BYJU'S EXAM PREP

The Most Comprehensive Preparation App For All Exams

TRIANGLE

Part-II

Agenda : Triangles Part 2

Leftover Part of Last class

+

Similarity

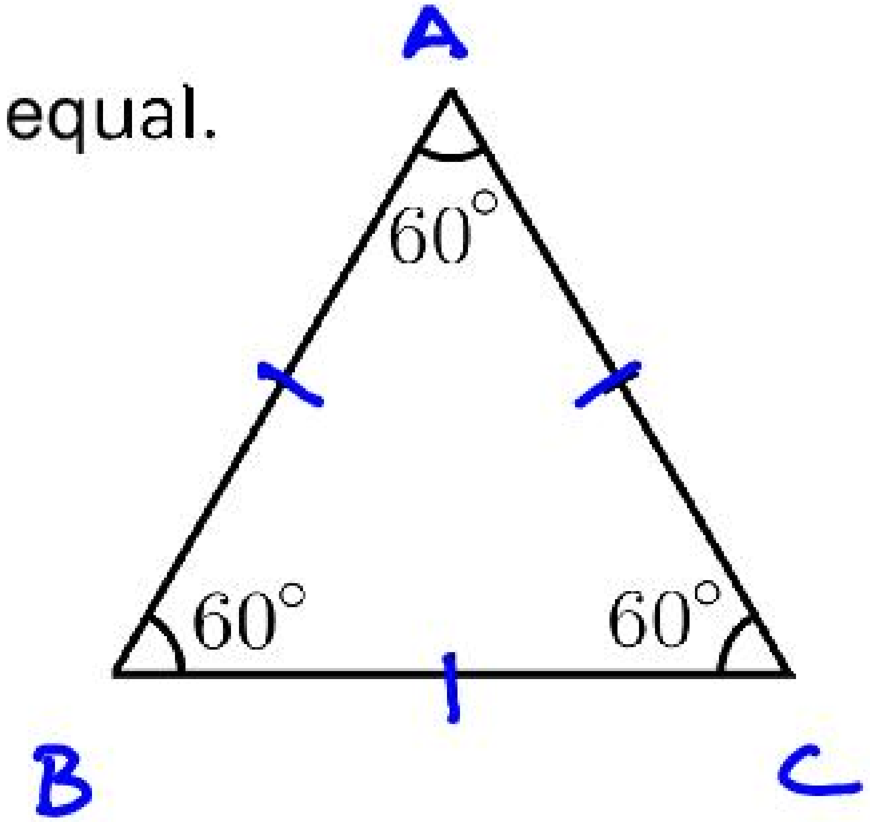
+

Congruency

$\boxed{11:02 - 12:40\,pm}$

EQUILATERAL TRIANGLE

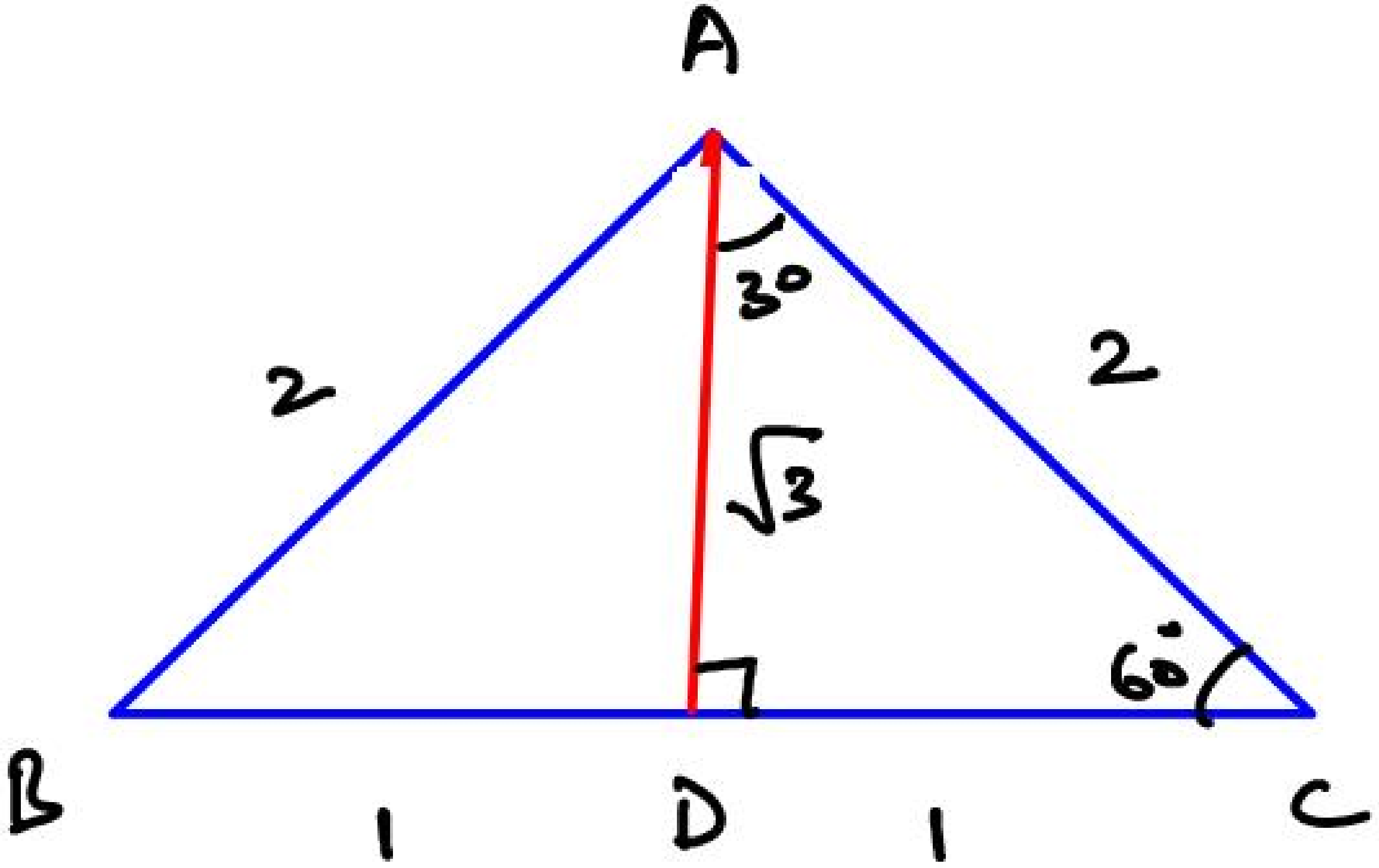
An **equilateral triangle** is a **triangle** in which all three sides are equal.



Imp

$$\textit{Height of equilateral } \Delta = \frac{\sqrt{3}}{2} \times S$$

$$\textit{Area of equilateral } \Delta = \frac{\sqrt{3}}{4} \times S^2$$



$$\frac{1}{2} \times S \times \frac{\sqrt{3}}{2} S$$

Side $\longrightarrow$ 2

Height $\longrightarrow$ $\sqrt{3}$

$$\boxed{\text{Height} \longrightarrow \frac{\sqrt{3}}{2} s}$$

Eg. If height of equilateral triangle = 12 cm.
Find area of equilateral triangle.

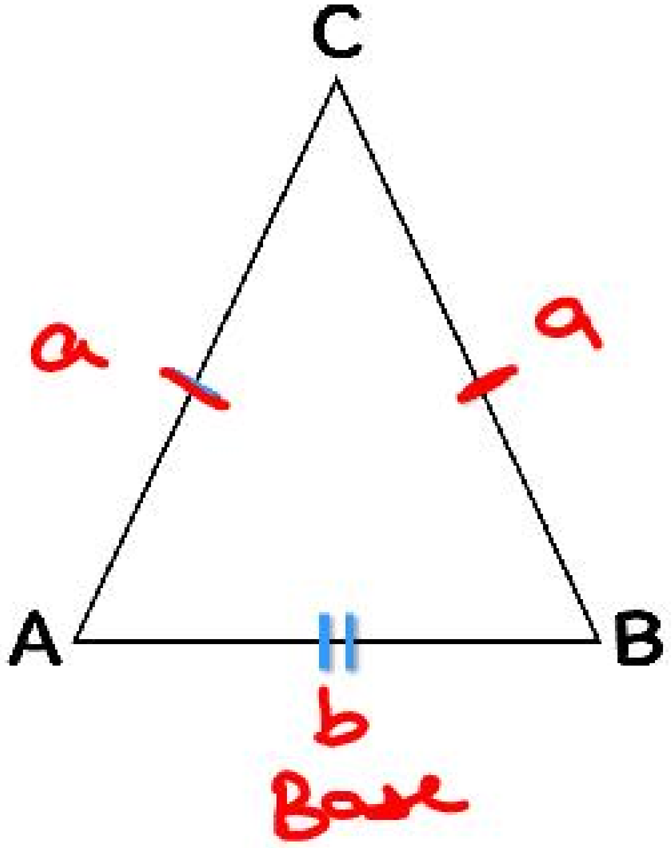
$$\frac{\sqrt{3}}{2} \cdot S = 12 \qquad S = 8\sqrt{3}$$

$$\text{Area} = \frac{\sqrt{3}}{\cancel{4}} \cdot \overset{2}{\cancel{8}}\sqrt{3} \cdot 8\sqrt{3}$$

$$\underline{\underline{48\sqrt{3}\ cm^2}}$$

ISOSCELES TRIANGLE

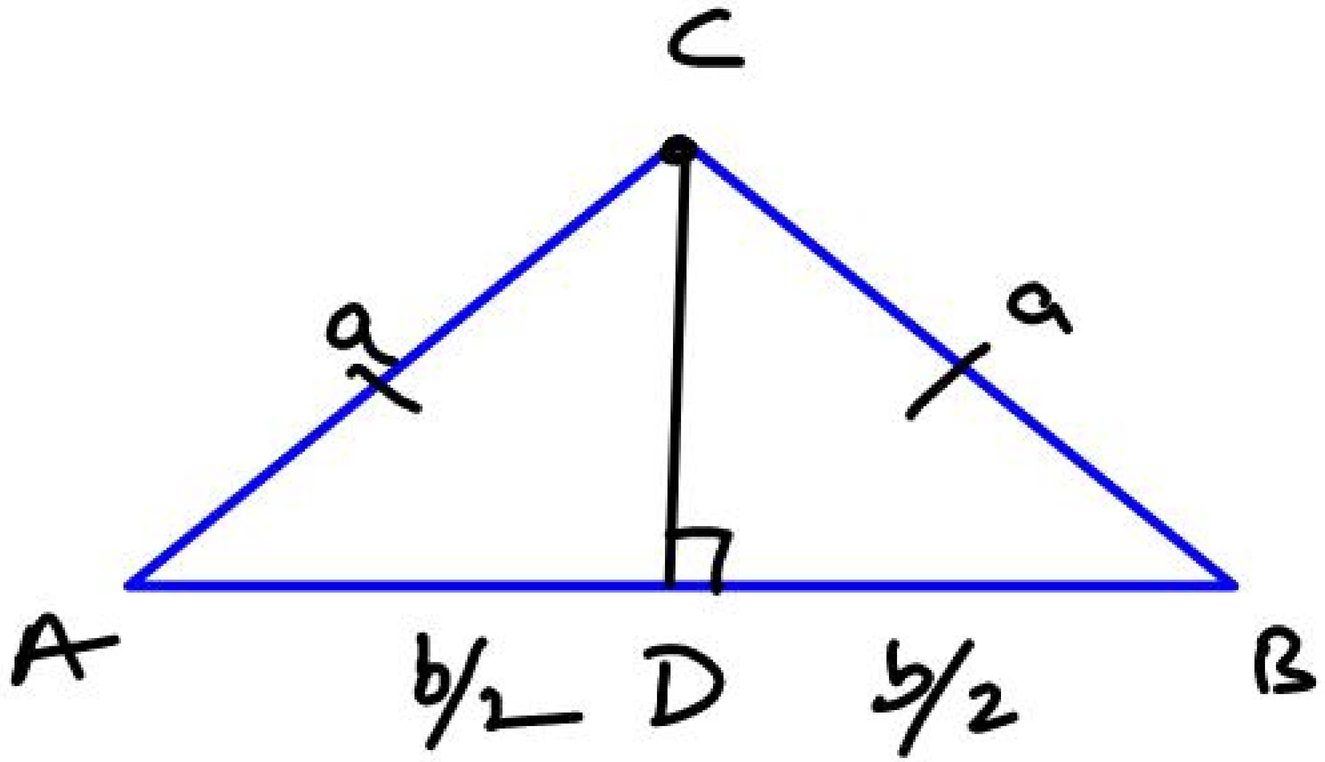
An **isosceles triangle** is a **triangle** that has two sides of equal length.



$$\boldsymbol{Area\ of\ isosceles\ \Delta = \frac{b}{4}\sqrt{4a^2 - b^2}}$$

$$\frac{1}{2} \times b \times \frac{\sqrt{4a^2-b^2}}{2}$$

Where, b is base of isosceles Δ.
and a is length of equal sides.



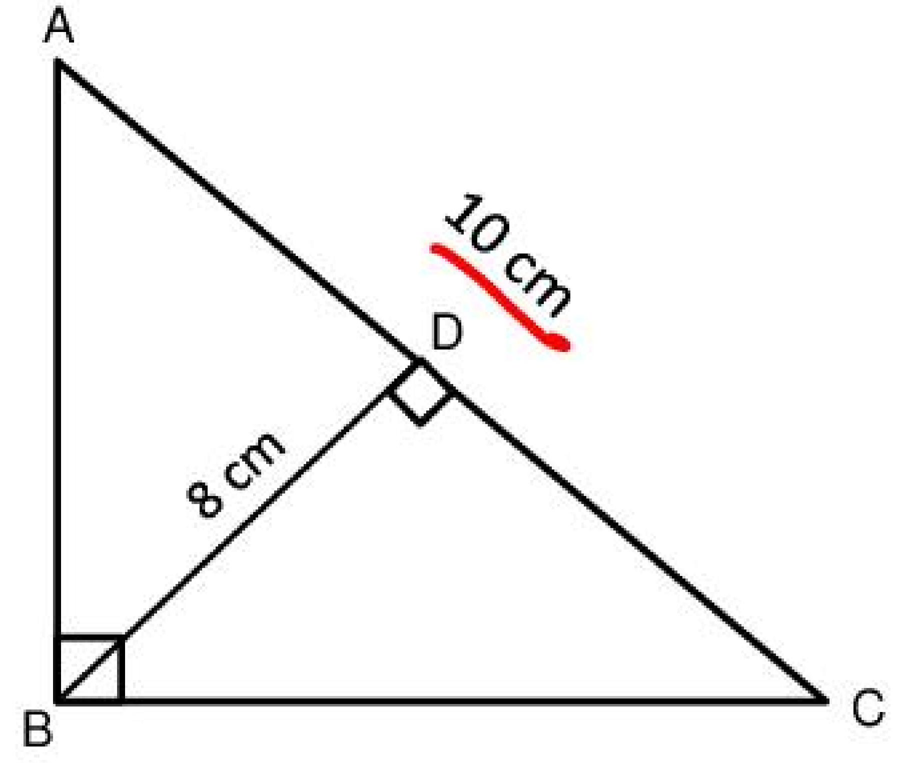
$$CD^2 + \left(\frac{b}{2}\right)^2 = a^2$$

$$CD^2 = a^2 - \frac{b^2}{4}$$

$$CD = \frac{\sqrt{4a^2-b^2}}{2}$$

BYJU'S EXAM PREP

Eg. In a ΔABC, AC = 10 cm ; BD = 8 cm
Find area of ΔABC.

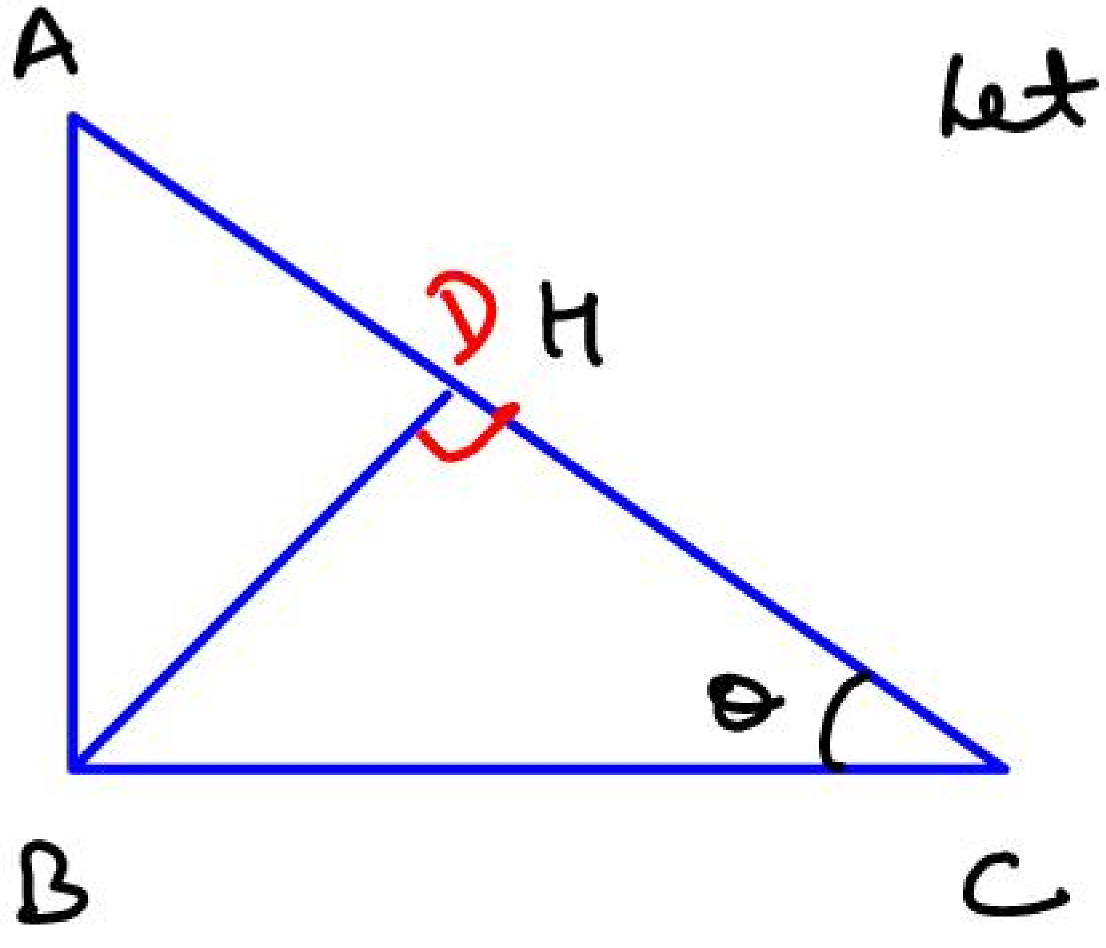


$\frac{1}{2} \times 10 \times 8 = 40 cm^2$

This is wrong

" DATA INCONSISTENT "

height can't be greater than 5



Let $AC = H$

$\sin\theta = \dfrac{AB}{H}$ $\qquad$ $\cos\theta = \dfrac{BC}{H}$

$AB = H\sin\theta$ $\qquad$ $BC = H\cos\theta$

Area of $\Delta = \dfrac{2}{2}\cdot\dfrac{1}{2} \times H\sin\theta \times H\cos\theta$

$= \dfrac{H^2 \sin 2\theta}{4}$

Area of $\Delta = H \cdot BD$

$\dfrac{1}{\cancel{2}}\cancel{H}\cdot BD = \dfrac{\cancel{H^2}\sin 2\theta}{\cancel{4}\ 2}$

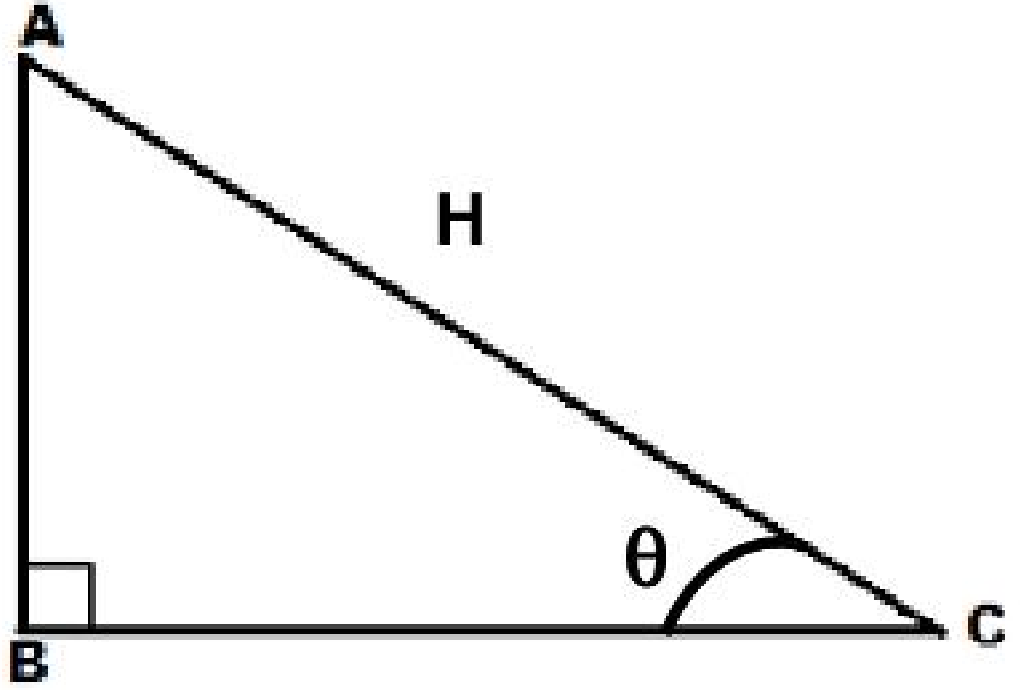
$$\boxed{(BD)_{max} \Rightarrow \dfrac{H}{2}}$$

RIGHT ANGLE TRIANGLE

$$Area\ of\ right\ angle\ \Delta = \frac{H^2}{4}\sin 2\theta$$

Where, H → Hypotenuse

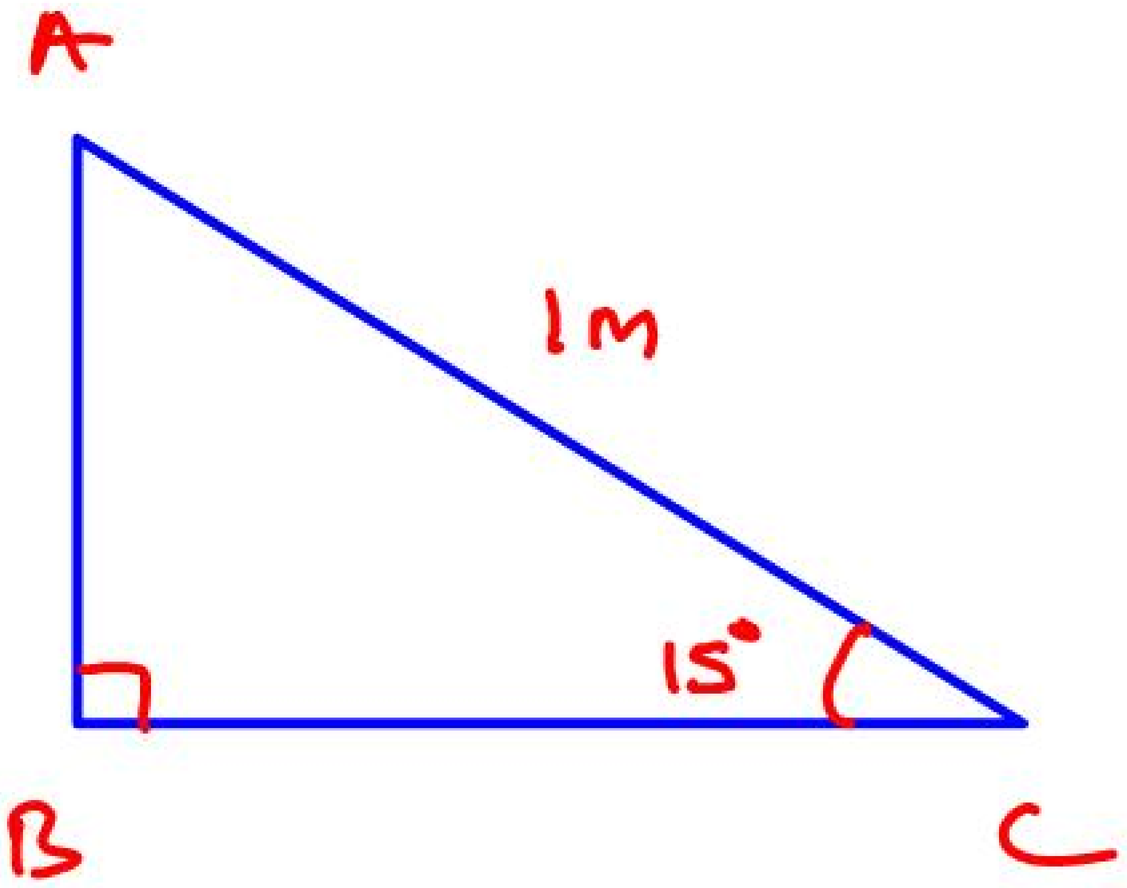
and, θ → one of the acute angle of right angle triangle.



Note : The max value of height drawn to hypotenuse is half of hypotenuse

BYJU'S EXAM PREP

v.imp eg



Find area of right angle Δ in (cm^2)

$$\text{Area} = \frac{(100)^2}{4} \cdot \sin 30$$

$$= \frac{(100)^2}{4} \cdot \frac{1}{2} \Rightarrow \underline{1250\, cm^2}$$

Eg. If hypotenuse of a right angle Δ is 10 cm. What can be its maximum area?

$$\text{Area} = \frac{H^2}{4}\sin 2\theta$$

$$\text{Max Area} \longrightarrow \frac{H^2}{4}$$

$$\frac{10^2}{4} = 25\,cm^2$$

SIMILARITY

Similarity means 'similar in terms of shape' & 'proportion in terms of size'.

Two squares are similar, when ?? Always

Two circles are similar, when ?? Always

Two line segments are similar, when ?? Always

Two polygons are similar, if

(i) their corresponding angles are equal.

(ii) the ratio of their corresponding sides are same.

Δ is a special polygon, even if, one of the condition exist then also triangles are similar.

$\Delta ABC \sim \Delta DEF$

↳ Symbol of similarity

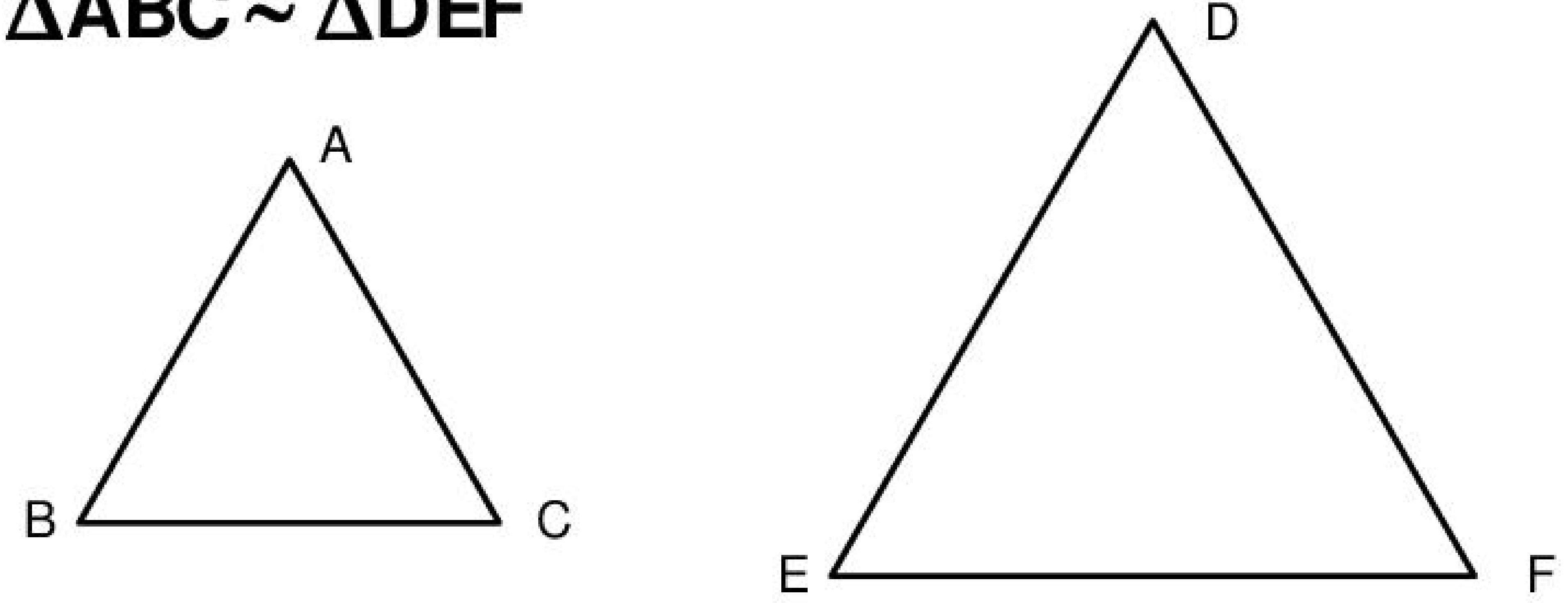
* $\angle A = \angle D, \quad \angle B = \angle E, \quad \angle C = \angle F$

** $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

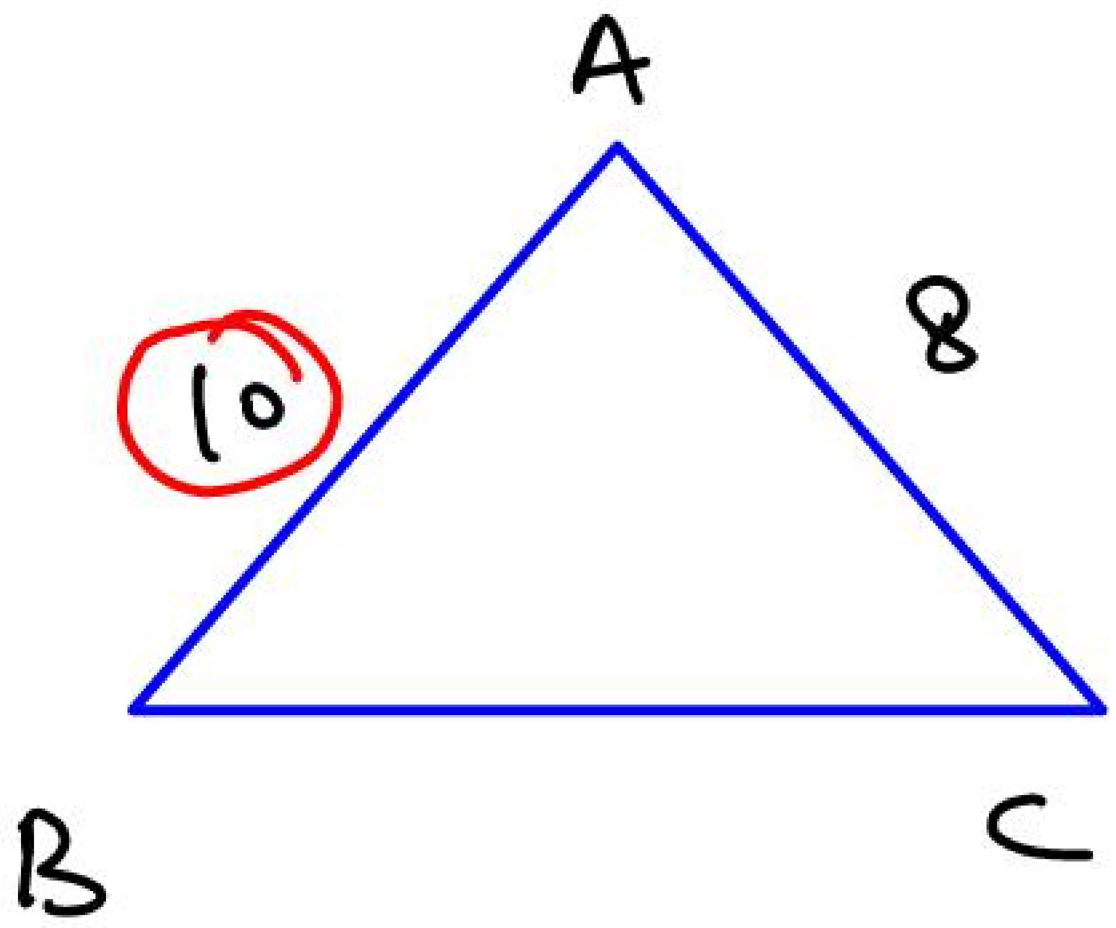
$\Delta ABC \sim \Delta DEF$

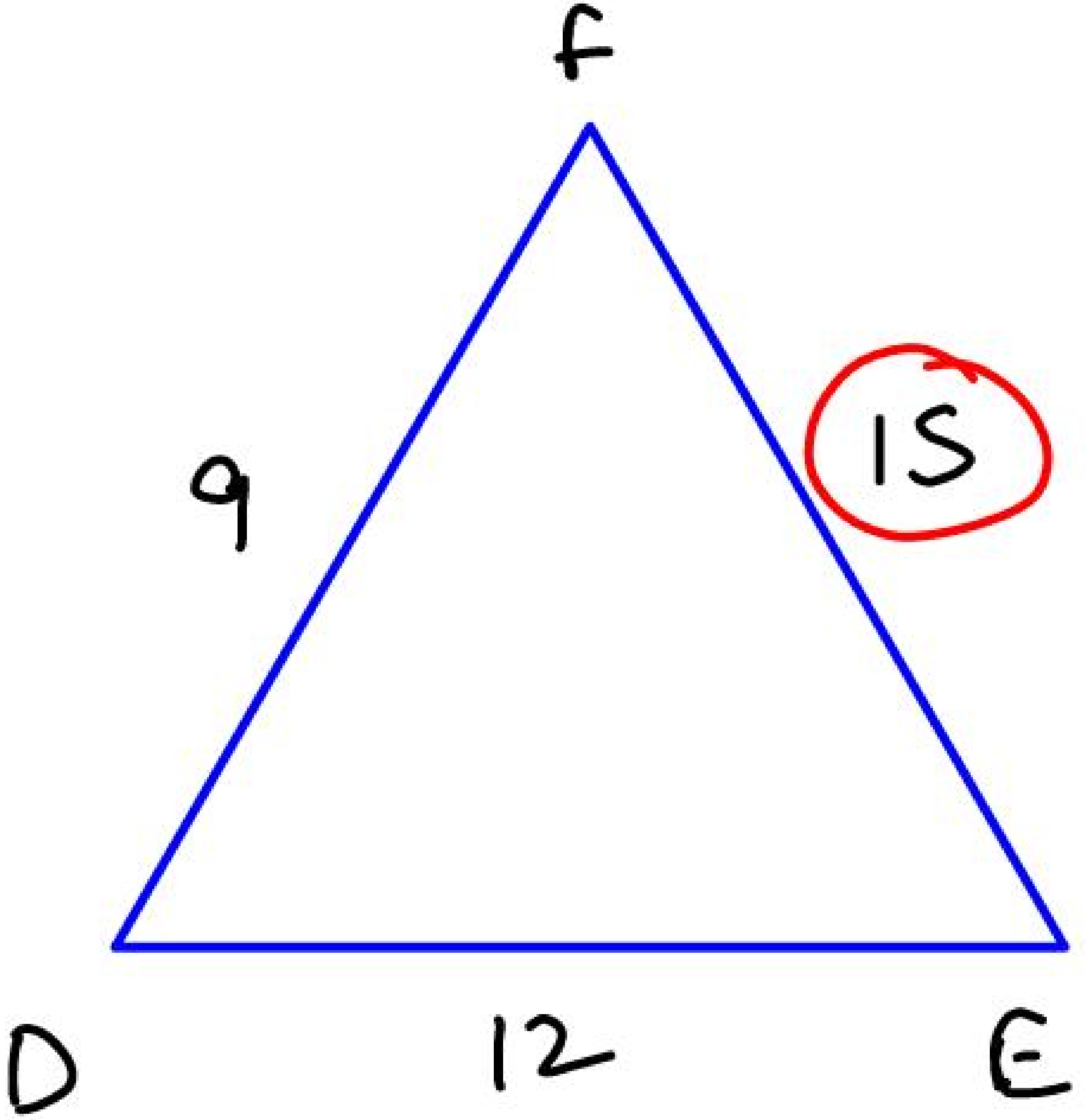
(i) $\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$

(ii) $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = K$



eg





$\Delta ABC \sim \Delta EFD$

$$\frac{\cancel{10}^{\,2}}{{}_3\cancel{15}} = \frac{BC}{9} = \frac{\cancel{8}^{\,2}}{\cancel{12}_{\,3}}$$

$BC = 6\,cm$

Eg1. $\Delta ABC \sim \Delta PQR$

If AB = 20 cm, BC = 12 cm, PQ = 8 cm

Find QR = ??

$$\frac{20}{8} = \frac{12}{QR}$$

$$QR = \frac{12 \cdot \cancel{8}^{2}}{\cancel{20}_{5}} \Rightarrow \underline{\underline{4.8\ cm}}$$

If two triangles are similar, then the ratio of their corresponding sides is equal and let the ratio be K

then, ratio of perimeter = K
altitudes = K
medians = K
length of angle bisector = K
inradius = K
circumradius = K

Ratio of Areas = K^2

BYJU'S EXAM PREP

Eg2. $\Delta ABC \sim \Delta DEF$

If $BC = 3$ cm, $EF = 4$ cm, Area of $\Delta DEF = 96\ cm^2$

Find area of ΔABC.

$$\frac{\text{area of } \Delta ABC}{\text{area of } \Delta DEF} = \left(\frac{BC}{EF}\right)^2$$

$$\frac{\text{area of } \Delta ABC}{\cancel{96}\ 6} = \frac{9}{\cancel{16}}$$

$$\text{area of } \Delta ABC = \underline{\underline{54\ cm^2}}$$

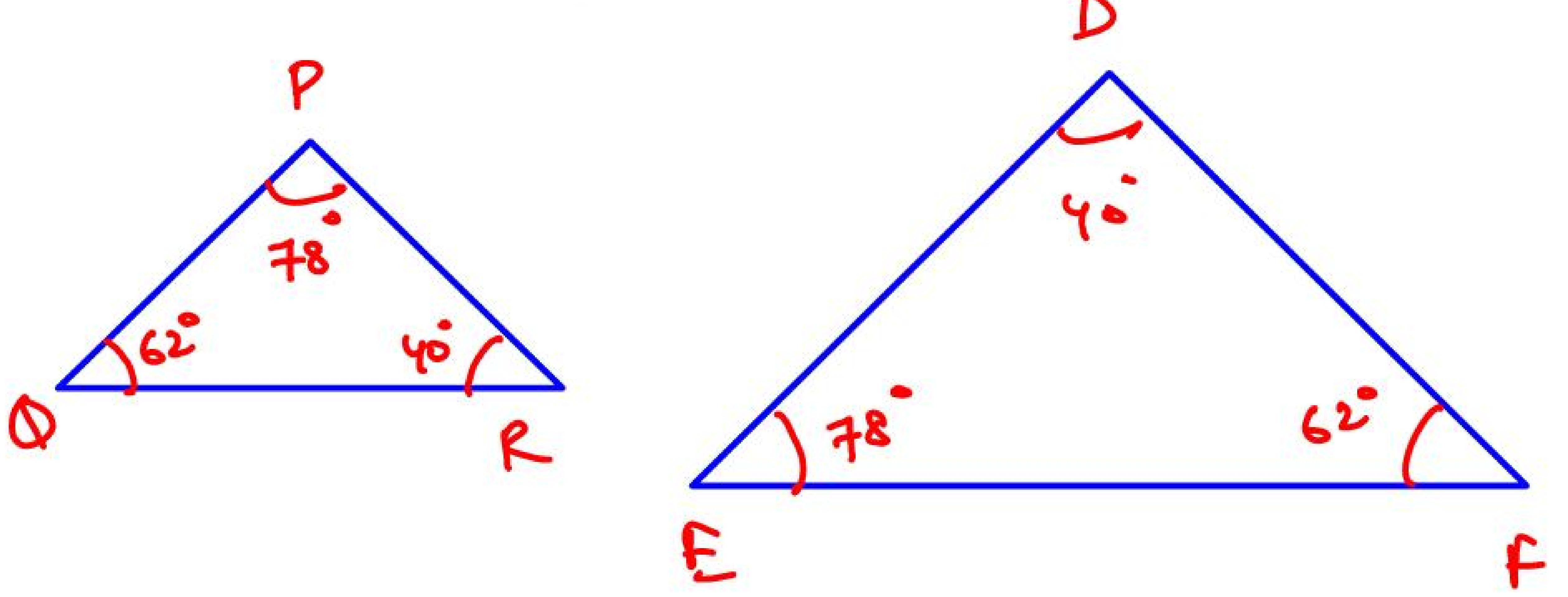
CONDITIONS OF SIMILARITY

(1) AAA or AA (Angle – Angle)

(2) SSS (Side – Side – Side)

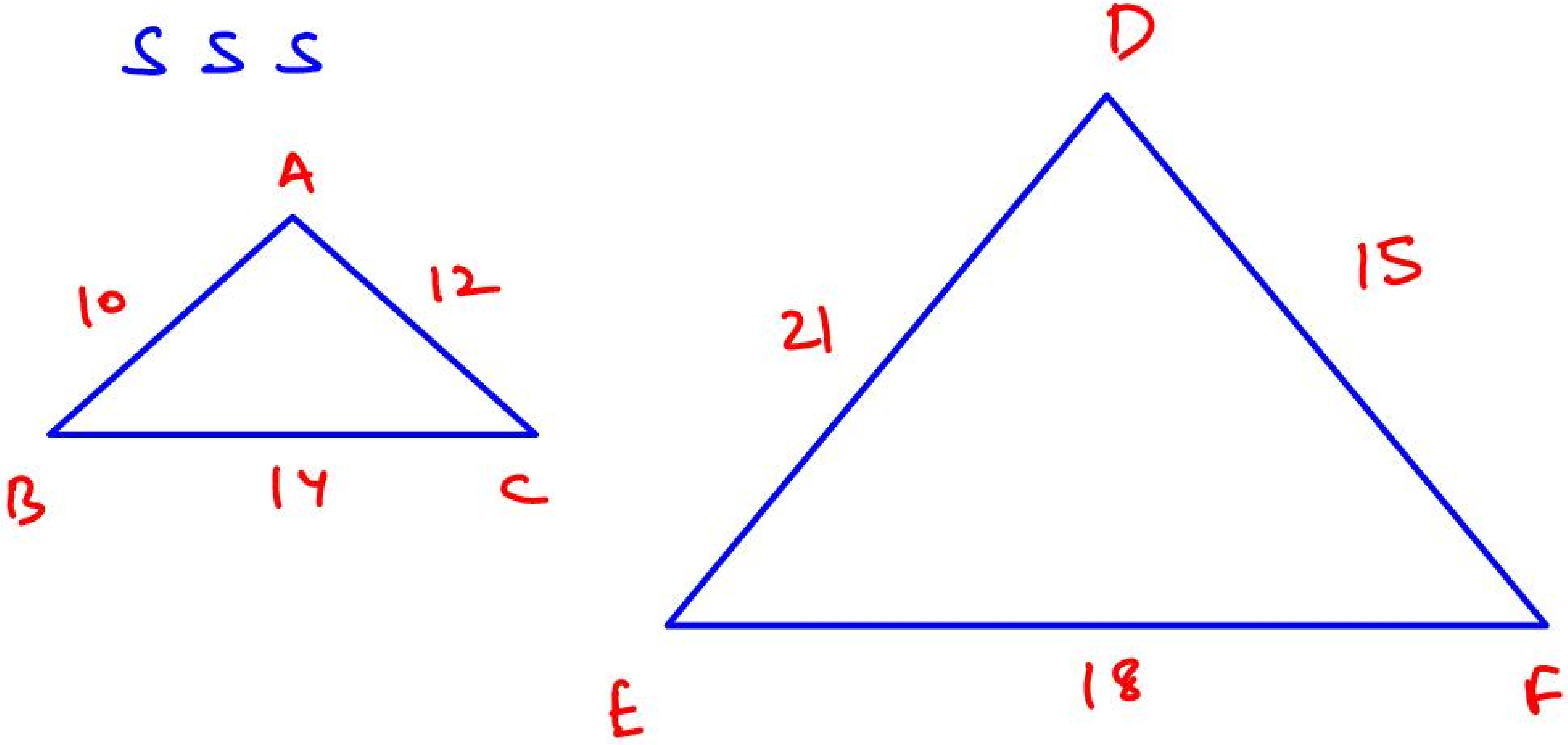
(3) SAS (Side – Angle – Side)

I A A A



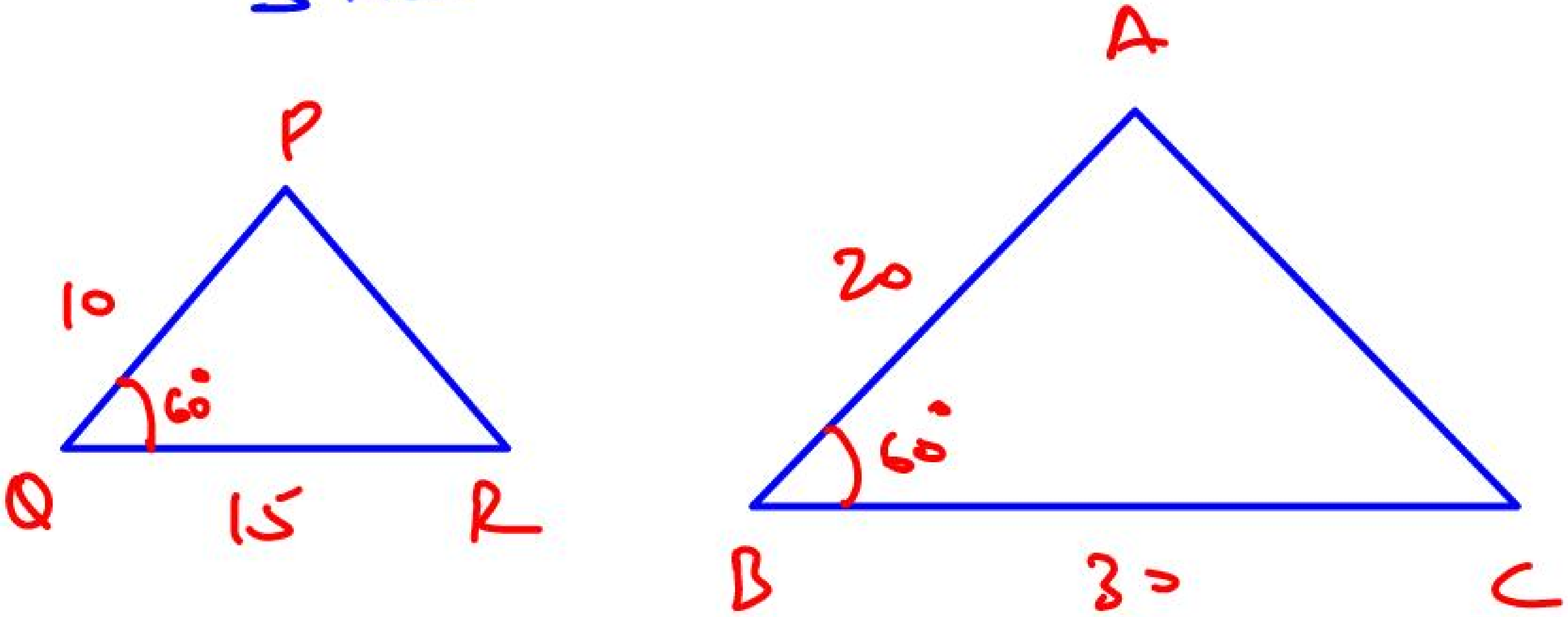
$\Delta PQR \sim \Delta EFD$

II SSS



$\triangle ABC \sim \triangle FDE$

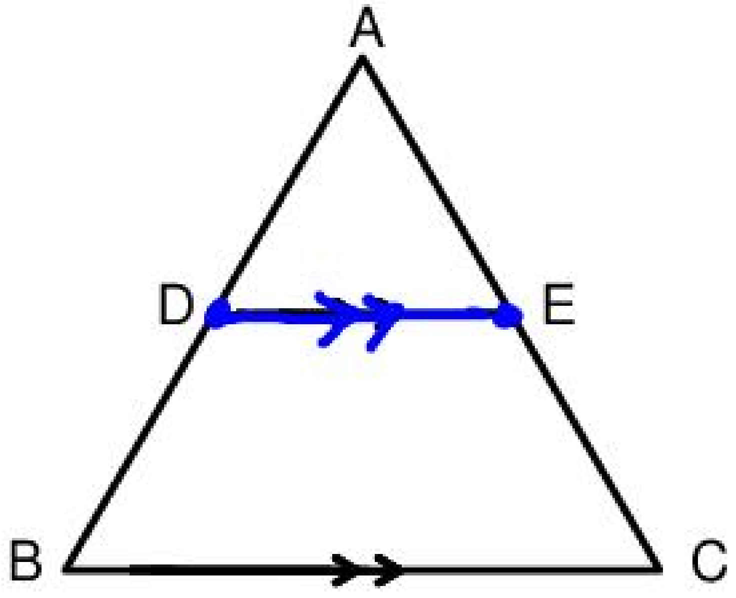
III SAS



$\Delta PQR \sim \Delta ABC$

BASIC PROPORTIONALITY THEOREM OR THALES THEOREM

V Imp



If a line is drawn parallel to one side of a triangle intersecting the other two sides then it divides the two sides in the same ratio.

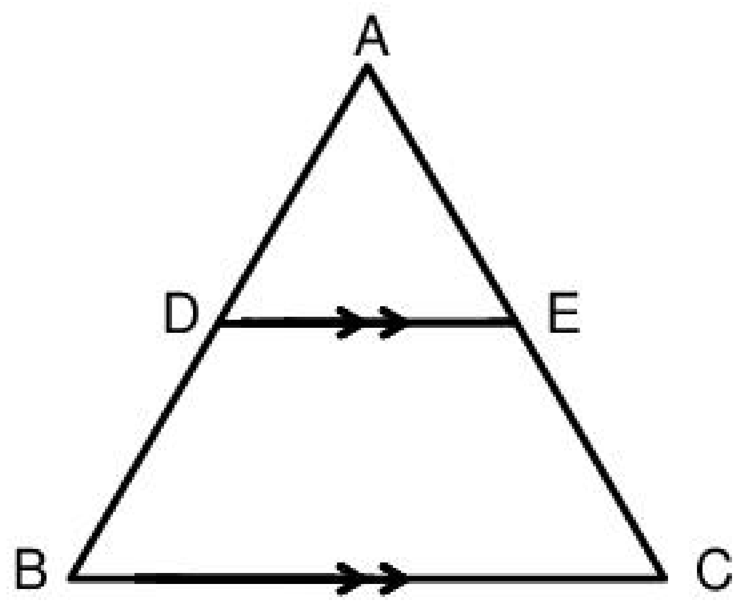
Given : D, E are points on AB and AC such that DE || BC

V Imp

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Proof of BPT or Thales:

Given : D, E are points on AB and AC such that DE || BC



To prove: $\frac{AD}{DB} = \frac{AE}{EC}$

Proof : In ΔADE & ΔABC

$\angle A = \angle A$ (Common)

$\angle ADE = \angle ABC$ (Corresponding angle)

$\therefore$ $\Delta ADE \sim \Delta ABC$

$$\frac{AD}{AB} = \frac{AE}{AC}$$

$$\frac{AB}{AD} - 1 = \frac{AC}{AE} - 1$$

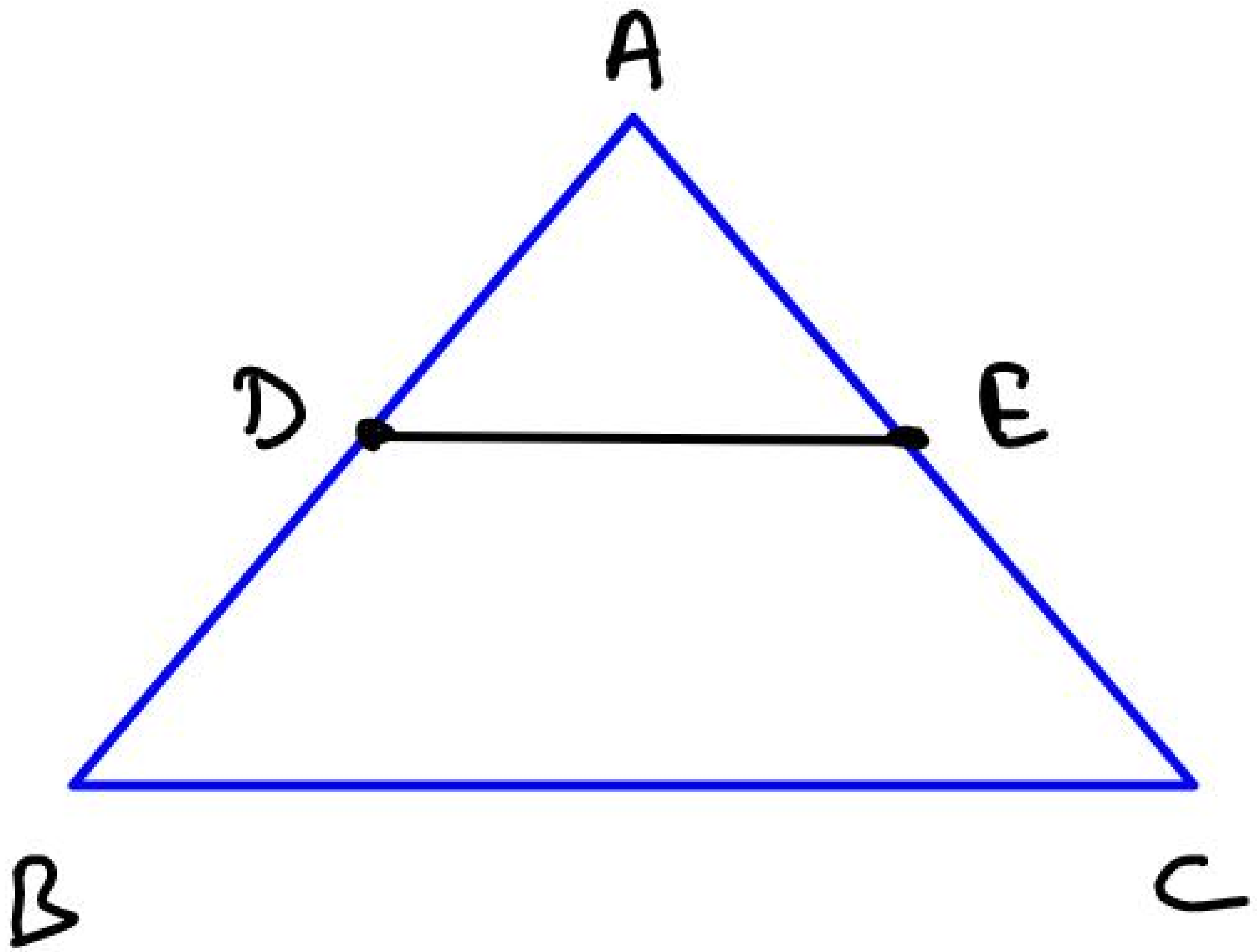
$$\frac{DB}{AD} = \frac{EC}{AE}$$

$$\frac{AD}{DB} = \frac{AE}{EC}$$

BYJU'S EXAM PREP

CONVERSE OF BASIC PROPORTIONALITY THEOREM

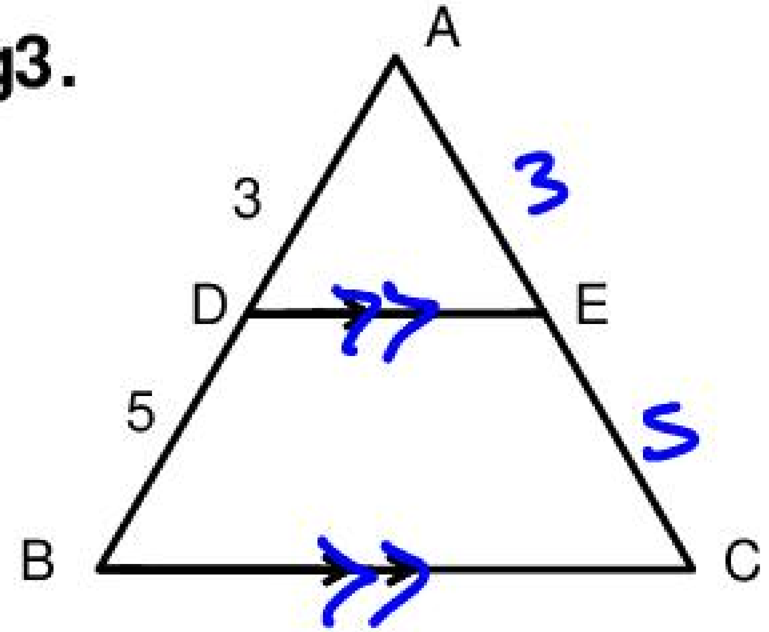
If a line divides any two sides of a triangle in the same ratio, then the line must be parallel to the third side.



Given $\frac{AD}{DB} = \frac{AE}{EC}$

$\longrightarrow DE \parallel BC$

Eg3.



Given DE || BC

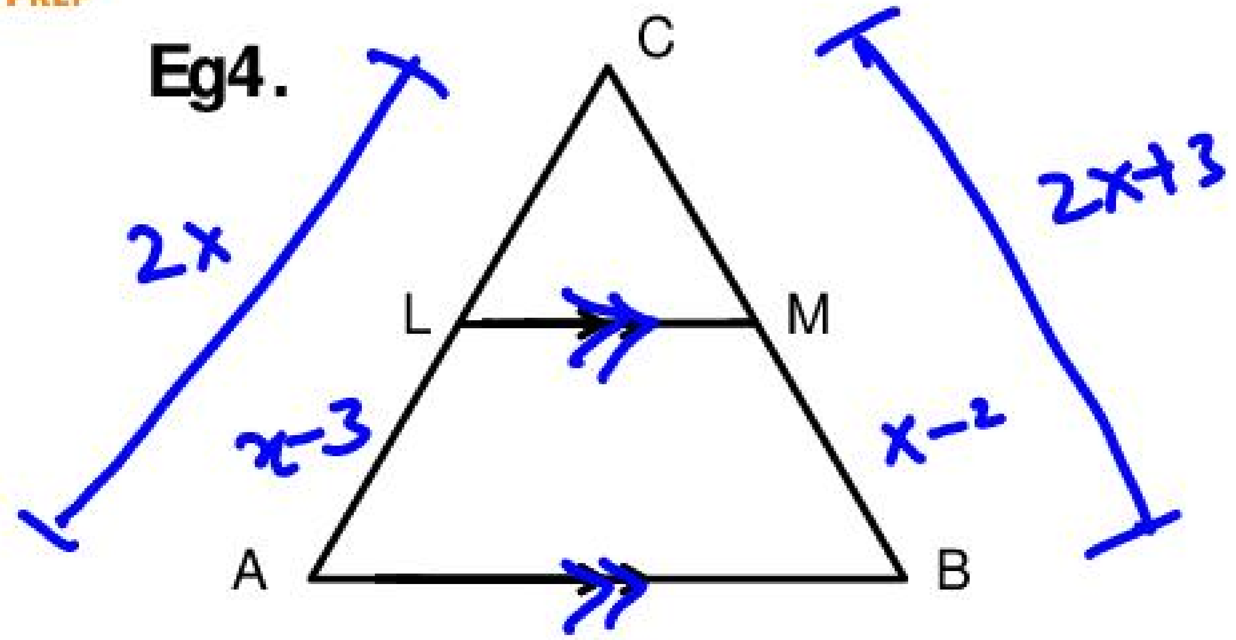
If $\frac{AD}{DB} = \frac{3}{5}$ **, AC = 11.2 cm**

Find AE = ??

$$\frac{3}{\cancel{8}} \times \overset{1.4}{\cancel{11.2}}$$

$$= \underline{4.2\,cm}$$

Eg4.



If LM || AB

AL = x – 3 , AC = 2x , BM = x – 2, BC = 2x + 3

then x = ??

$$\frac{x-3}{2x} = \frac{x-2}{2x+3}$$

$$\cancel{2x^2} - 3x - 9 = \cancel{2x^2} - 4x$$

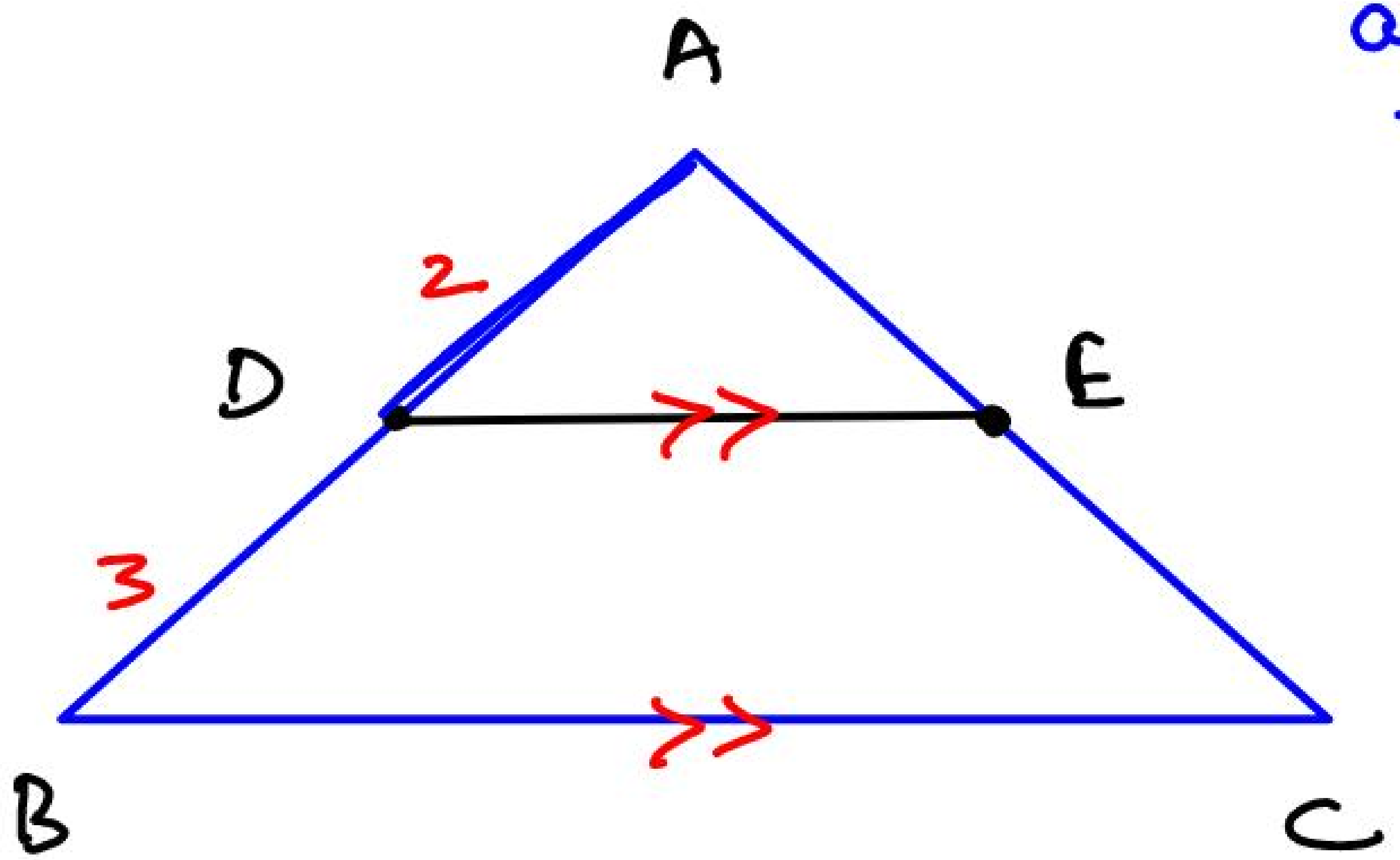
$$\boxed{x = 9}$$

V. imp

Eg5. In a ΔABC, D and E are taken on AB & AC in such a way that DE || BC and $\frac{AD}{DB} = \frac{2}{3}$.

Find : $\frac{Area\ of\ \Delta ADE}{Area\ of\ quadrilateral\ DECB} = ??$

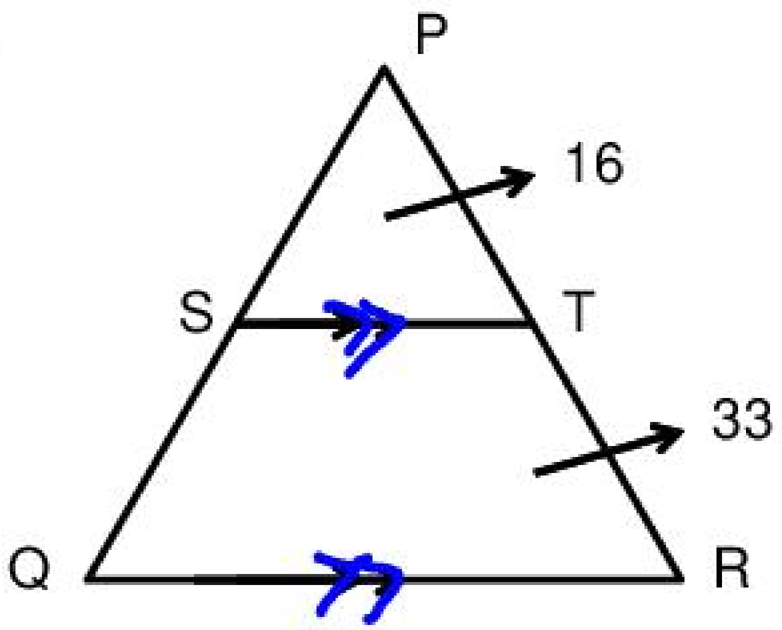
$\Delta \overline{ADE} \sim \Delta \overline{ABC}$



$\frac{\text{area of } \Delta ADE}{\text{area of } \Delta ABC} = \frac{4}{25}$

$\frac{\text{area of } \Delta ADE}{\text{area of quad } DECB} = \frac{4}{21}$

Eg6.



If $\dfrac{\textit{Area of } \triangle PST}{\textit{Area of quadrilateral } STRQ} = \dfrac{16}{33}$

Find : $\dfrac{PS}{SQ} = ??$ $\dfrac{4}{3}$

$\dfrac{\text{Area of } \triangle PST}{\text{Area of } \triangle PQR} = \dfrac{16}{49}$

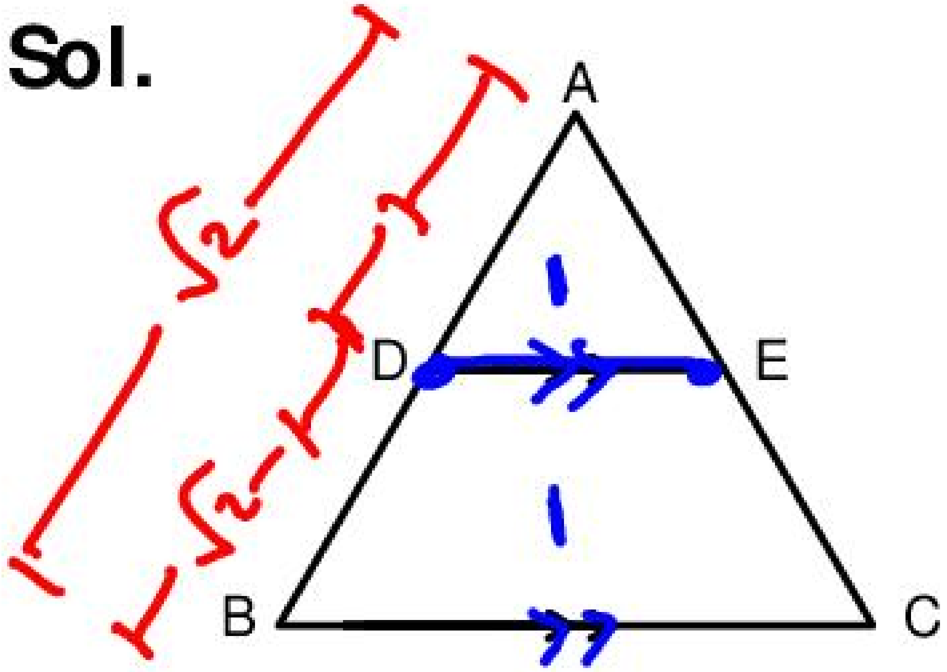
$\left(\dfrac{PS}{PQ}\right)^2 = \dfrac{16}{49}$

$\dfrac{PS}{PQ} = \dfrac{4}{7}$

BYJU'S EXAM PREP

Eg7. In a ΔABC, points D and E are taken on AB & AC in such that DE || BC and it divides the triangle in two equal areas. find AD : DB.

Sol.



$\Delta ADE \sim \Delta ABC$

$$\frac{\text{area of } \Delta ADE}{\text{area of } \Delta ABC} = \frac{1}{2}$$

$$\left(\frac{AD}{AB}\right)^2 = \frac{1}{2}$$

$$\frac{AD}{AB} = \frac{1}{\sqrt{2}}$$

$$\frac{AD}{DB} = \frac{1}{\sqrt{2}-1}$$

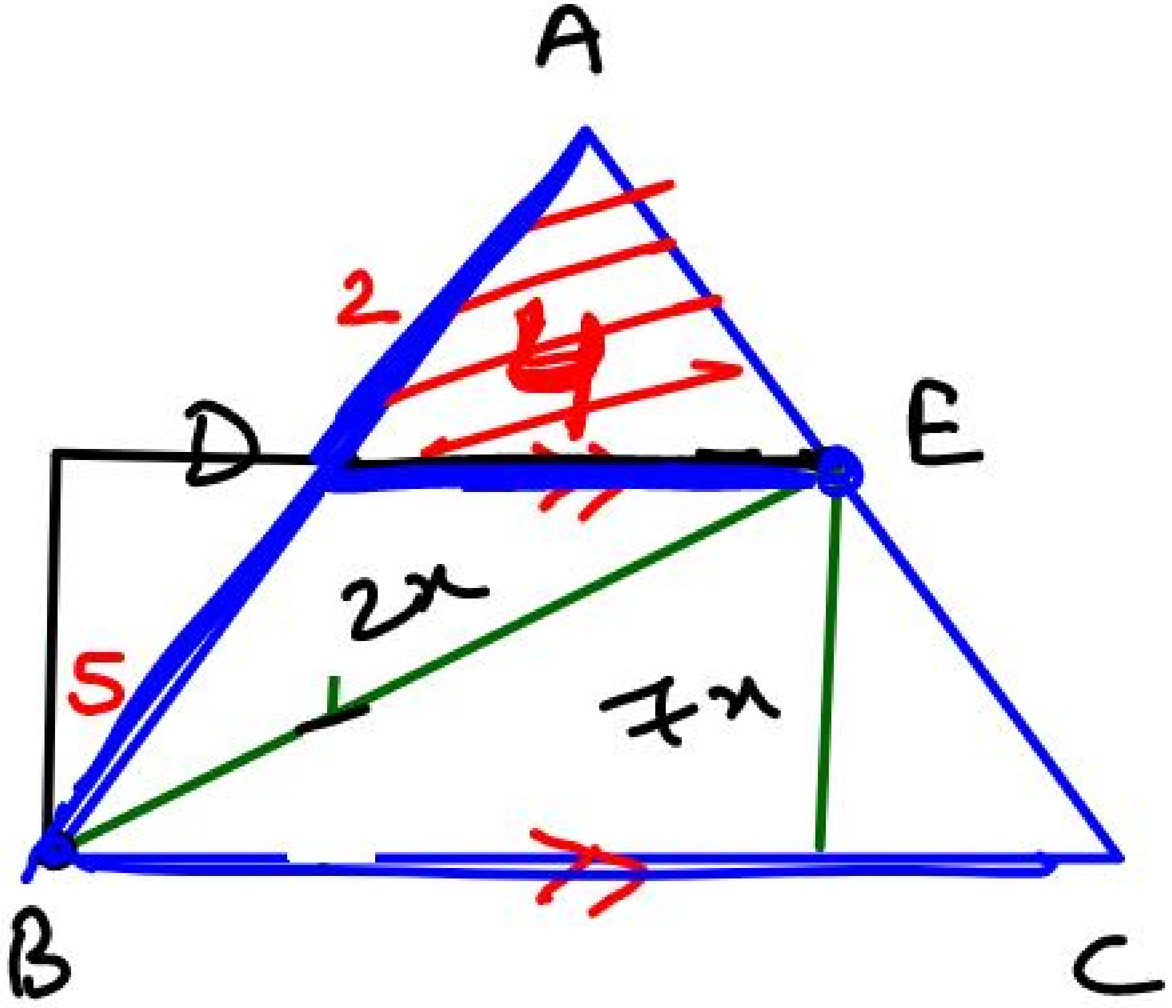
V.imp

Eg8. In a ΔABC, points D and E are taken on AB & AC in such that DE || BC.

4 : 10 : 35

If $\frac{AD}{DB} = \frac{2}{5}$, find (Area of ΔADE : Area of ΔDEB : Area of ΔBEC)

2 min



$$\frac{\text{area of } \Delta ADE}{\text{area of } \Delta ABC} = \frac{4}{49}$$

If area of ΔADE → 4 units

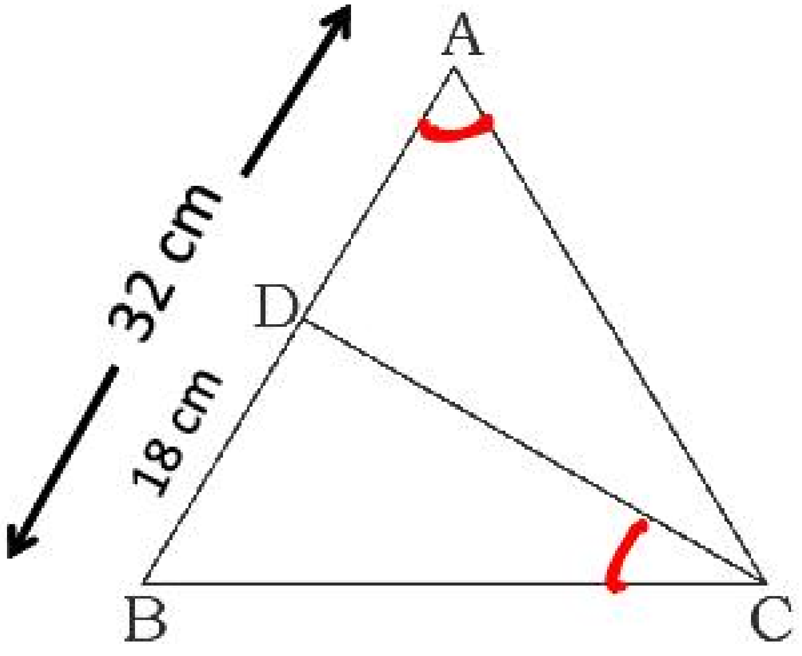
area of ΔDEB + ΔBEC → 45 units

$$\frac{\text{area of } \Delta DEB}{\text{area of } \Delta BEC} = \frac{DE}{BC} = \frac{2}{7}$$

9x → 45 units   x → 5 units

PYQ

Eg9. In the given figure, $\angle BAC = \angle BCD$, AB = 32 cm and BD = 18 cm, then the ratio of perimeter of ΔBDC and ΔABC is:



(a) 4 : 3 (b) 8 : 5

(c) 5 : 8 (d) 3 : 4

$\Delta BDC \sim \Delta BCA$

$$\frac{18}{BC} = \frac{BC}{32}$$

$$\boxed{BC = 24}$$

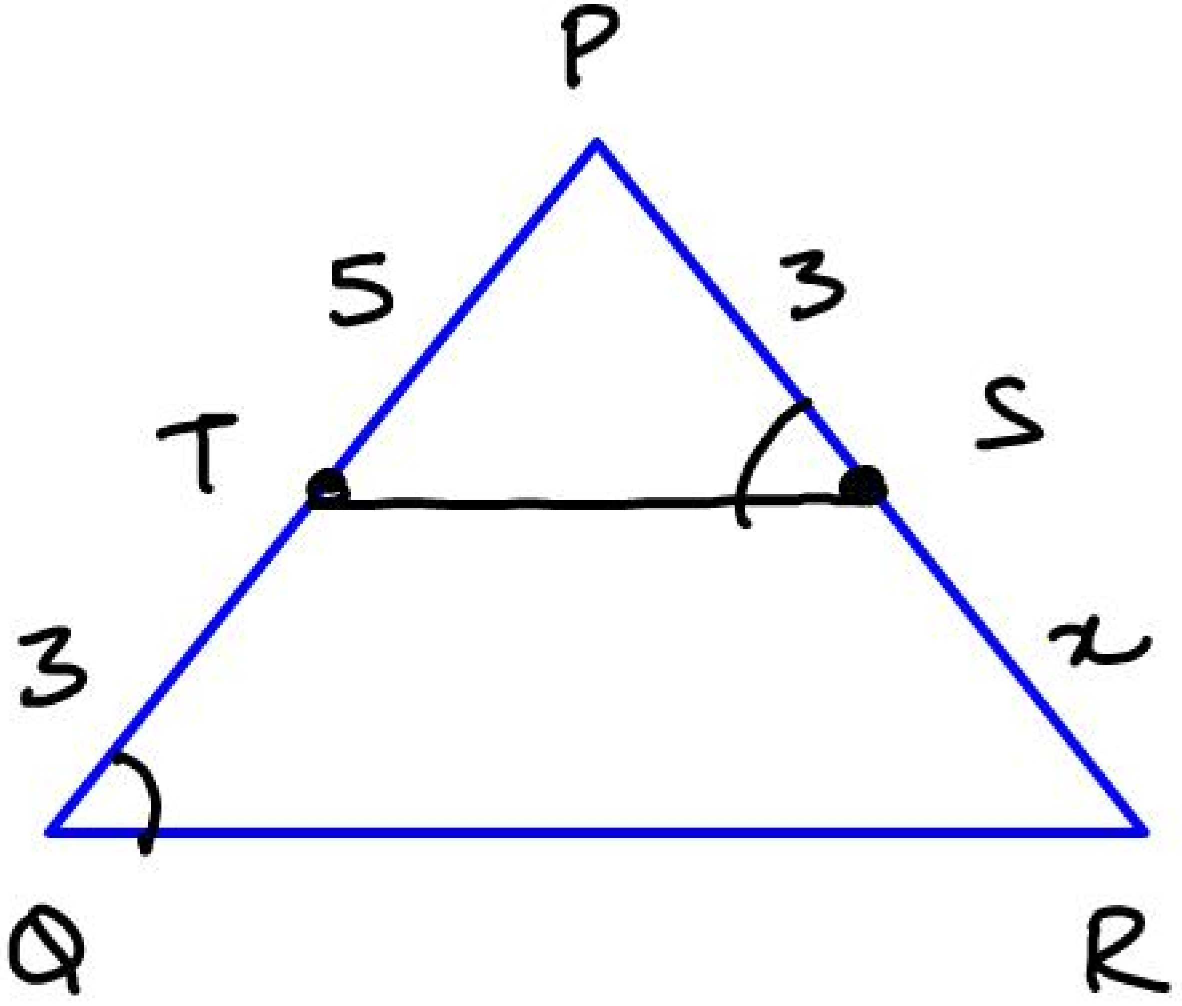
$$\frac{\cancel{18}\ 3}{\cancel{24}\ 4}$$

V. Imp

Eg10. In ΔPQR, S and T are points on side PR and PQ respectively such that, $\angle PQR = \angle PST$. If PT = 5 cm, PS = 3 cm and TQ = 3 cm, then length of SR is

(a) 5 cm (b) 6 cm

(c) $\frac{31}{3}$ cm (d) $\frac{41}{3}$ cm

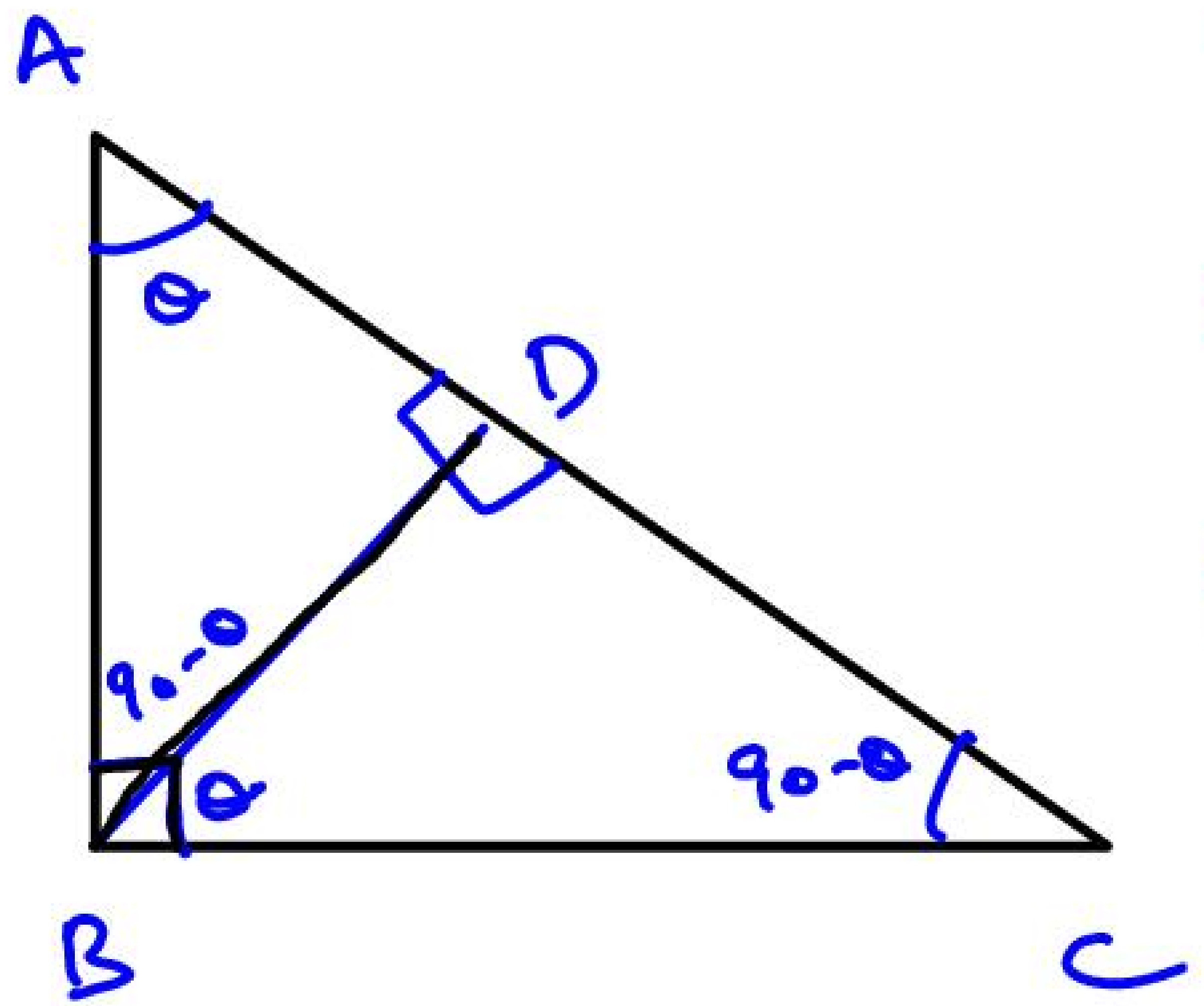


$\Delta PST \sim \Delta PQR$

$$\frac{3}{8} = \frac{5}{3+x}$$

$$9 + 3x = 40$$

$$\boxed{x = \frac{31}{3}}$$



θ 90 90-θ

ADB

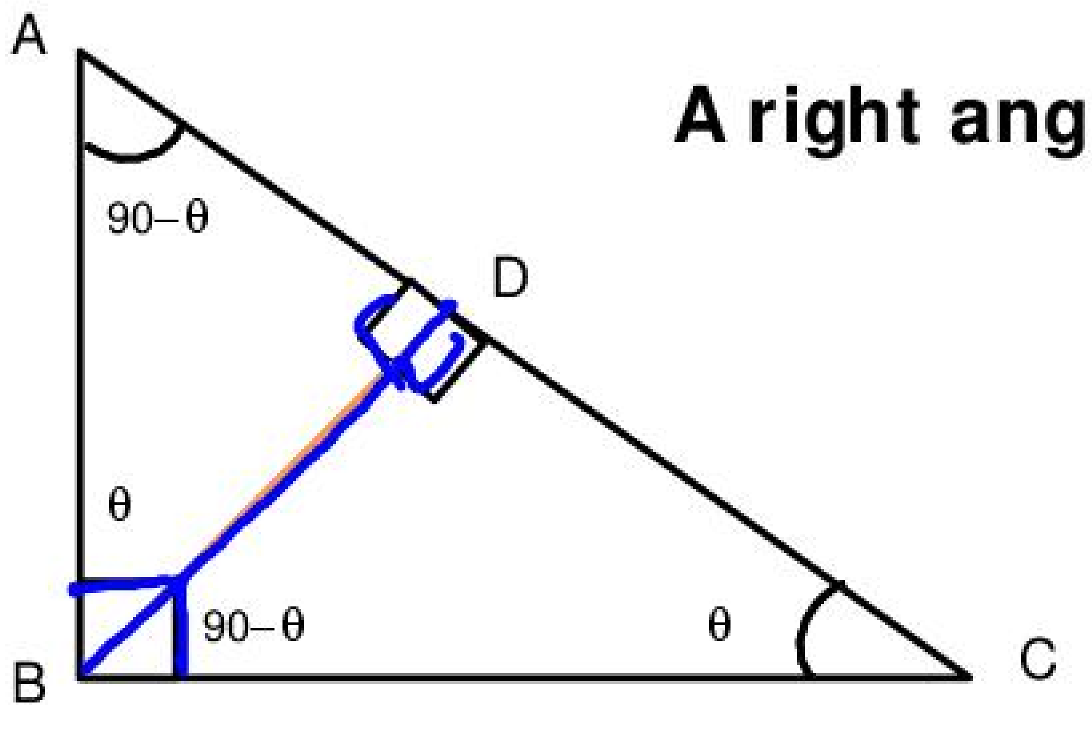
θ 90 90-θ

BDC

θ 90 90-θ

ABC

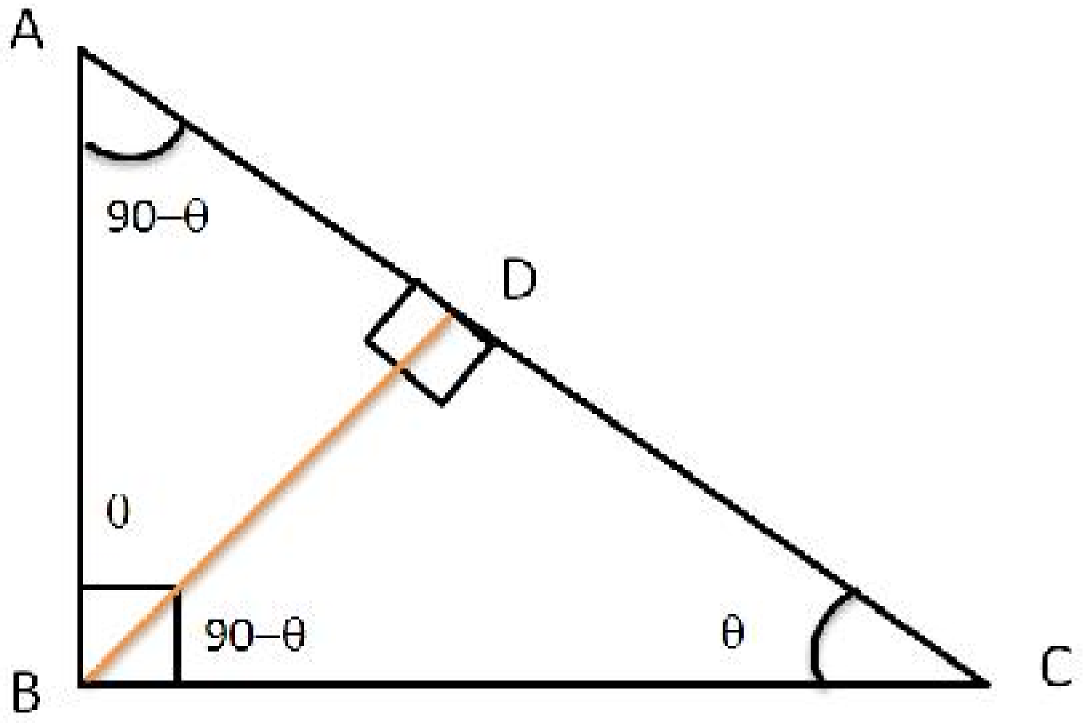
SIMILARITY IN RIGHT ANGLE TRIANGLE



A right angle Δ, right angle at B and BD is perpendicular to AC.

$$\Delta ABC \sim \Delta ADB \sim \Delta BDC$$

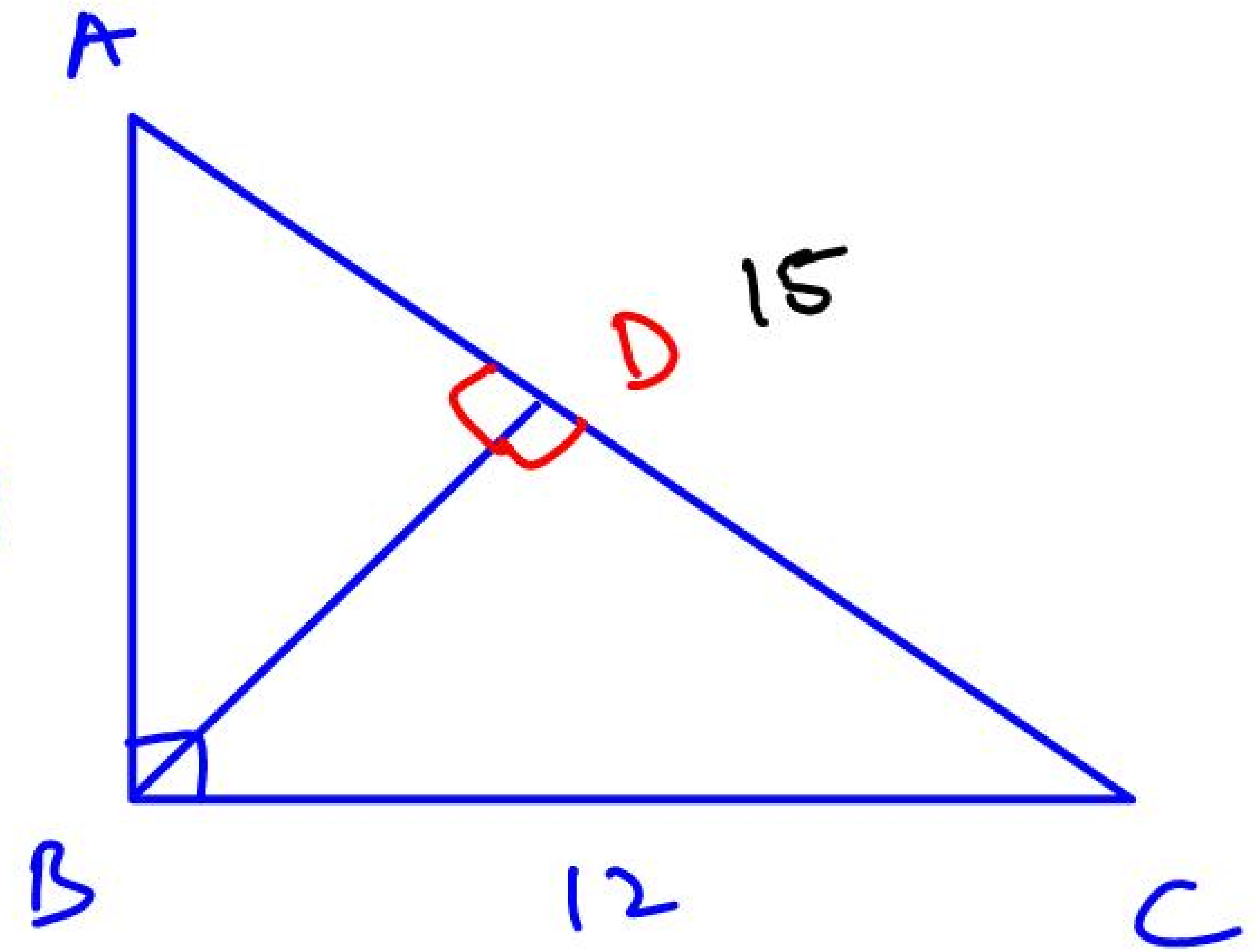
(1) A right angle Δ, right angle at B and BD is perpendicular to AC.



$$\cancel{\frac{1}{2}}(BC)(AB) = \cancel{\frac{1}{2}}(AC)(BD)$$

$$\boxed{BC \cdot AB = AC \cdot BD}$$

eg



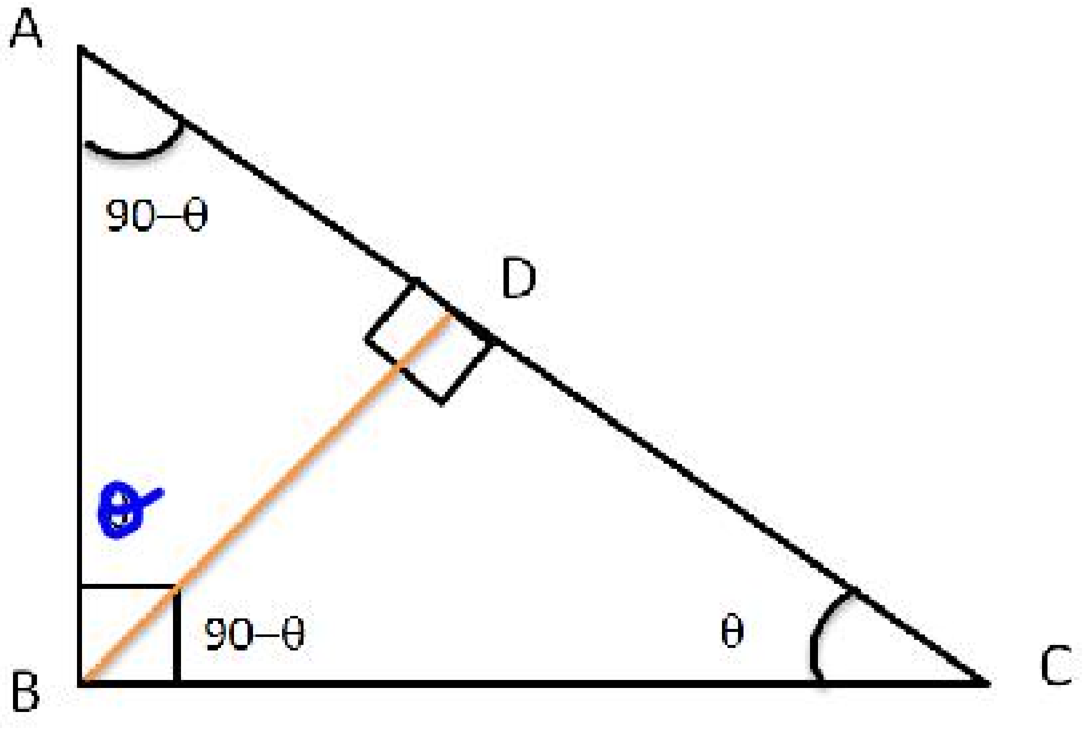
Find BD = ??

$9 \cdot 12 = 15 \cdot BD$

$BD = \frac{9 \cdot \cancel{12}^{4}}{\cancel{15}_{5}}$

$\frac{36}{5}$

(2) A right angle Δ, right angle at B and BD is perpendicular to AC.

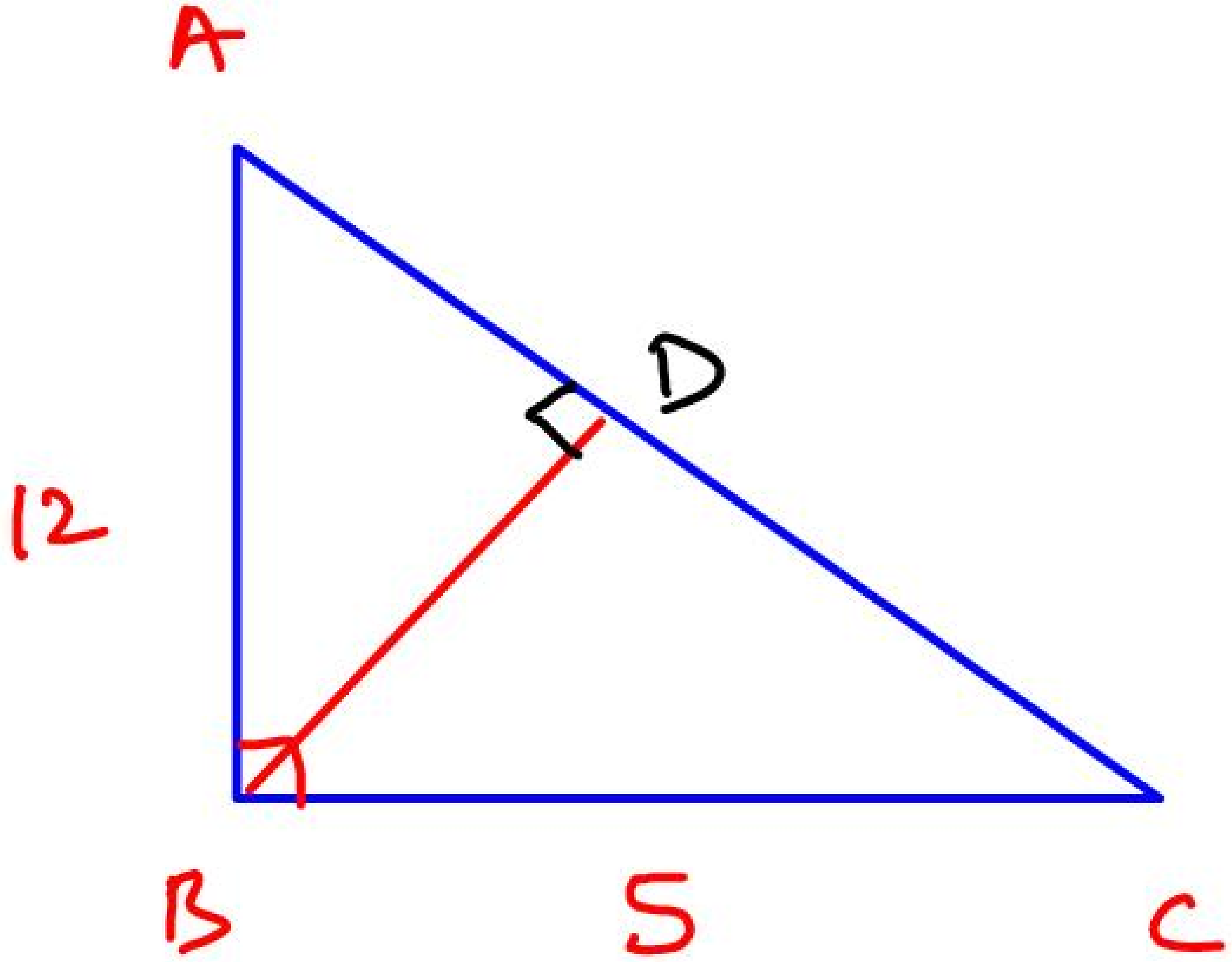


$$\Delta ABC \sim \Delta ADB$$

$$\frac{AB}{AD} = \frac{AC}{AB}$$

$$\boxed{AB^2 = AC \cdot AD}$$

eg



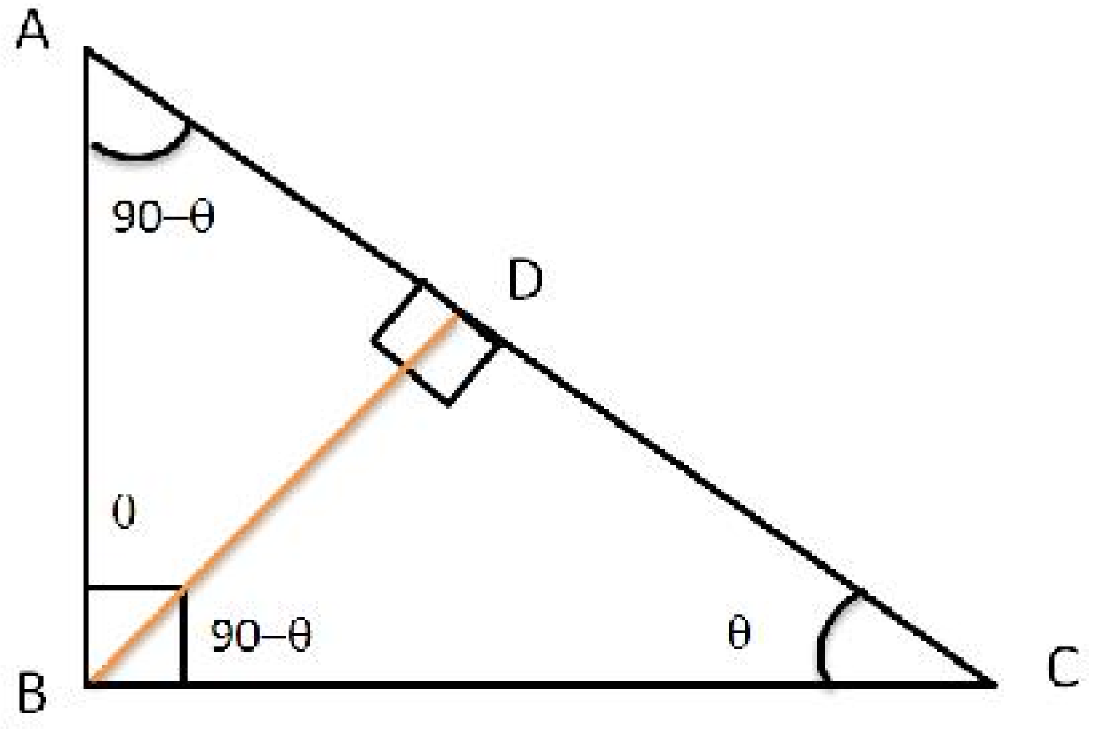
Find AD

$$BA^2 = AD \times AC$$

$$144 = AD \times 13$$

$$AD = \frac{144}{13}$$

(3) A right angle Δ, right angle at B and BD is perpendicular to AC.

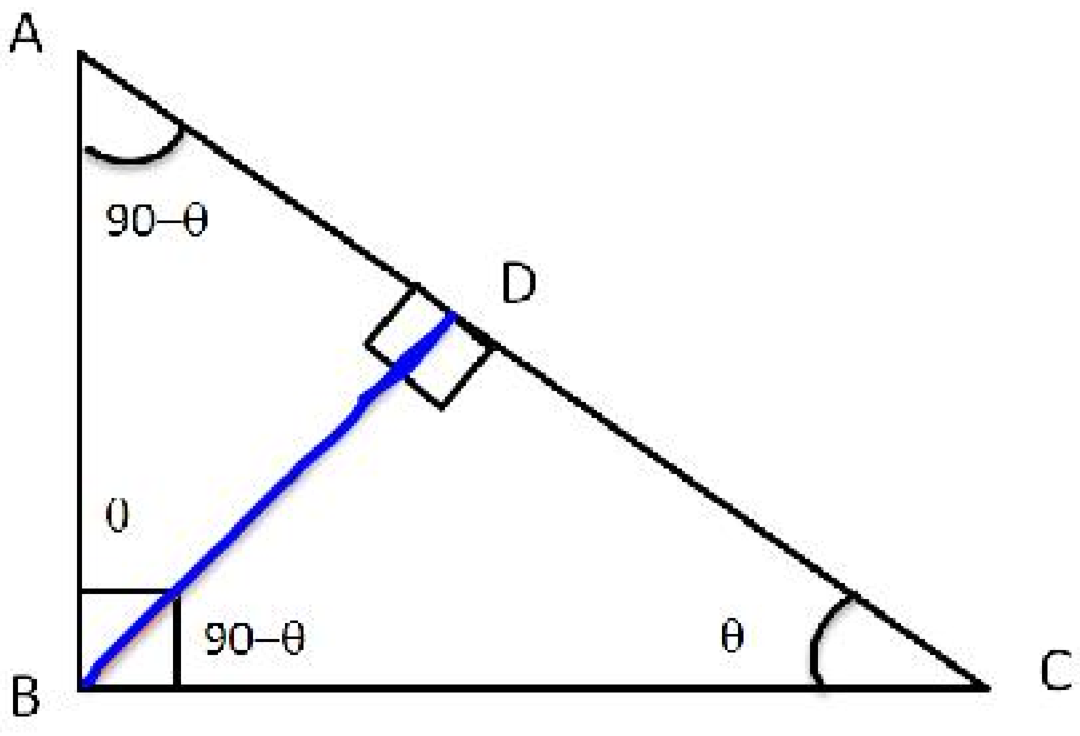


$$\Delta A\overset{\frown}{BC} \sim \Delta B\overset{\frown}{DC}$$

$$\frac{AC}{BC} = \frac{BC}{DC}$$

$$\boxed{BC^2 = CD \cdot CA}$$

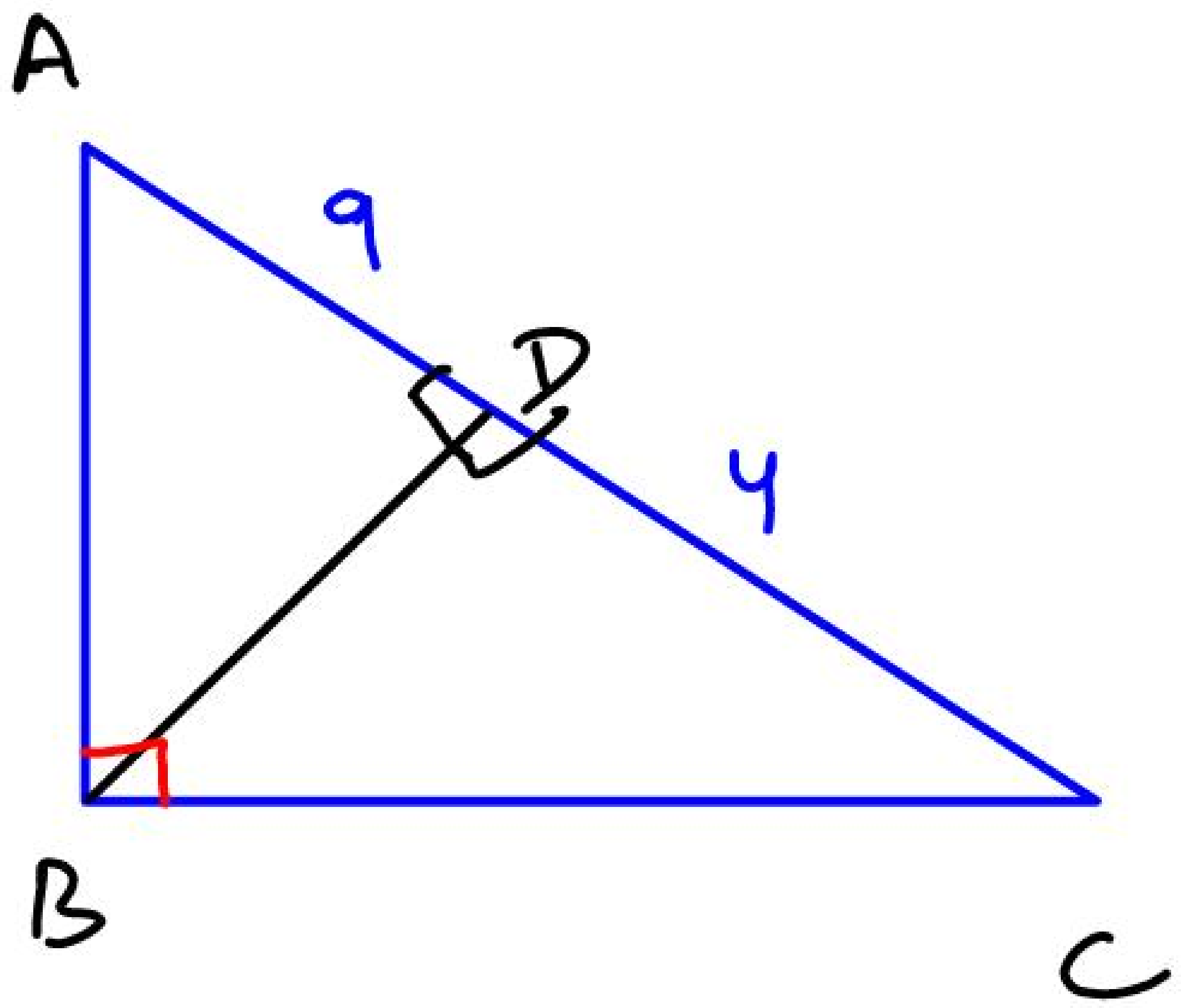
(4) A right angle Δ, right angle at B and BD is perpendicular to AC.



$\Delta A\overline{DB} \sim \Delta B\overline{DC}$

$$\frac{AD}{BD} = \frac{DB}{DC}$$

$$\boxed{(BD)^2 = (AD)(DC)}$$



Find BD = ??

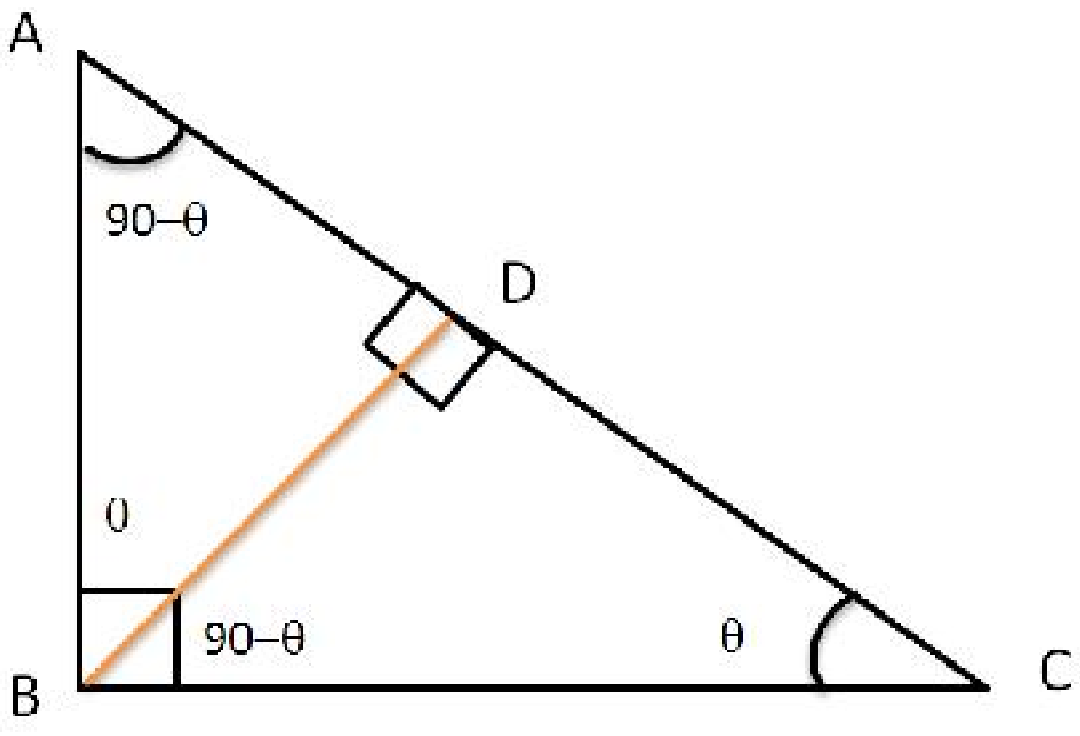
$BD^2 = (DA)(DC)$

$BD^2 = 36$

$BD = 6$

BYJU'S EXAM PREP

(5) A right angle Δ, right angle at B and BD is perpendicular to AC.



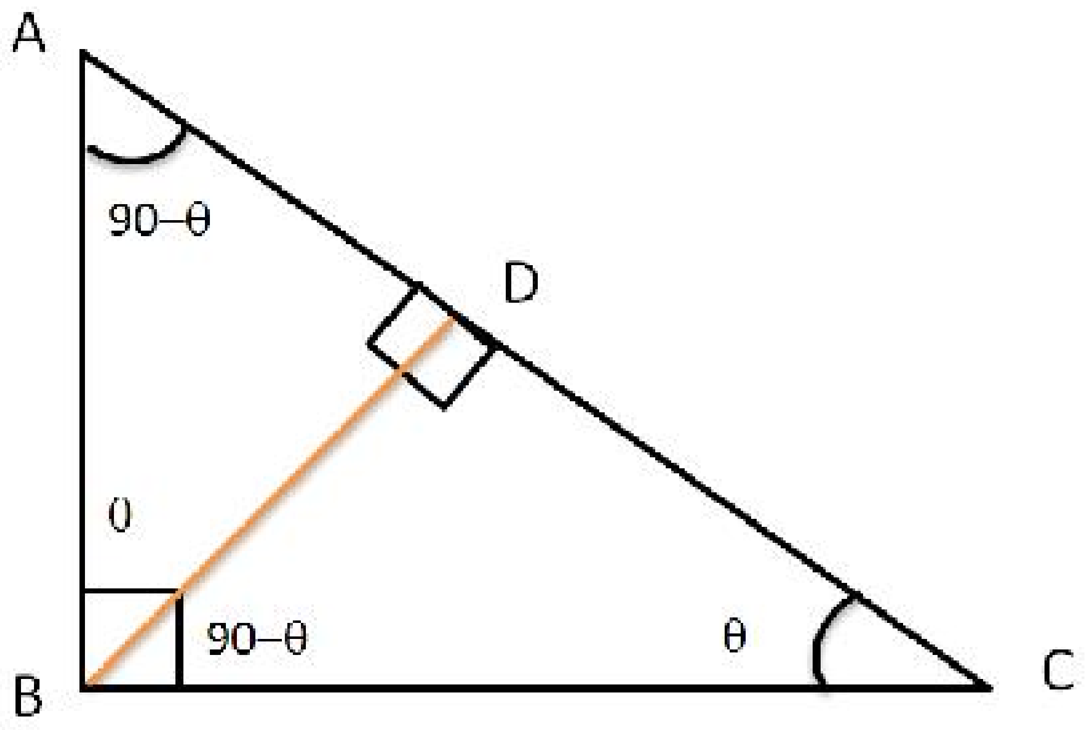
$$(BC)(AB) = (AC)(BD)$$

$$(BC)^2 (AB)^2 = (AC)^2 (BD)^2$$

$$\frac{1}{(BD)^2} = \frac{(AC)^2}{(BC)^2 (AB)^2}$$

$$= \frac{(AB)^2 + (BC)^2}{(AB)^2 \cdot (BC)^2}$$

$$\boxed{\frac{1}{(BD)^2} = \frac{1}{(AB)^2} + \frac{1}{(BC)^2}}$$



(1) $AB \times BC = AC \cdot BD$

(2) $BA^2 = AD \cdot AC$

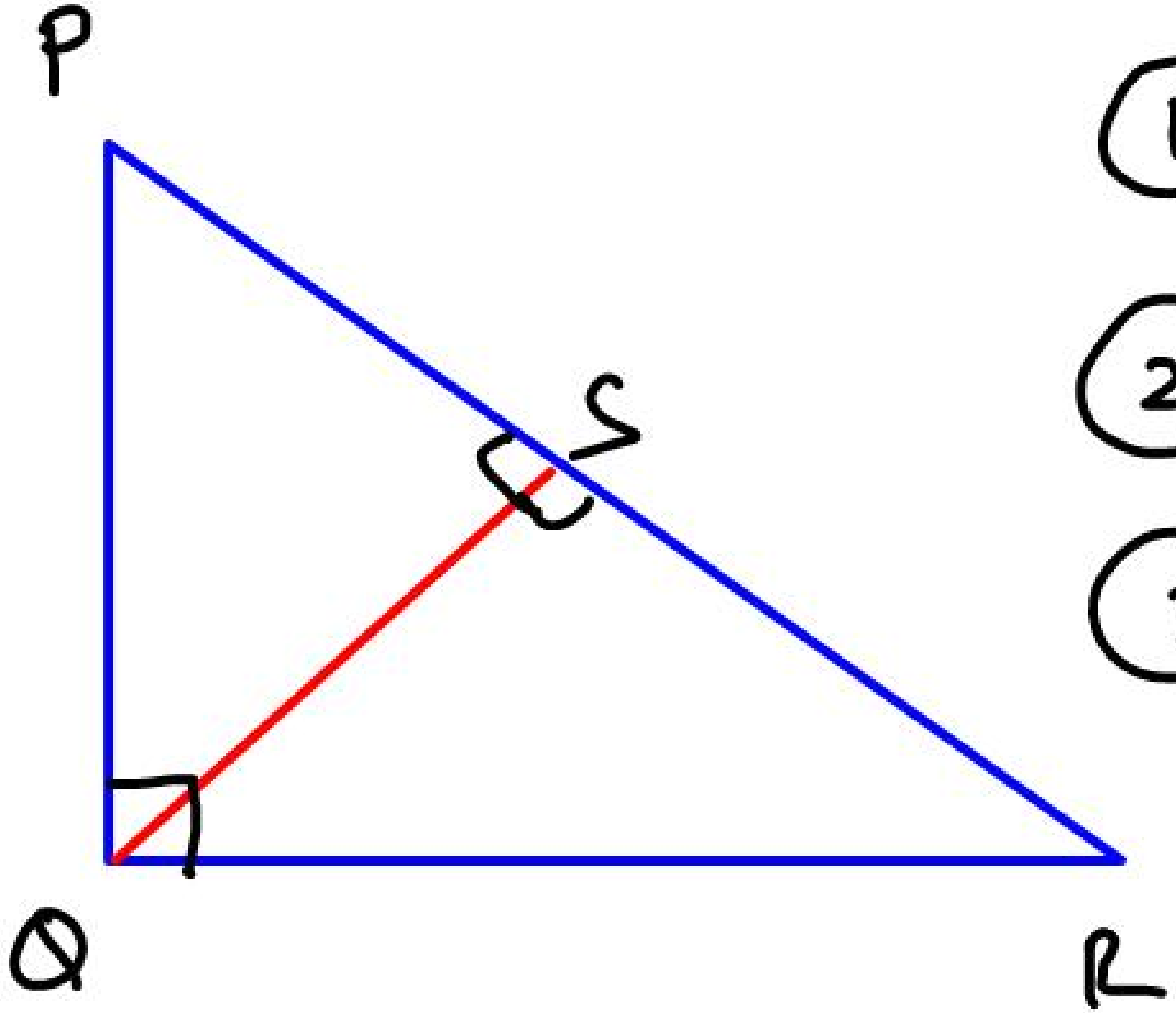
(3) $BC^2 = CD \cdot CA$

(4) $BD^2 = DA \cdot DC$

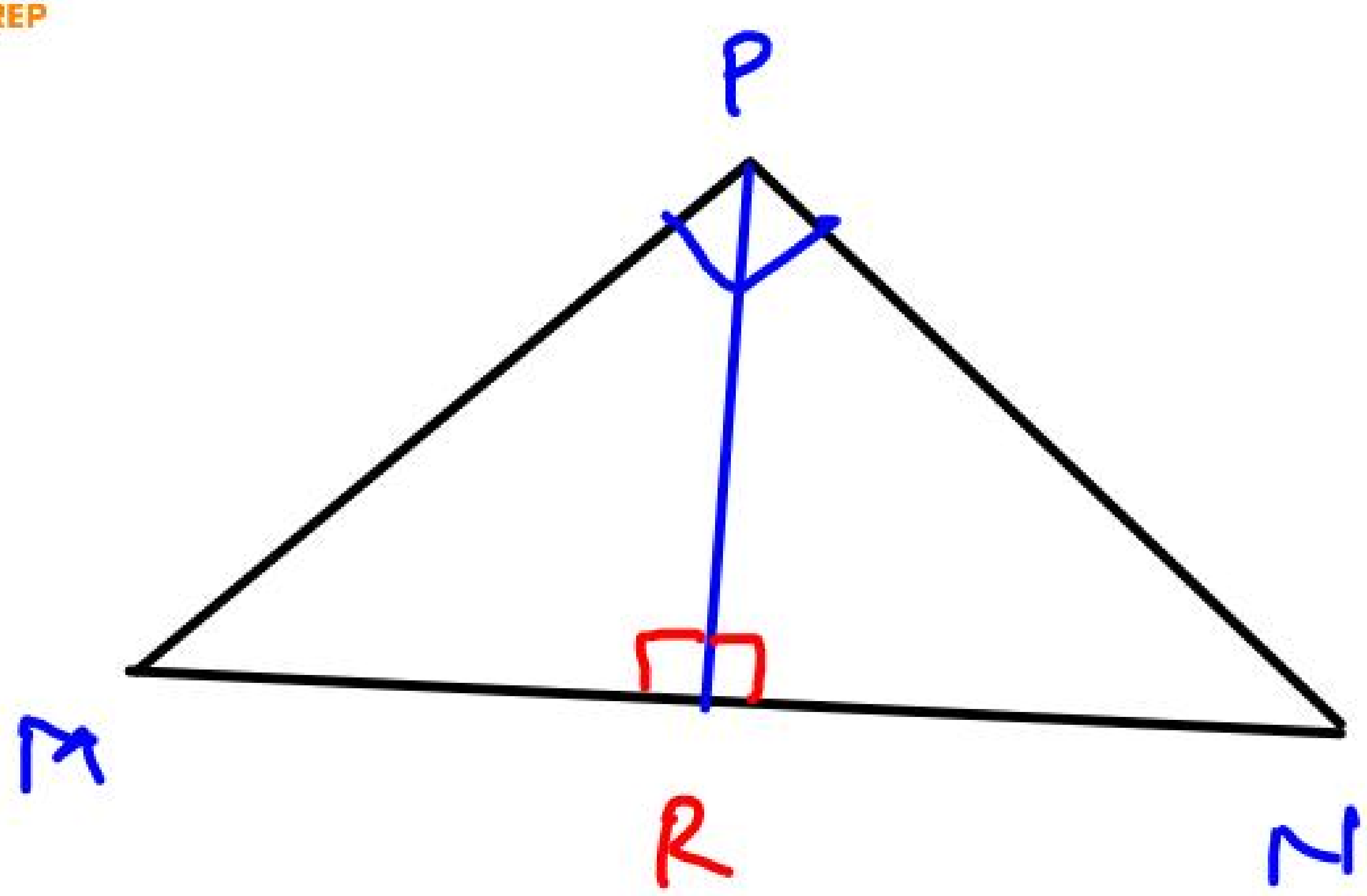
(5) $\dfrac{1}{BD^2} = \dfrac{1}{AB^2} + \dfrac{1}{BC^2}$

BYJU'S EXAM PREP

EXAMPLES ON SIMILARITY IN RIGHT ANGLE Δ



1. $(PQ)(QR) = (PR)(QS)$
2. $(QP)^2 = (PS)(PR)$
3. $(QR)^2 = (RS)(RP)$
4. $(QS)^2 = (SP)(SR)$
5. $\dfrac{1}{(QS)^2} = \dfrac{1}{(PQ)^2} + \dfrac{1}{(QR)^2}$



1. $(MP)(NP) = (MN)(PR)$

2. $(PN)^2 = (NR)(NM)$

3. $(PM)^2 = (MR)(MN)$

4. $(PR)^2 = (RM)(RN)$

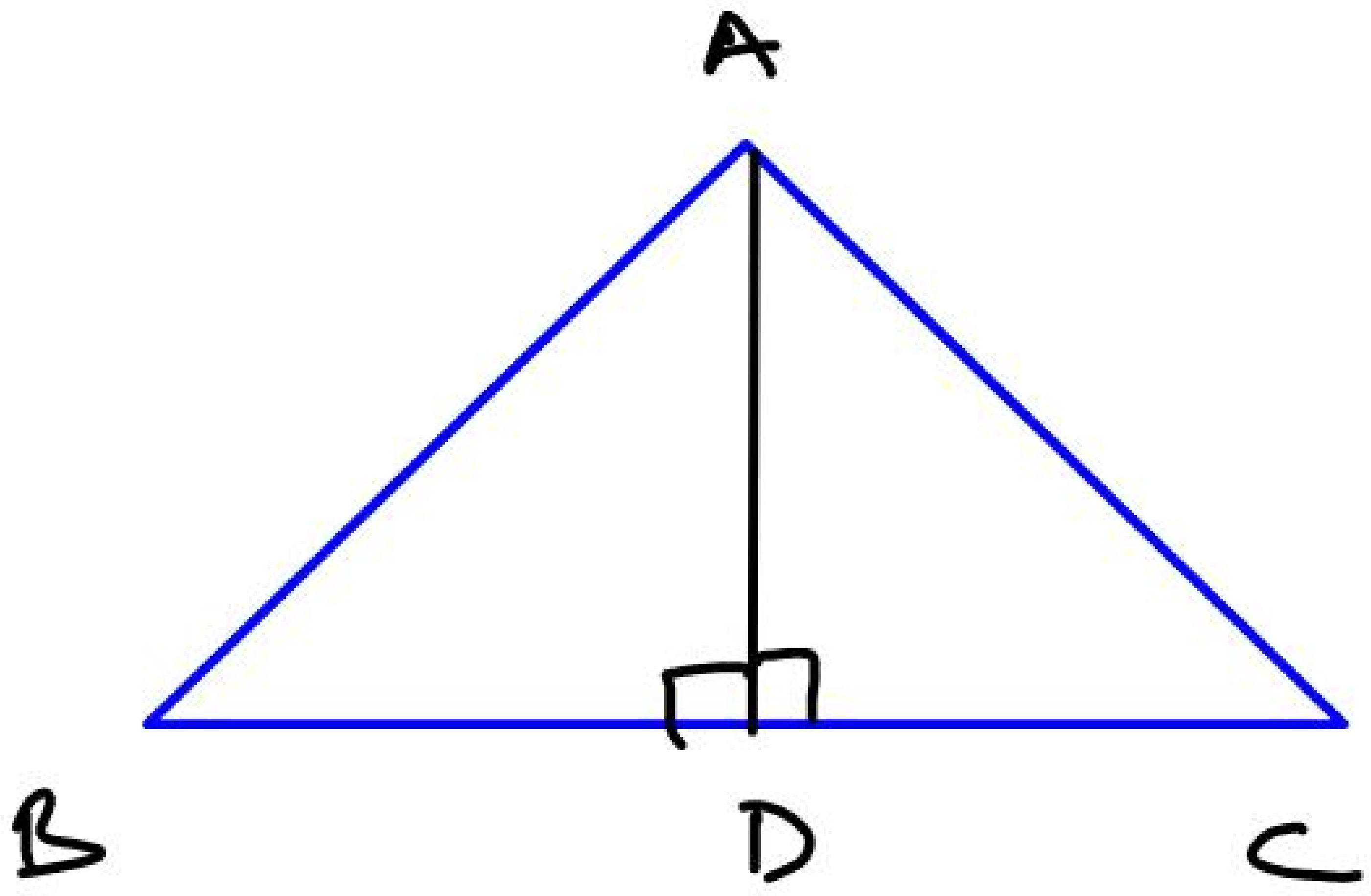
5. $$\frac{1}{(PR)^2} = \frac{1}{(PM)^2} + \frac{1}{(PN)^2}$$

BYJU'S EXAM PREP

Eg11. In a ΔABC,

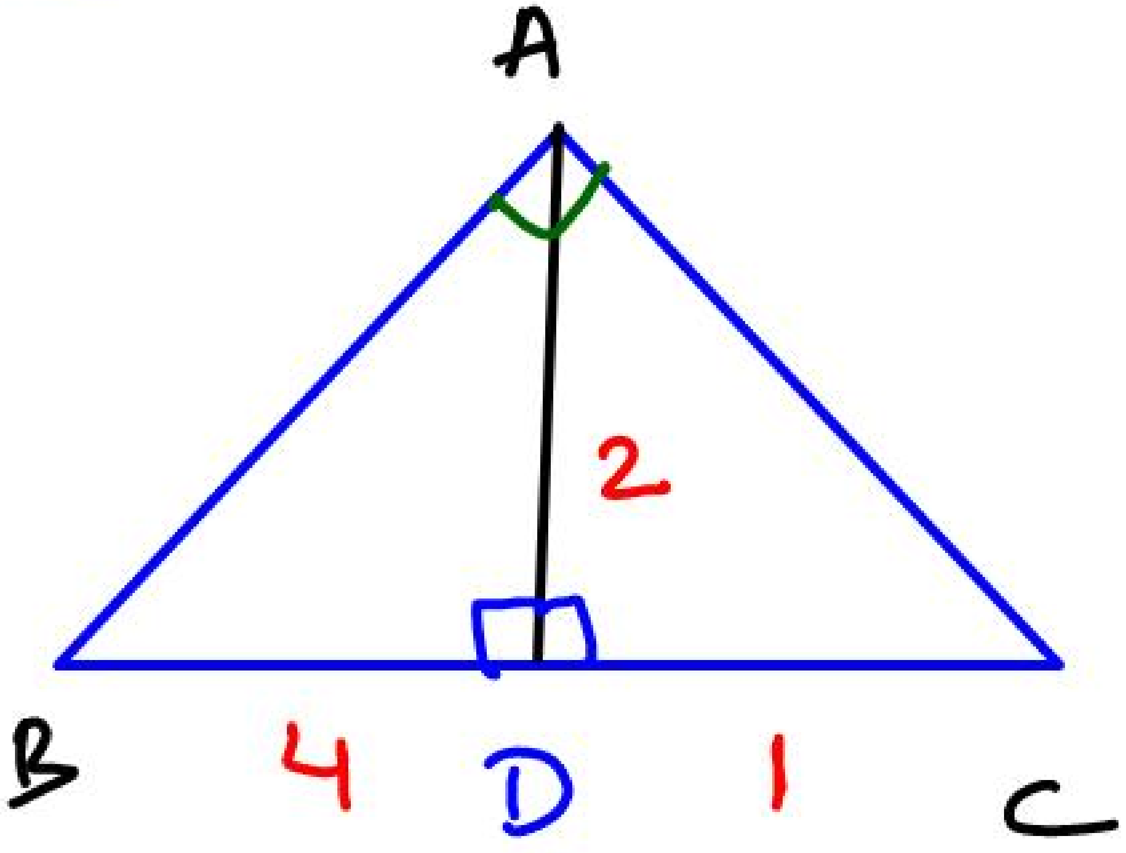
$AD \perp BC$ & $AD^2 = BD \cdot DC$

Find $\angle BAC = ??$



$\angle BAC \longrightarrow 90^\circ$

BYJU'S EXAM PREP



II^{nd}

$$\boxed{AD^2 = BD \cdot DC}$$

Find $\angle BAC$

Let $AD = 2$

$BD = 4$

$DC = 1$

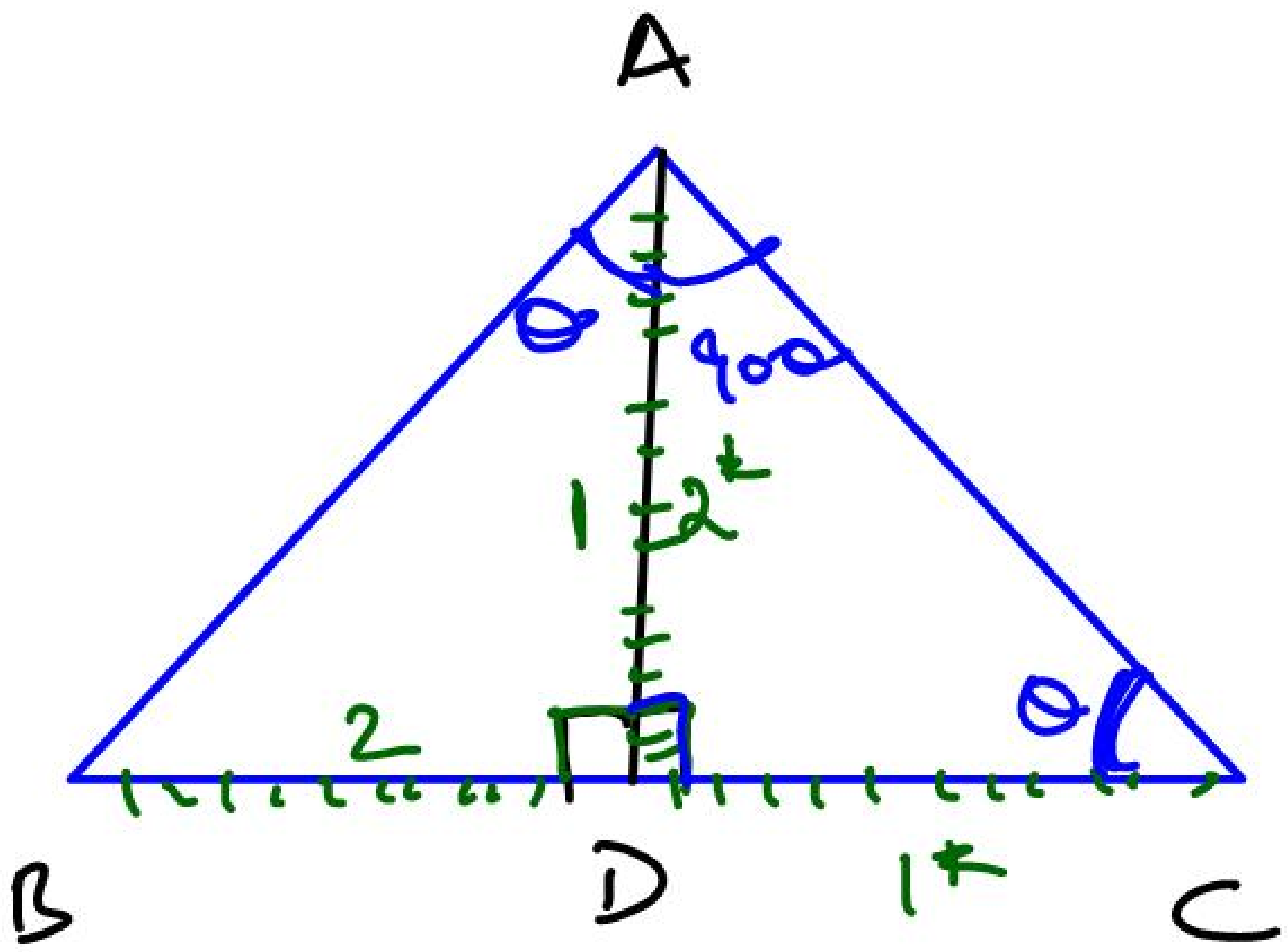
$AB = \sqrt{20}$

$AC = \sqrt{5}$

$BC = 5$

$(AB)^2 + (AC)^2 = (BC)^2$

$\angle A = 90^\circ$



$AD^2 = BD \cdot DC$

Solⁿ

$$\frac{AD}{BD} = \frac{DC}{AD}$$

$\Delta ADB \sim \Delta CDA$

By SAS

$\angle A \rightarrow 90$

MID-POINT THEOREM

If we join mid-points of any 2 sides of a Δ by a line segment then that line segment will be parallel to the third side and half of it.

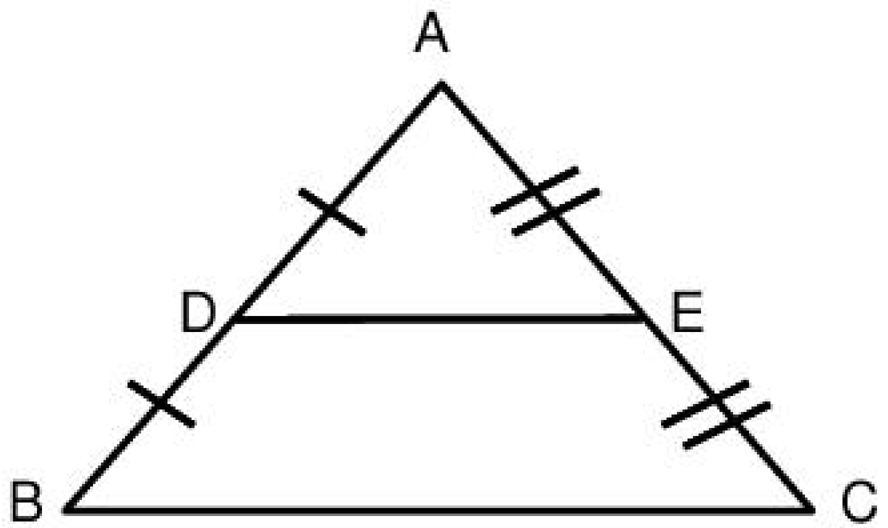
Eg12. Given,

D is mid-point of AB.

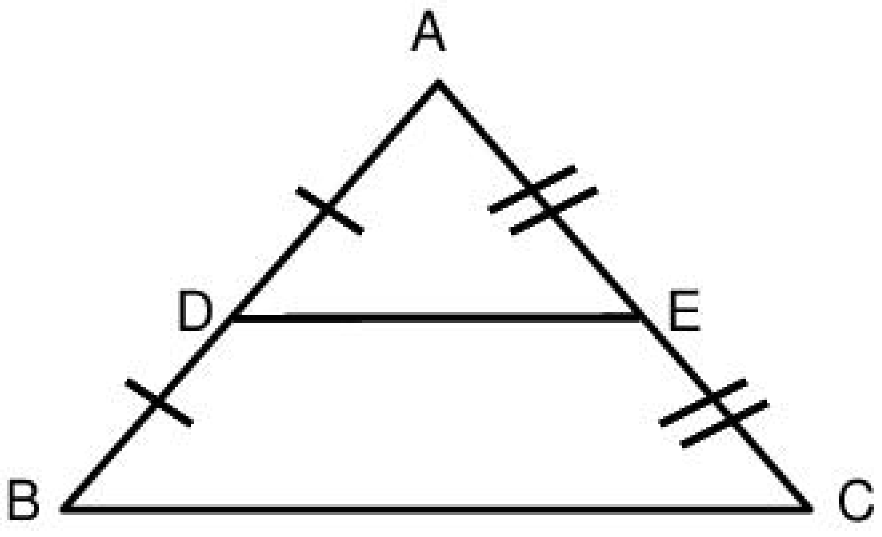
E is mid-point of AC.

$$\boldsymbol{DE \mid\mid BC}$$

$$\boldsymbol{DE = \frac{1}{2} BC}$$



Proof of Mid-point theorem:



Given, D, E are mid-point of AB & AC.

To prove: (i) $DE \parallel BC$

(ii) $DE = \frac{1}{2} BC$

Proof: AD : AB = 1 : 2

AE : AC = 1 : 2

$\Delta ADE \sim \Delta ABC$ (SAS similarity)

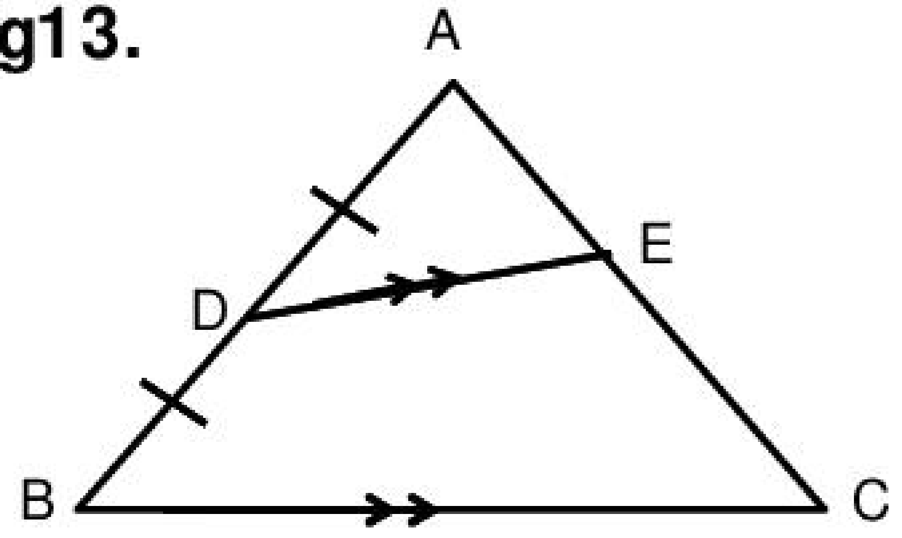
$\angle ADE = \angle ABC$ (Corresponding angles)

$DE \parallel BC$

$DE = \frac{1}{2} BC$

CONVERSE OF MID-POINT THEOREM

Eg13.



Given,

D is mid-point of AB.

DE || BC

E is mid-point of AC.

CONGRUENCY

Two figures are said to be congruent, if they are exactly same in every aspect.

2 line segments are congruent ?

2 circles are congruent ?

2 squares are congruent ?

$$\Delta ABC \cong \Delta DEF$$

Then

CONDITIONS OF CONGRUENCY

(1) SSS

(2) SAS

(3) ASA

(4) AAS

(5) RHS

SSS (Side – Side – Side)

SAS (Side – Angle – Side)

ASA (Angle – Side – Angle)

AAS (Angle – Angle – Side)

RHS (Right – Hypotenuse – Side)

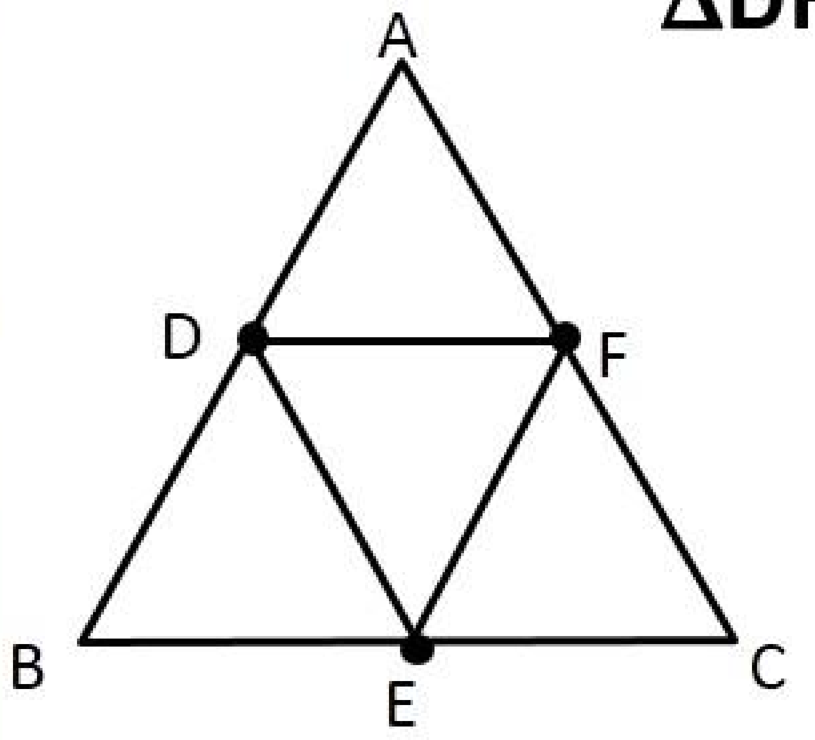
BYJU'S EXAM PREP

AAA & SSA does not guarantee congruency.

If D, E & F are midpoints of the sides AB, BC, CA

Then,

$$\Delta DFE \cong \Delta FDA \cong \Delta EBD \cong \Delta CEF$$



$$\text{Area of } \Delta DFE = \frac{1}{4} (\text{Area of } \Delta ABC)$$

BYJU'S EXAM PREP

If Congruent	→	Similar
If Similar	→	Congruent
If Congruent	→	Area same
If Area same	→	Congruent
Similar + Area same	→	Congruent

BYJU'S EXAM PREP

Eg14. AD is perpendicular to the internal bisector of $\angle ABC$ of ΔABC. DE is drawn through D parallel to BC to meet AC at E. If the length of AC is 12 cm, then the length of AE (in cm.) is :

(a) 8 (b) 6 (c) 3 (d) 4

BYJU'S EXAM PREP

Ans. (b)

PRACTICE QUESTIONS