3.



Given,

D is mid-point of AB.

DE | | BC

E is mid-point of AC.

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CONGRUENCY

Two figures are said to be congruent, if they are exactly same in every aspect.

2 line segments are congruent ? when their length are same

2 circles are congruent ? when their radius are same

2 squares are congruent ? when them sides are same

$$\Delta ABC \cong \Delta DEF$$

Then

↓

Symbol of Congruency

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CONDITIONS OF CONGRUENCY

(1) SSS (side - side - side)

(2) SAS (side - Angle - side)

(3) ASA (Angle - side - Angle)

(4) AAS (Angle - Angle - side)

(5) RHS (Right - Hypotenuse - side)

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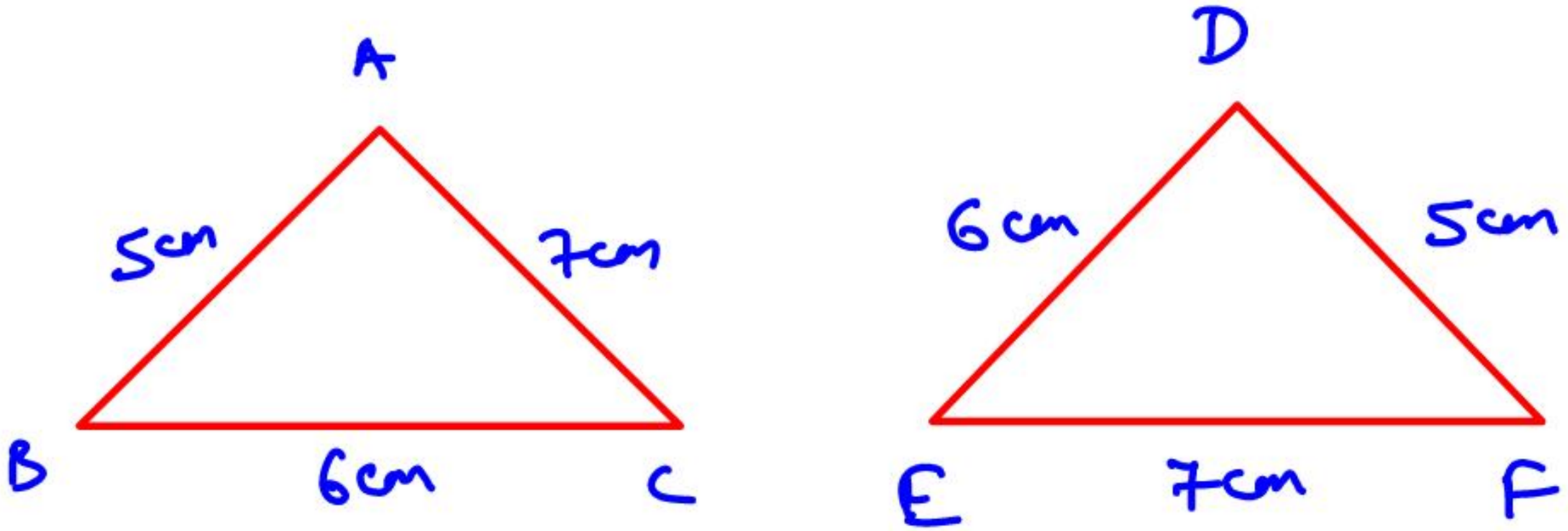
SSS (Side – Side – Side)



$\triangle ABC \cong \triangle FDE$

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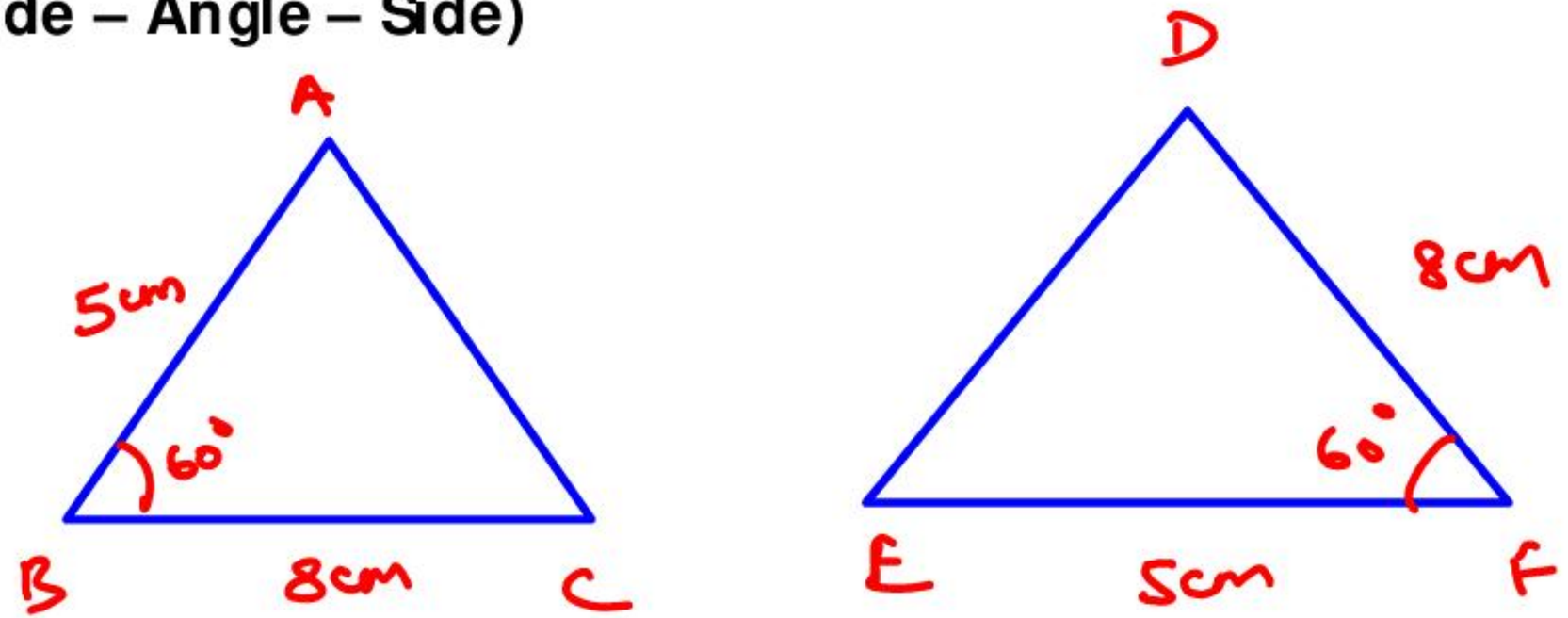
SAS (Side – Angle – Side)



$\Delta ABC \cong \Delta EFD$

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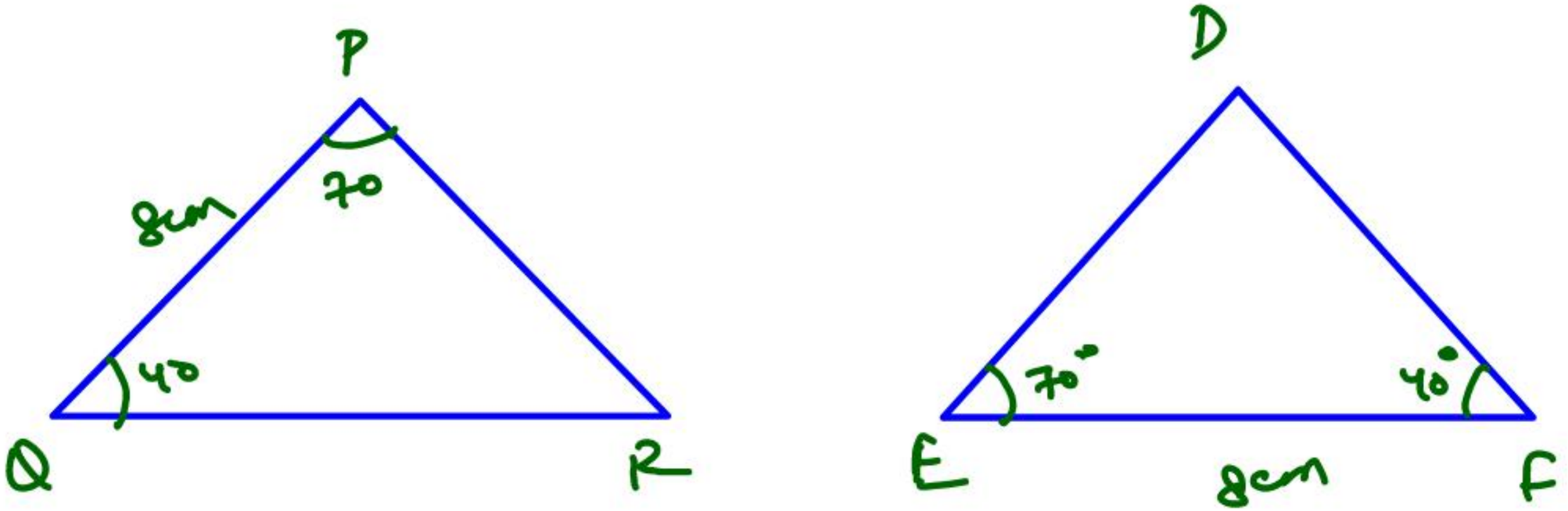
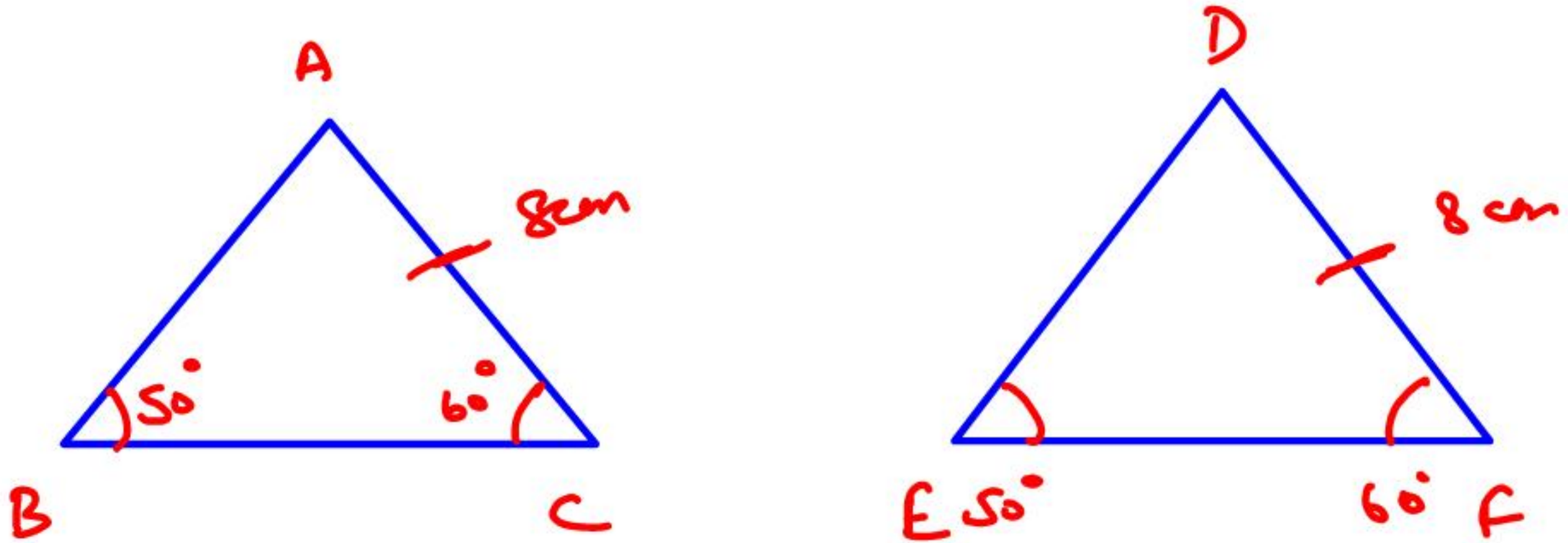
ASA (Angle – Side – Angle)



$\Delta PQR \cong \Delta EFD$

AAS (Angle – Angle – Side)



$\Delta ABC \cong \Delta DEF$

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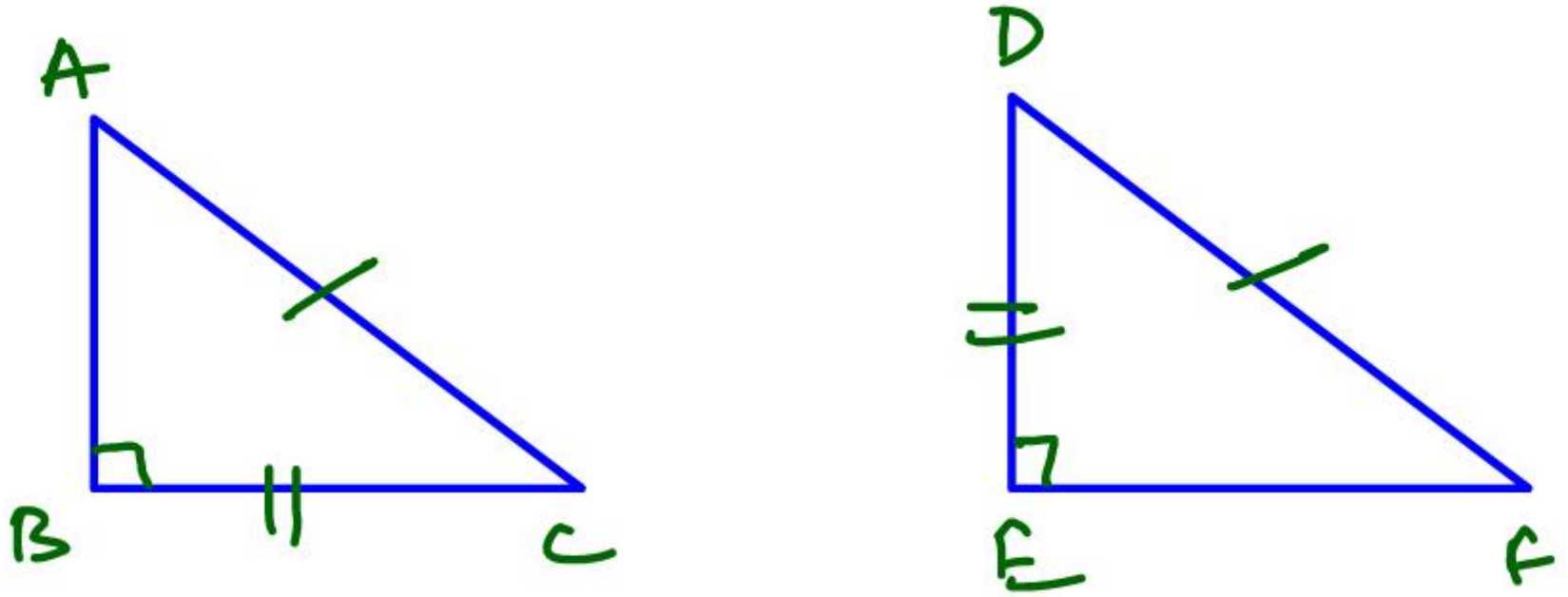
RHS (Right – Hypotenuse – Side)



$\Delta ABC \cong \Delta FED$

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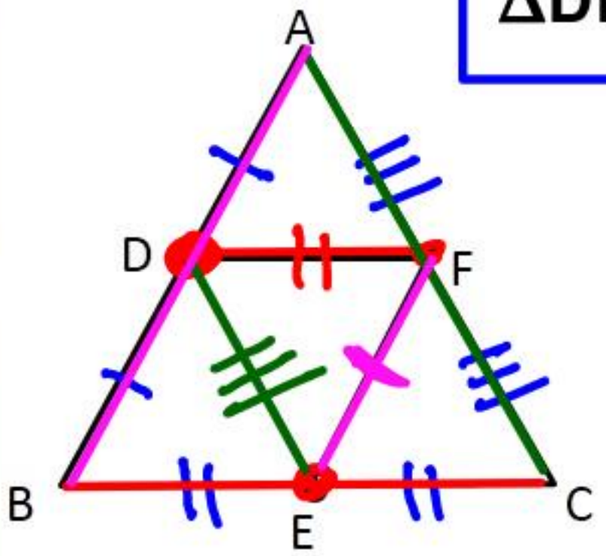
AAA & SSA does not guarantee congruency.

If D, E & F are midpoints of the sides AB, BC, CA

Then,

$\Delta DFE \cong \Delta FDA \cong \Delta EBD \cong \Delta CEF$

In all Δ's



Area of $\Delta DFE = \frac{1}{4}$ (Area of ΔABC)

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If Congruent	→	Similar	Yes
If Similar	→	Congruent	may or may not
If Congruent	→	Area same	Yes
If Area same	→	Congruent	may or may not
Similar + Area same	→	Congruent	Yes

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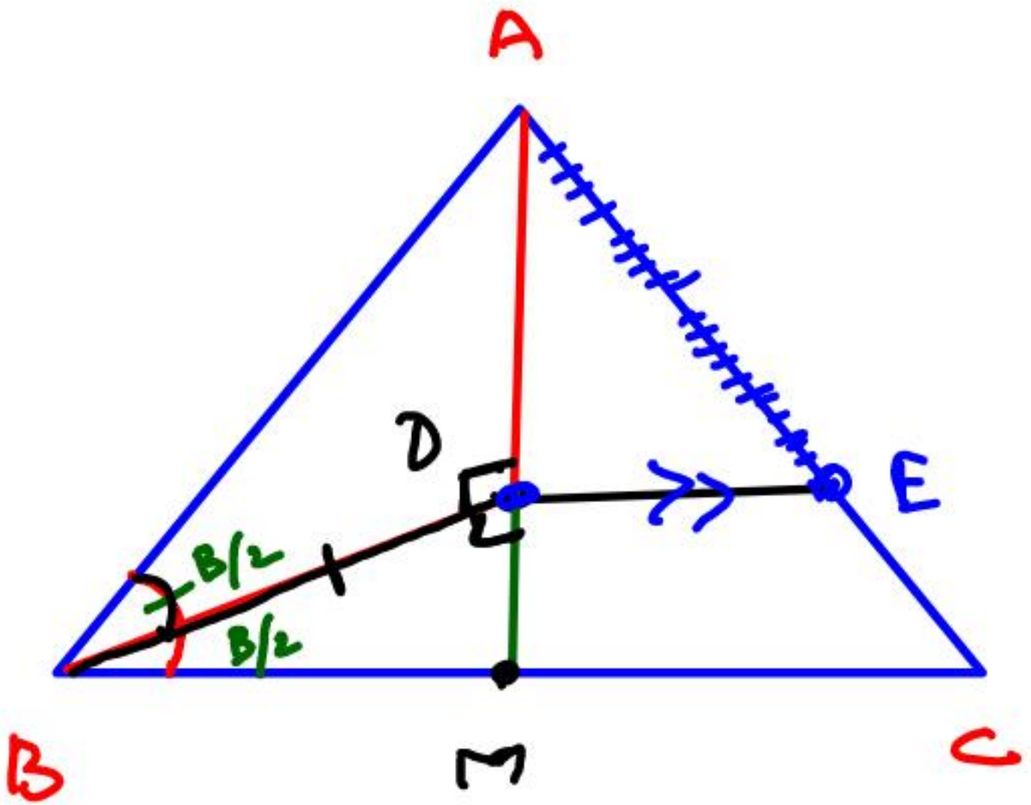
V. Imp

Eg14. AD is perpendicular to the internal bisector of ∠ABC of ΔABC. DE is drawn through D parallel to BC to meet AC at E. If the length of AC is 12 cm, then the length of AE (in cm.) is :

(a) 8 (b) 6 (c) 3 (d) 4



$\Delta \overline{A}DB \cong \Delta \overline{M}DB$ (ASA)

$AD = MD$

D is m.p of AM

In ΔAMC

$\because$ DE ∥ MC

E is m.p of AC [Converse of Mid pt

Ans. (b)

Centres of $\triangle$

1 → Orthocentre

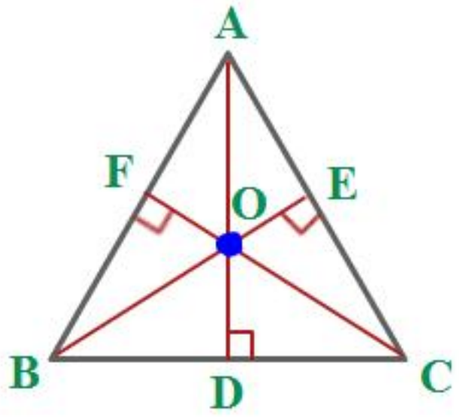
2 → Circumcentre

3 → Incentre

4 → Centroid

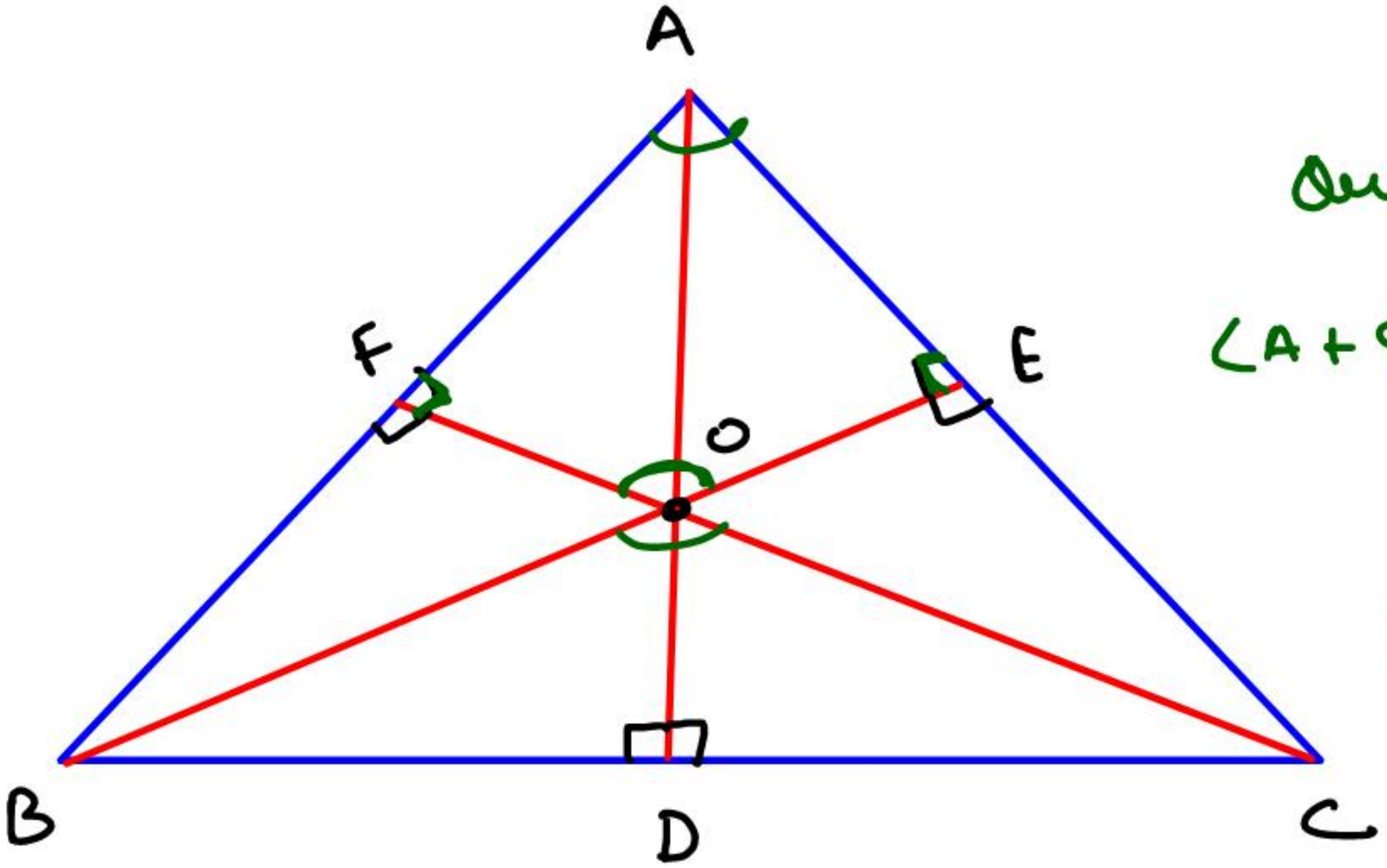
ORTHOCENTRE

Def: Meeting point of all altitudes



AD, BE and CF are altitudes of triangle.

O → Orthocentre

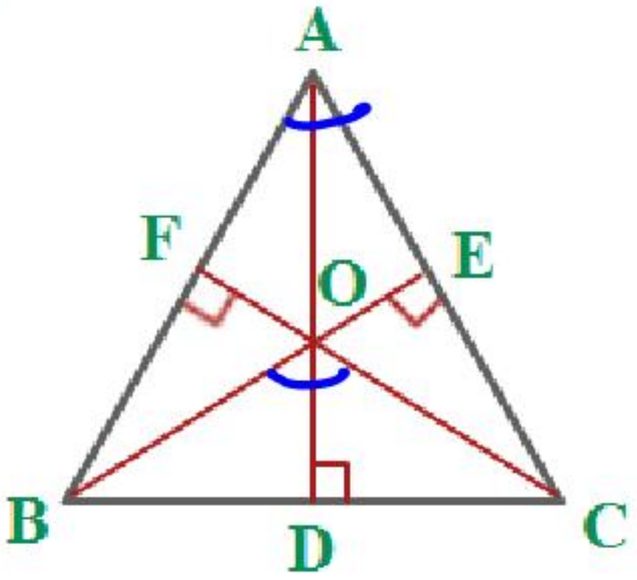


Quad AFOE

$\angle A + 90 + \angle O + 90 = 360$

$\angle A + \angle O = 180^\circ$

$\boxed{\angle A + \angle BOC = 180^\circ}$



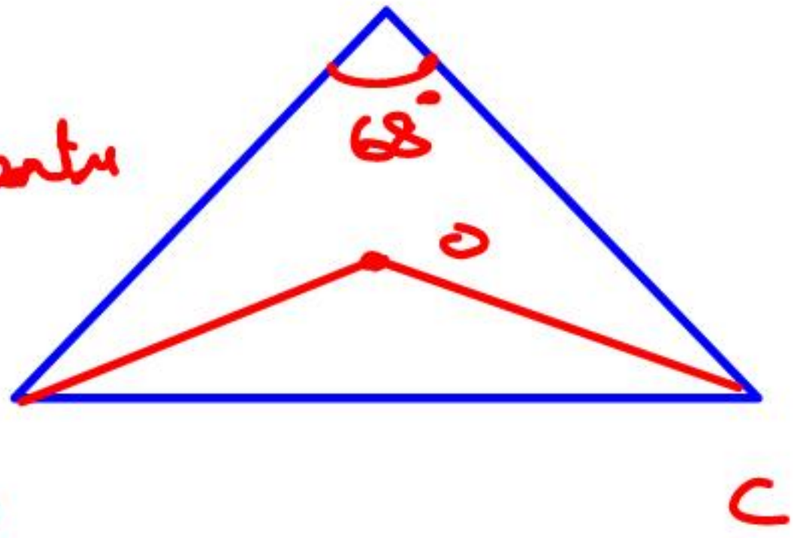
$\angle A + \angle BOC = 180°$

Reason ??

→ I have discussed

eg

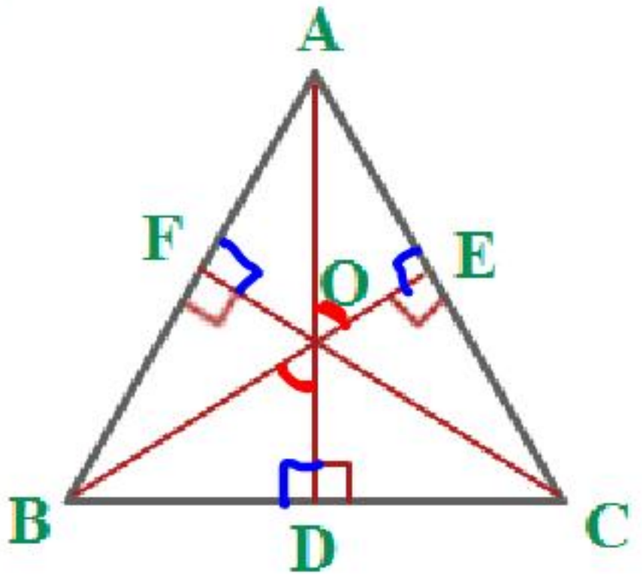
O → orthocentre



Find $\angle BOC$

$68 + \angle BOC = 180$

$\angle BOC = 112°$



$$AO \cdot OD = BO \cdot OE = CO \cdot OF$$

Reason ??

$\angle AOE = \angle BOD$ [vertically opp angles]

$\triangle AOE \sim \triangle BOD$ (AAA)

$$\frac{AO}{BO} = \frac{OE}{OD}$$

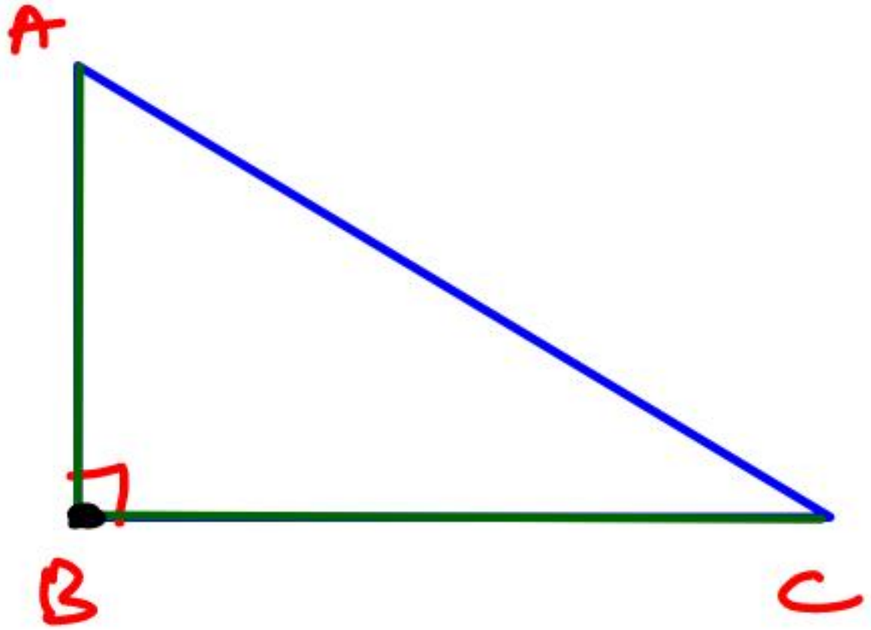
$(AO)(OD) = (BO)(EO$

POSITION OF ORTHOCENTRE

1. **Acute Angle Triangle**

→ lies inside the Δ

2. **Right Angle Triangle**



Orthocentre → B

↓

vertex where 90° is formed

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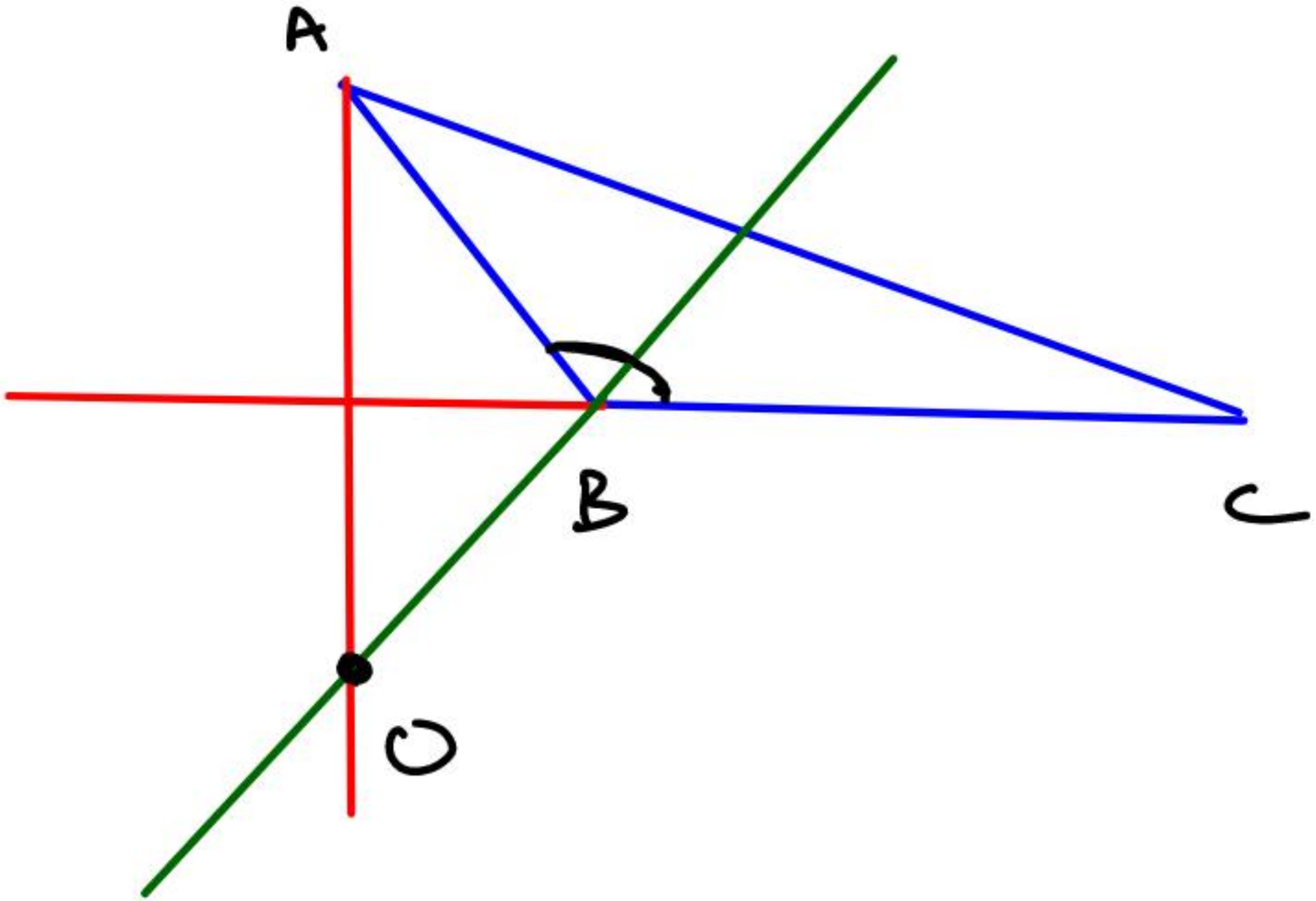
3. Obtuse Angle Triangle

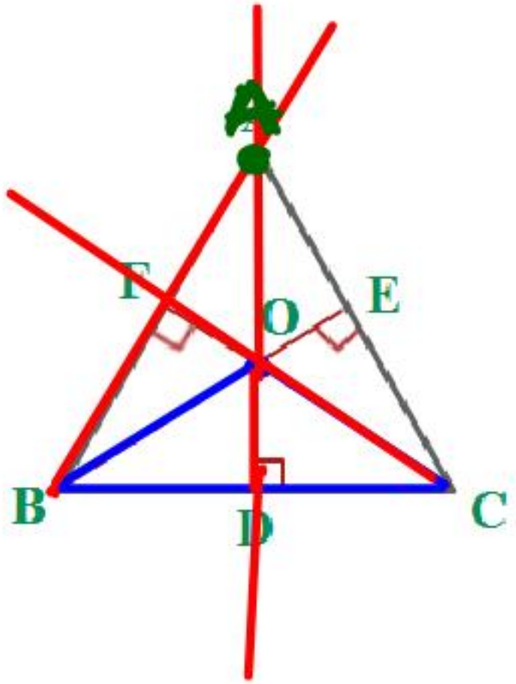


∠B is obtuse

Outside the Δ

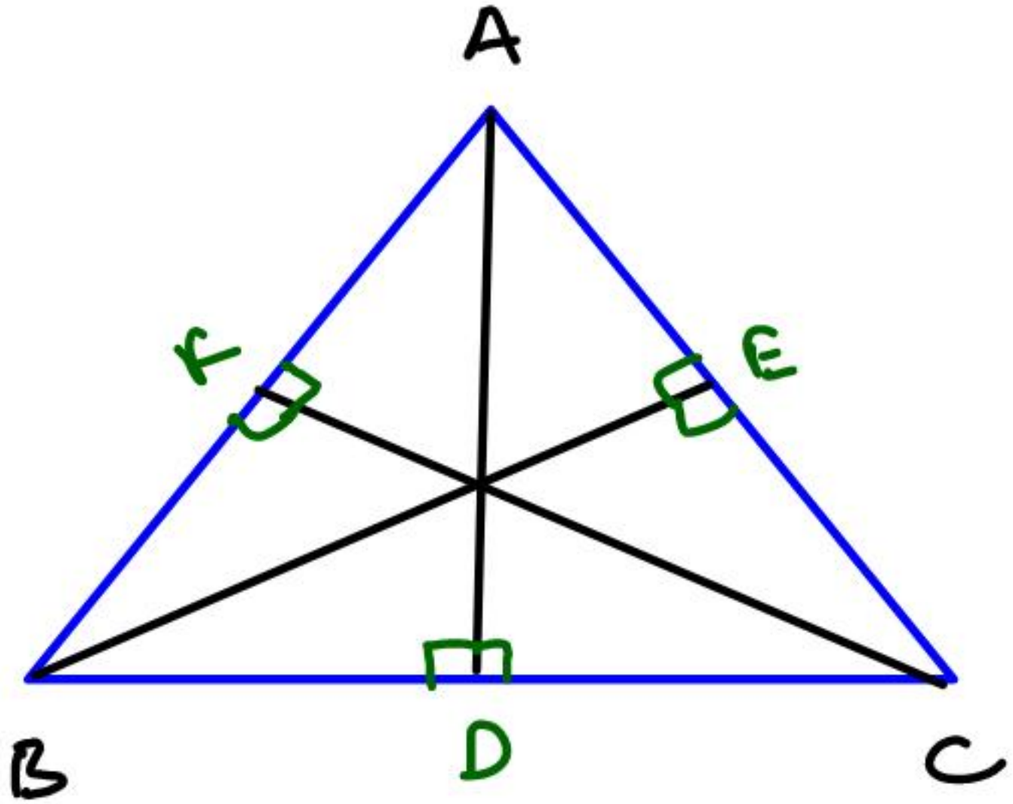
In a ΔABC, O is the orthocentre. Which point is the orthocentre of ΔBOC?

Orthocentre of ΔBOC is at A



Sum of all altitudes of a triangle is less than perimeter of triangle.

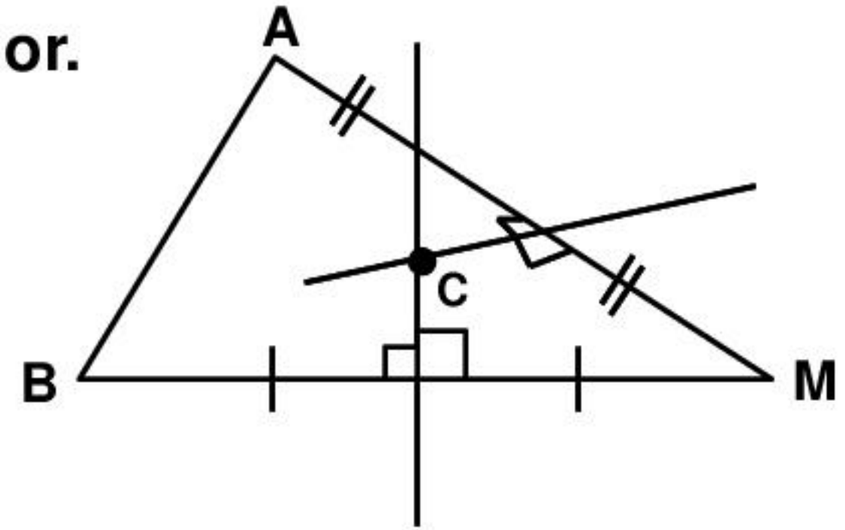
किसी त्रिभुज के सभी शीर्षलंबों का योग त्रिभुज के परिमाप से कम होता है।

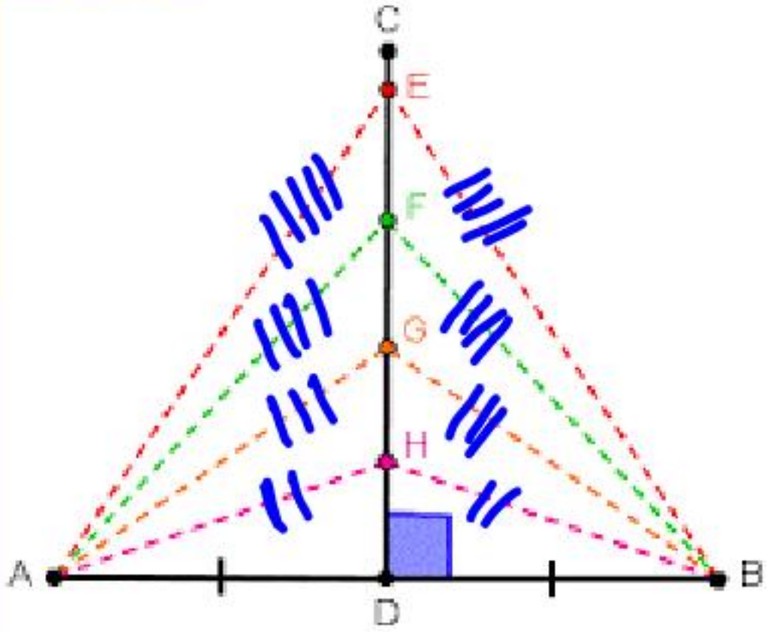


$(AD+BE+CF < AB+BC+CA)$

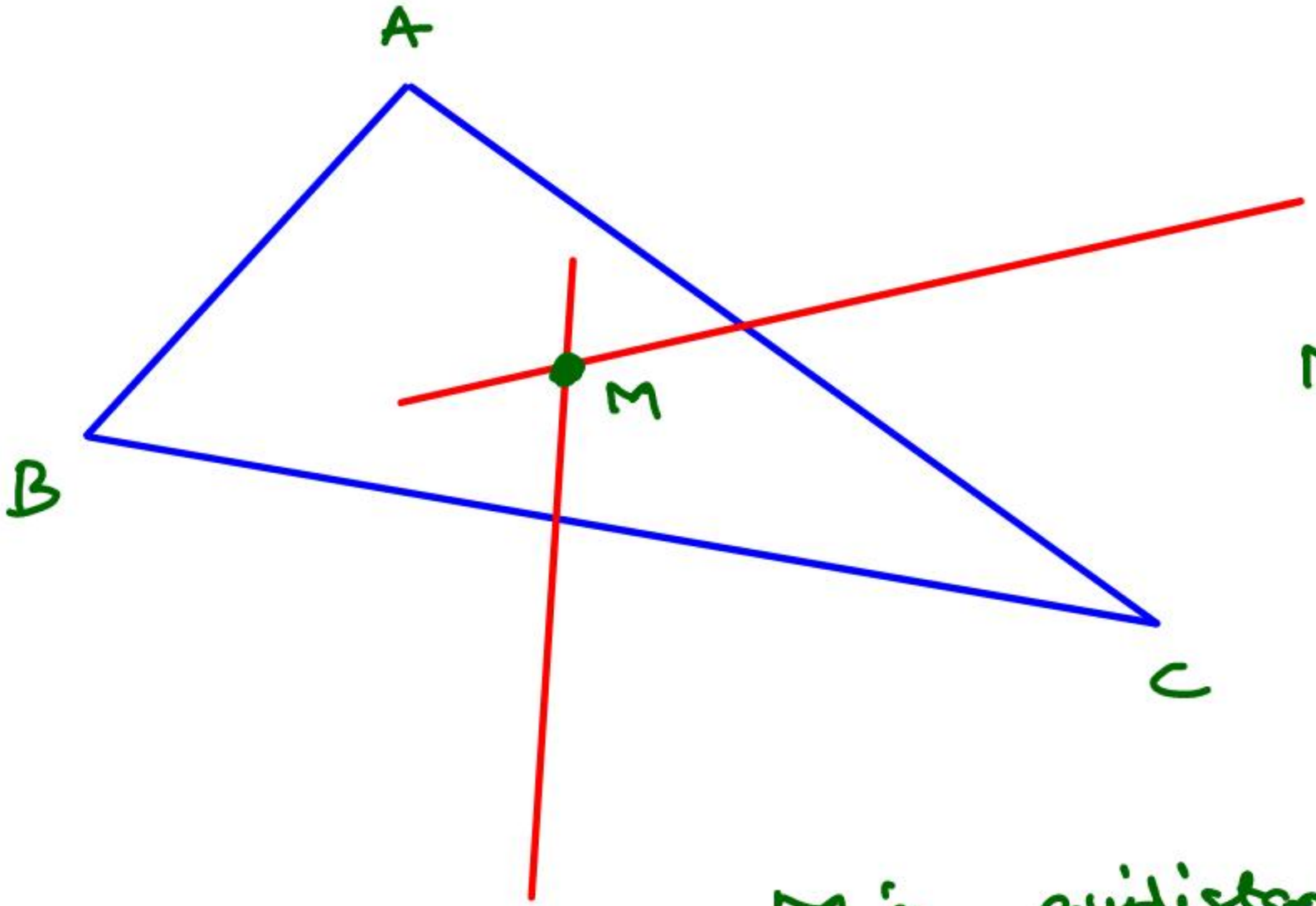
CIRCUM CENTRE

Def: Meeting point of all perpendicular bisector.





Any point on perpendicular bisector of AB, is equidistant from the end points of line segment AB.

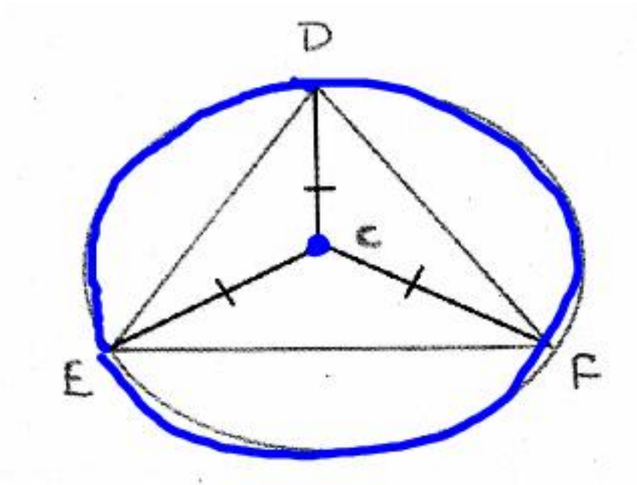


M is equidistant from B & C

M is eq from A & C

M is equidistant from A / B & C

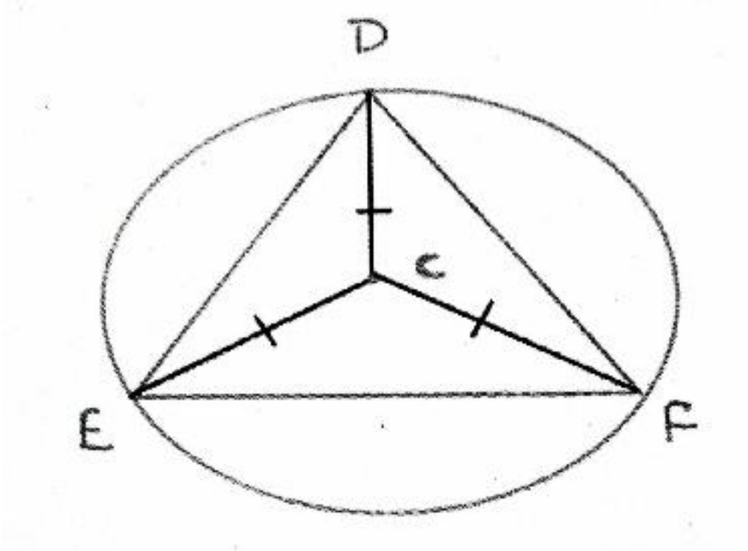
Circumcentre is equidistant from the vertices of triangle.



ΔDEF

Δ C is the Circumcentre

$DC = EC = FC = R$



$$\text{CIRCUM RADIUS (R)} = \frac{abc}{4\Delta}$$

Equilateral (R) = $\frac{\text{side}}{\sqrt{3}}$

Right angle Δ = $\frac{\text{Hypotenuse}}{2}$

$$R = \frac{a \cdot b \cdot c}{4 \times \text{Area of } \Delta}$$

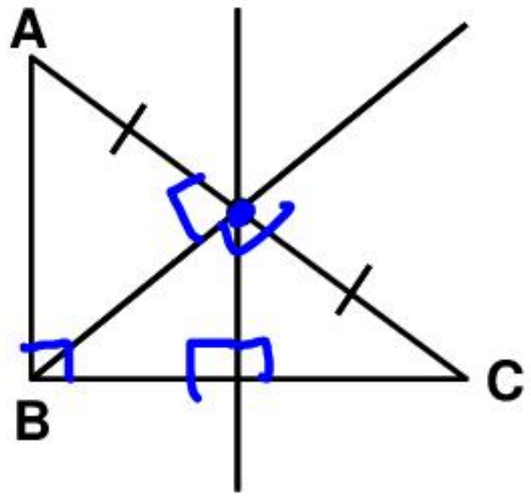
For eqΔ $$R = \frac{s \cdot s \cdot s}{4 \cdot \frac{\sqrt{3}}{4} s \cdot s} \Rightarrow \frac{\text{side}}{\sqrt{3}}$$

For Rightangle Δ $$R = \frac{B \cdot P \cdot H}{2 \times \frac{1}{2} B \cdot P} = \frac{\text{Hyp}}{2}$$

POSITION OF CIRCUM CENTRE

(1) Acute angle Δ → lies inside the Δ

(2) Right angle Δ → mid-point of hypotenuse



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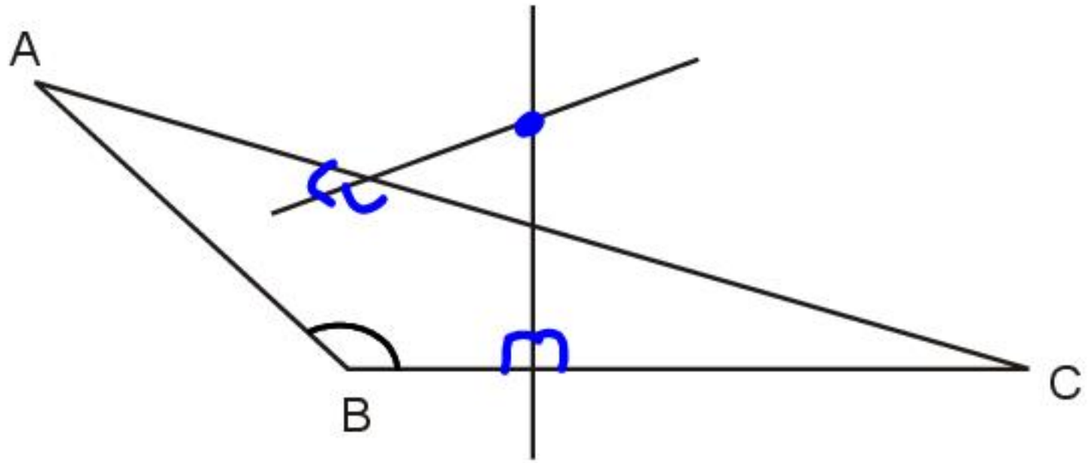
(3) Obtuse angle Δ → outside the Δ



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In all acute angle ΔABC, if the O is the circumcentre then $\angle BOC = 2\angle A$

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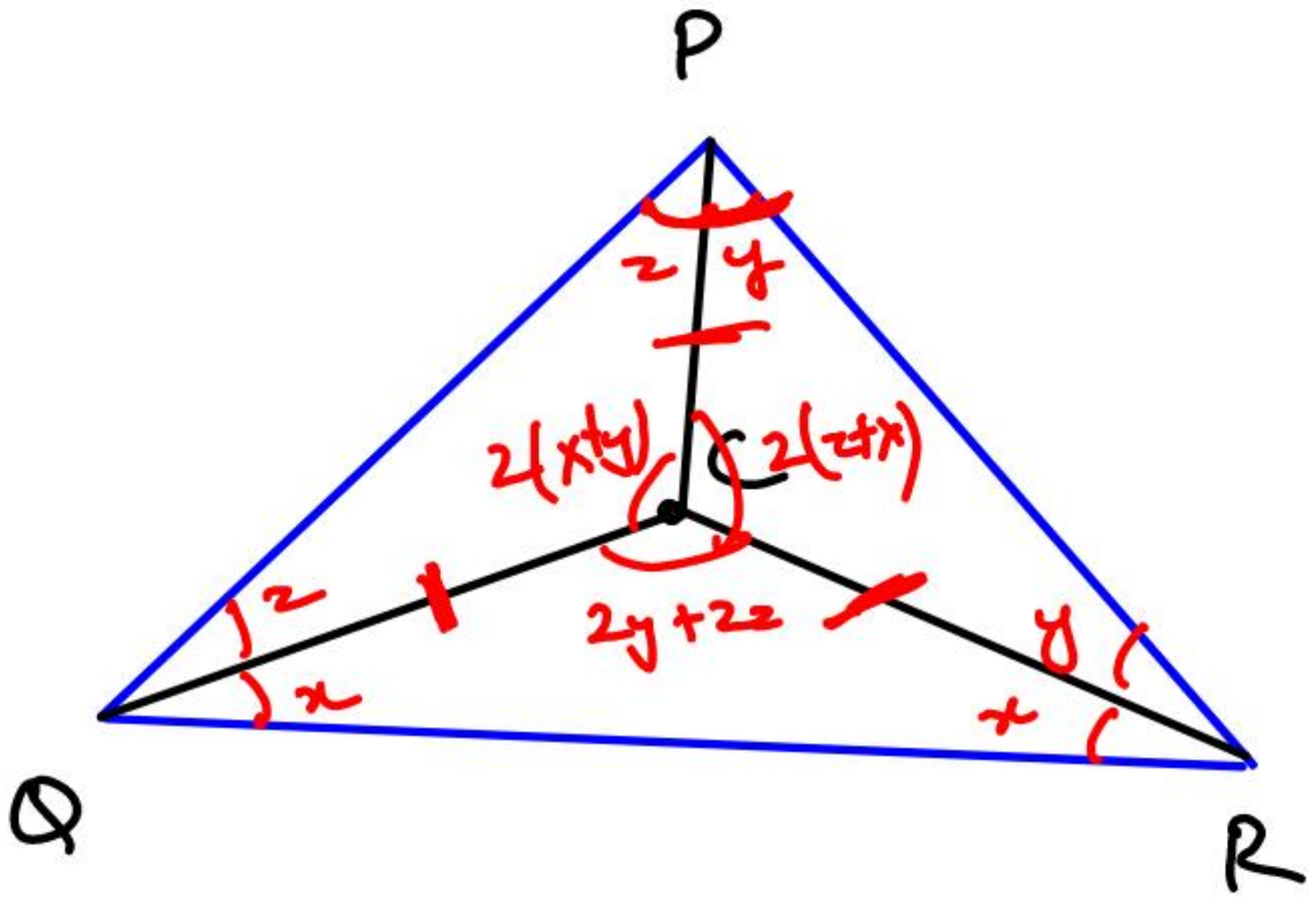


C is circumcentre of

$\triangle PQR$

$2(x+y+z) = 180$

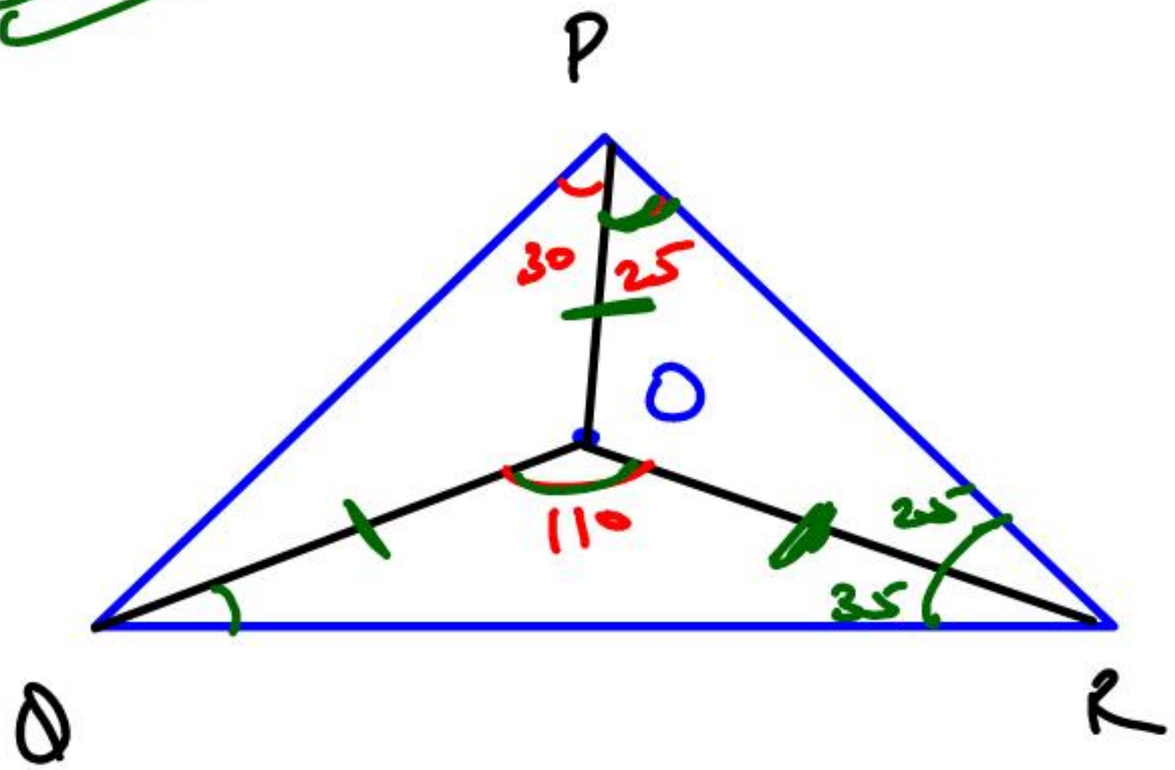
Eg. If O be the circumcentre of a triangle PQR and ∠QOR =110, ∠OPR = 25, the measure of ∠PRQ is :

(a) 65

(b) 50

(c) 60

(d) 55



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PRACTICE QUESTIONS

Q1. The area of two similar triangles are in the ratio 9 : 16. Their corresponding sides will be in the ratio :

(a) 3 : 5 (b) 3 : 4

(c) 4 : 5 (d) 4 : 3

Ans. (b)

Q2. If the three side of one triangle are equal to the corresponding sides of the other triangle then the triangle are :

(a) Congruent

(b) Similar

(c) Congruent and similar

(d) None of these

Ans. (c)

Q3. Two line segments PQ and RS intersect at X in such a way that XP = XR. If $\angle PSX = \angle RQX$, then one must have.

(a) PR = QS

(b) PS = RQ

(c) $\angle XSQ = \angle XRP$

(d) ar (ΔPXR) = ar(ΔQXS)

Ans. (b)

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Q4. In ΔABC, points D and E are on the sides, AB and AC respectively such that AD is 80% of AB and EC is 20% of AC. What percentage of the area of the triangle ΔABC does that of ΔADE form ?

(a) 32%
(b) 40%
(c) 64%
(d) 80%

Ans. (c)

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Q5. In ΔABC, D and E are points on AB and AC respectively such that DE || BC. If area of $\Delta ABC = 50\ cm^2$ and AD : DB = 2 : 3 then area of ΔBDE is :

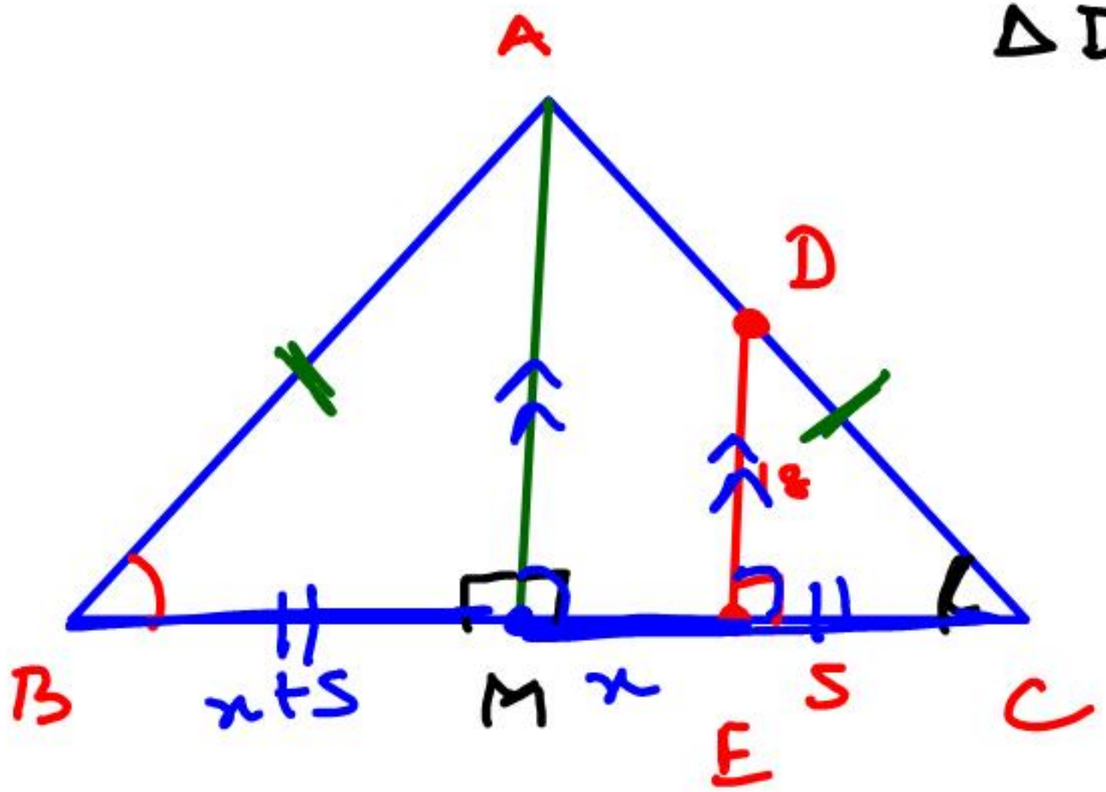
(a) $8\ cm^2$ (b) $12\ cm^2$

(c) $20\ cm^2$ (d) $30\ cm^2$

Ans. (b)

Q8. D and E are two points on the sides AC and BC respectively of ΔABC such that DE = 18 cm, CE = 5 cm and $\angle DEC = 90°$. If $\tan \angle ABC = 3.6$, then AC : CD

(a) BC : 2CE
(b) 2CE : BC
(c) 2BC : CE
(d) CE : 2BC



ΔDEC $\tan \angle C = \frac{18}{5} = 3.6$

$\angle B = C$

ΔAMC $DE \parallel AM$

$\frac{CD}{CA} = \frac{2CE}{2CM}$

$\frac{CD}{CA} = \frac{2CE}{BC}$

Ans. (a)

Q7. ABC is a right angled triangle, right angled at C and p is the length of the perpendicular from C to AB. If a, b and c are the length of the sides BC, CA and AB respectively, then

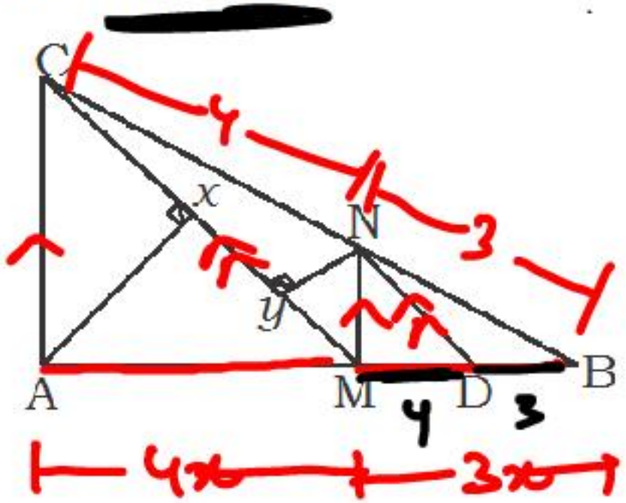
(a) $\frac{1}{p^2} = \frac{1}{b^2} - \frac{1}{a^2}$ (b) $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$

(c) $\frac{1}{p^2} = \frac{1}{a^2} = -\frac{1}{b^2}$ (d) $\frac{1}{p^2} = \frac{1}{a^2} - \frac{1}{b^2}$

Ans. (b)

Q8. In the given figure, M is an point on AB such that AM : MB = 4 : 3, MN is parallel to AC and ND is parallel to CM. In ΔAXC, ∠AXC = 90° and in ΔMNY, ∠MYN = 90°. The length of NY is 6 cm. What is ratio AM : MD ?

(a) 3 : 7
(b) 12 : 7
(c) 7 : 3
(d) 7 : 5



AM : MB = 4 : 3 AM : MD

Δ ABC MN ∥ AC

ΔCAB ~ ΔNMB $\frac{AC}{MN} = \frac{AB}{MB} = \frac{BC}{BN} = \frac{7}{3}$ — (1)

$MD = \frac{4 \cdot 3x}{7}$

DN ∥ CM

ΔBDN ~ Δ BMC $\frac{BD}{BM} = \frac{BN}{NC} = \frac{3}{4}$

$4x : \frac{4 \cdot 3x}{7}$

7 : 3

Ans. (c)

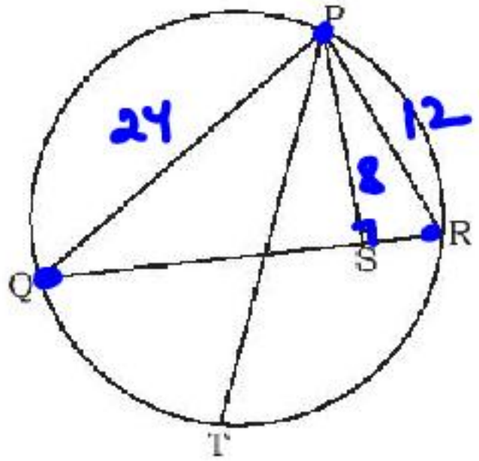
Q9. In a right angled ΔABC, $\angle ABC = 90°$, BN is perpendicular to AC, AB = 6 cm, AC = 10 cm. Then AN : NC is-

(a) 3 : 4
(b) 9 : 16
(c) 3 : 16
(d) 1 : 4

Ans. (b)

Q10. In the given figure, PQR is a triangle in which, PQ = 24 cm, PR = 12 cm and altitude PS = 8 cm. If PT is the diameter of the circum-circle, then what is the length (in cm) of circum-radius ?

(a) 15 (b) 18
(c) 20 (d) 21



$$R = \frac{a \cdot b \cdot c}{4 \text{ Area}}$$

$$= \frac{24 \cdot 12 \cdot QR}{2 \cdot 24 \cdot \frac{1}{2} \cdot QR \cdot 8}$$

$$= 18$$

Ans. (b)

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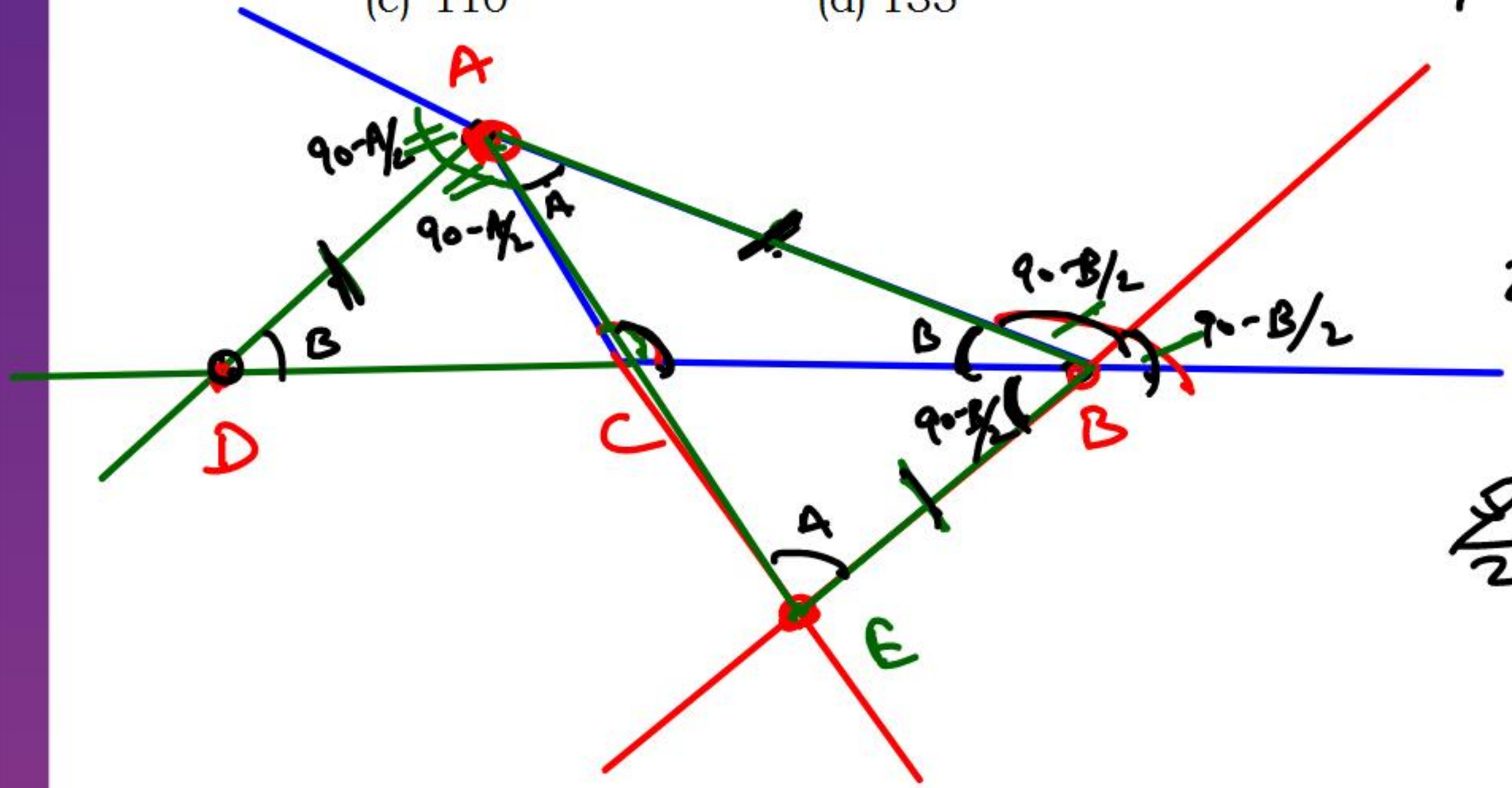
Q11. D is a point on the side BC of a triangle ABC such that AD $\perp$ BC. E is a point on AD for which AE : ED = 5 : 1. If $\angle BAD = 30^\circ$ and $\tan(\angle ACB) = 6 \tan(\angle DBE)$, then $\angle ACB =$

(a) 15° (b) 60°
(c) 45° (d) 30°

Ans. (b)

Q12. In ΔABC, $\angle C$ is an obtuse angle. The bisectors of the exterior angles at A and B meet BC and AC produced at D and E respectively. If AB = AD = BE, then $\angle ACB$ = ?

(a) 105°
(b) 108°
(c) 110°
(d) 135°



$90 - A/2 + A + B + B = 180$

$2B + \frac{A}{2} = 90$ — (1)

$2A + \frac{B}{2} = 90$ — (2)

$\frac{3}{2}(A+B) = 180$

$A + B = 72$

Ans. (b)

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Q13. In a triangle ABC, $\angle B = \angle C = 76°$. D and E are two points on side AB and AC such that $\angle BCD = 52°$ and $\angle CBE = 28°$. Find the $\angle CDE$ = ?

(a) 9°
(b) 12°
(c) 8°
(d) 14°

Ans. (d)

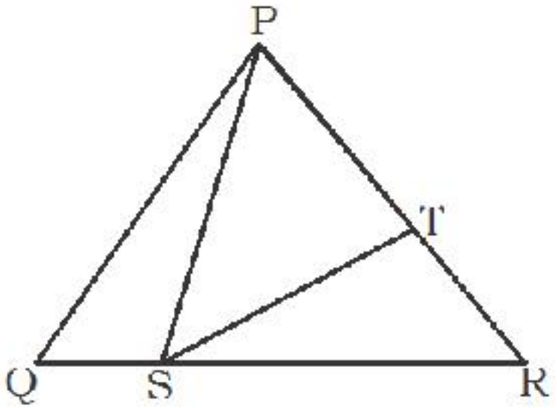
Q14. In ΔPQR three point K, L and M are on side PQ, PR and QR respectively such that QM = KM and MR = LM. If $\angle QPR = 50°$. Find $\angle KML$ = ?

(a) 150°
(b) 80°
(c) 100°
(d) 50°

Ans. (b)

Q15. In the figure shown, PQR is an isosceles triangle with PQ = PR, S is a point on QR such that PS = PT and ∠QPS = 30°, then find value of ∠RST.

(a) 15° (b) 45° (c) 30° (d) 20°



Ans. (a)

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Q16. The sides of a triangle are in geometric progression with common ratio $r < 1$. If the triangle is a right angled triangle the square of common ratio is given by :

(a) $\frac{\sqrt{5}+1}{2}$ (b) $\frac{\sqrt{5}-1}{2}$ (c) $\frac{\sqrt{3}+1}{2}$ (d) $\frac{\sqrt{3}-1}{2}$

Ans. (b)

Q17. An isosceles triangle ABC is right angled at B. D is a point inside the triangle ABC. P and Q are the feet of the perpendiculars drawn from D on the side AB and AC respectively of ΔABC. If AP = a cm, AQ = b cm and $\angle BAD = 15°$, $\sin 75° = ?$

(a) $\frac{2b}{\sqrt{3}a}$ (b) $\frac{a}{2b}$ (c) $\frac{\sqrt{3}a}{2b}$ (d) $\frac{2a}{\sqrt{3}b}$

Ans. (c)

Q18. In a ΔABC, AB = AC and BA is produced to D such that AC = AD, then the $\angle BCD$ is –

(a) 100° (b) 60° (c) 80° (d) 90°

Ans. (d)

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