BYJU'S EXAM PREP

The Most Comprehensive
Preparation App For All Exams

QUADRILATERAL

Part-II

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Agenda

41 min

- Rhombus
- Rectangle

] → (40 – 42) min

8 min

- Square

] → (8 – 10) min

- Trapezium
- Isosceles Trapezium

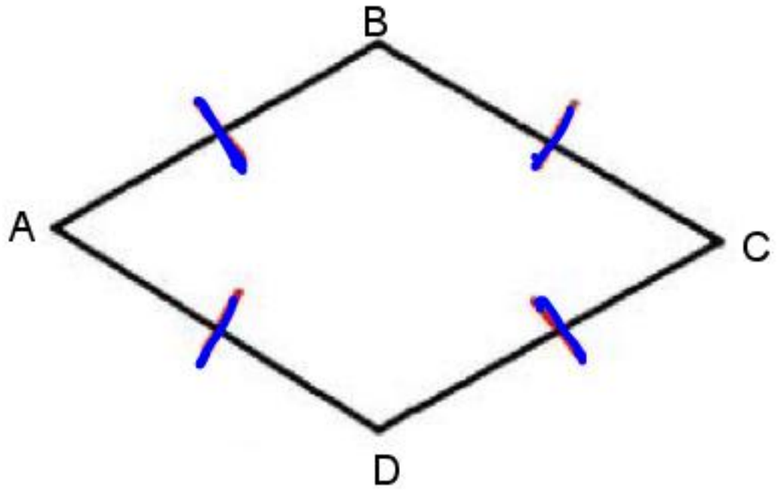
] (50 – 54) min

- KITE

] → (8 – 10)

RHOM BUS

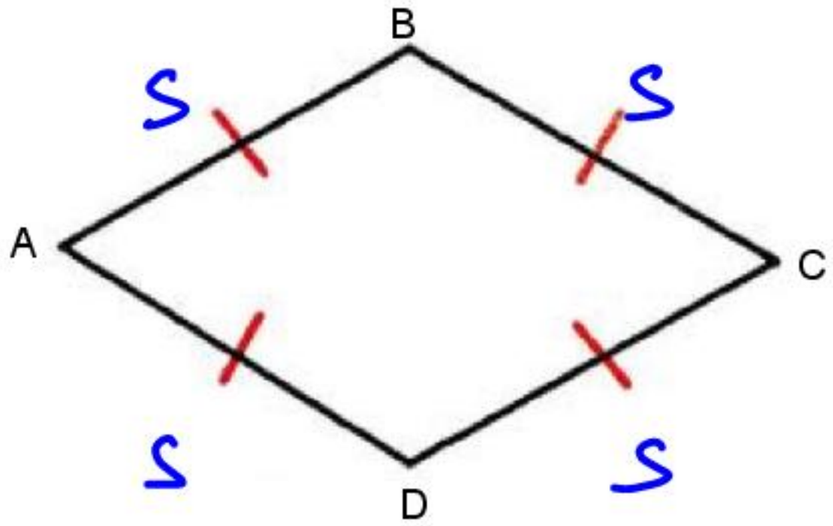
Def: Rhombus is a parallelogram in which adjacent sides are equal.



Def :- ||gm + AB = BC

PROPERTIES OF RHOM BUS

1. All sides of rhombus are equal.



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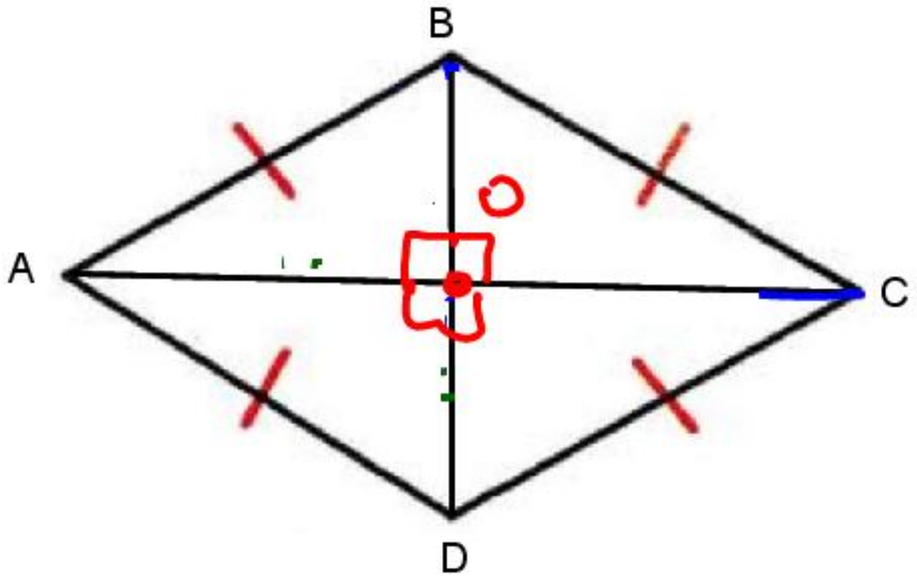
2. (i) Diagonals of a rhombus bisect each other at 90°.



$\angle AOB = \angle BOC = \angle COD = \angle DOA$

$= 90^\circ$

Reason

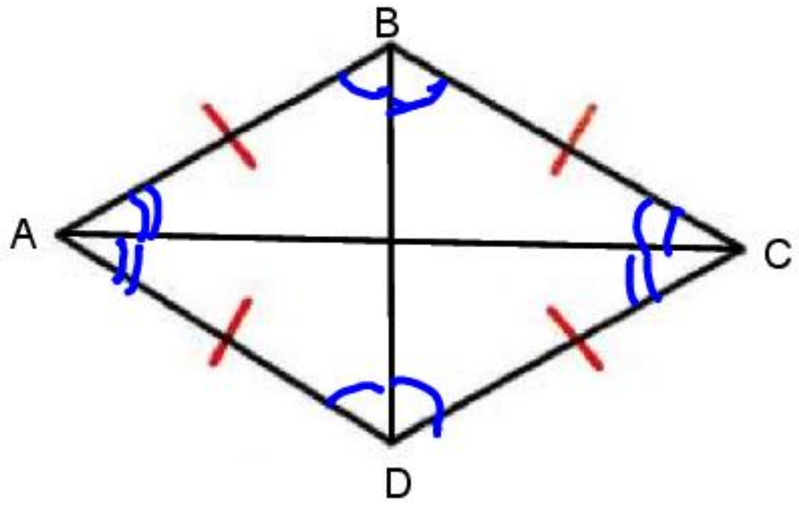
$\Delta BOC \cong \Delta BOA$

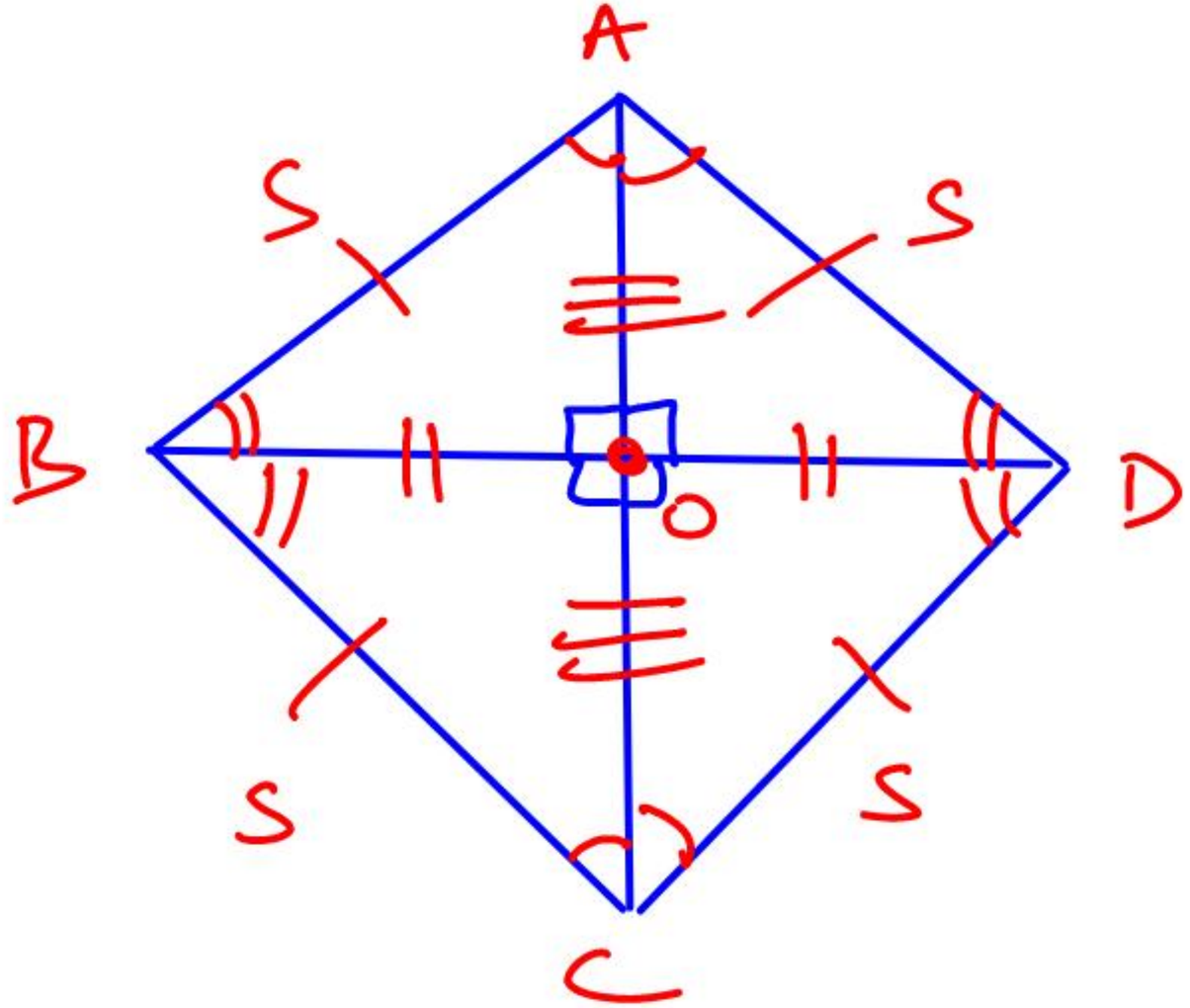
$\angle BOC = \angle BOA = \theta$

$\theta + \theta = 180$

$\theta = 90^\circ$

(ii) Diagonals of a rhombus are angle bisector.




A
S
S
B
O
D
S
S
C

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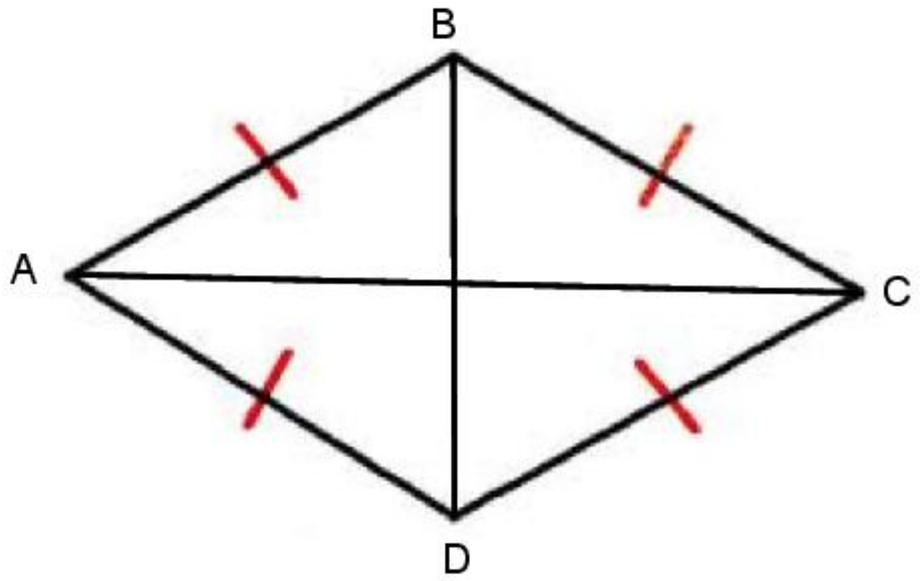
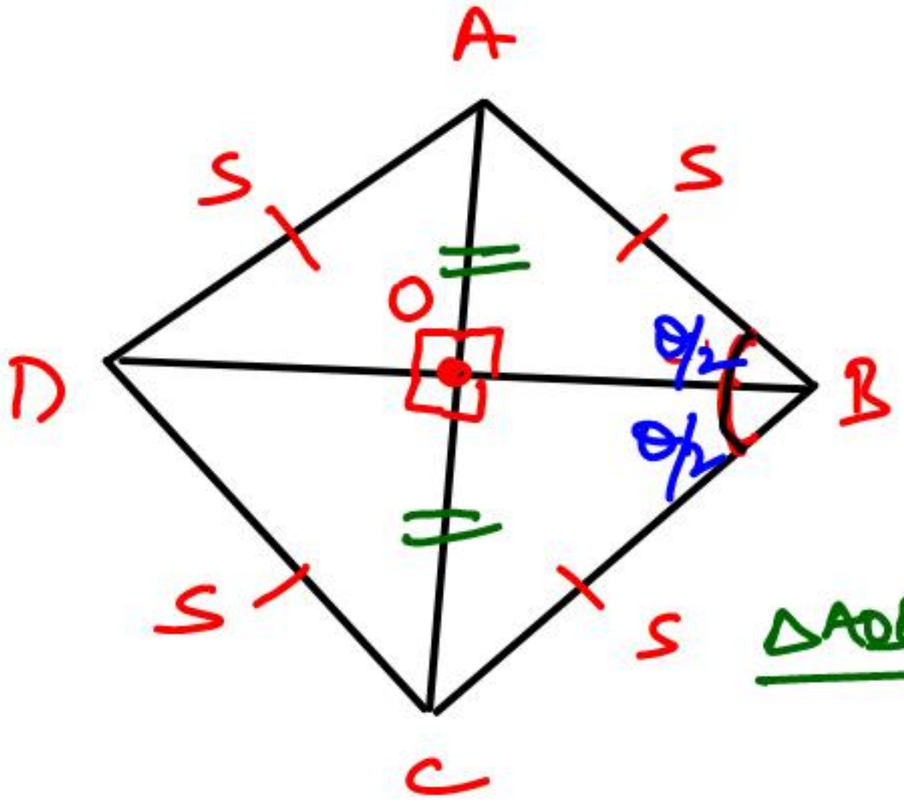
3. Diagonals of a rhombus need not be equal.



$D_1 \neq D_2$



Let $\angle B = \theta$

where $\underline{\theta}$ is an Acute angle

$0 < \theta < 90$

$0 < \frac{\theta}{2} < 45$

ΔAOB $\sin\theta/2 = \frac{AO}{S}$ $AO = S\sin\theta/2$

Smaller

$AC = 2S\sin\theta/2$

$\cos\theta/2 = \frac{BO}{S}$ $BO = S\cos\theta/2$

Larger

$BD = 2S\cos\theta/2$

4. ABCD is a rhombus and one of the angle of rhombus is θ, where $0° < \theta < 90°$

Length of longer diagonal = $2s\cos\frac{\theta}{2}$

Length of shorter diagonal = $2s\sin\frac{\theta}{2}$

Eg7. If perimeter of rhombus is 40 cm and one of its angle is 120°. Find the length of longer diagonal.

$4 \cdot S = 40$ $\quad S = 10$

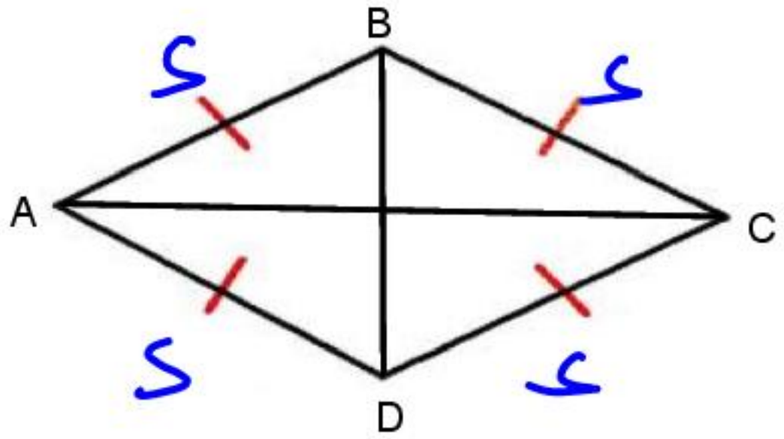
Longer Diagonal $= 2S \cos\theta/2$

$= 2 \cdot 10 \cdot \cos 30$

$= 2 \cdot 10 \cdot \frac{\sqrt{3}}{2}$

$10\sqrt{3}$ cm

5. ABCD is a rhombus and D_1 and D_2 are the diagonals of rhombus and S is the side of rhombus.



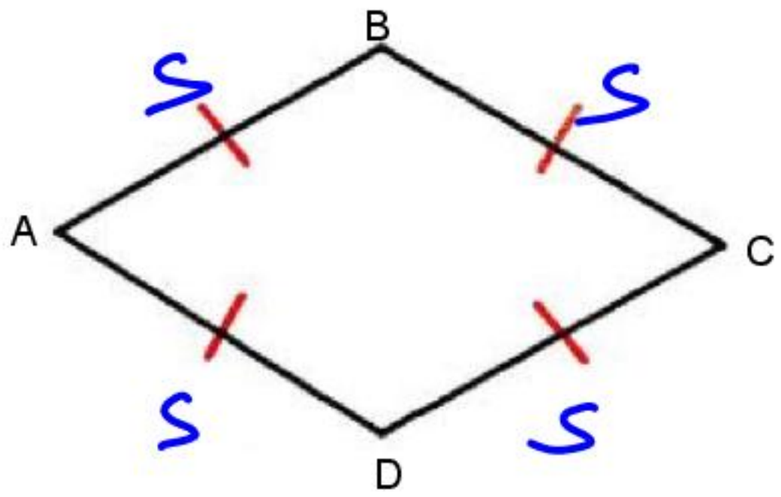
$$D_1^2 + D_2^2 = 4S^2$$

In ||gm $D_1^2 + D_2^2 = 2(x^2 + y^2)$

$= 2(s^2 + s^2)$

$= 4s^2$

6.



Perimeter of Rhombus = 4S

$$\text{Area of Rhombus} = \frac{1}{2} D_1 D_2$$

$$= S^2 \cdot \sin\theta$$

Where, θ is one of the angle of rhombus.

Area of Quad $\rightarrow$ $\frac{1}{2} D_1 D_2 \sin\theta$

$\rightarrow$ $\frac{1}{2} D_1 D_2 \sin 90$

$\rightarrow$ $\frac{1}{2} D_1 D_2$

If Perimeter of Rhombus = 40 cm

and One of angles = 60°

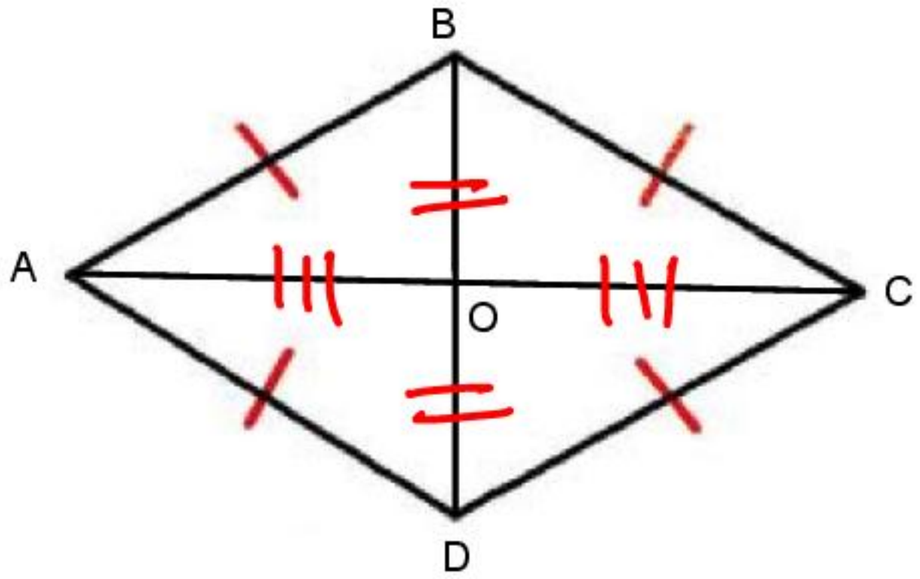
Area of Rhombus = ??

$S = 10$

Sol^n $\longrightarrow$ $S^2 \sin\theta$

$100 \cdot \frac{\sqrt{3}}{2} \Rightarrow 50\sqrt{3}\ cm^2$

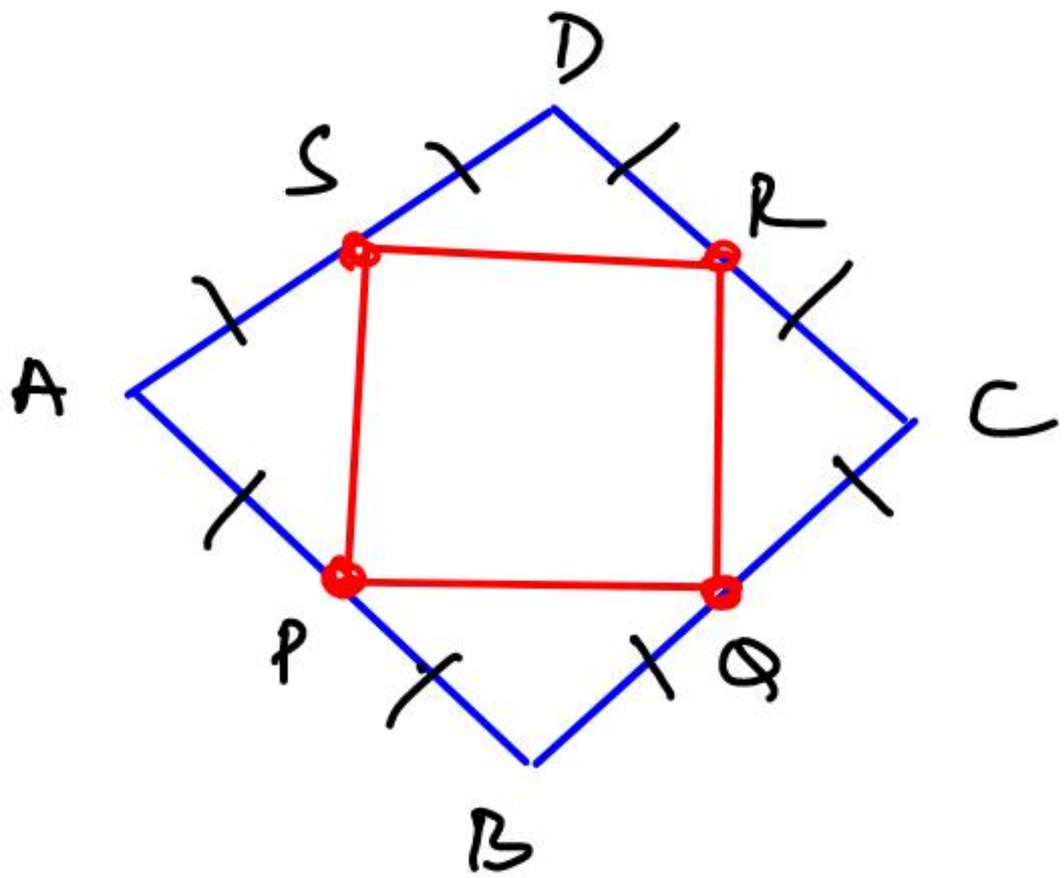
7.



[redacted]DOA

$\Delta AOB \cong \Delta COB \cong \Delta COD \cong \Delta AOD$

8. Figure formed by joining the mid point of all sides of a rhombus is RECTANGLE.



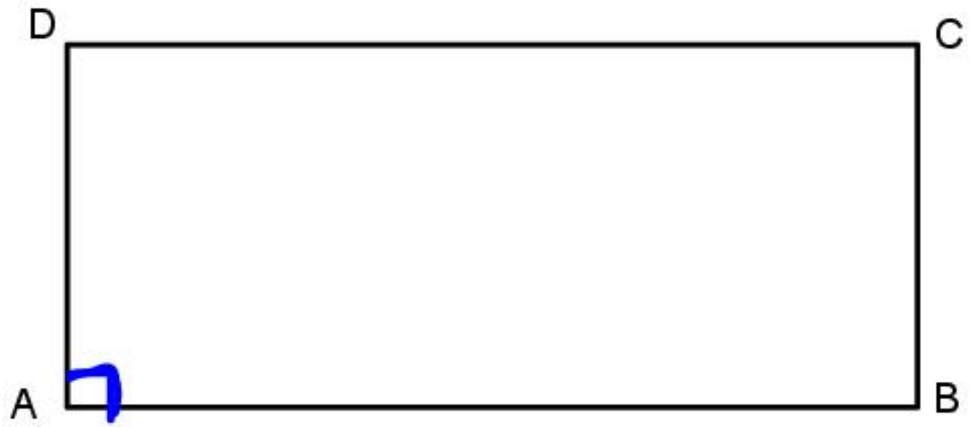
If ABCD is a Rhombus

P, Q, R & S are mid pts of AB, BC, CD & DA

PQRS → Rectangle

RECTANGLE

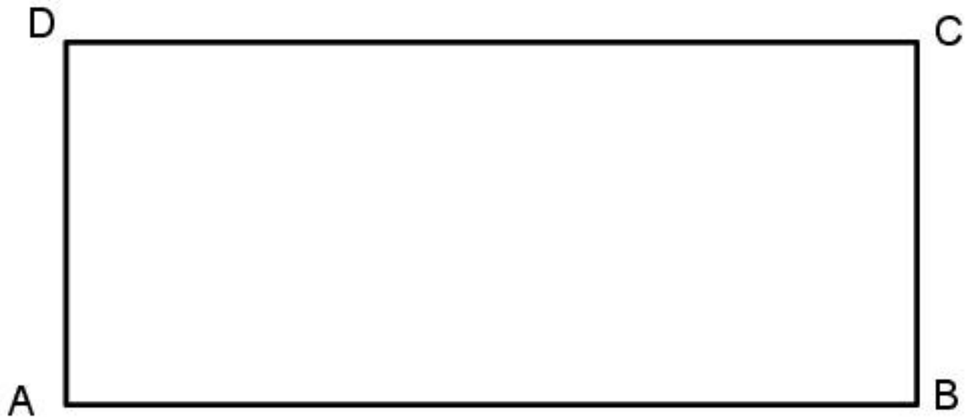
Def: A parallelogram in which one angle is 90°.



$\parallel gm + One\ Angle = 90°$

PROPERTIES OF RECTANGLE

1. All angles of a rectangle are right angle.



$\angle A = \angle B = \angle C = \angle D$

$= 90^\circ$

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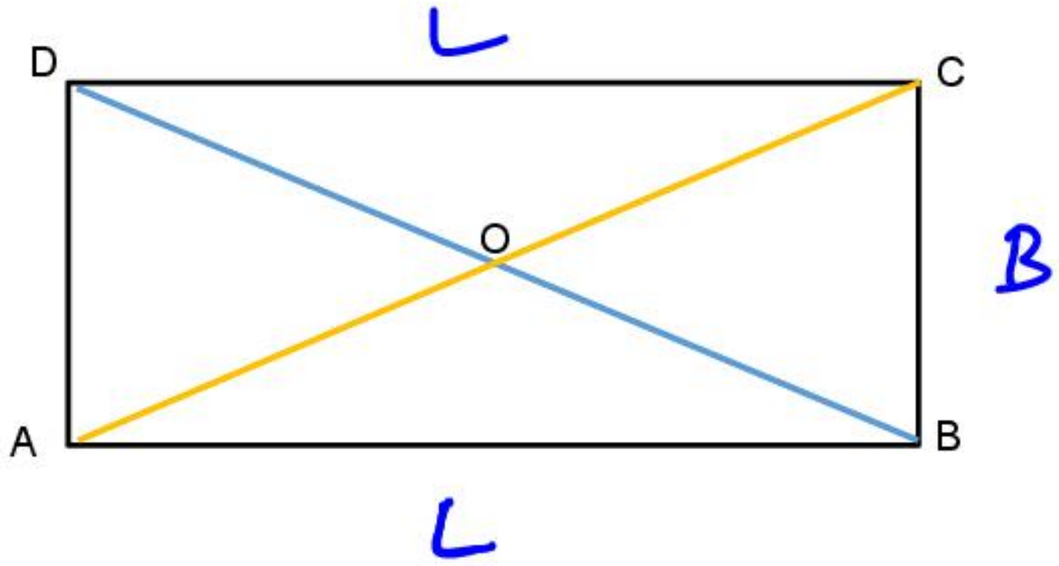
2. Diagonals of a rectangle are equal.

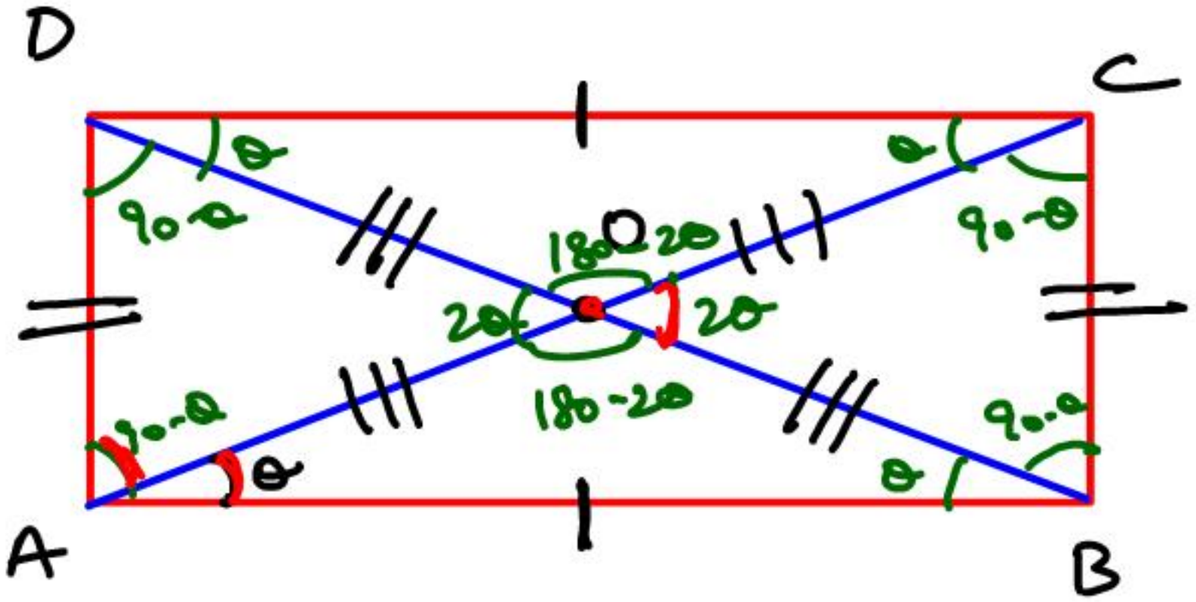


$$AC = \sqrt{L^2+B^2}$$

$$BD = \sqrt{L^2+B^2}$$

$$AC = BD$$

$$\boxed{D_1 = D_2}$$

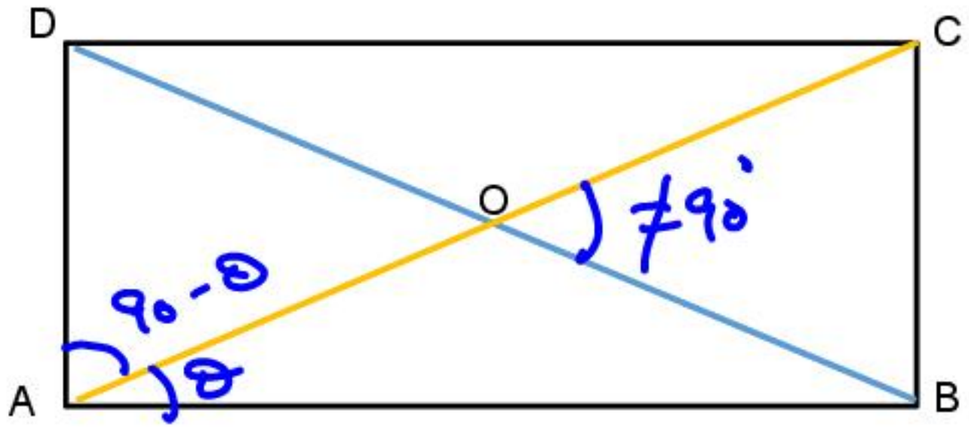


* Diagonals bisect each other but not at 90°

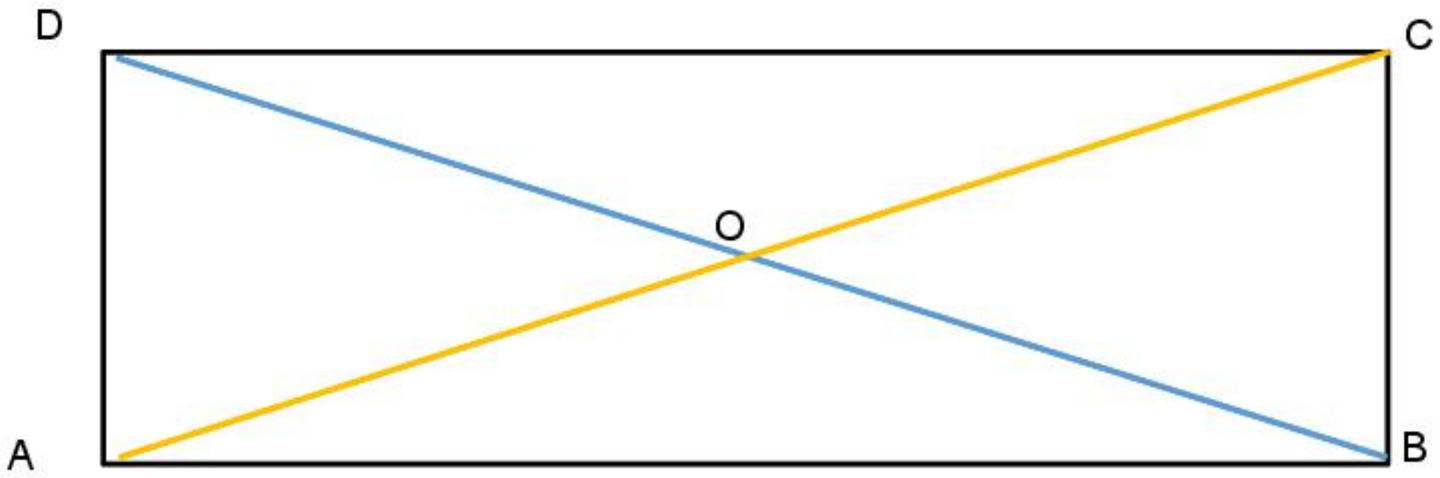
* Diagonals are not angle bisector

3. (i) Diagonals of a rectangle bisect each other but not necessarily at 90°.

(ii) Diagonals of a rectangle need not be angle bisector.

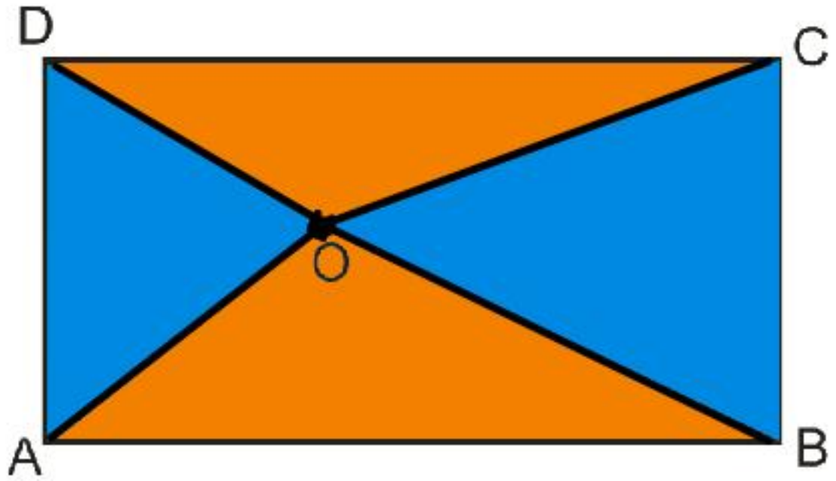


UNDERSTANDING OF A RECTANGLE FIGURE



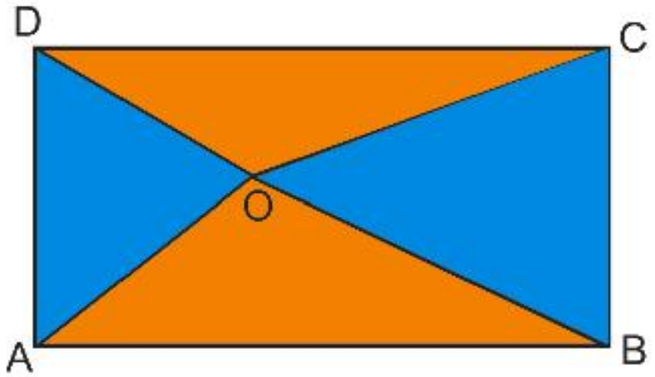
4. If O is any point in the interior of rectangle ABCD, then

$(OA)^2 + (OC)^2 = (OB)^2 + (OD)^2$



Proof → Pythagoras Theorem

Eg8. ABCD is a rectangle and O is only point in the interior of rectangle ABCD. If OA = 3 cm, OB = 4 cm, OC = 5 cm, find the value of OD = ??



$$(OA)^2 + (OC)^2 = (OB)^2 + (OD)^2$$

$$9 + 25 = 16 + (OD)^2$$

$$(OD)^2 = 18$$

$$OD = 3\sqrt{2}\ cm$$

5.

Perimeter of rectangle (P)	= 2 (L + B)
Area of rectangle (A)	= L · B
Diagonal of rectangle (D)	$=\sqrt{L^2 + B^2}$

Important relationship between P, A & D of rectangle.

$$P^2 = 4\left(D^2 + 2A\right)$$

Eg9. If diagonal of rectangle is 14 cm and its area is 68 cm^2. Find its perimeter.

$$P^2 = 4(D^2 + 2A)$$

$$P^2 = 4(14^2 + 2 \cdot 68)$$

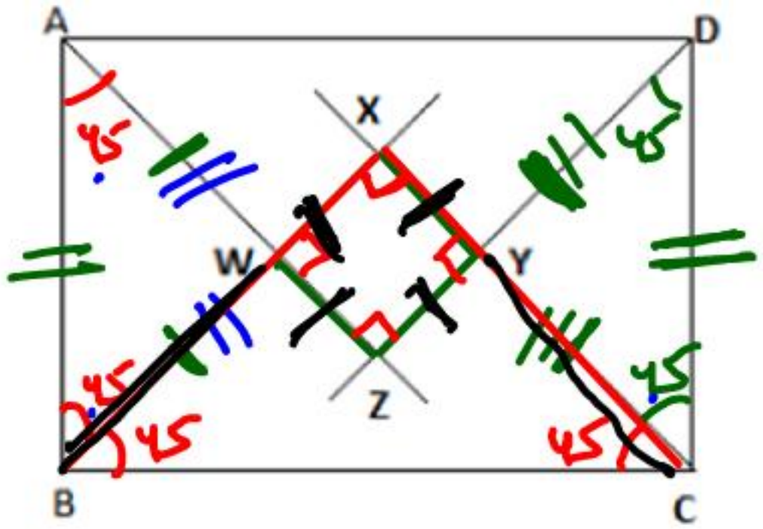
$$P = 2\sqrt{196 + 136}$$

$$= 2\sqrt{332}$$

$$2 \cdot 2\sqrt{83}$$

$$4\sqrt{83}\ cm$$

6. Angle bisectors of a rectangle forms a square.



ΔAWB & ΔDYC

$\Delta AWB \cong \Delta DYC$ [ASA

ΔBXC (Isosceles)

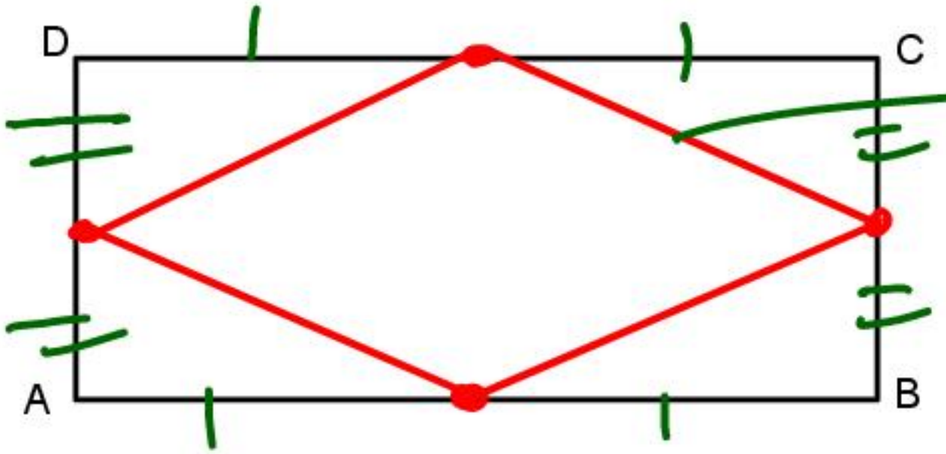
$BX = XC$ — (1)

$BW = CY$ — (2)

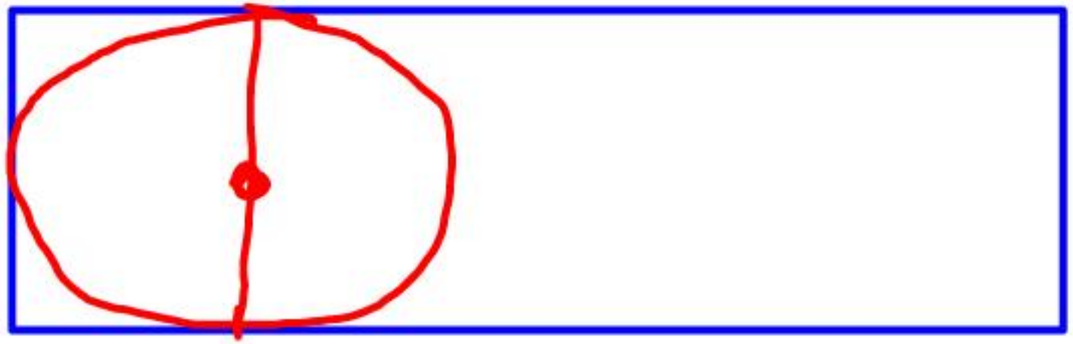
(1) − (2)

$WX = XY$

7. Figure formed by joining the mid-point of all sides of a rectangle is rhombus.



Rhombus



Diameter of circle = Breadth of Rectangle

SQUARE

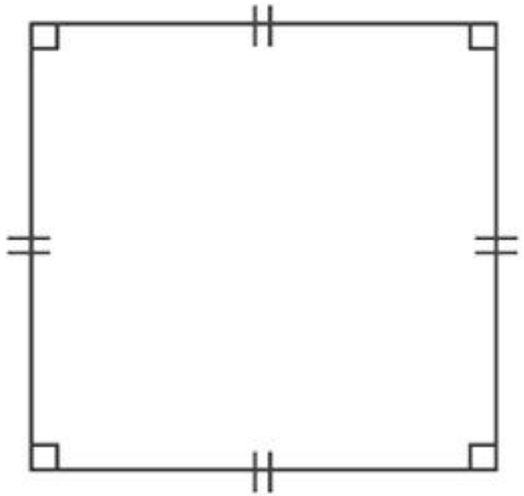
Def :

(1) Quadrilateral + all sides are equal + all angles are equal.

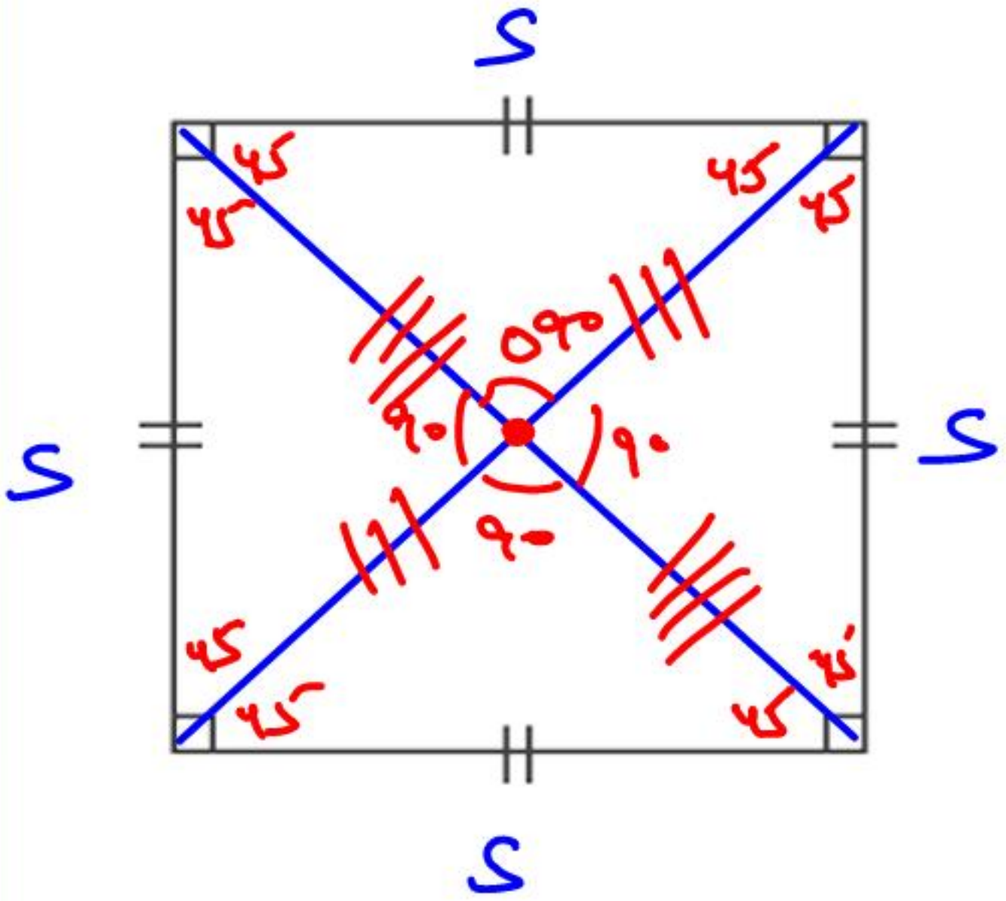
(2) Regular polygon of 4 sides.

(3) Rectangle in which adjacent sides are equal.

(4) Rhombus + one angle = 90°



DETAILED ANALYSIS OF SQUARE FIGURE



Perimeter = $4 \cdot S$

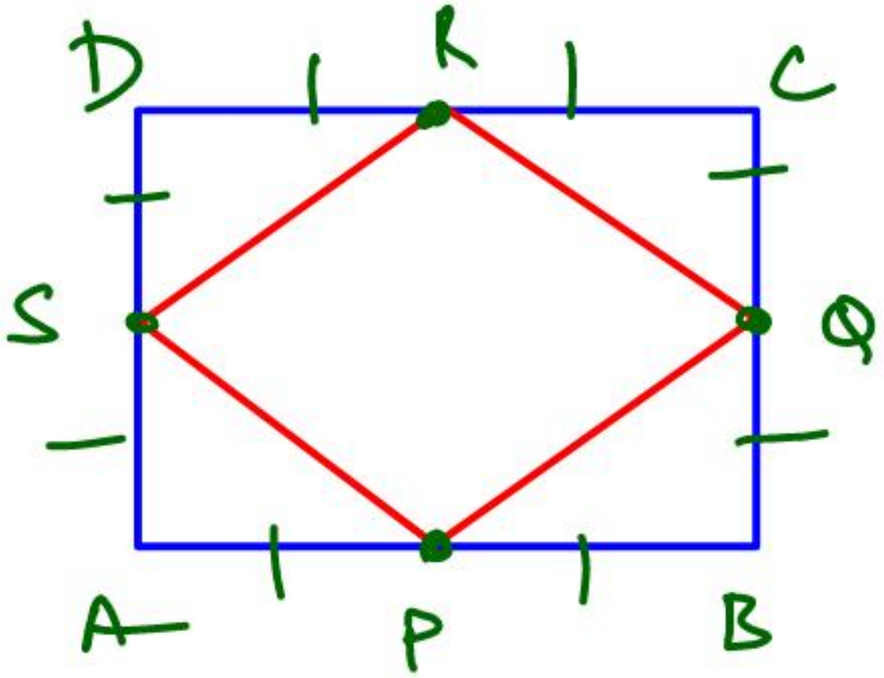
Area = S^2

$= \frac{(D)^2}{2}$

Diagonal $(D) = \sqrt{2} \cdot S$

Figure formed by joining the mid-points of all sides of a square is a square.

ABCD → square



PQRS → square

For a given perimeter of a quadrilateral, square will have maximum area.

Eg. A quadrilateral whose perimeter = 120 cm
Find maximum area of quadrilateral .

It has to be square

$4S = 120 \quad S = 30$

Area = $900\ cm^2$

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Property	Rhombus	Rectangle	Square
Diagonals bisect each other	✓	✓	✓
Diagonals bisect each other at 90°	✓	✗	✓
Diagonals are angle bisector	✓	✗	✓
Diagonals are equal	✗	✓	✓

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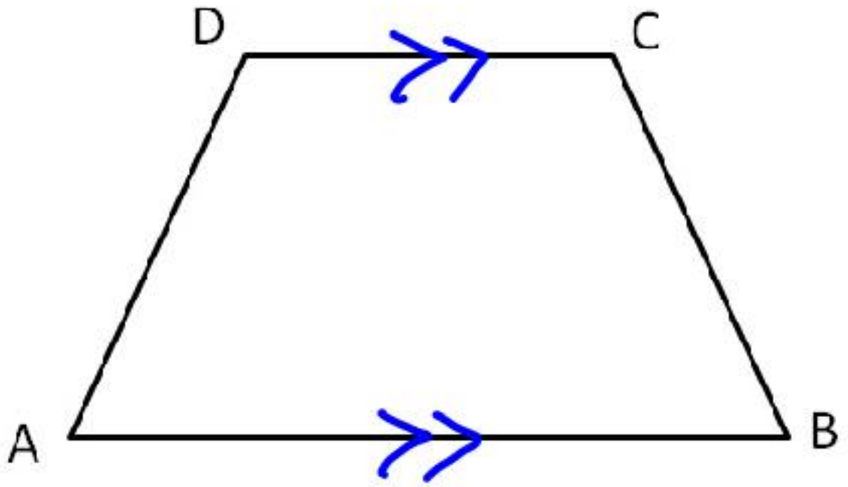
Figure formed by joining mid-points of all sides of a:

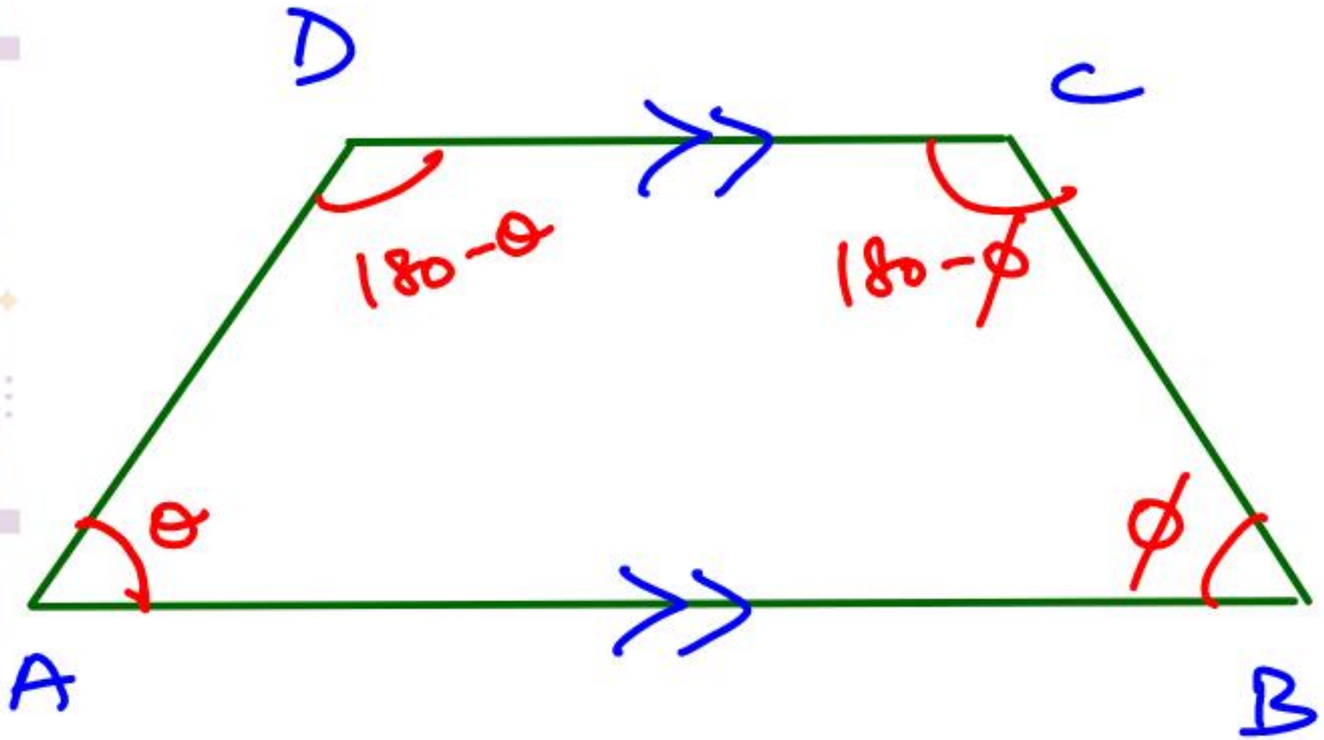
Quadrilateral	→	Parallelogram
Parallelogram	→	Parallelogram
Rhombus	→	Rectangle
Rectangle	→	Rhombus
Square	→	Square

TRAPEZIUM

Def: A quadrilateral in which one pair of side is parallel.

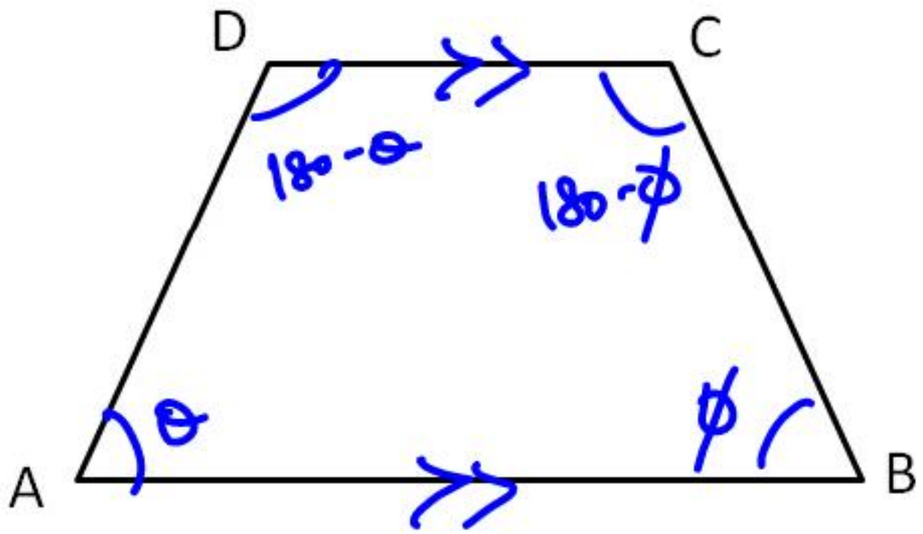


Def → Quad ABIICD


D
C
180-θ
180-ϕ
θ
ϕ
A
B

1. In a trapezium ABCD, if AB || CD, then

$\angle A + \angle D = \angle B + \angle C = 180^\circ$



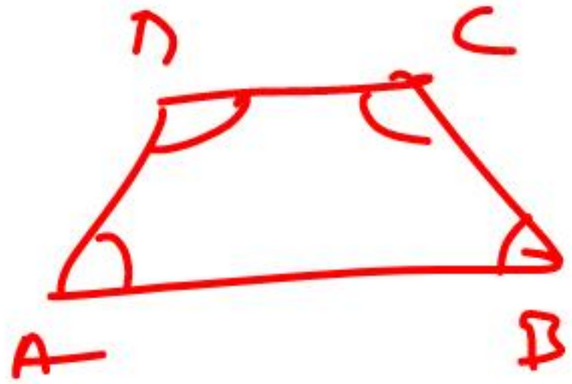
Eg9. If 4 angles of a quadrilateral are in the ratio 5:8:13:16, then what can be the name of the quadrilateral?

(a) Parallelogram

(b) Rectangle

(c) Trapezium

(d) None of these



Total

$42x$

$360°$

$\angle A$ $\angle B$ $\angle C$ $\angle D$

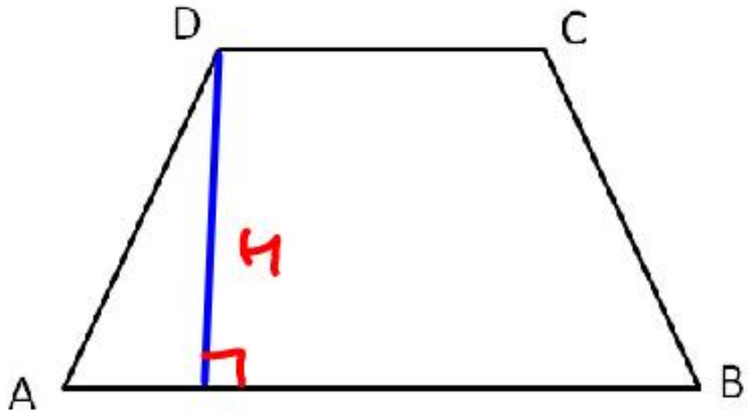
$5x$ $8x$ $13x$ $16x$

$180°$

$180°$

2. Area of trapezium = ½ × (Sum of parallel sides) × Distance between them

= ½ × (AB + CD) × H



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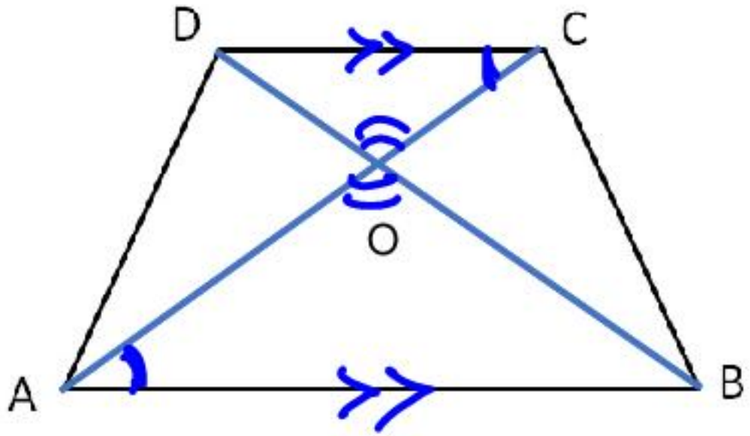
V. Imp

3. If diagonals AC and BD of a trapezium intersect each other at O, where AB| | CD, then ΔAOB ~ ΔCOD.



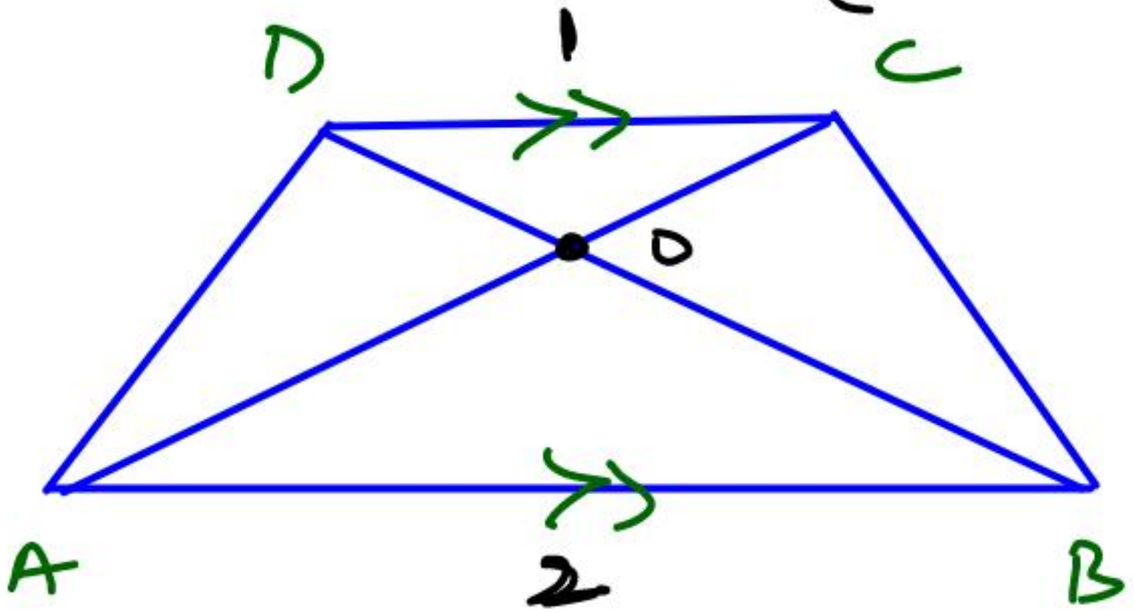
$\Delta AOB \sim \Delta COD$

* $\frac{AO}{CO} = \frac{BO}{DO} = \frac{AB}{CD}$

* $\frac{\text{area of } \Delta AOB}{\text{area of } \Delta COD} = \left(\frac{AB}{CD}\right)^2$

Eg10. In a trapezium ABCD (AB || CD), diagonals AC & BD intersect each other at O and AB = 2 CD.

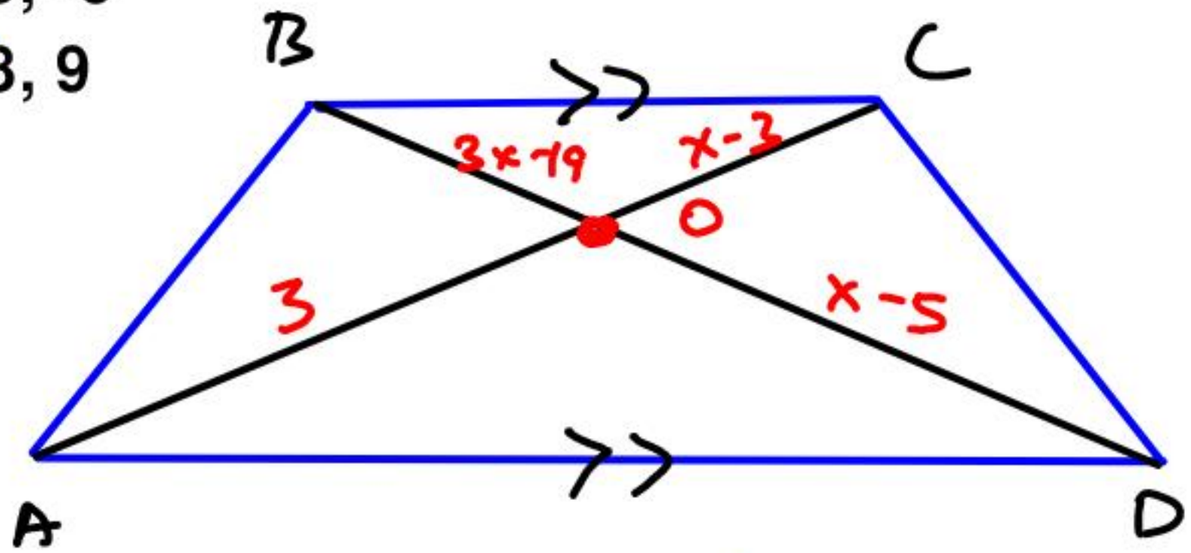
Find : $\frac{\textit{Area of } \triangle AOB}{\textit{Area of } \triangle COD}$.



Eg11. ABCD is a trapezium where AD || BC. The diagonals AC and BD intersect each other at a point O. If AO = 3, CO = x – 3, BO = 3x – 19 and DO = x – 5, the value of x is:

(a) -8, 9 (b) 8, -9

(c) -8, -9 (d) 8, 9



$\frac{AO}{CO} = \frac{DO}{BO}$

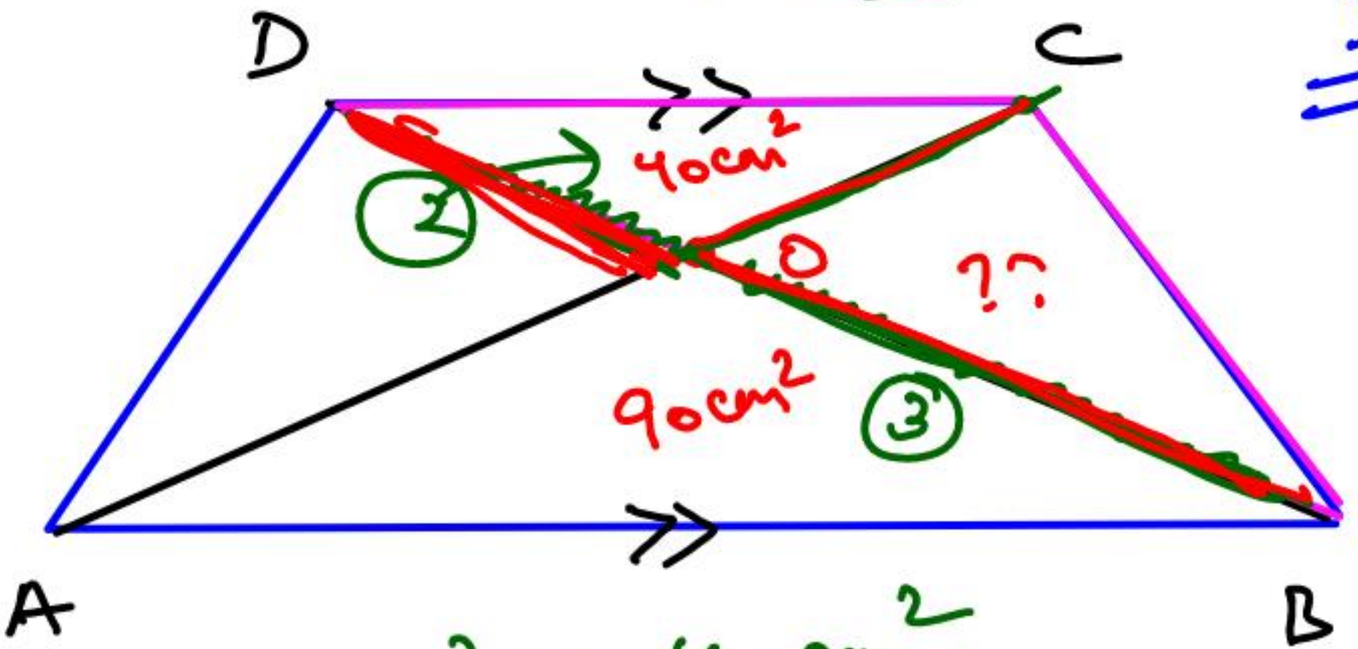
$\frac{3}{x-3} = \frac{x-5}{3x-19}$

$9x - 57 = x^2 - 8x + 15$

$x^2 - 17x + 72 = 0$

$x = 8, 9$

Eg12. ABCD is a trapezium, AB || CD and AC and BD intersect each other O. If area of $\Delta AOB = 90\ cm^2$ and area of $\Delta COD = 40\ cm^2$. Find area of ΔBOC.



$\rightarrow 60\ cm^2$

I^st

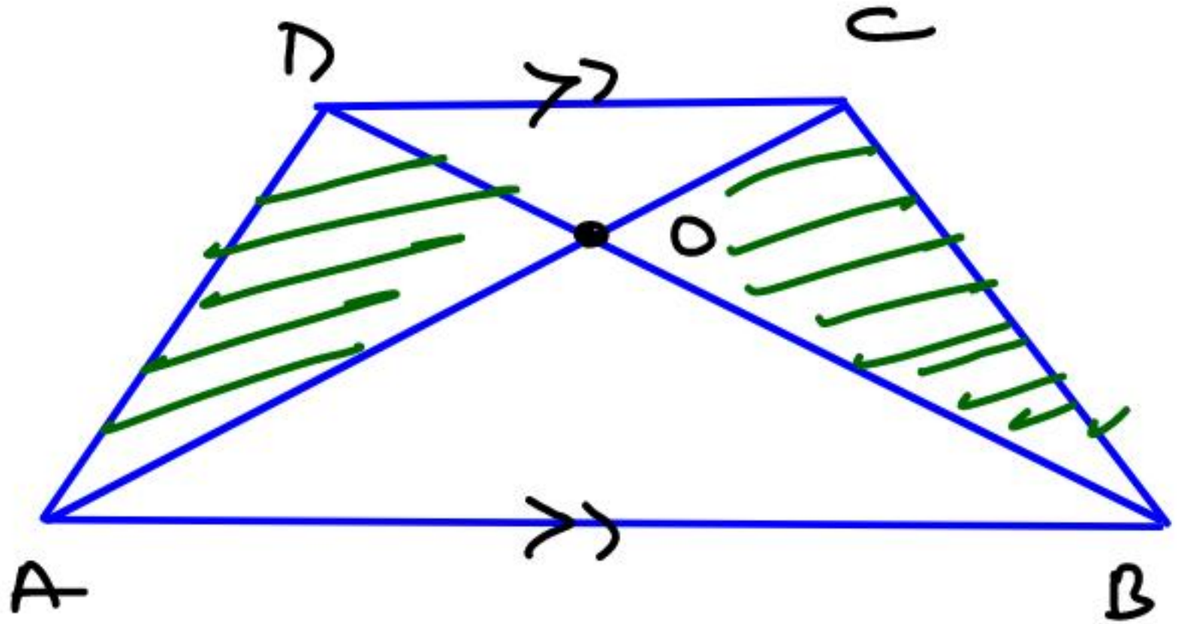
$\frac{\text{area of } \Delta AOB}{\text{area of } \Delta COD} = \frac{90}{40}$

$\frac{(BO)^2}{(DO)^2} = \frac{9}{4}$

$\frac{BO}{DO} = \frac{3}{2}$

$2 \rightarrow 40\ cm^2$

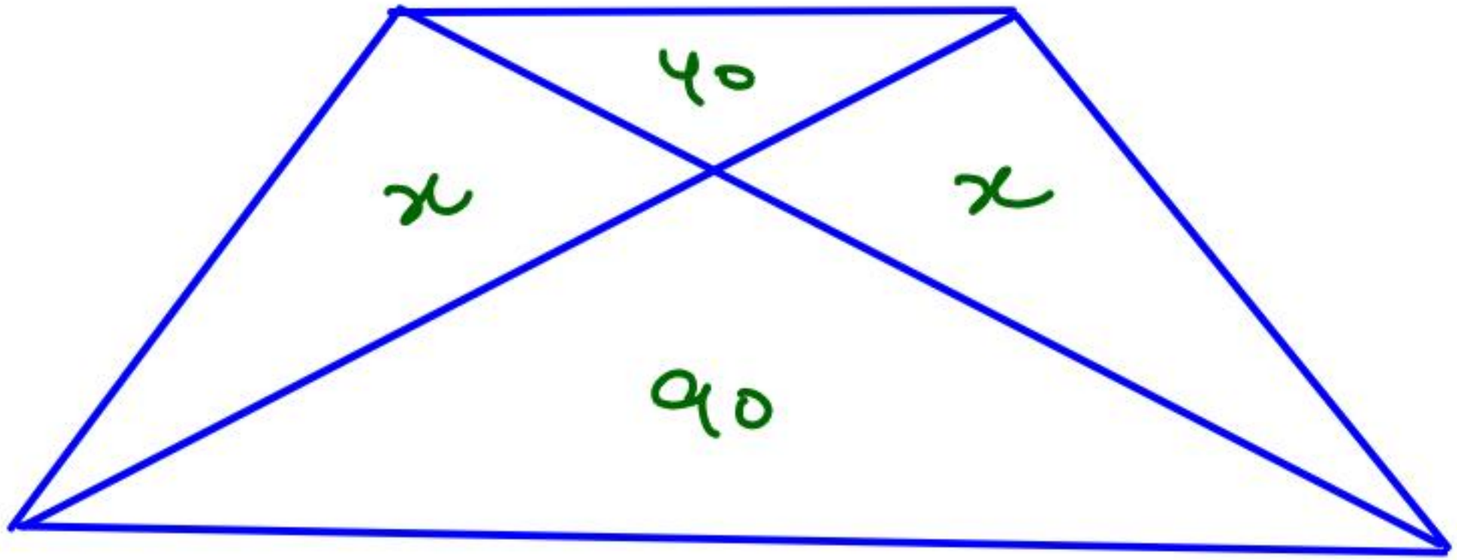
$3 \rightarrow 60\ cm^2$



$$\text{ar}(\Delta ABD) = \text{ar}(\Delta ABC)$$

$$-\Delta AOB \qquad -\Delta AOB$$

$$\Delta AOD = \Delta BOC$$

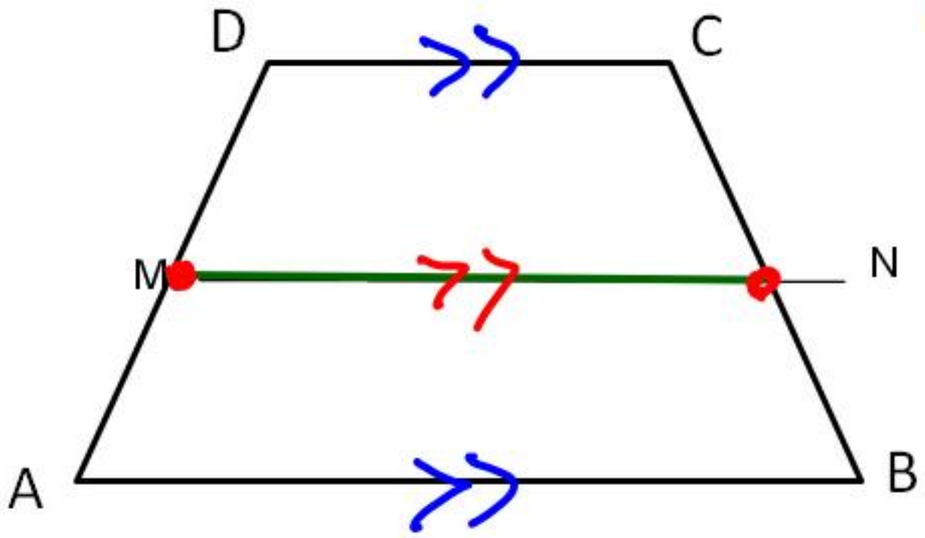


$$x^2 = 90 \cdot 40$$

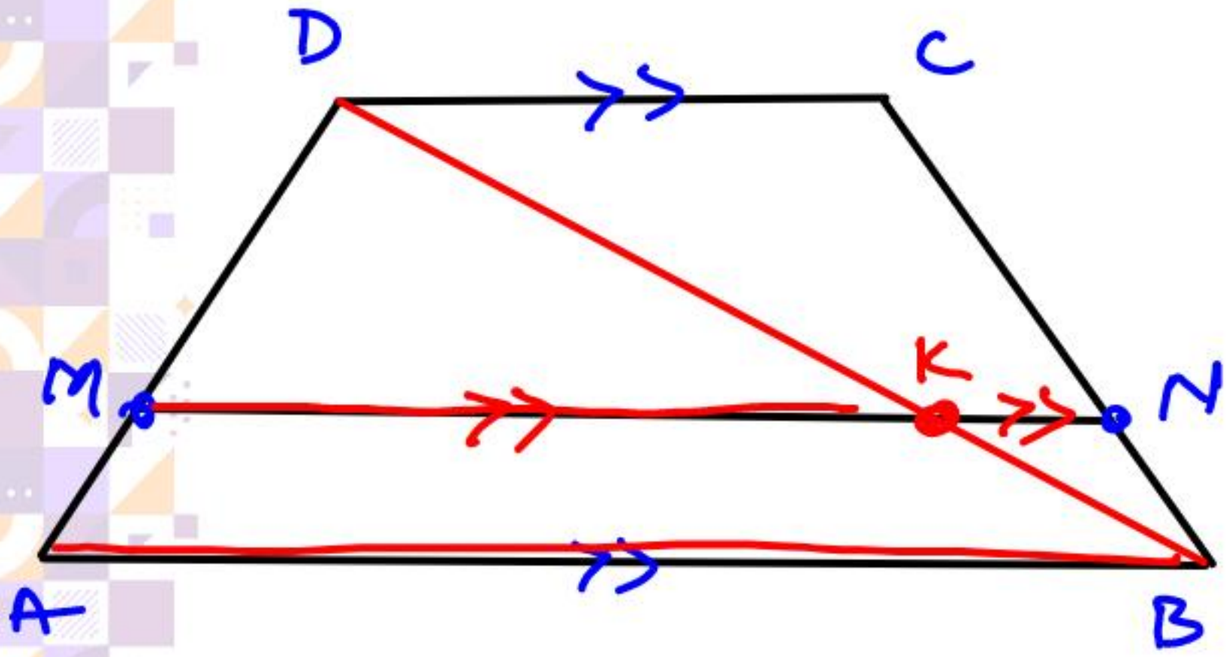
$$x = 60 \text{ cm}^2$$

4. ABCD is a trapezium where AB || CD.

M, N are points on AD and BC in such a way that MN || AB.



then $\frac{DM}{MA} = \frac{CN}{NB}$



Given $AB \parallel CD$

$MN \parallel AB$

To prove $\frac{DM}{MA} = \frac{CN}{NB}$

Const → Join BD

Proof $\triangle DAB$

$\triangle DCB$ $KN \parallel CD$

$\triangle DMK \sim \triangle DAB$

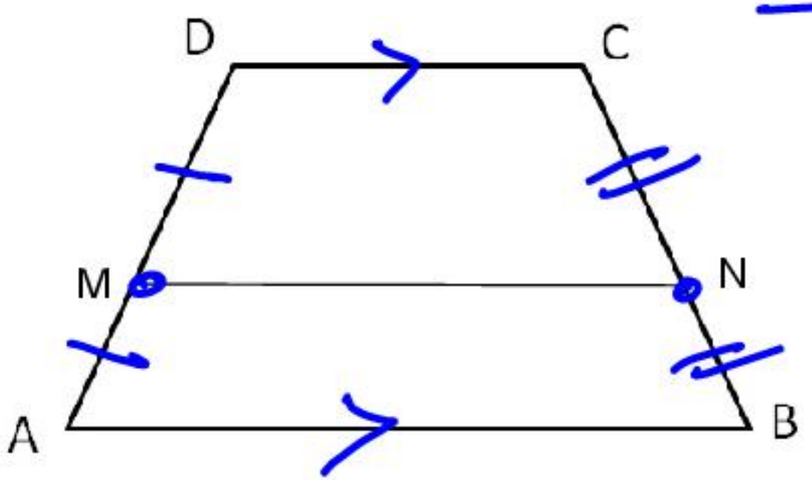
$$\frac{DM}{MA} = \frac{DK}{KB} \quad -(1)$$

$$\frac{BN}{NC} = \frac{BK}{KD}$$

$$\frac{NC}{NB} = \frac{DK}{KB} \quad -(2)$$

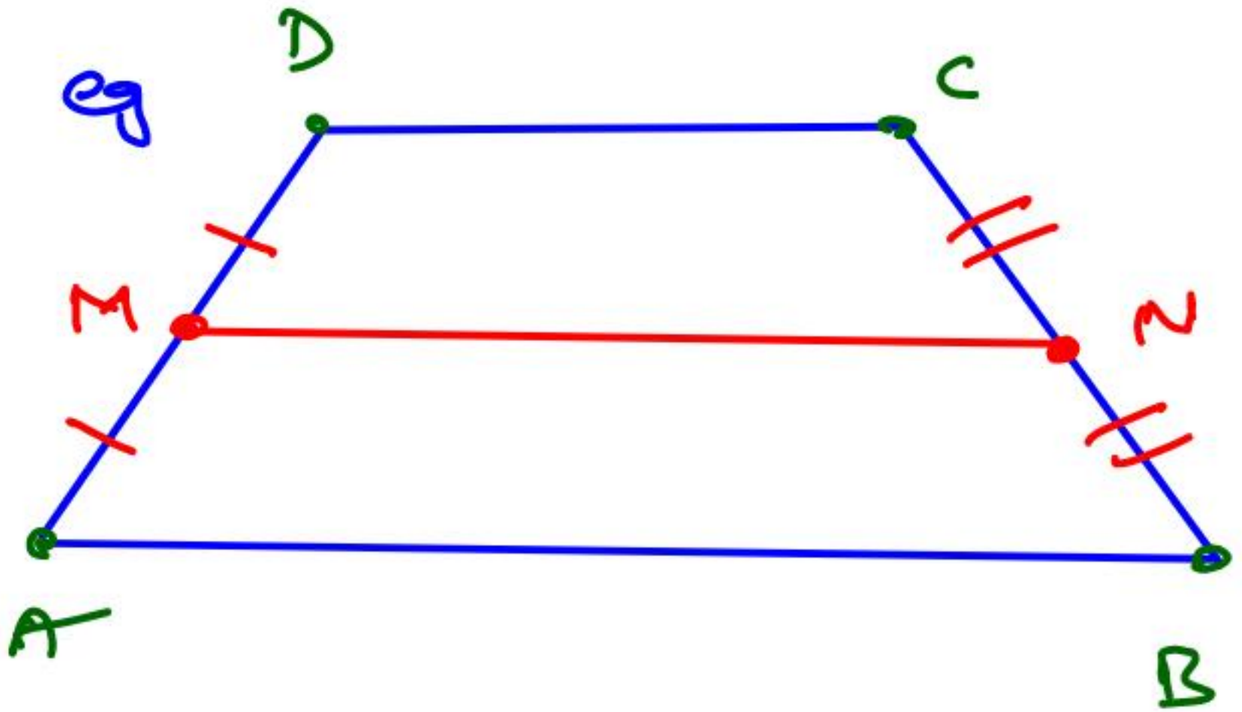
BYJU'S EXAM PREP

5. ABCD is a trapezium where AB || CD.
M, N are mid-points on AD and BC



then (i) **MN || AB**

(ii) $MN = \frac{1}{2}(AB + CD)$



If AB || CD

M, N are mid pt of

AD & BC

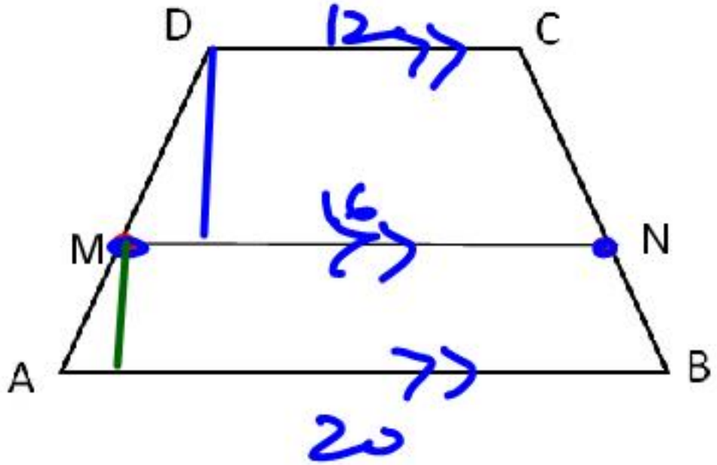
AB = 20 cm, CD = 12cm

Find MN = ??

$$MN \rightarrow \frac{1}{2}(20+12)$$

$$= 16 \text{ cm}$$

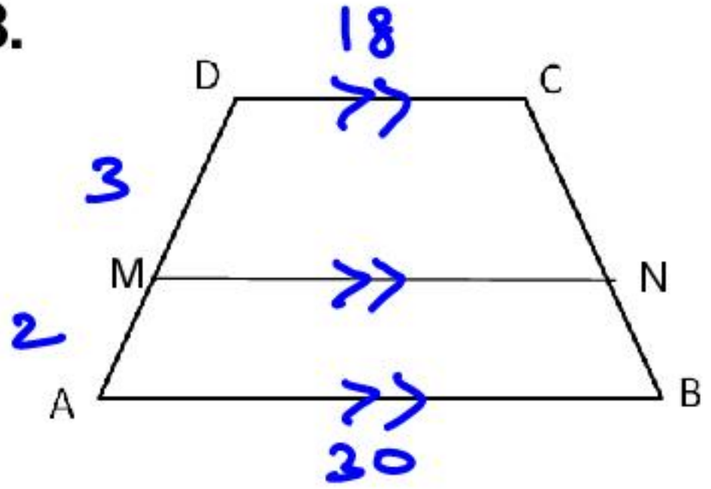
Eg13. ABCD is a trapezium where AB || CD.
M, N are mid-points on AD and BC.
If AB = 20 cm and CD = 12 cm.
Find Area of DCNM : Area of MNBA



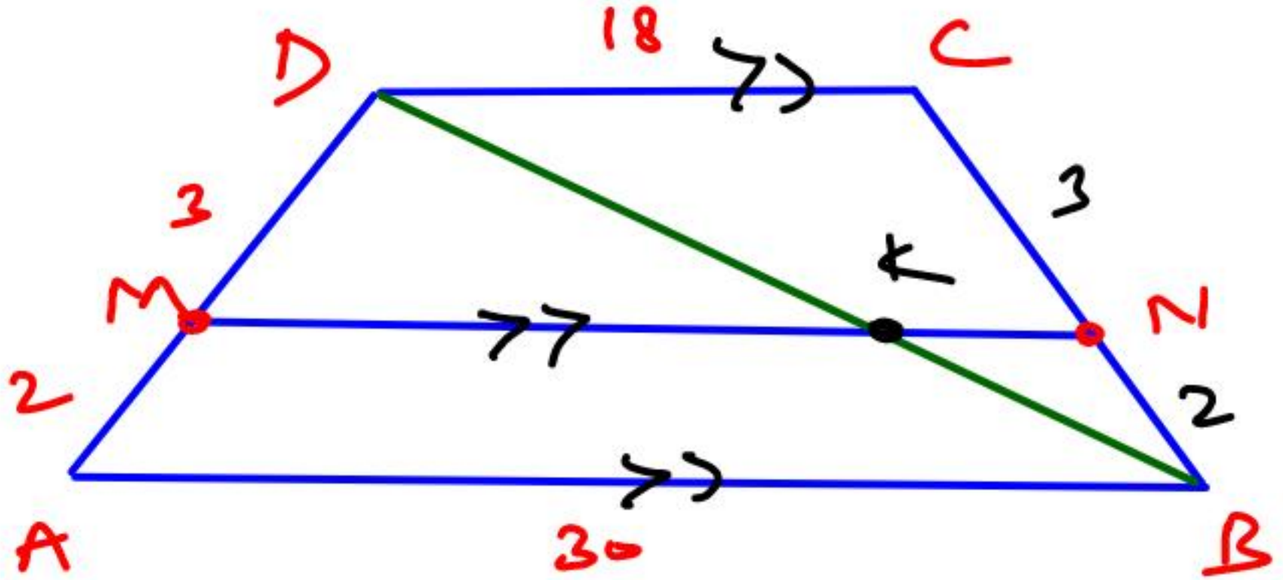
$MN = \frac{1}{2}(20+12) = 16\text{ cm}$

$$\frac{\text{area of DCNM}}{\text{area of MNBA}} = \frac{\frac{1}{2}(12+16)\cdot 4}{\frac{1}{2}(16+20)\cdot 4} = \frac{28}{36} = \frac{7}{9}$$

Eg14. ABCD is a trapezium where AB || CD. M, N are points on AD and BC in such a way that MN || AB.
If DM : MA = 3 : 2, DC = 18 cm, AB = 30 cm.
Find the value of MN.



Detailed



In ΔDAB

$$\frac{DM}{DA} = \frac{MK}{AB}$$

$$\frac{3}{5} = \frac{MK}{30}$$

$$MK = 18\text{cm}$$

In ΔDBC

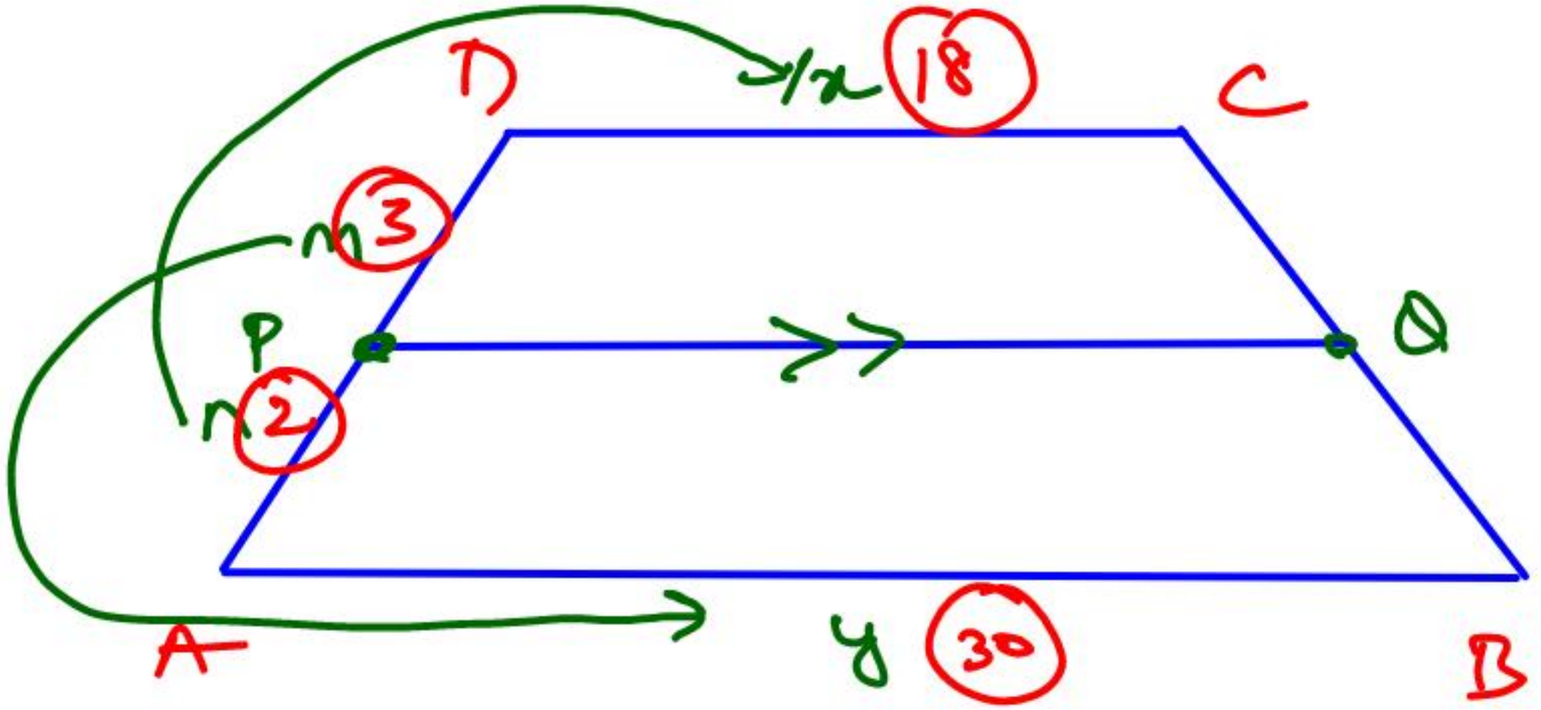
$$\frac{BN}{BC} = \frac{KN}{CD}$$

$$\frac{2}{5} = \frac{KN}{18}$$

$$KN = 7.2\text{cm}$$

$MN = 25.2\text{cm}$

IInd formula Approach



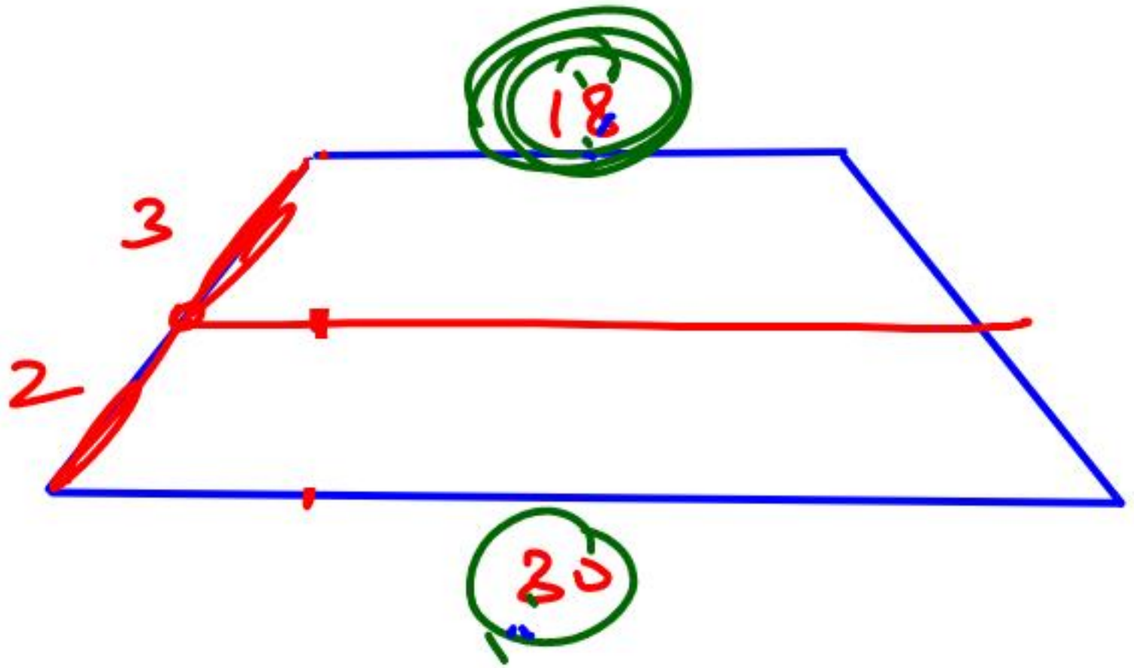
$$PQ = \frac{nx + my}{m+n}$$

$$PQ = \frac{2 \cdot 18 + 3 \cdot 30}{5}$$

$$\frac{126}{5} = 25.2\text{cm}$$

III

Logical

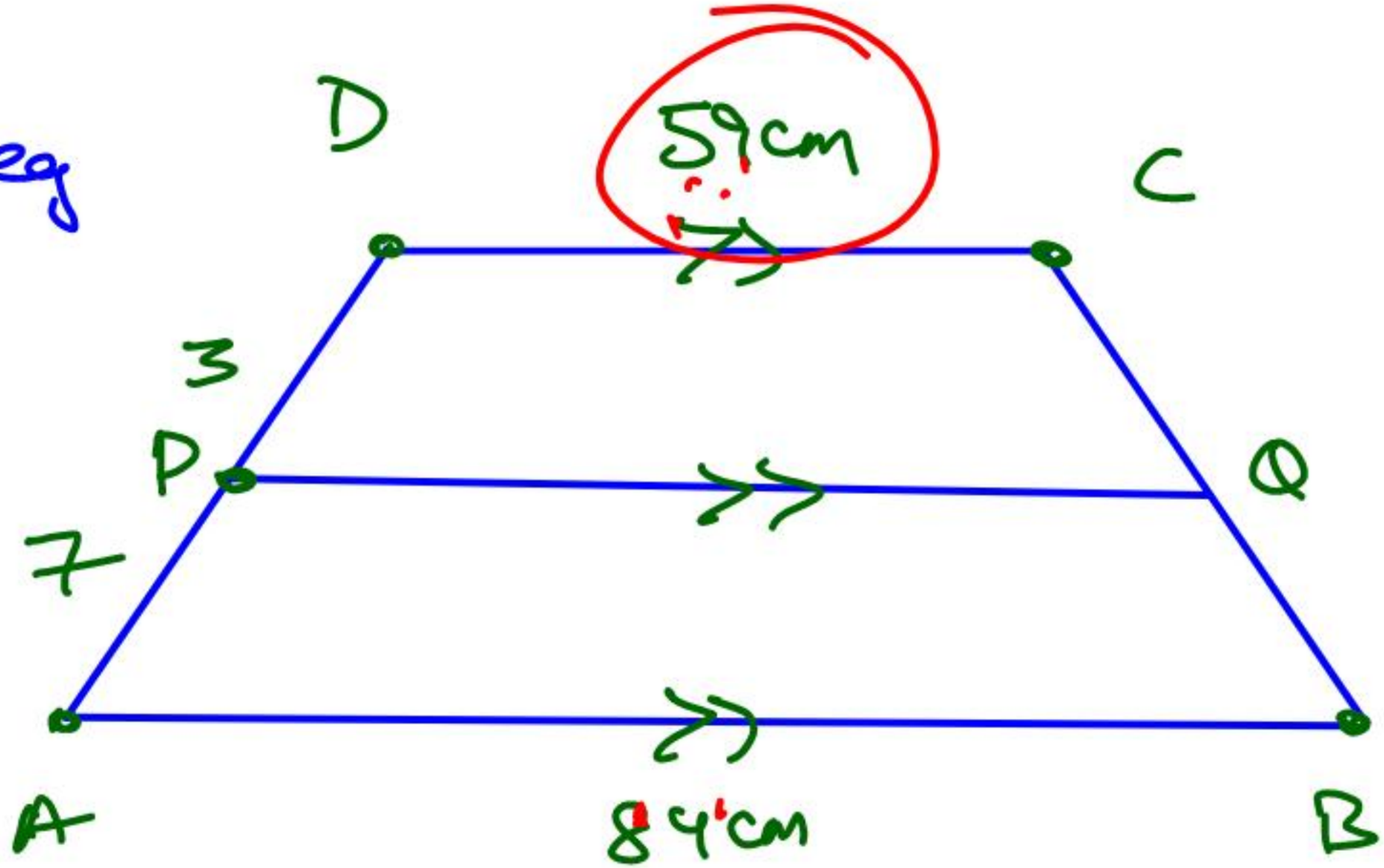


$12 \times \frac{3}{5} = 7.2$

$18 + 7.2$

$= 25.2\ cm$

eg



PQ $\rightarrow$

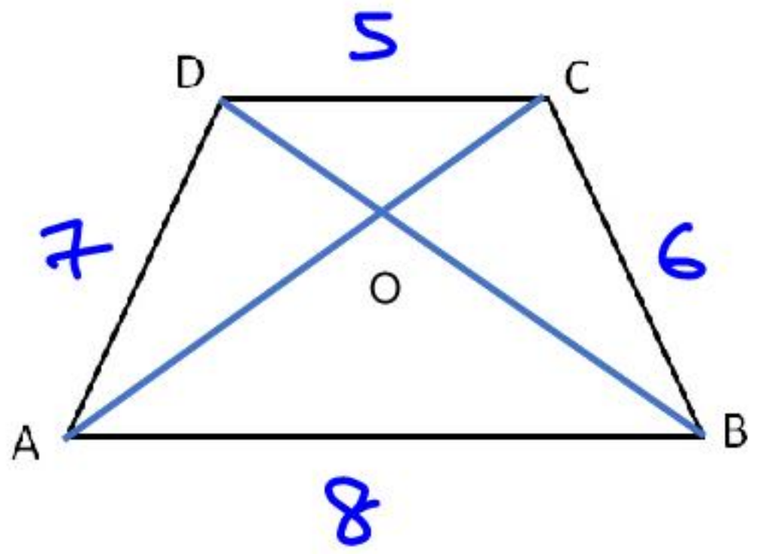
66.5 cm

$\frac{3}{10} \times 25 = 7.5$

BYJU'S EXAM PREP

6. $(AC)^2 + (BD)^2 = (AD)^2 + (BC)^2 + 2(AB)(CD)$

Sum of square of diagonals = Sum of squares of non-parallel sides + 2 (product of parallel sides)



$AC^2 + BD^2 = ??$

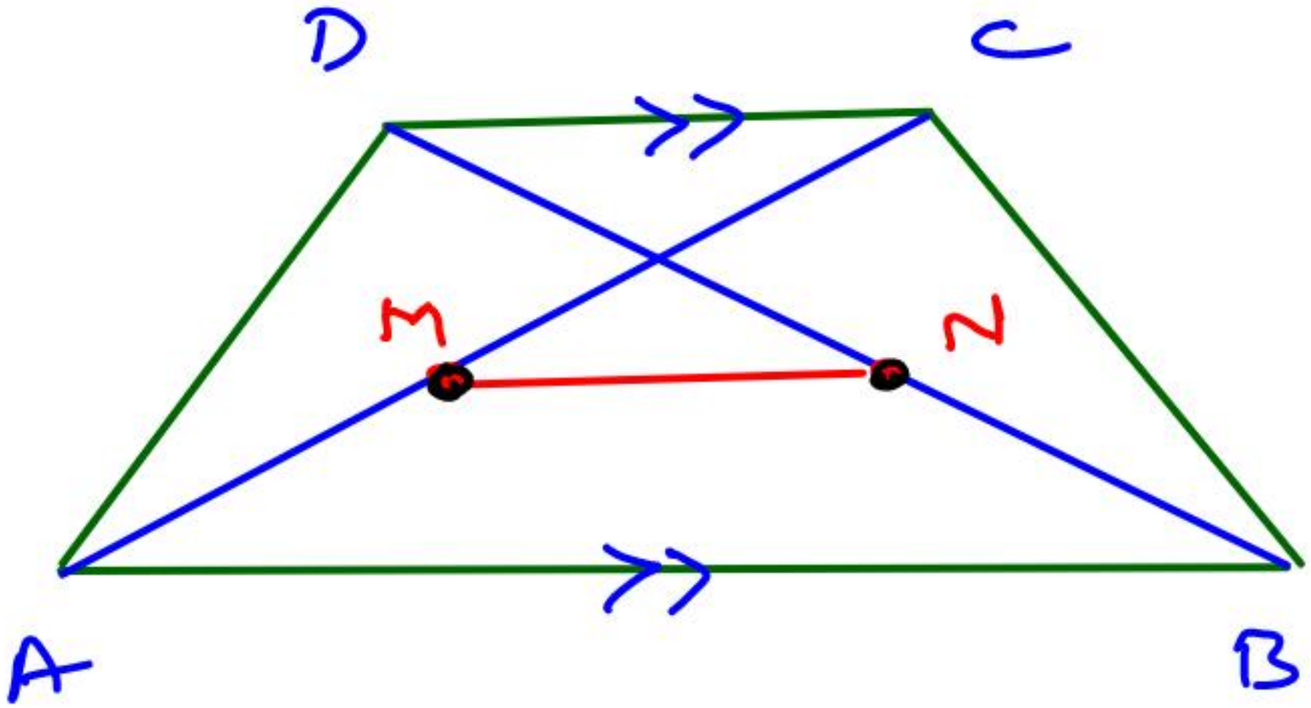
$7^2 + 6^2 + 2 \cdot 8 \cdot 5$

$85 + 80$

$= 165$

7. ABCD is a trapezium, where AB || CD.
M, N are mid-points of AC and BD,

then $MN = \frac{1}{2}|AB - CD|$



MN=

eg ABCD is a trapezium where AB || CD

& M, N are mid pts of AC & BD

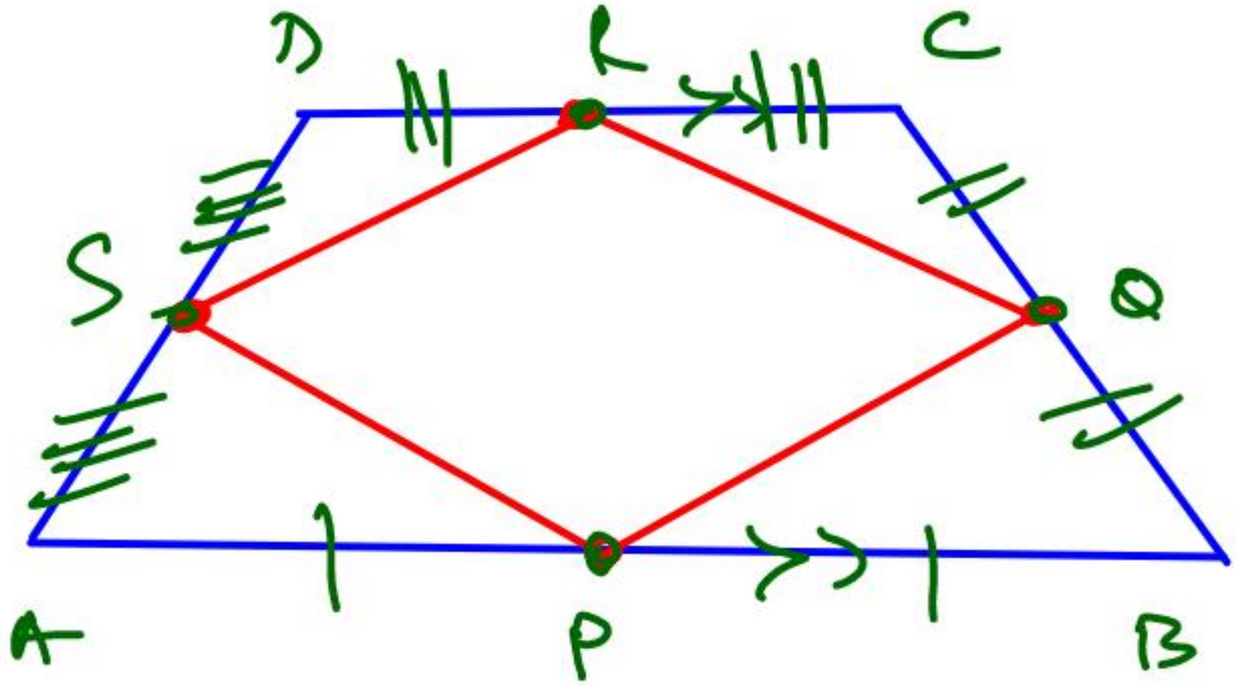
Find MN where AB = 12cm

CD = 8cm

$$MN = \frac{1}{2}(AB - CD)$$

$$= \frac{1}{2}(4) = \underline{\underline{2\text{cm}}}$$

8. Figure formed by joining mid-point of all sides of the trapezium is a parallelogram.



If ABCD is a trapezium

AB || CD

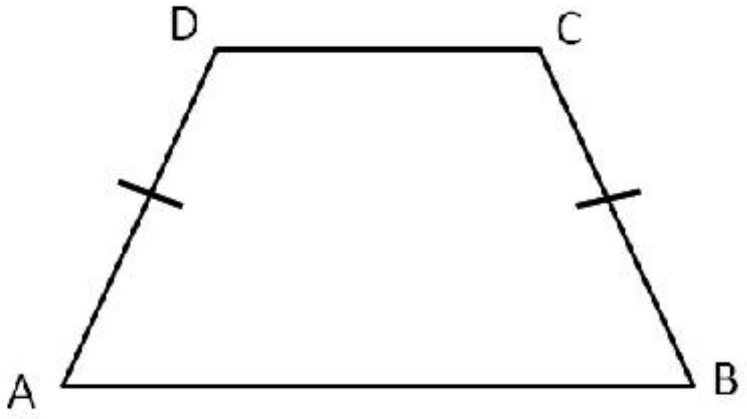
PQRS

↓

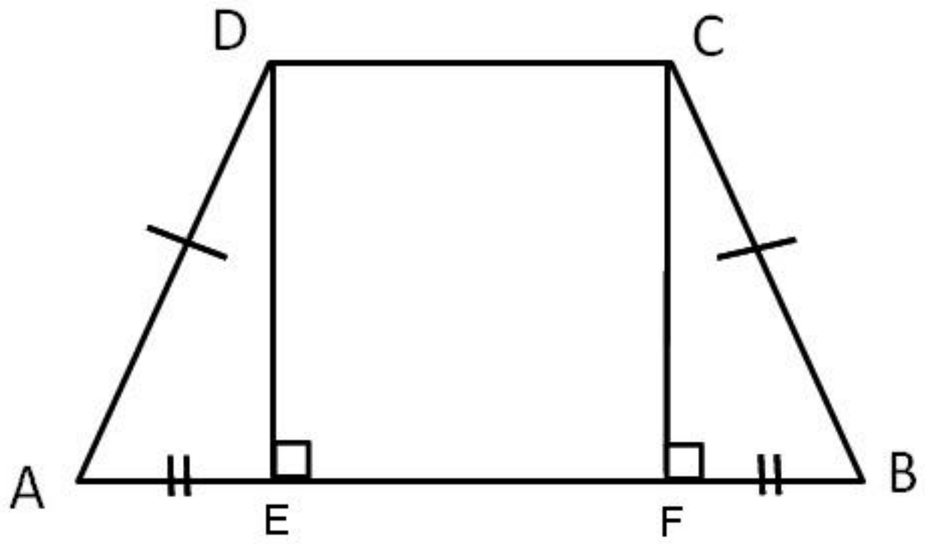
||gm

ISOSCELES TRAPEZIUM

Def: A trapezium in which non-parallel sides are equal.



AD = BC



In Isosceles trapezium where AB || CD

(1) AD = BC

(2) AE = BF

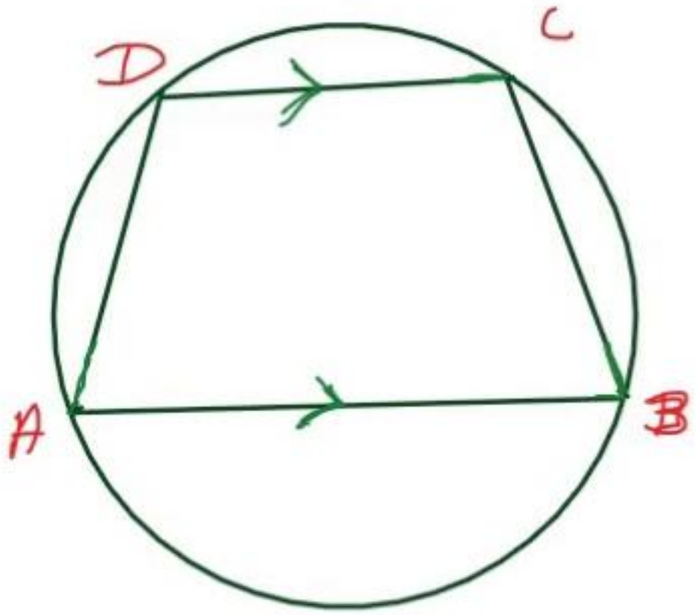
(3) AC = BD

(4) ∠D = ∠C

(5) ∠A = ∠B

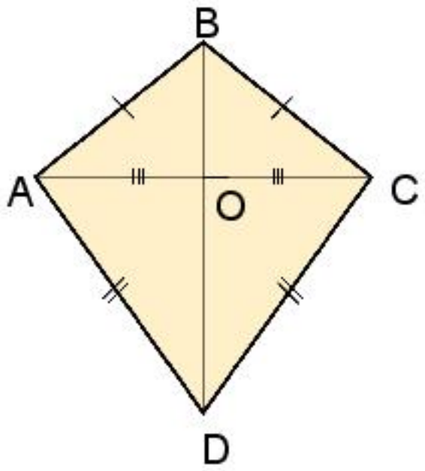
BYJU'S EXAM PREP

Cyclic trapezium is always an Isosceles Trapezium.



KITE

Kite is a quadrilateral in which two pairs of adjacent sides are of equal length and the diagonals intersect each other at right angles.



(1) AB = BC

AD = CD

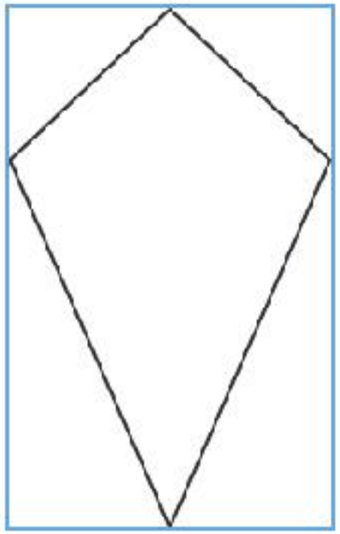
(2) $\angle AOB = \angle BOC = \angle COD = \angle DOA = 90°$

(3) AO = OC (The longer diagonal bisects the shorter diagonal.)

(4) $\angle A = \angle C$

$$\textit{Area of Kite} = \frac{1}{2} D_1 D_2$$

Eg15. The area of the rectangle is 80 cm^2, what is the area of the kite?



Eg16. HATS is a kite with diagonals that intersect at C.
$\angle TSC = 32^\circ$. Find $\angle SHC$.